

Journal of the Quadri Figures Group 2014

Digital Edition

Chris van Tienhoven et al.

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Messages #394 - #916

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1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at <https://groups.io/g/Quadri-and-Poly-Geometry>. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at <https://groups.io/g/Quadri-Figures-Group>.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at <https://www.chrisvantienhoven.nl/>, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Bernard Gibert
- Bernard Keizer
- Chris van Tienhoven
- Dao Thanh Oai
- Eckart Schmidt
- Randy Hutson
- Seiichi Kirikami
- Systems Manager

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.

Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location

Lives in Greece.

Year of Birth / Generation

1952.

Short Biography

Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: <http://www.anthrakitis.blogspot.com/>

Chris van Tienhoven

Global Location

Living in the Netherlands.

Year of Birth

1950.

Short Biography

Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks

Website: www.chrisvantienhoven.nl

Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location

Living in Germany.

Year of Birth / Generation

1939.

Short Biography

Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n -gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halkes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Seiichi Kirikami (1949?–2023)

Location

Japan.

Year of Birth / Generation

Exact year unknown; passed away on 11 December 2023.

Short Biography

In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions

Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes

Colleagues remembered Seiichi with deep affection:

- “Geometry is the poorer of his death.” — A. P. Hatzipolakis
- “He was a great expert in projective and triangle geometry, and a very kind and helpful person.” — E. Suppa
- “He had a way of seeing things from a different angle and presenting them simply.” — C. van Tienhoven
- “He inspired many of my problems.” — A. Altıntaş
- “He wrote many interesting and innovative messages in ADGEOM.” — F. J. García Capitán
- “He helped me in my beginnings with wise and generous advice.” — C. E. Lozada
- “He contributed to OEIS and had important work in epidemiology.” — P. Moses

Additional Remarks

Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

3 Subjects

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4.2 Messages

Message: #394

Date: 04/1/2014 5:47:49

From: yeuemtrondoitb85

Subject: Fomular for quadrilateral

Dear Mr Chris, Mr Eckart and all Friends

Happy New Year To Everybody.

I found following formula:

Let four points A,B,C,D lie on a plane.

AB meets CD at F. AC meets DB at E.

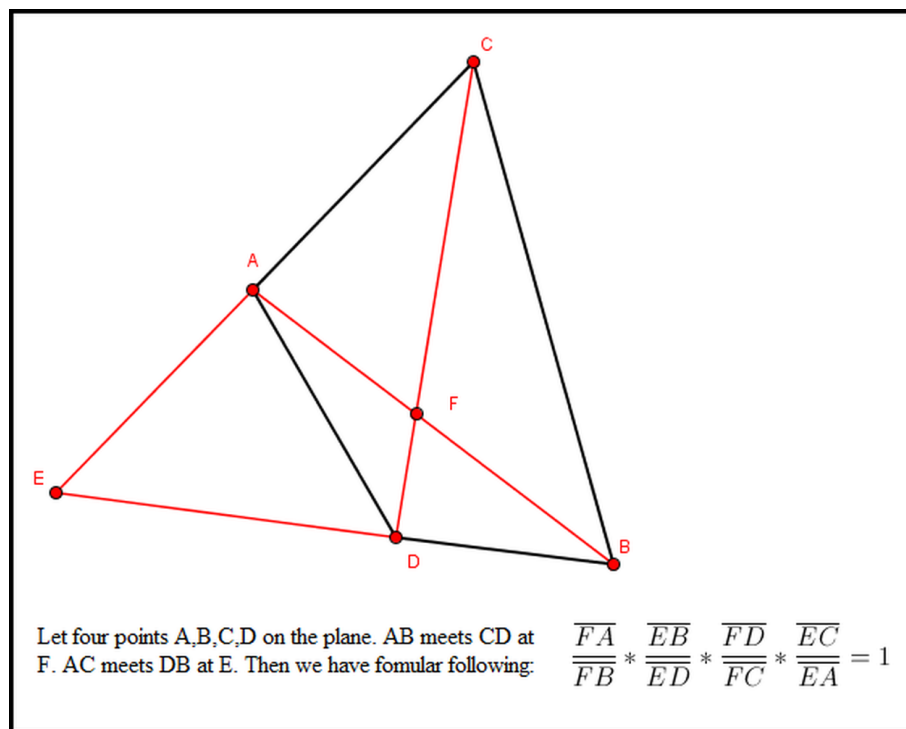
Then we have following formula:

$$FA/FB * EB/ED * FD/FC * EC/EA = 1$$

Best regards

Sincerely

Dao Thanh Oai



fomular with Quadrilateral.png

Message: #395
Date: 04/1/2014 9:24:18
From: eckart_schmidt@t-online.de
Subject: Fomular for quadrilateral

Dear Mr. Dao Thanh Oai,
there is a general property: Every conic through the
intersections of opposite sides of a quadrigon cuts the sides in
ratios with product 1.
Best regards Eckart

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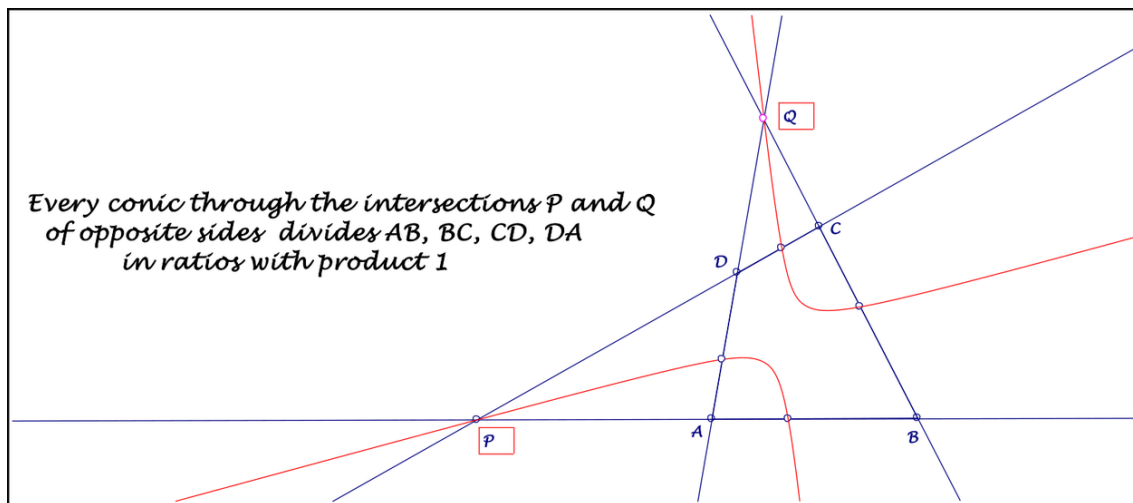
Message: #396
Date: 04/1/2014 9:45:59
From: Dao Thanh Oai
Subject: Fomular for quadrilateral

Dear Mr Eckart,
I don't understand your genaral.
Please share the picture.
Thank to you.
Best regards
Sincerely
Dao Thanh Oai

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #397
Date: 05/1/2014 9:34:06
From: eckart_schmidt@t-online.de
Subject: Fomular for quadrilateral

Dear Mr. Dao Thanh Oai,
I think, the attached picture will help.
Best regards Eckart

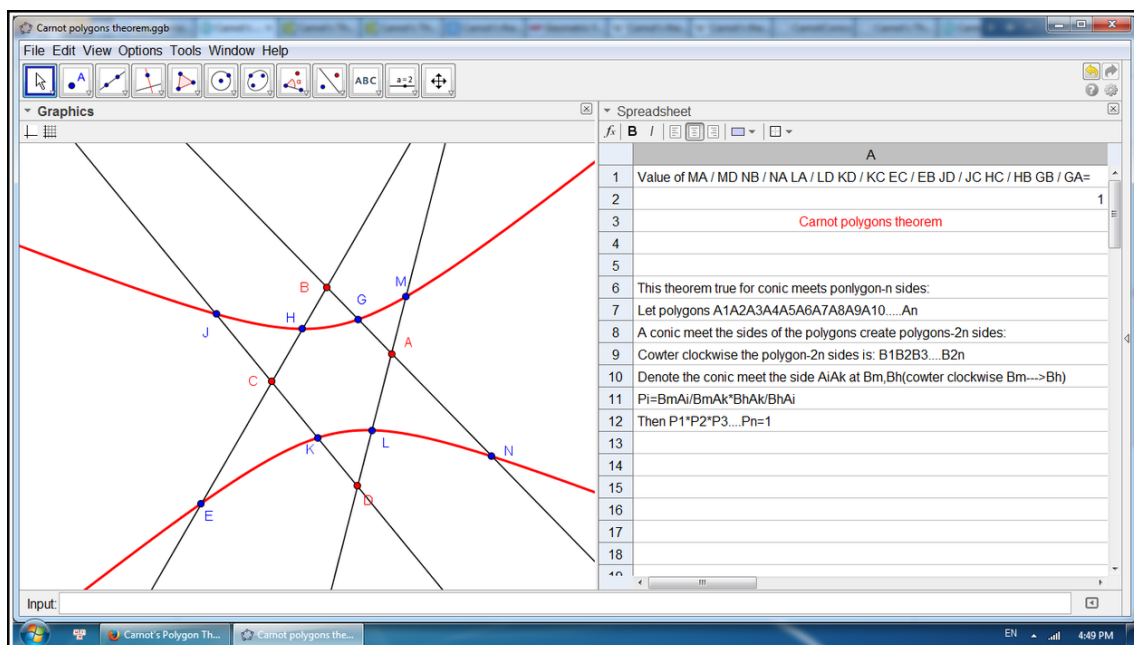


Oai-14-01-05.pdf

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Message: #398
Date: 05/1/2014 10:54:18
From: yeuemtrondoitb85
Subject: Fomular for quadrilateral

Dear Mr. Eckart,
Thank to you very much,
Now I have just generalization with n-gon. But I search in internet, I also have just known the result is Carnot polygon theorem.
Best regards
Sincerely
Dao Thanh Oai



Carnot polygons theorem.png

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Message: #399
Date: 05/1/2014 11:18:27
From: Chris van Tienhoven
Subject: Fomular for quadrilateral

Dear Eckart, Dear Dao,
It's a beautiful property.
You forgot to tell Eckart that your document: "09-06
Ceva-Dreiecke und Ceva-Vierecke" deals about the subject.
See: <http://eckartschmidt.de/>
Best regards,
Chris

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Message: #400
Date: 05/1/2014 11:20:20
From: yeuemtrondoitb85
Subject: 1st, 2nd seven circles problems

Dear Mr. Eckart, and Mr Chris
I have a lemma "isosceles trapezoid in cyclic quadrilateral
"for: "2nd seven circles problems". I think you can solution.
Please see the file pdf attachment.
<http://www.geogebraTube.org/student/m67456>
<http://www.geogebraTube.org/material/show/id/66877>
Best regards
sincerely
Dao Thanh Oai

New problem of seven circles

Dao Thanh Oai

November 16, 2013

Problem 1. Let six points: $A_b, A_c, B_c, B_a, C_a, C_b$ lie on a circle.

A_cB_c meets C_bA_b at A .

A_cB_c meets B_aC_a at B .

B_aC_a meets C_bA_b at C .

C_bC_a meets B_cB_a at X .

C_bC_a meets A_cA_b at Y .

A_cA_b meets B_cB_a at Z .

Denote: $A_1, Z_1, B_1, X_1, C_1, Y_1$ are circumcenter of six circles $(AA_bA_c), (ZA_cB_c), (BB_cB_a), (XB_aC_a), (CC_aC_b), (YC_bA_b)$ respectively. Then A_1X_1, B_1Y_1, C_1Z_1 are concurrent.

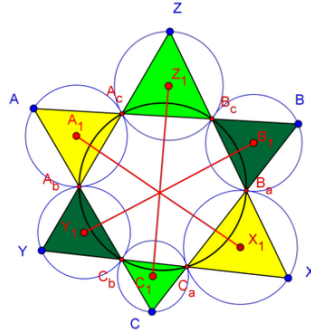
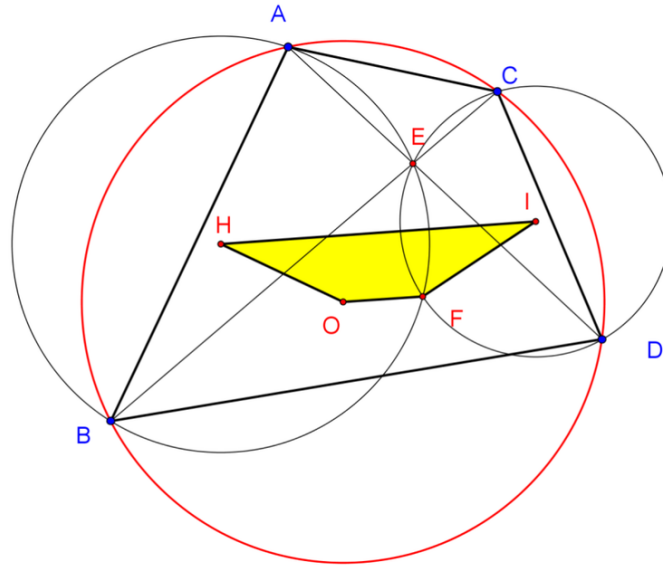


Figure 1: New problem of seven circles

Dao Thanh Oai, Quang Trung-Kien Xuong-Thai Binh-Viet Nam
Email: oaidt.slhpc@gmail.com

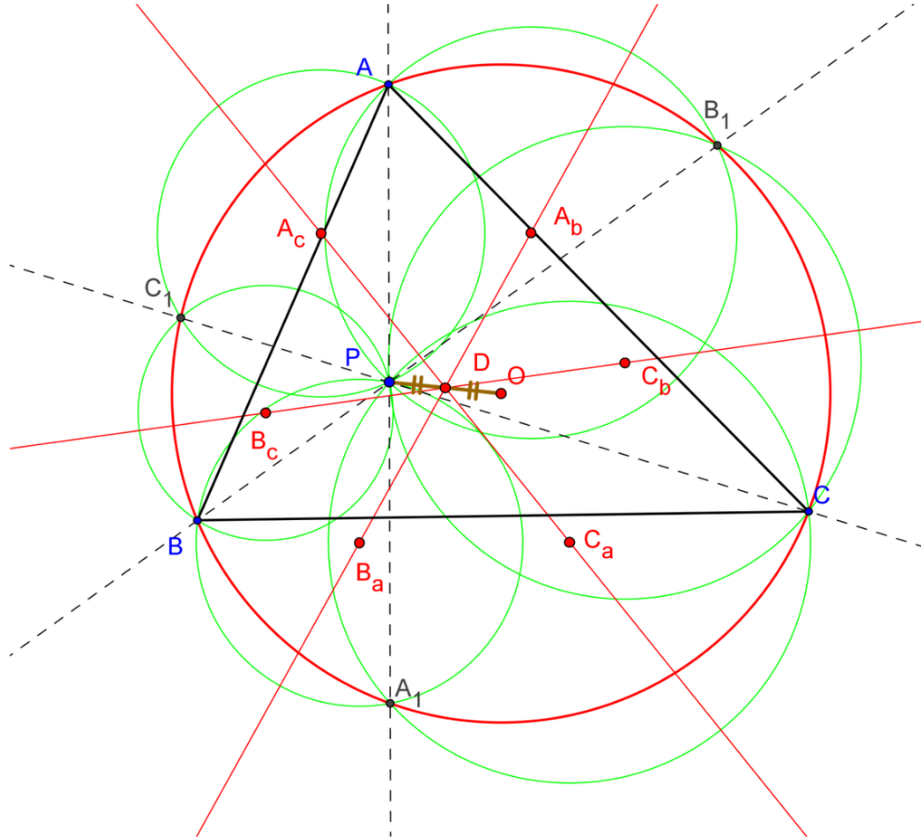
Isosceles trapezoid in cyclic quadrilateral and 2nd another seven circle problem

Problem 1 (*isosceles trapezoid in cyclic quadrilateral*) : Let cyclic quadrilateral $ABCD$, AD meet BC at E . Circle (AEB) meet circle (CED) again at F . Denote H, I respectively are circumcircle of (AEB) and (CED) then $HOIF$ is isosceles trapezoid. (Figure 1)



Another_seven_circle_problem.pdf

Problem 2: Let ABC be a triangle, P be any point on the plane. AP, BP, CP meet circumcircle (ABC) again respectively at A_1, B_1, C_1 . O is circumcenter of (ABC) . Define A_c, A_b respectively are circumcenter of circles $(ADC_1), (ADB_1)$. Define C_b, C_a, A_b, B_c cyclically. Prove that $A_cC_a, A_bB_a, B_cC_b, PO$ concurrent at midpoint of them. (Figure 2)



Dao Thanh Oai-03/Jan/2014

Quang Trung-Kien Xuong-Thai Binh-Viet Nam

Another_seven_circle_problem.pdf

Message: #401
Date: 06/1/2014 8:42:31
From: eckart_schmidt@t-online.de
Subject: Some "splitter" IV

Dear Chris,

my desk has to be cleaned:

- * QA-Cu1 is invariant under the three versions of QL-Tf1.
- * A point on QA-Cu1 and its three QL-Tf1 images give a quadrangle on QA-Cu1 with Isogonal Center on QA-Cu1 (and the same Miquel Triangle as the reference quadrangle).
- * The point at infinity of the asymptote of QA-Cu1 is the QA-Tf2 image of QA-P3.
- * For the intersection of QA-Cu1 and its asymptote the Simson line wrt the Miquel triangle is perpendicular to the asymptote.
- * For a quadrangle the fixed points of two QL-Tf1 transformations lie on a circle round an excenter of the Miquel Triangle.
- * QA-Cu1 is invariant wrt the reflections in these three circles (anallagmatic curve).
- * For a point P on QA-Cu1 its isogonal conjugate P* and the residuals of P wrt the Miquel triangle give a quadrangle on QA-Cu1 with Isogonal Center on QA-Cu1 (and the same Miquel triangle as the reference quadrangle).
- * (Well known: The Miquel Triangle and the QA-Diagonal Triangle are perspective wrt QA-P3.) The angle bisectors of the Miquel Triangle cut the corresponding sides of the QA-Diagonal Triangle in points with a Ceva point, which lies on the circumconic of the quadrangle through QA-P4.
- * For a quadrilateral and a point the trilinear poles (wrt QL-DT) of the perpendicular lines wrt the four lines and three diagonals lie on a QL-DT circumconic.
- * For the two circumscribed parabolas of a quadrangle tangents to one parabola parallel to the axis of the other parabola cut in QA-P22.
- * For quadrigons QL-Cu1 is the locus of points P, for which the conics with foci in opposite vertices touch in P.
- * For iterated circumcenter quadrangles the Isogonal Centers QA-P4 are equal and the Euler-Poncelet Points QA-P2 lie alternately on two lines: QA-P4.QA-P2 and QA-P4.QA-P3.
- * For a quadrigon the Thales Circles about opposite sides have radical axes with intersection QG-P10.

I hope there are no mistakes! If the properties are already mentioned or not relevant, excuse.

Best regards Eckart

Message: #402
Date: 12/1/2014 6:51:26
From: eckart_schmidt@t-online.de
Subject: Alternating sum of distances

Dear friends,

I have gathered my EQF-Notes for the last year on my homepage:
<http://eckartschmidt.de>.

There is a new note in the attachment, concerning the loci of points X wrt a quadrigon $P_1P_2P_3P_4$ with the distance properties:

1. $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = 0$
2. $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = 0$
3. $XP_1 - XP_2 + XP_3 - XP_4 = 0$
4. $XL_1 - XL_2 + XL_3 - XL_4 = 0$

I know, that Seiichi has discussed distance problems last year, but I don't remember these properties. Perhaps someone knows, whether the results are already published anywhere.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Alternating Sum of Distances

For quadrilaterals $P_1P_2P_3P_4$ with the sidelines L_1, L_2, L_3, L_4 here are discussed the loci for points X with the distance properties:

1. $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = 0$,
2. $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = 0$,
3. $XP_1 - XP_2 + XP_3 - XP_4 = 0$,
4. $XL_1 - XL_2 + XL_3 - XL_4 = 0$.

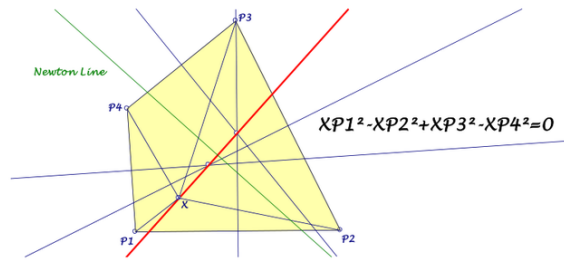
1. The locus for points X with the property

$$XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = 0$$

is a line, perpendicular to the Newton Line $QL-LI$, containing the intersections of the perpendicular bisectors of P_1P_2, P_3P_4 and P_2P_3, P_4P_1 .

This line can easily be constructed and calculated.

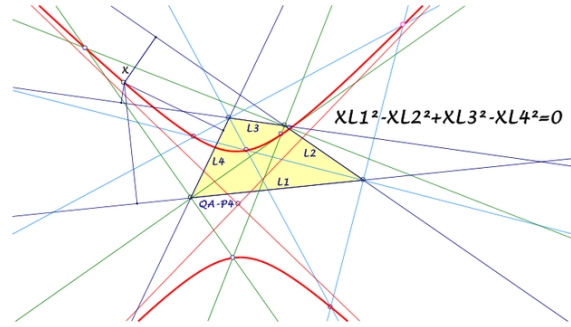
Additional remark: For points X on a line, perpendicular to the Newton Line, the alternating sum is constant.



2. The locus for points X with the property

$$XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = 0$$

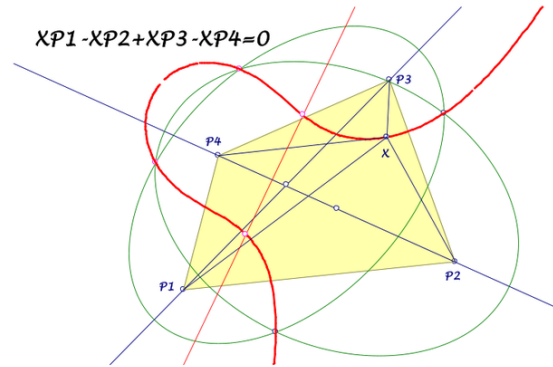
is a rectangular hyperbola centered in the Isogonal Center $QA-P4$. The hyperbola contains the intersections of the inner or outer angle bisectors at opposite vertices of the quadrilateral (8 points, see 4.). So there is no problem to construct the curve and calculate its equation.



3. The locus for points X with the property

$$XP_1 - XP_2 + XP_3 - XP_4 = 0$$

can be constructed in the following way: Consider ellipses with foci P_1, P_3 and P_2, P_4 with the same length of the principal axis. The intersections give points of a curve, which seems to be a cubic, but a calculation gives a (very extensive) equation of degree 5 (Mathematica gives no further factorization).



The intersections of the perpendicular bisectors of P_1P_2 , P_3P_4 and P_2P_3 , P_4P_1 (see 1.) lie on the curve. The corresponding line is parallel to the asymptote of the curve.

4. The loci for points X with the property

$$XL_1 - XL_2 + XL_3 - XL_4 = 0$$

are sections of lines, connecting intersections of angle bisectors of opposite vertices (see 2.).

If we denote the angle bisectors at P_i with g_i^+ and g_i^- , we get the intersections

$$P_{++}, P_{+-}, P_{-+}, P_{--} \quad \text{e.g.} \quad P_{+-} = g_1^+ \cap g_3^-,$$

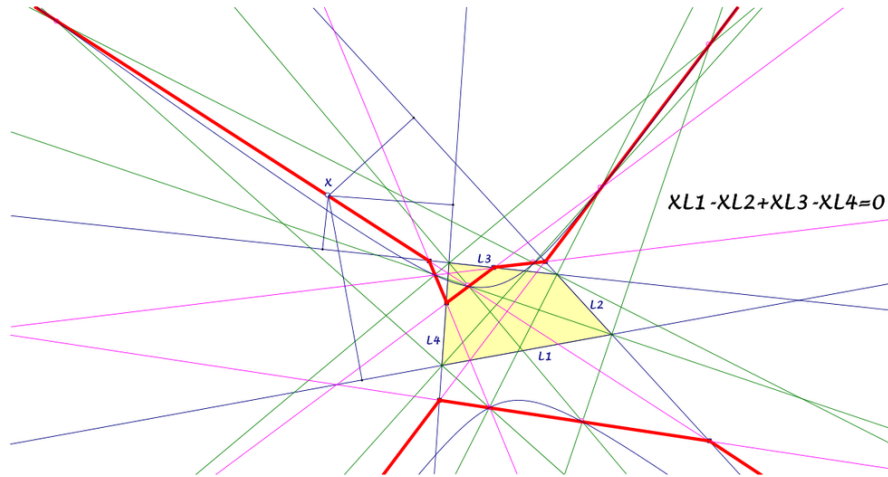
$$Q_{++}, Q_{+-}, Q_{-+}, Q_{--} \quad \text{e.g.} \quad Q_{+-} = g_2^+ \cap g_4^-.$$

These are the points on the rectangular hyperbola of 2, also satisfying the condition 4.

On each of the six lines

$$P_{++}Q_{--}, P_{+-}Q_{-+}, P_{-+}Q_{+-}, P_{-+}Q_{+-}, P_{-+}Q_{+-}, P_{-+}Q_{+-}$$

we find one section for the points X , bordered by intersections with sidelines of the quadrigon. In the other sections another sign constellation in the sum gives zero.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-01-12.pdf

Message: #403
Date: 14/1/2014 11:04:31
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Chris,

are the following properties already mentioned?

... QL-Cu1 is the locus of points,
whose reflections in the circumcircles
of the QL-quadrignon components are concyclic.
... Alternative construction:
Let F1 and F2 be the fixed points of QL-Tf1
and X variable points on the Newton Line QL-L1,
then the intersections of the angle bisector

Best regards Eckart

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Message: #404
Date: 16/1/2014 12:47:25
From: rhutson2
Subject: conic through reflections

Dear Eckart,

What is the perspector of the QA-diagonal triangle and the
triangle formed by the tangents to the curve at the vertices of
the QA-diagonal triangle?

Best regards,
Randy

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Message: #405

Date: 16/1/2014 2:06:42

From: yeuemtrondoitb85

Subject: Tangential quadrilateral in quadrilateral well know?

Dear Mr Chris, and Mr Eckart and all Friends

Let ABCD be a quadrilateral. Let four points E,G,H,F lie on two angle bisector of diagonal (please see the figure attachment), then intersection of four line are reflection of AC on AH,FG and BD on HE,GF at I,J,K,L then IJKL are tangential quadrilateral.

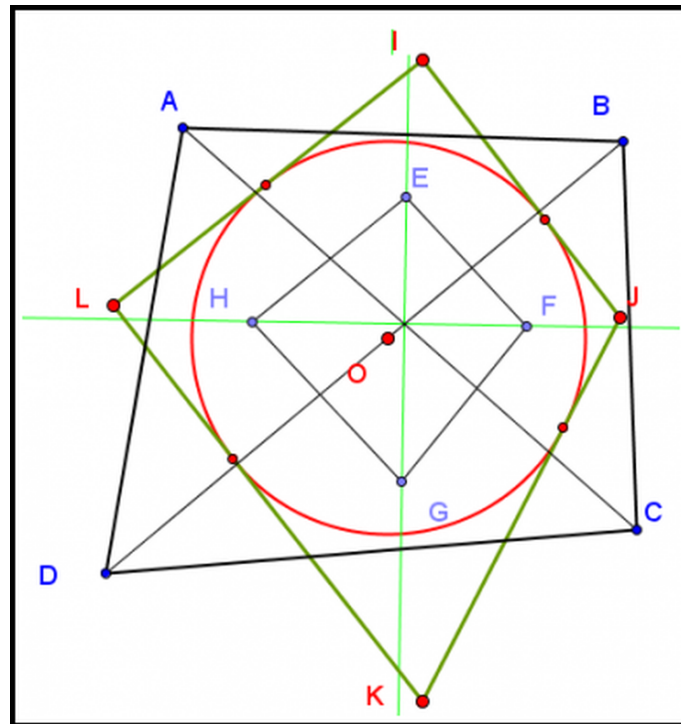
<http://www.geogebra.org/m69513>

The property above is well know?

Best regards

Sincerely

Dao Thanh Oai



Tangential quadrilateral in any quadrilateral.png

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Message: #406
Date: 16/1/2014 2:18:44
From: yeuemtrondoitb85
Subject: Tangential quadrilateral in quadrilateral well know?

Dear all Members
It is very easily
Best regards
Sincerely
Dao Thanh Oai

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Message: #407
Date: 16/1/2014 8:34:30
From: Chris
Subject: Precisions concerning the curve QL-Cu1

Dear Eckart,
These properties were new to me.
I will include them in EQF.
However the site of EQF will be renewed to new standards.
So it will take some time.
Best regards,
Chris
>
> --- In Quadri-Figures-Group@yahoogroups.com ,
> "eckart_schmidt@..." wrote:
> > Dear Chris,
> > are the following properties already mentioned?
> > ... QL-Cu1 is the locus of points, whose reflections in the
> > circumcircles of the QL-quadrignon components are concyclic.
> > ... Alternative construction: Let F1 and F2 be the fixed
> > points of QL-Tf1 and X variable points on the Newton Line
> > QL-L1, then the intersections of the angle bisector
> > Best regards Eckart
>

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Message: #408
Date: 16/1/2014 2:01:38
From: eckart_schmidt@t-online.de
Subject: Conic through reflections

Dear Randy,
sorry, I can't find a perspector of QA-DT and the triangle formed by the tangents to the curve at the QA-DT vertices. I think, that these tangents can be constructed in the following way:

Consider a QG component of the quadrangle.
The QL-Tf1 image of the circle through QL-P1, QA-P4 centered on QA-L2 will be the tangent in the diagonal crosspoint QG-P1.

Background:

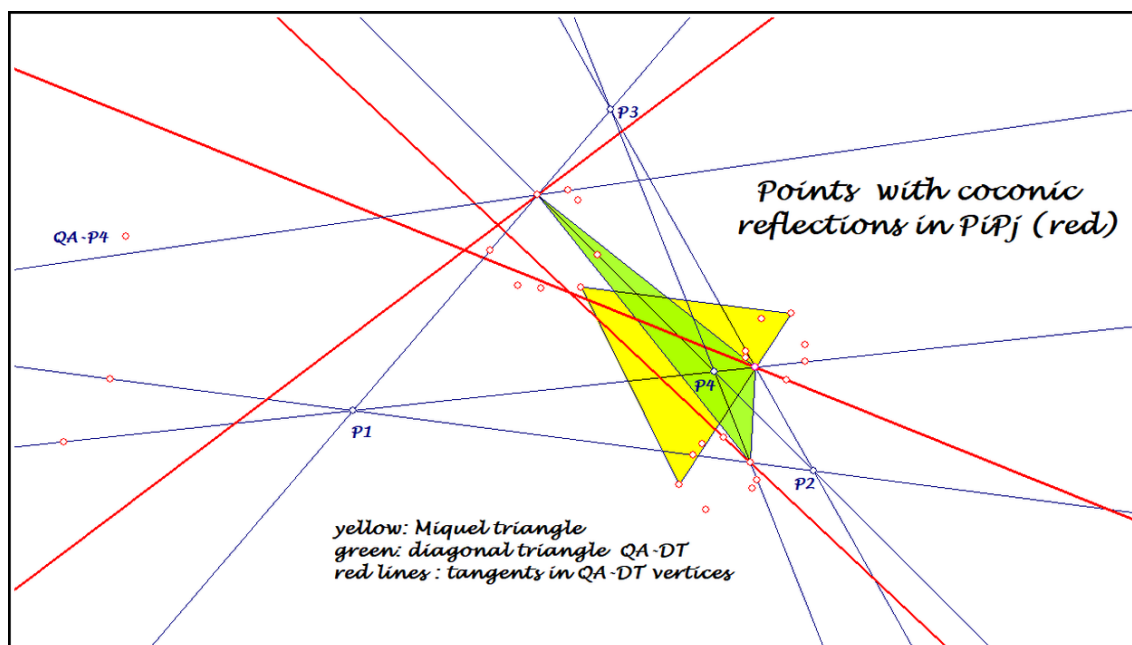
The tangent in QG-P1 touches the curve (evident). The transformation QL-Tf1 is angle-true.

The QL-Tf1 image of the curve is the curve again.

The QL-Tf1 image of the tangent is a circle through QL-P1 and QA-P4, touching the curve in QA-P4 (the QL-Tf1 image of QG-P1). The tangent in QA-P4 is perpendicular to QA-L2.

I hope, the construction is right. There is a Cabri file in the attachment.

Best regards Eckart



2014-01-15-fig.png

Message: #409

Date: 16/1/2014 3:26:07

From: yeuemtrondoitb85

Subject: Another four incenter lie on a circle, well know?

Dear Mr Eckart, Mr Chris and all Friends

Mr Tony Gacia share me the link,

http://www.cut-the-knot.org/cgi-bin/dcforum/forumctk.cgi?az=printer_format&om=383&forum=DCForumID6

I inspired and construct and found some problems following:

Let ABCD be a tangential quadrilateral. F,G,H,I are intersection of perpendicular bisector of four sideline AB,BC,CAD,DA (show in the figure).

1- Prove that FHGI are tangential quadrilateral.

2- Four incircle of four in triangles ALB,BLC,CLD,DLA are cyclic quadrilateral.

3- Denote O is center of inscribed circle of quadrilateral FHFI, then four incenter of four triangle AOB,BOC,COD,DOA lie on a circle.

4- If F lie on AD then incenter of three circle AFB, FBC,FCD and F lie on circle.

Please see the picture and see geogebra tube

<http://www.geogebra tube.org/student/m69759>

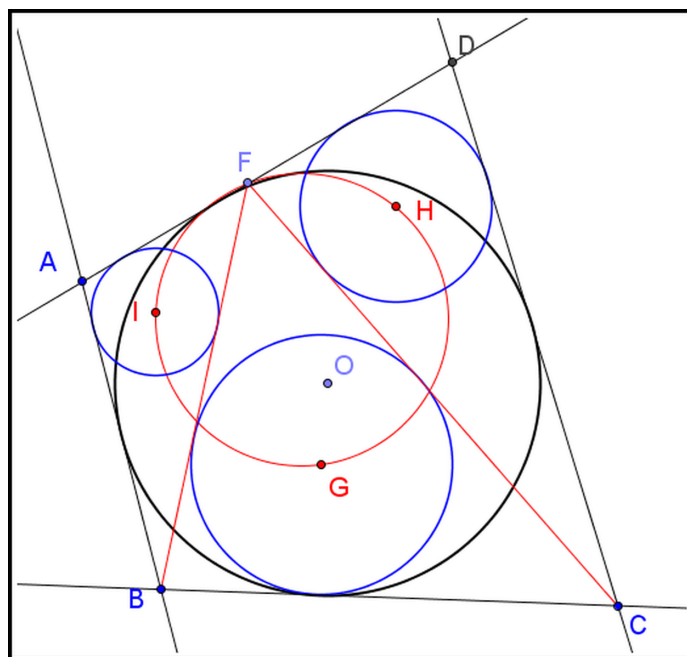
<http://www.geogebra tube.org/student/m69722>

<http://www.geogebra tube.org/student/m69739>

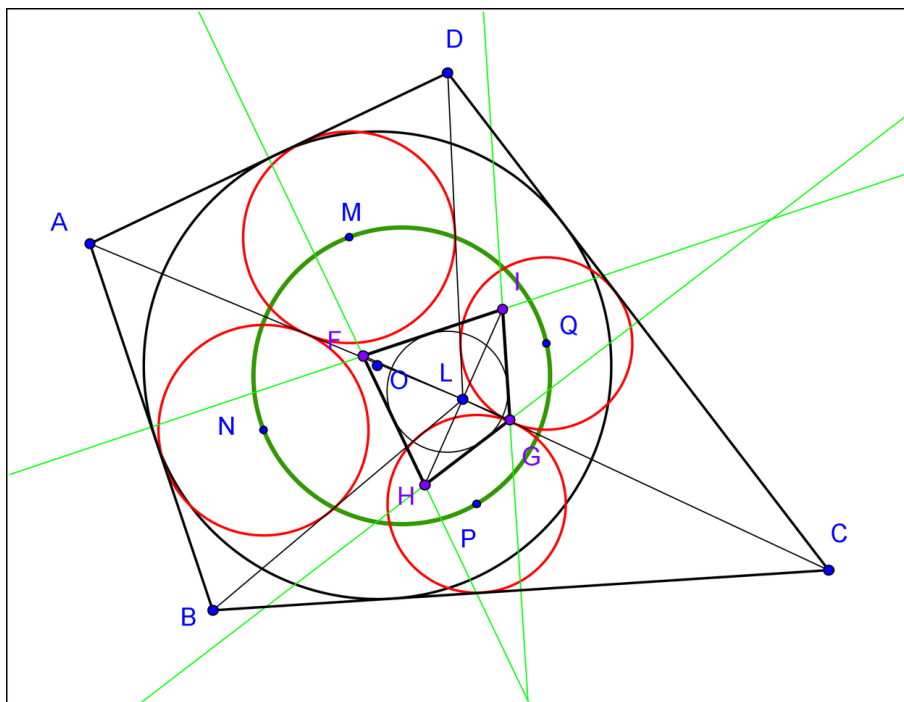
Best regards

Sincerely

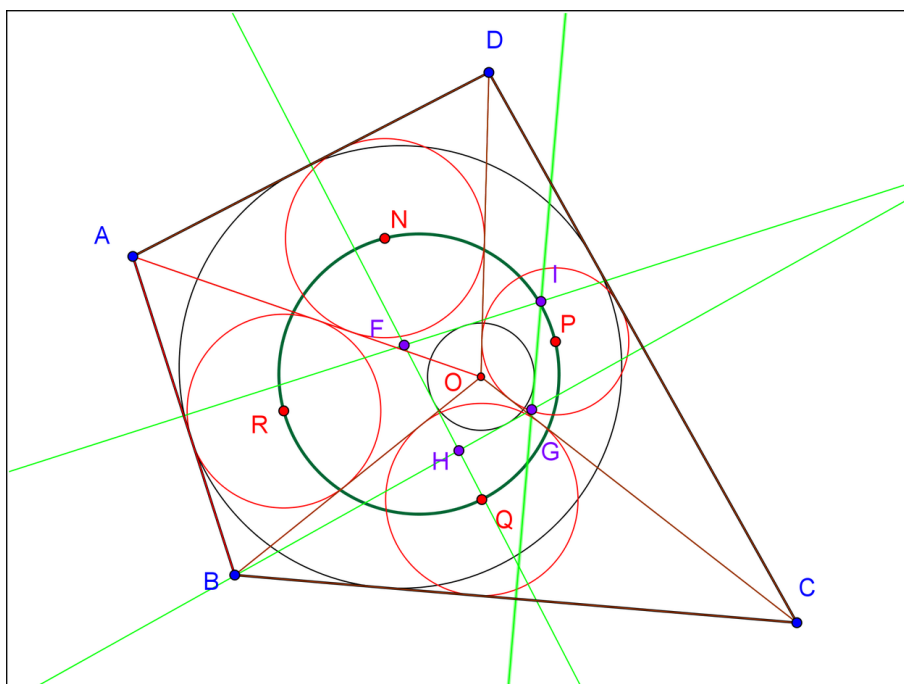
Dao Thanh Oai



Three incenter and a vertex lie on a circle.png



Another four incenter lie on a circle 1.png



Another four incenter lie on a circle 0.png

Message: #410
Date: 19/1/2014 12:15:20
From: bernard.keizer
Subject: stelloïd, hessian and cayleyan

Dear Chris, dear Eckart, dear Bernard G., dear members of the Quadri-Figures-Group
First of all, happy new year to all of you and your families and a lot of new properties and curves for EQF.
As I told Chris at the end of last year, I spent part of my energy to read Bernard G.'s paper on Eckart's cubic and J. Marchand's papers on the QL's conjugated stelloïd, it's hessian and it's cayleyan and I have some remarks and many questions. As Bernard had mentionned in an old message, it happens to be the same curve.
It's hard and beautiful stuff, which needs the intervention of an expert in cubics, but it is very promising
Best regards
Bernard

Remarks and question about the stelloïd associated to the QL, it's hessian and it's cayleyan

1) Remarks on Bernard Gibert's paper about inscribed cardioids and Eckart's cubic QL-Cu2

- Page 3 : if the barycentric coordinates of the center O are (u,v,w) , the barycentric coordinates of the cusp K are $(u^3b^2c^2, v^3a^2c^2, w^3a^2b^2)$; if we deal with the trilinear coordinates, the coordinates of the center are $(u/a, v/b, w/c)$ and those of the cusp $(u^3/a^3, v^3/b^3, w^3/c^3)$. In fact, if $OS = 3r$ and the angle between the bitangent and the tangent is 3θ , the distance from O to the tangent is equal to $3r \cos\theta$ and the distance from R to the tangent is $4r \cos^3\theta$, which explains the calculation (see message 61) ...
- Page 3 figure 2 : the sides of the Morley triangles are 3 sets of 3 parallel lines, forming equilateral triangles, the direction of the lines being the mean direction of the 3 sides of the triangle ($m^\circ \pi/3$) ; for example, if θ_i are the angles between the Steiner Line and the lines L_i , the angle between the Steiner line and the Morley lines will be, for the triangle T_i , $\frac{1}{3} \sum \theta_j$ ($m^\circ \pi/3$), with $j = 1$ to 4 and $j \neq i$
- Page 8 : the asymptotes of QL-Cu2 are parallel to the axes of the deltoid. In fact, if we consider the parabola tangent to the 4 lines (QL-Co1) and the cardioid, tangent to the 4 circles circumscribed to the 4 reference triangles of the QL (QL-Qu1), with the same definition of θ_i , the angle between the axis of the cardioid and the axis of the parabola is $\sum \theta_i$ (with $i = 1$ to 4) and the angle of the 3 axes of the deltoid and the axis of the parabola is $\frac{1}{3} \sum \theta_i$ ($m^\circ \pi/3$). The asymptotes of QL-Cu2 are the trisectors of the angle between the 2 axes of the parabola QL-Co1 and the cardioid QL-Qu2
- Page 16 figure 16 : the intersection points between $E(L)$ and $H(L)$ are inflexion points of the 2 curves (property mentioned by Eckart in message 281) and are on a line
- Page 17 figure 17 : marvelous construction ! Are there special interesting positions (for example if P_1 is chosen on the lines L_1 or L_2) ?

Does this construction allow, by any chance, an easy construction of $E(L)$?

2) Stelloïd associated to the QL

- In an earlier Quadriforum message, Bernard G. told that Eckart's cubic was in fact the curve J. Marchand called the QL's conjugated stelloïd
- J. Marchand wrote 2 articles, a short one in 1927, Sur un théorème de Steiner (reference 16 in Bernard's paper) and a long one in 1937, Géométrie du quadrilatère complet, quoted in the message 171 from Bernard, of which a summary in german was mentioned in the message 172 from Andreas.
- My understanding is following :
 - a given QL has 4 lines and 6 vertices
 - the inscribed conics have their centers on the Newton Line and their foci on the curve QL-Cu1 and form a tangential set ; the degenerated conics of this set are reduced to a pair of 2 foci
 - the rectangular hyperbolas harmonically circumscribed to the inscribed conics or having the QL as polar quadrilateral (cutting each side of the diagonal triangle in 2 points harmonic conjugates of the 2 vertices on this side) form a second set ;

the degenerated conics of this set are made of pairs of orthogonal lines (for example 2 angle bisectors in a vertex, a side and the corresponding altitude of the diagonal triangle or the Newton Line and the infinity line)

- the QL's conjugated stelloid is the cubic for which the first set of the inscribed conics is the set of the apolar conics and the second set of the rectangular hyperbolas the set of the polar conics (curve QL-Cu2)
- the hessian of the stelloid is the locus of the foci of the inscribed conics (curve QL-Cu1), which are Clawson-Schmidt conjugate points
- the cayleyan of the stelloid is the envelop of the axes of the inscribed conics (lines through the pairs of conjugated points forming the degenerated apolar conics) and of the pairs of orthogonal lines forming the degenerated polar conics
- the 2 sets of circles with F1 and F2 as characteristic points (basis points for one and Poncelet points for the other) are 2 sets of autopoloconics wrt the stelloid and each of the 2 is a set of autopoloconics wrt another cubic (the 8 circles through the in- and excenters as mentioned in the Steiner Angle Bisector octet QL-8P1 belong to these 2 sets of circles)
- the cayleyan is the cayleyan of the stelloid and of these 2 other cubics

I suppose all this stuff needs new investigations, barycentric calculations and drawings of many figures by an expert in conics ...

3) Less known or unknown properties of the QL deduced from these 3 curves

- Miquel point and infinity point on the Newton Line are Clawson-Schmidt conjugates : the isogonal line of the line through Miquel point and a vertex of the QL wrt the 2 lines of the QL through this vertex is parallel to the Newton Line
- The axis of the parabola QL-Co1 cuts all the rectangular hyperbolas of the set of the polar conics of the stelloid, in particular the pair of orthogonal lines forming the degenerated polar conics (see above) in 2 points symmetric wrt the Miquel point
- 2 rectangular hyperbolas of the set cut in 4 points formic an orthocentric quadrilateral (vertices of a triangle and it's orthocenter) ; the Euler circles of these triangles pass all through the Miquel point (for example, the 4 circumscribed circles of the reference triangles of the QL are Euler triangles of the excentral triangles, they all pass through the Miquel point, as the Euler triangle of the diagonal triangle ...)
- The line through the center of a rectangular hyperbola of the set and the middle of the segment cut on the Newton Line pass through the Miquel point (in the case of the degenerated ones, we obtain rectangular triangles and their medians)
- 2 Clawson-Schmidt conjugate points belong to 2 circles, one in each set of circles mentioned above (they are on a circle through the 2 points F1 and F2 and on a circle centered on the line F1F2) ; having the 6 vertices of a QL and the Miquel point, this simple and amazing property gives an easy construction of the points F1 and F2
- The 2 axes of any inscribed conic are harmonic conjugates wrt the 2 points F1 and F2 (These 2 points deserve to be named in EQF, as they play a very important role in the QL)

It is remarkable that J. Marchand didn't show any figure of all these properties !

Cayleyan.docx

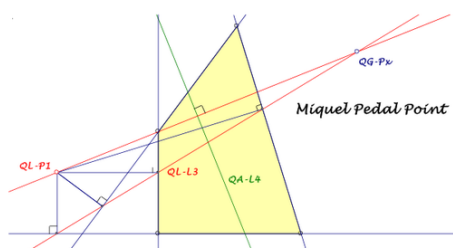
Message: #411
Date: 20/1/2014 9:21:05
From: eckart_schmidt@t-online.de
Subject: Miquel Pedal Point

Dear Chris, dear friends,
there is a new QG-point described in the attachment. The name is
my proposal, perhaps not the best. But I was astonished about
the relationships for pedal points of the Miquel Point.
Best regards Eckart

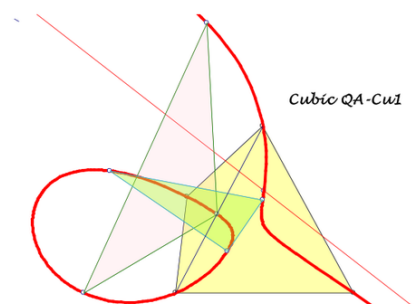
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

The Miquel Pedal Point

For the Miquel Point of a quadrilateral the pedal points on the sidelines are collinear on the Pedal Line $QL-L3$. For a quadrangle a special point on this line will be discussed: the intersection of $QL-L3$ and the perpendicular line through the Miquel Point wrt $QA-L4$.

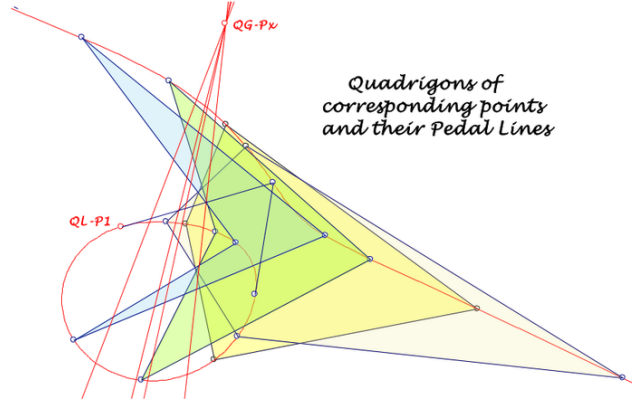


Background for this point is the cubic $QA-Cu1$ for quadrangles, a pivotal isogonal cubic wrt the Miquel Triangle $QA-Tr1$. The asymptote is parallel $QA-L4$. The cubic contains the vertices of the quadrangle, the vertices of the QA -Diagonal Triangle $QA-Tr1$ and the vertices of the Miquel Triangle $QA-Tr2$.



This cubic is invariant under the three versions of the Clawson-Schmidt Conjugate $QL-Tf1$. Any point on the cubic has three images wrt $QL-Tf1$, forming a “quadrangle of corresponding points” on the cubic with a common tangential point on the cubic and the same Miquel Triangle as the reference quadrangle. Examples: Evidently the vertices of the quadrangle, further the in- and excenters of the Miquel Triangle, the vertices of the QA -Diagonal Triangle and the Isogonal Center $QA-P4$, the vertices of the Miquel Triangle $QA-Tr2$ and the point at infinity of the asymptote.

Now we consider a quadrigon: Taken as quadrilateral, we get the Miquel Point $QL-P1$. Taken as quadrangle, we get the cubic $QA-Cu1$ with quadrangles of corresponding points. Each quadrangle of corresponding points has a quadrigon component with the same Miquel Point as the reference quadrigon, only these quadrigons of corresponding points shall be considered:



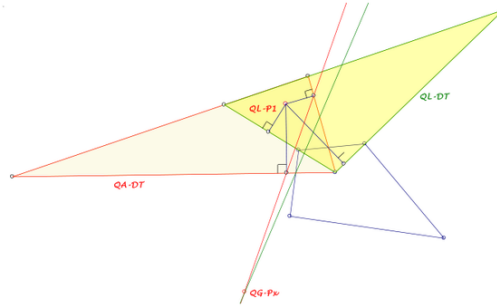
The Pedal Lines ($QL-L3$) for quadrilaterons of corresponding points on the cubic $QA-Cu1$ of a reference quadrigon have a common point.

If we take the QL -Diagonal Triangle $QL-Tr1$ as reference triangle for barycentric coordinates with $L_4 = (l, m, n)$, this point is

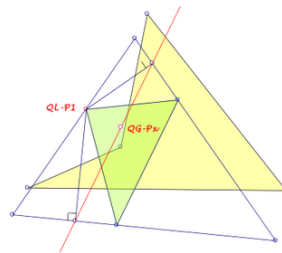
$$\begin{aligned}
 & (M^2S_B^2 + N^2S_C^2 - L(N-L)S_AS_B + 2N^2S_BS_C - NLS_AS_C) \\
 & : -(LS_A - NS_C)^2 - MS_B(LS_A - 2MS_B + NS_C) \\
 & : L^2S_A^2 + M^2S_B^2 - N(L-N)S_BS_C + 2L^2S_AS_B - NLS_AS_C) \\
 & \text{with } L = m^2 - n^2, \quad M = n^2 - l^2, \quad N = l^2 - m^2.
 \end{aligned}$$

Properties

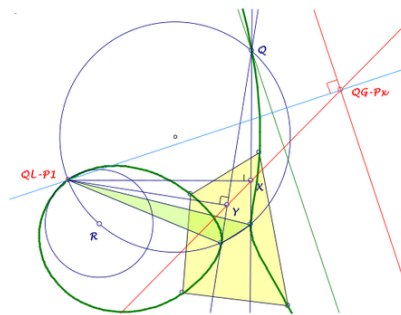
- $QG-Px$ lies on the Pedal Line $QL-L3$ of the quadrigon.
- $QG-Px$ lies on a perpendicular line through the Miquel Point wrt $QA-L4$.
- $QG-Px$ is a point on the line through the pedal points of the Miquel Point wrt the legs of the QA -Diagonal Triangle.
- $QG-Px$ is a point on the perpendicular bisector of the pedal points of the Miquel Point wrt the legs of the QL -Diagonal Triangle.



- The Miquel Point is a point on a sideline of the excenter triangle of the Miquel Triangle. The line through the pedal points of the Miquel Point wrt the other two sidelines contains $QG-Px$.



- Let Q be the intersection of $QA-Cu1$ and its asymptote and X, Y the pedal points of the Miquel Point to the connections of Q with the other two vertices of the Miquel Triangle, then $QG-Px$ is a point of XY .
- Let R be the reflection of Q in the circumcenter of the Miquel Triangle and Ci a circle round R through the Miquel Point, then the Clawson-Schmidt Conjugate $QL-Tf1$ of Ci is a line through $QG-Px$ (parallel to the asymptote) and $QG-Px$ is the pedal point of $QL-P1$ wrt this line.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-01-20.pdf

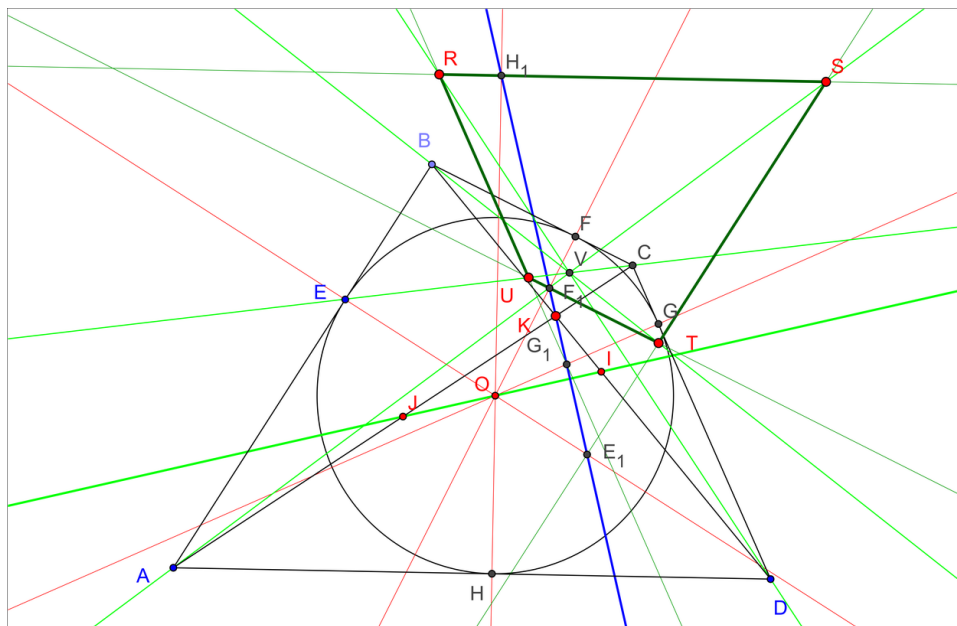
Message: #412
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Messages

Message number #412 is not available in Yahoo groups.

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Message: #413
Date: 22/1/2014 3:07:58
From: yeuemtrondoitb85
Subject: Homothetic in tangential quadrilateral

Dear Mr Chris, and Mr Eckart, and all Friends
Let ABCD be a tangential quadrilateral. O is circumscribed of ABCD. E, F, G, H are projection of O to four sidelines. (tangent points of ABCD with (O)).
The line perpendicular with Newton Line meet OE, OF, OG, OH at E_1, F_1, G_1, H_1 . R, S, T, U are intersection of four lines perpendicular with OE, OF, OG, OH at E_1, F_1, G_1, H_1 .
Then ABCD and RSTU are similar tangential quadrilateral, homothetic at V .
Please see the figure attachment.
Best regards Sincerely Dao Thanh Oai



A property of tangential quadrilateral.png

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Message: #414
Date: 23/1/2014 4:35:19
From: eckart_schmidt@t-online.de
Subject: QG-Cubic with $XP1 \times XP3 = XP2 \times XP4$

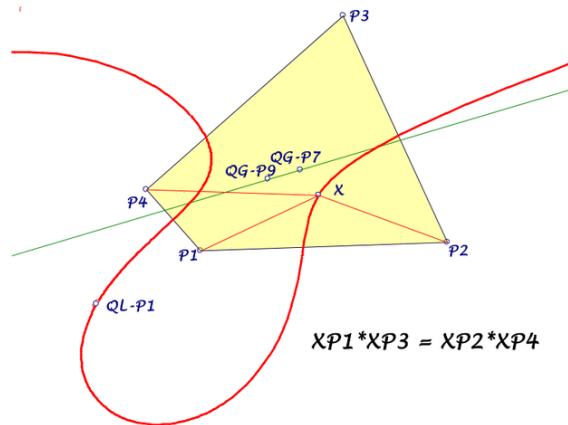
Dear friends,
the Miquel Point has the property, that the products of distances to opposite vertices of a quadrigon are equal. In general: Points with this property lie on a cubic, invariant under the Clawson-Schmidt Conjugate QL-Tf1. A construction of the cubic is possible related to the line QG-P7.QG-P9 (see attachment).
Best regards Eckart

EQF-Note 2013-01-23

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

QG-Cubic with $XP_1 \cdot XP_3 = XP_2 \cdot XP_4$

The Miquel Point has the property, that the products of distances to opposite vertices of a quadrilateral are equal. In general: Points with this property lie on a cubic, invariant under the Clawson-Schmidt Conjugate QL-Tr1. A construction of the cubic is possible related to the line QG-P7, QG-P9.



Points X in the QG -environment with the property

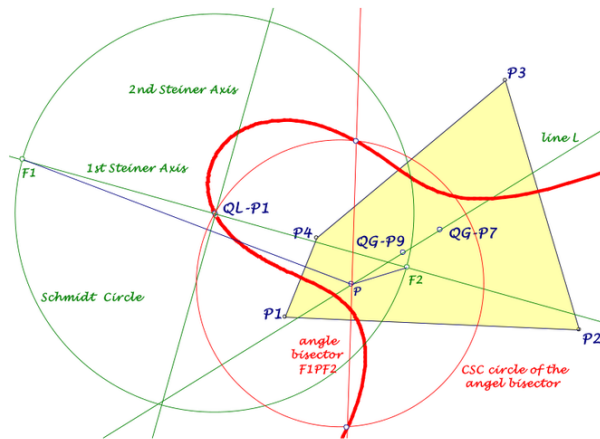
$$XP_1 \cdot XP_3 = XP_2 \cdot XP_4$$

lie on a cubic. Using the QL -Diagonal Triangle $QL-Tr1$ as reference triangle for barycentric coordinates, the equation of the cubic can easily be calculated, but it is rather extensive and here not printed. There is one point at infinity

$$(-a^2l^2 + S_Cm^2 + S_Bn^2 : S_Cl^2 - b^2m^2 + S_An^2 : S_Bl^2 + S_Am^2 - c^2n^2),$$

which is the point at infinity of the line $L = QG-P7.QG-P9$.

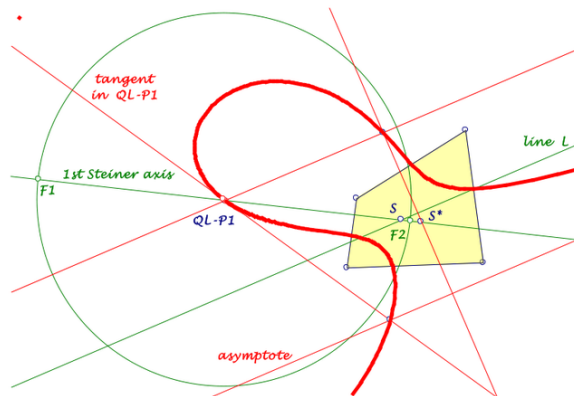
For a construction of the cubic we need (see *EQF*, *QL-Tr1*) the Clawson-Schmidt Conjugate (*CSC*) and its fixed points F_1 and F_2 , the Schmidt Circle, the 1st and 2nd Steiner Axis and the Miquel Point $QL-P1$. For variable points P on the line $L = QG-P7.QG-P9$ the intersections of the angle bisector of $\angle F_1PF_2$ and its *CSC*-image (a circle through $QL-P1$) are points of the cubic.



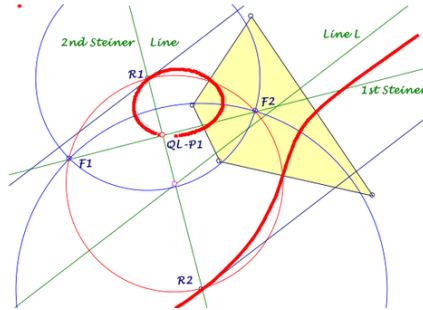
Properties

Let S be the intersection of the line $L = QG-P7, QG-P9$ and the 1st Steiner Axis and S^* its reflection in the Schmidt Circle.

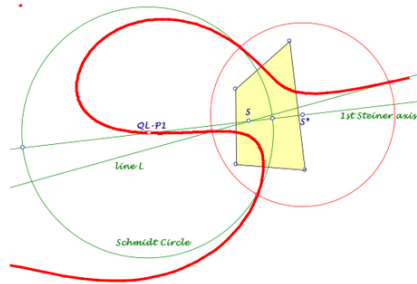
1. The cubic contains the Miquel Point $QL-P1$.
2. The cubic is invariant wrt the Clawson-Schmidt Conjugate $QL-Tf1$.
3. The cubic is unipartite, if S lies inside F_1F_2 ; the cubic is bipartite, if S lies outside F_1F_2 .
4. The asymptote is a parallel to the line L through the reflection of the Miquel Point in the line L .
5. A parallel to the line L through the Miquel point, reflected in the 1st Steiner Axis gives the tangent to the cubic in the Miquel Point.
6. The tangent in the Miquel Point and the asymptote intersect on the cubic.
7. A parallel to the line L through the Miquel Point and a perpendicular to L through S^* intersect on the cubic.



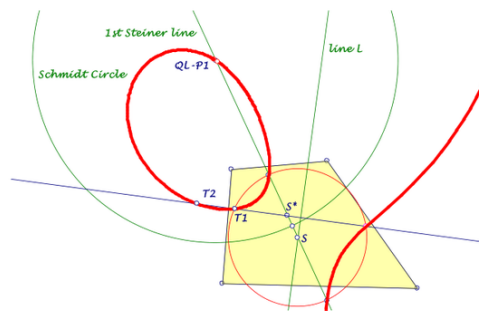
8. The intersections R_1 and R_2 of the cubic and the 2nd Steiner Line lie on a circle through F_1 and F_2 round the intersection of the line L and the 2nd Steiner Line.
9. The cubic is anallagmatic, invariant under reflections in circles round R_1 or R_2 through F_1 and F_2 .



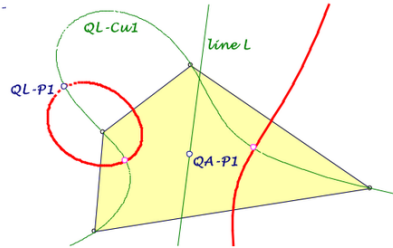
10. If the cubic is unipartite, there is a circle perpendicular to the Schmidt Circle and centered in S^* , containing the intersections of the cubic and the Schmidt Circle and the intersections of the cubic and the line L .



11. If the cubic is bipartite, there is a circle perpendicular to the Schmidt Circle and centered in S , containing the intersections of the cubic and the 1st Steiner Axis (without $QL-P1$) and the intersections T_1 and T_2 of the cubic and the perpendicular to L through S^* (without the point out of 7).



12. There are alternative constructions of the cubic: If S is inside F_1F_2 , circles round variable points X on L and perpendicular to the circle out of 10 cut the connections of X and the Miquel Point in points of the cubic. If S is outside F_1F_2 , circles round variable points X on L through T_1 and T_2 cut the connections of X and the Miquel Point in points of the cubic.
13. The cubic intersects the QL -Quasi Isogonal Cubic QL - $Cu1$ perpendicular in three points, the Miquel Point and two CSC -partners symmetric wrt $QA-P1$.



Eckart Schmidt
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2014-01-23.pdf

Message: #415
Date: 24/1/2014 7:59:10
From: Chris
Subject: stelloïd, hessian and cayleyan

Dear Bernard K.,
About your document I got these questions.
1) second and third point: do you have references for the mean angle properties?
3) forelast point: could you explain this construction?

Best regards,

Chris

>

> --- In Quadri-Figures-Group@yahoogroups.com , wrote:

> > Dear Chris, dear Eckart, dear Bernard G., dear members of
> > the Quadri-Figures-Group

> > First of all, happy new year to all of you and your families
> > and a lot of new properties and curves for EQF.

> > As I told Chris at the end of last year,

> > I spent part of my energy to read Bernard G.'s paper

> > on Eckart's cubic and J. Marchand's papers on the

> > QL's conjugated stelloïd,

> > it's hessian and it's cayleyan and I have some

> > remarks and many questions.

> > As Bernard had mentionned in an old message,

> > it happens to be the same curve.

> > It's hard and beautiful stuff,

> > which needs the intervention of an expert in cubics,

> > but it is very promising

> > Best regards

> > Bernard

>

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Message: #416

Date: 24/1/2014 8:25:55

From: Chris

Subject: Tangential quadrilateral in quadrilateral well know?

Dao: Let ABCD be a quadrilateral. Let four points E,G,H,F lie on two angle bisector of diagonal, then intersection of four line are reflection of AC on AH,FG and BD on HE,GF at I,J,K,L then IJKL are tangential quadrilateral.

Dear Dao,

Interesting configuration!

When we pinpoint the points E,G,H,F as the intersection points of the angle bisectors and the four sides of ABCD we have a special tangential quadrilateral with its center on

QG-P1.QG-P17, where:

QG-P1 = Diagonal Crosspoint

QG-P17 = Projection of QG-P1 on 3rd diagonal of ABCD

Best regards,

Chris

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Message: #417

Date: 24/1/2014 9:55:03

From: Chris

Subject: Angle property for a cyclic quadrilateral

Dear friends,

I noticed this property for a cyclic quadrilateral.

Given Cyclic Quadrilateral ABCD.

Let O be the CircumCenter of ABCD.

Let I = Midpoint AC and J = Midpoint BD.

Now the two angles between the lines OI and OJ correspond with $(\text{Angle AOB} + \text{Angle COD})/2$ and $(\text{Angle BOC} + \text{Angle DOA}) / 2$.

Are there any references for this property?

Best regards,

Chris

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Message: #418
Date: 24/1/2014 3:45:46
From: eckart_schmidt@t-online.de
Subject: Angle property of a cyclic quadrilateral

Dear Chris,
the property can easily be proved (XOY orientated angles):
(1) $\angle COI = \angle IOA$, $\angle BOJ = \angle JOD$
(2) $\angle JOD + \angle DOC + \angle COI = \angle JOI$
(3) $\angle AOB + \angle BOJ + \angle JOI + \angle IOA = 360^\circ$
This gives:
 $2\angle JOI = 360^\circ - \angle AOB - \angle COD = \angle BOC + \angle DOA$.
The property also holds for an arbitrary quadrigon, if O is the intersection of the perpendicular bisectors of the diagonals.
Best regards Eckart

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Message: #419
Date: 24/1/2014 7:17:43
From: yeuemtrondoitb85
Subject: Tangential quadrilateral in quadrilateral(well known??)

Dear Mr Chris, and Mr Eckart
Let ABCD be a any quadrilateral. E,F,G,H are intersection of angle bisector of four vertex $\angle A$, $\angle B$, $\angle C$, $\angle D$ (Show in the figure attachment. M,N,P,Q are intersection of four line through E,F,G,H and perpendicular with AB, AD, DC,CB (show in the figure).
<http://www.geogebraTube.org/student/m79112> (<http://www.geogebraTube.org/student/m79112>)
1-Prove that MNPQ are tangential quadrilateral.
2-Prove that if ABCD is cyclic, then MNPQ is a point T, and center of circle (ABCD),T and intersection of AC,BD are collinear
Best regards
Sincerely
Dao Thanh Oai

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Message: #420
Date: 24/1/2014 10:15:51
From: Chris
Subject: Angle property of a cyclic quadrilateral

Dear Eckart,

I was looking for generalization in a Quadrangle.
QA-P4 has Circumcenter function in a Quadrangle, see:
<http://www.chrisvantienhoven.nl/quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/25-qa-p4.html>
But the angle property wasn't valid for this point.
You are right the circumcenter function at Quadrigon level is the answer. The perpendicular bisectors of the diagonals you mention is the 1st Quasi Circumcenter QG-P5 and this point has a Circumcenter function in a Quadrigon, see:
<http://www.chrisvantienhoven.nl/quadrigon-objects/16-mathematics/encyclopedia-of-quadri-figures/quadrigon-objects/artikelen-qg/170-qg-p5.html>

Best regards,
Chris

>
> --- In Quadri-Figures-Group@yahoogroups.com ,
> "eckart_schmidt@..." wrote:
> > Dear Chris,
> > the property can easily be proved (XOY orientated angles):
> > (1) $\angle COI = \angle IOA$, $\angle BOJ = \angle JOD$
> > (2) $\angle JOD + \angle DOC + \angle COI = \angle JOI$
> > (3) $\angle AOB + \angle BOJ + \angle JOI + \angle IOA = 360^\circ$
> > This gives:
> > $2\angle JOI = 360^\circ - \angle AOB - \angle COD = \angle BOC + \angle DOA$.
> > The property also holds for an arbitrary quadrigon, if O is
> > the intersection of the perpendicular bisectors
> > of the diagonals.
> > Best regards Eckart
>

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Message: #421
Date: 26/1/2014 6:57:54
From: eckart_schmidt@t-online.de
Subject: Some "splitter" V

Dear Chris,

some single properties, perhaps of interest:

- ... Let $P_1P_2P_3P_4$ be a quadrigon and X_{ij} the pedal points of X wrt P_iP_j . The trilinear poles of XX_{ij} wrt QL-DT lie on a circumconic of QL-DT.
- ... For points X on QA-Cu1 the lines $X_{12}X_{34}$, $X_{23}X_{41}$, $X_{13}X_{24}$ have a common point.
- ... The Diagonal Triangle and the Miquel Triangle are point-perspective wrt QA-P3 (well known). They are also line-perspective wrt the QA-DT trilinear polar of the reflection of QA-P4 in QA-P3, that is the QA-Tr3 trilinear polar of the intersection $QA-P3.QA-P4 \wedge QA-P1.QA-P2$.
- ... Let S_i and M_i be corresponding vertices for the Diagonal and the Miquel Triangle of a quadrangle. The circumcircles of S_iM_iQA-P4 have a common point in the Involutary Conjugate of QA-P4 (tangential point of QA-P4 wrt QA-Cu1 on QA-Cu1).
- ... For a quadrigon triangles P_1XP_3 and P_2XP_4 have equal areas for points X on two lines through QG-P1, Involutary Conjugates with points at infinity in the Involutary Conjugates of the intersections of QA-Co1 and the reflection of QG-L1 in QG-P1.

Best regards Eckart

PS. It seems, that there is a typo in QA-Cu1, point 4: ... asymptote ... // QA-L4

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Message: #422
Date: 26/1/2014 11:57:14
From: Chris
Subject: interesting new Quadrangle Line

Dear friends,
I found an interesting new Quadrangle Line in the document [36]
"Central Points of the Complete Quadrangle - Milan. J. Math., 75
(2007) 333-356." of Benedetto Scimemi.

QA-L8 is the line which is the Isogonal Conjugate of the
Nine-point Conic QA-Co1 wrt the QA-Diagonal Triangle. This line
is mentioned with several properties by Benedetto Scimemi in
[36] page 347. It is also used for the construction of the axes
/ asymptotes of QA-Co1.

1st DT-coefficient QA-L8: p^2/a^2
1st DT-coordinate Infinity point QA-L8:
 $a^2 (b r + c q) (b r - c q)$

Properties:

- * The QA-DT-Isogonal Conjugate of QA-P3 is the projection point
of QA-P11 on QA-L8. See [36] 3.4.3 and 3.4.4 at page 347.
- * The QA-DT-Isogonal Conjugate of QA-P2 is the infinity point of
QA-L8. See [36] 3.4.3 and 3.4.4 at page 347.
- * The reflections of QA-P2 in the sides of QA-DT are collinear
on the line QA-P4.QA-P12, which is a line parallel to QA-L8. See
[36] 3.4.3 and 3.4.4 at page 347.
- * The line QA-P13.QA-P28 is also perpendicular to QA-L8.

QA-L8 is also used for the construction of the axes / asymptotes
of the Nine-point Conic QA-Co1.

When the Reference Quadrangle is convex/crossed QA-L8 and QA-Ci1
(circumcircle Diagonal Triangle) will intersect and QA-Co1 will
be a hyperbola.

Let U and V be the intersection points $QA-L8 \wedge QA-Ci1$.

The lines through QA-P1 parallel to QA-P2.U and QA-P2.V are the
asymptotes of QA-Co1.

The lines through QA-P1 parallel to the internal/external angle
bisectors between lines QA-P2.U and QA-P2.V are the axes of
hyperbola QA-Co1.

When the Reference Quadrangle is concave QA-L8 and QA-Ci1 will
not intersect and QA-Co1 will be an ellipse.

Now let U and V be the intersection points of QA-Ci1 with the
polar of QA-P3* wrt QA-Ci1 (QA-P3* = the Isogonal Conjugate of
QA-P3 wrt QA-DT).

The lines through QA-P1 parallel to the internal/external angle bisectors between lines QA-P2.U and QA-P2.V are the axes of the ellipse QA-Co1.

See [36] 3.4.4 at page 347.

Best regards,
Chris

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Message: #423

Date: 27/1/2014 9:05:36

From: Chris

Subject: Miquel Pedal Point

Dear Eckart,
I did not find (despite the nice properties) any incidences with other known points / curves for your new QG-point.
Did you?
Best regards,
Chris

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Message: #424

Date: 28/1/2014 8:43:02

From: eckart_schmidt@t-online.de

Subject: Interesting new Quadrigon Line

Dear Chris,
I have studied your interesting description of the new QA-line;
some remarks:

- ... The QA-DT-isogonal conjugate of QA-P3
is the intersection of QA-L8 and QA-P4.QA-P10.
- ... There is a typo: QA-P4.PA-P12 is PERPENDICULAR to QA-L8.
- ... You mention three perpendicular lines wrt QL-L8:
QA-P11.QA-P3*, QA-P4.QA-P12, QA-P13.QA-P28.
The last one is the middle parallel of the first two.
- ... There is a further perpendicular line to QA-L8:
QA-P6.QA-P36 (QA-DT Simson line of QA-P2).
- ... QA-L8 is the trilinear polar of QA-P16*,
the QA-DT isogonal conjugate of QA-P16.

Best regards Eckart

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Message: #425
Date: 28/1/2014 11:14:34
From: Chris
Subject: Interesting new Quadrigon Line

Dear Eckart,

You mentioned:

... The QA-DT-isogonal conjugate of QA-P3
is the intersection of QA-L8 and QA-P4.QA-P10.

It even is more prominent.

The QA-DT-Isogonal Conjugate of QA-P3
= the QA-DT-Complement of QA-P4.

And so:

QA-P3 = The QA-DT-Isogonal Conjugate
of the QA-DT-Complement of QA-P4.

QA-P4 = The QA-DT-Anticomplement
of the QA-DT-Isogonal Conjugate of QA-P3.

And these are remarkable relationships for these well known
points QA-P3 and QA-P4.

Checking these properties in EQF, I found they are already
mentioned!

Best regards,
Chris

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Message: #426
Date: 28/1/2014 11:28:55
From: eckart_schmidt@t-online.de
Subject: Precisions concerning the curve QL-Cu1

Dear Chris,

a further property of QL-Cu1:

I don't know, whether it is already mentioned.

... Let P1P2P3P4 be a quadrigon:

For points X on QL-Cu1 the centers of the circumcircles
for P1XP2, P2XP3, P3XP4, P4XP1 are concyclic.

Best regards Eckart

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Message: #427

Date: 28/1/2014 6:24:46

From: bernard.keizer

Subject: a new (?) simple construction of QL-Cu1

Dear Chris,

In answer to your message, the angle properties are, for the direction of the sides of the Morley triangles, in Morley's article Extensions of the Clifford's Chain Theorem and in his book Inversive Geometry and, for the directions of the axes of the deltoïd, in J. Lemaire's book Hypocycloïdes et épicycloïdes.

By the way, J.P. Ehrmann made a lot of metric calculations with the same angles θ between the Steiner Line and the lines L_i ; for example, if r is the radius of the Miquel and of the Hervey circle, the distance from the focus (Miquel point QL-P1) to the directrix of the parabola (Steiner Line QL-L2) is $p = 8r \cdot \cos(\theta)$ and the radius of the inversion circle (Schmidt circle) is $4r \cdot \sqrt{\cos(\theta)}$.

For the last point, using the property of the invariance of any circle through F1 and F2 in the Clawson-Schmidt transformation, I've found an easy way to find these 2 points and an easy construction of the hessian curve QL-Cu1 (see attached file).

I would be pleased if you, Eckart or Bernard G. could check this construction and give a validation. I don't know if it is an old well-known property, but for me at last it was completely new !

Best regards
Bernard

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Message: #428

Date: 29/1/2014 11:01:48

From: eckart_schmidt@t-online.de

Subject: a new (?) simple construction of QL-Cu1

Dear Bernard K,

it took some time, to understand your construction.

Here a summary:

... w point on the 2nd Steiner Axis

... Ci circle round w through F1, F2

... w' pole of F1F2 wrt Ci

... Thales circle about ww'

... intersections m of the Thales circle with QL-L1

... intersections of the lines mw' and Ci on QL-Cu1

The construction was new for me. An advantage: It is independent whether the cubic is unicursal or bicursal. (So I don't understand your last remark.) It can also happen, that there are four intersections in the last step of construction (in the bicursal case).

Thanks for this new aspect and best regards Eckart

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Message: #429

Date: 30/1/2014 9:08:49

From: eckart_schmidt@t-online.de

Subject: Orthocentric Pedal Quadrangles

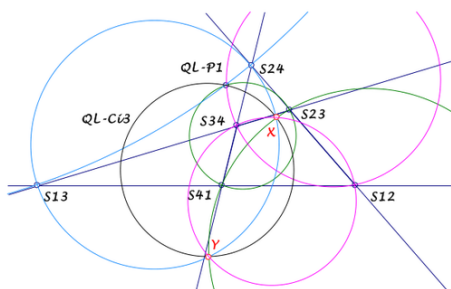
Dear friends,

for a quadrigon the pedal quadrangle of the Isogonal Center QA-P4 is a parallelogram; for a quadrilateral the pedal quadrangle of the Miquel Point QL-P1 degenerates collinear on QL-L3. For a quadrilateral there are two points, whose pedal quadrangle are orthocentric. These points - without their property - are already mentioned by Clawson. See attachment. Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

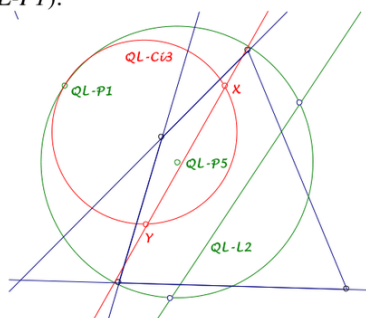
Orthocentric Pedal Quadrangles

For a quadrigon the pedal quadrangle of the Isogonal Center $QA-P4$ is a parallelogram; for a quadrilateral the pedal quadrangle of the Miquel Point $QL-P1$ degenerates collinear on $QL-L3$. Here for a quadrilateral two points are described, whose pedal quadrangle are orthocentric. These points – without their property – are already mentioned by Clawson (Ref. EQF [22], page 248 (38)).

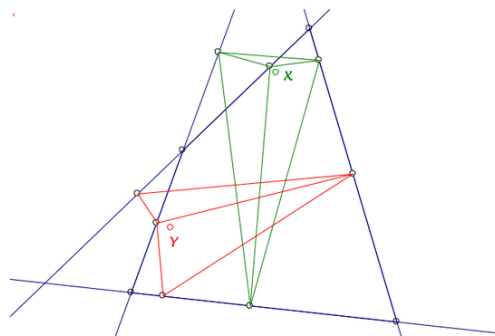


Clawson describes two points X, Y on the “circumcentric circle” – in EQF Miquel Circle $QL-Ci3$ – as common intersections of circles through $S_{i,j}$ and $S_{k,l}$, orthogonal to the circumcircle through the Miquel Point $QL-P1$ and $S_{i,j}$ and $S_{k,l}$ ($S_{i,j}$ intersection of L_i and L_j).

You get these points X and Y also as Clawson-Schmidt Conjugate $QL-Tf1$ of the Plücker Pair of Points $QL-2P1$ (intersections of the Steiner Line $QL-L2$ and a circle round $QL-P5$ through $QL-P1$).



The points X and Y have orthocentric pedal quadrangles.



This property is only Cabri-controlled; a calculation with barycentric coordinates is very extensive.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-01-30.pdf

Message: #430
Date: 30/1/2014 11:52:26
From: Chris
Subject: Orthocentric Pedal Quadrangles

Dear Eckart,

Clawson had his own insights and he was not hindered by our modern tools!

I agree the coordinates of both points X and Y are extensive. However the CT-coefficients of the line XY are relatively simple.

First CT-Coefficient is:

$$\begin{aligned} & b^2 c^2 l (m - n) \\ & (-2 a^2 l + a^2 m + b^2 m - c^2 m + a^2 n - b^2 n + c^2 n) \\ & (-a^2 b^2 l + b^4 l - a^2 c^2 l - 2 b^2 c^2 l + c^4 l + a^2 b^2 m - b^4 m + b^2 c^2 m + a^2 c^2 n + b^2 c^2 n - c^4 n) \end{aligned}$$

First some definitions:

1. Let $CSC(QL-Px)$ = Clawson-Schmidt Conjugate of QL-Px
2. Since the Clawson Schmidt Conjugate of a Line is a Circle we can define:
 $CSCe(\text{Line})$ = Center of the Clawson Schmidt Conjugate of "Line".

Now the remarks:

- * XY is the perpendicular bisector of $QL-P1.CSC(QL-P5)$.
- * X and Y are collinear with $CSCe(\text{Newton Line})$ and $CSCe(QL-P3.QL-P4)$,
where $CSCe(QL-P3.QL-P4)$ is a point on the Steiner Line $QL-L2$ and $CSCe(\text{Newton Line})$ is the reflection of $QL-P1$ in XY.
- * The midpoint of X and Y lies on the line $CSC(QL-P5).CSC(QL-P7)$.

Best regards,
Chris

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Message: #431
Date: 30/1/2014 2:27:43
From: eckart_schmidt@t-online.de
Subject: Orthocentric Pedal Quadrangles

Dear Chris,
wonderful properties, thank you very much! In addition:
Let L be a parallel to $QL-P3.QL-P4$ through $QL-P1$, let L' be the reflection of L in the 1st Steiner Axis, then the QL -Line Isoconjugate $QL-Tf2$ of L' is XY .
Best regards Eckart

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Message: #432
Date: 30/1/2014 11:19:36
From: bernard.keizer
Subject: a new (?) simple construction of $QL-Cu1$

Dear Eckart,

Thanks a lot for having checked and validated my construction.
Your summary is perfect!
In fact, it holds for uni- as well as for bicursal, without remark ...
It seems the curve is unicursal if the Newton Line cuts the segment $F1F2$ and bicursal if it cuts the line through $F1$ and $F2$ outside of the segment?

Best regards
Bernard K

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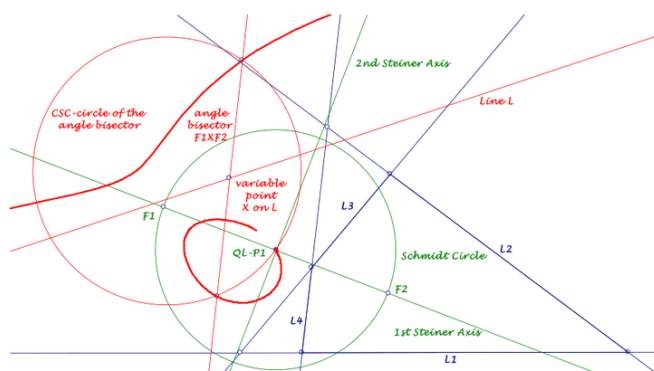
Message: #433
Date: 31/1/2014 12:12:28
From: eckart_schmidt@t-online.de
Subject: Cubics related to lines

Dear friends,
for lines L in the QL-environment a special construction gives
an anallagmatic cubic through the Miquel Point $QL-P1$ with an
asymptote parallel to L , invariant wrt the Clawson-Schmidt
Conjugate $QL-Tf1$. For example: The line $QL-L1$ gives the
Quasi-Isogonal Cubic $QL-Cu1$ (see message #403). Another example
is in message #414. For lines through $F1$, $F2$ the cubic is a
strophoid (see attachment).
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Cubics related to Lines

For lines a special construction in the QL-environment gives a cubic through the Miquel Point $QL-P1$, invariant wrt the Clawson-Schmidt Conjugate $QL-Tf1$. For example: The Newton Line $QL-L1$ gives the QL -Quasi Isogonal Cubic $QL-Cu1$. – The relationships are only CABRI-controlled.



The Construction

We consider a reference quadrilateral with the lines L_1, L_2, L_3, L_4 and a further line L . For the construction of a cubic for the line L we need (see *EQF, QL-Tf1*) the Clawson-Schmidt Conjugate (CSC) and its fixed points F_1 and F_2 , the Schmidt Circle, the 1st and 2nd Steiner Axis and the Miquel Point $QL-P1$ of the quadrilateral.

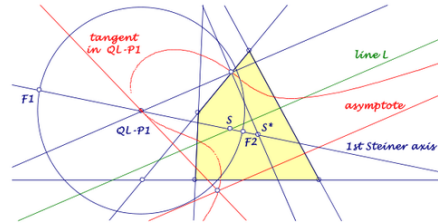
For variable points X on the line L the intersections of the angle bisector of $\angle F_1XF_2$ and its CSC-image (a circle through F) are points of the cubic.

Properties

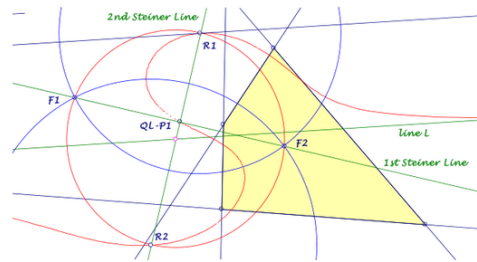
Let S be the intersection of the line L and the 1st Steiner Axis and S^* its reflection in the Schmidt Circle.

1. The cubic contains the Miquel Point $QL-P1$.
2. The cubic is invariant wrt the Clawson-Schmidt Conjugate $QL-Tf1$.

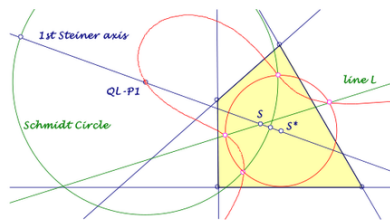
3. The cubic is unipartite, if S lies inside F_1F_2 ; the cubic is bipartite, if S lies outside F_1F_2 .
4. The asymptote is a parallel to the line L through the reflection of the Miquel Point in the line L .
5. A parallel to the line L through the Miquel point, reflected in the 1st Steiner Axis gives the tangent to the cubic in the Miquel Point.



6. The tangent in the Miquel Point and the asymptote intersect on the cubic.
7. A parallel to the line L through the Miquel Point and a perpendicular to L through S^* intersect on the cubic.
8. The intersections R_1 and R_2 of the cubic and the 2nd Steiner Line lie on a circle through F_1 and F_2 round the intersection of the line L and the 2nd Steiner Line.

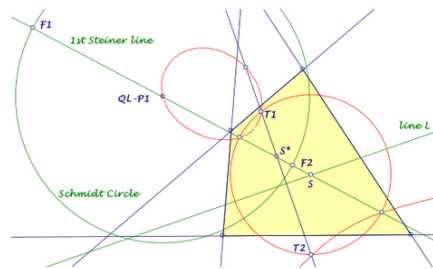


9. The cubic is anallagmatic, invariant under reflections in circles round R_1 or R_2 through F_1 and F_2 .
10. If the cubic is unipartite, there is a circle perpendicular to the Schmidt Circle and centered in S^* , containing the intersections of the cubic and the Schmidt Circle and the intersections of the cubic and the line L .

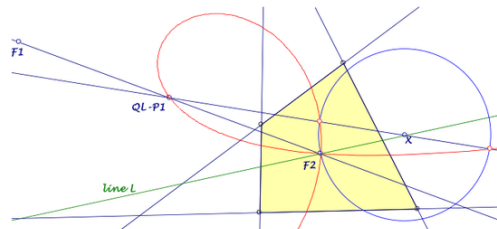


11. If the cubic is bipartite, there is a circle perpendicular to the Schmidt Circle and centered in S , containing the

intersections of the cubic and the 1st Steiner Axis (without $QL-PI$) and the intersections T_1 and T_2 of the cubic and the perpendicular to L through S^* (without the point out of 7).



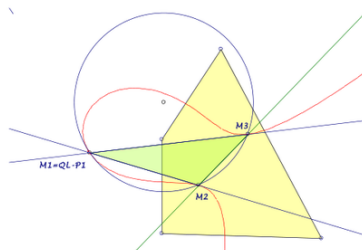
12. There are alternative constructions of the cubic: If S is inside F_1F_2 , circles round variable points X on L and perpendicular to the circle out of 10 cut the connections of X and the Miquel Point in points of the cubic. If S is outside F_1F_2 , circles round variable points X on L through T_1 and T_2 out of 11 cut the connections of X and the Miquel Point in points of the cubic.
13. For the 1st Steiner Axis the cubic degenerates to the Schmidt Circle and the 1st Steiner Line.
14. For the Newton Line $QL-L1$ the cubic is the QL -Quasi Isogonal Cubic $QL-Cu1$.
15. For lines L through one of the fixed points F_i , the cubic is a strophoid of the line L with the pole $QL-PI$ and the fixed point F_i . That means, that the intersections of circles through F_i round variable points X on the line L and the connections of X with the Miquel Point generate the cubic.



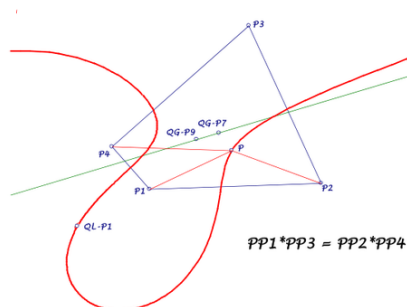
16. For lines L through the Miquel Point the cubic is symmetric wrt $QL-PI$, cutting the 2nd Steiner Line on the Schmidt Circle.

Here are some remarks for quadrilaterals, considering the Miquel Triangle $M_1M_2M_3$ (QA-Tr2) with Miquel Point M_1 .

- For the sideline M_2M_3 of the Miquel Triangle we get a circumcubic of the Miquel Triangle.
- This cubic is the locus for points, whose Clawson-Schmidt Conjugate $QL-Tf1$ is the isogonal conjugate wrt the Miquel Triangle.



- Cubic related to $QG-P7.QG-P9$**



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Message: #434
Date: 31/1/2014 12:13:26
From: eckart_schmidt@t-online.de
Subject: a new (?) simple construction of Q

Dear Bernard K.,

your criterium for uni- or bicursal character of QL-Cu1 is right. It can be generalized for "Cubics Related to Lines" (see my last message).
Perhaps the remarks give further properties for QL-Cu1.
I think, your remark at the end of the message #410 is right:
The fixed points F1, F2 of CSC are of value, to be mentioned in EQF.

A further property:
... Let P1P2P3P4 be a quadrigon. F1 and F2 are the points X with touching circumcircles of P1XP2 and P3XP4 as well as P2XP3 and P4XP1 (each pair of these circles consists of CSC partners).

Best regards Eckart

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Message: #435
Date: 31/1/2014 7:07:57
From: bernard.keizer
Subject: Orthocentric Pedal Quadrangles

Dear Chris, dear Eckart

The Clawson-Schmidt conjugation is a really fascinating toy and I had great fun to read your messages #429 and #430 and to draw new figures!

1) I can understand on the first page from Eckart's attached file the construction of the 2 points X and Y as CSC-conjugates on the 2 Plücker points QL-2P1 with CSC conjugation of Steiner Line and of the 3 circles with diameters $S_{ij}Sk_l$.

But I absolutely don't see why the pedal quadrangles of these 2 points are orthocentric! How did you find this property? More generally, which points do have that property?

2) I've appreciated the new concept of $CSCe(\text{Line})$ = center of the circle through QL-P1 $CSC(\text{Line})$ and the property $CSC(CSCe(\text{Line}))$ = reflexion of QL-P1 in the line.

I can understand again why XY is the perpendicular bisector of $P1CSC(P5)$ - I suppose there is a mistake: the reflexion of P1 in XY is $CSC(P5)$ and not $CSCe(\text{Newton Line})$ - why $CSCe(\text{Newton Line})$ is on XY (the circle $CSC(\text{Newton Line})$ is orthogonal to QL-Ci3 and pass through P1 and $CSC(P5)$) and why $CSCe(P3P4)$ is on the Steiner Line (as $CSC(P4)$ is the reflexion of P1 in the Steiner Line), but I don't see immediately why the middle of XY is on $CSC(5)CSC(P7)$...

Did you calculate these properties or is there a synthetic explanation?

May be it could be interesting to give systematically in EQF for each point it's CSC conjugate and for each line it's $CSCe$ (like Clark Kimberling does in ETC). It could be also interesting to find for other circles (for example Hervey circle) or even for other curves their CSC conjugates (for example the CSC conjugate of the deltoid is a curve tangent to 5 circles CSC of the 5 lines tangent to the deltoid).

Best regards

Bernard

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Message: #436
Date: 01/2/2014 9:54:30
From: Chris
Subject: Orthocentric Pedal Quadrangles

Dear Bernard K, dear Eckart and friends,

I surely agree that the Clawson-Schmidt Conjugate is fascinating and full of pleasant surprises.

To answer your questions Bernard the result I presented in message #430 were all algebraic results.

The CSCe transformation I found for over a year ago. I did register then some CSCe-points in my algebraic system for later use. And now it paid off!

So all I did was looking for collinearities in an algebraic Mathematica program and I presented it with some interpretations. You made the deduction: $\text{CSC}(\text{CSCe}(\text{Line})) =$ reflexion of QL-P1 in the line. Very well!

I also noticed there was more to explore. I saw some similarity with QL-P26.

QL-P26 lies on the perpendicular bisector of QL-P1.CSC(QL-P6).

Applying this to your rule:

$\text{CSC}(\text{CSCe}(\text{Line-x})) = \text{Reflection of QL-P1 in Line-x,}$

where $\text{Line-x} = \text{QL-P26.CSC}(\text{QL-P17}).\text{CSC}(\text{QL-P24}),$ and

where $\text{CSC}(\text{Line-x}) = \text{Dimidium Circle QL-Ci6 with center QL-P6.}$

Moreover this line Line-x intersects the Steiner Line QL-L2 in the same point $\text{CSCe}(\text{QL-P3.QL-P4})$ also occurring on the line XY we were exploring.

This point also has relatively short Quadrilateral coordinates.

1st CT-coordinate:

$$\begin{aligned} & a^2 (1 - m) (1 - n) m n (a^4 (b - c) (b + c) (1 - m) (1 - n) \\ & - a^2 (b^2 (1 - m) + c^2 (-1 + n)) (b^2 (1 + m - 2 n) \\ & + c^2 (1 - 2 m + n)) + (b - c) (b + c) (m - n) (b^4 (1 - m) \\ & + c^4 (-1 + n) + 2 b^2 c^2 (-m + n))) \end{aligned}$$

It is the Clawson-Schmidt Conjugate of the 2nd intersection point of this pencil of circles QL-Ci3, QL-Ci5 and QL-Ci6. First intersection point is of course the Miquel Point QL-P1.

Summarizing these results I wonder what we can say about occurrences of these CSCe-situations. When do they happen? In which circumstances?

About your remark to make a list of instances of CSC and CSCe I can say there is already a list at the page of QL-Tf1 for CSC-examples:

When you have further examples of CSC transformations or CSCe transformations please let me know.

Best regards,
Chris

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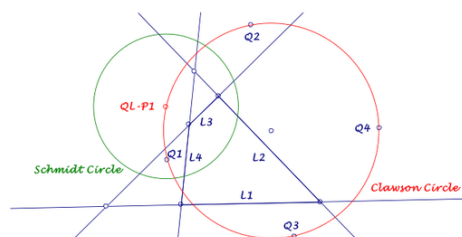
Message: #437
Date: 01/2/2014 10:22:13
From: eckart_schmidt@t-online.de
Subject: Clawson's Polar Reciprocation

Dear friends,
there is a further interesting aspect in Clawson's work: The poles for the lines of a quadrilateral wrt a circle round the Miquel Point give a cyclic quadrangle with the Miquel Point on its circumcircle. This constellation is discussed for the Schmidt Circle in the attachment.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Clawson's Polar Reciprocation

Clawson describes for quadrilaterals the pole-polar-relation wrt circles centered in the Miquel Point $QL-P1$. For these circles the poles for the lines of the quadrilateral give a cyclic quadrangle with the Miquel Point on its circumcircle (Ref. EQF [22], page 257). This constellation shall here be studied for the Schmidt Circle (see $QL-Tf1$). – The results are only Cabri-controlled.



Let L_1, L_2, L_3, L_4 define a quadrilateral and Q_1, Q_2, Q_3, Q_4 be the poles of L_1, L_2, L_3, L_4 wrt the Schmidt Circle (see EQF, $QL-Tf1$). As a result of Clawson the Q -quadrangle is cyclic with the Miquel Point on its circumcircle. This circle here shall be named as Clawson Circle.

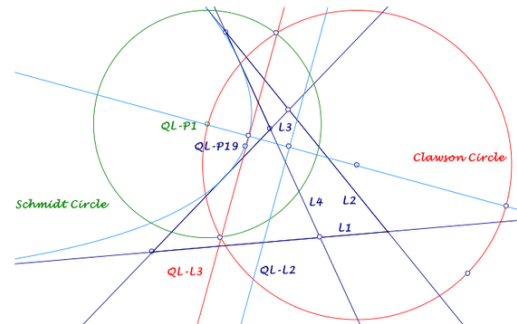
Properties:

- The Clawson Circle contains the Miquel Point $QL-P1$.
- The Clawson Circle is the reflection of the Pedal Line $QL-L3$ in the Schmidt Circle.
- The CSC-image of the Clawson Circle is the reflection of the Pedal Line $QL-L3$ in the 1st Steiner Axis.
- The reflection of $QL-P1$ (on $QL-L3$) in the Schmidt Circle is a point of the Clawson Circle.

There are significant relationships between the Clawson Circle and the Inscribed Parabola $QL-Co1$.

- The midpoint of the Clawson Circle is the reflection of the intersection of the axis and the directrix of $QL-Co1$ in the Schmidt Circle.
- The vertex of $QL-Co1$, reflected in the Schmidt Circle, gives the second intersection of the axis of $QL-Co1$ and the Clawson Circle.

- The Clawson Circle is the locus for the poles of QL - CoI -tangents wrt the Schmidt Circle.
- The Newton Line cuts QL - CoI in a point, for which the pole (wrt the Schmidt Circle) of its tangent is the reflection of QL - $P19$ in the Schmidt Circle.



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2014-02-01.pdf

Message: #438
Date: 02/2/2014 8:44:32
From: eckart_schmidt@t-online.de
Subject: Orthocentric Pedal Quadrangles

Dear Chris, dear Bernard K.,

I just studied your CSC discussion and found a (new?) simple construction for QL-Cu1:

Let L be lines parallel the Newton Line QL-L1. Then the intersections of the circles CSC(L) and the reflections of L in QL-L1 give points of the cubic QL-Cu1.

This construction can be generalized for "cubics related to lines" (see my message #433).

Remark: The special circle CSC(QL-L1) is already mentioned by Clawson (EQF Ref. 22, page 248 (33)). Clawson defines "Apollonian points" for QL-P1 and pairs of opposite points of the quadrilateral as second intersections of the circumcircle and the Apollonian circle through QL-P1. The three "Apollonian points" define a circle through QL-P1.

This circle cuts QL-Ci3 perpendicular and is tangent to the cubic QL-Cu1 in QL-P1 with intersections in QL-2P2.

Best regards Eckart

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Message: #439

Date: 04/2/2014 3:21:21

From: Chris

Subject: a new (?) simple construction of QL-Cu1

Dear Bernard K., Dear Eckart,

There still is one item that is bothering me about the apparently simple construction of the 1st Steiner Axis. The 1st Steiner Axis is needed in the construction of the Clawson-Schmidt Conjugate as well as with the fixed Schmidt points F1 and F2.

The 1st Steiner Axis is now defined as the common internal angle bisector of the angle $P_{ij}.QL-P_1.P_{kl}$, where P_{ij} and P_{kl} are the intersection points resp. $L_i \wedge L_j$ and $L_k \wedge L_l$, where (i,j,k,l) is a permutation of the number set $(1,2,3,4)$.

The 2nd Steiner Axis is the external angle bisector.

However normally in the environment of Quadrilaterals there actually is no such notion as internal/external. There just are two angle bisectors that are mutually perpendicular. Because of the Clawson Schmidt Conjugate we know that there is a 1st Steiner Axis with the Fixed Schmidt points on it.

Realizing this I figured out this construction using elements of Bernard K.'s construction.

1. Let L_a/L_b be the two angle bisectors of the angle $P_{12}.QL-P_1.P_{34}$.
2. Let L_c be the perpendicular bisector of $P_{12}.P_{34}$.
3. Let $S_a = L_a \wedge L_c$ and $S_b = L_b \wedge L_c$.
4. Let C_{ia} be the circle with center S_a through P_{12} and P_{34} .
Let C_{ib} be the circle with center S_b through P_{12} and P_{34} .
5. Now either C_{ia} will intersect L_b or C_{ib} will intersect L_a (I have no proof of this).

The real intersection points are F1 and F2 and the line of these intersection points will be the 1st Steiner Axis.

Can I get your comments please. Do you think this is a valid construction or is working with the notion of the internal angle bisector also valid? Or some other method?

Best regards,
Chris

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Message: #440
Date: 04/2/2014 9:08:10
From: eckart_schmidt@t-online.de
Subject: a new (?) simple construction of QL-Cu1

Dear Chris,
take one of the QG-components of the QL and its Miquel Triangle.
The (inner) angle bisector at QL-P1 is the 1st Steiner Axis.
Best regards Eckart

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Message: #441
Date: 05/2/2014 3:48:55
From: eckart_schmidt@t-online.de
Subject: QA-P4 - QL-P1 - QG-P1 - Circumcubic

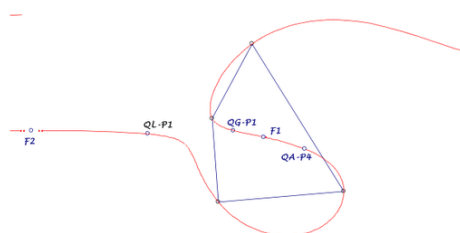
Dear friends,
in the attachment you find a special QG-circumcubic,
containing QA-P4, QL-P1, QG-P1, the main points of quadrangle,
quadrilateral and quadrigon.
This cubic has interesting properties, I was fascinated!
Best regards Eckart

EQF-Note 2014-02-05

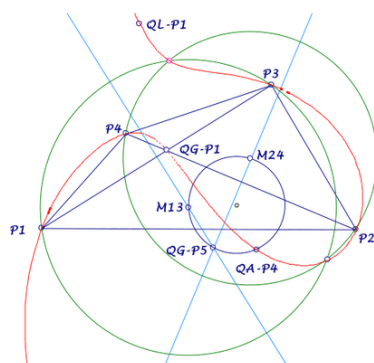
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

QA-P4-QL-P1-QG-P1-Circumcubic

For lines L through the Diagonal Crosspoint $QG-P1$ of a quadrigon the intersections with their Clawson-Schmidt Conjugate ($QL-Tf1$) give a special QG -circumcubic, combining the geometry of quadrangle, quadrilateral and quadrigon.

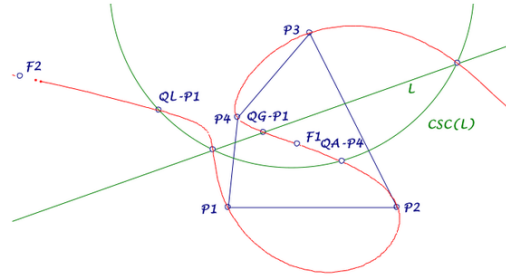


We start with a construction for the cubic independent of the Clawson-Schmidt Conjugate. The Isogonal Center $QA-P4$ is the second intersection of circles through the Miquel Point $QL-P1$ and opposite vertices of the quadrigon (see EQF), . The QG -Quasi Circumcenter $QG-P5$ is the intersection of the perpendicular bisectors of the diagonals (see EQF). Considering circles through $QA-P4$ and $QG-P5$ we get two points M_{13} , M_{24} on the perpendicular bisectors of the diagonals as centers of circles through opposite vertices. The intersections of these circles give points of the cubic.



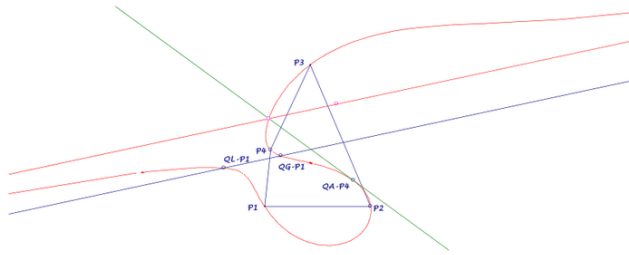
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Using the Clawson-Schmidt Conjugate $QL-TfI$ – shortened CSC – we consider lines L through $QG-P1$ and their intersections with $CSC(L)$, which are circles through $QL-P1$ and $QA-P4 = CSC(QG-P1)$.

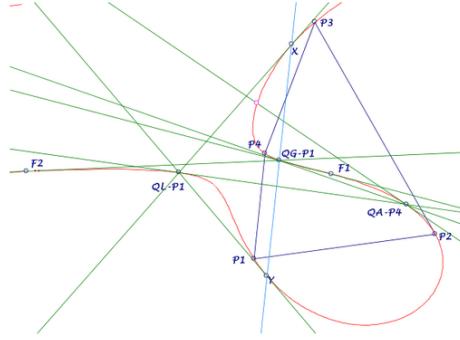


Properties:

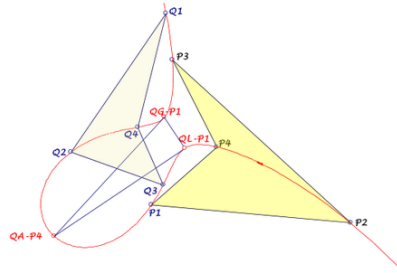
- The cubic contains the vertices of the quadrigon, the Miquel Point $QL-P1$, the Diagonal Crosspoint $QG-P1$ and the Isogonal Center $QA-P4 = CSC(QG-P1)$ as well as the CSC -fixed points F_1, F_2 (see $QL-TfI$).
- The cubic is CSC -invariant with pivot $QG-P1$.
- The asymptote is a parallel to $QG-P1.QL-P1$ through the reflection of $QA-P4$ in $QG-P1.QL-P1$.
- The asymptote cuts the cubic on the tangent at $QA-P4$ to the cubic (see below).



- The tangent at $QL-P1$ is $QA-P4.QG-P1$.
- The tangent at $QG-P1$ contains $QA-P4$.
- The tangents in F_1, F_2 intersect in $QG-P1$.
- The tangent in $QA-P4$ is the CSC -image of a circle through $QL-P1$, touching $QG-P1.QA-P4$ in $QG-P1$.
- The angle bisector L of $QL-P1, QG-P1, QA-P4$ cuts $CSC(L)$ in two points X and Y on the cubic with tangents through $QL-P1$.



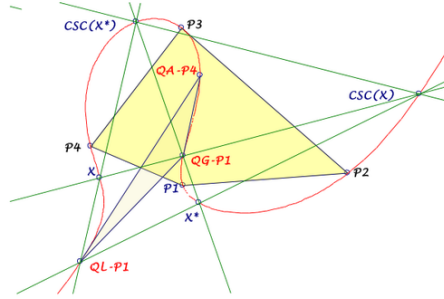
- The cubic is isogonal invariant wrt the triangle $\Delta = QA-P4.QL-P1.QG-P1$. (This is a very curious property! For example: F_1 and F_2 are isogonal conjugate.)
- Wrt the triangle Δ the cubic contains the isogonal conjugates Q_i of the vertices P_i . The P - and Q -quadrilaterons are perspectiv wrt $QL-P1$ and have the same Diagonal Crosspoint, Miquel Point and Isogonal Center.



- Finally the equation in DT -barycentric coordinates wrt $QL-DT$.

$$\begin{aligned}
 & a^2 (-\lambda SA + \mu SB + \nu SC) z (m^2 y^2 - n^2 z^2) - c^2 (\lambda SA + \mu SB - \nu SC) x (l^2 x^2 - m^2 y^2) \\
 & + 2 m^2 b^2 (\lambda SA - \mu SB + \nu SC) x y z \\
 & - (\lambda^2 SA^2 + \mu (2 l^2 + n^2) SB^2 - \nu m^2 SC^2 + (\mu m^2 + 2 \mu n^2 - \nu n^2) SA SB \\
 & \quad - (\lambda l^2 - 2 \mu m^2 + 2 \nu m^2) SA SC - (\lambda l^2 - 2 \mu m^2 + 2 \nu l^2) SB SC) x^2 z \\
 & + (\lambda m^2 SA^2 - \mu (l^2 + 2 n^2) SB^2 + \nu^2 SC^2 + (\lambda n^2 - 2 \mu m^2 - \mu n^2) SA SB \\
 & \quad + (2 \lambda m^2 - 2 \mu m^2 + \nu n^2) SA SC + (\lambda l^2 - 2 \mu l^2 - \mu m^2) SB SC) x z^2 = 0 \\
 & \text{with } \lambda := m^2 - n^2; \mu := n^2 - l^2; \nu := l^2 - m^2
 \end{aligned}$$

- Additional remark: For a triangle ABC we can consider a transformation, consisting of a reflection in the angle bisector at B and a reflection in a circle round B , so that A and C changes. This transformation shall be the “ ABC -inversion wrt B ”. For example the CSC -mapping is of this type wrt the Miquel Triangle.



The considered cubic is invariant under the Δ -inversions wrt $QA-P4$, $QL-P1$ and $QG-P1$. Let X be a point on the cubic:

Δ -inversion of X wrt $QL-P1$ is $CSC(X)$, collinear with X and $QG-P1$.

Δ -inversion of X wrt $QA-P4$ is the Δ -isogonal conjugate X^* of X , collinear with $CSC(X)$ and $QL-P1$.

Δ -inversion of X wrt $QG-P1$ is $CSC(X^*)$, collinear with X and $QL-P1$.

Further: XX^* parallel $CCS(X).CCS(X^*)$.

Final conclusion: The cubic depends only on the points $QA-P4$, $QL-P1$, $QG-P1$. If we take these points as A , B , C of a reference triangle ABC for barycentric coordinates, the CSC -mapping becomes

$$(x:y:z) \rightarrow (-a^2z : \frac{a^2yz + b^2zx + c^2xy}{x+y+z} : -c^2x)$$

and the cubic has the equation

$$x(a^2yz + b^2zx + c^2xy) + a^2yz(x+y+z) = 0.$$

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2014-02-05.pdf

Message: #442

Date: 07/2/2014 6:15:08

From: bernard.keizer

Subject: Orthocentric Pedal Quadrangles

Dear Chris, dear Eckart, dear fellows

Sorry to have delayed my answer, but I'm working on different things about the QL.

I suppose it's better to handle step by step!

First, at the end of my message #435, I told I didn't understand the collinearity between the middle of XY, $CSC(P5)$ and $CSC(P7)$; in fact, $CSCe$ (Newton Line) is the pole of the segment joining $P1$ and the reflexion of $P1$ in the line $P3P4$ and it is therefore not difficult to show that $CSC(P5)$ and $CSC(P7)$ are inverse wrt the circle of diameter XY.

I have always the question to Eckart on the property of orthocentric QA: where does that come from? (I only know that the vertices of an orthocentric QA are on a set of rectangular hyperbolas ...)

Then your questions: I read in EQF that $CSC(P26)$ is on the Dimidium circle $Ci6$, like $P17$ and $P24$; it's then normal that $P26$ and $CSC(P17)$ and $CSC(P24)$ are on a line.

Let's consider now the line $P3P4P5P6$ (Euler Line of the inscribed QA formed by the orthocenters of the reference triangles); CSC of this line is a circle through $CSC(P3)$, $CSC(P4)$, $CSC(P5)$ and $CSC(P6)$ and $CSCe$ of this line is CSC of the reflexion of $P1$ in this line. This $CSCe$ is then on the perpendicular bisector of any cord of the circle; as the Steiner Line is CSC of the Miquel circle $Ci3$, $P4$ is $CSCe$ (Steiner Line) and $CSC(P4)$ is the reflexion of $P1$ in the Steiner Line. The same way, the line XY is CSC of the Plücker circle $Ci5$, $P5$ is $CSCe$ (XY) and $CSC(P5)$ is the reflexion of $P1$ in XY; the line $CSC(P17)CSC(P24)$ is the CSC of the Dimidium circle $Ci6$, $P6$ is $CSCe[CSC(P17)CSC(P24)]$ and $CSC(P6)$ is the reflexion on of $P1$ in the line $CSC(P17)CSC(P24)$.

So far so good, but I don't know why $P26$ is on the Dimidium circle; I intend to send you another message with another definition of $P26$, given by Léon Ripert, who calls this point the Lemoine point of the QL, as it has the same property as the Lemoine point for the triangle.

Two last remarks on the CSC transformation:

first, for the QL, any of the 4 lines is the CSC of the circumcircle of a reference triangle, $CSCe$ of this line is the circumcenter of this triangle and the CSC of this $CSCe$ is the reflexion of $P1$ in the line; as $P1$ is on the 4 circumcircles, the reflexions of $P1$ are on a line, which is the Steiner Line and the 4 circumcircles are on the CSC of this line, which is the Miquel circle and $P1$ is on this circle.

Second, any QL has it's own CSC-transformation and, for example, for the diagonal QL formed by the diagonal triangle and the Newton Line, it's CSC transformation swaps the circumscribed circle $Ci1$ and the Newton Line and $CSc(N\text{ewton Line})$ $P9$ with the reflexion of it's Miquel point $P17$ in the Newton Line: we have the Miquel point and 2 conjugates and we could find the 2 invariant points ...

Best regards
Bernard K.

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Message: #443
Date: 07/2/2014 7:01:09
From: bernard.keizer
Subject: Orthocentric Pedal Quadrangles

Dear Eckart, dear Chris
Once you have your 2 points, on the reflexion of L in $L1$, if you draw the lines between $P1$ and the 2 points, which cut $L1$ in 2 points and the reflexions of the 2 initial points in these 2 new points are 2 other points of $Cu1$, which are on L the CSC conjugates of the 2 first points ...

Best regards
Bernard K

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Message: #444
Date: 08/2/2014 11:26:18
From: eckart_schmidt@t-online.de
Subject: Orthocentric Pedal Quadrangles

Dear Bernard K.
my result wrt orthocentric pedal quadrangles is only Cabri-controlled. I often prefer the "joy of finding" in the EQF-network than the "necessarity of proving and calculating". Thank you very much for the numerous CSC-relationships in message #442 and the additonal rermark to the QL-Cu1-construction in message #443.
A short CSC-remark (surely without successful application):
Using an unusual reference triangle ABC with B = QL-P1 and A, C two opposite vertices of QL, the CSC-transformation has the simple form:
 $(x : y : z) \rightarrow (-a^2z : (a^2yz + b^2zx + c^2xy)/(x+y+z) : -c^2x)$
with the fixed points $F=(a : -a-c+ \sqrt{(a+b+c)(a-b+c)}) : c)$.
Best regards Eckart

P.S: Wrt orthocentric QA:
Every vertex is orthocenter of the remaining vertices.

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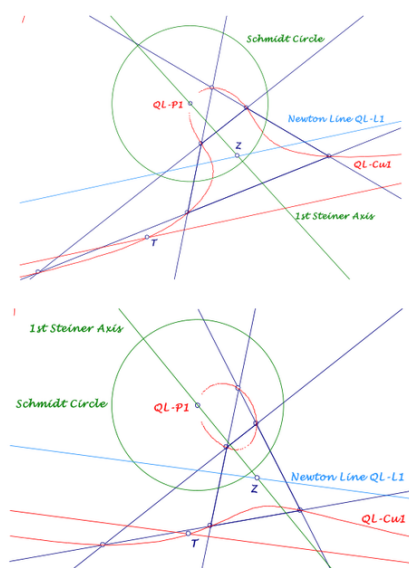
Message: #445
Date: 08/2/2014 2:19:39
From: eckart_schmidt@t-online.de
Subject: Pendant to QL-2P2

Dear friends,
QL-2P2 is a pair of foci of an inscribed conic, centered in the intersection of the Newton Line QL-L1 and QL-L6. In the attachment are discussed inscribed conics, centered in the intersection of the Newton Line and the 1st Steiner Axis with foci on the 1st Steiner Axis or on the Schmidt Circle. It is a comparison of the geometry of the cubic QL-Cu1 in the unipartite and bipartite case.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Pendant to $QL-2P2$

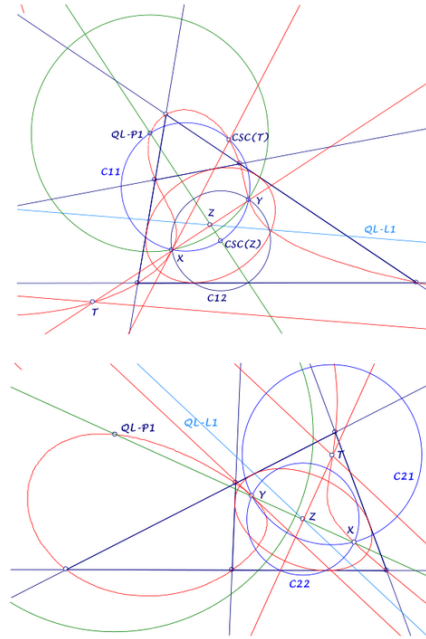
$QL-2P2$ is a pair of foci of an inscribed conic, centered in the intersection of the Newton Line $QL-L1$ and $QL-L6$. Here we discuss the foci of an inscribed conic, centered in the intersection of the Newton Line and the 1st Steiner Axis (see $QL-Tf1$). It is a comparison of the geometry of the cubic $QL-Cu1$ in the unipartite and bipartite case.



First remarks: The 1st Steiner Axis is the angle bisector at the Miquel Point $QL-P1$ wrt two opposite vertices of the quadrilateral. The Schmidt Circle is a circle round $QL-P1$, radius geometrical mean of the distances of $QL-P1$ to opposite vertices (see $QL-Tf1$). The reflection in the 1st Steiner Axis, followed by a reflection in the Schmidt Circle is the Clawson-Schmidt Conjugate $QL-Tf1$ (shortened CSC). Considering parallels L to the Newton Line, the CSC -image circles of L cut the reflections of L in the Newton Line on a cubic $QL-Cu1$, locus for the foci of inscribed conics of the quadrilateral. The cubic $QL-Cu1$ is unipartite, if the Newton Line cuts the 1st Steiner Line inside the Schmidt Circle, and bipartite, if the intersection is outside. The asymptote is a parallel to the Newton Line through the reflection of the Miquel Point in the Newton Line. The asymptote cuts $QL-Cu1$ in T on the tangent of $QL-P1$.

Foci on $QL-Cu1$

- Let Z be the intersection of the Newton Line $QL-L1$ and the 1st Steiner Axis. Z is the center of an inscribed conic:
 ... unipartite: The minor axis is the 1st Steiner Axis, the main axis cuts $QL-Cu1$ in T on the asymptote.
 ... bipartite: The main axis is the 1st Steiner Axis, the minor axis cuts $QL-Cu1$ in T on the asymptote.

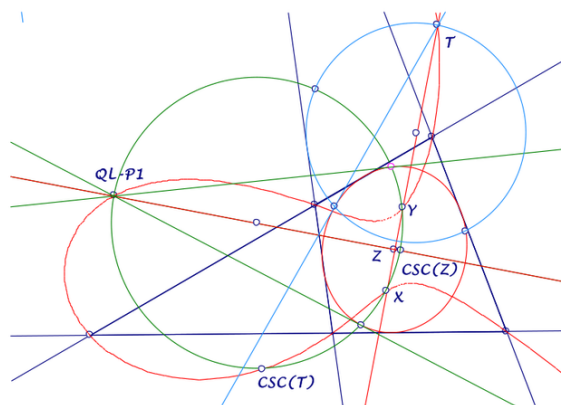


- The foci X, Y of the Z -centered inscribed conic lie on $QL-Cu1$ CSC-associated:
 ... unipartite: The foci lie on the Schmidt Circle.
 The foci lie on the CSC-image of the main axis, that is the Thales circle $C_{1,1}$ about $QL-P1$ and $CSC(Z)$ (through $CSC(T)$).
 The foci lie on a circle $C_{1,2}$ round $CSC(Z)$, perpendicular to the Schmidt Circle.
 ... bipartite: The foci lie on the 1st Steiner Axis.
 The foci lie on a circle $C_{2,1}$ round T perpendicular to the Schmidt Circle.
 The foci lie on a circle $C_{2,2}$ round Z , perpendicular to the Schmidt Circle.
- The circles $C_{1,1}$ and $C_{2,1}$ contain also the foci of the inscribed conic, centered in the intersection of $QL-L6$ and the Newton Line (see $QL-2P2$).

- Tangents for $QL-CuI$ at the foci X, Y :
 ... unipartite: The tangents at the foci X, Y intersect in $CSC(T)$.
 ... bipartite: The tangents at the foci X, Y are parallel to the asymptote (parallel to $QL-LI$).

Further properties:

- The Thales circle about $QL-P1$ and $CSC(Z)$ (through $CSC(T)$) contains also the tangential points of $QL-P1$ for the inscribed conic.



- The tangential points of T for the inscribed conic lie on a circle through T and the reflection of $CSC(T)$ in the 1st Steiner Axis, centered ...
 ... unipartite: ... on the main axis.
 ... bipartite: ... on the minor axis.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-02-08.pdf

Message: #446

Date: 10/2/2014 7:01:59

From: bernard.keizer

Subject: a new (?) simple construction of QL-Cu1

Dear Chris, dear Eckart

I hope you are convinced with the notion of internal bisector!
I have now improved my construction and I imagine it could be useful

1) Properties

- a) any circle centered on the 2nd Steiner axis is globally invariant in CLS
- b) 2 CS conjugates X and X' are on a circle through F_1F_2 with center I on the 2nd Steiner axis
- c) XX' cuts this axis in J and the middle m of XX' is on a 2nd circle with diameter IJ (I and J are inverse wrt the first circle and the second circle is a Thales circle wrt the first)

2) Constructions

- a) the CS conjugate X' of a point X is the intersection between the reflexion of P_1X in the 1st Steiner axis and the circle through F_1 , F_2 and X (in fact, there are 2 points, but only one is with 1st Steiner axis as internal bisector of XP_1X')
- b) given a point m , it's possible to find the 2 CS conjugates points with m as middle (circle through F_1 , F_2 and m gives I and J on the 1st Steiner axis ...)

Remark: the construction a) offers the possibility of drawing the CS conjugate of any curve (for example Qu_1 as CS conjugate of Co_1) or to find the locus of the middle of 2 points on the curve and it's CS conjugate (in my example, the result is interesting)

the construction b) offers the possibility of drawing new curves by letting m describe a given locus (Cu_1 if the locus is the Newton Line but many other cubics for other lines or many other curves if the locus is a circle or a conic)

3) Suggestions

In order to avoid CSC conjugate of ..., I suggest to call simply CX or CSX the CL conjugate of any element X (CSP_x as conjugate of the point P_x or CSL_x for the conjugate of a line, the same for a circle or a curve); for example, we would say that X and Y described by Eckart are $2CS2P_1$ and that CSL_2 is Ci_3 or $CSCo_1$ is Qu_1 ... Steve Sigur used coX for isogonal conjugate of X and $isoX$ for isotomic conjugate in the triangle and even $isoco$ for isotomic of isogonal and $coiso$ for isogonal of isotomic and he made then marvellous figures.

