Journal of the Quadri Figures Group 2015

Digital Edition

Chris van Tienhoven et al.

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1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at https://groups.io/g/Quadri-and-Poly-Geometry. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at https://groups.io/g/Quadri-Figures-Group.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Ángel Montesdeoca
- Antonio Sánchez Briones:
- Antreas Hatzipolakis
- Benedetto Scimemi
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- Eckart Schmidt
- Emmanuel José García
- Francisco Javier García Capitán
- Ken Pledger
- Ngo Quang Duong
- Seiichi Kirikami
- Systems Manager

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

Ángel Montesdeoca:
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Antonio Sánchez Briones::
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Antreas Hatzipolakis:
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Benedetto Scimemi:
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Bernard Keizer:
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Chris van Tienhoven:
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César Lozada:
#1008

Eckart Schmidt:
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Emmanuel José García:
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Francisco Javier García Capitán:
#1038 #1112

Ken Pledger:
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Ngo Quang Duong:
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Seiichi Kirikami:
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Systems Manager:
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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Benedetto Scimemi (1938–2023)

Location
Italy

Year of Birth / Generation
1938–2023.

Short Biography
Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.esteri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter's 60th birthday (the "A5 Meeting", named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See CO-Tf3 in EPG.

- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Francisco Javier García Capitán

Location
Priego de Córdoba, Andalucía, Spain.

Year of Birth / Generation
1963.

Short Biography
Francisco Javier García Capitán is a mathematician and long-time secondary school teacher with a strong interest in geometry, elementary mathematics, and computational approaches. He is an active explorer of barycentric coordinates and the author of *Baricentricas.nb*. His work bridges classical geometric insight with modern computational tools.

Themes and Interests

- Elementary mathematics
- Geometry
- Mathematica and hobby programming
- Barycentric coordinates

Selected Publications
International Journal of Geometry

- (with Paul Yiu) *Three mutually tangent congruent circles…*, 5 (2016), 15–18.
- *A structure on the circumcircle*, 10 (2021), 71–83.
- *Infinite points and isogonal conjugate*, 12 (2023), 127–134.
- *Isotomic conjugate and parallelism*, 12 (2023), 89–100.

Forum Geometricorum

- *Means as chords*, 8 (2008), 99–101.
- *Trilinear polars of Brocardians*, 9 (2009), 297–300.
- *Collinearity of the first trisection points…*, 11 (2011), 217–221.
- (with Ehrmann & Myakishev) *Construction of circles…*, 11 (2011), 261–268.
- (with Dergiades & Lim) *On six circumcenters…*, 11 (2011), 269–275.
- *Some simple results on cevian quotients*, 13 (2013), 227–231.
- *A simple construction of an inconic*, 14 (2014), 387–388.
- *Lemniscates and a locus…*, 15 (2015), 123–125.
- *Another construction of the Simson lines…*, 15 (2015), 173–176.

- *Locus of centroids of similar inscribed triangles*, 16 (2016), 257–267.
- *A Family of Triangles…*, 18 (2018), 79–82.

Additional Remarks

- Website: www.garciacapitan.epizy.com
- Blog: www.garciacapitan.blogspot.com

Ngo Quang Duong

Location
Living in Hanoi, Vietnam.

Year of Birth / Generation
Born in 1998.
Generation Z (approx. 1997–2012).

Short Biography
Ngo Quang Duong studied at the Vietnam National University, where he initially majored in Mathematics before switching to Software Engineering. He has since been working professionally in the software field, while continuing to pursue his interest in Mathematics in his free time. His background combines formal mathematical training with practical experience in computing and problem solving, giving his work a distinctive blend of theoretical insight and computational intuition.

Themes and Interests

- Geometry, Topology, Analysis, and their interactions
- Classical Geometry, especially triangle geometry and quadri-figure geometry
- Contributions to QFG, including n-angle centers and new uses of coordinate systems
- General mathematical exploration and independent study

Selected Publications

- (with T. T. Vu) *A Generalization of the Droz Farny Line Theorem with Orthologic Triangles*, Forum Geometricorum, Volume 16 (2016), 415–418.
- *Generalization of Musselman's theorem. Some Properties of Isogonal Conjugate Points*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 15–29.
- (with O. T. Dao and P. Yiu) *Golden Sections in an Isosceles Triangle and Its Circumcircle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 93–97.
- *Generalizations of Lester Circle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 10 (2021), 49–61.

Additional Remarks
He has not actively returned to Classical Geometry for some time, but remains mathematically engaged through online communities. He is active on Math StackExchange (MSE), where he contributes under the profile: https://www.math.stackexchange.com/users/821868/duong-ngo

Seiichi Kirikami (1949?–2023)

Location
Japan.

Year of Birth / Generation
Exact year unknown; passed away on 11 December 2023.

Short Biography
In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions
Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes
Colleagues remembered Seiichi with deep affection:

- "Geometry is the poorer of his death." — A. P. Hatzipolakis
- "He was a great expert in projective and triangle geometry, and a very kind and helpful person." — E. Suppa
- "He had a way of seeing things from a different angle and presenting them simply." — C. van Tienhoven
- "He inspired many of my problems." — A. Altıntaş
- "He wrote many interesting and innovative messages in ADGEOM." — F. J. García Capitán
- "He helped me in my beginnings with wise and generous advice." — C. E. Lozada
- "He contributed to OEIS and had important work in epidemiology." — P. Moses

Additional Remarks
Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

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subject: 1st Orthocenter and Summary:
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subject: 2 Quadrangle Curves of QA-P1:
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subject: 2n-CSC-Transformation:
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subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle:
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subject: 3rd Quartic for a QG:
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subject: 4 Anticenters on Pedal Line (L3):
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subject: 4 Points problem:
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subject: 4th Construction of QL-Cu1:
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subject: 64 axes of a QL:
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subject: 64 cyclologic points accompanying QA-Miquel center:
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subject: A conic accompanying van Aubel quadrangle:
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subject: A conic made from isogonal center (QA-P4):
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subject: A conic made from the isogonal center (QA-P4):
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subject: Another construction method of Steiner angle bisector center octet:
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subject: Anticenters in Quadri-Figures Geometry:
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subject: Anticenters of pentangle and hexangle:
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4.2 Messages

Message: #917
Date: 02/1/2015 12:52:00
From: Chris van Tienhoven
Subject: 1st Orthocenter and Summary

Dear Eckart,
Best wishes for 2015 !
I added your remark in my summary about Morley's first Orthocenter at page 1.
Thank you!
Besides that I made some additions/corrections at page 5.
That's why I send also a revised version attached now of the document.
Best regards,
Chris

> Van:
> Quadri-Figures-Group@yahoogroups.com
> [mailto:Quadri-Figures-Group@yahoogroups.com]
> Verzonden: zaterdag 27 december 2014 19:12
> Aan: -Group, Quadri-Figures
> Onderwerp: [Quadri-Figures-Group]
> Re: 1st Orthocenter and Summary
> Dear Chris,
> your detailed paper " Morley's first orthocenter" shows,
> that Bernard's interpretation was the key for understanding.
> I have to correct my summary,
> for there is a false description!
> In your paper I miss Morley's primary idea, to generalize the
> orthocenter of a triangle for n-lines as common point of
> perpendiculars to the lines (see also page 5 of his paper).
> For me it is amazing, that
> ... the ratiopoint of (2n+1)L-p(n-1) and the centroid of
> 2nL-p(n-1) with ratio -(2n+1)/(n+1)
> is the same point as
> ... the common point for perpendiculars of 2nL-p(n-1)
> wrt the line left out.
> Interesting new properties for me are the two orthogonal axes!
> Best regards Eckart

Morley's first Orthocenter

Morley describes a so called ***first Orthocenter*** in his document "Orthocentric properties of the plane n-Line" [1]. Morley's paper was published in the year 1902. Morley proofs all his results algebraically using calculations in the complex plane. He explains his methods at [2]. First mentioned document is very compact and often hard to understand. It was "decoded" at the end of 2014 by Bernard Keizer, Eckart Schmidt and the author of this document in a discussion at the Yahoo Quadri-Figures Group [5], especially messages 826-917.

Global description
An n-Line is supposed to be a system of n random lines, where n is a natural number > 1. This first Orthocenter is equally dependent on these n Lines.
Note that Morley's 1st Orthocenter only exists for n=odd.
In Morley's description he describes a recursive method of constructing this point, using intermediate points pi, where 'i' is a number in the range 1, … , (n-1)/2. Ultimately pi, where 'i' has reached the highest value (n-1)/2 is Morley's 1st Orthocenter.
In this document pi will be prefixed with "nL-" in order to specify the level at which pi occurs.
Morley's 1st Orthocenter in a 3-Line is X(4), the triangle orthocenter [3].
In general Morley's 1st Orthocenter in an n-Line (n=odd) is nL-pi, where i=(n-1)/2.
In a 3-Line Morley's 1st Orthocenter will be 3L-p1, in a 5-Line it will be 5L-p2, in a 7-Line it will be 7L-p3, etc.

Morley's Ortho Directrix
There is no nL-first Orthocenter for n=even but there are n (n-1)L-first Orthocenters on the level below n. They will be collinear on the so called Morley's nL-Ortho directrix.
When n=4, then QL-L2 (Steiner Line) is the 4L-Ortho directrix, containing the Orthocenters X(4) of the 4 Component triangles. See [4].

What makes Morley's first Orthocenter so special?
To prevent many abstract notations I will just mention special properties for Morley's first Orthocenter in a 9-Line being 9L-p4. The general principle works accordingly.

1. Morley's 1st Orthocenters applied for all 10 Component 9-Lines in a 10-Line will be collinear on the so called 10L-***Morley's Ortho directrix***.
2. Morley's 1st Orthocenter is the ***common point of the perpendiculars of 8L-p3 on the omitted line*** (by regularly omitting a line there are 9 component 8-Lines in a 9-Line).
3. Using 9L-p4 as origin, the segments 9L-p4.8L-p4 and 9L-p4.8L-p3 have a ***fixed ratio*** for all 9 occurrences of 8L-p4 and 8L-p3. In general this property is also valid using lower level points 9L-P3, 9L-P2, 9L-P1 as origin and connecting them with their lower level points.
4. There are ***two orthogonal axes*** at Morley's first Orthocenter X bisecting the 9 versions of angles 8L-p4.X.8L-p3. In general this property is only valid for n=odd and i=(n-1)/2 (new property).

Morley's recursive nL-pi points
nL-pi (i = 1, .. ,(n-1)/2) is defined in a recursive way:
nL-pi = ratiopoint of
nL-p(i-1) and the
centroid of n points (n-1)L-p(i-1)
with ratio n : (i-n).

Morleys 1st Orthocenter-03.pdf

By applying this formula to an increasingly lower level finally the level is reached unto some nL-p0, and nL-p0 is defined as follows:
nL-p0 = Center of the nL-Centric Circle of an n-Line, also called the nL-Centric Center.
The nL-Centric Center now in turn is defined as the center of the circle through the n (n-1)L-Centric Centers (general property of an n-Line).
By applying this definition to an increasingly lower level finally the level is reached of the 3L-Centric Center, which simply is the center of the circumcircle of a triangle.
This is the only known item and can be rolled up (wherever it appears) upwardly to the required point nL-pi. Basically nL-pi is built from large quantities of triangle circumcenters. These triangles being formed by the different combinations of the basic lines of the n-Line.

Definition Ratiopoint
A *Ratiopoint* of two points Q1 and Q2 is a collinear point X defined by the relative distances (m:n) to the two points.
Both m and n are positive numbers unless,
* when X is not positioned at the same side of Q1 as Q2, then m is negative,
* when X is not positioned at the same side of Q2 as Q1, then n is negative.

Behaviour nL-Pi points for different values of i

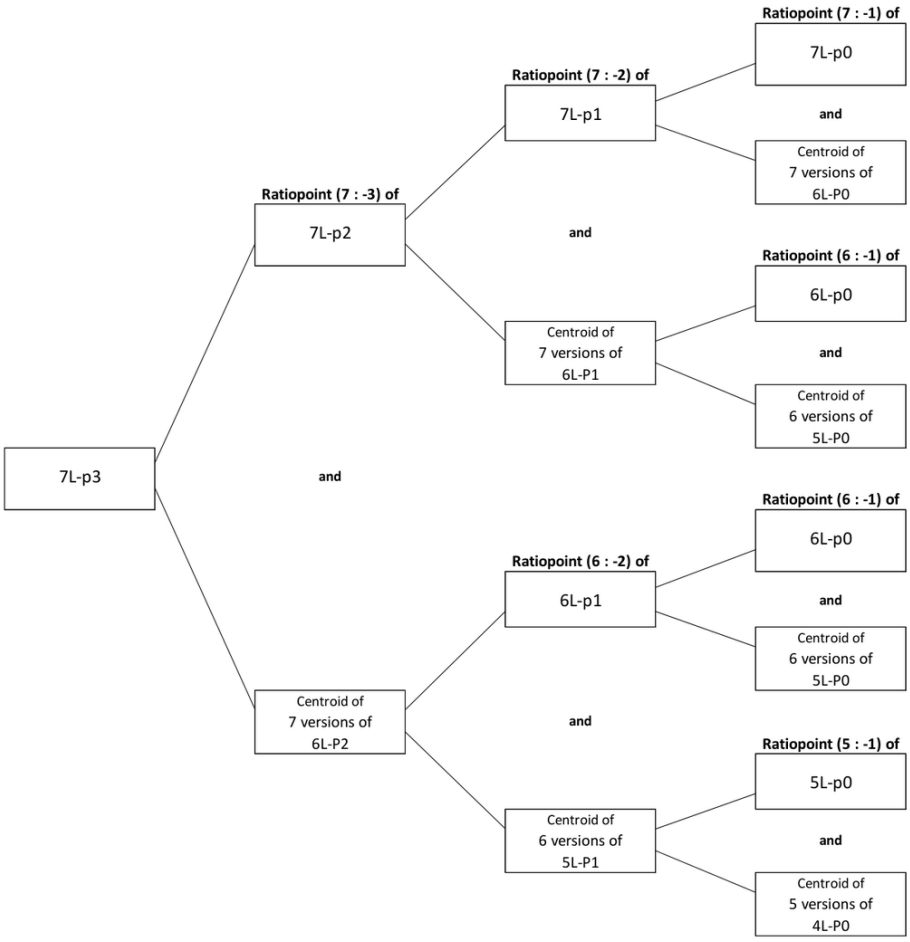
These properties are valid for Morley's nL-pi points **when n = odd** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , (n+1)/2, but not for higher values.
- There are two orthogonal reflective axes for i=(n-1)/2, bisecting angles with lower level (n-1)L-pi and (n-1)L-p(i-1).
- nL-pi for i = 1,2,3, ... ,n culminates in a point nL-pn, which will be the same point as the centroid of the n lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) =nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

These properties are valid for nL-pi points **when n = even** :

- nL-pi has a fixed distance ratio with the n versions of mL-pi & mL-p(i-1), when i=1, ... , n/2, but not for higher values. This distance ratio =1, when i = n/2 - 1. As a consequence nL-p(n/2-1) will be the common point of the perpendicular bisectors of the n pairs (n-1)L-p(n/2-1), (n-1)L-p(n/2-2).
- When n=even there are no reflective axes at any nL-pi bisecting angles with lower level mL-pi and mL-p(i-1).
- nL-pi for i = 1,2,3, ... ,n culminates in a point nL-pn, which will be the same point as the centroid of the lower level points (n-1)L-p(n-1). Indeed when i=n then nL-g(n-1) =nL-pn (ratiopoint (n:0)).
- When i > n the outcome will produce the same point nL-pn because the end of iteration has been reached.

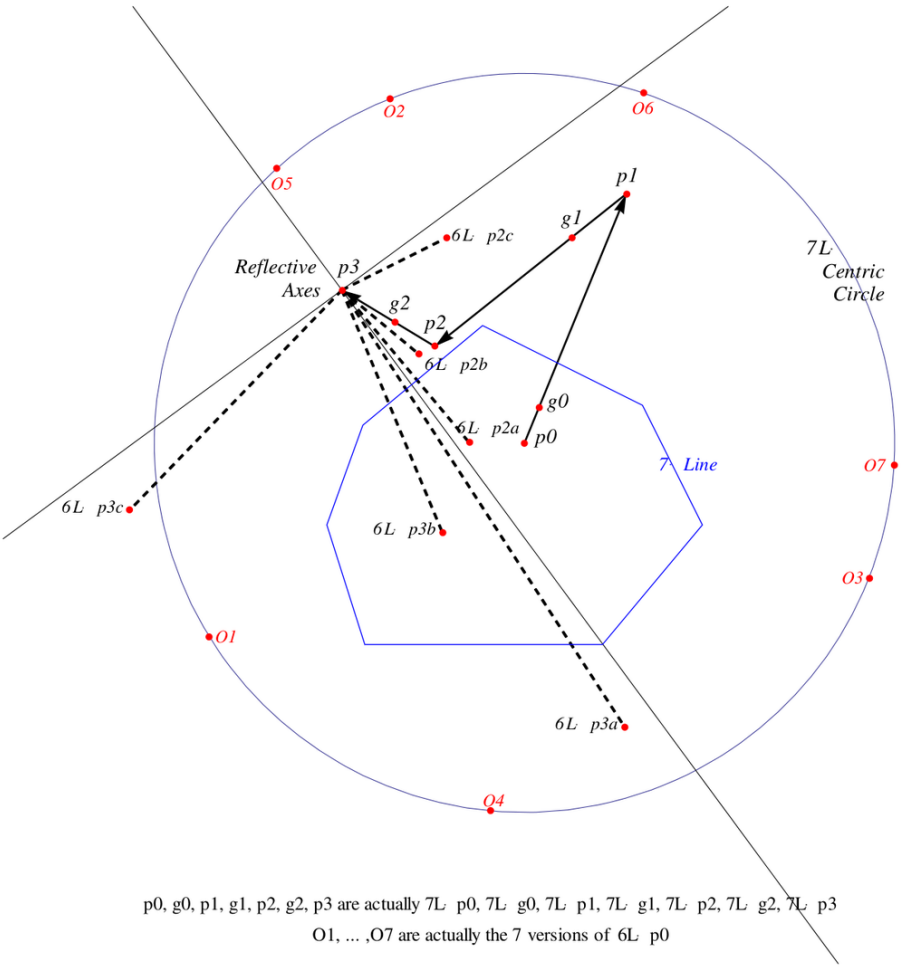
Example recursive 7L-p3 point structure:



Morleys 1st Orthocenter-03.pdf

Example Morley's first Orthocenter in a 7-Line:

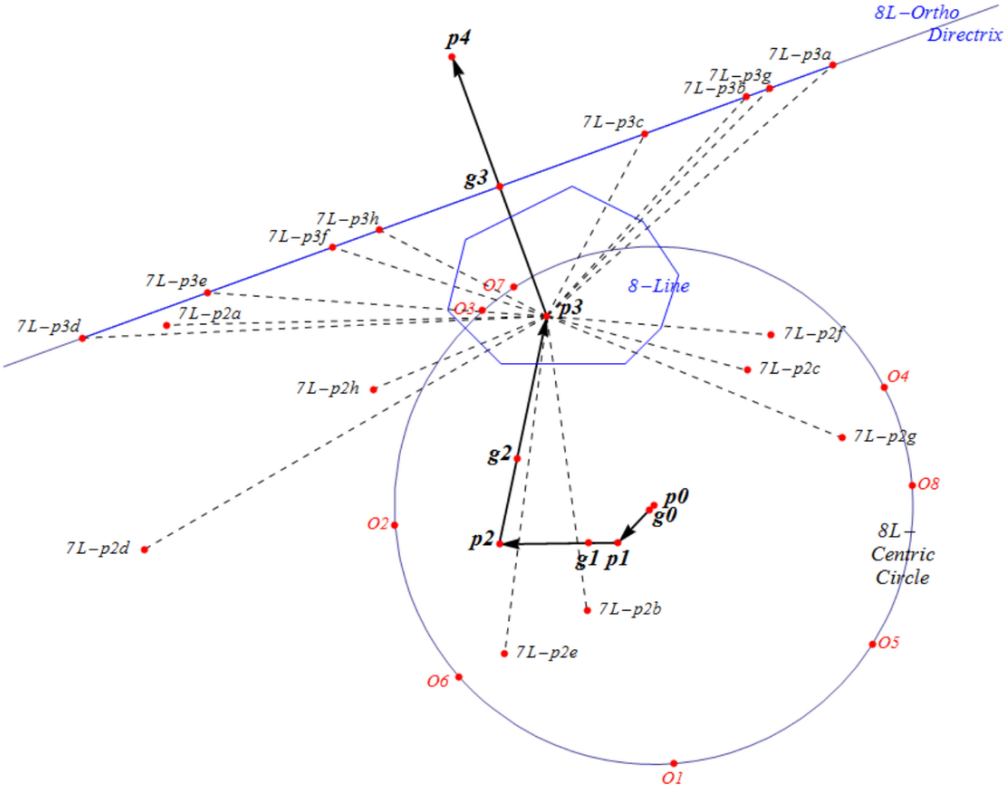
The sides of the blue 7-Gon represent the basic lines of the 7-Line.
7L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 7-Line.
It is constructed via g0 to p1 (1:7), via g1 to p2 (2:7), via g2 to p3 (3:7).
g0 = centroid of 7 points 6L-p0
g1 = centroid of 7 points 6L-p1
g2 = centroid of 7 points 6L-p2
Only three of the seven 6L-p3i points and 6L-p2i points have been drawn for checking the fixed relationship of distances 7L-p3 unto 6L-p3i and 6L-p2i.
In this picture 7L-p3.6L-p3i : 7L-p3.7L-p2i = 2.6167 for all i, where i=a,b,c, . . . (total 7).



Morleys 1st Orthocenter-03.pdf

Example Morley's Ortho directrix in an 8-Line

The sides of the blue 8-Gon represent the basic lines of the 8-Line.
8L-p3 (p3 in the picture) is Morley's 1st orthocenter of the 8-Line.
It is constructed via g0 to p1 (1:8), via g1 to p2 (2:8), via g2 to p3 (3:8) , via g3 to p4 (4:8).
g0 = centroid of 8 points 7L-p0
g1 = centroid of 8 points 7L-p1
g2 = centroid of 8 points 7L-p2
g3 = centroid of 8 points 7L-p3.
In this picture where n=even one of the pi has a prominent place: 8L-p3, which also is the common point of the perpendicular bisectors of all component 7-Line segments 7L-p2.7L-p3.
In general when n=even, then nL-p(n-1)=Center of the inscribed EnnaDeltoid. See [1], §4.

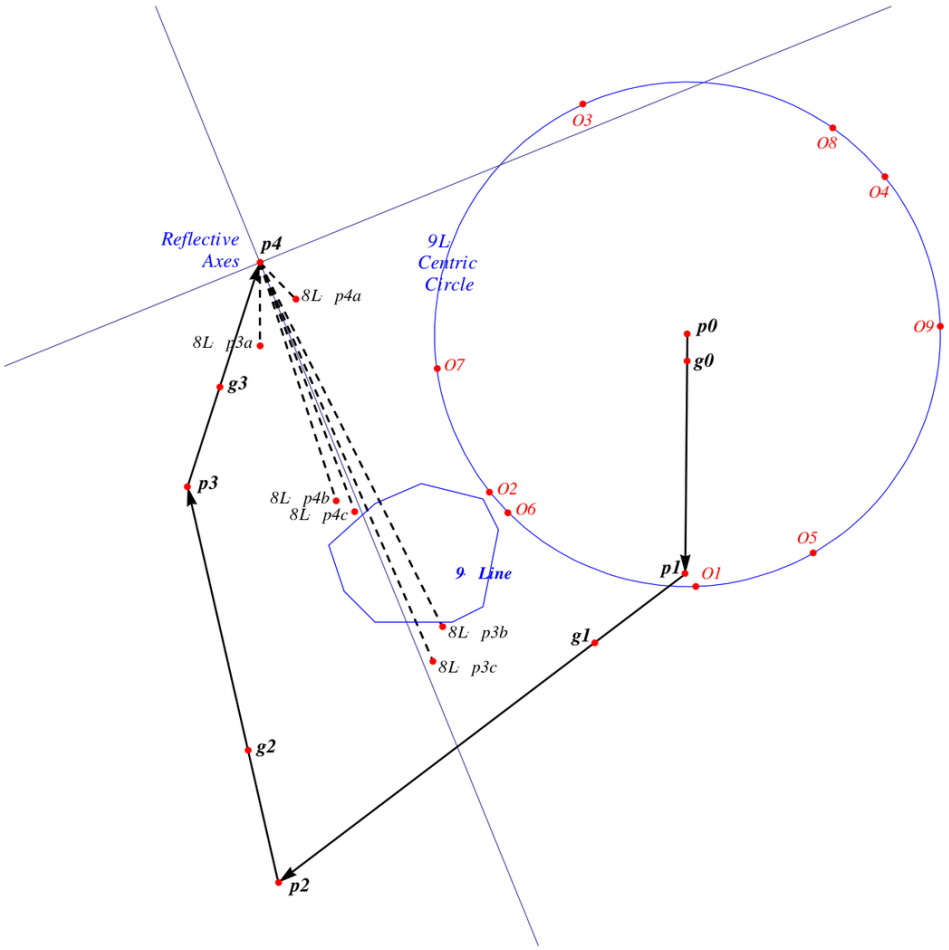


O1, ... ,O8 are actually the 8 versions of 7L−p0, being the Centric Centers of the 8 Component 7−Lines.
p0, g0, p1, g1, p2, g2, p3, g3, p4 are actually 8L−p0, 8L−g0, 8L−p1, 8L−g1, 8L−p2, 8L−g2, 8L−p3, 8L−g3, 8L−p4
The line segments 8L−p3.7L−p2x and 8L−p3.7L−p3x (x = a,b, ... ,h) are of equal size.
The 8L−Ortho Directrix is the line through 7L−p3a, ... ,7L−p3h, being the 1st Orthocenters of the 8 Component 7−Lines.
It also passes through 8L−g3, being the Centroid of 7L−p3a, ... ,7L−p3h.
It also is the perpendicular bisector of 8L−p3.8L−p4.

Morleys 1st Orthocenter-03.pdf

Example Morley's first Orthocenter in a 9-Line:

The sides of the blue 9-Gon represent the basic lines of the 9-Line.
9L-p4 (p4 in the picture) is Morley's 1st orthocenter of the 9-Line.
It is constructed via g0 to p1 (1:9), via g1 to p2 (2:9), via g2 to p3 (3:9), via g3 to p4 (4:9).
g0 = centroid of 9 points 8L-p0
g1 = centroid of 9 points 8L-p1
g2 = centroid of 9 points 8L-p2
g3 = centroid of 9 points 8L-p3
Only three of the nine 8L-p4i points and 8L-p3i points are drawn for checking the fixed relationship of distances 9L-p4 unto 8L-p4i and 8L-p3i.
In this picture 9L-p4.8L-p4i : 9L-p4.8L-p3i = 0.614571 for all i, where i=a,b,c, . . . (total 9).



Morleys 1st Orthocenter-03.pdf

References:

[1] F. Morley, *Orthocentric properties of the plane n-Line*, Trans Amer Math Soc, 4 (1903) 1-12
available at:
http://www.ams.org/journals/tran/1903-004-01/S0002-9947-1903-1500618-2/S0002-9947-1903-1500618-2.pdf

[2] F. Morley, *On the Metric Geometry of the N-Line*, Trans Amer Math Soc, 1 (1900) 97-115
available at:
http://www.ams.org/journals/tran/1900-001-02/S0002-9947-1900-1500528-8/S0002-9947-1900-1500528-8.pdf

[3] C. Kimberling, *Encyclopedia of Triangle Centers*, available at
http://faculty.evansville.edu/ck6/encyclopedia/ETC.html.

[4] Chris van Tienhoven, Encyclopedia of Quadri-Figures, available at
http://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia

[5] Quadri-Figures-Group, Internet forum for discussions on topics related to Quadrilaterals, Quadrangles, etc.
available at: http://tech.groups.yahoo.com/group/Quadri-Figures-Group/

Chris van Tienhoven
www.chrisvantienhoven.nl
van10hoven@gmail.com

Message: #918
Date: 02/1/2015 3:43:15
From: eckart_schmidt@t-online.de
Subject: Morley’s Limacon

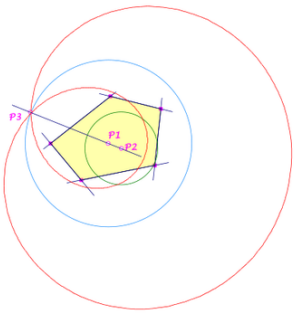
Dear Chris, dear BernardK,
perhaps of interest: Some remarks wrt Morley's limacon in "On the metric geometry of the plane n-line".
Best regards Eckart

EQF-Note 2015-01-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Morley's Limaçon as Inverse of a Conic

Morley described in "On the metric geometry of the plane n-line" a limaçon for n-lines. Here are gathered some properties of this curve for a 5-line. Finally result is a conic for n-lines inverse to the limaçon.



P1 Clifford point
P2 midpoint center-circle
P3 node of the 5-line
green center-circle
blue Clifford circle

Notations

Clifford chain (§4): There is a recursive definition of Clifford points for an even number of lines and of Clifford circles for an odd number of lines, beginning with the circumcircle for 3-lines.
Clifford point of *2n*-lines is the common point of the Clifford circles for *2n-1* of *2n* lines (Miquel point *QL-P1* for 4-lines).
Clifford circle of *(2n+1)*-lines is the circumcircle of the Clifford points for *2n* of *2n+1* lines.

Center-circles (§2): A recursive definition begins with the circumcircle for 3-lines.
The center-circle of *n*-lines is the circumcircle of the midpoints of the center-circles for *n-1* of *n* lines (Miquel circle *QL-Ci3* for 4 lines).

Node of a *n*-line (§3)**:** Common point of the center-circles for *n-1* of *n* lines (Miquel point *QL-P1* for 4-lines).

Morley's limaçon for *n*-lines is the envelope of circles through the node, centered on the center-circle.

These circles are Morley's "penosculants" (p.102), which he studies in §5.

"Limaçon" of a 4-line

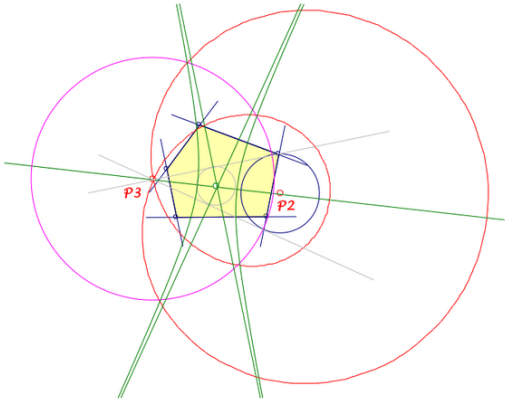
For a 4-line the node is the Miquel point *QL-P1* on the center-circle *QL-Ci3*. So the limaçon degenerates to a cardioïd.
Circles round points on *QL-Ci3* through *QL-P1* envelop the cardioïd *QL-Qu1*.
This cardioïd can also be considered …
… as locus of the reflections of *QL-P1* in tangents at *QL-Ci3*.
… as *CSC*-image of the inscribed parabola *QL-Co1* (see *EQF*).
… as pedal curve: Consider a circle through *QL-P1* and centered in the reflection of *QL-P1* in *QL-P4*. The pedal points of *QL-P1* wrt tangents at this circle give the cardioïd.
… as catacaustic of a circle round *QL-P4* through a point, which divides *QL-P1.QL-P4* with ratio *-4/3*. Rays from this point envelop with their reflections at the circle the cardioïd.

Limaçon of a 5-line

We shall use the following notation wrt a 5-line (see *QFG* message 710):

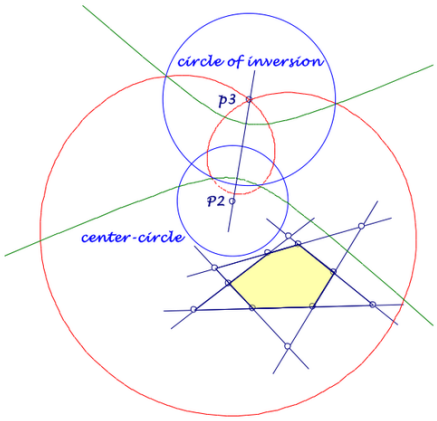
P_1 midpoint Clifford circle,
P_2 midpoint center-circle,
P_3 node of the 5-line.

- The node P_3 is the inverse of P_1 wrt the center-circle.
- The limaçon is the envelope of circles through the node P_3, centered on the center-circle (see above).
- The limaçon is the envelope of circles, centered on a circle round P_2 through P_3, which orthogonally intersect the Clifford circle.
- The limaçon is the locus of the reflections of P_3 in tangents at the center-circle.
- The limaçon is invariant under an inversion wrt the Clifford circle.
- The vertices of the limaçon are the reflections of the node P_3 in the intersections of P_1P_2 and the center-circle.
- The limaçon is the pedal curve of P_3 wrt the tangents of the Thales circle about the vertices.
- The limaçon is the inverse of a special conic: Foci of the conic are P_2 and P_3, center the midpoint. The circle about the vertices of the conic contacts the tangents from P_3 to the center-circle. Circle for inversion is a circle round P_3, orthogonal intersecting the center-circle.



Generalisation

Morley's limaçon of a n-line depends only on the center-circle (with midpoint $nL\text{-}P_2$) and the common point $nL\text{-}P_3$ of the center-circles of $n\text{-}1$ of n lines.
Morley's limaçon is the inverse of a conic ($n>4$):
… The circle for inversion is centered in $nL\text{-}P_3$ orthogonal intersecting the center-circle.
… The conic is the envelope of the perpendicular bisectors of $nL\text{-}P_3$ and points of the center circle.
Or let X be points on the center circle:
… The conic is the locus for intersections of perpendicular bisectors of $X.nL\text{-}P_3$ and $X.nL\text{-}P_2$.



Final question:
What is the relation of the conic to the reference n-line?

Eckart Schmidt
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2015-01-02.pdf

Message: #919
Date: 03/1/2015 3:47:58
From: eckart_schmidt@t-online.de
Subject: Morley’s penosculants of the limacon

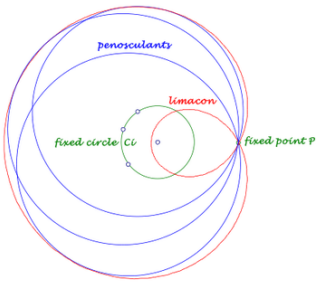
Dear Chris, dear BernardK,
here is a further note wrt Morley's paper "On the metric geometry of the plane n-line". Perhaps helpful.
Best regards Eckart

EQF-Note 2015-01-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

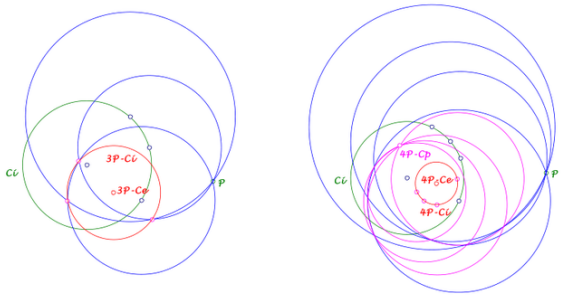
Morley's Penosculants of the Limaçon

Morley describes in "On the metric geometry of the plane n-line" the geometry of circles centered on a fixed circle through a fixed point – named "penosculants of the limaçon". The geometry of n penosculants corresponds to the geometry of n-lines. Here are gathered some figures in addition to Morley's calculations.



Penosculants of the Limaçon

In §5 Morley describes penosculants as circles through a point *P* and centered on a circle *Ci*, their envelope is a limaçon.
… If we consider three penosculants, there are three 2nd intersections with circumcircle *3P-Ci* and center *3P-Ce*.
… For four penosculants there are four *3P-Ci* with common point *4P-Cp* and centers on a circle *4P-Ci* with center *4P-Ce*.

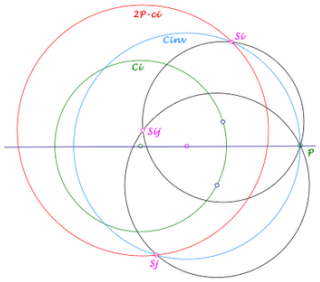


For five penosculants there are five *4P-Ci* with common point *5P-Cp* and centers on a circle *5P-Ci* with center *5P-Ce*…
This gives a chain analog to Morley's center-circle theorem for n-lines (p.107).

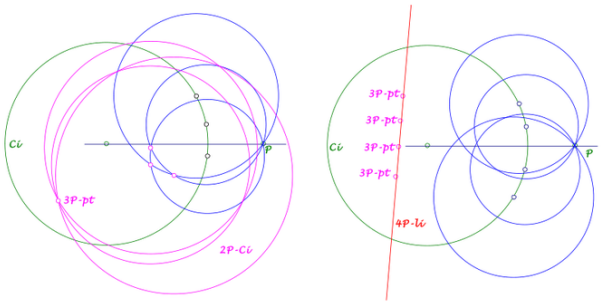
If we take for *Ci* the center-circle of a n-line and as penosculants the n center-circles of the *(n-1)*-lines and as *P* their common point, then there will be further points and circles for n-lines.

Inversion of the Limaçon

The limaçon is invariant under an inversion. The circle of inversion *Cinv* contains the node *P* and has its center in the inverse of *P* wrt the fixed circle *Ci*.
… If we consider two penosculants p_i and p_j, there is a 2nd intersection $S_{i,j}$ and 2nd intersections S_i and S_j with the circle of inversion *Cinv*. The circle round $S_{i,j}$ through S_i and S_j shall be named *2P-ci*.



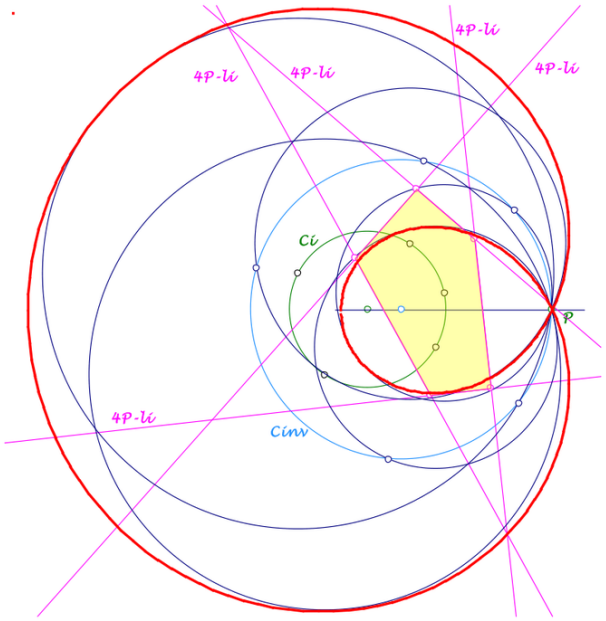
… If we consider three penosculants, there are three circles *2P-ci* with a common point *3P-pt*.
… For four penosculants there are four points *3P-pt* on a line *4P-li*.



… For five penosculants there are five lines *4P-li,* defining a 5-line with the following properties:

- The five Miquel Circles *QL-Ci3* of this 5-line are the five penosculants with common point *P*.
- The five Miquel Circumcenter *QL-P4* for this 5-line are the midpoints of the penosculants.
- The five Miquel Points *QL-P1* for this 5-line lie on the circle of inversion *Cinv* in the 2nd intersections with the penosculants.

- Morley's center-circle of this 5-line is the fixed circle *Ci* of the penosculants.



These last properties are not explicit mentioned in Morley's paper, but they will be the background of his final remark (p.106):
"Naturally then five penosculants will give five lines and thus we get the MIQUEL figure as completed by KANTOR."

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2015-01-03.pdf

Message: #920
Date: 05/1/2015 3:39:33
From: eckart_schmidt@t-online.de
Subject: nL-CSC-Transformation?

Dear Chris,
In §6 of his paper "On the metric geometry of the plane n-line" Morley researched 2n-lines with an inscribed conic. Here are cited his results:
“This involution has the following properties:
(i) Its center is the Clifford point of the 2p-lines.
(ii) The foci are a pair of the involution.
(iii) The Clifford point of 2q lines and that of the remaining 2(p-q) lines are a pair of the involution. The Clifford point of two lines means merely their intersection.
(iv) The Clifford circle of 2q-1 lines and that of the remaining lines are partners. The Clifford circle of a line is merely the line itself.”
For a 4-line, which has always an inscribed conic, this involution is the CSC-transformation.
For 6-lines with an inscribed conic this involution is the transformation 6L-CSC, mentioned in QFG #784, #861, #864.
Remember: The CSC-images of a point X wrt the 4-lines of a 5-line lie on a circle, these circles for the 5-lines of a 6-line have a common point, the 6L-CSC-image of the point X.
For an arbitrary 6L the transformation 6L-CSC holds Morley's properties (ii) and (iii):
wrt (ii): 6L-CSC swaps the foci of the inscribed conics of 5 of 6 lines.
wrt (iii): 6L-CSC swaps the intersection of 2 of 6 lines and the Miquel point of the remaining 4 of 6 lines.
But I don't see any generalization for 2n-CSC!
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #921
Date: 06/1/2015 8:44:08
From: chris.vantienhoven
Subject: nL-CSC-Transformation?

Dear Eckart,
Morley surprises us again with this transformation.
He gives us an answer to our earlier question about generalization.
He shows us that the 4L-CSC tansformation can be generalized to n-Lines (only?) when circumscribing a conic.
Thanks for all your study and descriptions of all these Morley papers (also about the Hodgson-properties and the limacon)!
Best regards,
Chris

← Previous Next → ↩ Message Index ↑ Subjects

Message: #922
Date: 08/1/2015 11:15:49
From: eckart_schmidt@t-online.de
Subject: 2n-CSC-Transformation

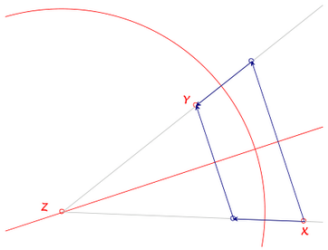
Dear Chris, dear BernardK,
now I can offer a generalization of CSC for 2n-lines, see attached file.
The discussion in #784, #861, #864 was not the right way.
The key for a generalization is Morley's involution for 2n-lines tangent to a conic (p.111 of his paper).
With a special inversion we can assign to the reference 2n-line a 2nd 2n-line with an inscribed conic. This inversion, followed by Morley's involution for the 2nd 2n-line and then the inversion again, this gives the 2n-CSC-transformation for a 2n-line.
Best regards Eckart

EQF-Note 2015-01-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

CSC-Transformation for 2n-Lines

The CSC-transformation QL-Tf1 is an interesting mapping for 4-lines (see EQF). Here is a generalization for 2n-lines, orientated at an involution, which Morley mentioned in "On the metric geometry of the plane n-line" for 2n lines tangent to a conic. With an inversion we can assign to the reference 2n-line a 2nd 2n-line with an inscribed conic. This inversion, followed by Morley's involution for the 2nd 2n-line and then the inversion again, this gives the 2n-CSC-transformation for a 2n-line.



Preliminary Remarks

The *CSC*-transformation is an involution, determined by a center *Z* and a pair *X, Y* of *CSC*-partners. This involution consists of a reflection in the angle bisector of $\angle XZY$ and an inversion wrt a circle round *Z* with radius $\sqrt{ZX \cdot ZY}$.
For a 4-line *Z* is the Miquel point *QL-P1* and *X, Y* can be two opposite intersections of the 4-line.

Morley's results

In §6 Morley researched 2n-lines with an inscribed conic. Here are cited his results:

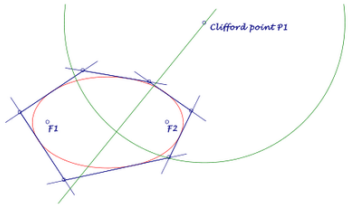
"This involution has the following properties:

(i) Its center is the Clifford point of the 2p-lines.

(ii) The foci are a pair of the involution I_1^2.

(iii) The Clifford point of 2q lines and that of the remaining 2(p-q) lines are a pair of I_1^2. The Clifford point of two lines means merely their intersection.

(iv) The Clifford circle of 2q-1 lines and that of the remaining lines are partners. The Clifford circle of a line is merely the line itself."



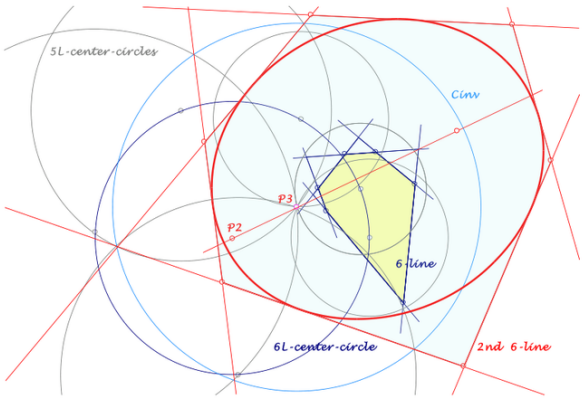
For a 4-line, which has always an inscribed conic, this involution is the *CSC*-transformation.

For 6-lines with an inscribed conic this involution is the transformation mentioned in *QFG* #784, #861, #864.

Generalization

For a 2n-line we shall use the following points (generalized notation of *QFG* #710) …
… P_1 Clifford point of the 2n-line,
… P_2 center of the 2n-center-circle,
… P_3 common point of the (2n-1)-center-circles of the 2n-line.

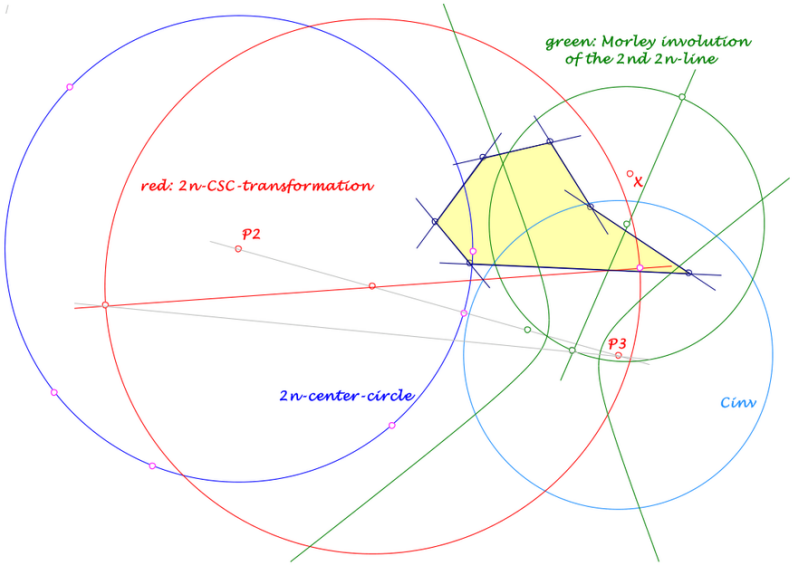
We consider a 2n-line and the 2n center-circles for 2n-1 of 2n lines.
… These (2n-1)-center-circles intersect in the point P_3 and their centers give the 2n-center-circle with center P_2.
… In addition we use an inversion wrt an arbitrary circle C_{inv} round *P3*.
… The images of the (2n-1)-center-circles are 2n lines tangent to a conic, defining a 2nd 2n-line.
… This conic has one focus P_3 (a special example in *QFG* # 918). For the 2nd focus take the inverse of P_3 wrt the 2n-center-circle and then the C_{inv}-inverse of this point.



… **The 2n-CSC-transformation is the following chain of mappings:**

Inversion wrt a circle C_{inv} round P_3,
Morley's involution wrt the 2nd 2n-line,
Inversion wrt the circle C_{inv} again.

… The 2n-*CSC*-transformation for a 4-line is the *CSC*-transformation *QL-Tf1*.
… The center of the 2n-*CSC*-transformation is the inverse of P_3 wrt the 2n-center-circle.
… A pair of 2n-*CSC*-partners are P_3 and the C_{inv}-inverse X of the Clifford point of the 2nd 2n-line.
… The fixed points of the 2n-*CSC*-transformation are the C_{inv}-inverses of the fixed points of the Morley involution for the 2nd 2n-line.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-01-08.pdf

Message: #923
Date: 08/1/2015 9:36:47
From: bernard.keizer
Subject: 2n-CSC-Transformation

```
Dear Eckart, dear Chris
This time, I'm very impressed !
It looks like the inversion which transforms the QL in an
inscribed QL.
I'm back from a short week in Neapel for the Newyear and the
first day in Paris was horrible, as you will have followed ...
Best regards
Bernard
```

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Message: #924
Date: 12/1/2015 12:15:42
From: bernard.keizer
Subject: Je suis Charlie

```
Dear Chris, dear Eckart
I'm back from the giant demonstration in Paris with the
participation of the french president François Hollande, the
dutch prime minister Mark Rutte and the german Kanzler Angela
Merkel.
It's as important for me to fight for democracy and liberty
against terrorism as to crack Morley's code !
My wife, my children and grand-children and I were particulary
honored and grateful for walking behind your leaders in this
popular demonstration.
Tonight, je suis Charlie
Best regards
Bernard
```

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Message: #925
Date: 12/1/2015 6:38:55
From: chris.vantienhoven
Subject: Je suis Charlie

```
Dear Bernard,
In Holland we sympathize very much with you and your country in
these dark hours.
Personally I watched all the programs about the terror and the
actions of the French government.
Your countrymen did a wonderful job and I am glad you had time
to commemorate what happened in the huge march in Paris.
All the best from Holland.
Best regards,
Chris
```

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Message: #926
Date: 12/1/2015 8:52:50
From: eckart_schmidt@t-online.de
Subject: Je suis Charlie

```
Dear Bernard,
it was an impressive demonstration against terror yesterday in
Paris! You have participated also in my name!
My hope is a citation of Albert Schweitzer: "Aus Ehrfurcht vor
dem Leben erwächst der Welt Frieden."
Best regards Eckart
```

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Message: #927
Date: 13/1/2015 9:31:08
From: eckart_schmidt@t-online.de
Subject: Line-Penosculant-Trapezoid of a n-Line

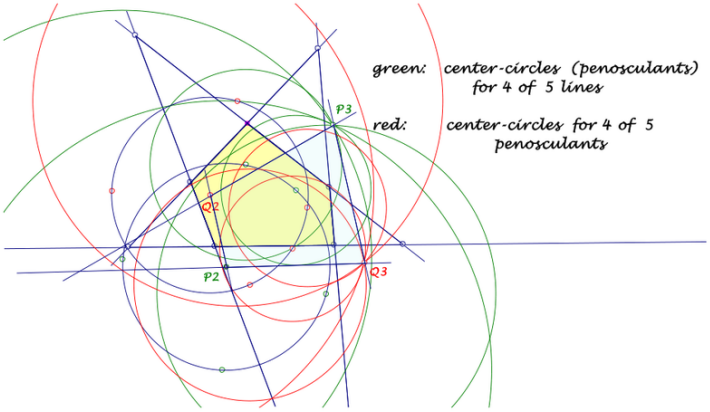
Dear Bernard, dear Chris,
attached there is a Cabri observation wrt n lines and corresponding n penosculants. I don't know, whether it is evident, but I do not see the background.
Best regards Eckart

EQF-Note 2015-01-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line-Penosculant-Trapezoid of a n-Line

In his paper "On the metric geometry of the plane n-line" Morley describes penosculants of a n-line as center-circles for n-1 of n lines. In §5 he proved the center-circle theorem for penosculants. So there are center-circles for the n lines and for the corresponding n penosculants. Let P_2 / Q_2 be the center of the center-circle for n lines / n penosculants and analogous P_3 / Q_3 the common point of the n center-circles of n-1 lines / n-1 penosculants, then P_2Q_2 is parallel P_3Q_3.



Center-circle for n lines (§2):
A recursive definition begins with the circumcircle for 3 lines.
The center-circle for *n* lines is the circumcircle of the midpoints of the center-circles for *n-1* of *n* lines.

Center-circle for n penosculants (§5):
The center-circles for *n-1* of *n* lines are the considered *n* penosculants.
A recursive definition begins with the circumcircle for the 3 2nd intersections of 3 penosculants.
The center-circle for *n* penosculants is the circumcircle of the midpoints of the center-circles for *n-1* of *n* penosculants.

Points:

P_2 center of the center-circle for n lines,
Q_2 center of the center-circle for n penosculants,
P_3 common point of the center-circles for n-1 of n lines,
Q_3 common point of the center-circles for n-1 of n penosculants.

Line-Penosculant-Trapezoid ($n>4$): P_2Q_2 is parallel P_3Q_3.

Final remark:
This constellation can be used for an involution with center in the intersection of P_2Q_3 and P_3Q_2 with partners P_2, P_3 or Q_2, Q_3.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-01-13.pdf

Message: #928
Date: 13/1/2015 6:09:27
From: bernard.keizer
Subject: Line-Penosculant-Trapezoid of a n-Line

Dear Eckart,
I suppose the answer is given by Morley at the end of §5.
The 5 green circles are penosculants of the same limaçon ; it would be helpful to put this limaçon on the same figure with node, focus and inversion circle ...
Best regards
Bernard
PS For the QL, the 2nd intersections 2 by 2 of 3 of the 4 circumcircles are 3 vertices on the omitted line and the 4 centers are at the infinity : P2 is QL-P3 and P3 is QL-P1, but what about Q2 and Q3 ?

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Message: #929
Date: 13/1/2015 10:53:37
From: chris.vantienhoven
Subject: polygons and similar conics

Dear friends,
Accidentally I found this site from
Sahatchai Wanawongsawad:
http://www.sahatchaiw.com/
He wrote two interesting documents on projective geometry,
polygons and and similar conics.
His first document deals about methods of constructing
perspective polygons:
http://sahatchaiw.com/page1/files/Perspectivepolygons.pdf
But also his second document on similar conics is certainly
worthwhile.
http://sahatchaiw.com/page1/files/Similar%20Conics.pdf
Best regards,
Chris

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Message: #930
Date: 14/1/2015 8:35:08
From: chris.vantienhoven
Subject: 6L-point

Dear Eckart and BernardK,
When constructing the Inscribed Conic Centers 6 times in a
6-Line they are coconic.
Consequently the center of this 6L-conic is a 6-Line Center.
When applied 7 times in a 7-Line alas these centers aren't
coconic.
Best regards,
Chris

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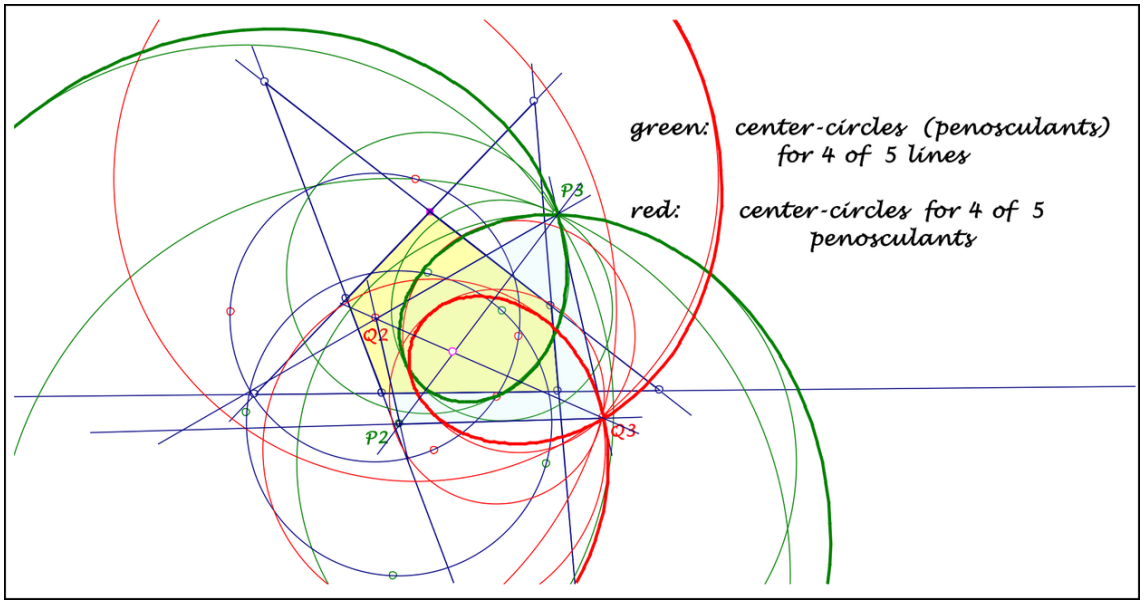
Message: #931
Date: 14/1/2015 10:05:05
From: eckart_schmidt@t-online.de
Subject: Line-Penosculant-Trapezoid of a n-Line

Dear Bernard,
it is evident, that the center-circles for n-1 of n lines and the corresponding center-circles for n-1 of n penosculants define limacons, but they are different (see attached file). Equal are the inverse of P3 wrt the center-circle of the n lines and the inverse of Q3 wrt the center-circle of the n penosculants. But this gives not naturally the trapezoid.
Best regards Eckart



2015-01-14.pdf

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Message: #932
Date: 15/1/2015 5:00:33
From: Benedetto Scimemi
Subject: Line-Penosculant-Trapezoid of a n-Line

Dear Friends,

Following Chris's kind invitation I have recently joined this quadri-figures group, assuming that my (mostly old, but unpublished) results on quadrangles may arise some interest and also that possible new contacts would encourage me to revisit this amusing research field.

Recently I have enjoyed receiving the group communications, but I feel somehow submerged by the high number of messages arriving from n<4 people who are working on n>4-gons!
I fully understand and sympathize with this run towards higher and higher n.
I hope, however, that somebody is still looking at quadri-figures, where, in my opinion, much has to be still explained and explored.
This is why I now decided to share with you the following notes (that Chris has already privately received) which collect some news (are they new?) on the quadrangle with an application to pentagons. By the way, all my proofs were analytical.

A THEOREM ON QUADRANGLES AND AN APPLICATION TO PENTAGONS (B.S.)
THEOREM. In a quadrangle *Q* let
* H=QA-P2 be the Euler-Poncelet point,
* O=QA-P4 the isoptic point.
Consider any central conic *C* circumscribed to *Q*.
Let C be its center (a point on QA-Co1),
    s its major axis (the line of the foci),
    a>b the axes lengths.
Then the vector CO is obtained by reflecting the vector CH on *s* and multiplying it by the factor
     (1) (a^2-b^2)/(a^2+b^2)   if *C* is an ellipse
     (2) (a^2+b^2)/(a^2-b^2)   if *C* is a hyperbola
In other words: for any point C on QA-Co, the major axis of the *Q*-circumscribed conic centered in C bisects the angle < OCH >, and the ratio |CO|/|CH| equals (cos α)^(±1), where α is
     (1) the angle under which a vertex views the minor axis of the ellipse *C*, or
     (2) the minor angle between the asymptotes of the hyperbola *C*.
This statement becomes trivial if *C* is a circle.
The possibility for *C* to be a parabola (b tends to 0) will be considered in one of the following Corollaries.

By choosing on QA-Co1 particular points for C (possibly at infinity) one has, for example:

Corollary 1. (C=QA-P3) In a non-cyclic quadrangle *Q* consider the Gergonne-Steiner conic *C* = QA-Co3.
Let J=QA-P3 be its center, *s* the major axis and a>b the axes lengths.
Then *s* bisects the angle < HJO > and |JO|/|JH|^(±1) = (a^2-b^2)/(a^2+b^2) according to *C* being an ellipse or a hyperbola (compare ref.36 in EQF, at p.344).

Corollary 2. (C a diagonal point) In a quadrangle *Q* consider a diagonal point C and let *s* be the inner bisector of the angle α between the sides that meet in C.
Then *s* bisects also the angle < OCH > and |CO|/|CH|=(cos α)^(±1) ....

Corollary 3. (C at infinity) In a convex quadrangle *Q* let C be one of the points at infinity of the hyperbola QA-Co1. Then there is a *Q* -circumscribed parabola *C* whose axis is parallel to an asymptote of QA-Co1. As for the central points of *Q*, O is obtained from H by a reflection on the axis *s* of *C*, followed by a translation which amounts to twice the distance focus-directix . (Then clearly (H+O)/2 belongs to *s* , see EQF ref.36, p.348).
In particular, if *Q* is a trapezoid (a diagonal point C goes to infinity and *C* degenerates into a pair of parallel lines), then OH is orthogonal to the parallel sides of *Q* , the points O and H being at the same distance from them.
When C = H = QA-P2, the distance ratio has no meaning.
By continuity, however, one gets:

Corollary 4. (C tends to H = QA-P2). In any quadrangle *Q* the tangent *t* to the Nine-point conic QA-Co-1 at the point H is reflected by the axes of the circumscribed rectangular hyperbola QA-Co2 into the line HO. (O=QA-P4).
Since the relation between H and O in the Theorem above only depends on the circumscribed conic *C* but is independent of the particular inscribed quadrangle, we also have

Corollary 5. Let *Q1* , *Q2* be two quadrangles inscribed in the same conic *C* centered in C.
If H1, H2 denote the two Euler-Poncelet points and O1, O2 the two isoptic points,
then the major axis of *C* bisects both the angles
< H1 C O1 >, < H2 C O2 >
and we have O1 → H1 and O2 → H2 in a same (Möbius) mapping, a product of a reflection and a commuting homothety.
The last Corollary suggests an application to pentagons

THEOREM. Given a non-cyclic pentagon *P* let *C* be the
circumscribed conic.
For each component quadrangle of *P* , say *Qi* (i= 1,...5),
consider
* the Euler-Poncelet point Hi (=QA-P2) and
* the isoptic point Oi (=QA-P5).
Then the five pairs of central points correspond in the same
negative similarity ζ: Hi → Oi.
If *C* has a center C, let *f* be the line of the foci, a>b the
axes lengths.
Then ζ is the product of the reflection on *f* and the homothety
centered in C whose scale factor is
     (a^2 − b^2)/(a^2 + b^2)^(±1)
according to *C* being an ellipse or a hyperbola.
If *C* is a parabola, then ζ : Hi → Oi is an isometry, indeed
a glisso-reflection whose fixed line is the axis *f* of *C* and
the parallel translation amounts to twice the distance
focus-directrix.
In a shorter and less precise form:
Let *P* be a pentagon and *C* the circumscribed conic. Let
O(P) and *H(P)* be the pentagons whose vertices are the
isoptic points and the Euler- Poncelet points of the five
component quadrangles . Then *C(P)* and *H(P)* are inversely
similar. (see the figure above)

END

This little contribution to n>4 problems makes me feel more
comfortable with the most active members of the group!
A happy 2015 to all of you and best regards.
Benedetto Scimemi

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Message: #933
Date: 16/1/2015 10:54:16
From: bernard.keizer
Subject: Line-Penosculant-Trapezoid of a n-Line

Dear Eckart,
This time, I give up ! I don't get your point.
Working on your messages 918, 919, 927 and 931, I can't see why the 2 penosculants should be different.
Please, just tell me what's wrong in my approach ...
I start with the 5 lines Li from i to m ; there are 10 vertices Aij and 10 circumcenters Oij being 4 by 4 on the 5 Miquel circles with centers Oi through the Miquel points Mi. The 5 Miquel circles are through a common point X, the 5 Oi are on the center-circle with center O, the 5 Mi are on a circle with center M through X and M and X are inverse wrt the center-circle.
But if I start with the 5 penosculants, Sij is the point Oij, Si and Sj are the points Mi and Mjj and Oij is the 2nd intersection of the 2 penosculants with centers Oi and Oj ; the circle with center Oij through Mi and Mj is one of your 2P-ci and one of the 10 circumcircles, the common point to the 3 circles 2P-ci, 2Pcj and 2P-ck id est your point 3P-pt is one of the 10 vertices Alm, the 4 points 3P-ct are the vertices Aim, Ajm, Akm and Alm and give the line Lm !
In other words, the 5-L are identical and the limaçon is the same ...
Best regards
Bernard

PS The limaçon is the inverse of a conic wrt any circle through one of the 2 foci (the cardioïd is the inverse of a parabola wrt any circle through the focus)
We need a transformation which swaps Aij, intersection of 2 lines, and Oij, circumcenter of the triangle formed by the 3 other lines, and each line with the Miquel circle of the QL with the 4 other lines : then it should swap the limaçon tangent to the 5 Miquel circles and the conic tangent to the 5 lines ...

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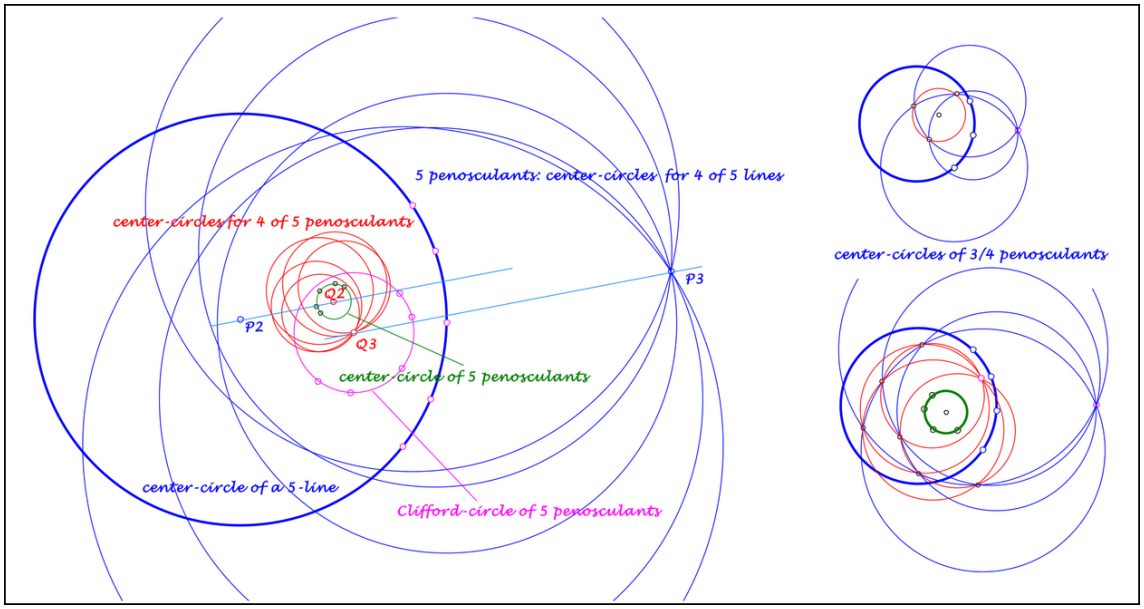
Message: #934
Date: 17/1/2015 11:00:54
From: eckart_schmidt@t-online.de
Subject: Line-Penosculant-Trapezoid of a n-Line

Dear Bernard,
thanks for your interest, but I had difficulties with your nomination. I try an answer with a drawing. You will not find a 5-line, but its center-circle, centered in 5L-P2 (your point O), and the common point 5L-P3 (your point X) for the 5 center-circles for 4 of 5 lines, which are the penosculants of the 5-line.
I prepare a paper for Clifford points / circles for n penosculants. You will find the Clifford circle for 5 penosculants in the drawing, centered in 5L-P3.
I hope, I could help.
Best regards Eckart



2015-01-17.pdf

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Message: #935
Date: 17/1/2015 11:32:29
From: benedetto.scimemi
Subject: component pentagons and alia

Editorial note: This message is identical to message #932.

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Message: #936
Date: 17/1/2015 11:00:46
From: benedetto.scimemi
Subject: figures for my previous message

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I apologize. These figures were supposed to be attached to my
previous message. B.S.
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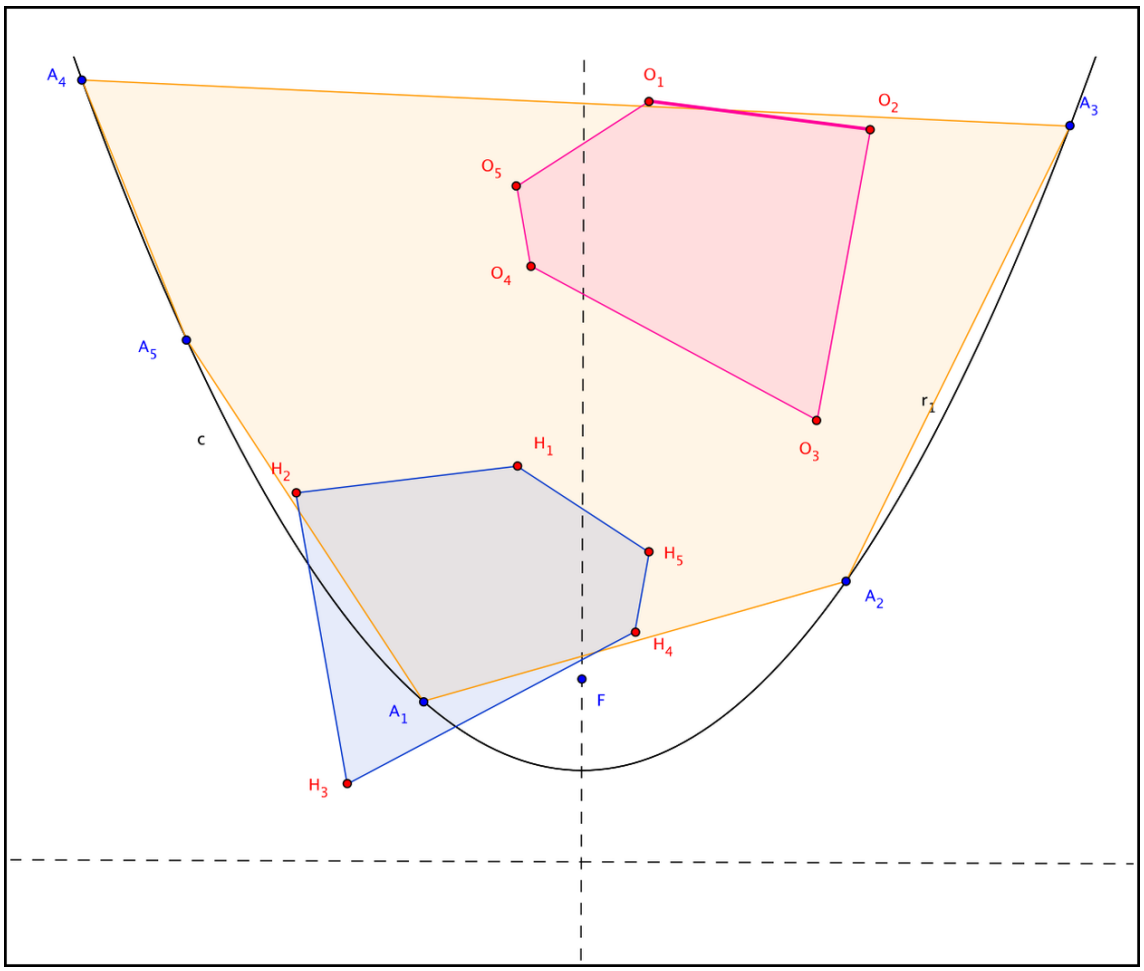

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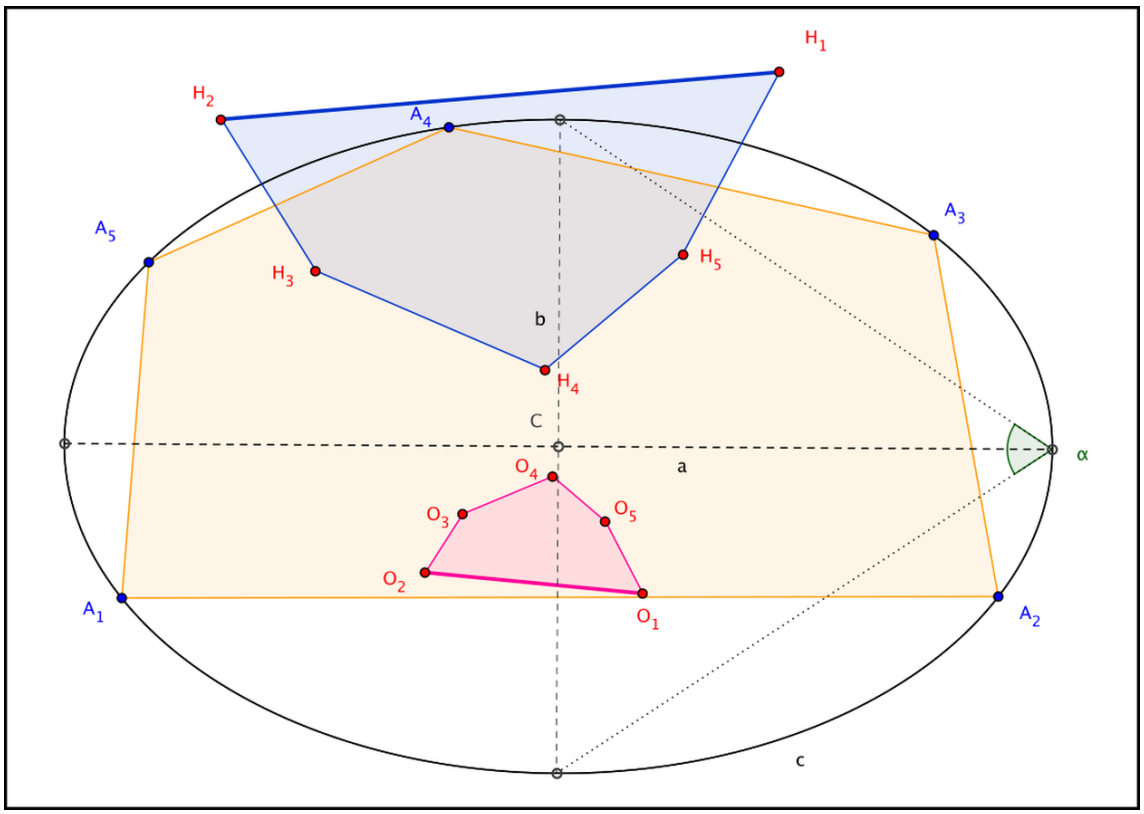

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Message: #937
Date: 18/1/2015 10:20:14
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain for n Lines and n Penosculants

---

Dear Bernard, dear Chris,
In his paper "On the metric geometry of the plane n-line" Morley treated center-circles and Clifford's chain for n lines as well as for n penosculants.Center-circles for n-1 of n lines are the penosculants of a n-line. So we can look for the following circles and points of n-lines:
center-circles: nL-cc and nP-cc,
Clifford circles (n odd): nL-Cc and nP-Cc,
midpoint Clifford circles (n odd): nL-P1 and nP-P1,
Clifford points (n even): nL-Cp and nP-Cp,
midpoint center circles: nL-P2 and nP-P2,
intersection of center-circles for n-1 of n lines: nL-P3 and nP-P3.
I think, you will have an understanding of the nomination (orientated at QFG#710 for 5-lines) and I hope, that there are no basic mistakes. Cabri observations have their limits, but perhaps the last guess holds.
Best regards Eckart

EQF-Note 2015-01-18

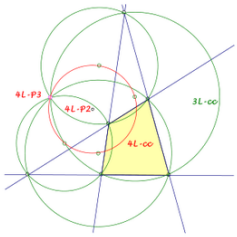
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Center-Circles and Clifford's Chain for n Lines and n Penosculants

In his paper "On the metric geometry of the plane n-line" Morley treated center-circles and Clifford's chain for n lines as well as for n penosculants. Penosculants are circles through a fixed point, centered on a fixed circle. Center-circles for n-1 of n lines are the penosculants of a n-line. Here is tried to describe relationships between points and circles for n lines and the corresponding penosculants.

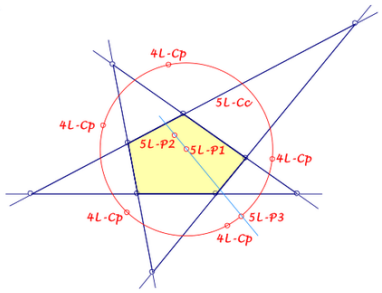
Center-circles of n-lines (§2):

A recursive definition begins with the circumcircle for 3-lines. The center-circle *nL-cc* of *n*-lines is the circumcircle of the midpoints of the center-circles for *n-1* of *n* lines. The midpoint of the center-circle *nL-cc* shall be *nL-P2*. The center-circles for *n-1* of n lines have a common point *nL-P3* (nomination as in *QFG #710* for 5-lines).



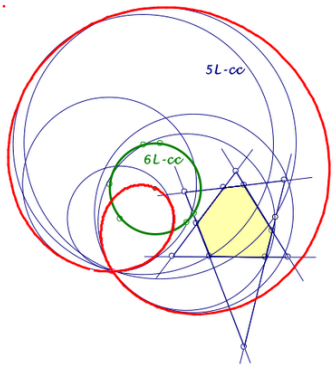
Clifford's chain (§4)

There is a recursive definition of Clifford points 2*nL-Cp* for an even number of lines and of Clifford circles *(2n+1)L-Cc* for an odd number of lines, beginning with the intersection of 2 lines. Clifford circle of *(2n+1)*-lines is the circumcircle of the Clifford points for *2n* of *2n+1* lines. Let *(2n+1)L-P1* be the midpoint of the Clifford circle. Clifford point of *2n*-lines is the common point of the Clifford circles for *2n-1* of *2n* lines.
Special: For a 3-line the circumcircle is the Clifford circle *3L-Cc* as well as the center-circle *3L-cc*. For a 4-line the Clifford point *4L-Cp = 4L-P3* is the Miquel point. For a 5-line the points *5L-P1, 5L-P2, 5L-P3* are collinear with *5L-P3* on the Clifford circle *5L-Cc* and *5L-P1* the inverse of *5L-P3* wrt the center-circle *5L-cc*.



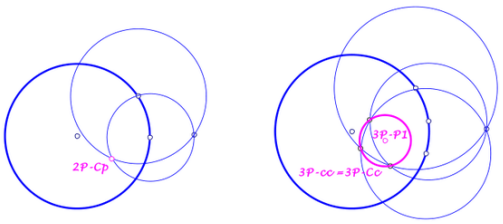
Penosculants (§3)

Morley defines penosculants as circles through a fixed point centered on a fixed circle. For example the center-circles for *n-1* of *n* lines give *n* penosculants. It is evident, that penosculants define a limaçon.

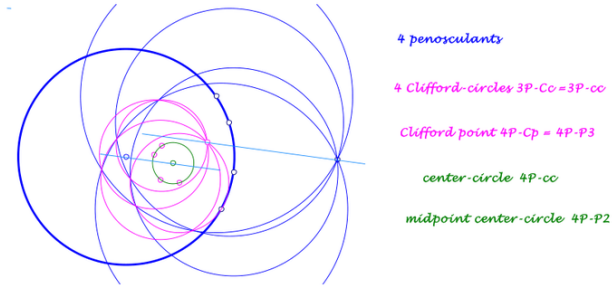


Center-circles and Clifford's chain for n penosculants (§5)

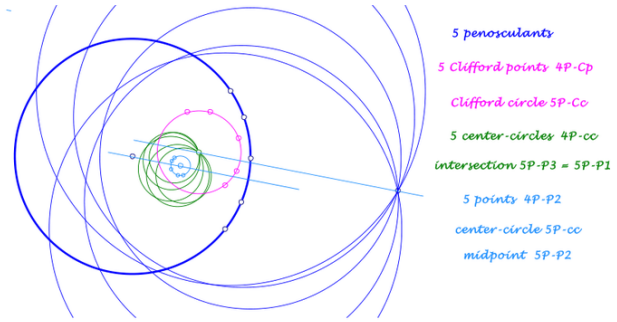
There is an analog recursive definition of center-circles *nP-cc* and of Clifford points *2nP-Cp* for an even number and of Clifford circles *(2n+1)P-Cc* for an odd number of penosculants, beginning with the 2nd intersection of 2 penosculants for the Clifford chain and with the circumcircle of the 2nd intersections of 3 penosculants for the center-circles. Analog we can consider points *nP-P1* (for odd *n*), *nP-P2* and *nP-P3*.



For 4 penosculants the four circles *3P-Cc = 3P-cc* have the Miquel point as common point *4P-Cp = 4P-P3* and midpoints on the center-circle *4P-cc*. This gives a new constellation for 4 penosculants, whose center-circles reproduce the reference fixed point and circle. Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.4P-P2* is parallel *P3.4P-P3*.

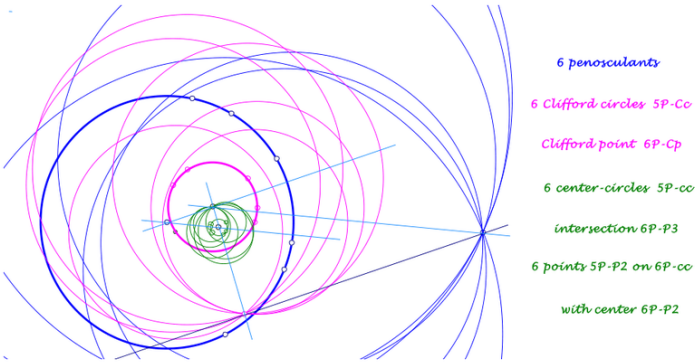


For 5 penosculants the five Clifford points *4P-Cp* give the Clifford circle *5P-Cc*. The five center-circles *4P-cc* intersect in the point *5P-P3*, which is the midpoint of the Clifford circle. The midpoints of the center-circles *4P-cc* give the center-circle *5P-cc* with midpoint *5P-P2*. Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.5P-P2* is parallel *P3.5P-P3*.



For 6 penosculants there is something new: The six Clifford circles *5P-Cc* are centered on a circle with midpoint *6P-P3*. This is not valid for the Clifford circles *6L-Cc* of 6 lines.
Let *P2* be the center of the fixed circle and *P3* the fixed point, then *P2.6P-P2* is parallel *P3.6P-P3*. This seems to be valid in general.
Further holds: The points *6P-P2, 6P-P3, 6P-Cp* are collinear and the lines *P2.6P-P3*, *P3.6P-Cp* are parallel.

So far examples for Morley's result in §5, that the center-circle theorem and Clifford's chain also exist for penosculants.

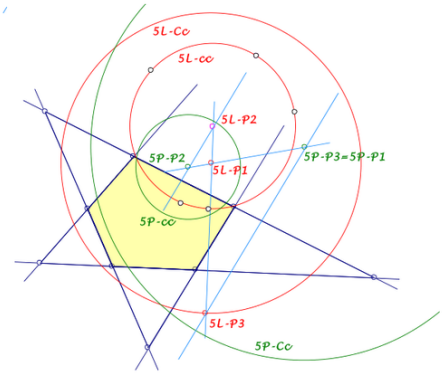


n-lines and their penosculants

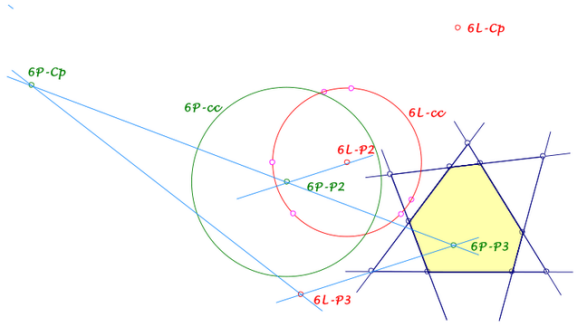
The *n* center-circles *(n-1)L-cc* of *n* lines are the penosculants of a *n*-line: *nL-P3* is the fixed point *P3,* the center-circle *nL-cc* is the fixed circle with center *nL-P2* = *P2*. We can have a look on the following points and circles:

center-circles:	*nL-cc and nP-cc*,
Clifford circles (*n* odd):	*nL-Cc and nP-Cc*,
midpoint Clifford circles (*n* odd):	*nL-P1 and nP-P1*,
Clifford points (*n* even):	*nL-Cp and nP-Cp*,
midpoint center circles:	*nL-P2 and nP-P2*,
intersection of *(n-1)*-center-circles:	*nL-P3 and nP-P3*.

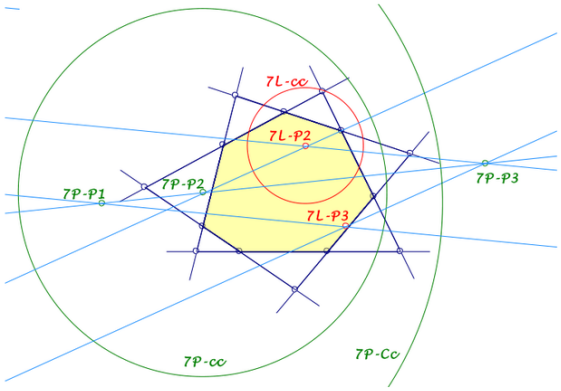
4-line:
4L-cc = *QL-Ci3, 4L-Cp* = *4L-P3* = *QL-P1, 4L-P2* = *QL-P4, 4P-cc* is the line at infinity, so *4P-Cp, 4P-P2, 4P-P3* are not defined.



5-Line
5L-P1, 5L-P2, 5L-P3 are the collinear points as defined in *QFG #710*. *5L-cc* is the *QL-P4*-circle, *5L-Cc* is the *QL-P1*-circle. Already mentioned: *5L-P2.5P-P2* is parallel *5L-P3.5P-P3* (see *QFG #931*). New is the circle *5P-Cc*, centered in *5P-P1* = *5P-P3* collinear with *5L-P1* and *5P-P2*.



6-line
6L-P2.6P-P2 is parallel *6L-P3.6P-P3* (see *QFG #931*).
6L-P2.6P-P3 is parallel *6L-P3.6P-Cp*.
6P-P2, 6P-P3 and *6P-Cp* are collinear.



7-line
7L-P2.7P-P2 is parallel *7L-P3.7P-P3* (see *QFG #931*).
7L-P2.7P-P3 is parallel *7L-P3.7P-P1*.
7P-P1, 7P-P2 and *7P-P3* are collinear.

Final guess
There are limits of CABRI observations, but perhaps holds:

… ***nL-P2.nP-P2*** **parallel** ***nL-P3.nP-P3***
… (***n*** **even)** ***nL-P2.nP-P3*** **parallel** ***nL-P3.nP-Cp***
… (***n*** **nod)** ***nL-P2.nP-P3*** **parallel** ***nL-P3.nP-P1.***

Eckart Schmidt
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2015-01-18.pdf

Message: #938
Date: 19/1/2015 11:18:58
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain for n Lines ...

Dear Bernard, dear Chris,
with the nominations of my last message ...
center-circles: nL-cc and nP-cc,
Clifford circles (n odd): nL-Cc and nP-Cc,
midpoint Clifford circles (n odd): nL-P1 and nP-P1,
Clifford points (n even): nL-Cp and nP-Cp,
midpoint center circles: nL-P2 and nP-P2,
intersection of (n-1)-center-circles: nL-P3 and nP-P3.
... there are further properties:
(1) The inverse of nL-P3 wrt nL-cc is the same as the inverse of nP-P3 wrt nP-cc.
(2) This point is the intersection of the diagonals of the line-penosculant-trapezoid nL-P2.nP-P2.nL-P3.nP-P3 (#931).
(3) A circle round this point through nL-P3 is the circle of inversion for the limacon of the n-line.
(4) For n=5 this point is 5L-P1, for n=6 this point is the center of the 6L-CSC-involution (#922), for n=7 this is a new point.
(5) The 6L-CSC-involution swaps 6L-P3 and 6P-Cp and therefore 6L-P2.6L-P3 and 6P-P2.6P-P3.
Best regards Eckart

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Message: #939
Date: 19/1/2015 6:20:21
From: bernard.keizer
Subject: Center Circles and Clifford's Chain for n Lines ...

Dear Eckart, dear Chris
I try to keep your notations, but I need more points !
Thanks for your patience, I think I've understood now more properties ...
Let's call Ci the 5 center-circles of 4/5 lines (with centers on the center-circle with center P2 and trough the same point P3) and C'i the 5 center-circles of 4/5 penosculants (with centers on the center-circle of the 5 penosculants with center Q2) and through the same point Q3.
Let's call Oij the intersection of Ci and Cj and O'ij the intersection of C'i and C'j (O'ij is the circumcenter of the triangle formed by the 3 points Okl, Okm and Olm). Mi is the common point of the 4 circles with centers Oij to Oim and N'i is the common point to the 4 circles with centers O'ij to O'im.
Let's call M the inverse of P3 wrt the center-circle of the 5-line; it's also the inverse of Q3 wrt the center-circle of the the 5 penosculants
The Clifford circle of the 5-L is the circle with center M through the Miquel points Mi and the point P3 and the Clifford circle of the 5 penosculants is with center Q3 through the points N'i, but not through M.
Nothing new until now !
1) The Clifford circle of the 5-L associated to the 5 penosculant is easily obtained by keeping the system of center-circles of the penosculants (the 10 points O'ij, the 5 points O'i and the point O') and by taking as Clifford circle the circle with center M through Q3, the 2nd intersection with the circle C'i being the Miquel point M'i. The circles with centers O'ij through M'i and M'j give as intersections the 10 vertices A'ij of a new 5-L.
2) *Lets consider the bisector of the angle P2MQ2*
There are a lot of Clawson-Schmidt transformations with this same bisector and different inversion circles, all centered in the point M.
Swapping Q2 and Q3 swaps the Mi and the M'i.
Swapping the 2 center-circles swaps the Oi and the O'i.
M, N'i and O'i are aligned and it's possible to swap the Oi and the N'i.
It's also possible to swap the Oij and the O'ij.
3)We have then a chain 5L - 5 penosculants - 5L - 5 penosculants - 5L ...

(As Oij is not the circumcenter of the triangle formed by the 3 points O'kl, O'km and O'lm, I don't think that the 3rd 5-L will be the same as the 1rst ...).
It's also possible to reverse the operation and to find a system of 5 penosculants for which the initial 5-L will be the associate ...
Best regards
Bernard

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Message: #940
Date: 21/1/2015 5:07:30
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain for n L ...

Dear Bernard,
sorry, again I have great difficulties, to understand your message:
wrt 1): What do you mean with "... the 5-L associated to the 5 penosculants ..."?
wrt 3): I think, the background is an understanding of 1).
Wrt 2) I have tested some CSC-involutions (for n=5, 6, 7) (not always wrt the bisector of the angle P2MQ2).
... center: inverse of nL-P3 wrt nL-cc
    (equal inverse of nP-P3 wrt nP-cc),
... partners nL-P2 <--> nL-P3: nL-cc will be invariant,
... partners nP-P2 <--> nP-P3: nP-cc will be invariant,
... partners nL-P2 <--> nP-P3 or nL-P3 <--> nP-P2:
    nL-cc <--> nP-cc,
... partners (n odd) nL-P3 <--> nP-P1: nL-cc <--> nP-Cc,
... partners (n odd) nP-P1 <--> nP-P3: nP-cc <--> nP-Cc,
... partners (n odd) nL-P2 <--> nP-P1: 5L-P3 <--> 5P-P2,
    5L-cc <--> 5P-cc, 7l-P3 <--> 7P-P3,
... partners nP-P1 <--> nP-P2: 5P-cc invariant, 7P-P3 fix,
... partners nL-P3 <--> nP-P3: 5L-cc <--> 5P-Cc,
    6L-P2 <--> 6P-Cp, 7L-P2 <--> 7P-P1.
Manyfold relationships and confusing! I hope, there are no typos!
Best regards Eckart

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Message: #941
Date: 22/1/2015 7:26:05
From: bernard.keizer
Subject: Center Circles and Clifford's Chain for nL ...

Dear Eckart,
It seems we are converging slowly, but surely!
I refer explicitly to your messages 918, 919, 927, 931 and 934.
In 918, you describe the conic inverse of the limaçon: on page 3, you say the circle of inversion intersects orthogonally the center-circle; what happens if P3 is Inside the center-circle (and P1 or M is outside)? Then you say the conic envelops the perpendicular bisectors of XP3, which seems true; last, you say, the conic is the locus of the intersections of perpendicular bisectors of XP3 and XP2, which is certainly wrong, as this locus would be the perpendicular bisector of P2P3. There is a typo somewhere ...
The 1rst 5-L has as center-circle the circle with center P2, inversion center of P1 and P3, and as Clifford-circle the circle with center P1 through P3.
In 934, you mention the center-circle of the 5 penosculants (with center Q2) and their Clifford circle with center Q3; in 919 and 931, you mention the 2nd 5-L (the lines are the 4P-li) with the same center-circle as the 5 penosculants and with Clifford-circle the circle with center P1 through Q3.
I called this 2nd 5-L the 5-L associated to the 5 penosculants of the 1rst one. Consider now the Clawson-Schmidt transformation which swaps P3 and Q3.
This transformation swaps the 2 Clifford circles with one another and the 5 penosculants (blue on figure in message 934) with the center-circles of 4/5 penosculants (red on the same figure); it swaps therefore Oij and O'ij (as intersections of 2 circles of each group) and Mi and M'i (as intersections between one circle of each group with the corresponding Clifford circle). Last, but certainly not least, it also sawps the 2 limaçons of the message 931.
The beauty of this property is that we only need one copple of conjugates to have the Cl-S; for example, if O12 is the circumcenter of the triangle A34, A35 and A45 (this circle is through the 2 Miquel points M1 and M2), O'12 will be the circumcenter of the triangle O34, O35 and O45; the rest comes easily! By doing this again with O'12 as circumcenter of the triangle O'34O'35O'45, we pursue the chain ...
1rst 5L-5 penosculants - 2nd 5L-5 penosculants - 3rd 5L ...
The point P1 is the same for all the 5L's !
Best regards Bernard

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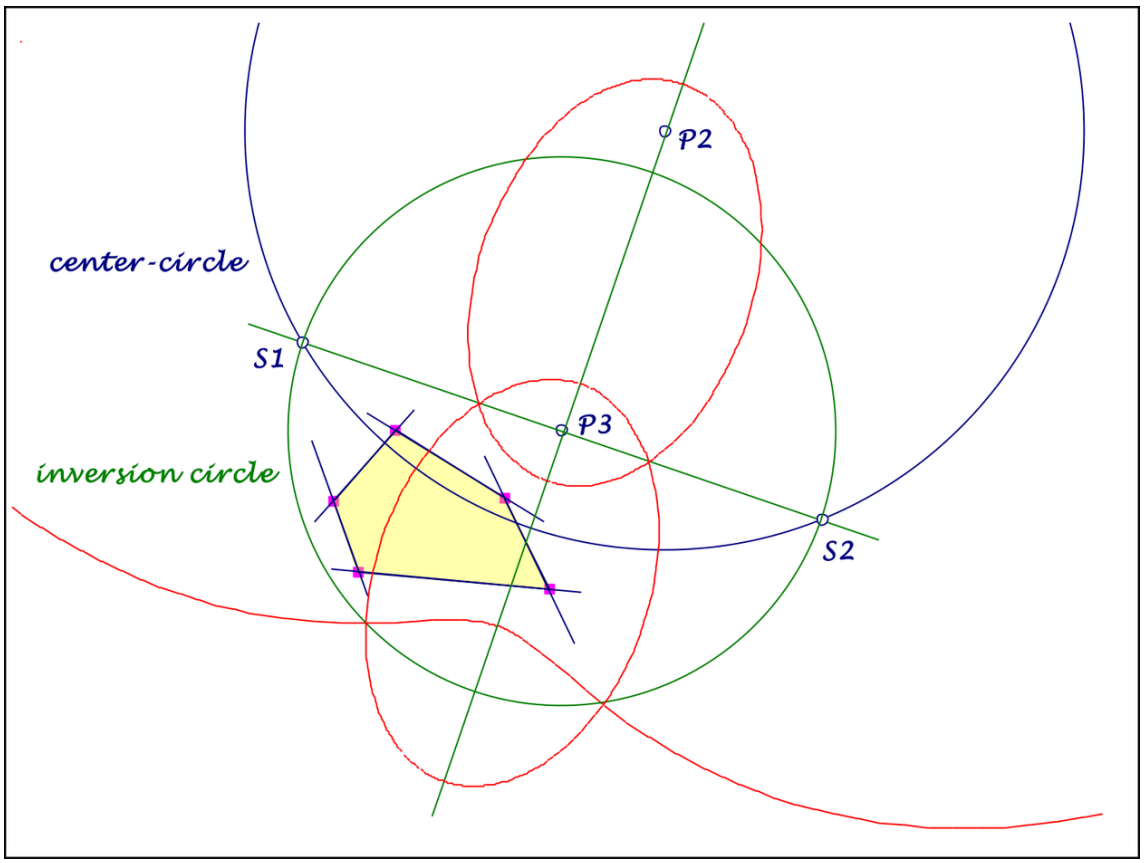
Message: #942
Date: 23/1/2015 10:09:49
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain ...

