Journal of the Quadri Figures Group 2013

Digital Edition

Chris van Tienhoven et al.

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Volume 1

(april 2013 - dec. 2013)

Messages #1 - #393

Contents

1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at https://groups.io/g/Quadri-and-Poly-Geometry. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at https://groups.io/g/Quadri-Figures-Group.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

↩ Table of Contents ↩ Message Index

2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Ángel Montesdeoca
- Antreas Hatzipolakis
- Bernard Gibert
- Bernard Keizer
- Chris van Tienhoven
- Dao Thanh Oai
- Eckart Schmidt
- Eisso J. Atzema
- Emmanuel Tsukerman
- Jean-Louis Ayme
- Randy Hutson
- Seiichi Kirikami
- Systems Manager

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

Emmanuel Tsukerman:
#80 #122 #131 #137

Jean-Louis Ayme:
#77

Randy Hutson:
#387 #390

Seiichi Kirikami:
#82 #87 #92 #93 #116 #117 #118 #121 #123 #126 #129 #134 #139 #140 #143 #145 #146 #148 #149 #150 #153 #156 #157 #163 #190 #191 #211 #218 #219 #227 #229 #240 #252 #298 #299 #302 #307 #315 #316 #318 #324 #330 #331 #334 #335 #342 #349

Systems Manager:
#3 #4 #5 #8 #9 #19 #36 #104

↩ Table of Contents ↩ Message Index

2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at:
www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines.* Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry.* Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke.* B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken.* MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten.* MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks.* PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles.* Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect.* Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen.* MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks.* MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Seiichi Kirikami (1949?–2023)

Location
Japan.

Year of Birth / Generation
Exact year unknown; passed away on 11 December 2023.

Short Biography
In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions
Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes
Colleagues remembered Seiichi with deep affection:

- “Geometry is the poorer of his death.” — A. P. Hatzipolakis
- “He was a great expert in projective and triangle geometry, and a very kind and helpful person.” — E. Suppa
- “He had a way of seeing things from a different angle and presenting them simply.” — C. van Tienhoven
- “He inspired many of my problems.” — A. Altıntaş
- “He wrote many interesting and innovative messages in ADGEOM.” — F. J. García Capitán
- “He helped me in my beginnings with wise and generous advice.” — C. E. Lozada
- “He contributed to OEIS and had important work in epidemiology.” — P. Moses

Additional Remarks
Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

↩ Table of Contents ↩ Message Index

4 Messages

4.1 Message Index

↩ Table of Contents

4.2 Messages

Message: #1
Date: 17/5/2013 1:12:01
From: Chris
Subject: Welcome

```
COPY FROM FORMER GOOGLE GROUP
Dear Friends,
The Group on Quadri-Figures has started.
Messages can be placed at: https://groups.google.com/forum/#!
newtopic/quadri-figures.
Files on figures, etc. can be attached.
Now all our messages will be retained for the future and can be
looked up easily.
Moreover new members easily can join our group.
Let's go on sharing our findings, questions and answers in
quadri-geometry!
Best regards,
Chris van Tienhoven
```

Next → ↔ Message Index ↑ Subjects

Message: #2
Date: 17/5/2013 1:15:27
From: Chris
Subject: EQF-Notes

```
COPY FROM FORMER GOOGLE GROUP
Dear friends,
since 2011 I have followed up the development of Chris´
Encyclopedia of Quadri-Figures.
Up to now there are many interesting questions.
Since jan. 2013 I have gathered some remarks in "EQF-Notes".
If you are interested, have a look on my homepage:
html://eckartschmidt.de .
Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #3 is not available in Yahoo groups.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #4
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #4 is not available in Yahoo groups.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #5
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #5 is not available in Yahoo groups.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #6
Date: 17/5/2013 1:39:08
From: Chris
Subject: a LSD point of a quadrilateral and its Newton line

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 1, 2013)
Given a quadrilateral P1P2P3P4 and its Newton line, the point
which has the property of least square distance (LSD) is given
by the following barycentric coordinates.
1st DT: a^2(2a^4(l-m)^2(l-n)^2mn+b^4(l-m)^2n(m-n)(5lm-3ln+mn)+c^
4(l-n)^2m(n-m)(5ln-3lm+nm)-a^2b^2(l-m)^2(l-n)n(3l(m-n)+m(4m-n))-
a^2c^2(l-n)^2(l-m)m(3l(n-m)+n(4n-m))-2b^2c^2(l-m)(l-n)(m-n)^2(3l
(m+n)-mn)).
I obtained the above result with the conic contour method, which
Mr. van Tienhoven showed me examples. Before computation, I have
confirmed that in case of a quadrilateral, the method gives
QL-P26 (LSD point) in EQF.
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #7
Date: 17/5/2013 1:40:44
From: Chris van Tienhoven
Subject: a LSD point of a quadrilateral and its Newton line

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 1, 2013)
1st DT should be replaced by 1st CT.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #8
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #8 is not available in Yahoo groups.
```

← Previous   Next →   ↩ Message Index   ↑ Subjects

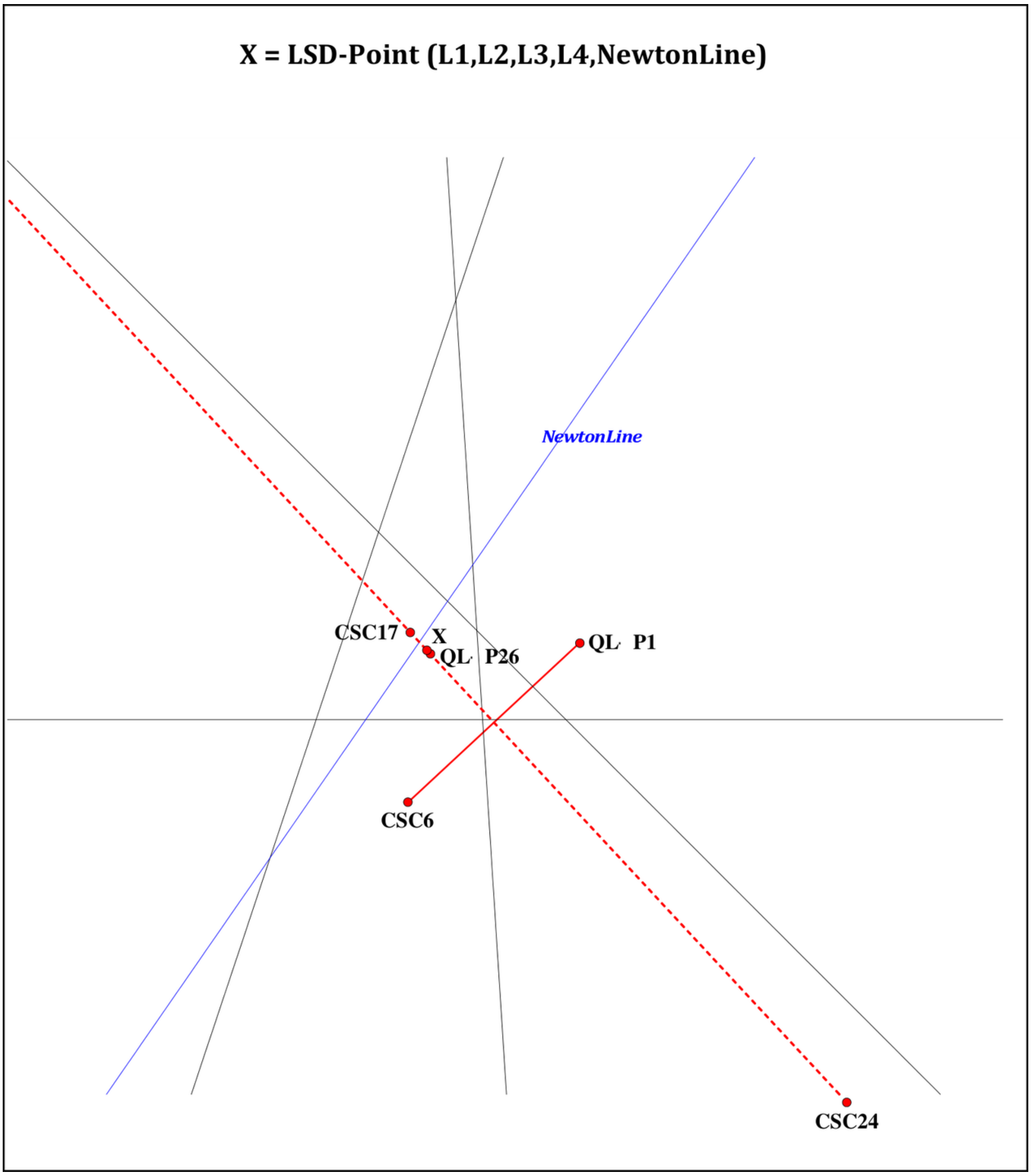
Message: #9
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #9 is not available in Yahoo groups.
```

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #10
Date: 17/5/2013 2:20:34
From: Chris van Tienhoven
Subject: a LSD point of a quadrilateral and its Newton line

```
COPY FROM FORMER GOOGLE GROUP from Chris van Tienhoven
(April 2, 2013)
Dear Seiichi,
Very nice!
This point LSD(L1,L2,L3,L4,NewtonLine) lies on the perpendicular
bisector of QL-P26.CSC(QL-P6), which is this line
QL-P26.CSC(QL-P17).CSC(QL-P24).
CSC stands for Clawson-SchmidtConjugate(QL-Tf1).
You can see these lines in the 1st picture of QL-P26 in EQF.
There are no other concurrencies with known lines or curves.
See attached PDF-file.
Best regards,
Chris
```



LSD-L1L2L3L4NL.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #11
Date: 17/5/2013 2:36:00
From: Chris
Subject: New names QL-P3 and QL-P5

COPY FROM FORMER GOOGLE GROUP from Chris van Tienhoven
(April 4, 2013)
Dear friends,
I renamed the points QL-P3 and QL-P5 in EQF.
Thanks to a message from Bernard Keizer it came out that in the document "More theorems on the Complete Quadrilateral" from J.W. Clawson these QL-points:
QL-P1, QL-P2, QL-P3, QL-P4, QL-P5, QL-P12, QL-P20 and QL-P22 were mentioned.
Also it came out that the name Kantor-Hervey Point belonged to QL-P3 and not to QL-P5.
So now
* QL-P3 = Kantor-Hervey Point, and
* QL-P5 = Clawson Center.
This all gave several additions and modifications in EQF.
See "Recently Added" in EQF.
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #12
Date: 17/5/2013 6:04:35
From: Chris van Tienhoven
Subject: EQF-Note 2013-04-05

COPY FROM FORMER GOOGLE GROUP from Eckart Schmidt
(April 5, 2013)
Dear friends,
if you are interested in Involutary Conjugates (QA-Tf2) of infinity points, there is a paper in the attachement.
Best regards Eckart

EQF-Note 2013-04-05

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Involutary Conjugate of Infinity Points

In EQF there are some QG-points Involutary Conjugates of infinity points of QG-lines (see tables for QA-Tf2). Here corresponding relationships are tested in the QA-environment.
The Involutary Conjugate QA-Tf2 is an isoconjugation wrt the QA-Diagonal Triangle QA-Tr1 with fixpoints in the vertices of the reference quadrangle. QA-Tf2 maps lines into circumscribed conics of QA-Tr1, especially the line at infinity in the Nine-point Conic QA-Co1. QA-Co1 is the locus of centers of circumscribed conics of the reference quadrangle. But in EQF there are only four circumscribed conics. Here the Involutary Conjugates of infinity points of QA-lines will be discussed wrt their collinearity with other QA-points. – Reference triangle for barycentric coordinates will be the QA-Diagonal Triangle QA-Tr1.

1. Circumscribed Conics of a Quadrangle

The Involutary Conjugate

$$(x:y:z) \quad \rightarrow \quad (p^2yz:q^2zx:r^2xy)$$

of infinity points are points on the Nine-point Conic *QA-Co1* with the equation

$$r^2xy+q^2zx+p^2yz=0$$

and the center

$$QA-P1=(p^2(-p^2+q^2+r^2):q^2(p^2-q^2+r^2):r^2(p^2+q^2-r^2)).$$

If we consider a line *L(e,f,g)*, the Involutary Conjugate of its infinity point

$$(\frac{p^2}{f-g}:\frac{q^2}{g-e}:\frac{r^2}{e-f})$$

will be a point of *QA-Co1* and a center of a circumscribed conic of the quadrangle. Up to now there are four circumscribed conics of a quadrangle in *EQF*:

***QA-Co2 QA*-Orthogonal Hyperbola**:
The center *QA-P2* is the Involutary Conjugate of infinity points of lines orthogonal to *QA-L2*.

2013-04-05.pdf

QA-Co3 **Gergonne-Steiner Conic:**
The center *QA-P3* is the Involutary Conjugate of the infinity point of *QA-L4*.

QA-2Co1a,b **Pair of Circumscribed *QA*-Parabolas:**
The axes of the *QA*-Parabolas are parallel to the asymptotes of *QA-Co1*; their infinity points are Involutary Conjugates.

There are two further circumscribed conics of a quadrangle (see my EQF-Note 2013-01-18):

Circumscribed conic through *QA-P5* and *QA-P10*:
The center, reflected in *QA-P1*, is the Involutary Conjugate of the infinity point of *QA-P1.QA-P16*.

Circumscribed conic through *QA-P1* and *QA-P16*:
The center, reflected in *QA-P1*, is the Involutary Conjugate of the infinity point of *QA-L3*.

2. Special Involutary Conjugates of Infinity Points

Considering *QA*-lines, the Involutary Conjugates of their infinity points on *QA-Co1* are sometimes collinear with *QA-P2* and another *QA*-point. For example: The Involutary Conjugate of the infinity point of *QA-P4.QA-P8* is the second intersection point of *QA-P2.QA-P7* and *QA-Co1*.

infinity point of line …	*Involutary Conjugate*
QA-L4	*QA-Co1 ^ QA-P2.QA-P3*
QA-P1.QA-P11	*QA-Co1 ^ QA-P2.QA-P37*
QA-P1.QA-P28	*QA-Co1 ^ QA-P2.QA-P15*
QA-L2	*QA-Co1 ^ QA-P2.QA-P23*
QA-P2.QA-P23	*QA-Co1 ^ QA-P2.QA-P6*
QA-P4.QA-P8	*QA-Co1 ^ QA-P2.QA-P7*
QA-P4.QA-P12	*QA-Co1 ^ QA-P2.QA-P11*
QA-P10.QA-P14	*QA-Co1 ^ QA-P2.QA-P14*
QA-P11.QA-P23	*QA-Co1 ^ QA-P2.QA-P36*

If the Involutary Conjugate of an infinity point is reflected in *QA-P1* – always a point on *QA-Co1* – there are corresponding properties wrt *QA-P3*.

infinity point of line …	*Involutary Conjugate reflected in QA-P1*
QA-L1	*QA-Co1 ^ QA-P3.QA-P4*
QA-L2	*QA-Co1 ^ QA-P3.QA-P32*
QA-L4	*QA-Co1 ^ QA-P3.QA-P2*
QA-P1.QA-P4	*QA-Co1 ^ QA-P3.QA-P8*
QA-P1.QA-P11	*QA-Co1 ^ QA-P3.QA-P12*
QA-P1.QA-P12	*QA-Co1 ^ QA-P3.QA-P30*

2013-04-05.pdf

In *QG*-environment there are some further results: As mentioned above the first three are to be found in *EQF* under *QA-Tf2*.

infinity point of line …	*Involutary Conjugate*
QG-P1.QG-P2	***QG-P13***
QG-P1.QG-P3	***QG-P14***
QL-L1	***QG-P15***
QG-L2	***QA-Co1 ^ QG-P1.QG-P2***
QG-L3	***refl. of QG-P14 in QA-P1***

3. Infinity Points of orthogonal Lines

All orthogonal hyperbolas circumscribed *QA-Tr1* contain the orthocenter *QA-P12*. The Involutary Conjugate of *QA-P12* is the Inscribed Square Axes Crosspoint *QA-P23*. Therefore *QA-P23* can be considered as the common point of all *QA-Tf2*-images of orthogonal hyperbolas circumscribed *QA-Tr1*.

For orthogonal lines the Involutary Conjugates of their infinity points lie on *QA-Co1* with a chord through *QA-P23*.

Remark: Consider chords of *QA-Co1* through *QA-P23*. The Thales-circle over a chord intersects *QA-Co1* in *QA-P2* and a further point *X*. The orthogonal hyperbola circumscribed *QA-Tr1* through *X* cuts the chord in points of an isocubic wrt *QA-Tr1* for pivot *QA-P23* and isoconjugation *QA-Tf2*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-04-05.pdf

Message: #13
Date: 17/5/2013 6:10:04
From: Chris
Subject: a LSD point made from 6 pedal points in QA- environment

---

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami (April
5,2013)
Dear friends,
Given a quadrangle P1P2P3P4, the following point P has the
property of least square distance (LSD) from 6 its pedal points
Pdij on PiPj.
1st CT: a^2(a^6q^2r^2(p+q)(p+r)+a^4(b^2r^2q(p+q)(-2p^2+2pq+r(2r+
q))+c^2q^2r(p+r)(-2p^2+2pr+q(2q+r)))+a^2(b^4p(p+q)r^2(4pq-q^2+3p
r+3r^2)+c^4p(p+r)q^2(4pr-r^2+3pq+3q^2)+2b^2c^2pqr(-2p^3-q^3-r^3+
3q^2r+3qr^2+p^2(q+r)+2p(q+r)^2))-c^6p^2q^2(p+r)(3q+2r)-b^6p^2r^2
(p+q)(2q+3r)+b^2c^4p^2q(3pq(2q+3r)+p^2(6q+4r)+r(7q^2+2qr-4r^2))+
b^4c^2p^2r(3pr(2r+3q)+p^2(6r+4q)+q(7r^2+2qr-4q^2)))
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #14
Date: 17/5/2013 6:11:33
From: Chris van Tienhoven
Subject: a LSD point made from 6 pedal points in QA- environment

---

```
COPY FROM FORMER GOOGLE GROUP from Chris van Tienhoven
(April 5,2013)
Dear Seiichi,
I calculated this Quadrangle-point before.
It has pretty long coordinates and it is not lying on known
lines or curves.
So I don't estimate it as a very special point.
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #15
Date: 17/5/2013 6:13:37
From: Chris
Subject: a LSD point made from 6 pedal points in QA- environment

---

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 5,2013)
Dear Chris,
I see. I hope that it will be something in future.
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #16
Date: 17/5/2013 6:17:08
From: Chris
Subject: New properties for QA-P23 and QA-Co1???

```
COPY FROM FORMER GOOGLE GROUP from Eckart Schmidt
(April 5, 2013)
Dear friends,
for a triangle an isoconjugation transforms lines into
circumconics
and QA-Tf2 is an isoconjugation wrt QA-DT:
The Involutary Conjugates of lines through QA-P23 are the
circumscribed orthogonal hyperbolas of QA-DT.
The Involutary Conjugates of tangents at QA-Co1 are the
circumscribed parabolas of QA-DT.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #17
Date: 17/5/2013 6:18:15
From: Chris
Subject: QA-P1 as a LSD point

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 3, 2013)
Dear friends,
Given a quadrangle P1P2P3P4 and its midpoints Mij of Pi and Pj,
QA-P1 has the property of least square distance(LSD) from P1,
P2, P3 and P4. And it also has the property of least square
distance(LSD) from M12, M13, M14, M23, M24 and M34.
LSD of the former:
    cyclic sum {a^2(2q^2+qr+2r^2+3pq+3pr+p^2}/4/(p+q+r)^2.
LSD of the latter:
    half of the above value.
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #18
Date: 17/5/2013 6:21:12
From: Chris
Subject: QA-P1 as a LSD point

```
COPY FROM FORMER GOOGLE GROUP from Chris van Tienhoven
(April 6, 2013)
Dear Seiichi,
I added this property now at QA-P1 in EQF.
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #19
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

```
Message number #19 is not available in Yahoo groups.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #20
Date: 17/5/2013 6:23:33
From: Chris
Subject: QA-P1 as a LSD point

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 7, 2013)
Dear Chris,
Thanks a lot!
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #21
Date: 17/5/2013 8:44:46
From: Chris van Tienhoven
Subject: EQF-Note 2013-04-08 Common point of five Miquel Circles

---

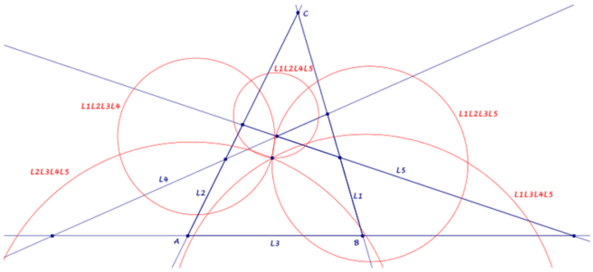
```
COPY FROM FORMER GOOGLE GROUP   from   Eckart Schmidt
(April 8, 2013)
Dear friends,
five lines give five quadrilaterals. Their Miquel circles have a
common point.
I don´t know, whether this is published anywhere.
In the attachement there are some examples in the
QL-environment.
Best regards Eckart
```

EQF-Note 2013-04-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Common Point of five Miquel Circles

Five lines give five quadr. Their Miquel circles have a common point. In this paper there are some examples for a quadrilateral and a further line. – The results are CABRI-controlled.



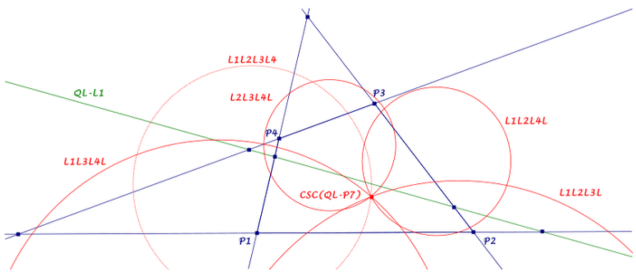
The Miquel circles (*QL-Ci3*) of the five quadrilaterals of five lines have a common point.

A calculation with barycentric coordinates is possible, taking three of the lines for a reference triangle. But the results can´t be handled.

We consider now a quadrilateral $L_1L_2L_3L_4$ and a further line L. Then the Miquel circles of the quadrilaterals

$L_1L_2L_3L$, $L_1L_2L_4L$, $L_1L_2L_4L$, $L_2L_3L_4L$ and $L_1L_2L_3L_4$

have a common point, here named *MC*-point of a line wrt a quadrilateral. The Clawson-Schmidt Conjugate (*CSC*) of the *MC*-points lie on *QL-L2* (*CSC* image of *QL-Ci3*).



2013-04-08.pdf

Examples:

QL-L1
MC-point is *CSC* of *QL-P7* (see above).

QL-L2,3
MC-point is *QL-P1*.

Lines orthogonal to *QL-L1*
MC-point is *QL-P1*.

QL-L4
MC-point is *CSC* of the reflection of *QL-P2* in *QL-P7*.

Line through *QL-P4*, parallel *QL-L1*
MC-point is *CSC* of *QL-P2*.

Lines parallel to *QL-L1*
MC-point is *CSC* of the reflection of the intersection of the line with *QL-L2* in *QL-P7*.

QG-P1.QG-P14
MC-point is the second intersection of *QL-Ci3* with the *CSC* image *QG-P1.QG-P14*.

Limit cases:

Lines parallel to the quadrangle sides
MC-point is *CSC* of the intersection of the sideline and *QL-L2*.

Diagonals of the quadrilateral
MC-point is *CSC* of the intersection of the diagonal and *QL-L2*.

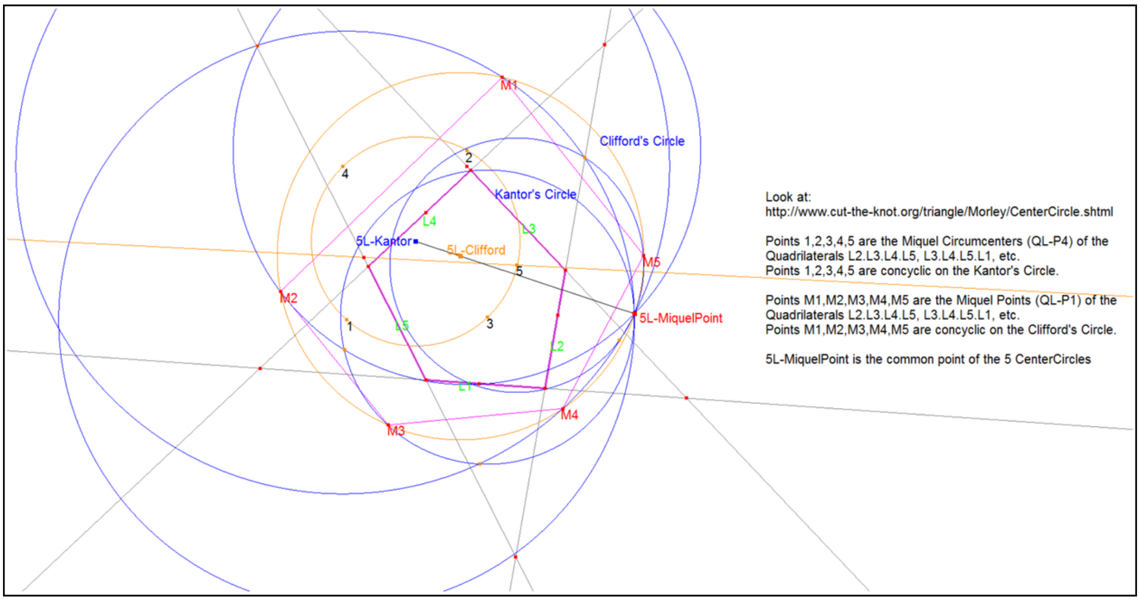
Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-04-08.pdf

Message: #22
Date: 17/5/2013 8:49:30
From: Chris van Tienhoven
Subject: EQF-Note 2013-04-08 Common point of five Miquel Circles [1 Attac

---

```
Dear Eckart,
The points/lines in a 5L-configuration are very interesting
indeed.
Last years I found this article:
http://www.cut-the-knot.org/triangle/Morley/CenterCircle.shtml
Also I produced attached Cabri-figure converted to PDF-file.
So I think the 5L-Miquel Points is already known.
However your applications with known QL-Lines is very
interesting!
Best regards,
Chris
```



5L-MiquelPoint-01.pdf

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #23
Date: 17/5/2013 8:49:32
From: Chris
Subject: a LSD point of a quadrilateral which is on Newton line

---

```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 9, 2013)
Dear friends,
Given a quadrilateral L1L2L3L4 and Newton line, the following
point on Newton line has the property of least squares distance
(LSD) from L1, L2, L3 and L4.
1st CT: a^2(b^4(l-m)^2(m-n)n(l(m-n)+mn)+c^4(l-n)^2(n-m)m(l(n-m)+
mn)-a^2(l-m)(l-n)(b^2(l-m)n(l(m-n)+mn)+c^2(l-n)m(l(n-m)+mn)+2b^2
c^2(l-m)(m-n)(n-l)(l(m+n)-mn)).
QL-P26, a LSD point of a quadrilateral and Newton line, a LSD
point of a quadrilateral which is on Newton line are colinear.
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #24
Date: 17/5/2013 8:51:22
From: Chris van Tienhoven
Subject: a LSD point of a quadrilateral which is on Newton line

---

```
COPY FROM FORMER GOOGLE GROUP from Chris van Tienhoven (April 9,
2013)
Dear Seiichi,
I do not quite understand.
You describe the point as "has the property of least squares
distance (LSD) from L1, L2, L3 and L4".
This is the point QL-P26.
Do I interprete it wrong?
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #25
Date: 17/5/2013 8:53:37
From: Chris
Subject: a LSD point of a quadrilateral which is on Newton line

---

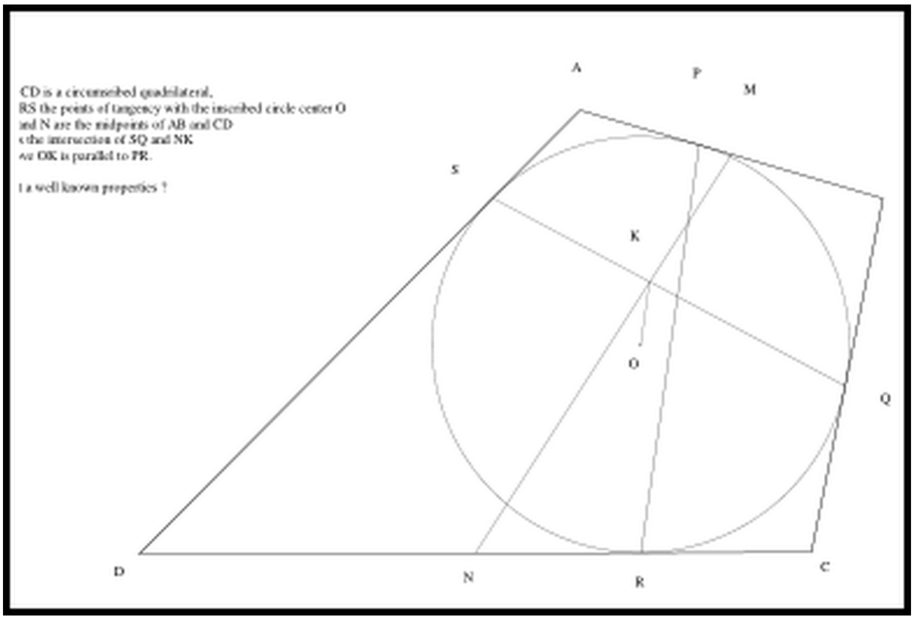
```
COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami (April 10,
2013)
Dear Chris,
I added the condition that a LSD point was on Newton line. So I
applied Lagrange's multiplier method to this problem. If you
need Mathematica-file, I will send you it.
Best regards,
Seiichi Kirikami
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #26
Date: 17/5/2013 8:56:53
From: Chris van Tienhoven
Subject: Tangential quadrilateral well known properties?

```
COPY FROM FORMER GOOGLE GROUP  from Chris van Tienhoven (April
10, 2013)
Dear Friends,
This mail for support came from:
Mosca Sebastiano
Maybe we can help him.
Chris

*Mosca Sebastiano:*
ABCD is a circumsribed quadrilateral,
PQRS the points of tangency with the inscribed circle center O
M and N are the midpoints of AB and CD
K is the intersection of SQ and NK
Prove OK is parallel to PR.
Is it a well known properties?
Can yo give me some reference?
```



quadrilateral properties.pdf

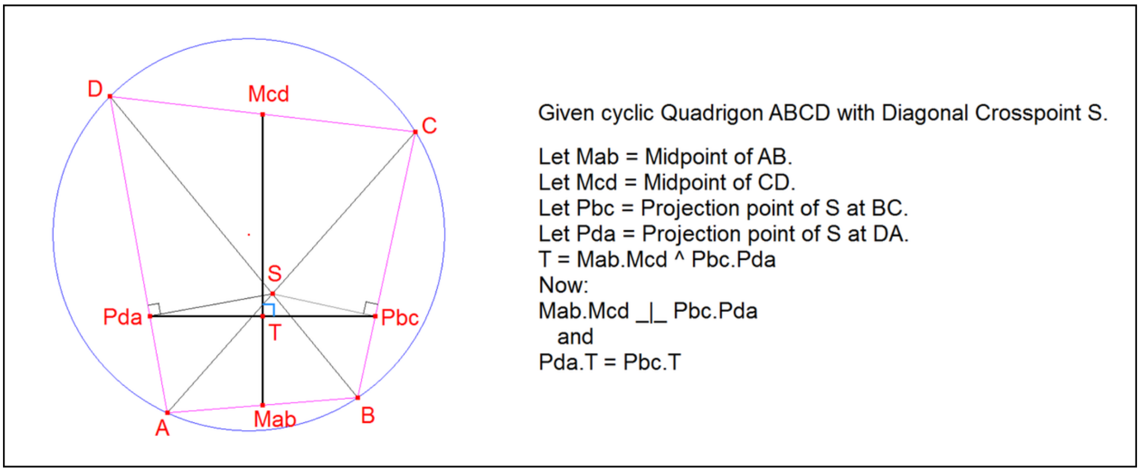
Message: #27
Date: 17/5/2013 8:58:38
From: Chris van Tienhoven
Subject: Tangential quadrilateral well known properties?

```
COPY FROM FORMER GOOGLE GROUP  from Eckart Schmidt
(April 11, 2013)
Dear Chris,
only a short remark to Mosca´s question:
Using polar coordinates P(cosP,sinP), Q(cosQ,sinQ), ...
the lines have the equations
   OK: y = -cot((P+R)/2) x
   PR: y = -cot((P+R)/2) x + cos((P-R)/2) / sin((P+R)/2)
Perhaps the angle relations in the equationes lead to a
synthetic proof.
Remarkable that the property doesn´t depend on the chord QS.
By the way: In Mosca´s message the definition of K must be:
K is the intersection of SQ and NM.
Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #28
Date: 17/5/2013 9:08:06
From: Chris van Tienhoven
Subject: Cyclic Quadrilateral

```
COPY FROM FORMER GOOGLE GROUP   from  Chris van Tienhoven
(April 12, 2013)
Dear friends,
Once I found this configuration for a Cyclic Quadrilateral.
I don't remember from where.
Given a Cyclic Quadrilateral (actually a Quadrigon) ABCD with
Diagonal Crosspoint S.
Let Mab = Midpoint of AB.
Let Mcd = Midpoint of CD.
Let Pbc = Projection point of S at BC.
Let Pda = Projection point of S at DA.
Let T = Mab.Mcd ^ Pbc.Pda
Now:
Mab.Mcd _|_ Pbc.Pda
Pda.T = Pbc.T
See attachment.  Is there a synthetic proof for this?
Best regards, Chris
```



CyclicQuadrilateral.pdf

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #29
Date: 17/5/2013 9:11:21
From: Chris
Subject: Cyclic Quadrilateral

```
COPY FROM FORMER GOOGLE GROUP from Eckart Schmidt
(April 12, 2013)
Dear Chris,
in my papers I found two references.
Bundeswettbewerb Mathemathik 2003, 2. Runde
Praxis der Mathematik 3/46. Jg. 2004 , S.135
The first one can be found per Google.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #30
Date: 17/5/2013 9:13:15
From: Chris van Tienhoven
Subject: EQF-Note 2013-04-12: QL-P5, QL-P12, QL-P20, QL-P22

```
COPY FROM FORMER GOOGLE GROUP   from  Eckart Schmidt
(April 12, 2013)
Dear friends,
for quadrilaterals with the same Inscribed parabola QL-Co1 and
the same QL-Diagonal Triangle QL-Tr1
the loci for the points QL-P5, QL-P12, QL-P20, QL-P22 are lines
with a common point.
There is a paper in the attachement. Perhaps someone can find
further properties of this interesting point.
Best regards Eckart
```

EQF-Note 2013-04-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

QL-P5, QL-P12, QL-P20, QL-P22

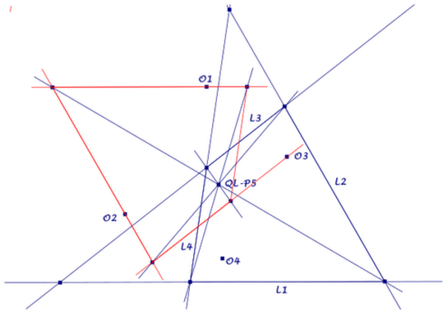
For quadrilaterals with the same Inscribed Parabola QL-Co1 and the same QL-Diagonal Triangle QL-Tr1 the loci for the points above are lines with a common point. This is the reason for the following research. – Reference triangle for barycentric calculation is QL-Tr1.

The Points

The four points above lie collinear on the Newton line *QL-L1* with rational distance ratios:

$$P5.P12 : P12.P22 : P22.P20 = 2:1:3\,.$$

These are the ratios of the Euler line. Background: Take the Euler lines of the triangle components, choose on each Euler line the same special point *X* and draw a parallel to the corresponding sideline of the quadrilateral, then the constructed quadrilateral is homothetic to the reference quadrilateral with center *Y*.



X	Y	
circumcenter	*QL-P5*	*(see figure)*
centroid	*QL-P12*	*(see EQF)*
nine-point center	*QL-P22*	*(see EQF)*
orthocenter	*QL-P20*	*(see EQF)*

The Lines

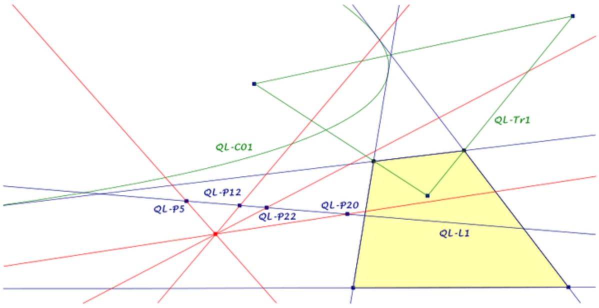
We consider the pencil of quadrilaterals with the same *QL-Diagonal Triangle QL-Tr1* and the same Inscribed Parabola *QL-Co1*. The inscribed parabola has the equation:

$$\frac{x^2}{m^2-n^2}+\frac{y^2}{n^2-l^2}+\frac{z^2}{l^2-m^2}=0.$$

For a shorter description the point at infinity of the parabola shall be

$$(u:v:w)=(m^2-n^2:n^2-l^2:l^2-m^2) \quad with \quad u+v+w=0.$$

With $l=\sqrt{m^2+w}$ *and* $n=\sqrt{m^2-u}$ we have a parameter m for the coefficients of the defining lines of the quadrilateral.



The loci of the considered points are lines with the equations:

QL-P5: $\sum_{cycl}(a^4vw+b^4wu+c^4uv+4a^2S_Avw)x=0$

This line through *QL-P5* is orthogonal to *QL-P1.QL-P9*.

QL-P12: $\sum_{cycl}vwx=0$

This line is the polar of *QL-P8* wrt the parabola.

QL-P20: $\sum_{cycl}(b^2w+c^2v)^2x=0$

Construction of this line: a parallel to *QL-L1* through *QL-P10* (*QL-DT*-orthocenter), intersection with the parabola, tangent at the parabola, a parallel through *QL-P20*.

QL-P22: $\sum_{cycl}(4S^2vw+(-S_Au+S_Bv+S_Cw)^2)x=0$

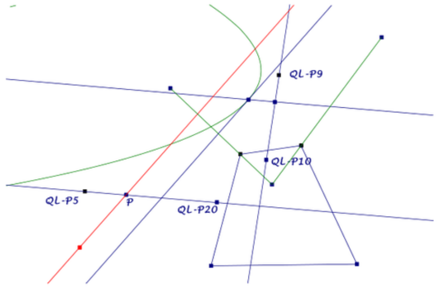
Construction of this line: a parallel to *QL-L1* through *QL-P11* (*QL-DT*-nine-point center), intersection with the parabola, tangent at the parabola, a parallel through *QL-P22*.

Within the meaning of the *X-Y*-list above the last two constructions can be generalized:

The locus of a point *P* on *QL-L1*, dividing *QL-P5.QL-P20* in the ratio κ, can be constructed as follows: a

2013-04-12.pdf

parallel to *QL-L1* through a point, dividing *QL-P9.QL-P10* in the same ratio κ, intersection with the parabola, tangent at the parabola, a parallel through *P*.



Equation of this line:

$$\sum_{cycl}(-a^4vw+b^4wu+c^4uv+2a^2(b^2+c^2)vw+2(c^2v+b^2w)^2\kappa)x)=0$$

All these lines have a common point.

For quadrilaterals with the same inscribed parabola and the same *QL*-diagonal triangle the loci for the points *QL-P5, QL-P12, QL-P20, QL-P22* are lines with a common point.

This common point has the coordinates:

$$(u(v(a^2w+c^2u)^2-w(a^2v+b^2u)^2)$$
$$:(v(w(b^2u+a^2v)^2-u(b^2w+c^2v)^2)$$
$$:(w(u(c^2v+a^2w)^2-v(c^2u+a^2w)^2)).$$

The polar of this point wrt the inscribed parabola is a line through *QL-P8*. The direction of the polar is orthogonal to a line through *QL-P10* and a point, which is the intersection of *QL-L2* and a perpendicular line to *QL-L7* through *QL-P3*.

− Unfortunately no further properties can be given. −

A special Case

Taking three lines for a reference triangle and a fourth line parallel to the Euler line of the reference triangle, then we get a quadrilateral, where the four considered points coincide.

For quadrilaterals with sidelines parallel to the Euler lines of their triangle components holds *QL-P5 = QL-P12 = QL-P20 = QL-P22*.

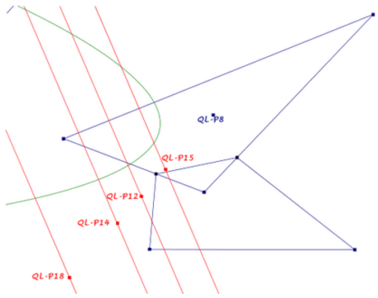
There are four quadrilaterals of this type among those with the same inscribed parabola and the same *QL*-diagonal triangle, taking lines

$$(\pm\sqrt{3c^4v^2+2(3S_A^2+S^2)vw+3b^4w^2},$$
$$\pm\sqrt{3a^4w^2+2(3S_B^2+S^2)wu+3c^4u^2},$$
$$\pm\sqrt{3b^4u^2+2(3S_C^2+S^2)uv+3a^4v^2}).$$

for defining the quadrilateral.

Final Remark

Among the quadrilaterals of the considered pencil there are four further collinear points, whose loci are lines with a common point. The common point is a point at infinity, so the lines are parallel. The first one is already mentioned.



QL-P12: $\sum_{cycl} vwx = 0$

This line is the polar of *QL-P8* wrt the parabola.
Polar distance from *QL-P8:*

$$d=\frac{S(uv+vw+wu)}{3\sqrt{S_Au^2(v-w)^2+S_Bv^2(w-u)^2+S_Cw^2(u-v)^2}}$$

QL-P14: $\sum_{cycl} (u^2-10vw)x = 0$

Parallel line with distance 4/3d from *QL-P8*.

QL-P15: $\sum_{cycl} (u^2+8vw)x = 0$

Parallel line with distance 2/3d from *QL-P8*.

QL-P18: $\sum_{cycl} (v-w)^2x = 0$

Parallel line with distance 2d from *QL-P8*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-04-12.pdf

Message: #31
Date: 17/5/2013 9:17:06
From: Chris van Tienhoven
Subject: EQF-Note 2013-04-15: Involutary Conjugate of the Newton Line

---

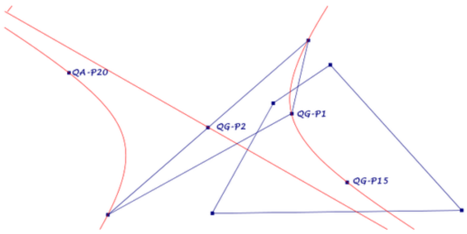
```
COPY FROM FORMER GOOGLE GROUP from Eckart Schmidt
(April 15, 2013)
Dear friends,
there is a conic for quadrigons in the attachement
with interesting properties in the QL-environment.
Best regards Eckart
```

EQF-Note 2013-04-15

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Involutary Conjugate of the Newton Line

For a quadrigon a QA-DT circumscribed conic with center QG-P2 through QG-P15 will be discussed. – Reference triangle for barycentric coordinates is QA-DT.



The Conic in *QG*-environment

In the *QG*-environment we consider the Newton Line

$$QL\text{-}L1: \quad q^2x+(p^2-r^2)y-q^2z=0.$$

Its Involutary Conjugate (*QA-Tf2*) is a *QA-DT* circumscribed conic with the equation

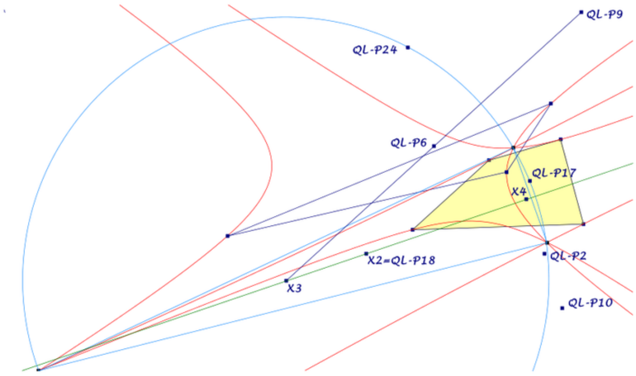
$$-r^2xy+(p^2-r^2)zx+p^2yz=0.$$

- The center of the conic is *QG-P2*.
- The conic contains the points
  QG-P15 (image of the point at infinity of *QL-L1*),
  QA-P20 (image of *QA-P1*),
  reflection of *QG-P1* in *QG-P2* (image of *QG-P12*),
  the reflections of *QG-P15, QA-P*20 in *QG-P2*.
- The asymptotes are parallel to the legs of the *QL*-diagonal triangle. Their points at infinity are the images of the midpoints of the diagonals of the quadrigon.
- The tangent in *QG-P1* is *QG-L2*,
  the tangent in *QG-P15* is *QG-P12.QG-P15*,
  the pole of *QG-L3* is *QG-P12*.
  The tangents in the endpoints of the 3rd diagonal are parallel *QG-P1.QG-P3*.
- The conic is the locus of *QG-P15* for all quadrigons with the same *QA*- and *QL*-diagonal triangle.
- The conic divides the sidelines in ratios with product 1.

2013-04-15.pdf

The Conic in *QL*-environment

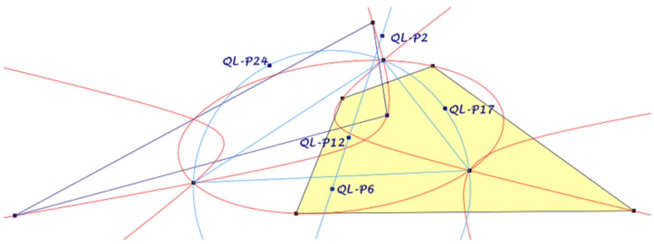
For a quadrangle there are three conics of the new type. Common points are the vertices of *QA-DT* and *QA-P20*. For a quadrilateral there are also three conics but with three common points. The calculation needs solutions of equations with degree 3. So the following interesting properties are only Cabri controlled.



- The three conics have three common points.
- The centroid *X2* of this triangle is *QL-P18* (reflection of *QL-P8* in *QL-P12*).
- The orthocenter *X4* of this triangle is the reflection of *QL-P10* in *QL-P2*.
- The circumcenter *X3* of this triangle is the reflection of *QL-P9* in *QL-P6*.
- The circumcircle of this triangle contains *QL-P17* and *QL-P24*.
- The Simson line of *QL-P17* wrt this triangle is a parallel to *QL-L6* half the distance to *QL-P17*.

Comparison with the Nine-point Conic *QA-Co1*

We can compare these properties with those of another conic. Taking the three Nine-Point Conics *QA-Co1* for a quadrilateral, we get also three common points (see *QL-P6* in *EQF*).



The following list shows centroid, circumcenter and orthocenter of the *QL*-diagonal triangle *QL-DT*, the triangle wrt *QA-Co1* and the triangle wrt *QG-Cox*.

	QL-DT	*QA-Co1-Δ*	*QG-Cox-Δ*
centroid	*QL-P8*	*QL-P12*	*QL-P18*
circumcenter	*QL-P9*	*QL-P6*	*see above*
orthocenter	*QL-P10*	*QL-P2*	*see above*

Reflecting one of the *QL-DT*-points in the corresponding point of *QA-Co1-Δ* we get the corresponding point of *QG-Cox-Δ*. The circumcircles of the three triangles contain *QL-P17* and *QL-P24*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-04-15.pdf

Message: #32
Date: 17/5/2013 9:21:20
From: Chris van Tienhoven
Subject: Cardioïd(s) tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP  from Bernard Keizer
(April 17, 2013)
Dear friends,
This first contribution is rather a question.
I would like to know how it is possible to find the cardioïd(s)
tangent to 4 lines.
Any synthetic or analytical approach or bibliographic reference
would be welcome ...
You will find in the attached file a short introduction to the
subject.
Best regards
Bernard
```

Cardioïds tangent to 4 lines

(Background for this note is Chris van Tienhoven : Encyclopedia of Quadrifigures)

- For a 4-lines or complete quadrilateral QL1, there is one parabola tangent to the 4 lines (QL-Co1), the focus being the Miquel point M (QL-P1), the directrix the Steiner Line (QL-L2) and the axis being parallel to the Newton Line QL-L1. There is also one deltoïd tangent to the 4 lines, the center being the Kantor-Hervey point H (QL-P3) and the inner circle the Hervey circle (QL-Ci4). If I call θi the angles between the Steiner Line and the lines Li, the angle of the 3 axes of the deltoïd and the axis of the parabola is $\frac{1}{3} \sum \theta i$ (to the modulus $\pi/3$). Last, there is a cardioïd tangent to the 4 circles Ci circumscribed to the 4 reference triangles Ti of the QL (QL-Qu1), the center being the Miquel circumcenter O (QL-P4), the cusp the Miquel point M (QL-P1) and the inner circle the Miquel circle (QL-Ci3). The parabola and the cardioïd are Clawson-Schmidt conjugates and the angle between the axis of the cardioïd and the axis of the parabola is $\sum \theta i$. I suppose these are more or less well-known properties.
- I consider now the 4 tangent lines to the cardioïd, which are also tangents to the circles Ci ; these 4 tangents to the cardioïd form a new complete quadrilateral QL2. The contact points with the cardioïd Ei are the reflections of the cusp M in the tangents to the Miquel circle in the points Oi, centers of the 4 reference circles ; we have MEi parallel to OOi. If r is the radius of the Miquel and the Hervey circles, the cardioïd has a bitangent perpendicular to it's axis in a point T at a distance 3r/2 from it's center O. Let's Ki be the intersections between this bitangent and each of the 4 tangent lines to the cardioïd. The main property of the cardioïd is that KiO trisects the angle TKiEi.
- Conversely, for a given cardioïd (with center O and cusp M and therefore bitangent) and 4 tangent lines forming the QL2, it's easy to find the points Ki as intersections of the tangents and the bitangent, which gives the Oi as reflections of the cusp M in the segments KiO. The reflections of M in OjOk are the vertices of the QL1.
- The problem is much more difficult if the cardioïd isn't given and we have only the 4 lines. If I believe Frank Morley, there are 8 cardioïds tangent to 4 lines not directed, but only one tangent to the 4 lines to which the deltoïd assigns a definite direction (as the cardioïd is a curve of direction).
- Morley proved also in his Morley's Miracle that the locus of the centers of cardioïds inscribed in a triangle is made of 3 sets of 3 parallel lines, forming equilateral triangles, the direction of the lines being the mean direction of the 3 sides of the triangle to the modulus $\pi/3$ (see American Journal of Mathematics, vol 51, n°3 : Extensions of Clifford's Chain-Theorem by F. Morley). For the QL, the centers of the cardioïds tangent to the 4 lines must therefore belong to the 4 groups of the 3 sets of 3 parallel lines for each of the 4 reference triangles of the QL ...

The question is precisely how to find the cardioïd(s) tangent to 4 given lines.

Synthetic or analytical approach and bibliographic references would be most welcome.

Bernard Keizer
(bernard.keizer@wanadoo.fr)

Cardio_ds tangent to 4 lines.doc

Message: #33
Date: 17/5/2013 9:24:20
From: Chris van Tienhoven
Subject: Cardioïd(s) tangent to 4 lines [1 Attac

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Dear Bernard,
in addition to your paper "Cardioids tangent to 4 lines"
there is a further property described in the attachement.
Best regards Eckart
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21.04.2013

Dear Bernard,

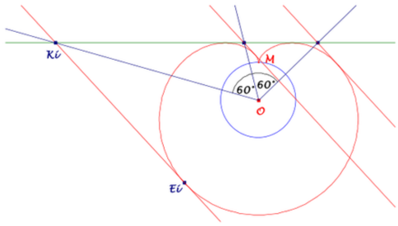
in addition to your paper **Cardioïds tangent to 4 lines:**

There is the following property for cardioïds in

H. Schmidt: Ausgewählte höhere Kurven.

Kesselringsche Verlagsbuchhandlung Wiesbaden, 1949, S.122.

„Die Tripel paralleler Tangenten der Kardioide schneiden die Doppeltangente in Tripeln von Punkten, die vom Mittelpunkt des Grundkreises aus unter dem Winkel 60° erscheinen.“



Not to be found there is a further property:

The sum of the orientated distances of these parallel tangents from the center O of the basic circle is zero.

We can start with a quadrilateral, for example described by the vertices of a quadrigon component in cartesian coordinates:

$$A(a,\frac{1}{a}),\quad B(b,\frac{1}{b}),\quad C(c,\frac{1}{c}),\quad D(d,\frac{1}{d})\ .$$

The cardioïd can be described by the center of the base circle $O(u,v)$ and the intersection $T(s,t)$ of the axis and the bitangent.

The four sidelines *AB, BC, CD, DA* are tangent to the cardioïd. Their parallel tangents can easily be calculated. The sums 0 of their orientated distances to *Z* lead to four equations for *u, v, s, t*.

So far so good …

These four equations (see beyond) with rational coefficients are up to degree 3 in every variable, but perhaps there are possible interpretations.

Best regards Eckart

Cardio_ds .doc

$$
\begin{aligned}
&a^3 + 3a^2b + 3ab^2 + b^3 - 3as^2 - 3bs^2 - 3a^3b^2s^2 - 3a^2b^3s^2 + 2s^3 + \\
&\quad 2a^2b^2s^3 + 2abs^2t + 2a^3b^3s^2t - 3at^2 - 3bt^2 - 3a^3b^2t^2 - \\
&\quad 3a^2b^3t^2 + 2st^2 + 2a^2b^2st^2 + 2abt^3 + 2a^3b^3t^3 - 3a^2u - 6abu - \\
&\quad 3b^2u + 6asu + 6bsu + 6a^3b^2su + 6a^2b^3su - 3s^2u - 3a^2b^2s^2u - \\
&\quad 4abstu - 4a^3b^3stu + t^2u + a^2b^2t^2u - 3a^3b^2u^2 - 3a^2b^3u^2 + \\
&\quad 2abtu^2 + 2a^3b^3tu^2 + a^2b^2u^3 - 3a^3bv - 6a^2b^2v - 3ab^3v + \\
&\quad abs^2v + a^3b^3s^2v + 6atv + 6btv + 6a^3b^2tv + 6a^2b^3tv - \\
&\quad 4stv - 4a^2b^2stv - 3abt^2v - 3a^3b^3t^2v + 6a^2buv + 6ab^2uv - \\
&\quad 2absuv - 2a^3b^3suv - 2tuv - 2a^2b^2tuv - 2abu^2v + a^3b^3u^2v - \\
&\quad 3av^2 - 3bv^2 + 2sv^2 + 2a^2b^2sv^2 + uv^2 - 2a^2b^2uv^2 + abv^3 \text{ Š } 0
\end{aligned}
$$

$$
\begin{aligned}
&b^3 + 3b^2c + 3bc^2 + c^3 - 3bs^2 - 3cs^2 - 3b^3c^2s^2 - 3b^2c^3s^2 + 2s^3 + \\
&\quad 2b^2c^2s^3 + 2bcs^2t + 2b^3c^3s^2t - 3bt^2 - 3ct^2 - 3b^3c^2t^2 - \\
&\quad 3b^2c^3t^2 + 2st^2 + 2b^2c^2st^2 + 2bct^3 + 2b^3c^3t^3 - 3b^2u - 6bcu - \\
&\quad 3c^2u + 6bsu + 6csu + 6b^3c^2su + 6b^2c^3su - 3s^2u - 3b^2c^2s^2u - \\
&\quad 4bcstu - 4b^3c^3stu + t^2u + b^2c^2t^2u - 3b^3c^2u^2 - 3b^2c^3u^2 + \\
&\quad 2bctu^2 + 2b^3c^3tu^2 + b^2c^2u^3 - 3b^3cv - 6b^2c^2v - 3bc^3v + \\
&\quad bcs^2v + b^3c^3s^2v + 6btv + 6ctv + 6b^3c^2tv + 6b^2c^3tv - \\
&\quad 4stv - 4b^2c^2stv - 3bct^2v - 3b^3c^3t^2v + 6b^2cuv + 6bc^2uv - \\
&\quad 2bcsuv - 2b^3c^3suv - 2tuv - 2b^2c^2tuv - 2bcu^2v + b^3c^3u^2v - \\
&\quad 3bv^2 - 3cv^2 + 2sv^2 + 2b^2c^2sv^2 + uv^2 - 2b^2c^2uv^2 + bcv^3 \text{ Š } 0
\end{aligned}
$$

$$
\begin{aligned}
&c^3 + 3c^2d + 3cd^2 + d^3 - 3cs^2 - 3ds^2 - 3c^3d^2s^2 - 3c^2d^3s^2 + 2s^3 + \\
&\quad 2c^2d^2s^3 + 2cds^2t + 2c^3d^3s^2t - 3ct^2 - 3dt^2 - 3c^3d^2t^2 - \\
&\quad 3c^2d^3t^2 + 2st^2 + 2c^2d^2st^2 + 2cdt^3 + 2c^3d^3t^3 - 3c^2u - 6cdu - \\
&\quad 3d^2u + 6csu + 6dsu + 6c^3d^2su + 6c^2d^3su - 3s^2u - 3c^2d^2s^2u - \\
&\quad 4cdstu - 4c^3d^3stu + t^2u + c^2d^2t^2u - 3c^3d^2u^2 - 3c^2d^3u^2 + \\
&\quad 2cdtu^2 + 2c^3d^3tu^2 + c^2d^2u^3 - 3c^3dv - 6c^2d^2v - 3cd^3v + \\
&\quad cds^2v + c^3d^3s^2v + 6ctv + 6dtv + 6c^3d^2tv + 6c^2d^3tv - \\
&\quad 4stv - 4c^2d^2stv - 3cdt^2v - 3c^3d^3t^2v + 6c^2duv + 6cd^2uv - \\
&\quad 2cdsuv - 2c^3d^3suv - 2tuv - 2c^2d^2tuv - 2cdu^2v + c^3d^3u^2v - \\
&\quad 3cv^2 - 3dv^2 + 2sv^2 + 2c^2d^2sv^2 + uv^2 - 2c^2d^2uv^2 + cdv^3 \text{ Š } 0
\end{aligned}
$$

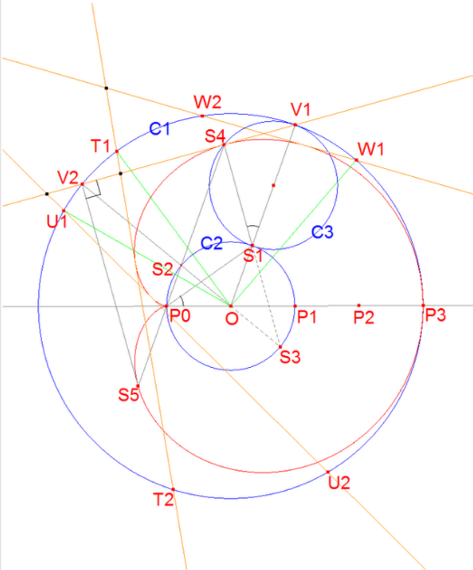
$$
\begin{aligned}
&a^3 + 3a^2d + 3ad^2 + d^3 - 3as^2 - 3ds^2 - 3a^3d^2s^2 - 3a^2d^3s^2 + 2s^3 + \\
&\quad 2a^2d^2s^3 + 2ads^2t + 2a^3d^3s^2t - 3at^2 - 3dt^2 - 3a^3d^2t^2 - \\
&\quad 3a^2d^3t^2 + 2st^2 + 2a^2d^2st^2 + 2adt^3 + 2a^3d^3t^3 - 3a^2u - 6adu - \\
&\quad 3d^2u + 6asu + 6dsu + 6a^3d^2su + 6a^2d^3su - 3s^2u - 3a^2d^2s^2u - \\
&\quad 4adstu - 4a^3d^3stu + t^2u + a^2d^2t^2u - 3a^3d^2u^2 - 3a^2d^3u^2 + \\
&\quad 2adtu^2 + 2a^3d^3tu^2 + a^2d^2u^3 - 3a^3dv - 6a^2d^2v - 3ad^3v + \\
&\quad ads^2v + a^3d^3s^2v + 6atv + 6dtv + 6a^3d^2tv + 6a^2d^3tv - \\
&\quad 4stv - 4a^2d^2stv - 3adt^2v - 3a^3d^3t^2v + 6a^2duv + 6ad^2uv - \\
&\quad 2adsuv - 2a^3d^3suv - 2tuv - 2a^2d^2tuv - 2adu^2v + a^3d^3u^2v - \\
&\quad 3av^2 - 3dv^2 + 2sv^2 + 2a^2d^2sv^2 + uv^2 - 2a^2d^2uv^2 + adv^3 \text{ Š } 0
\end{aligned}
$$

Cardio_ds .doc

← Previous Next → ↔ Message Index ↑ Subjects

Message: #34
Date: 17/5/2013 9:29:14
From: Chris van Tienhoven
Subject: Cardioïd(s) tangent to 4 lines [1 Attac

Dear Friends,
I already had an earlier discussion with Bernard about these Cardiods.
I like the approach of Eckart. However the 4 equations he found seem pretty tough to solve.
I had another approach.
But again the algebraic way as well as the synthetic way of solving its simplification shown in attached file didn't pay off.
Maybe others can use clues in this approach.
See attachment.
Best regards,
Chris van Tienhoven

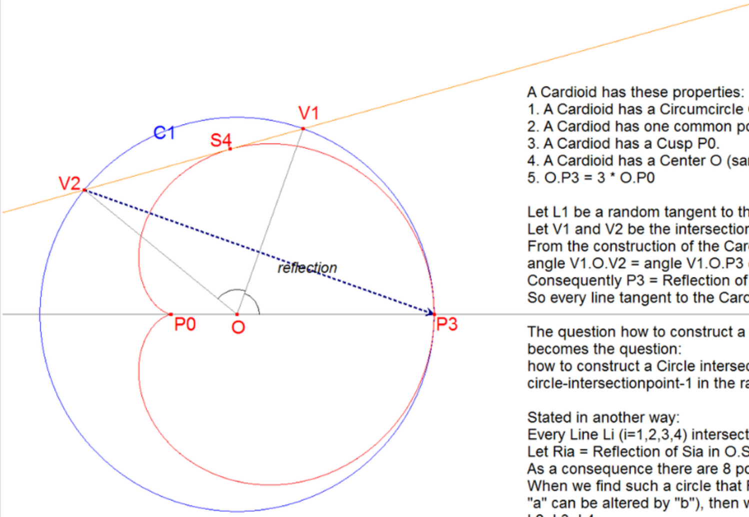


Construction Cardiod

Draw circle C1 with center O
Choose referencepoint P3 on this circle.
Divide OP in 3 equal parts with dividing points P1 and P2 (P1 nearest O).
Draw circle C2 with center O through P1.
Let P0 be reflection of P1 in O.
Choose variable point V1 on circle C1.
Let V2 be reflection of P3 in line O.V1.
Let S1 be the intersection point of circle C2 with line segment O.V1.
Let S2 be the intersection point of circle C2 with line segment O.V2.
Let S3 be the reflection of S1 in O.
Let S4 be the reflection of S3 in S1.
Let S5 be the reflection of S4 in S2.
The locus of S4 with variable point P1 is the CARDIOD.

Now V1.V2 is tangent to the cardiod at S4 !
And the perpendicular of V1.V2 at V2 is also tangent at the cardiod at S5.

From Xah Lee:
http://xahlee.org/SpecialPlaneCurves_dir/specialPlaneCurves.html



A Cardioid has these properties:
1. A Cardioid has a Circumcircle C1.
2. A Cardiod has one common point P3 with its Circumcircle.
3. A Cardiod has a Cusp P0.
4. A Cardiod has a Center O (same as the Center of its Circumcircle).
5. O.P3 = 3 * O.P0

Let L1 be a random tangent to the Cardiod at S4.
Let V1 and V2 be the intersectionpoints of L1 with circumcircle C1.
From the construction of the Cardiod it is evident that:
angle V1.O.V2 = angle V1.O.P3 (or angle V2.O.P3).
Consequently P3 = Reflection of V2 in O.V1 (or Reflection of V1 in O.V2).
So every line tangent to the Cardioid determines 2 possible common points P3.

The question how to construct a Cardioid tangent to 4 lines L1, L2, L3, L4 now becomes the question:
how to construct a Circle intersecting these 4 lines so that the Reflectionpoints of circle-intersectionpoint-1 in the radius defined by circle-intersectionpoint-2 coincide.

Stated in another way:
Every Line Li (i=1,2,3,4) intersects some Circle with Center O in Sia and Sib (i=1,2,3,4).
Let Ria = Reflection of Sia in O.Sib and Rib = Reflection Sib in O.Sia (i=1,2,3,4).
As a consequence there are 8 points R1a, R1b, R2a, R2b, R3a, R3b, R4a, R4b.
When we find such a circle that R1a = R2a = R3a = R4a (where moreover every suffix "a" can be altered by "b"), then we have found a Cardiod tangent to the the 4 lines L1, L2, L3, L4.

Cardiod-Construction-02-03.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #35
Date: 17/6/2013 2:37:36
From: Chris
Subject: A classification of QA-points in EQF

COPY FROM FORMER GOOGLE GROUP from Seiichi Kirikami
(April 24, 2013)
Dear friends,
[1] An idea of classification of QA-points:
Given a triangle ABC and a point P{p, q, r}, cevian points are D{0, q, r}, E{p, 0, r} and F{p, q, 0}. Naturally AD, BE and CF are concurrent.
Likewise, given a triangle ABC, a point P{p, q, r} and QA-point Px{f(p, q, r), g(p, q, r), h(p, q, r)}, 3 points are determined as D{f(0, q, r), g(0, q, r), h(0, q, r)}, E{f(p, 0, r), g(p, 0, r), h(p, 0, r)} and F{f(p, q, 0), g(p, q, 0), h(p, q, 0)}.
According to the colinear determinant of AD, BE and CF, QA-point Px is classified as follows.
[2] Classification
(1) vertex type: D, E and F degenerate to A, B and C respectively. Examples, QA-P4, P5, P7, P17, P41.
(2) Decomposition type: the colinear determinant consists of the simple product of p, q, r, (p+-q), (q+-r), (r+-p). Examples, QA-P1, P10, P16, P18 to P22, P25 to P28, P38.
(3) Orthocentral type: D, E and F are on AH, BH and CH, where H means the orthocenter of ABC. Examples, QA-P2, P6, P8, P11 to P13, P23, P24, P30, P32, P33, P36, P37, P39.
(4) Cubic type: the main portion of the colinear determinant is cubic. Examples, QA-P3, P29, P34, P35.
(5) Other higher curve type: the main portion of the colinear determinant is higher than cubic. Examples, QA-P9, P14, P15, P31, P40.
Best regards,
Seiichi Kirikami

← Previous Next → ↩ Message Index ↑ Subjects

Message: #36
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

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Message number #36 is not available in Yahoo groups.
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #37
Date: 17/6/2013 3:11:01
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

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COPY FROM FORMER GOOGLE GROUP  by Eckart Schmidt
(April 25, 2013)
Dear friends,
before starting for holidays a "short" message:
I have realized my concept for calculation cardioids tangent to
4 lines:
The centers of these cardioids lie on cubics.
You find the equations for cartesian and barycentric coordinates
in the attachement as Mathematica file.
It seems, that no QL-point is on the cubic.
Best regards Eckart
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Cardioids tangent to 4 lines

The centers of cardioids tangent to for lines lie on cubics with the following equations :

Cartesian coordinates : A $(a; \frac{1}{a})$, B $(b; \frac{1}{b})$, C $(c; \frac{1}{c})$, D $(d; \frac{1}{d})$

with lines AB, BC, CD, DA

$$
\begin{aligned}
&\text{cub[\{x\_, y\_, 1\}]} := -a^2+b^2-a^4 b^2+a^2 b^4-4 a c+a^4 b c-a^3 b^2 c+3 a^2 b^3 c+a b^4 c-c^2+a^3 b c^2+\\
&a^2 b^2 c^2+3 a b^3 c^2+b^4 c^2-a^4 b^4 c^2+a^2 b c^3-a b^2 c^3+a^4 b^3 c^3-a^3 b^4 c^3+a b c^4-b^2 c^4+a^3 b^3 c^4-\\
&a^2 b^4 c^4+4 b d-a^4 b d-3 a^3 b^2 d+a^2 b^3 d-a b^4 d+a^4 c d+2 a^3 b c d-2 a b^3 c d-b^4 c d+a^3 c^2 d+\\
&2 a^2 b c^2 d+b^3 c^2 d-a^4 b^3 c^2 d-3 a^3 b^4 c^2 d+a^2 c^3 d+2 a b c^3 d-3 b^2 c^3 d+a^4 b^2 c^3 d+2 a^3 b^3 c^3 d-\\
&3 a^2 b^4 c^3 d+a c^4 d-b c^4 d+a^3 b^2 c^4 d-a^2 b^3 c^4 d+d^2-a^4 d^2-3 a^3 b d^2-a^2 b^2 d^2-a b^3 d^2+\\
&a^4 b^4 d^2-a^3 c d^2-2 a b^2 c d^2-b^3 c d^2+3 a^4 b^3 c d^2+a^3 b^4 c d^2+a^2 c^2 d^2-b^2 c^2 d^2+a^4 b^2 c^2 d^2-\\
&a^2 b^4 c^2 d^2-a c^3 d^2-3 b c^3 d^2+a^4 b c^3 d^2+2 a^3 b^2 c^3 d^2+a b^4 c^3 d^2-c^4 d^2+a^3 b c^4 d^2+a^2 b^2 c^4 d^2+\\
&3 a b^3 c^4 d^2+b^4 c^4 d^2-a^4 b^4 c^4 d^2+a^2 b d^3-a b^2 d^3+a^4 b^3 d^3-a^3 b^4 d^3+3 a^2 c d^3-2 a b c d^3-\\
&b^2 c d^3+3 a^4 b^2 c d^3-2 a^3 b^3 c d^3-a^2 b^4 c d^3+3 a c^2 d^3+b c^2 d^3-a^4 b c^2 d^3-2 a^2 b^3 c^2 d^3-\\
&a b^4 c^2 d^3+a^4 c^3 d^3+2 a^3 b c^3 d^3-2 a b^3 c^3 d^3-b^4 c^3 d^3+a^3 c^4 d^3-a^2 b c^4 d^3+3 a b^2 c^4 d^3+\\
&b^3 c^4 d^3-4 a^4 b^3 c^4 d^3+a^2 d^4-a b d^4+a^4 b^2 d^4-a^3 b^3 d^4+a c d^4-b c d^4+a^3 b^2 c d^4-a^2 b^3 c d^4+\\
&c^2 d^4-a^4 c^2 d^4-3 a^3 b c^2 d^4-a^2 b^2 c^2 d^4-a b^3 c^2 d^4+a^4 b^4 c^2 d^4-a^3 c^3 d^4-3 a^2 b c^3 d^4+\\
&a b^2 c^3 d^4-b^3 c^3 d^4+4 a^3 b^4 c^3 d^4-a^2 c^4 d^4+b^2 c^4 d^4-a^4 b^2 c^4 d^4+a^2 b^4 c^4 d^4+3 a x-3 b x+\\
&3 a^3 b^2 x-3 a^2 b^3 x+3 c x-3 a^3 b c x-3 a^2 b^2 c x-3 a b^3 c x-3 a^2 b c^2 x-3 a b^2 c^2 x-3 b^3 c^2 x+\\
&3 a^3 b^4 c^2 x-3 a b c^3 x+3 b^2 c^3 x-3 a^3 b^3 c^3 x+3 a^2 b^4 c^3 x-3 d x+3 a^3 b d x+3 a^2 b^2 d x+\\
&3 a b^3 d x-3 a^3 c d x-3 a^2 b c d x+3 a b^2 c d x+3 b^3 c d x-3 a^2 c^2 d x-3 a b c^2 d x+3 b^2 c^2 d x+\\
&3 a^3 b^3 c^2 d x+6 a^2 b^4 c^2 d x-3 a c^3 d x+3 b c^3 d x-3 a^3 b^2 c^3 d x+3 a^2 b^3 c^3 d x+3 a^3 d^2 x+\\
&3 a^2 b d^2 x+3 a b^2 d^2 x-3 a^4 b^3 d^2 x-3 a^2 c d^2 x+3 a b c d^2 x+3 b^2 c d^2 x-6 a^4 b^2 c d^2 x-\\
&3 a^3 b^3 c d^2 x-3 a c^2 d^2 x+3 b c^2 d^2 x-3 a^3 b^2 c^2 d^2 x+3 a^2 b^3 c^2 d^2 x+3 c^3 d^2 x-3 a^3 b c^3 d^2 x-\\
&3 a^2 b^2 c^3 d^2 x-3 a b^3 c^3 d^2 x-6 a b^2 c^4 d^2 x-3 b^3 c^4 d^2 x+3 a^4 b^3 c^4 d^2 x-3 a^2 d^3 x+3 a b d^3 x-\\
&3 a^4 b^2 d^3 x+3 a^3 b^3 d^3 x-3 a c d^3 x+3 b c d^3 x-3 a^3 b^2 c d^3 x+3 a^2 b^3 c d^3 x-3 c^2 d^3 x+\\
&3 a^3 b c^2 d^3 x+3 a^2 b^2 c^2 d^3 x+3 a b^3 c^2 d^3 x-3 a^3 c^3 d^3 x+3 a^2 b c^3 d^3 x-3 a b^2 c^3 d^3 x+\\
&3 b^3 c^3 d^3 x+6 a^4 b^3 c^3 d^3 x-6 a^3 b^4 c^3 d^3 x-3 b^2 c^4 d^3 x+3 a^4 b^2 c^4 d^3 x+6 a^3 b^3 c^4 d^3 x+\\
&3 a^3 c^2 d^4 x+6 a^2 b c^2 d^4 x-3 a^3 b^4 c^2 d^4 x+3 a^2 c^3 d^4 x-6 a^3 b^3 c^3 d^4 x-3 a^2 b^4 c^3 d^4 x+\\
&3 a^2 b c x^2+3 a b^2 c x^2+3 a b c^2 x^2-3 a^2 b^4 c^2 x^2-3 a^2 b d x^2-3 a b^2 d x^2+3 a^2 c d x^2-3 b^2 c d x^2+\\
&3 a c^2 d x^2-3 b c^2 d x^2-6 a^2 b^3 c^2 d x^2-3 a b d^2 x^2+3 a^4 b^2 d^2 x^2+3 a c d^2 x^2-3 b c d^2 x^2+\\
&6 a^3 b^2 c d^2 x^2+6 a b^2 c^3 d^2 x^2-3 a^4 b^3 c^3 d^2 x^2+3 a^3 b^4 c^3 d^2 x^2+3 b^2 c^4 d^2 x^2-3 a^3 b^3 c^4 d^2 x^2-\\
&6 a^2 b c^2 d^3 x^2-3 a^4 b^3 c^2 d^3 x^2+3 a^3 b^4 c^2 d^3 x^2-3 a^4 b^2 c^3 d^3 x^2+3 a^2 b^4 c^3 d^3 x^2-3 a^3 b^2 c^4 d^3 x^2-\\
&3 a^2 b^3 c^4 d^3 x^2-3 a^2 c^2 d^4 x^2+3 a^3 b^3 c^2 d^4 x^2+3 a^3 b^2 c^3 d^4 x^2+3 a^2 b^3 c^3 d^4 x^2-a b c x^3+\\
&a^2 b^3 c^2 x^3+a b d x^3-a c d x^3+b c d x^3+a^2 b^2 c^2 d x^3-a^3 b^2 d^2 x^3-a^2 b^2 c d^2 x^3+a^2 b c^2 d^2 x^3-\\
&a b^2 c^2 d^2 x^3+a^4 b^3 c^2 d^2 x^3-a^3 b^4 c^2 d^2 x^3-b^2 c^3 d^2 x^3+a^3 b^3 c^3 d^2 x^3-a^2 b^4 c^3 d^2 x^3+a^2 b^3 c^4 d^2 x^3+\\
&a^2 c^2 d^3 x^3+a^4 b^2 c^2 d^3 x^3-a^3 b^3 c^2 d^3 x^3+a^3 b^2 c^3 d^3 x^3-a^2 b^3 c^3 d^3 x^3+a^2 b^2 c^4 d^3 x^3-\\
&a^3 b^2 c^2 d^4 x^3-a^2 b^2 c^3 d^4 x^3+3 a^2 c y+6 a b c y-3 a^2 b^4 c y+3 a c^2 y-6 a^2 b^3 c^2 y-3 a b^4 c^2 y-\\
&6 a b d y-3 b^2 d y+3 a^4 b^2 d y+6 a c d y-6 b c d y-3 a^4 b c d y+3 a^3 b^2 c d y-3 a^2 b^3 c d y+\\
&3 a b^4 c d y-3 a^3 b c^2 d y-3 a^2 b^2 c^2 d y-3 a b^3 c^2 d y+3 a^4 b^4 c^2 d y-3 a^2 b c^3 d y+3 a b^2 c^3 d y-\\
&3 a^4 b^3 c^3 d y+3 a^3 b^4 c^3 d y-3 a b c^4 d y+3 b^2 c^4 d y-3 a^3 b^3 c^4 d y+3 a^2 b^4 c^4 d y-3 b d^2 y+\\
&3 a^4 b d^2 y+6 a^3 b^2 d^2 y+3 a^3 b c d^2 y+3 a^2 b^2 c d^2 y+3 a b^3 c d^2 y-3 a^4 b^4 c d^2 y-3 a^2 b c^2 d^2 y+\\
&3 a b^2 c^2 d^2 y-3 a^4 b^3 c^2 d^2 y+3 a^3 b^4 c^2 d^2 y+3 a b c^3 d^2 y+6 b^2 c^3 d^2 y-3 a^4 b^2 c^3 d^2 y-\\
&3 a^3 b^3 c^3 d^2 y+3 a^2 b^4 c^3 d^2 y+3 b c^4 d^2 y-3 a^3 b^2 c^4 d^2 y-3 a^2 b^3 c^4 d^2 y-3 a b^4 c^4 d^2 y-\\
&3 a^2 b c d^3 y+3 a b^2 c d^3 y-3 a^4 b^3 c d^3 y+3 a^3 b^4 c d^3 y-6 a^2 c^2 d^3 y-3 a b c^2 d^3 y-3 a^4 b^2 c^2 d^3 y+\\
&3 a^3 b^3 c^2 d^3 y+3 a^2 b^4 c^2 d^3 y-3 a^4 b c^3 d^3 y-3 a^3 b^2 c^3 d^3 y+3 a^2 b^3 c^3 d^3 y+3 a b^4 c^3 d^3 y-\\
&3 a^3 b c^4 d^3 y-3 a^2 b^2 c^4 d^3 y-3 a b^3 c^4 d^3 y+3 a^4 b^4 c^4 d^3 y-3 a^2 c d^4 y+3 a b c d^4 y-3 a^4 b^2 c d^4 y+\\
&3 a^3 b^3 c d^4 y-3 a c^2 d^4 y+3 a^4 b c^2 d^4 y+3 a^3 b^2 c^2 d^4 y+3 a^2 b^3 c^2 d^4 y+3 a^3 b c^3 d^4 y+\\
&3 a^2 b^2 c^3 d^4 y+3 a b^3 c^3 d^4 y-3 a^4 b^4 c^3 d^4 y+3 a^2 b c^4 d^4 y-3 a b^2 c^4 d^4 y+3 a^4 b^3 c^4 d^4 y-\\
&3 a^3 b^4 c^4 d^4 y-6 a c x y+6 a^2 b^3 c x y+6 a^2 b^2 c^2 x y+6 a b^3 c^2 x y+6 b d x y-6 a^3 b^2 d x y+\\
&6 a^3 b c d x y-6 a b^3 c d x y+6 a^2 b c^2 d x y-6 a^3 b^4 c^2 d x y+6 a b c^3 d x y-6 b^2 c^3 d x y+\\
&6 a^3 b^3 c^3 d x y-6 a^2 b^4 c^3 d x y-6 a^3 b d^2 x y-6 a^2 b^2 d^2 x y-6 a b^2 c d^2 x y+6 a^4 b^3 c d^2 x y+
\end{aligned}
$$

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$6 a^2 c^2 d^2 x y - 6 b^2 c^2 d^2 x y + 6 a^4 b^2 c^2 d^2 x y - 6 a^2 b^4 c^2 d^2 x y - 6 b c^3 d^2 x y + 6 a^3 b^2 c^3 d^2 x y +$
$6 a^2 b^2 c^4 d^2 x y + 6 a b^3 c^4 d^2 x y + 6 a^2 c d^3 x y - 6 a b c d^3 x y + 6 a^4 b^2 c d^3 x y - 6 a^3 b^3 c d^3 x y +$
$6 a c^2 d^3 x y - 6 a^2 b^3 c^2 d^3 x y + 6 a^3 b c^3 d^3 x y - 6 a b^3 c^3 d^3 x y + 6 a b^2 c^4 d^3 x y - 6 a^4 b^3 c^4 d^3 x y -$
$6 a^3 b c^2 d^4 x y - 6 a^2 b^2 c^2 d^4 x y - 6 a^2 b c^3 d^4 x y + 6 a^3 b^4 c^3 d^4 x y - 3 a^2 b^2 c x^2 y - 3 a b^2 c^2 x^2 y +$
$3 a^2 b^2 d x^2 y - 3 a^2 b c d x^2 y + 3 a b^2 c d x^2 y - 3 a b c^2 d x^2 y + 3 b^2 c^2 d x^2 y + 3 a^2 b^4 c^2 d x^2 y +$
$3 a^2 b d^2 x^2 y - 3 a^2 c d^2 x^2 y + 3 a b c d^2 x^2 y - 3 a^4 b^2 c d^2 x^2 y - 3 a c^2 d^2 x^2 y + 3 b c^2 d^2 x^2 y -$
$3 a^3 b^2 c^2 d^2 x^2 y + 3 a^2 b^3 c^2 d^2 x^2 y - 3 a^2 b^2 c^3 d^2 x^2 y - 3 a b^2 c^4 d^2 x^2 y + 3 a^2 b^2 c^2 d^3 x^2 y +$
$3 a^4 b^3 c^3 d^3 x^2 y - 3 a^3 b^4 c^3 d^3 x^2 y + 3 a^3 b^3 c^4 d^3 x^2 y + 3 a^2 b c^2 d^4 x^2 y - 3 a^3 b^3 c^3 d^4 x^2 y -$
$3 a^2 b c y^2 - 3 a b^2 c y^2 - 3 a b c^2 y^2 + 3 a^2 b^4 c^2 y^2 + 3 a^2 b d y^2 + 3 a b^2 d y^2 - 3 a^2 c d y^2 +$
$3 b^2 c d y^2 - 3 a c^2 d y^2 + 3 b c^2 d y^2 + 6 a^2 b^3 c^2 d y^2 + 3 a b d^2 y^2 - 3 a^4 b^2 d^2 y^2 - 3 a c d^2 y^2 +$
$3 b c d^2 y^2 - 6 a^3 b^2 c d^2 y^2 - 6 a b^2 c^3 d^2 y^2 + 3 a^4 b^3 c^3 d^2 y^2 - 3 a^3 b^4 c^3 d^2 y^2 - 3 b^2 c^4 d^2 y^2 +$
$3 a^3 b^3 c^4 d^2 y^2 + 6 a^2 b c^2 d^3 y^2 + 3 a^4 b^3 c^2 d^3 y^2 - 3 a^3 b^4 c^2 d^3 y^2 + 3 a^4 b^2 c^3 d^3 y^2 - 3 a^2 b^4 c^3 d^3 y^2 +$
$3 a^3 b^2 c^4 d^3 y^2 + 3 a^2 b^3 c^4 d^3 y^2 + 3 a^2 c^2 d^4 y^2 - 3 a^3 b^3 c^2 d^4 y^2 - 3 a^3 b^2 c^3 d^4 y^2 - 3 a^2 b^3 c^3 d^4 y^2 +$
$3 a b c x y^2 - 3 a^2 b^3 c^2 x y^2 - 3 a b d x y^2 + 3 a c d x y^2 - 3 b c d x y^2 - 3 a^2 b^2 c^2 d x y^2 +$
$3 a^3 b^2 d^2 x y^2 + 3 a^2 b^2 c d^2 x y^2 - 3 a^2 b c^2 d^2 x y^2 + 3 a b^2 c^2 d^2 x y^2 - 3 a^4 b^3 c^2 d^2 x y^2 +$
$3 a^3 b^4 c^2 d^2 x y^2 + 3 b^2 c^3 d^2 x y^2 - 3 a^3 b^3 c^3 d^2 x y^2 + 3 a^2 b^4 c^3 d^2 x y^2 - 3 a^2 b^3 c^4 d^2 x y^2 -$
$3 a^2 c^2 d^3 x y^2 - 3 a^4 b^2 c^2 d^3 x y^2 + 3 a^3 b^3 c^2 d^3 x y^2 - 3 a^3 b^2 c^3 d^3 x y^2 + 3 a^2 b^3 c^3 d^3 x y^2 -$
$3 a^2 b^2 c^4 d^3 x y^2 + 3 a^3 b^2 c^2 d^4 x y^2 + 3 a^2 b^2 c^3 d^4 x y^2 + a^2 b^2 c y^3 + a b^2 c^2 y^3 - a^2 b^2 d y^3 +$
$a^2 b c d y^3 - a b^2 c d y^3 + a b c^2 d y^3 - b^2 c^2 d y^3 - a^2 b^4 c^2 d y^3 - a^2 b d^2 y^3 + a^2 c d^2 y^3 - a b c d^2 y^3 +$
$a^4 b^2 c d^2 y^3 + a c^2 d^2 y^3 - b c^2 d^2 y^3 + a^3 b^2 c^2 d^2 y^3 - a^2 b^3 c^2 d^2 y^3 + a^2 b^2 c^3 d^2 y^3 + a b^2 c^4 d^2 y^3 -$
$a^2 b^2 c^2 d^3 y^3 - a^4 b^3 c^3 d^3 y^3 + a^3 b^4 c^3 d^3 y^3 - a^3 b^3 c^4 d^3 y^3 - a^2 b c^2 d^4 y^3 + a^3 b^3 c^3 d^4 y^3$

Barycentric coordinates : Reference triangle QL - Diagonal Triangle,

generating line : $l x + m y + n z = 0$

cube[{x_, y_, z_}] := $l^2 m^4 Sa^3 x^3 - 2 l^2 m^2 n^2 Sa^3 x^3 + l^2 n^4 Sa^3 x^3 + 2 l^4 m^2 Sa^2 Sb x^3 + 2 l^4 n^2 Sa^2 Sb x^3 +$
$2 l^2 m^2 n^2 Sa^2 Sb x^3 + 2 l^2 n^4 Sa^2 Sb x^3 + l^6 Sa Sb^2 x^3 + 6 l^4 n^2 Sa Sb^2 x^3 + l^2 n^4 Sa Sb^2 x^3 +$
$2 l^4 m^2 Sa^2 Sc x^3 + 2 l^2 m^4 Sa^2 Sc x^3 + 2 l^4 n^2 Sa^2 Sc x^3 + 2 l^2 m^2 n^2 Sa^2 Sc x^3 + 2 l^6 Sa Sb Sc x^3 +$
$6 l^4 m^2 Sa Sb Sc x^3 + 6 l^4 n^2 Sa Sb Sc x^3 + 2 l^2 m^2 n^2 Sa Sb Sc x^3 + 4 l^6 Sb^2 Sc x^3 + 4 l^4 n^2 Sb^2 Sc x^3 +$
$l^6 Sa Sc^2 x^3 + 6 l^4 m^2 Sa Sc^2 x^3 + l^2 m^4 Sa Sc^2 x^3 + 4 l^6 Sb Sc^2 x^3 + 4 l^4 m^2 Sb Sc^2 x^3 + 3 l^2 m^4 Sa^2 Sb x^2 y +$
$18 l^2 m^2 n^2 Sa^2 Sb x^2 y + 3 l^2 n^4 Sa^2 Sb x^2 y + 6 l^4 m^2 Sa Sb^2 x^2 y + 6 l^4 n^2 Sa Sb^2 x^2 y +$
$6 l^2 m^2 n^2 Sa Sb^2 x^2 y + 6 l^2 n^4 Sa Sb^2 x^2 y + 3 l^6 Sb^3 x^2 y - 6 l^4 n^2 Sb^3 x^2 y + 3 l^2 n^4 Sb^3 x^2 y +$
$12 l^2 m^4 Sa^2 Sc x^2 y + 12 l^2 m^2 n^2 Sa^2 Sc x^2 y + 18 l^4 m^2 Sa Sb Sc x^2 y + 6 l^2 m^4 Sa Sb Sc x^2 y +$
$6 l^4 n^2 Sa Sb Sc x^2 y + 18 l^2 m^2 n^2 Sa Sb Sc x^2 y + 6 l^6 Sb^2 Sc x^2 y + 6 l^4 m^2 Sb^2 Sc x^2 y +$
$6 l^4 n^2 Sb^2 Sc x^2 y + 6 l^2 m^2 n^2 Sb^2 Sc x^2 y + 12 l^4 m^2 Sa Sc^2 x^2 y + 12 l^2 m^4 Sa Sc^2 x^2 y + 3 l^6 Sb Sc^2 x^2 y +$
$18 l^4 m^2 Sb Sc^2 x^2 y + 3 l^2 m^4 Sb Sc^2 x^2 y + 3 m^6 Sa^3 x y^2 - 6 m^4 n^2 Sa^3 x y^2 + 3 m^2 n^4 Sa^3 x y^2 +$
$6 l^2 m^4 Sa^2 Sb x y^2 + 6 l^2 m^2 n^2 Sa^2 Sb x y^2 + 6 m^4 n^2 Sa^2 Sb x y^2 + 6 m^2 n^4 Sa^2 Sb x y^2 + 3 l^4 m^2 Sa Sb^2 x y^2 +$
$18 l^2 m^2 n^2 Sa Sb^2 x y^2 + 3 m^2 n^4 Sa Sb^2 x y^2 + 6 l^2 m^4 Sa^2 Sc x y^2 + 6 m^6 Sa^2 Sc x y^2 + 6 l^2 m^2 n^2 Sa^2 Sc x y^2 +$
$6 m^4 n^2 Sa^2 Sc x y^2 + 6 l^4 m^2 Sa Sb Sc x y^2 + 18 l^2 m^4 Sa Sb Sc x y^2 + 18 l^2 m^2 n^2 Sa Sb Sc x y^2 +$
$6 m^4 n^2 Sa Sb Sc x y^2 + 12 l^4 m^2 Sb^2 Sc x y^2 + 12 l^2 m^2 n^2 Sb^2 Sc x y^2 + 3 l^4 m^2 Sa Sc^2 x y^2 +$
$18 l^2 m^4 Sa Sc^2 x y^2 + 3 m^6 Sa Sc^2 x y^2 + 12 l^4 m^2 Sb Sc^2 x y^2 + 12 l^2 m^4 Sb Sc^2 x y^2 + m^6 Sa^2 Sb y^3 +$
$6 m^4 n^2 Sa^2 Sb y^3 + m^2 n^4 Sa^2 Sb y^3 + 2 l^2 m^4 Sa Sb^2 y^3 + 2 l^2 m^2 n^2 Sa Sb^2 y^3 + 2 m^4 n^2 Sa Sb^2 y^3 +$
$2 m^2 n^4 Sa Sb^2 y^3 + l^4 m^2 Sb^3 y^3 - 2 l^2 m^2 n^2 Sb^3 y^3 + m^2 n^4 Sb^3 y^3 + 4 m^6 Sa^2 Sc y^3 + 4 m^4 n^2 Sa^2 Sc y^3 +$
$6 l^2 m^4 Sa Sb Sc y^3 + 2 m^6 Sa Sb Sc y^3 + 2 l^2 m^2 n^2 Sa Sb Sc y^3 + 6 m^4 n^2 Sa Sb Sc y^3 + 2 l^4 m^2 Sb^2 Sc y^3 +$
$2 l^2 m^4 Sb^2 Sc y^3 + 2 l^2 m^2 n^2 Sb^2 Sc y^3 + 2 m^4 n^2 Sb^2 Sc y^3 + 4 l^2 m^4 Sa Sc^2 y^3 + 4 m^6 Sa Sc^2 y^3 +$
$l^4 m^2 Sb Sc^2 y^3 + 6 l^2 m^4 Sb Sc^2 y^3 + m^6 Sb Sc^2 y^3 + 12 l^2 m^2 n^2 Sa^2 Sb x^2 z + 12 l^2 n^4 Sa^2 Sb x^2 z +$
$12 l^4 n^2 Sa Sb^2 x^2 z + 12 l^2 n^4 Sa Sb^2 x^2 z + 3 l^2 m^4 Sa^2 Sc x^2 z + 18 l^2 m^2 n^2 Sa^2 Sc x^2 z +$
$3 l^2 n^4 Sa^2 Sc x^2 z + 6 l^4 m^2 Sa Sb Sc x^2 z + 18 l^4 n^2 Sa Sb Sc x^2 z + 18 l^2 m^2 n^2 Sa Sb Sc x^2 z +$
$6 l^2 n^4 Sa Sb Sc x^2 z + 3 l^6 Sb^2 Sc x^2 z + 18 l^4 n^2 Sb^2 Sc x^2 z + 3 l^2 n^4 Sb^2 Sc x^2 z + 6 l^4 m^2 Sa Sc^2 x^2 z +$
$6 l^2 m^4 Sa Sc^2 x^2 z + 6 l^4 n^2 Sa Sc^2 x^2 z + 6 l^2 m^2 n^2 Sa Sc^2 x^2 z + 6 l^6 Sb Sc^2 x^2 z + 6 l^4 m^2 Sb Sc^2 x^2 z +$
$6 l^4 n^2 Sb Sc^2 x^2 z + 6 l^2 m^2 n^2 Sb Sc^2 x^2 z + 3 l^6 Sc^3 x^2 z - 6 l^4 m^2 Sc^3 x^2 z + 3 l^2 m^4 Sc^3 x^2 z +$
$3 m^6 Sa^3 x y z - 3 m^4 n^2 Sa^3 x y z - 3 m^2 n^4 Sa^3 x y z + 3 n^6 Sa^3 x y z + 9 l^2 m^4 Sa^2 Sb x y z +$
$6 l^2 m^2 n^2 Sa^2 Sb x y z + 9 m^4 n^2 Sa^2 Sb x y z + 9 l^2 n^4 Sa^2 Sb x y z + 6 m^2 n^4 Sa^2 Sb x y z +$
$9 n^6 Sa^2 Sb x y z + 9 l^4 m^2 Sa Sb^2 x y z + 9 l^4 n^2 Sa Sb^2 x y z + 6 l^2 m^2 n^2 Sa Sb^2 x y z +$
$6 l^2 n^4 Sa Sb^2 x y z + 9 m^2 n^4 Sa Sb^2 x y z + 9 n^6 Sa Sb^2 x y z + 3 l^6 Sb^3 x y z - 3 l^4 n^2 Sb^3 x y z -$
$3 l^2 n^4 Sb^3 x y z + 3 n^6 Sb^3 x y z + 9 l^2 m^4 Sa^2 Sc x y z + 9 m^6 Sa^2 Sc x y z + 6 l^2 m^2 n^2 Sa^2 Sc x y z +$

$6 m^4 n^2 Sa^2 Sc x y z + 9 l^2 n^4 Sa^2 Sc x y z + 9 m^2 n^4 Sa^2 Sc x y z + 18 l^4 m^2 Sa Sb Sc x y z +$
$18 l^2 m^4 Sa Sb Sc x y z + 18 l^4 n^2 Sa Sb Sc x y z - 12 l^2 m^2 n^2 Sa Sb Sc x y z + 18 m^4 n^2 Sa Sb Sc x y z +$
$18 l^2 n^4 Sa Sb Sc x y z + 18 m^2 n^4 Sa Sb Sc x y z + 9 l^6 Sb^2 Sc x y z + 9 l^4 m^2 Sb^2 Sc x y z +$
$6 l^4 n^2 Sb^2 Sc x y z + 6 l^2 m^2 n^2 Sb^2 Sc x y z + 9 l^2 n^4 Sb^2 Sc x y z + 9 m^2 n^4 Sb^2 Sc x y z +$
$9 l^4 m^2 Sa Sc^2 x y z + 6 l^2 m^4 Sa Sc^2 x y z + 9 m^6 Sa Sc^2 x y z + 9 l^4 n^2 Sa Sc^2 x y z + 6 l^2 m^2 n^2 Sa Sc^2 x y z +$
$9 m^4 n^2 Sa Sc^2 x y z + 9 l^6 Sb Sc^2 x y z + 6 l^4 m^2 Sb Sc^2 x y z + 9 l^2 m^4 Sb Sc^2 x y z + 9 l^4 n^2 Sb Sc^2 x y z +$
$6 l^2 m^2 n^2 Sb Sc^2 x y z + 9 m^4 n^2 Sb Sc^2 x y z + 3 l^6 Sc^3 x y z - 3 l^4 m^2 Sc^3 x y z - 3 l^2 m^4 Sc^3 x y z +$
$3 m^6 Sc^3 x y z + 12 m^4 n^2 Sa^2 Sb y^2 z + 12 m^2 n^4 Sa^2 Sb y^2 z + 12 l^2 m^2 n^2 Sa Sb^2 y^2 z +$
$12 m^2 n^4 Sa Sb^2 y^2 z + 3 m^6 Sa^2 Sc y^2 z + 18 m^4 n^2 Sa^2 Sc y^2 z + 3 m^2 n^4 Sa^2 Sc y^2 z + 6 l^2 m^4 Sa Sb Sc y^2 z +$
$18 l^2 m^2 n^2 Sa Sb Sc y^2 z + 18 m^4 n^2 Sa Sb Sc y^2 z + 6 m^2 n^4 Sa Sb Sc y^2 z + 3 l^4 m^2 Sb^2 Sc y^2 z +$
$18 l^2 m^2 n^2 Sb^2 Sc y^2 z + 3 m^2 n^4 Sb^2 Sc y^2 z + 6 l^2 m^4 Sa Sc^2 y^2 z + 6 m^6 Sa Sc^2 y^2 z + 6 l^2 m^2 n^2 Sa Sc^2 y^2 z +$
$6 m^4 n^2 Sa Sc^2 y^2 z + 6 l^4 m^2 Sb Sc^2 y^2 z + 6 l^2 m^4 Sb Sc^2 y^2 z + 6 l^2 m^2 n^2 Sb Sc^2 y^2 z +$
$6 m^4 n^2 Sb Sc^2 y^2 z + 3 l^4 m^2 Sc^3 y^2 z - 6 l^2 m^4 Sc^3 y^2 z + 3 m^6 Sc^3 y^2 z + 3 m^4 n^2 Sa^3 x z^2 -$
$6 m^2 n^4 Sa^3 x z^2 + 3 n^6 Sa^3 x z^2 + 6 l^2 m^2 n^2 Sa^2 Sb x z^2 + 6 l^2 n^4 Sa^2 Sb x z^2 + 6 m^2 n^4 Sa^2 Sb x z^2 +$
$6 n^6 Sa^2 Sb x z^2 + 3 l^4 n^2 Sa Sb^2 x z^2 + 18 l^2 n^4 Sa Sb^2 x z^2 + 3 n^6 Sa Sb^2 x z^2 + 6 l^2 m^2 n^2 Sa^2 Sc x z^2 +$
$6 m^4 n^2 Sa^2 Sc x z^2 + 6 l^2 n^4 Sa^2 Sc x z^2 + 6 m^2 n^4 Sa^2 Sc x z^2 + 6 l^4 n^2 Sa Sb Sc x z^2 +$
$18 l^2 m^2 n^2 Sa Sb Sc x z^2 + 18 l^2 n^4 Sa Sb Sc x z^2 + 6 m^2 n^4 Sa Sb Sc x z^2 + 12 l^4 n^2 Sb^2 Sc x z^2 +$
$12 l^2 n^4 Sb^2 Sc x z^2 + 3 l^4 n^2 Sa Sc^2 x z^2 + 18 l^2 m^2 n^2 Sa Sc^2 x z^2 + 3 m^4 n^2 Sa Sc^2 x z^2 +$
$12 l^4 n^2 Sb Sc^2 x z^2 + 12 l^2 m^2 n^2 Sb Sc^2 x z^2 + 3 m^4 n^2 Sa^2 Sb y z^2 + 18 m^2 n^4 Sa^2 Sb y z^2 +$
$3 n^6 Sa^2 Sb y z^2 + 6 l^2 m^2 n^2 Sa Sb^2 y z^2 + 6 l^2 n^4 Sa Sb^2 y z^2 + 6 m^2 n^4 Sa Sb^2 y z^2 + 6 n^6 Sa Sb^2 y z^2 +$
$3 l^4 n^2 Sb^3 y z^2 - 6 l^2 n^4 Sb^3 y z^2 + 3 n^6 Sb^3 y z^2 + 12 m^4 n^2 Sa^2 Sc y z^2 + 12 m^2 n^4 Sa^2 Sc y z^2 +$
$18 l^2 m^2 n^2 Sa Sb Sc y z^2 + 6 m^4 n^2 Sa Sb Sc y z^2 + 6 l^2 n^4 Sa Sb Sc y z^2 + 18 m^2 n^4 Sa Sb Sc y z^2 +$
$6 l^4 n^2 Sb^2 Sc y z^2 + 6 l^2 m^2 n^2 Sb^2 Sc y z^2 + 6 l^2 n^4 Sb^2 Sc y z^2 + 6 m^2 n^4 Sb^2 Sc y z^2 +$
$12 l^2 m^2 n^2 Sa Sc^2 y z^2 + 12 m^4 n^2 Sa Sc^2 y z^2 + 3 l^4 n^2 Sb Sc^2 y z^2 + 18 l^2 m^2 n^2 Sb Sc^2 y z^2 +$
$3 m^4 n^2 Sb Sc^2 y z^2 + 4 m^2 n^4 Sa^2 Sb z^3 + 4 n^6 Sa^2 Sb z^3 + 4 l^2 n^4 Sa Sb^2 z^3 + 4 n^6 Sa Sb^2 z^3 +$
$m^4 n^2 Sa^2 Sc z^3 + 6 m^2 n^4 Sa^2 Sc z^3 + n^6 Sa^2 Sc z^3 + 2 l^2 m^2 n^2 Sa Sb Sc z^3 + 6 l^2 n^4 Sa Sb Sc z^3 +$
$6 m^2 n^4 Sa Sb Sc z^3 + 2 n^6 Sa Sb Sc z^3 + l^4 n^2 Sb^2 Sc z^3 + 6 l^2 n^4 Sb^2 Sc z^3 + n^6 Sb^2 Sc z^3 +$
$2 l^2 m^2 n^2 Sa Sc^2 z^3 + 2 m^4 n^2 Sa Sc^2 z^3 + 2 l^2 n^4 Sa Sc^2 z^3 + 2 m^2 n^4 Sa Sc^2 z^3 + 2 l^4 n^2 Sb Sc^2 z^3 +$
$2 l^2 m^2 n^2 Sb Sc^2 z^3 + 2 l^2 n^4 Sb Sc^2 z^3 + 2 m^2 n^4 Sb Sc^2 z^3 + l^4 n^2 Sc^3 z^3 - 2 l^2 m^2 n^2 Sc^3 z^3 + m^4 n^2 Sc^3 z^3$

Eckart Schmidt
eckart_schmidt@ t - online.de
http : // eckartschmidt.de

Message: #38
Date: 17/6/2013 3:14:30
From: Chris
Subject: Cardioids tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven
(April 26, 2013)
Dear Eckart,
This looks like a very interesting result.
Could you share with us (after your holiday) how you came to
this result?
Do you have a CT-equation of this cubic?
Then I can try to draw the cubic.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #39
Date: 17/6/2013 3:21:56
From: Chris van Tienhoven
Subject: Varignon points (temporarily named)

COPY FROM FORMER GOOGLE GROUP by Seiichi Kirikami
(April 22, 2013)
Dear friends, Welcome! Bernard. I am sorry that this is not about Cardioid here.
[1] Varignon points: Given a quadrangle P1P2P3P4, the midpoints Mij of PiPj and the midpoints Tikj of Mij Mjk, then 4 circles through the following 3 points have a common point.
(1) T134M13T243, T243M24T132, T132M13T241, T241M24T134.
(2) T134M24T243, T243M13T132, T132M24T241, T241M24T134.
(3) T123M12T342, T342M34T124, T124M12T341, T341M34T123.
(4) T123M34T342, T342M12T124, T124M34T341, T341M12T123.
(5) T234M23T142, T142M14T231, T231M23T143, T143M14T234.
(6) T234M14T142, T142M23T231, T231M14T143, T143M23T234.
[2] Example
(1) Case(1) in CT- environment:
Given a quadrangle P1P2P3, the midpoints M12, M23, M34, M14, M13 and M24, the midpoints T132, T134, T241 and T243 of M12M23, M14M34, M12M14 and M23M34, then 4 circles through T134M13T243, T243M24T132, T132M13T241 and T241M24M134 have a common point.
See the attached pdf. See Mathematica-file for the coordinates.
My idea in Mathematica is as follows. We know that M13 is the common point of the circles through T134M13T243 and T132M13T241.
We compute the centers of these circles. The other intersection is given as the reflection point of M13 across the line through the centers of these circles.
Similarly we compute the other intersection as the reflection point of M24.The former other intersection coincides with the latter other one. Since the coordinates are too complicated to be written here, the above Mathematica-file is attached.
(2) Case(1) in DT- environment: Change pt1{0,1,0}, pt2{0,0,1}, pt3{1,0,0} and pt4{p, q, r} to {p, -q, r}, {p, q, -r}, {-p, q, r} and {p, q, r}.
(3) Case(2) in CT and DT- environment: Interchange M13 and M24.
Quadrigonal other cases (3), (4), (5) and (6) are similarly computed.
P. S. I received the email from Chris that he received my message personally, but there was not my message in the mailing list of the group. So I post it again. If you receive the 2nd message of the same contents personally, I wish that you do not mind erasing it.
Best regards, Seiichi Kirkami.

Message: #40
Date: 17/6/2013 3:26:09
From: Chris
Subject: Varignon points (temporarily named)

```
COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven (May 12,
2013)
Dear Seiichi,
I checked your calculations and they look allright to me.
There should be 6 Varignon Points. Is that correct?
What can be done more with these 6 points?
Are they cyclic?
What is their Centroid?
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #41
Date: 17/6/2013 3:28:42
From: Chris
Subject: New property of QA-Tf2 (Involutary Conjugate)

```
COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 13, 2013)
Dear friends,
the six conics through
... two vertices of the quadrangle,
... two diagonal intersections (not collinear with the vertices)
... and a point P
have a second common point,
which is the QA-Tf2 image of P.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #42
Date: 17/6/2013 3:31:24
From: Chris
Subject: 3 LSD-points on Newton line in QL-environment

```
COPY FROM FORMER GOOGLE GROUP by Seiichi Kirikami
(May 5, 2013)
Dear friends,
Given a quadrilateral L1L2L3L4 and their intersections s12, s13,
s14, s23, s24 and s34, the LSD point P1 from s14, s12, s23, s34,
the LSD P2 point from s14, s13, s23, s24 and the LSD point P3
from s12, s13, s24, s34 are on Newton line of the quadrilateral.
Best regards,
Seiichi Kirikami
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #43
Date: 17/6/2013 3:33:12
From: Chris
Subject: 3 LSD-points on Newton line in QL-environment

```
COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven
(May 12, 2013)
Dear Seiichi,
This result is not quite unexpected.
The 3 LSD-points (points with Least Sum of Squared Distances to
the 4 designated points) are the LSD-points of the 3 Component
Quadrigons of the Reference Quadrilateral.
It is known that the LSD-point of 4 points is the Centroid
QA-P1.
It is also known that the Centroids of the 3 Component
Quadrigons lie on the Newton Line.
So the 3 LSD-points of your 3 triangles coincide with the 3
Centroids of the 3 QL-Component Quadrigons and lie on the Newton
Line.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #44
Date: 17/6/2013 3:34:48
From: Chris
Subject: 3 LSD-points on Newton line in QL-environment

```
COPY FROM FORMER GOOGLE GROUP by Seiichi Kirikami
(May 13, 2013)
Dear Chris,
Thank you for your information. After I computed LSD point to 6
points of the quadrilateral, I found that it was QL-P12 on
Newton line. Then I knew what you said below.
I found a weighted LSD problem in a book written in Japanese. If
I can formulate it for EQF, I will post it.
Best regards,
Seiichi Kirikami.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #45
Date: 17/6/2013 3:41:04
From: Chris van Tienhoven
Subject: Conic poles of a quadrangle wrt its circumcubics

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 15, 2014)
Dear friends,
consider a quadrangle, a circumcubic and the pencil of circumscribed conics with further intersections P and Q with the cubic.
The lines PQ have a common point on the cubic, called "conic pole" of the quadrangle.
(H.M.Cundy and C.F.Parry. Some cubic curves associated with the triangle. Journal of Geometry 53 (1995), p. 45).
Examples:
   QA-Cu1: QA-Tf2 of QA-P4
   QA-Cu2: QA-P17
   QA-Cu3: QA-P16
   QA-Cu4: QA-P18
   QA-Cu5: QA-P20
   QA-Cu6: QA-P22.
The first five cubics are pivotal isocubics of QA-DT wrt the isoconjugation QA-Tf2.
For these cubics the conic pole of the quadrangle is the common point of the tangents in the vertices of QA-DT.
With this property the points are already mentioned in EQF.
New will be the conic pole QA-P22 for a quadrangle and its circumcubic QA-Cu6.
Best regards Eckart

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #46
Date: 17/6/2013 3:42:37
From: Chris
Subject: Conic poles of a quadrangle wrt its circumcubics

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 16, 2014)
Dear friends,
sorry, I have forgotten the following property:
For a pivotal circumscribed isocubic of a quadrangle (wrt QA-DT and QA-Tf2) the conic pole is the QA-Tf2 image of the pivot.
Best regards Eckart

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #47
Date: 17/6/2013 3:45:10
From: Chris van Tienhoven
Subject: What is the necessary and sufficient condition of quadrifigure funct

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```
COPY FROM FORMER GOOGLE GROUP  by  Seiichi Kirikami
(may 17, 2013)
Dear friends,
After a while when I knew EQF, I wondered what the necessary and
sufficient condition of quadrifigure function would be. As it
seems to me that there is no literature about this theme, I post
this subject as a question.
An example of the necessary condition of quadrangle: If the
quadrangle is a square, a=1, b=1, c=sqrt[2] and p=1, q=1, r=-1,
then the quadrangle point has the absolute coordinates
   {1/2, 1/2, 0}.
If we denote the quadrangle function by
   {f(a,b,c; p,q,r), g(a,b,c; p,q,r), h(a,b,c; p,q,r)},
then the necessary condition is h(1,1,sqrt[2]; 1,1,-1)=0.
So h(p,q,r)=p+q+2r (QA-P1) can be a quadrangle function because
h(1,1,-1)=0. h(p,q,r)=p+q+3r is not a quadrangle function. I
would like to know the form of quadrifigure function,
especially, for the sufficient condition.
Best regards, Seiichi Kirikami.
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #48
Date: 17/6/2013 3:47:41
From: Chris van Tienhoven
Subject: EQF-Note 2013-05-17: A Cubic, a Circle and a Line wrt a Quadrilatera

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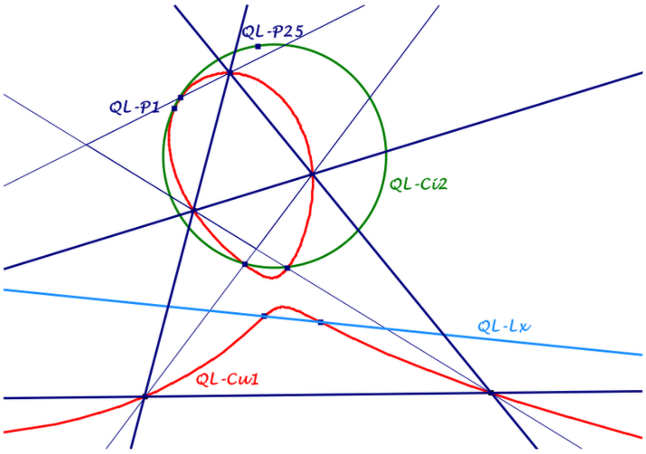
```
COPY FROM FORMER GOOGLE GROUP  by  Eckart Schmidt
(May 17, 2013)
Dear friends,
there is a short essay about a new line for a quadrilateral
in the attachment.
Best regards Eckart
```

EQF-Note 2013-05-17

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Cubic, a Circle and a Line wrt a Quadrilateral

The cubic will be QL-Cu1, which contains the foci of inscribed conics, the circle will be QL-Ci2, the nine-point circle of the QL-diagonal triangle, and the line will be the Clawson-Schmidt Conjugate of the circle QL-Ci2. This line is not mentioned in EQF up to now. – Reference triangle for barycentric coordinates is QL-DT.



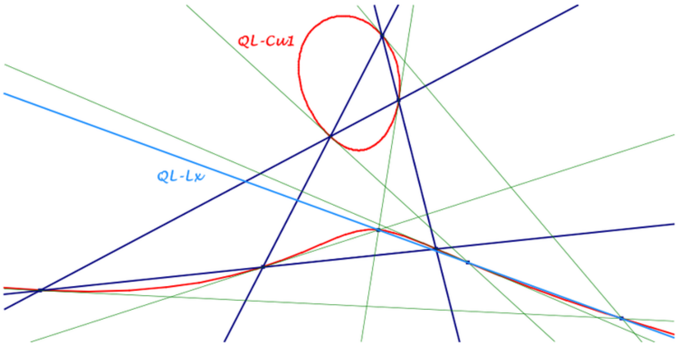
We start with the cubic *QL-Cu1*. The vertices of the orthic triangle of *QL-DT* lie on this cubic as well as on the nine-point circle *QL-Ci2* of *QL-DT*. The Clawson-Schmidt Conjugate *QL-Tf1* of *QL-Ci2* is a line *QL-Lx*, which cuts the cubic in the *QL-Tf1* images of the vertices of the orthic triangle. Other points of *QL-Lx* are the *QL-Tf1* images of the Miquel Point *QL-P1* and *QL-P25*.

The equation of the line *QL-Lx*:

$$\sum_{cycl}(l^4a^2(S^2+3S_BS_C)+m^4b^4S_A+n^4c^4S_A+2m^2n^2S_A(S^2-S_A^2)$$
$$+2l^2m^2(a^2b^2S_A+2S^2S_C)+2l^2n^2(a^2c^2S_A+2S^2S_B))x)=0$$

2013-05-17.pdf

But there is a further connection to the cubic *QL-Cu1*: The tangents in opposite points of the quadrilateral intersect on the cubic in three collinear points on *QL-Lx* (which are the *QL-Tf1* images of the vertices of the orthic triangle).



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-05-17.pdf

Message: #49
Date: 17/6/2013 3:50:51
From: Chris
Subject: EQF-Note 2013-05-17: A Cubic, a Circle and a Line wrt a Quadrila

---

```
COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 17, 2013)
Dear friends,
sorry, just found out: The new QL-line is the connection of the
three collinear QG-P18 for the quadrigon components, which lie
on the cubic QL-Cu1.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #50
Date: 17/6/2013 3:54:09
From: Chris van Tienhoven
Subject: a quadrangle identity derived from a tetrahedron identity

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```
COPY FROM FORMER GOOGLE GROUP  by  Seiichi Kirikami
(May 24, 2013)
Dear friends,
The following may be well known, but I post it because it
relates a tetrahedron to a quadrangle.
Given a tetrahedron P1P2P3P4 and the midpoints of its edges
      P1P2, P2P3, P3P1, P1P4, P2P4, P3P4 are denoted by
      m12,  m23,  m31,  m14,  m24,  m34  respectively.
Then  P1P2^2+P2P3^2+P3P1^2+P1P4^2+P2P4^2+P3P4^2
      = 4*(m12m34^2+m23m14^2+m31m24^2).
If we compress the tetrahedron to the quadrangle, then we
obtain an identity of quadrangle.
Best regards,
Seiichi Kirikami.
Show trimmed content
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #51
Date: 17/6/2013 3:57:24
From: Chris
Subject: a quadrangle identity derived from a tetrahedron identity

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COPY FROM FORMER GOOGLE GROUP by Seiichi Kirikami
(May 27, 2014)
I add 2 other relations.
(1) QA-environment: Given a quadrangle P1P2P3P4, a point (QA-P1)
and their midpoints M12, M23, M31, M14, M24, M34, SD(square
distances) from P to Pi and Mij has the relation:
   PP1^2+PP2^2+PP3^2+PP4^2
   = 2*(PM12^2+PM23^2+PM31^2+PM14^2+PM24^2+PM34^2).
(2) QL-environment: Given a quadrilateral L1L2L3L4, a point
(QL-P26) and pedal points Pi on Li, SD( square distances) from P
to Pi and of PiPj has the relation:
   P1P2^2+P2P3^2+P3P1^2+P1P4^2+P2P4^2+P3P4^2
   = 4*(PP1^2+PP2^2+PP3^2+PP4^2).
Case(1) is from my previous post. I learned Case(2) from Eckart.
Best regards,
Seiichi Kirikami.
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #52
Date: 17/6/2013 4:06:03
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(April 26, 2013)
Dear friends,
Chris has asked for a CT-equation for the cubic for the centers of the cardioids tangent to 4 lines.
It was a great surprise: The equation is very simple, see in the Mathematica file in the attachement.
Further: The asymptotes cut under an angle of 60° !
If Chris can draw the cubic, he can say, wether the asymptotes intersect in one point.
Perhaps this point lies on the cubic.
Best regards Eckart

Cardioids tangent to 4 lines

The centers of cardioids tangent to four lines lie on cubics with the following equations :

Barycentric coordinates : Reference triangle
L1 (1, 0, 0), L2 (0, 1, 0), L3 (0, 0, 1),
fourth line L4 (l, m, n)

card[{x_, y_, z_}] := $b^2 c^2 l (-b^2 (-l+m)(m-n) - c^2 (-l+n)(-m+n) + a^2 (l m + l n - m n)) x^3 +$
$a^2 c^2 m (-a^2 (l-m)(l-n) - c^2 (-l+n)(-m+n) + b^2 (l m - l n + m n)) y^3 +$
$a^2 b^2 n (-a^2 (l-m)(l-n) - b^2 (-l+m)(m-n) + c^2 (-l m + l n + m n)) z^3 +$
$3 a^2 b^2 c^2 (l x + m y)(l x + n z)(m y + n z)$

The asymptotes intersect with angles of 60 °.

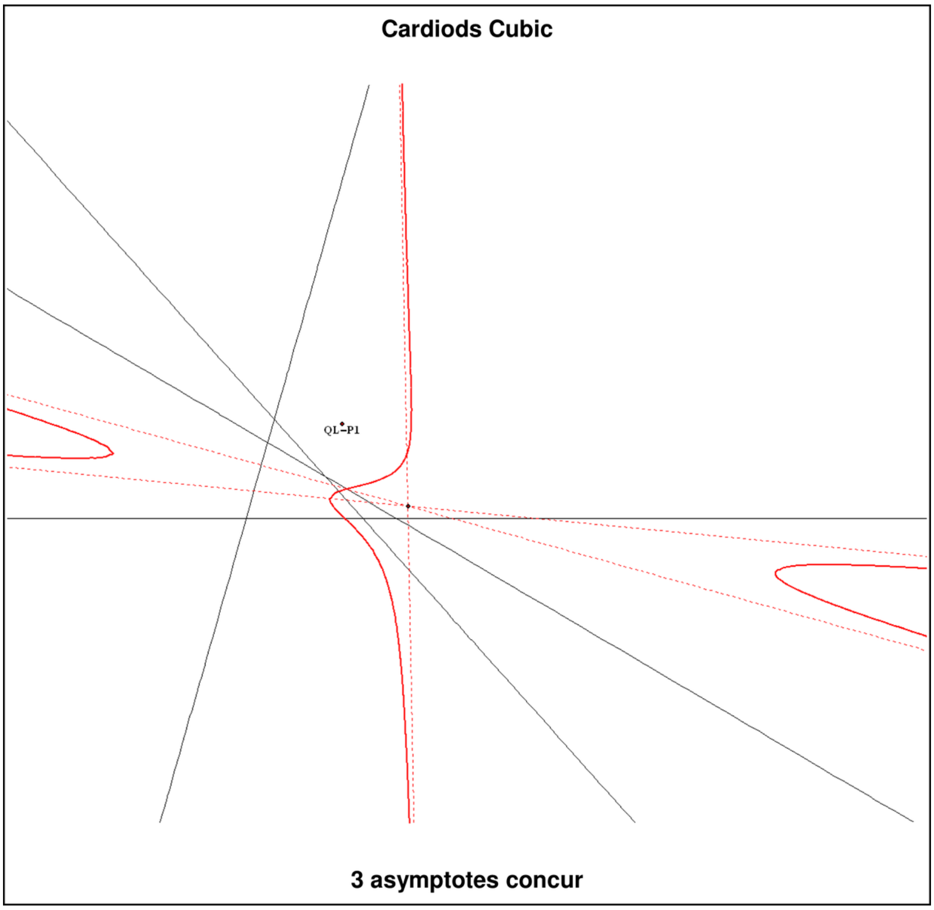
Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

Card-nb.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #53
Date: 17/6/2013 4:09:40
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven
(April 26, 2013)
Dear Eckart,
Wonderful this equation in CT-format!
It is not surprising it is more simple in CT-format, because the problem is about 4 lines and there's no relationship with the Diagonal Triangle.
Attached is the picture of the Cardioids Cubic.
The 3 asymptotes concur in an unknown point.
I did NOT see the asymptotes cutting under an angle of 60° .
This far I didn't manage to calculate the coordinates of this interesting point.
Maybe someone else can?
Can you please tell me how you discovered the cubic and what exactly is the relationship with the 8 Cardioids?
Chris



Cardiods Cubic.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #54
Date: 17/6/2013 4:12:35
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

Dear Eckart and Chris,
Thank you for your efforts!
I wasn't able to read Eckart's attached file and I
discovered the drawing of the cubic with surprise!
In fact, I join Chris in his questions: where does this cubic
come from and what's the link to the 8 searched cardioïds?
I'm sure there are 8 cardioïds tangent to 4 lines and only but
one for oriented lines.
May be the answer to a simple question could help: if I have a
cardioïd and 4 tangent lines, is it possible to find the other 7
cardioïds tangent to the same lines?
As all cardioïds are similar to one another, we need for each
cardioïd 4 parameters, for exemple coordinates of center and
cusp, or center and dopple tangent, or coordinates of the center
of similarity, angle and ratio of the similarity with another
known cardioïd ...
And again, the 8 centers belong to 4 groups of 3*3 lines (one
group for each reference triangle); is it possible to calculate
the equation of those lines and find the centers as intersection
of the 4 groups (meaning a maximum of 81 possible points ...)?
Anyhow, it's not the end of the question: if we had the centers,
we still would have to find then the corresponding cusps in
order to draw the cardioïds tangent to the 4 lines; it's another
question, for which the answer could be useful.
In the same way, the cusps belong to 4 groups of 3*3 cubics (
one for each reference triangle) ...
Best regards
Bernard

← Previous Next → ↩ Message Index ↑ Subjects

Message: #55
Date: 17/6/2013 4:18:11
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP  by  Eckart Schmidt
(May 7, 2013)
Dear friends,
... back from holiday: You ask, how I got the equation of the
cubic for the centers of the cardioids tangent to four lines.
I have described the way already in my first message wrt this
theme. The following file was damaged,
so you find an update in the attachment with a possibility to
calculate the cusp for a given center.
Best regards Eckart
PS. Dear Bernard, can you give me a short description of the
deltoid mentioned in your paper?
Thank you Eckart
```

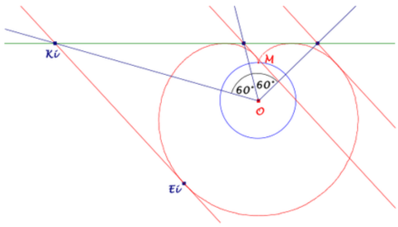
07.05.2013

Cardioïds tangent to 4 lines
- A cubic for the centers and a formula for the cusps -

There are two properties for tangents of cardioïds:

„Die Tripel paralleler Tangenten der Kardioide schneiden die Doppeltangente in Tripeln von Punkten, die vom Mittelpunkt des Grundkreises aus unter dem Winkel 60° erscheinen.“

H. Schmidt: Ausgewählte höhere Kurven.
Kesselringsche Verlagsbuchhandlung Wiesbaden, 1949, S.122.



Not to be found there is a further property:

For the three parallel tangents of a cardioïd the sum of orientated distances from the center O is zero.