There is now a lot to look at by searching systematically the conjugates of all points, lines, circles, conics or quartics of EQF (I really would like to know what $CSQu_2$ would look like ...)

Chris, you told you had already calculated a lot of such
elements; are your results available?
My hope is to find new colinearities or cocircularities with the
CS conjugates ...
Best regards
Bernard K

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Message: #447

Date: 11/2/2014 2:02:53

From: Seiichi Kirikami

Subject: van Aubel quadrangle and CSC conjugate

Dear friends,

Given a quadrangle $P_1P_2P_3P_4$, its midpoints M_{ij} and one of its inner or outer van Aubel quadrangle (for example, 1st outer one for quadrigon $P_1P_2P_3P_4$), we denote the vertices of 1st outer van Aubel quadrangle by T_{12R} , T_{23R} , T_{34R} and T_{14L} . The midpoints M_{31} is transformed to M_{24} by CSC conjugate and vice versa. See the attached Cabri picture for the notation.

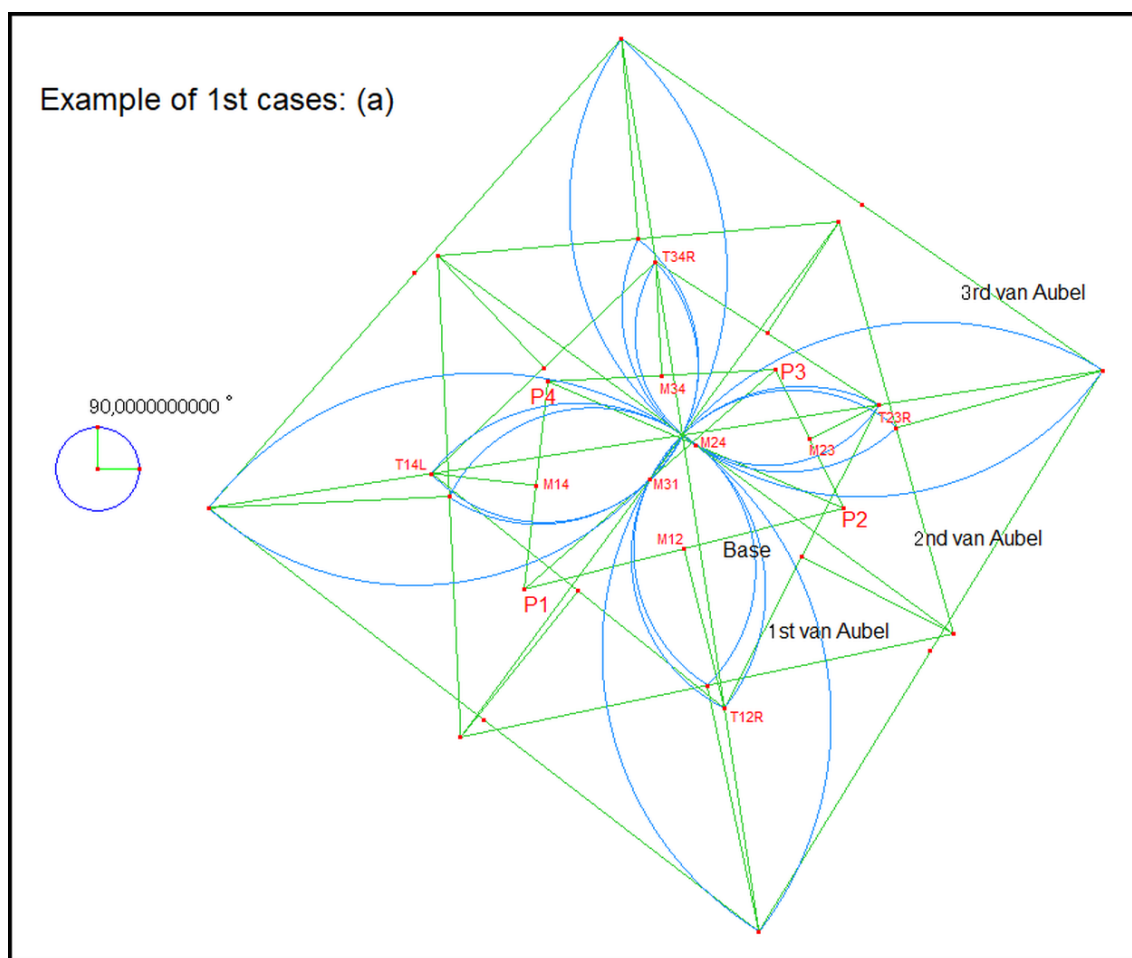
This is an interpretation of message #116 with CSC conjugate.

I am seeking for van Aubel quadrangle of 2nd kind whose CSC conjugate transform, for example, N_{13} to N_{24} . See message #149.

I hope any suggestions.

Best regards,

Seiichi.



Aubel-SK-01a-fig.png

Message: #448

Date: 12/2/2014 4:54:05

From: Seiichi Kirikami

Subject: Graphical confirmation of van Aubel quadrangle of 2nd kind

Dear friends,

I confirmed the existence of van Aubel quadrangle of 2nd kind graphically.

Given a reference quadrangle $P_1P_2P_3P_4$ and the isogonal center(QAP_4), we denote the pedal points of QAP_4 on P_iP_j by N_{ij} .

(1) I made an arbitrary reference QA and transformed it to a cyclic one with the help of Ptolemy's theorem. Then N_{ij} coincides with the midpoint M_{ij} of the reference QA.

(2) As the reference QA is cyclic, it is natural that the vertices of van Aubel QA of 2nd kind coincide with those of van Aubel QA of 1st kind (usual one).

(3) I assume that the vertices P_{ij} of van Aubel QA of 2nd kind lie on the lines perpendicular to P_iP_j through N_{ij} .

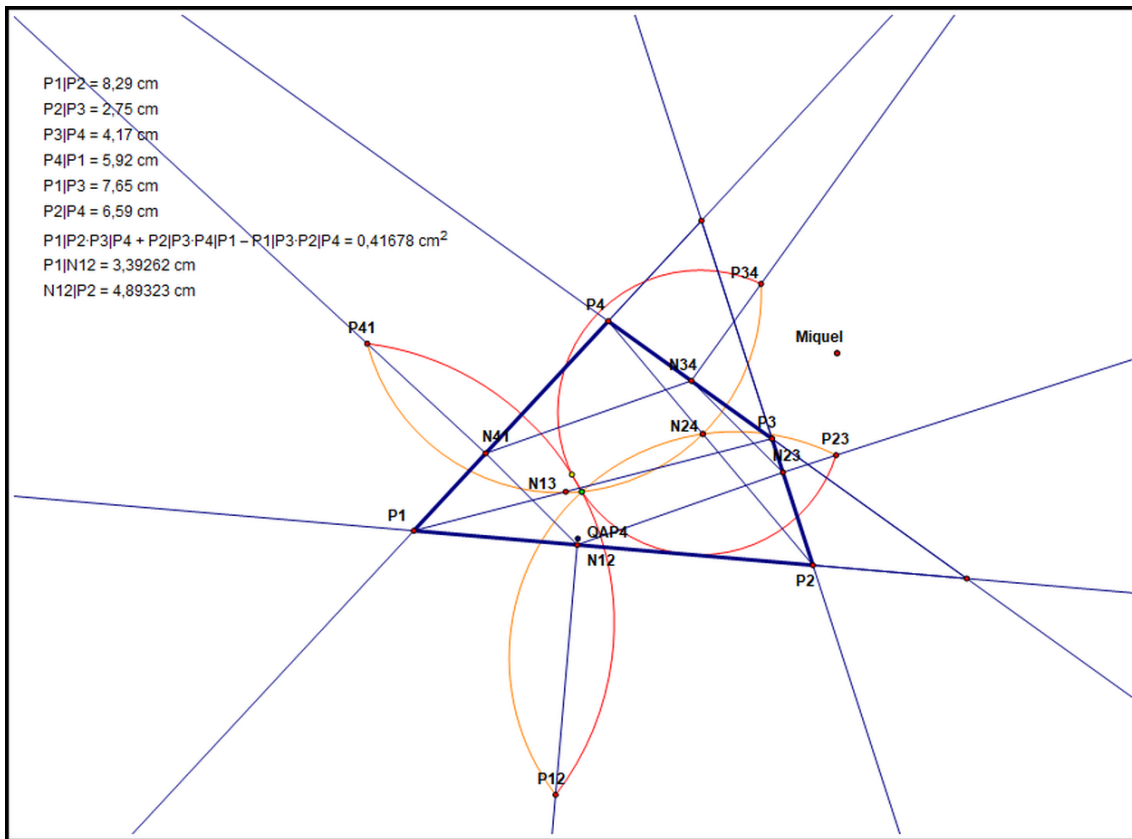
(4) The reference QA become not cyclic by the translation of one of its vertices.

(5) The intersection of arc $P_{12}N_{24}P_{23}$ and arc $P_{34}N_{24}P_{41}$ is indicated by the green point (GP). The intersection of $P_{12}GPP_{41}$ and $P_{23}GPP_{34}$ is indicated by the yellow point (YP). The latter is easily coincidable with N_{13} by the translation of P_{ij} on the lines perpendicular to P_iP_j through N_{ij} .

(6) Conclusion: I think that this shows the existence of van Aubel quadrangle of 2nd kind, though it is not unique.

See the attached gsp.

Best regards,
Seiichi.



vanAubel(2nd)-gsp.png

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Message: #449
Date: 12/2/2014 11:19:22
From: bernard.keizer
Subject: Orthocentric Pedal Quadrangles

Dear Eckart, dear Chris

This is surely a minor property, but ...

As A and A' are CLS conjugates and X and Y are the conjugates of the Plücker points, the 3 circles CS conjugates of the circles with diameters AA' , BB' and CC' pass through A , A' , X and Y , B , B' , X and Y and C , C' , X and Y and are coaxial with XY as radical axis, the centers being on the perpendicular bisector of XY through P_4 , which is parallel to $P_1CSC(P_5)$.

Best regards

Bernard K

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Message: #450

Date: 12/2/2014 4:12:36

From: eckart_schmidt@t-online.de

Subject: a new (?) simple construction of QL-Cu1

Dear Bernard K.,

I had difficulties to understand your constructions, but I think, background are the following facts:

Circles, centered on the 1st Steiner Axis
perpendicular to the Schmidt Circle and
circles, centered on the 2nd Steiner Axis through F1, F2
... are CSC-invariant,
... intersect perpendicular in CSC-partners X, X'
... and the line XX' is the angle bisector at the middle m
of X.X' wrt F1, F2.

You describe three possibilities, to get new curves wrt a reference curve:

1. X --> X':
Example: QL-Co1 --> QL-Qu1 (as you mentioned).
2. m --> X, X':
Example: Line QG-P7.QG-P9 --> Cubic
with $XP1 \times XP3 = XP2 \times XP4$ (see message #414)
3. XX' --> m for CSC-invariant curves:
Example: QL-Cu1 --> QL-L1.

For a quadrigon the third possibility is of interest for the reference Cubic QA-Cu1.

QA-Cu1 contains several CSC-partners out of EQF:

- * opposite vertices,
- * intersections of opposite lines,
- * the vertices of the Miquel Triangle unequal QL-P1,
(QG-P1, QA-P4), (QG-P17, QG-P18), (QG-P16, QA-P3).

The midpoints lie on a new cubic ...

I hope my summary is in your sence.

Best regards Eckart

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Message: #451

Date: 12/2/2014 10:26:31

From: bernard.keizer

Subject: a new (?) simple construction of QL-Cu1

Dear Eckart,

I must apologise

I understand you had difficulties with my construction, as I realise I made a mistake in my message: in the construction b) I and J are on the 2nd Steiner axis and not the 1st!

Then Im cuts the circle with center J through F1 and F2 in 2 points and Jm cuts the circle with center I through F1 and F2 in 2 other points ...

(I draw for example 2 Cu with m describing the Newton Line and the Steiner Line and the 2 cubics cut in 3 points, the 2 CL-conjugates for m being in the Newton-Steiner point P7 and in P1)

In the 3 possibilities, 1 and 2 are what I thought, but 3 is possible for any curve, not only for CSC- invariant curves: for example, if X describes QL-C01 and the conjugate X' describes at the same time QL-Cu1, the middle m describes an amusing curve with a nodus. The construction makes easy to draw such curves with Cabri (or Geogebra for me ...)

Best regards

Bernard K

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Message: #452
Date: 13/2/2014 9:41:47
From: eckart_schmidt@t-online.de
Subject: CSC in QA-environment

Dear friends,

the Clawson-Schmidt Conjugate QL-Tf1 - shortened CSC - is a transformation for quadrilaterals, but it has also applications for quadrangles, considering the three quadrigon components. The three CSC-versions give

- ... for a QA-vertice the other vertices,
- ... for QA-P4 the vertices of the QA-diagonal triangle,
- ... for the incenter of the Miquel triangle the excenters.

The cubic QA-Cu1 is an isogonal pivotal circular isocubic wrt the Miquel triangle M1M2M3 and pivot in the point at infinity of QA-L4.

- ... QA-Cu1 is CSC-invariant wrt the quadrigon components (see message #401).

Now let X be a point on the cubic QA-Cu1 and Y_i its CSC-images (wrt Miquel point M_i).

- ... The circles through X , M_i , Y_i have a second common point T on QA-Cu1.
- ... The point T is the common tangential point of X and Y_i .
- ... The CSC-images of T are the 3rd intersections of XY_i and QA-Cu1.
- ... The lines M_iY_i intersect on the cubic in the isogonal conjugate of X .

Best regards Eckart

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Message: #453
Date: 16/2/2014 10:00:54
From: eckart_schmidt@t-online.de
Subject: CSCe-parabolas

Dear Chris, dear Bernard K.,

you have defined $\text{CSCe}(L)$ as CSC of the reflection of $QL-P1$ in L .
Let L be tangents to a circle through $QL-P1$, centered in an arbitrary point M :

- ... $\text{CSCe}(L)$ give a parabola
- ... with focus $QL-P1$
- ... and axis $QL-P1.CSC(M)$
- ... and directrix CSC-image of the circle,
that is the perpendicular bisector of $QL-P1.CSC(M)$.

Example: $M=QL-P4$ (midpoint of the Miquel Circle $QL-Ci3$):
Inscribed Parabola $QL-Co1$.

- ... For $M=QL-P2, QL-P7, QL-P9$ the directrices intersect in $QL-P4$.
- ... For $M=QL-P6$ the directrix contains $QL-P26$ and for $QL-P26$ the directrix contains $QL-P6$.
- ... For circle centers M on a line g the directrices have a common point $\text{CSCe}(g)$.
- ... For circle centers M on a line g the intersections of directrix and axis lie on the Thales circle about $QL-P1.CSCe(g)$.
- ... The CSC-images of these parabolas are cardioids with cusp $QL-P1$ and center M (example $QL-Qu1$).

Let L be tangents to an arbitrary circle, centered in a point M :

- ... $\text{CSCe}(L)$ give a conic
- ... with one focus $QL-P1$
- ... and axis $QL-P1.CSC(M)$
- ... and second focus in CSC of the reflection of $QL-P1$ in the circle.

You can get the same results, if you start with points P on the circle and consider the envelope of the perpendicular bisectors of $QL-P1.CSC(P)$.

Best regards Eckart

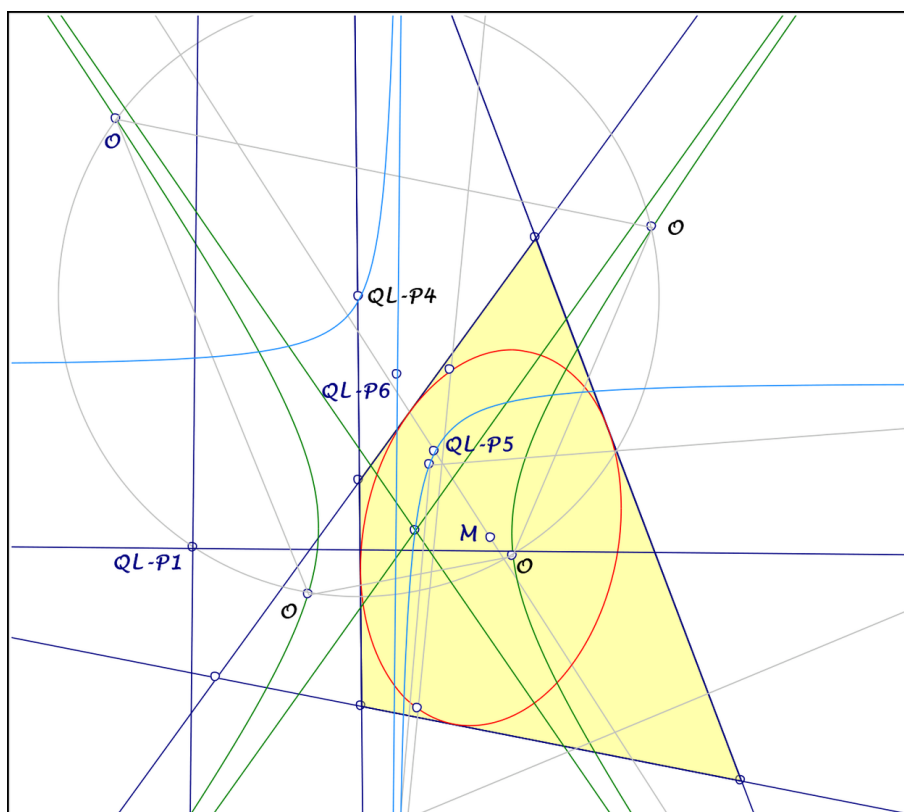
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Message: #454
Date: 17/2/2014 9:12:31
From: eckart_schmidt@t-online.de
Subject: CSCe-parabolas

Dear Chris, dear Bernard K.,
here is a further generalisation of the CSCe-transformation for lines (see attachment):
Let L be tangents to an inscribed conic Co , centered in M on the Newton line.

- ... CSCe(L) give a conic
- ... through the circumcenters O_i of the QL-triangle components
- ... with center CSCe(polar of QL-P1 wrt Co)
- ... and centered on an orthogonal hyperbola with center QL-P6 through QL-P4 and QL-P5, circumscribed the diagonal triangle of the O -quadrangle with asymptotes parallel to the Steiner axes.

Best regards Eckart



2014-02-15.pdf

Message: #455
Date: 17/2/2014 3:20:07
From: bernard.keizer
Subject: CSce-parabolas

Dear Eckart, dear Chris
Wunderbar!
Beautiful constructions, indeed ...
Best regards
Bernard
(By the way,
I know now what CSC(Qu2) looks like, it's rather strange ...)

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Message: #456
Date: 18/2/2014 10:53:35
From: eckart_schmidt@t-online.de
Subject: CSce-parabolas

Dear Bernard K, dear Chris,
in addition to my messages #453, #454 a further aspect of the CSce-transformation for lines:
Let L be tangents to an arbitrary conic through QL-P1, then CSce(L) gives a parabola.
Two interesting examples:
Let Co be an inscribed QL-conic through QL-P1. There are two possibilities: the tangent in QL-P1 can be the 1st or 2nd Steiner Axis.
Let L be tangents to the conic Co :
... CSC(L) gives a parabola
... through the circumcenters O_i of the QL-triangle components
... with axis through QL-P6 perpendicular to the corresponding Steiner axis.
Best regards Eckart
PS: Thanks to Bernard for his positive feedback!

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Message: #457
Date: 19/2/2014 10:07:47
From: bernard.keizer
Subject: Dimidium circle and Lemoine point: P6, P13, P17 , P24 and P26 revis

Dear Chris, dear Eckart
Here are, as I earlier promised, some (new I hope) developments about the Dimidium circle and the Lemoine point. I found the construction of P26 in Léon Ripert Notes sur le quadrilatère 1901; the rest, in particular the D-points on the Dimidium circle and the link with the conics through the S-points, is my own deduction ...
Best regards
Bernard

Dimidium circle and Lemoine point : P6, P13, P17, P24 and P26 revisited

A. Dimidium circle

(cf figure1 Dimidium circle)

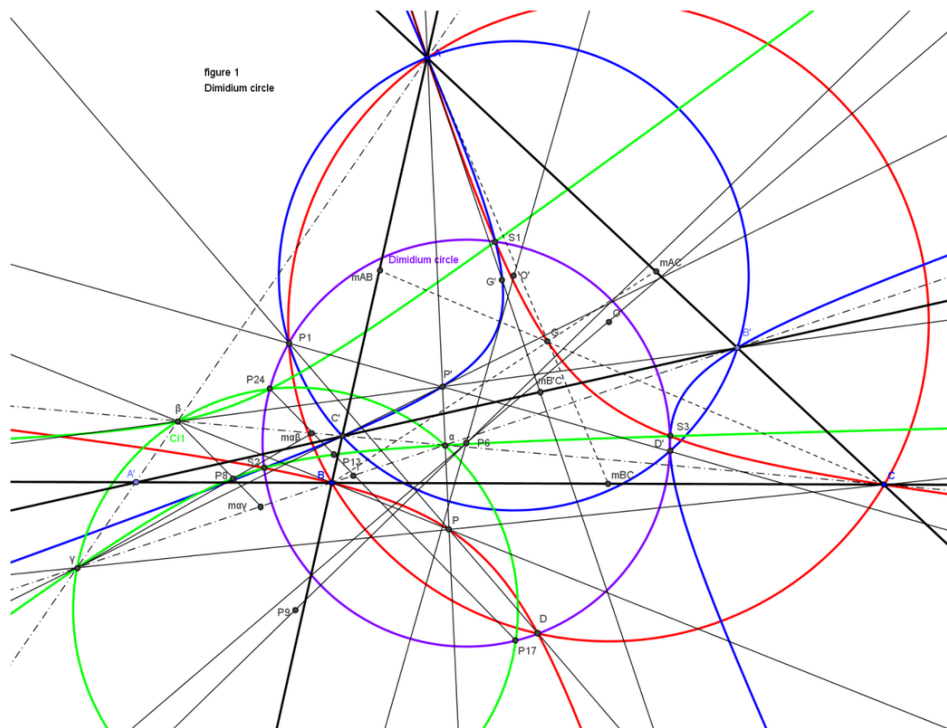
For the reference triangle ABC with centroid G and circumcenter O, let P be the pole of the 4th line of equation $lx + my + nz = 0$; P has coordinates $1/l, 1/m, 1/n$ and is the perspector of the triangle ABC and the diagonal triangle $\alpha\beta\gamma$ with circumcenter P9 and centroid P8.

The isotomic conjugate of P, IsoP, has coordinates l, m, n and its polar is the line with equation $x/l + y/m + z/n = 0$, which is parallel to the Newton Line L1.

Let D be the second intersection of the line P1P with the circumcircle of ABC ; D has coordinates $[a^2(l-n) + b^2(m-n)][a^2(l-m) + c^2(n-m)]$.

D belongs to the line P1P, the circumcircle of ABC, the Dimidium circle and the conic through A, B, C, G and P, with equation $(m-n)yz + (n-l)xz + (l-m)xy = 0$; D is the reflexion of P1 in P6O and is the 4th intersection between the Dimidium circle and the conic.

There are 4 conics like this and 4 points D like this : they are all on the Dimidium circle as 4th intersection with the 4 conics, the 3 others being the 3 well-known S-points and P6 is on the 4 perpendicular bisectors of the lines P1P.



Dimidium and Lemoine.docx

It's particularly interesting that the same property goes for the diagonal triangle : P17 is the Miquel point of the diagonal QL, formed by the 3 diagonals and the Newton Line, P13 is the pole of the Newton Line and P24 is the second intersection of P17P13 with the circumcircle of the diagonal triangle $\alpha\beta\gamma$ Ci1.

P24 belongs to the line P17P13, the circumcircle Ci1, the Dimidium circle and the conic through α , β , γ , P8 and P13.

P24 is on the Dimidium circle as 4th intersection with the conic, the 3 others being the 3 well-known S-points and P6 is on the perpendicular bisector of the line P17P24.

On this way, P6 is stronger than P3, as the Kantor-Hervey points belongs to 4 lines (the perpendicular bisectors of the Euler segments) whereas the point P6 belongs to 5 lines (the perpendicular bisectors of the lines P1P and P17P13).

As we already know (cf my message 164 on diagonal quadrilaterals), the sides of the DT-QL are the diagonals of the QL and it's 4th line is the Newton Line of the QL. The equation of the Newton Line of the QL is the same as the equation of the 4th line in CT-coordinates with l^2 , m^2 and n^2 instead of l , m and n ; the DT-coordinates of P17, P13 and P24 are the same as the CT-coordinates of P1, P and D and the equation of the conic associated to the DT-QL is the same as the CT-coordinates of the conic associated to the QL with the same modification.

B. Lemoine point

(cf figure2 Lemoine point)

The centroid G and the Lemoine point K of ABC have respectively coordinates $(1,1,1)$ and (a^2, b^2, c^2) ; the centroid G' and the Lemoine point K' of $AB'C'$ have respectively coordinates $[-, l(n-l), l(m-l)]$ and $[-, b^2/l(n-l), c^2/l(m-l)]$.

The equation of the symmedian of $AB'C'$ in A is $(n-l)y/b^2 + (l-m)z/c^2 = 0$.

The symmedian of $AB'C'$ in A , of $A'BC'$ in B and of $A'B'C$ in C cut respectively BC , AC and AB in 3 points Q , R and S ; these 3 points are on a line of equation $(m-n)x/a^2 + (n-l)y/b^2 + (l-m)z/c^2 = 0$, which contains the point K , the point CoP , isogonal conjugate of P , of coordinates a^2l, b^2m, c^2n and the point CoD , isogonal conjugate of D , as infinity point.

The point $P26$ or Lemoine point of the QL is the intersection of the 4 lines $KCoP$ (if the 4th line is through K , the point $P26$ is in K , as the distance to the 4th line is 0 and the point which minimise the sum of the squares of the distances to the 3 sides of ABC is precisely the Lemoine point K of the triangle).

The lines $KCoP$ are the isogonal conjugates of the 4 conics we met in the 1st part.



Dimidium and Lemoine.docx

Message: #458
Date: 20/2/2014 12:24:44
From: eckart_schmidt@t-online.de
Subject: Dimidium circle and Lemoine point

Dear Bernard K.,
I have studied your paper about "Dimidium circle and Lemoine point". It was interesting for me, to work with the reference triangle $L_1L_2L_3$ as usual in publications, for I prefer the QL-diagonal triangle.
One question to the last section: Is it really true, that the lines $G_{Iso}P$ intersect in a point? I cannot verify this property with CABRI.
Best regards Eckart

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Message: #459

Date: 20/2/2014 4:08:53

From: eckart_schmidt@t-online.de

Subject: Two points on the Newton Line @ QL-Cu1 revisited

Dear Chris, dear friends,

if QL-Cu1 is unipartite, QL-2P2 are the foci of an inscribed QL-conic centered on the Newton Line (see EQF).

Here is a further property of these points:

QL-2P2 are points X, which are the center of a conic, containing the pedal points of X.

The locus for points X with this property is a sextic with CT-equation in the attachment.

The sextic contains the QL-points and QL-P1 with degenerated conics. The QL-points seems to be nodal points ...

I am sure, Chris will find further properties.

Best regards Eckart

*Locus for points X, which are the center of a conic,
containing the pedal points of X wrt the QL-sides.
CT-coordinates*

```
sext[{x_, y_, z_}] := a^4 (1 - m) (n - 1) y^2 z^2 (m y + n z) (2 l x + m y + n z)
+ b^4 (1 - m) (m - n) x^2 z^2 (l x + n z) (l x + 2 m y + n z)
+ c^4 (n - 1) (m - n) x^2 y^2 (l x + m y) (l x + m y + 2 n z)
+ (a^4 (1 - m) (n - 1) (l m + l n - m n)
+ b^4 (1 - m) (m - n) (l m - l n + m n)
+ c^4 (n - 1) (m - n) (-l m + l n + m n)
+ 2 (1 - m) (m - n) (n - 1) (a^2 b^2 (1 - m) + b^2 c^2 (m - n) + a^2 c^2 (n - 1))) x^2 y^2 z^2 = 0
```

Eckart Schmidt

eckart_schmidt@t-online.de

<http://eckartschmidt.de>

Chris-2014-02-20-nb.png

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Message: #460
Date: 20/2/2014 6:15:20
From: bernard.keizer
Subject: Dimidium circle and Lemoine point

Dear Eckart,
Thanks for your quick answer
I didn't check the property with Geogebra
It's purely conjectural for me, but it seems true: if K, CoP and P26 are colinear for each of the 4 triangles (this property is proved and checked with Geogebra), then IsoCoK, IsoCoCoP and IsoCoP26 must be colinear. IsoCoK is IsoG and G itself, IsoCoCoP is IsoP And IsoCoP26 is the point with the coordinates of P26 divided respectively by a2, b2 and c2; the calculation of the colinearity is immediate, as it is the same as for P26,
I hope you will find the solution and tell me soon about this new point...
Best regards
Bernard

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Message: #461
Date: 20/2/2014 9:14:26
From: chris.vantienhoven
Subject: Two points on the Newton Line @ QL-Cu1 revisited

Dear Eckart, dear friends,
I am sorry that I wasn't able to react on the last very interesting messages.
I am going through a very very busy period and hardly have time for geometry.
Eckart, regarding your sextic I noticed that 3 terms are of 4th degree (like $x^2 y^2$) and term is of 6th degree ($x^2 y^2 z^2$).
Maybe you can check this. Or did I overlook something?
Best regards,
Chris

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Message: #462

Date: 20/2/2014 9:35:48

From: chris.vantienhoven

Subject: Two points on the Newton Line @ QL-Cu1 revisited

Dear Eckart,

Indeed I overlooked something: The sextic is overall of 6th degree.

I checked if I could find other points than QL-P1 and QL-2P2a, QL-2P2b.

All I found is that the circular points at infinity lie on the sextic. This makes it a circular sextic.

Best regards,

Chris

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Message: #463

Date: 21/2/2014 1:36:48

From: bernard.keizer

Subject: Dimidium circle and Lemoine point

Dear Eckart,

I tried this morning to draw the figures with Geogebra.

In fact, you are right and I must apologise, the 4 points IsoCoP26 are colinear with G and IsoP for each reference triangle, but they are not the same point!

P26 is colinear with K and CoP for each reference triangle and the 4 points are the same.

By the way, are there other points than P26 which belong the property that the points with the same coordinates in the 4 reference triangles are the same point? It's an amazing property, isn't it?

May be Chris could answer this question?

Best regards

Bernard

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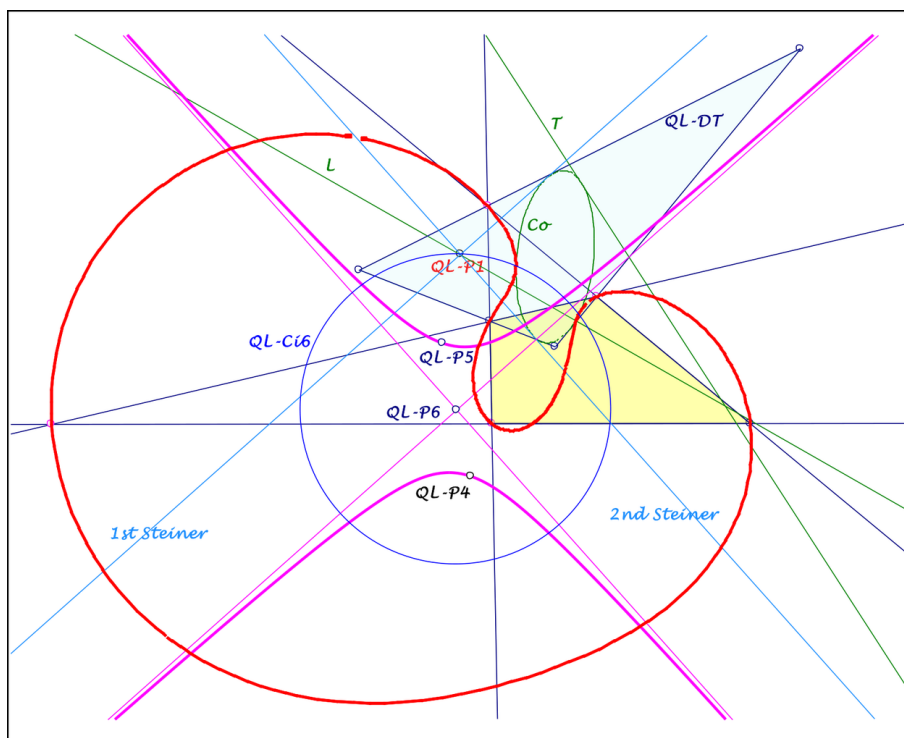
Message: #464
Date: 21/2/2014 5:52:39
From: bernard.keizer
Subject: Dimidium circle and Lemoine point

Dear Eckart,
I'm sorry again, the idea was perhaps right, but the formulation is wrong!
All the EQF points have the same property of course.
The question is rather following: what makes the difference between local points, same coordinates in each reference triangle, but different points and QL-points, same local definition and coordinates and same point in the end. For example, the Kantor-Hervey point P3 is the intersection of the 4 bisectors of the Euler segments or the Lemoine point P26 is the intersection of the 4 lines KCoP, but IsoCoP, as we saw, is on every local line GIsOP, but it's not the same point ...
Are there obvious criteria for good points and bad points?
I'd rather stop here for a while, but I would appreciate your comments and those of Chris.
Thanks in advance
Best regards
Bernard

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Message: #465
Date: 22/2/2014 1:27:38
From: eckart_schmidt@t-online.de
Subject: CSCe-parabolas

Dear friends,
there is a further interesting example for the line transformation $\text{CSCe}(L)$ (see attachment):
Consider lines L through the Miquel Point $QL-P1$:
... The QL -Line Isoconjugate ($QL-Tf2$) of L
envelope a QL -DT-inscribed conic Co
tangent to the Steiner Axes and $QL-L2$.
... For tangents T at this conic Co $\text{CSCe}(T)$ gives an orthogonal
hyperbola with center $QL-P6$ through $QL-P4$ and $QL-P5$,
circumscribed the diagonal triangle of the O -quadrangle
with asymptotes parallel to the Steiner axes
(see message #454).
... The intersections of T and $\text{CSC}(T)$ give the quartic
described in message #364.
... The conic Co is tangent to the quartic at the intersections
with the Dimidium Circle $QL-Ci6$.
Best regards Eckart



2014-02-22.pdf

Message: #466

Date: 27/2/2014 4:10:58

From: eckart_schmidt@t-online.de

Subject: Nodal QL-Cubic wrt the Miquel Point

Dear friends,

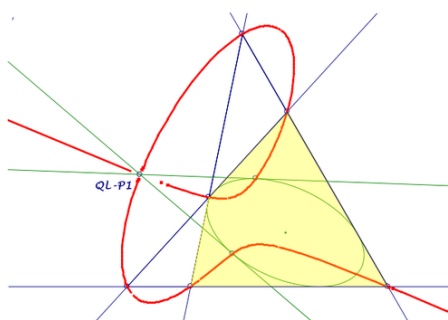
if you consider a quadrilateral and the tangents from the Miquel Point to inscribed conics, the points of tangency will give a cubic, containing the six points of the quadrilateral and the Miquel Point as nodal point. The cubic is a conico-pivotal isocubic (see attachment).

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Nodal QL -Cubic wrt the Miquel Point

If we consider a quadrilateral and the tangents from the Miquel Point to inscribed conics, the points of tangency will give a cubic, containing the six points of the quadrilateral and the Miquel Point as nodal point. The cubic is a conico-pivotal isocubic.



Let the lines L_1, L_2, L_3, L_4 define a quadrilateral with a reference triangle L_1, L_2, L_3 for barycentric coordinates and a fourth line $L_4(l, m, n)$.

We consider inscribed conics of the quadrilateral, centered on the Newton Line, and the tangents from the Miquel Point $QL-P1$ to these conics. The points of tangency give a cubic, which shall be discussed here.

Properties:

- The equation of the cubic:

$$\begin{aligned}
 &2lmnLMN(a^2b^2N + a^2c^2M + b^2c^2L)xyz \\
 &+ a^4mnM^2N^2(my + nz)yz \\
 &+ b^4nlN^2L^2(nz + lx)zx \\
 &+ c^4lmL^2M^2(lx + my)xy = 0 \\
 &\text{with } L = m - n, M = n - l, N = l - m.
 \end{aligned}$$

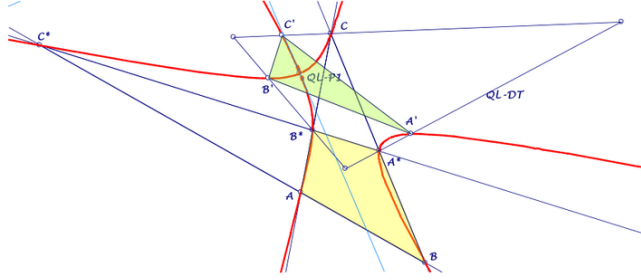
- The cubic contains the six intersections of the four lines of the quadrilateral.
- The cubic contains the Miquel Point $QL-P1$.
- The Miquel Point $QL-P1$ is a nodal point with orthogonal tangents.
- The tangents at the Miquel Point are the Steiner Axes (see $QL-Tf1$).

- The cubic contains the vertices of the Ceva triangle $Tr = A'B'C'$ of $QL-P1$ wrt the QL -Diagonal Triangle $QL-DT$.

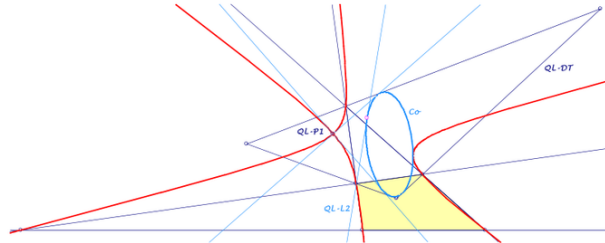
$$A' \left(\frac{2a^2 MN + b^2 LN + c^2 LM}{(c^2 M - b^2 N) L}, -\frac{1}{m}, \frac{1}{n} \right)$$

$$B' \left(\frac{1}{l}, \frac{a^2 MN + 2b^2 LN + c^2 LM}{(a^2 N - c^2 L) m}, -\frac{1}{n} \right)$$

$$C' \left(-\frac{1}{l}, \frac{1}{m}, \frac{a^2 MN + b^2 LN + 2c^2 LM}{(b^2 L - a^2 M) n} \right)$$



- The cubic is invariant under an isoconjugation * wrt the reference triangle $Tr = A'B'C'$ and the fixed point $QL-P1$.
- The isoconjugation * swaps the points of tangency for tangents from $QL-P1$ to inscribed conics.
- The isoconjugation * swaps opposite points of the quadrilateral.
- The polars of $QL-P1$ wrt inscribed conics are tangents of an inscribed conic Co of the QL -Diagonal Triangle $QL-DT$.



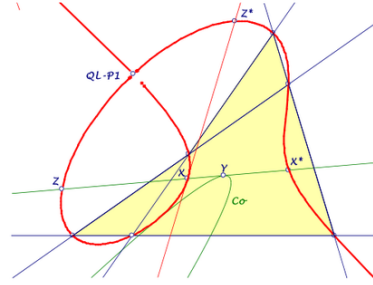
- Co is also the envelope of the $QL-Tf2$ images of lines through $QL-P1$.
- Equation of Co :

$$\begin{aligned} & l^2 L^2 (b^2 N - c^2 M)^2 x^2 + m^2 M^2 (a^2 N - c^2 L)^2 y^2 + n^2 N^2 (a^2 M - b^2 L)^2 z^2 \\ & - 2 l m L M (a^2 b^2 N^2 + 3 b^2 c^2 N L + 3 a^2 c^2 N M + c^4 M L) x y \\ & - 2 l n L N (3 a^2 b^2 M N + 3 b^2 c^2 L M + a^2 c^2 M^2 + b^4 L N) x z \\ & - 2 m n M N (3 a^2 b^2 L N + b^2 c^2 L^2 + 3 a^2 c^2 L M + a^4 M N) y z = 0 \end{aligned}$$

- Special tangents of Co are the Steiner Axes (see above) and the Steiner Line $QL-L2$. Point of tangency wrt $QL-L2$:

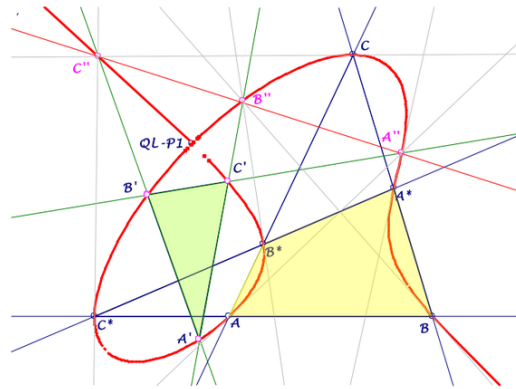
$$\begin{aligned} & (m n M N (a^2 (b^2 N + c^2 M) + 2 S_b S_c L), \\ & n l N L (b^2 (a^2 N + c^2 L) + 2 S_a S_c M), \\ & l m L M (c^2 (a^2 M + b^2 L) + 2 S_a S_b N)) \end{aligned}$$

- If the tangents from $QL-P1$ to the cubic contact in X and X^* and XX^* is tangent in Y to the conic Co , then the fourth harmonic point Z is the third intersection of XX^* with the cubic.



- Two isoconjugate points X and X^* on the cubic have the same tangential, which is the isoconjugate Z^* of the third intersection of XX^* with the cubic.
- The third intersections A'', B'', C'' of the sidelines of the triangle $Tr = A'B'C'$ with the cubic are collinear:

$$\begin{aligned} A'' & \left(\frac{a^4 m n M^2 N^2 (-c^2 M + b^2 N)}{2 a^2 N M + b^2 N L + c^2 M L}, b^4 n l N^2 L, -c^4 l m L M^2 \right) \\ B'' & \left(-a^4 m n M N^2, \frac{b^4 n l N^2 L^2 (-a^2 N + c^2 L)}{a^2 N M + 2 b^2 N L + c^2 M L}, c^4 l m L^2 M \right) \\ C'' & \left(a^4 m n M^2 N, -b^4 n l N L^2, \frac{c^4 l m L^2 M^2 (-b^2 L + a^2 M)}{a^2 N M + b^2 N L + 2 c^2 M L} \right) \end{aligned}$$



- The points A'', B'', C'' are the tangentials of opposite points of the quadrilateral.
- These properties show, that the cubic is a nonpivotal isocubic; more precisely: The cubic is a conico-pivotal isocubic cK with pivotal-conic Co (see *B. Gibert: Special Isocubics*, 1.5 and 8.1).
- Finally: The cubic has one inflection point; its isoconjugate gives a point of tangency between the cubic and its pivotal-conic Co .

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2014-02-27.pdf

Message: #467
Date: 28/2/2014 2:59:10
From: eckart_schmidt@t-online.de
Subject: Nodal QL-Cubic wrt the Miquel Point

Dear friends,
in addition to my last message:
The cubic is the locus for the intersections of lines through
QL-P1 and their QL-Tf2-image.
Best regards Eckart

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Message: #468
Date: 28/2/2014 10:59:28
From: chris.vantienhoven
Subject: Nodal QL-Cubic wrt the Miquel Point

Dear Eckart,
Very nice cubic with very special properties!
I wonder what the coordinates of the inflexion point are.
Best regards,
Chris

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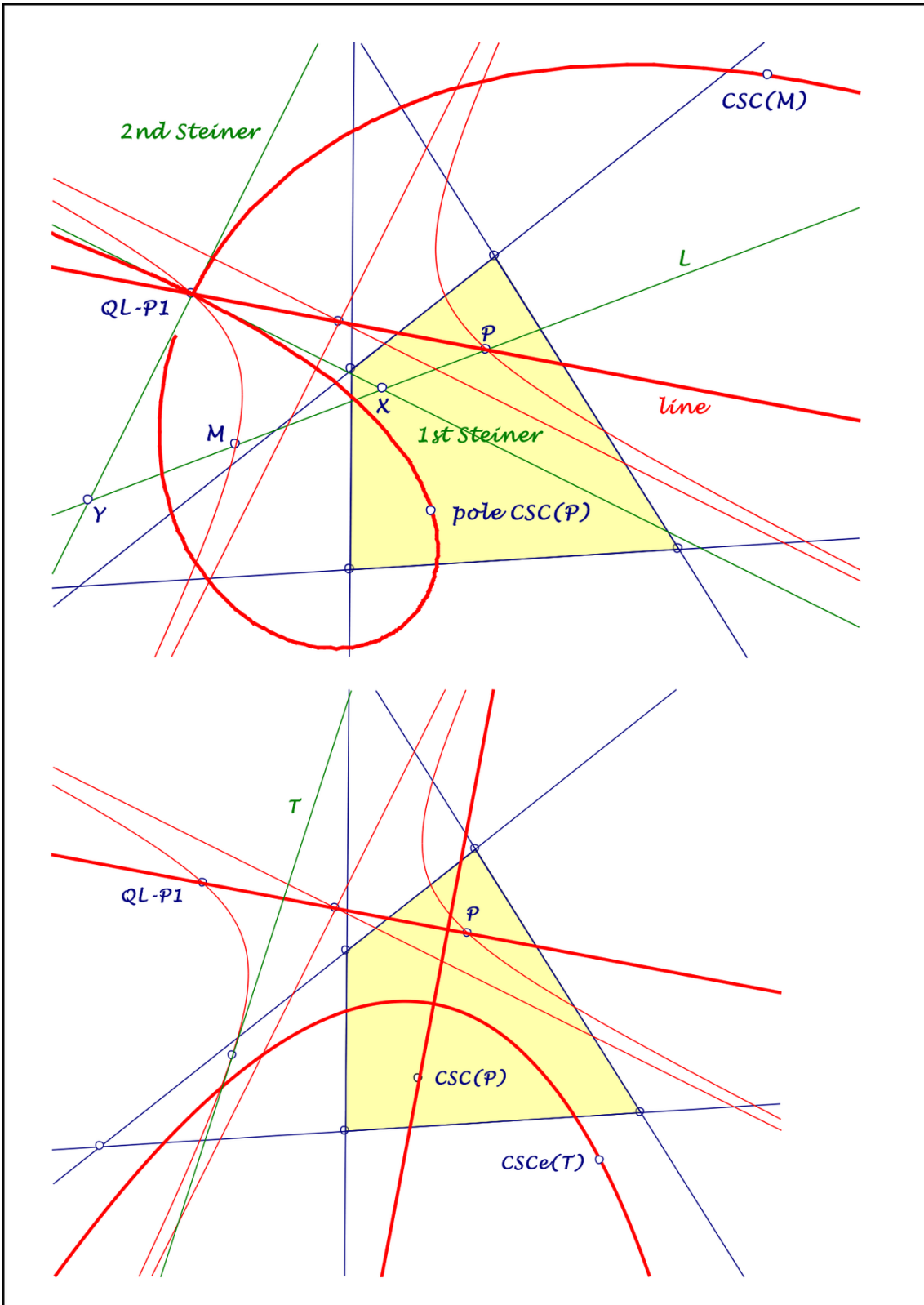
Message: #469
Date: 01/3/2014 10:00:55
From: eckart_schmidt@t-online.de
Subject: Nodal QL-Cubic wrt the Miquel Point

Dear Chris,
to calculate the inflexion point, it is necessary to find the
real solutions of equations of degree three. Background for this
property is a footnote in B. Gibert: Special Isocubics, 8.1.
Best regards Eckart

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Message: #470
Date: 01/3/2014 3:38:55
From: eckart_schmidt@t-online.de
Subject: Strophoids wrt the Miquel Point

Dear friends,
take the line pencil for a point P with lines L , consider the intersections X and Y of L with the Steiner Axes (see QL-Tf1): The locus of the midpoints M of X, Y is an orthogonal hyperbola through the Miquel Point $QL-P1$ with asymptotes parallel to the Steiner Axes and centered in the midpoint of $P.QL-P1$. The Clawson-Schmidt Conjugate QL-Tf1 of this hyperbola gives a strophoid of the line $P.QL-P1$ with pole $CSC(P)$ and fixed point $QL-P1$ (see attachment). The Steiner Axes are tangents in $QL-P1$. For points P on the angle bisectors of the Steiner Axes the strophoids are symmetric.
Let T be tangents to the hyperbola and $CSCe(T)$ the CSC- image of the reflection of $QL-P1$ in T : $CSCe(T)$ gives a parabola (see message #456) with focus $CSC(P)$ and directrix $P.QL-P1$ (see attachment).
Best regards Eckart



2014-03-01.pdf

Message: #471
Date: 04/3/2014 12:12:06
From: eckart_schmidt@t-online.de
Subject: A Network of QL-Transformations

Dear friends,
perhaps of interest: An excursion in QL-transformations, showing
somewhat confusing interactions (see attachment).
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

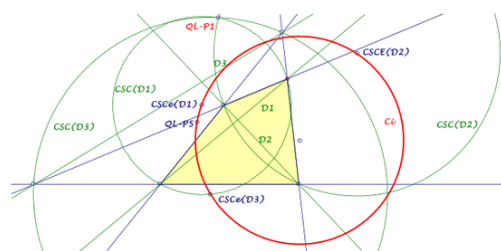
A Network of QL-Transformations

CSC is a shortcut for the transformation QL-Tf1 (Clawson-Schmidt Conjugate). CSCe is a transformation, giving for a line the CSC-image of the reflection of the Miquel Point QL-P1 in the line (see QFG messages 435, 436). Here are gathered some relationships to the QL-Line Conjugate QL-Tf2.

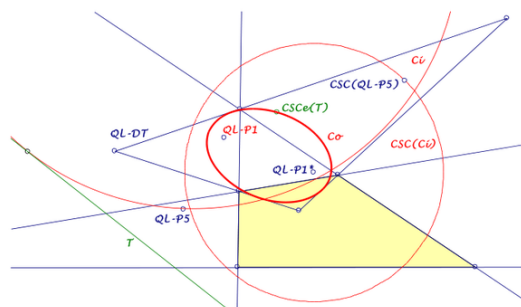
CSC is a point-to-point transformation, which swaps opposite points of a quadrilateral. Lines will give circles through the Miquel Point QL-P1. The four lines of a quadrilateral give circumcircles of the triangle components.

CSCe is a line-to-point transformation. For a line of the quadrilateral CSCe gives the circumcenter of the remaining triangle component (on the Miquel Circle QL-Ci3). A line pencil wrt a point gives a line, whose CSCe-image is the point again.

QL-Tf2 is a line-to-line transformation with the lines of the quadrilateral as fixed lines.

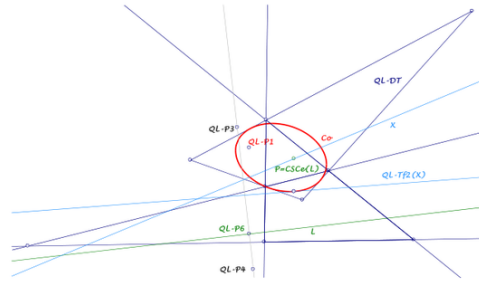


- The CSCe-images of the QL-diagonals lie on a circle Ci through the Clawson Center QL-P5. (These points are the circumcenters for the CSC-image-circles of the diagonals.)



- $CSC(Ci)$ gives a circle through $CSC(QL-P5)$ centered in the isogonal conjugate $QL-P1^*$ of the Miquel Point wrt $QL-DT$.
- $CSC(QL-P1^*)$ is the reflection of $QL-P1$ in Ci .
- The $CSCe$ -images of tangents T at the circle Ci give an inscribed conic Co of $QL-DT$ with foci $QL-P1$ and $QL-P1^*$.
- The conic Co is also the envelope of the perpendicular bisectors of $QL-P1$ and points of $CSC(Ci)$.
- The equation of Co in DT -coordinates:

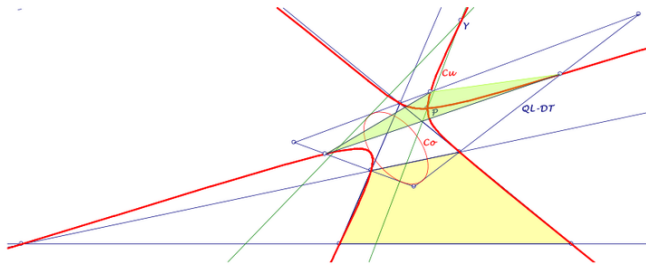
$$\begin{aligned}
&U^2 x^2 + V^2 y^2 + W^2 z^2 - 2UVxy - 2VWyz - 2WUzx = 0, \\
&U = L \quad (-L Sa + M Sb + N Sc) \\
&(L^2 Sa^2 + M^2 Sb^2 + N^2 Sc^2 + 2(M-N) Sa (M Sb - N Sc) - 2MN Sb Sc), \\
&V = M \quad (L Sa - M Sb + N Sc) \\
&(L^2 Sa^2 + M^2 Sb^2 + N^2 Sc^2 + 2(N-L) Sb (N Sc - L Sa) - 2NL Sc Sa), \\
&W = N \quad (L Sa + M Sb - N Sc) \\
&(L^2 Sa^2 + M^2 Sb^2 + N^2 Sc^2 + 2(L-M) Sc (L Sa - M Sb) - 2LM Sa Sb), \\
&L = m^2 - n^2, \quad M = n^2 - l^2, \quad N = l^2 - m^2.
\end{aligned}$$



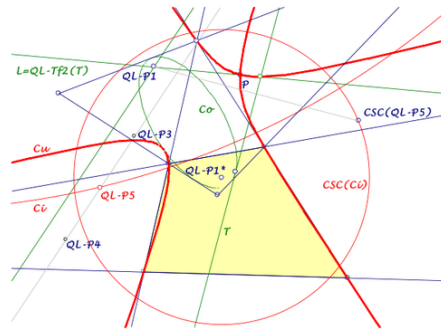
- The $QL-DT$ inscribed conic Co is also the envelope of the $QL-Tf2$ -images of lines through a point P , which is the $CSCe$ -image of a line through $QL-P6$ perpendicular to $QL-P3, QL-P4, QL-P5, QL-P6$.
- The DT -coordinates of point P :

$$\begin{aligned}
&\{m^2 n^2 L (c^2 M + b^2 N) (2 a^2 (b^2 + c^2) MN - a^4 NM + b^4 NL + c^4 LM), \\
&n^2 l^2 M (a^2 N + c^2 L) (2 b^2 (c^2 + a^2) NL + a^4 NM - b^4 NL + c^4 LM), \\
&l^2 m^2 N (b^2 L + a^2 M) (2 c^2 (a^2 + b^2) LM + a^4 NM + b^4 NL - c^4 LM)\}
\end{aligned}$$

- The conic Co is also the envelope of the polars of point P wrt inscribed conics of the quadrilateral.
- The intersections Y of lines L through P and their $QL-Tf2$ -image give a conico-pivotal isocubic Cu (see B. Gibert: Special Isocubics, 8.1):
 - ... reference triangle: Ceva triangle of P wrt $QL-DT$,
 - ... isoconjugation with fixed point P ,
 - ... pivotal conic Co .
 - ... If there are real tangents from P to Co , they are also tangent to the isocubic Cu and P is a nodal point.



- The isocubic Cu is also the locus for the contacts X and Y of tangents from P to inscribed conics of the quadrilateral; the isoconjugation of Cu swaps X and Y .



- History of point $QL-P5$: $QL-P5$ is a point on Ci ; the perpendicular bisector T of $QL-P1$ and $CSC(QL-P5)$ is tangent Co ; $QL-Tf2(T)$ is the line $L = P.QL-P1$; $CSC(L)$ is a parallel of $QL-P3.QL-P4.QL-P5.QL-P6$ through $QL-P1$ and the intersection of T and L is a point on the cubic Cu .

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2014-03-04.pdf

Message: #472
Date: 04/3/2014 4:28:03
From: eckart_schmidt@t-online.de
Subject: Some "splitter" VI

Dear Chris,

perhaps some properties will be already mentioned or evident,
then excuse!

(1) The midpoints of isogonal conjugated QA-Cu1-points wrt the Miquel Triangle (e.g. the midpoint of QA-P3 and QA-P4) lie on an orthogonal hyperbola. One asymptote is the QA-Cu1-asymptote, the other passes QA-P9. The center lies on the circumcircle of the Miquel Triangle in the second intersection with the QA-Cu1-asymptote (first intersection is the common point of QA-Cu1 and its asymptote).

(2) The centers of orthogonal hyperbolas through QL-P1 and circumscribed a triangle component lie on QL-L3.

(3) The pedal points of QL-Cu1-points lie on a circle (see EQF) centered on the Newton-Line QL-L1.

(4) The Miquel Circle QL-Ci3 is the locus of points, which lie on the orthogonal hyperbola of its pedal quadrangle (except points on the four lines of the quadrilateral).

(5) The QL-Circumcircle Diagonal Triangle QL-Ci1 is the locus of the Poncelet Points of quadrangles with vertices in the contact points of QL-inscribed conics.

Best regards Eckart

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Message: #473
Date: 05/3/2014 1:51:35
From: eckart_schmidt@t-online.de
Subject: Orthocentric Pedal Quadrangles

Dear Bernard K., dear Chris,
there is a further aspect for the QL-points X, Y with
orthocentric pedal quadrangles:
... For a quadrigon the locus of points, whose pedal quadrigon
has orthogonal diagonals, is a circle C_i .
... Center of the circle C_i is the reflection of QG-P2
in the circumcircle of QG-Tr3.
... The circle C_i contains the intersections QG-2P2
of opposite sides of the quadrigon.
... The circle C_i is the QG-Tf2- image or the QL-Tf1-image
of the Thales circle about QG-2P2.
... For a quadrilateral the three circles C_i have the common
points X and Y with orthocentric pedal quadrangles.
Best regards Eckart

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Message: #474
Date: 06/3/2014 10:20:28
From: bernard.keizer
Subject: A Network of QL-Transformations

Dear Eckart,
Wunderbar again!
Simple beginning with the diagonals and amazing properties
through the Clawson-Schmidt transformation.
The property concerning the point P5 is really beautiful.
I've tried to find a synthetic explanation, but I haven't
succeeded yet. Have you any idea?
On page 2, about $CSC(QL-P1^*)$, why do you mean by reflexion in a
circle?
Beautiful too is this conic C_o inscribed in the QL-DT ...
Best regards
Bernard

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Message: #475
Date: 07/3/2014 12:20:06
From: eckart_schmidt@t-online.de
Subject: A Network of QL-Transformations

Dear Bernard K.
thanks for your message. My English is not the best: perhaps
"inversion wrt a circle" is better than "reflexion in the
circle".
Sorry for no synthetic explination.
After experiences with CSC perhaps there are similar
possibilities with the QL-transformations $L(P)$ and $P(L)$ in my
message #151.
Best regards Eckart

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Message: #476

Date: 13/3/2014 4:01:22

From: Seiichi Kirikami

Subject: The midpoints and crosspoints of van Aubel quadrangles

Dear friends,

In connection to message #378 by Chris, I drew 3 pictures for van Aubel quadrangles.

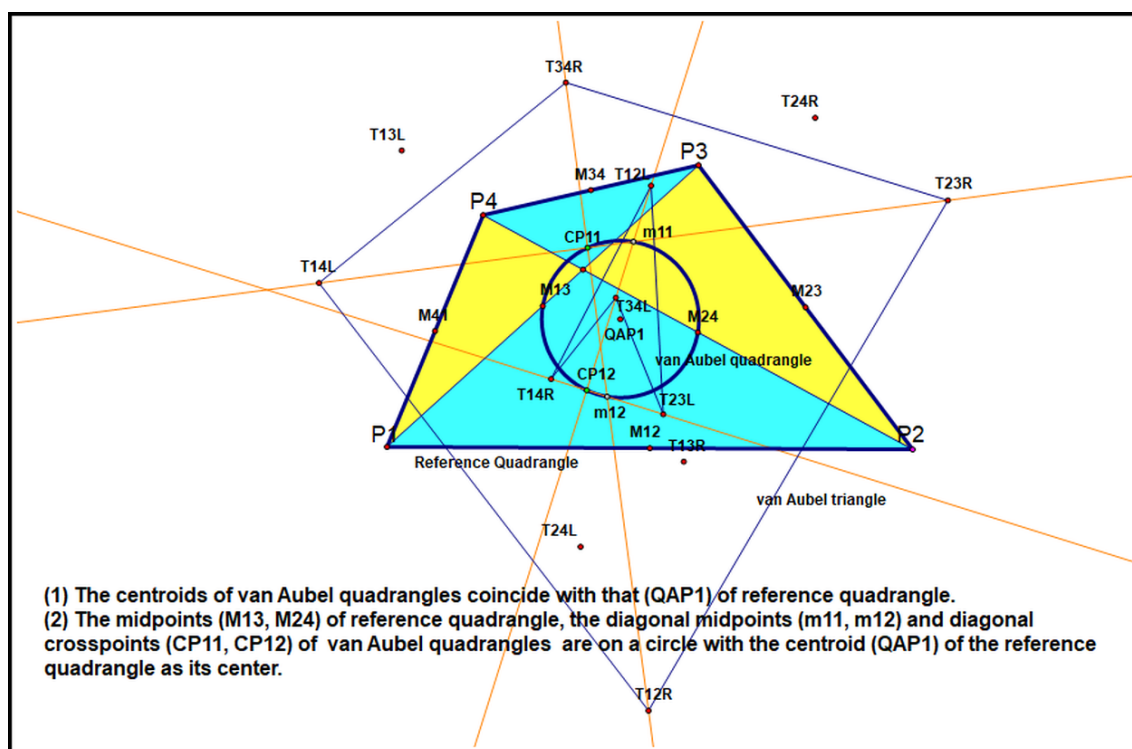
Given a reference quadrangle and its van Aubel quadrangles, 2 midpoints of the former, 2 diagonal midpoints and 2 crosspoints of the latter are on a circle with the centroid (QAP1) of the reference quadrangle as its center.

See the attached gsp.

Best regards,

Seiichi.

P.S. Point K2 in OQ-2P99-VanAubelPoints-08a-circumsquares.png coincides with the crosspoint of internal van Aubel quadrangle.



vanAubelQuadrangles-gsp.png

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Message: #477
Date: 13/3/2014 11:32:30
From: bernard.keizer
Subject: A Network of QL-Transformations

Dear Eckart,
Thanks for your answer, in fact I understand better the inversion in the circle ...
As you say at the end of your paper, the perpendicular bisector of P_1 and $CSC(P_5)$ is tangent to Co ; it's precisely your line XY (CSC of the Plücker circle) in an older message. The circle with center P_1^* the isogonal conjugate of P_1 through $CSC(P_5)$ is $CSC(C_i)$ and is through the CSC of the $CSCe$ of the 3 diagonals, id est the reflexions of P_1 in the 3 diagonals; this circle is the homothetic of the pedal circle of both P_1 and P_1^* wrt the diagonal triangle in the homotethy of center P_1 and ratio 2. The foot of the perpendicular from P_1 to XY is on the pedal circle of P_1 wrt the diagonal triangle.
I don't see why, but anyhow, it's beautiful!
I have other ideas, but it will take more time ...
Best regards
Bernard

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Message: #478

Date: 14/3/2014 12:03:04

From: eckart_schmidt@t-online.de

Subject: The midpoints and crosspoints of van Aubel quad ...

Dear Seiichi,

three years ago Fukushima: I thought of you, for the news here in Germany are not so good. I hope you and your family are well. Back to "van Aubel quadrangles": I could not open your attachment, but I think, that the property is already mentioned in Chris' message #378 in the attachment.

By the way: is already known, that the radical axis of the circumcircles of the latter van Aubel quadrangles contains the Miquel Point QL-P1?

Best regards Eckart

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Message: #479

Date: 16/3/2014 10:05:57

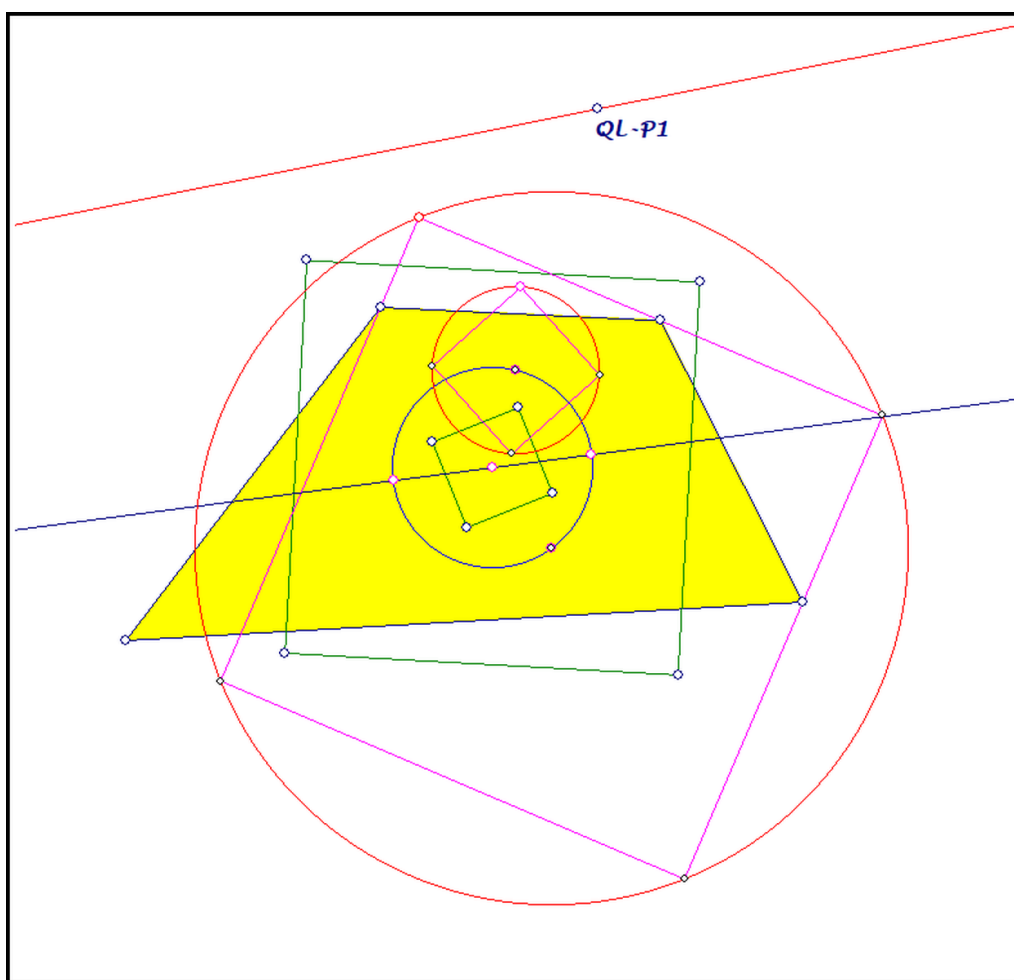
From: eckart_schmidt@t-online.de

Subject: The midpoints and crosspoints of van Au ...

Dear Chris,

I have used Seiichi's description. "Circumcircles of the latter van Aubel quadrangles" are the circumcircles of the circumscribed quadrangles. I think, the attachment will give clearness.

Best regards Eckart



2014-03-16-a-fig.png

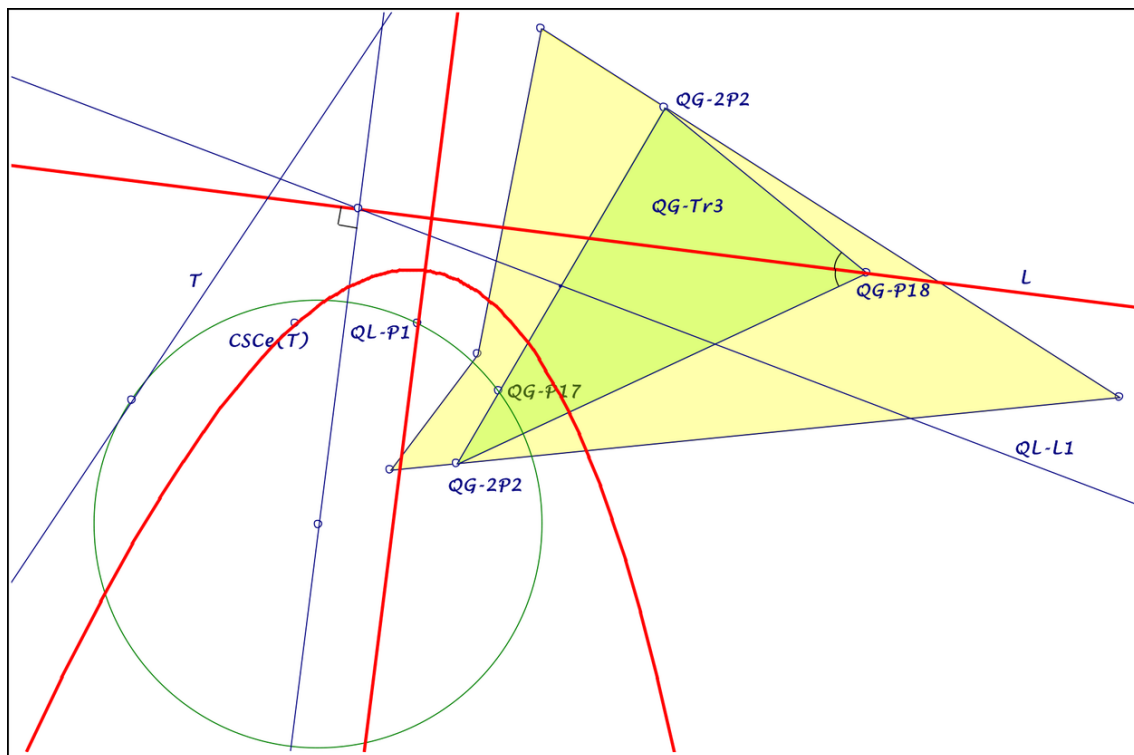
Message: #480
Date: 16/3/2014 10:13:55
From: eckart_schmidt@t-online.de
Subject: A Network of QL-Transformations

Dear Bernard K.,

thank you for additional CSC-remarks. There are permanent new CSC-surprises, here a further example:

... Consider a quadrigon and the angle bisector L
for $QG-Tr3$ at $QG-P18$.
... $CSC(L)$ is a circle C_i through $QL-P1$ and $QL-P17$
... centered on a line perpendicular to L
through the intersection of L and $QL-L1$.
... Let T be tangents at C_i , then $CSCe(T)$ give a parabola
... with focus $QL-P1$ and directrix L .
See attachment.

Best regards Eckart



2014-03-16.pdf

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Message: #481

Date: 16/3/2014 11:07:11

From: eckart_schmidt@t-online.de

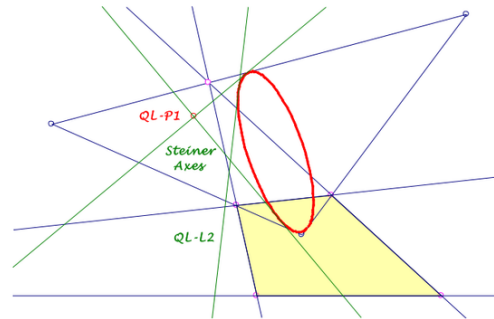
Subject: Inscribed QL-DT-Conic tangent to the Steiner Axes

Dear Bernard K., dear friends,
in message #465 and #466 I have mentioned a conic, inscribed
QL-DT and tangent to the Steiner Axes.
Sorry, the equation of this conic in message #466 is false!
There are many properties of the tangents and polars for this
conic.
So I have gathered these aspects in a new EQF-Note (see
attachment).
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

**Inscribed QL-DT-Conic
tangent to the Steiner Axes**

*There is a special conic Co for a quadrilateral, inscribed in the QL -Diagonal Triangle and tangent to the Steiner Axes and the Steiner Line $QL-L2$. The Steiner Axes are the angle bisectors at the Miquel Point $QL-P1$ wrt two opposite points of the quadrilateral (see EQF $QL-Tf1$). This conic Co is already mentioned in QFG -message 465 and in EQF -Note 2014-02-27 - *but with false equation!* – Reference triangle for barycentric coordinates is $QL-DT$.*



Properties: It is worth to have a special look on tangents and polars wrt this conic Co ; there seem to be no relevant EQF -points on Co . Some properties are already mentioned in QFG -message 465 and in EQF -Note 2014-02-27.

- Defining tangents of Co : The diagonals of the quadrilateral and the Steiner Axes (see $QL-Tf1$).
- Equation of Co wrt reference triangle $QL-DT$ and one line (l, m, n) :

$$U^2 x^2 + V^2 y^2 + W^2 z^2 - 2 V W y z - 2 W U z x - 2 U V x y = 0;$$

$$U := 1^2 (m^2 - n^2) (b^2 (1^2 - m^2) + c^2 (n^2 - 1^2));$$

$$V := m^2 (n^2 - 1^2) (c^2 (m^2 - n^2) + a^2 (1^2 - m^2));$$

$$W := n^2 (1^2 - m^2) (a^2 (n^2 - 1^2) + b^2 (m^2 - n^2)).$$

By the way, the coordinates of the Miquel Point are:

$$\left\{ \frac{U}{1^2}, \frac{V}{m^2}, \frac{W}{n^2} \right\}$$

- The Steiner Line $QL-L2$ is tangent to Co . Point of tangency:

$$\{1^4 (m^2 - n^2)^2 v w, m^4 (n^2 - 1^2)^2 w u, n^4 (1^2 - m^2)^2 u v\}$$

- The center of Co

$$\{v + w, w + u, u + v\}$$

lies diametral to $QL-P6$ on the circumcircle of $QL-P1$,
 $QL-P6$, $QL-P17$.

The Brianchon point of Co wrt $QL-DT$ is

$$\{v w, w u, u v\}$$

- The polars of $QL-P1$ wrt inscribed conics of the quadrilateral envelope the conic Co .
- Co is also the envelope of the $QL-Tf2$ images of lines through $QL-P1$. Example: The line parallel $QL-P3.4.5.6$ through $QL-P1$ gives $QL-L2$ as tangent for Co .
- Consider a quadrigon component $S_{12}S_{23}S_{34}S_{41}$ of the quadrilateral with Miquel Point $M_1=QL-P1$ and the further Miquel Points M_2 of $S_{12}S_{23}S_{41}S_{34}$ and M_3 of $S_{12}S_{34}S_{23}S_{41}$, then the lines M_2S_{24} and M_3S_{13} are tangent to Co .
- The polar of $QL-P1$ wrt Co is the Clawson-Schmidt Conjugate $QL-Tf1$ (shortened CSC) of the Dimidium Circle $QL-Ci6$.
- The polar of $QL-P1$ wrt Co contains the Least Squares Point $QL-P26$ and $CSC(QL-P17)$, $CSC(QL-P24)$. Coefficients of the polar:

$$\begin{aligned} &\{u (-m^2 n^2 u^2 + n^2 1^2 v^2 + 1^2 m^2 w^2), \\ &\quad v (m^2 n^2 u^2 - n^2 1^2 v^2 + 1^2 m^2 w^2), \\ &\quad w (m^2 n^2 u^2 + n^2 1^2 v^2 - 1^2 m^2 w^2)\} \end{aligned}$$

- The CSC -images for reflections of $QL-P1$ in tangents at Co give an orthogonal hyperbola, centered in $QL-P6$ with asymptotes parallel to the Steiner Axes containing $QL-P4$ and $QL-P5$.
- The perpendicular bisector of $QL-P1.CSC(QL-P5)$ is tangent to Co . This line is the CSC -image of the Plücker Circle $QL-Ci5$.

- Consider inscribed conics of a quadrilateral centered in points on $QL-LI$: The points of tangency lie with each pair of opposite points of the quadrilateral on a conic. The polars of $QL-PI$ wrt these three conics have a common point and the locus for this point is the discussed conic Co .
- The intersections of tangents T at Co and $CSC(T)$ give a conico-pivotal isocubic (see *EQF-Note 2014-02-27*).

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-03-17.pdf

Message: #482

Date: 18/3/2014 3:42:27

From: Seiichi Kirikami

Subject: The midpoints and crosspoints of van Aubel quad ...

Dear Eckart,

Sorry for not quick response. See the attached Word-file.

I could not reach your statement "the radical axis of the circumcircles of the latter van Aubel quadrangles contains the Miquel Point QL-P1".

I hope that you will teach me it graphically (desirably).

And I hope your comments for the following.

(1) The point CP11 is the concurrency of 4 circumcircles T12RM13T23R, T23RM24T34R, T34RM13T14L and T14LM24T12R. CP11 coincides with the diagonal crosspoint of T23RT14L and T12RT34R of van Aubel quadrangle. So I think that it is important for van Aubel quadrangle as for the reference quadrangle. But its barycentrics are depressingly complicated in terms of a , b , c and p , q , r . I would like to know your comment whether this is inescapable or inevitable.

(2) There is another point of the concurrency of T12RM24T23R, T23RM13T34R, T34RM24T14L and T14LM13T12R. I tried in vain to find connections to known points. If you have suggestions for it, I would like to know it.

P.S. 3 years ago I could see the scenes of debries everywhere with curiosity. But now when I see them in TV, they are too tragic for me to look at.

Best regards,

Seiichi.

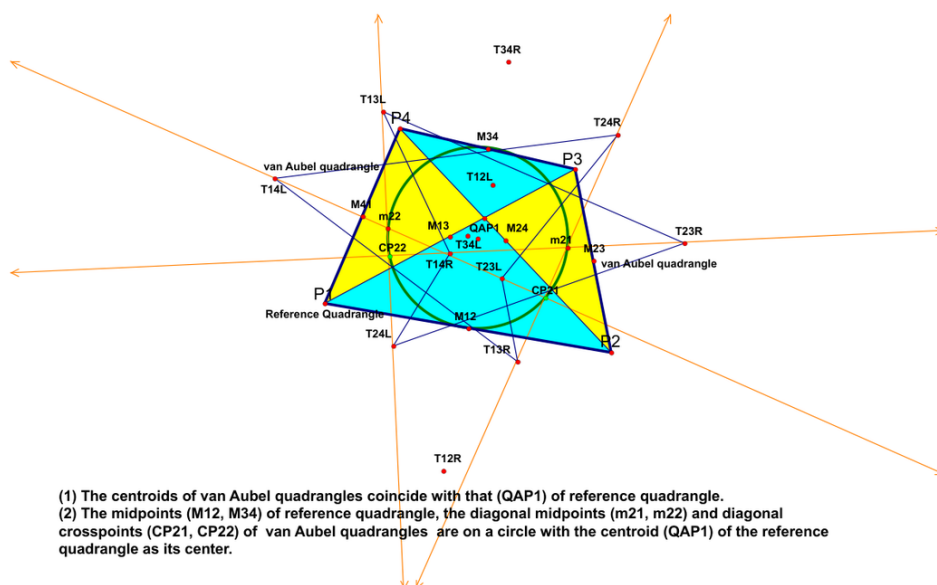
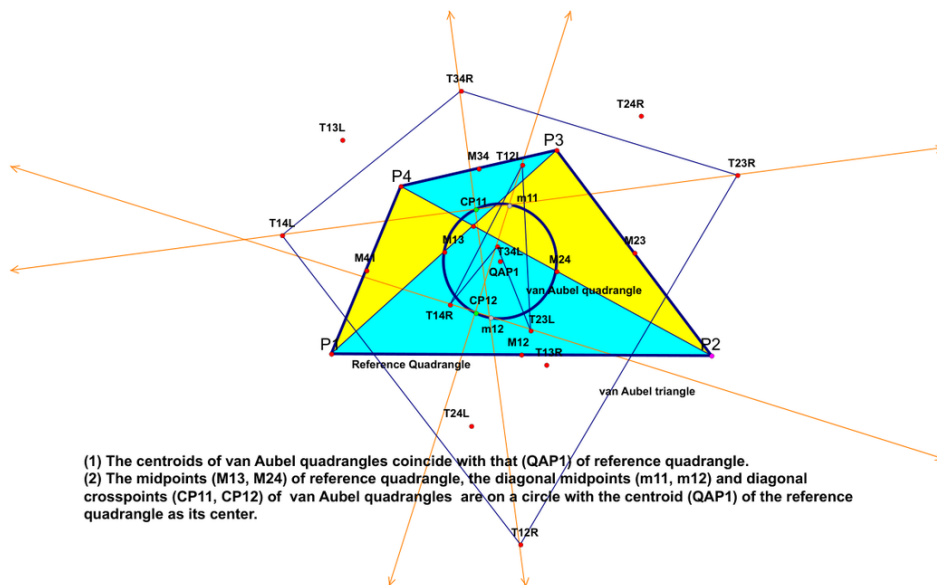
On Friday, March 14, 2014 8:03 PM, "eckart_schmidt@t-online.de" wrote:

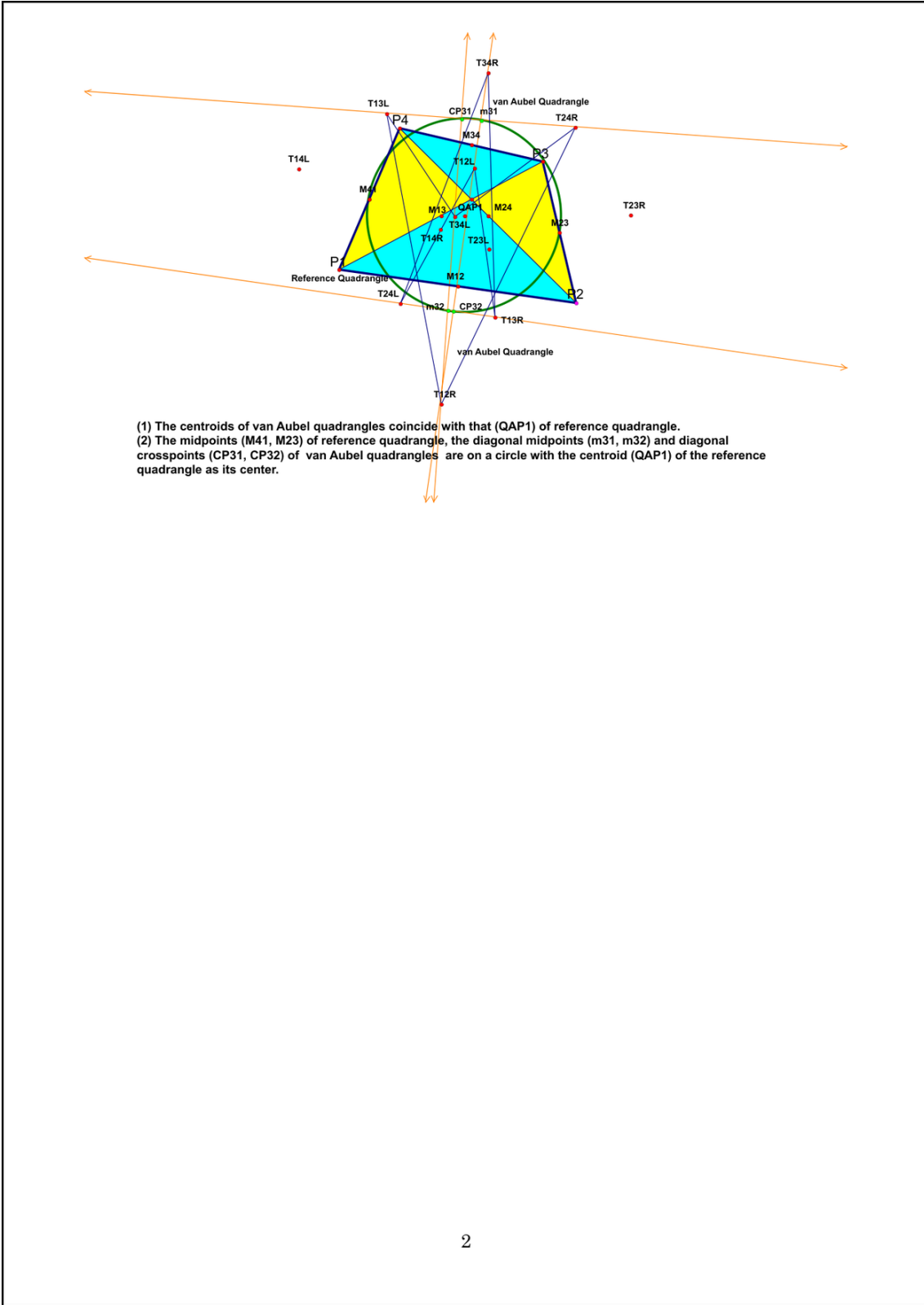
Dear Seiichi,

three years ago Fukushima: I thought of you, for the news here in Germany are not so good. I hope you and your family are well. Back to "van Aubel quadrangles": I could not open your attachment, but I think, that the property is already mentioned in Chris' message #378 in the attachment.

By the way: is already known, that the radical axis of the circumcircles of the latter van Aubel quadrangles contains the Miquel Point QL-P1?

Best regards Eckart





Message: #483

Date: 18/3/2014 12:04:05

From: eckart_schmidt@t-online.de

Subject: The midpoints and crosspoints of van Au ..

Dear Seiichi, dear Chris,
thanks to Seiichi for the Word files wrt "van Aubel
quadrangles". Now I see, that I used false definitions for "van
Aubel quadrangles", please excuse! My remark wrt the Miquel
point is only valid for the circumscribed squares in the
attachment of Chris' message #378. Now I shall study Seiichis
questions.
Best regards Eckart

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Message: #484

Date: 18/3/2014 2:14:23

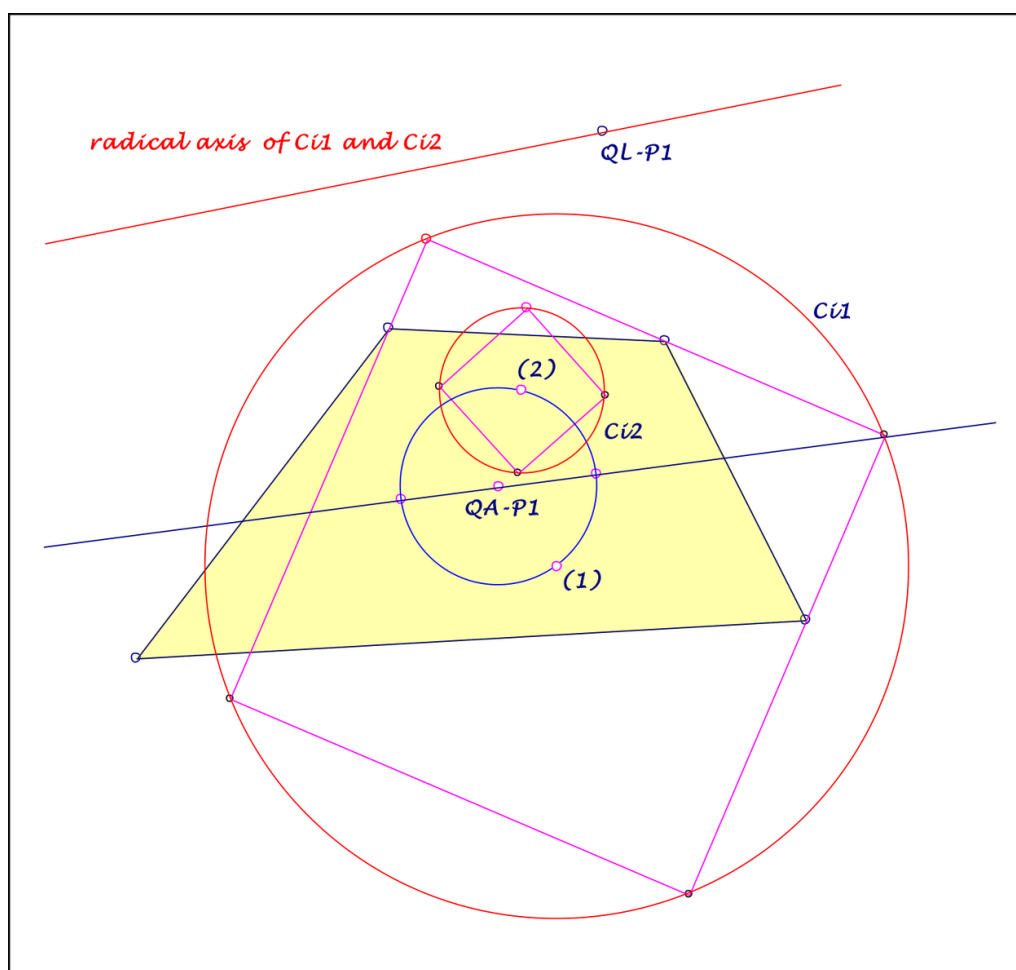
From: eckart_schmidt@t-online.de

Subject: The midpoints and crosspoints of van Au ..

Dear Seiichi,

a short reply to your points (1) and (2): They are the centers of the two circumscribed squares of a quadrigon (see attachment).

Best regards Eckart



Seiichi-2014-03-18.pdf

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Message: #485

Date: 18/3/2014 8:41:05

From: eckart_schmidt@t-online.de

Subject: The midpoints and crosspoints of van Au

Dear Seiichi,

there is a word file in the attachment with the DT-coordinates of the points (1) and (2) in your message #482. I have tried, to minimize the presentation.

Best regards Eckart

18.03.2014

DT-Coordinates of the points (1) and (2) in message 482

$$\begin{aligned} & \begin{matrix} 2 & m^2 & S & SB & 1 & m^2 & n & SB & M & 2 & N & SC \\ 1 & 2 & L^2 & SA & 1 & 2 & n^2 & L & M & 2 & n^2 & N^2 & SC \\ & 1 & n & L & SA & SB & n^2 & N & SB & SC & L & N & SA \\ & m^2 & n^2 & N & S & SB & 1 & m^2 & n & SB & M & 2 & N & SC \end{matrix} \end{aligned}$$

with

SA, SB, SC, S Conway abbreviations,

$$L=m^2-n^2, M=n^2-l^2, N=l^2-m^2$$

For the second point change the sign of S!

Eckart Schmidt

<http://eckartschmidt.de>

eckart_schmidt@t-online.de

Seiichi-2014-03-18.docx

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Message: #486
Date: 19/3/2014 11:19:35
From: bernard.keizer
Subject: CSCe: a general statement

Dear Eckart, dear Chris

Working on Eckart's conics Co, I found the following property:
Given any circle C_i with center O , its CSC is another circle C'_i with center O' , called $CSCe(C_i)$; the CSC of this second center, called $CSC[CSCe(C_i)]$, is a point o , which is the inverse of P_1 wrt the circle C_i .

It's easy to prove the property by considering the tangents P_1K and P_1K' from P_1 to C_i and C'_i ; P_1 is inversion center and homothety center between C_i and the reflexion of C_i in the Steiner axis and $P_1K.P_1K' = P_{10}.P_{1o}$ with $P_1K/P_1K' = P_{10}/P_{10}'$; it follows $P_{10}P_{1o} = P_1K^2$ and $OP_1.Oo = OK^2$.

If the circle C_i is through P_1 , C'_i is a line, its center is the infinity point of P_{10} and o is in P_1 .

If the circle C_i is a line, its center is the infinity point of the perpendicular through P_1 to the line and the inverse of P_1 wrt the circle-line is the reflexion of P_1 in the line.

May be this was obvious, but as Monsieur de Talleyrand used to say " Ce qui va sans dire va aussi bien en le disant " (What goes without saying goes as well with saying).

Best regards
Bernard

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Message: #487
Date: 20/3/2014 12:58:01
From: seiichikiri
Subject: The midpoints and crosspoints of van Au

Dear Eckart,
I see the radical axis of the van Aubel quadrangles. The coordinates are simple in DT expression!
Thank you very much.
Best regards,
Seiichi.

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Message: #488
Date: 22/3/2014 12:11:52
From: eckart_schmidt@t-online.de
Subject: Special Conics for QG, QL, QA

Dear friends,
inscribed conics of a quadrigon lead to other conics:
1. The boundary points of an inscribed conic and the intersections of opposite sides give a new conic.
2. The polars of a point wrt inscribed conics envelope a further conic.
These conics are described for quadrigon, quadrilateral and quadrangle (see attachment).
Best regards Eckart

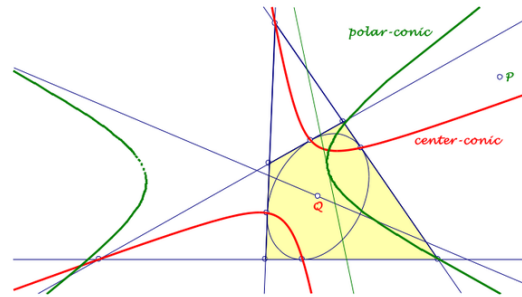
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Special Conics for QG, QL, QA

The inscribed conics of a quadrigon lead to other conics:

1. The boundary points of an inscribed conic and the intersections of opposite sides give a new conic.
2. The polars of a point wrt inscribed conics envelope a further conic.

These conics will be discussed for quadrigon, quadrilateral and quadrangle. – Barycentric DT-coordinates are used for a QL-quadrigon.



Center-Conics

Consider an inscribed conic of a quadrigon, centered in $Q(\alpha : \beta : \gamma)$ on the Newton Line $QL-LI$ with the equation

$$\gamma(l^2\alpha + n^2\gamma)x^2 - m^2\alpha\gamma^2y^2 + \alpha(l^2\alpha + n^2\gamma)z^2 = 0.$$

This conic has the boundary points

$$(\pm lm\alpha : \pm(l^2\alpha + n^2\gamma) : \pm mn\gamma).$$

In connection with the intersections of opposite sides they give a new conic with the equation

$$l^2(l^2\alpha + n^2\gamma)x^2 - m^2(l^2\alpha - n^2\gamma)y^2 - n^2(l^2\alpha + n^2\gamma)z^2 = 0$$

and the center

$$(m^2n^2(l^2\alpha - n^2\gamma) : -n^2l^2(l^2\alpha + n^2\gamma) : -l^2m^2(l^2\alpha - n^2\gamma)).$$

Such a conic here shall be named as **center-conic of Q** , defined by the center $Q(\alpha : \beta : \gamma)$ of an inscribed conic.

Polar-Conics

Consider the polars of an arbitrary point $P(u : v : w)$ wrt inscribed conics of a quadrigon. These polars have the equation

$$\gamma u(l^2\alpha + n^2\gamma)x - \alpha\gamma m^2vy + \alpha w(l^2\alpha + n^2\gamma)z = 0$$

and envelope a conic with the equation

$$l^4 u^2 x^2 + m^4 v^2 y^2 + n^4 w^2 z^2 - 2l^2 m^2 uvxy - 2m^2 n^2 vwy z - 2n^2 l^2 wuzx = 0$$

and the center

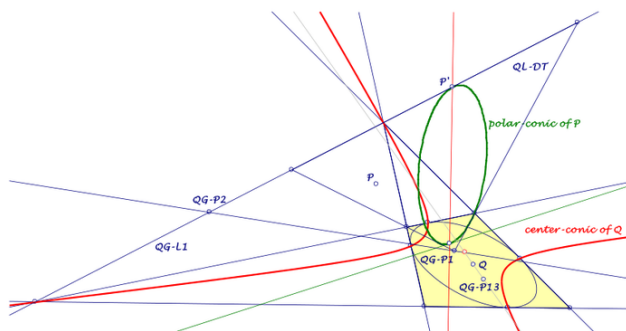
$$(m^2 v + n^2 w : n^2 w + l^2 u : l^2 u + m^2 v)$$

tangent to the polar in the point

$$(\alpha^2 l^2 m^2 vw : (l^2 \alpha + n^2 \gamma)^2 wu : m^2 n^2 uv \gamma^2).$$

Such a conic here shall be named as **polar-conic of P** , defined by the polars of a point $P(u : v : w)$ wrt inscribed conics.

Properties wrt a Quadrigon:

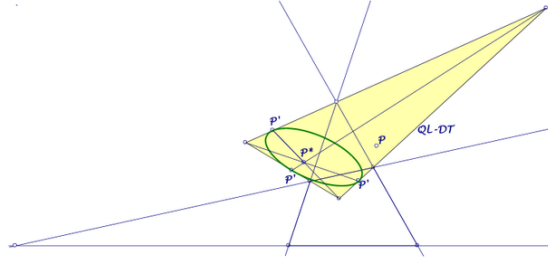


- The centers of center-conics are collinear on the line $QG-P1, QG-P2$.
- The polar of P wrt an inscribed conic and the polar of P wrt the corresponding center-conic intersect on the polar conic of P .
- The center Q of an inscribed conic and the center of the corresponding center-conic are collinear with $QG-P13$.
- The polars of a point $P(u : v : w)$ wrt center-conics have a common point $P'(n^2 w : 0 : l^2 u)$ on the third diagonal $QG-L1$ of the quadrigon.
- The point P' is the intersection of $QG-L1$ and the trilinear polar of P wrt the QA -Diagonal Triangle $QA-DT$.
- The point P' is the boundary point of the third diagonal and the polar-conic of P .
- The point P' is the common pole of $P, QG-P1$ for center-conics.
- Points P on a line through $QG-P1$ have the same P' :
 ... for $QG-L2$ the point P' is $QG-P3$;
 ... for a parallel to $QG-L1$ through $QG-P1$ the point P' is $QG-P2$;
 ... for $QG-P1, QG-P2$ the point P' is the point at infinity of $QG-L1$;
 ... for $QG-P1, QG-P3$ the point P' is the intersection of $QG-L1$ and $QG-L2$.

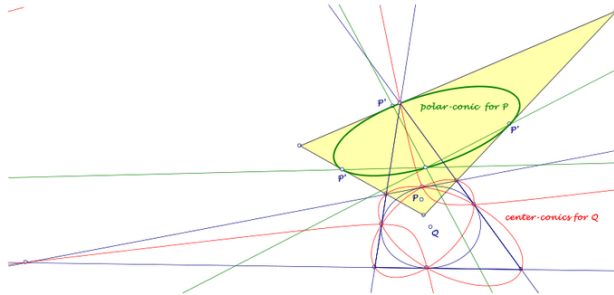
Properties wrt a Quadrilateral and its 3 QG-versions:

The polar-conic of a point P is an inscribed conic of the QL -Diagonal Triangle $QL-DT$.

- The polar-conic of a point P is the envelope of the QL - $Tf2$ -images of lines through P .
- Let $*$ be a QL - DT -isoconjugation with fixed points in the trilinear poles of the QL -lines. For a point P the polar-conic is a QL - DT inscribed conic with Brianchon point P^* . The boundary points are the three points P' wrt the 3 QG -versions of the quadrilateral.



- For $P = QL-P1$ the polar conic is a QL - DT inscribed conic with asymptotes in the Steiner Axes (see $QL-Tf1$). This conic is detailed worked out in *EQF-Note 2014-03-17*.
- For $P = QL-P13$ the polar conic is the QL - DT inscribed Steiner ellipse.
- For $P = QL-P8$ the polar conic is the QL - DT inscribed conic with Brianchon point $QL-P13$.
- Let G be the Gergonne point of QL - DT , then $P = G^*((-a+b+c)m^2n^2 : (a-b+c)n^2l^2 : (a+b-c)l^2m^2)$ gives the incircle of QL - DT as polar-conic.



For a center Q of a QL -inscribed conic there are three center-conics wrt the three QG -versions of the quadrilateral.

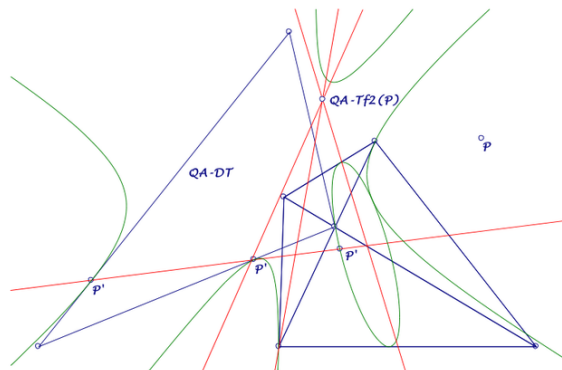
- The polars of a point P wrt such a trio of center-conics have a common point on the polar-conic of P .

- This point is the boundary point of the polar-conic and the polar of P wrt the inscribed QL -conic centered in Q .
- This point is the image of P wrt a QL - DT -isoconjugation with fixed points in the boundary points of the inscribed QL -conic centered in Q .

Properties wrt a Quadrangle and its 3 QG-versions:

For a point P there are three polar-conics wrt the three QG -versions of the quadrangle.

- The polars of P wrt its three polar-conics have a common point in the Involutory Conjugate QA - $Tf2$ of P .



- The three points P' wrt the QG -versions of a quadrangle are collinear on the trilinear polar of P wrt the QA -Diagonal Triangle $QA-DT$.
- The three points QL - PI' for the QG -versions of a quadrangle are collinear on the trilinear polar of the Gergonne-Steiner Point $QA-P3$ wrt $QA-DT$. This line is perpendicular $QA-P11$, $QA-P32$ and contains $QA-P16$.

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2014-03-22.pdf

Message: #489
Date: 22/3/2014 9:00:19
From: eckart_schmidt@t-online.de
Subject: CSCe: a general statement

Dear Bernard K.,

thanks for your further CSC-remarks. In addition with the same constellation:

Consider the pencil of circles C_i centered in O :

- ... the centers of the circles $CSC(C_i)$
lie on the line $QL-P1.CSC(O)$,
- ... inner / outer tangents of C_i and $CSC(C_i)$
intersect on the Steiner axes,
- ... the inversion of $QL-P1$ wrt $CSC(C_i)$ is $CSC(O)$,
- ... the inversions of O wrt $CSC(C_i)$
give a circle through $QL-P1$, O , $CSC(O)$,
- ... the polars of O wrt $CSC(C_i)$ have a common point
on the circumcircle of $QL-P1$, O , $CSC(O)$ diametral to O ,
- ... the intersections of C_i and $CSC(C_i)$ lie on a CSC-invariant
quartic through the fixed points of CSC,
- ... the radical axes of C_i and $CSC(C_i)$ envelop a parabola
- tangent to the Steiner axes
- with focus $CSC(O)$
- and directrix $O.QL-P1$.
- ... For tangents T at this parabola the points $CSCe(T)$
will give an orthogonal hyperbola through O and $QL-P1$
centered in the midpoint of $O.QL-P1$
with asymptotes parallel to the Steiner axes.
- ... The CSC-image of this hyperbola is the strophoid of $O.QL-P1$
wrt the pole $CSC(O)$ and the fixed point $QL-P1$.

I think, I should shut up!

Best regards Eckart

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Message: #490

Date: 24/3/2014 4:12:34

From: bernard.keizer

Subject: Inscribed QL-DT-Conic tangent to the Steiner Axes

Dear Eckart,

I might say Wunderbar again, as I said already to your messages #454 and #471!

You make too many Wunder at the the same time and I hardly find enough time to follow ...

This new conic Co (same name as in #471) is really a great thing and I feel it's linked to many properties I mentioned in my message #410.

The couple of Steiner axes forms, as the 6 couples of bisectors of the angles in the 6 vertices, the 3 couples side + altitude of the diagonal triangle and the couple Newton Line + infinity line, a degenerated rectangular hyperbola and belongs to the set of conics having the QL as polar QL, it means the 2 lines cut each diagonal of the DT in 2 points harmonically conjugates of the 2 vertices on this diagonal. The rectangular hyperbolas having this property are the polar conics of your curve QL-Cu2 or QL's conjugated stelloid (the inscribed conics in the QL are the apolar conics of the same curve and the Van Rees curve is the hessian of the curve)!

Back to your paper:

- you mention 3 times as reference your EQF note 2014-02-27, but, if I'm not wrong, this note is not yet on your website
- amazing is the symmetric role played by the 2 points P4 and P5 and the 2 lines perpendicular bisectors of P1.CSC(P4) and P1.CSC(P) and CSC-images of the Plücker and Miquel circles ...
- the last curve of the note (locus of the intersections between the tangents (T) to Co and their CSC-images CSC(T)) is described as a conico-pivotal-isocubic, but the same curve was described in the messages #364 and #465 as a quartic.

Best regards

Bernard

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Message: #491

Date: 25/3/2014 11:03:42

From: eckart_schmidt@t-online.de

Subject: Inscribed QL-DT-Conic tangent to the Steine ...

Dear Bernard K.,
thank you for positive feedback! You can find my EQF-Notes also under the same date in QFG-messages. But today I have updated my homepage.

Special thanks for the notice of my typo in the last point of EQF-Note 2014-03-17: Not CSC(T), but QL-Tf2(T) is the right transformation!

For I am not so familiar with "polar conics" and "hessians", I cannot comment links to your explicite summary in message #410. But I shall have a look on the conics Co wrt your remarks.
Best regards Eckart

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Message: #492

Date: 25/3/2014 11:05:30

From: eckart_schmidt@t-online.de

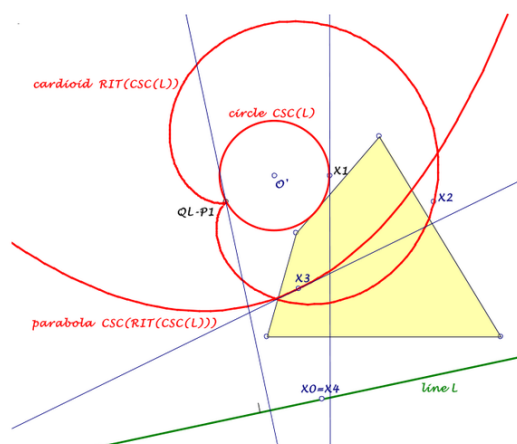
Subject: A Ring of Transformations

Dear Bernard K, dear Chris,
Two transformations will be considered (see attachment):
CSC(curve): Clawson-Schmidt Conjugates QL-Tf1 of points of the curve.
RIT(curve): Reflections of the Miquel Point QL-P1 in tangents at the curve.
Let L be a line, then holds $RIT(CSC(RIT(CSC(L)))) = L$.
Bernard and Chris will realize, that $CSC(RIT(L))$ can be interpreted as $CSCe(L)$.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

A Ring of Transformations

Two transformations will be considered:
 $CSC(\text{curve})$: **Clawson-Schmidt Conjugates** $QL-Tf1$
of points of the curve.
 $RIT(\text{curve})$: **Reflections of the Miquel Point** $QL-P1$
In Tangents at the curve.
Let L be a line, then holds:
 $RIT(CSC(RIT(CSC(L)))) = L$.



L : Let L be a line.

$CSC(L)$ is a circle through $QL-P1$ with midpoint O' in the CSC -image of the reflection of $QL-P1$ in L .

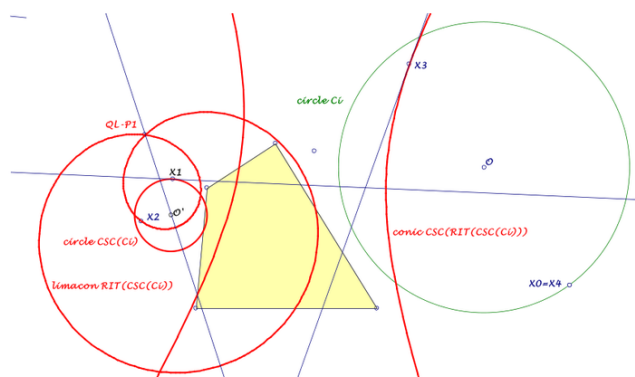
$RIT(CSC(L))$ is a cardioid with center O' and cusp $QL-P1$.

$CSC(RIT(CSC(L)))$ is a parabola with focus $QL-P1$ and directrix L .

$RIT(CSC(RIT(CSC(L))))$ is the line L again.

For example:

Let L be the Steiner Line $QL-L2$:
 $CSC(QL-L2)$ is the Miquel Circle $QL-Ci3$,
 $RIT(QL-Ci3)$ is the QL -Cardioid $QL-Qu1$,
 $CSC(QL-Qu1)$ is the Inscribed Parabola $QL-Co1$,
 $RIT(QL-Co1)$ is the Steiner Line $QL-L2$ again.



We can also start with a circle.

Ci : Let Ci be a circle with center O (not containing $QL-P1$).

$CSC(Ci)$ is a circle again with midpoint O' in the CSC -image of the inversion of $QL-P1$ wrt Ci .

$RIT(CSC(Ci))$ is a limaçon with center O' and cusp $QL-P1$.

$CSC(RIT(CSC(L)))$ is a conic centered in the midpoint of $O, QL-P1$ with foci O and $QL-P1$.

$RIT(CSC(RIT(CSC(L))))$ is the circle Ci again.

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2014-03-25.pdf

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Message: #493
Date: 26/3/2014 2:56:01
From: seiichikiri
Subject: van Aubel quadrangle of 2nd kind

Dear Eckart,
Seeing many transformations in messages, I would like to hear your opinion of the following transformation.
In message #149, I proposed van Aubel quadrangle of 2nd kind. As one of the possibilities of solving this problem, I considered a transformation.
A transformation: M_{ij} is transformed to N_{ij} . The vertices of the quadrangle are transformed to themselves. In case of the cyclic quadrangle, N_{ij} coincides with M_{ij} .
My hope is that by this transformation, the vertices of van Aubel quadrangle would be transformed to those of van Aubel quadrangle of 2nd kind.
Best regards,
Seiichi.

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Message: #494
Date: 01/4/2014 10:24:46
From: eckart_schmidt@t-online.de
Subject: van Aubel quadrangle of 2nd kind

Dear Seiichi,

excuse my late answer, but I am working on another question. I have studied your messages wrt "van Aubel quadrangle of 2nd kind", but - sorry - I don't know, whether I have understood this term in the right way. Is there any definition or construction? You consider van Aubel quadrangles connected with the Varignon parallelogram and search for "van Aubel quadrangle of 2nd kind" connected with the QA-.P4-parallelogram (... is that right?). But these parallelograms have different origins: The first is related to midpoints, the second is related to pedal points.

My sight: Every point can be the center of an inscribed parallelogram (PG) and these PG can have different properties: The PG can have sides parallel to the diagonals with half the length (Varignon PG) or the PG can be a pedal quadrangle (QA-P4-PG),

- ... or the PG can be collinear degenerated
(center on QA-Co1),

- ... or the PG has sides parallel to the diagonals
(center on the QL-L1),

- ... or the PG has a pair of sides parallel to QG-L1
(center on a diagonal),

- ... or the PG has diagonals parallel to the legs of QA-DT
(center on QG-L1),

- ... or the PG is a rectangle or a rhombus
(centers on special conics, see my homepage 11.2).

Special:

- The QA-P4-PG is a representativ of similar PG,
centered on the bisector of QA-P4.QA-P2
(the angle between the PG-diagonals
is the angle between the legs of QA-DT).

Best regards Eckart

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Message: #495
Date: 03/4/2014 1:50:46
From: seiichikiri
Subject: van Aubel quadrangle of 2nd kind

Dear Eckart,
Sorry for disturbing your work. Thank you very much for the information.
I saw an analogy of birefringence for van Aubel quadrangle from physics of material.
cyclic quadrangle corresponds to isotropic material. General quadrangle corresponds to anisotropic material. Van Aubel quadrangle which coincides in cyclic quadrangle split into 2 van Aubel quadrangles in general quadrangle. But I think that almost every one say that my idea digresses greatly from quadrifigures. If I have something interesting, I will post it.
P.S. Early this morning I was awakened by the announcement of the city, "The tsunami of 2 m height maximum will come at 5:30 a. m. from Chile's earthquake yesterday. Evacuate from the coastline." Happily any change of sea level is not reported here at 8:30 a.m.
Best regards,
Seiichi.

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Message: #496
Date: 03/4/2014 9:57:48
From: eckart_schmidt@t-online.de
Subject: Foci of QA-Circumscribed Conics

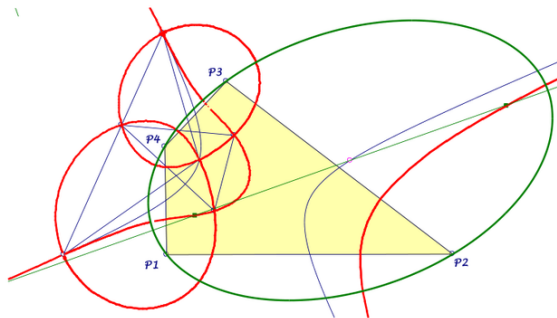
Dear friends,
the foci of inscribed conics of a quadrilateral lie on QL-Cu1. The locus for the foci of circumscribed conics of a quadrangle is described in the attachment. The locus is a sextic, built up of three isogonal invariant curves wrt the orthic triangle of the QA-diagonal triangle.
Perhaps someone knows, where this is published.
Best regards Eckart

EQF-Note 2014-04-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Foci of QA-Circumscribed Conics

The foci of inscribed conics of a quadrilateral lie on the QL-Quasi Isogonal Cubic QL-Cu1. In this note the locus for the foci of circumscribed conics of a quadrangle is described. The locus is a sextic, built up of three isogonal invariant curves wrt the orthic triangle of the QA-diagonal triangle.



Let QA be a quadrangle, DT its diagonal triangle and OT the orthic triangle of DT .

1. The centers of QA -circumscribed conics lie on the Nine-point Conic $QA-Co1$ (see *EQF*).
2. The foci of QA -circumscribed conics are isogonal conjugated wrt OT .
3. The foci of a QA -circumscribed conic with center Z lie on an isogonal pivotal isocubic wrt the reference triangle OT and pivot Z .
4. The midpoints for the OT -pedal circles of the considered foci lie on $QA-Co1$.
(This property gives the possibility for construction and calculation.)
5. For $QA-DT$ as reference triangle the locus for the considered foci has the equation (DT -notation):

2014-04-02.pdf

$$\begin{aligned}
& \frac{p^2 y z}{-SA x^2 + SB y (x + z) + SC z (x + y)} \\
& + \frac{q^2 x z}{SA x (y + z) - SB y^2 + SC z (x + y)} \\
& + \frac{r^2 x y}{SA x (y + z) + SB y (x + z) - SC z^2} = 0
\end{aligned}$$

6. This sextic for the considered foci consists of three *OT*-isogonal invariant curves, each containing two vertices of *DT* and the corresponding vertices of *OT*.
7. Construction: Let *U*, *V*, *W* be the vertices of *OT*.
 - *C_i* circle with variable radius, centered in a point *P* on the conic *QA-Co1*,
 - *P_{u,v,w}* in each case one intersection of *C_i* and the sides *u*=*VW*, *v*=*WU*, *w*=*UV*,
 - *L_i* perpendicular in *P_i* wrt the corresponding side,
 - *Q_i* intersections of *L_j* and *L_k*,
 - *Co_i* conic as locus of *Q_i* wrt variable radius of *C_i*:
 - The common two points of *Co_u*, *Co_v*, *Co_w* are the searched foci for a *QA*-circumscribed conic centered in *P*.
8. Special points:
 - the vertices of *DT* and *OT*,
 - the infinity points of *QA-Co1*,
 - the foci of the circumscribed parabolas *QA-2Co1* (on the *OT*-circumcircle as *OT*-isogonal conjugates of the infinity points of *QA-Co1*),
 - the reflections of the *OT*-vertices in the intersections of the corresponding mid-parallel of *OT* and *QA-Co1* (these six points lie on a conic, they are the partner-foci for the *OT*-vertices),
 - the foci of the *QA*-Orthogonal Hyperbola *QA-Co2* and the foci of the Gergonne-Steiner Conic *QA-Co3*,
 - the foci for the Circumscribed Harmonic Conics *QG-Co2* and the foci for the *M3D* Hyperbolas *QG-Co3* of the three quadrigon components.

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2014-04-02.pdf

Message: #497
Date: 03/4/2014 10:23:31
From: chris.vantienhoven
Subject: van Aubel quadrangle of 2nd kind

Dear Seiichi,
It would be very nice to find an application of the Van Aubel configuration in physics!
I did not quite understand, but do you think that (parts of) birefringence could be described by using properties of the Van Aubel configuration?
Best regards,
Chris

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Message: #498
Date: 04/4/2014 2:40:19
From: seiichikiri
Subject: van Aubel quadrangle of 2nd kind

Dear Chris,
Sorry, I do not have any intension of application of quadrifigures to birefringence. In order to intensify my belief in the existence of geometric objects, I use drawing, numerical computation and analogy.
Best regards,
Seiichi

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Message: #499
Date: 07/4/2014 10:28:52
From: bernard.keizer
Subject: Urqhart's theorem

Dear Chris, dear Eckhart, dear friends,
I found this theorem in the french publication Quadrature and I
sent them a short commentar.

The theorem states:

if $AP + PB = AR + RB$, then $AQ + QB = AS + SB$

I thought it could interest some of you.

Best regards

Bernard

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Message: #500
Date: 10/4/2014 1:12:13
From: bernard.keizer
Subject: Cayleyan

Dear Eckart, dear Chris, dear friends,
I have a question about the cayleyan of the stelloïd cubic associated to the QL.
At the end of his message #491, Eckart said he wasn't a specialist of hessians and polar cubics ...
I'm not either!
But Eckart and Chris have already found the cubic locus of the centers of the 27 inscribed cardioïds and Bernard Gibert told us this curve was the stelloïd cubic associated to the QL and it's hessian was the curve QL-Cu1, locus of the foci of the inscribed conics (the locus of the centers of the inscribed conics being the Newton Line).
I'd like to know now what the cayleyan of the stelloïd is: this curve is defined by J. Marchand as the envelope of the lines joining the pairs of foci or main axes of the inscribed conics. (This definition includes the 4 lines of the QL, the Newton Line, the 2nd Steiner axis and a few other lines like ST described in EQF at QL-Cu1 ...).
Among or beside the many cubics and quartics described by Eckart, is it possible with algebraic calculations to find the curve responding to this definition?
Many thanks in advance for your inquiry
Best regards
Bernard

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Message: #501

Date: 11/4/2014 4:30:26

From: bernard.keizer

Subject: Dimidium associated quadrilaterals

Dear Chris, dear Eckart, dear friends

I had the idea of these 2 Dimidium-associated QLs after reading Eckart's message #481, but it took me more time than I thought to check the properties. It's more a recreation, but I found it interesting.

I hope you will enjoy it too.

Best regards

Bernard

Dimidium-associated QLs

1. Introduction

- Let's call QL the complete quadrilateral, VR the Van Rees circular isogonal cubic QL-Cu1 and CI-S the Clawson-Schmidt transformation QL-Tf1
1 QL determines at the same time 1 VR and 1 CI-S
1 VR determines 1 CI-S
VR is given by P1, the Newton Line, the Steiner axes and the Schmidt circle
CI-S is given by P1, the Steiner axes and the Schmidt circle
For a VR, there are an infinity of QLs and for a CI-S, there are an infinity of VR and an infinity² of QL's
- In order to have a QL, it's VR and it's CI-S, we need 5 elements, for example a reference triangle and a line or the diagonal triangle and the Newton Line or 5 independant points

2. Converse construction

- If we have P1, P4, P5, we have 3 independant points with the Miquel and Plücker circles.
The 2 circles cut in P1 and another point, which is the reflexion of P1 in P4P5.
If we have the conjugate of this point, which is CSCe(P4P5), this gives the CI-S and we are able to find the Steiner axes and the Schmidt circle with the points F1 and F2.
The perpendicular bisector of the segment joining P1 to CSC(P4) is the Steiner Line, which cuts the Plücker circle in the Plücker points and the perpendicular bisector of the segment joining P1 to CSC(P5) is the CSC of the Plücker circle and cuts the Miquel circle in 2 points, which are the CSC X and Y of the Plücker points.
- Now the choice of the Newton Line gives the 5th element and at the same time the VR and the QL :
 - If we choose the ordinary Newton Line, which is the perpendicular bisector of the segment joining the Plücker points, we find the original QL with P4 as Miquel point and P5 as Clawson center
 - If we swap now the points P4 and P5 and the Miquel and Plücker circles, we swap the same way the Plücker points and their CSC X and Y and have as Newton Line the perpendicular bisector of XY and as Steiner Line the line XY.

3. Properties of the 2 QLs

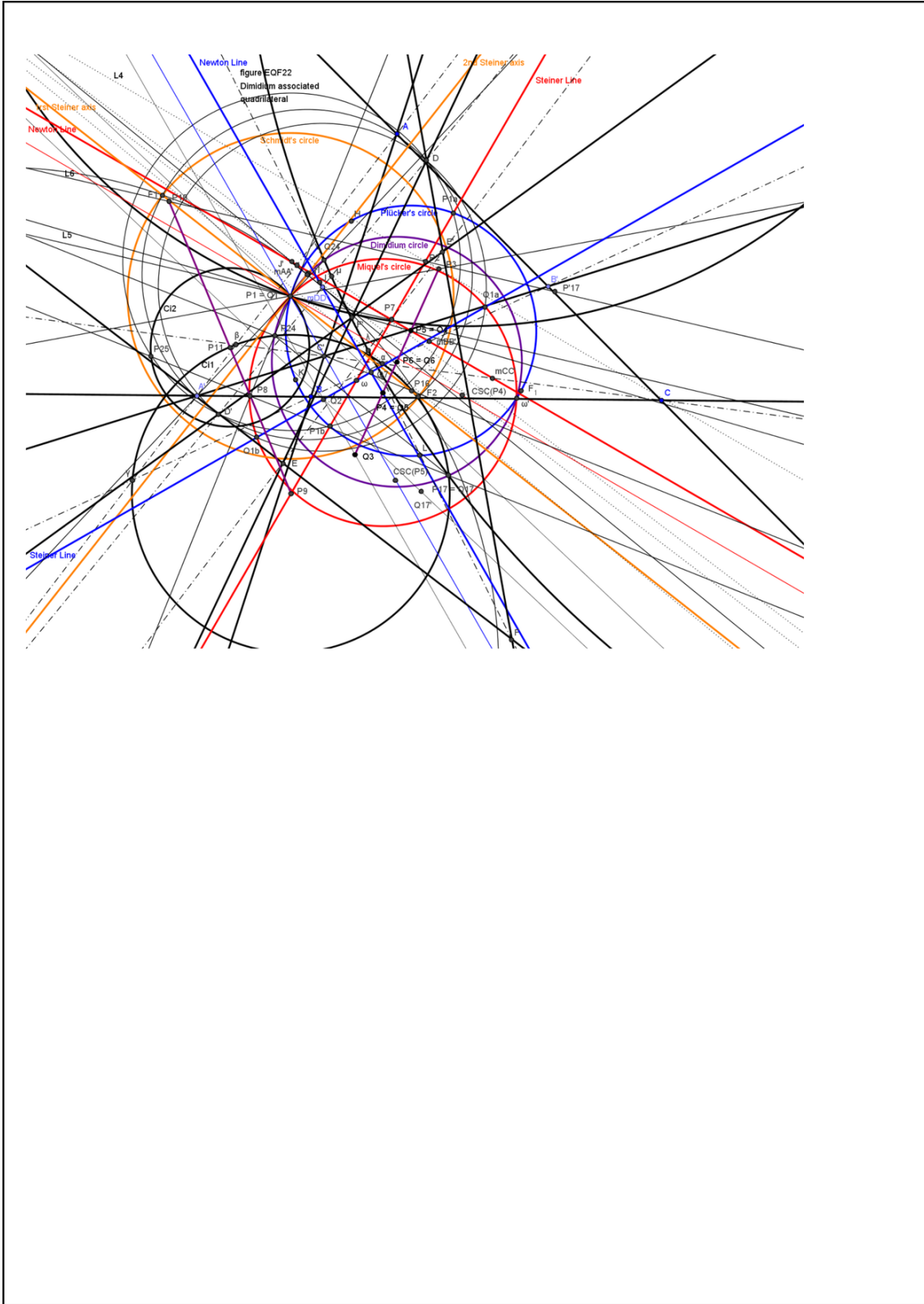
- The 2 QLs share
 - The CI-S with P1, the Steiner axes and the Schmidt circle
 - The Dimidium point P6 and the Dimidium circle, the rectangular hyperbola with center P6 through P4 and P5 and axes parallel to the Steiner axes.
 - The point P17 and therefore the 2nd conic Co (message 481) with the quartic (messages 364 and 465), which passes through the vertices of the 2QLs and through the Plücker points and their CSC X and Y.