Dear Bernard,
thanks for your review of my message 918 wrt a conic inverse of the limacon of an n-line.
You are right, my description is only valid, if P3 lies outside the center-circle!
... with a typo as you supposed: ...bisector, not bisectors.
Here a better formulation for the last property:
"The conic is the locus for intersections of X.nL-P2 and the bisector of X.nL-P3."
If P3 lies inside the center-circle, the geometry is another (see attached file):
... Let S1, S2 be the intersections of the center-circle and a perpendicular in P3 wrt P2P3.
... Consider a circle for inversion round P3 through Si.
... The inverse of the Limacon is a conic.
... The reflection of this conic in S1S2 gives a 2nd conic.
... The 2nd conic has the properties, described for the conic in the case P3 outside the center-circle:
... ... foci P2, P3,
... ... envelope of the perpendicular bisectors of P3
and points on the center-circles,
... ... and so on.
There is a further property for these conics:
... Circles through P3, centered on the conic are tangent to the center-circle.
Best regards Eckart
PS: Your further remarks shall be answered in another message, thanks for interest.



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Message: #943
Date: 23/1/2015 12:06:04
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain ...

Dear Bernard,
thank you for an explanation of the 2nd 5-line. Observations with Cabri show, that this is the reference 5-line again. The five 4P-li are the five lines, the ten 3P-pt are the ten vertices of the 5-line and the ten 2P-ci are the circumcircles of the ten 3-lines.
Is there a misunderstanding on my side?
Best regards Eckart

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Message: #944
Date: 23/1/2015 2:19:00
From: chris.vantienhoven
Subject: figures for my previous message

Dear Benedetto,

Thank you very much for your message about “A THEOREM ON QUADRANGLES AND AN APPLICATION TO PENTAGONS” that you presented at the Bloomington meeting in Feb. 2005.
I am sorry I didn’t react earlier. I have been very busy.
In your document I see 2 interesting transformations:
* A Quadrangle Transformation (4P or QA), transforming QA-P2 into QA-P4 and vice versa.
At first I thought it was QA-Tf3 (QA-Orthopole Transformation), but after studying it in detail I concluded it isn’t.
* A Pentangle Transformation (5P), transforming QA-P2 into QA-P4 and vice versa for all component Quadrangles. Because not many 5P-points are known (yet) no examples can be found for the 5P-transformation.
Like you noted the transformations are actually transformations purely dependent on the conic circumscribing the Quadrangle/Pentangle.
That’s why it also could be applied in a dual way to line-configurations for Quadrilateral (4L or QL) and Pentangle (5L), because there is always a conic inscribing a Quadrilateral/Pentangle.
I had a quick view if this gave nice point-to-point relations in a 5L-configuration, but I couldn’t find any. Still I think it is a remarkable transformation and it shows that transformations for conics can be applied to 4P-/5P-/4L-/5L-configurations.

Best regards,
Chris

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Message: #945
Date: 23/1/2015 5:46:34
From: bernard.keizer
Subject: Center Circles and Clifford's Chain ...

Dear Eckart,
This time, it's becoming a kind of a joke !
I gave you all the explanations on your own figures (red circles and blue circles ...).
The 2nd 5-line is certainly not the same as the first as the points P2 and Q2 are not the same as P3 and Q3 !
The center-circles and the Clifford-circles and the limaçons are different too ...
The only same point is the inverse P1.
There must be somewhere a misunderstanding.
Please read again my last message.
Thanks in advance
Best regards
Bernard

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Message: #946
Date: 24/1/2015 10:19:32
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain ...

Dear Bernard,
sorry, I can't laugh, I miss the joke! I cite your message 941:
"The 1rst 5-L has as center-circle the circle with center P2, inversion center of P1 and P3, and as Clifford-circle the circle with center P1 through P3.
In 934, you mention the center-circle of the 5 penosculants (with center Q2) and their Clifford circle with center Q3 ; in 919 and 931, you mention the 2nd 5-L (the lines are the 4P-li) with the same center-circle as the 5 penosculants and with Clifford-circle the circle with center P1 through Q3.
I called this 2nd 5-L the 5-L associated to the 5 penosculants of the 1rst one."
If the 2nd 5-line has the five lines 4P-li, its center-circle is the fixed circle of the penosculants (see #919) and this circle is the center- circle of the reference 5-line and not the center-circle of the five penosculants as you say. The Clifford circle of the 2nd 5-line is the circle of inversion for the limacon (see my # 919) with center P1 through P3 and not through Q3 as you say.
Have you drawn the constellation?
Best regards Eckart

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Message: #947
Date: 24/1/2015 1:46:44
From: Benedetto Scimemi
Subject: figures for my previous message

Dear Chris, thanks for your message and comments.
I can imagine how, busy as you have been and still are with Morley circles, you have found little time for writing me!
As for your 1. and 2. points, only the latter deals with a 5P-transformation, say PA-Tf?, and, as you say, it would be nice to find pairs of transformed central points for pentagons.
But, besides the circumscribed conic center (which is fixed by definition) I could only think of the gravity center G. Its image is a new 5-central point, but until we do not know any other of its properties, this idea makes little sense.
Excellent idea to look for a dual for 5-lines, but for the time being I cannot help at all.
As four your 1. notice that one does not find any QA-Tf unless another point C (the center of the 4P-circumscribed conic) is chosen, and this must lie on the nine-point conic QA-Co1.
Thus a full family of QA-Tf is born, parametrized by C. Indeed my corollaries refer to all significant choices I could imagine for C on QA-Co1.
By the way, I plan to write you more on QA-Tf3 and QA-Tf4.
Ciao Benedetto

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Message: #948
Date: 24/1/2015 2:52:19
From: chris.vantienhoven
Subject: figures for my previous message

Dear Benedetto,
I am looking forward to your further analyses on QA-Tf3 and QA-Tf4.
I love these tranformations very much.
Best regards,
Chis

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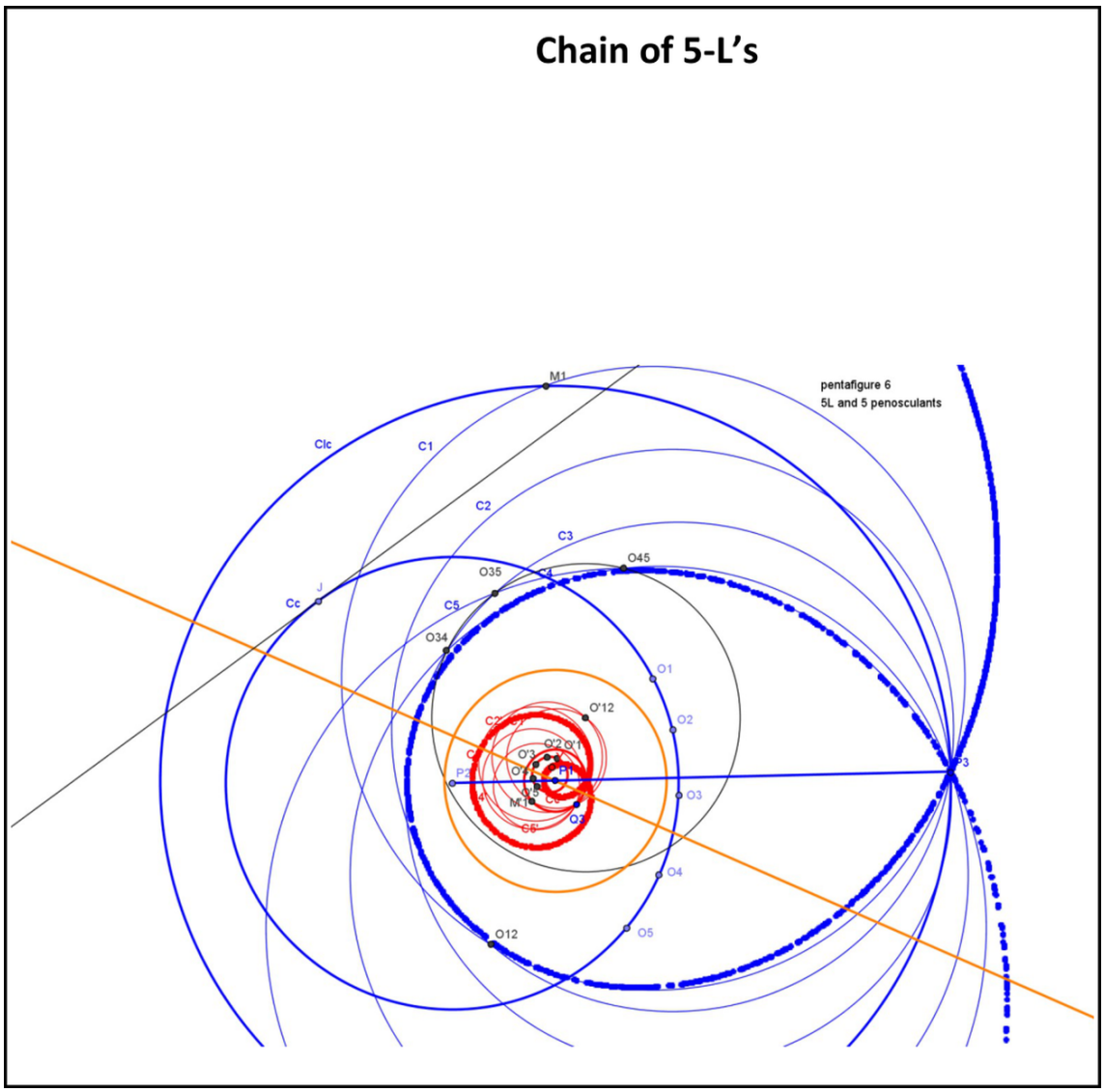
Message: #949
Date: 25/1/2015 5:47:45
From: bernard.keizer
Subject: Center Circles and Clifford's Chain ...

Dear Eckart,
I'm really sad that we aren't always able to understand each other better or quicker ...
I was very grateful to you that you took time to explain your construction, as I didn't get it at the beginning, but now I think I've understood, but it's my turn to have difficulties to explain mine to you.
Of course I made dozens of drawings!
I try again another way and I join a figure wich
looks approximatively like yours your figure in message 934.
Starting with an ordinary 5-L with center-circle centered in P2 and Clifford-circle centered in P1 through P3 inverse of P1 wrt center-circle, let's name Oi the 5 centers of the Miquel circles Ci of the 5 QL's, Oij the intersections of Ci and Cj, centers of the 10 circumcenters of the 10 circumcircles of the triangles formed by 3 lines and Mi the intersections of Ci with the Clifford circle.
Conversely, the 10 circles through the 10 circumcenters Oij through 2 Miquel points Mi and Mj have as intersections the 10 vertices of the 5-L.
The blue circles Ci are the 5 Miquel circles of the 1rst 5-L.
Let's now draw as you do on the figure 934 the red circles or center-circles for 4 of 5 penosculants ; it gives the center-circle of 5 penosculants with center P3 and the Clifford-circle of the 5 penosculants with center Q3.
Let's now consider these 5 red circles as the Miquel circles of the 2nd 5-L and the Clifford circle as the circle with center P3 through Q3.
The Clawson-Schmidt transformation which swaps P3 and Q3 swaps also the 5 blue center-circles for 4/5 lines with the 5 red center-circles for 4/5 circles and the blue Clifford circle with the red and therefore all the blue intersections with the red ones: Oij with O'ij and Mi with Mj. Last but not least, it swaps also the blue limaçon with the red one!
Now, it's easy to see that O'ij is the circumcenter of the triangle formed by the intersections of the 3 circles omitting Ci and Cj and one copple (O12 and O'12 on my figure) gives the Cl-S transformation.
The circles through the 10 circumcenters O'ij through the 2 Miquel points M'i and M'j intersect in the 10 vertices of the 2nd 5-L. I hope this will be a little bit clearer.
Best regards
Bernard



Chain of 5-L's.docx

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Message: #950
Date: 26/1/2015 1:51:57
From: bernard.keizer
Subject: Center Circles and Clifford's Chain ...

```
Dear Eckart,
I realise I forgot to put on the figure the other
Clifford-circle (of the 5 penosculants) to follow completely
your figure in message 934.
It's done now !
Best regards
Bernard
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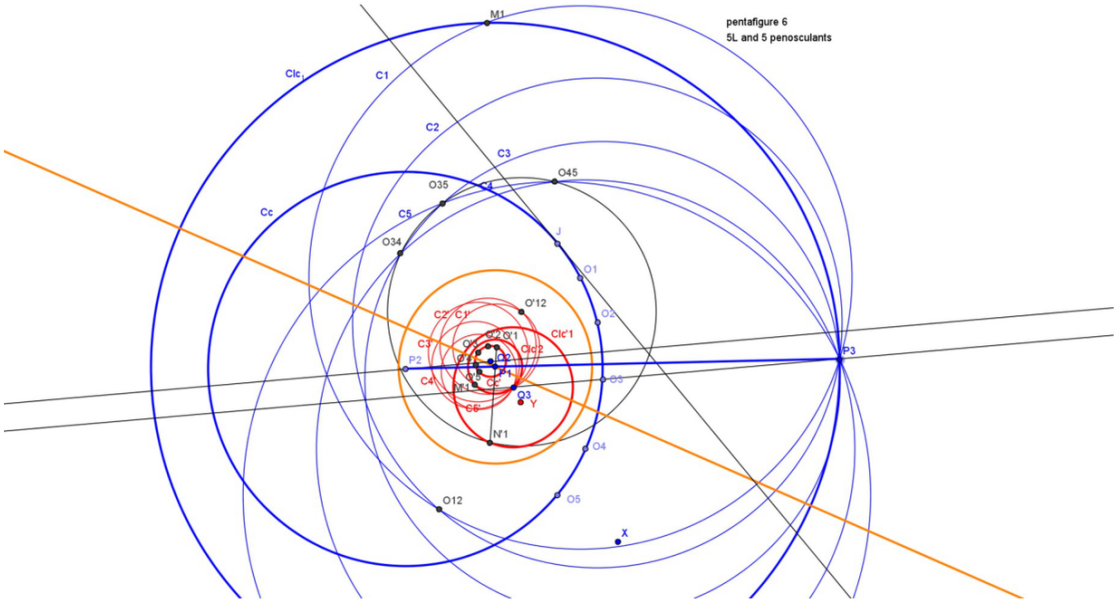
Chain of 5-L's



In blue : the 5 Miquel circles of the 1rst 5-L or penosculants C1 to C5

the center-circle Cc of the 1rst 5-L

the Clifford-circle Cl-c of the 1rst 5-l

In red : the center-circles for 4/5 penosculants, which are also the 5 Miquel circles of the 2nd 5-L

C'1 to C'5, inverse of the C1 to C5

the center-circle of the 5 penosculants Cc', which is also the center-circle of the 2nd 5-L

the Clifford-circle Cl-c'1 of the 5 penosculants, with center Q3 ; it's homothetic from the center-circle Cc' in the homothety with center P1 which swaps Q2 in Q3

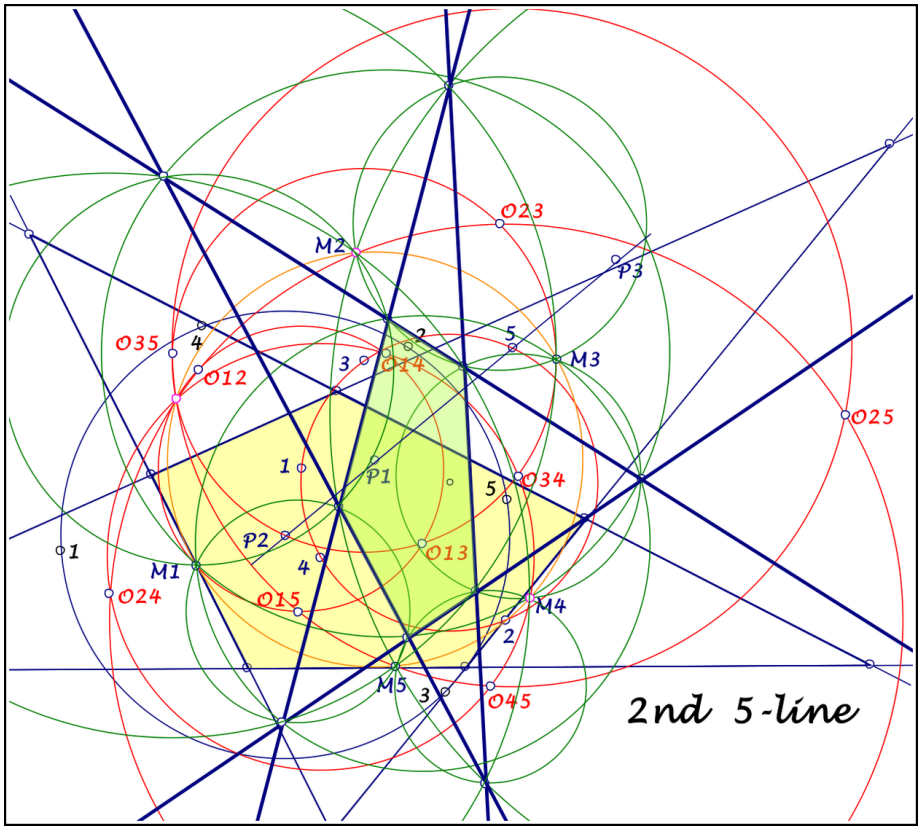
the Clifford-circle Cl-c'2 of the 2nd 5-L with center P1 through Q3

Chain of 5-L's-950.docx

Message: #951
Date: 26/1/2015 9:16:39
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain ...

Dear Bernard,
thanks for explanation again! Perhaps I have understood.
The 2nd 5-line is another as that described in #941 (wrt the lines 4P-li).
I think, you use the fact, that a 5-line is completely determined by its center-circles and its Clifford circle. That is your description with Oi,Oij and Mi.
Taking for center-circles the center-circles of the 5 penosculants and for Clifford circle the Clifford circle of the 5 penosculants, you get the 2nd 5-line.
I have made a construction, see attached file.
I think, you will look through the chaos.
Now I shall study this 2nd 5-line.
Best regards Eckart

PS. Wrt your last message: We use different Pi, Qi! In line 10 and 20 I think "... circles round (not: through) the 10 circumcenters ..." would be right.



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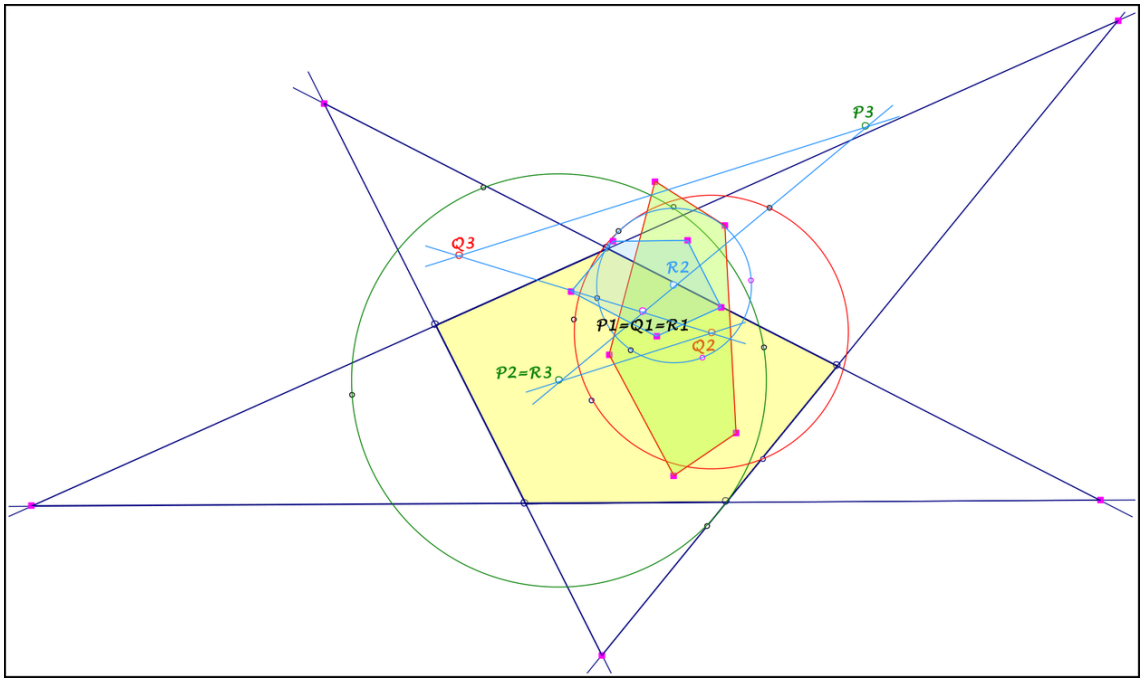
Message: #952
Date: 26/1/2015 11:39:01
From: eckart_schmidt@t-online.de
Subject: Center Circles and Clifford's Chain ...

Dear Bernard,
excuse, there is something not quite correct in my last message, better:
" ... that a 5-line is completely determined by the center-circles QL-Ci3 for 4 of 5 lines."
Their common point is P3, the center of their midpoints is P2 and the inverse of P3 wrt the circumcircle of their midpoints is P1.
Let Q1, Q2, Q3 be the analog points for the 5 penosculants and for the 2nd 5-line, R1, R2, R3 for the 3rd 5-line, ...
Then holds:
... P1 = Q1 = R1 = ...,
... P2 = R2,
... the 3rd 5-line is homothetic wrt the 1st 5-line wrt ratio -P1.P2 : P1.P3.
You can get the next 5-line by taking the 4P-li (see #919) of the center-circles of the preceding 5 penosculants.
(My first interpretation ... taking the 4P-li of the preceding 5 penosculants ... wrt your #941 was not right.)
Best regards Eckart



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← Previous Next → ↩ Message Index ↑ Subjects

Message: #953
Date: 26/1/2015 5:53:44
From: bernard.keizer
Subject: Center Circles and Clifford's Chain ...

Dear Eckart,
I'm really glad that we understand eachother again !
Nice property that the 3rd 5-L is homothetic to the 1rst one.
As P3Q3 is parallel to P2Q2 and the same way Q3R3 to Q2R2, with P2 = R3, this looks like the beginning of a beautiful cobweb, rapidly converging in the point P1 = Q1 = R1 = ...
In the Cl-S transformation, the bisector is the same and the 2 inversion circles have the same center and the product of the 2 Cl-S gives a homothety.
Best regards
Bernard

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Message: #954
Date: 26/1/2015 6:29:49
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris, dear Eckart
Many topics have been now described on the 5-L. An important one remains unsolved.
I'm always looking for the 64 axes of the QL.
Have you tried again to read the last Morley's article ?
I have tried many constructions, but I'm not able to find what he meant.
It must be relatively simple, as he writes " All we need, apart of the euclidian geometry is the n-section of the angles "
Every angle has 2 axes, the 2 bisectors and their direction is the mean direction of the 2 lines m° pi/2
Every triangle has 9 axes, the Morley lines, obtained by trisecting the 3 angles and the direction of the 9 axes is the mean direction of the 3 lines m° pi/3. Every angle has 6 trisectors and the Morley lines can be obtained by combining intersections of the trisectors.
According to Morley, every QL must have 64 axes, obtained by quadrisecting each angle and the direction must be the mean direction of the 4 lines m° pi/4. Now, every angle has 8 quadrisectors and the axes must be obtained by combining intersections of the quadrisectors.
I tried for example to draw the intersections of the quadrisectors of the 3 copples of 2 opposite vertices of the QL, which gives 3* 64 points and to search if by any chance 3 points with one in each group could be on a line of one of the 4 given directions. Maybe you have had some kinds of ideas or different ones ...
As Cabri is certainly more powerful than Geogebra, I hope that some of these approachs could one day succeed on your side!
Then the intersections of the 5*64 axes for the 5-L should give the 256 centers of the tetracardioïds tangent to the 5 lines (one center intersection of 5 axes, one of each set of 64, and every axis carrying 4 centers).
And as the 4 in- and excenters, centers of the 4 circles tangent to 3 lines, are on rectangular hyperbola, which is a conic stelloïd, and the 27 centers of the 27 cardioïds tangent to 4 lines, I'm always convinced that the 256 centers of the tetracardioïds tangent to the 5 lines will be on a quartic stelloïd ...
Best regards
Bernard

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Message: #955
Date: 26/1/2015 7:37:54
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard,
I agree with you. We should find the 64 axes and the 256 centers in a 5-Line as described by Morley.
I already tried 4-secting all angles in a 4-Line several times without finding any line through more than 2 intersection points.
As for now I am very busy writing a first start of the new pioneering Encyclopedia of Polygon Figures as a result of our joint efforts.
When I wouldn't continue now it probably would never come to an end.
So I continue doing so and please forgive me not reacting too much.
Of course I am looking to all interesting messages of our group.
Best regards,
Chris

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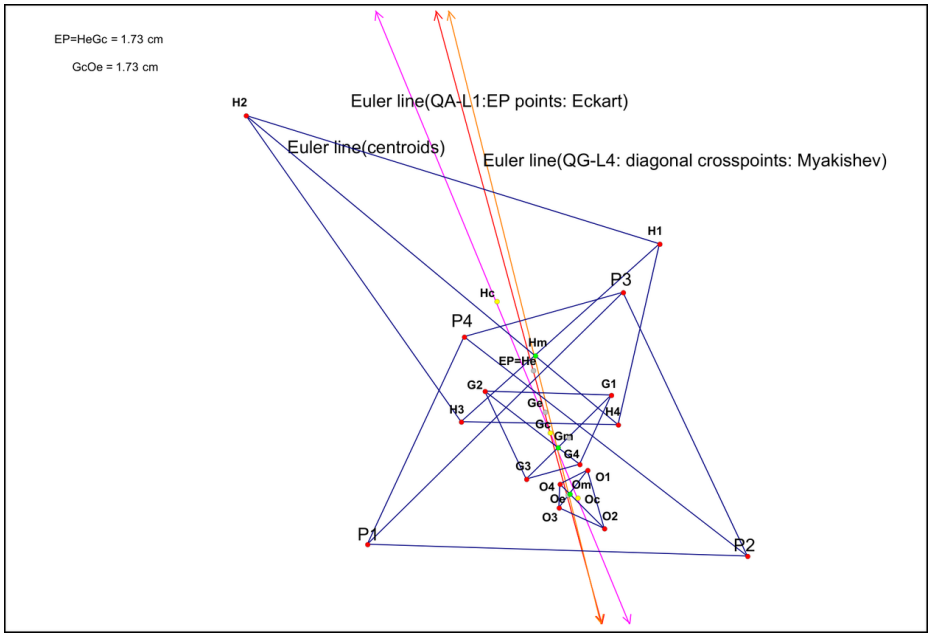
Message: #956
Date: 29/1/2015 10:17:11
From: Seiichi Kirikami
Subject: Some Euler lines

Dear Chris, dear Angel,
Thank you very much for the information.
I must correct Anopolis message #2371 †) as follows:
(1) http://forumgeom.fau.edu/FG2006volume6/FG200634.pdf (Alexei Myakishev, On Two Remarkable Lines Related to a Quadrilateral). QG-L4 in EQF. His Euler line is through the diagonal crosspoints of quadrangles made from 4 centroids, 4 circumcenters, and 4 orthocenters of its component triangles.
(2) http://eckartschmidt.de/ (Eckart Schmidt, Themen 2004/04-3/ Euler-Gerade eines Vierecks). QA-L1 in EQF.
His Euler line is through Euler-Poncelet points of quadrangles made from 4 centroids, 4 circumcenters, and 4 orthocenters of its component triangles.
(3) There is another Euler line, which is through the centroids of quadrangles made from 4 centroids, 4 circumcenters, and 4 orthocenters of its component triangles.adrangles. Is this Euler line already described in EQF?
See the attached picture. I think that Angel's 1st and 2nd computation used (3) and (1) respectively.
P.S. The description of QG-L4 in EQF mentions Ref-15i, 4th line from top. I think that this mention is not necessary.
Best regards, Seiichi.



QFEulerLines.docx

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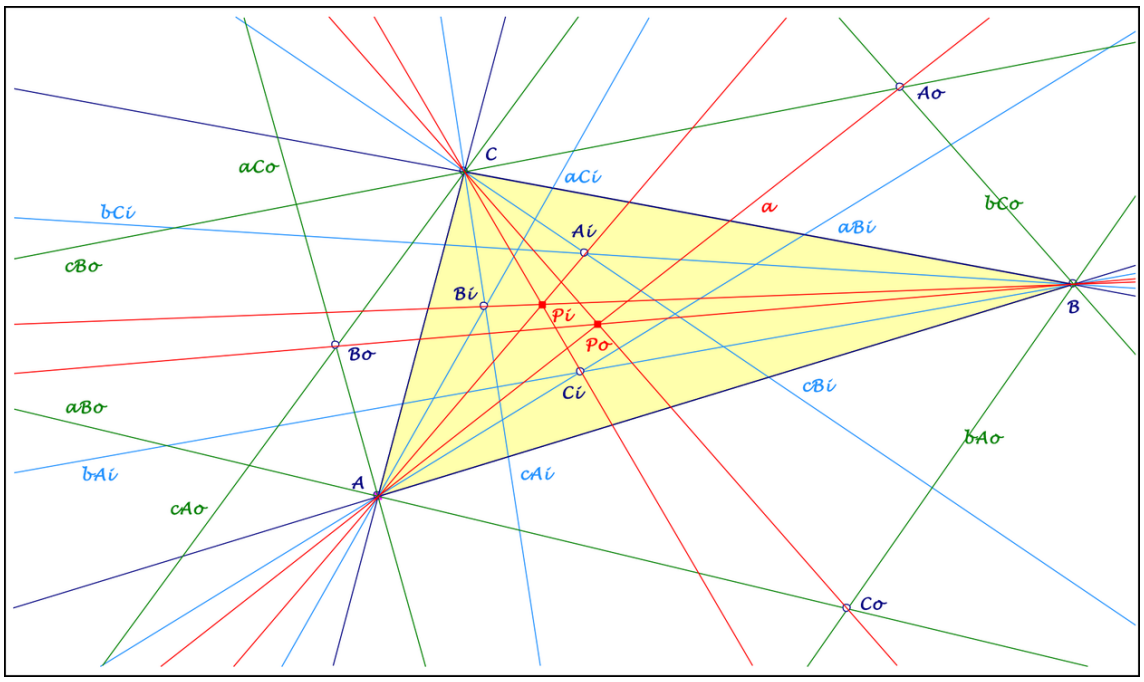
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #957
Date: 29/1/2015 1:18:05
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,dear Chris,

I can offer a configuration of 4 x 16 parallels for a 4-line
connected with quadrisecting angles, perhaps Morley's 64 axes.
... The direction of the axes must be the mean direction
    of the 4 lines m° pi/4 (Morley).
... A 4-line has four 3-lines. Every 3-line gives
    4 special points (see below).
... Lines through these points with the 4 directions
    are perhaps the axes.
The special 4 points of a triangle ABC (see attached file):
... Consider inside (index i) and outside (index o) the
    angle-quadrisectors without the angle-bisectors.
... Let aBi / aBo be the quadrisectors through A near B
    and aCi / aCo the quadrisectors through A near C, ...
... Let Ci be the intersection of aBi and bAi,
    Co the intersection of aBo and bAo, ...
... The lines A.Ai, B.Bi, C.Ci intersect in Pi
    and the lines A.Ao, B.Bo, C.Co intersect in Po.
... Analog: Let Ci' be the intersection of aCi and bCi,
    Co' the intersection of aCo and bCo, ...
... The lines A.Ai', B.Bi', C.Ci' intersect in Pi'
    and the lines A.Ao', B.Bo', C.Co' intersect in Po'.
... Pi, Po, Pi', Po' are the four special points of a triangle.
I need your advice: How can I prove, whether these lines are
Morley's axes?

Best regards Eckart



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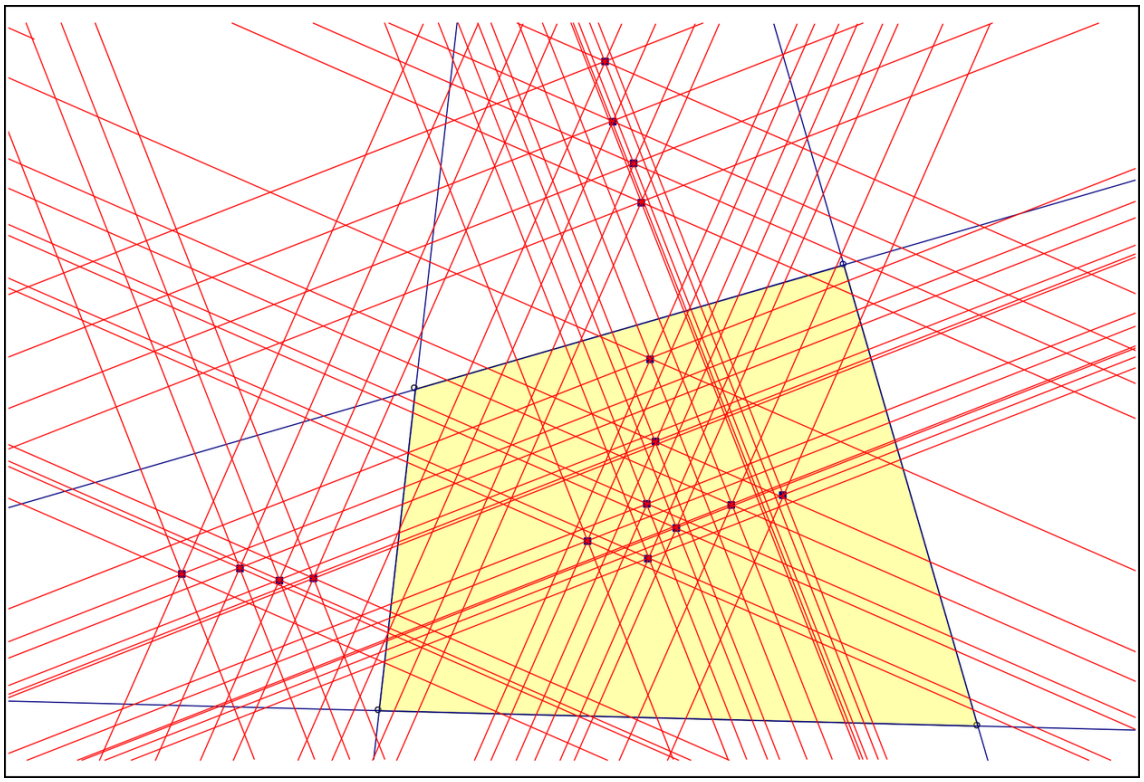

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Message: #958
Date: 29/1/2015 3:39:39
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Eckart,

You can check the legitimacy of your proposal by constructing in a 5-line all axes from the five 4-Lines:
“that is the 5x64 axes of the component 4-lines (should) meet in the 256 incenters,
there being 5 axes on a point and 4 points on an axis.”
(quote from Morley’s ‘Extensions of Clifford’s Chain Theorem’, page 470)
Hard work, but it seems worthwhile to me!

Best regards,
Chris

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Message: #959
Date: 29/1/2015 8:01:11
From: chris.vantienhoven
Subject: Some Euler lines

Dear Seiichi,

The Eulerline through the centroids of quadrangles made from 4 centroids, 4 circumcenters, and 4 orthocenters of its component triangles is the line through resp. QA-P1, QA-P32 and QA-P33.
I think this another nice Quadri-Eulerline to investigate wrt your earlier question.

Best regards,
Chris

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Message: #960
Date: 30/1/2015 8:59:17
From: eckart_schmidt@t-online.de
Subject: 64 axes for the QL

```
Dear Bernard, dear Chris,

excuse, forget my last message, it wasn't a good idea, for there
is no generalization.

Best regards Eckart
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Message: #961
Date: 30/1/2015 12:19:37
From: bernard.keizer
Subject: 64 axes for the QL

```
Dear Eckart,

I had in fact some doubts with your construction.
But the fact that the lines AAi, BBi and CCi converge in a point
Pi for any n-section remains really interesting !
For n = 2, Pi and Po are both the incenter.
For n = 3, AiBiCi is the 1rst Morley triangle and Pi and Po
cannot be on the 27 axes.
But the fight goes on and I'm glad you joined this battle !

Best regards
Bernard
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Message: #962
Date: 08/2/2015 6:11:19
From: seiichikiri
Subject: Orthologies among quadrigonal triangles

Dear Chris,

The following triangles of 3 quadrigonal points seem orthologic.
1) OGP1 and OGP4,
2) OGP1 and OGP5,
3) OGP4 and OGP5,
4) OGP4 and OGP6,
Have you ever studied orthologies among quadrigonal triangles?

Best Regards,
Seiichi.

P.S. The diagonal triangle (OGP1) and the triangle of the symmedian points Ki of PjPkPl are not orthologic.

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Message: #963
Date: 08/2/2015 9:44:43
From: chris.vantienhoven
Subject: Orthologies among quadrigonal triangles

Dear Seiichi,
Nice to hear some Quadrangle news from you again!
About your question if I ever studied orthologic questions among quadrigonal triangles, I have to admit I didn't.
At first I didn't quite understand what you meant with "triangles of 3 quadrigonal points".
After studying it a bit I understand that you mean the Triple Triangles as described in QA-1:
Systematics for describing QA-points.
They are triangles formed by constructing points of Quadrigons or Quadrilaterals in a Quadrangle.
Since there are 3 versions of them they form a triangle.
Mathworld gives this definition of Orthologic Triangles:
Two triangles A1B1C1 and A2B2C2 are orthologic if the perpendiculars from the vertices A1, B1, C1 on the sides B2C2, A2C2, and A2B2 are concurrent.
Furthermore, if this is the case, then the perpendiculars from the vertices A2, B2, C2 on the sides B1C1, A1C1, and A1B1 are also concurrent, as shown by Steiner in 1827.
The point of concurrence is known as the orthology center of A1B1C1 with respect to A2B2C2.
I determined the Orthology Centers from your examples.

OGP1 and OGP4
Orthology Center of QG-P1-Triple Triangle wrt QG-P4-Triple Triangle = QA-P12.
Orthology Center of QG-P4-Triple Triangle wrt QG-P1-Triple Triangle = QA-P1.QA-P12 ^ QA-P5.QA-P11.

OGP1 and OGP5
Orthology Center of QG-P1-Triple Triangle wrt QG-P5-Triple Triangle = QA-P20.
Orthology Center of QG-P5-Triple Triangle wrt QG-P1-Triple Triangle = QA-P13.QA-P40 (3:-2) being the AntiComplement of QA-P13 wrt the QL-P6-Triple Triangle as well as the QG-P9-Triple Triangle.

OGP4 and OGP5
Orthology Center of QG-P4-Triple Triangle wrt QG-P5-Triple Triangle = QA-P1.QA-P5 (1:2)

Orthology Center of QG-P5-Triple Triangle wrt QG-P4-Triple Triangle = QA-P13.QA-P40 (3:-2) being the AntiComplement of QA-P13 wrt the QL-P6-Triple Triangle as well as the QG-P9-Triple Triangle.

OGP4 and OGP6
Orthology Center of QG-P4-Triple Triangle wrt QG-P5-Triple Triangle = QA-P15.QA-P24 (-1:4)
Orthology Center of QG-P5-Triple Triangle wrt QG-P4-Triple Triangle = QA-P24

I feel pretty sure there are more of these Orthologic Triangles examples.
Maybe you or someone else has the opportunity to look for those examples.
Your last remark: “The diagonal triangle (OGP1) and the triangle of the symmedian points Ki of PjPkPl are not orthologic.” I do not understand.

Best regards,
Chris

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Message: #964
Date: 09/2/2015 3:59:20
From: seiichikiri
Subject: Orthologies among quadrigonal triangles

Dear Chris,
Sorry for the wrong terminology.
QG-P1-Triple Triangle and QG-P7-Triple Triangle are orthologic.
QG-P1-Triple Triangle and Triple Triangle of 1st Quasi Symmedian point are not orthologic. This is what I meant in P.S. and may show the importance of QG-L4.
I think that there are things or properties to be found in QF.
Best regards,
Seiichi.

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Message: #965
Date: 09/2/2015 6:08:51
From: seiichikiri
Subject: Cyclology of QG-P1 and P15-Triple Triangles

Dear Chris,
QG-P1-Triple Triangle is congruent to QG-P15-Triple Triangle in rotation.
So, according to Hyacinthos message #9381 †), they are cyclologic.
I hope that 2 cyclologic centers will be determined.
Best regards,
Seiichi.

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #966
Date: 09/2/2015 7:41:52
From: chris.vantienhoven
Subject: Cyclology of QG-P1 and P15-Triple Triangles

Dear Seiichi,
I recollect the definition of Cyclologic from Hyacinthos #9381 and #22721 †):
Triangles ABC and A'B'C' are *cyclologic* when circles (AB'C'), (BC'A'), (CA'B') and also the circles (A'BC), (B'CA), (C'AB) are concurrent.
The example you choose was well chosen:
The Cyclologic Center of the QG-P1 Triple Triangle wrt the QG-P15 Triple Triangle is QA-P2.
The Cyclologic Center of the QG-P15 Triple Triangle wrt the QG-P1 Triple Triangle is QA-P3.
Do you have more examples of Cyclologic Triangles?
Especially when they are not homothetic.
Best regards,
Chris

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Message: #967
Date: 09/2/2015 10:34:20
From: Antreas Hatzipolakis
Subject: Cyclic quadrangle centers on the circumcircle

Which centers we know (or could find) lying on the circumcircle of cyclic quadrangle?
Such a center has no interest from the view of Quadri-Geometry (since we want centers of general quadrangles), but we can use it in quadrilateral's Miquel cyclic quadrangle ie the quadrangle formed with the four circumcenters of the component triangles of the quadrilateral.
aph

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #968
Date: 09/2/2015 3:22:41
From: chris.vantienhoven
Subject: Cyclic quadrangle centers on the circumcircle

Dear Antreas,
There are several cases of 4 concyclic points in EQF.
The easiest way to find them is by going to the SEARCH option
and search on keyword "concyclic".
What application did you have in mind?
Best regards,
Chris

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Message: #969
Date: 09/2/2015 7:11:46
From: Antreas Hatzipolakis
Subject: Cyclic quadrangle centers on the circumcircle

Dear Chris,
I asked for centers on the circumcircle of a cyclic quadrangle, but in fact we can ask same question for any center of cyclic quadrangle.
Let me explain with an example:
Let ABCD be a cyclic quadrangle and Sa, Sb, Sc, Sd the Simson lines of A,B,C,D wrt triangle BCD,CDA, DAB, ABC resp.
Sa. Sb, Sc, Sd are concurrent.
Call the point of concurrence as S-point of the cyclic quadrangle.
Now, we can apply this to any cyclic quadrangle of a quadrilateral or quadrangle.
For example: Which is the S-point of the cyclic quadrangle formed with the circumcenters of the four component triangles of a quadrilateral?
APH

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Message: #970
Date: 09/2/2015 9:50:57
From: chris.vantienhoven
Subject: Cyclic quadrangle centers on the circumcircle

Dear Antreas,
[AH] Which is the S-point of the cyclic quadrangle formed with the circumcenters of the four component triangles of a quadrilateral?
In this case the S-point is QL-P5: Clawson Center.
When you take the Component Orthocenters of the Quadrangle formed by the 4 Circumcenters you mentioned then there is another cyclic Quadrangle again with S-point QL-P5.
Nice Cyclic-Simsonline-property! I will also keep it in mind for future cases.
Best regards,
Chris

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Message: #971
Date: 10/2/2015 1:39:26
From: amontes1949
Subject: Cyclic quadrangle centers on the circumcircle

Dear Chris,
Antreas Notation:
Let ABC be a triangle and D on the circumcircle.
Sa, Sb, Sc, Sd the Simson lines of A,B,C,D wrt triangle BCD,CDA, DAB, ABC resp.
Sa, Sb, Sc, Sd are concurrent. Call the point of concurrence as S-point of the cyclic quadrangle ABCD.
*** The S-point of the cyclic quadrangle ABCD is the anticomplement of the antipode of D.
Two immediate examples in Encyclopedia of Quadri-Figures:
The S-point of the cyclic quadrangle formed with S1, S2, S2 (the vertices of the QA-Circumcircle Diagonal Triangle) and QA-P2 is QA-P36=Compl. of QA-P30 wrt QA-Diagonal Triangle.
The S-point of the cyclic quadrangle formed with S1, S2, S2 (the vertices of the QA-Circumcircle Diagonal Triangle) and QA-P30 is QA-P29= Compl. of QA-P2 wrt QA-Diagonal Triangle.
Best regards,
Angel M.

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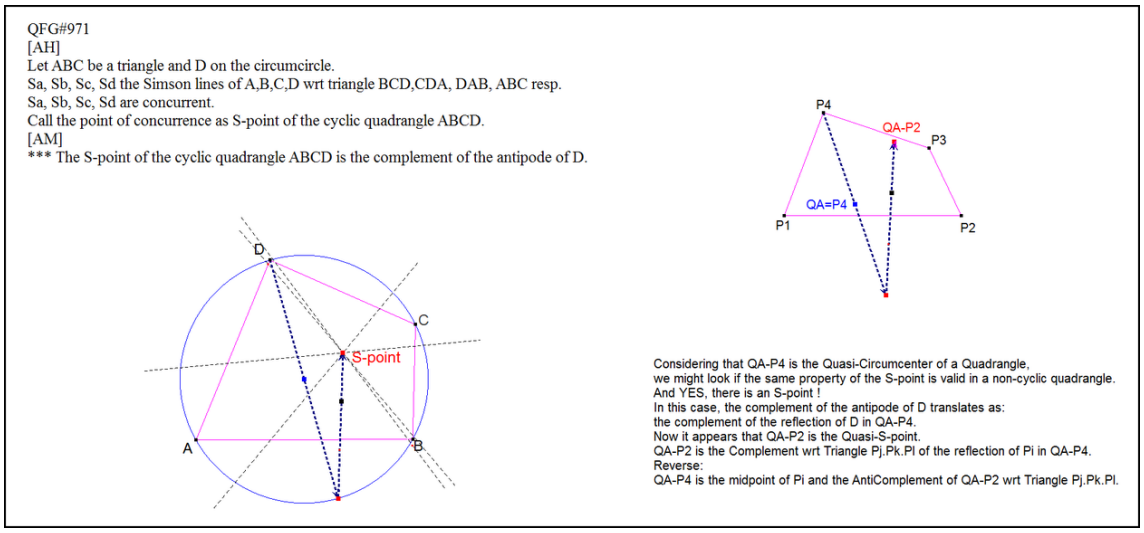
Message: #972
Date: 10/2/2015 9:22:44
From: Chris van Tienhoven
Subject: Cyclic quadrangle centers on the circumcircle

Dear Angel,
[AM]
*** The S-point of the cyclic quadrangle ABCD is the anticomplement of the antipode of D.
Very nice observation!
(I suppose you mean the complement instead of the anticomplement)
Considering that QA-P4 is the Quasi-Circumcenter of a Quadrangle, we might look if the same property of the S-point is valid in a non-cyclic quadrangle. And YES, there is an S-point !
The complement of the antipode of D translates as the complement of the reflection of D in QA-P4.
Now it appears that QA-P2 is the Quasi-S-point.
QA-P2 is the Complement wrt Triangle Pj.Pk.Pl of the reflection of Pi in QA-P4.
Reverse:
QA-P4 is the midpoint of Pi and the AntiComplement of QA-P2 wrt Triangle Pj.Pk.Pl.
See attachment.
Best regards, Chris
p.s. The Quadri-Figures Group works perfect!
Seiichi starts a discussion on orthologic and cyclologic triangles.
Antreas continues on cyclic quadrangles.
Angel observes special property.
And then there is a generalization, etc. What's next . . . .



QA-P2-S-point-property-01.png

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Message: #973
Date: 10/2/2015 9:39:57
From: Antreas Hatzipolakis
Subject: Cyclic quadrangle centers on the circumcircle

Dear Chris,
I was wondering who discovered the Simson lines theorem in the cyclic quadrangle. This is what I found but probably the theorem is older.
1507. The Simson Lines of a Cyclic Quadrilateral
R. F. Cyster
The Mathematical Gazette Vol. 25, No. 263 (Feb., 1941), pp. 56-58.
See
http://www.jstor.org/discover/10.2307/3606490?sid=21105827958553&uid=70&uid=2&uid=2129&uid=4&uid=3738128
APH

1507. *The Simson lines of a cyclic quadrilateral.*
If P, Q, R, S are the mid-points of the sides of the cyclic quadri-

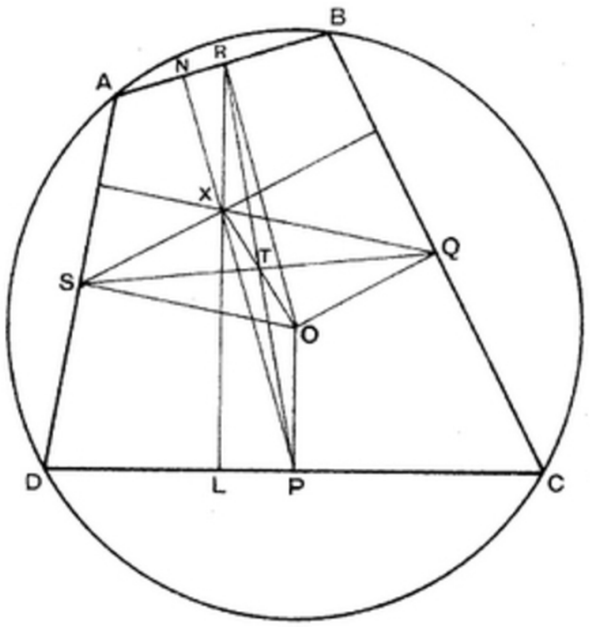

Fig. 1.

lateral $ABCD$, the following propositions can be proved by elementary geometry :

(*a*) the four perpendiculars from P, Q, R, S to the opposite side are concurrent at a point X ;
(*b*) the Simson line of A with respect to the triangle BCD and the other three lines similarly obtained are concurrent at X ;
(*c*) if the straight lines PR, QS intersect at T, then T is the mid-point of OX.

simson.jpg

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Message: #974
Date: 10/2/2015 10:03:17
From: Chris van Tienhoven
Subject: Cyclic quadrangle centers on the circumcircle

```
Dear Antreas,
I have no idea. Maybe someone else in our group knows?
Best regards,
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Message: #975
Date: 10/2/2015 10:29:39
From: amontes1949
Subject: Cyclic quadrangle centers on the circumcircle

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[Chris]:
I suppose you mean the complement instead of the anticomplement
Of course you are right.
My apology
Angel M.
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Message: #976
Date: 10/2/2015 10:42:34
From: Antreas Hatzipolakis
Subject: Cyclic quadrangle centers on the circumcircle

```
Dear Chris,
The Simson lines concur at Poncelet - Euler point of the c.
quadrangle.
http://www.cut-the-knot.org/Curriculum/Geometry/FourSimsons.shtm
l
aph
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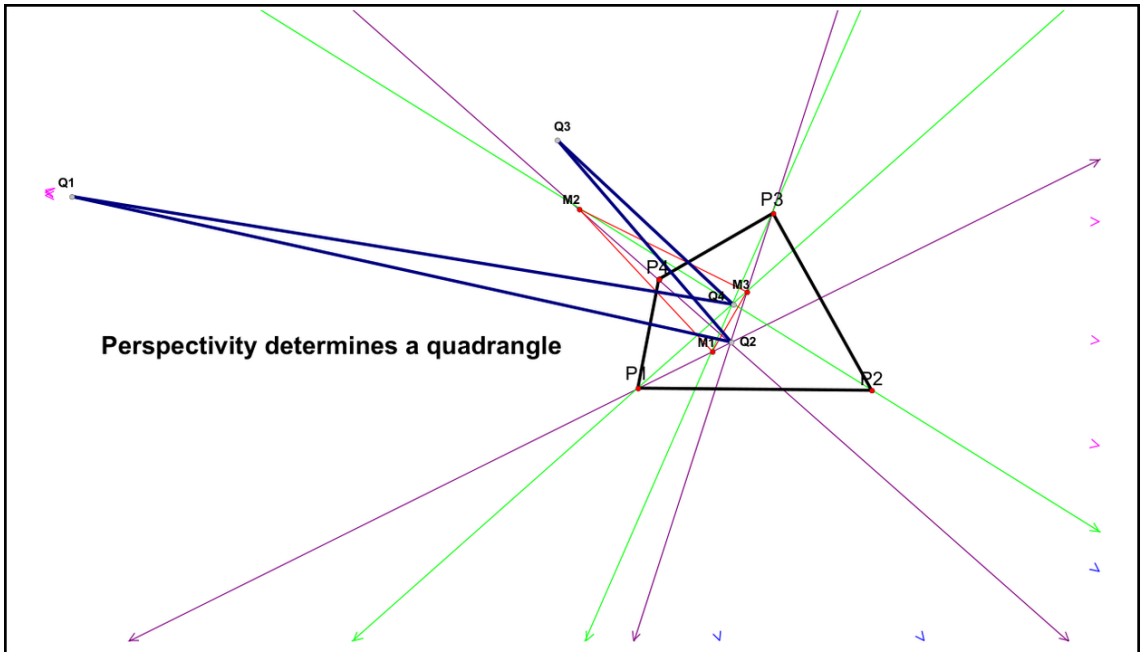
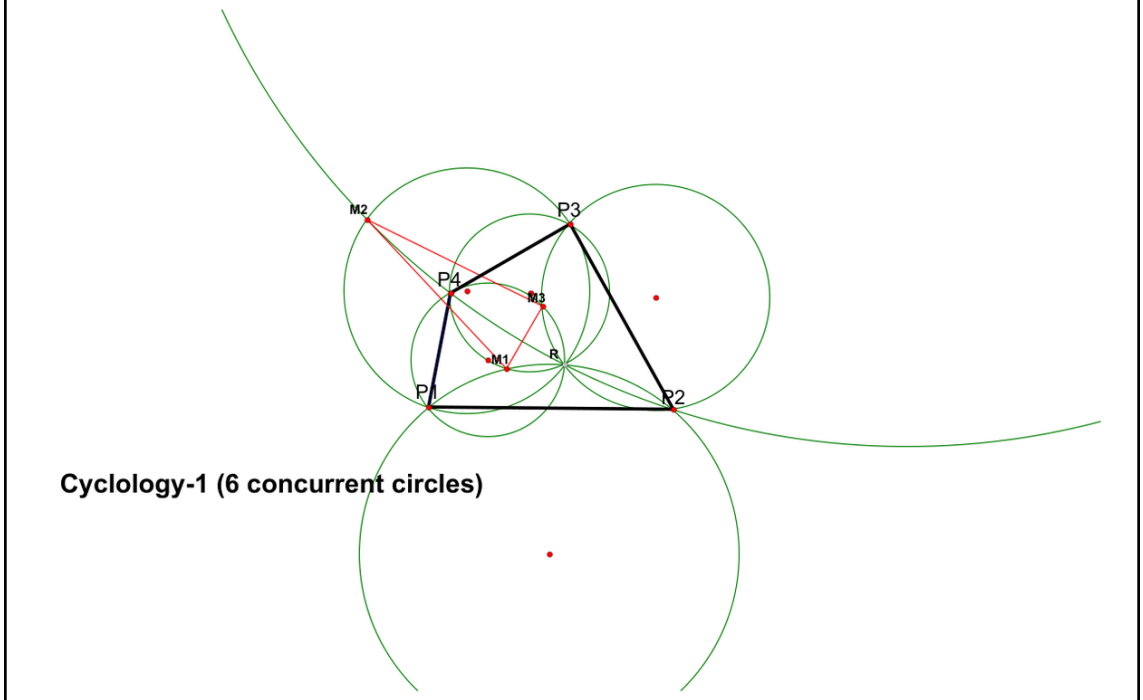
Message: #977
Date: 11/2/2015 6:37:48
From: Seiichi Kirikami
Subject: Miquel points generate another quadrangle

Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
M1= Miquel point of P1P2P3P4,
M2= Miquel point of P1P3P2P4,
M3= Miquel point of P1P2P4P3,
as shown in QA-P9 in EQF.
1) M1M2M3 and P2P3P4 are perspective in a point Q1.
2) M1M2M3 and P1P4P3 are perspective in a point Q2.
3) M1M2M3 and P4P1P2 are perspective in a point Q3.
4) M1M2M3 and P3P2P1 are perspective in a point Q4.
Q1Q2Q3Q4 determines a quadrangle.
See the attached picture.
5) M1M2M3 and P2P3P4 are cyclologic in points R(QA-P4) and S1.
6) M1M2M3 and P1P4P3 are cyclologic in points R(QA-P4) and S2.
7) M1M2M3 and P4P1P2 are cyclologic in points R(QA-P4) and S3.
8) M1M2M3 and P3P2P1 are cyclologic in points R(QA-P4) and S4.
See the attached picture.
P.S. Chris’s request (message #966) led me to the above, though it is a bit different from what he meant.
Best regards,
Seiichi.





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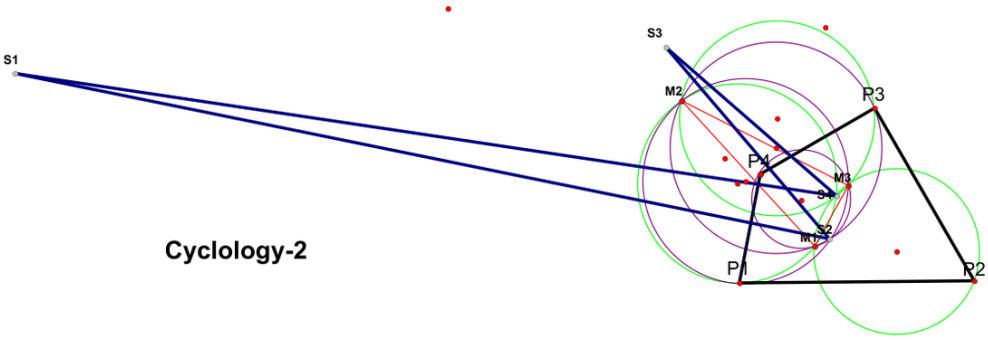
Perspectivity and cyclology.docx



2

Perspectivity and cyclology.docx

Message: #978
Date: 11/2/2015 10:36:02
From: Chris van Tienhoven
Subject: Miquel points generate another quadrangle

Dear Seiichi,
Remarkable construction!
It is very special that the Miquel Triangle (QA-Tr2) is perspective with all Component Triangles of the Reference Quadrangle. I wasn't aware of that.
Here are some remarks.

Collinearities
I made a diagram of the collinearities involved:

	M1	M2	M3	M0
Q1	P3	P2	P4	P1
Q2	P4	P1	P3	P2
Q3	P1	P4	P2	P3
Q4	P2	P3	P1	P4

where
M0 = Infinity Point QA-P3.QA-P4 = Infinity Point asymptote QA-Cu1.
M1 = Miquel Point (QL-P1) of Quadrigon P1.P2.P3.P4
M2 = Miquel Point (QL-P1) of Quadrigon P1.P3.P2.P4
M3 = Miquel Point (QL-P1) of Quadrigon P1.P2.P4.P3
Quadrangles P1.P2.P3.P4 and Q1.Q2.Q3.Q4 are point perspective in M0.

Inscribed Miquel Quadrilateral
Lines PiPj and QiQj and MkMl coincide in points Sij
delivering 6 points S12,S23,S34,S41,S23,S24,S34.
being the vertices of an inscribed Miquel Quadrilateral:
L1(S23,S34,S24),
L2(S13,S34,S14),L3(S12,S24,S14),L4(S12,S23,S13).
I wonder what QL-points of this inscribed Miquel Quadrilateral can play a role in the Reference Quadrangle.

Points S1,S2,S3,S4
I noticed that the Cyclologic Centers S1,S2,S3,S4 coincide with R1,R2,R3,R4.

Best regards,
Chris

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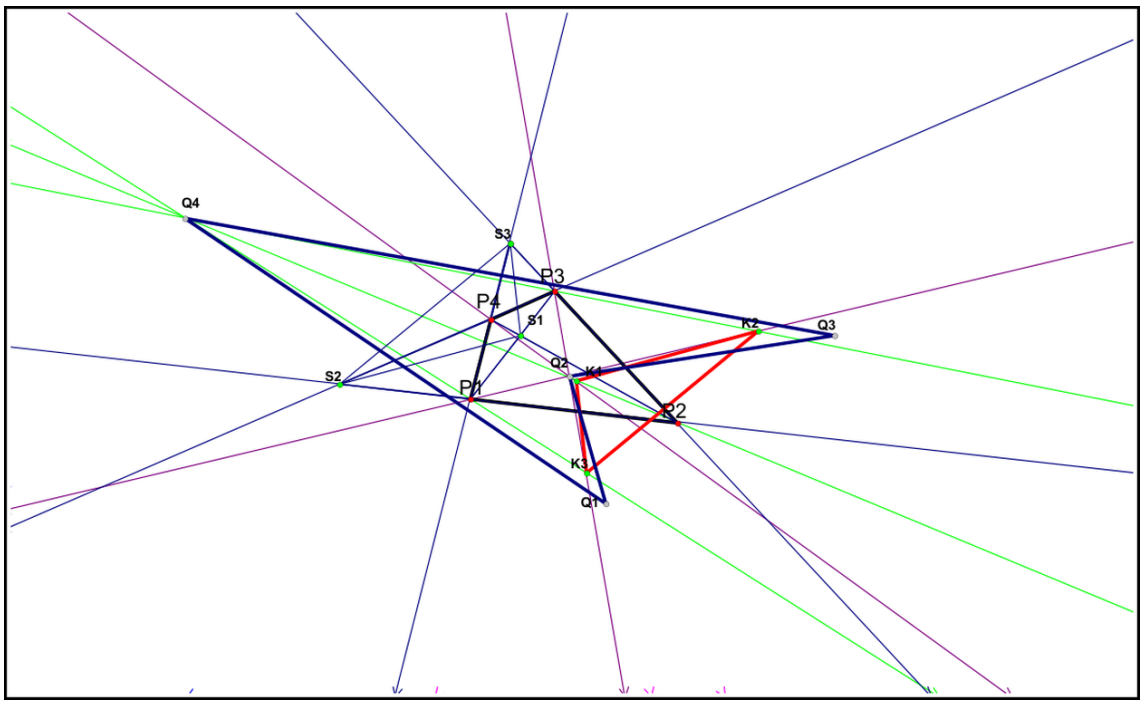
Message: #979
Date: 12/2/2015 2:43:18
From: Seiichi Kirikami
Subject: Miquel points generate another quadrangle

Dear Chris,
Let P1P2P3P4 be a quadrangle.
Denote :
K1= QG-P15 of P1P2P3P4,
K2= QG-P15 of P1P3P2P4,
K3= QG-P15 of P1P2P4P3,
1) K1K2K3 and P3P2P4 are perspective in a point Q1.
2) K1K2K3 and P4P1P3 are perspective in a point Q2.
3) K1K2K3 and P1P4P2 are perspective in a point Q3.
4) K1K2K3 and P2P3P1 are perspective in a point Q4.
If we call the above property “quadri-perspective”, Miquel Triangle(QA-Tr2) and QG-P15 Triple Triangle are quadri-perspective to the Reference Quadrangle (See the attached picture).
I confirmed that QG-P4 Triple Triangle is not quadri-perspective to the Reference Quadrangle. I think that quadri-perspectivity is not rare and it is a property of the triangle and the quadrangle.
Your properties are very interesting, especially the 2nd one for me.
Which points do you mean by R1, R2, R3, and R4?
P.S. QG-P15 did not have the cyclologic property with the Reference Quadrangle.
Best regards,
Seiichi.



QG-P15 quadri-perspective.docx

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Message: #980
Date: 13/2/2015 6:33:33
From: Seiichi Kirikami
Subject: Quadri-orthology of the reference quadrangle and its Morley triangle

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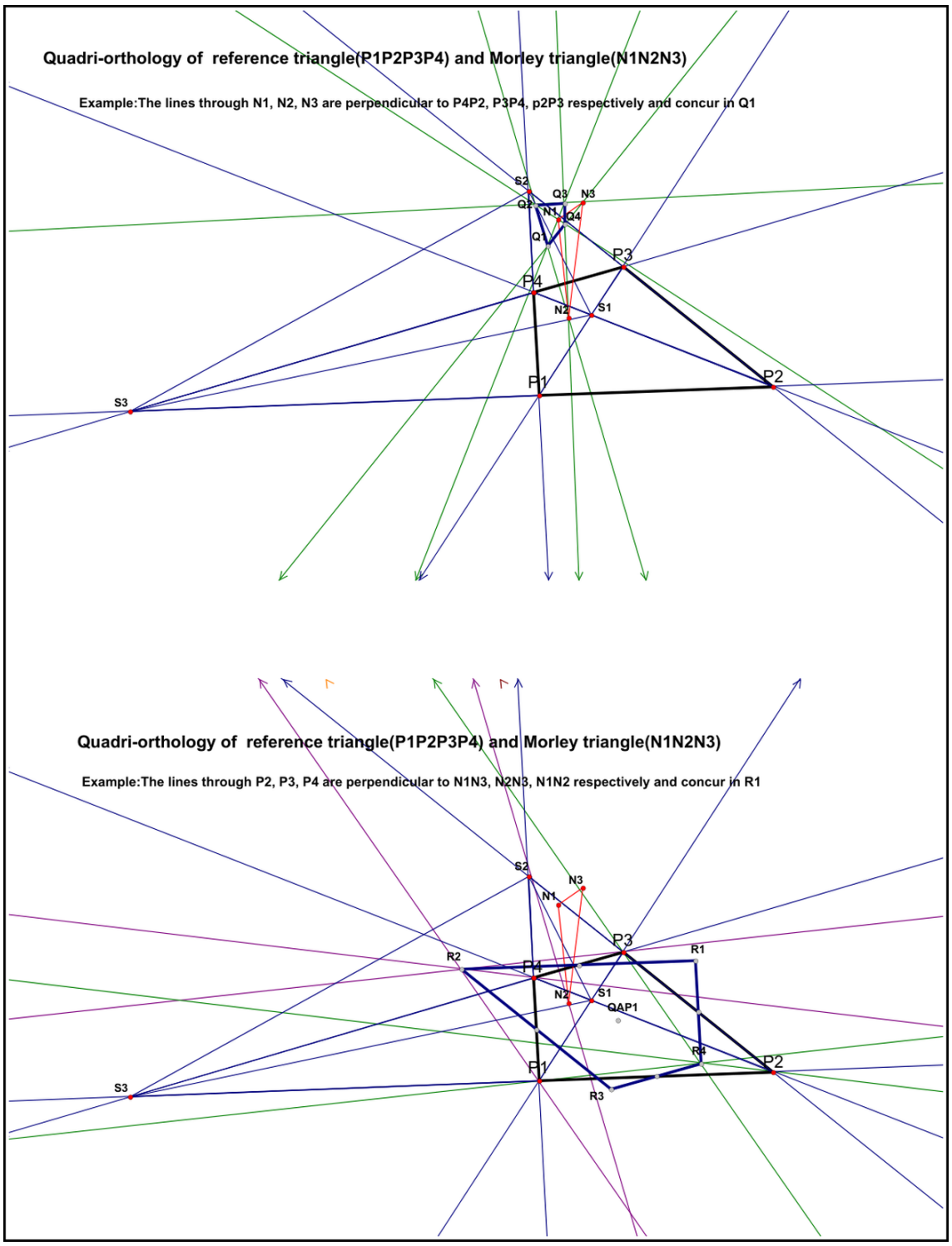
Dear friends, dear Chris,
Let P1P2P3P4 be a quadrangle.
Denote:
N1= Morley point of P1P2P3P4,
N2= Morley point of P1P3P2P4,
N3= Morley point of P1P2P4P3,
1) N1N2N3 and P3P2P4 are orthologic in points Q1 and R1.
2) N1N2N3 and P4P1P3 are orthologic in points Q2 and R2.
3) N1N2N3 and P1P4P2 are orthologic in points Q3 and R3.
4) N1N2N3 and P2P3P1 are orthologic in points Q4 and R4.
The reference quadrangle and its Morley triangle (QA-Tr3) are quadri-orthologic.
See the attached picture.
Best regards,
Seiichi.



quadriorthology.docx

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Message: #981
Date: 13/2/2015 1:22:21
From: chris.vantienhoven
Subject: Quadri-orthology of the reference quadrangle and its Morley tria

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Dear Seiichi,
It seems that lots of Triangle-logics also are logical for a Quadri-figure in an entangled way.
Very well. I am still trying to get an overview of the Quadri-perspectivity. That is already spectacular enough.
Nevertheless I am following your progress!
Best regards,
Chris

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Message: #982
Date: 14/2/2015 2:25:21
From: Seiichi Kirikami
Subject: Quadri-orthology of the reference quadrangle and its Morley tria

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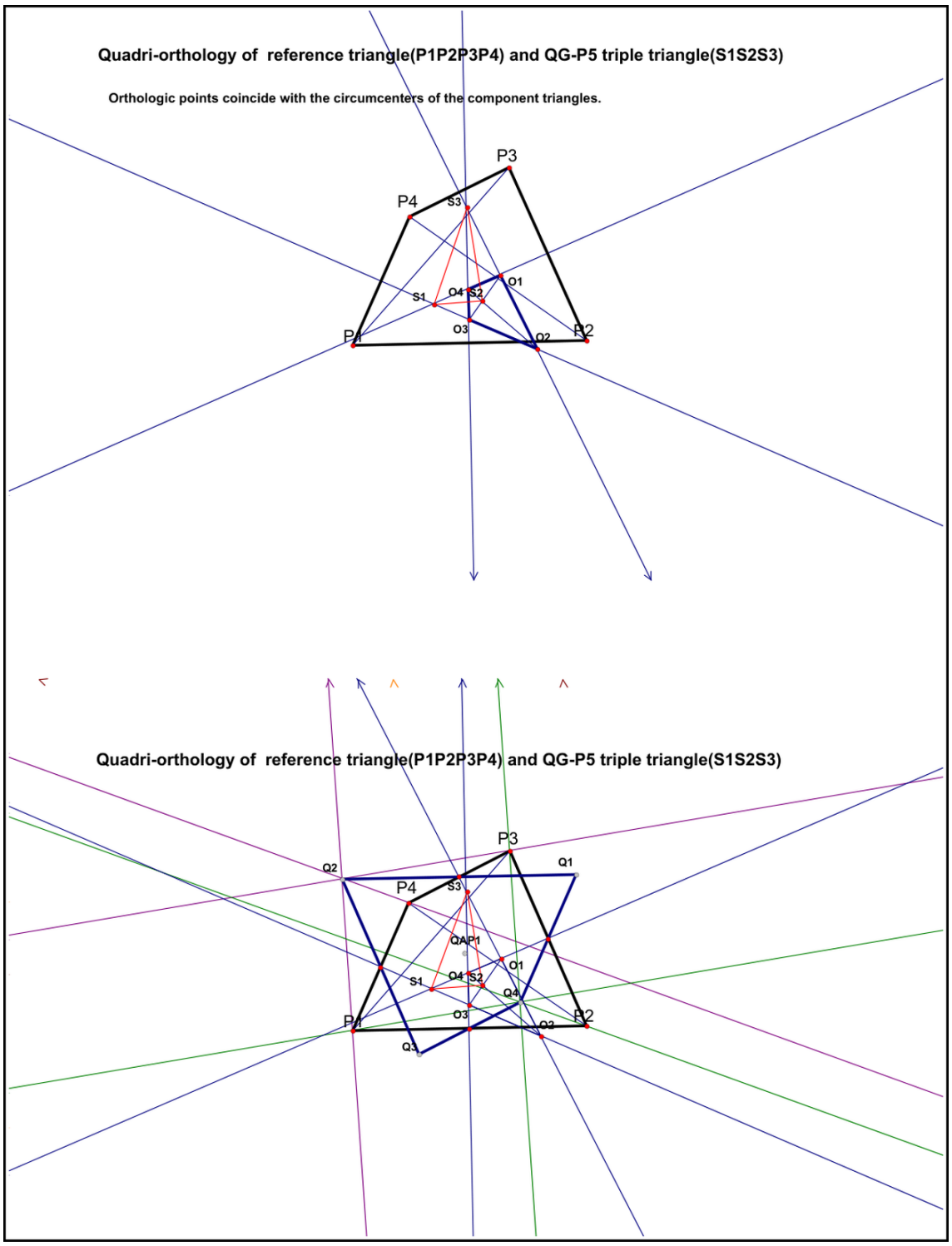
Dear Chris,
Attached are the pictures of quadri-orthology of the reference triangle and QG-P5 triple triangle. They are not so interesting. The last one can be easily drawn.
Best regards,
Seiichi.



QG-P5quadriorthology.docx

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Message: #983
Date: 15/2/2015 1:08:01
From: Seiichi Kirikami
Subject: Quadri-orthology of the reference quadrangle and its Morley tria

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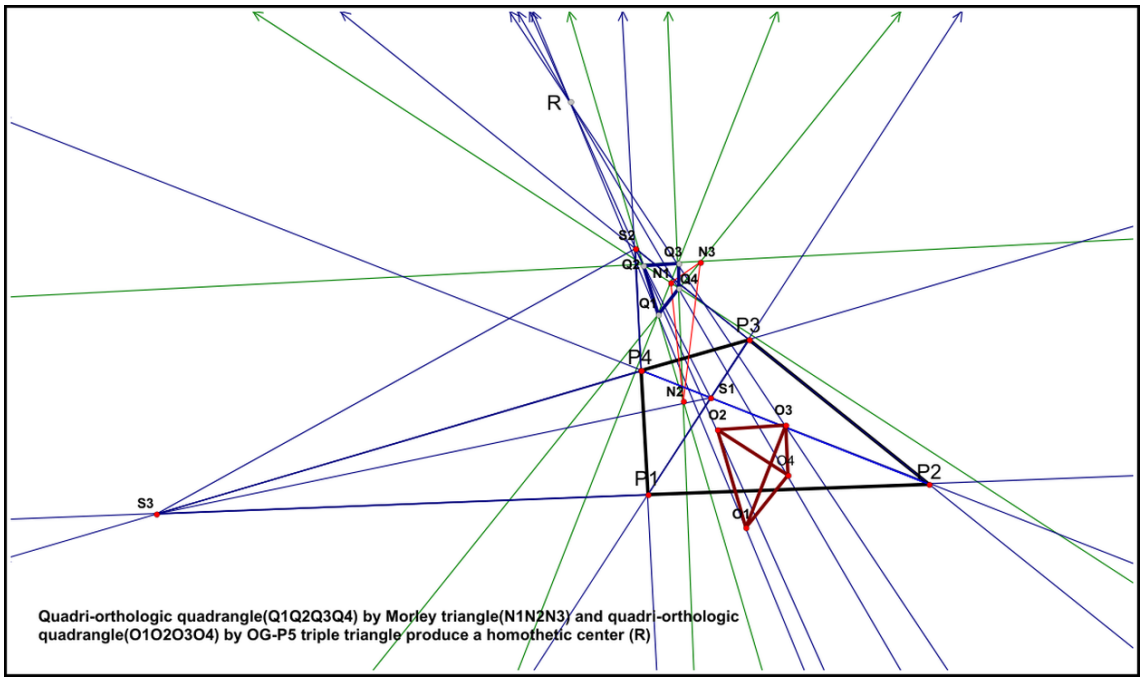
Addition: A homothetic center of quadrangles
Dear Chris,
We have two examples of quadri-orthology.
1) Quadri-orthology of the projection from the vertices of the quadrangle to the sides of the triangle produce the same quadrangle.
2) Quadri-orthology of the projection from the vertices of the triangle to the sides of the quadrangle produce the homothetic quadrangles: Q1Q2Q3Q4 for Morley triangle and O1O2O3O4(the circumcenters of component triangles) for QG-P5 triple triangle.
3) So we obtain a homothetic center (R) of quadrangles.
See the attached picture.
Best regards,
Seiichi.



homothetic center.docx

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Message: #984
Date: 15/2/2015 11:25:25
From: chris.vantienhoven
Subject: Quadri-orthology of the reference quadrangle and its Morley tria

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Dear Seiichi,

Very well!
The homothetic center (R) you describe is QA-P24: AntiComplement
of QA-P1 wrt the Morley Triangle.

Best regards,
Chris
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Message: #985
Date: 16/2/2015 3:24:55
From: seiichikiri
Subject: Quadri-orthology of the reference quadrangle and its Morley tria

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Dear Chris,
Thank you very much! After sending the previous message, I
found a description in QG-P5, equivalent to what I found.
Best regards,
Seiichi.
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Message: #986
Date: 19/2/2015 8:58:00
From: chris.vantienhoven
Subject: Desmic Triple Triangles

Dear Seiichi, dear friends,
Seiichi noticed a very special feature in a Quadrangle.
I studied it in detail now. It really is stunning!
Let QA-P = P1.P2.P3.P4 be the Reference Quadrangle.
There are 3 component Quadrigons in P1.P2.P3.P4.
Let QG2=P1.P3.P2.P4, QG3=P1.P2.P3.P4 and QG4=P1.P2.P4.P3 be the 3 component Quadrigons in QA-P. Note that the Quadrigon number is determined by the vertice number opposite to P1. Let K2 = QG-P15 wrt QG2, K3 = QG-P15 wrt QG3, K2 = QG-P15 wrt QG3.
Seiichi noticed that Triple Triangle K2.K3.K4 is perspective with all 4 component triangles of P1.P2.P3.P4.
First of all K2.K3.K4 is perspective with P2.P3.P4 but also with P1.P4.P3, P4.P1.P2, P3.P2.P1 (vertices in these orders). Let Q1,Q2,Q3,Q4 be the respective perspectors of these perspective pairs of triangles. Now P1.Q1, P2.Q2, P3.Q3 and P4.Q4 are concurrent in a point K1.
Herewith we have 3 Quadrangles P1.P2.P3.P4, Q1.Q2.Q3.Q4 and K1.K2.K3.K4.
Theorem about these 3 Quadrangles:
All Component Triangles of a chosen 1 st Quadrangle are perspective with all Component Triangles of a chosen 2 nd Quadrangle having perspectors being the vertices of the 3 rd Quadrangle.
I have a generic algebraic proof of this.
Later I found several references.
This configuration is also called a DESMIC configuration. See http://en.wikipedia.org/wiki/Desmic_system.
An extra property is that there is an involutary QA-DT-cubic passing through the 3x4 vertices with a pivot point that can be constructed form the 3x4 vertices (method Hyacinthos #2451 †)).
This cubic already was mentioned earlier by Barry Wolk at Hyacinthos #462 †).
Like Seiichi mentioned this construction not only works for the QG-P15 Triple Triangle, but also for the QL-P1 Triple triangle and probably more Triple Triangles.
I even think the same type of construction could pop up in QUADRILATERALS.
Who will be the first one to find any Desmic Triple Triangle in a Quadrilateral?
Best regards, Chris

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #987
Date: 19/2/2015 12:14:22
From: bernard.keizer
Subject: A particular 5-L

Dear Chris, dear Eckart, dear friends
While looking always at the 64 axes of the QL, I've found a curiosity mentioned by Raoul Bricard " Sur un système remarquable de 5 droites".
Let's consider an ordinary QL with 4 lines Li (i = 1 to 4).
It's well known that the point QL-P20 on the Newton Line is the center of an inscribed conic tangent to a 5th line which is the directrix QL-L2 of the inscribed parabola (see EQF).
The reflexions of the Li in QL-P20 are 4 lines L'i (i = 1 to 4) through the orthocenters Hi on QL-P2 and form a QL which is the reflexion of the initial QL in QL-P20.
Let's now name L5 the reflexion of QL-L2 in QL-P20 and L'5 the line QL-L2.
The lines Li (i = 1 to 5) form a 1rst 5-L and the lines L'i (1 = 1 to 5) a 2nd one, which is the reflexion of the 1rst in QL-P20.
Both 5-L's have the same properties:
° the point QL-P20 is the same for the 10 QL's of the 2 5-L's
° the 10 lines Li and L'i are tangent to the same conic with center QL-P20
° the lines QL-L2, directrices of the parabolas tangent to 4/5 lines of one 5-L, are the reflexions of the omitted 5th line in the same 5-L in QL-P20 and the 5th line of the other 5-L
° the 10 vertices Aij (or A'ij) of one 5-L are the orthocenters H'ij (or Hij) of the 10 triangles of the other 5-L
° QL-P20 is the middle of the 10 segments joining the corresponding vertices Aij and A'ij of the 2 5-L's (or the vertices Aij of one 5-L to the orthocenters Hij of the triangles formed by the 3 other lines of the same 5-L).
The beauty of this construction is that, starting with 4 lines forming a QL, it's always possible to find a 5th line (reflexion of QL-L2 in QL-P20) such as the 4 other QL's of the 5-L share the same property, that the directrix of the parabola tangent to 4 lines is the reflexion of the omitted 5th line in the commun point QL-P20.
Best regards
Bernard

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Message: #988
Date: 21/2/2015 4:57:13
From: seiichikiri
Subject: Desmic Triple Triangles

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Dear Chris,
Thank you very much for your appreciation.
I would like to see all 3 quadrangles.
Not to mention other new findings, I think that QF geometry is
an old, but very new field of geometries.
Best regards,
Seiichi.
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Message: #989
Date: 23/2/2015 12:25:25
From: seiichikiri
Subject: Orthologies among quadrigonal triangles

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Dear Chris,
If we interpret "desmic" in a broader sense, can we say that
triple triangles of QG-P1, P4, P5, P6 are desmically orthologic?
Best regards,
Seiichi.
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Message: #990
Date: 23/2/2015 7:21:07
From: chris.vantienhoven
Subject: Orthologies among quadrigonal triangles

Dear Seiichi,
I think that in principle Triple Triangles can be Quadri Perspective (Desmic) as well as Quadri Orthologic ("Quadri" = meaning that this property is valid wrt all Component Triangles).
But I didn't find any example yet.
I found these examples of Quadri Perspective (or Desmic) Triangles:

- QG-P1 Triple Triangle
- QG-P2 Triple Triangle
- QG-P15 Triple Triangle
- QG-P16 Triple Triangle
- QG-P17 Triple Triangle
- QG-P18 Triple Triangle
- QG-P19 Triple Triangle
- QL-P1 Triple Triangle

And these examples of Quadri Orthologic Triangles:

- QG-P5 Triple Triangle
- QG-P10 Triple Triangle
- QL-P2 Triple Triangle

But I didn't find any example of a Triple Triangle having both properties.
Best regards,
Chris

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Message: #991
Date: 23/2/2015 11:38:00
From: seiichikiri
Subject: Orthologies among quadrigonal triangles

Dear Chris,
It is written in Wikipedia "Desmic System" that "desmic" means "band or chain" in Greek.
Triple triangles QG-Pi and QG-Pj are orthologic if i=1, 4, 5, 6, 7 and j=1, 4, 5, 6, 7 except for i=j. The combination number of the orthologic cases is 10.
So I would like to say that triple triangles QG-P1, P4, P5, P6 and P7 are desmically orthologic (a band of orthologies).
Best regards,
Seiichi.

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Message: #992
Date: 23/2/2015 12:00:11
From: chris.vantienhoven
Subject: Orthologies among quadrigonal triangles

Dear Seiichi,
I see the Orthologic properties are intriguing for you too.
I made a list of the Orthologic cases of QA-Triple Triangles that occur in EQF.
QA-Px means “non-EQF-center”.
QA-P90.. means “non-EQF-center” but inofficially registered by me.

TripleTriangles	OrthologicCenters
{QG-P1, QG-P2,	QA-P12, QA-P11},
{QG-P1, QG-P4,	QA-P12, QA-Px},
{QG-P1, QG-P5,	QA-P20, QA-Px},
{QG-P1, QG-P6,	QA-Px, QA-P24},
{QG-P1, QG-P7,	QA-P3, QA-P9009},
{QG-P1, QG-P8,	QA-P12, QA-Px},
{QG-P1, QG-P9,	QA-P3, QA-P32},
{QG-P1, QG-P10,	QA-P20, QA-Px},
{QG-P1, QG-P11,	QA-Px, QA-Px},
{QG-P1, QG-P15,	QA-P12, QA-P37},
{QG-P1, QG-P17,	QA-P11, QA-P12},
{QG-P1, QL-P2,	QA-P20, QA-P9009},
{QG-P1, QL-P6,	QA-P3, QA-Px},
{QG-P1, QL-P10,	QA-Px, QA-P12},
{QG-P1, QL-P12,	QA-P12, QA-Px},
{QG-P2, QG-P4,	QA-P11, QA-Px},
{QG-P2, QG-P5,	QA-P1, QA-Px},
{QG-P2, QG-P6,	QA-Px, QA-P24},
{QG-P2, QG-P7,	QA-Px, QA-P9009},
{QG-P2, QG-P8,	QAP11, QA-Px},
{QG-P2, QG-P9,	QA-Px, QA-P32},
{QG-P2, QG-P10,	QA-P1, QA-Px},
{QG-P2, QG-P11,	QA-Px, QA-Px},
{QG-P2, QG-P15,	QA-P11, QA-P37},
{QG-P2, QG-P17,	QA-P13, QA-P12},
{QG-P2, QL-P2,	QA-P1, QA-P9009},
{QG-P2, QL-P6,	QA-Px, QA-Px},
{QG-P2, QL-P10,	QA-Px, QA-P12},
{QG-P2, QL-P12,	QA-P11, QA-Px},
{QG-P3, QL-P5,	QA-Px, QA-Px},
{QG-P3, QL-P15,	QA-Px, QA-Px},
{QG-P3, QL-P18,	QA-Px, QA-Px},
{QG-P4, QG-P5,	QA-Px, QA-Px},
{QG-P4, QG-P6,	QA-Px, QA-P24},
{QG-P4, QG-P7,	QA-Px, Q-AP9009},
{QG-P4, QG-P8,	QA-Px, QA-Px},

```
{QG-P4, QG-P9,   QA-Px, QA-P32},
{QG-P4, QG-P10,  QA-Px, QA-Px},
{QG-P4, QG-P11,  QA-Px, QA-Px},
{QG-P4, QG-P15,  QA-Px, QA-P37},
{QG-P4, QG-P17,  QA-Px, QA-P12},
{QG-P4, QL-P2,   QA-Px, QA-P9009},
{QG-P4, QL-P6,   QA-Px, QA-Px},
{QG-P4, QL-P10,  QA-Px, QA-P12},
{QG-P4, QL-P12,  QA-Px, QA-Px},
{QG-P5, QG-P6,   QA-Px, QA-Px},
{QG-P5, QG-P7,   QA-Px, QA-Px},
{QG-P5, QG-P8,   QA-Px, QA-P10},
{QG-P5, QG-P9,   QA-Px, QA-Px},
{QG-P5, QG-P10,  QA-Px, QA-Px},
{QG-P5, QG-P11,  QA-Px, QA-Px},
{QG-P5, QG-P12,  QA-P32, QA-P1},
{QG-P5, QG-P14,  QA-P32, QA-P5},
{QG-P5, QG-P15,  QA-Px, QA-P5},
{QG-P5, QL-P2,   QA-Px, QA-P15},
{QG-P5, QL-P3,   QA-Px, QA-P15},
{QG-P5, QL-P4,   QA-P9, QA-Px},
{QG-P5, QL-P5,   QA-Px, QA-P1},
{QG-P5, QL-P6,   QA-Px, QA-Px},
{QG-P5, QL-P7,   QA-Px, QA-P1},
{QG-P5, QL-P12,  QA-Px, QA-P1},
{QG-P5, QL-P20,  QA-Px, QA-P1},
{QG-P5, QL-P22,  QA-Px, QA-P1},
{QG-P5, QL-P23,  QA-Px, QA-P1},
{QG-P5, QL-P27,  QA-Px, QA-Px},
{QG-P5, QL-P29,  QA-Px, QA-P15},
{QG-P5, QL-P30,  QA-Px, QA-Px},
{QG-P5, QA-CT1,  QA-Px, QA-Px},
{QG-P6, QG-P7,   QA-Px, QA-Px},
{QG-P6, QG-P8,   QA-P24, QA-Px},
{QG-P6, QG-P9,   QA-Px, QA-Px},
{QG-P6, QG-P10,  QA-Px, QA-Px},
{QG-P6, QG-P11,  QA-Px, QA-Px},
{QG-P6, QG-P15,  QAP24, QA-Px},
{QG-P6, QL-P2,   QA-Px, QA-Px},
{QG-P6, QL-P6,   QA-Px, QA-Px},
{QG-P6, QL-P12,  QA-P24, QA-Px},
{QG-P7, QG-P8,   QA-P9009, QA-P34},
{QG-P7, QG-P9,   QAP15, QA-Px},
{QG-P7, QG-P10,  QA-Px, QA-Px},
{QG-P7, QG-P11,  QA-Px, QA-Px},
{QG-P7, QG-P15,  QAP9009, QA-P2},
{QG-P7, QL-P1,   QA-Px, QA-P3},
{QG-P7, QL-P2,   QA-Px, QA-Px},
{QG-P7, QL-P6,   QAP15, QA-Px},
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{QG-P7, QL-P12,  QAP9009, QA-Px},
{QG-P8, QG-P9,   QAP34, QA-P32},
{QG-P8, QG-P10,  QAP10, QA-Px},
{QG-P8, QG-P11,  QA-Px, QA-Px},
{QG-P8, QG-P15,  QA-Px, QA-P37},
{QG-P8, QG-P17,  QA-Px, QA-P12},
{QG-P8, QL-P2,   QA-P10, QAP-9009},
{QG-P8, QL-P6,   QA-P34, QA-Px},
{QG-P8, QL-P10,  QA-Px, QA-P12},
{QG-P8, QL-P12,  QA-Px, QA-Px},
{QG-P9, QG-P10,  QA-Px, QA-Px},
{QG-P9, QG-P11,  QA-Px, QA-Px},
{QG-P9, QG-P15,  QA-P32, QA-P2},
{QG-P9, QL-P1,   QA-P9007, QA-P3},
{QG-P9, QL-P2,   QA-Px, QA-Px},
{QG-P9, QL-P6,   QA-Px, QA-Px},
{QG-P9, QL-P12,  QAP32, QA-Px},
{QG-P10, QG-P11, QA-Px, QA-Px},
{QG-P10, QG-P12, QA-P9009, QA-P1},
{QG-P10, QG-P14, QA-P9009, QA-P5},
{QG-P10, QG-P15, QA-Px, QA-P5},
{QG-P10, QL-P2,  QA-Px, QA-P15},
{QG-P10, QL-P3,  QA-Px, QA-P15},
{QG-P10, QL-P4,  QA-Px, QA-Px},
{QG-P10, QL-P5,  QA-Px, QA-P1},
{QG-P10, QL-P6,  QA-Px, QA-Px},
{QG-P10, QL-P7,  QA-Px, QA-P1},
{QG-P10, QL-P12, QA-Px, QA-P1},
{QG-P10, QL-P20, QA-Px, QA-P1},
{QG-P10, QL-P22, QA-Px, QA-P1},
{QG-P10, QL-P23, QA-Px, QA-P1},
{QG-P10, QL-P27, QA-Px, QA-Px},
{QG-P10, QL-P29, QA-Px, QA-P15},
{QG-P10, QL-P30, QA-Px, QA-Px},
{QG-P10, QA-CT1, QA-Px, QA-Px},
{QG-P11, QG-P15, QA-Px, QA-Px},
{QG-P11, QL-P2,  QA-Px, QA-Px},
{QG-P11, QL-P6,  QA-Px, QA-Px},
{QG-P11, QL-P12, QA-Px, QA-Px},
{QG-P12, QG-P14, QA-Px, QA-Px},
{QG-P12, QL-P2,  QA-P1, QA-Px},
{QG-P14, QL-P2,  QA-P5, QA-Px},
{QG-P15, QG-P17, QA-P9030, QA-P12},
{QG-P15, QL-P2,  QA-P5, QAP-9009},
{QG-P15, QL-P6,  QA-P2, QA-Px},
{QG-P15, QL-P10, QA-Px, QA-P12},
{QG-P15, QL-P12, QA-P37, QA-Px},
{QG-P17, QG-P18, QA-Px, QA-P23},
{QG-P17, QL-P12, QA-P12, QA-Px},
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{QL-P1, QL-P6,   QA-P3, QA-Px},
{QL-P2, QL-P3,   QA-Px, QA-P15},
{QL-P2, QL-P4,   QA-Px, QA-Px},
{QL-P2, QL-P5,   QA-Px, QA-P1},
{QL-P2, QL-P6,   QA-Px, QA-Px},
{QL-P2, QL-P7,   QA-Px, QA-P1},
{QL-P2, QL-P12,  QA-P9009, QA-P1},
{QL-P2, QL-P20,  QA-Px, QA-P1},
{QL-P2, QL-P22,  QA-Px, QA-P1},
{QL-P2, QL-P23,  QA-Px, QA-P1},
{QL-P2, QL-P27,  QA-Px, QA-Px},
{QL-P2, QL-P29,  QA-Px, QA-P15},
{QL-P2, QL-P30,  QA-Px, QA-Px},
{QL-P2, QA-CT1,  QA-Px, QA-Px},
{QL-P3, QL-P29,  QA-Px, QA-Px},
{QL-P4, QL-P22,  QA-Px, QA-Px},
{QL-P4, QL-P30,  QA-Px, QA-Px},
{QL-P5, QL-P15,  QA-Px, QA-Px},
{QL-P5, QL-P18,  QA-Px, QA-Px},
{QL-P6, QL-P12,  QA-Px, QA-Px},
{QL-P10, QL-P12, QA-P12, QA-Px},
{QL-P10, QL-P23, QA-Px, QA-Px},
{QL-P15, QL-P18, QA-Px, QA-Px},
{QL-P22, QL-P30, QA-Px, QA-Px}
I was baffled that so many Orthologic cases occur.
Best regards,
Chris
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Message: #993
Date: 23/2/2015 11:57:22
From: chris.vantienhoven
Subject: Orthologies among quadrigonal triangles

Dear Seiichi, dear friends,

I realized that maybe I wasn't very clear about the table with Orthologic Triangles for those not familiar with Triple Triangles. After all it is new material, so I will explain it now.