Let O be the center of the cardioïd, M its cusp, t its bitangent and l_0, l_1, l_2 parallel tangents.

… the angle property gives the intersections of the tangents l_0, l_1, l_2 with the bitangent t.

… the distance property gives an equation for further interpretations.

The necessary calculations are very hard and the terms extreme extensive! Using barycentric coordinates the reference triangle will be the triangle of three sidelines of the quadrilateral with a fourth line $l(l,m,n)$. The balance of orientated distances can be done with the intersections of the parallels with a sideline of the reference triangle.

2013-05-07 .pdf

The three equations for the sidelines of the reference triangle lead to a calculation of the cusp M for a center O. This is also valid for inscribed cardioïds of a triangle.

A cardioïd tangent to three lines with center O(u:v:w) has the cusp $M(b^2c^2u^3 : c^2a^2v^3 : a^2b^2w^3)$ **.**

The four equations for the sidelines of a quadrilateral lead to a cubic, locus for the centers of inscribed cardioïds.

The centers of cardioïds tangent to four lines lie on a cubic with the equation

$$\sum_{cycl} b^2c^2 l(-a^2(lm+ln-mn)+b^2(m-n)(m-l)+c^2(n-l)(n-m))x^3$$

$$=3a^2b^2c^2(lx+my)(my+nz)(nz+lx) \ .$$

The asymptotes of this cubic intersect with angles of 60°.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-05-07 .pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #56
Date: 17/6/2013 4:19:40
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 9, 2013)
Dear friends,
in addition to my last message:
The asymptotes of the cubic for the centers of cardioids tangent to 4 lines
are parallel to the axes of the deltoid tangent to 4 lines (see Bernard´s message).
Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #57
Date: 17/6/2013 4:24:51
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven
(May 12, 2013)
Dear Eckart, dear friends,

What a wonderful cubic !!!
What a wonderful result that the asymptotes of the cubic are parallel to the axes of the Deltoid !!!
I also have a nice result: the asymptotes of the cubic concur in the Miquel Point QL-P1!
Let me tell how I proceeded using Eckart's results:

1. When we have in a triangle a Cardioid with center (u: v: w) tangent to all sidelines,
then the cusp has coordinates (b2c2u3: a2c2v3: a2b2w3).
Not every point however in the Triangle Plane can function as the center of a tangent Cardioid.
Still there are infinite point in the Triangle Plane that can function as the center of a tangent Cardioid.

2. To diminish the number of points with possibility to function as the center of a tangent Cardioid we introduce a 4th line that also has to be tangent to the Cardioid. Now we deal with a Quadrilateral instead of a Triangle.

3. The Quadrilateral has 4 Component Triangles (CT). Given a center P, then for each Component Triangle the calculation of the cusp has to deliver the same point. I did draw this in Cabri and indeed when 2 CT-cusps concur, automatically all 4 CT-cusps concur.

4. When we equalize the calculated coordinates of the cusps in 2 Component Triangles with a given point P, then your Cubic rolls out.

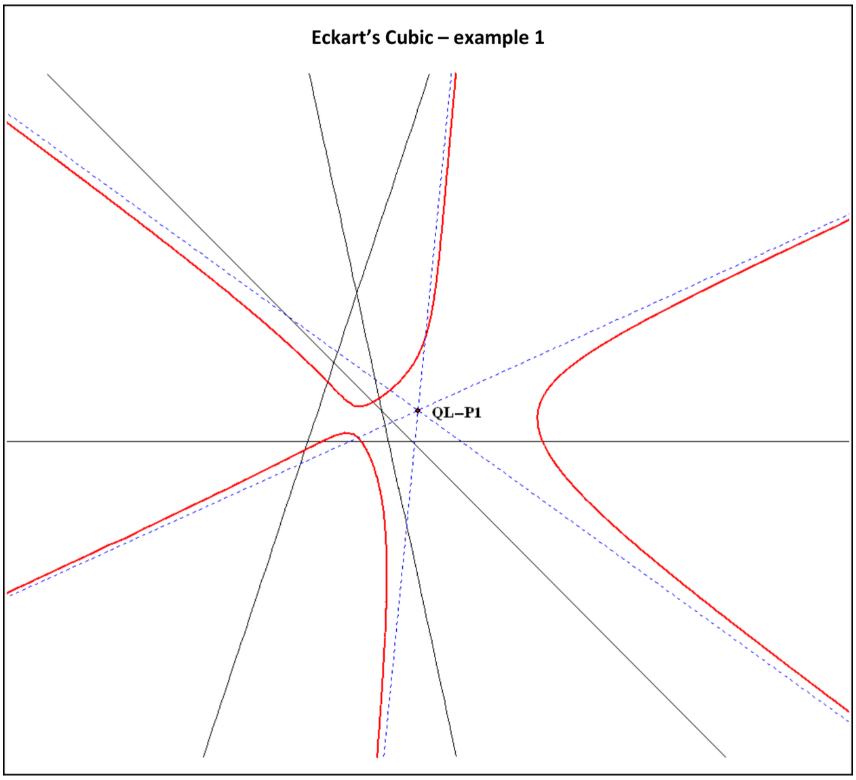
5. It is important to realize that this cubic only show the points P(u: v: w) for which the transformed points (b 2c2u3: a2c2v3: a2b2w3) for the 4 Component Triangles concur. Not each point on the cubic is the center of a tangent Cardioid. However when there is a center of a tangent Cardioid then it should lie on this cubic.

6. Enclosed you will find a new picture of the Cubic and indeed it looks like the asymptotes concur in a point and when I am not wrong this point is the Miquel Point QL-P1 !!!
I checked it in several numerical examples and the coordinates of QL-P1 and the Asymptote Crosspoint are exactly the same.

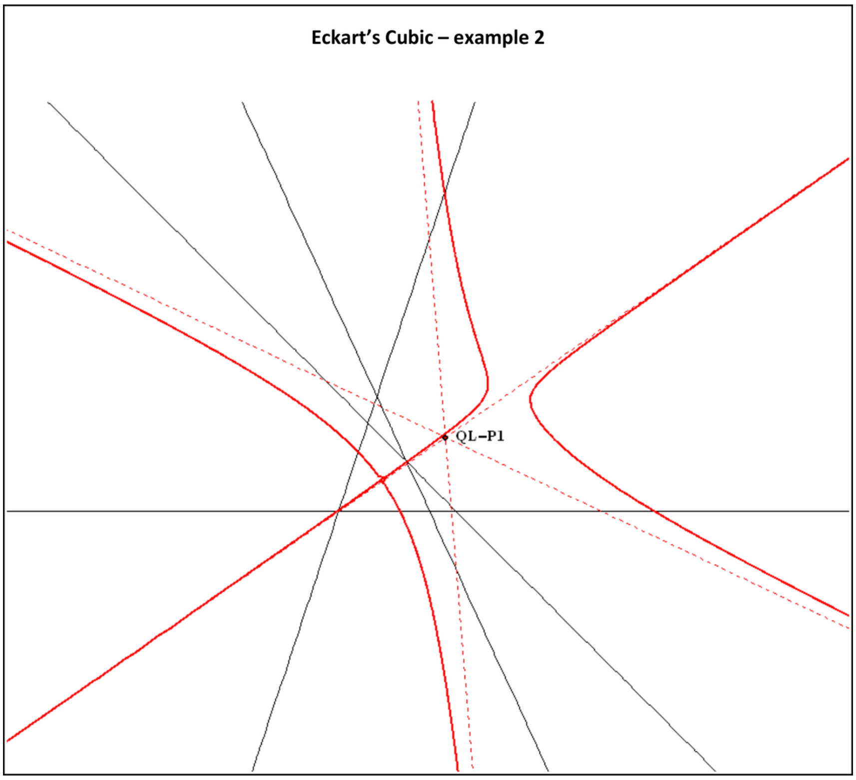
7. Moreover the picture also shows –like Eckart told- the asymptotes cross under angles of 60 degrees.

8. Just like Eckart I already earlier had calculated the Deltoid however again the equations of the axes are very long. Maybe I can try to include the Deltoid in the same picture as Eckarts cubic later.
Eckart, some questions:
· Could you tell me how you calculated the cusp coordinates (b2c2u3: a2c2v3: a2b2w3)?
· Did you find simple equations of the asymptotes or the infinity points of the asymptotes? I found the equations however far from simple.
· How did you prove the asymptotes crossing under angles of 60 degrees?
· How did you calculate the Deltoid and its axes?

Best regards,
Chris



EckartsCubic.pdf



EckartsCubic.pdf

Message: #58
Date: 17/6/2013 4:27:21
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP   by Eckart Schmidt
(May 12, 2013)
Dear Chris, dear friends,
thank you very much Chris for the new picture of the cubic for the centers of cardioids tangent to four lines.
It will be a nice result, that the asymptotes intersect in QL-P1!
To your questions:
The coordinates for the cusps are a consequence of the four equations mentioned in my last message,
describing the bitangent with O and M.
If you take the infinity point (1,f,-1-f), you get with the cubic an equation for f,
which leads to the crossing angles of 60°.
The properties of the deltoid I only have proved with Cabri, using the description of Bernard.
The real reason for my message is, there will be 27 cardioids tangent to four lines
. The centers of the inscribed cardioids of the reference triangle lie on 3 sets of 3 parallel lines.
Each line has 3 points of intersection with the cubic for the centers,
so there are 27 possible centers.
I have calculated an example (see the Mathematica file in the attachment), drawing most of the cardioids with Cabri
(using the calculated center and cusp) and it seems alright! The four groups of three sets of three parallel lines
for the centers of the cardioids can be found.
Best regards Eckart

Centers of Cardioids Tangent to 4 Lines, an Example (12.05.2013)

Reference triangle and fourth line:

```
a:=6;b:=5;c:=4;l:=-1;m:=-2;n:=2
```

For the reference triangle there are three sets of three parallel lines for the centers of inscribed cardioids, intersecting with angles of 60° (see Bernard`s message).
For a quadrilateral the centers of inscribed cardioids lie on a cubic (see my last messages).
Each line cuts the cubic in three points,
so there are 9 x 3 = 27 centers for inscribed cubics.

Centers of cardioids:

```
O1:={-59.78173625283743`,42.165047287904294`,1};
```

```
O2:={-0.203511113184554`,-0.332074102406412`,1};
```

```
O3:={0.060606252681044546`,-0.48627193182838463`,1}
```

```
O4:={0.46707804440590894`,-0.7759758006335196`,1};
```

```
O5:={1.101895148780922`,1.071079720158494`,1};
```

```
O6:={1.4669799267249983`,0.8246623879358206`,1};
```

```
O7:={1.576555916219293`,0.6206153569597764`,1};
```

```
O8:={3.1347898750709837`,-2.2810514619917557`,1}
```

```
O9:={6.800318699532346`,7.196195725014429`,1}
```

```
O10:={-118.11486997562572`,0.`,1}
```

```
O11:={-16.291106305040064`,-52.702072298551705`,1}
```

```
O12:={-5.630704830920701`,3.990640240900943`,1}
```

```
O13:={-2.295603569683266`,-0.04916986488667343`,1}
```

```
O14:={-2.0265143243867794`,-0.0026242615925480264`,1}

O15:={-1.7369505064696762`,1.496911448196238`,1}

O16:={-1.4284127080083318`,0.03475928403102694`,1}

O17:={-1.1406814741361457`,1.2731330086853312`,1}

O18:={-0.9699454275148954`,1.2321324378106324`,1}

O19:={-0.5496842851246039`,-0.1657206649926386`,1}

O20:={0.3951953482797242`,-3.308535439200701`,1}

O21:={0.5219728461181129`,1.158069062394834`,1}

O22:={0.6657709472051169`,-0.962307875435281`,1}

O23:={0.8156503060159306`,-2.077717398028317`,1}

O24:={1.6346115236594532`,-0.5806351452160409`,1}

O25:={2.0879758424326775`,-1.4692625981628826`,1}

O26:={2.340553042869985`,-1.683799644726599`,1}

O27:={22.888718892371514`,-3252.4707475677624`,1}
```

Cusps of cardioids

```
M1 := {-94956.14051740553`, 47977.51491845804`, 1};

M2 := {-0.003746121607258228`, -0.023436081318746842`, 1};

M3 := {0.0000989395152867481`, -0.07358979293768209`, 1}

M4 := {0.045288281516972485`, -0.29903671073929927`, 1};

M5 := {0.5946183329443193`, 0.7864033653062752`, 1};

M6 := {1.4031044294890394`, 0.35892898907724097`, 1};

M7 := {1.7415887784074824`, 0.15298453388442712`, 1};

M8 := {13.691240144445958`, -7.596004686931817`, 1}

M9 := {139.76720536496666`, 238.5002698914323`, 1}

M10 := {-732371.1205364267`, 0.`, 1}
```

2013-05-13-nb.pdf

```
M11 := {-1921.6275421056544`, -93683.48787269817`, 1}
M12 := {-79.34247922520326`, 40.673140482795155`, 1}
M13 := {-5.376605303659266`, -0.00007608114126702285`, 1}
M14 := {-3.6988482617096423`, -1.1566483617190364`*^-8, 1}
M15 := {-2.3290553665460507`, 2.146684910110426`, 1}
M16 := {-1.295324553091914`, 0.000026877720962941166`, 1}
M17 := {-0.6596455644224091`, 1.3206912754310147`, 1}
M18 := {-0.40556398529447424`, 1.1971598622027408`, 1}
M19 := {-0.07381717919302891`, -0.002912795372660838`, 1}
M20 := {0.02743171461076124`, -23.178607790321074`, 1}
M21 := {0.06320642318203891`, 0.9939930618404149`, 1}
M22 := {0.1311571589284251`, -0.5703245862452914`, 1}
M23 := {0.2411734479114258`, -5.740363617722843`, 1}
M24 := {1.9411591827455632`, -0.12528236207403975`, 1}
M25 := {4.045702005879118`, -2.0299168273264825`, 1}
M26 := {5.698662609732382`, -3.055281389233865`, 1}
M27 := {5329.44442871325`, -2.2020144862990997`*^10, 1}
```

Four groups of three sets of three parallel lines (not completely):

group 1: set1: koll[O1,O5,O6] // koll[O12,O13,O20] ...
set2: koll[O2,O3,O8] ...
set3: koll[O4,O7,O9] ...

group 2: set1: koll[O4,O20,O21] // koll[O2,O17,O23] ...
set2: koll[O7,O13,O14] // koll[O3,O19,O26] ...
set3: koll[O1,O18,O24] // koll[O8,O24,...] ...

group3: set1: koll[O2,O13,O24] // koll[O4,O16,O25] ...
set2: koll[O5,O15,O21] // koll[O7,O17,O18] // koll[O3,O12,O22]
set3: koll[O8,O20,...] // koll[O9,O26,...] ...

group4: set1: koll[O21,O24,O26] // koll[O6,O7,O8] ...
set2: koll[O1,O3,O4] // koll[O2,O22,...] ...
set3: koll[O2,O5,O9] // koll[O18,O19,O20] ...

Eckart Schmidt
http://eckartschmidt.de

2013-05-13-nb.pdf

Message: #59
Date: 17/6/2013 4:27:49
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven
(May 12, 2013)
Dear Eckart,

I am not quite surprised.
I already had calculated in a triangle the locus of possible centers of cardiods that were tangent to the sides of the triangle.
Two loci came out, one of the 11th degree and one of the 13th degree!
Pretty complicated. But in Mathematica we have the possibility to draw these functions.
When I drew them I saw overlapping structures.
I also saw several lines under angles of 60 degrees but also complicated curved lines.
So the functions probably could have been factorized in lines and another part.
However even Mathematica could not deal with these huge equations.
That's why I am not quite sure yet if your 27 solutions are the only solutions.
But maybe you have good reasoning for your 27 centers.
Let me know please.
These 27 points can be divided in 9 points per quadrigon?

Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #60
Date: 17/6/2013 4:34:30
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP  by  Chris van Tienhoven
(May 23, 2013)
Dear friends,
Thank you for the beautiful information about the amazing
QL-Cardioids.
It is a pleasure developing it together.
Although most of it comes from Eckart.
I am  trying right now to find the equation of a cardiod in
barycentrics.
I already managed to find an equation for a very specific case
in reasonable format (4th degree).
Eckart: can you give me more information about the 27 lines?
How you found them? Do you have CT-coefficients? If so, could
you send them to me in a Mathematica-notebook-file (even when
they are huge)?
Tomorrow morning I leave for 2 days then I will be back for half
a day and leave again for 5 days.
So I can’t react  during these days.
As soon as I find time I will try to make a picture of the
Cardioids.
Best regards,
Chris
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

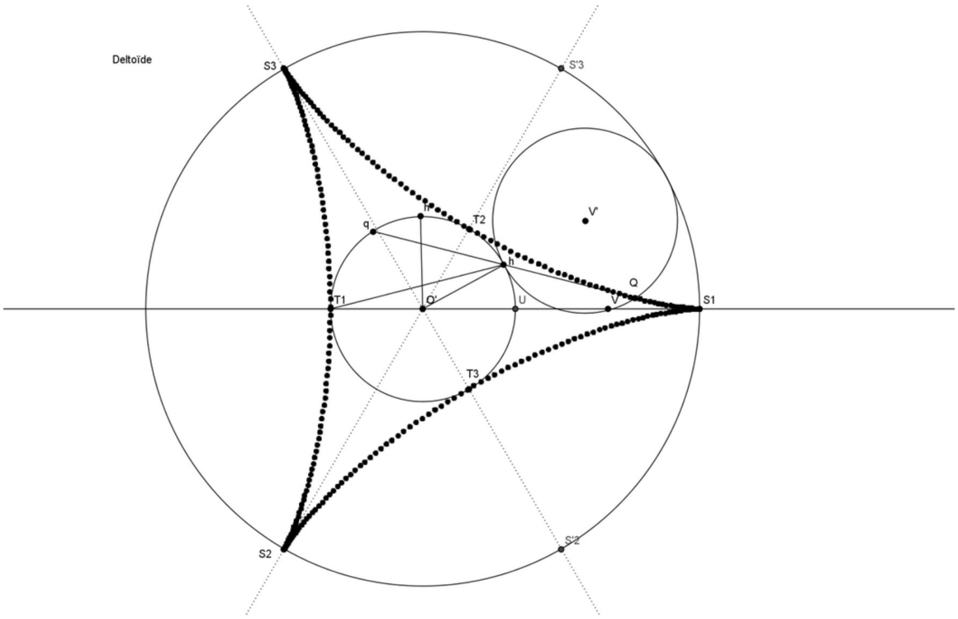
Message: #61
Date: 17/6/2013 4:35:16
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP   by Bernard Keizer
(May 21, 2013)
Dear friends,

I had a lot of problems in the last days, technical (I change my computer in the Dordogne and had great difficulties to get back all my files) as well as familal ...
It seems, you have worked hard during this time!
I'm still not able to read Eckart's attached files ...
I was intrigued by the barycentric coordinates of the cusp, I have found an explanation (see attached file).
I have tried to put in the same file all I know about deltoïd (to answer Eckart's question) and cardioïd.
I'm now totally convinced with the 27 cardioïds of Eckart, it's simply great ! The mistake was that a line to a cardioïd has not 2, but 3 possible directions (m° 3pi).
I'm really impatient now to see some drawings and I hope you will be able to show the possible cardioïds.
I still have a question about what Morley and Bogomolny mean by the unique cardioïd tangent to the 4 directed lines, the direction being given by the deltoïd; may be, it will be obvious with your calculations and drawings ...

Best regards
Bernard

Deltoïd



qQ is one tangent to the deltoïd : T1q = -2 T1h (this goes for T2 and T3) : h is the primary point, q the secondary point and Q the symmetric of q in h is the contact point ; the tangent is the symmetric of T1h in the perpendicular to the axis in h and the angle of the tangent with the axis is $\pi - \theta$.

If θ = OT1h, UOh = T1hq = 2θ, Ohq = Oqh = 3θ and T1Oq = 4θ.

The distance from O to the tangent is r cos3θ.

If we consider 4 lines Li tangent to the deltoid, the QL formed by the 4 primary points hi is cyclic by construction ; we have hiOhj = - 2(Di,Dj) and hihkhj = hihlhj = - (Di,Dj).

To find the center O, we have to inscribe this QL in the 4 lines ; each triangle hjhkhl is inversely similar to the reference triangle Ti built by the lines Dj, Dk and Dl or directly similar to its medial triangle. For each triangle Ti, all the triangles hjhkhl are similar, the center of similarity being the center of the circumscribed circle to the triangle Ti and the locus of the centers of the circumscribed circles to the triangles hjhkhl is the perpendicular bisector of the segment joining the center of similarity and its isogonal conjugate (the orthocenter of the triangle).

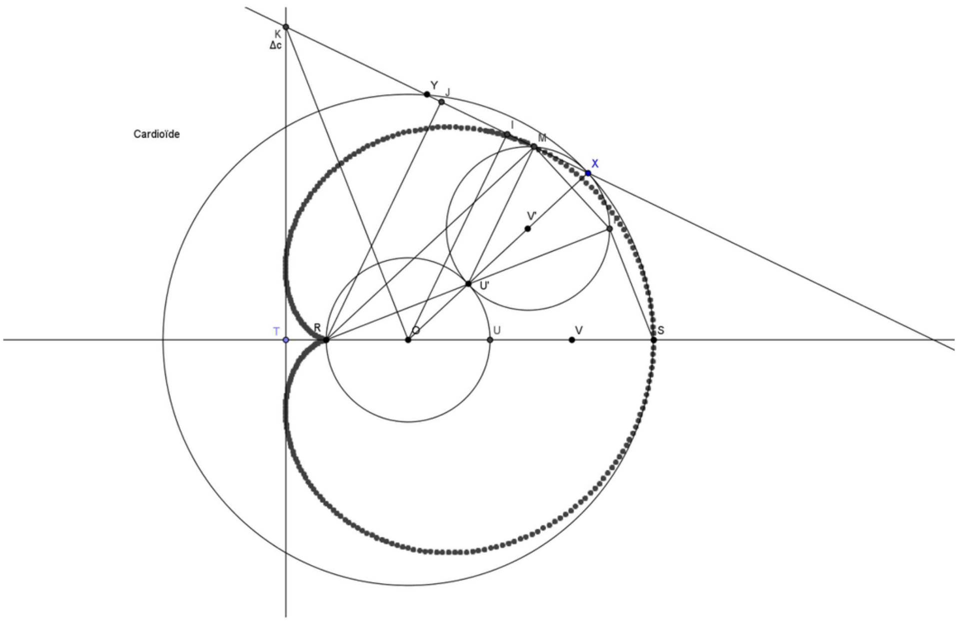
The center of the inscribed circle in the deltoid is therefore on the 4 perpendicular bisectors of the Euler segments ; that's the Kantor-Hervey point (theorem of Hervey) and the inscribed circle of the deltoid is in fact the Hervey circle …

All that is well known, we have 4 tangents Li and the symmetric of the axis of the parabola tangent to the 4 lines in the Newton Line (asymptote of the van Rees cubic) is a 5^{th} tangent to the deltoid.

The important thing here is to note that there is only but one way to inscribe a cyclic QL of a given shape on 4 lines.

Answer to Chris-Eckart.docx

Cardioïd



XY is a tangent to the cardioid, X is the primary point, Y, the symmetric of S in OX is the secondary point and M, at the third of XY next to X, is the contact point. O is the center of the cardioid, R its cusp and S its apse ; the angle between the tangent and the axis is $\pi/2 + 3\theta$.

If the barycentric coordinates of the center are (u,v,w), the barycentric coordinates of the cusp are ($u^3b^2c^2$,$v^3a^2c^2$,$w^3u^2v^2$) ; if we deal with the trilinear coordinates used by Clark Kimberling for the triangle, the coordinates of the center are (u/a, v/b, w/c) and those of the cusp (u^3/a^3, v^3/b^3, w^3/c^3).

If TKO = θ, SRU' = θ, SOX = 2θ and SOI = SRJ = 3θ.

OX = 3r and the distance from O to the tangent is OI = 3r cosθ ; RU' = 2r cosθ, RS = 4r, RM = 4r $\cos^2\theta$ and the distance from R to the tangent is RJ = 4r $\cos^3\theta$, which explains the calculation …

If we consider 4 lines Li tangent to the cardioid, the QL formed by the 4 primary points Xi is always cyclic, but we have this time XiOXj = 2/3 (Di,Dj) and hihkhj = hihlhj = 1/3 (Di,Dj) .

The question is now to determine the number of QL from different shapes to inscribe in the 4 lines : it seems in fact that there are 27 possible QL (and again one solution corresponding to one QL). Each angle is modulo π/3 and we have to choose independently 3 of the 4 angles (for example (Oh1,Oh2), (Oh2,Oh3) and (Oh3,Oh4)).

For one given triangle, the locus of the centers of the inscribed cardioids is one set of 3*3 lines (the mean direction of the 3 sides of the triangle modulo π/3). The locus of the cusps is one set of 3*3 cubics. For the 4 lines, the centers are at the intersection of 4 sets (considering the intersection of 2 sets, the maximum is 9*9 = 81). There are in fact 27 points, 3 on each line of each set and each on 4 lines (one of each set) and all the 27 points belong to Eckart's cubic !

Answer to Chris-Eckart.docx

← Previous Next → ↔ Message Index ↑ Subjects

Message: #62
Date: 17/6/2013 4:36:10
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

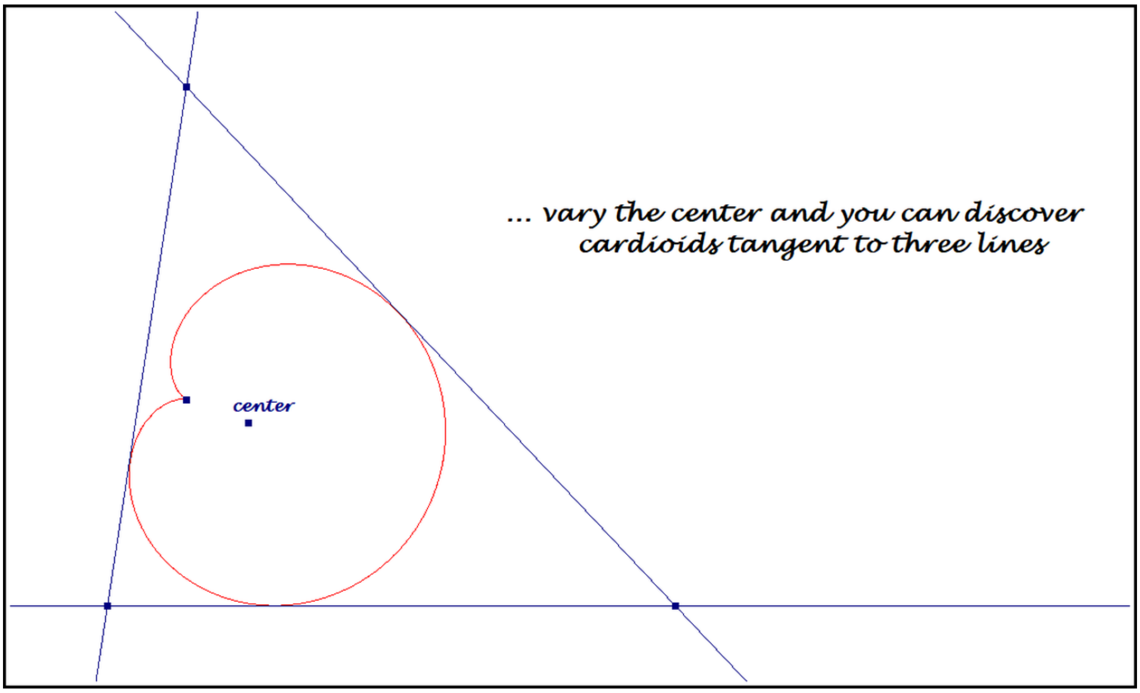
```
COPY FROM FORMER GOOGLE GROUP  by  Bernard Keizer
(May 24, 2013)
Dear Eckart,
I would like to share your enthousiasm and I hoped to see at
last those figures I have dreamed of so long!
But unfortunately, as I already told, I'm not able to open your
files (I suppose you use a program, for which I have no licence
...).
Could you send it again in another file (I use Geogebra) or
Adobe Reader?
By the way, where have you been teaching in Germany? (I spent 4
years in Bonn between 1976 and 1980 in the french embassy ...).
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

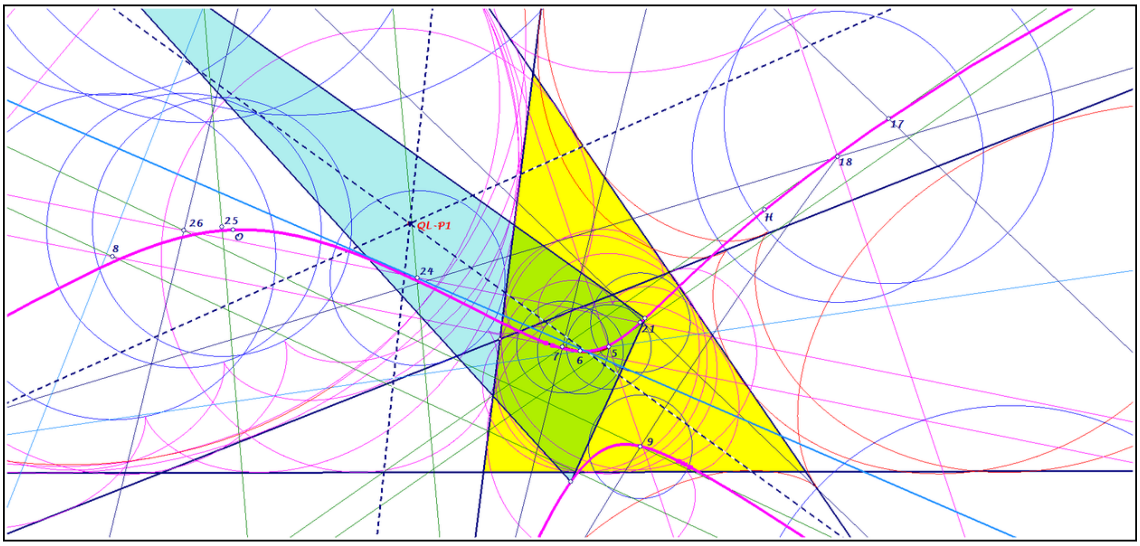
Message: #63
Date: 17/6/2013 4:36:44
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(May 24, 2013)
Dear friends,
thanks to Bernard for the description of the deltoid.
I also do not understand Morley´s remark wrt directed lines.
For me there is up to now no sign for a special cardioid.
There are new properties for my cubic of the centers:
The asymptotes cut the cubic in collinear points on a parallel of the Newton Line.
I have proved this for several numeral examples. This is a property of the McCay Cubic
(see B. Gibert´s catalogue). Maybe it holds for all pK60+, I don´t know.
I am positive about: The cubic is a McCay Cubic.

The McCay Cubic is an isogonal pivotal isocubic with pivot in the circumcenter of the reference triangle.
The problem is the reference triangle for my cubic: The only property I know is, that the centroid is the Miquel Point.
... What about this reference triangle? ...
You can find all these properties in a Cabri file in the attachment.
It is a drawing to my calculated example, not constructed, the calculated points drawn by hand!!!
So it is somewhat unexact and confuse, only a view for reflecting.
But it contains about 20 cardioids with their centers, the 3 sets of parallels for the reference triangle,
the asymptotes and the cubic, drawn as McCay Cubic of a tested reference triangle.
... just for fun: There is a further Cabri file for discovering cardioids tangent to 3 lines.
Best regards
Eckart



cusp-fig.png



cardioids-fig.png

← Previous Next → ↩ Message Index ↑ Subjects

Message: #64
Date: 17/6/2013 4:38:03
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP  by Eckart Schmidt
(May 25, 2013)
Dear friends,
Chris asked for equations to calculate the centers and cusps of
cardioids tangent to 4 lines.
There is a Mathematica file in the attachment.
In addition to the speculation, that my cubic for the centers is
a McCay Cubic:
The asymptotes of the cubic are parallel to the axes of the
deltoid,
that is the envelope of the Simson lines of the reference
triangle of the McCay cubic.
Best regards Eckart
```

Basic equations for the centers of cardioids tangent to 4 lines

For a cardioid with center (u:v:w) and bitangent (e,f,g) a line (λ,μ,ν) is tangent, if the following Term is zero:

```
T[{λ_, μ_, ν_}] :=
 (-f^4 Sa^2 u^3 λ^3 + 4 f^3 g Sa^2 u^3 λ^3 - 6 f^2 g^2 Sa^2 u^3 λ^3 + 4 f g^3 Sa^2 u^3 λ^3 - g^4 Sa^2 u^3 λ^3 + e^2 f^2 Sa Sb u^3 λ^3 -
   2 e^2 f g Sa Sb u^3 λ^3 + 4 e f^2 g Sa Sb u^3 λ^3 + e^2 g^2 Sa Sb u^3 λ^3 - 8 e f g^2 Sa Sb u^3 λ^3 -
   2 f^2 g^2 Sa Sb u^3 λ^3 + 4 e g^3 Sa Sb u^3 λ^3 + 4 f g^3 Sa Sb u^3 λ^3 - 2 g^4 Sa Sb u^3 λ^3 - 3 e^2 g^2 Sb^2 u^3 λ^3 +
   4 e g^3 Sb^2 u^3 λ^3 - g^4 Sb^2 u^3 λ^3 + e^2 f^2 Sa Sc u^3 λ^3 + 4 e f^3 Sa Sc u^3 λ^3 - 2 f^4 Sa Sc u^3 λ^3 -
   2 e^2 f g Sa Sc u^3 λ^3 - 8 e f^2 g Sa Sc u^3 λ^3 + 4 f^3 g Sa Sc u^3 λ^3 + e^2 g^2 Sa Sc u^3 λ^3 +
   4 e f g^2 Sa Sc u^3 λ^3 - 2 f^2 g^2 Sa Sc u^3 λ^3 + e^2 f^2 Sb Sc u^3 λ^3 - 8 e^2 f g Sb Sc u^3 λ^3 +
   4 e f^2 g Sb Sc u^3 λ^3 + e^2 g^2 Sb Sc u^3 λ^3 + 4 e f g^2 Sb Sc u^3 λ^3 - 2 f^2 g^2 Sb Sc u^3 λ^3 -
   3 e^2 f^2 Sc^2 u^3 λ^3 + 4 e f^3 Sc^2 u^3 λ^3 - f^4 Sc^2 u^3 λ^3 + 6 e f^3 Sa Sb u^2 v λ^3 - 12 e f^2 g Sa Sb u^2 v λ^3 +
   6 e f g^2 Sa Sb u^2 v λ^3 - 6 e^2 f g Sb^2 u^2 v λ^3 + 6 e f g^2 Sb^2 u^2 v λ^3 + 6 e f^3 Sa Sc u^2 v λ^3 -
   12 e f^2 g Sa Sc u^2 v λ^3 + 6 e f g^2 Sa Sc u^2 v λ^3 - 6 e^2 f^2 Sb Sc u^2 v λ^3 + 6 e f^3 Sb Sc u^2 v λ^3 -
   6 e^2 f g Sb Sc u^2 v λ^3 + 6 e f g^2 Sb Sc u^2 v λ^3 - 6 e^2 f^2 Sc^2 u^2 v λ^3 + 6 e f^3 Sc^2 u^2 v λ^3 +
   3 f^4 Sa Sb u v^2 λ^3 - 6 f^3 g Sa Sb u v^2 λ^3 + 3 f^2 g^2 Sa Sb u v^2 λ^3 - 3 e^2 f^2 Sb^2 u v^2 λ^3 +
   3 f^2 g^2 Sb^2 u v^2 λ^3 + 3 f^4 Sa Sc u v^2 λ^3 - 6 f^3 g Sa Sc u v^2 λ^3 + 3 f^2 g^2 Sa Sc u v^2 λ^3 -
   6 e^2 f^2 Sb Sc u v^2 λ^3 + 3 f^4 Sb Sc u v^2 λ^3 + 3 f^2 g^2 Sb Sc u v^2 λ^3 - 3 e^2 f^2 Sc^2 u v^2 λ^3 +
   3 f^4 Sc^2 u v^2 λ^3 - 2 e f^3 Sb^2 v^3 λ^3 + 2 f^3 g Sb^2 v^3 λ^3 - 4 e f^3 Sb Sc v^3 λ^3 + 2 f^4 Sb Sc v^3 λ^3 +
   2 f^3 g Sb Sc v^3 λ^3 - 2 e f^3 Sc^2 v^3 λ^3 + 2 f^4 Sc^2 v^3 λ^3 + 6 e f^2 g Sa Sb u^2 w λ^3 - 12 e f g^2 Sa Sb u^2 w λ^3 +
   6 e g^3 Sa Sb u^2 w λ^3 - 6 e^2 g^2 Sb^2 u^2 w λ^3 + 6 e g^3 Sb^2 u^2 w λ^3 + 6 e f^2 g Sa Sc u^2 w λ^3 -
   12 e f g^2 Sa Sc u^2 w λ^3 + 6 e g^3 Sa Sc u^2 w λ^3 - 6 e^2 f g Sb Sc u^2 w λ^3 + 6 e f^2 g Sb Sc u^2 w λ^3 -
   6 e^2 g^2 Sb Sc u^2 w λ^3 + 6 e g^3 Sb Sc u^2 w λ^3 - 6 e^2 f g Sc^2 u^2 w λ^3 + 6 e f^2 g Sc^2 u^2 w λ^3 +
   6 f^3 g Sa Sb u v w λ^3 - 12 f^2 g^2 Sa Sb u v w λ^3 + 6 f g^3 Sa Sb u v w λ^3 - 6 e^2 f g Sb^2 u v w λ^3 +
   6 f g^3 Sb^2 u v w λ^3 + 6 f^3 g Sa Sc u v w λ^3 - 12 f^2 g^2 Sa Sc u v w λ^3 + 6 f g^3 Sa Sc u v w λ^3 -
   12 e^2 f g Sb Sc u v w λ^3 + 6 f^3 g Sb Sc u v w λ^3 + 6 f g^3 Sb Sc u v w λ^3 - 6 e^2 f g Sc^2 u v w λ^3 +
   6 f^3 g Sc^2 u v w λ^3 - 6 e f^2 g Sb^2 v^2 w λ^3 + 6 f^2 g^2 Sb^2 v^2 w λ^3 - 12 e f^2 g Sb Sc v^2 w λ^3 +
   6 f^3 g Sb Sc v^2 w λ^3 + 6 f^2 g^2 Sb Sc v^2 w λ^3 - 6 e f^2 g Sc^2 v^2 w λ^3 + 6 f^3 g Sc^2 v^2 w λ^3 +
   3 f^2 g^2 Sa Sb u w^2 λ^3 - 6 f g^3 Sa Sb u w^2 λ^3 + 3 g^4 Sa Sb u w^2 λ^3 - 3 e^2 g^2 Sb^2 u w^2 λ^3 +
   3 g^4 Sb^2 u w^2 λ^3 + 3 f^2 g^2 Sa Sc u w^2 λ^3 - 6 f g^3 Sa Sc u w^2 λ^3 + 3 g^4 Sa Sc u w^2 λ^3 -
   6 e^2 g^2 Sb Sc u w^2 λ^3 + 3 f^2 g^2 Sb Sc u w^2 λ^3 + 3 g^4 Sb Sc u w^2 λ^3 - 3 e^2 g^2 Sc^2 u w^2 λ^3 +
   3 f^2 g^2 Sc^2 u w^2 λ^3 - 6 e f g^2 Sb^2 v w^2 λ^3 + 6 f g^3 Sb^2 v w^2 λ^3 - 12 e f g^2 Sb Sc v w^2 λ^3 +
   6 f^2 g^2 Sb Sc v w^2 λ^3 + 6 f g^3 Sb Sc v w^2 λ^3 - 6 e f g^2 Sc^2 v w^2 λ^3 + 6 f^2 g^2 Sc^2 v w^2 λ^3 -
   2 e g^3 Sb^2 w^3 λ^3 + 2 g^4 Sb^2 w^3 λ^3 - 4 e g^3 Sb Sc w^3 λ^3 + 2 f g^3 Sb Sc w^3 λ^3 + 2 g^4 Sb Sc w^3 λ^3 -
   2 e g^3 Sc^2 w^3 λ^3 + 2 f g^3 Sc^2 w^3 λ^3 - 2 e^3 f Sa Sb u^3 λ^2 μ + 2 e^3 g Sa Sb u^3 λ^2 μ - 2 e^3 f Sa Sc u^3 λ^2 μ -
   6 e^2 f^2 Sa Sc u^3 λ^2 μ + 2 e^3 g Sa Sc u^3 λ^2 μ + 12 e^2 f g Sa Sc u^3 λ^2 μ - 6 e^2 g^2 Sa Sc u^3 λ^2 μ -
   2 e^3 f Sb Sc u^3 λ^2 μ + 8 e^3 g Sb Sc u^3 λ^2 μ - 6 e^2 g^2 Sb Sc u^3 λ^2 μ + 6 e^3 f Sc^2 u^3 λ^2 μ -
   6 e^2 f^2 Sc^2 u^3 λ^2 μ - 3 f^4 Sa^2 u^2 v λ^2 μ + 12 f^3 g Sa^2 u^2 v λ^2 μ - 18 f^2 g^2 Sa^2 u^2 v λ^2 μ +
   12 f g^3 Sa^2 u^2 v λ^2 μ - 3 g^4 Sa^2 u^2 v λ^2 μ - 9 e^2 f^2 Sa Sb u^2 v λ^2 μ + 12 e^2 f g Sa Sb u^2 v λ^2 μ +
   12 e f^2 g Sa Sb u^2 v λ^2 μ - 3 e^2 g^2 Sa Sb u^2 v λ^2 μ - 24 e f g^2 Sa Sb u^2 v λ^2 μ - 6 f^2 g^2 Sa Sb u^2 v λ^2 μ +
   12 e g^3 Sa Sb u^2 v λ^2 μ + 12 f g^3 Sa Sb u^2 v λ^2 μ - 6 g^4 Sa Sb u^2 v λ^2 μ + 6 e^3 g Sb^2 u^2 v λ^2 μ -
   15 e^2 g^2 Sb^2 u^2 v λ^2 μ + 12 e g^3 Sb^2 u^2 v λ^2 μ - 3 g^4 Sb^2 u^2 v λ^2 μ - 9 e^2 f^2 Sa Sc u^2 v λ^2 μ -
   6 f^4 Sa Sc u^2 v λ^2 μ + 12 e^2 f g Sa Sc u^2 v λ^2 μ + 12 f^3 g Sa Sc u^2 v λ^2 μ - 3 e^2 g^2 Sa Sc u^2 v λ^2 μ -
   6 f^2 g^2 Sa Sc u^2 v λ^2 μ + 12 e^3 f Sb Sc u^2 v λ^2 μ - 9 e^2 f^2 Sb Sc u^2 v λ^2 μ + 6 e^3 g Sb Sc u^2 v λ^2 μ -
   12 e^2 f g Sb Sc u^2 v λ^2 μ + 12 e f^2 g Sb Sc u^2 v λ^2 μ - 3 e^2 g^2 Sb Sc u^2 v λ^2 μ - 6 f^2 g^2 Sb Sc u^2 v λ^2 μ +
   12 e^3 f Sc^2 u^2 v λ^2 μ - 9 e^2 f^2 Sc^2 u^2 v λ^2 μ - 3 f^4 Sc^2 u^2 v λ^2 μ - 6 e f^2 g Sa Sb u v^2 λ^2 μ +
   6 e f g^2 Sa Sb u v^2 λ^2 μ + 6 e^3 f Sb^2 u v^2 λ^2 μ - 12 e^2 f g Sb^2 u v^2 λ^2 μ + 6 e f g^2 Sb^2 u v^2 λ^2 μ -
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$6 f^{4} Sa\, Sc\, u\, v^{2} \lambda^{2} \mu - 6 e f^{2} g\, Sa\, Sc\, u\, v^{2} \lambda^{2} \mu + 12 f^{3} g\, Sa\, Sc\, u\, v^{2} \lambda^{2} \mu + 6 e f g^{2} Sa\, Sc\, u\, v^{2} \lambda^{2} \mu -$
$6 f^{2} g^{2} Sa\, Sc\, u\, v^{2} \lambda^{2} \mu + 12 e^{3} f\, Sb\, Sc\, u\, v^{2} \lambda^{2} \mu - 12 e^{2} f g\, Sb\, Sc\, u\, v^{2} \lambda^{2} \mu + 6 e f g^{2} Sb\, Sc\, u\, v^{2} \lambda^{2} \mu -$
$6 f^{2} g^{2} Sb\, Sc\, u\, v^{2} \lambda^{2} \mu + 6 e^{3} f\, Sc^{2} u\, v^{2} \lambda^{2} \mu - 6 f^{4} Sc^{2} u\, v^{2} \lambda^{2} \mu + f^{4} Sa\, Sb\, v^{3} \lambda^{2} \mu -$
$4 f^{3} g\, Sa\, Sb\, v^{3} \lambda^{2} \mu + 3 f^{2} g^{2} Sa\, Sb\, v^{3} \lambda^{2} \mu + 3 e^{2} f^{2} Sb^{2} v^{3} \lambda^{2} \mu - 6 e f^{2} g\, Sb^{2} v^{3} \lambda^{2} \mu +$
$3 f^{2} g^{2} Sb^{2} v^{3} \lambda^{2} \mu + f^{4} Sa\, Sc\, v^{3} \lambda^{2} \mu - 4 f^{3} g\, Sa\, Sc\, v^{3} \lambda^{2} \mu + 3 f^{2} g^{2} Sa\, Sc\, v^{3} \lambda^{2} \mu +$
$6 e^{2} f^{2} Sb\, Sc\, v^{3} \lambda^{2} \mu + f^{4} Sb\, Sc\, v^{3} \lambda^{2} \mu - 6 e f^{2} g\, Sb\, Sc\, v^{3} \lambda^{2} \mu - 4 f^{3} g\, Sb\, Sc\, v^{3} \lambda^{2} \mu +$
$3 f^{2} g^{2} Sb\, Sc\, v^{3} \lambda^{2} \mu + 3 e^{2} f^{2} Sc^{2} v^{3} \lambda^{2} \mu - 3 f^{4} Sc^{2} v^{3} \lambda^{2} \mu - 6 e^{2} f g\, Sa\, Sb\, u^{2} w \lambda^{2} \mu +$
$6 e^{2} g^{2} Sa\, Sb\, u^{2} w \lambda^{2} \mu - 6 e^{2} f g\, Sa\, Sc\, u^{2} w \lambda^{2} \mu - 12 e f^{2} g\, Sa\, Sc\, u^{2} w \lambda^{2} \mu + 6 e^{2} g^{2} Sa\, Sc\, u^{2} w \lambda^{2} \mu +$
$24 e f g^{2} Sa\, Sc\, u^{2} w \lambda^{2} \mu - 12 e g^{3} Sa\, Sc\, u^{2} w \lambda^{2} \mu + 6 e^{3} g\, Sb\, Sc\, u^{2} w \lambda^{2} \mu - 6 e^{2} f g\, Sb\, Sc\, u^{2} w \lambda^{2} \mu +$
$12 e^{2} g^{2} Sb\, Sc\, u^{2} w \lambda^{2} \mu - 12 e g^{3} Sb\, Sc\, u^{2} w \lambda^{2} \mu + 6 e^{3} g\, Sc^{2} u^{2} w \lambda^{2} \mu + 6 e^{2} f g\, Sc^{2} u^{2} w \lambda^{2} \mu -$
$12 e f^{2} g\, Sc^{2} u^{2} w \lambda^{2} \mu - 6 e f^{2} g\, Sa\, Sb\, u\, v\, w \lambda^{2} \mu + 6 e g^{3} Sa\, Sb\, u\, v\, w \lambda^{2} \mu + 6 e^{3} g\, Sb^{2} u\, v\, w \lambda^{2} \mu -$
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$24 f^{2} g^{2} Sa\, Sc\, u\, v\, w \lambda^{2} \mu + 6 e g^{3} Sa\, Sc\, u\, v\, w \lambda^{2} \mu - 12 f g^{3} Sa\, Sc\, u\, v\, w \lambda^{2} \mu + 12 e^{3} g\, Sb\, Sc\, u\, v\, w \lambda^{2} \mu +$
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$6 e f^{2} g\, Sc^{2} u\, v\, w \lambda^{2} \mu - 12 f^{3} g\, Sc^{2} u\, v\, w \lambda^{2} \mu - 6 f^{2} g^{2} Sa\, Sb\, v^{2} w \lambda^{2} \mu + 6 f g^{3} Sa\, Sb\, v^{2} w \lambda^{2} \mu +$
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$6 f g^{3} Sa\, Sc\, v^{2} w \lambda^{2} \mu + 12 e^{2} f g\, Sb\, Sc\, v^{2} w \lambda^{2} \mu + 6 e f^{2} g\, Sb\, Sc\, v^{2} w \lambda^{2} \mu - 12 e f g^{2} Sb\, Sc\, v^{2} w \lambda^{2} \mu -$
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$6 f^{2} g^{2} Sa\, Sc\, u\, w^{2} \lambda^{2} \mu + 6 e g^{3} Sa\, Sc\, u\, w^{2} \lambda^{2} \mu + 12 f g^{3} Sa\, Sc\, u\, w^{2} \lambda^{2} \mu - 6 g^{4} Sa\, Sc\, u\, w^{2} \lambda^{2} \mu +$
$12 e^{2} g^{2} Sb\, Sc\, u\, w^{2} \lambda^{2} \mu - 6 e f g^{2} Sb\, Sc\, u\, w^{2} \lambda^{2} \mu - 6 g^{4} Sb\, Sc\, u\, w^{2} \lambda^{2} \mu + 12 e^{2} g^{2} Sc^{2} u\, w^{2} \lambda^{2} \mu -$
$6 e f g^{2} Sc^{2} u\, w^{2} \lambda^{2} \mu - 6 f^{2} g^{2} Sc^{2} u\, w^{2} \lambda^{2} \mu - 3 f^{2} g^{2} Sa\, Sb\, v\, w^{2} \lambda^{2} \mu + 3 g^{4} Sa\, Sb\, v\, w^{2} \lambda^{2} \mu +$
$3 e^{2} g^{2} Sb^{2} v\, w^{2} \lambda^{2} \mu - 6 e g^{3} Sb^{2} v\, w^{2} \lambda^{2} \mu + 3 g^{4} Sb^{2} v\, w^{2} \lambda^{2} \mu - 3 f^{2} g^{2} Sa\, Sc\, v\, w^{2} \lambda^{2} \mu +$
$3 g^{4} Sa\, Sc\, v\, w^{2} \lambda^{2} \mu + 6 e^{2} g^{2} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu + 12 e f g^{2} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu - 3 f^{2} g^{2} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu -$
$6 e g^{3} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu - 12 f g^{3} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu + 3 g^{4} Sb\, Sc\, v\, w^{2} \lambda^{2} \mu + 3 e^{2} g^{2} Sc^{2} v\, w^{2} \lambda^{2} \mu +$
$12 e f g^{2} Sc^{2} v\, w^{2} \lambda^{2} \mu - 15 f^{2} g^{2} Sc^{2} v\, w^{2} \lambda^{2} \mu - 2 f g^{3} Sa\, Sb\, w^{3} \lambda^{2} \mu + 2 g^{4} Sa\, Sb\, w^{3} \lambda^{2} \mu -$
$2 f g^{3} Sa\, Sc\, w^{3} \lambda^{2} \mu + 2 g^{4} Sa\, Sc\, w^{3} \lambda^{2} \mu + 6 e g^{3} Sb\, Sc\, w^{3} \lambda^{2} \mu - 2 f g^{3} Sb\, Sc\, w^{3} \lambda^{2} \mu -$
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$3 e^{2} g^{2} Sa^{2} u^{3} \lambda \mu^{2} + e^{4} Sa\, Sb\, u^{3} \lambda \mu^{2} - 4 e^{3} g\, Sa\, Sb\, u^{3} \lambda \mu^{2} + 3 e^{2} g^{2} Sa\, Sb\, u^{3} \lambda \mu^{2} + e^{4} Sa\, Sc\, u^{3} \lambda \mu^{2} +$
$6 e^{2} f^{2} Sa\, Sc\, u^{3} \lambda \mu^{2} - 4 e^{3} g\, Sa\, Sc\, u^{3} \lambda \mu^{2} - 6 e^{2} f g\, Sa\, Sc\, u^{3} \lambda \mu^{2} + 3 e^{2} g^{2} Sa\, Sc\, u^{3} \lambda \mu^{2} +$
$e^{4} Sb\, Sc\, u^{3} \lambda \mu^{2} - 4 e^{3} g\, Sb\, Sc\, u^{3} \lambda \mu^{2} + 3 e^{2} g^{2} Sb\, Sc\, u^{3} \lambda \mu^{2} - 3 e^{4} Sc^{2} u^{3} \lambda \mu^{2} + 3 e^{2} f^{2} Sc^{2} u^{3} \lambda \mu^{2} +$
$6 e f^{3} Sa^{2} u^{2} v \lambda \mu^{2} - 12 e f^{2} g\, Sa^{2} u^{2} v \lambda \mu^{2} + 6 e f g^{2} Sa^{2} u^{2} v \lambda \mu^{2} - 6 e^{2} f g\, Sa\, Sb\, u^{2} v \lambda \mu^{2} +$
$6 e f g^{2} Sa\, Sb\, u^{2} v \lambda \mu^{2} + 12 e f^{3} Sa\, Sc\, u^{2} v \lambda \mu^{2} - 12 e f^{2} g\, Sa\, Sc\, u^{2} v \lambda \mu^{2} - 6 e^{2} g^{2} Sa\, Sc\, u^{2} v \lambda \mu^{2} +$
$6 e f g^{2} Sa\, Sc\, u^{2} v \lambda \mu^{2} - 6 e^{4} Sb\, Sc\, u^{2} v \lambda \mu^{2} + 12 e^{3} g\, Sb\, Sc\, u^{2} v \lambda \mu^{2} - 6 e^{2} f g\, Sb\, Sc\, u^{2} v \lambda \mu^{2} -$
$6 e^{2} g^{2} Sb\, Sc\, u^{2} v \lambda \mu^{2} + 6 e f g^{2} Sb\, Sc\, u^{2} v \lambda \mu^{2} - 6 e^{4} Sc^{2} u^{2} v \lambda \mu^{2} + 6 e f^{3} Sc^{2} u^{2} v \lambda \mu^{2} +$
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$9 e^{2} f^{2} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} + 12 e^{2} f g\, Sa\, Sb\, u\, v^{2} \lambda \mu^{2} + 12 e f^{2} g\, Sa\, Sb\, u\, v^{2} \lambda \mu^{2} - 6 e^{2} g^{2} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} -$
$24 e f g^{2} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} - 3 f^{2} g^{2} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} + 12 e g^{3} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} + 12 f g^{3} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} -$
$6 g^{4} Sa\, Sb\, u\, v^{2} \lambda \mu^{2} - 3 e^{4} Sb^{2} u\, v^{2} \lambda \mu^{2} + 12 e^{3} g\, Sb^{2} u\, v^{2} \lambda \mu^{2} - 18 e^{2} g^{2} Sb^{2} u\, v^{2} \lambda \mu^{2} +$
$12 e g^{3} Sb^{2} u\, v^{2} \lambda \mu^{2} - 3 g^{4} Sb^{2} u\, v^{2} \lambda \mu^{2} - 9 e^{2} f^{2} Sa\, Sc\, u\, v^{2} \lambda \mu^{2} + 12 e f^{3} Sa\, Sc\, u\, v^{2} \lambda \mu^{2} +$
$12 e^{2} f g\, Sa\, Sc\, u\, v^{2} \lambda \mu^{2} - 12 e f^{2} g\, Sa\, Sc\, u\, v^{2} \lambda \mu^{2} + 6 f^{3} g\, Sa\, Sc\, u\, v^{2} \lambda \mu^{2} - 6 e^{2} g^{2} Sa\, Sc\, u\, v^{2} \lambda \mu^{2} -$
$3 f^{2} g^{2} Sa\, Sc\, u\, v^{2} \lambda \mu^{2} - 6 e^{4} Sb\, Sc\, u\, v^{2} \lambda \mu^{2} - 9 e^{2} f^{2} Sb\, Sc\, u\, v^{2} \lambda \mu^{2} + 12 e^{3} g\, Sb\, Sc\, u\, v^{2} \lambda \mu^{2} +$
$12 e f^{2} g\, Sb\, Sc\, u\, v^{2} \lambda \mu^{2} - 6 e^{2} g^{2} Sb\, Sc\, u\, v^{2} \lambda \mu^{2} - 3 f^{2} g^{2} Sb\, Sc\, u\, v^{2} \lambda \mu^{2} - 3 e^{4} Sc^{2} u\, v^{2} \lambda \mu^{2} -$
$9 e^{2} f^{2} Sc^{2} u\, v^{2} \lambda \mu^{2} + 12 e f^{3} Sc^{2} u\, v^{2} \lambda \mu^{2} - 2 e f^{3} Sa\, Sb\, v^{3} \lambda \mu^{2} + 2 f^{3} g\, Sa\, Sb\, v^{3} \lambda \mu^{2} -$
$2 e f^{3} Sa\, Sc\, v^{3} \lambda \mu^{2} + 8 f^{3} g\, Sa\, Sc\, v^{3} \lambda \mu^{2} - 6 f^{2} g^{2} Sa\, Sc\, v^{3} \lambda \mu^{2} - 6 e^{2} f^{2} Sb\, Sc\, v^{3} \lambda \mu^{2} -$
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$6 e f^{2} g\, Sc^{2} u^{2} w \lambda \mu^{2} + 6 f^{3} g\, Sa^{2} u\, v\, w \lambda \mu^{2} - 12 f^{2} g^{2} Sa^{2} u\, v\, w \lambda \mu^{2} + 6 f g^{3} Sa^{2} u\, v\, w \lambda \mu^{2} -$
$6 e^{2} f g\, Sa\, Sb\, u\, v\, w \lambda \mu^{2} + 6 f g^{3} Sa\, Sb\, u\, v\, w \lambda \mu^{2} - 6 e^{2} f g\, Sa\, Sc\, u\, v\, w \lambda \mu^{2} + 12 e f^{2} g\, Sa\, Sc\, u\, v\, w \lambda \mu^{2} +$
$12 f^{3} g\, Sa\, Sc\, u\, v\, w \lambda \mu^{2} - 12 f^{2} g^{2} Sa\, Sc\, u\, v\, w \lambda \mu^{2} - 12 e g^{3} Sa\, Sc\, u\, v\, w \lambda \mu^{2} + 6 f g^{3} Sa\, Sc\, u\, v\, w \lambda \mu^{2} -$
$12 e^{3} g\, Sb\, Sc\, u\, v\, w \lambda \mu^{2} - 6 e^{2} f g\, Sb\, Sc\, u\, v\, w \lambda \mu^{2} + 24 e^{2} g^{2} Sb\, Sc\, u\, v\, w \lambda \mu^{2} - 12 e g^{3} Sb\, Sc\, u\, v\, w \lambda \mu^{2} +$
$6 f g^{3} Sb\, Sc\, u\, v\, w \lambda \mu^{2} - 12 e^{3} g\, Sc^{2} u\, v\, w \lambda \mu^{2} - 6 e^{2} f g\, Sc^{2} u\, v\, w \lambda \mu^{2} + 12 e f^{2} g\, Sc^{2} u\, v\, w \lambda \mu^{2} +$
$6 f^{3} g\, Sc^{2} u\, v\, w \lambda \mu^{2} - 6 e f^{2} g\, Sa\, Sb\, v^{2} w \lambda \mu^{2} + 6 f^{2} g^{2} Sa\, Sb\, v^{2} w \lambda \mu^{2} - 6 e f^{2} g\, Sa\, Sc\, v^{2} w \lambda \mu^{2} +$
$6 f^{3} g\, Sa\, Sc\, v^{2} w \lambda \mu^{2} + 12 f^{2} g^{2} Sa\, Sc\, v^{2} w \lambda \mu^{2} - 12 f g^{3} Sa\, Sc\, v^{2} w \lambda \mu^{2} - 12 e^{2} f g\, Sb\, Sc\, v^{2} w \lambda \mu^{2} -$

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$6 e f^2 g\, Sb\, Sc\, v^2 w \lambda \mu^2 + 24 e f g^2 Sb\, Sc\, v^2 w \lambda \mu^2 + 6 f^2 g^2 Sb\, Sc\, v^2 w \lambda \mu^2 - 12 f g^3 Sb\, Sc\, v^2 w \lambda \mu^2 -$
$12 e^2 f g\, Sc^2 v^2 w \lambda \mu^2 + 6 e f^2 g\, Sc^2 v^2 w \lambda \mu^2 + 6 f^3 g\, Sc^2 v^2 w \lambda \mu^2 + 3 f^2 g^2 Sa^2 u w^2 \lambda \mu^2 -$
$6 f g^3 Sa^2 u w^2 \lambda \mu^2 + 3 g^4 Sa^2 u w^2 \lambda \mu^2 - 3 e^2 g^2 Sa\, Sb\, u w^2 \lambda \mu^2 + 3 g^4 Sa\, Sb\, u w^2 \lambda \mu^2 -$
$3 e^2 g^2 Sa\, Sc\, u w^2 \lambda \mu^2 + 12 e f g^2 Sa\, Sc\, u w^2 \lambda \mu^2 + 6 f^2 g^2 Sa\, Sc\, u w^2 \lambda \mu^2 - 12 e g^3 Sa\, Sc\, u w^2 \lambda \mu^2 -$
$6 f g^3 Sa\, Sc\, u w^2 \lambda \mu^2 + 3 g^4 Sa\, Sc\, u w^2 \lambda \mu^2 - 3 e^2 g^2 Sb\, Sc\, u w^2 \lambda \mu^2 + 3 g^4 Sb\, Sc\, u w^2 \lambda \mu^2 -$
$15 e^2 g^2 Sc^2 u w^2 \lambda \mu^2 + 12 e f g^2 Sc^2 u w^2 \lambda \mu^2 + 3 f^2 g^2 Sc^2 u w^2 \lambda \mu^2 - 6 e f g^2 Sa\, Sb\, v w^2 \lambda \mu^2 +$
$6 f g^3 Sa\, Sb\, v w^2 \lambda \mu^2 - 6 e f g^2 Sa\, Sc\, v w^2 \lambda \mu^2 + 12 f^2 g^2 Sa\, Sc\, v w^2 \lambda \mu^2 - 6 g^4 Sa\, Sc\, v w^2 \lambda \mu^2 -$
$6 e^2 g^2 Sb\, Sc\, v w^2 \lambda \mu^2 - 6 e f g^2 Sb\, Sc\, v w^2 \lambda \mu^2 + 12 e g^3 Sb\, Sc\, v w^2 \lambda \mu^2 + 6 f g^3 Sb\, Sc\, v w^2 \lambda \mu^2 -$
$6 g^4 Sb\, Sc\, v w^2 \lambda \mu^2 - 6 e^2 g^2 Sc^2 v w^2 \lambda \mu^2 - 6 e f g^2 Sc^2 v w^2 \lambda \mu^2 + 12 f^2 g^2 Sc^2 v w^2 \lambda \mu^2 -$
$2 e g^3 Sa\, Sb\, w^3 \lambda \mu^2 + 2 g^4 Sa\, Sb\, w^3 \lambda \mu^2 - 2 e g^3 Sa\, Sc\, w^3 \lambda \mu^2 + 6 f g^3 Sa\, Sc\, w^3 \lambda \mu^2 -$
$4 g^4 Sa\, Sc\, w^3 \lambda \mu^2 - 2 e g^3 Sb\, Sc\, w^3 \lambda \mu^2 + 2 g^4 Sb\, Sc\, w^3 \lambda \mu^2 - 6 e g^3 Sc^2 w^3 \lambda \mu^2 + 6 f g^3 Sc^2 w^3 \lambda \mu^2 -$
$2 e^3 f\, Sa^2 u^3 \mu^3 + 2 e^3 g\, Sa^2 u^3 \mu^3 + 2 e^4 Sa\, Sc\, u^3 \mu^3 - 4 e^3 f\, Sa\, Sc\, u^3 \mu^3 + 2 e^3 g\, Sa\, Sc\, u^3 \mu^3 +$
$2 e^4 Sc^2 u^3 \mu^3 - 2 e^3 f\, Sc^2 u^3 \mu^3 - 3 e^2 f^2 Sa^2 u^2 v \mu^3 + 3 e^2 g^2 Sa^2 u^2 v \mu^3 + 3 e^4 Sa\, Sb\, u^2 v \mu^3 -$
$6 e^3 g\, Sa\, Sb\, u^2 v \mu^3 + 3 e^2 g^2 Sa\, Sb\, u^2 v \mu^3 + 3 e^4 Sa\, Sc\, u^2 v \mu^3 - 6 e^2 f^2 Sa\, Sc\, u^2 v \mu^3 +$
$3 e^2 g^2 Sa\, Sc\, u^2 v \mu^3 + 3 e^4 Sb\, Sc\, u^2 v \mu^3 - 6 e^3 g\, Sb\, Sc\, u^2 v \mu^3 + 3 e^2 g^2 Sb\, Sc\, u^2 v \mu^3 +$
$3 e^4 Sc^2 u^2 v \mu^3 - 3 e^2 f^2 Sc^2 u^2 v \mu^3 - 6 e f^2 g\, Sa^2 u v^2 \mu^3 + 6 e f g^2 Sa^2 u v^2 \mu^3 +$
$6 e^3 f\, Sa\, Sb\, u v^2 \mu^3 - 12 e^2 f g\, Sa\, Sb\, u v^2 \mu^3 + 6 e f g^2 Sa\, Sb\, u v^2 \mu^3 + 6 e^3 f\, Sa\, Sc\, u v^2 \mu^3 -$
$6 e^2 f^2 Sa\, Sc\, u v^2 \mu^3 - 6 e f^2 g\, Sa\, Sc\, u v^2 \mu^3 + 6 e f g^2 Sa\, Sc\, u v^2 \mu^3 + 6 e^3 f\, Sb\, Sc\, u v^2 \mu^3 -$
$12 e^2 f g\, Sb\, Sc\, u v^2 \mu^3 + 6 e f g^2 Sb\, Sc\, u v^2 \mu^3 + 6 e^3 f\, Sc^2 u v^2 \mu^3 - 6 e^2 f^2 Sc^2 u v^2 \mu^3 -$
$3 f^2 g^2 Sa^2 v^3 \mu^3 + 4 f g^3 Sa^2 v^3 \mu^3 - g^4 Sa^2 v^3 \mu^3 + e^2 f^2 Sa\, Sb\, v^3 \mu^3 + 4 e^2 f g\, Sa\, Sb\, v^3 \mu^3 -$
$2 e f^2 g\, Sa\, Sb\, v^3 \mu^3 - 2 e^2 g^2 Sa\, Sb\, v^3 \mu^3 - 8 e f g^2 Sa\, Sb\, v^3 \mu^3 + f^2 g^2 Sa\, Sb\, v^3 \mu^3 +$
$4 e g^3 Sa\, Sb\, v^3 \mu^3 + 4 f g^3 Sa\, Sb\, v^3 \mu^3 - 2 g^4 Sa\, Sb\, v^3 \mu^3 - e^4 Sb^2 v^3 \mu^3 + 4 e^3 g\, Sb^2 v^3 \mu^3 -$
$6 e^2 g^2 Sb^2 v^3 \mu^3 + 4 e g^3 Sb^2 v^3 \mu^3 - g^4 Sb^2 v^3 \mu^3 + e^2 f^2 Sa\, Sc\, v^3 \mu^3 + 4 e^2 f g\, Sa\, Sc\, v^3 \mu^3 -$
$8 e f^2 g\, Sa\, Sc\, v^3 \mu^3 - 2 e^2 g^2 Sa\, Sc\, v^3 \mu^3 + 4 e f g^2 Sa\, Sc\, v^3 \mu^3 + f^2 g^2 Sa\, Sc\, v^3 \mu^3 -$
$2 e^4 Sb\, Sc\, v^3 \mu^3 + 4 e^3 f\, Sb\, Sc\, v^3 \mu^3 + e^2 f^2 Sb\, Sc\, v^3 \mu^3 + 4 e^3 g\, Sb\, Sc\, v^3 \mu^3 - 8 e^2 f g\, Sb\, Sc\, v^3 \mu^3 -$
$2 e f^2 g\, Sb\, Sc\, v^3 \mu^3 - 2 e^2 g^2 Sb\, Sc\, v^3 \mu^3 + 4 e f g^2 Sb\, Sc\, v^3 \mu^3 + f^2 g^2 Sb\, Sc\, v^3 \mu^3 -$
$e^4 Sc^2 v^3 \mu^3 + 4 e^3 f\, Sc^2 v^3 \mu^3 - 3 e^2 f^2 Sc^2 v^3 \mu^3 - 6 e^2 f g\, Sa^2 u^2 w \mu^3 + 6 e^2 g^2 Sa^2 u^2 w \mu^3 +$
$6 e^3 g\, Sa\, Sc\, u^2 w \mu^3 - 12 e^2 f g\, Sa\, Sc\, u^2 w \mu^3 + 6 e^2 g^2 Sa\, Sc\, u^2 w \mu^3 + 6 e^3 g\, Sc^2 u^2 w \mu^3 -$
$6 e^2 f g\, Sc^2 u^2 w \mu^3 - 6 e f^2 g\, Sa^2 u v w \mu^3 + 6 e g^3 Sa^2 u v w \mu^3 + 6 e^3 g\, Sa\, Sb\, u v w \mu^3 -$
$12 e^2 g^2 Sa\, Sb\, u v w \mu^3 + 6 e g^3 Sa\, Sb\, u v w \mu^3 + 6 e^3 g\, Sa\, Sc\, u v w \mu^3 - 12 e f^2 g\, Sa\, Sc\, u v w \mu^3 +$
$6 e g^3 Sa\, Sc\, u v w \mu^3 + 6 e^3 g\, Sb\, Sc\, u v w \mu^3 - 12 e^2 g^2 Sb\, Sc\, u v w \mu^3 + 6 e g^3 Sb\, Sc\, u v w \mu^3 +$
$6 e^3 g\, Sc^2 u v w \mu^3 - 6 e f^2 g\, Sc^2 u v w \mu^3 - 6 f^2 g^2 Sa^2 v^2 w \mu^3 + 6 f g^3 Sa^2 v^2 w \mu^3 +$
$6 e^2 f g\, Sa\, Sb\, v^2 w \mu^3 - 12 e f g^2 Sa\, Sb\, v^2 w \mu^3 + 6 f g^3 Sa\, Sb\, v^2 w \mu^3 + 6 e^2 f g\, Sa\, Sc\, v^2 w \mu^3 -$
$6 e f^2 g\, Sa\, Sc\, v^2 w \mu^3 - 6 f^2 g^2 Sa\, Sc\, v^2 w \mu^3 + 6 f g^3 Sa\, Sc\, v^2 w \mu^3 + 6 e^2 f g\, Sb\, Sc\, v^2 w \mu^3 -$
$12 e f g^2 Sb\, Sc\, v^2 w \mu^3 + 6 f g^3 Sb\, Sc\, v^2 w \mu^3 + 6 e^2 f g\, Sc^2 v^2 w \mu^3 - 6 e f^2 g\, Sc^2 v^2 w \mu^3 -$
$6 e f g^2 Sa^2 u w^2 \mu^3 + 6 e g^3 Sa^2 u w^2 \mu^3 + 6 e^2 g^2 Sa\, Sc\, u w^2 \mu^3 - 12 e f g^2 Sa\, Sc\, u w^2 \mu^3 +$
$6 e g^3 Sa\, Sc\, u w^2 \mu^3 + 6 e^2 g^2 Sc^2 u w^2 \mu^3 - 6 e f g^2 Sc^2 u w^2 \mu^3 - 3 f^2 g^2 Sa^2 v w^2 \mu^3 + 3 g^4 Sa^2 v w^2 \mu^3 +$
$3 e^2 g^2 Sa\, Sb\, v w^2 \mu^3 - 6 e g^3 Sa\, Sb\, v w^2 \mu^3 + 3 g^4 Sa\, Sb\, v w^2 \mu^3 + 3 e^2 g^2 Sa\, Sc\, v w^2 \mu^3 -$
$6 f^2 g^2 Sa\, Sc\, v w^2 \mu^3 + 3 g^4 Sa\, Sc\, v w^2 \mu^3 + 3 e^2 g^2 Sb\, Sc\, v w^2 \mu^3 - 6 e g^3 Sb\, Sc\, v w^2 \mu^3 +$
$3 g^4 Sb\, Sc\, v w^2 \mu^3 + 3 e^2 g^2 Sc^2 v w^2 \mu^3 - 3 f^2 g^2 Sc^2 v w^2 \mu^3 - 2 f g^3 Sa^2 w^3 \mu^3 + 2 g^4 Sa^2 w^3 \mu^3 +$
$2 e g^3 Sa\, Sc\, w^3 \mu^3 - 4 f g^3 Sa\, Sc\, w^3 \mu^3 + 2 g^4 Sa\, Sc\, w^3 \mu^3 + 2 e g^3 Sc^2 w^3 \mu^3 - 2 f g^3 Sc^2 w^3 \mu^3 +$
$2 e^3 f\, Sa\, Sb\, u^3 \lambda^2 \nu - 6 e^2 f^2 Sa\, Sb\, u^3 \lambda^2 \nu - 2 e^3 g\, Sa\, Sb\, u^3 \lambda^2 \nu + 12 e^2 f g\, Sa\, Sb\, u^3 \lambda^2 \nu -$
$6 e^2 g^2 Sa\, Sb\, u^3 \lambda^2 \nu + 6 e^3 g\, Sb^2 u^3 \lambda^2 \nu - 6 e^2 g^2 Sb^2 u^3 \lambda^2 \nu + 2 e^3 f\, Sa\, Sc\, u^3 \lambda^2 \nu -$
$2 e^3 g\, Sa\, Sc\, u^3 \lambda^2 \nu + 8 e^3 f\, Sb\, Sc\, u^3 \lambda^2 \nu - 6 e^2 f^2 Sb\, Sc\, u^3 \lambda^2 \nu - 2 e^3 g\, Sb\, Sc\, u^3 \lambda^2 \nu +$
$6 e^2 f^2 Sa\, Sb\, u^2 v \lambda^2 \nu - 12 e f^3 Sa\, Sb\, u^2 v \lambda^2 \nu - 6 e^2 f g\, Sa\, Sb\, u^2 v \lambda^2 \nu + 24 e f^2 g\, Sa\, Sb\, u^2 v \lambda^2 \nu -$
$12 e f g^2 Sa\, Sb\, u^2 v \lambda^2 \nu + 6 e^3 f\, Sb^2 u^2 v \lambda^2 \nu + 6 e^2 f g\, Sb^2 u^2 v \lambda^2 \nu - 12 e f g^2 Sb^2 u^2 v \lambda^2 \nu +$
$6 e^2 f^2 Sa\, Sc\, u^2 v \lambda^2 \nu - 6 e^2 f g\, Sa\, Sc\, u^2 v \lambda^2 \nu + 6 e^3 f\, Sb\, Sc\, u^2 v \lambda^2 \nu + 12 e^2 f^2 Sb\, Sc\, u^2 v \lambda^2 \nu -$
$12 e f^3 Sb\, Sc\, u^2 v \lambda^2 \nu - 6 e^2 f g\, Sb\, Sc\, u^2 v \lambda^2 \nu + 6 e f^3 Sa\, Sb\, u v^2 \lambda^2 \nu - 6 f^4 Sa\, Sb\, u v^2 \lambda^2 \nu -$
$6 e f^2 g\, Sa\, Sb\, u v^2 \lambda^2 \nu + 12 f^3 g\, Sa\, Sb\, u v^2 \lambda^2 \nu - 6 f^2 g^2 Sa\, Sb\, u v^2 \lambda^2 \nu + 12 e^2 f^2 Sb^2 u v^2 \lambda^2 \nu -$
$6 e f^2 g\, Sb^2 u v^2 \lambda^2 \nu - 6 f^2 g^2 Sb^2 u v^2 \lambda^2 \nu + 6 e f^3 Sa\, Sc\, u v^2 \lambda^2 \nu - 6 e f^2 g\, Sa\, Sc\, u v^2 \lambda^2 \nu +$
$12 e^2 f^2 Sb\, Sc\, u v^2 \lambda^2 \nu - 6 f^4 Sb\, Sc\, u v^2 \lambda^2 \nu - 6 e f^2 g\, Sb\, Sc\, u v^2 \lambda^2 \nu + 2 f^4 Sa\, Sb\, v^3 \lambda^2 \nu -$
$2 f^3 g\, Sa\, Sb\, v^3 \lambda^2 \nu + 6 e f^3 Sb^2 v^3 \lambda^2 \nu - 6 f^3 g\, Sb^2 v^3 \lambda^2 \nu + 2 f^4 Sa\, Sc\, v^3 \lambda^2 \nu -$
$2 f^3 g\, Sa\, Sc\, v^3 \lambda^2 \nu + 6 e f^3 Sb\, Sc\, v^3 \lambda^2 \nu - 4 f^4 Sb\, Sc\, v^3 \lambda^2 \nu - 2 f^3 g\, Sb\, Sc\, v^3 \lambda^2 \nu -$
$3 f^4 Sa^2 u^2 w \lambda^2 \nu + 12 f^3 g\, Sa^2 u^2 w \lambda^2 \nu - 18 f^2 g^2 Sa^2 u^2 w \lambda^2 \nu + 12 f g^3 Sa^2 u^2 w \lambda^2 \nu -$
$3 g^4 Sa^2 u^2 w \lambda^2 \nu - 3 e^2 f^2 Sa\, Sb\, u^2 w \lambda^2 \nu + 12 e^2 f g\, Sa\, Sb\, u^2 w \lambda^2 \nu - 9 e^2 g^2 Sa\, Sb\, u^2 w \lambda^2 \nu -$
$6 f^2 g^2 Sa\, Sb\, u^2 w \lambda^2 \nu + 12 f g^3 Sa\, Sb\, u^2 w \lambda^2 \nu - 6 g^4 Sa\, Sb\, u^2 w \lambda^2 \nu + 12 e^3 g\, Sb^2 u^2 w \lambda^2 \nu -$
$9 e^2 g^2 Sb^2 u^2 w \lambda^2 \nu - 3 g^4 Sb^2 u^2 w \lambda^2 \nu - 3 e^2 f^2 Sa\, Sc\, u^2 w \lambda^2 \nu + 12 e f^3 Sa\, Sc\, u^2 w \lambda^2 \nu -$
$6 f^4 Sa\, Sc\, u^2 w \lambda^2 \nu + 12 e^2 f g\, Sa\, Sc\, u^2 w \lambda^2 \nu - 24 e f^2 g\, Sa\, Sc\, u^2 w \lambda^2 \nu + 12 f^3 g\, Sa\, Sc\, u^2 w \lambda^2 \nu -$

$9 e^2 g^2 Sa\, Sc\, u^2 w \lambda^2 \nu + 12 e f g^2 Sa\, Sc\, u^2 w \lambda^2 \nu - 6 f^2 g^2 Sa\, Sc\, u^2 w \lambda^2 \nu + 6 e^3 f\, Sb\, Sc\, u^2 w \lambda^2 \nu -$
$3 e^2 f^2 Sb\, Sc\, u^2 w \lambda^2 \nu + 12 e^3 g\, Sb\, Sc\, u^2 w \lambda^2 \nu - 12 e^2 f g\, Sb\, Sc\, u^2 w \lambda^2 \nu - 9 e^2 g^2 Sb\, Sc\, u^2 w \lambda^2 \nu +$
$12 e f g^2 Sb\, Sc\, u^2 w \lambda^2 \nu - 6 f^2 g^2 Sb\, Sc\, u^2 w \lambda^2 \nu + 6 e^3 f\, Sc^2 u^2 w \lambda^2 \nu - 15 e^2 f^2 Sc^2 u^2 w \lambda^2 \nu +$
$12 e f^3 Sc^2 u^2 w \lambda^2 \nu - 3 f^4 Sc^2 u^2 w \lambda^2 \nu + 6 e f^3 Sa\, Sb\, u v w \lambda^2 \nu - 12 f^3 g\, Sa\, Sb\, u v w \lambda^2 \nu -$
$6 e f g^2 Sa\, Sb\, u v w \lambda^2 \nu + 24 f^2 g^2 Sa\, Sb\, u v w \lambda^2 \nu - 12 f g^3 Sa\, Sb\, u v w \lambda^2 \nu + 6 e^3 f\, Sb^2 u v w \lambda^2 \nu +$
$12 e^2 f g\, Sb^2 u v w \lambda^2 \nu - 6 e f g^2 Sb^2 u v w \lambda^2 \nu - 12 f g^3 Sb^2 u v w \lambda^2 \nu + 6 e f^3 Sa\, Sc\, u v w \lambda^2 \nu -$
$6 e f g^2 Sa\, Sc\, u v w \lambda^2 \nu + 12 e^3 f\, Sb\, Sc\, u v w \lambda^2 \nu - 12 e^2 f^2 Sb\, Sc\, u v w \lambda^2 \nu + 6 e f^3 Sb\, Sc\, u v w \lambda^2 \nu +$
$12 e^2 f g\, Sb\, Sc\, u v w \lambda^2 \nu - 12 f^3 g\, Sb\, Sc\, u v w \lambda^2 \nu - 6 e f g^2 Sb\, Sc\, u v w \lambda^2 \nu + 6 e^3 f\, Sc^2 u v w \lambda^2 \nu -$
$12 e^2 f^2 Sc^2 u v w \lambda^2 \nu + 6 e f^3 Sc^2 u v w \lambda^2 \nu + 3 f^4 Sa\, Sb\, v^2 w \lambda^2 \nu - 3 f^2 g^2 Sa\, Sb\, v^2 w \lambda^2 \nu +$
$3 e^2 f^2 Sb^2 v^2 w \lambda^2 \nu + 12 e f^2 g\, Sb^2 v^2 w \lambda^2 \nu - 15 f^2 g^2 Sb^2 v^2 w \lambda^2 \nu + 3 f^4 Sa\, Sc\, v^2 w \lambda^2 \nu -$
$3 f^2 g^2 Sa\, Sc\, v^2 w \lambda^2 \nu + 6 e^2 f^2 Sb\, Sc\, v^2 w \lambda^2 \nu - 6 e f^3 Sb\, Sc\, v^2 w \lambda^2 \nu + 3 f^4 Sb\, Sc\, v^2 w \lambda^2 \nu +$
$12 e f^2 g\, Sb\, Sc\, v^2 w \lambda^2 \nu - 12 f^3 g\, Sb\, Sc\, v^2 w \lambda^2 \nu - 3 f^2 g^2 Sb\, Sc\, v^2 w \lambda^2 \nu + 3 e^2 f^2 Sc^2 v^2 w \lambda^2 \nu -$
$6 e f^3 Sc^2 v^2 w \lambda^2 \nu + 3 f^4 Sc^2 v^2 w \lambda^2 \nu + 6 e f^2 g\, Sa\, Sb\, u w^2 \lambda^2 \nu - 6 e f g^2 Sa\, Sb\, u w^2 \lambda^2 \nu -$
$6 f^2 g^2 Sa\, Sb\, u w^2 \lambda^2 \nu + 12 f g^3 Sa\, Sb\, u w^2 \lambda^2 \nu - 6 g^4 Sa\, Sb\, u w^2 \lambda^2 \nu + 6 e^3 g\, Sb^2 u w^2 \lambda^2 \nu -$
$6 g^4 Sb^2 u w^2 \lambda^2 \nu + 6 e f^2 g\, Sa\, Sc\, u w^2 \lambda^2 \nu - 6 e f g^2 Sa\, Sc\, u w^2 \lambda^2 \nu + 12 e^3 g\, Sb\, Sc\, u w^2 \lambda^2 \nu -$
$12 e^2 f g\, Sb\, Sc\, u w^2 \lambda^2 \nu + 6 e f^2 g\, Sb\, Sc\, u w^2 \lambda^2 \nu - 6 f^2 g^2 Sb\, Sc\, u w^2 \lambda^2 \nu + 6 e^3 g\, Sc^2 u w^2 \lambda^2 \nu -$
$12 e^2 f g\, Sc^2 u w^2 \lambda^2 \nu + 6 e f^2 g\, Sc^2 u w^2 \lambda^2 \nu + 6 f^3 g\, Sa\, Sb\, v w^2 \lambda^2 \nu - 6 f^2 g^2 Sa\, Sb\, v w^2 \lambda^2 \nu +$
$6 e^2 f g\, Sb^2 v w^2 \lambda^2 \nu + 6 e f g^2 Sb^2 v w^2 \lambda^2 \nu - 12 f g^3 Sb^2 v w^2 \lambda^2 \nu + 6 f^3 g\, Sa\, Sc\, v w^2 \lambda^2 \nu -$
$6 f^2 g^2 Sa\, Sc\, v w^2 \lambda^2 \nu + 12 e^2 f g\, Sb\, Sc\, v w^2 \lambda^2 \nu - 12 e f^2 g\, Sb\, Sc\, v w^2 \lambda^2 \nu + 6 f^3 g\, Sb\, Sc\, v w^2 \lambda^2 \nu +$
$6 e f g^2 Sb\, Sc\, v w^2 \lambda^2 \nu - 12 f^2 g^2 Sb\, Sc\, v w^2 \lambda^2 \nu + 6 e^2 f g\, Sc^2 v w^2 \lambda^2 \nu - 12 e f^2 g\, Sc^2 v w^2 \lambda^2 \nu +$
$6 f^3 g\, Sc^2 v w^2 \lambda^2 \nu + 3 f^2 g^2 Sa\, Sb\, w^3 \lambda^2 \nu - 4 f g^3 Sa\, Sb\, w^3 \lambda^2 \nu + g^4 Sa\, Sb\, w^3 \lambda^2 \nu +$
$3 e^2 g^2 Sb^2 w^3 \lambda^2 \nu - 3 g^4 Sb^2 w^3 \lambda^2 \nu + 3 f^2 g^2 Sa\, Sc\, w^3 \lambda^2 \nu - 4 f g^3 Sa\, Sc\, w^3 \lambda^2 \nu +$
$g^4 Sa\, Sc\, w^3 \lambda^2 \nu + 6 e^2 g^2 Sb\, Sc\, w^3 \lambda^2 \nu - 6 e f g^2 Sb\, Sc\, w^3 \lambda^2 \nu + 3 f^2 g^2 Sb\, Sc\, w^3 \lambda^2 \nu -$
$4 f g^3 Sb\, Sc\, w^3 \lambda^2 \nu + g^4 Sb\, Sc\, w^3 \lambda^2 \nu + 3 e^2 g^2 Sc^2 w^3 \lambda^2 \nu - 6 e f g^2 Sc^2 w^3 \lambda^2 \nu +$
$3 f^2 g^2 Sc^2 w^3 \lambda^2 \nu - 6 e^2 f^2 Sa^2 u^3 \lambda \mu \nu + 12 e^2 f g\, Sa^2 u^3 \lambda \mu \nu - 6 e^2 g^2 Sa^2 u^3 \lambda \mu \nu -$
$2 e^4 Sa\, Sb\, u^3 \lambda \mu \nu + 4 e^3 f\, Sa\, Sb\, u^3 \lambda \mu \nu + 4 e^3 g\, Sa\, Sb\, u^3 \lambda \mu \nu - 6 e^2 g^2 Sa\, Sb\, u^3 \lambda \mu \nu -$
$2 e^4 Sa\, Sc\, u^3 \lambda \mu \nu + 4 e^3 f\, Sa\, Sc\, u^3 \lambda \mu \nu - 6 e^2 f^2 Sa\, Sc\, u^3 \lambda \mu \nu + 4 e^3 g\, Sa\, Sc\, u^3 \lambda \mu \nu -$
$8 e^4 Sb\, Sc\, u^3 \lambda \mu \nu + 4 e^3 f\, Sb\, Sc\, u^3 \lambda \mu \nu + 4 e^3 g\, Sb\, Sc\, u^3 \lambda \mu \nu - 12 e f^3 Sa^2 u^2 v \lambda \mu \nu +$
$24 e f^2 g\, Sa^2 u^2 v \lambda \mu \nu - 12 e f g^2 Sa^2 u^2 v \lambda \mu \nu + 6 e^2 f^2 Sa\, Sb\, u^2 v \lambda \mu \nu + 12 e^2 f g\, Sa\, Sb\, u^2 v \lambda \mu \nu -$
$6 e^2 g^2 Sa\, Sb\, u^2 v \lambda \mu \nu - 12 e f g^2 Sa\, Sb\, u^2 v \lambda \mu \nu - 6 e^4 Sb^2 u^2 v \lambda \mu \nu + 12 e^3 g\, Sb^2 u^2 v \lambda \mu \nu -$
$6 e^2 g^2 Sb^2 u^2 v \lambda \mu \nu - 12 e f^3 Sa\, Sc\, u^2 v \lambda \mu \nu + 12 e^2 f g\, Sa\, Sc\, u^2 v \lambda \mu \nu - 6 e^4 Sb\, Sc\, u^2 v \lambda \mu \nu -$
$12 e^3 f\, Sb\, Sc\, u^2 v \lambda \mu \nu + 6 e^2 f^2 Sb\, Sc\, u^2 v \lambda \mu \nu + 12 e^2 f g\, Sb\, Sc\, u^2 v \lambda \mu \nu - 6 f^4 Sa^2 u v^2 \lambda \mu \nu +$
$12 f^3 g\, Sa^2 u v^2 \lambda \mu \nu - 6 f^2 g^2 Sa^2 u v^2 \lambda \mu \nu + 6 e^2 f^2 Sa\, Sb\, u v^2 \lambda \mu \nu + 12 e f^2 g\, Sa\, Sb\, u v^2 \lambda \mu \nu -$
$12 e f g^2 Sa\, Sb\, u v^2 \lambda \mu \nu - 6 f^2 g^2 Sa\, Sb\, u v^2 \lambda \mu \nu - 12 e^3 f\, Sb^2 u v^2 \lambda \mu \nu + 24 e^2 f g\, Sb^2 u v^2 \lambda \mu \nu -$
$12 e f g^2 Sb^2 u v^2 \lambda \mu \nu + 6 e^2 f^2 Sa\, Sc\, u v^2 \lambda \mu \nu - 12 e f^3 Sa\, Sc\, u v^2 \lambda \mu \nu - 6 f^4 Sa\, Sc\, u v^2 \lambda \mu \nu +$
$12 e f^2 g\, Sa\, Sc\, u v^2 \lambda \mu \nu - 12 e^3 f\, Sb\, Sc\, u v^2 \lambda \mu \nu + 12 e f^2 g\, Sb\, Sc\, u v^2 \lambda \mu \nu +$
$4 e f^3 Sa\, Sb\, v^3 \lambda \mu \nu - 2 f^4 Sa\, Sb\, v^3 \lambda \mu \nu + 4 f^3 g\, Sa\, Sb\, v^3 \lambda \mu \nu - 6 f^2 g^2 Sa\, Sb\, v^3 \lambda \mu \nu -$
$6 e^2 f^2 Sb^2 v^3 \lambda \mu \nu + 12 e f^2 g\, Sb^2 v^3 \lambda \mu \nu - 6 f^2 g^2 Sb^2 v^3 \lambda \mu \nu + 4 e f^3 Sa\, Sc\, v^3 \lambda \mu \nu -$
$8 f^4 Sa\, Sc\, v^3 \lambda \mu \nu + 4 f^3 g\, Sa\, Sc\, v^3 \lambda \mu \nu - 6 e^2 f^2 Sb\, Sc\, v^3 \lambda \mu \nu + 4 e f^3 Sb\, Sc\, v^3 \lambda \mu \nu -$
$2 f^4 Sb\, Sc\, v^3 \lambda \mu \nu + 4 f^3 g\, Sb\, Sc\, v^3 \lambda \mu \nu - 12 e f^2 g\, Sa^2 u^2 w \lambda \mu \nu + 24 e f g^2 Sa^2 u^2 w \lambda \mu \nu -$
$12 e g^3 Sa^2 u^2 w \lambda \mu \nu + 12 e^2 f g\, Sa\, Sb\, u^2 w \lambda \mu \nu - 12 e g^3 Sa\, Sb\, u^2 w \lambda \mu \nu - 6 e^2 f^2 Sa\, Sc\, u^2 w \lambda \mu \nu +$
$12 e^2 f g\, Sa\, Sc\, u^2 w \lambda \mu \nu - 12 e f^2 g\, Sa\, Sc\, u^2 w \lambda \mu \nu + 6 e^2 g^2 Sa\, Sc\, u^2 w \lambda \mu \nu -$
$6 e^4 Sb\, Sc\, u^2 w \lambda \mu \nu - 12 e^3 g\, Sb\, Sc\, u^2 w \lambda \mu \nu + 12 e^2 f g\, Sb\, Sc\, u^2 w \lambda \mu \nu + 6 e^2 g^2 Sb\, Sc\, u^2 w \lambda \mu \nu -$
$6 e^4 Sc^2 u^2 w \lambda \mu \nu + 12 e^3 f\, Sc^2 u^2 w \lambda \mu \nu - 6 e^2 f^2 Sc^2 u^2 w \lambda \mu \nu - 6 f^4 Sa^2 u v w \lambda \mu \nu +$
$12 f^3 g\, Sa^2 u v w \lambda \mu \nu - 12 f^2 g^2 Sa^2 u v w \lambda \mu \nu + 12 f g^3 Sa^2 u v w \lambda \mu \nu - 6 g^4 Sa^2 u v w \lambda \mu \nu -$
$12 e^2 f^2 Sa\, Sb\, u v w \lambda \mu \nu + 36 e^2 f g\, Sa\, Sb\, u v w \lambda \mu \nu + 36 e f^2 g\, Sa\, Sb\, u v w \lambda \mu \nu -$
$12 e^2 g^2 Sa\, Sb\, u v w \lambda \mu \nu - 48 e f g^2 Sa\, Sb\, u v w \lambda \mu \nu - 12 f^2 g^2 Sa\, Sb\, u v w \lambda \mu \nu +$
$12 e g^3 Sa\, Sb\, u v w \lambda \mu \nu + 12 f g^3 Sa\, Sb\, u v w \lambda \mu \nu - 12 g^4 Sa\, Sb\, u v w \lambda \mu \nu - 6 e^4 Sb^2 u v w \lambda \mu \nu +$
$12 e^3 g\, Sb^2 u v w \lambda \mu \nu - 12 e^2 g^2 Sb^2 u v w \lambda \mu \nu + 12 e g^3 Sb^2 u v w \lambda \mu \nu - 6 g^4 Sb^2 u v w \lambda \mu \nu -$
$12 e^2 f^2 Sa\, Sc\, u v w \lambda \mu \nu + 12 e f^3 Sa\, Sc\, u v w \lambda \mu \nu - 12 f^4 Sa\, Sc\, u v w \lambda \mu \nu +$
$36 e^2 f g\, Sa\, Sc\, u v w \lambda \mu \nu - 48 e f^2 g\, Sa\, Sc\, u v w \lambda \mu \nu + 12 f^3 g\, Sa\, Sc\, u v w \lambda \mu \nu -$
$12 e^2 g^2 Sa\, Sc\, u v w \lambda \mu \nu + 36 e f g^2 Sa\, Sc\, u v w \lambda \mu \nu - 12 f^2 g^2 Sa\, Sc\, u v w \lambda \mu \nu -$
$12 e^4 Sb\, Sc\, u v w \lambda \mu \nu + 12 e^3 f\, Sb\, Sc\, u v w \lambda \mu \nu - 12 e^2 f^2 Sb\, Sc\, u v w \lambda \mu \nu +$
$12 e^3 g\, Sb\, Sc\, u v w \lambda \mu \nu - 48 e^2 f g\, Sb\, Sc\, u v w \lambda \mu \nu + 36 e f^2 g\, Sb\, Sc\, u v w \lambda \mu \nu -$
$12 e^2 g^2 Sb\, Sc\, u v w \lambda \mu \nu + 36 e f g^2 Sb\, Sc\, u v w \lambda \mu \nu - 12 f^2 g^2 Sb\, Sc\, u v w \lambda \mu \nu -$
$6 e^4 Sc^2 u v w \lambda \mu \nu + 12 e^3 f\, Sc^2 u v w \lambda \mu \nu - 12 e^2 f^2 Sc^2 u v w \lambda \mu \nu + 12 e f^3 Sc^2 u v w \lambda \mu \nu -$
$6 f^4 Sc^2 u v w \lambda \mu \nu + 12 e f^2 g\, Sa\, Sb\, v^2 w \lambda \mu \nu - 12 f g^3 Sa\, Sb\, v^2 w \lambda \mu \nu - 12 e^2 f g\, Sb^2 v^2 w \lambda \mu \nu +$
$24 e f g^2 Sb^2 v^2 w \lambda \mu \nu - 12 f g^3 Sb^2 v^2 w \lambda \mu \nu - 6 f^4 Sa\, Sc\, v^2 w \lambda \mu \nu + 12 e f^2 g\, Sa\, Sc\, v^2 w \lambda \mu \nu -$

$$12 f^3 g\, Sa\, Sc\, v^2 w \lambda \mu \nu + 6 f^2 g^2 Sa\, Sc\, v^2 w \lambda \mu \nu - 6 e^2 f^2 Sb\, Sc\, v^2 w \lambda \mu \nu - 12 e^2 f g\, Sb\, Sc\, v^2 w \lambda \mu \nu +$$
$$12 e f^2 g\, Sb\, Sc\, v^2 w \lambda \mu \nu + 6 f^2 g^2 Sb\, Sc\, v^2 w \lambda \mu \nu - 6 e^2 f^2 Sc^2 v^2 w \lambda \mu \nu + 12 e f^3 Sc^2 v^2 w \lambda \mu \nu -$$
$$6 f^4 Sc^2 v^2 w \lambda \mu \nu - 6 f^2 g^2 Sa^2 u w^2 \lambda \mu \nu + 12 f g^3 Sa^2 u w^2 \lambda \mu \nu - 6 g^4 Sa^2 u w^2 \lambda \mu \nu +$$
$$6 e^2 g^2 Sa\, Sb\, u w^2 \lambda \mu \nu + 12 e f g^2 Sa\, Sb\, u w^2 \lambda \mu \nu - 12 e g^3 Sa\, Sb\, u w^2 \lambda \mu \nu - 6 g^4 Sa\, Sb\, u w^2 \lambda \mu \nu -$$
$$12 e f^2 g\, Sa\, Sc\, u w^2 \lambda \mu \nu + 6 e^2 g^2 Sa\, Sc\, u w^2 \lambda \mu \nu + 12 e f g^2 Sa\, Sc\, u w^2 \lambda \mu \nu -$$
$$6 f^2 g^2 Sa\, Sc\, u w^2 \lambda \mu \nu - 12 e^3 g\, Sb\, Sc\, u w^2 \lambda \mu \nu + 12 e f g^2 Sb\, Sc\, u w^2 \lambda \mu \nu - 12 e^3 g\, Sc^2 u w^2 \lambda \mu \nu +$$
$$24 e^2 f g\, Sc^2 u w^2 \lambda \mu \nu - 12 e f^2 g\, Sc^2 u w^2 \lambda \mu \nu + 12 e f g^2 Sa\, Sb\, v w^2 \lambda \mu \nu + 6 f^2 g^2 Sa\, Sb\, v w^2 \lambda \mu \nu -$$
$$12 f g^3 Sa\, Sb\, v w^2 \lambda \mu \nu - 6 g^4 Sa\, Sb\, v w^2 \lambda \mu \nu - 6 e^2 g^2 Sb^2 v w^2 \lambda \mu \nu + 12 e g^3 Sb^2 v w^2 \lambda \mu \nu -$$
$$6 g^4 Sb^2 v w^2 \lambda \mu \nu - 12 f^3 g\, Sa\, Sc\, v w^2 \lambda \mu \nu + 12 e f g^2 Sa\, Sc\, v w^2 \lambda \mu \nu - 12 e^2 f g\, Sb\, Sc\, v w^2 \lambda \mu \nu -$$
$$6 e^2 g^2 Sb\, Sc\, v w^2 \lambda \mu \nu + 12 e f g^2 Sb\, Sc\, v w^2 \lambda \mu \nu + 6 f^2 g^2 Sb\, Sc\, v w^2 \lambda \mu \nu - 12 e^2 f g\, Sc^2 v w^2 \lambda \mu \nu +$$
$$24 e f^2 g\, Sc^2 v w^2 \lambda \mu \nu - 12 f^3 g\, Sc^2 v w^2 \lambda \mu \nu + 4 e g^3 Sa\, Sb\, w^3 \lambda \mu \nu + 4 f g^3 Sa\, Sb\, w^3 \lambda \mu \nu -$$
$$8 g^4 Sa\, Sb\, w^3 \lambda \mu \nu - 6 f^2 g^2 Sa\, Sc\, w^3 \lambda \mu \nu + 4 e g^3 Sa\, Sc\, w^3 \lambda \mu \nu + 4 f g^3 Sa\, Sc\, w^3 \lambda \mu \nu -$$
$$2 g^4 Sa\, Sc\, w^3 \lambda \mu \nu - 6 e^2 g^2 Sb\, Sc\, w^3 \lambda \mu \nu + 4 e g^3 Sb\, Sc\, w^3 \lambda \mu \nu + 4 f g^3 Sb\, Sc\, w^3 \lambda \mu \nu -$$
$$2 g^4 Sb\, Sc\, w^3 \lambda \mu \nu - 6 e^2 g^2 Sc^2 w^3 \lambda \mu \nu + 12 e f g^2 Sc^2 w^3 \lambda \mu \nu - 6 f^2 g^2 Sc^2 w^3 \lambda \mu \nu +$$
$$6 e^3 f\, Sa^2 u^3 \mu^2 \nu - 6 e^3 g\, Sa^2 u^3 \mu^2 \nu + 2 e^4 Sa\, Sb\, u^3 \mu^2 \nu - 2 e^3 g\, Sa\, Sb\, u^3 \mu^2 \nu - 4 e^4 Sa\, Sc\, u^3 \mu^2 \nu +$$
$$6 e^3 f\, Sa\, Sc\, u^3 \mu^2 \nu - 2 e^3 g\, Sa\, Sc\, u^3 \mu^2 \nu + 2 e^4 Sb\, Sc\, u^3 \mu^2 \nu - 2 e^3 g\, Sb\, Sc\, u^3 \mu^2 \nu +$$
$$12 e^2 f^2 Sa^2 u^2 v \mu^2 \nu - 6 e^2 f g\, Sa^2 u^2 v \mu^2 \nu - 6 e^2 g^2 Sa^2 u^2 v \mu^2 \nu - 6 e^4 Sa\, Sb\, u^2 v \mu^2 \nu +$$
$$6 e^3 f\, Sa\, Sb\, u^2 v \mu^2 \nu + 12 e^3 g\, Sa\, Sb\, u^2 v \mu^2 \nu - 6 e^2 f g\, Sa\, Sb\, u^2 v \mu^2 \nu - 6 e^2 g^2 Sa\, Sb\, u^2 v \mu^2 \nu -$$
$$6 e^4 Sa\, Sc\, u^2 v \mu^2 \nu + 12 e^2 f^2 Sa\, Sc\, u^2 v \mu^2 \nu - 6 e^2 f g\, Sa\, Sc\, u^2 v \mu^2 \nu + 6 e^3 f\, Sb\, Sc\, u^2 v \mu^2 \nu -$$
$$6 e^2 f g\, Sb\, Sc\, u^2 v \mu^2 \nu + 6 e f^3 Sa^2 u v^2 \mu^2 \nu + 6 e f^2 g\, Sa^2 u v^2 \mu^2 \nu - 12 e f g^2 Sa^2 u v^2 \mu^2 \nu -$$
$$12 e^3 f\, Sa\, Sb\, u v^2 \mu^2 \nu + 6 e^2 f^2 Sa\, Sb\, u v^2 \mu^2 \nu + 24 e^2 f g\, Sa\, Sb\, u v^2 \mu^2 \nu - 6 e f^2 g\, Sa\, Sb\, u v^2 \mu^2 \nu -$$
$$12 e f g^2 Sa\, Sb\, u v^2 \mu^2 \nu - 12 e^3 f\, Sa\, Sc\, u v^2 \mu^2 \nu + 12 e^2 f^2 Sa\, Sc\, u v^2 \mu^2 \nu + 6 e f^3 Sa\, Sc\, u v^2 \mu^2 \nu -$$
$$6 e f^2 g\, Sa\, Sc\, u v^2 \mu^2 \nu + 6 e^2 f^2 Sb\, Sc\, u v^2 \mu^2 \nu - 6 e f^2 g\, Sb\, Sc\, u v^2 \mu^2 \nu + 6 f^3 g\, Sa^2 v^3 \mu^2 \nu -$$
$$6 f^2 g^2 Sa^2 v^3 \mu^2 \nu - 6 e^2 f^2 Sa\, Sb\, v^3 \mu^2 \nu + 2 e f^3 Sa\, Sb\, v^3 \mu^2 \nu + 12 e f^2 g\, Sa\, Sb\, v^3 \mu^2 \nu -$$
$$2 f^3 g\, Sa\, Sb\, v^3 \mu^2 \nu - 6 f^2 g^2 Sa\, Sb\, v^3 \mu^2 \nu - 6 e^2 f^2 Sa\, Sc\, v^3 \mu^2 \nu + 8 e f^3 Sa\, Sc\, v^3 \mu^2 \nu -$$
$$2 f^3 g\, Sa\, Sc\, v^3 \mu^2 \nu + 2 e f^3 Sb\, Sc\, v^3 \mu^2 \nu - 2 f^3 g\, Sb\, Sc\, v^3 \mu^2 \nu + 3 e^2 f^2 Sa^2 u^2 w \mu^2 \nu +$$
$$12 e^2 f g\, Sa^2 u^2 w \mu^2 \nu - 15 e^2 g^2 Sa^2 u^2 w \mu^2 \nu + 3 e^4 Sa\, Sb\, u^2 w \mu^2 \nu - 3 e^2 g^2 Sa\, Sb\, u^2 w \mu^2 \nu +$$
$$3 e^4 Sa\, Sc\, u^2 w \mu^2 \nu - 6 e^3 f\, Sa\, Sc\, u^2 w \mu^2 \nu + 6 e^2 f^2 Sa\, Sc\, u^2 w \mu^2 \nu - 12 e^3 g\, Sa\, Sc\, u^2 w \mu^2 \nu +$$
$$12 e^2 f g\, Sa\, Sc\, u^2 w \mu^2 \nu - 3 e^2 g^2 Sa\, Sc\, u^2 w \mu^2 \nu + 3 e^4 Sb\, Sc\, u^2 w \mu^2 \nu - 3 e^2 g^2 Sb\, Sc\, u^2 w \mu^2 \nu +$$
$$3 e^4 Sc^2 u^2 w \mu^2 \nu - 6 e^3 f\, Sc^2 u^2 w \mu^2 \nu + 3 e^2 f^2 Sc^2 u^2 w \mu^2 \nu + 6 e f^3 Sa^2 u v w \mu^2 \nu +$$
$$12 e f^2 g\, Sa^2 u v w \mu^2 \nu - 6 e f g^2 Sa^2 u v w \mu^2 \nu - 12 e g^3 Sa^2 u v w \mu^2 \nu + 6 e^3 f\, Sa\, Sb\, u v w \mu^2 \nu -$$
$$12 e^3 g\, Sa\, Sb\, u v w \mu^2 \nu + 24 e^2 g^2 Sa\, Sb\, u v w \mu^2 \nu - 6 e f g^2 Sa\, Sb\, u v w \mu^2 \nu - 12 e g^3 Sa\, Sb\, u v w \mu^2 \nu +$$
$$6 e^3 f\, Sa\, Sc\, u v w \mu^2 \nu - 12 e^2 f^2 Sa\, Sc\, u v w \mu^2 \nu + 12 e f^3 Sa\, Sc\, u v w \mu^2 \nu - 12 e^3 g\, Sa\, Sc\, u v w \mu^2 \nu +$$
$$12 e f^2 g\, Sa\, Sc\, u v w \mu^2 \nu - 6 e f g^2 Sa\, Sc\, u v w \mu^2 \nu + 6 e^3 f\, Sb\, Sc\, u v w \mu^2 \nu - 6 e f g^2 Sb\, Sc\, u v w \mu^2 \nu +$$
$$6 e^3 f\, Sc^2 u v w \mu^2 \nu - 12 e^2 f^2 Sc^2 u v w \mu^2 \nu + 6 e f^3 Sc^2 u v w \mu^2 \nu + 12 f^3 g\, Sa^2 v^2 w \mu^2 \nu -$$
$$9 f^2 g^2 Sa^2 v^2 w \mu^2 \nu - 3 g^4 Sa^2 v^2 w \mu^2 \nu - 3 e^2 f^2 Sa\, Sb\, v^2 w \mu^2 \nu + 12 e f^2 g\, Sa\, Sb\, v^2 w \mu^2 \nu -$$
$$6 e^2 g^2 Sa\, Sb\, v^2 w \mu^2 \nu - 9 f^2 g^2 Sa\, Sb\, v^2 w \mu^2 \nu + 12 e g^3 Sa\, Sb\, v^2 w \mu^2 \nu - 6 g^4 Sa\, Sb\, v^2 w \mu^2 \nu -$$
$$3 e^4 Sb^2 v^2 w \mu^2 \nu + 12 e^3 g\, Sb^2 v^2 w \mu^2 \nu - 18 e^2 g^2 Sb^2 v^2 w \mu^2 \nu + 12 e g^3 Sb^2 v^2 w \mu^2 \nu -$$
$$3 g^4 Sb^2 v^2 w \mu^2 \nu - 3 e^2 f^2 Sa\, Sc\, v^2 w \mu^2 \nu + 6 e f^3 Sa\, Sc\, v^2 w \mu^2 \nu - 12 e f^2 g\, Sa\, Sc\, v^2 w \mu^2 \nu +$$
$$12 f^3 g\, Sa\, Sc\, v^2 w \mu^2 \nu - 6 e^2 g^2 Sa\, Sc\, v^2 w \mu^2 \nu + 12 e f g^2 Sa\, Sc\, v^2 w \mu^2 \nu - 9 f^2 g^2 Sa\, Sc\, v^2 w \mu^2 \nu -$$
$$6 e^4 Sb\, Sc\, v^2 w \mu^2 \nu + 12 e^3 f\, Sb\, Sc\, v^2 w \mu^2 \nu - 3 e^2 f^2 Sb\, Sc\, v^2 w \mu^2 \nu + 12 e^3 g\, Sb\, Sc\, v^2 w \mu^2 \nu -$$
$$24 e^2 f g\, Sb\, Sc\, v^2 w \mu^2 \nu + 12 e f^2 g\, Sb\, Sc\, v^2 w \mu^2 \nu - 6 e^2 g^2 Sb\, Sc\, v^2 w \mu^2 \nu + 12 e f g^2 Sb\, Sc\, v^2 w \mu^2 \nu -$$
$$9 f^2 g^2 Sb\, Sc\, v^2 w \mu^2 \nu - 3 e^4 Sc^2 v^2 w \mu^2 \nu + 12 e^3 f\, Sc^2 v^2 w \mu^2 \nu - 15 e^2 f^2 Sc^2 v^2 w \mu^2 \nu +$$
$$6 e f^3 Sc^2 v^2 w \mu^2 \nu + 6 e f^2 g\, Sa^2 u w^2 \mu^2 \nu + 6 e f g^2 Sa^2 u w^2 \mu^2 \nu - 12 e g^3 Sa^2 u w^2 \mu^2 \nu +$$
$$6 e^3 g\, Sa\, Sb\, u w^2 \mu^2 \nu - 6 e^2 g^2 Sa\, Sb\, u w^2 \mu^2 \nu + 6 e^3 g\, Sa\, Sc\, u w^2 \mu^2 \nu - 12 e^2 f g\, Sa\, Sc\, u w^2 \mu^2 \nu +$$
$$12 e f^2 g\, Sa\, Sc\, u w^2 \mu^2 \nu - 12 e^2 g^2 Sa\, Sc\, u w^2 \mu^2 \nu + 6 e f g^2 Sa\, Sc\, u w^2 \mu^2 \nu + 6 e^3 g\, Sb\, Sc\, u w^2 \mu^2 \nu -$$
$$6 e^2 g^2 Sb\, Sc\, u w^2 \mu^2 \nu + 6 e^3 g\, Sc^2 u w^2 \mu^2 \nu - 12 e^2 f g\, Sc^2 u w^2 \mu^2 \nu + 6 e f^2 g\, Sc^2 u w^2 \mu^2 \nu +$$
$$6 f^3 g\, Sa^2 v w^2 \mu^2 \nu - 6 g^4 Sa^2 v w^2 \mu^2 \nu + 6 e^2 f g\, Sa\, Sb\, v w^2 \mu^2 \nu - 6 e^2 g^2 Sa\, Sb\, v w^2 \mu^2 \nu -$$
$$6 e f g^2 Sa\, Sb\, v w^2 \mu^2 \nu + 12 e g^3 Sa\, Sb\, v w^2 \mu^2 \nu - 6 g^4 Sa\, Sb\, v w^2 \mu^2 \nu + 6 e^2 f g\, Sa\, Sc\, v w^2 \mu^2 \nu -$$
$$12 e f^2 g\, Sa\, Sc\, v w^2 \mu^2 \nu + 12 f^3 g\, Sa\, Sc\, v w^2 \mu^2 \nu - 6 e^2 g^2 Sa\, Sc\, v w^2 \mu^2 \nu + 6 e^2 f g\, Sb\, Sc\, v w^2 \mu^2 \nu -$$
$$6 e f g^2 Sb\, Sc\, v w^2 \mu^2 \nu + 6 e^2 f g\, Sc^2 v w^2 \mu^2 \nu - 12 e f^2 g\, Sc^2 v w^2 \mu^2 \nu + 6 f^3 g\, Sc^2 v w^2 \mu^2 \nu +$$
$$3 f^2 g^2 Sa^2 w^3 \mu^2 \nu - 3 g^4 Sa^2 w^3 \mu^2 \nu + 3 e^2 g^2 Sa\, Sb\, w^3 \mu^2 \nu - 4 e g^3 Sa\, Sb\, w^3 \mu^2 \nu +$$
$$g^4 Sa\, Sb\, w^3 \mu^2 \nu + 3 e^2 g^2 Sa\, Sc\, w^3 \mu^2 \nu - 6 e f g^2 Sa\, Sc\, w^3 \mu^2 \nu + 6 f^2 g^2 Sa\, Sc\, w^3 \mu^2 \nu -$$
$$4 e g^3 Sa\, Sc\, w^3 \mu^2 \nu + g^4 Sa\, Sc\, w^3 \mu^2 \nu + 3 e^2 g^2 Sb\, Sc\, w^3 \mu^2 \nu - 4 e g^3 Sb\, Sc\, w^3 \mu^2 \nu +$$
$$g^4 Sb\, Sc\, w^3 \mu^2 \nu + 3 e^2 g^2 Sc^2 w^3 \mu^2 \nu - 6 e f g^2 Sc^2 w^3 \mu^2 \nu + 3 f^2 g^2 Sc^2 w^3 \mu^2 \nu + 3 e^2 f^2 Sa^2 u^3 \lambda \nu^2 -$$
$$6 e^2 f g\, Sa^2 u^3 \lambda \nu^2 + 3 e^2 g^2 Sa^2 u^3 \lambda \nu^2 + e^4 Sa\, Sb\, u^3 \lambda \nu^2 - 4 e^3 f\, Sa\, Sb\, u^3 \lambda \nu^2 +$$
$$3 e^2 f^2 Sa\, Sb\, u^3 \lambda \nu^2 - 6 e^2 f g\, Sa\, Sb\, u^3 \lambda \nu^2 + 6 e^2 g^2 Sa\, Sb\, u^3 \lambda \nu^2 - 3 e^4 Sb^2 u^3 \lambda \nu^2 +$$
$$3 e^2 g^2 Sb^2 u^3 \lambda \nu^2 + e^4 Sa\, Sc\, u^3 \lambda \nu^2 - 4 e^3 f\, Sa\, Sc\, u^3 \lambda \nu^2 + 3 e^2 f^2 Sa\, Sc\, u^3 \lambda \nu^2 +$$

$e^4 Sb\,Sc\,u^3 \lambda \nu^2 - 4 e^3 f\,Sb\,Sc\,u^3 \lambda \nu^2 + 3 e^2 f^2 Sb\,Sc\,u^3 \lambda \nu^2 + 6 e f^3 Sa^2 u^2 v \lambda \nu^2 -$
$12 e f^2 g\,Sa^2 u^2 v \lambda \nu^2 + 6 e f g^2 Sa^2 u^2 v \lambda \nu^2 - 12 e^2 f^2 Sa\,Sb\,u^2 v \lambda \nu^2 + 6 e f^3 Sa\,Sb\,u^2 v \lambda \nu^2 +$
$6 e^2 f g\,Sa\,Sb\,u^2 v \lambda \nu^2 - 12 e f^2 g\,Sa\,Sb\,u^2 v \lambda \nu^2 + 12 e f g^2 Sa\,Sb\,u^2 v \lambda \nu^2 - 12 e^3 f\,Sb^2 u^2 v \lambda \nu^2 +$
$6 e^2 f g\,Sb^2 u^2 v \lambda \nu^2 + 6 e f g^2 Sb^2 u^2 v \lambda \nu^2 - 6 e^2 f^2 Sa\,Sc\,u^2 v \lambda \nu^2 + 6 e f^3 Sa\,Sc\,u^2 v \lambda \nu^2 -$
$6 e^2 f^2 Sb\,Sc\,u^2 v \lambda \nu^2 + 6 e f^3 Sb\,Sc\,u^2 v \lambda \nu^2 + 3 f^4 Sa^2 u v^2 \lambda \nu^2 - 6 f^3 g\,Sa^2 u v^2 \lambda \nu^2 +$
$3 f^2 g^2 Sa^2 u v^2 \lambda \nu^2 - 3 e^2 f^2 Sa\,Sb\,u v^2 \lambda \nu^2 - 12 e f^3 Sa\,Sb\,u v^2 \lambda \nu^2 + 3 f^4 Sa\,Sb\,u v^2 \lambda \nu^2 +$
$12 e f^2 g\,Sa\,Sb\,u v^2 \lambda \nu^2 - 6 f^3 g\,Sa\,Sb\,u v^2 \lambda \nu^2 + 6 f^2 g^2 Sa\,Sb\,u v^2 \lambda \nu^2 - 15 e^2 f^2 Sb^2 u v^2 \lambda \nu^2 +$
$12 e f^2 g\,Sb^2 u v^2 \lambda \nu^2 + 3 f^2 g^2 Sb^2 u v^2 \lambda \nu^2 - 3 e^2 f^2 Sa\,Sc\,u v^2 \lambda \nu^2 + 3 f^4 Sa\,Sc\,u v^2 \lambda \nu^2 -$
$3 e^2 f^2 Sb\,Sc\,u v^2 \lambda \nu^2 + 3 f^4 Sb\,Sc\,u v^2 \lambda \nu^2 - 2 e f^3 Sa\,Sb\,v^3 \lambda \nu^2 - 4 f^4 Sa\,Sb\,v^3 \lambda \nu^2 +$
$6 f^3 g\,Sa\,Sb\,v^3 \lambda \nu^2 - 6 e f^3 Sb^2 v^3 \lambda \nu^2 + 6 f^3 g\,Sb^2 v^3 \lambda \nu^2 - 2 e f^3 Sa\,Sc\,v^3 \lambda \nu^2 +$
$2 f^4 Sa\,Sc\,v^3 \lambda \nu^2 - 2 e f^3 Sb\,Sc\,v^3 \lambda \nu^2 + 2 f^4 Sb\,Sc\,v^3 \lambda \nu^2 + 6 e f^2 g\,Sa^2 u^2 w \lambda \nu^2 -$
$12 e f g^2 Sa^2 u^2 w \lambda \nu^2 + 6 e g^3 Sa^2 u^2 w \lambda \nu^2 - 6 e^2 f^2 Sa\,Sb\,u^2 w \lambda \nu^2 + 6 e f^2 g\,Sa\,Sb\,u^2 w \lambda \nu^2 -$
$12 e f g^2 Sa\,Sb\,u^2 w \lambda \nu^2 + 12 e g^3 Sa\,Sb\,u^2 w \lambda \nu^2 - 6 e^4 Sb^2 u^2 w \lambda \nu^2 + 6 e g^3 Sb^2 u^2 w \lambda \nu^2 -$
$6 e^2 f g\,Sa\,Sc\,u^2 w \lambda \nu^2 + 6 e f^2 g\,Sa\,Sc\,u^2 w \lambda \nu^2 - 6 e^4 Sb\,Sc\,u^2 w \lambda \nu^2 + 12 e^3 f\,Sb\,Sc\,u^2 w \lambda \nu^2 -$
$6 e^2 f^2 Sb\,Sc\,u^2 w \lambda \nu^2 - 6 e^2 f g\,Sb\,Sc\,u^2 w \lambda \nu^2 + 6 e f^2 g\,Sb\,Sc\,u^2 w \lambda \nu^2 + 6 f^3 g\,Sa^2 u v w \lambda \nu^2 -$
$12 f^2 g^2 Sa^2 u v w \lambda \nu^2 + 6 f g^3 Sa^2 u v w \lambda \nu^2 - 12 e f^3 Sa\,Sb\,u v w \lambda \nu^2 - 6 e^2 f g\,Sa\,Sb\,u v w \lambda \nu^2 +$
$6 f^3 g\,Sa\,Sb\,u v w \lambda \nu^2 + 12 e f g^2 Sa\,Sb\,u v w \lambda \nu^2 - 12 f^2 g^2 Sa\,Sb\,u v w \lambda \nu^2 +$
$12 f g^3 Sa\,Sb\,u v w \lambda \nu^2 - 12 e^3 f\,Sb^2 u v w \lambda \nu^2 - 6 e^2 f g\,Sb^2 u v w \lambda \nu^2 + 12 e f g^2 Sb^2 u v w \lambda \nu^2 +$
$6 f g^3 Sb^2 u v w \lambda \nu^2 - 6 e^2 f g\,Sa\,Sc\,u v w \lambda \nu^2 + 6 f^3 g\,Sa\,Sc\,u v w \lambda \nu^2 - 12 e^3 f\,Sb\,Sc\,u v w \lambda \nu^2 +$
$24 e^2 f^2 Sb\,Sc\,u v w \lambda \nu^2 - 12 e f^3 Sb\,Sc\,u v w \lambda \nu^2 - 6 e^2 f g\,Sb\,Sc\,u v w \lambda \nu^2 + 6 f^3 g\,Sb\,Sc\,u v w \lambda \nu^2 -$
$6 f^4 Sa\,Sb\,v^2 w \lambda \nu^2 - 6 e f^2 g\,Sa\,Sb\,v^2 w \lambda \nu^2 + 12 f^2 g^2 Sa\,Sb\,v^2 w \lambda \nu^2 - 6 e^2 f^2 Sb^2 v^2 w \lambda \nu^2 -$
$6 e f^2 g\,Sb^2 v^2 w \lambda \nu^2 + 12 f^2 g^2 Sb^2 v^2 w \lambda \nu^2 - 6 e f^2 g\,Sa\,Sc\,v^2 w \lambda \nu^2 + 6 f^3 g\,Sa\,Sc\,v^2 w \lambda \nu^2 -$
$6 e^2 f^2 Sb\,Sc\,v^2 w \lambda \nu^2 + 12 e f^3 Sb\,Sc\,v^2 w \lambda \nu^2 - 6 f^4 Sb\,Sc\,v^2 w \lambda \nu^2 - 6 e f^2 g\,Sb\,Sc\,v^2 w \lambda \nu^2 +$
$6 f^3 g\,Sb\,Sc\,v^2 w \lambda \nu^2 - 3 f^4 Sa^2 u w^2 \lambda \nu^2 + 12 f^3 g\,Sa^2 u w^2 \lambda \nu^2 - 15 f^2 g^2 Sa^2 u w^2 \lambda \nu^2 +$
$6 f g^3 Sa^2 u w^2 \lambda \nu^2 - 6 e^2 f^2 Sa\,Sb\,u w^2 \lambda \nu^2 + 12 e^2 f g\,Sa\,Sb\,u w^2 \lambda \nu^2 - 9 e^2 g^2 Sa\,Sb\,u w^2 \lambda \nu^2 -$
$12 e f g^2 Sa\,Sb\,u w^2 \lambda \nu^2 - 3 f^2 g^2 Sa\,Sb\,u w^2 \lambda \nu^2 + 12 e g^3 Sa\,Sb\,u w^2 \lambda \nu^2 + 6 f g^3 Sa\,Sb\,u w^2 \lambda \nu^2 -$
$3 e^4 Sb^2 u w^2 \lambda \nu^2 - 9 e^2 g^2 Sb^2 u w^2 \lambda \nu^2 + 12 e g^3 Sb^2 u w^2 \lambda \nu^2 - 6 e^2 f^2 Sa\,Sc\,u w^2 \lambda \nu^2 +$
$12 e f^3 Sa\,Sc\,u w^2 \lambda \nu^2 - 6 f^4 Sa\,Sc\,u w^2 \lambda \nu^2 + 12 e^2 f g\,Sa\,Sc\,u w^2 \lambda \nu^2 - 24 e f^2 g\,Sa\,Sc\,u w^2 \lambda \nu^2 +$
$12 f^3 g\,Sa\,Sc\,u w^2 \lambda \nu^2 - 9 e^2 g^2 Sa\,Sc\,u w^2 \lambda \nu^2 + 12 e f g^2 Sa\,Sc\,u w^2 \lambda \nu^2 - 3 f^2 g^2 Sa\,Sc\,u w^2 \lambda \nu^2 -$
$6 e^4 Sb\,Sc\,u w^2 \lambda \nu^2 + 12 e^3 f\,Sb\,Sc\,u w^2 \lambda \nu^2 - 6 e^2 f^2 Sb\,Sc\,u w^2 \lambda \nu^2 - 9 e^2 g^2 Sb\,Sc\,u w^2 \lambda \nu^2 +$
$12 e f g^2 Sb\,Sc\,u w^2 \lambda \nu^2 - 3 f^2 g^2 Sb\,Sc\,u w^2 \lambda \nu^2 - 3 e^4 Sc^2 u w^2 \lambda \nu^2 + 12 e^3 f\,Sc^2 u w^2 \lambda \nu^2 -$
$18 e^2 f^2 Sc^2 u w^2 \lambda \nu^2 + 12 e f^3 Sc^2 u w^2 \lambda \nu^2 - 3 f^4 Sc^2 u w^2 \lambda \nu^2 - 12 f^3 g\,Sa\,Sb\,v w^2 \lambda \nu^2 -$
$6 e f g^2 Sa\,Sb\,v w^2 \lambda \nu^2 + 12 f^2 g^2 Sa\,Sb\,v w^2 \lambda \nu^2 + 6 f g^3 Sa\,Sb\,v w^2 \lambda \nu^2 - 12 e^2 f g\,Sb^2 v w^2 \lambda \nu^2 +$
$6 e f g^2 Sb^2 v w^2 \lambda \nu^2 + 6 f g^3 Sb^2 v w^2 \lambda \nu^2 - 6 e f g^2 Sa\,Sc\,v w^2 \lambda \nu^2 + 6 f^2 g^2 Sa\,Sc\,v w^2 \lambda \nu^2 -$
$12 e^2 f g\,Sb\,Sc\,v w^2 \lambda \nu^2 + 24 e f^2 g\,Sb\,Sc\,v w^2 \lambda \nu^2 - 12 f^3 g\,Sb\,Sc\,v w^2 \lambda \nu^2 - 6 e f g^2 Sb\,Sc\,v w^2 \lambda \nu^2 +$
$6 f^2 g^2 Sb\,Sc\,v w^2 \lambda \nu^2 - 6 f^2 g^2 Sa\,Sb\,w^3 \lambda \nu^2 - 2 e g^3 Sa\,Sb\,w^3 \lambda \nu^2 + 8 f g^3 Sa\,Sb\,w^3 \lambda \nu^2 -$
$6 e^2 g^2 Sb^2 w^3 \lambda \nu^2 + 6 e g^3 Sb^2 w^3 \lambda \nu^2 - 2 e g^3 Sa\,Sc\,w^3 \lambda \nu^2 + 2 f g^3 Sa\,Sc\,w^3 \lambda \nu^2 -$
$6 e^2 g^2 Sb\,Sc\,w^3 \lambda \nu^2 + 12 e f g^2 Sb\,Sc\,w^3 \lambda \nu^2 - 6 f^2 g^2 Sb\,Sc\,w^3 \lambda \nu^2 - 2 e g^3 Sb\,Sc\,w^3 \lambda \nu^2 +$
$2 f g^3 Sb\,Sc\,w^3 \lambda \nu^2 - 6 e^3 f\,Sa^2 u^3 \mu \nu^2 + 6 e^3 g\,Sa^2 u^3 \mu \nu^2 - 4 e^4 Sa\,Sb\,u^3 \mu \nu^2 -$
$2 e^3 f\,Sa\,Sb\,u^3 \mu \nu^2 + 6 e^3 g\,Sa\,Sb\,u^3 \mu \nu^2 + 2 e^4 Sa\,Sc\,u^3 \mu \nu^2 - 2 e^3 f\,Sa\,Sc\,u^3 \mu \nu^2 +$
$2 e^4 Sb\,Sc\,u^3 \mu \nu^2 - 2 e^3 f\,Sb\,Sc\,u^3 \mu \nu^2 - 15 e^2 f^2 Sa^2 u^2 v \mu \nu^2 + 12 e^2 f g\,Sa^2 u^2 v \mu \nu^2 +$
$3 e^2 g^2 Sa^2 u^2 v \mu \nu^2 + 3 e^4 Sa\,Sb\,u^2 v \mu \nu^2 - 12 e^3 f\,Sa\,Sb\,u^2 v \mu \nu^2 - 3 e^2 f^2 Sa\,Sb\,u^2 v \mu \nu^2 -$
$6 e^3 g\,Sa\,Sb\,u^2 v \mu \nu^2 + 12 e^2 f g\,Sa\,Sb\,u^2 v \mu \nu^2 + 6 e^2 g^2 Sa\,Sb\,u^2 v \mu \nu^2 + 3 e^4 Sb^2 u^2 v \mu \nu^2 -$
$6 e^3 g\,Sb^2 u^2 v \mu \nu^2 + 3 e^2 g^2 Sb^2 u^2 v \mu \nu^2 + 3 e^4 Sa\,Sc\,u^2 v \mu \nu^2 - 3 e^2 f^2 Sa\,Sc\,u^2 v \mu \nu^2 +$
$3 e^4 Sb\,Sc\,u^2 v \mu \nu^2 - 3 e^2 f^2 Sb\,Sc\,u^2 v \mu \nu^2 - 12 e f^3 Sa^2 u v^2 \mu \nu^2 + 6 e f^2 g\,Sa^2 u v^2 \mu \nu^2 +$
$6 e f g^2 Sa^2 u v^2 \mu \nu^2 + 6 e^3 f\,Sa\,Sb\,u v^2 \mu \nu^2 - 12 e^2 f^2 Sa\,Sb\,u v^2 \mu \nu^2 - 12 e^2 f g\,Sa\,Sb\,u v^2 \mu \nu^2 +$
$6 e f^2 g\,Sa\,Sb\,u v^2 \mu \nu^2 + 12 e f g^2 Sa\,Sb\,u v^2 \mu \nu^2 + 6 e^3 f\,Sb^2 u v^2 \mu \nu^2 - 12 e^2 f g\,Sb^2 u v^2 \mu \nu^2 +$
$6 e f g^2 Sb^2 u v^2 \mu \nu^2 + 6 e^3 f\,Sa\,Sc\,u v^2 \mu \nu^2 - 6 e^2 f^2 Sa\,Sc\,u v^2 \mu \nu^2 + 6 e^3 f\,Sb\,Sc\,u v^2 \mu \nu^2 -$
$6 e^2 f^2 Sb\,Sc\,u v^2 \mu \nu^2 - 3 f^4 Sa^2 v^3 \mu \nu^2 + 3 f^2 g^2 Sa^2 v^3 \mu \nu^2 + 3 e^2 f^2 Sa\,Sb\,v^3 \mu \nu^2 -$
$4 e f^3 Sa\,Sb\,v^3 \mu \nu^2 + f^4 Sa\,Sb\,v^3 \mu \nu^2 - 6 e f^2 g\,Sa\,Sb\,v^3 \mu \nu^2 + 6 f^2 g^2 Sa\,Sb\,v^3 \mu \nu^2 +$
$3 e^2 f^2 Sb^2 v^3 \mu \nu^2 - 6 e f^2 g\,Sb^2 v^3 \mu \nu^2 + 3 f^2 g^2 Sb^2 v^3 \mu \nu^2 + 3 e^2 f^2 Sa\,Sc\,v^3 \mu \nu^2 -$
$4 e f^3 Sa\,Sc\,v^3 \mu \nu^2 + f^4 Sa\,Sc\,v^3 \mu \nu^2 + 3 e^2 f^2 Sb\,Sc\,v^3 \mu \nu^2 - 4 e f^3 Sb\,Sc\,v^3 \mu \nu^2 +$
$f^4 Sb\,Sc\,v^3 \mu \nu^2 - 6 e^2 f^2 Sa^2 u^2 w \mu \nu^2 - 6 e^2 f g\,Sa^2 u^2 w \mu \nu^2 + 12 e^2 g^2 Sa^2 u^2 w \mu \nu^2 -$
$6 e^4 Sa\,Sb\,u^2 w \mu \nu^2 - 6 e^2 f g\,Sa\,Sb\,u^2 w \mu \nu^2 + 12 e^2 g^2 Sa\,Sb\,u^2 w \mu \nu^2 - 6 e^4 Sa\,Sc\,u^2 w \mu \nu^2 +$
$12 e^3 f\,Sa\,Sc\,u^2 w \mu \nu^2 - 6 e^2 f^2 Sa\,Sc\,u^2 w \mu \nu^2 + 6 e^3 g\,Sa\,Sc\,u^2 w \mu \nu^2 - 6 e^2 f g\,Sa\,Sc\,u^2 w \mu \nu^2 +$
$6 e^3 g\,Sb\,Sc\,u^2 w \mu \nu^2 - 6 e^2 f g\,Sb\,Sc\,u^2 w \mu \nu^2 - 12 e f^3 Sa^2 u v w \mu \nu^2 - 6 e f^2 g\,Sa^2 u v w \mu \nu^2 +$
$12 e f g^2 Sa^2 u v w \mu \nu^2 + 6 e g^3 Sa^2 u v w \mu \nu^2 - 12 e^3 f\,Sa\,Sb\,u v w \mu \nu^2 + 6 e^3 g\,Sa\,Sb\,u v w \mu \nu^2 -$
$6 e f^2 g\,Sa\,Sb\,u v w \mu \nu^2 - 12 e^2 g^2 Sa\,Sb\,u v w \mu \nu^2 + 12 e f g^2 Sa\,Sb\,u v w \mu \nu^2 +$

$12 e g^3 Sa\, Sb\, u v w \mu \nu^2 + 6 e^3 g Sb^2 u v w \mu \nu^2 - 12 e^2 g^2 Sb^2 u v w \mu \nu^2 + 6 e g^3 Sb^2 u v w \mu \nu^2 -$
$12 e^3 f Sa\, Sc\, u v w \mu \nu^2 + 24 e^2 f^2 Sa\, Sc\, u v w \mu \nu^2 - 12 e f^3 Sa\, Sc\, u v w \mu \nu^2 + 6 e^3 g Sa\, Sc\, u v w \mu \nu^2 -$
$6 e f^2 g Sa\, Sc\, u v w \mu \nu^2 + 6 e^3 g Sb\, Sc\, u v w \mu \nu^2 - 6 e f^2 g Sb\, Sc\, u v w \mu \nu^2 - 6 f^4 Sa^2 v^2 w \mu \nu^2 +$
$6 f g^3 Sa^2 v^2 w \mu \nu^2 - 6 e^2 f^2 Sa\, Sb\, v^2 w \mu \nu^2 + 6 e^2 f g Sa\, Sb\, v^2 w \mu \nu^2 - 12 e f g^2 Sa\, Sb\, v^2 w \mu \nu^2 +$
$12 f g^3 Sa\, Sb\, v^2 w \mu \nu^2 + 6 e^2 f g Sb^2 v^2 w \mu \nu^2 - 12 e f g^2 Sb^2 v^2 w \mu \nu^2 + 6 f g^3 Sb^2 v^2 w \mu \nu^2 -$
$6 e^2 f^2 Sa\, Sc\, v^2 w \mu \nu^2 + 12 e f^3 Sa\, Sc\, v^2 w \mu \nu^2 - 6 f^4 Sa\, Sc\, v^2 w \mu \nu^2 + 6 e^2 f g Sa\, Sc\, v^2 w \mu \nu^2 -$
$6 e f^2 g Sa\, Sc\, v^2 w \mu \nu^2 + 6 e^2 f g Sb\, Sc\, v^2 w \mu \nu^2 - 6 e f^2 g Sb\, Sc\, v^2 w \mu \nu^2 - 12 e f^2 g Sa^2 u w^2 \mu \nu^2 +$
$6 e f g^2 Sa^2 u w^2 \mu \nu^2 + 6 e g^3 Sa^2 u w^2 \mu \nu^2 - 12 e^3 g Sa\, Sb\, u w^2 \mu \nu^2 + 12 e^2 g^2 Sa\, Sb\, u w^2 \mu \nu^2 -$
$6 e f g^2 Sa\, Sb\, u w^2 \mu \nu^2 + 6 e g^3 Sa\, Sb\, u w^2 \mu \nu^2 - 12 e^3 g Sa\, Sc\, u w^2 \mu \nu^2 + 24 e^2 f g Sa\, Sc\, u w^2 \mu \nu^2 -$
$12 e f^2 g Sa\, Sc\, u w^2 \mu \nu^2 + 6 e^2 g^2 Sa\, Sc\, u w^2 \mu \nu^2 - 6 e f g^2 Sa\, Sc\, u w^2 \mu \nu^2 + 6 e^2 g^2 Sb\, Sc\, u w^2 \mu \nu^2 -$
$6 e f g^2 Sb\, Sc\, u w^2 \mu \nu^2 - 3 f^4 Sa^2 v w^2 \mu \nu^2 - 9 f^2 g^2 Sa^2 v w^2 \mu \nu^2 + 12 f g^3 Sa^2 v w^2 \mu \nu^2 -$
$6 e^2 f^2 Sa\, Sb\, v w^2 \mu \nu^2 + 12 e f^2 g Sa\, Sb\, v w^2 \mu \nu^2 - 3 e^2 g^2 Sa\, Sb\, v w^2 \mu \nu^2 - 12 e f g^2 Sa\, Sb\, v w^2 \mu \nu^2 -$
$9 f^2 g^2 Sa\, Sb\, v w^2 \mu \nu^2 + 6 e g^3 Sa\, Sb\, v w^2 \mu \nu^2 + 12 f g^3 Sa\, Sb\, v w^2 \mu \nu^2 - 3 e^4 Sb^2 v w^2 \mu \nu^2 +$
$12 e^3 g Sb^2 v w^2 \mu \nu^2 - 15 e^2 g^2 Sb^2 v w^2 \mu \nu^2 + 6 e g^3 Sb^2 v w^2 \mu \nu^2 - 6 e^2 f^2 Sa\, Sc\, v w^2 \mu \nu^2 +$
$12 e f^3 Sa\, Sc\, v w^2 \mu \nu^2 - 6 f^4 Sa\, Sc\, v w^2 \mu \nu^2 - 3 e^2 g^2 Sa\, Sc\, v w^2 \mu \nu^2 + 12 e f g^2 Sa\, Sc\, v w^2 \mu \nu^2 -$
$9 f^2 g^2 Sa\, Sc\, v w^2 \mu \nu^2 - 6 e^4 Sb\, Sc\, v w^2 \mu \nu^2 + 12 e^3 f Sb\, Sc\, v w^2 \mu \nu^2 - 6 e^2 f^2 Sb\, Sc\, v w^2 \mu \nu^2 +$
$12 e^3 g Sb\, Sc\, v w^2 \mu \nu^2 - 24 e^2 f g Sb\, Sc\, v w^2 \mu \nu^2 + 12 e f^2 g Sb\, Sc\, v w^2 \mu \nu^2 - 3 e^2 g^2 Sb\, Sc\, v w^2 \mu \nu^2 +$
$12 e f g^2 Sb\, Sc\, v w^2 \mu \nu^2 - 9 f^2 g^2 Sb\, Sc\, v w^2 \mu \nu^2 - 3 e^4 Sc^2 v w^2 \mu \nu^2 + 12 e^3 f Sc^2 v w^2 \mu \nu^2 -$
$18 e^2 f^2 Sc^2 v w^2 \mu \nu^2 + 12 e f^3 Sc^2 v w^2 \mu \nu^2 - 3 f^4 Sc^2 v w^2 \mu \nu^2 - 6 f^2 g^2 Sa^2 w^3 \mu \nu^2 +$
$6 f g^3 Sa^2 w^3 \mu \nu^2 - 6 e^2 g^2 Sa\, Sb\, w^3 \mu \nu^2 + 8 e g^3 Sa\, Sb\, w^3 \mu \nu^2 - 2 f g^3 Sa\, Sb\, w^3 \mu \nu^2 -$
$6 e^2 g^2 Sa\, Sc\, w^3 \mu \nu^2 + 12 e f g^2 Sa\, Sc\, w^3 \mu \nu^2 - 6 f^2 g^2 Sa\, Sc\, w^3 \mu \nu^2 + 2 e g^3 Sa\, Sc\, w^3 \mu \nu^2 -$
$2 f g^3 Sa\, Sc\, w^3 \mu \nu^2 + 2 e g^3 Sb\, Sc\, w^3 \mu \nu^2 - 2 f g^3 Sb\, Sc\, w^3 \mu \nu^2 + 2 e^3 f Sa^2 u^3 \nu^3 - 2 e^3 g Sa^2 u^3 \nu^3 +$
$2 e^4 Sa\, Sb\, u^3 \nu^3 + 2 e^3 f Sa\, Sb\, u^3 \nu^3 - 4 e^3 g Sa\, Sb\, u^3 \nu^3 + 2 e^4 Sb^2 u^3 \nu^3 - 2 e^3 g Sb^2 u^3 \nu^3 +$
$6 e^2 f^2 Sa^2 u^2 v \nu^3 - 6 e^2 f g Sa^2 u^2 v \nu^3 + 6 e^3 f Sa\, Sb\, u^2 v \nu^3 + 6 e^2 f^2 Sa\, Sb\, u^2 v \nu^3 -$
$12 e^2 f g Sa\, Sb\, u^2 v \nu^3 + 6 e^3 f Sb^2 u^2 v \nu^3 - 6 e^2 f g Sb^2 u^2 v \nu^3 + 6 e f^3 Sa^2 u v^2 \nu^3 -$
$6 e f^2 g Sa^2 u v^2 \nu^3 + 6 e^2 f^2 Sa\, Sb\, u v^2 \nu^3 + 6 e f^3 Sa\, Sb\, u v^2 \nu^3 - 12 e f^2 g Sa\, Sb\, u v^2 \nu^3 +$
$6 e^2 f^2 Sb^2 u v^2 \nu^3 - 6 e f^2 g Sb^2 u v^2 \nu^3 + 2 f^4 Sa^2 v^3 \nu^3 - 2 f^3 g Sa^2 v^3 \nu^3 + 2 e f^3 Sa\, Sb\, v^3 \nu^3 +$
$2 f^4 Sa\, Sb\, v^3 \nu^3 - 4 f^3 g Sa\, Sb\, v^3 \nu^3 + 2 e f^3 Sb^2 v^3 \nu^3 - 2 f^3 g Sb^2 v^3 \nu^3 + 3 e^2 f^2 Sa^2 u^2 w \nu^3 -$
$3 e^2 g^2 Sa^2 u^2 w \nu^3 + 3 e^4 Sa\, Sb\, u^2 w \nu^3 + 3 e^2 f^2 Sa\, Sb\, u^2 w \nu^3 - 6 e^2 g^2 Sa\, Sb\, u^2 w \nu^3 +$
$3 e^4 Sb^2 u^2 w \nu^3 - 3 e^2 g^2 Sb^2 u^2 w \nu^3 + 3 e^4 Sa\, Sc\, u^2 w \nu^3 - 6 e^3 f Sa\, Sc\, u^2 w \nu^3 + 3 e^2 f^2 Sa\, Sc\, u^2 w \nu^3 +$
$3 e^4 Sb\, Sc\, u^2 w \nu^3 - 6 e^3 f Sb\, Sc\, u^2 w \nu^3 + 3 e^2 f^2 Sb\, Sc\, u^2 w \nu^3 + 6 e f^3 Sa^2 u v w \nu^3 -$
$6 e f g^2 Sa^2 u v w \nu^3 + 6 e^3 f Sa\, Sb\, u v w \nu^3 + 6 e f^3 Sa\, Sb\, u v w \nu^3 - 12 e f g^2 Sa\, Sb\, u v w \nu^3 +$
$6 e^3 f Sb^2 u v w \nu^3 - 6 e f g^2 Sb^2 u v w \nu^3 + 6 e^3 f Sa\, Sc\, u v w \nu^3 - 12 e^2 f^2 Sa\, Sc\, u v w \nu^3 +$
$6 e f^3 Sa\, Sc\, u v w \nu^3 + 6 e^3 f Sb\, Sc\, u v w \nu^3 - 12 e^2 f^2 Sb\, Sc\, u v w \nu^3 + 6 e f^3 Sb\, Sc\, u v w \nu^3 +$
$3 f^4 Sa^2 v^2 w \nu^3 - 3 f^2 g^2 Sa^2 v^2 w \nu^3 + 3 e^2 f^2 Sa\, Sb\, v^2 w \nu^3 + 3 f^4 Sa\, Sb\, v^2 w \nu^3 -$
$6 f^2 g^2 Sa\, Sb\, v^2 w \nu^3 + 3 e^2 f^2 Sb^2 v^2 w \nu^3 - 3 f^2 g^2 Sb^2 v^2 w \nu^3 + 3 e^2 f^2 Sa\, Sc\, v^2 w \nu^3 -$
$6 e f^3 Sa\, Sc\, v^2 w \nu^3 + 3 f^4 Sa\, Sc\, v^2 w \nu^3 + 3 e^2 f^2 Sb\, Sc\, v^2 w \nu^3 - 6 e f^3 Sb\, Sc\, v^2 w \nu^3 +$
$3 f^4 Sb\, Sc\, v^2 w \nu^3 + 6 e f^2 g Sa^2 u w^2 \nu^3 - 6 e f g^2 Sa^2 u w^2 \nu^3 + 6 e^3 g Sa\, Sb\, u w^2 \nu^3 +$
$6 e f^2 g Sa\, Sb\, u w^2 \nu^3 - 6 e^2 g^2 Sa\, Sb\, u w^2 \nu^3 - 6 e f g^2 Sa\, Sb\, u w^2 \nu^3 + 6 e^3 g Sb^2 u w^2 \nu^3 -$
$6 e^2 g^2 Sb^2 u w^2 \nu^3 + 6 e^3 g Sa\, Sc\, u w^2 \nu^3 - 12 e^2 f g Sa\, Sc\, u w^2 \nu^3 + 6 e f^2 g Sa\, Sc\, u w^2 \nu^3 +$
$6 e^3 g Sb\, Sc\, u w^2 \nu^3 - 12 e^2 f g Sb\, Sc\, u w^2 \nu^3 + 6 e f^2 g Sb\, Sc\, u w^2 \nu^3 + 6 f^3 g Sa^2 v w^2 \nu^3 -$
$6 f^2 g^2 Sa^2 v w^2 \nu^3 + 6 e^2 f g Sa\, Sb\, v w^2 \nu^3 + 6 f^3 g Sa\, Sb\, v w^2 \nu^3 - 6 e f g^2 Sa\, Sb\, v w^2 \nu^3 -$
$6 f^2 g^2 Sa\, Sb\, v w^2 \nu^3 + 6 e^2 f g Sb^2 v w^2 \nu^3 - 6 e f g^2 Sb^2 v w^2 \nu^3 + 6 e^2 f g Sa\, Sc\, v w^2 \nu^3 -$
$12 e f^2 g Sa\, Sc\, v w^2 \nu^3 + 6 f^3 g Sa\, Sc\, v w^2 \nu^3 + 6 e^2 f g Sb\, Sc\, v w^2 \nu^3 - 12 e f^2 g Sb\, Sc\, v w^2 \nu^3 +$
$6 f^3 g Sb\, Sc\, v w^2 \nu^3 - f^4 Sa^2 w^3 \nu^3 + 4 f^3 g Sa^2 w^3 \nu^3 - 3 f^2 g^2 Sa^2 w^3 \nu^3 - 2 e^2 f^2 Sa\, Sb\, w^3 \nu^3 +$
$4 e^2 f g Sa\, Sb\, w^3 \nu^3 + 4 e f^2 g Sa\, Sb\, w^3 \nu^3 + e^2 g^2 Sa\, Sb\, w^3 \nu^3 - 8 e f g^2 Sa\, Sb\, w^3 \nu^3 +$
$f^2 g^2 Sa\, Sb\, w^3 \nu^3 - e^4 Sb^2 w^3 \nu^3 + 4 e^3 g Sb^2 w^3 \nu^3 - 3 e^2 g^2 Sb^2 w^3 \nu^3 - 2 e^2 f^2 Sa\, Sc\, w^3 \nu^3 +$
$4 e f^3 Sa\, Sc\, w^3 \nu^3 - 2 f^4 Sa\, Sc\, w^3 \nu^3 + 4 e^2 f g Sa\, Sc\, w^3 \nu^3 - 8 e f^2 g Sa\, Sc\, w^3 \nu^3 +$
$4 f^3 g Sa\, Sc\, w^3 \nu^3 + e^2 g^2 Sa\, Sc\, w^3 \nu^3 - 2 e f g^2 Sa\, Sc\, w^3 \nu^3 + f^2 g^2 Sa\, Sc\, w^3 \nu^3 - 2 e^4 Sb\, Sc\, w^3 \nu^3 +$
$4 e^3 f Sb\, Sc\, w^3 \nu^3 - 2 e^2 f^2 Sb\, Sc\, w^3 \nu^3 + 4 e^3 g Sb\, Sc\, w^3 \nu^3 - 8 e^2 f g Sb\, Sc\, w^3 \nu^3 +$
$4 e f^2 g Sb\, Sc\, w^3 \nu^3 + e^2 g^2 Sb\, Sc\, w^3 \nu^3 - 2 e f g^2 Sb\, Sc\, w^3 \nu^3 + f^2 g^2 Sb\, Sc\, w^3 \nu^3 -$
$e^4 Sc^2 w^3 \nu^3 + 4 e^3 f Sc^2 w^3 \nu^3 - 6 e^2 f^2 Sc^2 w^3 \nu^3 + 4 e f^3 Sc^2 w^3 \nu^3 - f^4 Sc^2 w^3 \nu^3)$

For the 4 lines of a quadrilateral (1,0,0), (0,1,0), (0,0,1), (l,m,n)
this gives 4 equations.
Eliminating e, f, g there remain two equations

for the centers (if the following terms are zero).