It's easy to check that the reflexion of P17 in the Newton Line is on the line L6 for the 2 QLs

- The 2 QLs have symmetrical properties
 - The pedal quadrangles of X and Y wrt the 1st QL are orthocentric ; it's the same for the Plücker points wrt the associated QL
 - The 1st conic Co described in message 471 is inscribed in the DT of the 1st QL and tangent to XY as perpendicular bisector of P1CSC(P5) ; the same conic inscribed in the DT of the associated QL is tangent to the initial Steiner Line, as perpendicular bisector of the initial P1CSC(P4), becoming precisely P1CSC(P5) of the associated QL.
- The 2 diagonal triangles of the QL's share the same Miquel point P17 and the same Cl-S transformation, the vertices of the DT being the CSC-conjugates of the middles of the diagonals and the circle circumscribed to the DT being the CSC-conjugate of the Newton Lines for the 2 QLs.

The figure is terribly complicate, but I hope the idea is clear : the 2 QLs have vertices AA', BB' and CC' and DD', EE' and FF', the 2 DT have vertices α , β and γ and λ , μ and ν .

Despite all my efforts, I didn't find an easy construction, so I used the search of 2 points A and A' (the same for D and D') such as the circle with diameter AA' and center on the Newton Line passes through the Plücker points P1a and b, and there are one circle centered on the 2nd Steiner axis through A and A' and F1 and F2 and it's easy to check that a 3rd circle centered on the other Newton Line passes through A and A' and the points Q1a and Q1b.



Dimidium-associated QLs.docx

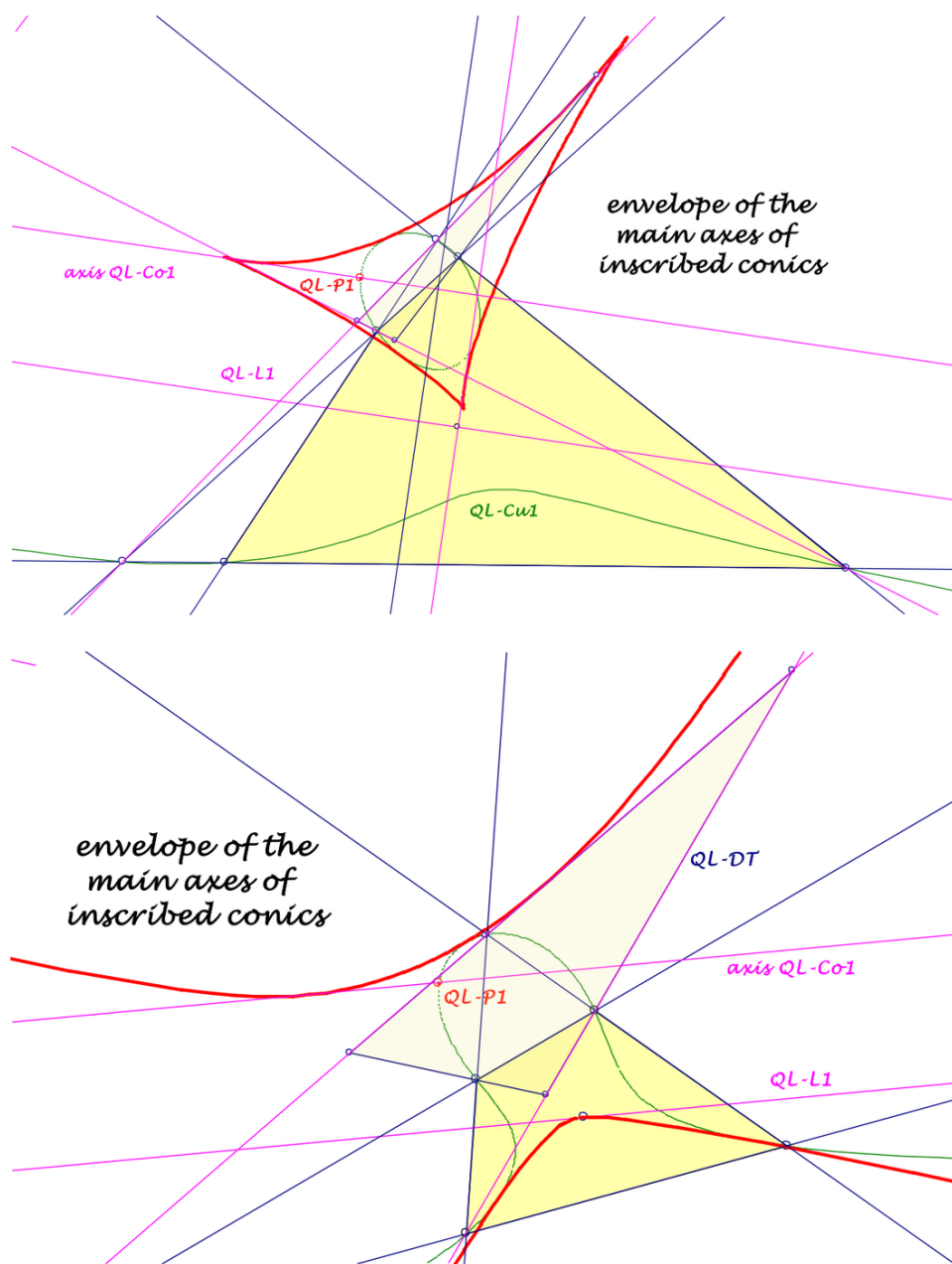
Message: #502
Date: 13/4/2014 9:11:04
From: eckart_schmidt@t-online.de
Subject: Cayleyan

Dear Bernard K.,

only some observations with Cabri wrt the envelope of main axes of QL-inscribed conics, see attachment:

- ... not the four lines of QL are tangent to the curve,
 but the sidelines of QL-DT,
- ... the Newton Line is only in the unipartite case of QL-Cu1
 tangent to the curve,
- ... in the bipartite case of QL-Cu1 a perpendicular to QL-L1
 in the intersection with QL-L6 is tangent to the curve,
- ... a further tangent is the axis of QL-Co1.

Best regards Eckart



2014-04-13.pdf

Message: #503
Date: 14/4/2014 3:39:03
From: eckart_schmidt@t-online.de
Subject: Conic through reflections

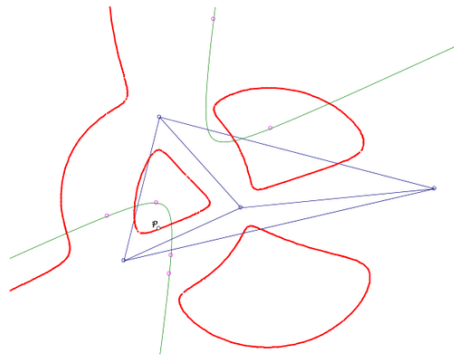
Dear Randy, dear Angel, dear Chris,
some efforts were necessary, but now I can construct the locus
of the points with coconic reflections in the six lines of a
quadrangle, see attachment.
Best regards Eckart

EQF-Note 2014-04-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Coconic Reflections of Points in P_iP_j

This problem is mentioned by Randy Hutson in QFG-message 387: What is the locus of points whose reflections in the six lines P_iP_j of a quadrangle lie on a conic? In QFG-message 391 Angel Montesdeca has given the very extensive equation for the curve of degree seven. For this curve a construction will be described.

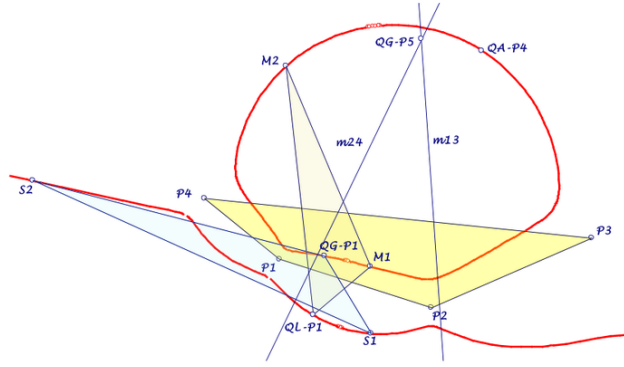


Let P_1, P_2, P_3, P_4 be the points of a quadrangle, but we shall describe the constellation wrt a quadrigon version $P_1P_2P_3P_4$ with

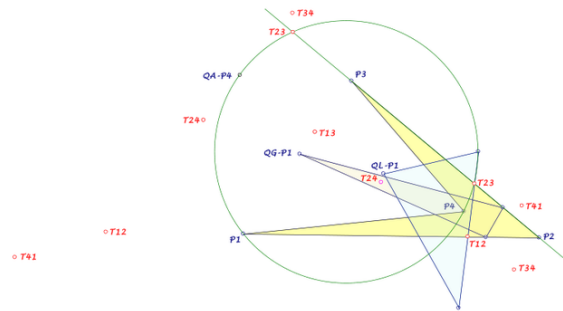
... the Diagonal Triangle $QG-P1, S_1, S_2$,
... the Miquel Triangle $QL-P1, M_1, M_2$.
... the perpendicular bisectors m_{13}, m_{24} of the diagonals,
... the QL -mapping CSC (Clawson-Schmidt Conjugate $QL-TfI$),
... the QG -mapping CIC , which gives for a point X the second intersection of the circles $Ci(X, P_1, P_3)$ and $Ci(X, P_2, P_4)$. This transformation is already mentioned in EQF-Note 2013-12-05.

Properties:

- The basic 7 points of the curve are (see QFG 388, 389)
 - ... the vertices of the Diagonal triangle,
 - ... the vertices of the Miquel Triangle,
 - ... the Isogonal Center $QA-P4$.
- The asymptote is perpendicular to the Steiner Line $QL-L2$ (see QGF 389).



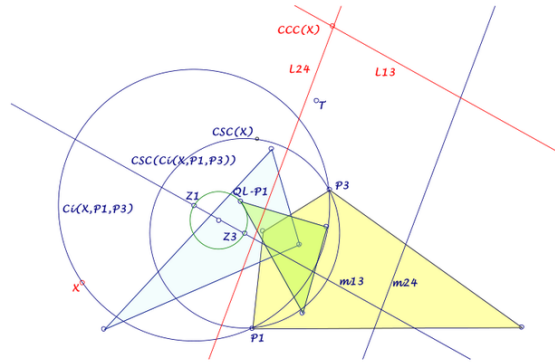
- $QA-P4$ lies either inside of all circumcircles of the triangle components or outside. In the first case the curve is bipartite, in the second case the curve is quadripartite.
- The curve is *CSC*-invariant (see *QFG* 388, 392):
 $\dots CSC(QG-P1) = QA-P4$, $CSC(S_{1,2}) = S_{2,1}$,
 $\dots CSC(QL-P1)$ not defined, $CSC(M_{1,2}) = M_{2,1}$,
 $\dots CSC(QA-P4) = QG-P1$.
- The curve is *CIC*-invariant:
 $\dots CIC(QG-P1)$ not defined, $CIC(S_{1,2}) = M_{2,1}$,
 $\dots CIC(QL-P1) = QA-P4$, $CIC(M_{1,2}) = S_{2,1}$,
 $\dots CIC(QA-P4) = QL-P1$.



- There are further 12 special points on the curve (not all real): $T_{ij} = P_i P_j \cap Ci(QA-P4, P_k, P_l)$ intersections of lines $P_i P_j$ and the circumcircle through the remaining vertices P_k, P_l and $QA-P4$.

Construction:

For construction we need a transformation *CCC* as follows:



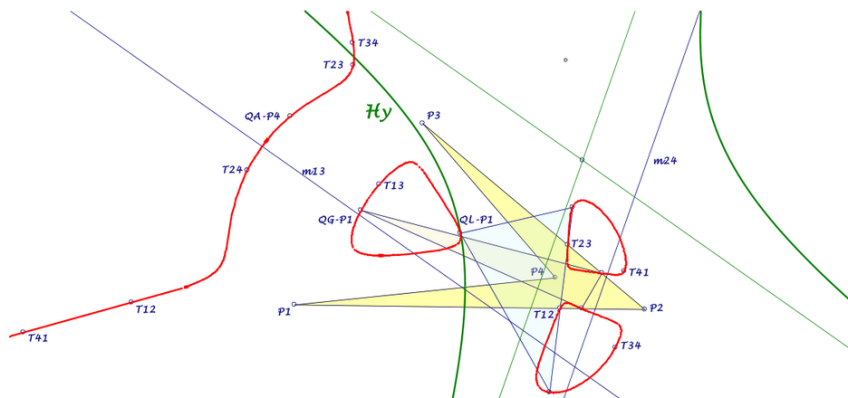
Let X be a variable point,
 ... Z_1 midpoint of the circle $Ci(X, P_1, P_3)$ on m_{13} ,
 ... Z_3 midpoint of $CSC(Ci(X, P_1, P_3))$ on m_{13} ,
 ... circle $Ci(Z_1, Z_3, QL-P1)$,
 ... CSC of this circle gives a line L_{13} parallel m_{13} .
 ... Analog we get a line L_{24} .
 ... The intersection of L_{13} and L_{24} is $CCC(X)$.

$$CCC(QG-P1) = CCC(QA-P4) = CCC(QL-P1) = QL-P1,$$

$$CCC(S_1) = CCC(S_2) = CCC(M_1) = CCC(M_2) = T \text{ (new point),}$$

$$CCC(T_{13}) = CCC(T_{24}) = QL-P1.$$

- The CCC -images of the 7 basic points and the 12 special points (see above) lie on a hyperbola Hy with asymptotes parallel to the perpendicular bisectors m_{13} , m_{24} of the diagonals.
- The inverse transformation of CCC for points on Hy give the locus for points with coconic reflections in P_iP_j .



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-04-14.pdf

Message: #504
Date: 14/4/2014 6:50:12
From: bernard.keizer
Subject: Co1 and Co3: some precisions

Dear Chris, dear Eckart
Most of these properties are well known, but I find interesting
the link with the diagonal QL.
Best regards
Bernard
What happened to the Encyclopedia of Quadrifigures?

Co1 and Co3 : some precisions

The parabola Co1 with P1 as focus and L2 as directrix is tangent to the 4 lines of the QL, to the line L3 and to the sides of the medial triangle of DT.

The parabola with P17 as focus and L6 as directrix is tangent to the 4 lines of the diagonal QL or DQL (3 sides of DT or diagonals of the QL and the Newton Line of the QL) see message 164.

The parabola Co3 with P25 as focus and the parallel to L6 in P9 as directrix is the complement wrt the DT (homothetic in the homothety of center P8 and ratio $-\frac{1}{2}$) of the previous parabola ; this parabola is tangent to the 3 sides of the medial triangle of DT and to the complement of the Newton Line of the QL wrt the DT.

As it is well known, the 2 parabolas Co1 and Co3 pass through the same 4 points, which are the contact points between the parabola Co1 and the 4 lines of the QL (the centroid of the 4 points is on the Newton Line) ; both parabolas are inscribed in the medial triangle of DT.

The 2 parabolas have therefore perfectly symmetric properties :

- Their tangential QLs pass through the same 4 points
- They share the same DT, which is the DT of the QA of the 4 points, and the same points P8, P9, P10 and P11
- They share the same medial triangle of DT
- The 2 foci of the parabolas are P1 and P25 and lie on the medial triangle of the DT
- The 2 directrices of the parabolas and Steiner Lines of the QLs meet in P9
- The 2 Newton Lines of the QLs meet in the centroid of the 4 contact points and intersection points of the 2 parabolas
- Each axis of a parabola and each Newton Line of its tangential QL are orthogonal to the line L5 or P1P7 of the other QL
- Each directrix of a parabola and Steiner Line of its tangential QL is parallel to the line P1P7 or L5 of the other QL
- Last, but not least, as it is the most important property, the complement of the diagonal QL (tangent to the diagonals and to the Newton Line of the QL) wrt the DT (homothetic in the homothety with center P8 and ratio $-\frac{1}{2}$) of one QL is circumscribed to the same parabola as the other QL and vice-versa.

Co1 and Co3.docx

Message: #505
Date: 14/4/2014 9:54:22
From: Antreas Hatzipolakis
Subject: QUADRILATERAL - CIRCLE

Let ABC be a triangle and L a line.

Denote:

A_a = the reflection of A in L

A_b = the reflection of A_a in AB

A_c = the reflection of A_a in AC

Similarly (Cyclically) B_c, B_a and C_a, C_b .

The triangles ABC , bounded by (A_bA_c, B_cB_a, C_aC_b) are orthologic.

The orthologic center $(ABC, (A_bA_c, B_cB_a, C_aC_b))$ lies on the

circumcircle of ABC . It is the antipode of the isogonal

conjugate

of the infinity point of L .

[ie if L_a, L_b, L_c are the parallels through A, B, C , resp., then the reflections of L_a, L_b, L_c in the respective angle bisectors of ABC concur on the circumcircle]

Note: If L passes through O , then A_bA_c, B_cB_a, C_aC_b concur at H .

Application to Quadrilateral:

Let (a, b, c, d) be a quadrilateral.

Denote:

1 = the antipode in the circumcircle of (b, c, d)
of the isogonal conjugate
of the infinity point of a wrt triangle (b, c, d)

2 = the antipode in the circumcircle of (c, d, a)
of the isogonal conjugate
of the infinity point of b wrt triangle (c, d, a)

3 = the antipode in the circumcircle of (d, a, b)
of the isogonal conjugate
of the infinity point of c wrt triangle (d, a, b)

4 = the antipode in the circumcircle of (a, b, c)
of the isogonal conjugate
of the infinity point of d wrt triangle (a, b, c)

The points 1, 2, 3, 4, Miquel_point are concyclic.

Is this circle known?

Antreas

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Message: #506
Date: 15/4/2014 10:03:32
From: Antreas Hatzipolakis
Subject: QUADRILATERAL - CIRCLE

This circle is the same with the circle Chris and I have discussed some time ago in Anopolis list.
The antipode in the circumcircle of the isogonal conjugate of the infinity point of line L can be constructed by this way: Let L_a, L_b, L_c be the reflections of L in the sidelines BC, CB, CA , resp.
The incenter (or an excenter) of the triangle bounded by (L_a, L_b, L_c) lies on the circumcircle and is the antipode of the isogonal conjugate of the infinity point of L ,

Antreas

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Message: #507
Date: 15/4/2014 1:03:20
From: Chris van Tienhoven
Subject: QUADRILATERAL - CIRCLE

Dear Antreas,

Input for your construction is

* a triangle ABC with sidelines a,b,c

* a random line d

The orthologic center (ABC, (AbAc, BcBa, CaCb)) like you describe is the antipode of the isogonal conjugate of the infinity point of L.

I checked if the circle through the 4 antipodes and the Miquel Point of quadrilateral a,b,c,d is known.

In my Cabri drawing it appears that this is the Miquel Circle also passing through the circumcenters of the 4 triangles a,b,c / b,c,d / c,d,a / d,a,b.

See:

<http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-mathematics/quadrilateral-objects/artikelen-ql/148-ql-ci3.html>

Now it also appears that the orthologic center (ABC, (AbAc, BcBa, CaCb)) simply can be constructed as the 2nd intersection point of the circumcircle of ABC and the Miquel Circle of Quadrilateral abcd.

Best regards,

Chris van Tienhoven

Van:

Quadri-Figures-Group@yahoogroups.com

[mailto:Quadri-Figures-Group@yahoogroups.com]

Namens Antreas Hatzipolakis

Verzonden: dinsdag 15 april 2014 10:04

Aan: Hyacinthos; anopolis@yahoogroups.com;

Quadri-Figures-Group@yahoogroups.com

Onderwerp: [Quadri-Figures-Group] Re: QUADRILATERAL - CIRCLE

[Antreas]:

Let ABC be a triangle and L a line.

Denote:

Aa = the reflection of A in L

Ab = the reflection of Aa in AB

Ac = the reflection of Aa in AC

Similarly (Cyclically) Bc,Ba and Ca,Cb.

The triangles ABC, bounded by (AbAc, BcBa, CaCb) are orthologic.

The orthologic center (ABC, (AbAc, BcBa, CaCb)) lies on the circumcircle of ABC. It is the antipode of the isogonal conjugate

of the infinity point of L .

[ie if L_a, L_b, L_c are the parallels through A, B, C , resp., then the reflections of L_a, L_b, L_c in the respective angle bisectors of ABC concur on the circumcircle]

Note: If L passes through O , then $AbAc, BcBa, CaCb$ concur at H .

Application to Quadrilateral:

Let (a, b, c, d) be a quadrilateral.

Denote:

1 = the antipode in the circumcircle of (b, c, d) of the isogonal conjugate of the infinity point of a wrt triangle (b, c, d)

2 = the antipode in the circumcircle of (c, d, a) of the isogonal conjugate of the infinity point of b wrt triangle (c, d, a)

3 = the antipode in the circumcircle of (d, a, b) of the isogonal conjugate of the infinity point of c wrt triangle (d, a, b)

4 = the antipode in the circumcircle of (a, b, c) of the isogonal conjugate of the infinity point of d wrt triangle (a, b, c)

The points 1, 2, 3, 4, Miquel_point are concyclic.

Is this circle known?

This circle is the same with the circle Chris and I have discussed

sometime ago in Anopolis list.

The antipode in the circumcircle of the isogonal conjugate of the infinity

point of line L can be constructed by this way:

Let L_a, L_b, L_c be the reflections of L in the sidelines BC, CB, CA , resp.

The incenter (or an excenter) of the triangle bounded by (L_a, L_b, L_c)

lies on the circumcircle and is the antipode of the isogonal conjugate of the infinity point of L ,

Antreas

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Message: #508
Date: 15/4/2014 5:42:00
From: Antreas Hatzipolakis
Subject: QUADRILATERAL - CIRCLE

Dear Chris,

A problem is:

For which fixed lines the antipodes are concyclic (on the Miquel circle or not)?

In details:

Let (a,b,c,d) be a quadrilateral and L a line.

Denote:

1 = the antipode in the circumcircle of (b,c,d) of the isogonal conjugate of the infinity point of L wrt triangle (b,c,d) .

Cyclically 2,3,4.

For which lines L the four points are concyclic?

(are they necessarily lying on the Miquel circle?)

For L = the Steiner line, the four points coincide with the Miquel point.

Sorry if we have already discussed it (I didn't check)

Antreas

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Message: #509
Date: 15/4/2014 9:01:56
From: eckart_schmidt@t-online.de
Subject: Quadrilateral-Circle

Dear Antreas,

some remarks wrt your cyclic quadrangle (prefix QA) for a
reference quadrilateral (prefix QL):
... the following points coincide:
 QA-P1, 6, 36 = QL-P6 or QA-P3, 4, 8, 12, 32 = QL-P4,
... the lines QA-L1 and QA-L2 are the same, containing
 QA-P1, 2, 3, 4, 6, 7, 8, 9, 12, 32, 33, 34, 36, 37;
 this line is:
 QL-P3.QL-P4.QL-P5.QL-P6 of the reference quadrilateral QL,
... for example:
 QA-P2,37 = QL-P5, QA-P7 divides QL-P3.QL-P4 in the ratio
 2:1,
 QA-P33 is the midpoint of QL-P3.QL-P5.

Remarkable:

... The Simson lines of the Miquel Point QL-P1
 wrt the triangle components of QA give a quadrilateral,
 homothetic to the reference QL with factor 1/2.

Best regards Eckart

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Message: #510
Date: 17/4/2014 10:53:55
From: bernard.keizer
Subject: Cayleyan

Vielen Dank, lieber Eckart
It's exactly what I hoped!
I only regret that the 2 figures are on the same page, 2 pages would give more space to think about and that the unipartite has almost P_1 on a diagonal, which makes this diagonal coincide with the 2nd Steiner axis.
Thanks for having corrected diagonals and not sides of the QL and Newton Line only in the unipartite case and for adding the axis of the parabola.
Among others, we have also the 2nd Steiner axis, the line ST described in $QL-Cu_1$ (S being the point where Cu_1 cuts its asymptote and T its conjugate on the intersection between the perpendicular to L_1 in the intersection with L_6 and the axis of the parabola) and the parallel to the 2nd Steiner axis through S (the 2 conjugate points are the intersections of Cu_1 with the Schmidt circle).
Such a little figure is already very interesting. It appears that the hessian Cu_1 and the cayleyan are tangent in 3 points which lie on a circle through P_1 ; the conjugates of these 3 points are also their tangentials and are therefore their own tangentials, which means that they are the inflexion points of the curve Cu_1 (as it's well known, these 3 points are on a line) as well as of the stelloid or Eckart's cubic Cu_2 .
(I used the Rechnungen auf der Zirkularkurve ...)
I'm convinced, there are many other properties to discover, but that's enough for today!
Best regards
Bernard

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Message: #511
Date: 18/4/2014 9:54:12
From: eckart_schmidt@t-online.de
Subject: Isogonal invariance of QL-Cu1

Dear Bernard K., dear Chris,
the cubic QL-Cu1 is isogonal invariant wrt the three triangle components of the quadrilateral as well as wrt the three QG-Tr3 of the quadrigon versions. But there are infinite further examples of triangles on QL-Cu1 with this property.

Let C_i be a circle through QL-P1, then there are three other intersections (not always real) of C_i and QL-Cu1, defining a triangle Tr. The cubic QL-Cu1 is isogonal invariant wrt Tr. More precisely: For points X on QL-Cu1 the isogonal conjugate wrt Tr is the isogonal conjugate of the triangle components. Example: Orthic triangle of QL-DT.

Further examples can easily be constructed: Let A and B be two points on QL-Cu1, then the third point of the triangle is $A.CSC(B) \wedge B.CSC(A)$.

An interesting example is the triangle of the contact points of QL-Cu1 and the Marchand envelope (see Bernard's messages #500 and #510), the circumcircle contains QL-P1.

Best regards Eckart

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Message: #512
Date: 18/4/2014 2:41:33
From: chris.vantienhoven
Subject: Isogonal invariance of QL-Cu1

Dear Eckart,
These are remarkable properties!
When I remember well I proposed the question to you some years ago for finding a general method in order to get a(nother) reference triangle in a Quadri-cubic. Then we could not solve it.
For this cubic QL-Cu1 you found a wonderful method for all kinds of reference triangles with all special properties you might expect from a reference triangle.
Maybe this property is applicable for more cubics knowing its leading conjugate.
Thanks a lot!
Chris

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Message: #513
Date: 18/4/2014 3:39:10
From: eckart_schmidt@t-online.de
Subject: Isogonal invariance of QL-Cu1

Dear Bernard K., dear Chris,
in addition to my last message: There is a further interesting triangle for isogonal invariance of QL-Cu1. One point of the triangle is QL-P1, the other are the foci of an inscribed conic with center $QL-L1 \wedge QL-L6$ (see QL-2P2). In the unipartite case of QL-Cu1 these foci lie on QL-L1; the corresponding isogonal conjugate for points P on QL-Cu1 is CSC(P). In the bipartite case of QL-Cu1 the foci lie on a perpendicular of QL-L1; the corresponding isogonal conjugate for points P on QL-Cu1 is the reflection of CSC(P) in $CSC(P).QL-P1 \wedge QL-L1$. In the bipartite case QL-Cu1 is an isogonal pivotal circular cubic wrt this triangle and pivot in the point at infinity of QL-L1 (see my homepage, 11.1).
Best regards Eckart

PS. Dear Chris, thanks for positive feedback. I remember your question wrt reference triangles for isocubics, so that they are self-isogonal (E-mail 22.12.2011), but I doubt, that there is a generalisation of the properties for QL-Cu1.

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Message: #514
Date: 18/4/2014 8:48:34
From: chris.vantienhoven
Subject: QUADRILATERAL - CIRCLE

Observations:

- * The points $Pa(L)$, $Pb(L)$, $Pc(L)$, $Pd(L)$ form a Quadrangle and are concyclic. Let us say on circle Cix .
- * The points $Pa(L)$, $Pb(L)$ are collinear with intersection point c^d .
This is cyclically true for all points $Pa(L)$,

The Miquel Point $QL-P1$ lies on Cix .

- * The center X of Cix lies on the Miquel Circle $QL-Ci3$.
- * The circles Cix envelope Morley's Mono Cardioid $QL-Qu1$.
- * The lines through X parallel to L envelope a Deltoid $QL-Qux$.
- * Because $QL-Qux$ is circumscribed around circle $QL-Ci3$ and because $QL-Qu2$ is circumscribed around circle $QL-Ci4$ and because $QL-Ci3$ and $QL-Ci4$ are of the same size, the deltoids $QL-Qu2$ and $QL-Qux$ are of the same size.

Moreover $QL-Qu2$ and $QL-Qux$ are positioned in the same direction.

Best regards,
Chris

p.s. $QL-P1$, $QL-Ci3$, $QL-Ci4$, $QL-Qu1$ and $QL-Qu2$ can be found in the Quadrilateral section of EQF.
(because I am working on a new format in EQF it is a bit messy)

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Message: #515
Date: 20/4/2014 4:03:13
From: Antreas Hatzipolakis
Subject: A quadrangle of quadrangle

I quote my message #21938 †) to Hyacinthos:
Let ABC be a triangle, P, P* two isogonal conjugate points.
Denote:
Ra = radical axis of (NPC_PBC), (NPC_P*BC)
Rb = radical axis of (NPC_PCA), (NPC_P*CA)
Rc = radical axis of (NPC_PAB), (NPC_P*AB)
Which is the locus of P such that Ra,Rb,Rc are concurrent?
The entire plane?
[NPC_PBC means the Nine Point Circle of the triangle PBC]
<https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/21938> †)

As it was proved the locus is indeed the whole plane.
We can apply this to a quadrangle ABCD to get another quadrangle.
Let D' be the point of concurrence of the above radical axes in triangle ABC and point D
and similarly C', B',A'.
Which properties has the quadrangle A'B'C'D'?
Are the lines AA', BB', CC', DD' concurrent?

Antreas

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #516
Date: 20/4/2014 9:39:17
From: chris.vantienhoven
Subject: A quadrangle of quadrangle

[AH] Let D' be the point of concurrence of the above radical axes in triangle ABC and point D and similarly C' , B' , A' . Which properties has the quadrangle $A'B'C'D'$? Are the lines AA' , BB' , CC' , DD' concurrent?

Dear Antreas,

I checked above question.
It's always interesting to check this question for any Triangle Transformation.

It transforms a Triangle Transformation into a Quadrangle Transformation.

I already checked this for many Triangle Transformations and only very few resulted into a perspector.

What you actually do is trying to transform a Triangle Transformation into a Quadrangle Transformation by producing a new Quadrangle.

It is often not the first generation Quadrangle that pays off, but the 2nd generation Quadrangle that pays off.

Examples of 1st generation perspectors:

- * QA-P5
- * QA-P38

Examples of 2nd generation perspectors:

- * QA-P4
- * QA-P7

However alas your P-NPC-Radical-Axis-Triangle Transformation doesn't produce a Quadrangle Perspector.

Neither for the 1st nor the 2nd generation.

Best regards,
Chris van Tienhoven

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Message: #517
Date: 20/4/2014 1:56:28
From: Antreas Hatzipolakis
Subject: A quadrangle of quadrangle

Dear Chris,

Thank you!

So, naturally we have to return to triangle geometry and ask for locus!

That is: Let ABC be a triangle and P a point.

Denote:

A_p, B_p, C_p = the isogonal conjugates of A, B, C , resp.
wrt triangles PBC, PCA, PAB , resp.

$rA1$ = the radical axis of NPC_{ABC} and NPC_{ApBC}

$rA2$ = the radical axis of NPC_{APC} and NPC_{ApPC}

$rA3$ = the radical axis of NPC_{APB} and NPC_{ApPB}

A' = the point of concurrence of $rA1, rA2, rA3$.

Similarly (cyclically) B', C' .

Which is the locus of P such that $ABC, A'B'C'$ are perspective?

Antreas

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Message: #518
Date: 20/4/2014 9:12:40
From: eckart_schmidt@t-online.de
Subject: Quadrilateral-Circle

Dear Chris, dear Antreas,

you discuss special QL-circles Cix through the Miquel Point QL-P1 related to lines L (see #514). Cix depends only on the direction of the line and degenerates for lines L perpendicular to the Newton Line QL-L1.

Some remarks wrt the "ring of transformations" (see #492) for lines L through QL-P1 and corresponding circles Cix:

The Clawson-Schmidt Conjugate QL-Tf1 (shortened CSC) of Cix is a line L' orthogonal to L.

L and L' intersect on QL-L3.

The reflections of QL-P1 in tangents at Cix give a cardioide Cd with cusp QL-P1 and center in the midpoint of Cix.

For a parallel to QL-L1 through QL-P1, Cd is homothetic to QL-Qu1 wrt QL-P1 and factor 2.

CSC(Cd) is a parabola Pb with axis L, dirextrix L' and focus QL-P1.

For a parallel to QL-L1 through QL-P1, Pb is homothetic to QL-Co1 wrt QL-P1 and factor 2.

The reflections of QL-P1 in tangents at Pb give the line L' again.

Best regards Eckart

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Message: #519
Date: 25/4/2014 4:42:07
From: yeuemtrondoitb85
Subject: Three points collinear again

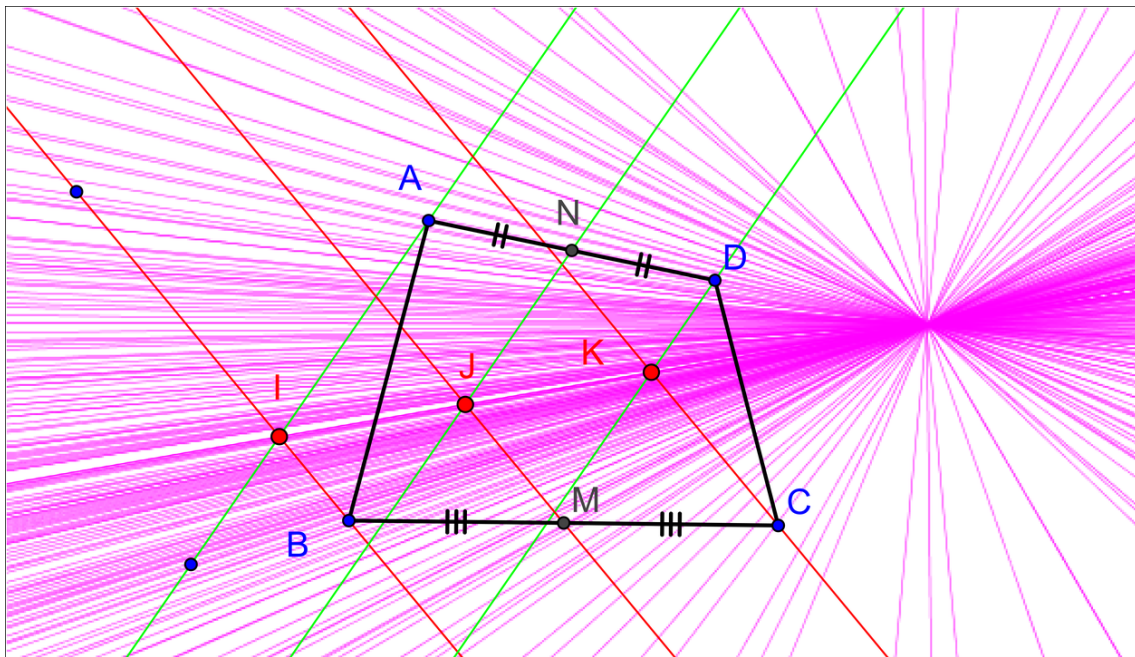
Dear Friends

Please see the picture attachments

Let $ABCD$ be quadrilateral, M, N are midpoints of AD, BC respectively.

Three red parallel lines through B, M, C respectively meet three blue parallel lines through A, ND respectively at I, J, K .
Show that I, J, K are collinear and the line IK through fixed point when three parallel line turn.

Best regards
Sincerely
Dao Thanh Oai



Three points are collinear.png

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Message: #520
Date: 28/4/2014 3:25:30
From: chris.vantienhoven
Subject: Three points collinear again

Dear Dao,

At first glance:

1. This fixed point X lies on AD .
 2. A line through X parallel to the red lines
is tangent to the unique parabola tangent to AB, BC, CD, DA .
- See QL-Co1 in EQF.
I have no proof (yet).

Best regards,
Chris

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Message: #521
Date: 28/4/2014 9:10:36
From: eckart_schmidt@t-online.de
Subject: QUADRILATERAL - CIRCLE

Dear Chris, dear Bernard K.,

a short remark before holiday: With great interest I have studied Chris' message #514, especially the properties of QL-Qux.

They make it possible, to construct the deltoid QL-Qu2 without trisectrices as in EQF.

The deltoid is then an envelope of lines, but is it possible to construct the axes of the deltoid?

I need these axes to construct my cubic QL-Cu2.

I don't know, whether there is an answer in Bernard K's paper, because my French isn't good.

Best regards Eckart

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Message: #522
Date: 02/5/2014 12:15:18
From: bernard.keizer
Subject: QUADRILATERAL - CIRCLE

Dear Chris, dear Eckart, dear Antreas, dear friends

It's a really interesting and amazing construction, which gives a direct link between the parabola and the deltoïd. I hadn't seen it before!

I made a few remarks and proposed a direct construction of Qu2. Sorry for Eckart, I didn't find a direct construction of the axes avoiding the trisection of angles, but I hope my remarks will help (may be with algebraic calculations) ...

Best regards
Bernard

PS Any comments to my messages #501 and #504?

For Eckart, could you make a big figure with stelloïd (your cubic Cu2), hessian (cubic Cu1) and cayleyan (envelop of the main axes of inscribed conics) and if possible with Steiner axes and Newton Line. Thanks in advance, it would help me greatly to pursue the reflexion on these matters ...

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Message: #523
Date: 02/5/2014 6:50:21
From: bernard.keizer
Subject: QUADRILATERAL - CIRCLE

Dear Chris, dear Eckart, dear Antreas and dear friends
Perhaps my last formulation wasn't clear enough.
Of course, the 3 points P on L2 and corresponding O on Ci3 are perfectly determined with trisection of angles.
The P1P are parallel to the axes of the deltoïd and make with the axis of the parabola oriented angles of $\frac{1}{3}$ of the angle made by the axis of the cardioïd with the axis of the parabola $m^\circ \pi/3$, which is the sum of the angles made by the 4 lines of the QL with the Steiner Line L2.
P1P and P1O are symmetric wrt the 1st Steiner axis like the axes of the parabola and of the cardioïd (as CSC-conjugates) and the P1O make with the axis of the cardioïd the same angles ($\frac{1}{3}$ of the sum $\dots m^\circ \pi/3$), but with opposite sign.
(see previous messages, for example #410)
But I didn't find a direct construction without using the trisection.
Best regards
Bernard

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Message: #524
Date: 04/5/2014 5:46:25
From: Antreas Hatzipolakis
Subject: Quadrangle

Let ABC be a triangle and $A'B'C'$ the cevian triangle of P .
Denote:
 Ab, Ac = The circumcenters of APB', APC' , resp.
 Bc, Ba = The circumcenters of BPC', BPA' , resp.
 Ca, Cb = The circumcenters of CPA', CPB' , resp.
 $M1a, M1b, M1c$ = The midpoints of $AbAc, BcBa, CaCb$, resp.
 $M2a, M2b, M2c$ = The midpoints of $BcCb, CaAc, AbBa$, resp.
 $M3a, M3b, M3c$ = The midpoints of $BaCa, CbAb, AcBc$, resp.
The Euler Lines of $M1aM1bM1c, M2aM2bM2c, M3aM3bM3c, M1aM2aM3a, M1bM2bM3b, M1cM2cM3c$ are concurrent (at the same centroid of the six triangles) (proof?)
(from my message Anopolis #1449 †)
Now, in a quadrangle $ABCD$ we have four points of concurrence Pa, Pb, Pc, Pd corresponding to triangles BCD, CDA, DAB, ABC .
Has the quadrangle $PaPbPcPd$ any interesting properties?

Antreas

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Message: #525
Date: 04/5/2014 8:27:25
From: chris.vantienhoven
Subject: Quadrangle

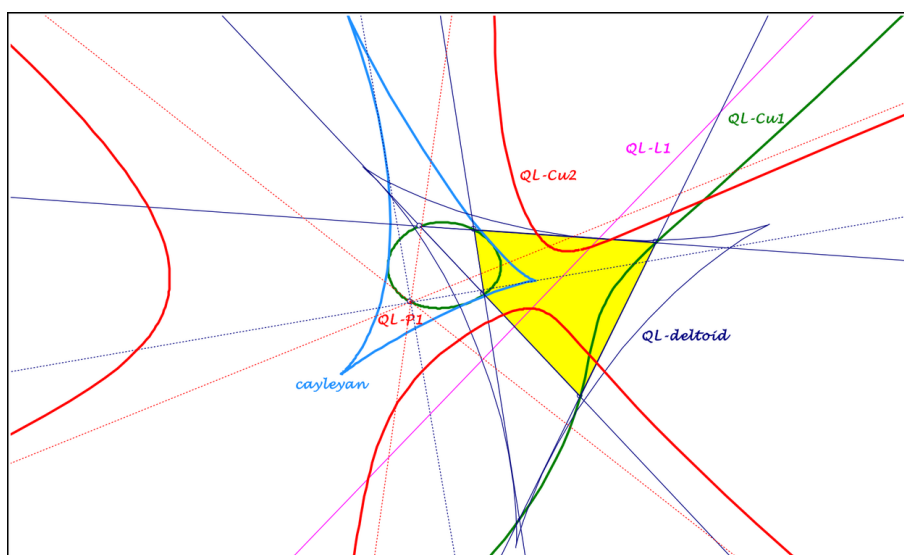
Dear Antreas,
Let $Q(x:y:z)$ = the concurring point of Euler lines.
When $P = X(2)$, then $Q=X(140)$.
When $P = X(4)$, then $Q=X(381)$.
I found no other ETC-matches in the range $P=X(1)-X(25)$.
Your question:
Has the quadrangle $PaPbPcPd$ any interesting properties?
I couldn't find any when taking $P=A$ wrt triangle BCD , etc.
Best regards, Chris

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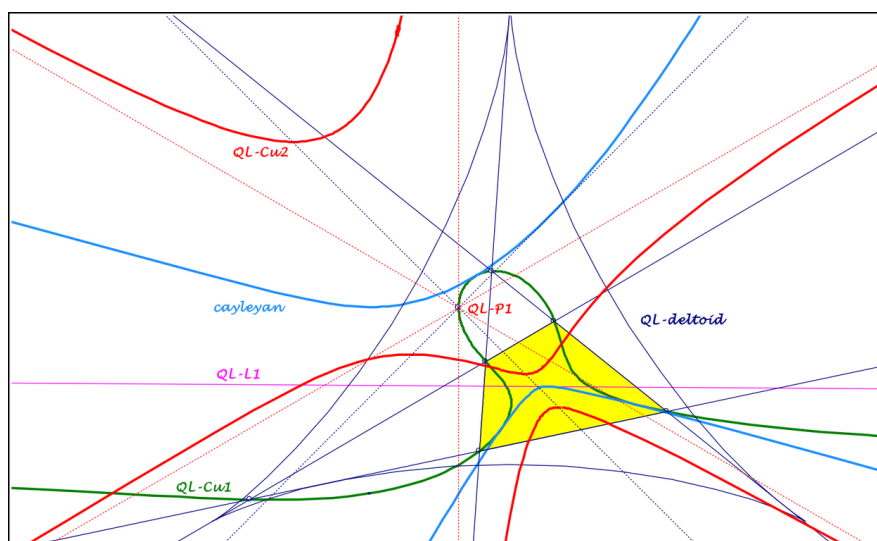
†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[33\]](#).

Message: #526
Date: 09/5/2014 10:52:54
From: eckart_schmidt@t-online.de
Subject: Quadrilateral-Circle

Dear Bernard K.,
back from holiday: Thank you for affirmation that trisection of angles is necessary for construction of the asymptotes of QL-Cu2.
Today only the desired figure. I hope there is no basic mistake in drawing the curves.
Now I have to study your interesting last papers.
Best regards Eckart



2014-05-09.docx



2014-05-09.docx

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Message: #527
Date: 10/5/2014 8:41:43
From: eckart_schmidt@t-online.de
Subject: Quadrilateral Circle

Dear Bernard K.,
remarkable pointwise construction of QL-Qu2 without trisection
of angles in your paper "Parabola and deltoid" (QFG-message
#522)!

This makes it possible, to construct the contact points of
QL-Qu2 and the QL-lines. But I haven't found any properties for
the quadrangle of contact points.

By the way: Is there also a pointwise construction for the
deltoid of the Simson lines for a triangle?

Best regards Eckart

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Message: #528
Date: 11/5/2014 9:27:59
From: eckart_schmidt@t-online.de
Subject: Drawing of QL-Cu2 with reference triangles

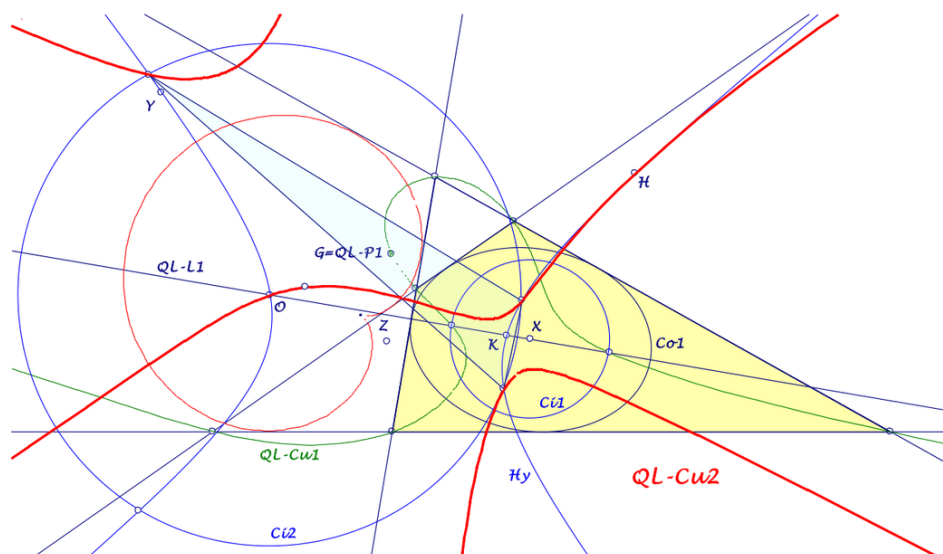
Dear friends,
up to now there is no general construction of the cubic QL-Cu2
for the centers of inscribed cardioids of a quadrilateral. I
have updated a possibility, to draw the cubic with reference
triangles in approximation (see QFG-message #161). You find
examples in the attachment.

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

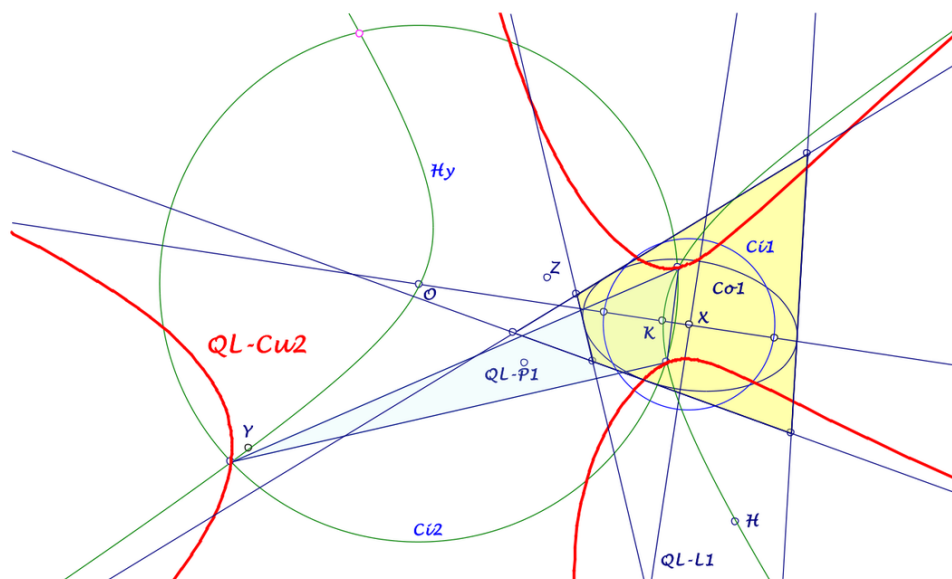
Drawing Eckart's Cubic $QL-Cu2$

If a quadrilateral is tangent to a circle, $QL-Cu2$ is a nodal cubic, described by Bernard Gibert (EQF-Ref.17d) with a construction. If the quadrilateral isn't tangent to a circle here is a possibility to draw the cubic by reference triangles in approximation (see QFG-message 161). Up to now there is no general construction of $QL-Cu2$.



- X is the intersection of $QL-L1$ and $QL-L6$.
- $Co1$ is a conic with center X and tangent to the QL -lines.
- $Ci1$ is the Thales circle for the foci of $Co1$.
- O is a variable point on the axis of $Co1$.
- $Ci2$ is a circle round O orthogonal $Ci1$.
- K is the inverse of X wrt $Ci2$.
- Let $G = QL-P1$, $H = hG, -2(O)$, $Y = hG, -2(K)$.

- H_y is the orthogonal hyperbola through O, H, K, Y with center Z .
- Chose O on the axis of $Co1$ so that the reflection of H in Z lies on $Ci2$.
- Then the further intersections of H_y and $Ci2$ give a reference triangle. G, O, H, K, Y are finally $X2, X3, X4, X6, X69$ of the reference triangle.
- If X lies outward the Schmidt Circle (see $QL-Tf1$), $QL-Cu2$ is the **McCay cubic K003**, if X lies inside the Schmidt Circle $QL-Cu2$ is the **Kjp cubic K024** of the reference triangle ($EQF-Ref.17b$).



Eckart Schmidt
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eckart_schmidt@t-online.de

2014-05-11.pdf

Message: #529

Date: 12/5/2014 8:45:10

From: eckart_schmidt@t-online.de

Subject: Drawing of QL-Cu2 with asymptotes

Dear friends,

in message #528 I have described a possibility to draw the cubic QL-Cu2 with reference triangles; attached here is a further possibility, if the asymptotes are known. It seems, that the asymptotes only can be constructed with trisection of angles, see message #523 of Bernard K. The asymptotes are parallel to the axes of the deltoid QL-Qu2 and intersect in QL-P1. There is a simple construction of QL-Qu2 by Bernard K. in his paper "Parabola and deltoid" attached in message #522, which gives a good approximation for the axes.

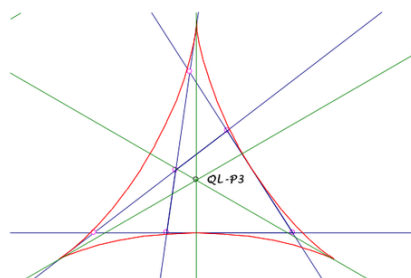
Drawing QL-Cu2 with its asymptotes has the advantage, that it is valid for the McCay type as well as for the Kjp type (without knowledge of these cubics) and also for quadrilaterals tangent to a circle.

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

“Construction” Eckart’s Cubic $QL-Cu2$

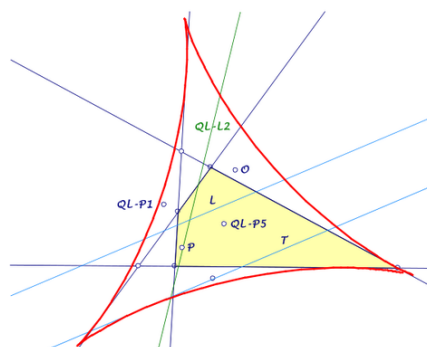
“Construction” in parentheses means: “When the asymptotes are given ...”. The asymptotes of $QL-Cu2$ intersect in the Miquel Point $QL-P1$ with angles of 60° . They are parallel to the axes of the Kantor-Hervey Deltoid $QL-Qu2$, which is tangent to the lines of the quadrilateral and centered in $QL-P3$. This deltoid can be constructed as envelope, so there is a good approximation for the asymptotes.



Kantor-Hervey Deltoid $QL-Qu2$

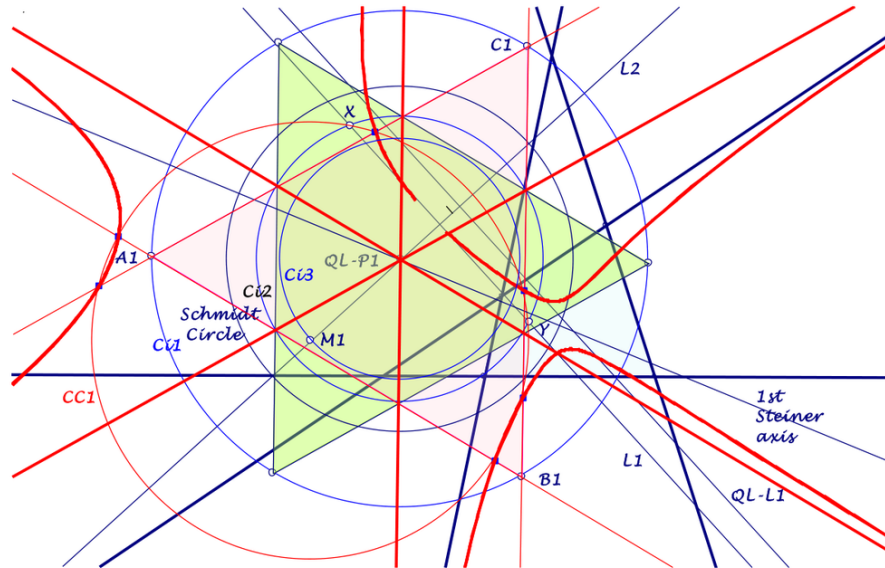
There is a construction of $QL-Qu2$ as envelope (but also pointwise) by Bernard Keizer in his paper “Parabola and deltoid” (*QFG*-message 522):

- P point on $QL-L2$,
- L bisector $P, QL-P1$,
- $O = QL-Tf1(P)$,
- T parallel to L through the reflection of O in $QL-P5$.
- The lines T envelope $QL-Qu2$.



“Construction” of $QL-Cu2$ with given asymptotes:

This “construction” is valid for the McCay type and the Kjp type of $QL-Cu2$ and also for quadrilaterals tangent to a circle.



- $Ci1$ variable circle round the Miquel Point $QL-P1$;
- $A_1B_1C_1$ and $A_2B_2C_2$ equilateral triangles, inscribed $Ci1$, with sides parallel to the given asymptotes;
- $Ci2$ circumcircle for the hexagon of the triangle intersections;
- $L1$ parallel to the Newton Line $QL-L1$ in $2/3$ distance wrt $QL-P1$;
- X, Y intersections of $L1$ and $Ci2$;
- $Ci3$ reflection of $Ci1$ in the Schmidt Circle (see $QL-Tf1$);
- $M1$ and $M2$ intersections of $Ci3$ and a perpendicular line $L2$ through $QL-P1$ wrt $QL-L1$, so that Mi and two points of A_i, B_i, C_i lie on one side of the 1st Steiner Axis (see $QL-Tf1$);
- CCi circles round Mi through X and Y .
- The intersections of CCi and the sidelines of $A_iB_iC_i$ are points of $QL-Cu2$.

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2014-05-12.pdf

Message: #530

Date: 13/5/2014 11:17:16

From: eckart_schmidt@t-online.de

Subject: Forum Geometricorum Volume 14 (2014) 15-29

Dear Chris,

perhaps of interest:

Paris Pamfilos: The Associated Harmonic Quadrilateral.

For a quadrigon ABCD there is described a related cyclic quadrigon M Z1 N Z2 as "associated harmonic quadrilateal" (chapter 2):

M, N midpoints of the diagonals and Z1, Z2 special centers of similarity.

... It is not mentioned, that Z1 and Z2

are the Miquel points of the quadrigons ABDC and ADCB.

The circumcircle of this "associated harmonic quadrilateral"

-not further discussed in the article- is already described in EQF-Note 2013-11-22 (QFG-message #347);

... midpoint QG-P9

The circle contains 8 EQF-points:

... the midpoints of the diagonals,

... two vertices of the Miquel triangle unequal QL-P1,

... the diagonal crosspoint QG-P1,

... QG-P5

... QA-P3,

... QL-P17.

Best regards Eckart

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Message: #531
Date: 13/5/2014 1:48:43
From: chris.vantienhoven
Subject: new release EQF

Dear friends,

I would like to inform you that the site of the Encyclopedia of Quadri-Figures (EQF) has been updated to a new major release. See: <http://chrisvantienhoven.nl/index.php/encyclopedia>. The website is now "responsive", meaning that all pages of the website will adapt themselves to the resolution of the screen of the visitor. This update is important because nowadays people not only look at the site from a laptop or standalone computer but also from a tablet or a smartphone.

Best regards,
Chris

p.s. Within limited time I will also restart updating EQF with new items.

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Message: #532
Date: 13/5/2014 8:32:13
From: Antreas Hatzipolakis
Subject: new release EQF

Dear Chris
I did not see a search possibility.
How about to add it? (powered by Google!)

APH

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Message: #533
Date: 15/5/2014 10:42:52
From: bernard.keizer
Subject: new release EQF

Dear Chris,
Alleluiah !
It's Christmas before Christmas !
I'm awaiting now with great impatience for the new items ...
Best regards
Bernard

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Message: #534
Date: 15/5/2014 10:55:40
From: bernard.keizer
Subject: Quadrilateral-Circle

Lieber Eckart,
Vielen Dank für die Figuren !
It will take now some time to study them ...
I've found new interesting references about cubics, hessians and cayleyans.
May be you know them already: Heinrich Schröter Die Theorie der ebenen Kurven dritter Ordnung and Heinrich Wieleitner Theorie des ebenen algebraischen Kurven höherer Ordnung
Best regards
Bernard

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Message: #535
Date: 15/5/2014 11:15:20
From: bernard.keizer
Subject: Drawing of QL-Cu2 with asymptotes

Dear Eckart, dear Chris and dear friends,
As it is necessary to have an exact drawing of the asymptotes of the curve QL-Cu2, I give you a construction adapted from a Chasles construction for the trisection of the angle. It's possible to use it with Cabri, may be it will be possible to calculate the exact coordinates with Mathematica ...
Best regards
Bernard

Trisection of the angle : the Chasles construction

Let's consider the following figure with Miquel circle, center P_4 and point P_1 on the circle.

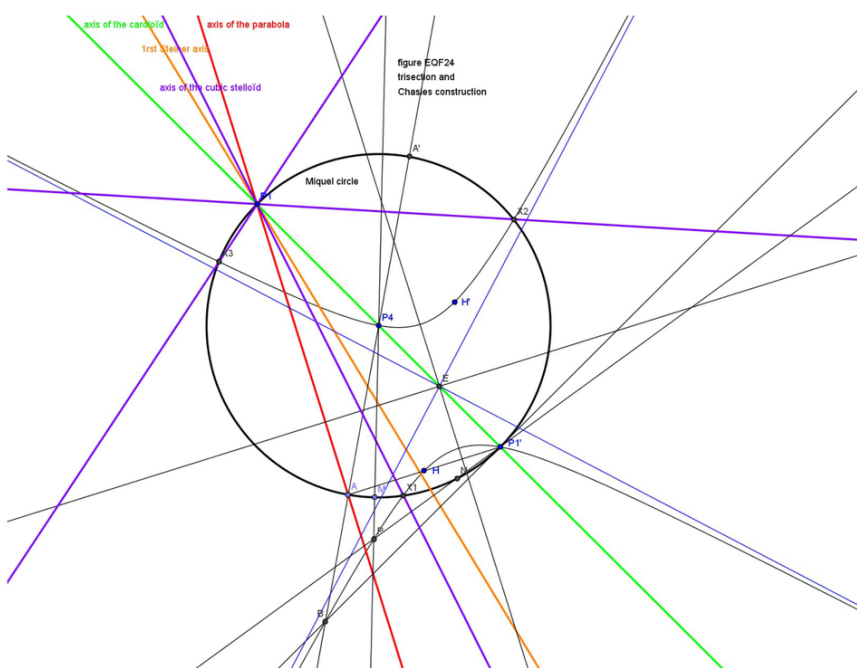
The axis of the parabola is P_1A , the axis of the cardioid is P_1P_1' with P_1' antipode of P_1 . The point B is the intersection between P_4A and the tangent to the circle in P_1' . E is the middle of P_4P_1' , H the middle of AP_1' and H' the middle of $P_1'A'$.

Let's now consider the following construction, adapted from Chasles : M is a point on the Miquel circle and N the point on the circle such as $\text{arc } NP_1' = 2 \text{ arc } AM$; P is the intersection point between P_4M and PP_1' .

The locus of P is a curve through the 3 points X_1 , X_2 and X_3 which trisect the angle AP_4P_1' and therefore the angle AP_1P_1' ; the curve pass through the points B (for M in A or A'), H (for M in the middle of the arc AP_1'), P_1' (for M in P_1'), P_4 (for M on the perpendicular to P_4A) and H' (for M in the middle of the arc $A'P_1'$). P_4 is the orthocenter of the triangle $X_1X_2X_3$ and $P_4HP_1'H'$ is a rectangle.

The locus of P is the rectangular hyperbola with center E , axes of symmetry the perpendicular bisectors of P_4H and P_4H' and asymptotes the bisectors of these axes.

It's possible with Cabri to draw the hyperbola through the 5 points P_4 , H , P_1' and H' and to find the intersection with the Miquel circle P_1' , X_1 , X_2 and X_3 .



Trisection of the angle.docx

P1X1, P1X2 and P1X3 are the asymptotes of the cubic stelloïd (Eckart's cubic QL-Cu2).

The axes of the deltoïd are the parallel to these 3 asymptotes through P3.

It's also easy to show that the parallels to these 3 asymptotes through P4 cut the Miquel circle in the 3 points which trisect the arc of circle P1A ; that proves that the reflexion of the axis of the parabola P1A in P5 (which is the asymptote of the hessian QL-Cu1) is trisected by the relexions of these 3 points in P5 and is therefore tangent to the deltoïd QL-Qu2.

Trisection of the angle.docx

Message: #536
Date: 15/5/2014 11:20:54
From: bernard.keizer
Subject: Quadrilateral Circle

Dear Eckart,
It's always the same construction: the intersection between the Simson Lines and the Euler circle of the triangle gives 2 points and the contact point with the Steiner deltoïd is the reflexion of one of the 2 points in the other.
You may, for example, choose 2 of the 4 lines L_i and L_j as sides of a main triangle of the deltoïd: the points h_i and h_j are the reflexions of the circumcenters O_i and O_j in P_5 , the points q_i and q_j are the other intersections of L_i and L_j with the Hervey circle, the reflexions of the intersection S_{ij} of L_i and L_j in q_i and q_j are the 2 other vertices from a main triangle of the deltoïd having the Hervey circle as Euler circle and the Simson Lines of this triangle envelop the deltoïd, the contact point being the reflexion of one point in the other.
Of course, the Steiner deltoïd is tangent to the 3 sides and the 3 altitudes of the triangle.
Best regards
Bernard

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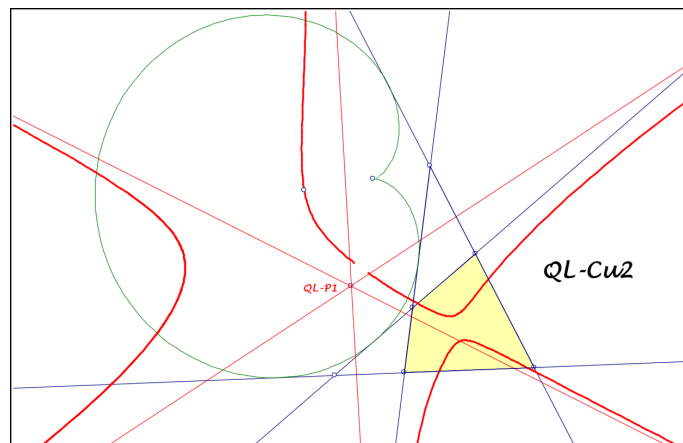
Message: #537
Date: 15/5/2014 2:55:07
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,
WONDERFUL! The Chasles construction was completely new for me.
Now there is a right construction for my cubic QL-Cu2. Thank you very much!
Best regards Eckart

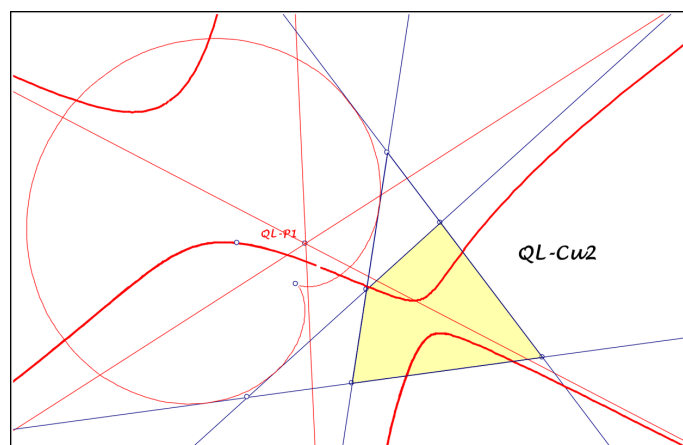
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Message: #538
Date: 16/5/2014 9:41:35
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,
still I am fascinated of the Chasles construction (attached two
now possible constructions for QL-Cu2)!
There is a typo in your paper, line 7: P is the intersection
point between P4M and NP1'.
Thanks also for the explanations in message #536.
First I had difficulties with the Euler circle, which is the
nine-point-circle.
In line 9 there is a typo: ... the reflections of the
intersection S_{ij} of L_i and L_j in h_i and h_j are ...
Best regards Eckart



2014-05-16.pdf



2014-05-16.pdf

Message: #539
Date: 16/5/2014 11:10:25
From: chris.vantienhoven
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
Beautiful pictures!
Only the essentials!
Chris

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Message: #540
Date: 16/5/2014 9:38:25
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
The figures are indeed fascinating, but lack of explanation!
I suppose the construction is the one explained in message #529?
ok for the typo in line 7, anyhow the figure was correct.
For the message #536, the nine-point-circle is also called Euler circle or Feuerbach circle (see Mathword).
For the other typo, you're wrong. If I call T_{ij} the described triangle, the perpendicular to L_i and L_j in h_i and h_j intersect in a point H_{ij} and the perpendicular to L_i and L_j in q_i and q_j intersect in a point O_{ij} . H_{ij} is the orthocenter and O_{ij} the circumcenter of T_{ij} and the middle of $H_{ij}O_{ij}$ is the point P_3 , which is the Kantor-Hervey point and the circumcenter of the Hervey circle, which is the Euler circle of T_{ij} . Therefore, the deltoïd Qu_2 is the Steiner deltoïd of T_{ij} (there are 6 such main triangles of the deltoïd, one for each vertice of the QL).
And the 2 other vertices of T_{ij} are the reflexions of S_{ij} in q_i and q_i , which are the middles of the 2 sides on L_i and L_j and lie on the Hervey circle or Euler circle of T_{ij} .
I hope this explanation is more clearer ...
Best regards
Bernard

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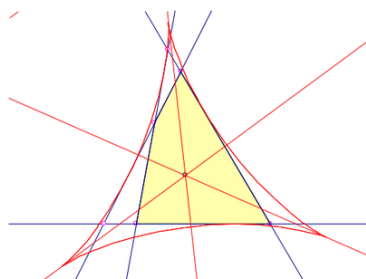
Message: #541
Date: 17/5/2014 10:58:33
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear friends,
I have updated my drawing of QL-Cu2 to a real construction, see attachment.
Thanks to Bernard Keizer!
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

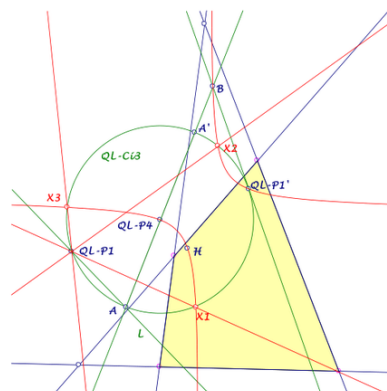
Construction of Eckart's Cubic $QL-Cu2$

$QL-Cu2$ is the cubic for the centers of 27 cardioids tangent to four lines. The construction of $QL-Cu2$ depends on the asymptotes, intersecting in the Miquel Point $QL-P1$ with angles of 60° . They are parallel to the axes of the Kantor-Hervey Deltoid $QL-Qu2$, which is tangent to the lines of the quadrilateral and centered in $QL-P3$.



Axes of the cubic $QL-Cu2$

There is a construction of the axes of $QL-Cu2$ by Bernard Keizer in his paper "Trisection of the angle: the Chasles construction" (*QFG*-message 535):

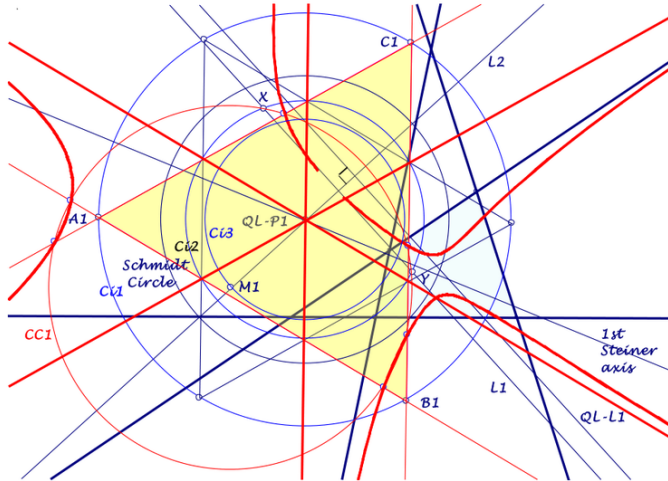


- $QL-Ci3$ Miquel Circle with midpoint $QL-P4$;
- $QL-P1$ Miquel Point on $QL-Ci3$ with antipode $QL-P1'$;
- L parallel to the Newton Line $QL-L1$ through $QL-P1$;
- A second intersection of L and $QL-Ci3$;

- H midpoint of A and $QL-P1$;
- B intersection of $A, QL-P4$ and the tangent in $QL-P1$ at $QL-Ci3$;
- H_y orthogonal hyperbola through $QL-P4, QL-P1', H, B$;
- $X1, X2, X3$ further intersections of H_y and $QL-Ci3$;
- $Xi, QL-P1$ are the asymptotes.

Construction of $QL-Cu2$ with given asymptotes

This construction is valid for the McCay type and the Kjp type of $QL-Cu2$ and also for quadrilaterals tangent to a circle.



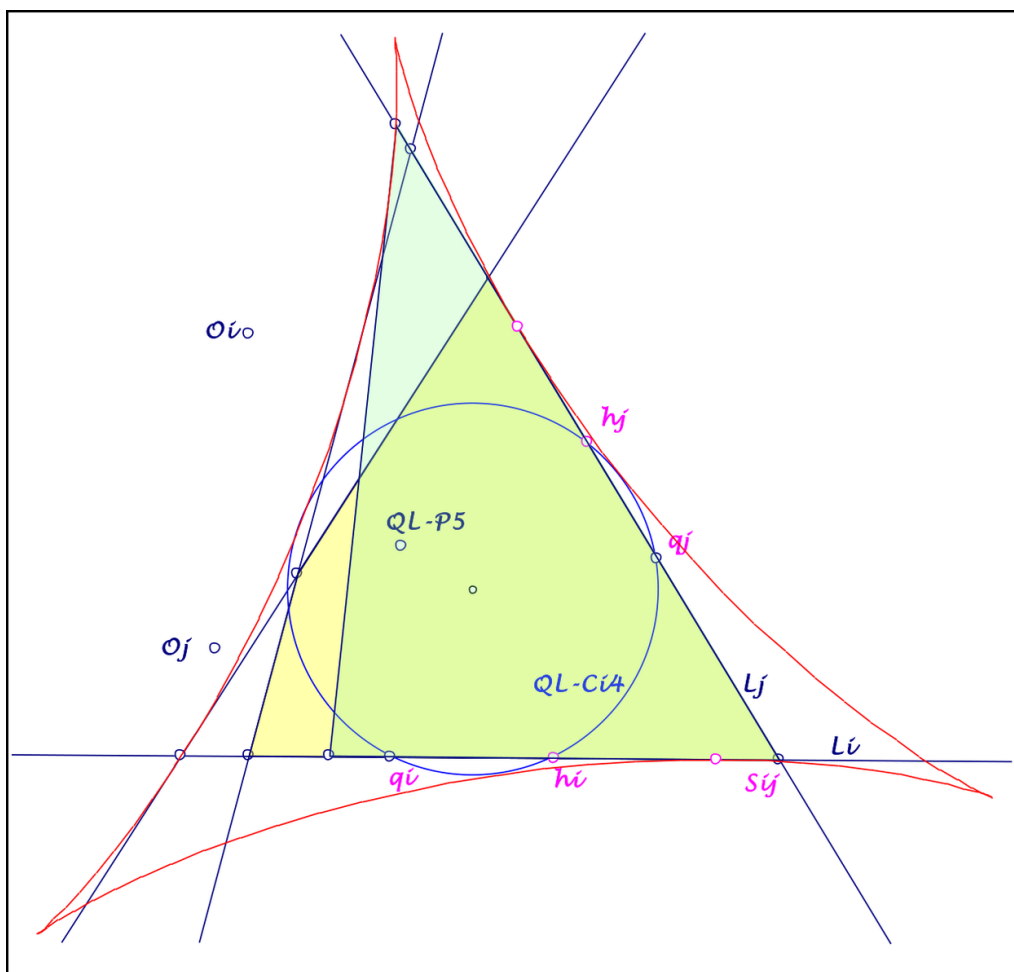
- $Ci1$ variable circle round the Miquel Point $QL-P1$;
- $A_1B_1C_1$ and $A_2B_2C_2$ equilateral triangles, inscribed $Ci1$, with sides parallel to the given asymptotes;
- $Ci2$ circumcircle for the hexagon of the triangle intersections;
- $L1$ parallel to the Newton Line $QL-L1$ in $2/3$ distance wrt $QL-P1$;
- X, Y intersections of $L1$ and $Ci2$;
- $Ci3$ reflection of $Ci1$ in the Schmidt Circle (see $QL-Tf1$);
- $M1$ and $M2$ intersections of $Ci3$ and a perpendicular line $L2$ through $QL-P1$ wrt $QL-L1$, so that Mi and two points of Ai, Bi, Ci lie on one side of the 1st Steiner Axis (see $QL-Tf1$);
- CCi circles round Mi through X and Y .
- The intersections of CCi and the sidelines of $AiBiCi$ are points of $QL-Cu2$.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-05-17.pdf

Message: #542
Date: 17/5/2014 11:56:53
From: eckart_schmidt@t-online.de
Subject: Quadrilateral circle

Dear Bernard K.,
wrt the typo in message #536: I have drawn my understanding in
the attached file.
Best regards Eckart



2014-05-17a.pdf

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Message: #543
Date: 17/5/2014 1:30:14
From: Bernard Gibert
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart and Bernard K.
I've added a simple construction of the asymptotes of $E(L)$, see §5.1 in my paper.
Maybe it corresponds to those you have proposed, I didn't check.
I'm still convinced that there is a relatively simple construction of the cubic...
Best regards
Bernard G.

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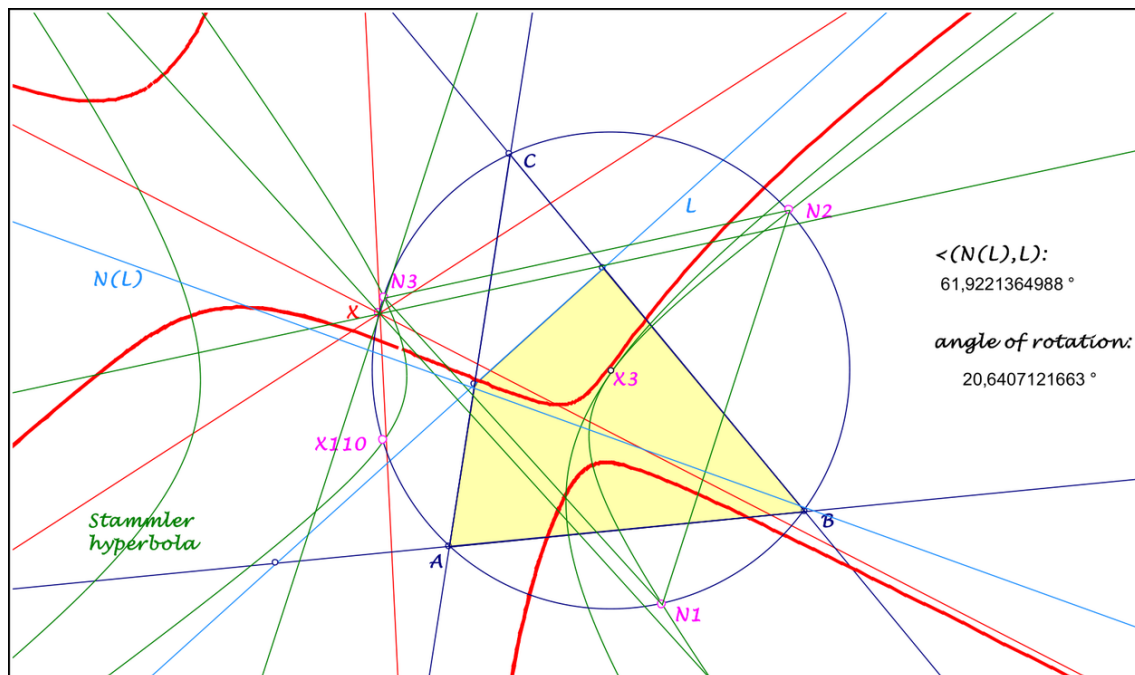
Message: #544
Date: 17/5/2014 6:34:39
From: chris.vantienhoven
Subject: new release EQF

Dear Antreas,
Thanks for your good idea.
I implemented the search option in the menu of my site.
I noticed that the SEARCH-option can be used:
* as cross-reference, searching where some EQF-item has been named.
* searching where some reference has been used in EQF. For example search "Ref-8".
* as quick search for an EQF-item, the items being presented without left Index column.
Best regards,
Chris.

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Message: #545
Date: 18/5/2014 9:34:24
From: eckart_schmidt@t-online.de
Subject: Drawing of QL-Cu2 with asymptotes

Dear Bernard G.,
sorry, I can not follow your construction of the asymptotes of $E(L)$ in point 3 of your description. I have made a drawing, attached. Can you please find my mistake? Thanks in advance.
Best regards Eckart



Bernard-G-2014-05-18.pdf

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Message: #546
Date: 18/5/2014 11:39:39
From: Bernard Gibert
Subject: Drawing of QL-Cu2 with asymptotes

Dear Eckart,

>> sorry, I can not follow your construction of the asymptotes
of $E(L)$ in
>> point 3 of your description. I have made a drawing, attached.
Can you
>> please find my mistake?

at first sight, it's not you but me who is responsible of a
mistake.
I observe that your angle of rotation is one third of the angle
 $N(L), L$.
I'll check that later on...

Best regards
Bernard G.

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Message: #547
Date: 18/5/2014 4:16:55
From: bernard.keizer
Subject: Quadrilateral circle

Lieber Eckart,
Ich fühle mich diesmal etwas geniert und bitte um
Entschuldigung, Du hast völlig Recht!
If I can't make the distinction between the middle of the side
and the foot of the altitude, it looks like most of the
Europeans, who can't make a distinction between their right and
their left! I'm afraid, the result next week will be disastrous
for Europe!
To be serious, the middles of the sides of T_{ij} are the h_i and h_j
and the feet of the altitudes the q_i and q_j .
I know where my confusion came from; my initial construction was
following. If you consider the 2 reference triangles with sides
on L_i and L_j , called T_k and T_l , the circumcenters are O_k and O_l
and the orthocenters H_k and H_l ; then H_{ij} is the intersection
between the perpendiculars to $S_{ij}H_k$ and $S_{ij}H_l$ in H_k and H_l and
 O_{ij} is the intersection between the perpendiculars to $S_{ij}O_k$ and
 $S_{ij}O_l$ in O_k and O_l . That was explained in my article on page 23,
but when I looked at it rapidly before answering you, I mixed
the H_i (orthocenters of the reference triangles of the QL) and
the h_i (orthocenters of the triangles $O_jO_kO_l$ and reflexions of
the O_i in P_5).
The 2 constructions give obviously the same result and the main
point is the the middle of $H_{ij}O_{ij}$ is P_3 , which makes the Hervey
circle the Euler circle of the triangle T_{ij} and the deltoïd Qu_2
it's Steiner deltoïd.
I apologise again and thank you for your patience
Best regards
Bernard
PS I will concentrate now on your construction of the stelloïd
 Cu_2 , the hessian Cu_1 and the cayleyan.

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Message: #548
Date: 19/5/2014 10:27:37
From: Antreas Hatzipolakis
Subject: Simson Lines - Quadrangle

In a recent discussion in my group Anopolis I asked a number of locus questions related to Poncelet Point.

This is related to quadrangle:

Let ABC be a triangle, P a point and P* the Poncelet point related to P

(ie the point the NPCs of PBC, PCA, PAB and ABC concur at).

Let S1, S2, S3 be the Simson lines wrt medial triangles of PBC, PCA, PAB, resp.

Which is the locus of P such that S1, S2, S3 are concurrent?

The locus is the entire plane. Cesar Lozada computed the point of concurrence Zm:

$$Z_m = (u \cdot v \cdot w \cdot (a \cdot (-3 \cdot S^2 + SB \cdot SC) \cdot u + b \cdot (SB \cdot (2 \cdot SB + SC) - S^2) \cdot v + c \cdot (SC \cdot (2 \cdot SC + SB) - S^2) \cdot w) + a^3 \cdot b \cdot c \cdot v^2 \cdot w^2 + a \cdot b \cdot c \cdot u^2 \cdot (SB \cdot v^2 + w^2 \cdot SC)) / a : : \\ \text{ETC-Pairs } (P, Z_m) = (2, 230), (7, 3911), (80, 1387), (99, 620), (100, 3035)$$

See fwded message to Hyacinthos #22304 †)

I do not know if this result is already known, since I do not have seen it before. Please let me know if you have references.

Now, and the Simson line S0 of P* wrt ABC passes through the point of concurrence of S1, S2, S3 as well.

So we have a point in a quadrangle ABCD.

(the point where the Simson lines of the Poncelet-Euler point of the quadrangle wrt medial triangles BCD, CDA, DAB, ABC concur)

Is it a known quadrangle point?

Antreas

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Message: #549
Date: 19/5/2014 12:51:06
From: eckart_schmidt@t-online.de
Subject: Simson Lines - Quadrangle

Dear Antreas,
the asked point is the Parabola Axes Crosspoint QA-P6.
Best regards Eckart

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #550
Date: 19/5/2014 1:02:19
From: Antreas Hatzipolakis
Subject: Simson Lines - Quadrangle

Dear Eckart
Thank you!
I should have made a search from EQF page (with keyword Simson),
APH

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Message: #551
Date: 19/5/2014 3:54:05
From: chris.vantienhoven
Subject: Simson Lines - Quadrangle

Dear Antreas,
QA-P2 = the common point of the medial circles of the Component Triangles of a Quadrangle.
Since the Simson lines only are produced from points on a circle it is a logical thing to investigate what the Simson lines do of the common point of 4 circles.
As Cesar and Antreas pointed out in a Quadrangle these 4 Simson lines concur.
As Eckart Schmidt pointed out this point happens to be QA-P6:
<http://www.chrisvantienhoven.nl/quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/27-qa-p6.html>
Now I wondered in a similar situation where the Miquel point (QL-P1) is a common point of the 4 circumcircles of the Component Triangles of a QUADRILATERAL (system of 4 lines) what the 4 Simson lines do.
I noticed that in all 4 cases these 4 Simson lines concur!
The concurring line is the Pedal Line QL-L3.
It is always thrilling what happens in geometry!
Best regards,
Chris

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Message: #552
Date: 20/5/2014 9:51:48
From: Antreas Hatzipolakis
Subject: Seebach-Walser Line

Dear Chris,
A search in EQF with keywords "Seebach-Walser Line" returned:
Zoekresultaten voor Seebach-Walser Line Er zijn geen resultaten gevonden
So I guess this line is not in EQF (at least with that name)
Here is a description:
178
The Seebach-Walser Line of a Quadrangle
Rudolf Fritsch and Gunter Pickert
This is the 1st of a three part series of articles investigating the centroid of vertices, centroid of perimeter and centroid of area for a plane noncrossed quadrangle. In this 1st installment, it is shown that the centroid of vertices, centroid of area and the point of intersection of the diagonals of a convex quadrilateral, not a parallelogram, are collinear
Crux 39: No 4 April / Avril 2013
http://cms.math.ca/crux/v39/n4/public_synopsis_v39n4.pdf
APH

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Message: #553
Date: 20/5/2014 12:47:25
From: eckart_schmidt@t-online.de
Subject: Seebach-Walser Line

Dear Antreas,
the Seebach-Walser Line is $QA-P1.QG-P4.QG-P1 = QG-L3$.
Best regards Eckart

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Message: #554
Date: 21/5/2014 7:17:47
From: chris.vantienhoven
Subject: Seebach-Walser Line

Dear Antreas, dear Eckart,
I will make a notification in EQF for this new name of QG-L3.
As a matter of fact in EQF this line simply is called the
QG-Centroids Line (QG-L3).
There are also the QA-Centroids Line (QA-L3) and the
QL-Centroids Line (QL-L8).
See:
QG-L3:
<http://www.chrisvantienhoven.nl/index.php/16-mathematics/encyclopedia-of-quadri-figures/quadrignon-objects/artikelen-qg/182-qg-l3>
QA-L3:
<http://www.chrisvantienhoven.nl/index.php/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-objects/artikelen-qa/69-qa-l3>
QL-L8: <http://www.chrisvantienhoven.nl/index.php/17-mathematics/encyclopedia-of-quadri-figures/quadrilateral-objects/artikelen-ql/144-ql-l8>
Showing very nice the differences between a Quadrilateral (QL),
Quadrangle (QA) and a Quadrignon (QG).
Best regards,
Chris

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Message: #555
Date: 21/5/2014 1:29:37
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

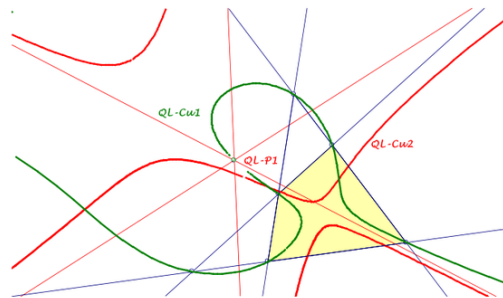
Dear friends,
in message #541 I had described a construction for QL-Cu2, here
is another construction, using the asymptotes and the Hessian
QL-Cu1.
Best regards Eckart

EQF-Note 2014-05-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Construction of Eckart's Cubic $QL-Cu2$ (2nd possibility)

$QL-Cu2$ is the cubic for the centers of 27 cardioids tangent to four lines. A first construction with the asymptotes is described in EQF-Note 2014-05-17 (QFG-message 541), here is a further possibility, using the asymptotes and the Hessian $QL-Quasi$ Isogonal Cubic $QL-Cu1$, which are constructible.

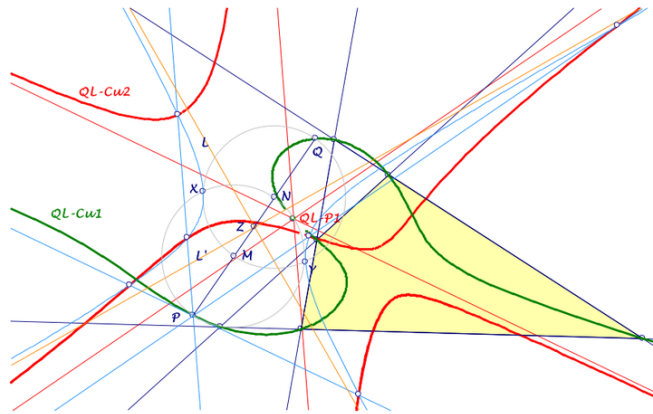


The asymptotes of $QL-Cu2$ intersect in the Miquel Point $QL-P1$ with angles of 60° . They are parallel to the axes of the Kantor-Hervey Deltoid $QL-Qu2$, which is tangent to the lines of the quadrilateral and centered in $QL-P3$. There is a construction of the asymptotes – related to Bernard Keizer – in EQF-Note 2014-05-17.

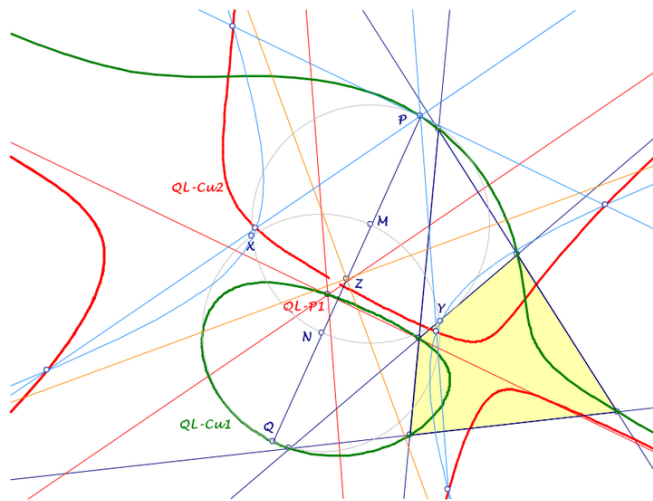
The Hessian of $QL-Cu2$ is $QL-Cu1$. There are several constructions discussed in QFG-messages. Here a short summary (see QFG-message 188). There are two possibilities, see brackets:

- Let $F1, F2$ be the foci of an inscribed conic with center $QL-L1 \cap QL-L6$,
- $C1$ circle with diameter $F1F2$.
- If the main axis of the inscribed conic is $QL-L1$ (orthogonal $QL-L1$),
- let O be a variable point on $QL-L1$
- as midpoint of a circle $C2$ perpendicular $C1$ (through $F1, F2$),
- then the lines $O, QL-P1$ cut $C2$ in points of $QL-Cu1$.

Construction of $QL-Cu2$



- P point on $QL-Cu1$,
- $Q = QL-Tf1(P)$ Clawson-Schmidt Conjugate of P ,
- Z midpoint $P.Q$,
- M, N points trisecting $P.Q$,
- X, Y intersections of circle round M through N and circle round N through M ,
- L, L' parallels through Z wrt the angle bisectors of $QL-L1$ and $QQL-P1$,
- Hy orthogonal hyperbola through X, Y with axes L, L' .
- Parallels to the asymptotes through P cut the orthogonal hyperbola Hy in points of $QL-Cu2$.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-05-21.pdf

Message: #556
Date: 21/5/2014 10:03:05
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
I had just finished to realise your first construction that you already propose a second one!
The first one worked and I wanted to tell you how happy I was to reproduce the different types McCay if the Newton Line cuts the Steiner axis between F1 and F2 (the invariant points of the Cl-S transformation) with the hessian unicursal, Kjp if it cuts outside with the hessian bicursal and even nodal with the hessian nodal too, the node being F1 or F2 for both curves. To be honest, the second one seems much easier! I'll check completely and make you a longer answer later.
I have also ideas concerning the cayleyan ...
Just 2 simple remarks on the construction of Cu1. First, it would be better to keep the same names in different papers: F1 and F2 are the intersection points between the Schmidt circle and the 1st Steiner axis, you could name your 2 points like Bernard Gibert R1 and R2 on the Newton Line for the unicursal curve and B1 and B2 on the perpendicular to the Newton Line in the intersection point with L6 for the bicursal curve (R1, R2, B1 and B2 coincide in F1 or F2 in the nodal case which is the limit position between the uni- and bicursal).
Your construction could be in both cases simplified as following: the contact points with the tangent from P1 to the circles centered on the perpendicular to the Newton Line in the intersection point with L6 are on Cu1 (the circles are through R1 and R2 in the unicursal and centered on B1B2 in the bicursal).
Please, let me enough time to study this second construction of Cu2 before proposing a third one!
Best regards
Bernard K.

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Message: #557
Date: 22/5/2014 11:33:04
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,

thanks for testing my first construction of QL-Cu2, only possible with your construction of the asymptotes. You are right, the second construction seems easier and better in understanding. Background is the following property of QL-Cu2:

- ... Parallels to the asymptotes through an arbitrary point P intersect with QL-Cu2 in points on an orthogonal hyperbola H_y , centered in the midpoint of P and $CSC(P)$.
- ... For points P on a line L through QL-P1 the axes of H_y are parallel to the angle bisectors of QL-L1 and $CSC(L)$.

Examples:

- ... For points P on the 1st Steiner Axis the axes of H_y are parallel to the angle bisectors of QL-L1 and the 1st Steiner Axis.
- ... For points P on a parallel to QL-L1 through QL-P1 the axes of H_y are parallel to the Steiner Axes.

Perhaps there are further constructions of QL-Cu2 related to these properties.

Best regards Eckart

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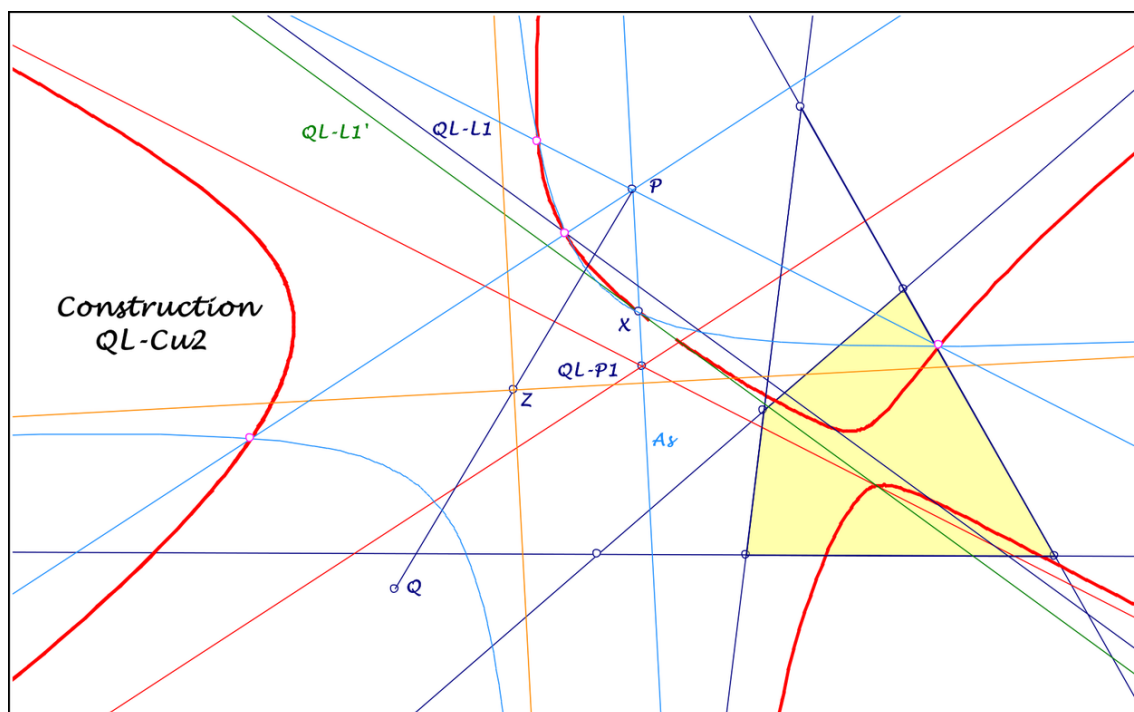
Message: #558
Date: 22/5/2014 4:32:23
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,

now I found a very simple construction of QL-Cu2, it is a simplification of my second construction in message #555, not using QL-Cu1:

- ... As asymptote,
- ... QL-L1' parallel to QL-L1 in $\frac{2}{3}$ distance to QL-P1,
- ... P point on As,
- ... $Q = \text{CSC}(P)$,
- ... Z midpoint PQ,
- ... L, L' parallels through Z
 wrt the angle bisectors of QL-L1 and Q.QL-P1,
- ... Hy orthogonal hyperbola through X with axes L, L'.
- ... Parallels to the asymptotes through P
 cut the orthogonal hyperbola Hy in points of QL-Cu2.

See attachment.
Best regards Eckart



2014-05-22.pdf

Message: #559
Date: 23/5/2014 12:36:05
From: chris.vantienhoven
Subject: new release EQF

Dear friends,

Recently I restarted updating EQF with several new items.
Especially I recommend these NEW items:

- * QA-L8: QA-DT-Isogonal Conjugate of QA-Co1
- * QA-L9: Perpendicular Bisector of QA-P2.QA-P4
- * QL-2P3: Schmidt Pair of Points
- * QL-2P4: Orthocentric Pedal Quadrangle Centers
- * QG-2Cu1: Perspective Squares Dual Cubic
- * QA-PF-1, QA-PF1: (new QA-Perspective Fields)

See: <http://www.chrisvantienhoven.nl/index.php/recently-added>

Best regards,
Chris

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Message: #560
Date: 23/5/2014 6:29:32
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
the second and third constructions work and are amazingly simple!
I have a problem and hope you will be able to help me. Geogebra doesn't allow me to draw a rectangular hyperbola with only 2 asymptotes and a point; it needs 5 points and I have only 4 with the reflections in the axes of symmetry.
Do you see any solution?
Thanks in advance
Best regards
Bernard K.

PS I have found a way to draw easily the cayleyan (see next message)
To draw the 3 curves, we need only the Schmidt circle, the Steiner axes and the Newton Line.

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Message: #561
Date: 23/5/2014 6:41:32
From: bernard.keizer
Subject: Properties of the inscribed QL's

Dear Chris, dear Eckart, dear friends
As Chris intends to put new items in EQF (and already did), I wanted to put down links between axes or asymptotes of different curves linked to a QL and conversely imposed to a family of QL's inscribed in the same curve QL-Cu1.
Best regards
Bernard

Properties of the QL's inscribed in a given Cu1

The fundamental property of the QL's inscribed in a given curve Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curve.

All the QL's share :

- the Miquel point P1
- the Schmidt circle, the Steiner axes and the invariant points F1 and F2
- the Newton Line L1, parallel to the asymptote of Cu1 at half a distance (P1 being the railwaywatcher of the 2 lines)
- the axis of the inscribed parabola
- the direction of the Steiner Line L2
- the points B1 and B2 (bicursal curve) or R1 and R2 (unicursal curve)
- therefore the middle of these 2 points where the line L6 cuts the Newton Line
- the axis of the inscribed parabola (parallel to the Newton Line through the Miquel point)
- the axis of the cardioïd Qu1 (tangent to Cu1 in the Miquel point, through the point where Cu1 cuts its asymptote, reflection of the axis of the parabola in the 1st Steiner axis and locus of the point P3)
- the direction of the axes of the inscribed deltoïd Qu2, which is the direction of the asymptotes of the stelloïd Cu2
the axes of the stelloïd trisect the oriented angle between the 1st Steiner axis and the axis of the parabola or the oriented angle between the axis of the parabola and the axis of the cardioïd (it's easy to draw them with the Chasles construction directly on the Schmidt circle with a rectangular hyperbola through P1 centered in the middle of P1A, where A is a point where the axis of the parabola cuts the Schmidt circle, with axes of symmetry parallel to the bisectors of the angle between the axis of the parabola and the 1st Steiner axis)
- the asymptote of the hessian as tangent to the inscribed deltoïd
- last, but not least, as discovered by Eckart and discussed in details by Bernard G., the centers of the 27 inscribed cardioïds lying on the stelloïd.

If we call R the fixed radius of the Schmidt circle, r the variable radius of the Miquel or the Hervey circle of a QL, p the distance from P1 to the Steiner Line and θ_i the angles between the lines L_i and the fixed direction of the Steiner Line, we have for all inscribed QL's :

- $\Sigma \theta_i$ equal to the angle between the axes of the parabola and the cardioïd or twice the angle between the 1st Steiner axis and the axis of the parabola
- $R^2 = 2pr = 16 r^2 \Pi \cos \theta_i$ ($p = 8r \Pi \cos \theta_i$)

Properties of the QL.docx

Message: #562

Date: 23/5/2014 6:49:24

From: bernard.keizer

Subject: Some properties of the stelloïd, it's hessian and it's cayleyan

Dear Chris, dear Eckart, dear friends

Since I had, thanks to Eckart and Bernard G., the opportunity of looking at the beautiful figures of these 3 curves, I tried to understand some visible properties. I submit these reflexions, maybe Bernard G. could bring some light in these complicate, but fascinating, matters ...

Best regards

Bernard K.

Some properties of the cubic stelloïd, it's hessian and it's cayleyan

The cubic stelloïd associated to a QL is the curve having as apolar conics the inscribed conics of the QL and as polar conics the rectangular hyperbolas cutting harmonically the sides of the diagonal triangle of the QL.

The hessian of the cubic is the locus of the points for which the rectangular hyperbola is degenerated in 2 orthogonal lines. 2 conjugate points of the hessian are the foci of an inscribed conic.

In each point of the hessian pass 2 orthogonal lines, which form the degenerated polar conic of it's conjugate. These 2 lines cut the stelloïd in the contact points of the tangent to the stelloïd from the conjugate point on the hessian.

For example, the Miquel point P1 is the conjugate of the infinity point, the 2 Steiner axes are the degenerated rectangular hyperbola of the infinity point and cut the stelloïd in points where the tangent is parallel to the Newton Line.

These 2 lines cut the axis of the parabola in 2 points, which are symmetric wrt the Miquel point P1, which makes easy to find the second one if we know the first.

The cayleyan is the envelope of all the lines forming these degenerated rectangular hyperbolas or the envelope of the lines through 2 conjugate points (main axis of the inscribed conics). The contact point is the harmonic conjugate of the 3rd intersection point of a line through 2 conjugates.

This makes easy to draw the curve : if we have 2 conjugate points, the line joining them cut the axis of the parabola in a point ; the perpendicular to this line in the reflection of this point in P1 gives the 3rd point of intersection. Both lines are the degenerated hyperbola of the tangential point of the 2 conjugate points, which is the conjugate of the 3rd intersection.

For example, the axis of the parabola as tangent to the cayleyan in a point, which is the harmonic conjugate of the infinity point, the reflection of of the 2nd intersection point in P1 and, in the unicursal curve, the Newton Line is tangent to the cayleyan in a point, which is the harmonic conjugate of the infinity point, the middle of R1R2. The second Steiner axis is tangent to the cayleyan in a point, which is the harmonic conjugate of P1 wrt the 2 points where this axis cuts the hessian.

Last, the cayleyan and the hessian are tangents in 3 points on a circle through P1 and the conjugate of the points are also their tangentials, the 3 inflexion points and intersection points of the stelloïd and the cayleyan. The tangent to the hessian and the cayleyan in the 3 contact points of the 2 curves is one of the tangents to the stelloïd in these 3 inflexion points and the other touch the stelloïd in points on the perpendicular to the 1st tangent in the contact point.

There are many other interesting properties to be found by looking at these beautiful figures ...

Some properties of the cubic stelloïd.docx

Message: #563
Date: 24/5/2014 10:07:30
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
Don't worry anymore!
The night brings often good ideas and I found this morning the solution to my little problem.
By reproducing the Chasles construction, I was able to find a 5th point which allows me to draw your rectangular hyperbola centered in Z and finally to draw the curve Cu2 with the different types.
Thanks a million time, this construction is amazingly simple and efficient!
Again, we just need now for the 3 curves the Schmidt circle, the Steiner axes and the Newton Line ...
Best regards
Bernard
PS How did you find the construction of the cayleyan by points (see my previous message)?

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Message: #564
Date: 24/5/2014 10:43:29
From: Antreas Hatzipolakis
Subject: Quadrangle - Concurrent Circles

I came across the following problem (*):
Given a convex quadrilateral ABCD. The point P and Q are the midpoints of the diagonals AC and BD respectively. The line PQ intersects the lines AB and CD at N and M respectively.
Prove that the circumcircles of triangles NAP, NBQ, MQD, and MPC have a common point.
The point of concurrence is not a center, but there is another one related to other pair of sides (AD, BC), so we have a central line passing through the two bicentric points.
Which is that line?
(*)
<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=590989&p3500572#p3500572>
APH

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Message: #565
Date: 24/5/2014 3:24:15
From: eckart_schmidt@t-online.de
Subject: Quadrangle Concurrent Circles

Dear Antreas,
the common point of the 4 circles is the Miquel point of ABDC, the other point is the Miquel point of ADBC.
Best regards Eckart

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Message: #566
Date: 24/5/2014 4:02:11
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,
back from my 55-year-old "Abitur", only a short message:
Thanks for testing the second and third version of the
construction of QL-Cu2.
To draw an orthogonal hyperbola out of four points, take as
fifth point an orthocenter of three of the four given points.
I shall study your papers in the next days. I do not know
whether it is only my problem (not the first time, not in
message #561, but in message #562): If I use the Windows
Internet Explorer for meeting QFG, the attached file is not
shown, if I use my E-mail Browser of Telecom the file is shown.
Best regards Eckart

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Message: #567
Date: 25/5/2014 9:53:22
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
Next year, it will be half a century that I passed my
baccalauréat. So I conclude that you are a little older than me;
I should call you brother born before me, as the Chinese say ...
Of course for the orthocenter, but in this case, my 4 points
were X and the reflections in the axes of symmetry and in Z; all
the triangles are rectangular and it doesn't give another point!
But I found a solution and a 5th point by using again this
marvelous Chasles construction ...
In fact, I don't see either the attached file in my message
#562! Strange, as the file appears in the list of attachments
... Anyhow, I put it here again.
I look forward to reading your comments soon.
Best regards
Bernard K.

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Message: #568
Date: 26/5/2014 8:51:36
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,
one point in my construction of QL-Cu2 with asymptotes can be simplified:

- ... As asymptote,
- ... QL-L1' parallel to QL-L1 in 2/3 distance to QL-P1,
- ... X intersection of As and QL-L1',
- ... P point on As,
- ... Q = CSC(P),
- ... Z midpoint PQ,
- ... L, L' lines through Z parallel and perpendicular wrt As,
- ... Hy orthogonal hyperbola through X with axes L, L'.
- ... Parallels to the asymptotes through P cut the orthogonal hyperbola Hy in points of QL-Cu2.

Best regards Eckart

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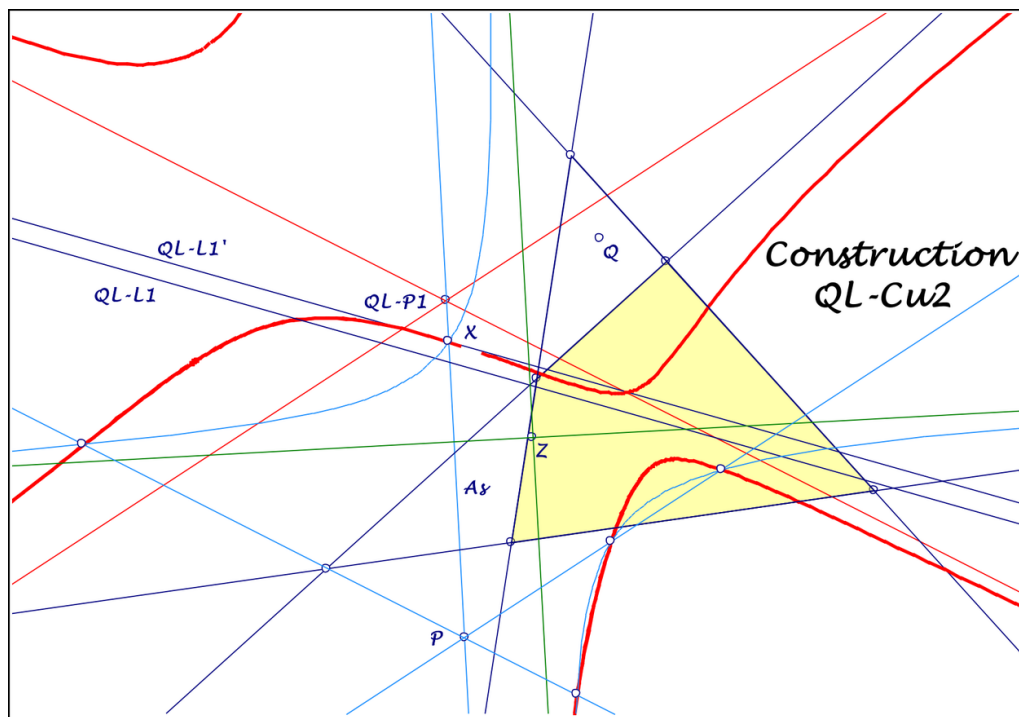
Message: #569
Date: 27/5/2014 10:47:34
From: bernard.keizer
Subject: Drawing QL-Cu2 with asymptotes

Dear Eckart,
If I'm not wrong, L and L' are then the asymptotes and not the axes of the rectangular hyperbola.
Best regards
Bernard

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Message: #570
Date: 27/5/2014 2:22:00
From: eckart_schmidt@t-online.de
Subject: Drawing QL-Cu2 with asymptotes

Dear Bernard K.,
thanks for correcting my construction! Unfortunately the typo was already in the first description (but not in the drawing). Here the correct form with an attached file:
... As asymptote,
... QL-L1' parallel to QL-L1 in 2/3 distance to QL-P1,
... X intersection of As and QL-L1',
... P point on As,
... Q = CSC(P),
... Z midpoint PQ,
... L, L' lines through Z parallel and perpendicular wrt As,
... Hy orthogonal hyperbola through X with asymptotes L, L'.
... Parallels to the asymptotes through P cut the orthogonal hyperbola Hy in points of QL-Cu2.
Best regards Eckart



2014-05-27.pdf

Message: #571

Date: 27/5/2014 3:44:27

From: eckart_schmidt@t-online.de

Subject: Some properties of the stelloid , it's hessian and it's cayleya

Dear Bernard K.,

for me your paper - attached in message #562 - was very, very

interesting! I have studied it point for point, only

reproducing. It will take some time to get a real view.

Remarkable is your pointwise construction of the cayleyan in the 7th section!

Only the property in the next-to-last section I could not reproduce:

"... the other touch the stelloid in points on the perpendicular to the 1st tangent in the contact point."

I can only verify

"... the other intersect on the perpendicular to the 1st tangent in the contact point."

These perpendiculars to the common tangents of hessian and cayleyan in the contact points are the QL-Line Conjugates (QL-Tf2) of the common tangents. As altitudes in the triangle of the common tangents they intersect in a point, what about his point?

Best regards Eckart

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Message: #572

Date: 28/5/2014 9:45:36

From: eckart_schmidt@t-online.de

Subject: Some properties of the stelloid, it's hessian ...

Dear Bernard K.,

an additional observation, perhaps wellknown:

The cayleyan is the envelope of lines with orthogonal QL-Tf2 image.

Or:

Let X and Y be the foci of an inscribed conic, then QL-Tf2(XY) is orthogonal XY with intersection Z in the third intersection of XY and QL-Cu1. These two orthogonal lines are the degenerated polar conic of QL-Tf1(Z).

Best regards Eckart

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Message: #573
Date: 28/5/2014 10:25:03
From: Bernard Gibert
Subject: Some properties of the stelloïd, it's he

Dear Eckart and Bernard K.
I have added a new (?) construction of $E(L)$ and its asymptotes in my paper.
<http://bernard.gibert.pagesperso-orange.fr/files/cardioids.html>
I await your comments and, although I proved and checked everything carefully, I hope there's still no mistake ...
Best regards
Bernard G.

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Message: #574
Date: 28/5/2014 4:03:44
From: eckart_schmidt@t-online.de
Subject: Some properties of the stelloïd, it's hessian ...

Dear Bernard G.,
Your construction of $QL-Cu2 = E(L)$ in 5.3 of your paper "Inscribed Cardioids and Eckart Cubics" has the same background as my construction (last version QFG-message #570):
... the point "omega1" in your construction is the Clawson-Schmidt Conjugate $QL-Tf1$ of the variable point M ,
... the asymptotes of $H2$ are parallel to the angle bisectors of the Newton Line $QL-L1$ and the connection of "omega1" with the Miquel Point $QL-P1$,
... to get a point of $H2$ depends on the curve Cv for the variable point M ,
... if Cv is $QL-Cu1 = H(L)$, see my message #555,
... if Cv is a line through the Miquel point $QL-P1$, the asymptotes of $H2$ are parallel,
... I chose for Cv an asymptote of $E(L)$, the intersection with $S(L)$ must be a point of $H2$,
... You chose the fourth line L of the quadrigon.
Best regards Eckart

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Message: #575

Date: 28/5/2014 4:59:48

From: bernard.keizer

Subject: Properties of the QL's and the cubic stelloïd

Dear Chris, dear Eckart and dear Bernard G.,

After reading Eckart's remarks, I have renewed my 2 messages about the properties of the QL's inscribed in the hessian and the 3 linked curves, the cubic stelloïd, it's hessian and it's cayleyan.

Thanks in advance for all your remarks, comments and critics. For Chris, I've used now pdf.files, I hope it will be better. For Eckart, we had discussed already the figure with inflexion points, but I didn't know the cayleyan then, it's all clearer now.

For Bernard G. what's the meaning of this angle property: constant sum of the directions of the 4 lines of all the QL's inscribed in the hessian?

Best regards

Bernard

Properties of the QL's inscribed in a given curve Cu1

The fundamental property of the QL's inscribed in a given curve Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curve.

All the QL's share :

- the Miquel point P1
- the Schmidt circle, the Steiner axes and the invariant points F1 and F2
- the Newton Line L1, parallel to the asymptote of Cu1 at half a distance (P1 being the railwaywatcher of the 2 lines)
- the axis of the inscribed parabola
- the direction of the Steiner Line L2
- the points B1 and B2 (bicursal curve) or R1 and R2 (unicursal curve)
- therefore the middle of these 2 points where the line L6 cuts the Newton Line
- the axis of the inscribed parabola (parallel to the Newton Line through the Miquel point)
- the axis of the cardioïd Qu1 (tangent to Cu1 in the Miquel point, through the point where Cu1 cuts its asymptote, reflection of the axis of the parabola in the 1st Steiner axis and locus of the point P3)
- the direction of the axes of the inscribed deltoïd Qu2, which is the direction of the asymptotes of the stelloïd Cu2
the axes of the stelloïd trisect the oriented angle between the 1st Steiner axis and the axis of the parabola or the oriented angle between the axis of the parabola and the axis of the cardioïd (it's easy to draw them with the Chasles construction directly on the Schmidt circle with a rectangular hyperbola through P1 centered in the middle of P1A, where A is a point where the axis of the parabola cuts the Schmidt circle, with axes of symmetry parallel to the bisectors of the angle between the axis of the parabola and the 1st Steiner axis)
- the asymptote of the hessian as tangent to the inscribed deltoïd
- last, but not least, as discovered by Eckart and discussed in details by Bernard G., the centers of the 27 inscribed cardioïds lying on the stelloïd.

If we call R the fixed radius of the Schmidt circle, r the variable radius of the Miquel or the Hervey circle of a QL, p the distance from P1 to the Steiner Line and θ_i the angles between the lines L_i and the fixed direction of the Steiner Line, we have for all inscribed QL's :

- $\sum \theta_i$ equal to the angle between the axes of the parabola and the cardioïd or twice the angle between the 1st Steiner axis and the axis of the parabola
- $R^2 = 2pr = 16 r^2 \prod \cos \theta_i$ ($p = 8r \prod \cos \theta_i$)

For a given reference QL, the inscribed conics and their confocals (of which the degenerated ones made of 2 conjugate points) form a tangential set. The QL's inscribed in the hessian are determined by the 4 common tangents to 2 conics of the set of these confocals (for example the 4 lines joining 2 pairs of conjugate points ; the reference QL is obtained by joining 2 pairs of opposite vertices)

Properties of the cubic stelloïd, it's hessian and it's cayleyan

1. The stelloïd QL-Cu2

The cubic stelloïd associated to a QL is the curve having as apolar conics the inscribed conics of the QL and as polar conics the rectangular hyperbolas cutting harmonically the sides of the diagonal triangle of the QL and the main axes of all inscribed conics (diagonals of QL's inscribed in the hessian)

2. The hessian QL-Cu1

The hessian of the cubic is the locus of the points for which the rectangular hyperbola is degenerated in 2 orthogonal lines. 2 conjugate points of the hessian are the foci of an inscribed conic and a pair of vertices of an infinity of QL's inscribed in the hessian.

In each point of the hessian pass 2 orthogonal lines, which form the degenerated polar conic of it's conjugate. These 2 lines cut the stelloïd in the contact points of the tangent to the stelloïd from the conjugate point on the hessian. The 2 orthogonal lines in a point P of the hessian are the bisectors of the angles XPX' for all the pairs of conjugate points X and X' . They are QL-Tf2 for all the QL's inscribed in the hessian.

For example, the Miquel point P1 is the conjugate of the infinity point, the 2 Steiner axes are the degenerated rectangular hyperbola of the infinity point wrt the stelloïd and cut the stelloïd in points where the tangent is parallel to the Newton Line.

The 2 perpendicular lines in each point cut the axis of the parabola (main axis of an inscribed conic) in 2 points, which are harmonic conjugates wrt the Miquel point and the infinity point and therefore symmetric wrt the Miquel point P1, which makes easy to find the second one if we know the first.

The 2 points F1 and F2, intersections of the Schmidt circle and the 1st Steiner axis are invariant in the Clawson-Schmidt transformation QL-Tf1; any circle through these 2 points is globally invariant in the same transformation and cuts the hessian in 2 conjugate points X and X' , the middle of XX' is on the Newton Line and XX' is the internal bisector of $F1ZF2$. These observations give an easy way to draw the hessian: for Z on the Newton Line, the external bisector of $F1ZF2$ cuts the 2nd Steiner axis in a point I and the internal bisector of $F1ZF2$ cuts the circle with center I through F1 and F2 in 2 conjugate points X and X' (the construction holds in both cases unicursal and bicursal, avoids the points B1 and B2 or R1 and R2 and gives directly 2 conjugate points).

3. The cayleyan

The cayleyan is the envelope of all the lines forming these degenerated rectangular hyperbolas or the envelope of the lines through 2 conjugate points (main axis of an inscribed conic). The contact point is the harmonic conjugate of the 3rd intersection point of a line through 2 conjugates.

This makes easy to draw the curve : the line joining 2 conjugate points cuts the axis of the parabola in a point ; the perpendicular to this line in the reflection of this point in P1 gives the 3rd point of intersection between the line and the hessian. Both lines are the degenerated hyperbola of the tangential point of the 2 conjugate points, which is the conjugate of the 3rd intersection.

For example, the axis of the parabola is tangent to the cayleyan in a point , which is the harmonic conjugate of the infinity point, the reflection of of the 2nd intersection point in P1 and, in the unicursal curve, the Newton Line is tangent to the cayleyan in a point, which is the harmonic conjugate of the infinity point, the middle of R1R2. The second Steiner axis is tangent to the cayleyan in a point, which is the harmonic conjugate of P1 wrt the 2 points where this axis cuts the hessian (centers of inversion for the hessian, which is an allagmatic curve).

Last, the cayleyan and the hessian are tangents in 3 points on a circle through P1 and the conjugates of these 3 points are also their tangentials, the 3 inflexion points and intersection points of the stelloïd and the hessian, which lie on a line conjugate of this circle. The 6 points are therefore the vertices of a QL inscribed in the hessian.

The tangent to the hessian and the the cayleyan in one of the 3 contact points of the 2 curves is one of the tangents to the stelloïd in one of the 3 inflexion points and intersection points of the stelloïd and the hessian and the 3 other tangents from this last point to the stelloïd touch the stelloïd in 3 another points, which lies on the perpendicular to the tangent to the hessian and the cayleyan in their contact point.

2 common tangents to the hessian and the cayleyan intersect in a point, which lies on the perpendicular to the 3rd common tangent in the contact point. The 3 tangents to the hessian in the 3 inflexion points and intersection points of the stelloïd and the hessian and the 3 perpendiculars to the common tangents to the hessian and the cayleyan in their contact points concur in point. The triangle formed by the 3 common tangents has therefore this point as orthocenter. This orthocenter is one of the excenters of the triangle formed by these 3 contact points between the hessian and the cayleyan, which is the orthic center of the triangle formed by the 3 common tangents to the 2 curves.

The 3 tangents to the hessian in the 3 inflexion points and intersection points of the stelloïd and the hessian concur in the same point

There are many other interesting properties to be found by looking at these beautiful figures ...

Message: #576

Date: 28/5/2014 5:06:56

From: Bernard Gibert

Subject: Some properties of the stelloïd, it's he

Dear Eckart,

>> ... the asymptotes of H2 are parallel to the angle bisectors
>> of the Newton Line QL-L1 and the connection of "omega1"
>> with the Miquel Point QL-P1,

I suspect you mean Omega2 here...

Best regards

Bernard

PS: your CSC is the transfo Phi in my paper which I hadn't realized...

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Message: #577
Date: 28/5/2014 5:07:54
From: bernard.keizer
Subject: Properties of the cubic stelloïd

Following of the previous message, the second file has been lost

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Message: #578
Date: 28/5/2014 5:27:12
From: Bernard Gibert
Subject: Properties of the QL's and the cubic stel

Dear Bernard K.

>> For Bernard G. what's the meaning of this angle property:
>> constant sum of the directions of the 4 lines
>> of all the QL's inscribed in the hessian?

you will find all you need in Numdam:
G. Humbert, orientation des systèmes de droites
NAM 3ème série, tome 12, 1893, p. 37-64 & 123-136
Best regards
Bernard G.
PS: I cannot open your second attachment

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Message: #579
Date: 28/5/2014 5:37:22
From: Bernard Gibert
Subject: Some properties of the stelloïd, it's he

Dear Eckart,
please ignore my previous message: the labels Ω_1 , Ω_2 are
inverted in the figure of my paper!
sorry, this will be corrected very soon.
Best regards
Bernard

>> Dear Eckart,
>>> ... the asymptotes of H_2 are parallel to the angle bisectors
>>> of the Newton Line $QL-L_1$ and the connection of " ω_1 "
>>> with the Miquel Point $QL-P_1$,
>> I suspect you mean Ω_2 here...

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Message: #580
Date: 29/5/2014 10:39:08
From: bernard.keizer
Subject: Some properties of the stelloïd , it's hessian and it's cayleya

Dear Eckart,
I hope you were able to open both attached files.
I realise, I've forgotten a last important property: your last
point, the orthocenter of the triangle of the common tangents to
the hessian and the cayleyan, lies also on the 3 tangents to the
hessian in the intersection points between the stelloïd and the
hessian and inflexion points of these 2 curves.
Best regards
Bernard

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Message: #581
Date: 29/5/2014 12:48:08
From: chris.vantienhoven
Subject: collinearity for QL-P26

Dear friends,
I always liked point QL-P26, the Least Sum of Squared Distances Point in a Quadrilateral.
Thus far the only way to find collinearities for this point was using the Clawson-Schmidt Conjugate QL-Tf2 to other points.
However now I stumbled about this collinearity:
QL-P26.QG-P12.QA-P42

where:
QL-P26: Least Squares Point
QA-P42: QA-Orthopole Center (also Pedal Quadrangle Homothetic Center)
QG-P12: Inscribed Harmonic Conic Center
Special is that this collinearity is valid at Quadrigon level.
Also special is that adding/subtracting the barycentric coordinates of QL-P26 and QA-P42 will deliver the coordinates of QG-P12.
Maybe someone has additions to this?

Best regards,
Chris

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Message: #582
Date: 29/5/2014 2:12:13
From: bernard.keizer
Subject: Some properties of the stelloïd , it's hessian and it's cayley

Dear Eckart,
This last property (message #580) is apparently true only in the unicursal case!
Best regards
Bernard

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Message: #583
Date: 30/5/2014 9:27:24
From: eckart_schmidt@t-online.de
Subject: collinearity for QL-P26

Dear Chris,
I think, the last "special" holds general: If three points are collinear, there exist presentations of the coordinates, so that one is the sum of the other.
Wrt QL-P26, there is an EQF-Note attached (I do not know, whether I have already sent this older paper to you). Perhaps there are connections to the collinearity.
Best regards Eckart

EQF-Note 2014-05-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

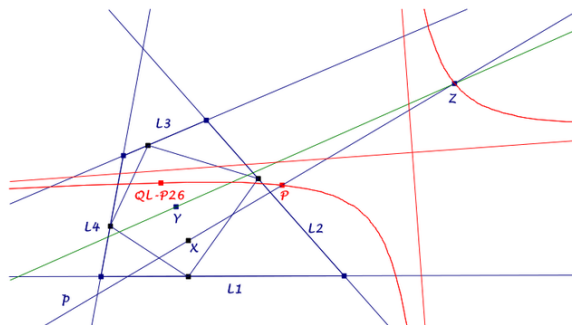
Orthogonal Hyperbolas through *QL-P26* (Least Squares Point)

In *EQF* there is a special orthogonal hyperbola through *QL-P26*;
here is a generalization. The studies are only CABRI controlled,
analytical calculations are too extensive. We start with

- ... a quadrilateral $L_1L_2L_3L_4$,
 - ... a line pencil for a point P (*), called pivot,
 - ... lines p through P with variable points X ,
- and construct
- ... the quadrangle of the pedal points of X wrt L_i ,
 - ... the centroid Y of the pedal quadrangle
- and consider
- ... the locus of Y for variable X , which is a line q ,
 - ... the intersection Z of p and q ,
 - ... the locus of Z for the lines p through P .