Quadrigons/Quadrilaterals/Quadrilaterals
It is well known that a Quadrangle P1.P2.P3.P4 has 4 Component Triangles P1.P2.P3, P2.P3.P4, P3.P4.P1, P4.P1.P2.
Less known is that a Quadrangle has 3 Component Quadrigons P1.P2.P3.P4, P1.P2.P4.P3 and P1.P3.P2.P4. They represent the 3 possible cyclic orders in a Quadrangle.
In fact they are Quadrigons or Tetragons, figures with 4 points and 4 connecting lines in a fixed order.

Triple Triangles
This is the reason that there are 3 versions of a Quadrigon point in a Quadrangle. They form together a so called Triple Triangle.
There are also 3 versions of a Quadrilateral point in a Quadrangle. They also form a Triple Triangle.

Orthologic Triangles
Two triangles A1B1C1 and A2B2C2 are Orthologic if the perpendiculars from the vertices A1, B1, C1 on the sides B2C2, A2C2, and A2B2 are concurrent.
Furthermore, if this is the case, then the perpendiculars from the vertices A2, B2, C2 on the sides B1C1, A1C1, and A1B1 are also concurrent, as shown by Steiner in 1827.
The point of concurrence is known as the Orthology Center of A1B1C1 with respect to A2B2C2.

Table of pairs of Orthologic Triple
Triangles
Now recently it appeared (thank you Seiichi) that QA-Triple Triangles often are mutually Orthologic related. There are even 158 occurrences of them.
I made a program to find all cases of Orthologic related Triple Triangles from which I showed the results in the QFG message before.
{QG-P1, QG-P2, QAP12, QAP11} means:

The Triple Triangle formed by the 3 versions of QG-P1 and the Triple Triangle formed by the 3 versions of QG-P2 are Orthologic where QA-P12 is the Orthology Centers of the QG-P1 Triple Triangle wrt the QG-P2 Triple Triangle and where QA-P11 is the Orthology Centers of the QG-P2 Triple Triangle wrt the QG-P1 Triple Triangle.
I hope this will clarify things.

Best regards,
Chris

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Message: #994
Date: 24/2/2015 12:47:23
From: seiichikiri
Subject: Orthologies among quadrigonal triangles

Dear Chris,
What a remarkable result it is!
I am very happy that I met you and QF geometry.
Best regards, Seiichi.

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Message: #995
Date: 26/2/2015 1:43:51
From: seiichikiri
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear friends,
Given a quadrangle P1P2P3P4, its Diagonal Triangle(QG-P1), its Miquel Triangle (QL-P1) and Triple Triangle of QG-P16,
1) Diagonal Triangle(QG-P1) and Miquel Triangle (QL-P1) are cyclologic,
2) Miquel Triangle (QL-P1) and Triple Triangle of QG-P16 are cyclologic,
3) Diagonal Triangle(QG-P1) and Triple Triangle of QG-P16 are cyclologic.
N. B. QG-P16: the isogonal of QG-P1 wrt Miquel Triangle.
Best regards, Seiichi.

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Message: #996
Date: 27/2/2015 10:14:48
From: chris.vantienhoven
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear Seiichi,
According to your examples the Triple Triangles of QG-P1, QG-P16 and QL-P1 are mutually cyclologic related.
To find more examples of "cyclologics" I made a program for all Cyclologic pairs of Triple Triangles.
I had some calculation problems, so it is possible there is an omitting pair.

Triple Triangles	Cyclologic Centers
{QG-P1, QG-P15,	QA-P2, QA-P3}
{QG-P1, QG-P16,	QA-P34, QA-Px}
{QG-P1, QG-P19,	QA-P2, QA-P4}
{QG-P1, QL-P1,	QA-P3, QA-P41}
{QG-P1, QL-P17,	QA-Px, QA-P3}
{QG-P2, QL-P1,	QA-Px, QA-P1}
{QG-P3, QL-P17,	QA-Px, QA-Px}
{QG-P4, QG-P8,	QA-P34, QA-Px}
{QG-P5, QG-P10,	QA-Px, QA-Px}
{QG-P5, QL-P1,	QA-P3, QA-Px}
{QG-P5, QL-P4,	QA-Px, QA-Px}
{QG-P6, QL-P21,	QA-Px, QA-Px}
{QG-P7, QG-P9,	QA-Px, QA-Px}
{QG-P9, QL-P1,	QA-P1, QA-Px}
{QG-P9, QL-P4,	QA-Px, QA-Px}
{QG-P9, QL-P5,	QA-Px, QA-Px}
{QG-P12, QL-P1,	QA-Px, QA-Px}
{QG-P16, QL-P1,	QA-P4, QA-Px}
{QG-P18, QG-P19,	QA-P41, QA-P4}
{QL-P1, QL-P17,	QA-Px, QA-P3}

Comment:
QA-Px means QA-point not registered in EQF.

Cyclologic Triangles
Two triangles A1B1C1 and A2B2C2 are Cyclologic if the circles defined by the triples (A1,B2,C2), (B1,C2,A2) and (C1,A2,B2) have a concurring point. This point is named the Cyclologic Center of triangle A1B1C1 wrt triangle A2B2C2.
Furthermore, if this is the case, then also the circles defined by the triples (A2,B1,C1), (B2,C1,A1) and (C2,A1,B1) have a concurring point. This point is named the Cyclologic Center of triangle A2B2C2 wrt triangle A1B1C1.

Table of pairs of Orthologic Triple Triangles
Example:
{QG-P1, QG-P15,   QA-P2, QA-P3}
means:
The Triple Triangle formed by the 3 versions of QG-P1 and the Triple Triangle formed by the 3 versions of QG-P15 are Cyclologic where QA-P2 is the Cyclologic Center of the QG-P1 Triple Triangle wrt the QG-P15 Triple Triangle and where QA-P3 is the Cyclologic Center of the QG-P15 Triple Triangle wrt the QG-P1 Triple Triangle.

Best regards,
Chris

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Message: #997
Date: 27/2/2015 5:33:43
From: Benedetto Scimemi
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear Chris (and other friends), in the rich list of "cyclologic triangles" of your last message, I found the tenth example specially interesting:
{QG-P5, QLP1} QAP3, QAPx
I want to remark that this QAPx is nothing new.
If one applies the Möbius transformation μ=QA-Tf4, it is easy to see that μ interchanges the points QG-P5 and QL-P1 and therefore (as circles go into circles) μ leaves invariant the cyclologic set { , }.
Therefore QAPx=μ(QAP3)=QAP9 !
About the circles passing through two QL-P1 and one QG-P5, it seems also remarkable that they can be defined in a quite different way; in fact they pass through a diagonal point QGP1 and the corresponding pair of opposite side-midpoints (of the reference quadrangle).
Incidentally, the triple triangles QG-P5 and QL-P1 happen to be also perspective.
Why don't I find this perspector in the EQF list of central points QA-Px?
Looking forward for your reaction,
best regards.
Benedetto

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Message: #998
Date: 28/2/2015 12:03:52
From: chris.vantienhoven
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear Benedetto,
That’s a wonderful connection you make between the Cyclological relation and the Möbius Transformation (QA-Tf4).
In a Cyclological relation we have 2x3 circles and when 3 points on one of the circles is transformed by a Möbius Transformation on another circle, then all points on the 1 st circle will be transformed on the other circle. Automatically an intersection point of 3 circles will transform on the intersection point of the other 3 circles.
Since QA-P3 is transformed by QA-Tf4 in QA-P9 the 2nd Cyclological Center should be QA-P9.
I checked it and it is true! Thank you!
[BS] About the circles passing through two QL-P1 and one QG-P5, it seems also remarkable that they can be defined in a quite different way; in fact they
pass through a diagonal point QGP1 and the corresponding pair of opposite side-midpoints (of the reference quadrangle).
Most remarkable property of these circles indeed!
Studying this I noticed that the circles passing through two QL-P5 and one QG-P5 also are special because they are the Miquel Circles (QL-Ci3) of the Quadrilaterals connected to the 3 QA-Quadrigons of the Reference Quadrangle. See 1st and 4th property QA-P9 at EQF.
[BS] Incidentally, the triple triangles QG-P5 and QL-P1 happen to be also perspective. Why don't I find this perspector in the EQF list of central points QA-Px ?
I already knew this point because of its perspectivity. However the coordinates are pretty long and there are no more geometric properties known yet about this point. Normally I preferably register new points when they have (at least) three special geometrical properties.
By the way here is a list of perspective Triple Triangles.

Triple Triangle-1 formed by the 3 QA-versions of	Triple Triangle-2 formed by the 3 QA-versions of	Perspector
QG-P1	QG-P2	QA-P10
QG-P1	QG-P4	QA-P1
QG-P1	QG-P8	QA-P1
QG-P1	QG-P12	QA-P16
QG-P1	QG-P13	QA-P16
QG-P1	QG-P15	QA-P1

QG-P1	QG-P16	QA-Px
QG-P1	QG-P17	QA-P12
QG-P1	QG-P18	QA-Px
QG-P1	QG-P19	QA-Px
QG-P1	QL-P1	QA-P3
QG-P1	QL-P10	QA-P12
QG-P1	QL-P12	QA-Px
QG-P1	QL-P13	QA-P16
QG-P1	All Component Triangles	Desmic
QG-P2	QG-P4	QA-P5
QG-P2	QG-P8	QA-Px
QG-P2	QG-P12	QA-P1
QG-P2	QG-P15	QA-P20
QG-P2	QL-P5	QA-P1
QG-P2	QL-P7	QA-P1
QG-P2	QL-P12	QA-P1
QG-P2	QL-P20	QA-P1
QG-P2	QL-P22	QA-P1
QG-P2	QL-P23	QA-P1
QG-P2	All Component Triangles	Desmic
QG-P3	QL-P15	QA-Px
QG-P4	QG-P8	QA-P1
QG-P4	QG-P12	QA-Px
QG-P4	QG-P15	QA-P1
QG-P4	QL-P12	QA-P10
QG-P5	QG-P10	QA-P1
QG-P5	QL-P1	QA-Px
QG-P5	QL-P2	QA-P24
QG-P7	QG-P9	QA-P1
QG-P7	QL-P2	QA-Px
QG-P7	QL-P6	QA-Px
QG-P8	QG-P12	QA-Px
QG-P8	QG-P15	QA-P1
QG-P8	QL-P12	QA-P26
QG-P9	QL-P6	QA-Px
QG-P10	QL-P2	QA-P14
QG-P10	QL-P10	QA-P14
QG-P12	QG-P13	QA-P16
QG-P12	QG-P14	QA-Px
QG-P12	QG-P15	QA-Px
QG-P12	QG-P17	QA-Px
QG-P12	QL-P5	QA-P1
QG-P12	QL-P7	QA-P1
QG-P12	QL-P12	QA-P1
QG-P12	QL-P13	QA-P16

QG-P12	QL-P20	QA-P1
QG-P12	QL-P22	QA-P1
QG-P12	QL-P23	QA-P1
QG-P12	QL-P26	QA-Px
QG-P13	QL-P13	QA-P16
QG-P13	QL-P17	QA-P3
QG-P14	QG-P15	QA-P5
QG-P15	QL-P12	QA-Px
QG-P15	All Component Triangles	Desmic
QG-P16	QL-P1	QA-P4
QG-P16	All Component Triangles	Desmic
QG-P17	QL-P10	QA-P12
QG-P17	All Component Triangles	Desmic
QG-P18	QG-P19	QA-Px
QG-P18	All Component Triangles	Desmic
QG-P19	All Component Triangles	Desmic
QL-P1	All Component Triangles	Desmic
QL-P2	QL-P3	QA-P15
QL-P2	QL-P10	QA-P14
QL-P2	QL-P29	QA-P15
QL-P3	QL-P29	QA-P15
QL-P5	QL-P7	QA-P1
QL-P5	QL-P12	QA-P1
QL-P5	QL-P20	QA-P1
QL-P5	QL-P22	QA-P1
QL-P5	QL-P23	QA-P1
QL-P7	QL-P12	QA-P1
QL-P7	QL-P20	QA-P1
QL-P7	QL-P22	QA-P1
QL-P7	QL-P23	QA-P1
QL-P12	QL-P20	QA-P1
QL-P12	QL-P22	QA-P1
QL-P12	QL-P23	QA-P1
QL-P20	QL-P22	QA-P1
QL-P20	QL-P23	QA-P1
QL-P22	QL-P23	QA-P1

Because I calculated these items numerical in a Mathematica program, there can be errors because of precision of numerical values.
They should be checked before using them seriously.
Best regards,
Chris

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Message: #999
Date: 28/2/2015 2:00:47
From: seiichikiri
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear Chris, dear Benedetto,
I highly appreciate Chris's determination of cyclologic pairs because it is sometimes very difficult to obtain the coordinates.
Algebraic proof or computation has merits in comparison with synthetic proof. It reduces a problem to programming and the computation time.
There are many problems difficult to compute in QF geometry, Penta-Figure geometry, or even in Triangle geometry.
I had a problem in Anopolis (#2009 and #2202) †) solved by a man of Hulinks, inc., a representative of Wolfram Research, Inc. in Japan. The specs of computation are as follows:
Windows7,
Intel(R) Core(TM)i5-2400 CPU @3.10GHz 3.10GHz.…(4 Core),
RAM:4.00GB/64Bit,
Mathematica 10.0.01,
Coordenadas Baricentricas by Professor Garcia Capitan.
He said that the computation time was about an hour.
It might be a solution to purchase a powerful PC. But its downgrade is price, although I see used, relatively cheap ones in the web.
A friend of mine in a university uses AnsysFluent, a software for thermal fluids. The specs of computation are as follows:
Windows7,
Model: Optiplex-9020
Intel(R) Core(TM)i7-4770 CPU @3.40GHz 3.40GHz….(4 Core)
RAM:16.00GB/64Bit.
He says that he run the program in the evening and obtain the solution in the morning next day, although I think that much time is consumed in process of iteration.
To run, for example, 12 hours might be a solution.
But it makes me reluctant to purchase a new powerful PC to think that I can solve a few more problems, but there remain most of problems unsolved.
I would like to hear your opinion about how to obtain algebraic solutions of difficult problems of Triangle, QF or PF geometries.
Sorry in advance if my message would be irrelevant. But this is always in my mind.
Best regards,
Seiichi.

P.S. Up to now we obtained many algebraic solutions for TG or QFG. But It seems to me that we can not compute many interesting coordinates or properties for PF geometry in the same way as TG or QFG. I do not mind that the solution is “too complicated to be written”.

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #1000
Date: 28/2/2015 8:11:57
From: chris.vantienhoven
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

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Dear Seiichi,
Computation time very often is a problem when you make programs. I even made programs that took many days to finish. Every morning I looked to the new results. Sometimes I finished it after one or two weeks. Sometimes I had enough information after a week although the job didn't finish.
I noticed that the way of programming is very important. Not working with files but working with data in the internal memory of the computer. Mathematica is wonderful in this.
For the rest my recent calculations about Orthologic pairs, Cyclologic pairs and Perspective pairs didn't take more than one minute calculation time.
I also noticed that for this way of working it is important to gather programming modules in a structured way and combining them later. Because I made Mathematica functions for each QFG-point based on their construction before, I was able to do my recent calculations about Orthologic pairs, Cyclologic pairs and Perspective pairs.
Working with recursive functions (like in Morley's n-Lines) is amazing. Often 3-5 programming rules do a hell of job. Analyzing to reduce things to recursive rules takes most of the time.
I am glad you mentioned Triangle, QF (=Quadri-Figures) as well as PF (=Perspective Fields) geometries. As a matter of fact I love the part about Perspective Fields most. In my spare time I work on it to extend and to simplify it more and more. I made (gathered) several beautiful Mathematica functions (like Francisco's Baricentricas) and the results are often so beautiful, simple and amazing! However they also need explanation. So I plan to make a 3 rd document on Perspective Fields sometime.
Fortunately I sometimes have some results with this theory, that couldn't be solved without using the theory of Perspective Fields, like the calculation of the barycentric coordinates of X(5390) and making a list of competitive ETC-points as functioning in the Thomson Triangle as well as the Reference Triangle.
It's a work in progress.
Nice to discuss these items too.
Best regards,
Chris

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Message: #1001
Date: 28/2/2015 9:02:01
From: chris.vantienhoven
Subject: Mediologic pairs of Triple Triangles

Dear friends,

I will mention here a new “logic relation”, the mediologic relation. When this relation already is known please let me know.
It is only valid for a pair of homothetic triangles. It is introduced here because in EQF so many homothetic Triple Triangles occur.
Two homothetic triangles A1B1C1 and A2B2C2 are Mediologic when the lines from the vertices A1, B1, C1 to the midpoints of sides B2C2, A2C2, and A2B2 are concurrent.
The point of concurrence will be called the Mediologic Center of A1B1C1 with respect to A2B2C2.
If this is the case, then the lines from the vertices A2, B2, C2 to the midpoints of sides B1C1, A1C1, and A1B1 are will be concurrent. This point of concurrence will be called the Mediologic Center of A2B2C2 with respect to A1B1C1.
Here is a list of Mediologic pairs of Triple Triangles in a Quadrangle.

Triple Tr-1 formed by 3 QA-versions of	Triple Tr-2 formed by 3 QA-versions of	Mediologic Center-1	Mediologic Center-2
QG-P1	QG-P2	QA-P10	QA-Px
QG-P1	QG-P4	QA-Px	QA-P5
QG-P1	QG-P8	QA-Px	QA-Px
QG-P1	QG-P15	QA-P5	QA-P20
QG-P1	QL-P12	QA-Px	QA-P1
QG-P2	QG-P4	QA-P1	QA-Px
QG-P2	QG-P8	QA-P1	QA-P20
QG-P2	QG-P15	QA-P1	QA-Px
QG-P2	QL-P12	QA-Px	QA-Px
QG-P3	QL-P15	QA-Px	QA-Px
QG-P4	QG-P8	QA-P10	QA-Px
QG-P4	QG-P15	QA-Px	QA-Px
QG-P4	QL-P12	QA-Px	QA-P1
QG-P5	QG-P10	QA-P24	QA-Px
QG-P5	QL-P2	QA-Px	QA-P1
QG-P7	QG-P9	QA-Px	QA-Px
QG-P7	QL-P6	QA-Px	QA-P1
QG-P8	QG-P15	QA-P20	QA-Px

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   QG-P8        QL-P12        QA-P26       QA-Px
   QG-P9        QL-P6         QA-Px        QA-Px
   QG-P10       QL-P2         QA-P14       QA-Px
   QG-P12       QG-P14        QA-Px        QA-Px
   QG-P15       QL-P12        QA-Px        QA-P1

QA-Px is a QA-point not registered in EQF.

Best regards,
Chris
p.s. We passed the number of 1.000 messages in our group.
Congratulations!
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Message: #1002
Date: 28/2/2015 9:46:57
From: Antreas Hatzipolakis
Subject: Mediologic pairs of Triple Triangles

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>Two homothetic triangles A1B1C1 and A2B2C2 are Mediologic when
> the lines from the vertices A1, B1, C1
> to the midpoints of sides B2C2, A2C2, and A2B2 are concurrent.

Dear Chris
The triangle formed with the midpoints of B2C2, A2C2, and A2B2
is the medial triangle of A2B2C2 and is always mediologic with
A1B1C1 if A1B1C1, A2B2C2 are homothetic, since the homothetic
property is transitive.
In other words: Any homothetic triangles are mediologic.
I have another logic problem.
Let A1B1C1, A2B2C2 be two homothetic triangles and T1, T2, their
orthic triangles, resp
If A1B1C1, T2 are perspective then are A2B2C2, T1 necessarily
perspective?
If the answer is "yes", then we have a logic relation.
APH
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Message: #1003
Date: 28/2/2015 10:24:12
From: chris.vantienhoven
Subject: Mediologic pairs of Triple Triangles

Dear Antreas,
You are right. It is not a "logic relation".
It is a straight forward property of homothetic triangles.
Still it gives interesting new points.
I made a figure of your proposal:
"Let A1B1C1, A2B2C2 be two homothetic triangles and T1, T2,
their orthic triangles, resp. if A1B1C1, T2 are perspective then
are A2B2C2, T1 necessarily perspective?"

it was not dificult to make a figure with two homothetic
triangles A1B1C1 and A2B2C2, where A1B1C1 and T2 are
perspective, however the reverse (A2B2C2 and T1 perspective)
appeared not to be true.

It is quite a challenge to find other "logic relations" !

Best regards,
Chris

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Message: #1004
Date: 01/3/2015 3:01:19
From: seiichikiri
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

---

Dear Chris,
It is a nice information for me that you computed a couple of weeks long.
I see that you have made one of the best databases for QF geometry. When I meet a difficult problem, I hope your kind help as before.
I see that you are preparing one of the best databases for >= Penta-Figure geometry. I would like to see its effectiveness, especially in PF geometry.
Incidentally I would like to request you to coin the words with the following reason:
Our statements in the messages are roughly classified into 4 categories.
1) Graphically confirmed.
2) Numerically confirmed.
3) Algebraically confirmed (or proved).
4) Synthetically confirmed (or proved).
For example, message #995 is a graphically confirmed one and message #556 is a algebraically confirmed one.
So it would be very convenient if the words would be added to the last line of the statement such as “Q. E. D.” for the statement of proof.
Because
1) it is often that a month after I can not remember whether my message is graphically confirmed or algebraically confirmed.
2) if the statement is shown as a graphically or numerically confirmed one, at a glance any member or anyone can prove it algebraically or synthetically.
If you agree with me, I hope that you will coin the corresponding words. I would be very glad that they would be dignified Latin words (Latin is Greek to me).
I expect your 3rd document with intension to use it.
Best regards,
Seiichi.

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Message: #1005
Date: 01/3/2015 7:41:31
From: chris.vantienhoven
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

---

Dear Seiichi,
I think it's a good idea to make the difference between:
1) Graphically confirmed.
2) Numerically confirmed.
3) Algebraically confirmed (or proved).
4) Synthetically confirmed (or proved).
And so to be clear in our messages about the type of confirmation we used.
It will improve the quality of our messages.
I hope to think about it when I write my new messages.
Don't hesitate to ask me when I forgot.
The abbreviation "q.e.d." I only use for synthetical or algebraic proofs, but I think it also can be omitted when the type of confirmation/proof is stated.
Best regards,
Chris

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Message: #1006
Date: 01/3/2015 8:46:03
From: seiichikiri
Subject: 3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple Triangle

---

Dear Chris,
I classify in a simple manner as follows:
1) G.D. (graphically demonstrated).
2) G.D. (numerically demonstrated).
3) A.D. (algebraically demonstrated).
4) S.D. (synthetically demonstrated).
I will always add these words at the last line of the statement.
Best regards,
Seiichi.

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Message: #1007
Date: 02/3/2015 12:46:10
From: seiichikiri
Subject: A conic made from isogonal center (QA-P4)

Dear friends,
Let a quadrangle be P1P2P3P4 and its isogonal center P.
Denote:
N12 = the projection of P on P1P2,
N13 = the projection of P on P1P3,
N14 = the projection of P on P1P4,
N23 = the projection of P on P2P3,
N24 = the projection of P on P2P4,
N34 = the projection of P on P3P4,
P, N12, N13, N14, N23, N24 and N34 are on a conic.
G.D. (graphically demonstrated.)
Is this conic already known?
Best regards,
Seiichi.

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Message: #1008
Date: 02/3/2015 5:58:15
From: César Lozada
Subject: A conic made from isogonal center (QA-P4)

```
Dear Seiichi:
Algebraically confirmed.
1st CT coordinate of the center of the conic is:
2*a*b*c*(a^2*y^2*z^2+x^2*(y^2*SB+z^2*SC)) -
x*y*z*((3*S^2-SB*SC)*a*x+(S^2-SB*SC-2*SB^2)*b*y+(S^2-SB*SC-2*SC^
2)*c*z)
I regret not being able to give you more information. I have not
built yet a database of EQF.
Note I use x,y,z instead of p,q,r because I'm afraid to confuse
r with inradius.
By the way, I wasted some time because I didn´t read carefully
the way in which QA-P4 is described in EQF.
I took the construction literally as specified, i.e,
intersecting lines joining Qi, the isogonal conjugate of Pi wrt
PjPkPl, and Qi*, the inverse of Qi wrt circle (Pj,Pk,Pl).
In this way calculus are horrendous. Algebraically is simpler to
find QA-P4 just by intersecting lines joining Qi with Oi, the
circumcenter of PjPkPl.
Best regards
César Lozada
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Message: #1009
Date: 02/3/2015 9:38:52
From: chris.vantienhoven
Subject: A conic made from isogonal center (QA-P4)

```
Dear Seiichi and Cesar,
I noticed
    QA-P4 lies on the conic and
    QA-P2 is the center (G.D.).
Being so I am not quite sure if Cesars calculation of the center
is correct.
Question: I wonder which is the locus of points for which the 6
reflections are coconic.
Best regards,
Chris
```

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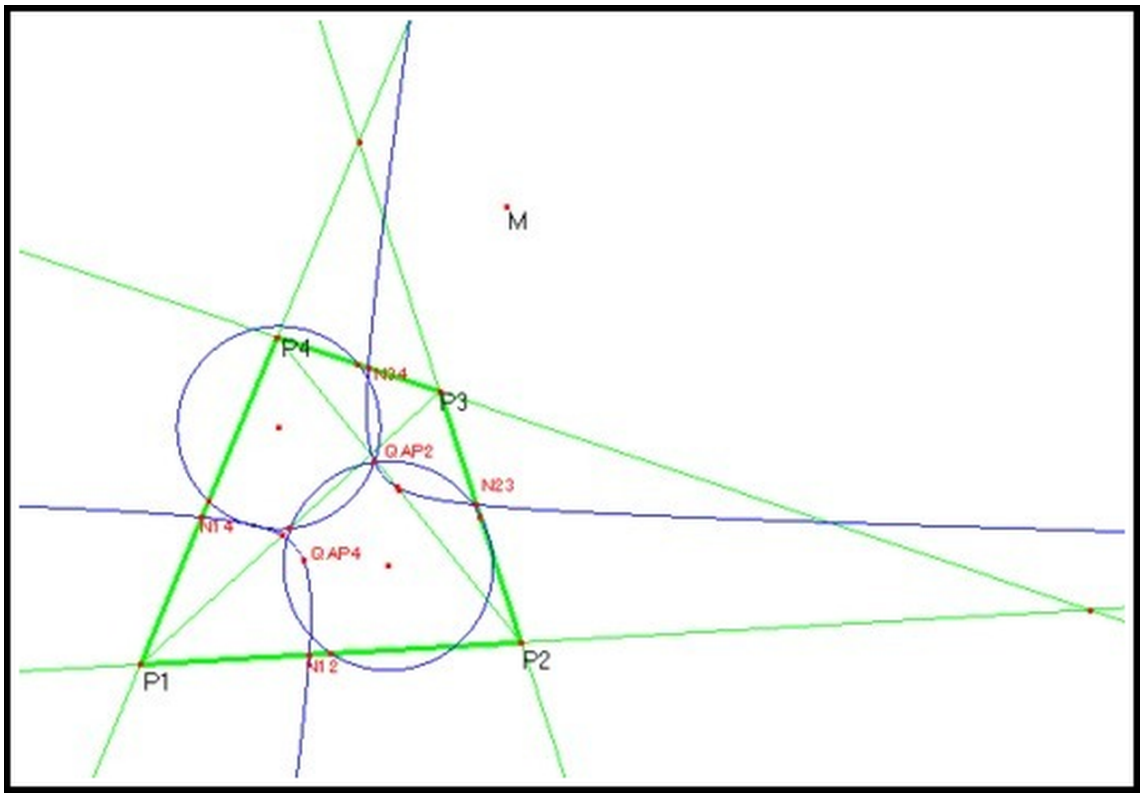
Message: #1010
Date: 02/3/2015 12:57:06
From: seiichikiri
Subject: A conic made from isogonal center (QA-P4)

Dear Cesar, dear Chris,
It seems to me that Cesar's point is new and QA-P2 is on the conic.
See the attached picture.
Best regards,
Seiichi.



QP-P4conic.png

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Message: #1011
Date: 02/3/2015 1:11:08
From: chris.vantienhoven
Subject: A conic made from isogonal center (QA-P4)

Dear Seiichi and Cesar,
Seiichi, you are right.
I made a conic from the reflections of QA-P4 in the sides of the Quadrangle.
They are also coconic.
Sorry for the mistake.
Best regards,
Chris

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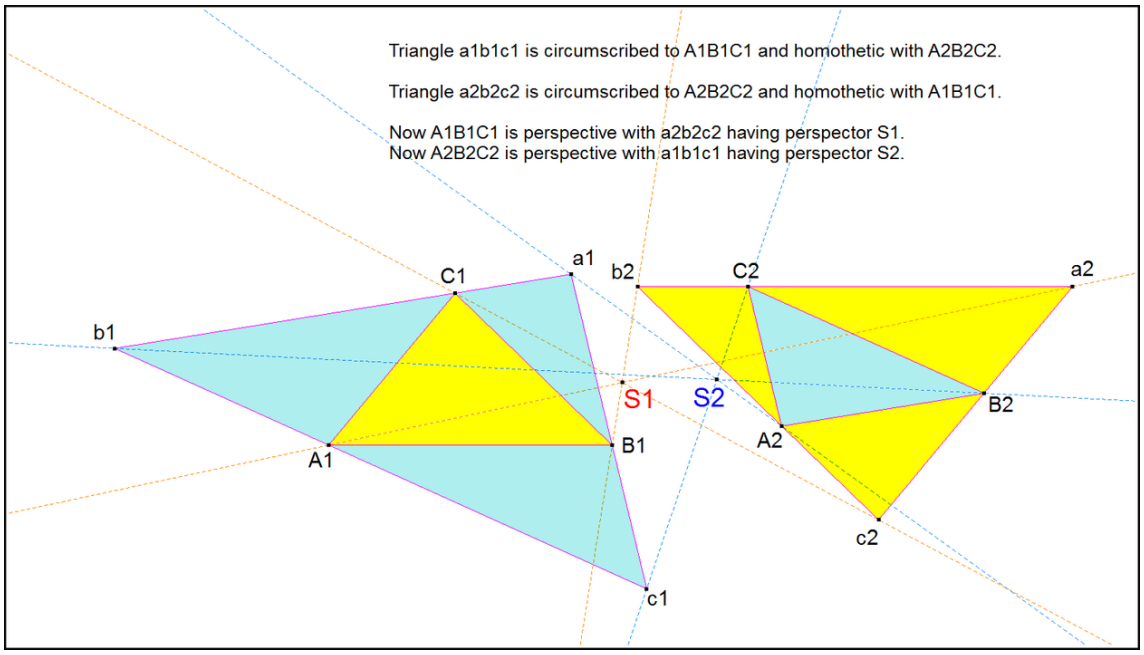
Message: #1012
Date: 03/3/2015 10:36:45
From: Chris van Tienhoven
Subject: Relations between Triple Triangles

Dear Antreas and Seiichi and friends,
I found another relation between two triangles A1B1C1 and A2B2C2 although it is not a “logic relation”. I don’t think it is quite new.
Let triangle a1b1c1 be circumscribed to A1B1C1 and homothetic with A2B2C2.
Let triangle a2b2c2 be circumscribed to A2B2C2 and homothetic with A1B1C1.
Now A1B1C1 is perspective with a2b2c2 having perspector S1.
Now A2B2C2 is perspective with a1b1c1 having perspector S2.
In a way you build Mutual AntiComplementary Triangles (MAT) wrt the other triangle.
Maybe you know a better name.
See attached file.
Of course it is only interesting when it delivers nice mutual centers. And it does!
This is what I found.

Triple Tr-1 formed by 3 QA-versions of	Triple Tr-2 formed by 3 QA-versions of	MAT Center-1	MAT Center-2
QG-P1	QG-P2	QA-P10	QA-Px
QG-P1	QG-P4	QA-Px	QA-P5
QG-P1	QG-P10	QA-P14	QA-Px
QG-P1	QG-P12	QA-Px	QA-P18
QG-P1	QG-P14	QA-Px	QA-P10
QG-P1	QG-P15	QA-P5	QA-P20
QG-P1	QL-P2	QA-P14	QA-Px
QG-P1	QL-P8	QA-Px	QA-P10
QG-P1	QL-P12	QA-Px	QA-P1
QG-P1	QL-P18	QA-Px	QA-P5
QG-P1	QL-P20	QA-Px	QA-P2
QG-P2	QG-P3	QA-Px	QA-P10
QG-P2	QG-P4	QA-P1	QA-Px
QG-P2	QG-P8	QA-P1	QA-P20
QG-P2	QG-P12	QA-P10	QA-P1
QG-P2	QG-P14	QA-Px	QA-P10
QG-P2	QG-P15	QA-P1	QA-Px
QG-P2	QG-P17	QA-Px	QA-P10
QG-P2	QL-P8	QA-Px	QA-P10
QG-P2	QA-CT1	QA-P10	QA-Px

QG-P3	QG-P4	QA-P1	QA-Px
QG-P3	QG-P8	QA-P1	QA-Px
QG-P3	QG-P15	QA-P1	QA-Px
QG-P4	QG-P8	QA-P10	QA-Px
QG-P4	QG-P12	QA-P1	QA-Px
QG-P4	QG-P17	QA-Px	QA-P1
QG-P4	QL-P8	QA-Px	QA-P10
QG-P4	QL-P12	QA-Px	QA-P1
QG-P4	QL-P18	QA-Px	QA-P25
QG-P4	QA-CT1	QA-P1	QA-Px
QG-P5	QG-P7	QA-P3	QA-P1
QG-P5	QG-P8	QA-Px	QA-P24
QG-P5	QG-P9	QA-P3	QA-P1
QG-P5	QG-P10	QA-P24	QA-Px
QG-P5	QL-P2	QA-Px	QA-P1
QG-P5	QL-P4	QA-P9	QA-Px
QG-P5	QL-P6	QA-P3	QA-Px
QG-P5	QL-P7	QA-Px	QA-P1
QG-P5	QL-P9	QA-Px	QA-P1



QA-Tr-MutualAntiComplemetaryCenters-01.png

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Message: #1013
Date: 04/3/2015 2:14:10
From: seiichikiri
Subject: Relations between Triple Triangles

Dear Chris, dear Antreas,
I think that your relation is a very clear one and MAT is a good name.
But I knew "MAT" several years ago in the field of gas turbine engineering.
See: http://www.sciencedirect.com/science/article/pii/S019689049␣
8000880
Dr. Utamura is a friend of mine and a successor of successors of successors of van der Waals.
Their fields are different. If there is no better name, I think that MAT is good.
Best regards,
Seiichi.

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Message: #1014
Date: 04/3/2015 12:38:59
From: seiichikiri
Subject: Relations between Triple Triangles

Dear Chris,
I hope that, given the vertex matrices of A1B1C1 and A2B2C2, the formulas of the coordinates of S1 and S2 will be shown.
Best regards,
Seiichi.

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Message: #1015
Date: 04/3/2015 9:03:57
From: chris.vantienhoven
Subject: Relations between Triple Triangles

```
Dear Seiichi,
Regarding the MAT centers.
Let
A1=(1:0:0), B1=(0:1:0), C1=(0:0:1)
A2=(x1:y1:z1), B2=(x2:y2:z2), C2=(x3:y3:z3)
then
S1 = (x2 x3 y1-x1 x3 y2+x2 x3 z1+x1 y3 z2-x1 x2 z3-x1 y2 z3 :
      x2 y1 y3-x1 y2 y3+x3 y2 z1+y1 y3 z2-x1 y2 z3-y1 y2 z3 :
      x3 z1 z2+y3 z1 z2+x2 y1 z3-x1 y2 z3-y2 z1 z3-x1 z2 z3 )
S2 = (x1/(x1+y1+z1) : y2/(x2+y2+z2) : z3/(x3+y3+z3))
Not too complicated I would say.
Best regards,
Chris
```

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Message: #1016
Date: 06/3/2015 8:54:33
From: chris.vantienhoven
Subject: Relations between Triple Triangles

```
Dear Antreas, Seiichi and friends,
I wrote about
Let triangle a1b1c1 be circumscribed to A1B1C1 and homothetic
with A2B2C2.
Let triangle a2b2c2 be circumscribed to A2B2C2 and homothetic
with A1B1C1.
Now A1B1C1 is perspective with a2b2c2 having perspector S1.
Now A2B2C2 is perspective with a1b1c1 having perspector S2.
I noticed that when inscribing triangles instead of
circumscribing triangles we also get two perspectors.
These perspectors are identical only mutually changed (AD/GD).
The synthetical proof is very simple.
Best regards,
Chris
```

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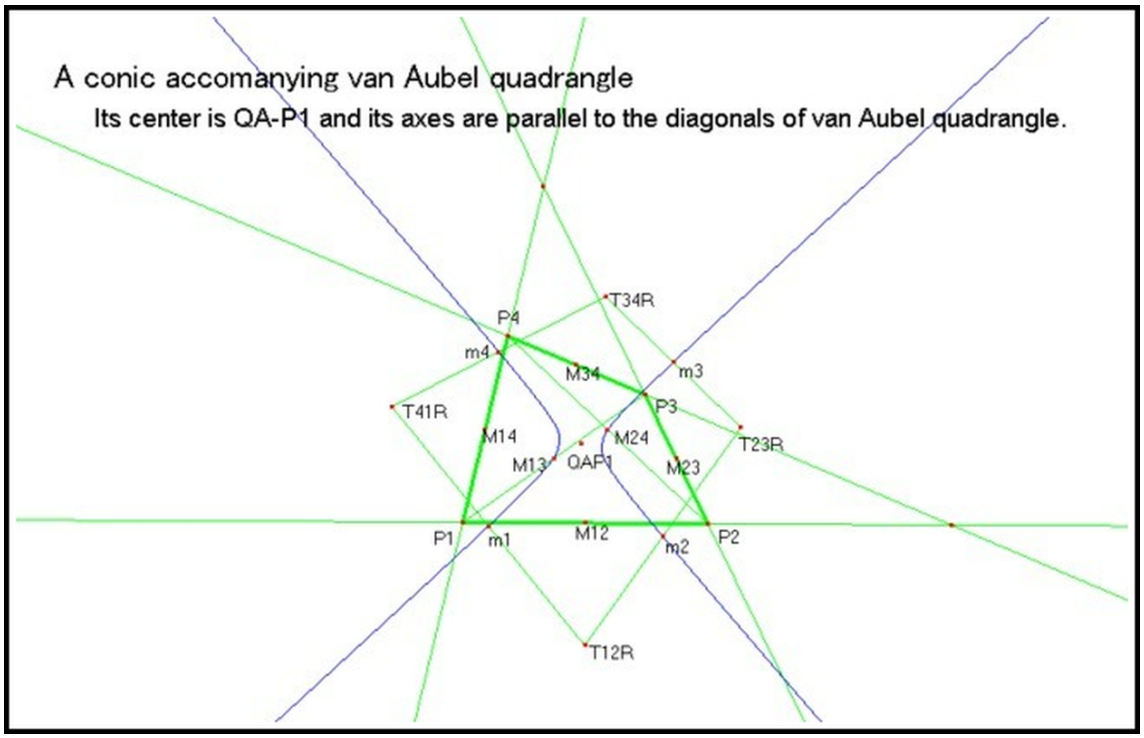
Message: #1017
Date: 08/3/2015 9:05:49
From: seiichikiri
Subject: A conic accompanying van Aubel quadrangle

Dear friends,
Let P1P2P3P4 be a quadrangle and its van Aubel quadrangle T12RT23RT34RT41R.
See the attached picture.
Denote:
M13, M24= the midpoints of P1P3, P2P4 respectively.
m1, m2, m3, m4= the midpoints of T41RT12R, T12RT23R, T23RT34R, T34RT41R respectively.
m1, m2, m3, m4, M13 and M24 are on a conic.
Its center is QA-P1 and its axes are parallel to the diagonals of van Aubel quadrangle. G.D. (graphically demonstrated.)
van Aubel quadrangle: see
http://mathworld.wolfram.com/vanAubelsTheorem.html
van Aubel quadrangle has its vertices of the centers of squares in the description of "van Aubels Theorem" in mathworld.wolfram.com.
Best regards,
Seiichi.
P.S. an addition to the properties of van Aubel quadrangle.



Conic-van Aubel.png

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Message: #1018
Date: 11/3/2015 3:15:04
From: seiichikiri
Subject: Translation of a locus property of a triangle to quadrangle curve

---

Dear friends,
We translate a locus property of a triangle curve to that of a quadrangle.
Given a triangle P1P2P3 and a point P, we denote the projections of P on P2P3, P1P3, P1P2 by P23, P13, P12 respectively.
K004 in CTC is the locus such that P1P23, P2P13, P3P12 are concurrent.
In a similar manner, given a quadrangle P1P2P3P4 and a point P, we denote the projections of P on P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by P12, P13, P14, P23, P24, P34 respectively.
The following is the locus such that P12P34, P13P24, P14P23 are concurrent.
Cyclic sum (a^2b^2p(p+q)+a^2c^2p(p+r)-a^4(p+q)(p+r))yz(ry-qz)
=0.
QA-P4 satisfies the curve and the concurrent point is QA-P6.
Best regards,
Seiichi.

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Message: #1019
Date: 11/3/2015 10:50:31
From: seiichikiri
Subject: 2 quadrangle curves of QA-P1

Dear friends,
Given a quadrangle P1P2P3P4 and a point P, we denote the midpoints of P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by M12, M13, M14, M23, M24, M34 respectively. We denote the intersections of P1P2 and PM34, P1P3 and PM24, P1P4 and PM23, P2P3 and PM14, P24 and PM13, P3P4 and PM12 by P12, P13, P14, P23, P24, P34 respectively.
The following are the loci of P such that P12P34, P13P24, P14P23 are concurrent.
Cyclic sum (2p+q+r)pyz(ry-qz)=0. (QA-Cu5).
Cyclic sum (2p+q+r)(q+r)pyz(y-z)=0.
QA-P1 satisfies the curve and the concurrent point is QA-P1.
Best regards,
Seiichi.

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Message: #1020
Date: 11/3/2015 11:04:41
From: chris.vantienhoven
Subject: Translation of a locus property of a triangle to quadrangle curv

---

Dear Seiichi,
This cubic is QA-Cu1.
Nice approach and nice new property of QA-Cu1 !
I cannot confirm that QA-P6 is on the cubic.
I will make a note of this property in EQF.
Best regards,
Chris

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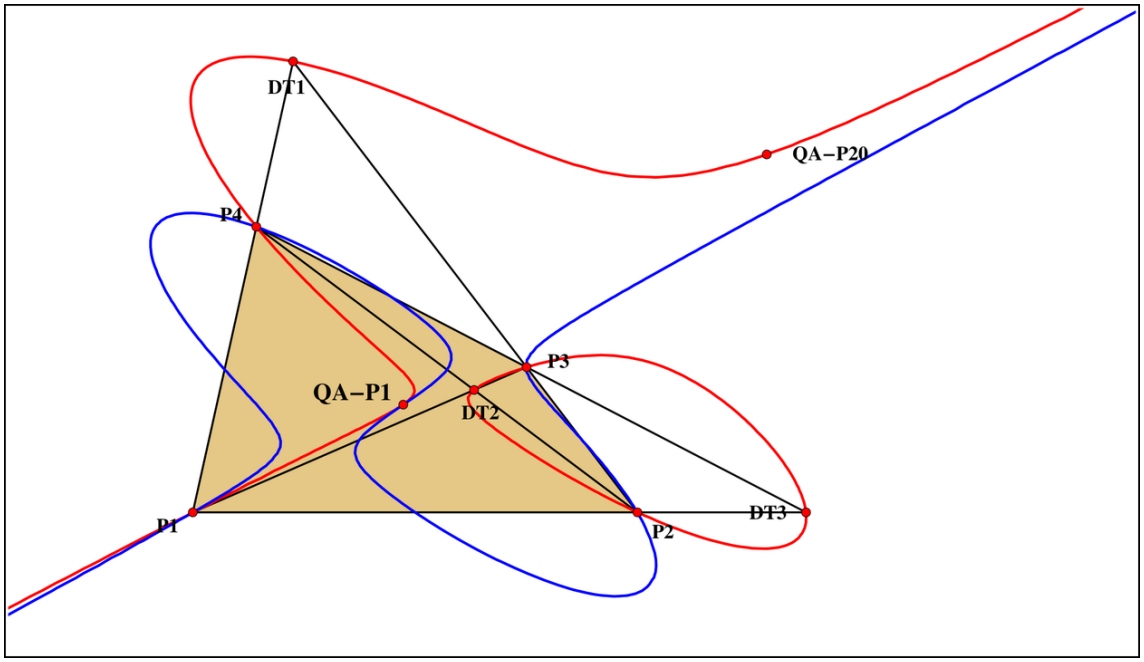
Message: #1021
Date: 11/3/2015 12:16:11
From: chris.vantienhoven
Subject: 2 quadrangle curves of QA-P1

Dear Seiichi.
Nice this Double Cubic.
The first cubic QA-Cu5 passes through:
* The 4 vertices of the Reference Quadrangle
* The 3 vertices of the Diagonal Triangle
* QA-P1, QA-P20
The second cubic passes through:
* The 4 vertices of the Reference Quadrangle
* The 6 midpoints of the Reference Quadrangle
* QA-P1
I made a picture of both cubics. See attachment.
Best regards,
Chris



QA-Cu5-and-other-Cubic.pdf

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Message: #1022
Date: 12/3/2015 6:24:52
From: seiichikiri
Subject: Translation of a locus property of a triangle to quadrangle curv

---

Dear Chris,
QA-P6 is not on the cubic. It is the image of QA-P4, as shown near the last lines of QA-P6 in EQF.
Addition to translation:
1) Triangle (K001): given a triangle P1P2P3 and a point P, we denote the reflections of P in P2P3, P1P3, P1P2 by P23, P13, P12 respectively. P1P23, P2P13, P3P12 are concurrent.
Its corresponding quadrangle: given a quadrangle P1P2P3P4 and a point P, we denote the reflections of P in P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by P12, P13, P14, P23, P24, P34 respectively. P12P34, P13P24, P14P23 are concurrent.
2) Triangle (K002): given a triangle P1P2P3 and a point P, we denote the projections of P on P2P3, P1P3, P1P2 by P23, P13, P12 respectively. We denote the symmetric points of P23, P13, P12 in P by Q23, Q13, Q12 respectively.
P1Q23, P2Q13, P3Q12 are concurrent.
Its corresponding quadrangle: given a quadrangle P1P2P3P4 and a point P, we denote the projections of P on P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by P12, P13, P14, P23, P24, P34 respectively. We denote the symmetric points of P12, P13, P14, P23, P24, P34 in P by Q12, Q13, Q14, Q23, Q24, Q34 respectively.
Q12Q34, Q13Q24, Q14Q23 are concurrent.
Best regards,
Seiichi.

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Message: #1023
Date: 12/3/2015 2:35:44
From: seiichikiri
Subject: Translation of a locus property of a triangle to quadrangle curv

---

Dear Chris,
Added 1) and 2) are not interesting. In case of quadrangle,
QA-Cu1 is also their solution.
Best regards,
Seiichi.

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Message: #1024
Date: 13/3/2015 9:29:29
From: seiichikiri
Subject: A quadrangle curve concerning QA-P2

Dear friends,
Given a quadrangle P1P2P3P4 and a point P, we denote the lines through P perpendicular to P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by L12, L13, L14, L23, L24, L34 respectively. We denote the intersections of P1P2 and L34, P13 and L24, P1P4 and L23, P23 and L14, P24 and L13, P34 and L12 by P12, P13, P14, P23, P24, P34 respectively.
The following is the locus such that P12P34, P13P24, P14P23 are concurrent.
Cyclic sum (a^2b^2p^2r^2(p+q)+a^2c^2p^2q^2(p+r)-a^2b^2p^2qr(p+q)
-a^2c^2p^2qr(p+r)-b^2c^2p^3(q+r)^2-2b^2c^2p^2qr(q+r)+b^4p^2(p+q)
(q+r)r+c^4p^2(p+r)(q+r)q)yz(ry-qz) = 0.
QA-P2 satisfies the curve. (A.D., algebraically demonstrated)
Best regards,
Seiichi.

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Message: #1025
Date: 15/3/2015 4:17:10
From: seiichikiri
Subject: Orthology between the diagonal triangle and a triple triangle

---

Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
S1, S2, S3 =the intersections of P1P3 and P2P4, P1P2 and P3P4, P1P4 and P2P3 respectively.
S1S2S3 is the diagonal triangle.
M12, M13, M14, M23, M24, M34 =the midpoints of P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 respectivly.
bm1, bm2, bm3 = the lines through M13 and M24, M12 and M34, M14 and M23 respectively.
N12, N13, N14, N23, N24, N34 =the projections of QA-P4 on P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 respectivly.
bp1, bp2, bp3 =the lines through N13 and N24, N12 and N34, N14 and N23 respectively.
T1, T2, T3 = the intersections of bm1 and bp1, bm2 and bp2, bm3 and bp3 respectively.
T1T2T3 is a triple triangle.
S1S2S3 and T1T2T3 are orthologic.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.

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Message: #1026
Date: 15/3/2015 4:38:12
From: chris.vantienhoven
Subject: Orthology between the diagonal triangle and a triple triangle

---

Dear Seiichi,
That's a nice way to produce a new Triple Triangle!
I looked for the Orthologic Centers involved (AD&GD = Algebraically and Graphically Determined)
The Orthologic Center of triangle T1.T2.T3 wrt the diagonal triangle is QA-P2.
The Orthologic Center of the Diagonal Triangle wrt triangle T1.T2.T3 is a point lying on:
* The Orthogonal Hyperbola QA-Co4
* The line QA-P3.QA-P4
* The line through QA-P12 parallel to QA-P2.QA-P23
* The line through QA-P20 parallel to QA-P1.QA-P2
* The line through QA-P30 parallel to QA-P1.QA-P32
Moreover
* Triangle T1.T2.T3 is also Orthologic with the Triple Triangles of QG-P2, QG-P4 till QG-11, QG-P15, QL-P6, QL-P12.
* Triangle T1.T2.T3 is Perspective with the Triple Triangles of QG-P1, QG-P12, QL-P5, QL-P7, QL-P12, QL-P20, QL-P22, QL-P23. All with perspector QA-P1.
* Triangle T1.T2.T3 is Cyclologic with the Triple Triangle of QL-P30, which is Morley's 2 nd Circle Center (will be mentioned later in EQF).
(AD = Algebraically Determined)
Best regards,
Chris

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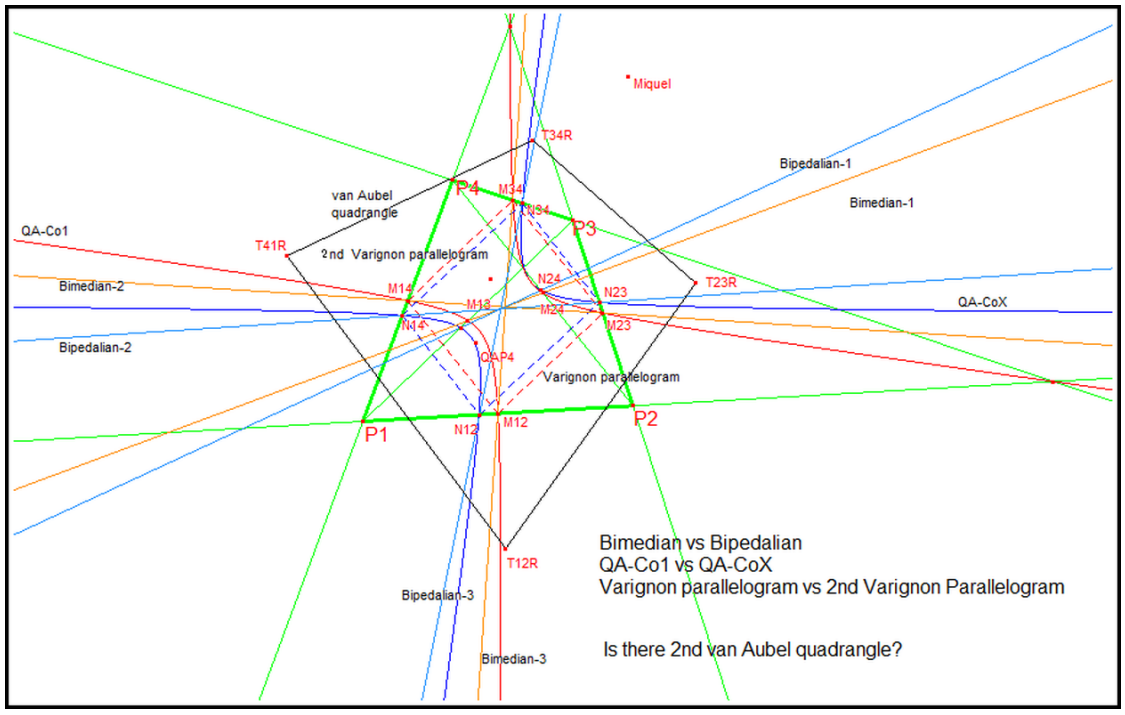
Message: #1027
Date: 16/3/2015 5:13:07
From: seiichikiri
Subject: Orthology between the diagonal triangle and a triple triangle

---

Dear Chris, dear friends,
Thank you very much for the elaborate computation.
I would like to propose the problem again, which I did in message #149.
Is there 2nd van Aubel quadrangle?
Circumstantial evidences:
1) We call the lines denoted by bm1, bm2 and bm3 “bimedian”. I would like to call the lines denoted by bp1, bp2 and bp3 “bipedalians”. Then, for bimedians, there are bipedalians.
2) For Varignon parallelogram, there is 2 nd Varignon parallelogram. See message #149.
3) For Conic QA-Co1in EQF, there is the conic shown in message #1007.
4) They coincide with each other when the quadrangle become cyclic. In the reverse expression, they split into two different ones when the cyclic quadrangle become a general one.
For van Aubel quadrangle, there must be the 2 nd van Aubel quadrangle.
See the attached Cabri picture.
Best regards,
Seiichi.



2nd van Aubel-fig.png

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Message: #1028
Date: 16/3/2015 12:19:28
From: seiichikiri
Subject: Orthology between the diagonal triangle and a triple triangle

---

Dear Chris, dear friends,
Addition of the requirement of 2 nd van Aubel quadrangle:
If we denote the vertices of 2 nd van Aubel quadrangle corresponding to T12R, T23R, T34R, T41R by U12R, U23R, U34R, U41R respectively, the second intersection of the circumcircles of U12RM24U23R and U34RM24U41R coincides with that of the circumcircles of U23RM13U34R and U41RM13U12R.
N. B. This statement is the same requirement as that of van Aubel quadrangle.
Best regards,
Seiichi.

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Message: #1029
Date: 17/3/2015 10:49:51
From: chris.vantienhoven
Subject: A conic made from isogonal center (QA-P4)

Dear Seiichi,
[SK]
Let a quadrangle be P1P2P3P4 and its isogonal center P.
Denote:
N12= the projection of P on P1P2,
N13= the projection of P on P1P3,
N14= the projection of P on P1P4,
N23= the projection of P on P2P3,
N24= the projection of P on P2P4,
N34= the projection of P on P3P4,
P, N12, N13, N14, N23, N24 and N34 are on a conic.
Is this conic already known?
This conic is new to me.
I noticed that QA-P2 and QA-P4 lie on this conic.
QA-P6 is its center. (GD = Graphically Demonstrated).
I am still working on its expression.
I also looked for the locus of points with the property that the 6 pedal points are coconic.
The locus is the line at infinity as well as a septic (7th degree) with large expression. (AD = Algebraically Demonstrated).
Points on this septic are QA-P4 and the vertices of the QA-Diagonal Triangle (QA-Tr1) as well as the vertices of the Miquel Triangle (QA-Tr2). (ND = Numerically Demonstrated).
Best regards,
Chris

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Message: #1030
Date: 18/3/2015 9:34:56
From: seiichikiri
Subject: A conic made from isogonal center (QA-P4)

Dear Chris,
I also computed the locus of coconicity of the 6 pedal points in message #1007, with the infinity line and a large septic expression. (A.D. algebraically demonstrated.)
I computed the locus of coconicity of the 6 points in message #1019, with a large dodecadic expression. (A.D. algebraically demonstrated.)
Best regards,
Seiichi.

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Message: #1031
Date: 18/3/2015 8:51:02
From: eckart_schmidt@t-online.de
Subject: A conic made from the isogonal center (QA-P4)

Dear Seiichi, dear Chris,
I think, you will find the septic and its construction in message 503 (EQF-Note 2014-04-14).
Best regards Eckart

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Message: #1032
Date: 19/3/2015 1:48:16
From: bernard.keizer
Subject: 64 axes of the QL

Dear Chris, dear Eckart, dear Seeichi
I see you are completely involved with the QA.
Congratulations for all these new properties I read regularly with interest !
During all this time, I was always trying to find the construction of the 64 axes of the QL and I've finally succeeded in cracking this last Morley code.
In fact, Morley gave the principle, which is very simple, but not the construction, which is a little bit more complicate !
All we have to do is to consider 4 lines tangent to the tetracardioïds with one double the same way we have considered a triangle with 3 sides tangent to the cardioïds one side being double in order to find the 9 Morley lines.
The difference is that a step is enough for the triangle with trisection of angles, whereas 2 steps are needed for the QL, with quadrisection of angles and double as well as triple tangents (see attached file)
I hope now you will be able to reproduce my construction and to calculate the coordinates of the 256 centers of the tetracardioïds tangent to 5 lines in order to find the equation of a curve to compare with the cubic stelloïd carrying the 27 centers of cardioïds tangent to the 4 lines of the QL.
Best regards
Bernard

64 axes of the QL

A. Theory

The fundamental property is mentionned by F. Morley in an article published in 1929 « Extensions of Clifford's Chain-Theorem » ; an analytical proof is given by R. Bricard in an article published in 1923 « Sur les droites moyennes d'un triangle ».
Considering a n-line, the mean lines of the n (n-1)-lines are concurring.
This property has 2 possible applications :

a) n different lines : there are $(n-1)^{n-1}$ incenters, centers of the ennacardioïds tangent to the n lines
b) n-1 different lines : there are $(n-1)^{n-2}$ lines, called axes of the (n-1)-lines, which are the locus of the centers of the same ennacardioïds tangent to the n-1 lines and bitangent to one of the n-1 lines
The points in a) are on the n sets of lines in b) with n axes on a point and n-1 points on an axis

B. Examples

1. QL : n = 4
   a) There are 27 centers of cardioïds tangent to 4 lines
   b) For each triangle, there are 9 Morley lines locus of the centers of the cardioïds tangent to the the 3 sides and bitangent to one of the 3 sides
   The 27 centers are on the 4*9 Morley lines of the 4 triangles with 4 lines in a point and 3 points on each line
2. 5-L : n = 5
   a) There are 256 centers of tetracardioïds tangent to 5 lines
   b) For each QL, there are 64 axes locus of the centers of the tetracardioïds tangent to the 4 lines and bitangent to one of the 4 lines
   The 256 centers are on the 5*64 axes of the 5 QL's with 5 axes on a point and 4 points on each axis

C. Construction

Very important remark : we know the direction of all the n-sectors or meanlines m° π/n.
Let's call m(Di, Dj, Dk …) the mean line of Di, Dj, Dk …

1. Triangle ABC

   Let's choose one side BC as double line and let's draw the 3 trisectors in B and C (near to BC, in one conterclockwise and in the other clockwise)
   We get 9 points called I11 to I33, forming equilateral triangles on 3 circles through the 2 points B and C
   In each of these 9 points, 4 lines concur : 2 trisectors in B and C and 2 meanlines of the 3 sides or Morley lines of the triangle
   In other words, considering the 4 lines as Di, Di, Dj and Dk (Di is a double line), the 2 trisectors are m(Di, Di, Dj) and m(Di, Di, Dk) and the 2 axes are 2 Morley lines m(Di, Dj, Dk)
   The 2 axes m(Di, Dj, Dk) are the lines joining the intersection point of the 2 trisectors to the 2 other vertices of the equilateral triangle on the same circle
   In order to have a complete figur, we have to do the same construction for the 2 other sides of the triangle.

2. QL

   We have to proceed this time in 2 steps
   Let's choose one line as double or triple line and let's draw the 4 quadrisectors in 2 vertices B and C on this line (near to BC, in B conterclockwise and in C clockwise)
   We get the same way 16 points called I11 to I44, forming 4 squares on 4 circles through the 2 points B and C

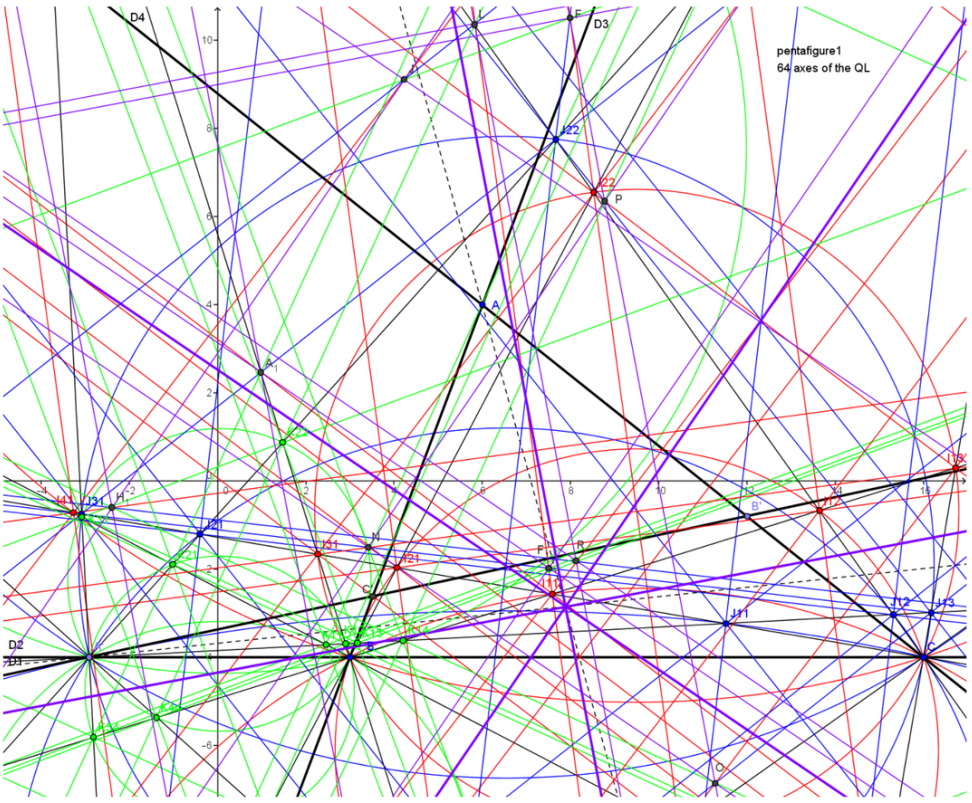
   a) 1rst step
      Consider the 5 lines Di, Di, Di, Dj and Dk (Di is a triple line)
      In each of these 16 points, 5 lines concur : 2 quadrisectors in B and C as m(Di, Di, Di, Dj) and m(Di, Di, Di, Dk) and 3 meanlines of the type m(Di, Di, Dj, Dk) (with Di as a double line this time)
      The 3 lines m(Di, Di, Dj, Dk) are the lines joining the intersection point of the 2 quadrisectors to the 3 other vertices of the square on the same circle through the 2 vertices B and C

   b) 2nd step
      There are 3 triangles having one side on this choosen line and there are the same way 16 points J11 to J44 and 16 points K11 to K44
      Consider now the 5 lines Di, Di, Dj, Dk and Dl (Di is a double line)
      We find new intersection points where 5 lines concur : 3 of the preceding mean lines m(Di, Di, Dj, Dk), m(Di, Di, Dj, Dl) and m(Di, Di, Dk, Dl) and 2 of the searched 64 axes of the QL m(Di, Dj, Dk, Dl)

64 axes of the QL.docx

It appears that there are on each mean line of the type m(Di, Di, Dj, Dk) belonging to one set 2 points like I, J or K and 3 points of intersection with the 2 other sets and on each of the axes of the QL again 3 of these intersection points
For one line choosen as double or triple line, there are 3*16 = 48 points like I, J or K and 3*24 = 72 mean lines in the 1rst step, which intersect in 72 points on 48 axes in the 2nd step (a group of 6 points give 4 axes making angles of π/4 or π/2 with 3 points on each axis)
In order to have a complete figur with 64 axes, we have to do the same construction for the 3 other lines of the QL.

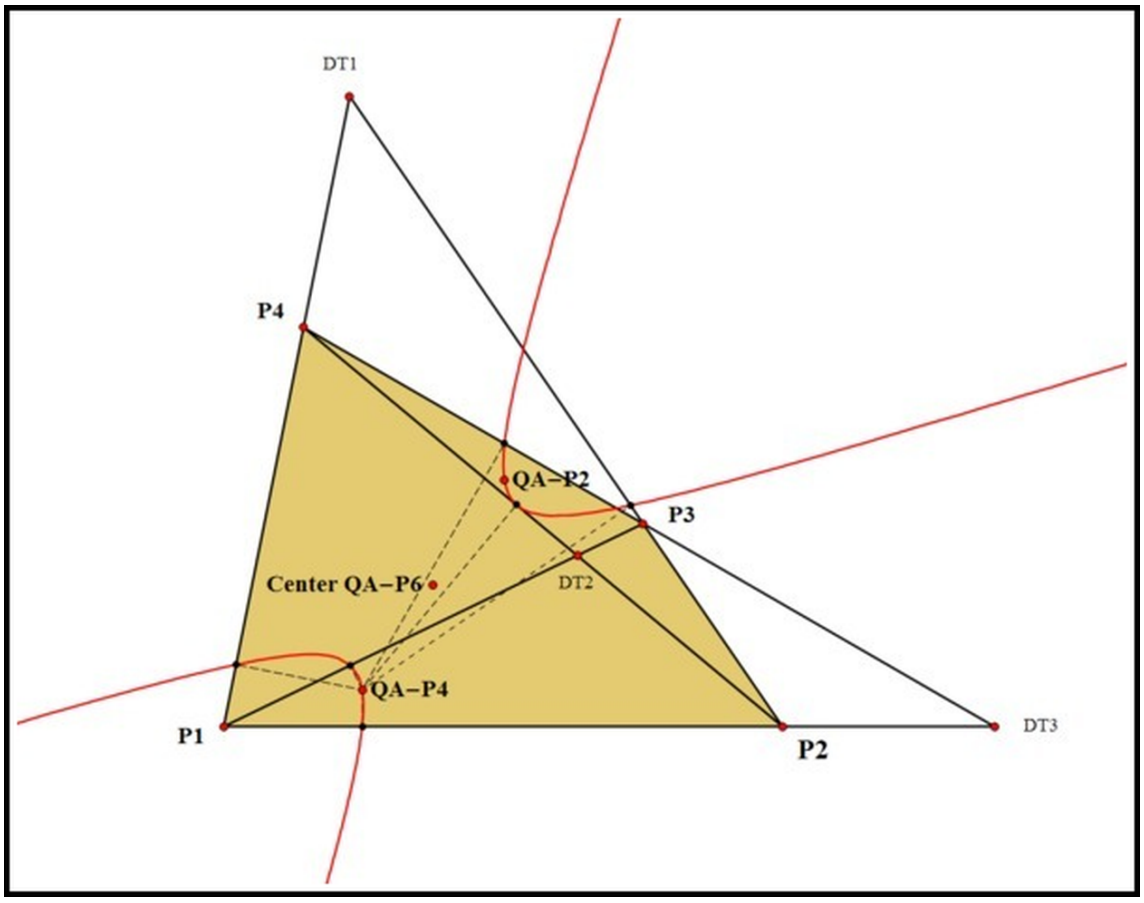
On the figur below, D2 is the double or triple tangent, the 3 sets are in red (ABC), blue (A'B'C) and green (A'BC') and the axes are in purple



64 axes of the QL.docx

Message: #1033
Date: 19/3/2015 1:48:40
From: chris.vantienhoven
Subject: A conic made from isogonal center (QA-P4)

Dear Seiichi and Eckart,
Wow it looks like that this septic is well investigated.
Eckart, good to see your paper about this conic again!
I had quite forgotten it.
I calculated the Pedal Conic of QA-P4 and it is quite elaborate and long.
However the picture is nice. See attachment.
Algebraically it comes out that QA-P2 and QA-P4 lie on the conic and that QA-P6 is its center.
Best regards,
Chris



QA-Co99-QAP4-Pedal-Conic-01.gif

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Message: #1034
Date: 20/3/2015 4:56:42
From: seiichikiri
Subject: A conic made from the isogonal center (QA-P4)

Dear Eckart, dear friends,
Thank you very much for your information.
I think that there are 4 interesting conics in a quadrangle, which pass the following points.
1) QA-Co1: the midpoints Mij of PiPj, the vertices of DT.
2) QA-CoX: the projections of QA-P4 on PiPj, QA-P4.
3) QA-CoY: the reflections of QA-P4 in Mij.
G.D. (graphically (Cabri) demonstrated.)
4) QA-CoZ: the reflections of QA-P4 in PiPj, QA-P4.
If we consider the loci of the coconic projections and the coconic reflections, the vertices of DT, Miquel triangles and QA-P4 are on them.
G.D. (graphically (Cabri) demonstrated.)
They seems very similar. I hope that their identity or difference will be confirmed, because I failed to compute the case of the reflections.
Best regards,
Seiichi.
B. S. If the quadrangle is cyclic, 1) and 2), 3) and 4) coincides respectively.

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Message: #1035
Date: 20/3/2015 7:15:59
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris, dear Eckart, dear Seiichi
I realise part of sentences were missing in my previous attached file, which made the formulation wrong : for the n-line, the axes are the locus of the centers of the ennacardioïds (curves defined as tangent to n lines) tangent to n-1 lines and they intersect at angles of pi/n in points which are the centers of the same curves bitangent to one of the lines.
This doesn't change the construction
I join the corrected attached file (maybe Chris could destroy the other one)
Best regards
Bernard

64 axes of the QL

A. Theory

The fundamental property is mentionned by F. Morley in an article published in 1929 « Extensions of Clifford's Chain-Theorem » ; an analytical proof is given by R. Bricard in an article published in 1923 « Sur les droites moyennes d'un triangle ».
Considering a n-line, the mean lines of the n (n-1)-lines are concurring.
This property has 2 possible applications :

a) n different lines : there are $(n-1)^{n-1}$ incenters, centers of the ennacardioïds tangent to the n lines
b) n-1 different lines : there are $(n-1)^{n-2}$ lines, called axes of the (n-1)-lines, which are the locus of the centers of the same ennacardioïds tangent to the n-1 lines ; the axes intersect at angles of π/n in points which are the centers of the ennacardioïds bitangent to one of the n-1 lines
The points in a) are on the n sets of lines in b) with n axes on a point and n-1 points on an axis

B. Examples

1. QL : n = 4
   a) There are 27 centers of cardioïds tangent to 4 lines
   b) For each triangle, there are 9 Morley lines locus of the centers of the cardioïds tangent to the the 3 sides ; the Morley lines intersect at angles of $\pi/3$ in 27 points which are the centers of the cardioïds bitangent to one of the 3 sides
   The 27 centers in a) are on the 4*9 Morley lines of the 4 triangles with 4 lines in a point and 3 points on each line
2. 5-L : n = 5
   a) There are 256 centers of tetracardioïds tangent to 5 lines
   b) For each QL, there are 64 axes locus of the centers of the tetracardioïds tangent to the 4 lines ; they intersect in points which are the centers of tetracardioïds bitangent to one of the 4 lines
   The 256 centers in a) are on the 5*64 axes of the 5 QL's with 5 axes on a point and 4 points on each axis

64 axes of the QL.pdf

C. Construction

Very important remark : we know the direction of all the n-sectors or meanlines m° π/n.
Let's call m(Di, Dj, Dk …) the mean line of Di, Dj, Dk …

1. Triangle ABC

   Let's choose one side BC as double line and let's draw the 3 trisectors in B and C (near to BC, in B conterclockwise and in C clockwise)
   We get 9 points called I11 to I33, forming equilateral triangles on 3 circles through the 2 points B and C
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   In other words, considering the 4 lines as Di, Di, Dj and Dk (Di is a double line), the 2 trisectors are m(Di, Di, Dj) and m(Di, Di, Dk) and the 2 axes are 2 Morley lines m(Di, Dj, Dk)
   The 2 axes m(Di, Dj, Dk) are the lines joining the intersection point of the 2 trisectors to the 2 other vertices of the equilateral triangle on the same circle
   In order to have a complete figur, we have to do the same construction for the 2 other sides of the triangle.

2. QL

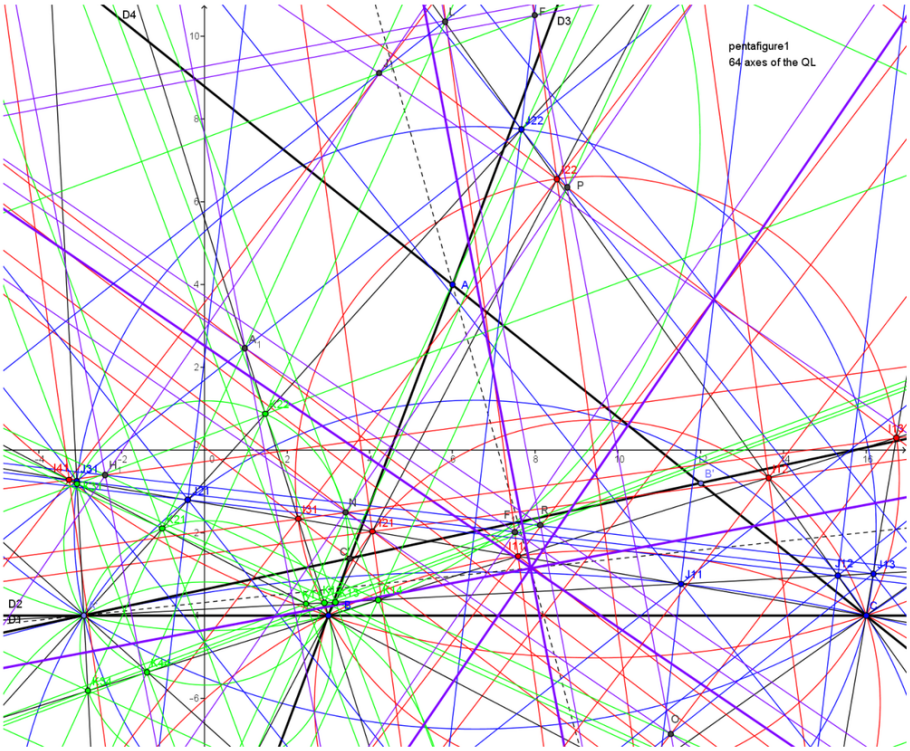
   We have to proceed this time in 2 steps
   Let's choose one line as double or triple line and let's draw the 4 quadrisectors in 2 vertices B and C on this line (near to BC, in B conterclockwise and in C clockwise)
   We get the same way 16 points called I11 to I44, forming 4 squares on 4 circles through the 2 points B and C

   a) 1rst step
      Consider the 5 lines Di, Di, Di, Dj and Dk (Di is a triple line)
      In each of these 16 points, 5 lines concur : 2 quadrisectors in B and C as m(Di, Di, Di, Dj) and m(Di, Di, Di, Dk) and 3 meanlines of the type m(Di, Di, Dj, Dk) (with Di as a double line this time)
      The 3 lines m(Di, Di, Dj, Dk) are the lines joining the intersection point of the 2 quadrisectors to the 3 other vertices of the square on the same circle through the 2 vertices B and C
   b) 2nd step
      There are 3 triangles having one side on this choosen line and there are the same way 16 points J11 to J44 and 16 points K11 to K44
      Consider now the 5 lines Di, Di, Dj, Dk and Dl (Di is a double line)

64 axes of the QL.pdf

We find new intersection points where 5 lines concur : 3 of the preceding mean lines m(Di, Di, Dj, Dk), m(Di, Di, Dj, Dl) and m(Di, Di, Dk, Dl) and 2 of the searched 64 axes of the QL m(Di, Dj, Dk, Dl)
It appears that there are on each mean line of the type m(Di, Di, Dj, Dk) belonging to one set 2 points like I, J or K and 3 points of intersection with the 2 other sets and on each of the axes of the QL again 3 of these intersection points
For one line choosen as double or triple line, there are 3*16 = 48 points like I, J or K and 3*24 = 72 mean lines in the 1rst step, which intersect in 72 points on 48 axes in the 2nd step (a group of 6 points give 4 axes making angles of π/4 or π/2 with 3 points on each axis)
In order to have a complete figur with 64 axes, we have to do the same construction for the 3 other lines of the QL.

On the figur below, D2 is the double or triple tangent, the 3 sets are in red (ABC), blue (A'B'C) and green (A'BC') and the axes are in purple



64 axes of the QL.pdf

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Message: #1036
Date: 21/3/2015 7:21:48
From: seiichikiri
Subject: A quadrangle image

Dear friends,
Let P1P2P3P4 be a quadrangle and P {x,y,z} a point.
Denote:
L12, L13, L14, L23, L24, L34= the lines through P parallel to P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 respectively.
P12, P13, P14, P23, P24, P34=the intersections of L12 and P34, L13 and P24, L14 and P23, L23 and P14, L24 and P13, L34 and P12 respectively.
P12P34, P13P24 and P14P23 concur in a point Q.
The 1 st barycentric of Q: x(qrx^2-pry(x+z)-pqz(x+y)).
G.D. and A.D. (graphically and algebraically demonstrated.)
Best regards,
Seiichi

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Message: #1037
Date: 21/3/2015 9:52:23
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
two months lost in the jungle of Morley's axes, but I think, you found the way out of the chaos! Congratulations! The key is your good interpretation of the 3L-constellation. I have reproduced your construction up to the 48 axes of a 4L wrt one reference line, fascinated of the intersections of the mean lines! I needed some time, to find the four squares for the 1st step.
My complimentand and best regards Eckart

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Message: #1038
Date: 21/3/2015 10:07:21
From: garciacapitan
Subject: A quadrangle image

Dear Seiichi,
The point Q is infinite when P lies on the nine point conic of the quadrilateral.
If we take an arbitrary line L instead of line at infinity, that is, we consider lines joining P with the intersections with L instead of parallels, the point Q lies on line L when P lies on the conic through the harmonic conjugates of the intersections of the quadrilateral and L (these are the midpoints of the sides when L is the line at infinity).
Best regards,
Francisco Javier.

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Message: #1039
Date: 21/3/2015 5:48:56
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard,
Magnificent solution!
I was looking myself in the same direction. So I had no problems understanding your solution. The only part that was missing for me is your “2nd step” 2b.
The description you actually describe in 1. is the Lighthouse Theorem.
See: http://www.cut-the-knot.org/Curriculum/Geometry/Lighthouse.shtml.
Look also at Hyacinthos †) with keyword “Lighthouse”.
Regarding your solution I have an outline for a general proof in my mind. However a proof always needs to be very accurate and specific. I don’t know if I have to patience to do that.
Anyway I will deepen myself in the matter coming days.
Best regards,
Chris

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #1040
Date: 22/3/2015 3:22:51
From: Benedetto Scimemi
Subject: A quadrangle image

Dear friends, the construction described yesterday by Sheiichi defines a very interesting quadrangle mapping
f : P -> Q :
1) f transforms a generic line r into a cubic with a double point on r
2) f transforms a quadrangle side into itself (the same line, counted 3 times) and therefore
3) f fixes the four quadrangle vertices; other fixed points are at infinity, i.e. the 6 sides directions
4) f transforms a side of the diagonal triangle into the 3 sides of its own medial triangle
5) f transforms the 9-point conic into the line at infinity (as noted by Francisco Xavier); in particular :
   a) a side midpoint goes into that side direction
   b) a diagonal point is singular; as a limit, it goes into the midpoint of its own opposite side
   c) QA-P3 goes into the direction QA-P3 QA-P4; QA-P2 normal to QA-P2 QA-P4.
I think this is enough for inserting f as a QA-Tf in Chris's list , but more properties will come.
I have not yet completed the algebraic proof, but it will not take me long.
Thanks to Sheiichi!
Best regards Benedetto
P.S. As for the various conics passing through central points which have been mentioned in recent messages and "seem" similar, they are indeed (modulo similarities) either the same or the conjugate conic.
I am preparing a page of explanation.

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Message: #1041
Date: 22/3/2015 8:29:11
From: chris.vantienhoven
Subject: A quadrangle image

Dear Seiichi, Francisco, Benedetto and friends,
Like Francisco and Benedetto commented
the construction described by Seiichi defines a very interesting quadrangle mapping f : P -> Q .
Here are some extra comments of mine.
* f(P) is a point on the tangent of the conic (P1,P2,P3,P4,P) at P.
* f(P) is actually the Involution Center on this tangent wrt the Reference Quadrangle. This is described in EQF at QA-Tf2 (Involution Center). See also QA-Tf1 QA-Line Involution and explanation below of Desargues' Involution Theorem.
* f(P) = Midpoint (P,QA-Tf2(P))
* f(QA-P1) = f(QA-P20) = QA-P22
* f(QA-P6) = f(QA-P30) = QA-P6.QA-P30 ^ QA-P11.QA-P38
Implications:
* f(P) can be used to construct the tangent at a conic when 4 other points on the conic are known: the tangent at P is the line P.f(P).
* f(P) can be used to construct the Involution Center of the involution on the tangent line wrt the Reference Quadrangle.
Best regards,
Chris
Desargues' Involution Theorem
A figure consisting of 4 points and their 6 connecting lines is called a (complete) Quadrangle.
The 4 points are the vertices.
The 6 connecting lines are the sides of the (complete) Quadrangle. Sides in a (complete) Quadrangle that have no vertices in common are called opposite sides.
Desargues' Involution theorem states that the points of intersection of a line with the three pairs of opposite sides of a complete Quadrangle and a conic section circumscribed about the complete quadrangle form the pairs of an involution.
So every line crossing a Quadrangle has an Involution Center and 2 Double Points.

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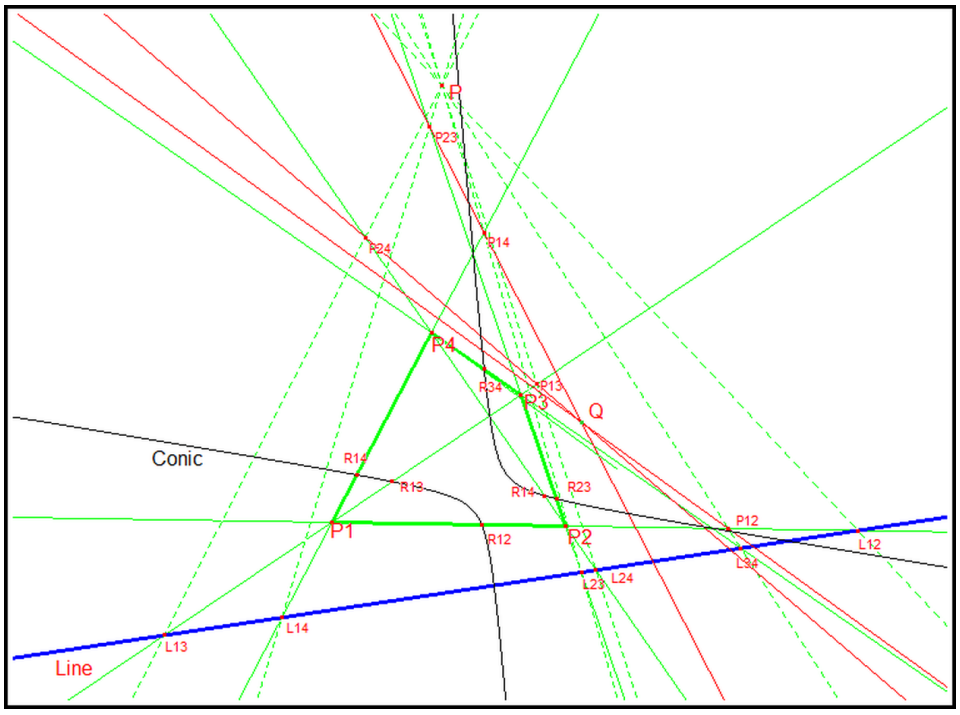
Message: #1042
Date: 23/3/2015 2:51:49
From: seiichikiri
Subject: A quadrangle image

Dear Francisco, dear Chris, dear Benedetto, dear friends,
Thank you very much for the information. Sorry for the late posting.
I rewrote as follows:
Let P1P2P3P4 be a quadrangle, P a point and L a line.
Denote:
L12, L13, L14, L23, L24, L34= the intersections of L and P1P2, L and P1P3, L and P1P4, L and P2P3, L and P2P4, L and P3P4 respectively.
P12, P13, P14, P23, P24, P34= the intersections of P1P2 and PL34, P1P3 and PQ24, P1P4 and PL23, P2P3 and PL14, P2P4 and PL13, P3P4 and PL12 respectively.
R12, R13, R14, R23, R24, R34= the harmonic conjugates of L12 in P1P2, L13 in P1P3, L14 in P1P4, L23 in P2P3, L24 in P2P3, L34 in P3P4 respectively.
1) P12P34, P13P24 and P14P23 concur in a point Q.
2) the harmonic conjugates are on a common conic.
3) If P is on the conic, then Q is on L.
See the attached Cabri picture.
Best regards,
Seiichi.



quadrangle image-fig.png

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Message: #1043
Date: 23/3/2015 10:58:50
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris, dear Eckart
Thank you for your positive reactions
I'm very glad to see I've rediscovered alone a theorem I had never heard about before !
Now I'm awaiting your drawings and calculations (equations of the lines, coordinates of the 256 centers and determination of the curve carrying them) ; it's the same as for the 27 centers of cardioïds, but a little bit more complicate, I'm afraid ...
Best regards
Bernard

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Message: #1044
Date: 23/3/2015 8:16:35
From: Benedetto Scimemi
Subject: A conic made from isogonal center (QA-P4)

Dear friends, I enjoy reading the various news regarding the quadrangle (conics, orthogonalities, prospectivities etc.).
Most news were observed from Seiichi and proved or commented by Chris. In order to to prevent both of them from further heavy work, I would like to explain why some of these observations can be made and deserve no proof.

FORWORD
All of us know the definition and main properties of the isogonic point of a quadrangle A=A1A2A3A$. For simplicity, let me call it O=Qa-P4.
Now let d the line through O whose direction is the mean of the four directions OAi (any of the two orthogonal quadrisectrices) ant let r be the mean (multiplicative) value of the four "radii" |OAi| (the positive4facts can be scovered" and deserve no proof and more "news" can be observed. Before they face the work of calculate I have the impression, however, that a more general standpoint and shorter symbols would help explaining (not only observing or proving) most of these facts. As I already pointed out in a message to Chris, in his admirable work EQD an important section seems to me missing: a list of "Quadrangle (-derived) QUADRANGLES".

```
For example, the following symbols were introduced in my paper
[]: as
   A   : the reference quadrangle
   o(A): the quadrangle whose vertices Oi
         are the circumcenters of the component triangles of A
   n(A): the quadrangle whose vertices Ni
         are the ninecenters of the component triangles of A
   c(A): the quadrangle whose vertices Ci
         are the isogonal conjugates of a vertex Ai
         on the component triangle AjAhAk of A
   p(A): the quadrangle whose vertices Pi
         are the centers of the pedal triangle of a vertex Ai
         on the component triangle AjAhAk of A
   s(A): the quadrangle whose side-midpoints Sij
         are the reflection of the point O on the side AiAj of A
   etc.

Some -but not all- of these quadrangles are actually found in
EQF as images of A under the action of the
"transformations" QATf1, 2, 3 . . . .
However, when composing these transformations (as product of
functions) symbols become cumbersome.
Moreover, there are some advantages in treating these
quadrangles as a result of an "operator" acting on the set of
all quadrangles, rather than a function acting on the plane
points.
"Composing" these operators is illuminating.
For example, the following properties look to me
self-explaining:
* c°o = o°c = i  i.e. c(o(A))=o(c(A))=A  or c and o are inverse
operators
* c(A) and o(A)  are  homothetic p =(i +c)/2,
                 i.e  Pi is the midpoint between Ai and Ci
                 A and p(A) are inversely similar
* n(A) and o(A) are inversely similar.
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Message: #1045
Date: 23/3/2015 8:23:19
From: Benedetto Scimemi
Subject: PLEASE DESTROY MY LAST MESSAGE!

Dear friends, please ignore the message that was sent you by mistake just one minute ago and contains some nonsenses. I plan to send tomorrow the correct version. I apologize. Benedetto

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Message: #1046
Date: 24/3/2015 8:24:23
From: chris.vantienhoven
Subject: A quadrangle image

Dear Seiichi,
Again a very nice item!
My comments:
* P.Q is tangent of the conic P1.P2.P3.P4.P at P.
  See also QA-L-1: 5th Point Tangents.
* The conic you describe is a conic through the vertices
  of the Diagonal Triangle,
  being the Involutary Conjugate (QA-Tf2) of L.
* When P is on this conic,
  then Q = the Involutary Conjugate of P.
* Let P = (x : y : z) and L = (l : m : n),
  then Q = (x(lx(qrx-pry-pqz)-p(mq+nr)yz) : :).
1,2,3 are Graphically Determined (GD) and 4 is Algebraically Determined (AD).
Best regards,
Chris

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Message: #1047
Date: 24/3/2015 11:41:36
From: bernard.keizer
Subject: QL and QA

Dear Chris, dear Eckart, dear Seiichi and dear fellows,
It's well known that the n-lines and the n-points are 2 dual figures.
For n = 3, the triangle is it's own dual, the QL (4 lines and 6 points) and the QA (4 points and 6 lines) are dual ...
Morley used a system of complex coordinates representing every line as the reflexion of a base point in the line. He told it was convenient to take the centric center as base point, the centric circle as base circle and the line joining the centric center and the common point of the n centric circles of the n (n-1)-lines as base line.
With this convention, we could associate to a triangle another triangle having as vertices the reflexions of the circumcenter in the 3 sides; the circumcenter of the 1rst triangle is obviously the orthocenter of the 2nd.
We could as well associate to any QL a QA having as vertices the reflexions of QL-P4 in the 4 lines ; now all the points, lines, curves or properties of this QA could be derived or calculated from the QL.
Conversely, I didn't find the meaning of QL-P4 for the derived QA, but I suppose there is one (what is x such as QL-P4=QA-Px ?)
Then it goes on, we could associate a 5-P to a 5-L and generally a n-P to a n-L always with reflexions of the centric center in the n lines ...
Did anyone try this approach ?
Best regards Bernard

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Message: #1048
Date: 24/3/2015 11:48:33
From: seiichikiri
Subject: A quadrangle image

Dear Chris,
I am very pleased that the quadrangle image and its extension are incorporated very well into quadrangle geometry.
Best regards, Seiichi.