```
T1[{u_, v_, w_}] :=
 (a^4 b^6 c^6 u^9 - 2 a^2 b^8 c^6 u^9 + b^10 c^6 u^9 - 2 a^2 b^6 c^8 u^9 - 2 b^8 c^8 u^9 + b^6 c^10 u^9 + 27 a^2 b^6 c^8 u^7 v^2 +
   3 a^6 b^4 c^6 u^6 v^3 - 6 a^4 b^6 c^6 u^6 v^3 + 3 a^2 b^8 c^6 u^6 v^3 - 6 a^4 b^4 c^8 u^6 v^3 + 21 a^2 b^6 c^8 u^6 v^3 +
   3 a^2 b^4 c^10 u^6 v^3 - 54 a^4 b^4 c^8 u^5 v^4 - 54 a^4 b^4 c^8 u^4 v^5 + 3 a^8 b^2 c^6 u^3 v^6 - 6 a^6 b^4 c^6 u^3 v^6 +
   3 a^4 b^6 c^6 u^3 v^6 + 21 a^6 b^2 c^8 u^3 v^6 - 6 a^4 b^4 c^8 u^3 v^6 + 3 a^4 b^2 c^10 u^3 v^6 + 27 a^6 b^2 c^8 u^2 v^7 +
   a^10 c^6 v^9 - 2 a^8 b^2 c^6 v^9 + a^6 b^4 c^6 v^9 - 2 a^8 c^8 v^9 - 2 a^6 b^2 c^8 v^9 + a^6 c^10 v^9 - 27 a^4 b^6 c^6 u^7 v w +
   27 a^2 b^8 c^6 u^7 v w + 27 a^2 b^6 c^8 u^7 v w - 27 a^4 b^6 c^6 u^6 v^2 w + 27 a^2 b^8 c^6 u^6 v^2 w + 54 a^2 b^6 c^8 u^6 v^2 w +
   27 a^6 b^4 c^6 u^5 v^3 w - 27 a^4 b^6 c^6 u^5 v^3 w - 27 a^4 b^4 c^8 u^5 v^3 w - 108 a^4 b^4 c^8 u^4 v^4 w -
   27 a^6 b^4 c^6 u^3 v^5 w + 27 a^4 b^6 c^6 u^3 v^5 w - 27 a^4 b^4 c^8 u^3 v^5 w + 27 a^8 b^2 c^6 u^2 v^6 w - 27 a^6 b^4 c^6 u^2 v^6 w +
   54 a^6 b^2 c^8 u^2 v^6 w + 27 a^8 b^2 c^6 u v^7 w - 27 a^6 b^4 c^6 u v^7 w + 27 a^6 b^2 c^8 u v^7 w + 27 a^2 b^8 c^6 u^7 w^2 -
   27 a^4 b^6 c^6 u^6 v w^2 + 54 a^2 b^8 c^6 u^6 v w^2 + 27 a^2 b^6 c^8 u^6 v w^2 + 54 a^6 b^4 c^6 u^4 v^3 w^2 -
   54 a^4 b^4 c^8 u^4 v^3 w^2 + 54 a^4 b^6 c^6 u^3 v^4 w^2 - 54 a^4 b^4 c^8 u^3 v^4 w^2 + 54 a^8 b^2 c^6 u v^6 w^2 -
   27 a^6 b^4 c^6 u v^6 w^2 + 27 a^6 b^2 c^8 u v^6 w^2 + 27 a^8 b^2 c^6 v^7 w^2 + 3 a^6 b^6 c^4 u^6 w^3 - 6 a^4 b^8 c^4 u^6 w^3 +
   3 a^2 b^10 c^4 u^6 w^3 - 6 a^4 b^6 c^6 u^6 w^3 + 21 a^2 b^8 c^6 u^6 w^3 + 3 a^2 b^6 c^8 u^6 w^3 + 27 a^6 b^6 c^4 u^5 v w^3 -
   27 a^4 b^8 c^4 u^5 v w^3 - 27 a^4 b^6 c^6 u^5 v w^3 + 54 a^6 b^6 c^4 u^4 v^2 w^3 - 54 a^4 b^8 c^4 u^4 v^2 w^3 -
   21 a^8 b^4 c^4 u^3 v^3 w^3 + 42 a^6 b^6 c^4 u^3 v^3 w^3 - 21 a^4 b^8 c^4 u^3 v^3 w^3 + 42 a^6 b^4 c^6 u^3 v^3 w^3 +
   42 a^4 b^6 c^6 u^3 v^3 w^3 - 21 a^4 b^4 c^8 u^3 v^3 w^3 - 54 a^8 b^4 c^4 u^2 v^4 w^3 + 54 a^6 b^6 c^4 u^2 v^4 w^3 -
   27 a^8 b^4 c^4 u v^5 w^3 + 27 a^6 b^6 c^4 u v^5 w^3 - 27 a^6 b^4 c^6 u v^5 w^3 + 3 a^10 b^2 c^4 v^6 w^3 - 6 a^8 b^4 c^4 v^6 w^3 +
   3 a^6 b^6 c^4 v^6 w^3 + 21 a^8 b^2 c^6 v^6 w^3 - 6 a^6 b^4 c^6 v^6 w^3 + 3 a^6 b^2 c^8 v^6 w^3 - 54 a^4 b^8 c^4 u^5 w^4 -
   108 a^4 b^8 c^4 u^4 v w^4 - 54 a^4 b^8 c^4 u^3 v^2 w^4 + 54 a^4 b^6 c^6 u^3 v^2 w^4 - 54 a^8 b^4 c^4 u^2 v^3 w^4 +
   54 a^6 b^4 c^6 u^2 v^3 w^4 - 108 a^8 b^4 c^4 u v^4 w^4 - 54 a^8 b^4 c^4 v^5 w^4 - 54 a^4 b^8 c^4 u^4 w^5 - 27 a^6 b^6 c^4 u^3 v w^5 -
   27 a^4 b^8 c^4 u^3 v w^5 + 27 a^4 b^6 c^6 u^3 v w^5 - 27 a^8 b^4 c^4 u v^3 w^5 - 27 a^6 b^6 c^4 u v^3 w^5 +
   27 a^6 b^4 c^6 u v^3 w^5 - 54 a^8 b^4 c^4 v^4 w^5 + 3 a^8 b^6 c^2 u^3 w^6 + 21 a^6 b^8 c^2 u^3 w^6 + 3 a^4 b^10 c^2 u^3 w^6 -
   6 a^6 b^6 c^4 u^3 w^6 - 6 a^4 b^8 c^4 u^3 w^6 + 3 a^4 b^6 c^6 u^3 w^6 + 27 a^8 b^6 c^2 u^2 v w^6 + 54 a^6 b^8 c^2 u^2 v w^6 -
   27 a^6 b^6 c^4 u^2 v w^6 + 54 a^8 b^6 c^2 u v^2 w^6 + 27 a^6 b^8 c^2 u v^2 w^6 - 27 a^6 b^6 c^4 u v^2 w^6 +
   3 a^10 b^4 c^2 v^3 w^6 + 21 a^8 b^6 c^2 v^3 w^6 + 3 a^6 b^8 c^2 v^3 w^6 - 6 a^8 b^4 c^4 v^3 w^6 - 6 a^6 b^6 c^4 v^3 w^6 +
   3 a^6 b^4 c^6 v^3 w^6 + 27 a^6 b^8 c^2 u^2 w^7 + 27 a^8 b^6 c^2 u v w^7 + 27 a^6 b^8 c^2 u v w^7 - 27 a^6 b^6 c^4 u v w^7 +
   27 a^8 b^6 c^2 v^2 w^7 + a^10 b^6 w^9 - 2 a^8 b^8 w^9 + a^6 b^10 w^9 - 2 a^8 b^6 c^2 w^9 - 2 a^6 b^8 c^2 w^9 + a^6 b^6 c^4 w^9)
```

```
T2[{u_, v_, w_}] := b^2 c^2 l (-b^2 (-l + m) (m - n) - c^2 (-l + n) (-m + n) + a^2 (l m + l n - m n)) u^3
  +a^2 c^2 m (-a^2 (l - m) (l - n) - c^2 (-l + n) (-m + n) + b^2 (l m - l n + m n)) v^3
  +a^2 b^2 n (-a^2 (l - m) (l - n) - b^2 (-l + m) (m - n) + c^2 (-l m + l n + m n)) w^3
  +3 a^2 b^2 c^2 (l u + m v) (l u + n w) (m v + n w)
```

The first term includes the 9 lines of the 3 sets of 3 parallels
for the reference triangle. All points on these lines are centers.
The second Term gives a cubic for the centers wrt a quadrilateral.
The intersections of the 9 lines with the cubic are centers of cardioids tangent to 4lines.

With this two equations you can calculate the centers for a numeral example:

```
a := 6; b := 5; c := 4; l := -1; m := -2; n := 2
```

```
w := 1; Solve[{T1[{u, v, w}] == 0, T2[{u, v, w}] == 0}, {u, v}] // N
```

```
{{v → 42.1651, u → -59.7817}, {v → -0.332074, u → -0.203511}, {v → -0.486272, u → 0.0606063},
 {v → -0.775976, u → 0.467078}, {v → 1.07108, u → 1.1019}, {v → 0.824662, u → 1.46698},
 {v → 0.620615, u → 1.57656}, {v → -2.28105, u → 3.13479}, {v → 7.1962, u → 6.80032},
 {v → 0., u → -118.115}, {v → -52.7021, u → -16.2911}, {v → 3.99064, u → -5.6307},
 {v → -0.0491699, u → -2.2956}, {v → -0.00262426, u → -2.02651}, {v → 1.49691, u → -1.73695},
 {v → 0.0347593, u → -1.42841}, {v → 1.27313, u → -1.14068}, {v → 1.23213, u → -0.969945},
 {v → -0.165721, u → -0.549684}, {v → -3.30854, u → 0.395195}, {v → 1.15807, u → 0.521973},
 {v → -0.962308, u → 0.665771}, {v → -2.07772, u → 0.81565}, {v → -0.580635, u → 1.63461},
 {v → -1.46926, u → 2.08798}, {v → -1.6838, u → 2.34055}, {v → -3252.47, u → 22.8887}}
```

```
Clear[a, b, c, u, v, w]
```

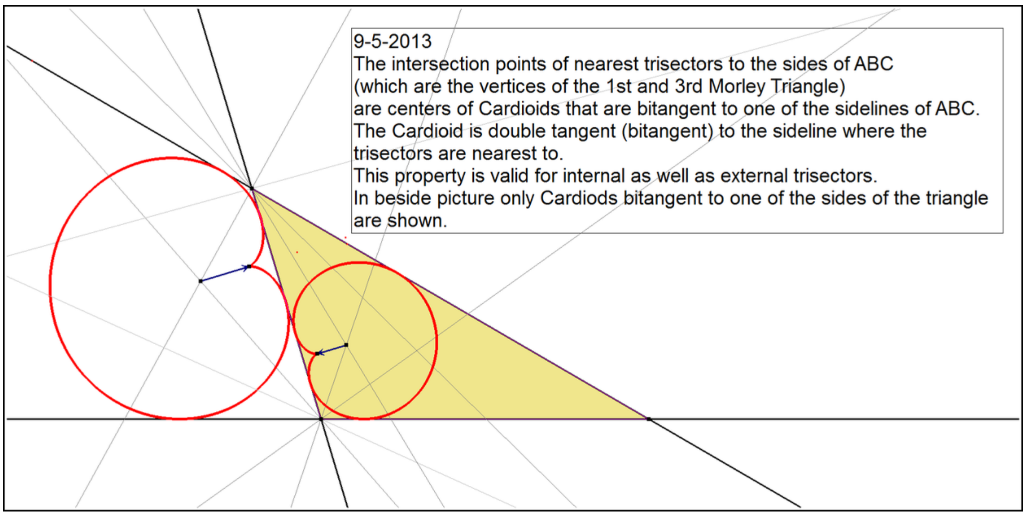
The cusps corresponding to a center (u:v:w) can be calculated as follows:

```
cusp[{u_, v_, w_}] := {b^2 c^2 u^3, a^2 c^2 v^3, a^2 b^2 w^3}
```

Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

Message: #65
Date: 17/6/2013 4:46:39
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

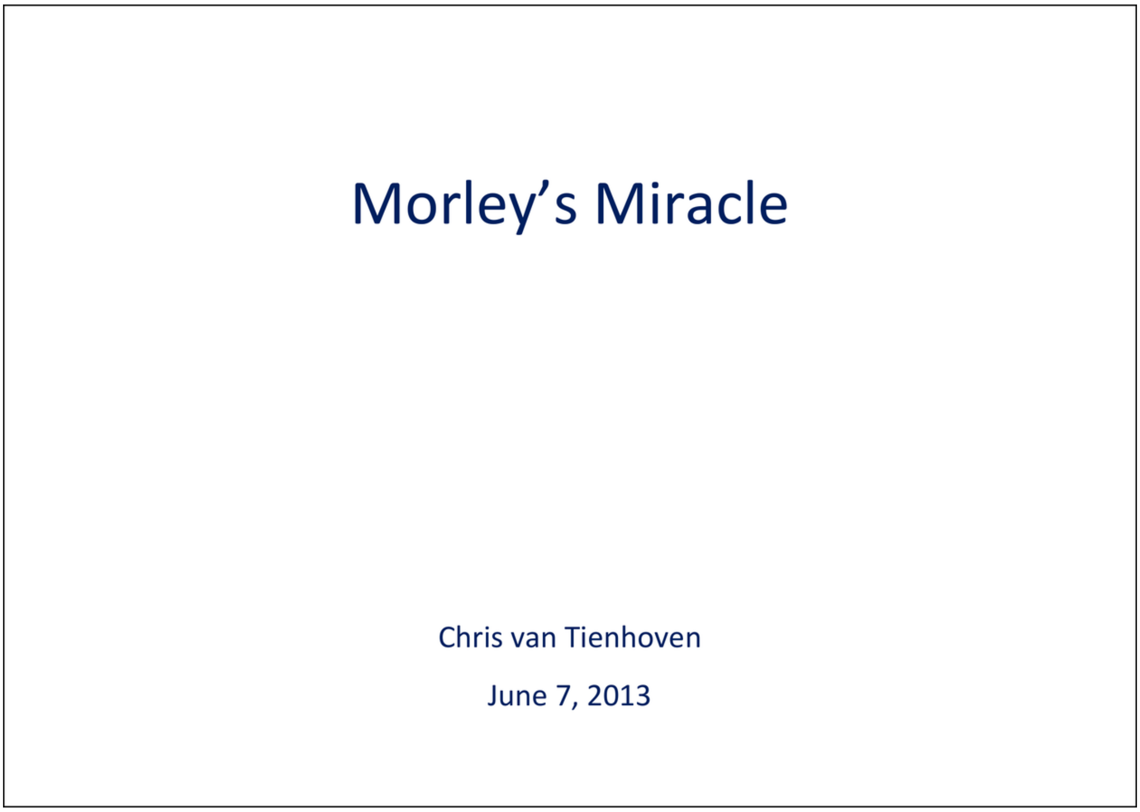
COPY FROM FORMER GOOGLE GROUP by Chris van Tienhoven (May 26, 2013)
Dear Friends,
Tonight I will be flying to Malta with my wife for a 5-day-holiday.
Before I go want to thank you for all further information.
Also I want to share with you what I found about Cardioids tangent to all sides of a triangles:
The intersection points of nearest trisectors to the sides of ABC
(which are the vertices of the 1st and 3rd Morley Triangle)
are centers of Cardioids that are bitangent to one of the sidelines of ABC.
The Cardioid is double tangent (bitangent) to the sideline where the trisectors are nearest to.
This property is valid for internal as well as external trisectors.
In beside picture only Cardiods bitangent to one of the sides of the triangle are shown.
I made a picture from this property . See attachment in different formats..
Does anyone of you know if this property is known? References?
When I come back I want to investigate further Cardioids tangent to all sides of a Quadrilateral and make pictures.
See you next week,
Chris



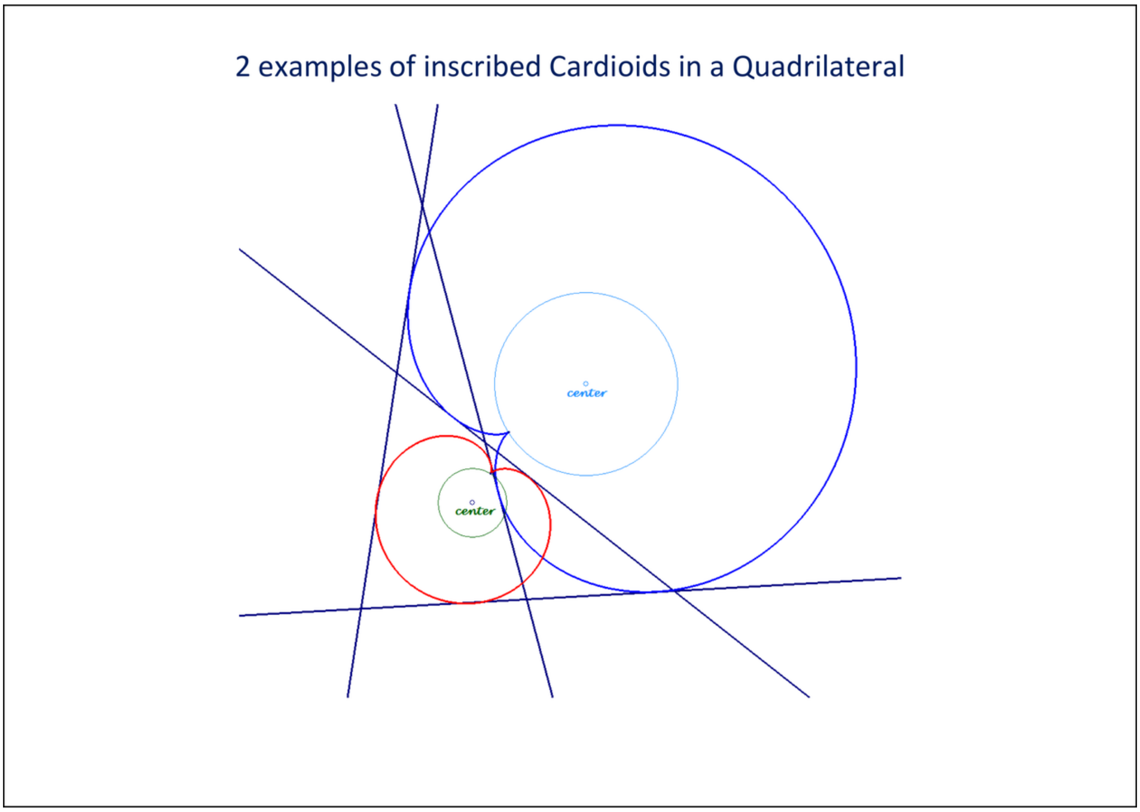
QL-Qu98-Cardioids-12-Tr.png

Message: #66
Date: 17/6/2013 5:16:42
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

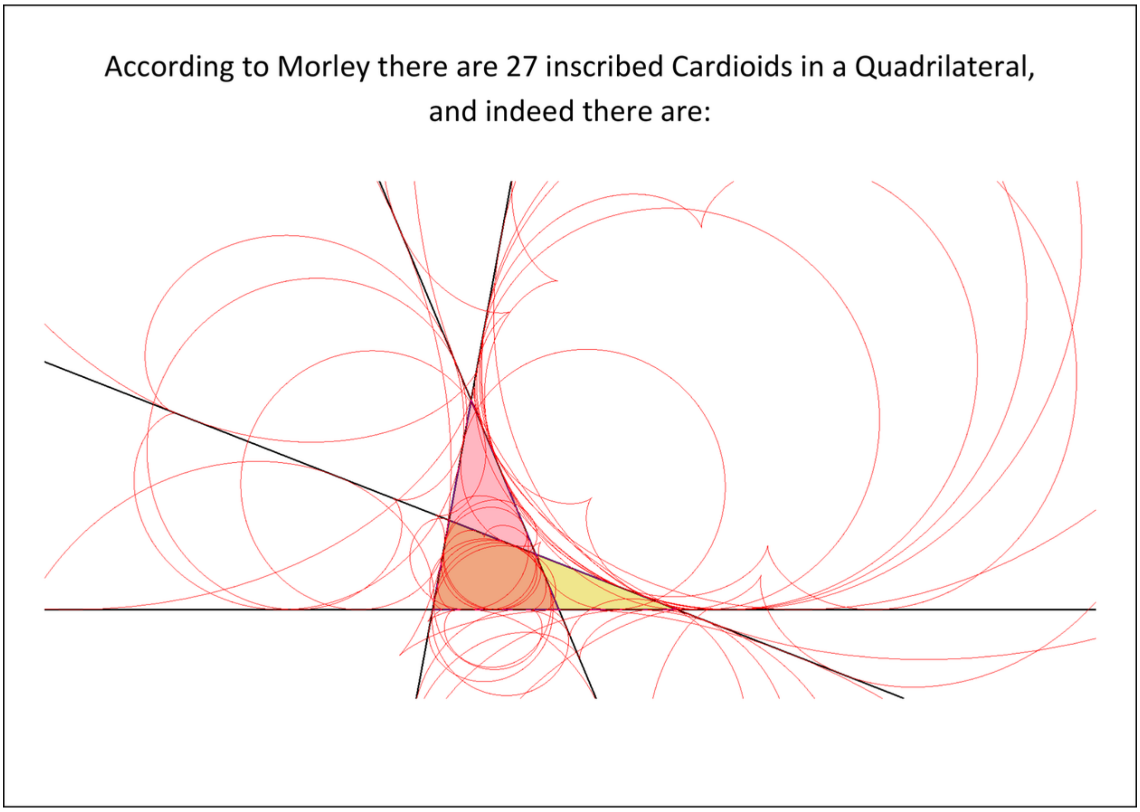
```
COPY FROM FORMER GOOGLE GROUP  by  Chris van Tienhoven
(June 7, 2013)
Dear friends,
I managed to make a picture of all possible 27 Cardioids
inscribed in a Quadrilateral.
I made a presentation of the process of constructing the
inscribed Cardioids.
See attachment.
Next step remains calculating the coordinates of the 27 Cardioid
centers.
Best regards,
Chris van Tienhoven
```



QL-Qu98-MorleyCardioids-01.pdf

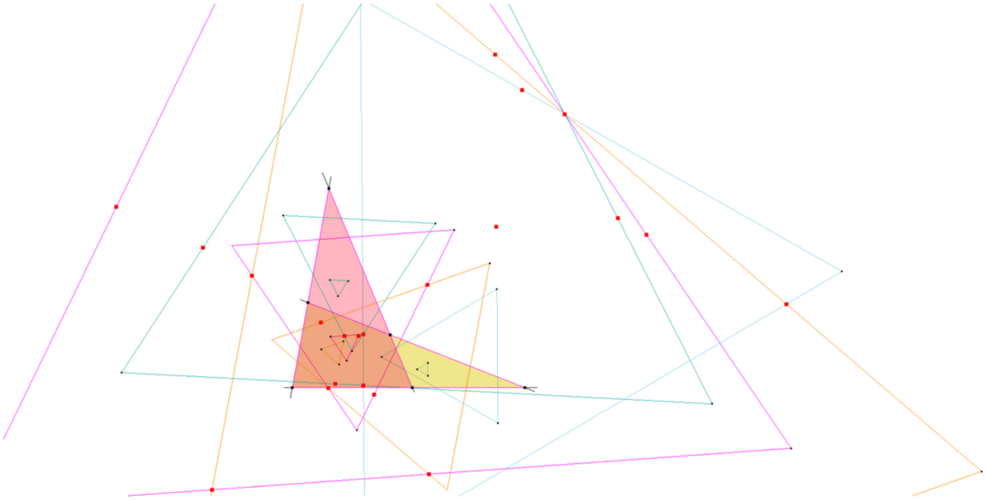


QL-Qu98-MorleyCardioids-01.pdf



QL-Qu98-MorleyCardioids-01.pdf

4 Times 3 equilateral triangles in a Quadrilateral are needed to construct them.



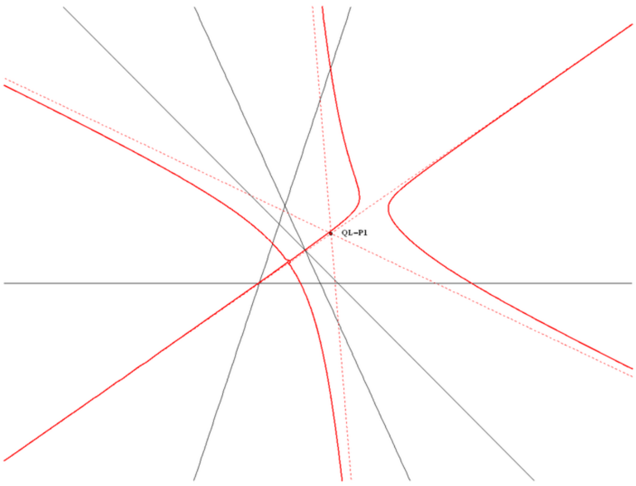
The 27 red intersections of the (extended) sides of the 4x3 equilateral triangles are the centers of the inscribed Cardioids.

Each center is the intersection point of 4 sidelines.

QL-Qu98-MorleyCardioids-01.pdf

The 27 Cardioid Centers lie on Eckart's Cubic

(found by Eckart Schmidt, May 2013)



The asymptotes are concurrent in the Miquel Point (QL-P1) and cross each other at 60 degrees

QL-Qu98-MorleyCardioids-01.pdf

Message: #67
Date: 17/6/2013 5:16:43
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP   by Chris van Tienhoven
(June 5, 2013)
Dear friends,
After a wonderful holiday at Malta I couldn’t wait for further exploring the Cardioids.
However I am very busy with all kinds of things and they all seem to be very important.
Anyway I took my time to read the paper of Frank Morley:
Extensions of Clifford’s Chain-Theorem.
At page 470 Morley says quite below:
“the n(n-1)^(n-2) axes of the component (n-1)-lines meet in the (n-1)^(n-1) incenters,
there being n axes on a point and (n-1) points on an axis.”
When I substitute n = 4, then it says:
“the 4*9 axes of the component 3-lines (triangles) meet in 27 incenters,
there being 4 axes on a point and 3 points on an axis.”
This is exactly what Eckart found out.
Stated in my own words:
In a Quadrilateral we have 4 Component Triangles. Each Component Triangle has its own set of 3 * 3 parallel axes.
Morley’s Miracle is that these 4 sets of 3*3 axes intersect in exactly 27 points. Each Axis contains 3 of these points.
Eckart found out that these points lie on a Eckart’s cubic.
It’s hard to understand how it can be, but it happens to be.
I myself started to calculate these 27 points and I think I just need some time to succeed.
Besides this Morley also wrote:
“n(n-2)^(n-2) incenters are on the (n-2)^(n-1) circles, there being n points on each circle, (n-2) circles on each point.
These (n-2) circles cut at the angle pi/(n-2).
The (n-3)^(n-1) cardioids arise from the n(n-3)^(n-2) circles;
the circles are osculants of the cardiods, each having n osculants C3’s;
And each C2 osculating (n-3) C3’s.”
When I substitute n = 4 and rephrase a bit, then it says:
“2*4 incenters are on the 8 circles, there being 4 points on each circle, 2 circles on each point.
These 2 circles cut at the angle pi/2. (this is Seiners theorem, rules 8 and 9! See QL-8P1.)
The 1 cardioid arising from the 4*1 circles; the circles are inscribed in the single cardioid;
And each circle osculating (is inscribed) in the single cardioid. (This is QL-Qu1) !”

Bernard, you already mentioned the property of the single
Cardiod that Morley mentioned! Thanks, it made me think.
Eckart, I wish you a beautiful and inspired holiday!
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #68
Date: 17/6/2013 5:16:44
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(June 5, 2013)
Dear friends,
before 10 days holiday a last message. Thanks to Bernard for his summary with new questions.
But calculations and constructions wrt the center cubic are difficult because of the trisection of angles in the background.
At most numeral examples can be tested.
But I am sure of the following properties for the reference triangle of the McCay Cubic:
... centroid is the Miquel Point QL-P1
... Brocard axis X3.X6=OK is the Newton Line QL-L1
... axes of the deltoid of the Simson lines are parallel to the axes of the deltoid of the quadrilarteral
... a parallel to the Newton Line with 2/3 distance to QL-P1 cuts the cubic on its asymptotes
There are so many properties for the McCay Cubic in B. Gibert`s catalogue perhaps with further relations to the problem.
Best regards Eckart
P.S. For Bernard: The 27 centers make no Morley set for the reference triangle of the McCay Cubic.

← Previous Next → ↔ Message Index ↑ Subjects

Message: #69
Date: 17/6/2013 5:16:47
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
COPY FROM FORMER GOOGLE GROUP  by Eckart Schmidt
(May 27, 2013)
Dear friends,
Chris has opened a new sight of the geometry of the centers of
inscribed cardioids of a triangle - thanks!
There are further properties wrt this question in the
attachment.
Best regards Eckart
```

27.05.2013

Inscribed cardioïds of a triangle
bitangent to one side

Morley proved, that the centers of inscribed cardioïds of a triangle lie on 3 sets of 3 parallel lines, forming equilateral triangles (see Bernard´s first message). The sidelines of the first and third Morley triangle belong to these sets. Every point on these lines can be a center of an inscribed cardioïd.

Here we examine those inscribed cardioïds, which are bitangent to one side: Chris (message 26.05) has worked out, that the vertices of the first and third Morley triangle belong to these centers. For one side of the triangle – further on AB – there will be nine centers. For a numeral example the centers $(u:v:w)$ can be calculated with the following equations (see my message 25.05.)

$$-c^4u^3+3c^2a^2uw^2+2a^2S_Bw^3=0,$$
$$-c^4v^3+3b^2c^2vw^2+2b^2S_Aw^3=0.$$

Now the geometry of these centers will be described: Let $A'B'C'$ be the first Morley triangle and $A''B''C''$ the third Morley triangle. Further points will be:

$$A^*=AC'\cap A'B',\quad B^*=BC'\cap A'B',$$
$$A^{**}=AC''\cap B'C',\quad B^{**}=BC''\cap A'C',$$
$$U=AB^*\cap BA^*,\quad V=AA^*\cap BB^{**},\quad W=BB^*\cap AA^{**}.$$

The centers of the inscribed cardioïds bitangent to AB are:

C', C'', A^*, B^*, A^{**}, B^{**}, U, V, W.

Collinear on the sides of the first Morley triangle:

A', B', A^*, B^* – B', C', A^{**} – C', A', B^{**}

Collinear on one side of the third Morley triangle:

A'', B'', V, W

Further collinear points:

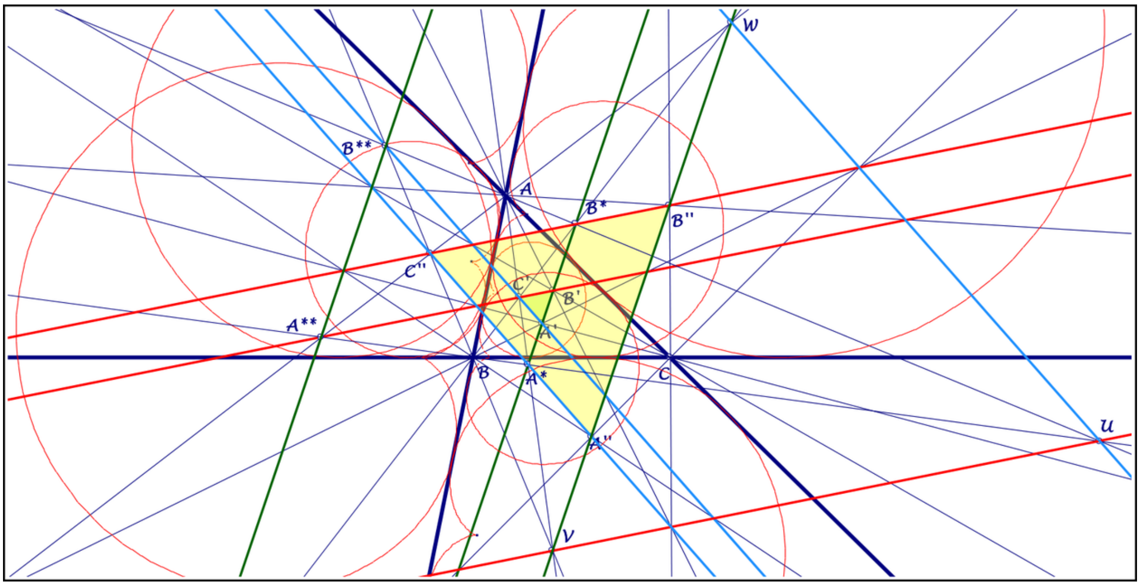
A, B^*, B^{**}, U – B, A^*, A^{**}, U

A, A^*, V – B, B^*, W – A, A^{**}, C'', W – B, B^{**}, C'', V

The 3 sets of 3 parallels for all centers are:

$A'C'$, $A''C''$, UW – $B'C'$, B^*C^*, UV – $A'B'$, $A''B''$, $A^{**}B^{**}$

Card-06 .pdf



Card-06b.pdf

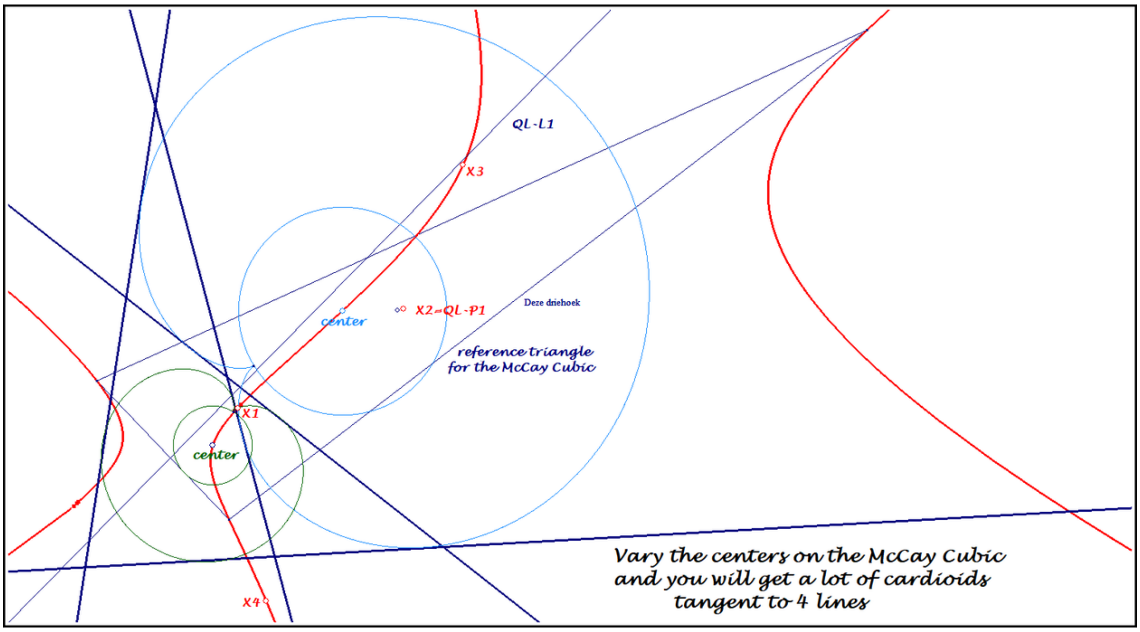
← Previous Next → ↩ Message Index ↑ Subjects

Message: #70
Date: 17/6/2013 5:16:49
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP   by   Bernard Keizer
(May 27, 2013)
Dear friends,
Thank you again for all the new discoveries
For Chris: in my first questionning paper, I wrote " The main property of the cardioïd is that the line KiO trisects the angle between the tangent and the bitangent ".
That's exactly how Morley discovered his Morley's theorem (known often as Morley's miracle: see Bogomolny ...)
For a given triangle, there are exactly 27 centers of cardioïds bitangent to one of the 3 sides: they are the vertices of 27 equilateral triangles (of which the first and the third Morley's triangles) and those 27 points are on 3*3 = 9 lines , whose direction is the mean direction (m° pi/3) of the 3 sides..
That's what we could call the Morley set for this triangle.
Back to the QL: the centers of the cardioïds tangent to the 4 lines belong to the 4 Morley's sets of the 4 reference triangles of the QL.
The discovery (for me, at last) is what Eckart proved: there are 27 points intersection of the 4 sets (each point on one line of each set and each of the 4*9 = 36 lines having 3 points) and those 27 points belong to the McKay cubic of a new triangle!
For Eckart: how do you find that there is a deltoïd envelopping the Simson lines of that triangle? If this is true, the triangle is a main triangle of the deltoïd and the deltoïd is the Steiner deltoïd (tangent to the the sides and the altitudes) of this triangle and the Euler point of the triangle is the center of the inscribed circle of the deltoïd. As you have the centroïd as the point QL-P1 and the circumcenter and the orthocenter on the cubic, I wonder where all those points are (you say the circumcenter, which is the pivot of the cubic is on the line QL-L1 ...)
My last question is on those 27 points: did I understand correctly that they are making a Morley set for the last triangle? In that case, the direction of the lines is again the mean direction of the 3 sides and therefore parallel to the 3 axes of the Steiner deltoïd of the last triangle (what's the link between this new deltoïd and the deltoïd tangent to the 4 lines?).
Best regards
Bernard

Message: #71
Date: 17/6/2013 5:18:30
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

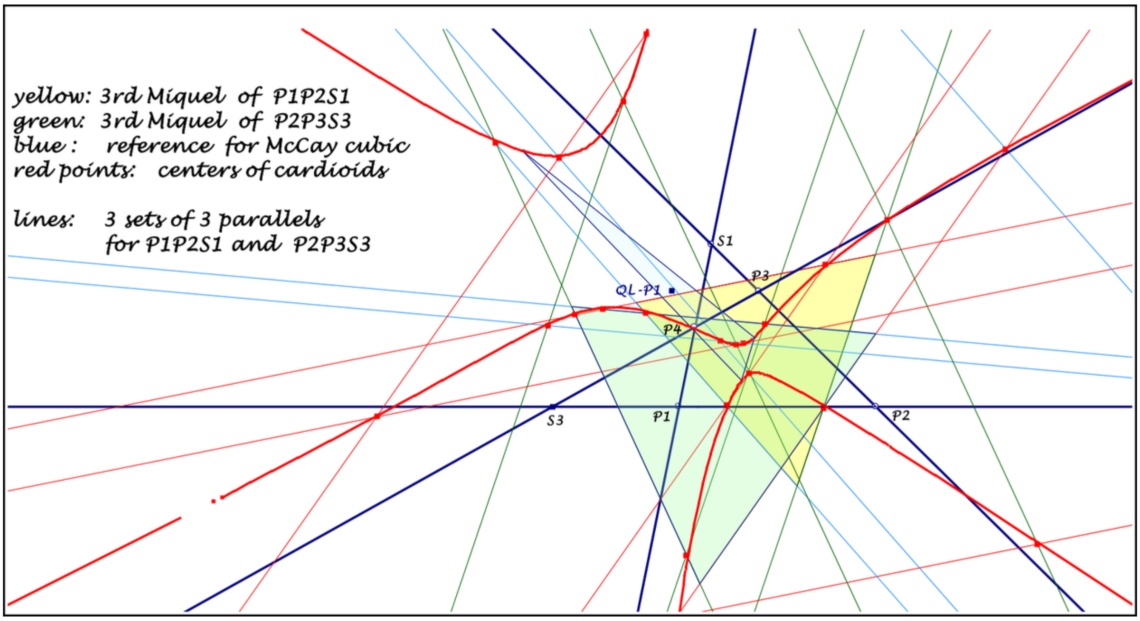
```
COPY FROM FORMER GOOGLE GROUP  by  Eckart Schmidt
(May 26, 2013)
Dear friends,
there is a Cabri file in the attachment for a quadrilateral and
its inscribed cardioids.
I have drawn ( free hand, not constructed
) a reference triangle for the McCay Cubic.
Vary the shown centers on the cubic and you can find most of the
27 inscribed cardioids.
The reference triangle for the McCay Cubic has centroid in QL-P1
and circumcenter on QL-L1.
The axes of the deltoid of its Simson lines are parallel to the
asymptotes of the cubic.
Best regards Eckart
```



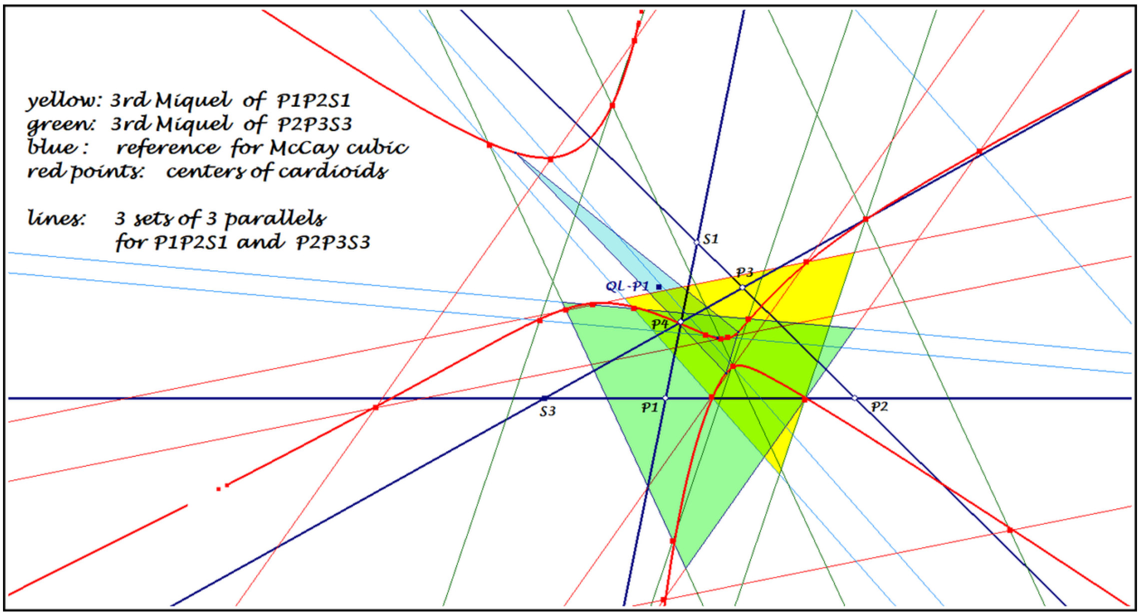
McCayCubic-fig.png

Message: #72
Date: 17/6/2013 5:18:55
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Eckart Schmidt
(may 28, 2013)
Dear friends,
there are problems with the exactness in my Cabri file (26.05), when reloaded (???).
So there is a new drawing of the centers for cardioids tangent to four lines in the attachment.
For two triangles of the quadrilateral the 3 sets of 3 parallels for the centers are drawn
and you can study the intersections, which are centers, with the macro in my Cabri file (24.05).
The McCay cubic is also shown.
Best regards Eckart



2013-05-27.pdf



13-05-28-fig.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #73
Date: 17/6/2013 5:23:19
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

COPY FROM FORMER GOOGLE GROUP by Bernard Keizer
(June 8, 2013)
Dear friends,
I'm really happy that the question of the cardioïds tangent to 4 lines has raised so much attention from your side.
First, I'll give you all the references I found on the subject:
4 articles by Alex Bogomolny in Cut the knot: Doodlings and Miracles, nov 98; Morley's Pursuit of Incidence, dec 98; Lines, Circles and Beyond jan 99; On Motivation and Understanding, fev 99.
On page 1 of the 4th text , I found the 8 cardioïds that touch 4 lines, that's the origin of my initial mistake, but I suppose the 8 cardioïds were tangent to 4 lines themselves tangent to a hypocycloïd with 5 cusps, as explained on page 4 of the 3rd text.
4 articles and a book by Frank Morley: On the metric geometry of the plane n-line, 1900; Orthocentric properties of the plane n-line, 1902; On reflexive geometry, 1907; Extensions of Clifford's Chain-Theorem, 1929 (followed by another interesting article on the same matter by Paul Smith Wagner) and last, but not least Inversive Geometry, 1929.
Encouraged by your discoveries, I tried again to draw more exactly my 4 Morley sets and I found also the 27 points (as Eckart says, it's really hard to draw it precisely, because of the trisection of all angles!). But of course, as I use Geogebra, my drawings are less beautiful than yours ...
I made an interesting observation, which is visible on the last drawing from Chris: each of the 36 lines (or extended sides of equilateral triangles) belonging to one set cuts each of the 3 other sets in 3 points, which are in 3 different directions (in other words, 2 points on the same line are never on 2 parallels). That explains why there are only 27 points and not 81 as intersections of 2 sets of 9 lines each; maybe it's obvious, but I don't see why ...
I still have my questions on the reference triangle of the MacCay cubic: I read the article on the cubic K03 in Bernard Gibert, but I don't see the link between the 4 reference triangles of the QL and this 5th triangle; if ti is the angle between the ortholine and the line Li of the QL, the angle between the axis of the parabola and the axis of the cardioïd tangent to the 4 circles is Sum of ti and the angle between the axis of the parabola and the axes of the deltoïd are 1/3 Sum of ti m° pi/3 ...

My last curiosity is the question of the directed lines: the circle is a curve of direction and there are 4 circles tangent to 3 lines (centers are incenter and excenters), but only one tangent to 3 directed lines; the same goes for the cardioïd, which is also a curve of direction and I wonder if you could see a regularity after drawing all your 27 cardioïds!
That's all for today
Best regards
Bernard

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #74
Date: 20/6/2013 3:37:58
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear friends,
I hope being successful in using the new Quadri-Figures-Group.
In the attachment (editorial note: the attachment is omitted because of very large size) there is the equation of cardioids tangent to 3 or 4 lines, when the center is given.
Perhaps someone can simplify this very extensive term!
Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #75
Date: 20/6/2013 9:41:57
From: Chris
Subject: New points QL-P28 and QL-P29

```
Dear friends,
Welcome to the new Yahoo Group!
Eckart has been on his holiday and we had a Group removal!
In the meantime Antreas and me had a discussion at his Anopolis
Group site. About several interesting Quadrilateral topics.
See message #392 "Is this Miquel point interesting?" and
following messages at:
http://tech.groups.yahoo.com/group/Anopolis/  †)
As a result two new interesting QL-points QL-P28 and QL-P29 were
born.
Seiichi found as a consequence 2 marvelous follow-op properties
of these points.
I leave it to him to mention them at the Quadri Group.
So far this time.
Best regards,  Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #76
Date: 21/6/2013 1:07:23
From: Antreas
Subject: New points QL-P28 and QL-P29

Dear Chris,
Very nice!!!!!!!!!!!!!!!!
I think there should be a dual problem: which similar lines (Euler lines, Brocard axes, in general XiXj-lines) of the component triangles of a quadrangle do envelope a circle?
And how about to return to Triangle Geometry?
Let ABC be a triangle and L a line.
For P = X186 or X265, the P-points of the triangles ABC and the triangles bounded by (AB,BC,L),(BC,CA,L), (CA,AB,L) are concyclic.
Which are the centers (with respect ABC) of the circles for special lines L = Euler Line, Brocard axis, etc
Antreas

← Previous Next → ↔ Message Index ↑ Subjects

Message: #77
Date: 21/6/2013 2:20:57
From: jeanlouisayme
Subject: With the Euler-Poncelet's point

Dear Geometers,
I propose this problem coming from Brianchon and Poncelet for which I haven't a synthetic proof (avoid conic...)
1. ABCD a quadrilateral
2. Po the Euler-Poncelet's point of ABCD,
3. E,F,G the points of intersection of AC and BD, AD and BC, AB and CD
4. (O) the circumcircle of the triangle XYZ.
Prove that (O) goes through Po.
Chris knows my problem. An elementary proof is welcome.
Sincerely
Jean-Louis

← Previous Next → ↔ Message Index ↑ Subjects

Message: #78
Date: 21/6/2013 6:38:49
From: Antreas
Subject: New points QL-P28 and QL-P29

```
Dear Chris
Also, with a good drawing program, is possible I think to draw
the locus of the centers of the circles as the Euler line (for
example) moves around the circumcenter O or the orthocenter H
(the most important points) of ABC.
Antreas
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #79
Date: 22/6/2013 8:00:27
From: Chris
Subject: New points QL-P28 and QL-P29

```
Dear Antreas,
When I transcribe your proposal I get this construction:
1. Let ABC be reference Triangle with bounding lines a,b,c.
2. Let d be a line through O.
3. Construct the P-points QL-P28 (based on X186), QL-P29 (based
on X265) or QL-P4 (based on X3) of the Quadrilateral abcd.
4. Determine the locus of the P-point with variable line d.
I found these results in Cabri:
   The locus of QL-P4 is a cubic through X(3).
   The locus of QL-P28 is a quartic.
   The locus of QL-P29 is a cubic through X(4)?
When changing d to a variable line through X(4) the results are:
   The locus of QL-P4 is a cubic.
   The locus of QL-P28 is a quartic.
   The locus of QL-P29 is a cubic.
Of course also loci could be determined with other points than
QL-P4, QL-P28 and QL-P29.
This is a nice example of combining Quadri-Geometry into
Triangle Geometry.
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #80
Date: 24/6/2013 4:59:57
From: emmanueltsukerman
Subject: Circumcenter of Mass and generalized Euler line

Dear friends,
For your interest, I wanted to bring to your attention the preprint "Circumcenter of Mass and generalized Euler line" which you may find at http://arxiv.org/abs/1301.0496 .
The preprint contains generalizations of QG-L4 (Quasi Euler line) and the points on it (e.g., QG-P5 Quasi Circumcenter, QG-P6 Quasi Orthocenter, QG-P7 Quasi Nine-point Center) to n-polygons and, more generally, simplicial polytopes in Euclidean, spherical and hyperbolic geometries.
From the abstract:
- We define and study a variant of the center of mass of a polygon and, more generally, of a simplicial polytope which we call the Circumcenter of Mass (CCM). The Circumcenter of Mass is an affine combination of the circumcenters of the simplices in a triangulation of a polytope, weighted by their volumes.
- Our motivation comes from the study of completely integrable discrete dynamical systems, where the CCM is an invariant of the discrete bicycle (Darboux) transformation and of recuttings of polygons.
- We show that the CCM satisfies an analog of Archimedes' Lemma, a familiar property of the center of mass.
- We define and study a generalized Euler line associated to any simplicial polytope, extending the previously studied Euler line associated to the quadrilateral. We show that the generalized Euler line for polygons consists of all centers satisfying natural continuity and homogeneity assumptions and Archimedes' Lemma.
- Finally, we show that CCM can also be defined in the spherical and hyperbolic settings.

← Previous Next → ↩ Message Index ↑ Subjects

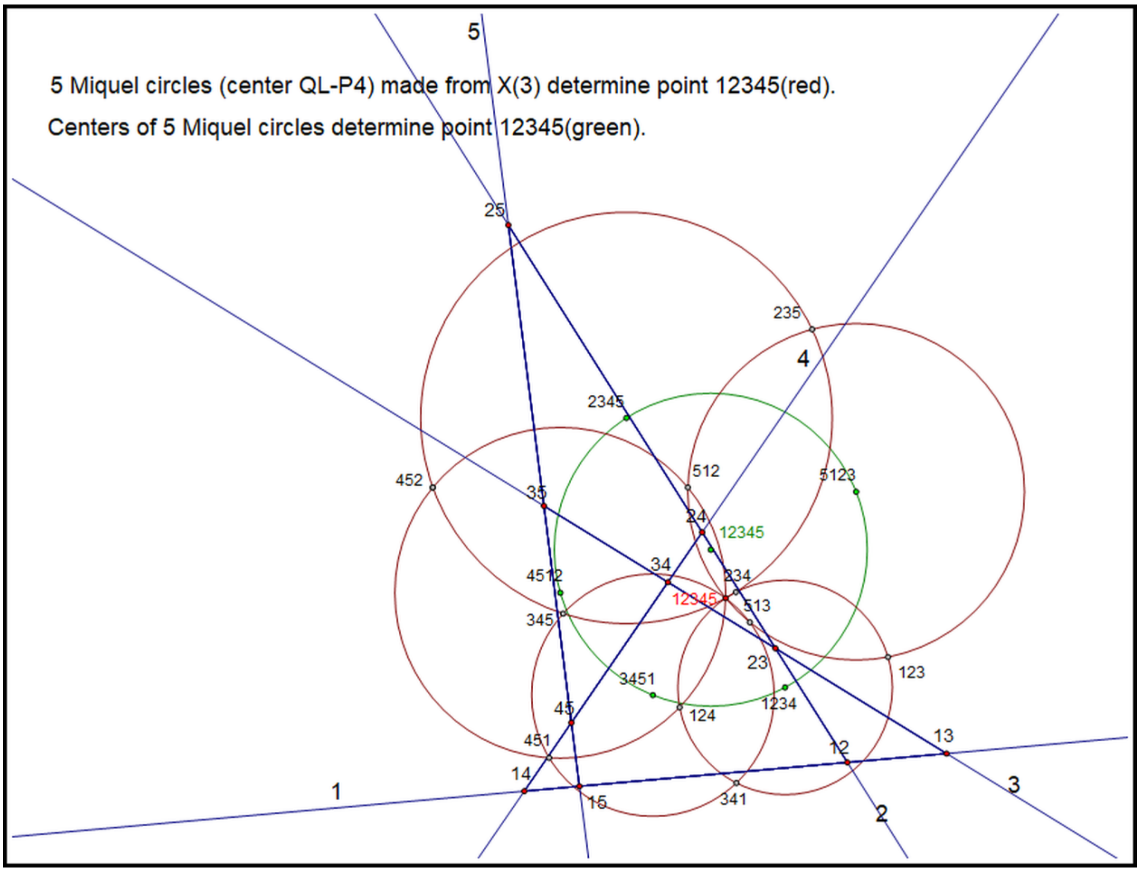
Message: #81
Date: 24/6/2013 1:00:56
From: Antreas
Subject: CONICS IN C. QUADRILATERAL

```
An old problem on conics was:
How many conics are tangent to five given conics?
See here the history of the problem and a special solution:
http://www.mat.ucm.es/serv/revmat/vol10-2/vol10-2h.pdf
Now, in a complete quadrilateral, we have a special case
of 5 concurrent conics (circles):
the circumcircles of the 4 component triangles and the Miquel
circle passing through the 4 circumcenters.
How many conics are tangent to these 5 conics (circles)
and which ones?
On the other hand, five points determine a conic.
Which is the conic passing through the centers of the
above 4+1 circumcircles?
APH
```

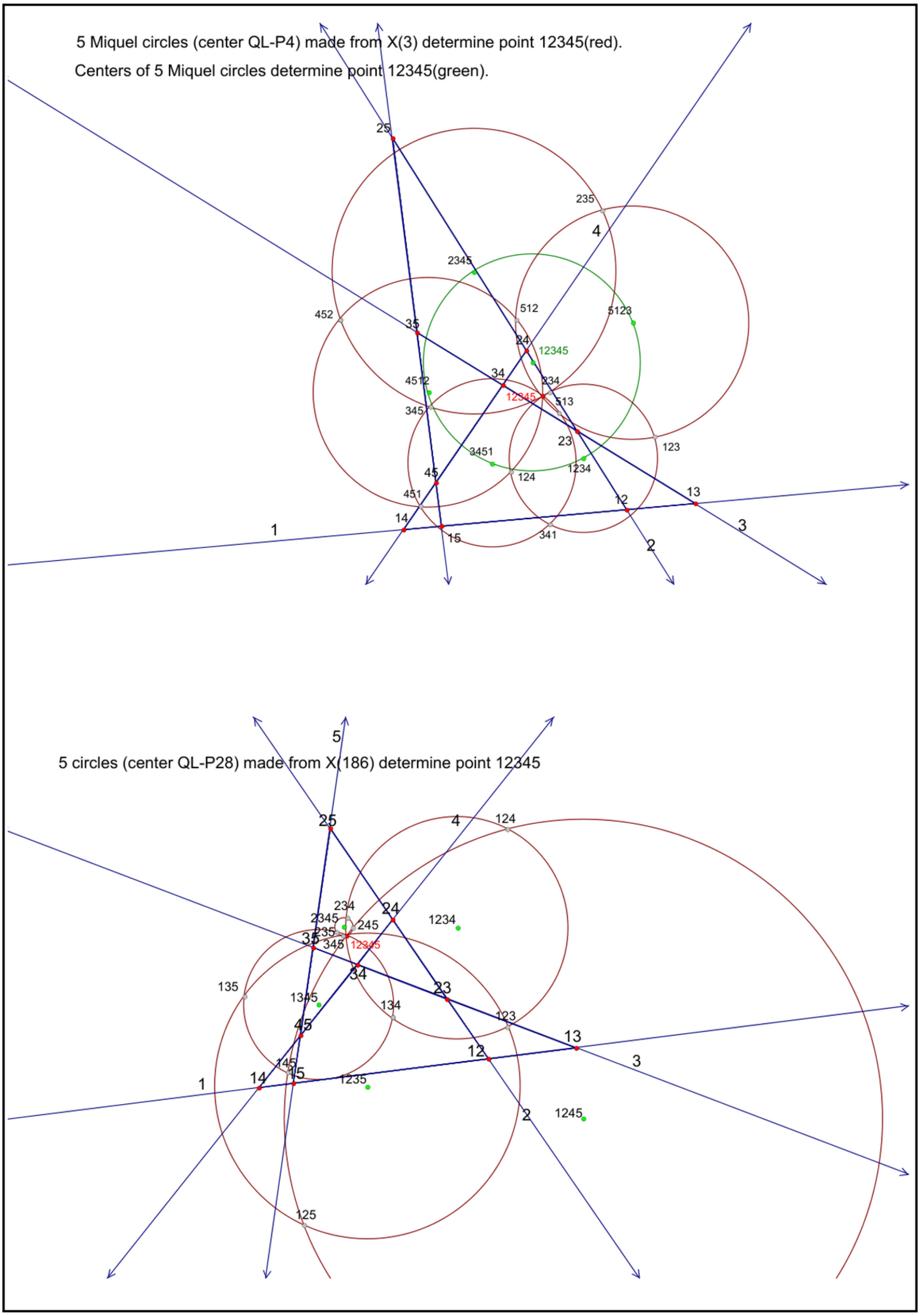
← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #82
Date: 25/6/2013 2:22:55
From: Seiichi Kirikami
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P4

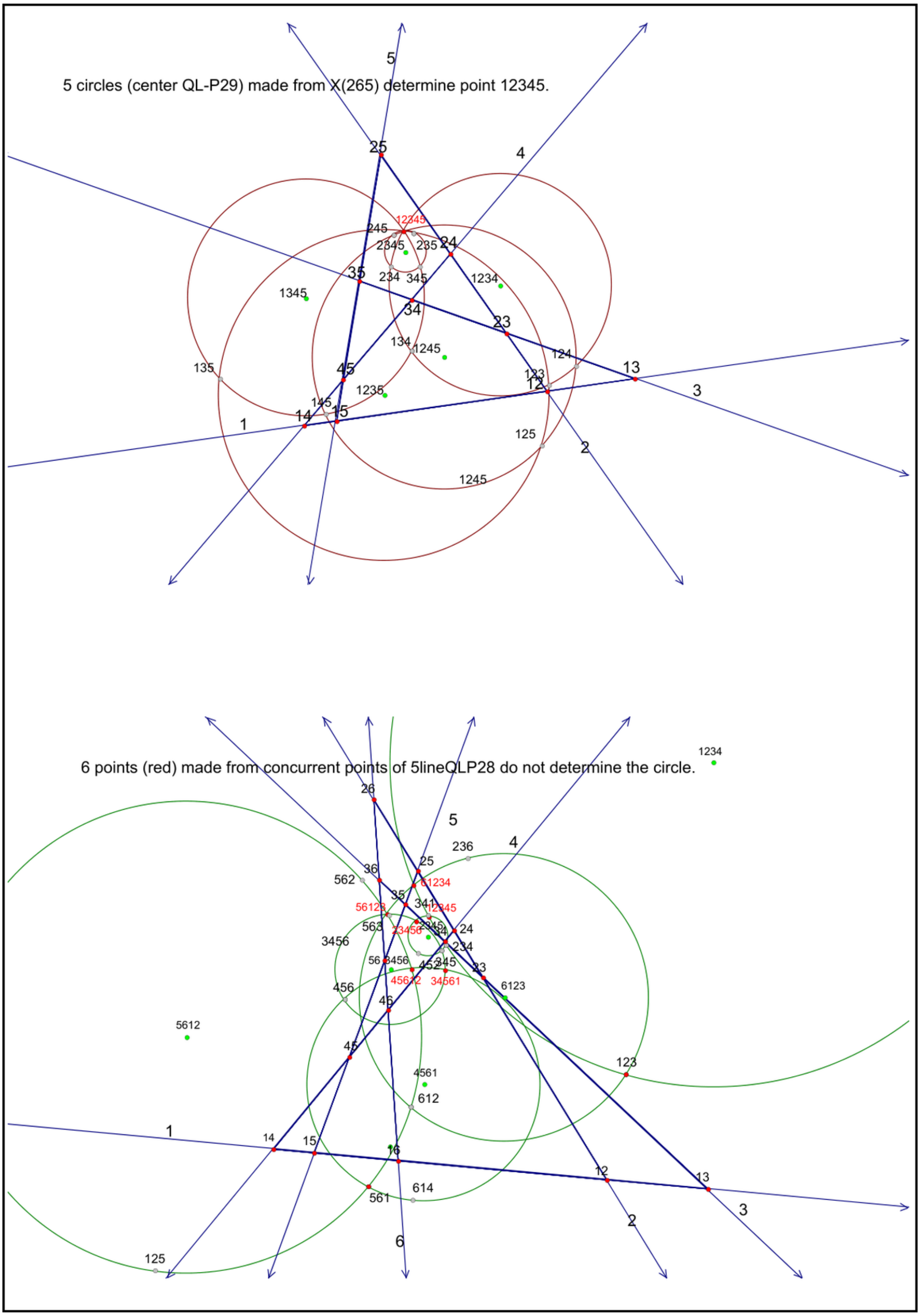
Dear friends,
Chris found QL-P28 and QL-P29 and their circles, which are determined by 4 concyclic points. Then I studied them graphically, expecting that there might be such chains as center-circle or Clifford's like the case of QL-P4 and its circle.
"Cut The Knot! Morley's Pursuit of Incidence by Alex Bogomolny, Dec. 1998" has description about center-circle and Clifford's chains.
I first describe the case of QL-P4 in comparison. Then QL-P28 and P29. The following notation is applied to these cases.
Given 5 lines 1, 2, 3, 4, 5, the intersection of 1,2 is denoted by 12. Other intersections similarly denoted. X(i) of triangle {12, 12, 23} is denoted by 123. Other points similarly denoted.
The points 123, 124, 341, 234 are on a circle with its center denoted by 1234. Other centers similarly denoted. i=3, 186 and 265. See the attached files(gsp or MS word).
(1) 5 line configuration of QL-P4 with the centers of Miquel circles denoted by 1234 and others:
These circles concur in point 12345(red). Their centers are on a circle with its center 12345(green).
(2)5 line configurations of QL-P28 and P29 with the centers of their circles denoted by 1234 and others:
These circles concur in point 12345(red). Their centers are not on a circle. I also confirmed that 6 line configurations of QL-P28 and P29 do not produce any concurrency of point 12345 and others.
My conclusion is as follows.
(1) 5 line configurations of QL-P28 and P29 do not have the same properties as QL-P4, but they rather determine pentalateral points.
(2) Center- circle and Clifford's chains with QL-P4 are rare phenomena.
Best regards,
Seiichi.



QLP4P28P29-gsp.png



QLP4P28P29.docx



QLP4P28P29.docx

Message: #83
Date: 25/6/2013 3:29:14
From: Quadri-Figures-Group@yahoogroups.com
Subject: No Subject

```
Hello,
This email message is a notification to let you know that
a file has been uploaded to the Files area of the
Quadri-Figures-Group
group.
File: /Equilateral triangle on angle bisectors.pdf
Uploaded by: yeuemtrondoitb85 < yeuemtrondoitb85@yahoo.com >
Description:
You can access this file at the URL:
http://groups.yahoo.com/group/Quadri-Figures-Group/files/Equilat␣
eral%20triangle%20on%20angle%20bisectors.pdf
To learn more about file sharing for your group, please visit:
http://help.yahoo.com/l/us/yahoo/groups/original/members/web/ind␣
ex.html
Regards,
yeuemtrondoitb85 < yeuemtrondoitb85@yahoo.com >
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #84
Date: 25/6/2013 3:30:14
From: yeuemtrondoitb85
Subject: D point and secon circle and ten equilateral triangles

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Dear group!
I'm sorry. I'm not mathematician so I introduce some problems
at:
http://tech.groups.yahoo.com/group/Quadri-Figures-Group/files/
I wish you proved it. Thanks very much
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #85
Date: 25/6/2013 10:15:06
From: Chris
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Seiichi,
When I say it in my own words:
* When we draw the points X(186) for all 4 Component Triangles
in a Quadrilateral (4L or system of 4 random lines) then these
points are concyclic on a circle. Let's call it C186. Its center
is QL-P28.
* When we draw the circles C186 for all 5 Component
Quadrilaterals in a Pentalateral (5L or system of 5 random
lines) then these 5 circles all pass through a common point.
The same is valid for X(265).
It is most remarkable indeed.
Congratulations! You discovered 2 brandnew 5L-points (actually
centers)!
One based on X(186) and one based of X(265).
And these 5L-points are very rare!
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #86
Date: 26/6/2013 8:47:29
From: eckart_schmidt@t-online.de
Subject: RE. Cardioids tangent to 4 lines

```
Dear friends,
sorry, I have to cancel my conjecture, that the cubic for the
centers of cardioids tangent to 4 lines generally is a McCay
Cubic.
Bernard Gibert has given me a counter-example: The cubic ...
"...   is generally not a McCay cubic for some triangle since it
can be a nodal cubic, try l:m:n = X190 for example."
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #87
Date: 26/6/2013 12:00:41
From: seiichikiri
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Chris, dear friends,
Thank you very much for your kind comment. I highly appreciate
your ability of Mathematica.
Congratulations for the anniversary of EQF! Though it was
several days earlier, I think.
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #88
Date: 26/6/2013 9:29:59
From: Antreas Hatzipolakis
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Friends
Is there an evidence whether there are more points with that
concyclicity property?
I would bet there are infinitely many (that is,  there is a
locus where they are lying on).
Probably some day Chris will check the rest 1000+ points in
ETC....  :-)
APH
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #89
Date: 28/6/2013 4:22:04
From: Chris
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Antreas, dear friends,
[APH}
>
> > Is there an evidence whether there are more points with that
> concyclicity property?
> > Probably some day Chris will check the rest 1000+ points in
ETC....:-)
I checked all ETC-points in the range X(1)-X(5372) and only
X(3), X(186) and X(265) give 4 concyclic points in the Component
Triangles of a Quadrilateral.
[APH]
>
> > I would bet there are infinitely many (that is, there is a
locus where
> they are lying on).
That's a very interesting question.
I am not so sure there will be a locus of points having this
property.
If so then there won't be an evidence that there aren't more of
these loci, because the locus probably would be dependent of
some construction method, and why wouldn't be there another
construction method also giving the concyclic-property?
It is a bit of speculating as long as we don't have a (general)
method to determine a locus.
Somebody with an idea?
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #90
Date: 28/6/2013 9:10:19
From: Chris
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Seiichi and Friends,
[SK]
>> Congratulations for the anniversary of EQF!
You are right Seiichi.
At June 15, 2012 I introduced the "Encyclopedia of
Quadri-Figures (EQF)" at the internet.
Since then lots of beautiful comments and additions came to me.
I had a hard time digesting it all.
The volume of EQF increased with 30% !!
And now we started up a Discussion Group related to
Quadrilaterals and n-gons.
I am glad how it worked out and hope we have lots of fun
together exploring this beautiful part of Geometry.
Best regards,
Chris van Tienhoven
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #91
Date: 28/6/2013 11:34:37
From: Antreas Hatzipolakis
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

```
Dear Chris
Although I do not know how to determine a locus in 4-lateral/gon
geometry,
I think there should be a method, since such loci exist!
An example, where the locus is the entire plane, is Orthopolar
circles
(from a discussion in Hyacinthos. I have the conjecture here:
http://anthrakitis.blogspot.gr/2013/03/orthopolar-circles-quadri⌋
lateral.html )
By the way, could someone provide a proof (synthetic or not)?
APH
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #92
Date: 28/6/2013 12:59:32
From: Seiichi Kirikami
Subject: Graphical observation: X(186) and X(265) in cyclic QA-environment

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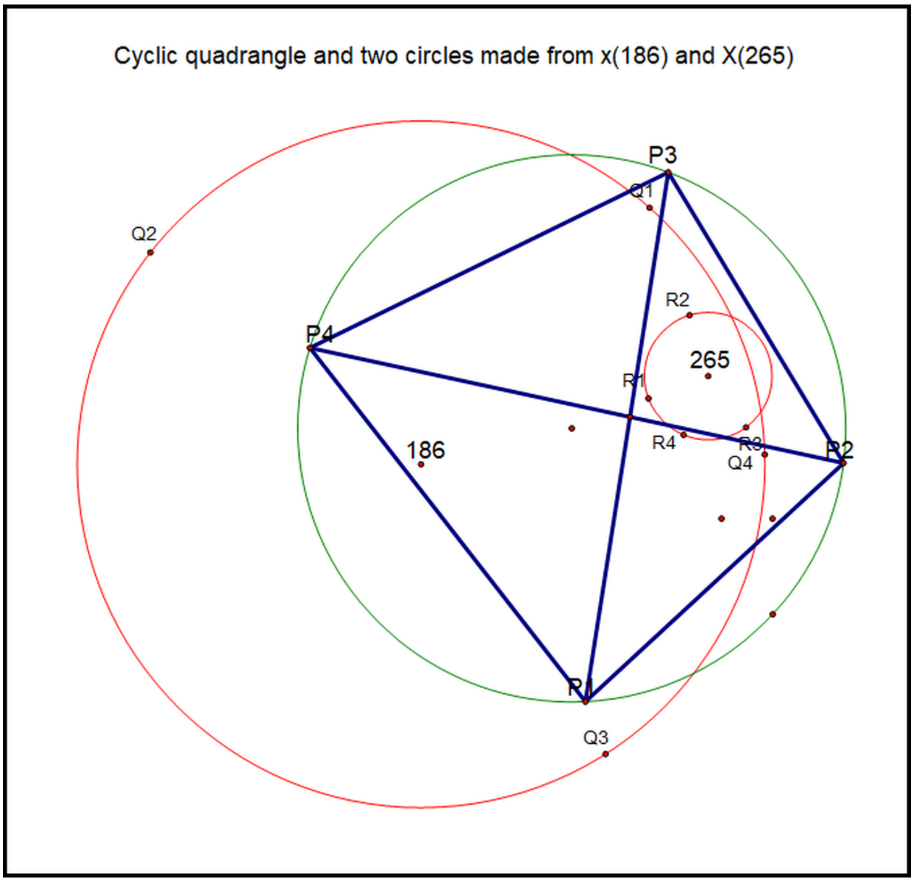
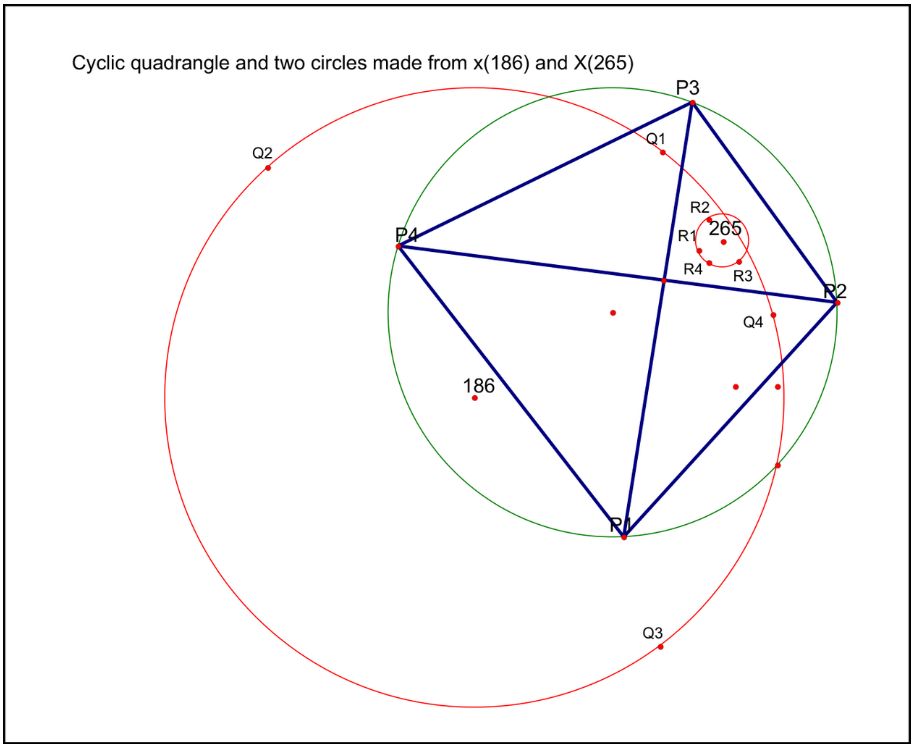
```
Dear friends,
This message is the addition of the message #82.
Given a cyclic quadrangle P1P2P3P4, let X(186) of triangle
PiPjPk be Ql. Then Q1, Q2, Q3 and Q4 are concyclic.
Given a cyclic quadrangle P1P2P3P4, let X(265) of triangle
PiPjPk be Rl. Then R1, R2, R3 and R4 are concyclic.
See the attached files (gsp or MS word).
This property is easier  for me to understand X(186) and X(265).
Best regards,
Seiichi.
```



CyclicQAX(186)X(265)-gsp.png



CyclicQAX(186)X(265).docx

Message: #93
Date: 28/6/2013 2:45:31
From: Seiichi Kirikami
Subject: Graphical observation: QL-P28 and QL-P29 in comparison with QL-P

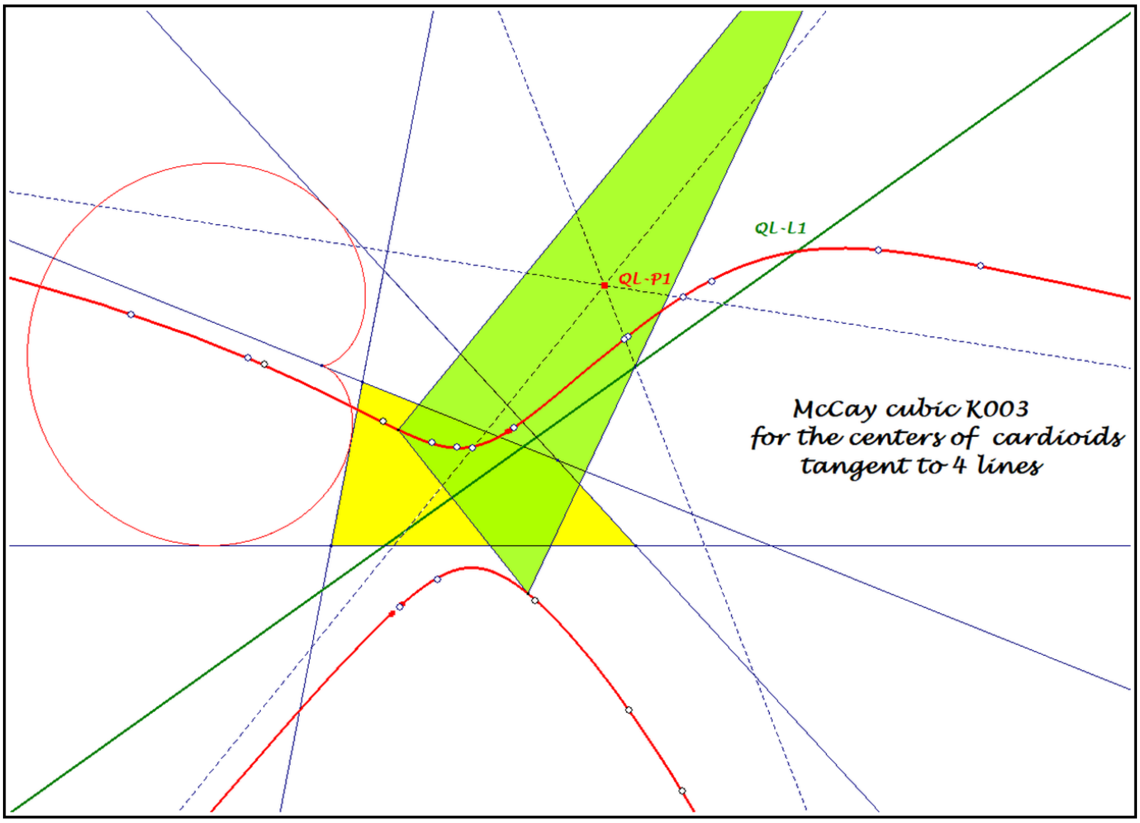
Dear Chris, dear friends,
You are the driver and conductor of the train of EQF! I see that your work is very, very hard. No one except for you can endure it. I think that, including me, many people are grateful to you.
Thanks a lot!
Best regards,
Seiichi.

← Previous Next → ↩ Message Index ↑ Subjects

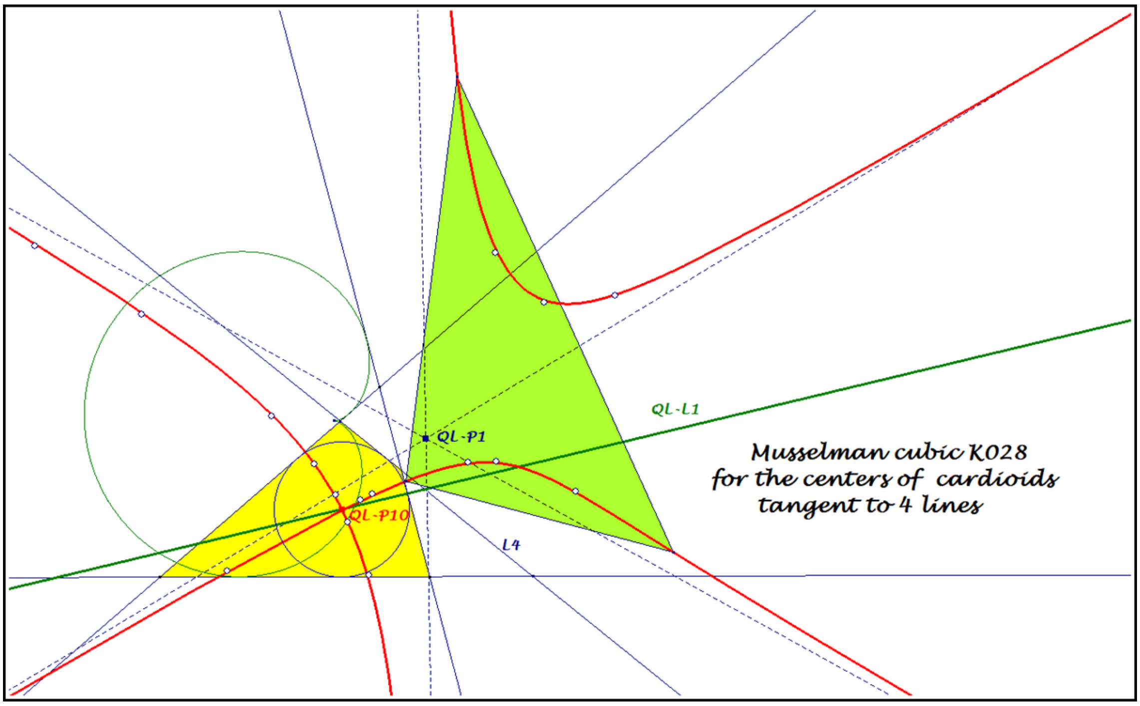
Message: #94
Date: 29/6/2013 10:42:37
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear friends,
after Bernard Gibert had given a counter-example I have to modify my conjecture for the cubic of the centers of cardioids tangent to 4 lines. I risk to sum up my new sight:
* The cubic is a K60+ . The three asymptotes are parallel to the axes of the deltoid of the quadrilateral and concur in in the Miquel Point QL-P1.
* The asymptotes cut the cubic collinear on a line parallel QL-L1 in 2/3 distance to QL-P1.
The Newton Line QL-L1 can have three intersections with the cubic:
* If there is only one real intersection, the cubic will be a McCay cubic K003. The reference triangle has its centroid in the Miquel point QL-P1 and its circumcenter is the point of intersection (QL-L1=X3.X6).
* If there are one and a double counted real intersection, the cubic will be a Musselman cubic K028. In this case the quadrilateral is tangent to a circle with center QL-P10, which is the nodal point of the cubic. The reference triangle for K028 has its orthocenter in QL-P10 and the midpoint of the orthocenter and the centroid (X381) is the Miquel point QL-P1.
* If there are three real intersections, the cubic will be a Kjp K024. The reference triangle has its centroid in the Miquel Point QL-P1 and the points of intersection are the centers of the Apollonial circles.

I hope, there are no basic faultes. You find Cabri files in the attachment.
Best regards Eckart



13-06-29-a-fig.png



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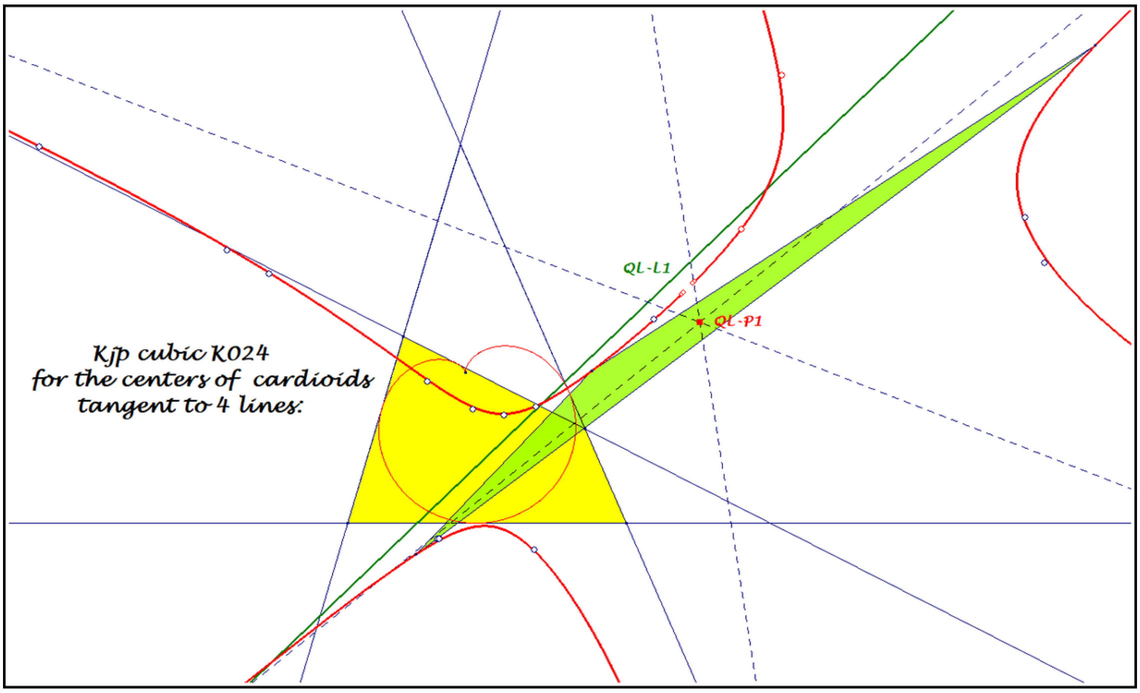


13-06-29-c-fig.png

Message: #95
Date: 29/6/2013 9:11:28
From: Chris
Subject: Cardioids tangent to 4 lines

```
Dear Eckart,
Very nice results!
I know this takes a lot of time to resolve.
However I wonder,
1. How a QL-Cubic can belong to different Triangle Cubic-types
dependant on circumstances. Or is it that actually these 3
Triangle Cubic-types belong to the same family?
2. Is there a standard Reference Triangle for these 3 types of
Triangle cubcis? What is the relation of this/these Reference
Triangle(s)?
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #96
Date: 29/6/2013 9:34:42
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Chris,
I wonder too and have the same questions, for my knowledge of
cubics is bounded.
Corresponding with Bernard Gibert, he announced connections of
the cubics in the next week.
A further question:
What is the geometrical background for having one or three real
intersections between QL-L1 and the cubic?
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #97
Date: 01/7/2013 10:06:35
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

Dear Chris and Eckart,

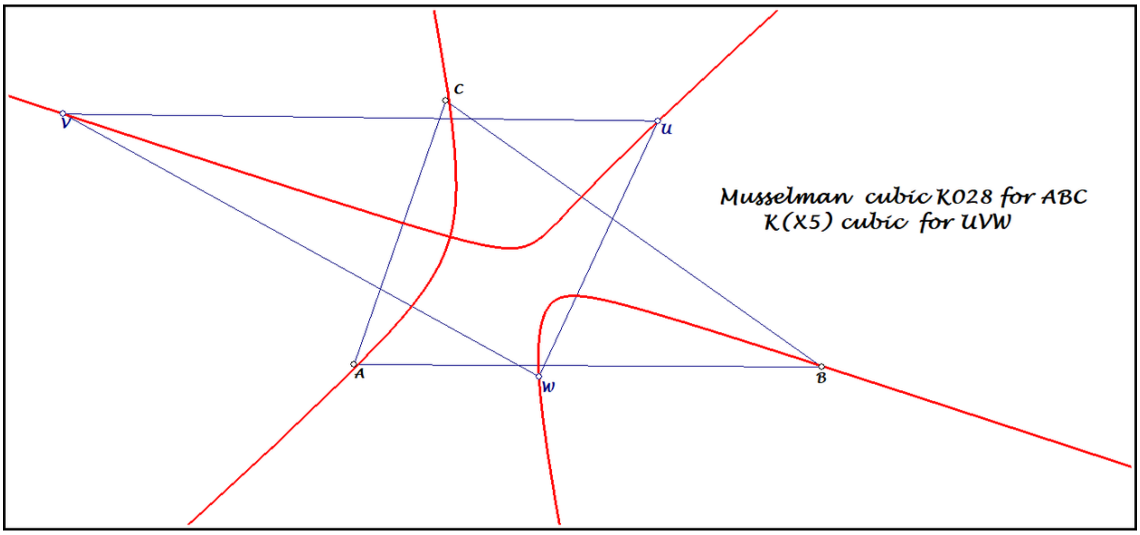
>> [Chris] What is the geometrical background for having one or three real
>> intersections between QL-L1 and the cubic?

I have more or less achieved the study of these cubics although a couple of details are still not fully completed.
I can answer the question above:
the Newton line QL-L1 is the orthic line of the hessian H(P) of the Eckart cubic E(P).
the number of real intersections depends of the position of the line L(P) lx+my+nz=0 wrt to the 4 in/excircles of ABC.
1. if L(P) cuts an even number of circles then H(P) is bipartite and E(P) meets QL-L1 thrice.
2. if L(P) cuts an odd number of circles then H(P) is unipartite and E(P) meets QL-L1 once.
3. if L(P) is tangent to one circle then H(P) and E(P) are nodal with node the center of the circle.
4. if L(P) is tangent to two circles then H(P) and E(P) must decompose into a line and a circle for H(P), a hyperbola with excentricity 2 for E(P).
I have a lot of additional informations and I wonder whether it could be worth a paper...
Best regards
Bernard

← Previous Next → ↩ Message Index ↑ Subjects

Message: #98
Date: 01/7/2013 10:36:24
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear Chris, dear friends,
perhaps there is a common sight of the three types (McCay K003, Musselman K028, Kjp K024) of my cubic:
In "Special Isocubics ..." from Bernard Gibert there is a "McCay-Kjp pencil" in table 22 with the property:
"The locus of M whose pedal circle cuts the nine-point circle under a given angle ... is a circum-cubic which is always a K60+..."
It is mentioned, that this pencil contains two nodal cubics.
Perhaps it is possible for my cubic to find reference triangles, so that this nodal cubics are the Musselman cubics K028 in the case of quadrilaterals tangent to a circle. I can´t test it, for I have no construction for these cubics.
Unexpected: I found, that on a Musselman cubic K028 there is a reference triangle, for which the cubic is a K(X5).
This cubic is described by Bernard Gibert in Forum Geometricorum, Volume 2 (2002), p.55.
There is a Cabri file in the attachment.
Best regards Eckart



13-06-30-a-fig.png

← Previous Next → ↔ Message Index ↑ Subjects

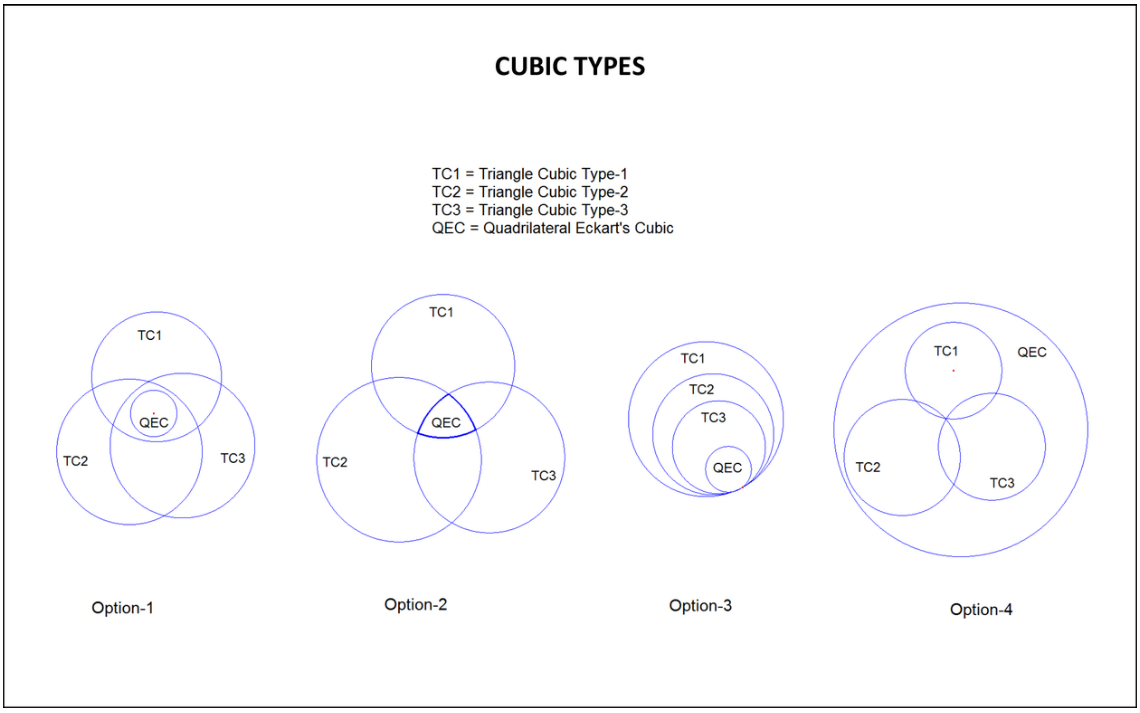
Message: #99
Date: 01/7/2013 2:49:55
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear Chris, dear Bernard Gibert,
thanks to Bernard for the answer on my question, I searched without success.
In addition to my last message:
On a Musselman cubic K028 there are reference triangles, so that K028 belongs to the McCay-Kjp-pencil (table 22):
Construction for a given K028 and its reference triangle:
* Take the midpoint X2.X4 as centroid G of a new reference triangle ABC.
* Take the nodal point X4 as a focus F for the inscribed Steiner ellipse of ABC.
* Choose a point A on the cubic with its homothetic Ma under h(G,-1/2).
* Construct the Steiner ellipse with center G, Focus F and point Ma.
* The tangents from A to the ellipse and the tangent in Ma give a triangle ABC on the cubic.
* For ABC as reference triangle the cubic belongs to the McCay-Kjp pencil.
So the three types for my cubic belong to the same pencil.
Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #100
Date: 01/7/2013 3:10:20
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, Dear Eckart,
Thanks for all information.
Bernard I hope it is worth a paper -like you suggested- for
clarity and for reference in EQF.
One question remains for my own clarity.
A QL-Cubic is a unit in itself. How can it be 3 types in the
Triangle environment?
If so, when we consider a cubic to belong to a set of similar
cubics, how do these sets of cubics, type 1 (McCay), type 2
(Musselman), type3 (Kjp) relate to Eckart's (type of) cubic?
I made a picture with set diagrams of 4 options. See attachment.
Does one option apply to this situation or is there a better set
diagram to be made?
Best regards,
Chris
```



CUBIC TYPES.pdf

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #101
Date: 01/7/2013 3:33:53
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Chris,

>> A QL-Cubic is a unit in itself. How can it be 3 types in the
Triangle
>> environment?