(*) For a point at infinity see below.

The result of this construction is an orthogonal hyperbola.



**These orthogonal hyperbolas contain always the pivot point
 P and the Least Squares Point $QL-P26$.**

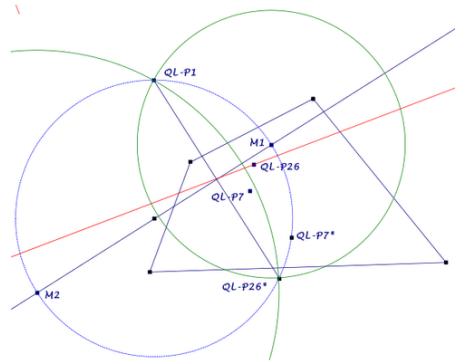
**For the pivot point $QL-P26$ this orthogonal hyperbola
degenerates into two orthogonal lines.**

**The asymptotes of these orthogonal hyperbolas
are parallel in pairs.**

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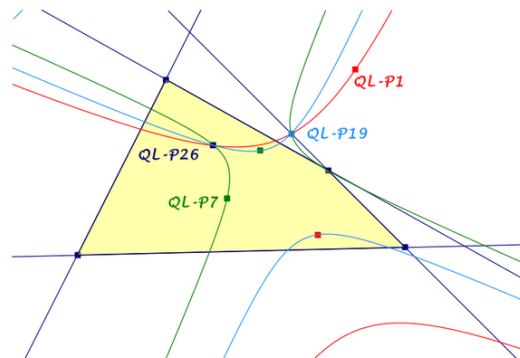
The two orthogonal lines through $QL-P26$, which are parallel to the asymptotes of the hyperbolas, can be constructed as follows. The Clawson-Schmidt Conjugate ($QL-Tf1$) will be abbreviated with CSC .

The circumcircle of $QL-P1$, $CSC(QL-P26)$ and $CSC(QL-P7)$ intersects the perpendicular bisector of $QL-P1.CSC(QL-P26)$ in M_1 and M_2 . The CSC -images of the circles round M_1 and M_2 through $QL-P1$ are the searched orthogonal lines through $QL-P26$.



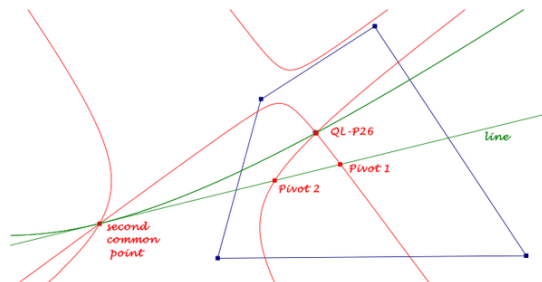
Taking for pivot QL -points, there are three special examples:

In addition to $QL-P26$ the orthogonal hyperbolas with pivot $QL-P1$ and $QL-P7$ contain $QL-P19$.



There are some generalizations:

**Pivots on a line have hyperbolas, which contain with $QL-P26$ a second common point on the line.
The line is tangent to the hyperbola for the second common point as pivot.**



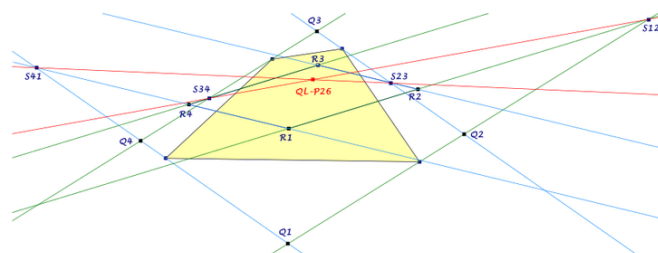
Example: For pivots on the line $QL-L6$ the second common point will be $QL-L6 \wedge QL-P26.QL-P19$.

For pivots on a line through $QL-P26$ the hyperbolas contact. For pivots on lines through a fixed point the second common points lie on the hyperbola of the fixed point.

Pivots on a line have hyperbolas with collinear centers.

(*) If the pivot is a point at infinity, the result of the construction at the beginning will not be a hyperbola but a line connecting $QL-P26$ with the second common points for pivots on parallel lines through the point at infinity. For the point at infinity of $QL-L5,6$ this line will be $QL-P26.QL-P19$.

This leads to the following clearly arranged construction of $QL-P26$:



**Take a quadrigon component of the quadrilateral
... with the circumscribed parallelogram $Q_1Q_2Q_3Q_4$,
... the centroids R_i of the pedal quadrangles of Q_i ,
... the intersections S_{ij} of R_iR_j and Q_iQ_j
... and $S_{12}.S_{34} \wedge S_{23}.S_{41} = QL-P26$.**

Eckart Schmidt
<http://eckartschmidt.de>
 eckart_schmidt@t-online.de

2014-05-30.pdf

Message: #584

Date: 30/5/2014 3:50:50

From: eckart_schmidt@t-online.de

Subject: Some properties of the stelloïd , it's hessian ...

Dear Bernard K.,

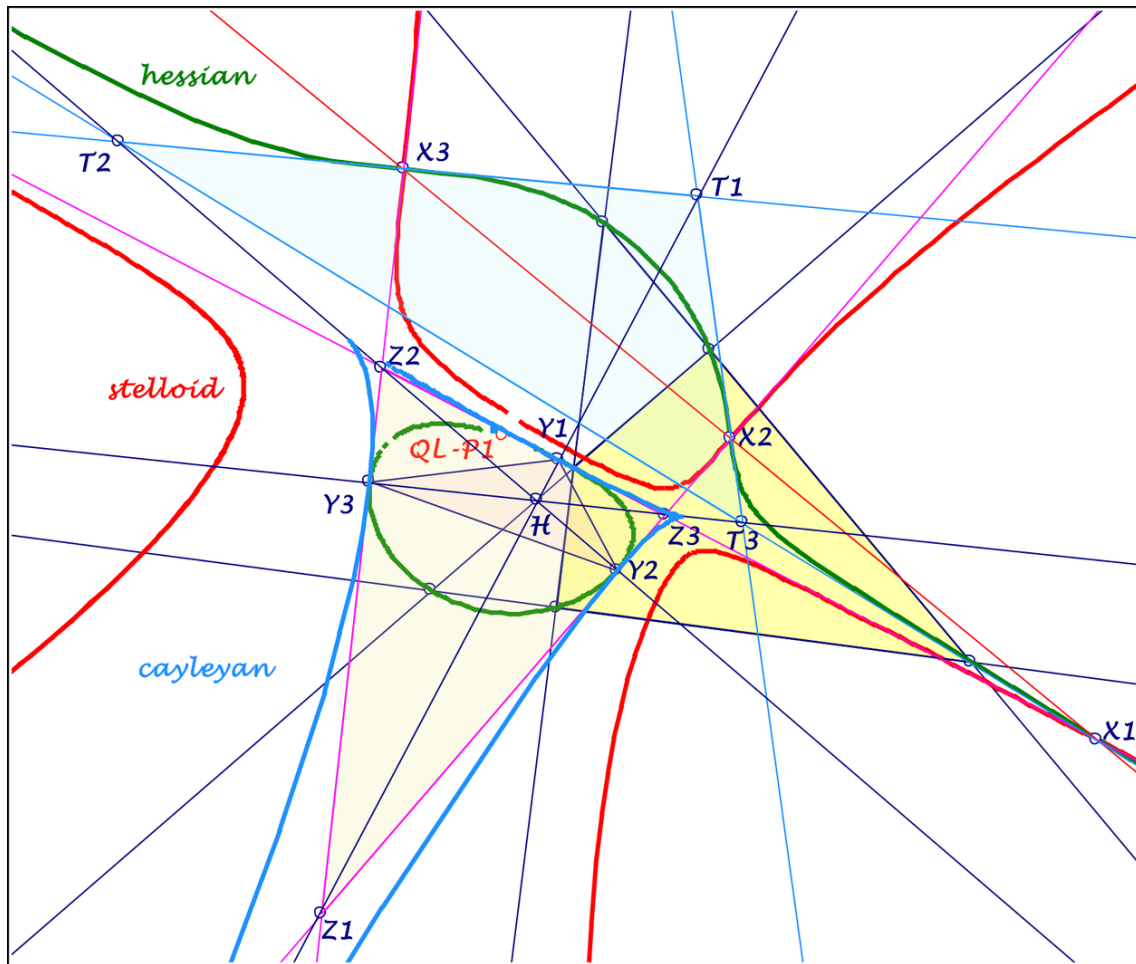
the differences in the unipartite and the bipartite case of QL-Cu1 are fatal! You are right: Only in the unipartite case the orthocenter of the triangle of the common tangents to stelloïd, hessian and cayleyan lies also on the 3 tangents to the hessian in the intersections of the stelloïd and the hessian. Remarkable remains, that the three curves have three common tangents.

A summary and some Cabri-observations (see attachment):

- ... let X_i be the inflection points of the stelloïd, collinear on L_x ,
- ... $Y_i = CSC(X_i)$ are the contact points of hessian and cayleyan,
- ... the common tangents are also tangent to the stelloïd in X_i ,
- ... let Z_i be the intersections of the common tangents of stelloïd, hessian and cayleyan,
- ... triangle Y_i is the orthic triangle of triangle Z_i ,
- ... the trilinear polar of the orthocenter H of the triangle Z_i is the line L_x ,
- ... in the unipartite case of the hessian H is the common point of the tangents in X_i to the hessian,
- ... in the bipartite case the tangents in X_i to the hessian intersect pairwise in T_i on the altitudes of the triangle Z_i ,
- ... in the bipartite case the trilinear polar of H wrt the triangle T_i is the line L_x ,
- ... in the bipartite case L_x is the perspective axis of triangle Z_i and triangle T_i ,
- ... in the bipartite case the altitudes of the Triangle Z_i cut the hessian in the pedal point Y_i and two CSC-partners U_i and V_i with common tangential in X_i ,
- ... in the bipartite case the altitudes of the triangle Z_i are tangent to the cayleyan (in the cusps?).

I hope, there are not too much obscurities!

Best regards Eckart



2014-05-31.pdf

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Message: #585
Date: 02/6/2014 11:55:10
From: bernard.keizer
Subject: Some properties of the stelloïd , it's hessian ...

Dear Eckart
Beautiful figure indeed!
As the X_i and Y_i are CSC conjugates, the 6 points form a QL inscribed in the hessian and X_i is in line with Y_j and Y_k ($i, j, k = 1, 2, 3$).
The bisectors of the angles $U_i X_i V_i$ form the degeneretad rectangular hyperbola and polar conic of Y_i , meaning the contact points of the tangents from Y_i to the stelloïd (including X_i) are on these 2 orthogonal lines. Conversely, the orthogonal lines $X_i Y_i$ and $Y_i Z_i$ form the degenerated rectangular hyperbola and polar conic of X_i , meaning the contact points of the tangents to the stelloïd from X_i (including X_i again) are on these 2 lines.
Best regards
Bernard K.
PS Did you read my 2 attached files?

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Message: #586
Date: 02/6/2014 2:49:02
From: bernard.keizer
Subject: Properties of th e QL's and the cubic ste

Dear Bernard G.
Thanks a lot for your reference.
I feel, I won't die stupid, after all!
Best regards
Bernard K.

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Message: #587

Date: 04/6/2014 3:25:49

From: eckart_schmidt@t-online.de

Subject: collinearity for QL-P26

Dear Chris,

I deal your interest in the Least Squares Point QL-P26.

Attached a file wrt points with constant sum of

QL-distance-squares.

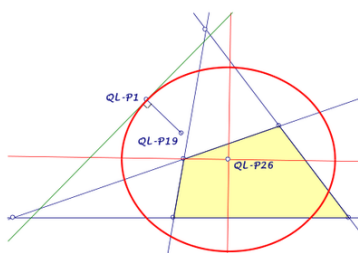
Result: Two interesting orthogonal lines through QL-P26.

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Constant sum of QL-Distance-Squares

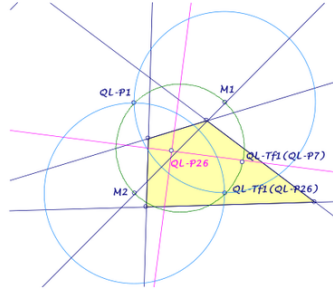
For the Least Squares Point QL-P26 the sum of the squares of the distances to the four lines of a quadrilateral is minimal. The loci for points with a constant sum of QL-distance-squares are homothetic ellipses with parallel axes, centered in QL-P26. – Calculations with barycentric coordinates are possible, but extensive and here omitted.



Let $d_i(P)$ be the distance of a point P to the line L_i of the quadrilateral, then we consider points X with

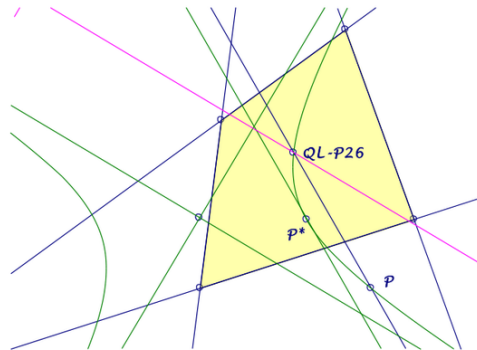
$$\sum_i d_i(P)^2 = \sum_i d_i(X)^2.$$

- The loci of these points are ellipses $E(P)$, centered in the Least Square Point QL-P26 (which is the centroid of its pedal quadrangle).
- If P is the Miquel Point QL-P1, the tangent in QL-P1 to $E(P)$ is perpendicular to QL-P1.QL-P19 (QL-P19 is the centroid of the collinear degenerated pedal quadrangle of QL-P1).
- In general: The tangent in P to $E(P)$ is perpendicular to the line, connecting P and the centroid P^* of its pedal quadrangle.
- The ellipses $E(P)$ are homothetic wrt QL-P26.
- The axes of $E(P)$: The circumcircle of QL-P1, QL-Tf1(QL-P26) and QL-Tf1(QL-P7) intersects the perpendicular bisector of QL-P1.QL-Tf1(QL-P26) in M_1 and M_2 . The QL-Tf1-images of the circles round M_1 and M_2 through QL-P1 are the searched orthogonal axes through QL-P26 (see also EQF-Note 2014-05-30 in QFG-message 583).



With these properties it is possible to construct the ellipses $E(P)$.

- The axes of $E(P)$ are the loci of points Q , so that Q , the centroid Q^* of the pedal quadrangle of Q and $QL-P26$ are collinear.
- The axes of $E(P)$ are parallel to the asymptotes of all orthogonal hyperbolas, constructed with $QL-P26$, an arbitrary point P , the centroid P^* of the pedal quadrangle of P with a tangent in P^* parallel $P.QL-P26$.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-06-04.pdf

Message: #588

Date: 04/6/2014 4:07:06

From: eckart_schmidt@t-online.de

Subject: Some properties of the stelloïd , it's hessian ...

Dear Bernard K.,

thanks for your feedback. I have read your interesting 2 papers,
last edition attached #575 and #586.

One remark to "QL's inscribed in QL-Cu1": These quadrilaterals
have the same stelloïd, but not the same centers for inscribed
cardioids.

Best regards Eckart

PS. ... in the same paper:

the axis of the parabola is mentioned twice.

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Message: #589
Date: 04/6/2014 8:43:23
From: bernard.keizer
Subject: Some properties of the stelloïd , it's hessian ...

Selbstverständlich, lieber Eckart
Perhaps, it would be useful to add: the Morley set of the $4 \times 9 = 36$ lines of every QL inscribed in the hessian cuts the stelloïd in 27 points (each line carries 3 points, but each point lies on 4 lines, one line in a Morley set of one of the 4 reference triangles) and every point is the center of a cardioïd inscribed in the QL.
Thanks for the axis of the parabola ...
Best regards
Bernard K.

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Message: #590
Date: 05/6/2014 1:58:49
From: eckart_schmidt@t-online.de
Subject: Van Aubel Cubic QG-Cu1

Dear Chris,

some remarks for QG-Cu1:
... It is not explicite mentioned,
 that QA-P1 is a point of QG-Cu1, but the figure shows it.
... Perhaps the self-intersecting point can shorter be described
 as intersection of QG-L3 and QL-L6.
... The Miquel Points of the Van Aubel quadrilaterals are collinear
 on a parallel to QG-P7.QG-P10.QG-P15 through QL-P1.
... The pair of orthogonal asymptotes are parallel
 to the angle bisectors of corresponding diagonals
 of the Van Aubel quadrilaterals for the ratios $+1/2$ and $-1/2$.

Best regards Eckart

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Message: #591
Date: 12/6/2014 10:11:47
From: Antreas Hatzipolakis
Subject: A circle in quadrangle

No much traffic lately ... : -)
Let me write a problem I found in an old Greek math. magazine:
Let ABC be a quadrangle. Denote:
 $O = AC \cap BD$
 M, N = the midpoints of AC, BD, resp.
 P = the other than O point of intersection of the circumcircles
of OAB, OCD
 S = the other than O point of intersection of the circumcircles
of OAD and OBC.
Prove that the points O, M, N, P, S are concyclic.
APH

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Message: #592
Date: 13/6/2014 12:00:15
From: chris.vantienhoven
Subject: A circle in quadrangle

Dear Antreas,
Tomorrow I will be leaving for a holiday.
So I have not much time to answer your question.
The circle you describe is a very interesting one.
In EQF it is shortly known as QG-Ci3.
See: <http://www.chrisvantienhoven.nl/other-quadrigon-objects/index.php/16-mathematics/encyclopedia-of-quadri-figures/quadrigon-objects/artikelen-qg/248-qg-ci4>
Many points lie on this circle.
So have a look at it.
Best regards,
Chris

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Message: #593

Date: 23/6/2014 3:31:55

From: seiichikiri

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Chris,

Im my memory, you made a QA point corresponding to Hiriart-Urruty minimizer X(5544).

But I can not remember its number in EQF.

I hope that you will make QA points or locus corresponding to Thomson-Gibert-Moses hyperbola shown in X(5642).

Sorry in advance if my idea would be wrong.

Best regards,

Seiichi.

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Message: #594

Date: 23/6/2014 10:22:49

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Seiichi,

The Hiriart-Urruty minimizer X(5544) calculates the point with minimal sum of squared distances to the vertices of the ref. triangle with added the sum of squared distances to the vertices of its pedal triangle.

We can do the same with quadrilaterals/quadrangles/quadrilaterals:

Quadrilateral (QL):

4 lines, 6 vertices, 1 pedal quadrangle with 4 vertices

Quadrangle (QA):

4 vertices, 6 sides, 1 pedal sextangle with 6 vertices

Quadrignon (QG):

4 vertices, 4 sides, 1 pedal quadrangle with 4 vertices

I calculated the corresponding minimizers in these three situations.

The minimizer in a Quadrilateral (QL) delivers this minimizer point with 1st CT-coordinate:

$$\begin{aligned} & a^2 (a^6 (1 - m)^2 (1 - n)^2 (m - n) - a^4 (1 - m) (1 - n) (b^2 \\ & (1^2 (8 m - 6 n) + m (-2 m^2 + 9 m n - 5 n^2) + 1 (-6 m^2 - 3 m \\ & n + 5 n^2)) + c^2 (1^2 (6 m - 8 n) + n (5 m^2 - 9 m n + 2 n^2) + \\ & 1 (-5 m^2 + 3 m n + 6 n^2))) + a^2 (-4 b^2 c^2 (1 - m) (1 - n) \\ & (m - n) (11 l^2 + 2 m^2 + 27 m n + 2 n^2 - 21 l (m + n)) + b^4 \\ & (1 - m)^2 (1^2 (7 m - 5 n) + n (-8 m^2 + 17 m n - 7 n^2) + 2 l \\ & (5 m^2 - 14 m n + 7 n^2)) + c^4 (1 - n)^2 (1^2 (5 m - 7 n) - 2 l \\ & (7 m^2 - 14 m n + 5 n^2) + m (7 m^2 - 17 m n + 8 n^2))) + (m - \\ & n) (c^6 (1 - n)^2 (5 l m - 3 m^2 - 8 l n + 6 m n) - b^6 (1 - \\ & m)^2 (1 (8 m - 5 n) + 3 n (-2 m + n)) + b^4 c^2 (1^3 (55 m - 52 \\ & n) + 3 m n (-40 m^2 + 37 m n + 2 n^2) + 1^2 (-141 m^2 + 40 m n + \\ & 92 n^2) + 1 (86 m^3 + 132 m^2 n - 203 m n^2 - 6 n^3)) + b^2 c^4 \\ & (1^3 (-52 m + 55 n) + 1^2 (92 m^2 + 40 m n - 141 n^2) + 3 m n (2 \\ & m^2 + 37 m n - 40 n^2) + 1 (-6 m^3 - 203 m^2 n + 132 m n^2 + 86 \\ & n^3)))) \end{aligned}$$

I found no relationships with known EQF-points or -curves.

The minimizer in a Quadrangle (QA) delivers this minimizer point with 1st CT-coordinate:

$$\begin{aligned} & -(q - r) \\ & (3 p^3 - 2 p^2 (q + r) + 4 q r (q + r) - p (3 q^2 + q r + 3 \\ & r^2)) \end{aligned}$$

I found no relationships with known EQF-points or -curves.
 The minimizer in a Quadrigon (QG) delivers this minimizer point
 with CT-coordinates:

$$\begin{aligned} &\{-(q-r)(-3p^2+2pq+3q^2-2pr+3qr), \\ &2p^2q+2pq^2-2q^3-3p^2r+2q^2r-3pr^2+2qr^2, \\ &(p-q)(3pq+3q^2-2pr+2qr-3r^2)\} \end{aligned}$$

Again I found no relationships with known EQF-points or -curves.
 Regarding the Thomson-Gibert-Moses hyperbola
 it lacks me in time to investigate this.
 Maybe you can give it a try?

Best regards,
 Chris

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Message: #595
Date: 23/6/2014 1:30:32
From: seiichikiri
Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Chris,

Sorry that I misunderstood X(5544).
 Did you compute the following LSD point?
 Let P1P2P3P4 be a quadrangle with its diagonal triangle S1S2S3.
 Find a point P
 which minimizes the sum =
 PP1^2+PP2^2+PP3^2+PP4^2+PS1^2+PS2^2+PS3^2.

Best regards,
 Seiichi.

P.S. I expect that the above case will give simpler solution
 than the case of the pedal triangle?

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Message: #596

Date: 23/6/2014 3:01:26

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Seiichi,

The point that minimizes

the sum = $PP_1^2 + PP_2^2 + PP_3^2 + PP_4^2 + PS_1^2 + PS_2^2 + PS_3^2$ is well known to me.

It has 1st CT-coordinate:

$(q + r) (2 p + q + r) (2 p^2 + 2 p q + 2 p r + q r)$

It is also the Perspector of the QA-Diagonal triangle and the QL-P12-Triple Triangle.

There is construction-method for this point.

See:

<http://www.xtec.cat/~qcastell/ttw/ttweng/resultats/r2028.html>

Let P be a point; A', B', C' its cevian traces. Let Ab, Ac be the midpoints of AB', AC' and define analogously Bc, Ba, Ca, Cb. Let A1, B1, C1 lbe the midpoints of AA', BB', CC'; A2, B2, C2 the midpoints of BcCb, CaAc, AbBa. The lines A1A2, B1B2, C1C2 concur.

Applied in a Quadrangle (triangle Pi.Pj.Pk with random point Pl for (i,j,k,l) elements of (1,2,3,4)) it produces a Quadrangle homothetic with the Reference Quadrangle giving Homothetic Center on QA-P1.QA-P10 (3:4).

This point is again

LSD-point P1-P2-P3-P4-S1-S2-S3,

the point that minimizes

the sum = $PP_1^2 + PP_2^2 + PP_3^2 + PP_4^2 + PS_1^2 + PS_2^2 + PS_3^2$.

It lies on these lines:

QGP1, QLP12

QGP3, QLP15

QA-P1, QA-P5

QA-P24, QA-P40

Best regards,

Chris

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Message: #597
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Message
597

Message number 597 is not available in Yahoo groups.

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Message: #598
Date: 23/6/2014 3:35:21
From: Antreas Hatzipolakis
Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Chris,

The construction in #596 is my "12 Midpoints" construction posted to Anopolis group where it was take from.

Cf:

ANOPOLIS' MESSAGES IN QC's TTW

<https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/22196> †)

However, probably it is a well-known one (any references are welcome!)

Here are two of the messages of the thread:

Anopolis #899 †)

<https://groups.yahoo.com/neo/groups/Anopolis/conversations/messages/899>

[APH]

Let ABC be a triangle, P a point and A'B'C' the cevian triangle of P.

Denote:

Ab, Ac = the midpoints of AB', AC', resp.

Bc, Ba = the midpoints of BC', BA', resp.

Ca, Cb = the midpoints of CA', CB', resp.

A1,B1,C1 = the midpoints of AA',BB',CC', resp.

A2,B2,C2 = the midpoints of BcCb, CaAc, AbBa, resp.

For which P's the triangles A1B1C1, A2B2C2 are perspective?

For all?

[Cesar Lozada]:

Yes. For all P in the plane of ABC.

If $P=u:v:w$ (trilinears) then the perspector is

$$Z(P) = (3u^2a^2 + b^2c^2 + 2a^2u(b^2+c^2)) \cdot (b^2+c^2)/a :$$

[P, Z(P)] = [X(I), X(J)] for these (I,J): (2,2), (7,3742), (8,3740), (69,3819)

$$Z(X(1)) = (3a^2+b^2+c^2) \cdot (b+c)/a = \text{midpoint of } (2,1962)$$

$$\text{ETC: } 2.255721648292953017400$$

$$Z(X(3)) = (3,275) \wedge (549,1154)$$

$$\text{ETC: } -7.458117798896139375776$$

$$Z(X(4)) = ((b^2+c^2) \cdot a^2 - c^4 + 4b^2c^2 - b^4) \cdot a = \text{Complement of } X(3917) = \text{Midpoint of } (2,51)$$

$$\text{ETC: } 1.374997187598107204997$$

$$Z(X(6)) = (3a^4 + 2(b^2+c^2) \cdot a^2 + b^2c^2) \cdot (b^2+c^2)/a = (2,732) \wedge (6,1799)$$

$$\text{ETC: } 1.950074078181595348448$$

$$Z(X(10)) = (a^2 + (5(b+c)) \cdot a + 5b^2 + 11b^2c + 5c^2) \cdot (2a+b+c)/a = (2,1051) \wedge (10,1255)$$

$$\text{ETC: } 2.801829039589988817933$$

Anopolis #905 †)

<https://groups.yahoo.com/neo/groups/Anopolis/conversations/messages/905>

[APH]:

Let ABCD be a quadrangle.

Denote:

$$A' = AD \wedge BC, B' = BD \wedge CA, C' = AC \wedge AB$$

M1,M2,M3 = the midpoints of AA', BB', CC'

Gd = the centroid of M1M2M3

Similarly Ga, Gb, Gc for the triangles BCD,CDA,DAB

The lines AGa, BGb, CGc, DGd are concurrent (??)

APH

*** The lines AGa, BGb, CGc, DGd are concurrent in the point X on QA-L3, dividing QA-P1 QA-P10 with ratio 1:4 (QA-P1X:XQA-P10 = 1:4), of 1st DT-Coordinate:

$$-13 p^4 - (q^2 - r^2)^2 + 14 p^2 (q^2 + r^2)$$

Angel Montesdeoca

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[33\]](#).

Message: #599

Date: 23/6/2014 3:43:57

From: Antreas Hatzipolakis

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

I apologize for not making clear that the author of TTW explicitly writes:

"TTW started taking shape in the 2002 Spring originated from a wide collection of previous notes, a great number of them borrowed from the Hyacinthos

(<http://groups.yahoo.com/group/Hyacinthos/>) mail list and, after, of its follower Anopolis

(<http://groups.yahoo.com/neo/groups/Anopolis/>). Let me thank all its members."

<http://www.xtec.cat/~qcastell/ttw/ttweng/forew1.html>

APH

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Message: #600

Date: 23/6/2014 5:00:34

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbo

Dear Antreas,

This makes extra clear how many assists (speaking in football terms) you gave!

Thank you for that.

Please go on!

Best regards,

Chris

p.s.

One of your nicest assists resulted in QA-Tf3, the remarkable QA-Orthopole Transformation, see:

<http://www.chrisvantienhoven.nl/other-quadrangle-objects/index.php/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-objects/artikelen-qa/229-qa-tf3.html>

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Message: #601

Date: 24/6/2014 2:26:50

From: seiichikiri

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbo

Dear Chris,

Thank you very much for your information. I would like to hear your opinion.

Like the description under X(5642) in ETC, is it worth while to study the locus or the point with the parameter t which minimizes or maximizes the sum

$$PP1^2+PP2^2+PP3^2+PP4^2+t(PS1^2+PS2^2+PS3^2)?$$

Already the locus determined?

Best regards,

Seiichi.

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Message: #602

Date: 24/6/2014 4:10:04

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbo

Dear friends,

I placed the point that minimizes

the sum = $PP1^2+PP2^2+PP3^2+PP4^2+PS1^2+PS2^2+PS3^2$,

(meaning the minimized sum of squared distances to the vertices of a reference quadrangle and the vertices of its diagonal triangle) in the Encyclopedia of Quadri-Figures (EQF) with identifier QA-P43.

See: <http://www.chrisvantienhoven.nl/quadrangle-objects/index.php/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-objects/artikelen-qa/249-qa-p43>

Best regards,

Chris

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Message: #603

Date: 27/6/2014 2:07:45

From: eckart_schmidt@t-online.de

Subject: QA points or locus corresponding to Thomson ...

Dear Chris,

perhaps of interest: two loci for your new point QA-P43:

(1) QG-line: parallel to the polar of QL-P8 wrt QL-Co1
in 6/7 distance to QG-P1,

(2) QG-hyperbola:

... asymptotes parallel to the diagonals,

... centered in a point Z, dividing QG-P1.QA-P10 with 6:1,

... further points dividing QG-P1.QA-P10 with 3:4 or -9:2.

Best regards Eckart

PS.: I am sure, it is well known, that points on a circle round QA-P43 have a constant sum of squared distances to the 7 points.

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Message: #604

Date: 27/6/2014 8:37:41

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson ...

Dear Eckart,

I analyzed your statements.

* Your statement (1) is valid for every line through QL-P12 because QA-P43 lies on

QG-P1.QL-P12 with ratio 6 : 1 and the Polar of QL-P8 wrt QL-Co1 happens to be a line through QL-P12.

* I do not understand this hyperbola. When you mean the hyperbola through QA-P43 with asymptotes parallel to the diagonals of a Reference Quadrignon with center $C_e = QG-P1.QA-P10$ (6:1), then there is no fixed ratio between the distances to resp. QG-P1 and QA-P10.

Your PS is well known to me but I suppose not to everyone.

Best regards,

Chris

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Message: #605
Date: 28/6/2014 9:48:57
From: eckart_schmidt@t-online.de
Subject: QA points or locus corresponding to Thomson ...

Dear Chris,
thank you for explanation wrt point 1. Wrt point 2 perhaps my description was not clear:
"... further points dividing QG-P1.QA-P10 with 3:4 or -9:2."
means, these two points are special points of the hyperbola.
Best regards Eckart

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Message: #606
Date: 29/6/2014 7:30:26
From: seiichikiri
Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear friends, dear Chris,
The parametric sum of square distances of 7 points produces the following simple results.
The barycentrics are given:
 $((q+r)(2p+q+r)(p^2(1+t)+p(q+r)(1+t)+qr); \dots)$.
For $t=-1$, it is QA-P5.
For $t=-2$, it is QA-P20.
For $t=-4$, it is QA-P22.
Best regards,
Seiichi.

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Message: #607

Date: 30/6/2014 8:54:07

From: eckart_schmidt@t-online.de

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Seiichi,

your results lead to the following property:

The points with minimum parametric sum of square distances of 7 points divide QA-P1.QA-P10 with ratio $3t/4$.

Best regards Eckart

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Message: #608

Date: 30/6/2014 12:20:27

From: seiichikiri

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Eckart,

Thank you very much for your comment.

I have realized the importance of QA-L3 once more. But I think that there was nothing interesting, regrettably.

Best regards,

Seiichi.

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Message: #609

Date: 03/7/2014 8:45:02

From: Antreas Hatzipolakis

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Chris,

Another assist :-)

<https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/22493> †)

If we apply it to a quadrangle ABCD (*), we get four circles but I do not know if they have any interesting properties.

(*)

Let ABCD be a quadrangle and O_a, O_b, O_c, O_d the circumcenters of BCD, CDA, DAB, ABC, resp. and Q a point.

The reflections of QO_a in the sidelines of BCD bound a triangle whose the incenter (or an excenter) J_a is lying on the circumcircle of BCD. Similarly J_b, J_c, J_d .

The points (J_a, J_b, J_c, D) are concyclic. The same is true for the points $(J_a, J_b, C, J_d), (J_a, B, J_c, J_d), (A, J_b, J_c, J_d)$

Antreas

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #610
Date: 03/7/2014 11:30:39
From: Antreas Hatzipolakis
Subject: Quadrilateral problem on orthocenters

Lemma:

https://www.facebook.com/photo.php?fbid=1531492960407110&set=o.9_6389292832&type=1&theater

Application to quadrilateral:

Let (a,b,c,d) be a quadrilateral and H_a, H_b, H_c, H_d the orthocenters of (b,c,d) , (c,d,a) , (d,a,b) , (a,b,c) , resp. (on the Steiner line).

Let a', b', c', d' be the parallels to a,b,c,d through H_a, H_b, H_c, H_d , resp.

Let M_a, M_b, M_c, M_d be the homothetic centers of $[(b,c,d), (b',c',d')]$, $[(c,d,a), (c',d',a')]$, $[(d,a,b), (d',a',b')]$ and $[(a,b,c), (a',b',c')]$, resp.

Do they coincide?

APH

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Message: #611
Date: 04/7/2014 10:36:30
From: eckart_schmidt@t-online.de
Subject: Quadrilateral problem on orthocenters

Dear Antreas,
the homothetic centers coincide in QL-P20.
Best regards Eckart

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Message: #612
Date: 09/7/2014 10:13:17
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
We denote the intersections of Miquel circles 1 and 2 by Q12.
Define Q23 and Q31 cyclically.
It seems to me that the lines through M1 and Q23, M2 and Q31, M3 and Q12 are concurrent.
Best regards,
Seiichi

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Message: #613
Date: 09/7/2014 11:21:09
From: chris.vantienhoven
Subject: A concurrent point derived from QA-P9

Dear Seiichi,
This point is well known to me.
I studied it 2 years ago.
It has very long coordinates.
Nevertheless there are two nice properties.
Let us call this point XX.
1. XX is also the perspector of the QL-P1 Triple Triangle & the QG-P5 Triple Triangle (your points Q12, Q23, Q31 are the vertices of the QG-P5 Triple Triangle)
2. QA-Tf3(XX) is collinear with QA-P1 and the Circumcenter of the QL-P1 Triple Triangle.
Best regards,
Chris

p.s.
This Circumcenter of the QL-P1 Triple Triangle is also a nice QA-point and collinear with the Orthocenter of the QL-P6 Triple Triangle, all 3 vertices of the QL-P5 Triple Triangle and infinity point QA-Tf2 (QA-P3).

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Message: #614
Date: 09/7/2014 11:57:49
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
Thank you very much for the properties of this point (XX).
I met XX when I tried to make triangle centers from
quadrifigures. As XX is a QA point, I tried to make a pure image
by parallel translation of 3 lines of concurrency to the
vertices P1, P2 and P3. But it has failed.
Best regards,
Seiichi.

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Message: #615
Date: 09/7/2014 4:56:58
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
Addition:
I think that the circumcenter of QG-P5 triple triangle
(Q12,Q23,Q31) is also a nice QA-point.
Because this triangle is easily constructed by the
perpendicularity of the midpoints of the sides.
Is this point already in EQF?
Best regards,
Seiichi.

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Message: #616

Date: 09/7/2014 7:55:48

From: chris.vantienhoven

Subject: A concurrent point derived from QA-P9

Dear Seiichi,

You are right the Circumcenter of the QG-P5 Triple triangle is a nice point.

I studied 2 years ago also the Centroid G5, Circumcenter O5, Orthocenter H5 of this QG-P5 Triple Triangle. Here are some results.

For QG-P5 (1st Quasi Circumcenter) see: <http://www.chrisvantienhoven.nl/quadrignon-objects/16-mathematics/encyclopedia-of-quadrignon-figures/quadrignon-objects/artikelen-qg/170-qg-p5.html>.

About G5, O5, H5:

H5 = QA-P4.QA-P5 $\wedge$ QA-P15.QA-P24

H5 = Reflection QA-P24 in QA-P15

O5 = Reflection QA-P15 in QA-P1

G5 = Reflection QA-P14 in QA-P1

G5 = Reflection QA-P10 in QA-P40

G5 lies on QA-P12.QA-P32 (4 : -1)

About the QG-P5 Triple Triangle:

QG-P5 Triple Triangle is homothetic with QA-Tr3, the QA-Morley Triangle (= QL-P2 Triple Triangle).

The Homothetic Center = QA-P24.

Area QG-P5 Triple Triangle = 4x Area QA-Tr3 (= QL-P2 Triple Triangle).

Best regards,

Chris

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Message: #617
Date: 10/7/2014 3:36:00
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
I made 3 pedal lines for the quadrangle and it seems to me that the triangle made from them is perspective to the QG-P5 triple triangle.
Sorry in advance if my idea would be wrong.
Best regards,
Seiichi.

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Message: #618
Date: 10/7/2014 10:32:50
From: chris.vantienhoven
Subject: A concurrent point derived from QA-P9

Dear Seiichi,

Very interesting. This expands your QFG-message #324.
Combining these messages I find:
There are 3 Homothetic triangles:
Tr2=QL-L2 Triple Triangle (QL-L2 is Steiner Line in a Quadrilateral)
Tr3=QL-L3 Triple Triangle (QL-L3 is Pedal Line in a Quadrilateral)
Tr5=QG-P5 Triple Triangle (QG-P5 is 1st Quasi Circumcenter in a Quadrigon)
(actually QL-P2 Triple Triangle is also homothetic with these triangles, but not further investigated here)
Areas Tr2 and Tr5 are equal so they are even congruent triangles.
Let HC23 = Homothetic Center Tr2 & Tr3
Let HC25 = Homothetic Center Tr2 & Tr5
Let HC35 = Homothetic Center Tr3 & Tr5
HC25 happens to be the QA-Centroid QA-P1.
HC23, HC25 and HC35 are collinear.
I didn't find other incidences for HC25 and HC35.
HC23 and HC35 have rather lengthy coordinates.
Best regards,
Chris

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Message: #619
Date: 10/7/2014 11:22:29
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
Thank you very much for your computation.
I think that the following might be the simplest one for QG-P5 TripleTriangle.
Given a quadrangle, 3 bimedians of it and its QG-P5 Triple Triangle(Tr5), the parallel translations of the bimedians to the corresponding vertices of Tr5 are concurrent.
Sorry for not computing the coordinates myself.
Best regards,
Seiichi.

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Message: #620
Date: 11/7/2014 1:44:42
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
Addition:
the parallel translations of the bimedians to the corresponding centers of Miquel circles are concurrent.
Best regards,
Seiichi.

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Message: #621
Date: 11/7/2014 8:42:17
From: chris.vantienhoven
Subject: A concurrent point derived from QA-P9

Dear Seiichi,
I would describe your point like this:
Translate the corresponding bimedians (the one connecting the diagonal midpoints) to the vertices of the QG-P5 Triple Triangle.
Let's call this point ptX.
It has these properties:
 $ptX = QA-P4.QA-P5 \wedge QA-P15.QA-P24$
 $ptX = \text{Reflection } QA-P24 \text{ in } QA-P15$
Interesting is that also these transformations play a role:
 QA-Tf3 = Orthopole Transformation, and
 QA-Tf4 = Moebius Conjugate
Because:
 $QA-Tf3(ptX) = QA-P15 = \text{Orthocenter Morley Triangle}$
 $QA-Tf4(ptX)$ is collinear with QA-P4 and QA-Tf4(QA-P5)
Moreover:
 $QA-P5.QA-P4 : QA-P4.ptX = QA-Tf4(ptX).QA-P4 :$
 $QA-P4.QA-Tf4(QA-P5)$
Very nice I think.
Best regards,
Chris

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Message: #622
Date: 11/7/2014 2:44:41
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
Thanks a lot for your detailed study, which is beyond my ability.
I would be glad if what I noticed would enrich EQF.
Best regards,
Seiichi.

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Message: #623

Date: 12/7/2014 9:19:12

From: chris.vantienhoven

Subject: A concurrent point derived from QA-P9

Dear Seiichi,

It looks like you found a new construction method for QA-points:
"Translated Bimedial Points".

The construction method is simple.

- * In a Quadrangle: let X be some QG-point or QL-Point.
- * In all 3 Component Quadrilaterals of the Reference Quadrangle a version of X can be constructed: X1, X2, X3.
- * X1, X2, X3 form a so called QA-Triple Triangle
- * In the same 3 Component Quadrilaterals a bimedian can be constructed connecting the midpoints of the 2 diagonals.
- * Draw lines through X1, X2, X3 parallel to the corresponding bimedians.

* When these parallel lines concur we have a new QA-point.

I examined your Translated Bimedial Points using QG-P5 in
QFG-message #621.

Now I also examined your Translated Bimedial Points using QL-P4
(like you suggested).

The combined results of both points can be seen in attached
file.

I wonder are there more Translated Bimedial Points using other
QG- and QL-points?

Could you investigate this?

Best regards,

Chris

Dear Seiichi,

I describe your points like this:

Translate the corresponding bimedians (the one connecting the diagonal midpoints) to the vertices of the QG-P5 Triple Triangle. They concur in a point ptX.

Translate the corresponding bimedians (the one connecting the diagonal midpoints) to the vertices of the QL-P4 Triple Triangle. They concur in a point ptY.

ptX and ptY have these properties:

-
- ptX lies on these lines:
 - QA-P4.QA-P5
 - QA-P15.QA-P24 $\text{ptX} = \text{Reflection QA-P24 in QA-P15}$
 - QA-P10.QA-P9043
 - QA-Tf3(ptX) = QA-P15
 - $\text{ptX.QA-Tf4(QA-P41)} \parallel \text{QA-P3.QA-P20}$
 - QA-Tf4(ptX) is collinear with QA-P4 and QA-Tf4(QA-P5)
Moreover: $\text{QA-P5.QA-P4} : \text{QA-P4.ptX} = \text{QA-Tf4(ptX).QA-P4} : \text{QA-P4.QA-Tf4(QA-P5)}$
-
- ptY lies on these lines:
 - QA-P1.QA-P15
 - QA-P40.QA-P9043
 - $\text{ptY.QA-P9} \parallel \text{QA-P1.QA-P2}$
 - QA-Tf3(ptY) lies on these lines:
 - QA-P1.QA-P28.QA-P9043
 - QA-P5.QA-P7
 - QA-Tf4(ptY) lies on these lines:
 - QA-P3.QA-P9043
 - QA-P4.QA-P9007

where:

QA-Tf3 = Orthopole Transformation

QA-Tf4 = Moebius Conjugate

QA-P9043 = QA-Orthocenter QL-P6-Triple Triangle

QA-P9007 = QA-Circumcenter QL-P1-Triple Triangle

Best regards,

Chris van Tienhoven

July 12, 2014

TranslatedBimedianPoints-01.pdf

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Message: #624

Date: 12/7/2014 7:27:36

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Antreas,

This again is another wonderfull assist.

I just can look to it in awe!

It is a wonderful construction!

Given a random Quadrangle and a random point Q we get four circles entangled in a wonderful way.

I do not see a special single object (like a point or another circle) coming out.

Maybe others can help us with enlightening ideas.

BTW, Do you know an unambiguous construction of the in-/ex-center point on the circumcircle?

Best regards,

Chris

Initial construction:

[Antreas]

Let ABCD be a quadrangle and O_a, O_b, O_c, O_d the circumcenters of BCD, CDA, DAB, ABC, resp. and Q a point.

The reflections of QO_a in the sidelines of BCD bound a triangle whose the incenter (or an excenter) J_a is lying on the circumcircle of BCD.

Similarly J_b, J_c, J_d .

The points (J_a, J_b, J_c, D) are concyclic.

The same is true for the points $(J_a, J_b, C, J_d), (J_a, B, J_c, J_d), (A, J_b, J_c, J_d)$

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Message: #625

Date: 12/7/2014 8:12:18

From: Antreas Hatzipolakis

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbola

Dear Chris,

You asked me:

BTW, Do you know an unambiguous construction of the in-/ex-center point on the circumcircle?

Yes!!! I know and I have only recently paid attention on it!

Long ago, before the discussion of incenter/excenters construction (in Hyacinthos) and when we were members in geometry-forum, Paul Yiu posted a very nice construction just by perspectivity!

Theorem

Given a line ℓ , denote by ℓ_a , ℓ_b , and ℓ_c its reflections about the sides of a triangle ABC. The triangle A'B'C' bounded by these three reflections is in perspective with triangle ABC, the perspector being a point on the circumcircle of ABC, diametrically opposite to the isogonal conjugate of the infinite point of the line ℓ .

Neither Paul nor anyone else had then noticed that the perspector is the incenter (or an excenter).

See Paul's original posting I have reposted to Hyacinthos:

<https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/22499> †)

Greetings Antreas

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Message: #626

Date: 13/7/2014 1:51:36

From: seiichikiri

Subject: A concurrent point derived from QA-P9

Dear Chris,

I see. I will examine the other QG and QL-points. I feel that translation method has some connection with Perspective Field Theory. But I can not say what it is.

Best regards, Seiichi.

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #627

Date: 13/7/2014 10:53:40

From: chris.vantienhoven

Subject: QA points or locus corresponding to Thomson-Gibert-Moses hyperbo

Dear Antreas,

Paul Yiu's construction makes it a lot easier. It is so simple!
Hard to understand that I overlooked it.

I repeat your general construction:

[Antreas]

Let ABCD be a quadrangle and O_a, O_b, O_c, O_d the circumcenters of BCD, CDA, DAB, ABC, resp. and Q a point.

The reflections of QO_a in the sidelines of BCD bound a triangle whose the incenter (or an excenter) J_a is lying on the circumcircle

of BCD. Similarly J_b, J_c, J_d .

The points (J_a, J_b, J_c, D) are concyclic. The same is true for the points $(J_a, J_b, C, J_d), (J_a, B, J_c, J_d), (A, J_b, J_c, J_d)$.

I still found some extra properties regarding your configuration.

Quadrangle

In a Quadrangle $P_1.P_2.P_3.P_4$ the points J_1, J_2, J_3, J_4 form a new Quadrangle.

I figured out that when $Q=QA-P_4$ then the quadrangles $P_1.P_2.P_3.P_4$ and $J_1.J_2.J_3.J_4$ are perspective.

Their perspector is $QA-P_3$!

Quadrilateral

Applying the reflection of lines in a Quadrilateral (system of 4 lines) by reflecting L_4 in the sides of triangle $L_1.L_2.L_3$ we also get a point J_4 .

Points J_1, J_2, J_3 are found analogously.

Now it appears that J_1, J_2, J_3, J_4 are concyclic.

They lie on the Miquel Circle $QL-Ci_3$ together with the Miquel point $QL-P_1$ and the circumcenters of the four Component Triangles of the Reference Quadrilateral.

This makes a connection with your Anopolis message #431 †).

Best regards,

Chris

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[33\]](#).

Message: #628
Date: 17/7/2014 4:29:43
From: eckart_schmidt@t-online.de
Subject: Special loci for EQF-points

Dear Chris, dear friends,

in 2012 I began to study loci of EQF-points for special pencils of quadrilaterals (Chris will remember).

1. Quadrilaterals (with the same Diagonal Crosspoint QG-P1), which have the same QA-Diagonal Triangle QA-DT and the same QL-Diagonal Triangle QL-DT.

2. Quadrilaterals with the same Diagonal Crosspoint QG-P1, which have the same QA-Diagonal Triangle QA-DT and the same QA-Orthogonal Hyperbola QA-Co2.

3. Quadrilaterals with the same Diagonal Crosspoint QG-P1, which have the same QL-Diagonal Triangle QL-DT and the same Inscribed Parabola QL-Co1.

Now I have gathered the results. Attached you will find a lot of new lines and conics, which will have further properties.

Perhaps you are interested. The collection is not complete, maybe there are mistakes and bad descriptions. Thank you in advance for corrections.

Best regards Eckart

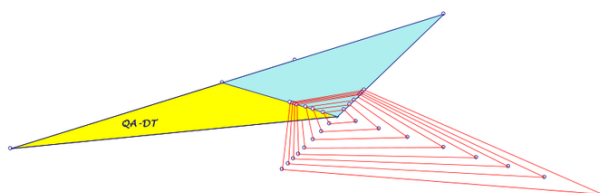
PS. I have chosen a word-file, for the translation in PDF was sometimes incomplete.

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

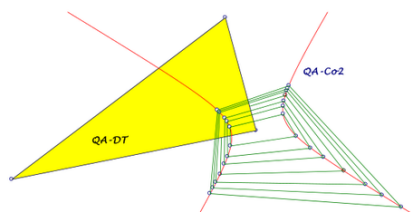
Special Loci for EQF-Points

Here are studied the loci of EQF-points for three pencils of quadrilaterals:

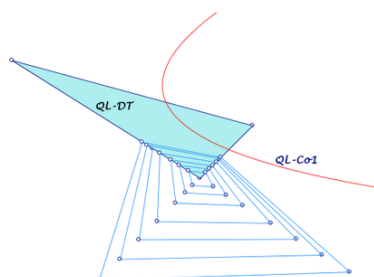
1. *Quadrilaterals (with the same Diagonal Crosspoint $QG-P1$), which have the same QA -Diagonal Triangle $QA-DT$ and the same QL -Diagonal Triangle $QL-DT$.*



2. *Quadrilaterals with the same Diagonal Crosspoint $QG-P1$, which have the same QA -Diagonal Triangle $QA-DT$ and the same QA -Orthogonal Hyperbola $QA-Co2$.*



3. *Quadrilaterals with the same Diagonal Crosspoint $QG-P1$, which have the same QL -Diagonal Triangle $QL-DT$ and the same Inscribed Parabola $QL-Co1$.*



Barycentric coordinates for the equations of the curves are DT-notations (see EQF).

For the second pencil the following shortcuts are used:

$$u = \frac{1}{b^2 r^2 - c^2 q^2}, \quad v = \frac{1}{c^2 p^2 - a^2 r^2}, \quad w = \frac{1}{a^2 q^2 - b^2 p^2},$$

and the orthogonal hyperbola has the equation

$$\frac{x^2}{u} + \frac{y^2}{v} + \frac{z^2}{w} = 0.$$

For the third pencil the following shortcuts are used:

$$u = m^2 - n^2, \quad v = n^2 - l^2, \quad w = l^2 - m^2,$$

and the inscribed parabola has the equation

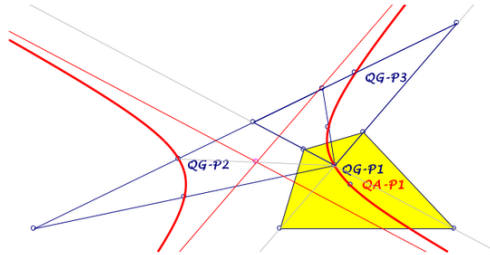
$$\frac{x^2}{u} + \frac{y^2}{v} + \frac{z^2}{w} = 0.$$

Some remarks: For every EQF-point three loci can be considered, but not all are discussed. Only loci are mentioned, which are a fixed point or a line or a circle or a conic, otherwise you find “-----”. If a locus is not completely described, you will find “???”. Considered are only QA-P1 – QA-P30, QL-P1 – QL-P26, QG-P1-QG-P19.

QA-P1 QA-Centroid or Quadrangle Centroid

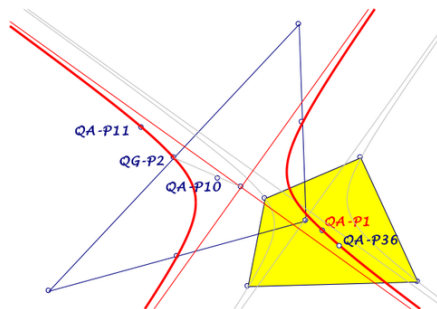
- Hyperbola:**

- ... centered in the midpoint of QG-P1, QG-P2,
- ... asymptotes parallel to the diagonals,
- ... through QG-P1, QG-P2, QG-P3,
- ... equation: $r^2 x(x-y) + (p^2 - r^2)zx + p^2 z(y-z) = 0$



- Orthogonal hyperbola:**

- ... centered in $T(QG-P2, QA-P10, -3:1)$,
- ... parallel asymptotes wrt QA-Co2,
- ... circumscribed QA-DT medial triangle,
- ... through QG-P2, QA-P11, QA-P36
- ... equation: $vw x(-x+y+z) + wuy(x-y+z) + uvz(x+y-z) = 0$



- Line $QG-L3$

QA-P2 Euler-Poncelet Point

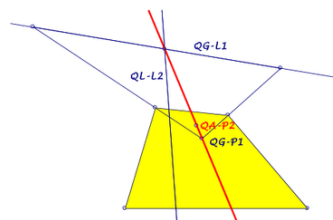
- Circle $QA-Ci1$
- Fixed point
- Line

...through $QG-P1$ and the intersection of $QG-L1$ and $QL-L2$.

... equation:

$$(-S_A u + S_B v + S_C w)x + (S_A u + S_B v - S_C w)z = 0$$

3



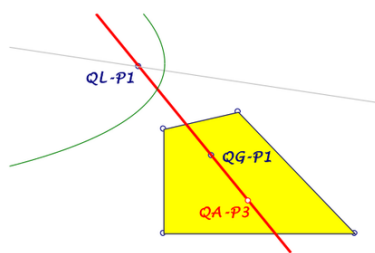
QA-P3 Gergonne-Steiner Point

- -----
- Orthogonal Hyperbola $QA-Co4$
- Line

... through $QG-P1$ and $QL-P1$

...equation:

$$(S_A u + S_B v - S_C t)wx - (-S_A u + S_B v + S_C w)rz = 0$$



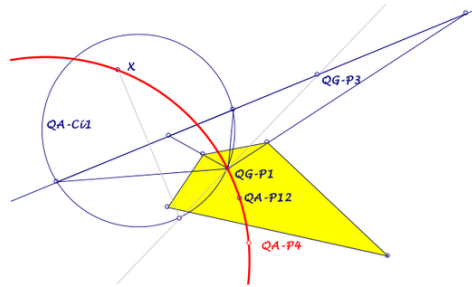
QA-P4 Isogonal Center

- **Circle**

... through $QG-P1$, $QA-P12$ and a point X , which is the reflection of the second intersection of $QG-P1.QG-P3$ and $QA-Ci1$ in $QG-L1$.

... equation:

$$2r^2a^2S_Ax^2 - 2p^2c^2S_Cz^2 + (S_A - S_C)(p^2c^2 + r^2a^2)zx + (p^2c^4 + r^2a^2(S_A - S_B))xy - (r^2a^4 - p^2c^2(S_B - S_C))yz = 0$$

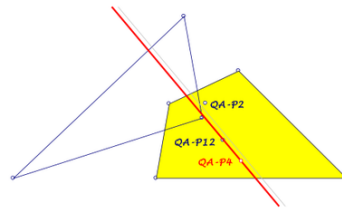


- **Line**

... through $QA-P12$,

... parallel to the Simson line of $QA-P2$ wrt $QA-DT$,

... equation: $a^2S_Avwx + b^2S_Bwuy + c^2S_Cuvz = 0$

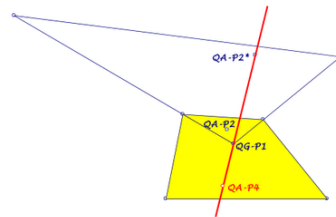


- **Line**

... through $QG-P1$ and the isogonal conjugate of $QA-P2$ wrt $QL-DT$,

... equation:

$$c^2(S_Au + S_Bv - S_Cw)x + a^2(-S_Au + S_Bv + S_Cw)z = 0$$



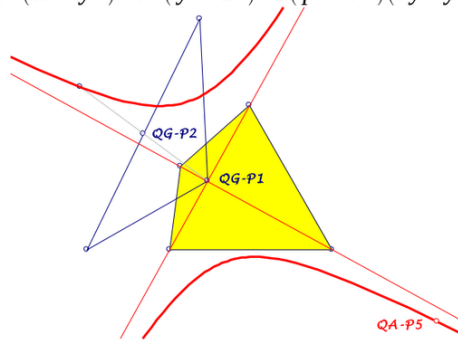
4

QA-P5 Isotomic Center

- **Hyperbola**

... centered in $QG-P1$,
 ... asymptotes are the diagonals,
 ... through the reflection of $QG-P1$ in $QG-P2$,
 ... equation:

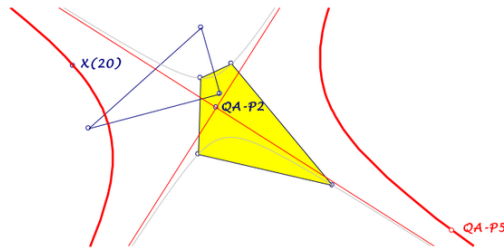
$$p^2(x^2+y^2) - r^2(y^2+z^2) + 2(p^2-r^2)(xy+yz+zx) = 0$$



- **Orthogonal hyperbola**

... asymptotes and center as $QA-Co2$,
 ... through $X(20)$ of $QA-DT$,
 ... equation:

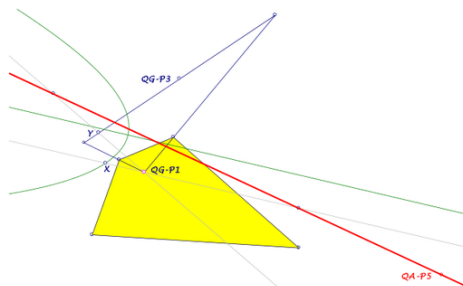
$$vw(y+z)(2x+y+z) + wu(z+x)(x+2y+z) + uv(x+y)(x+y+2z) = 0$$



- **Line**

... through $T(QG-P1, X, -4:5)$ with X intersection of $QL-Co1$ and a parallel to $QL-L1$ in $QG-P1$,
 ... through $T(QG-P1, Y, -2:1)$ with Y intersection of $QG-L1$ and the polar of $QG-P3$ wrt $QL-Co1$,
 ... equation: $(u-v)wx + 2wuy + (w-v)uz = 0$

5



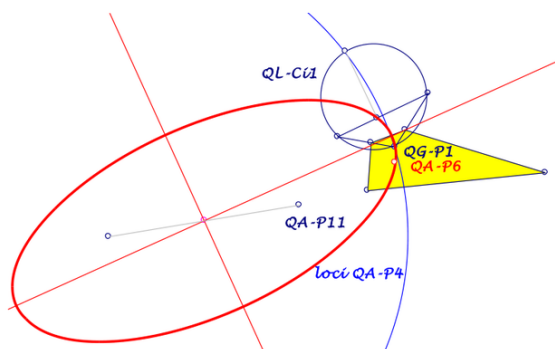
QA-P6 Parabola Axes Crosspoint

- **Conic**

... centered in the midpoint of $QA-P11$ and the center of the circle, which is the loci for $QA-P4$,
 ... axes parallel to the angle bisectors of $QL-DT$ in $QG-P1$,
 ... through $QG-P1$ and the pedal point on $QG-L1$ of the second intersection of $QL-Ci1$ and the circle, which is the loci of $QA-P4$,
 ... equation:

$$r^4 a^2 (r^2 a^2 - p^2 S_B) x^2 - p^4 c^2 (p^2 c^2 - r^2 S_B) z^2 + p^2 r^4 a^2 S_A xy - p^4 r^2 c^2 S_C yz + p^2 r^2 (r^2 a^2 (c^2 + S_B) - p^2 c^2 (a^2 + S_B)) zx = 0$$

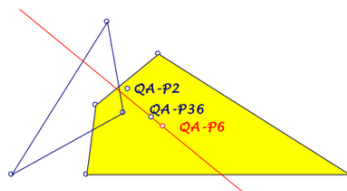
6



- **Line**

... Simson line of $QA-P2$ wrt $QA-DT$,
 ... through $QA-P36$
 ... equation:

$$\frac{vwx}{b^2 w(u+v) - c^2 v(u+w)} + \frac{wuy}{c^2 u(v+w) - a^2 w(v+u)} + \frac{uvz}{a^2 v(w+u) - b^2 u(w+v)} = 0$$



- **Line**
 ... through $QG-P1$ and a point Z , constructed as follows:
 let X be the intersection of $QG-L1$ and $QL-L2$,
 let Y be the intersection of $QG-L3$ and a parallel to $QA-L1$ through X ,
 let Z be the intersection of a parallel to $QA-L4$ through Y
 and a parallel to $QA-L2$ through X ,
 ... equation: $c^2(a^2v+b^2u)x+a^2(b^2w+c^2v)z=0$

QA-P7 QA-Nine-point Homothetic Center

- **Hyperbola**
 ... through $QG-P1$,
 ... ???
- **Hyperbola**
 ...through $QA-P11$,
 ... ???
- **Line**
 ...through $QG-P1$,
 ... ???

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QA-P8 Midray Homothetic Center

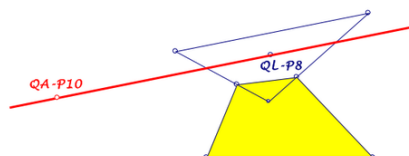
- **Hyperbola**
 ... through $QG-P1$,
 ... ???
- **Hyperbola**
 ...centered in $QA-P13$,
 ... through $QA-P11$, $QA-P12$,
 ...???
- **Line**
 ... through $QG-P1$,
 ... ???

QA-P9 QA-Miquel Center

- **Line**
 ...???

QA-P10 Centroid of the QA -Diagonal Triangle

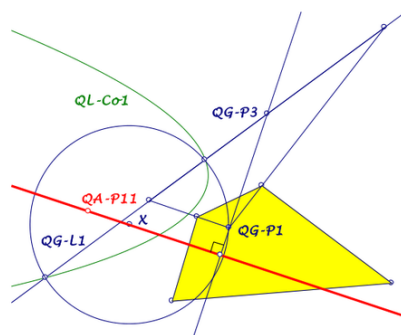
- **Fixed point**
- **Fixed point**
- **Line**
 - ...parallel to $QG-L1$,
 - ...through $QL-P8$,
 - ...equation: $x - 2y + z = 0$



QA-P11 Circumcenter of the QA -DT Triangle

- **Fixed point**
- **Fixed point**
- **Line**
 - ...orthogonal to $QG-P1.QG-P3$,
 - ...through circumcenter X for $QG-P1$ and the intersections of $QL-Co1$ and $QG-L1$,
 - ...equation: $c^2x - S_B y + a^2z = 0$

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QA-P12 Orthocenter of the QA -DT Triangle

- **Fixed point**
- **Fixed point**
- **Line**
 - ...through $QG-P1$ perpendicular $QG-L1$

QA-P13 Nine-Point Center of the QA -DT Triangle

- **Fixed point**

- **Fixed point**

- **Line**

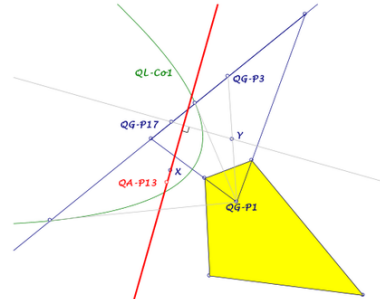
... through the nine-point center X of the triangle $QG-P1$

and the intersections of $QL-Co1$ and $QG-L1$,

... orthogonal to $Y.QG-P17$ with Y midpoint of $QG-P1.QG-P3$,

... equation:

$$(S^2 - 2S_A^2 + S_A S_C)x - (3S^2 + S_A S_C)y + (S^2 - 2S_C^2 + S_A S_C)z = 0$$



QA-P14 Centroid of the Morley Triangle

- **Hyperbola**

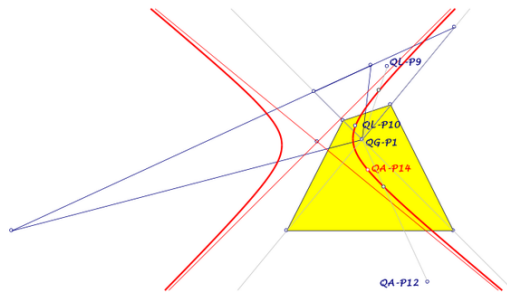
... asymptotes orthogonal to the diagonals,

... through $QL-P10$, $T(QG-P1, QL-P9, 2:1)$, $T(QG-P1,$

$QA-P12, 1:2)$,

... equation:

$$\begin{aligned} & (p^2 c^2 S_A + r^2 (S^2 + S_B S_C))x^2 - (p^2 (S^2 + S_A S_B) + r^2 a^2 S_C)z^2 \\ & + S_B (p^2 c^2 - r^2 a^2)y^2 - (p^2 (S^2 + 2S_A^2) - r^2 (S^2 + 2S_C^2))xz \\ & - (2p^2 c^4 - r^2 (S^2 + 2S_B^2))xy - (p^2 (S^2 + 2S_B^2) - 2r^2 a^4)yz = 0 \end{aligned}$$



- **Orthogonal hyperbola**

... parallel asymptotes wrt $QA-Co2$,

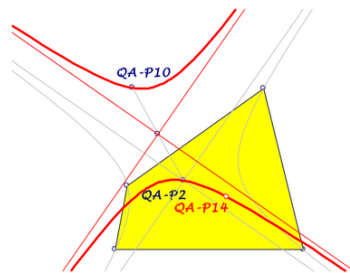
... centered in the midpoint of $QA-P2.QA-P10$,

... through $QA-P2, QA-P10$,

... equation:

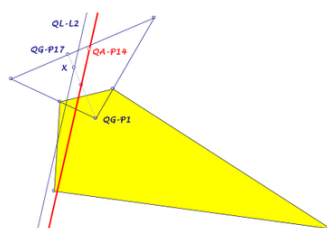
$$2vwx^2 + 2wuy^2 + 2uvz^2 - w(u+v)xy - v(w+u)zx - u(v+w)yz = 0$$

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- **Line**
 ... parallel $QL-L2$,
 ... through $T(QG-P1, X, 2:1)$ with X intersection of $QG-P1$, $QG-P17$ and $QL-L2$,
 ... equation:

$$(-2S_A u + 2S_B v + S_C w)x + (S_A u - S_B v + S_C w)y + (S_A u + 2S_B v - 2S_C w)z = 0$$



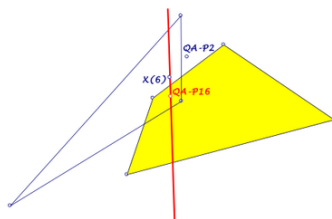
10

QA-P15 Orthocenter of the Morley Triangle

- **Line**
 ...???

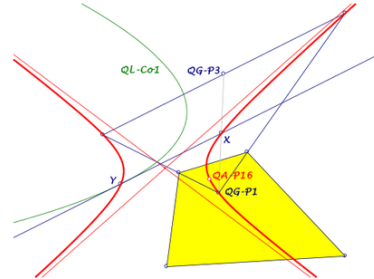
QA-P16 QA -Harmonic Center

- **Line** $QG-L2$
- **Line**
 ... trilinear polar of $QA-P2$ wrt $QA-DT$,
 ... through $X(6)$ of $QA-DT$,
 ... equation: $vwx + wuy + uvz = 0$



- **Hyperbola**

- ... circumscribed $QL-DT$,
- ... through midpoint X of $QG-P1.QG-P3$,
- ... Y contact point of the tangent at $QL-Co1$ through X (parallel to $QG-L1$),
- ... equation: $uyz + 2vzx + wxy = 0$



QA-P17 Involutory Conjugate of $QA-P5$



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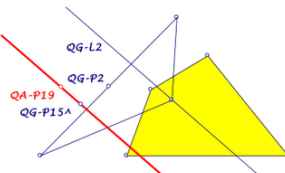
QA-P18 Involutory Conjugate of $QA-P19$



QA-P19 AntiComplement of $QA-P16$ wrt $QA-DT$

- **Line**

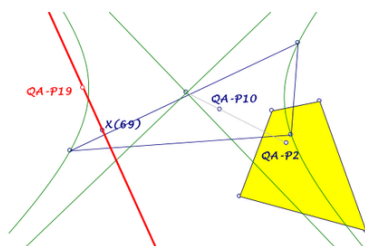
- ... reflection of $QG-L2$ in $QG-P2$,
- ... through isotomic conjugate of $QG-P15$ wrt $QA-DT$,
- ... equation: $p^2x + (p^2 - r^2)y - r^2z = 0$



- **Line**

- ... isotomic conjugate of an orthogonal hyperbola, circumscribed $QA-DT$, centered in $T(QA-P2, QA-P10, -3:1)$ through $X(69)$ of $QA-DT$,

... equation: $u(v+w)x+v(w+u)y+w(u+v)z=0$



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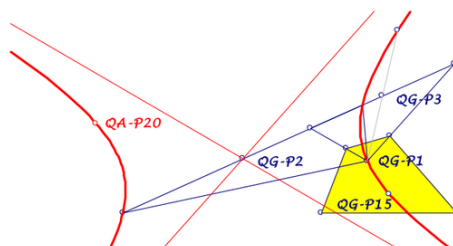
QA-P20 Reflection of QA-P5 in QA-P1

- **Hyperbola**

... QA-DT-isotomic conjugate of the locus of QA-P19,
 ... asymptotes parallel to the diagonals,
 ... centered in QG-P2,
 ... through QG-P1, QG-P15, reflection of QG-P1 in QG-P3,

... equation: $p^2(x+y)z-r^2(y+z)x=0$

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- **Orthogonal Hyperbola QA-Co4**

- **Line**

... parallel QL-L1 through QG-P1

QA-P21 Reflection of QA-P16 in QA-P1

• ----

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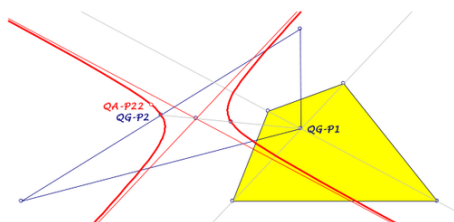
• ----

QA-P22 Midpoint QA-P1 and QA-P20

- **Hyperbola**

... asymptotes parallel to the diagonals,
 ... centered in $T(QG-P1, QG-P2, 3:1)$,
 ... through QG-P2 and midpoint QG-P1.QG-P2
 ... equation:

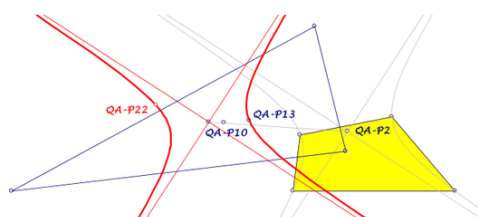
$$p^2(x+y-z)(x+y-3z)-r^2(-x+y+z)(-3x+y+z)=0$$



- **Orthogonal hyperbola**

- ... parallel asymptotes wrt $QA-Co2$,
- ... centered $T(QA-P2, QA-P10, -9:1)$,
- ... through $QA-P13$,
- ... equation:

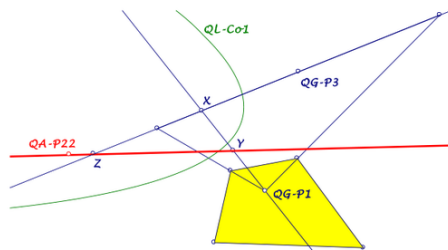
$$(uv+uw+3vw)x^2+(vw+vu+3wu)y^2+(wu+wv+3uv)z^2+2(vw-2u(v+w))yz+2(wu-2v(w+u))zx+2(uv-2w(u+v))xy=0$$



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- **Line**

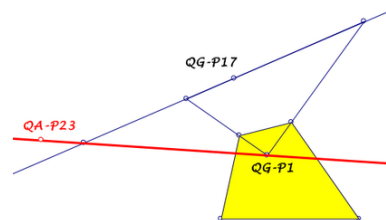
- ... through the midpoint Y of $QG-P1$ and X with X intersection of $QG-L1$ and the polar of $QG-P3$ wrt $QL-Co1$,
- ... through the 4th harmonic point Z of X wrt $QL-2P3$,
- ... equation: $(w-u)(wx-uz)+2wuy=0$



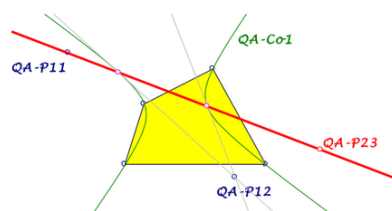
QA-P23 Inscribed Square Axes Crosspoint

- **Line**

- ... through $QG-P1$ and the 4th harmonic point of $QG-P17$ wrt $QG-2P3$,
- ... equation: $p^2S_Az-r^2S_Cx=0$



- **Line**
 ... polar of $QA-P12$ wrt $QA-Co2$,
 ... through $QA-P11$,
 ... equation: $vwS_B S_C x + wuS_C S_A y + uvS_A S_B z = 0$

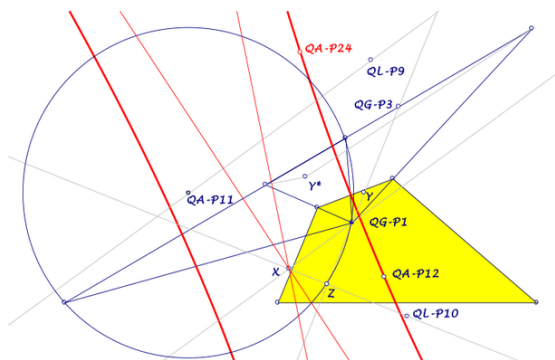


- **Line $QA-L4$**

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QA-P24 Anticomplement of $QA-P1$ wrt Morley Triangle

- **Hyperbola**
 ... centered in the intersection X of a parallel to $QA-P11$. $QL-P9$ through $QG-P1$ and a perpendicular line to $QG-P1$. $QG-P3$ through $QL-P10$,
 ... asymptotes orthogonal wrt the legs of a triangle about $QL-2P2$ with the vertex Y^* , which is the $QL-DT$ -isogonal conjugate of the fourth harmonic point Y of the second intersection Z of $QG-P1$. $QG-P3$ and $QA-Ci1$ wrt $QG-P1$. $QG-P3$
 ... through $QA-P12$

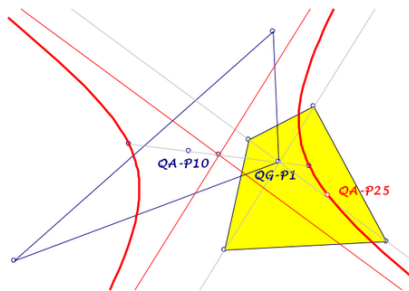


- **Hyperbola**
... centered in $QA-P2$,
... through $QA-P12$,
... ???
- **Line**
... ???

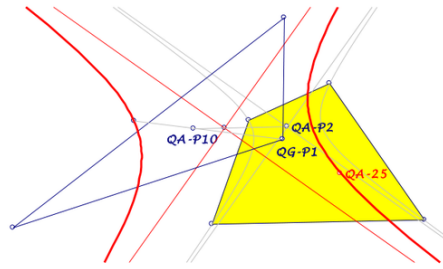
QA-P25 1st QA-Quasi Centroid

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- **Hyperbola**
... asymptotes parallel to the diagonals,
... centered in $T(QG-P1, QA-P10, 2:1)$,
... through $T(QG-P1, QA-P10, -1:4)$ and $T(QG-P1, QA-P10, -5:2)$,
... equation:
$$p^2(5x+5y-4z)(x+y+10z) - r^2(-4x+5y+5z)(10x+y+z) = 0$$



- **Orthogonal hyperbola**
... asymptotes parallel wrt $QA-Co2$,
... centered $T(QA-P2, QA-P10, 2:1)$,
... through $T(QG-P1, QA-P10, -5:2)$,
... equation:
$$vw(-4x+5y+5z)(10x+y+z) + wu(5x-4y+5z)(x+10y+z) + uv(5x+5y-4z)(x+y+10z) = 0$$



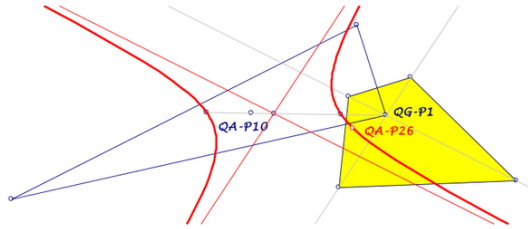
• -----

QA-P26 2nd QA-Quasi Centroid

- **Hyperbola**

- ... asymptotes parallel to the diagonals,
- ... centered in $T(QG-P1, QA-P10, 5:1)$,
- ... through $T(QG-P1, QA-P10, 1:2)$ and $T(QG-P1, QA-P10, -4:1)$,
- ... equation:

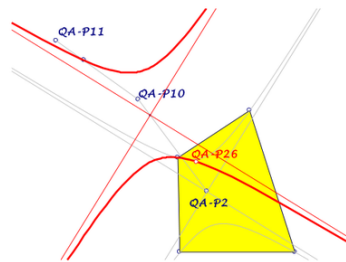
$$p^2(x+y-8z)(4x+4y-5z)-r^2(-5x+4y+4z)(-8x+y+z)=0$$



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- **Orthogonal hyperbola**

- ... parallel asymptotes wrt QA-Co2,
 - ... centered in $T(QA-P2, QA-P10, 5:1)$,
 - ... through $T(QA-P10, QA-P11, 2:1)$,
 - ... equation:
- $$vw(-5x+4y+4z)(-8x+y+z)+wu(4x-5y+4z)(x-8y+z)+uv(4x+4y-5z)(x+y-8z)=0$$



• -----

QA-P27 M3D Center

- -----
- -----
- -----

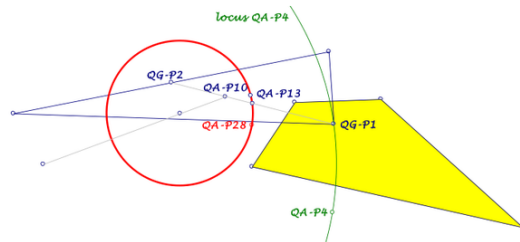
QA-P28 Midpoint of the Foci of the QA-Parabolas

• **Circle**

... $h_{QA-P10}(Ci, 1/4)$ with the circle Ci locus for $QA-P4$,
 ... through $QA-P13$ and midpoint $QG-P1.QG-P2$,
 ... equation:

$$r^2 a^2 (2 S_A x (-3x + y + z) - S_B (x - 3y + z)(x - y + z) - S_C (x + y - 3z)(x + y - z))$$

$$+ p^2 c^2 (S_A (-3x + y + z)(-x + y + z) + S_B (x - 3y + z)(x - y + z) - 2 S_C z (x + y - 3z)) = 0$$



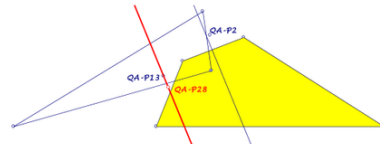
17

• **Line**

... parallel to the Simson line of $QA-P2$ wrt $QA-DT$,
 ... through $QA-P13$,
 ... equation:

$$(b^2 c^2 u (v + w) - 2 a^2 S_A v w) x + (c^2 a^2 v (w + u) - 2 b^2 S_B w u) y$$

$$+ (a^2 b^2 w (u + v) - 2 c^2 S_C u v) z = 0$$



- -----

QA-P29 Complement of $QA-P2$ wrt $QA-DT$

• **Circle QA-Ci2**

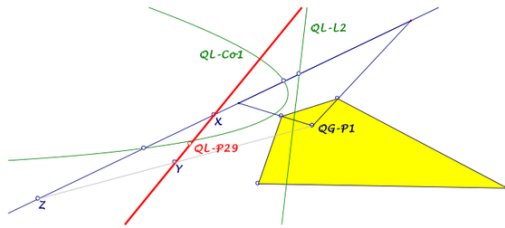
• **Fixed point**

• **Line**

... through the midpoint X of the intersections of $QG-L1$ and $QL-Co1$,

... through the midpoint Y of $QG-P1$ and the 4th harmonic point Z of $QG-L1 \cap QL-L2$ wrt $QG-2P3$,
 ... equation:

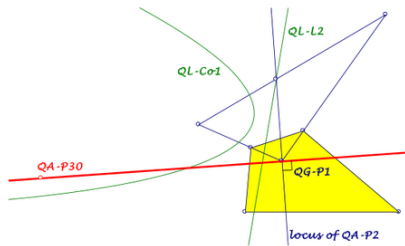
$$2S_B(wx-uz) - (a^2w - c^2u)y = 0$$



QA-P30 Reflection of $QA-P2$ in $QA-P11$

- **Circle $QA-Ci1$**
 - **Fixed point**
 - **Line**
- ... through $QG-P1$,
 ... orthogonal to the line, which is the locus of $QA-P2$,
 ... equation: $(S_A^2u - S^2w + S_A S_B u - S_B S_C w)x - (S_C^2w - S^2u - S_A S_B u + S_B S_C w)z = 0$

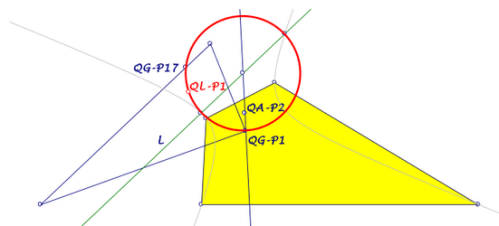
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QL-P1 Miquel point

- **Circle $QL-Ci2$**
 - **Circle**
- ... centered in the intersection of $QG-P1$, $QA-P2$ and a parallel L to $QG-L1$ half the distance to $QG-P1$,
 ... through $QG-P1$, $QG-P17$, intersections of $QA-Co2$ and L ,
 ... equation:

$$wS_A x^2 + uS_C z^2 - (uc^2 + wS_B)xy - (wa^2 + uS_B)yz - (uS_A + wS_C)zx = 0$$



- **Fixed point**

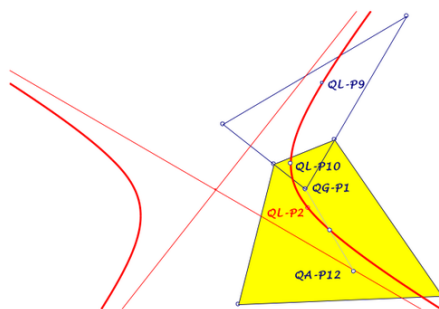
QL-P2 Morley Point

- **Hyperbola**

... asymptotes orthogonal to the diagonals,
 ... through $QL-P9$, $QL-P10$ and midpoint of $QG-P1.QA-P12$,

... equation:

$$l^2 S_A (S_B - S_C) x^2 + S_B (l^2 a^2 - n^2 c^2) y^2 + n^2 S_C (S_A - S_B) z^2 - (l^2 (S_A a^2 + S_B (S_B - S_C)) - n^2 S_B c^2) xy + (l^2 S_A a^2 - n^2 S_C c^2) zx - (l^2 S_B a^2 - n^2 (S_C c^2 + S_B (S_B - S_A))) yz = 0$$



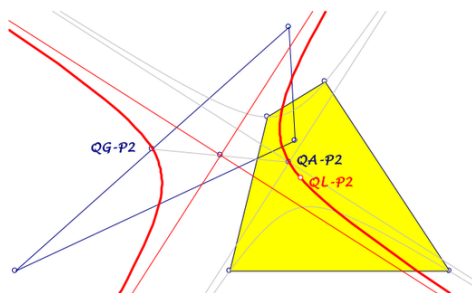
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- **Orthogonal hyperbola**

... parallel asymptotes wrt $QA-Co1$,
 ... centered in midpoint $QA-P2.QG-P2$,
 ... through $QA-P2$, $QG-P2$,

$$\frac{x^2}{u} + \frac{2y^2}{v} + \frac{z^2}{w} - \frac{x(y+z)}{u} - \frac{z(x+y)}{w} = 0$$

... equation:



- Line $QL-L2$

QL-P3 Kantor-Hervey Point

• ----

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- Line

...orthogonal to $QL-L7$,

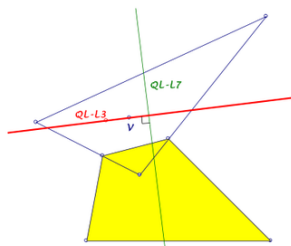
... through point V (see remark $QL-P22$),

... equation:

$$(8S^2S_Auvw+T)x+(8S^2S_Buvw+T)y+(8S^2S_Cuvw+T)z=0$$

$$\text{with } T=(a^2v+b^2u)(b^2w+c^2v)(c^2u+a^2w)$$

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QL-P4 Miquel Circumcenter

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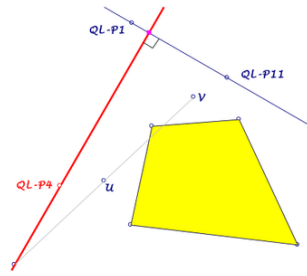
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- Line

... orthogonal to $QL-P1, QL-P11$,

... through $T(U, V, -1:2)$ with U, V see remark $QL-P22$,

... equation very extensive

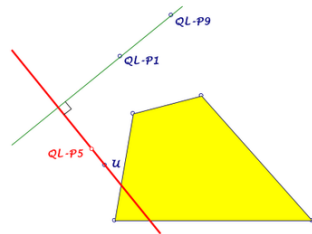


QL-P5 Clawson Center

- - - - -
- - - - -
- Line

... orthogonal to $QL-P1, QL-P9$,
 ... through point U (see remark $QL-P22$),
 ... equation:

$$(a^4vw + b^4wu + c^4uv + 4a^2S_Avw)x + (a^4vw + b^4wu + c^4uv + 4b^2S_Bwu)y + (a^4vw + b^4wu + c^4uv + 4c^2S_Cuv)z = 0$$



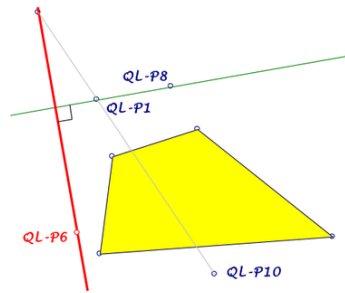
21

QL-P6 Dimidium Point

- - - - -
- - - - -
- Line

... orthogonal $QL-P1, QL-P8$,
 ... through $T(QL-P1, QL-P10, -1:3)$,
 ... equation:

$$\begin{aligned} &((S_A - 3S_B)c^2v^2 + 2(S_A^2 + 3S^2)vw + (S_A - 3S_C)b^2w^2)x \\ &+ ((S_B - 3S_A)c^2u^2 + 2(S_B^2 + 3S^2)wu + (S_B - 3S_C)a^2w^2)y \\ &+ ((S_C - 3S_A)b^2u^2 + 2(S_C^2 + 3S^2)uv + (S_C - 3S_B)a^2v^2)z = 0 \end{aligned}$$



QL-P7 Newton-Steiner Point

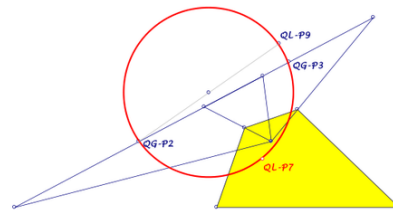
- **Circle**

... Thales circle about $QG-P2$. $QL-P9$,

... through $QG-P3$,

... equation:

$$b^2(z-x)(l^2x+n^2z)+(n^2c^2-l^2a^2)y^2 + (l^2(c^2-2S_C)-n^2c^2)xy + (l^2a^2-n^2(a^2-2S_A))yz=0$$



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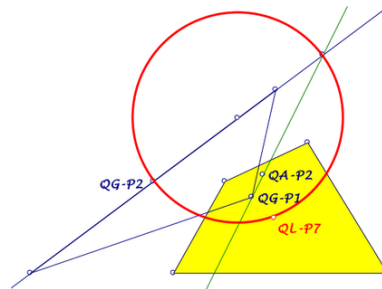
- **Circle**

... Thales circle about $QG-P2$ and the intersection of

$QG-L1$ and $QG-P1$. $QA-P2$,

... equation:

$$b^2v(x-z)(wx-uz)+(S_Bv(u+w)-b^2wu)y^2 + (3S_Cw-S_Au-b^2w)vxy + (3S_Au-S_Cw-b^2u)vyz=0$$



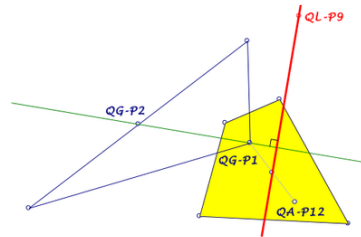
- **Line QL-L2**

QL-P8 Centroid of QL-Diagonal Triangle

- **Fixed point**
- **Line**
 - ... parallel to $QG-L1$ through $QA-P10$,
 - ... equation: $x - 2y + z = 0$
- **Fixed point**

QL-P9 Circumcenter QL-Diagonal Triangle

- **Fixed point**
- **Line**
 - ... orthogonal to $QG-P1.QG-P2$,
 - ... through midpoint of $QG-P1.QA-P12$,
 - ... equation: $c^2x - 2S_By + a^2z = 0$



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- **Fixed point**

QL-P10 Orthocenter QL-Diagonal Triangle

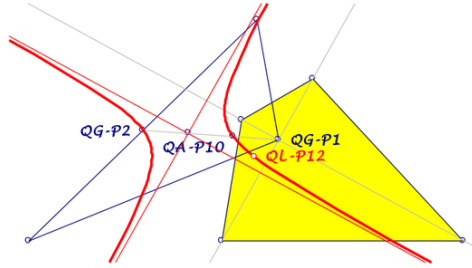
- **Fixed point**
- **Line** $QG-P1.QA-P12$
- **Fixed point**

QL-P11 Nine-Point Center QL-Diagonal Triangle

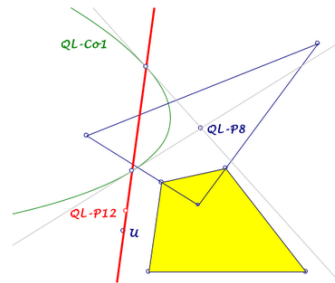
- **Fixed point**
- **Line**
 - ... ???
- **Fixed point**

QL-P12 QL-Centroid or Lateral Centroid

- **Hyperbola**
 - ... centered in $QA-P10$,
 - ... asymptotes parallel to the diagonals,
 - ... through $QG-P2$ and midpoint $QG-P1.QA-P10$,
 - ... equation:
- $$2(l^2x + n^2z)^2 - l^2n^2(x^2 - y^2 + z^2) + l^4(2y - z)x + n^4(2y - x)z = 0$$



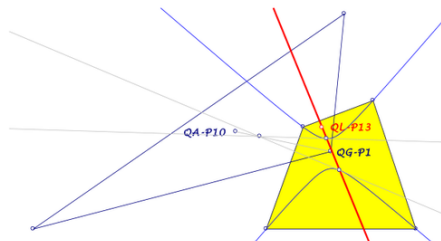
- **Orthogonal hyperbola**
... asymptotes parallel to those of $QA-Co2$,
... through $QG-P2$,
... ???
- **Line**
... polar of $QL-P8$ wrt $QL-Co1$,
... through point U (see remark $QL-P22$),
... equation: $vwx + wuy + uvz = 0$



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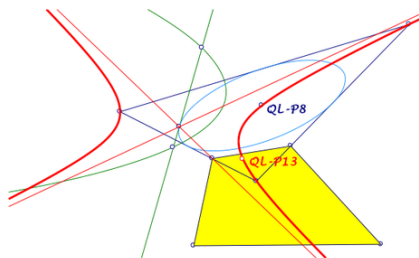
QL-P13 Lateral Harmonic Center

- **Line $QG-L2$**
- **Line**
... polar of $T(QG-P1, QA-P10, 3:1)$ wrt $QA-Co2$
... equation: $vwx + 2wuy + uvz = 0$



- **Hyperbola**
... circumscribed $QL-DT$,

... centered in the contact point of the polar of $QL-P8$
wrt $QL-Co1$ and the inscribed Steiner ellipse of $QL-DT$,
... through $QL-P8$
... equation: $wxy + uyz + vzx = 0$



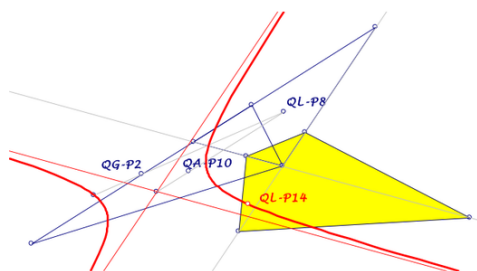
QL-P14 1st QL-Quasi Centroid

- **Hyperbola**

... asymptotes parallel to the diagonals,
... centered in $T(QA-P10, QL-P8, -1:4)$,
... through $T(QG-P2, QL-P8, -4:1)$,
... equation:

$$\begin{aligned} &-2(-35l^4 + 10l^2n^2 + n^4)x^2 + (7l^4 + 34l^2n^2 + 7n^4)y^2 \\ &-2(l^4 + 10l^2n^2 - 35n^4)z^2 + (5l^4 + 14l^2n^2 + 77n^4)yz \\ &-(13l^4 - 122l^2n^2 + 13n^4)zx + (77l^4 + 14l^2n^2 + 5n^4)xy = 0 \end{aligned}$$

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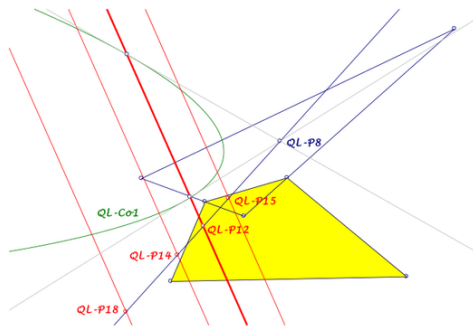


• -----

- **Line**

... parallel to the polar of $QL-P8$ wrt $QL-Co1$ (see $QL-P12$),
... $4/3$ distance wrt $QA-P8$,
... equation:

$$(u^2 - 10vw)x + (v^2 - 10wu)y + (w^2 - 10uv)z = 0$$



QL-P15 2nd QL-Quasi Centroid

- **Hyperbola**

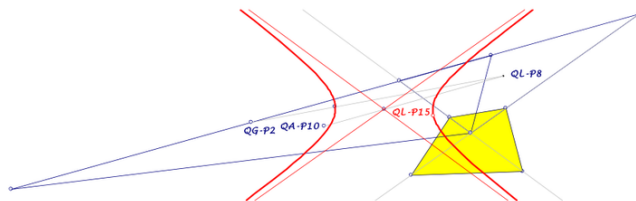
... asymptotes parallel to the diagonals,.

... centered in $T(QA-P10, QL-P8, 1:2)$,

... through $T(QG-P2, QL-P8, 1:2)$,

... equation:

$$4(10l^4 - 8l^2n^2 + n^4)x^2 - 5(l^4 - 22l^2n^2 + 5n^4)y^2 + 4(l^4 - 8l^2n^2 + 10n^4)z^2 - (l^4 + 10l^2n^2 - 35n^4)yz - (37l^4 - 98l^2n^2 + 37n^4)zx - (-35l^4 + 10l^2n^2 + n^4)xy = 0$$



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- -----

- **Line**

... parallel to the polar of $QL-P8$ wrt $QL-Co1$ (see $QL-P12, 14$),

... 2/3 distance wrt $QA-P8$,

... equation: $(u^2 + 8vw)x + (v^2 + 8wu)y + (w^2 + 8uv)z = 0$

QL-P16 QL-Quasi Circumcenter

- **Circle $QL-Ci1$**

- -----

- **Fixed point**

QL-P17 QL-Adjunct Quasi Circumcenter

- **Circle $QL-Ci1$**

- **Kreis**

... centered on $QG-L1$,

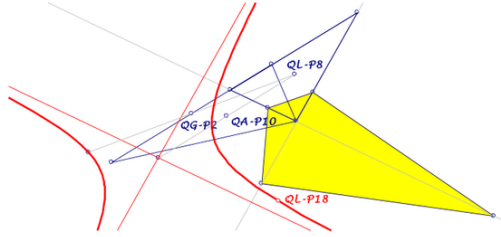
- **Circle $QL-Ci1$**

QL-P18 Reflection of $QL-P8$ in $QL-P12$

- **Hyperbola**

... asymptotes parallel to the diagonals,
 ... centered in the reflection of $QL-P8$ in $QA-P10$,
 ... through the reflection of $QL-P8$ in $QG-P2$,
 ... equation:

$$4l^4x^2 + (l^2 + n^2)y^2 + 4n^4z^2 + (l^4 + 2l^2n^2 + 5n^4)yz \\ + (l^4 + 6l^2n^2 + n^4)zx + (5l^4 + 2l^2n^2 + n^4)xy = 0$$



- **Line**

... parallel to the polar of $QL-P8$ wrt $QL-Co1$ (see $QL-P12$, 14, 15),
 ... double distance wrt $QA-P8$,
 ... equation: $(v-w)^2x + (w-u)^2y + (u-v)^2z = 0$

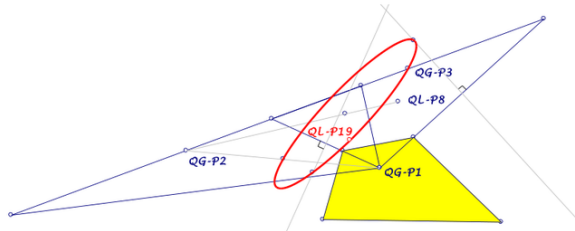
27

QL-P19 Midpoint of $QL-P1$ and $QL-P7$

- **Ellipse**

... centered in $T(QG-P2, QL-P8, 3:1)$,
 ... through $QG-P3$, midpoint $QG-P1.QG-P2$ and its
 pedal points on the bisectors of the $QL-DT$ -legs,
 ... equation:

$$n^6c^2(-x+y+z)^2 - l^6a^2(x+y-z)^2 \\ + l^2n^4(-x+y+z)(2S_Ax + S_B(3x+y-3z)) \\ - l^4n^2(x+y-z)(2S_Cz + S_B(-3x+y+3z)) = 0$$



- **Ellipse**

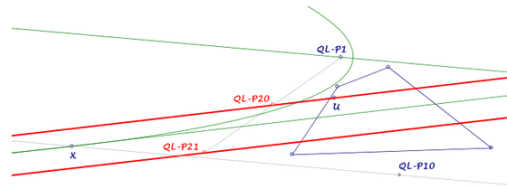
... axes parallel to the asymptotes of $QA-Co2$,
 ... through intersection of $QG-L1$ and $QG-P1.QA-P2$,
 ... ???

- **Line $QL-L3$**

QL-P20 Orthocenter Homothetic Center

- **Line**
- ... through U (see remark $QL-P22$),
- ... parallel to the tangent in X at $QL-Co1$ with X intersection of $QL-Co1$ and a parallel to $QL-L1$ through $QL-P10$,
- ... equation:

$$(b^2w+c^2v)^2x+(c^2u+a^2w)^2y+(a^2v+b^2u)^2z=0$$



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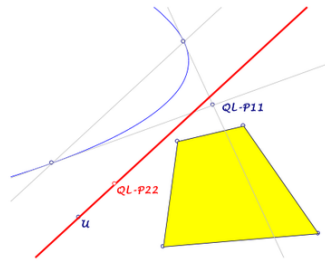
QL-P21 Adjunct Orthocenter Homothetic Center

- **Line**
- ... parallel to the locus-line of $QL-P20$,
- ... with double distance to $QL-P1$

QL-P22 QL-Nine-Point Center Homothetic Center

- **Line**
- ... through U (see remark),
- ... parallel to the polar of $QL-P11$ wrt $QL-Co1$
- ... equation:

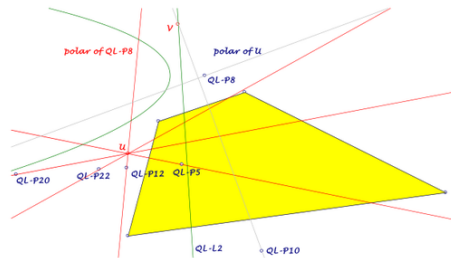
$$(4S^2vw+(-S_Au+S_Bv+S_Cw)^2)x+(4S^2wu+(S_Au-S_Bv+S_Cw)^2)y+(4S^2uv+(S_Au+S_Bv-S_Cw)^2)z=0$$



Remark wrt U and V : The points $QL-P5$, $QL-P12$, $QL-P20$, $QL-P22$ are collinear on $QL-L1$. Their loci are lines with a common point U on the polar of $QL-P8$ wrt $QL-Co1$:

$$U(u(v(a^2w+c^2u)^2-w(a^2v+b^2u)^2) : v(w(a^2v+b^2u)^2-u(b^2w+c^2v)^2) : w(u(b^2w+c^2v)^2-v(a^2w+c^2u)^2))$$

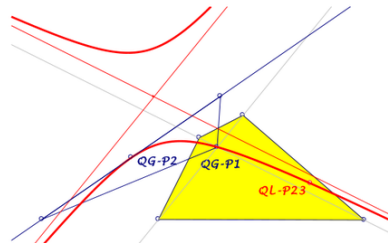
A perpendicular line to the polar of U wrt $QL-Co1$ cuts $QL-L2$ in the point V .



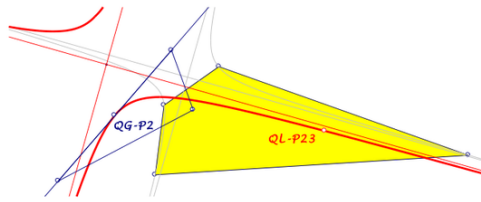
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QL-P23 Center of the Inscribed Midline Hyperbola

- **Hyperbola**
 - ... asymptotes parallel to the diagonals,
 - ... through $QG-P1$, $QG-P2$,
 - ... $QG-L1$ is tangent in $QG-P2$,
 - ... equation: $l^4(x+y)x+2l^2n^2xz+n^4(y+z)z=0$

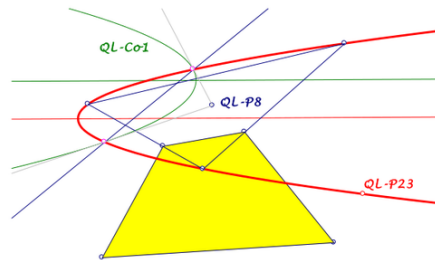


- **Orthogonal hyperbola**
 - ... parallel asymptotes wrt $QA-Co2$,
 - ... through $QG-P2$,
 - ... $QG-L1$ tangent in $QG-P2$



- **Parabola**

- ... circumscribed $QL-DT$,
- ... axis parallel to the axis of $QL-Co1$, half the distance to $QL-P8$ on the other side,
- ... through the intersections of $QL-Co1$ and the polar of $QL-P8$,
- ... equation: $u^2 yz + v^2 zx + w^2 xy = 0$



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QL-P24 Intersection $QL-P1.QL-P8 \wedge QL-P13.QL-P17$

- **Circle $QL-Ci1$**
- -----
- **Fixed point**

QL-P25 2nd QL -Parabola Focus

- **Circle $QL-Ci2$**
- -----
- **Circle $QL-Ci2$**

QL-P26 Least Squares Point

- -----
- -----
- -----

QG-P1 Diagonal Crosspoint

- **Fixed point**
- **Fixed point**
- **Fixed point**

QG-P2 Midpoint 3rd *QA*-Diagonal

- **Fixed point**
- **Fixed point**
- **Line *QG-L1***

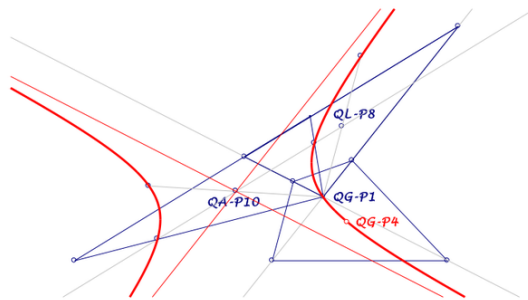
QG-P3 Midpoint 3rd *QL*-Diagonal

- **Fixed point**
- **Line *QG-L1***
- **Fixed point**

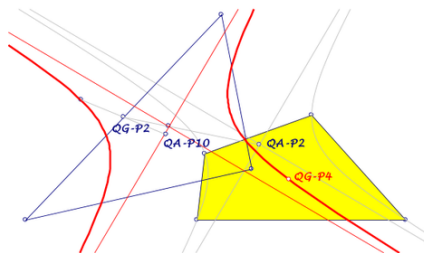
QG-P4 1st *QG*-Quasi Centroid

- **Hyperbola**
 - ... asymptotes parallel to the diagonals,
 - ... centered in *QA-P10*,
 - ... through *QG-P1* and reflections of *QG-P1* in *QA-P10* or *QL-P8*,
 - ... intersections of *QA-P10*, *QL-P8* and the legs of *QA-DT*,
 - ... equation:

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- **Orthogonal hyperbola**
 - ... parallel asymptotes wrt *QA-Co2*,
 - ... centered in $T(QA-P2, QG-P2, 2:1)$,
 - ... through the reflection of *QA-P10* in *QG-P2*,
 - ... equation:
- $$3vwx(-x+2v+2w)+3uvz(2x+2y-z) + uw(x-2y+z)(x+4y+z)=0$$



- Line $QG-L3$

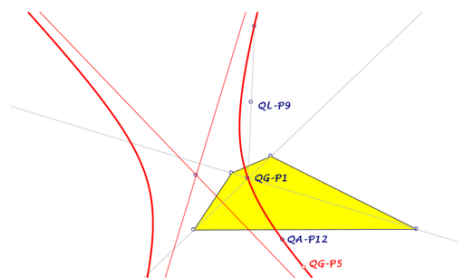
QG-P5 1st QG-Quasi Circumcenter

- **Hyperbola**

... asymptotes orthogonal to the diagonals,
 ... through $QG-P1$, $QA-P12$ and the reflection of $QG-P1$
 in $QL-P9$,
 ... equation:

$$\begin{aligned} & -(p^2c^2 - r^2a^2)(S_Ax^2 + S_Cz^2) \\ & + (p^2c^2(S_B + c^2) + r^2(a^2S_A - 2S_B^2))xy \\ & - (p^2(S^2 - S_A^2) - r^2(S^2 - S_C^2))zx \\ & - (r^2a^2(S_B + a^2) + p^2(c^2S_C - 2S_B^2))yz = 0 \end{aligned}$$

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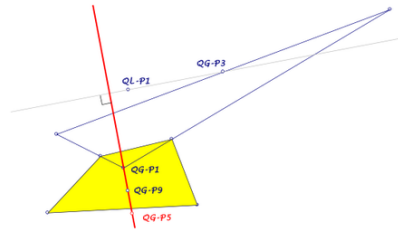


- **Hyperbola**

... centered in $QG-P2$,
 ... through $QA-P12$,
 ... ???

- **Line**

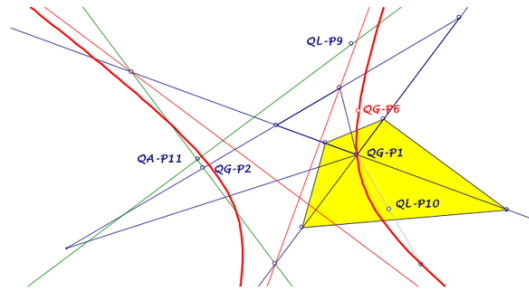
... orthogonal $QG-P3$, $QL-P1$,
 ... through $QG-P1$, $QG-P9$
 ... equation: $c^2(S_Cw - S_Bv)x + a^2(S_Au - S_Bv)z = 0$



QG-P6 1st QG-Quasi Orthocenter

- **Hyperbola**

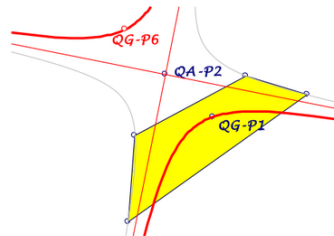
- ... asymptotes orthogonal to the diagonals,
 - ... an asymptote cuts the non-orthogonal diagonal on an orthogonal line through $QG-P2$ wrt $QA-P11$. $QL-P9$,
 - ... through $QG-P1$, reflection of $QG-P1$ in $QL-P10$,
 - ... equation:
- $$(p^2 c^2 S_B - r^2 (S^2 + a^2 S_B)) x^2 - (r^2 a^2 S_B - p^2 (S^2 + c^2 S_B)) z^2 + (p^2 c^2 - r^2 a^2) ((a^2 + S_B) yz + (a^2 + c^2) zx + (c^2 + S_B) xy) = 0$$



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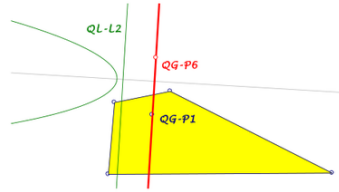
- **Orthogonal hyperbola**

- ... asymptotes and center as $QA-Co2$,
 - ... through $QG-P1$,
 - ... equation:
- $$w(u-v)x^2 + u(w-v)z^2 + 2wu(xy + yz + zx) = 0$$



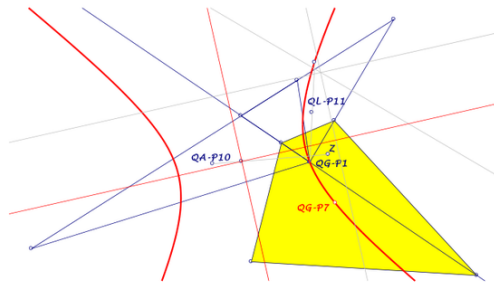
- **Line**

- ... parallel to $QL-L2$,
 - ... through $QG-P1$,
 - ... equation:
- $$(S_A u - S_B v)x + (S_C w - S_B v)z = 0$$



QG-P7 1st QG-Quasi Nine-point Center

- **Conic**
... axes: parallel to the angle bisectors of the bisectors of the $QL-DT$ -legs,
... center $T(QA-P10, Z, 1:3)$ with Z center of the locus-hyperbola of $QG-P6$,
... through $QG-P1$ and its reflection in $QL-P11$.



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- **Hyperbola**
...through $QA-P11$,
...???
- **Line** $QG-P1.QG-P11$

QG-P8 2nd QG-Quasi Centroid

- **Hyperbola**
... $h_{QG-P1}(\text{locus } QG-P4, 1/2)$
- **Orthogonal hyperbola**
... $h_{QG-P1}(\text{locus } QG-P4, 1/2)$
- **Line** $QG-L3$

QG-P9 2nd QG-Quasi Circumcenter

- **Hyperbola**
... $h_{QG-P1}(\text{locus } QG-P5, 1/2)$
- **Hyperbola**
... $h_{QG-P1}(\text{locus } QG-P5, 1/2)$
- **Line**
... $h_{QG-P1}(\text{locus } QG-P5, 1/2)$

QG-P10 2nd QG-Quasi Orthocenter

- **Hyperbola**
... $h_{QG-P1}(\text{locus } QG-P6, 1/2)$
- **Orthogonal hyperbola**
... $h_{QG-P1}(\text{locus } QG-P6, 1/2)$
- **Line**
... $h_{QG-P1}(\text{locus } QG-P6, 1/2)$

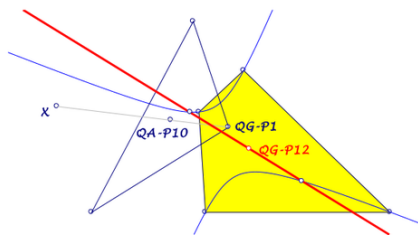
QG-P11 2nd QG-Quasi Nine-point Center

- **Conic**
... $h_{QG-P1}(\text{locus } QG-P7, 1/2)$
- **Hyperbola**
... $h_{QG-P1}(\text{locus } QG-P7, 1/2)$
- **Line**
... $h_{QG-P1}(\text{locus } QG-P7, 1/2)$

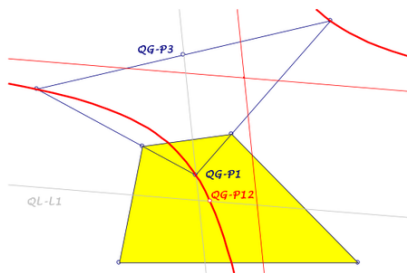
QG-P12 Inscribed Harmonic Conic Center

- **Line QG-L2**
- **Line**
... polar of $X = T(QG-P1, QA-P10, -3:2)$ wrt $QA-Co2$,
... equation: $vwx - wuy + uvz = 0$

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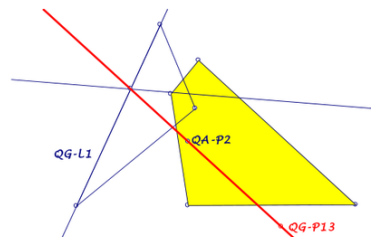


- **Hyperbola**
... circumscribed $QL-DT$,
... asymptotes: parallels to $QG-P1, QG-P3$ and $QL-L1$,
... equation: $uyz - 2vzx + wxy = 0$



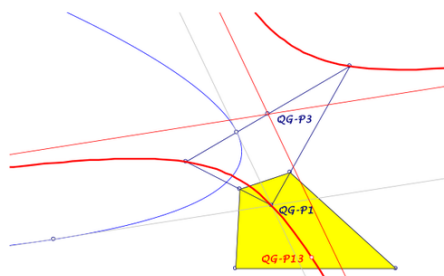
QG-P13 Circumscribed Harmonic Conic Center

- **Line QG-L2**
- **Line**
 - ... through QA-P2 and the intersection of QG-L1 with the trilinear polar of QA-P2 wrt QA-DT,
 - ... equation: $vw x - 2 w u y + u v z = 0$



- **Hyperbola**
 - ... circumscribed QL-DT,
 - ... centered in QG-P3,
 - ... asymptotes parallel to the tangents from QG-P1 to QL-Co1,
 - ... equation: $u y z - v z x + w x y = 0$

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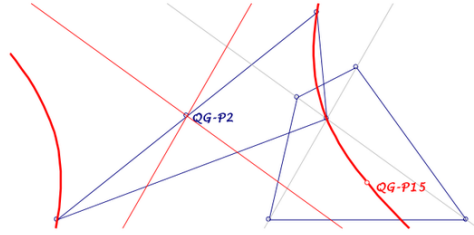


QG-P14 Center of the M3D Hyperbola

- **Line**
 - ... parallel QG-L1 through QG-P1
- **Line**
 - ... parallel QG-L1 through QG-P1
- **Line**
 - ... parallel QG-L1 through QG-P1

QG-P15 Kirikami Center

- **Hyperbola**
 - ... circumscribed QA-DT,
 - ... centered in QG-P2,
 - ... asymptotes parallel to the diagonals

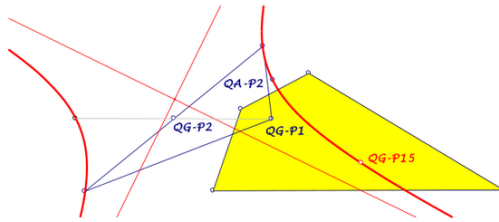


- **Orthogonal hyperbola**

... points: $QG-2P2$, $QA-P2$, reflection of $QG-P1$ in $QG-P2$,

... equation:

$$(u-v)wxy - (w+u)vzx + (w-v)uyz + 2uwy^2 = 0$$



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- **Line $QG-L3$**

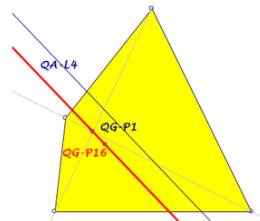
QG-P16 Schmidt Point

- -----
- -----
- **Line**

... parallel to $QA-L4$,

... through $QG-P1$,

... equation: $w(a^2v+b^2u)x + u(b^2w+c^2v)z = 0$

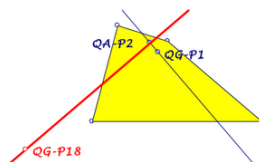


QG-P17 Projection Point of $QG-P1$ on $QG-L1$

- **Fixed point**
- **Fixed point**
- **Fixed point**

QG-P18 Quasi Isogonal Crosspoint

- **Line**
... ???
- **Line**
... orthogonal to $QG-P1.QA-P2$
... through $QA-P2$,
... equation:
$$S_C(c^2q^2-b^2r^2)x+b^2(c^2p^2-a^2r^2)y+S_A(b^2p^2-a^2q^2)z=0$$



- **Line**
... ???

QG-P19 Quasi Isogonal Conjugate of QG-P1

- **Conic**
... circumscribed $QA-DT$, ???
- **Conic**
... through $QG-2P2a,b$,
... ???
- **Conic**
... through $QG-P1$

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Eckart Schmidt
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eckart_schmidt@t-online.de

Message: #629
Date: 17/7/2014 8:26:10
From: chris.vantienhoven
Subject: Special loci for EQF-points

Dear Eckart,
Quite a job you did!
This gives insight about so many more relevant lines and curves.
Could you send me the CT-equations?
Then I can check them all in my system.
Who knows what else pops up.
Best regards,
Chris

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Message: #630
Date: 20/7/2014 9:18:02
From: eckart_schmidt@t-online.de
Subject: Some "splitter" VII

Dear Chris,

perhaps of interest, perhaps in EQF or evident:

- (1) The Simson Line of QL-P1 wrt QG-Tr3 is orthogonal QL-L1.
- (2) QA-Tf2 images of lines through QA-P23 give orthogonal hyperbolas QA-circumscribed.
- (3) QA-Cu7 is the locus of points, whose QG-Tf2 images wrt the quadrigon versions of the QA are equal.
- (4) QL-Cu1 is the locus of points, which contains in their line pencil an orthogonal pair of QL-Tf2 partners.
- (5) For QA-Cu1 as 20th point QA-P41.
- (6) QL-L3 is the Simson line of QL-P1 wrt the medial triangle of QL-DT.
- (7) QL-Tf2(L) contains the pole of L wrt QL-Co1.
- (8) QG-L3 is the polar of QG-P3 wrt QL-Co1.
- (9) QG-P7.QG-P9 // QL-P7.QL-P9.
- (10) The midpoint of the intersections of QA-Co2 and QG-L1 is a point on QL-L2.
- (11) The midpoint of the intersections of QA-Co2 and QG-P1.QA-P10 is a point on QL-L6.
- (12) QA-L4 is tangent to a QA-DT-inscribed conic, centered in $T(QA-P2, QA-P10, 3:1)$. Point of tangency is the intersection with QA-P3.QA-P20.
- (13) A parallel to QG-L1 through QG-P1 cuts QA-Co2 äquidistant to QG-P1.
- (14) QL-L9 is tangent to a parabola, homothetic QL-Co1 wrt QL-P8 with factor 4. Point of tangency divides QL-P18.QL-P23 with ratio $-2/3$.

Best regards Eckart

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Message: #631

Date: 20/7/2014 9:33:35

From: eckart_schmidt@t-online.de

Subject: Re. Special loci for EQF-points

Dear Chris,

the geometry of the three pencils of quadrilaterals is per definition closely connected with the diagonal triangles QA-DT and QL-DT. So I have chosen these triangles as reference triangles for barycentric coordinates. You can get the CT notation of the equations, following the description of the loci (... if it is complete).

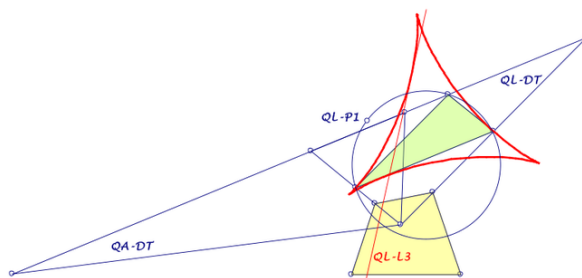
A further aspect are the envelopes for the EQF-lines for the three pencils of quadrilaterals. There are unexpected deltoids for QL-L3, see attachment. Some new points wrt line pencils are listed in "Splitter VI".

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

Two Deltoids wrt the Pedal Line QL-L3

For two special pencils of quadrilaterals the Pedal Lines QL-L3 envelope deltoids, which will be described as envelopes of Simson lines.



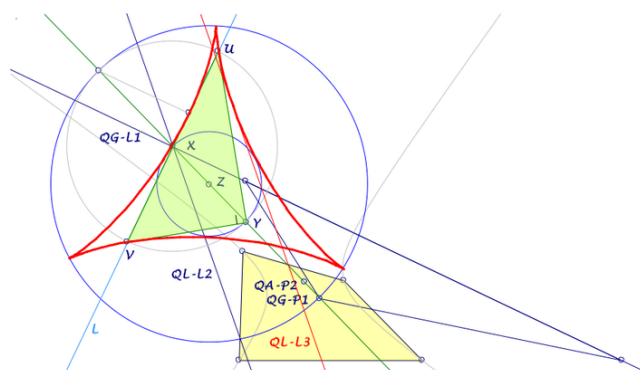
The quadrilaterals of the first pencil have the same diagonal triangles $QA-DT$ and $QL-DT$.

$QL-L3$ is the Simson line of the Miquel Point $QL-P1$ wrt the medial triangle of $QL-DT$, whose circumcircle contains $QL-P1$.

The Pedal Lines $QL-L3$ envelope a deltoid, which is the envelope of the Simson lines of the medial triangle of $QL-DT$.

The quadrilaterals of the second pencil have the same Diagonal Triangle $QA-DT$ with the same Diagonal Crosspoint $QG-P1$ and the same Orthogonal Hyperbola $QA-Co2$.

The Pedal Lines $QL-L3$ envelope a deltoid, which is the envelope of the Simson lines of a triangle, which can be constructed as follows:



... Let X be the common point of $QG-L1$, $QL-L2$ and $QG-P1.QA-P2$ (also midpoint of the intersections of $QG-L1$ and $QA-Co2$).

... Let Y be the midpoint of $QG-P1$ and X .

... Let Z be the midpoint of X and Y .

... Let C_1 be a circle round Z through $QG-P1$ and C_2 a circle round Z through X . C_1 is the circumcircle and C_2 is the incircle of the deltoid.

... A perpendicular line L wrt $QG-L1$ through X is tangent to the deltoid. The point of tangency is the pedal point of the reflection of $QG-P1$ in Z .

... The circle round X through Y cuts L in U and V on the deltoid. The tangents in U and V at the deltoid intersect in Y perpendicular.

... The Simson lines of the orthogonal triangle UVY envelope the deltoid.

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2014-07-20.pdf

Message: #632
Date: 21/7/2014 1:08:09
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Eckart,
After a long silence, you come back with in fact an enormous and remarkable work!
I tried to check all your constructions with the 3rd pencil (QL).
I just give you very few remarks and a question.
First, the remarks:
* page 20 above (on the figure): P3 and not L3
* page 21 about P6: the locus is the perpendicular bisector of P1P24
* page 24 about P13: the hyperbola passes also through P24 and through the 3 S-points on the Dimidium circle (therefore the locus of these 3 S-points is the same hyperbola)
* page 28 about P22 at the end of the remark wrt U and V: the perpendicular line ... through P10 ...
The locus of the invariant points F1 and F2 of the (variable) Cl-S transformation is a orthogonal hyperbola centered in P1 through the vertices of the DT and through P10 and P16
Then the question

Apparently, the points U and V (which, by the way, are new points for the QL) depend only from the DT and the point P1 or simpler from the 4 points on the Euler circle: 3 vertices of the medial DT and the point P1.
(The parabola is then given, as it is inscribed in the medial DT with focus P1).
What are these 2 points U and V for the QA formed by these 4 points on the Euler circle of the DT?