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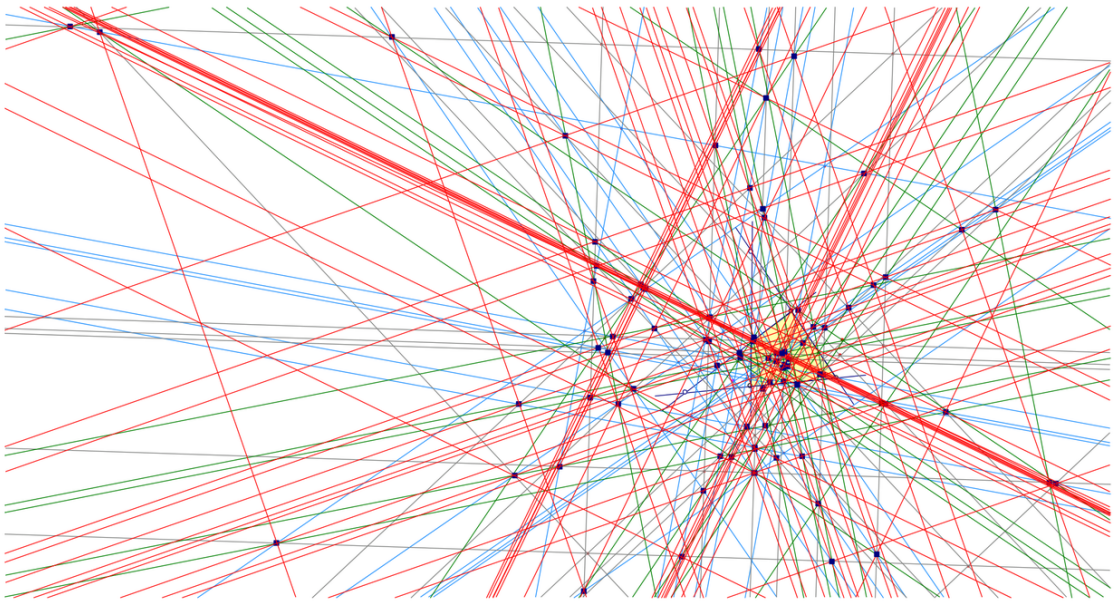
Message: #1049
Date: 25/3/2015 2:34:27
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
I have constructed the 64 axes of the QL (a part in the attached file), but I think, there must be a correction in the final results of your paper:
" ... there are 3*16 = 48 points like I, J or K and 3*24 = 72 mean lines in the 1st step, which intersect in 96 points on 64 axes in the 2nd step ...".
These 64 axes will be the searched axes of the QL and no further construction is necessary. Now I try, to get a macro, to prove the property for the 5 QL of a 5-line.
Best regards Eckart



64-4L-axes.pdf

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Message: #1050
Date: 25/3/2015 7:28:48
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart,
I'm very glad you succeeded in reproducing my construction.
Thank you for having corrected my mistake.
There are finally 4 intersection points on each mean line of the 1rst step and therefore 96 points in the 2nd step and the 64 axes, each point on 2 axes and 3 points on each axis.
If I understand well, these 96 points are centers of tetracardioïds tangent to the 4 lines and bitangent the one chosen ; this means, by doing the same for the 4 lines, that we will have 4*96 = 364 centers of tetracardioïds tangent to the 4 lines and bitangent to one of the 4 lines.
The 364 points are each on 2 axes and 12 points on each axis ; in theory, each axis cuts the 3 other sets of axes in 3*16 = 48 points and only one in 4 points seems to be one of the centers of the considered tetracardioïds.
(For the triangle, each Morley line cuts the 2*3 = 6 other lines in 6 points, each point on 2 Morley lines and 6 points on a line, which makes 27 cardioïds tangent to the 3 sides and bitangent to one side).
Do you agree with this interpretation ?
Best regards
Bernard
PS I'm now impatient to see what the locus of the 256 points for the 5-L looks like ; I'm always convinced it will be a quartic stelloïd ...

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Message: #1051
Date: 26/3/2015 12:56:40
From: chris.vantienhoven
Subject: A quadrangle image

Dear Seiichi, Francisco, Benedetto and friends,
Regarding the discussed quadrangle mapping I added this new Transformation to EQF: QA-Tf5.
We have now these transformations related to an Involution Center in the environment of a Quadrangle.
* QA-Tf1: QA-Line Involution Center (already existed but now reserved for Involution Center on a random line)
* QA-Tf5: QA-Involution Center of 5th point tangent (new transformation for Involution Center on the 5 th point tangent of a random point)
Some information is taken from QA-Tf2 (QA-Involutary Conjugate) and deleted there.
Best regards,
Chris

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Message: #1052
Date: 27/3/2015 8:26:57
From: chris.vantienhoven
Subject: QL and QA

Dear Bernard,
I never tried this approach before.
I found that QL-P4 in your derived QL-P4-reflective QA doesn't concur with the points QA-P1 till QA-P22 (GD=Graphically Determined).
There are more dual approaches of course.
For example the coefficient-triple of a line can be considered as the coordinate-triple of a point.
Once I tried this in a triangle with special results. See my file Perspective Fields-Part1 pages 40-42, http://www.chrisvantienhoven.nl/index.php/mathematics/perspective-fields
I remember there is very limited overlap of known lines in the line-system with known points in the point-system.
Nevertheless I belief in investigating these approaches.
Best regards,
Chris

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Message: #1053
Date: 27/3/2015 11:52:38
From: seiichikiri
Subject: A quadrangle image

Dear Chris, dear Francisco, dear Benedetto, dear friends,
I am very glad that a kind of team work produced a nice result.
I would like to think that there would be many other things
which we do not know now.
Best regards,
Seiichi.

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Message: #1054
Date: 28/3/2015 7:10:58
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
in addition to your paper "64 axes of the QL" I have a
restriction to 12 special axes, which show the geometry of
Morley's axes. Perhaps there is a generalization, to get the 256
"incenters" of a 5-line, showing the quadric you search for.
Best regards Eckart
PS: If you miss the reference n-lines in the two last drawings,
they have to be zoomed very small, to show the properties.

EQF-Note 2015-03-28

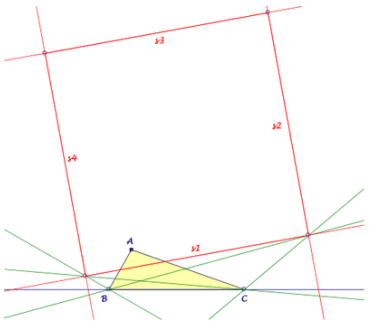
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

12 Special Morley-Axes for a Quadrilateral

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes. Their directions are well known, but Morley doesn't mention further properties. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032). Here 12 special axes – out of the construction of Bernard Keizer – are considered and CABRI-tested.

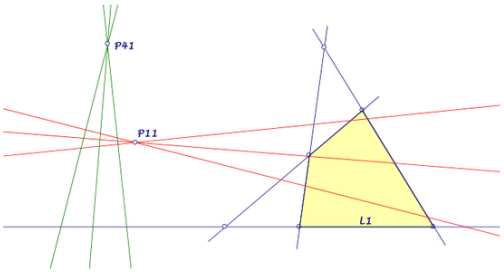
A special square for a triangle wrt a basic line

Three lines of a quadrilateral give a triangle *ABC*; let *BC* be the basic line. Morley mentioned wrt the axes, that only quadrisecting of angles is necessary. Consider the quadrisectors in *B* and *C* near to the basic line and the two intersections of an inner and an outer quadrisector. These two points are vertices of a square with circumcircle through *B* and *C*. Let s_1, s_2, s_3, s_4 be the sidelines of this square.



The 12 special Morley-axes

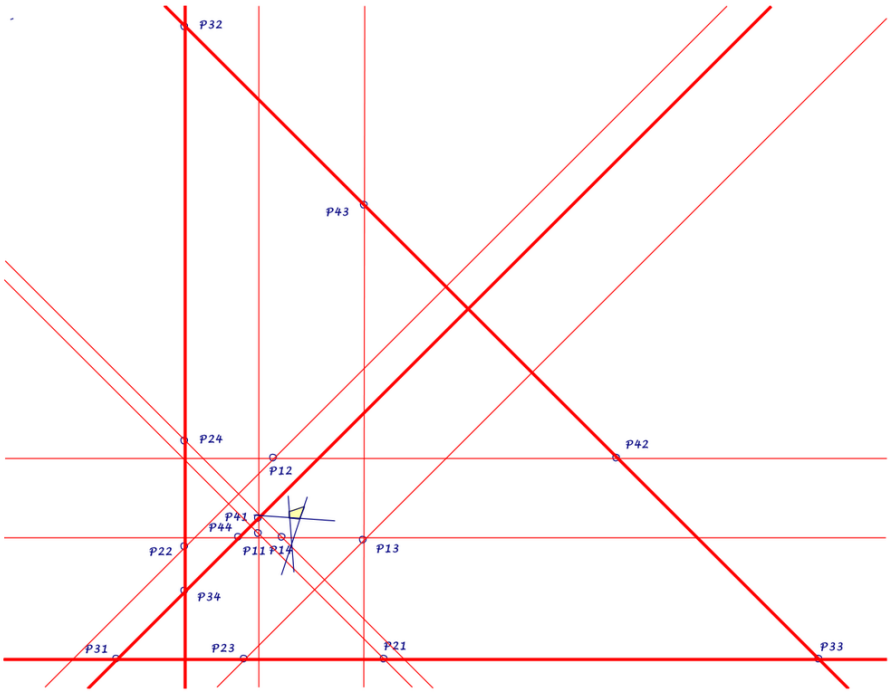
If we consider for a quadrilateral three triangles with the same basic line L_j, their lines $s_{i,j}$ have a common point $P_{i,j}$.



So we get 16 points $P_{i,j}$ and the following 12 lines (first 16, but 4 are counted twice):

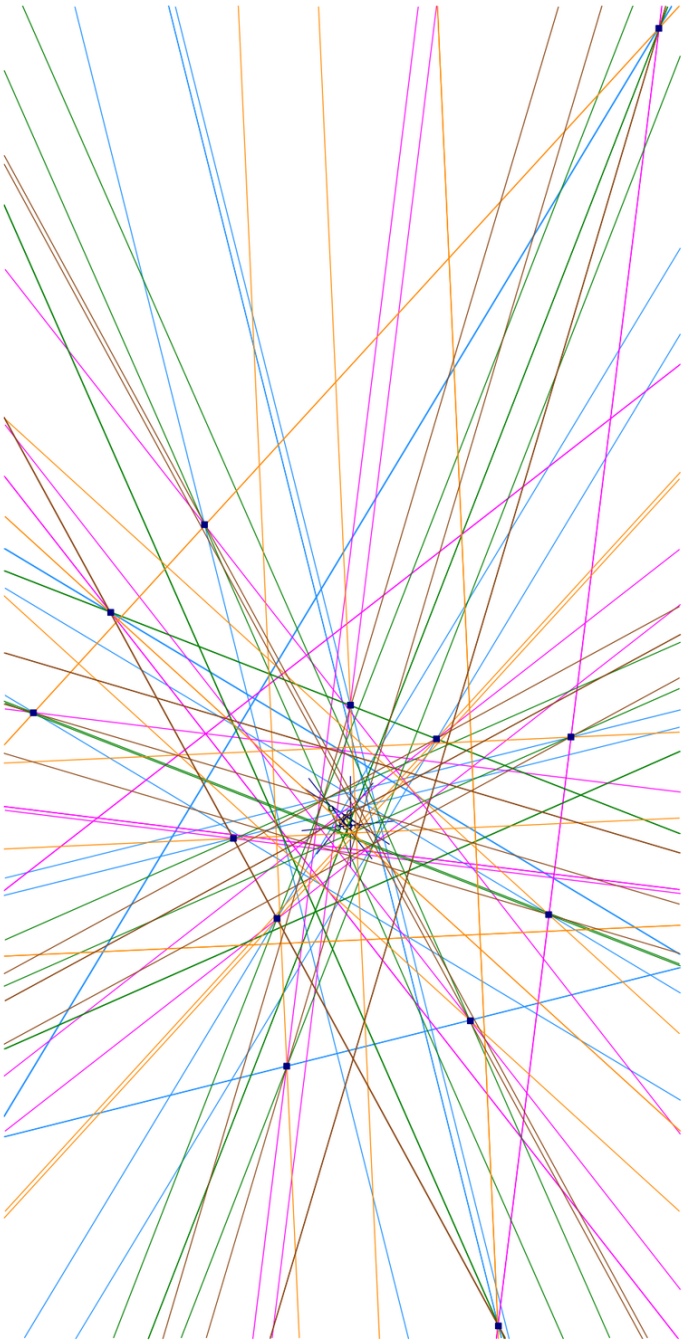
$$P_{1,j}P_{2,j},\ P_{2,j}P_{3,j},\ P_{3,j}P_{4,j},\ P_{4,j}P_{1,j} \text{ for } j=1,2,3,4.$$

These lines are four sets of three parallels, intersecting with angles equal modulo 45°. Further their directions satisfy Morley's condition "…clinant of an axis is a geometric mean of the clinants of the n lines. …". In accord with Bernard Keizer's construction these lines are Morley-axes of the quadrilateral.



The 4L-axes for a 5-line

For a 5-line Morley describes 4^4 "incenters" as intersections of the 5*64 axes for the five 4-lines: In our case there are 5*12 such axes for a 5-line. The corresponding constellation shows in Morley's sense five special axes with four points, where five axes intersect. These 15 intersections, where five axes intersect, will be special "incenters" of the 5-line.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-03-28.pdf

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Message: #1055
Date: 31/3/2015 7:39:55
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart,
1) This is a very interesting pedagogical presentation, which shows the logic of the intersection points : the sij concur in the Pij (same i, same j) ; you could even add the diagonals of the squares s5 and s6. The property is true for the 4 squares, always the same sides or diagonals of the different squares intersect ! This helps greatly to find these intersections in a very complicate figur ! Thank you for this clarification.
2) By considering the 4 squares, there are 4 points Pij, Qij, Rij and Sij on each elementary side and 2 axes in each point, wich makes 8 axes with 2 in each of the 4 possible directions. For one basic line, there are then 4*24 = 96 points (as you said) and the 64 axes, with 3 points on each axis (6 points give 4 lines forming a QL with angles at pi/4 or pi/2). For the 4 lines of the QL, there are 4*96 = 384 points with 12 points on each axis.
3) You make at the same time a simplification in considering only one square instead of 4 and a generalisation in taking the 4 lines instead of one ; I don't understand why it gives 12 lines and not 16 as expected.
4) At the end, I must confess I'm a little bit disappointed as you show only 5 axes (there are 5*64 = 320 and yourself consider 5*12 = 60) and only15 points (there are 256). Why do you call them special axes and points ?
I'm afraid you loose too much information with this simplification.
Anyhow, I hope you will improve this a little and maybe find the property of the curve ...
To answer an old question of Chris, the curve is obviously a quartic, as it is cut in 4 points by the 320 axes !
Best regards
Bernard

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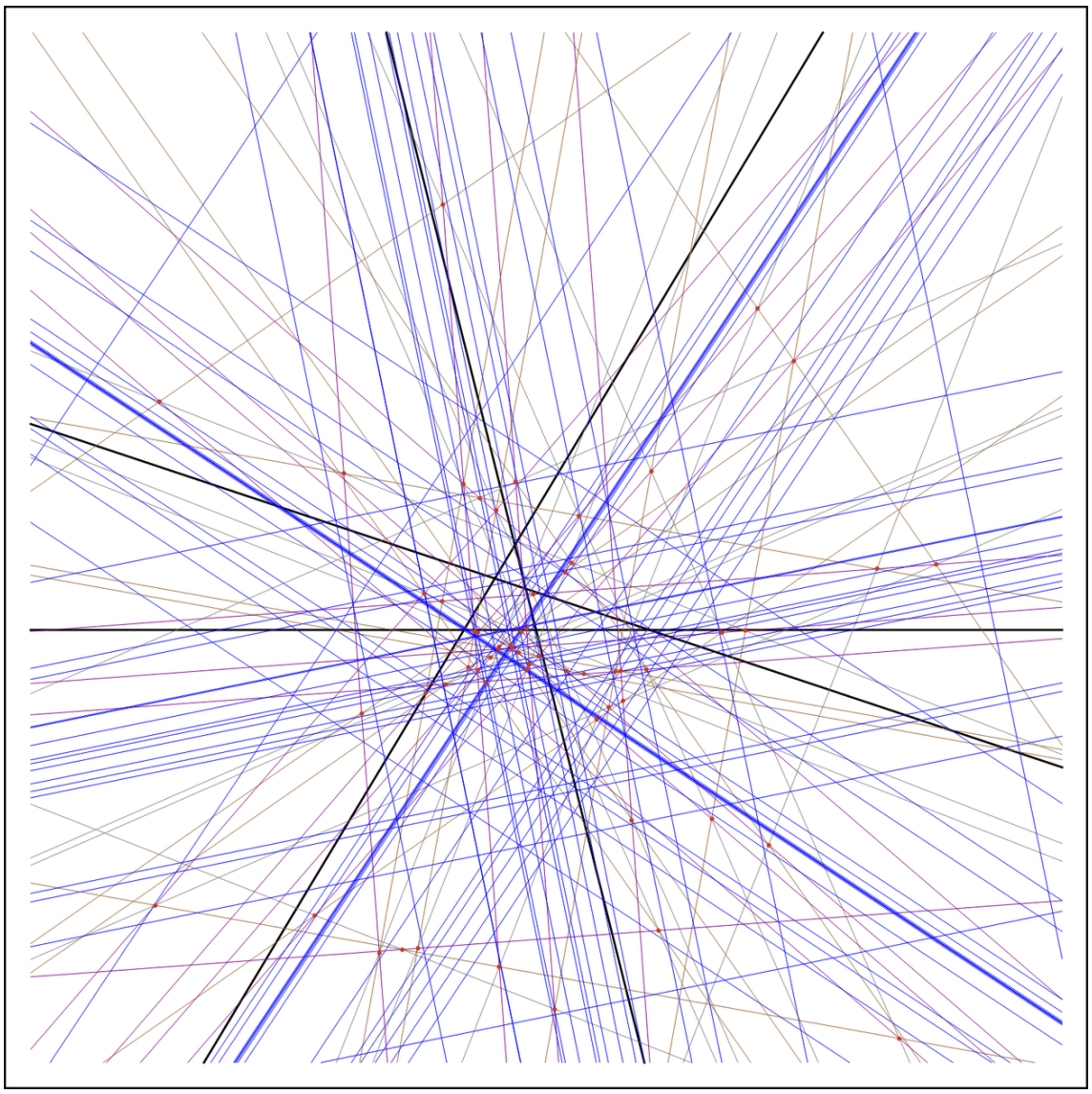
Message: #1056
Date: 31/3/2015 11:08:31
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard, Dear Eckart,
It took me quite a while to program the 64 Morley axes in a Quadrilateral in Mathematica.
Attached you will find the picture with 64 axes and 48 crosspoints of mean lines.
Lots of algorithms were involved.
Now I can start summarizing and further analyzing the results (both of you already started with) and look forward to next questions.
I think our different approaches complement each other very well.
We really did a fine job in cracking Morley's code.
Best regards,
Chris



4L-64 Morley-Axes-03.gif

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Message: #1057
Date: 01/4/2015 12:10:30
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris,
It would be nice if you gave a little more explanations about your construction !
We are able to see the 64 axes, but what are exactly your 48 crosspoints ?
There are in a complete figur normally 384 such crosspoints with 4 on each of the 288 meanlines and 12 on each of the 64 axes, each point being on 3 meanlines and on 2 axes.
We can check 12*64 = 2*384 = 768 and 4*288 = 3*384 = 1152.
Thanks in advance for your precisions.
Indeed, we achieved a fine job in cracking this last Morley's code!
Best regards
Bernard
PS By the way, the 384 crosspoints are on 4 sets of 96 points being the centers of ennacardioïds tangent to the 4 lines of the QL and bitangent to one of the 4 lines ; each set of 96 points belongs to a quartic and there are logically 4 such quartics for the QL. I hope to see the 64 axes and these 4 quartics some day in EQF ...

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Message: #1058
Date: 01/4/2015 5:05:14
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
thanks for your remarks. I think, it is not only a "pedagogical presentation", it is a real "special" restriction.
Wrt 1): Adding the diagonals of the square, there are more intersections, but no more axes.
You write "The property is true for the 4 squares ...", but for the other squares my constructions didn't show this (therefore "special").
The corresponding lines si of other squares for three triangles with the same basic line must not have a common point.
Wrt 3): At the moment I have also no explination, that the 16 points Pij give only 12 axes, 8 with 2 intersections Pij, 4 with four intersections Pij.
Wrt 4): With my restriction for the five 4L of a 5L, there are only 5 axes with four intersections of five axes (see my figur).
I have made a drawing for the 320 axes in the general case and up to now I marked about 140 of the 256 incenters of a 5L. But I see no chance, to find all the rest of over 100 further intersections of 5 axes. Perhaps my macro doesn't hold for all 4L. The structure of a curve isn't also to make out.
Best regards Eckart

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Message: #1059
Date: 02/4/2015 9:56:24
From: bernard.keizer
Subject: QL and QA

Dear Chris,
This item of perspective fields is indeed fascinating and opens new perspectives !
A few years ago, I spent hours with ETC aligments specially on figures drawn by Steve Sigur.
For example, I started with 3 points I1, O3 and H4 (in-, circum- and orthocenter) and found all important ETC points and only at the end the triangle ABC using the Feuerbach hyperbola ...
Now I see plenty of diagonal triangles of particular QA's of ETC points giving as vertices other ETC points.
I'll continue to dig these links between triangle, QA and QL, but it will certainly take time ...
Best regards
Bernard

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Message: #1060
Date: 02/4/2015 10:26:58
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart,
My caracterisation of your "pedagogical presentation" was rather positive in my eyes!
Of course, taking the diagonals s5 and s6 gives mores points, but no more axes, but the presentation is more beautiful then, as it shows with completing with P5j and P6j how 6 points Pij with i from 1 to 6 give 4 axes (P52 for example is the intersection of P12P22 with P32P42 and P62 the intersection of P12P42 and P22P32).
And I'm sure, with numerating the sides of the 3 other squares the same way as you did the 1rst one (s1 to s4 counterclockwise and adding s5 and s6) that the property holds for the 6 lines of the 4 squares ; that's why I told you it helped me to identify quickly on my figure the missing points !
And that's also why I do not understand your restriction, as each starting square would give the same result.
Anyhow, if you have already 140 points, it's better than 15 and I would be really happy, if it is possible, to have a look at it.
The same way, as I told Chris in the previous message, the 96 points (instead of 256) obtained by taking one basic line must also give a quartic (and there will be 4 quartics). Maybe your macro could hold for only one QL.
Best regards
Bernard

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Message: #1061
Date: 02/4/2015 10:34:30
From: bernard.keizer
Subject: 64 axes of a QL

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Dear Chris,
A huge work you did with calculating all these lines and points!
please read tetracardioïds instead of ennacardioïds
Best regards
Bernard
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Message: #1062
Date: 02/4/2015 6:00:15
From: chris.vantienhoven
Subject: QL and QA

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Dear Bernard,
I am glad with your interest in Perspective Fields.
Perspective Fields are one of my favorite subjects.
I think it is a new area in geometry with lots of perspectives.
It also can be applied in triangle geometry, quadri geometry and
I suppose also in other geometric realms.
Best regards,
Chris
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Message: #1063
Date: 03/4/2015 3:33:06
From: seiichikiri
Subject: Desmic image

Dear friends,
Let P1P2P3P4 be a quadrangle and P{x,y,z} a point.
Denote:
S1, S2, S3= the intersections of P1P3 and P2P4, P1P2 and P3P4, P2P3 and P1P4 respectively.
D, E, F= the intersections of PS1 and S2S3, PS2 and S1S3, PS3 and P1P2 respectively. (the vertices of the cevian triangle of P with respect to S1S2S3).
We use the DT coordinates:
S1={1,0,0}; S2={0,1,0}; S3={0,0,1}.
P1={p,q,-r}; P2={-p,q,r}; P3={p,-q,r}; P4={p,q,r}.
1) DP3, EP2, FP4 concur in Q1.
2) DP4, EP1, FP3 concur in Q2.
3) DP1, EP4, FP2 concur in Q3.
2) DP2, EP3, FP1 concur in Q4.
Q1={p(rxy-qxz+pyz), q(rxy+qxz-pyz), r(rxy+qxz+pyz)}.
Q2={p(rxy+qxz+pyz), q(-rxy+qxz+pyz), r(rxy-qxz+pyz)}.
Q3={p(rxy+qxz-pyz), q(rxy-qxz+pyz), r(-rxy+qxz+pyz)}.
Q4={p(-rxy+qxz+pyz), q(rxy+qxz+pyz), r(rxy+qxz-pyz)}.
Best regards,
Seiichi.

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Message: #1064
Date: 03/4/2015 11:17:54
From: chris.vantienhoven
Subject: Desmic image

Dear Seiichi,
All Desmic examples are wonderful and this one is unexpected!
So every Cevian Triangle wrt the Diagonal Triangle in a Quadrangle is a Desmic Triple Triangle.
We concluded before that when there is a Desmic Triple Triangle there is a 4 th point Q being the intersection P1.Q1^P2.Q2^P3.Q3^P4.Q4 which actually complements the Triple Triangle into a Quadrangle.
Now we have three Quadrangles P1.P2.P3.P4, Q1.Q2.Q3.Q4 and D.E.F.Q where any Component Triangle of any Quadrangle is perspective with any Component Triangle of any other Quadrangle.
Also we have a transformation f(P) -> Q.
This is a Triangle Transformation as well as a Quadrangle Transformation.
Observing this point Q in a Quadrangle it appears that it is the Involutary Conjugate of P. (AD=Algebraically Determined)
In a triangle it is the
P4-Ceva Conjugate of P.
Best regards,
Chris

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Message: #1065
Date: 04/4/2015 1:02:56
From: seiichikiri
Subject: Desmic image

Dear Chris,
I also obtained the involutory conjugate of P.
Q=(p^2yz,q^2zx,r^2xy) in the DT-coordinates.
If we require that Q1, Q2, Q3 and Q4 are concyclic, we obtain 16th degree polynomial.
Best regards,
Seiichi.

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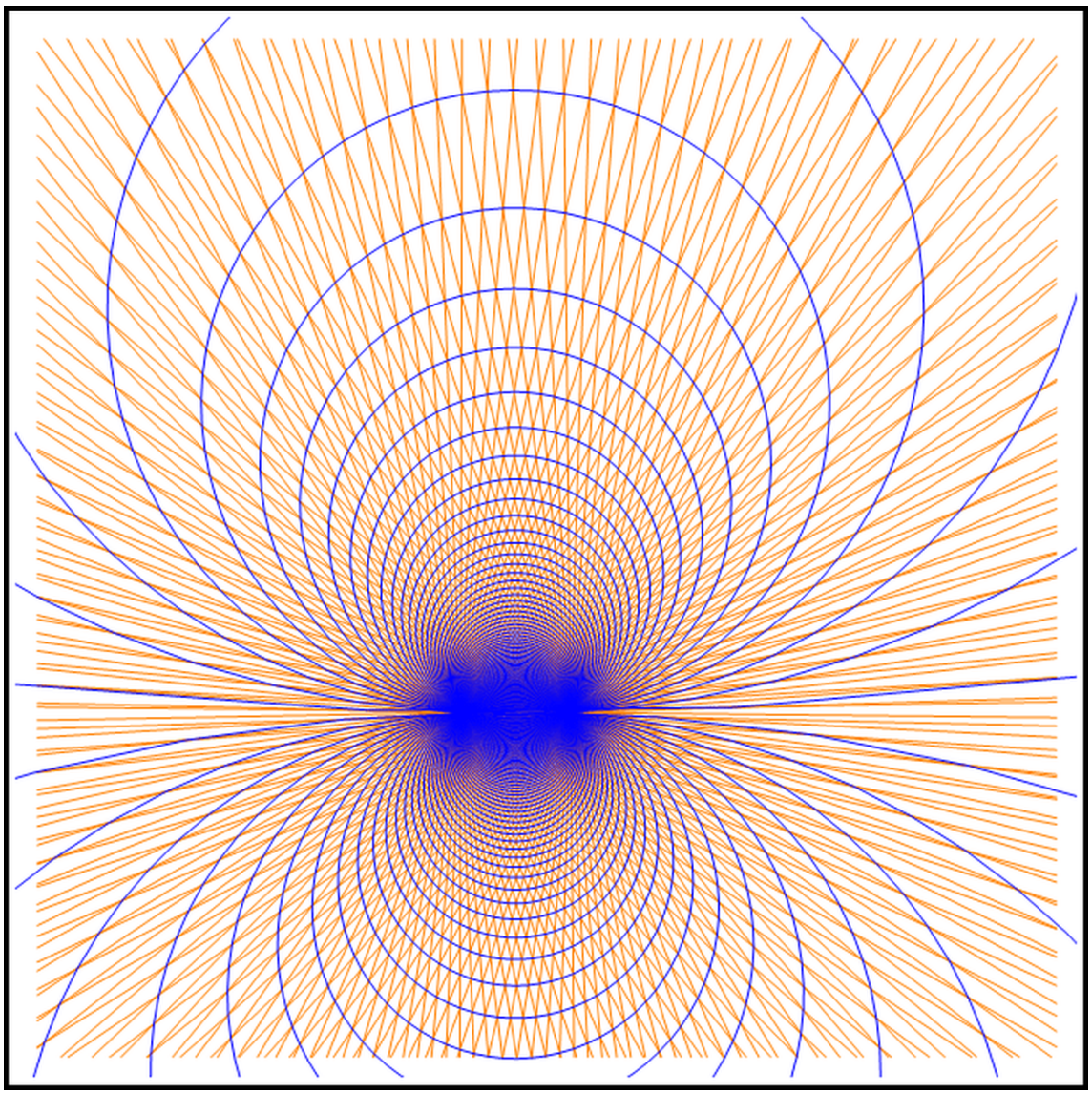
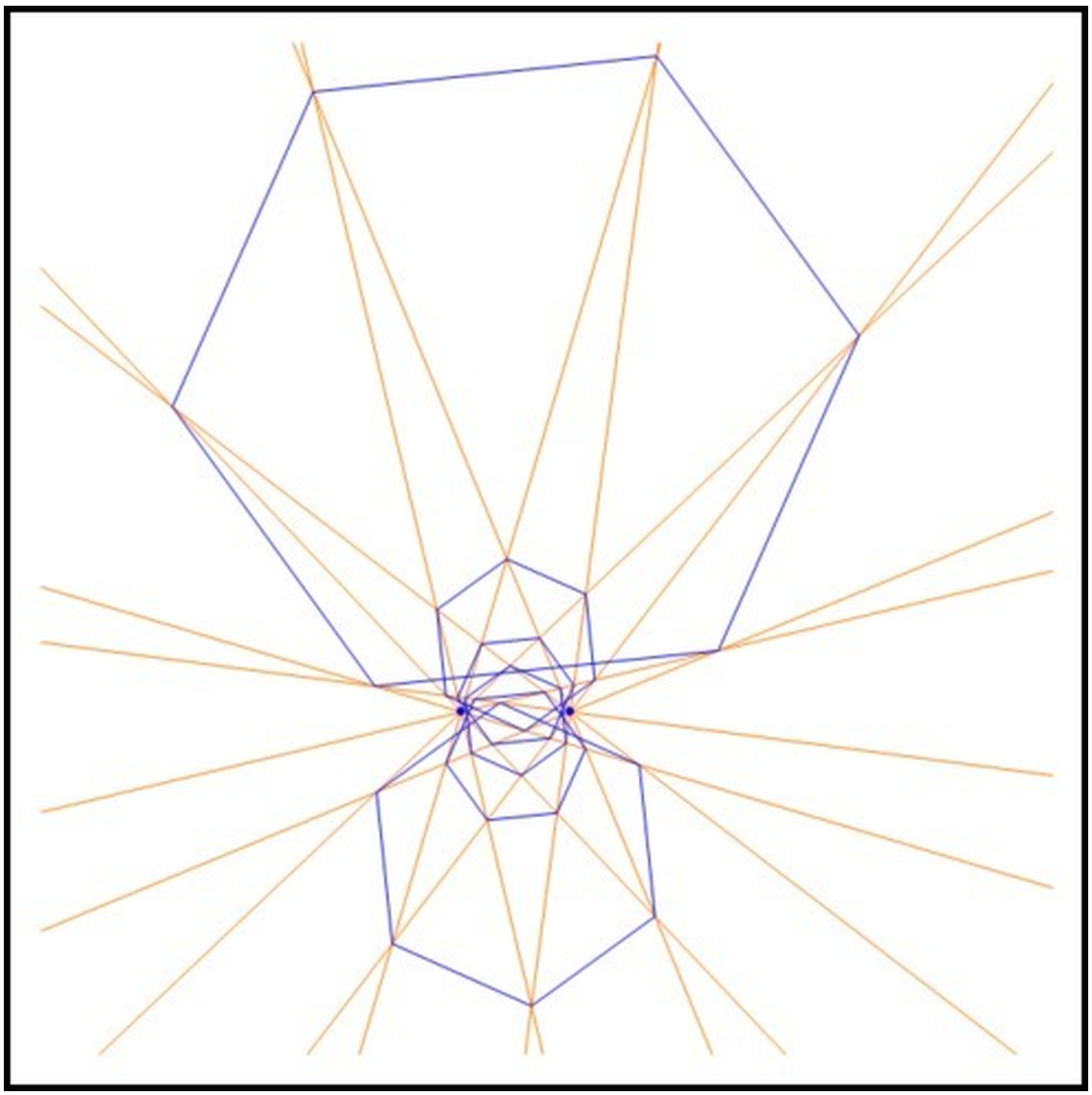
Message: #1066
Date: 04/4/2015 8:20:05
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard, dear Eckart,
Here is another picture of the 64 axes with explanation.
The Reference Quadrilateral is presented by the 4 black bold lines.
The horizontal line is the baseline L1 with three intersection points with L2, L3, L4.
The quadri-sectors are the orange lines. According to the Lighthouse Theorem they are the rays from the Lighthouses.
There are three sets of quadri-sectors (lighthouse rays) resp. through L1^L2, L1^L3 / L1^L4, L1^L3 / L1^L4, L1^L2. Using resp. anti-clockwise rays and clockwise rays per intersection point I noticed that the internal order per sets of intersection points is critical. For example L1^L2, L1^L3 / L1^L3, L1^L2 / L1^L4, L1^L3 give results and L1^L3, L1^L2 / L1^L4, L1^L3 / L1^L4, L1^L2 not.
The intersection points of the quadri-sectors are black points.
The mean lines connecting these points have different colors per set:
Gray, Brown and Pink.
These mean lines intersect in 64 Red colored points (48 finite, 16 infinite), where each Red point is intersected by 3 lines of colors Gray, Brown and Pink.
These 48 finite points are the bearers of the 64 axes.
The 64 axes are Blue colored.
About the Lighthouse Theorem:
* We always have 2 lighthouses represented by 2 points A and B. Their connecting line is the baseline.
* There are 2 starting lines La, Lb through resp. these points A and B.
* Let n be an integer > 1.
* La is rotated (n-1) times over Pi/n in A. Lb is rotated (n-1) times over Pi/n in B.
* Now we have n rays through A and n rays through B intersecting in n*n points.
* The Lighthouse Theorem states that these n*n points are the vertices of n regular n-gons!
I figured out that:
* Let
La1, La2, La3, . . . , Lan be the respective rays through A (noted clockwise or anticlockwise),
and Lb1, Lb2, Lb3, . . . , Lbn be the respective rays through B (noted other direction as rays through A),
Let Pij = Lai ^ Lbj (i,j = 1,2,3, … , n).

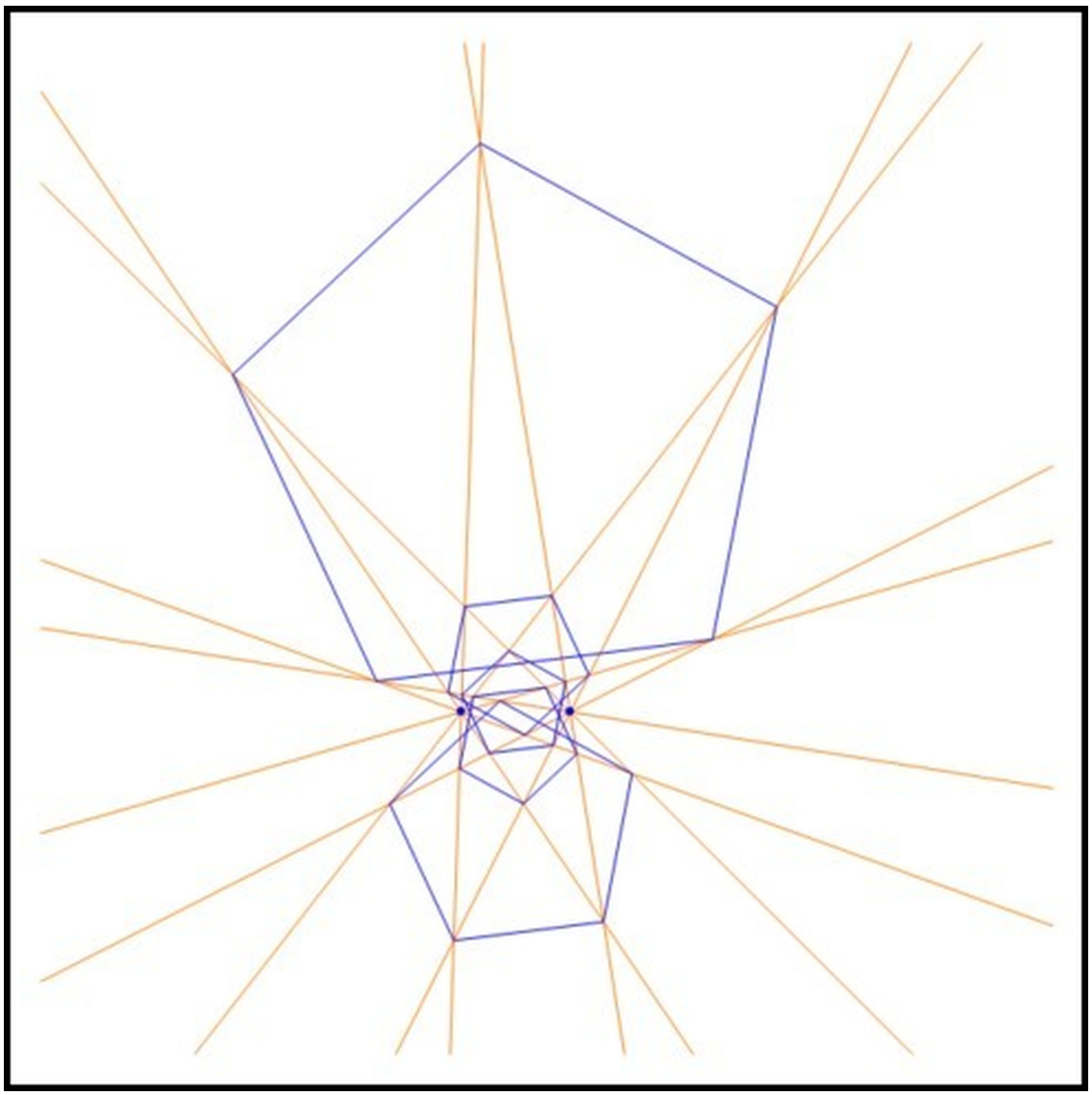
* Then:
the points Pij for which (i+j)modulo n = 0 form a regular n-gon,
the points Pij for which (i+j)modulo n = 1 form a regular n-gon,
…
the points Pij for which (i+j)modulo n = (n-1) form a regular n-gon.
* These are n regular n-gons.
The sides of the n-gons are the mean lines we were talking about.
This last property makes it possible to program the n-gons in Mathematica.
I enclose several examples.
So far this time.
Best regards,
Chris



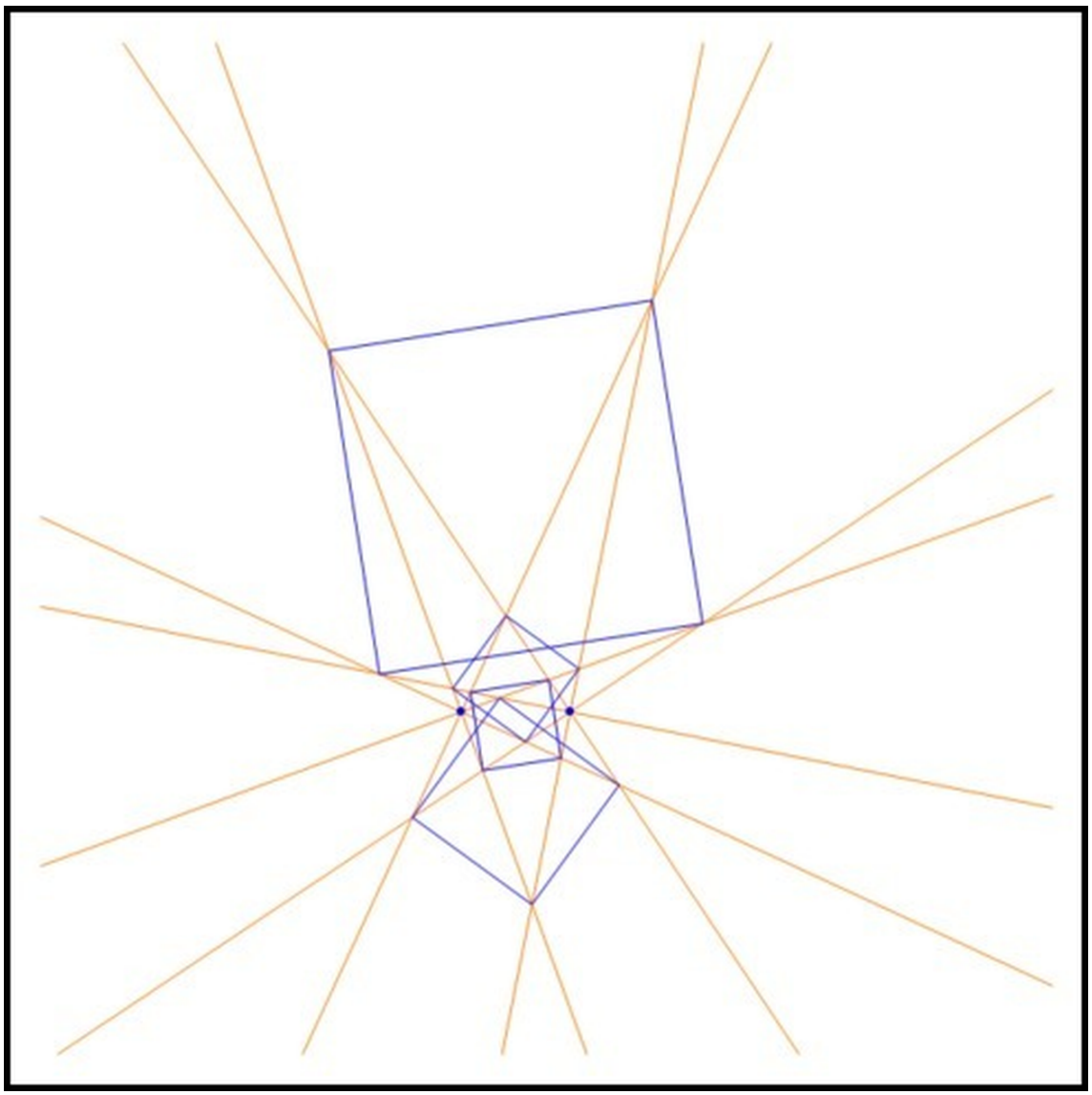
Lighthouse-77Rays.gif



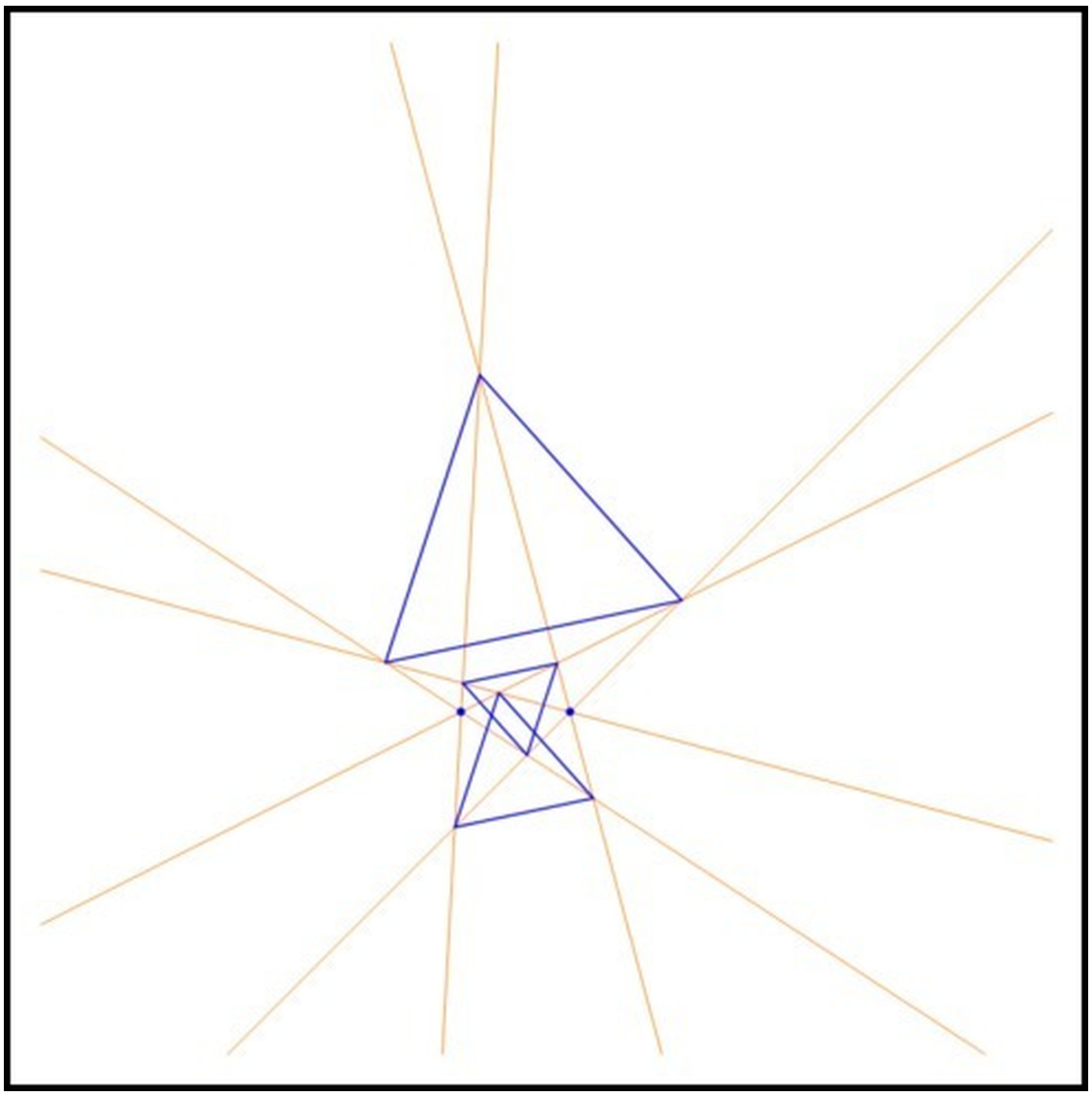
Lighthouse-6Rays.gif



Lighthouse-5Rays.gif



Lighthouse-4Rays.gif



Lighthouse-3Rays.gif

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Message: #1067
Date: 14/4/2015 7:26:12
From: eckart_schmidt@t-online.de
Subject: Sets of Morley's 4L-axes and 5L-incenters

Dear Bernard, dear Chris,
in addition to my last message (#1054) I studied sets of
Morley's 4L-axes related to the four squares for the mean lines.
There are unexpected Cabri observations and I hope someone can
attest these results. I reached 44 axes, what about the
remaining 20 axes?
Best regards Eckart

EQF-Note 2015-04-14

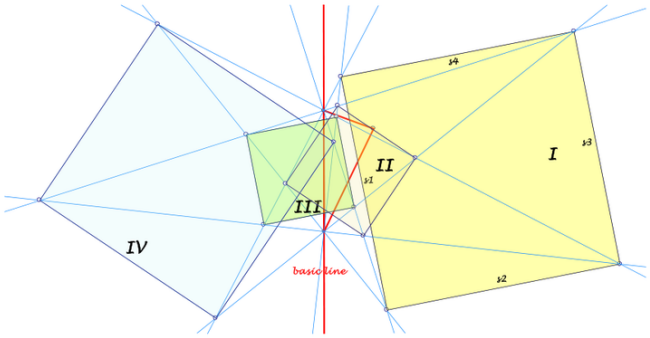
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Sets of Morley's 4L-Axes and 5L-Incenters

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes. Their directions are well known, but Morley doesn't mention further properties. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors. Each square gives a set of 4L-axes and for a 5-line a set of 5L-incenters in Morley's sense.

The squares for a triangle wrt a basic line

For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.

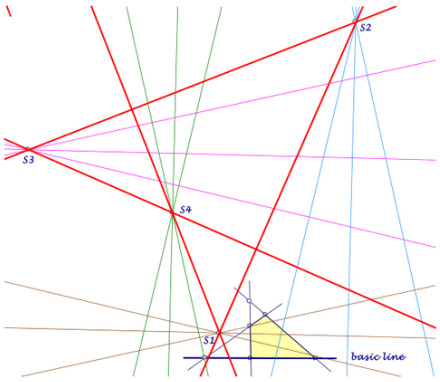


Set Ia and set Ib wrt the square I

Square I shall be the first with special properties (see *QFG*-message 1054).

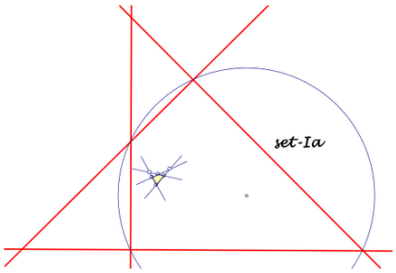
- If we consider for a quadrilateral three triangles with the same basic line, their lines s_i have a common point S_i.

- The lines S_1S_2, S_2S_3, S_3S_4, S_4S_1 are Morley-axes, satisfying Morley's condition "…clinant of an axis is a geometric mean of the clinants of the n lines …".



- Wrt the four basic lines of a quadrilateral, there are 12 axes (first 16, but four double counted): four sets of three parallels.

Definition: These 4 double axes give set Ia.
The 8 single axes give set Ib.



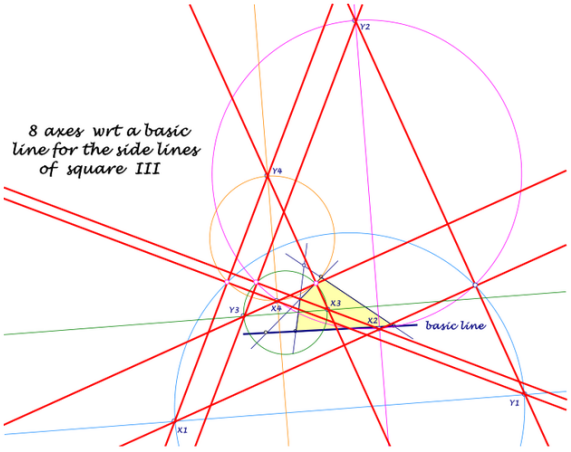
Sets wrt the other squares

Now let s_i be the side lines of another square. We consider once more three triangles of a quadrilateral with the same basic line, but their lines s_i have not a common point. Taking a sideline s_i for the three triangles, there are three intersections, giving an obtuse angled triangle. We take only the vertices X_i and Y_i with acuted angles.

- The vertices X_i and Y_i are intersections of Morley-axes.

Let X_{i+1}, Y_{i+1} the corresponding elements wrt the next sideline s_{i+1}.

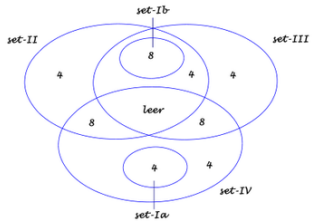
- The lines X_iY_i ($i=1,2,3,4$) give a square.
- The lines X_iX_{i+1} and Y_iY_{i+1} are orthogonal Morley-axes.
- In this way we get 8 Morley axes wrt one basic line.



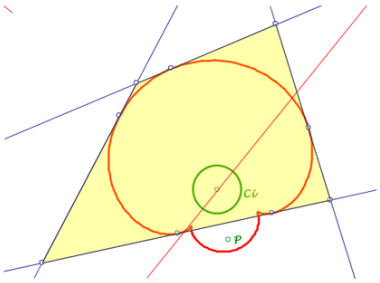
- Wrt the four basic lines of a quadrilateral there are 24 Morley axes (first 32, but 8 counted twice).

Definition:
These 24 axes for each other square give set II, set III, set IV.

- Set II, III and IV give 44 of 64 Morley-axes. Set IV contains set Ia. Set Ib is part of set II and III. The intersecting set of set II, set III and set IV is vacant.



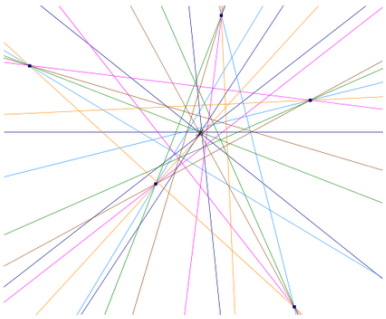
Remark: The discussed points S_i, X_i, Y_i for a quadrilateral wrt a basic line are centers of Morley's tetracardioids, tangent to the sidelines, twice for the basic line (see interpretation of Bernard Keizer in *QFG*-message 1050). A tetracardioid can be defined with a point and a circle (see *QFG*-message 831), the midpoint of the circle is the center.



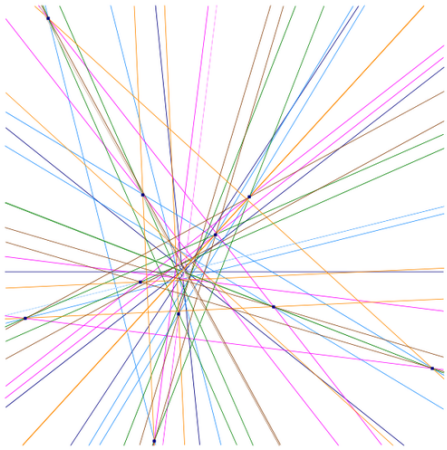
Sets of Morley-incenters for a 5-line

If we consider the axes of a quadrilateral for the five 4-lines of a 5-line, then there are 256 intersections of five, four on each axis. These are Morley-incenters of a 5-line. For the described sets of 4L-axes there are sets of 5L-incenters. Some examples:

- The set Ia (4 axes) of the five 4-lines give a constellation with five intersections of five (see figure).



- The 8 double axes of set-IV (set IV without set II and set III) for the five 4-lines give 10 intersections of five (see figure).



- Set Ia and Ib (12 axes) of the five 4-lines give 15 intersections of five (see figure in *QFG*-message 1054).

- Set IV (24 axes) of the five 4-lines give 25 intersections of five.

- Set Ia, Ib, II, III, IV (all considered 44 axes) for the five 4-lines give 92 intersections of five.

2015-04-14.pdf

The last constellation for a 5-line contains 5*44 = 220 axes, 20 sets of 11 parallels. Every set of 11 parallels has 2 lines with 4 points, 6 lines with 2 points and 3 lines with one point, that are 23 points. 20 sets of 23 points give 92 points counted 5 times.

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2015-04-14.pdf

Message: #1068
Date: 16/4/2015 12:41:47
From: eckart_schmidt@t-online.de
Subject: Construction of Morley’s 4L-axes

Dear Bernard, dear Chris,
I have described Bernard's construction in another way. It seems easier to handle for constructing special axes. Perhaps this construction is a better background to study sets of axes .
Besrt regards Eckart

EQF-Note 2015-04-16

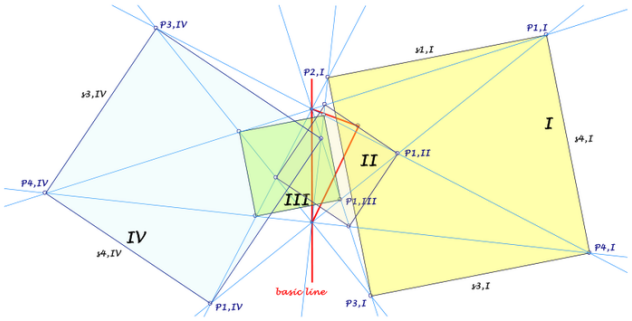
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Construction of Morley's 4L-axes

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes, but Morley doesn't mention a construction. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors. The following is only a concrete procedure for this construction.

The squares for a triangle wrt a basic line

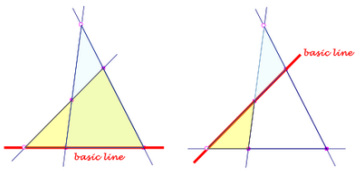
(1) For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.



(2) Chose an anticlockwise nomination for the vertices of the squares:

$P_{i,J}$ with $i = 1, 2, 3, 4$ and $J = I, II, III, IV$,

so that for example $P_{1,J}$ for $J = I, II, III, IV$ lie on the same quadrisector. Let $s_{i,J} = P_{i,J}P_{i+1,J}$ be the side lines of the squares.



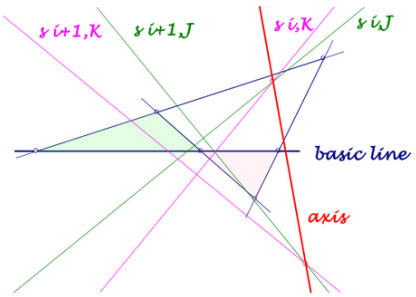
(3) Consider for a quadrilateral a basic line. There are three triangles with this basic line, but we take only two: If

the three triangles lie on one side of the basic line, we use the two, which intersect in the convex quadrigon of the quadrilateral. If the three triangles lie on different sides of the basic line, we use the two, which have no intersection with the convex quadrigon of the quadrilateral.

(4) Take for a basic line of a quadrilateral the two triangles in the sense of (3). Draw for one triangle $s_{i,J}$ and $s_{i+1,J}$, for the other $s_{i,K}$ and $s_{i+1,K}$, then:

$s_{i,J} \cap s_{i,K} \cdot s_{i+1,J} \cap s_{i+1,K}$ **is a Morley-axis**

$i \in \{1, 2, 3, 4\}$, J and $K \in \{I, II, III, IV\}$.



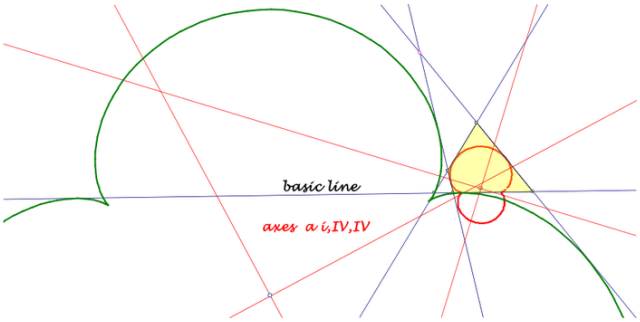
Remarks:

- Let $a_{i,J,K}$ be the so constructed axis wrt a basic line, then the four possibilities for *i, J, K* show, that there are 4^3 = 64 Morley-axes for a quadrilateral.

- For $J = K$ wrt a basic line there are four axes, that gives wrt four basic lines 16 axes, but four are counted twice. For $J = K = I$ these 12 axes are described in *QFG*-message 1054, further in *QFG*-message 1067 as set Ia and set Ib. For $J = K = II, III, IV$ these axes give set II, III, IV in *QFG*-message 1067.

- The points $s_{i,J} \cap s_{i,J}$ are intersections of axes, which will be the centers for Morley's tetracardioids, tangent to the sidelines of the quadrilateral, twice for the basic line.

- Final figure as Cabri observation to prove: The four axes $a_{i,IV,IV}$ wrt a basic line are two pairs of orthogonal axes. The intersection of each pair is a center of a tetracardioid, tangent to the four sides and twice to the basic line in symmetry.



Eckart Schmidt
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2015-04-16.pdf

Message: #1069
Date: 17/4/2015 11:44:30
From: bernard.keizer
Subject: Construction of Morley's 4L-axes

Dear Chris, dear Eckart
I wasn't home for 10 days and I find your last messages.
If I understand correctly, Chris has already begun to calculate points and lines whereas Eckart continues to observe the properties with Cabri.
The 2 methods should converge at the end !
Many thanks to Chris for the explanations about the Lighthouse Theorem and to Eckart for showing the tetracardioïds tangent to the 4 lines and bitangent to one, called the basic line.
I personnally regret the need of simplification in the 2 last presentations.
I can only repeat my last message : 1 basic line gives 3 sets of 4 squares having the same properties, one square counts 6 mean lines (sides and diagonals), the intersections of these 72 mean lines give 96 points, 4 points on each mean line and each point on 3 meanlines (one in ech set) and the 96 points are on the 64 axes, 3 points on each axis and each point on 2 axes (6 points give 4 axes forming a QL with angles at pi/4 or pi/2).
For the 5-L, we have 256 intersection points of the 5*64 = 320 axes, 4 on each of the 320 axes and each point on 5 axes (one for each QL).
What interests me now is to see and to understand the quartic of the 96 points for each basic line of the QL (there are 4 quartics like this for a QL) and the quartic of the 256 points of the 5-L.
My observations with Geogebra were sufficient to understand the construction of the 64 axes but I'm not able to make calculations as you do with Mathematica ; I hope you will succeed in solving this last mystery !
Best regards
Bernard
PS for Chris : I spent time in the perspective fields and found again with the same pleasure all this beautiful aligments in ETC points, which had fascinated me some years ago (for example your explanation on page 30 of the 2nd part with copples of points aligned with the symmedian point X6 and the harmonic conjugate of X6 wrt the 2 points being systematically on the Euler Line)

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Message: #1070
Date: 17/4/2015 11:46:32
From: chris.vantienhoven
Subject: Construction of Morley's 4L-axes

Dear Eckart, dear Bernard,
Thanks for your messages.
It is good to have some friends with the same interest.
I am describing now the 64 axes in a paper also with proofs why it works like it works.
There will be some overlap with all information coming from you lately.
But every time I think I reach completion a new question pops up.
I think I will need considerably more time to make a proper paper of it. So I take my time.
One of my problems is that when we work with a baseline L1 and find 64 axes, that we should find the same 64 axes when constructing the whole thing with baseline L2. After all, these 64 axes are described by Morley for a general 4-Line. But so far I did not succeed in producing the same 64 axes when I change the baseline, neither in Cabri, neither in Mathematica. Did you both have better results?
Your last remark Eckart
(The four axes ai,IV,IV wrt a basic line are two pairs of orthogonal axes. The intersection of each pair is a center of a tetracardioid, tangent to the four sides and twice to the basic line in symmetry) confirms Morleys statement
that the axes are the loci of the Incenters C1 of curves Cn.
Another observation is that the 64 axes do not consist of 16 squares like I hoped it to be.
Best regards,
Chris
p.s. Bernard, I am glad you enjoyed all the beautiful aligments as shown in Perspective Fields!

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Message: #1071
Date: 18/4/2015 7:36:27
From: seiichikiri
Subject: Desmic locus of pedal points

Let P1P2P3P4 be a quadrangle and P{x,y,z} a point.
Denote:
S1, S2, S3= the intersections of P1P3 and P2P4, P1P2 and P3P4, P2P3 and P1P4 respectively. (the vertices of the diagonal triangle of the quadrangle).
R1, R2, R3= the pedal points of P on S2S3, S1S3, S1S2 respectively. (the vertices of the pedal triangle of P with respect to S1S2S3).

We use the DT coordinates:
   S1={1,0,0}; S2={0,1,0}; S3={0,0,1}.
   P1={p,q,-r}; P2={-p,q,r}; P3={p,-q,r}; P4={p,q,r}.

If P is on the following curve: (a^6 - 2 a^4 b^2 + a^2 b^4 + 2 a^4 c^2 + 2 a^2 b^2 c^2 - 3 a^2 c^4) y^2 z + (-a^6 - 2 a^4 b^2 + 3 a^2 b^4 + 2 a^4 c^2 - 2 a^2 b^2 c^2 - a^2 c^4) y z^2 + x^2 ((a^4 c^2 + 2 a^2 b^2 c^2 - 2 a^2 c^4 + 3 b^2 c^2 (-b^2 + c^2) + c^4 (-b^2 + c^2)) y + (-a^4 b^2 + 2 a^2 b^4 - 2 a^2 b^2 c^2 + b^4 (-b^2 + c^2) + 3 b^2 c^2 (-b^2 + c^2)) z) + x ((3 a^4 c^2 - 2 a^2 b^2 c^2 - 2 a^2 c^4 + b^2 c^2 (-b^2 + c^2) - c^4 (-b^2 + c^2)) y^2 + (-3 a^4 b^2 + 2 a^2 b^4 + 2 a^2 b^2 c^2 - b^4 (-b^2 + c^2) + b^2 c^2 (-b^2 + c^2)) z^2)=0.

   1) R1P3, R2P2, R3P4 concur in Q1.
   2) R1P4, R2P1, R3P3 concur in Q2.
   3) R1P1, R2P4, R3P2 concur in Q3.
   2) R1P2, R2P3, R3P1 concur in Q4.

For Q1, Q2, Q3, Q4, see the attached text.
P1Q1, P2Q2, P3Q3 and P4Q4 are concurrent at R4:
R4={p^2((-a^2+b^2+c^2)z+c^2y)((a^2-b^2+c^2)x+2a^2z),
    q^2((a^2-b^2+c^2)z+2c^2x)((a^2-b^2+c^2)x+2a^2z),
    r^2((a^2-b^2+c^2)z+2c^2x)((a^2+b^2-c^2)x+2a^2y)}.
Best regards,
Seiichi.

```
ptQ1=-(p*(a^4*z*(-(r*(x + 2*y)) + p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(b^2*(p + q +
r)*z - c^2*(q*(2*x + z) + r*(2*x + z) - p*(2*y + z))) + 2*a^2*(-(b^2*z*(-(r*y) + p*(x + z) +
q*(x + z))) - c^2*(r*(x^2 + 2*x*y + y*z) - q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2)))))
q*(a^4*z*(r*(x + 2*y) + p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(b^2*(p + q - r)*z + c^2*(2
*r*x + 2*p*y + p*z + r*z - q*(2*x + z))) + 2*a^2*(-(b^2*z*(r*y + p*(x + z) + q*(x + z))) + c^2
*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) - p*(x*y + 2*y*z + z^2))))
r*(a^4*z*(r*(x + 2*y) - p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(-(b^2*(p - q + r)*z) - c^2
*(q*(2*x + z) - r*(2*x + z) + p*(2*y + z))) + 2*a^2*(b^2*z*(-(r*y) + p*(x + z) - q*(x + z)) +
c^2*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))

ptQ2=p*(a^4*z*(r*(x + 2*y) - p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(-(b^2*(p - q + r)*z) -
c^2*(q*(2*x + z) - r*(2*x + z) + p*(2*y + z))) + 2*a^2*(b^2*z*(-(r*y) + p*(x + z) - q*(x + z))
+ c^2*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))
q*(-(a^4*z*(r*(x + 2*y) + p*(x + 2*z) - q*(x + 2*z))) + (b^2 - c^2)*x*(b^2*(-p + q + r)*z - c^2
*(2*r*x + 2*p*y + p*z + r*z + q*(2*x + z))) + 2*a^2*(b^2*z*(r*y + p*(x + z) - q*(x + z)) + c^2
*(-(r*(x^2 + 2*x*y + y*z)) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))
r*(-(a^4*z*(-(r*(x + 2*y)) + p*(x + 2*z) + q*(x + 2*z))) - (b^2 - c^2)*x*(b^2*(p + q + r)*z -
c^2*(q*(2*x + z) + r*(2*x + z) - p*(2*y + z))) + 2*a^2*(b^2*z*(-(r*y) + p*(x + z) + q*(x + z))
+ c^2*(r*(x^2 + 2*x*y + y*z) - q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))

ptQ3=p*(a^4*z*(r*(x + 2*y) + p*(x + 2*z) - q*(x + 2*z)) - (b^2 - c^2)*x*(b^2*(-p + q + r)*z -
c^2*(2*r*x + 2*p*y + p*z + r*z + q*(2*x + z))) - 2*a^2*(b^2*z*(r*y + p*(x + z) - q*(x + z)) +
c^2*(-(r*(x^2 + 2*x*y + y*z)) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))
-(q*(a^4*z*(r*(x + 2*y) - p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(-(b^2*(p - q + r)*z) - c^
2*(q*(2*x + z) - r*(2*x + z) + p*(2*y + z))) + 2*a^2*(b^2*z*(-(r*y) + p*(x + z) - q*(x + z)) +
c^2*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2)))))
-(r*(a^4*z*(r*(x + 2*y) + p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(b^2*(p + q - r)*z + c^2
*(2*r*x + 2*p*y + p*z + r*z - q*(2*x + z))) + 2*a^2*(-(b^2*z*(r*y + p*(x + z) + q*(x + z))) +
c^2*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) - p*(x*y + 2*y*z + z^2)))))

ptQ4=p*(a^4*z*(r*(x + 2*y) + p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(b^2*(p + q - r)*z + c^
2*(2*r*x + 2*p*y + p*z + r*z - q*(2*x + z))) + 2*a^2*(-(b^2*z*(r*y + p*(x + z) + q*(x + z))) +
c^2*(r*(x^2 + 2*x*y + y*z) + q*(x^2 + 3*x*z + z^2) - p*(x*y + 2*y*z + z^2))))
-(q*(a^4*z*(-(r*(x + 2*y)) + p*(x + 2*z) + q*(x + 2*z)) + (b^2 - c^2)*x*(b^2*(p + q + r)*z - c^
2*(q*(2*x + z) + r*(2*x + z) - p*(2*y + z))) + 2*a^2*(-(b^2*z*(-(r*y) + p*(x + z) + q*(x +
z))) - c^2*(r*(x^2 + 2*x*y + y*z) - q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2)))))
r*(-(a^4*z*(r*(x + 2*y) + p*(x + 2*z) - q*(x + 2*z))) + (b^2 - c^2)*x*(b^2*(-p + q + r)*z - c^2
*(2*r*x + 2*p*y + p*z + r*z + q*(2*x + z))) + 2*a^2*(b^2*z*(r*y + p*(x + z) - q*(x + z)) + c^2
*(-(r*(x^2 + 2*x*y + y*z)) + q*(x^2 + 3*x*z + z^2) + p*(x*y + 2*y*z + z^2))))
```

ptQ1234-txt.png

There will be some overlap with all information coming from you lately.

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Message: #1072
Date: 18/4/2015 9:09:25
From: chris.vantienhoven
Subject: Desmic locus of pedal points

Dear Seiichi,
Earlier (QFG # 1063-1065) you contributed that every P-cevian Triangle wrt QA-DT is a Desmic Triple Triangle.
Now you say that every P-pedal Triangle wrt QA-DT also is a Desmic Triple Triangle when P lies on the Darboux Cubic K004 of the Diagonal Triangle QA-DT.
Analyzing it, it is not quite strange because K004 = Locus of point P whose pedal triangle is a cevian triangle.
Best regards,
Chris

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Message: #1073
Date: 18/4/2015 9:45:34
From: seiichikiri
Subject: Desmic locus of pedal points

Dear Chris,
You are right. The curve is K004.
One thing which I can not understand: R4 is not symmetric by permutation.
Best regards,
Seiichi.

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Message: #1074
Date: 18/4/2015 10:37:45
From: chris.vantienhoven
Subject: Desmic locus of pedal points

```
Dear Seiichi,
Your equation of R4 =
   {p^2 (2 SB x + 2 a^2 z) (c^2 y + 2 SA z),
    q^2 (2 SB x + 2 a^2 z) (2 c^2 x + 2 SB z),
    r^2 (2 SC x + 2 a^2 y) (2 c^2 x + 2 SB z)}
which is not symmetric indeed.
I think you made some typo. It should be symmetric.
Best regards,
Chris
```

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Message: #1075
Date: 18/4/2015 11:03:27
From: eckart_schmidt@t-online.de
Subject: Construction of Morley's 4L-axes

```
Dear Chris,
the macro, I use for the 64 axes, gives for each line Li of the
4L the same result. Wrt my axes ai,IV,IV I describe the special
case of symmetrical
touchment of the basic line. In addition, this holds also for
the axes ai,I,I, ai,II,II, ai,III,III.
Best regards Eckart
```

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Message: #1076
Date: 18/4/2015 12:11:32
From: bernard.keizer
Subject: Construction of Morley’s 4L-axes

Dear Chris,
Yes, the 4 basic lines give the same 64 axes, but with different groups of 3 points and there are alltogether 12 points on each axis, 3 for each basic line ; these 4*96 = 384 points are on 4 quartics, each group of 96 points belonging to one quartic.
(Theorically, each of the 64 axes cuts 48 other in finite points and only 12 of these 48 are centers of tetracardioïds tangent to the 4 lines and bitangent to one).
As Eckart, you and I agree now on the 1rst step (vertices of the squares), I'm afraid something is wrong in your 2nd step : I tried to look at your 1rst picture in message 1066, but it's not easy to recongnize the construction.
Maybe you could draw the 12 squares for one basic line in the same colors as the meanlines (gray, brown and pink). There should be 4 black intersections on each quadrisector (vertices of 4 different squares).
Then these 72 meanlines (24 in each color, side or diagonal of the squares) should intersect in 96 finite red points, 4 on each meanline and each counted 3 times. These 96 red points are on the 64 blue axes (6 points give 4 axes). In other words, through each red point shoud intersect 5 lines, 3 meanlines, one in each color gray, brown and pink and 2 blue axes, making an angle of pi/4 or pi/2.
I hope this will be some help for you ...
Best regards
Bernard
PS I send you again my initial attached file with some corrections due to Eckart's remarks

64 axes of the QL

A. Theory

The fundamental property is mentionned by F. Morley in an article published in 1929 « Extensions of Clifford's Chain-Theorem » ; an analytical proof is given by R. Bricard in an article published in 1923 « Sur les droites moyennes d'un triangle ».
Considering a n-line, the mean lines of the n (n-1)-lines are concurring.
This property has 2 possible applications :

a) n different lines : there are $(n-1)^{n-1}$ incenters, centers of the ennacardioïds tangent to the n lines
b) n-1 different lines : there are $(n-1)^{n-2}$ lines, called axes of the (n-1)-lines, which are the locus of the centers of the same ennacardioïds tangent to the n-1 lines ; the axes intersect at angles of π/n in points which are the centers of the ennacardioïds bitangent to one of the n-1 lines
The points in a) are on the n sets of lines in b) with n axes on a point and n-1 points on an axis

B. Examples

1. QL : n = 4
   a) There are 27 centers of cardioïds tangent to 4 lines
   b) For each triangle, there are 9 Morley lines locus of the centers of the cardioïds tangent to the the 3 sides ; the Morley lines intersect at angles of $\pi/3$ in 27 points which are the centers of the cardioïds bitangent to one of the 3 sides
   The 27 centers in a) are on the 4*9 Morley lines of the 4 triangles with 4 lines in a point and 3 points on each line
2. 5-L : n = 5
   a) There are 256 centers of tetracardioïds tangent to 5 lines
   b) For each QL, there are 64 axes locus of the centers of the tetracardioïds tangent to the 4 lines ; they intersect in points which are the centers of tetracardioïds bitangent to one of the 4 lines
   The 256 centers in a) are on the 5*64 axes of the 5 QL's with 5 axes on a point and 4 points on each axis

64 axes of the QL-1076.pdf

C. Construction

Very important remark : we know the direction of all the n-sectors or meanlines m° π/n.
Let's call m(Di, Dj, Dk …) the mean line of Di, Dj, Dk …

1. Triangle ABC

   Let's choose one side BC as double line and let's draw the 3 trisectors in B and C (near to BC, in B conterclockwise and in C clockwise)
   We get 9 points called I11 to I33, forming equilateral triangles on 3 circles through the 2 points B and C
   In each of these 9 points, 4 lines concur : 2 trisectors in B and C and 2 meanlines of the 3 sides or Morley lines of the triangle
   In other words, considering the 4 lines as Di, Di, Dj and Dk (Di is a double line), the 2 trisectors are m(Di, Di, Dj) and m(Di, Di, Dk) and the 2 axes are 2 Morley lines m(Di, Dj, Dk)
   The 2 axes m(Di, Dj, Dk) are the lines joining the intersection point of the 2 trisectors to the 2 other vertices of the equilateral triangle on the same circle
   In order to have a complete figur, we have to do the same construction for the 2 other sides of the triangle.

2. QL

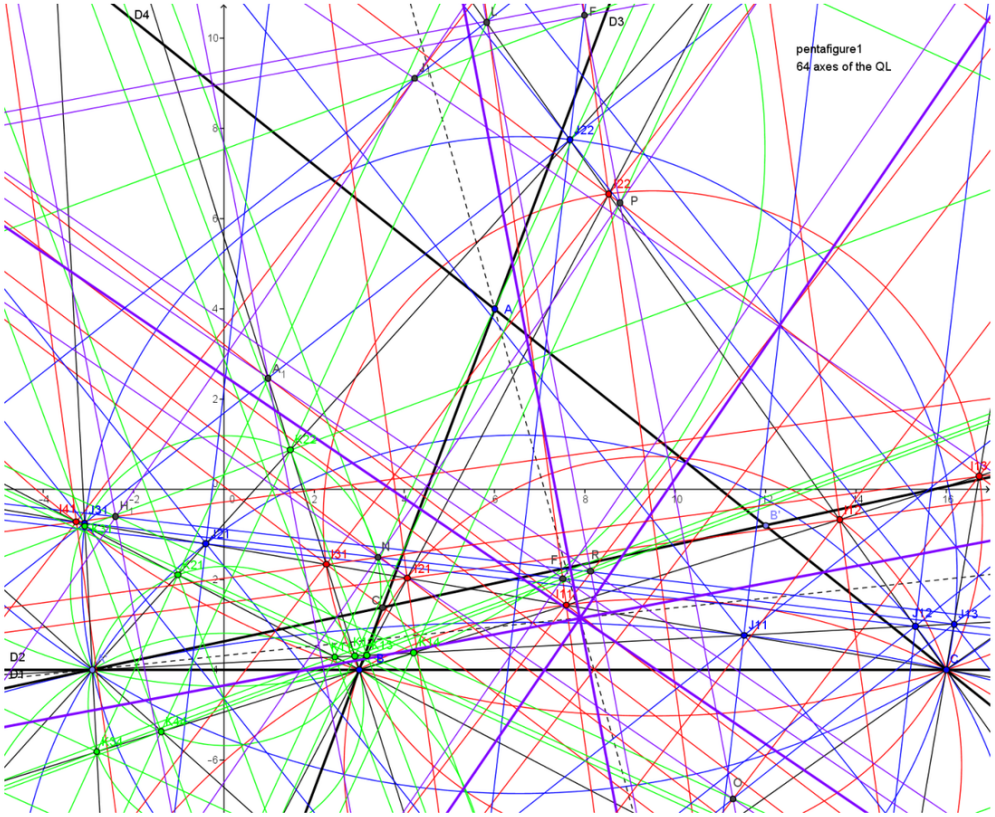
   We have to proceed this time in 2 steps
   Let's choose one line as double or triple line and let's draw the 4 quadrisectors in 2 vertices B and C on this line (near to BC, in B conterclockwise and in C clockwise)
   We get the same way 16 points called I11 to I44, forming 4 squares on 4 circles through the 2 points B and C

   a) 1rst step
      Consider the 5 lines Di, Di, Di, Dj and Dk (Di is a triple line)
      In each of these 16 points, 5 lines concur : 2 quadrisectors in B and C as m(Di, Di, Di, Dj) and m(Di, Di, Di, Dk) and 3 meanlines of the type m(Di, Di, Dj, Dk) (with Di as a double line this time)
      The 3 lines m(Di, Di, Dj, Dk) are the lines joining the intersection point of the 2 quadrisectors to the 3 other vertices of the square on the same circle through the 2 vertices B and C
   b) 2nd step
      There are 3 triangles having one side on this choosen line and there are the same way 16 points J11 to J44 and 16 points K11 to K44
      Consider now the 5 lines Di, Di, Dj, Dk and Dl (Di is a double line)

64 axes of the QL-1076.pdf

We find new intersection points where 5 lines concur : 3 of the preceding mean lines m(Di, Di, Dj, Dk), m(Di, Di, Dj, Dl) and m(Di, Di, Dk, Dl) and 2 of the searched 64 axes of the QL m(Di, Dj, Dk, Dl)
It appears that there are on each mean line of the type m(Di, Di, Dj, Dk) belonging to one set 2 points like I, J or K and 4 points of intersection with the 2 other sets and on each of the axes of the QL 3 of these intersection points or centers of the ennacardioïds
For one line choosen as double or triple line, there are 3*16 = 48 points like I, J or K and 3*24 = 72 mean lines in the 1rst step, which intersect in 4*24 = 96 points on the 64 axes in the 2nd step (a group of 6 points give 4 axes making angles of π/4 or π/2 with 3 points on each axis)
In order to have a complete figur, we have to do the same construction for the 3 other lines of the QL ; it appears then that every axis carries 4*3 = 12 points and that there are 4*96 = 384 centers of ennacardioïds tangent to 4 lines and bitangent to one.

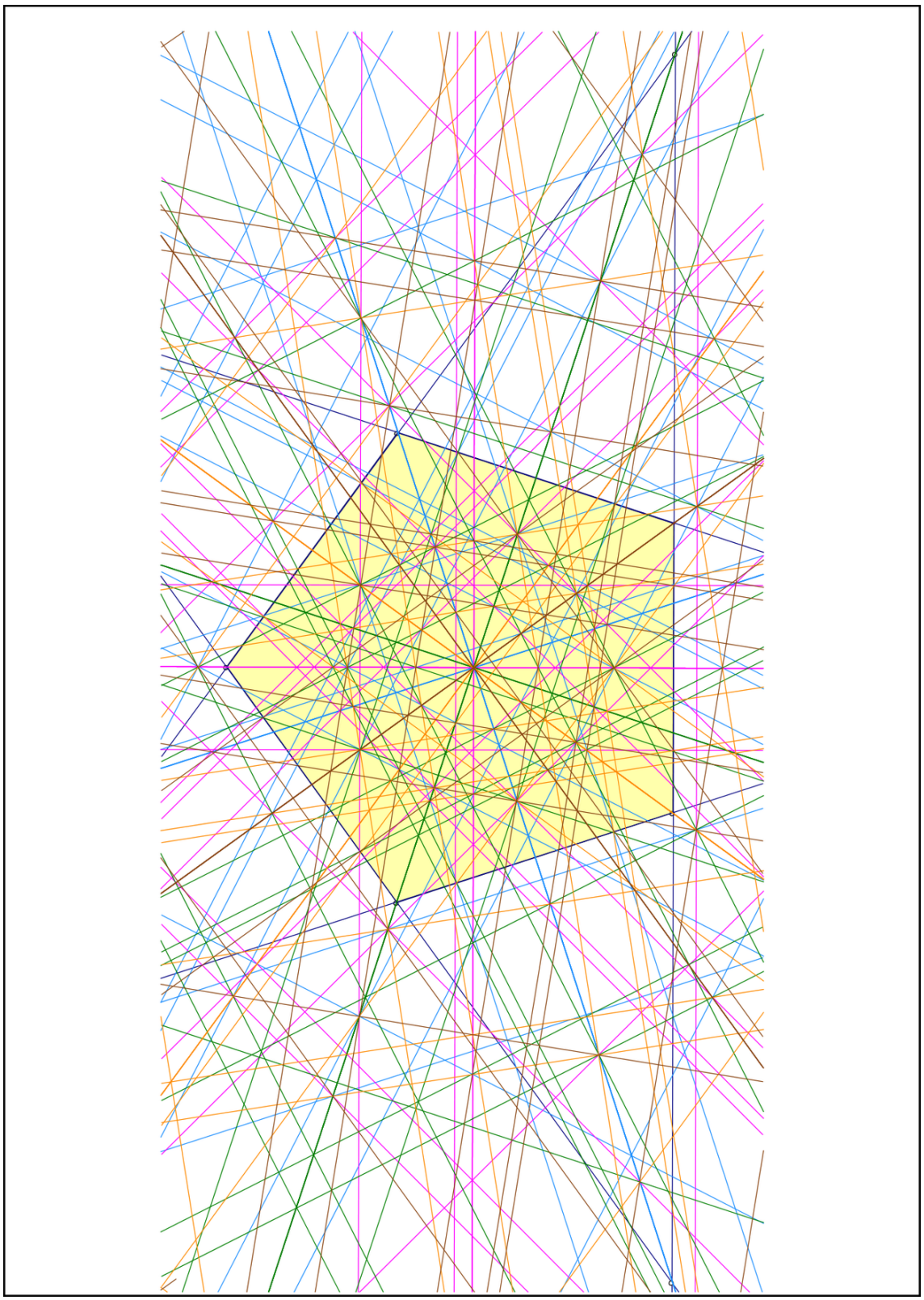
On the figur below, D2 is the double or triple tangent, the 3 sets are in red (ABC), blue (A'B'C) and green (A'BC') and the axes are in purple



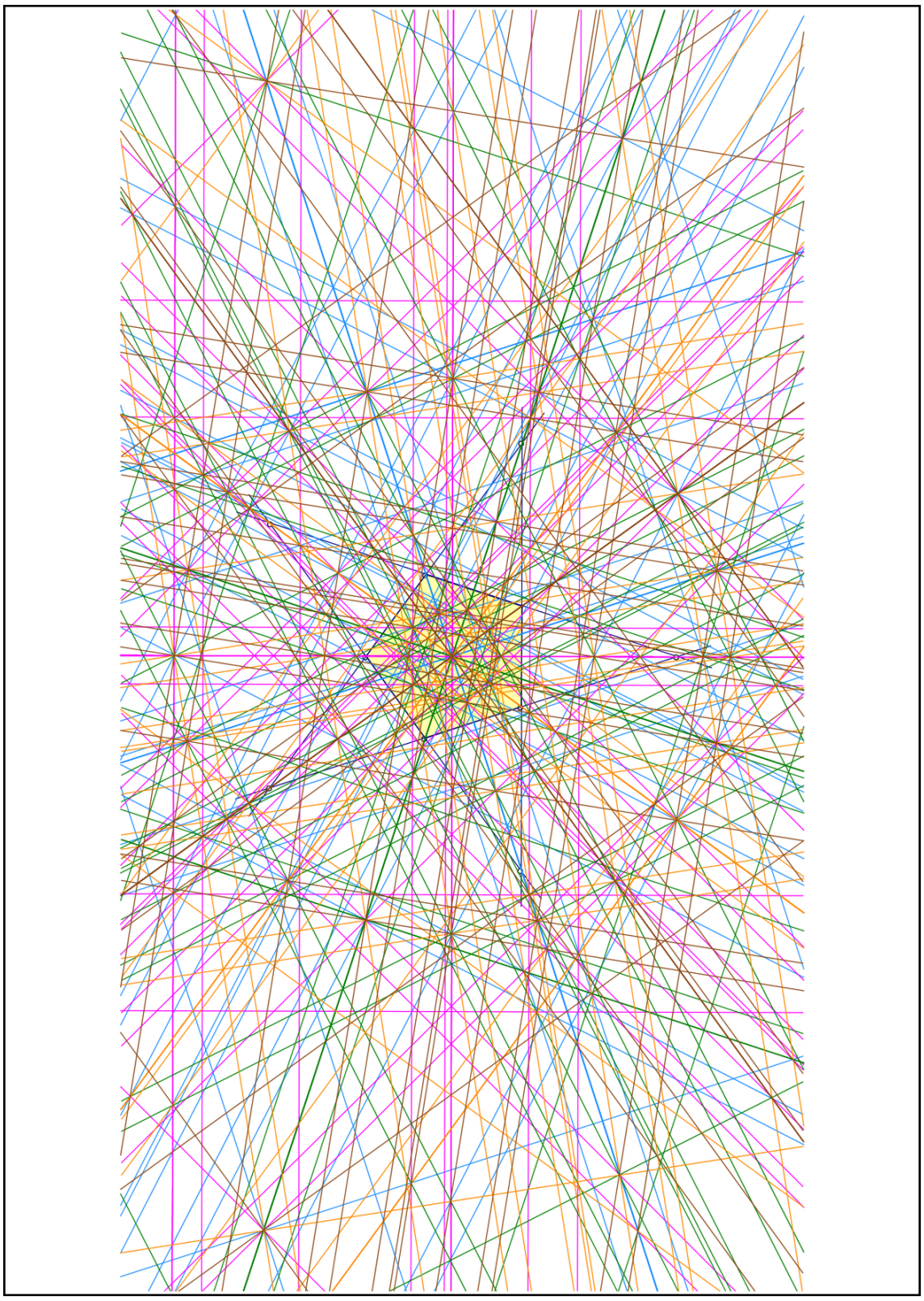
64 axes of the QL-1076.pdf

Message: #1077
Date: 18/4/2015 1:58:05
From: eckart_schmidt@t-online.de
Subject: Incenters for a regular 5L

Dear Bernard, dear Chris,
just for fun: the 5 x 64 axes for a regular 5-line. What are the incenters?
Best regards Eckart



5L-incenters.pdf



5L-incenter.pdf

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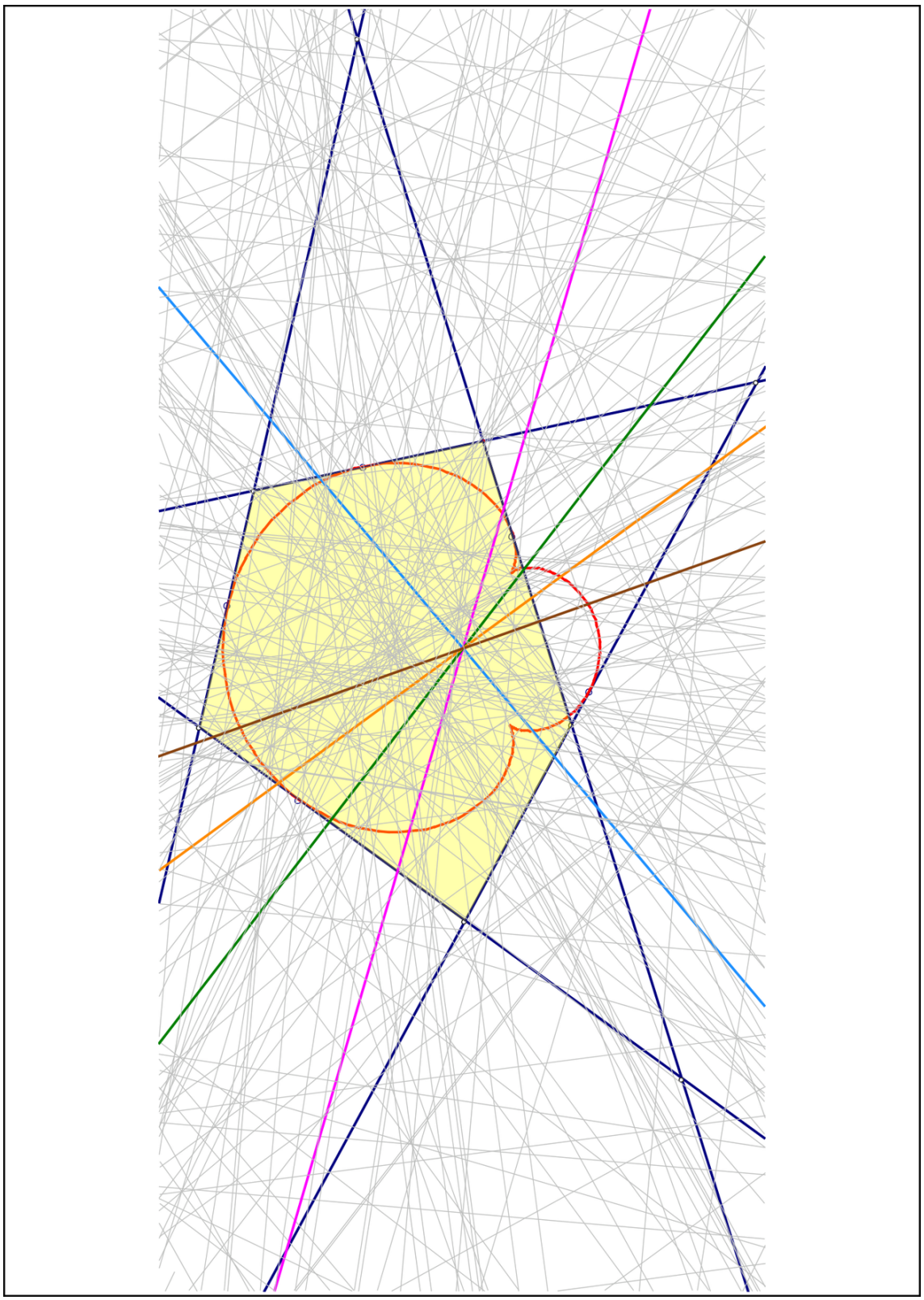
Message: #1078
Date: 18/4/2015 9:13:18
From: eckart_schmidt@t-online.de
Subject: Inscribed 5L-tetracardioid

Dear Bernard, dear Chris,
in the attached file you find a construction for a tetracardioid, five tangents, 320 axes for the five 4L and coloured the five axes for the center of the tetracardioid.
Best regards Eckart



5L-tetracard.pdf

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Message: #1079
Date: 19/4/2015 1:37:51
From: seiichikiri
Subject: Desmic locus of pedal points

Dear Chris,
If P is on K004, the coordinates of the pedal points have the cevian relations:
1) (a^2+b^2-c^2)y+2b^2x=(a^2-b^2+c^2)z+2c^2x.
2) (-a^2+b^2+c^2)z+2c^2y=(a^2+b^2-c^2)x+2a^2y.
3) (a^2-b^2+c^2)x+2a^2z=(-a^2+b^2+c^2)y+2b^2z.
Then R4 is transformed into the symmetric expression:
R4={p^2/((a^2+b^2-c^2)y+2b^2x), q^2/((-a^2+b^2+c^2)z+2c^2y), r^2/((a^2-b^2+c^2)x+2a^2z)}.
Best regards,
Seiichi.