All depends of the reference triangle you choose.
A nK in one triangle can be a pK in another triangle.
Actually, any non-singular cubic is, in infinitely many ways, a
pK wrt another triangle but this latter triangle is not always
real.
It will be if you can draw four real tangents to the cubic from
one of its points.
In particular, if the cubic is circular with an oval, this is
always possible with a point on the oval.
See this paper for more information:
http://bernard.gibert.pagesperso-orange.fr/files/bitripk.html
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #102
Date: 01/7/2013 5:58:03
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Correction:
>>
>> this is always possible with a point NOT
>> on the oval.
>>
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #103
Date: 02/7/2013 9:24:41
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear Bernard, dear friends
can it be, that the Hessian of my cubic is the QL-Quasi Isogonal Cubic QL-Cu1, containing the foci of all inscribed conics of the quadrilateral? Then Bernards criterion for intersections of QL-L1 and my cubic can be formulated wether QL-Cu1 is bipartite, unipartite or nodal.
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #104
Date: 2020-02-20
From: Systems Manager
Subject: Deleted Messages

Message number #104 is not available in Yahoo groups.

← Previous Next → ↩ Message Index ↑ Subjects

Message: #105
Date: 03/7/2013 3:57:42
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear friends,
meanwhile I know, how to get the Hessian of my cubic. I think I´m right:
The Hessian of my cubic is QL-Cu1 (QL-Quasi Isogonal Cubic).
By the way, for a quadrilateral tangent to a circle the cubic QL-Cu1 is a strophoid of the line QL-L1, pole QL-P1 and fixed point QL-P10 (see "A book of curves" by E.H.Lockwood). Let S1, S2, S3 be the further intersections of the circle round QL-P10 through QL-P1 with the strophoid, then the lines QL-P1.Si are the asymptotes of my cubic.
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #106
Date: 04/7/2013 11:14:27
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear friends,
please excuse, that my last message appears twice. I am not perfect in computering. Perhaps Chris can cut one message.
There is a further criterium for the types of my cubic:
Consider the conic C tangent to the four lines of the quadrilateral with center QL-L1^QL-L6; one axis is QL-L1 (seeEQF, QL-Co/1).
If the conic C is a circle (center QL-P10), then ...
QL-Cu1 is nodal,
QL-Cu1 is the strophoid of QL-L1 with pole QL-P1 and fixed point QL-P10,
my cubic has a nodal point (QL-P10) and a further real intersection with QL-L1,
my cubic can be considered as a Musselman cubic K028 or a K(X5) cubic or a cubic of the McCay-Kjp pencil (depending on corresponding reference triangles).
If the principal axis of the conic C is QL-L1, then
QL-Cu1 is unipartite,
my cubic has one real intersection with QL-L1,
my cubic is a McCay cubic (for a corresponding reference triangle).
If the principal axis of the conic C is orthogonal QL-L1, then
QL-Cu1 is bipartite,
QL-Cu1 is a strophoid of QL-L1 with pole QL-P1 and fixed points in the foci of C,
my cubic has three distinct real intersections with QL-L1,
my cubic is a Kjp cubic K024 (for a corresponding reference triangle).
If QL-L1=QL-L6, the quadrilateral is tangent to two circles (see Bernard´s message), then ...
QL-Cu1 decomposes into QL-L1 and a circle Ci with center QL-P1 and the intersections of the four lines not on QL-L1, my cubic decomposes into QL-L1 and a hyperbola with eccentricity 2 (see Bernards message) centered in QL-P1 with main circle Ci.
By the way: In the first and third case QL-Cu1 can be described as a strophoid in the definition of E. H. Lockwood: A Book of Curves (p.135). In the second case QL-Cu1 is a modified strophoid in the following sense: The line will be QL-L1 again, also the pole QL-P1, but the circles centered at QL-L1 have not a fixed point, they have to be orthogonal to the Thales circle wrt the foci of C. Best regards Eckart

---

Message: #107
Date: 05/7/2013 8:05:27
From: Chris
Subject: New transformation QA-Tf3 and related new point QA-P42

Dear Friends,
Thanks to Antreas' message #91, I once again looked at his orthopole circle at http://anthrakitis.blogspot.gr/2013/03/orthopolar-circles-quadrilateral.html
Already when we discussed this feature at Hyacinthos I found it most remarkable.
Summarized:
Let O1, O2, O3, O4 be the circumcenters of the Component Triangles of Reference Quadrangle P1.P2.P3.P4.
Let P be some random point.
Let OPi be the Orthopole of line P.Pi wrt Component Triangle Pj.Pk.Pl, where (i,j,k,l) □¸ (1,2,3,4). The 4 Orthopoles OP1, OP2, OP3, OP4 are concyclic.
The circle through OP1, OP2, OP3, OP4 also passes through QA-P2, the Euler-Poncelet Point.
The center of this circle appears to be a linear transformation of starting point P. I called it QA-Tf3. It has very interesting features.
One of them is that the consecutive points generated from P diverge oscillating from another interesting fixed point: QA-P42.
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #108
Date: 06/7/2013 2:51:28
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Chris and Eckart,
>>
>> I know this takes a lot of time to resolve.
>>

Indeed, and a very big margin as well!...
I know how to draw the reference triangle for a non-singular
Eckart cubic when it meets the orthic line of its hessian at one
real point (giving a McCay cubic) or at three real points
(giving a Kjp cubic).
The method is difficult and involves all the heavy artillery
related to general cubic curves: hessian, prehessians,
poloconics, etc.
Naturally, these constructions are not possible with ruler and
compass.
I will post a figure with the Kjp  configuration.
Best regards
Bernard
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #109
Date: 06/7/2013 7:21:20
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Chris and Eckart,
here are two figures with K003 and K024 types of Eckart's
cubics.
Best regards
Bernard
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #110
Date: 07/7/2013 8:44:26
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, dear Eckart,
Thanks for all your effort!
It is a tough, challenging and wonderful cubic we are dealing
with
Bernard, you said you are using all the *heavy artillery*
related to general cubic curves.
I think you act completely in the style of your famous
countrymen Poncelet and Brianchon, who were both *soldiers in
artillery* in Napoleon's army.
About the possible Reference Triangle I wonder if there is only
one or possibly more of them.
What are the geometric properties of such triangle?
Since the basis of Eckart's Cubic is a Quadrilateral, being a
system of 4 random lines, it is not unreasonable to think of a
Reference Triangle, being a system of 3 lines (instead of 3
points) with an extra random line to complete the system.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #111
Date: 07/7/2013 11:44:04
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Chris,
>>
>> About the possible Reference Triangle I wonder
>> if there is only one or possibly more of them.
>>

when the Eckart cubic is elliptic (= not singular = without
node), there is only one such triangle.
I'm still struggling with nodal cubics which have only one
prehessian and the geometry is quite different.
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #112
Date: 08/7/2013 9:57:10
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

Dear Bernard, dear friends,
I am very glad, that my conjectures seem right. Gratulation to Bernard for constructing the reference triangles for the McCay and Kjp cubic. But up to now I cannot reproduce the construction.
A remark to the nodal case:
There are infinite reference triangles, so that the cubic belongs to the McCay-Kjp-pencil. Here the construction of an example, if the axes of the deltoid of the quadrilateral are known (see also #99):
* The parallels to the deltoid-axes through QL-P1 cut QL-L1 in three points. Their homothetics wrt h(QL-P1 ,2/3) are points of the cubic.
* Take one of these points as vertice A of the reference triangle.
* Let QL-P1 be the centroid of the reference triangle.
* The homothetic of A wrt h(QL-P1,-1/2) is the middle Ma of BC.
* Construct the (Steiner) ellipse with center QL-P1, focus QL-P10 (nodal point) and point Ma.
* The tangents from A to the ellipse and the tangent in Ma give a triangle ABC on the cubic.
* For ABC as reference triangle the cubic belongs to the McCay-Kjp pencil.
Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #113
Date: 09/7/2013 11:55:39
From: bernard.keizer
Subject: Cardioids tangent to 4 lines

Dear Eckart, dear Chris and dear Bernard

Although I'm not a specialist in cubics, I had several times visited Bernard'site and I'm glad to see he joined his efforts to help us to understand the wunderful cubic of the 27 centers of cardioïds tangent to 4 lines discovered by Eckart. I must confess I was a little lost with your previous messages, but this one is more precise and I think I begin to see the possible construction of the reference triangle of the MacCay cubic ...
We are now far from the initial Morley sets of the 4 reference triangles! I noticed that each line from one set cuts a second set in 9 points (as there are 3*3 parallels = 9 lines in each set), but only three of them are centers of cardioïds tangent to the 4 lines , one point in each group of parallels. the other points are centers of 2 different cardioïds , one tangent to 3 lines and the other also to 3 lines, but 2 lines only are commun to both.
And by drawing cardioïds, I was wondering what the locus of the 27 cusps could be (also a cubic?)

Best regards
Bernard

I begun to read other messages from the group, I saw the group is expanding, I noticed in particular that Jean-Louis Aymé had joined and I am happy of that because he is the one who advised me to contact Chris. (I haven't found yet a synthetic proof to his problem with the Euler-Poncelet point belonging to the circle circumscribed to the diagonal triangle of a quadrangle)

← Previous Next → ↔ Message Index ↑ Subjects

Message: #114
Date: 10/7/2013 2:59:52
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, dear friends,
it seems accepted, that my cubic belongs to the McCay-Kjp pencil
(see table 22). These cubics are defined (wrt a reference
triangle) by an angle "theta" , which is the angle of
intersection of the nine-point circle and the pedal circles of
points on the cubic. For the McCay cubic "theta" is 0°, for a
Kjp cubic "theta" is 90°. Here is a general construction of
these cubics, knowing the angle "theta". I have not found this
construction on table 22, it is a generalisation of the
construction for the Kjp cubic:
Let P be a point on the circumcircle of the reference triangle
ABC,
...  let L(P) be the parallel at O to the Simson line of P;
... L(P) cuts the circumcircle in two points,
... consider the rectangular hyperbola
    through these two points and B, C,
... consider the line AP rotated by 90°+"theta" at A,
... the intersections of this line and the rectangular hyperbola
    will be points of the cubic.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #115
Date: 10/7/2013 8:28:50
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Eckart,
>
>> it seems accepted,
>> that my cubic belongs to the McCay-Kjp pencil
>>
I am more and more convinced that this is not true.
K003 and K024 are two very special cases of apolar isogonal
cubics with same radial center G and their hessians are focal
cubics with focus G.
Conversely, if you consider a pencil of focal isogonal nKs with
same focus F on the circumcircle, the (real) prehessians of
these cubics do not generally form a pencil!
So, the problem is not that simple and certainly much more
challenging than expected!
We need time and reflexion, and above all, PROOFS!
Best regards
Bernard
PS: are you aware of the paper by Sister Mary Gervase on
cardioids?
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #116
Date: 11/7/2013 12:43:27
From: Seiichi Kirikami
Subject: van Aubel points

Dear friends,

(1) Preparation of van Aubel quadrangle.
Given a quadrangle P1P2P3P4, we denote the midpoint of the segment PiPj by Mij or Mji. We erect the vertical segment of half length of its side at Mij in both direction of it. We denote the endpoint of the vertical segment by TijR or TijL, if we proceed from Pi to Pj and see the endpoint at the right or left direction respectively. So TijR and TjiL indicate the same endpoint. van Aubel quadrangles can be constructed indefinitely.

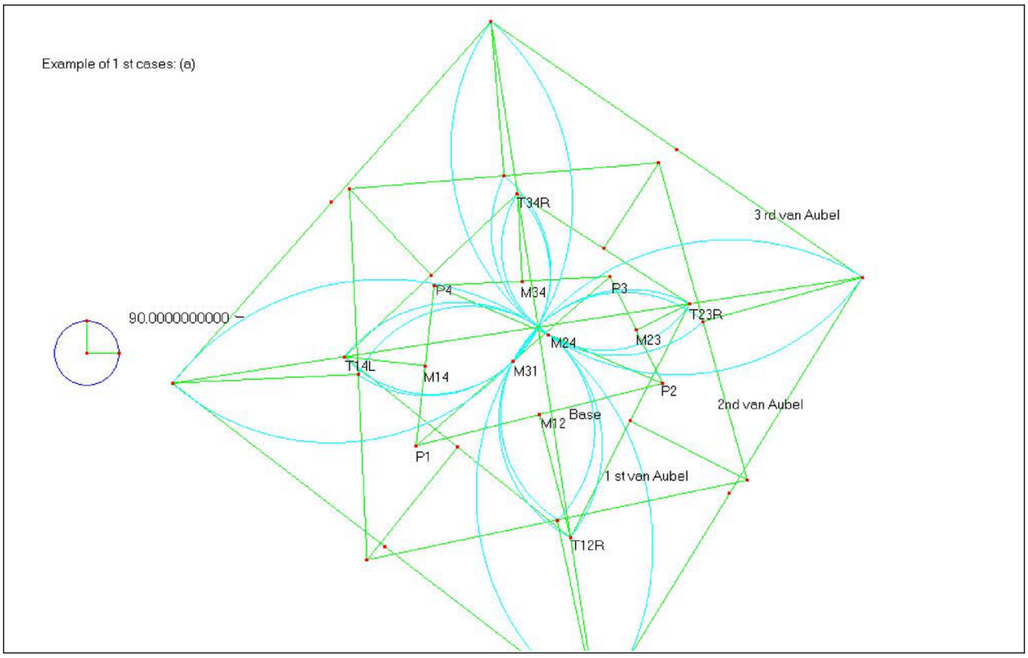
(2) 4 circles through the following 3 points are concurrent.
(2.1) 1st cases: See van Aubel points (Aubel-SK-01a)
(a)T12RM31T23R, T23RM24T34R, T34RM31T41R, T41RM24T12R
(b)T12LM31T23L, T23LM24T34L, T34LM31T41L, T41LM24T12L
(c)T12RM14T24R, T24RM23T43R, T43RM14T31R, T31RM23T12R
(d) T12LM14T24L, T24LM23T43L, T43LM14T31L, T31LM23T12L
(e) T23RM12T31R, T31RM34T14R, T14RM12T42R, T42RM34T23R
(f) T23LM12T31L, T31LM34T14L, T14LM12T42L, T42LM34T23L

(2.2) 2nd cases: See van Aubel points (Aubel-SK-02a)
(a)T12RM24T23R, T23RM31T34R, T34RM24T41R, T41RM31T12R
(b)T12LM24T23L, T23LM31T34L, T34LM24T41L, T41LM31T12L
(c)T12RM23T24R, T24RM14T43R, T43RM23T31R, T31RM14T12R
(d) T12LM23T24L, T24LM14T43L, T43LM23T31L, T31LM14T12L
(e) T23RM34T31R, T31RM12T14R, T14RM34T42R, T42RM12T23R
(f) T23LM34T31L, T31LM12T14L, T14LM34T42L, T42LM12T23L

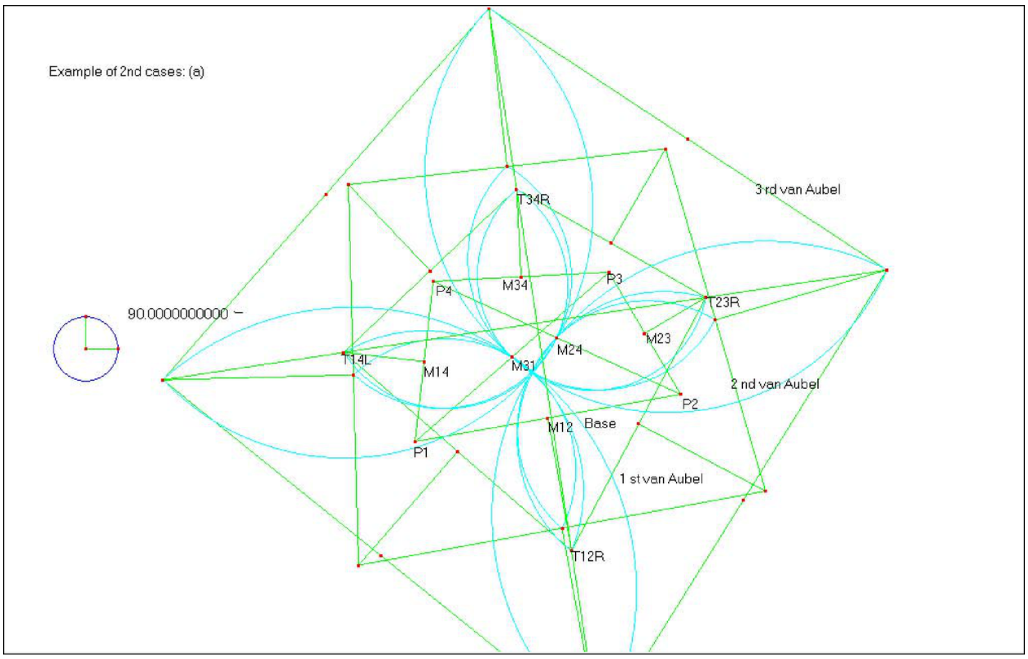
(2.3) 3rd cases: See van Aubel points (Aubel-SK-03a)
(a)S124M31S123, S123M24S234, S234M31S134, S134M24S124 (outer)
(b)S124M31S123, S123M24S234, S234M31S134, S134M24S124 (inner)
(c)S123M14S124, S124M23S234, S234M14S134, S134M23S123 (outer)
(d) S123M14S124, S124M23S234, S234M14S134, S134M23S123 (inner)
(e) S234M12S123, S123M34S134, S134M12S124, S124M34S234 (outer)
(f) S234M12S123, S123M34S134, S134M12S124, S124M34S234 (inner)
Note: Sijk from (a) to (f) are different points.

(2.4) 4th cases: See van Aubel points (Aubel-SK-04a)
(a)S124M24S123, S123M31S234, S234M24S134, S134M31S124 (outer)
(b)S124M24S123, S123M31S234, S234M24S134, S134M31S124 (inner)
(c)S123M23S124, S124M14S234, S234M23S134, S134M14S123 (outer)
(d) S123M23S124, S124M14S234, S234M23S134, S134M14S123 (inner)
(e) S234M34S123, S123M12S134, S134M34S124, S124M12S234 (outer)
(f) S234M34S123, S123M12S134, S134M34S124, S124M12S234 (inner)

```
Note: Sijk from (a) to (f) are different points.
N th quadrangle has concurrent points, which coincide with those
of <N th quadrangles.
Now I have no Mathematica evidence of concurrency.
Sorry in advance if the above description were false.
Seiichi.
```



van Aubel points.pdf



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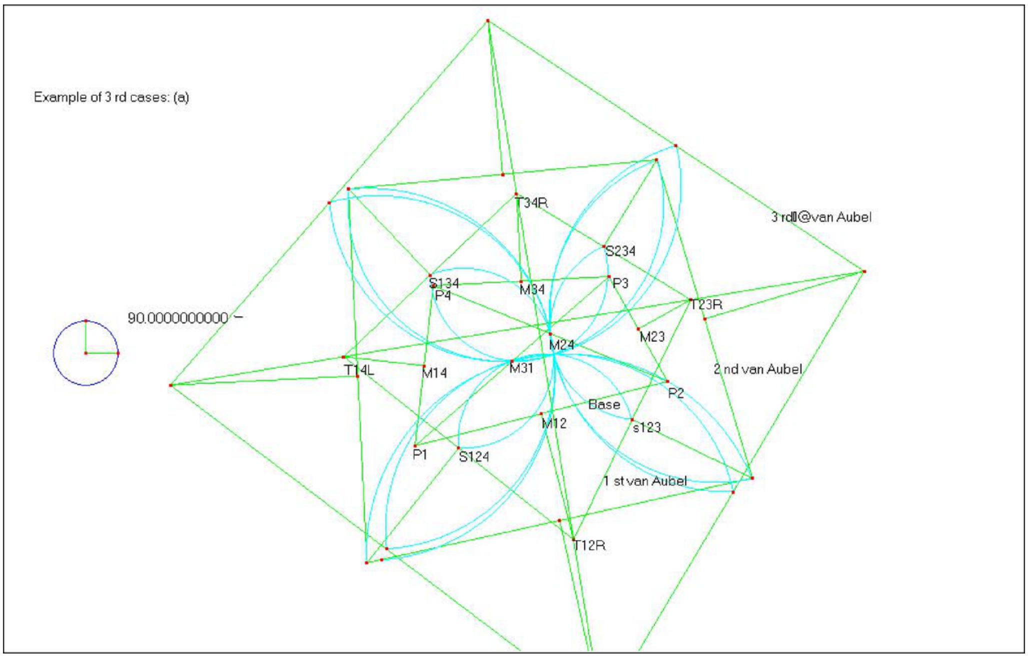

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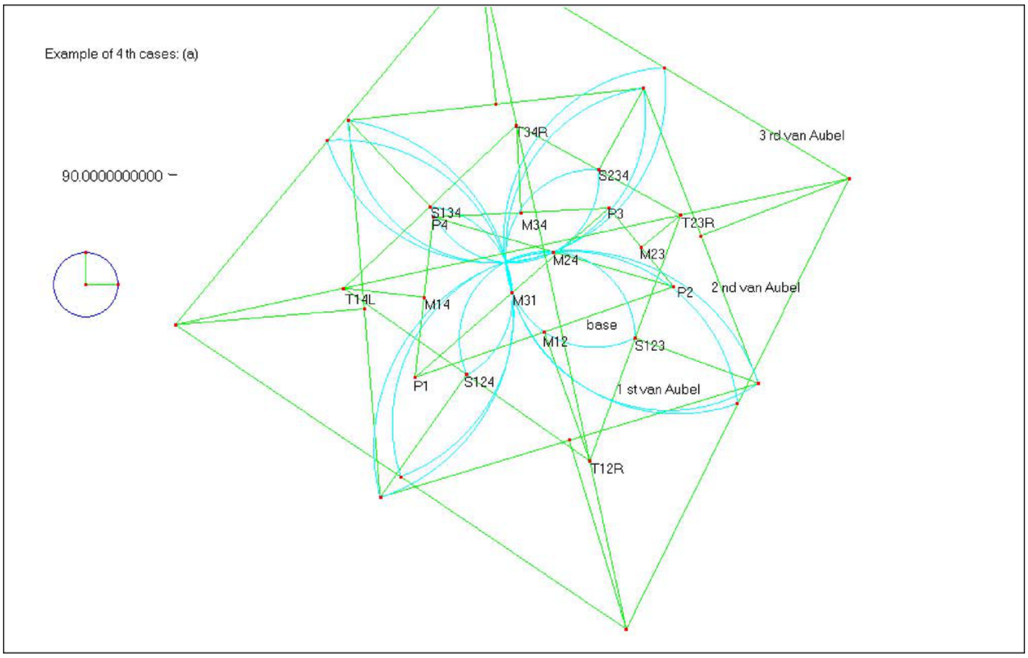


van Aubel points.pdf

← Previous Next → ↔ Message Index ↑ Subjects

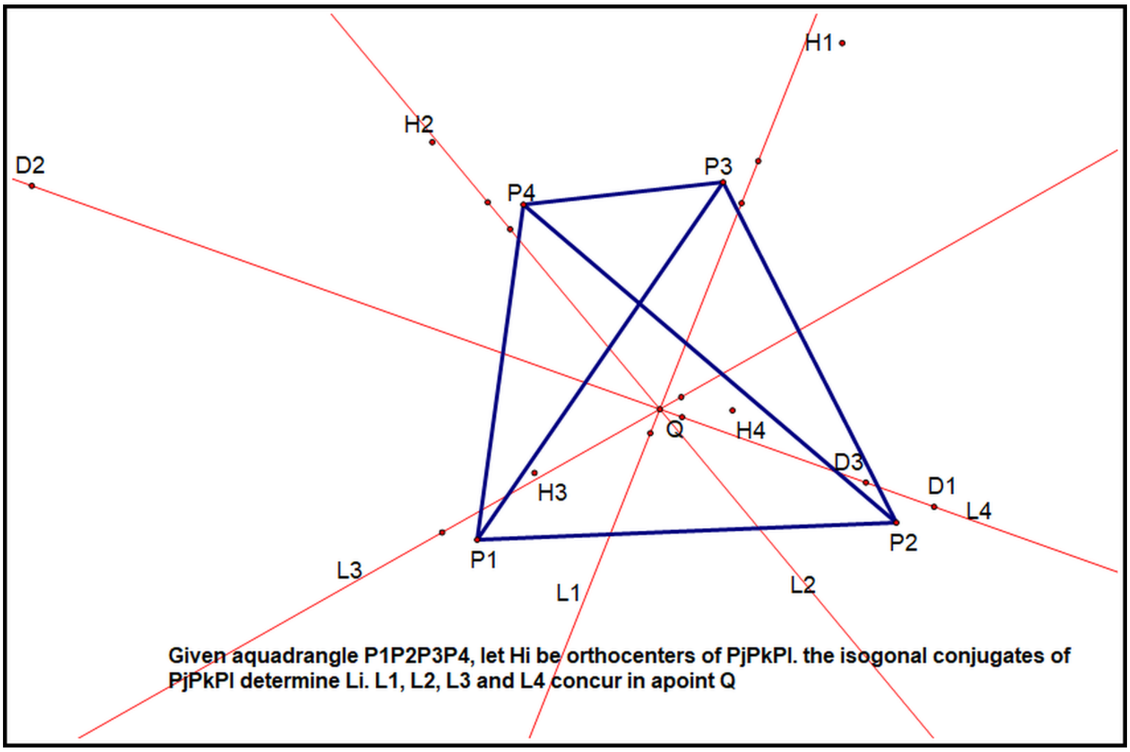
Message: #117
Date: 14/7/2013 3:02:23
From: Seiichi Kirikami
Subject: A quadrangle point

```
Dear friends,

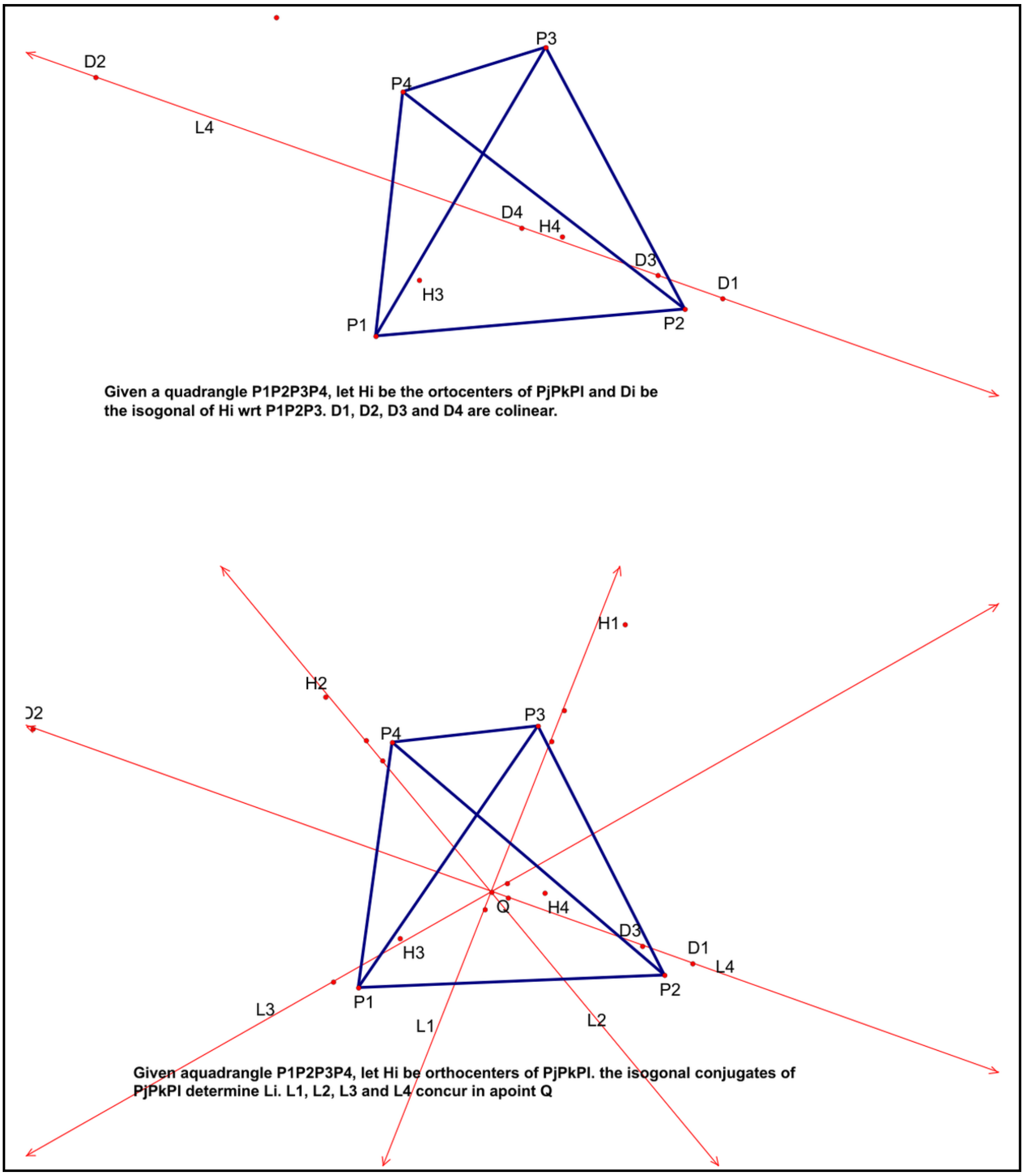
[1] Given a quadrangle P1P2P3P4, let Hi be the orthocenters of
PjPkPl and Di be the isogonal of Hi wrt P1P2P3. D1, D2, D3 and
D4 are colinear.
See Anopolis message #537.
See the attached files (gsp or MSword).

[2] Given a quadrangle P1P2P3P4, let Hi be the orthocenters of
PjPkPl. The isogonals of PjPkPl determine Li. L1, L2, L3 and L4
concur in a point Q.
See the attached files (gsp or MSword).
Sorry in advance if the above were wrong.
Sorry Chris that I can not send the coordinates. Now I am
learning copy and run program of "ConjugadoIsogonal" by
Professor GarciaCapitan.
Is there a point in EQF which may coincide the above one?

Best regards,
Seiichi.
```



QuadranglePoint-gsp.png



QuadranglePoint.docx

← Previous Next → ↩ Message Index ↑ Subjects

Message: #118
Date: 14/7/2013 8:21:36
From: seiichikiri
Subject: A quadrangle point

```
Dear friends,
This point is QA-P4(isogonal center), because I confirmed
numerically that, for example, angleP1P3P2+angleP1P4P2=
angleP1QP2.
Best regards,
Seiichi.
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #119
Date: 14/7/2013 5:35:21
From: Chris
Subject: A quadrangle point

```
Dear Seiichi,
This is a most remarkable construction of QA-P4.
I never saw this type of construction before.
Thanks to an observation of Antreas and your implementation in a
Quadrangle. Congratulations!
I tried some other Triangle points with the same construction,
however without success.
By the way QA-P4 surprises me with its many ways of beautiful
construction all the time.
I mentioned your new construction as a new property in EQF.
See:
http://www.chrisvantienhoven.nl/quadrangle-objects/15-mathematic
s/quadrangle-objects/artikelen-qa/25-qa-p4.html
Best regards,
Chris
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #120
Date: 15/7/2013 10:47:11
From: Antreas
Subject: A quadrangle point

```
Dear Seiichi and Chris
Seiichi has noticed that it seems collinearity
appears also for isotomic conjugates.
See:
http://tech.groups.yahoo.com/group/Anopolis/message/544
If it is true: Are, in a quadrangle, the four lines
concurrent as well?
APH
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #121
Date: 15/7/2013 12:42:38
From: seiichikiri
Subject: A quadrangle point

```
Dear Antreas,
I tried the isotomic case. But there was no colinearity. I would
also like to post something interesting in Anopolis.
Dear Chris,
Thank you very much for the inclusion in EQF. When I drew the
figure, it gave me some kind of dejavu feeling and it took some
time to find that it was QA-P4. But as you say, it is a
beautiful point.
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #122
Date: 15/7/2013 7:37:06
From: emmanueltsukerman
Subject: A quadrangle point

Hello Friends,
I hope I don't disappoint anyone if I point out a reference that considers the QA-P4 construction mentioned: "Central Points of the Complete Quadrangle" by Benedetto Scimemi.
You may also find related content in
http://forumgeom.fau.edu/FG2012volume12/FG201214.pdf
involving QA-P4, isogonal conjugation and collinearity, but this time via the circumcenters.
Best Regards,
Emmanuel

← Previous Next → ↩ Message Index ↑ Subjects

Message: #123
Date: 16/7/2013 11:48:41
From: seiichikiri
Subject: A quadrangle point

Dear Emmanuel, dear friends,
Thank you very much, Emmanuel, for your information. Could I post a problem which I noticed when I knew QA-P4? There may be literatures about it or it may be already in EQF or it may not exist.
Given a quadrangle P1P2P3P4 and a point P, P has the property:
angleP1P4P+anglePP3P2=angleP1PP2.
angleP2P1P+anglePP4P3=angleP2PP3.
angleP3P2P+anglePP1P4=angleP3PP4.
angleP4P3P+anglePP2P1=angleP4PP1.
In case of QA-P4, the 1st relation is written:
angleP1P4P2+angleP1P3P2=angleP1PP2, as shown in Corollary 10 of FG201214.
whether it exists or not, it remains a problem for me.
Is the Simson line in Fig.18 of FG201214 different from QL-L3(pedal line) in EQF?
Best regards,
Seiichi.

← Previous Next → ↩ Message Index ↑ Subjects

Message: #124
Date: 17/7/2013 9:10:59
From: eckart_schmidt@t-online.de
Subject: A quadrangle point

Dear Seiichi, dear friends,
I had a short look on your discussion about the new property of QA-P4 (but i am still blocked with cardioids).
There is a synthetic proof for the last angle property of QA-P4.
Jean Louis Ayme asked for a proof and I posted the letter in the attachment.
Best regards Eckart.

01.03.2013

Dear Mister Ayme,

Chris van Tienhoven told me, that you need a synthetic proof for the following property of *T=QA-P4*:

$$(*) \quad \angle ACB + \angle ADB = \angle ATB$$

for all permutations of the vertices of the quadrangle *ABCD*. I try to describe a possibility, but my English isn´t the best … et mon Francais plus mauvais.

Perhaps you are interested in the following information:
The property (*) is discussed in

Praxis der Mathematik in der Schule 1/44,. Jg 2002, 19-27
(see the scan of the beginning).

Ein merkwürdiger Punkt des Vierecks

Roland Stärk und Daniel Baumgartner

Gegeben sind in der Ebene vier Punkte A, B, C, D. Man zeige, dass es einen Punkt T gibt, der die folgenden Winkelbedingungen (orientierte Winkel modulo 180°) erfüllt:

$$\measuredangle ATB = \measuredangle ADB + \measuredangle ACB, \quad \measuredangle BTC = \measuredangle BAC + \measuredangle BDC,$$
$$\measuredangle CTD = \measuredangle CBD + \measuredangle CAD, \quad \measuredangle DTA = \measuredangle DCA + \measuredangle DBA.$$

In chapter 2 Stärk gives an analytical proof with special cartesian coordinates for the following property (see scan), which is the definition of *QA-P4* in *EQF*.

Satz: (Fig. 5): Der zur Ecke A des Vierecks $ABCD$ bezüglich des Dreiecks BCD isogonalkonjugierte Punkt A^* und der Punkt T des Vierecks sind invers bezüglich des Umkreises des Dreiecks BCD.

Now my idea for a synthetic proof, that *QA-P4* of a quadrangle *ABCD* has the property (*). *QA-P4=T* shall be defined as the invers of the isogonal conjugate of a vertice *D* wrt the residue triangle *ABC*.
Angles:

$$\angle BAD = \alpha, \quad \angle CBA = \beta, \quad \angle ACB = \varphi_1, \quad \angle ADB = \varphi_2.$$

Ayme-TPkt.pdf

These angles define the quadrangle.

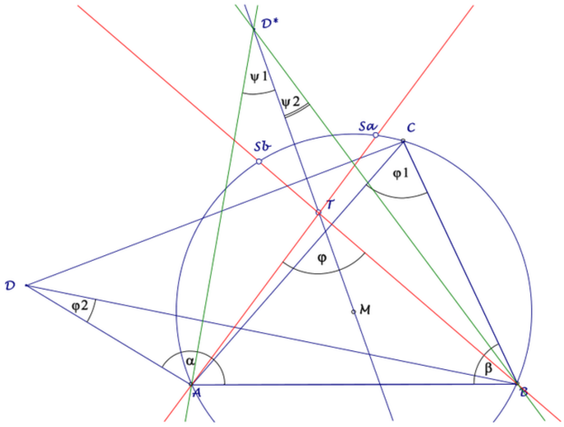
Points:

M midpoint of the circumcircle of ABC,
D^* isogonal conjugate of D wrt ABC,
T=QA-$P4$ reflection of D^* in the circumcircle of ABC.

Angles:

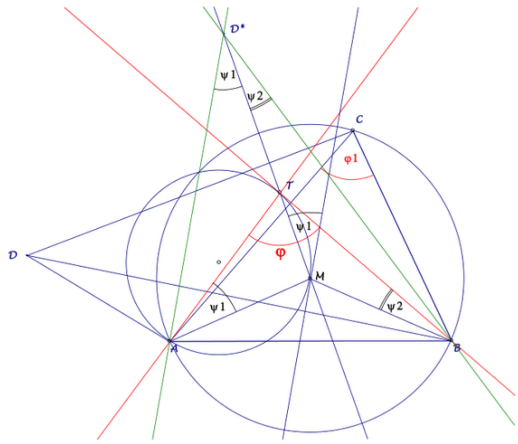
$$\angle ATB = \varphi, \quad \angle AD^*T = \psi_1, \quad \angle TD^*B = \psi_2.$$

To prove: $\varphi = \varphi_1 + \varphi_2$.



1. The reflection in a circle is angle-true. Reflecting AD^* in the circumcircle of ABC, we get a circumcircle of ATM with a tangent in M parallel AD^*. The angle ψ_1 becomes chord-tangent-angle of MT and equals the peripheral-angle:

$$\angle MAT = \psi_1 \quad \textit{and in the same way:} \quad \angle TBM = \psi_2.$$



Ayme-TPkt.pdf

Angle balance in *ATBM* gives with $\angle AMB = 2\varphi_1$

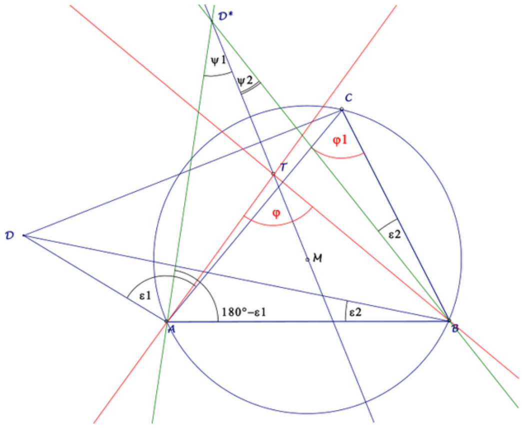
$$(1) \quad \varphi = 2\varphi_1 - \psi_1 - \psi_2 \ .$$

2. The isogonal conjugate wrt *ABC* gives

$$\angle CAD = \angle D^* AB = \varepsilon_1, \quad \angle DBA = \angle CBD^* = \varepsilon_2$$

with the angle balance in *ABD**

$$(2) \quad \psi_1 + \psi_2 = \varepsilon_1 + \varepsilon_2 - \beta \ .$$



3. Angle balance in *ABD:*

$$(3) \quad \varepsilon_2 = 180° - \alpha - \varphi_2 .$$

Angle balance in *ABC:*

$$(4) \quad \varepsilon_1 = -180° + \alpha + \beta + \varphi_1 .$$

From (1), (2), (3), (4) follows directly:

$$\varphi = \varphi_1 + \varphi_2 \qquad \blacksquare$$

Yours sincerely
Eckart Schmidt

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Ayme-TPkt.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #125
Date: 17/7/2013 12:13:18
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

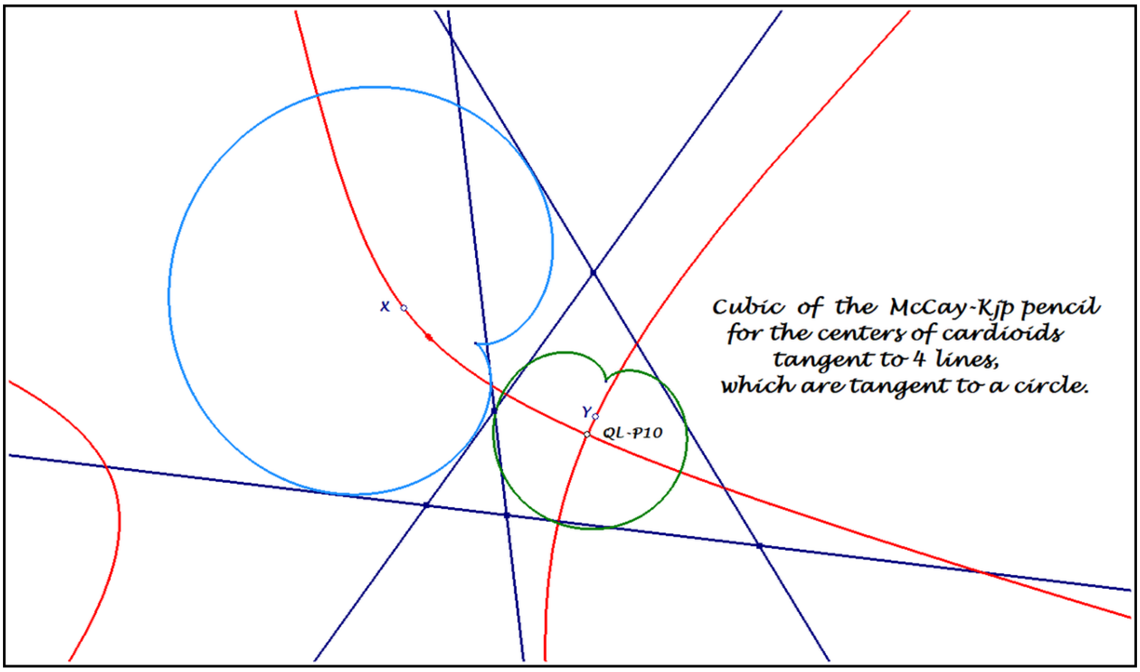
```
Dear Bernard, dear friends,
ut to now I cannot reproduce Bernard´s construction of reference
triangles for my cubic. Here is my own way, how to find the
reference triangles in the McCay or Kjp case:
#   X is the intersection of QL-L1 and QL-L6.
#   Co1 is the conic with center X and tangent to the lines
    of the quadrilateral.
#   Ci1 is the Thales circle for the foci of Co1.
#   O  is a variable point on the secondary axis of Co1.
#   Ci2 is the circle round O perpendicular Ci1.
#   Point L is the reflection of X in Ci2.
#   Point  A is a variable point on Ci2, the corresponding
    triangle ABC has circumcircle Ci2 and centroid QL-P1.
#   Ci3 is a circle as locus of the Lemoine points L' of ABC.
#   Chose O on QL-L1, that Ci3 contains L.
#   Chose A on Ci2, that L' = L.
#   If the secondary axis of Co1 is QL-L1,
    my cubic is the McCay cubic.

If the secondary axis of Co1 is perpendicular QL-L1, my cubic is
the Kjp cubic.
This method degenerates for a quadrilateral tangent to 4 lines:
Co1 is a circle, Ci1 is the point X = QL-P10, the secondary axis
can be QL-L1 (or a perpendicular line). With the order next to
last the reference triangle degenerates collinear and my cubic
is the limit McCay (or Kjp) cubic..
Bernard has doubts, that for a quadrilateral tangent to 4 lines
my cubic belongs to the McCay-Kjp pencil.

Here is a construction, knowing the deltoid axes of the
quadrilateral:
#  Consider parallels to the deltoid axes through QL-P1 and
   their intersections with a line parallel to QL-L1 in 2/3
   distance from QL-P1. These points lie on the cubic.
#  Chose one of these points as A and construct a special
reference triangle:
   * Take QL-P1 as centroid G of  the reference triangle ABC.
   * Take QL-P10 as a focus F for the inscribed Steiner ellipse
     of ABC.
   * Let Ma be the homothetic of A under h(G,-1/2).
   * Construct the Steiner ellipse with center G,
     Focus F and point Ma.
   * The tangents from A to the ellipse and the tangent in Ma
```

```
     give a special reference triangle ABC.
#  Take another point of the cubic (see above) and construct
   its pedal circle wrt ABC. Let "theta" be the angle of
   intersection of this pedal circle and the nine-point circle.
#  Construct the cubic of the McCay-Kjp pencil
   with the angle "theta" (see message #114)
   * Let P be a point on the circumcircle
     of the reference triangle ABC,
   * ... let L(P) be the parallel at O to the Simson line of P;
   * ... L(P) cuts the circumcircle in two points,
   * ... consider the rectangular hyperbola
         through these two points and B, C,
   * ... consider the line AP rotated by 90°+"theta" at A,
   * ... the intersections of this line and the rectangular
         hyperbola will be points of the cubic.

In the attachment there is a Cabri file with an quadrilateral
tangent to 4 lines and my cubic constructed in the described
way. You can test the variable centers on the cubic for
cardioids tangent to 4 lines (the cubic is constructed in two
parts, therefore the centers X and Y).
I hope, there are no basic mistakes.
Best regards Eckart
```



McCay-Kjp-fig.png

Message: #126
Date: 17/7/2013 1:46:26
From: Seiichi Kirikami
Subject: 4 concurrent circles points similar to Euler-poncelet and Varignon p

---

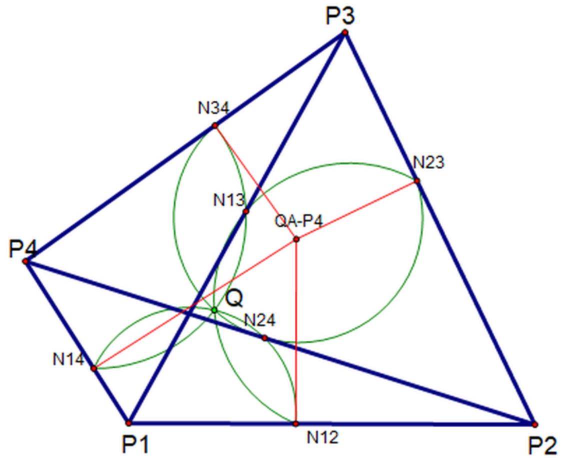
```
Dear Emmanuel, dear friends,

Your paper FG201214 led me to the following 4 concurrent circles
points. See the attached files (gsp or MSword).
[1] Euler-Poncelet like point:
Given a quadrangle P1P2P3P4, QA-P4 and pedal Nijfrom QA-P4 on
PiPj, the circles through N12N14N24, N23N24N34, N12N13N23 and
N13N14N34 concur in a point Q.

[2]Gergonne-Steiner like point:
Given a quadrangle P1P2P3P4, QA-P4 and pedal Nijfrom QA-P4 on
PiPj, the circles through N12N13N14, N13N23N34, N14N24N34 and
N12N24N23 concur in a point QA-P4. This is not interesting.

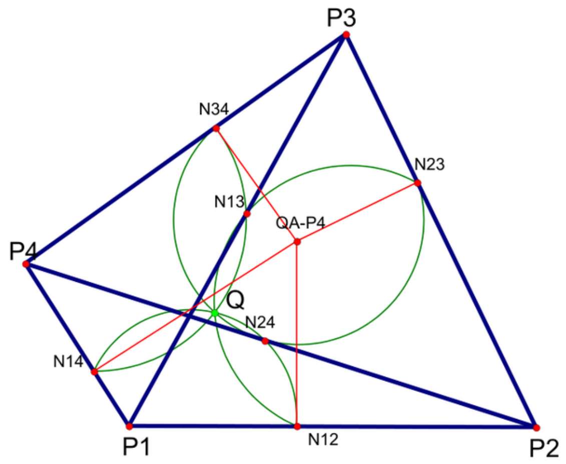
[3] Varignon like points(1)
Similar to Varignon points. See the message#39(Varignon Points).

[4] Varignon like points(2)
Similar to Varignon points. See the message#39(Varignon Points).
I will make the coordinates of [1].
Best regards,
Seiichi.
```

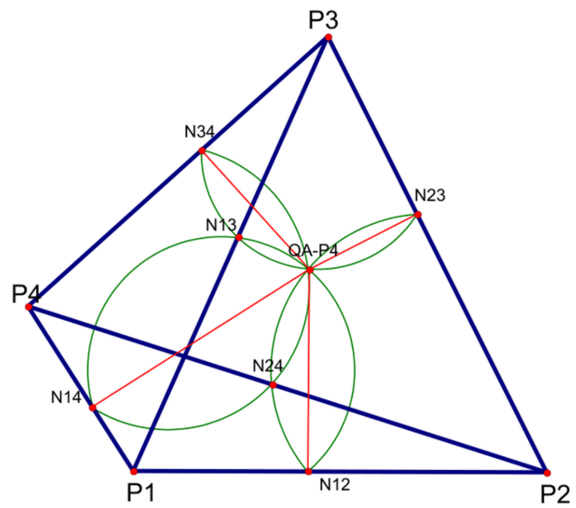


Given a quadrangle P1P2P3P4, QA-P4 and pedal points Nij from QA-P4 on PiPj, the circles through N12N14N24, N23N24N34, N12N13N23 and N13N14N34 concur in a point Q.

4ConcurrentCircles-gsp.png

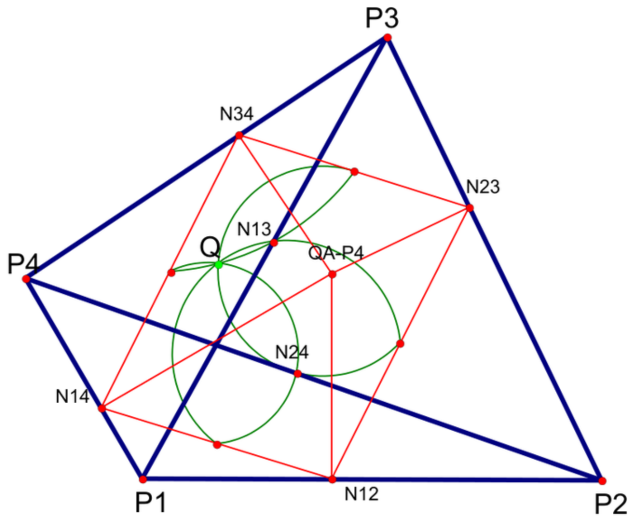


Given a quadrangle P1P2P3P4, QA-P4 and pedal points Nij from QA-P4 on PiPj, the circles through N12N14N24, N23N24N34, N12N13N23 and N13N14N34 concur in a point Q.

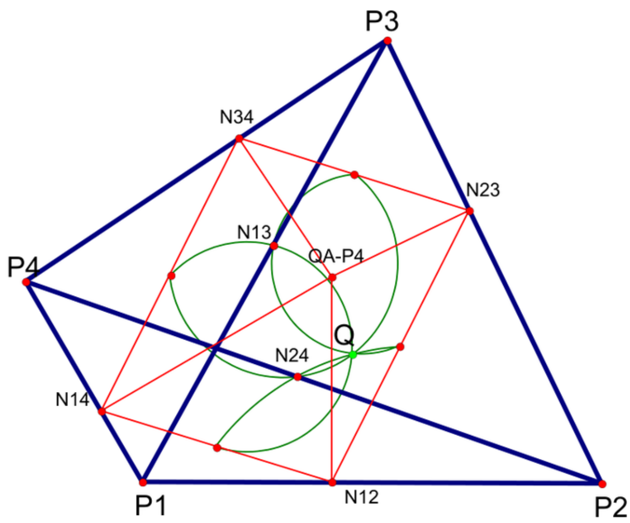


Given a quadrangle P1P2P3P4, QA-P4 and pedal points Nij from QA-P4 on PiPj, the circles through N12N13N14, N13N23N34, N14N24N34 and N12N24N23 concur in a point QA-P4.

4ConcurrentCircles.docx



See message #39 (Varignon points)



See message #39 (Varignon points)

4ConcurrentCircles.docx

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #127
Date: 18/7/2013 8:14:57
From: Chris
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

```
Dear Seiichi,
Interesting!
These points are made of restricted permutations of points.
Restated:
Let Nij be the feet of the perpendiculars from QA-P4 to Pi.Pj.
[1] The 4 versions of circles through Nij, Njk, Nki (for all
permutations (i,j,k) ∈ (1,2,3,4)) have one common point, which
is QA-P2.
[2] The 4 versions of circles through Nij, Nik, Nil (for all
permutations (i,j,k,l) ∈ (1,2,3,4)) have one common point, which
is QA-P4.
[3] and [4} are actually Quadrigon points. However their
midpoint is Quadrangle point QA-P6.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #128
Date: 18/7/2013 10:01:02
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugate quadrigon of a point

Dear friends,

Emmanuel led me to the paper FG201214, I had a short look and found on the last pages the following definition:
* "Let P be a point on the plane of ABCD. Let lA, lB, lC, lD be the reflections of the lines AP, BP, CP, DP in the bisectors of
* Definition. Let PA= lA^lB, PB=lB^lC, PC=lC^lD, PD=lD^lA. The quadrilateral BA.PB.PC.PD will be called the isogonal conjugate of P wrt to ABCD ..."

Wrt Chris classification, this is a definition for quadrigons.
I have tested the question, for which points the isogonal conjugate quadrigon degenerates into a four times counted point.
Here the results:
* If the isogonal conjugate quadrigon of P degenerates, the four counted point Q is the QG-Tf2 image of P, or the isogonal conjugated of P wrt QG-Tr3.
* The points P, for which the isogonal conjugate quadrigon degenerates, and the four counted point Q lie on a cubic with the equation
* $2\ SA\ z\ (q^2x^2-p^2y^2) + 2\ SC\ x\ (r^2y^2-q^2z^2) + (c^2q^2+b^2r^2)\ x^2\ y - (b^2p^2+a^2q^2)\ y\ z^2 + (a^2r^2-c^2p^2)\ y^3 = 0$ (u
sing the diagonal triangle QA-DT as reference triangle for barycentric coordinates).
* This cubic is the QL-Quasi Isogonal Cubic QL-Cu1.
* The points P and Q are foci of inscribed conics of the corresponding quadrilateral.
* The three quadrigons of a quadrilateral have the same points, for which the isogogonal conjugate quadrigon degenerates.

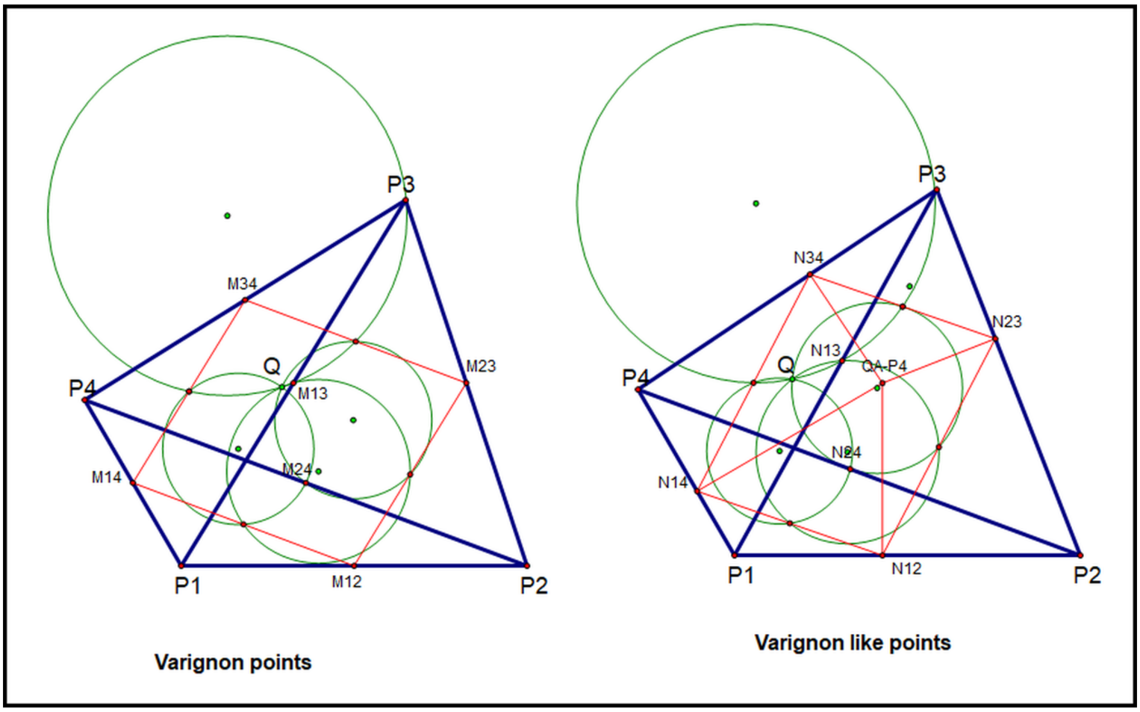
Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #129
Date: 18/7/2013 1:20:45
From: Seiichi Kirikami
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

```
Dear Chris,
Thank you very much for your identification. I confirmed QA-P2.
What I found was not Euler Poncelet like point, but Euler
Poncelet point itself. I could not help laughing.
As for [3] and [4], I could not understand "However, their
midpoint is Quadrangle point QA-P6".
I attached the file. I hope your detailed explanation.
Eckart's results about FG201214: Splendid!
Best regards,
Seiichi.
```



4ConcurrentCirclesR1-gsp.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #130
Date: 18/7/2013 3:32:03
From: Chris
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

Dear Seiichi,
The Varignon-like points deal with the midpoints of the sides of the Varignon Parallelogram. It is good to realize that the sides of the Varignon Parallelogram are parallel to 2 of the 3 diagonals of a Quadrangle. In a Quadrigon we also work with 2 diagonals. That's why the construction of the Varignon-like points are Quadrigon points.
You work actually in the Quadrigon P1.P2.P3.P4 (order is significant).
There are also Quadrigons P1.P2.P4.P3 and P1.P3.P2.P4.
When you construct the same duo of Varignon-like points for Quadrigons P1.P2.P4.P3 and P1.P3.P2.P4 then you will find other points.
However the connecting lines of the 3 duo's intersect in one point, which is QA-P6.
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #131
Date: 18/7/2013 6:54:46
From: emmanueltsukerman
Subject: A quadrangle point

Dear Seiichi,
Thank you for your question. That's correct - the Simson line in Fig.18 of FG201214 is QL-L3(pedal line) in EQF.
Best Regards,
Emmanuel

← Previous Next → ↔ Message Index ↑ Subjects

Message: #132
Date: 19/7/2013 9:59:28
From: Chris
Subject: Finding point X on QA-Cu1 Cubic

```
Dear friends,
Given random point P and Quadrangle P1.P2.P3.P4.
Let Rij = Reflection of P in Pi.Pj, where i,j can have values
1,2,3,4.
The locus where lines R12.R34, R13.R24, R14.R23 are concurrent
is the cubic QA - Cu1.
See: http://www.chrisvantienhoven.nl/other-quadrangle-objects/15
-mathematics/quadrangle-objects/artikelen-qa/83-qa-cu1.html
Let's call this concurrency point the QA-Reflect Image.
Example:
QA-P2 is the QA-Reflect-Image of QA-P4.
I observed that QA-P38 is the QA-Reflect-Image of some point X
on QA-Cu1.
Anyone who can help me finding the coordinates of this point X?
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #133
Date: 20/7/2013 11:20:07
From: eckart_schmidt@t-online.de
Subject: Finding Point X on QA-Cu1 Cubic

```
Dear Chris, dear friends,
the "QA-Reflect Image" is only defined for points on QA-Cu1. The
connection of a point on QA-Cu1 and its image is parallel to
QA-P2.QA-P4 or QA-P11.QA-P38. So the searched point is the
intersection of QA-P11.QA-P38 with the cubic.
First DT-coordinate:
p^2 (b^2 r^4 (b^4 p^2 - a^4 q^2) + c^2 q^4 (c^4 p^2 - a^4 r^2) +
b^2 c^2 p^2 q^2 r^2 (2a^2 - b^2 - c^2)
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #134
Date: 20/7/2013 12:21:24
From: seiichikiri
Subject: A quadrangle point

```
Dear Emmanuel, dear friends,
Concerning another Simson line, I would like to mention Eckart's
Probe(Themen 2005, 05-1, Simson-Gerade eines Kreisvierecks).
http://eckartschmidt.de/
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #135
Date: 20/7/2013 1:49:26
From: Chris
Subject: Finding Point X on QA-Cu1 Cubic

Dear Eckart, dear friends,

Thanks Eckart!
I used the same trick as you did, however I was not able to find simple coordinates as an intersection of a cubic and a line. I wonder how you did.
Now that the coordinates are known it is so much simpler to find other properties of this point.
I found it lies on:
* The line QA-P11.QA-P38,
* The line through QA-P41 parallel to QA-P3.QA-P4,
* The perpendicular bisector of QA-P2.QA-P4,
* Cubic QA-Cu1.

The perpendicular bisector of QA-P2.QA-P4 is an old acquaintance of ours. We (Eckart, Seiichi and me) discussed it before in mails before this Quadri-Figures-Group existed.

I quote:
*** Mail 8-1-2013 from 13:22 Seiichi Kirikami
"Montesdeoca-Hutson point (QA-P38) is determined by the diagonal lines of the hexagon s12s14s13s34s23s24, which I call MH hexagon.
The intersections of the lines s12.s14*s23.s34, s13.s14*s23.s24 and s13.s34*s12.s24 determine a Pascal line."

*** Mail 9-1-2013 12:25 from Eckart Schmidt
"Further properties of the MH Pascal line:
 ... polar of QA-P38 wrt QA-Ci1
 ... QA-Tf2 image of QA-Ci1
 ... perpendicular bisector of QA-P2.QA-P4
The line is already mentioned under QA-Ci1, where the last two properties are to be found."

I also will mention now some other properties related to the QA-Reflect transformation, that only is valid for points on the QA-Cu1 cubic.
1. The line through 2 Involutary Conjugated points on QA-Cu1 passes through QA-P4.
2. The line through a point on QA-Cu1 and its QA-Reflect image is parallel to QA-P2.QA-P4.
3. The QA-Reflect-images of a point on QA-Cu1 and its Involutary Conjugate pass through the Reflection of QA-P4 in QA-P2.

4. The line through a point on QA-Cu1 and the QA-Reflect image
   of its Involutary Conjugate pass through QA-P2.

It looks like that QA-P2 and QA-P4 are pretty dominant for this transformation.
Knowing these properties I wonder if it is possible to construct the reverse of the QA-Reflect transformation in a simple way.
Any Suggestions?

Best regards,
Chris

← Previous Next → ↩ Message Index ↑ Subjects

Message: #136
Date: 21/7/2013 3:09:07
From: Chris
Subject: A quadrangle point

Dear Emmanuel, dear friends,
I looked through the paper of Benedetto Scimemi.
Thanks Emmanuel for the reference.
I found all kinds of interesting properties regarding QA-P1 (Centroid), QA-P2 (Euler-Poncelet Point/ nine-Circle Point) and QA-P4 (Isogonal Center/ Isoptic Point), but not the one we were discussing.
So as yet I consider it to be revealed by Antreas and Seiichi.
Best regards,
Chris

← Previous Next → ↩ Message Index ↑ Subjects

Message: #137
Date: 22/7/2013 7:15:46
From: emmanueltsukerman
Subject: A quadrangle point

Dear Chris,
That's excellent! I'm glad you found the reference interesting and I'm glad my memory served me wrong here.
Best Regards,
Emmanuel

← Previous Next → ↔ Message Index ↑ Subjects

Message: #138
Date: 22/7/2013 11:56:48
From: Chris
Subject: What is the necessary and sufficient condition of quadrifigure f

---

Dear Seiichi,
I studied your original thoughts about the necessary and sufficient condition of quadrifigure function.
When I see it right your premise is that when a Quadrangle point is placed into a Square it should be in the center of the square.
And -I checked it- that is true for all points QA-P1 till QA-P42.
However I still wonder if we can say this true for all Quadrangle points.
There is an equivalence in a triangle.
When we place triangle points in an equilateral triangle almost all triangle points end up in the center.
However not all of them.
You can see this when you make a triangle almost equilateral and see where the points "fly".
See: http://sites.paideiaschool.org/steve_sigur/resources/all%20points%20web/all-points.html
Especially Yao Liu's observations (at 30% from the beginning of the page).
How do you see this?
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #139
Date: 23/7/2013 3:34:39
From: seiichikiri
Subject: What is the necessary and sufficient condition of quadrifigure f

---

```
Dear Chris,
Thank you very much for your pointing out the complexity of the
center of almost equilateral triangle. I vaguely remember the
case that as the triangle becomes almost equilateral, the
center{p, q, r} limits to {0, 0, 0}. I think that in some cases
the points fly somewhere out of the center. I must correct the
contents of message#47. What I proposed is not the necessary
condition. It is one of many methods to check the possibility of
a quadrangle point.
Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

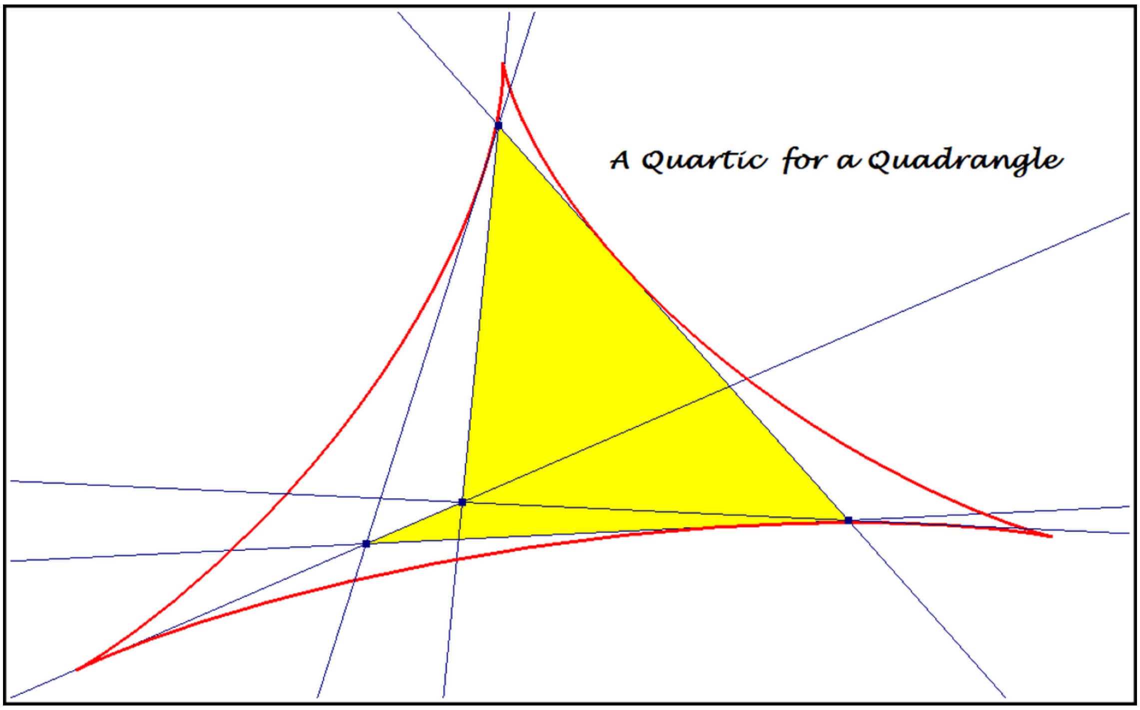
Message: #140
Date: 23/7/2013 4:29:17
From: seiichikiri
Subject: a property of QA-P2

```
Dear friends,
I learned the following property from Mr. Peter J. C. Moses.
Given a quadrangle P1P2P3P4 and the antigonal image Qi of Pi wrt
PjPkPl, QA-P2 is the midpoint of PiQi.
I changed the property of a triangle to that of a quadrangle.
Best regards,
Seiichi.
```

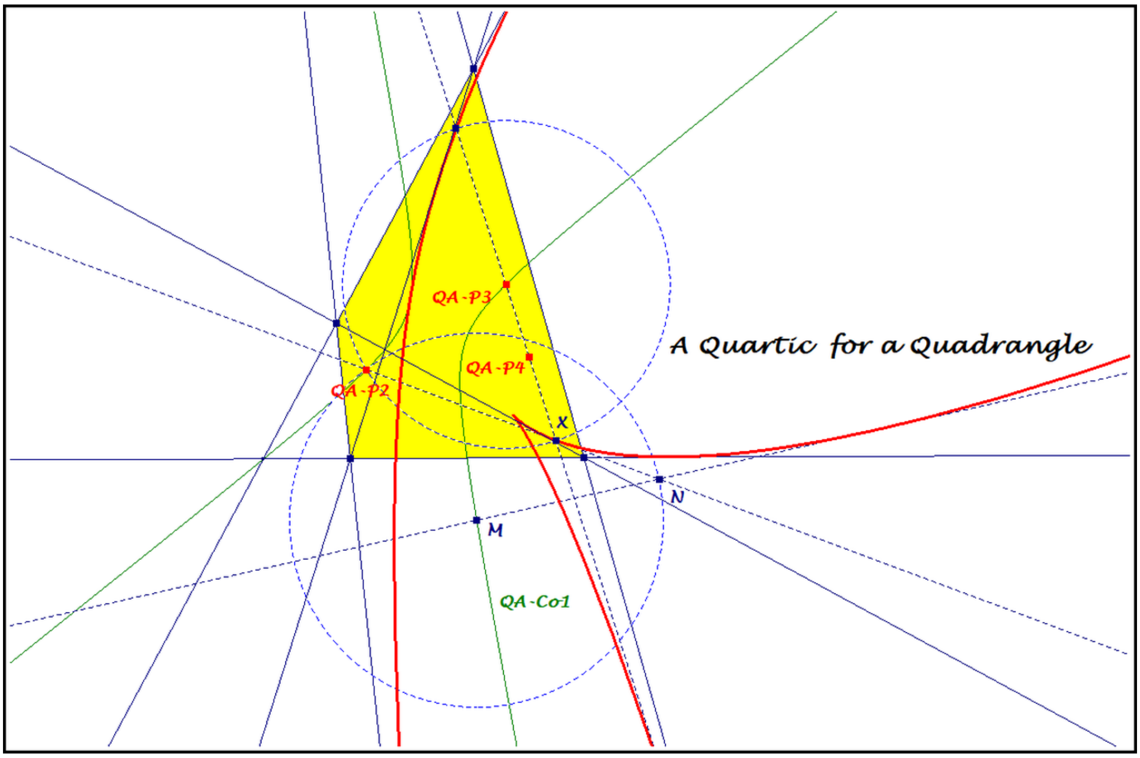
← Previous Next → ↩ Message Index ↑ Subjects

Message: #141
Date: 24/7/2013 9:10:37
From: eckart_schmidt@t-online.de
Subject: A Quartic for a Quadrangle

```
Dear friends,
I have found a quartic tangent to the six lines of a quadrangle.
Geometrical background, equation and construction are described
in a Mathematica file, two examples are to be found in Cabri
files in the attachment.
Not the points of the quartic, but rather the tangents are
interesting.
Best regards Eckart
```



quartic02-fig.png



quartic01-fig.png

A Quartic for a Quadrangle

Not difficult to prove:
 Let P be a point in the plane of the quadrilateral L1L2L3L4 (with the intersections Sij).
 Let Lij be the reflection of PSij in the bisector of <LiLj.
 The points L12^L34, L23^L41, L13^L24 coincide for points P on QL-Cu1 in the QL-Tf1 image of P.

More interesting, changing points and lines:
 Let L be a line in the plane of the quadrangle P1P2P3P4 (with connecting lines Lij).
 Let Sij be the reflections of L^Lij in the midpoint of PiPj.
 The lines S12.S34, S23.S41, S13.S24 will be the same for lines L(e,f,g) (DT-coefficients) with

$$\frac{e\,p^2}{f-g}+\frac{f\,q^2}{g-e}+\frac{g\,r^2}{e-f}==0$$

If a line belongs to this pencil, the three counted image line belongs also to the pencil.
Example1: QA-P3.QA-P4 and QA-P2.QA-Tf2(QA-P2).
Example 2: Parallel to QG-P1.QG-P2 through QG-P13 and

$$\{(q^2-4r^2)(3p^2-q^2+r^2),\ 2(p^2-r^2)^2,\ (q^2-4p^2)(3r^2+p^2-q^2)\}$$

Through every point there are up to three lines of the pencil.
For any direction (u:v:w) with u + v + w = 0 there is only one line of the pencil:

$$L_{uvw}:=\{u(q^2w^2-r^2v^2),\ v(r^2u^2-p^2w^2),\ w(p^2v^2-q^2u^2)\}$$

The lines envelope a quartic with the equation:

$$\begin{aligned}
&\text{quad}[\{x_,y_,z_\}]:=4q^2(q^2-r^2)^2r^2x^4+4p^2(p^2-r^2)^2r^2y^4+4p^2(p^2-q^2)^2q^2z^4\\
&+4q^2r^2(3p^2q^2+5p^2r^2+(q^2-r^2)^2)x^3y\\
&+4q^2r^2(3p^2r^2+5p^2q^2+(q^2-r^2)^2)x^3z\\
&+4p^2r^2(3p^2q^2+5q^2r^2+(p^2-r^2)^2)xy^3\\
&+4p^2r^2(3q^2r^2+5p^2q^2+(p^2-r^2)^2)y^3z\\
&+4p^2q^2(3p^2r^2+5q^2r^2+(p^2-q^2)^2)xz^3\\
&+4p^2q^2(3q^2r^2+5p^2r^2+(p^2-q^2)^2)yz^3\\
&+p^2(p^6-2p^4q^2+p^2q^4-2p^4r^2+38p^2q^2r^2+12q^4r^2+p^2r^4+12q^2r^4)y^2z^2\\
&+q^2(p^4q^2-2p^2q^4+q^6+12p^4r^2+38p^2q^2r^2-2q^4r^2+12p^2r^4+q^2r^4)x^2z^2\\
&+r^2(12p^4q^2+12p^2q^4+p^4r^2+38p^2q^2r^2+q^4r^2-2p^2r^4-2q^2r^4+r^6)x^2y^2\\
&+2q^2r^2(11p^4+24p^2q^2+q^4+24p^2r^2-2q^2r^2+r^4)x^2yz\\
&+2p^2r^2(p^4+24p^2q^2+11q^4-2p^2r^2+24q^2r^2+r^4)xy^2z\\
&+2p^2q^2(p^4-2p^2q^2+q^4+24p^2r^2+24q^2r^2+11r^4)xyz^2
\end{aligned}$$

The point of tangency of the line L_{uvw} will be:

$$\begin{aligned}
&\{q^2r^2u^4-p^4v^2w^2+p^2u(v-w)(r^2v^2-q^2w^2),\\
&\ p^2r^2v^4-q^4u^2w^2+q^2v(u-w)(r^2u^2-p^2w^2),\ p^2q^2w^4-r^4u^2v^2+r^2w(u-v)(q^2u^2-p^2v^2)\}
\end{aligned}$$

The other two points of intersection with the quartic have midpoint:

$$\{p^2vw,\ q^2uw,\ r^2uv\}$$

This is the QA-Tf2 image of the point at infinity of L_{uvw}.
That means that the further intersections of the tangents to the quartic have midpoint on the

QA-DT circumconic, which is the QA-Tf2 image of the line at infinity.

The quartic is tangent to the lines Lij.
The points of tangency Tij divide Pi.Pj in the following ratios:

$$\tau12 := \frac{-p+q+r}{p-q+r};\ \tau23 := \frac{p-q+r}{p+q-r};\ \tau34 := \frac{p+q-r}{p+q+r};\ \tau41 := \frac{p+q+r}{-p+q+r};$$

$$\tau13 := \frac{-p+q+r}{p+q-r};$$

$$\tau24 := \frac{p-q+r}{p+q+r};$$

The lines T12.T34, T23.T41, T13.T24 have a common point:

$$\{p\,(p^6 - 2p^5 q - p^4 q^2 + 4p^3 q^3 - p^2 q^4 - 2p q^5 + q^6 - 2p^5 r - 6p^4 q r - 4p^3 q^2 r + 4p^2 q^3 r + 6p q^4 r + 2q^5 r - p^4 r^2 - 4p^3 q r^2 + 10p^2 q^2 r^2 - 4p q^3 r^2 - q^4 r^2 + 4p^3 r^3 + 4p^2 q r^3 - 4p q^2 r^3 - 4q^3 r^3 - p^2 r^4 + 6p q r^4 - q^2 r^4 - 2p r^5 + 2q r^5 + r^6),$$
$$q\,(p^6 - 2p^5 q - p^4 q^2 + 4p^3 q^3 - p^2 q^4 - 2p q^5 + q^6 + 2p^5 r + 6p^4 q r + 4p^3 q^2 r - 4p^2 q^3 r - 6p q^4 r - 2q^5 r - p^4 r^2 - 4p^3 q r^2 + 10p^2 q^2 r^2 - 4p q^3 r^2 - q^4 r^2 - 4p^3 r^3 - 4p^2 q r^3 + 4p q^2 r^3 + 4q^3 r^3 - p^2 r^4 + 6p q r^4 - q^2 r^4 + 2p r^5 - 2q r^5 + r^6),$$
$$r\,(p^6 + 2p^5 q - p^4 q^2 - 4p^3 q^3 - p^2 q^4 + 2p q^5 + q^6 - 2p^5 r + 6p^4 q r - 4p^3 q^2 r - 4p^2 q^3 r + 6p q^4 r - 2q^5 r - p^4 r^2 + 4p^3 q r^2 + 10p^2 q^2 r^2 + 4p q^3 r^2 - q^4 r^2 + 4p^3 r^3 - 4p^2 q r^3 - 4p q^2 r^3 + 4q^3 r^3 - p^2 r^4 - 6p q r^4 - q^2 r^4 - 2p r^5 - 2q r^5 + r^6)\}$$

It seems, that no QA-point lies on this quartic!
Not the points, but rather the tangents are interesting!

Finally a simple construction of the quartic:

Consider the line QA-P2.X, when X is one of the intersections of QA-P3.QA-P4 and the circle round QA-P3 through QA-P2 (X is a point of the quartic).
Let M be a variable point on QA-Co1 as center of a circle through QA-P2.
This circle has a second intersection N with QA-P2.X.
The lines MN envelope the quartic.

Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

Quartic-nb.pdf

Message: #142
Date: 24/7/2013 12:55:44
From: Chris
Subject: A Quartic for a Quadrangle

Dear Eckart,
Very nice!
What are the coordinates of:
* the Cusp
* the point of tangency Quartic and NinePointConic?
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #143
Date: 25/7/2013 3:36:20
From: seiichikiri
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

Dear Chris,
I see quadrigons. I confirmed the following.
[1] Varignon parallelogram:the lines through 3 duos determined by concurrent circles concur in QA-P1.
[2] Varignon-like parallelogram: the lines through 3 duos determined by concurrent circles concur in QA-P6.
Incidentally I hope that Chris will give a proper name to Varignon-like parallelogram on the basis of literatures, as it is inconvenient to always call it Varignon-like.
Best regards,
Seiichi.

← Previous Next → ↔ Message Index ↑ Subjects

Message: #144
Date: 25/7/2013 7:39:03
From: Chris
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

```
Dear Seiichi,
I think that the predicate "Varignon-like" is very
understandable.
I don't know of better terminology in literature with the same
meaning.
Maybe anyone else has a suggestion?
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #145
Date: 28/7/2013 1:08:34
From: seiichikiri
Subject: 4 concurrent circles points similar to Euler-poncelet and Varign

---

```
Dear Chris,
I think that you should give a proper name on the basis of 1st
appearence in the literatures.
For example, someone will find the same Varignon-like after 10
years and it will be named something. Then there will be
possibility of negotiation of names.
Other naming: Varignon parallelograms of the 1st kind and 2nd
kind.
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #146
Date: 29/7/2013 7:32:55
From: seiichikiri
Subject: A property of QA-P4

Dear friends,
Given a quadrangle P1P2P3P4 and the antigonal image Ai of Pi wrt PjPkPl, the isogonal conjugates of Ai wrt PjPkPl coincide with each other, QA-P4.
Best regards,
Seiichi.

← Previous Next → ↔ Message Index ↑ Subjects

Message: #147
Date: 29/7/2013 9:06:53
From: Chris
Subject: A property of QA-P4

Dear Seiichi,
This is not quite surprising, because:
The Antigonal of a point P is the isogonal conjugate of the inverse in the circumcircle of the isogonal conjugate of QA-vertex Pi (i=1,2,3,4).
So Antigonal = g.i.g.Pi
When you take the isogonal conjugate of the antigonal this is the result: X = g.g.i.g.Pi
Since 2 times Isogonal Conjugate is Identity X = i.g.P.
In other words X = QA-P4 = the Inverse of the Isogonal Conjugate of Pi.
This property is already known.
Best regards,
Chris
notations: g = isogonal, i = inverse

← Previous Next → ↔ Message Index ↑ Subjects

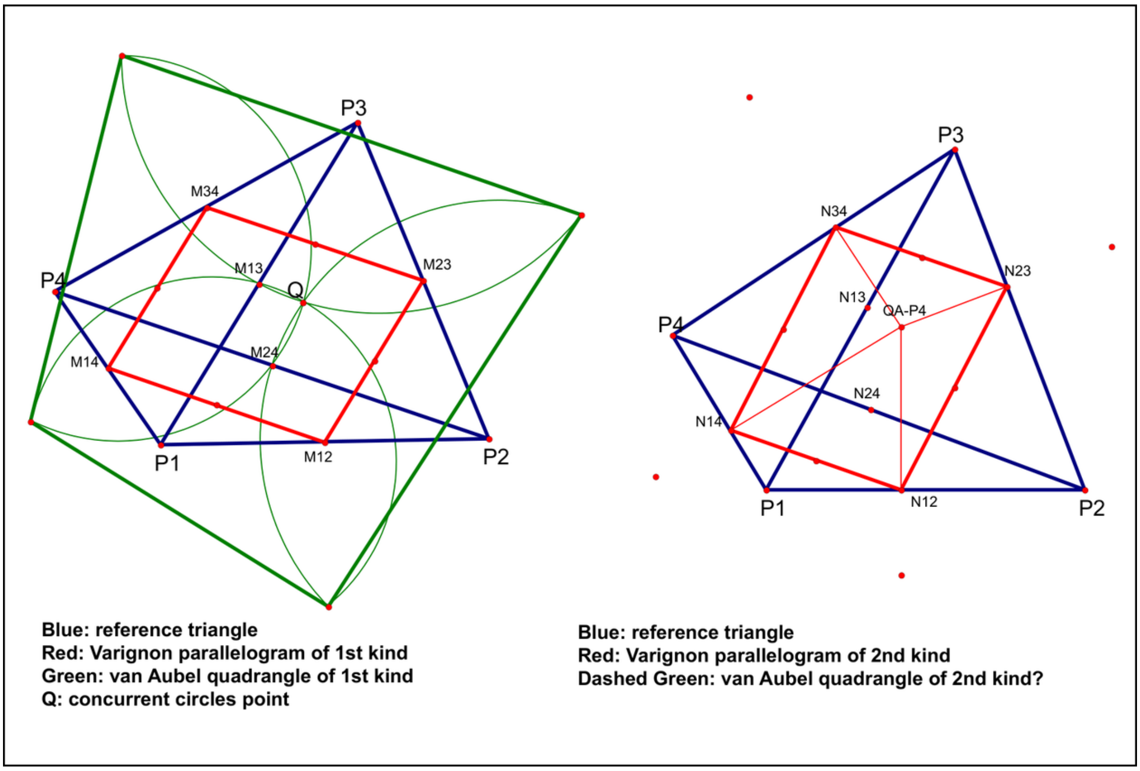
Message: #148
Date: 29/7/2013 10:18:25
From: seiichikiri
Subject: A property of QA-P4

Dear Chris,
I see. When I saw the description of QA-P4 precisely, what I found is not beyond it.
My experience: antigonal image produces something, but syngonal image does not produce anything.
Best regards,
Seiichi.

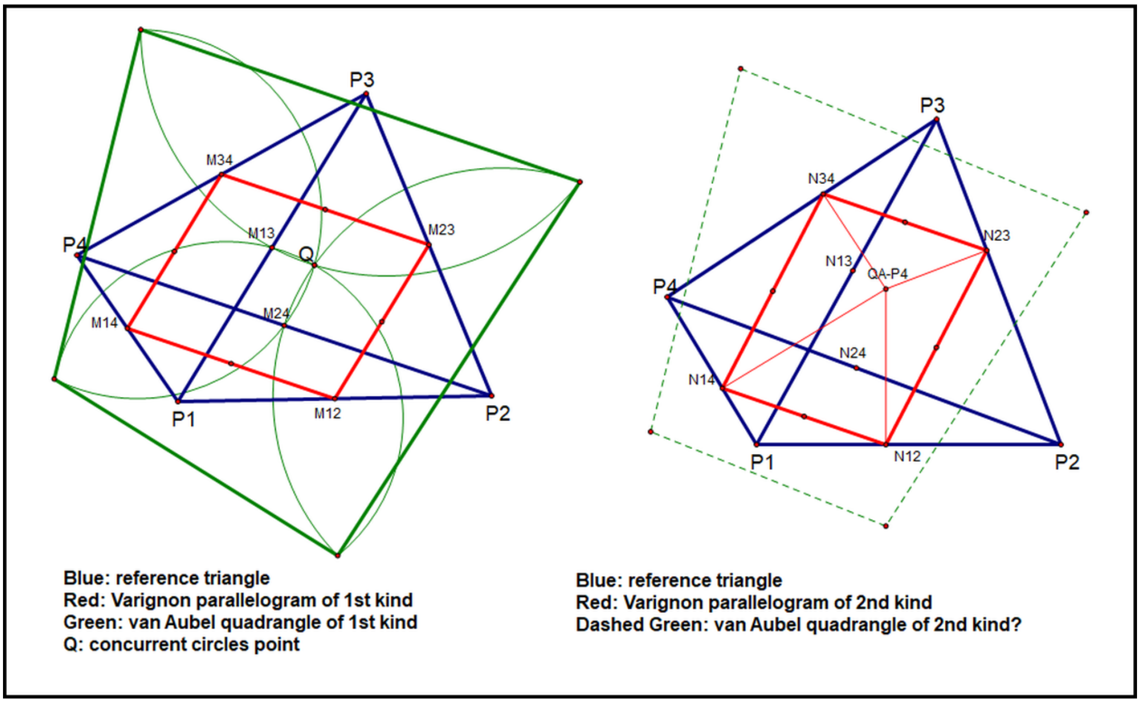
← Previous Next → ↔ Message Index ↑ Subjects

Message: #149
Date: 30/7/2013 12:20:02
From: Seiichi Kirikami
Subject: Are there van Aubel quadrangle and points of 2nd kind?

Dear friends,
I would like to propose a problem about van Aubel quadrangle and points of 2nd kind.
Sorry in advance if there were no such things as van Aubel quadrangle and points of 2nd kind.
[1] van Aubel quadrangle of 1st kind: See Matheworld"van Aubel's theorem". For example, the centers of the squares placed outwardly on each side of a quadrangle determine its van Aubel quadrangle of 1st kind.
[2] van Aubel points of 1st kind: These points accompany van Aubel quadrangle of 1st kind. See the message #116.
[3] Varignon parallelogram of 1st kind: See Matheworld "Varignon parallelogram".
[4] Varignon parallelogram of 2nd kind: See the description of QA-P4 in EQF.
They([3] and [4]) coincide with each other in a cyclic quadrangle when vertices are the same.
[5] I think that van Aubel quadrangle and points of 2nd kind accompany Varignon parallelogram of 2nd kind.
See the attached files(gsp or MSword).
Best regards,
Seiichi.



vanAubel2nd.docx



van Aubel 2nd-gsp.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #150
Date: 30/7/2013 12:43:08
From: seiichikiri
Subject: What is the necessary and sufficient condition of quadrifigure f

---

```
Dear Chris, dear friends,

I saw the figure of Yao Liu's almost equilateral triangle.
My observation is as follows.
(1) the triangle centers which are not constrained on a circle
or other figure limit to the center of the equilateral triangle,
(1,1,1).
(2) the triangle centers which are constrained on a circle or
other figure are on the same circle or other figure.

Examples of the circumcircle, X(107), X(162); NP circle, X(123),
X(134); the circumcircle of the anticomplementary triangle,
X(147), X(148).
In these limiting cases, the coordinates are indefinite,
(0,0,0). If they are finite other than (1,1,1), it is not a
triangle center.

My generalization is as follows.
(1) the quadrangle point which is not constrained limits to the
center of the square. f(-1,1,1,sqrt(2),1,1)=0.
(2) If the quadrangle point is constrained on a figure, its
coordinates are indefinite, (0,0,0). If they have finite values
other than (0,1/2,1/2), it is not a quadrangle point.
In any case, f(-1,1,1, sqrt(2),1,1) = 0.

Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #151
Date: 31/7/2013 10:54:43
From: eckart_schmidt@t-online.de
Subject: Modified Involutary Conjugate QA-Tf2 for Quadrigons

Dear friends,

Involutary Conjugate (*) is an isoconjugation for the QA-Diagonal Triangle with fixed points in the vertices of the quadrangle. Here for quadrigons a modified transformation (^) will be considered, which swaps opposite vertices. There are some interesting aspects in the QL-environment. – Reference triangles for barycentric coordinates are QA-DT or QL-DT.

The Modified Involutary Conjugate (shorted MIC) X^ is the fourth harmonic point of the Involutary Conjugate X* wrt QG-P1 and the intersection of X*.QG-P1 with QG-L1:

(x:y:z) --> (p^2 y z: -q^2 z x: r^2 x y).

Some MIC-images in QG-environment:
QG-P12^: QA-P10
QG-P13^: midpoint of QG-P1.QG-P2
QG-P14^
QG-P15^: intersection of QG-P2.QA-P16
and a QG-L1-parallel half the distance to QG-P1
QG-P16^: fourth harmonic point of QL-P1 on the QG-P1-traversal

In QA-environment a point P has three MIC-images, which are the vertices of the QA-DT-anticevian triangle of the Involoutary Conjugate P*.
In QL-environment the three MIC -images of a point P(u:v:w) are collinear on a line L(P) with simple coefficients

L(P) = (l^2 u , m^2 v , n^2 w)

The line L(P) is the image of the trilinear polar of P wrt the “isoconjugation for lines” (see my homepage, 12.3).

For an intersection P of two lines of the quadrilateral L(P) is the connection of the opposite point with the noncollinear vertice of QL-DT.
L(QL-P8) is QL-L1,
for all points on QA-P8.QA-P13 L(P) is a parallel to QL-L1,
and for all points on QL-L1 the line L(P) contains QL-P8.

The last property can be generalized: For points P on a line L(e,f,g)
the lines L(P) have a common point

P(L) = (m^2 n^2 e: n^2 l^2 f: l^2 m^2 g) ,

which can be interpreted as follows:

Taking the QL-DT -isoconjugation with fixed points in the
trilinear poles of the sidelines of the quadrilateral,
then P(L) is the image of the trilinear pole of L.

For these QL-transformations L(P) (points into lines) and P(L)
(lines into points) holds P(L(P))=P and L(P(L))=L.

Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #152
Date: 31/7/2013 11:23:37
From: eckart_schmidt@t-online.de
Subject: Modified Involutary Conjugate QA-Tf2 for Quadrigons

Dear friends,
excuse the outward appearance of my last message and the missing
MIC-image of QG-P14:
    QG-P14 ^: midpoint of QG-P1.QG-P3
Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #153
Date: 31/7/2013 11:48:59
From: seiichikiri
Subject: QA-P4 as an image of a triangle and a point

Dear friends,
The following image D coincides with QA-P4 in EQF. See Anopolis
message #724.
[APH&AM]
Let ABC be a triangle, P a point and Ha, Hb, Hc the orthocenters
of PBC, PCA, PAB respectively. Da, Db, Dc = the isogonal
conjugates of Ha, Hb, Hc(wrt the triangle ABC).
The circumcircles of DaBC, DbCA, DCAB are concurrent.
If P=(u, v, w), the circumcircles of DaBC, DbCA, DcAB are
concurrentin D=(a^2v*w(a^2(u+v)(u+w)-u(b^2(u+v)+c^2(u+w)));;).
Best regards,
Seiichi.

← Previous Next → ↔ Message Index ↑ Subjects

Message: #154
Date: 31/7/2013 4:16:28
From: Chris
Subject: QA-P4 as an image of a triangle and a point

```
Dear Seiichi,
Nice method for constructing QA-P4.
I suppose you also noticed that D, Da, Db, Dc and X(3) are
collinear.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #155
Date: 01/8/2013 11:39:09
From: eckart_schmidt@t-online.de
Subject: Circular QG-cubic

```
Dear friends,
for triangles the locus of points, whose pedal triangle is a
cevian triangle is the Darboux cubic. In the attachment there is
an analogon for quadrigons. I think, the geometrical background
is interesting, but there are rarely relationships to
QG-geometry. Perhaps someone can  find further properties.
Best regards Eckart
```

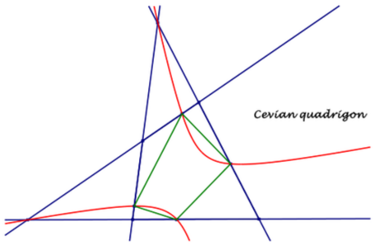
A Pivotal Isogonal Circular QG-Cubic

For a triangle the locus of points, whose pedal triangles are cevian triangles is the Darboux cubic. Defining cevian quadrigons in a corresponding way there is a pivotal isogonal circular cubic for quadrigons. – The following information is a summary of my paper 12.2 on my homepage.

Definition: A cevian quadrigon is an inscribed quadrigon, dividing the sides in ratios with product 1.

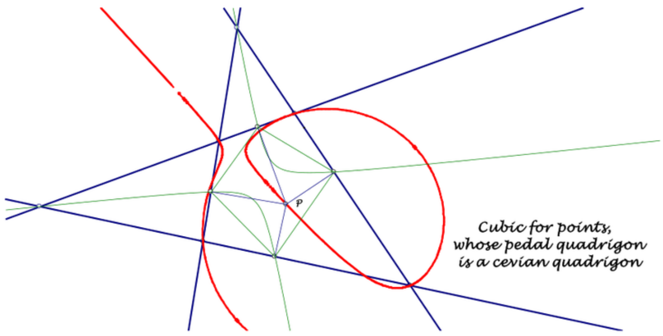
For example inscribed conics of a quadrigon give cevian quadrigons.

Property: The vertices of a cevian quadrigon and the intersections of opposite sides of the reference quadrigon lie on a conic.



With this property we get easily the equation for the points, whose pedeal quadrigon is a cevian quadrigon. But the equation is a bit extensive.

Theorem: The locus of points, whose pedal quadrigon is a cevian quadrigon, is a circular cubic, containing the Miquel point with an asymptote parallel to the bisector of the intersections of opposite sides.

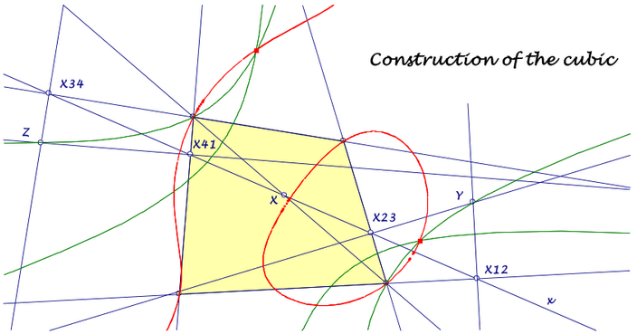


QG-cubic.pdf

A reference triangle can be constructed, so that the cubic is a pivotal isogonal circular cubic.

Finally a possible construction for the cubic:

Let $P_1P_2P_3P_4$ be the quadrigon. Consider points X on a diagonal and lines x through X, which cut the sidelines in X_{12}, X_{23}, X_{34}, X_{41}. Let Y be the intersection of perpendiculars in X_{12} and X_{23} to the corresponding sidelines, let Z be the intersection of perpendiculars in X_{34} and X_{41} to the corresponding sidelines. The loci of Y and Z changing the lines x through X are conics, the intersections of these conics are points of the cubic.



Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

QG-cubic.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #156
Date: 01/8/2013 1:41:21
From: seiichikiri
Subject: An inequality of the bimedian segments of a quadrangle

---

```
Dear friends,
Given a quadrangle P1P2P3P4 and the midpoints Mij on PiPj, the
sum of the lengths of the bimedian segments equals or less than
the square root of the square sum of the sidelengths of the
quadrangle multiplied by Sqrt(3)/2.
M12M34+M13M24+M14M23
   <= Sqrt(3)/2*Sqrt(P1P2^2+P2P3^2+P3P4^2+P4P1^2+P1P3^2+P2P4^2).
A simple application of Cauchy Schwarts inequality gives the
result.
Best regards,
Seiichi.
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #157
Date: 03/8/2013 7:53:41
From: seiichikiri
Subject: An inequality of the bimedian segments of a quadrangle

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```
Dear friends,
Numerical experiment of a linear two sided inequality of a
quadrangle:
(P1P2+P2P3+P3P4+P4P1+P1P3+P2P4)/4
   <= M12M34+M13M24+M14M23<=(P1P2+P2P3+P3P4+P4P1+P1P3+P2P4)/2.
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #158
Date: 03/8/2013 8:55:20
From: Chris
Subject: New items in EQF

```
Dear friends,
I placed several new items in EQF.
See Recently added:
http://www.chrisvantienhoven.nl/index.php/recently-added.html
Especially:
* A QA-Moebius Conjugate QA-Tf4 at
http://www.chrisvantienhoven.nl/other-quadrangle-objects/index.p
hp/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-obje
cts/artikelen-qa/230-qa-tf4.html
* A QL-Line Isoconjugate QL-Tf2 at
http://chrisvantienhoven.nl/other-quadrilateral-objects/index.ph
p/17-mathematics/encyclopedia-of-quadri-figures/quadrilateral-ob
jects/artikelen-ql/231-ql-tf2.html
* New properties QL-Cardioid QL-Qu1 at
http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-m
athematics/quadrilateral-objects/artikelen-ql/157-ql-qu1.html
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #159
Date: 03/8/2013 11:53:28
From: Bernard Gibert
Subject: Cardioids and Eckart Cubics

```
Dear Chris, Eckart and friends,
I've written a draft and put it on my web site. Although it's
far from being completed, feel free to comment and correct
possible mistakes.
http://bernard.gibert.pagesperso-orange.fr/files/Resources/eckar
t.pdf
Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #160
Date: 03/8/2013 3:32:56
From: eckart_schmidt@t-online.de
Subject: Circle for a quadrigon

```
Dear friends,
here the short geometry of a circle for a quadrigon: QG-Cix
round QG-P9 through QG-P1
* Points on the circle: QG-P1, QG-P5, QA-P3, QL-P17 and the
vertices of QA-Tr2 unequal QL-P1
* QG-Cix is the QL-Tf1 image of a circle through QA-P4, QG-P16
and the vertices of QA-Tr2 unequal QL-P1
* QG-Cix is the QA-Tf4 image of QL-Ci3 (Miquel Circle)
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #161
Date: 06/8/2013 11:42:16
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, dear friends,

thanks to Bernard for the reference to your paper. You describe
our previous efforts on a higher level. Up to now I miss an
explination for the construction of the reference triangles for
the cubic (message #109). I try it once more, to describe my
"construction". The description in message #125 contains
unfortunately a mix-up of the axes of Co1!
Here is a better version of my own way , how to find the
reference triangles for my cubic, if the quadrilateral is not
tangent to a circle:
#   X is the intersection of QL-L1 (Newton line) and QL-L6.
#   Co1 is the conic with center X and tangent
    to the lines of the quadrilateral.
#   Ci1 is the Thales circle for the foci of Co1.
#   O  is a variable point on the  axis of Co1.
#   Ci2 is the circle round O perpendicular Ci1.
#   Point K is the reflection of X in Ci2.
#   Let G = QL-P1 (Miquel point) and H = hG,-2(O)
    and Y = hG,-2(K).
#   Consider the rectangular hyperbola
    through  O, H, K, Y with center Z.
#   Chose O on the axis of Co1 so that the reflection
    of H in Z lies on the circle Ci2.
#   Then the further intersections of the hyperbola
    and Ci2 give the reference triangle.
#   If the axis of Co1 is QL-L1, the cubic is the McCay cubic.
    If the axis of Co1 is perpendicular QL-L1,
    the cubic is the Kjp cubic.

Remark: G, O, H, K are finally X2, X3, X4, X6 of the reference
triangle.

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #162
Date: 06/8/2013 11:47:23
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Eckart,

>> thanks to Bernard for the reference to your paper.
>> You describe our previous efforts on a higher level.
>> Up to now I miss an explanation for the construction of
>> the reference triangles for the cubic (message #109).

It's on its way! Please leave me some more time since it's
difficult, at least for me.
Best regards
Bernard

PS: I will probably post an updated version of my draft paper
this evening.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #163
Date: 07/8/2013 2:18:38
From: seiichikiri
Subject: What is the necessary and sufficient condition of quadrifigure f

---

```
Dear friends,
The following Q is an example which satisfies f(-1,1,1;
sqrt(2),1,1)=0, but is not a quadrangle point.
   Q = (u((v+w)(w+u-v)(u+v-w)+2uvw) : : )
from Advanced Plane Geometry message #443 (AMM11615)  †).
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #164
Date: 08/8/2013 6:27:33
From: bernard.keizer
Subject: Diagonal quadrilaterals and conics through the S-points of the Dimid

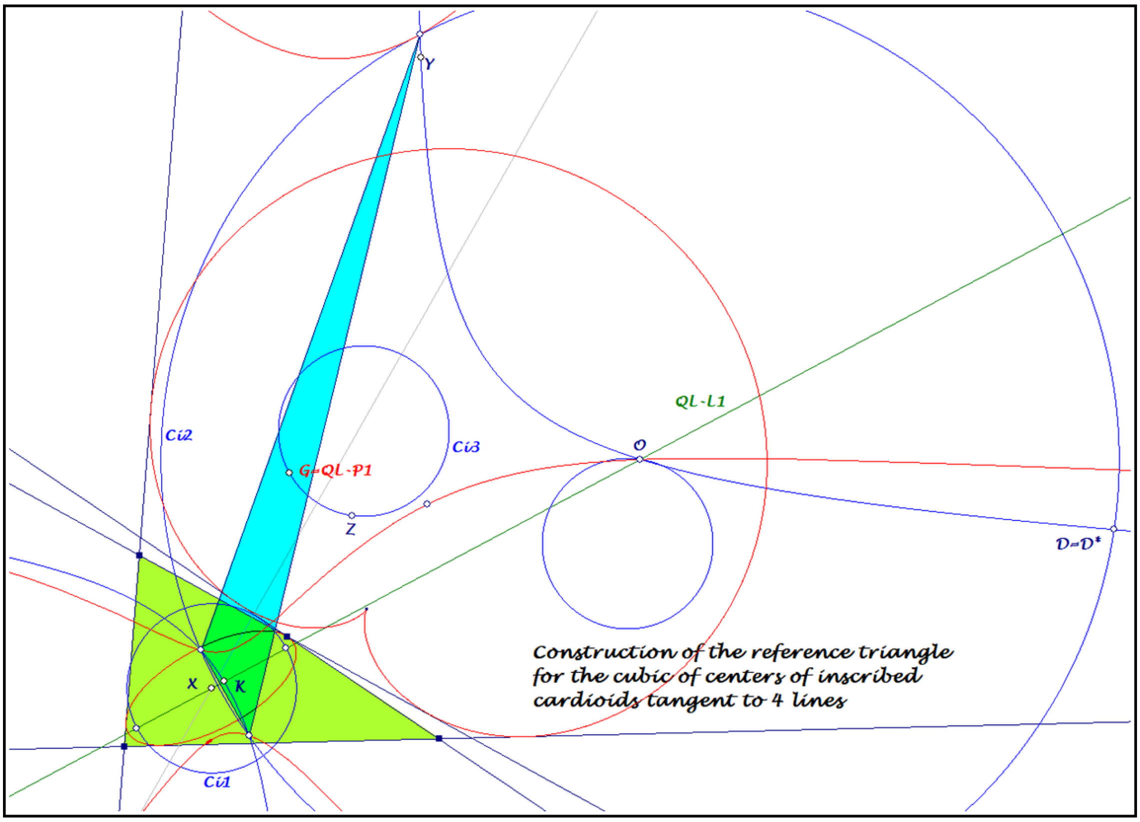
---

```
Dear Chris, dear Eckart and dear friends of the Quadri-group
When I came to the quadri-group, I was not only interested in
cardioïds, but in plenty of other things; among them are the
S-points of the Dimidium circle and I was and am still intrigued
by these 3 points.
I've tried to make a link with what I call the first and second
diagonal triangles and the first and second diagonal
quadrilaterals of the QL. I hope it could interest you and, may
be, lead to new definitions or new points.
Best regards
Bernard Keizer
I put the attached file in the files, I hope it will work ...
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #165
Date: 09/8/2013 9:30:44
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, dear Chris, dear friends,
I have updated my  "construction" for the reference triangles of
my cubic, if the quadrilateral is not  tangent to 4 lines: It
needs only an intersection point of a cubic and a quartic
-which can be marked by Cabri- to construct the reference
triangles.
#  X is the intersection of QL-L1 (Newton line) and QL-L6.
#  Co1 is the conic with center X and tangent
   to the lines of the quadrilateral.
#  Ci1 is the Thales circle for the foci of Co1.
#  O is a variable point on the  axis of Co1.
#  Ci2 is the circle round O perpendicular Ci1.
#  Point K is the inverse of  X wrt Ci2.
#  Let G = QL-P1 (Miquel point) and H = hG,-2(O)
   and Y = hG,-2(K).
#  The intersections of GO and Ci2 give the Hessian QL-Cu1
   of the cubic.
#  Consider the rectangular hyperbola through  O, H, K, Y
   with center Z.
#  Let D be the reflection of H in Z.
#  Changing O on the axis of Co1,
   ... the locus of Z is a circle  Ci3 (through QL-P1),
   ... the locus of D is a cubic (through the foci of Co1),
   ... the locus of the intersections of the hyperbola and Ci2
       is a quartic (through the foci of Co1).
#  Let D* be the further intersection of the cubic
   and the quartic.
#  Reflecting D* in points Z of Ci3 you get  circle Ci4.
#  The homothetic circle hG-1/2(Ci4) cuts the Newton line
   in two points.
#  Let O be finally that  intersection point with D=D*.
#  Then the  intersections of the hyperbola and Ci2 -unequal D-
   give the reference triangle ABC.
#  G, O, H, K and Y are finally X2, X3, X4, X6 and X69
   of the reference triangle.
#  The rectangular hyperbola is the polar conic of H.
#  If the axis of Co1 is QL-L1, the cubic is the McCay cubic.
   If the axis of Co1 is perpendicular QL-L1,
   the cubic is the Kjp cubic.
I hope there are no basic mistakes. The Cabri file in the
attachment can be an orientation. Perhaps Bernard can lighten
the background for the circles and curves.
Best regards Eckart
```



cardMcCay-fig.png

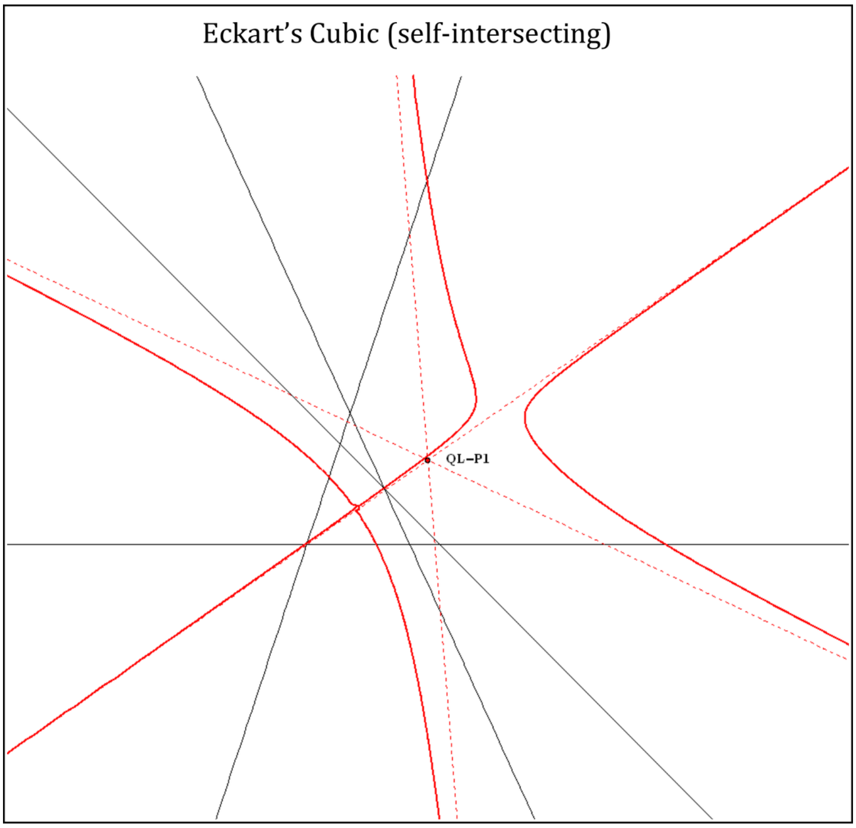
Message: #166
Date: 10/8/2013 12:10:56
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear friends,
sorry, it is better, to cut out the following remaks in my last
message:
#  The intersections of GO and Ci2
   give the Hessian QL-Cu1 of the cubic.
#  The rectangular hyperbola is the polar conic of H.
They are only valid, if the axis of Co1 is QL-L1.
(If the axis of  Co1 is perpendicular to QL-L1, circles with
midpoint M on QL-L1 through the foci of Co1 cut MG in points of
the Hessian of the cubic.)
Best regards Eckart
```

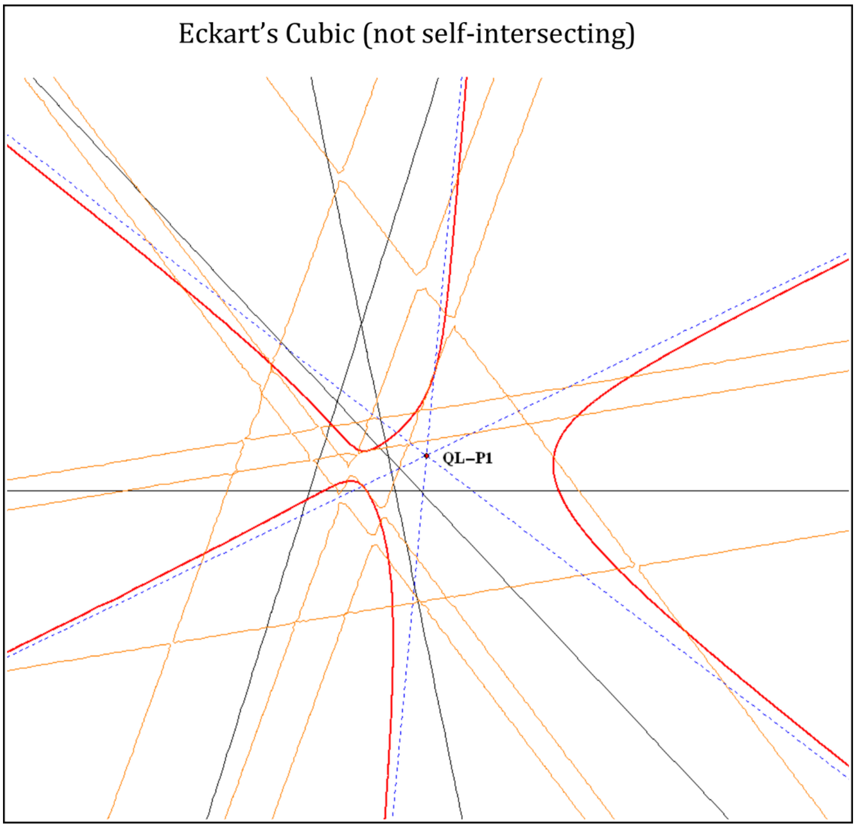
← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #167
Date: 10/8/2013 7:31:10
From: Chris van Tienhoven
Subject: Cardioids tangent to 4 lines

```
Dear Eckart, dear friends,
Sometimes Eckart’s Cubic is self-intersecting.
Often this indicates a special point, even when it is not
self-intersecting.
Is already known which point the self-intersecting point of
Eckart's cubic is?
What are the coordinates of this point?
Best regards,
Chris
See attachment.
```



EckartsCubic2.pdf



EckartsCubic2.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #168
Date: 10/8/2013 8:07:49
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Chris,

>> Sometimes Eckart’s Cubic is self-intersecting.
>> Often this indicates a special point, even when it is not
>> self-intersecting.
>> Is already known which point the self-intersecting point
>> of Eckart's cubic is?
>> What are the coordinates of this point?

This happens when the quadrilateral circumscribes a circle.
This is detailed in the draft paper available on my web site.

Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #169
Date: 11/8/2013 10:57:59
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard,
trying to construct the nodal E(L), I succeeded in the
alternative construction (p.12), but I failed in the first
construction (p.11).
In my construction wrt 6. the points P1, P2, P3 are not the
vertices of an equilateral triangle and their focus line is not
E(L).
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #170
Date: 11/8/2013 11:50:14
From: Bernard Gibert
Subject: Cardioids tangent to 4 lines

```
Dear Eckart,

>> trying to construct the nodal E(L),
>> I succeeded in the alternative, construction (p.12),
>> but I failed in the first construction (p.11).

it's only a typo in item 5: the center of (Ho) is the midpoint
of X1 Go and not X1 Ho.
sorry!
thank you for checking my paper
please feel free to comment and criticize
Best regards
Bernard
PS: item 1 is also incoherent: forget the circumcircle of ABC...
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #171
Date: 14/8/2013 8:12:31
From: Bernard Gibert
Subject: interesting paper?

```
Dear friends,
here's a paper by Jules Marchand I cannot find on the internet.
It could be of interest...
Marchand, J. Géométrie du quadrilatère complet. (French) Bull.
Soc. Vaudoise Sci. nat. 59, 251-270. Published: 1937
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #172
Date: 14/8/2013 9:59:30
From: Antreas Hatzipolakis
Subject: interesting paper?

```
> On Wed, Aug 14, 2013 at 9:12 AM, Bernard Gibert <
bg42@orange.fr >
> wrote:
>> Dear friends,
>> here's a paper by Jules Marchand
>> I cannot find on the internet. It could be of interest...
>> Marchand, J. Géométrie du quadrilatère complet. (French)
>> Bull. Soc. Vaudoise Sci. nat. 59, 251-270. Published: 1937
>> Best regards
>> Bernard
>> (
>> http://groups.yahoo.com/group/Quadri-Figures-Group/post;_ylc=
X3oDMTJwdDc1M3FxBF9TAzk3MzU5NzE0BGdycElkAzg3Njg2NjExBGdycHNwSWQD
MTcwNTA4MzM4NgRtc2dJZAMxNzEEc2VjA2Z0cgRzbGsDcnBseQRzdGltZQMxMzc2
NDYwNzUx?act=reply&messageNum=171
>> )
Review in German:
http://www.zentralblatt-math.org/zmath/en/search/?q=an:02523907&
type=pdf&format=complete
```

aph

← Previous Next → ↔ Message Index ↑ Subjects

Message: #173
Date: 14/8/2013 11:24:07
From: Antreas Hatzipolakis
Subject: QUADRILATERAL Bibliography

```
---------- Forwarded message ----------
From: Antreas < anopolis72@gmail.com >
Date: Thu, Aug 15, 2013 at 12:06 AM
Subject: [EGML] QUADRILATERAL Bibliography
To: Anopolis@yahoogroups.com

Thebault, V.
Sur le quadrilatere complet. (French)
C. R. Acad. Sci., Paris 217, 97-99 (1943).
http://gallica.bnf.fr/ark:/12148/bpt6k31698/f97.image
http://gallica.bnf.fr/ark:/12148/bpt6k31698/f98.image
http://gallica.bnf.fr/ark:/12148/bpt6k31698/f99.image

Thebault, V.
Sur la géométrie du quadrilatere complet. (French)
C. R. Acad. Sci., Paris 218, 97-99 (1944).
http://gallica.bnf.fr/ark:/12148/bpt6k3170r/f97.image
http://gallica.bnf.fr/ark:/12148/bpt6k3170r/f98.image
http://gallica.bnf.fr/ark:/12148/bpt6k3170r/f99.image
```

aph

← Previous Next → ↩ Message Index ↑ Subjects

Message: #174
Date: 18/8/2013 11:10:35
From: Antreas Hatzipolakis
Subject: QUADRILATERAL NEWTON LINE (A medial line)

```
---------- Forwarded message ----------
From: *Antreas < anopolis72@gmail.com >
Date: Mon, Aug 19, 2013 at 12:07 AM
Subject: [EGML] QUADRILATERAL NEWTON LINE (Re: A medial line)
To: Anopolis@yahoogroups.com
[APH]:
> > Let ABC be a triangle and A'B'C' the medial triangle.
> > A line L intersects BC,CA,AB at P1,P2,P3, resp.
> > Let M1, M2, M3 be the midpoints of P2P3, P3P1, P1P2, resp.
> > and A*, B*, C* the midpoints of A'M1, B'M2, C'M3, resp.
> > The points A*,B*,C* are collinear.
> > The line A*B*C* is the Newton line
> > of the complete quadrilateral
> > (AB, BC, CA, L) ie the line passing through the midpoints
> > of AP1, BP2, CP3.

Let (a,b,c,d) be a complete quadrilateral.
The A*,B*,C* points, as described above, coincide
for the four triangles having as L-line the fourth line
ie the triangles
   (b,c,d, L=a), (c,d,a, L=b), (d,a,b, L=c), (a,b,c, L=d).
So, for a complete quadrilateral we have three new (?)
points on the Newton line.
At first glance, I did not find them in Chris' page
http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-m
athematics/quadrilateral-objects/artikelen-ql/137-ql-l1.html
Sorry if they are listed.
APH
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #175
Date: 19/8/2013 9:20:48
From: Chris
Subject: QUADRILATERAL Bibliography

Dear friends,
In the paper of V. Thebault "Sur le quadrilatere complet" page 98 "Dans les quatre triangles formés par quatre droites d'un plan, prises trois par trois, les perpendiculaires élevées au milieu de la distance du centre du cercle circonscrit a l'orthocentre par un même point"
The described point is QL-P3, the Kantor-Hervey Point.
The circle described right after is QL-Ci5, The Plücker circle.
At page 97 there was a special description "Les extrémités . . ."
Transcribed in English:
The ends of the diameters (perpendicular to the corresponding line Li) of the circumcircles of the component triangles Lj.Lk.Ll lie on two perpendicular axes through the Miquel Point F.
Each of these axes is the angle bisector of A.F.A', etc.
where A', B', C' are the intersection points of 1st Component Triangle ABC.
This construction as well as the axes were new to me.
These axes are other lines than the perpendicular axes from Steiner (described in QL-8P1 and QL-Tf1).
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #176
Date: 19/8/2013 9:59:52
From: Chris
Subject: QUADRILATERAL NEWTON LINE (A medial line)

```
Dear Antreas,
The Newton Line is defined as the line through the 3 midpoints
of the diagonals of a Quadrilateral.
Let's name these 3 midpoints Mid1, Mid2, Mid3.
The points you mentioned are related points. They are:
* Midpoint(Mid1,Mid2)
* Midpoint(Mid2,Mid3)
* Midpoint(Mid3,Mid1)
I wasn't familiar with these points.
Very nice!
The Tripolar Centroid of Mid1, Mid2, Mid3 is QL-P12.
The Tripolar Centroid of your points is also QL-P12.
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #177
Date: 20/8/2013 8:42:18
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard, dear friends,
there are some EQF-elements in the constructions in Bernard's
paper, perhaps of interest:
* The point  "Omega" on page 13 is QL-L1 ^ QL-L6.
* The circle "Gamma" on page 16 is the Schmidt Circle
  (see EQF QL-Tf1).
* The conjugation "Phi"(M) on page 15
  is the Clawson-Schmidt Conjugate QL-Tf1 in EQF.
* The points F1, F2 on page 16 are the fixed points of QL-Tf1.
* The lines L1, L2 are the angle bisectors of the lines from the
  Miquel point to to two oppsite points of the quadrilateral.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #178
Date: 20/8/2013 10:05:28
From: Chris
Subject: QUADRILATERAL Bibliography

```
Dear friends,

I made a mistake with the construction of V. Thebault with 2
axes through the Miquel Point.
They still are identical with The Steiner Axes as described in
QL-8P1 (Steiner Octet) and QL-Tf1 (Clawson-Schmidt Conjugate).

Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #179
Date: 21/8/2013 9:35:35
From: Chris
Subject: Two points on the Newton Line

```
Dear friends,
There are exactly 2 points on the Newton Line that are each
other's Isogonal Conjugate wrt all 4 QL-Component Triangles.
These points lie on QL-Cu1.
Their midpoint is the intersection point of the Newton Line and
the Quasi Ortholine: QL-L1^QL-L6.
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #180
Date: 21/8/2013 11:09:12
From: Antreas Hatzipolakis
Subject: Two points on the Newton Line

Dear Chris

A quiz:
Which points are the antigonal conjugates of the center of the
Miquel circle
wrt the four component triangles of a complete quadrilateral?

APH

> On Wed, Aug 21, 2013 at 10:35 PM, Chris <van10hoven@gmail.com>
> wrote:
>> Dear friends,
>> There are exactly 2 points on the Newton Line that are each
>> other's Isogonal Conjugate wrt all 4 QL-Component Triangles.
>> These points lie on QL-Cu1.
>> Their midpoint is the intersection point of the Newton Line
>> and the Quasi Ortholine: QL-L1^QL-L6.
>> Best regards,
>> Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #181
Date: 21/8/2013 11:11:37
From: Antreas Hatzipolakis
Subject: Two points on the Newton Line

> On Thu, Aug 22, 2013 at 12:09 AM,
> Antreas Hatzipolakis < anopolis72@gmail.com
> wrote:
>> Dear Chris
>> A quiz:
>> Which points are the antigonal conjugates
>> of the center of the Miquel circle

Sorry: Miquel point

← Previous Next → ↔ Message Index ↑ Subjects

Message: #182
Date: 22/8/2013 8:14:03
From: Chris
Subject: Two points on the Newton Line

Dear Antreas,

The Antigonal Conjugate of some point on the circumcircle of a triangle coincides with its Orthocenter.
The Miquel Point of a Quadrilateral is the common point of the circumcircles of its 4 QL-Component Triangles.
That's why the Antigonal Conjugate of the Miquel Point wrt the 4 QL-Component Triangles coincide with the Orthocenters of the 4 QL-Component Triangles.
As already known these 4 Orthocenters are collinear on the Steiner Line QL-L2.

Best regards,
Chris

Message: #183
Date: 22/8/2013 11:25:42
From: Antreas
Subject: Two points on the Newton Line

Dear Chris,

So we have a circle (of infinity radius, Steiner line) where
are lying the four antigonal conj. points of a point (Miquel
point) wrt the component triangles.

Question:
Which other points have the same property?
Either one point or four points respective
to component triangles.

To make it clear:
1. Let P be a point on the plane of a c. 4lateral.
Let Pa,Pb,Pc,Pd be the antigonal conjugates of
P wrt the four component triangles.
For which points P the points Pa,Pb,Pc,Pd are concyclic?
2. Let P be a point with h. coordinates (x:y:z) wrt some
reference triangle..
Let Pa,Pb,Pc,Pd be the points with h. coordinates
(x:y:z) wrt the four component triangles of a c. 4lateral.

For which P's the Pa,Pb,Pc,Pd are concyclic?
I guess the general problem is quite complicated.
Probably by some program one could investigate
the points P (centers X(i)) listed in ETC.

APH

← Previous Next → ↔ Message Index ↑ Subjects

Message: #184
Date: 22/8/2013 11:32:25
From: Antreas
Subject: Two points on the Newton Line

--- In Quadri-Figures-Group@yahoogroups.com , "Antreas" wrote:

> > Dear Chris,
> > So we have a circle (of infinity radius, Steiner line) where
> > are lying the four antigonal conj. points of a point
> > (Miquel point) wrt the component triangles.
> > Question:
> > Which other points have the same property?
> > Either one point or four points respective
> > to component triangles.
> > To make it clear:
> > 1. Let P be a point on the plane of a c. 4lateral.
> > Let Pa,Pb,Pc,Pd be the antigonal conjugates of
> > P wrt the four component triangles.
> > For which points P the points Pa,Pb,Pc,Pd are concyclic?
> > 2. Let P be a point with h. coordinates (x:y:z) wrt some
> > reference triangle..
> > Let Pa,Pb,Pc,Pd be the points with h. coordinates
> > (x:y:z) wrt the four component triangles of a c. 4lateral.

****
Let Qa,Qb,Qc,Qd be the antigonal conjugates of Pa,Pb,Pc,Pd
wrt the four component triangles.
****

> > For which P's the Pa,Pb,Pc,Pd are concyclic?

****
Read instead:
For which P's the Qa,Qb,Qc,Qd are concyclic.
****

Sorry!
aph

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #185
Date: 22/8/2013 4:31:23
From: bernard.keizer
Subject: Two points on the Newton Line

Dear Chris,
These 2 points are each other's Clawson-Schmidt conjugate.
The Clawson-Schmidt conjugate of the Newton Line is a circle through the 2 points and QL-P1.
One property of the curve QL-Cu1 is that 2 CS-conjugate points on the curve have the same tangential point (point where the tangent to the curve recuts the curve)and that the CS-conjugate of this tangential is in line with the 2 points.
The tangential point of the 2 points on the Newton Line is QL-P1 and the conjugate of QL-P1 is the infinity point of the Newton Line.
The tangential point of QL-P1 and this infinity point of the Newton Line is the point where QL-Cu1 cuts it's asymptote and is on the line QL-P1 QL-P4 (axis of the mono-cardioïd); the CS-conjugate of this point is on the axis of the parabola (parallel to the Newton Line).
The 2 points of the curve on the Newton Line are the 2 caracteristic points of 2 orthogonal sets of circles (the centers of one set being on the Newton Line and the centers of the other set on the perpendicular bisector of the 2 points).
The first set is composed of circles of which a diameter is through QL-P1 and the second is composed of circles such as the cord through the 2 contact points of the 2 tangents from QL-P1 to the circle is through the tangential of QL-P1.
If the first set is with the 2 points as basis-points (and the other with the 2 points as Poncelet points), the curve QL-Cu1 is made of 2 parts and the 2 points are on the circle having as diameter QL-P1 and it's tangential.
In the reverse case (2 points as Poncelet points of the first set and basis-points of the second), the curve QL-Cu1 is in one piece.
In this case, it's easy to draw the curve as the locus of the contact points of the tangents from QL-P1 to the circles having their center on the perpendicular bisector of the 2 points.
Best regards
Bernard

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #186
Date: 22/8/2013 7:23:53
From: Bernard Gibert
Subject: Two points on the Newton Line

```
Dear Chris and Bernard K.,

>> There are exactly 2 points on the Newton Line that are each
>> other's Isogonal Conjugate wrt all 4 QL-Component Triangles.
>> These points lie on QL-Cu1.
>> Their midpoint is the intersection point of the Newton Line
>> and the Quasi Ortholine: QL-L1^QL-L6.

All this stuff is directly related with focal cubics.
This is explained and detailed in my draft paper
http://bernard.gibert.pagesperso-orange.fr/files/cardioids.html
and also in
http://bernard.gibert.pagesperso-orange.fr/files/isocubics.html

Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #187
Date: 22/8/2013 10:24:20
From: Chris
Subject: Two points on the Newton Line

```
Dear Bernard G. and Bernard K.,
Thanks for all information!
I also calculated that the line through the Miquel Point (QL-P1)
and the Center of the Miquel Circle (QL-P4) is tangent to the
QL-Cu1-cubic.
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #188
Date: 23/8/2013 8:52:06
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton line

```
Dear Bernard K., dear friends,
it seems, that Bernard's (K) construction of QL-Cu1 has the same
background as the "strophoid"-construction in my message #106.
A short summary (two posibilities, see brackets):
Let F1, F2 be the foci of an inscribed conic
with center QL-L1 ^ QL-L6
... and C1 the circle with diameter F1F2.
... If the axis of the inscribed conic is QL-L1
    (orthogonal QL-L1),
... let M be a variable point on QL-L1
... as midpoint of a circle C2 perperdicular C1
    (through F1, F2),
... then the lines M.QL-P1 cut C2 in points of QL-Cu1.
Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #189
Date: 23/8/2013 11:26:22
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard (G), dear friends,
in addition to Bernard's paper "Inscribed Cardioids ...":
For a quadrilateral tangent to a circle the cubic  E(L)  can be
considered as Musselman cubic K028 (see my message #94).
Every point A on E(L) can easily be completed to a reference
triange ABC for K028 with
    X4 = QL-P10,
    X2 = reflection of X4 in QL-P1,
    X3 = reflection of QL-P1 in X2.
Well known points are for example:
... the common midpoint X3 of the circumcircle,
... the intersection of QL-L1
    with the perpendicular bisector of X3.X4,
... the homothetic  h(QL-P1,-2) of the last  point.
The reference triangle degenerates for the intersections of
QL-Cu1 and the circle with diameter X2.X3.
If one vertice is QL-P10, there are two rectangular reference
triangles.
In addition I cite my message #99:
On a Musselman cubic K028 there are reference triangles, so that
K028 belongs to the McCay-Kjp-pencil (table 22):
Construction for a given K028 and its reference triangle:
* Take the midpoint X2.X4 as centroid G
  of a new reference triangle ABC.
* Take the nodal point X4 as a focus F
  for the inscribed Steiner ellipse of ABC.
* Choose a point A on the cubic
  with its homothetic Ma under h(G,-1/2).
* Construct the Steiner ellipse
  with center G, Focus F and point Ma.
* The tangents from A to the ellipse and the tangent
  in Ma give a triangle ABC on the cubic.
* For ABC as reference triangle the cubic
  belongs to the McCay-Kjp pencil.
Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #190
Date: 24/8/2013 8:17:09
From: Seiichi Kirikami
Subject: An area problem in QG- environment

```
Dear friends,

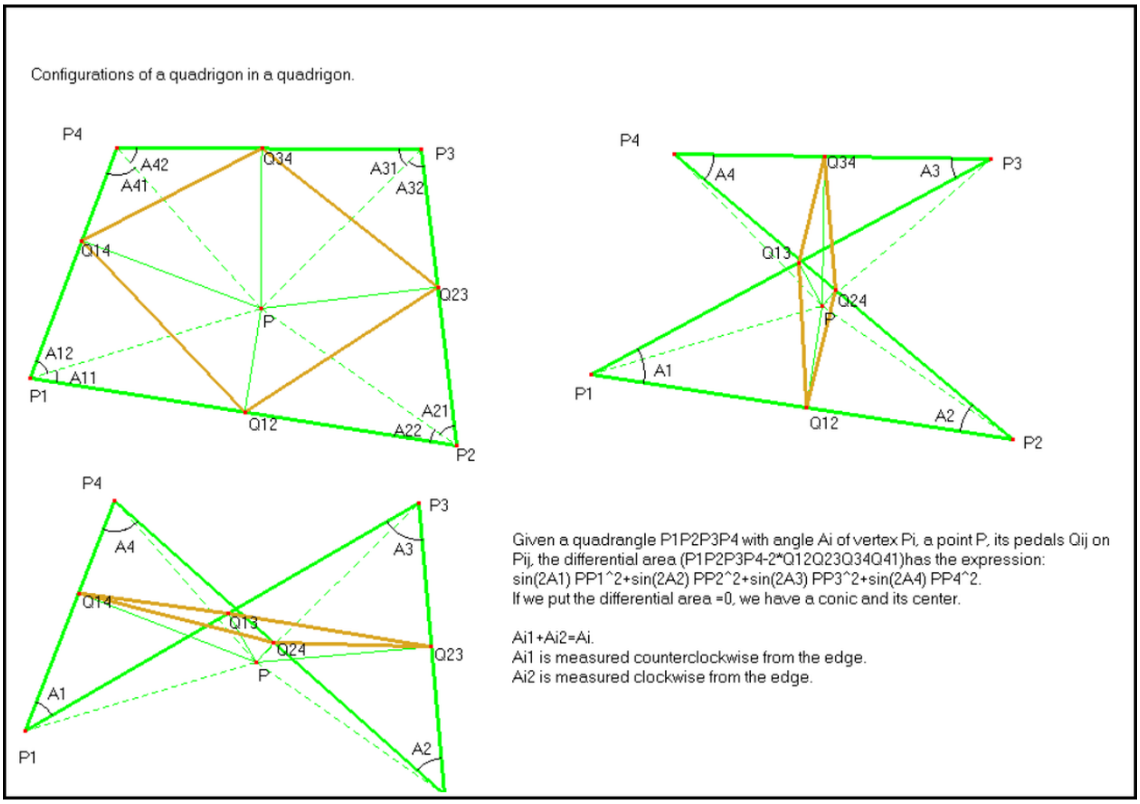
The following is what I found as an optimum problem in a
Japanese book and I changed a bit.

(1) Quadrigon P1P2P3P4: P1(0,1,0), P2(0,0,1), P3(1,0,0),
P4(p,q,r).
Given a quadrigon P1P2P3P4 with angle Ai of vertex Pi,
a point P and its pedals Qij on PiPj, the differential area
of P1P2P3P4-2*Q12Q23Q34Q41 has the expression:
   Sin[2A1]PP1^2+Sin[2A2]PP2^2+Sin[2A3]PP3^2 +Sin[2A4]PP4^2
If area (P1P2P3P4)=2*area(Q12Q23Q34Q41),
   we have a conic and its center as follows.
   x1 = a^2(q(SAq+b^2r)
           (2SBpr+c^2p^2+a^2r^2)+b^2pr(r(SAp+SBq-SCr)-c^2pq)):
   y1 = b^2(p(SBp+a^2r)
           (2SAqr+b^2r^2+c^2q^2)+a^2qr(r(SAp+SBq-SCr)-c^2pq)):
   z1 = SC (2SAqr+b^2r^2+c^2q^2)
           (2SBpr+c^2p^2+a^2r^2)+a^2b^2r^2(r(SAp+SBq-SCr)-c^2pq)

(2) Quadrigon P1P2P4P3: P3(1,0,0), P1(0,1,0), P2(0,0,1),
P4(p,q,r).
    x2 = y1/.{a->c,b->a, c->b, p->r,q->p,r->q}.
    y2 = z1/.{a->c,b->a, c->b, p->r,q->p,r->q}.
    z2 = x1/.{a->c,b->a, c->b, p->r,q->p,r->q}.

(3) Quadrigon P1P3P2P4: P2(0,0,1), P3(1,0,0), P1(0,1,0),
P4(p,q,r).
    x3 = z1/.{a->b,b->c, c->a, p->q,q->r,r->p}.
    y3 = x1/.{a->b,b->c, c->a, p->q,q->r,r->p}.
    z3 = y1/.{a->b,b->c, c->a, p->q,q->r,r->p}.

Best regards,
Seiichi.
```



quadrigon in a quadrigon.docx

← Previous Next → ↩ Message Index ↑ Subjects

Message: #191
Date: 24/8/2013 12:40:02
From: Seiichi Kirikami
Subject: The sufficient conditions of a quadrangle point

Dear friends,

The sufficient conditions of a quadrangle point are as follows.
A quadrangle point is denoted by P1P2P3P4 such that
P3={1,0,0}, P1={0,1,0}, P2={0,0,1}, P4={p, q, r},
where p+q+r=1, based on the triangle system P1P2P3.
P1P2^2 = a^2,
P2P3^2 = b^2,
P1P3^2 = c^2,
P1P4^2 = a^2r^2+c^2p^2+2SBrp,
P2P4^2 = b^2p^2+a^2q^2+2SCpq,
P3P4^2 = c^2q^2+b^2r^2+2SAqr,
A point{x,y,z} is a quadrangle one if the absolute area PPjPj+1 of the triangle system Pj-1PjPj+1 equals the absolute area PPjPj+1 of the triangle system PjPj+1Pj+2 and the sum of the absolute area PPjPj+1 equals the absolute area of P1P2P3P4.

If we suppose a quadrangle function {f(a^2, b^2, c^2; p, q, r);;}, the above conditions are as follows.
(1) f(b^2,c^2,a^2; q, r, p)/cyclic sum f(b^2,c^2,a^2; q, r, p) = qf(b^2, c^2q^2+b^2r^2+2SAqr, b^2p^2+a^2q^2+2SCpq; 1,-r,-p)/cyclic sum f(b^2, c^2q^2+b^2r^2+2SAqr, b^2p^2+a^2q^2+2SCpq; 1,-r,-p).

(2) qf(c^2q^2+b^2r^2+2SAqr, b^2p^2+a^2q^2+2SCpq, b^2; -r, -p, 1)/cyclic sum f(c^2q^2+b^2r^2+2SAqr, b^2p^2+a^2q^2+2SCpq, b^2; -r, -p, 1) = -rf(c^2q^2+b^2r^2+2SAqr, a^2r^2+c^2p^2+2SBrp, c^2; q, p, -1)/cyclic sum f(c^2q^2+b^2r^2+2SAqr, a^2r^2+c^2p^2+2SBrp, c^2; q, p, -1).

(3) -rf(a^2r^2+c^2p^2+2SBrp, c^2, c^2q^2+b^2r^2+2SAqr; p, -1, q)/cyclic sum f(a^2r^2+c^2p^2+2SBrp, c^2, c^2q^2+b^2r^2+2SAqr; p, -1, q) = pf(a^2r^2+c^2p^2+2SBrp, a^2, b^2p^2+a^2q^2+2SCpq; -r, 1, -q)/cyclic sum f(a^2r^2+c^2p^2+2SBrp, a^2, b^2p^2+a^2q^2+2SCpq; -r, 1, -q).

(4) pf(a^2, b^2p^2+a^2q^2+2SCpq, a^2 r^2+c^2p^2+2SBrp; 1, -q, -r)/cyclic sum f(a^2, b^2p^2+a^2q^2+2SCpq, a^2 r^2+c^2p^2+2SBrp; 1, -q, -r) = f(a^2, b^2, c^2; p, q, r)/cyclic sum f(a^2, b^2, c^2; p, q, r).

(5) 1 - r = the sum of the right handed sides of (1), (2), (3) and (4).

```
If the quadrangle function is independent of the sides a^2, b^2
and c^2, then the following formulae hold.

(6)    f( q, r, p) /cyclic sum f(q, r, p)
    = qf(1, -r, -p)/cyclic sum f(1,-r,-p).

(7) qf(-r, -p, 1)  /cyclic sum f(-r, -p,  1)
    = -rf(q, p, -1)/cyclic sum f( q,  p, -1).

(8) -rf(p, -1, q)  /cyclic sum f( p, -1, q)
    = pf(-r, 1, -q)/cyclic sum f(-r, 1, -q).

(9) pf(1, -q, -r)  /cyclic sum f(1, -q, -r)
    =   f( p, q, r)/cyclic sum f(p,  q,  r).

(10) 1-r = the sum of the right handed sides of (6), (7), (8)
and (9).
A result which I obtained from the above conditions:
If the quadrangle function is independent of the sides,
there is no other linear quadrangle point
except for QA-P1(x = 2p+q+r;; ).

Best regards,
Seiichi.
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #192
Date: 25/8/2013 11:43:39
From: eckart_schmidt@t-online.de
Subject: Cardioids tangent to 4 lines

```
Dear Bernard (G),
In my message #99 I already used your "construction of groups of
pivots"  for the nodal case to find reference triangles, so that
E(L) belongs to the McCay-Kjp-pencil. This holds also in
general: The triangles P1P2P3 constructed as in 7 of your paper
are reference triangles so that E(L) belongs to the
McCay-Kjp-pencil.
Best regards Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #193
Date: 25/8/2013 6:31:16
From: Chris van Tienhoven
Subject: Two points on the Newton Line QL-Cu1 revisited

Dear friends,
Here are some remarks joining the remarks of Bernard Keizer.

Conjugates on QL-Cu1
The Clawson-Schmidt Conjugate is very important on the cubic QL-Cu1.
It is very special that this conjugate has the same performance for points on QL-Cu1as
the Isogonal Conjugate of P wrt   the Orthic Triangle of the QL-Diagonal Triangle (note Eckart Schmidt).
the Quasi Isogonal Conjugate (QG-Tf2) of P wrt any Quadrigon of the Reference Quadrilateral (note Eckart Schmidt).

Easy construction method of a CS-conjugated point of any point on QL-Cu1
Any point X on QL-Cu1 can be seen as the intersection point of the line Ln parallel to the Newton line and the connecting line Lp=QL-P1.X.
Now CSC(X) can be constructed as the intersection point of La’ and Lb’, where:
Ln’ = Ln Reflected in the Newton Line
Lp’ = Lp reflected in the 1st Steiner Axis (see QL-Tf1 in EQF)

The 1st Steiner Axis easily can be constructed as:
1. an internal angle bisector of Sij.QL-P1.Skl, where Sij=Li^Lj, Skl=Lk^Ll for all (i,j,k,l) ∈ (1,2,3,4), or
2. the internal angle bisector if QL2P2a.QL-P1.QL-2P2b,
where QL-2P2a and QL-2P2b are the two points on the Newton Line discussed before.
As a consequence, when X1 and X2 are CS-conjugated points, then the line through X1 parallel to QL-L1 and the line X2.QL-P1 will cross in another point on QL-Cu1. X1 and X2 can be interchanged here.

Two new interesting points on QL-Cu1
* S = the intersection point of the QL-Cu1 and its asymptote. It can be constructed as the intersection of the asymptote and line QL-P1.QL-P4.
* T = the intersection point of the QL-Cu1 and the line through QL-P1 parallel to the Newton Line QL-L1. It can be constructed as the perpendicular bisector of QL-2P2a/b and the line // QL-L1 through QL-P1.
* S and T are each other's Clawson-Schmidt Conjugate

* and so the intersection point W of their tangents at QL-Cu1 also lies at QL-Cu1.

Tangents at QL-Cu1
* The tangent at QL-Cu1 in QL-P1 = QL-P1.QL-P4.
This line is also tangent in QL-P1 at the circle (QL-P1,QL-2P2a,QL-2P2b),
which is the Clawson-Schmidt Conjugate of the Newton Line QL-L1
* The tangent at QL-Cu1 in T is S.T reflected in the perpendicular bisector of line segment QL-2P2a.QL-2P2b.
* How to construct the tangent in S ?????

Open questions:
I think and I feel there should be a simple construction method for constructing tangents at QL-Cu1, but how??
CS-conjugated pairs of points on QL-Cu1 lie at different sides of the Newton line at equal distances. There is a maximal distance from the Newton Line. What are the coordinates of the two points (on both sides of the Newton Line) on QL-Cu1 with maximal distance from the Newton Line?
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #194
Date: 25/8/2013 9:22:48
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton Line QL-Cu1 revisited

Dear Chris,

on my homepage I have described QL-Cu1 under 11.1. So I can answer your first question:
Construction of the tangent in a point P on QL-Cu1:
* Let P* be the isogonal conjugate of P
* ... and Co1 the isogonal conjugate of PP*
      wrt the triangle L1,L2,L3
* ... and Co2 the isogonal conjugate of PP*
      wrt the triangle L2,L3,L4.
* Let Q be the fourth intersection of Co1 and Co 2.
* Then PQ is the tangent in P wrt QL-Cu1.

Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #195
Date: 26/8/2013 11:58:55
From: Antreas Hatzipolakis
Subject: QUADRANGLE POINT? (12 MIDPOINTS ......)

```
--- In Anopolis@yahoogroups.com , "Antreas" wrote:

>> Let ABC be a triangle, P a point and A'B'C'
>> the cevian triangle of P.
>> Denote:
>> Ab, Ac    = the midpoints of AB', AC', resp.
>> Bc, Ba    = the midpoints of BC', BA', resp.
>> Ca, Cb    = the midpoints of CA', CB', resp.
>> A1,B1,C1 = the midpoints of AA',BB',CC', resp.
>> A2,B2,C2 = the midpoints of BcCb, CaAc, AbBa, resp.
>> For which P's the triangles A1B1C1, A2B2C2 are perspective?
>> For all?

Conjecture 1:  For all P's the triangles are perspective.
Now, probably a new point in quadrangle (or a new construction
of an old point),
if my conjectures are true!!! :-)
Let ABCD be a quadrangle.
Denote:
Da, Db, Dc, Dd the above described perspectors
    wrt (BCD), (CDA), (DAB), (ABC), resp.

Conjecture 2:
The Lines ADa, BDb, CDc, DDd are concurrent.

Antreas
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #196
Date: 27/8/2013 1:37:20
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
> On Mon, Aug 26, 2013 at 11:43 PM,
> Antreas <anopolis72@gmail.com>
> wrote:
>> Let ABC be a triangle, P a point and A'B'C'
>> the cevian triangle of P.
>> Denote:
>> Ab, Ac   = the midpoints of AB', AC', resp.
>> Bc, Ba   = the midpoints of BC', BA', resp.
>> Ca, Cb   = the midpoints of CA', CB', resp.
>> A1,B1,C1 = the midpoints of AA',BB',CC', resp.
>> A2,B2,C2 = the midpoints of BcCb, CaAc, AbBa, resp.
>> For which P's the triangles A1B1C1, A2B2C2 are perspective?
>> For all?

We can also take a line L intersecting BC, CA,AB at A',B',C',
resp.
The triangles A1B1C1, A2B2C2 are perspective on the Newton line
of the complete quadrilateral (AB,BC,CA, L)
If we consider a complete quadrilateral (a,b,c,d) we get 4
points on the Newton line of the quadrilateral,
corresponding to triangles (b,c,d, L=a), (c.d,a, L=b), (d,a,b,
L=c), (a,b,c, L=d).
But it's now too late here (in Greece) to draw a figure! :-)
Antreas
```

Message: #197
Date: 27/8/2013 8:40:22
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
[APH]

>> Let ABC be a triangle, P a point and A'B'C'
>> the cevian triangle of P.
>> Denote:
>> Ab, Ac   = the midpoints of AB', AC', resp.
>> Bc, Ba   = the midpoints of BC', BA', resp.
>> Ca, Cb   = the midpoints of CA', CB', resp.
>> A1,B1,C1 = the midpoints of AA',BB',CC', resp.
>> A2,B2,C2 = the midpoints of BcCb, CaAc, AbBa, resp.
>> For which P's the triangles A1B1C1, A2B2C2 are perspective?
>> For all?
>> Antreas

[ César Lozada]:
A1B1C1, A2B2C2 are perspective because A2B2C2 is the medial
triangle of A1B1C1 !!!
So, their perspector is the centroid of A1,B1,C1.
*************************************************

Dear Cesar,

Very good!!!
So we need 3 only midpoints!!
"Quadranglization": Let ABCD be a quadrangle.

Denote:
A' = AD /\ BC, B' = BD /\ CA, C' = AC /\ AB
M1,M2,M3 = the midpoints of AA', BB', CC'
Gd = the centroid of M1M2M3
Similarly Ga, Gb, Gc for the triangles BCD,CDA,DAB
The lines AGa, BGb, CGc, DGd are concurrent  (??)

APH
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← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #198
Date: 27/8/2013 8:44:57
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
Sorry for the typo:
>> A' = AD /\ BC, B' = BD /\ CA, C' = AC /\ AB
Read:
C' = CD /\ AB

APH
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #199
Date: 27/8/2013 9:25:55
From: eckart_schmidt@t-online.de
Subject: 12 Midpoints

```
Dear Andreas,
the intersection of the lines in your last message is a point on
QA-L3, dividing QA-P1.QA-P10 with ratio 3:4.
Best regards Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #200
Date: 27/8/2013 12:24:00
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
[APH]:
Equivalently:
Let ABC be a triangle, D a point and A'B'C' the cevian
triangle of D. (ABCD is a quadrangle)
Denote:
M1,M2,M3 = the midpoints of AA',BB',CC', resp.
Ma,Mb,Mc = the midpoints of DA', DB', DC', resp.
Ga,Gb,Gc,Gd = the centroids of BCD,CDA,DAB,ABC, resp.
The lines AGa, BGb, CGc, DGd are concurrent.
Antreas
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #201
Date: 27/8/2013 12:29:57
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
[APH]:
Equivalently:
Let ABC be a triangle, D a point and A'B'C' the cevian
triangle of D. (ABCD is a quadrangle)
Denote:
M1,M2,M3 = the midpoints of AA',BB',CC', resp.
Ma,Mb,Mc = the midpoints of DA', DB', DC', resp.
Ga,Gb,Gc,Gd = the centroids of MaM2M3,M1MbM3,M1M2Mc,M1M2M3,
resp.
The lines AGa, BGb, CGc, DGd are concurrent.
Antreas
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #202
Date: 27/8/2013 7:34:34
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

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[APH]
Let (1,2,3,4,5,6) be a hexagon.
Consider the set {A,B,C}, subset of  {1,2,3,4,5,6}, and the
complementary {A',B',C'}.
Denote:
Bc,Ba      = the midpoints of  BC', BA', resp.
Ca, Cb     = the midpoints of CA', AB', resp.
Ab, Ac     = the midpoints of AB', AC', resp.
Ma, Mb, Mc = the midpoints of  BcCb, CaAc, AbBa, resp.
M1, M2, M3 = the midpoints of AA', BB', CC', resp.

The lines MaM1, MbM2, McM3 are concurrent at a point P. (Proof?)
The point P is the same for all sets {A,B,C}, subsets of
{1,2,3,4,5,6}
So we have a point in a sexangle.
Is this point known?

Note: Two points may coincide and then we have a point P in a
quintangle.
Antreas

P.S. Notice in the terminology:
The prefices quadra-, quinta-, sexa - septa- etc come from
Latin,
while the prefices tetra-, penta-, hexa, hepta- etc from Greek.
John H. Conway and I had worked on the names of the regular
polygons and polyhedra
http://mathforum.org/dr.math/faq/faq.polygon.names.html
APH
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #203
Date: 27/8/2013 10:31:05
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
The point P is the centroid of M1M2M3 [Cesar Lozada, Anopolis
#899]
So a simpler form is:
Let (1,2,3,4,5,6) be a sexangle.
Let {A,B,C} be a subset of the set {1,2,3,4,5,6}
and {A',B',C'} the complementary of {A,B,C} with respect
{1,2,3,4,5,6} .
Let M1,M2,M3 be the midpoints of AA', BB', CC', resp.
The centroid of M1M2M3 is a fixed point, ie it is independent
from the choice of {A,B,C}.
Antreas
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #204
Date: 27/8/2013 11:24:48
From: Antreas Hatzipolakis
Subject: 12 MIDPOINTS ......

```
[APH:
>> The point P is the centroid of M1M2M3
>> [Cesar Lozada, Anopolis #899]
>> So a simpler form is:
>> Let (1,2,3,4,5,6) be a sexangle.
>> Let {A,B,C} be a subset of the set {1,2,3,4,5,6}
>> and {A',B',C'} the complementary of {A,B,C}
>> with respect {1,2,3,4,5,6} .
>> Let M1,M2,M3 be the midpoints of AA', BB', CC', resp.
>> The centroid of M1M2M3 is a fixed point, ie
>> it is independent from the choice of {A,B,C}.

For octangle:
Let 12345678 be an octangle.
Let (A,B,C,D} be a subset of the set X =: {1,2,3,4,5,6,7,8}
and {A',B',C',D'} the complementary of {A,B,C,D} with respect X.
Let M1,M2,M3,M4 be the midpoints of AA', BB', CC', DD', resp.
The centroid of the quadrangle M1M2M3M4 is a fixed point, ie
independent from the choice of {A,B,C,D}
Antreas
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #205
Date: 28/8/2013 2:32:49
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton line QL-Cu1 revisited

```
Dear friends,

just for fun:
Let F1, F2 be the foci of the inscribed conic of a quadrilateral
with center QL-L1 ^ QL-L6.
... Consider the circumcribed conics of the triangle components
    through F1, F2.
... These conics have a common point in QL-P1.

Let F1, F2 be the foci of an inscribed conic of a quadrilateral.
... Consider the circumcribed conics of the triangle components
    through F1, F2.
... These conics have a common point  X on QL-Cu1.
... X is the isogonal conjugate of the third intersection
    of F1F2 and QL-Cu1.
... F1, F2, X and QL-P1 are concyclic.

Let F1, F2 be a QL-Tf1 pair .
... Consider the circumcribed conics of the triangle components
    through F1, F2.
... These conics have a common point  X .
... F1, F2, X and QL-P1 are concyclic.

Let L be a line.
... Consider  the  isogonal conjugates of L
    wrt the triangle components.
... The common points of these conics lie on QL-Cu1.
... Consider a line pencil and you have a construction
    for QL-Cu1.

Best regards Eckart
```

Message: #206
Date: 28/8/2013 3:13:12
From: Angel
Subject: A Quadrangle Centers

Dear Members of the group Quadri-Figures-Group

(Adopting the notation of Chris van Tienhoven in
http://www.chrisvantienhoven.nl/)

Let P1, P2, P3, P4 be the defining Quadrangle Points.
Let S1 = P1.P2 /\ P3.P4, S2 = P1.P3 /\ P2.P4 and S3 = P1.P4 /\ P2.P3.

Now S1 S2 S3 is the QA-Diagonal Triangle of the Reference Quadrangle.
QA-DT-Coordinate system, where the QA-Diagonal Triangle is defined as the Reference Triangle with barycentric vertice coordinates S1 =(1:0:0), S2 = (0:1:0), S3 = (0:0:1).
An arbitrary point of the Quadrangle is defined as P4=(p:q:r).

The other 3 points now form the Anticevian triangle of Pi wrt the QA-Diagonal Triangle and have vertices P1=(-p: q: r), P2=(p: -q: r), P3=(p: q: -r).
Let V =(u:v:w) be a point and Ci is the center of the conic through points S1, S2, S3, Pi and V (i=1,2,3,4).
The lines PiCi (i=1,2,3,4) are concurrent if and only if P=(x:y:z) lies on the circumconic that passes through the vertices of the QA-Diagonal Triangle of perspector the barycentric square of P4=(p,q,r), together with the quartic that passes through the P1, P2, P3, P4, S1, S2, S3, QA-P10(=Centroid of the QA-Diagonal Triangle), ..., of equation

(DT-Coordinate):
p^2(q^2-r^2)y^2z^2 + q^2(r^2-p^2)z^2x^2 + r^2(p^2-q^2)x^2y^2=0.

Particular case:
If V=QA-P10 (Centroid of the QA-Diagonal Triangle)
The lines PiCi (i=1,2,3,4) intersect at Quadrangle Centers of
1st DT-Coordinate: p^2(q^2+r^2)

Best regards
Angel Montesdeoca

← Previous Next → ↔ Message Index ↑ Subjects

Message: #207
Date: 28/8/2013 6:20:12
From: Chris
Subject: A Quadrangle Centers

Dear Angel,

Very nice construction and very nice point.
The point you found is collinear with QA-P1, QA-P16, QA-P21 and also the QA-Involution Center (QA-Tf1) of the line QA-P1.QA-P16.

I will study the involved conic and quartic later.
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #208
Date: 28/8/2013 11:54:27
From: Chris
Subject: A Quadrangle Centers

Dear Angel,

Some extra properties:
The conic is the Ninepoint Conic QL-Co1.
The Quartic also passes through QA-P20.
There are four known QA-points X that are on these curves for which the lines PiCi (i=1,2,3,4) are concurrent in some point Y.
When X = QA-P2 then Y=QA-P1.QA-P6 ^ QA-P11.QA-P38, which is the Involution Center of the line QA-P1.QA-P6.
When X = QA-P3 then Y= QA-P1.QA-Tf2(QA-P2) ^ QA-P22.QA-P29
When X = QA-P10 then Y=the Involution Center of the line QA-P1.QA-P16.
When X = QA-P20 then Y= QA-P22, which is the Involution Center of the line QA-P1.QA-P5.
All these points Y lie on the cubic QA-Cu6 because they are all Involution Centers of some line through QA-P1.
I do not quite see why. Do you?

Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #209
Date: 29/8/2013 10:31:16
From: eckart_schmidt@t-online.de
Subject: A quadrangle center

```
Dear Angel, dear Chris,
interesting new curves! My sight: (
Let X be points for which the lines PiCi are concurrent in
points Y.)
One locus for X is the QA-DT circumconic QA-Co1, which is the
image of the line at infinity under the isoconjugation QA-Tf1
(see Chris' message). So it is easy to study the locus for the
corresponding Y points.
It is a cubic with a nodal point in QA-P1 and the equation
      q^2r^2x^2(q^2-r^2)(-x+y+z)
   +  r^2p^2y^2(r^2-p^2)( x-y+z)
   +  p^2q^2z^2(p^2-q^2)( x+y-z)
   + 2p^2q^2r^2 (-y)(y-z)(z-x) = 0
The second locus for X is a quartic. It is the isotomic
conjugate of a conic with the equation
      p^2x^2(q^2-r^2)+q^2y^2(r^2-p^2)+r^2z^2(p^2-q^2) = 0
This conic contains with a point also the vertices of the
anticevian triangle. So it is easy to construct the quartic:
Take QA-P10 and QA-P20 on the quartic (see Chris' message) and
their isotomic images QA-P10 and QA-P19 and for QA-P19 the
vertices of its anticevian triangle, then the conic can be
drawn. Its isotomic conjugate is the quartic. The quartic
contains the vertices of the anticomplementary triangle. The
locus for the Y points is the same as above.
So far, there will be more properties.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #210
Date: 29/8/2013 11:53:40
From: eckart_schmidt@t-online.de
Subject: A quadrangle center

```
Dear Angel, dear Chris,
sorry there is a typo in the first equation of my last message.
Chris already mentioned this cubic as QA-Cu6.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #211
Date: 30/8/2013 6:57:41
From: seiichikiri
Subject: The sufficient conditions of a quadrangle point

```
Dear friends,

About a quadratic quadrangle function independent of the
sidelengths of the reference triangle.
The conditions without the restriction p+q+r=1 are as follows:
 (6) (p+q+r)f(q,r,p) /cyclic sum f(p,q,r)
     =qf(p+q+r,-r,-p)/cyclic sum f(p+q+r,-r,-p).
 (7) qf(-r,-p,p+q+r) /cyclic sum f(-r,-p,p+q+r)
     =-rf(q,p,-p-q-r)/cyclic sum f(q,p,-p-q-r).
 (8) -rf(p,-p-q-r,q) /cyclic sum f(p,-p-q-r,q)
     =pf(-r,p+q+r,-q)/cyclic sum f(-r,p+q+r,-q).
 (9) pf(p+q+r,-q,-r) /cyclic sum f(p+q+r,-q,-r)
     =(p+q+r)f(p,q,r)/cyclic sum f(p,q,r).
(10) p+q=the sum of the right handed sides
     of (6), (7), (8) and (9).

From (9), f(p,q,r) must be p(hp+kq+lr), because the denominators
do not have factors p or p+q+r owing to their symmetry of
variables. Cyclic sum f(p,q,r) and f(p+q+r,-q,-r) must equal
each other.
Then we have QA-P16={p(2p+q+r) : : }.
It seems to me that there is no other quadratic quadrangle
function except for QA-P16 when it is independent of the
sidelengths of the reference triangle.

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #212
Date: 01/9/2013 10:37:50
From: Antreas Hatzipolakis
Subject: Conjecture - Concyclic points

---------- Forwarded message ----------
From: Antreas <anopolis72@gmail.com>
Date: Sun, Sep 1, 2013 at 11:35 AM
Subject: [EGML] Re: Conjecture - Concyclic points
To: Anopolis@yahoogroups.com
[César Lozada]:
>> Let P be a point in the plane of ABC. The perpendicular
bisector
>> of [BC] cuts the circumcircle of PBC at A', with A' and A in
>> the same side w/r to (BC). Build B' and C' cyclically.
>> Conjectures:
>> 1) The points A', B', C' and P are concyclic.
>> 2) If Z(P) is the center of the circle(P, A', B', C') then:
>> 2.1) if P is on the circumcircle of ABC then Z(P)=O
>> 2.2) Z(O) = O, Z(H) = X(355), Z(X(13))=G=X(2),
>> Z (X(14))= X(628) and maybe others

Dear César,

This is very nice and very good for quadrangle loci!
Now, let ABCD be a quadrangle and P a point.
Denote:
aZ(P) = the center Z(P) for the triangle BCD and point P
bZ(P) = the center Z(P) for the triangle CDA and point P
cZ(P) = the center Z(P) for the triangle DAB and point P
dZ(P) = the center Z(P) for the triangle ABC and point P
For which points P we have:
aZ(P)A, bZ(P)B, cZ(P)C, dZ(P)D are concurrent?
Also, for which P's the four centers are concyclic?

Antreas

← Previous Next → ↔ Message Index ↑ Subjects

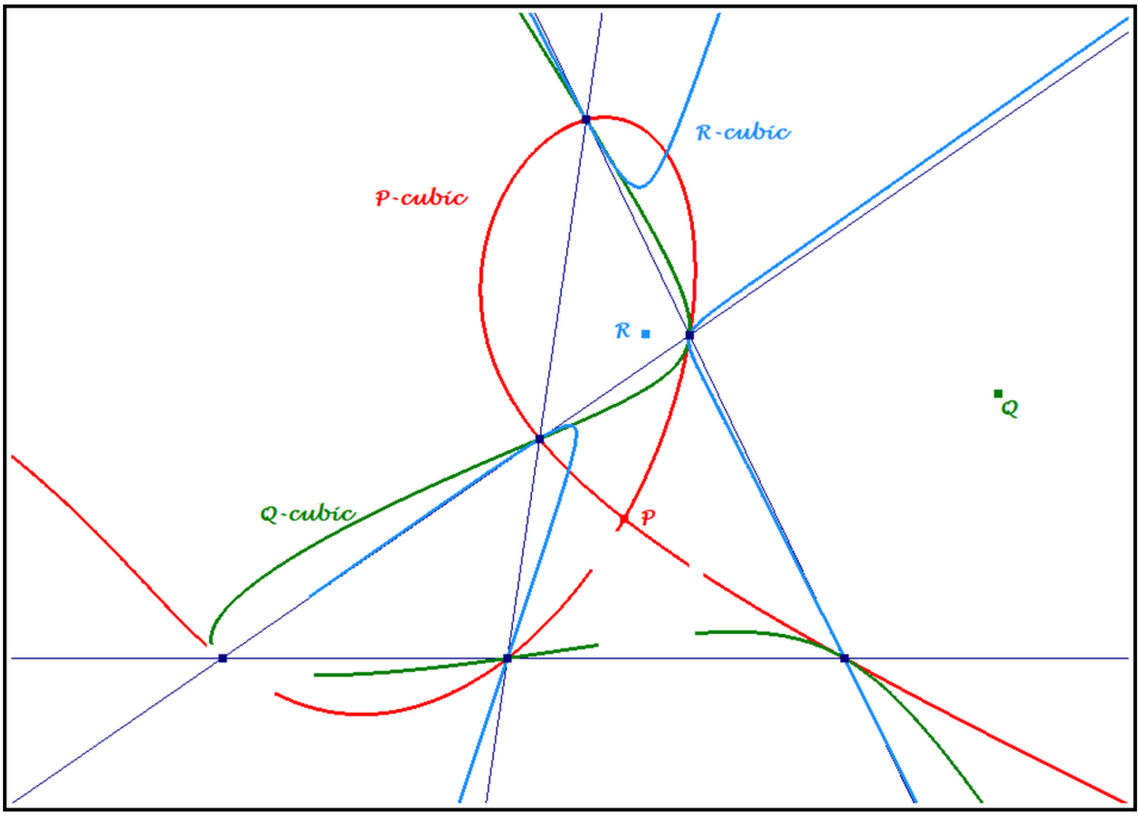
Message: #213
Date: 01/9/2013 11:38:55
From: eckart_schmidt@t-online.de
Subject: Six-point QL-cubic wrt a point

```
Dear friends,
here is an QL-analogon to QA-cubics of type 1 (see QA-Cu/1).
These QA-cubics (e.g. QA-Cu1,2,3,4,5) are pivotal isocubics wrt
QA-Tf2 and can easily be constructed in the following way: For a
pivot P these cubics are the locus of the tangential points of P
wrt circumscribed conics of the quadrangle (Is this anywhere
described?). This construction can be generalized for
quadrilaterals:
For a given point P the cubic is the locus of the tangential
points of P wrt inscribed conics of the quadrilateral.
Another construction: Lines through P cut their QL-Tf2 image in
points of the cubic.
Properties:
1. The cubic contains the six points of the quadrilateral.
2. For P(u:v:w) (CT-notation) the cubic has the equation
     m n x (w^2 y^2 + v^2 z^2)
   + n l y (u^2 z^2 + w^2 x^2)
   + l m z (v^2 x^2 + u^2 y^2)
   + 2 (l m u v+n l w u+m n v w) x y z = 0
3. Tangents to the cubic in opposite points
   of the quadrilateral intersect on the cubic.
4. The three intersection points are collinear.
5. The equation of this line is
     m n v^2 w^2 (m v + n w) (2 l n + m v +n w) x + ... = 0
6. Tangents to the cubic in the tangential points of P wrt an
inscribed conic  intersect on the cubic.

Observation: The cubic has a nodal point in P, if P lies in the
"convex heart" of the quadrilateral. Changing the region about a
line, the cubic will lose its nodal character; changing the
region about a point, the cubic will hold its nodal character...

Remark: Using QL-points for P, I found no relationships to other
QL-points. Perhaps Chris will find some.
There is a Cabri file in the attachment showing three examples.

Best regards Eckart
```



13-08-31-a-fig.png

Message: #214
Date: 01/9/2013 12:01:12
From: Bernard Gibert
Subject: Six-point QL-cubic wrt a point

```
Dear Eckart,

>> 2. For P(u:v:w) (CT-notation) the cubic has the equation
>> m n x (w^2 y^2 + v^2 z^2) + n l y (u^2 z^2 + w^2 x^2)
>> + l m z (v^2 x^2 + u^2 y^2)
>> + 2 (l m u v+n l w u+m n v w) x y z = 0
There's a typo in your equation since this cubic is a cK with
node P and root the tripole of L.
See SITP §8 (and also §1.4.1)

Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #215
Date: 01/9/2013 4:07:04
From: eckart_schmidt@t-online.de
Subject: Six-points QL-cubic wrt a point

```
Dear Bernard,

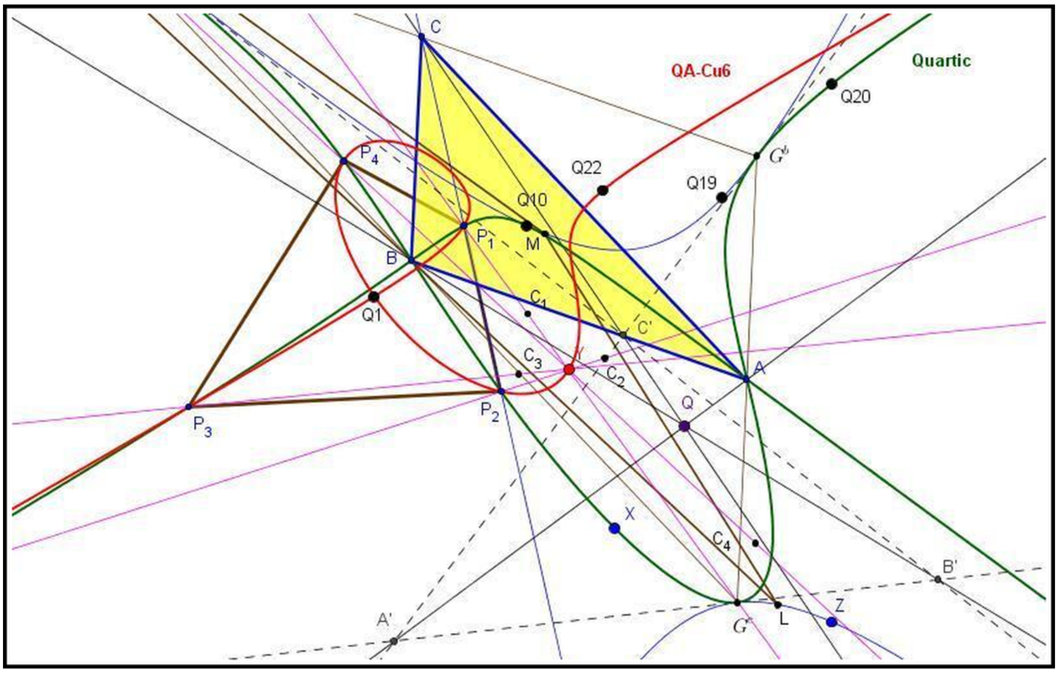
thanks for correction. The right equation of the cubic must be:
     m n x (w^2 y^2 + v^2 z^2)
   + n l y (u^2 z^2 + w^2 x^2)
   + l m z (v^2 x^2 + u^2 y^2)
   - 2(l m u v + n l w u + m n v w) x y z = 0
Thanks also for a classification as a conico-pivotal isocubic
(cK). So the cubic is invariant under an isoconjugation with
fixed point P.
The pivotal conic of the cubic (see SITP, §8.1) is an inscribed
conic of QL-DT as envelope of the QL-Tf2 images of lines through
P.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #216
Date: 02/9/2013 2:00:45
From: amontes
Subject: A Quadrangle Center

Dear Chris
Response to post [Quadri-Figures-Group] #208.
An additional Quadrangle Center:
(Eckart Schmidt
http://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/209)
The quartic that passes through the P1, P2, P3, P4, S1, S2, S3, QA-P10,
QA-P20 is the isotomic conjugate of a conic with the equation
p^2x^2(q^2-r^2)+q^2y^2(r^2-p^2)+r^2z^2(p^2-q^2)=0.
This conic contains with a point also the vertices of the anticevian triangle: QA-P10 and QA-P19 and the vertices of its anticevian triangles.
Thus, the quartic passes through the vertices of the triangle antimedial of the diagonal triangle.
The conic and the quartic are tangent in the vertices of the antimedial triangle.
The three tangents bound a triangle A'B'C' perspective with the diagonal triangle, its perspector Q has 1st DT-Coordinate: 1 / (p^2(q^2-r^2))
Best regards
Angel Montesdeoca



QAPnuevoQuartic.jpg

Message: #217
Date: 05/9/2013 4:08:02
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton Line

```
Dear Chris,
the discussed points are now QL-2P2a,b in EQF. Here is a further
property, I haven't found there:
#  These points are QL-Tf1 images as well as QG-Tf2 images.
#  These points are not only foci of an inscribed conic of the
   quadrilateral but also foci of inscribed conics for the
   triangles QG-Tr3 of the corresponding quadrigon components.
#  These QG-conics touch QG-L1 in QG-P17.
#  If there are intersections of the three QG-conics for a
   quadrilateral, they are orthogonal.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #218
Date: 07/9/2013 1:15:52
From: seiichikiri
Subject: A parametric expression of QA-L3

```
Dear friends,
While I examined QA-points independent of the sidelengths of the
reference triangle with the conditions of message #211,
I noticed a parametric expression of QA-L3 as follows.
QA-L3={(q+r)(2p+q+r)(mp(p+q+r)+nqr);; }.
(1) m=n=1      gives QA-P1.
(2) m=0,  n=1  gives QA-P5.
(3) m=1,  n=0  gives QA-P10.
(4) m=-1, n=1  gives QA-P20.
(5) m=3,  n=-1 gives QA-P22.
(6) m=2,  n=3  gives QA-P25.
(7) m=5,  n=3  gives QA-P26.
Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #219
Date: 08/9/2013 6:27:38
From: seiichikiri
Subject: A parametric expression of QA-L3

```
Dear friends,

Addition: QAP31 extended.
QAP31extended = {p(q+r)(2p+q+r)(mp^2+mpq+mpr+mqr+nq^2+nr^2);; }.

Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #220
Date: 08/9/2013 9:48:07
From: Chris
Subject: Six-point QL-cubic wrt a point

```
Dear Eckart,

I found these points / asymptotes:
* Asymptotes QL-P5, QL-P7, QL-P12, QL-P20, QL-P22, QL-P23-cubics
  equal or parallel to Newton Line:
* Asymptotes QL-P23-cubic equal or parallel
  to 2nd asymptote QL-Co2
* QL-P8-cubic passes through the 3 QL-versions of QG-P3.

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #221
Date: 09/9/2013 9:35:05
From: eckart_schmidt@t-online.de
Subject: Six-point QL-cubic wrt a point

```
Dear Chris,
in addition to your last message:
For all points P on the Newton line there is an asymptote for
the cubic equal or parallel the Newton Line.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #222
Date: 09/9/2013 8:50:54
From: bernard.keizer
Subject: Extension to QL-8P1

Reference for this is F. Loud 1900 Sundry metric theorems concerning n lines in a plane
Considering the 8 circles described in the 8th point of Steiner, the first points of intersection are the 4*4=16 in- and excenters of the reference triangles. The second points of intersection of the same circles are 16 other points belonging to the same circles. But they belong to 12 other circles too, 4 on each circle and 3 circles on each point; these 12 circles are all through the point QL-P1 and form 6 pairs of orthogonal circles, the second points of intersection of the pairs being the 6 vertices of the QL. The centers of the 12 circles are 2 on each perpendicular bisector of the segments joining QL-P1 to the 6 vertices, which are the sides and diagonals of the inscribed quadrilateral formed by the circumcenters of the reference triangles. As there are 3 circles on each point and all circles are through QL-P1, there are 16 alignements of 3 centers on the perpendicular bisectors of the segments joining QL-P1 to the 16 points.
Best regards
Bernard Keizer

← Previous Next → ↔ Message Index ↑ Subjects

Message: #223
Date: 10/9/2013 3:07:26
From: eckart_schmidt@t-online.de
Subject: Circumscribed conics of the QA-DT Medial Triangle

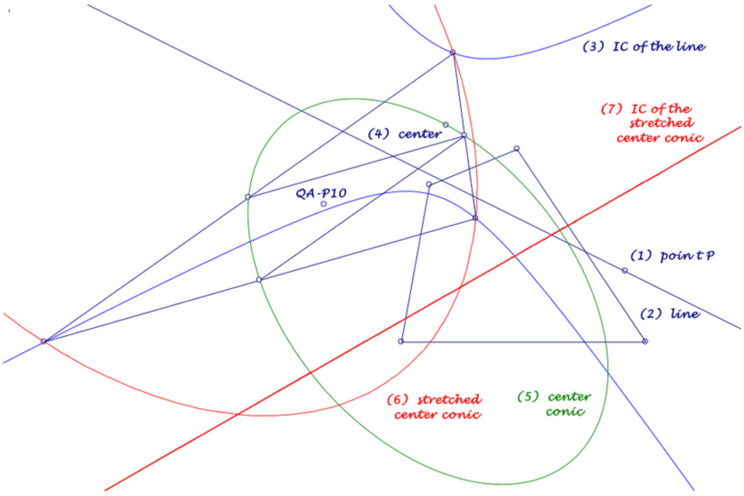
Dear friends,
a special construction leads from an arbitrary point P to a circumscribed conic of the QA-DT medial triangle. Stretching this conic to a circumscribed conic of QA-DT it is the Involutary Conjugate of a line L. There are a lot of EQF relationships, see the attachment.
Best regards Eckart

EQF-Note 2013-09-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Circumscribed Conics of the *QA*-DT Medial Triangle

A special construction leads from an arbitrary point P to a circumscribed conic of the QA-DT medial triangle. Stretching this conic to a circumscribed conic of QA-DT it is the Involutary Conjugate of a line L. Here several examples are listened without right to completeness. – Reference triangle for barycentric coordinates is QA-DT.



Construction

- Line pencil of an arbitrary point $P(u:v:w)$.
- Involutary conjugates of the lines are *QA-DT* circumscribed conics.
- Locus of the centers is a circumscribed conic of the medial triangle of *QA-DT*:

$$q^2r^2ux^2 + r^2p^2vy^2 + p^2q^2wz^2$$
$$-p^2(q^2w + r^2v)yz - q^2(r^2u + p^2w)zx - r^2(p^2v + q^2u)xy = 0$$
with center
$$(2p^2vw + q^2wu + r^2uv : p^2vw + 2q^2wu + r^2uv : p^2vw + q^2wu + 2r^2uv).$$

2013-09-10.pdf

- The anticomplement of the center conic is a circumscribed conic of *QA-DT*:

$$p^2(q^2w+r^2v)yz+q^2(r^2u+p^2w)zx+r^2(p^2v+q^2u)xy=0$$

with center

$$(u(q^2w+r^2v):v(r^2u+p^2w):w(p^2v+q^2u)).$$

- The Involutary Conjugate of the stretched center conic is a line:

$$(q^2w+r^2v)x+(r^2u+p^2w)y+(p^2v+q^2u)z=0.$$

Examples

P = QA-P1

The center conic contains *QA-P29* and has center *QA-P22*.
The stretched center conic is the Nine-point Conic *QA-Co1*.
The final line is the line at infinity.

P = QA-P2

The center conic and the stretched center conic are parabolas with axis orthogonal to *QA-L2*.
The final line contains *QA-P2* and is orthogonal to *QA-P2.QA-P23*.

P = QA-P3

The center conic and the stretched center conic are parabolas with axis parallel to *QA-L4*.
The final line contains *QA-P3* and is orthogonal to *QA-P3.QA-P32*.

P = QA-P6

The center conic is an orthogonal hyperbola through *QA-P11* and *QA-P29* with center in the midpoint of *QA-P11.QA-P29.*
The stretched center conic is an orthogonal hyperbola through *QA-P12* and has the center *QA-P36*.
The final line contains *QA-P23* and is orthogonal to *QA-P1.QA-P32*.

P = QA-P10

The center conic has a center in the midpoint of *QA-P16.QA-P31*.
The center of the stretched center conic is *QA-P31*.
The final line is the *QA-DT* trilinear polar of the isotomic conjugate of *QA-P31* (orthogonal to *QA-P12.QA-P32*).

P = QA-P16

The center conic is the *QA-DT* inscribed Steiner ellipse with center *QA-P10*.
The stretched center conic is the *QA-DT* circumscribed Steiner ellipse with center *QA-P10*.
The final line is the *QA-DT* trilinear polar of *QA-P16*.

QA-P17

The center of the center conic is *QA-P1*.
The center of the stretched center conic is *QA-P20*.
The final line is a *QA-DT* trilinear polar of a point on *QA-P1.QA-P17…*

QA-P18

The center of the center conic is *QA-P31*.
The center of the stretched center conic is *QA-P16*.
The final line is the trilinear polar of *QA-P20*.

QA-P19

The center of the center conic divides *QA-P10.QA-P18* with ratio *1:3*.
The center of the stretched center conic divides *QA-P10.QA-P18* with ratio *-1:3*.

QA-P20

The center of the center conic is the midpoint of *QA-P1.QA-P22*.
The center of the stretched center conic is *QA-P22*.
The final line is orthogonal to *QA-P12.QA-32…*

QA-P23

The center conic is the circumcircle *QA-Ci2* of the *QA-DT* medial triangle.
The stretched center conic is the circumcircle *QA-Ci1* of *QA-DT*.
The final line is the connection of *QA-P6* and the reflection of *QA-P38* in *QA-Ci1* (orthogonal to *QA-P1.QA-P32* and *QA-P11.QA-P38*). It is the *QA-DT* trilinear polar of a point which is the isotomic conjugate of the isogonal conjugate of *QA-P16*:

$$a^2q^2r^2x + b^2r^2p^2y + c^2p^2q^2z = 0 \ .$$

QA-P27

The center of the center conic is the midpoint of *QA-P1.QA-P31* (or divides *QA-P10.QA-P21* with ratio *1:3*). The center of the stretched center conic divides *QA-P10.QA-P21* with ratio *-1:3*.

QA-P30

The center of the center conic divides *QA-P10.QA-P6* with ratio *1:3*. The center of the stretched center conic divides *QA-P10.QA-P6* with ratio *-1:3*.

QA-P36

The final line is orthogonal to *QA-P12.QA-P29*.

In the *QG*-environment the center conic always contains *QG-P2* and the stretched center conic always contains *QG-P1* (further not mentioned). The parallel to *QG-L1* half the distance to *QG-P1* shall be denoted as *QG*-mid-parallel.

QG-P1

The construction degenerates.

QG-P2

The center of the center conic is the midpoint of *QG-P1.QG-P2*. The center conic contains *QG-P1* and the intersection of *QG-L1* and *QG-L2*.
The stretched center conic has center *QG-P2*.
The final line is a parallel to *QG-L1* through *QG-P12*.

QG-P3

The center conic degenerates into two lines: *QG-P1.QG-P2* and *QG*-mid-parallel.

QG-P4

The center of the stretched center conic lies on *QG-P2.QG-P12*. It is the reflection of the intersection of *QG-P2.QG-P12* and the *QG*-mid-parallel in *QA-P1*.
The final line is parallel to *QG-P1.QG-P3* through the reflection of *QG-P1* in *QG-P15*.

QG-P12

The Involutary Conjugate of all lines through *QG-P12* are *QA-DT* circumscribed conics with center *QG-P2*. So the construction degenerates.

QG-P13

The center conic is a parabola with an axis parallel to *QG-P1.QG-P2* and *QG-L1* tangent in *QG-P2*.
The stretched center conic is a parabola with an axis parallel to *QG-P1.QG-P2* and *QG-P1.QG-P14* tangent in *QG-P1*.
The final line is *QG-P3.QG-P13*.

QG-P14

The center conic is a parabola through *QG-P3* with an axis parallel to *QG-P1.QG-P3*.
The stretched center conic is a parabola through the *QA-DT*-anticomplement of *QG-P3* with an axis parallel to to *QG-P1.QG-P3*.
The final line contains *QG-P14*.

QG-P15

The center conic is a parabola with an axis parallel to *QG-P2.QG-P12*.
The stretched center conic is a parabola through *QG-P14* with an axis parallel to *QG-P2.QG-P12*.
The final line is parallel to *QG-P1.QG-P3* through *QG-P15*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-09-10.pdf

Message: #224
Date: 11/9/2013 2:03:45
From: eckart_schmidt@t-online.de
Subject: Simple QL-Cu1 construction

```
Dear friends,
there is a simple QL-Cu1 construction:
   #  lines L through QL-P1
   #  lines L' = QL-Tf1(L)
      = reflections of L in the 1st Steiner axis (see QL-Tf1)
   #  conics Co = QG-Tf2(L)
   #  intersections of L' and Co give points on QL-Cu1.

Background is the following geometry:
Let P be an arbitrary point and L a line through P.
QL-Tf1(L) is a circle, QG-Tf2(L) is a conic.
There are  4  intersections of the circle and the conic.
3 points lie on QL-Cu1, one point on the line
QL-Tf1(P).QG-Tf2(P).
This line cuts QL-Cu1 in 3 points: The QL-Tf2 images of these
points lie concyclic with QL-P1 and P.

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #225
Date: 12/9/2013 8:55:14
From: eckart_schmidt@t-online.de
Subject: Similar construction for QA-Cu1

```
Dear friends,
consider a quadrigon component of a quadrangle with lines L
through QG-P1.
The line QA-Tf2(L) and the circle QL-Tf1(L) intersect on QA-Cu1.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #226
Date: 13/9/2013 8:58:44
From: Chris
Subject: Similar construction for QA-Cu1

```
Dear Eckart,
1. This again shows how a Quadrilateral and Quadrangle meet in a
Quadrigon. Very nice!
2. Your method also gives a new construction method of the cubic
QA-Cu1: the intersection QA-Tf2(L) ^ QL-Tf1(L), where L = random
line through QG-P1 (intersection point diagonals Quadrigon).
Best regards,
Chris
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #227
Date: 14/9/2013 8:51:16
From: seiichikiri
Subject: Algebraic quadrangle points independent of the sidelengths of the re

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Dear friends,
I confirmed that quadrangle points independent of the
sidelengths of the reference triangle satisfied five conditions
of message#211 and derived "algebraic quadrangle points from
them. "Algebraic quadrangle point" means one which does not have
its corresponding geometry now:

[1] Quadrangle points ordered according to their polynomial
degree.
 (1) QAP1:  x=2p+q+r.
 (2) QAP16: x=p(2p+q+r).
 (3) QAP10: x=p(q+r)(2p+q+r).
 (4) QAP21: x=(2p+q+r)(q^2+qr+r^2).
 (5) QAP5:  x=qr(q+r)(2p+q+r).
 (6) QAP20: x=(q+r)(2p+q+r)(-p^2-pq-pr+qr).
 (7) QAP22: x=(q+r)(2p+q+r)(-3p^2-3pq-3pr+qr).
 (8) QAP25: x=(q+r)(2p+q+r)(2p^2+2pq+2pr+3qr).
 (9) QAP26: x=(q+r)(2p+q+r)(5p^2+5pq+5pr+3qr).
(10) QAP19: x=p(2p+q+r)(q^3+q^2r+qr^2+r^3).
(11) QAP31: x=p(q+r)(2p+q+r)(p^2+pq+pr+qr+2q^2+2r^2).
(12) QAP18: x=p(2p+q+r)(p^2+pq+pr-qr)(q^2+r^2).
(13) QAP17: x=qr(2p+q+r)
              (-p^2q^2-pq^3-pq^2r+q^3r-p^2r^2-pqr^2+2q^2r^2-pr^3
              +qr^3).
(14) QAP27: x=(q+r)(2p+q+r)(q^2+qr+r^2)
              (-p^4-2p^3q-2p^2q^2-pq^3-2p^3r+pq^2r+q^3r-2p^2r^2+
              pqr^2+q^2r^2-pr^3+qr^3).

[2] Algebraic quadrangle points
    (1) x=p(q+r)(2p+q+r)(p^2+pq+pr+qr).
    (2) x=p(q+r)(2p+q+r)(q^2+r^2).
quadrangle points may contain more than an algebraic point.

Best regards,
Seiichi.
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #228
Date: 14/9/2013 3:02:57
From: Chris
Subject: Algebraic quadrangle points independent of the sidelengths of th

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Dear Seiichi,

Interesting way of finding new QA-points!
In Triangle Geometry it is relatively easy to make up new
points.
Just find a new function f(a,b,c) and there you are . . .
In Quadrangle Geometry there are several conditions to fulfill
when you have a new function f(a,b,c,p,q,r).
Like you pointed out the most important is that substitution in
the x-coordinate with
    {a -> Sqrt[2], b -> 1, c -> 1, p -> -1, q -> 1, r -> 1}
should give zero.
This implies that every function with factor (2p+q+r) makes a
QA-point.
About your two new points:
    (1) x=p(q+r)(2p+q+r)(p^2+pq+pr+qr).
    (2) x=p(q+r)(2p+q+r)(q^2+r^2).
I checked nr (1) and there are no linear connections with other
QA-points, neither does this point lie on known conics or
cubics.
Nr (2) happens to be QA-P19.

Best regards,
Chris
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← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #229
Date: 15/9/2013 2:17:49
From: seiichikiri
Subject: Algebraic quadrangle points independent of the sidelengths of th

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Dear Chris, dear friends,
Thank you very much Chris for your comments.
I have a plan to make the conditions for quadrilateral points. I
wonder why there are no linear or quadratic points (independent
of the sidelengths). They are 3*n degree polynomials.
Best regards,
Seiichi.
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #230
Date: 16/9/2013 4:42:06
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

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Dear Chris,
I've just discovered the new properties recently added to
QL-Cu1;
I must apologise, I should visit the site EQF more often ...
I wonder if you could a few more properties and may be other
interesting points.

Dear Eckart, dear Bernard G.,
As I know I'm dealing with experts, I thank you in advance to
check these propositions and if possible to answer my 3 final
questions.
Best regards
Bernard K.
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A) Properties

- 2 CL-S conjugate points are isogonal wrt any triangle inscribed in the curve QL-Cu1 ; this is a generalisation of the remark concerning the orthic triangle of the diagonal triangle. This is true in particular for the triangle QL-P1QL-2P2aQL-2P2b.
- The point where the tangents to QL-Cu1 in 2 CL-S conjugates points intersect is the tangential of the 2 points.
- Any circle through QL-P1 cuts QL-Cu1 in 3 other points : the CL-S conjugates of the 3 points are on a line and the tangentials are on a line too ; the converse is true.
- In particular, any circle through 2 CL-S conjugate points and their tangential is through QL-P1 and the line through the 2 CL-S conjugate points is the CL-S transform of the circle and cuts QL-Cu1 in a 3rd point, which is the CL-S conjugate of the tangential (the vertices of the orthic triangle of the diagonal triangle are the CL-S conjugates of the tangentials of the 3 pairs of CL-S conjugate points forming the vertices of the QL).
- Any line through QL-P1 cuts QL-Cu1 in 2 points and the middle of the segment joining the 2 points is on the Newton Line (even if the points are not CL-S conjugates).
- Any line through the point S (which is the tangential of QL-P1) cuts the curve QL-Cu1 in 2 points on equal distance from QL-P1.

B) New points

- After S and T, you could add the points U and V, the points where the circle of inversion of the CL-S transformation or Schmidt circle cuts the curve QL-Cu1 ; U and V are CL-S conjugates, symmetric of one another wrt the 1st Steiner axis and are on the line perpendicular to the 1st Steiner axis trough the point S. T is the tangential of U and V and T, U, V and QL-P1 are concyclic. QL-2P2a, QL-2P2b, U and V are concyclic, the center of the circle being the intersection between the 1st Steiner axis and the perpendicular bisector of QL-2P2a and QL-2P2b.
- UQL-2P2a and VQL2P2b intersect in a point A and VQL-2P2a and UQL2P2b intersect in a point A' ; A and A' are CL-S conjugates on the curve QL-Cu1 and have for tangential the infinity point of the Newton Line (meaning that the tangents to QL-Cu1 in A and A' are parallel to the Newton Line).
- Now we can have 2 pairs of points B and B' and C and C' : B and B' are CL-S conjugates , lie on the bisector of QL-P1QL-2P2aQL-2P2b, have QL-2P2a for middle and QL-2P2b for tangential and the circle through B, B' and QL-P1 is through QL-2P2b (the center of this circle is the intersection between the perpendicular bisector of QL-P1QL-2P2b and the circle circumscribed to QL-P1, QL-2P2a and QL-2P2b) ; the same construction goes for C and C' by inversing the points Ql-2P2a and QL-2P2b. A, B and C' are on a line, A, C and B' and A', B' and C' are on lines too and this QL formed by the 6 points (the only one with 2 tangents to QL-Cu1 parallel to the Newton Line) could be named the main QL of QL-Cu1.

C) Three questions

- The curve QL-Cu1 has 3 inflexion points (points where the curve cuts it's tangent) ; these points are therefore their own tangential. From each of the 3 points one can draw 3 other tangents to the curve. The 3 inflexion points are on a line and for each of them, the 3 contact points with the 3 other tangents are on a line. There are totally 12 points on 4 different lines. How is it possible to find the points and lines for a given QL-Cu1 curve ?

Properties of QL-Cu1.docx

- The points S (where the curve cuts it's asymptote) and it's CL-S conjugate T have the same tangential (4th intersection of the curve and the circle through S, T and QL-P1). Does this point have particular properties ?
- In general, the main QL described above has no other properties than to have 2 tangents to it's van Rees circular focal cubic parallel to the Newton Line. What condition must the points QL-P1, QL-2P2a and QL-2P2b observe if we want the diagonals BB' and CC' be orthogonal to the sides AC and AB ? In this case, we have a triangle and the 4th line as the line through the feet of 2 altitudes of the triangle …

Properties of QL-Cu1.docx

Message: #231
Date: 16/9/2013 11:23:48
From: Chris
Subject: A Quadrangle Center

```
Dear Angel,
Nice point Q.
I checked this point in relationship with other QA-objects.
However I couldn't find any collinearity with existing
QA-points,
neither does it lie on any known QA-Curve.
Best regards,
Chris
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← Previous Next → ↩ Message Index ↑ Subjects

Message: #232
Date: 17/9/2013 12:38:50
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

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Dear Bernard,

I am just studying your interesting paper about QL-Cu1.
First remarks:
  #  If the curve is bipartite, the points T, U, V, ...
     are not real.
  #  If the curve is unipartite, the "main QL of QL-Cu1"
     has the same QL-P1, QL-2P2a,b, QL-L1 and QL-Cu1
     as the reference quadrilateral (perhaps more analogies).
  #  QL-Cu1 is an isogonal circular isocubic wrt QL-P1
     QL-2P2a QL-2P2b (I think up to now not mentioned).
  #  If the curve is bipartite,
     there is a pivot in the point at infinity of QL-L1
     (see my homepage 11.1).

So far, best regards Eckart

PS. My homepage: http://eckartschmidt.de
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← Previous Next → ↩ Message Index ↑ Subjects

Message: #233
Date: 18/9/2013 12:39:08
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Bernard,
there is a mistake in my last message: If QL-Cu1 is bipartite, point T exists as CL-S conjugate of S (not as tangential of U, V, as mentioned in your paper).

If QL-Cu1 is unipartite, the "main QL of QL-Cu1" can be shortened described as the quadrilateral for the points, where QL-Cu1 cuts the following lines:
   outer bisector QL-2P2a QL-P1 QL-2P2b and
   inner bisectors of QL-P1 QL-2P2a QL-2P2b and
   QL-2P2a QL-2P2b QL-P1 (not equal QL-.P1, QL-2P2a, QL-2P2b).

If QL-Cu1 is bipartite, these bisectors cut the cubic in the centers of the in- and excircles of QL-2P2a QL-P1 QL-2P2b.

If QL-Cu1 is bipartite every point P1 on the cubic can be completed by P2, P3, P4 on the cubic, so that the quadrigon P1P2P3P4 has the same QL-P1, QL-2P2a,b, QL-L1, QL-Cu1 as the reference quadrilateral (in analogy to the "main QL" in the unipartite case) in the following way:

For every triangle there is a transformation t(P) wrt a reference vertice, composed of a reflection in the angle bisector at the reference vertice and an inversion wrt a circle round the reference vertice, so that the other two vertices change.

Consider now the triangle
QL-2P2a QL-P1 QL-2P2b and the images t(P1) for the reference vertices
QL-2P2a, QL-P1, QL-2P2b as P2, P3, P4.

Background: The triangle QL-2P2a QL-P1 QL-2P2b is the Miquel Triangle QA-Tr2 of the quadrangle P1P2P3P4.

Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #234
Date: 19/9/2013 10:19:25
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

```
Dear Bernard,

in addition to my last message:
The last construction of the quadrigon P1P2P3P4 on QL-Cu1 also
holds for the unipartite case.
Wrt question 2 in your paper:
   Let X be the tangential of S and T on the cubic.
   # Point X is the CL-S conjugate
     of the third intersection R of ST with QL-Cu1.
     (This point is the second intersection of ST
      with the circle through T, QL-2P2a,b.)
   # Point X is the isogonal conjugate of R
     wrt QL-2P2a QL-P1 QL-2P2b.
   # Point X is the second intersection of the circles
     S, T, QL-P1 and S, QL-2Pa, QL-2P2b.
     (First circle already mentioned in your paper.)
   # The tangent in S to QL-Cu1 is also tangent in S
     to the circle through R, S, QL-P1.
   # The tangent in T to QL-Cu1 is also tangent in T
     to the circle through R, T, QL-P1.
   # The tangent in S to QL-Cu1 is the isogonal conjugate
     wrt QL-2P2a QL-P1 QL-2P2b of the conic
     through R, T, QL-P1, QL-2P2a, QL-2P2b.
   # The tangent in T to QL-Cu1 is the isogonal conjugate
     wrt QL-2P2a QL-P1 QL-2P2b
     of the conic through R, S, QL-P1, QL-2P2a, QL-2P2b.

Best regards Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #235
Date: 19/9/2013 7:07:01
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

Dear Eckart,
Thanks a lot for your commentars and explanations!
For the part A) Properties in my paper, I suppose you didn't find errors ... I suggest to add at the end of the first one your development about the transformation t(P) (reflexion + inversion ...), which in fact holds for the unipartite case too.
For the second part B) New points, I suggest to add to the points S, T, U and V the tangential W of S and T with some of the properties you pointed out (I personnally prefer the intersection of the circles S, T and QL-P1 and S, QL-2P2a and QL-2P2b).
Those developments were meant for EQF, but it belongs to Chris to decide about that ...
There is a mistake at the end of this part B: this main QL is obviously not the only one with 2 tangents to QL-Cu1 parallel to the Newton Line,but the only one one to have 2 middles of diagonals in the points QL-2P2a and QL2P2b.
The part C) three questions was more personal: you answered marvellously the second one (point W), but not the first (inflexion points).
I would like to precise the last question. There are a lot of QL's having the points A and A' as vertices, QL-P1 as Miquel point and QL-2P2a and QL-2P2b as intersection points of QL-Cu1 with the Newton Line (any circle through QL-P1 and A gives 2 other points B and C on the curve ...). Two are particularly interesting: the first one is what I called the main QL, with the 2 points QL2P2a and QL-2P2b as middles of 2 diagonals; the second one has the 2 diagonals perpendicular to the sides (the orthocenter of the triangle ABC is on the bisector of the angle QL-2P2aQL-P1QL-2P2b) and we could call it orthogonal QL. I 'm curious to know in which conditions these 2 QL-s coincide.
To be honest, I started with a triangle ABC and the altitudes BB' and CC' and have the orthogonal QL as the sides of the triangle ABC and the line through the feet B' and C' as the 4th line, which cuts BC in A'; I soon discovered that the points QL-2P2a and 2P2b weren't the middles of the diagonals and that it was possible to form the main QL of the same QL-Cu1 with same Miquel point and Newton Line ...
Thanks again for your attention
Best regards
Bernard

← Previous Next → ↩ Message Index ↑ Subjects

Message: #236
Date: 20/9/2013 4:54:25
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Bernard,

thanks for your extensive answer. Some remarks:

If QL-Cu1 is unipartite, your "main QL" is a special one, for B, B', C, C' lie on a circle. The midpoint is the center of the QL-P1 corresponding excircle of QL-2P2a QL-P1 QL-2Pb. This holds also for the further constructions in your last message. The midpoint of the circle for your "orthogonal QL" seems to be the midpoint of BC.

If QL-Cu1 is bipartite and you define a "main QL" analogue with the intersections of the cubic with the outer bisector at QL-P1 and the inner bisectors at QL-2P2a,b wrt QL-2P2a QL-P1 QL-2P2b, then the points of the "main QL" are QL-2P2a,b and the in- and excenters of QL-2P2a QL-P1 QL-2P2b. A very special QL. Sorry, but I cannot find an analogue for the "orthogonal QL".

I prefer the following sight:
Consider quadrigons P1P2P3P4 on QL-Cu1 of a reference quadrilateral, constructed with the reference triangle QL-2P2a QL-P1 QL-2P2b
as described in message #233.
 # Opposite points of P1P2P3P4 as quadrilateral
 are CL-S conjugate on the cubic.
 # QL-Cu1 is an isogonal isocubic
 wrt the reference triangle.
 # P1P2P3P4 as quadrilateral has the same
 QL-Cu1 (QL-L1, QL-P1, QL-2P2a,b)
 as the reference quadrilateral.
 # If QL-Cu1 is bipartite,
... QL-Cu1 is a pivotal isogonal circular cubic
 wrt the reference triangle with pivot
 in the point at infinity of QL-L1,
... P1, P2, P3, P4 have the same tangential,
... the reference triangle is the Miquel Triangle QA-Tr2
 of the quadrangle P1P2P3P4,
... QL-Cu1 of the reference quadrilateral is QA-Cu1
 of P1P2P3P4 as quadrangle.
 # If QL-Cu1 is unipartite, P1P2P3P4 are special quadrigons:
... opposite vertices of the quadrigon P1P2P3P4
 are isogonal conjugate,
... the diagonals are parallel,

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... QA-P4 = QL-P1,
... intersections of opposite sides are QL-2P2a,b.

I hope, there are no basic mistakes!

Best regards Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #237
Date: 21/9/2013 1:22:05
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

```
Dear Bernard, dear Chris,
I think it's remarkable, that QL-Cu1 is isogonal invariant wrt
the reference triangle QL-2P2a QL-P1 QL-2P2b = QL-Trx.
Let ^ denote the Clawson-Schmidt conjugate and * the isogonal
conjugate wrt QL-Trx.
#  If X is a point on QL-Cu1, then holds X* = X^.
#  If X is not a point on QL-Cu1,
   the line X^.X* contains QL-P1.
#  For a line L the image L^ is a circle through QL-P1
   and L* is a conic through QL-P1,
   ... both have 4 common points on QL-Cu1
       (one of them QL-P1),
   ... the Simson line of QL-P1 wrt the further three common
       points is perpendicular QL-L1.
#  For a side line Li the image Li* is a common circumconic
   of the triangles QL-Trx and  LjLkLl.
#  X^* = X*^ and X.X^* = X.X*^ contains QL-P1.
#  For X and  X^=X*  on QL-Cu1
   ... the circumconics of the triangle components through these
       two points have a further common point Y on the cubic,
   ... X, Y, X^=X*, and QL-P1 lie concyclic,
   ... Y^=Y* is the third intersection of  X*X^ with QL-Cu1.
# For X not on QL-Cu1 (X^ unequal X*)
   ... the circumconics of the triangle components
       through X and X^ have a further common point Y,
   ... X, X^, Y and QL-P1 are concyclic,
       X, X^, Y^ are collinear,
   ... the circumconics of the triangle components
       through X and X* have a further common point Z,
   ... X, X*, Z* are collinear.
There will be more properties.

Best regards Eckart
```

Message: #238
Date: 21/9/2013 2:14:25
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

```
Dear Bernard, dear Chris,

sorry, the properties in my last message are only valid in the
unipatite case of QL-Cu1.
In the bipartite case there are significant corrections!
I will describe it later.
But QL-Cu1 is also in the bipartite case isogonal invariant wrt
QL-2P2a QL-P1 QL-2P2b.

Excuse, Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #239
Date: 22/9/2013 12:07:43
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

```
Dear Bernard, dear Chris,

I repeat: It is remarkable, that QL-Cu1 is isogonal invariant
wrt the reference triangle QL-2P2a QL-P1 QL-2P2b = QL-Trx.

The unipartite case of QL-Cu1 (see also my last message):
Let ^ denote the Clawson-Schmidt conjugate and * the isogonal
conjugate wrt QL-Trx.
# If X is a point on QL-Cu1, then holds X* = X^.
# If X is not a point on QL-Cu1,
  the line X^.X* contains QL-P1.
# For a line L the image L^ is a circle through QL-P1
  and L* is a conic through QL-P1,
  ... both have 4 common points on QL-Cu1 (one of them QL-P1),
  ... the Simson line of QL-P1
      wrt the further three common points is perpendicular
      QL-L1.
# For a side line Li the image Li* is a common circumconic
  of the triangles QL-Trx and  LjLkLl.
# X^* = X*^ collinear with X, QL-P1.
# For X and  X^=X*  on QL-Cu1
  ... the circumconics of the triangle components through these
      two points have a further common point Y on the cubic,
  ... X, Y, X^=X*, and QL-P1 lie concyclic,
  ... Y^=Y* is the third intersection of  X*X^ with QL-Cu1.
# For X not on QL-Cu1 ( X^ unequal X*)
  ... the circumconics of the triangle components
      through X and X^ have a further common point Y,
  ... X, X^, Y and QL-P1 are concyclic, X, X^, Y^ are collinear,
  ... the circumconics of the triangle components
      through X and X* have a further common point Z,
  ... X, X*, Z*  are collinear.

The bipartite case of QL-Cu1:
# The line X^.X* contains QL-P1.
# For a line L the image L^ is a circle through QL-P1
  and L* is a conic through QL-P1,
  ... the Simson line of QL-P1 wrt the further three
      common points of L^ and L* is parallel QL-L1.
# For a side line Li the image Li* is a circumconic of QL-Trx.
  Further intersections of these conics lie on QL-Cu1
  as third intersections of the the lines through QL-P1
  and a point of the quadrilateral.
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```
# X^* = X*^ collinear with X, QL-P1.
# For X, X^, X* on QL-Cu1
  ... the circumconics of the triangle components
      through X and X^ have a further common point Y on the
      cubic,
  ... X, Y, X^, and QL-P1 lie concyclic,
  ... the circumconics of the triangle components
      through X and X* have a further common point
      in QL-P1 on the cubic,
# For X, X^, X* not on QL-Cu1
  ... the circumconics of the triangle components
      through X and X^ have a further common point Y.
  ... X, Y, X^ and QL-P1 are concyclic,
  ... the circumconics of the triangle components
      through X and X* have a further common point Z
  ... Apparently there are more interesting properties
      in the unipartite case.

Best regards Eckart
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #240
Date: 23/9/2013 3:57:26
From: seiichikiri
Subject: colinearity and cyclic sums of quadrangle points

```
Dear friends,

I observed the following about colinearity  and cyclic sums of
quadrangle points.
Cyclic sum means f(p,q,r)+f(q,r,p)+f(r,p,q).

[1] the following points are colinear as QA-L3. Cyclic sums:
    (1) QAP1: 4(p+q+r).
    (2) QAP10: 3(p+q)(q+r)(r+p).
    (3) QAP5: (p+q)(q+r)(r+p)(p+q+r)
    (4) QAP20: -2(p+q)(q+r)(r+p)(p+q+r).
    (5) QAP22: -8(p+q)(q+r)(r+p)(p+q+r).
    (6) QAP25: 9(p+q)(q+r)(r+p)(p+q+r).
    (7) QAP26: 18(p+q)(q+r)(r+p)(p+q+r).

[2] the following points are colinear. Cyclic Sums:
    (1) QAP10: 3(p+q)(q+r)(r+p).
    (2) QAP16: 2(p^2+q^2+r^2+qr+rp+pq).
    (3) QAP19: (p+q)(q+r)(r+p)(p^2+q^2+r^2+qr+rp+pq).
    (4) QAP31: 4(p+q)(q+r)(r+p)(p^2+q^2+r^2+qr+rp+pq).

We can think that colinear points are derived from 2 points of
the simplest cyclic sums.

Best regards,
Seiichi.
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← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #241
Date: 23/9/2013 5:08:12
From: yeuemtrondoitb85
Subject: A generalization Brahmagupta's Theorem

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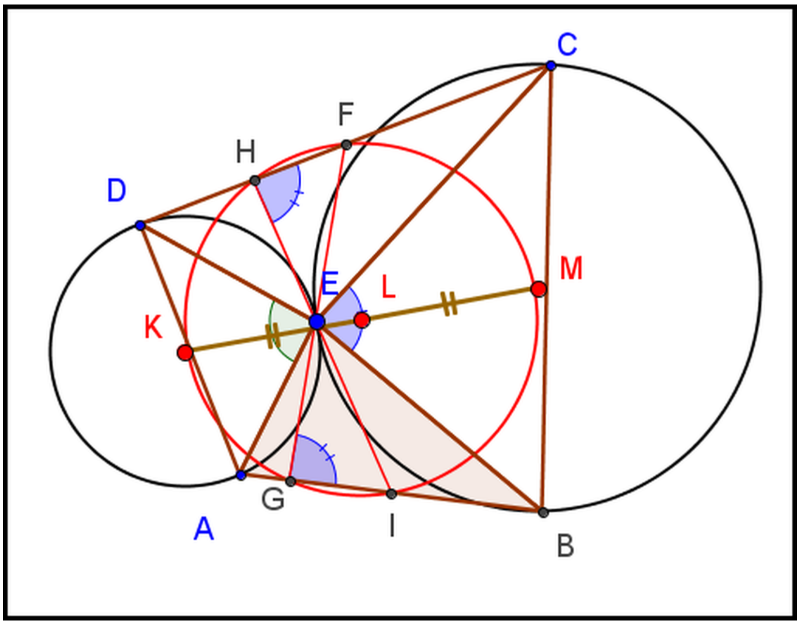
Dear all Member!

Let a quadrilateral ABCD,
let E point and Angle(DEA)+Angle(CEB)=180 deg and DE*EB=EC*EA.
F is midpoint of DC.
EF meet AB at G.
F, I are midpoints of CD,AB respectively.
K,L,M are circumcircle of (EDA),(EIG),(ECB)

Prove that
1- Angle DEA = Angle AGE
2- L is midpoints of KM
I'm not arithmetician, I want you proved and publish it.

Best regard
Dao Thanh Oai
```



A generalization Brahmagupta's Theorem.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #242
Date: 23/9/2013 5:10:10
From: yeuemtrondoitb85
Subject: A generalization Brahmagupta's Theorem

```
Dear all Member!

Let a quadrilateral ABCD,
let E point and
angle DEA + angle CEB = 180 deg and
DE * EB = EC * EA.
F is midpoint of DC.
EF meet AB at G. F, I are midpoints of CD,AB respectively.
K,L,M are circumcircle of (EDA),(EIG),(ECB)

Prove that
1- angle DEA = angle AGE
2- L is midpoints of KM

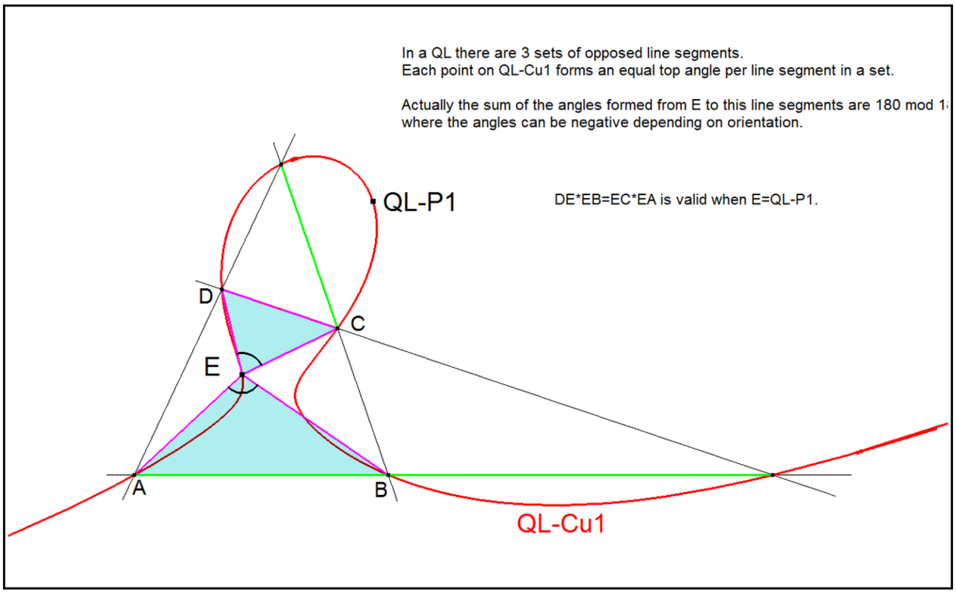
I'm not arithmetician, I want you proved and publish it.

Best regard
Dao Thanh Oai
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #243
Date: 23/9/2013 10:20:30
From: Chris van Tienhoven
Subject: A generalization Brahmagupta's Theorem

Dear Dao Thanh Oai,
Your generalization of Brahmagupta's Theorem is very interesting.
I checked the locus of points with the property
    angleDEA + angleCEB = 180 degree.
Surprisingly it is the cubic QL-Cu1.
Note that this is a property belonging to a Quadrilateral (system of 4 random lines) and not to a Quadrangle (system of 4 random points).
When we define a Reference Quadrilateral as a system of 4 random line, then we can discern 3 x 2 sets of 2 opposite line segments.
For each 2 sets of 2 opposite line segments (e.g. AB/CD, AC/BD in your example) we have the rule that the sum of the angles formed from E to this line segments are 180 mod 180, where the angles can be negative depending on orientation.
The property for points E that:
*   DE*EB=EC*EA is valid for some selected points on QL-Cu1.
The Miquel Point QL-P1 is one of these points!
The property for points E that:
*   Angle BGE = Angle CHE = Angle CEB is valid for another selection of points on QL-Cu1.
Best regards,
Chris van Tienhoven



QL-Cu1-BrahmaguptaTheorem-01.png

Message: #244
Date: 23/9/2013 10:49:25
From: eckart_schmidt@t-online.de
Subject: A generalization Brahmaguptas Theorem

Dear mister Dao Thanh Oai, dear Chris,

only a short remark: Points E of a quadrigon with the angle
property <DEA+<BEC=180° lie on a section of QL-Cu1.
This will give a new property of QL-Cu1 wrt a quadrigon as locus
for points X with <P1XP2 + <P3XP4 = 0° or 180°.

Best regards Eckart

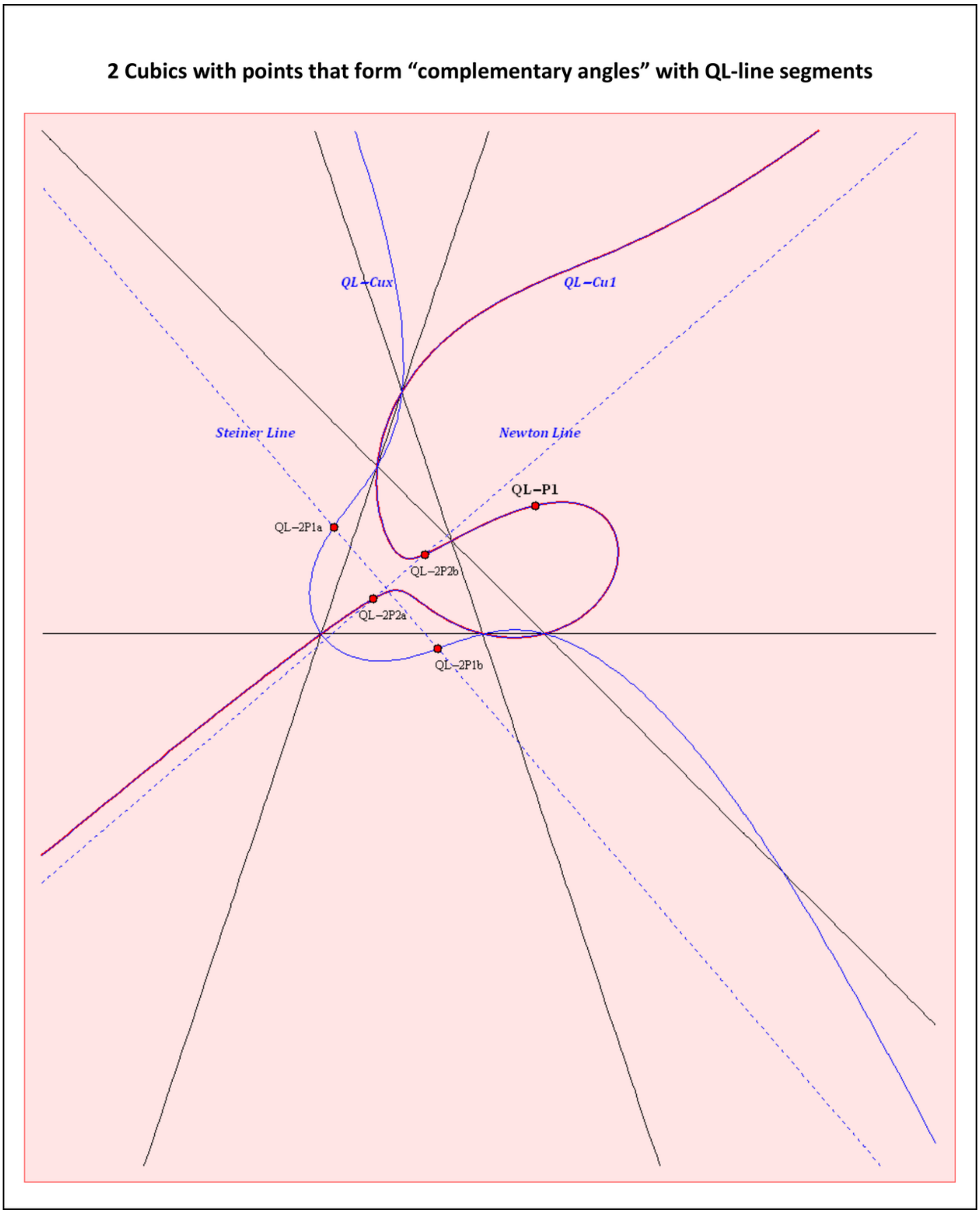
← Previous Next → ↔ Message Index ↑ Subjects

Message: #245
Date: 23/9/2013 9:27:34
From: Chris van Tienhoven
Subject: A generalization Brahmagupta's Theorem

Dear friends,

There are 2 different cubics with the property that
the sum of the angles formed from E to the opposite QL-line
segments are 180 mod 180.
I checked the cubics with the property that Cos[E1]^2 -
Cos[E2]^2 = 0.
The first one is the well-known cubic QL-Cu1.
The 2nd one is a new cubic QL-Cux with CT-equation:
-c^2 l n x^2 y - c^2 m n x y^2 + l (b^2 m + a^2 n - b^2 n - c^2
n) x^2 z + m (a^2 l + b^2 l - c^2 l + a^2 n - b^2 n - c^2 n) x
y z + a^2 l m y^2 z + (a^2 l + b^2 l - c^2 l - b^2 m) n x z^2 +
a^2 l n y z^2

Somehow the equation is not quite symmetric although it is valid
for all 3 QL-Quadrigons.
But it passes through the Plücker Pair of points QL-2P1a and
QL-2P1b and also through QG-P1 (this looks like a separate
point).
The Plücker Pair of points QL-2P1a and QL-2P1b are easy to
verify that the formed angles are “complementary”.
See attachment.
Best regards,
Chris



QL-Cux-01.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #246
Date: 23/9/2013 9:48:55
From: Eisso J. Atzema
Subject: interesting paper?

```
Dear Bernard,
In 1937, Jules Marchand also had a paper with the exact same
title in the following less obscure publication by the
University of Lausanne (where Marchand was a professor at the
time):
Recueil de travaux publiés à l'occasion du quatrième centenaire
de la fondation de l'Université juin MCMXXXVIII. Faculté des
Sciences (Lausanne, 1937)
I bet this is the same article. There are several libraries in
France that have a copy of the Recueil (Paris, Toulouse,
Montpellier, Dijon) (see Sudoc.fr). I can see if I can order a
copy of Marchand's paper through Interlibrary Loan as there are
at least six libraries in the US that own the Recueil.
best,
Eisso
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #247
Date: 24/9/2013 8:55:37
From: Dao Thanh Oai
Subject: A generalization Brahmagupta's Theorem

```
Dear Chris van Tienhoven

Let a quadrilateral ABCD,
let E point and angleDEA+angleCEB = 180 deg and DE*EB=EC*EA.
F is midpoint of DC.
EF meet AB at G.
F, I are midpoints of CD,AB respectively.
K,L,M are circumcircle of (EDA),(FIG),(ECB)

Prove that
1-angle DEA = angle AGE
2-L is midpoints of KM

I'm not mathematicians.
I want you publish it.
Because I don't know QL-cu1.
Its is yours

Best regard
Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #248
Date: 25/9/2013 9:26:11
From: yeuemtrondoitb85
Subject: May be a new property of cyclic quadrilateral?

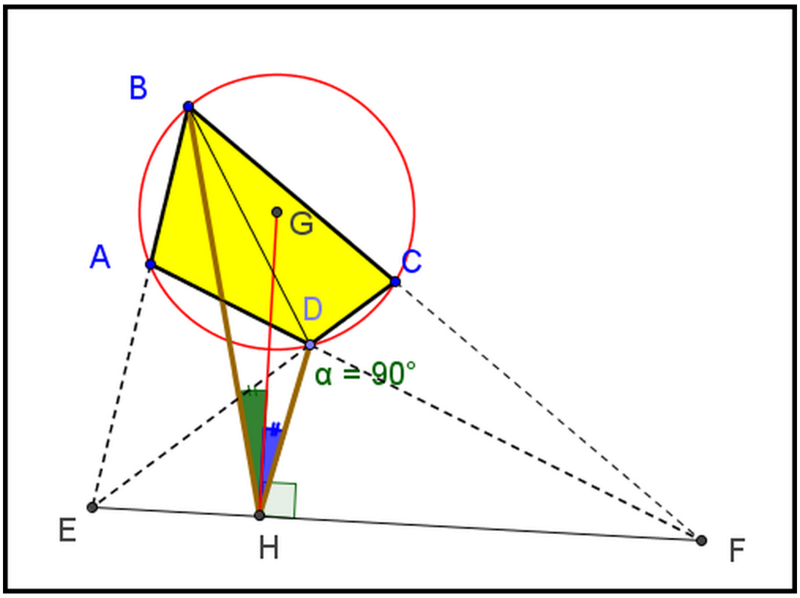
```
Dear all Member!

I have a question to you for this problem:
Let a cyclic quadrilateral ABCD and G is circumcircle of (ABCD).
BC meets AD at F.
CD meets BA at E.
H is on EF and GH perpendicular with EF.

Prove  that HG is angle bisetor of angle BHD

Maybe a new property of cyclic quadrilateral?

Best regards
Dao Thanh Oai
```



May be a new property of cyclic quadrilateral.png

Message: #249
Date: 25/9/2013 11:46:58
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

Dear Eckart,

1) In the unipartite case, for the main QL in fact, B, B', C and C' are concyclic and the circumcenter is on 3 remarkable circles: trough QL-P1, B, B' and QL-2P2b, through QL-P1, C, C' and QL-2P2a and through QL2P2a, QL-2P2b, U and V; on this last circle, it's the opposite of the incenter of the triangle QL-P1QL-2P2aQL-2P2b and, as you say, it's one of the excenters of the same triangle. For the orthogonal QL, this point is the middle of BC, but I haven't found yet in which conditions these 2 QL's coincide ...

2) If I'm not wrong, in the bipartite case, the points QL-2P2a and QL-2P2b aren't real ...

3) Do you know a way to draw the inflexion points of QL-Cu1 in the unipartite case?

Best regards
Bernard

← Previous Next → ↩ Message Index ↑ Subjects

Message: #250
Date: 26/9/2013 1:35:55
From: Antreas Hatzipolakis
Subject: REFERENCE

Dear Chris
The paper in your reference:
[16] Daniel Baumgartner, Roland Stärk, Ein merkwürdiger Punkt des Vierecks, available at
http://mathematicus.ch/geometrie/empdv.pdf
is no longer available.
Daniel Baumgartner's web site is at:
http://www.geometria.ch/
and the paper is available at:
http://www.geometria.ch/geometrie/empdv.pdf
Antreas

← Previous Next → ↩ Message Index ↑ Subjects

Message: #251
Date: 26/9/2013 11:22:24
From: bernard.keizer
Subject: May be a new property of cyclic quadrilateral?

```
Dear Dao Thanh Oai,

For a cyclic quadrilateral, the circumcenter is the orthocenter
of the diagonal triangle.

From the vertices of the orthic triangle of this diagonal
triangle, you see the sides AB and CD under the same angle and
these vertices belong to the cubic QA-Cu1 ...

You may find demos and developments in Cyclic Quadrilaterals -
The Big Picture Yufei Zhao

Best regards
Bernard Keizer
>
> ---In Quadri-Figures-Group@yahoogroups.com, wrote:
> Dear all Member!
> I have a question to you for this problem:
> Let a cyclic quadrilateral ABCD and G is circumcircle of
> (ABCD). BC meets AD at F. CD meets BA at E.
> H is on EF and GH perpendicular with EF.
> Prove  that HG is angle bisetor of angle BHD
> May be a new property of cyclic quadrilateral?
> Best regards
> Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #252
Date: 26/9/2013 12:42:50
From: seiichikiri
Subject: colinearity and cyclic sums of quadrangle points

```
Dear friends,

Addition (an algebraic QA point derived from QAP17)

(1) QAP17: {qr(2p+q+r)(-p^2q^2-pq^3-pq^2r+q^3r-p^2r^2-pqr^2+2q^2
r^2-pr^3+qr^3);; }.
Cyclic sum =
-(p+q+r)(p^2+p(q+r)-qr)(q^2+q(r+p)-rp)(r^2+r(p+q)-pq).

(2) QAPX: {p(2p+q+r)(p^2(q-r)^2+p(q-r)^2(q+r)-qr(q^2+r^2)).
Cyclic sum = -3(p^2+p(q+r)-qr)(q^2+q(r+p)-rp)(r^2+r(p+q)-pq).

The factor(p+q+r) of the cyclic sum of QAP17 are eliminated.
See the attached Mathematica file.

Best regards,
Seiichi.
>
> ---In quadri-figures-group@yahoogroups.com, wrote:
> Dear friends,
> I observed the following about colinearity  and cyclic sums of
quadrangle
> points.
> Cyclic sum means f(p,q,r)+f(q,r,p)+f(r,p,q).
> [1] the following points are colinear as QA-L3. Cyclic sums:
> (1) QAP1:  4(p+q+r).
> (2) QAP10: 3(p+q)(q+r)(r+p).
> (3) QAP5:  (p+q)(q+r)(r+p)(p+q+r)
> (4) QAP20: -2(p+q)(q+r)(r+p)(p+q+r).
> (5) QAP22: -8(p+q)(q+r)(r+p)(p+q+r).
> (6) QAP25:  9(p+q)(q+r)(r+p)(p+q+r).
> (7) QAP26:  18(p+q)(q+r)(r+p)(p+q+r).
> [2] the following points are colinear. Cyclic Sums:
> (1) QAP10:  3(p+q)(q+r)(r+p).
> (2) QAP16:  2(p^2+q^2+r^2+qr+rp+pq).
> (3) QAP19:  (p+q)(q+r)(r+p)(p^2+q^2+r^2+qr+rp+pq).
> (4) QAP31:  4(p+q)(q+r)(r+p)(p^2+q^2+r^2+qr+rp+pq).
> We can think that colinear points are derived from 2 points of
> the simplest cyclic sums.
> Best regards,
> Seiichi.
```

```
(* confirmation of 5 conditions *)
(* QAP17 *)
f1[1] = q r (2 p + q + r)
   (-p^2 q^2 - p q^3 - p q^2 r + q^3 r - p^2 r^2 - p q r^2 + 2 q^2 r^2 - p r^3 + q r^3);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}] // Factor // Simplify
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}] // Factor // Simplify
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}] // Factor // Simplify
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}] // Factor // Simplify
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}] // Factor // Simplify
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}] // Factor // Simplify
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}] // Factor // Simplify
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}] // Factor // Simplify
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Simplify
```

$(p+q+r)\left(p^2-qr+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$p\left(p^2-qr+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$q\left(p^2-qr+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$(p+q+r)\left(p^2-qr+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$-r\left(p^2-qr+p(q+r)\right)(p(-q+r)-q(q+r))(p(-q+r)+r(q+r))$

$q\left(p^2-qr+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

algebraic QA-Point-nb.pdf

```
p (p^2 - q r + p (q + r)) (p (q - r) + q (q + r)) (p (q - r) - r (q + r))

-r (p^2 - q r + p (q + r)) (p (-q + r) - q (q + r)) (p (-q + r) + r (q + r))

0

0

0

0

0

(* QAPX *)
f1[1] = p (2 p + q + r) (p^2 (q - r)^2 + p (q - r)^2 (q + r) - q r (q^2 + r^2));
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}] // Factor // Simplify
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}] // Factor // Simplify
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}] // Factor // Simplify
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}] // Factor // Simplify
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}] // Factor // Simplify
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}] // Factor // Simplify
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}] // Factor // Simplify
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}] // Factor // Simplify
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Simplify
```

algebraic QA-Point-nb.pdf

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

$3\left(p^2-q r+p(q+r)\right)(p(q-r)+q(q+r))(p(q-r)-r(q+r))$

0

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algebraic QA-Point-nb.pdf

Message: #253
Date: 26/9/2013 10:07:58
From: Chris
Subject: colinearity and cyclic sums of quadrangle points

```
Dear Seiichi,
This point QAPX:
     {p(2p+q+r)(p^2(q-r)^2+p(q-r)^2(q+r)-q r(q^2+r^2))
   is:
     QA-P1.QA-P17 ^ QA-P10.QA-P16 ^ QA-P18.QA-P21
Accidentally this point is already mentioned in QA/5:
     List of QA-Crosspoints in EQF.
Best regards,
Chris
```

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #254
Date: 27/9/2013 9:05:43
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Bernard, dear Chris,

back from some days holiday in Denmark, I have only an
explanation wrt Bernard's second question:
Only now I have seen, that Chris' definition of QL-2P2a,b is not
my use of this points. If we take the first line of the
definition in EQF, Bernard is right: These points are not real
in the bipartite case.
Up to now I interpreted QL-2P2a,b in the sense of the fourth
line as foci of an inscribed conic with center in the
intersection of QL-L1 and QL-L6. In this interpretation you have
to read my messages.
In the bipartite case these foci lie on a perpendicular line wrt
QL-L1 through the intersection of QL-L1 and QL-L6.
I would appreciate, if Chris changes the definition.
These two foci F1, F2 are always real and of significance for
simple "strophoid" constructions for QL-Cu1 (see earlier
messages):
If F1, F2 are points on QL-L1 (QL-Cu1 unipartite), the
intersections of of circles round points M on QL-L1 orthogonal
to the Thales circle about F1F2 cut the line M.QL-P1 in points
of QL-Cu1.
If F1, F2 are not points on QL-L1 (QL-Cu1 bipartite), the
intersections of of circles round points M on QL-L1 through F1,
F2 cut the line M.QL-P1 in points of QL-Cu1.

Best regards Eckart

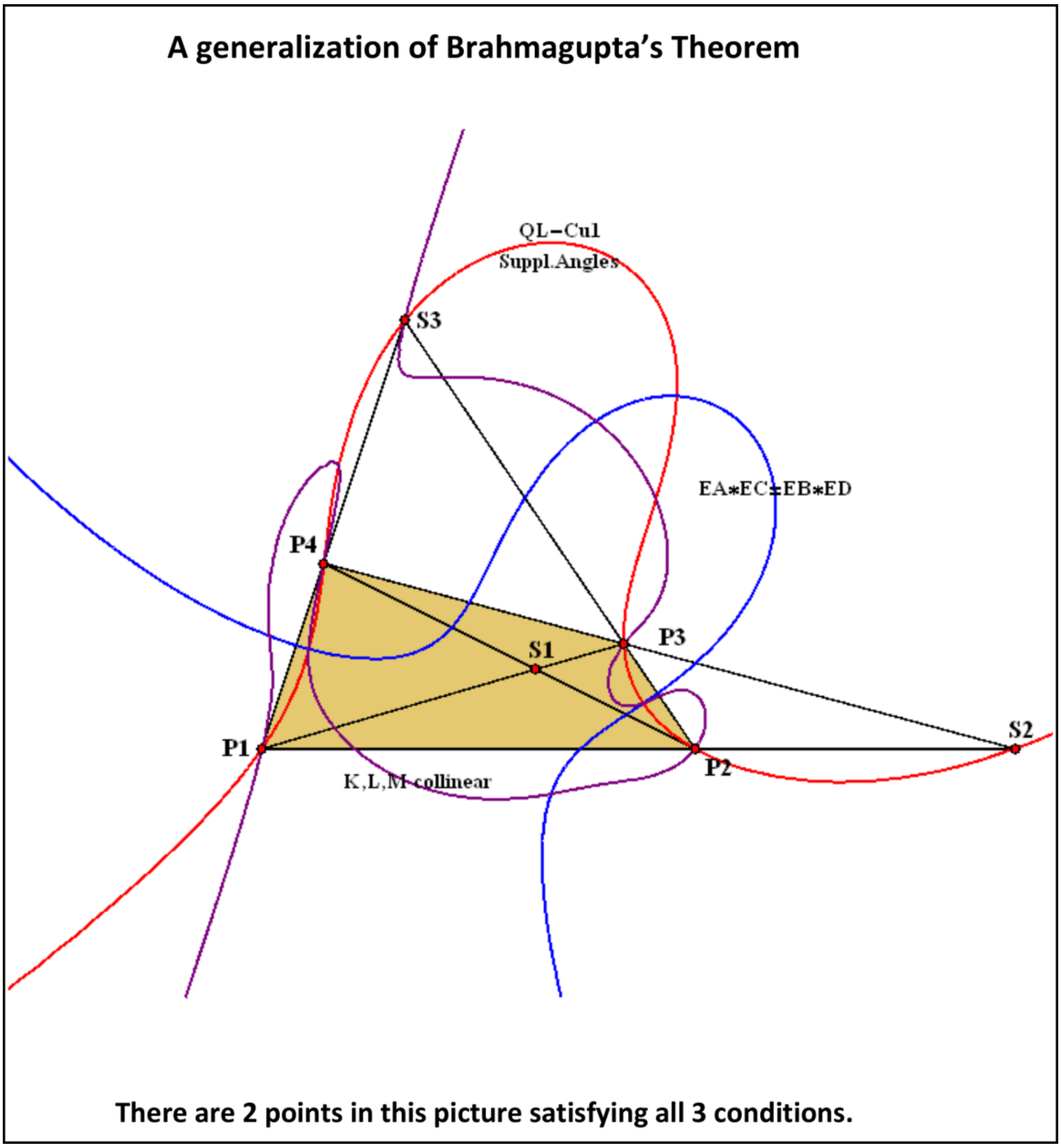
← Previous Next → ↔ Message Index ↑ Subjects

Message: #255
Date: 27/9/2013 1:57:31
From: Chris van Tienhoven
Subject: A generalization Brahmagupta's Theorem

```
Dear Dao,

Unfortunately I am not able to solve your problem synthetically
yet.
Probably when I could I also would find a construction method.
When I would have a construction method I probably would be able
to solve this problem synthetically.
The point(s) E you describe have these properties:
they form supplementary angles with opposite line segments of a
quadrilateral.
EA * EC = EB * ED, where A,B,C,D, are concyclic.
Circumcenters K,M,L of resp. triangles EDA, EBC and FGHI
are collinear and LM=LK.
Condition 1 tells us that point E lies on cubic QL-Cu1.
Condition 2 tells us that point E lies on another cubic.
Condition 3 tells us that point E should lie on a curve of 5th
degree.
See attached picture for these curves and how they coincide in 2
points.
The question is how to construct these 2 (or more) points with
these conditions.
Algebraically I can't find a way either to solve this problem.
Anyone else with a good idea?

Best regards,
Chris
```



A generalization of Brahmagupta's Theorem.pdf

← Previous Next → ↩ Message Index ↑ Subjects

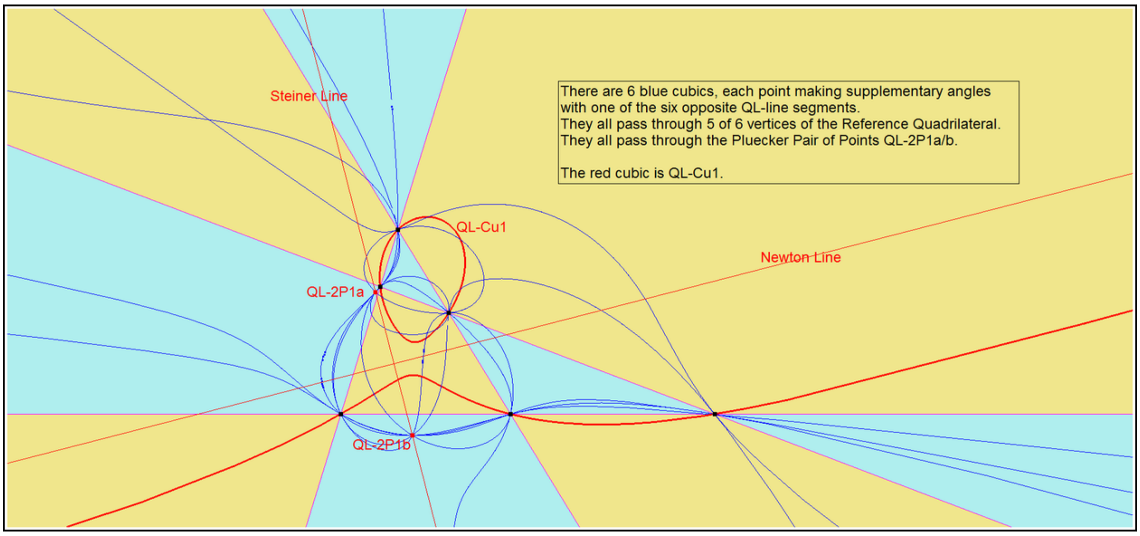
Message: #256
Date: 27/9/2013 2:02:07
From: Chris van Tienhoven
Subject: A generalization Brahmagupta's Theorem

```
Dear Friends,

I explored the new cubic a bit further.
I already told that the equation wasn't symmetric.
Now it appears that it is a sextet of QL-cubics.
They can be constructed in a similar way as QL-Cu1.

Construction:
Let L1, L2, L3, L4 be the 4 defining lines of a Reference
Quadrilateral.
Define a pair of opposite QL-line segments = the line segments
on Li, Lj intersected by Lk,Ll, where (i,j,k,l) are elements of
(1,2,3,4).
There are 6 pairs of opposite QL-line segments in a
Quadrilateral (2 pairs per QL-Quadrigon).
Erect isosceles triangles at opposite pair of QL-line segments
with same top-angle.
Define V-circle = circle centered at the toppoint of this
triangle and going through the endpoints of the QL-line segment.
Construct a V-circle-1 at one QL-line segment.
Construct similar V-circles-3 and 4 at both sides of the
opposite QL-line segment.
The locus of V-circle-1 intersected with V-circles-3 and 4 will
produce two different cubics. One cubic will be QL-Cu1.
The other one is one version of a sextet of cubics QL-6Cu1a.
All 6 cubics are making equal/supplementary angles with the
opposite QL-line segments.
They all pass through 5 of 6 vertices of the Reference
Quadrilateral.
Through each vertex of the Reference Quadrilateral are passing 5
versions of QL-6Cu1.
They all pass through the Pluecker Pair of Points QL-2P1a/b.
See attachment.

Best regards,
Chris
```

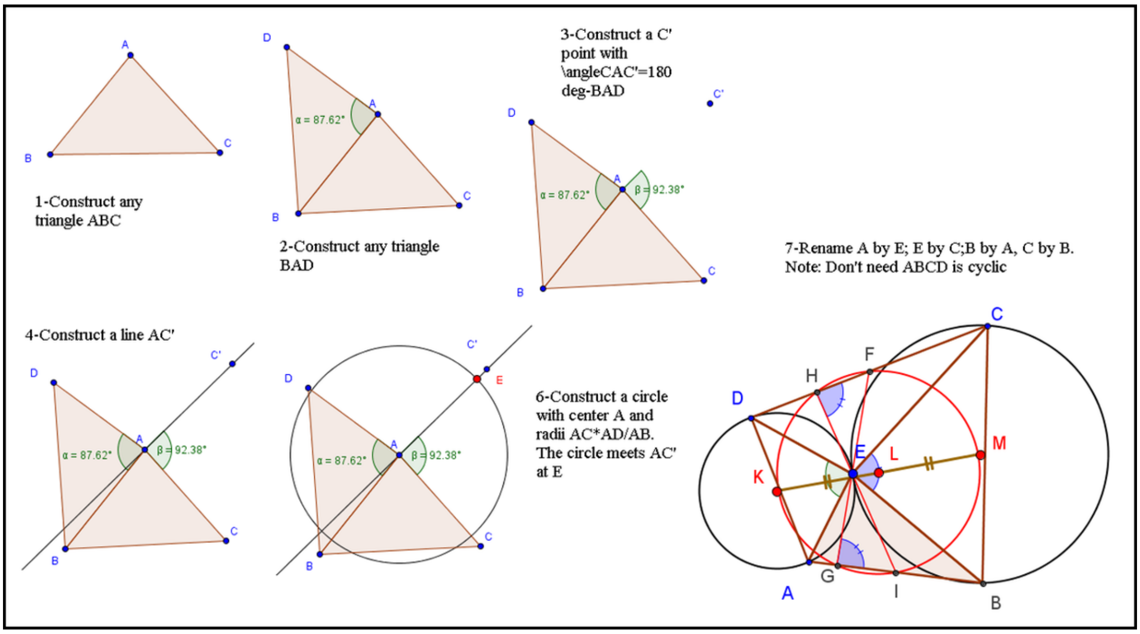


QL-Cu1-Quasi Isogonal Cubic-60-extraCubics.png

Message: #257
Date: 27/9/2013 3:32:22
From: Dao Thanh Oai
Subject: A generalization Brahmagupta's Theorem

```
Thank you Chris van Tienhoven very very very much!
You can construct converse for this problem.
   Step 1-Construct a triangle ABC
   Step 2-Construct a triangle ABD
   Step 3-Construct C' and \anleCAC'=180deg-\angleBAD
   Step 4-Construct a line through AC'
   Step 5-Construct a circle with center A and radii AD*AC/AB
   Step 6-The circle meet AC' at E
   Step 7-Rename A by E, B by A, C by B, E by C
Note ABCD don't need cyclic
You see it at:
http://data.webdien.com/photo/up/873493f895fd58900b3698809df41a4d.bmp
or Figure attachment.

Best regard
Dao Thanh Oai
```



Converse construct A generalization Brahmagupta's Theorem-bmp.png

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← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #258
Date: 27/9/2013 4:29:05
From: eckart_schmidt@t-online.de
Subject: A generalisation Brahmagupta's Theorem

```
Dear Dao Thanh Oai, dear Chris,
there is synthetical proof for the first property in the
attachment.
Best regards Eckart
```

A generalization Brahmagupta's Theorem

Quadri-Figures Group message #241 (22.09.2013)

Dear Chris van Tienhoven

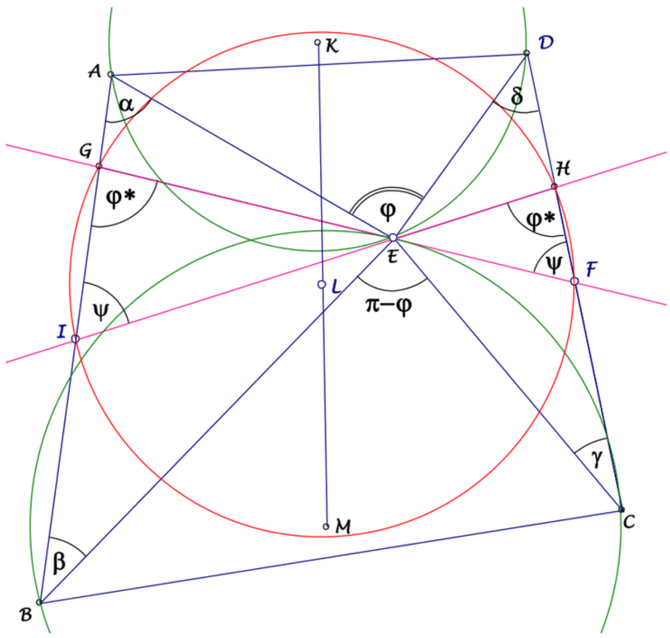
*Let a quadrilateral ABCD, let E point and angleDEA+angleCEB=180 deg and DE*EB=EC*EA. F is midpoint of DC. EF meet AB at G. F, I are midpoints of CD,AB respectively. K,L,M are circumcircle of (EDA),(FIG),(ECB)*
Prove that
1-angleDEA=angleAGE
2-L is midpoints of KM

I'm not mathematicians. I want you publish it.

Best regard

Dao Thanh Oai

A synthetical prove of the first property:



Brahmagupta.pdf

1. $EA \cdot EC = EB \cdot ED \quad \Leftrightarrow \quad \sin\alpha \cdot \sin\gamma = \sin\beta \cdot \sin\delta$ (sine rule)
2. $\varphi + \varphi^* = \alpha + \delta + \psi = 180° - \beta - \gamma + \psi$ (angle balance for *AGFD* and *BGFC*)
3. $\frac{\sin(\alpha+\psi)}{\sin\alpha} = \frac{AI}{EI}$ and $\frac{\sin(\psi-\beta)}{\sin\beta} = \frac{BI}{EI}$ (sine rule)
4. … give with $AI = BI$ $\frac{\sin(\alpha+\psi)}{\sin(\psi-\beta)} = \frac{\sin\alpha}{\sin\beta} = \frac{\sin\delta}{\sin\gamma}$
5. Using 2.: $\frac{\sin(\varphi+\varphi^*-\delta)}{-\sin(\varphi+\varphi^*+\gamma)} = \frac{\sin\delta}{\sin\gamma}$

   … gives $\sin(\gamma+\delta) \cdot \sin(\varphi+\varphi^*) = 0$
6. For $\sin(\gamma+\delta) \neq 0$ the sum $\varphi+\varphi^*$ must be 180°.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Brahmagupta.pdf

Message: #259
Date: 27/9/2013 7:08:11
From: Chris van Tienhoven
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear Eckart,

Beautiful construction of a tangent to QL-Cu1!
I used your construction for a construction of the tangential of
two conjugated points at QL-Cu1 without using conics.
So you don’t have to construct the 4 th intersection point of
two conics.
The tangential of a point at QL-Cu1 is the intersection point of
the tangent of this point and the tangent of its conjugate at
QL-Cu1.
This point also lies on QL-Cu1.
Construction Tangential Q of a point P on QL-Cu1
Let P* be the Clawson-Schmidt Conjugate of P.
Let Sij be a random intersection point of Li ^ Lj (i,j are
different numbers from set (1,2,3,4))
Let Sik* be the Isogonal Conjugate of Sik wrt triangle Sij.P.P*.
Let Sil* be the Isogonal Conjugate of Sil wrt triangle Sij.P.P*.
Let Sjk* be the Isogonal Conjugate of Sjk wrt triangle Sij.P.P*.
Let Sjl* be the Isogonal Conjugate of Sjl wrt triangle Sij.P.P*.
Now Q = the isogonal Conjugate of intersection point Sik*.Sil* ^
Sjk*.Sjl* wrt triangle Sij.P.P*.
 Q  is the Tangential of P as well as P*.
PQ  is the tangent in P at QL-Cu1.
P*Q is the tangent in P* at QL-Cu1.

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #260
Date: 28/9/2013 6:17:11
From: Chris
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear friends,

Here is a simple method to construct the tangents at the 6
vertices Sij to the cubic QL-Cu1.
The tangents at Sij to QL-Cu1 are the reflection of the diagonal
Sij.Skl in the (internal or external) bisector of lines Li,Lj.
i,j are different numbers from the set (1,2,3,4)

Best regards,
Chris
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #261
Date: 28/9/2013 6:44:33
From: Chris
Subject: A generalization Brahmagupta's Theorem

```
Dear Dao, dear friends,

Thanks for the converse construction method.
This made it possible to me to verify that the SuperConjugate of
point E has exactly the same properties as E itself:
* supplemetary angles with QL-line segmants
* EA*EC = EB*ED
* K,L,M are collinear

SuperConjugate =
* Clawson-Schmidt Conjugate, as well as
* Isogonal Conjugate wrt the Orthic Triangle of the QL-Diagonal
Triangle of ABCD, as well as
* Isogonal Conjugate wrt any Component Triangle of the
Quadrilateral defined by ABCD

It would be wonderful to find a simple construction of these two
special points from basic points A,B,C&D and to find their
coordinates.
Like you observed ABCD doesn't have to be cyclic at all.
It looks like this is a special property at the level of a
Quadrigon.
See:
http://www.chrisvantienhoven.nl/index.php/quadrigon-objects.html

Best regards,
Chris
```

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #262
Date: 28/9/2013 4:40:47
From: eckart_schmidt@t-online.de
Subject: A generalization Brahmagupta's Theorem

```
Dear Chris,
for a quadrigon the two "SuperConjugate" points E and E' on
QL-L1 lie symmetric to QA-P1, that means E and E' are the foci
of an inscribed conic with center in the centroid of the
quadrigon.
Let the lines E.QL-P1 and E'.QL-P1 cut the cubic in further
points F and F', and the lines E.E' and F.F' cut the cubic in
E'' and F'', then then E, E'', F', F'' and E', E'', F, F'' lie
concyclic .
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #263
Date: 29/9/2013 11:01:31
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear Chris,

...  & QL-Cu1 revisited:
I think, there is a further property for QL-Cu1:
QL-Cu1 is the locus of points P,
for which the second intersection of the circles through P,
L1^L2, L3^L4 and P, L2^L3, L4^L1 are Clawson-Schmidt Conjugates.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #264
Date: 29/9/2013 11:14:20
From: Chris
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear Eckart and friends,

I think I will have to rewrite the EQF-page of QL-Cu1.
There are so many additions!
There remains this open question:
What are the coordinates of the inflexion points?
Bernard G, do you got a solution for this?