Best regards
Bernard

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Message: #633
Date: 23/7/2014 8:39:02
From: eckart_schmidt@t-online.de
Subject: Special loci for EQF-points

Dear Bernard K.,
thank you very much for your corrections and additional remarks!
They are correct!
But the best of all, the locus of QL-2P3. I hadn't researched these points. The interesting orthogonal hyperbola has the following asymptotes: The Simson lines of QL-DT wrt the intersections of QL-L2 and QL-Ci1. Furthermore the CSC-image of this orthogonal hyperbola is a lemniscate.
Your question wrt the geometry of U and V: You are correct, they only depend on QL-DT and QL-P1. But I can't see the geometric connection.
Best regards Eckart

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Message: #634
Date: 24/7/2014 3:52:39
From: seiichikiri
Subject: A concurrent point derived from QA-P9

Dear Chris,
I investigated OGP1 to 19 and found that the following numbers have their translated bimedial points.
OGP1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15.
Best regards,
Seiichi.
P.S. I will send Math-file by email.

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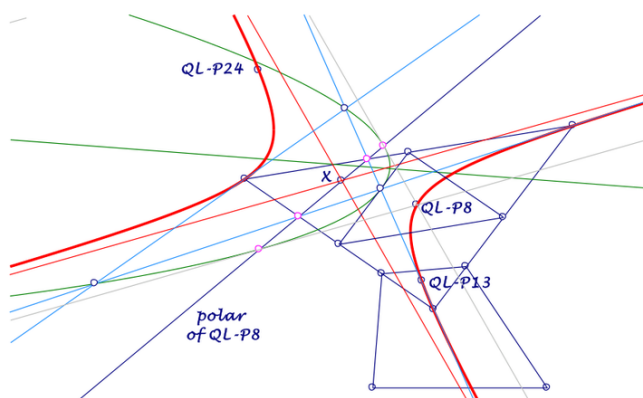
Message: #635
Date: 25/7/2014 8:49:10
From: eckart_schmidt@t-online.de
Subject: QL-Hyperbola wrt QL-P8

Dear friends,
a special QL-DT-isoconjugation maps the polar of QL-P8 wrt
QL-Co1 into a QL-DT-circumscribed hyperbola through QL-P8,
QL-P13 and QL-P24. The geometry of the isoconjugation and the
hyperbola are worked out in the attached file.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

QL-Hyperbola wrt QL-P8

A special QL-DT-isoconjugation maps the polar of QL-P8 wrt QL-Co1 into a QL-DT-circumscribed hyperbola through QL-P8, QL-P13 and QL-P24. The geometry of the isoconjugation and the hyperbola is worked out. – Reference triangle for barycentric coordinates is QL-DT.



Preliminary remarks

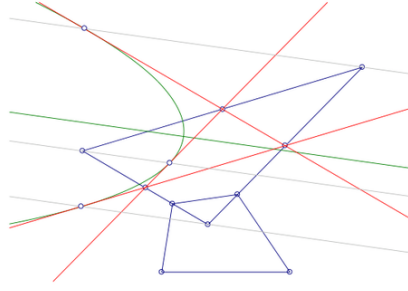
- The inscribed parabola $QL-Co1$ of a quadrilateral is not only tangent to the lines of the quadrilateral, but also to the sidelines of the medial triangle of $QL-DT$.
- $QL-Co1$ has the equation
$$(l^2 - m^2)(l^2 - n^2)x^2 + (m^2 - n^2)(m^2 - l^2)y^2 + (n^2 - l^2)(n^2 - m^2)z^2 = 0.$$

The points of tangency wrt the medial triangle of $QL-DT$ are

$$(\pm(m^2 - n^2) : \pm(n^2 - l^2) : \pm(l^2 - m^2))$$

without the point with coordinates of equal sign, which is the point of infinity of $QL-Co1$.

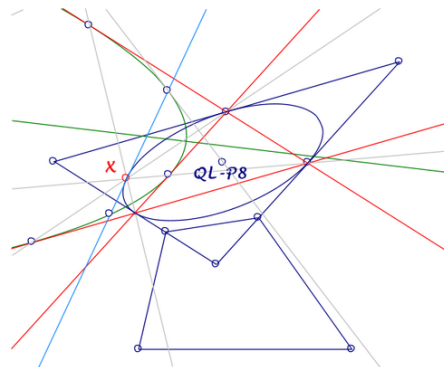
- The points of tangency are the intersections of the three QG -Centroids Lines $QG-L3$.
- The points of tangency are the intersections of $QL-Co1$ with parallels to $QL-L1$ through the vertices of $QL-DT$.



The QL-DT-isoconjugation

- We consider an isoconjugation wrt $QL-DT$ with fixed points in the points of tangency and the point at infinity of $QL-Co1$ as described above:
 $(x : y : z) \rightarrow ((m^2 - n^2)^2 yz : (n^2 - l^2)^2 zx : (l^2 - m^2)^2 xy) .$
- This isoconjugation maps the Miquel Point $QL-P1$ in the intersection of $QL-L2$ and $QL-P1, QL-P10$.
- The pole X of the isoconjugation (image of $QL-P8$)
 $X((m^2 - n^2)^2 : (n^2 - l^2)^2 : (l^2 - m^2)^2)$
 is the Brianchon point of $QL-Co1$ wrt the $QL-DT$ -medial triangle.
- The point X lies on the inscribed Steiner ellipse of $QL-DT$.
- The tangent at X to the Steiner ellipse is the polar of $QL-P8$ wrt $QL-Co1$ with the equation

$$\frac{x}{m^2 - n^2} + \frac{y}{n^2 - l^2} + \frac{z}{l^2 - m^2} = 0 .$$



The QL-DT circumscribed hyperbola

- The isoconjugation maps the polar of $QL-P8$ wrt $QL-Co1$ in a circumscribed hyperbola of $QL-DT$ with the equation
 $(l^2 - m^2)xy + (m^2 - n^2)yz + (n^2 - l^2)zx = 0 .$

- The hyperbola contains the points $QL-P8$, $QL-P13$, $QL-P24$.
- The hyperbola is centered in the Brianchon point X described above (see first figure).
- The asymptotes are parallel to the tangents through $QL-P8$ wrt $QL-Co1$.
- The tangents to the hyperbola in the $QL-DT$ -vertices are the QG -Centroids Lines $QG-L3$ of the three quadrigon versions.
- The $QL-DT$ -isotomic image of the hyperbola is its tangent in $QL-P8$.
- The tangent in $QL-P8$ at the hyperbola cuts the asymptotes on the $QL-DT$ -inscribed Steiner ellipse.

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2014-07-24.pdf

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Message: #636
Date: 25/7/2014 4:27:45
From: eckart_schmidt@t-online.de
Subject: Special loci for EQF-points

Dear Bernard K.,
here is a construction for the new QL-point U (see remark under QL-P22 in my paper) as intersection of two lines:
1. (already mentioned) polar of QL-P8 wrt QL-Co1,
2. perpendicular bisector of QL-P1.X with X second intersection of QL-P1.QL-P9 and QL-Ci5.
Best regards Eckart

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Message: #637
Date: 26/7/2014 2:18:07
From: seiichikiri
Subject: A QA point with Miquel points

Dear Chris,
It seems to me that [1](4) in Anopolis message #1808 †) is a QA point because it uses QA Miquel points and DT(diagonal triangle).
I hope your examination.
Best regards,
Seiichi.

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[33\]](#).

Message: #638
Date: 26/7/2014 2:33:37
From: Antreas Hatzipolakis
Subject: A QA point with Miquel points

Seiichi wrote:

>> Dear Chris,
>> It seems to me that [1](4) in Anopolis message #1808
>> is a QA point
>> Anopolis #1808 = Hyacinthos #22545 †)

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Message: #639
Date: 26/7/2014 8:53:47
From: chris.vantienhoven
Subject: A QA point with Miquel points

Dear Seiichi, dear Antreas,

Interesting configuration!

Here are my remarks.

The cubic QA-Cu1 passes through 12 of your points:

A,B,C,D,E,F,Ma,Mb,Mc,Q,S,T.

The cubic QA-Cu7 passes through 5 of your points: D,E,F,S,T.

Points Q and R are no Quadrangle Centers.

But points S and T are Quadrangle Centers QA-P4 and QA-P41.

Regarding to point $U = (q+r : r+p : p+q)$.

These coordinates are the coordinates of point P in triangle DEF, the cevian triangle of P.

Stated in another way:

barycentric coordinates of U in ref.triangle ABC = barycentric coordinates of P in ref.triangle DEF

Point V is Quadrangle Center QA-P1 (Quadrangle Centroid)

Best regards,

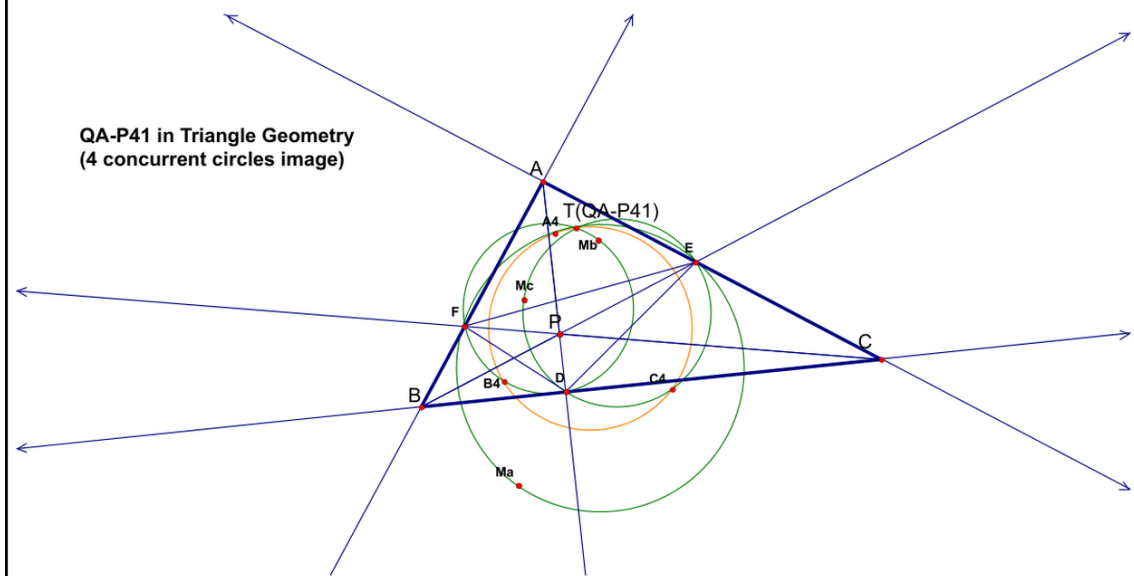
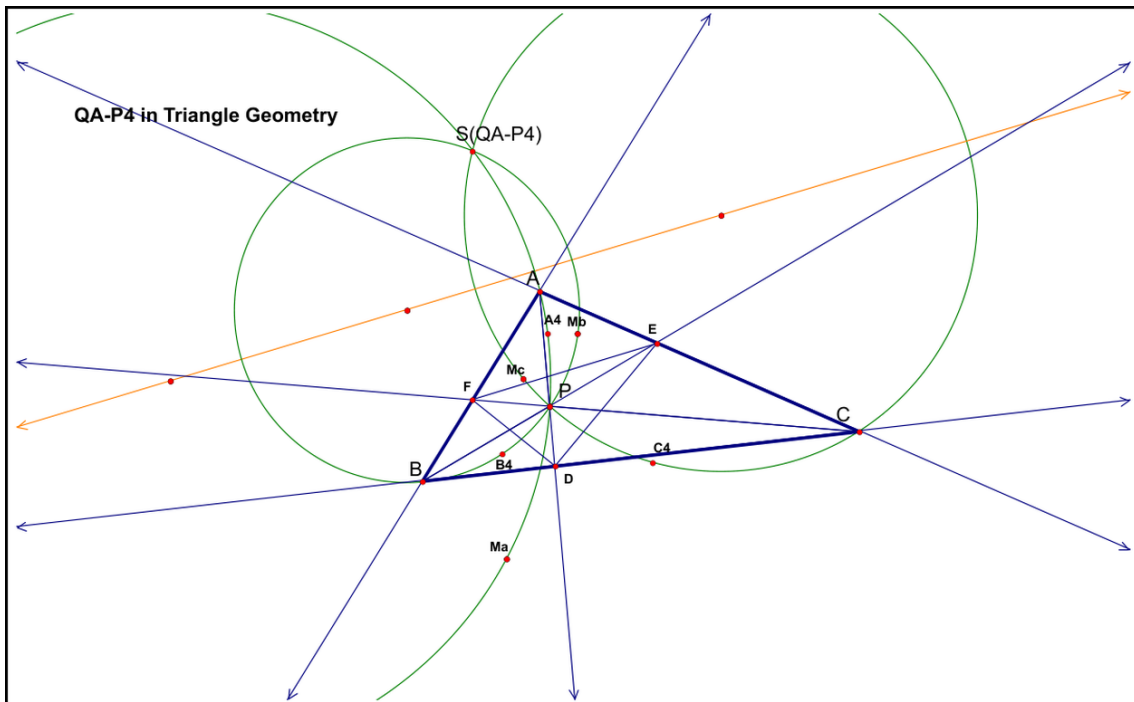
Chris van Tienhoven

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #640
Date: 27/7/2014 1:51:27
From: Seiichi Kirikami
Subject: A QA point with Miquel points

Dear Chris, dear Antreas,
In the environment of Triangle Geometry, $S(QAP4)$ looks very different.
See the attached file.
If we denote $QAP4$'s of $AEPF$, $BFPD$ and $CDPE$ by $A4$, $B4$, and $C4$ respectively, $T(QA-P41)$ is on the circumcircle of $A4B4C4$. So T is a 4 concurrent circles image in the environment of Triangle Geometry.
See the same file.
Best regards,
Seiichi.



QApointsInTriangleGeometry.docx

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Message: #641
Date: 27/7/2014 4:58:18
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Eckart,
I have constructed easily the points U , V and V' (reflexion of V in U), which are new possible points for the QL , but I don't see an obvious signification starting only with the elements of the DT and the parabola!
In fact, U is on the polar of $P8$ wrt the parabola and V comes from U with elements of the parabola.
If we had a second line depending only from the DT or the parabola, it would be perfect.
BTW, I didn't find either the locus of $P26$...
Best regards
Bernard

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Message: #642
Date: 27/7/2014 5:19:40
From: bernard.keizer
Subject: QL-Hyperbola wrt QL-P8

Dear Eckart,

This new conjugation allows more interesting details about the center of the hyperbola as the Brianchon point, the contact points of the medial triangle with the parabola ...

As already known, the same construction is possible for each reference triangle of the QL, the hyperbola through the vertices, the centroid G , the perspector P with the DT and the second intersection D of $P1P$ with the circumscribed circle: the equation is the same with l , m and n instead of l_2 , m_2 and n_2 and the hyperbola is the isotomic image of its tangent in G . For the DT, we have the vertices, the centroid P_8 , the perspector P_{13} and the D-point P_{24} .

The 5 D-points are on the Dimidium circle and each hyperbola cuts the Dimidium circle in 4 points, one D-point for each parabola and the 3 strange S-points common for the 5 hyperbolas (these 3 points have, as you already mentioned, P_5 as circumcenter, P_{12} as centroid and P_2 as orthocenter).

This similarity between the DT and the reference triangles is curious, as, after all, the parabola is inscribed in each reference triangle, but not in the DT, but in the medial DT!

Nevertheless, I was curious and tried to look at the centers of the 4 other hyperbolas, then at the polar of the centroids G wrt the parabola, but I didn't find anything interesting ...

Best regards
Bernard

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Message: #643
Date: 27/7/2014 10:11:13
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Eckart,
This time, I think I have it!
If I'm not wrong, the 2nd intersection of P_1P_9 with Ci_5 is the inverse of P_1 wrt the circumcircle of DT .
Therefore, the locus of P_5 is the parallel to the polar of P_1 wrt the circumcircle of DT (perpendicular to P_1P_9) at half the distance from P_1 (P_1 being the railwaywatcher between the 2 lines ...).
This property for P_5 is totally new for me; it could perhaps interest EQF (Chris, what do you think of that?).
Best regards
Bernard

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Message: #644
Date: 27/7/2014 11:53:48
From: chris.vantienhoven
Subject: Special loci for EQF-points

Dear Bernard,
I placed a new property at $QL-Ci_5$ in EQF mentioning this special point on $QL-Ci_5$.
See:
<http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-mathematics/quadrilateral-objects/artikelen-ql/150-ql-ci5.html>
Best regards,
Chris

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Message: #645
Date: 27/7/2014 11:57:36
From: Chris van Tienhoven
Subject: A concurrent point derived from QA-P9

Hi Seiichi,
Tomorrow I go on holiday for about 10 days.
After this I will study these Translated Bimedial Points further.
Best regards,
Chris

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Message: #646
Date: 28/7/2014 12:17:57
From: chris.vantienhoven
Subject: Special loci for EQF-points

Dear Bernard,
Let QL-Px be the inverse of the Miquel Point QL-P1 wrt the circumcircle of the QL-Diagonal Triangle (QL-Ci1).
The Clawson-Schmidt Conjugate (CSC or QL-Tf1) of QL-Px lies on these lines:
* QL-P1.CSC(QL-P9)
* CSCe(QL-P3.QL-P4).CSCe(QL-L1).
where CSCe(Line) = the circumcenter of the circle image of CSC(Line).
Best regards,
Chris

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Message: #647
Date: 28/7/2014 10:35:03
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Chris,
For any circle C_i , CSC(inverse of P_1 in the circle C_i) is the center of the CSC image of the circle C_i .
The point you mention is therefore by definition the center of the CSC image of the circumcircle of DT , which cuts the line CSC(Dimidium circle) in the 2 points CSC(P_{17}) and CSC(P_{24}).
The second line you mention is the line through $QL-2P_4$, which is the CSC of the Plücker circle C_{i5} .
Best regards
Bernard

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Message: #648
Date: 29/7/2014 9:10:28
From: eckart_schmidt@t-online.de
Subject: Special loci for EQF-points

Dear Bernard K.,
interesting properties for the second intersection of $QL-P_9$, $QL-P_1$ and $QL-C_{i5}$! In addition: this point lies on a circle round the curious point U (see remark under $QL-P_{22}$ in QFG-message #628), orthogonal to the $QL-DT$ -circumcircle, containing $QL-P_1$.
This curious point U has my special interest! You find a new aspect in the attached file.
Best regards Eckart

EQF-Note 2014-07-27

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://chrisvantienhoven.nl/>

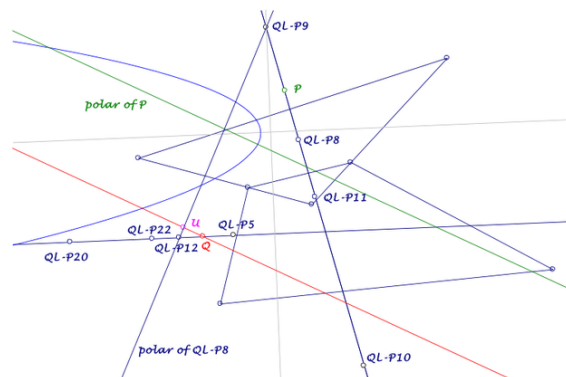
A Curious QL-Point

This point is already described in EQF-Note 2013-04-12 (see QFG-message 30) and under QL-P22 in EQF-Note-2014-07-17 (see QFG-message 628). Independent of this here is another curious aspect related to correspondent points on the Newton-Line QL-L1 and the QL-DT-Euler-Line QL-L7.

On QL-L7 and QL-L1 we find points with the same distance ratios:

$$\begin{array}{l} \text{QL-L7: } \text{QL-P9} \quad \text{QL-P8} \quad \text{QL-P11} \quad \text{QL-P10} \\ \quad \quad \quad 2 \quad : \quad 1 \quad : \quad 3 \\ \text{QL-L1: } \text{QL-P5} \quad \text{QL-P12} \quad \text{QL-P22} \quad \text{QL-P20} . \end{array}$$

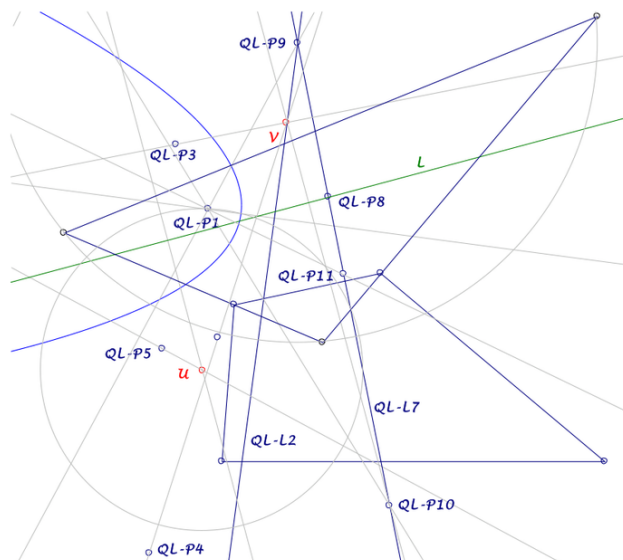
A point P on QL-P7 and a point Q on QL-L1 shall be named “correspondent” points, if P divides QL-P9.QL-P8 in the same ratio as Q divides QL-P5. QL-P12.



- **Parallels to the polars of P wrt the Inscribed Parabola QL-Co1 through the correspondent points Q have a common point U .**
- The common point U is the intersection of the polar of QL-P8 wrt QL-Co1 and a perpendicular line to QL-P1.QL-P9 through QL-P5.
- The common point U has the DT-coordinates:

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$$X(u(v(a^2w+c^2u)^2-w(a^2v+b^2u)^2) \\ : v(w(a^2v+b^2u)^2-u(b^2w+c^2v)^2): w(u(b^2w+c^2v)^2-v(a^2w+c^2u)^2)) \\ \text{with } u=m^2-n^2, \quad v=n^2-l, \quad w=l^2-m^2.$$



- Let V be the intersection of $QL-L2$ and a perpendicular line to $QL-L7$ through $QL-P3$. Let L be a line through $QL-P8$ perpendicular to $V, QL-P10$. The pole of L wrt $QL-CoI$ is the point U .
- The point U lies on a perpendicular line to $QL-P1, QL-P11$ through the midpoint of V and $QL-P4$.
- A circle round U through $QL-P1$ cuts the $QL-DT$ -circumcircle orthogonal.

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2014-07-27.pdf

Message: #649
Date: 04/8/2014 2:32:47
From: eckart_schmidt@t-online.de
Subject: CSC of QL-diagonals

Dear Chris,

I hope, you and your family had nice holiday, we had sun in the garden.

Here are some remarks wrt CSC-circles of QL-diagonals:

(1) The CSC-images of the three QL-diagonals are circles C_i , each through QL-P1, two opposite points of QL, one QG-P18, one QA-P4.

The midpoints of the circles C_i form a triangle Tr with ...

- (2) ... QL-P4 as orthocenter,
- (3) ... QL-P5 on the circumcircle,
- (4) ... QL-P6 on the nine-point circle,
- (5) ... QL-P6 on the Simson line of QL-P5,
- (6) ... QL- C_{i3} as polar circle, containing the intersections of circumcircle and nine-point circle,
- (7) ... sidelines as CSC-images of circles round QL-diagonal crosspoints through QL-P1.
- (8) The triangle Tr is perspective QL-DT wrt a point P and a line L .
- (9) The line L is the perpendicular bisector of QL-P1.CSC(P).
The point P is the midpoint of the circle CSC(L).

Curious properties!

Best regards Eckart

PS: The DT-coordinates of the center of perspectivity can be given.

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Message: #650
Date: 04/8/2014 8:48:07
From: Chris van Tienhoven
Subject: CSC of QL-diagonals

Dear Eckart,
I am still on holiday. I am in Andorra now with my wife.
Beautiful and special properties you describe. I will study them
further when I am at home.
Best regards,
Chris

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Message: #651
Date: 05/8/2014 1:57:01
From: seiichikiri
Subject: The properties of 3QA versions of QL points

Dear Chris,
How is the weather in Andorra?
It is scorching hot here in eastern Japan, while it rains cats
and dogs in western Japan.
I observed the following with Geometer`s Sketchpad.
(1) 3 QA-versions of QL-P2 and QL-P4 have their translated
bimedial points.
(2) 3 QA-versions of QL-P5 are colinear.
Best regards,
Seiichi

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Message: #652
Date: 13/8/2014 1:53:36
From: bernard.keizer
Subject: CSC of QL-diagonals

Dear Eckart, dear Chris

A new stimulating message, remarkable during the summer holidays with low activity!

This triangle Tr deserves a description in EQF.

I found particularly interesting the points (8) and (9) with the perspective.

Three remarks:

1) The described triangle Tr is the DT of the QA formed by the circumcenters of the 4 reference triangles of the QL (as the vertices of the QL are each the second intersection between 2 circumcircles and therefore the reflexions of $P1$ in the line joining 2 circumcenters).

For this QA inscribed in QL-Ci3, QA-P1 is QL-P6, QA-P2 is QL-P5 and QA-P3 is QL-P4, which explains the points (2) to (5). The 3 points Sa are on the 9 points conic QA-Co1 with center QL-P6, asymptotes parallel to the Steiner axes and through $P4$ and $P5$. For Eckart, you described this conic already in earlier messages (#454 and #465).

2) Properties of $C1-S$

Let A and A' be 2 opposite vertices of the QL, a , b and c the vertices of QL-DT. The CSC of Sa is the point $S'a$ reflexion of $P1$ in the diagonal AA' (carrying b and c). $CSC(a)$ is the second intersection of the circles with centers Sb and Sc and therefore the reflexion of $P1$ in $SbSc$.

CircumDT is orthogonal to the circles with diameters AA' ; $CSC(\text{circumDT})$ is orthonal to the CSC of these circles through A , A' and the points QL-2P4a and b , CSC of the Plücker points QL-2P1a and b ; therefore, the center of $CSC(\text{circumDT})$ (CSC of the inverse of $P1$ wrt circumDT) is on the line QL-2P4aQL-2P4b, which explains why the inverse of $P1$ wrt circumDT is on the CSC of this line, the circle with center $P5$ through $P1$.

3) Conics

The center of the circle through the $S'a$ is the isogonal conjugate $P1^*$ of $P1$ wrt DT; as the circle through the Sa contains $P5$ (property of the QA), the circle through the $S'a$ contains $CSC(P5)$ and the conic with foci $P1$ and $P1^*$ inscribed in DT is tangent to QL-2P4aQL-2P4b and to any perpendicular bisector of a line joining $P1$ to a point X of the circle through the $S'a$.

For Eckart again, you described this conic Co in message #471.

The center of the circle $CSC(circumDT)$ is the isogonal conjugate of $P1$ wrt the described triangle Tr ; the same way, the conic with foci $P1$ and this point and inscribed in Tr is tangent to any perpendicular bisector of a line joining $P1$ to a point Y on the circle $CSC(circumDT)$ (for example, Y could be $CSC(P16)$, $CSC(P17)$ or $CSC(P24)$).

Best regards
Bernard

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Message: #653
Date: 14/8/2014 9:40:38
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Eckart,
Sorry for reacting so slowly (I wasn't home in the last 2 weeks).
For any point lying on the perpendicular bisector on the segment joining a given point and it's inverse in a given circle, the circle centered in the variable point and passing through the givenpoint and the inverse is orthogonal to the given circle. (theorem of Pythagora applied to the rectangular triangle formed by the 2 circumcenters and the middle of the segment joining the 2 inverse points).
Considering the point $P1$ and it's inverse in the $circumDT$, this is in particular true for the point U and for the point $P5$ on the perpendicular bisector between $P1$ and it's inverse (the Plücker circle is orthogonal to the $circumDT$).
Best regards
Bernard

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Message: #654
Date: 15/8/2014 9:39:23
From: eckart_schmidt@t-online.de
Subject: CSC of QL-Diagonals

Dear Bernard K.,
no new results on my side, but thanksgiving to you for detailed interesting reactions on my messages! Special thanks for message #652: Your result, that the considered triangle Tr is the diagonal triangle of the QA formed by the circumcenters of the 4 QL-triangles, is the background for my "curious" properties. I have studied your further results with interest. It remains the question of EQF-relationships for the point or line of perspectivity for Tr and QL-DT (point (9) in my message #649 is evident). Perhaps you see connections to other EQF-points.
Best regards Eckart
PS: Thanks also for your messages #642 and #653, but I cannot add new aspects.

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Message: #655
Date: 15/8/2014 11:54:32
From: bernard.keizer
Subject: CSC of QL-Diagonals

Dear Eckart,

Thanks for your kind message!

I didn't find anything interesting for your perspectivity point. But instead went back to your messages #465 and #481 with the conic Co, the quartic and the rectangular hyperbola with center P6 and I found plenty of ideas.

2 preliminary remarks:

1) there are an infinity of QLs inscribed in the quartic and sharing the points P1, P6 and P17, the Cl-S conjugation, the conic Co and the rectangular hyperbola; in other words, for a given Cl-S (which gives the invariant points and their middle P1) and a given rectangular hyperbola with center P6 and asymptotes parallel to the Steiner axes, the point P17 is given (of course it is the reflexion of P1 in the segment joining P6 to the center of the conic Co). I tried many constructions, but I haven't found yet how to determinate it easily.

2) if I call Cl-Sdiag the Cl-S which swaps the vertices of DT with the middles of the diagonal segments and the circle circumDT with the Newton Line, the center is P17 as well known; it appears that the Cl-S conjugate of P1 and the Cl-Sdiag of P17 are the same point: $CSC(P17) = CSCdiag(P1)$.

All the QL's inscribed in the quartic share also the same Cl-Sdiag.

I found 2 possible constructions:

1) taking a point Sa on the rectangular hyperbola, the intersections between the circle centered in Sa through P1 and the perpendicular bisector of P1CSC(Sa) give 2 conjugate vertices. The perpendicular bisectors of P1A and P1A' cut the rectangular hyperbola in 2 points (other than Sa) which are the middles of 2 pairs of circumcenters of the reference triangles; the perpendicular to these lines in these points intersect on the rectangular hyperbola in the point P4. The circle with center P4 through P1 gives the 4 circumcenters ...

It's easy to check that all circles circumDT pass through a fixed point P17.

Knowing P17 and CSCdiag, we could also determine one vertex a of DT as CSCdiag of the middle of AA' and the points Sb and Sc as intersections of the perpendicular bisector of P1CSC(a), which gives directly the points B and B' and C and C' the same way as A and A'.

2) taking a point P_4 on the rectangular hyperbola and it's reflexion in P_6 as P_5 , the intersections between the 2 lines $P_1CSC(P_4)$ and $P_1CSC(P_5)$ with the circles with centers P_4 and P_5 through P_1 give 8 points P and P' as Plücker points, Q and Q' their CSC conjugates, R and R' and S and S' which are 2 pairs of Cl-S conjugates. We can check that the Euler Lines of the QLs formed by the 4 lines PQ , $P'Q'$, RR' and SS' pass through a fixed point, the center of the conic Co . Then we are able to determine the Euler Line of the searched QL as perpendicular bisector of PP' , the circle circumDT as $CSCdiag(Euler\ Line)$ through P_{17} ; the other intersection with the Dimidium circle is P_{24} .

Then I used the construction of Chris and determined the 2 parabolas (with axes parallel to the asymptotes of the rectangular hyperbola through P_9) through the circumcenters of the reference triangles ...

I hope you will find something interesting in all this!

Best regards
Bernard

PS As I told Chris, I will have my 5 grand-children at home for the 2 next weeks and it will let me little time for the geometry of the QL ...

---In Quadri-Figures-Group@yahoogroups.com, wrote:

Dear Bernard K., no new results on my side, but thanksgiving to you for detailed interesting reactions on my messages! Special thanks for message #652: Your result, that the considered triangle Tr is the diagonal triangle of the QA formed by the circumcenters of the 4 QL-triangles, is the background for my "curious" properties. I have studied your further results with interest. It remains the question of EQF-relationships for the point or line of perspectivity for Tr and QL-DT (point (9) in my message #649 is evident) . Perhaps you see connections to other EQF-points. Best regards Eckart PS: Thanks also for your messages #642 and #653, but I cannot add new aspects.

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Message: #656
Date: 15/8/2014 7:52:06
From: bernard.keizer
Subject: CSC of QL-Diagonals

Dear Eckart,
I realise I've made a slight error in my message.
I suppose you will have corrected by yourself.
In the second construction, the first parabola is the one with P_1 as focus and PP' as line L_2 or directrix.
The second one is through the contact points of this parabola with the contact points with the 4 lines forming the searched QL: the directrix is parallel to P_1P_7 through P_9 and the focus the complement of P_{17} wrt DT .
Best regards
Bernard

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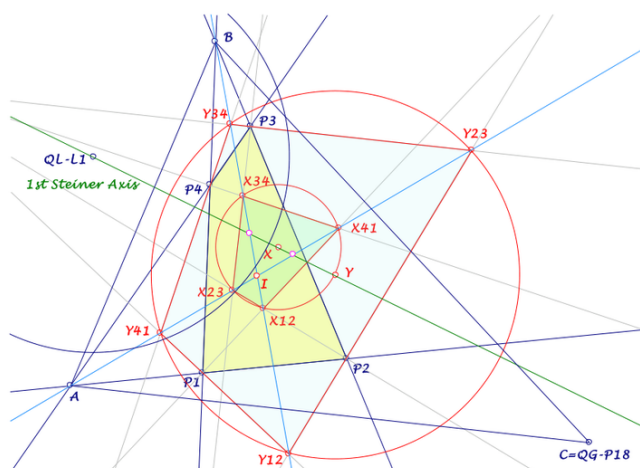
Message: #657
Date: 16/8/2014 3:11:02
From: eckart_schmidt@t-online.de
Subject: Angle Bisectors of a Quadrigon

Dear friends,
I think, it is remarkable and well-known, that the inner / outer angle bisectors of a quadrigon give cyclic quadrigons. But there was no reaction on my message #345. So I have tried, to describe the EQF-geometry of this constellation with calculations in barycentric coordinates wrt a new reference triangle $QG-Tr_3$ (see attached EQF-Note).
Best regards Eckart

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Angle Bisectors of a Quadrigon

It is well-known, that the angle bisectors of a quadrigon give a cyclic quadrigon. Here is described the EQF-geometry of this constellation with calculations in barycentric coordinates. Reference triangle is the Quasi Isogonal Triangle $QG-Tr3$ (see QFG-messages 343 and 345).



The Reference Triangle

Here an unusual reference triangle is used: $QG-Tr3$ with vertices A and B in the intersections of opposite QG -sidelines and the Quasi Isogonal Crosspoint $C=QG-P18$ (see QFG-message 343). A quadrigon then is defined by one vertex such as $P_4(u:v:w)$:

P_2 is the isogonal conjugate of P_4 wrt $QG-Tr3$,

P_1 is the intersection of $A.P_4$ and $B.P_2$,

P_3 is the intersection of $A.P_2$ and $B.P_4$.

In this way the quadrigon gets the vertices:

$$P_1 = (a^2w^2 : c^2uv : c^2uw), \quad P_2 = (a^2vw : b^2wu : c^2uv),$$

$$P_3 = (c^2uv : b^2w^2 : c^2vw), \quad P_4 = (u : v : w).$$

Using the abbreviations

$$A = c^2v^2 + S_A vw + b^2w^2, \quad B = c^2u^2 + 2S_B uw + a^2w^2,$$

the Miquel Point $QL-P1$ has the coordinates

$$QL-P1 = (a^2vB(uA+vB) : b^2uA(uA+vB) : -c^2uvAB)$$

and the 1st Steiner Axis (see $QL-Tf1$) gets the equation

$$-bc^2uAx + ac^2vBy - ab(a-b)(uA+vB)z = 0.$$

The Schmidt Circle (see *QL-TfI*) has the equation

$$\begin{aligned} & c^2(buA - avB)(buAx^2 - avBy^2) \\ & + ab(ab(uA + vB)^2 - c^2uvAB)z^2 \\ & - 2c^2uvAB(S_Cxy + ab(xy + yz + zx)) \\ & + 2(uA + vB)(b^2uS_BAx + a^2vS_ABy)z = 0 \end{aligned}$$

and the fixed points *QL-2P3* of the Clawson-Schmidt Conjugate *QL-TfI*, which are the intersections of the Schmidt Circle and the 1st Steiner Axis, have the coordinates

$$\left(-a \pm \frac{a\sqrt{2vB(ab - S_C)}}{(a-b)\sqrt{uA}} : b \pm \frac{b\sqrt{2uA(ab - S_C)}}{(a-b)\sqrt{vB}} : \frac{c^2}{a-b} \right).$$

Also the Clawson- Schmidt Conjugate (*CSC*) is acceptable:

$$\begin{aligned} (x : y : z) & \rightarrow \\ & (a^2vB(uA(c^2xy + (a^2 - b^2)yz - b^2z^2) - vB(c^2y^2 + 2S_Ayz + b^2z^2) \\ & : (b^2uA(vB(c^2xy - (a^2 - b^2)xz - a^2z^2) - uA(c^2x^2 + 2S_Bxz + a^2z^2) \\ & : -c^2uvAB(a^2yz + b^2zx + c^2xy)) . \end{aligned}$$

The Two Cyclic Quadrigons

The inner / outer angle bisectors of the quadrigon $P_1P_2P_3P_4$ give two cyclic quadrigons with vertices in the incenters and excenters of the *QL*-triangle components (excenters wrt A, B):

$$\begin{aligned} X_{12} &= In(P_1, P_2, B), & Y_{12} &= Ex(P_1, P_2, B), \\ X_{23} &= In(P_2, P_3, A), & Y_{23} &= Ex(P_2, P_3, A), \\ X_{34} &= Ex(P_3, P_4, B), & Y_{34} &= In(P_3, P_4, B), \\ X_{41} &= Ex(P_4, P_1, A), & Y_{41} &= In(P_4, P_1, A), \end{aligned}$$

with coordinates:

$$\begin{aligned} X_{12}, Y_{12} &= (a(aw\sqrt{A} \mp (cv - bw)\sqrt{B}) : bcu\sqrt{A} : c^2u\sqrt{A}), \\ X_{23}, Y_{23} &= (acv\sqrt{B} : b(bw\sqrt{B} \mp (cu - aw)\sqrt{A}) : c^2v\sqrt{B}), \\ X_{34}, Y_{34} &= (cu\sqrt{A} \pm (cv - bw)\sqrt{B} : bw\sqrt{A} : cw\sqrt{A}), \\ X_{41}, Y_{41} &= (aw\sqrt{B} : cv\sqrt{B} \pm (cu - aw)\sqrt{A} : cw\sqrt{B}). \end{aligned}$$

(1) The two cyclic quadrigons have common diagonals.

These common diagonals are angle bisectors of opposite sidelines of the reference quadrigon, intersecting in the incenter $I(a : b : c)$ of the reference triangle *QG-Tr3*.

(2) The two cyclic quadrigons have centers on the 1st Steiner Axis.

These centers are

$$\begin{aligned} X, Y &= (\mp acv(-a + b + c)(cu - aw)\sqrt{AB} \\ & - a(a - b + c)(cv - bw)\sqrt{B}((a - b)(c^2uv(u + v + w) + w(a^2vw + b^2wu + c^2uv)) - cvB) \\ & : bcu(a - b + c)(cv - bw)A\sqrt{B} \end{aligned}$$

$$\pm b(-a+b+c)(cu-aw)\sqrt{A}((b-a)(c^2uw(u+v+w)+w(a^2vw+b^2wu+c^2uv))-cuA) \\ : \mp c^2v(-a+b+c)(cu-aw)\sqrt{AB}+c^2u(a-b+c)(cv-bw)A\sqrt{B})$$

(3) The two cyclic quadrilaterons have circumcircles perpendicular to the Schmidt Circle. They are CSC-invariant. The radical axis is the 2nd Steiner Axis.

(4) The homothetic centers of the two circumcircles are the intersections of the common diagonals and the 1st Steiner Axis.

These two centers are CSC-partners with the coordinates:

$$(-a((a-b)uA-(-a+b+c)vB:bcuA:c^2uA), \\ (acvB:b((a-b)vB+(a-b+c)uA:c^2vB).$$

They lie harmonic with the midpoints of the circumcircles of the cyclic quadrilaterons.

(5) The 3rd diagonals of the cyclic quadrilaterons intersect in the excenter I_c of the reference triangle.

(6) The Morley Points ($QL-P2$) of the cyclic quadrilaterons lie on the Newton Line $QL-L1$ of the reference quadrigon.

(7) The Miquel Points ($QL-P1$) of the cyclic quadrilaterons lie concyclic ...

... with the Schmidt Pair of Points $QL-2P3$,

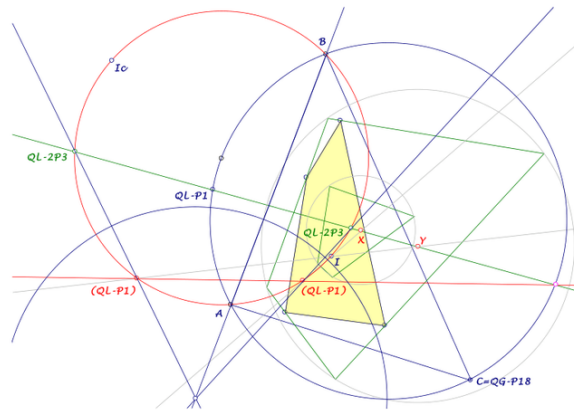
... with the vertices $QG-2P2$ of $QG-Tr3$,

... with the incenter and the excenter I_c of $QG-Tr3$ as diametral points.

This interesting QG -circle is CSC-invariant with the equation:

$$c^2xy+(a-b)(ay-bx)z-abz^2=0.$$

The circle is also perpendicular to the two circumcircles of the cyclic quadrilaterons.



(8) The Miquel Points of the cyclic quadrilaterals lie collinear with the second intersection of the 1st Steiner Axis and the circumcircle of $QG-Tr3$ (midpoint of the excenters I_a and I_b).

The Miquel Points are the second intersections of XI and YI with this circle under (7).

The coordinates of the Miquel Points are:

$$\begin{aligned} & (a\sqrt{B}(cv-bw)(-(a-b)\sqrt{A}(cu-aw) \mp (a-b+c)\sqrt{B}(cv-bw)) \\ & : b\sqrt{A}(cu-aw)((a-b)\sqrt{B}(cv-bw) \mp (-a+b+c)\sqrt{A}(cu-aw)) \\ & : \sqrt{A}\sqrt{B}c^2(cu-aw)(cv-bw)). \end{aligned}$$

There is an inversion, changing the Schmidt Pair of Points $QL-2P3$ and these Miquel Points. The corresponding circle is centered on the 3rd diagonal of the reference quadrilateral in

$$(a\sqrt{a-b+c})B\sqrt{v}(cv-bw) : b\sqrt{-a+b+c})A\sqrt{u}(cu-aw) : 0).$$

(9) The centers X, Y of the cyclic quadrilaterals are the Double Points of a QA -Line Involution $QA-Tf1$ on the 1st Steiner Axis.

1st pair of points:

intersections S and S' of the 1st Steiner Axis and the angle bisectors of $QG-Tr3$ at A and B (see (1), (4)).

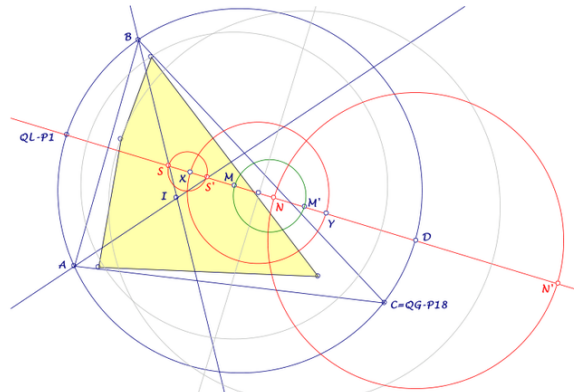
2nd pair of points:

... let M and M' be the midpoints of circles centered on the 1st Steiner Axis and containing opposite vertices of the quadrilateral,

... let D be the second intersection of the 1st Steiner Axis and the circumcircle of $QG-Tr3$ (see (8)),

... let Ci be a circle round D orthogonal to the Thales circle of MM' ,

... the intersections N and N' of Ci and the 1st Steiner Axis are the 2nd pair of points.



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2014-08-16.pdf

Message: #658
Date: 18/8/2014 12:58:51
From: bernard.keizer
Subject: Angle Bisectors of a Quadrigon

Dear Eckart,

I'm not really familiar with QG-environnement.
I observe that A, B, P1, P2 P3 and P4 form a QL.

1) In this QL, C is the common tangential from A and B on the hessian Cu1 and the bisectors of the triangle Tr in the angles A and B are the bisectors of the 4 reference triangles of the QL in A and B. The construction can be made obviously with the 3 pairs of vertices.

2) if you take the intersections of the inner/inner and outer/outer bisectors, you find 8 points and 2 circles. But if you take also the inner/outer and outer/inner bisectors, you find 8 other points. The 16 points lie 4 on each of the 12 bisectors (2 in each vertice of the QL), but each point is on 3 bisectors. These 16 points are on the 8 Steiner circles described in Gergonne's Annales (see EQF QL-8P1); 4 circles are centered on the 1st Steiner axis and 4 on the 2nd Steiner axis through the 2 points 2P3, each circle carries 4 points and each point belong to 2 circles, one of each set. (The 2 circles you describe in details are 2 of the 4 circles of the 1st set).

3) the same way, if you take the intersection of inner/inner and outer/outer bisectors in A and B, you find 2 points I and Ic and one circle centered on the 2nd Steiner axis through A and B, I and Ic and the 2 points 2P3. But if you take also the intersections of inner/outer and outer/inner bisectors in A and B, you find 2 other points Ia and Ib and another circle centered on the 1st Steiner axis through A and B, Ia and Ib. The 4 points I, Ia, Ib and Ic form an orthocentric quadrangle and your triangle Tr is the orthic triangle of IaIbIc.

4) This last property is generalizable to any pair of conjugate points X and X' on the hessian; the 2 points X and X' are on 2 orthogonal circles centered on the Steiner axes (the one centered on the 2nd Steiner axis pass through the 2 points $2P3$). In fact, if we have another pair of conjugate points Y and Y' (for example 2 opposite vertices of the initial QL), the bisectors of any angle YXY' form the degenerated rectangular hyperbola which is the polar conic of the conjugate X' and, conversely, the bisectors of the angles $YX'Y'$ form the degenerated rectangular hyperbola which is the polar conic of X . The 4 bisectors in X and X' intersect in 4 points forming an orthocentric QA; the Euler circle pass through $P1$, 2 points are on a circle through X and X' and the 2 points $2P3$ and the 2 other points are on a circle through X and X' and centered on the 1st Steiner axis.

I hope you will be able to transform part of these remarks in barycentric calculations and wish you a lot of fun!

Best regards
Bernard

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Message: #659
Date: 18/8/2014 2:56:36
From: eckart_schmidt@t-online.de
Subject: A new aspect of QL-Cu1

Dear friends,
background for the attached file is the following property of QL-Cu1, which gives a further construction.
QL-Cu1 is the locus of the vertices of quadrilaterals with the same triangle QG-Tr3 and the same point QL-P1.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

A new aspect of QL-Cu1

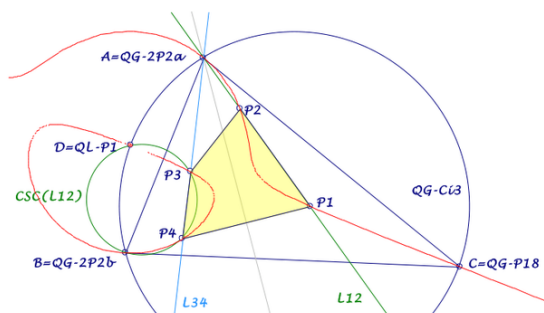
Here the *QL-Quasi Isogonal Cubic QL-Cu1* is considered wrt the *Quasi Isogonal Triangle QG-Tr3* and the *Miquel Point QL-P1*. Background is the following property:

QL-Cu1 is the locus for the vertices of quadrilaterals with the same triangle QG-Tr3 and the same point QL-P1.

The cubic *QL-Cu1* of a quadrilateral $P_1P_2P_3P_4$ has a clearly arranged equation, taking *QG-Tr3* as reference triangle for barycentric coordinates (see also *QFG-message 657*) and giving P_4 the coordinates u, v, w :

$$(c^2y^2 + 2S_A yz + b^2z^2)(c^2u^2 + 2S_B wu + a^2w^2)vx - (c^2v^2 + 2S_A vw + b^2w^2)(c^2x^2 + 2S_B zx + a^2z^2)uy = 0$$

So it is obvious, to study the cubic wrt the reference triangle *QG-Tr3* and the Miquel Point *QL-P1*, which is a point on the circumcircle. *QL-Cu1* is isogonal invariant wrt *QG-Tr3* (see *EQF*).



Construction of quadrilaterals $P_1P_2P_3P_4$ with the same reference triangle *QG-Tr3* and the same *QL-P1*:

- ... let ABC be the reference triangle (later *QG-Tr3* with $A, B = QG-2P2a, b$ and $C = QG-P18$),
- ... let D (later *QL-P1*) be a point on the circumcircle of ABC ,
- ... then the later *CSC*-transformation (*QL-Tf1*) is defined by D and A, B (reflection in the angle bisector of

$\angle ADB$ and inversion wrt a circle round D with radius $\sqrt{DA \cdot DB}$),

... let L_{12} be an arbitrary line through A and L_{34} the reflection in the angle bisector of $QG-Tr3$ at A ,

... let C_i be the circle, which is the CSC -image of L_{12} ,

... then the intersections of C_i and L_{34} give the vertices P_3 and P_4 and their isogonal conjugates are P_1 and P_2 .

Varying the line L_{12} through A , the vertices of the quadrilaterals give $QL-Cu1$. This gives a further construction of $QL-Cu1$.

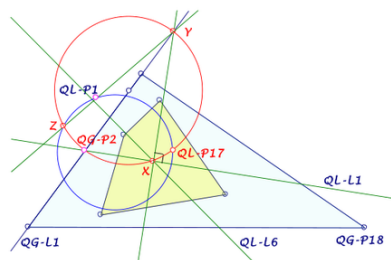
Considering the pencil of quadrilaterals with the same $QG-Tr3$ and $QL-P1$ we can study loci of QL -points. Only some examples:

... The Newton Line $QL-L1$ and $QG-P1.QG-P17$ are fixed lines.

... The locus of $QL-P4$ is the tangent in $QL-P1$ for $QL-Cu1$.

... The locus of $QL-P11$ is the perpendicular bisector of $QL-P1.QG-P17$.

... The locus of $QA-P4$ is a circle: CSC -image of $QG-P1.QG-P17$.



... The locus of $QL-P17$ is a circle centered on $QG-L1$ through

... $QL-P17$, $QG-P2$,

... the intersection X of $QL-L1$ and $QL-L6$,

... the intersection Y of $QG-L1$ and a perpendicular line to $QL-L1$ through X ,

... the second intersection Z of $QL-Ci6$ and $Y.QL-P1$.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

Message: #660
Date: 20/8/2014 9:20:22
From: eckart_schmidt@t-online.de
Subject: Angle Bisectors of a Quadrigon

Dear Bernard K.,
thank you for your detailed additional remarks, using the corresponding QL as point of view.

In 1) you describe the property, that QG-P18.QG-2P2
are tangents to QL-Cu1 (I think not mentioned in EQF).

In 2) you expand the intersections of angle bisectors for a QL
and describe the connections with Steiner's Theorems.

In 3) you developpe another circle through A, B
centered on the 1st Steiner Axis
in the CSC-image of the intersection with QG-L1.

In 4) you make an interesting generalization for two
CSC-partners X, X' on QL-Cu1. Some remarks:

- ... The common tangential T of X, X' is the CSC-image
of the 3rd intersection of XX' and QL-Cu1.
- ... The cubic is isogonal invariant wrt XTX'.
- ... The circumcircle of XTX' is CSC(XX') and contains QL-P1.
- ... The Simson line of QL-P1 wrt XTX' is parallel to QL-L2.
- ... In- and excenters of XTX' lie on your orthogonal circles
through X, X' and QL-2P3.
- ... All quadrigons with QG-Tr3 = XTX' and the same Miquel Point
as the reference QL (construction in my message #659)
have vertices on QL-Cu1.

Sorry, that I am not fit in polar conics!

Some further remarks for two of this triangles XT_xX' and YT_yY'
with X, Y collinear with QL-P1:

- ... The triangles XT_xX' and YT_yY' have a common circumconic.
- ... T_x, T_y and T_x', T_y' are collinear with S, the intersection
of QL-Cu1 and its asymptote.
- ... The perpendicular bisectors of T_x, T_y and T_x', T_y'
intersect in QL-P1.
- ... The circumcircles of XT_xX' and YT_yY' have a second
intersection on the Thales circle about S.QL-P1.
- ... The inner (outer) angle bisectors at T_x and T_y intersect
on a perpendicular line to QL-L1 in the intersection
of QL-L1 and QL-L6.

Excuse that I made no barycentric calculations!

Best regards Eckart

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Message: #661
Date: 20/8/2014 7:30:01
From: bernard.keizer
Subject: A new aspect of QL-Cu1

Dear Eckart,
Interesting construction, a little bit self-defining!
A QL-Cu1 is defined by a Cl-S and a Newton Line.
A, B and P1 give the Cl-S; a point C is chosen on the circle circumscribed to A, B and P1.
If $C' = CSC(C)$ (on AB), the line through the middles of the segments AB and CC' is the Newton Line.
It works even if C is in P1, A and B are then the points QL-2P2 (in this case, QL-Cu1 is monocursal) or if the middle of CC' is on the perpendicular bisector of AB (in this case, QL-Cu1 is bicursal).
Best regards
Bernard

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Message: #662
Date: 24/8/2014 3:59:44
From: eckart_schmidt@t-online.de
Subject: CSC of QL-Diagonals

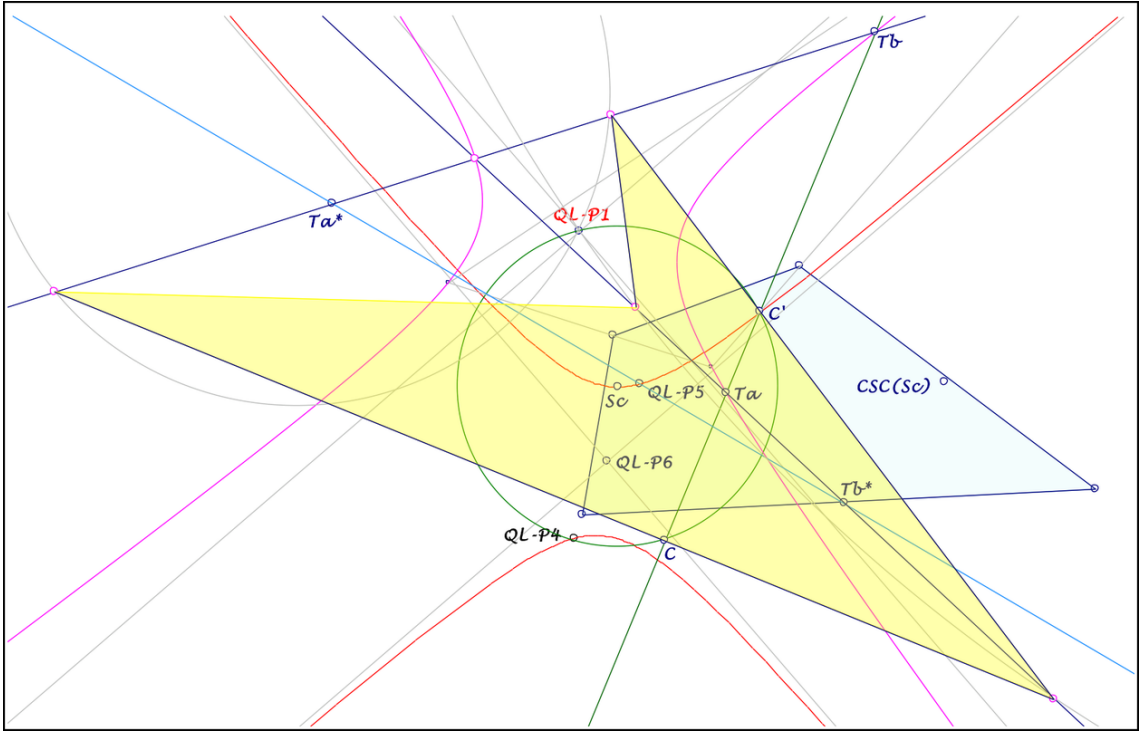
Dear Bernard K.,

I have studied your interesting message #655. The transformation CSCdiag was completely new for me. Is it you, who found it? I have reproduced your constructions of QLs inscribed in the quartic with the mentioned properties. Here is a modification of your 1st construction (see attached file):

Let QG be a quadrigon version of the reference QL

- ... let Hi1 be the orthogonal hyperbola, centered in QL-P6 through QL-P4, QL-P5 with asymptotes parallel to the Steiner Axes,
- ... let Sc be a point on Hi1: the intersections of the perpendicular bisector of QL-P1.CSC(Sc) and the circle round Sc through QL-P1 give C and C'; these are QG-2P2 of the searched QG (so far your construction),
- ... let Hi2 be the orthogonal hyperbola centered in QL-P1 circumscribed QL-DT,
- ... let Ta, Tb be the intersections of CC' and Hi2; these are the points QG-2P3 of the searched QG,
- ... let Ta*, Tb* be the CSCdiag-images of Ta, Tb; these are the diagonal midpoints of the searched QG, then TaTb*, Ta*Tb are the diagonals of the searched QG,
- ... the intersections of the diagonals and their CSC-circles give the opposite vertices A, A' and B, B' of the searched QG.

Best regards Eckart



2014-08-24.pdf

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Message: #663
Date: 26/8/2014 3:42:47
From: bernard.keizer
Subject: CSC of QL-Diagonals

Dear Eckart,

1) I found the CSCdiag transformation myself: it's the logical following of the DQL in an old message, the QL formed by DT and the Newton Line is an ordinary QL and DT is one of it's reference triangles ...

I was very surprised when I found that $CSC(P17) = CSCdiag(P1)$

2) Your 2nd rectangular hyperbola is very interesting and new for me; it passes through F1 and F2 (the QL-2P3 points). Do you know it's asymptotes?

It carries the vertices of DT's and points P16 of all QL's inscribed in your quartic (and by the way it cuts harmonically all their diagonals).

It makes my second construction much easier: starting with a point P4 on the 1st rectangular hyperbola, P5 it's reflexion in P6, we get the Plücker points and the Newton Line as their perpendicular bisector; now, $CSCdiag(\text{Newton Line})$ is the circumcircle of DT, which cuts the 2nd rectangular hyperbola in 4 points, the vertices of DT and P16 (reflexion of P10 in P1).

The rest is like in your message ...

It's a real pleasure to progress this way together step by step!

Best regards

Bernard

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Message: #664
Date: 27/8/2014 10:29:41
From: eckart_schmidt@t-online.de
Subject: CSC for QL-Diagonals

Dear Bernard K,

1) I think your transformation CSCdiag is an interesting one;
some examples:

... CSCdiag(QL-P16) = QL-L1 $\wedge$ QL-L6,
... CSCdiag(QL-P9) = QL-L6 $\wedge$ QL-L9,
... CSCdiag(QL-P23) = QL-Ci1 $\wedge$ QL-P1.QL-P24 (2nd intersection),
... CSCdiag(QL-P7) = 2nd intersection of QL-Ci1
and the line QL-P7.(QL-L6 $\wedge$ QL-L9),
... CSCdiag(QL-L6) = circle through QL-P9, QL-P17, QL-Ci1 $\wedge$
QL-P1.QL-P24 (orthogonal to the circle
above).

2) There is a generalization of CSCdiag: Using the notation
CSC(X) = CSC(P1,X) and

CSCdiag(X) = CSC(P17,X) with CSC(P1,P17) = CSC(P17,P1)

you can define CSC(P,X) with CSC(P1,P) = CSC(P,P1) for an
arbitrary point P.

For example: Let P be a point on QL-Ci1, then CSC(P,QL-L1) is a
circle through P, QL-P16, centered in CSC(P,P') with P'
reflection of P in QL-P1.

3) Wrt your question for the asymptotes of the orthogonal
circumhyperbola of QL-DT, centered in QL-P1:

The points at infinity are the QL-DT-isogonal images of the
intersections QL-Ci1 $\wedge$ QL-L2.

Best regards Eckart

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Message: #665
Date: 29/8/2014 8:40:11
From: eckart_schmidt@t-online.de
Subject: False property QG-Ci3

Dear Chris,

it seems that the last property of QG-Ci3 is false,
unfortunately cited with my name:
Right is: $QL-Tf1(QG-Ci3) = QG-L1$ and the $QL-Tf1$ -image of the
circle through QG-2P2, QG-P19, QA-P4 is QA-Ci1.

Further property:

QA-Tf2(QG-Ci3) is a hyperbola through QG-2P2, QG-P19, QA-P4,
QA-P16.

(Points at infinity of the asymptotes are the QA-Tf2-images of
the intersections of QG-Ci3 and the QA-DT-circumconic through
QG-P13 and QG-P15).

Additional remark:

There is an orthogonal circumhyperbola of QG-Tr3 through QG-P19
(QG-Tr3-isogonal image of the QG-Ci3-diameter-line through
QG-P1).

The 3 hyperbolas of this type for a QL have 2 common points.
What about these points?

Best regards Eckart

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Message: #666
Date: 30/8/2014 10:49:18
From: bernard.keizer
Subject: CSC for QL-Diagonals

Dear Eckart,

1) This transformation CSCdiag is in fact even more interesting as I thought myself when I found it!

CSCdiag(P16) is the middle of the points QL-2P2

CSCdiag(P9), intersection of L6 and L9, is the reflexion of P17 in the Newton Line.

CSCdiag(P23) is the reflexion of P16 in P9.

CSC(P7) is in line with P7 and CSCdiag(P9); you call it 2nd intersection with Ci1, what is the 1st?

2) The generalization is particularly interesting when the transformation swaps a circle and a line (for example Ci1 and L2); I suppose, it could give other properties ...

3) Thanks for the asymptotes.

Best regards

Bernard

PS For Chris, I suppose some of these properties could find place in EQF (for example the reflexion of P17 in the Newton Line being the intersection between L6 and L9).

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Message: #667
Date: 30/8/2014 4:10:05
From: eckart_schmidt@t-online.de
Subject: CSC for QL-Diagonals

Dear Bernard K.,

you are right, there is an incorrect description of

CSCdiag(QL-P7). The right text must be:

... CSCdiag (QL-P7) = 2nd intersection of QL-Ci1 and the circle through QL-P7, QL-P9, QL-P17

(CSC(QL-P7) is collinear with QL-P7 and $QL-L6 \wedge QL-L9$).

Best regards Eckart

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Message: #668
Date: 01/9/2014 10:39:14
From: bernard.keizer
Subject: CSC for QL-Diagonals

Dear Eckart,

Very amusing property!

CSCdiag(QL-P7) is the 2nd intersection of QL-Ci1 as CSCdiag(L1) and the circle CSCdiag(L2), orthogonal to Ci1 through QL-P17, CSCdiag(QL-P2), CSCdiag(QL-P9) and CSCdiag(Plücker points). As CSCdiag(QL-P9) is the reflexion of QL-P17 in the Newton Line, this circle is centered on the Newton Line; more than that, it passes through the Plücker points themselves!

The 3rd circle you describe has for center the middle of the segment joining the centers of these 2 circles; this 3rd circle passes through the 2 centers of the circles (one being QL-P9) and, as the 2 circles are orthogonal), through QL-P17 and CSCdiag(QL-P7); it happens that it passes also through QL-P7, as it is the CSCdiag conjugate of the line QL-P7, CSCdiag(QL-P7) and CSCdiag(QL-P9).

Why these 3 points are in line is not so obvious ...

Best regards

Bernard

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Message: #669
Date: 01/9/2014 12:24:38
From: Chris van Tienhoven
Subject: Special loci for EQF-points

Dear Eckart and Bernard K. and Seiichi,
Finally I found time to study your messages.
My wife and me had in a small VW-camper our own Tour de France.
We visited France and surrounding countries Spain, Andorra and Italy.
After our holiday there were lots of other things to do.
But now is the time for Quadrilateral Geometry.
Eckart, I studied your points U and V as well as their CSC-images.
Especially point U is a point with nice properties.
I could not find any incidences with other QL-points.
The only incidence I found is:
* CSC(V) lies on QL-Ci3, the Miquel Circle.
However this is obvious because V lies on the Steiner Line and the CSC-image of the Steiner Line is the Miquel Circle.
I keep your points in my database for investigations for future incidences.
Best regards,
Chris

>Van:
>Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
>Verzonden: dinsdag 29 juli 2014 9:10
>Aan: Quadri-Figures
>Onderwerp: [Quadri-Figures-Group] Re: Special loci for EQF-points [1 Attachment]
>[Attachment(s) (#TopText)
>from eckart_schmidt@t-online.de included below]
>Dear Bernard K.,
>interesting properties for the second intersection
>of QL-P9.QL-P1 and QL-Ci5!
>In addition: this point lies on a circle round
>the curious point U
>(see remark under QL-P22 in QFG-message #628),
>orthogonal to the QL-DT-circumcircle, containing QL-P1.
>This curious point U has my special interest!
>You find a new aspect in the attached file.
>Best regards Eckart

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Message: #670
Date: 01/9/2014 3:15:21
From: bernard.keizer
Subject: CSC for QL-Diagonals

Dear Eckart,
I almost forgot this beautiful and amazing property that the CSCdiag transformation, which swaps the circle C_{i1} circumscribed to DT and the Newton Line L_1 , with center in P_{17} also swaps the Plücker points; each Plücker point is the CSCdiag conjugate of the other!
Best regards
Bernard

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Message: #671
Date: 01/9/2014 5:49:01
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Chris,
I'm really happy that you had a nice tour in France (wie Gott in Frankreich, sagen die Deutschen).
I live myself 15 km from Sarlat, near the Dordogne and surrounded by a lot of nice castles ...
For the interesting points U , V and V' (reflexion of V in U), they depend only from 4 points, vertices of DT and point P_1 (on the Euler circle of DT) and they belong more to the geometry of the QA than the QL !
So far, the 4 points give the points P_8 to P_{11} as well as P_{16} and P_{24} ; L_2 is the Steiner Line of P_1 wrt the medial DT and the parabola is given.
Now, consider the inverse X of P_1 in the circle $\text{circum}DT$; U is the intersection of the perpendicular bisector of P_1X and the polar of P_8 wrt the parabola and V is the intersection between L_2 and the perpendicular through P_{10} to polar of U wrt the parabola (which passes through P_8).
Always following Eckart's constructions, P_3V is perpendicular to P_9P_{10} (the intersection is a fixed point Y), P_4V' is perpendicular to P_1P_{11} (the intersection is another fixed point Z) and P_5U is the perpendicular bisector of P_1X .
I'm not surprised that you didn't find incidences with other QL -points, as these 5 points U , V , X , Y and Z depend only from the 4 initial points (vertices of DT and P_1).
Best regards
Bernard

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Message: #672
Date: 01/9/2014 6:15:19
From: Chris van Tienhoven
Subject: Special loci for EQF-points

Dear Bernard K.,
In the Netherlands we say "leven als God in Frankrijk".
Probably with the same meaning.
Thanks for your explanation.
I already wondered why there were so little connections with
known QL-features.
Best regards,
Chris

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Message: #673
Date: 02/9/2014 10:14:41
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Chris, dear Eckart
I've tried to make a short presentation of the CSC
transformations linked to the QL, simply called CSC of QL, and
to it's diagonal QL, called CSCdiag of DQL.
I'm convinced that the systematic use of the barycentric
calculations could lead to plenty of other properties.
I hope, this beginning will be promising.
Thanks to Eckart for having shared my first reflexions in this
matter.
I hope, the CSCdiag will be worth a short notice in EQF, but
even without this, some properties are interesting.
Best regards
Bernard

QL, DQL, CSC and CSCdiag

For an ordinary QL, the CSC transformation swaps the lines L_i and the circumscribed circles C_i .

The diagonal QL is formed by the vertices of DT and the Newton Line and is called DQL ; for this DQL, DT is one of the 4 reference triangles with the Newton Line as the menelian ; the CSCdiag swaps the Newton Line and the circle circumscribed to DT.

Obviously, QL is the same way the DQL of 4 another QL's, called dQL.

DQL and dQL are ordinary QL's with all properties and elements of QL ; we call DX the image of an element X of QL the same element for DQL and conversely dX the antecedent of an element of QL the same element for dQL. CSC and CSCdiag are properties linked to QL and DQL and $\text{CSCdiag}(DX) = \text{DCSC}(X)$. In other words, the CSCdiagconjugate of an element DX of DQL is the image for DQL of the CSCconjugate of the element X of QL.

The important point, already mentioned in an old message 164 quoted in EQF (see QL-P17) is that the CT coefficient of the 4th line is l and the DT coefficient of the Newton Line is l^2 . It follows immediately that the DT-coordinates of an element of DQL are simply the same as the CT-coordinates of the same element of QL with l^2 , m^2 and n^2 instead of l , m and n . The converse is true and the CT-coordinates of an element of dQL are the same as the DT-coordinates of the same element of QL with l , m and n instead of l^2 , m^2 and n^2 .

Now, some points already mentioned in EQF are clearly either for the QL or for the DQL, but some are for both and have an image and an antecedent. This leads easily to new points or new properties for old points.

For example, P8 to P11 are DQL elements and have for antecedents G, O, H and N the centroid, circumcenter, orthocenter and nine-point center of the reference triangle of QL. $Ci1$ has for antecedent the circumscribed circle of the reference triangle, its CSCdiag conjugate is the Newton Line and has for antecedent the 4th line of the reference triangle. Points of $Ci1$ have antecedents on the circumscribed circle of the reference triangle.

P17 is known as the image of P1 : it is the Miquel point of DQL and center of the CSCdiag.

The CSCdiag swaps the 2 Plücker points of QL (**new property**) ; it follows that $\text{CSCdiag}(L2)$ is a circle orthogonal to $Ci1$ through the Plücker points and P17 (and through $\text{CSCdiag}(P2)$). The 2nd intersection between the 2 circles is $\text{CSCdiag}(P7)$; this point is also on a circle through P7, P9 and P17, which is $\text{CSCdiag}(P7\text{CSCdiag}(P7))$, therefore the point $\text{CSCdiag}(P9)$ lies on $P7\text{CSCdiag}(P7)$. (**new property, first mentioned by Eckart**)

P16 is the 2nd intersection between the circle $Ci1$ and the Miquel circle of DQL, which is $\text{CSCdiag}(L6)$, the circle through P9, P16 and P17 (**new property for P16**). This point has for antecedent in QL the 2nd intersection X between the circumcircle and the Miquel circle of QL $Ci3$; it is well known that this point is a perspector between the 2 similar triangles reference triangle and triangle formed by the 3 circumcenters of the 3 other reference triangles of QL. $\text{CSCdiag}(P16)$ is the intersection of the lines $L1$ and $L6$ (middle of the segment joining the points 2P2) and CSCconjugate of the antecedent is the intersection between $L2$ and the 4th line of QL.

P24 is the 2nd intersection between $P13P17$ and the circle $Ci1$, P13 being the pole of the Newton Line wrt $Ci1$ and the perspector between DT and its own DT, called DDT ; it has for antecedent the point D, 2nd intersection between $P1P$, where P is the pole of the 4th line and the perspector between the reference triangle and DT. There are 4 such poles or perspectors P and 4 such points D, which lie, like P24, on the Dimidium circle (see message 457). (**new property for the Dimidium circle**)

The hyperbola through the vertices of DT, P8, P13 and P24 has for antecedent the hyperbola through the vertices of the reference triangle of QL, through the centroid G, the perspector P and the point D. The 4 hyperbolas of the reference triangles as well as the hyperbola of DQL cut the Dimidium circle in the 3 S-points described in EQF.

Lets come now to P1 : it has 1st CT-coordinate $a^2mn/(m-n)$ and 1st DT-coordinate $b^2/(l^2-n^2) - c^2/(l^2-m^2)$. Following the above rule, P1 has an image, which is the Miquel point of DQL P17 and has as 1st DT-coordinate $a^2m^2n^2/(m^2-n^2)$; but it has also an antecedent, which has as 1st CT-coordinate $b^2/(l-n) - c^2/(l-m)$ and is the middle of the segment joining the orthocenter H to the 2nd intersection X between the circumcenter of the reference triangle and the Miquel circle (exactly like P1 is the middle of P10P16).

We could find the same way for P2 an image DP2, which is the middle of P2P10 and an antecedent dP2, which is the reflexion of the orthocenter H of the reference triangle of QL in P2 (as P2 is the middle of HdP2).

The Kantor-Hervey point of DQL is DP3, circumcenter of the circle through P2P9P10 ; P3, center of the Hervey circle, is also the center of the circle through O, H and the point dP2.

The image of the deltoid Qu2 is a deltoid with inner circle the Hervey circle of DT (reflexion of the Miquel circle of DQL in DP5) ; it is inscribed in DT and tangent to the Newton Line.

The circumcenter of the Miquel circle of DQL is the point DP4 ; as seen above, the Miquel circle of DQL is the circle through P9, P16 and P17, CSCdiag(L6).

DP5 is the middle of DP3DP4 and DP6 is the middle of DP4DP5 and also the middle of P6P9.

DP12 is the middle of P8P12.

The Newton Line L1 is the 4th line of DQL, image of the 4th line of the QL.

DL1, the Newton Line of DQL is the perpendicular to L6 through DP5 or DP12.

The line L6 is the image for DQL of L2 for QL, the Steiner Line and CSCdiag(L6) is the Miquel circle of DQL.

Therefore, the image of the parabola Co1 is the parabola inscribed in DT and tangent to the Newton Line with focus P17 and directrix L6 ; it is the anticomplement of the parabola Co3 described in EQF.

The image of the monocardioid Qu1 is the cardioid with center DP4 and cusp P17 ; it is tangent to the circle Ci1.

The line L9 is the image for DQL of the perpendicular to L2 in the reflexion of P1 wrt the 4th line, which is the point CSC(circumcenter O) on L2 ; the intersection of L6 and L9 is the point CSCdiag(P9), which is the reflexion of P17 in the Newton Line. (**new property for L9**).

CSCdiag(L9) is a circle orthogonal to the Miquel circle of DQL through P9, P17 and CSCdiag(P23), which is the reflexion of P16 in P9. The antecedent of P23 for the QL is the reflexion of the 2nd intersection between the circumcircle of the reference triangle and the Miquel circle in the circumcenter. 1st DT-coefficient of L9 is $l^2(m^2+n^2)$, 1st CT-coefficient of the antecedent would be $l(m+n)$.

There are surely plenty of other properties to be found ...

CSC's of QL and DQL.docx

Message: #674
Date: 02/9/2014 10:25:41
From: bernard.keizer
Subject: Special loci for EQF-points

Dear Chris, dear Eckart
In fact, it's better to start with an inscribed QA formed by the 4 fixed points vertices of DT and P16; P1 is simply the middle of P10 and P16.
That's look like QA properties, particularly interesting as this QA is inscribed in the circle Ci1.
There are then plenty of circles Ci1, Ci2, polar circle of DT ... and rectangular hyperbolas through the 4 points and P10 or through the middles of the sides and the vertices of the DT of the QA ... or the conic inscribed in DT and having P1 and it's isogonal conjugate wrt DT as foci ...
Best regards
Bernard

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Message: #675
Date: 03/9/2014 12:24:45
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Chris, dear Eckart,
I realised I forgot some interesting properties (in particular the alignment indicated by Eckart of P7, CSCdiag(P7) and CSCdiag(P9)) and a few other elements.
So I made a new version (maybe it's possible to destroy the 1st one).
Best regards
Bernard

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Message: #676
Date: 03/9/2014 5:14:46
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Chris, dear Eckart,
I've found now the explanation to the beautiful alignment
mentionned by Eckart.
CSCdiag(L1) is Ci_1 , centered on L2; CSCdiag(L2) is a circle
orthogonal to Ci_1 , passing through P17 and the Plücker points
and therefore centered on L1 (as CSCdiag swaps the Plücker
points).
Both circles intersect in P17 and in CSCdiag(P7).
The circle with diameter the segment joining the 2 centers (if
I'm not wrong, Eckart calls it Thales circle) is through the 2
centers and the 2 points of intersection CSCdiag(P7) and P17;
but it is also through P7, as one center is on L1 and the other
in L2.
CSCdiag of this 3rd circle is obviously the line $P7CSC(P7)$,
which passes through CSCdiag(P9), which is the reflexion of P17
in L1 and CSCdiag of the second center, which is the reflexion
of P17 in L2 and the 2nd intersection with the circle Ci_1 .
The 3 circles can be defined without knowing CSCdiag and give
new properties to existing points of EQL.
This time, I will stop for a while and remain looking forward to
reading you soon on these matters.
Best regards
Bernard
PS I've also forgotten to mention the property $CSC(P17) =$
 $CSCdiag(P1)$, which could also be taken as a definition of
CSCdiag with center P17, once knowing CSC with center P1.

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Message: #677

Date: 04/9/2014 4:20:53

From: eckart_schmidt@t-online.de

Subject: CSC's of QL and DQL

Dear Bernard K,

thanks for all the informations. While I haven't studied all your messages, here a remark wrt my generalisation of CSCdiag, mentioned in message #664:

CSC(X) is centered in QL-P1 and swaps opposite points of a quadrilateral, CSCdiag(X) is centered in QL-P17 and swaps QL-P1 and CSC(QL-P17), this can be generalized for arbitrary points P: CSC(P,X) shall be centered in P and swap QL-P1 and CSC(P).

My example in message #664: Let P be a point on QL-Ci1, then CSC(P,QL-L1) is a circle through P, QL-P16, centered in CSC(P,P') with P' reflection of P in QL-P1.

Here another aspect: QL-Cu1 is invariant under CSC(X):

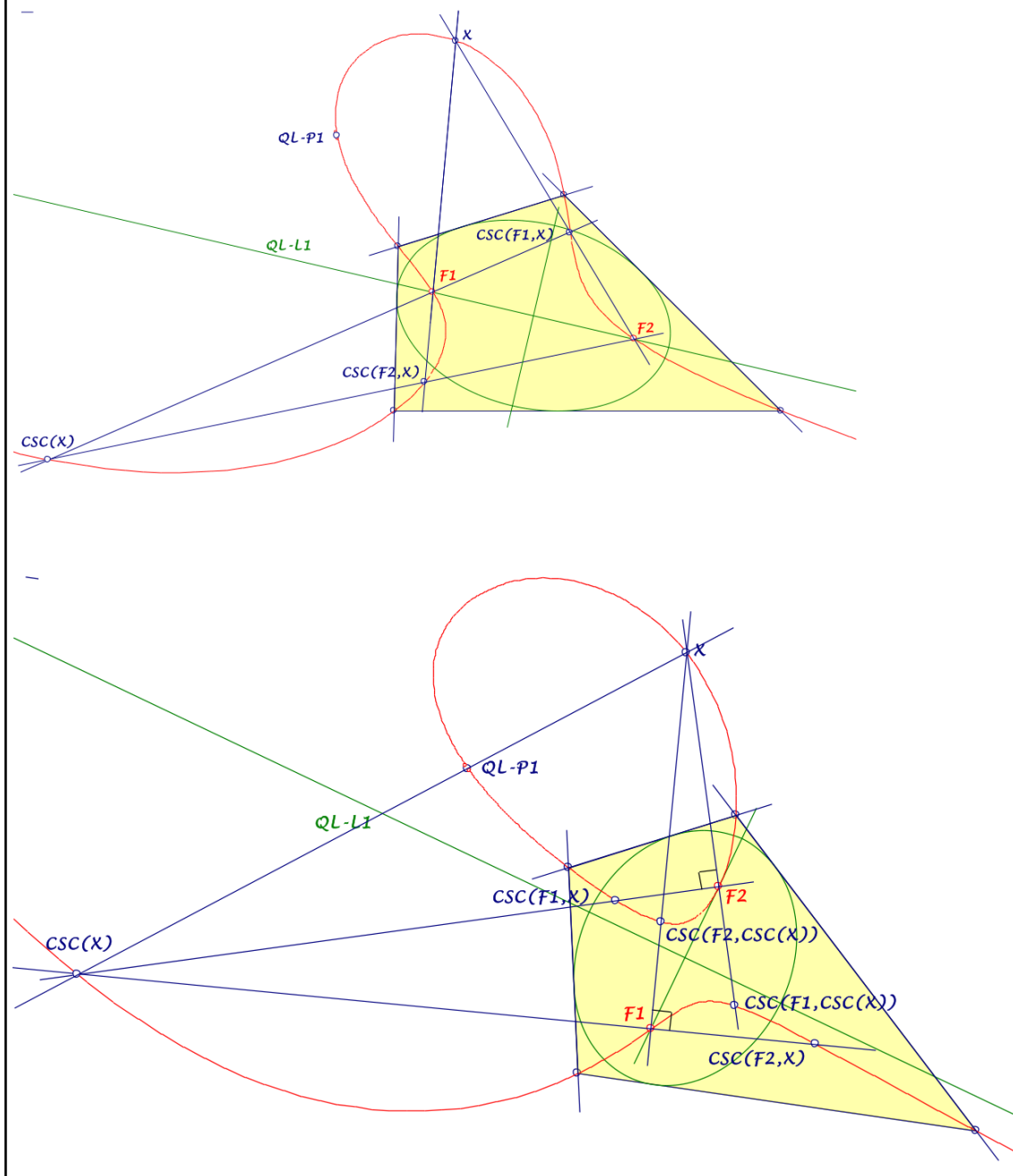
... QL-Cu1 is also invariant under CSC(P,X),
if P is a foci F1, F2 of the inscribed conic,
centered in QL-L1 $\wedge$ QL-L6 (if QL-Cu1 is unipartite,
these are the points QL-2P2).
... For points X on QL-Cu1 CSC(F1,X) and CSC(F2,X)
are CSC partners (see attached file).
... In the unipartite case of QL-Cu1 are collinear:
X, F1, CSC(F2,X) and X, F2, CSC(F1,X) and
CSC(X), F1, CSC(F1,X) and CSC(X), F2, CSC(F2,X),
... in the bipartite case are collinear X, F1, CSC(F2,CSC(X))
and X, F2, CSC(F1,CSC(X)) and CSC(X),
F1, CSC(F2,X) and CSC(X), F2, CSC(F1,X).
... In the unipartite case CSC(F1,QL-L1) and CSC(F2,QL-L1)
are the tangents in F1 and F2 at QL-Cu1 (with tangetial
QL-P1),

in the bipartite case CSC(F1,QL-L1) and CSC(F2,QL-L1) are
circles round QL-P1 through F1, F2.

In the unipartite case $\text{CSC}(F1,\text{CSC}(X)) = \text{CSC}(F2,X)$ and
 $\text{CSC}(F2,\text{CSC}(X)) = \text{CSC}(F1,X)$,

Best regards Eckart

Attachment message 677



2014-09-04.pdf

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Message: #678

Date: 06/9/2014 3:40:19

From: eckart_schmidt@t-online.de

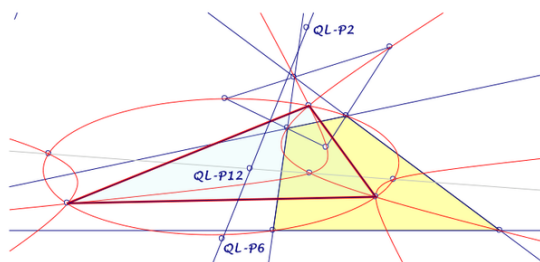
Subject: Remarkable QL-Triangle

Dear Chris, dear friends,
the QL-points QL-P2, QL-P6, QL-P12 are collinear in Euler
constellation. Attached the geometry of a triangle with this
Euler line is researched. Vertices are the common points of the
three versions of the Nine-Point Conic QA-Co1 for a
quadrilateral.
Perhaps of interest.
Best regards Eckart

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

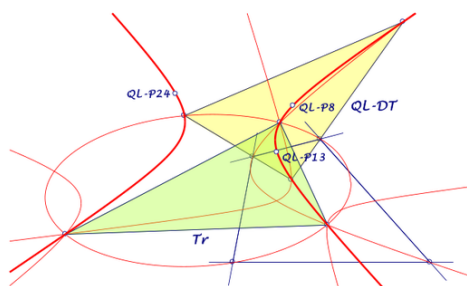
Remarkable QL-Triangle

The QL-points QL-P2, QL-P6, QL-P12 are collinear in Euler constellation. Here the geometry of a triangle with this Euler line is researched. Vertices are the common points of the three versions of the Nine-Point Conic QA-Co1 for a quadrilateral.

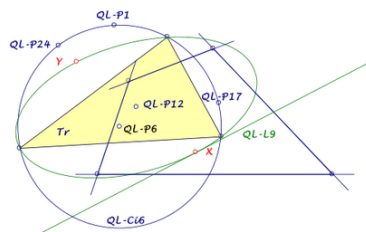


For a quadrilateral a version of *QA-Co1* contains two opposite *QL*-points, two midpoints of diagonals and one vertex of *QL-DT*. The double intersections of the three *QA-Co1* versions are the midpoints of the diagonals on the Newton Line *QL-L1*. The triple intersections give the vertices of a triangle *Tr*, which shall be tested here.

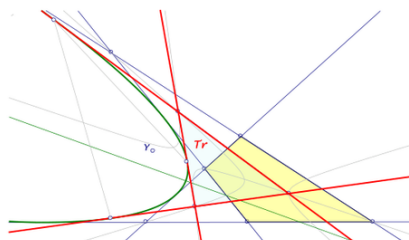
- The Euler line of *Tr* is *QL-P2*.*QL-P6*.*QL-P12* with centroid *QL-P2*, circumcenter *QL-P6*, orthocenter *QL-P12*.
- The circumcircle of *Tr* is the Dimidium Circle *QL-Ci6*.



- The triangles Tr and $QL-DT$ have a common circumconic Co containing $QL-P8$, $QL-P13$, $QL-P24$.
- There are two special points X and Y on the Tr -Steiner ellipse:
... **point X** : Tr -trilinear pole of $QL-L1$ and contact point of $QL-L9$ and the Tr -Steiner ellipse,
... **point Y** : Tr -isotomic conjugate of the point at infinity of $QL-L1$ (diametral to point X on the Tr -Steiner ellipse).



- The sidelines of Tr are tangent to the Inscribed Parabola $QL-Co1$ of the reference quadrilateral.
- Brianchon point of $QL-Co1$ wrt Tr is the point Y .



Trilinear poles and polars wrt Tr :

- The Tr -trilinear pole of XY is the Tr -isotom conjugate of the point at infinity of $QL-L9$.
- The Tr -trilinear poles of the $QL-DT$ -sidelines are collinear on XY .
- The Tr -trilinear polars of the $QL-DT$ -vertices have a common point U , which is the same for all points on Co .
- The Tr -trilinear poles of the QL -lines are collinear on the Tr -trilinear polar of Y .
- The Tr -trilinear polars of the QL -points give six lines, defining a quadrangle: The Tr -trilinear polars of the four quadrangle points have a common point V on $QL-L1$.

In the following properties let $^\circ$ be an arbitrary isoconjugation wrt Tr (such as isogonal or isotomic conjugation):

- The Tr -isoconjugates of the QL - DT -vertices are collinear on the line

$$L_1 = QL-P8^\circ.QL-P13^\circ.QL-P24^\circ.$$
- The Tr -isoconjugates of the QA - CoI -versions are three lines
 ... with Tr -trilinear poles on a line L_2 ,
 ... giving a triangle Δ perspective $QL-DT$,
 ... with center of perspectivity on Co in the Tr -isoconjugate of the intersection of $QL-L1$ and L_1 .
- The Tr -isoconjugates of the vertices of a QL -triangle component are collinear:
 ... the four resulting lines give a quadrilateral with diagonal triangle Δ ,
 ... the Tr -trilinear poles of the four lines are collinear on a line L_3 with Tr -trilinear pole on L_2 and its isoconjugate is point V on $QL-L1$ (independent of the isoconjugation).
- The three lines L_1 , L_2 , L_3 have a common point, dependent on the isoconjugation. Its isoconjugate is the 2nd intersection Z of XY and Co , independent of the isoconjugation.

There are special properties for Tr -isogonal and Tr -isotomic conjugations:

- Examples of Tr -isogonal conjugates $*$:
 ... $QL-P1^*$ is the point at infinity of $QL-L1$,
 ... $QL-P17^*$ is the point at infinity of $QL-L9$,
 ... $QL-P24^*$ is the point at infinity of L_1 .
- Examples of Tr -isotomic conjugates $^\wedge$:
 ... $X^\wedge$ is the point at infinity of $U.QL-P12$,
 ... $Y^\wedge$ is the point at infinity of $QL-L1$.
- Simson lines Sl for points P on $QL-Ci6$ wrt Tr :
 ... In general: $Sl(P)$ contains the midpoint of $P.QL-P2$ and is parallel $Q.QL-P1$ with Q as 2nd intersection of $QL-Ci6$ and a line through P perpendicular $QL-L1$.
 Special:
 ... $Sl(QL-P1)$ is $QL-L3$,
 ... $Sl(QL-P17)$ – also the Simson line of $QL-P17$ wrt $QL-DT$ – is parallel $QL-L5$.

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2014-09-06.pdf

Message: #679
Date: 06/9/2014 8:35:44
From: eckart_schmidt@t-online.de
Subject: Some "splitter" VIII

Dear Chris,

excuse, if a property is evident or already mentioned.
... QL-P1.QL-2P2 tangent to QL-Cu1 in QL-2P2,
... QG-P18.QG-2P2 is tangent to QL-Cu1 in QG-2P2,
... 2 points on QL-Ci6: (1) 2nd intersection of QL-P17.QL-P23
and the QL-Co1-axis, (2) intersection of QL-L5
and a perpendicular line to QL-L1 through QL-P17,
... QL-P17 reflected in QL-L1 is the intersection QL-L6 $\wedge$ QL-L9,
... Simson line of QL-P16 wrt QL-DT is the axis of QL-Co1,
... Simson line of QL-P17 wrt QL-DT is a parallel to QL-L5
through the midpoint of QL-P2.QL-P17,
... Simson line of QL-P1 wrt QL-medial triangle is QL-L3,
... the isogonal conjugate of QL-P1 wrt QG-Tr3
is the point at infinity of QL-L1, or:
the Simson line of QL-P1 wrt QG-Tr3 is perpendicular QL-L1,
... QG-circle through QG-2P2, QL-2P3,
incenter of QG-Tr3 and excenter wrt QG-P18,
... the collinear QG-P18 of a QL lie on CSC(QL-Ci2),
... polar of QL-P7 wrt QL-Co1 is a parallel to QL-L9
through QL-P1,
tangent to the Thales circle about QL-P1.QL-P7,
... the tangent at QL-Co1 in the intersection of QL-L1 and
QL-Co1 is a parallel to QL-L9 in 1/4 distance to QL-P8.

Best regards Eckart

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Message: #680
Date: 07/9/2014 10:15:23
From: eckart_schmidt@t-online.de
Subject: Remarkable QL-Triangle

Dear Chris, dear friends,
this triangle is mentioned in EQF as S1S2S3 under QL-Ci6 .
Best regards Eckart

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Message: #681
Date: 07/9/2014 3:01:47
From: eckart_schmidt@t-online.de
Subject: Remarkable QL-Triangle

Dear Bernard K., dear friends,

up to now I hadn't a look for the circumhyperbolas of the QL-triangle components, containing the centroid and the tripole of the 4th line. In Bernard's paper "QL, DQL, CSC and CSCdiag" I found the property, that their common points for a QL are the 3 S-points on QL-Ci6, which are the vertices of the triangle Tr, I considered.

Let's have a look on these 4 hyperbolas and an arbitrary Tr-isoconjugation:

- ... the isoconjugates of the hyperbolas are 4 lines,
- ... the Tr-tripoles of these lines are 4 collinear points,
- ... the Tr-tripole of the corresponding line gives a point,
- ... whose isoconjugate is a point V on QL-L1
(independent of the isoconjugation).

Two properties of this point V are already mentioned in my paper under point 11 and 14.

Now I found, that the point V is the midpoint of QL-P12 and the intersection of its QL-DT-tripolar and QL-L1.

Best regards Eckart

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Message: #682
Date: 07/9/2014 5:09:39
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Eckart,
In the unipartite case, the points you call X and $CSC(X)$ are the 2 centers of inversion in which $QL-Cu1$ is invariant and $F1, F2$ and $CSC(F1,X)$ and $CSC(F2,X)$ are concyclic.
The line $XCSC(X)$ is the 2nd Steiner axis.
The points you call $CSC(F2,X)$ and $CSC(F1,X)$ are CSC-conjugates, as you say, and are on the Schmidt circle, symmetric wrt the 1st Steiner axis and in line with the point where $QL-Cu1$ meets its asymptote.
I suppose similar properties exist in the bicursal case.
Best regards
Bernard

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Message: #683
Date: 07/9/2014 5:34:44
From: bernard.keizer
Subject: Remarkable QL-Triangle

Dear Eckart,
At last, I began to desperate!
At the risk to be insistant, the circumhyperbola contains also what I call the D-point, 2nd intersection of P1P with the circumcenter (antecedent of D24) and this seems to me as an interesting new property for EQF that these 4 D-points lie, as D24, on the Dimidium circle.
In fact the vertices of your triangle Tr are mentionned 3 times in EQF, 1st as S1, S2 and S3 in Ci6, 2nd as S1, S2 and S3 in Ci6 and 3rd as N1, N2 and N3 in Co1.
By the way, the centroid is P12(and not P2) and the orthocenter is P2 (and not P12).
Most of the mentionned properties were already known (circumcircle, circumhyperbola and inscription in the parabola Co1).
For the other circumhyperbolas, you gave yourself the equations in CT and DT coordinates in an old note of yours! And I found another last one, already mentionned in an old message on QL, DQL and DDQL through the vertices of DDT, 4 poles/perspectors and the 3 S-points.
To calculate the coordinates of the S-points leads to an equation of the 3rd degree and considering the sum of the roots, it's possible to show that the centroid is P12.
The biggest interest of your message is to prove that the 3 S-points deserve a description on their own, as a triplet QL-3P1!
Best regards
Bernard

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Message: #684
Date: 08/9/2014 11:25:56
From: eckart_schmidt@t-online.de
Subject: CSC's of QL and DQL

Dear Bernard K.,
if I am right, you describe a special case, while in my
description the point X is an arbitrary point on QL-Cu1.
Best regards Eckart

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Message: #685
Date: 08/9/2014 11:41:43
From: eckart_schmidt@t-online.de
Subject: Remarkable QL-Triangle

Dear Bernard K.,
thanks for comment and correction of the typo wrt QL-P2 and
QL-P12. Too late I noticed, that the triangle Tr has the 3
S-points as vertices. So the main properties of the triangle Tr
are already discussed. Excuse, if you found no references.
The combination of isoconjugates and tripols has the following
background:
For a triangle and 3 circumscribed conics the tripoles of the
isoconjugate image-lines are collinear, if the 3 conics have
collinear 4th intersections. Wrt 4 circumscribed conics the 4th
intersections must be the points of a QL.
Best regards Eckart

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Message: #686
Date: 09/9/2014 2:05:45
From: eckart_schmidt@t-online.de
Subject: CSC's of QL and DQL

Dear Bernard K.,
another sight of your D-points on QL-Ci6 (perhaps already mentioned):
The three QG-P16-points of a QL are collinear on a line $L = \text{CSC}(\text{QL-Ci6})$ (see EQF).
The CSC-images for the intersections of L and the QL-lines are your D-points.
Best regards Eckart

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Message: #687
Date: 10/9/2014 12:29:37
From: eckart_schmidt@t-online.de
Subject: CSC's of QL and DQL

Dear Bernard K.,
I had once more a look in your paper "Dimidium circle and Lemoine point" in message #457.
Interested in the point "IsoCoP26", proposed for a new EQF-point in the last section, I cannot reproduce, that the 4 lines "GISO" for the QL-triangle components intersect in one point.
Best regards Eckart

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Message: #688
Date: 11/9/2014 10:35:39
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Eckart,
In fact, they don't! It was a mistake.
I already apologised in the message #463.
Best regards
Bernard

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Message: #689
Date: 11/9/2014 10:45:29
From: bernard.keizer
Subject: CSC's of QL and DQL

Dear Eckart,
No, it wasn't already mentionned; thank you very much.
It's obvious, as the D-points are the intersections between $Ci6$ and the circumcircles.
What goes without saying goes better with saying!
Of course, L carries also $CSC(P17)$ and $CSC(P24)$, intersections with the circle $CSC(Ci1)$; the center of this circle is the CSC-conjugate of the inverse of $P1$ in $Ci1$.
Best regards
Bernard

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Message: #690
Date: 11/9/2014 10:48:00
From: bernard.keizer
Subject: CSC's of QL and DQL

Der Eckart,
In fact, you are right!
Best regards
Bernard

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Message: #691
Date: 14/9/2014 4:22:42
From: eckart_schmidt@t-online.de
Subject: Three QG-Cubics for a QL and a QA

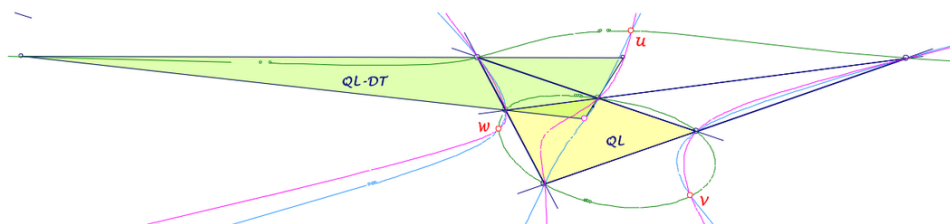
Dear friends,
this is an extension of EQF-Note 2013-01-10 (see my homepage),
where a pivotal isogonal cubic for the reference triangle QG-Tr3
with pivot QG-P1 is described. The three versions of this cubic
for a quadrilateral give three common points, whose trilinear
polars wrt QL-DT coincide. For a quadrangle there are six double
intersections on a conic (see attached file).
Best regards Eckart

EQF-Note 2014-09-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

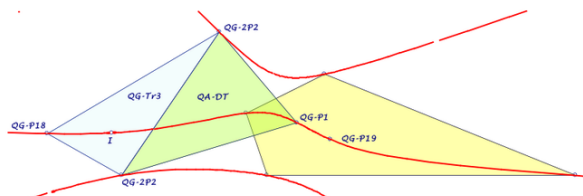
Three QG-Cubics for a QL and a QA

In EQF-Note 2013-01-10 (see my homepage) a pivotal isogonal cubic for the reference triangle $QG-Tr3$ with pivot $QG-P1$ is described. The three versions of this cubic for a quadrilateral give three common points, whose trilinear polars wrt $QL-DT$ have a common point. For a quadrangle there are six double intersections on a conic.



The QG-cubic

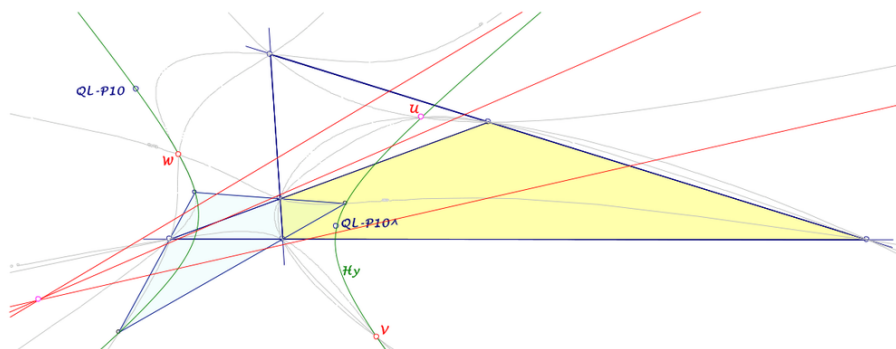
- This QG -cubic is the pivotal isogonal cubic wrt $QG-Tr3$ and pivot $QG-P1$.
- This QG -cubic is also an isocubic wrt $QA-DT$: isoconjugation is $QA-Tf2$ and pivot $QG-P18$.



- This QG -cubic contains ...
 - ... the vertices of the quadrangle,
 - ... the vertices of $QG-Tr3$,
 - ... the vertices of $QA-DT$,
 - ... the point $QG-P19$,
 - ... the in- and excenters of $QG-Tr3$.
- Tangents in the vertices of the quadrangle intersect in $QG-P18$.

The QG-cubics for a quadrilateral

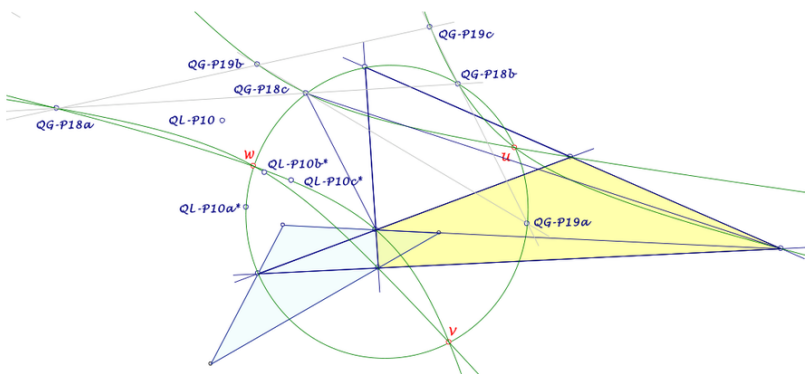
- The three QG -cubics for a quadrilateral have three common points U , V , W (beside the points of the quadrilateral).



- The common points U , V , W lie on an orthogonal hyperbola Hy ...
...circumscribed $QL-DT$
...through $QL-P10$ and its image $QL-P10^A$ wrt an $QL-DT$ -isoconjugation with fixed points in the $QL-DT$ -trilinear poles of the QL -lines.

These common points can be constructed as intersections of conics:

- The three $QG-P18$ points lie collinear (see EQF) on the sidelines of the $QG-P19$ -triangle.



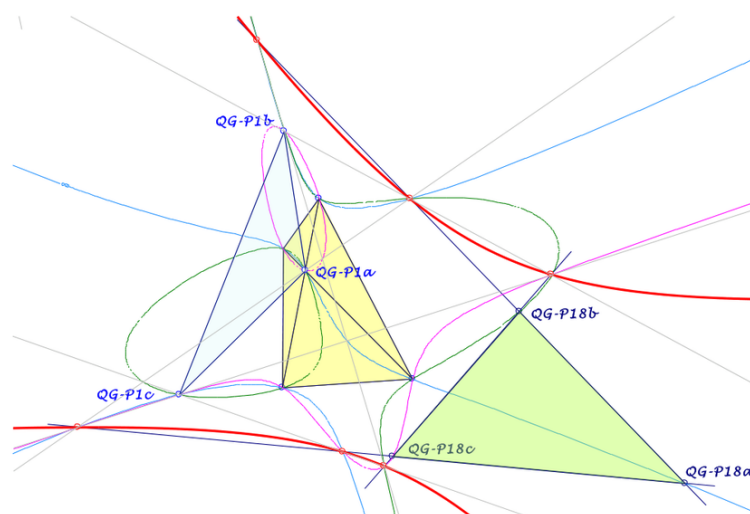
- The common points U , V , W lie on a conic Co ...
...through $QG-2P2$, $QG-P19$ of one QG -version
...and $QG-P18$ of the two other QG -versions
...(and the isogonal conjugate $QL-P10^*$ wrt $QG-Tr3$ of the 1st QG -version).

The intersections of these three conics give the common points U, V, W , which not always will be real.

- The QL - DT -trilinear polars of U, V, W have a common point with DT -coordinates
 $(S_A(m^2S_C^2 - n^2S_B^2) : S_B(n^2S_A^2 - l^2S_C^2) : S_C(l^2S_B^2 - m^2S_A^2))$.
- The QL - DT -trilinear poles of lines through this common point give the orthogonal hyperbola Hy .

The QG-cubics for a quadrangle

- The common points of the three QG -cubics for a quadrangle are the QA -vertices and the QA - DT -vertices.
- The double intersections of the cubics lie on the sidelines of the QG - $P18$ -triangle as QA - $Tf2$ -partners.



- Construction of the double intersections: Intersections of the line $QG-P18a.b$ with the QA - DT circumconic through $QG-P19a,b, \dots$
- The six double intersections lie on a conic.
- There are further collinear relations indicated in the figure.

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2014-09-14.pdf

Message: #692

Date: 19/9/2014 9:25:05

From: Antreas Hatzipolakis

Subject: Morley-Napoleon-Miquel Points applied to Quadrilateral

Dear Chris

Regarding Tran Quang Hung's Problem (*):

We have two points in a triangle, and therefore 4+4 in a quadrilateral

or quadrangle

(2 for each component triangle)

I think it will be interesting to investigate whether the points of each group are concyclic (or what properties have they anyways! :-))

(*)

<http://www.artofproblemsolving.com/Forum/viewtopic.php?f=47&t=606857&p=3605571#p3605571>

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Message: #693

Date: 21/9/2014 1:45:57

From: eckart_schmidt@t-online.de

Subject: QG-conics wrt a perspective row on QG-L2

Dear Chris, dear Bernard K.,

in EQF a perspective row of points on QG-L2 is described.

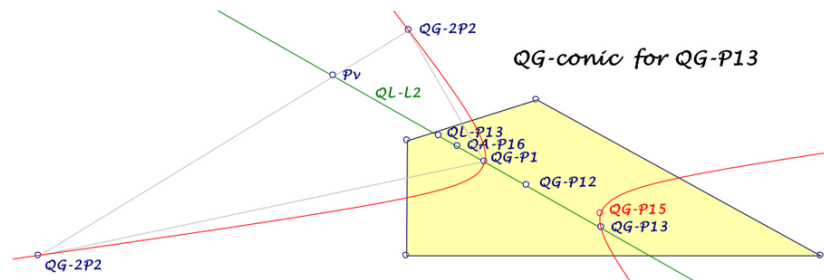
QA-DT-circumscribed conics through QG-P15 and a point of this row lead to QG-conics, whose three versions for a quadrilateral give three common points. The geometry of the corresponding QL-triangles is worked out.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

QG-conics wrt a perspective row on QG-L2

In EQF a perspective row of points on QG-L2 is described. QA-DT-circumscribed conics through QG-P15 and a point of this row lead to QG-conics, whose three versions for a quadrilateral give three common points. The geometry of the corresponding QL-triangles is worked out.



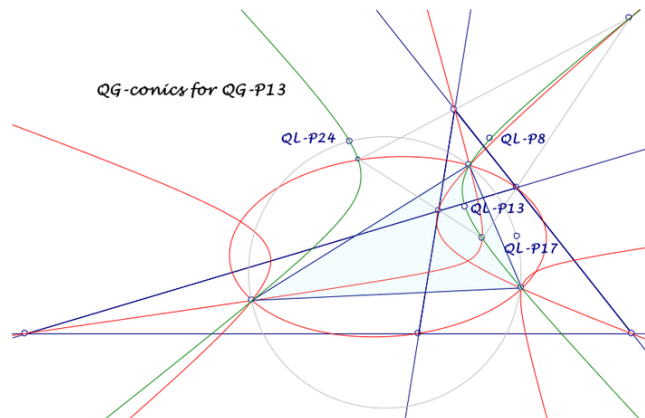
The perspective row of points on QG-L2 has the vanishing point

$$P_v = QG-L1 \cap QG-L2.$$

Let $P_0 = QG-P1$ and $P_1 = QA-P16$, then we get as further points $P_2 = QL-P13$, $P_{-1} = QG-P12$, $P_{-2} = QG-P13$.

We consider QA-DT-circumconics Co_n through QG-P15 and a point P_n ($n \neq 0$) of the row.

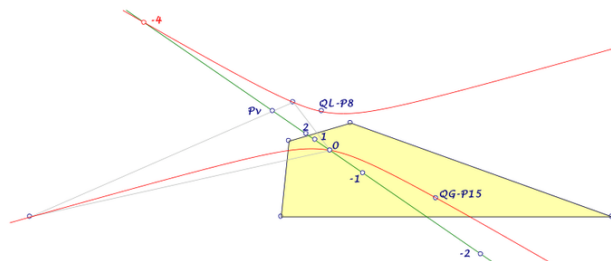
- For a quadrilateral the three versions of Co_n have three common points (for $P_{-2} = QL-P13$ the constellation degenerates).



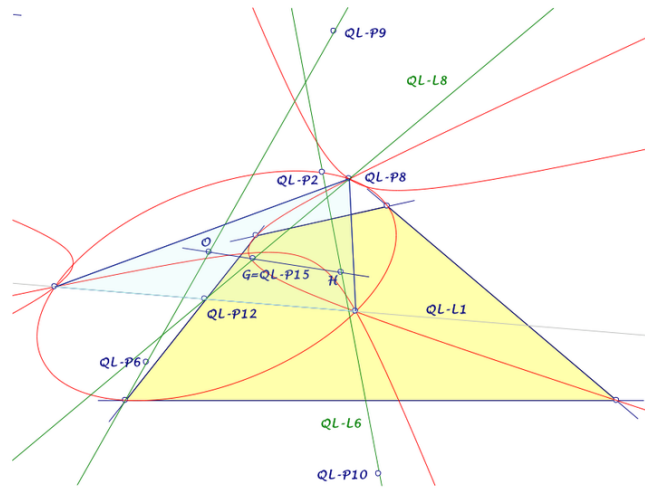
- The three common points lie on a QL - DT -circumscribed conic through QL - $P8$, QL - $P13$, QL - $P24$.
- The circumcircle of the three common points contains QL - $P17$, QL - $P24$.
- For $P_2 = QG$ - $P13$ the common three points are the S -points on QL - $Ci6$ (see EQF).
- For P_v the three common points are the vertices of QL - DT (degenerated conics).
- The triangle of the three common points has its centroid on QL - $L8$, dividing QL - $P12$. QL - $P8$ with ratio $-(n+2):4$.
- The triangle of the three common points has its orthocenter on QL - $L6$, dividing QL - $P2$. QL - $P10$ with the same ratio $-(n+2):4$.
- The triangle of the three common points has its circumcenter on QL - $P6$. QL - $P9$, dividing this line also with ratio $-(n+2):4$.
- The triangles of the three common points have the same Simson line for QL - $P17$: parallel to QL - $L6$ half the distance to QL - $P17$.

Example for P_4 :

This point is the 2nd intersection of QG - $L2$ and the QA - DT -circumconic through QG - $P15$ and QL - $P8$.



The triangle of the three common points has the vertices QL - $P8$ and the intersections of QL - $L1$ with the circle through QL - $P8$, QL - $P17$, QL - $P24$. The centroid is QL - $P15$ dividing QL - $P12$. QL - $P8$ with ratio 1:2, the orthocenter divides QL - $P2$. QL - $P10$ with ratio 1:2 and the circumcenter divides QL - $P6$. QL - $P9$ with ratio 1:2.



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Message: #694
Date: 23/9/2014 4:15:18
From: eckart_schmidt@t-online.de
Subject: Easy QL-Cu1 construction

Dear friends,
let X be a point on QL-L1 and L the angle bisector of QL-P1, X,
CSC(X), then the intersections of L and CSC(L) give points on
QL-Cu1.
Best regards Eckart

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Message: #695
Date: 28/9/2014 10:01:02
From: eckart_schmidt@t-online.de
Subject: QG-circles coinciding for QA/QL

Dear Chris,
perhaps of interest: Let Cix be the circumcircle of QG-2P2 and
QG-Px: The three QA / QL-versions of Cix have a common point ...
... for QG-P12 and a QA in a point with DT-coordinates
 $(p^2(-p^2+q^2+r^2)/(-a^2q^2r^2+b^2r^2p^2+c^2p^2q^2):$
 $q^2(p^2-q^2+r^2)/(a^2q^2r^2-b^2r^2p^2+c^2p^2q^2):$
 $r^2(p^2+q^2-r^2)/(a^2q^2r^2+b^2r^2p^2-c^2p^2q^2)),$
... for QG-P15 and a QA in QA-P3,
... for QG-P15 and a QL in QL-P17,
... for QG-P16 and a QA in a point on QA-Cu1, the Involutary
Conjugate of the intersection Q2 of QA-Cu1 and its asymptote,
collinear with Q2 and QA-P4,
... for QG-P16 and a QL in CSC(QL-P17),
... for QG-P18 and a QA in QA-P41,
... for QG-P18 and a QL in QL-P1,
... for QG-P19 and a QA in QA-P4.
In addition: The circumcircles of QG-2P3 and QG-P13 for a QL
have also a common point with DT-coordinates
 $(m^2n^2(l^2m^2-m^2n^2+n^2l^2)/(-a^2l^2+b^2m^2+c^2n^2):$
 $n^2l^2(l^2m^2+m^2n^2-n^2l^2)/(a^2l^2-b^2m^2+c^2n^2):$
 $l^2m^2(-l^2m^2+m^2n^2+n^2l^2)/(a^2l^2+b^2m^2-c^2n^2)).$
I am sure, you will find incidences of the two points with
coordinates.
Best regards Eckart

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Message: #696
Date: 28/9/2014 10:03:44
From: bernard.keizer
Subject: QG-conics wrt a perspective row on QG-L2

Dear Eckart,
This is an interesting generalisation of the famous S-points!
Best regards
Bernard

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Message: #697
Date: 28/9/2014 10:24:58
From: bernard.keizer
Subject: Easy QL-Cu1 construction

Dear Eckart,
First, I didn't pay very much attention, as you already mentioned this construction in an older message (#433).
The, I looked attentively and I'm glad to have done it, as your construction gives exactly the same result as the one I gave in an old message too (#475)!
In fact, if I'm not wrong, the intersections of L and $CSC(L)$ give 2 conjugate points Y and Y' on $Cu1$; the middle of YY' is therefore X on the Newton Line. The perpendicular in X to YY' is the perpendicular bisector of YY' and the external bisector of F_1XF_2 ; it contains the center of the circle through P_1 , Y and Y' . This line cuts the second Steiner axis in a point, which is the center of another circle through F_1 , F_2 , Y and Y' (one of the 1st set of the so-called Steiner circles).
As my attached file in the message #475 has disappeared, I try to put it again.
Best regards
Bernard
PS I'm concentrating now again on the stelloïd, the hessian and the cayleyan and I hope to come back soon with new developments
...

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Message: #698
Date: 28/9/2014 8:25:20
From: eckart_schmidt@t-online.de
Subject: Easy QL-Cu1-construction

Dear Bernard K.,
you are right, my constructions in #433 and #694 use the same
lines L through points X on QL-L1.
But it is not evident for me, that the angle bisectors of
F1.X.F2 and QL-P1.X.CSC(X) are the same line.
For my last construction you don't need the fixed points of CSC.
Best regards Eckart
PS: It seems, that you can't mean #475.

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Message: #699
Date: 28/9/2014 9:19:26
From: chris.vantienhoven
Subject: QG-circles coinciding for QA/QL

Dear Eckart,
[ES] QFG#695
Let Cix be the circumcircle of QG-2P2 and QG-Px: The three QA /
QL-versions of Cix have a common point .
For QG-P12 and a QA in a point with DT-coordinates
$$\begin{aligned} & (p^2(-p^2+q^2+r^2)/(-a^2q^2r^2+b^2r^2p^2+c^2p^2q^2)) : \\ & q^2(p^2-q^2+r^2)/(a^2q^2r^2-b^2r^2p^2+c^2p^2q^2) : \\ & r^2(p^2+q^2-r^2)/(a^2q^2r^2+b^2r^2p^2-c^2p^2q^2), \end{aligned}$$

The circumcircles of QG-2P3 and QG-P13 for a QL have also a
common point with DT-coordinates
$$\begin{aligned} & (m^2n^2(1^2m^2-m^2n^2+n^2l^2)/(-a^2l^2+b^2m^2+c^2n^2)) : \\ & n^2l^2(1^2m^2+m^2n^2-n^2l^2)/(a^2l^2-b^2m^2+c^2n^2) : \\ & l^2m^2(-1^2m^2+m^2n^2+n^2l^2)/(a^2l^2+b^2m^2-c^2n^2). \end{aligned}$$

I am sure, you will find incidences of the two points with
coordinates. I studied both points. With these circumstances I
suspected (just like you) lots of (co)incidences with this new
QA-point and new QL-point. However to my utter surprise I
couldn't find any. No collinearities, no curves to be found
upon.
Even not for the Involuntary Conjugate of the QA-point or the
Clawson-Schmidt Conjugate of the QL-point.
Maybe I made mistakes/typos?
I'll keep them in mind.
Best regards, Chris

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Message: #700
Date: 28/9/2014 9:29:32
From: bernard.keizer
Subject: Easy QL-Cu1-construction

Dear Eckart,
I confirm: it's extremely simple to check that the 2 angle bisectors of F_1X_F2 and $P_1XCSC(X)$ are the same line. I haven't found a simple explanation, but there must be one!
As soon as you have a CSC transformation, you have implicitly the points F_1 and F_2 (with middle in P_1) ...
Best regards
Bernard
PS My attached file has disappeared again from the message, but you can find it in the mails ...

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Message: #701
Date: 29/9/2014 9:46:21
From: bernard.keizer
Subject: Easy QL-Cu1-construction

Dear Eckart,
I've found the explanation.
Elementary, my dear Watson would have said Sherlock Holmes!
 F_1F_2 is the angle bisector of $X'P_1X$ and $P_1X'/P_1F_2 = P_1F_2/P_1X$.
The triangles $P_1X'F_2$ and P_1F_2X are similar.
Now the triangles $X'P_1F_1$, F_1P_1X and $X'F_2X$ are also similar (the angles in P_1 and in F_2 are equal to the supplementary of $1/2$ $X'P_1X$ and $P_1X'/P_1F_1 = P_1F_1/P_1X = X'F_2/XF_2$)
It follows that the angles F_1XP_1 and $X'XF_2$ are equal.
Best regards
Bernard

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Message: #702
Date: 29/9/2014 10:03:20
From: eckart_schmidt@t-online.de
Subject: QG-circles coinciding for QA / QL

Dear Chris,
thanks for proving, here another unknown QL-point:
Let Cix be the circumcircle of the diagonal midpoints and a point QG-Px of a quadrigon.
The three QA / QL-versions have a common point ...
... for QG-P1 or QG-P5 and a QA in QA-P3,
... for QG-P1 or QG-P5 and a QL in QL-P17,
... for QG-P10 or QG-P15 and a QA in QA-P2,
... for QG-P10 or QG-P15 and a QL in QL-L6 ^ QL-L9,
... for QG-P13 and a QL in a new point X:
Geometry of point X:
... DT-coordinates in the attached Mathematica-file (Cabri controlled),
... the centers of the three QL-versions of Cix give a triangle Tr, whose circumcircle contains X,
... the Simson line of X wrt Tr is parallel QL-L1 half the distance.
Best regards Eckart

QG-circles through diagonal midpoints and QG-P13 intersect for a quadrilateral in a point with DT-coordinates:

$$\{m^2 n^2 (l^2 m^2 + w n^2) \times (a^2 (m^4 n^4 + l^4 u^2) v w + b^2 (-m^4 n^4 + l^4 u^2 + 2 m^4 n^2 v) u w + c^2 (-m^4 n^4 + l^4 u^2 - 2 m^2 n^4 w) u v), l^2 n^2 (m^2 n^2 + u l^2) \times (a^2 (-l^4 n^4 - 2 l^4 n^2 u + m^4 v^2) v w + b^2 (l^4 n^4 + m^4 v^2) u w + c^2 (-l^4 n^4 + m^4 v^2 + 2 l^2 n^4 w) u v), l^2 m^2 (l^2 n^2 + v m^2) \times (a^2 (-l^4 m^4 + 2 l^4 m^2 u + n^4 w^2) v w + b^2 (-l^4 m^4 - 2 l^2 m^4 v + n^4 w^2) u w + c^2 (l^4 m^4 + n^4 w^2) u v)\}$$
$$u := m^2 - n^2; v := n^2 - l^2; w := l^2 - m^2;$$

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Chris-2014-09-29-nb.png

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Message: #703
Date: 29/9/2014 10:51:39
From: chris.vantienhoven
Subject: QG-circles coinciding for QA / QL

Dear Eckart,
I checked the QL-point.
Again no EQF-incidences.
Best regards,
Chris

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Message: #704
Date: 30/9/2014 10:47:55
From: chris.vantienhoven
Subject: new Pentangle Point

Dear friends,

I found a nice Pentangle point using Quadrangle Geometry.

Let $P1.P2.P3.P4.P5$ be a Pentangle (system of 5 independent random points).

Let $Q1$ = Involutary Conjugate of $P1$ wrt Quadrangle $P2.P3.P4.P5$.

Let $Q2$ = Involutary Conjugate of $P2$ wrt Quadrangle $P3.P4.P5.P1$.

Let $Q3$ = Involutary Conjugate of $P3$ wrt Quadrangle $P4.P5.P1.P2$.

Let $Q4$ = Involutary Conjugate of $P4$ wrt Quadrangle $P5.P1.P2.P3$.

Let $Q5$ = Involutary Conjugate of $P5$ wrt Quadrangle $P1.P2.P3.P4$.

Next:

Let $R1$ = Involutary Conjugate of $Q1$ wrt Quadrangle $Q2.Q3.Q4.Q5$.

Let $R2$ = Involutary Conjugate of $Q2$ wrt Quadrangle $Q3.Q4.Q5.Q1$.

Let $R3$ = Involutary Conjugate of $Q3$ wrt Quadrangle $Q4.Q5.Q1.Q2$.

Let $R4$ = Involutary Conjugate of $Q4$ wrt Quadrangle $Q5.Q1.Q2.Q3$.

Let $R5$ = Involutary Conjugate of $Q5$ wrt Quadrangle $Q1.Q2.Q3.Q4$.

Now Pentangle $P1.P2.P3.P4.P5$ is point perspective with

$R1.R2.R3.R4.R5$.

I proved this algebraically and the perspector $5P-Px$ is a regular Pentangle Center.

When $P1=(1:0:0)$, $P2=(0:1:0)$, $P3=(0:0:1)$, $P4=(p:q:r)$, $P5=(x:y:z)$, the 1st barycentric coordinate of $5P-Px$ is:

$$\begin{aligned} &((-r y + q z) (p^2 q^2 r^6 x^6 y^4 - p^3 q r^6 x^5 y^5 + 2 q^5 r^5 x^8 y z - 5 p q^4 r^5 x^7 y^2 z - 2 p^2 q^3 r^5 x^6 y^3 z + \\ &5 p^3 q^2 r^5 x^5 y^4 z + 2 p^4 q r^5 x^4 y^5 z - p^5 r^5 x^3 y^6 z - 5 p q^5 r^4 x^7 y z^2 + 15 p^2 q^4 r^4 x^6 y^2 z^2 - 20 p^4 q^2 r^4 x^4 y^4 z^2 + 5 p^5 q r^4 x^3 y^5 z^2 + p^6 r^4 x^2 y^6 z^2 - \\ &2 p^2 q^5 r^3 x^6 y z^3 + 10 p^4 q^3 r^3 x^4 y^3 z^3 - 2 p^6 q r^3 x^2 y^5 z^3 + p^2 q^6 r^2 x^6 z^4 + 5 p^3 q^5 r^2 x^5 y z^4 - 20 p^4 q^4 r^2 x^4 y^2 z^4 + 15 p^6 q^2 r^2 x^2 y^4 z^4 - \\ &5 p^7 q r^2 x y^5 z^4 - p^3 q^6 r x^5 z^5 + 2 p^4 q^5 r x^4 y z^5 + 5 p^5 q^4 r x^3 y^2 z^5 - 2 p^6 q^3 r x^2 y^3 z^5 - \\ &5 p^7 q^2 r x y^4 z^5 + 2 p^8 q r y^5 z^5 - p^5 q^5 x^3 y z^6 + p^6 q^4 x^2 y^2 z^6 : :) \end{aligned}$$

Best regards,

Chris van Tienhoven

for explanation Involutary Conjugate see:

<http://www.chrisvantienhoven.nl/other-quadrangle-objects/15-mathematics/quadrangle-objects%20/artikelen-qa/94-qa-tf2.html>

Message: #705
Date: 30/9/2014 12:03:25
From: eckart_schmidt@t-online.de
Subject: QG-circles coinciding for QA / QL

Dear Chris,
the two new points in message #695 can be described as follows:
... The QA-point (reference triangle QA-DT) is the image of the anticomplement of the isogonal conjugate of QA-P16 wrt an isoconjugation, that swaps QA-P1 and QA-P10.
... The QL-point (reference triangle QL-DT) is the image of the anticomplement of the isogonal conjugate of QL-P13 wrt an isoconjugation, that swaps QL-P13 and its anticomplement.
Best regards Eckart

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Message: #706
Date: 30/9/2014 5:19:44
From: chris.vantienhoven
Subject: QG-circles coinciding for QA / QL

Dear Eckart,

Can you explain this Isoconjugation?