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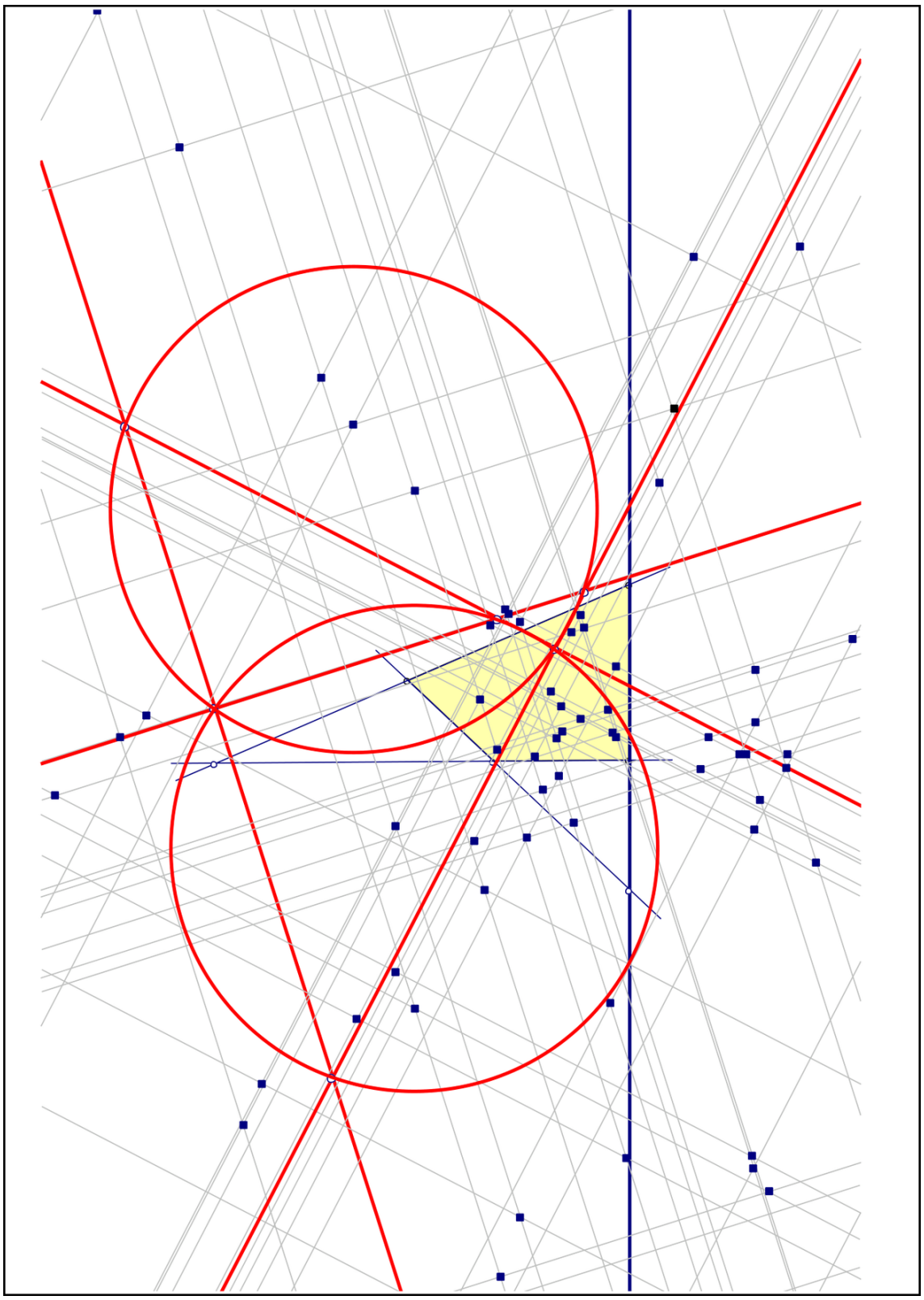
Message: #1080
Date: 19/4/2015 10:02:05
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard, dear Chris,
in the attached file you find a 4-line with 64 Morley-axes and 96 intersections wrt a basic line, containing 16 pairs of orthogonal circles through six intersections on four axes perpendicular in pairs (only one example is drawn).
Best regards Eckart



2015-04-19.pdf

Message: #1081
Date: 21/4/2015 2:46:44
From: seiichikiri
Subject: A desmic point of the excentral triangle of the diagonal triangle

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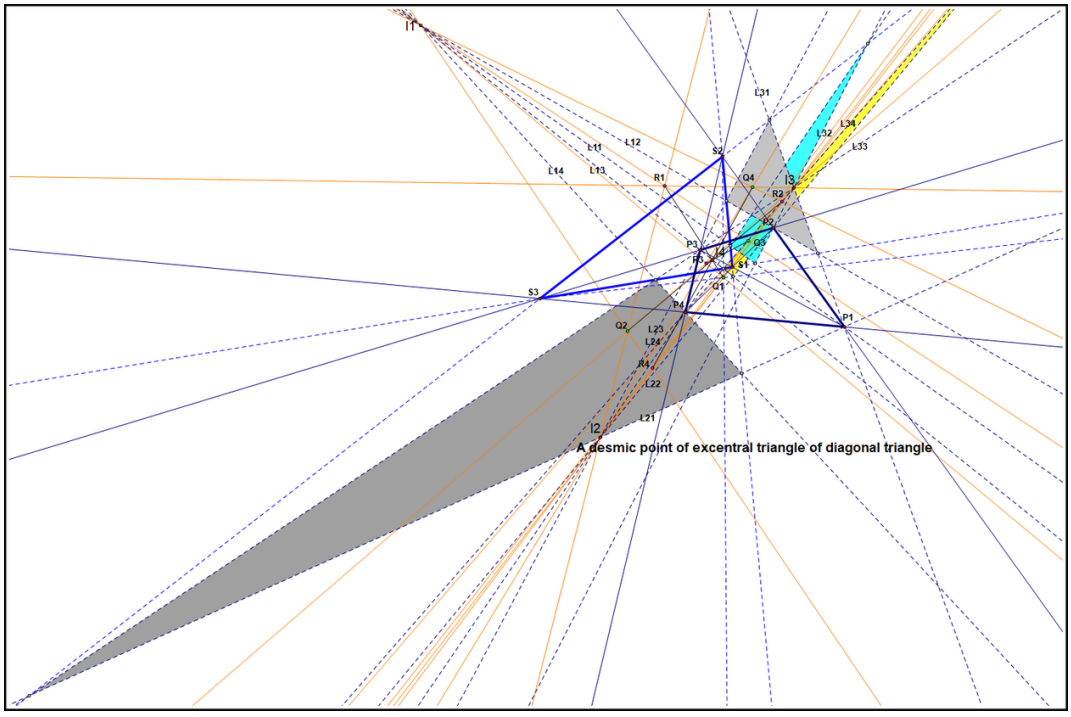
Dear friends,
Let P1P2P3P4 be a quadrangle.
S1, S2, S3 =the intersections of P1P3 and P2P4, P1P2 and P3P4, P2P3 and P1P4 respectively. (the vertices of the diagonal triangle of P1P2P3P4.)
Let I1I2I3 be the excentral triangle of S1S2S3. See the attached gsp.
Denote:
Lij =the line through Ii and Pj for i=1,2,3 and j=1,2,3,4.
Q1 = the perspector of S1S2S3 and the trilateral L13.L22.L34.
Q2 = the perspector of S1S2S3 and the trilateral L14.L21.L33.
Q3 = the perspector of S1S2S3 and the trilateral L11.L24.L32.
Q4 = the perspector of S1S2S3 and the trilateral L12.L23.L31.
1) I1Q3, I2Q2, I3Q4 concur in R1.
2) I1Q4, I2Q1, I3Q3 concur in R2.
3) I1Q1, I2Q4, I3Q2 concur in R3.
4) I1Q2, I2Q3, I3Q1 concur in R4
Q1R1, Q2R2, Q3R3 and Q4R4 concur in I4.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.



DesmicExcentralDT-gsp.png

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Message: #1082
Date: 21/4/2015 7:10:04
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart, dear Chris
The 2 last messages 1078 and 1080 confirm plainely the theory, the 1rst one with this strange curve called tetracardioïd tangent to 5 lines and the 2nd one with the 96 points on the 64 axes of the QL, generalising the cardioïd tangent to 4 lines and the 27 points on the 9 Morley axes for the triangle.
Huge and beautiful teamwork we achieved !
Congratulations and best regards to both of you
Bernard
PS What do you think for the curve through the 96 points ?

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Message: #1083
Date: 22/4/2015 8:40:28
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard, dear Chris,
among Bernard's 96 points of a 4-line wrt a basic line there are 32 intersections of orthogonal Morley-axes. These points give an interesting constellation, see attached file for first remarks.
I am shure, there will be further properties.
Best regards Eckart

EQF-Note 2015-04-22

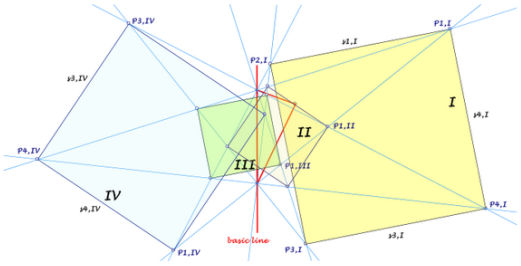
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Orthogonal Morley-4L-axes

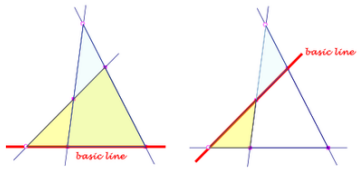
Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes, but Morley doesn't mention a construction. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes (see QFG-message 1032), using four squares related to quadrisectors and 96 points wrt a basic line. Here the constellation of 32 of these points is researched, where Morley-axes intersect orthogonal.

The squares for a triangle wrt a basic line

(1) For a triangle with one side as basic line there are four squares, generated by quadrisectors and described in *QFG*-messages 1032 and 1066.



(2) Chose an anticlockwise nomination for the vertices of the squares:
$P_{i,J}$ with $i = 1, 2, 3, 4$ and $J = I, II, III, IV$,
so that for example $P_{1,J}$ for $J = I, II, III, IV$ lie on the same quadrisector. Let $s_{i,J} = P_{i,J}P_{i+1,J}$ be the side lines of the squares.



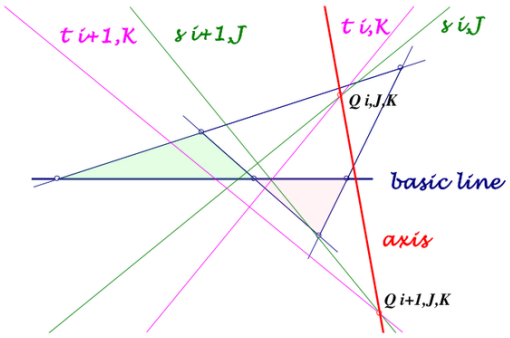
(3) Consider for a quadrilateral a basic line. There are three triangles with this basic line, but we take only two: If

the three triangles lie on one side of the basic line, we use the two, which intersect in the convex quadrigon of the quadrilateral. If the three triangles lie on different sides of the basic line, we use the two, which have no intersection with the convex quadrigon of the quadrilateral.

(4) Take for a basic line of a quadrilateral the two triangles in the sense of (3), distinguished by s and t. Draw for one triangle $s_{i,J}$ and $s_{i+1,J}$, for the other $t_{i,K}$ and $t_{i+1,K}$ and consider the intersections $s_{i,J} \cap t_{i,K} = Q_{i,J,K}$ and $s_{i+1,J} \cap t_{i+1,K} = Q_{i+1,J,K}$.

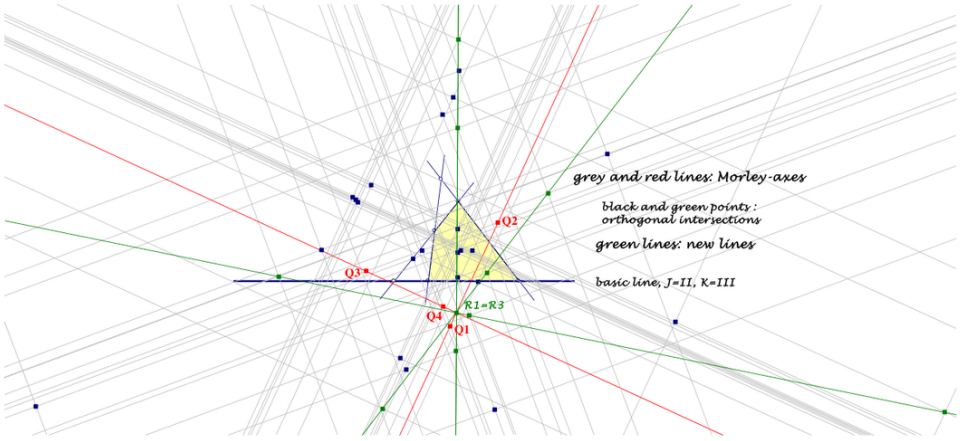
$Q_{i,J,K}\, Q_{i+1,J,K}$ is a Morley-axis

$i \in \{1, 2, 3, 4\}$, J and $K \in \{I, II, III, IV\}$.

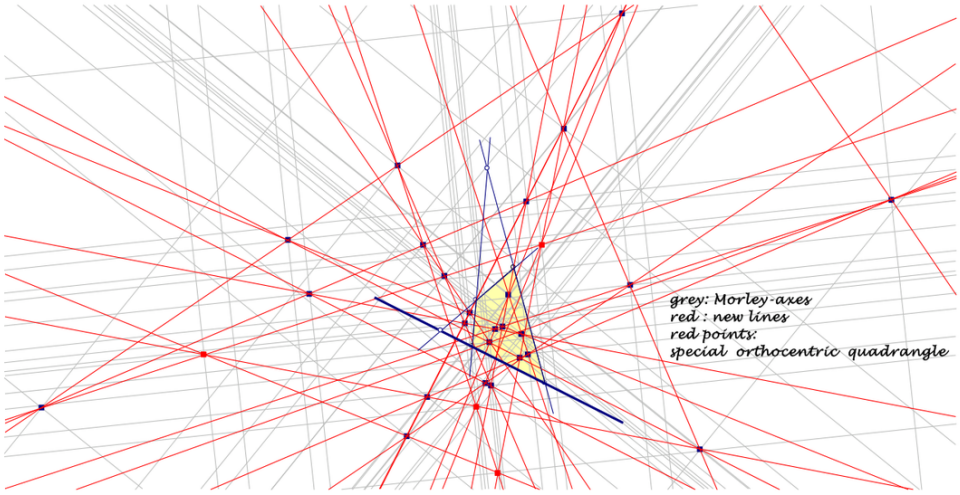


- The four possibilities for i, J, K show, that there are $4^3 = 64$ Morley-axes wrt a basic line for a quadrilateral. The four basic lines give the same Morley-axes.
- The 64 points $Q_{i,J,K}$ are intersections of the Morley-axes $Q_{i-1,J,K}\, Q_{i,J,K}$ and $Q_{i,J,K}\, Q_{i+1,J,K}$.
- The Morley-axes $Q_{i,J,K}\, Q_{i+1,J,K}$ and $Q_{i+2,J,K}\, Q_{i+3,J,K}$ are orthogonal.
- There are 32 intersections $R_{i,J,K} = R_{i+2,J,K} = Q_{i,J,K}\, Q_{i+1,J,K} \cap Q_{i+2,J,K}\, Q_{i+3,J,K}$ of such orthogonal Morley-axes.
- If we consider also the diagonals $s_{\alpha,J} = P_{1,J}P_{3,J}$ and $s_{\beta,J} = P_{2,J}P_{4,J}$ of the squares, we can get the points $R_{i,J,K}$ as $s_{\alpha,J} \cap t_{\alpha,K}$ for $i = 1or3$ and as $s_{\beta,J} \cap t_{\beta,K}$ for $i = 2or4$.

Among Bernhard Keizer's 96 points of a 4-line (wrt a basic line) these 32 intersections $R_{i,J,K}$ give a very special constellation:



... Sets of 4 of these points are collinear (green).
... The corresponding lines intersect by 3 in one of these points.
... These are 24 new lines: 12 pairs of parallels, each pair with a perpendicular pair.
… The constellation contains in different ways orthocentric quadrangles, their lines bear 16 of the 32 points.
... For the 4 points on a bearer line the angles between the other two lines are equal.



Bernhard Keizer's 96 points of a 4-line are the centers of Morley's tetracardioids, tangent to the four lines and twice to the basic line.

The 32 special orthogonal intersections $R_{i,J,K}$ of Morley-axes are the centers of tetracardioids tangent to the four lines and twice in symmetry to the basic line.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-04-22.pdf

Message: #1084
Date: 23/4/2015 9:51:26
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart,
I don't know if this coulp help you.
The 96 points form 16 QL with 4 lines and angles at pi/2 or pi/4.
Each QL with 6 points is easely completed by a 7th point, which is the intersection of the 3rd side and the 3rd altitude of a triangle formed by a side/axis with 3 of the 6 points and a 4th point of the 6 on the altitude/perpendicular axis.
These 4 points of the 6 form obviously an orthocentric QA ; the 2 others and the 7th point are the feet of the altitudes in the triangle.
I suppose there are 16 such 7th point.
I observe that these 7th points are the intersections of 3 circles through the vertices of the squares and of 2 of the 3 points on the basic line (each circle in a different reference triangle).
Best regards
Bernard

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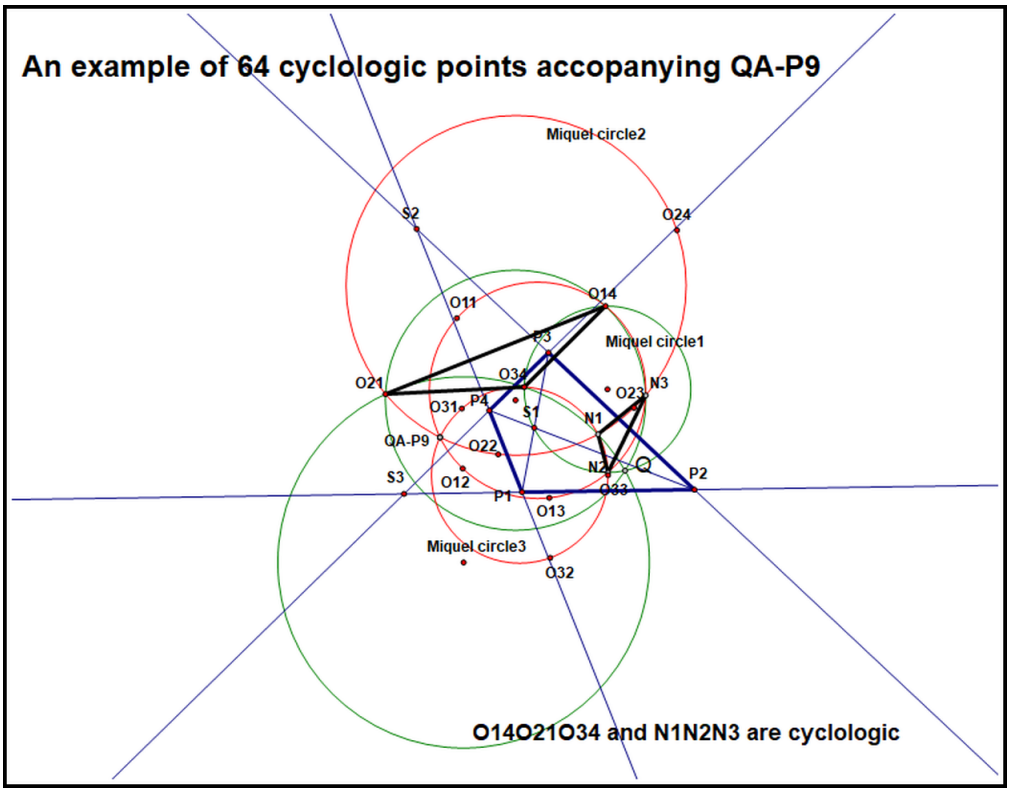
Message: #1085
Date: 23/4/2015 11:47:05
From: seiichikiri
Subject: 64 cyclologic points accompanying QA-Miquel center

Dear friends,
QA-Miquel center (QA-P9 inEQF) is constructed from 3 Miquel circles 1, 2, 3 through 4 circumcenters of the component triangles respectively.
Denote:
N1, N2, N3 =the intersections of Miquel circle 2 and Miquel circle 3, Miquel circle 3 and Miquel circle 1, Miquel circle 1 and Miquel circle 2 respectively.
O14= one of 4 circumcenters of Miquel circle 1.
O21= one of 4 circumcenters of Miquel circle 2.
O34= one of 4 circumcenters of Miquel circle 3.
The circumcircles of N1O21O34, N2O34O14 and N3O14O21 concur in a point Q.
There are 4^3=64 cyclologic points accompanying QA-P9. See an example of them in the attached gsp.
For QA-Miquel center, see QA-P9.
For 4 circumcenters of the component triangles, see QL-P4.
Best regards,
Seiichi.



64cyclologic-gsp.png

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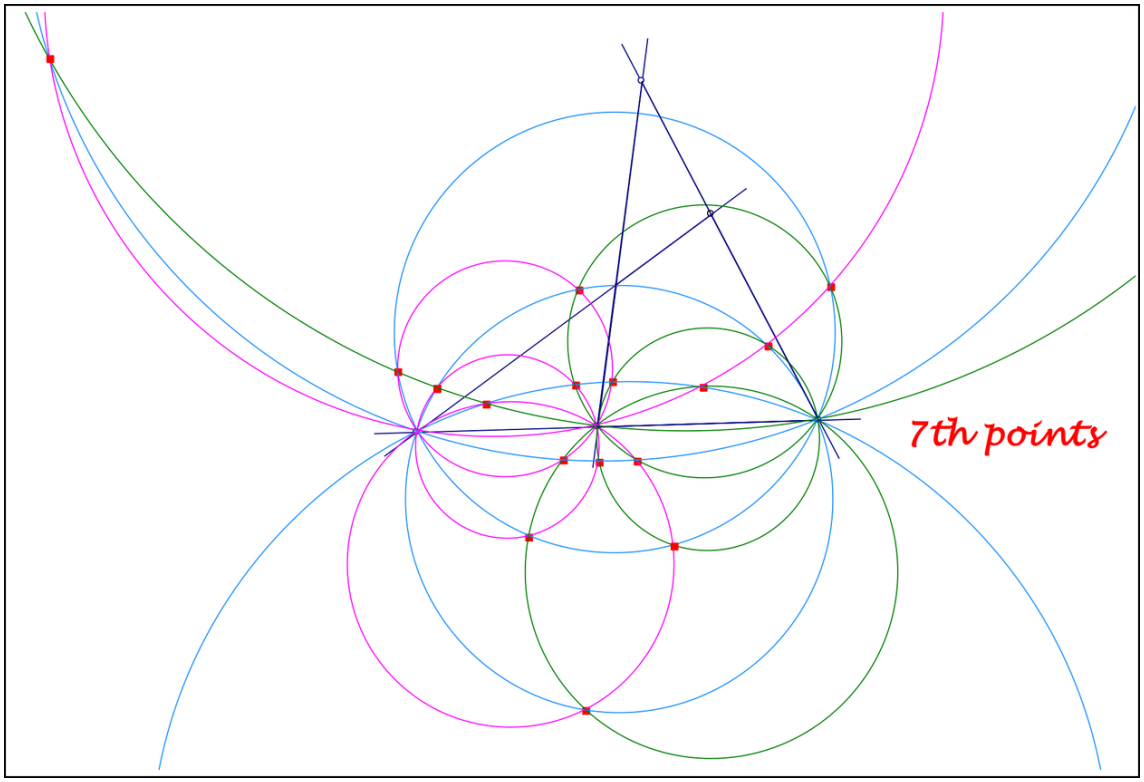
Message: #1086
Date: 23/4/2015 2:43:39
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
thanks for the interesting constellation: Each axis bears 3 of your 96 points wrt a basic line. These points are intersections of two axes, one point with orthogonal intersection. Taking the 3*2 axes through these three points, you get a 4-line with orthogonal triangles, whose circumcircles have a common point: your 7th point of the 4-line. There are 16 7th points. A circle through two basic points and a 7th point contains three other 7th points. Each circumcircle of a square for a triangle to the basic line contains four 7th points. These are the same 12 circles.
Best regards Eckart



2015-04-23.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1087
Date: 23/4/2015 4:21:20
From: chris.vantienhoven
Subject: 64 cyclologic points accompanying QA-Miquel center

Dear Seiichi,
Summarizing I would say that Triangle N1.N2.N3 is cyclologic with all possible Triangles O1i.O2j.O3k (i,j,k=1,2,3,4).
That is a special relationship indeed.
Actually you points N1, N2, N3 are also the points QG-P5 of the three QA-Quadrigons.
Best regards,
Chris

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Message: #1088
Date: 24/4/2015 9:30:08
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart,
In this item of 7th points and circumcircles of squares for a basic line, we perfectly agree : there are 16 7th points, each point on 3 circles and 4 points on each of the 12 circles.
Now, if i'm not wrong, a conic is entirely defined by 5 points, a cubic by 9 points and a quartic by 14 points (a curve of degree n by n(n + 3)/2 points).
Are there by any chance in all these groups of points 96 or 32 or now 16 quartics to identify ?
I suppose you are able to draw such curves with Macros in Cabri ...
Thanks in advance and best regards
Bernard

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Message: #1089
Date: 28/4/2015 8:56:27
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

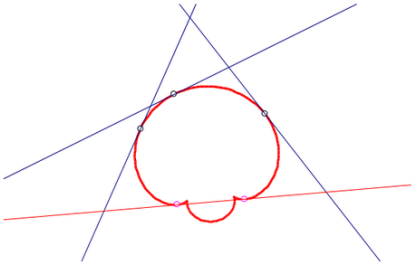
Dear Bernard, dear Chris,
I have researched tetracardioids double tangent in symmetry wrt a line. In the attached file are gathered my CABRI observations. There is a simple construction for the loci of centers of tetracardioids, tangent to the sides of a triangle and twice to one in symmetry.
Best regards Eckart

EQF-Note 2015-04-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

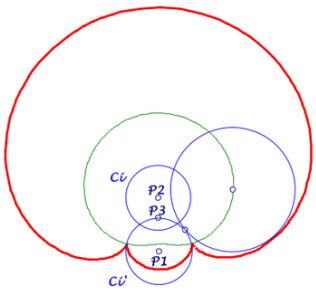
Double Tangent Tetracardioïds

Morley mentioned in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 96 tetracardioïds tangent to the four lines and to one line twice. Centers of these tetracardioïds are special intersections of Morley's 64 axes for a 4-line. Here a set of 32 tetracardioïds is researched, which contact a line twice in symmetry.



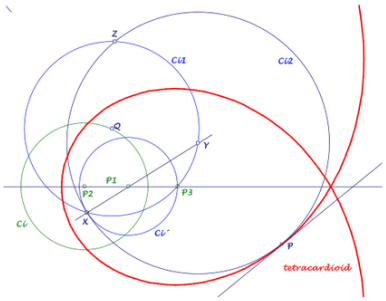
Tetracardioïds

A limaçon is the envelope of circles centered on a circle and through a fixed point. A tetracardioïd can be described as envelope of circles centered on a limaçon and tangent to a circle. Let P_1 be a point, Ci a circle with center P_2 and P_3 the inverse of P_1 wrt Ci. Consider the limaçon wrt Ci and P_3 and a circle Ci' round P_1 through P_3. Circles centered on the limaçon and tangent to Ci' envelope a tetracardioïd. If P_1 lies inside Ci, let Ci' be inside the contacting circles, if P_1 lies outside Ci, let Ci' be outside the contacting circles.

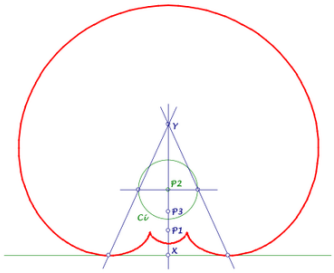


In this way a tetracardioïd is defined by a point P_1 and a circle Ci. P_2 is the center of the tetracardioïd. If P_1 lies inside the

circle, the tetracardioïd is limaçon-similar, if P_1 lies outside the circle the tetracardioïd has two cusps. A construction can be done as follows:
... Let Q be a variable point on the circle Ci,
... let Ci_1 be a circle round Q through $P3$,
... let X be the second intersection of Ci_1 and the circle $Ci´$,
... let Y be the second intersection of XP_1 and Ci_1,
... let Ci_2 be a circle round Y through X,
... let Z be the second intersection of Ci_1 and Ci_2,
... then Z reflected in Y is a point P of the tetracardioïd
…with the same tangent to Ci_2.



Tetracardioïd with a given double tangent

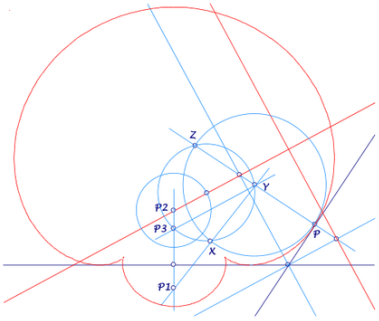


We consider only double tangents in symmetry. If the double tangent is given, every circle Ci with center P_2 gives a corresponding tetracardioïd, constructed as follows:
… Let X be the pedal point of P_2 wrt the double tangent,
… let P_3 divide P_2X in the ratio 1:2,
… let P_1 be the inverse of P_3 wrt Ci,
… further construction see above.

The contact points can be constructed as follows:
… Let Y be the reflection of X in P_2,
… let S_1 and S_2 be the intersections of a perpendicular line in P_2 wrt XY,
… then YS_1 and YS_2 intersect the double tangent in the contact points.

Tetracardioïds with a given double tangent and a 2nd tangent

Let the double tangent be given and a 2nd tangent with contact point *P*. The centers of corresponding tetracardioïds lie on two lines:
… Let *u* and *u´* be the angle bisectors of the given tangents,
… let *v* be a perpendicular in *P* wrt the 2nd tangent,
… then the perpendiculars wrt the angle bisectors in the intersections with *v* are the loci for centers of tetracardioïds contacting the 2nd tangent in *P*.

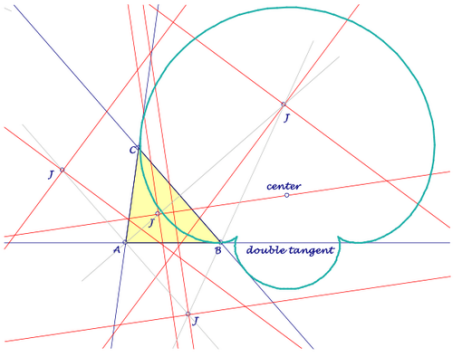


If we chose a center P_2 on these lines, the tetracardioïd can be constructed in a way back:
… Let P_3 divide the distance between P_2 and its pedal point on the double tangent with ratio 1:2,
… a parallel to the locus line of the centers through P_3 intersects the perpendicular in *P* in the point *Y*,
…let Ci_2 be a circle round *Y* through *P* and *Z* the reflection of *P* in *Y*,
… let Ci_1 be the circumcircle of *Y*, *Z*, P_3 with second intersection *X* with Ci_2,
… the line *XY* intersects the perpendicular through P_2 wrt the double tangent in P_1,
... then the circle *Ci* for the tetracardioid is a circle round P_2 so that P_1 and P_3 lie inverse.

Tetracardioïds with a given double tangent and two other tangents

This constellation gives a triangle *ABC*: Let *AB* be the double tangent and *AC, BC* two tangents. The described properties and constructions give the following CABRI observation:

- **Let *J* be an in- or excenter of *ABC*, then the angle bisectors of <*AJB* are the loci for centers of tetracardioïds tangent to *AC, BC* and double tangent to *AB* in symmetry.**

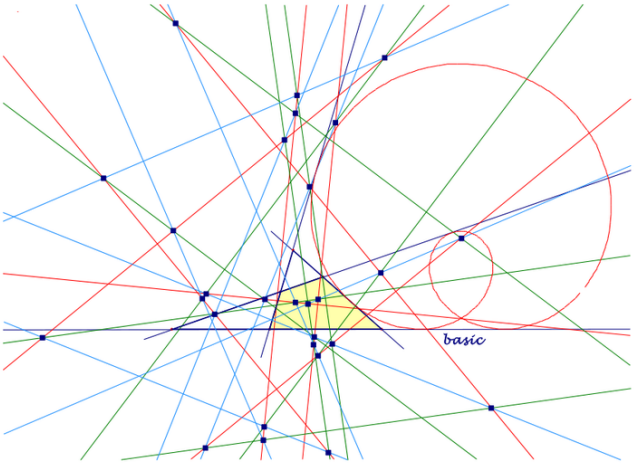


There are 8 center lines: 4 pairs of parallels or 4 pairs of orthogonal lines, intersecting in the in- and excenters.

Tetracardioïds with a given double tangent and three other tangents

Background now is Morley's constellation for 96 inscribed tetracardioïds of a 4-line, double tangent to one line. But here are only considered the double contacts in symmetry. If one line of the 4-line is declared as double tangent or basic line, there are three triangles with this side line.

- **The centers of inscribed tetracardioïds for a 4-line, double tangent to a basic line in symmetry, are the 32 triple intersections of the 8 center lines for the 3 triangles at the basic line.**



On each center line of the three triangles are 4 centers of tetracardioïds. This constellation is already described in *EQF*-message 1083 with the result:

- **The 32 centers of inscribed tetracardioids of a 4-line, double tangent to a basic line in symmetry, are the intersections of orthogonal Morley-axes among Bernard Keizer's 96 points of a 4-line wrt a basic line.**

Wrt to 4 possible basic lines for a 4-line there are 96 centers of inscribed tetracardioïds, tangent twice to one side in symmetry.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-04-28.pdf

Message: #1090
Date: 30/4/2015 8:33:17
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

```
Dear Bernard, dear Chris,
sorry, there is a clear typo in the last sentence of my paper
"Double tangent tetracardioids": 4 x 32 = 128 and not 96. Please
excuse.
Best regards Eckart
```

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Message: #1091
Date: 02/5/2015 12:14:41
From: seiichikiri
Subject: A property of Steiner line

```
Dear friends,

Is the following property already known?
The reflections of QL-P1 (Miquel point) in L1, L2, L3 and L4 lie
on Steiner line (QL-L2).

Best regards,
Seiichi.
```

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Message: #1092
Date: 02/5/2015 9:38:45
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

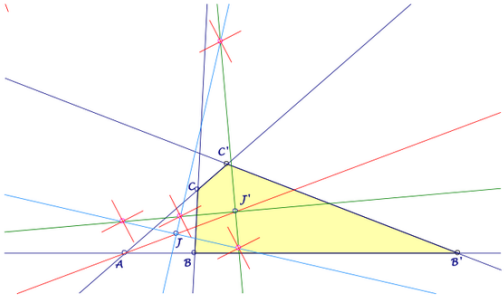
```
Dear Bernard, dear Chris,
I think, there is a very simple construction for the Morley-axes
of a 4-line in the attached file, orientated at orthogonal
intersections of the axes.
Best regards Eckart
```

EQF-Note 2015-05-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Construction of Morley's 4L-axes II

Morley describes in his paper "Extensions of Clifford's Chain-Theorem" for a 4-line 64 axes, but Morley doesn't mention a construction. In his paper "64 axes of the QL" Bernard Keizer gives an interpretation and a construction of these axes, see QFG-message 1032. Further constructions are described in QFG-message 1056 and 1068. Here is a new – very simple – construction, orientated at orthogonal intersections of the axes.



The orthogonal intersections

Consider a 4-line and two perspective triangle components ABC and $AB'C'$ wrt a common vertex A (see figure: J incenter of ABC, $J´$ incenter of $AB´C´$):
… Let J_i and J_i' corresponding in-/ex-centers of ABC and $AB´C´$.
… An angle bisector at A contains two in-/ex-centers J_i of ABC and two in-/ex-centers J_i' of $AB'C'$.
… The intersections of the angle bisectors of $\sphericalangle AJ_iB$ and $\sphericalangle AJ_j'B'$ give 4 intersections of orthogonal Morley-axes with the same directions.
… For the two possibilities of $i = j$ there are 8 intersections of orthogonal Morley-axes with the same directions, analog for $i \neq j$ wrt the other possible orthogonal directions.
… Wrt one angle bisector at A we get 16 intersections, taking the other angle bisector at A, we get further 16 orthogonal intersections.
… The intersecting orthogonal axes in these 32 points are all 64 Morley-axes.

The orthogonal directions

Each direction of a line has a mean direction wrt the 4 lines of a quadrilateral. This mean direction is one of the four possible directions of Morley-axes. We consider the orthogonal mean directions of the angle bisectors at A as primary directions and the other two as alternative directions.
The directions of the orthogonal axes in the intersections of the angle bisectors of $\sphericalangle AJ_iB$ and $\sphericalangle AJ_j'B'$ depend on whether $i = j$ or $i \neq j$ and whether J_i and J_j' lie on an inner or outer angle bisector at A.

Ji and Jj´	$i = j$	$i \neq j$
... on inner angle bisector at A	*primary directions*	*alternative directions*
... on outer angle bisector at A	*alternative directions*	*primary directions*

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #1093
Date: 02/5/2015 12:32:36
From: seiichikiri
Subject: A characteristic of QL-Cu1(QL-QuasiIsogonal Cubic)

Dear friends,
It is written in EQF that QL-Cu1 is the locus of all points for which the feet of the perpendiculars from P to the 4 quadrilateral lines are concyclic.
Also QL-Cu1 is the locus of all points for which the reflections of P in the 4 quadrilateral lines are concyclic. (a commonplace result.)
A.D. (algebraically demonstrated.)
Best regards,
Seiichi.

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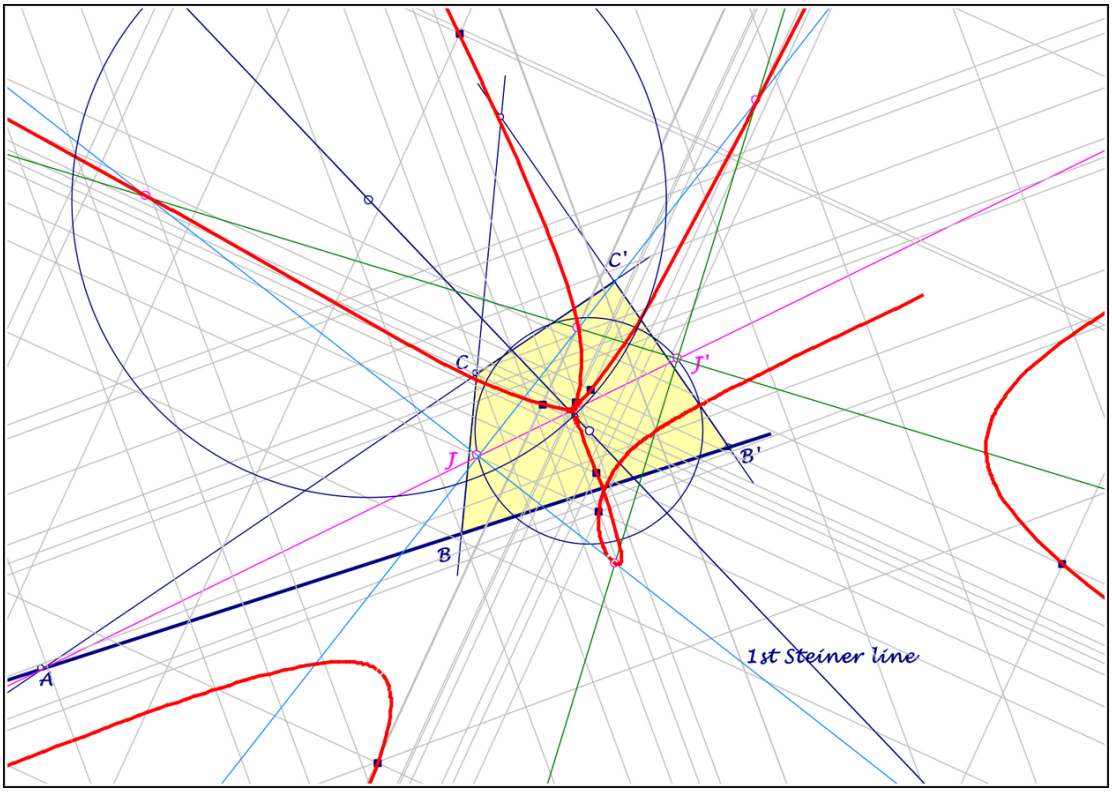
Message: #1094
Date: 03/5/2015 1:38:32
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
perhaps a first step wrt curves for your 96 intersections of Morley-axes wrt a basic line. In the attached file there is a curve, which contains 8 of your 96 points, where Morley-axes intersect orthogonal with the same directions. You get the curve - using the nomination of my last message - as follows:
... Take circles, centered on the 1st Steiner-axis
    and orthogonal wrt the Schmidt-circle,
... they intersect the angle bisector at A in J and J´,
... then the further construction of my last message
    -in the right assignment- gives the curve, but clearly no
    quartic.
Or:
... Take circles, centered on the 2nd Steiner-axis
    through the CSC-fixed points,
... see above, but that is another curve.
Or:
... Take the other angle bisector at A.
Best regards Eckart



2015-05-03.pdf

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Message: #1095
Date: 04/5/2015 4:15:34
From: seiichikiri
Subject: A property of Steiner line

```
Dear friends,
Addition:
The reflection lines of Steiner line (QL-L2) in L1, L2, L3 and
L4 pass Miquel point (QL-P1).
Best regards,
Seiichi.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1096
Date: 04/5/2015 4:47:25
From: seiichikiri
Subject: A property of Steiner line

```
Dear friends,
Addition:
Let L1L2L3L4 be a quadrilateral.
Denote:
i=1, 2, 3, 4, J=1, 2, 3, 4 and i does not equal j.
Mi= the reflection line of QL-Pedal line (QL-L3) in Li.
sij=the intersection of Li and Lj.
tij=the intersection of Mi and Mj.
The line sij tij passes Miquel point (QL-P1).
G.D. (graphically demonstrated.)
Best regards,
Seiichi.
```

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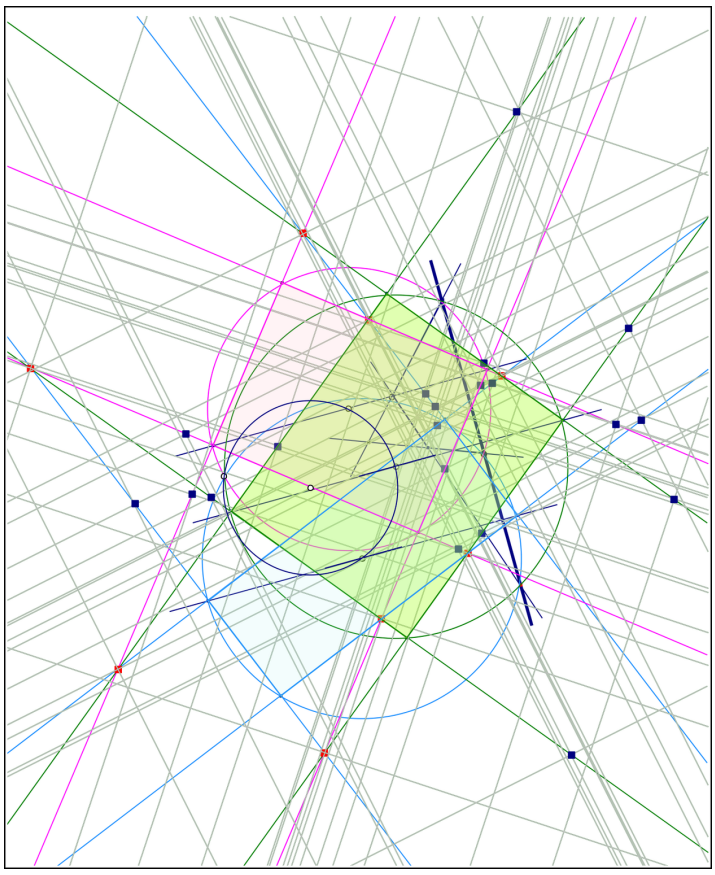
Message: #1097
Date: 04/5/2015 9:32:41
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard, dear Chris,
among Bernard's 96 intersections of Morley-axes (wrt a basic line) there are 32 orthogonal intersections (see #1083 and #1089) with an interesting constellation (see attached file):
... these 32 points lie by four
    on the sidelines of three rectangles,
... 8 points are triple intersections of sidelines,
... the rectangles belong to three triangles ABC
    (AB common basic line): one pair of opposite rectangle vertices
    are the excenters Ja and Jb, the other pair of opposite rectangle
    vertices are points on the bisector of AB,
... the circumcircles of the rectangles have a common point
    on the circumcircle of their centers,
    which contains also the Miquel point.
Best regards Eckart



2015-05-04.pdf

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Message: #1098
Date: 07/5/2015 2:54:55
From: seiichikiri
Subject: Another construction method of Steiner angle bisector center octet

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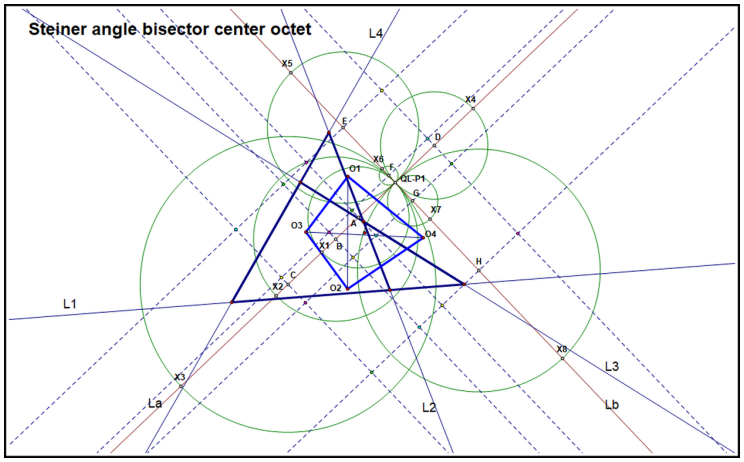
Dear friends,
Let L1L2L3L4 be a quadrilateral and QL-P1 its Miquel point.
We denote the circumcenters of trileterals L2L3L4, L1L3L4, L1L2L4, L2LL3L4 by O1, O2, O3, O4 respectively.
(1) Each of the 4 component triangles O1O2O3O4 has 1 incircle and 3 excircles. The centers of these 16 circles lie, 4 by 4, on new 8 lines.
(2) These 8 new lines form 2 sets of 4. Each line of 1 set is orthogonal to each line of the other set.
We denote the lines through QL-P1 orthogonal and parallel to 1 set by La and Lb respectively.
We denote the intersections of La and the lines of 1 set by A, B, C, D respectively.
We denote the intersections of Lb and the lines of the other set by E, F, G, H respectively.
(3) The circles of their centers on A, B, C, D and radii AQL-P1, BQL-P1, CQL-P1, DQL-P1 intersect La at X1, X2, X3, X4 (4 of Steiner angle bisector center octet(QL-8P1)) respectively.
(4) The circles of their centers on E, F, G, H and radii EQL-P1, FQL-P1, GQL-P1, HQL-P1 intersect La at X5, X6, X7, X8 (4 of Steiner angle bisector center octet(QL-8P1)) respectively.
G.D. (graphically demonstrated.)
See the attached gsp.
For Steiner angle bisector octet, see:
http://chrisvantienhoven.nl/quadrilateral-objects/17-mathematics/quadrilateral-objects/133-ql-8p1.html
Best regards,
Seiichi.



SteinerABCOctet-gsp.png

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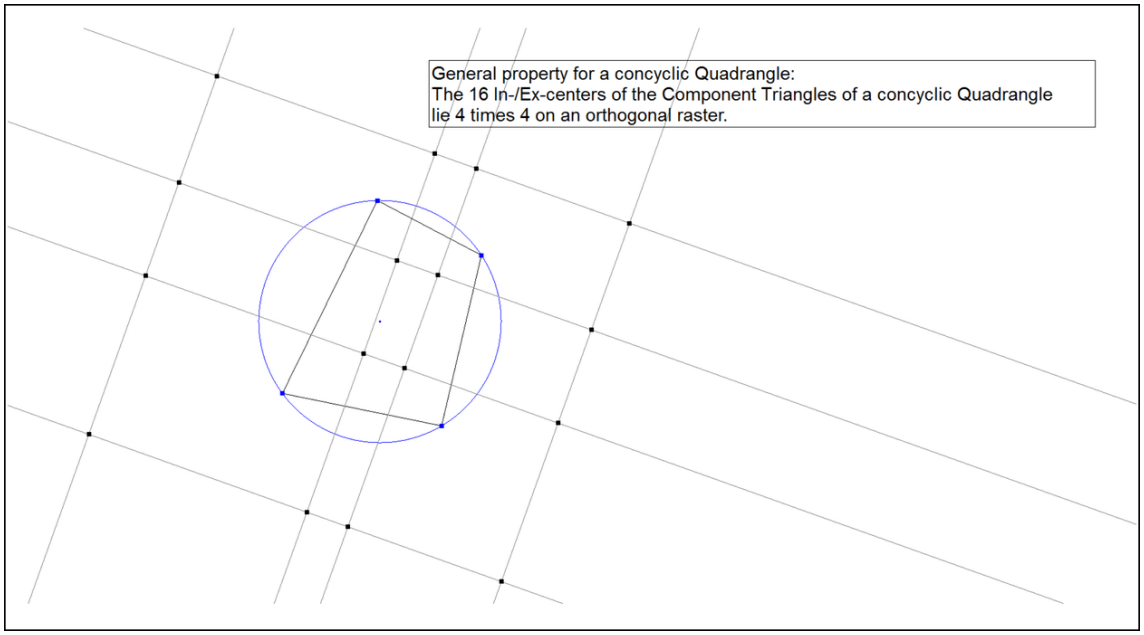
Message: #1099
Date: 07/5/2015 9:37:29
From: chris.vantienhoven
Subject: Another construction method of Steiner angle bisector center oct

---

Dear Seiichi,
The most remarkable thing in this construction is that the 16 In-/Excenters you describe form 2 orthogonal sets of 4 parallel lines.
I noticed that this is a general property of a concyclic Quadrangle:
The 16 In-/Ex-centers of the Component Triangles of a concyclic Quadrangle lie 4 times 4 on an orthogonal raster, each line containing one in-/excenter of each Component Triangle.
(GD=Graphically Demonstrated)
See attachment.
Moreover this can be applied to construct the Steiner Octet QL-8P1 like you showed.
Very nice!
Best regards,
Chris



ConcyclicQuadrangel CT-InExcenters on Raster.pdf

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Message: #1100
Date: 07/5/2015 10:42:56
From: bernard.keizer
Subject: Another construction method of Steiner angle bisector center oct

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Dear Seiichi, dear Chris
If I'm not wrong, this property is linked to the Japanese Theorem, which appears in old sangakus.
I observe that in the ordinary QL :
L3 is the Simson Line and L2 the Steiner Line (through the orthocenters) of the same Miquel point P1 wrt the 4 reference triangles
The line Li is the Steiner Line of this same Miquel point P1 wrt the triangle OjOkOl and the vertices Aij are the reflexions of P1 in the lines OjOk ...
The fact that the Oi and the point P1 are on the same Miquel circle helps to understand the beautiful properties described by Seiichi.
Best regards
Bernard

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Message: #1101
Date: 07/5/2015 10:59:20
From: bernard.keizer
Subject: 64 axes of a QL

Dear Eckart, dear Chris
I observe with the greatest interest your efforts in finding new properties to all these points:
256 points for a 5-L.
96 + 16 points (3 groups of 32 and the group of the 7th points) and I hope always that you will finally be able to find curves through all these points (or parts of them) !
My intuition that it should be quartics lied only on the fact that there are 320 lines (5*64) cutting the curve of the 256 points in 4 points (each point on 5 lines), exactly as the were 4*9 = 36 lines cutting the cubic stelloïd in 3 points (each point on 4 lines)!
I'm forced to admit that intuitions and conjectures are not demonstrations ...
Best regards
Bernard

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Message: #1102
Date: 12/5/2015 10:14:10
From: seiichikiri
Subject: The orthopoles of a quadrilateral

Dear friends,
1) Let L1L2L3L4 be a quadrilateral.
The orthopoles of the line parallel to Newton line (QL-L1) wrt 4 component triangles lie on Pedal line (QL-L3).
2) Let L1L2L3L4 be a quadrilateral.
The orthopoles of Pedal line (QL-L3) wrt 4 component triangles lie on Steiner line (QL-L2).
There are at least 16 points on Steiner line because of orthocenters, reflections and orthopoles of Li for i=1,2,3,4 and of Pedal line.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.

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Message: #1103
Date: 12/5/2015 1:40:45
From: chris.vantienhoven
Subject: The orthopoles of a quadrilateral

Dear Seiichi, dear friends,
I made notes in EQF for all the remarks wrt QL-L2 and QL-L3.
Besides that I am catching up in EQF with all kinds of earlier findings in the Quadri-Figures-Group.
Best regards,
Chris

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Message: #1104
Date: 12/5/2015 7:46:51
From: bernard.keizer
Subject: The orthopoles of a quadrilateral

Dear Seichi,
The properties are linked again with the Simson and Steiner Lines :
1) The orthopole of a line wrt a triangle lies on the Simson Line which is perpendicular to it.
For any line parallel to the Newton Line L1, the Simson Line wrt each of the 4 reference triangles perpendicular to L1 is precisely L3, which is the common Simson Line of P1 wrt the 4 reference triangles.
2) Taking a triangle formed by 3 tangents to a parabola
a) the focus of the parabola is on the circumcircle of the triangle
b) the orthocenter is on the directrix of the parabola, which is the Steiner Line of the focus wrt the triangle
c) the orthopole of any other tangent is on this directrix
The orthopole of each Li wrt the triangle of the 3 other lines lie on L2, which is the common Steiner Line of P1 wrt the 4 reference triangles.
The property holds obviously for L3, which is the tangent to the parabola in it's apse (the reflection of P1 in L3 lies on L2).
The orthopoles of L3 wrt the 4 reference triangle are on L2
Taking L3 as a 5th line tangent to the parabola like the Li and combining whith 2 of the 4 Li, you may find 6 more orthocenters on the directrix L2, which is the common Steiner Line of the focus P1 wrt the 10 triangles (4 reference and 6 more), 6 more circumcircles through the point P1 and 12 more orthopoles on L2.
Best regards
Bernard

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Message: #1105
Date: 13/5/2015 11:24:37
From: bernard.keizer
Subject: The orthopoles of a quadrilateral

Dear Chris,
I had a quick glance on the items recently added in EQF.
About QL-L3, I'm surprised by the formulation that the Simson Lines of QL-P1 wrt the 4 reference triangles concur on QL-L3.
In fact, QL-L3 is the common Simson Line of QL-P1 wrt the 4 reference triangles.
The same way, QL-L2 is the common Steiner Line of QL-P1 wrt the 4 reference triangles (whithout mistake of my side, this property doesn't appear clearly).
The explanation is that QL-P1 belongs to the 4 circumcircles of the reference triangles.
For other properties of the Simson and Steiner Lines, please see my previous message to Seiichi (reflections, orthocenters and orthopoles).
I'm now awaiting with impatience the construction of the 64 axes of the QL.
Did you achieve the calculations ?
Best regards
Bernard
PS I was glad to see your new reference 43 with my article ...

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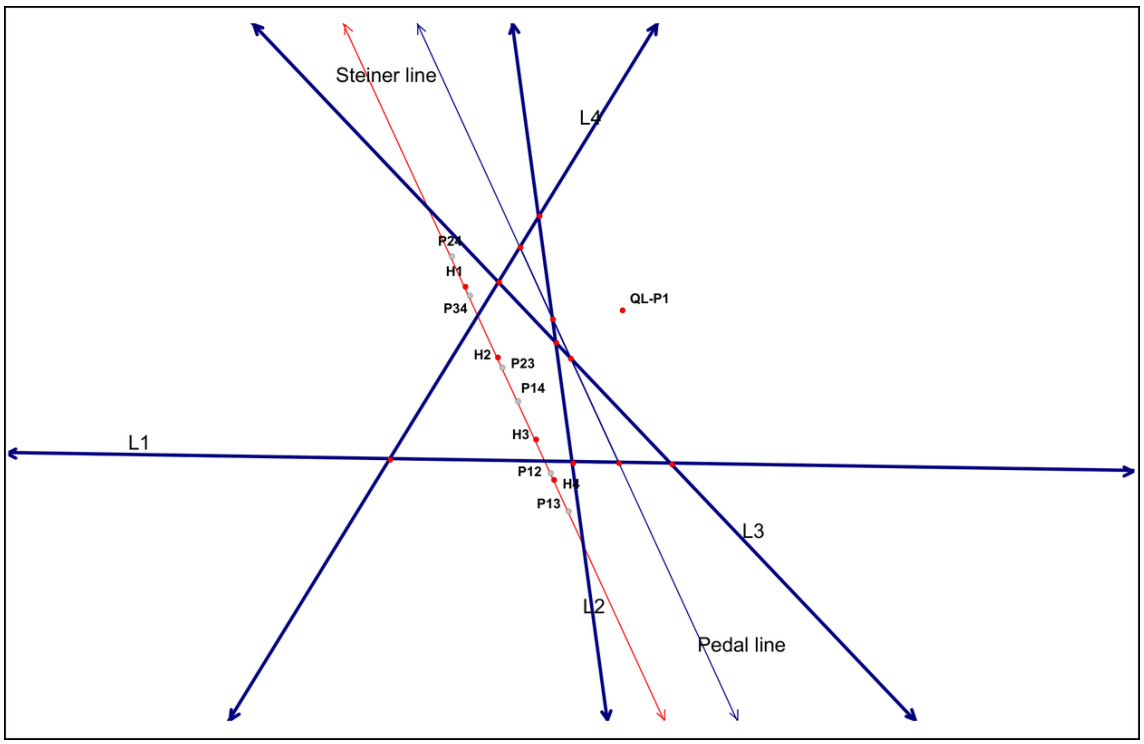
Message: #1106
Date: 13/5/2015 2:46:34
From: seiichikiri
Subject: The orthopoles of a quadrilateral

Dear Bernard,
According to your instruction, I added 6 more orthocenters on QL-P2. See the attached picture.
Pij means the orthocenter of trilateral (Li, Lj and Pedal line).
As for orthopoles, I will make a picture later.
Best regards,
Seiichi.



SteinerLine.docx

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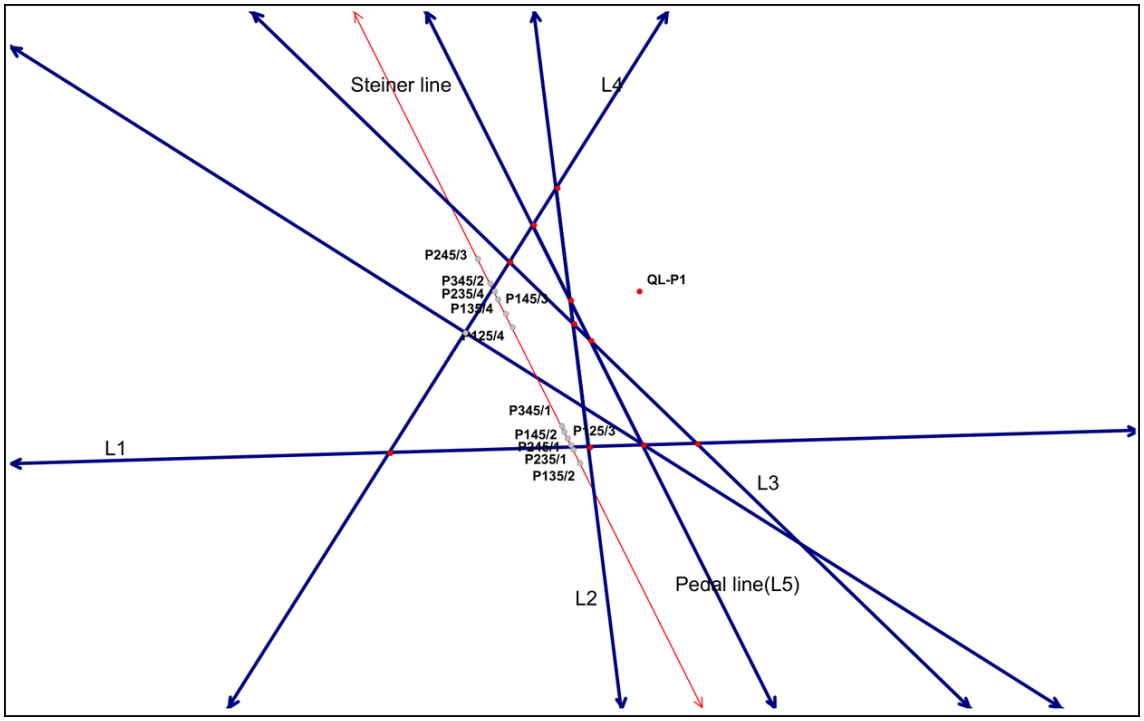
Message: #1107
Date: 14/5/2015 12:21:58
From: seiichikiri
Subject: The orthopoles of a quadrilateral

Dear Bernard,
I made a picture of 12 more orthopoles. See the attachment.
Pedal line is denoted by L5.
For example, P235/4 means the orthopoles of L4 wrt trilateral (L2, L3 and L5).
Your parabolic explanation is natural and refreshing for me.
Best regards,
Seiichi.



Steinerline-a.docx

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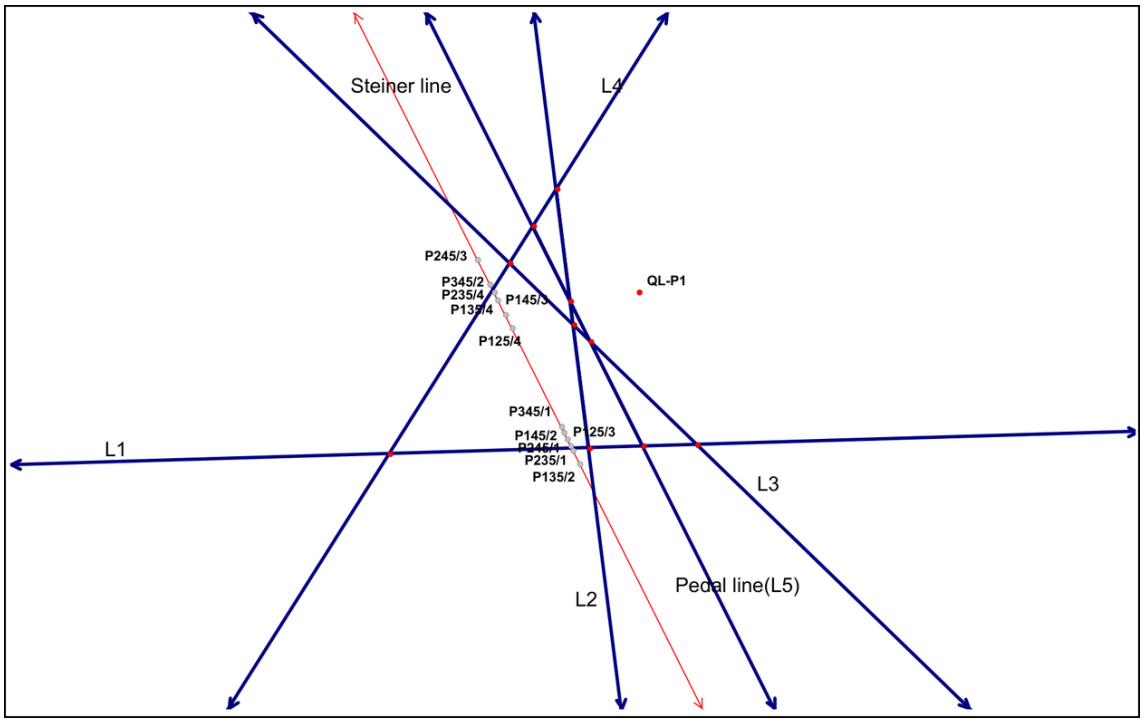
Message: #1108
Date: 14/5/2015 12:34:02
From: seiichikiri
Subject: The orthopoles of a quadrilateral

Dear Bernard,
Sorry that a perpendicular line remained in the previous attachment.
See the attachment "Steinerline-b".
Best regards,
Seiichi.



Steinerline-b.docx

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Message: #1109
Date: 14/5/2015 9:39:42
From: bernard.keizer
Subject: The orthopoles of a quadrilateral

Dear Seiichi,
Thanks for your pictures !
Of course, all the mentionned properties are true for any 5 tangents to a parabola (id est if L5 is not necessary the Simson/pedal line QL-L3 but any other tangent to the parabola).
There are 5 QL's sharing the same Miquel point P1, the same Simson/pedal Line QL-L3 and the same Steiner Lines QL-L2.
The 5 Newton Lines QL-L1 are parallel.
The 10 circumcircles are through QL-P1, the 5 reflections of QL-P1 in the Li, the 10 orthocenters and the 20 orthopoles are on QL-L2.
The Miquel circle of the 5-L (circle through the 5 Miquel points) is reduced to a point on the centric circle.
Best regards
Bernard

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Message: #1110
Date: 14/5/2015 4:58:46
From: chris.vantienhoven
Subject: The orthopoles of a quadrilateral

Dear Bernard,
Thanks for checking the modifications in EQF.
You are right about my description of the Simson Lines. They do not concur on a point of QL-L3,
they concur with QL-L3. I changed EQF correspondingly and adopted your formulation.
I also added at QL-L2 the corresponding property regarding the Steiner Line.
I am still busy with a paper on the 64 Morley Axes which proofs the construction.
I found several nice new properties needed for proving.
Unfortunately it takes lots of time.
Best regards,
Chris

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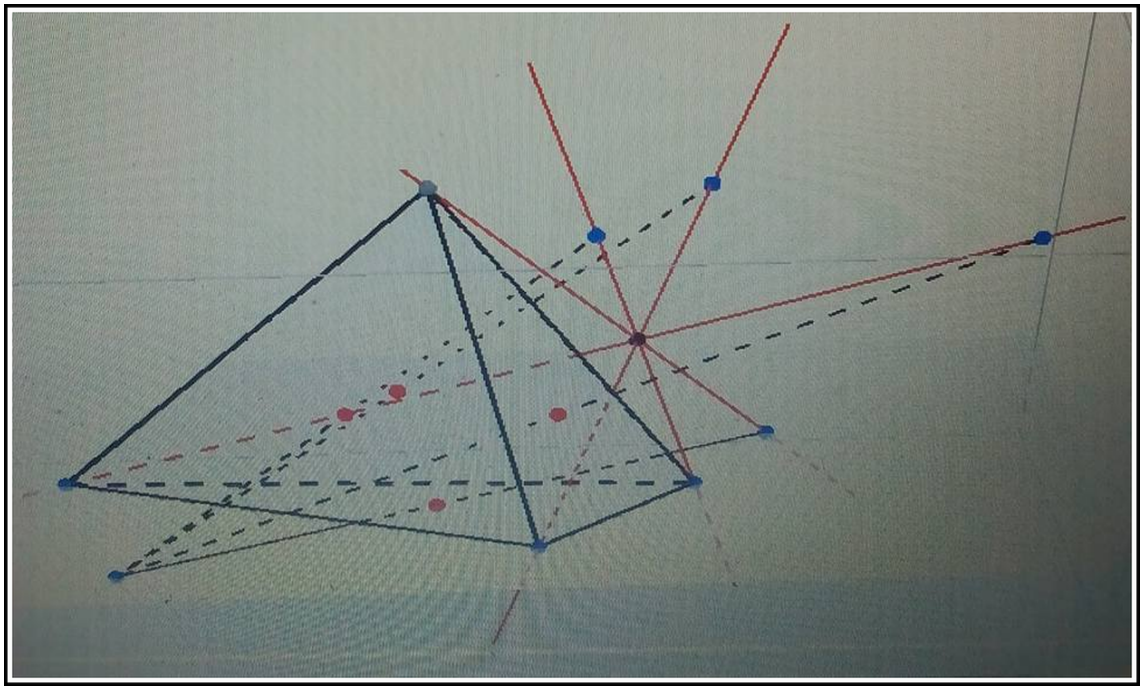
Message: #1111
Date: 14/5/2015 9:26:12
From: emmanuelantoniojosgarca
Subject: A property of Tetrahedron

Dear Mr Seiichi, Mr Chris and friends,
I have found a new property of the Tetrahedron. In fact, this is a 3D generalization of theorem 1 in my paper "A Note on Reflection", published by Forum Geometricorum.
Consider any Tetrahedron ABCD. Take an arbitrary point P on the space. Now, reflect P around the four centroids of the four triangular faces of the Tetrahedron.Then, the line segments joining the vertices with the symmetry image of P corresponding to the opposite faces of the vertices are concurrent.
I took a photo with my smarphone since I could not find a way save the diagram in my pc.
Best regards,
Emmanuel José García



Tetraedro.jpg

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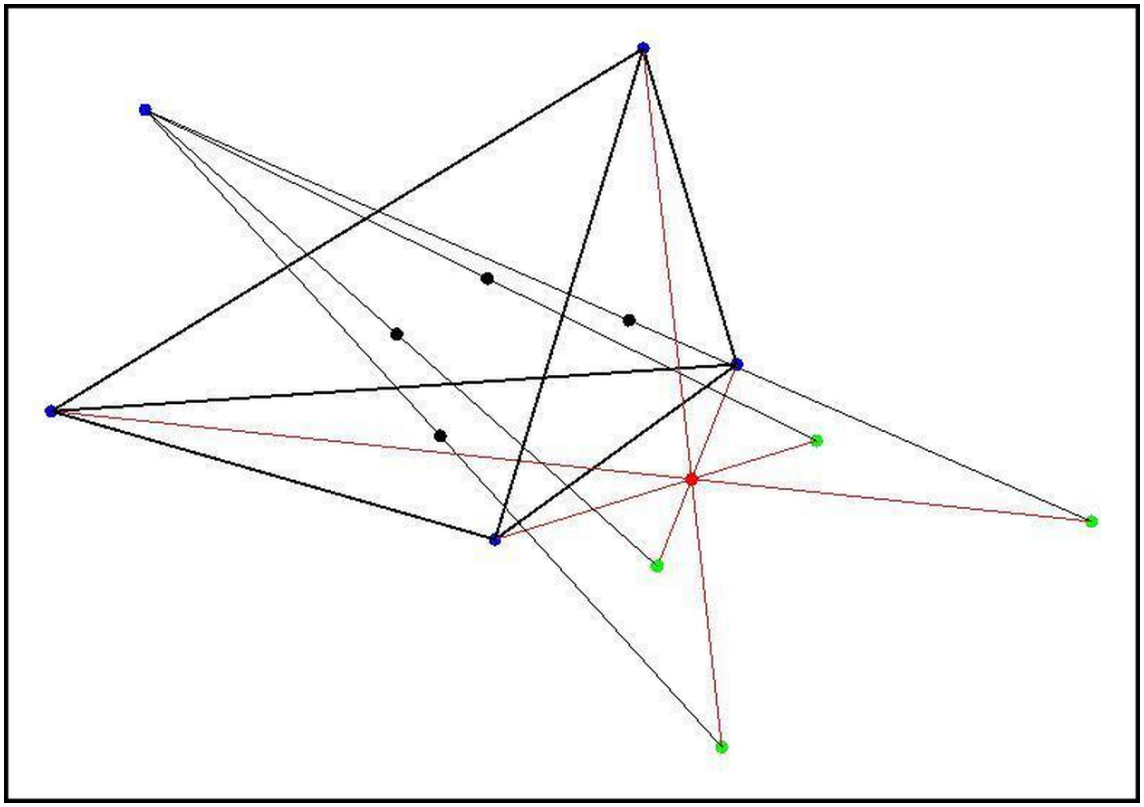
Message: #1112
Date: 14/5/2015 10:58:31
From: garciacapitan
Subject: A property of Tetrahedron

Dear Enmanuel,
This is quite straightforward using mass point geometry:
Let G, Ga be the centroids of tetrahedron ABCD and triangle BCD.
Then we have 4G=A+B+C+D and 3Ga=B+C+D
If Qa is the reflection of P on Ga, then Qa=2Ga-P and we have
3Qa=6Ga-3P = 2B+2C+2D -3P
2A + 3Qa = 2(A+B+C+D) - 3P = 8G -3P
therefore the line AQa contains the point that divides the segment
GP in the ratio -3:8, and the same for BQb, CQc, DQd.



tetra.jpg

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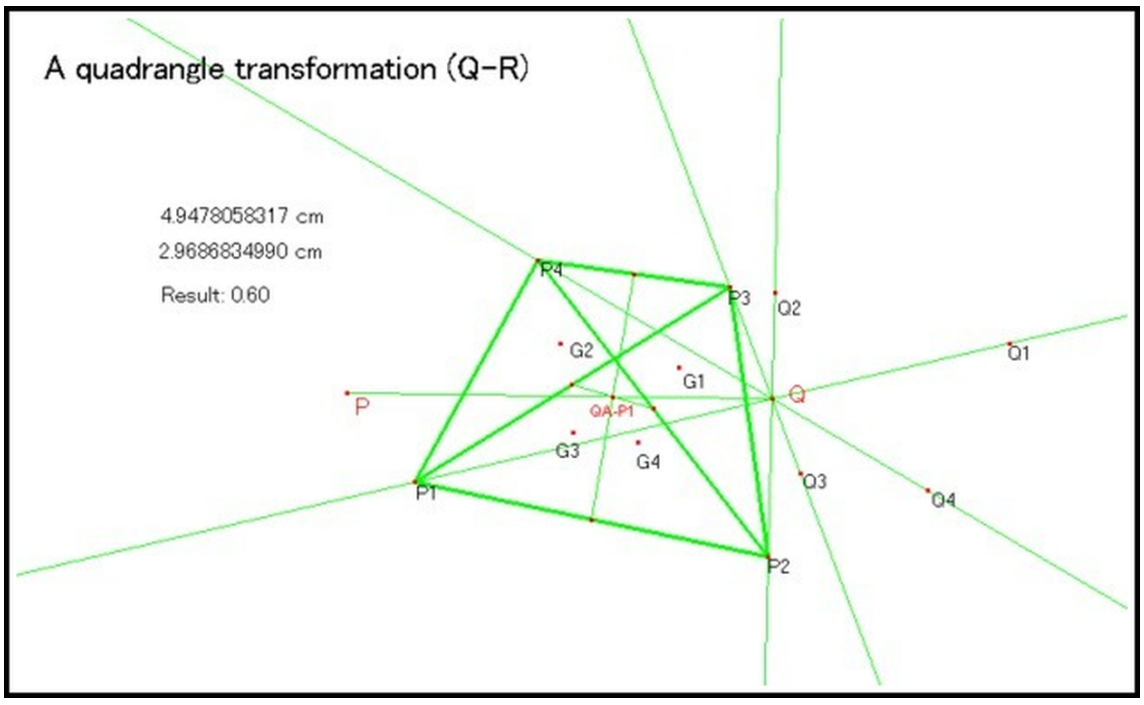
Message: #1113
Date: 15/5/2015 2:18:57
From: seiichikiri
Subject: A property of Tetrahedron

Dear Emmanuel, Dear Francisco,
The property of a tetrahedron reduces to that of a quadrangle.
Let P1P2P3P4 be a quadrangle and P an arbitrary point.
Denote:
Gi= the centroid of PjPkPl.
Qi= the symmetric point of P in Gi.
QA-P1= the centroid of P1P2P3P4.
P1Q1, P2Q2, P3Q3 and P4Q4 concur in Q.
Barycentrics of Q={-3(p+q+r)x+2(2p+q+r)(x+y+z), , }.
Length(Q QA-P1)/Length(P QA-P1)=0.6.
See the attachement.
I also tried the circumcenter, the orthocenter and NP center with no good result.
Best regards,
Seiichi.



EJGarcia.png

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Message: #1114
Date: 15/5/2015 3:36:12
From: emmanuelantoniojosgarca
Subject: A property of Tetrahedron

Dear Francisco, dear Seiichi,
Thank you very much for your contributions.
Reflect the four symmetry images around the centroid-corresponding-planes, this is to say, if Qa is the symmetry image of P around the centroid of BCD, then, reflect Qa around the plane containing BCD. The same for the other three images.
Conjecture: these four new images, including P lie on the same sphere.
Best regards,
Emmanuel

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Message: #1115
Date: 15/5/2015 9:22:39
From: chris.vantienhoven
Subject: A property of Tetrahedron

Dear Emmanuel,
Like Seiichi described the 2D-projection of your 3D-reflection can be seen as a Quadrangle Transformation QA-Tfx, where QA-Tfx(P) = P.QA-P1 (8:-3).
I looked if this gave nice results with known EQF centers.
I only found these results:
QA-Tfx (QA-P1) = QA-P1
QA-Tfx (QA-P14) = QA-P20.QA-P40 ^ QA-P32.QA-P37
QA-Tfx (QA-P34) = homothetic center of 2nd Centroid Quadrangle and 1 st Antigonal Quadrangle.
It is remarkable that almost all involved QA-points have connections with centroids.
There probably will be more QA-points that can be seen as the projection of a 3D-picture.
At least QA-P1 is such a point.
Best regards,
Chris

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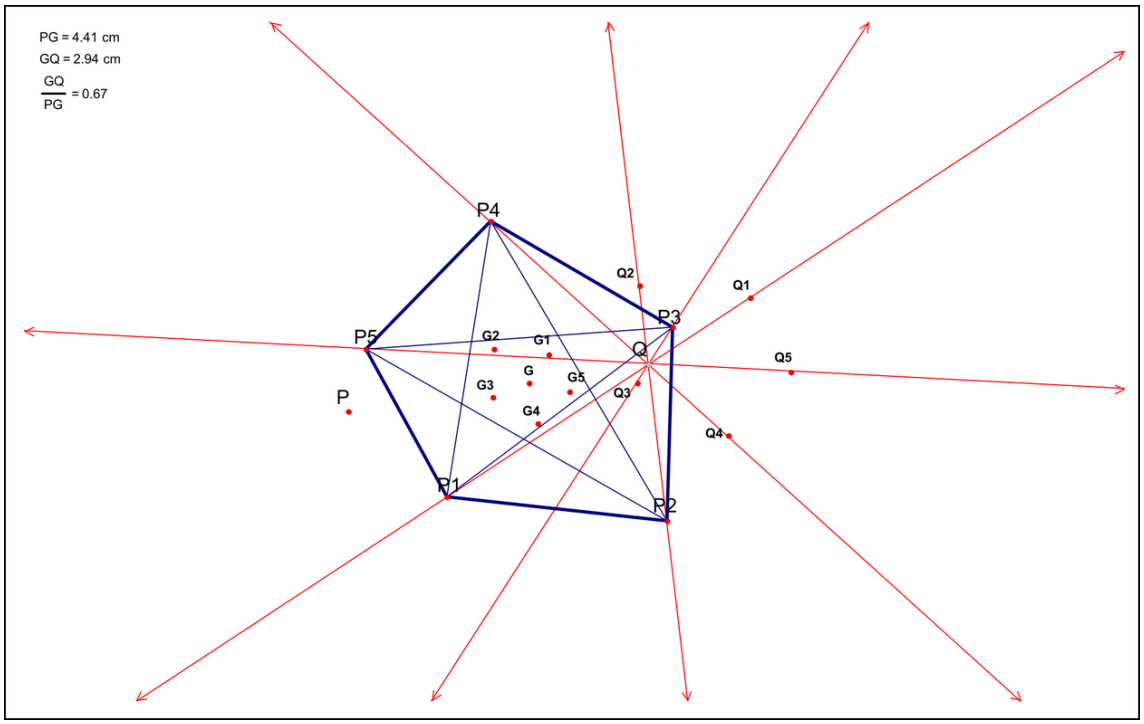
Message: #1116
Date: 15/5/2015 10:18:16
From: seiichikiri
Subject: A property of pentangle

Dear friends,
A pentangle has a property similar to that of quadrangle shown in message #1111, 1112, 1113.
Let P1P2P3P4P5 be a pentangle and P an arbitrary point.
Denote:
G1, G2, G3, G4, G5= the centroids of P2P3P4P5, P1P3P4P5, P1P2P4P5, P1P2P3P5, P1P2P3P4 respectively.
Qi= the symmetric point of P in Gi for i=1, 2, 3, 4, 5.
G = the centroid of P1P2P3P4P5.
P1Q1, P2Q2, P3Q3, P4Q4 and P5Q5 concur in Q.
Length(Q G)/Length(P G)=2/3.
See the attachement.
G.D. (Graphically Demonstrated.)
Best regards,
Seiichi.



pentangle image.docx

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Message: #1117
Date: 15/5/2015 4:04:16
From: seiichikiri
Subject: Generalizations of complement of triangle

Dear friends,
The images or transformations in message #1113 and 1116 are generalizations of “complement of triangle”.
1) triangle:
Let P1P2P3 be a triangle and P an arbitrary point.
G1, G2, G3= the centroids (or midpoints) of P2P3, P3P1, P1P2 respectively.
Qi = the symmetric point of P in Gi for i=1, 2, 3.
G = the centroids of P1P2P3.
P1Q1, P2Q2 and P3Q3 concur in Q.
PG = 2 QG.
2) quadrangle:
Let P1P2P3P4 be a quadrangle and P an arbitrary point.
G1, G2, G3, G4= the centroids of P2P3P4, P1P3P4, P1P2P4, P1P2P3 respectively.
Qi = the symmetric point of P in Gi for i=1, 2, 3, 4.
G= the centroid of P1P2P3P4.
P1Q1, P2Q2, P3Q3 and P4Q4 concur in Q.
PG = 5/3 QG.
3) pentangle:
Let P1P2P3P4P5 be a pentangle and P an arbitrary point.
G1, G2, G3, G4, G5= the centroids of P2P3P4P5, P1P3P4P5, P1P2P4P5, P1P2P3P5, P1P2P3P4 respectively.
Qi = the symmetric point of P in Gi for i=1, 2, 3, 4, 5.
G = the centroid of P1P2P3P4P5.
P1Q1, P2Q2, P3Q3, P4Q4 and P5Q5 concur in Q.
PG = 3/2 QG.
Best regards,
Seiichi.

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Message: #1118
Date: 15/5/2015 4:17:12
From: emmanuelantoniojosgarca
Subject: Generalizations of complement of triangle

Dear Mr Seiichi,
Please see proposition 1 here
http://forumgeom.fau.edu/FG2014volume14/FG201414index.html
Best regards
Emmanuel

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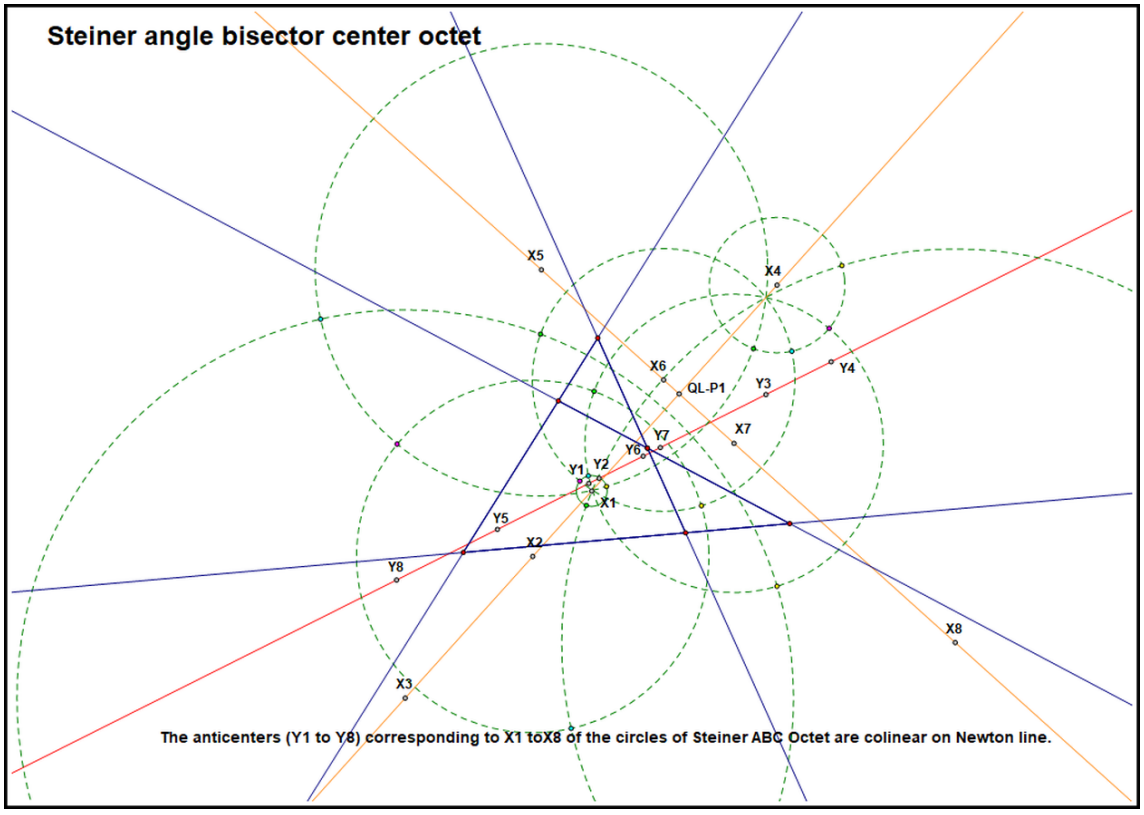
Message: #1119
Date: 18/5/2015 2:13:11
From: seiichikiri
Subject: Anticenters in Quadri-Figures Geometry

Dear Chris, dear friends,
1) The anticenters of 4 points on Miquel circles of QA-P9 are colinear.
2) The anticenters (Y1 to Y8) corresponding to X1 to X8 of the circles of Steiner ABC Octet are colinear on Newton line.
G.D. (graphically demonstrated.)
See the attachment (gsp).
I think that other anticenters will be examined.
Best regards,
Seiichi.



Anticenters-gsp.png

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Message: #1120
Date: 18/5/2015 11:20:23
From: bernard.keizer
Subject: Anticenters in Quadri-Figures Geometry

Dear Seiichi,
The second property is a very simple and beautiful one for the 8 Steiner circles of the QL with centers on either one or the other Steiner axis and anticenters of each group of 4 in- or excenters on the Newton Line.
Congratulations !
Bernard

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Message: #1121
Date: 19/5/2015 12:56:19
From: seiichikiri
Subject: Anticenters in Quadri-Figures Geometry

Dear Bernard,
I could revise Steiner's results with you. Thank you very much.
Best regards,
Seiichi.

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Message: #1122
Date: 19/5/2015 9:50:43
From: Antreas Hatzipolakis
Subject: N – Locus

Dear Chris,
I am wondering how could we use the property quoted below in qudrangles.
Let ABCD be a quadrangle
Denote:
Na, Nb, Nc, Nd = the NPC centers of BCD, CDA, DAB, ABC, resp.
Nda, Ndb, Ndc = the NPC centers of NdBC, NdCA, NdAB, resp.
Denote La the homothetic center of ABC, NdaNdbNdc
Cyclically we get Lb, Lc, Ld.
So we get a quadrangle LaLbLcLd.
Whch properties has it?
Sorry I did not draw a figure.
APH
>
>> [APH]:
>> Let ABC be a triangle and P a point.
>> Denote:
>> Na, Nb, Nc = the NPC centers of PBC, PCA, PAB, resp.
>> N1, N2,N3 = the NPC centers of NNbNc, NNcNa, NNaNb, resp.
>> Which is the locus of P such that ABC, N1N2N3 are perspective?
>> The entire plane ?
>> APH
>> [CL]:
>> Yes. And they are homothetic always.
>> For P = u:v:w (trilinears) the homothetic center Z(P) is:
>> Z(P) = (a^3*b*c*v*w
>> +c*((2*b^2+c^2)*a^2-(b^2-c^2)^2)*u*w
>> +b*((2*c^2+b^2)*a^2-(b^2-c^2)^2)*u*v
>> -a*(a^4-3*(b^2+c^2)*a^2+2*(b^2-c^2)^2)*u^2)/(a*u) : :
>> ETC-pairs (P,Z(P)):
>> (1,5443), (3,6143), (4,5), (523,476), (1138,3258), (3459,5)
>> Others:
>> Z(X(2)) = (3*(b^2+c^2)*a^2-2*(b^2-c^2)^2)/a : :
>> = On lines: (2,187), (5,39), (6,5055), (30,3055), (32,1656), (216,2072),
>> (230,547), (232,7577) and more
>> = (1.118075803607456, 0.93477301231647, 2.477478948638764)
>> Z(X(5)) =
(2*cos(A)*cos(B-C)+3*cos(2*(B-C))-cos(2*A)+7/2)*sec(B-C) : :
>> = On lines: (5,49), (252,1656), (1157,3628)
>> = (1.036723436416172, -1.11290425012558, 3.932648915186932)
>> Z(X(6)) =

```
>>
(11-4*cos(2*A))*cos(B-C)+2*cos(A)*cos(2*(B-C))+6*cos(A)-cos(3*A)
: :
>> = On lines: (2,51), (5,399), (23,2916), (125,5643),
(182,7533),
>> (370,5313)(?), (1112,6143),
>> (3410,5422), (3580,7570), (5056,5645), (5480,7496)
>> = ( 1.405585317866251, 0.82165275079938, 2.423096276954230 )
>> Regards
>> César Lozada
```

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Message: #1123
Date: 19/5/2015 10:02:13
From: Antreas Hatzipolakis
Subject: N – Locus