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #265
Date: 29/9/2013 11:21:26
From: Bernard Gibert
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear Chris,

>> There remains this open question:
>> What are the coordinates of the inflexion points?
>> Bernard G, do you got a solution for this?

when the class is 6 (non singular cubic) there are 9 inflexion
points and 3 are real, hence no chance to find them!

Best regards
Bernard

PS: if you rewrite the QL-Cu1 page, it would be good to speak of
focus and its polar conic...
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #266
Date: 30/9/2013 10:19:57
From: Chris
Subject: A generalization Brahmagupta's Theorem

Dear Dao, Eckart and friends,

Thanks Eckart.
This actually defines the Brahmagupta's points E and E' in a Quadrigon (4-gon).
They can be constructed now like this:
1. Given 4 consecutive points P1,P2,P3,P4 connected by 4 lines L12, L23, L34, L41.
2. Construct the Newton Line (QL-L1) of the Quadrilateral defined by L12,L23,L34,L41.
3. Construct the Quadrangle Centroid (QA-P1) of the Quadrangle defined by P1, P2, P3, P4. This point lies on the Newton Line,
4. Construct the inscribed Conic within the the Quadrilateral defined by lines L12,L23,L34,L41.
5. Construct the foci of this conic. They are Brahmagupta's points E and E'.

Regarding 4., this an easy way to construct an inscribed conic in a Quadrilateral knowing its Center (per definition on the Newton Line).
1. Determine Centerpoint Ce on the Newton Line of Quadrilateral L1.L2.L3.L4.
2. Let Ca be the Anticomplement of Ce wrt some QL-Component Triangle L1.L2.L3.
3. Let Pe be the Isotomic Conjugate of Ca wrt QL-Component Triangle L1.L2.L3.
Pe is the Perspector of the inconic of QL-Component Triangle L1.L2.L3.
4. The traces of Pe on L1.L2.L3 lie on the inscribed conic we are looking for.
5. Also the Reflections of the traces in Ce lie on the conic we are looking for.
6. With these 6 points we can construct the conic.

The question stays open: what are the coordinates of the Brahmagupta's points?
I already had a try, but it is not easy to calculate the foci of a given conic.
Anyone can help?
Best regards,
Chris

Message: #267
Date: 30/9/2013 1:36:22
From: eckart_schmidt@t-online.de
Subject: A generalization Brahmagupta's Theorem

```
Dear Chris,
The coordinates of the points E are horrible (see attachment).
Best regards Eckart
```

A generalization Brahmagupta's Theorem

CT - Coordinates for points E

Let

$$z := 1$$

and

$$y := -\big(a^2 l n + a^2 m n - 2 a^2 n^2 + 2 a^2 l m x - c^2 l m x - a^2 l n x + c^2 l n x - a^2 m n x + 2 c^2 m n x - 2 c^2 n^2 x +$$
$$c^2 l m x^2 - c^2 l n x^2\big) / \big(2 a^2 l m - a^2 l n - a^2 m n - c^2 l m x + c^2 l n x + 2 c^2 m n x - 2 c^2 n^2 x\big)$$

and x one of the following values

$$\{\{x :=$$
$$-\big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2$$
$$n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 -$$
$$2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big) /$$
$$\big(4 c^2 l\,(l + m - 2 n)\,(m - n)\,n\big) - \frac{1}{2}\sqrt{}\Big(\frac{2 a^2}{c^2} + \big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n -$$
$$3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 +$$
$$7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 +$$
$$2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big) \wedge 2 / \big(4 c^4 l^2\,(l + m - 2 n)^2\,(m - n)^2\,n^2\big) -$$
$$\big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n +$$
$$10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 +$$
$$c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 +$$
$$a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 +$$
$$8 b^2 c^2 m n^3 - 8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\big) / \big(c^4 l\,(l + m - 2 n)\,(m - n)\,n\big)\Big) -$$
$$\frac{1}{2}\sqrt{}\Big(-\frac{2 a^2}{c^2} + \big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +$$
$$a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big) \wedge 2 / \big(2 c^4 l^2\,(l + m - 2 n)^2\,(m - n)^2\,n^2\big) -$$
$$\big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n +$$
$$10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 +$$
$$c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 +$$
$$a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 +$$
$$8 b^2 c^2 m n^3 - 8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\big) / \big(c^4 l\,(l + m - 2 n)\,(m - n)\,n\big) -$$
$$\Big(-\big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -$$
$$5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big) \wedge 3 / \big(c^6 l^3\,(l + m - 2 n)^3\,(m - n)^3\,n^3\big) -$$
$$\Big(8\,\big(2 a^4 l^2 m^2 + 2 a^2 b^2 l^2 m^2 - a^2 c^2 l^2 m^2 - a^4 l^2 m n - 3 a^2 b^2 l^2 m n + a^2 c^2 l^2 m n +$$

Chris-16-nb.pdf

```
a^4 l m^2 n - 5 a^2 b^2 l m^2 n + a^2 c^2 l m^2 n + a^2 b^2 l^2 n^2 - 5 a^4 l m n^2 + 7 a^2 b^2 l m n^2 +
a^2 c^2 l m n^2 - a^4 m^2 n^2 + 2 a^2 b^2 m^2 n^2 + 2 a^2 c^2 m^2 n^2 + 2 a^4 l n^3 - 2 a^2 b^2 l n^3 -
2 a^2 c^2 l n^3 + 2 a^4 m n^3 - 2 a^2 b^2 m n^3 - 6 a^2 c^2 m n^3 + 4 a^2 c^2 n^4)) /
(c^4 l (l + m - 2 n) (m - n) n) + (4 (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n -
3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 +
7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 -
2 c^2 l n^3 + 2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) (4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 -
3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n + 4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n +
2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n +
4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 -
2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 -
3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 -
8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4)) / (c^6 l^2 (l + m - 2 n)^2 (m - n)^2 n^2)) /
(4 Sqrt[(2 a^2 / c^2 + (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +
    a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
    a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -
    2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) ^2 / (4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2) -
  (4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +
    4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +
    4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -
    a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -
    8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -
    4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +
    4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4) / (c^4 l (l + m - 2 n) (m - n) n))])},
{x := - (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -
      5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 -
      5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 +
      2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 -
      2 c^2 l n^3 + 2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 +
      4 c^2 n^4) / (4 c^2 l (l + m - 2 n) (m - n) n) -
  1/2 Sqrt[(2 a^2 / c^2 + (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +
        a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
        a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -
        2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) ^2 / (4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2) -
      (4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +
        4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n +
        10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 +
        c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 +
        a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 +
        8 b^2 c^2 m n^3 - 8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4) / (c^4 l (l + m - 2 n) (m - n) n))] +
  1/2 Sqrt[(- 2 a^2 / c^2 + (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +
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a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -
2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) ^2 / (2 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2) -
(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +
 4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n +
 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 +
 c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 +
 a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 +
 8 b^2 c^2 m n^3 - 8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4) / (c^4 l (l + m - 2 n) (m - n) n) -
(-(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -
     5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
     a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -
     2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) ^3 / (c^6 l^3 (l + m - 2 n)^3 (m - n)^3 n^3) -
  (8 (2 a^4 l^2 m^2 + 2 a^2 b^2 l^2 m^2 - a^2 c^2 l^2 m^2 - a^4 l^2 m n - 3 a^2 b^2 l^2 m n + a^2 c^2 l^2 m n +
     a^4 l m^2 n - 5 a^2 b^2 l m^2 n + a^2 c^2 l m^2 n + a^2 b^2 l^2 n^2 - 5 a^4 l m n^2 + 7 a^2 b^2 l m n^2 +
     a^2 c^2 l m n^2 - a^4 m^2 n^2 + 2 a^2 b^2 m^2 n^2 + 2 a^2 c^2 m^2 n^2 + 2 a^4 l n^3 - 2 a^2 b^2 l n^3 -
     2 a^2 c^2 l n^3 + 2 a^4 m n^3 - 2 a^2 b^2 m n^3 - 6 a^2 c^2 m n^3 + 4 a^2 c^2 n^4)) /
   (c^4 l (l + m - 2 n) (m - n) n) + (4 (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n -
     3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 +
     7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 -
     2 c^2 l n^3 + 2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) (4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 -
     3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n + 4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n +
     2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n +
     4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 -
     2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 -
     3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 -
     8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4)) / (c^6 l^2 (l + m - 2 n)^2 (m - n)^2 n^2)) /
 (4 Sqrt[(2 a^2/c^2 + (2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +
        a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
        a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -
        2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4) ^2 / (4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2) -
      (4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +
        4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +
        4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -
        a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -
        8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -
        4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +
        4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4) / (c^4 l (l + m - 2 n) (m - n) n))]))},
{x := -(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -
     5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 -
     5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -
     a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 +
     2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 +
     2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 +
     4 c^2 n^4) / (4 c^2 l (l + m - 2 n) (m - n) n) +
```

$$\begin{aligned}
&\frac{1}{2}\sqrt{\Bigg(\frac{2\,a^2}{c^2}} + \big(2\,a^2\,l^2\,m^2 + 2\,b^2\,l^2\,m^2 - c^2\,l^2\,m^2 - a^2\,l^2\,m\,n - 3\,b^2\,l^2\,m\,n + c^2\,l^2\,m\,n + \\
&\quad a^2\,l\,m^2\,n - 5\,b^2\,l\,m^2\,n + c^2\,l\,m^2\,n + b^2\,l^2\,n^2 - 5\,a^2\,l\,m\,n^2 + 7\,b^2\,l\,m\,n^2 + c^2\,l\,m\,n^2 - \\
&\quad a^2\,m^2\,n^2 + 2\,b^2\,m^2\,n^2 + 2\,c^2\,m^2\,n^2 + 2\,a^2\,l\,n^3 - 2\,b^2\,l\,n^3 - 2\,c^2\,l\,n^3 + 2\,a^2\,m\,n^3 - \\
&\quad 2\,b^2\,m\,n^3 - 6\,c^2\,m\,n^3 + 4\,c^2\,n^4\big)\,{}^\wedge 2 \big/ \left(4\,c^4\,l^2\,(l+m-2\,n)^2\,(m-n)^2\,n^2\right) - \\
&\big(4\,a^4\,l^2\,m^2 - 4\,a^2\,b^2\,l^2\,m^2 - 3\,a^2\,c^2\,l^2\,m^2 - b^2\,c^2\,l^2\,m^2 + c^4\,l^2\,m^2 - 4\,a^4\,l^2\,m\,n + \\
&\quad 4\,a^2\,b^2\,l^2\,m\,n + 4\,a^2\,c^2\,l^2\,m\,n + 2\,b^2\,c^2\,l^2\,m\,n - 2\,c^4\,l^2\,m\,n - 4\,a^4\,l\,m^2\,n + \\
&\quad 4\,a^2\,b^2\,l\,m^2\,n + 10\,a^2\,c^2\,l\,m^2\,n + 4\,b^2\,c^2\,l\,m^2\,n - 4\,c^4\,l\,m^2\,n + a^4\,l^2\,n^2 - \\
&\quad a^2\,b^2\,l^2\,n^2 - b^2\,c^2\,l^2\,n^2 + c^4\,l^2\,n^2 + 2\,a^4\,l\,m\,n^2 - 2\,a^2\,b^2\,l\,m\,n^2 - 12\,a^2\,c^2\,l\,m\,n^2 - \\
&\quad 8\,b^2\,c^2\,l\,m\,n^2 + 8\,c^4\,l\,m\,n^2 + a^4\,m^2\,n^2 - a^2\,b^2\,m^2\,n^2 - 3\,a^2\,c^2\,m^2\,n^2 - \\
&\quad 4\,b^2\,c^2\,m^2\,n^2 + 4\,c^4\,m^2\,n^2 + 4\,b^2\,c^2\,l\,n^3 - 4\,c^4\,l\,n^3 + 8\,b^2\,c^2\,m\,n^3 - 8\,c^4\,m\,n^3 + \\
&\quad 4\,a^2\,c^2\,n^4 - 4\,b^2\,c^2\,n^4 + 4\,c^4\,n^4\big) \big/ \left(c^4\,l\,(l+m-2\,n)\,(m-n)\,n\right)\Bigg) - \\
&\frac{1}{2}\sqrt{\Bigg(-\frac{2\,a^2}{c^2}} + \big(2\,a^2\,l^2\,m^2 + 2\,b^2\,l^2\,m^2 - c^2\,l^2\,m^2 - a^2\,l^2\,m\,n - 3\,b^2\,l^2\,m\,n + c^2\,l^2\,m\,n + \\
&\quad a^2\,l\,m^2\,n - 5\,b^2\,l\,m^2\,n + c^2\,l\,m^2\,n + b^2\,l^2\,n^2 - 5\,a^2\,l\,m\,n^2 + 7\,b^2\,l\,m\,n^2 + c^2\,l\,m\,n^2 - \\
&\quad a^2\,m^2\,n^2 + 2\,b^2\,m^2\,n^2 + 2\,c^2\,m^2\,n^2 + 2\,a^2\,l\,n^3 - 2\,b^2\,l\,n^3 - 2\,c^2\,l\,n^3 + 2\,a^2\,m\,n^3 - \\
&\quad 2\,b^2\,m\,n^3 - 6\,c^2\,m\,n^3 + 4\,c^2\,n^4\big)\,{}^\wedge 2 \big/ \left(2\,c^4\,l^2\,(l+m-2\,n)^2\,(m-n)^2\,n^2\right) - \\
&\big(4\,a^4\,l^2\,m^2 - 4\,a^2\,b^2\,l^2\,m^2 - 3\,a^2\,c^2\,l^2\,m^2 - b^2\,c^2\,l^2\,m^2 + c^4\,l^2\,m^2 - 4\,a^4\,l^2\,m\,n + \\
&\quad 4\,a^2\,b^2\,l^2\,m\,n + 4\,a^2\,c^2\,l^2\,m\,n + 2\,b^2\,c^2\,l^2\,m\,n - 2\,c^4\,l^2\,m\,n - 4\,a^4\,l\,m^2\,n + 4\,a^2\,b^2\,l\,m^2\,n + \\
&\quad 10\,a^2\,c^2\,l\,m^2\,n + 4\,b^2\,c^2\,l\,m^2\,n - 4\,c^4\,l\,m^2\,n + a^4\,l^2\,n^2 - a^2\,b^2\,l^2\,n^2 - b^2\,c^2\,l^2\,n^2 + \\
&\quad c^4\,l^2\,n^2 + 2\,a^4\,l\,m\,n^2 - 2\,a^2\,b^2\,l\,m\,n^2 - 12\,a^2\,c^2\,l\,m\,n^2 - 8\,b^2\,c^2\,l\,m\,n^2 + 8\,c^4\,l\,m\,n^2 + \\
&\quad a^4\,m^2\,n^2 - a^2\,b^2\,m^2\,n^2 - 3\,a^2\,c^2\,m^2\,n^2 - 4\,b^2\,c^2\,m^2\,n^2 + 4\,c^4\,m^2\,n^2 + 4\,b^2\,c^2\,l\,n^3 - 4\,c^4\,l\,n^3 + \\
&\quad 8\,b^2\,c^2\,m\,n^3 - 8\,c^4\,m\,n^3 + 4\,a^2\,c^2\,n^4 - 4\,b^2\,c^2\,n^4 + 4\,c^4\,n^4\big) \big/ \left(c^4\,l\,(l+m-2\,n)\,(m-n)\,n\right) + \\
&\Big(-\big(2\,a^2\,l^2\,m^2 + 2\,b^2\,l^2\,m^2 - c^2\,l^2\,m^2 - a^2\,l^2\,m\,n - 3\,b^2\,l^2\,m\,n + c^2\,l^2\,m\,n + a^2\,l\,m^2\,n - \\
&\qquad 5\,b^2\,l\,m^2\,n + c^2\,l\,m^2\,n + b^2\,l^2\,n^2 - 5\,a^2\,l\,m\,n^2 + 7\,b^2\,l\,m\,n^2 + c^2\,l\,m\,n^2 - \\
&\qquad a^2\,m^2\,n^2 + 2\,b^2\,m^2\,n^2 + 2\,c^2\,m^2\,n^2 + 2\,a^2\,l\,n^3 - 2\,b^2\,l\,n^3 - 2\,c^2\,l\,n^3 + 2\,a^2\,m\,n^3 - \\
&\qquad 2\,b^2\,m\,n^3 - 6\,c^2\,m\,n^3 + 4\,c^2\,n^4\big)\,{}^\wedge 3 \big/ \left(c^6\,l^3\,(l+m-2\,n)^3\,(m-n)^3\,n^3\right) - \\
&\quad \Big(8\,\big(2\,a^4\,l^2\,m^2 + 2\,a^2\,b^2\,l^2\,m^2 - a^2\,c^2\,l^2\,m^2 - a^4\,l^2\,m\,n - 3\,a^2\,b^2\,l^2\,m\,n + a^2\,c^2\,l^2\,m\,n + \\
&\qquad a^4\,l\,m^2\,n - 5\,a^2\,b^2\,l\,m^2\,n + a^2\,c^2\,l\,m^2\,n + a^2\,b^2\,l^2\,n^2 - 5\,a^4\,l\,m\,n^2 + 7\,a^2\,b^2\,l\,m\,n^2 + \\
&\qquad a^2\,c^2\,l\,m\,n^2 - a^4\,m^2\,n^2 + 2\,a^2\,b^2\,m^2\,n^2 + 2\,a^2\,c^2\,m^2\,n^2 + 2\,a^4\,l\,n^3 - 2\,a^2\,b^2\,l\,n^3 - \\
&\qquad 2\,a^2\,c^2\,l\,n^3 + 2\,a^4\,m\,n^3 - 2\,a^2\,b^2\,m\,n^3 - 6\,a^2\,c^2\,m\,n^3 + 4\,a^2\,c^2\,n^4\big)\Big) \big/ \\
&\quad\ \left(c^4\,l\,(l+m-2\,n)\,(m-n)\,n\right) + \Big(4\,\big(2\,a^2\,l^2\,m^2 + 2\,b^2\,l^2\,m^2 - c^2\,l^2\,m^2 - a^2\,l^2\,m\,n - \\
&\qquad 3\,b^2\,l^2\,m\,n + c^2\,l^2\,m\,n + a^2\,l\,m^2\,n - 5\,b^2\,l\,m^2\,n + c^2\,l\,m^2\,n + b^2\,l^2\,n^2 - 5\,a^2\,l\,m\,n^2 + \\
&\qquad 7\,b^2\,l\,m\,n^2 + c^2\,l\,m\,n^2 - a^2\,m^2\,n^2 + 2\,b^2\,m^2\,n^2 + 2\,c^2\,m^2\,n^2 + 2\,a^2\,l\,n^3 - 2\,b^2\,l\,n^3 - \\
&\qquad 2\,c^2\,l\,n^3 + 2\,a^2\,m\,n^3 - 2\,b^2\,m\,n^3 - 6\,c^2\,m\,n^3 + 4\,c^2\,n^4\big)\,\big(4\,a^4\,l^2\,m^2 - 4\,a^2\,b^2\,l^2\,m^2 - \\
&\qquad 3\,a^2\,c^2\,l^2\,m^2 - b^2\,c^2\,l^2\,m^2 + c^4\,l^2\,m^2 - 4\,a^4\,l^2\,m\,n + 4\,a^2\,b^2\,l^2\,m\,n + 4\,a^2\,c^2\,l^2\,m\,n + \\
&\qquad 2\,b^2\,c^2\,l^2\,m\,n - 2\,c^4\,l^2\,m\,n - 4\,a^4\,l\,m^2\,n + 4\,a^2\,b^2\,l\,m^2\,n + 10\,a^2\,c^2\,l\,m^2\,n + \\
&\qquad 4\,b^2\,c^2\,l\,m^2\,n - 4\,c^4\,l\,m^2\,n + a^4\,l^2\,n^2 - a^2\,b^2\,l^2\,n^2 - b^2\,c^2\,l^2\,n^2 + c^4\,l^2\,n^2 + 2\,a^4\,l\,m\,n^2 - \\
&\qquad 2\,a^2\,b^2\,l\,m\,n^2 - 12\,a^2\,c^2\,l\,m\,n^2 - 8\,b^2\,c^2\,l\,m\,n^2 + 8\,c^4\,l\,m\,n^2 + a^4\,m^2\,n^2 - a^2\,b^2\,m^2\,n^2 - \\
&\qquad 3\,a^2\,c^2\,m^2\,n^2 - 4\,b^2\,c^2\,m^2\,n^2 + 4\,c^4\,m^2\,n^2 + 4\,b^2\,c^2\,l\,n^3 - 4\,c^4\,l\,n^3 + 8\,b^2\,c^2\,m\,n^3 - \\
&\qquad 8\,c^4\,m\,n^3 + 4\,a^2\,c^2\,n^4 - 4\,b^2\,c^2\,n^4 + 4\,c^4\,n^4\big)\Big) \big/ \left(c^6\,l^2\,(l+m-2\,n)^2\,(m-n)^2\,n^2\right)\Big) \Big/ \\
&\quad \Bigg(4\sqrt{\Bigg(\frac{2\,a^2}{c^2}} + \big(2\,a^2\,l^2\,m^2 + 2\,b^2\,l^2\,m^2 - c^2\,l^2\,m^2 - a^2\,l^2\,m\,n - 3\,b^2\,l^2\,m\,n + c^2\,l^2\,m\,n + \\
&\qquad\quad a^2\,l\,m^2\,n - 5\,b^2\,l\,m^2\,n + c^2\,l\,m^2\,n + b^2\,l^2\,n^2 - 5\,a^2\,l\,m\,n^2 + 7\,b^2\,l\,m\,n^2 + c^2\,l\,m\,n^2 - \\
&\qquad\quad a^2\,m^2\,n^2 + 2\,b^2\,m^2\,n^2 + 2\,c^2\,m^2\,n^2 + 2\,a^2\,l\,n^3 - 2\,b^2\,l\,n^3 - 2\,c^2\,l\,n^3 + 2\,a^2\,m\,n^3 -
\end{aligned}$$

$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\Big)\,\hat{}\,2 \Big/ \Big(4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2\Big) -$$
$$\Big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +$$
$$4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -$$
$$a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -$$
$$8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -$$
$$4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +$$
$$4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\Big) \Big/ \Big(c^4 l (l + m - 2 n) (m - n) n\Big)\Big)\Big)\Big)\Big\},$$

$$\Big\{x := -\Big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -$$
$$5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 -$$
$$5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 +$$
$$2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 +$$
$$2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 +$$
$$4 c^2 n^4\Big) \Big/ \Big(4 c^2 l (l + m - 2 n) (m - n) n\Big) +$$

$$\frac{1}{2}\sqrt{\vphantom{X}}\Big(\frac{2 a^2}{c^2} + \Big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +$$
$$a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\Big)\,\hat{}\,2 \Big/ \Big(4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2\Big) -$$
$$\Big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +$$
$$4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -$$
$$a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -$$
$$8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -$$
$$4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +$$
$$4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\Big) \Big/ \Big(c^4 l (l + m - 2 n) (m - n) n\Big)\Big) +$$

$$\frac{1}{2}\sqrt{\vphantom{X}}\Big(-\frac{2 a^2}{c^2} + \Big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +$$
$$a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\Big)\,\hat{}\,2 \Big/ \Big(2 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2\Big) -$$
$$\Big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +$$
$$4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -$$
$$a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -$$
$$8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -$$
$$4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +$$
$$4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\Big) \Big/ \Big(c^4 l (l + m - 2 n) (m - n) n\Big) +$$
$$\Big(-\Big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n -$$
$$5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\Big)\,\hat{}\,3 \Big/ \Big(c^6 l^3 (l + m - 2 n)^3 (m - n)^3 n^3\Big) -$$
$$\Big(8 \Big(2 a^4 l^2 m^2 + 2 a^2 b^2 l^2 m^2 - a^2 c^2 l^2 m^2 - a^4 l^2 m n - 3 a^2 b^2 l^2 m n + a^2 c^2 l^2 m n +$$
$$a^4 l m^2 n - 5 a^2 b^2 l m^2 n + a^2 c^2 l m^2 n + a^2 b^2 l^2 n^2 - 5 a^4 l m n^2 + 7 a^2 b^2 l m n^2 +$$

$$a^2 c^2 l m n^2 - a^4 m^2 n^2 + 2 a^2 b^2 m^2 n^2 + 2 a^2 c^2 m^2 n^2 + 2 a^4 l n^3 - 2 a^2 b^2 l n^3 -$$
$$2 a^2 c^2 l n^3 + 2 a^4 m n^3 - 2 a^2 b^2 m n^3 - 6 a^2 c^2 m n^3 + 4 a^2 c^2 n^4\big)\big) \Big/$$
$$\left(c^4 l (l + m - 2 n) (m - n) n\right) + \Big(4 \big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n -$$
$$3 b^2 l^2 m n + c^2 l^2 m n + a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 +$$
$$7 b^2 l m n^2 + c^2 l m n^2 - a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 -$$
$$2 c^2 l n^3 + 2 a^2 m n^3 - 2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big) \big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 -$$
$$3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n + 4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n +$$
$$2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n + 4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n +$$
$$4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 - a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 -$$
$$2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 - 8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 -$$
$$3 a^2 c^2 m^2 n^2 - 4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 -$$
$$8 c^4 m n^3 + 4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\big)\Big) \Big/ \left(c^6 l^2 (l + m - 2 n)^2 (m - n)^2 n^2\right)\Big) \Big/$$
$$\Bigg(4 \sqrt{\Bigg(\frac{2 a^2}{c^2} + \big(2 a^2 l^2 m^2 + 2 b^2 l^2 m^2 - c^2 l^2 m^2 - a^2 l^2 m n - 3 b^2 l^2 m n + c^2 l^2 m n +}$$
$$a^2 l m^2 n - 5 b^2 l m^2 n + c^2 l m^2 n + b^2 l^2 n^2 - 5 a^2 l m n^2 + 7 b^2 l m n^2 + c^2 l m n^2 -$$
$$a^2 m^2 n^2 + 2 b^2 m^2 n^2 + 2 c^2 m^2 n^2 + 2 a^2 l n^3 - 2 b^2 l n^3 - 2 c^2 l n^3 + 2 a^2 m n^3 -$$
$$2 b^2 m n^3 - 6 c^2 m n^3 + 4 c^2 n^4\big)\hat{}2 \Big/ \left(4 c^4 l^2 (l + m - 2 n)^2 (m - n)^2 n^2\right) -$$
$$\big(4 a^4 l^2 m^2 - 4 a^2 b^2 l^2 m^2 - 3 a^2 c^2 l^2 m^2 - b^2 c^2 l^2 m^2 + c^4 l^2 m^2 - 4 a^4 l^2 m n +$$
$$4 a^2 b^2 l^2 m n + 4 a^2 c^2 l^2 m n + 2 b^2 c^2 l^2 m n - 2 c^4 l^2 m n - 4 a^4 l m^2 n +$$
$$4 a^2 b^2 l m^2 n + 10 a^2 c^2 l m^2 n + 4 b^2 c^2 l m^2 n - 4 c^4 l m^2 n + a^4 l^2 n^2 -$$
$$a^2 b^2 l^2 n^2 - b^2 c^2 l^2 n^2 + c^4 l^2 n^2 + 2 a^4 l m n^2 - 2 a^2 b^2 l m n^2 - 12 a^2 c^2 l m n^2 -$$
$$8 b^2 c^2 l m n^2 + 8 c^4 l m n^2 + a^4 m^2 n^2 - a^2 b^2 m^2 n^2 - 3 a^2 c^2 m^2 n^2 -$$
$$4 b^2 c^2 m^2 n^2 + 4 c^4 m^2 n^2 + 4 b^2 c^2 l n^3 - 4 c^4 l n^3 + 8 b^2 c^2 m n^3 - 8 c^4 m n^3 +$$
$$4 a^2 c^2 n^4 - 4 b^2 c^2 n^4 + 4 c^4 n^4\big) \Big/ \left(c^4 l (l + m - 2 n) (m - n) n\right)\Bigg)\Bigg)\Bigg)\Big\}\Big\}$$

... then two points are real,

that are the searched E - points, two points are not real.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Chris-16-nb.pdf

Message: #268
Date: 30/9/2013 2:38:55
From: Dao Thanh Oai
Subject: A generalization Brahmagupta's Theorem

```
Dear Chris van Tienhoven an Eckart!

Thank to you very much! (I want chat many word but my english is
not good)
You can see reference: Ellipse with two foci tangent pedal
circle and tangent three side of a triangle at here: http://www.
artofproblemsolving.com/Forum/viewtopic.php?f=47&t=554936.
Maybe giving to you your idear about ellipse tangent fouf sides
quadrilateral.

Best regard
Dao Thanh Oai
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← Previous Next → ↔ Message Index ↑ Subjects

Message: #269
Date: 30/9/2013 3:02:46
From: Chris van Tienhoven
Subject: Precisions concerning the curve QL-Cu1

Dear Eckart,

To quote Shakespeare: "There is something special in Denmark" (statement a bit adapted by me).
QL-Cu1 has real brain teasing aspects.
You are right, the foci of the inscribed conic with center QL-L1^QL-L6 can be on QL-L1 as well as the perpendicular line wrt QL-L1 through QL-L1^QL-L6.

My analyses is as follows.
The foci of a conic –when it is an ellipse– always reside on the main axis.
The main axis of the inscribed conic with center QL-L1^QL-L6 obviously can coincide with QL-L1 as well as its perpendicular line through QL-L1^QL-L6 (depending on the shape of the Reference Quadrilateral).
That's why the described foci can reside on alternate lines.
I am wondering if the following is true: An ellipse has 4 foci, 2 of them real, 2 of them imaginary. Maybe others can confirm or correct this.
(just like 2 circles always have 4 intersection points, 2 of them real or imaginary and 2 of them always being the imaginary intersection points at infinity)
This would also explain why it is so difficult to define a simple method for constructing the foci of an ellipse unless you know which axis is the main axis.
When my conjecture is right, then the foci on QL-L1 as well as the foci on the perpendicular line through QL-L1^QL-L6 are both valid, although per pair alternately real and imaginary.
My conclusion is anyway that they are not identical points because they reside on different lines and have distinct coordinates.
That's why I am reluctant to give them a corporate name.
But thanks to your correct comment I will make a remark about the special character of QL-2P2 in EQF regarding being focus or not.
If you have another opinion please let me know. I am open minded.

Best regards,
Chris

Message: #270
Date: 30/9/2013 5:13:41
From: bernard.keizer
Subject: Tangential quadrilateral well known properties?

Dear Chris, dear Eckart, dear friends

May be you remember this problem with ABCD circumscribed quadrilateral, forming the tangential quadrilateral of a cyclic quadrilateral PQRS in circle center O, M, N, M' and N' middles of the sides of the first one.
We have to prove that MN cuts QS in a point K on a parallel through O to PR (and the same with M'N' cutting PR in a point K' on a parallel trough O to QS).
I first tried the idea of Eckart with the polar coordinates of P (cosP, sinP), but it's rather boring ...
The well known property asked by Chris is that the tangential QL and the cyclic QA have the same diagonal triangle EFG self polar wrt the circle (AC, BD, PR and QS cut in a point G and the inverse of G wrt the circle is the Miquel point of the inscribed QL). O is on the Newton Line of the tangential QL.
In fact, the property is true for a conic and is mentioned in EQF for the quadrangle formed by the 4 contact points of the inscribed parabola QL-Co1 tangent to 4 lines forming it's tangential QL.
So, OKGK' is a parallelogam ...
Despite all my efforts, I'm not able to find a synthetic proof of this property and I didn't find any reference ...
Could someone help me?

Best regards
Bernard Keizer

← Previous Next → ↔ Message Index ↑ Subjects

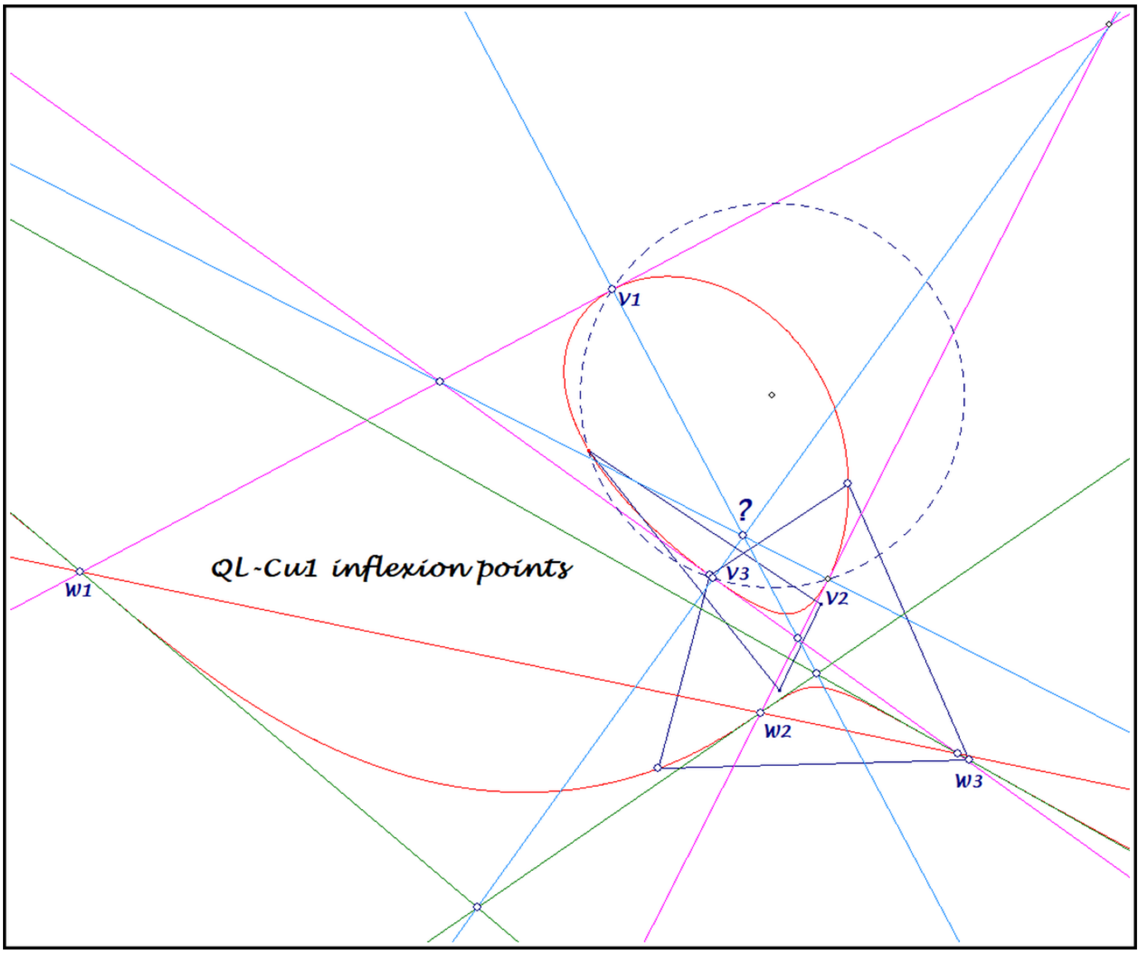
Message: #271
Date: 01/10/2013 9:02:53
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Chris,

I don't know, whether I have understood your last message. Even if it is possible, to consider 4 foci for a conic, there will be no common sight of the uni- and bipartite case of QL-Cu1. For example wrt the "strophoid"-construction: On the one hand we need circles centered on QL-L1 through the foci of the inscribed conic with center QL-L1 ^ QL-L6, on the other hand we need circles orthogonal to the Thales circle about these foci.
I would prefer to define QL-2P2a,b as the addressed foci, for QL-Cu1 is isogonal invariant wrt the triangle with the vertices QL-P1 and these foci (independent whether the foci are on QL-L1 or not).

Another point: Even if Bernard gives no chance in calculation, some remarks wrt the inflexion points of QL-Cu1 (see attachment):
... QL-Cu1 and QL-Cu2 have the same inflexion points
    (see Bernard's paper 8.1).
... The three inflexion points Wi lie collinear on a line L,
... The Clawson-Schmidt Conjugate (QL-Tr1) of L is a circle
    through QL-P1 and the isogonal conjugate of L (see above)
    is a conic through QL-P1. The three intersections Vi
    unequal QL-P1 are points on QL-Cu1.
... The tangentials of Vi wrt QL-Cu1 are their QL-Tr1 images,
    which are the inflexion points Wi. So the inflexion points
    are the tangentials of its QL-Tf1 image.
... The Simson line of QL-P1 wrt the V-triangle
    is perpendicular QL-L1.
... The normal at Vi
    contains the intersection of the tangents in Vj, Vk.
... The normal at Vi
    contains the intersection of the tangents in Wj, Wk.
... The normals at Vi, Vj, Vk have a common point.
What about this point???

Best regards
Eckart



13-09-29-e-fig.png

← Previous Next → ↩ Message Index ↑ Subjects

Message: #272
Date: 02/10/2013 10:27:08
From: eckart_schmidt@t-online.de
Subject: Tangential quadrilateral well known properties?

Dear Bernard,

there is even not a synthetic proof, but a
 Mathematica-calculation in the attachment wrt your problem for
an inscribed conic.

Best regards Eckart

Dear Bernard,

wrt your message "Re: Tangential quadrilateral well known properties?":

Dear Chris, dear Eckart, dear friends

May be you remember this problem with ABCD circumscribed quadrilateral, forming the tangential quadrilateral of a cyclic quadrilateral PQRS in circle center O,M,N,M' and N' middles of the sides of the first one.We have to prove that MN cuts QS in a point K on a parallel through O to PR (and the same with M'N' cutting PR in a point K' on a parallel trough O to QS).
I first tried the idea of Eckart with the polar coordinates of P (cosP,sinP), but it's rather boring...
...
In fact,the property is true for a conic and is mentionned in EQF for the quadrangle formed by the 4 contact points of the inscribed parabola QL-Co1 tangent to 4 lines forming it's tangential QL.
So, OKGK' is a parallelogam...
Despite all my efforts, I'm not able to find a synthetic proof of this property and I didn't find any reference...
Could someone help me?
Best regards
Bernard Keizer

... here is not a synthetic proof, but a calculation in CT-coordinates for an inscribed conic:

The quadrigon:

```
P1 := {1, 0, 0}; P2 := {0, 1, 0}; P3 := {0, -n, m}; P4 := {n, 0, -l}
```

Let Z be the center of an inscribed conic on the Newton line of a quadrigon with parameter κ

$$Z := \left\{\kappa, \frac{l m - l n - m n - (l m + l n - m n)\kappa}{l m - l n + m n}, 1\right\}$$

... then the Brianchon point T of the conic wrt the reference triangle is the isotomic conjugate of the anticomplement of Z:

$$T := \{m n (n + l\kappa) (-m - l\kappa + m\kappa), l n (m + l\kappa - m\kappa) (-m + n + m\kappa), -l m (n + l\kappa) (-m + n + m\kappa)\}$$

This gives the contact points

$$P := \{-m (n + l\kappa), l (-m + n + m\kappa), 0\}$$

$$S := \{n (-m - l\kappa + m\kappa), 0, -l (-m + n + m\kappa)\}$$

$$Q := \{0, -n (-m - l\kappa + m\kappa), -m (n + l\kappa)\}$$

Analogue you get the Brianchon point T' of the conic wrt the reference trian gle P2 P3 P1P2^P3P4

$$T' := \{m n (n + l\kappa) (-m + n + m\kappa), -l n \left(m^2 - m n + n^2 - \left(-l m + 2 m^2 - l n - m n\right)\kappa + \left(l^2 - l m + m^2\right)\kappa^2\right), l m (n + l\kappa) (m + l\kappa - m\kappa)\}$$

Keizer-01-nb.pdf

This gives the contact point

$R := \{-m\,n\,(-m+n+m\,\kappa),\ l\,n\,(n+l\,\kappa),\ -l\,m\,(m+l\,\kappa-m\,\kappa)\}$

The intersection K of QS and MN is

$$K := \{-n\,(-2\,l\,m\,(m-n)-(l-m)\,(l\,m-l\,n-m\,n)\,\kappa),\ -n\,(-l^2\,m-l\,m^2+l^2\,n+m^2\,n-(l-m)\,(l\,m-l\,n-m\,n)\,\kappa),\ -(2\,l\,m-l\,n-m\,n)\,(l\,m-l\,n+m\,n)\}$$

The points at infinity of PR and OK are the same

$\{2\,m\,n-2\,n^2+(l\,m-l\,n-m\,n)\,\kappa,\ l\,m-l\,n-m\,n+2\,n^2-(l\,m-l\,n-m\,n)\,\kappa,\ -l\,m+l\,n-m\,n\}$

So PR and OK are parallel.

Best regards Eckart

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← Previous Next → ↔ Message Index ↑ Subjects

Message: #273
Date: 03/10/2013 9:28:42
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear friends,

sorry there is something wrong in my message #263. Correct is:
For every point P in the QL-environment  the second
intersections of the circles through P, L1^L2, L3^L4 and P,
L2^L3, L4^L1 are Clawson-Schmidt Conjugates (QL-Tf1).
If P is a point on QL-Cu1 and P' its QL-Tf1 image and Q, Q' the
intersections of the circles, then these points on QL-Cu1 give a
quadrigon PQP'Q' with the same point QL-P1, the same line QL-L1
and the same cubic QL-Cu1 as the reference quadrilateral.

Correcting my message the following must hold:
QL-Cu1 is the locus of points P, for which the second
intersections of the circles through P, L1^L2, L3^L4 and P,
L2^L3, L4^L1 are
... isogonal conjugate wrt a QL triangle component,
... or isogonal conjugate wrt the triangle with the vertices
    QL-P1 and the foci of the inscribed conic
    with center QL-L1^QL-L6,
... or isogonal conjugate wrt the orthic triangle
    of QL Diagonal Triangle,
... or QG-Quasi Isogonal Conjugate (QG-Tf2)
    wrt a quadrigon component.

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #274
Date: 03/10/2013 10:42:14
From: eckart_schmidt@t-online.de
Subject: A generalisation Brahmagupta's Theorem

```
Dear Dao Thanh Oai, dear Chris,
I have completed my synthetical proof, see attachment.
Best regards Eckart
```

A generalization Brahmagupta's Theorem

Quadri-Figures Group message #241 (22.09.2013)

Dear Chris van Tienhoven

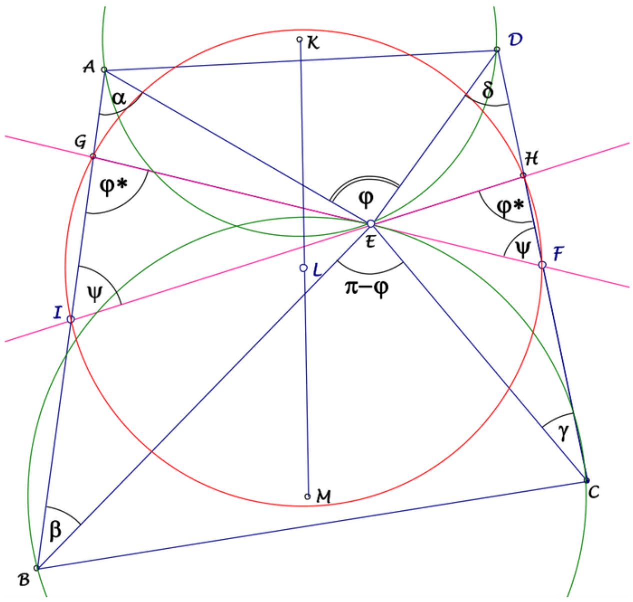
*Let a quadrilateral ABCD, let E point and angleDEA+angleCEB=180 deg and DE*EB=EC*EA. F is midpoint of DC. EF meet AB at G. F, I are midpoints of CD,AB respectively. K,L,M are circumcircle of (EDA),(FIG),(ECB)*
Prove that
1-angleDEA=angleAGE
2-L is midpoints of KM

I'm not mathematicians. I want you publish it.

Best regard

Dao Thanh Oai

A synthetical prove of the first property:

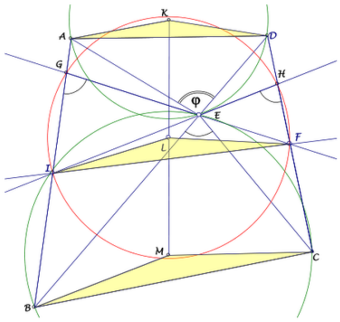


Brahmagupta-274.pdf

1. $EA \cdot EC = EB \cdot ED \quad \Leftrightarrow \quad \sin\alpha \cdot \sin\gamma = \sin\beta \cdot \sin\delta$ (sine rule)
2. $\varphi + \varphi^* = \alpha + \delta + \psi = 180° - \beta - \gamma + \psi$ (angle balance for *AGFD* and *BGFC*)
3. $\dfrac{\sin(\alpha+\psi)}{\sin\alpha} = \dfrac{AI}{EI}$ and $\dfrac{\sin(\psi-\beta)}{\sin\beta} = \dfrac{BI}{EI}$ (sine rule)
4. … give with $AI = BI$ $\dfrac{\sin(\alpha+\psi)}{\sin(\psi-\beta)} = \dfrac{\sin\alpha}{\sin\beta} = \dfrac{\sin\delta}{\sin\gamma}$
5. Using 2.: $\dfrac{\sin(\varphi+\varphi^*-\delta)}{-\sin(\varphi+\varphi^*+\gamma)} = \dfrac{\sin\delta}{\sin\gamma}$

   … gives $\sin(\gamma+\delta) \cdot \sin(\varphi+\varphi^*) = 0$
6. For $\sin(\gamma+\delta) \neq 0$ the sum $\varphi + \varphi^*$ must be 180°.

Prove of the second property, using the first property:



The isosceles triangles *ADK, IFL, BCM* are similar, for central angles are twice the peripheral angles.
The fact, that *I* is midpoint of *AB* and *F* is midpoint of *DC* implies of course for similar triangles *ADK, IFL, BCM*, that *L* is midpoint of *KM*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Brahmagupta-274.pdf

Message: #275
Date: 03/10/2013 11:32:20
From: bernard.keizer
Subject: Tangential quadrilateral well known properties?

```
Dear Eckart,

I spent hours to try to get a free version of Mathematica, but I
didn't succeed!
Can't you send me your file in pdf or Adobe Reader?
Did you also find that the circumcenter O was on the Newton Line
of the tangential quadrilateral and that KK' was orthogonal to
this Newton Line?

Best regards
Bernard
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #276
Date: 03/10/2013 12:54:33
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

Dear Chris, dear Eckart

My understanding of the van Rees focal circular cubic and the 2 points QL-2P2a,b is following:

* there are 2 ways of drawing the curve: the first one is to determinate the contact points with the tangencies from QL-P1 to a first set of circles and the second one is to determinate the extremities of the diameters through QL-P1 of a second set of circles

* these 2 sets are orthogonal and have the same 2 points QL-2P2a,b as reference points, basis points for one and Poncelet points for the other

* if the points are basis points for the first one, they are on the Newton Line, the centers of the circles of this set are on the perpendicular bisector of the segment joining the 2 points, the curve is unicursal and the Newton Line cuts the curve in these 2 points

* if the points are basis points for the second one, they are on a perpendicular to the Newton Line, the centers of the circles of this set are on the Newton Line (one of those circles through the 2 points is for example the circle with diameter the points QL-P1 and S), the curve is bicursal and the Newton Line doesn't cut the curve

Thanks to Eckart for the inflexion points, but I couldn't open this file either ...

Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

Message: #277
Date: 03/10/2013 6:16:42
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

```
Dear Eckart, dear Chris
It's a real pleasure to work this way with you!
Thanks to Eckart, I was able to see your figure.
If we call O the center of the circle through the Vi and QL-P1,
I the common point? of the normals in the Vi and Ki the
intersection points of the tangents in Vj and Vk, I is the
incenter and the Ki the excenters of the V-triangle and the line
of the Wi is the orthic axis, perpendicular to OI and the points
ViVjWk are on a line perpendicular to OKi.
The middles of the segments ViWi are on the Newton Line QL-L1,
which explains that the Simson- and Steiner-Line of QL-P1 wrt
the V-triangle is perpendicular to this line.
Last point: the circle through Vi, Wi and QL-P1 is tangent to
the curve QL-Cu1 in Wi
All this should bring new ideas about an easy way to construct
those points an lines for a given QL-Cu1 ...
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #278
Date: 03/10/2013 11:30:38
From: Chris
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear friends,
I made some typos in the use of indices i,j,k,l wrt the
construction of the Tangential at QL-Cu1.
Here is the right formulation:
Construction Tangential Q of a point P on QL-Cu1
Let P* be the CSC of P.
Let Sij be a random intersection point of Li ^ Lj (i,j are
different numbers from set (1,2,3,4))
Let Ski* be the Isogonal Conjugate of Ski wrt triangle Sij.P.P*.
Let Skj* be the Isogonal Conjugate of Skj wrt triangle Sij.P.P*.
Let Sli* be the Isogonal Conjugate of Sli wrt triangle Sij.P.P*.
Let Slj* be the Isogonal Conjugate of Slj wrt triangle Sij.P.P*.
Now Q = the isogonal Conjugate of intersection point Ski*.Skj* ^
Sli*.Slj* wrt triangle Sij.P.P*.
Q is the Tangential of P as well as P*.
PQ is tangent in P at QL-Cu1. P*Q is tangent in P* at QL-Cu1.
Best regards,
Chris
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #279
Date: 04/10/2013 9:29:44
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

```
Dear friends,
There is a light mistake in my message, for which I apologise.
Please read as following:
... and the points ViVjWk are on a line. Let Ti be the
intersection between ViKi and VjVk; then TjTk is perpendicular
to TjTk.
The rest seems to hold ...
For Eckart: did you try your drawing with a unicursal curve,
with Ql-P1 and Newton Line?
Best regards
Bernard
```

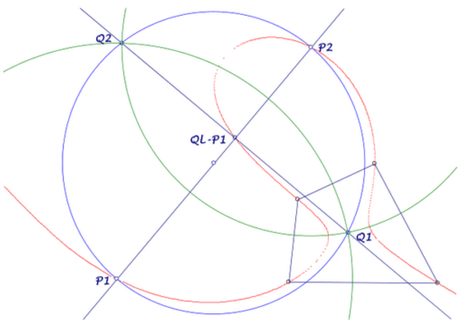
← Previous   Next →   ↔ Message Index   ↑ Subjects

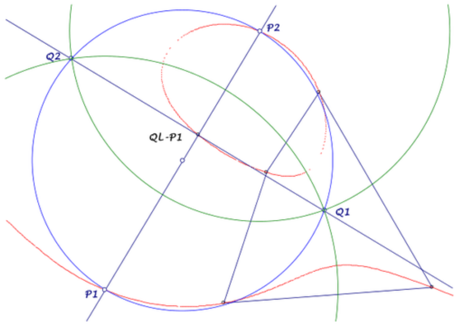
Message: #280
Date: 04/10/2013 12:12:14
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Bernard, dear Chris,
Bernard's understanding of QL-2P2a,b has the same background as my understanding. I hope, Chris can change the definition.
If Chris rewrites QL-Cu1, perhaps a remark, that QL-Cu1 is anallamatic (that means invariant under an inversion): see attachment.
Last not least: For a quadrilateral tangent to a circle QL-Cu1 is a strophoid and for a parallelogram the foci of inscribed conics lie on the QA-Orthogonal Hyperbola QA-Co2 (see http://eckartschmidt.de/Focus.pdf and http://eckartschmidt.de/Strtv.pdf))
Best regards Eckart

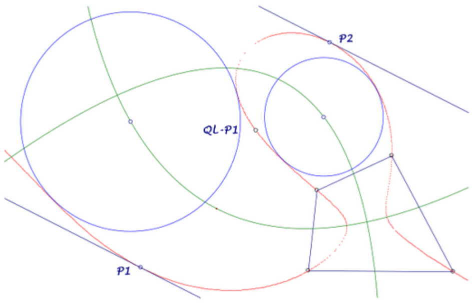
QL-Cu1 as anallagmatic cubic

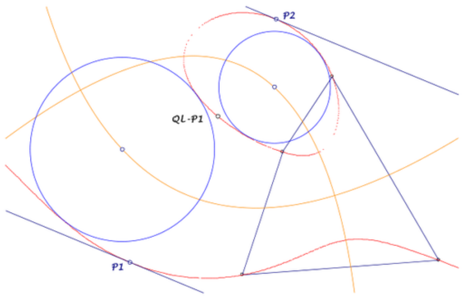
A curve is anallagmatic, if it is invariant under an inversion. For *QL-Cu1* there are two inversions:
Let Q_1 and Q_2 be the fixed points of *QL-Tf1* and P_1, P_2 the intersections of *QL-Cu1* with the perpendicular bisector of Q_1Q_2. The points P_1, P_2, Q_1, Q_2 lie concyclic with center on *QL-L1*. Then *QL-Cu1* is invariant under reflections in circles round P_1 or P_2 through Q_1, Q_2 (see http://eckartschmidt.de/Focus.pdf).





An allagmatic curve is also envelope of two sets of circles: The centers of these circles lie on two parabolas with focus *QL-P1* and directrix in the tangent at *P1* or *P2* parallel *QL-L1*.



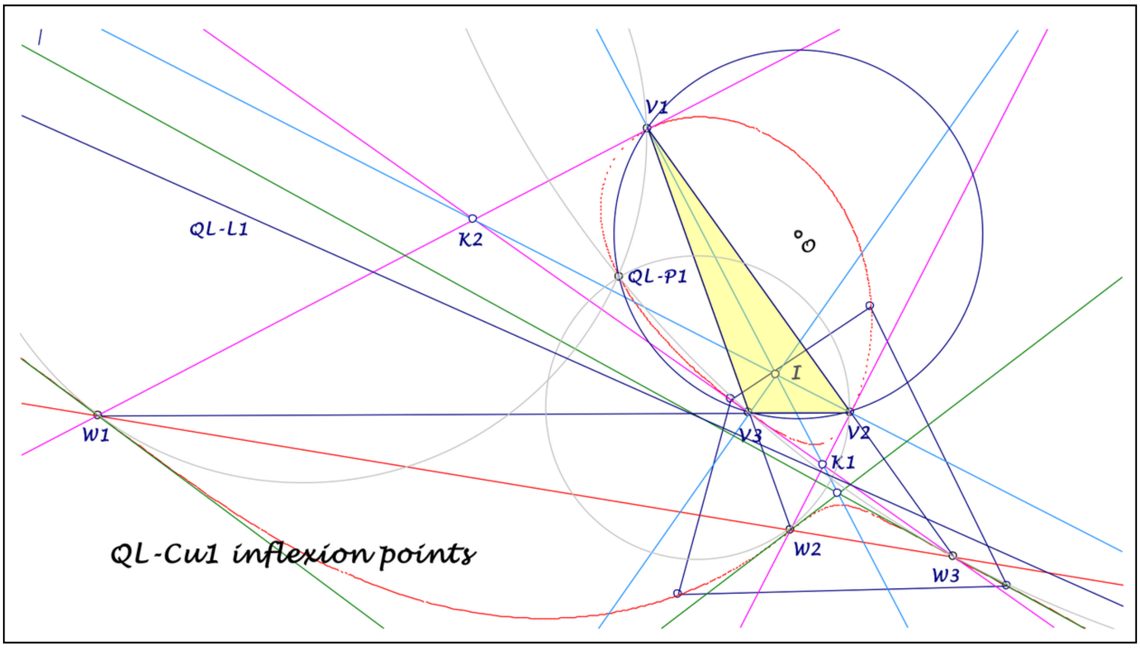


Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

QL-Cu1-an.pdf

Message: #281
Date: 04/10/2013 4:03:13
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

```
Dear Bernard,
whow, good additional properties for the QL-Cu1 inflexion points
constellation!
There are drawings for the unicursal and the bicursal case of
QL-Cu1 in pdf-format (see attachment).
Your last correction I haven't understood: "... then TjTk  is
perpendicular TjTk."
But I disbelieve, that there will be real construction. I think
Bernard Gibert will be right: "... no chance", for the inflexion
points of QL-Cu1 are also inflexion points of QL-Cu2!
Best regards Eckart
```



Keizer-06.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #282
Date: 04/10/2013 5:16:29
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

```
Dear Eckart,
Thanks for your 2 figures!
Sorry again, it was TjTk is perpendicular to OKi ...Best regards
Bernard
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #283
Date: 06/10/2013 10:55:55
From: eckart_schmidt@t-online.de
Subject: A pair of QG-Cubics

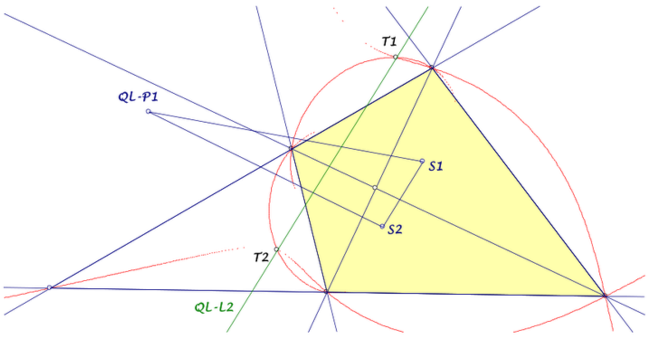
```
Dear friends,
the loci for points, which have the same angle of view (mod
180°) for the diagonals of a quadrigon, are two cubics (see
attachment).
Best regards Eckart
```

EQF-Note 2013-10-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Pair of QG-Cubics

The loci for points, which have the same angle of view (mod 180°) for the diagonals of a quadrigon, are two cubics. – Reference triangle for barycentric coordinates is QA-Diagonal Triangle QA-Tr1.



Constructing about and under the diagonals of a quadrigon the circles for the same peripheral angle, there are four points of intersection, which have as loci two cubics with the equations

$$2S_A y(r^2x^2 - p^2z^2) - 2S_B x(r^2y^2 - q^2z^2)$$
$$+(c^2q^2+b^2r^2)x^2z - (c^2p^2+a^2r^2)y^2z - (b^2p^2-a^2q^2)z^3 = 0,$$
$$2S_B y(q^2x^2 - p^2y^2) - 2S_C y(r^2x^2 - p^2z^2)$$
$$+(b^2p^2+a^2q^2)xz^2 - (c^2p^2+a^2r^2)xy^2 - (b^2r^2-c^2q^2)x^3 = 0.$$

The points at infinity are

$$(p^2-q^2+r^2 : -p^2+q^2+r^2 : -2r^2)$$
$$(-2p^2 : p^2+q^2-r^2 : p^2-q^2+r^2).$$

From the points of these cubics the diagonals of the quadrigon are seen under angles mod 180°.

2013-10-06.pdf

Properties:

- Both cubics are circumcubics of the quadrigon.
- Both cubics contain the diagonal crosspoint *QG-P1*.
- Each cubic contains one of the intersections *QG-2P2a,b* of opposite sides.
- Further intersections of the two cubics are the intersections *T1, T2* of the Thales circles about the diagonals.
- The line *T1T2* is *QL-L2*.
- Each of the cubics contains one of the vertices *S1, S2* of the Miquel triangle *QA-Tr2* unequal *QL-P1*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-10-06.pdf

Message: #284
Date: 06/10/2013 2:24:31
From: Chris van Tienhoven
Subject: A pair of QG-Cubics

```
Dear Eckart,

It looks like your cubics are the same as the ones I described
in QFG Message #256.
I described 6 cubics for a Quadrilateral, meaning 2 cubics per
QL-Quadrigon.
I also described the way how to construct them.
New for me is that one point of   the Miquel Triangle lie on
these cubics.
The points T1 and T2 are the Pluecker pair of points QL-2P1a/b.

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #285
Date: 07/10/2013 4:11:36
From: eckart_schmidt@t-online.de
Subject: QL-DT Circumscribed Conics

```
Dear friends,
in EQF there are no QL-DT circumscribed conics (without the
circumcircle). In the attachment you find some examples.
Best regards Eckart
```

EQF-Note 2013-10-07

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

QL-DT Circumscribed Conics

For a triangle circumscribed conics are isoconjugates of lines. In the QA-environment circumscribed conics of QA-DT are the Involutary Conjugates (QA-Tf2) of lines, for this transformation is an isoconjugation wrt QA-DT with fixed points in the vertices of the quadrangle. Examples: QA-Co1, QA-Co4, QA-Co5. In the QL-environment no circumscribed conics of QL-DT (without the circumcircle) are mentioned in EQF. Here are given some examples. – Reference triangle for barycentric coordinates is QL-DT.

QL-Isoconjugation

Wrt *QL-DT* there is an isoconjugation with fixed points in the trilinear poles of the side lines of the quadrilateral:

$$(-mn : nl : lm),\ (mn : -nl : lm),\ (mn : nl : -lm),\ (mn : nl : lm).$$

This *QL*-isoconjugation (for points) is the mapping

$$(x : y : z) \quad \rightarrow \quad (m^2n^2yz : n^2l^2zx : l^2m^2xy)$$

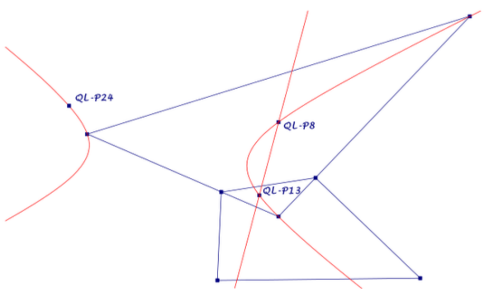
with the special example *QL-P8* $\rightarrow$ *QL-P13*.

Lines will be mapped in *QL*-circumscribed conics, but there are only a few interesting examples.

QL-L1 The *QL-DT* circumscribed conic is the Steiner ellipse.

QL-L7,8 The *QL-DT* circumscribed conics contain *QL-P13*.

QL-P8.QL-P13 The *QL-DT* circumscribed conic contains beside *QL-P8* and *QL-P13* further *QL-P24*.



2013-10-07.pdf

Another Way to *QL-DT* Circumscribed Conics

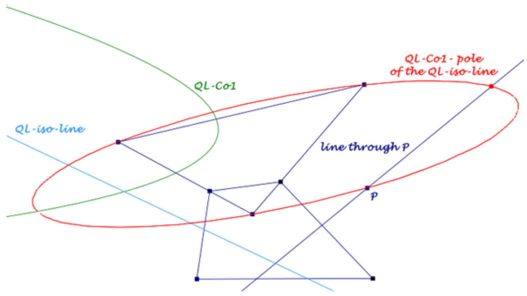
Wrt *QL-DT* there is an isoconjugation for lines (*QL-Tf2*). Fixed lines are the side lines of the quadrilateral. This *QL*-isoconjugation for lines is the mapping

$$(e, f, g) \;\; \rightarrow \;\; (l^2 fg, m^2 ge, n^2 ef).$$

You get the image line taking the trilinear pole, its *QL*-isoconjugate (for points) and then the trilinear polar.

Construction of a special *QL-DT* circumscribed conic through an arbitrary point *P(u:v:w)*:
Consider the line pencil of *P* and the images of the lines wrt the *QL*-isoconjugation for lines. The poles wrt the Inscribed Parabola *QL-Co1* are points on the lines again. The locus of these points is a *QL-DT* circumscribed conic with the equation

$$(l^2 - m^2)n^2 w\, xy + (m^2 - n^2)l^2 u\; yz + (n^2 - l^2)m^2 v\; zx = 0.$$



Some Examples:

P = QL-P2 The *QL-DT* circumscribed conic is an orthogonal hyperbola through *QL-P2* and *QL-P10*.

P = QL-P7 The *QL-DT* circumscribed conic contains *QL-P7* and the point at infinity of *QL-L1* (one asymptote parallel to *QL-L1*).

P = QL-P10 The *QL-DT* circumscribed conic is an orthogonal hyperbola through *QL-P10*.

P = QL-P13 The *QL-DT* circumscribed conic contains *QL-P8, QL-P13, QL-P24* (see example above).

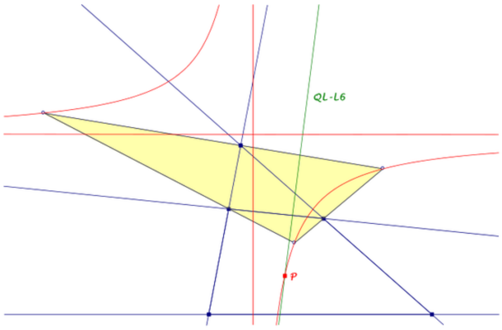
P = QL-P17 The *QL-DT* circumscribed conic is the circumcircle *QL-Ci1*.

P = QL-P23 The *QL-DT* circumscribed conic is a parabola with an axis parallel *QL-L1*.

All points on the line *QL-L6* give *QL-DT* circumscribed orthogonal hyperbolas.

2013-10-07.pdf

Example: The point at infinity gives an orthogonal hyperbola through *QL-P10* with asymptotes parallel to *QL-L6* and *QL-L9*.

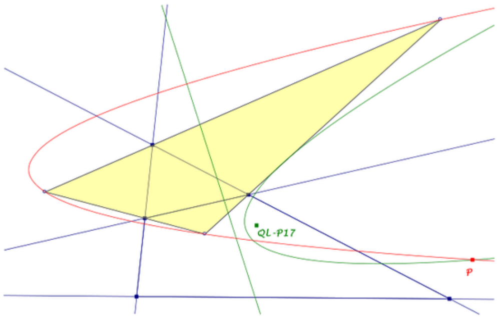


All points on the parabola with focus *QL-P17* and directrix *QL-L6* give *QL*-circumscribed parabolas.

The reference parabola has the equation

$$\sum_{cycl}(l^4(m^2-n^2)^2x^2-2(l^2-m^2)(l^2-n^2)m^2n^2yz)=0.$$

It is an inscribed conic of *QL-DT*. *QL-L1* is tangent to this parabola in *QL-P23*. Mapping the parallels to *QL-L1* by the *QL*-isoconjugation for lines (see above), the envelope gives this parabola.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-10-07.pdf

Message: #286
Date: 09/10/2013 4:34:45
From: eckart_schmidt@t-online.de
Subject: Some "splitter"

Dear Chris,

here are some "splitter", found on my desk: perhaps unimportant, perhaps already mentioned:
* QG-P12 is a point on the QG-circumconic through QA-P1.
* QL-P18 is the centroid of the three collinear points QG-P15 on QL-L9.
* QG-Co2: QL-P1 lies on the polar of QG-P16 (and invers).
* The polar of QG-P12 is parallel QG-L1 (QG-P1 railway watcher).
* The tangents in the intersections with QG-L2 are parallel QG-L1.
* The tangents in the intersections with QG-P13.QG-P15 are pependicular QL-L2.
* The polar of QG-P3 is QG-P1.QG-P2.
* The 3 QL-versions of QG-Co2 touch in the 6 QL-points.
* The Miquel points QL-L1 of tangential quadrilaterals wrt circumconics of a quadrangle lie on QA-Ci2.
* Center of QA-Co5: Let X be the QA-DT isotomic conjugate of QA-P16, let Y be the point at infinity of the trilinear polar of X wrt QA-DT, let Z be the isoconjugate of QA-P20 wrt a QA-DT isoconjugation with fixed point Y, then Z is the center of QA-Co5.
* Center of QA-Co5 is a point on a conic through the side midpoints of QA-DT with center QA-P22 (this conic contains QA-P29).
* A new cubic: QA-DT-P23 Cubic (containing QA-P12): Consider the QA-DT circumscribed orthogonal hyperbolas and the intersections with their QA-Tf2 image lines. This leads to a cubic with pivot QA-P23 invariant wrt the isoconjugation QA-Tf2.
I hope there are no basic errors!

Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #287
Date: 10/10/2013 9:47:23
From: Chris van Tienhoven
Subject: Some "splitter"

Dear Eckart,

Nice Splitters!
Some remarks:
* QG-P12 is a point on the QG-circumconic through QA-P1.
This conic is even a QA-circumconic. It passes though the 3
QA-versions of QG-P12 and also through QA-P16.
Its tangent at QA-P1 is the Centroids line QA-L3.
Its tangent at QA-P16 is the line through
QA-P10,QA-P16,QA-P19,QA-P31.
* QG-Co2: The tangents in the intersections with QG-L2 are
parallel QG-L1.
* The tangents in the intersections with QG-P13.QG-P15 are
pependicular QL-L2.
I cannot confirm this with Cabri.
Regarding the properties wrt QG-Co2, are there similar
properties wrt QG-Co1?
* Center of QA-Co5: Let X be the QA-DT isotomic conjugate of
QA-P16, let Y be the point at infinity of the trilinear polar of
X wrt QA-DT, let Z be the isoconjugate of QA-P20 wrt a QA-DT
isoconjugation with fixed point Y, then Z is the center of
QA-Co5.
I do not understand the part: "Let Z be the Isoconjugate of
QA-P20 wrt ……………………"

Best regards,
Chris

Message: #288
Date: 10/10/2013 2:16:14
From: eckart_schmidt@t-online.de
Subject: Some ”splitter”

```
Dear Chris,

I think, these properties of QG-Co2 are right (the first one
holds also for QG-Co1):
* QG-Co2: The tangents in the intersections with QG-L2
  are parallel QG-L1.
* The tangents in the intersections with QG-P13.QG-P15
  are pependicular QL-L2.
  An isoconjugation wrt a reference triangle
  and fixed point (u:v:w)
  is a transformation (x:y:z) -> (u^2 y z:: v^2 z x: w^2 x y).

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #289
Date: 11/10/2013 9:22:09
From: yeuemtrondoitb85
Subject: A generalization Brahmagupta's Theorem

```
Dear Mr Eckart!

I'm sorry, I don't know your proof about: "A generalization
Brahmagupta's Theorem"

Best regard
Dao Thanh Oai
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #290
Date: 11/10/2013 10:05:00
From: Dao Thanh Oai
Subject: A generalization Brahmagupta's Theorem

```
Dear Eckart!

I didn't understood your proof about: A generalization
Brahmagupta theorem!

Best regard
Dao Thanh Oai
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #291
Date: 11/10/2013 8:31:05
From: eckart_schmidt@t-online.de
Subject: A generalization Brahmagupta's Theorem

```
Dear Mr. Dao Thanh Oai,

for a better understanding of my proof in message #274 Chris has
proposed, to add the following phrases:

2. All angles in the constellation can be  calculated without
EAD and ADE with sum 180°-phi as well as ECB and CBE with sum
phi.
Using that the sum of all angles in AGFD and BGFC are 360
degrees it follows that:

4. Combining 1 and 3 gives:
When AI =BI, then. . . . .

5. Combining 2 & 4 and using the formula
sin(a+b) = sin(a).cos(b) + cos(a).sin(b)
gives:

I hope, this will be helpful.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #292
Date: 11/10/2013 8:45:27
From: Dao Thanh Oai
Subject: A generalization Brahmagupta's Theorem

```
Thank you Eckart!
I didn't notice AI=BI
Best regard
Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #293
Date: 13/10/2013 10:24:52
From: eckart_schmidt@t-online.de
Subject: A Curious QA-Transformation

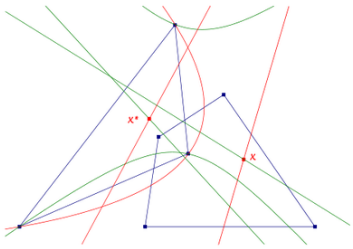
Dear friends,
there is a new type of QA-transformation: not an involution,
but holding fixed the vertices of a quadrangle as well as the
six connection lines. If you are interested, see attachment.
Best regards Eckart

EQF-Note 2013-10-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Curious *QA*-Transformation

This is a new type of QA-transformation: not an involution but holding fixed the vertices of the quadrangle as well as the six connection lines. Further properties are discussed. – Reference triangle for barycentric coordinates is the QA-Diagonal Triangle QA-DT.



Definition *QA-Tfx*:

Let *X* be a point, consider lines through *X* and their Involutary Conjugates (*QA-Tf2*), which are *QA-DT* -circumconics. The polars of *X* wrt these conics have a common point *X.**

For $X=(x:y:z)$ the image point is

$$X^*=(x(-q^2r^2x^2+r^2p^2y^2+p^2q^2z^2)$$
$$:y(q^2r^2x^2-r^2p^2y^2+p^2q^2z^2):z(q^2r^2x^2+r^2p^2y^2-p^2q^2z^2)).$$

Properties:

1. *QA-Tfx* is not an involution.
2. The vertices P_i of the quadrangle as well as the vertices S_i of *QA-DT* are fixed points.
3. The lines P_iP_j of the quadrangle as well as the sidelines S_iS_j of *QA-DT* are invariant wrt *QA-Tfx*.
4. All points on the trilinear polar of P_i wrt *QA-DT* have the image P_i.
5. All points X, X^* on a sideline S_iS_j of *QA-DT* divide $S_i.S_j$ harmonic.

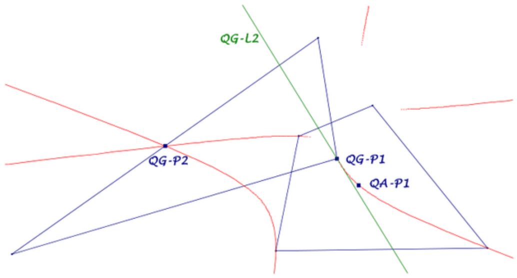
2013-10-13.pdf

6. Circumscribed conics of the quadrangle are invariant under *QA-Tfx*.
7. *QA-Tfx* mappings of lines unequal P_iP_j and S_iS_j are circumcubics of the quadrangle. For a line with the coefficients *e, f, g* The cubic has the equation

$$eq^2r^2(f^2q^2-g^2r^2)x^3+fr^2p^2(g^2r^2-e^2p^2)y^3+gp^2q^2(e^2p^2-f^2q^2)z^3$$
$$+fq^2r^2(2e^2p^2-f^2q^2+g^2r^2)x^2y+ep^2r^2(e^2p^2-2f^2q^2-g^2r^2)xy^2$$
$$+gq^2r^2(-2e^2p^2-f^2q^2+g^2r^2)x^2z+ep^2q^2(e^2p^2-f^2q^2-2g^2r^2)xz^2$$
$$+gp^2r^2(e^2p^2+2f^2q^2-g^2r^2)y^2z+fp^2q^2(-e^2p^2+f^2q^2-2g^2r^2)yz^2=0$$

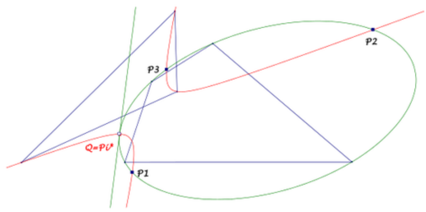
Example *QA-Tfx(QG-L2)* with equation:

$$(p^2-r^2)(p^2z-r^2x)y^2-q^2(z-x)^2(p^2z+r^2x)=0$$



This circumscribed cubic of the quadrangle contains *QG-P1* as inflexion point, *QG-P2* as nodal point and *QA-P1*.

8. For a point *Q* on a circumconic of a quadrangle there are three points *Pi* with *QA-Tfx*-image *Q*. All four points lie on a circumconic of *QA-DT*, which is the *QA-Tf2* image of the tangent at *Q* to the *QA*-circumconic.



9. For a line *L* unequal P_iP_j and S_iS_j there is a cubic, whose *QA-Tfx*-image is the line *L*.
If the line has the coefficients *e, f, g*, the equation of the cubic is

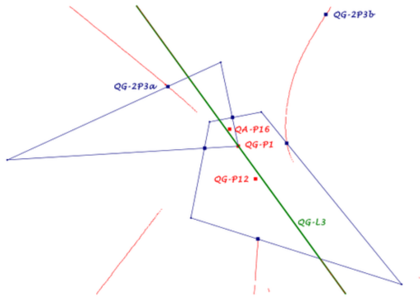
$$-eq^2r^2x^3-fr^2p^2y^3-gp^2q^2z^3+fq^2r^2x^2y+er^2p^2xy^2$$
$$+gp^2r^2y^2z+fp^2q^2yz^2+ep^2q^2xz^2+fp^2q^2yz^2=0\,.$$

The cubic can be constructed in the following way: Let *X* be a point of the line *L* and Co_1 the *QA*-circumconic through *X*. The *QA-Tf2*-image of the tangent in *X* at Co_1 is a *QA-DT*-circumconic Co_2. The intersections of Co_1 and *Co2* are points of the cubic.

2013-10-13.pdf

Example: *QG*-Cubic with *QA-Tfx*-image *QG-L3* and equation

$$p^2(-p^2+q^2+r^2)(q^2r^2x^2+r^2p^2y^2-p^2q^2z^2)z$$
$$+r^2(p^2+q^2-r^2)(-q^2r^2x^2+r^2p^2y^2+p^2q^2z^2)x=0$$



This cubic contains *QG-P1* as inflexion point, *QG-2P3a,b*, *QA-P16* and *QG-P12*. Further points on the cubic are the intersections of the lines *QG-P1.QG-2P2a,b* with the sidelines of the quadrigon.

Final remark

There are analogous relationships in the *QL*-environment, defining the following transformation for lines:

Definition *QL-Tfx*:

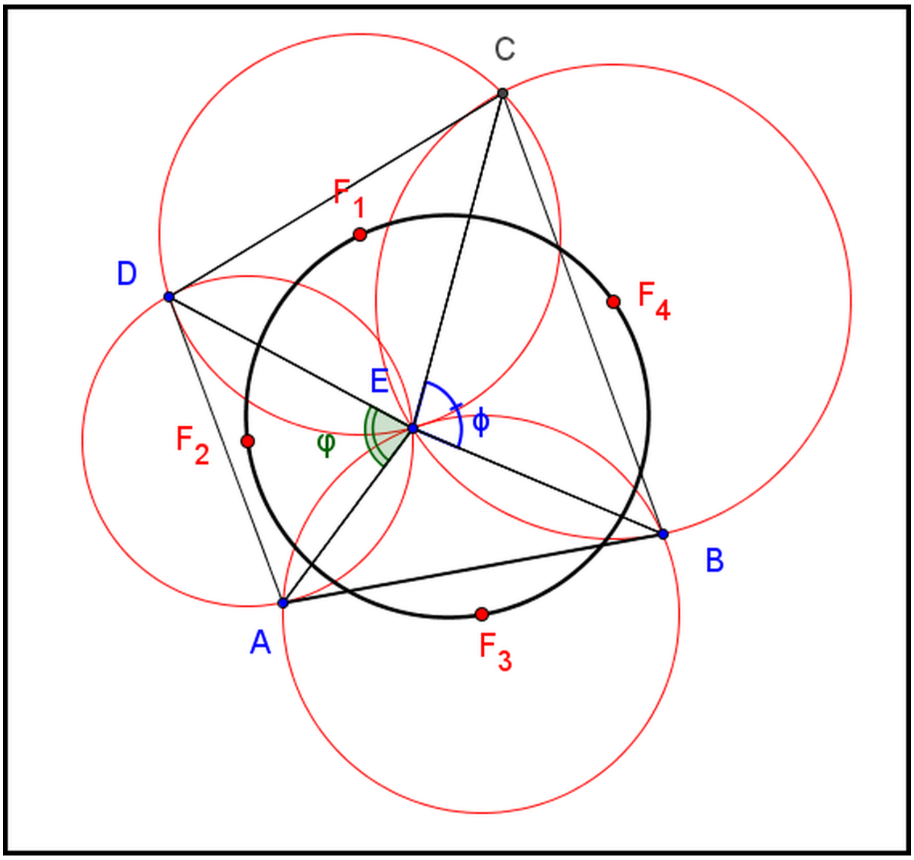
Let *L* be a line, consider points *X* on *L* and for the lines through *X* the *QL*-Line Isoconjugates (*QL-Tf2*), which envelope *QL-DT*-inconics. The poles of *L* wrt these conics are collinear on *L*.*

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-10-13.pdf

Message: #294
Date: 13/10/2013 1:57:02
From: Dao Thanh Oai
Subject: A generalisation Brahmagupta's Theorem

```
Dear Eckart and Chris!
I discovered one problem: Centre of four circle AEB,BEC,CED,DEA
are cyclic.
Best regard
Dao Thanh Oai
```



Centre of four circle are cyclic.png

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #295
Date: 13/10/2013 2:44:44
From: Dao Thanh Oai
Subject: A generalisation Brahmagupta's Theorem

```
Dear Chris and Eckart!
I'm sorry because last property very easily.
Best regard
Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #296
Date: 13/10/2013 10:54:52
From: Chris
Subject: A Curious QA-Transformation

```
Dear Eckart,
Very nice QA-transformation!
I found these examples:
Let:
X1= InvolutionCenter QA-P1.QA-P16 (see QA-Tf1)
X2= InvolutionCenter QA-P1.QA-P31 (see QA-Tf1)
X3= QA-DT-ConicPerspector of QA-Co5 (see QA-Co/1)
X4= QA-DT-ConicPerspector of QA-DT-Conic through P5-P17-P18-P27
    (see QA-Co/1)
X5= Infinity point QA-P1.QA-P5
QA-Tfx(QA-P1)  lies on QA-P1.X3};
QA-Tfx(QA-P1)  lies on QA-P16. QA-P22;
QA-Tfx(QA-P5)  lies on QA-P5.X4;
QA-Tfx(QA-P5)  lies on QA-P10.X3;
QA-Tfx(QA-P10) lies on QA-P5.X3;
QA-Tfx(QA-P10) lies on QA-P10.X1;
QA-Tfx(QA-P16) = QA-P1;
QA-Tfx(QA-P17) lies on QA-P17.X4;
QA-Tfx(QA-P18) lies on QA-P10.X2
QA-Tfx(QA-P18) lies on QA-P16.QA-P17;
QA-Tfx(QA-P20) lies on QA-P20.X3;
QA-Tfx(QA-P22) lies on X3.X5;
It is special that only points on the the Centroids Line QA-L3
(apart from QA-P16) are transformed such that they lie on lines
with other known points.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #297
Date: 13/10/2013 11:24:00
From: Chris
Subject: A Curious QA-Transformation

```
Dear Eckart,
Also:
* QA-Tfx(QA-P17) lies on cubic QA-Cu2
* QA-Tfx(QA-P18) lies on cubic QA-Cu4
* QA-Tfx(QA-P20) lies on cubic QA-Cu5
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #298
Date: 15/10/2013 4:18:32
From: Seiichi Kirikami
Subject: Algebraic computation of quadratic, cubic and quartic quadrangle poi

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Dear friends,

5 conditions of message #211 are as follows:
(1) pf(p+q+r,-q,-r)/cyclic sum f(p+q+r,-q,-r)=(p+q+r)f(p,q,r)/cyclic sum f(p,q,r).
(2) (p+q+r)f(q,r,p)/cyclic sum f(q,r,p)=qf(p+q+r,-r,-p)/cyclic sum f(p+q+r,-r,-p).
(3) qf(-r,-p,p+q+r)/cyclic sum f(-r,-p,p+q+r)=-rf(q,p,-p-q-r)/cyclic sum f(q,p,-p-q-r).
(4) -rf(p,-p-q-r,q)/cyclic sum f(p,-p-q-r,q)=pf(-r,p+q+r,-q)/cyclic sum f(-r,p+q+r,-q).
(5) p+q= the sum of the right-handed sides of (1), (2), (3) and (4).
The 1st condition is most important. The other conditions are for confirmation.
The 1st condition is an identity. The arbitrary coordinates of the 4th point P4 {p,q,r} satisfy it.
We choose P4 {1,n,n}, n=2, 3,... We suppose the factor (2p+q+r) in f(p,q,r).
The following solutions are obtained by Mathematica 8. See the attached files.

[1] Quadratic cases:
Assumption: f(p,q,r)=(2p+q+r)(ap+b(q+r)).
Solutions: (1) {b->0}, (2) {b->a}.
Reductions:
(1) f(p,q,r)=(2p+q+r)p, (QAP16).
The other case degenerates to linear case 2p+q+r, (QAP1).

[2] Cubic cases:
Assumption: f(p,q,r)=(2p+q+r)(ap^2+b(q^2+r^2)+cp(q+r)+dqr).
Solutions: (1) {b->a,c->d}, (2) {c->a(a+b)/(a-2b), d->-2b}, (3) {c->a,d->b}, (4) {a->0, b->0, c->d}, (5) {a->0,b->0,d->0}, (6) {b->a/2, c->a, d->a/2}.
Reductions:
(3) f(p,q,r)=(2p+q+r)(ap(p+q+r)+b(q^2+r^2+qr)).
a=0 simplifies f(p,q,r)=(2p+q+r)(q^2+r^2+qr), (QAP21).
b=0 simplifies f(p,q,r)=(2p+q+r)p, (QAP16).
(5) f(p,q,r)=(2p+q+r)cp(q+r), simplifies to (2p+q+r)p(q+r), (QAP10).
The other cases degenerates to linear or quadratic cases.

[3] Quartic cases:
Assumption: f(p,q,r)=(2p+q+r)(ap^3+bp^2(q+r)+cp(q^2+r^2)+dpqr+e(q^3+r^3)+fqr(q+r)).
Solutions: (1) {a->c-2e, b->c-2e, f->-e}, (2) {a->e,b->c, f->c}, (3) {b->a+c-e, d->(-a^2+ac-2c^2-2ae+4ce-2e^2)/(-c+e),f->(a^2-ac+ae+2ce-2e^2)/a}, (4) {b->(-a^2+ac)/2/e, d->(-a^3+a^2c+2a^2e-2ace+2ae^2+4ce^2)/2/(a-e)/e, f->-e}, (5) {a->0, b->0, d->-2c, f->-e}, (6) {a->c-2e, b->c-2e, d->-2c, f->-e}, (7) {a->0, b->c, d->2c, e->0}, (8) {a->0, b->c, e->0, f->c}, (9) {a->0, d->2(b^2-bc+c^2)/(2b-c),e->0, f->0}, (10) {a->c, b->c, e->0, f->0}, (11) {a->c, d->b, e->0,f->0}, (12) {b->a+c, d->(a^2-ac+c^2)/c, e->0, f->a-c}, (13) {a->c/3, b->c/3, e->c/3, f->-c/3}, (14) {a->c/3, b->c, e->c/3, f->c}, (15) {a->0, b->0, e->c/2, f->-c/2}, (16) {a->0, b->0, e->c, f->2c^2/d}, (17) {a--c, b->-c, e->c, f->-c}.
Reductions:
(1) With additional condition (e=0),
f(p,q,r)=(2p+q+r)p(c(p^2+p(q+r)+q^2+r^2)+dqr).
(7) f(p,q,r)=(2p+q+r)(q+r)(cp(p+q+r)+fqr).
c=0 simplifies f(p,q,r)=(2p+q+r)(q+r)qr, (QAP5).
f=0 simplifies f(p,q,r)=(2p+q+r)p(q+r), (QAP10).
(16) With additional condition (c=d),
f(p,q,r)=(2p+q+r)(p(q^2+r^2+qr)+q^3+r^3+2qr(q+r)).
The other cases degenerates to less than quartic cases or do not satisfy the other conditions.
As for the quintic case, Mathematica 8 did not finish itself in 30 minutes. So I obtained no results.

[4] Summary
(1) quadratic: f(p,q,r)=(2p+q+r)p.
(2) cubic: f(p,q,r)=(2p+q+r)(ap(p+q+r)+b(q^2+r^2+qr)).
f(p,q,r)=(2p+q+r)p(q+r).
(3) quartic: f(p,q,r)=(2p+q+r)p(c(p^2+p(q+r)+q^2+r^2)+dqr).
f(p,q,r)=(2p+q+r)(q+r)(cp(p+q+r)+fqr).
f(p,q,r)=(2p+q+r)(p(q^2+r^2+qr)+q^3+r^3+2qr(q+r)).

Best regards,
Seiichi.

```
(* Quadratic QA-points *)
f1[1] = (2 p + q + r) (a p + b (q + r));
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}] // Simplify
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}] // Simplify
(p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 2, r → 2} // Simplify
(p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 3, r → 3} // Simplify
```

$2\,a\,\left(p^2+q^2+q\,r+r^2+p\,(q+r)\right)+2\,b\,\left(p^2+q^2+3\,q\,r+r^2+3\,p\,(q+r)\right)$

$2\,\left(b\,\left(p^2-q^2-q\,r-r^2-p\,(q+r)\right)+a\,\left(p^2+q^2+q\,r+r^2+p\,(q+r)\right)\right)$

$2016\,(a-b)\,b$

$15\,360\,(a-b)\,b$

QA-Points-nb.pdf

```
(* Cubic QA-points *)
f1[1] = (2 p + q + r) (a p^2 + b (q^2 + r^2) + c p (q + r) + d q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}] // Simplify
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}] // Simplify
poly1 = (p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 2, r → 2} // Simplify
poly2 = (p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 3, r → 3} // Simplify
poly3 = (p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 4, r → 4} // Simplify
poly4 = (p + q + r) f1[1] F2 - p f2[1] F1 /. {p → 1, q → 5, r → 5} // Simplify
(* solutions *)
Solve[poly1 == 0 && poly2 == 0 && poly3 == 0 && poly4 == 0, {a, b, c, d}]
```

$(p+q+2r)\left(d\,p\,q+b\left(p^2+q^2\right)+r\left(c\,(p+q)+a\,r\right)\right)+(p+2q+r)\left(a\,q^2+d\,p\,r+c\,q\,(p+r)+b\left(p^2+r^2\right)\right)+(2p+q+r)\left(a\,p^2+d\,q\,r+c\,p\,(q+r)+b\left(q^2+r^2\right)\right)$

$(2p+q+r)\left(d\,q\,r-c\,(q+r)\,(p+q+r)+a\,(p+q+r)^2+b\left(q^2+r^2\right)\right)+(p-r)\left(a\,r^2-c\,r\,(p+r)-d\,q\,(p+q+r)+b\left(q^2+(p+q+r)^2\right)\right)+(p-q)\left(a\,q^2-c\,q\,(p+q)-d\,r\,(p+q+r)+b\left(r^2+(p+q+r)^2\right)\right)$

$-1008\left(5a^2+8b^2+2b\,(9c-2d)+4\,(c-d)\,d-a\,(13b+5c+9d)\right)$

$-7680\left(7a^2+18b^2+b\,(32c-9d)+9\,(c-d)\,d-a\,(25b+7c+16d)\right)$

$-31200\left(9a^2-a\,(41b+9c+25d)+2\left(16b^2+b\,(25c-8d)+8\,(c-d)\,d\right)\right)$

$-92160\left(11a^2+50b^2+b\,(72c-25d)+25\,(c-d)\,d-a\,(61b+11c+36d)\right)$

Solve::svars : 方程式はすべての "solve" 変数に対しては解を与えない可能性があります. ≫

$\left\{\{b\to a,\ c\to d\},\ \left\{c\to \frac{a\,(a+b)}{a-2b},\ d\to -2b\right\},\ \{c\to a,\ d\to b\},\ \{a\to 0,\ b\to 0,\ c\to d\},\ \{a\to 0,\ b\to 0,\ d\to 0\},\ \left\{b\to \frac{a}{2},\ c\to a,\ d\to \frac{a}{2}\right\}\right\}$

QA-Points-nb.pdf

```
(* Confirmation of solutions *)
(* {b→a,c→d} *)
f1[1] = (2 p + q + r) (a p^2 + a (q^2 + r^2) + d p (q + r) + d q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Simplify
0

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```

QA-Points-nb.pdf

```
(* {c→a (a+b)/(a-2 b),d→-2 b} *)
f1[1] = (2 p + q + r) (a p^2 + b (q^2 + r^2) + a (a + b) p (q + r) / (a - 2 b) - 2 b q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
```

$$\Big(3\,(a-2b)\,(a-b)\,b\,(q-r)^2\,(q+r)\,(2p+q+r)^2\,\big(a\,p\,(p+q+r)+b\,(p^2-3qr+p\,(q+r))\big)\Big)\Big/$$
$$\Big(2\,\big(-b^2\,(2p^3+2q^3+q^2r+qr^2+2r^3+p^2\,(q+r)+p\,(q^2-12qr+r^2))+$$
$$a\,b\,(-p^3+p^2\,(q+r)-(q-r)^2\,(q+r)+p\,(q^2-3qr+r^2))+$$
$$a^2\,(p^3+q^3+2q^2r+2qr^2+r^3+2p^2\,(q+r)+p\,(2q^2+3qr+2r^2))\big)$$
$$\big(a^2\,p\,(p^2+q^2+qr+r^2+p\,(q+r))-a\,b\,(p^3+4p^2\,(q+r)+9qr\,(q+r)+$$
$$p\,(4q^2+13qr+4r^2))-b^2\,(2p^3+5p^2\,(q+r)-9qr\,(q+r)+p\,(5q^2-4qr+5r^2))\big)\Big)$$

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$$
\begin{aligned}
& -(3\,(a-2\,b)\,(a-b)\,b\,(p-r)^2\,(p+r)\,(p+2\,q+r)^2\,(a\,q\,(p+q+r)+b\,(p\,(q-3\,r)+q\,(q+r))))\,/ \\
& \quad (2\,(-b^2\,(2\,p^3+2\,q^3+q^2\,r+q\,r^2+2\,r^3+p^2\,(q+r)+p\,(q^2-12\,q\,r+r^2))+ \\
& \qquad a\,b\,(-p^3+p^2\,(q+r)-(q-r)^2\,(q+r)+p\,(q^2-3\,q\,r+r^2))+ \\
& \qquad a^2\,(p^3+q^3+2\,q^2\,r+2\,q\,r^2+r^3+2\,p^2\,(q+r)+p\,(2\,q^2+3\,q\,r+2\,r^2))) \\
& \quad\; (a^2\,q\,(p^2+q^2+q\,r+r^2+p\,(q+r))-b^2\,(p^2\,(5\,q-9\,r)+p\,(5\,q^2-4\,q\,r-9\,r^2)+ \\
& \qquad\quad q\,(2\,q^2+5\,q\,r+5\,r^2))-a\,b\,(q\,(q+2\,r)^2+p^2\,(4\,q+9\,r)+p\,(4\,q^2+13\,q\,r+9\,r^2))))
\end{aligned}
$$

$$
\begin{aligned}
& -(3\,(a-2\,b)\,(a-b)\,b\,(q-r)^2\,(q+r)\,(2\,p+q+r)^2\,(a\,q\,r+b\,(-3\,p^2+q\,r-3\,p\,(q+r))))\,/ \\
& \quad (2\,(a^2\,r\,(p^2+q^2+q\,r+r^2+p\,(q+r))+ \\
& \qquad b^2\,(p^2\,(9\,q-5\,r)+p\,(9\,q^2+4\,q\,r-5\,r^2)-r\,(5\,q^2+5\,q\,r+2\,r^2))- \\
& \qquad a\,b\,(r\,(2\,q+r)^2+p^2\,(9\,q+4\,r)+p\,(9\,q^2+13\,q\,r+4\,r^2))) \\
& \quad\; (a^2\,q\,(p^2+q^2+q\,r+r^2+p\,(q+r))-b^2\,(p^2\,(5\,q-9\,r)+p\,(5\,q^2-4\,q\,r-9\,r^2)+ \\
& \qquad\quad q\,(2\,q^2+5\,q\,r+5\,r^2))-a\,b\,(q\,(q+2\,r)^2+p^2\,(4\,q+9\,r)+p\,(4\,q^2+13\,q\,r+9\,r^2))))
\end{aligned}
$$

$$
\begin{aligned}
& (3\,(a-2\,b)\,(a-b)\,b\,(p-r)^2\,(p+r)\,(p+2\,q+r)^2\,(a\,p\,r+b\,(p\,(-3\,q+r)-3\,q\,(q+r))))\,/ \\
& \quad (2\,(a^2\,r\,(p^2+q^2+q\,r+r^2+p\,(q+r))+ \\
& \qquad b^2\,(p^2\,(9\,q-5\,r)+p\,(9\,q^2+4\,q\,r-5\,r^2)-r\,(5\,q^2+5\,q\,r+2\,r^2))- \\
& \qquad a\,b\,(r\,(2\,q+r)^2+p^2\,(9\,q+4\,r)+p\,(9\,q^2+13\,q\,r+4\,r^2)))\,(a^2\,p\,(p^2+q^2+q\,r+r^2+p\,(q+r))- \\
& \qquad a\,b\,(p^3+4\,p^2\,(q+r)+9\,q\,r\,(q+r)+p\,(4\,q^2+13\,q\,r+4\,r^2))- \\
& \qquad b^2\,(2\,p^3+5\,p^2\,(q+r)-9\,q\,r\,(q+r)+p\,(5\,q^2-4\,q\,r+5\,r^2))))
\end{aligned}
$$

$$
\begin{aligned}
& (3\,(a-2\,b)\,(a-b)\,b\,(p-q)\,(p+q)\,(p+q+2\,r) \\
& \quad (a^5\,p\,(p-q)\,q\,r\,(p^2+2\,p\,q+q^2+3\,p\,r+3\,q\,r+2\,r^2)\,(p^2+q^2+q\,r+r^2+p\,(q+r))^2+ \\
& \qquad a\,b^4\,(-p^8\,(9\,q^2+4\,q\,r+39\,r^2)-p^7\,(48\,q^3+51\,q^2\,r+116\,q\,r^2+201\,r^3)- \\
& \qquad\quad p^6\,(165\,q^4+97\,q^3\,r-243\,q^2\,r^2+461\,q\,r^3+336\,r^4)- \\
& \qquad\quad p^5\,(192\,q^5+98\,q^4\,r-702\,q^3\,r^2-1077\,q^2\,r^3+390\,q\,r^4+171\,r^5)+ \\
& \qquad\quad 3\,r^2\,(q+r)^2\,(3\,q^6+2\,q^5\,r+20\,q^4\,r^2+56\,q^3\,r^3+16\,q^2\,r^4-11\,q\,r^5-5\,r^6)+ \\
& \qquad\quad p^4\,(21\,q^6+290\,q^5\,r+96\,q^4\,r^2+873\,q^3\,r^3+1624\,q^2\,r^4+323\,q\,r^5+45\,r^6)+ \\
& \qquad\quad p^3\,(111\,q^7+538\,q^6\,r-51\,q^5\,r^2-2028\,q^4\,r^3-819\,q^3\,r^4+1041\,q^2\,r^5+371\,q\,r^6-27\,r^7)+ \\
& \qquad\quad p^2\,(39\,q^8+228\,q^7\,r+156\,q^6\,r^2-1392\,q^5\,r^3-2161\,q^4\,r^4- \\
& \qquad\qquad 531\,q^3\,r^5+141\,q^2\,r^6-227\,q\,r^7-141\,r^8)+p\,r \\
& \qquad\quad\; (4\,q^8+53\,q^7\,r+20\,q^6\,r^2+123\,q^5\,r^3+640\,q^4\,r^4+553\,q^3\,r^5-145\,q^2\,r^6-297\,q\,r^7-87\,r^8))+ \\
& \qquad a^2\,b^3\,(p^8\,(27\,q^2-44\,q\,r+9\,r^2)+p^7\,(90\,q^3-69\,q^2\,r-202\,q\,r^2+117\,r^3)+ \\
& \qquad\quad p^6\,(171\,q^4-122\,q^3\,r-564\,q^2\,r^2-124\,q\,r^3+387\,r^4)+ \\
& \qquad\quad p^5\,(225\,q^5+20\,q^4\,r-969\,q^3\,r^2-762\,q^2\,r^3+276\,q\,r^4+594\,r^5)- \\
& \qquad\quad 9\,r^2\,(q+r)^2\,(3\,q^6+8\,q^5\,r+14\,q^4\,r^2+11\,q^3\,r^3+4\,q^2\,r^4+4\,q\,r^5+r^6)+ \\
& \qquad\quad p^4\,(180\,q^6+322\,q^5\,r-396\,q^4\,r^2-1023\,q^3\,r^3-349\,q^2\,r^4+355\,q\,r^5+459\,r^6)+ \\
& \qquad\quad p^3\,(45\,q^7+257\,q^6\,r+258\,q^5\,r^2+168\,q^4\,r^3+189\,q^3\,r^4+72\,q^2\,r^5-20\,q\,r^6+135\,r^7)+p^2 \\
& \qquad\quad\; (-9\,q^8+78\,q^7\,r+123\,q^6\,r^2+249\,q^5\,r^3+691\,q^4\,r^4+720\,q^3\,r^5+117\,q^2\,r^6-241\,q\,r^7-36\,r^8)+ \\
& \qquad\quad p\,r\,(44\,q^8+67\,q^7\,r-11\,q^6\,r^2-15\,q^5\,r^3+158\,q^4\,r^4+218\,q^3\,r^5-47\,q^2\,r^6-162\,q\,r^7-36\,r^8))+ \\
& \qquad a^3\,b^2\,(p^8\,(9\,q^2+19\,q\,r+15\,r^2)+p^7\,(24\,q^3+72\,q^2\,r+143\,q\,r^2+105\,r^3)+ \\
& \qquad\quad p^6\,(69\,q^4+85\,q^3\,r+303\,q^2\,r^2+566\,q\,r^3+258\,r^4)+ \\
& \qquad\quad p^5\,(114\,q^5+74\,q^4\,r+294\,q^3\,r^2+1038\,q^2\,r^3+1053\,q\,r^4+297\,r^5)+ \\
& \qquad\quad 3\,r^2\,(q+r)^2\,(-3\,q^6-10\,q^5\,r-28\,q^4\,r^2-34\,q^3\,r^3-8\,q^2\,r^4+q\,r^5+r^6)+ \\
& \qquad\quad p^4\,(57\,q^6-62\,q^5\,r-120\,q^4\,r^2+735\,q^3\,r^3+1490\,q^2\,r^4+946\,q\,r^5+171\,r^6)+ \\
& \qquad\quad p^3\,(-15\,q^7-202\,q^6\,r-669\,q^5\,r^2-540\,q^4\,r^3+315\,q^3\,r^4+762\,q^2\,r^5+442\,q\,r^6+63\,r^7)-
\end{aligned}
$$

$$
\begin{aligned}
&p^2 (15 q^8 + 129 q^7 r + 534 q^6 r^2 + 1047 q^5 r^3 + 1235 q^4 r^4 + 756 q^3 r^5 + \\
&\quad 21 q^2 r^6 - 161 q r^7 - 39 r^8) - p r (19 q^8 + 152 q^7 r + 449 q^6 r^2 + \\
&\quad 906 q^5 r^3 + 1153 q^4 r^4 + 706 q^3 r^5 + 101 q^2 r^6 - 63 q r^7 - 21 r^8)) + \\
&a^4 b (-p^8 (9 q^2 + 4 q r + 3 r^2) - p^7 (39 q^3 + 42 q^2 r + 17 q r^2 + 30 r^3) - \\
&\quad p^6 (75 q^4 + 97 q^3 r + 63 q^2 r^2 + 101 q r^3 + 84 r^4) - \\
&\quad p^5 (75 q^5 + 98 q^4 r + 36 q^3 r^2 + 165 q^2 r^3 + 237 q r^4 + 117 r^5) + \\
&\quad 3 r^2 (q + r)^2 (3 q^6 + 5 q^5 r + 5 q^4 r^2 + 5 q^3 r^3 + 4 q^2 r^4 + 4 q r^5 + r^6) - \\
&\quad 2 p^4 (21 q^6 + 8 q^5 r - 66 q^4 r^2 + 151 q^2 r^4 + 131 q r^5 + 45 r^6) - \\
&\quad p^3 (6 q^7 - 52 q^6 r - 273 q^5 r^2 - 285 q^4 r^3 + 63 q^3 r^4 + 246 q^2 r^5 + 124 q r^6 + 27 r^7) + \\
&\quad p^2 (3 q^8 + 39 q^7 r + 210 q^6 r^2 + 336 q^5 r^3 + 188 q^4 r^4 - 18 q^3 r^5 - 39 q^2 r^6 + 25 q r^7 + 12 r^8) + \\
&\quad p r (4 q^8 + 62 q^7 r + 146 q^6 r^2 + 150 q^5 r^3 + 91 q^4 r^4 + 58 q^3 r^5 + 71 q^2 r^6 + 54 q r^7 + 12 r^8)) + \\
&b^5 (-2 p^8 (9 q^2 - 32 q r + 15 r^2) + p^7 (-75 q^3 + 135 q^2 r + 251 q r^2 - 183 r^3) + \\
&\quad p^6 (-192 q^4 + 211 q^3 r + 213 q^2 r^2 + 341 q r^3 - 381 r^4) + \\
&\quad p^5 (-228 q^5 + 56 q^4 r + 69 q^3 r^2 - 825 q^2 r^3 + 639 q r^4 - 351 r^5) + \\
&\quad 3 r^2 (q + r)^2 (6 q^6 + 11 q^5 r + 29 q^4 r^2 + 50 q^3 r^3 + 16 q^2 r^4 - 2 q r^5 - 2 r^6) - \\
&\quad p^4 (60 q^6 + 80 q^5 r - 240 q^4 r^2 + 1245 q^3 r^3 + 1075 q^2 r^4 - 1189 q r^5 + 153 r^6) + \\
&\quad p^3 (57 q^7 + 23 q^6 r + 681 q^5 r^2 + 855 q^4 r^3 - 630 q^3 r^4 + 42 q^2 r^5 + 964 q r^6 - 72 r^7) + \\
&\quad p^2 (30 q^8 - 21 q^7 r + 249 q^6 r^2 + 843 q^5 r^3 + 565 q^4 r^4 - 54 q^3 r^5 + 42 q^2 r^6 + 152 q r^7 - 78 r^8) - \\
&\quad p r (64 q^8 + 233 q^7 r + 575 q^6 r^2 + 933 q^5 r^3 + 775 q^4 r^4 + \\
&\qquad 436 q^3 r^5 + 272 q^2 r^6 + 126 q r^7 + 42 r^8)))) / \\
&(2 (-b^2 (2 p^3 + 2 q^3 + q^2 r + q r^2 + 2 r^3 + p^2 (q + r) + p (q^2 - 12 q r + r^2)) + \\
&\quad a b (-p^3 + p^2 (q + r) - (q - r)^2 (q + r) + p (q^2 - 3 q r + r^2)) + \\
&\quad a^2 (p^3 + q^3 + 2 q^2 r + 2 q r^2 + r^3 + 2 p^2 (q + r) + p (2 q^2 + 3 q r + 2 r^2))) \\
&(a^2 r (p^2 + q^2 + q r + r^2 + p (q + r)) + \\
&\quad b^2 (p^2 (9 q - 5 r) + p (9 q^2 + 4 q r - 5 r^2) - r (5 q^2 + 5 q r + 2 r^2)) - \\
&\quad a b (r (2 q + r)^2 + p^2 (9 q + 4 r) + p (9 q^2 + 13 q r + 4 r^2))) \\
&(a^2 p (p^2 + q^2 + q r + r^2 + p (q + r)) - \\
&\quad a b (p^3 + 4 p^2 (q + r) + 9 q r (q + r) + p (4 q^2 + 13 q r + 4 r^2)) - \\
&\quad b^2 (2 p^3 + 5 p^2 (q + r) - 9 q r (q + r) + p (5 q^2 - 4 q r + 5 r^2))) \\
&(a^2 q (p^2 + q^2 + q r + r^2 + p (q + r)) - \\
&\quad b^2 (p^2 (5 q - 9 r) + p (5 q^2 - 4 q r - 9 r^2) + q (2 q^2 + 5 q r + 5 r^2)) - \\
&\quad a b (q (q + 2 r)^2 + p^2 (4 q + 9 r) + p (4 q^2 + 13 q r + 9 r^2))))
\end{aligned}
$$

```
(* {c→a (a+b)/(a-2 b),d→-2 b},{b→a} *)
f1[1] = (2 p + q + r) (a p^2 + a (q^2 + r^2) + a (a + a) p (q + r) / (a - 2 a) - 2 a q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
0

0

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0
```

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```
(* {c→a,d→b} *)
f1[1] = (2 p + q + r) (a p^2 + b (q^2 + r^2) + a p (q + r) + b q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
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QA-Points-nb.pdf

```
(* {a→0,b→0,c→d} *)
f1[1] = (2 p + q + r) (d p (q + r) + d q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
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QA-Points-nb.pdf

```
(* {a→0,b→0,d→0} *)
f1[1] = (2 p + q + r) (c p (q + r) );
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
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(* {b→a/2,c→a,d→a/2} *)
f1[1] = (2 p + q + r) (a p^2 + a / 2 (q^2 + r^2) + a p (q + r) + a / 2 q r);
f1[2] = f1[1] /. {p → q, q → r, r → p};
f1[3] = f1[2] /. {p → q, q → r, r → p};
F1 = Sum[f1[i], {i, 3}];
f2[1] = f1[1] /. {p → p + q + r, q → -q, r → -r};
f2[2] = f1[2] /. {p → p + q + r, q → -q, r → -r};
f2[3] = f1[3] /. {p → p + q + r, q → -q, r → -r};
F2 = Sum[f2[i], {i, 3}];
f3[1] = f1[1] /. {p → p + q + r, q → -r, r → -p};
f3[2] = f1[2] /. {p → p + q + r, q → -r, r → -p};
f3[3] = f1[3] /. {p → p + q + r, q → -r, r → -p};
F3 = Sum[f3[i], {i, 3}];
f4[1] = f1[1] /. {p → q, q → r, r → p};
f4[2] = f1[2] /. {p → q, q → r, r → p};
f4[3] = f1[3] /. {p → q, q → r, r → p};
F4 = Sum[f4[i], {i, 3}];
f5[1] = f1[1] /. {p → q, q → p, r → -p - q - r};
f5[2] = f1[2] /. {p → q, q → p, r → -p - q - r};
f5[3] = f1[3] /. {p → q, q → p, r → -p - q - r};
F5 = Sum[f5[i], {i, 3}];
f6[1] = f1[1] /. {p → -r, q → -p, r → p + q + r};
f6[2] = f1[2] /. {p → -r, q → -p, r → p + q + r};
f6[3] = f1[3] /. {p → -r, q → -p, r → p + q + r};
F6 = Sum[f6[i], {i, 3}];
f7[1] = f1[1] /. {p → -r, q → p + q + r, r → -q};
f7[2] = f1[2] /. {p → -r, q → p + q + r, r → -q};
f7[3] = f1[3] /. {p → -r, q → p + q + r, r → -q};
F7 = Sum[f7[i], {i, 3}];
f8[1] = f1[1] /. {p → p, q → -p - q - r, r → q};
f8[2] = f1[2] /. {p → p, q → -p - q - r, r → q};
f8[3] = f1[3] /. {p → p, q → -p - q - r, r → q};
F8 = Sum[f8[i], {i, 3}];
(p + q + r) f1[1] / F1 - p f2[1] / F2 // Factor // Simplify
q f3[1] / F3 - (p + q + r) f4[1] / F4 // Factor // Simplify
-r f5[1] / F5 - q f6[1] / F6 // Factor // Simplify
p f7[1] / F7 - (-r) f8[1] / F8 // Factor // Simplify
p + q - (p + q + r) f1[1] / F1 - q f3[1] / F3 - (-r) f5[1] / F5 - p f7[1] / F7 // Factor //
 Simplify
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QA-Points-nb.pdf

Message: #299
Date: 16/10/2013 1:39:05
From: seiichikiri
Subject: Algebraic computation of quadratic, cubic and quartic quadrangle

---

```
Dear friends,

Sorry!
Correction:
The last quartic quadrangle point degenerates to QAP21,
because (p(q^2+r^2+qr)+q^3+r^3+2qr(q+r)) = (q^2+r^2+qr)(p+q+r).

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #300
Date: 17/10/2013 9:04:04
From: eckart_schmidt@t-online.de
Subject: Tangential quadrilateral well known properties?

```
Dear Bernard K.,

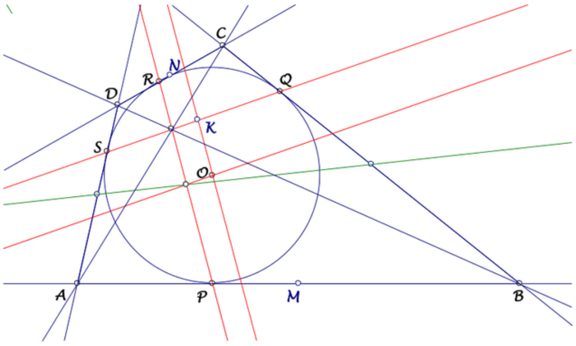
up to now I haven't found a synthetic prove for the discussed
property of a tangential quadrigon, but  "... the evening crowns
the day."
In the attachment there is a clearly laid out calculation with
barycentric coordinates, perhaps of interest.

Best regards Eckart
```

Re: Tangential quadrilateral well known properties?

(*QFG* message 26)

Mosca Sebastiano:
ABCD is a circumscribed quadrilateral,
PQRS the points of tangency with the inscribed circle center *O*,
M and *N* are the midpoints of *AB* and *CD*,
K is the intersection of *SQ* and *MN*
Prove *OK* is parallel to *PR*.



Here is a simple calculation with barycentric coordinates, using the *QA*-Diagonal Triangle of the quadrigon *PQRS* as reference triangle:

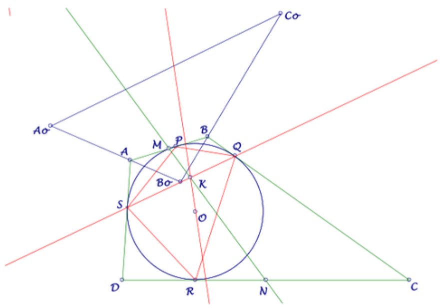
Let $A_0B_0C_0$ be the reference triangle for barycentric coordinates, obtuse-angled at B_0.
Use the polar circle ([1], p.37) of the reference triangle with the equation

$$S_A x^2 + S_B y^2 + S_C z^2 = 0$$

for a cyclic quadrigon *PQRS*:

$$P(u:v:w),\quad Q(-u:v:w),\quad R(u:-v:w),\quad S(u:v:-w)$$

$$\text{with}\quad v^2 = \frac{S_A u^2 + S_C w^2}{-S_B}\ .$$



Mosca.pdf

The tangents in P, Q, R, S have the intersections

$$A(S_B v : -S_A u : 0), \quad B(0 : S_C w : -S_B v),$$
$$C(S_B v : S_A u : 0), \quad D(0 : S_C w : S_B v) \ .$$

The midpoints of AB and CD are

$$M(S_B v(S_B v - S_C w) : -S_A S_B uv + 2S_A S_C uw - S_B S_C vw : S_B v(S_B v - S_A u))$$
$$N(S_B v(S_B v + S_C w) : S_A S_B uv + 2S_A S_C uw + S_B S_C vw : S_B v(S_B v + S_A u)) \, .$$

This gives the intersection of QS and MN

$$K(-u : \frac{S_A S_B u^2 + (S^2 + S_A S_C) uw + S_B S_C w^2}{S_B(S_A u - S_C w)} : w) \, .$$

The center of the polar circle is the orthocenter of the reference triangle $O(S_B S_C : S_C S_A : S_A S_B)$.

The common point at infinity of KO and PR is $(u : -u - w : w)$, so KO and PR are parallel.

[1] http://forumgeom.fau.edu/FG2004volume4/FG200405.pdf

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Mosca.pdf

Message: #301
Date: 17/10/2013 10:06:15
From: Chris
Subject: Algebraic computation of quadratic, cubic and quartic quadrangle

---

Dear Seiichi,

Very interesting approach.
Nice to see which QA-points can be deduced using a systematic algebraic approach for composing barycentric coordinates.
It appears that most "simple" points are QA-P1, QA-P10 and QA-P16.
Your quartic case
f(p,q,r)=(2p+q+r)(p(q^2+r^2+qr)+q^3+r^3+2qr(q+r)) can be simplified to f(p,q,r)=(p+q+r)(2p+q+r)(q^2+qr+r^2), which is QA-P21.
Not strange that also QA-P21 pops up (Reflection of QA-P16 in QA-P1).
For as far as I can see you only constructed points as a function f(p,q,r) and not as f(p,q,r,a,b,c).
Could be interesting to investigate these kind of points too.

Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #302
Date: 17/10/2013 1:09:07
From: seiichikiri
Subject: Algebraic computation of quadratic, cubic and quartic quadrangle

---

```
Dear Chris,

Thank you very much for your comments.
I have very clear aims:
(1) to make an example of an infinite sequence of QA-points.
(2) to find some complex QA-points.
As for the latter aim, the equations which I obtained are the
quadratic ones of many variables. At present, I think that no
one can say that there are no complex QA-points. So, for my
aims, I use a simpler function f(p, q, r). But there are many
possibilities that what I said turns out blunders.

Best regards,
Seiichi.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #303
Date: 19/10/2013 10:40:17
From: bernard.keizer
Subject: Tangential quadrilateral well known properties?

Dear Eckart,

Thank you for your calculations.
Is it possible, with the same referential, to prove following properties:
O is the orthocenter of the diagonal triangle see my message #251.
The QA PQRS and the tangential QL (4 lines are the sides of the QA ABCD) have the same diagonal triangle (see my message #270).
The Miquel point of the inscribed QA is the inverse of B0 wrt the inscribed circle and lies on A0C0
O is on the Newton Line of the tangential QL.
M' and N' being the middles of AD and BC and K' being the intersection of PR and M'N', KK' is orthogonal to this Newton Line (see my message #275).

Best regards
Bernard K.

← Previous Next → ↔ Message Index ↑ Subjects

Message: #304
Date: 19/10/2013 11:06:28
From: bernard.keizer
Subject: Precisions concerning the curve QL-Cu1

Dear Eckart,

I've just realised that my two points A and A', which have as tangential the infinity point of the Newton Line (and QL-P1 as the CL-S conjugate of this infinity point lies on AA') are precisely the points you called P1 and P2, the centers of the 2 inversions in which QL-Cu1 is invariant (anallagmatic curve).
For any QL having A and A' as opposite vertices, the 4 other vertices B, B' C and C' are concyclic and the circumcenter lies on the bisector of the CL-S transformation, orthogonal to AA' in QL-P1.
Among all these QL's is the one I called main QL of the curve QL-Cu1, with the points QL-2Pa and b as middles of BB' and CC' and the circumcenter as excenter of the triangle QL-P1 and QL-2P2a and b, as you told me in a former message.
Among these QL's is also the one I called orthogonal QL, with diagonal orthogonal to the sides; this time, the circumcenter, always on the bisector of the CL-S transformation, is the middle of BC. But there are at least 2 solutions, maybe more ...
I haven't found on which conditions the main is also orthogonal, but it must be a special QL-Cu1, as it isn't true an ordinary one.

Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

Message: #305
Date: 19/10/2013 11:24:33
From: bernard.keizer
Subject: La Géométrie du Quadrilatère Complet

```
Dear Chris, dear Eckart, dear friends of the Quadri-forum,
The technical part took finally more time than I thought, but
now it's done ...
I've just put on the web an article about The Geometry of the
QL.
The link is http://bernardkeizer.blogspot.fr/
You may recognise some items already discussed in this forum as
Eckart'cubic of the centers of the 27 cardioïds inscribed in the
QL and many references to EQF.
Thanks in advance for all your remarks and critics.
Best regards
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #306
Date: 20/10/2013 11:52:22
From: eckart_schmidt@t-online.de
Subject: Tangential quadrilateral well known properties?

Dear Bernard,

congratulations on your remarkable paper "La Géométrie du Quadrilatère Complet"! Shame on me, that I had only a look on some chapters.
Your properties wrt the tangential quadrigon can easily be proved with my calculation:

"O is on the Newton Line of the tangential QL"
... evident, for the Newton line is the focus of centers
    of inscribed conics.

"The QA PQRS and the tangential QL (4 lines are the sides of the QA ABCD) have the same diagonal triangle (see my message #270)"
... have a look on the coordinates of P, Q, R, S, A, B, C, D.

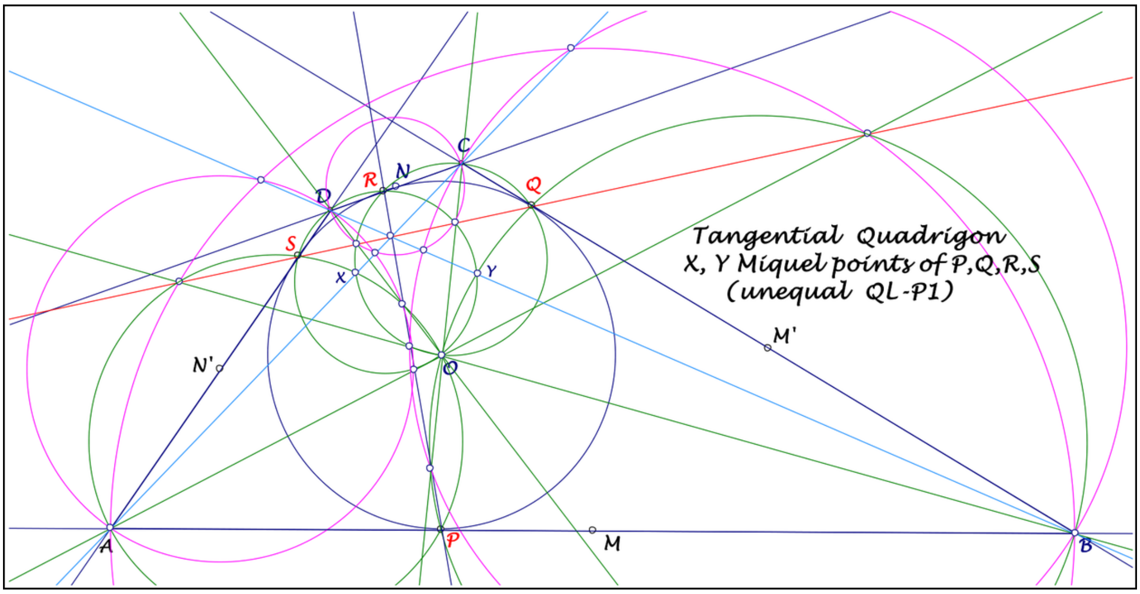
"O is the orthocenter of the diagonal triangle see my message #251"
... already mentioned in my calculation.

"The Miquel point of the inscribed QA is the inverse of B0 wrt the inscribed circle and lies on A0C0"
... this is a basic property of cyclic quadrigons. The Miquel point is QG-P17 of PQRS.

"M' and N' being the middles of AD and BC andK' being the intersection of PR and M'N',
KK' is orthogonal to this Newton Line (see my message #275)"
... this seems somewhat remarkable! If the circle is replaced by a conic, KK' and the Newton line give conjugated diameters.

It's a pity, that there are no connections with midpoints!
But there is a glimmer of hope (see attachment): There are a lot of intersections wrt a tangential quadrigon, if you consider the following elements:
Lines: Angle bisectors and diagonals of the quadrigon ABCD and diagonals of the quadrigon PQRS.
Circles: Incircle of ABCD and Thales circles of AB, BC, CD, DA, OA, OB, OC, OD.
There are 15 new intersections of more than two elements. Among these points are the Miquel points of P,Q,R,S (unequal QL-P1 of the quadrigon PQRS). The corresponding angle bisector of the Miquel triangle of P,Q,R,S contains the diagonal crosspoint of ABCD and the center O of the incircle.