Here are some remarks:

* Denote XX = the anticomplement of the isogonal conjugate of QA-P16.

Knowing that QA-P16 = the Complement of the Isotomic Conjugate of the AntiComplement of QA-P1, we find

XX = the Anticomplement of the Isogonal Conjugate of the Complement of the Isotomic Conjugate of the AntiComplement of QA-P1.

* Since an Isoconjugation also represents a projective transformation we also can say:

QA-Px = (XX , QA-P10) Projective Transformation of QA-P1.

* Since QA-P10 = the Involutary Conjugate of QA-P16 we can say:

QA-Px = (the Anticomplement of the Isogonal Conjugate of the Complement of the Isotomic Conjugate of the AntiComplement of QA-P1, the Involutary Conjugate of the Complement of the Isotomic Conjugate of the AntiComplement of QA-P1) Projective Transformation of QA-P1.

And so it all depends on QA-P1.

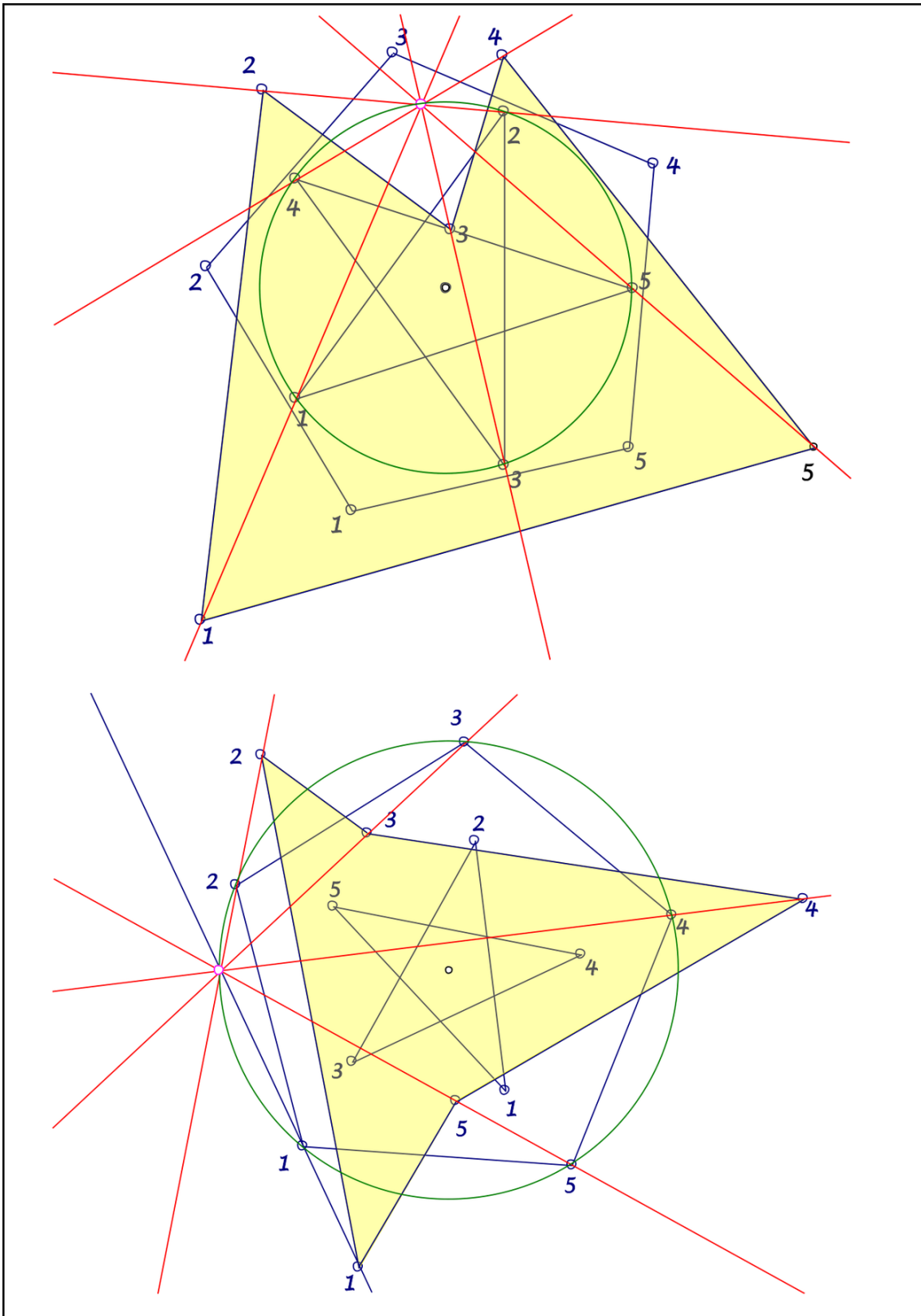
If there was no Involutary Conjugate involved it almost was a triangle construction.

Best regards,
Chris

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Message: #707
Date: 30/9/2014 9:20:47
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear Chris,
a new window is opened: looking for 5-gons! I remember my studies at the university (about 50 years ago):
Every 5-gon $P_1P_2P_3P_4P_5$ is unique the analytic sum of a pentagram and a pentagon (all with the same centroid).
Drawing this with Cabri (see attached file), I just found an interesting point, new for me.
Let P_i be the vertices of the 5-gon, U_i be the vertices of the pentagon and V_i the vertices of the pentagram.
If the pentagram and the pentagon have the same circuit, the lines P_iV_i coincide on the circumcircle of the pentagram.
If the pentagram and the pentagon have contrary circuit, the lines P_iU_i coincide on the circumcircle of the pentagon.
The angle between neighboring lines is 36° .
Only a first feedback, perhaps evident.
Best regards Eckart



Chris-2014-09-30.pdf

Message: #708
Date: 02/10/2014 10:50:52
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear friends,
please ignore my message #707, for there is a serious mistake:
The right theorem is:
Every 5-gon $P_1P_2P_3P_4P_5$ is unique the analytic sum of an AFFIN
REGULAR pentagram and an AFFIN REGULAR pentagon (all with the
same centroid).
So the 5-gons in the attached file are special 5-gons and the
properties not valid in general!
Excuse my overhaste!
Best regards Eckart

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Message: #709
Date: 03/10/2014 4:21:17
From: seiichikiri
Subject: new Pentangle Point

Dear Chris,
It is very good to have a pentangle point or pentalateral one
in EQF!
Does EQF already include the case of 5 lines in Morley's
Extensions of Clifford's Chain-Theorem?
Given 5 lines and their 5 Miquel points, the latters are on a
circle. I think that the center of the circle is a pentalateral
point.
Sorry if my old memory would be wrong.
Best regards,
Seiichi.

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Message: #710
Date: 03/10/2014 9:53:20
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Seiichi, dear friends,
Your memory is serving you well Seiichi!
A very good article on this can be found at:
<http://www.cut-the-knot.org/triangle/Morley/CenterCircle.shtml>
I try to summarize it for a system of 5 random lines: a 5-Line.
A 5-Line contains 5 Quadrilaterals also called 4-Lines (by Morley).
* So a 5-Line has 5 QL-P1: Miquel Points.
They are concentric and their center is 5L-P1-Clifford's Center.
* So a 5-Line has 5 QL-P4: Miquel Circumcenters.
They are concentric and their center is 5L-P2-Kantor's Center.
* So a 5-Line has 5 QL-Ci3: Miquel Circles.
They have one common point, I call here 5L-P3-Miquel Point.
Special is that 5L-P1, 5L-P2 and 5L-P3 are collinear. The 1st collinearity in a Pentangle

Then there are other points/centers to be mentioned:
* 5L-P4-Center of Inscribed Conic* of the 5-Line
* 5L-P5-LSD-point of the 5-Line (Least Squared Distances Point)
* 5L-P6-Centroid (can be generated like QL-P12.
* 5L-P7-X186-Circle Point: the X186-circles of the 4-Lines (described at QL-P28: Circumcenter QL-X(186)-Quadrangle in the 5L-figure have one point in common. (found by Seiichi Kirikami, June 16, 2013).
* 5L-P8-X265-Circle Point: the X265-circles of the 4-Lines: (described at QL-P29: Circumcenter QL-X(265)-Quadrangle in the 5L-figure have one point in common. (found by Seiichi Kirikami, June 16, 2013).

Further all QL-points generate a new point in a 5-Line: the Center of the Conic through the 5 versions of the QL-point in the 5-Line.
It looks like we have a good start for a Pentangle Encyclopedia! Questions arise like:

- * Which system is best for describing coordinates?
- * Is there a corresponding transformation like the QL-Tf1: Clawson-Schmidt Conjugate?

Any suggestions are welcome.
Best regards, Chris

Message: #711
Date: 03/10/2014 12:58:47
From: seiichikiri
Subject: new Pentangle Point

Dear Chris,
Thank you very much for your appreciation.
I have found a pentangle point, which perhaps does not coincide with any of your mentions.
I am struggling for the computation of its coordinates.
It seems to me that the computability of a pentangle or pentalateral point is generally worse.
As for your questions, it is very difficult to answer.
I think that a pentagon is a very interesting figure, because $P_1P_2P_3P_4P_5$ is a pentagon and $P_1P_3P_5P_2P_4$ is also a pentagon. (two fold figure?)
Best regards,
Seiichi.

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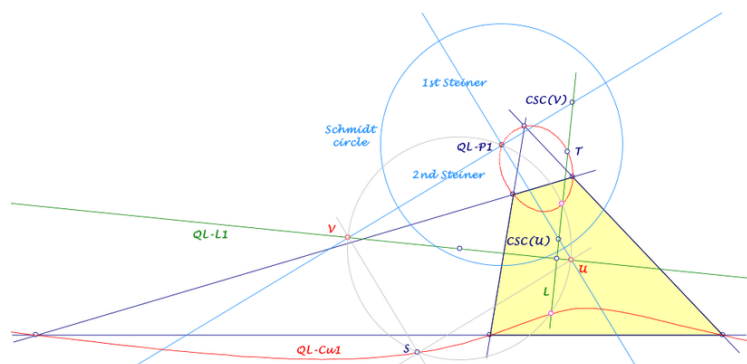
Message: #712
Date: 04/10/2014 9:37:19
From: eckart_schmidt@t-online.de
Subject: Steiner Axes, Newton Line and QL-Cu1

Dear Bernard K.,
if you are not engaged in 5-Lines, here are further properties wrt QL-Cu1. Perhaps some new aspects wrt the intersections of the Steiner axes and the Newton line.
Best regards Eckart
PS. Thanks for the synthetic justification in #701.

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Steiner Axes, Newton Line and QL-Cu1

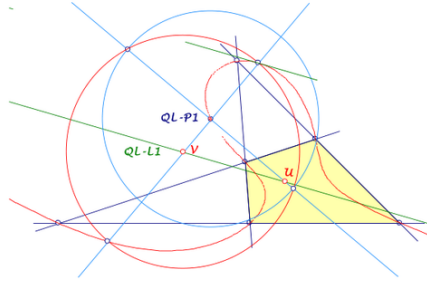
The intersections of the Steiner axes and the Newton Line $QL-L1$ are tested wrt the QL -Quasi Isogonal Cubic $QL-Cu1$.



Preliminary remarks: The Steiner axes and the Schmidt circle are described in *EQF* wrt the transformation $QL-Tf1$ (shortened *CSC*). Some points on the cubic $QL-Cu1$ are already in *EQF*: beside $QL-P1$ the point S as intersection of $QL-Cu1$ and its asymptote and point $T = CSC(S)$ on a line L through $QL-L1 \cap QL-L6$ perpendicular $QL-L1$. The point $QL-L1 \cap QL-L6$ is the center of a QL -inscribed conic with foci F_i , which are in the unipartite case of $QL-Cu1$ the *EQF*-points $QL-2P2$.

Let U and V be the intersections of the 1st and 2nd Steiner axis with the Newton line $QL-L1$.

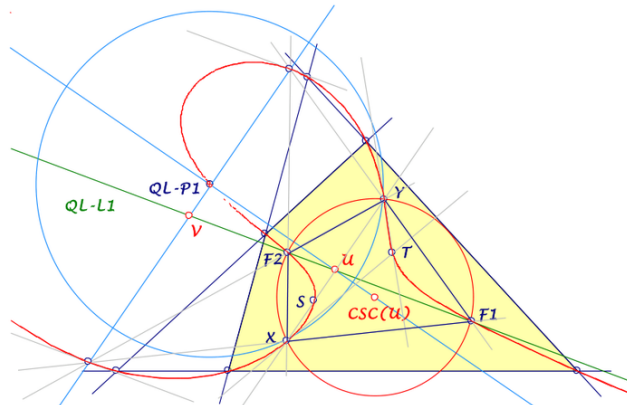
- U and V lie on a circle round $QL-L1 \cap QL-P1, QL-P4$ through $QL-P1$.
- This circle contains in the bipartite case also the foci F_i collinear with T on L .
- The 4th rectangle point of $U, QL-P1$ and V is S .
- A circle round V through the *CSC*-fixed points on the 1st Steiner axis cuts the 2nd Steiner axis on $QL-Cu1$ in points with tangents parallel to the asymptote (parallel $QL-L1$).



- The *CSC*-images of U and V are the intersections of the Steiner axes and the line L (which is the *CSC*-image of the circle above).

The unipartite case of $QL-Cu1$

- If $QL-Cu1$ is unipartite, the point U is an inner point of the Schmidt circle.

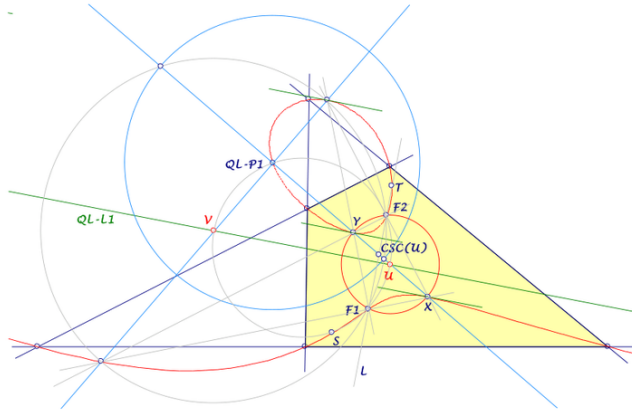


- A circle round $CSC(U)$ orthogonal wrt the Schmidt circle intersects $QL-L1$ on $QL-Cu1$ in the foci $F_i = QL-2P2$.
- This circle is *CSC*-invariant; *CSC*-partners on the circle lie collinear with U .
- This circle has two further intersections X and Y with $QL-Cu1$: these are the intersections of the line US and the Schmidt circle, *CSC*-partners with common tangential T .
- The lines, connecting F_i with X, Y intersect with the 2nd Steiner axis on $QL-Cu1$ in points with a tangent parallel to the asymptote.

- A circle round $CSC(V)$ through the CSC -fixed points $QL-2P3$ contains the points F_i . This circle is also CSC -invariant; CSC -partners on the circle lie collinear with V .

The bipartite case of $QL-P1$

- If $QL-Cu1$ is bipartite, the point U is an outer point of the Schmidt circle.
- A circle round U orthogonal wrt the Schmidt circle intersects the line L on $QL-Cu1$ in the foci F_i .
- The circle is CSC -invariant; CSC -partners on the circle lie collinear with $CSC(U)$.



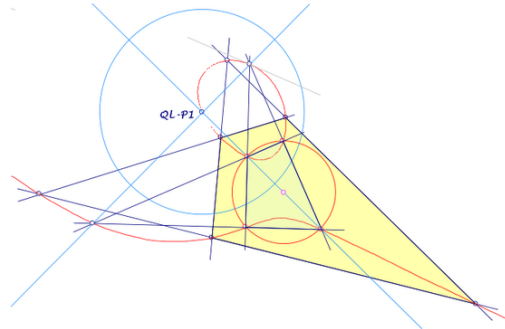
- The foci F_i are also points of the circumcircle of $U, V, QL-P1$ (see above) and have the common tangential S .
- The circle has two further intersections X and Y with $QL-Cu1$ on the 1st Steiner axis, CSC -partners with tangents parallel to the asymptote.
- The lines, connecting the foci F_i with X, Y intersect the 2nd Steiner axis on $QL-Cu1$ in CSC -partners with tangents parallel to the asymptote.
- A circle round V through the CSC -fixed points $QL-2P3$ contains the foci F_i . This circle is also CSC -invariant; CSC -partners on the circle are collinear with $CSC(V)$.

Final remark

The circle round U in the bipartite case of $QL-Cu1$ or round $CSC(U)$ in the unipartite case, which is orthogonal to the Schmidt circle, intersects $QL-Cu1$ in four points: the foci F_i and

the points X, Y (as described above). The cyclic quadrigon F_1XF_2Y and the reference quadrilateral have the same Miquel Point $QL-P1$, the same Newton Line $QL-L1$, the same Steiner axes, the same Schmidt circle and the same cubic $QL-Cu1$.
Background:

- Every circle orthogonal to the Schmidt circle and centered on the 1st Steiner axis gives two pairs of CSC-partners on $QL-Cu1$ as opposite vertices of a cyclic quadrigon. Opposite sides of this quadrigon intersect with the 2nd Steiner axis on $QL-Cu1$ in points with tangents parallel to the asymptote.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-10-04.pdf

Message: #713
Date: 04/10/2014 1:27:20
From: eckart_schmidt@t-online.de
Subject: 5L-Tfx???

Dear Chris,
my first experiences with 5-Lines:
Consider an arbitrary point X and its CSC-images wrt the 5 QL of a 5-Line. These points are concyclic with center X' , which shall be the 5L-Tfx- image of X .
... 5L-Tfx is not involutory,
... 5L-Tfx maps lines to conics,
... the 5L-Tfx-image of 5L-P1.5L-P2 is a conic, centered in 5L-P4, containing 5L-P1,
... the 5L-Tfx-image of any line contains 5L-P1.
These are only first observations with CABRI.
Best regards Eckart
PS: 5L-P3 is concyclic with the Miquel points of the QL-components.

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Message: #714
Date: 04/10/2014 2:37:04
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Seiichi,

[SK]

I am struggling for the computation of its coordinates.

There is at least one method for calculating in a Pentangle.

Denote $L1=(1:0:0)$, $L2=(0:1:0)$, $L3=(0:0:1)$, $L4=(l1,m1,n1)$,

$L5=(l2,m2,n2)$.

Any Pentangle Center should have symmetric coordinates calculating with these basics.

For example the Pentangle Centroid has 1st barycentric coordinate:

$$(m1 - n1) (m2 - n2) ((l2 - m2) (l1^2 + 3 m1 n1 - 2 l1 (m1 + n1)) (l2 - n2) (-m2 n1 + l2 (-m1 + n1) + l1 (m2 - n2) + m1 n2) + (l1 - m1) (l1 - n1) (m2 n2 (-3 m2 n1 + 2 l1 (m2 - n2) + 3 m1 n2) + l2^2 (-n1 (2 m2 + n2) + m1 (m2 + 2 n2)) + l2 (2 m2^2 n1 + 4 m2 (-m1 + n1) n2 - 2 m1 n2^2 + l1 (-m2^2 + n2^2)))).$$

The other coordinates can be cyclically derived.

However the expression still looks a bit obscure.

When we denote $L1=(1:0:0)$, $L2=(0:1:0)$, $L3=(0:0:1)$, $L4=(l,m,n)$,

$L5=(L,M,N)$, then the expression looks like this:

$$(m - n) (M - N) ((L - M) (l^2 + 3 m n - 2 l (m + n)) (L - N) (-M n + L (-m + n) + l (M - N) + m N) + (l - m) (l - n) (M N (-3 M n + 2 l (M - N) + 3 m N) + L^2 (-n (2 M + N) + m (M + 2 N)) + L (2 M^2 n + 4 M (-m + n) N - 2 m N^2 + l (-M^2 + N^2)))).$$

As you can see in this notation it becomes much clearer which variables belong to $L4$ and which belong to $L5$ as well as which coordinates are corresponding l -/ m -/ n -coordinates.

So I tend to use this barycentric notation for pragmatic reasons.

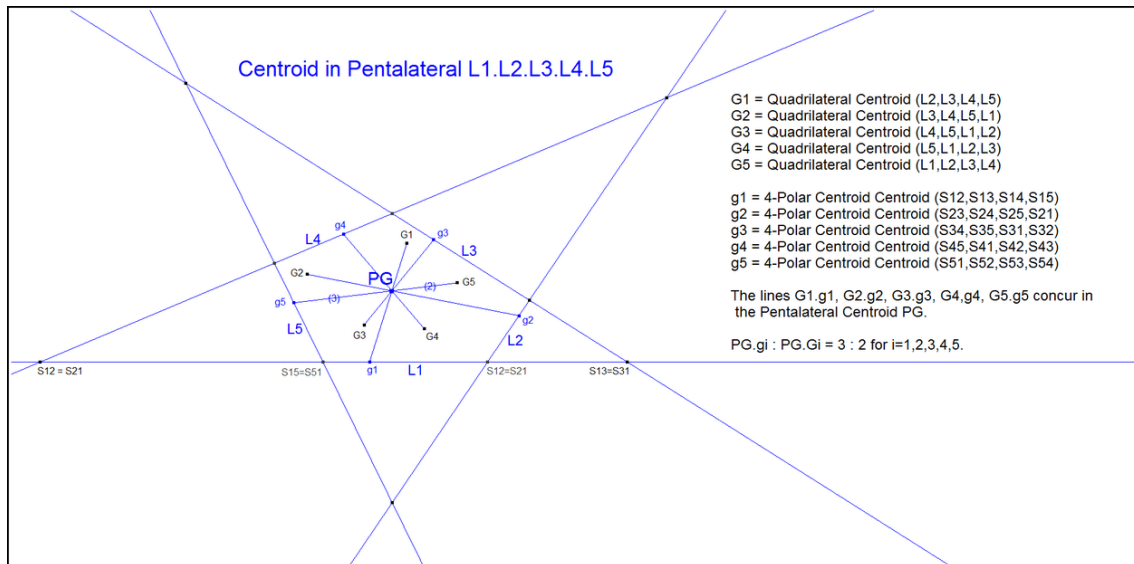
When we work with 6-Lines we have to make up new tricks.

Attached I show how the Pentangle Centroid can be constructed.

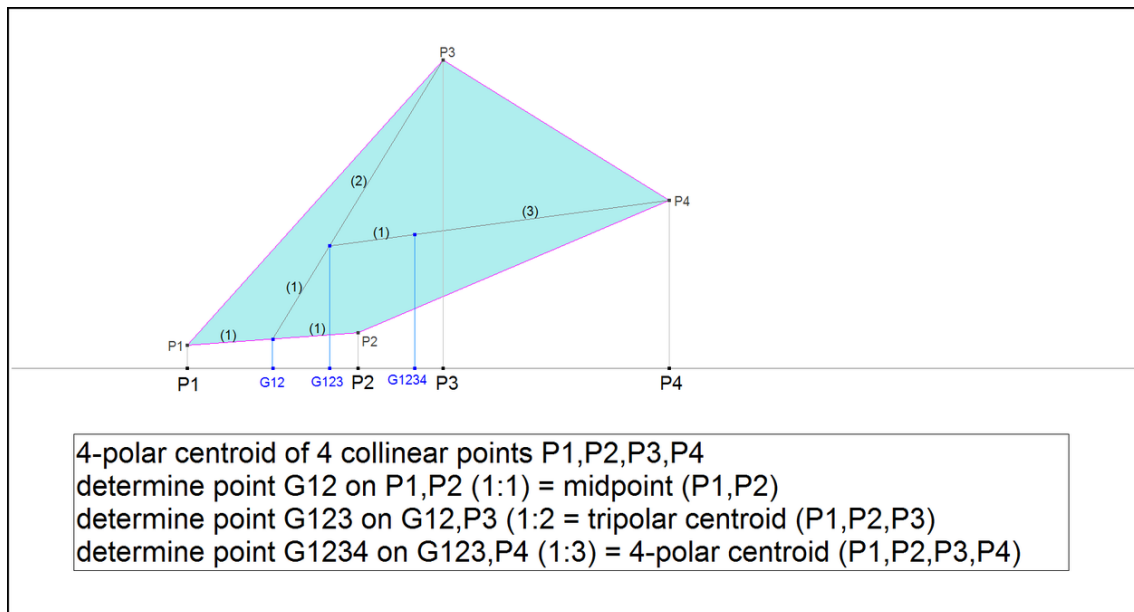
As a matter of fact there is also a kind of "Clifford chain" to construct the centroid in an n -Line using the results from the $(n-1)$ -Line.

So far this time.

Chris



5L-P4-Centroid-02.png



4P-4-polar Centroid-01.png

Message: #715
Date: 04/10/2014 4:24:10
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Eckart, Seiichi and all,

Construct line QL-L1 (The Newton Line) in each 4-Line of the Reference 5-Line.

They concur in one point.

I call this point the 5L-Newton Point.

Construct line QL-L4 (Morley Line) in each 4-Line of the Reference 5-Line.

They form the 1st generation Morley 5-Line.

Do the same process in this 1st generation Morley 5-Line.

This leads to the 2nd generation Morley 5-Line consisting of 5 concurrent lines.

This point is also the point QL-P3 (Kantor-Hervey Point) of all sets of 4 lines of the 1st generation Morley 5-Line.

That's why I think 5L-Kantor-Hervey Point will be a fine name for this point.

It doesn't look like there are any collinearities related to earlier described points in the 5-Line.

Best regards,
Chris

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Message: #716
Date: 04/10/2014 5:53:58
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris, dear Eckart, dear Seiichi, dear fellows,

It seems you open a new battlefield!

I give you one first contribution.

For references, page 51 of my article on the web, 4 articles by Morley (reference 9 on page 55), 4 by Bogomolny (Morley miracle and followings), one by Wagner (reference 20) and one by Goormaghtigh (reference 22).

The inversive geometry is a heavy peace of cake, but I hope other will read the same articles and help me to understand those beautiful properties.

The 5 Miquel circles of the QL's have a commun point, which is the node of the limaçon tangent to these circles (as this point doesn't generally lie on the Kantor circle, in a difference to P1 for the QL, that's why the limaçon becomes a cardioïd for the QL): we shall call it, as Morley, the node of the 5-L (or the focus like P1 for the QL).

The point constructed as the orthocenter for the triangle or P4 for the QL (sum of the vectors with origin in 5L-P2 to the 5 points P4) is called 1st orthocenter of 5-L: it's the generalization of the orthocenter or point QL-P4. The distances from these orthocenter to the 5 copples of points P3 and P4 are in the same ratio, equal to the ratio of the distance between 5L-P2 and each point P4 (radius of the Kantor circle) and the distance between 5L-P2 and the node of the 5L.

Last, but not least, the 5 perpendiculars drawn from the points P3 to the 5th line are tangent to a new deltoïd.

Near the centric circle (QL Miquel circle), Morley defines a second circle with radius $1/n-1$ radius of the centric circle, id est $1/2$ for the triangle and $1/3$ for the QL.

The perpendiculars drawn from the centers of these circles of the $n-1$ lines to the omitted line of the n -line concur in the 2nd orthocenter of the n -line, which is the external center of homothety of the 1st and the 2nd circles of the n -line.

Practically, for the triangle, the 2nd circle is the Euler circle (or Feuerbach circle) and the 1st and 2nd orthocenters coincide in the orthocenter of the triangle. For the QL, the perpendicular lines drawn from the Euler circumcenters concur in the point QL-P2, which is the 2nd orthocenter of the QL. The center of the 2nd circle for the QL is therefore at the third on the segment P2P4 of the QL. If I'm not wrong in my interpretation, the 5 perpendiculars drawn from these 5 points of the QL's to the 5th line should concur in the 2nd orthocenter of the 5-L.

2)Curves

For the QL, we had 16 in- or excenters on 8 Steiner circles, 1 monocardioid envelopping the 4 circumcircles and 27 multicardioids tangent to the 4 lines.

For the 5-L, the $5 \cdot 8 = 40$ Steiner circles are envelopped by 16 cardioids (each cardioid touching 5 circles and each circle touching 2 cardioids), the 5 monocardioids are envelopped by a nephroid, each cardioid passing through the 2 cusps of the nephroid and the $5 \cdot 27 = 135$ centers of the multicardioids are on 81 circles (5 centers on each circle and each center on 3 circles).

Dear Chris, did you try to make a picture of these 5-L?

Best regards

Bernard

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Message: #717

Date: 04/10/2014 6:00:54

From: bernard.keizer

Subject: new Pentangle Point

Dear Chris,

The point where the Newton Lines concur is the center of the unique conic tangent to the 5 lines.

The same way, the 5 QL-Cu1 pass through 2 points which are the foci of this conic.

Best regards

Bernard

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Message: #718
Date: 04/10/2014 7:13:32
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Bernard,
Nice reading your study and point of view!
Yes I think we made new relevant steps into this new world.
I think we have grown into it since we disclosed the
Quadrilateral- /Quadrangle- /Quadrigon-domain.
Now we can use the same principles for Pentalaterals, Pentangles
and Pentagons.
And it is inevitable that we also have some bycatch according to
the principles described as Clifford's chain and Morley's
general description of n-Lines.
Upon closer inspection it is evident that the 5 Newton Lines in
a 5-Line are concurrent in the Inscribed Conic Center of a
5-Line.
After all Newton's line is called after Newton because he
revealed that the centers of all conics inscribed in the 4 lines
of a Quadrilateral/4-Line lie on one line, called by us the
Newton Line.
Since there is only one Inscribed conic in a 5-Line there is
only one Conic Center
and the Newton Lines of all 5 Component Quadrilaterals of a
5-Line should pass through it.
Some time ago it was quite a job for me to make a picture of all
27 Cardioids in a Quadrilateral.
I do not see myself making a picture of even more corresponding
figures in a Pentalateral right now.
But you never know in the future!
Best regards,
Chris

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Message: #719
Date: 05/10/2014 10:29:48
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris,

Thank you for your nice comments!

1) In my message, I forgot to precise that the node of the limaçon or common point of the 5 Miquel circles of the QL's is the point 5L-P3 of your message #710

2) It should be possible to check if the point mentionned in the construction with the lines QL-L4 is the 1st orthocenter of the 5L (a sum of 5 vectors with origin in 5L-P2). In this case, the name of Kantor-Hervey would be fully appropriate!

For the figure, I didn't mean of couse the 135 cardioïds, but at least some relevant points like the 5L-P1, P2 , P3, P4, P6 and perhaps the 5 parabolas and cardioïds (as I suppose you don't know how to determinate P5).

I guess the 2 common points of the 5 cardioïds belong to a circle (Clifford?)

Best regards

Bernard

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Message: #720
Date: 05/10/2014 10:34:03
From: chris.vantienhoven
Subject: new Pentangle Point

Dear friends,

Here is a better expression of the 1st barycentric coordinate of the Centroid of a Pentalateral (5-Line) when $L1=(1:0:0)$, $L2=(0:1:0)$, $L3=(0:0:1)$, $L4=(l:m:n)$, $L5=(L:M:N)$. It shows that coefficients $(l:m:n)$ and $(L:M:N)$ can be interchanged preserving the equation.

$$\begin{aligned} & (m - n) (M - N) \\ & ((L - M) (L - N) \\ & (-L m + l M + L n - M n - l N + m N) (l^2 - 2 l m - 2 l n + 3 m n) \\ & + (l - m) (l - n) \\ & (-L m + l M + L n - M n - l N + m N) (L^2 - 2 L M - 2 L N + 3 M N) \\ & + (l - m) (l - n) \\ & (L m - l M - L n + l N) (L - M) (L - N)) \end{aligned}$$

Best regards,
Chris

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Message: #721
Date: 06/10/2014 6:35:41
From: Seiichi Kirikami
Subject: 2 would-be pentangle points

Dear friends,

(1) Given a pentangle $P_1P_2P_3P_4P_5$, we denote the intersection of P_iP_{i+2} and $P_{i-1}P_{i+1}$ by Q_{ii+1} . We denote the circumcircle of $P_iQ_{ii+1}P_{i+1}$ by C_{ii+1} . We denote the second intersection of C_{i-1i} and C_{ii+1} by R_i . Then P_iR_i concur in a point P .

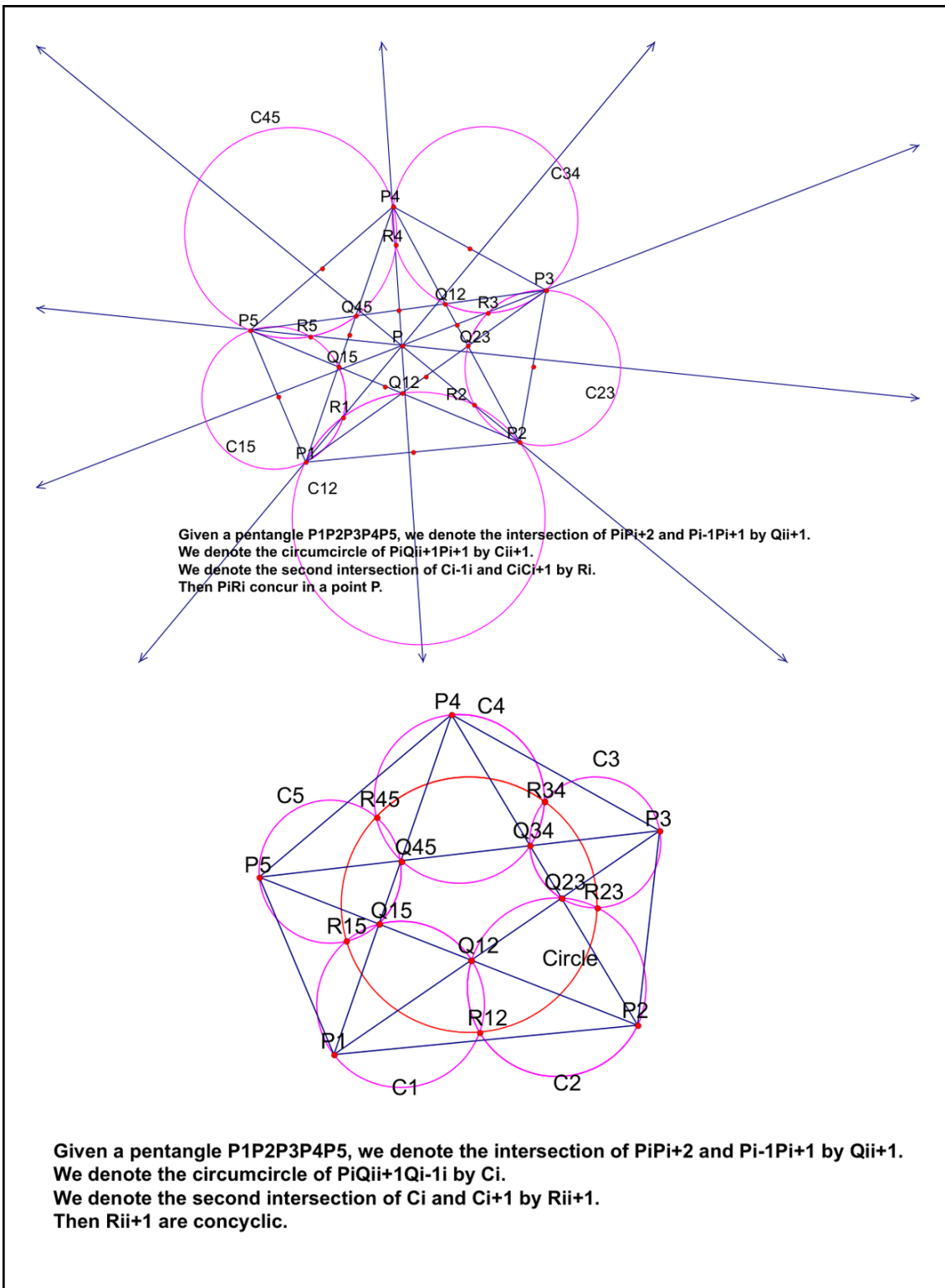
(2) Given a pentangle $P_1P_2P_3P_4P_5$, we denote the intersection of P_iP_{i+2} and $P_{i-1}P_{i+1}$ by Q_{ii+1} . We denote the circumcircle of $P_iQ_{ii+1}Q_{i-1i}$ by C_i . We denote the second intersection of C_i and C_{i+1} by R_{ii+1} . Then R_{ii+1} are concyclic.

(This is the same as the concyclicity of 5 Miquel points?)

See the attached file.

I am sorry that I could not produce their coordinates for too much time of the computation.

Best regards,
Seiichi.



PentanglePoints.docx

Message: #722
Date: 06/10/2014 11:53:51
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear Chris,

before holiday some remarks:

- ... 5L-P3 is a point on the QL-P1-circle, inversion of 5L-P1 wrt the QL-P4-circle,
- ... 5L-P7 is a point of the QL-P4 circle,

Additional properties to my curious transformation in #713:

- ... 5L-Tfx swaps the foci of the inscribed conic,
- ... the image of 5L-P1 is the reflection in 5L-P4,
- ... the image of the QL-P1-circle is the line at infinity,
- ... the image of a QL-P1-point is the point at infinity of a perpendicular line to the corresponding QL-line,
- ... the image of the QL-P4-circle is a hyperbola,
- ... the image of the main axis of the inscribed conic is a conic through the foci and 5L-P1, symmetric to the second axis.

But what about the inverse of 5L-Tfx?

I hope, there are no false CABRI-observations.

Best regards Eckart

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Message: #723
Date: 06/10/2014 11:34:45
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Bernard,

[BK]

> It should be possible to check if the point mentioned in the
> construction with the lines QL-L4 is the 1st orthocenter
> of the 5L.

According to Morley's document "On the plane L-line" the perpendiculars from the centers of the Quadri-Deltoids (QL-P3) to the omitted line of the 5-Line will be concurrent in "the first orthocenter of the 5-line".

I verified that the perpendiculars from the 5 versions of QL-P3 on resp. L1, L2, L3, L4, L5 concur indeed in one point. This point is different from the points mentioned yet. So we have another point 5L-P10.

I did not see any collinearities with the other points.

The Kantor-Hervey point I described earlier will be 5L-P9.

[BK]

> For the figure, I didn't mean of course the 135 cardioids, but
> at least some relevant points like the 5L-P1, P2, P3, P4, P6
> and perhaps the 5 parabolas and cardioids
> (as I suppose you don't know how to determinate P5).

Well I was afraid you wanted me to draw 135 cardioids in a single picture. About the other points I already started with a description.

Some years ago I already told Eckart that I also wanted to make a description of points, lines and curves related to n-gons. So now I started fulfilling an old wish. I am glad that we can do it together!

I think I will incorporate these items in an "Encyclopedia of Polygon Figures". Of course this is long term project that will take its time.

[BK]

> (as I suppose you don't know how to determinate P5)
> 5L-P5 is the Least Squared Distance Point.

I already constructed it according to the method of

[25] J.L. Coolidge, Harvard University,

Two geometrical applications of the method of least squares , available at:

JSTOR: The American Mathematical Monthly, Vol. 20, No. 6 (Jun., 1913), pp. 187-190 (<http://www.jstor.org/stable/2973072>)

Journal Description The Monthly publishes articles, as well as notes and other features, about mathematics and the profession.

See also QL-P26: Least Squares Point.

Further:

I give these points a name and code for the present, so that we can talk about them easy in this preliminary stage.

When I publish the new "Encyclopedia of Polygon Figures" it is conceivable that I renumber them.

Especially taking into account that there will be many corresponding points in n-Lines etc. with $n > 5$. I want these points first as well as the most important points. After this start new items will be numbered in order of entry.

Best regards,

Chris

p.s.

I mention here Morley's statements about a 5-Line, found in Morley: Extensions of Clifford's Chain Theorem, page 470:

"that is the 320 axes of the component 4-lines meet in the 256 incenters,

there being 5 axes on a point and 4 points on an axis.

The circles arise from the incenters of the preceding column.

That is the 135 incenters are on the 81 circles,

there being 5 points on each circle, 3 circles on each point.

These 3 circles cut at the angle $\pi/3$.

The 16 cardioids arise from the 40 circles;

The circles are osculants of the cardioids,

each C3 having 4 osculant C2's; and each C2 osculating 1 C3's."

Note: osculants are circles inscribed in/touching a cardioid.

For a better understanding I am afraid one should read the whole paper, which is absolutely worth while.

A must for those interested in n-Lines (figure consisting of n random lines).

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Message: #724
Date: 07/10/2014 12:41:47
From: chris.vantienhoven
Subject: 2 would-be pentangle points

Dear Seiichi.

[SK]

(1) Given a pentangle $P_1P_2P_3P_4P_5$,

(2) Given a pentangle $P_1P_2P_3P_4P_5$,

It looks like these points are 5-Gon points, because in the construction points are not chosen random but in order. So I suppose they are 5-Gon points. Maybe even a bicentric pair. I will make a note of them.

Best regards,

Chris

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Message: #725
Date: 07/10/2014 12:46:17
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Eckart,

[ES]

Additional properties to my curios transformation in #713:

This 5L-Tfx looks very promising.

I am curious for your further analyses and description.

Have a nice holiday.

Best regards,

Chris

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Message: #726
Date: 07/10/2014 2:21:04
From: Seiichi Kirikami
Subject: 2 would-be pentangle points

Dear Chris,
I see. Attached is the concurrent point of 5 lines through the midpoints.

(3) Given a pentagon $P_1P_2P_3P_4P_5$, we denote the midpoints of P_iP_{i+1} by M_{i+3} . we denote the intersection of P_iP_{i+2} and $P_{i-1}P_{i+1}$ by Q_{i+3} . We denote the midpoints of Q_iQ_{i+2} by m_i . The lines $M_i m_i$ concur in a point P .

(1), (2) and (3) are collinear in drawing.

(2) is called "Miquel Five Circles Theorem" in Mathworld.

Best regards,
Seiichi.

On Tuesday, October 7, 2014 7:41 AM, "van10hoven@gmail.com [Quadri-Figures-Group]" wrote:

Dear Seiichi.

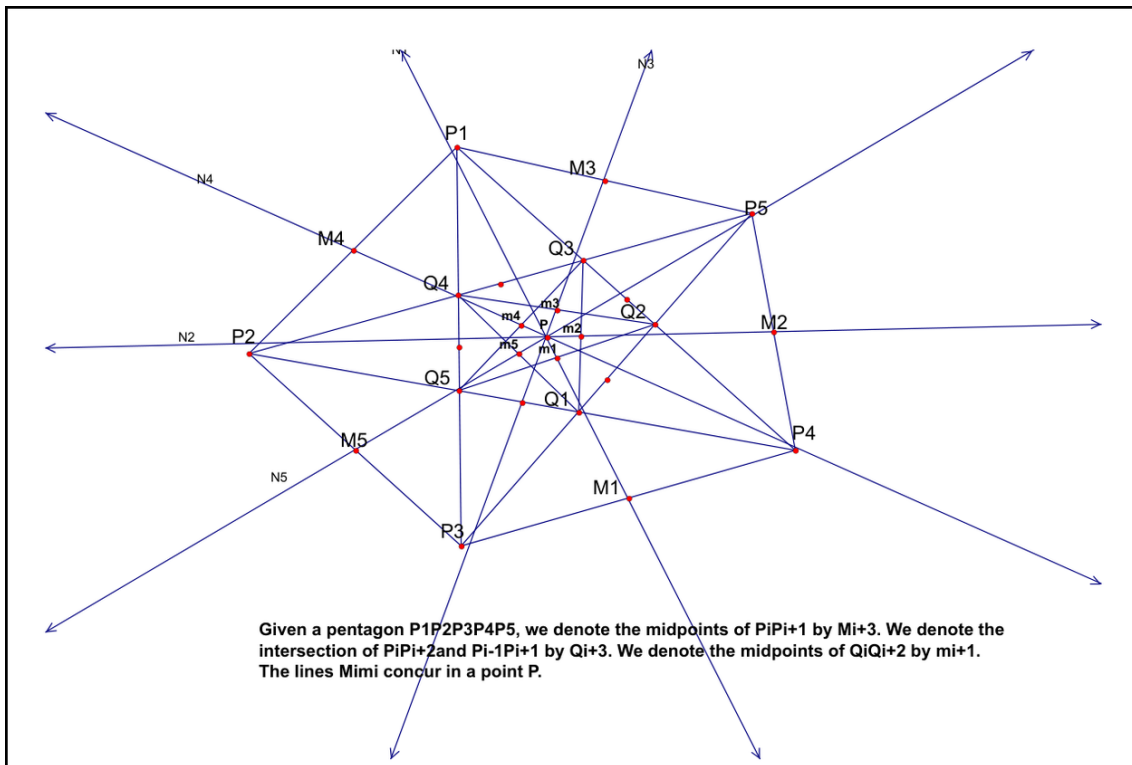
[SK]

(1) Given a pentangle $P_1P_2P_3P_4P_5$,

(2) Given a pentangle $P_1P_2P_3P_4P_5$,

It looks like these points are 5-Gon points, because in the construction points are not chosen random but in order. So I suppose they are 5-Gon points. Maybe even a bicentric pair. I will make a note of them.

Best regards,
Chris



5NewtonLines.docx

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Message: #727

Date: 07/10/2014 7:03:29

From: chris.vantienhoven

Subject: 2 would-be pentangle points

Dear Seiichi,

Very nice.

Do you think these 5G-points (1), (2) and (3) also are applicable in 7-Gons, 9-Gons, etc.?

What about corresponding features in even Gons: 6-Gons, 8-Gons, etc?

Best regards,

Chris

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Message: #728

Date: 07/10/2014 9:43:37

From: bernard.keizer

Subject: Steiner Axes, Newton Line and QL-Cu1

Dear Eckart,

I am now unfortunately already engaged in the 5-Line!

Thanks for your summary about theses properties.

You could add a few details more, for example:

S is the common tangentiel of P1 and it's conjugate the infinity point; SP1 is tangent to Cu-1 and P4 is on SP1; the other tangent in S is the asymptote of Cu-1, parallel to the Newton Line and tangent to the deltoïd.

T is also on the parallel line to the Newton Line through P1 (as conjugate of S, it's the 3rd intersection of the line through P1 and it's conjugate the infinity point)

any line through P1 cuts Cu-1 in 2 points and the middle of these points lies on the Newton Line

any line through S cuts Cu-1 in 2 points, which are equidistant from P1.

The 2 points with tangent parallel to the Newton Line are CSC-conjugates, their middle is the point V and they are the 2 centers of inversion of the anallagmatic curve Cu-1.

It seems there is never an end to all these beautiful properties!

I wish you a nice holiday

Best regards

Bernard

PS I hope I will find time to finalise something new about the cayleyan for your come-back.

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Message: #729
Date: 07/10/2014 9:53:48
From: bernard.keizer
Subject: new Pentangle Point

Dear Eckart,
I've found the 1st property very interesting (and totally new for me).
5L-P3 can generally not be on the QL-P4 circle; it's the node of a limaçon envelopping the 5 Miquel circles.
(The limaçon is a curve studied by Pascal, mathematician and father of Blaise Pascal).
For the QL, P1 lies on the Miquel circle and the limaçon envelopping the 4 circles becomes a cardioïd, the node and the focus being the same point or cusp.
Best regards
Bernard

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Message: #730
Date: 08/10/2014 3:00:33
From: seiichikiri
Subject: The pentangle point corresponding to QA-P1

Dear friends,
Given a pentangle $P_1P_2P_3P_4P_5$ and a point P , the point which minimizes $PP_1^2+PP_2^2+PP_3^2+PP_4^2+PP_5^2$ has the following coordinates.
 $Barycentrics=\{(v+w)(2p+q+r)+u(3p+2(v+w)) : : \}$,
where $P_1=(0,1,0)$, $P_2=(0,0,1)$, $P_3=(1,0,0)$, $P_4=(p,q,r)$ and $P_5=(u,v,w)$.
If we denote the midpoint of P_iP_j by M_{ij} , the point which minimizes $PM_{12}^2+PM_{13}^2+PM_{14}^2+PM_{15}^2+PM_{23}^2+PM_{24}^2+PM_{25}^2+PM_{34}^2+PM_{35}^2+PM_{45}^2$ has the same coordinates.
If we denote the centroid of $P_iP_jP_k$ by G_{lm} , the point which minimizes $PG_{12}^2+PG_{13}^2+PG_{14}^2+PG_{15}^2+PG_{23}^2+PG_{24}^2+PG_{25}^2+PG_{34}^2+PG_{35}^2+PG_{45}^2$ has the same coordinates.
I think that we can easily differentiate $P_5=(u,v,w)$ from $P_4=(p,q,r)$.
Best regards,
Seiichi.

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Message: #731

Date: 08/10/2014 3:53:29

From: chris.vantienhoven

Subject: The pentangle point corresponding to QA-P1

Dear Seiichi,

You are right this point minimizing distances is also the Centroid of the Pentangle.

I found a simple "chain" for calculating centroids of consecutive n-Point figures.

1-Point: $G_1 = \text{Point } P_1$

2-Point: $G_2 = \text{Midpoint } (P_1, P_2)$

= point dividing G_1, P_2 with ratio 1 : 1

3-Point: $G_3 = \text{Centroid } (P_1, P_2, P_3)$

= point dividing G_2, P_3 with ratio 1 : 2

4-Point: $G_4 = \text{Centroid } (P_1, P_2, P_3, P_4)$

= point dividing G_3, P_4 with ratio 1 : 3

5-Point: $G_5 = \text{Centroid } (P_1, P_2, P_3, P_4, P_5)$

= point dividing G_4, P_5 with ratio 1 : 4

6-Point: $G_6 = \text{Centroid } (P_1, P_2, P_3, P_4, P_5, P_6)$

= point dividing G_5, P_6 with ratio 1 : 5

etc.

Here are some extra related 5P-Centroid points I found some years ago:

* Let G be the Centroid of 5P.

* Let $PiCe$ be the center of the Conic through $P_1.P_2.P_3.P_4.P_5$.

* Let $GiCe$ be the center of the Conic through $G_1.G_2.G_3.G_4.G_5$.
(G_i are centroids of the 5 component Quadrangles).

* Now G , $PiCe$ and $GiCe$ are collinear.

* And G divides $PiCe.GiCe$ in segments 4 : 1.

* The perimeter of the Pi -conic = 4 * perimeter of Gi -Conic.

* The area of the Pi -conic = 16 * area of Gi -Conic.

Best regards,

Chris

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Message: #732

Date: 08/10/2014 5:37:53

From: Antreas Hatzipolakis

Subject: Quadrangle and cubics passing through eight points

Every cubic curve C_1 on an algebraically closed field (http://en.wikipedia.org/wiki/Algebraically_closed_field) that passes through a given set of eight points $P_1, \dots, P_8$ also passes through a certain (fixed) ninth point P_9 , counting multiplicities.

http://en.wikipedia.org/wiki/Cayley%E2%80%93Bacharach_theorem

Now, let $ABCD$ be a quadrangle and M_1, M_2, M_3, M_4 the midpoints of AB, BC, CD, DA , resp.

Which is the point the cubics through $A, B, C, D, M_1, M_2, M_3, M_4$ are passing through?

In general, in an octangle we have a point related to cubics which pass through its vertices.

APH

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Message: #733

Date: 08/10/2014 5:50:48

From: Bernard Gibert

Subject: Quadrangle and cubics passing through eight points

>> [APH] Which is the point the cubics through $A, B, C, D, M_1, M_2, M_3, M_4$ are passing through?

$X = M_1M_3 \wedge M_2M_4$

which is totally trivial if you consider the matrix

	A	B	M ₁	
	D	C	M ₃	
	M ₄	M ₂	X	

Bernard

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Message: #734
Date: 09/10/2014 2:38:34
From: seiichikiri
Subject: The pentangle point corresponding to QA-P1

Dear Chris,
It may be that the centroid and similar centers are more important in Pentangle than in Quadrangle.
It seems to me that other centers tend to be too big to be written.
Best regards,
Seiichi.

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Message: #735
Date: 09/10/2014 5:06:10
From: seiichikiri
Subject: A question of LSD (least square distances)

Dear friends,
I have a question.
The sum of the square distances from the vertices of a quadrangle to a point P determines an ellipse.
The center of the ellipse is QA-P1.
How about its foci?
Is there any relation between its foci, QA-P2 and QA-P3?
Is there any relation between the directions of its axes and the line passing QA-P2 and QA-P3?
Best regards,
Seiichi.

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Message: #736

Date: 09/10/2014 6:05:00

From: chris.vantienhoven

Subject: Quadrangle and cubics passing through eight points

Dear Antreas, Dear Bernard,
This Cayley-Bacharach theorem
is astonishing!

It actually means that 8 random coplanar points define a unique
9th point.

In Quadri Geometry we often have 2 sets of Quadrangles.
According to the Cayley-Bacharach theorem they define a unique
point.

Is there anything known about how to construct this point
(preferably by ruler and compass) and/or how to calculate this
point algebraically in a relative easy way?

Best regards,
Chris

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Message: #737

Date: 09/10/2014 11:46:04

From: Antreas Hatzipolakis

Subject: Quadrangle and cubics passing through eight points

[Chris]:

>> This Cayley-Bacharach theorem is astonishing!
>> It actually means that 8 random coplanar points define a unique 9th point.
>> In Quadri Geometry we often have 2 sets of Quadrangles.
>> According to the Cayley-Bacharach theorem they define a unique point.
>> Is there anything known about how to construct this point (preferably by
>> ruler and compass) and/or how to calculate this point algebraically in a
>> relative easy way?
>> __,_.-.,_

Dear Chris

And how about quadrilaterals?

A possibly interesting point defined by cubics:

In a quadrilateral (a,b,c,d), a set of eight points:

(a/\b), (a/\c), (a/\d), (b/\c), (b/\d), (c/\d), Miquel_Point,
Miquel_circle_center

APH

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Message: #738
Date: 09/10/2014 11:59:56
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris,

Sorry for having delayed my answer, but I wanted to make my own figure and check some properties.

We have 3 different and fundamental points:

- 1) the 1st orthocenter, with lines from the 5 P3 to the 5th line.
- 2) the 2nd orthocenter, with lines from the 5 points situated on the segments P2P4 at the 3rd from P2 (these points are called centers of the 2nd circle of the QL) to the 5th line.
- 3) the Kantor-Hervey point as resultant of the sum of the 5 vectors with origin the centric circle of the 5-L (point 5L-P2) and with extremities the 5 points P4; it's easy to check that the distances from this point to the 5 versions of P4 and P3 are in the same ratio, equal to the ratio between the distance from 5L-P2 to 5L-P3 and the radius of the centric circle (common distance from 5L-P2 to the 5 points P4)

The 1st orthocenter exists only for n odd; for n even, there is a line of the orthocenters of the $n-1$ -line.

For $n=3$, the 3 definitions give the same point, the orthocenter being the center of similitude between the circumcircle and the Feuerbach circle (second circle of the triangle).

For $n=4$, there is no 1st orthocenter, but a line of the 4 orthocenters of the triangles (Steiner Line L2). The 2nd orthocenter is the Morley point P2 and the Kantor-Hervey point is P3.

For $n=5$, there are 3 points.

For $n=6$, there will be again 2 points and a line of the 6 1st orthocenters of the 5-lines.

and so on ...

For $n=4$, we had P12 at the 1rd of P2P6 from P6.

For $n=5$, I couldn't find the same kind of property between the 2nd orthocenter, the centroid of the 5L and the centroid of the 5 points P4.

Best regards

Bernard

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Message: #739

Date: 09/10/2014 1:29:49

From: chris.vantienhoven

Subject: The pentangle point corresponding to QA-P1

Dear Seiichi,

[SK] It may be that the centroid and similar centers are more important in Pentangle than in Quadrangle. It seems to me that other centers tend to be too big to be written.

There is at least one other 5P-point with short coordinates, the 5P-center of the circumscribed conic:

Let $P1=(1:0:0)$, $P2=(0:1:0)$, $P3=(0:0:1)$, $P4=(p:q:r)$, $P5=(P:Q:R)$.

The conic through $P1$, $P2$, $P3$, $P4$, $P5$ is:

$-(P\ q - p\ Q)\ r\ R\ x\ y + q\ Q\ (P\ r - p\ R)\ x\ z - p\ P\ (Q\ r - q\ R)\ y\ z$

The 1st coordinate of the center of the conic is:

$P\ q\ r\ (Q - R) + p\ Q\ R\ (q - r) - P\ p\ (Q\ r - q\ R)$

So now we these three 5-Point Centers;

1. 5P-P1 = 5P-Centroid
2. 5P-P2 = 5P-Center of circumscribed conic
3. 5P-P3 = 5P-Involuntary Center (see QFG#704)

What will be the next?

Best regards,
Chris

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Message: #740
Date: 09/10/2014 9:10:06
From: seiichikiri
Subject: The pentangle point corresponding to QA-P1

Dear Chris,
I proposed a question of LSD in message #735. It is for the extension of QA-P2 and P3 in Quadrangle to those in Pentangle. Instead of the LSD ellipse, it would be interesting that there would be a special or characteristic ellipse with the center QA-P1 and 2 foci QA-P2 and P3, which allows us to make the similar ones in Pentangle.
I think that 2 foci of 5-point conic are also pentangle ones.
Best regards,
Seiichi.

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Message: #741
Date: 09/10/2014 11:20:52
From: chris.vantienhoven
Subject: The pentangle point corresponding to QA-P1

Dear Seiichi,
[SK] it would be interesting that there would be a special or characteristic ellipse with the center QA-P1 and 2 foci QA-P2 and P3, which allows us to make the similar ones in Pentangle. I looked if I could find such a characteristic ellipse in a Quadrangle.
However I do not see such thing right away.
Maybe you manage.
Best regards,
Chris

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Message: #742
Date: 10/10/2014 1:37:01
From: seiichikiri
Subject: The pentangle point corresponding to QA-P1

Dear Chris,
Another idea:
In a pentangle $P_1P_2P_3P_4P_5$, there are 5 QA-P1's, P2's and P3's for quadrangles $P_1P_2P_3P_4$, $P_1P_2P_3P_5$, $P_1P_2P_4P_5$, $P_1P_3P_4P_5$ and $P_2P_3P_4P_5$. 5 QA-P1's, P2's and P3's determine their conics respectively.
Their centers are pentangle points. Are they collinear?
Best regards,
Seiichi.

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Message: #743
Date: 10/10/2014 10:49:55
From: chris.vantienhoven
Subject: The pentangle point corresponding to QA-P1

Dear Seiichi,
[SK] In a pentangle $P_1P_2P_3P_4P_5$, there are 5 QA-P1's, P2's and P3's for quadrangles $P_1P_2P_3P_4$, $P_1P_2P_3P_5$, $P_1P_2P_4P_5$, $P_1P_3P_4P_5$ and $P_2P_3P_4P_5$. 5 QA-P1's, P2's and P3's determine their conics respectively. Their centers are pentangle points. Are they collinear?
In principle all Quadrangle Centers applied in the 5 Component Quadrangles in a 5-Point system generate 5 points defining a conic which center is a 5-Point Center.
I tried to calculate the coordinates of the center of the conic defined by the 5 Euler-Poncelet Points in many ways. Thus far many Mathematica messages like:
No more memory available.
Mathematica kernel has shut down.
Try quitting other applications and then retry.
The corresponding Pentangle Centers of QA-P1, QA-P2, QA-P3 are not collinear according to my Cabri-pictures.
Best regards,
Chris

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Message: #744
Date: 11/10/2014 1:47:28
From: seiichikiri
Subject: The pentangle point corresponding to QA-P1

Dear Chris,
Thank you very much for your computation and drawing. I also meet the message " No more memory available" sometimes.
Allow me to add the following 2 cases.
[1] In a pentangle, there is a conic through its vertices. There is also a conic through the centroids of its component quadrangles. The major and minor axes of the latter determine 2 poles with respect to the former.
[2] In a pentangle, there is a conic through its vertices. The conic has its center and 2 foci. The number of the points is $5+1+2=8$. Is there another point according to Antreas's message #732 (Caley-Bacharach theorem)?
Best regards,
Seiichi.

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Message: #745
Date: 11/10/2014 8:14:49
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Bernard,
[BK]
> 1) the 1st orthocenter, with lines from the 5 P3
> to the 5th line.
> 2) the 2nd orthocenter, with lines from the 5 points
> situated on the segments P2P4 at the 3rd from P2
> (these points are called centers of the 2nd circle
> of the QL) to the 5th line.
In which documents (of Morley and/or Clifford?) did you find the definitions of the 1st orthocenter and the 2nd orthocenter?
Best regards,
Chris

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Message: #746
Date: 11/10/2014 10:31:04
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris,
the 3 points I mention are called p_1 , h and p_2 in Morley's article Orthocentric properties of the plan n -line.
Apparently, you checked yourself the construction of p_1 .
For h , it's not really clear, you have to guess this center of 2nd circle, but I checked the construction I give.
For p_2 , it is better explained in Goormaghtigh The Hervey point of the general n -line and I checked also the ratio of distances I mention.
Best regards
Bernard

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Message: #747
Date: 11/10/2014 12:55:26
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear Bernard K.,
in #729 you write and explain " $5L$ - P_3 can generally not be on the QL - P_4 circle ...". But I have not mentioned this in #722.
Best regards Eckart

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Message: #748
Date: 11/10/2014 3:17:22
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear friends,
I tried a bit geometry with the new 5L-points:
Consider the 5L of the QL-Co1-axes of the QL-components:
... the Miquel points of the QL-components of this 5L
 are collinear on a line 5L-Lx,
... the point 5L-P2 of this 5L is also a point on this line,
... the points 5L-P2 and 5L-P3 of this 5L are the same.
Best regards Eckart

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Message: #749
Date: 11/10/2014 3:53:37
From: bernard.keizer
Subject: new Pentangle Point

Dear Eckart,
But I didn't say you have mentionned this property!
On a contrary, I wanted to say precisely that I understood,
thanks to you, why 5L-P3 could not be on the QL-P4 circle, as it
is the inverse wrt this circle of a point which is not on this
circle!
The rest with the limaçon holds!
Best regards
Bernard

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Message: #750
Date: 12/10/2014 2:37:37
From: seiichikiri
Subject: 2 would-be pentangle points

Dear friends,

The point of message #726(3)
(the concurrent point of 5 lines) has the following coordinates.
Barycentrics =
 $\{p(2P + Q) + P(q + r), Pq + pQ + q(2Q + R), Pr + qR\}$,
where
 $P_1=(0,1,0)$, $P_2=(0,0,1)$, $P_3=(1,0,0)$, $P_4=(p,q,r)$ and $P_5=(P,Q,R)$.

Best regards,
Seiichi.

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Message: #751
Date: 12/10/2014 8:08:23
From: seiichikiri
Subject: 2 would-be pentangle points

Dear friends,

The point of message # 721(1) has the following coordinates.
Barycentrics= $\{p(P+Q), (p+q)Q, -rR\}$,
Where $P_1=(0,1,0)$, $P_2=(0,0,1)$, $P_3=(1,0,0)$, $P_4=(p,q,r)$ and
 $P_5=(P,Q,R)$.

The point of message # 721(2) (Miquel Five Circles Theorem) has
the coordinates too complicated to be written.

Correction: The points of (1), (2) and (3) in messages #721,
#726 are not collinear.

Best regards,
Seiichi.

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Message: #752
Date: 12/10/2014 10:31:33
From: chris.vantienhoven
Subject: new Pentangle Point

Dear Bernard K,

[BK]

> Apparently, you checked yourself the construction of p1.

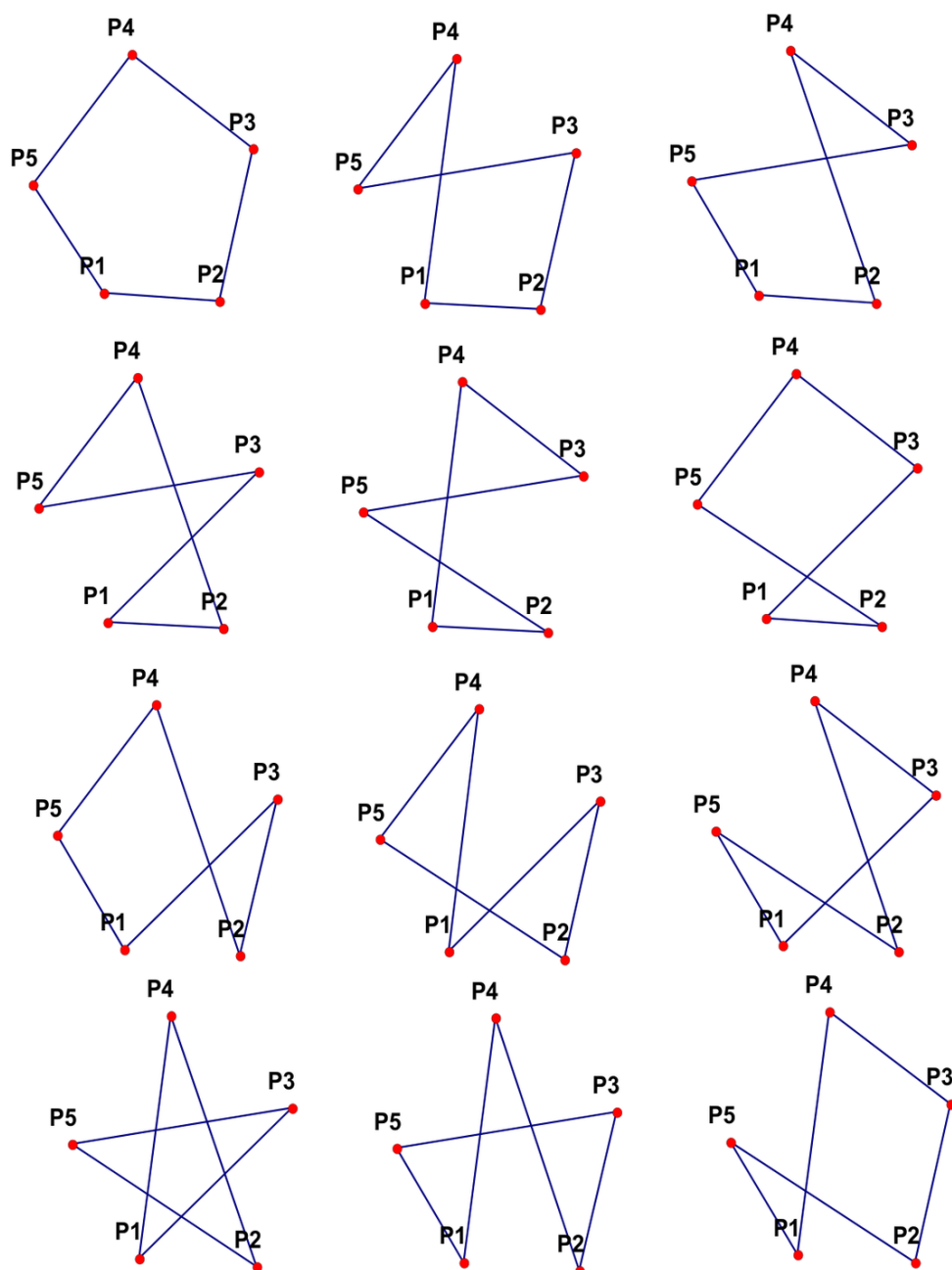
Yes I read about p1 in Morley's document "On the plane n-Line".
I started now with the document you recommended "Orthocentric properties of the plane n-Line".
It is pretty hard stuff to read these documents.
But the results are very worth while.
Do you understand his reasoning and use of equations?
I only can guess how it works.
Best regards, Chris

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Message: #753
Date: 13/10/2014 2:37:33
From: Seiichi Kirikami
Subject: The system of the coordinates of Pentagon Geometry

Dear Chris, dear friends,
You proposed 2 questions in Message #710.
1st question: which system is best for describing coordinates?
It seems to me that the question is very important in Pentagon Geometry.
Because I think that the vertices of a pentagon is ordered in 12 ways.
See the attached file.
So the coordinates in message #750 and #751 are incomplete for the solutions of the problems.
If we denote $P_i=(p_i,q_i,r_i)$, $i=1,\dots,5$ and make the coordinates of 1-2-3-4-5, we obtain other coordinates by changing the suffixes.
But then we will receive the message "No more memory available" more often.
I would like to hear how you think about the system of the coordinates of Pentagon Geometry.
Best regards,
Seiichi.

Variations of a pentagon



Variations of a pentagon.docx

Message: #754
Date: 13/10/2014 9:55:53
From: eckart_schmidt@t-online.de
Subject: new Pentangle point

Dear friends,

in message #713 and #722 I have described a transformation 5L-Tfx, here some further properties:
Remember: The 5 CSC-images of a point X lie concyclic on the so called CSC-circle of X with center 5L-Tfx(X).
... The CSC-circle of an intersection of 2 5L-lines is the circumcircle of the triangle of the remaining 3 5L-lines.
... The 5L-Tfx-image of the QL-P1-circle is the line at infinity (see #722).
... The CSC-circles of points on the QL-P1-circle degenerate to lines, tangent to the inscribed conic of the 5L.
... For the QL-P1-points the degenerated CSC-circles are the lines Li of the 5L.
... For intersections of a line Li and the QL-P1-circle the degenerated CSC-circle contains the 2nd intersection and QL-P1 of the remaining QL.
... For a point X on the QL-P1-circle the degenerated CSC-circle cuts the QL-P1-circle in 2 points, whose connections with X are tangent to the inscribed conic.
... For 2 diametral points on the QL-P1-circle the degenerated CSC-circles intersect on a line perpendicular 5L-P1.5L-P4.
... The CSC-circle of 5L-P7 contains 5L-P8 and the CSC-circle of 5L-P8 contains 5L-P7.
... For 2 points inverse wrt the QL-P1-circle the 5L-Tfx-images are symmetric wrt 5L-P4.

Up to now these properties are a bit curious and incoherent for me, but worth to be studied anymore.

Best regards Eckart

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Message: #755
Date: 13/10/2014 11:39:05
From: chris.vantienhoven
Subject: new Pentangle point

Dear Eckart,
Marvelous properties all together!
I still got some questions:
1. Do you have an algebraic expression for this transformation?
2. You transferred the 4L-CSC-conjugate
to the 5L-CSC-transformation.
Is there a way to generalize this to 6L-CSC-transformation,
then 7L-CSC-transformation, etc.?
3. How to describe the inverse of the 5L-CSC-transformation?
4. Which points are transformed onto itself?
Best regards,
Chris

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Message: #756
Date: 13/10/2014 11:15:11
From: chris.vantienhoven
Subject: The system of the coordinates of Pentagon Geometry

Dear Seiichi,
[SK] I would like to hear how you think about the system of the
coordinates of Pentagon Geometry.
I think that just as with Quadri Geometry we have to
distinguish:
* Quadrilaterals can be compared with n-Lines
* Quadrangles can be compared with n-Points
* Quadrigons can be compared with n-Gons
an n-Line has $(n-1)!/2$ Component n-Gons
an n-Point has also $(n-1)!/2$ Component n-Gons.
When we work with Component Triangles as Reference Triangle we
will get symmetric coordinates for Points in n-Lines and
n-Points, but not for points in n-Gons.
Just like you showed with your 5-Gon-points # 721(1) and
#726(3).
Best regards,
Chris

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Message: #757

Date: 14/10/2014 1:35:40

From: seiichikiri

Subject: The system of the coordinates of Pentagon Geometry

Dear Chris,

It will be convenient if the points of message #721(1), (2) and #726(3) will have their numbers.

I hope that #726(3), #721(1), #721(2) (Miquel Five Circles Theorem) have P1, P2, P3 respectively.

#726 (3) is a very docile point for computation. If there are any other points, I hope your addition.

Best regards,

Seiichi.

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Message: #758

Date: 14/10/2014 8:53:11

From: chris.vantienhoven

Subject: The system of the coordinates of Pentagon Geometry

Dear Seiichi,

[SK]

It will be convenient if the points of message #721(1), (2) and #726(3) will have their numbers. I hope that #726(3), #721(1), #721(2) (Miquel Five Circles Theorem) have P1, P2, P3 respectively.

I am fine to have preliminary point numbers in the 5-Gon.

I kept your order and made up some names.

5G-p1 5G-Midlines Crosspoint

5G-p2 5G-BiCircle Crosspoint

5G-p3 5G-Miquel Five Circles Center

When you know better names please let me know.

When I publish all Polygon points in an Encyclopedia of Polygon Figures, I will make up a new logical order.

After that I will number on in sequential order.

I propose to use the preliminary point numbers with prefix small letter "p", like 5G-p1 (instead of 5G-P1). Also for 5L- and 5P-points.

Best regards,

Chris

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Message: #759

Date: 14/10/2014 9:25:43

From: chris.vantienhoven

Subject: The system of the coordinates of Pentagon Geometry

Dear Seiichi,

I think that 5G-p3 also has been mentioned as the center in Miquel's Pentagram Theorem:

Consider a convex pentagon and extend the sides to a pentagram. Externally to the pentagon, there are five triangles. Construct the five circumcircles. Each pair of adjacent circles intersect at a vertex of the pentagon and a second point. Then Miquel's pentagram theorem states that these five second points are concyclic.

This theorem is sometimes referred to as Jiang Zemin's problem, as this former president of China talked about the theorem in the end of 1999 as he visited Macau.

Found at:

Best regards,

Chris

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Message: #760
Date: 14/10/2014 9:40:57
From: chris.vantienhoven
Subject: The system of the coordinates of Pentagon Geometry

Dear Seiichi,
In an earlier mail of
july 2, 2012 you already mentioned these points:
(3) Given 5 points P_1, P_2, P_3, P_4 and P_5 , we denote the
intersection of P_1P_3 and P_2P_5 by P_{12} . Similarly P_{23}, P_{34}, P_{45}
and P_{51} are defined. 5 Miquel points of $P_1P_{12}P_2P_4, P_2P_{23}P_3P_5,$
 $P_3P_{34}P_4P_1, P_4P_{45}P_5P_2$ and $P_5P_{51}P_1P_3$ are concyclic. (Miquel
pentagram theorem)
(4) Given 5 points P_1, P_2, P_3, P_4 and P_5 , we denote the
intersection of P_1P_3 and P_2P_5 by P_{12} . Similarly P_{23}, P_{34}, P_{45}
and P_{51} are defined. 5 Newton lines of $P_1P_{12}P_2P_4, P_2P_{23}P_3P_5,$
 $P_3P_{34}P_4P_1, P_4P_{45}P_5P_2$ and $P_5P_{51}P_1P_3$ have a common point. This
point seems the center of the circle of (3).
According to me (3) and (4) are different points. Can you check
this?
Best regards,
Chris

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Message: #761
Date: 14/10/2014 10:20:21
From: seiichikiri
Subject: The system of the coordinates of Pentagon Geometry

Dear Chris,
Thank you very much!
Though temporary, they are fine names!
Best regards,
Seiichi.

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Message: #762
Date: 14/10/2014 11:34:33
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear Chris, dear friends,
wrt 5L-Tfx (see #713, #722, #754) further properties:
Remember: 5L-Tfx maps a point X to the center of the concyclic 5
CSC-images of X.
... Let F1 and F2 be the foci of the inscribed conic of 5L,
swapped by 5L-Tfx (see #722).
... Let $F1^\circ$ and $F2^\circ$ be their inverses wrt the QL-P1-circle,
then $5L-Tfx(Fi^\circ) = Fi$.
... So there are two points F1 and $F2^\circ$ with 5L-Tfx-image F2 ...
... Let X be the intersection of $F1.F2^\circ$ and $F1^\circ.F2$,
then holds $5L-Tfx(X) = 5L-P1$.
... Lines through X have a line through 5L-P1 as 5L-Tfx-image...
... with intersections on an orthogonal hyperbola Hy ...
... through X, 5L-P1, $F1^\circ$ and $F2^\circ$ and a tangent in 5L-P1
through 5L-P4.
... (The inverse of Hy wrt the QL-P1-circle is the strophoid of
5L-P1.5L-P4 with pole 5L-P1 and fixed point Y,
which is the inverse wrt the QL-P1-circle
of the reflection of 5L-P1 in the center of Hy.)
... The transformation 5L-Tfx has three fixed points.
They are the intersections of the orthogonal hyperbola Hy
and its 5L-Tfx-image (a curve of degree 5).
... Generally there are two preimages P' and P''
of a point P wrt 5L-Tfx.
... The line P'P'' contains the point X (see above).
... The CSC-circles of P' and P'' are inverse wrt a circle $Ci(P)$
round P with radius $\sqrt{PF1*PF2}$.
... Construction of the second Point P'', if P and P' are known:
Let CSC- $Ci(P')$ be the CSC-circle of P', then the inverse wrt
 $Ci(P)$ is CSC- $Ci(P'')$ and the common point of the CSC-images
of CSC- $Ci(P'')$ is P''.
The properties are only CABRI-observations!
Best regards Eckart

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Message: #763
Date: 14/10/2014 12:48:38
From: seiichikiri
Subject: The system of the coordinates of Pentagon Geometry

Dear Chris,
They are clearly different. (4) is 5G-p1.
Best regards,
Seiichi.

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Message: #764
Date: 19/10/2014 8:08:29
From: chris.vantienhoven
Subject: Morley's miracle in a 5-line

Dear Bernard K and Eckart and Bernard G,
In Morley's document: Extensions of Clifford's Chain Theorem,
page 470 it says:
· a 4-line has 27 incenters
· a 5-line has 256 incenters.
We found a year ago that these 27 incenters of the 4-line are
the incenters of 27 inscribed Cardioids: QL-27Cu1. Morley's
miracle!
The Cardioid is a figure with one cusp and Morley denotes these
Cardioids with C3.
The Nephroid is a figure with two cusps and Morley denotes these
Nephroids with C4.
Am I interpreting it right that according to Morley the 256
incenters will be the incenters of 256 inscribed Nephroids?
Meanwhile I tried to construct an inscribed Nephroid in a
5-line, however up to now I only managed to construct it in a
4-line.
Best regards,
Chris

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Message: #765
Date: 19/10/2014 8:20:00
From: chris.vantienhoven
Subject: Morley's miracle in a 5-line

Dear Bernard K and Eckart and Bernard G,
The 27 incenters in a 4-line lie on Eckart's Cubic.
A follow-up question is:
Do these 256 incenters in a 5-line lie on a similar cubic?
Best regards,
Chris

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Message: #766
Date: 19/10/2014 9:17:04
From: chris.vantienhoven
Subject: Morley's miracle in a 5-line

Some extra information:
Morley gives his definition at pages 467 and 468 of his document: Extensions of Clifford's Chain Theorem. Among other he states at page 467:
"A 3-line has four inscribed circles. Their centers are the incenters of the 3-line.
We ask then for the curves C_n which touch $n+1$ given lines, and more particularly for their centers.
These centers we call the incenters of the $(n+1)$ -line."

See: <http://www.jstor.org/discover/10.2307/2370734?uid=3738736&uid=2&uid=4&sid=21102545290233>.

Last year we noticed that a 4-line has 27 inscribed cardioids C_3 .

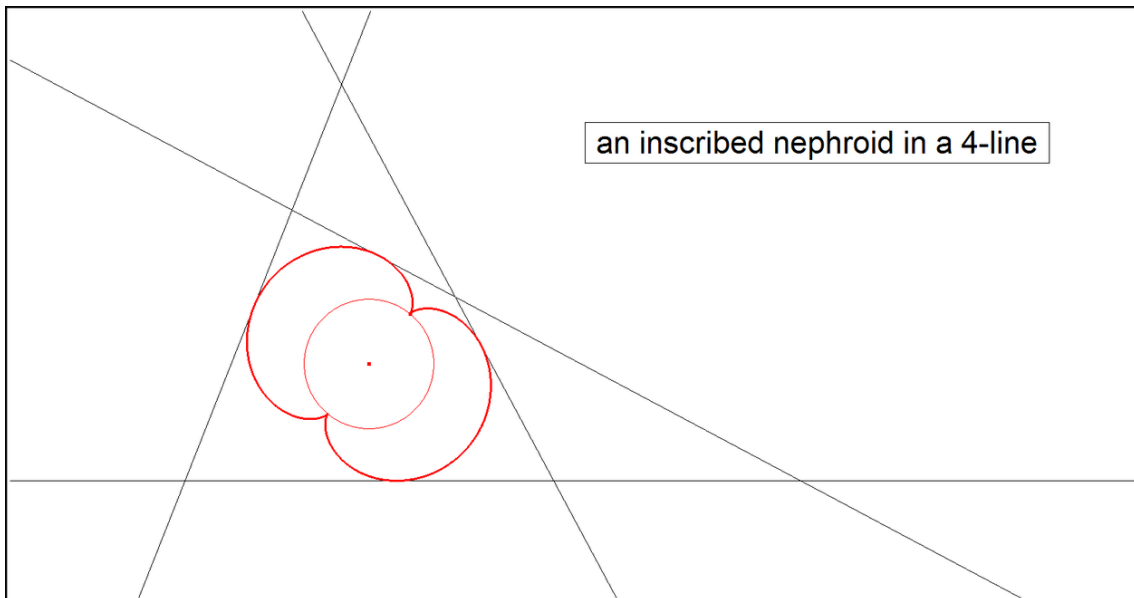
This is in accordance with the number of incenters Morley predicted at page 470.

So I wondered if we can deduct that a 5-line has 256 nephroids C_4 (again predicted at page 470).

And also if their incenters lie on a nice curve like Eckart's Cubic.

I attach an example of a nephroid in a 4-line.

Best regards,
Chris



5L-Construction-Nehproid-30-in QL-.png

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Message: #767
Date: 21/10/2014 2:41:18
From: seiichikiri
Subject: A point of pentagon

Dear friends,
Let $P_1P_2P_3P_4P_5$ be a pentagon such that
 $P_1=(1,0,0)$, $P_2=(0,1,0)$, $P_3=(0,0,1)$, $P_4=(p,q,r)$ and
 $P_5=(P,Q,R)$.
We denote the midpoints of P_1P_2 , P_2P_3 , P_3P_4 , P_4P_5 and P_5P_1
by M_{12} , M_{23} , M_{34} , M_{45} and M_{51} respectively.
The coordinates of the LSD point from M_{12} , M_{23} , M_{34} , M_{45} and M_{51}
are as follows.
$$\begin{aligned} \text{Barycentrics} = \{ & -(q+r)(2P+Q+R) - p(3P+2(Q+R)), \\ & -3qQ - 2Qr - P(2q+r) - 2qR - rR - p(P+2Q+R), \\ & -qQ - 2Qr - P(q+2r) - 2qR - 3rR - p(P+Q+2R) \}. \end{aligned}$$

Let $P_1P_2P_3P_4P_5$ be a pentagon.
We denote the midpoints of P_1P_3 , P_3P_5 , P_5P_2 , P_2P_4 and P_4P_1
by M_{13} , M_{35} , M_{52} , M_{24} and M_{41} respectively.
The coordinates of the LSD point from M_{13} , M_{35} , M_{52} , M_{24} and M_{41}
are the same as the barycentrics above.
There are the centers of conics through five midpoints such as
above.
But the expressions are too complicated to be written here.
Best regards,
Seiichi.

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Message: #768
Date: 21/10/2014 2:59:24
From: seiichikiri
Subject: A point of pentagon

Dear friends,
Addition:
For 12 ordered configurations of a pentagon, the point has the
same coordinates.
Best regards,
Seiichi.

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Message: #769

Date: 21/10/2014 8:59:25

From: eckart_schmidt@t-online.de

Subject: 5L-CSC-Transformation

Dear Chris, dear friends

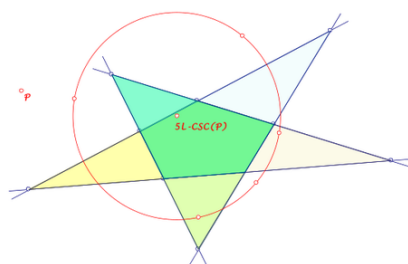
I have gathered my Cabri-observations wrt the 5L-transformation, described in #713, #722, #754, #762. There was a mistake in #762: The transformation has 5 (not 3) fixed points. If there are more mistakes, let me know.

Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

The CSC-Transformation for Pentalaterals

The CSC-images of a point wrt the 5 quadrilaterals of a Pentalateral are concyclic. The midpoint of the circle shall be the image of the point wrt the 5L-CSC-transformation. The geometry of this – not involutory – transformation here will be tested with CABRI.



Some 5L-points will be of importance for the geometry of this transformation (see QFG-message 710):

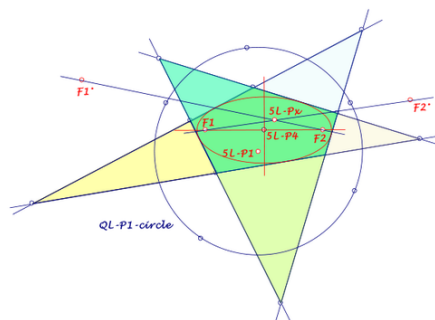
... The Miquel points of the 5 QL-components of a 5L are concyclic on the QL-P1-circle.

... 5L-P1 is the center of the QL-P1-circle.

... 5L-P4 is the center of the inscribed conic.

... F_1 and F_2 are the foci of the inscribed conic and F_1° and F_2° their inverses wrt the QL-P1-circle.

... 5L-Px shall be the intersection of $F_1^\circ.F_2$ and $F_1.F_2^\circ$.



Definition:

5L-CSC(P) is the center of the 5 concyclic CSC-images of P.

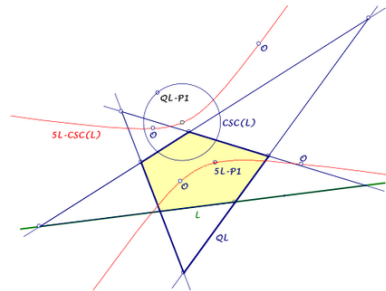
The CSC-circle of P is the corresponding circle.

(wrt CSC see EQF under QL-Tf1)

Properties:

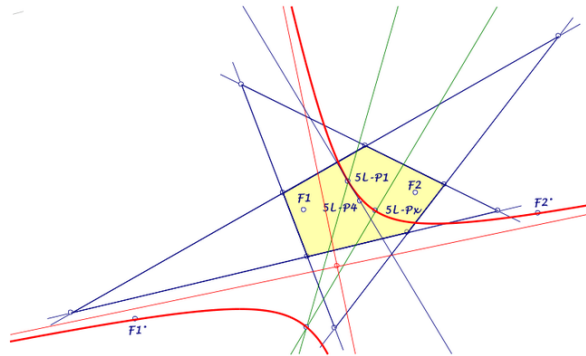
5L-CSC-images:

1. 5L-CSC is not involuntary.
2. 5L-CSC swaps F_1 and F_2 .
3. The 5L-CSC-image of $L_i \cap L_j$ is the circumcenter O_{ij} for the triangle of the remaining three 5L-lines.
4. The 5L-CSC-image of 5L-P1 is the reflection in 5L-P4.
5. The 5L-CSC-image of 5L-Px is 5L-P1.
6. The 5L-CSC-image of the QL-P1-circle is the line at infinity.
7. The image of a QL-P1-point is the point at infinity of a line, perpendicular to the remaining 5L-Line.
8. For two points inverse wrt the QL-P1-circle the 5L-CSC-images are symmetric wrt 5L-P4.
9. The 5L-CSC-image of a line is a conic through 5L-P1.
10. The 5L-CSC-image of a 5L-line L_i is a conic
... through 5L-P1,
...the center of $CSC(L_i)$ wrt the corresponding QL,
...the four circumcenters O_{ij} (see 3).



11. The 5L-CSC-images of tangents at the QL-P1-circle are parabolas through 5L-P1.
12. For lines through 5L-Px the 5L-CSC-image degenerates to a line through 5L-P1.
13. Examples: $F_1.F_2^\circ \rightarrow F_2.F_2^\circ, F_2.F_1^\circ \rightarrow F_1.F_1^\circ,$
 $5L-P4.5L-Px \rightarrow 5L-P1.5L-Px \rightarrow 5L-P1.5L-P4,$

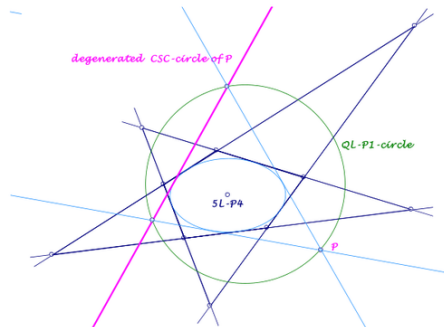
14. Lines through $5L-Px$ and their $5L-CSC$ -line intersect on an orthogonal hyperbola H_y through $5L-P1$, $5L-Px$, $F1^\circ$ and $F2^\circ$, centered in the midpoint of $F1^\circ.F2^\circ$ and tangent to $5L-P1.5L-P4$.



15. The $5L-CSC$ -image of a circle is a conic.

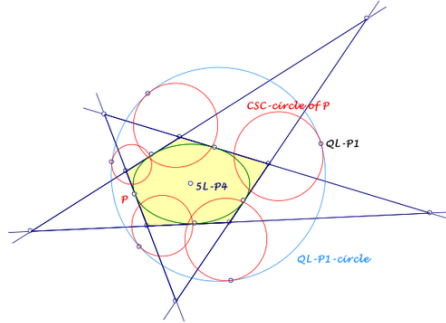
CSC-circles:

16. The CSC -circle of an intersection of two $5L$ -lines is the circumcircle of the triangle of the remaining three $5L$ -lines.
17. The CSC -circles of points on the $QL-P1$ -circle degenerate to lines tangent to the inscribed conic of the $5-L$.



18. For a point P on the $QL-P1$ -circle the degenerated CSC -circle cuts the $QL-P1$ -circle in two points, whose connections with P are also tangents to the inscribed conic.
19. For the $QL-P1$ -points the degenerated CSC -circles are the lines L_i of the $5-L$.

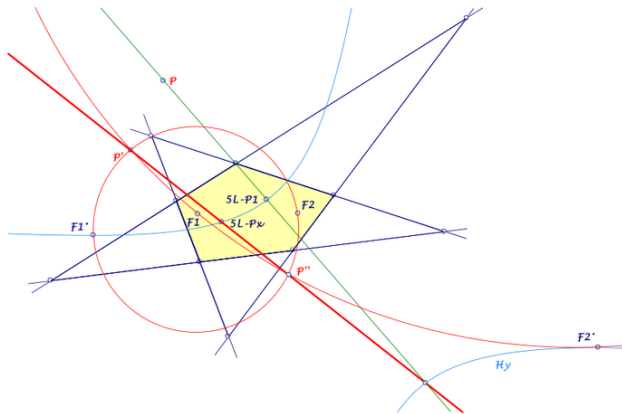
20. For intersections of a line L_i and the $QL-P1$ -circle the degenerated CSC -circles contain the 2nd intersection and $QL-P1$ of the remaining QL .
21. For two diametral points on the $QL-P1$ -circle the degenerated CSC -circles intersect on the polar of $5L-Px$ wrt the inscribed conic (perpendicular $5L-P1.5L-P4$).
22. The CSC -circles of points on the inscribed conic are tangent to the $QL-P1$ -circle (for the contact points of the $5L$ -lines the CSC -circles touch the $QL-P1$ -circle in the $QL-P1$ points).



23. The CSC -circle of $5L-P7$ contains $5L-P8$ and the CSC -circle of $5L-P8$ contains $5L-P7$ (wrt the points see QFG -message 710).

5L-CSC-preimages:

24. Generally there are two preimages P' and P'' of a point P .

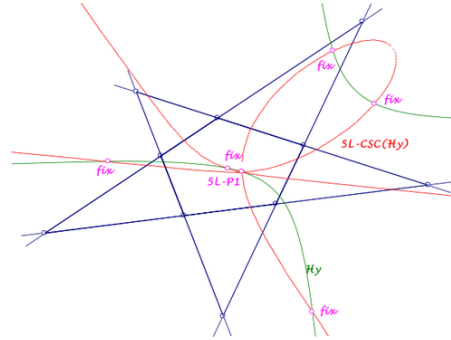


25. The two preimages P' and P'' of P are collinear with $5L-Px$.

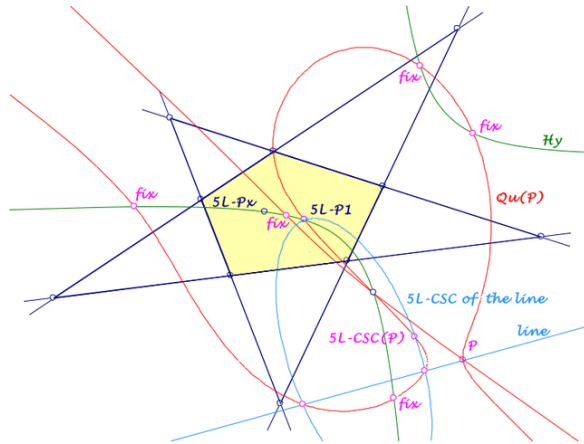
26. The line PP'' is the connection of $5L-Px$ and the 2nd intersection of $P.5L-PI$ and the hyperbola Hy (see 14).
27. The two preimages P' and P'' of P are concyclic with F_1 and F_2° or with F_1° and F_2 .
28. F_1 and F_1° have the same $5L-CSC$ -image F_2 ; F_2 and F_2° the same $5L-CSC$ -image F_1 .
29. $5L-PI$ and the inverse of $5L-Px$ wrt the $QL-PI$ -circle have the same $5L-CSC$ -image (reflection of $5L-PI$ in $5L-P4$).
30. The CSC -circles of P' and P'' are inverse wrt a circle round P with radius $\sqrt{PF_1 \cdot PF_2}$.

5L-CSC-fixed points

31. The $5L-CSC$ -transformation has 5 fixed points (not necessary real) on the hyperbola Hy (see 14).



32. These fixed points are the intersections of Hy with its $5L-CSC$ -image (without $5L-PI$).
33. Lines through a point P cut their $5L-CSC$ -image – a conic through $5L-PI$ – in two points, whose locus is a quartic $Qu(P)$, containing the fixed points of $5L-CSC$ (beside P , the $5L-CSC$ -image of P , $5L-PI$ and the intersection of Hy and $P.5L-Px$).
34. For points X on PP' or PP'' the quartics $Qu(X)$ contain P .



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-10-21.pdf

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Message: #770
Date: 21/10/2014 5:43:11
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris,
I realize I haven't answered this message!
In fact, I don't see what to say more
1) it's very hard stuff
2) it's really worth making efforts
I spent many dozens of hours trying to understand before I saw any light.
The 4 papers of Bogomolny could help ...
Did you finally check the 2 other points 2nd orthocenter h (P_2 for QL) and Kantor-Hervey point (P_3 for QL)?
Theoretically, any circle suits to Morley's calculations, but it is convenient to take the centric circle as reference: for example, in this case the KH-point is given by sum of t_i (vectorial sum) with t_i the famous turn represented by the centers of the centric circles of the $n-1$ lines.
You have now a dozen of points, if I'm not wrong.
Best regards
Bernard

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Message: #771
Date: 21/10/2014 5:58:48
From: bernard.keizer
Subject: Morley's miracle in a 5-line

Dear Chris,
I think you're right!
There are, I suppose, one mononephroid tangent to the 5
cardioids and 256 multinephroids tangent to the 5 lines.
The 256 incenters are on the set of axes coming from the 5 QL's;
to find them, you have to quadrisect each angle (as you had to
trisect the angles of the 4 reference triangles in order to find
the Morley sets).
You have also 16 cardioids envelopping the $5 \cdot 8 = 40$ Steiner
circles of the QL's (each cardioid touching 5 circles and each
circle 2 cardioids) and 81 circles carrying the centers of the
 $5 \cdot 27 = 135$ cardioids tangent to the 5 QL's.
Best regards
Bernard

PS I suppose you know the construction of the nephroid with a
circle center O and fixed point S; a variable point A on the
circle, B is the reflexion in OA of the reflexion A' of A in OS
(the angle SOB is the triple of SOA). AB is tangent to a
nephroid and the contact point is at the quarter from A.

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Message: #772
Date: 21/10/2014 6:18:22
From: bernard.keizer
Subject: Morley's miracle in a 5-line

Dear Chris,

Certainly not!

I'm ready to bet that they are on a quartic, more precisely a stelloïd quartic (locus of the points such as the sum of the directions to 4 points of the curve taken as pivots wrt a fixed line is constant).

For the triangle or 3-lines, the axes are the bisectors of the 3 angles in the vertices and the 4 intersections form an orthocentric quadrilateral or the basis points of a set of rectangular hyperbolas. The rectangular hyperbola is a conic stelloïd: for a point on the curve and any pair of points P1 and P2 of the curve symmetric wrt the center, the bisectors of the angle P1XP2 are parallel to the asymptotes of the rectangular hyperbola.

For the QL or 4-lines, the axes are the Morley lines of the 4 reference triangles and the intersection points are on the cubic stelloïd (locus of points such as the sum of the directions to 3 points of the curve taken as pivots - see Bernard Gibert's paper- wrt a fixed line is constant).

The quadrisection of angles gives angles of $\pi/2$ or $\pi/4$...

Best regards

Bernard

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Message: #773
Date: 21/10/2014 6:35:53
From: Chris van Tienhoven
Subject: A point of pentagon

Dear Seiichi,
It looks like this is the 5L-Centroid.
Best regards,
Chris

Op 21 okt. 2014 02:41 schreef " seiichikiri@yahoo.com
[Quadri-Figures-Group]" < Quadri-Figures-Group@yahoogroups.com

>:

>> Dear friends,
>> Let $P_1P_2P_3P_4P_5$ be a pentagon such that $P_1=(1,0,0)$,
>> $P_2=(0,1,0)$, $P_3=(0,0,1)$, $P_4=(p,q,r)$ and $P_5=(P,Q,R)$.
>> We denote the midpoints of P_1P_2 , P_2P_3 , P_3P_4 ,
>> P_4P_5 and P_5P_1 by M_{12} , M_{23} , M_{34} , M_{45} and M_{51} respectively.
>> The coordinates of the LSD point from M_{12} , M_{23} , M_{34} , M_{45} and
>> M_{51} are as follows.
>> Barycentrics=
>> $\{-(q+r)(2P+Q+R) - p(3P+2(Q+R)), -3qQ - 2Qr$
>> $- P(2q+r) - 2qR - rR - p(P+2Q+R), -qQ - 2Qr -$
>> $P(q+2r) - 2qR - 3rR - p(P+Q+2R)\}$.
>> Let $P_1P_2P_3P_4P_5$ be a pentagon.
>> We denote the midpoints of P_1P_3 , P_3P_5 , P_5P_2 ,
>> P_2P_4 and P_4P_1 by M_{13} , M_{35} , M_{52} , M_{24} and M_{41} respectively.
>> The coordinates of the LSD point from M_{13} , M_{35} , M_{52} , M_{24}
>> and M_{41} are the same as the barycentrics above.
>> There are the centers of conics through five midpoints
>> such as above. But the expressions are too complicated
>> to be written here.
>> Best regards,
>> Seiichi.

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Message: #774
Date: 21/10/2014 6:53:05
From: bernard.keizer
Subject: quartic for QL's and CSC and CSCdiag

Dear Eckart,
I don't know if you are always interested in the QL (ordinary 4-lines)!
I found totally accidentally that the quartic described in your messages #364 and #465 is not only CSC invariant, as you well know, but also CSCdiag invariant and even $CSC * CSCdiag = CSCdiag * CSC$ invariant!
CSCdiag swaps the Plücker points and also their CSC conjugates. If you consider the 4 points together, the properties are simple to understand.
The quartic is through the invariant points of CSC and also through the invariant points of CSCdiag.
Best regards
Bernard

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Message: #775
Date: 22/10/2014 2:33:38
From: seiichikiri
Subject: A point of pentagon

Dear Chris,
I would like to confirm your statement. The png picture of the 5L-Centroid is attached in message #714. So I would like you to send me its Cabri picture.
If you have your confirmation, please send me it.
Best regards,
Seiichi.

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Message: #776
Date: 22/10/2014 7:40:27
From: Chris van Tienhoven
Subject: A point of pentagon

Dear Seiichi,
Right now I am on holiday with my wife in the Camargue, (southern France). I can send you the Cabri picture later.
Best regards,
Chris

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Message: #777
Date: 23/10/2014 2:57:47
From: seiichikiri
Subject: A point of pentagon

Dear Chris,
You do not need send me the Cabri picture.
The coordinates of my computation is that of the pentangle centroid.
In a pentangle, 5 midpoints are sufficient instead of 10 (all combinations of 5 points).
5 midpoints produce the same pentangle centroid.
Best regards,
Seiichi.

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Message: #778

Date: 23/10/2014 6:01:18

From: Seiichi Kirikami

Subject: Euler lines in a pentagon

Dear friends,

According to Euler-Gerade eines Vierecks by Eckart in 2004, I drew 2 pictures by 5 component triangles instead of 5 quadrangles.

(1) $P_1P_2P_3$, $P_2P_3P_4$, $P_3P_4P_5$, $P_4P_5P_1$ and $P_5P_1P_2$.

(2) $P_1P_3P_4$, $P_2P_4P_5$, $P_3P_5P_1$, $P_4P_1P_2$ and $P_5P_2P_3$.

It seems that there are at least 2 Euler lines through its centroid (G) in a pentagon.

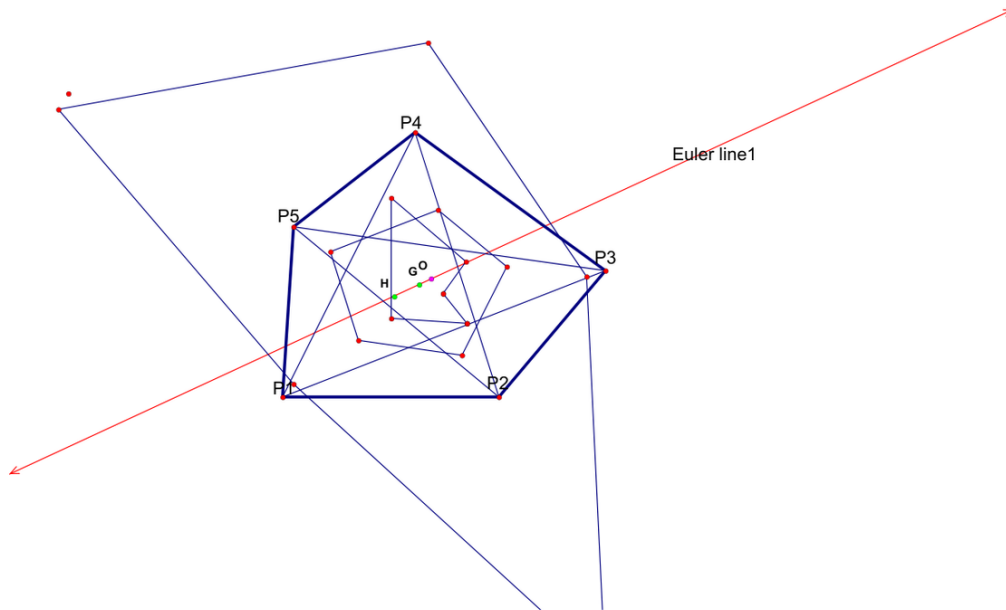
See the attached file.

Best regards,

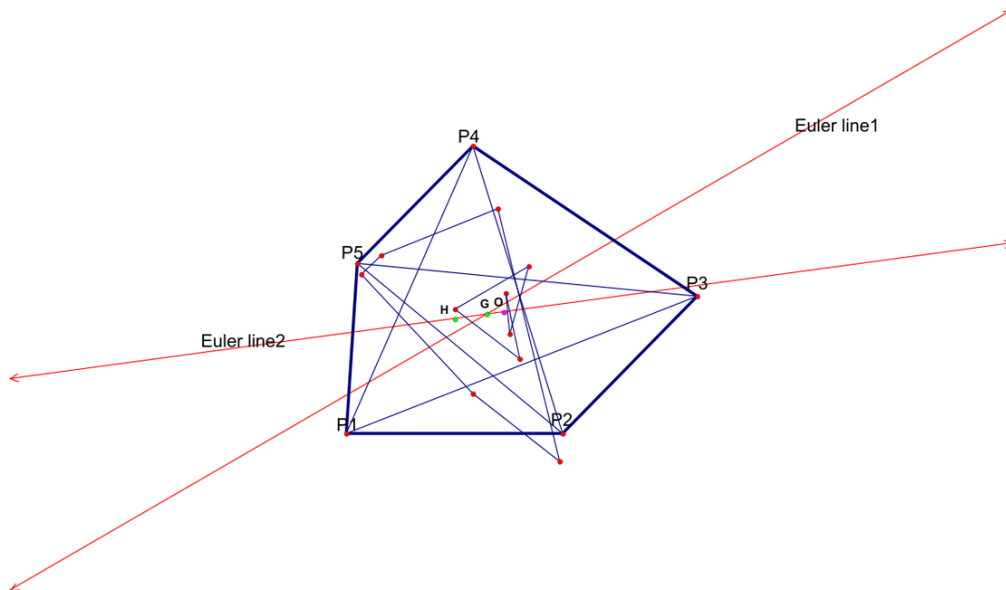
Seiichi.

Euler lines

(1) Component triangles: $P_1P_2P_3$, $P_2P_3P_4$, $P_3P_4P_5$, $P_4P_5P_1$, $P_5P_1P_2$.



(2) Component triangles: $P_1P_3P_4$, $P_2P_4P_5$, $P_3P_5P_2$, $P_4P_1P_2$, $P_5P_2P_3$.



Euler lines.docx

Message: #779
Date: 23/10/2014 12:06:37
From: bernard.keizer
Subject: 5L-CSC-Transformation

Dear Eckart, dear Chris
I find your developments about this transformation really amazing!
I'm just beginning to understand the first properties.
F1 and F2 are CSC-conjugates in the 5 CSC and therefore in 5L-CSC.
They are the intersections of the 5 curves QL-Cu1 and their middle is the intersection of the 5 Newton Lines.
Does the CSC-circle of 5L-P4 have a particular property (the 5 CSC are on the 5 circles circumscribed to the DT's)?
The CSC conjugates of a vertice of the 5L is either one of the vertices of 3 QL's or on the circumscribed circle through these 3 points as conjugate of a point on the 4th line of the 2 other QL's.
But there are plenty other properties for which I don't see any reason!
Congratulations
Bernard

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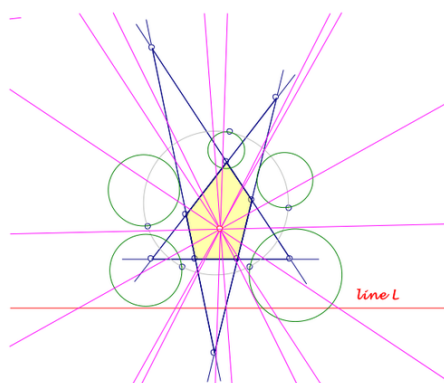
Message: #780
Date: 23/10/2014 3:10:26
From: eckart_schmidt@t-online.de
Subject: 5L-CSC-Line-Transformation

Dear friends,
in the attached file a 5L-CSC-line-transformation is Cabri-tested, which maps a line to a point.
Background: The 5 CSC-images of a line (or a circle) give 5 circles with a common radical center ...
Best regards Eckart

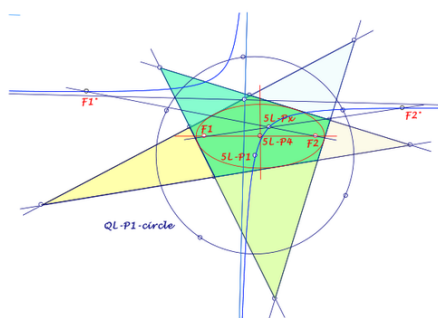
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

CSC-Line-Transformation for Pentalaterals

This CSC-related transformation maps a line to a point: For a line L the 5 CSC-circles (wrt the QL -components) have pairwise radical axes with a common point P . The geometry of this mapping is here tested with CABRI.



Some $5L$ -elements will be of importance for the geometry of this transformation (see *QFG*-message 710):

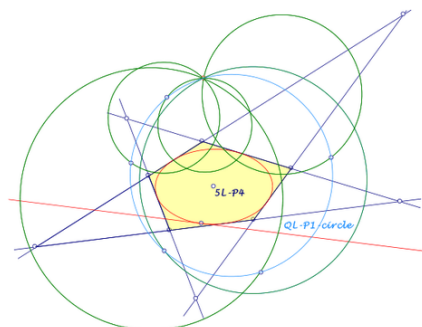


- ... The Miquel points of the 5 QL -components are concyclic on the QL - $P1$ -circle.
- ... $5L$ - $P1$ is the center of the QL - $P1$ -circle.
- ... $5L$ - $P4$ is the center of the inscribed conic.
- ... F_1 and F_2 are the foci of the inscribed conic and F_1° and F_2° their inverses wrt the QL - $P1$ -circle.
- ... $5L$ - Px shall be the intersection of $F_1^\circ.F_2$ and $F_1.F_2^\circ$.
- ... H_y orthogonal hyperbola through $5L$ - $P1$, $5L$ - Px , F_1° and F_2° , centered in the midpoint of $F_1^\circ.F_2^\circ$ (see *QFG* # 762).

Properties

Images of lines:

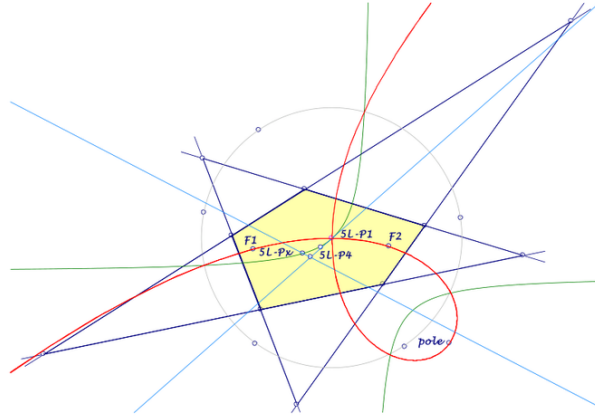
1. A line L_i of the $5L$ will be mapped to the Miquel point of the remaining QL .
2. Lines through one focus F_i of the inscribed conic will be mapped to the other focus F_j . (In this case the image is a common point of the 5 circles.)
3. The 1st Steiner axes of the QL -components will be mapped to their 2nd intersection with the QL - PI -circle.
4. $5L$ - PI . $5L$ - $Px \rightarrow$ point at infinity of $5L$ - PI . $5L$ - $P4$.
5. $5L$ - $P4$. $5L$ - $Px \rightarrow$ inverse of $5L$ - Px wrt the QL - PI -circle.
6. Tangents to the inscribed conic will be mapped to points on the QL - PI -circle. (In this case the image is a common point of the 5 circles.)



7. Tangents at the QL - PI -circle in the intersections with the orthogonal hyperbola Hy will be mapped to their contact point.

Images of line pencils

8. Lines through $5L$ - $P4$ are mapped on collinear points on the polar of $5L$ - Px wrt the QL - PI -circle. The image of this line is $5L$ - $P4$ again.
9. Lines through $5L$ - Px are mapped to points on the strophoid of $5L$ - PI . $5L$ - $P4$ with fixed point $5L$ - PI and pole in the CSC -line-transformation image for the connection of $5L$ - Px and the reflection of $5L$ - PI in $5L$ - $P4$. This strophoid contains F_1 , F_2 and is the inverse of the orthogonal hyperbola Hy wrt the QL - PI -circle.



Generalisation:

Lines are special circles. If C_i is a circle, then the 5 *CSC*-images wrt the *QL*-components have also a common radical center, which shall be the image of the circle.

10. The image of the *QL-P1*-circle is the reflection of *5L-Px* in *5L-P4*.
11. A circle through the focus F_i is mapped to the other focus.
12. The circumcircle wrt 3 *5L*-lines is mapped to the intersection of the remaining 2 *5L*-lines.
13. Circles round *5L-P4* have collinear images on *5L-P1.5L-Px*.
14. Circles round *5L-P1* have images on an orthogonal hyperbola through *5L-P1*, *5L-Px*, F_1 , F_2 (centered in *5L-P4*, asymptotes parallel to those of H_y).
15. The inverse of this orthogonal hyperbola (wrt the *QL-P1*-circle) is the strophoid of a line, connecting *5L-P1* and the center of H_y , with fixed point *5L-P1* and pole in the inverse (wrt *QL-P1*-circle) of the reflection of *5L-P1* in *5L-P4*.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-10-23.pdf

Message: #781
Date: 24/10/2014 9:05:14
From: eckart_schmidt@t-online.de
Subject: 5L-CSC-Transformation

Dear Bernard K.,
thanks for interest in the 5L-CSC-transformation. Wrt the
CSC-circle of 5L-P4: I only know, that the midpoint lies on
5L-P1.5L-Px. But there is a curious further property: The
5L-CSC-image of this circle is a conic, centered in 5L-P4,
similar to the 5L-inscribed conic, but turned around 90° ???
Best regards Eckart
PS: Thanks also for your additional remarks to the quartic in
message #774.

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Message: #782
Date: 26/10/2014 4:43:57
From: eckart_schmidt@t-online.de
Subject: new Pentangle Point

Dear Chris,
Camargue ... Les Saintes Maries de la Mer ... promener sur la
digue vers le phare de Gacholle ... merveilleux souvenirs.
I hope, you and your wife had a good holiday!
Back to 5L-geometry:
(1) The 5 QL-P20 of a 5L give an orthogonal hyperbola,
containing 5L-P4.
(2) A new point: The pedal points of the foci of the inscribed
conic wrt the 5L-lines are concyclic on a circle, whose 5
CSC-image-circles have a common radical center. This center lies
on 5L-P1.5L-Px (wrt 5L-Px see #769 and #780).
Best regards Eckart

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Message: #783
Date: 26/10/2014 6:19:50
From: Chris van Tienhoven
Subject: new Pentangle Point

Dear Eckart,
Indeed Sainte Marie de la Mer, that's where I was for three days. With our Vw camper we headed for Italy and now we are in San Remo, again at the Mediteranian. Very nice temperature. About your remarks.
I am glad you tried out QL-P20. Unexpected property!
Also the new 5L-point from the focii of the inscribed conic. It shows that we just started disclosing this 5L-domain.
Best regards,
Chris

Op 26 okt. 2014 16:43 schreef "' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de [Quadri-Figures-Group]" <Quadri-Figures-Group@yahoogroups.com >:
>> Dear Chris,
>> Camargue ... Les Saintes Maries de la Mer ...
>> promener sur la dique vers
>> le phare de Gacholle ... merveilleux souvenirs.
>> I hope, you and your wife had a good holiday!
>> Back to 5L-geometry:
>> (1) The 5 QL-P20 of a 5L give an orthogonal hyperbola,
>> containing 5L-P4.
>> (2) A new point: The pedal points of the foci
>> of the inscribed conic wrt
>> the 5L-lines are concyclic on a circle,
>> whose 5 CSC-image-circles have a
>> common radical center. This center lies on 5L-P1.5L-Px
>> (wrt 5L-Px see #769 and #780).
>> Best regards Eckart

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Message: #784
Date: 01/11/2014 10:30:24
From: eckart_schmidt@t-online.de
Subject: 5L-/6L-Cabri-observations

Dear Chris, dear friends,

some observations with Cabri:

- (1) In #704 Chris described a 5A-point using twice the transformation QA-Tf2 (Involuntary Conjugate) for a 5A-point wrt the remaining QA.
There is an analog 5L-line wrt QL-Tf2 (QL-Line-Conjugate):
Let L_i be a 5L-line and $M_i = QL-Tf2(L_i)$
(wrt the remaining QL of the L-5L) and then $N_i = QL-Tf2(M_i)$
(wrt the remaining QL of the M-5L).
The 5 intersections of L_i and N_i are collinear.
- (2) For a 6L the 6 QL-P1-circles (see #710, 1) of the 5L-components have a common point.
- (3) For a 6L the 6 QL-P4-circles (see #710, 2) of the 5L-components have a common point.
- (4) The 6 5L-P4-points of a 6L lie on a conic.
- (5) The 6 5L-P9-points of a 6L lie on a conic.
- (6) The 6 CSC-circles of an arbitrary point wrt the 5L-components (see #769) of a 6L have a common point.
(Perhaps this will give a 6L-transformation.)

Best regards Eckart

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Message: #785

Date: 03/11/2014 7:26:16

From: Chris van Tienhoven

Subject: The system of the coordinates of Pentagon Geometry

Dear Seiichi,

[SK] They are clearly different.

(4) is 5G-p1.

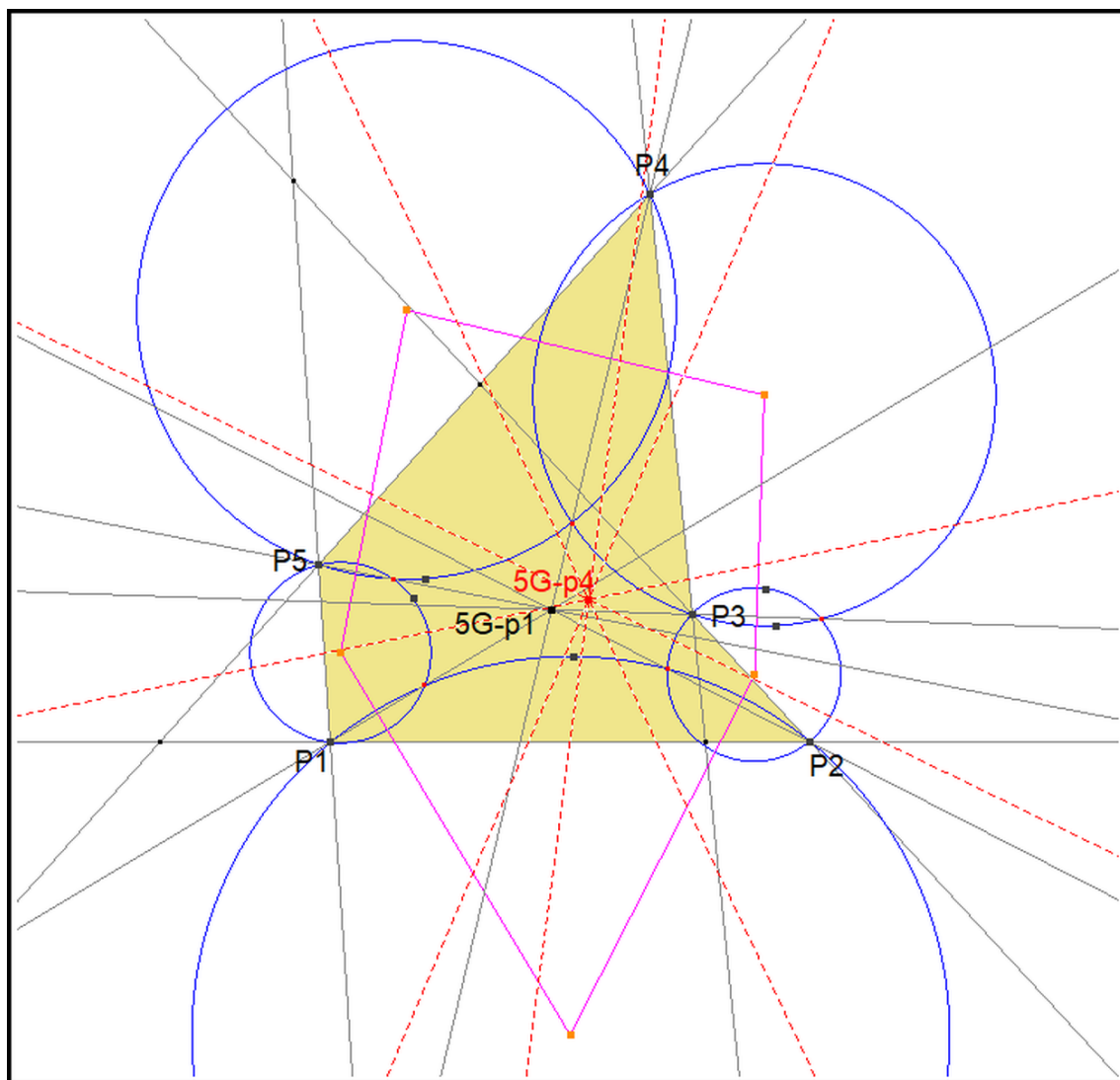
According to my Cabri drawing (4) is unequal 5G-p1.

See attached drawing.

Could you check this?

Best regards,

Chris



5G-p4-NewtonLinePoint-01-fig.png

Message: #786
Date: 04/11/2014 12:42:22
From: bernard.keizer
Subject: 5L-/6L-Cabri-observations

Dear Eckart,

There are plenty of properties in this short message!

(1) is too complicate for me

(2) is the Clifford's chain: for the QL , 4 circles through the Miquel point, for the 5-L 5 Miquel points on a circle, for the 6-L 6 circles through a point, for the 7-L 7 points on a circle and so on ... The chain ends always with a point or a circle, depending if n is odd or even.

(3) is a completely different property: in a n-line, the centric circles of the n n-1lines a) have their centers cocyclic on the centric circle of the n-line b) have a common point, which is the node of the limaçon tangent to the n centric circles of the n-1lines (for the QL, the limaçon is a cardioïd).

(4) it's an interesting generalisation of the conic inscribed in the 5-L.

(5) it seems I missed one message: what is 5L-P9?

(6) this is really the most important property of the message, as it seems to be the generalisation of the Clifford's chain : if I understand well, 1 arbitrary point , 5 CSC give a circle and the center is the conjugate in the 5-L, the 6 circles have a common point, which is the conjugate in the 6-L.

Taking the intersection of 2 lines, the conjugate for the QL is the intersection of the 2 remaining lines, the conjugate for the 5-L is the circumcenter of the remaining triangle and the conjugate in the 6-L is the Miquel point of the remaining QL.

Taking the infinity point, this gives the point 2) with the Miquel points and the Clifford's chain.

Does this function the same way for an arbitrary point?

Now taking the circumcenter of 3 lines, the conjugate in the QL is the reflexion of the Miquel point in the remaining line, the conjugate in the 5-L is the intersection of the 2 remaining lines.

Is by any chance the conjugate in the 6-L the circumcenter of the 3 remaining lines and the conjugate in the 7-L the Miquel point of the remainig QL?

Best regards

Bernard

---In Quadri-Figures-Group@yahoogroups.com, wrote:

Dear Chris, dear friends,
some observations with Cabri:

(1) In #704 Chris described a 5A-point using twice the transformation QA-Tf2 (Involuntary Conjugate) for a 5A-point wrt the remaining QA. There is an analog 5L-line wrt QL-Tf2 (QL-Line-Conjugate): Let L_i be a 5L-line and $M_i = \text{QL-Tf2}(L_i)$ (wrt the remaining QL of the L-5L) and then $N_i = \text{QL-Tf2}(M_i)$ (wrt the remaining QL of the M-5L). The 5 intersections of L_i and N_i are collinear.

(2) For a 6L the 6 QL-P1-circles (see #710, 1) of the 5L-components have a common point.

(3) For a 6L the 6 QL-P4-circles (see #710, 2) of the 5L-components have a common point.

(4) The 6 5L-P4-points of a 6L lie on a conic.

(5) The 6 5L-P9-points of a 6L lie on a conic.

(6) The 6 CSC-circles of an arbitrary point wrt the 5L-components (see #769) of a 6L have a common point. (Perhaps this will give a 6L-transformation.)