```
I wrote:
> >Denote La the homothetic center of ABC, NdaNdbNdc

It should be Ld
APH
```

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Message: #1124
Date: 19/5/2015 11:19:18
From: bernard.keizer
Subject: Anticenters in Quadri-Figures Geometry

Dear Seiichi, dear Chris, dear Eckart,
In fact, your property of anticenters is even more beautiful as I thought !
I gave in message 840, to which noone answered, a generalisation of the Steiner circles.
For any copple of Cl-S conjugate points X and X' on the hessian QL-Cu1, let's call Y, Y' Z and Z' the poles of the line XX' wrt the cubic stelloïd QL-Cu2 (they are the intersections of the bisectors in X and X' and the 4 fixed points of the set of the rectangular hyperbolas which form the polar conics of the points of XX').
The points X, X', Y and Y' on one side, X, X', Z and Z' on the other side are concyclic on 2 circles centered on either one or the other Steiner axis.
These are the generalised Steiner circles (in particular, this consctruction works for the 3 copples of conjugate vertices of the QL, whith the intersections of the inner and outer bisectors)
These circles belong to the 2 sets of autopoloconics common to the cubic stelloïd QL-Cu2 and to one of the 2 other cubics sharing the same cayleyan (Y and Y' describe one of these 2 cubics and Z and Z' the other).
Your property, dear Seiichi, is true for all these circles (the anticenter of the QA's formed by the 2 groups of 4 points lie on the Newton Line)
Best regards
Bernard

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Message: #1125
Date: 19/5/2015 11:40:44
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard, dear Chris,
excuse, if there is no more interest in Morley's 4L-axes, but here is a better description as in message 1092 for constructing the 32 orthogonal intersections among Bernard's 96 points for a 4-line wrt a basic line, which are the centers of tetracardioids tangent to the 4 lines and twice in symmetry to the basic line:
... For a 4-line there are 3 triangles ABC with AB basic line.
... Consider the angle bisectors at A and B: 4 lines intersecting in the in- and excenters.
... The angle bisectors of these angle bisectors give 8 lines for each triangle ABC, forming two rectangles.
... The triple intersections of these 8 lines for the 3 triangles with the same basic line give the searched 32 points.
Best regards
Eckart

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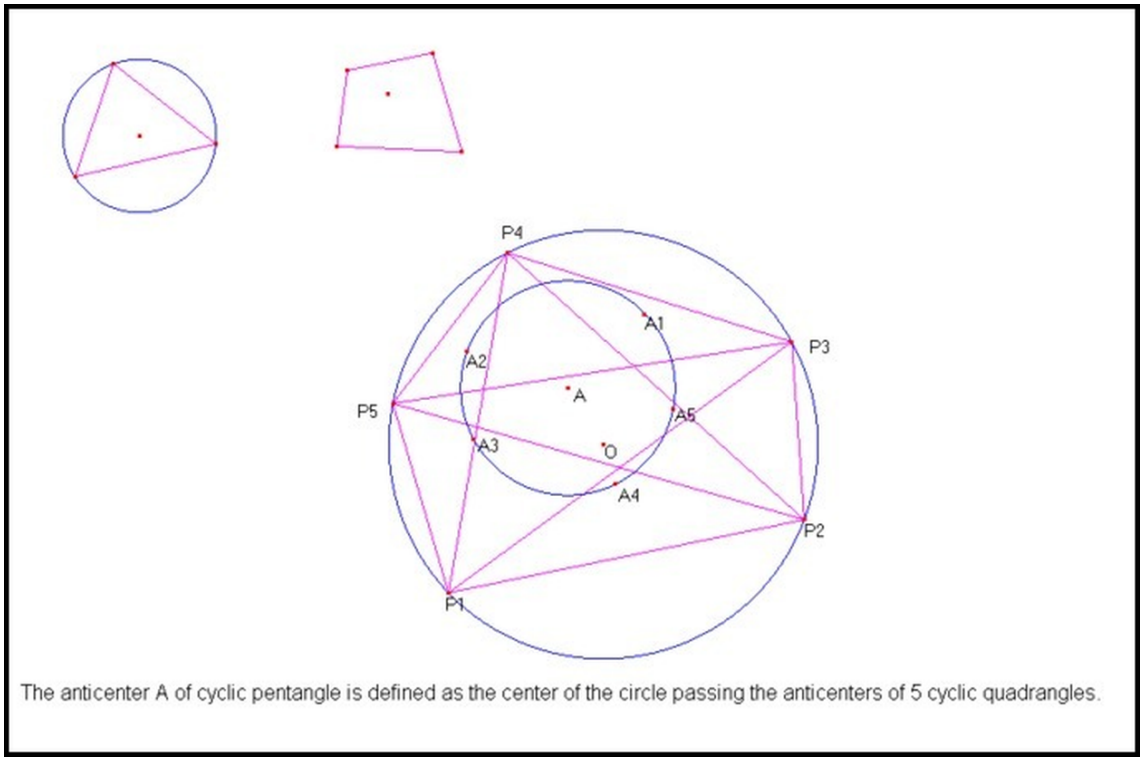
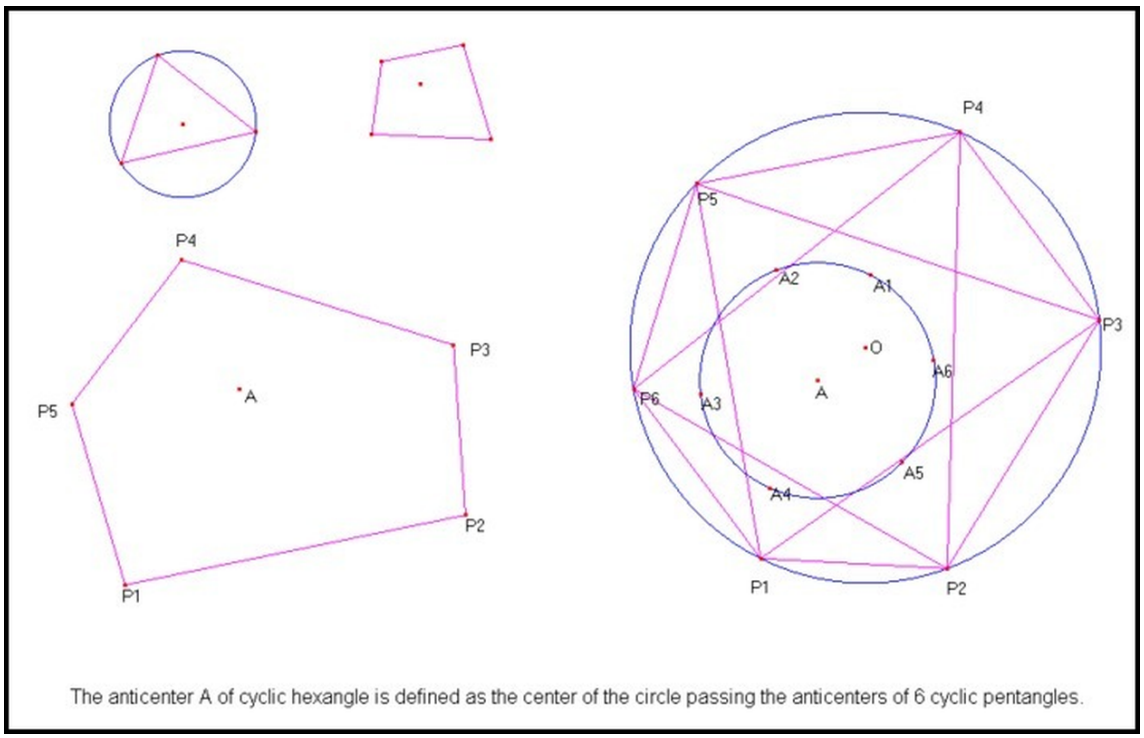
Message: #1126
Date: 19/5/2015 3:23:45
From: seiichikiri
Subject: Anticenters of pentangle and hexangle

Dear friends,
Is the following already known?
1) The anticenter of cyclic pentangle is defined as the center of the circles passing the anticenters of 5 cyclic quadrangles.
2) The anticenter of cyclic hexangle is defined as the center of the circles passing the anticenters of 6 cyclic pentangles.
See the attached pictures.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.



Anticenter-pentangle.png



Anticenter-hexangle.png

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Message: #1127
Date: 20/5/2015 11:20:04
From: bernard.keizer
Subject: Anticenters in Quadri-Figures Geometry

Dear friends,
Please note this correction : Y and Y' describe the hessian of one of these 2 cubics and Z and Z' the hessian of the other.
By combining the 4 bisectors in X and X', you get 6 possible combinations, which give the degenerated polar conics of the 6 points wrt their respective cubic.
All the lines XX', YY', ZZ', XY, XY', X'Y, X'Y' are tangents to the cayleyan.
Best regards
Bernard

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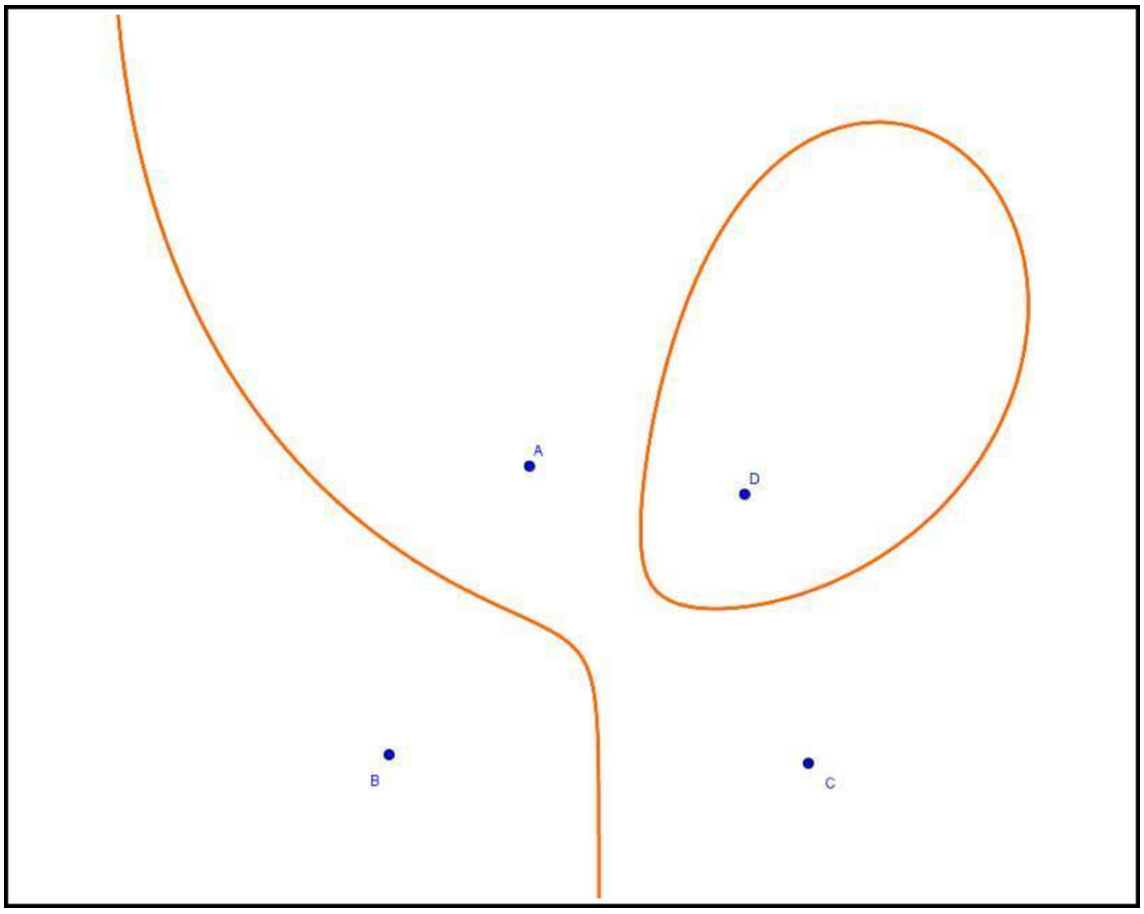
Message: #1128
Date: 20/5/2015 5:49:40
From: Antreas Hatzipolakis
Subject: A Locus

Let ABCD be a quadrangle and P a point.
Which is the locus of P such that: PA x PC=PB x PD
(or PA x PD = PC x PB, PA x PB = PC x PD) ?
Problem and figure by Telv Cohl (
https://www.facebook.com/photo.php?fbid=350881248452886&set=a.316620011879010.1073741868.100005930409730&type=1&theater)
Note:
In quadrilateral we can ask a similar question:
Let (a,b,c,d) be a quadrilateral and P a point.
Denote P(a), P(b), P(c), P(d) = the distances from P to a,b,c,d, resp.
Which is the locus of P such that:
P(a) x P(c) = P(b) x P(d) etc ?
APH



cohl.jpg

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Message: #1129
Date: 22/5/2015 8:45:55
From: eckart_schmidt@t-online.de
Subject: A Locus

```
Dear Antreas,
I don't know, whether the folowing properties are already
mentioned:
Let ABCD be a quadrigon and P a point.
... The locus of P such that PA x PC=PB x PD is a cubic .
... The asymptote is perpendicular wrt the Newton line QL-L1.
... The Miquel point QL-P1 is a point on the cubic.
... The cubic is CSC-invariant (invariant wrt QL-Tf1).
Best regards Eckart
```

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Message: #1130
Date: 22/5/2015 10:57:21
From: bernard.keizer
Subject: 64 axes of a QL

```
Dear Eckart,
I think I already proved my interest in this item !
I can only repeat my last previous messages (see 1088 or 1101)
1) I don't find the 32 points with orthogonal intersection more
or less attractive as the 64 other or the 16 7th points (if you
deal with pi/5 for example, you woudn't say that 2pi/5 is more
interesting than 3pi/5)
2) I'm now awaiting with hope and patience that Chris or you can
calculate these points and their curve(s) with Mathematica (I'm
not able to do that with Geogebra) and that Chris explains the
construction in EQF.
Best regards
Bernard
```

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Message: #1131
Date: 22/5/2015 12:03:44
From: eckart_schmidt@t-online.de
Subject: A Locus

```
Dear Antreas,
I just realized, that I have worked out this cubic in message
#414.
Best regards Eckart
```

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Message: #1132
Date: 22/5/2015 12:10:24
From: Antreas Hatzipolakis
Subject: A Locus

```
Dear Eckart
Thank you !
How about the case of the distances
to opposite sides instead of vertices ?
APH
--
http://anopolis72000.blogspot.com/
```

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Message: #1133
Date: 22/5/2015 7:33:21
From: bernard.keizer
Subject: Apollonius circles

Dear Seiichi,
As you seem interested in discovering or rediscovering properties of the QL, I propose you the following one, which I found in rereading J. W. Clawson:
1) consider first 2 of the 6 vertices of the QL, not conjugates, let say Aik and Ail and their conjugates Ajl and Ajk.
2) consider then the triangles P1AikAil and P1AjkAjl (P1 being the Miquel point)
3) draw now the 2 P1-Apollonius circles of these 2 triangles
4) the 2nd intersection of these 2 circles (other than P1) lies on the Steiner Line L2.
This gives 6 new points on the Steiner Line.
Best regards
Bernard
PS If I'm not wrong, this simple property is not yet in EQF and could interest Chris

---

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Message: #1134
Date: 23/5/2015 9:58:54
From: eckart_schmidt@t-online.de
Subject: A Locus

Dear Antreas,
wrt the second question: Let ABCD be a quadrigon and P a point. Denote P(AB), P(BC), P(CD), P(DA) = the distances from P to the sides.
The locus of P such that
P(AB) x P(CD) = P(BC) x P(DA)
... is a conic through the vertices A, B, C, D and the intersections of the inner / outer angle bisectors for opposite vertices.
Best regards Eckart

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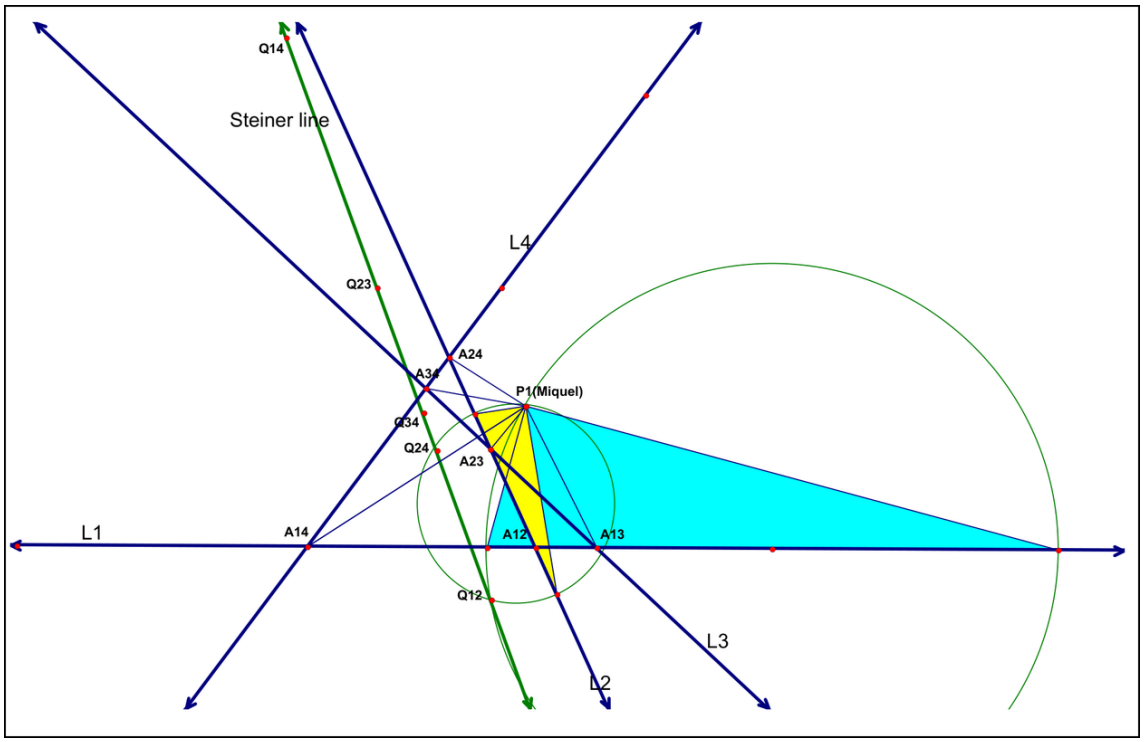
Message: #1135
Date: 23/5/2015 11:24:08
From: seiichikiri
Subject: Apollonius circles

Dear Bernard,
I drew the picture. See the attached one. I used Geometer's Sketchpad, exported it with wmf format and pasted it on Microsoft Word. So some points on the attahced one are not just on the Steiner line.
Best regards,
Seiichi.



SteinerLine-1.docx

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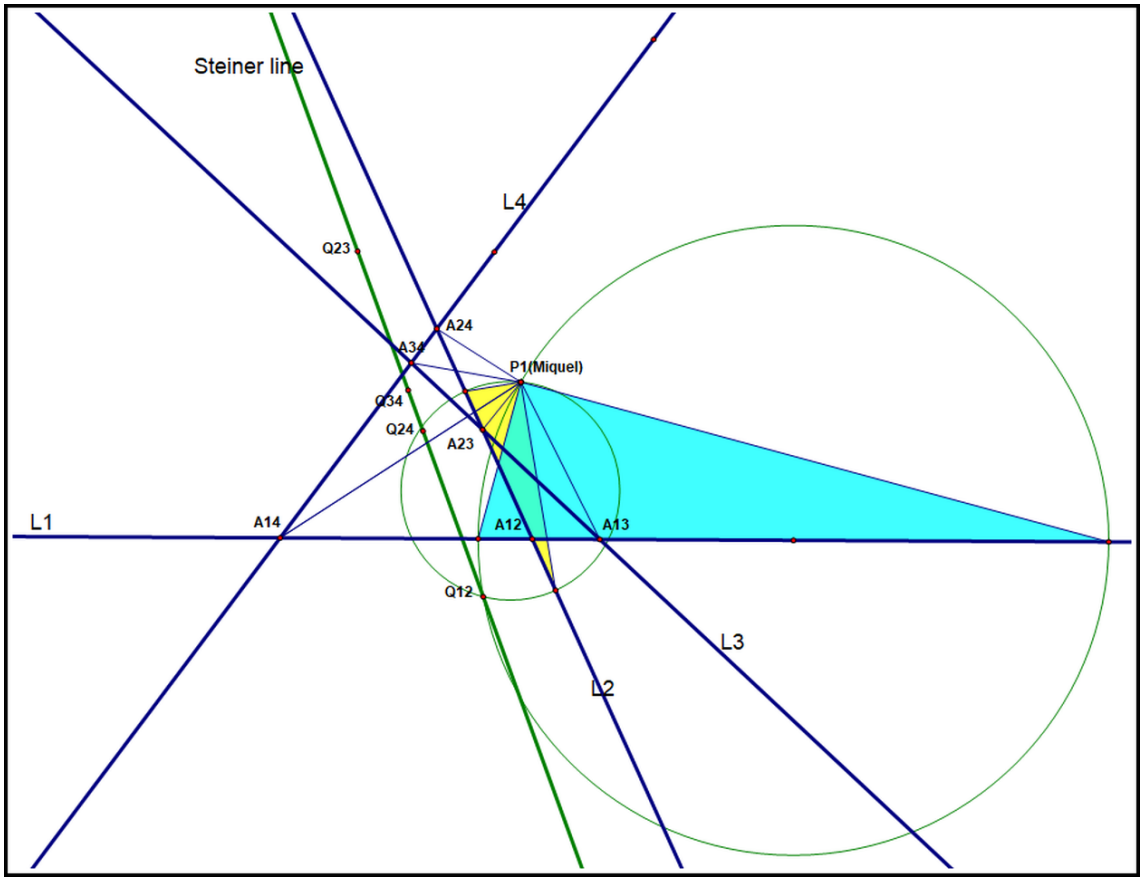
Message: #1136
Date: 23/5/2015 12:19:15
From: seiichikiri
Subject: Apollonius circles

Dear Bernard,
Attached is the picture (gsp).
Best regards,
Seiichi.



SteinerLine-1-gsp.png

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Message: #1137
Date: 23/5/2015 7:36:10
From: bernard.keizer
Subject: Apollonius circles

Dear Seiichi,
I couln't open your 2nd picture, I'm able to read only word- or pdf- files.
But I could open the 1rst one.
Isn't it a nice property ?
I haven't found a synthetic proof yet ...
Best regards
Bernard

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Message: #1138
Date: 23/5/2015 7:41:48
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard, dear Eckart,

Attached 3 pictures and a description all about Morley's Axes in an n-Line.
* 64 Morley Axes in a random 4-Line (mutual angles 45 0)
* 625 Morley Axes in a random 5-Line (mutual angles 36 0)
* 7776 Morley Axes in a random 6-Line (mutual angles 30 0)
* A short description of the method for constructing Morley Axes in n n-Line.

The programming of one Morley line in an n-Line is calculated by this Mathematica Module:

```
MorleyLine[L0_, LL_, n_] :=
Module[{nLL, LLx1, LLx2, LLy1, LLy2, Lb, mline}, nLL =
Length[LL];
If[nLL == 1, {mline} = LL,
LLx1 = Delete[LL, nLL]; LLx2 = Map[LineRotation[Cross[L0, #],
Pi/n, #] &, LLx1];
LLy1 = Delete[LL, nLL - 1]; LLy2 = Map[LineRotation[Cross[L0,
#], Pi/n, #] &, LLy1];
mline = Cross[Cross[MorleyLine[L0, LLx1, n], MorleyLine[L0,
LLy1, n]], Cross[MorleyLine[L0, LLx2, n], MorleyLine[L0, LLy2,
n]]]; mline]
```

where L0=the baseline, LL= set of initial (n-1) n-sectors to be rotated, n=number of n-Line.

By rotating stepwise the initial n-sectors per intersection point (L0^Li) the calculated Morley Line varies and the n n-1 Morley Axis are reached.
It is always surprising for me
that a long hidden problem can be coded so shortly.
I intent to make more detailed descriptions.

Best regards,
Chris

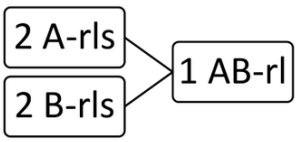
p.s. Let me know when you want me to make other drawings (and in which way).

Morley's Axes in an n-Line

The following description of the construction of Morley's Axes in an n-Line is building upon the principles of the Lighthouse Theorem.
A new designation is introduced, the "rimline".

An extension to the Lighthouse Theorem
It's all about *rimlines*.
A rimline is defined here as a sideline of a polygon constructed from beams of Lighthouses.
Actually the beams themselves are considered to be rimlines of the first generation.
They are noted as A-rl(s) or B-rl(s) meaning the rimline(s)/beam(s) emanated from A or B.
Actually the beams from a point P emanated around in a regular pattern can be considered as the sides of a polygon with radius 0. So the rimlines of the following generations will be sidelines of regular polygons with radius > 0.

The way to get one rimline from 2 Lighthouses.
This the situation of the original Lighthouse Theorem.

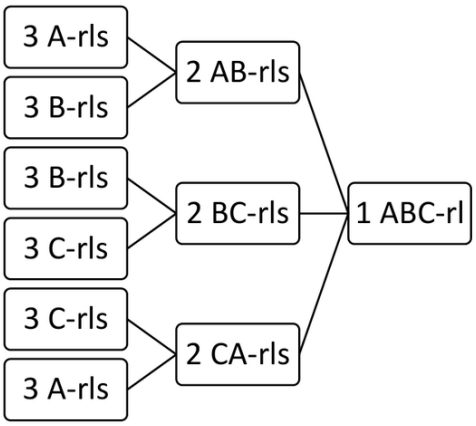


Meaning: 2 A-rimlines and 2 B-rimlines interconnected produce 1 AB-rimline.

Let a1, a2 be the 2 A-rimlines (beams from A) and let b1, b2 be the 2 B-rimlines (beams from B).
Now the 1 AB-rimline is the line (a1^b1)^(a2^b2).
This AB-rimline is a sideline of one of the polygons generated according to the Lighthouse Theorem.

Morleys Axes in an n-Line.pdf

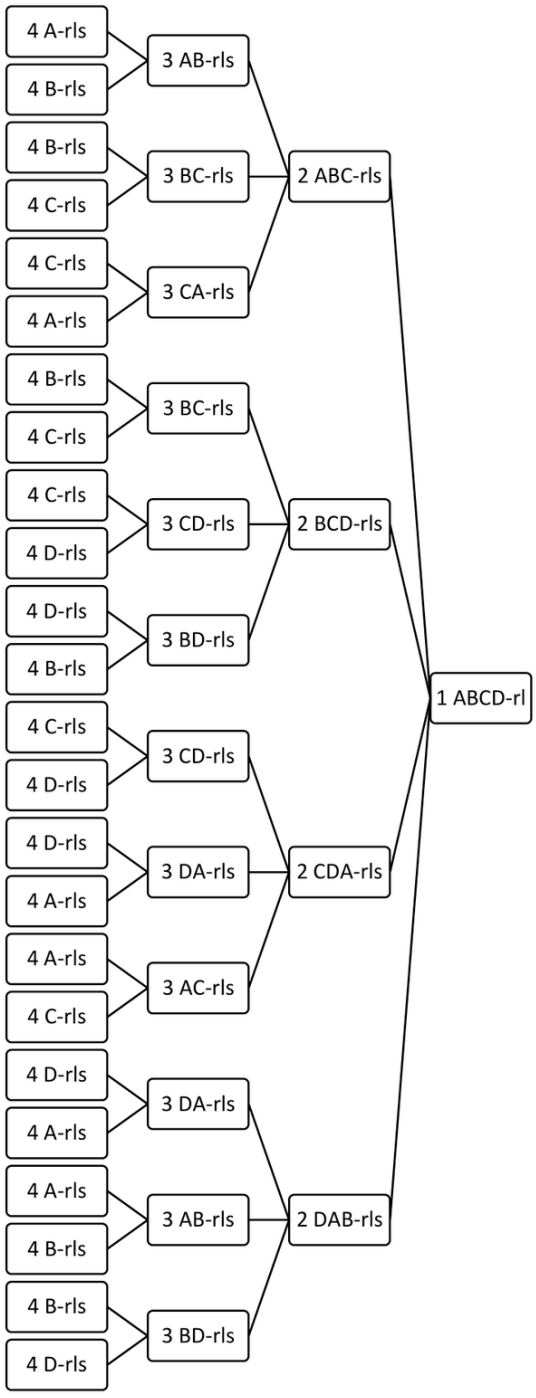
The way to get one rimline from 3 collinear Lighthouses:



Meaning:
3 A-rimlines and 3 B-rimlines interconnected produce 2 AB-rimlines.
3 B-rimlines and 3 C-rimlines interconnected produce 2 BC-rimlines.
3 C-rimlines and 3 A-rimlines interconnected produce 2 CA-rimlines.
2 AB-rimlines & 2 BC-rimlines & 2 CA-rimlines interconnected produce 1 ABC-rimline.

Let a1, a2, a3 be the 3 A-rimlines (beams from A) and let b1, b2, b3 be the 3 B-rimlines (beams from B) and let c1, c2, c3 be the 3 C-rimlines (beams from C).
Now the 2 AB-rimlines are ab12=(a1^b1)^(a2^b2) and ab23=(a2^b2)^(a3^b3).
The 2 BC-rimlines and the 2 CA-rimlines are constructed accordingly.
Then the 1 ABC-rimline is (ab12^bc12^ca12)^(ab23^bc23^ca23).
As can be seen from the notation the sets of rimlines (ab12, bc12, ca12) and (ab23, bc23, ca23) are considered to be concurrent lines. And that's what they are. Will be proved later.

The way to get one rimline from 4 collinear Lighthouses:



Morleys Axes in an n-Line.pdf

Meaning:
3 A-rimlines and 3 B-rimlines interconnected produce 2 AB-rimlines.
3 B-rimlines and 3 C-rimlines interconnected produce 2 BC-rimlines.
3 C-rimlines and 3 A-rimlines interconnected produce 2 CA-rimlines.
2 AB-rimlines & 2 BC-rimlines & 2 CA-rimlines interconnected produce 1 ABC-rimline.

Let a1, a2, a3, a4 be the 4 A-rimlines (beams from A) and let b1, b2, b3, b4 be the 4 B-rimlines (beams from B) and let c1, c2, c3, c4 be the 4 C-rimlines (beams from C) and let d1, d2, d3, d4 be the 4 D-rimlines (beams from D).
The 3 AB-rimlines are ab12=(a1^b1)^(a2^b2), ab23=(a2^b2)^(a3^b3) and ab34=(a3^b3)^(a4^b4).
The 3 BC-rimlines, 3 CA-rimlines, 3 CD-rimlines, 3 DB-rimlines and the 2 DA-rimlines are constructed accordingly.
Then the 2 ABC-rimlines are abc123=(ab12^bc12^ca12)^(ab23^bc23^ca23) and abc234=(ab23^bc23^ca23)^(ab34^bc34^ca34).
The 2 BCD-rimlines, 2 CDA-rimlines and the 2 DAB-rimlines are constructed accordingly.

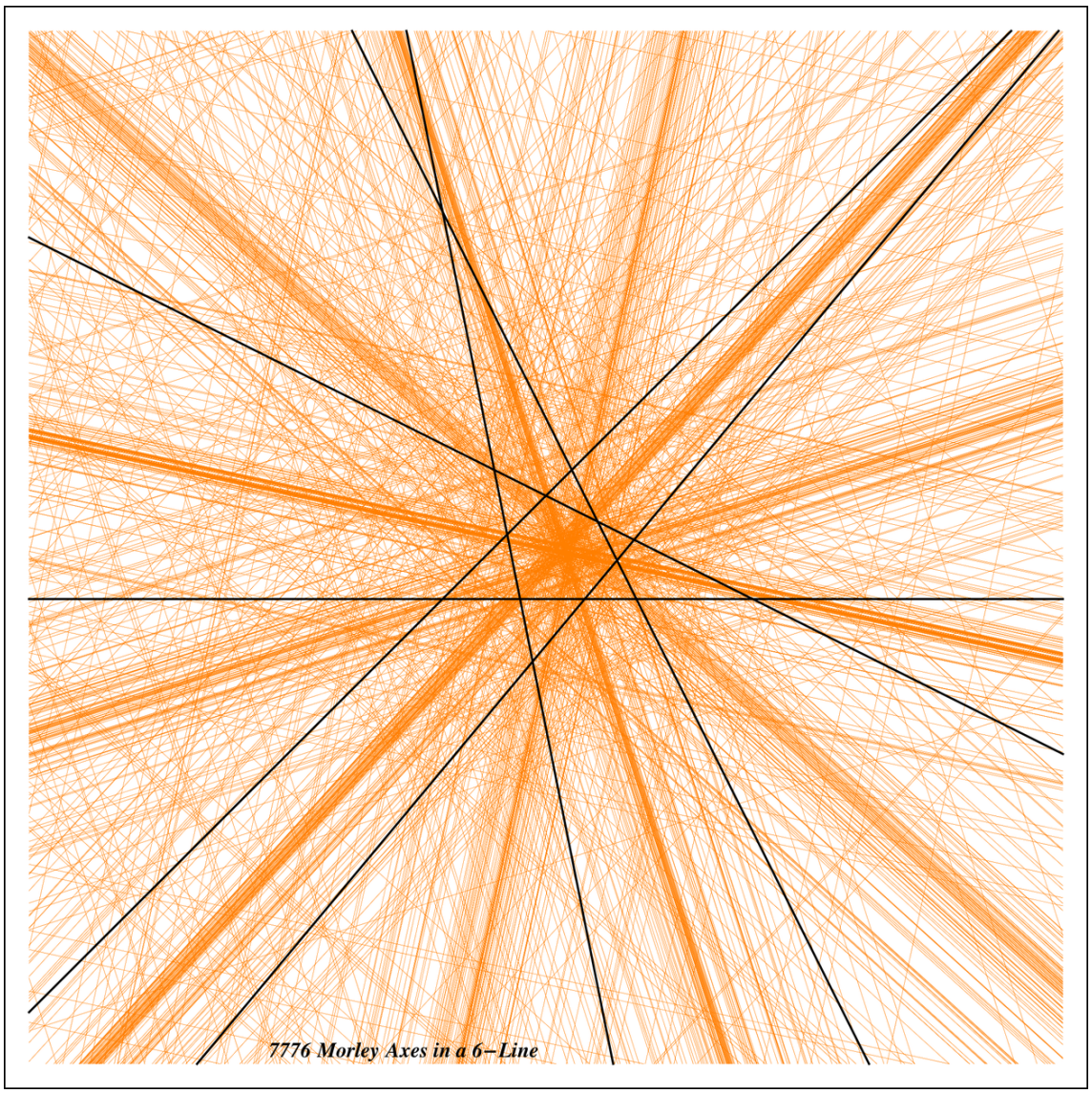
Finally the 1 ABCD-rimline is (abc123^bcd123^cda123^dab123)^(abc234^bcd234^cda234^dab234).
As can be seen from the notation the sets of rimlines (abc123,bcd123,cda123,dab123) and (abc234,bcd234,cda234,dab234) are considered to be concurrent lines. And that's what they are.
Will be proved later.

The way to get one rimline from n collinear Lighthouses:
The way to get one rimline from n collinear Lighthouses works accordingly.
For every generation step n -> n+1 one level in the scheme is added.
And the numbers of needed rimlines at each level is added with 1.

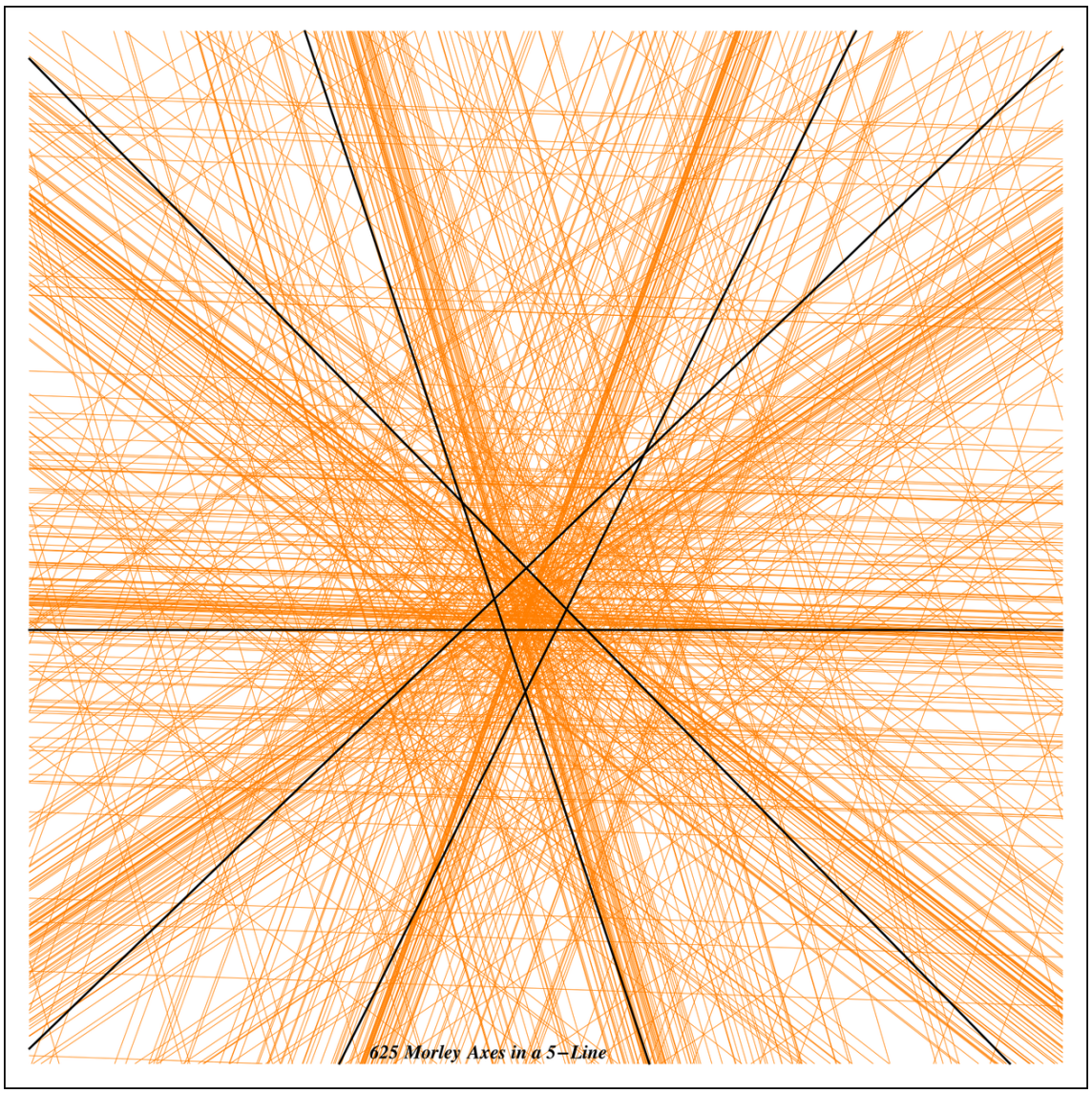
May 23, 2015

Chris van Tienhoven
www.chrisvantienhoven.nl
van10hoven@gmail.com

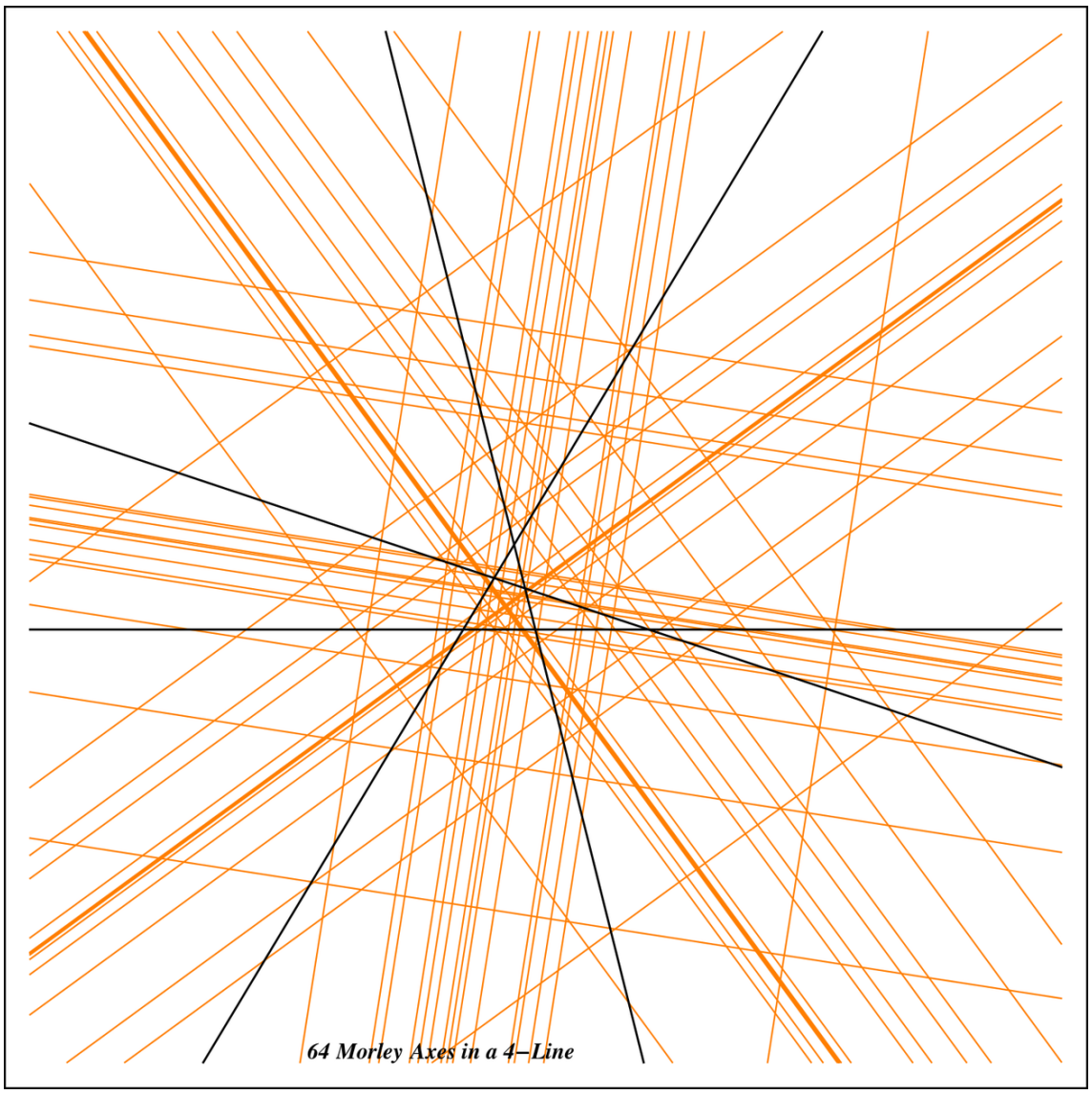
Morleys Axes in an n-Line.pdf



6Line-7776-Morley-Axes.pdf



5Line-625-Morley-Axes.pdf



4Line-64-Morley-Axes.pdf

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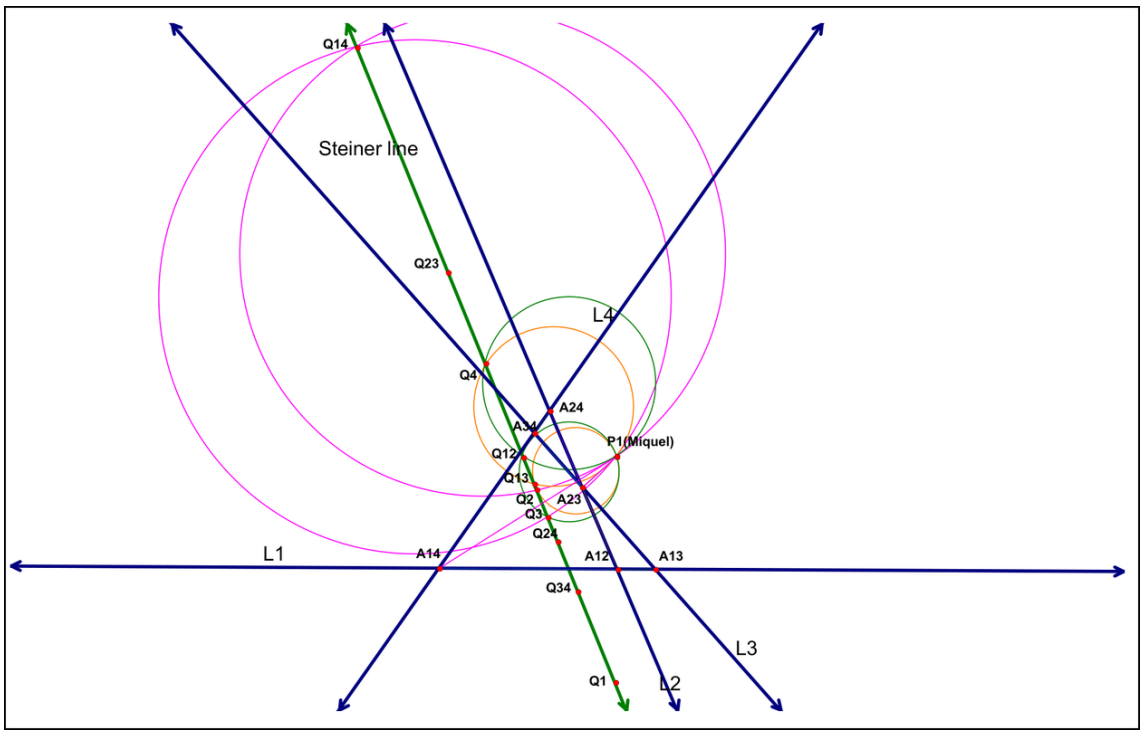
Message: #1139
Date: 24/5/2015 8:13:45
From: seiichikiri
Subject: Apollonius circles

Dear Bernard,
Your configuration contains 4 more points!
See the attached picture.
6 points on Steiner line are as follows.
Q12: the intersection of Apollonius circles of P1A24A14 and P1A13A23. (green).
Q13: the intersection of Apollonius circles of P1A14A34 and P1A12A23. (orange).
Q14: the intersection of Apollonius circles of P1A24A12 and P1A13A24. (pink).
Q23: the intersection of Apollonius circles of P1A12A13 and P1A24A34. (dashed green).
Q24: the intersection of Apollonius circles of P1A23A34 and P1A14A12. (dashed orange)
Q34: the intersection of Apollonius circles of P1A23A24 and P1A14A13. (dashed pink).
4 more points on Steiner line are as follows.
Q1: the intersection of Apollonius circles of P1A12A13, P1A13A14 and P1A14A12.
Q2: the intersection of Apollonius circles of P1A12A23, P1A23A24 and P1A24A12.
Q3: the intersection of Apollonius circles of P1A13A23, P1A23A34 and P1A34A13.
Q4: the intersection of Apollonius circles of P1A14A24, P1A24A34 and P1A34A14.
We obtained fine results.
Best regards,
Seiichi.



SteinerLine-2.docx

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Message: #1140
Date: 24/5/2015 9:05:22
From: eckart_schmidt@t-online.de
Subject: A Locus

```
Dear Antreas,
in addition to my last message: The equation of the conic with
A(1,0,0), B(0,1,0), C(0,0,1), D(p,q,r):
  a*sqrt(c^2*q^2+2*SA*q*r+b^2*r^2)*(q*x-p*y)*z
- c*sqrt(b^2*p^2+2*SC*p*q+a^2*q^2)*(q*z-r*y)*x=0

You get an equation without squareroots wrt the reference
triangle QG-Tr3 (see message #657):
 a(c^2*u^2+2SB*u*w+a^2*w^2)(v*z-w*y)(b^2*w*z-c^2*v*y)
-b(c^2*v^2+2SA*v*w+b^2*w^2)(u*z-w*x)(a^2*w*z-c^2*u*x)=0

Best regards Eckart
```

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Message: #1141
Date: 24/5/2015 4:13:22
From: seiichikiri
Subject: 4 anticenters on Pedal Line (L3)

```
Dear friends,
Let L1L2L3L4 be a quadrilateral and P1 Miquel point.
Denote:
Sij= the intersection of Li and Lj for i, j=1, 2, 3, 4 except
for i= j.
A1= the anticenter of the circle of P1S23S24S34.
A2= the anticenter of the circle of P1S13S14S34.
A3= the anticenter of the circle of P1S12S14S24.
A4= the anticenter of the circle of P1S12S13S23.
A1, A2, A3 and A4 are on Pedal Line (L3).
G.D. ( graphically demonstrated.)
Best regards,
Seiichi.
```

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Message: #1142
Date: 25/5/2015 10:37:30
From: bernard.keizer
Subject: Apollonius circles

Dear Seiichi,
As you probably have already noticed, I'm generally more interested in the explanation of the reasons of a property than in the property itself.
I've found the synthetical proof : it's rather obvious !
The Apollonius circles of a triangle are orthogonal to the circumcircle.
Here, the 12 Apollonius circles are centered on a line Dj and orthogonal to the circle Ci (as P1 is on the 4 reference circles).
Therefore, the Cl-S conjugate of one Apollonius circle is a line (as the circle is through P1) perpendicular to the circle Dj (a diameter) and to the line Di : it's the perpendicular to Di through Oj.
In other words, the 12 Apollonius circles are the Cl-S conjugates of the 12 perpendicular bisectors of the 4 reference triangles.
The intersect by pairs in points and the Cl-S conjugates of these points are the intersections of the perpendiculars to Di through Oj and to Dj to Oi ; the angle being equal to the angle between Di and Dj is also equal to the angle OiP1Oj, as the Oi and P1 are concyclic on the Miquel circle.
At the end, the intersections of the 2 lines perpendicular to Di through Oj and to Dj through Oi are on the Miquel circle (6 interesting points) and their Cl-S conjugates are the 6 intersections Qij between pairs of Apollonius circles and lie on the Steiner Line.
The Apollonius circles intersect also 3 by 3 in your points Qi, which are not new points, but simply the Cl-S conjugates of the circumcenters Oi, id est the reflections of P1 in the 4 lines Di, which lie on the Steiner Line too.
Best regards
Bernard

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Message: #1143
Date: 25/5/2015 7:24:26
From: bernard.keizer
Subject: 64 axes of a QL

I read your short paper with great pleasure !
It contains the generalisation of the construction of the Morley axes of the n-line. I give you only a quick answer, as you say it's not complete yet.
1) Taking the paper as it is, I have only 2 remarks
a) concerning the text, there are 2 typos on page 4 (the last one):
above, the meaning is not the right one, you made a copy of the meaning of the rimline for 3 collinear Lighthouses, it should start with 4 A-rimlines and 4B-rimlines ... and end with 1 ABCD-rimline.
on the 10th line at the end, read 3 (instead of 2) DA-rimlines
b) concerning the figures, you could pedagogically start with the 2 axes of the 2-lines (angle of 90°) and the 9 axes of the triangle/3-lines (angle of 60°)
2) In a complete paper, I suppose you intend to develop the property that for any n-line the mean-lines of the n (n-1)-lines are concurrent and to introduce the strange curves of Morley (circle, cardioïd, tetracardioïd and generalised ennacardioïds ...), the axes being the locus of the centers of these curves tangent to the n lines...
3) As you invite us to express wishes, I would be greatly interested in seing the 4 sets of 96 intersection points of the 64 axes in 4 different colors (or at least one of the sets).
Best regards
Bernard
PS As I'm not practicing Mathematica, I'm not able to give a judgment on the performance, but I'm astonished by the shortness of your coding!

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Message: #1144
Date: 25/5/2015 9:19:59
From: Chris van Tienhoven
Subject: 64 axes of a QL

Dear Bernard,
Thanks for the remarks.
I will make the new picture as soon as I come back from a short holiday.
In the paper I am writing I will limit myself to the Morley Axes and the Lighthouse theorem. My hands are fill with that
Maybe later there will be time for (enna-)Cardioids.
I really love the beauty of the n-Line configuration of the morley axes and the intermediate rimlines.
Today driving the car in your beautiful country France, I noticed my mind dwelling over these beatifull geometrical matters and also designing a new program for constructing and printing all axes.
Till next time,
Chris

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Message: #1145
Date: 26/5/2015 1:51:10
From: seiichikiri
Subject: Apollonius circles

Dear Bernard,
your proof is always instructive to me.
Synthetic proof is like an elaborate cooking, which I must learn.
To date, I am tempted to cook with a microwave oven because of sparing time.
But I think at the same time that, as we have different careers, it is not so bad that we have different tendencies.
Best regards,
Seiichi.

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Message: #1146
Date: 28/5/2015 8:53:20
From: eckart_schmidt@t-online.de
Subject: Directions of Morley-4L-axes

```
Dear Bernard, dear Chris,

Morley describes the directions of his 4L-axes with mean
directions of the 4 lines wrt a basic line. Here is given
another simple construction for the directions of
Morley-4L-axes.
Consider a quadrigon of the QL
... and its triangle QG-Tr3 = ABC with C = QG-P18.
... Let the Thales circle about the excenters Ja and Jb
intersect the bisector of AB in S1 and S2,
... let the Tales circle about the incenter J and the excenter
Jc intersect the bisector of AB in T1 and T2,
... then  S1JaS2Jb and T1JT2Jc are rectangles
... with sidelines parallel  to Morley axes.
The directions depend only on the triangle QG-Tr3.
Taking this triangle as reference triangle for barycentric
coordinates, the points at infinity are:
    (a(2ab+2b^2-ac-bc-c^2)+(a+b-c)W,
    -b(2a^2+2ab+ac+bc-c^2)-(a+b+c)W,
    c(a^2+2ab+b^2+ac-bc+2W))
    (-a(2ab+2b^2+ac+bc-c^2)-(a+b+c)W,
    b(2a^2+2ab-ac-bc-c^2)+(a+b-c)W,
    c(a^2+2ab+b^2-ac+bc+2W))
with W=sqrt(ab(a+b-c)(a+b+c))

    (-a(2ab-2b^2-ac+bc+c^2)-(a-b+c)V,
    b(2a^2-2ab-ac+bc-c^2)-(-a+b+c)V,
    c(-a^2+2ab-b^2+ac+bc+2V))
    (-a(2ab-2b^2+ac-bc+c^2)-(-a+b+c)V,
    b(2a^2-2ab+ac-bc-c^2)-(a-b+c)V,
    c(a^2-2ab+b^2+ac+bc+2V))
with V=sqrt(ab(a-b+c)(-a+b+c))

Best regards Eckart
```

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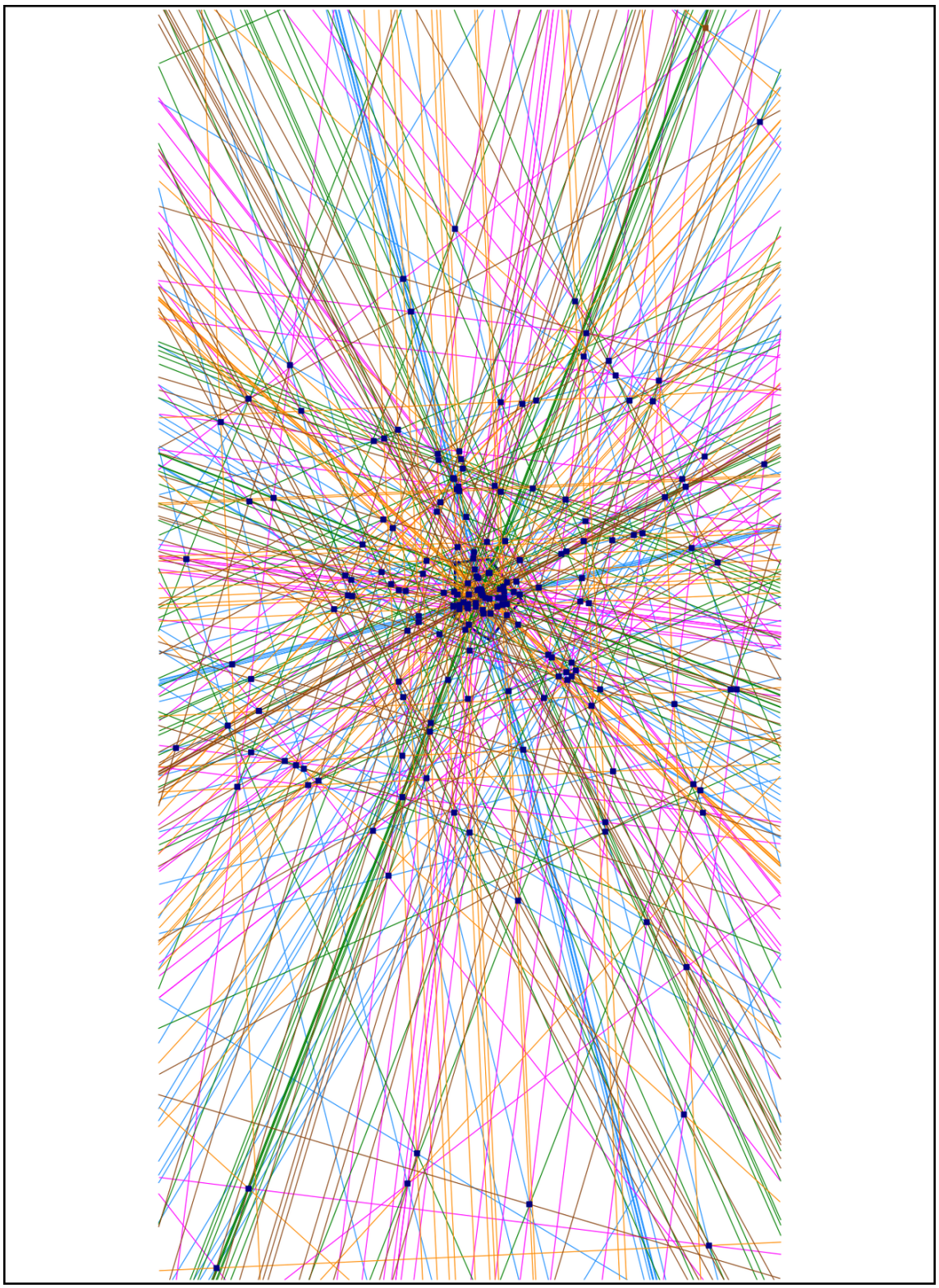
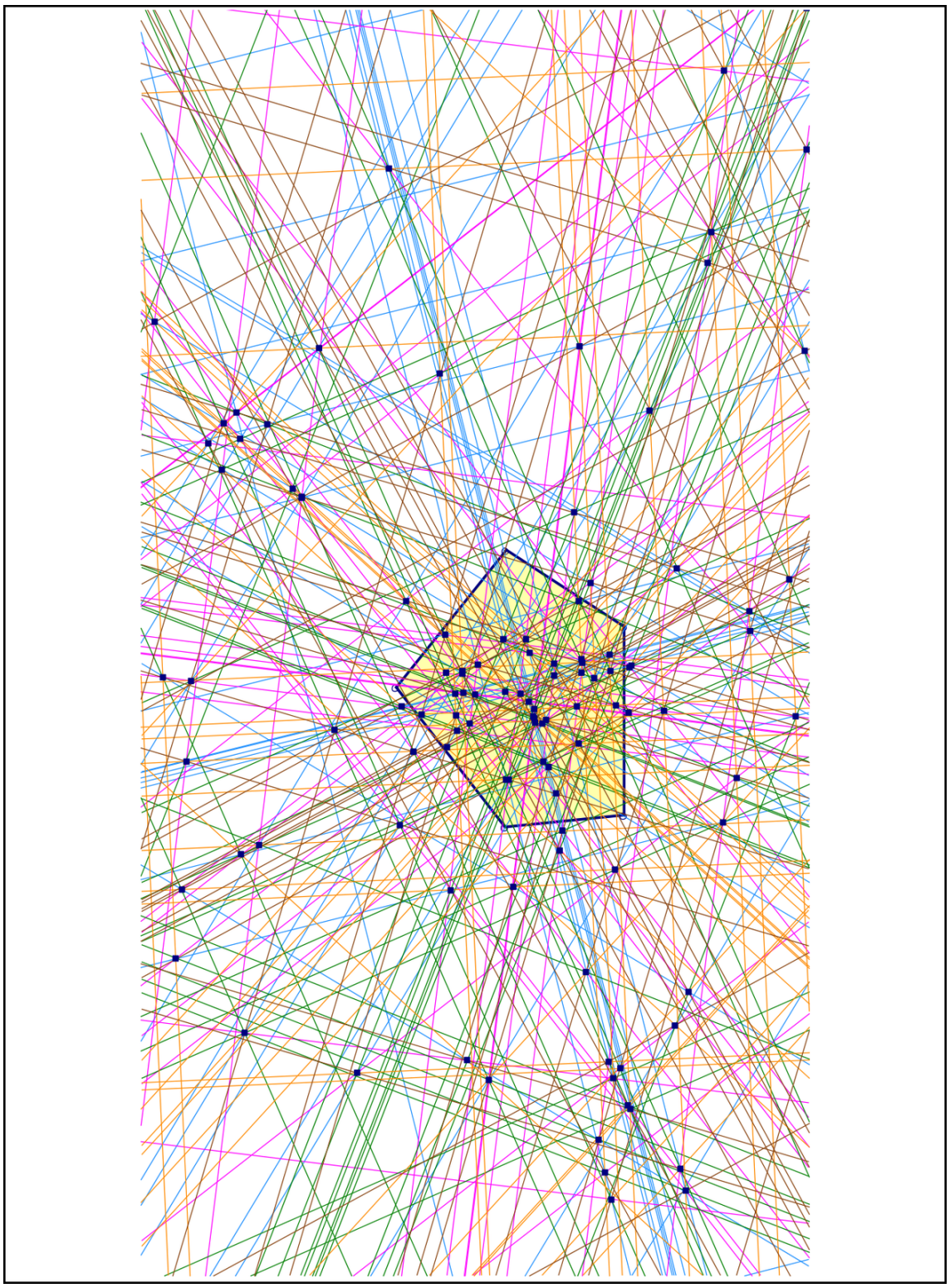
Message: #1147
Date: 29/5/2015 11:08:08
From: eckart_schmidt@t-online.de
Subject: : 64 axes of a QL

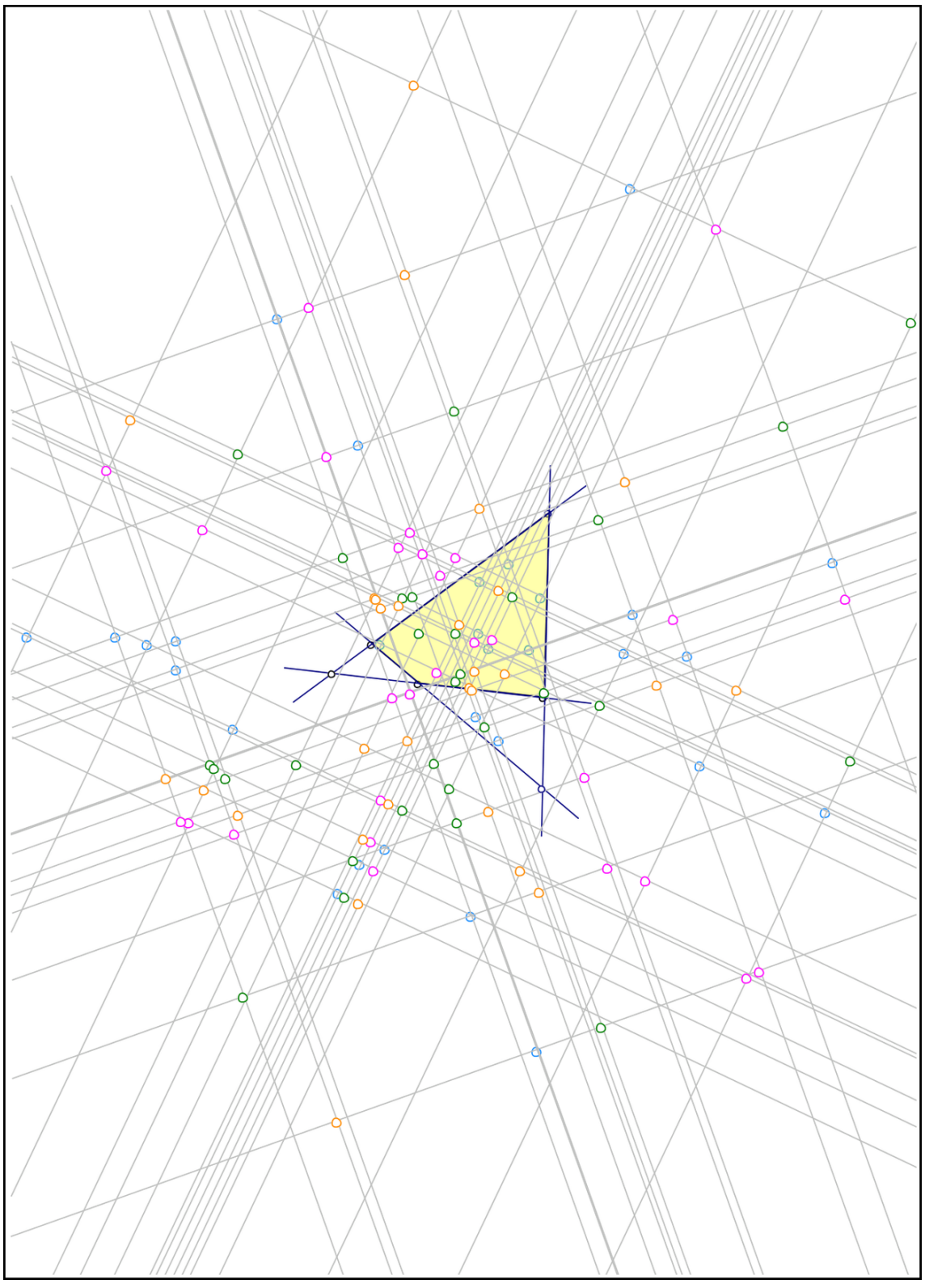
Dear Bernard,
wrt the quartics - you look for - here two remarks:
...In #1143 you asked for a figure for your 4 x 96 intersections of Morley-axes for a QL. Attached there is a drawing of 4 x 32 orthogonal intersections among your 4 x 96.
...There are two further drawings of the 5 x 64 axes for a 5-line. I have marked about 240 of the 256 intersections of 5, which are the centers of inscribed tetracardioids.
In both figures I can't discover quartics, perhaps you.
Best regards Eckart
PS: Cabri-files transformed in pdf-format show inexact points, sorry.



2015-05-29z.pdf



2015-05-29y.pdf



2015-05-29x.pdf

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Message: #1148
Date: 29/5/2015 8:31:02
From: Chris van Tienhoven
Subject: Directions of Morley-4L-axes

```
Dear Eckart,
This construction is quite remarkable.
Do you have an explanation for it?
Best regards,
Chris
```

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Message: #1149
Date: 30/5/2015 1:14:12
From: bernard.keizer
Subject: Directions of Morley-4L-axes

```
Dear Eckart, dear Chris
You find it simple!
There are other ways to obtain these directions.
The simplest way remains to draw the bisectors in 2 opposite
vertices, which intersect in 4 points and to draw the bisectors
of 2 of the previous bisectors (one in each vertice) in any of
these 4 points ...
The construction is the same with every copple of conjugate
points on the hessian QL-Cu1, as the mean direction is the same!
Another, perhaps even more simple, is to draw the bisector
between the axis of the parabola and the 1rst Steiner axis ...
Best regards
Bernard
```

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Message: #1150
Date: 30/5/2015 9:01:11
From: eckart_schmidt@t-online.de
Subject: Directions of Morley-4L-axes

```
Dear Bernard,
please cut the adjective "simple" in  #1149.
Best regards Eckart
```

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Message: #1151
Date: 31/5/2015 9:29:11
From: eckart_schmidt@t-online.de
Subject: Direction of Morley-axes