```
Best regards Eckart
```



Keizer-07.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #307
Date: 21/10/2013 8:33:19
From: seiichikiri
Subject: QA-points at infinity

```
Dear friends,

[1] QA-point: f(p,q,r)=(2p+q+r)(q+r)(h(p+q)(p+r)+kqr).
Examples:
(1) QA-P25: (2p+q+r)(q+r)(2p^2+2pq+2pr+3qr).
(2) QA-P26: (2p+q+r)(q+r)(5p^2+5pq+5pr+3qr).
Cyclic sum of f(p,q,r)=(4h+k)(p+q)(q+r)(r+p)(p+q+r). If 4h+k=0,
it becomes zero.
A point at infinity: (2p+q+r)(q+r)(p^2+pq+pr-3qr).

[2] QA-point: f(p,q,r)=(2p+q+r)p(q+r)(h(p+q)(p+r)+k(q^2+r^2).
h=1 and k=2 gives QA-P31.
Cyclic sum of
f(p,q,r)=(2h+k)(p+q)(q+r)(r+p)(p^2+q^2+r^2+qr+rp+pq). If 2h+k=0,
it becomes zero.
A point at infinity: (2p+q+r)p(q+r)(p^2+pq+pr+qr-2q^2-2r^2).

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #308
Date: 22/10/2013 10:27:44
From: eckart_schmidt@t-online.de
Subject: Tangential quadrilateral well known properties?

```
Dear Bernard K.,

some remarks perhaps a help for a synthetic prove:

*Mosca Sebastiano:*
ABCD is a circumscribed quadrilateral,
PQRS the points of tangency with the inscribed circle center O
M and N are the midpoints of AB and CD
K is the intersection of SQ and NM
T is the common diagonal crosspoint of ABCD and PQRS
Prove OK is parallel to PR.

Consider the angles of ABCD
and let the radius of the incircle be 1:
... QS = 2 sin(A+B)/2) = sin(C+D)/2)
... TQ /TS = (cos(B/2) cos(C/2)) / (cos(D/2) cos(A/2))
... KQ /KS = cos((B-C)/2) / cos((D-A)/2)
... KT = / cos((A+D)/2) / sin((B+D)/2) / = ...
... KO = / cos((A+B)/2) / sin((B+D)/2) / = ...
... Let A', B' be the intersections of the Thales circle
    about AB with QS;
    let C', D' be the intersections of the Thales circle
    about CD with QS:
    KA' / KB' = KC' / KD' = K'N' / K'M' = DA / BC =
                     (sin(B/2) sin(C/2)) / (sin(D/2) sin(A/2))
... The midpoints of the concyclic points
    P,M,A',B' and R,N,C',D' are collinear with K.
These properties can be calculated with cartesian coordinates as
in message #27 or proved with CABRI.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

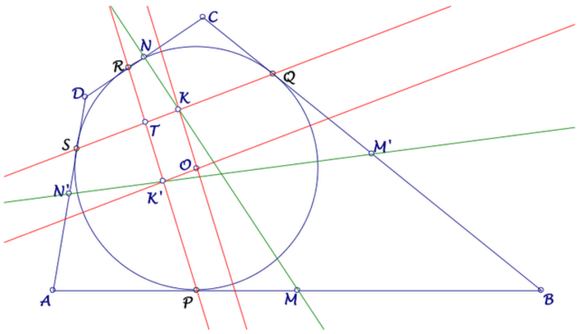
Message: #309
Date: 26/10/2013 1:34:36
From: eckart_schmidt@t-online.de
Subject: Tangential quadrilateral well known properties?

```
Dear Bernard K., dear Chris,
in the attachment you find a synthetic proof for the discussed
properties. I hope, there are no basic mistakes. Perhaps someone
can make it shorter!
Best regards Eckart
```

Re: Tangential quadrilateral well known properties?

(*QFG* message 26)

Mosca Sebastiano:
ABCD is a circumscribed quadrilateral,
PQRS the points of tangency with the inscribed circle center *O*,
M and *N* are the midpoints of *AB* and *CD*,
K is the intersection of *SQ* and *MN*
Prove *OK* is parallel to *PR*.



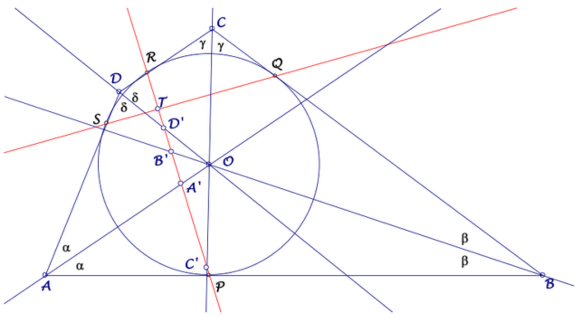
Synthetic Proof

(1) Let the radius of the incircle be 1 and the angles of *ABCD* be $2\alpha, 2\beta, 2\gamma, 2\delta$ with $\alpha+\beta+\gamma+\delta=180°$.

(2) Side lengths of *ABCD*:
$AB = AP + PB = \cot\alpha + \cot\beta$,
$BQ = BQ + QC = \cot\beta + \cot\gamma$, ...

(3) Side lengths of *PQRS*:
$PQ = 2\cos\beta$, $QR = 2\cos\gamma$, ...

(4) Angles:
$\angle TPA = 180° - \delta - \alpha$, $\angle TQB = 180° - \alpha - \beta$, ...
and $\angle PTQ = \alpha + \gamma$.



Mosca-synth.pdf

(5) First distances on *PR* (law of sines):

$$PT=\frac{2\cos\alpha\cos\beta}{\sin(\alpha+\gamma)},\ RT=\frac{2\cos\gamma\cos\delta}{\sin(\alpha+\gamma)},\ PR=2\sin(\beta+\gamma).$$

(6) Further points on *PR*. These points will be used, to calculate *PK'*, *RK'*.

$A'=PR\cap OA,\quad B'=PR\cap OB,\quad ...$

(7) Distances of these points on *PR*:

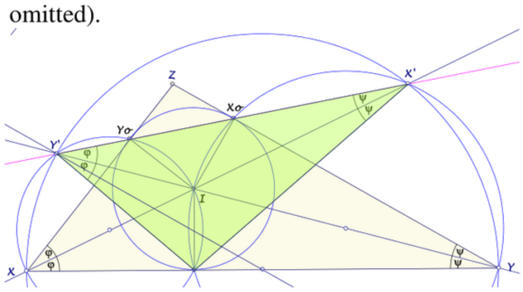
In the triangle *POA'* with

$OP=1,\quad \angle A'OP=90°-\alpha,\quad \angle PA'O=180°-\delta$

the law of sines gives $PA'=\frac{\cos\alpha}{\sin\delta}$.

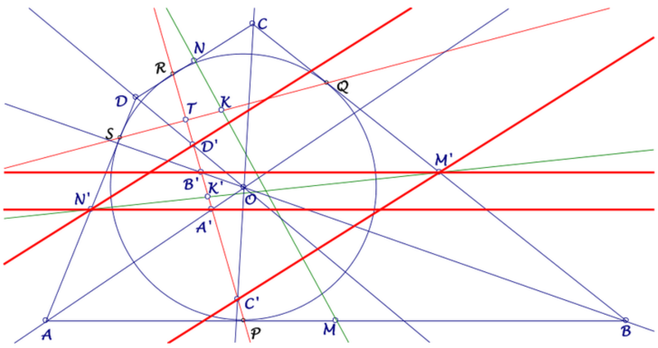
Analog: $PB'=\frac{\cos\beta}{\sin\gamma},\quad RC'=\frac{\cos\gamma}{\sin\beta},\quad RD'=\frac{\cos\delta}{\sin\alpha}$.

(8) Lemma: In a triangle *XYZ* with incircle (or excircle), touching the sides in X_o, Y_o, Z_o, the angle bisectors *XI / YI* cut the line X_oY_o in points *X' / Y'*, which lie on the Thales circle about *XY* as well as on the Thales circle about *YI / XI*. The radii of *X' / Y'* wrt the Thales circle about *XY* are parallel to the sidelines *XZ / YZ*. This can be verified with help of the following drawing (here omitted).



(9) Projecting these properties in the constellation of the tangential quadrigon *ABCD*, we get the following parallels:

$M'B'$ *parallel* $N'A'$ and $M'C'$ *parallel* $N'D'$.



(10) Intercept theorems give:

$$\frac{K'A'}{K'B'} = \frac{K'N'}{K'M'} = \frac{K'D'}{K'C'}.$$

(11) Further distances on PR: Replace in (10)

$K'A' = PK' - PA'$, $K'B' = PB' - PK'$,

$K'C' = RC' - RK'$, $K'D' = RK' - RD'$

and $PK' + RK' = PR$.

Then we get with (5) and (7):

$$PK' = \frac{\cos(\alpha-\beta)}{\sin(\alpha+\gamma)} \quad and \quad RK' = \frac{\cos(\gamma-\delta)}{\sin(\alpha+\gamma)}.$$

(12) An interesting consequence of (10) and (11) is:

$$\frac{K'N'}{K'M'} = \frac{DA}{BC}.$$

(13) Now we can calculate $TK' = RK' - RT$ (see (11) and (5)); analog TK:

$$\boxed{TK' = \frac{\cos(\alpha+\beta)}{\sin(\alpha+\gamma)} \quad and \quad TK = \frac{\cos(\beta+\gamma)}{\sin(\alpha+\gamma)}}.$$

(14) The law of chords give $K'O^2 = 1 - PK' \cdot RK'$ (see (11)); analog KO^2:

$$\boxed{K'O^2 = \frac{\cos^2(\beta+\gamma)}{\sin^2(\alpha+\gamma)} \quad and \quad KO^2 = \frac{\cos^2(\alpha+\beta)}{\sin^2(\alpha+\gamma)}}$$

This completes the proof.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

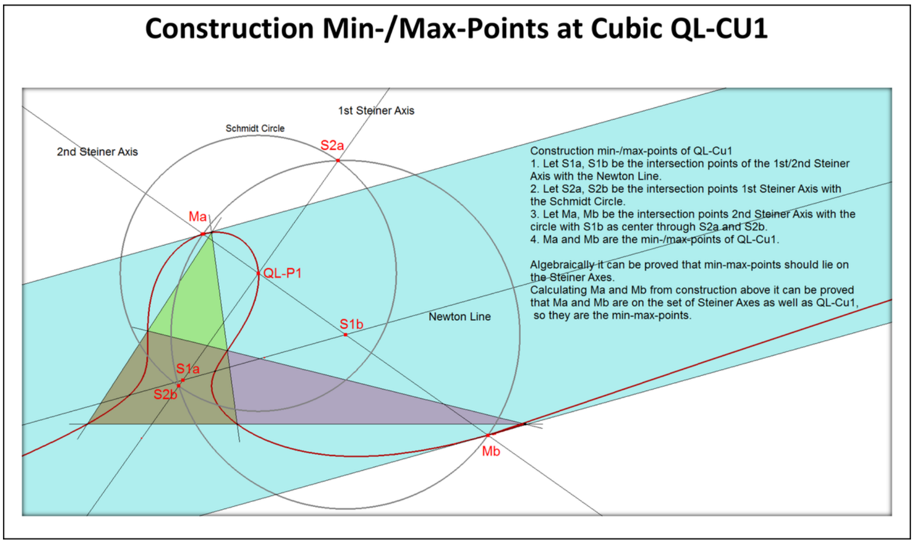
Mosca-synth.pdf

Message: #310
Date: 27/10/2013 4:02:06
From: Chris van Tienhoven
Subject: Two points on the Newton Line QL-Cu1 revisited

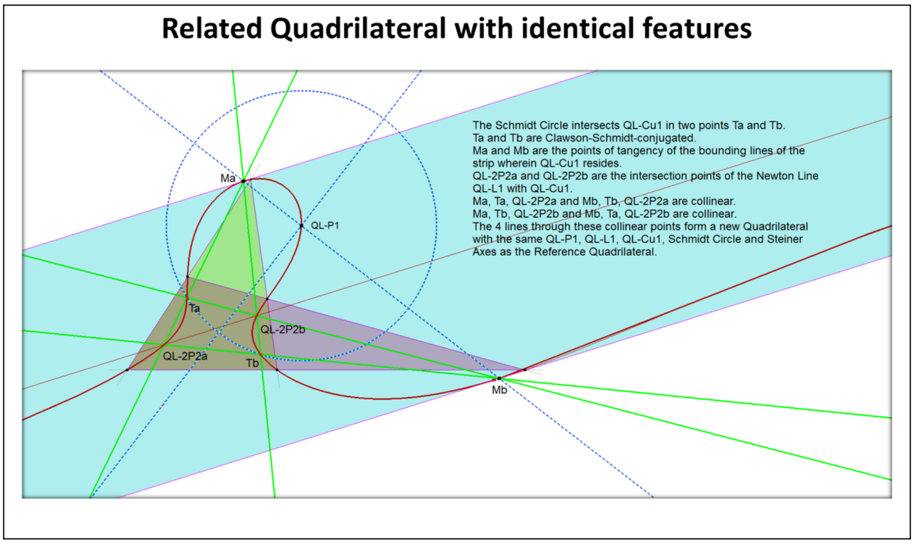
Dear friends,

All points on QL-Cu1 lie within a stroke bounded by two lines parallel to the Newton Line on equal distances of the Newton Line.
These two bounding lines touch QL-Cu1 in 2 points.
In the last phrase of message #193 I wondered how to identify these points.
I found a method for constructing these points.
They lie on the 2nd Steiner Axis as well as a special circle.
See QL-Tf1 for explanation *Steiner Axes and Schmidt Circle*:
http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-mathematics/quadrilateral-objects/artikelen-ql/159-ql-tf1.html.
See also attachment.
I calculated the CT-coordinates for these points. Unfortunately they are very lengthy.
They are not collinear with other QL-Points (except QL-P1), neither do they occur on other known QL-lines or QL-curves (except one of the Steiner Axes).
Last but not least, here is an extra method for constructing Clawson-Schmidt Conjugates (QL-Tf1) of points P on QL-Cu1.
* Let Ln = line through P parallel to the Newton Line QL-L1.
* Let Lm = line connecting P and the Miquel Point QL-P1
* Let Ln’ = Ln reflected in the Newton Line QL-L1.
* Let Lm’ = Lm reflected in the 1st Steiner Line.
* Now QL-Tf1(P) = Ln’ ^ Lm’.
Note: This method is only valid for points on QL-Cu1.

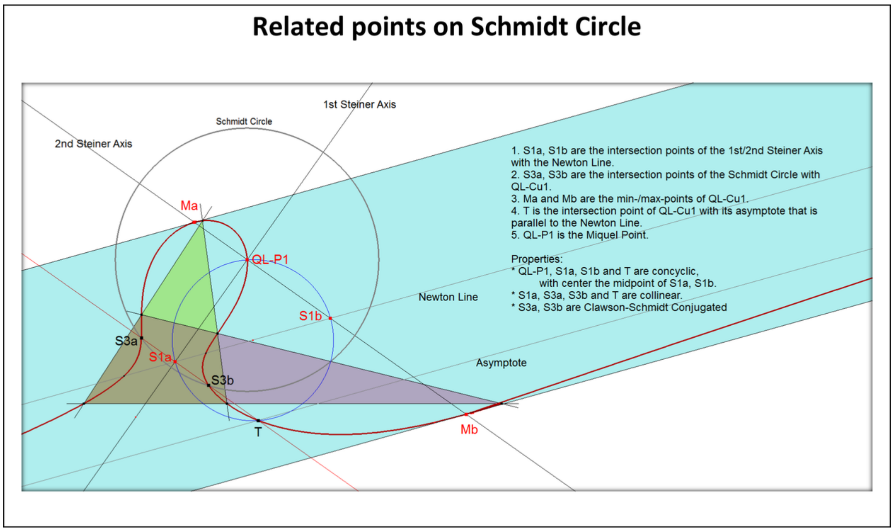
Best regards,
Chris



QL-Cu1-MinMax-points.pdf



QL-Cu1-MinMax-points.pdf



QL-Cu1-MinMax-points.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #311
Date: 28/10/2013 3:49:33
From: eckart_schmidt@t-online.de
Subject: Inscribed Parallelograms of a Quadrigon

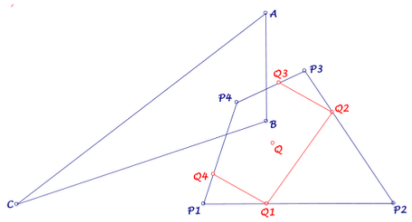
Dear friends,
I think, it can be of interest: Inscribed parallelograms as rhombi, rectangles, squares. So I have made a summary of an article on my homepage.
Best regards Eckart

EQF-Note 2013-10-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Inscribed Parallelograms of a Quadrigon

For any point P in the plane of a quadrigon there is an inscribed parallelogram with center P. The centers of inscribed rhombi and inscribed rectangles lie on special conics. Their intersections give the centers of inscribed squares (see QG-L6 in EQF). This paper is a summary of
http://eckartschmidt.de/Pgive.pdf
Reference triangle for barycentric coordinates is QA-DT.



Inscribed Parallelograms

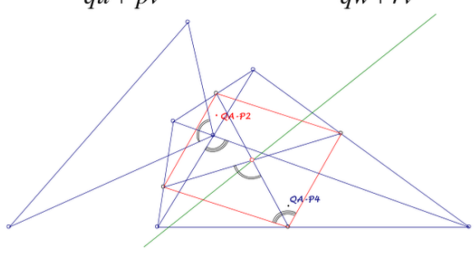
Let $P_1P_2P_3P_4$ be a quadrigon with diagonal triangle ABC as reference triangle for barycentric coordinates:

$$P_1(-p,q,r),\quad P_2(p,-q,r),\quad P_3(p,q,-r),\quad P_4(p,q,r)\,.$$

For a parallelogram center $Q(u:v:w)$ the inscribed parallelogram $Q_1Q_2Q_3Q_4$ can be constructed and calculated as follows: The intersection of P_3P_4 with the reflection of P_1P_2 in Q gives Q_3, and so on:

$$Q_1(p:-q:\frac{q^2u+p^2v+pqw}{qu-pv}),\quad Q_2(\frac{r^2v+q^2w+qru}{qw-rv}:-q:r)$$

$$Q_3(p:q:\frac{-q^2u-p^2v+pqw}{qu+pv}),\quad Q_4(\frac{-r^2v-q^2w+qru}{qw+rv}:q:r)\,.$$



2013-10-28.pdf

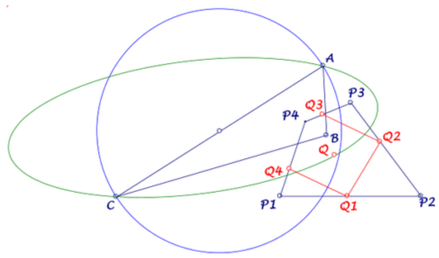
A special example is mentioned in *EQF*: The pedal quadrigon of the Isogonal Center *QA-P4* is a parallelogram. This parallelogram is a representative of parallelograms with center on the perpendicular bisector of *QA-P2.QA-P4*. These parallelograms have angles as in the intersection of the diagonals of the quadrigon.
The inscribed parallelogram degenerates for centers on the Nine-point Conic *QA-Co1*.

Inscribed Rhombi

A rhombus is a parallelogram with equal side lengths. The centers of inscribed rhombi lie on a conic with equation

$$q^2(p^2S_A yz - q^2S_B zx + r^2S_C uv) + r^2p^2b^2y^2 = 0\,.$$

This conic is the Involutary Conjugate *QA-Tf2* of the Thales circle about the third diagonal *AC* of the quadrigon.

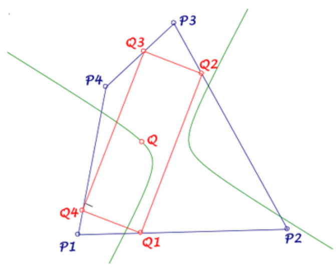


Special examples are two inscribed rhombi with sides parallel to the diagonals; the vertices divide the sides of the quadrigon in the ratio of the diagonal lengths.

Inscribed Rectangles

For rectangles the right angularity gives the equation for the centers:

$$a^2q^4r^2x^2 + b^2p^2r^2(p^2 - r^2)y^2 - c^2p^2q^4z^2 + 2p^2q^2r^2y(S_Cx - S_Az) = 0$$



This conic contains the midpoints of the diagonals, for which the rectangles degenerate collinear. The center of the conic is the midpoint of the Euler-Poncelet Point *QA-P2* and the Isogonal Center *QA-P4*.

2013-10-28.pdf

Inscribed Squares

There are two real intersections of the conics for the centers of inscribed rhombi and rectangles:

$$Q^{\pm}(p(S_A p \pm Sr : S_B q^2 : r(S_C r \pm Sp)).$$

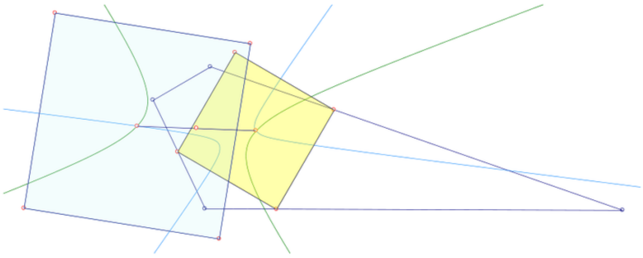
These centers of inscribed squares lie symmetric to the center of the conic for the centers of inscribed rhombi. Their connection is the Inscribed Square Axis *QG-L6* (see *EQF*). The vertices of the inscribed squares are:

$$Q_1^{\pm}(p:-q:\frac{S_C r^2 + c^2 pq \pm Sr(p+q)}{S_A p - S_B q \pm Sr}),$$

$$Q_2^{\pm}(\frac{S_A p^2 + a^2 qr \pm Sp(q+r)}{S_C r - S_B q \pm Sp}:-q:r),$$

$$Q_3^{\pm}(p:q:\frac{S_C r^2 - c^2 pq \pm Sr(p-q)}{S_A p + S_B q \pm Sr}),$$

$$Q_4^{\pm}(\frac{S_A p^2 - a^2 qr \pm Sp(r-q)}{S_C r + S_B q \pm Sp}:q:r).$$



For further properties have a look on my homepage 11.2.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-10-28.pdf

Message: #312
Date: 29/10/2013 9:47:36
From: eckart_schmidt@t-online.de
Subject: Two points on the Newton Line QL-Cu1 revisited

```
Dear Chris,

if QL-Cu1 is bipartite, there are two further points on QL-Cu1
with tangents parallel and symmetric to the Newton line. These
four points lie two by two on the 1st and 2nd Steiner line.They
are the in- and excenters of a triangle QL-P1.F1.F2, when F1, F2
are the foci of an inscribed conic with center QL-L1 ^ QL-L6.
Background is, that QL-Cu1 in the bipartite case is an isogonal
pivotal circular cubic with reference triangle
QL-P1.F1.F2.

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #313
Date: 31/10/2013 3:10:46
From: eckart_schmidt@t-online.de
Subject: Involutary Conjugate QA-Tf2

```
Dear Chris,

what about this construction for QA-Tf2?
Let P be a point and P1, P2, P3, P4 the points of a quadrangle
with P2P3^P4P1=Si and P1P2^P3P4=Sj,
then the fourth intersection of the conics through P,P1,P3,Si,Sj
and P,P2,P4,Si,Sj is QA-Tf2(P).

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #314
Date: 31/10/2013 7:16:50
From: Chris van Tienhoven
Subject: Involutary Conjugate QA-Tf2

```
Very nice!
Our insight and knowledge is growing every day!
Best regards,  Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #315
Date: 02/11/2013 6:15:51
From: Seiichi Kirikami
Subject: Numerically confirmed quadrangle points with a property of angles

---

```
Dear friends,

I studied quadrangle points with a property similar to, but a
bit different from QA-P4 in EQF.
Given a quadrangle P1P2P3P4 and a point P{x, y, z}, P has the
property:
   angle P1PP2 = angleP1P4P+anglePP3P2,
   angle P2PP3 = angleP2P1P+anglePP4P3,
   angle P3PP4 = angleP3P2P+anglePP1P4,
   angle P4PP1 = angleP4P3P+anglePP2P1.
In order to determine P, I used the tangent method as shown in
p4 of "Ein merkwuergiger Punkt des Vierecks" by Daniel
baumgartner and Roland Staerk, Praxis der Mathematik 1/44 Jg,
2002 of message #250 and obtained 4 quartic equations
(kr1,kr2,kr3 and kr4) of x, y and z.
With 2 equations (for example, kr1and kr2) and x+y+z=1, I
computed the coordinates of P numerically and confirmed that
they coincided with each other.
I used the values (a=6, b=9, c=13, p=3, q=23, r=-10) and
obtained 4 vertices, 2 real coordinates and 2 complex
coordinates as solutions. I confirmed the relations of angles by
Cabri in case of real solutions. See the attached file.

Best regards,
Seiichi.
```

```
(* QA-PX similar to, a bit different from QAP4 *)
Simplificar[lista_] := Simplify[lista / Apply[PolynomialGCD, lista]];
Punto[r_, s_] := Simplificar[Cross[r, s]];
Recta[P_, Q_] := Simplificar[Cross[P, Q]];
DividirRazon[ptP_, ptQ_, m_, n_] := Simplificar[n Tr[ptQ] ptP + m Tr[ptP] ptQ];
Medio[ptP_, ptQ_] := DividirRazon[ptP, ptQ, 1, 1];
PuntoInfinito[{p_, q_, r_}] := {q - r, r - p, p - q};
PuntoInfinitoPerpendicular[{f_, g_, h_}] := PuntoInfinito[{SA f, SB g, SC h}] /.
   {SA → (b^2 + c^2 - a^2)/2, SB → (c^2 + a^2 - b^2)/2, SC → (a^2 + b^2 - c^2)/2};
Perpendicula[P_, r_] := Recta[P, PuntoInfinitoPerpendicular[PuntoInfinito[r]]];
Perpendic[P_, Q_, R_] := Perpendicula[P, Recta[Q, R]];
Mediatriz[ptP_, ptQ_] := Perpendic[Medio[ptP, ptQ], ptP, ptQ];
Circuncentro[ptA_, ptB_, ptC_] :=
  Punto[Mediatriz[ptA, ptB], Mediatriz[ptA, ptC]];
Tg[r_, s_] := (S Tr[Cross[r, s]]) / ((r[[2]] - r[[3]]) (s[[2]] - s[[3]]) SA +
      (r[[3]] - r[[1]]) (s[[3]] - s[[1]]) SB + (r[[1]] - r[[2]]) (s[[1]] - s[[2]]) SC) /.
   {S → Sqrt[2 a^2 b^2 + 2 b^2 c^2 + 2 c^2 a^2 - a^4 - b^4 - c^4] / 2,
    SA → (b^2 + c^2 - a^2)/2, SB → (c^2 + a^2 - b^2)/2, SC → (a^2 + b^2 - c^2)/2};

(* 4 vertices *)
pt1 = {0, 1, 0};
pt2 = {0, 0, 1};
pt3 = {1, 0, 0};
pt4 = {p, q, r};
ptX = {x, y, z};

(* lines *)
rt14 = Recta[pt1, pt4]
rt12 = Recta[pt1, pt2]
rt23 = Recta[pt2, pt3]
rt34 = Recta[pt3, pt4]
rt24 = Recta[pt2, pt4]
rt13 = Recta[pt1, pt3]
rt1X = Recta[pt1, ptX]
rt2X = Recta[pt2, ptX]
rt3X = Recta[pt3, ptX]
rt4X = Recta[pt4, ptX]

(* curves *)

kr1 = Numerator[Factor[Tg[rt1X, rt2X] - (Tg[rt14, rt4X] + Tg[rt3X, rt23]) / (1 - Tg[rt14, rt4X] × Tg[rt3X, rt23])]] /.
   {a → 6, b → 9, c → 13, p → 3, q → 23, r → -10} // Simplify
kr2 = Numerator[Factor[Tg[rt2X, rt3X] - (Tg[rt12, rt1X] + Tg[rt4X, rt34]) / (1 - Tg[rt12, rt1X] × Tg[rt4X, rt34])]] /.
```

QA-PX-nb.pdf

```
    {a → 6, b → 9, c → 13, p → 3, q → 23, r → -10} // Simplify
kr3 = Numerator[Factor[Tg[rt3X, rt4X] - (Tg[rt23, rt2X] + Tg[rt1X, rt14])/(1 - Tg[rt23, rt2X] × Tg[rt1X, rt14])]] /.
    {a → 6, b → 9, c → 13, p → 3, q → 23, r → -10} // Simplify
kr4 = Numerator[Factor[Tg[rt4X, rt1X] - (Tg[rt34, rt3X] + Tg[rt2X, rt12])/(1 - Tg[rt34, rt3X] × Tg[rt2X, rt12])]] /.
    {a → 6, b → 9, c → 13, p → 3, q → 23, r → -10} // Simplify
N[Solve[kr1 == 0 && kr2 == 0 && x + y + z == 1, {x, y, z}] // Factor // Simplify, 20]
N[Solve[kr2 == 0 && kr3 == 0 && x + y + z == 1, {x, y, z}] // Factor // Simplify, 20]
N[Solve[kr3 == 0 && kr4 == 0 && x + y + z == 1, {x, y, z}] // Factor // Simplify, 20]
N[Solve[kr4 == 0 && kr1 == 0 && x + y + z == 1, {x, y, z}] // Factor // Simplify, 20]
```

{r, 0, -p}

{1, 0, 0}

{0, 1, 0}

{0, -r, q}

{-q, p, 0}

{0, 0, 1}

{z, 0, -x}

{-y, x, 0}

{0, -z, y}

{-r y + q z, r x - p z, -q x + p y}

$-32\sqrt{35}\,\left(-81\,x^3\,(27\,040\,y+13\,911\,z)+x^2\left(1\,245\,361\,y^2-23\,733\,y\,z-154\,062\,z^2\right)+108\,y\,z\left(467\,y^2+3107\,y\,z+1296\,z^2\right)+3\,x\left(78\,923\,y^3+625\,661\,y^2\,z+419\,337\,y\,z^2+79\,299\,z^3\right)\right)$

$32\sqrt{35}\,\left(48\,281\,x^3\,(169\,y+81\,z)-108\,y\,z\left(2237\,y^2+634\,y\,z-4099\,z^2\right)+x^2\left(1\,245\,361\,y^2+9\,851\,858\,y\,z+3\,619\,161\,z^2\right)-36\,x\left(27\,040\,y^3+7493\,y^2\,z-104\,971\,y\,z^2-29\,808\,z^3\right)\right)$

$-32\sqrt{35}\,\left(-81\,x^3\,(27\,040\,y+13\,911\,z)+x^2\left(1\,245\,361\,y^2-23\,733\,y\,z-154\,062\,z^2\right)+108\,y\,z\left(467\,y^2+3107\,y\,z+1296\,z^2\right)+3\,x\left(78\,923\,y^3+625\,661\,y^2\,z+419\,337\,y\,z^2+79\,299\,z^3\right)\right)$

$32\sqrt{35}\,\left(48\,281\,x^3\,(169\,y+81\,z)-108\,y\,z\left(2237\,y^2+634\,y\,z-4099\,z^2\right)+x^2\left(1\,245\,361\,y^2+9\,851\,858\,y\,z+3\,619\,161\,z^2\right)-36\,x\left(27\,040\,y^3+7493\,y^2\,z-104\,971\,y\,z^2-29\,808\,z^3\right)\right)$

QA-PX-nb.pdf

```
{{x → 0, y → 0, z → 1.0000000000000000000},
 {x → 1.0000000000000000000, y → 0, z → 0}, {x → 0, y → 1.0000000000000000000, z → 0},
 {x → 0.18750000000000000000, y → 1.4375000000000000000, z → -0.62500000000000000000},
 {x → -0.75382293859371643960, y → 0.71837161774204917366, z → 1.0354513208516672659},
 {x → 0.35431835598286679878, y → 0.95098681673900111121, z → -0.30530517272186790999},
 {x → -0.19975229130542482041 - 0.21591078091826634944 i,
  y → 0.83467921724052514243 - 0.88436359840345461817 i,
  z → 0.36507307406489967798 + 1.1002743793217209676l i},
 {x → -0.19975229130542482041 + 0.21591078091826634944 i,
  y → 0.83467921724052514243 + 0.88436359840345461817 i,
  z → 0.36507307406489967798 - 1.1002743793217209676l i}}
```

```
{{x → 0, y → 0, z → 1.0000000000000000000},
 {x → 1.0000000000000000000, y → 0, z → 0}, {x → 0, y → 1.0000000000000000000, z → 0},
 {x → 0.18750000000000000000, y → 1.4375000000000000000, z → -0.62500000000000000000},
 {x → -0.75382293859371643960, y → 0.71837161774204917366, z → 1.0354513208516672659},
 {x → 0.35431835598286679878, y → 0.95098681673900111121, z → -0.30530517272186790999},
 {x → -0.19975229130542482041 - 0.21591078091826634944 i,
  y → 0.83467921724052514243 - 0.88436359840345461817 i,
  z → 0.36507307406489967798 + 1.1002743793217209676l i},
 {x → -0.19975229130542482041 + 0.21591078091826634944 i,
  y → 0.83467921724052514243 + 0.88436359840345461817 i,
  z → 0.36507307406489967798 - 1.1002743793217209676l i}}
```

```
{{x → 0, y → 0, z → 1.0000000000000000000},
 {x → 1.0000000000000000000, y → 0, z → 0}, {x → 0, y → 1.0000000000000000000, z → 0},
 {x → 0.18750000000000000000, y → 1.4375000000000000000, z → -0.62500000000000000000},
 {x → -0.75382293859371643960, y → 0.71837161774204917366, z → 1.0354513208516672659},
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 {x → -0.19975229130542482041 - 0.21591078091826634944 i,
  y → 0.83467921724052514243 - 0.88436359840345461817 i,
  z → 0.36507307406489967798 + 1.1002743793217209676l i},
 {x → -0.19975229130542482041 + 0.21591078091826634944 i,
  y → 0.83467921724052514243 + 0.88436359840345461817 i,
  z → 0.36507307406489967798 - 1.1002743793217209676l i}}
```

```
{{x → 0, y → 0, z → 1.0000000000000000000},
 {x → 1.0000000000000000000, y → 0, z → 0}, {x → 0, y → 1.0000000000000000000, z → 0},
 {x → 0.18750000000000000000, y → 1.4375000000000000000, z → -0.62500000000000000000},
 {x → -0.75382293859371643960, y → 0.71837161774204917366, z → 1.0354513208516672659},
 {x → 0.35431835598286679878, y → 0.95098681673900111121, z → -0.30530517272186790999},
 {x → -0.19975229130542482041 - 0.21591078091826634944 i,
  y → 0.83467921724052514243 - 0.88436359840345461817 i,
  z → 0.36507307406489967798 + 1.1002743793217209676l i},
 {x → -0.19975229130542482041 + 0.21591078091826634944 i,
  y → 0.83467921724052514243 + 0.88436359840345461817 i,
  z → 0.36507307406489967798 - 1.1002743793217209676l i}}
```

QA-PX-nb.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #316
Date: 02/11/2013 8:56:56
From: seiichikiri
Subject: Numerically confirmed quadrangle points with a property of angle

---

```
Dear friends,
The attached file will open by Mathematica, if you add ".nb" to
it.
Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #317
Date: 03/11/2013 9:52:54
From: eckart_schmidt@t-online.de
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Seiichi,
your discussed points are the fixed points of the
Clawson-Schmidt Conjugate QL-Tf1 for a quadrigon, that are the
intersections of the 1st Steiner axis and the Schmidt circle.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #318
Date: 03/11/2013 1:28:07
From: seiichikiri
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Eckart,

I am very glad that you have determined the points, which I
could only compute numerically, such that they are suitable in
EQF.

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #319
Date: 04/11/2013 9:47:47
From: Eisso J. Atzema
Subject: Question about a Quadrilateral

Colleagues,

I apologize beforehand for triple posting, but I have a somewhat urgent question. While working on a paper, I found a curious 16th century Indian attempt at a derivation for the area formula of a cyclic quadrilateral. The derivation is incorrect, but it is fairly easy to fix it using modern means. The resulting proof is quite interesting, but I just think I have seen it before. I only cannot remember where. Can anyone help me out? The proof goes as follows:
Let ABCD be a (convex) cyclic quadrilateral, with AB of length a, BC of length b and so on. Now, going either clockwise or counter-clockwise, let A' on AB be such that AA'=(a+c)/2, B' on BC such that BB'=(b+c)/2 and so on. Then, it is straightforward to see that A'B'C'D' is a rectangle with the same area as ABCD. Furthermore, it can be shown that the sides of this rectangle equal e/f*sqrt{(s-a)(s-c)} and f/e*sqrt{(s-b)(s-d)}, with e is the length of AC and f the length of BD (where I might have switched the role of e and f). Thus, the area of ABCD equals sqrt{(s-a)(s-b)(s-c)(s-d)}.
Has anyone seen this proof before? Please provide me with a reference, however vague. Also, basically my verification for the lengths of the rectangle is a tedious computation. Might there be a nicer and shorter way to verify these lengths?

Thanks so much,
Eisso

← Previous Next → ↔ Message Index ↑ Subjects

Message: #320
Date: 04/11/2013 9:56:13
From: eckart_schmidt@t-online.de
Subject: 3 QG-circumconics for a QL and a fixed point

Dear friends,

for a quadrilateral with its 3 quadrigon components there are 3 QG-circumconics through a fixed point. These conics have 3 collinear fourth intersections. The geometry of the corresponding line is discussed in the attachment, leading to a new type of QL-cubics.

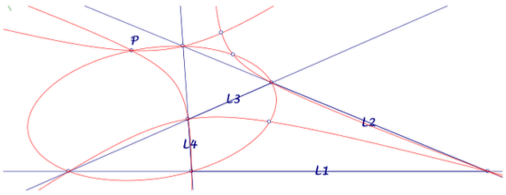
Best regards Eckart

EQF-Note 2013-11-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Three *QG*-Circumconics for a Quadrilateral

For the three QG-components of a quadrilateral there are circumconics through a fixed point with three collinear fourth intersections. The corresponding line will be discussed here, leading to a new type of QL-cubics. − Reference triangle for barycentric coordinates is QL-DT.



The three *QG*-Circumconics and their Intersections

For a quadrilateral with lines

$$L_1(-l,m,n),\quad L_2(l,-m,n),\quad L_3(l,m,-n),\quad L_4(l,m,n)$$

there are the intersections

$$Q_{12}(m:l:0),\quad Q_{23}(0:n:m),\quad Q_{34}(-m:l:0),$$
$$Q_{41}(0:-n:m),\quad Q_{13}(n:0:l),\quad Q_{24}(-n:0:l)$$

and the quadrigon components

$$Q_{12}Q_{23}Q_{34}Q_{41},\quad Q_{12}Q_{24}Q_{34}Q_{13},\quad Q_{23}Q_{24}Q_{41}Q_{13}.$$

Let $P(u:v:w)$ be an arbitrary point, then the *QG*-circumconics through *P* have the equations

$$(l^2u^2-m^2v^2+n^2w^2)zx-(l^2x^2-m^2y^2+n^2z^2)wu=0,$$
$$(l^2u^2+m^2v^2-n^2w^2)xy-(l^2x^2+m^2y^2-n^2z^2)uv=0,$$
$$(-l^2u^2+m^2v^2+n^2w^2)yz-(-l^2x^2+m^2y^2+n^2z^2)vw=0.$$

The fourth intersections of two of these conics give

$$(m^2uv:l^2u^2-n^2w^2:-m^2vw),$$
$$(n^2uw:-n^2vw:l^2u^2-m^2v^2),$$
$$(n^2w^2-m^2v^2:-l^2uv:l^2wu).$$

These intersections are the Involutary Conjugates *QA-Tf2* of *P* wrt the *QG*-components. They are collinear on a line with the coefficients

$$L_P(l^2u(-l^2u^2+m^2v^2+n^2w^2),$$
$$m^2v(l^2u^2-m^2v^2+n^2w^2),n^2w(l^2u^2+m^2v^2-n^2w^2)).$$

2013-11-04.pdf

Three Examples

P* = *QL-P1: The coefficients of $L_{QL\text{-}P1}$ are extensive. The line is the Clawson-Schmidt Conjugate *QL-Tf1* of the Dimidium Circle *QL-Ci6* containing *QL-P26*.

P* = *QL-P8: The line $L_{QL\text{-}P8}$ has the coefficients

$$L_{QL-P8}(l^2(-l^2+m^2+n^2), m^2(l^2-m^2+n^2), n^2(l^2+m^2-n^2))$$

and is the second asymptote of *QL-Co2* containing *QL-P23*.

P* = *QL-P13: The line $L_{QL\text{-}P13}$ has the coefficients

$$L_{QL-P13}(l^2m^2-m^2n^2+n^2l^2, l^2m^2+m^2n^2-n^2l^2, -l^2m^2+m^2n^2+n^2l^2)$$

and is a parallel to *QL-L9* through *QL-P19*.

For points *P* on L_i the line L_P is L_i again. For the vertices of the diagonal triangle *QL-DT* the line is the opposite sideline of *QL-DT*. For the midpoints of the sides of *QL-DT* the line containes the opposite vertex and *QL-P13*.

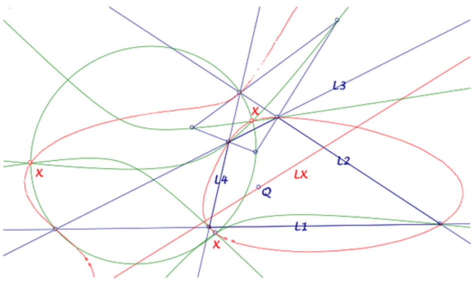
Further Properties

For a line *L* there are always three points *X* with $L_X = L$. These points are the common intersections of the conics, which are the *QA-Tf2* images of *L* wrt the *QG*-components.

For all lines through a fixed point $Q(x_o : y_o : z_o)$ these three points lie on a cubic with the equation

$$l^4x_ox^3 + m^4y_oy^3 + n^4z_oz^3$$
$$-l^2m^2(x_oy+xy_o)xy - m^2n^2(y_oz+yz_o)yz - n^2l^2(z_ox+zx_o)zx = 0.$$

This cubic is the locus for all points *X*, whose L_X containes *Q*, passing through the six points of the quadrilateral.



If *Q* is a vertex of *QL-DT*, this conic is *QG-Co2* of the correspondent *QG*-component.

If *Q* = *QL-P13*, this cubic has the equation

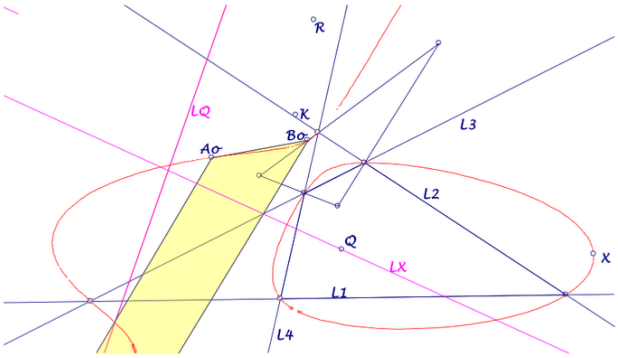
$$l^2(-x+y+z)x^2 + m^2(x-y+z)y^2 + n^2(x+y-z)z^2 = 0.$$

and is a circumcubic of the medial triangle of *QL-DT* , isotomic invariant wrt this triangle.

A new Type of *QL*-Cubics

The general case of the locus for points X, whose L_X containes a fixed point $Q(x_o : y_o : z_o)$

- … is a nonpivotal isocubic of type nK containing the points of the quadrilateral.
- The reference triangle $A_oB_oC_o$ for this cubic has its vertices in the third intersections of the cubic with the sidelines of *QL-DT*:
  $A_o(0:n^2z_o:m^2y_o)$, $B_o(n^2z_o:0:l^2x_o)$, $C_o(m^2y_o:l^2x_o:0)$.
  $A_oB_oC_o$ is the cevian triangle wrt *QL-DT* of a point
  $$K(m^2n^2y_oz_o:l^2n^2z_ox_o:l^2m^2x_oy_o),$$
  which is the image of Q wrt a *QL-DT*-isoconjugation with fixed points in the trilinear poles of L_i.
- The isoconjugation wrt $A_oB_oC_o$ for the cubic has fixed point K.
- The root R of the cubic is the trilinear pole of L_Q wrt $A_oB_oC_o$. The coordinates are extensive.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-11-04.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #321
Date: 04/11/2013 9:58:20
From: Eisso J. Atzema
Subject: Question about a Quadrilateral (Minor Correction)

```
> On Mon, Nov 4, 2013 at 9:47 AM, Eisso J. Atzema <
atzema@math.umaine.edu >
> wrote:
>> Colleagues,
>> I apologize beforehand for triple posting, but I have a
>> somewhat urgent question. While working on a paper,
>> I found a curious 16th century Indian attempt at a derivation
>> for the area formula of a cyclic quadrilateral.
>> The derivation is incorrect, but it is fairly easy to fix it
>> using modern means. The resulting proof is quite interesting,
>> but I just think I have seen it before.
>> I only cannot remember where.
>> Can anyone help me out? The proof goes as follows:
>> Let ABCD be a (convex) cyclic quadrilateral, with AB of
>> length a, BC of length b and so on. Now, let A' on the ray
>> from A to B be such that AA'=(a+c)/2, B' on the ray
>> from B to C such that BB'=(b+c)/2 and so on.
>> Then, it is straightforward to see that A'B'C'D' is a
>> rectangle with the same area as ABCD. Furthermore,
>> it can be shown that the sides of this rectangle equal
>> e/f*sqrt{(s-a)(s-c)} and f/e*sqrt{(s-b)(s-d)},
>> with e is the length of AC and f the length of BD
>> (where I might have switched the role of e and f).
>> Thus, the area of ABCD equals sqrt{(s-a)(s-b)(s-c)(s-d)}.
>> Has anyone seen this proof before? Please provide me with a
>> reference, however vague. Also, basically my verification for
>> the lengths of the rectangle is a tedious computation.
>> Might there be a nicer and shorter way to verify these
>> lengths?
>> Thanks so much,
>> Eisso
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #322
Date: 05/11/2013 12:15:21
From: eckart_schmidt@t-online.de
Subject: Question about a Quadrilateral

Dear Mister Atzema,

indeed no reference, but perhaps a help: see attachment.

Best regards Eckart

Re: Question about a Quadrilateral

(*QFG* message 319, 321)

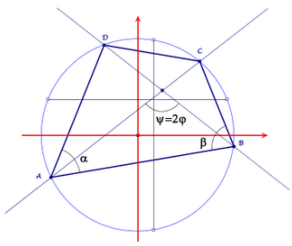
"Let ABCD be a (convex) cyclic quadrilateral, with AB of length a, BC of length b and so on. Now, let A' on the ray from A to B be such that AA'=(a+c)/2, B' on the ray from B to C such that BB'=(b+c)/2 and so on. Then, it is straightforward to see that A'B'C'D' is a rectangle with the same area as ABCD."

Let *ABCD* be a cyclic quadrigon wrt a circle of radius 1. The quadrigon shall be defined by the following angles:

$$\angle DAB = \alpha, \quad \angle ABC = \beta, \quad \angle ASB = \psi = 2\varphi,$$

$$(\alpha \le \beta \le 90°,\ S \text{ diagonal crosspoint}).$$

We can choose a cartesian coordinate system, orientated at the midpoints of the arcs (see figure). Advantage: The axes will be parallel to the sides of your rectangle.



The vertices have the coordinates:

$$A(-\cos(\beta-\varphi);-\sin(\beta-\varphi)), \quad \mathrm{B}(\cos(\alpha-\varphi);-\sin(\alpha-\varphi)),$$

$$C(-\cos(\beta+\varphi);\sin(\beta+\varphi)), \quad D(\cos(\alpha+\varphi);\sin(\alpha+\varphi))$$

with the following distances

$$a = 2\cos(\frac{\alpha+\beta-\psi}{2}), \quad b = 2\cos(\frac{-\alpha+\beta+\psi}{2}),$$

$$c = -2\cos(\frac{\alpha+\beta+\psi}{2}), \quad d = 2\cos(\frac{\alpha-\beta+\psi}{2}),$$

$$e = 2\sin\beta, \quad f = 2\sin\alpha.$$

The vertices of the discussed rectangle can be calculated:

$$A'(-\cos\beta\cos\varphi+\sin\alpha\sin\varphi;-\sin\beta\cos\varphi+\cos\alpha\sin\varphi),$$

$$B'(-\cos\beta\cos\varphi+\sin\alpha\sin\varphi;\sin\beta\cos\varphi+\cos\alpha\sin\varphi),$$

$$C'(-\cos\beta\cos\varphi-\sin\alpha\sin\varphi;\sin\beta\cos\varphi+\cos\alpha\sin\varphi),$$

$$D'(-\cos\beta\cos\varphi-\sin\alpha\sin\varphi;-\sin\beta\cos\varphi+\cos\alpha\sin\varphi).$$

The side lengths of the rectangle are

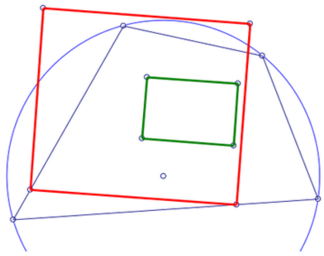
$$AB = CD = 2\sin\beta\cos\varphi, \quad BC = DA = 2\sin\alpha\sin\varphi$$

and the area $2\sin\alpha\sin\beta\sin\psi$.

Atzema.pdf

Additional remark: There is a further rectangle with parallel sides wrt the discussed rectangle, built up of the incenters of the triangles *ABC, BCD, CDA, DAB*. This rectangle has the side lengths

$$\sin\frac{\alpha+\beta}{2}-\cos\varphi \quad and \quad \cos\frac{\alpha-\beta}{2}-\sin\varphi.$$



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Atzema.pdf

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #323
Date: 08/11/2013 4:00:06
From: eckart_schmidt@t-online.de
Subject: Some ”splitter”

Dear Chris, dear friends,

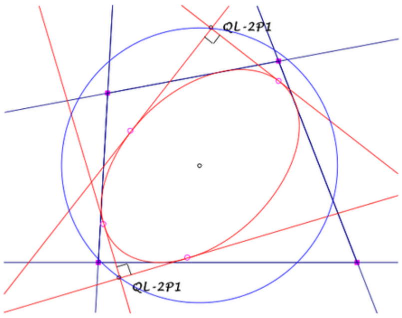
I had a look in "Encyklopädie der Mathematischen Wissenschaften"
and got some suggestions, concerning:
* further properties of the Plücker Points QL-2P1,
* angle bisectors of pairs of tangents wrt QL-inscribed conics
* inscribed QL-conics with maximum product of axes.
... see attachment.

Best regards Eckart

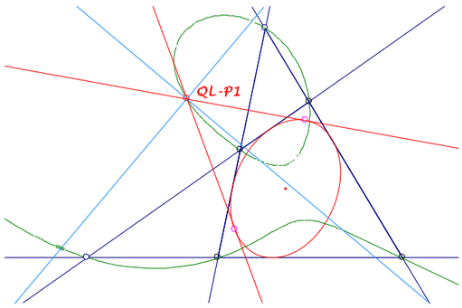
Some „Splitter“

(08.11.2013)

- The Plücker Points *QL-2P1* (which are the common points of the Thales circles about the *QL*-diagonals) are the intersections of the directrix circles of the inscribed *QL*-conics.
  For the Plücker Points the tangents to inscribed conics are orthogonal.

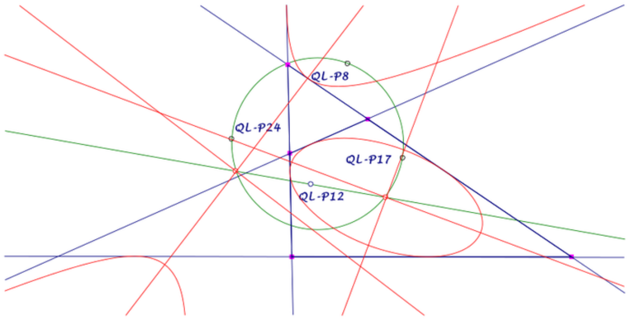


- Pairs of tangents from a fixed point on *QL-Cu1* to *QL*-inscribed conics have the same angle bisectors. For *QL-P1* these bisectors are the *1st* and *2nd* Steiner axis (see *QL-Tf1*).



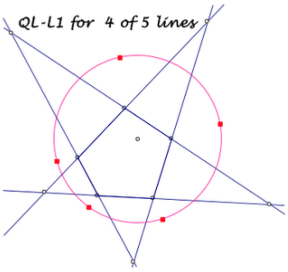
- There are two inscribed *QL*-conics (an ellipse and a hyperbola) with maximum product of axes. The centers lie symmetric to *QL-P12* in the intersections of *QL-L1* and a circle through *QL-P8, QL-P17, QL-P24*.

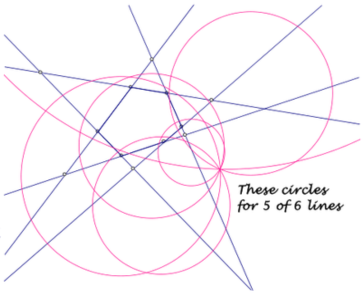
Splitter 13-11-08.pdf



- (Special for Chris, extending *EQF* for n = 5, 6, …, perhaps well known?):

  2 lines give a point (intersection).
  These points for 2 of 3 lines give a circle (circumcircle).
  These circles for 3 of 4 lines give a point (Miquel point).
  These points for 4 of 5 lines give a circle (…).
  These circles for 5 of 6 lines give a point (…) …





Suggestions out of

Dingeldey, Friedrich,
Kegelschnitte und Kegelschnittsysteme.
(Encyklopädie der Mathematischen Wissenschaften,
Bd.III/2, Heft 1).
Teubner Leipzig,1903

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Splitter 13-11-08.pdf

Message: #324
Date: 09/11/2013 10:53:23
From: Seiichi Kirikami
Subject: The perspector of 2 triangles made from 3 QA version Pedal lines and

---

Dear friends,

Given a quadrangle P1P2P3P4, 2 triangles made from 3 QA version Pedal lines and Steiner lines has a perspector.
Its barycentrics: See the last line of the attached file.
It might be too complicated.

Best regards,
Seiichi.

P. S. If the attachment does not have ".nb", add ".nb" to it.

```
(* Perspector of triangles of Pedal and Steiner lines *)
Simplificar[lista_] := Simplify[lista / Apply[PolynomialGCD, lista]];
Punto[r_, s_] := Simplificar[Cross[r, s]];
Recta[P_, Q_] := Simplificar[Cross[P, Q]];
DividirRazon[ptP_, ptQ_, m_, n_] := Simplificar[n Tr[ptQ] ptP + m Tr[ptP] ptQ];
Medio[ptP_, ptQ_] := DividirRazon[ptP, ptQ, 1, 1];
PuntoInfinito[{p_, q_, r_}] := {q - r, r - p, p - q};
PuntoInfinitoPerpendicular[{f_, g_, h_}] := PuntoInfinito[{SA f, SB g, SC h}] /.
   {SA → (b^2 + c^2 - a^2)/2, SB → (c^2 + a^2 - b^2)/2, SC → (a^2 + b^2 - c^2)/2};
Perpendicula[P_, r_] := Recta[P, PuntoInfinitoPerpendicular[PuntoInfinito[r]]];
Perpendic[P_, Q_, R_] := Perpendicula[P, Recta[Q, R]];
Mediatriz[ptP_, ptQ_] := Perpendic[Medio[ptP, ptQ], ptP, ptQ];
Circuncentro[{ptA_, ptB_, ptC_}] :=
  Punto[Mediatriz[ptA, ptB], Mediatriz[ptA, ptC]];
Ortocentro[{ptA_, ptB_, ptC_}] :=
  Punto[Perpendic[ptA, ptB, ptC], Perpendic[ptB, ptC, ptA]];
Pie[P_, r_] := Punto[Perpendicula[P, r], r];
SimetriaAxial[P_, r_] := DividirRazon[P, Pie[P, r], 2, -1];
SimetriaAxiala[P_, Q_, R_] := SimetriaAxial[P, Recta[Q, R]];

(* 4 vertices *)
pt1 = {0, 1, 0};
pt2 = {0, 0, 1};
pt3 = {1, 0, 0};
pt4 = {p, q, r};

(* lines and cross points *)
rt12 = Recta[pt1, pt2];
rt13 = Recta[pt1, pt3];
rt14 = Recta[pt1, pt4];
rt23 = Recta[pt2, pt3];
rt24 = Recta[pt2, pt4];
rt34 = Recta[pt3, pt4];
ptS1 = Punto[rt13, rt24];
ptS2 = Punto[rt12, rt34];
ptS3 = Punto[rt23, rt14];

(* circumcenters *)
ct11 = Circuncentro[{pt2, pt3, ptS2}];
ct12 = Circuncentro[{pt3, pt4, ptS3}];
ct21 = Circuncentro[{pt4, pt2, ptS3}];
ct22 = Circuncentro[{pt2, pt3, ptS1}];
ct31 = Circuncentro[{pt3, pt4, ptS1}];
ct32 = Circuncentro[{pt4, pt2, ptS2}];

(* Miquel points *)
ptM1 = SimetriaAxiala[pt3, ct11, ct12];
ptM2 = SimetriaAxiala[pt2, ct21, ct22];
```

PedalSteinerLines-nb.pdf

```
ptM3 = SimetriaAxiala[pt4, ct31, ct32];

(* perpendicular *)
ppdM11 = Pie[ptM1, rt23];
ppdM12 = Pie[ptM1, rt34];
ppdM21 = Pie[ptM2, rt24];
ppdM22 = Pie[ptM2, rt23];
ppdM31 = Pie[ptM3, rt34];
ppdM32 = Pie[ptM3, rt24];

(* Pedal lines *)
ped1 = Recta[ppdM11, ppdM12];
ped2 = Recta[ppdM21, ppdM22];
ped3 = Recta[ppdM31, ppdM32];

(* Pedal triangle *)
ped12 = Punto[ped1, ped2];
ped23 = Punto[ped2, ped3];
ped31 = Punto[ped3, ped1];

(* Orthocenters *)
ot11 = Ortocentro[{pt2, pt3, ptS2}];
ot12 = Ortocentro[{pt3, pt4, ptS3}];
ot21 = Ortocentro[{pt4, pt2, ptS3}];
ot22 = Ortocentro[{pt2, pt3, ptS1}];
ot31 = Ortocentro[{pt3, pt4, ptS1}];
ot32 = Ortocentro[{pt4, pt2, ptS2}];

(* Steiner lines *)
stn1 = Recta[ot11, ot12];
stn2 = Recta[ot21, ot22];
stn3 = Recta[ot31, ot32];

(* Steiner triangle *)
stn12 = Punto[stn1, stn2];
stn23 = Punto[stn2, stn3];
stn31 = Punto[stn3, stn1];

(* connecting lines *)
rtA = Recta[ped12, stn12]
rtB = Recta[ped23, stn23]
rtC = Recta[ped31, stn31]
Det[{rtA, rtB, rtC}] // Factor // Simplify
Punto[rtA, rtB]
```

PedalSteinerLines-nb.pdf

$$
\begin{aligned}
&\{-a^4 (p+q) r (p+r)^2 - \\
&\quad (q+r) \left(c^4 (p+r)^2 (2p+2q+r) + b^4 (p+q) \left(2p^2+4pq+2q^2+5pr+6qr+5r^2\right) + \right. \\
&\qquad \left. b^2 c^2 \left(4p^3+2q^3+4q^2 r+q r^2+r^3+p^2 (8q+6r)+3p\left(2q^2+3qr+r^2\right)\right)\right) + \\
&\quad a^2 \left(c^2 (p+r)^2 \left(2pq+2q^2+3pr+4qr+r^2\right) + b^2 \left(p^3 (2q+r) + p^2 \left(6q^2+6qr-r^2\right) + \right.\right. \\
&\qquad\qquad \left.\left. p\left(6q^3+11q^2 r+2q r^2-2r^3\right) + q\left(2q^3+6q^2 r+3q r^2-2r^3\right)\right)\right), \\
&\ -a^4 (p+r) \left(p^3+2p^2 (q+2r) + qr (3q+5r) + p\left(q^2+7qr+5r^2\right)\right) + \\
&\quad (q+r) \left(b^4 (p+q) (p-r) (p+q+r) + c^4 (p+r) \left(5p^2 + r (q+r) + p (3q+4r)\right) + \right. \\
&\qquad \left. b^2 c^2 \left(2p^3 + p^2 (3q-r) + p\left(q^2+qr-2r^2\right) + r\left(q^2-r^2\right)\right)\right) + \\
&\quad a^2 \left(-c^2 \left(p^2-r^2\right) \left(p^2+3pq+2q^2+4pr+3qr+r^2\right) + b^2 \right. \\
&\qquad\quad \left. \left(p^4 + p^3 (q+2r) + p^2 \left(-q^2+qr+r^2\right) - qr\left(q^2+3qr+2r^2\right) - p\left(q^3+2q^2 r+2q r^2+2r^3\right)\right)\right), \\
&\ a^4 (p+q) (p+r)^2 (p+2(q+r)) + (q+r) \\
&\quad\ \left(c^4 p (p+r)^2 + b^4 (p+q) \left(5p^2+2(q+r)^2+p(6q+5r)\right) + \right. \\
&\qquad \left. b^2 c^2 \left(2p^3 + p^2 (-3q+r) - 2q (q+r)^2 - p\left(6q^2+5qr+r^2\right)\right)\right) + \\
&\quad a^2 \left(-c^2 (p+r)^2 \left(p^2+2q(q+r)+p(4q+3r)\right) + \right. \\
&\qquad \left. b^2 (p+q) \left(p^3 + p^2 (q+3r) + p\left(4q^2+9qr+6r^2\right) + 2\left(q^3+3q^2 r+4q r^2+2r^3\right)\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
&\{a^4 q (p+q)^2 (p+r) + \\
&\quad (q+r) \left(b^4 (p+q)^2 (2p+q+2r) + c^4 (p+r) \left(2p^2+5pq+5q^2+4pr+6qr+2r^2\right) + \right. \\
&\qquad \left. b^2 c^2 \left(4p^3+q^3+q^2 r+4q r^2+2r^3+p^2 (6q+8r)+3p\left(q^2+3qr+2r^2\right)\right)\right) - \\
&\quad a^2 \left(b^2 (p+q)^2 \left(3pq+q^2+2pr+4qr+2r^2\right) + c^2 \left(p^3 (q+2r) + p^2 \left(-q^2+6qr+6r^2\right) + \right.\right. \\
&\qquad\qquad \left.\left. r\left(-2q^3+3q^2 r+6q r^2+2r^3\right) + p\left(-2q^3+2q^2 r+11q r^2+6r^3\right)\right)\right), -a^4 (p+q)^2 (p+r) \\
&\quad\ (p+2(q+r)) - (q+r) \left(b^4 p (p+q)^2 + c^4 (p+r) \left(5p^2+2(q+r)^2+p(5q+6r)\right) + \right. \\
&\qquad \left. b^2 c^2 \left(2p^3 + p^2 (q-3r) - 2r (q+r)^2 - p\left(q^2+5qr+6r^2\right)\right)\right) + \\
&\quad a^2 \left(b^2 (p+q)^2 \left(p^2+2r(q+r)+p(3q+4r)\right) - \right. \\
&\qquad \left. c^2 (p+r) \left(p^3 + p^2 (3q+r) + p\left(6q^2+9qr+4r^2\right) + 2\left(2q^3+4q^2 r+3q r^2+r^3\right)\right)\right), \\
&\ a^4 (p+q) (p+r) \left(p^2 + p (4q+r) + q (5q+3r)\right) - \\
&\quad (q+r) \left(c^4 (p-q) (p+r) (p+q+r) + b^4 (p+q) \left(5p^2 + q (q+r) + p (4q+3r)\right) + \right. \\
&\qquad \left. b^2 c^2 \left(2p^3 - q^3 - p^2 (q-3r) + q r^2 + p\left(-2q^2+qr+r^2\right)\right)\right) + \\
&\quad a^2 \left(b^2 \left(p^2-q^2\right) \left(p^2+4pq+q^2+3pr+3qr+2r^2\right) - \right. \\
&\qquad \left. c^2 (p+r) \left(p^3 + 2p^2 q + p\left(q^2-qr-r^2\right) - q\left(2q^2+3qr+r^2\right)\right)\right)\}
\end{aligned}
$$

$$
\begin{aligned}
&\{a^4 (p+q) (q-r) (p+r) (p+q+r) - (q+r) \left(b^2 c^2 (q-r) \left(2p^2+q^2+4qr+r^2+3p(q+r)\right) - \right. \\
&\qquad c^4 \left(p^2 (3q+r) + r\left(5q^2+4qr+r^2\right) + p\left(5q^2+7qr+2r^2\right)\right) + \\
&\qquad \left. b^4 \left(p^2 (q+3r) + q\left(q^2+4qr+5r^2\right) + p\left(2q^2+7qr+5r^2\right)\right)\right) + a^2 \\
&\quad\ \left(b^2 \left(-p^3 (q+r) - p^2 \left(q^2+2qr+3r^2\right) + p\left(q^3+q^2 r-2q r^2-2r^3\right) + q\left(q^3+2q^2 r+q r^2-2r^3\right)\right) + \right. \\
&\qquad c^2 \left(p^3 (q+r) + p^2 \left(3q^2+2qr+r^2\right) + \right. \\
&\qquad\qquad \left.\left. p\left(2q^3+2q^2 r-q r^2-r^3\right) - r\left(-2q^3+q^2 r+2q r^2+r^3\right)\right)\right), \\
&\ -a^4 (p+q) (p+r) \left(2p^2+4pq+2q^2+6pr+5qr+5r^2\right) - \\
&\quad \left(b^2-c^2\right) (q+r)^2 \left(b^2 (p+q) r - c^2 (p+r) (2p+2q+r)\right) + \\
&\quad a^2 \left(-c^2 (p+r) \left(2p^3+4q^3+6q^2 r+3q r^2+r^3+p^2 (6q+4r)+p\left(8q^2+9qr+r^2\right)\right) + b^2 \left(2p^4 + \right.\right. \\
&\qquad\qquad \left.\left. 6p^3 (q+r) + qr\left(q^2-qr-2r^2\right) + p^2\left(6q^2+11qr+3r^2\right) + 2p\left(q^3+3q^2 r+q r^2-r^3\right)\right)\right), \\
&\ a^4 (p+q) (p+r) \left(2p^2+6pq+5q^2+4pr+5qr+2r^2\right) + \\
&\quad \left(b^2-c^2\right) (q+r)^2 \left(-c^2 q (p+r) + b^2 (p+q) (2p+q+2r)\right) + \\
&\quad a^2 \left(-c^2 (p+r) \left(2p^3 + p^2 (6q+4r) + q\left(-2q^2-qr+r^2\right) + p\left(3q^2+5qr+2r^2\right)\right) + \right. \\
&\qquad \left. b^2 (p+q) \left(2p^3+q^3+3q^2 r+6q r^2+4r^3+p^2 (4q+6r)+p\left(q^2+9qr+8r^2\right)\right)\right)\}
\end{aligned}
$$

$$0$$

$$\{-a^8 (p+q)^2 (p+r)^2$$
$$(p^2 (q^2+6 q r+r^2) + q r (11 q^2+26 q r+11 r^2) + p (q^3+19 q^2 r+19 q r^2+r^3)) - 2 a^6 (p+q)$$
$$(p+r) (c^2 (p+r) (p^4 (3 q+r) + p^3 (10 q^2+7 q r - r^2) + p^2 (11 q^3+4 q^2 r - 14 q r^2 - 5 r^3) +$$
$$p (4 q^4 - 6 q^3 r - 28 q^2 r^2 - 21 q r^3 - 5 r^4) - r (4 q^4+9 q^3 r+10 q^2 r^2+7 q r^3+2 r^4)) +$$
$$b^2 (p+q) (p^4 (q+3 r) + p^3 (-q^2+7 q r+10 r^2) + p^2 (-5 q^3 - 14 q^2 r+4 q r^2+11 r^3) -$$
$$p (5 q^4+21 q^3 r+28 q^2 r^2+6 q r^3 - 4 r^4) - q (2 q^4+7 q^3 r+10 q^2 r^2+9 q r^3+4 r^4))) +$$
$$(b^2 - c^2) (q+r)^3 (-c^6 (p+r)^2 (2 p^3 + q r (q+r) + p^2 (7 q+3 r) + p (3 q^2+6 q r+r^2)) +$$
$$b^6 (p+q)^2 (2 p^3 + q r (q+r) + p^2 (3 q+7 r) + p (q^2+6 q r+3 r^2)) +$$
$$b^2 c^4 (p+r) (6 p^4 + 3 p^3 (7 q+5 r) + p^2 (29 q^2+43 q r+12 r^2) +$$
$$3 p (6 q^3+12 q^2 r+9 q r^2+r^3) + q (4 q^3+10 q^2 r+11 q r^2+5 r^3)) -$$
$$b^4 c^2 (6 p^5+21 p^4 (q+r) + p^3 (27 q^2+64 q r+29 r^2) + q r (5 q^3+11 q^2 r+10 q r^2+4 r^3) +$$
$$p^2 (15 q^3+70 q^2 r+65 q r^2+18 r^3) + p (3 q^4+32 q^3 r+47 q^2 r^2+28 q r^3+4 r^4))) +$$
$$a^4 (q+r) (c^4 (p+r)^2 (10 p^5 + p^4 (43 q+23 r) + 7 p^3 (9 q^2+8 q r+3 r^2) +$$
$$q r (2 q^3+4 q^2 r+3 q r^2+r^3) + p^2 (36 q^3+41 q^2 r+32 q r^2+7 r^3) +$$
$$p (6 q^4+12 q^3 r+17 q^2 r^2+8 q r^3 - r^4)) +$$
$$b^4 (p+q)^2 (10 p^5+p^4 (23 q+43 r) + 7 p^3 (3 q^2+8 q r+9 r^2) + q r (q^3+3 q^2 r+4 q r^2+2 r^3) +$$
$$p^2 (7 q^3+32 q^2 r+41 q r^2+36 r^3) + p (-q^4+8 q^3 r+17 q^2 r^2+12 q r^3+6 r^4)) +$$
$$2 b^2 c^2 (p+q) (p+r) (6 p^5+2 q^5+9 q^4 r+16 q^3 r^2+16 q^2 r^3+9 q r^4+2 r^5 +$$
$$23 p^4 (q+r) + p^3 (31 q^2+54 q r+31 r^2) + p^2 (19 q^3+51 q^2 r+51 q r^2+19 r^3) +$$
$$p (7 q^4+29 q^3 r+46 q^2 r^2+29 q r^3+7 r^4))) -$$
$$2 a^2 (q+r) (b^6 (p+q)^2 (4 p^5 + q^2 r (q+r)^2 + 4 p^4 (3 q+4 r) + 2 p^3 (8 q^2+17 q r+9 r^2) +$$
$$2 p q (q^3+6 q^2 r+8 q r^2+3 r^3) + p^2 (10 q^3+33 q^2 r+28 q r^2+5 r^3)) +$$
$$c^6 (p+r)^2 (4 p^5 + q r^2 (q+r)^2 + 4 p^4 (4 q+3 r) + 2 p^3 (9 q^2+17 q r+8 r^2) +$$
$$2 p r (3 q^3+8 q^2 r+6 q r^2+r^3) + p^2 (5 q^3+28 q^2 r+33 q r^2+10 r^3)) -$$
$$b^2 c^4 (p+r) (4 p^6+4 p^5 (3 q+5 r) + 2 p^4 (6 q^2+35 q r+23 r^2) + p^3 r (83 q^2+136 q r+53 r^2) -$$
$$q (q+r)^2 (2 q^3+q^2 r - q r^2 - 3 r^3) + p^2 (-11 q^4+26 q^3 r+119 q^2 r^2+110 q r^3+28 r^4) +$$
$$p (-9 q^5 - 10 q^4 r+26 q^3 r^2+57 q^2 r^3+35 q r^4+5 r^5)) -$$
$$b^4 c^2 (4 p^7+12 p^6 (2 q+r) + 2 p^5 (33 q^2+41 q r+6 r^2) + p^4 q (99 q^2+206 q r+95 r^2) +$$
$$q r (q+r)^2 (3 q^3+q^2 r - q r^2 - 2 r^3) + p^3 (81 q^4+246 q^3 r+202 q^2 r^2+26 q r^3 - 11 r^4) +$$
$$p^2 (33 q^5+145 q^4 r+176 q^3 r^2+52 q^2 r^3 - 21 q r^4 - 9 r^5) +$$
$$p (5 q^6+38 q^5 r+64 q^4 r^2+30 q^3 r^3 - 13 q^2 r^4 - 14 q r^5 - 2 r^6)))),$$
$$a^8 (p+q)^2 (p+r)^3 (p^2 (q+r) + p (3 q^2+6 q r+r^2) + q (2 q^2+7 q r+3 r^2)) -$$
$$2$$
$$a^6$$
$$(p+q)$$
$$(p+r)$$
$$(b^2 (p+q) (p^4 (2 q+r) + 2 p^3 (5 q^2+6 q r+r^2) + p^2 (16 q^3+33 q^2 r+16 q r^2+r^3) +$$
$$2 p q (6 q^3+17 q^2 r+14 q r^2+3 r^3) + q^2 (4 q^3+16 q^2 r+18 q r^2+5 r^3)) +$$
$$c^2 (p+r)^2 (4 q^4+14 q^3 r+16 q^2 r^2+9 q r^3+2 r^4+p^3 (2 q+3 r) +$$
$$p^2 (8 q^2+17 q r+6 r^2) + p (10 q^3+28 q^2 r+20 q r^2+5 r^3))) -$$
$$(q+r)^2 (-c^8 (p+r)^3 (p^2 (3 q+r) + q (2 q^2+3 q r+r^2) + p (7 q^2+6 q r+r^2)) +$$
$$b^8 (p+q)^2 (q r^2 (q+r) + p^3 (q+11 r) + p r (6 q^2+19 q r+11 r^2) + p^2 (q^2+19 q r+26 r^2)) +$$
$$2 b^6 c^2 (4 p^5 (q-r) + p^4 (15 q^2 - 10 q r - 9 r^2) + p^3 (21 q^3 - 2 q^2 r - 37 q r^2 - 10 r^3) +$$

$$p^2 (13 q^4 + 11 q^3 r - 42 q^2 r^2 - 31 q r^3 - 7 r^4) + q r (q^4 - q^3 r - 5 q^2 r^2 - 5 q r^3 - 2 r^4) +$$
$$p (3 q^5 + 8 q^4 r - 15 q^3 r^2 - 26 q^2 r^3 - 12 q r^4 - 2 r^5)) -$$
$$b^4 c^4 (2 p^5 (3 q + r) + 6 p^4 (6 q^2 + 3 q r + r^2) + p^3 (63 q^3 + 77 q^2 r + 29 q r^2 + 7 r^3) +$$
$$q r (10 q^4 + 23 q^3 r + 21 q^2 r^2 + 7 q r^3 - r^4) + p^2 (43 q^4 + 119 q^3 r + 73 q^2 r^2 + 25 q r^3 + 4 r^4) +$$
$$p (10 q^5 + 66 q^4 r + 77 q^3 r^2 + 39 q^2 r^3 + 7 q r^4 + r^5)) +$$
$$2 b^2 c^6 (p^4 (5 q^2 + 6 q r + r^2) + p^3 (18 q^3 + 33 q^2 r + 22 q r^2 + 3 r^3) +$$
$$2 q r (2 q^4 + 6 q^3 r + 8 q^2 r^2 + 5 q r^3 + r^4) + p^2 (16 q^4 + 52 q^3 r + 61 q^2 r^2 + 28 q r^3 + 3 r^4) +$$
$$p (4 q^5 + 28 q^4 r + 50 q^3 r^2 + 43 q^2 r^3 + 14 q r^4 + r^5))) +$$
$$2 a^2 (q + r) (-c^6 (p + r)^3 (2 p^4 + 2 q (q + r)^2 (2 q + r) + p^3 (9 q + 5 r) +$$
$$2 p^2 (8 q^2 + 10 q r + 3 r^2) + p (14 q^3 + 28 q^2 r + 17 q r^2 + 3 r^3)) +$$
$$b^6 (p + q)^2 (2 p^5 - q r (q + r)^2 (3 q + 4 r) + p^4 (5 q + 7 r) + p^3 (5 q^2 + 21 q r + 10 r^2) +$$
$$p^2 (q^3 + 14 q^2 r + 28 q r^2 + 9 r^3) - p (q^4 + 7 q^3 r + 4 q^2 r^2 - 6 q r^3 - 4 r^4)) +$$
$$b^4 c^2 (p + q) (p + r) (2 p^5 + p^4 (7 q + 9 r) + p^3 (19 q^2 + 29 q r + 16 r^2) +$$
$$(q + r)^2 (6 q^3 + 11 q^2 r + 3 q r^2 + 2 r^3) + p^2 (31 q^3 + 51 q^2 r + 46 q r^2 + 16 r^3) +$$
$$p (23 q^4 + 54 q^3 r + 51 q^2 r^2 + 29 q r^3 + 9 r^4)) -$$
$$b^2 c^4 (p + r) (2 p^6 + p^5 (9 q + 5 r) - 2 p^3 r (13 q^2 + 13 q r + 2 r^2) + p^4 (11 q^2 + 10 q r + 3 r^2) -$$
$$q (q + r)^2 (4 q^3 + 12 q^2 r + 18 q r^2 + 5 r^3) - p^2 (12 q^4 + 83 q^3 r + 119 q^2 r^2 + 57 q r^3 + 7 r^4) -$$
$$p (12 q^5 + 70 q^4 r + 136 q^3 r^2 + 110 q^2 r^3 + 35 q r^4 + 3 r^5))) +$$
$$a^4 (p + r) (-b^4 (p + q)^2 (p^4 (q - r) - p^3 (7 q^2 + 8 q r + 3 r^2) - p^2 (21 q^3 + 32 q^2 r + 17 q r^2 + 4 r^3) -$$
$$p (23 q^4 + 56 q^3 r + 41 q^2 r^2 + 12 q r^3 + 2 r^4) - q (10 q^4 + 43 q^3 r + 63 q^2 r^2 + 36 q r^3 + 6 r^4)) +$$
$$c^4 (p + r)^2 (p^4 (7 q + 9 r) + 3 p^3 (11 q^2 + 20 q r + 7 r^2) + p^2 (56 q^3 + 135 q^2 r + 94 q r^2 + 21 r^3) +$$
$$q (12 q^4 + 42 q^3 r + 56 q^2 r^2 + 33 q r^3 + 7 r^4) +$$
$$p (42 q^4 + 128 q^3 r + 135 q^2 r^2 + 60 q r^3 + 9 r^4)) +$$
$$2 b^2 c^2 (p^6 (5 q + 3 r) + p^5 (33 q^2 + 38 q r + 7 r^2) + p^4 (81 q^3 + 145 q^2 r + 64 q r^2 + 4 r^3) +$$
$$p^3 (99 q^4 + 246 q^3 r + 176 q^2 r^2 + 30 q r^3 - 3 r^4) +$$
$$p^2 (66 q^5 + 206 q^4 r + 202 q^3 r^2 + 52 q^2 r^3 - 13 q r^4 - 5 r^5) +$$
$$p (24 q^6 + 82 q^5 r + 95 q^4 r^2 + 26 q^3 r^3 - 21 q^2 r^4 - 14 q r^5 - 2 r^6) +$$
$$q (4 q^6 + 12 q^5 r + 12 q^4 r^2 - 11 q^2 r^4 - 9 q r^5 - 2 r^6))),$$
$$a^8 (p + q)^3 (p + r)^2 (p^2 (q + r) + r (3 q^2 + 7 q r + 2 r^2) + p (q^2 + 6 q r + 3 r^2)) -$$
$$2$$
$$a^6$$
$$(p + q)$$
$$(p + r)$$
$$(b^2 (p + q)^2 (2 q^4 + 9 q^3 r + 16 q^2 r^2 + 14 q r^3 + 4 r^4 + p^3 (3 q + 2 r) +$$
$$p^2 (6 q^2 + 17 q r + 8 r^2) + p (5 q^3 + 20 q^2 r + 28 q r^2 + 10 r^3)) +$$
$$c^2 (p + r) (p^4 (q + 2 r) + 2 p^3 (q^2 + 6 q r + 5 r^2) + r^2 (5 q^3 + 18 q^2 r + 16 q r^2 + 4 r^3) +$$
$$2 p r (3 q^3 + 14 q^2 r + 17 q r^2 + 6 r^3) + p^2 (q^3 + 16 q^2 r + 33 q r^2 + 16 r^3))) + (q + r)^2$$
$$(-c^8 (p + r)^2 (q^2 r (q + r) + p^3 (11 q + r) + p^2 (26 q^2 + 19 q r + r^2) + p q (11 q^2 + 19 q r + 6 r^2)) +$$
$$b^8 (p + q)^3 (p^2 (q + 3 r) + r (q^2 + 3 q r + 2 r^2) + p (q^2 + 6 q r + 7 r^2)) +$$
$$2 b^2 c^6 (4 p^5 (q - r) + p^4 (9 q^2 + 10 q r - 15 r^2) + p^3 (10 q^3 + 37 q^2 r + 2 q r^2 - 21 r^3) +$$
$$p^2 (7 q^4 + 31 q^3 r + 42 q^2 r^2 - 11 q r^3 - 13 r^4) + q r (2 q^4 + 5 q^3 r + 5 q^2 r^2 + q r^3 - r^4) +$$
$$p (2 q^5 + 12 q^4 r + 26 q^3 r^2 + 15 q^2 r^3 - 8 q r^4 - 3 r^5)) -$$
$$2 b^6 c^2 (p^4 (q^2 + 6 q r + 5 r^2) + p^3 (3 q^3 + 22 q^2 r + 33 q r^2 + 18 r^3) +$$
$$2 q r (q^4 + 5 q^3 r + 8 q^2 r^2 + 6 q r^3 + 2 r^4) + p^2 (3 q^4 + 28 q^3 r + 61 q^2 r^2 + 52 q r^3 + 16 r^4) +$$
$$p (q^5 + 14 q^4 r + 43 q^3 r^2 + 50 q^2 r^3 + 28 q r^4 + 4 r^5)) +$$

$$b^4 c^4 \left(2 p^5 (q+3 r)+6 p^4 \left(q^2+3 q r+6 r^2\right)+p^3 \left(7 q^3+29 q^2 r+77 q r^2+63 r^3\right)+\right.$$
$$q r \left(-q^4+7 q^3 r+21 q^2 r^2+23 q r^3+10 r^4\right)+p^2 \left(4 q^4+25 q^3 r+73 q^2 r^2+119 q r^3+43 r^4\right)+$$
$$\left.p \left(q^5+7 q^4 r+39 q^3 r^2+77 q^2 r^3+66 q r^4+10 r^5\right)\right)\Big)-$$
$$2 a^2 (q+r) \left(b^6 (p+q)^3 \left(2 p^4+2 r (q+r)^2 (q+2 r)+p^3 (5 q+9 r)+\right.\right.$$
$$\left.2 p^2 \left(3 q^2+10 q r+8 r^2\right)+p \left(3 q^3+17 q^2 r+28 q r^2+14 r^3\right)\right)-$$
$$c^6 (p+r)^2 \left(2 p^5-q r (q+r)^2 (4 q+3 r)+p^4 (7 q+5 r)+p^3 \left(10 q^2+21 q r+5 r^2\right)+\right.$$
$$\left.p^2 \left(9 q^3+28 q^2 r+14 q r^2+r^3\right)+p \left(4 q^4+6 q^3 r-4 q^2 r^2-7 q r^3-r^4\right)\right)-$$
$$b^2 c^4 (p+q) (p+r) \left(2 p^5+p^4 (9 q+7 r)+p^3 \left(16 q^2+29 q r+19 r^2\right)+\right.$$
$$(q+r)^2 \left(2 q^3+3 q^2 r+11 q r^2+6 r^3\right)+p^2 \left(16 q^3+46 q^2 r+51 q r^2+31 r^3\right)+$$
$$\left.p \left(9 q^4+29 q^3 r+51 q^2 r^2+54 q r^3+23 r^4\right)\right)+$$
$$b^4 c^2 \left(2 p^7+p^6 (7 q+9 r)+p^5 \left(8 q^2+19 q r+11 r^2\right)-p^4 q \left(q^2+16 q r+15 r^2\right)-\right.$$
$$q r (q+r)^2 \left(5 q^3+18 q^2 r+12 q r^2+4 r^3\right)-p^3 \left(11 q^4+83 q^3 r+145 q^2 r^2+83 q r^3+12 r^4\right)-$$
$$p^2 \left(10 q^5+92 q^4 r+229 q^3 r^2+219 q^2 r^3+82 q r^4+12 r^5\right)-$$
$$\left.\left.p \left(3 q^6+40 q^5 r+138 q^4 r^2+189 q^3 r^3+116 q^2 r^4+32 q r^5+4 r^6\right)\right)\right)+$$
$$a^4 (p+q) \left(c^4 (p+r)^2 \left(p^4 (q-r)+p^3 \left(3 q^2+8 q r+7 r^2\right)+p^2 \left(4 q^3+17 q^2 r+32 q r^2+21 r^3\right)+\right.\right.$$
$$\left.r \left(6 q^4+36 q^3 r+63 q^2 r^2+43 q r^3+10 r^4\right)+p \left(2 q^4+12 q^3 r+41 q^2 r^2+56 q r^3+23 r^4\right)\right)+$$
$$b^4 (p+q)^2 \left(p^4 (9 q+7 r)+3 p^3 \left(7 q^2+20 q r+11 r^2\right)+p^2 \left(21 q^3+94 q^2 r+135 q r^2+56 r^3\right)+\right.$$
$$r \left(7 q^4+33 q^3 r+56 q^2 r^2+42 q r^3+12 r^4\right)+$$
$$\left.p \left(9 q^4+60 q^3 r+135 q^2 r^2+128 q r^3+42 r^4\right)\right)+$$
$$2 b^2 c^2 \left(p^6 (3 q+5 r)+p^5 \left(7 q^2+38 q r+33 r^2\right)+p^4 \left(4 q^3+64 q^2 r+145 q r^2+81 r^3\right)+\right.$$
$$p^3 \left(-3 q^4+30 q^3 r+176 q^2 r^2+246 q r^3+99 r^4\right)+p^2 \left(-5 q^5-13 q^4 r+52 q^3 r^2+\right.$$
$$\left.202 q^2 r^3+206 q r^4+66 r^5\right)+r \left(-2 q^6-9 q^5 r-11 q^4 r^2+12 q^2 r^4+12 q r^5+4 r^6\right)+$$
$$\left.\left.\left.p \left(-2 q^6-14 q^5 r-21 q^4 r^2+26 q^3 r^3+95 q^2 r^4+82 q r^5+24 r^6\right)\right)\right)\right\}$$

PedalSteinerLines-nb.pdf

← Previous Next → ←→ Message Index ↑ Subjects

Message: #325
Date: 09/11/2013 1:14:48
From: Eisso J. Atzema
Subject: {Disarmed} Question about a Quadrilat

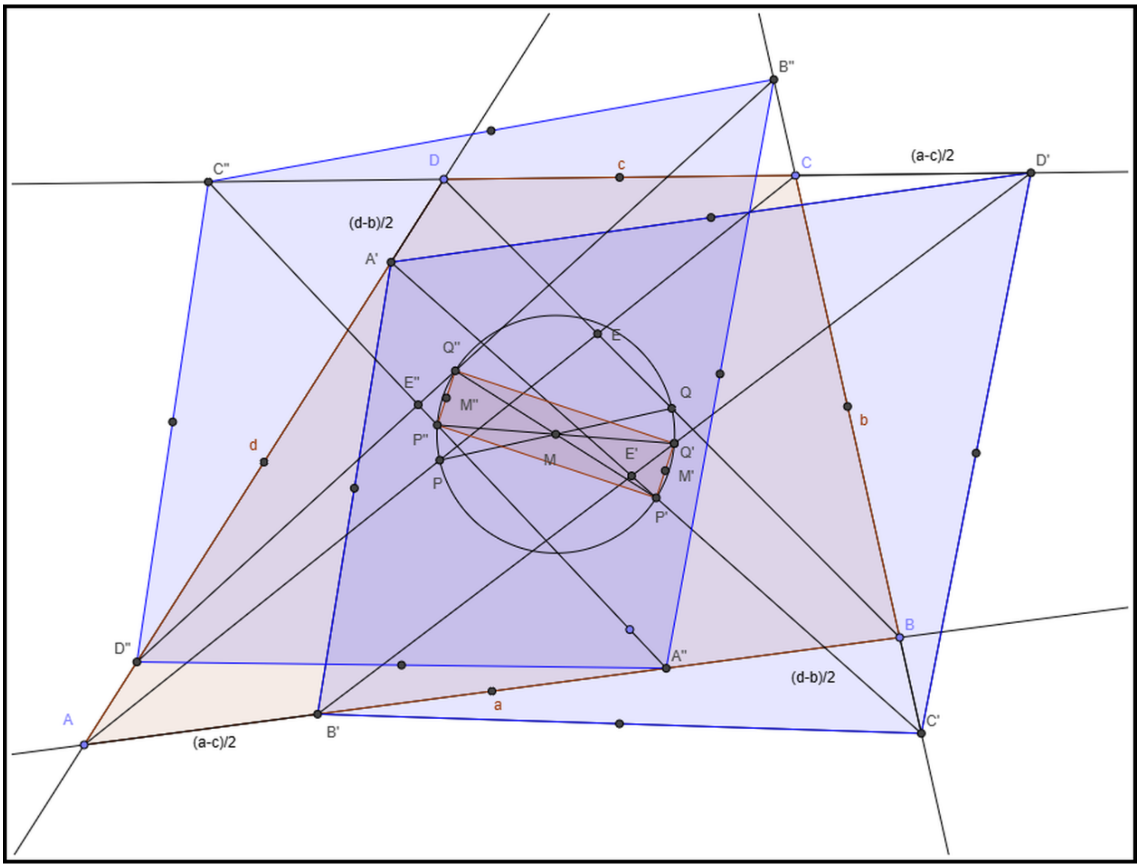
Sehr geehrter Herr Schmidt,

vielen Dank für Ihren Beweis. Der meinige ist ungefähr gleich lang (aber weniger trigonometrisch). Mittelerweile habe ich auch noch einige andere Eigenschaften der Konstruktion gefunden:

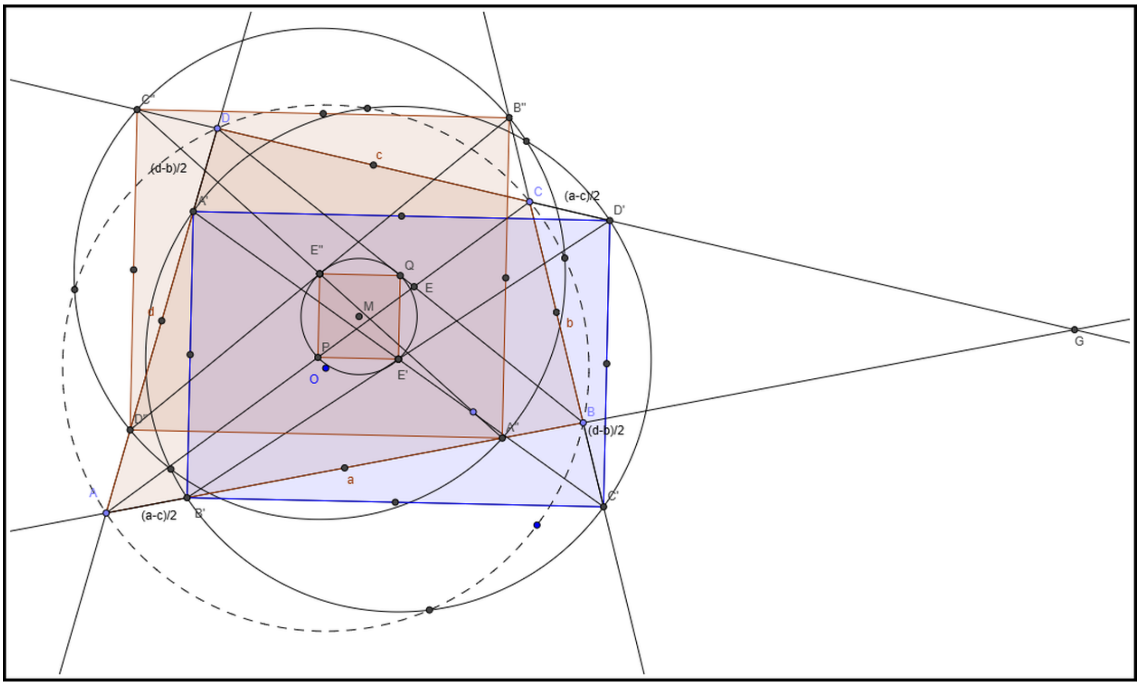
* Nebst dem von mir angedeuteten Rechteck gibt es noch ein zweites, das man bekommt wenn man in die andere Richtung vorgeht: Statt A' auf der Halbgerade AB mit AA'=(a+c)/2, nehmen wir einfach B'' auf der Halbgerade BA mit BB''=(a+c)/2 usw. Nennen wir die Mittelpunkte von A'B'C'D' und A''B''C''D'', E' bzw E'', so sind E' und E' Eckpunkten eines Rechteckes deren anderen zwei Eckpunkten die Mittelpunkte P, Q der Diagonalen AC und BD sind und deren Seiten den Seiten von A'B'C'D' und A''B''C''D'' parallel sind.

* Falls ABCD ein algemeines Kreisviereck ist, können A'B'C'D' und A''B''C''D'' noch immer konstruiert werden. Natürlich sind A'B'C'D' und A''B''C''D'' i.A. keine Rechtecke mehr und auch haben sie eine andere Flächeninhalt als ABCD. Einiges kann man aber doch sagen. Auf Grund einer Skizze, sieht es aus als ob P, Q und deren analoge Punkten P', Q', P'', Q'' auf einem Kreis liegen und zwar so dass PQ, P'Q'' und P''Q' Diametern sind. Anders gesagt, PQ, P'Q'' und P''Q' sind gleichlang und haben den gleichen Mittelpunkt M. Demnach wären auch P'Q' und P''Q'' gleichlang (und zwar gleich null falls ABCD ein Kreisviereck ist). Der Punkt M is auch der Mittelpunt von M'M'', wo M der Mittelpunt von P'Q', M'' der Mittelpunt von P''Q'' ist.
Ich hänge zwei Geogebra Dateien an...

Mit freundlichen Grüssen,
Eisso



ganesh2-ggb.png



ganesh1-ggb.png

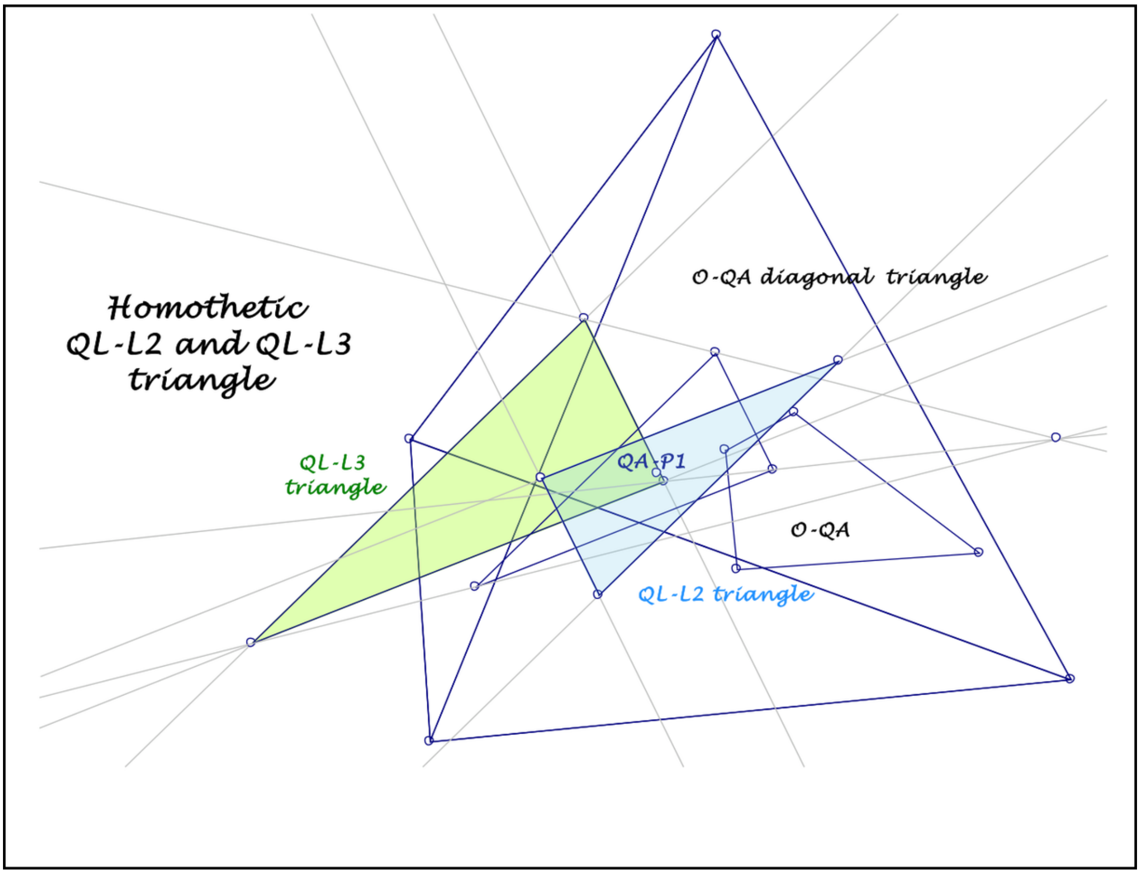
← Previous Next → ↔ Message Index ↑ Subjects

Message: #326
Date: 10/11/2013 10:21:41
From: eckart_schmidt@t-online.de
Subject: The perspector of 2 triangles made from 3 QA ...

Dear Seiichi,

interesting constellation, but extensive coordinates. Some remarks for the geometrical background:
Consider a quadrangle QA and its O-QA (circumcenters of the triangle components) and the diagonal triangle of O-QA:
* The QL-L2 triangle is the reflection of the O-QA diagonal triangle in the centroid of the reference quadrangle.
* The QL-L3 triangle is homothetic to the O-QA diagonal triangle.
... see attachment

Best regards Eckart



Seiichi-13-11-10.pdf

Message: #327
Date: 10/11/2013 11:50:40
From: Chris
Subject: The perspector of 2 triangles made from 3 QA version Pedal lines

---

```
Dear Seiichi,

Interesting construction.
I found the same CT-coordinates.
Your point QA-Px as well as QA-Tf2[QA-Px], QA-Tf3[QA-Px],
QA-Tf4[QA-Px] have no collinearities with known points, neither
are they lying on known curves.

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #328
Date: 10/11/2013 12:35:44
From: Chris
Subject: {Disarmed} Question about a Quadrilateral

Dear Eisso, dear Eckart,

GE: Das Punkt M ist das Schwerpunkt (QA-P1) des Vierecks ABCD.
EN: The point M is the centroid (QA-P1) of the quadrangle ABCD.
NL: Het punt M is het zwaartepunt (QA-P1) van de vierhoek ABCD.

Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #329
Date: 10/11/2013 1:09:28
From: Chris
Subject: Some ”splitter”

Dear Eckart,
Very nice properties from an old source!
1. Do you have references to persons who were the "finders" of these properties?
2. About the inscribed conics (3rd point): what is the max. product of axes of an hyperbola?
3. The implications for n-gons I hope to implement them once beside of EQF although I think this can take a lot of time in the future.
Best regards,
Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #330
Date: 11/11/2013 11:12:41
From: seiichikiri
Subject: The perspector of 2 triangles made from 3 QA ...

```
Dear Eckart,

They have beautiful relations! I think that I have  understood
them more deeply.
Thanks a lot!

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #331
Date: 11/11/2013 11:19:38
From: seiichikiri
Subject: The perspector of 2 triangles made from 3 QA version Pedal lines

---

```
Dear Chris,
Thank you very much for your computation and confirmation.
I hope that you will appreciate my progress of computation by
Mathematica, compared with that one year ago.
Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #332
Date: 11/11/2013 1:03:46
From: eckart_schmidt@t-online.de
Subject: Some ”splitter”

```
Dear Chris,

the background for my "splitter" are pages 106 -108 in
http://dfg-viewer.de/v2/?set%5Bimage%5D=124&set%5Bmets%5D=http%3
A%2F%2Fgdz.sub.uni-goettingen.de%2Fmets_export.php%3FPPN%3DPPN36
0609856&set%5Bzoom%5D=default&set%5Bdebug%5D=0&set%5Bdouble%5D=0
&set%5Bstyle%5D=.
... and for the last point page 1010 in
http://dfg-viewer.de/v2/?set%5Bimage%5D=252&set%5Bmets%5D=http%3
A%2F%2Fgdz.sub.uni-goettingen.de%2Fmets_export.php%3FPPN%3DPPN36
0609767&set%5Bzoom%5D=default&set%5Bdebug%5D=0&set%5Bdouble%5D=0
&set%5Bstyle%5D=
I am sure, there are more suggestions.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #333
Date: 11/11/2013 1:37:36
From: eckart_schmidt@t-online.de
Subject: The perspector of 2 triangles made from ...

```
Dear Seiichi,

thanks for your message. A further remark:
The QL-L2 triangle is the circumscribed triangle of the Morley
Triangle QA-Tr3 with vertices in QG-P10 of the QG-components.

Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #334
Date: 11/11/2013 2:25:41
From: seiichikiri
Subject: The perspector of 2 triangles made from ...

```
Dear Eckart,
Your description of nesting triangles is very interesting for
me. As QL-3 triangle is homothetic to O-QA diagonal triangle, I
will deduce the homothetic point later. If it had simpler
coordinates, I would be very happy.
Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #335
Date: 12/11/2013 2:43:54
From: seiichikiri
Subject: The perspector of 2 triangles made from ...

```
Dear Eckart,

I obtained the coordinates of the homothetic center or point.
It is also complicated, including (..a^8(p, q or r)^9..),
while those of the former case include (.. a^8(p, q or r)^8..).

Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #336
Date: 14/11/2013 9:30:05
From: eckart_schmidt@t-online.de
Subject: 3 QG-circumconics for a QL II

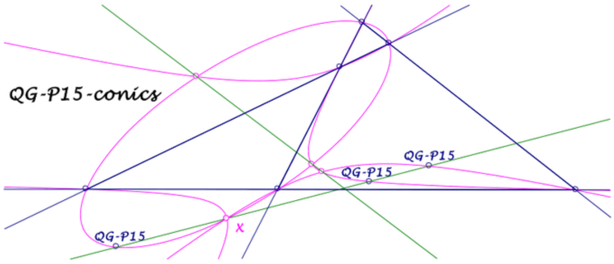
```
Dear friends,
here is another aspect as in message #320 of three
QG-circumconics for a quadrilateral.
If interested, see attachment.
Best regards Eckart
```

EQF-Note 2013-11-15

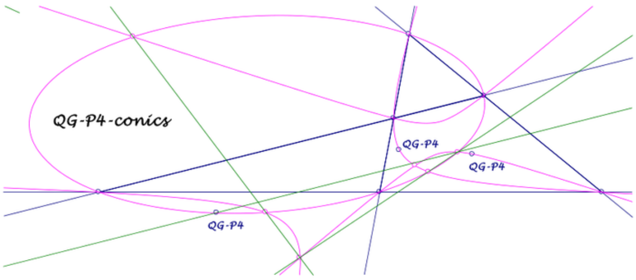
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Three *QG*-Circumconics for a Quadrilateral II

Here another aspect is worked out as in EQF-Note 2013-11-04. We consider circumconics of quadrigons through a fixed quadrigon point QG-Px. For a quadrilateral there are three of these conics. We discuss the cases, that these conics have a common point. − Reference triangle for barycentric coordinates is QL-DT.



For quadrigons we consider circumconics through a fixed quadrigon point *QG-Px*. For the *QG*-components of a quadrilateral there are three of these *QG-Px*-conics, each containing four of the six *QL*-points. In pairs they have four intersections: two *QL*-points and two further (not always real) intersections. So there are six further intersections on four lines defining a new quadrilateral (see figure below).



In the following only examples are worked out, where three intersections of the conics coincide (see first figure):

If the three points *QG-Px* for a quadrilateral are collinear, the three *QG-Px*-conics have a common point (collinear with the three *QG-Px*) and three further collinear intersections.

2013-11-15.pdf

QG-P2
The common point of the three *QG-P2*-conics is the point at infinity of the Newton-Line *QL-L1:*

$$X(m^2-n^2:n^2-l^2:l^2-m^2).$$

The three further intersections lie on the *M3D* Line *QL-L9*.

QG-P12
The common point of the three *QG-P12*-conics is the *QL*-Harmonic Center *QL-P13*.
The three further intersections lie on a parallel to *QL-L9* through *QL-P19*.

QG-P15 (see first figure)
The common point of the three *QG-P15*-conics is the intersection of *QL-L9* and *QL-P8.QL-P13*:

$$X(m^2n^2(m^2n^2-l^4):n^2l^2(n^2l^2-m^4):l^2m^2(l^2m^2-n^4)).$$

The three further intersections lie on a line with the equation

$$(m^2n^2+l^4)x+(n^2l^2+m^4)y+(l^2m^2+n^4)z=0,$$

containing the intersection of *QL-L9* and its *QL-Tf2* image.

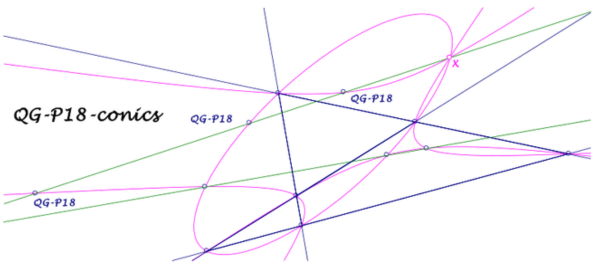
QG-P16
The common point of the three *QG-P16*-conics lies collinear with the *QG-P16*-points and *QL-P26* on the *QL-Tf1* image of the Dimidium Circle *QL-Ci6*:

$$X(m^2n^2(a^2M+b^2L)(a^2N+c^2L)(a^2MN+(m-n)(b^2nN+c^2mM))$$
$$(a^2MN+(m+n)(-b^2nN+c^2mM)):...)$$

(only the first coordinate with $L=m^2-n^2, M=n^2-l^2, N=l^2-n^2$).

QG-P18
The common point of the three *QG-P18*-conics lies collinear with the *QG-P18*-points on the *QL-Tf1* image of the *QL-DT* Medial Circle *QL-Ci2*:

$$X((m^2S_C^2-n^2S_B^2)(l^2(S_BS_C+S^2)+S_A(m^2b^2+n^2c^2))$$
$$:(n^2S_A^2-l^2S_C^2)(m^2(S_AS_C+S^2)+S_B(n^2c^2+l^2a^2))$$
$$:(l^2S_B^2-m^2S_A^2)(n^2(S_AS_B+S^2)+S_C(l^2a^2+m^2b^2))).$$



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-11-15.pdf

Message: #337
Date: 15/11/2013 3:43:08
From: eckart_schmidt@t-online.de
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Seiichi,
if you use for the fixed CSC-points the Miquel triangle as
reference triangle ABC (with C in the Miquel point),
then the barycentric coordinates are
a(a+b+sqrt((a+b)^2-c^2):b(a+b+sqrt((a+b)^2-c^2):-c^2),
a(a+b-sqrt((a+b)^2-c^2):b(a+b-sqrt((a+b)^2-c^2):-c^2).
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #338
Date: 15/11/2013 4:34:49
From: eckart_schmidt@t-online.de
Subject: Clawson-Schmidt Conjugate QL-Tf1

```
Dear Chris,
if we take the Miquel triangle as reference triangle ABC (C in
the Miquel point), then the Clawson-Schmidt Conjugate QL-Tf1
becomes a very simple form for quadrigons:
(x: y: z)
   -> (-a^2 y: -b^2 x: (a^2 y z+b^2 z x+c^2 x y) / (x+y+z)).

Let L3^L4 = P4 = (u: v: w),
then
    L2^L3 = P3 = (a^2 w: T: c^2 u),
    L1^L2 = P2 = (a^2 v: b^2 u: T),
    L4^L1 = P1 = (T: b^2 w: c^2 v)
with T = -(a^2 v w+b^2 w u+c^2 u v) / (u+v+w).

The diagonal crosspoints have not so simple coordinates.

Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #339
Date: 15/11/2013 8:50:25
From: Chris van Tienhoven
Subject: Clawson-Schmidt Conjugate QL-Tf1

```
Dear Eckart,
When the Miquel Triangle is reference triangle, the question
arises how to construct a corresponding reference Quadrangle.
With your triple transformation:
(u: v: w)
--->((a^2 w: T: c^2 u), (a^2 v: b^2 u: T), (T: b^2 w: c^2 v)),
where T = -(a^2 v w+b^2 w u+c^2 u v) / (u+v+w).
Simplified P1 -> (P2,P3,P4) this becomes clear:
Every point P1 can be the vertice of a Quadrilateral with ABC as
Miquel Triangle.
Because P1, P2, P3, P4 represent a Quadrilateral with ABC as
Miquel Triangle.

Further when we apply this triple transformation strictly to a
triangle, then:
* The incenter is transformed into the 3 excenters
* An excenter is transformed into the incenter + other 2
excenters.
* A triangle vertice is transformed into the other 2 triangle
vertices + some undefined point (0:0:0).

This leads us to the solution of how to construct the triple
transformation in a triangle.
1. Let P be some point to be triple transformed wrt some
triangle ABC.
2. Construct the incenter I and excenters E1,E2,E3 of ABC.
I.E1.E2.E3 forms a Quadrangle (system of 4 points), representing
3 possible Quadrilaterals:
   Consecutive lines from Quadrigon I.E1.E2.E3
   Consecutive lines from Quadrigon I.E1.E3.E2
   Consecutive lines from Quadrigon I.E2.E1.E3
3. Construct the Clawson-Schmidt Conjugate of P wrt the 3
Quadrilaterals with consecutive lines of resp. I.E1.E2.E3 ,
I.E1.E3.E2 , I.E2.E1.E3 .
These 3 points are the triple transformed points of P.

Best regards,
Chris
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #340
Date: 15/11/2013 11:30:57
From: Chris
Subject: Clawson-Schmidt Conjugate QL-Tf1

```
Dear Eckart,

An extra property of P1 when triple transformed into (P2,P3,P4):
* P2.P3.P4 is perspective with ABC.
* The perspector is the isogonal conjugate of P1.

Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #341
Date: 16/11/2013 10:31:54
From: eckart_schmidt@t-online.de
Subject: Clawson-Schmidt Conjugate QL-Tf1

```
Dear Chris,
you describe a "triple transformation in a triangle". I would
describe it in the following way:
For a vertex A of a triangle ABC (incenter I) there is a
transformation TA (analogue TB, TC):
... first reflection in AI,
... then reflection in a circle round A with radius sqrt(b c).
TA(P), TB(P), TC(P) are the "triple transformation" of P wrt
ABC.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #342
Date: 16/11/2013 11:42:26
From: seiichikiri
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Eckart,
Thank you very much for your computation of 2 fixed points.
I think that it is very important to obtain the simple
solution and you showed its example on the Miquel triangle.
Best regards,
Seiichi
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #343
Date: 17/11/2013 2:21:37
From: eckart_schmidt@t-online.de
Subject: QG-Tr3 a new Reference Triangle

Dear Chris, dear Seiichi,
the not successful efforts to calculate the fixed points of the Clawson-Schmidt Conjugate QL-Tf1 have led me to QG-Tr3 as new reference triangle. I think, it is an acceptable way to calculate the Miquel point, the 1st Steiner line, the Schmidt circle, the transformation QL-Tf1 and its fixed points. See Mathematica file in the attachment.
Best regards Eckart

QG-Tr3 a new Reference Triangle

QG-Tr3 shall be the reference triangle ABC for barycentric coordinates:

A=QG-2P2a, B=QG-2P2b, C=QG-P18.

If we chose a point P4(u:v:w), we get the other vertices of the quadrigon in the following way:

P2 is the isogonal conjugate of P4 wrt QG-Tr3
P1 is the intersection of A.P4 and B.P2,
P3 is the intersection of A.P2 and B.P4.

$$pP1 := \{-a^2 w^2, -c^2 u v, -c^2 u w\};$$
$$pP2 := \{a^2 v w, b^2 u w, c^2 u v\};$$
$$pP3 := \{c^2 u v, b^2 w^2, c^2 v w\};$$
$$pP4 := \{u, v, w\}$$

Then the Miquel point QL-P1 is

$$pMiqu := \{a^2 v B (u A + v B), b^2 u A (u A + v B), -c^2 u v A B\}$$

... with the short cuts

$$A := (c^2 v^2 + 2 SA v w + b^2 w^2) ; B := (c^2 u^2 + 2 SB u w + a^2 w^2)$$

The 1st Steiner Line has the coefficients

$$gSt1 := \{-b c^2 u A, a c^2 v B, -a b (a - b) (u A + v B)\}$$

The fixed points of the Clawson Schmidt Conjugate are the intersections of the 1st Steiner Line and the Schmidt Circle. The Schmidt Circle has the equation

$$krsS[\{x\_, y\_, z\_\}] := -c^2 (b u A - a v B) (b u A x^2 - a v B y^2) + 2 A B c^2 (a b + SC) u v x y$$
$$-2 a B v (a A SA u + a B SA v - b c^2 A u) y z - 2 b A u (b A SB u + b B SB v - a B c^2 v) x z$$
$$-a b (a b (u A + v B)^2 - A B c^2 u v) z^2$$

The fixed points of the Clawson Schmidt Conjugate lie also on a circle through A=QG-2P2a, B=QG-2P2b and the incenter of QG-Tr3 with the equation

Chris17-nb.pdf

```
krs[{x_, y_, z_}] := a^2 y z + b^2 x z + c^2 x y - a b z (x + y + z)
```

The fixed points of the Clawson Schmidt Conjugate are

$$pfix1 := \left\{-a + \frac{a\sqrt{v B}\ \sqrt{c^2 - (a-b)^2}}{(a-b)\ \sqrt{u A}},\ b + \frac{b\sqrt{u A}\ \sqrt{c^2 - (a-b)^2}}{(a-b)\ \sqrt{v B}},\ \frac{c^2}{(a-b)}\right\}$$

$$pfix2 := \left\{-a - \frac{a\sqrt{v B}\ \sqrt{c^2 - (a-b)^2}}{(a-b)\ \sqrt{u A}},\ b - \frac{b\sqrt{u A}\ \sqrt{c^2 - (a-b)^2}}{(a-b)\ \sqrt{v B}},\ \frac{c^2}{a-b}\right\}$$

Also the Clawson-Schmidt Conjugate QL-Tf1 gets an acceptable form:

```
QLTf1[{x_, y_, z_}] :=
 {-a^2 B v (-B c^2 v y^2 + A c^2 u x y + (a^2 A u - A b^2 u - 2 B SA v) y z - b^2 (A u + B v) z^2),
  -b^2 A u (-A c^2 u x^2 + B c^2 v x y + (-a^2 B v + b^2 B v - 2 A SB u) x z - a^2 (A u + B v) z^2),
  c^2 u v A B (c^2 x y + b^2 x z + a^2 y z)}
```

Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

Chris17-nb.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #344
Date: 17/11/2013 7:47:58
From: Chris
Subject: QG-Tr3 a new Reference Triangle

```
Dear Eckart,
Very nice approach (change of reference triangle) for dealing
with items concerning the Clawson-Schmidt Conjugates (CSC)!
The new QG-circle you mention (through QG-2P2a/b, Incenter of
QG-Tr3 and fixed CSC-points looks special. The only special
thing I found about this circle is that the 3 QL-versions are
coaxial with axis the 1st Steiner Axis and with common points
the fixed CSC-points.
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #345
Date: 18/11/2013 1:30:10
From: eckart_schmidt@t-online.de
Subject: Angle bisectors of a quadrigon

```
Dear friends,

it is well known, that the intersections of inner / outer angle
bisectors of a quadrigon give cyclic quadrigons. But there are
rarely connections with EQF.

Some observations with CABRI:
* The two cyclic quadrigons have the same diagonal lines (angle
  bisectors of opposite side lines of the reference quadrigon).
* The midpoints of the circumcircles for the cyclic quadrigons
  lie on the 1st Steiner axis (see QL-Tf1).
* The circumcircles of the cyclic quadrigons
  cut the Schmidt circle (see QL-Tf1) orthogonal.
* The radical axis of the two circumcircles
  is the 2nd Steiner axis (see QL-Tf1).
* The circumcircles of the cyclic quadrigons
  are invariant wrt the Clawson-Schmidt Conjugate QL-Tf1.
* The Euler-Poncelet points of the cyclic quadrigons
  lie on the Newton line QL-L1 of the reference quadrigon.
* Taking QG-Tr3 as reference triangle (see my message #343),
  the vertices of the inner cyclic quadrigon can be calculated:
  P12(a (-(c v-b w)sqrt(B)+a w sqrt(A)) :
                          b c u sqrt(A): c^2 u sqrt(A)),

  P23(a c v sqrt(B) :
      b(-(c u-a w)sqrt(A)+ b w sqrt(B)) : c^2 v sqrt(B)),

  P34((c v-b w)sqr(B)+c u sqrt(A) :
                          b w sqrt(A)  : c w sqrt(A)),

  P41(a w sqrt(B) :
           c v sqrt(B)+(c u-a w)sqrt(A) : c w sqrt(B)).

Not mentioned are general properties of cyclic quadrigons.
Perhaps someone can find further properties.
Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #346
Date: 20/11/2013 9:46:34
From: eckart_schmidt@t-online.de
Subject: A new circle for quadrigons

Dear Chris,

I have collected some properties of the new QG-circle, I
mentioned in my message #343.
... see attachment

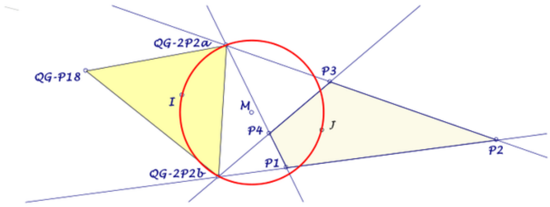
Best regards Eckart

EQF-Note 2013-11-20

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A new Circle for Quadrigons

Here a new QG-circle will be described connected with the angle bisectors of a quadrigon. – Reference triangle is the Quasi Isogonal Triangle QG-Tr3. But most of the properties are only tested with CABRI.



Preliminary Remarks

Reference triangle is the Quasi Isogonal Triangle *QG-Tr3* with vertices in the intersections of opposite sides *QG-2P2a,b* and the Quasi Isogonal Crosspoint *QG-P18*. We use the Incenter *I*, the excenter *J* wrt *QG-P18* and the corresponding isogonal conjugate (*QG-Tf2*).

The new *QG*-circle is the Thales circle about *IJ*.

If we chose the reference triangle *ABC* with

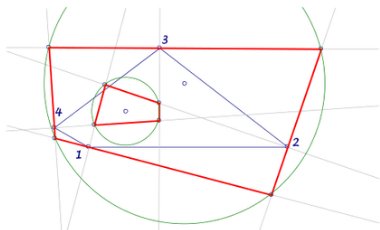
$$A = QG-2P2a, \quad B = QG-2P2b, \quad C = QG-P18$$

and give P_4 the coordinates *u, v, w*, then we get

$$P_1(a^2w : c^2uv : c^2uw), \quad P_2(a^2vw : b^2uw : c^2uv),$$
$$P_3(c^2uv : b^2w^2 : c^2vw), \quad P_4(u : v : w)$$

and the equation of the new *QG*-circle is

$$a^2yz + b^2zx + c^2xy - abz(x + y + z) = 0.$$



Furthermore we consider the two cyclic quadrigons, constructed with the inner and outer angle bisectors of the quadrigon.

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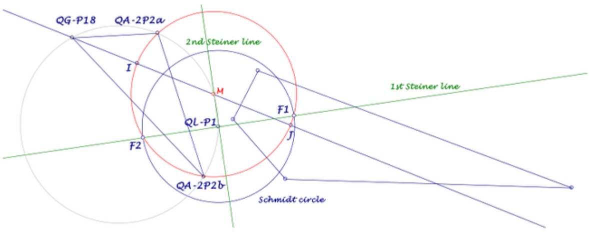
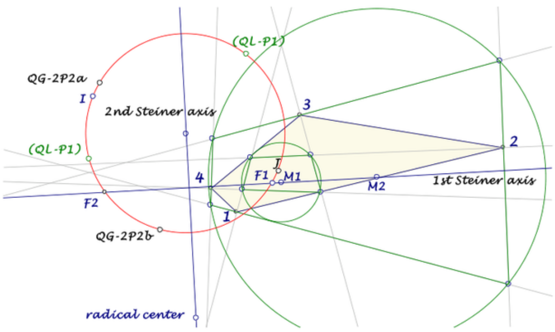
Properties of the new *QG*-Circle *QG-Cix*

- *QG-Cix* contains the intersections *I* and *J* of the inner and outer angle bisectors of opposite sidelines of the quadrigon.
- *QG-Cix* contains *QG-2P2a,b* the intersections of opposite sidelines of the quadrigon.
- *QG-Cix* contains the fixed points F_1, F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.
- The midpoint *M* of *QG-Cix* is the intersection of the 2nd Steiner axis (see *QL-Tf1*) and the circumcircle of *QG-Tr3* (beside *QL-P1*).



- *QG-Cix* is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.
- *QG-Cix* is invariant wrt the Isogonal Conjugate for *QG-Tr3* (*QG-Tf2*).
- *QG-Cix* cuts the circumcircles of the cyclic angle-bisector-quadrigons orthogonal.
- *QG-Cix* contains the Miquel points (*QL-P1*) of the cyclic angle-bisector-quadrigons.
- The radical center of *QG-Cix* and the circumcircles of the angle-bisector-quadrigons is the intersection of *QG-L1* and the 2nd Steiner axis.
- The polar of the radical center wrt *QG-Cix* is the 1st Steiner axis.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-11-20.pdf

Message: #347
Date: 22/11/2013 4:02:42
From: eckart_schmidt@t-online.de
Subject: Another quadrigon circle

Dear friends,

in the attachment there is another quadrigon circle described, connected with diagonal points.

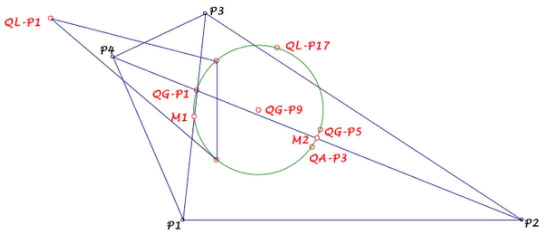
Best regards Eckart

EQF-Note 2013-11-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Another Quadrigon Circle

Here another QG-circle will be described connected with diagonal points.



This *QG*-circle is the circumcircle of the Diagonal Crosspoint *QG-P1* and the midpoints of the diagonals.

If we use *QL-Tr1* as reference triangle, this circle has the equation

$$a^2(l^2-m^2)(n^2x+m^2y+n^2z)z-c^2(m^2-n^2)(l^2x+m^2y+l^2z)x$$
$$+b^2(l^2-m^2)(m^2-n^2)zx=0$$

with the midpoint

2nd *QG*-Quasi Circumcenter *QG-P9*

and contains …

… the Diagonal Crosspoint *QG-P1*,
… the diagonal midpoints M_1 and M_2,
… the 1st *QG*-Quasi Circumcenter *QG-P5*,
… the Gergonne-Steiner Point *QA-P3*,
… the *QL*-Adjunct Quasi Circumcenter *QL-P17*
… two vertices of the Miquel Triangle unequal *QL-P1*.

Furthermore there is a transformation, consisting of a reflection in the angle bisector at *QG-P1* (wrt *QL-Tr1*) and a reflection in a circle round *QG-P1*, so that the diagonal midpoints M_1 and M_2 change:

$$(x:y:z) \quad \rightarrow \quad (a^2m^4z(x+y+z)$$
$$:-(m^2n^2-n^2l^2+l^2m^2)(a^2z^2+2S_Bzx+c^2x^2)-m^4(a^2yz+b^2zx+c^2xy)$$
$$:c^2m^4x(x+y+z))$$

This transformation gives the new *QG*-circle as image of the Newton Line *QL-L1*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-11-22.pdf

Message: #348
Date: 24/11/2013 10:58:13
From: eckart_schmidt@t-online.de
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Seiichi, dear Chris,
there are further properties for the CSC-fixed points F1 and F2
... for a quadrigon:
... P1, P3, F1, F2 and P2, P4, F1, F2 are concyclic.
... circumcircles of P1P2Fi and P3P4Fi or circumcircles
    of P2P3Fi and P4P1Fi are tangent in Fi.
... for a quadrilateral:
... F1, F2 points, which are the Isogonal Center (QA-P4)
    of their pedal quadrangle.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #349
Date: 24/11/2013 12:30:17
From: seiichikiri
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Eckart,
I confirmed the concyclicity and tangency by Cabri pictures. It
is really interesting!!!
Best regards,  Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #350
Date: 24/11/2013 2:17:57
From: Chris
Subject: Numerically confirmed quadrangle points with a ...