Best regards Eckart

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Message: #787
Date: 04/11/2014 12:54:22
From: bernard.keizer
Subject: new Pentangle Point

Dear Chris,
Now you're back from France and Italy, you have certainly gathered energy for the 5-L.
Did you finally checked the construction of the 2nd orthocenter and of the Kantor-Hervey point?
Best regards
Bernard

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Message: #788
Date: 04/11/2014 1:07:37
From: bernard.keizer
Subject: Morley's miracle in a 5-line

Dear Chris,
Did you find the construction of the mononephroid, tangent to the 5 cardioïds?
(I suppose it's center is Kantor's center QL-P2)
Did you find the construction of the 64 axes for the QL (by quadrisecting the angles)?
There are then $5 \cdot 64 = 320$ lines for the 5-L and the 256 centers of the multinephroids tangent to the 5 lines are on these 320 lines, each center as intersection of 5 lines (one in each set) and 4 centers on each line.
I was able myself with Geogebra to draw the Morley lines of the 4 triangles of a QL by trisecting the angles and to find the 27 centers of the multicardioïds, but I didn't try this operation ...
I suppose Cabri is more powerful!
Best regards
Bernard

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Message: #789
Date: 05/11/2014 5:01:16
From: eckart_schmidt@t-online.de
Subject: 5L / 6L-Cacri-observations

Dear Bernard,
thanks for your comments wrt my message #784.
Wrt (5): 5L-P9 is for me the Cantor-Hervey point as described by you in message #738.
Wrt (6): The last conjecture does not hold, sorry.
But I observed an interesting property of the 6L-conjugate:
It is involutory!
Best regards Eckart

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Message: #790

Date: 06/11/2014 2:45:42

From: eckart_schmidt@t-online.de

Subject: CSC-Circles for Pentalaterals

Dear friends,

the attached file is an excursion in the CSC-geometry of a pentalateral. Perhaps of interest.

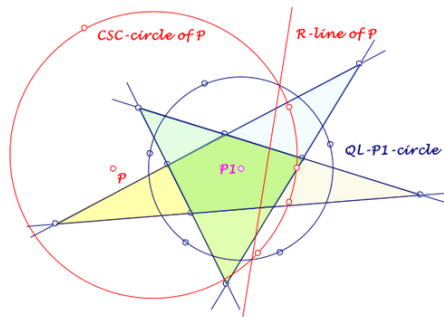
Best regards Eckart

EQF-Note 2014-11-06

Background for these notes is:
 Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

CSC-Circles for Pentalaterals

The transformation $QL-Tf1$ – shortened CSC – is the central transformation for quadrilaterals. For a pentalateral with 5 QL -components there are 5 concyclic CSC-images of a point. The geometry of these CSC-circles will be Cabri-researched, considering points on lines and circles wrt a pentalateral.



Preliminary remarks

Some $5L$ -elements will be of importance for the geometry of CSC -circles (see QFG -message 710):

... The Miquel points of the 5 QL -components of a $5L$ are concyclic on the $QL-P1$ -circle.

... $5L-P1 = P_1$ is the center of the $QL-P1$ -circle.

... $5L-P4 = P_4$ is the center of the inscribed conic.

... F_1 and F_2 are the foci of the inscribed conic and F_1° and F_2° their inverses wrt the $QL-P1$ -circle.

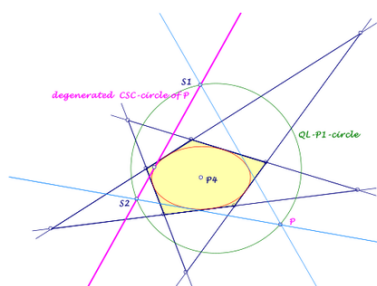
... $5L-Px = P_x$ shall be the intersection of $F_1^\circ.F_2$ and $F_1.F_2^\circ$.

... Hy is the orthogonal hyperbola through $P_1, P_x, F_1^\circ, F_2^\circ$ with center Z .

For basic properties of CSC -circles see EQF -Note 2014-10-21 in QFG -message 769. Here we need the special property:

- The CSC -circle for a point on the $QL-P1$ -circle degenerates to a line, tangent to the inscribed conic.

This line can be easily constructed: Consider the tangents from P (on the QL - PI -circle) to the inscribed conic and their 2nd intersections S_1 and S_2 with the QL - PI -circle, then S_1S_2 is the degenerated CSC -circle of P .



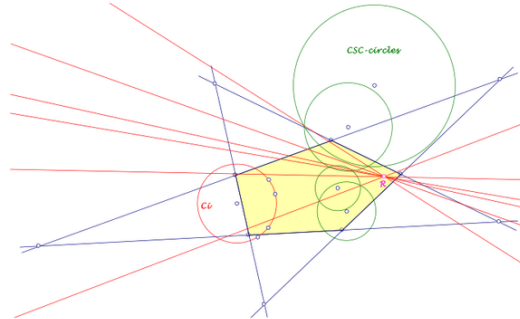
The R -line of a point

Definition: The R -line of a point P is the radical axis of the CSC -circle of P and the QL - PI -circle.

- The R -line for an intersection of two $5L$ -lines is the line between the corresponding QL - PI -points.
- The R -line for the QL - PI -point of a QL -component is the remaining $5L$ -line.
- The R -line of a point P on the QL - PI -circle is the degenerated CSC -circle of P .
- The R -line of a point P on the inscribed conic is tangent to the QL - PI -circle.
- The R -line of a contact-point P of the inscribed conic is the tangent to the QL - PI -circle in the corresponding QL - PI -point.
- The R -line for a focus F_i of the inscribed conic is the perpendicular bisector of the other focus F_j and its reflection in the QL - PI -circle.
- The R -line of P_I is the polar of P_x wrt the inscribed conic.
- The R -line of P_4 is a perpendicular line to $P_I P_x$ through the inverse of P_x wrt the QL - PI -circle.

The R -point of a line or circle

Definition: The CSC -circles of points on a given line or circle have radical axes with a common point (R -point of a line or circle).

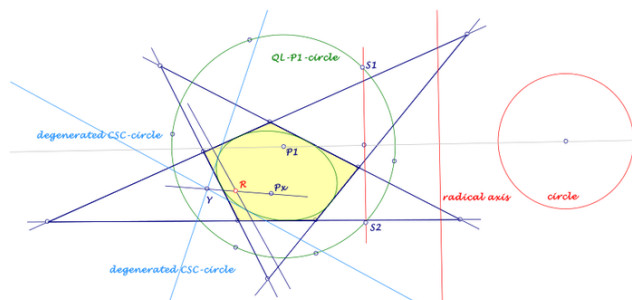


Construction of the R -point for a circle:

If there are two intersections of the circle and the QL - PI -circle, the corresponding degenerated CSC -circles give the R -point.

If there are not two intersections of the circle and the QL - PI -circle, consider the radical axis and a parallel through its pole wrt the QL - PI -circle, which cuts the QL - PI -circle in S_1 and S_2 , whose degenerated CSC -circles intersect in Y . The intersection of YP_x and the polar of Y wrt the inscribed conic give the R -point of the circle.

The R -point of a line can be constructed in the same way, using the line as radical axis.



- The R -line of a point and the R -point of a line are inverse transformations.
- The R -point of the CSC -circle of P is P again.
- All circles with the same radical axis wrt the QL - PI -circle have the same R -point.

- The R -point of a $5L$ -line is the QL - PI -point for the remaining QL -component.
- The R -point of a tangent in P at the QL - PI -circle is the contact-point of the degenerated CSC -circle of P and the inscribed conic.
- The R -point of a tangent in P at the inscribed conic is the contact-point of the CSC -circle of P and the QL - PI -circle.
- The R -point of the line $L = 5L-P1.5L-P2.5L-P3$ is the pole (wrt the inscribed conic) of the line through P_x and the 2nd intersection of L and the hyperbola Hy .

Further example: Consider the circle of the 5 QL - $P4$ -points (see *QFG*-message 710): The R -point of this circle has a CSC -circle, centered on the line $5L-P1.5L-P2.5L-P3$.

R -points for Hy -tangents

- The R -points for tangents at the orthogonal hyperbola Hy (through $P_l, P_x, F_1^\circ, F_2^\circ$, centered in Z) give a parabola: directrix P_lP_x , focus is the R -point of a perpendicular line to P_lZ in Z .

R -points of concentric circles

- The R -points for concentric circles with center P give a line through P_x and the pole (wrt the inscribed conic) of the R -line of P .

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2014-11-06.pdf

Message: #791

Date: 07/11/2014 5:25:37

From: seiichikiri

Subject: The system of the coordinates of Pentagon Geometry

Dear Chris,

In your message #758, you wrote:

5G-p1 5G-Midlines Crosspoint,

5G-p2 5G-BiCircle Crosspoint,

5G-p3 5G-Miquel Five Circles Center.

According to my interpretation, it seems to me that 5G-p1 is for 5G-p4 in your Cabri and 5G-p2 is for 5G-p1 in your Cabri.

So I wrote that (4) in message #763 were 5G-p1.

Sorry for the late response!

Best regards,

Seiichi.

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Message: #792
Date: 07/11/2014 11:04:19
From: Chris van Tienhoven
Subject: "lost" Morley-documents

Dear Seiichi,

This is about two "lost documents" (or hard to get) of Frank Morley published in Japanese Journals.

Document 1

In Hyacinthos #5105 †) Alexander Bogomolny says:

> Math Assoc of Japan for Secondary Mathematics,

> > 6(1924) [sic - with no title or page nos.]

I wonder if any one has ever seen the paper, because I too has never seen it referred to by the title.

Is there any chance that you could discover this document in Japanese archives?

However I noticed there is a reference in the document "The theorem of Morley - J.W. Peters (university of Illinois)":
15 F.Morley, Journal for the Mathematical Association of Japan for Secondary Education V.6 (1924) pp. 260-262.

Document 2

Then I stumbled across this document of F. Morley:

81CC._____, On the intersections of the trisectors of the angles of a triangle,

J. Math. Assoc. Japan, Sec. Ed., 6 (1924) 260-262. See [53].

reference from

THE MORLEY TRISECTOR THEOREM

CLETUS O. OAKLEY AND JUSTINE C.BAKER

Could you find this document too?

Best regards,
Chris

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†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[11\]](#).

Message: #793
Date: 07/11/2014 11:12:21
From: Bernard Gibert
Subject: "lost" Morley-documents

Dear Chris,
here is the Oakley-Baker paper.
I don't have the other but would be interested as well.
Best regards
Bernard

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Message: #794
Date: 07/11/2014 11:27:31
From: Antreas Hatzipolakis
Subject: "lost" Morley-documents

Dear Chris and Seiichi
The first is available on the web but registration required.
Please look at:
http://ci.nii.ac.jp/vol_issue/nels/AN00195268_en.html
http://ci.nii.ac.jp/vol_issue/nels/AN00195268/ISS0000229063_en.html
<http://ci.nii.ac.jp/naid/110003852328/en>
APH

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Message: #795
Date: 07/11/2014 11:38:09
From: Antreas Hatzipolakis
Subject: "lost" Morley-documents

Additionally:
searched the archive with keyword trisectors.
Here is the result
<http://ci.nii.ac.jp/search?q=trisectors&range=0&count=20&sortorder=1&type=0>
APH

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Message: #796
Date: 07/11/2014 8:54:14
From: chris.vantienhoven
Subject: "lost" Morley-documents

Dear Bernard, dear Antreas,
Thanks for all information.
I just downloaded the "lost" document of F. Morley from the Japanese site.
Best regards,
Chris

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Message: #797
Date: 08/11/2014 6:52:08
From: Antreas Hatzipolakis
Subject: Applying the Orthopole Transformation to Pentangle

Telv Cohl applied the Orthopole Transformation(*) to pentangle as follows (my wording):
Let ABCDE be a pentangle and P a point.
Denote:
Oae, Obe, Oce, Ode the circumcenters of BCD, CDA, DAB, ABC, resp
Tae, Tbe, Tce, Tde = the orthopoles of OaeP, ObeP, OceP, OdeP wrt triangles BCD, CDA, DAB, ABC, resp.
Similarly the orthopoles Tab, Tac, Tad, Tbc, Tbd, Ted.
Denote:
(Wa) = the circle (Tab, Tac, Tad, Tae)
 [we get by the orthopole transformation]
and similarly (Wb), (Wc), (Wd), (We).
1. (Wa), (Wb), (Wc), (Wd), (We) are concurrent at a point Q.
2. If P moves around a circle, then the locus of Q is also a circle.
Reference:
<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=613101&p3646902&p3646902>
Antreas
(*)
<http://chrisvantienhoven.nl/index.php/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-objects/artikelen-qa/229-qa-tf3>

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Message: #798
Date: 10/11/2014 12:11:29
From: chris.vantienhoven
Subject: Morley's miracle in a 5-line

Did you find the construction of the mononephroïd, tangent to the 5 cardioïds?
(I suppose it's center is Kantor's center QL-P2)
Did you find the construction of the 64 axes for the QL (by quadrisecting the angles)?
3 intersection points of these quadri-sectors.

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Message: #799
Date: 10/11/2014 12:15:19
From: Chris van Tienhoven
Subject: Morley's miracle in a 5-line

Dear Bernard,
I forgot the attachment.
I have enclosed it now.
Best regards,
Chris

Van:
Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
Verzonden: maandag 10 november 2014 12:11
Aan: Quadri-Figures-Group@yahoogroups.com
Onderwerp: Re: Morley's miracle in a 5-line
Dear Bernard,

[BK]

> Did you find the construction of the mononephroid,
> tangent to the 5 cardioïds?
> (I suppose it's center is Kantor's center QL-P2)

No I didn't find the construction of the mononephroid yet.
After that I returned from holiday I hardly had time to continue
my research.
I hope to go on as soon as possible.

[BK]

> Did you find the construction of the 64 axes for the QL
> (by quadrisecting the angles)?
> There are then $5 \cdot 64 = 320$ lines for the 5-L
> and the 256 centers of the multinephroids
> tangent to the 5 lines are on these 320 lines,
> each center as intersection of 5 lines (one in each set)
> and 4 centers on each line.

This is exactly what I tried to do one year ago already and some
weeks ago again.
I drew all quadri-sectors in a 4-Line and tried to find lines
through at least 3 intersection points of these quadri-sectors.
However I did not find any at all!
So I am really doubting if the 64 axes can be found this way.
So I started constructing nephroids because the axes are
supposed to be the loci of the centers of these nephroids.
First in a given triangle.

In a first attempt I noticed that the locus of the centers of these nephroids inscribed in a triangle consisted of 8 lines pairwise crossing in a point on the sidelines (4 cross points per sideline).

See attachment.

Later I noticed there were even more nephroids.

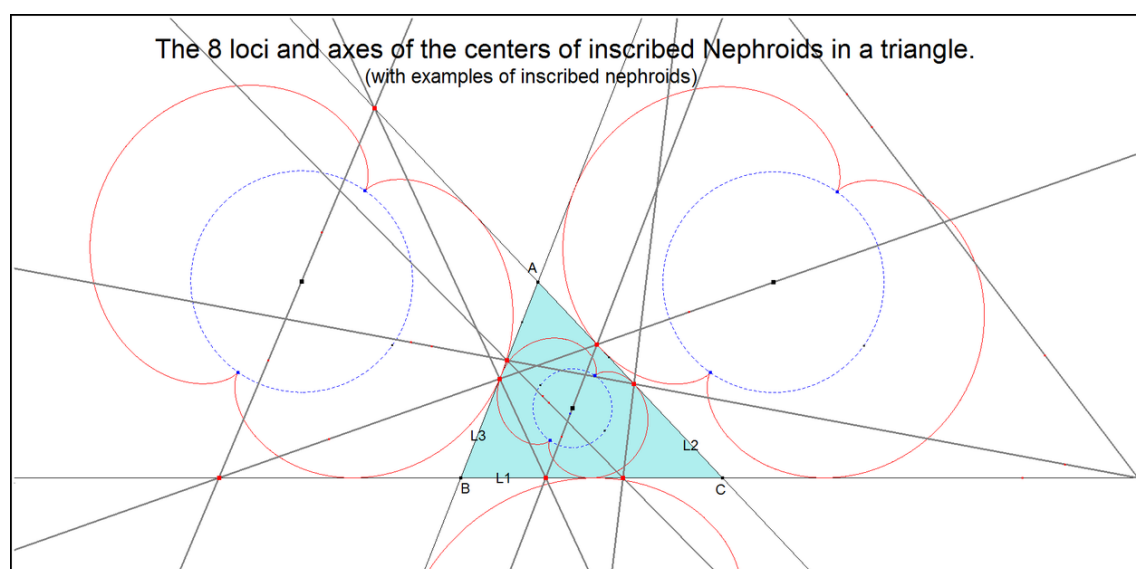
Meanwhile I had a discussion with Bernard Gibert about the subject and he confirmed (algebraically) that there were even 16 lines pairwise crossing at the sidelines.

Remarkable: the lines weren't crossing at angles of $n \cdot \pi/4$ like described by Morley.

Then I tried to construct a nephroid in a 4-Line. When I remember it right I only managed to find 8 nephroids inscribed in a 4-Line. So now I have to make the step to a 5-Line.

That's how far I got. It really intrigues me (despite the discrepancies with Morley's theorems) so I am not ready with it yet.

Best regards,
Chris



5L-Construction-Nehproid-25-in TR-.png

Message: #800

Date: 10/11/2014 12:21:16

From: chris.vantienhoven

Subject: Applying the Orthopole Transformation to Pentangle

Dear Antreas,

Very nice construction. Telv Cohl actually constructs the QA-Orthopole Circles (with circumcenter QA-Tf3) of the 5 Component Quadrangles in this 5-Point construction.

The result is a real 5P-center and there are not so many of them known yet.

n-Gons always have n Component (n-1)-Gons. Somehow when we have circles in the (n-1)-Gon they often concur at the level of an n-Gon.

Best regards,

Chris van Tienhoven

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Message: #801
Date: 10/11/2014 7:26:24
From: eckart_schmidt@t-online.de
Subject: 5L-Limacon

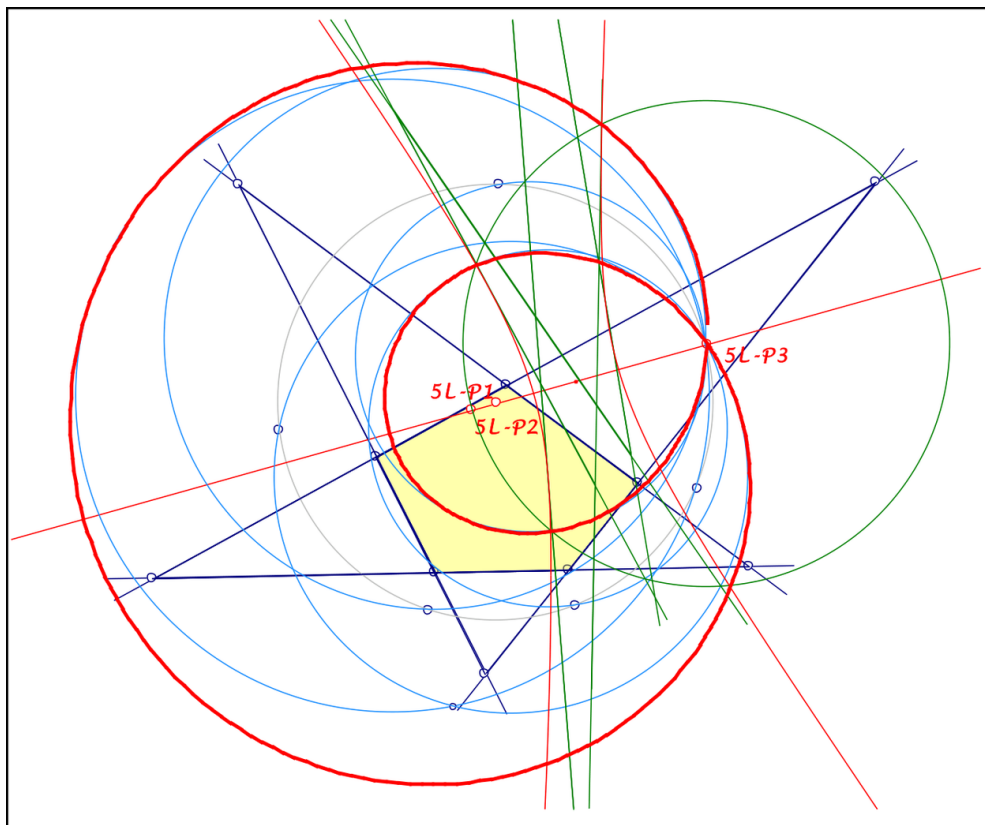
Dear Bernard K.,

I am not familiar with the references, you mentioned in message #716: Is there already a construction of the limaçon, tangent to the 5 Miquel circles of the QL-components? I found the following construction:

Take an arbitrary circle C_i round 5L-P3,
... consider the inverses of the 5 Miquel circles wrt C_i ,
... which give a new 5L,
... with inscribed conic C_o ,
... whose inverse wrt C_i give the limaçon:
... node is 5L-P3,
... axis is 5L-P1.5L-P2.5L-P3,
... center is 5L-P2,
... distance of the two cusps is the diameter of the QL-P4-circle.

If this is well known, please let me know, where it is published.

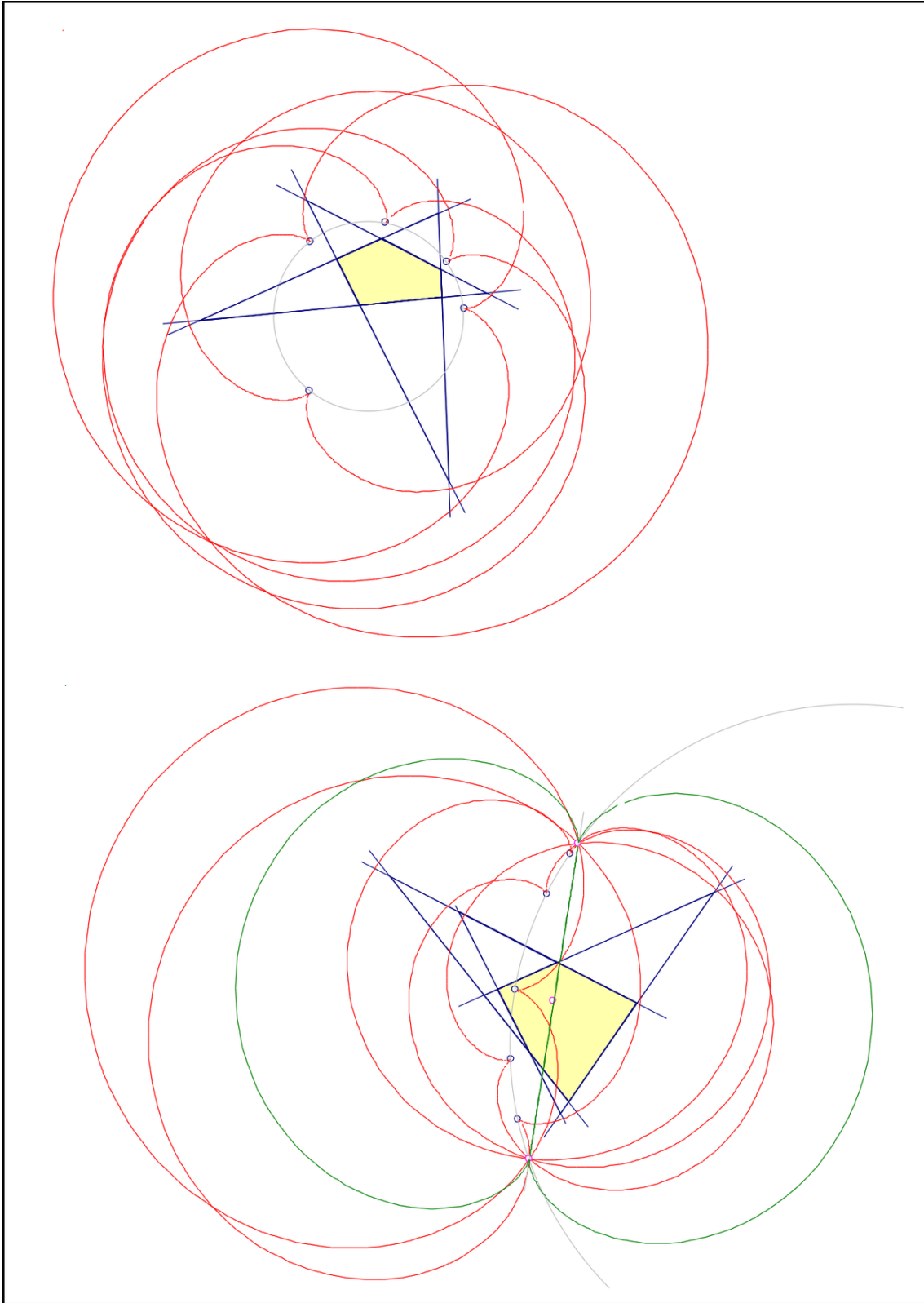
Best regards Eckart



2014-11-10.pdf

Message: #802
Date: 11/11/2014 11:54:51
From: eckart_schmidt@t-online.de
Subject: new Pantangle Point

Dear Bernard K.,
sorry I need your advice. In message #716 you wrote:
... the 5 monocardioids are envelopped by a nephroid, each
cardioid passing through the 2 cusps of the nephroid, ...
Drawing with Cabri, there are 5L, where this does not hold. On
the other hand there are 5L, where the 5 cardioids have 2 common
points on the QL-P1-circle symmetric wrt 5L-P1.5L-P2.5L-P3, but
these are not the cusps of an envelopping nephroid (see attached
file).
I don't find my error.
Best regards Eckart



2014-11-11.pdf

Message: #803
Date: 11/11/2014 12:18:15
From: bernard.keizer
Subject: 5L-Limacon

Dear Eckart,
Beautiful and simple construction!
5L-P3 is a focus of the hyperbola inscribed in your 2nd 5L
1) The references given in my article are almost all available on the web.
For example, I've just checked On the metric geometry of the plane n-line on page 102 and followings, Morley describes the limaçon of the n-line
2) Morley doesn't give a construction
3) Every n-line has the property that the n centric circles of the n-1 lines
a) have their centers concyclic
b) have a common point
4) Your construction is available for any n-line, if you are able to draw the n centric circles of the n-1 lines.
All the lines obtained in the inversion are tangent to the same conic.
Best regards Bernard

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Message: #804
Date: 11/11/2014 12:37:44
From: bernard.keizer
Subject: new Pantangle Point

Dear Eckart,
I'm sorry, I hope you didn't loose too much time!
In fact, you're right, the 5 cardioïds have generally not 2 common points.
Did you succeed in drawing the nephroïd?
I suppose it's center is 5L-P2 ...
By the way, there was another error in my message #716: the point described with equal ratio between the points P3 and P4 is the Kantor-Hervey point of the 5-L and not the 1st orthocenter, but I've corrected in another message with the 1st and the 2nd orthocenter and the K-H point
Best regards Bernard

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Message: #805
Date: 11/11/2014 11:04:32
From: bernard.keizer
Subject: new Pantangle Point

Dear Eckart,
I've found where I took this idea with the cardioïds through the cusps of the nephroïd!
It wasn't an invention from me.
The reference is Morley On reflexive geometry page 20
"The cardioïds of 4 from 5 lines are osculants of a C4; and in particular meet at two points (the cusps)."
Best regards
Bernard

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Message: #806
Date: 13/11/2014 12:36:14
From: bernard.keizer
Subject: Morley's miracle in a 5-line

Dear Chris,

Thank you for your long message!

First, I was desperate, then I revisited all my knowledge and documentation about the QL.

Let's concentrate on page 25 of my article on the web and on the references 25, 19 and 9 of the bibliography on page 55

1) It's true that the locus of the centers of the nephroïds inscribed in the triangle is made of 16 lines.

For any epi-or hypocycloïd with definition 2 points on a circle at different speeds p and q envelop the curve, the number of lines is $(p+q)^2$. For the deltoïd, $q = -1$ and $p = 2$, there is one line (perpendicular bisector of the Euler segment, for the cardioïd, $q = 1$ and $p = 2$, there are $3^2 = 9$ lines and for the nephroïd, $q = 1$ and $p = 3$, there are $4^2 = 16$ lines.

More interesting, the number of curves inscribed in 4 lines is $(p+q)^3$, meaning one deltoïd, 27 cardioïds and 64 nephroïds.

The reason is that there is only one solution for inscribing a QA from a given shape in a QL and the lines through 2 primary points make angles from $1/(p+q)$ of the angles of the tangents.2)

2) I'm always convinced that there are 256 incenters on the 320 lines ...

See pages 253 and 254 of Bricard's article (available on the web)

3) The conclusion of the 1st point is that there are generally no nephroïds tangent to 5 lines!

I'm not surprised that you found only nephroïds tangent to 4 lines.

In other words, the 256 curves C4 cannot be nephroïds!

In fact, Morley doesn't speak of nephroïds, but of curves C_n , which he calls ennacardioïds ...

Maybe we have to revisit all Morley in order to understand.

I hope this short contribution will be somehow useful for you.

Best regards

Bernard

PS You have surely already noticed in the last exchange with Eckart an error in Morley about the 5 cardioïds through the 2 cusps of the nephroïd!

Message: #807
Date: 13/11/2014 2:04:08
From: chris.vantienhoven
Subject: Morley's miracle in a 5-line

Dear Bernard K,
Yesterday I was contemplating in my car and reached the same conclusion as you did.
My reasoning was as follows.
I will use the names 3L-figure or 3-Lines for a Triangle and 4L-figure or 4-Lines for a Quadrilateral.
The centers of all 3L-inscribed nephroids lie on one of 16 axes. There are no centers outside these 16 axes.
When I made a picture in a 4L-figure drawing these 16 axes for all component 3-Lines I noticed there was no overlap of axes. They all were crossing each other. So eventual Nephroids in a 4-Line had to lie at the intersection points of axes originating from different Component 3-L figure. I noticed that there were (probably) 64 points that lie on 4 axes each originating from a different Component 3-Line.
This means that the locus of centers of Nephroids inscribed in a 4L-figure consists of a limited set of separate points.
When we have a configuration with 4 Lines and an inscribed Nephroid and we add a random extra line, then it is not likely that it will be tangent to this Nephroid. Of course this is valid for all 4L-inscribed Nephroids.

So my conclusion was the same as yours:
So this question remains: What did Morley mean with his Cn-curves ???
And do the centers of these Cn-curves lie on sets of axes that intersect at angles $i \cdot \pi / n$???
And are they constructible from 4-sectors in 4L-figure, 5-sectors in a 5L-figure, etc. ???
Anyway there is enough of stuff to think about
Best regards,
Chris

p.s. the part of your message about "epi-or hypocycloid with definition 2 points on a circle at different speeds p and q"
epi-or hypocycloid with definition 2 points on a circle at different speeds p and q

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Message: #808
Date: 13/11/2014 9:52:05
From: Antreas Hatzipolakis
Subject: A problem with Circles in Quadrilateral

Four lines meet each other in six distinct points. Six circles centered at these points are constructed. It is known that three circles centered on any of the first three lines share a common point. Prove that the circles centered on the fourth line also share a common point or do not intersect each other at all.

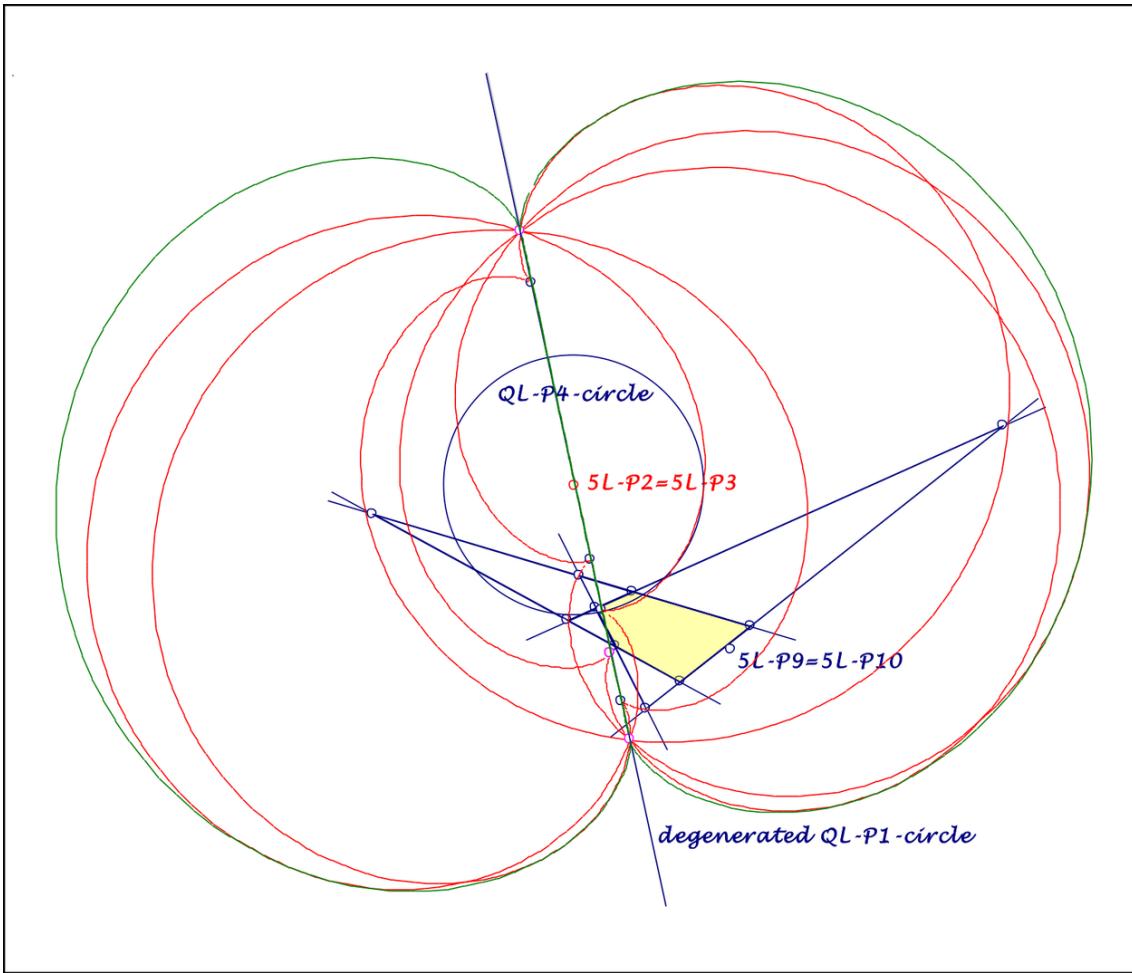
Source:

<http://www.artofproblemsolving.com/Forum/viewtopic.php?t=600073&p3651479&#p3651479>
aph

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Message: #809
Date: 14/11/2014 8:49:36
From: eckart_schmidt@t-online.de
Subject: A special 5L-mono-nephroid

Dear Bernard K., dear Chris,
for special 5Ls I found the nephroid tangent to the 5 QL-cardioids:
Consider a 5L with the property $5L-P2 = 5L-P3$,
... then the QL-Ci4-circles are identic,
... the midpoint of the common QL-Ci4-circle
coincides with $5L-P9$ and $5L-P10$, ,
... the QL-P1-circle degenerates in a line,
... the 5 cardioids have two common points
(this is always the case, when $5L-P2$ and $5L-P3$
lie on the same side of $5L-P1$),
... the two points lie on the degenerated QL-P1-circle
with midpoint $5L-P2 = 5L-P3$
and distance double the diameter of the QL-P4-circle,
... these two points are the cusps of the nephroid.
These are Cabri observations! I hope, you can confirm these properties.
Best regards Eckart
PS: Perhaps here are the same doubts in nephroids, as you discuss wrt the multi-nephroids.



2014-11-14.pdf

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Message: #810
Date: 14/11/2014 1:13:25
From: bernard.keizer
Subject: A special 5L-mono-nephroid

Dear Eckart, dear Chris

The case you describe is a special case, very interesting as we have the property mentioned by Morley as general that the 5 cardioids are on the 2 cusps of the nephroid which envelops them.

In this special case, the 5 lines are tangent to a deltoid:

- ... then the QL-Ci4 are identic (it's the Hervey circle inscribed in the deltoid) as well as the QL-P3
- ... then the Miquel circles have all the same radius (equal to the radius of the Hervey circle)
- ... then the center of the P4 circle
is necessary the common point to the 5 circles,
id est $5L-P2 = 5L-P3$ and the radius of this circle is also equal
to the radius of the common Hervey circle and of the Miquel circles.
- ... then the center of the Miquel circle is the infinity point,
as the inverse of the center of the circle of the P4 in this circle,
id est the 5 Miquel points are on a line.

I don't see immediately why $5L-P9$ is the common QL-P3.

I don't know the point $5L-P10$

I suppose that the deltoid is inscribed in the nephroid (as the length between the cusps is twice the diameter of the Hervey circle).

That's all for today ...

Best regards

Bernard

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Message: #811
Date: 14/11/2014 5:33:04
From: bernard.keizer
Subject: A special 5L-mono-nephroid

Dear Eckart, dear Chris

I forgot an interesting and obvious detail (visible on your figure)

As the 5Miquel circles have the same radius, the 5 cardioïds have the same size.

The cord through the cusps of the nephroïd is for each cardioïd through it's own cusp and has the length 4 times the radius of the inner circle (well-known property of the cardioïd)

I've found another property in Morley: take 2 circumscribed circles in the QL, they are tangent to the cardioïd and cut in 2 points the Miquel point P1 and one vertice of the QL; take a variable line through this vertice, it cuts the 2 circles in 2 other points and the tangent to the 2 circles in these points intersect on the cardioïd.

After Morley, the same goes on in chain: take one of the 10 circles of the 5-L, it's tangent to 2 cardioïds; any tangent to the circle cuts the cardioïds in 2 points and the tangents in these points to the 2 cardioïds intersect on the so called nephroïd and so on ...

I hope I didn't make an error in the interpretation, but anyhow, I have checked the property for the cardioïd (which gives another method to draw it); maybe you will be able to use the property for drawing the nephroïd ...

Good luck

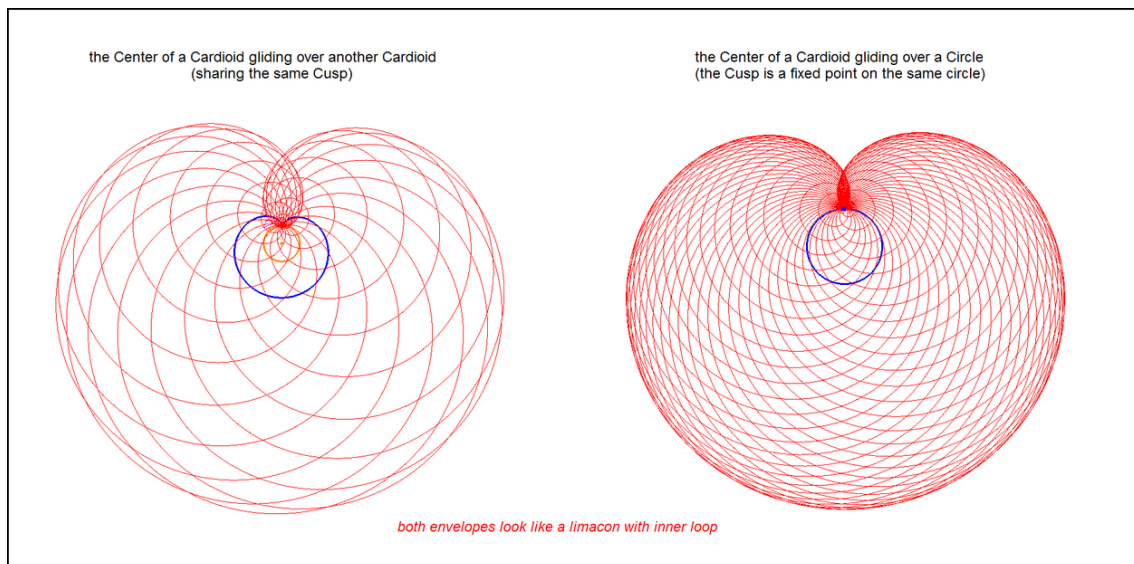
Best regards

Bernard

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Message: #812
Date: 15/11/2014 9:45:28
From: Chris van Tienhoven
Subject: Morley's miracle in a 5-line

Dear Bernard K, Bernard G, Eckart,
I did some research wrt the supposed Nephroid.
In a Quadrilateral/4Line the Mono Cardioid is an envelope of circles with its center on a reference circle.
Following Morley's theory what can we expect in a 5Line?
I thought about two possibilities:
1. an envelope of Cardioids with its center gliding over a circle.
2. an envelope of Cardioids with its center gliding over another cardioid.
I made pictures of both situations.
The outcome is certainly not a Nephroid, but both produce envelopes looking like a limaçon with inner loop. See enclosed picture. See also <http://en.wikipedia.org/wiki/Lima%C3%A7on>.
The question stays: Is this what Morley meant?
Bernard K, you mentioned that Morley himself used the word Ennardioids.
I found two references at Google on this word;
Morley's document "on the Reflexive Geometry".
surprisingly this document: <http://quod.lib.umich.edu/u/umhistmath/ABR0258.0001.001?rgn=main;view=fulltext>
On the pentacardioid ... by Phillips, Edward Charles
(a contemporary of Morley)
Summarizing I think we should let go the idea of Nephroids as being used in Morley's theories.
Now we can investigate the ennardioids.
I really wonder of what degree this curve is and how it looks.
Can we catch it in a Pentalateral?
Etc.
Best regards,
Chris



5L-Qu1-MonoLimacon-12.png

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Message: #813
Date: 15/11/2014 9:54:56
From: chris.vantienhoven
Subject: A special 5L-mono-nephroid

Dear Bernard K,

[BK]

> I've found another property in Morley: take 2 circumscribed circles in the QL, they are tangent to the cardioïd and cut in 2 points the Miquel point P1 and one vertice of the QL; take a variable line through this vertice, it cuts the 2 circles in 2 other points and the tangent to the 2 circles in these points intersect on the cardioïd.

> After Morley, the same goes on in chain: take one of the 10 circles of the 5-L, it's tangent to 2 cardioïds; any tangent to the circle cuts the cardioïds in 2 points and the tangents in these points to the 2 cardioïds intersect on the so called nephroïd and so on ...

You say "any tangent to the circle cuts the cardioids in 2 points".

In my drawings they cut each cardioid in 2 points, that makes 2x2 points.

Which one to choose?

Another problem is that in a 4Line we have 4 circles with a common point QL-P1.

However the 5 mono Cardioids in a 5Line do not have a common point I noticed from a picture.

Still the method you mention can be useful when interpreted in the right way.

Which way is the right way (philosophical question too).

Best regards,

Chris

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Message: #814
Date: 15/11/2014 10:21:21
From: bernard.keizer
Subject: A special 5L-mono-nephroid

Dear Chris,
I leave home for a few days and I will be in Paris next week;
Shortly, I just gave you what I found, my reference was the one
you mentionned on the reflexive geometry 1907 by Morley.
I also noticed that there were 4 points! I don't know the right
way!
2 curves C_{n-1} are tangent to one C_{n-2} and to C_n .
For the cardioïd C_3 , the 2 C_2 are the circles and C_1 is the
point (other than P_1).
For the C_4 , the 2 C_3 are the cardioïds and the C_2 is the circle
tangent to the cardioïd.
Morley uses also the words paranephroïd and tetracardioïd for
the same curve C_4 .
Best regards
Bernard

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Message: #815

Date: 16/11/2014 8:59:43

From: eckart_schmidt@t-online.de

Subject: Construction of Morley's C4 for a 5L

Dear Bernard K., dear Chris,

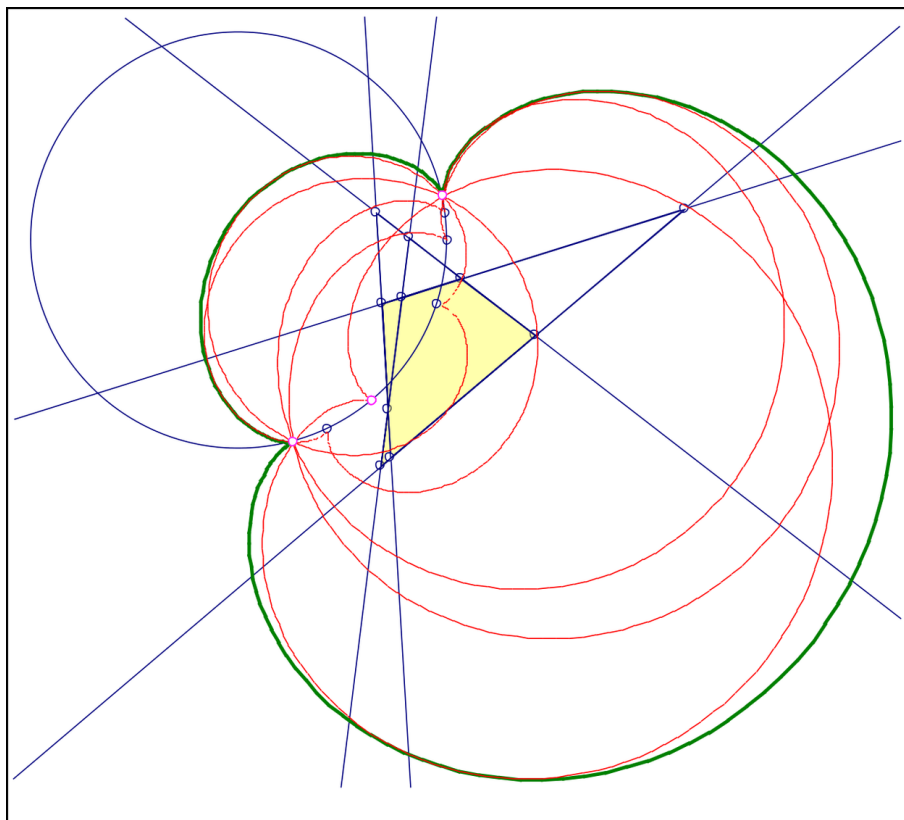
I think, the following construction gives Morleys "ennacardioid" for a 5L:

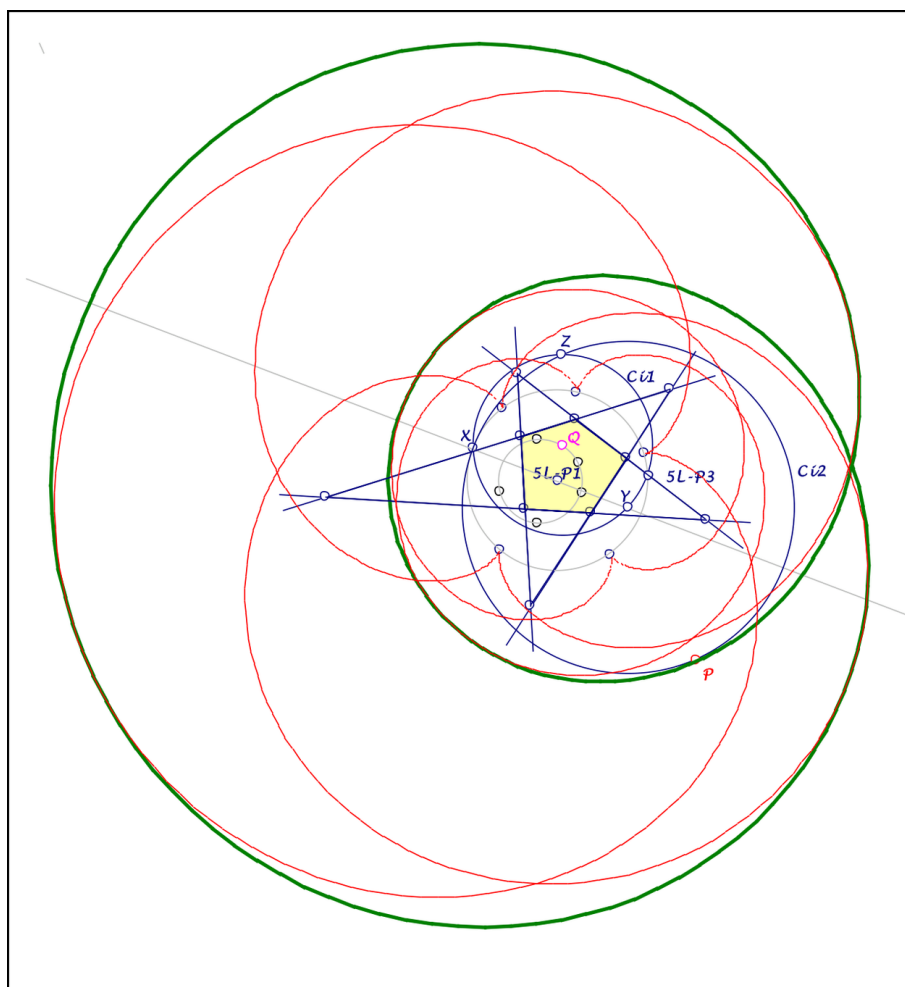
- ... Let Q be a variable point on the QL-P4-circle,
- ... let Ci1 be a circle round Q through 5L-P3,
- ... let X be the second intersection of Ci1 and the QL-P1-circle,
- ... let Y be the second intersection of X.5L-P1 and Ci1,
- ... let Ci2 be a circle round Y through X,
- ... let Z be the second intersection of Ci1 and Ci2,
- ... then Z reflected in Y is a point P of Morleys "ennacardioid" (see attached files).

Background:

- ... Q is a generalization of QL-P4.
- ... Morley's "On Reflexive Geometry".

Best regards Eckart



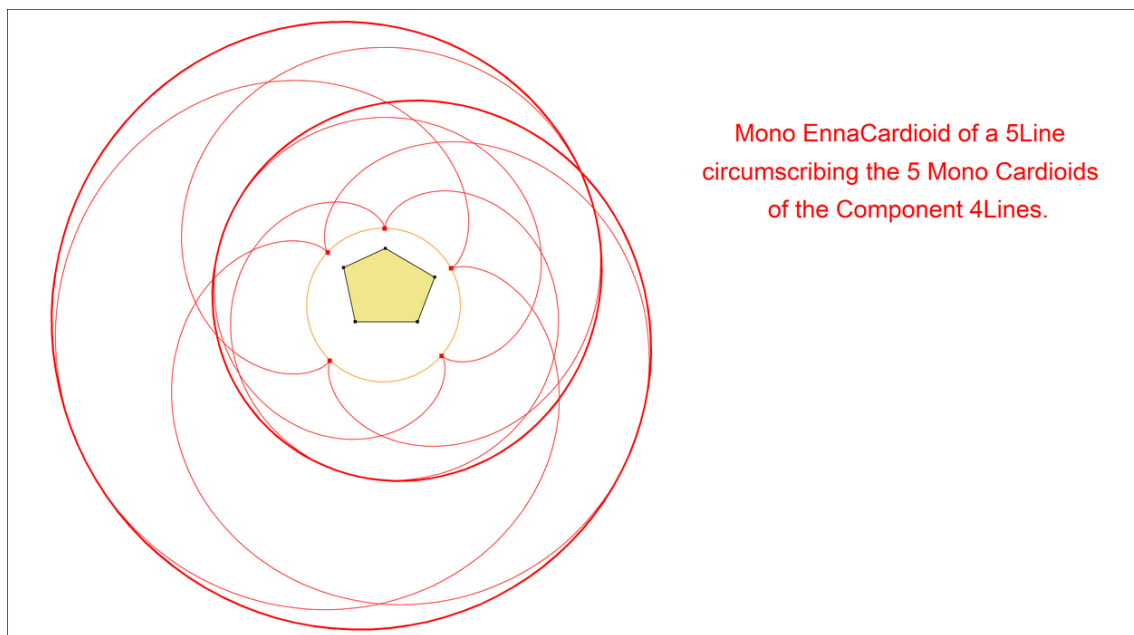


2014-11-16a.pdf

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Message: #816
Date: 16/11/2014 11:29:57
From: Chris van Tienhoven
Subject: Construction of Morley's C4 for a 5L

Dear Eckart,
I was working on it all day too. But you finished the job.
Congratulations !!!
It is good to see that is a limaçon(-like) figure with inner loop indeed.
I made a picture too and it is so beautiful!
See attached picture.
It is good to realize that we now found the Mono EnnaCardioid in a 5Line.
Like Morley predicted there are 256 Multiple EnnaCardioids in a 5Line.
I wonder if they are look alikes of the Mono EnnaCardioids.
Also I wonder what can be said about the cusp of the Mono EnnaCardioid.
Best regards,
Chris



5L-Qu1-EnnaCardioid-00.png

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Message: #817
Date: 17/11/2014 9:24:46
From: eckart_schmidt@t-online.de
Subject: Construction of Morley's C4 for a 5L

Dear Chris,
wrt the cusps of the Mono EnnaCardioid:
... There are real cusps,
 if 5L-P1 lies outward the QL-P4-circle.
... The cusps lie on the QL-P1-circle.
... The contact points of the tangents from 5L-P1 to the
 QL-P4-circle as Q (see my construction) give the cusps.
Best regards Eckart

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Message: #818
Date: 17/11/2014 9:23:11
From: eckart_schmidt@t-online.de
Subject: Construction of Morley's C4 for a 5L

Dear Chris, dear Bernard K.,
I think, the following description of the Mono EnnaCardioid of a 5L is more in the sence of Morley, using his "osculant circles", which had to be tangent to the QL-P1-circle.
Consider first another limacon: Envelope of circles through 5L-P3 centered on the QL-P4-circle.
Now use circles centered on this limacon and tangent to the QL-P1-circle *) and the envelope is the Mono EnnaCardioid.
*) There are two possibilities for circles tangent to the QL-P1-circle. 5L-P1 and 5L-P3 are inverse wrt the QL-P4-circle. If 5L-P3 lies outward the QL-P4-circle, the first limacon has a loop: Then take circles with 5L-P3 inside and the envelope is a limacon. If 5L-P3 lies inside the QL-P4-circle, the first limacon has no loop: Then take circles with 5L-P3 outside and the envelope has two cusps.
Best regards Eckart

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Message: #819
Date: 18/11/2014 6:58:36
From: bernard.keizer
Subject: Construction of Morley's C4 for a 5L

Dear Chris, lieber Eckart

Dear friends, as I told you, I'm now in Paris with a computer, but without my library on the QL and the n-lines.

Wunderbar, the problem has been solved during the week-end! I remain totally bluffed!

I have one remark about the naming and one question about the construction.

1) the naming: see my message #814

Ennacardioids is the generic name used by Morley for the family of curves linked to the n-lines, C_n for $n+1$ -lines ...

For example, Philips, who was a pupil of Morley (and later became Father Philips, if I'm not wrong), uses the word pentacardioid for the 6-lines. (Thank you to Chris for the reference of the article).

Nephroid was misleading, but Morley uses paranephroid (I can understand by looking at the 2nd picture of Eckart, as it looks like a nephroid with a compressed side and an expanded side) and tetracardioid, which seems the most appropriate name.

2) the construction: I can understand that Q is a point P_4 and X a point P_1 , so that the tetracardioid is the envelop of the cardioids with the center on the circle of the P_4 and cusp on the circle of the P_1 , under the condition that $QX = Q_5L - P_3$; every such cardioid is tangent in a point P

a) to the tetracardioid

b) to a circle with center Y through X; the circle C_{i1} is the Miquel circle and the circle C_{i2} one of the 4 reference circles of a QL ...

As I don't want to die stupid, I wonder where you found this construction? (it's not the one I found in Morley)

By the way, could Eckart tell me which point is 5L-P10 and Chris tell me if he found the 2nd orthocenter.

Congratulations again

Best regards

Bernard

PS We have now to face the problem of the 320 axes and the 256 little tetracardioids, if we believe Morley!

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Message: #820

Date: 18/11/2014 7:12:58

From: Antreas Hatzipolakis

Subject: Construction of Morley's C4 for a 5L

>
> On Tue, Nov 18, 2014 at 7:58 PM, bernard.keizer@yahoo.com
[Quadri-Figures-Group]
> < Quadri-Figures-Group@yahoogroups.com >
> wrote:
>> I have one remark about the naming and one question about the
>> construction.
>> 1) the naming: see my message #814
>> Ennacardioids is the generic name used by Morley for the
family of curves
>> linked to the n-lines, Cn for n+1-lines ...
>> For example, Philips, who was a pupil of Morley (and later
became Father
>> Philips, if I'm not wrong), uses the word pentacardioid for
the 6-lines.
>> (Thank you to Chris for the reference of the article).
>> Nephroid was misleading, but Morley uses paranephroid (I can
understand by
>> looking at the 2nd picture of Eckart, as it looks like a
nephroid with a
>> compressed side and and expanded side) and tetracardioid,
which seems the
>> most appropriate name.

And I have a remark about the naming.
Ennacardioid is wrong, if the enna- is meant as the prefix of
the number 9.
The correct is enneacardioid.
The Greek number prefices (1-10) are:
mono-, di-, tri-, tetra-, penta-, hexa-, hepta-, octa-,
ennea-, deca-
APH

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Message: #821
Date: 18/11/2014 11:22:04
From: chris.vantienhoven
Subject: Construction of Morley's C4 for a 5L

Dear friends,
Pretty cute all this interest in the name of the new curve.
First about your remark Antreas.
I don't think there is a link with the number 9.
So I think the name given by Frank Morley "ennacardioid" is a good one.
All the more because the figure found by Eckart is probably of the 5th degree (according to Cabri).
Like Bernard K mentioned the name Ennacardioid comes from Morley in his document "On Reflexive Geometry". In this very document (page 15) Morley writes:
* C1 = point
* C2 = circle
* C3 = cardioid
* C4 = tetracardioid (he also uses also the name paranephroid)
* a Cn can have (n-2) cusps (page 16)
In the document "On the pentacardioid" from his student Edward Charles Philips is written at page 1:
* C5 = pentacardioid
So Morley and Philips use the name Ennacardioid as a generic word for a type of curves Cn circumscribing other Ennacardioids of a lower level. The specific Cn curves get specific names with greek number prefix.
Following this method the newly found curve occurring in a system of 5 lines should be a C4 being a tetracardioid.
I used in a former message the word limacon with inner loop to explain the form of the expected curve.
However the tetracardioid is not a limacon, because the definition of a limacon is too simple compared with the forms the tetracarioid adopts.
Best regards,
Chris

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Message: #822

Date: 19/11/2014 11:35:54

From: eckart_schmidt@t-online.de

Subject: Construction of Morley's C4 for a 5L

Dear Bernard K., dear Chris,
thanks for your messages. How to find the construction: The deciding step was, that the QL-P1-circle as locus for the cusps of the cardioids is also the envelope of the "osculant circles" of C4 (see Morley "On Reflexive Geometry", page 18).

Further remarks wrt Morley's article:

(1) I found two QL-properties (p.22):

... QL-Ci3 is the locus of points, which lie with its reflections in the 4 lines on an orthogonal hyperbola.

... QL-Qu1 is the locus of points, which lie with its reflections in the 4 lines on a parabola.

(2) Morley's constellation (see attached file):

... A 5L has 10 circumcircles of the 3L-components,

... a 5L has 5 cardioids QL-Qu1 of its 4L-components,

... each circumcircle contacts two cardioids.

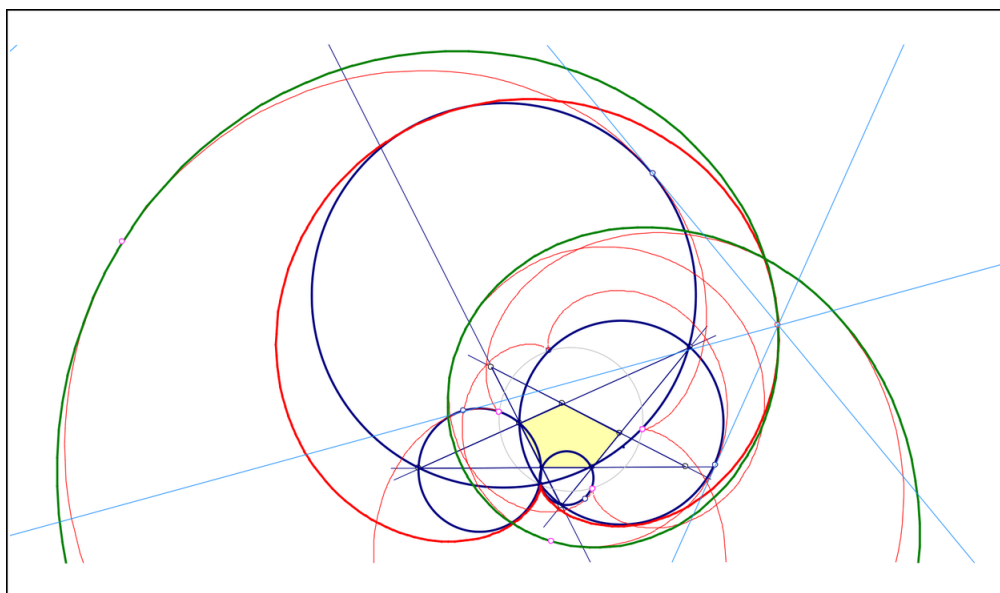
... A fixed cardioid has contact points with four circles,

... the tangents in the 2nd contact points wrt another cardioid coincide in the contact point of the fixed cardioid and the C4,

... my construction (see #815) gives this contact point for $Q = QL-P4$ of the corresponding 4L.

Best regards Eckart

PS for Bernard: 5L-P10 is the 1st orthocenter.



2014-11-19.pdf

Message: #823
Date: 19/11/2014 12:27:06
From: bernard.keizer
Subject: Construction of Morley's C4 for a 5L

Dear Chris
Long live the tetracardioid!
For Antreas, the ennacardioids belong to the history since more than a century!
Maybe Morley searched simply a name sounding greek and meaning any number of lines ...
Best regards
Bernard

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Message: #824
Date: 19/11/2014 3:41:05
From: Antreas Hatzipolakis
Subject: Construction of Morley's C4 for a 5L

> On Wed, Nov 19, 2014 at 1:27 PM, bernard.keizer@yahoo.com
[Quadri-Figures-Group]
> < Quadri-Figures-Group@yahoogroups.com >> wrote:
>> Dear Chris
>> Long live the tetracardioid!
>> For Antreas, the ennacardioids belong to the history
>> since more than a century!
>> Maybe Morley searched simply a name sounding greek
>> and meaning any number of lines ...
>> Best regards
>> Bernard

Yes, Bernard Morley writes:
"They might be called cardioids, or better ennacardioids.
Thus C\ is the tetracardioid."
So enna- is not the ennea - [9].
But I am wondering what he had in his mind "sounding Greek"! (in other words, why he has chosen that word (prefix) and not holo-, for example, which means "all" Cf. holomorphic)
APH

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Message: #825

Date: 19/11/2014 9:48:34

From: eckart_schmidt@t-online.de

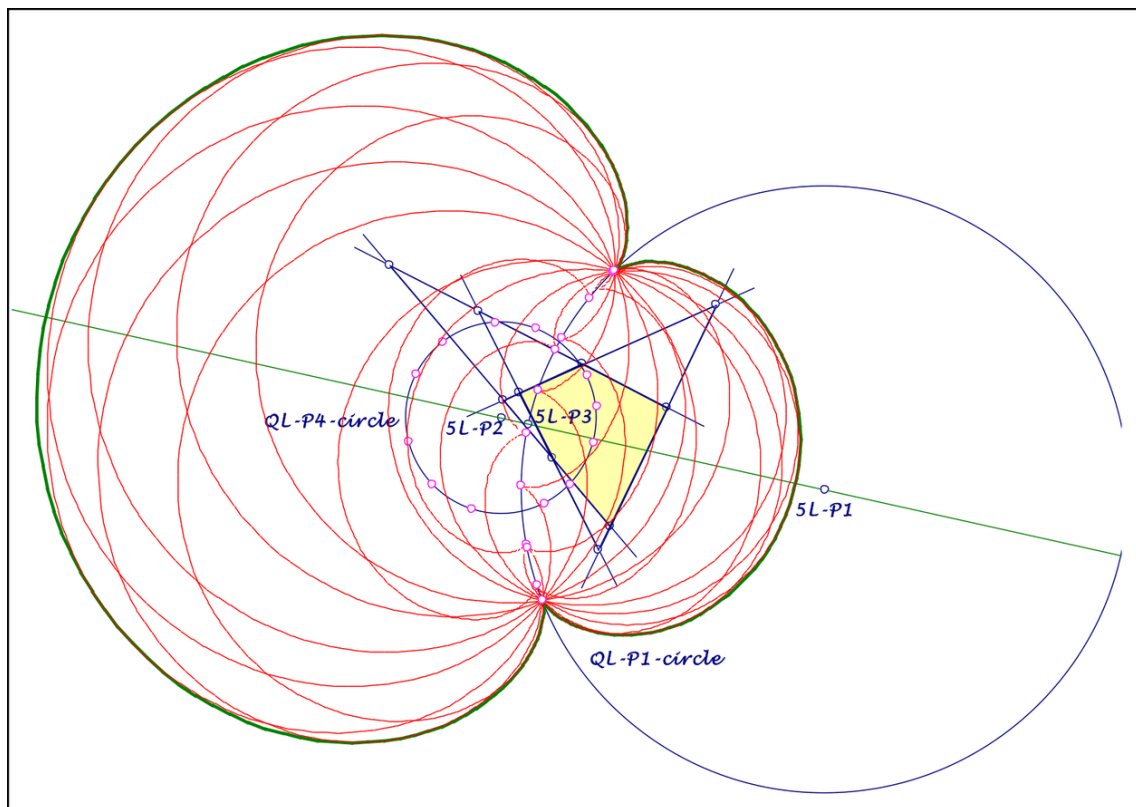
Subject: Construction of Morley's C4 for a 5L

Dear Chris,

in message #812 you tried to get C4 as "envelope of cardioids with center gliding over a circle ...".

This is possible: Let Q on the QL-P4-circle be the center of cardioids with cusps X in the 2nd intersection of the circle round Q through 5L-P3 and the QL-P1-circle, then the envelope of the cardioids is C4 (see attached file).

Best regards Eckart



2014-11-19-825.pdf

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Message: #826

Date: 21/11/2014 8:56:31

From: Chris van Tienhoven

Subject: 5L-Kantor-Hervey Point

Dear Bernard K. and friends,

I constructed Morley's 2nd Orthocenter and it is a point unequal to 5L-p1 - 5L-p10.

Because all Morley's points in his document "Orthocentric properties of the plan n-line" are related I will send a structured document about this later.

There is another 5L-point I am very enthusiastic about.

The earlier mentioned 5L-Kantor Hervey point 5L-p9.

Here are some very special properties:

Draw the perpendiculars from the 5 points QL-P3 (Kantor-Hervey Point) to the omitted line in the 5Line.

They concur in 5L-p9. See attachment.

The 5 lines QL-L4 (The Morley Line) form a new 5Line.

The 5 points QL-P3 in this new 5Line coincide in 5L-p9.

The 5 lines QL-L4 (The Morley Line) form a 1st generation 5Line.

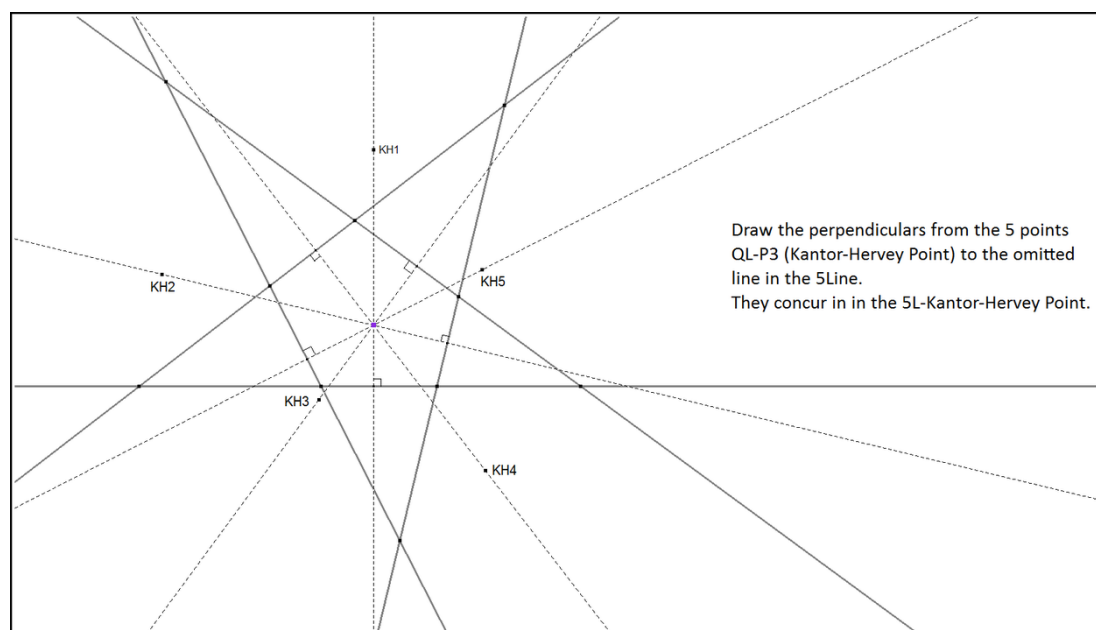
The 2nd generation 5Line consists of 5 lines concurring in 5L-p9.

The 5 points 5L-p9 in a 6Line are collinear!

I constructed 7 versions of this 6L-line, 1st and 2nd generation in a 7Line however no special properties came out (I hope I didn't make errors).

These were the results of yesterday.

Best regards, Chris



5L-KantorHerveyPoint-03.png

Message: #827
Date: 21/11/2014 12:36:03
From: bernard.keizer
Subject: 5L-Kantor-Hervey Point

Dear Chris,
I fear there is some confusion!
The point you describe is not the K-H point, but the 1st orthocenter.
See your own old message #723 and my message #738 about Morley's definitions.
(the property that the 6 points are in line belongs to the 1st orthocenter)
Perhaps it would be useful to send a figure with the 5 lines and the 10 intersection points with the main points already described ...
Best regards
Bernard

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Message: #828
Date: 21/11/2014 9:56:13
From: chris.vantienhoven
Subject: 5L-Kantor-Hervey Point

Dear Bernard K.

You are quite right. This point is Morley's 1st orthocenter.
Somehow I got lost with the logic of this 1st orthocenter.
Morley tells:

- * In a 4Line we have a center circle
with four 3Line-circumcenters.
- * With this construction
we make a Sum Vector Point (SVP): QL-P3.
- * Somehow this SVP (QL-P3)
is the center of the inscribed Deltoid of a 4Line.
- * When we draw the perpendiculars from the 5 versions of QL-P3
in a 5Line to the omitted line then these perpendiculars
concur in Morley's 1st orthocenter.
- * Then Morley states: The 5 5L-first orthocenters in a 6Line
lie on a line.
- * Then he concludes: we found an orthocenter for an odd number
of lines, a directrix for an even number of lines.

My main question is:

How to find in general a 1st orthocenter in a n-Line, where
 $n = \text{odd}$?

For $n=5$ apparently we use 4Line-point QL-P3 in a perpendicular
construction.

I tried this for $n=7$ by:

- * drawing the center circle in a 6Line
and the related SumVectorPoint (6L-SVP)
- * then drawing 7 times 6L-SVP
and corresponding perpendiculars at the omitted line
- * Somehow they do not concur.

Can you see the general logic for finding the nL-1st orthocenter
($n = \text{odd}$)?

Best regards,
Chris

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Message: #829

Date: 22/11/2014 6:29:48

From: Antreas Hatzipolakis

Subject: Van Aubel and Petr-Douglas-Neumann theorems

1. By the Van Aubel theorem we get a point in a quadrangle.
(intersection of the lines joining the opposite centers of squares)

Is it in EQF?

http://en.wikipedia.org/wiki/Van_Aubel%27s_theorem

2. By Petr-Douglas-Neumann theorem we get regular polygons associated with n-angles [quadrangles, pentagles etc].

Are the centers of these reg. polygons interesting for studying and including in EQF?

http://en.wikipedia.org/wiki/Petr%E2%80%93Douglas%E2%80%93Neumann_theorem

APH

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Message: #830

Date: 22/11/2014 8:41:17

From: chris.vantienhoven

Subject: Van Aubel and Petr-Douglas-Neumann theorems

1. By the Van Aubel theorem we get a point in a quadrangle.
QG-Cu1: Van Aubel Cubic

2. By Petr-Douglas-Neumann theorem we get regular polygons associated with n-angles [quadrangles, pentagles etc].

The centers of these reg. polygons are the centroids of the reference polygon and of course they are known.

Encyclopedia of Polygon Figures (EPF).

This theorem is very beautiful. It shows even more how interesting and many-sided polygons are.

Best regards,

Chris

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Message: #831

Date: 23/11/2014 4:08:07

From: eckart_schmidt@t-online.de

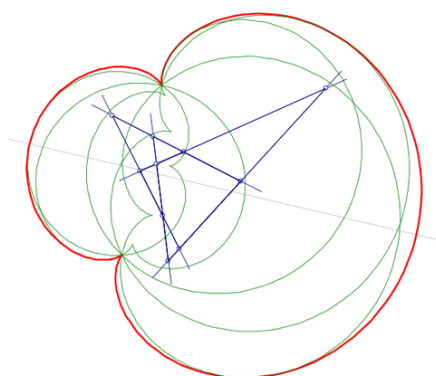
Subject: Construction of Morley's C4 for a 5L

Dear Bernard K, dear Chris,
following my construction, I have got an equation for the
tetracardioid of a 5-L, using cartesian coordinates wrt the
symmetry. I have gathered the results in the attached file.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Morley's Tetracardioid of a 5-Line (Construction and Equation)

Morley describes for a 5-line a tetracardioid C_4 , contacting the 5 cardioids $QL-Qu1$ of the 4-line-components [1]. Here is given a construction of this curve and its equation.



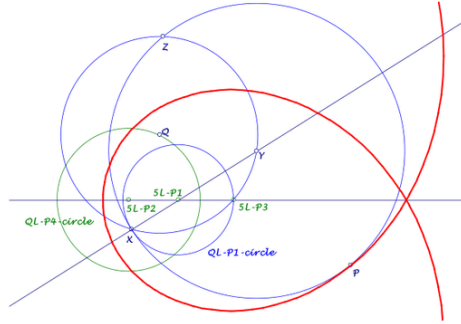
Construction

The tetracardioid is determined by a point and a circle: The point is $5L-P1$, the center of the 5 concentric Miquel points $QL-P1$ of the 4-L. The circle is the circumcircle of the 5 concentric Miquel circumcenters $QL-P4$ of the 4-L with center $5L-P2$. The inverse of $5L-P1$ wrt the $QL-P4$ -circle is the point $5L-P3$. The circle round $5L-P1$ through $5L-P3$ is the $QL-P1$ -circle (see *QFG* #710).

- **The tetracardioid of a 5-Line is determined by the point $5L-P1$ and the $QL-P4$ -circle.**

Construction (see *QFG*-message 815):

... Let Q be a variable point on the $QL-P4$ -circle,
... let $Ci1$ be a circle round Q through $5L-P3$,
... let X be the second intersection of $Ci1$ and the $QL-P1$ -circle,
... let Y be the second intersection of $X.5L-P1$ and $Ci1$,
... let $Ci2$ be a circle round Y through X ,
... let Z be the second intersection of $Ci1$ and $Ci2$,
... then Z reflected in Y is a point P of Morley's tetracardioid.



- The tetracardioid can be a limaçon-like curve or a curve with two real cusps.

Equation

We use Cartesian coordinates:

5L-P2(0,0), 5L-P1(0,p), r radius of the QL-P4-circle.

Following the construction we get the equation:

$$4 p x^4 (16 p^3 x^2 - 9 p x^4 - 54 p^2 x^2 y + 27 x^4 y + 27 p^3 y^2) - 3 x^4 (16 p^4 - 60 p^2 x^2 + 27 x^4 + 36 p^3 y) (x^2 + y^2) + 6 p^2 x^2 (2 p^2 + 3 x^2) (x^2 + y^2)^2 + p^4 (x^2 + y^2)^3 = 0$$

Discussion:

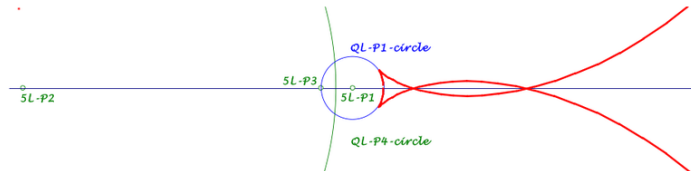
- The tetracardioid is limaçon-like, if 5L-P1 lies inside the QL-P4-circle ($p \leq r$).

Node $N(0, \frac{3r^2 + r\sqrt{9r^2 - 8p^2}}{2p})$.

Vertices $V_1(0, \frac{-4pr - 3r^2}{p})$ and $V_2(0, \frac{4pr - 3r^2}{p})$.

The contact points of the double-tangent $T_{1,2}(\pm 2r, \frac{3r^2}{p})$.

- The tetracardioid has two cusps, if 5L-P1 lies outside the QL-P4-circle ($p > r$).
- The cusps lie on the QL-P1-circle.



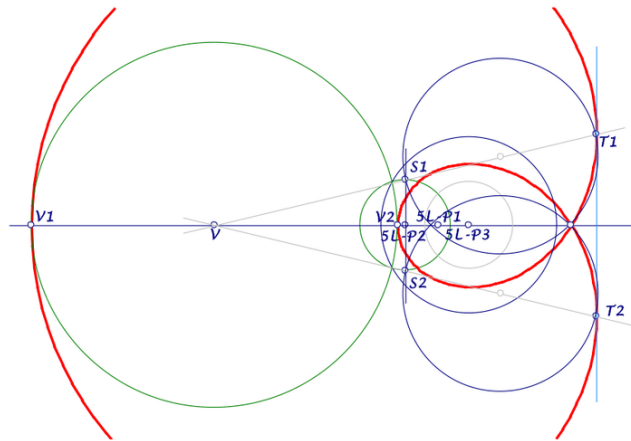
$$N_{1,2}(0, \frac{3r^2 \pm r\sqrt{9r^2 - 8p^2}}{2p})$$

Vertices (see above) $V_1(0, \frac{-4pr-3r^2}{p})$ and $V_2(0, \frac{4pr-3r^2}{p})$.

Cusps $C_{1,2}(\pm \frac{2r}{p^3} \sqrt{(p^2-r^2)^3}, \frac{r^2(3p^2-2r^2)}{p^3})$.

Construction of special points

... Take a circle round $5L-P3$, so that $5L-P1$ and $5L-P2$ lie inverse and double the radius for a further concentric circle,
 ... wrt this circle draw the inverse of the $QL-P4$ -circle,
 ... the last circle cuts the line $5L-P1.5L-P2$ in the vertices V_1, V_2 of the tetracardiod.
 ... Consider the intersections S_1, S_2 of the $QL-P4$ -circle and a perpendicular line through $5L-P2$ wrt $5L-P1.5L-P2$,
 ... reflect the midpoint V of the vertices in S_1 and S_2 ,
 ... these are the contact points T_1, T_2 of the double-tangent.
 ... The Thales circles about $S_i T_i$ contain the nodes.



[1] F. Morley: On Reflexive Geometry, Transactions of the American Mathematical Society, Volume 8, 1907.

Eckart Schmidt
<http://eckartschmidt.de>
 eckart_schmidt@t-online.de

Message: #832

Date: 23/11/2014 4:58:26

From: Antreas Hatzipolakis

Subject: Van Aubel and Petr-Douglas-Neuman

>> Dear Antreas,
>> [AH] 1. By the Van Aubel theorem we get a point in a
>> quadrangle. (intersection of the lines joining the opposite
>> centers of squares)
>> Is it in EQF?
>> The Van Aubel theorem is briefly mentioned in EQF at the item
>> QG-Cu1: Van Aubel Cubic.
>> [AH] 2. By Petr-Douglas-Neumann theorem we get regular
>> polygons associated with n-angles
>> [quadrangles, pentangles etc].
>> Are the centers of these reg. polygons interesting for
>> studying and including in EQF?
>> The centers of these reg. polygons are the centroids of the
>> reference polygon and of course they are known.
>> I am very glad to hear about this theorem. I didn't know it.
>> There are only very few theorems about properties of n-gons
>> applicable for all values of $n > 2$.
>> When you know more of these kind of theorems please let me
>> know. I can include them in a future Encyclopedia of
>> Polygon Figures (EPF).
>> This theorem is very beautiful.
>> It shows even more how interesting and
>> many-sided polygons are.
>> Best regards,
>> Chris

Dear Chris,

Note that we can combine Petr-Douglas-Neumann theorem (
PDN-theorem) with
Cayley-Bacharach theorem (on cubics passing through 8 points) to
get new points.

For quadrangle:

8 points: the 4 vertices of quadrangle + the 4 vertices of the
square (by *PDN-theorem*)

For heptagon:

8 points: the 7 vertices of regular heptagon (by *PDN-theorem*)
+ the center of r. heptagon.

I would like to see this point (the 9th point the cubics pass
through), but probably the

center of the r. heptagon is a double point of the cubic.

APH

Message: #833

Date: 23/11/2014 11:09:43

From: chris.vantienhoven

Subject: 5L-Kantor-Hervey Point + another mysterious Morley curve

Dear Bernard K.,

[CvT] My main question is:

How to find in general a 1st orthocenter in a n -Line, where n =odd?

I looked to the subject again.

Morley describes in his document "Orthocentric properties of the plane n -LINE" the 1st orthocenter in a 4Line as the center of the inscribed deltoid in the 4Line.

Then he says:

The peculiar appropriateness of the deltoid for the metrical theory of four lines makes it desirable to have an analogous curve for $2n$ -lines.

Apparently the center of a " $2n$ -Deltoid-like curve" will be the 1st orthocenter of the $2n$ -Line.

Here is another mysterious Morley curve.

The question remains how to construct its center!

Best regards,

Chris

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Message: #834
Date: 24/11/2014 2:58:49
From: seiichikiri
Subject: A pentagon with parallel sides

Dear friends,

A simple construction (similar to the idea of the anticomplementary triangle):

Let $P_1P_2P_3P_4P_5$ be a pentagon. We denote the line through P_i parallel to $P_{i+2}P_{i+3}$ by L_i , $i=1,\dots,5$, modulo 5.

We denote the intersection of L_i and L_{i+1} by Q_{i+3} , $i=1,\dots,5$, modulo 5.

Strangely to me, the pentagon $Q_1Q_2Q_3Q_4Q_5$ has its centroid different from that of $P_1P_2P_3P_4P_5$.

So the other pentagonal points of the former must be different from those of the latter.

Best regards,

Seiichi.

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Message: #835

Date: 24/11/2014 11:42:30

From: bernard.keizer

Subject: 5L-Kantor-Hervey Point + another mysterious Morley curve

Dear Chris,

I would like to be able to help you, but I'm not.

I've read this article by Morley many times, but there are always things I do not understand ...

First, this sery of points p_0 , p_1 , p_2 , and so on

P_0 is the centric center

P_1 is the Hervey point (and not the 1st orthocenter)

P_2 is the 1st orthocenter

P_3 ...

There are plenty of centroid properties.

The distances from $P_a(n)$ to $P_{a(n-1)}$ and $P_{a-1(n-1)}$ are in the same ratio for the n $n-1$ lines

I've checked this property for the Hervey point (why do you call it SVP?) of the 5-L

But it must be true for the other points ...

In particular, Morley says at one place that P_2 exists only for n odd and at another place that it's possible to have P_2 for the 6-L!

Anyhow, the constructions you give in message #715 and message #723 as 2 different points give the same point as you mention in message #826. (and that's the 1st orthocenter)

In theorem 5, if $n = 3$, this gives a point P_2 for the 6-L equidistant from the 6 copples of P_1 and P_2 of the 5_L's.

2 remarks more

if the 5 lines L_4 give the same point P_3 , it means that they are tangent to a deltoïd (see my message #810); it's not surprising that the 1st orthocenter of this 5-L is the common Kantor-Hervey point of the 5 QL's.

theorem 7 from Morley is also interesting for the 5-L: 5

tangents the 5 deltoïds of the QL's and perpendicular to the 5th line touch a 6th deltoïd: I don't know if the center of this deltoïd is a new point or one of the old points.

As you can see, I spent time on these questions

I hope it will be helpful

Best regards

Bernard

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Message: #836
Date: 24/11/2014 3:11:37
From: chris.vantienhoven
Subject: 5L-Kantor-Hervey Point

Dear Bernard K.,
Thanks for your efforts.
I am at the point now to quit the subject of the first orthocenter for the time being.
But I will first give you my last insights.
In a 3-Line the 3L-first-orthocenter is $X(4)$.
In a 4-Line these 3L-first-orthocenters are collinear on a 4L-ortholine (QL-L2=Steiner Line).
In a 5-Line the 5L-first-orthocenter is 5L-Kantor-Hervey point.
In a 6-Line these 5L-first-orthocenters are collinear on a 6L-ortholine.
etc.
The nL-first-orthocenter only exists for n=odd.
The nL-ortholine only exists for n=even.
It is not clear how to construct the nL-first-orthocenter.
Knowing the nL-first-orthocenter it is easy to construct the (n+1)L-ortholine.
Morley defines the nL-first-orthocenter for n=4 as the center of the inscribed Deltoid in a 4-Line.
To find this point he mentions that the locus of Inscribed-Deltoid-centers in a 3-Line is the perpendicular bisector of Circumcenter O and Orthocenter H. So in a 4Line the center of the inscribed Deltoid is the common point of the perpendicular bisectors of OH of the Component Triangles. Indeed these lines concur. This point happens to be QL-P3 the Kantor-Hervey point.
Then he tells in Theorem 3:
"If from the center of the inscribed deltoid of 4 of 5 lines a perpendicular be drawn to the line left out, the 5 perpendiculars meet at a point".
So the method for finding the 5L-first-orthocenter from the 3L-first-orthocenter is:
* Find the intermediate point X in a 4-Line by intersecting the perpendicular bisectors of OH of all Component Triangles in a 4-Line.
* Draw perpendiculars for all Component 4-Lines in a 5-Line from X to the omitted line.
They concur in the 5L-first-orthocenter.
However doing this same construction to find the 7L-first-orthocenter it gets stuck already at step 1.
I tried different definitions for the point H without result. No common point!

Then I thought: well this perpendicular bisector is the locus of centers of the inscribed deltoid.

So let's look at the special deltoid for $n > 3$. Maybe this locus isn't a line at all?

Fortunately Morley states at page 9:

Theorem 8. The locus of centers of curves Delta 5 of which 5 lines are given is a line.

And so for Delta $2n-1$.

So my conclusion is that the nL -first-orthocenter (for n =odd) can be constructed as soon as we know the linear locus of curves Delta $2n-1$.

Which line is this ??? How can it be constructed ???

Best regards,

Chris

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Message: #837

Date: 25/11/2014 8:42:34

From: eckart_schmidt@t-online.de

Subject: 5L-Kantor-Hervey Point + another mysterious ...

Dear Benard K.,

I need your advice, maybe I am somewhat confused:

In "Cut the knot" I found under "Morley's Pursuit of Incidence" the following statement:

"Back to the center-circles that intersect at a point (say P) and whose centers lie on a circle - the next center-circle in the hierarchy of circles. The point P is also incident to this latter center-circle."

My sight: Center circle of 4-L is QL-Ci3. Their intersection for a 5-L is 5L-P3, their centers lie on the QL-P4-circle, but 5L-P3 is not a point of the QL-P4-circle. Where is my mistake?

Thanks in advance, best regards Eckart

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Message: #838

Date: 25/11/2014 11:56:22

From: bernard.keizer

Subject: 5L-Kantor-Hervey Point + another mysterious ...

Dear Eckart,

Thanks for your confidence

May be I don't understand the word incidence.

For me, that's very clear in Morley.

There is always a centric circle for any n -lines with the centers of the n centric circles of the $n-1$ lines.

These n circles have a common point, which generally isn't on the last circle.

The envelop of the circles through a point and the centers on a circle is a limaçon with cusp in this point.

Yourself found a method to draw this limaçon.

If $n = 4$, the point P_1 lies on the 4 circles and the particular limaçon is then a cardioïd with the loop reduced to the cusp.

Best regards

Bernard

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Message: #839
Date: 25/11/2014 12:29:22
From: bernard.keizer
Subject: 5L-Kantor-Hervey Point

Dear Chris,
I regret you quit the subject!
I also sometimes wonder if you really read my messages completely ...
In my message #835, I precisely explained that the point p1 from Morley is the Hervey point of the n-L
(I gave you several times the reference of R. Goormaghtigh the Hervey point of the plane n-line) and that the point p2 from Morley is the 1st orthocenter!
These are 2 different points!
It happens that these 2 points coincide for the triangle (the orthocenter is the point p1 defined by the sum of 3 vectors and also the point p2 as intersection of the 3 altitudes).
For the QL, the point p1 is the Kantor-Hervey point, at the same distance from the points p0 (circumcenters) and p1 (orthocenters) and we don't know p2.
But they generally do not, and particularly not for the 5-L, where we know the 2 points.
The I gave you the idea of theorem 5 from Morley: for $n = 3$, I hoped you could check it on your figure of the 6-L. (I wasn't able to do this with Geogebra, the figures are too complicate)
Last, I indicated the theorem 7, which gives a 6th deltoïd for the 5_L; I hoped you could determine it's center and check if it was a new point or one of the old ones ...
I have no new ideas for the moment (in particular no idea about this curve Delta5)
Best regards
Bernard

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Message: #840
Date: 25/11/2014 1:33:40
From: bernard.keizer
Subject: Back to the cayleyan

Dear Chris, dear Eckart, dear Bernard G.

Working on the cayleyan, I've realized that I gave a wrong definition of this curve.

It's not, as I told to Eckart, the envelop of the main axes of the inscribed conics, but the envelop of all lines forming the degenerated polar conics.

This means for the cubic stelloïd both axes of the inscribed conics, the main axis through 2 conjugate points of the hessian, but also the minor axis is est the perpendicular bisector of the segment through the 2 points.

As put in an old message, in each point X of the hessian, the bisectors of the angles AXA' (where A and A' are any other couple of conjugate points on the hessian) form the degenerated polar conic of the conjugate X' wrt the cubic stelloïd.

The 2 bisectors in X and in X' intersect in 4 points Y and Y' and Z and Z', which form an orthocentric quadrilateral and are the 4 poles of the line XX'.

Y and Y' and Z and Z' describe 2 cubics, which are the Hessians of 2 other cubics having the same cayleyan as the cubic stelloïd.

There are 2 interesting circles, one through X, X', Y and Y' and through the 2 invariant points of the CSC conjugation and the other through Y, Y', Z and Z' and orthogonal to the other. These 2 circles are the generalization of the Steiner circles, they belong to the 2 sets of the autopoloconics common to the cubic stelloïd and to one of the 2 other cubics having the same cayleyan.

I suppose all this is obvious for Bernard G. but it took me time to understand these properties ...

Best regards
Bernard

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Message: #841
Date: 25/11/2014 11:58:11
From: Chris van Tienhoven
Subject: 5L-Kantor-Hervey Point

Dear Bernard K.,

[BK] I regret you quit the subject!

I couldn't resist working all day on the subject again!
Moreover I was reading Goormaghtigh's paper.

[BK] I also sometimes wonder if you really read my messages completely ...

I read your messages very well, especially those about Morley's papers! Sometimes I even reread your messages when I am totally involved in a subject. I am very grateful for your thorough studies!

[BK] These are 2 different points!

I am very much aware of that as well as the other topics you are mentioning. For over a month I am making an analyses of all points described by Morley. The only major point I cannot get grip of is the first orthocenter.

[BK] The I gave you the idea of theorem 5 from Morley: for $n = 3$, I hoped you could check it on your figure of the 6-L. (I wasn't able to do this with Geogebra, the figures are too complicate)

It happened that I was doing this all day! Finally I succeeded.
I enclose a picture.

[BK] I indicated the theorem 7, which gives a 6th deltoïd for the 5_L; I hoped you could determine it's center and check if it was a new point or one of the old ones ...

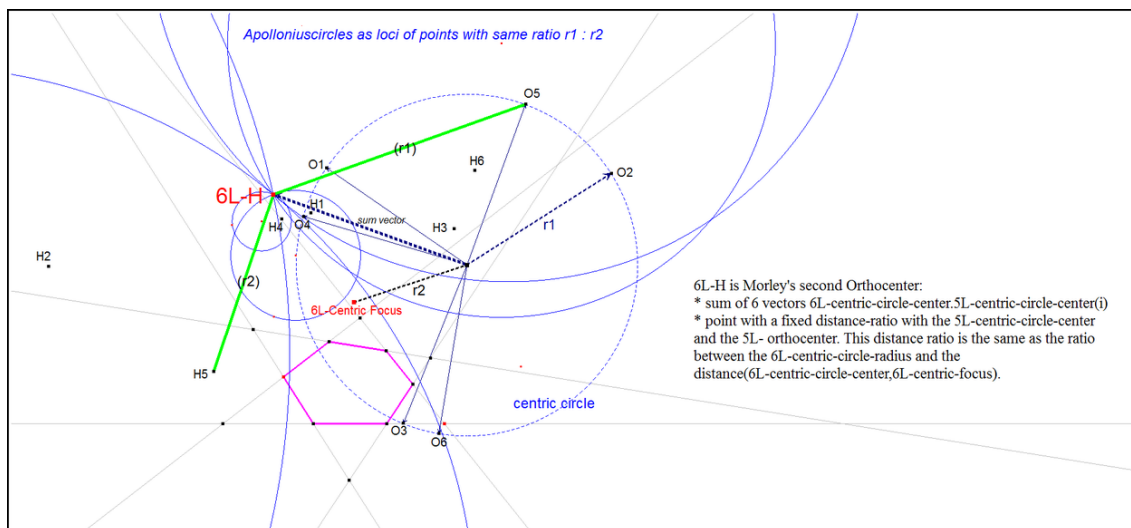
This has no priority for me yet. Maybe a nice challenge for Eckart?

[BK] I have no new ideas for the moment (in particular no idea about this curve Delta5)

I have been thinking about this Delta5. It looks like Morley is describing another generation of Deltoids just like he described another generation of Cardioids.

Maybe we can call these Deltoids of a higher level Ennadeltoids, perfectly in style with Morley's other name givings. Eckart, interesting for you?

Best regards,
Chris



6L-Morleys 2nd Orthocenter-01.png

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Message: #842
Date: 26/11/2014 11:17:29
From: bernard.keizer
Subject: 5L-Kantor-Hervey Point

Dear Chris,

Thank you very much for your message.

I'm very glad you spent more time on the subject and I thank you for the beautiful figure with the Apollonius circles.

In fact, Morley mentions n points p , from p_0 to p_{n-1} , as there are n coefficients a for the n -line, with barycentric properties ...

He names p_0 the centric center and p_2 the 1st orthocenter.

He doesn't give a name to p_1 and also no names to the points after p_2 .

Since the beginning, I call p_1 the Hervey point of the n -line after R. Goormaghtigh.

On your picture, you call it this time 2nd orthocenter, I suppose it is to make a difference with the 1st.

Unfortunately, Morley calls another point the 2nd orthocenter (at the end of his article); this point h is the center of the Euler circle for the triangle and the point P_2 or Morley point for the QL.

For the 1st orthocenter p_2 , as I told you, Morley himself is contradictory: sometimes it doesn't exist for the $2n$ -lines, sometimes it's possible to find it with the p_1 and p_2 of the $2n-1$ lines or with theorem 5 in the $2n$ -lines, p_{n-1} is equidistant from the p_{n-1} and the p_{n-2} of the $2n-1$ lines for $n=2$, this gives p_1 equidistant from the p_1 and the p_0 of the triangles (which is the Kantor-Hervey theorem) for $n=3$ this gives p_2 equidistant from the p_2 and the p_1 of the 5-L on your figure, the p_1 are the points H_1 to H_6 , you didn't put the p_2 (1st orthocenters); this would perhaps give a solution ...

I hope this will help.

Best regards

Bernard

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Message: #843
Date: 26/11/2014 11:38:32
From: bernard.keizer
Subject: New article

Dear Chris, dear Eckart, dear Bernard G., dear Antreas, dear Seiichi and dear fellows of the Quadriforum,
As I told Chris a few months ago, before leaving the QL, I wanted to write a follow up to my first article integrating the new knowledge about the cubic stelloïd and showing the deep unity of all these figures (deltoïd and cardioïds, Morley triangles, cubic stelloïd and rectangular hyperbolas, which are conic stelloïds) which is given by the trisection of angles. This is now done and I put on the web a new article *Le Quadrilatère Complet et la Stelloïde cubique*. There are 2 ways to get to it: the same link as the 1st article <http://bernardkeizer.blogspot.fr> or simply search on Google *stelloïde cubique*, it appears directly. By doing this, I noticed that the article *Géométrie du quadrilatère complet* by J. Marchand was now numerised and also available on the web. So, you may have 2 articles for the price of one! (unfortunately, both are in french!) I thank you in advance for all your comments or critics. For Antreas, this could perhaps interest also the members of Anopolis
Best regards to all of you
Bernard K.

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Message: #844
Date: 26/11/2014 12:12:22
From: Antreas Hatzipolakis
Subject: New article

[Bernard K.]:

> For Antreas,
> this could perhaps interest also the members of Anopolis

Dear Bernard K,

Yes !! I have Fw-ed your message to both Anopolis and Hyacinthos lists.

There is also the possibility to share it to Facebook and twitter.

I have shared it to my FB profile and to FB math. group "Short mathematical idea"

Best wishes to you and your nice math. work !!

APH

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Message: #845
Date: 26/11/2014 12:30:30
From: Antreas Hatzipolakis
Subject: New article

Dear Bernard K.

As I wrote I shared it to a FB math. group and a member asks:

Telv Cohl

(<https://www.facebook.com/telv.cohl?fref=ufi>)

I don't know how to download it ...

I think it is not downloadable.

But there is way (by some programs available in the Internet)
to download the ENTIRE blog for off-line reading.

APH

> On Wed, Nov 26, 2014 at 1:12 PM, Antreas Hatzipolakis <
anopolis72@gmail.com
> wrote:
>> [Bernard K.]:
>> For Antreas, this could perhaps interest also the members of
>> Anopolis
>> Dear Bernard K,
>> Yes !! I have Fw-ed your message to both Anopolis and
>> Hyacinthos lists.
>> There is also the possibility to share it to Facebook and
>> twitter.
>> I have shared it to my FB profile and to FB math. group
>> "Short mathematical idea"
>> Best wishes to you and your nice math. work !!
>> APH

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Message: #846
Date: 27/11/2014 3:48:41
From: Chris van Tienhoven
Subject: 5L-Kantor-Hervey Point

Dear Bernard K.

It happened!

Finally I constructed Morley's 1st Morley Orthocenter!

It was like cracking Morley's code or like cracking the code on Rosetta's stone.

See attached file for the construction in a 7Line.

See also the construction of the needed intermediate point in a 6Line.

Thank you for all your advice. Without this I wouldn't have been able to construct this.

Despite you described the way I will say it now in my own words.

According to Morley there is a 1st Orthocenter in n-Lines where n is odd. However to construct it you need an intermediate point in an n-Line for n is even.

I call this intermediate point the nL-Intermediate 1st Morley's Orthocenter (n=even).

The nL-Intermediate 1st Morley's Orthocenter (let's call it nL-IM01 for n=even) is the common point of the n perpendicular bisectors of:

the (n-1)L-Center of the Center Circle

the (n-1)L-1st Morley's Orthocenter.

Then the (n+1)L-Morley's 1st Orthocenter (let's call it (n+1)L-M01 for n=even) is the common point of the (n+1) perpendiculars through nL-IM01 on the omitted line of the (n+1)-Line.

When n=3 then nL-M01 = X(4).

When n=4 then nL-IM01 = QL-P3 (QL-Kantor-Hervey Point).

When n=5 then nL-M01 = 5L-p9 (5L-Kantor-Hervey Point,
see QFG #826).

When n=6 then nL-IM01 = 6L-Intermediate 1st Morley's
Orthocenter.

When n=7 then nL-M01 = 7L-1st Morley's Orthocenter.

Etc.

All this information comes from Frank Morley's paper "Orthocentric properties of the plane n-LINE" and is decoded now.

One final question.

Frank Morley's paper ends with these phrases:

"Whereas the orthocenter of Paragraph 2 applied only to an odd number of lines the present one applies to any number. We have then for an odd n -line two solutions of our problem, except when $n=3$, in which case the points h and p coincide.

But when we have two orthocenters, we have a whole line of orthocenters, since evidently a perpendicular to one of the n lines, dividing the join of the two known points of the remaining $n-1$ -lines in a fixed ratio, we will divide the join of the two orthocenters in the same fixed ratio.

Thus we found, for an even number of lines, one orthocenter; for an odd number of lines, a line of orthocenters; for an even number of lines, one directrix."

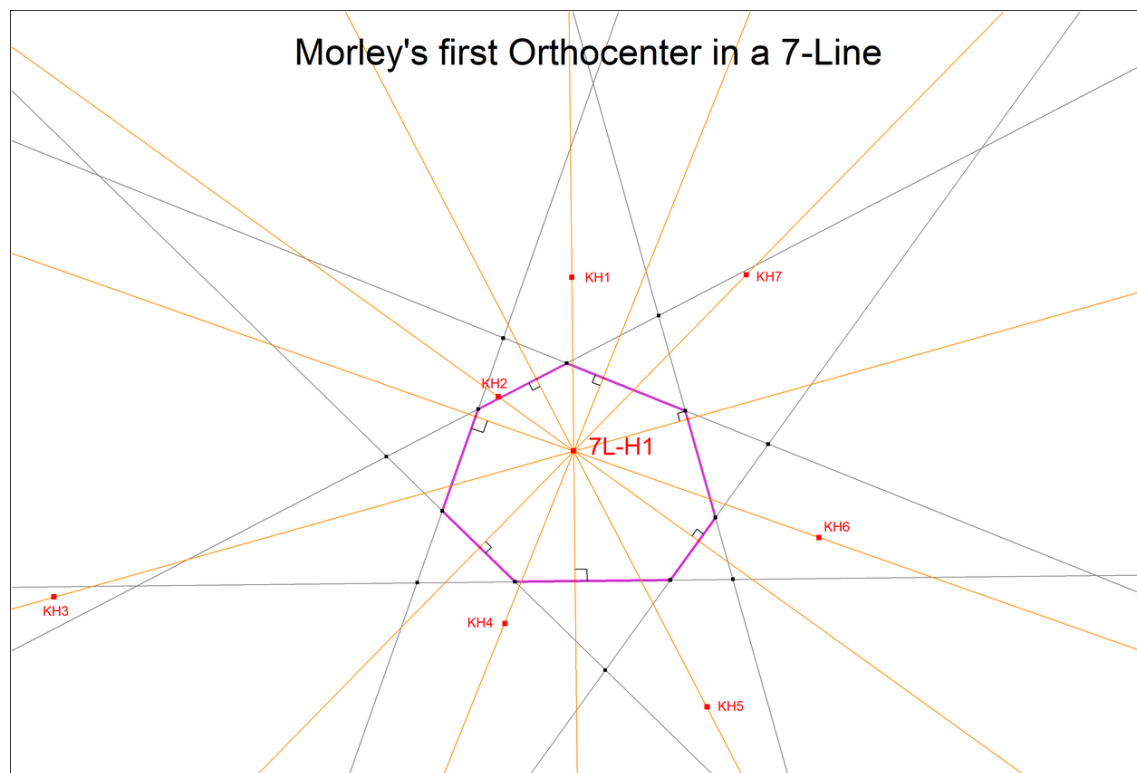
Bernard K, my question to you is:

Do you understand this fully? I don't!

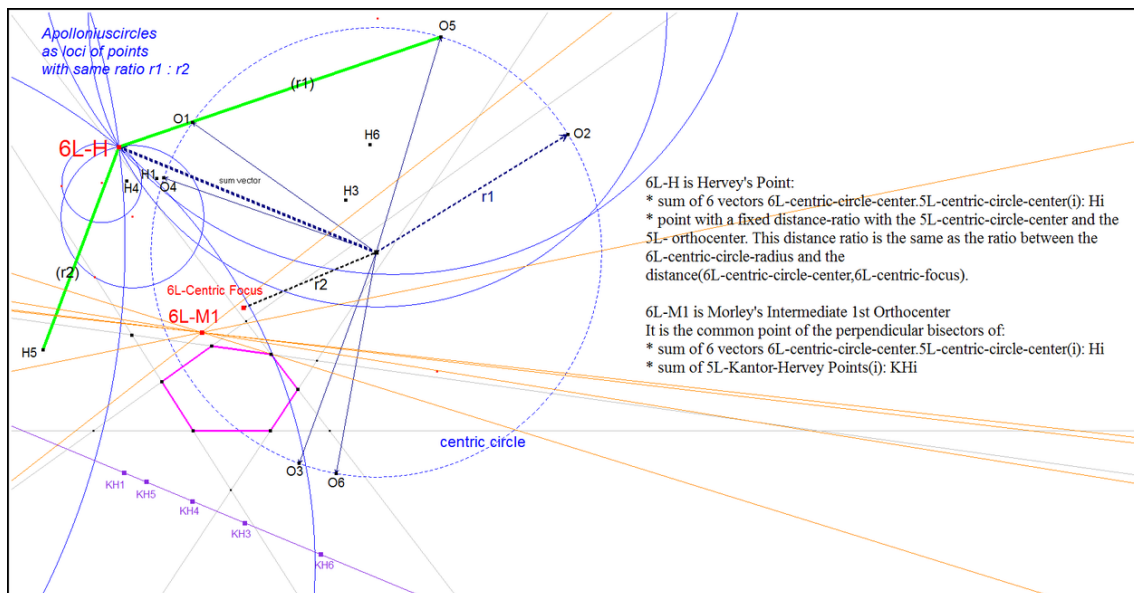
Especially: what is this "line of orthocenters"?

Best regards,

Chris



7L-1st Morleys Orthocenter-01.png



6L-Morleys Intermediate 1st Orthocenter-02.png

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Message: #847

Date: 27/11/2014 7:29:24

From: Antreas Hatzipolakis

Subject: Four Concurrent Euler Lines in Quadrangle

Let ABCD be a quadrangle. If the Euler lines of three of the component triangles

are concurrent, then the Euler lines of all four component triangles (ie ABC, BCD, CDA, DAB) are concurrent. (*)

Synthetic roof?

(*) The same is also true for other lines (OK, OI, etc). For proofs with tripolar coordinates see the paper (

<http://forumgeom.fau.edu/FG2001volume1/FG200109.pdf>)

APH (<http://www.artofproblemsolving.com/Forum/viewtopic.php?f=46&t=615558&p=3664276>)

Nice Proof of Telv Cohl (<http://www.artofproblemsolving.com/Forum/viewtopic.php?f=46&t=615558&p=3664276#p3664276>)

Thanks, Telv! :-)

APH

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Message: #848

Date: 27/11/2014 8:41:18

From: eckart_schmidt@t-online.de

Subject: 5L-Kantor-Hervey Point

Dear Chris,

sorry, I cannot follow your construction of Morley's 1st

Orthocenter for odd n. Please tell me my mistakes:

n=5: For a 3L the center of the Center Circle is X3, the 1st Orthocenter is X4. Then the 4L-Intermediate Point is QL-P2, not QL-P3 as you write ...

n=7: For a 5L the center of the Center Circle is 5L-P2, the 1st Orthocenter is 5L-P10. Then the perpendicular bisectors for the 5L-components of a 6L don't coincide ...

Having a look in the attached files, I find SVP-points, not mentioned in the text. I have to study the attached files more precisely.

Best regards Eckart

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Message: #849
Date: 27/11/2014 9:20:59
From: eckart_schmidt@t-online.de
Subject: Morley's point p2 for 6L

Dear Bernard K., dear Chris,
Morley's point p2 for a 6L can be constructed by Morley's theorem 5 as Bernard already mentioned.
Taking these points for 6Ls of a 7L and their perpendicular lines wrt the remaining line, there is a common point.
Is this point the 2nd orthocenter?
For a 8L they give a line.
Best regards Eckart

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Message: #850

Date: 27/11/2014 9:57:25

From: Chris van Tienhoven

Subject: 5L-Kantor-Hervey Point

Dear Eckart,

I made a construction of the 4L-Intermediate 1st Morley Orthocenter and I made a construction of the 5L-1st Morley Orthocenter. See attached files.

If still things are unclear please let me know.

Sorry I forgot to tell what SVP means.

It is my own abbreviation for Sum Vector Point.

It is used by Morley as a construction for a new point being the sum of vectors formed by the n vectors $0.01, 0.02, \dots, 0.0n$, where 0 = nL -center of the center circle and $01, 02, \dots, 0n$ are the circumcenters of the lower level $(n-1)$ -Lines lying on the nL -center circle.

This is another type of orthocenter mentioned by Morley, not to be confused with Morley's 1st and 2nd Orthocenter.

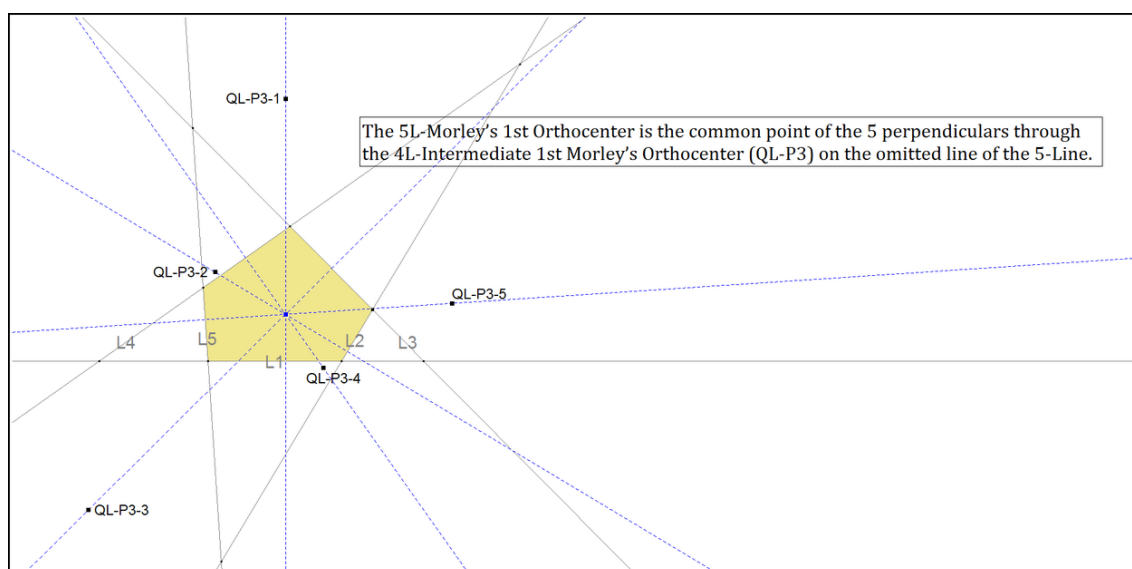
For me it was a totally new construction for centers with lots of potential for other applications.

For example I figured out, there is a QL-point X on the line $QL-L4$ for which the Sum Vector of $X.H1, X.H2, X.H3, X.H4$ is $QL-P3$. It is the point $QL-P2.QL-P3 (-1 : 4)$, as well as $QL-P12.QL-P27 (-3:4)$.

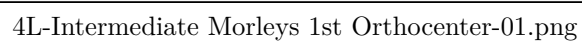
($H1, H2, H3, H4$ being the orthocenters of the 4 Component Triangles of the QL)

Best regards,

Chris



5L-Morleys 1st Orthocenter-01.png



556

Message: #851
Date: 28/11/2014 6:43:41
From: eckart_schmidt@t-online.de
Subject: 5L-Kantor-Hervey Point

Dear Chris,
thanks for your message #850. Wrt the construction of the 4L-Intermediate 1st Morley's Orthocenter, there was a mistake on my side: I changed QL-P2 and QL-P3, excuse.
But wrt 6L-IM01 I do not understand your description in the message and the attached file. Studying the figure, I will tell my sight:
Oi are the points 5L-P2, Hi are the points 5L-P9, KHi are the points 5L-P10 for the 5L of a 6L.
The perpendicular bisectors of 5L-P9.5L-P10 are the lines giving 6L-IM01.
In addition wrt my message #849: Your 6L-IM01 is Morley's point p2 for a 6L
and your 7L-1st Morley's Orthocenter is the point I described in my message #849.
Best regards Eckart

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Message: #852
Date: 28/11/2014 8:09:25
From: bernard.keizer
Subject: 5L-Kantor-Hervey Point

Dear Eckart, dear Chris,
Alleluiah !

Welcome to Eckart in the club of the Morley's code crackers.
For Chris, the cracking of the Enigma codes had for Alan Turing
a sad end, but that's another story ...

Of course, Eckart, you're right!

I try desperately since several messages to convince Chris that
the naming was misleading, but I haven't succeeded yet.

For me, again, see my message #842, the Morley's p_0 are the
centric centers $nL-0$, the Morley's p_1 are the Hervey or
Kantor-Hervey points or SVP points like the 6L-H, easy to
construct like Chris did with the Apollonius circles and the p_2
are the 1st orthocenters for n odd or qasi-1st orthocenters for
 n even.

We haven't find the points p_3 , p_4 and so on yet, nor the
centroid properties mentioned by Morley ...

For $n=5$, the 5L-P9 should be named the KH points and the 5LP10
the H points.

On the figure of the 6L, in fact the 6L-H should be named 6L-KH,
like the H_1 to H_6 the 5L-KHi. The center is 6L-0 and the O_1 to
 O_6 are the 5L-Oi. The line of KH is the directrix of the 6-L,
carrying the 6 1st orthocenters, which should be 5L-H1 to H_6 .
Congratulations to Chris for the construction of the 6L-M1,
which is, as Eckart says the 6L-quasi 1st orthocenter. Then it
gives the 7L-1st orthocenter and, for the 8L a directrix again
of the 8 1st orthocenters of the 8 7L's as said in message #849.
For Chris again, the end of the orthocentric properties is very
simple: for n even one orthocenter, which is the point called h
or the 2nd orthocenter, for n odd, a line of orthocenters, which
is the line through p and h , the 1st and the 2nd orthocenters,
for n even one directrix, carrying the 1st orthocenters of the
 $n-1$ lines.

The property mentioned is the application of the Thales theorem
(the perpendiculars to the last omitted line cut the segment ph
in the same ration as every segment of the $n-1$ line).

I have to stop now as I have guests for dinner, but I will
complete this tomorrow.

Best regards
Bernard

Message: #853
Date: 29/11/2014 10:47:29
From: eckart_schmidt@t-online.de
Subject: 5L-Kantor-Hervey Point

Dear Chris,
only a question: what is the "6L-Centric Focus" in your
construction of the 6L-IM01?
Best regards Eckart

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Message: #854
Date: 29/11/2014 11:02:34
From: bernard.keizer
Subject: Morley's 2nd orthocenter

Dear Chris, dear Eckart

As you understand, the application of theorem 5 gives in fact the point p_1 for the QL, with $n=2$ and it happens that this point coincide with the quasi- p_2 for the QL. For the 6-L, with $n=3$, you get directly the quasi- p_2 , which explains why the construction works for the 7L 1st orthocenter.

But, for the 8-L, with $n=4$, you will find the point p_3 and I fear some complications; did you check your construction?

For the 2nd orthocenters, Morley gives on page 11 all the needed explanations for drawing them from one nL to the next $(n+1)L$: the radius of the 2nd circle is $1/(n-1)$ the radius of the 1st the 2nd orthocenter is the external center of similitude of both circles of the nL and

is the intersection of the lines perpendicular to the omitted line through the center of the 2nd circle of the $(n-1)L$.

Application: for $n=3$, the 1st circle is the circumcircle, the 2nd is the Feuerbach circle, with radius hlf of the 1st, and the center of similitude is the orthocenter (which is therefore at the same time p_1 , as SVP point, p_2 , as intersection of the altitudes, and h).

For the QL, the searched point h is the intersection of the lines perpendicular to the 4th line through the Feuerbach points, the Morley point QL-P2 and the line of orthocenters is QL-P2QL-P3 or QL-L4.

In return, the center of the 2nd circle is a new point on the segment QL-P2QL-P4 at the third from QL-P2.

For the 5-L, the searched point h is the intersection of the lines perpendicular to the 5th line trough these new points, the point Chris named, if I'm not wrong, 5L-P11 and the line of orthocenters is 5L-P105L-P11.

In return, this gives on the line 5L-P115L-P2 the center of the 2nd circle at the quarter from 5L-P11.

I'm sorry if this was a little long, but I hope it will be clear.

Best regards to both of you

Bernard

PS Do you understand the calculations with centroids on page 6?

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Message: #855
Date: 29/11/2014 11:04:25
From: chris.vantienhoven
Subject: 5L-Kantor-Hervey Point

Dear Eckart,
It is a name from the document "The Hervey Point of the General
n-Line" from R. Goormaghtigh.
It is actually the 5L-p3 for the 6Line.
Best regards,
Chris

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Message: #856
Date: 29/11/2014 11:06:22
From: bernard.keizer
Subject: 5L-Kantor-Hervey Point

Dear Eckart,
This is the common point of the 6 centric circles of the 5L.
(The name was given by Goormaghtigh by analogy with the focus of
the parabola QL-P1)
Best regards
Bernard

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Message: #857
Date: 29/11/2014 1:45:02
From: eckart_schmidt@t-online.de
Subject: 5L-Kantor-Hervey Point

Dear Bernard K., dear Chris,
thanks for your descriptions of the "6L-Centric Focus", but my questions have no end:
For Bernard: With your description I cannot verify Chris' construction.
For Chris: What is "the 5L-p3 for a 6Line" and how to construct?
Excuse my misunderstanding!
Best regards Eckart

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Message: #858
Date: 29/11/2014 2:10:30
From: chris.vantienhoven
Subject: 5L-Kantor-Hervey Point

Dear Eckart,
[ES] For Chris: What is "the 5L-p3 for a 6Line" and how to construct?
I have the same definition as Bernard K: "the common point of the 6 centric circles of the 5Line".
This point you do not need for constructing 6L-IM01.
6L-IM01 = 6L-Morley's Intermediate 1st Orthocenter = the common point of the perpendicular bisectors of:
* vectorsum of 6 vectors
6L-centric-circle-center.5L-centric-circle-center(i), where $i=1,2,3,4,5,6$
* 5L-Morley's 1st Orthocenter (earlier called 5L-Kantor-Hervey Point by me) =
the common point of the 5 perpendiculars through the 4L-Intermediate 1st Morley's Orthocenter (QL-P3) on the omitted line of the 5-Line.
It looks like you go through the same pitfalls as I did.
Success!
Chris

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Message: #859
Date: 29/11/2014 2:11:24
From: eckart_schmidt@t-online.de
Subject: 5L-Kantor-Hervey Point

Dear Bernard K., dear Chris,
the solution: With Bernard's description of the "6L-Center Focus" I can follow Chris' construction, but replacing "5L-orthocenter" with 5L-P9.
Excuse once more my irritating questions!
Best regards Eckart

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Message: #860
Date: 30/11/2014 9:04:31
From: eckart_schmidt@t-online.de
Subject: 7L-1st Orthocenter

Dear Bernard K., dear Chris,
my questions have no end, can someone test my doubts (nL-pi are Morley's points pi):
We have constructed the Morley point 6L-p2 with the additional remark to Theorem 5 for 2n-lines and in consequence the 7L-1st orthocenter with perpendicular lines...
Taking this point as 7L-p2 it doesn't hold Theorem 5, which says, that the distances from 7L-p2 to 6L-p2 and 6L-p1 have a fixed ratio for included 6-L of the 7-L.
Perhaps this is the second solution for an odd n-line as Morley mentioned on page 12. I think Bernard can judge this. I have to study Morley's second circle. Hopefully there is something true in my message.
Best regards Eckart

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Message: #861
Date: 02/12/2014 8:01:22
From: chris.vantienhoven
Subject: nL-CSC Tranformation?

Dear Eckart,
In QFG#784 you mention:
(6) The 6 CSC-circles of an arbitrary point wrt the
5L-components (see #769) of a 6L have a common point. (Perhaps
this will give a 6L-transformation.)
When I summarize this for the different levels of n-Lines:
* in a 4-Line we have the 4L-Clawson-Schmidt Conjugate QL-Tf1
* in a 5-Line we have the 5L-CSC Transformation
* in a 6-Line we have another 6L-CSC Transformation
Is it possible for you to explore if this CSC Transformation is
a recursive thing in n-Lines for all n (eventually even/odd)?
Best regards,
Chris

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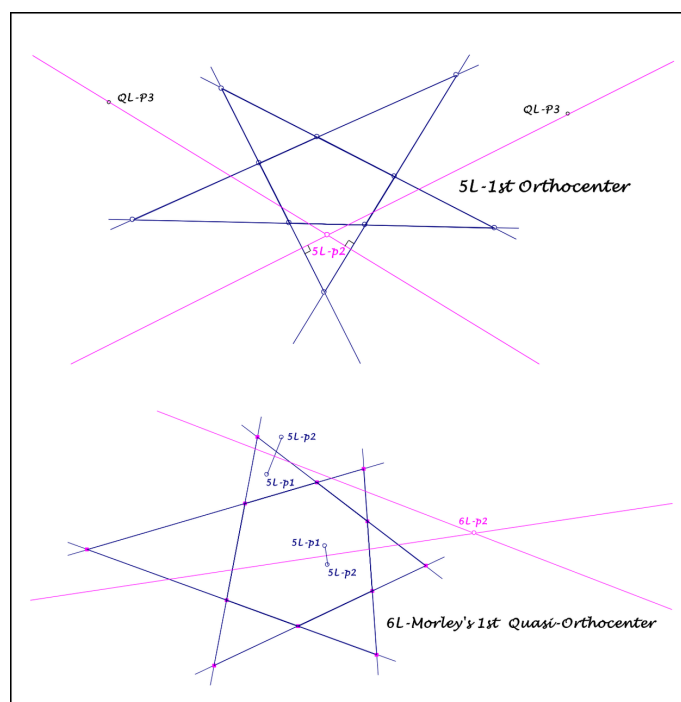
Message: #862
Date: 02/12/2014 11:12:50
From: bernard.keizer
Subject: 7L-1st Orthocenter

Dear Eckart, dear Chris
It's hard to say something without any figure.
I'm convinced it's not the 2nd orthocenter, as the logic is
completely different.
Maybe you have to check again step by step, the p_0 or centric
centers, which is easy, then the p_1 as sum of vectors, which is
possible and last the p_2 , which begins to be pretty hard with
over 6 or 7 lines!
Good luck
Best regards
Bernard

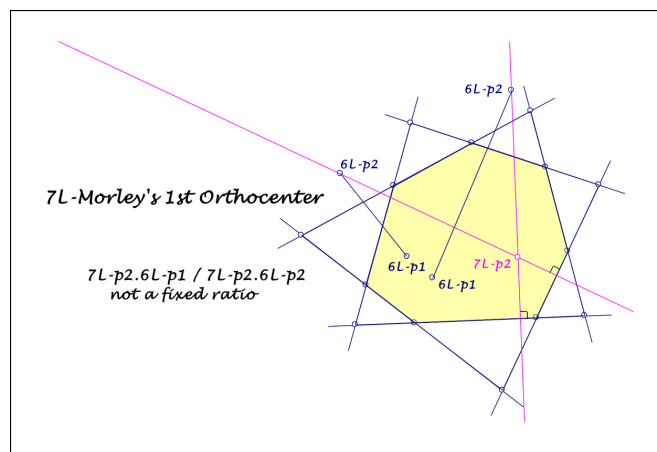
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Message: #863
Date: 02/12/2014 1:25:19
From: eckart_schmidt@t-online.de
Subject: 7L-1st Orthocenter

Dear Bernard K.,
I have demonstrated my constructions in 3 figures, see attached files. But the so constructed 7L-1st Orthocenter as 7L-p2 doesn't hold Morley's Theorem 5.
I use the names nL-pi for Morley's points pi.
Perhaps you can find my mistake, thanks in advance.
Best regards Eckart



2014-12-02a.pdf



2014-12-02b.pdf

Message: #864
Date: 03/12/2014 4:04:57
From: eckart_schmidt@t-online.de
Subject: nL-CSC Transformation?