```
Dear Bernard,
sorry, there is a typo in my last message:
Please cut the adjective "simple" in my #1146.
Best regards Eckart
```

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Message: #1152
Date: 31/5/2015 10:42:18
From: bernard.keizer
Subject: Direction of Morley-axes

Dear Eckart, dear Chris
I had understood ...
If I'm not wrong, you search in QG-Tr3 the direction of the angle bisectors between angle bisectors a and b as QG-2P2a and 2b are the 2 last vertices of the QL ; I suppose this explains your construction.
Now back to one of my favorite subjects.
A QL determines one Cl-S transformation and one Newton Line and following one cubic stelloïd QL-Cu2 and it's hessian QL-Cu1 and the usual curves parabola, deltoïd and cardioïd.
Conversely, given one Cl-S transformation (circle of inversion with center P1, invariant points F1 and F2 and 1rst Steiner axis F1F2) and one Newton Line, the parallel to the Newton Line through P1(axis of the parabola) cuts the inversion circle in 2 diametral points I and J.
For any copple of conjugate points (not necessary on the hessian) X and X', the perpendicular asymptotes of the rectangular hyperbolas forming the conic polar of X (centered in X') and of X' (centered in X) cut the line IJ in 2 points respectively Xi and Xj and X'i and X'j symmetric wrt P1 ; therefore these points are the intersections of the line IJ with the circles with center P1 through X and X'. The bisectors of XXi (or XXj) and X'X'i (orX'X'j) give the direction of the Morley axes, the same for every copple X and X'.
This is also true for a copple of Cl-S conjugate points on the hessian, as I put in message 1149 ; in this case, the rectangular hyperbolas are degenerated in their 2 asymptotes.
But this is also true for the 2 invariant points F1 and F2 and the direction is therefore simply F1I or F1J or F2I and F2J, which are the same as the bisectors of the axis of the parabola and the 1rst Steiner axis (as I put in message 1149) and their bisectors.
As I put it already in several previous messages, this angle property is for me one of the most beautiful properties of this figure that the mean direction of the 4 lines of a QL inscribed in a curve QL-Cu2 is always the same and therefore, they have the same direction for the axes of the parabola and the cardioïd and for the asymptotes of the deltoïd and now for the Morley axes.
Thanks for your attention
Best regards
Bernard

Message: #1153
Date: 31/5/2015 10:48:53
From: chris.vantienhoven
Subject: Directions of Morley-4L-axes

Dear Eckart, dear Bernard,
Now that I am home again after a short holiday I made pictures of both your beautiful constructions regarding the mean QL-angle.
Bernard, your 1st method seems explainable to me. Your 2nd method I can't explain but still is remarkable. Since the Axis of the QL-parabola is parallel with the Newton Line the angle bisector of the 1st Steiner Axis and the Newton Line also has a mean QL-angle.
Then I found some interesting overlaps. Both graphically demonstrated (Cabri-proof).
* The CSC-invariant points (lying on the 1 st Steiner Axis) lie on the Thales Circle (J,Ja), used in Eckart's method.
* The AB(1st Steiner Axis, Newton Line) coincides with AB(AB(P1,P3),AB(P2,P4)). AB meaning Angle Bisector. This combines both of Bernards methods.
And so it comes out again that in geometry all is connected.
Best regards,
Chris

Message: #1154
Date: 31/5/2015 11:06:11
From: bernard.keizer
Subject: : 64 axes of a QL

Dear Eckart,
Please have first many thanks for these beautiful figures.
Unfortunately, I don't see either any quartic there !
To be serious, as you know, I use only Geogebra, which doesn't allow the use of macros as Cabri and the possibility of calculations like Mathematica and I hoped you would be able to draw or to calculate a quartic through 14 of these points, just in order to prove that my conjecture is wrong.
I'm a little bit obstinous or like St Thomas, I believe only what I see ...
I'm full of admiration for Morley and others who imagined their properties and made their demonstrations almost whithout figures!
Best regards
Bernard

PS Do you know by any chance a reference on the web (in english or in german) where I could find clear definitions of polar or apolar conics of a curve wrt a cubic. For example, I suppose the polar curve of a curve wrt a cubic is the envelop of the polar lines wrt the polar conics of the points of the curve (therefore, any cubic is it's own polar as for any point on a cubic, it's polar conic is tangent to the cubic). Thanks in advance!

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Message: #1155
Date: 31/5/2015 1:04:28
From: bernard.keizer
Subject: Directions of Morley-4L-axes

Dear Chris, dear Eckart
Please read again my messages 840, 1124 and the correction in 1127 and of course the last one 1152.
The Thales circles described by Eckart are a special case for 2 conjugate vertices of the generalized Steiner circles for 2 conjugate points X and X' on the hessian QL-Cu1 : one is through X and X' and the 2 invariant points and centered on the 2nd Steiner axis and the other is orthogonal to the 1rst one, through X and X' and centered on the 1rst Steiner axis.
In this special case of 2 vertices of the QL, the points J and Ja are my points Y and Y' and the points Jb and Jc are my points Z and Z'.
Again, Y, Y', Z and Z' are the 4 poles of the line XX' wrt the cubic stelloïd QL-Cu2 and Y and Y' describe the hessian of one of the 2 other cubics sharing the same cayleyan and Z and Z' the hessian of the other.
Best regards
Bernard
PS I hope this marvelous angle property will come one day in EQF under any possible form !

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Message: #1156
Date: 2020-02-21
From: Systems Manger
Subject: Deleted Messages

Message number 1156 is not available in Yahoo groups.

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Message: #1157
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Messages

Message number 1157 is not available in Yahoo groups.

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Message: #1158
Date: 01/6/2015 3:39:35
From: eckart_schmidt@t-online.de
Subject: Directions of Morley-4L-axes

Dear Bernard, dear Chris,
I think, the following description will be in your sense:
All quadrigons with the same triangle QG-Tr3 = ABC (C=QG-P18)
have the same directions for their Morley-axes.
You get these quadrigons, taking an arbitrary point P1, its
isogonal P3 wrt ABC and P2=A.P1 ^ B.P3, P4=A.P3 ^ B.P1.
Best regards Eckart

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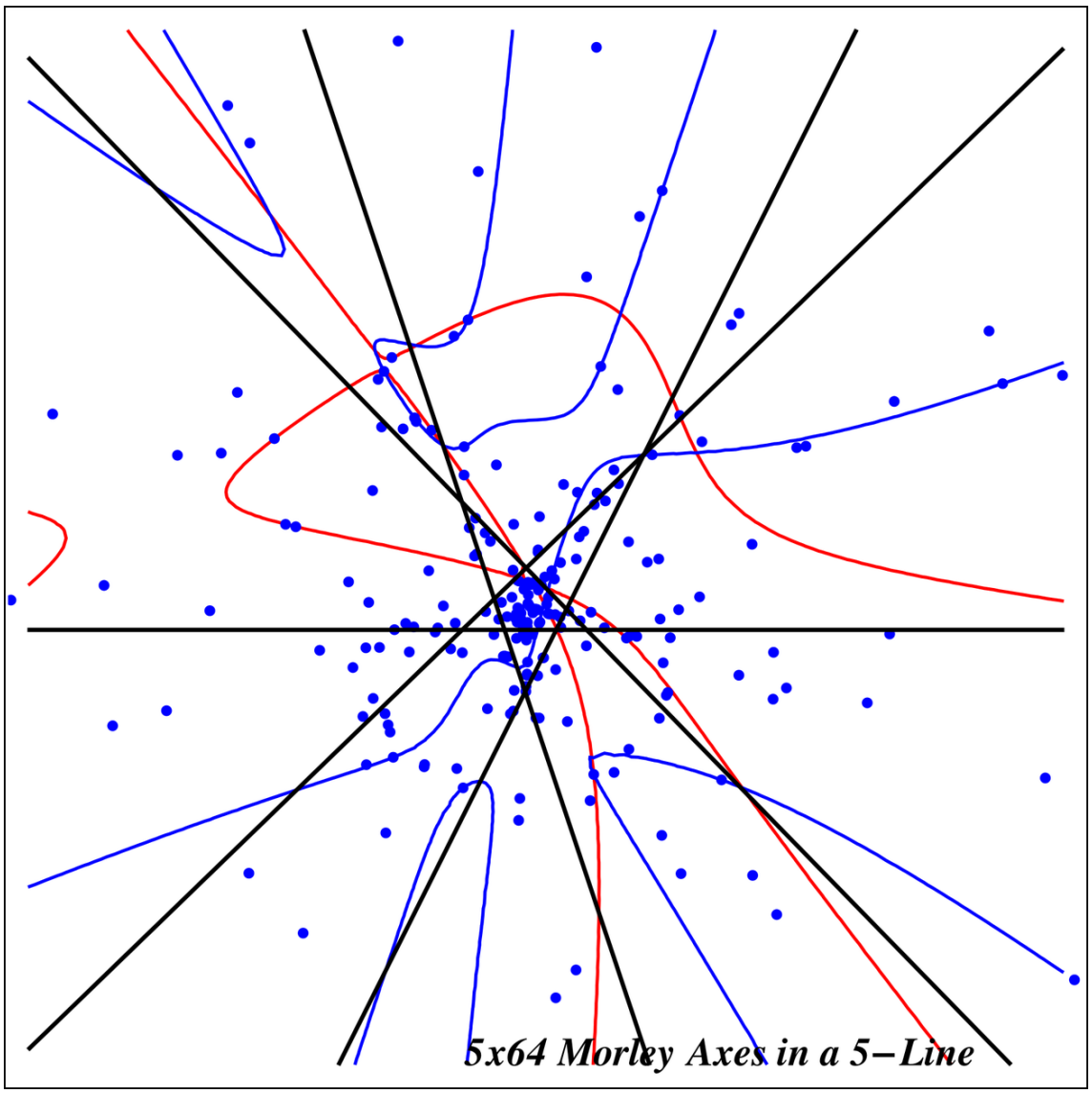
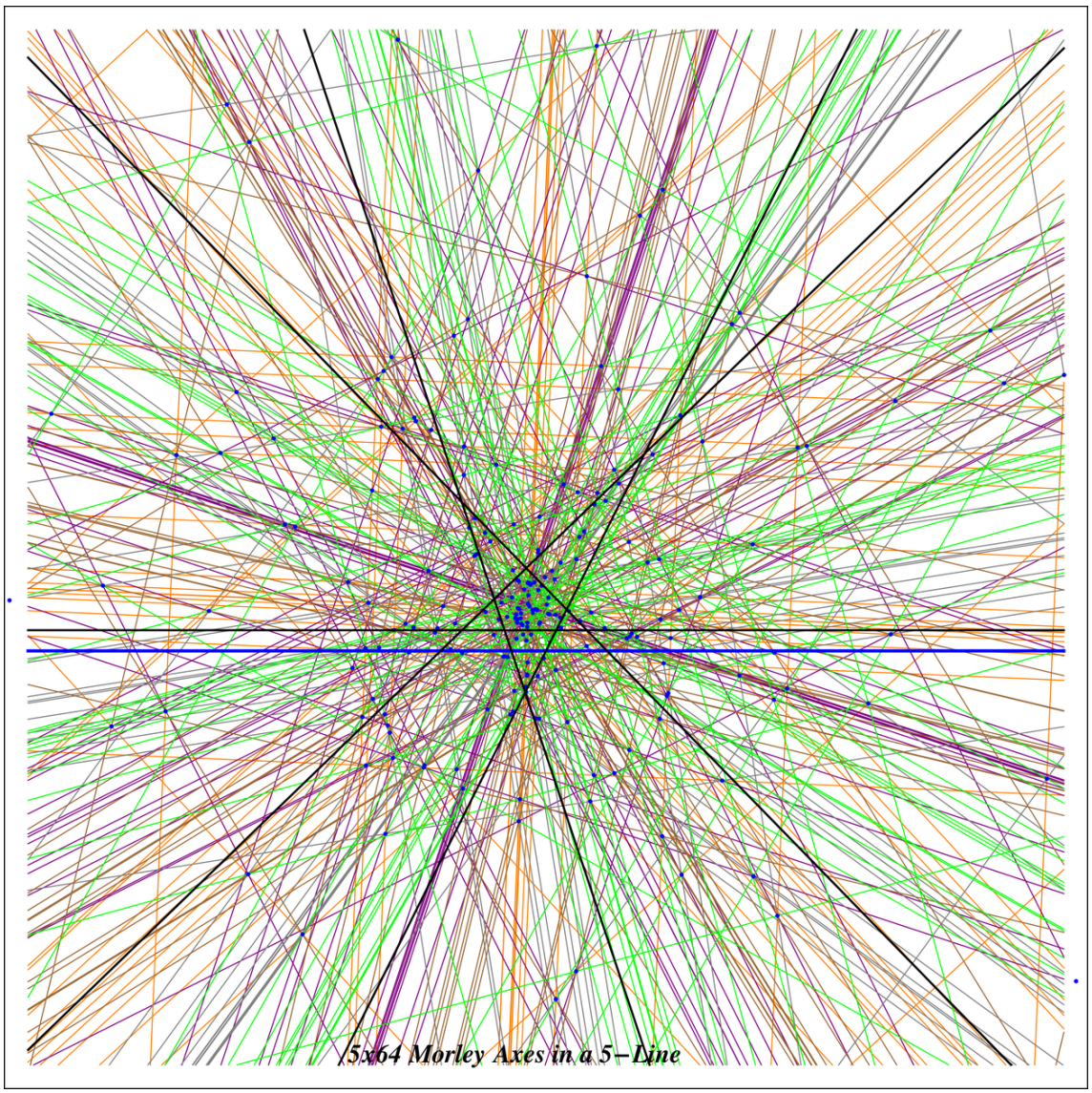
Message: #1159
Date: 02/6/2015 10:41:19
From: chris.vantienhoven
Subject: : 64 axes of a QL

Dear Bernard, dear Eckart,
I made a picture in Mathematica of a numerical case of 5x64
Morley Axes in a 5-Line as well as the predicted 256
intersection points with 5 axes crossing each point and 4 points
on an axis.
Bernard, like you suggested I made a second picture with a
quartic through 14 (no 3 of which collinear) of these 256
points. See red curve. Also I made a quintic through 20 (no 3 of
which collinear) of these 256 points. See blue curve.
Mathematica wasn't able to calculate a sextic through 27 points.
Like you can see the quartic and the quintic do not cover the
other points. So it seems that the 256 points can’t be caught by
a quartic or quintic.
However this is not a prove and there can be mistakes in my
calculations.
Best regards,
Chris



5L-5x64 MorleyAxes-256 IntersectionPoints-02.pdf



5L-5x64 MorleyAxes-256 IntersectionPoints-01.pdf

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Message: #1160
Date: 02/6/2015 10:56:42
From: bernard.keizer
Subject: Directions of Morley-4L-axes

Dear Eckart, dear Chris
Both constructions are indeed interesting, but different. At the end, obviously, they are converging, as "tous les chemins mènent à Rome!"
1) In your construction, A, B, P1, P2, P3 and P4 form a QL.
A and B are 2 fixed points, Cl-S conjugates wrt a variable Cl-S.
C is a fixed point and is the tangential point of both A and B wrt the variable QL-Cu1.
The variable Miquel point QL-P1 is on the circumcircle of ABC.
The variable QL-Cu1 is through A, B, C and QL-P1.
The fixed points J, Ja, Jb and Jc, in- and excenters of the triangle ABC, are the poles of the fixed line AB wrt the variable cubic QL-Cu2; this means that the polar conic of any point of AB wrt QL-Cu2 is a rectangular hyperbola centered in the Cl-S conjugate of the point (variable according to the Cl-S transformation) through these 4 fixed points.
The direction of the Newton Line (a line through the fixed middle of A) is variable ; the direction of the 1rst Steiner axis is variable (bisector of the angle).
It is remarkable that the bisector of these 2 lines remains parallel to the bisector of the 2 bisectors in A and B and has a fixed direction!
(That's precisely the direction of the Morley axes m° pi/4)
2) In my construction, QL-Cu1 is given (with QL-P1, 1rst Steiner axis, inversion circle, Cl-S, Newton Line and of course QL-Cu2).
There are no fixed vertices.
All the QL's inscribed in this QL-Cu1 (meaning the 6 vertices are 3 copples of Cl-S conjugate points) have the same direction, not only for Morley axes, but also for axes of parabola, cardioïd and deltoïd.
Best regards
Bernard

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Message: #1161
Date: 02/6/2015 1:46:55
From: eckart_schmidt@t-online.de
Subject: Directions of Morley-4L-axes

Dear Bernard,
thanks for your detailed remarks. My last message should not be a new description for constructing the directions of Morley-axes, but underline the property, that these directions depend only on the triangle QG-Tr3. If we take QG-Tr3 as reference triangle, calculations will be easier possible (see already #657).
Best regards Eckart
PS: Sorry, I can't give any reference for polar conics wrt a cubic.

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Message: #1162
Date: 03/6/2015 7:00:51
From: chris.vantienhoven
Subject: 64 axes of a QL

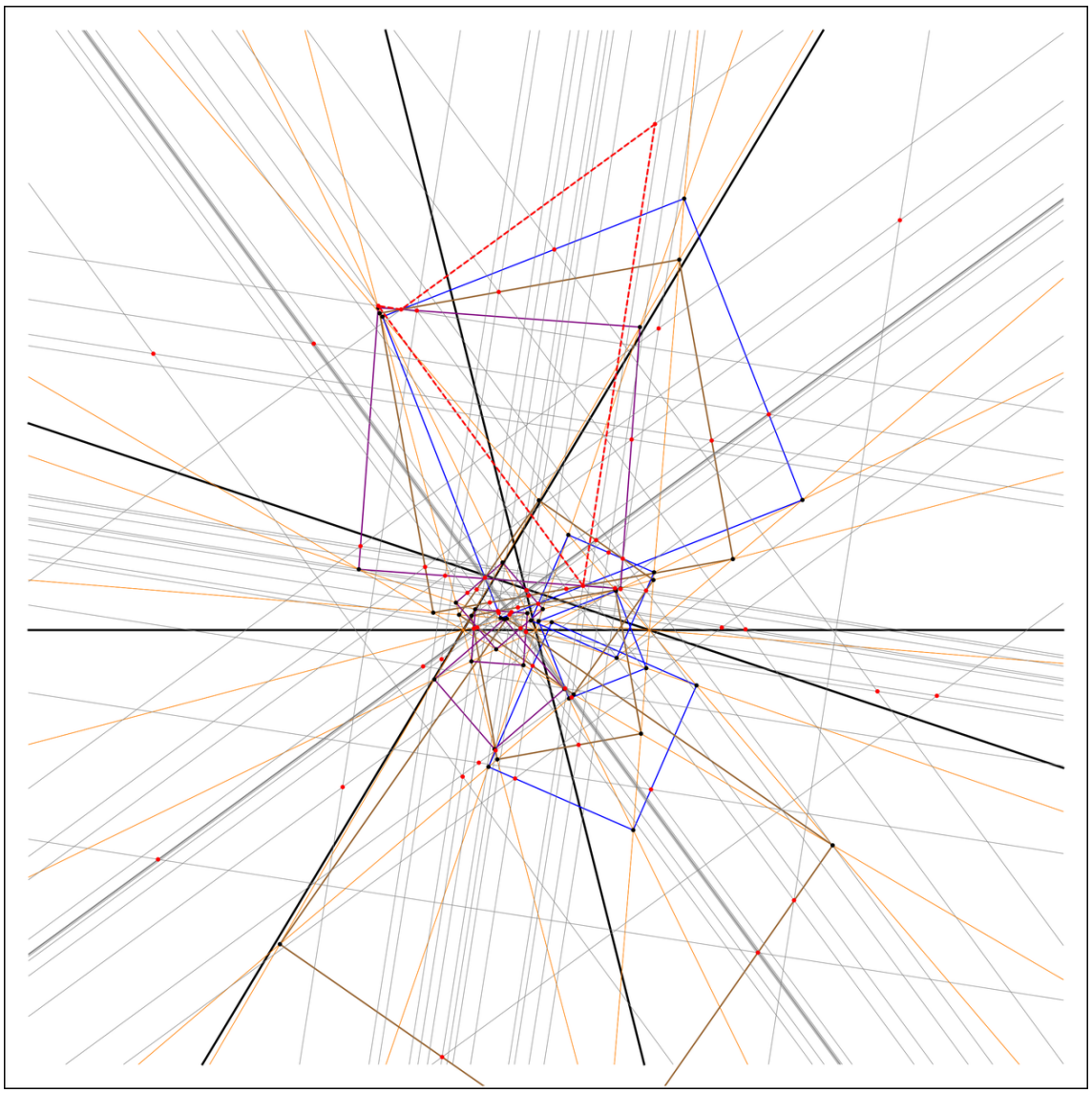
[#1143, BK]
I would be greatly interested in seeing the 4 sets of 96 intersection points of the 64 axes in 4 different colors (or at least one of the sets).
Dear Bernard,
I made some pictures of the intersection points.
You asked for 96 intersection points, however in the construction of the Morley Axes only 64 points are needed. That's why I never draw 96 intersection points but only 64 intersection points. It would enhance pretty much effort to add the extra points, so I hope you forgive me for pictures with only 64 points per set.
I learned from the exercise with constructing the Morley Axes in an n-Line that it's not really the intersection of n-sectors that are important but the intersection of n-gons generated from "Lighthouses" per baseline, they are important.
The method is that with 2 Lighthouses we get n regular n-gons, each having a starting point and its sides in a certain order. Because they all have a starting point we can intersect the corresponding sides of different n-gons.
By intersecting corresponding sides of two regular n-gons from different sets of Lighthouses we get n relevant intersection points which form a semi-regular n-gon.
In the case of a 4-Line we get sets of squares from which we intersect only the corresponding sides and not the diagonals, which by the way would produce the extra 32 points.
Per crossing of 2 squares we get 4 intersection points in a certain order which produces a figure of 4 Morley Axes, each crossing at 45 0 , which I call here a semi-square. Since we have 4 sets x 4 sets of squares to be crossed we end up with 16 semi-squares.
As I mentioned before the 64 Morley Axes in a 4-Line aren't composed of 16 squares.
However it can be said that the 64 Morley Axes in a 4-Line are composed of 16 semi-squares.

Semi-squares
Note that a semi-square is a special 4-sided figure with special properties:
* Opposite sides are perpendicular (unlike a regular square where the adjacent sides are perpendicular).
* Adjacent sides cross at 45 0 /135 0.
* The diagonals are equal of size (like a regular square).

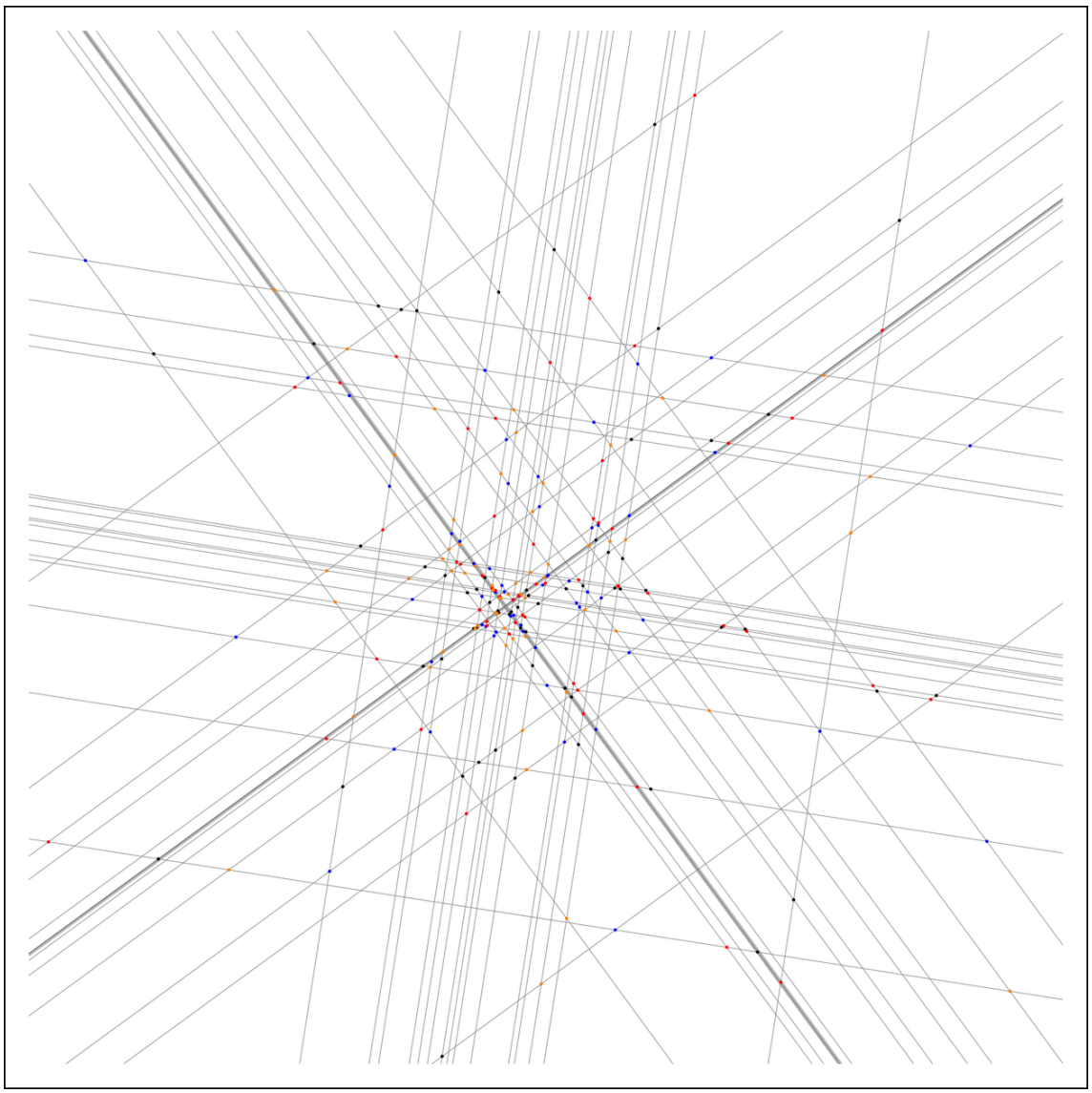
* The diagonals are perpendicular (like a regular square).
Semi-squares exist as a cross-product of 2 perspective squares. Squares are considered to be perspective when the connecting lines of corresponding square vertices are concurrent in a perspector. This perspector will be an intersection point of the circumcircles of both squares. The other intersection point of both circles will be the intersection point of the diagonals of the generated semi-square.
This all is a foreword for the pictures I am sending you.
In the picture based on one baseline L1 you see:
* One baseline L1 (Black) and 3 Lighthouse points (Orange)
* The other three QL-lines L2, L3, L4 (Black)
* Orange coloured beams from lighthouses (or 4-sectors)
* Per set of 2 Lighthouses 4 squares are generated. There are 3 sets of 2 Lighthouses so there are 3 sets of 4 squares.
Set-colours: Blue, Purple, Brown.
* As can be seen Squares of different colours cross in 4 points which consecutively form a semi-square whose sides are Morley Axes. One semi-square (Red coloured and dashed) is shown.
Further I enclose a picture like you asked with the 64 axes and 4*64 (instead of 4*96) intersection points. The points per set are in colours black, blue, red, and orange.
Each axis has 2 points of each colour.

I also made a picture without the reference 4-Line.
This one looks like a painting of Mondriaan! Mondriaan (1872-1944) was a Dutch painter who painted lots of geometric figures. See Piet Mondrian - Wikipedia, the free encyclopedia (http://en.wikipedia.org/wiki/Piet_Mondrian).

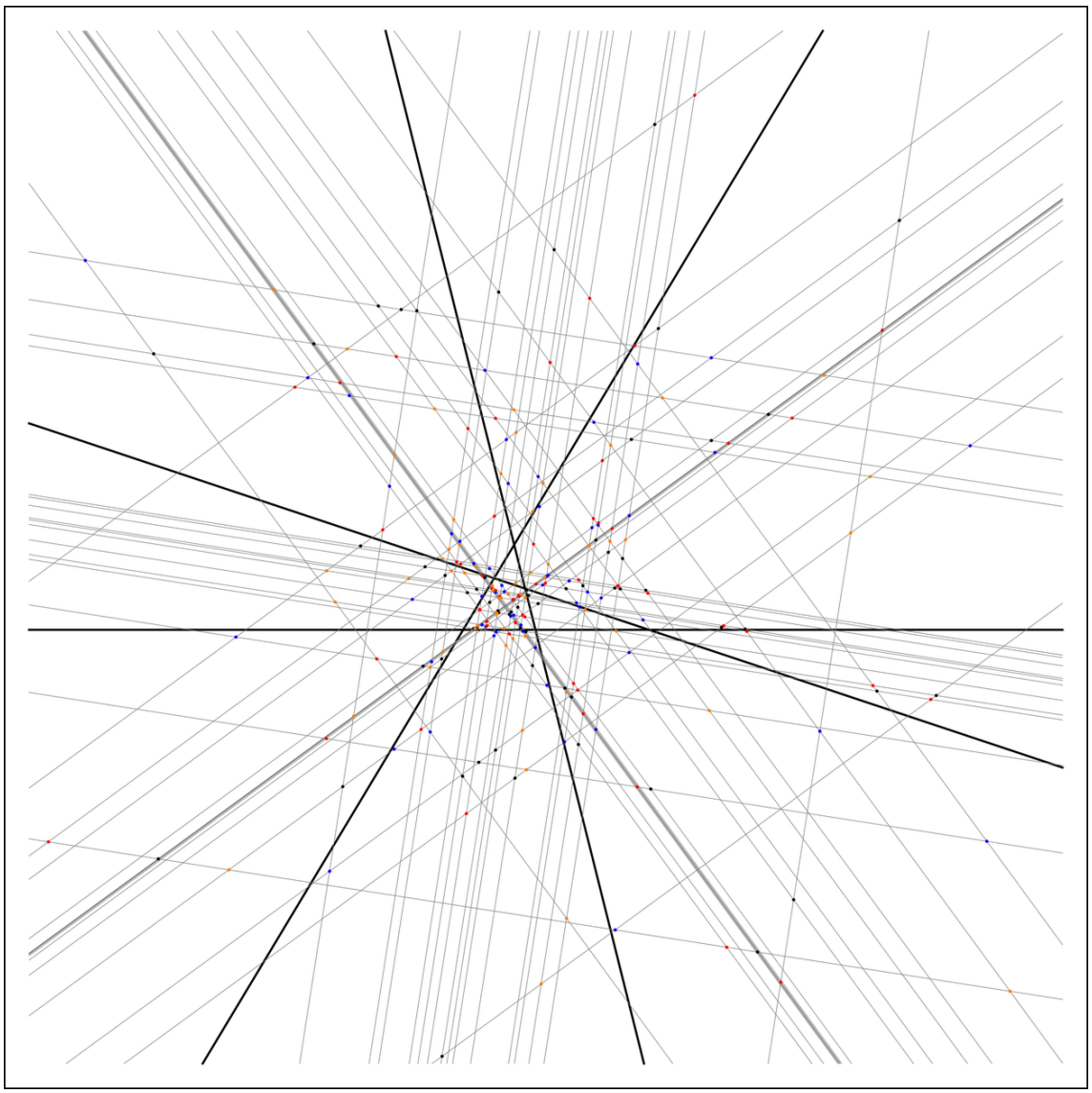
Nice combination Morley and Mondriaan! They lived about at the same time. When you look at their dwellings they could have met.
I hope this gives some more insight.
Best regards,
Chris



4L-64 MorleyAxes-64 IntersectionPoints-SemiSquare-01.pdf



4L-64 MorleyAxes-4x64 IntersectionPoints-02.pdf



4L-64 MorleyAxes-4x64 IntersectionPoints-01.pdf

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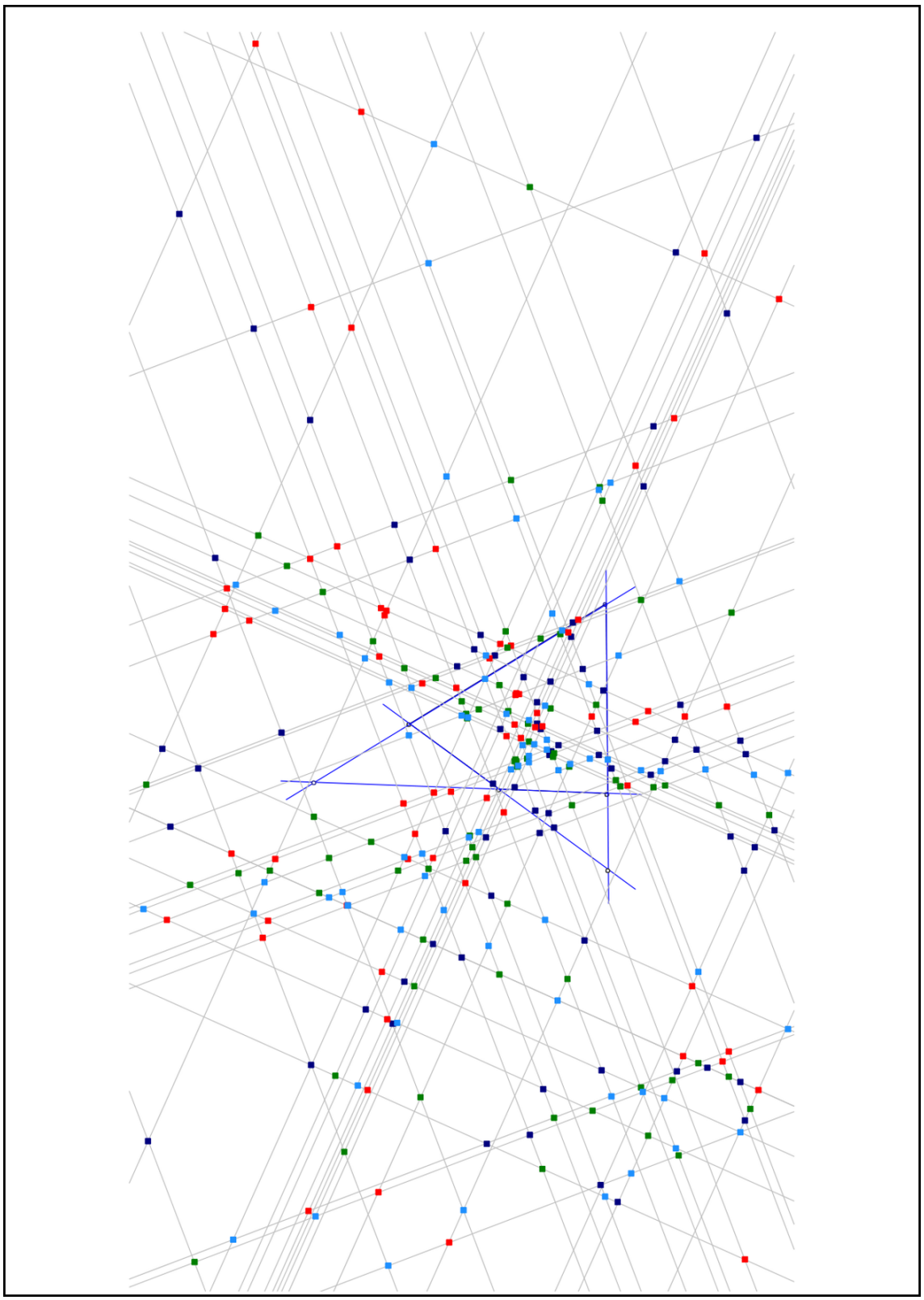
Message: #1163
Date: 03/6/2015 9:31:33
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Bernard,
before holiday and in addition to Chris' drawings: 4 x 96 intersections of Morley-axes in different colours, only in detail (word file).
Best regards Eckart



2015-06-03.docx

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Message: #1164
Date: 08/6/2015 1:33:04
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris, lieber Eckart,
I send you this quick message before long weeks of geometrical unactivity (holiday with my wife or occupation with my 6 grand-children, the 6th one was born 2 weeks ago)
1) Many thanks and Vielen Dank for these drawings and comments! I've highly appeciated all your last lessages (1147, 1159, 1162 and 1163).
The comment of Chris about Piet Mondrian was particularly appropriate (2 years ago was a beautiful exhibition in Paris of Mondrian's paintings).
Unfortunately, there is no quartic or even quintic in these points (thanks to Chris for giving the analytical proof) and it is for me like Wagner's "Götterdämmerung" !
As you know, I had a strong intuition that a curve cut in 4 points by 320 lines could and should be a quartic !
2) I try now to concentrate on the properties of the cubics QL-Cu1 and QL-Cu2 (the stelloïd and it's hessian) ; it's not really easy!
I regret that you never answered my messages 840, 1124 and 1127 and 1152 about the generalized Steiner circles (and the beautiful property of their anticenters mentionned by Seiichi for the 8 original Steiner circles).
Could you just tell me if these properties with poloconics or autopoloconics of the cubic stelloïd are uninteresting or too complicate ...
3) The property mentionned by J. W. Clawson about the Apollonius circles for which I gave a synthetic proof in message 1142 was on a contrary very simple, but remained unanswered too (apart of Seiichi).
Best regards
Bernard

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Message: #1165
Date: 12/6/2015 2:51:30
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Bernard,
[BK] Could you just tell me if these properties with poloconics or autopoloconics of the cubic stelloïd are uninteresting or too complicate ...
These items hardly can be uninteresting when you are so enthusiastic about them!
However sometimes I choose not to study certain portions because of limited time and other interests. I am afraid these items are amongst of them.
Nevertheless I enjoy all your messages and your great involvement!
I hope you will have wonderful holidays with your family!
Best regards,
Chris

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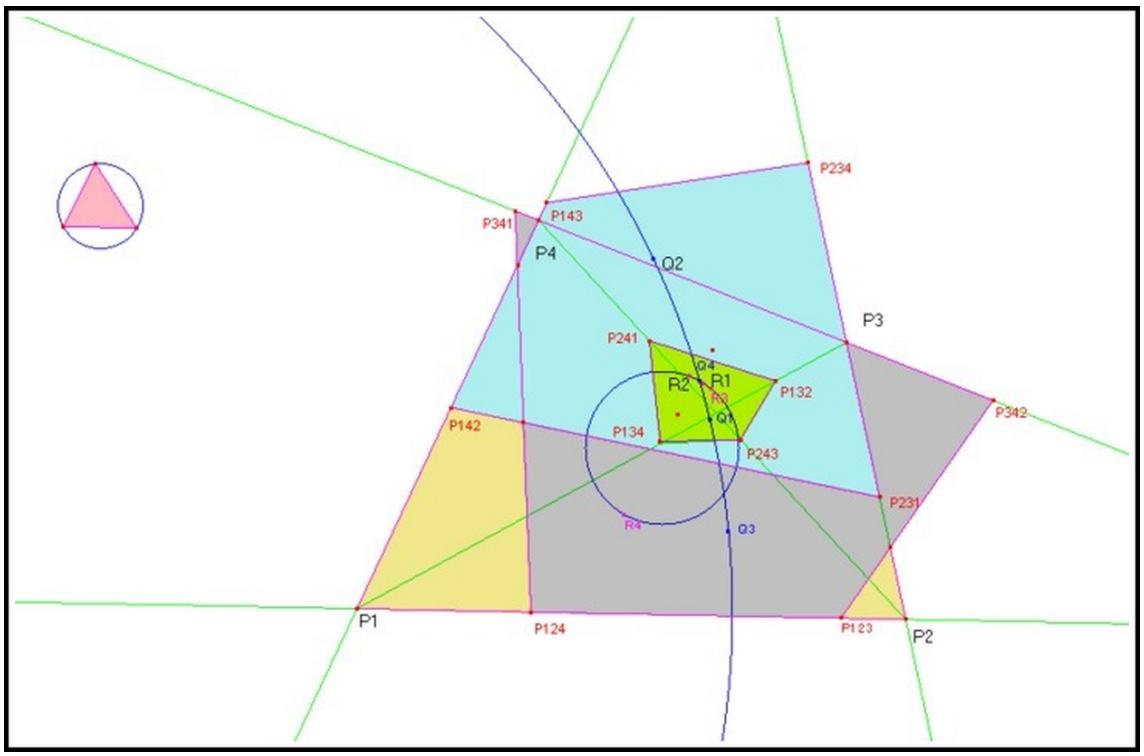
Message: #1166
Date: 14/6/2015 5:05:29
From: seiichikiri
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote :
P132 = the orthogonal projection of P2 on P1P3.
P134 = the orthogonal projection of P4 on P1P3.
P241 = the orthogonal projection of P1 on P2P4.
P243 = the orthogonal projection of P3 on P2P4.
P231 = the orthogonal projection of P1 on P2P3.
P234 = the orthogonal projection of P4 on P2P3.
P142 = the orthogonal projection of P2 on P1P4.
P143 = the orthogonal projection of P3 on P1P4.
P123 = the orthogonal projection of P3 on P1P2.
P124 = the orthogonal projection of P4 on P1P2.
P341 = the orthogonal projection of P1 on P3P4.
P342 = the orthogonal projection of P2 on P3P4.
Q1, R1= Euler-Poncelet point (QAP2) and
        Gergonne Steiner point (QAP3)
        of P134P243P132P241 respectively.
Q2, R2= Euler-Poncelet point (QAP2) and
        Gergonne Steiner point (QAP3)
        of P142P231P234P143 respectively.
Q3, R3= Euler-Poncelet point (QAP2) and
        Gergonne Steiner point (QAP3)
        of P124P123P342P341 respectively.
Q4, R4= Euler-Poncelet point (QAP2) and
        Gergonne Steiner point (QAP3)
        of P1P2P3P4 respectively.
Q1, Q2, Q3 and Q4 are concyclic and R1, R2, R3 and R4 are concyclic.
See the attached picture.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.



QAP2-P3.png

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Message: #1167
Date: 16/6/2015 10:32:49
From: seiichikiri
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear friends,
Addition:
The isogonal centers (QA-P4) of P134P243P132P241,
P142P231P234P143 and P124P123P342P341 coincide with
Euler-Poncelet point (QA-P2) of P1P2P3P4.
G.D. (graphically demonstrated.)
Best regards,
Seiichi.

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Message: #1168
Date: 18/6/2015 1:21:10
From: Benedetto Scimemi
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear Seiichi, you may like to know some history about the quadrangles mentioned in your last 2 messages.
They can be called "the 3 pedal quadrangles" (of the reference quadrangle A) and have been studied by M.Happach in Zeitschrift für Math. und Nat. Unterricht 43 (1912) p.175. Each of them is inversely similar to A; one of these similarities fixes a diagonal point D of A and can be easily factored as follows: a reflexion on the bisector of the two sides of A meeting in D and a D-centered dilatation of factor cos ∂, the angle between those two sides. Then, by the angular properties of the central ponts, one finds that these 3 (negative) similarities all map QA-P4 into QA-P2. Hence the 3 pedal quadrangles are similar to each other and the (positive) similarities between them have QA-P2 as a common fixed point, which therefore plays for the 3 pedal quadranglethe the common role of QA-P4 (this was your last message).
The existence of a common center for the 3 positive similarities was Happach's main discovery (he never mentioned Euler or Poncelet) and this is why I used H to denote QA-P2.
Now consider three circles, passing through the 3 Gergonne-Steiner points (of the 3 pedal quadrangles), the three Euler-Poncelet and the three gravity centers. You will notice that they belong to pencil, their centers being collinear, and the last center is the midpoint of the first two. All of this can be explained if one takes into account that the "circumcenter" of the 3 negative similarities above is just the quadripole transformation QA-Tf3, as was recently described in Chris's EQF.
Incidentally, applying geometric compositions to mapping as one woul do with points (mappings "midpoint" is another interesting composition) provides an intriguing chapter of geometry which - in my opinion - would deserve a more general study.
Best regards
Benedetto

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Message: #1169
Date: 18/6/2015 4:23:51
From: eckart_schmidt@t-online.de
Subject: QL-circle wrt QL-Cu1

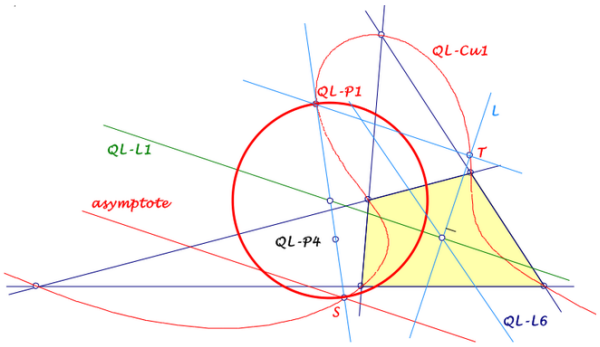
```
Dear Bernard, dear Chris,
attached you find a circle related to QL-Cu1 with diameter
Miquel-point and intersection S of QL-Cu1 and its asymptote.
Perhaps of interest:  CSC-orientated construction of QL-Cu1:
... Let L be the Newton line QL-L1 and Ci its CSC-circle
    and L' an orthogonal line through QL-L1^QL-L6 with
    CSC-circle Ci'.
... If L' cuts Ci' in two points (QL-Cu1 will be bipartite),
    all circles through the two points, centered in X on L,
    intersect X.QL-P1 on QL-Cu1.
... If L cuts Ci in two points (QL-Cu1 will be unipartite),
    all circles through the two points, centered in Y on L',
    intersect a perpendicular line to Y.QL-P1 through S on
    QL-Cu1.
Best regards Eckart
```

EQF-Note 2015-06-18

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Circle wrt QL-Cu1

This circle has as diameter the Miquel-point QL-P1 and the intersection of the cubic QL-Cu1 and its asymptote.



Intersection *S* of *QL-Cu1* and its asymptote

This point is already mentioned in *EQF* wrt *QL-Cu1*. It is the intersection of the asymptote and the tangent for the Miquel-point *QL-P1* wrt the cubic *QL-Cu1*. The asymptote is a parallel to the Newton-line *QL-L1* through the reflection of *QL-P1* in *QL-L1*. The tangent in *QL-P1* is *QL-P1.QL-P4*. Let *T* be the *CSC*-image (*QL-Tf1*) of *S*. *T* is a point on *QL-Cu1* as intersection of a parallel to *QL-L1* through *QL-P1* and a perpendicular line *L* wrt *QL-L1* through the intersection of *QL-L1* and *QL-L6*.

The new circle *Ci*

Let *Ci* be the circle with diameter *S.QL-P1*:

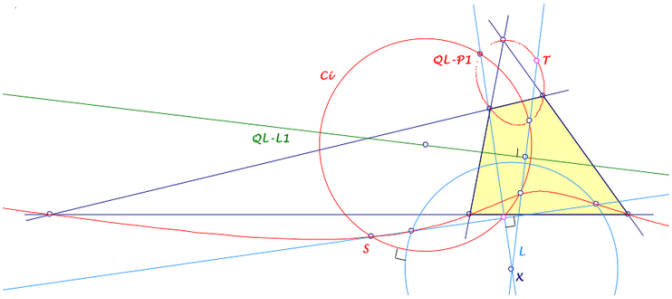
- *Ci* is the *CSC*-image of a line *L* orthogonal *QL-L1*:
  $L = T.QL\text{-}L1 \cap QL\text{-}L6$.
- Center of *Ci* is *QL-P1.QL-P4* ∩ *QL-L1*, which is the *CSC*-image of the reflection of *QL-P1* in *L*.
- The angle bisectors at *S* wrt two *CSC*-partners on *QL-Cu1* intersect *QL-L1* in two points of *Ci*.

- ***Ci* is the locus for the midpoints of the intersections of *QL-Cu1* and lines through *S*.**

- *Ci* intersects *L* on *QL-Cu1*, if *QL-Cu1* is bipartite.

- *Ci* is the locus for the reflections of *QL-P1* in circles through *QL-2P2*, if *QL-Cu1* is unipartite.

- The circle *Ci* can be used for a simple construction of *QL-Cu1*:
  Circles centered in *X* on *L* and orthogonal wrt *Ci* intersect lines through *S* orthogonal to *X.QL-P1* in points of the cubic *QL-Cu1*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #1170
Date: 18/6/2015 6:09:09
From: chris.vantienhoven
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear Seiichi, dear Benedetto,
Quite interesting this construction of 3 pedal quadrangles similar with the reference quadrangle.
I have not many additions.
The circle through R1, R2, R3, R4 and Q4 is the Thales Circle of Q4.R4 (in EQF: QL-P2.QL-P3) with center QA-P1.
The circle through Q1, Q2, Q3, Q4 is the Circle with center QA-P23 through QA-P2.
These circles share the point QA-P2 and have a second intersection point lying on QA-P3.QA-P4. It is also the reflection of QA-P2 in QA-P1.QA-P23.
Both intersection points are also the common points of the pencil of circles described by Benedetto.
Further I didn't find incidences.
Best regards,
Chris

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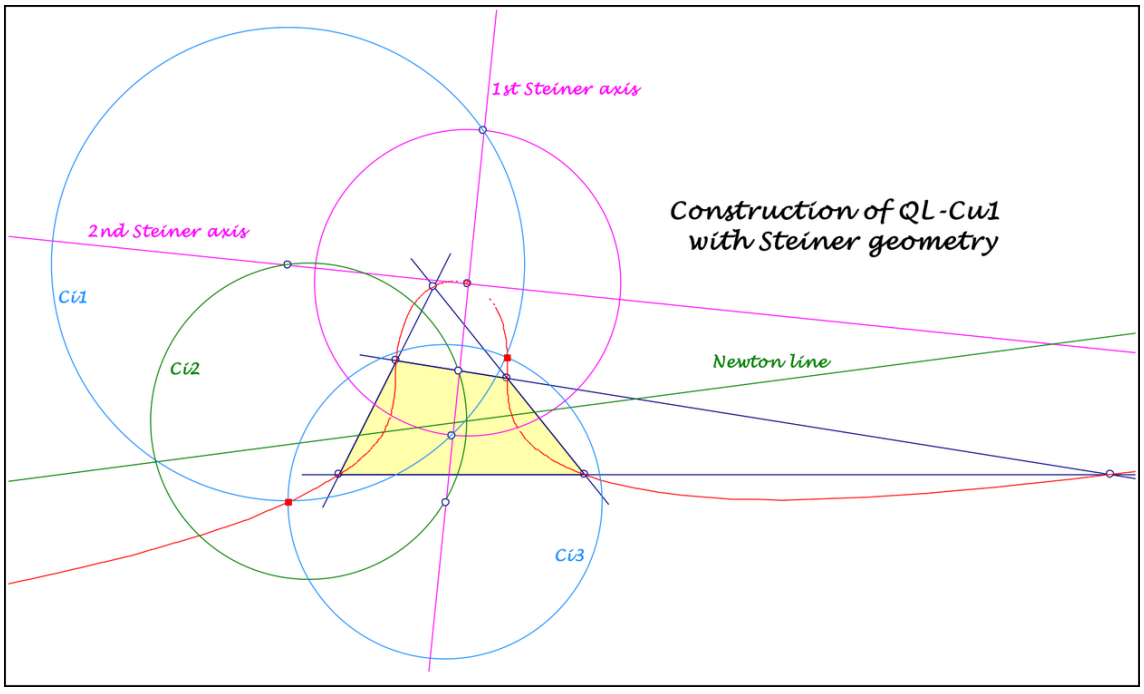
Message: #1171
Date: 19/6/2015 11:54:54
From: eckart_schmidt@t-online.de
Subject: Further simple construction of QL-Cu1

Dear Bernard, dear Chris,
studying Bernard's generalisation of the Steiner circles, I found a simple construction for QL-Cu1 with Steiner geometry:
... Let Ci1 be a circle through QL-2P3
... and Ci2 the inverse of the Newton line wrt Ci1.
... Circles Ci3, centered in an intersection of Ci2
    with the 1st Steiner axis,
    intersect Ci1 orthogonal in two CSC-partners on QL-Cu1.
Ci1 and Ci3 are generalized Steiner circles (see Bernard's #840).
Best regards Eckart



2015-06-19.pdf

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Message: #1172
Date: 19/6/2015 4:32:13
From: seiichikiri
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear Benedetto, dear Chris,
Thank you very much for your kind information.
I had known that in a cyclic quadrangle, its 3 pedal quadrangles are cyclic, have a common center and their 3 anticenters are colinear.
The center in a cyclic quadrangle corresponds to Gergonne-Steiner point or isogonal center. The anticenter corresponds to Euler-Poncelet point.
I could extend the properties of a cyclic quadrangle to those of a general one.
By the way, 3 years have passed since the appearance of EQF. Congratulations!!!
I would like to hear Chris's opinion about where Quadrifigure (or higher) Geometry will go since now.
Best regards, Seiichi.

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Message: #1173
Date: 19/6/2015 6:51:21
From: chris.vantienhoven
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear Seiichi,
Thanks for your congratulations!
Since we started with EQF 3 years ago the information more than doubled.
Not only by Eckart, you and me (who started the discussions), but also thanks to contributions of Bernard Keizer, Bernard Gibert, Angel Montesdeoca, Randy Hutson, Antreas Hatzipolakis, Benedetto Scimemi, Francisco Javier García Capitán and possibly even more persons I forgot.
With quadrilateral geometry we generalized the triangle geometry.
It is a small step now to generalize quadrilateral geometry to polygon geometry (n-lines, n-points, n-gons). We already started that in our discussions of last year. I already started with a script for an Encyclopedia of Polygon-Figures (about 40 pages now, but there is much more). Very exciting.
Again unknown territory waiting to be explored!
I am glad we can do this together. We all have different angles of view and it all helps to discover and unfold all these wonderful new parts of geometry!
Have a good weekend.
Best regards,
Chris

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Message: #1174
Date: 20/6/2015 1:12:30
From: seiichikiri
Subject: Concyclicities of 4 QA-P2 and 4 QA-P3 points

Dear Chris, dear friends,
I would like to add to your message:
Quadri-Figures Geometry is the base camp of Polygon-Figures Geometry.
Best regards,
Seiichi.

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Message: #1175
Date: 20/6/2015 6:35:28
From: eckart_schmidt@t-online.de
Subject: QL-Tf2 and QL-Cu1

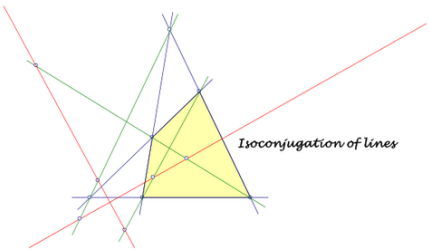
Dear Chris,
The isoconjugation for lines QL-Tf2 has rarely feedback in EQF, but there are interesting properties up to cubics as QL-Cu1. So you find attached a short summary of my paper 12.3 on my homepage. Perhaps of interest.
Best regards Eckart

EQF-Note 2015-06-20

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Tf2 and QL-Cu1

The isoconjugation for lines QL-Tf2 has rarely feedback in EQF, but there are interesting properties up to cubics as QL-Cu1. Here is a short summary of my paper [1], which gives barycentric calculations for the following results.



The Involutary Conjugate *QA-Tf2* is an isoconjugation for points wrt the *QA*-Diagonal Triangle *QA-Tr1*. The *QL*-Line Isoconjugate *QL-Tf2* is its analogon for lines wrt the *QL*-Diagonal triangle *QL-Tr1*. The *QL-Tf2*-image of a line is the locus of its poles wrt the inscribed conics of the quadrilateral.

But here we start with a simple construction for *QL-Tf2*, based on a theorem of Chasles [2]:

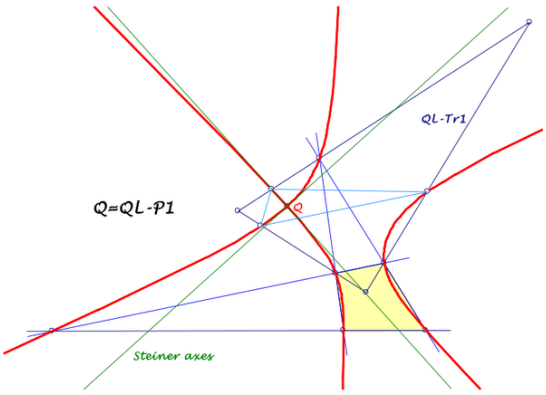
- **Let a line intersect the diagonals of a quadrilateral, then the 4th harmonic points on the diagonals are collinear.**

This line transformation *QL-Tf2* has the lines of the quadrilateral as fixed lines. The *QL-Tf2*-image of the Newton line is the line at infinity. The *QL-Tf2*-image of *QL-L2* is a parallel to *QL-P3.P4.P5.P6* through *QL-P1*. The Steiner axes are *QL-Tf2*-partners.

- **The *QL-Tf2*-images of lines through a fixed point envelope an inscribed conic of the *QL*-Diagonal Triangle *QL-Tr1*.**

For *QL-P13* we get the inscribed Steiner ellipse of *QL-Tr1*. For points on the Newton line these conics are inscribed parabolas of *QL-Tr1*. For the Miquel point *QL-P1* we get a special inscribed conic of *QL-Tr1*, which contacts the Steiner axes and the line *QL-L2* (see *QFG*-message 481).

- **The intersections of lines through a fixed point *Q* and their *QL-Tf2*-image give a cubic through …**
  … the six vertices of the quadrilateral,
  … the vertices of the Ceva triangle of *Q* wrt *QL-Tr1*.



Example: Let *Q* be the Miquel point *QL-P1*, which will be a knot of the cubic with the Steiner axes as tangents. This cubic is invariant wrt an isoconjugation with fixed point *QL-P1* and its Ceva triangle wrt *QL-Tr1* as reference triangle.

If *Q* is a *CSC*-fixed point *QL-2P3*, the cubic contains *QL-P1* and is *CSC*-invariant.

Of special interest are points, which have in their line pencil an orthogonal pair of *QL-Tf2*-partners. The locus for these points will be *QL-Cu1*.

- ***QL-Cu1* is the locus for points, whose angle bisectors wrt two opposite vertices are *QL-Tf2*-partners.**

Finally:

- **The intersections for *QL-Tf2*-images of perpendicular lines of a pencil are collinear.**

Example: For the line pencil of *QL-P1* we get a parallel to *QL-P3.P4.P5.P6* through *QL-P1*.

References:

[1] http://eckartschmidt.de/Isofl.pdf
[2] M. Chasles, Ann. de math. 18 (1827-1828), p. 297.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-06-20.pdf

Message: #1176
Date: 21/6/2015 10:23:57
From: Antreas Hatzipolakis
Subject: 4 points problem

I am FWDing a message on quadrangle posted to Anopolis †) by Quang Dương.
APH
[Quang Dương]:
Given 4 points A,B,C,D, any three of them aren't collinear.
A line d intersects AB,CD,AC,DB,AD,BC at
M_AB,M_CD,M_AC,M_DB,M_AD,M_BC
H_AB,H_CD,H_AC,H_DB,H_AD,H_BC is orthogonal projections of M_CD, M_AB,M_DB,M_AC,M_BC,M_AD on AB,CD,AC,DB,AD,BC
Then H_AB H_CD, H_AC H_DB, H_AD H_BC are concurrent.

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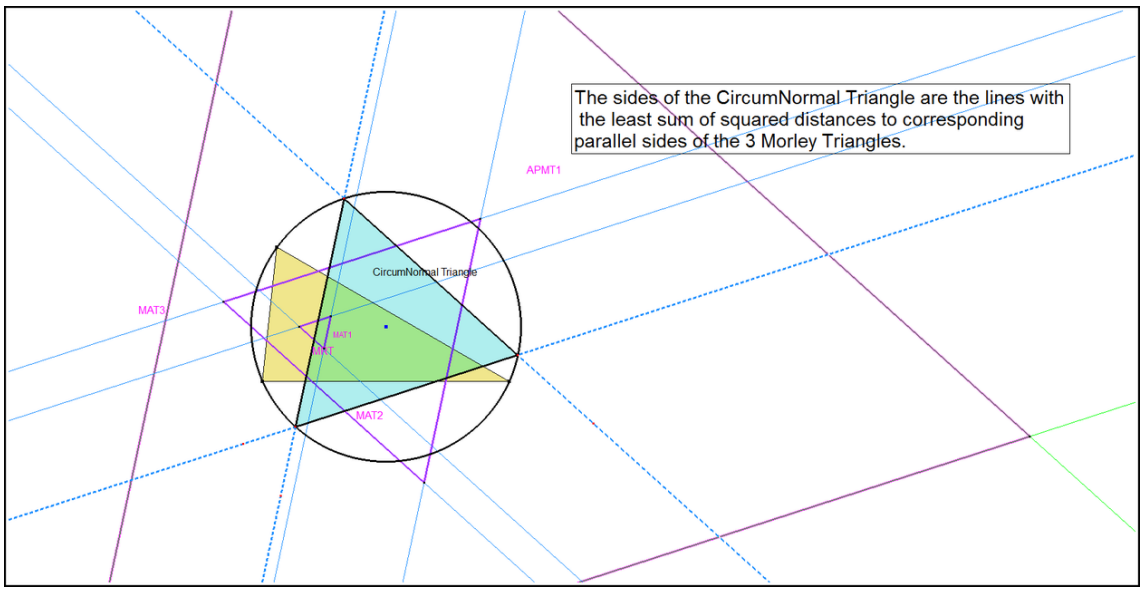
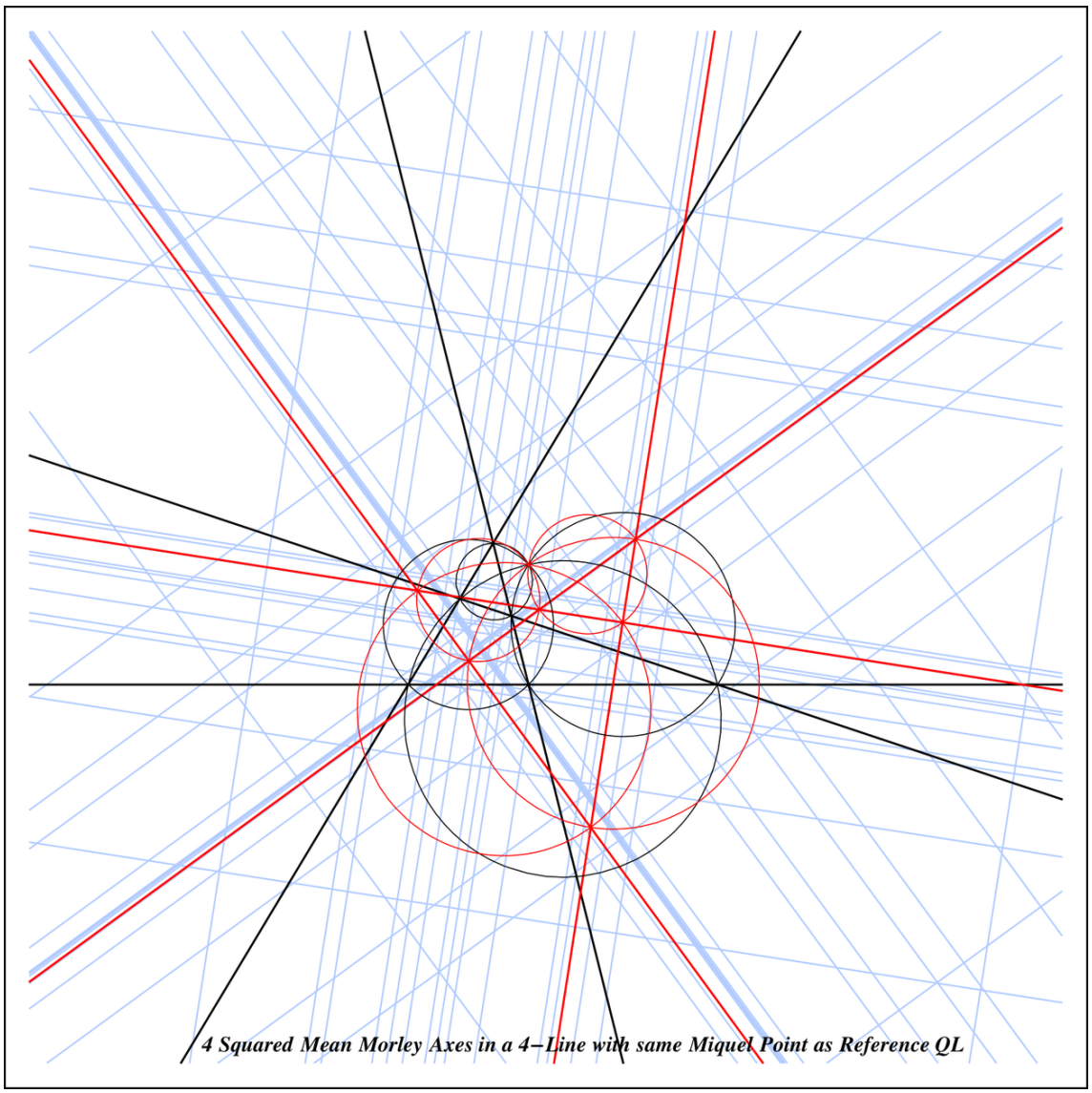
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #1177
Date: 21/6/2015 11:16:11
From: chris.vantienhoven
Subject: 64 axes of a QL

Dear Eckart, Dear Bernard,
We investigated a lot about the 64 Morley Axes in a Quadrilateral.
Since there are 4 sets of 16 Axes with the same direction I wondered how the 4 Mean Axes of these sets would look like.
I defined a mean Axis of parallel lines as the axis with the least sum of squared distances to the 16 parallel axes.
Attached you will find a picture of this constellation.
The black figures form the reference constellation.
The red figures form the Mean-Axes constellation.
It appears that they share the same Miquel Point (Numerical Determination).
We can do the same in a triangle.
In a triangle we have 3x3 Morley Axes forming the 3 Morley Triangles.
It appears that the mean Morley Axes is forming an equilateral triangle inscribed in the circumcircle of the triangle.
It actually is the CircumNormal Triangle (Graphical Determination).
See http://mathworld.wolfram.com/CircumnormalTriangle.html
Needless to say that they share the same circumcenter.
I haven’t checked it in an n-Line but I wouldn’t be surprised when the reference n-Line and the corresponding mean-Axes n-Line would be sharing either the nL-Miquel Point (Centric Focus) or the nL-Center of the Center Circle (Centric Center).
Best regards,
Chris



Morley-CircumNormal Triangle-02.png



4L-64 Morley-Axes-71-MeanAxes.pdf

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Message: #1178
Date: 21/6/2015 3:24:59
From: eckart_schmidt@t-online.de
Subject: 4 points problem

Dear Antreas,
your message 1176 gives a line to point transformation, here first Cabri observations:
... All lines through QA-P4 give QA-P2,
... a line pencil gives a circle through QA-P2,
... the line pencil of QA-P1 gives a circle through QA-P2, centered in the midpoint of QA-P1.QA-P23,
... the line pencil of QA-P2 gives a circle through QA-P2, centered in QA-P23.
Best regards Eckart

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Message: #1179
Date: 21/6/2015 4:14:57
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

Dear Chris,
wrt your message 1177:
If you take the following 4 mean lines, they will have also the same Miquel point:
Let a line intersect the 16 parallel Morley axes of a set and consider a further parallel through the centroid of the 16 intersections.
Best regards Eckart

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Message: #1180
Date: 21/6/2015 7:00:25
From: eckart_schmidt@t-online.de
Subject: 64 axes of a QL

```
Dear Chris,
please forget my last message.
Excuse, that are the same mean lines.
Best regards Eckart
```

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Message: #1181
Date: 21/6/2015 8:31:13
From: chris.vantienhoven
Subject: 4 points problem

Dear Quang Duong, Antreas and Eckart,

A surprising transformation this is. I didn't know it.
Maybe someone else has a reference?
The transformation makes me think of the conic perspector.
Lines are connected between points on opposite Quadrangle sides and then these lines concur.
See: QA-Co-1
This I found about this special transformation (algebraically determined):
* QA-L1 (line through QA-P1, QA-P2) is transformed into an unknown point on QA-P3.QA-P4.
* QA-L2 (line through QA-P2, QA-P4, QA-P6) is transformed into QA-P2.
* QA-L4 (line through QA-P1, QA-P6, QA-P23)
is transformed into an unknown point on the line through QA-P23 parallel to QA-P1.QA-P2.
* QA-L5 (Euler line of the Diagonal Triangle) is transformed into an unknown point on the circumcircle of the Diagonal triangle.
* QA-L9 (perpendicular bisector of QA-P2.QA-P4) is transformed into QA-P23.
When A=(1:0:0), B=(0:1:0), C=(0:0:1), D=(p:q:r), Line=(x:y:z), then the first coordinate of the transformed point is:

```
- a^2 (a^2 p q - b^2 p q + c^2 p q + a^2 p r + b^2 p r - c^2 p r
+ 2 a^2 q r) x^2
- b^2 (a^2 - b^2 + c^2 ) p q y^2
- c^2 (a^2 + b^2 - c^2 ) p r z^2
- (a^2 c^2 q + b^2 c^2 q - c^4 q + a^2 b^2 r - b^4 r + b^2 c^2
r) p y z
+ (a^4 p q - b^4 p q + 2 b^2 c^2 p q - c^4 p q + 2 a^2 b^2 p r +
a^4 q r + a^2 b^2 q r - a^2 c^2 q r) x y
+ (2 a^2 c^2 p q + a^4 p r - b^4 p r + 2 b^2 c^2 p r - c^4 p r +
a^4 q r - a^2 b^2 q r + a^2 c^2 q r) x z
```

Best regards,
Chris van Tienhoven

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Message: #1182
Date: 21/6/2015 10:53:46
From: Antreas Hatzipolakis
Subject: 4 points problem

After Quang Duong's problem I was trying for possible variations.
This is a problem I am wondering if is it already known.
Let P1P2P3P4 be a quadrangle and P a point
Denote:
P12, P34, P23, P41, P13, P24 same points Q's of the triangles PP1P2, PP3P4, PP2P3, PP4P1, PP1P3, PP2PP4, resp.
(ie all Q's circumcenters or NPC centers or orthocenters etc of the triangles).
Let X,Y,Z be the midpoints of P12P34, P23P41, P13P24.
For which Q's the X,Y,Z are collinear?
For Q's = centroids, circumcenters, NPC centers, they are.
How about other points?
APH

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Message: #1183
Date: 21/6/2015 11:01:44
From: seiichikiri
Subject: Concyclicity of QA-P3 and 3 QG-P15 points

Dear friends,
Let P1P2P3P4 be a quadrangle.
Its Gergonne Steiner point (QA-P3) and 3 QG-P15 points are concyclic.
G.D. (graphically determined.)
Best regards, Seiichi.

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Message: #1184
Date: 22/6/2015 1:06:23
From: chris.vantienhoven
Subject: QL-Tf2 and QL-Cu1

```
Dear Eckart,
Quite a mix of beautiful properties of QL-Tf2 you gave!
You are right QL-Tf2 was undervalued for a long time.
After all a quadrilateral is composed of lines and the main
objects to be acted upon should be lines.
I enclosed your properties
in EQF.
Best regards,
Chris
```

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Message: #1185
Date: 22/6/2015 5:00:52
From: Benedetto Scimemi
Subject: 4 points problem

```
Dear friends,
as for the Quang Duong's transformation from a(ny) line r into a
point R,
here are 2 equivalent fast constructions of R from r and the
central points:
1) Let: O = QA-P4 (isoptic), p = QA-Tf3 (orthopole tranform)
Let O^r  be the reflection of O on r. Then R = p(O^r), the image
of O^r under p.
2) Let H = Qa-P2 (Euler-Poncelet). Let p(r) be the image of  r
under the orth. transf.
Then R = H^p(r), the reflection of H on p(r).
This should explain many of your observations.
Best regards.
Benedetto
```

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Message: #1186
Date: 23/6/2015 12:06:19
From: chris.vantienhoven
Subject: 4 points problem

```
Dear friends,
Another observation about
http://mathworld.wolfram.com/OrthopolarLine.html.
```

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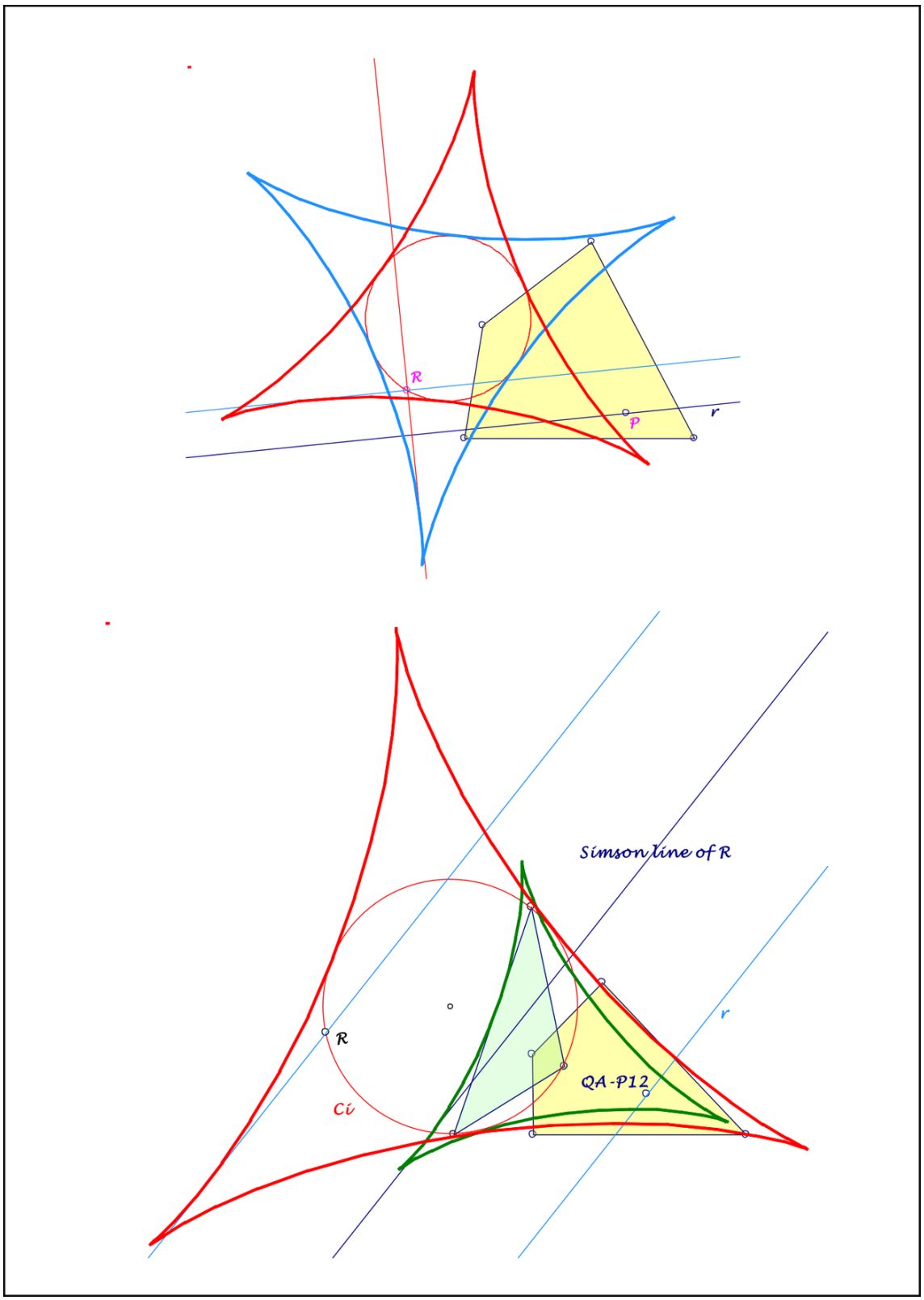
Message: #1187
Date: 23/6/2015 9:02:15
From: eckart_schmidt@t-online.de
Subject: 4 points problem

```
Dear friends,
a further property of the Quang Duong transformation (QDT):
Let r be a variable line through a fixed point P,
... R its image wrt the QDT-transformation,
... r1 a perpendicular line wrt r through R,
    r2 a parallel wrt r through R,
... then the locus of R is a circle Ci through QL-P2,
... the envelopes of r1 and r2 are deltoids
    with the incircle Ci.
... For P=QA-P4 the constellation degenerates in point QL-P2.
Example: Let P be the orthocenter QA-P12 of the QA-diagonal
triangle,
... then Ci is  the circumcircle
    of the QA-diagonal triangle QA-Ci1
... and the deltoid of r2 is homothetic to the deltoid
    for the Simson lines of the QA-diagonal triangle
    (wrt QA-P12 and factor 2).
Best regards Eckart
```



2015-06-23.pdf

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Message: #1188
Date: 23/6/2015 9:34:55
From: Antreas Hatzipolakis
Subject: 4 points problem

[Quang Dương]:
> > Given 4 points A,B,C,D, any three of them aren't collinear.
> > A line d intersects AB,CD,AC,DB,AD,BC at
M_AB,M_CD,M_AC,M_DB,M_AD,M_BC
> > H_AB,H_CD,H_AC,H_DB,H_AD,H_BC is orthogonal projections of
M_CD,
> M_AB,M_DB,M_AC,M_BC,M_AD on AB,CD,AC,DB,AD,BC
> > Then H_AB H_CD, H_AC H_DB, H_AD H_BC are concurrent.
I have found 5 pencils of circles
(XHabHcd),(XHacHdb),(XHadHbc)are coaxial, X=A,B,C,D.
Let A',B',C',D' be the second intersections of these pencils of
circles.
HabHcd,HacHdb,HadHbc are concurrent at a point R, then
(RAA'),(RBB'),(RCC'),(RDD') are coaxial.
Quang Dương

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Message: #1189
Date: 23/6/2015 11:53:44
From: Antreas Hatzipolakis
Subject: Just a reference....

BOYAN ZLATANOV: AN ETUDE ON ONE SHARYGIN'S PROBLEM
Global Journal of Advanced Research on Classical and Modern
Geometries
ISSN: 2284-5569, Vol.3, (2014), Issue 2, pp.50-61
Abstract:
By the methods of the synthetic geometry we investigate
properties of objects generated from a complete quadrangle and a
line, which lies in its plane. We start with a problem from the
book of Sharygin "Problems in Plane Geometry".
We generalize this problem with the help of Pappus, Desargues
and Pascal's Theorems and we discover new concurrent lines,
collinear points, and conic sections
http://gjarcmg.geometry-math-journal.ro/index/
Also from Boyan Zlatanov Page (
http://fmi-plovdiv.org/index.jsp?id=1035&ln=1)
http://fmi-plovdiv.org/GetResource?id=1784
APH

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Message: #1190
Date: 23/6/2015 3:34:13
From: chris.vantienhoven
Subject: 4 points problem

[QD] I have found 5 pencils of circles
(XHabHcd),(XHacHdb),(XHadHbc)are coaxial, X=A,B,C,D. Let
A',B',C',D' be the second intersections of these pencils of
circles.HabHcd,HacHdb,HadHbc are concurrent at a point R, then
(RAA'),(RBB'),(RCC'),(RDD') are coaxial
Dear Quang Dương,
Nice extension to your transformation.
Here two extra items:
1. ABCD is perspective with A'B'C'D' at a perspector P.
2. And then R, R' and P are collinear.
Best regards,
Chris van Tienhoven

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Message: #1191
Date: 24/6/2015 8:46:46
From: eckart_schmidt@t-online.de
Subject: 4 points problem

```
Dear friends,
some further properties of the Quang-Duong-Transformation (QDT):
... a perpendicular line to QA-L2 through QA-P6 gives QA-P23,
... a perpendicular line to QA-L4 through QA-P3 gives QA-P3,
... lines of a pencil give a circle through QA-P2,
... parallel lines give a line through QA-P2,
... perpendiculars of QA-L4 give QA-L1,
... perpendiculars of QA-L1 give a parallel to QA-L4
    through QA-P2,
... perpendiculars of QA-L2 give QA-P2.QA-P23,
... perpendiculars of QA-P2.QA-P23 give QL-L2,
... perpendiculars of a line L give a line L1 and perpendiculars
    of L1 give a parallel to L through QA-P2 (examples with
    QA/3).
... Tangents to circles round QA-P4 give circles round QA-P2,
... The intersections of the two circles give another circle
    through QA-P3 centered in QA-P2.QA-P4 ^ QA-P3.QA-P32.
... The locus for the QDT-images of the radical axes
    of the two circles is QA-P2.QA-P23.
... Tangents to parabolas with focus QA-P4 give a line,
    which is the perpendicular bisector of QA-P2
    and the QDT-image of the directrix.
... Tangents to a conic with focus QA-P4 give a circle,
    centered in the QDT-image of the secondary axis of the
    conic.
Best regards Eckart
```

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Message: #1192
Date: 24/6/2015 9:28:38
From: Antreas Hatzipolakis
Subject: CONJECTURE (was: 4 points problem)

```
[APH]:
CONJECTURE.
Let ABCD be a quadrangle, P a point and Q a point on the Euler
line of ABC.
Denote:
Qab = the Q-point of PAB
Qcd = the Q-point of PCD
Qbc = the Q-point of PBC
Qad = the Q-point of PAD
Qac = the Q-point of PAC
Qbd = the Q-point of PBD
ie Q, Qab, Qcd, etc are same points on the Euler lines of ABC,
PAB, PCD, etc
(all are orthocenters or circumcenters or centroids etc)
X = the midpoint of QabQcd
Y = the midpoint of QbcQad
Z = the midpoint of QacQbd
X,Y, Z are collinear for all P's (on fhe plane) and Q's (on the
Euler line).
True?
APH
**************************************************
Assuming my conjecture is true, there are interesting envelope
problems!
Rephrasing it:
Let ABCD be a quadrangle and P a point.
Denote:
Ouv, Huv = Circumcenter, Orthocenter of PUV, resp., where U,V e
{A,B,C,D}, U=/=V
Quv = Point on the Euler line OuvHuv of PUV such that QuvOuv /
QuvHuv = t.
X = the midpoint of QabQcd
Y= the midpoint of QbcQad
Z = the midpoint of QacQbd
X,Y,Z are collinear (conjecture).
1. If P is fixed point,
   which is the envelope of the line XYZ as t varies?
2. If t is fixed number,
   which is the envelope of XYZ
   as P moves on a line, a circle, a conic ..... ?
APH
```

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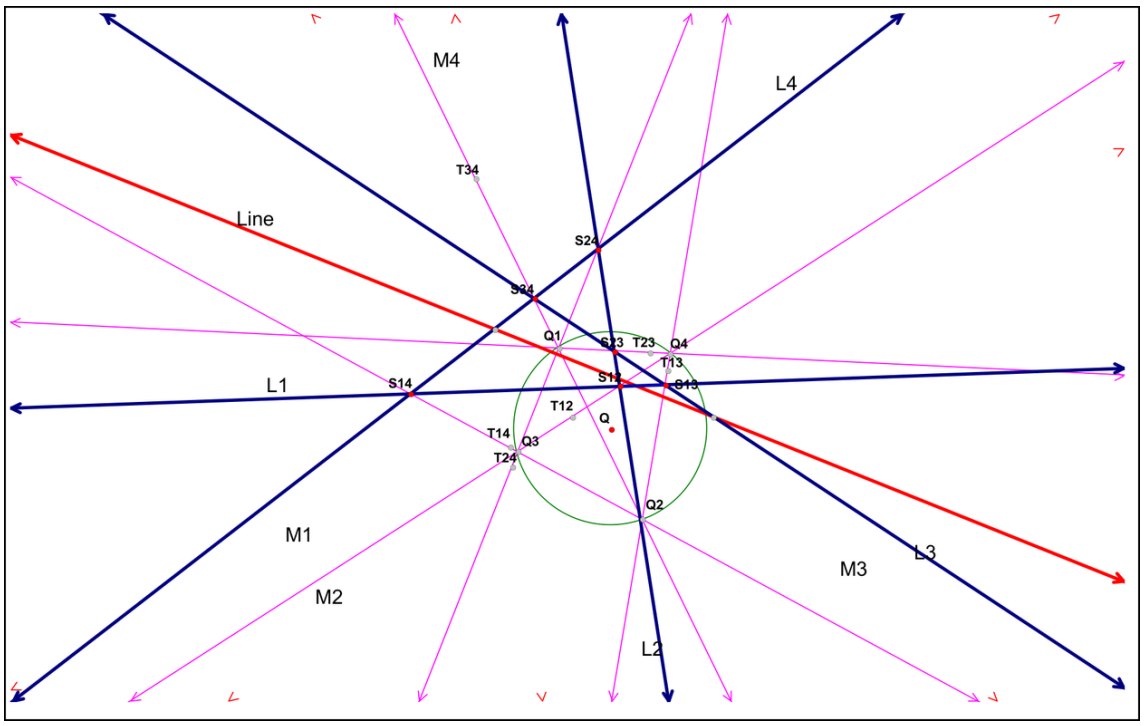
Message: #1193
Date: 25/6/2015 1:49:52
From: seiichikiri
Subject: A line transformation- reflection

Dear friends,
Let L1L2L3L4 be a quadrilateral and L a line.
Denote:
Sij= the intersection of Li and Lj.
Mi= the reflection of L in Li.
Tij= the intersection of Mi and Mj.
S23T23, S24T24, S34T34 concur in Q1.
S13T13, S14T14, S34T34 concur in Q2.
S24T24, S14T14, S12T12 concur in Q3.
S13T13, S23T23, S12T12 concur in Q4.
Q1, Q2, Q3 and Q4 are concyclic.
See the attached picture. G.D. (graphically demonstrated.)
Best regards, Seiichi.



LineTransformationReflection.docx

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Message: #1194
Date: 25/6/2015 9:51:02
From: eckart_schmidt@t-online.de
Subject: A line transformation- reflection

```
Dear Seiichi,
first observations with CABRI:
... The image-circle of a line contains the Miquel point QL-P1.
... Parallel lines give the same circle.
... For lines perpendicular to the Newton line the image-circle
degenerates in QL-P1.
... Lines of a pencil give always circles, which envelope the
cardioid QL-Qu1.
Best regards Eckart
```

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Message: #1195
Date: 25/6/2015 10:46:43
From: eckart_schmidt@t-online.de
Subject: A line transformation- reflection

```
Dear Seiichi,
as consequence of my last message:
... The image-circles are centered on QL-Ci3 and contain QL-P1.
... CSC of the image-circle
    of a line L is a perpendicular line L'.
... Wrt a line pencil these perpendiculars
    envelope the inscribed parabola QL-Co1.
Best regards Eckart
```

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Message: #1196
Date: 25/6/2015 12:01:11
From: seiichikiri
Subject: A line transformation- reflection

Dear Eckart,
I also obtained that "The image-circles are centered on QL-Ci3 and contain QL-P1."
This transformation is a rather simple one, I think.
Thank you very much.
Best regards, Seiichi.

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Message: #1197
Date: 26/6/2015 3:03:09
From: seiichikiri
Subject: A line transformation- reflection

Dear friends,
Addition:
If L is Newton line, its image-circle touches QL-Ci3 at QL-P1 and has twice the diameter of QL-Ci3.
Best regards,
Seiichi.

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Message: #1198
Date: 26/6/2015 6:20:39
From: Ngo Quang Duong
Subject: Về: 4 points problem

Another property: If d changes such that always passes a fix point then the concurrency point R lies on a fix circle.
I also have generalized my problem:
Cause the circles with diameter MabMcd, MacMdb,MadMbc are coaxial,
If (Oa) passes through MbcMad, (Ob) passes through McaMdb then we can construct (Oc) passes through MabMdc such that Oa, Ob, Oc are collinear. Furthermore, (Oa), (Ob), (Oc) are coaxial.
(Oa) cuts BC,AD again at Nbc, Nad
(Ob) cuts CA, BD again at Nca, Ndb
(Oc) cuts AB, CD again at Nab, Ncd
ThenNbcNad, NcaNdb, NabNcd are concurrent.
Some properties that we have found are preserved
[Quang DÆ°Æ¡ng]
Và o ngà y 1:46 Thá»© NÄƒm, 25 tháng 6 2015, "'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de [Quadri-Figures-Group]" Ä‘ã viáº¿t:
Dear friends,
some further properties of the Quang-Duong-Transformation (QDT):
... a perpendicular line to QA-L2 through QA-P6 gives QA-P23,
... a perpendicular line to QA-L4 through QA-P3 gives QA-P3,
... lines of a pencil give a circle through QA-P2,
... parallel lines give a line through QA-P2,
... perpendiculars of QA-L4 give QA-L1,
... perpendiculars of QA-L1 give a parallel to QA-L4 through QA-P2,
... perpendiculars of QA-L2 give QA-P2.QA-P23,
... perpendiculars of QA-P2.QA-P23 give QL-L2,
... perpendiculars of a line L give a line L1 and perpendiculars of L1 give a parallel to L through QA-P2 (examples with QA/3).
... Tangents to circles round QA-P4 give circles round QA-P2,
... The intersections of the two circles give another circle through QA-P3 centered in QA-P2.QA-P4 ^ QA-P3.QA-P32.
... The locus for the QDT-images of the radical axes of the two circles is QA-P2.QA-P23.
... Tangents to parabolas with focus QA-P4 give a line, which is the perpendicular bisector of QA-P2 and the QDT-image of the directrix.
... Tangents to a conic with focus QA-P4 give a circle, centered in the QDT-image of the secondary axis of the conic.
Best regards Eckart

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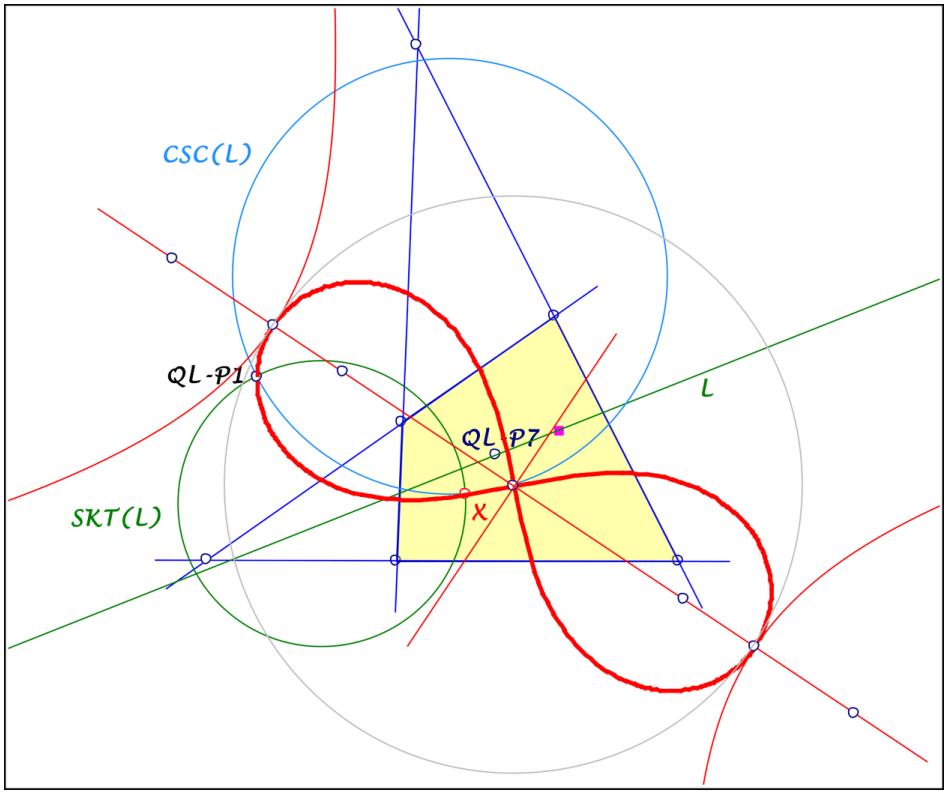
Message: #1199
Date: 26/6/2015 8:46:14
From: eckart_schmidt@t-online.de
Subject: A line transformation- reflection

Dear Seiichi,
your line transformation (SKT) leads to a lemniscate as follows:
Let L be a line of the pencil of the Newton-Steiner Point QL-P7.
... The 2nd intersections X of the circles CSC(L) and SKT(L)
    give a lemniscate (1st intersection is QL-P1).
... Center of the lemniscate is the center of SKT(QL-L5)
    or CSC(QL-P7) on QL-Ci3.
... Construction of the axes: A circle centered on QL-L2
    through QL-P1 and QL-P7 intersects the perpendicular
    bisector
    of QL-P1.QL-P7 in the midpoints of two circles
    through QL-P1 and QL-P7, whose CSC-images are the axes.
... Perpendiculars to X.QL-P7 through X
    envelope an orthogonal hyperbola inverse to the lemniscate.
... The vertices of the hyperbola
    are the vertices of the lemniscate.
... The foci of the hyperbola and of the lemniscate lie inverse
    wrt the Thales circle about the vertices.
Best regards Eckart



2015-06-26.pdf

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Message: #1200
Date: 26/6/2015 9:24:34
From: eckart_schmidt@t-online.de
Subject: A line transformation- reflection

```
Dear Seiichi,
excuse, I forgot the generalisation:
Not only QL-P7, but all points P on QL-L2 lead to a lemniscate,
centered in CSC(P) on QL-Ci3.
Best regards Eckart
```

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Message: #1201
Date: 27/6/2015 5:42:43
From: seiichikiri
Subject: A line transformation- reflection

```
Dear Eckart,
Thank you very much for your deduction.
For the first time, cardioid has become a natural object for me.
CSC transformation is effective enough to produce lemniscate.
I am glad of both!
Best regards,
Seiichi.
```

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Message: #1202
Date: 27/6/2015 10:44:04
From: eckart_schmidt@t-online.de
Subject: Conjecture

```
Dear Antreas,
wrt  #1192: Your conjecture is true, but I have found no
relations to EQF-objects (Chris will have better methods).
One observation wrt 1.: If P is a fixed point, the lines XYZ for
variable t are parallels.
Best regards Eckart
```

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Message: #1203
Date: 30/6/2015 3:19:25
From: seiichikiri
Subject: A property of Euler Poncelet point (QA-P2)

Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
O1= the circumcenter of P2P3P4.
O2= the circumcenter of P1P3P4.
O3= the circumcenter of P1P2P4.
O4= the circumcenter of P1P2P3.
O= isogonal center (QA-P4).
Gr1=Griffith point of P2P3P4 wrt OO1.
Gr2=Griffith point of P1P3P4 wrt OO2.
Gr3=Griffith point of P1P2P4 wrt OO3.
Gr4=Griffith point ofP1 P2P3 wrt OO4.
Gr1, Gr2, Gr3 and Gr4 coincide with Euler Poncelet point (QA-P2).
G.D. (graphically demonstrated.)
Griffith point: see
http://mathworld.wolfram.com/GriffithsTheorem.html
Best regards, Seiichi.

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Message: #1204
Date: 30/6/2015 5:59:09
From: seiichikiri
Subject: A property of Euler Poncelet point (QA-P2)

Dear friends,
Addition:
If we replace O by an arbitrary point P, then Grifiith points (Gri, i=1, 2, 3, 4) and Euler Poncelet point are concyclic.
G.D. (graphically demonstrated.)
Best regards, Seiichi.

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Message: #1205
Date: 30/6/2015 3:05:58
From: eckart_schmidt@t-online.de
Subject: Line-circle transformation for a QL

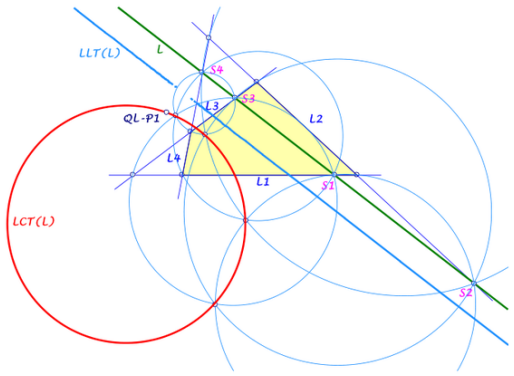
Dear friends,
in the last time several transformations were discussed.
Attached my input.
Best regards Eckart

EQF-Note 2015-06-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line-Circle Transformation for a QL

Starting with a Line-Circle Transformation (LCT), which gives circles, containing the Miquel Point QL-P1, and using the CSC-transformation QL-Tf1, there is a Line-Line Transformation (LLT) with relevant properties for EQF-objects. – The results are only CABRI-controlled.



Let L_1, L_2, L_3, L_4 be the lines of a quadrilateral and S_{ij} their intersections. Let L be a line and S_i its intersections with the lines L_i of the quadrilateral.

- **The circumcircles of S_i, S_j, S_{ij} have four concyclic triple intersections.**

Beside the already known transformation *QL-Tf1 = CSC* this property leads to another line-circle transformation *LCT* with the following properties:

- **The circle *LCT(L)* contains the Miquel Point *QL-P1*.**
- **The circle *LCT(L)* contacts the circle *CSC(L)* in *QL-P1*.**
- **For tangents *L* at the inscribed parabola *QL-Co1* the circle *LCT(L)* degenerates in *QL-P1*.**
- **The center of the circle *LCT(L)* is collinear with *QL-P1* and the *CSC*-image of the reflection of *QL-P1* in *L*.**

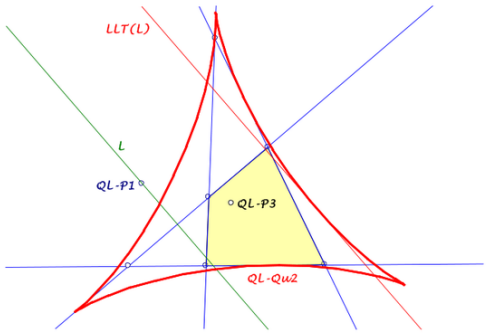
The circle *LCT(L)* has the point *QL-P1*, so its *CSC*-image is a line and we get a line-line transformation *LLT(L)* = *CSC(LCT(L))*.

- **The line *LLT(L)* is a parallel to *L*.**
- **The line-line transformation *LLT* is involuntary.**
- **The Newton Line *QL-L1* is a fixed line of *LLT*.**
- **For parallels *L* to *QL-L1* the *LLT*-image is the reflection of *L* in *QL-L1*.**
- ***LLT(QL-L9)* is a parallel through *QL-P13*.**
- **For *L* = *LLT(T)* − *T* line through *QL-P1* – the circle *LCT(L)* degenerates to a line tangent in *QL-P1* to the circle *LCT(T)*.**
- **For lines *L* parallel to a line L_i of the quadrilateral holds *LLT(L)* = L_i and *LCT(L)* is the circumcircle of the remaining triangle component.**
- **The asymptotes of inscribed hyperbolas of the quadrilateral are fixed lines of *LLT*.**
- **For a line *L* with *LCT(L)* = *Ci* holds *LLT(CSC(Ci))* = *L*.**

Now we consider lines *L* of a pencil with common point *QL-P1*:

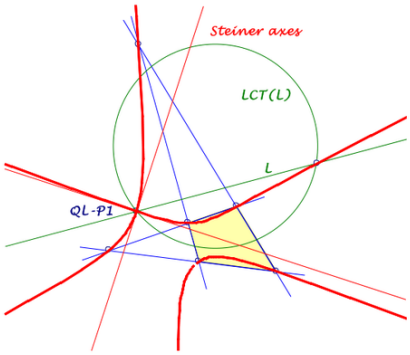
- **For lines *L* of the pencil of the Miquel Point *QL-P1* the lines *LLT(L)* envelope the Kantor-Hervey Deltoid *QL-Qu2*.**

The axes of *QL-Qu2* are lines through *QL-P3*, whose *LLT*-image contains *QL-P1*.

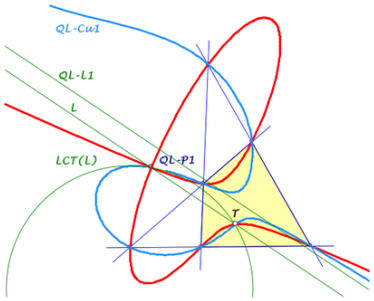


- **For lines *L* of the pencil of the Miquel Point *QL-P1* the 2nd intersection with the circle *LCT(L)* gives a cubic**

  … through the six intersections of the lines of the quadrilateral,
  … through the Miquel-Point *QL-P1* as knot
  … with the Steiner axes as tangents,
  … through the intersection *T* with *QL-Cu1*.
  … Lines *L* through *QL-P1* with *L* = *LLT(L)* are parallel to the asymptotes of the cubic.



Wrt the point *T*: The here discussed cubic and the *QL*-Quasi-Isogonal Cubic *QL-Cu1* have 8 common points: the 6 vertices of the quadrilateral, the Miquel Point *QL-P1* and a further intersection *T*, which is the *CSC*-image of the intersection of *QL-Cu1* and its asymptote (see *EQF*). On the other hand *T* is the 2nd intersection of a parallel to *QL-L1* through *QL-P1* and its *LCT*-circle.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-06-30.pdf

Message: #1206
Date: 01/7/2015 10:11:25
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

Dear Chris, dear Bernard,
here is a further characteristic for QL-Cu1:
QL-Cu1 is the locus of points X, which have the same or perpendicular angle bisectors wrt opposite intersections of the lines of the quadrilateral.
Best regards Eckart

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Message: #1207
Date: 01/7/2015 5:54:18
From: bernard.keizer
Subject: QL-Cu1

Dear Eckart,
The property shoud be generalised.
For any fixed point X on QL-Cu1, for any pair of variable Cl-S conjugate points Y and Y' (including obviously the opposite vertices of the QL), the direction of the bisectors of the angle YXY' is the same.
Best regards
Bernard

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Message: #1208
Date: 01/7/2015 6:00:03
From: bernard.keizer
Subject: 64 axes of a QL

Dear Chris,
Very interesting and intriguing configuration indeed !
Best regards
Bernard

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Message: #1209
Date: 01/7/2015 6:17:50
From: bernard.keizer
Subject: Further simple construction of QL-Cu1

Dear Eckart, dear Chris
A lot seems to hve been done while I wasn't home, it's hard to catch up !
I tried to summarize the constructions of QL-Cu1 I knew :
the 1rst one is the same as yours, but a little simplified, the 2nd was in Bernard Gibert's paper on QL-Cu2, the 3rd one is also the same as in one of your previous messages.
I discovered a link between the points determined in the 3 constructions.
I tried to find the locus of the center of the circle through X, X', X1 and X2, but I haven't succeeded ; maybe you will be more Lucky ...
I suggest for Chris that the inversion centers of QL-Cu1 QL-2P5a and b should be introduced in EQF.
Last, but not least, the autopoloconics of the cubic stelloïd cut QL-Cu1 in 3 copples of CSC conjugate points ; if one of these copples is formed by the 2 cyclical points, the conic becomes a socalled generalised Steiner circle.
Best regards
Bernard

QL-Cu1 : points, properties and constructions

I suppose the points P1, 2P2a and b, 2P3a and b and S and T known (see EQF) ; we need the 2 points I call 2P5a and b, which are Cl-S conjugates on the 2nd Steiner axis and are the 2 centers of inversion for the curve QL-Cu1 (anallagmatic curve)

1. Properties

   For any point X on Cu1 :

   a. It's Cl-S conjugate X' is on Cu1 ; the middle μ of XX' is on the Newton Line and XX' is the internal bisector of the angle 2P3aμ2P3b
   b. P1X cuts the Newton Line in a point mXX1 and the reflection of X in this point is a point X1 on Cu-1
   c. SX cuts the circle with center P1 through X in a 2nd point X2
   d. 2P5aX and 2P5bX' intersect in X5 and 2P5aX' and 2P5bX in X'5 ; X, 2P5b and X5 and X', 2P5a and X'5 are on circles through P1
   e. The 3rd intersection of XX' with Cu-1 is the complementary point X0 of X and X' ; XX' cuts the axis of the parabola in a point m and the perpendicular to XX' in X0 cuts the same axis of the parabola in a point m', which is the reflection of m in P1.
   f. X and X' have the same tangential Xt on Cu-1 and Xt is the Cl-S of X0 ; X, X', Xt are on a circle through P1, which is the Cl-S conjugate of the line XX'

2. Constructions

   a. Taking a point μ on the Newton Line, the external bisector of the angle 2P3aμ2P3b cuts the Steiner axes in 2 points I (on the 1rst) and J (on the 2nd) ; the 3 circles C1 with center I and orthogonal to the circle with diameter 2P3a2P3b, C2 with center J and through 2P3a and b and C3 with diameter IJ intersect in 2 conjugate points X and X'. XX' is the internal bisector of the angle 2P3aμ2P3b.
      The points X5 and X'5 mentioned above in the point d. are on the circle C1.
      The circle C3 is the Cl-S conjugate of XX' through P1 and Xt mentioned in the point f.
   b. Consider the circle through P1 Cl-S conjugate of the Newton Line ; any circle C4 centered on the Newton Line and orthogonal to this circle give 2 points X and X1 on Cu-1. As X, X', X1 and X'1 form an inscribed QL with P1 and the infinity point of the Newton Line, X'X1 and XX'1 are parallel to the Newton Line.
   c. Consider the circle with diameter P1S ; the contact points of the tangents from P1 to any circle C5 centered in the perpendicular to the Newton Line in T and orthogonal to this circle give 2 points X and X2 on Cu-1. As X, X', S, T, X2 and X'2 form another inscribed QL, S, X' and X'2, T, X and X'2 and T, X' and X2 are aligned.
      (Obviously, the circle with diameter P1S is a particular circle of b. and the circle conjugate of the Newton Line is a particular circle of c. ; both circles are orthogonal and the 2 points 2P2a and b are on one or the other circle, depending if Cu-1 is monocursal or bicursal)

      These 3 constructions of Cu-1 are classical, the new property is that the points X, X', X1 and X2 are on a circle C6 and the points X, X', X'1 and X'2 are on a second circle C'6 ; C6 and C'6 are Cl-S conjugate circles.

Constructions of QL-Cu1.pdf

Constructions of QL-Cu1.pdf

Message: #1210
Date: 02/7/2015 10:04:33
From: bernard.keizer
Subject: QL-Tf2 and QL-Cu1

Dear Eckart, dear Chris
Very interesting properties of this transformation QL-Tf2.
I have just a generalising remark for the link with QL-Cu1 as the locus of points, whose angle bisectors wrt 2 opposite vertices are QL-Tf2 partners.
A line and it's QL-Tf2 partner line form a degenerated conic.
It's possible to study directly the conics which cut the diagonals harmonically, but it's another study ...
Every pair of Cl-S conjugate points X and X' form a degenerated conic, which is apolar to the cubic stelloïd QL-Cu2 and to all it's polar conics, which are conic stelloïds or rectangular hyperbolas.
That means that every rectangular hyperbola, which is the polar conic of a point wrt the cubic stelloïd, cuts harmonically every line XX' with X and X' CSC partners (including obviously the diagonals of the QL, but also an infinity of other lines, all tangent to the cayleyan of the stelloïd).
QL-Cu1 being the hessian of QL-Cu2 is the locus of the points for which the rectangular hyperbola degenerates in 2 orthogonal lines ; these 2 lines intersect in the conjugate X' of the point X and are the common bisectors of all the angles YX'Y', where Y and Y' are any other copple of CSC partners (including the opposite vertices of the QL)
This explanation leads to plenty of interesting properties ; let's take a few examples :
a) the polar conic of the Miquel point QL-P1 is made of the infinity line and the Newton Line which is the locus of the middles of the segments XX' as the harmonics of the infinity points
b) the polar conic of the infinity point of the Newton Line is made of the 2 Steiner axes (the polar conics of other infinity points are rectangular hyperbolas through the 2 points 2P3a and b)
c) for any copple of CSC partners X and X', the tangential Xt is the CSC partner of the complementary point X0, 3rd intersection of XX' with QL-Cu1
The polar conic of Xt is made of the line XX' and it's perpendicular in X0
d) for any point X on QL-Cu1, the 2 orthogonal lines forming it's polar conic (intersecting in X' and bisectors of all angles YX'Y') cut the axis of the parabola in 2 points symmetric wrt the Miquel point QL-P1, as it's harmonic is the infinity point of the Newton Line.

This is the case, for example, for XX' and it's perpendicular in X0
There are surely plenty of other properties ...
Best regards
Bernard

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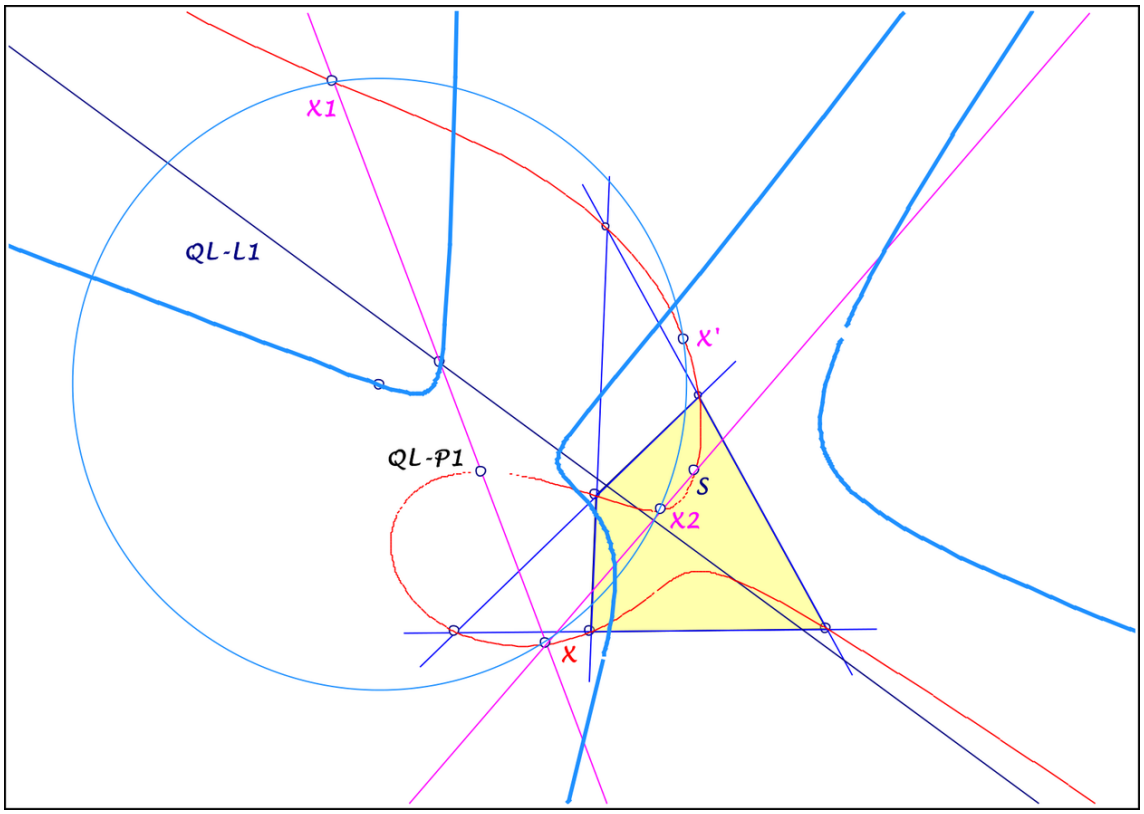
Message: #1211
Date: 02/7/2015 4:09:13
From: eckart_schmidt@t-online.de
Subject: Further simple construction of QL-Cu1

Dear Bernard,
with interest I have reproduced your modifications of earlier constructions of QL-Cu1. The second is very simple!
Attached you find a figur with the locus for the centers of the circumcircles of
... a variable point X on QL-Cu1
... its CSC-image X',
... X1 3rd intersection of X.QL-P1 with QL-Cu1,
    which is the 2nd point on QL-Cu1 with midpoint X.X1 on QL-L1,
... X2 3rd intersection of X.S with QL-Cu1,
    which is the intersection of X.S and X'.T.
The locus seems to be a quartic.
Best regards Eckart
PS. Your points QL-2P5 are already described in 11.1 on my homepage.



2015-07-02.pdf

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Message: #1212
Date: 03/7/2015 8:24:46
From: eckart_schmidt@t-online.de
Subject: QL-Tf2 and QL-Cu1

```
Dear Bernard, dear Chris,
the cubics described in #1175 and #1205 are the same:
... The intersections of lines through QL-P1
    and their QL-Tf2-image (#1175).
... The 2nd intersections of lines through QL-P1
    and their LCT-image (#1205).
This cubic is invariant wrt an isoconjugation with fixed point
QL-P1 and its Ceva triangle wrt QL-Tr1.
Best regards Eckart
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Message: #1213
Date: 03/7/2015 9:13:23
From: eckart_schmidt@t-online.de
Subject: New cubic for a QL

```
Dear Bernard, dear Chris,
in message 1212 I described a cubic for a QL in two different
ways:
... The intersections of lines through QL-P1 and their
QL-Tf2-image (#1175).
... The 2nd intersections of lines through QL-P1 and their
LCT-image (#1205).
*This cubic is the locus for the contact points of tangents from
QL-P1 to inscribed conics of the QL.*
The contact points are partner
wrt an isoconjugation with fixed point QL-P1 and its Ceva
triangle wrt QL-Tr1.
Best regards Eckart
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Message: #1214
Date: 03/7/2015 1:47:28
From: seiichikiri
Subject: A property of Euler Poncelet point (QA-P2)

Dear friends,
Addition:
This transformation coincides with QA-Tf3 (orthopole transformation).
Best regards, Seiichi.

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Message: #1215
Date: 04/7/2015 3:15:42
From: eckart_schmidt@t-online.de
Subject: New cubic for a QL

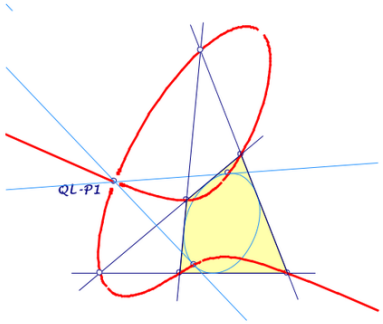
Dear friends,
attached you find a summary of properties of the new cubic.
There are parallels to QL-Cu1, this is only a first study.
Best regards Eckart

EQF-Note 2015-07-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A new Cubic for a Quadrilateral

This cubic is already mentioned in 12.3 on my homepage. There are further aspects in QFG-messages 1175, 1205, 1212 and 1213. Here is a summary of its properties. – Reference triangle for barycentric calculations is the QL-Diagonal Triangle QL-Tr1.



There are three possibilities, to describe the cubic:

- **The cubic is the locus for the contact points of tangents from the Miquel Point *QL-P1* to inscribed conics of the quadrilateral (# 1213).**
- **The cubic is the locus for the intersections of lines through *QL-P1* and their *QL-Tf2*-image (#1175).**
- **The cubic is the locus for the 2nd intersections of lines through *QL-P1* and their *LCT*-image (#1205).**

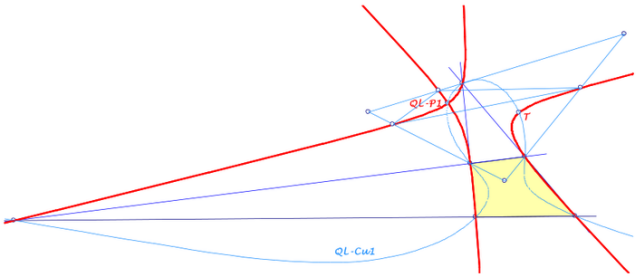
For the *QL*-Diagonal Triangle *QL-Tr1=ABC* as reference triangle for barycentric coordinates and a line (l,m,n) of the quadrilateral:

- **The cubic has the equation**

$$\frac{l^2x}{wy-vz}+\frac{m^2y}{uz-wx}+\frac{n^2z}{vx-uy}=0$$

with $QL-P1=(u:v:w)$.

- **The cubic contains the following points:**
  … the Miquel Point *QL-P1* as knot,
  … the six intersections of the lines of the quadrilateral,
  … the vertices of the Ceva triangle of *QL-P1* wrt *QL-Tr1* (this is the diagonal triangle of *A, B, C, QL-P1*),
  … the intersection *T* with *QL-Cu1* (*CSC*-image of the intersection of *QL-Cu1* and its asymptote).



- **Tangents to the cubic in *QL-P1* are the Steiner axes.**

- **The cubic is invariant wrt an isoconjugation * with fixed point *QL-P1* and the Ceva triangle of *QL-P1* wrt *QL-Tr1*:**

$$X(x:y:z) \quad \rightarrow$$
$$X^*((-vwx+wuy+uvz)x$$
$$:(vwx-wuy+uvz)y$$
$$:(vwx+wuy-uvz)z)$$

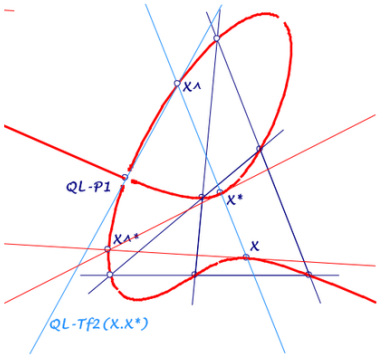
- **This isoconjugation …**
  … has the fixed points *QL-P1* and the vertices of *QL-Tr1*,
  … swaps opposite intersections of the lines of the quadrilateral,
  … swaps the contact points of tangents from *QL-P1* to inscribed conics
  … and gives for the line at infinity the Nine Point Conic of *A, B, C, QL-P1*.

- **The angle bisectors at *QL-P1* wrt two conjugate points *X* and *X** on the cubic are the Steiner axes.**

- **For points of the cubic the lines *XX** envelope an inscribed conic tangent to the Steiner axes and *QL-L2* (see #481, #488):**

$$u^2l^4x^2+v^2m^4y^2+w^2n^4z^2$$
$$-2(uvl^2m^2xy+vwm^2n^2yz+wun^2l^2zx)=0\ .$$

- **This conic is also the locus of the 4th harmonic points of *X.X** wrt the 3rd intersections with the cubic.**
- **The 3rd intersection *X^* of *X.X** with the cubic is the intersection with *QL-Tf2(X.X*)*.**
- **The tangents in *X* and *X** at the cubic intersect in the isoconjugate * of *X^* on the cubic.**



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2015-07-04.pdf

Message: #1216
Date: 04/7/2015 4:13:08
From: bernard.keizer
Subject: 4th construction of QL-Cu1

Dear Eckart, dear Chris
Working again on the points on QL-Cu1, I (re)discovered an old property of the cubic QL-Cu1 and, following, I found a new construction of this curve.
Maybe the 2 points QL-2P5a and b, the 2 parabolas and the 2 circles of inversion as well as the property and the construction deserve a description in EQF...
Best regards
Bernard

QL-Cu1

4th construction

Let's consider again the 4 points X, X', X3 and X'3 on the same circle through I orthogonal to the inversion circle with diameter 2P5a2P5b.

X, X', X3, X'3, 2P5a and 2P5b form a QL inscribed in QL-Cu1, X3 being the intersection of 2P5aX with 2P5bX' and X'3 the intersection of 2P5bX with 2P5aX'.

Let's consider now the 2 lines through 2P5a and 2P5b parallel to the Newton Line (they are symmetric wrt the Newton Line) and the 2 parabolas with common focus the Miquel point QL-P1 and directrices these 2 lines.

Last, let's consider the 2 inversion circles of QL-Cu1 with centers in 2P5a and 2P5b ; they are orthogonal and pass through the 2 points 2P3a and 2P3b.

We associate the circle with center 2P5a with the parabola with directrix the line through 2P5b and the circle with center 2P5b with the parabola with directrix the line through 2P5a.

Property :

The perpendicular bisectors of X and X'3 and of X' and X3 envelop the parabola with directrix the line through 2P5a and any circle through X and X'3 or through X' and X3 is orthogonal to the inversion circle with center 2P2b

The perpendicular bisectors of X and X3 and of X' and X'3 envelop the parabola with directrix the line through 2P5b and any circle through X and X3 or through X' and X'3 is orthogonal to the inversion circle with center 2P2a.

In particular, the contact points K1 and K2 of the tangents from I to the 1rst parabola are centers of circles bitangent to QL-Cu1 in X and X'3 and in X' and X3 and the contact points K3 and K4 of the tangents from I to the 2nd parabola are centers of circles bitangent to QL-Cu1 in X and X3 and in X' and X'3.

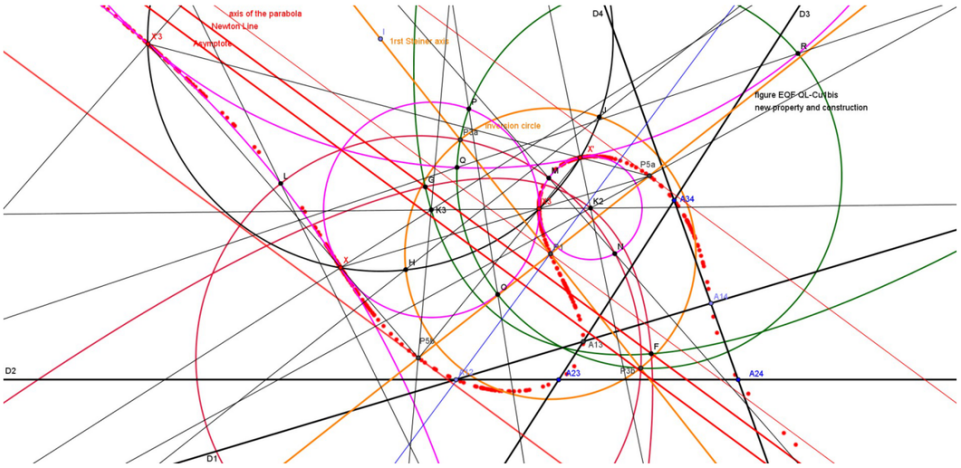
The cubic QL-Cu1 may be seen as the envelop of circles centered on one or the other of the 2 parabolas and orthogonal to the corresponding circle.

Construction :

For any variable point I on the 1rst Steiner axis, let's draw the 4 tangents to the 2 parabolas and the 4 contact points K1, K2, K3 and K4 ; let's draw the 4 circles with centers these 4 points and orthogonal to the corresponding circles of the 2 parabolas.

These 4 circles are by pairs tangent in the 4 points X, X', X3 and X'3 (X on K1K3, X' on K2K4, X3 on K2K3 and X'3 on K1K4) ; it's easy to check that the 4 points lie on the same circle with center I).

With I describing the 1rst Steiner axis, the 4 points X, X', X3 and X'3 describe QL-Cu1.



4th construction of QL-Cu1.docx

Message: #1217
Date: 05/7/2015 9:06:42
From: eckart_schmidt@t-online.de
Subject: New Cubic for a QL

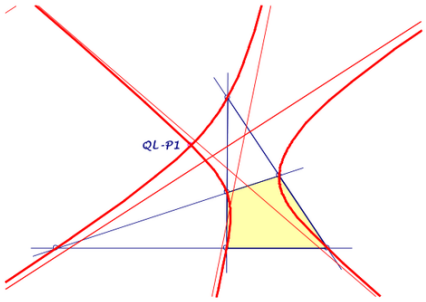
Dear friends,
in addition to my last message: attached a construction for the asymptotes and their intersections with the cubic.
Best regards Eckart

EQF-Note 2015-07-05

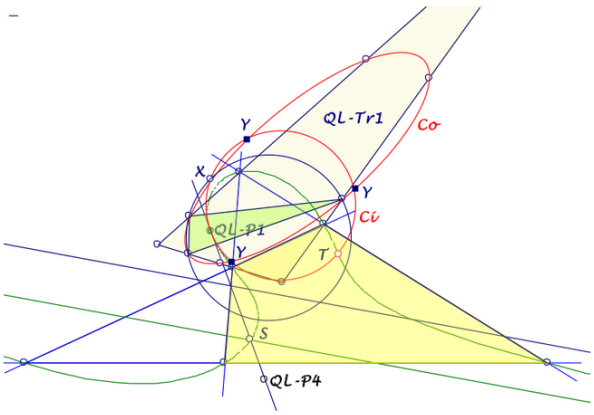
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A new Cubic for a Quadrilateral II (the Asymptotes)

This is an addition to EQF-Note 2015-07-04 in QFG-message 1215, giving a construction for the one or three asymptotes of the cubic. The results are only CABRI- controlled.



Remember the isoconjugation * for the Ceva triangle Δ of *QL-P1* wrt the *QL*-Diagonal Triangle *QL-Tr1=ABC* with the Miquel Point *QL-P1* as fixed point. This isoconjugation maps the line at infinity into the Nine-point Conic *QA-Co1=Co* of *A, B, C* and *QL-P1*, which is the circumconic of Δ and the medial triangle of *QL-Tr1*.
On the other hand we consider the *CSC*-image *Ci* of the asymptote of *QL-Cu1* (a parallel to the Newton Line *QL-L1* through the reflection of *QL-P1* in *QL-L1*). This circle *Ci* contains *QL-P1* and *T* (*CSC*-image of the intersection *S* of *QL-Cu1* and its asymptote) and is tangent to *QL-P1.QL-P4*.



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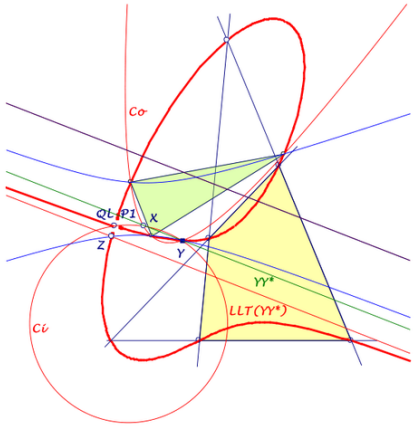
There can be 4 intersections of the circle *Ci* and the conic *Co*. One intersection *X* lies on the circumcircle of Δ. The other intersections *Y* lie on the cubic and give with their isoconjugate *Y** the directions of the asymptotes. So there can be one or three asymptotes.
The line-line transformation *LLT* – discussed in *EQF*-Note 2015-06-30 (*QFG*- message 1205) – gives the …

…asymptotes *LLT(YY*)*.



The isoconjugate of *YY** gives a hyperbola with one asymptote parallel *YY**. The intersection

$$(YY^*)^* \cap LLT(YY^*)$$

is the intersection *Z* of the cubic and its asymptote.

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2015-07-05.pdf

Message: #1218
Date: 06/7/2015 5:08:17
From: minhtenladuong
Subject: Mapping in quadrilateral

Given a quadrilateral ABCD and a point P pedal circles of P WRT BCD,CDA,DAB,ABC are concurrent at a point Q.
What is the name of mapping P--->Q

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Message: #1219
Date: 06/7/2015 8:16:03
From: chris.vantienhoven
Subject: Mapping in quadrilateral

Dear Quang Duong,
Again a very nice transformation!
It is not described in EQF.
Maybe someone knows another reference for this point?
In a quick and limited search I saw these properties:
* Every point on a side of a Quadrangle is transformed in the projection point on the opposite side of the Quadrangle.
* QA-P4 is transformed into QA-P2
I think there will be many more properties.
When P1=(1:0:0), P2=(0:1:0), P3= (0:0:1), P4=(p:q:r), P=(x:y:z), then the 1 st coordinate of the transformed point Q is:
p (r y - q z) (2 b^2 c^2 p r x^3 y + a^2 c^2 q r x^3 y + b^2 c^2 q r x^3 y - c^4 q r x^3 y + a^2 c^2 p r x^2 y^2 + b^2 c^2 p r x^2 y^2 - c^4 p r x^2 y^2 + 2 a^2 c^2 q r x^2 y^2 - 2 b^2 c^2 p q x^3 z - a^2 b^2 q r x 3 z + b^4 q r x 3 z - b^2 c^2 q r x 3 z - 2 a^2 c^2 p q x^2 y z - 2 b^2 c^2 p q x^2 y z + 2 c^4 p q x^2 y z + 2 a^2 b^2 p r x^2 y z - 2 b^4 p r x^2 y z + 2 b^2 c^2 p r x^2 y z - 2 a^2 c^2 p q x y^2 z + a^4 p r x y^2 z - b^4 p r x y^2 z + 2 b^2 c^2 p r x y^2 z - c^4 p r x y^2 z + a^4 q r x y^2 z - a^2 b^2 q r x y^2 z + a^2 c^2 q r x y^2 z - a^2 b^2 p q x^2 z^2 + b^4 p q x^2 z^2 - b^2 c^2 p q x^2 z^2 - 2 a^2 b^2 q r x^2 z^2 - a^4 p q x y z^2 + b^4 p q x y z^2 - 2 b^2 c^2 p q x y z^2 + c^4 p q x y z^2 + 2 a^2 b^2 p r x y z^2 - a^4 q r x y z^2 - a^2 b^2 q r x y z^2 + a^2 c^2 q r x y z^2 - a^4 p q y^2 z^2 + a^2 b^2 p q y^2 z^2 - a^2 c^2 p q y^2 z^2 + a^4 p r y^2 z^2 + a^2 b^2 p r y^2 z^2 - a^2 c^2 p r y^2 z^2)
Best regards,
Chris

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Message: #1220
Date: 06/7/2015 10:36:01
From: eckart_schmidt@t-online.de
Subject: New Cubic for a QL