```
Dear Eckart, dear Seiichi,
This brings me to this property:
QA-P9 is the Miquel Point of the Antipedal Quadrilateral of
QA-P4.
Best regards,  Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #351
Date: 25/11/2013 10:42:50
From: Chris
Subject: Inscribed Harmonic Quadrilateral

```
Dear Friends,
It is possible to construct an Inscribed Harmonic Quadrilateral
in a Quadrangle.
This can be done as follows:
Let P1, P2, P3, P4 be 4 random points forming our Reference
Quadrangle.
Let S1, S2, S3 be the vertices of the Diagonal Triangle.
   S1=intersection point P4.P1^P2.P3
   S2=intersection point P4.P2^P1.P3
   S3=intersection point P4.P3^P1.P2
Let
   M12 = P1.P2 ^ S1.S2,
   M34 = P3.P4 ^ S1.S2,
   M23 = P2.P3 ^ S2.S3,
   M41 = P4.P1 ^ S2.S3,
   M13 = P1.P3 ^ S1.S3,
   M24 = P2.P4 ^ S1.S3.
These 6 points lie on 4 lines:
   line M13.M34.M41
   line M12.M24.M41
   line M12.M23.M13
   line M23.M34.M24
So these 4 lines actually form a Complete Quadrilateral.
We will call this Quadrilateral the Inscribed Harmonic
Quadrilateral IH-QL.
and the Reference Quadrangle we will call Ref-QA.
Surprisingly IH-QL-DT = Ref-QA-DT (DT=Diagonal Triangle).
   IH-QL-P8  = Ref-QA-P10 (Centroid Diagonal Triangle)
   IH-QL-P9  = Ref-QA-P11 (Circumcenter Diagonal Triangle)
   IH-QL-P10 = Ref-QA-P12 (Orthocenter Diagonal Triangle)
   IH-QL-P11 = Ref-QA-P13 (Nine-point center Diagonal Triangle)
   IH-QL-P13 = Ref-QA-P16 (so the Harmonic Centers Coincide)
   IH-QL-P18 = Ref-QG-P4 of Reference Quadrigon
               (Quadrigon Centroid)
   IH-QL-P12 = Ref-QL-P12 of Reference Quadrigon
               (Quadrilateral Centroid)
I don't think I will be surprised with more interesting
properties for the Inscribed Harmonic Quadrilateral in a
Quadrangle
Best regards,
Chris
```

Message: #352
Date: 26/11/2013 7:47:45
From: Chris
Subject: Inscribed Harmonic Quadrilateral

```
Dear friends,
I made some false calculations.
About the coincidences only these are true:
IH-QL-P8 = Ref-QA-P10 (Centroid Diagonal Triangle)
IH-QL-P9 = Ref-QA-P11 (Circumcenter Diagonal Triangle)
IH-QL-P10 = Ref-QA-P12 (Orthocenter Diagonal Triangle)
IH-QL-P11 = Ref-QA-P13 (Nine-point center Diagonal Triangle)
IH-QL-P13 = Ref-QA-P16 (so the Harmonic Centers Coincide)
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #353
Date: 27/11/2013 12:32:24
From: eckart_schmidt@t-online.de
Subject: Inscribed Harmonic Quadrilateral

```
Dear Chris,
if you consider an IH-QL for a quadrigon, the points M12, M23,
M34, M41 give an IH-QG. The vertices of this IH-QG are the
points of tangency for QG-Co1 and divide the sides in ratios
with product 1.
Further relationships between IH-QL and Ref-QA can be related
wrt the same diagonal triangle. For Example: IH-QL-L7=Ref-QA-L5.
Perhaps new:
IH-QL-L6 contains Ref-QA-P32
IH-QL-Ci2 contains Ref-QA-P29 and Ref-QA-P36 .
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #354
Date: 27/11/2013 1:55:28
From: Chris
Subject: Another quadrigon circle

```
Dear Eckart,
A small contribution:
The angle bisector in the QL-Diagonal Triangle at QG-P1 is also
angle bisector between lines QG-P1.QG-P6.QG-P10 and
QG-P1.QG-P5.QG-P9.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #355
Date: 27/11/2013 8:54:12
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 for quadrigons

```
Dear Chris,

if you rewrite QL-Cu1 in EQF, here another aspect wrt
quadrigons:
... QL-Cu1 is the locus of points, whose antipedal quadrigon is
cyclic (if not degenerated).
... The antipedal quadrigons degenerate into triangles for
points on the circumcircles of the triangle components.
... A CT-equation of QL-Cu1:
        a^2 y z (q z - r y) + c^2 x y (p y - q x)
    -  (b^2 r+2 q SA) x^2 z + (b^2 p + 2 q SC) x z^2
    + 2(p SA - r SC)  x y z = 0

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #356
Date: 29/11/2013 9:25:27
From: eckart_schmidt@t-online.de
Subject: A curious QG-parabola

```
Dear friends,
consider a quadrigon, divide the sides in the same ratio and you
get an inscribed quadrigon I-QG.
Some properties ( EQF-elements of these inscribed quadrigons
will be set in parentheses):
... all I-QG have the same centroid (QA-P1)
    and the same Newton line (QL-L1) as the reference quadrigon,
... the Miquel points (QL-P1) are collinear on QL-P1.QG-P3,
... (QL-P19) are collinear on a line through QL-P19
    perpendicular QG-P5.QG-P10,
... (QG-P3) are collinear on QA-P1.QG-P3.
The NSM Lines (QL-L5) envelope a parabola (tangent to QL-L1 and
QL-L5).
What about this parabola?
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #357
Date: 29/11/2013 9:57:15
From: eckart_schmidt@t-online.de
Subject: Some ”splitter” III

```
Dear Chris,
some properties, not found in EQF:
# QG-P15: area(Pi,Pi+1,QG-P15) = area(P1+2,Pi+3,QG-P15)
# QL-Ci1: ... locus of Euler-Poncelet points of QA with vertices
              in the boundary points of QL-tangential conics.
# QL-P18: ... centroid of the three collinear QG-P15 points
              on QL-L9.
# QG-P9 : ... QL-P17 lies (with QL-P9 and QL-P16)
              on the circle through the 3 QL-versions of QG-P9.
# QA-Tr3: ... for a QG: QG-P10 is the 4. parallelogram point
              of QA-Tr3.
# QA-Tr3: ... for a QL: The three QL versions have parallel
              opposite sides of QL-P2 with midpoints on QL-L6.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #358
Date: 29/11/2013 11:59:42
From: Chris
Subject: Some "splitter" III

Dear Eckart,
I like your splitter!
Regarding your last item (The three QL versions of QA-Tr3 have parallel sides, opposite to QL-P2 with midpoints on QL-L6):
These opposite sides are parallel to the Steiner line QL-L2.
Best regards, Chris

← Previous Next → ↔ Message Index ↑ Subjects

Message: #359
Date: 30/11/2013 10:52:27
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 for quadrigons

Dear Chris,
in addition to my message #355 there is a further property for QL-Cu1 for quadrigons:
For quadrigons P1P2P3P4 the QL-Quasi Isogonal Cubic QL-Cu1 is the locus of points P, for which the circumcenters of PPiPi+1 are concyclic.
Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #360
Date: 01/12/2013 9:45:36
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 for quadrigons

Dear Chris,
QL-Cu1 for quadrigons stays on interesting:
For quadrigons P1P2P3P4 and points P on QL-Cu1 the angle bisectors of
For convex quadrigons these angle bisectors are identic for outer points and perpendicular for inner points.
Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

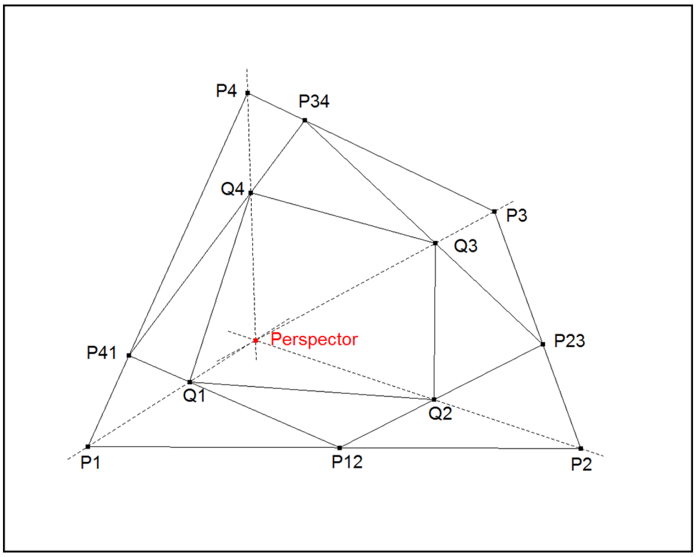
Message: #361
Date: 01/12/2013 12:03:33
From: Chris van Tienhoven
Subject: Quad Perimeter Problem

```
Dear friends,

Let P1.P2.P3.P4 be Quadrigon/Tetragon Quad-1 with perimeter
peri-1.
Construct points P12, P23, P34, P41 on AB, BC, CD, DA such that:
P1.P12 + P1.P41
    = P2.P12 + P2.P23 = P3.P23 + P3.P34 = P4.P34 + P4.P41
    = peri-1 / 4.
Let P12.P23.P34.P41 be Quadrigon/Tetragon Quad-2 with perimeter
peri-2.

Now construct in a similar way an inscribed Quad-3 Q1.Q2.Q3.Q4
such that:
P12.Q1 + P12.Q2
    = P23.Q2 + P23.Q3 = P34.Q3 + P34.Q4 = P41.Q4 + P41.Q1
    = peri-2/4.

Questions:
1. Is it possible to construct in a general way inscribed
Quadrigons Quad-2 and Quad-3 such that Quad-1 and Quad-3 are
perspective (see attachment)?
2. Is it possible to construct in a general way inscribed Quad-2
such that P12.P34 is perpendicular to P23.P41?
Best regards,
Chris
```



QG-P97-PerimeterPoint-01.png

Message: #362
Date: 05/12/2013 9:34:00
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrigons

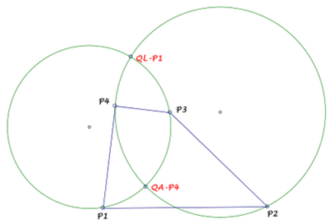
Dear friends,
consider for a quadrigon circles through opposite vertices, the boundary points give an interesting quartic: QG-circumscribed, QL-Tf1 invariant, anallagmatic (see attachment). The constructions are simple, the calculation extensive. But perhaps another reference triangle will give simpler expressions.
Best regards Eckart

EQF-Note 2013-12-05

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Quartic for Quadrigons

The fixed points of a simple quadrigon transformation give an interesting quartic, anallagmatic and invariant wrt the Clawson-Schmidt Conjugate QL-Tf1. Construction and equation shall be given.



The transformation

Consider for a quadrigon circles through opposite vertices and a given point *P*. The second intersection of these circles shall be the image of *P*.

Let $P_1P_2P_3P_4$ be the quadrigon, $P_1P_2P_3$ the reference triangle for barycentric coordinates and $P_4(p:q:r)$. For a point $P(x:y:z)$ the image has somewhat extensive coordinates:

$$\left\{\frac{b^2\,(p+q+r)\,z\,(r\,x-p\,z)+c^2\,(p+r)\,X\,y+(a^2\,p+a^2\,r-b^2\,p)\,Z\,y}{a^2\,r\,Y\,z+b^2\,r\,X\,z+c^2\,r\,(p+q+r)\,x\,y-c^2\,p\,q\,z\,(x+y+z)},\right.$$
$$\frac{b^2\,y\,(x+y+z)}{c^2\,x\,y+b^2\,x\,z+a^2\,y\,z},$$
$$\left.\frac{b^2\,(p+q+r)\,x\,(p\,z-r\,x)+a^2\,(p+r)\,Z\,y+(-b^2\,r+c^2\,r+c^2\,p)\,X\,y}{b^2\,p\,Z\,x+c^2\,p\,Y\,x+a^2\,p\,(p+q+r)\,y\,z-a^2\,q\,r\,x\,(x+y+z)}\right\}$$

with

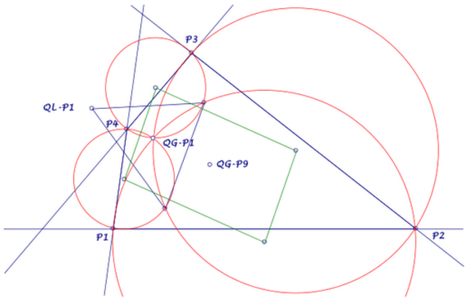
$$X:=x\,(q+r)-p\,(y+z);\ Y:=y\,(r+p)-q\,(z+x);\ Z:=z\,(p+q)-r\,(x+y)$$

Some properties:

- Fixed points of the transformation give a quartic (see below).
- The image of the Miquel Point *QL-P1* is the Isogonal Center *QA-P4* (see above).

2013-12-05.pdf

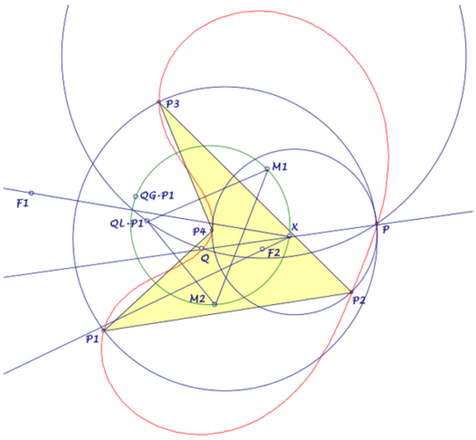
- The images of the intersections of opposite sidelines are vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel Triangle *QA-Tr2*.
- The diagonals are invariant wrt this transformation.
- The images of the sidelines are circles, containing the vertices of the opposite side, the Diagonal Crosspoint *QG-P1* and a vertex (unequal *QL-P1*) of the Miquel Triangle.
- The midpoints of these circles give a parallelogram with center *QG-P9* and sidelines perpendicular to the diagonals of the quadrigon.



The Quartic of the Fixed Points

For fixed points *P* of the transformation the circles through opposite vertices of the quadrigon are tangent in *P*. These points give a quartic with the equation (*X, Y, Z* see above):

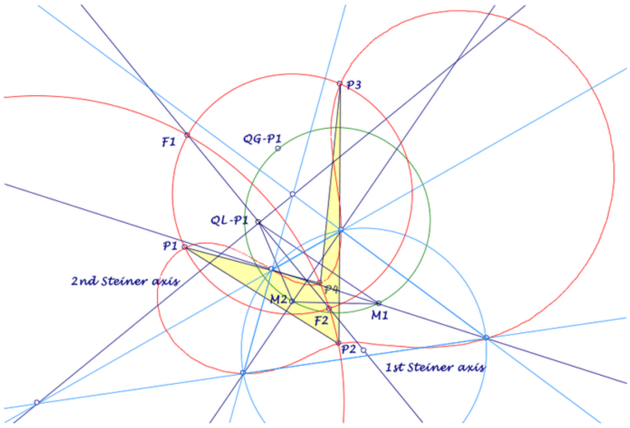
$$\begin{aligned}&(p+r)\,y^2\,(a^4 z Z + c^4 x X) + a^2 c^2 (p+r)\,y^2\,(x Z + z X)\\ &\quad - b^4 x z\,(r\,(x+y)\,X + p\,(y+z)\,Z)\\ &\quad - a^2 b^2 z\,(p Z y\,(z+y-x) + r\,(Z y\,(z-x) + Y\,(x+z)\,(y+x)))\\ &\quad - b^2 c^2 x\,(r X y\,(x+y-z) + p\,(X y\,(x-z) + Y\,(x+z)\,(y+z))) == 0\end{aligned}$$



Construction of the quartic: For the construction we need the vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel triangle and the fixed points F_1 and F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.

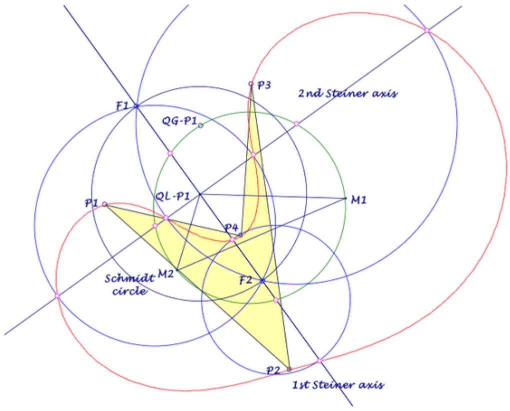
1. Circle through *QG-P1*, M_1, M_2 with a point X
2. L_x angle bisector of $\angle F_1XF_2$
3. P, Q as intersections of L_x and its image-circle wrt *QL-Tf1* are points of the quartic.

Properties:



- The quartic is the locus of points, where circles through opposite vertices of the quadrigon are tangent.
- The quartic is a circumquartic of the quadrigon.
- The quartic is invariant wrt *QL-Tf1*.
- For *QL-Tf1* partners on the quartic the midpoints lie on the circumcircle of M_1, M_2, *QG-P1*.
- The angle bisectors of the Miquel triangle at $M_{1,2}$ cut their *QL-Tf1* image on the quartic in points of a cyclic quadrigon.
- Midpoint of the circumcircle of this cyclic quadrigon is the excenter wrt *QL-P1* of the Miquel triangle.
- The quartic is anallagmatic, that means: The quartic is invariant wrt a reflection in a circle. There are two those circles with midpoints in the intersections of the opposite sidelines of the cyclic quadrigon (on the 2^{nd} Steiner axis), containing F_1 and F_2.
- The 1^{st} Steiner axis cuts the circle through M_1, M_2, *QG-P1* in two points. One point is the center of a circle orthogonal to the Schmidt circle, which cuts the 1^{st} Steiner line on the quartic.
- The 2^{nd} Steiner axis cuts the circle through M_1, M_2, *QG-P1* in two points. They are centers of circles through F_1 and F_2, which cut the 2^{nd} Steiner axis on the quartic.

2013-12-05.pdf



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-05.pdf

Message: #363
Date: 06/12/2013 12:06:57
From: eckart_schmidt@t-online.de
Subject: A Quartic for a Quadrigon

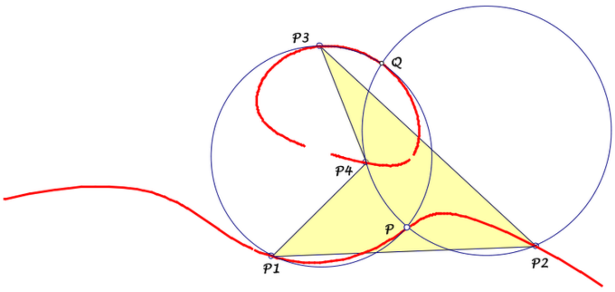
Dear friends,
in my last message I have described a quartic for a quadrigon as locus for points, where circles through opposite vertices are tangent. In the attachment is a further quartic as locus of points, where these circles cut perpendicular.
Best regards Eckart

EQF-Note 2013-12-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Second Quartic for Quadrigons

For a quadrigon circles through opposite vertices are tangent in points on a quartic (see EQF-Note 2013-12-05). Here another quartic is described as locus of points, where circles through opposite vertices intersect perpendicular.



Let $P_1P_2P_3P_4$ be a quadrigon and $P_1P_2P_3$ the reference triangle for barycentric coordinates with $P_4(p:q:r)$. Then the locus for points, where circles through opposite vertices intersect perpendicular, is a quartic with the equation

$$\begin{aligned}&(p+r)\,(a^4\,z\,Z\,y^2+a^2\,c^2\,y^2\,(X\,z+x\,Z)+c^4\,x\,X\,y^2)\\&-a^2\,b^2\,z\,(p\,Z\,y\,(-x+y+z)\;-r\,(X\,y\,z-x\,Y\,(x+y+z)+x\,y\,Z))\\&-b^2\,c^2\,x\,(r\,X\,y\,(x+y-z)-p\,(X\,y\,z-Y\,z\,(x+y+z)+x\,y\,Z))\\&-b^4\,x\,z\,(r\,X\,(x+y)+p\,Z\,(y+z))=0\end{aligned}$$

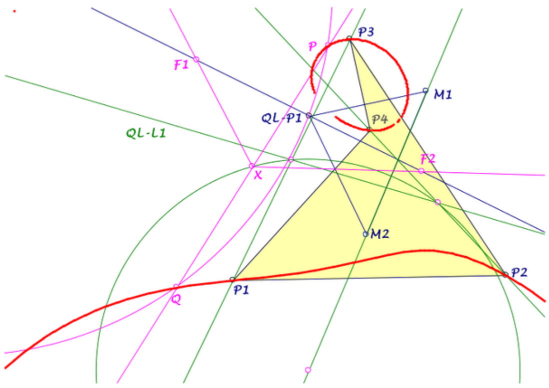
with

$$X:=x\,(q+r)-p\,(y+z)\,;\;Y:=y\,(r+p)-q\,(z+x)\,;\;Z:=z\,(p+q)-r\,(x+y)$$

Construction of the quartic: For the construction we need the vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel triangle and the fixed points F_1 and F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.

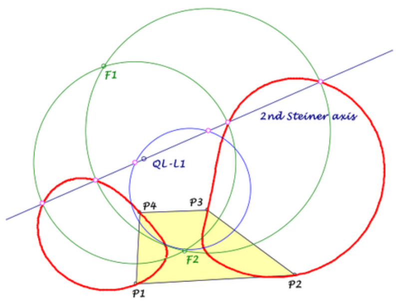
1. Circle through the intersections of the Newton Line *QL-L1* and the diagonals with center on M_1M_2.
2. For points X on this circle consider the angle bisector L_x of $\angle F_1XF_2$.
3. P, Q as intersections of L_x and its image-circle wrt *QL-Tf1* are points of the quartic.

2013-12-06.pdf



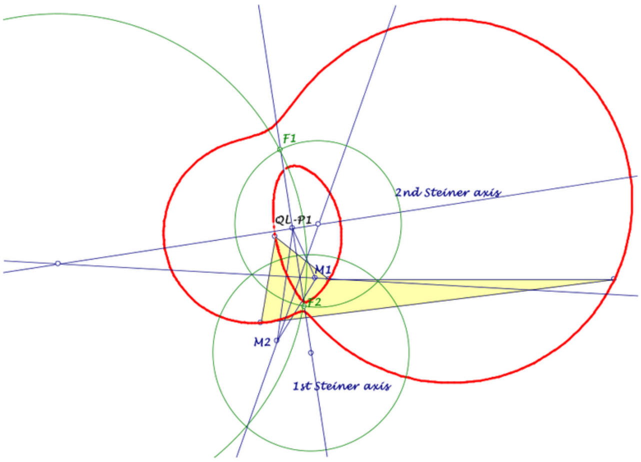
Properties:

- The quartic is the locus of points, where circles through opposite vertices of the quadrigon intersect perpendicular.
- The quartic is a circumquartic of the quadrigon.
- The quartic is invariant wrt *QL-Tf1*.
- The quartic is invariant wrt the transformation described in *EQF-Note 2013-12-05*.
- For *QL-Tf1* partners on the quartic the midpoints lie on a circle through the intersections of *QL-L1* and the diagonals, centered on M_1M_2.
- The 2^{nd} Steiner axis cuts this circle in two points not always real). Circles round these points through F_1, F_2 cut the 2^{nd} Steiner axis on the quartic.



- The quartic is anallagmatic, that means: The quartic is invariant wrt a reflection in a circle. There are three those circles (not always real): Two circles with midpoints in the intersections of the 2^{nd} Steiner axis with the angle bisectors of the Miquel triangle at M_1 and M_2, containing F_1 and F_2, and one circle perpendicular to the Schmidt circle with midpoint in the excenter of the Miquel triangle corresponding *QL-P1*.

2013-12-06.pdf



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-06.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #364
Date: 08/12/2013 11:29:16
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

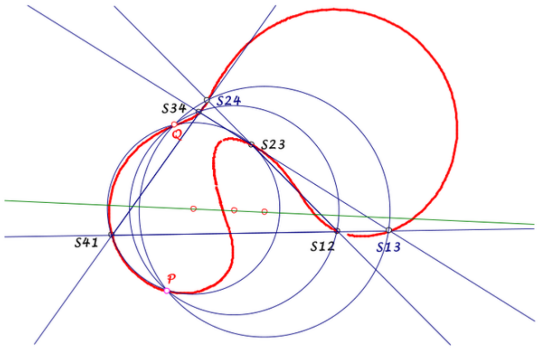
Dear friends,
for a quadrilateral there are three pairs of "opposite" points and for each pair a circle through a given point. The points, for which the three circles are coaxal give a quartic (see attachment).
Best regards Eckart

EQF-Note 2013-12-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Quartic for Quadrilaterals

For a quadrilateral there are three pairs of intersections and for each pair a circle through a given point. The points, for which these circles are coaxal give a quartic.



Let L_1, L_2, L_3, L_4 define a quadrilateral, then there are three pairs of intersections:

$$S_{12} = L_1 \cap L_2, S_{34} = L_3 \cap L_4 \quad and \quad S_{23} = L_2 \cap L_3, S_{41} = L_4 \cap L_1$$

$$and \quad S_{13} = L_1 \cap L_3, S_{24} = L_2 \cap L_4 \ .$$

We take for each pair the circumcircle through a point P and consider the locus of P for collinear centers of these three circles. This curve give a quartic.

If we chose the diagonal triangle *QL-DT* as reference triangle for barycentric coordinates and give L_4 the coefficients l, m, n, then the quartic has the equation

$$\begin{aligned} & a^2 b^2 (l^2 - m^2) z (l^2 n^2 x X + m^2 n^2 y Y - n^4 z Z + 2 l^2 m^2 x y (x + y)) \\ & + b^2 c^2 (m^2 - n^2) x (-l^4 x X + l^2 m^2 y Y + l^2 n^2 z Z + 2 m^2 n^2 y z (y + z)) \\ & + a^2 c^2 (n^2 - l^2) y (l^2 m^2 x X - m^4 y Y + m^2 n^2 z Z + 2 l^2 n^2 x z (x + z)) \\ & + a^4 (l^2 - m^2) (n^2 - l^2) y z (-m^2 y (x + y) + n^2 z (x + z)) \\ & + b^4 (l^2 - m^2) (m^2 - n^2) x z (-n^2 z (y + z) + l^2 x (x + y)) \\ & + c^4 (m^2 - n^2) (n^2 - l^2) x y (-l^2 x (x + z) + m^2 y (y + z)) == 0 \end{aligned}$$

with

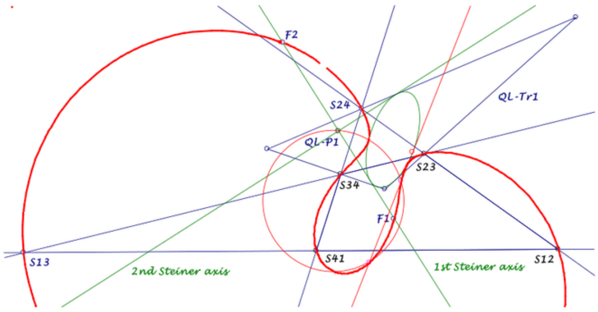
$$X := 2 y z + x (x + y + z); \ Y := 2 x z + y (x + y + z);$$

$$Z := 2 x y + z (x + y + z)$$

2013-12-08.pdf

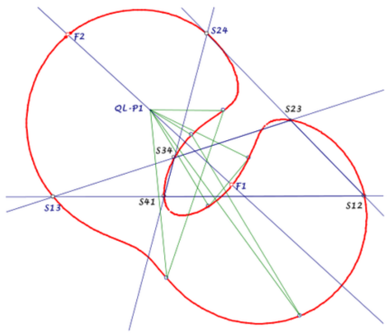
Construction of the quartic: For the construction we need the *1st* and *2nd* Steiner axis (see *QL-Tf1*):

1. Construct an inscribed conic of the Diagonal Triangle *QL-Tr1* with the *1st* and *2nd* Steiner axis (and Steiner Line *QL-L2*) as tangents.
2. The intersections of tangents at this conic and their image-circle wrt the Clawson-Schmidt Conjugate *QL-Tf1* will give points of the quartic.



Properties:

- This quartic contains the six intersections of the quadrilateral.
- The quartic contains the vertices M_1, M_2 (unequal *QL-P1*) of the Miquel Triangles *QA-Tr2* of the three quadrigon components.
- The quartic contains the fixed points F_1, F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.



- The quartic is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.
- The quartic is invariant wrt the transformation described in *EQF-Note 2013-12-05*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-08.pdf

Message: #365
Date: 08/12/2013 1:12:31
From: Chris van Tienhoven
Subject: An excellent book of geometry problems

```
Dear friends,
See message below from Francisco Javier Garcia Capitan.
He recommends a most special document of
Jose Maria Pedret.
I went through it.
Pages in the document 67-75, 292-334 (pages in PDF-file 69-77,
296-338)
all deal about quadrilaterals!
This is very special!
Best regards,
Chris

*Van:*
AdvancedPlaneGeometry@yahoogroups.com
[mailto:AdvancedPlaneGeometry@yahoogroups.com] *Namens*
garciacapitan@gmail.com
*Verzonden:* vrijdag 6 december 2013 8:52
*Aan:* AdvancedPlaneGeometry@yahoogroups.com
*Onderwerp:* [ADGEOM] An excellent book of geometry problems
Dear friends, here is an excellent book of geometry problems
written in 1955.
The text it is in Spanish but it contains many figures that can
help.
My friend Jose Maria Pedret bought it in a second hand bookshop
and carefully scanned it. I have the permission from the author
to distribute it.
Enjoy the book!
More information: http://garciacapitan.99on.com/jsb/
Direct download: https://drive.google.com/file/d/0B640DPhcZB-mcX
Y4b05ZQ3NPLUE/edit?usp=sharing
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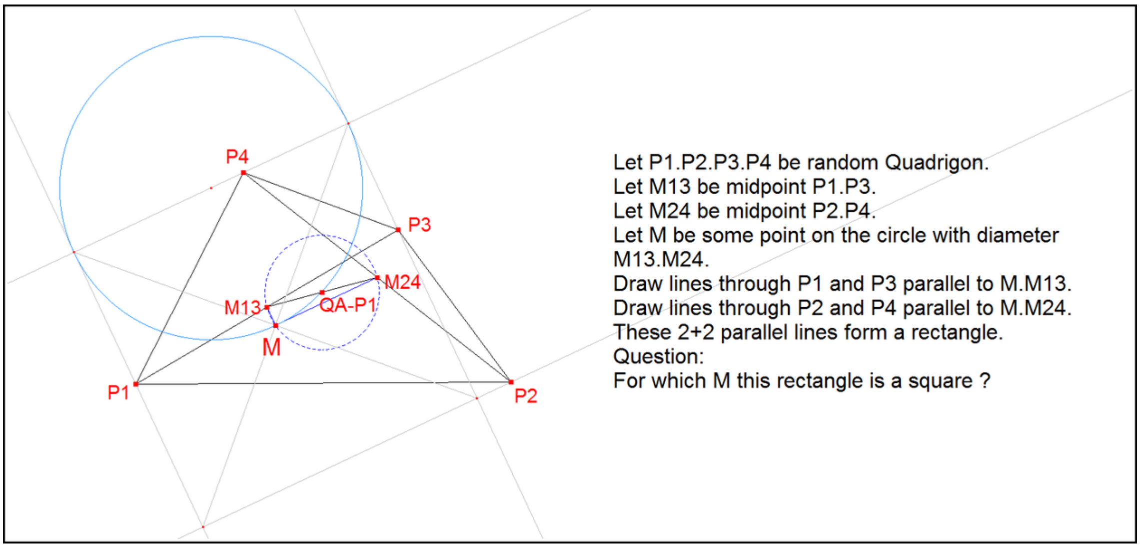
← Previous Next → ↔ Message Index ↑ Subjects

Message: #366
Date: 08/12/2013 1:21:17
From: Chris van Tienhoven
Subject: problem about circumscibed square of a quadrigon

```
Dear friends,
Let P1, P2, P3, P4 be the vertices of a Quadrigon.
Let M13 be midpoint P1.P3.
Let M24 be midpoint P2.P4.
Let M be some point on the circle with diameter M13.M24.
Draw lines through P1 and P3 parallel to M.M13.
Draw lines through P2 and P4 parallel to M.M24.
These 2+2 parallel lines form a rectangle.
Question:
How to construct and possibly calculate point M such that this
rectangle is a square?
Best regards,
Chris
See attachment
```

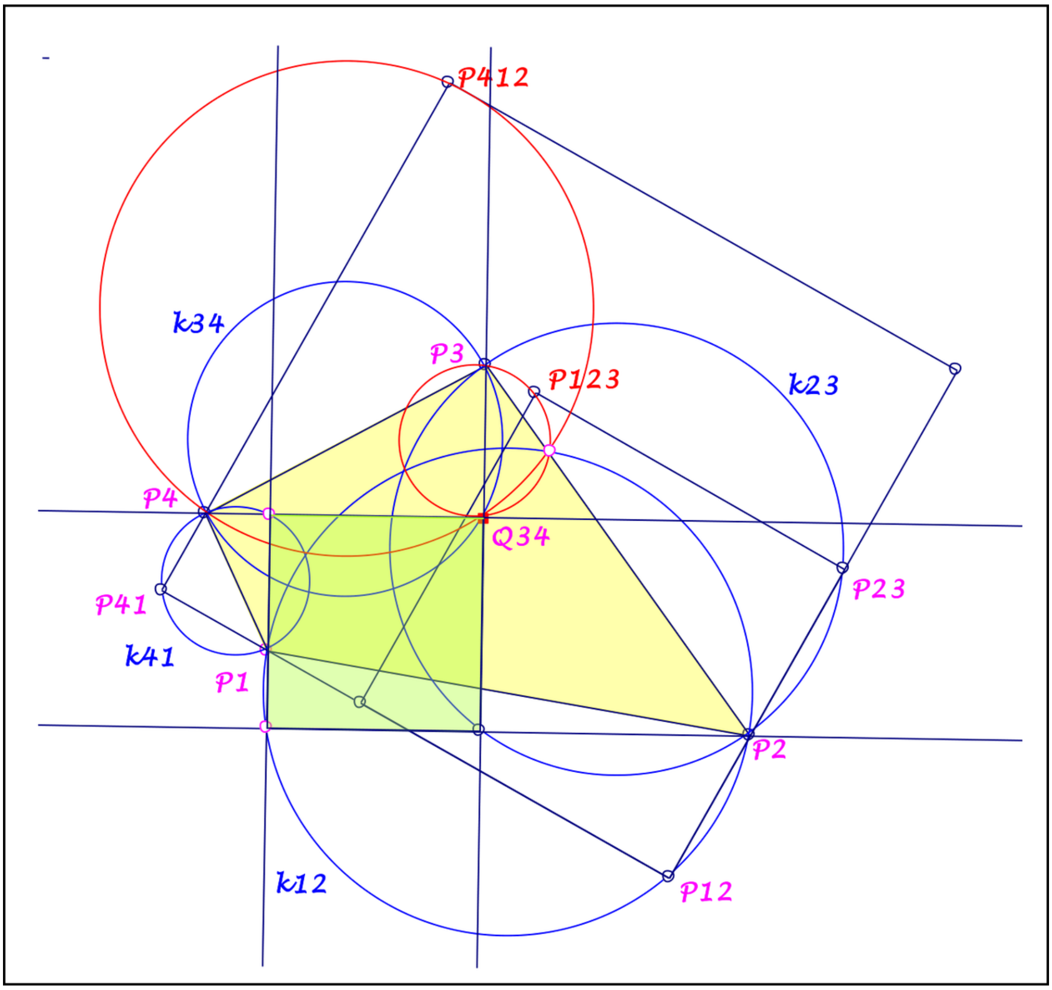


QG-Qu-Circumscribed Square-01.png

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #367
Date: 09/12/2013 9:12:27
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

```
Dear Chris,
here is a construction of the described square (see attachment):
Let P1P2P3P4 be a quadrigon and Cij the Thales circles about
PiPj with
... P12 point on C12,
    P41=P1P12^C41 and P23=P12P2^C23 (second intersections).
Construct a square about P41P12 with P412
    as opposite point of P12
... and a square about P12P23 with P123
    as opposite point of P12.
The loci of these opposite points are circles C412 and C123.
One of the intersections of these circles lies on C34.
This point is a vertex Q34 of the searched quadrigon ...
Best regards Eckart
```

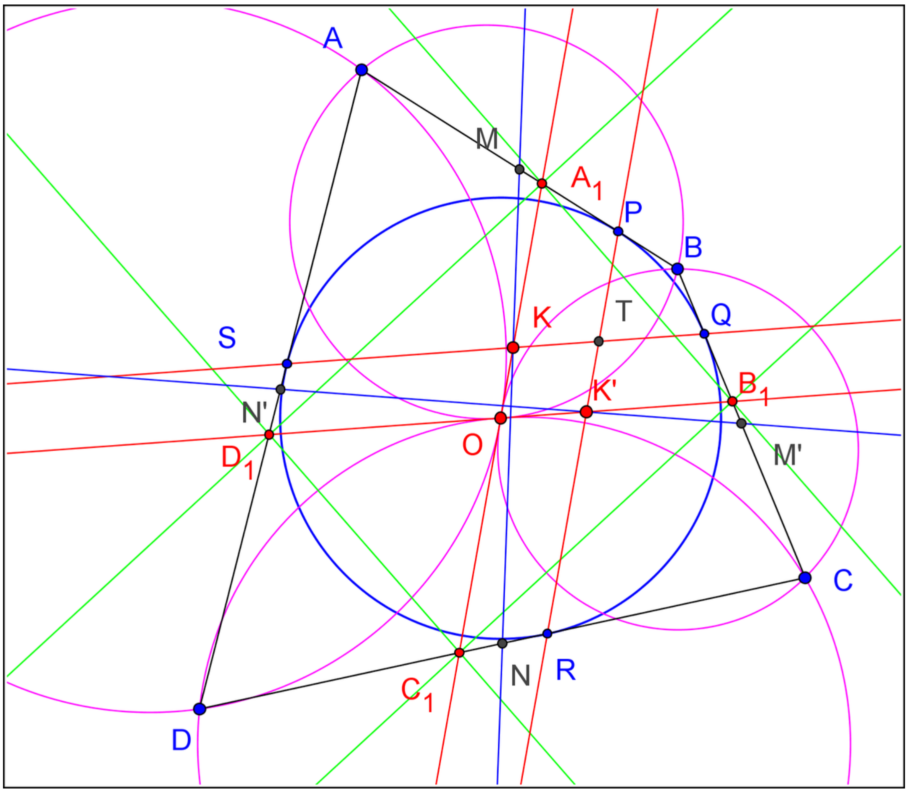


Square.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #368
Date: 09/12/2013 5:39:46
From: yeuemtrondoitb85
Subject: Tangential quadrilateral well known properties?

Dear Mr Eckart, Mr. Bernard K, Dear Mr. Chris,
(Denote similar file pdf of Mr Eckart)
ABCD is a circumscribed quadrilateral,
PQRS the points of tangency with the inscribed circle center O,
M and N are the midpoints of AB and CD,
M' and N' are the midpoints of BC and DA
Construct four circle (OAB), (OBC), (OCD), (ODA).
We have some result following:
1-Two tangents of (OAB), (OCD) at O is one line (coincedent line), this line meets BC,DA respectively at B1,D1. Two tangents of (OBC), (ODA) at O is one line (coincedent line), this line meets AB,CD at A1,C1
2-A1B1C1D1 is a parallelogram
3-QS//B1D1 and PR//A1C1
4-A1B1,MN,QS are concurrent at K; similar B1D1, M'N', RP are concurrent at K'
Best regards
Sincerely
Dao Thanh Oai



Tangential quadrilateral.png

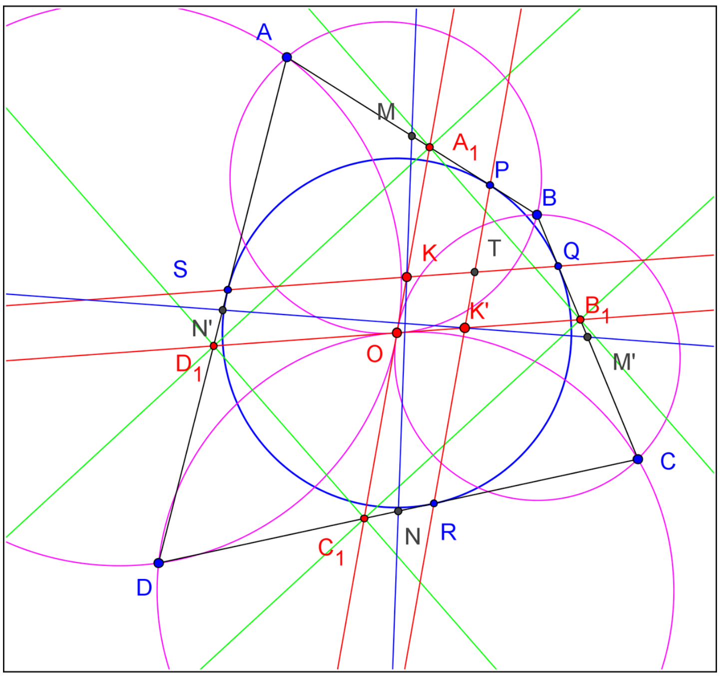
← Previous Next → ↔ Message Index ↑ Subjects

Message: #369
Date: 09/12/2013 5:50:23
From: yeuemtrondoitb85
Subject: Tangential quadrilateral well known properties?

```
Dear Mr Eckart, Mr. Bernard K, Dear Mr. Chris,

(Denote similar file pdf of Mr Eckart)
ABCD is a circumscribed quadrilateral,
PQRS the points of tangency with the inscribed circle center O,
M and N are the midpoints of AB and CD,
M' and N' are the midpoints of BC and DA
Construct four circle (OAB), (OBC), (OCD), (ODA).
We have some result following:
1-Two tangents of (OAB), (OCD) at O is one line (coincedent
line), this line meets BC,DA respectively at B1,D1. Two tangents
of (OBC), (ODA) at O is one line (coincedent line), this line
meets AB,CD at A1,C1
2-A1B1C1D1 is a parallelogram
3-QS//B1D1 and PR//A1C1
4-A1B1,MN,QS are concurrent at K; similar B1D1, M'N', RP are
concurrenr at K'
6-A1PRC1; and QB1D1S are isosceles trapezoid

Best regards
Sincerely
Dao Thanh Oai
```



Tangential quadrilateral-369.png

Message: #370
Date: 09/12/2013 11:40:36
From: Chris van Tienhoven
Subject: Quadri-Figures-Group] problem about circumscibed square of a qua

---

```
Dear Eckart,

Very nice construction of a circumscribed square around a
"quadrilateral"!
I took the liberty to alter your construction a bit. See also
attached files.

Construction two circumscribed squares around a quadrigon:
Let P1P2P3P4 be a quadrigon and Cij the Thales circles about
PiPj. C12 is intersected by the perpendicular bisector of P1.P2
in 2 points.
Let P12in  be the inward point (direction QA-Centroid).
Let P12out be the outward point (other direction).
Let S1     be 2nd intersection point of circles C12 and C14.
Let S2     be 2nd intersection point of circles C12 and C23.
Let C14in  be the circle through S1, P4 and P12in.
Let C23in  be the circle through S2, P3 and P12in.
Let V34in  be the 2nd intersection point of C14in and C23in.
V34in = the Vertex of the 1st circumscribed square connecting P3
and P4.
Let C14out be the circle through S1, P4 and P12out.
Let C23out be the circle through S2, P3 and P12out.
Let V34out be the 2nd intersection point of C14out and C23out.
V34out = the Vertex of the 2nd circumscribed square connecting
P3 and P4.
V34in and V34out are the constructed vertices of the
circumscribed squares.

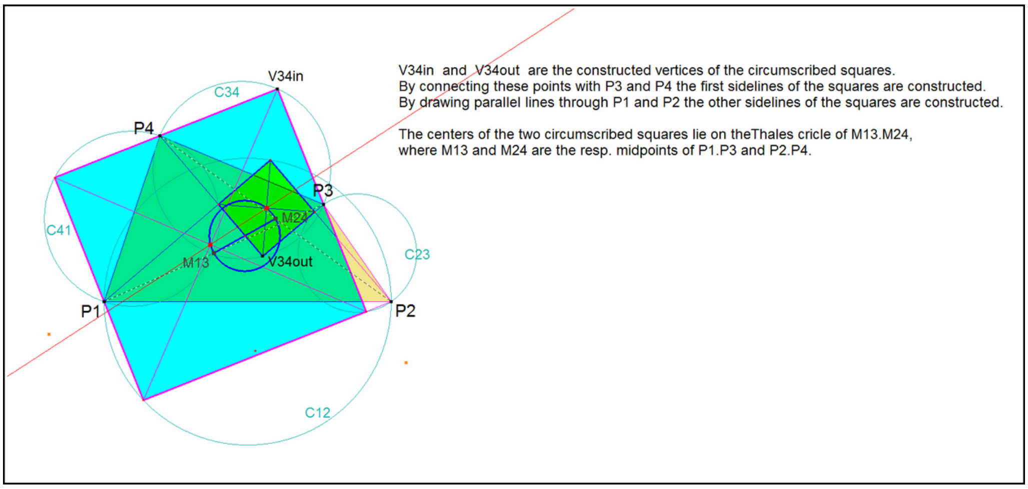
By connecting these points with P3 and P4 the first sidelines of
the squares are constructed.
By drawing parallel lines through P1 and P2 the other sidelines
of the squares are constructed.
Now we have 2 circumscribed squares.
The centers of the squares lie indeed on the Thales Circle of
M13.M24, where M13 and M24 are the midpoints of the two
diagonals of the Quadrigon.

Some other interesting properties:
* The diagonals of the two circumscribed squares produce 2 other
  intersection points, also on the Thales Circle of M13.M24.
* The connecting line of the two centers of the circumscribed
  squares passes through this point:
```

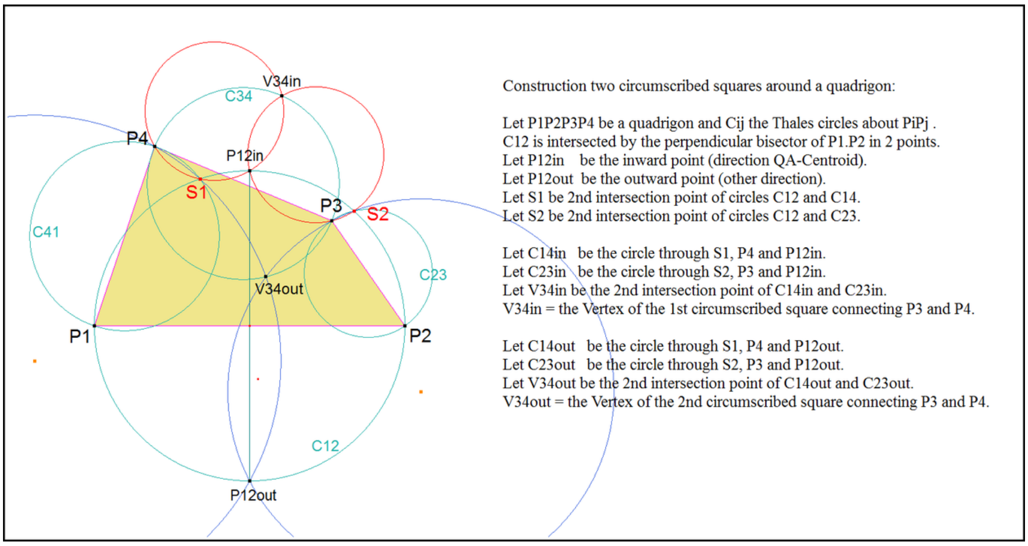
QG-P1.QG-P6 ^ QG-P5.QG-P8 ^ QA-P1.QG-P11,
which is a point on the Nine-point Conic QA-Co1.

See also QG/5: http://www.chrisvantienhoven.nl/quadrigon-objects/16-mathematics/encyclopedia-of-quadri-figures/quadrigon-objects/artikelen-qg/165-qg-5.html
I think there will be more of these intersection points on this circumscribed-square-center line.
* A first check didn't produce concurrencies of the QG-lines
of the 3 QA-Quadrigons or the 3 QL-Quadrigons.

Best regards,
Chris



QG-P9007-CircumscribedSquareCenter-12.png



QG-P9007-CircumscribedSquareCenter-10.png

Message: #371
Date: 10/12/2013 11:16:21
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

```
Dear Chris,
thank you for the description of the construction for the second
circumscribed square. I had supposed it, but wasn't able to
find! Reason: The calculation for point M gave two solutions.
Now I can give you the coefficients of the
circumscribed-square-center line (see attached Mathematica
file).
Best regards Eckart
```

Circumscribed-square-center line (message 370)

Reference triangle for barycentric coordinates:

P1P2P3 with P4:={p,q,r}.

$$
\begin{aligned}
\{&(q+r)\,(2p^2+3pq+2q^2+3pr+4qr+2r^2)\,SA^2\\
&-(p+q)\,r\,(p+q+2r)\,SC^2\\
&+(p^3+2p^2q+4p^2r+4pqr+pr^2-2qr^2-2r^3)\,SA\,SB\\
&+r\,(p^2-2pr-4qr-3r^2)\,SB\,SC\\
&-(p^3+4p^2q+5pq^2+2q^3+4pqr+3q^2r+2pr^2+2qr^2+r^3)\,SA\,SC,\ -p\,(q+r)\,(2p+q+r)\,SA^2\\
&-2\,(p+r)^2\,(p+2q+r)\,SB^2\\
&-(p+q)\,r\,(p+q+2r)\,SC^2\\
&+(p^3+4p^2q+10pq^2+4q^3+2p^2r+12pqr+10q^2r+3pr^2+8qr^2+2r^3)\,SA\,SB\\
&+(2p^3+8p^2q+10pq^2+4q^3+3p^2r+12pqr+10q^2r+2pr^2+4qr^2+r^3)\,SB\,SC\\
&+(3p+4q+3r)\,(p^2+2pq+q^2+pr+2qr+r^2)\,SA\,SC,\\
&-p\,(q+r)\,(2p+q+r)\,SA^2\\
&+(p+q)\,(2p^2+4pq+2q^2+3pr+3qr+2r^2)\,SC^2\\
&-p\,(3p^2+4pq+2pr-r^2)\,SA\,SB\\
&-(2p^3+2p^2q-p^2r-4pqr-4pr^2-2qr^2-r^3)\,SB\,SC\\
&-(p^3+2p^2q+3pq^2+2q^3+2p^2r+4pqr+5q^2r+4qr^2+r^3)\,SA\,SC\}
\end{aligned}
$$

Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

Chris-13-12-09-nb.pdf

Message: #372
Date: 12/12/2013 2:29:54
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

```
Dear Chris,
there is a very simple construction of the circumscribed squares
of a quadrigon, using the van Aubel constellation:
Given a quadrigon, place a square outward on each side, and
connect the centers of opposite squares. The second
intersections of these lines with the Thales circles over the
sides are the vertices of an circumscribed square. Placing the
squares inside the quadrigon, we get the second circumscribed
square.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #373
Date: 12/12/2013 7:25:24
From: Chris
Subject: problem about circumscibed square of a quadrigon

```
Dear Eckart,
Remarkably simple indeed!
I already had noticed long before the relation with the Van
Aubel configuration. But this simple construction is new to me.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #374
Date: 13/12/2013 9:04:36
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

Dear Chris,
there is a further aspect wrt the circumscribed squares of a quadrigon:
Reflecting a random point round the Varignon parallelogram of a quadrigon we get a quadrigon with the same midpoints of the sides as the reference quadrigon. All these quadrigons have circumscribed squares of the same size and parallel sides (see attachments).
Best regards Eckart

Circumscribed-squares of a Quadrigon

Reference triangle for barycentric coordinates:

P1P2P3 with P4:={p,q,r}.

Centroids of the circumscribed squares:

(For the second centroid change the sign of S)

$$\{-2 p^2 S\,SA - p q S\,SA - p r S\,SA - 2 p^2 S\,SB - 2 p r S\,SB + 3 p^2 SA\,SB + 2 p q SA\,SB + 3 p r SA\,SB - 2 p^2 S\,SC - 4 p q S\,SC - 2 q^2 S\,SC - 4 p r S\,SC - 3 q r S\,SC - r^2 S\,SC + 3 p^2 SA\,SC + 3 p q SA\,SC + 4 p r SA\,SC + 4 p^2 SB\,SC + 4 p q SB\,SC + 5 p r SB\,SC + 2 q r SB\,SC + r^2 SB\,SC + p r SC^2 + q r SC^2,$$

$$(-p S + r S - q SA - r SA + p SC + q SC)\ (p SA - r SC),\ -p^2 S\,SA - 3 p q S\,SA - 2 q^2 S\,SA - 4 p r S\,SA - 4 q r S\,SA - 2 r^2 S\,SA + p q SA^2 + p r SA^2 - 2 p r S\,SB - 2 r^2 S\,SB + p^2 SA\,SB + 2 p q SA\,SB + 5 p r SA\,SB + 4 q r SA\,SB + 4 r^2 SA\,SB - p r S\,SC - q r S\,SC - 2 r^2 S\,SC + 4 p r SA\,SC + 3 q r SA\,SC + 3 r^2 SA\,SC + 3 p r SB\,SC + 2 q r SB\,SC + 3 r^2 SB\,SC\}$$

Coefficients of the circumscribed-square-center line

$$\{(q + r)\ (2 p^2 + 3 p q + 2 q^2 + 3 p r + 4 q r + 2 r^2)\ SA^2$$
$$- (p + q)\ r\ (p + q + 2 r)\ SC^2$$
$$+ (p^3 + 2 p^2 q + 4 p^2 r + 4 p q r + p r^2 - 2 q r^2 - 2 r^3)\ SA\,SB$$
$$+ r\ (p^2 - 2 p r - 4 q r - 3 r^2)\ SB\,SC$$
$$- (p^3 + 4 p^2 q + 5 p q^2 + 2 q^3 + 4 p q r + 3 q^2 r + 2 p r^2 + 2 q r^2 + r^3)\ SA\,SC,\ -p\ (q + r)\ (2 p + q + r)\ SA^2$$
$$- 2\ (p + r)^2\ (p + 2 q + r)\ SB^2$$
$$- (p + q)\ r\ (p + q + 2 r)\ SC^2$$
$$+ (p^3 + 4 p^2 q + 10 p q^2 + 4 q^3 + 2 p^2 r + 12 p q r + 10 q^2 r + 3 p r^2 + 8 q r^2 + 2 r^3)\ SA\,SB$$
$$+ (2 p^3 + 8 p^2 q + 10 p q^2 + 4 q^3 + 3 p^2 r + 12 p q r + 10 q^2 r + 2 p r^2 + 4 q r^2 + r^3)\ SB\,SC$$
$$+ (3 p + 4 q + 3 r)\ (p^2 + 2 p q + q^2 + p r + 2 q r + r^2)\ SA\,SC,$$
$$-p\ (q + r)\ (2 p + q + r)\ SA^2$$
$$+ (p + q)\ (2 p^2 + 4 p q + 2 q^2 + 3 p r + 3 q r + 2 r^2)\ SC^2$$
$$- p\ (3 p^2 + 4 p q + 2 p r - r^2)\ SA\,SB$$
$$- (2 p^3 + 2 p^2 q - p^2 r - 4 p q r - 4 p r^2 - 2 q r^2 - r^3)\ SB\,SC$$
$$- (p^3 + 2 p^2 q + 3 p q^2 + 2 q^3 + 2 p^2 r + 4 p q r + 5 q^2 r + 4 q r^2 + r^3)\ SA\,SC\}$$

Side-length of a circumscribed square (For the second square change sign of S)

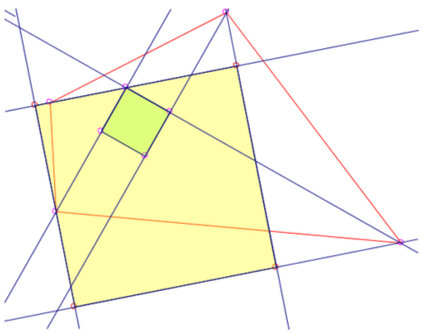
$$(p SA - r SC) / \sqrt{(-2\ (p + q + r)\ (p S + r S - p SA - r SC) + (q + r)^2 SA + (p + r)^2 SB + (p + q)^2 SC)}$$

Eckart Schmidt
eckart_schmidt@ t - online.de
http : // eckartschmidt.de

Chris-13-12-09-374-nb.pdf

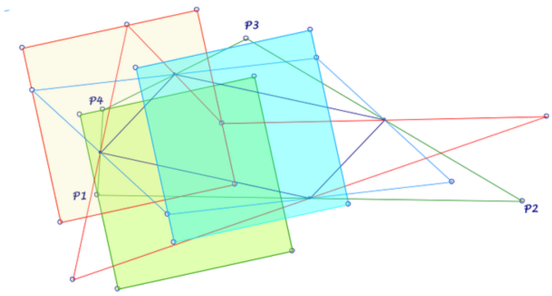
Some remarks wrt circumscribed squares of a quadrigon

For a quadrigon there are two circumscribed squares, see message 366, 370. Possible constructions see message 367, 370, 372. Properties of the circumscribed-square-center line see message 370. Results in calculation see attached Mathematica file.



New aspect:

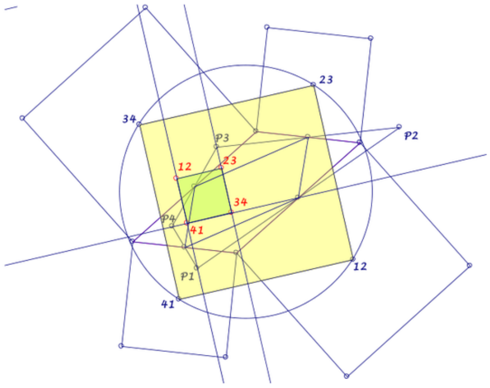
Consider the Varignon parallelogram of the quadrigon. Reflecting a random point *P* round this parallelogram we get a quadrigon with the same midpoints of the sides as the reference quadrigon. All these quadrigons have circumscribed squares of the same size and parallel sides. (The drawing shows only one square component).



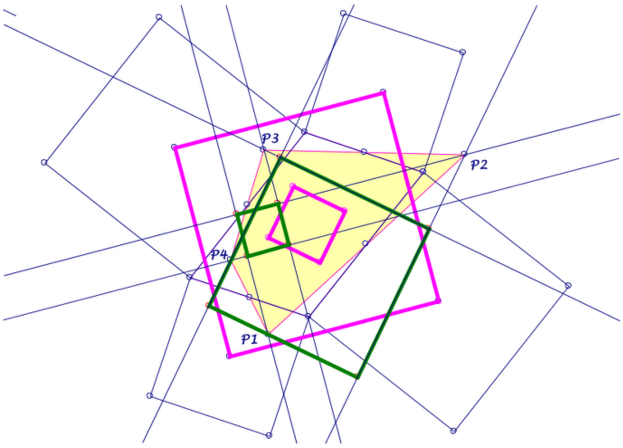
(If the point *P* runs along a circle round a vertex of the reference quadrigon, the centers of the circumscribed squares run along circles with the same radius round the centers of the circumscribed squares of the reference quadrigon.)

2013-12-13.pdf

Now we consider the parallelogram with the same Varignon parallelogram as the reference quadrigon and draw squares over / under the sides. The centers of these squares give again a square (Napoleon square). The sides of such a Napoleon square are parallel to those of that circumscribed square of the reference quadrigon with opposite orientation.



The ratio of side-lengths of circumscribed squares of a quadrigon equals the ratio of side-lengths of the Napoleon squares described in the last paragraph. This ratio depends only on the Varignon parallelogram of the reference triangle.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-13.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #375
Date: 13/12/2013 2:08:47
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

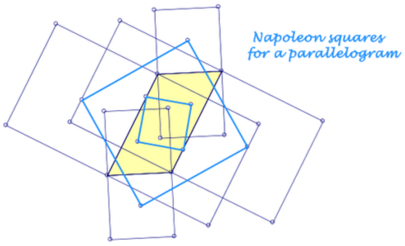
```
Dear Chris,
the Varignon parallelogram was the background for my examination
paper at university. So I have described a further aspect wrt
the circumscribed squares of a quadrigon (see attachment).
Best regards Eckart
```

Circumscribed squares of a quadrigon
− an additional aspect –

This is a reference to my professor
Prof. Dr. Friedrich Bachmann

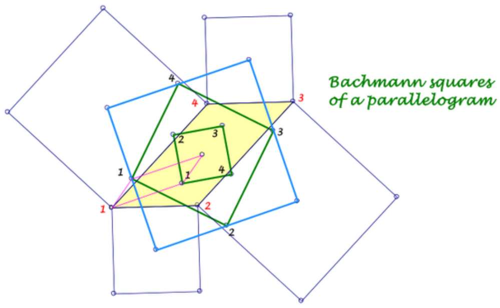
Napoleon squares of a parallelogram

Given a parallelogram, place a square outward / inside of each side, the centers of these squares give two new squares, here called the Napoleon squares (see [1], [2]).



Bachmann squares of a parallelogram

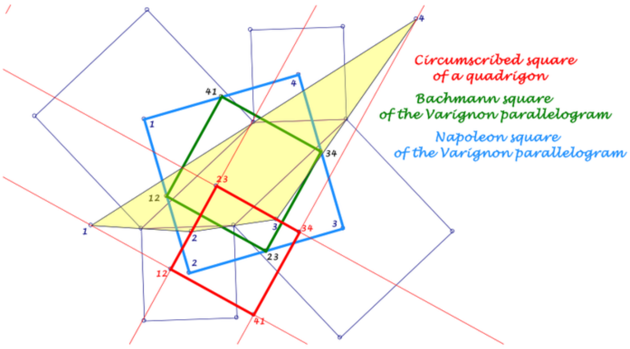
The midpoints of the sides of the Napoleon squares of a parallelogram give two further squares, here called Bachmann squares. The parallelogram is the vectorial sum of these Bachmann squares (see [1], [3]).



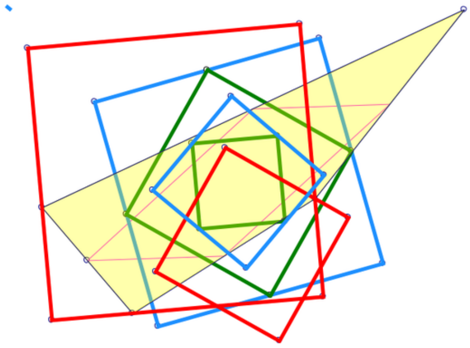
Circumscribed squares of a quadrigon

For a quadrigon there are two circumscribed squares (see QFG message 370).
For all quadrigons with the same Varignon parallelogram these circumscribed squares have the same size. Their sides are parallel to the sides of the Bachmann squares for the Varignon parallelogram.

2013-12-14.pdf



Finally: There are three pairs of squares, which have the same ratio of size.



References

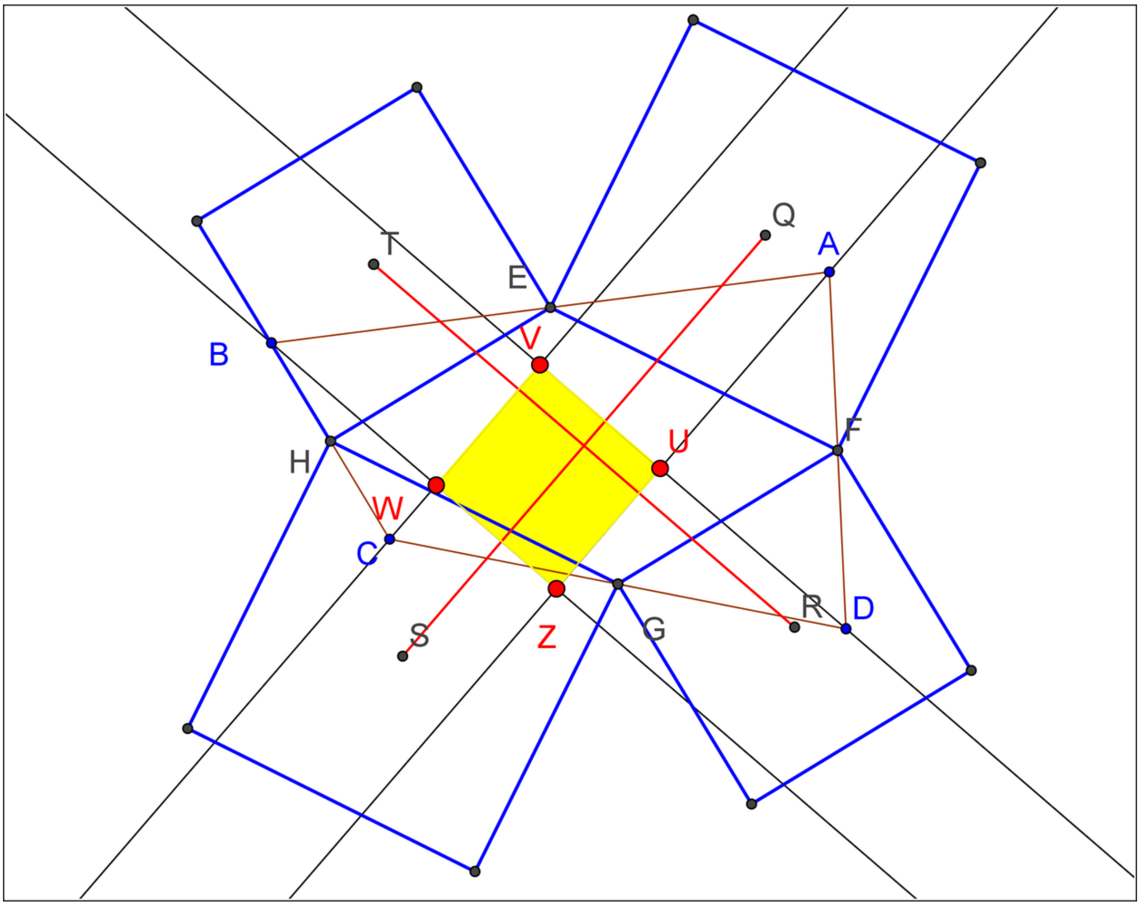
[1] E. Schmidt: Affin-reguläre n-Ecke und ihre regulären Komponenten. – MNU XXXIX,4.
[2] My homepage 05-5.
[3] F. Bachmann / E. Schmidt: n-Ecke.
– B.I - Hochschultaschenbücher 471/471a,
Bibliographisches Institut Mannheim 1970

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-14.pdf

Message: #376
Date: 13/12/2013 7:25:01
From: yeuemtrondoitb85
Subject: Problems: Square

```
Dear Mr Chris and Mr Eckart,
ABCD are a quadrilateral
EFGH are a parallelogram
TQRS is a square
then VWZU are squar
Best regards
Sincerely
Dao Thanh Oai
```



square.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #377
Date: 14/12/2013 8:13:12
From: Chris van Tienhoven
Subject: Problems: Square

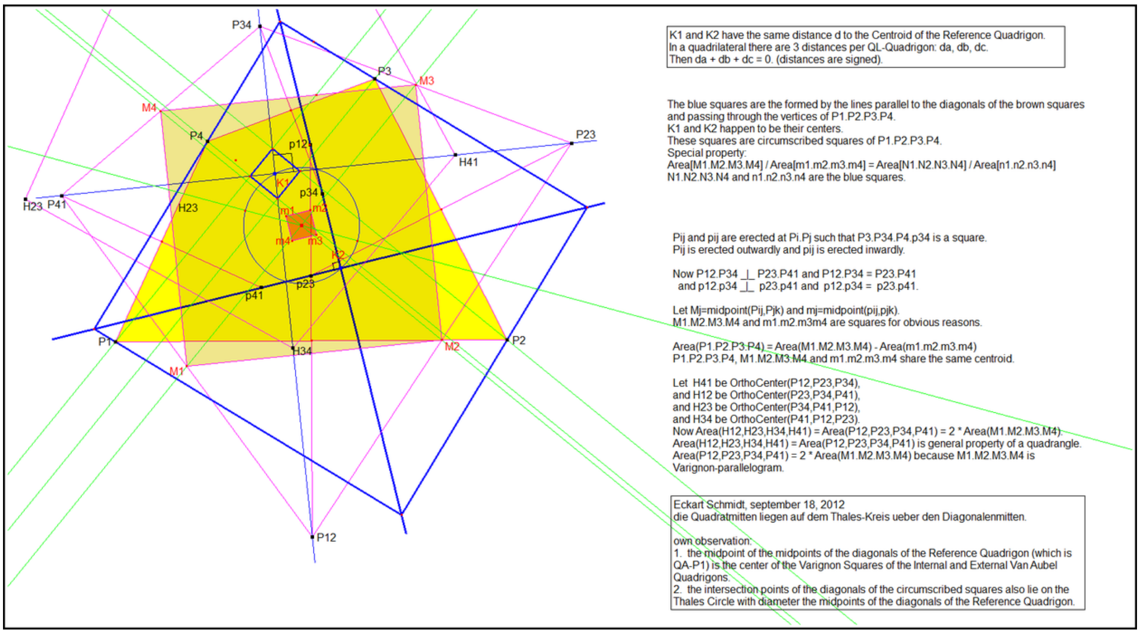
```
Dear Dao, Dear Eckart,
Very nice properties of the circumscribed square of a
quadrilateral.
Thank you!
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #378
Date: 14/12/2013 9:02:43
From: Chris van Tienhoven
Subject: problem about circumscibed square of a quadrigon

Dear Friends,
I found some old notes of mine about squares related to a Quadrigon and the Van Aubel Configuration.
Here are the notes. See also attached figure.
Notes:
Pij and pij are erected at Pi.Pj such that P3.P34.P4.p34 is a square.
Pij is erected outwardly and pij is erected inwardly.
Now P12.P34 _|_ P23.P41 and P12.P34 = P23.P41
and p12.p34 _|_ p23.p41 and p12.p34 = p23.p41.
Let Mj=midpoint(Pij,Pjk) and mj=midpoint(pij,pjk).
M1.M2.M3.M4 and m1.m2.m3m4 are squares for obvious reasons.
Area(P1.P2.P3.P4) = Area(M1.M2.M3.M4) - Area(m1.m2.m3.m4)
P1.P2.P3.P4, M1.M2.M3.M4 and m1.m2.m3.m4 share the same centroid.
Let H41 be OrthoCenter(P12,P23,P34),
and H12 be OrthoCenter(P23,P34,P41),
and H23 be OrthoCenter(P34,P41,P12),
and H34 be OrthoCenter(P41,P12,P23).
Now Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4).
Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) is general property of a quadrangle.
Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4) because M1.M2.M3.M4 is Varignon-parallelogram.
The blue squares in the figure are formed by the lines parallel to the diagonals of the brown squares and passing through the vertices of P1.P2.P3.P4.
K1 and K2 happen to be their centers.
These squares are circumscribed squares of P1.P2.P3.P4.
Special property:
Area[M1.M2.M3.M4] / Area[m1.m2.m3.m4] = Area[N1.N2.N3.N4] / Area[n1.n2.n3.n4]
N1.N2.N3.N4 and n1.n2.n3.n4 are the blue squares.
K1 and K2 have the same distance d to the Centroid of the Reference Quadrigon.
In a quadrilateral there are 3 distances per QL-Quadrigon: da, db, dc.
Then da + db + dc = 0. (distances are signed).
1. the midpoint of the midpoints of the diagonals of the Reference Quadrigon (which is QA-P1) is the center of the Varignon Squares of the Internal and External Van Aubel Quadrigons.

2. the intersection points of the diagonals of the circumscribed squares also lie on the Thales Circle with diameter the midpoints of the diagonals of the Reference Quadrigon.
Best regards,
Chris van Tienhoven

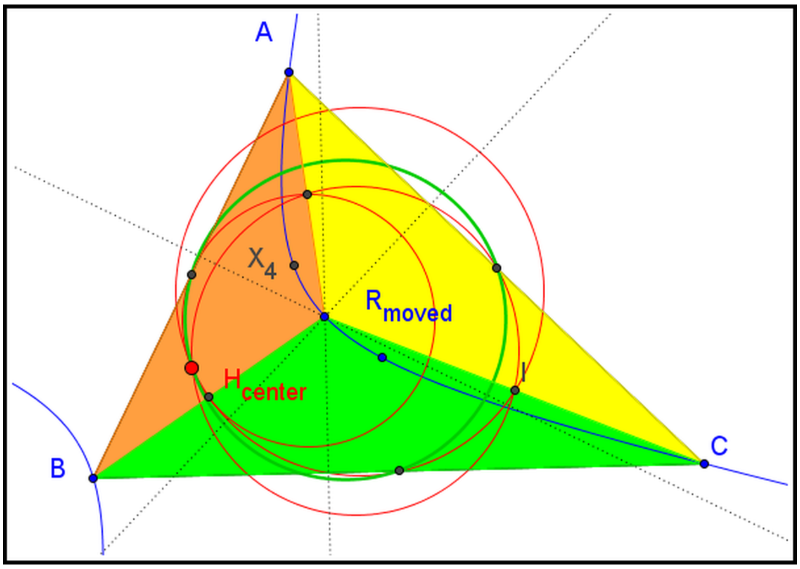


QG-2P99-VanAubelPoints-08a-circumsquares.png

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #379
Date: 15/12/2013 8:15:16
From: yeuemtrondoitb85
Subject: Poncelet point and rectangular hyperbola

```
Dear Mr. Chris and Mr. Eckart
Let ABC be a triangle, Let rectangular hyperbola through
A,B,C,X4 (X4 is the orthocenter of the triangle ABC). Denote D
is any point lie on the hyperbola. Then Poncelet point of four
points A,B,C,D is center of the rectangular hyperbola. (Show
that when D moved on the hyperbola then Poncelet point of ABCD
is fixed)
Best regards
Sincerely
Dao Thanh Oai
```



Poncelet and rectangular hyperbola.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #380
Date: 15/12/2013 11:07:51
From: Chris
Subject: Poncelet point and rectangular hyperbola

```
Dear Dao,
At first sight a remarkable property.
As often occurs at second sight it is evident.
Taking four points on an Orthogonal Hyperbola we have a
Quadrangle.
In a Quadrangle only one Orthogonal Hyperbola can be
constructed.
So the Orthogonal Hyperbola it is spanned in, is the only one.
The Poncelet Point (QA-P2) of a Quadrangle is the Center of the
only Orthogonal Hyperbola of the Quadrangle.
So it is evident that a Quadrangle that is spanned on 4 points
of an Orthogonal Hyperbola is a fixed point being the Center of
the Orthogonal Hyperbola.
Seen from the Triangle point of view the Orthogonal Hyperbola
can be defined by A,B,C,H and a 5th point P5. So the point you
describe can be seen as the transformation of random point P5.
It is the Center of the Orthogonal Hyperbola (A,B,C,H,P5).
Best regards,
Chris van Tienhoven
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #381
Date: 15/12/2013 11:49:34
From: eckart_schmidt@t-online.de
Subject: Problems: Square

```
Dear Dao,
with your simple construction of the circumscribed squares of a
quadrigon it is easy to realize, that for all quadrigons with
the same Varignon parallelogram these circumscribed squares have
parallel corresponding sides and the same size (already
mentioned in message #374, #375).
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #382
Date: 15/12/2013 11:59:36
From: yeuemtrondoitb85
Subject: Problems: Square

```
Dear Mr. Chris, and Mr. Eckart.
Thank to you very much
Best regards
Sincerely
Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

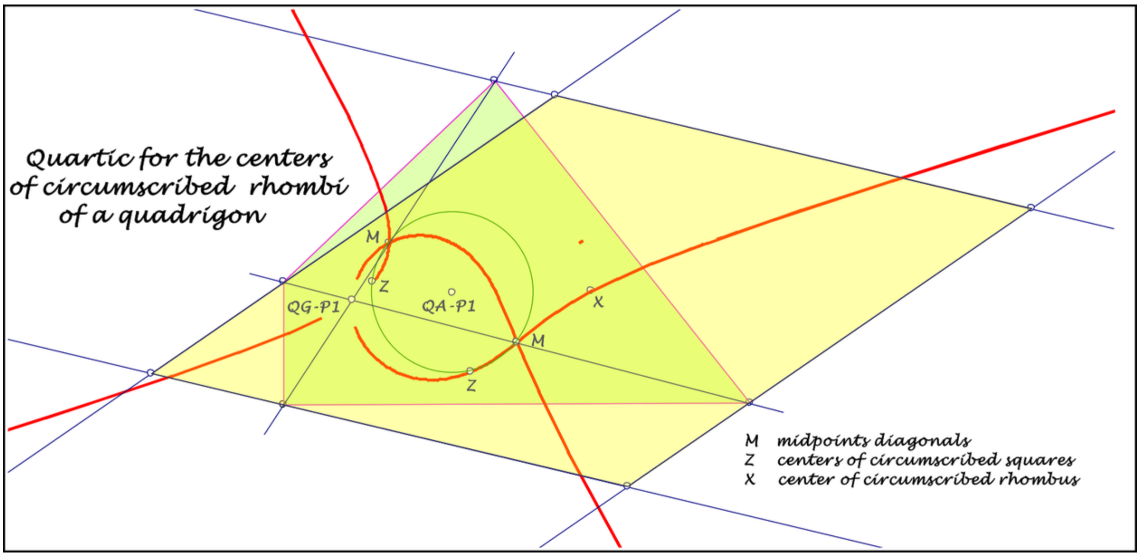
Message: #383
Date: 15/12/2013 12:03:09
From: yeuemtrondoitb85
Subject: Poncelet point and rectangular hyperbola

```
Dear Mr Chris,
I generalization one theorem at here: Vonk, Jan (2009), "The
Feuerbach point and reflections of the Euler line" http://forumg
eom.fau.edu/FG2009volume9/FG2009Volume9.pdf#page=51, Forum
Geometricorum *9*: 47-55
Best regards
Sincerely
Dao Thanh Oai
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #384
Date: 15/12/2013 3:00:59
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

```
Dear Chris,
in addition to circumscribed squares of a quadrigon:
The centers of circumscribed rectangles lie on the Thales circle
about the midpoints of the diagonals.
The centers of circumscribed rhombi lie on a quartic (see
attachment). The quartic contains the diagonal crosspoint QG-P1,
the midpoints M of the diagonals and the centers Z of the
circumscribed squares. Construction and equation can be given.
Best regards Eckart
```



2013-12-15.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #385
Date: 15/12/2013 4:17:10
From: Chris
Subject: problem about circumscibed square of a quadrigon

```
Dear Eckart,
Nice quartic with these familiar points on it.
I tried to construct the circumscribed rhombi.
In my picture I found 4 circumscribed rhombi.
Do you have a construction method?
I am interested in the equation too.
Best regards,
Chris
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #386
Date: 16/12/2013 10:20:50
From: eckart_schmidt@t-online.de
Subject: problem about circumscibed square of a quadrigon

```
Dear Chris,
attached a description of the construction for the circumscribed
rhombi of a quadrigon (I'm sure, there will be a simpler way).
The equation of the quartic is easy to calculate, see
attachment.
Best regards Eckart
```

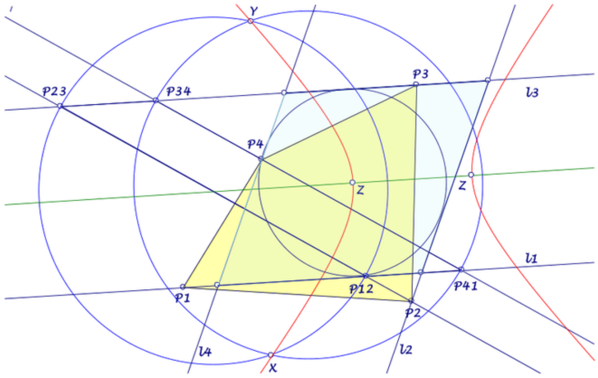
Equation of the quartic for the centers of circumscribed rhombi of a quadrigon (reference triangle P1P2P3 with P4(p:q:r))

```
quad[{x_, y_, z_}] := -b^2 (r^2 x^4 + p^2 z^4) + (c^2 p^2 + a^2 p r - b^2 p r + c^2 p r + a^2 r^2) y^4
+2 b^2 (p + r) x z (r x^2 + p z^2) - 2 (a^2 - c^2) y (r^2 x^3 - p^2 z^3) - b^2 (p^2 + 4 p r + r^2) x^2 z^2
+ (p + 2 q + 3 r) (c^2 p + 2 c^2 q - a^2 r + b^2 r) x^2 y^2
+ (3 p + 2 q + r) (b^2 p - c^2 p + 2 a^2 q + a^2 r) y^2 z^2
-2 (c^2 p^2 + 2 c^2 p q + 2 c^2 p r + a^2 q r - b^2 q r + c^2 q r - b^2 r^2 + c^2 r^2) x y^3
-2 (a^2 p^2 - b^2 p^2 + a^2 p q - b^2 p q + c^2 p q + 2 a^2 p r + 2 a^2 q r + a^2 r^2) y^3 z
+2 (a^2 - c^2) x y z (2 p r x + r^2 x - p^2 z - 2 p r z) +
  2 (a^2 p^2 - b^2 p^2 + 3 a^2 p q - 3 b^2 p q + c^2 p q + 2 a^2 q^2 - 2 b^2 q^2 + 2 c^2 q^2 +
     3 a^2 p r - 4 b^2 p r + 3 c^2 p r + a^2 q r - 3 b^2 q r + 3 c^2 q r - b^2 r^2 + c^2 r^2) x y^2 z
```

Eckart Schmidt
eckart_schmidt@ t - online.de
http : // eckartschmidt.de

Chris-13-12-16-nb.pdf

Construction of circumscribed rhombi for a quadrigon



Let $P_1P_2P_3P_4$ be a quadrigon.

- l_1 variable line through P_1 with a variable point P_{12},
- l_3 parallel to l_1 through P_3,
- $l_2 = P_{12}P_2$,
- l_4 parallel to l_2 through P_4,
- $P_{12}P_{23}P_{34}P_{41}$ circumscribed parallelogram.
- Thales circles over $P_{12}P_{23}$ and $P_{34}P_{41}$ with intersections X, Y.
- Changing P_{12} on l_1 the locus of X, Y is a conic.
- $Z_{1,2}$ intersections of the conic and the mid-parallel of l_1 and l_3, centers of circumscribed rhombi.
- Circle round Z_1 or Z_2 tangent to l_1, l_3.
- A tangent through P_2 at this circle and a parallel through P_4 give a circumscribed rhombus.
- Changing the line l_1 through P_1 the centers Z_i give the quartic for the centers of circumscribed rhombi.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2013-12-15-386.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #387
Date: 18/12/2013 5:41:06
From: rhutson2
Subject: conic through reflections

Dear friends,
I have been away from quadri-figures for awhile, so forgive me if this has come up before.
Let P1P2P3P4 be a quadrangle and Q a point. Let Qij be the reflection of Q in line PiPj for all i,j in {1,2,3,4}, i<>
1) What is the locus of Q such that Q12, Q13, Q14, Q23, Q24, Q34 all lie on a common conic? The locus would include QA-P4, but no other listed QA-points.
2) Is there a unique point Q such that the conic is a circle?
3) Is there a unique point Q such that the conic is a rectangular hyperbola?
4) Is there a unique point Q such that Q is also the center of the conic?
If Q = QA-P4, the conic has center at QA-P2 (but is not QA-Co2).
Conjecture for Q = QA-P4: if P1P2P3P4 is concave, the conic is always an ellipse; if convex, it is always a hyperbola, and rectangular if cyclic.
Best regards,
Randy Hutson

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #388
Date: 19/12/2013 10:03:43
From: eckart_schmidt@t-online.de
Subject: conic through reflections

Dear Randy,
interesting questions! Here some remarks to question 1 out of first experiences with Cabri:
* The locus of Q seems to be a cubic...
* ... circumscribed the diagonal triangle QA-Tr1,
* ... circumscribed the Miquel triangle QA-Tr2,
* ... invariant wrt the three versions of QL-Tf1.
Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #389
Date: 19/12/2013 3:25:01
From: Chris
Subject: conic through reflections

```
Dear Randy, Dear Eckart,
The locus of all points for which the reflections in the 6
connecting lines in a Quadrangle are coconic (all laying on a
conic) also passes through the Involutory Conjugate of QA-P2
(Euler-Poncelet Point), which is an infinity point.
Also the circular points at infinity lie on the locus, so it is
circular.
Best regards,
Chris
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #390
Date: 19/12/2013 5:04:12
From: rhutson2
Subject: conic through reflections

```
Another observation I forgot to mention: when Q = QA-P4, the
conic also passes through QA-P4 (and, of course, its antipode,
the reflection of QA-P4 in QA-P2).
Best regards,
Randy
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #391
Date: 20/12/2013 4:38:01
From: amontes
Subject: conic through reflections

Dear Randy, Dear Eckart, Dear Chris,

[Randy Hutson QFG#387]:
Let P1P2P3P4 be a quadrangle and Q a point. Let Qij be the
reflection
of Q in line PiPj for all i,j in {1,2,3,4}, i<>
1.) What is the locus of Q such that Q12, Q13, Q14, Q23, Q24,
Q34 all
lie on a common conic? The locus would include QA-P4.
If Q = QA-P4, the conic has center at QA-P2.
[Eckart QFG#388]:
The locus of Q is circumscribed the Miquel triangle QA-Tr2
[Chris van Tienhoven QFG#389]:
The locus of all points for which the reflections in the 6
connecting
lines in a Quadrangle are coconic (all laying on a conic) also
passes
through the Involutory Conjugate of QA-P2 (Euler-Poncelet
Point), which
is an infinity point.

[Randy Hutson QFG#390]:
Another observation I forgot to mention: when Q = QA-P4, the
conic also
passes through QA-P4.
*** The locus of Q is the septic with singularities ("isolated
points")
P1,P2,P3 and P4. With the properties outlined above by Randy,
Eckart and
Chris.
Equation of septic, in DT-notation (very complicated, see
attached file
Sextic-QFG#387.txt)
Particular cases:

==== If P1P2P3P4 be the quadrangle IIaIbIc (incenter and
excenters),
the septic is decomposed into the product of the line at
infinity and a
sextic with I, Ia, Ib and Ic only real points ("isolated
points"):
-b^2 c^4 x^4 y^2 - a^2 c^4 x^3 y^3 - b^2 c^4 x^3 y^3 + c^6 x^3
y^3 -
a^2 c^4 x^2 y^4 + a^2 b^2 c^2 x^4 y z - b^4 c^2 x^4 y z -

b^2 c^4 x^4 y z + a^2 b^2 c^2 x^3 y^2 z - b^4 c^2 x^3 y^2 z +
b^2 c^4 x^3 y^2 z - a^4 c^2 x^2 y^3 z + a^2 b^2 c^2 x^2 y^3 z +
a^2 c^4 x^2 y^3 z - a^4 c^2 x y^4 z + a^2 b^2 c^2 x y^4 z -
a^2 c^4 x y^4 z - b^4 c^2 x^4 z^2 + a^2 b^2 c^2 x^3 y z^2 +
b^4 c^2 x^3 y z^2 - b^2 c^4 x^3 y z^2 + 6 a^2 b^2 c^2 x^2 y^2
z^2 +
a^4 c^2 x y^3 z^2 + a^2 b^2 c^2 x y^3 z^2 - a^2 c^4 x y^3 z^2 -
a^4 c^2 y^4 z^2 - a^2 b^4 x^3 z^3 + b^6 x^3 z^3 - b^4 c^2 x^3
z^3 -
a^4 b^2 x^2 y z^3 + a^2 b^4 x^2 y z^3 + a^2 b^2 c^2 x^2 y z^3 +
a^4 b^2 x y^2 z^3 - a^2 b^4 x y^2 z^3 + a^2 b^2 c^2 x y^2 z^3 +
a^6 y^3 z^3 - a^4 b^2 y^3 z^3 - a^4 c^2 y^3 z^3 - a^2 b^4 x^2
z^4 -
a^4 b^2 x y z^4 - a^2 b^4 x y z^4 + a^2 b^2 c^2 x y z^4 -
a^4 b^2 y^2 z^4=0.

=== If P1P2P3P4 be the quadrangle GGaGbGc (the centroid and the
vertices
of triangle antimedial), the equation of septic is (in
DT-notation):
a^8 x^6 y - 6 a^6 b^2 x^6 y + 9 a^4 b^4 x^6 y - 2 a^2 b^6 x^6 y
-
3 a^6 c^2 x^6 y + 12 a^4 b^2 c^2 x^6 y - 6 a^2 b^4 c^2 x^6 y +
3 a^4 c^4 x^6 y - 6 a^2 b^2 c^4 x^6 y - 2 a^2 c^6 x^6 y +
a^8 x^5 y^2 - 8 a^6 b^2 x^5 y^2 + 15 a^4 b^4 x^5 y^2 +
4 a^2 b^6 x^5 y^2 - 2 b^8 x^5 y^2 + 4 a^6 c^2 x^5 y^2 +
3 a^4 b^2 c^2 x^5 y^2 + 6 a^2 b^4 c^2 x^5 y^2 - 2 b^6 c^2 x^5
y^2 -
9 a^4 c^4 x^5 y^2 + 6 a^2 b^2 c^4 x^5 y^2 + 4 a^2 c^6 x^5 y^2 -
2 b^2 c^6 x^5 y^2 - 2 c^8 x^5 y^2 - 3 a^8 x^4 y^3 +
10 a^6 b^2 x^4 y^3 + 18 a^2 b^6 x^4 y^3 - 5 b^8 x^4 y^3 +
15 a^6 c^2 x^4 y^3 - 12 a^4 b^2 c^2 x^4 y^3 +
9 a^2 b^4 c^2 x^4 y^3 + 6 b^6 c^2 x^4 y^3 - 6 a^4 c^4 x^4 y^3 +
6 b^4 c^4 x^4 y^3 - 3 a^2 c^6 x^4 y^3 - 2 b^2 c^6 x^4 y^3 +
3 c^8 x^4 y^3 - 5 a^8 x^3 y^4 + 18 a^6 b^2 x^3 y^4 +
10 a^2 b^6 x^3 y^4 - 3 b^8 x^3 y^4 + 6 a^6 c^2 x^3 y^4 +
9 a^4 b^2 c^2 x^3 y^4 - 12 a^2 b^4 c^2 x^3 y^4 +
15 b^6 c^2 x^3 y^4 + 6 a^4 c^4 x^3 y^4 - 6 b^4 c^4 x^3 y^4 -
2 a^2 c^6 x^3 y^4 - 3 b^2 c^6 x^3 y^4 + 3 c^8 x^3 y^4 -
2 a^8 x^2 y^5 + 4 a^6 b^2 x^2 y^5 + 15 a^4 b^4 x^2 y^5 -
8 a^2 b^6 x^2 y^5 + b^8 x^2 y^5 - 2 a^6 c^2 x^2 y^5 +
6 a^4 b^2 c^2 x^2 y^5 + 3 a^2 b^4 c^2 x^2 y^5 + 4 b^6 c^2 x^2
y^5 +
6 a^2 b^2 c^4 x^2 y^5 - 9 b^4 c^4 x^2 y^5 - 2 a^2 c^6 x^2 y^5 +
4 b^2 c^6 x^2 y^5 - 2 c^8 x^2 y^5 - 2 a^6 b^2 x y^6 +
9 a^4 b^4 x y^6 - 6 a^2 b^6 x y^6 + b^8 x y^6 -
6 a^4 b^2 c^2 x y^6 + 12 a^2 b^4 c^2 x y^6 - 3 b^6 c^2 x y^6 -
6 a^2 b^2 c^4 x y^6 + 3 b^4 c^4 x y^6 - 2 b^2 c^6 x y^6 +
a^8 x^6 z - 3 a^6 b^2 x^6 z + 3 a^4 b^4 x^6 z - 2 a^2 b^6 x^6 z
-

6 a^6 c^2 x^6 z + 12 a^4 b^2 c^2 x^6 z - 6 a^2 b^4 c^2 x^6 z +
9 a^4 c^4 x^6 z - 6 a^2 b^2 c^4 x^6 z - 2 a^2 c^6 x^6 z +
2 a^8 x^5 y z - 10 a^6 b^2 x^5 y z + 8 a^2 b^6 x^5 y z -
4 b^8 x^5 y z - 10 a^6 c^2 x^5 y z + 12 a^4 b^2 c^2 x^5 y z +
12 a^2 b^4 c^2 x^5 y z - 4 b^6 c^2 x^5 y z +
12 a^2 b^2 c^4 x^5 y z + 8 a^2 c^6 x^5 y z - 4 b^2 c^6 x^5 y z -
4 c^8 x^5 y z + 2 a^8 x^4 y^2 z - 16 a^6 b^2 x^4 y^2 z -
3 a^4 b^4 x^4 y^2 z + 17 a^2 b^6 x^4 y^2 z - 7 b^8 x^4 y^2 z +
4 a^6 c^2 x^4 y^2 z - 12 a^4 b^2 c^2 x^4 y^2 z +
18 a^2 b^4 c^2 x^4 y^2 z + 10 b^6 c^2 x^4 y^2 z -
21 a^4 c^4 x^4 y^2 z + 9 a^2 b^2 c^4 x^4 y^2 z +
18 b^4 c^4 x^4 y^2 z - 4 a^2 c^6 x^4 y^2 z + 2 b^2 c^6 x^4 y^2 z
+
c^8 x^4 y^2 z - 2 a^8 x^3 y^3 z - 2 a^6 b^2 x^3 y^3 z -
2 a^2 b^6 x^3 y^3 z - 2 b^8 x^3 y^3 z + 22 a^6 c^2 x^3 y^3 z -
12 a^4 b^2 c^2 x^3 y^3 z - 12 a^2 b^4 c^2 x^3 y^3 z +
22 b^6 c^2 x^3 y^3 z - 12 a^2 b^2 c^4 x^3 y^3 z -
14 a^2 c^6 x^3 y^3 z - 14 b^2 c^6 x^3 y^3 z + 10 c^8 x^3 y^3 z -
7 a^8 x^2 y^4 z + 17 a^6 b^2 x^2 y^4 z - 3 a^4 b^4 x^2 y^4 z -
16 a^2 b^6 x^2 y^4 z + 2 b^8 x^2 y^4 z + 10 a^6 c^2 x^2 y^4 z +
18 a^4 b^2 c^2 x^2 y^4 z - 12 a^2 b^4 c^2 x^2 y^4 z +
4 b^6 c^2 x^2 y^4 z + 18 a^4 c^4 x^2 y^4 z +
9 a^2 b^2 c^4 x^2 y^4 z - 21 b^4 c^4 x^2 y^4 z +
2 a^2 c^6 x^2 y^4 z - 4 b^2 c^6 x^2 y^4 z + c^8 x^2 y^4 z -
4 a^8 x y^5 z + 8 a^6 b^2 x y^5 z - 10 a^2 b^6 x y^5 z +
2 b^8 x y^5 z - 4 a^6 c^2 x y^5 z + 12 a^4 b^2 c^2 x y^5 z +
12 a^2 b^4 c^2 x y^5 z - 10 b^6 c^2 x y^5 z +
12 a^2 b^2 c^4 x y^5 z - 4 a^2 c^6 x y^5 z + 8 b^2 c^6 x y^5 z -
4 c^8 x y^5 z - 2 a^6 b^2 y^6 z + 3 a^4 b^4 y^6 z - 3 a^2 b^6
y^6 z +
b^8 y^6 z - 6 a^4 b^2 c^2 y^6 z + 12 a^2 b^4 c^2 y^6 z -
6 b^6 c^2 y^6 z - 6 a^2 b^2 c^4 y^6 z + 9 b^4 c^4 y^6 z -
2 b^2 c^6 y^6 z + a^8 x^5 z^2 + 4 a^6 b^2 x^5 z^2 -
9 a^4 b^4 x^5 z^2 + 4 a^2 b^6 x^5 z^2 - 2 b^8 x^5 z^2 -
8 a^6 c^2 x^5 z^2 + 3 a^4 b^2 c^2 x^5 z^2 + 6 a^2 b^4 c^2 x^5
z^2 -
2 b^6 c^2 x^5 z^2 + 15 a^4 c^4 x^5 z^2 + 6 a^2 b^2 c^4 x^5 z^2 +
4 a^2 c^6 x^5 z^2 - 2 b^2 c^6 x^5 z^2 - 2 c^8 x^5 z^2 +
2 a^8 x^4 y z^2 + 4 a^6 b^2 x^4 y z^2 - 21 a^4 b^4 x^4 y z^2 -
4 a^2 b^6 x^4 y z^2 + b^8 x^4 y z^2 - 16 a^6 c^2 x^4 y z^2 -
12 a^4 b^2 c^2 x^4 y z^2 + 9 a^2 b^4 c^2 x^4 y z^2 +
2 b^6 c^2 x^4 y z^2 - 3 a^4 c^4 x^4 y z^2 +
18 a^2 b^2 c^4 x^4 y z^2 + 18 b^4 c^4 x^4 y z^2 +
17 a^2 c^6 x^4 y z^2 + 10 b^2 c^6 x^4 y z^2 - 7 c^8 x^4 y z^2 +
6 a^8 x^3 y^2 z^2 - 16 a^6 b^2 x^3 y^2 z^2 -
18 a^4 b^4 x^3 y^2 z^2 - 24 a^2 b^6 x^3 y^2 z^2 +
8 b^8 x^3 y^2 z^2 - 16 a^6 c^2 x^3 y^2 z^2 -
30 a^4 b^2 c^2 x^3 y^2 z^2 - 12 a^2 b^4 c^2 x^3 y^2 z^2 -
4 b^6 c^2 x^3 y^2 z^2 - 18 a^4 c^4 x^3 y^2 z^2 -

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12 a^2 b^2 c^4 x^3 y^2 z^2 + 12 b^4 c^4 x^3 y^2 z^2 -
24 a^2 c^6 x^3 y^2 z^2 - 4 b^2 c^6 x^3 y^2 z^2 + 8 c^8 x^3 y^2
z^2 +
8 a^8 x^2 y^3 z^2 - 24 a^6 b^2 x^2 y^3 z^2 -
18 a^4 b^4 x^2 y^3 z^2 - 16 a^2 b^6 x^2 y^3 z^2 +
6 b^8 x^2 y^3 z^2 - 4 a^6 c^2 x^2 y^3 z^2 -
12 a^4 b^2 c^2 x^2 y^3 z^2 - 30 a^2 b^4 c^2 x^2 y^3 z^2 -
16 b^6 c^2 x^2 y^3 z^2 + 12 a^4 c^4 x^2 y^3 z^2 -
12 a^2 b^2 c^4 x^2 y^3 z^2 - 18 b^4 c^4 x^2 y^3 z^2 -
4 a^2 c^6 x^2 y^3 z^2 - 24 b^2 c^6 x^2 y^3 z^2 + 8 c^8 x^2 y^3
z^2 +
a^8 x y^4 z^2 - 4 a^6 b^2 x y^4 z^2 - 21 a^4 b^4 x y^4 z^2 +
4 a^2 b^6 x y^4 z^2 + 2 b^8 x y^4 z^2 + 2 a^6 c^2 x y^4 z^2 +
9 a^4 b^2 c^2 x y^4 z^2 - 12 a^2 b^4 c^2 x y^4 z^2 -
16 b^6 c^2 x y^4 z^2 + 18 a^4 c^4 x y^4 z^2 +
18 a^2 b^2 c^4 x y^4 z^2 - 3 b^4 c^4 x y^4 z^2 +
10 a^2 c^6 x y^4 z^2 + 17 b^2 c^6 x y^4 z^2 - 7 c^8 x y^4 z^2 -
2 a^8 y^5 z^2 + 4 a^6 b^2 y^5 z^2 - 9 a^4 b^4 y^5 z^2 +
4 a^2 b^6 y^5 z^2 + b^8 y^5 z^2 - 2 a^6 c^2 y^5 z^2 +
6 a^4 b^2 c^2 y^5 z^2 + 3 a^2 b^4 c^2 y^5 z^2 - 8 b^6 c^2 y^5
z^2 +
6 a^2 b^2 c^4 y^5 z^2 + 15 b^4 c^4 y^5 z^2 - 2 a^2 c^6 y^5 z^2 +
4 b^2 c^6 y^5 z^2 - 2 c^8 y^5 z^2 - 3 a^8 x^4 z^3 +
15 a^6 b^2 x^4 z^3 - 6 a^4 b^4 x^4 z^3 - 3 a^2 b^6 x^4 z^3 +
3 b^8 x^4 z^3 + 10 a^6 c^2 x^4 z^3 - 12 a^4 b^2 c^2 x^4 z^3 -
2 b^6 c^2 x^4 z^3 + 9 a^2 b^2 c^4 x^4 z^3 + 6 b^4 c^4 x^4 z^3 +
18 a^2 c^6 x^4 z^3 + 6 b^2 c^6 x^4 z^3 - 5 c^8 x^4 z^3 -
2 a^8 x^3 y z^3 + 22 a^6 b^2 x^3 y z^3 - 14 a^2 b^6 x^3 y z^3 +
10 b^8 x^3 y z^3 - 2 a^6 c^2 x^3 y z^3 - 12 a^4 b^2 c^2 x^3 y
z^3 -
12 a^2 b^4 c^2 x^3 y z^3 - 14 b^6 c^2 x^3 y z^3 -
12 a^2 b^2 c^4 x^3 y z^3 - 2 a^2 c^6 x^3 y z^3 +
22 b^2 c^6 x^3 y z^3 - 2 c^8 x^3 y z^3 + 8 a^8 x^2 y^2 z^3 -
4 a^6 b^2 x^2 y^2 z^3 + 12 a^4 b^4 x^2 y^2 z^3 -
4 a^2 b^6 x^2 y^2 z^3 + 8 b^8 x^2 y^2 z^3 - 24 a^6 c^2 x^2 y^2
z^3 -
12 a^4 b^2 c^2 x^2 y^2 z^3 - 12 a^2 b^4 c^2 x^2 y^2 z^3 -
24 b^6 c^2 x^2 y^2 z^3 - 18 a^4 c^4 x^2 y^2 z^3 -
30 a^2 b^2 c^4 x^2 y^2 z^3 - 18 b^4 c^4 x^2 y^2 z^3 -
16 a^2 c^6 x^2 y^2 z^3 - 16 b^2 c^6 x^2 y^2 z^3 +
6 c^8 x^2 y^2 z^3 + 10 a^8 x y^3 z^3 - 14 a^6 b^2 x y^3 z^3 +
22 a^2 b^6 x y^3 z^3 - 2 b^8 x y^3 z^3 - 14 a^6 c^2 x y^3 z^3 -
12 a^4 b^2 c^2 x y^3 z^3 - 12 a^2 b^4 c^2 x y^3 z^3 -
2 b^6 c^2 x y^3 z^3 - 12 a^2 b^2 c^4 x y^3 z^3 +
22 a^2 c^6 x y^3 z^3 - 2 b^2 c^6 x y^3 z^3 - 2 c^8 x y^3 z^3 +
3 a^8 y^4 z^3 - 3 a^6 b^2 y^4 z^3 - 6 a^4 b^4 y^4 z^3 +
15 a^2 b^6 y^4 z^3 - 3 b^8 y^4 z^3 - 2 a^6 c^2 y^4 z^3 -
12 a^2 b^4 c^2 y^4 z^3 + 10 b^6 c^2 y^4 z^3 + 6 a^4 c^4 y^4 z^3
+
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9 a^2 b^2 c^4 y^4 z^3 + 6 a^2 c^6 y^4 z^3 + 18 b^2 c^6 y^4 z^3 -
5 c^8 y^4 z^3 - 5 a^8 x^3 z^4 + 6 a^6 b^2 x^3 z^4 +
6 a^4 b^4 x^3 z^4 - 2 a^2 b^6 x^3 z^4 + 3 b^8 x^3 z^4 +
18 a^6 c^2 x^3 z^4 + 9 a^4 b^2 c^2 x^3 z^4 - 3 b^6 c^2 x^3 z^4 -
12 a^2 b^2 c^4 x^3 z^4 - 6 b^4 c^4 x^3 z^4 + 10 a^2 c^6 x^3 z^4
+
15 b^2 c^6 x^3 z^4 - 3 c^8 x^3 z^4 - 7 a^8 x^2 y z^4 +
10 a^6 b^2 x^2 y z^4 + 18 a^4 b^4 x^2 y z^4 + 2 a^2 b^6 x^2 y
z^4 +
b^8 x^2 y z^4 + 17 a^6 c^2 x^2 y z^4 + 18 a^4 b^2 c^2 x^2 y z^4
+
9 a^2 b^4 c^2 x^2 y z^4 - 4 b^6 c^2 x^2 y z^4 -
3 a^4 c^4 x^2 y z^4 - 12 a^2 b^2 c^4 x^2 y z^4 -
21 b^4 c^4 x^2 y z^4 - 16 a^2 c^6 x^2 y z^4 + 4 b^2 c^6 x^2 y
z^4 +
2 c^8 x^2 y z^4 + a^8 x y^2 z^4 + 2 a^6 b^2 x y^2 z^4 +
18 a^4 b^4 x y^2 z^4 + 10 a^2 b^6 x y^2 z^4 - 7 b^8 x y^2 z^4 -
4 a^6 c^2 x y^2 z^4 + 9 a^4 b^2 c^2 x y^2 z^4 +
18 a^2 b^4 c^2 x y^2 z^4 + 17 b^6 c^2 x y^2 z^4 -
21 a^4 c^4 x y^2 z^4 - 12 a^2 b^2 c^4 x y^2 z^4 -
3 b^4 c^4 x y^2 z^4 + 4 a^2 c^6 x y^2 z^4 - 16 b^2 c^6 x y^2 z^4
+
2 c^8 x y^2 z^4 + 3 a^8 y^3 z^4 - 2 a^6 b^2 y^3 z^4 +
6 a^4 b^4 y^3 z^4 + 6 a^2 b^6 y^3 z^4 - 5 b^8 y^3 z^4 -
3 a^6 c^2 y^3 z^4 + 9 a^2 b^4 c^2 y^3 z^4 + 18 b^6 c^2 y^3 z^4 -
6 a^4 c^4 y^3 z^4 - 12 a^2 b^2 c^4 y^3 z^4 + 15 a^2 c^6 y^3 z^4
+
10 b^2 c^6 y^3 z^4 - 3 c^8 y^3 z^4 - 2 a^8 x^2 z^5 -
2 a^6 b^2 x^2 z^5 - 2 a^2 b^6 x^2 z^5 - 2 b^8 x^2 z^5 +
4 a^6 c^2 x^2 z^5 + 6 a^4 b^2 c^2 x^2 z^5 + 6 a^2 b^4 c^2 x^2
z^5 +
4 b^6 c^2 x^2 z^5 + 15 a^4 c^4 x^2 z^5 + 3 a^2 b^2 c^4 x^2 z^5 -
9 b^4 c^4 x^2 z^5 - 8 a^2 c^6 x^2 z^5 + 4 b^2 c^6 x^2 z^5 +
c^8 x^2 z^5 - 4 a^8 x y z^5 - 4 a^6 b^2 x y z^5 -
4 a^2 b^6 x y z^5 - 4 b^8 x y z^5 + 8 a^6 c^2 x y z^5 +
12 a^4 b^2 c^2 x y z^5 + 12 a^2 b^4 c^2 x y z^5 +
8 b^6 c^2 x y z^5 + 12 a^2 b^2 c^4 x y z^5 - 10 a^2 c^6 x y z^5
-
10 b^2 c^6 x y z^5 + 2 c^8 x y z^5 - 2 a^8 y^2 z^5 -
2 a^6 b^2 y^2 z^5 - 2 a^2 b^6 y^2 z^5 - 2 b^8 y^2 z^5 +
4 a^6 c^2 y^2 z^5 + 6 a^4 b^2 c^2 y^2 z^5 + 6 a^2 b^4 c^2 y^2
z^5 +
4 b^6 c^2 y^2 z^5 - 9 a^4 c^4 y^2 z^5 + 3 a^2 b^2 c^4 y^2 z^5 +
15 b^4 c^4 y^2 z^5 + 4 a^2 c^6 y^2 z^5 - 8 b^2 c^6 y^2 z^5 +
c^8 y^2 z^5 - 2 a^6 c^2 x z^6 - 6 a^4 b^2 c^2 x z^6 -
6 a^2 b^4 c^2 x z^6 - 2 b^6 c^2 x z^6 + 9 a^4 c^4 x z^6 +
12 a^2 b^2 c^4 x z^6 + 3 b^4 c^4 x z^6 - 6 a^2 c^6 x z^6 -
3 b^2 c^6 x z^6 + c^8 x z^6 - 2 a^6 c^2 y z^6 -
6 a^4 b^2 c^2 y z^6 - 6 a^2 b^4 c^2 y z^6 - 2 b^6 c^2 y z^6 +
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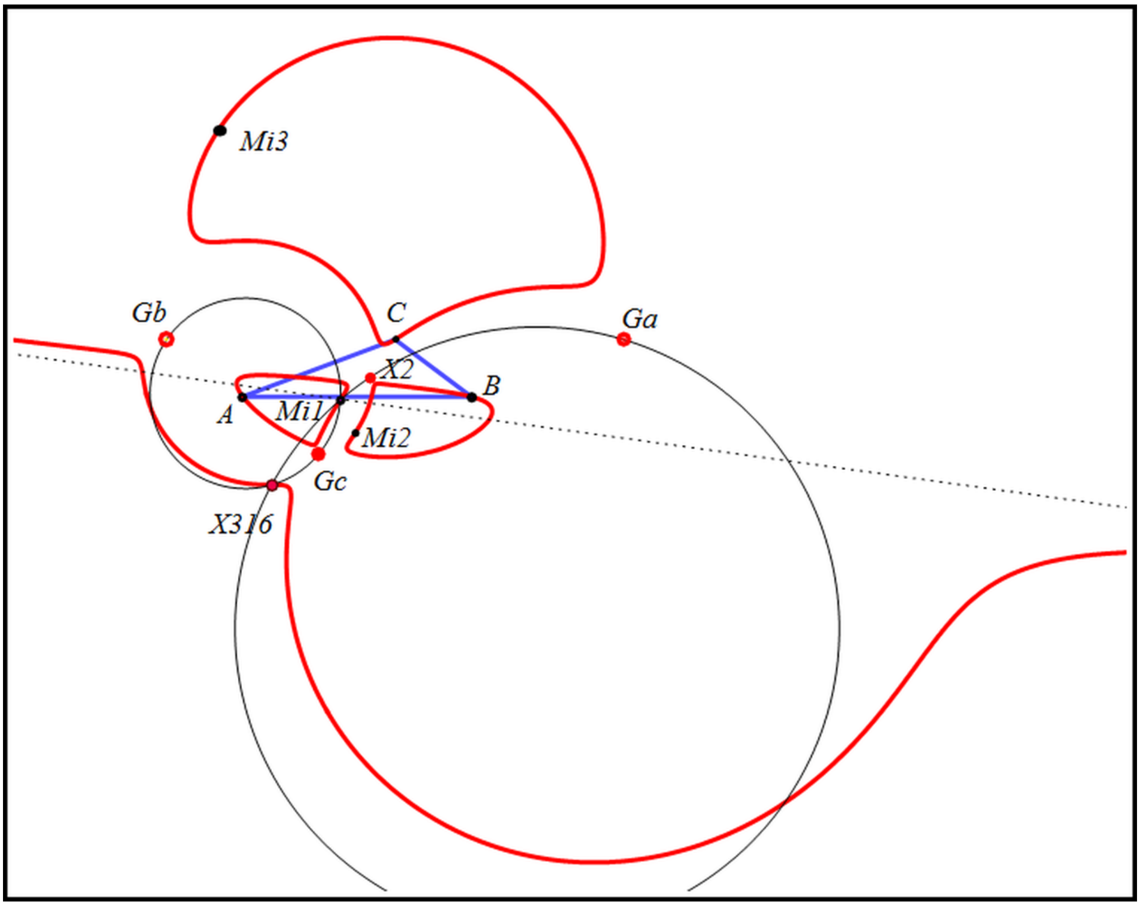
3 a^4 c^4 y z^6 + 12 a^2 b^2 c^4 y z^6 + 9 b^4 c^4 y z^6 -
3 a^2 c^6 y z^6 - 6 b^2 c^6 y z^6 + c^8 y z^6=0.

Is circumscribed the Miquel triangle QA-Tr2 =
{ Mi1=(a^2+b^2+c^2 : -a^2-b^2+2c^2 : -a^2 +2b^2-c^2) , Mi2, Mi3}.

Passes through X(523)= the Involutory Conjugate of QA-P2=X(99) (Euler-Poncelet Point), which is an infinity point.

If Q = QA-P4=X(316)= DROUSSENT PIVOT, the conic has center at QA-P2=X(99).
(see attached file Quadri-Figures-Group387.png)

Best regards,
Angel Montesdeca



Quadri-Figures-Group387.png

Message: #392
Date: 23/12/2013 11:48:49
From: eckart_schmidt@t-online.de
Subject: conic through reflections

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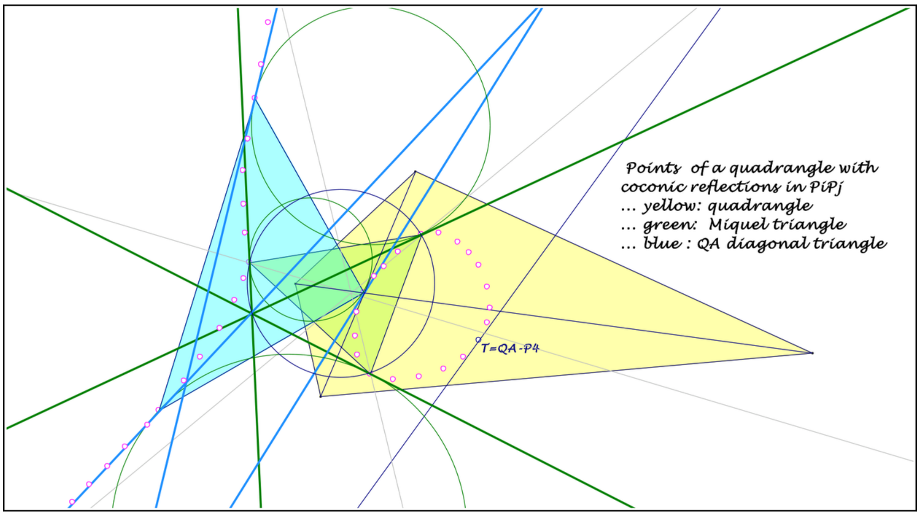
Dear Randy, Chris, Angel,

my speculation, that the discussed curve is a cubic was wrong,
sorry. But the invariance wrt the three QL-Tf1 versions (CSC)
should hold.
I think, the following properties will also be true. For a
quadrigon component of the quadrangle let M be the Miquel point,
F the point at infinity of perpendicular lines of QA-L2 (see
Chris' message) and T the Isogonal Center QA-P4 (see Randy's
message), all points of the curve as already mentioned.
* The reflection of MF in the corresponding angle bisector
  of the Miquel triangle is tangent to the curve at M.
* These tangents have a common point in the isogonal conjugate
  of F wrt the Miquel triangle (on the circumcircle).
* TF is tangent to the curve at T.
* The CSC images of the tangent in T are circles tangent to the
  curve in a Miquel point and a diagonal crosspoint.
* The tangents to these circles in the diagonal crosspoints
  are tangents to the curve.
There is a drawing in the attachment with some "hand-plotted"
points of the curve. Perhaps you can understand my first
speculation for a cubic, but there are other constellations with
three closed curve components and a further curve.

Merry Christmas
Eckart
```



2013-12-23.pdf

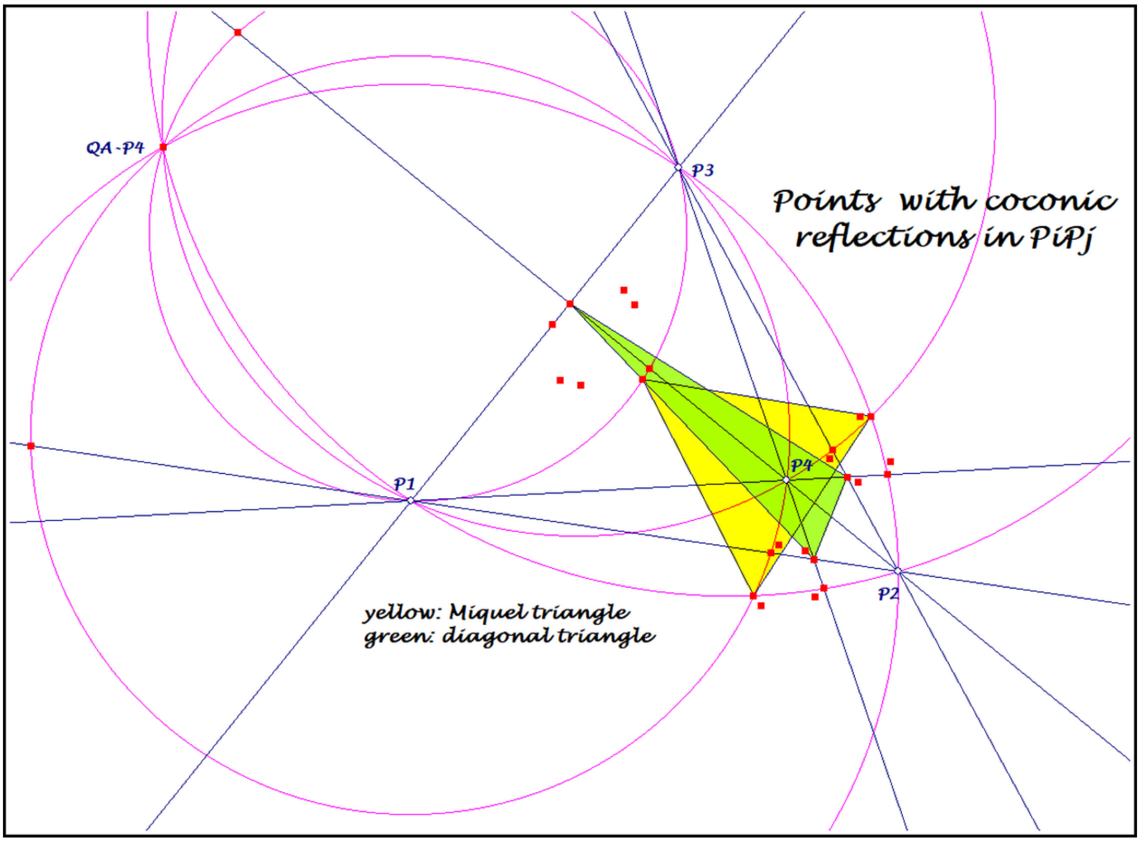
← Previous Next → ↔ Message Index ↑ Subjects

Message: #393
Date: 30/12/2013 11:46:08
From: eckart_schmidt@t-online.de
Subject: conic through reflections

```
Dear friends,

here is a construction for up to 24 special further points with
coconic reflections in PiPj of a quadrangle P1P2P3P4:
Consider the intersections of PiPj and the circumcircle of Pk,
Pl and QA-P4. These points and their images wrt the three
versions of QL-Tf1 have the discussed property (see attachment).

Best regards and a Happy New Year
Eckart
```



2013-12-30-fig.png

← Previous ↔ Message Index ↑ Subjects

5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

↩ Table of Contents ↩ Message Index

6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393