Dear Chris,
for a 7L the seven 6L-CSC-images of a point are not concyclic,
so I don't see a possibility for generalization. The 5L-CSC is
not involutory, the 6L-CSC is involutory. These transformations
seem to be different types of mappings.
Best regards Eckart

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Message: #865
Date: 04/12/2014 12:48:04
From: bernard.keizer
Subject: 7L-1st Orthocenter

Dear Eckart,
Thanks again for your confidence, but I'm only an ordinary
Morley's reader.
Since the beginning, we try to guess what Morley said!
I find your nL-pi very convenient, as they avoid any discussion
about the naming.
I read Morley again: so far, we are sure of the nL-p0 and nL-p1
for any n and we have checked the property of the distances from
nL-p1 to (n-1)L-p1 and (n-1)L-p0.
For nL-p2, we are sure only of n=5 (theorem 3) and n=6 (theorem
5); it must be possible to check the property of the distances
from 6L-p2 to 5L-p2 and 5L-p1.
For the QL (n = 4), we don't know 4L-p2 and are not able check
the property.
For 7L, the point constructed is very interesting, but if it
doesn't hold the searched property, the only logical conclusion
is that it is not 7L-p2!
Best regards
Bernard

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Message: #866
Date: 04/12/2014 1:51:12
From: eckart_schmidt@t-online.de
Subject: 7L-1st Orthocenter

Dear Bernard K.,
you have gathered precisely my thoughts, thanks!
Playing with Cabri, there seems to be another point, whose
distances to 6L-p1 and 6L-p2 have the same ratio, but I don't
see any properties and Cabri has its limits.
Best regards Eckart

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Message: #867
Date: 04/12/2014 6:20:57
From: bernard.keizer
Subject: 7L-1st Orthocenter

Dear Eckart,
I made a little error in my message: of course, the property for
the distances 6L-p2 to 5Lp2 and 5L-p1 has been checked, as it's
precisely the construction of 6L-p2 (intersection of the 6
perpendicular bisectors of 5L-p15L-p2).
Best regards
Bernard

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Message: #868
Date: 04/12/2014 7:00:16
From: bernard.keizer
Subject: Morley's centroïds

Dear Chris, dear Eckart

This time, I'm very excited, as I think , pursuing the cracking of Morley's code, that I have understood the general nL -pi construction.

It's written in the Bible: "They have eyes, but they don't see!" Morley's équations (14) on page 5 and (15) on page 6 deal with vectors.

I name SVP1 the sum of vectors with origin in nL -p0 and extremity $(n-1)L$ -p0, SVP2 the sum of vectors with the same origin and extremity $(n-2)L$ -p0 and so on ...

For example, for the 5-L, we have three levels, first the centers of the Miquel circles QL-P, 2nd the centers of the 10 circumcircles and 3rd the 10 vertices.

Then 1st equation gives nL -p1 = SVP1,

2nd equation gives nL -p2 - nL -p1 = SVP2 - $(n-1)$ SVP1

3rd equation gives nL -p3 function of SVP1, SVP2 and SVP3 and so on ...

Let's for example concentrate on nL -p1 and nL -p2:

nL -p1 = SVP1 is, as we know, the generalization of the orthocenter (O_iH_i = sum of 3 vectors O_iA_{ij}) or the Kantor-Hervey point (QL -P4 QL -P3 = sum of 4 vectors QL -P4 O_i) ...

For nL -p2, once given the sum of 10 vectors SVP2, if we name X the point which divides nL -p0SVP2 by $(n-1)$, then nL -p2 is the point such as nL -p1 nL -p2 = $(n-1)$ nL -p1X.

The construction is general: I've checked it directly for $n=5$. For $n = 4$, it's not too complicate to find that it gives as QL -p2 the reflexion of QL -P3 in QL -P2: the point $4L$ -p2 lies on the line QL -L4 and the perpendicular bisector of QL -p1 QL -p2 is the Steiner line QL -L2.

I've also checked directly that the distances from $5L$ -p2 to the 2 points QL -p1 and QL -p2 are in the same ratio for the 5 QL 's of the 5L.

This could give a new point for the QL in EQF ...

It is therefore extremely simple to have without any doubt all the nL -p2; it only takes time and patience!

Best regards to both of you and I hope you will be able to draw soon beautiful figures for the 7-L!

Bernard

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Message: #869
Date: 04/12/2014 9:24:27
From: chris.vantienhoven
Subject: Morley's centroids

Dear Bernard K,
Very well !!!
I did some tryouts myself in a 4-Line.
As you mentioned the 4L-p2 should be the reflection of QL-P3 in QL-P2.
This point happens to be QL-P29: Circumcenter
QL-X(265)-Quadrangle.

So I hope you are right, because the circumcircle of this point applied in a 5-Line produces 5L-P8: 5L-X(265)-Circle Point. That would be a nice coincidence with possibly nice consequences.
When I understand you right 4L-p2 is the Sum Vector Point of the vectors with QL-P4 as origin and as extremities the 6 intersection points of the 4 reference lines of the 4-Line.
However when I construct this then I do not find this point.
Is my assumption for this construction wrong?
Fortunately I found something very special while I was experimenting with Sum Vector Points.
I noticed that the centroid of an n-Line is the only point X that coincides with its Sum Vector Point, with X as origin and as extremities all intersection points of the n reference lines in the n-Line.
An appealing definition of the centroid of an n-Line!!!
Last question: do you know why and how Morley relates to centroids at page 6 of his document?
Best regards,
Chris

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Message: #870
Date: 04/12/2014 9:52:59
From: eckart_schmidt@t-online.de
Subject: Morley's centroids

Dear Bernard K., dear Chris,
here is an alternative construction of $nL-p_2$, proved up to $n=7$:
Let $nL-p_x$ be the centroid of the $n(n-1)L-p_1$.
Divide $nL-p_1.nL-p_x$ in the ratio $-n/(n-2)$ and you get $nL-p_2$.
Best regards Eckart
PS: I have not studied Bernard's message, but there will be the
same background.

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Message: #871
Date: 05/12/2014 11:28:32
From: eckart_schmidt@t-online.de
Subject: Morley's centroids

Dear Bernard K., dear Chris,
this seems a recursive definition for the Morley points p_i
(proved for $n=6$ with theorem 5):
 $nL-p_{i+1}$: Let $nL-p_x$ be the centroid of the $n(n-1)L-p_i$ and divide
 $nL-p_i.nL-p_x$ in the ratio $-n/(n-1-i)$.
Best regards Eckart

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Message: #872
Date: 05/12/2014 3:08:30
From: bernard.keizer
Subject: Morley's centroids

Dear Eckart,
Sehr schön, in Wahrheit!
But I checked my calculations and figures and, if I'm not wrong,
it is $n/(n-1-i)$ and not $-n/(n-1-i)$.
For $n=4$ and $i=1$, we have, as I told, p_2 = reflexion of QL-P3 in
QL-P2, which is, as mentioned by Chris, the point QL-P29.
Best regards
Bernard

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Message: #873
Date: 05/12/2014 3:24:34
From: bernard.keizer
Subject: Morley's centroids

Dear Chris,
Thanks for your precision about QL-P29.
You read too quickly, I'm afraid!
The SVP with QL-P4 as origin ... is, as it's name tells it,
QL-SVP2 and not QL-p2; then you have to find the point X and
only finally QL-p2. But Eckart's approach allows to avoid
intermediate steps.
The special property you mention is simply the definition of the
centroid of the vertices of the nL .
For your last question, it's the generalization of the sums and
combinations of vectors like
 $\text{sum of } 00ij = \text{sum of } 00i + \text{sum of } 0i0kl$ at different levels ...
Best regards
Bernard

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Message: #874
Date: 05/12/2014 5:40:38
From: eckart_schmidt@t-online.de
Subject: Morley's centroids

Dear Bernard K.,
I think our constructions of $nL-p_2$ differ in the following way:
You was orientated in line 3 on page 6 and I was orientated in
line 2 on page 6 of Morley's paper.
Wrt the sign of the ratio: If $QL-P_{29}$ is the reflection of $QL-P_3$
in $QL-P_2$, $QL-P_{29}$ divides $QL-P_3QL-P_2$ in the ratio -2 .
Best regards Eckart

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Message: #875
Date: 06/12/2014 9:37:40
From: eckart_schmidt@t-online.de
Subject: Morley's 2nd circle center

Dear Bernard K., dear Chris,
this seems to be a recursive construction of Morley's 2nd circle
center, here named $nL-pc$:
 $3L-pc = X_5$ (see Morley's paper p.10).
For a n -line the perpendicular lines of $(n-1)L-pc$ wrt the
omitted line coincide in the 2nd orthocenter of the n -line, here
named $nL-ph$. This point is a center of similitude for the
 nL -center circle and the nL -2nd circle (theorem 10).
(*) The center of the nL -2nd circle divides $nL-p_0.nL-ph$ in the
ratio $(n-2):1$.
The radius of the nL -2nd circle is $1/(n-1)$ of the radius of the
 nL -center circle (theorem 9).
If (*) is already mentioned in further messages, excuse!
Best regards Eckart

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Message: #876
Date: 06/12/2014 9:42:30
From: bernard.keizer
Subject: Morley's centroids

Dear Eckart,
Our constructions don't differ really, it's always the same kind of calculations.
But I've realized only lately that all these points $p_0, p_1, p_2, p_i \dots$ were indeed vectors!
That's why I said 'The have eyes, but they don't see!'
Then it was easy ...
For the sign, it's a simple question of definition: for me, if a point M has barycentric coordinates x and y wrt 2 points A and B, M divides the segment in the ratio $-y/x$; if x and y are the same sign, M is inside the segment and divides it in a negative ratio und umgekehrt.
Essential remains that we both agree that the point $nL-p_2$ lies outside of the segment joining $nL-p_1$ and the centroid $(n-1)L-g_1$ of the n $(n-1)L-p_1$ with the length $nL-p_1nL-p_2/n = (n-1)L-g_1nL-p_2/(n-2) = nL-p_1(n-1)-Lg_1/2$.
For $n=4$, the centroid of the 4 orthocenters is the Morley-point $QL-P_2$ on the Steiner Line $QL-L_2$ and the point $QL-p_2$ is the reflection of the Kantor-Hervey point $QL-P_3$ in the Morley point $QL-P_2$, id est the point $QL-P_{29}$.
What did you finally find as $7L-p_2$?
Best regards
Bernard
PS The last open question is that of the 64 axes of the QL and the 256 centers of tetracardioids tangent to 5 lines on the $5*64 = 320$ axes ! Any idea?

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Message: #877
Date: 06/12/2014 10:26:04
From: bernard.keizer
Subject: Morley's 2nd circle center

Dear Eckart,
If I'm not wrong, this is exactly my construction in message
#854!
Best regards
Bernard

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Message: #878

Date: 07/12/2014 8:48:29

From: Chris van Tienhoven

Subject: nL-Centroid

Dear BernardK and Eckart,

I am still inwardly processing all beautiful information of last messages.

Meanwhile I won't withhold you some pretty properties of the nL-Centroid.

I have been looking all the time to fit the nL-Centroid in the picture sketched by Morley, despite Morley isn't mentioning the nL-Centroid at all. After all in a triangle it is very easy to construct the 3L-Centroid knowing the 3L-CenterCircleCenter (X3) and the 3L-Orthocenter (X4).

Here are the special properties of nL-G:

nL-G = Point where Sum of vectors to all nL-vertices ($L_i \wedge L_j$) is zero vector.

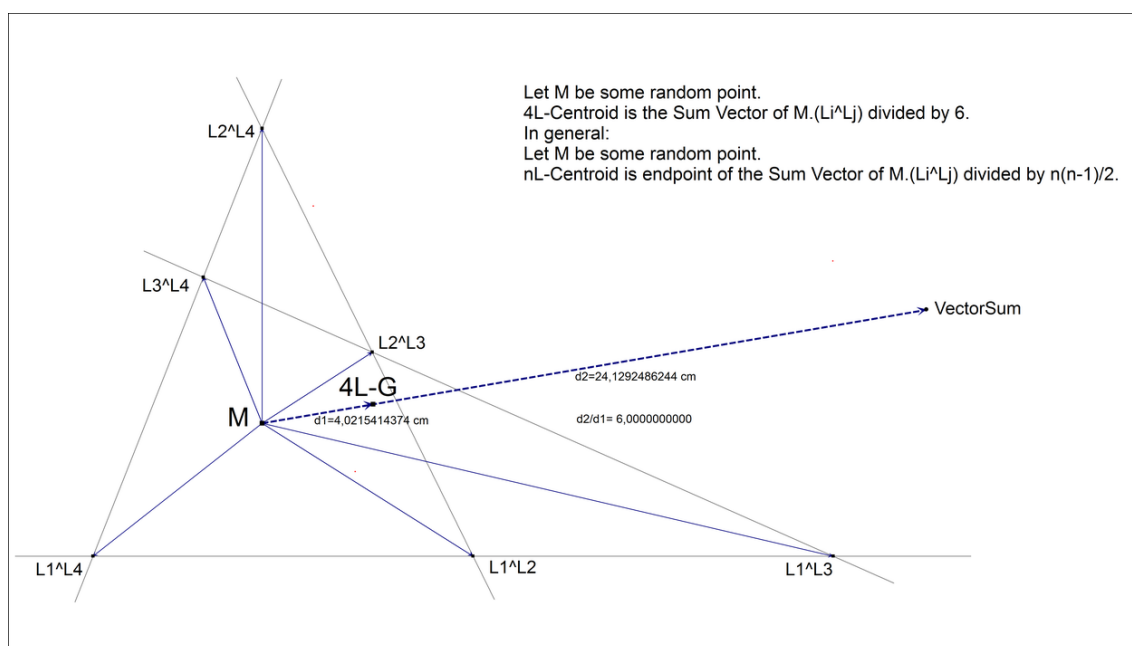
nL-G = SumVector X.($L_i \wedge L_j$) divided by $n(n-1)/2$, where X = some random point.

See attached file with an example in a 4-Line.

In BernardK's message #868 he is mentioning among other things the SVP (Sum Vector Point) of nL-p0 to the vertices of the n-Line. Actually this vector divided by $n(n-1)/2$ produces the vector with endpoint the nL-Centroid.

Best regards,

Chris



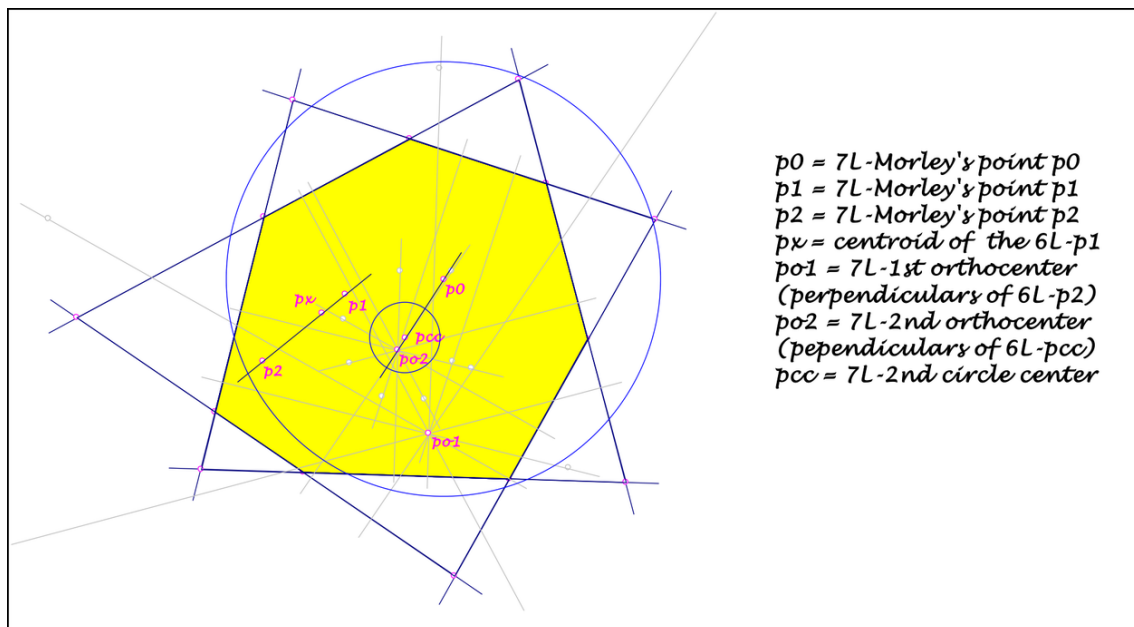
nL-Centroid-01-example in 4-Line.png

Message: #879
Date: 07/12/2014 11:01:07
From: eckart_schmidt@t-online.de
Subject: Morley's centroids

Dear Bernard K,
as sign of good cooperation I have drawn a 7-line with relevant Morley points (hard work, limits of Cabri, see attached word-file).

Best regards Eckart

PS. Wrt #877: You are right, that you describe in message #854 Morley's construction of the 2nd circle center, but I could not find there the property (*).



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Message: #880
Date: 07/12/2014 3:38:31
From: eckart_schmidt@t-online.de
Subject: nL-Quasi-Euler-Line

Dear Bernard K., dear Chris,
Morley gave a generalisation of the circumcenter and the orthocenter of a triangle, here is a generalization of the Euler line:

Let nL-X3 be the midpoint of the nL-center circle, nL-X4 the nL-2nd orthocenter, nL-X5 the nL-2nd circle center.

For n=3 holds $3L-X_i = X_i$ (see ETC),

For n=4 holds: $4L-X_3 = QL-P_4$, $4L-X_4 = QL-P_2$, $4L-X_5$ divides $QL-P_4.QL-P_2$ in the ratio 2:1.

Generalization: $(nL-X_3.nL-X_5) / (nL-X_5.nL-X_4) = (n-2)/1$

Let nL-X2 be the "nL-quasi-centroid", which is the homothetic center of the reference nL and the nL of the parallels to L_i through $(n-1)L-X_5$. For n=3 holds $3L-X_2 = X_2$, for n=4 holds $4L-X_2 = QL-P_{22}$.

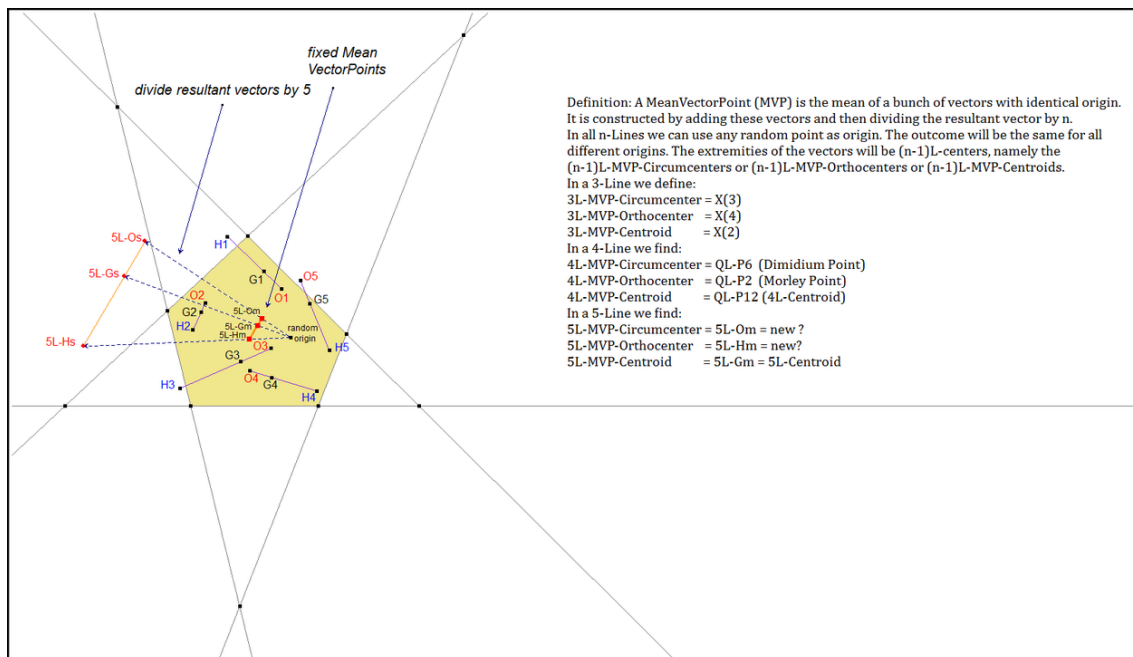
Generalization: $(nL-X_3.nL-X_2) / (nL-X_2.nL-X_4) = (n-2)/2$.

Best regards Eckart

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Message: #881
Date: 07/12/2014 8:27:16
From: Chris van Tienhoven
Subject: nL-MeanVectorPoints

Dear Eckart and BernardK,
Checking all Morley's tricks I found a new concept.
I myself find the results amazing.
Definition: A MeanVectorPoint (MVP) is the mean of a bunch of vectors with identical origin. It is constructed by adding these vectors and then dividing the resultant vector by n .
In all n -Lines we can use any random point as origin. The outcome will be the same for all different origins. The extremities of the vectors will be $(n-1)L$ -centers, namely the $(n-1)L$ -MVP-Circumcenters or $(n-1)L$ -MVP-Orthocenters or $(n-1)L$ -MVP-Centroids.
In a 3-Line we define:
 3L-MVP-Circumcenter = X(3)
 3L-MVP-Orthocenter = X(4)
 3L-MVP-Centroid = X(2)
In a 4-Line we find:
 4L-MVP-Circumcenter = QL-P6 (Dimidium Point)
 4L-MVP-Orthocenter = QL-P2 (Morley Point)
 4L-MVP-Centroid = QL-P12 (4L-Centroid)
In a 5-Line we find:
 5L-MVP-Circumcenter = new?
 5L-MVP-Orthocenter = new?
 5L-MVP-Centroid = 5L-Centroid
In a 6-Line we find:
 6L-MVP-Circumcenter = new?
 6L-MVP-Orthocenter = new?
 6L-MVP-Centroid = 6L-Centroid
Etc.
It is special that for all $n > 2$ these centers are collinear and the mutual distance ratios remain 2:1, just like in a triangle. Of course these nL -Points create for each n an nL -MVP-Eulerline.
Best regards,
Chris



nL-5L-MVPs-with-RandomOrigin-01.png

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Message: #882
Date: 08/12/2014 9:25:42
From: eckart_schmidt@t-online.de
Subject: nL-MeanVectorPoints

Dear Chris,
that is an interesting method to get new points nL-MVP- X_i out of triangle points X_i , for the ratios of distances remain constant, if they are not dependent of the form of the triangle. It will be interesting, which QL-points are 4L-MVP- X_i and which nL-MVP-points are follower of QL-points.
By the way: The 5L-MVP-Orthocenter divides Morley's 5L-1st Orthocenter and 5L-2nd Orthocenter in the ratio 9:1. I will research this for $n=7$.
So far, best regards Eckart

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Message: #883
Date: 08/12/2014 11:30:05
From: bernard.keizer
Subject: Morley's centroids

Dear Eckart,
Thanks a lot for this huge work.
I appreciate really this good cooperation!
Before forgetting, in fact the property (*) wasn't mentioned explicitly in my message #854.
Now, let's come to the essential!
As you and I feared, 7L-p2 and so-called 7L-1st orthocenter are not the same point.
But which is the right 1st orthocenter? Which are in line in the 8L?
Morley doesn't mention the construction with perpendiculars of 6L-p2, it was only guessing by analogy (but Kantor-Hervey point is p1 and not p2 for the QL!)
And as there are 2 points 6Lp2 and 6Lpcc with perpendiculars to the omitted line, they form a line of orthocenters ...
I'm afraid there will be never and end to these questions ...
Best regards
Bernard

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Message: #884
Date: 08/12/2014 11:36:27
From: bernard.keizer
Subject: nL-MeanVectorPoints

Dear Chris,
Amazing property indeed, this Euler Line of the nL!
It's Christmas before Christmas ...
If it's true for the nL, it's also true for the (n+1)L (a kind of recurrence).
It's true for the triangle and the QL, theefore it's always true.
Now, the points are not entirely new:
as you mention, MVP-centroid is the centroid of the vertices of the nL.
MVP-circumcenter is the centroid of the circumcenters of the (n-2)L, it's what Morley names g2 and is what I named SVP2, divided by $n(n-1)/2$ (g1 being SVP1 divided by n).
Surprising is this MVP-orthocenter (in fact the 2nd orthocenter) on the line of orthocenters!
Best regards
Bernard

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Message: #885
Date: 08/12/2014 12:45:47
From: eckart_schmidt@t-online.de
Subject: Morley's centroids

Dear BernardK,
you get the nL-1st orthocenter for odd n with the perpendiculars of (n-1)L-p2. For 8L the 7L-1st orthocenters are collinear.
You get the nL-2nd orthocenter with the perpendiculars of the (n-1)L-2nd circle centers.
But how to get further orthocenters of Morley's "line of orthocenters for odd n"?
Best regards Eckart

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Message: #886
Date: 08/12/2014 12:57:26
From: chris.vantienhoven
Subject: nL-MeanVectorPoints

Dear Eckart and BernardK,
Bernard, it feels like Christmas indeed.
This property is so amazing, I am really enthusiastic about it!
I checked several points (X1,X2,X3,X4,X5,X6,X7,X356,X5914) and
they all have a corresponding MeanVectorPoint in a 4-Line,
5-Line, etc.
So it looks like that (almost) all ETC-points can be "upgraded" and
have corresponding points in a 4-Line. These 4-Line points
can be used to construct corresponding points in a 5-Line. Etc.
Like you say Eckart, it is interesting now to investigate for
4L-points which 3L-points originate.
And for 5L-points which 4L-points originate. Etc.
Meanwhile I also investigated if this MVP principle is
applicable in an n-Point configuration.
And yes it is applicable in an n-Point system too!
For example:
X(2) -> QA-P1
 -> 5L-Centroid
 -> 6L-Centroid
 -> etc.
X(3) -> QA-P32
 -> 5L-0
 -> 6L-0
 -> etc.
X(4) -> QA-P33
 -> 5L-H
 -> 6L-H
 -> etc.
X(5) -> Midpoint (QA-P32,QA-P33)
 -> 5L-N
 -> 6L-N
 -> etc.
X(6) -> unknown QA-point
 -> 5L-K
 -> 6L-K
 -> etc.
Eulerline -> line QA-P1.QA-P32.QA-P33
etc.
Again ratios are preserved for different n!
Best regards,
Chris

Message: #887
Date: 08/12/2014 1:58:23
From: eckart_schmidt@t-online.de
Subject: nL-MeanVectorPoints

Dear Chris,
sorry, the 7L-MVP-Orthocenter is not collinear with Morley's
7L-1st and 2nd Orthocenter.
Best regards Eckart

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Message: #888
Date: 08/12/2014 4:51:35
From: eckart_schmidt@t-online.de
Subject: nL-MeanVectorPoints

Dear Chris,
the MVP of QL-P3 in a 5-line divides 5L-p1.5L-p2 (Morley points)
in the ratio 2:3.
Best regards Eckart

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Message: #889
Date: 08/12/2014 10:08:08
From: Chris van Tienhoven
Subject: nL-MeanVectorPoints

Dear Eckart and BernardK,

I made a picture of 4L-MVP points wrt triangle points:

X2, X3, X4, X5, X6,

X13, X14, X15, X16,

X61, X62, X381, X395, X396.

See attachment.

Normally in a triangle there are lots of collinearities between these points. However in the 4L-picture only the points on the Eulerline are collinear. Amongst the remaining 4L-MVP-points there is no more collinearity at all.

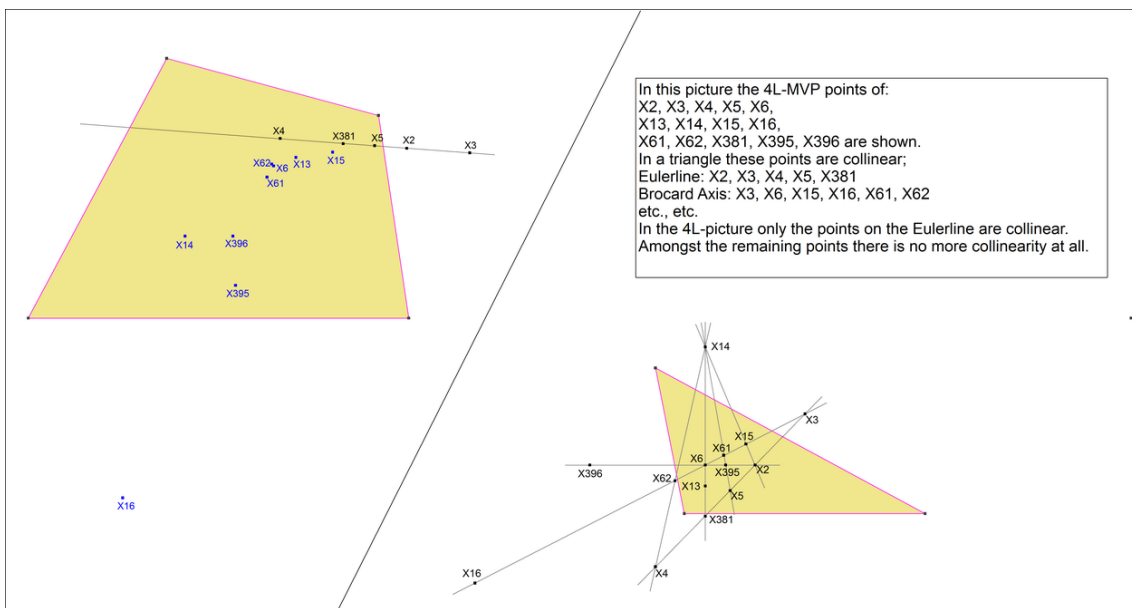
This gives insight in the MVP transformation to higher n-levels.

It looks like there is a schism around the Eulerline.

I wonder what kind of deformation occurs at both sides of the Eulerline.

Best regards,

Chris



4L-MVP-points-X2-X396-01.png

Message: #890
Date: 09/12/2014 10:22:51
From: eckart_schmidt@t-online.de
Subject: nL-MeanVectorPoints

Dear Chris,
MVP of Morley points nL-pi (see #871):
MVP of (n-1)L-pi-1 for n-lines divide nL-pi-1.nL-pi with ratio
 $i/(n-i)$.
Best regards Eckart

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Message: #891
Date: 09/12/2014 11:34:48
From: bernard.keizer
Subject: nL-MeanVectorPoints

Dear Eckart,
Very beautiful property and concentrate formula!
Immediate application was your message #888 with Kantor-Hervey
point 4L-p1.
Best regards
Bernard

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Message: #892
Date: 09/12/2014 12:13:26
From: bernard.keizer
Subject: Morley's centroids

Dear Eckart,
I knew since the beginning that Morley was a big and heavy peace of cake!
Now I begin to feel a little less stupid ...
In fact, the p_i are only helps for finding the 1st orthocenters!
(But $5L-p_2$ coincide with $5L$ -1st orthocenter)
In theorem 5, for $n=2$, we get $4L-p_1$ on the perpendicular bisectors of the $3L-p_0$ and $3L-p_1$ (well known Kantor-Hervey theorem) and $5L$ -1st orthocenter as concurrent point of the perpendicular from the $4L-p_1$ to the 5th line; for $n=2$, we get the same way $6L-p_2$ and $7L$ 1st orthocenter and for $n=3$, I suppose we will find $8L-p_3$ and $9L$ 1st orthocenter. I hope you agree with this interpretation!
I was disturbed by the fact that Kantor-Hervey point is $4L-p_1$ and that we use then $6L-p_2$...
Anyhow, back to your question.
I've already explained what I understand in message #852.
As soon as you have in each $(n-1)L$ 2 points with the same definition for which the perpendiculars to the omitted line concur, any point of the segment joining the 2 points with the same definition (dividing the segment in the same ratio) will have the same property that the perpendiculars to the omitted line concur in a point: this point divides the segment through the 2 1st points of concurrence in the same ratio (application of the Thales theorem). Morley calls this line the line of orthocenters, I suppose to distinguish of the directrix of orthocenters.
For example, on your beautiful figure of message #879, for any point in the $6-L$ dividing the segment $6L-p_2.6L-p_{cc}$ in the same ratio, the perpendiculars to the 7th line will concur in a point dividing the segment of the 2 orthocenters in the same ratio.
Best regards
Bernard

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Message: #893
Date: 09/12/2014 7:54:12
From: chris.vantienhoven
Subject: Morley's centroids

Dear BernardK, Eckart,
Today looking again at the construction methods of Morley's 1st Orthocenter.
Bernard is writing today about it too.
BernardK writes at QFG#868:
1st equation gives $nL-p1 = SVP1$,
2nd equation gives $nL-p2 - nL-p1 = SVP2 - (n-1) SVP1$
3rd equation gives $nL-p3$ function of $SVP1$, $SVP2$ and $SVP3$
and so on
Eckart writes at QFG#870:
Let $nL-px$ be the centroid of the $n (n-1)L-p1$.
Divide $nL-p1.nL-px$ in the ratio $-n/(n-2)$ and you get $nL-p2$.
Both methods are derived from Morley's statements at page 6 of his document and do properly produce the 1st Orthocenter.
The funny thing is that in this construction method Morley's 1st Orthocenter no longer is a point exclusively for $n=odd$!
For example the 1st Orthocenter in a 4-Line is $QL-P29$. And a 4-Line is not an odd n -Line.
Do you agree?
What are the consequences?
Bernard, I do not understand your comment: "3rd equation gives $nL-p3$ function of $SVP1$, $SVP2$ and $SVP3$ ".
Can you be a bit more specific?
Best regards,
Chris

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Message: #894
Date: 10/12/2014 12:47:10
From: bernard.keizer
Subject: Morley's centroids

Dear Chris,
It's precisely what I tried to explain in my last message!
 $nL-p_2$ is generally not the same point as $nL-1st$ orthocenter!
 $QL-P_{29}$ is $4L-p_2$, but not there isn't a $4L-1st$ orthocenter ...
For $n=5$, apparently $5L-p_2$ coincide with $5L-1st$ orthocenter.
For $n=6$, there is a $6L-p_2$, but no $6L-1st$ orthocenter.
For $n=7$, there are a $7L-p_2$ and a $7L-1st$ orthocenter, which are 2 different points.
and so on
Best regards
Bernard

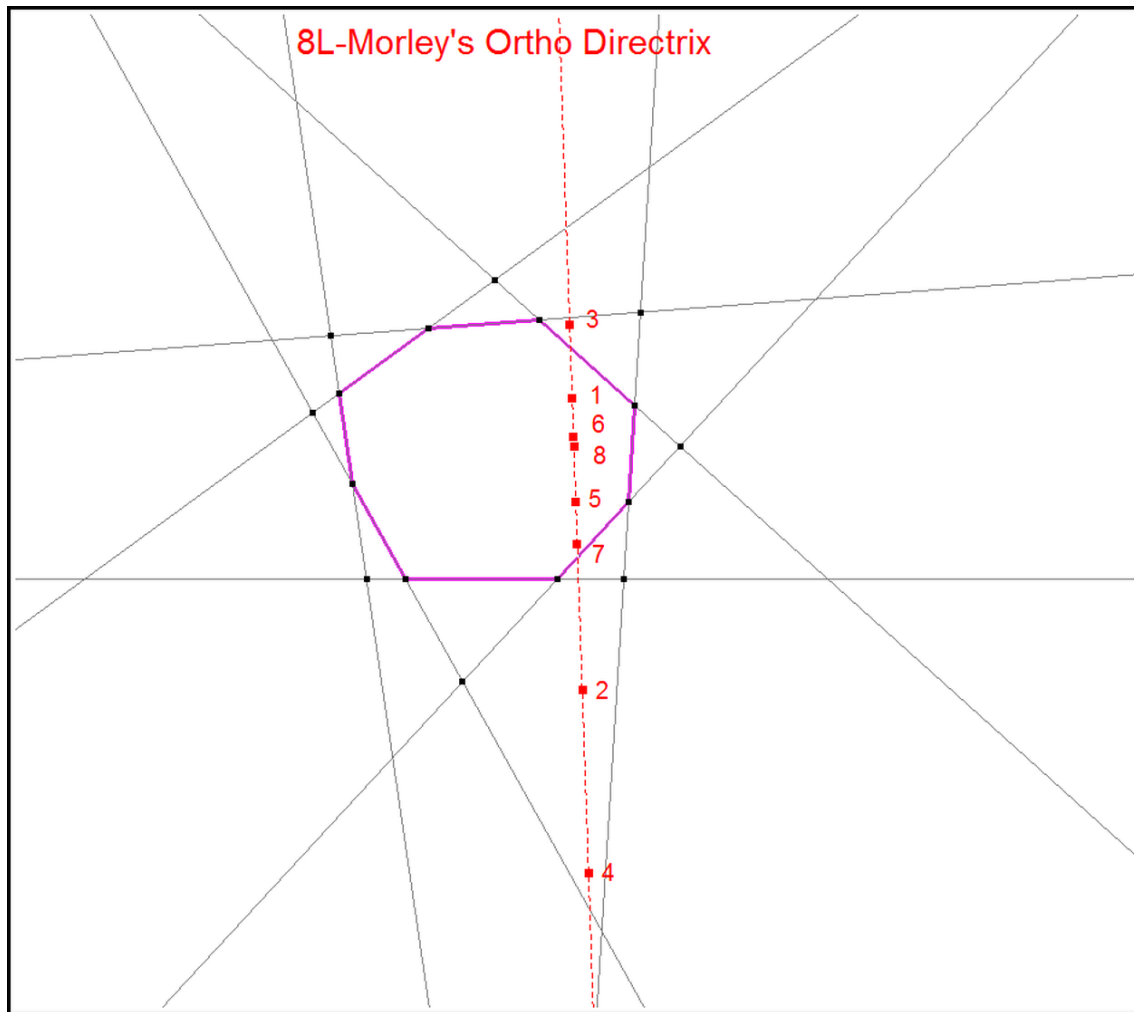
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Message: #895
Date: 10/12/2014 9:44:40
From: Chris van Tienhoven
Subject: 1st Orthocenter and Summary

Dear BernardK and Eckart,
It is a huge surprise party.
Every time we think we figured it all out and then inescapably there is a setback.
I will try to organize things now according to my latest insights.
Morley talks about lots of Orthocenters in an n-Line. Because even Morley's own coding doesn't seem consistent I start with my own coding now.
See attachment for a summary of all recursive nL-points. It is a document in progress (comments are welcome). I am still adapting it on a daily basis.
I use infixes in the coding, meaning o=odd-n, e=even-n, n=all-n. I do this because I want to number these different types of points separately.
Also I use "mL-" meaning "(n-1)L", referring to the n-Line of the first level below n.
In my numbering system I took into account wrt the most important points:
 1=Miquel Point,
 2=Centroid,
 3=Circumcenter,
 4=Orthocenter,
 5=Ninepoint Center,
 6=Symmedian Point(=LSD point).
This is a mixture of the coding in ETC and EQF.
Now back to the 1st Orthocenter. We have:
nL-nP7 = nL-Hervey Point.
nL-oP1 = nL-Morley's 1st Orthocenter
 = Common point Perpendiculars
 through mL-eP1(i) on omitted line.
nL-eP1 = nL-Morley's Intermediate 1st Orthocenter
 = Common Point Perpendicular bisectors mL-oP1.mL-nP7 (i).
I checked that in an 8-Line the seven 7L-oP1's are collinear. So this should be Morley's 1st Orthocenter.
See attachment.
Then there are Morley's p i 's about which we thought they were mentioned as an alternative construction for the 1st Orthocenter. However they are not. First, because the 1st Orthocenter is said to be an odd point. Second, because at level n=7 there is a discrepancy.

Eckart cracked Morley's code with the nL-Centroid of the mL-Hervey Points leading us to a new point coinciding with the 1st Orthocenter at levels $n=3$ and 5 , but not at level 7 . This new point at level 7 is not collinear at level 8 . So it is not the 1st Orthocenter, but let us say the Quasi 1st Orthocenter. About the p_i 's I am still wondering if p_2, p_3, p_4 , etc. can be of any significance. Maybe you have ideas about this. I hope this structures many of our thoughts. BernardK, thank you for pointing out so many things! Without this we wouldn't be where we are now!

Best regards,
Chris



8L-Morley's Ortho Directrix-01.png

Recursive points/lines/circles in n-Lines

Morley- notation	General nL-Points	n = 3	n = 4	Ref	Construction
nL-nP1	nL-Miquel Point / Centric Focus	--	QL-P1	[1,4]	Common point n mL-Center circles (nL-nCi1)
nL-nP2	nL-Centroid	X(2)	QL-P12	[3] #869 #878 #873 #881	Node Mono EnnaCardioid circumscribing n mL-EnnaCardioids Point where Sum of vectors to all nL-vertices is zero vector SumVector nL-nP3.(Li^Lj) divided by n(n-1)/2 nP-Centroid of all nL-vertices (Li^Lj) MeanVectorPoint(=SumVectorPoint/n) of mL-Centroid
nL-nP3	a1/p0 nL-Centercircle Center	X(3)	QL-P4	[2]	Center of nL-nCi1. Also called Kantor's point.
nL-nP4	h nL-Morley's 2nd Orthocenter	X(4)	QL-P2	[2]	Common point perpendiculars from mL-nP5 on omitted line
nL-nP5	nL-Second Circle Center	X(5)	QL-P30	[2]	External Homothetic Center of nL-nCi1 & nL-nCi2
nL-nP6	nL-Least Squared Distances Points	X(6)	QL-P26	[5]	Ratiopoint nL-nP3.nL-nP4 ((n-2) : 1)
nL-nP7	p1 nL-Hervey Point	X(4)	QL-P3	[2]	Point with Least Sum of squared Distances to reference lines
nL-nP8	gi nL-Centroid mL-Hervey Points	X(2)	QL-P2	[2] #870	Sum of Vectors nL-nP3.mL-nP3(i), where i=1,...,n. [2,4] Fixed Ratiopoint [mL-nP3(i), mL-nP4(i)], where i=1,...,n. nP-centroid of mL-nP7(i), where i=1,...,n.
nL-nP9	p2 nL-Morley's Quasi 1st Orthocenter	X(4)	QL-P29	#868,8	Ratiopoint nL-oP1.nL-nP7 ((n-2) : 2) Ratiopoint nL-nP7.nL-nP8 (n : (2-n)) Differs from nL-oP1 from level n=7.
nL-nP10	nL-Morley Centroid	X(2)	QL-P22	#880	Ratiopoint nL-nP3.nL-nP4 ((n-2) : 2)
nL-nP11	nL-MVP Circumcenter	X(3)	QL-P6	#881	MeanVectorPoint(=SumVectorPoint/n) of mL-MVP Circumcenter
nL-nP12	nL-MVP Orthocenter	X(4)	QL-P2	#881	MeanVectorPoint(=SumVectorPoint/n) of mL-MVP Orthocenter
nL-nP13	nL-MVP Nine-pointcenter	X(5)	QL-Px	#881	MeanVectorPoint(=SumVectorPoint/n) of mL-MVP Nine-pointcenter
General nL-Circles/Lines					
nL-nCi1	nL-Centercircle (center nL-nP3)	CircumCircle	QL-Ci3	[2]	Circle of concyclic points mL-nP3. Also called Kantor's Circle.
nL-nCi2	nL-Second Circle (center nL-nP5)	EulerCircle	QL-Cix	[2]	Radius nL-nCi2 = nL-R / (n-1), where nL-R = radius nL-nCi1.
nL-nL1	nL-Morley Eulerline	EulerLine	QL-P2.P4	#880	Line through nL-nP3, nL-nP4, nL-nP5, nL-nP9.
nL-nL2	nL-MVP Eulerline	EulerLine	QL-P2.P6	#880	Line through nL-nP2, nL-nP10, nL-nP11, nL-nP12.
n=Even/Odd Points/Lines/Circles					
nL-oP1	nL-Morley's 1st Orthocenter	X(4)	--	[2]	Common point Perpendiculars through mL-eP1(i) on omitted line
nL-eP1	nL-Morley's Intermediate 1st Orthocenter	--	QL-P3	[2]	Common Point Perpendicular bisectors mL-oP1.mL-nP7 (i)
nL-oL1	nL-Morley's Line of Orthocenters	--	--	[2]	Center of nL-Inscribed EnnaDeltoid
nL-eL1	nL-Morley's Ortho Directrix	--	QL-L2	[2]	Line connecting nL-oP1 and nL-nP4 Line of mL-Morley's 1st Orthocenters, when n = even
nL-eP2	nL-Clifford's Point	--	QL-P1		Common point n mL-Clifford's circles (nL-oCi1)
nL-oP2	nL-Clifford's Circle Center	X(3)	--		Center of n concyclic mL-Clifford's Points (nL-eP2)
nL-oCi1	nL-Clifford's Circle (center nL-oP2)	CircumCircle	--		Circle of n concyclic mL-Clifford's Points (nL-eP2)
infix:					
n	= point/figure occurring for all n>1			[1]	Morley's paper: On the metric Geometry of the Plane n-Line
e	= point/figure occurring only for n=even			[2]	Morley's paper: Orthocentric properties of the Plane n-line
o	= point/figure occurring only for n=odd			[3]	Morley's paper: Extensions of Clifford's Chain Theorem
s	= point/figure occurring only for some special n			[4]	R. Goormaghtigh: The Hervey Point of the general n-LINE
				[5]	J.L. Coolidge: Two geometrical applications of the method of least squares
				#nnn	Refers to Yahoo Quadri-Figures Group message # nnn
References:					
Abbreviations/Terminology:					
n-Line A system of n random coplanar lines					
nL- indicates item that is refering to an n-Line					
mL- means (n-1)L: referring to the nLine of the first level below n					
omitted line an n-Line has n m-Lines					
When refering to an m-Line there is always an omitted line.					
SVP Sum Vector Point = Endpoint of n Summarized radial Vectors					
MVP Mean Vector Point = Endpoint SumVector/n					
Ratiopoint Let X=Ratiopoint Q1.Q2 (m:n)					
Let d1=distance(X,Q1), d2=distance(X,Q2)					
d1 is positive when X lies on the same side of Q1 as Q2					
d1 is negative when X and Q2 lie at different sides of Q1					
Corresponding properties for d2. Now m:n = d1:d2.					
MVP properties:					
#888 The MVP of QL-P3 in a 5-line divides 5L-p1.5L-p2 (Morley points) in the ratio 2:3 (ES)					
#882 The 5L-MVP-Orthocenter divides Morley's 5L-1st Orthocenter and 5L-2nd Orthocenter in the ratio 9:1 (ES)					

Summary of all recursive nL-points-01.pdf

Message: #896
Date: 10/12/2014 10:38:31
From: eckart_schmidt@t-online.de
Subject: 1st Orthocenter and Summary

Dear Chris,
the summary a hard work for you, but it is difficult to handle.
Only a short remark: 8L-ep1 doesn't exist!
Best regards Eckart

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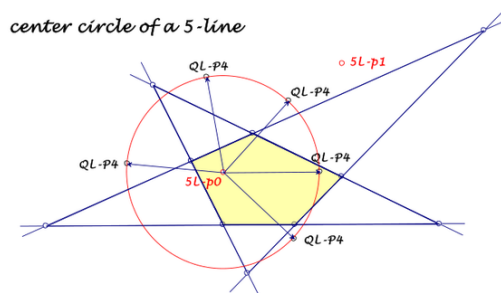
Message: #897
Date: 10/12/2014 11:22:56
From: eckart_schmidt@t-online.de
Subject: 1st Orthocenter and Summary

Dear BernardK, dear Chris,
on my desk lies already some days my summary. But there remain difficulties wrt the 1st orthocenter. I regret, that I can't prove Morley's definition for $n=9$: perpendiculars of 8L-p2 wrt the omitting lines. My Cabri has limits at 8L-p2.
I hope, that there are no basic mistakes in my summary.
Best regards Eckart

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Morley Points for n-Lines

Morley tried, to generalise the triangle orthocenter for n-lines. In his paper "Orthocentric Properties of the Plane n-Line"(1902) we find ambitious analytic calculations for special points and circles, but no explicit constructions. Here is a summary of results out of a discussion in QFG with Bernard Keizer and Chris van Tienhoven.



Center Circle and Morley Point p_0

Beginning with a 3-line we have the circumcircle and its center as Morley point $3L-p_0$. For a 4-line we get – omitting one line – four concyclic $3L-p_0$. The corresponding circle will be the center circle of the 4-line – that is the Miquel Circle $QL-Ci3$ – and its midpoint $4L-p_0 = QL-P4$.

In this way we get a recursive definition for the Morley point $nL-p_0$ as center of the n concyclic $(n-1)L-p_0$ of the n -line, omitting one line.

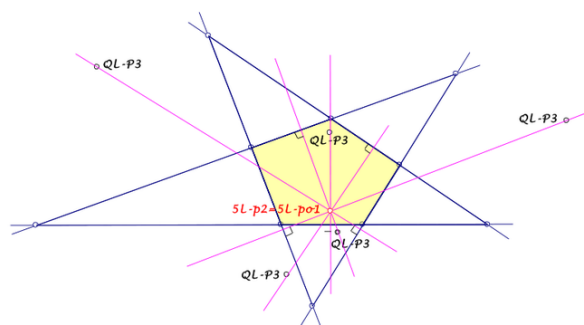
Morley point p_1

Adding the vectors with origin $nL-p_0$ and extremities $(n-1)L-p_0$ we get the Morley point $nL-p_1$. For a 3-line $3L-p_1$ is the orthocenter and for a 4-line we find $4L-p_1 = QL-P3$.

Morley point p_2

We give a recursive definition: Let $nL-p_x$ be the centroid of the n points $(n-1)L-p_1$ and divide $nL-p_1, nL-p_x$ in the ratio $-n/(n-2)$, then you get $nL-p_2$. For a 3-line let $3L-p_2$ be the orthocenter. For a 4-line we get $4L-p_2$ as reflection of $QL-P3$ in $QL-P2$. For a 5-

line $5L-p_2$ is the common point of the perpendiculars through $4L-p_1 = QL-P3$ wrt the omitting line. For a 6-line $6L-p_2$ is the common point of the perpendicular bisectors of $5L-p_1, 5L-p_2$.

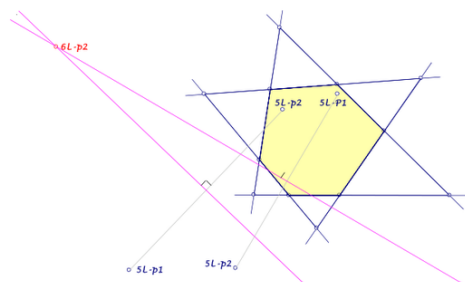


Morley points p_i

It seems, that the following recursive definition holds for $nL-p_{i+1}$: Let $nL-px$ be the centroid of the n $(n-1)L-p_i$ and divide $nL-p_i, nL-px$ in the ratio $-n/(n-i-1)$.

This can be controlled with Morley's theorem 5, which says, that $nL-p_i$ has distances with a fixed ratio to $(n-1)L-p_i$ and $(n-1)L-p_{i-1}$ of the included $(n-1)$ -lines.

As further control Morley mentioned in a remark to theorem 5 for an even number of lines: $2nL-p_{n-1}$ is equidistant of $(2n-1)L-p_{n-1}$ and $(2n-1)L-p_{n-2}$.



Morley's 1st Orthocenter p_{o1}

Morley's 1st orthocenter is only defined for odd n , beginning with $n=5$. Morley describes this point $nL-p_{o1}$ as common point of the perpendiculars of $(n-1)L-p_2$ wrt the omitting line.

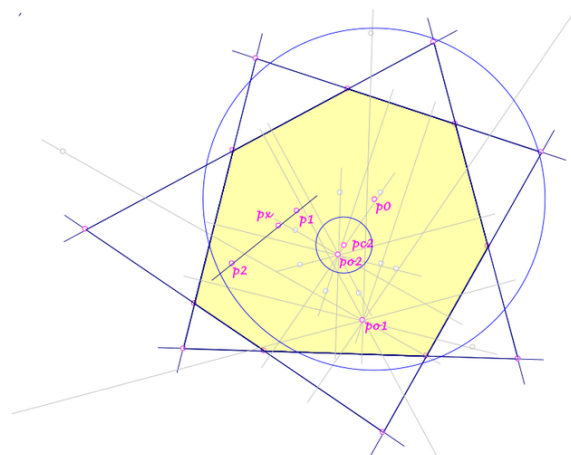
For $n=5$ holds $5L-p_{o1} = 5L-p_2$, but for $n=7$ these are two different points!

For even n the first orthocenters $(n-1)L-p_{o1}$ are collinear on the so called "directrix".

Morley's 2nd Circle Center p_{c2} and 2nd Orthocenter p_{o2}

The 1st circle will be the center circle. For a 3-line Morley's 2nd circle is the nine-point circle with center $X5 = 3L-p_{c2}$. For a n -line the perpendicular lines through $(n-1)L-p_{c2}$ wrt the omitted line will coincide in Morley's 2nd orthocenter $nL-p_{o2}$. This point – in Morley's paper h – is the external center of similitude for the 1st and 2nd circle (theorem 10). Then the center of the 2nd circle divides $nL-p_0, nL-p_{o2}$ in the ratio $(n-2):1$. The radius of the 2nd circle is $1/(n-1)$ of the radius of the 1st circle (theorem 9).

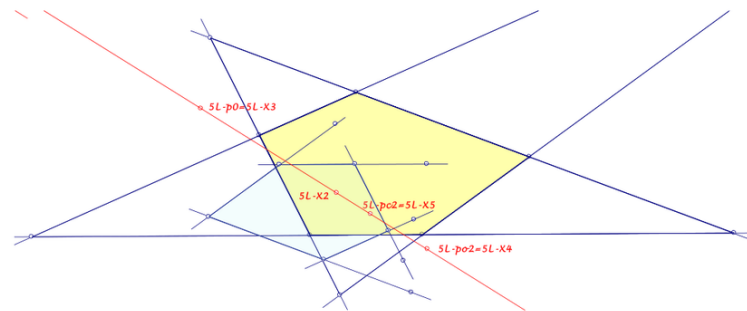
For a 4-line holds $4L-p_{o2} = QL-P2$ and $4L-p_{c2}$ divides $QL-P2, QL-P4$ with ratio 1:2.



$p0$ = 7L-Morley's point $p0$
 $p1$ = 7L-Morley's point $p1$
 $p2$ = 7L-Morley's point $p2$
 $pκ$ = centroid of the 6L- $p1$
 $po1$ = 7L-1st orthocenter
 (perpendiculars of 6L- $p2$)
 $po2$ = 7L-2nd orthocenter
 (perpendiculars of 6L- $pc2$)
 $pc2$ = 7L-2nd circle center

nL-Quasi Euler Line

Morley gave a generalisation of the circumcenter and the orthocenter of a triangle, here is proposed a generalisation of the Euler line:



Let be:

$nL-X3 = nL-p_0$ the midpoint of the center circle,
 $nL-X4 = nL-p_{o2}$ the 2nd orthocenter,
 $nL-X5 = nL-p_{c2}$ the 2nd circle center.

For $n=3$ holds $3L-X_i = X_i$ (see *ETC*), for $n=4$ holds: $4L-X_3 = QL-P_4$, $4L-X_4 = QL-P_2$, $4L-X_5$ divides $QL-P_4.QL-P_2$ in the ratio 2:1.

Generalisation:
$$\frac{nL-X_3.nL-X_5}{nL-X_5.nL-X_4} = \frac{n-2}{1}$$

Now let $nL-X_2$ be the "nL-quasi-centroid", which is the homothetic center of the reference n-line and the n-line of the parallels to L_i through $(n-1)L-X_5$.

For $n=3$ holds $3L-X_2 = X_2$, for $n=4$ holds $4L-X_2 = QL-P_{22}$.

Generalisation:
$$\frac{nL-X_3.nL-X_2}{nL-X_2.nL-X_4} = \frac{n-2}{2}$$

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2014-12-10.pdf

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Message: #898
Date: 10/12/2014 12:33:32
From: bernard.keizer
Subject: 1st Orthocenter and Summary

Dear Eckart, dear Chris
Remarkable works indeed!
I will read carefully your 2 summaries.
Before that, I react immediately to your last remark wrt 8L-p2 and 9L-1st orthocenter.
I'm not surprised that it doesn't work.
Please read again my messages #892 and #894!
I've asked you if you shared my interpretation, but you didn't answer.
I think the definition perpendiculars of $(n-1)L-p2$ is wrong
To continue my message #854, it would give
for $n = 8$, there is a 8L-p2 and a 8L-p3, but not a 8L-1st orthocenter
for $n = 9$, there are a 9L-p2 and a 9L-1st orthocenter, but you have to consider this time the perpendicular from the 8L-p3 and not the 8L-p2!
4L-p1 for 5L-1st orthocenter
6L-p2 for 7L-1st orthocenter
8L-p3 for 9L-1st orthocenter
generally the $2n-L-p_{n-1}$ for the $(2n+1)L$ -1st orthocenter and so on
There are no quasi- or intermediate 1st orthocenter, but only $nL-p_i$ points on one side (for any n) and 1st orthocenters on the other side (only for odd n) and 2nd orthocenter (again for any n). Each of the 3 groups has it's own rule of construction.
Best regards
Bernard

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Message: #899
Date: 11/12/2014 9:51:58
From: eckart_schmidt@t-online.de
Subject: 1st Orthocenter and Summary

Dear BernardK,
it seems, that we have to read your messages more precisely! I think, you will be right with your interpretation of the 1st orthocenter. It would wipe away my difficulties with this point. Morley's "... and so on" on page 5 is as you say: perpendiculars of 4L-p1 for a 5-line, perpendiculars of 6L-p2 for a 7-line, perpendiculars of 8L-p3 for a 9-line, ...
I try to find a construction for 8L-p3 and I have to correct my summary.
Best regards Eckart

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Message: #900
Date: 11/12/2014 10:38:49
From: bernard.keizer
Subject: 1st Orthocenter and Summary

Dear Eckart,
Thank you for your message, I appreciate your remark!
For your summary, I wouldn't define $5L-p_2$ with the
perpendiculars through $4L-p_1$: for me, this is the general
definition of the 1st orthocenter po_1 .
It happens that for $n=3$ p_1 , p_2 , po_1 and po_2 coincide and for $n=5$ p_2 and po_1 also coincide.
For 1st orthocenter po_1 , I would begin with $n=3$, as the
orthocenter is also the common point of the altitudes
(perpendiculars through the vertices $2L-p_0$ to the opposite
side).
Then it holds that $2nL-p_{n-1}$ is equidistant from $(2n-1)L-p_{n-1}$ and
 $(2n-1)L-p_{n-2}$ and that the perpendiculars from $2nL-p_{n-1}$ to the
omitted line intersect in the $(2n+1)L-po_1$: $2L-p_0$ gives $3L-po_1$,
 $4L-p_1$ equidistant from $3L-p_0$ and $3L-p_1$ (Kantor-Hervey's theorem)
and gives $5L-po_1$, $6L-p_2$ equidistant from $5L-p_1$ and $5L-p_2$ and
gives $7L-po_1$, $8L-p_3$ equidistant from $7L-p_2$ and $7L-p_3$ and gives
 $9L-po_1$ and so on ...
As you point yourself, you have now to find the construction of
the $nL-p_3$!
Best regards
Bernard

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Message: #901
Date: 11/12/2014 11:17:09
From: bernard.keizer
Subject: 1st Orthocenter and Summary

Dear Chris,
Reading your summary, I realise I already made the main remarks!
There are 3 series of different points: the $nL-p_i$, the 1st and the 2nd orthocenters.
Each serie has it's own rule.
Yes, I'm convinced that p_1 , p_2 , p_3 and other p_i have their own significance and there are as many g_i as p_i . (By the way, $nL-nP_8$ is g_1 and not g_i)
They are anyhow necessary to construct the serie of the 1st orthocenters, as explained again in my message #900 and we don't need the quasi- or intermediate 1st orthocenters. The rule of construction given for $nL-eP_1$ doesn't hold and it would be better to withdraw this line and to modify the rule of construction of $nL-oP_1$ as follows (see message #900): for $(2n+1)L-oP_1$, take the perpendiculars through the $2nL-p_{n-1}$, equidistant from $(2n-1)L-p_{n-1}$ and $(2n-1)L-p_{n-2}$.
Last detail: in the rule of construction given for $nL-P_7$, the fixed ratio is between $mL-nP_3$ and $mL-nP_7$ (and not $mL-nP_4$).
A suggestion to conclude: why not add a column with the points for $n=5$?
(It's nice to see on your summary for example that X_4 is at the same for the triangle h , p_1 , p_2 and oP_1 ; it would appear the same way that $5L-p_2$ is the same point as $5L-oP_1$)
Best regards
Bernard

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Message: #902
Date: 11/12/2014 9:02:14
From: eckart_schmidt@t-online.de
Subject: 1st Orthocenter and Summary

Dear BernardK.,
I have constructed 7L-p3 with the recursive definition in my paper. I could verify, that the perpendicular bisectors of 7L-p2 and 7L-p3 for a 8-line coincide in a point, which will be 8L-p3! But the simplest construction in this figure needs with Cabri a quarter of a hour! Saving the file refused ... Cabri gave up. So I cannot prove the 1st orthocenter for a 9-line.
Best regards Eckart

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Message: #903
Date: 11/12/2014 9:57:16
From: chris.vantienhoven
Subject: 1st Orthocenter and Summary

Dear Eckart,
Congratulations with the construction of 8L-p3!
What a pity you weren't able to save the results.
However it shows we are on the right way.
I don't know if we can draw conclusions now.
I myself wasn't able to work on the subject last two days because of migraine.
I intent trying to construct 8L-p3 and other pictures using Mathematica.
But I am afraid this will take some time to set this up.
Best regards,
Chris

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Message: #904
Date: 12/12/2014 11:22:25
From: chris.vantienhoven
Subject: 1st Orthocenter and Summary

Dear Eckart and BernardK,

Eckart, I read your summary and it is really excellent!
You write very concise exactly how everything is structured.
There still are some details I am studying an checking.
Congratulations!

[BK] A suggestion to conclude: why not add a column with the
points for $n=5$?

I will add a column with earlier codes we used in a pentangle.
I suppose they won't be needed as soon as there will be general
codes for points in n -Lines in an Encyclopedia of
Polygon-Figures.
I will make a new summary as soon as soon as other things have
been crystallized in my head.

Best regards, Chris

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Message: #905

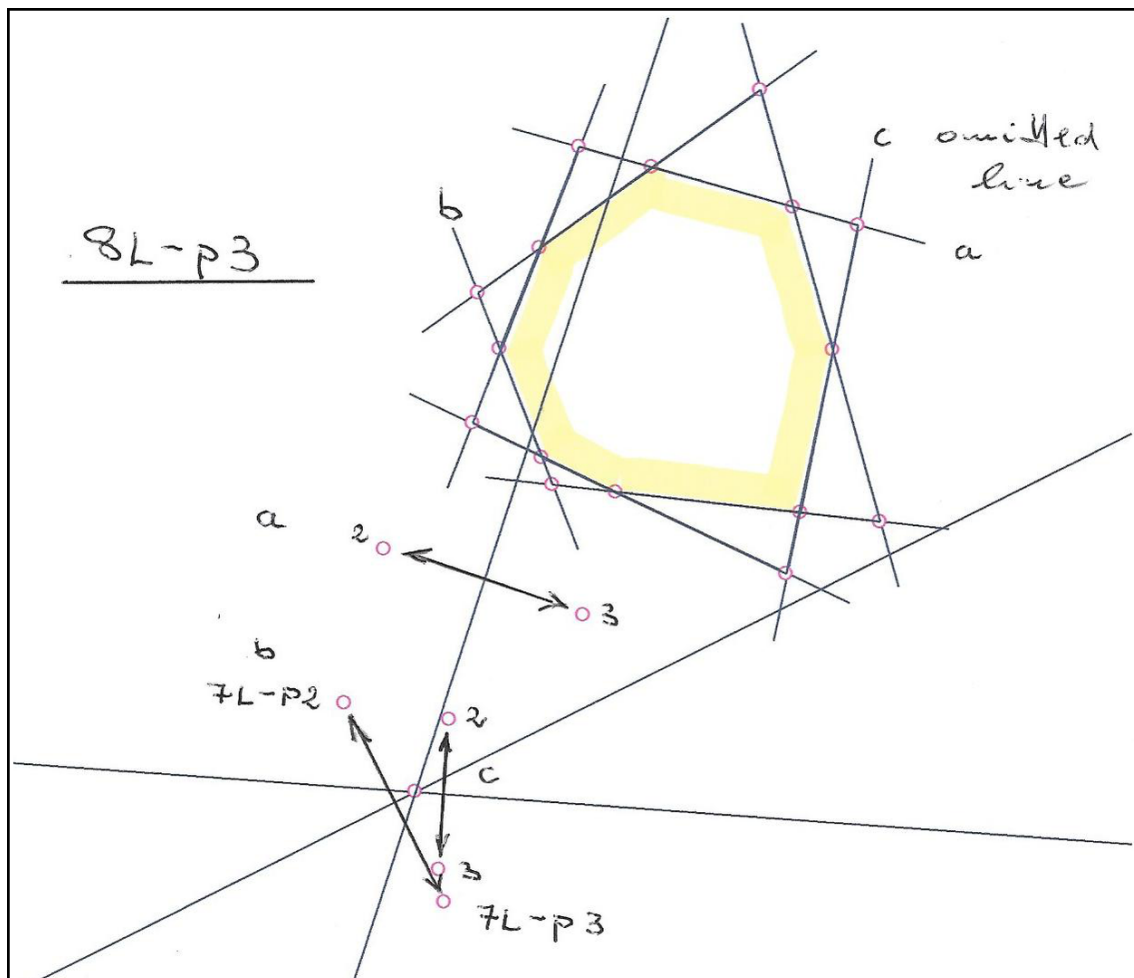
Date: 12/12/2014 11:42:32

From: eckart_schmidt@t-online.de

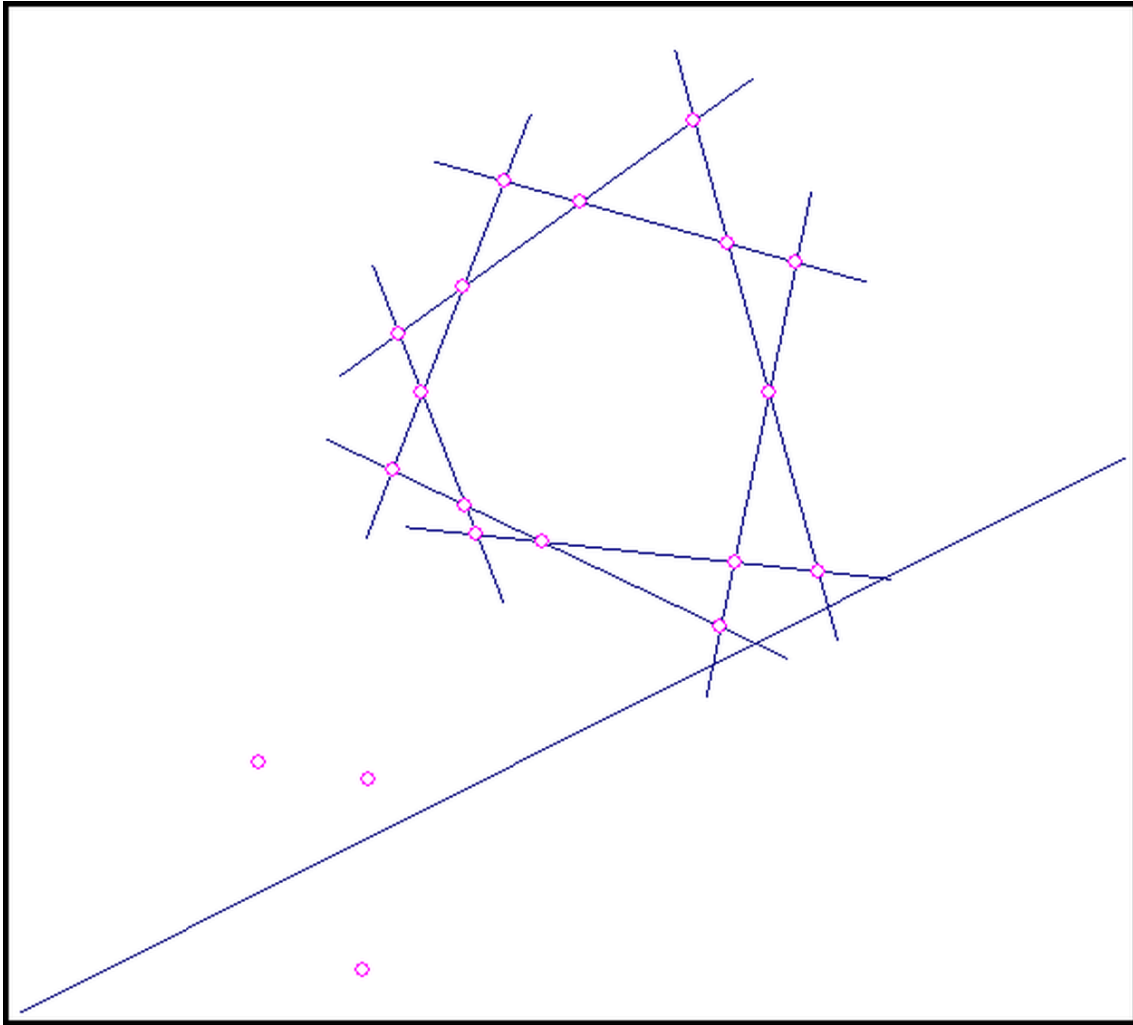
Subject: 1st Orthocenter and Summary

Dear BernardK, dear Chris,
I have tried a "hand-colored" figur for my construction of 8L-p3
(my Cabri would take for the names again hours, if there is
enough store). For Chris I add a Cabri file, but I could not
open it again with all points. On my side no hope for testing
9L-po1.

Best regards Eckart



8L-p3.pdf



2014-12-11c-fig.png

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Message: #906
Date: 13/12/2014 9:39:44
From: Antreas Hatzipolakis
Subject: A Morley's result

Dear BernardK, Chris, Eckart
Two others discuss a Morley's result on algebraic geometry
"In his short paper [19] Morley computed the degree of the
Luroth hypersurface
L by introducing some interesting ideas which seem to have been
forgotten, maybe because a few arguments are somehow obscure. In
this paper we put Morley's result and method on a solid
foundation by reconstructing his proof as faithfully as
possible."
I do not know whether is related or useful for your discussions
the paper:
Giorgio Ottaviani, Edoardo Sernesi, On the hypersurface of
Luroth quartics,
<http://arxiv.org/pdf/0903.5149.pdf>
aph

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Message: #907
Date: 13/12/2014 12:16:22
From: bernard.keizer
Subject: A Morley's result

Dear Antreas,
Thanks a lot for your reference!
This Lüroth quartic through the 10 vertices of the complete
pentalateral seems very interesting.
I fear it's quite hard to handle ...
Best regards
Bernard
PS Do you know by any chance a reference for the construction of
the 64 axes of the QL similar to the construction of the 27 axes
for a triangle through the Morley triangles (the direction of
the axes is obviously the mean direction of the 4 lines of the
QL $m^\circ \pi/4$, as it is for the sides of the Morley triangles the
mean direction of the 3 sides of the triangle $m^\circ \pi/3$)

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Message: #908

Date: 16/12/2014 1:13:36

From: eckart_schmidt@t-online.de

Subject: Hodgson's directed n-lines

Dear BernardK, dear Chris,

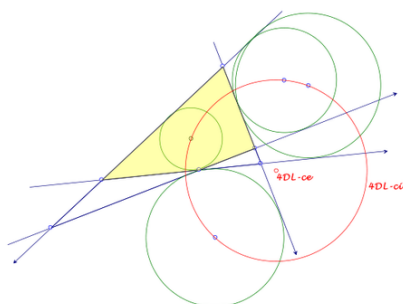
I had a look in an article of J. E. Hodgson: "Orthogonal Properties of the Plane Directed n-Line", but I cannot follow the analytic calculations. So I have tried to verify the results with constructions (see attached file). But there are two final results, which cannot be true by construction, but they would be of interest for 5-lines and 6-lines. Perhaps Bernard can read such articles better or Chris test the constructions.

Best regards Eckart

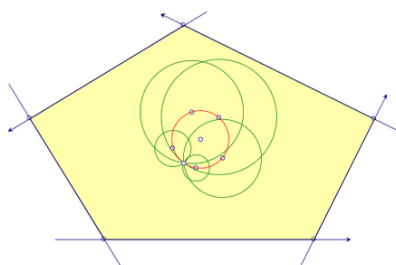
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

Hodgson's Directed n-Lines

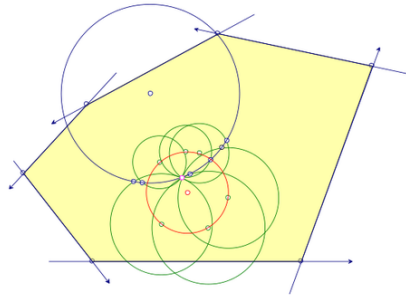
These are first remarks wrt an article of J. E. Hodgson: "Orthocentric Properties of the Plane Directed n-Line" (1912).



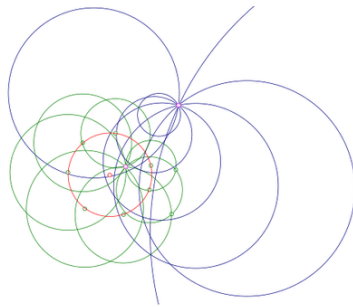
- (1) If the lines of a 3-line have a direction, there is a unique point (p.200) equidistant to the 3 lines as center of a circle contacting the lines. This is the first step for a chain of centers and center circles of directed n-lines (shortened $nDL-ce$ and $nDL-ci$).
- (2) For a directed 4-line there are four $3DL-ce$ on the center circle $4DL-ci$ (p.203). This circle is one of the 8 Steiner circles of a quadrilateral.



- (3) For a directed 5-line there are five $4DL-ce$ on the center circle $5DL-ci$ (p.227). Not mentioned by Hodgson: The five center circles $4DL-ci$ have a common point $5DL-px$ on the center circle $5DL-ci$.

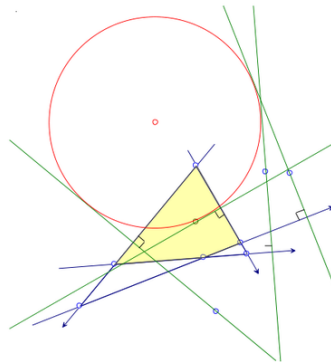


- (4) For a directed 6-line there are six $5DL-ce$ on the center circle $6DL-ci$. Not mentioned by Hodgson: The six center circles $5DL-ci$ have a common point $6DL-px$. This common point is concyclic with the six points $5DL-px$ of the directed 5-lines on a circle $6DL-cix$.

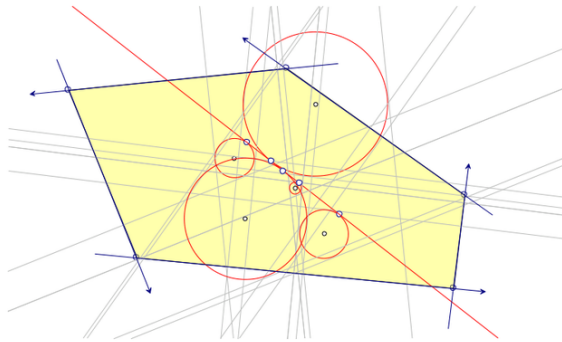


- (5) For a directed 7-line (not in the figure) there are seven $6DL-ce$ on the center circle $7DL-ci$ (red). Not mentioned by Hodgson: The seven center circles $6DL-ci$ (green) have a common point as well as the seven circles $6DL-cix$ (blue).

...



- (6) For directed 4-lines Hodgson mentioned a second circle (p.204) inscribed the perpendiculars of the *3DL-ce* wrt the line left out.



- (7) For a directed 5-line the five second circles of the directed 4-lines have a common tangent (p.207).

...

- (8) A n -line has 2^{n-1} directed n -lines (reversal of all directions shall give the same directed n -line). So a 4-line has 8 directed 4-lines and a 5-line 16 directed 5-lines.
- (9) For a 4-line the 8 center circles *4DL-ci* are the Steiner circles with centers on two perpendicular lines (1st and 2nd Steiner axis).

Final questions:

For a 5-line Hodgson proved, that the centers of the center circles of the 16 directed 4-lines are on a rectangular hyperbola (p.230). This can be verified by construction for 5 tangents of a parabola (p.228) but not in general.

For a 6-line Hodgson proved, that the centers of the center circles of the 32 directed 5-lines are on two perpendicular lines (p.231). This cannot be verified by construction.

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2014-12-16.pdf

Message: #909
Date: 17/12/2014 7:16:04
From: bernard.keizer
Subject: Hodgson's directed n-lines

Dear Eckart, dear Chris
Thank you for this new reference in the Morley's world!
It seems you have already taken the best from it.
Laguerre defined also curves of direction as the circle, of course, but also the cardioïd and the deltoïd and Morley took as direction for the 4 lines of the QL the direction given by the deltoïd.
I have no time now to study this new article, maybe next year ...
In the meantime, Merry Christmas and Happy Newyear to all the members of the Quadrigroup!
Best regards
Bernard

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Message: #910
Date: 19/12/2014 1:27:33
From: Chris van Tienhoven
Subject: 1st Orthocenter and Summary

Dear Eckart and BernardK,

Because of the limitations of Cabri pictures I made several pictures of Morley's 1st Orthocenter as well as Morley's Ortho directrix using Mathematica.

It took me some time making pictures this way not using barycentric coordinates.

Also it was a real challenge to make recursive Mathematica functions.

Doing so the document grew continuously and I also added some extra information (I investigated the behavior of the pi points for $n=\text{odd}$ as well as $n=\text{even}$).

Two extra results:

1. for $n=\text{odd}$ there are two orthogonal reflective axes crossing at Morley's 1st Orthocenter.
(a bit similar like the QL-1st and 2nd Steiner Axes)
 2. for $n=\text{even}$ it appeared that $nL-g(n/2) = nL-g(n/2 + 1)$.
- Eckart, now I will have time to study your latest document.

Best regards,
Chris

p.s.

If you would like me to make another picture regarding the subject, please let me know.

I am quite involved in the subject now.

Morley's first Orthocenter

Morley describes a so called **first Orthocenter** in his document "Orthocentric properties of the plane n-Line" [1]. Morley's paper was published in the year 1902. Morley proofs all his results algebraically using calculations in the complex plane. He explains his methods at [2]. First mentioned document is very compact and often hard to understand. It was "decoded" at the end of 2014 by Bernard Keizer, Eckart Schmidt and the author of this document in a discussion at the Yahoo Quadri-Figures Group [5], especially messages 826-905.

Global description

An n-Line is supposed to be a system of n random lines, where n is a natural number > 1 . This first Orthocenter is equally dependent on these n Lines.

Note that Morley's 1st Orthocenter only exists for $n=\text{odd}$.

In Morley's description he describes a recursive method of constructing this point, using intermediate points p_i , where 'i' is a number in the range $1, \dots, (n-1)/2$. Ultimately p_i , where 'i' has reached the highest value $(n-1)/2$ is Morley's 1st Orthocenter.

In this document p_i will be prefixed with "nL-" in order to specify the level at which p_i occurs.

Morley's 1st Orthocenter in a 3-Line is $X(4)$, the triangle orthocenter [3].

In general Morley's 1st Orthocenter in an n-Line ($n=\text{odd}$) is $nL-p_i$, where $i=(n-1)/2$.

In a 3-Line Morley's 1st Orthocenter will be $3L-p_1$, in a 5-Line it will be $5L-p_2$, in a 7-Line it will be $7L-p_3$, etc.

Morley's Ortho Directrix

There is no nL -first Orthocenter for $n=\text{even}$ but there are $n(n-1)L$ -first Orthocenters on the level below n. They will be collinear on the so called Morley's nL -Ortho directrix.

When $n=4$, then $QL-L_2$ (Steiner Line) is the $4L$ -Ortho directrix, containing the Orthocenters $X(4)$ of the 4 Component triangles. See [4].

What makes Morley's first Orthocenter so special?

To prevent many abstract notations I will just mention special properties for Morley's first Orthocenter in a 9-Line being $9L-p_4$. The general principle works accordingly.

1. Morley's 1st Orthocenters applied for all 10 Component 9-Lines in a 10-Line will be collinear on the so called **10L-Morley's Ortho directrix**.
2. Using $9L-p_4$ as origin, the segments $9L-p_4.8L-p_4$ and $9L-p_4.8L-p_3$ have a **fixed ratio** for all 9 occurrences of $8L-p_4$ and $8L-p_3$. In general this property is also valid using lower level points $9L-P_3$, $9L-P_2$, $9L-P_1$ as origin and connecting them with their lower level points.
3. There are **two orthogonal axes** at Morley's first Orthocenter X bisecting the 9 versions of angles $8L-p_4.X.8L-p_3$. In general this property is only valid for $n=\text{odd}$ and $i=(n-1)/2$ (new property).

Morley's recursive nL-pi points

$nL-p_i$ ($i = 1, \dots, (n-1)/2$) is defined in a recursive way:

$nL-p_i$ = ratiopoint of
 $nL-p(i-1)$ and the
 centroid of n points $(n-1)L-p(i-1)$
 with ratio $n : (i-n)$.

By applying this formula to an increasingly lower level finally the level is reached unto some $nL-p_0$, and $nL-p_0$ is defined as follows:

$nL-p_0$ = Center of the nL -Centric Circle of an n-Line, also called the nL -Centric Center.

The nL-Centric Center now in turn is defined as the center of the circle through the n (n-1)L-Centric Centers (general property of an n-Line).

By applying this definition to an increasingly lower level finally the level is reached of the 3L-Centric Center, which simply is the center of the circumcircle of a triangle.

This is the only known item and can be rolled up (wherever it appears) upwardly to the required point nL-pi. Basically nL-pi is built from large quantities of triangle circumcenters. These triangles being formed by the different combinations of the basic lines of the n-Line.

Definition Ratiopoint

A *Ratiopoint* of two points Q1 and Q2 is a collinear point X defined by the relative distances (m:n) to the two points.

Both m and n are positive numbers unless,

* when X is not positioned at the same side of Q1 as Q2, then m is negative,

* when X is not positioned at the same side of Q2 as Q1, then n is negative.

Behaviour nL-Pi points for different values of i

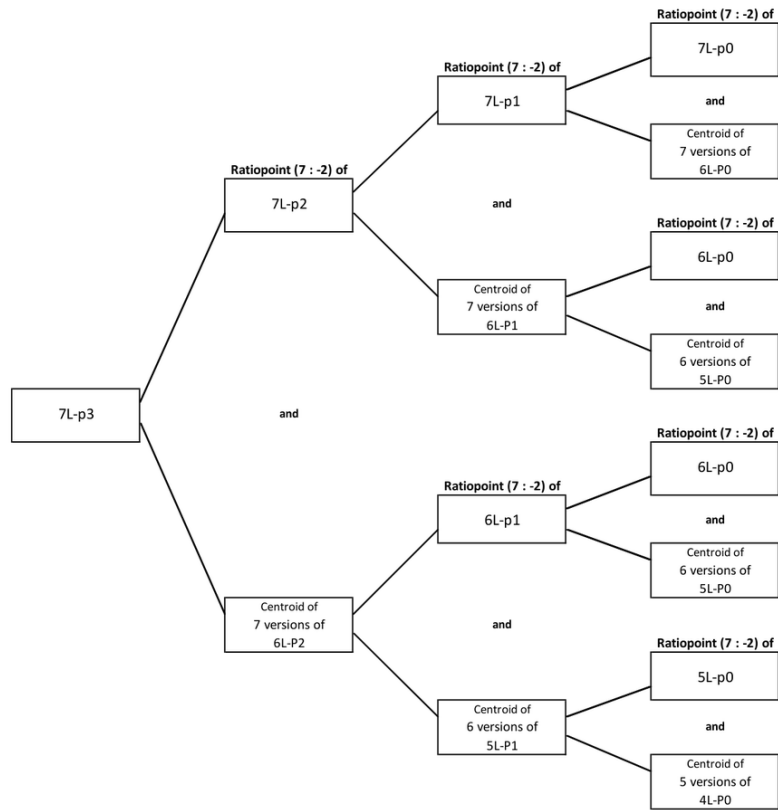
These properties are valid for Morley's nL-pi points **when n = odd** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , (n+1)/2, but not for higher values.
- There are two orthogonal reflective axes for i=(n-1)/2, bisecting angles with lower level (n-1)L-pi and (n-1)L-p(i-1).
- nL-pi for i = 1,2,3, ... ,n culminates in a point nL-pn, which will be the same point as the centroid of the n lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) =nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

These properties are valid for nL-pi points **when n = even** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , n/2, but not for higher values. This distance ratio =1, when i = n/2 - 1. As a consequence nL-p(n/2-1) will be the common point of the perpendicular bisectors of the n pairs (n-1)L-p(n/2-1), (n-1)L-p(n/2-2).
- nL-g(n/2) = nL-g(n/2 + 1)
- When n=even there are no reflective axes at any nL-pi bisecting angles with lower level mL-pi and mL-p(i-1).
- nL-pi for i = 1,2,3, ... ,n culminates in a point nL-pn, which will be the same point as the centroid of the lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) =nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

Example recursive 7L-p3 point structure:



Example Morley's first Orthocenter in a 7-Line:

The sides of the blue 7-Gon represent the basic lines of the 7-Line.

7L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 7-Line.

It is constructed via g0 to p1 (1:7), via g1 to p2 (2:7), via g2 to p3 (3:7).

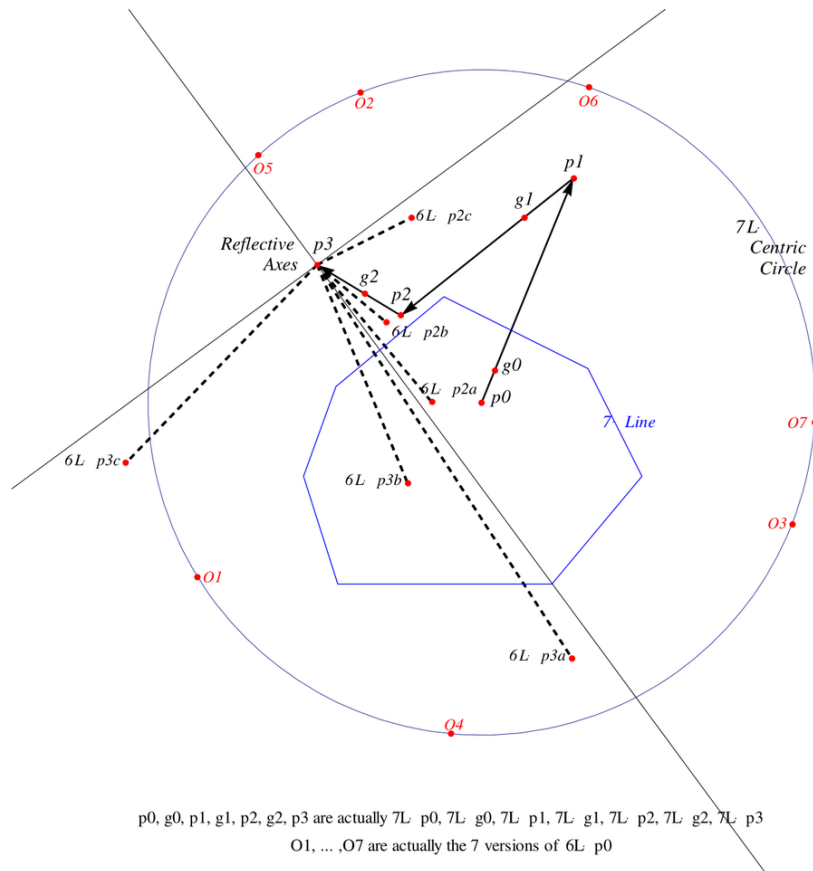
g0 = centroid of 7 points 6L-p0

g1 = centroid of 7 points 6L-p1

g2 = centroid of 7 points 6L-p2

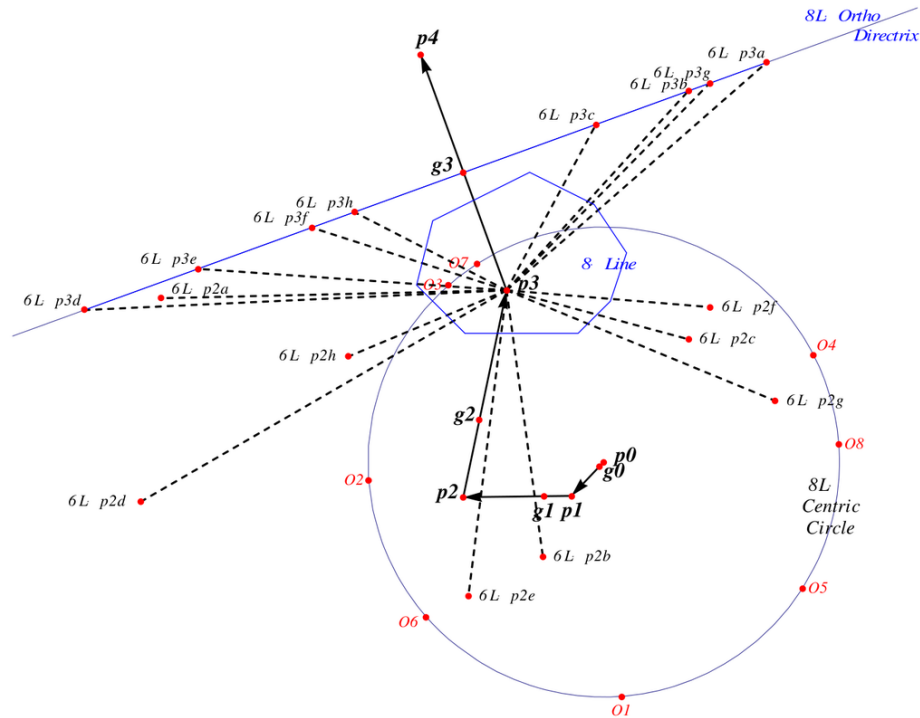
Only three of the seven 6L-p3i points and 6L-p2i points have been drawn for checking the fixed relationship of distances 7L-p3 unto 6L-p3i and 6L-p2i.

In this picture $7L\text{-}p3.6L\text{-}p3i : 7L\text{-}p3.7L\text{-}p2i = 2.6167$ for all i, where $i=a,b,c, \dots$ (total 7).



Morleys 1st Orthocenter-01.pdf

The sides of the blue 8-Gon represent the basic lines of the 8-Line.
 8L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 8-Line.
 It is constructed via g0 to p1 (1:8), via g1 to p2 (2:8), via g2 to p3 (3:8) , via g3 to p4 (4:8).
 g0 = centroid of 8 points 7L-p0
 g1 = centroid of 8 points 7L-p1
 g2 = centroid of 8 points 7L-p2
 g3 = centroid of 8 points 7L-p3.



O1, ..., O8 are actually the 8 versions of 7L p0, being the Centric Centers of the 8 Component 7- Lines.
p0, g0, p1, g1, p2, g2, p3, g3, p4 are actually 8L p0, 8L g0, 8L p1, 8L g1, 8L p2, 8L g2, 8L p3, 8L g3, 8L p4
The line segments 8L p3.6L p2x and 8L p3.6L p3x | x: a, b, ..., h are of equal size.
The 8L Ortho Directrix is the line through 6L p3a, ..., 6L p3h, being the 1st Orthocenters of the 8 Component 7- Lines.
It also passes through 8L g3, being the Centroid of 6L p3a, ..., 6L p3h.
It also is the perpendicular bisector of 8L p3.8L p4.

Example Morley's first Orthocenter in a 9-Line:

The sides of the blue 9-Gon represent the basic lines of the 9-Line.

9L-p4 (p4 in the picture) is Morley's 1st orthocenter of the 9-Line.

It is constructed via g0 to p1 (1:9), via g1 to p2 (2:9), via g2 to p3 (3:9), via g3 to p4 (4:9).

g0 = centroid of 9 points 8L-p0

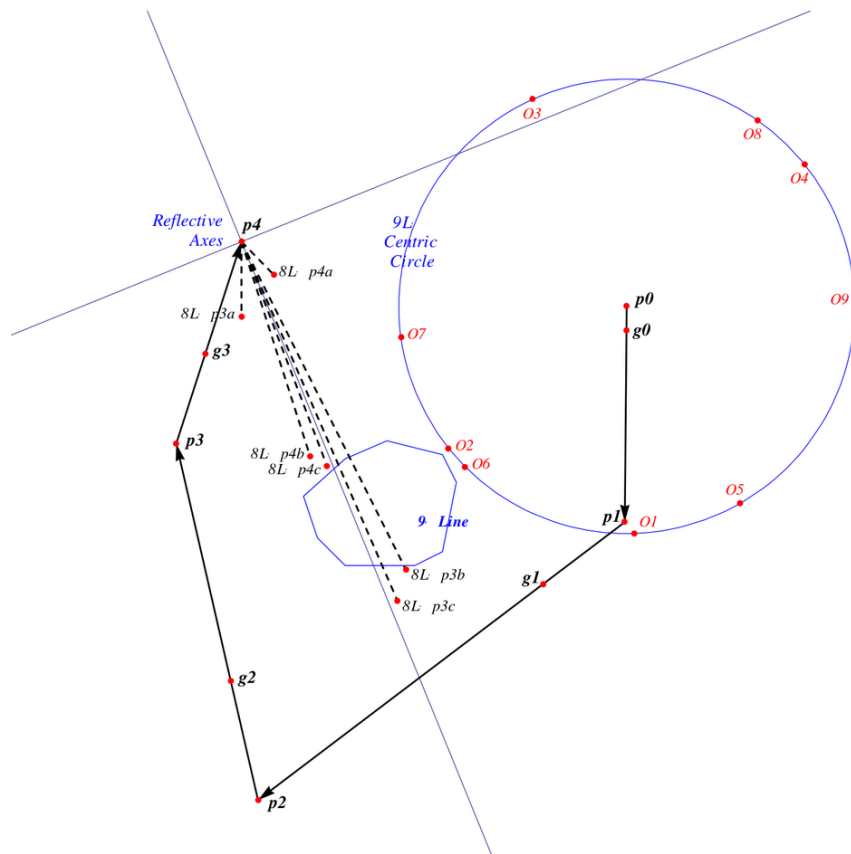
g1 = centroid of 9 points 8L-p1

g2 = centroid of 9 points 8L-p2

g3 = centroid of 9 points 8L-p3

Only three of the nine 8L-p4i points and 8L-p3i points are drawn for checking the fixed relationship of distances 9L-p4 unto 8L-p4i and 8L-p3i.

In this picture $9L-p4.8L-p4i : 9L-p4.8L-p3i = 0.614571$ for all i, where i=a,b,c, . . . (total 9).



Morleys 1st Orthocenter-01.pdf

References:

- [1] F. Morley, *Orthocentric properties of the plane n -Line*, Trans Amer Math Soc, 4 (1903) 1-12
available at:
<http://www.ams.org/journals/tran/1903-004-01/S0002-9947-1903-1500618-2/S0002-9947-1903-1500618-2.pdf>
- [2] F. Morley, *On the Metric Geometry of the N -Line*, Trans Amer Math Soc, 1 (1900) 97-115
available at:
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- [3] C. Kimberling, *Encyclopedia of Triangle Centers*, available at
<http://faculty.evansville.edu/ck6/encyclopedia/ETC.html>.
- [4] Chris van Tienhoven, *Encyclopedia of Quadri-Figures*, available at
<http://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia>
- [5] Quadri-Figures-Group, Internet forum for discussions on topics related to Quadrilaterals, Quadrangles, etc.
available at: <http://tech.groups.yahoo.com/group/Quadri-Figures-Group/>

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Morleys 1st Orthocenter-01.pdf

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Message: #911

Date: 19/12/2014 9:03:50

From: eckart_schmidt@t-online.de

Subject: Hodgson's directed n-lines

Dear BernardK, dear Chris,
if you study Hodgsons paper, there are further remarks and
figures wrt the 2nd circles in the attached file, perhaps
helpful.

Best regards and "Frohe Weihnacht"

Eckart

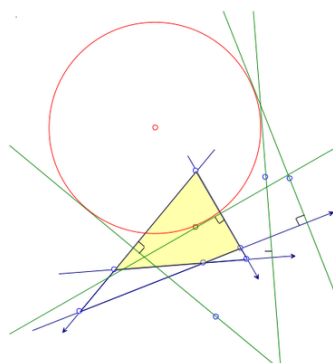
EQF-Note 2014-12-19

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
<http://www.chrisvantienhoven.nl/>

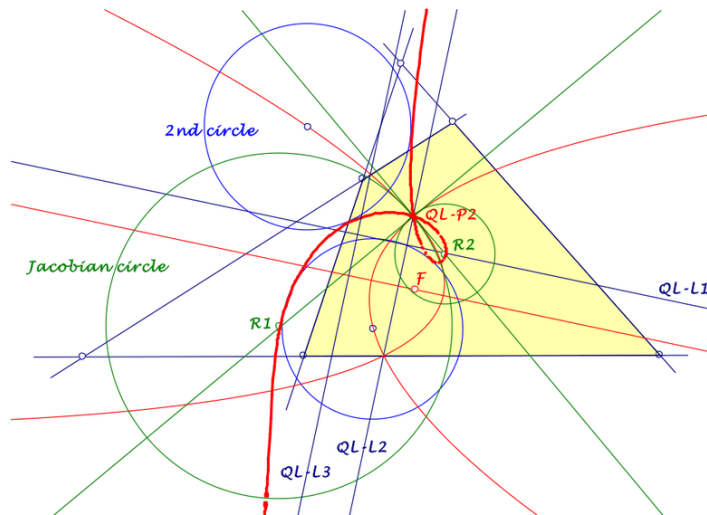
Hodgson's Directed n-Lines II

These are further remarks wrt an article of J. E. Hodgson: "Orthocentric Properties of the Plane Directed n-Line" (1912) (see also EQF-Note 2014-12-15 in #908).

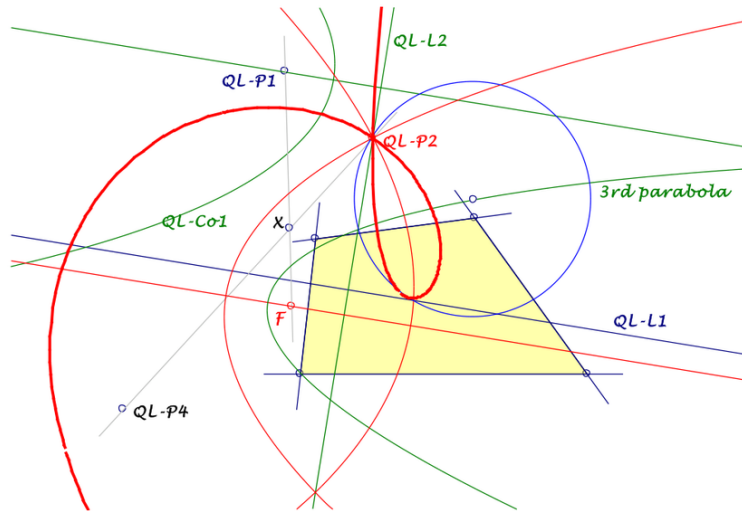
2nd Circles of directed 4-lines



- (1) For directed 4-lines Hodgson mentioned a second circle (p.204) inscribed the four perpendiculars through the equidistant points (of three lines) wrt the line left out.
- (2) A quadrilateral has eight directed 4-lines, so there are eight 2nd circles. They are divided into two groups of four. Take one directed 4-line for reference and three other changing two directions, then you get one group; the other group contains the directed 4-lines with one direction changed (p.220).
- (3) Each group has a common point $R_{1,2}$ for radical axes.
- (4) The centers of group-circles are with the Morley Point $QL-P2$ (point c by Hodgson on p. 222) on a parabola.
- (5) The two parabolas have the same axis parallel to the Newton Line $QL-L1$ with the same focus (p.222). They intersect orthogonally on the Steiner Line $QL-L2$ (e.g. in $QL-P2$).



- (6) The points $R_{1,2}$ lie on the orthogonal tangents in $QL-P2$ to the two parabolas.
- (7) Circles round $R_{1,2}$ through $QL-P2$ (Jacobian circles p.225) intersect the circles of a group orthogonally.
- (8) The envelope of all circles, centered on one parabola and orthogonal intersecting the correspondent Jacobian circle give a cubic, which is the same for both parabolas (p.226).
- (9) This cubic has a double point in $QL-P2$, contains the points $R_{1,2}$ and its asymptote is the Pedal Line $QL-L3$ (p.226). The cubic is anallagmatic, invariant to inversions wrt the Jacobian circles.
- (10) Not mentioned by Hodgson: The common focus of the two parabolas can be simply constructed as follows:
 ... let X divide $QL-P2.QL-P4$ with ratio 1:2,
 ... the common focus F divides $QL-P1.X$ with ratio -3:1.
- (11) Hodgson mentioned a third parabola (p.227):
 Circles through $QL-P2$ centered on this parabola envelop the cubic above.
- (12) Not mentioned by Hodgson: This third parabola has the same focus as the two other parabolas and is homothetic to the inscribed parabola $QL-Co1$ of the quadrilateral: center point X (see above), factor $-1/2$.



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2014-12-19.pdf

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Message: #912
Date: 20/12/2014 10:21:08
From: Chris van Tienhoven
Subject: 1st Orthocenter and Summary

Dear Eckart and BernardK,
I have 2 corrections on my last message #910.
The property that $nL-g(n/2) = nL-g(n/2 + 1)$ for $n=\text{even}$ is not true (typo in calculations).
The diagram in the paper at page 3 is corrected. See new paper in attachment.
Best regards,
Chris

Van:
Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
Verzonden: vrijdag 19 december 2014 13:27
Aan: Quadri-Figures-Group@yahoogroups.com
Onderwerp: [Quadri-Figures-Group] Re: 1st Orthocenter and Summary [1 Attachment]
[Attachment(s) (#TopText)
from Chris van Tienhoven included below]
1. for $n=\text{odd}$ there are two orthogonal reflective axes crossing at Morley's 1st Orthocenter.
(a bit similar like the QL-1st and 2nd Steiner Axes)
2. for $n=\text{even}$ it appeared that $nL-g(n/2) = nL-g(n/2 + 1)$.
p.s.
If you would like me to make another picture regarding the subject, please let me know.
I am quite involved in the subject now.

Morley's first Orthocenter

Morley describes a so called *first Orthocenter* in his document "Orthocentric properties of the plane n-Line" [1]. Morley's paper was published in the year 1902. Morley proofs all his results algebraically using calculations in the complex plane. He explains his methods at [2]. First mentioned document is very compact and often hard to understand. It was "decoded" at the end of 2014 by Bernard Keizer, Eckart Schmidt and the author of this document in a discussion at the Yahoo Quadri-Figures Group [5], especially messages 826-910.

Global description

An n-Line is supposed to be a system of n random lines, where n is a natural number > 1 . This first Orthocenter is equally dependent on these n Lines.

Note that Morley's 1st Orthocenter only exists for $n=\text{odd}$.

In Morley's description he describes a recursive method of constructing this point, using intermediate points p_i , where 'i' is a number in the range $1, \dots, (n-1)/2$. Ultimately p_i , where 'i' has reached the highest value $(n-1)/2$ is Morley's 1st Orthocenter.

In this document p_i will be prefixed with "nL-" in order to specify the level at which p_i occurs.

Morley's 1st Orthocenter in a 3-Line is $X(4)$, the triangle orthocenter [3].

In general Morley's 1st Orthocenter in an n-Line ($n=\text{odd}$) is $nL-p_i$, where $i=(n-1)/2$.

In a 3-Line Morley's 1st Orthocenter will be $3L-p_1$, in a 5-Line it will be $5L-p_2$, in a 7-Line it will be $7L-p_3$, etc.

Morley's Ortho Directrix

There is no nL-first Orthocenter for $n=\text{even}$ but there are $n(n-1)L$ -first Orthocenters on the level below n. They will be collinear on the so called Morley's nL-Ortho directrix.

When $n=4$, then QL-L2 (Steiner Line) is the 4L-Ortho directrix, containing the Orthocenters $X(4)$ of the 4 Component triangles. See [4].

What makes Morley's first Orthocenter so special?

To prevent many abstract notations I will just mention special properties for Morley's first Orthocenter in a 9-Line being $9L-p_4$. The general principle works accordingly.

1. Morley's 1st Orthocenters applied for all 10 Component 9-Lines in a 10-Line will be collinear on the so called 10L-*Morley's Ortho directrix*.
2. Using $9L-p_4$ as origin, the segments $9L-p_4.8L-p_4$ and $9L-p_4.8L-p_3$ have a *fixed ratio* for all 9 occurrences of $8L-p_4$ and $8L-p_3$. In general this property is also valid using lower level points $9L-P_3$, $9L-P_2$, $9L-P_1$ as origin and connecting them with their lower level points.
3. There are *two orthogonal axes* at Morley's first Orthocenter X bisecting the 9 versions of angles $8L-p_4.X.8L-p_3$. In general this property is only valid for $n=\text{odd}$ and $i=(n-1)/2$ (new property).

Morley's recursive nL-pi points

$nL-p_i$ ($i = 1, \dots, (n-1)/2$) is defined in a recursive way:

$nL-p_i$ = ratiopoint of

$nL-p(i-1)$ and the

centroid of n points $(n-1)L-p(i-1)$

with ratio $n : (i-n)$.

By applying this formula to an increasingly lower level finally the level is reached unto some $nL-p_0$, and $nL-p_0$ is defined as follows:

$nL-p_0$ = Center of the nL-Centric Circle of an n-Line, also called the nL-Centric Center.

The nL-Centric Center now in turn is defined as the center of the circle through the n (n-1)L-Centric Centers (general property of an n-Line).

By applying this definition to an increasingly lower level finally the level is reached of the 3L-Centric Center, which simply is the center of the circumcircle of a triangle.

This is the only known item and can be rolled up (wherever it appears) upwardly to the required point nL-pi. Basically nL-pi is built from large quantities of triangle circumcenters. These triangles being formed by the different combinations of the basic lines of the n-Line.

Definition Ratiopoint

A *Ratiopoint* of two points Q1 and Q2 is a collinear point X defined by the relative distances (m:n) to the two points.

Both m and n are positive numbers unless,

* when X is not positioned at the same side of Q1 as Q2, then m is negative,

* when X is not positioned at the same side of Q2 as Q1, then n is negative.

Behaviour nL-Pi points for different values of i

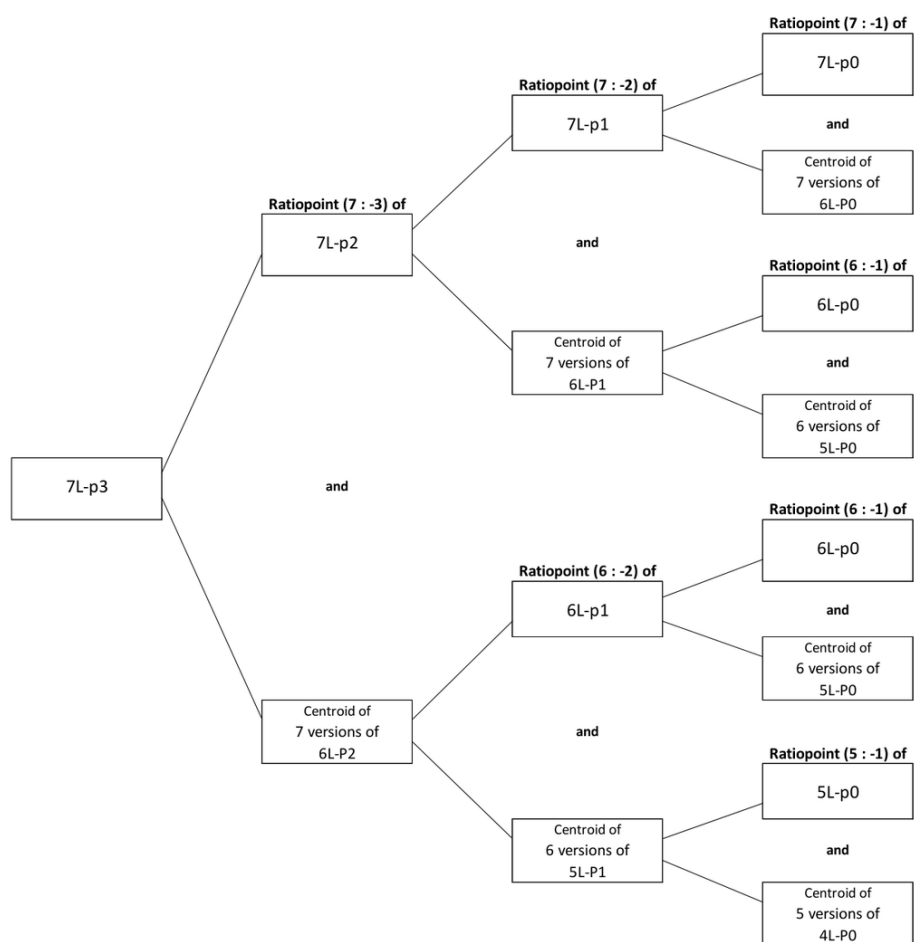
These properties are valid for Morley's nL-pi points **when n = odd** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , (n+1)/2, but not for higher values.
- There are two orthogonal reflective axes for i=(n-1)/2, bisecting angles with lower level (n-1)L-pi and (n-1)L-p(i-1).
- nL-pi for i = 1,2,3, ... , n culminates in a point nL-pn, which will be the same point as the centroid of the n lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) = nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

These properties are valid for nL-pi points **when n = even** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , n/2, but not for higher values. This distance ratio =1, when i = n/2 - 1. As a consequence nL-p(n/2-1) will be the common point of the perpendicular bisectors of the n pairs (n-1)L-p(n/2-1), (n-1)L-p(n/2-2).
- When n=even there are no reflective axes at any nL-pi bisecting angles with lower level mL-pi and mL-p(i-1).
- nL-pi for i = 1,2,3, ... , n culminates in a point nL-pn, which will be the same point as the centroid of the lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) = nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

Example recursive 7L-p3 point structure:



Example Morley's first Orthocenter in a 7-Line:

The sides of the blue 7-Gon represent the basic lines of the 7-Line.

7L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 7-Line.

It is constructed via g0 to p1 (1:7), via g1 to p2 (2:7), via g2 to p3 (3:7).

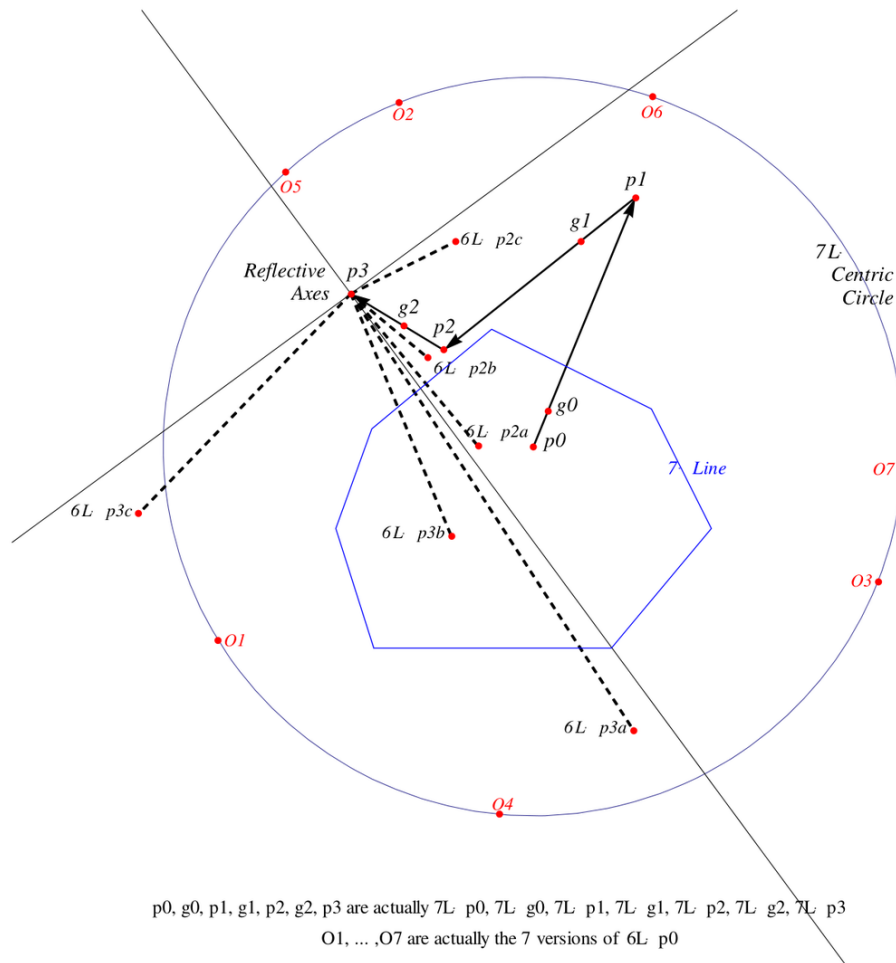
g0 = centroid of 7 points 6L-p0

g1 = centroid of 7 points 6L-p1

g2 = centroid of 7 points 6L-p2

Only three of the seven 6L-p3i points and 6L-p2i points have been drawn for checking the fixed relationship of distances 7L-p3 unto 6L-p3i and 6L-p2i.

In this picture $7L-p3.6L-p3i : 7L-p3.7L-p2i = 2.6167$ for all i, where $i=a,b,c, \dots$ (total 7).



Morleys 1st Orthocenter-02.pdf

Example Morley's Ortho directrix in an 8-Line

The sides of the blue 8-Gon represent the basic lines of the 8-Line.

8L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 8-Line.

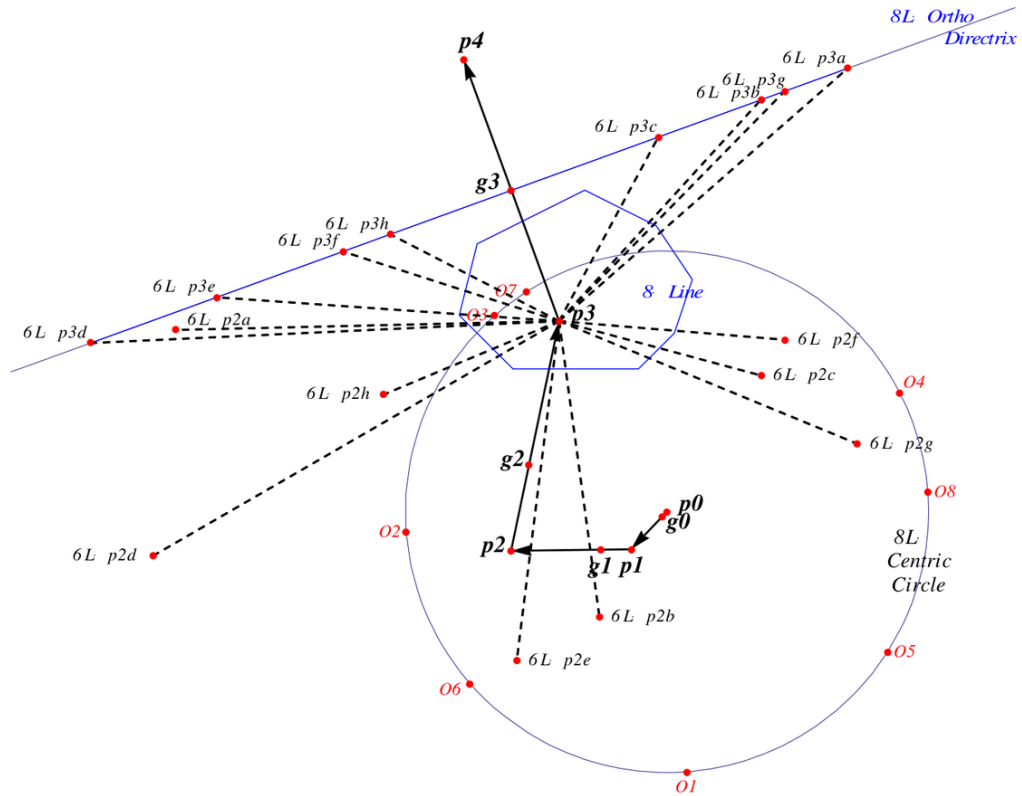
It is constructed via g0 to p1 (1:8), via g1 to p2 (2:8), via g2 to p3 (3:8), via g3 to p4 (4:8).

g0 = centroid of 8 points 7L-p0

g1 = centroid of 8 points 7L-p1

g2 = centroid of 8 points 7L-p2

g3 = centroid of 8 points 7L-p3.



O1, ... ,O8 are actually the 8 versions of 7L p0, being the Centric Centers of the 8 Component 7 Lines.

p0, g0, p1, g1, p2, g2, p3, g3, p4 are actually 8L p0, 8L g0, 8L p1, 8L g1, 8L p2, 8L g2, 8L p3, 8L g3, 8L p4

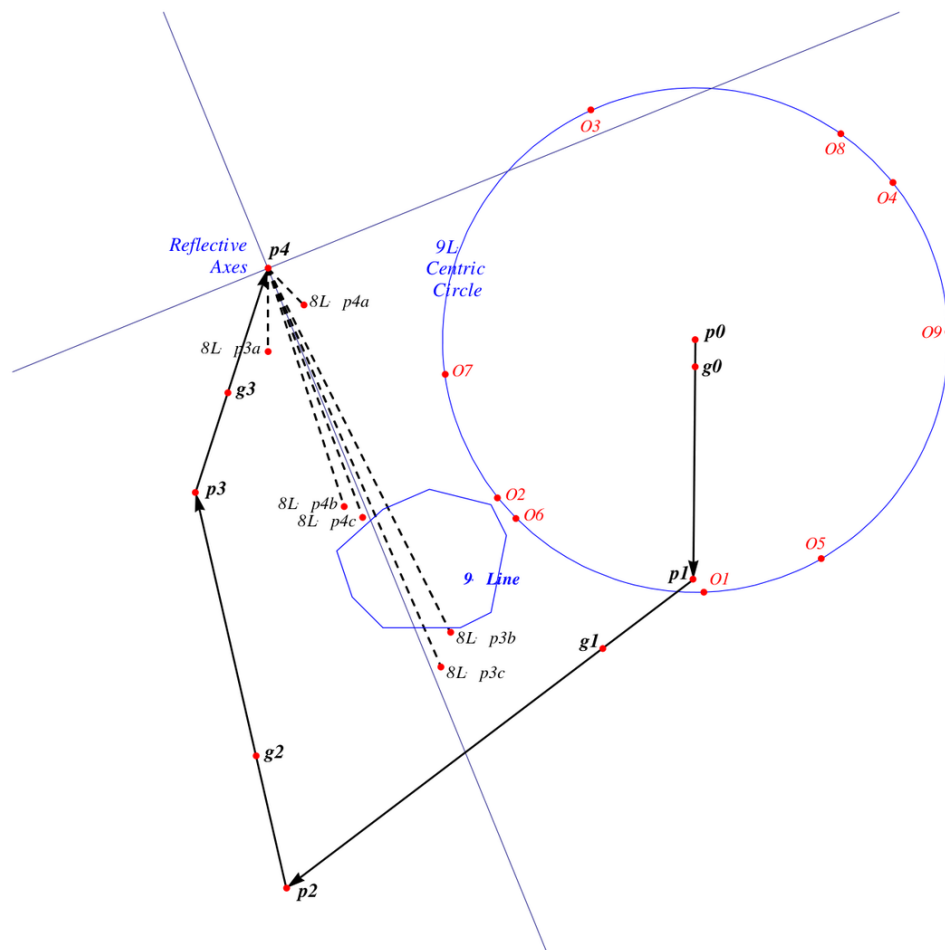
The line segments 8L p3.6L p2x and 8L p3.6L p3x | x: a,b, ... ,h are of equal size.

The 8L Ortho Directrix is the line through 6L p3a, ... ,6L p3h, being the 1st Orthocenters of the 8 Component 7 Lines.

It also passes through 8L g3, being the Centroid of 6L p3a, ... ,6L p3h.

It also is the perpendicular bisector of 8L p3.8L p4.

The sides of the blue 9-Gon represent the basic lines of the 9-Line.
 9L-p4 (p4 in the picture) is Morley's 1st orthocenter of the 9-Line.
 It is constructed via g0 to p1 (1:9), via g1 to p2 (2:9), via g2 to p3 (3:9), via g3 to p4 (4:9).
 g0 = centroid of 9 points 8L-p0
 g1 = centroid of 9 points 8L-p1
 g2 = centroid of 9 points 8L-p2
 g3 = centroid of 9 points 8L-p3
 Only three of the nine 8L-p4i points and 8L-p3i points are drawn for checking the fixed
 relationship of distances 9L-p4 unto 8L-p4i and 8L-p3i.
 In this picture $9L-p4.8L-p4i : 9L-p4.8L-p3i = 0.614571$ for all i, where i=a,b,c, . . . (total 9).



Morleys 1st Orthocenter-02.pdf

References:

- [1] F. Morley, *Orthocentric properties of the plane n -Line*, Trans Amer Math Soc, 4 (1903) 1-12
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<http://faculty.evansville.edu/ck6/encyclopedia/ETC.html>.
- [4] Chris van Tienhoven, *Encyclopedia of Quadri-Figures*, available at
<http://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia>
- [5] Quadri-Figures-Group, Internet forum for discussions on topics related to Quadrilaterals, Quadrangles, etc.
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Morleys 1st Orthocenter-02.pdf

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Message: #913
Date: 27/12/2014 7:12:19
From: eckart_schmidt@t-online.de
Subject: 1st Orthocenter and Summary

Dear Chris,
your detailed paper "Morley's first orthocenter" shows, that Bernard's interpretation was the key for understanding. I have to correct my summary, for there is a false description! In your paper I miss Morley's primary idea, to generalize the orthocenter of a triangle for n -lines as common point of perpendiculars to the lines (see also page 5 of his paper). For me it is amazing, that
... the ratiopoint of $(2n+1)L-p(n-1)$ and the centroid of $2nL-p(n-1)$ with ratio $-(2n+1)/(n+1)$ is the same point as
... the common point for perpendiculars of $2nL-p(n-1)$ wrt the line left out.
Interesting new properties for me are the two orthogonal axes!
Best regards Eckart

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Message: #914
Date: 31/12/2014 11:44:15
From: eckart_schmidt@t-online.de
Subject: Some "splitter" IX

Dear Chris,
are you further interested in my "splitters", there was no reaction the last times.
... The QG-P19 triangle of a quadrilateral is perspective to QL-DT.
... The Steiner axes of a quadrilateral are the angle bisectors of QL-P1.QL-P4 and the axis of the inscribed parabola.
Surprisingly for me the last property!
In message #665 I asked you for correcting in EQF a mistake wrt QG-Ci3 unknown cited with my name. Why is it not possible?
Best regards and all the best for 2015
Eckart

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Message: #915
Date: 31/12/2014 2:09:45
From: chris.vantienhoven
Subject: Some "splitter" IX

Dear Eckart,
Of course I am still interested in your "splitter". I read them always very interestedly, because there are real jewels between them.
However I hope you understand that it is not always easy to select and to process all these new items that come from QFG. Actually I still have a list of items to add in EQF to be done in spare time.
Thank you for remembering the correction. I changed it some minutes ago.
Only the remark of QG-P19 already existed as the 5 th property of QG-P19.
Ich wünsche dir und deine Frau ein glückliches und erfolgreiches Neujahr!
Best regards,
Chris

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Message: #916
Date: 31/12/2014 7:32:37
From: chris.vantienhoven
Subject: Morley's line of Orthocenters

Dear Eckart and BernardK,
Right at the end of the year a brain-teaser.
Denote in an n -Line:

$nP4$ = Morley's 2nd Orthocenter
 $nP5$ = Morley's 2nd Circle Center
 $oP1$ = Morley's 1st Orthocenter (exists only when n = odd)
 $eP1$ = Morley's (Enna-)Deltoid Center (exists only when n = even)

Let $oL2$ = Line $oP1.nP4$, also called by Morley the line of Orthocenters (exists only when n = odd).

Let $eL2$ = Line $eP1.nP5$ (exists only when n = even).

There is a 1-1 relationship between ratiopoints on $oL2$ and $eL2$ preserving the ratio wrt basic points on these lines.

nL - $oL2$ -ratiopoints can be constructed as the common point of the perpendiculars through corresponding $(n-1)L$ - $eL2$ -ratiopoints on the omitted line (like Bernard K explained so well).

I checked this in Mathematica for $n=5,7,9$ with several numerical examples and it works!

Is there a corresponding construction to produce nL - $eL2$ -ratiopoints from the underlying $(n-1)L$ - $oL2$ -ratiopoints? (I mean not the one constructing the points $eP1$ and $nP5$ and then the ratiopoint)

I wish all QFG members a wonderful astounding 2015 in many ways!

Best regards,
Chris

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6 Colophon

Sources & Contact

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- Volume 4 (2016), messages #1403–#2169
- Volume 5 (2017), messages #2170–#2799
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- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60
- Volume 2 (2020), messages #61–#630
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- Volume 4 (2022), messages #1295–#1544
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