```
Dear friends,
it seems, I lost control: the cubic is already described in
EQF-Note 2014-02-27 in #466.
Attention: the calculations  for the corresponding conic contain
mistakes, excuse!
Best regards Eckart
```

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Message: #1221
Date: 06/7/2015 11:29:06
From: eckart_schmidt@t-online.de
Subject: 4th construction of QL-Cu1

```
Dear Bernard,
your 4th construction is based on the anallagmatic character of
QL-Cu1. The property, you describe bold, is already mentioned in
11.1 on my homepage.
Best regards Eckart
```

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Message: #1222
Date: 06/7/2015 2:43:31
From: Ngo Quang Duong
Subject: Mapping in quadrilateral

Dear Chris van Tienhoven
That transformation has been posted on Vietnames math magazine called Mathematics and Youth Magazine, you can see it here:
>From a problem about concyclic in Mathematics and Youth magazine (https://nguyenvanlinh.wordpress.com/2011/08/22/from-a-problem-about-concyclic-in-mathematics-and-youth-magazine/)
Problem 1 (M&Y): Given 4 points $latex A_1, A_2, A_3, A_4$ in the plane and an arbitrary point $latex P$ such that $latex P$ is not lie on any circumcircle of thes...

It is quite old and very few care about it
Best regard
Ngo Quang Duong

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Message: #1223
Date: 06/7/2015 10:51:57
From: chris.vantienhoven
Subject: Mapping in quadrilateral

Dear Quang Duong and friends,
Not only the pedal circles of P wrt BCD,CDA,DAB,ABC are concurrent at a point Q.
I noticed that also the ANTIPEDAL CIRCLES of P wrt BCD,CDA,DAB,ABC are concurrent at a point R.
The coordinates of the transformation of some point P(x:y:z) into R give huge coordinates in x,y,z (15th degree).
Still I noticed that QA-P4 is transformed into QA-P9 (graphically determined). It makes me think of QA-Tf4 (Moebius Conjugate) where QA-P3 is transformed into QA-P9.
Maybe this AntiPedal Transformation can be described as the combination of QA-Tf4 and another transformation (transforming QA-P3 into QA-P4).
Best regards,
Chris

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Message: #1224
Date: 06/7/2015 11:13:07
From: bernard.keizer
Subject: 4th construction of QL-Cu1

Lieber Eckart,
Ich bitte um Entschuldigung, aber ich kannte diese Referenz 11.1 noch nicht.
I have found the mention of a deferent curve (here the 2 parabolas) in mathcurves (unfortunately in french), but without any other precision and it took me some hours of drawings to find the focus and the 2 directrices !
And I have imagined then this 4th construction I have described (a point I variable on the 1rst Steiner axis, it's tangents to the 2 parabolas, the 4 contact points and the circles centered in these points and orthogonal to the 2 inversion circles).
If I'm not wrong, this construction wasn't in your paper, nor the fact that the 4 circles are *bitangent* to the curve QL-Cu1.
Anyhow remains the fact that the points QL-2P5a and b, the 2 parabolas, the 2 inversion circles, the property and the construction deserve a desription in EQF, with reference to your paper of course ...
Best regards
Bernard

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Message: #1225
Date: 07/7/2015 7:26:19
From: chris.vantienhoven
Subject: 4th construction of QL-Cu1

Dear Bernard and Eckart,
I tried to calculate the coordinates of these inversion centers.
But what I found was very complicated.
Maybe Eckart can find better (DT-)coordinates?
Or even coordinates related to the Miquel Triangle?
Best regards,
Chris

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Message: #1226
Date: 07/7/2015 9:54:08
From: bernard.keizer
Subject: 4th construction of QL-Cu1

Dear Chris,
I'm very glad you join the discussion!
As you know, I don't use Mathematica and I don't imagine how you try to find the coordinates.
Geometrically, it's rather simple: if you call I the point where the Newton Line cuts the 2nd Steiner axis, the circle with center I through the 2 points 2P-3a and b cuts this 2nd Steiner axis in the 2 points 2P-5a and b.
(these 2 points are CSC partners and the tangent to QL-Cu1 in these 2 points are parallel to the Newton Line)
Best regards
Bernard

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Message: #1227
Date: 07/7/2015 12:36:06
From: eckart_schmidt@t-online.de
Subject: 4th Construction of QL-Cu1

Dear Bernard, dear Chris,
attached a short Mathematica file for Chris and a pdf-file for Bernard with the barycentric coordinates of QL-2P5 but for the reference triangle QG-Tr3 (see also #657, #659).
Best regards Eckart

Barycentric coordinates of the inversion centers QL-2P5 of QL-Cu1:
reference triangle QG-Tr3, one point of the quadrigon {u,v,w}:

$$pP5a := \left\{-a\,B\,v\left((a+b)^2(A\,u+B\,v)+c^2(A\,u-B\,v)+\sqrt{(a+b)^2-c^2}\,\sqrt{-c^2(A\,u-B\,v)^2+(a+b)^2(A\,u+B\,v)^2}\right),\right.$$
$$\left.-A\,b\,u\left((a+b)^2(A\,u+B\,v)-c^2(A\,u-B\,v)-\sqrt{(a+b)^2-c^2}\,\sqrt{-c^2(A\,u-B\,v)^2+(a+b)^2(A\,u+B\,v)^2}\right),\ 2A\,(a+b)\,B\,c^2\,u\,v\right\}$$

$$pP5b := \left\{-a\,B\,v\left((a+b)^2(A\,u+B\,v)+c^2(A\,u-B\,v)-\sqrt{(a+b)^2-c^2}\,\sqrt{-c^2(A\,u-B\,v)^2+(a+b)^2(A\,u+B\,v)^2}\right),\right.$$
$$\left.-A\,b\,u\left((a+b)^2(A\,u+B\,v)-c^2(A\,u-B\,v)+\sqrt{(a+b)^2-c^2}\,\sqrt{-c^2(A\,u-B\,v)^2+(a+b)^2(A\,u+B\,v)^2}\right),\ 2A\,(a+b)\,B\,c^2\,u\,v\right\}$$

$$A := c^2v^2+2\,Sa\,v\,w+b^2w^2;\quad B := c^2u^2+2\,Sb\,u\,w+a^2w^2$$

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2015-07-07a-nb.png

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Message: #1228
Date: 07/7/2015 3:16:43
From: chris.vantienhoven
Subject: 4th Construction of QL-Cu1

Dear Bernard and Eckart,
Studying the inversion centers wrt QL-Cu1 I realized that I mentioned these points before as min/max-points of QL-Cu1. See QFG#310.
All points on QL-Cu1 lie within a stroke bounded by two lines parallel to the Newton Line on equal distances of the Newton Line. These two bounding lines touch QL-Cu1 in 2 points (min/max points) which happen to be the inversion centers.
Then I also calculated these points and found that in EQF they only lie on QL-Cu1 and the second Steiner Axis.
At that time I didn't realize that Eckart mentioned them way before me as inversion centers in his document 11-1 Die Brennpunktkurve eines Vierecks - Eckart Schmidt, page 7.
Best regards,
Chris

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Message: #1229
Date: 07/7/2015 5:43:46
From: minhtenladuong
Subject: Orthopolar line

I have found a point on orthopolar line of a line d WRT ABCD. Please tell me this is new or not? If yes, have anyone had a synthetic proof for it?
AB,AC,AD intersects CD,DB,BC at P,Q,R. Then orthopole of the anticomplementary triangle of PQR lies on orthopolarline of d WRT ABCD.
Thank you,
Ngô Quang Dương

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Message: #1230
Date: 08/7/2015 7:44:10
From: chris.vantienhoven
Subject: Orthopolar line

Dear Quang Duong,
[NQD] I have found a point on orthopolar line of a line d WRT ABCD. Please tell me this is new or not? If yes, have anyone had a synthetic proof for it? AB,AC,AD intersects CD,DB,BC at P,Q,R. Then orthopole of the anticomplementary triangle of PQR lies on orthopolarline of d WRT ABCD.
I don't know this transformation mapping a line d into a point X.
There are some correlations with ETC and EQF.
When line d = Eulerline of PQR (QL-L5), then X = a point on QA-Ci1, the circumcircle of the Diagonal Triangle PQR. Actually it is ETC-point X(110) of the Diagonal Triangle. Numerically and graphically determined.
When line d = QL-L9, then X = point on QA-P23.QA-P33.
Algebraically determined.
Best regards,
Chris

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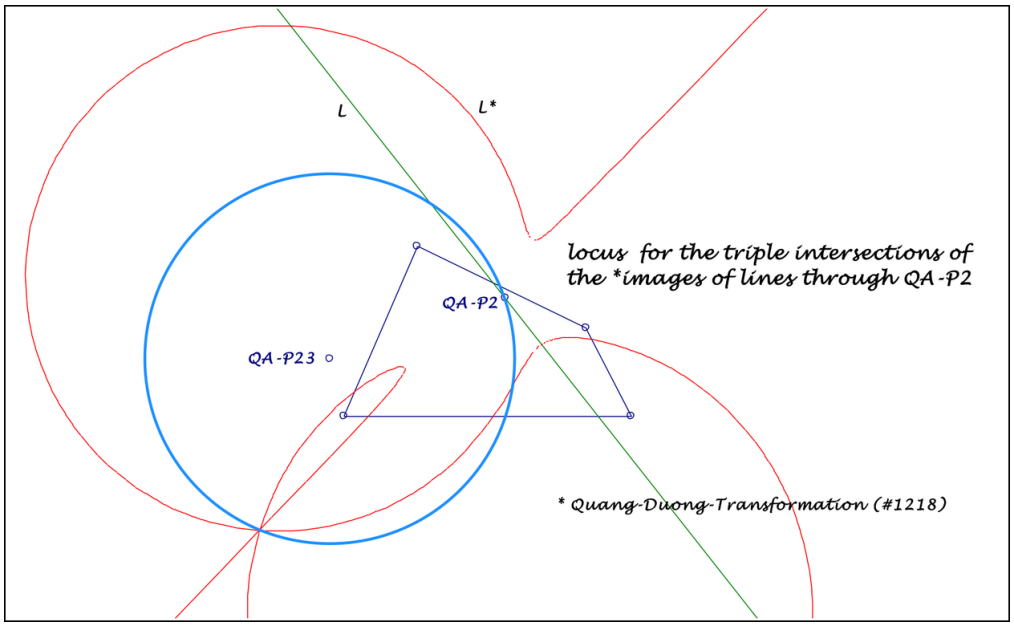
Message: #1231
Date: 09/7/2015 11:40:38
From: eckart_schmidt@t-online.de
Subject: Mapping in quadrilateral

Dear Quag Duong, dear Chris,
some remarks in addition to Chris wrt your transformation, which is very curious, but interesting:
Your transformation
... is a transformation for quadrangles,
... is not involutary,
... has the vertices of the QA-Diagonal Triangle
    as fixed points,
... maps not only QA-P4 into QA-P2,
    but all points of the QA-Orthogonal Hyperbola QA-Co2,
... maps lines into curves of higher order,
    which can have triple intersections.
For lines through a fixed point P the triple intersections of their images lie on a circle.
Examples:
... Let P be a point of the quadrangle,
    the circle is the pedal circle wrt the remaining triangle.
... Let P be a QA-P2, the circle is centered in QA-P23
    and contains QA-P2 (see attached file).
... Let P be QA-P4, the triple intersections
    degenerate in QA-P2.
... In general, the circle contains QA-P2 and the image of P.
The properties are only CABRI-controlled!
Best regards Eckart



2015-07-09.pdf

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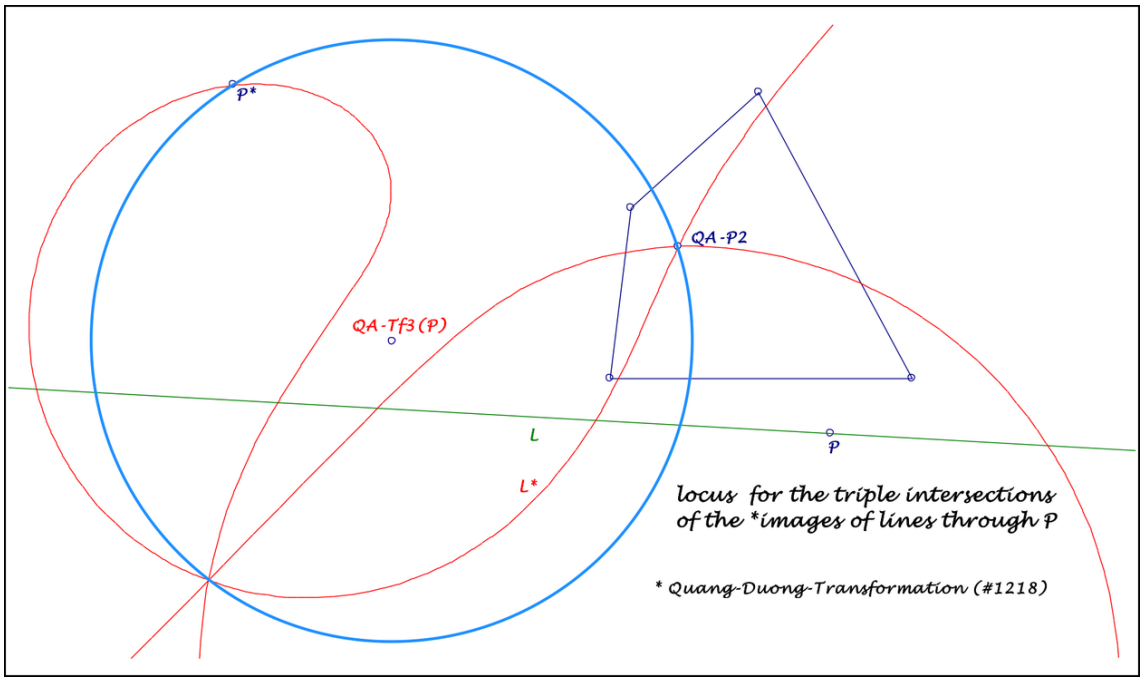
Message: #1232
Date: 09/7/2015 2:41:01
From: eckart_schmidt@t-online.de
Subject: Mapping in quadrilateral

Dear Quang Duong, dear Chris,
just found the center for the circle of the triple intersections (see # 1231):
The circle for the triple intersections is centered in QA-Tf3(P) and contains QA-P2 and P*.
Best regards Eckart



2015-07-09a.pdf

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Message: #1233
Date: 10/7/2015 7:41:44
From: minhtenladuong
Subject: Concurrency of orthotransversal

Community - Art of Problem Solving (http://www.artofproblemsolving.com/community/u239653h1106953p5076972)
On AoPS, VŠ© Thanh Tùng posted a problem:
Quadrilateral ABCD and a point P.
Then orthotransversals of P WRT BCD,CDA,DAB,ABC are concurrent at a point Q.
Is this transformation new? If it is old, anybody know a reference, and what is the name of the transformation P --> Q
Best regard
Ngô Quang DÆ°Æ¡ng

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1234
Date: 10/7/2015 9:25:55
From: eckart_schmidt@t-online.de
Subject: Two QA-Simson Lines

Dear Chris,
the Simson line ...
... of QA-P2 wrt QA-Tr1 is QA-P6.QA-P36,
... of QA-P9 wrt QA-Tr2: Let Q2 be the intersection of QA-Cu1 and its asymptote (see EQF at QA-Cu1). The Simson line is the QA-Tr2-isogonal conjugate of a circumconic of QA-Tr2 through Q2 with tangent Q2.QA-P9.
The Simson line is parallel to the asymptote of QA-Cu1 and cuts the circumconic on the cubic in two QA-Tr2-isogonal conjugated points.
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1235
Date: 12/7/2015 7:28:24
From: eckart_schmidt@t-online.de
Subject: QL-2P3

Dear friends,
consider a QL: The CSC-image of a line Li is the circumcircle of the remaining triangle component, cutting the line Li in points, which are the Isogonal Center QA-P4 of their (degenerated) pedal-QA.
There are two other points, the CSC-fixed points QL-2P3, which are QA-P4 of their (non degenerated) pedal-QA.
Take the QG-versions of the pedal-QA of QL-2P3 wrt a reference QL, and the pedal-QGs of QL-2P3 are parallelograms.
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1236
Date: 13/7/2015 10:05:39
From: eckart_schmidt@t-online.de
Subject: QL-Geometry for a QA

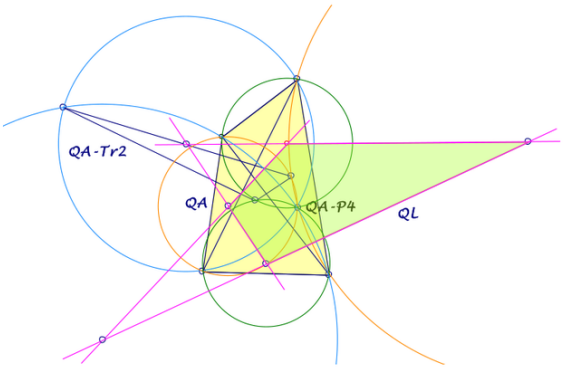
Dear friends,
The bisectors of vertices of a quadrangle QA and the Isogonal Center QA-P4 lead to a quadrilateral QL. Attached some relationships between the geometry of QA and QL are presented.
Best regards Eckart

EQF-Note 2015-07-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Geometry for a Quadrangle

The bisectors of vertices of a quadrangle QA and the Isogonal Center QA-P4 lead to a quadrilateral QL. Here some relationships between the geometry of QA and QL are presented, but only CABRI-controlled.



For a quadrangle *QA* the six circles through P_i, P_j, *QA-P4* intersect in the vertices of the Miquel Triangle *QA-Tr2*. The midpoints of these circles lie on four lines, defining a quadrilateral *QL*. The lines of the *QL* are the bisectors of the vertices of *QA* and *QA-P4*.

It will be interesting, to compare the geometry of the *QL* with the geometry of the *QA*. Here some aspects (The prefix *QA* and *QL* belong here to different objects!):

- **The Miquel Point *QL-P1* is the midpoint of *QA-P4* and *QA-P9*.**
- **The Newton Line *QL-L1* is the bisector of *QA-P3* and *QA-P4*.**
- **The Steiner Line *QL-L2* is a parallel to *QA-L4* through the center of the circumcircle of the Miquel Triangle *QA-Tr2*.**

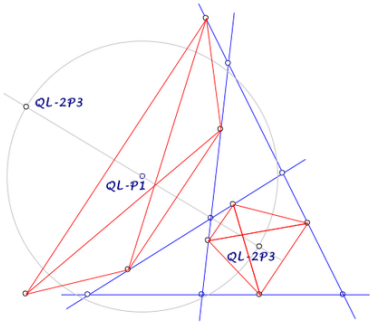
- **The Pedal Line *QL-L3* is a parallel to *QA-L4* through the midpoint of the midpoint of *QA-P4.QA-P9* and the center of the circumcircle of *QA-Tr2*.**

- **The 1st Steiner Axis of the *QL* is *QA-P4.QA-P9*.**

- **The *CSC*-fixed points *QL-2P3* are *QA-P4* and *QA-P9*.**

- **The cubics *QL-Cu1* and *QA-Cu1* have orthogonal asymptotes.**

Final remark: If we begin with a quadrilateral *QL* and consider the *CSC*-fixed points *QL-2P3*, these points are the Isogonal Centers *QA-P4* of their pedal-quadrangles wrt *QL*. The *QA*-Miquel Centers *QA-P9* of these pedal-quadrangles coincide in the Miquel point *QL-P1* and the Gergonne-Steiner Points *QA-P3* lie on the Newton Line *QL-L1* of the reference quadrilateral.



Eckart Schmidt
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eckart_schmidt@t-online.de

2015-07-13.pdf

Message: #1237
Date: 22/7/2015 8:21:13
From: eckart_schmidt@t-online.de
Subject: Pivotal CSC-cubics for a QL

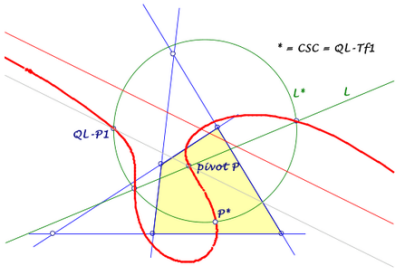
Dear Bernard, dear Chris,
well-known are “pivotal isocubics”, defined wrt a triangle, an isoconjugation and a pivot. Here an analogon is considered wrt a quadrilateral, the CSC-transformation (QL-Tf1) and a pivot.
Perhaps of interest, see attachment.
Best regards Eckart

EQF-Note 2015-07-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Pivotal *CSC*-Cubics for a Quadrilateral

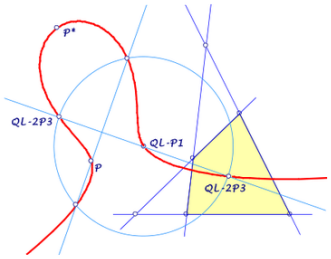
Well-known are "pivotal isocubics", defined wrt a triangle, an isoconjugation and a pivot [1]. Here an analogon is considered wrt a quadrilateral, the CSC-transformation (QL-Tf1) and a pivot. General properties are presented, but only CABRI-controlled.



A pivotal *CSC*-cubic of a quadrilateral shall be the locus for intersections of lines *L* through a pivot point *P* and their *CSC*-circles *L.**

- **A pivotal *CSC*-cubic degenerates for a pivot *P* on the Steiner Axes.**

- **A pivotal *CSC*-cubic with pivot *P* contains the following points:**
  … the Miquel Point *QL-P1*,
  … the *CSC*-fixed points *QL-2P3*,
  … the pivot *P* and its *CSC*-partner *P,**
  … the intersections of the Schmidt Circle and a perpendicular line wrt the 1st Steiner Axis through *P*.

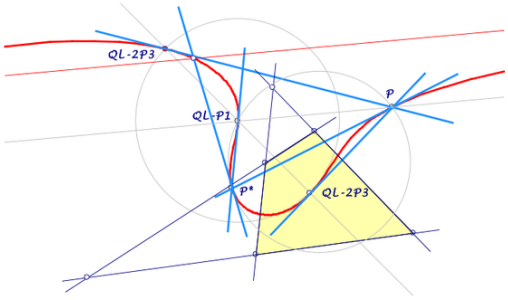


2015-07-22.pdf

- **A pivotal *CSC*-cubic with pivot *P* is the locus for the intersections of circles through *QL-P1*, centered on the bisector of *P*.QL-P1*, and their *CSC*-lines.**

- **A pivotal *CSC*-cubic is *CSC*-invariant. *CSC*-partners are collinear with the pivot.**

- **The asymptote is a parallel to *P.QL-P1* through the reflection of *P** in *P.QL-P1*.**



- **Tangents at the cubic in the *CSC*-fixed points *QL-2P3* intersect in the pivot point *P*.**

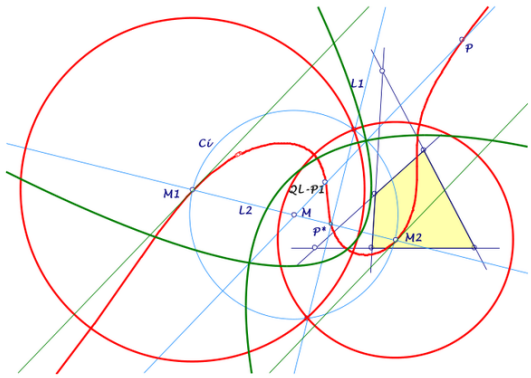
- **The tangent in *P* at the cubic is *PP**.**

- **The tangent in *P** at the cubic is the tangent in *P** at the circumcircle of *P, P** and *QL-P1*.**

- **The intersection of the cubic and its asymptote is the intersection with the tangent at *P**.**

- **A pivotal *CSC*-cubic is an anallagmatic curve.**

The cubic is invariant wrt two inversions. The inversion circles can be constructed as follows:

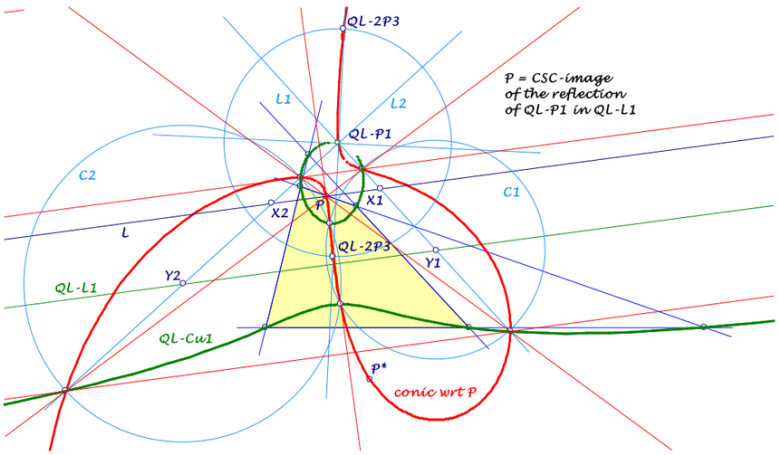


… L_1 : angle bisector at P^* wrt P, *QL-P1*,
… L_2 : perpendicular line wrt L_1 through P^*,
… M : intersection of L_2 and *P.QL-P1*,
… Ci : inversion circle round M wrt P and *QL-P1*,
… M_i : intersections of Ci and L_2.
Circles round M_i through the intersections of L_1 and Ci are inversion circles for the cubic. The tangents in M_i are parallel to the asymptote, that means parallel to *P.QL-P1*.

As anallagmatic curve the cubic is twice the envelope of circles, centered on a parabola and orthogonal to one inversion circle. The parabolas have the same focus in P^* and as directrices the tangents in M_1 and M_2.

Final example:

Taking for pivot the *CSC*-image of the reflection of *QL-P1* in *QL-L1*, the cubic is the locus for intersections of circles through *QL-P1* and centered on *QL-L1* with their *CSC*-lines. This cubic has intersections with the *QL*-Quasi Isogonal Cubic *QL-Cu1* on the angle bisectors of the Steiner Axes.
QL-P1 is one intersection of the here discussed cubic and *QL-Cu1*, but there are further three pairs of intersections, not always real. They can be constructed as follows:



… P : *CSC*-image of the reflection of *QL-P1* in *QL-L1*,
… L_1, L_2 : angle bisectors of the Steiner Axes,
… L : parallel to *QL-L1* through P,
… X_i : intersections $L \cap L_i$,
… Y_i : intersections $L_i \cap$ *QL-L1*,
... C_i : inversion circles round Y_i wrt X_i and *QL-P1* (*CSC*-partners),
… $C_i \cap L_i$: intersections of the cubics (not always real),
… $C_1 \cap C_2$: intersections of the cubics, if *QL-Cu1* is bipartite.

The first four intersections ($C_i \cap L_i$) lie in pairs on äquidistant parallels wrt *QL-L1*. The six intersections of the cubics lie by four on the circles C_i and in pairs of *CSC*-partners on lines through *P*.

References:

[1] Jean-Pierre Ehrmann and Bernard Gibert: Special Isocubics in the Triangle Plane.

Eckart Schmidt
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2015-07-22.pdf

Message: #1238
Date: 24/7/2015 10:26:14
From: eckart_schmidt@t-online.de
Subject: Generalization of CSC (QL-Tf1)

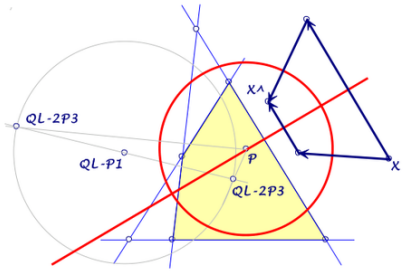
Dear Bernard, dear Chris,
I found a generalization of CSC, which seems a better transformation for pivotal QL-cubics. There are relevant connections with QL-Cu1.
Best regards Eckart

EQF-Note 2015-07-24

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Generalization of *CSC (QL-Tf1)*

*The CSC-transformation is determined by a **P**oint (Miquel Point QL-P1), a **L**ine (1st Steiner Axis) and a **C**ircle (Schmidt Circle). A generalization gives a **PLC**-transformation, which leads to pivotal QL-cubics analog to "pivotal isocubics" for a triangle, replacing the isoconjugation by PLC-transformation for a quadrilateral. General properties are presented, but only CABRI-controlled.*



***PLC*-Transformations**

A *PLC*-transformation for a quadrilateral is defined wrt a point *P* as center, a line through *P* as axis and a circle round *P* as inversion circle. The construction is orientated at the *CSC*-transformation * and its fixed points *QL-2P3*.

A *PLC*-transformation wrt a center *P* is the product of …
… a reflection in the angle bisector at *P* wrt *QL-2P3a,b*
… and an inversion wrt a circle round *P* with radius
$\sqrt{P.QL-2P3a}\sqrt{P.QL-2P3b}$.

The definition fails for centers *P* = *QL-2P3*. Properties of *PLC* are similar to those of *CSC*: The center *P* has no *PLC*-image. *PLC*-image of a line *L* is a circle through *P*, degenerating in a line, if *P* on *L*. *PLC*-image of a circle *Ci* is a circle, degenerating in a line, if *P* on *Ci*.

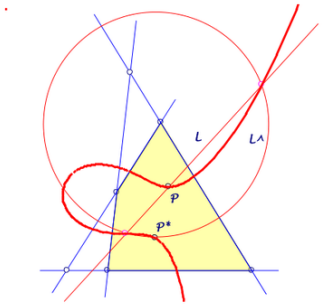
- **The *PLC*-transformation swaps *QL-2P3a,b* and *QL-P1* with *P****.
- **For center *P* = *QL-P17* holds *PLC(QL-L1)* = *QL-Ci1*.**

- **The *QL*-Quasi Isogonal Cubic *QL-Cu1* is invariant under special *PLC*-transformations …**
  … for centers in *QL-2P2* (intersections of *QL-L1* and its *CSC*-circle), if *QL-Cu1* is unipartite,
  … for centers in intersections of a perpendicular line to *QL-L1* at the intersection with *QL-L6* and its *CSC*-circle, if *QL-Cu1* is bipartite.

Pivotal *PLC*-Cubics

For pivotal *CSC*-cubics see *QFG*-message 1237. Here is an analog definition for pivotal *PLC*-cubics, using…

… **a *PLC*-transformation ^ with center *P**** **for the pivot *P***:

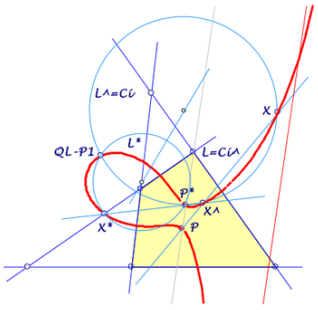


A pivotal *PLC*-cubic of a quadrilateral shall be the locus for intersections of lines *L* through a pivot point *P* and their *PLC*-circles $L^{\wedge}$.

The definition fails for pivots *P* on the Steiner Axes.

- **A pivotal *PLC*-cubic with pivot *P* and lines *L* through *P**** **is the locus for intersections of the circles *L**** **and the lines $L^{\wedge}$.**

- **Pivotal *PLC*-cubics with pivots *P*** **and *P**** **are the same.**

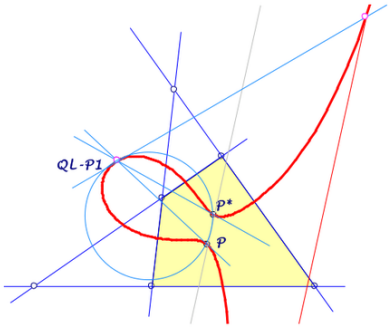


- **A pivotal *PLC*-cubic with pivot *P* is the locus for intersections of circles *Ci* through *QL-P1*, centered on the bisector of *P*.QL-P1*, and lines *Ci*^.**

- **A pivotal *PLC*-cubic with pivot *P* contains the points *P, P** and *QL-P1* with *P*^ = *QL-P1*.**

- **For points *X* on a pivotal *PLC*-cubic with pivot *P*…**
  … *X, X*^ and *P* are collinear,
  … *X*^, *X and *P** are collinear.**

- **A pivotal *PLC*-cubic with pivot *P* is invariant wrt the *CSC*-transformation and the *PLC*-transformations with center *P* and *P*.***



- **The asymptote is a parallel to *PP** through the reflection of *QL-P1* in *P* (or *P**).**

- **The tangents in *P* and *P** at the cubic intersect in *QL-P1*.**

- **The tangent in *QL-P1* at the cubic is the tangent in *QL-P1* at the circumcircle of *P, P** and *QL-P1*.**

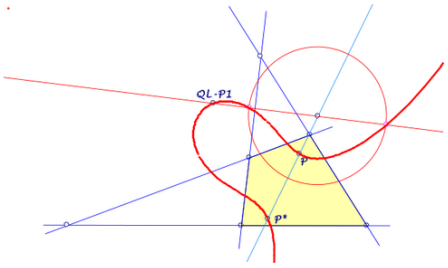
- **The intersection of the cubic and its asymptote is the intersection with the tangent at *QL-P1* (see above).**

- **The fixed points of ^ lie on the cubic as *CSC*-partners.**

- **An unipartite *QL*-Quasi Isogonal Cubic *QL-Cu1* is a pivotal *PLC*-cubic for pivots in *QL-2P2*.**

The fact, that *P* and *P** as pivot give the same cubic leads to a simple construction without *PLC*-transformation:

- **Inversion circles for the pivots *P* and *P** cut the lines from their center to *QL-P1* on the cubic.**

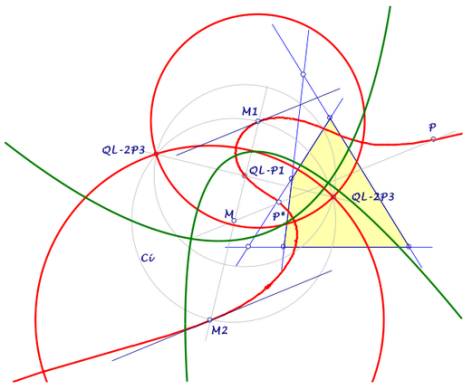


- **A pivotal *PLC*-cubic is an anallagmatic curve.**

The cubic is invariant wrt two inversions. The inversion circles can be constructed as follows:

… *M* : intersection of the 2nd Steiner Axis and *PP**,
… *Ci* : circle round *M* through *QL-2P3*,
… M_i : intersections of *Ci* and the 2nd Steiner Axis.
… Circles round M_i through *QL-2P3* are inversion circles for the cubic.

The tangents in M_i are parallel to the asymptote, that means parallel to *P.P**.

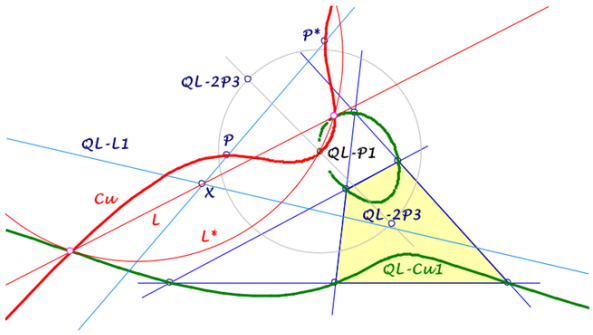


As anallagmatic curve the cubic is twice the envelope of circles, centered on a parabola and orthogonal to one inversion circle. The parabolas have the same focus in *QL-P1* and as directrices the tangents in M_1 and M_2.

- **A pivotal *PLC*-cubic cuts *QL-Cu1* in *QL-P1* and two *CSC*-partners.**

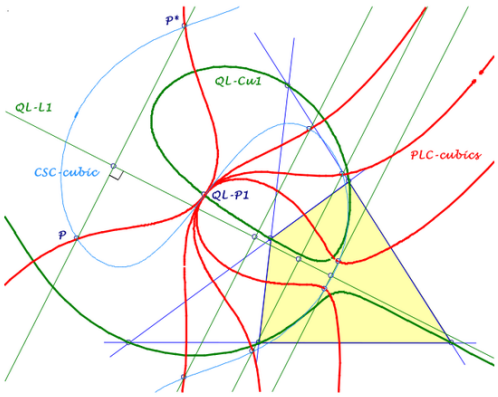
Intersections of pivotal *PLC*-cubics *Cu* and *QL-Cu1* (beside *QL-P1*) can be constructed as follows:

… *X* : intersection of *PP** and *QL-L1*,
… *L* : angle bisector at *X* wrt *QL-2P3*,
… intersections $L \cap L^*$ are the intersections of *Cu* and *QL-Cu1*.



- **All *PLC*-cubics with pivots *P*, *P**** **and *PP**** **orthogonal *QL-L1* cut the cubic *QL-Cu1* orthogonal.**

The pivots of these *PLC*-cubics lie on a *QL-P1*-symmetric *CSC*-cubic (see #1237) with pivot in the point at infinity of perpendiculars of *QL-L1*.



Example: Let *P* and *P** be the intersections of *QL-L2* and its *CSC*-circle *QL-Ci3*. The *PLC*-cubic with pivots *P*, *P** intersects the cubic *QL-Cu1* orthogonal in the intersections of the angle bisector at *QL-P7* wrt *QL-2P3* and its *CSC*-circle.

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2015-07-24.pdf

Message: #1239
Date: 24/7/2015 3:36:10
From: chris.vantienhoven
Subject: Malfatti Circles

Dear friends,
I got this message from Antonio Sánchez Briones.
He asked me to pose his question at the Quadri-Forum.
Dear sir: Thanks for your great work.
I am just an enthusiast, amateur, fan of Geometry; not a mathematician at all. For sure you know the circles of Malfatti in a triangle. My question and my quest is how to construct something similar in a quadrilateral: 4 circles inside a quadrilateral, each tangent to two of the other 3 and each tangent to two adjacent sides of the quadrilateral (i.e. each taking up one inner angle).
I have spent many hours in many weeks trying to solve the problem. The only thing given is the quadrilateral in which the 4 circles must be inserted.
I know that not for all quadrilaterals is the problem solvable. If you can show me a way to construct it with synthetic methods (points, lines, conics) or perhaps by a function I will be most grateful. Or at least providing me a hint.
Thank you very much.
P-S- Though I couldn't understand quite completely the analytical part of your magnificent work about perspective fields, I found it astonishing and the best innovation in geometry I have had notice of in much time. I tried to convert the x6-x15-x2-x13 quad. in its corresponding normalization to a square, but, frankly, I don't know how to map that. I think it would be a good idea to include the geometrical method to get it.
About the first question, I think there is lots more to say about the Malfatti circles in a quadrilateral.
About the second question I already got some ideas which I will post later.
Best regards,
Chris

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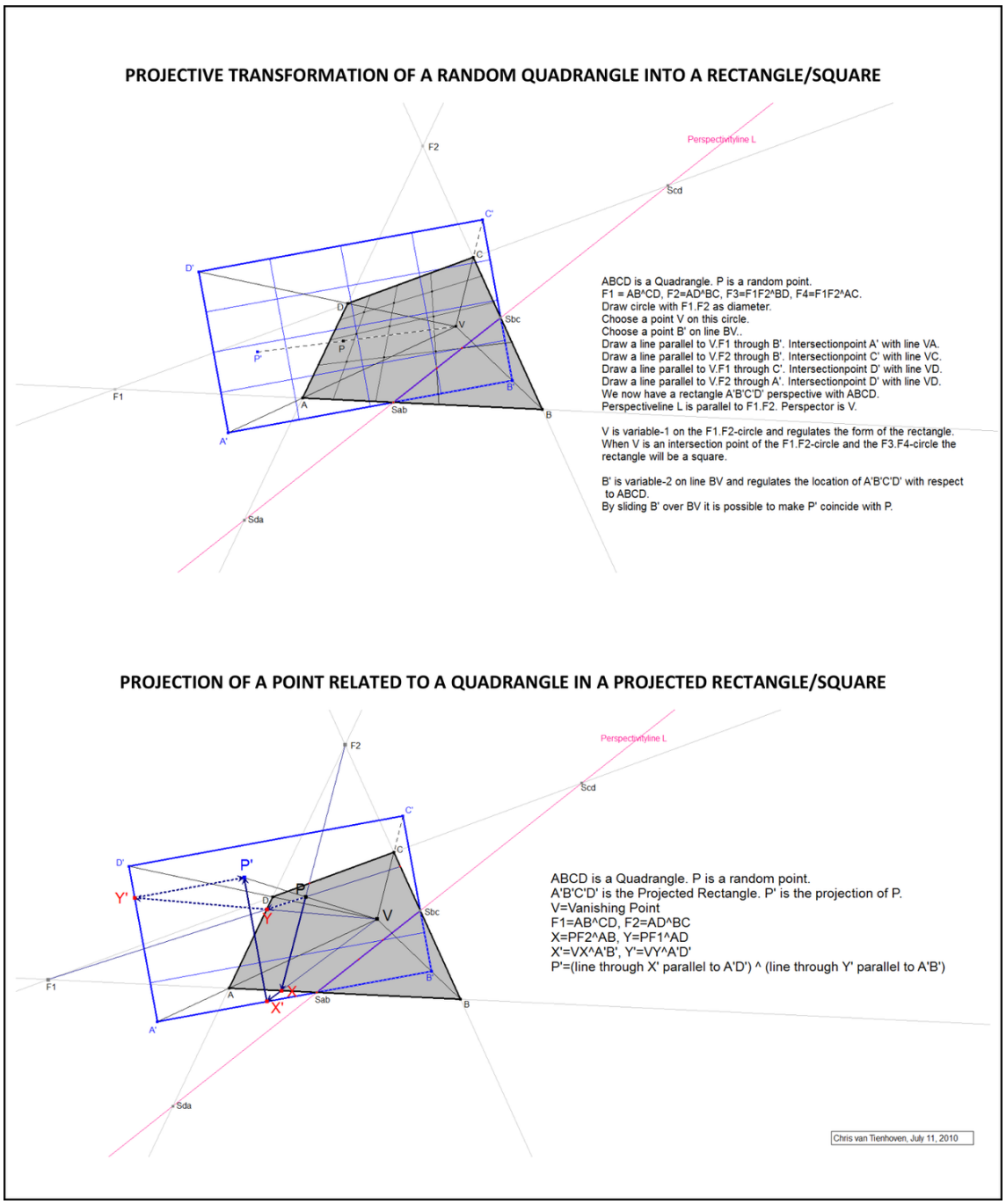
Message: #1240
Date: 24/7/2015 11:16:41
From: chris.vantienhoven
Subject: Malfatti Circles / Normalization Quad to a Square

Dear friends,
Regarding Antonio’s 2nd question:
“I tried to convert the x6-x15-x2-x13 quad. in its corresponding normalization to a square, but, frankly, I don't know how to map that. I think it would be a good idea to include the geometrical method to get it.”
I attach a method for normalizing/transforming/mapping a random quadrangle into a square.
Examples of the normalization of the X(6)-X(15)-X(2)-X(13) quad can be found in the file Perspective Fields-part 1, available at:
http://www.chrisvantienhoven.nl/index.php/2-uncategorised/15-downloads
Best regards,
Chris



PROJ-TRANSFORM OF A QUADR INTO A RECTANGLE-01.pdf

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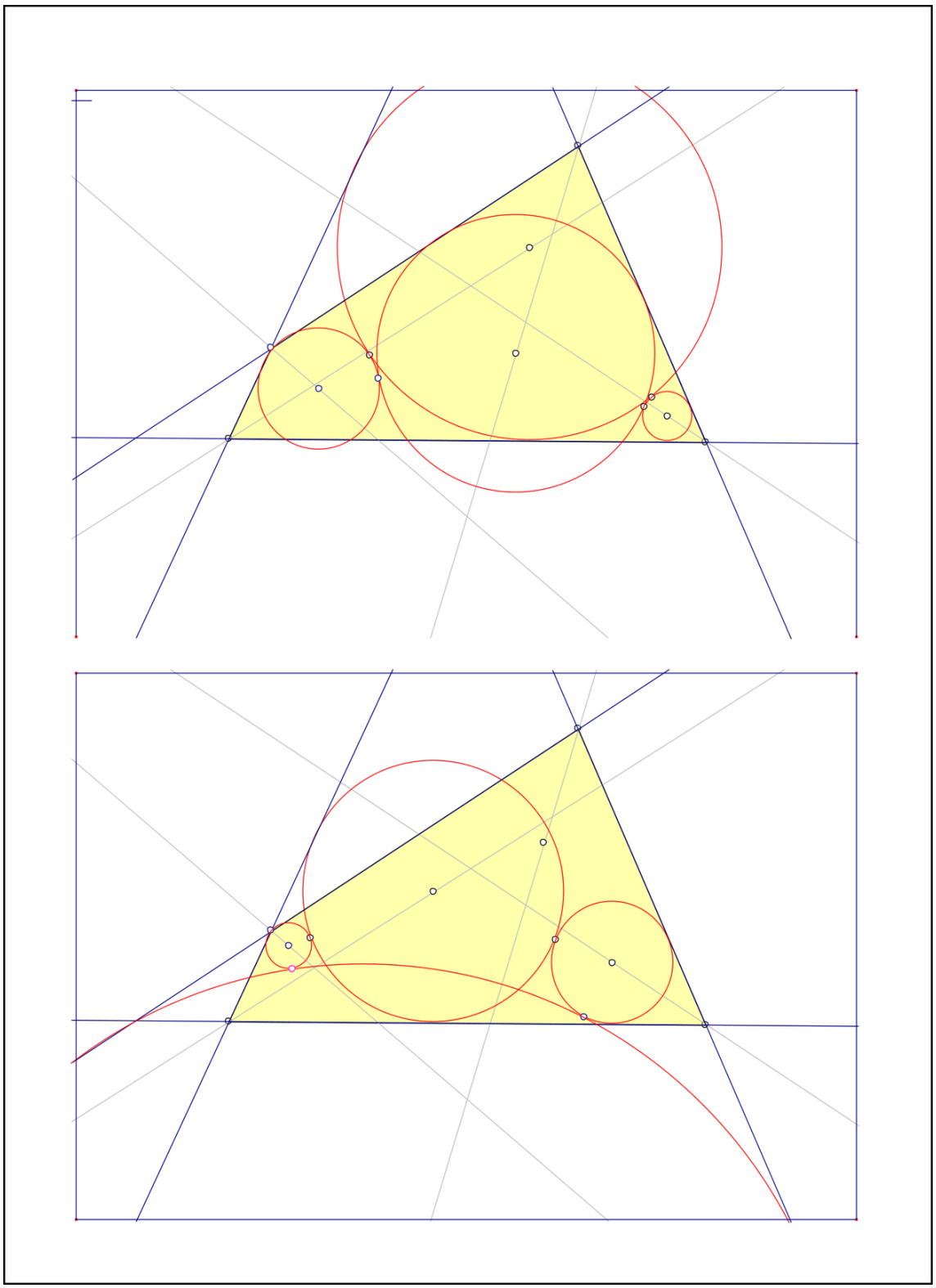
Message: #1241
Date: 26/7/2015 3:40:56
From: eckart_schmidt@t-online.de
Subject: Malfatti Circles

Dear Sir Sánchez Briones,
I think, there will be several possibilities (perhaps 8) for the 4 special circles in a quadrangle. Attached you find two for the same quadrangle. In each drawing there is one contact, which is not a perfect construction. Background: For a circle Ci tangent to a line L there are two circles tangent to Ci and L and another given line.
Best regards Eckart



2015-07-26.pdf

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1242
Date: 30/7/2015 4:19:02
From: eckart_schmidt@t-online.de
Subject: Pedal-Cevian Quadrigons

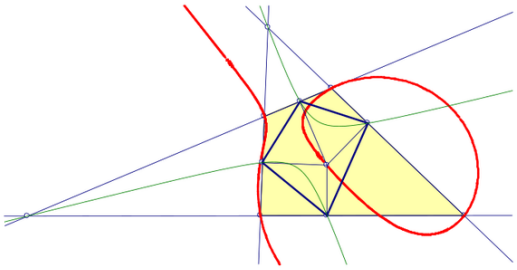
Dear friends,
I try it oncemore:
For a triangle the locus of points, whose pedal triangles are cevian triangles, is the Darboux cubic. Defining cevian quadrigons in a corresponding way there is a pivotal isogonal circular cubic for quadrigons. – Here is a new summary of paper 12.2 on my homepage (see also QFG-message 155).
I think, this cubic is worth, to be mentioned in EQF. It is a direct generalization of a triangle theorem to quadrigon geometry. The cubic can be constructed, calculated and characterized. I have made a new summary as help for testing.
Best regards Eckart

EQF-Note 2015-07-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Pedal-Cevian Quadrigons

For a triangle the locus of points, whose pedal triangles are cevian triangles, is the Darboux cubic. Defining cevian quadrigons in a corresponding way there is a pivotal isogonal circular cubic for quadrigons. – Here is a new summary of paper 12.2 on my homepage (see also QFG-message 155).

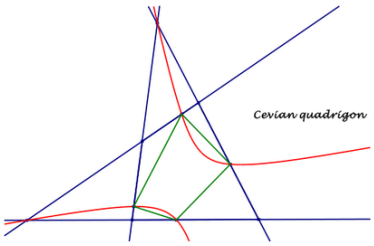


The vertices of a cevian triangle divide the sides of the reference triangle in ratios with product 1. Analog we can define cevian quadrigons:

Definition: A cevian quadrigon is an inscribed quadrigon, dividing the sides in ratios with product 1.

Properties of cevian quadrigons can be found in *09.6* on my homepage. Here some examples:

- **Inscribed conics of a quadrigon give with their contact points cevian quadrigons.**
- **The pedal points for the crosspoints of opposite sides give a cyclic-cevian quadrigon.**
- **Lines from a chosen point to the crosspoints of opposite sides cut the sidelines in vertices of a cevian quadrigon.**
- **The vertices of a cevian quadrigon and the crosspoints of opposite sides lie on a conic …**
  … and conics through the crosspoints generate cevian quadrigons.

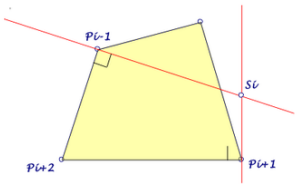


- **The pedal quadrangle of the Miquel Point *S* degenerates collinear to a cevian quadrangle.**

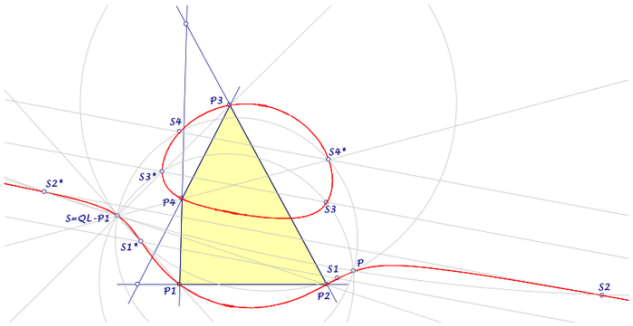
We consider now points, whose pedal quadrigon is a cevian quadrigon. With the 2nd property we can get the equation for a cubic (see *12.2* on my homepage).

Theorem: The locus of points, whose pedal quadrigon is a cevian quadrigon, is a cubic through the Miquel point with an asymptote parallel to the bisector of the crosspoints of opposite sides.

To study the cubic for a quadrangle $P_1P_2P_3P_4$, we use points S_i, for which the pedal points wrt $P_{i-1}P_{i+2}$ and $P_{i+1}P_{i+2}$ are P_{i-1} and P_{i+1}. The conic through the pedal points of S_i and the crosspoints of opposite sides degenerates in two lines.



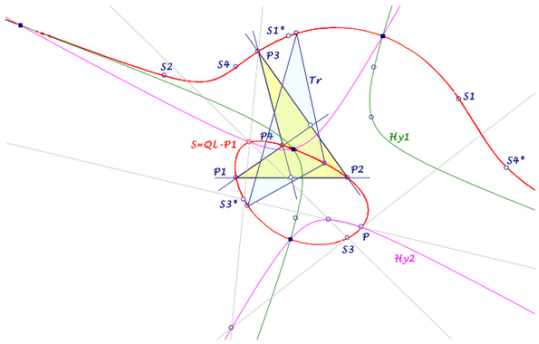
Let L be the bisector of the crosspoints of opposite sides …
… and S_i* the intersection of $S.P_i$ and a parallel to L through S_i
… and P the common point of the circumcircles of S, S_i, S_i*.



- **The points S, S_i, S_i* and P lie on the cubic.**

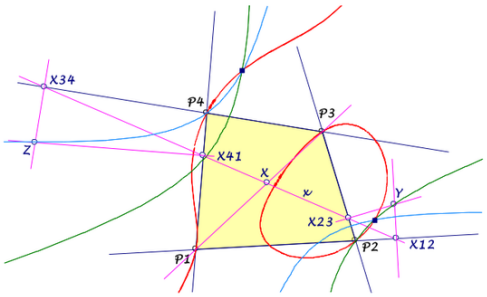
Let Hy_1 be the orthogonal hyperbola through the midpoints of $S_i.S_i$* …
… and Hy_2 the orthogonal hyperbola through S, P, $S.S_i \cap P.S_i$*, $S.S_i$*$\cap P.S_i$ …
… and QA the orthocentric system of their intersections, …
… which are the in- and excenters of a triangle Tr.

- **The triangle Tr is the reference triangle for the cubic as pivotal isogonal circular cubic.**



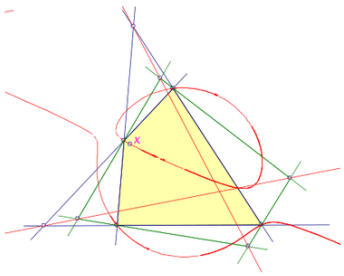
The pivot is the point at infinity for the bisector L of the crosspoints of opposite sides of the quadrigon. The used nomination * is isogonal conjugate wrt Tr. In this way the cubic can be constructed as locus for intersections of parallels to L and their Tr-isogonal conjugated image-conic.

On the other hand there is a simple construction without reference triangle and isoconjugation:



Let $P_1P_2P_3P_4$ be the quadrigon. Consider points X on the diagonal $P_1.P_3$ and lines x through X, which cut the sidelines in X_{12}, X_{23}, X_{34}, X_{41}. Let Y be the intersection of perpendiculars in X_{12} and X_{23} to the corresponding sidelines, let Z be the intersection of perpendiculars in X_{34} and X_{41} to the corresponding sidelines. The loci of Y and Z changing the lines x through X are two conics, the intersections of these conics are points of the cubic.

Final property:



- **The cubic is the locus for points X, whose antipedal quadrigon has diagonals through the crosspoints of opposite sides of the reference quadrigon.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-07-30.pdf

Message: #1243
Date: 31/7/2015 1:28:20
From: Antonio Sánchez Briones
Subject: Malfatti Circles / Normalization Quad to a Square

THANK YOU VERY MUCH! I WILL "PLAY" WITH THAT. WITH RESPECT TO THE "MALFATTI" PROBLEM APLIED TO QUADs. I CAN SAY THAT:
1. THE 4 CONTACT POINTS BETWEEN DE 2 PAIRS OF CIRCLES ARE CONCYCLIC.
2. BY INVERSION YOU CAN GET A MALFATTI STRUCTURE IN QUADS. PARTING FROM 4 EQUAL (?) CIRCLES INSCRIBED IN A SQUARE. THE PROBLEM IS HOW TO DO IT REVERSELY OR FIND OUT WHICH INVERSION CIRCLE IS VALID TO INVERT A SQUARE INTO A GIVEN QUAD.

2015-07-24 23:16 GMT+02:00 van10hoven@gmail.com [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >:
>> [Attachment(s) (#14ec1ece1cd12df5_TopText) from van10hoven@gmail.com [Quadri-Figures-Group]
>> included below]
>> Dear friends,
>> Regarding Antonio’s 2nd question: “
>> I tried to convert the x6-x15-x2-x13 quad. in its corresponding
>> normalization to a square, but, frankly, I don't know how to map that. I
>> think it would be a good idea to include the geometrical method to get it.
>> ”I attach a method for normalizing/transforming/mapping a random quadrangle
>> into a square.
>> Examples of the normalization of the X(6)-X(15)-X(2)-X(13) quad can be
>> found in the file Perspective Fields-part 1, available at:
>> http://www.chrisvantienhoven.nl/index.php/2-uncategorised/15-downloads
>> Best regards,
>> Chris

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Message: #1244
Date: 04/8/2015 10:09:59
From: eckart_schmidt@t-online.de
Subject: Pseudocircumcenter of a QA

```
Dear Chris,
perhaps of interest: In Forum Geometricorum Volume 14, p.145-153
a "pseudocircumcenter" of a QA is defined, this point is the
Gergonne-Steiner Point QA-P3 in EQF. The circles in theorem 9
are QG-Ci4.
Best regards Eckart
```

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Message: #1245
Date: 04/8/2015 8:31:31
From: eckart_schmidt@t-online.de
Subject: Radical Centers

```
Dear Chris,
here are some observations wrt the radical center of 3 circles.
I have only listened results, I didn't find in EQF. (???) means:
point not found in EQF:
QL-Ci1 for a QA:  QA-P16,
QA-Ci1 for a QL:  QL-P13,
QG-Ci1 for a QA:  QA-P12,
QG-Ci2 for a QA:  2 common points (???) inverse wrt QA-Ci1,
QG-Ci2 for a QL:  QL-P10,
circumcircles QG-2P2, QG-P12 for a QA:  one common point (???),
circumcircles QG-2P2, QG-P15 for a QA:  QA-P3,
circumcircles QG-2P2, QG-P15 for a QL:  one common point QL-P17,
circumcircles QG-2P2, QG-P16 for a QA:  one common point (???),
circumcircles QG-2P2, QG-P16 for a QL:  one common point
CSC(QL-P17),
circumcircles QG-2P2, QG-P19 for a QA:  one common point QA-P4,
circumcircles QG-2P2, QG-P19 for a QL:  QL-P26,
circumcircles QG-2P3, QG-P12 for a QA:  QA-P1,
circumcircles QG-2P3, QG-P13 for a QL:  one common point (???).
I shall study the unknown points.
Best regards Eckart
PS: I hope, there are no typos!
```

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Message: #1246
Date: 05/8/2015 1:05:12
From: chris.vantienhoven
Subject: Radical Centers

Dear Eckart,
Thabks for the nice study of radical centers of QL-/QG-circles in the QA-environment als well as QA-/QG-circles in the QL-environment. I will mention these radical centers in EQF.
I am looking forward to your study of the unknown points!
Best regards,
Chris

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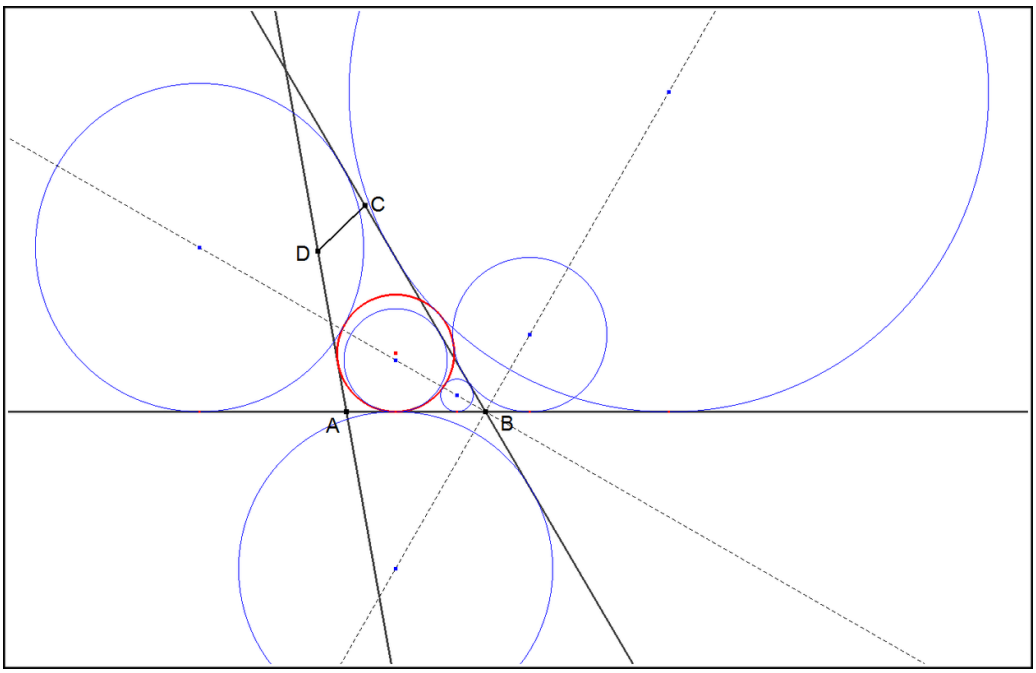
Message: #1247
Date: 05/8/2015 1:07:47
From: chris.vantienhoven
Subject: Malfatti Circles

Dear Antonio, dear Eckart, dear friends,
I am still searching for the Malfatti problem in a quadrilateral.
I wondered when we have a fixed circle Ci1 tangent at 2 sidelines AD and AB, how many circles possibly can touch Ci1, AB and AC?
By calculating and trying I found that at least 6 tangent circles are possible. I made a picture of it. See attached file.
All of these 6 B-circles can be fixed circles for possibly another 6 C-circles tangent at B-circle, BC, CD delivering 36 C-circles.
All of these 36 C-circles can be fixed circles for possibly another 6 D-circles tangent at C-circle, CD, DA, delivering 216 D-circles.
All of these 216 D-circles can be fixed circles for possibly another 6 A-circles tangent at D-circle, DA, AB, delivering 1296 A-circles.
We are looking for those situations where one of these 1296 A-circles equalizes Ci1.
Maybe this helps a bit for further research.
Best regards,
Chris



QG-Malfatti-Problem-01d.png

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Message: #1248
Date: 08/8/2015 2:00:07
From: eckart_schmidt@t-online.de
Subject: QL-Tf2 and QL-Cu1

```
Dear Bernard, dear Chris,
before holiday a last (but not least) remark to QL-Cu1:
Let L1 and L2 be axes of QL-inscribed conics:
... Li and QL-Tf2(Li) are orthogonal.
... QL-Tf2(Li) intersect on QL-L1.
... Li and QL-Tf2(Li) intersect collinear with QL-P1.
... QL-Cu1 is the locus for intersections
    of the lines Li and QL-Tf2(Li).
Let T1 and T2 be the asymptotes of QL-inscribed hyperbolas:
... Ti and QL-Tf2(Ti) are parallel.
... QL-Tf2(Ti) intersect on a line,
    which is (beside QL-L1) the 2nd asymptote
    of an inscribed hyperbola centered in QL-P23.
Best regards Eckart
```

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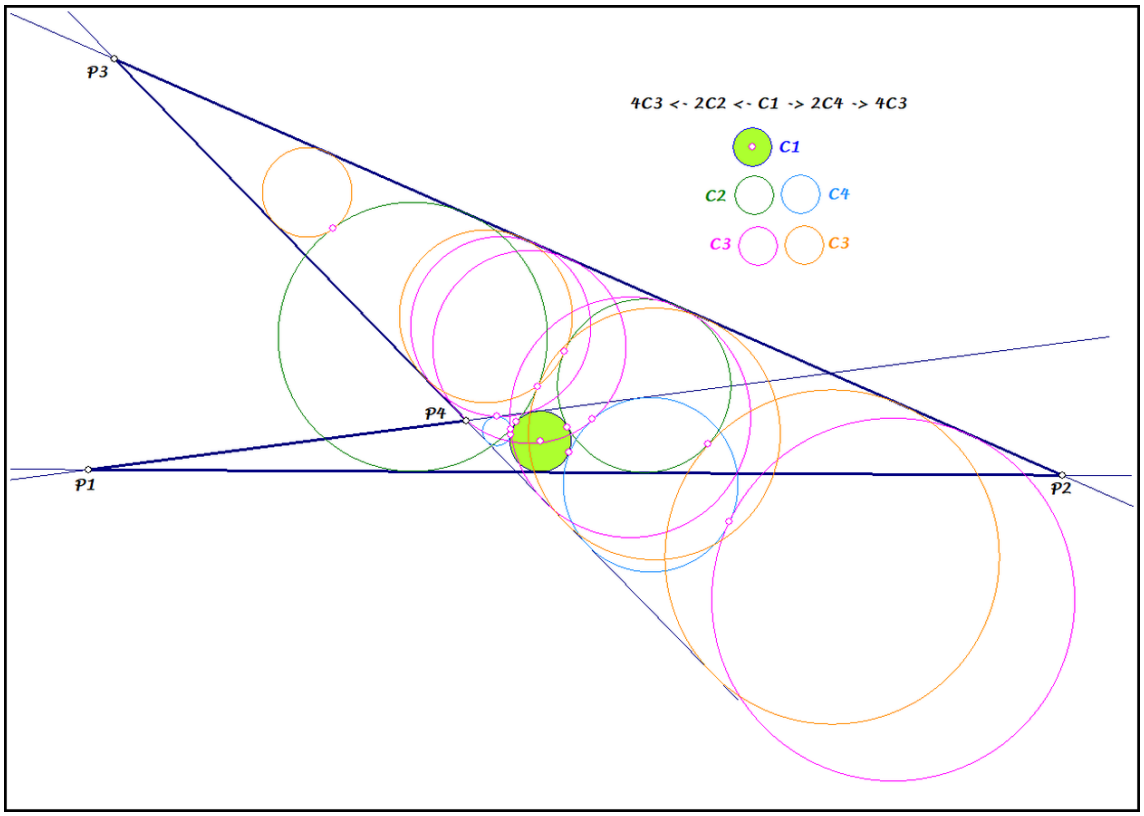
Message: #1249
Date: 08/8/2015 9:24:20
From: eckart_schmidt@t-online.de
Subject: Malfatti Circles

Dear Antonio, dear Chris,
before holiday an observation wrt the Malfatti Circles in a reduced case:
Let P1 P2 P3 P4 be a quadrigon and Ci circles, ...
... centered on the angle bisector of Pi-1, Pi, Pi+1
... and tangent to Pi-1Pi and PiPi+1
... and outside tangent to Ci-1 and Ci+1.
Then there are for a beginning circle C1 two constructable circles C2 and two circles C4 with the properties above. In the next step there are four circles C3 wrt C2 and four circles C3 wrt C4. Changing C1 I observed 8 possibilities, that C3 of the two sets coincide and give a Malfatti-constellation.
For Chris there is a Cabri-file attached. Please move the center of C1.
Best regards Eckart



2015-08-08-c-fig.png

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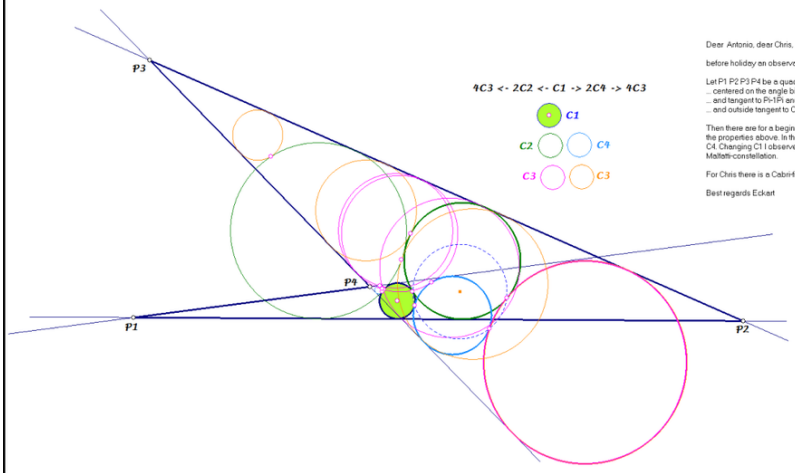
Message: #1250
Date: 09/8/2015 9:12:13
From: chris.vantienhoven
Subject: Malfatti Circles

Dear Eckart, dear Antonio,
Eckart, very nice next step for this problem!
Some comments:
* Let's call the first circle from which other circles are derived the "base circle"□.
* Only the base circle with its center on one angle bisector is drawn. There is a second angle bisector perpendicular to the first one. So probably there are more solutions,
* Only 2 direct circles tangent the base circle are drawn. Like I showed in QFG-message # 1247 there are at least 4 possible tangent circles. Possibly this generates more solutions.
For Antonio I made some pictures of 6 possible solutions I found using Eckart's Cabri-picture. See attachment.
The relevant circles that are mutually tangent are displayed bold. I also drew the circle through the relevant points of tangency (dotted circle) like predicted by Antonio in QFG-message # 1243.
Just like Eckart I will be leaving on holiday soon, but I will look during my holiday for more messages.
Best regards,
Chris
p.s. Antonio, I liked your construction where the circles were transformed by an inversion. Unfortunately it looks like this transformation doesn't lead to a direct solution. However it showed me the relationship with the "tangency-circle"□ (circle through the mutual points of tangency of involved circles).

6 Pictures of Malfatti Configurations based on Cabri-drawing of Eckart Schmidt

(the dotted circle is the circle through the 4 points of tangency of the 4 Malfatti circles in a quadrangle)



4C3 <- 2C2 <- C1 -> 2C4 -> 4C3

Dear Antonio, dear Chris,

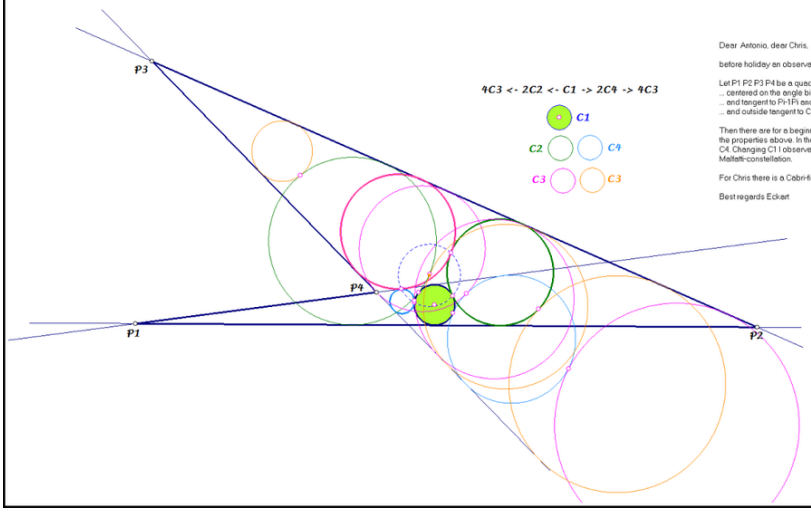
before holiday an observation wrt the Malfatti Circles in a reduced case:

Let P1 P2 P3 P4 be a quadrigon and Ci circles, ...
... centered on the angle bisector of Pi-1, Pi, Pi+1
... and tangent to Pi-1Pi and PiPi+1
... and outside tangent to Ci-1 and Ci+1.

Then there are for a beginning circle C1 two constructable circles C2 and two circles C4 with the properties above. In the next step there are four circles C3 wrt C2 and four circles C3 wrt C4. Changing C1 I observed 8 possibilities, that C3 of the two sets coincide and give a Malfatti-constellation.

For Chris there is a Cabri-file attached. Please move the center of C1.

Best regards Eckart



4C3 <- 2C2 <- C1 -> 2C4 -> 4C3

Dear Antonio, dear Chris,

before holiday an observation wrt the Malfatti Circles in a reduced case:

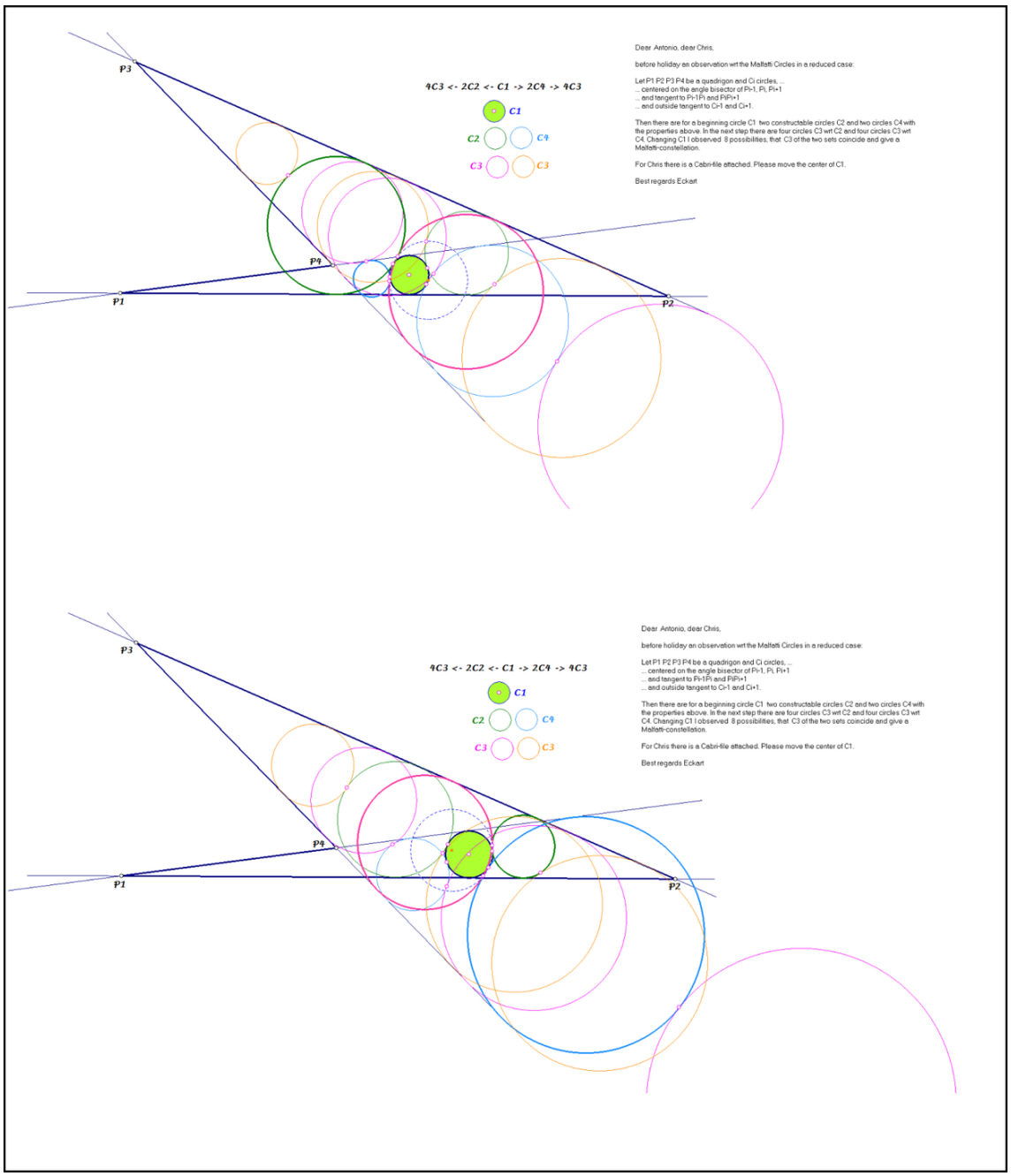
Let P1 P2 P3 P4 be a quadrigon and Ci circles, ...
... centered on the angle bisector of Pi-1, Pi, Pi+1
... and tangent to Pi-1Pi and PiPi+1
... and outside tangent to Ci-1 and Ci+1.

Then there are for a beginning circle C1 two constructable circles C2 and two circles C4 with the properties above. In the next step there are four circles C3 wrt C2 and four circles C3 wrt C4. Changing C1 I observed 8 possibilities, that C3 of the two sets coincide and give a Malfatti-constellation.

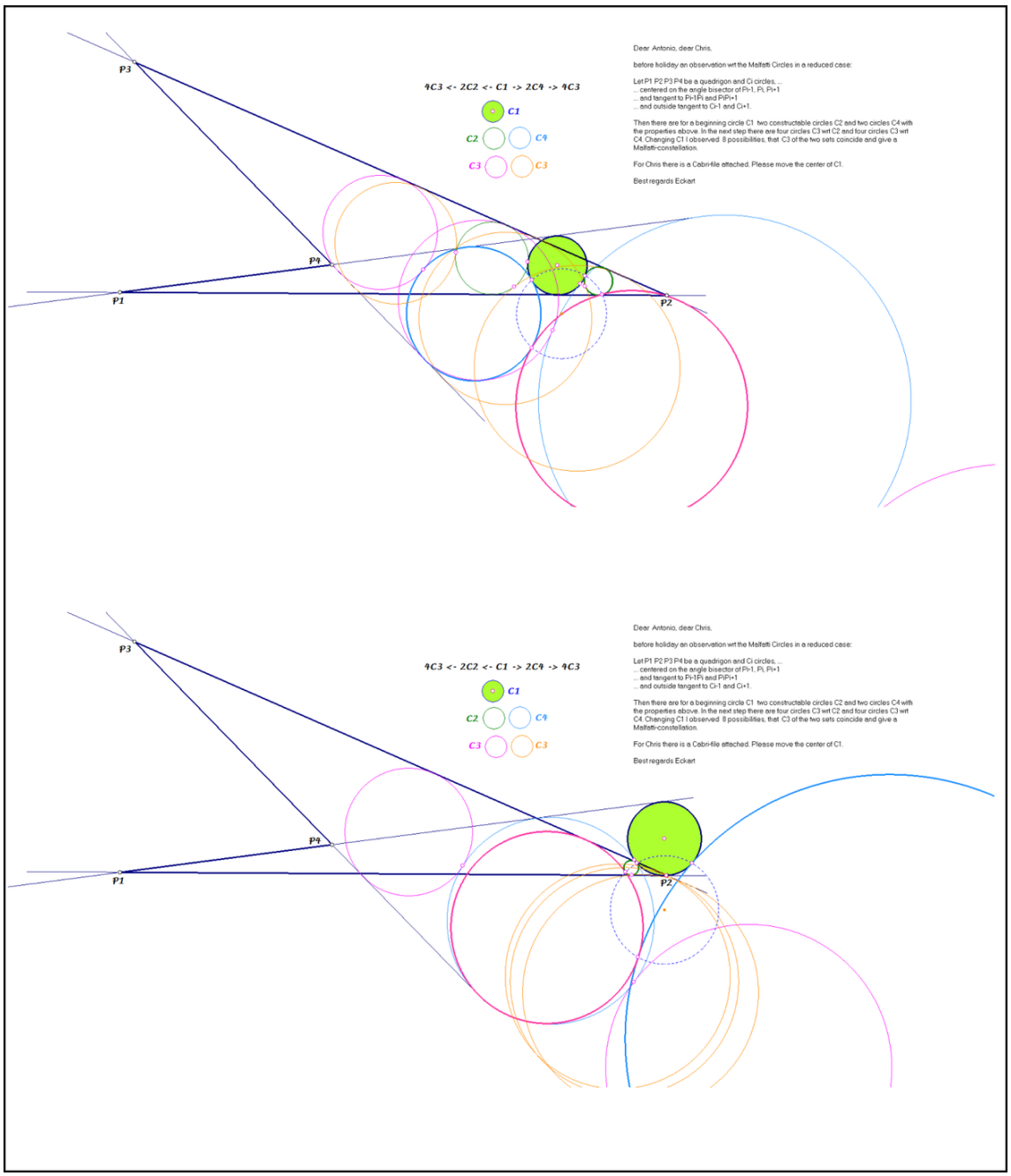
For Chris there is a Cabri-file attached. Please move the center of C1.

Best regards Eckart

6 Pictures of Malfatti Configurations.pdf



6 Pictures of Malfatti Configurations.pdf



6 Pictures of Malfatti Configurations.pdf

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Message: #1251
Date: 19/8/2015 4:46:09
From: seiichikiri
Subject: Concyclic antigonal images

Dear friends,
Let L1L2L3L4 be a quadrilateral.
Denote:
O1= the circumcenter of L2L3L4.
O2= the circumcenter of L1L3L4.
O3= the circumcenter of L1L2L4.
O4= the circumcenter of L1L2L3.
The antigonal images of O1, O2, O3 and O4 wrt L2L3L4, L1L3L4, L1L2L4 and L1L2L3 respectively are concyclic.
G.D. (graphically demonstrated.)
Best regards, Seiichi.

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Message: #1252
Date: 22/8/2015 4:47:57
From: Benedetto Scimemi
Subject: Two ”inverse” quadrangle transformations

Dear friends, in our recent QF correspondence, two quadrangle transformations have been pointed out by Ngo Quang Dong (July 6 and 10) and promptly examined by Chris and Eckart.
The first one, let me call it T, is obtained by intersecting the orThoTransversals of P on the component triangles of the quadrangle; the second one, let me call it L, is obtained by intersecting the pedaLcircLes of P on the component triangles.
I found both on AoPS (International Olympics Problems), with nice synthetic proofs of their being well-defined.
I recently examined them both graphically and analitycally and found some strict connections between them:
1) The actions P --> T and P--> L are the same for different quadrangles if the circumscribed conics C passing through P are the same; 2) When P moves on a given circumscribed conic, both T and L act as negative similarities, inverse of each other;
3) These similarities are easily described by the conic center, the eccentricity or the main central points of the quadrangle;
4) L induces on C the same mapping I described in a letter to Chris of Nov.14, 2014, ending with some old results on pentagons.
At that time I had not yet joined this Yahoo-group, but Chris did report a few details from my letter in his EQF, at page QA-Co3.
I am now attaching a revised version of those notes (sections 1-2) as well as an updating regarding T and L (sections 3-4-5), plus a pdf figure.
Incidentally, in my representation T and L produce rational functions of degree 3/2 and 5/4 resp., not too difficult to deal with.
More complicated -but perhaps less interesting- seems the third transformation (anti-pedal) to which Chris dedicated a few words in a following message.
I only found that QA-P4 is transformed into QA-P9 (well-known) and the circumcircles of the component triangles are fixed.
Best regards
Benedetto

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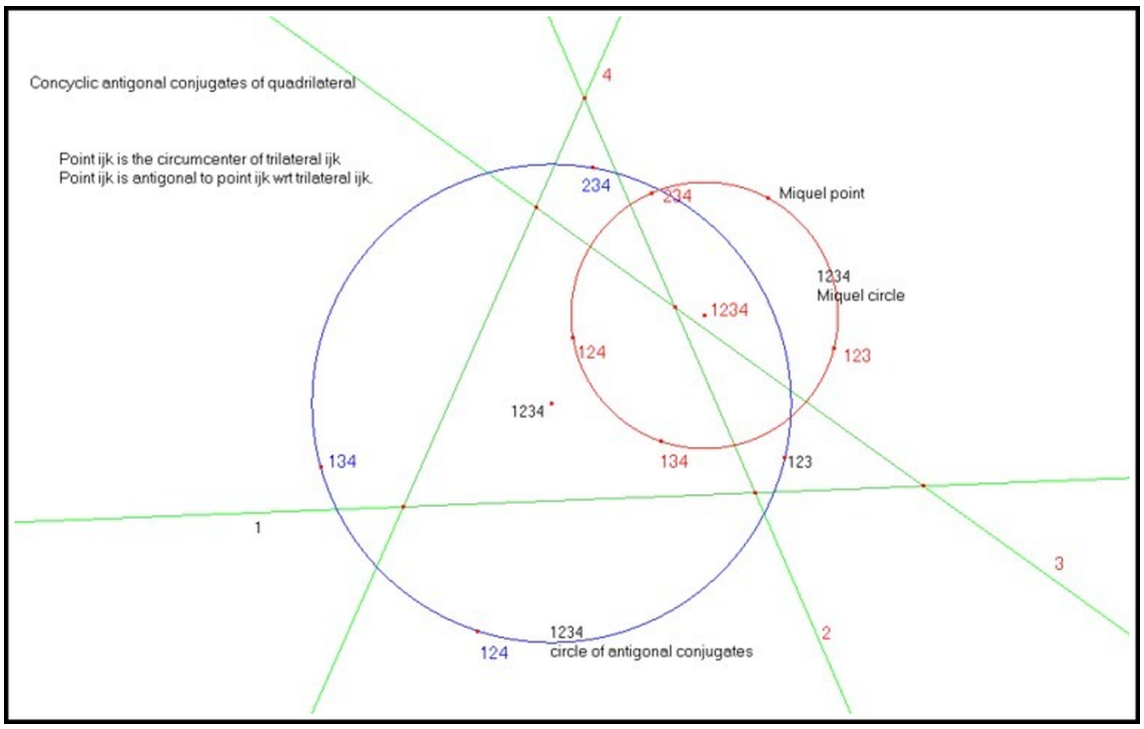
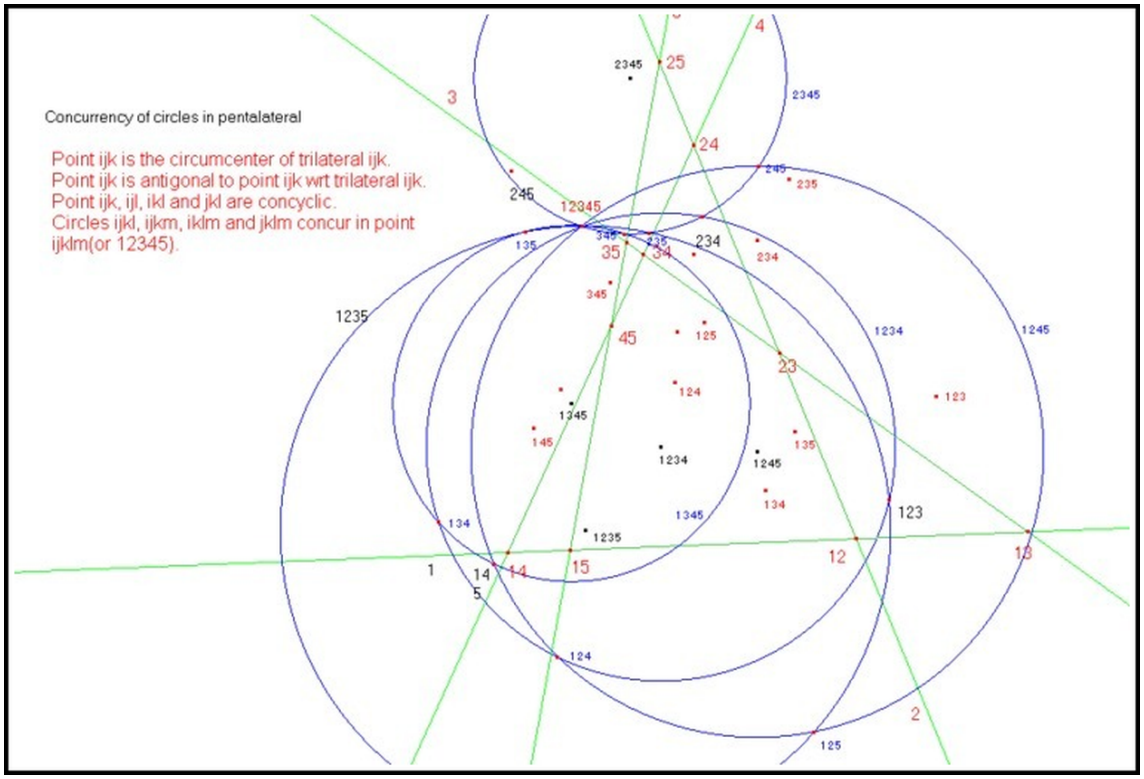
Message: #1253
Date: 24/8/2015 6:08:30
From: seiichikiri
Subject: Concurrency of circles in pentalateral

Dear friends,
In tripolar coordinates, the circumcenter X(3) satisfies the following equations.
   a^2=y^2+z^2-2yz cos(2A).
   b^2=z^2+x^2-2zx cos(2B).
   c^2=x^2+y^2-2xy cos(2C).
X(265) also satisfies them. X(265) is the antigonal conjugate of X(3).
I showed in message #1251 that in quadrilateral (4 lines), there is a circle through X(265)s of component trilaterals, corresponding to Miquel circle through X(3)s of component trilaterals.
Pentalateral (5 lines) case is as follows.
Let i (i=1,2,3,4,5) be five lines.
Denote:
ij= the intersection of lines i and j.
red ijk= X(3) (circumcenter) of trilateral ijk.
blue ijk=X(265) of trilateral ijk, antigonal to X(3).
blue ijkl= the circle through blue ijk, ijl, ikl and jkl.
circle ijkl, ijkm, ijlm, iklm and jklm concur in point ijklm(or 12345).
See the attached picture.
G.D.(graphically demonstrated.)
Best regards, Seiichi.

P.S. Also quadrilateral picture is attached.



antigonal conjugates in quadrilateral.png



antigonal conjugates in pentalateral.png

Message: #1254
Date: 24/8/2015 10:54:49
From: eckart_schmidt@t-online.de
Subject: Concyclic antigonal images

```
Dear Seiichi,
in message 1251 you found, that the intigonal images of the
circumcenters Oi wrt a QL are concyclic. The corresponding
circle has its center in the reflection of QL-P3 in QL-P2 on
QL-L4 and contains CSC(QL-P4).
Best regards Eckart
```

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Message: #1255
Date: 24/8/2015 12:32:55
From: eckart_schmidt@t-online.de
Subject: Concurrency of circles in pentalateral

```
Dear Seiichi,
in message 1253 you describe a 5L-point. This point is 5L-P8
(see message 710) and lies on the CSC-circle of 5L-P7 (see
message 769, 23).
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1256
Date: 25/8/2015 2:10:26
From: seiichikiri
Subject: Concurrency of circles in pentalateral

```
Dear Eckart, dear friends,
I found message #82, Jun 25,2013, which described almost the
same as my recent messages #1251, 1253.
Sorry for posting the irrelevant ones!
Best regards, Seiichi.
```

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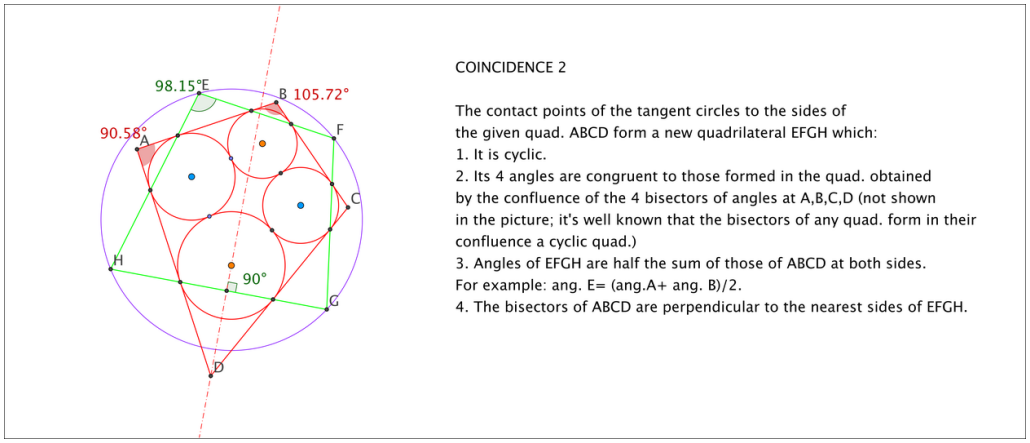
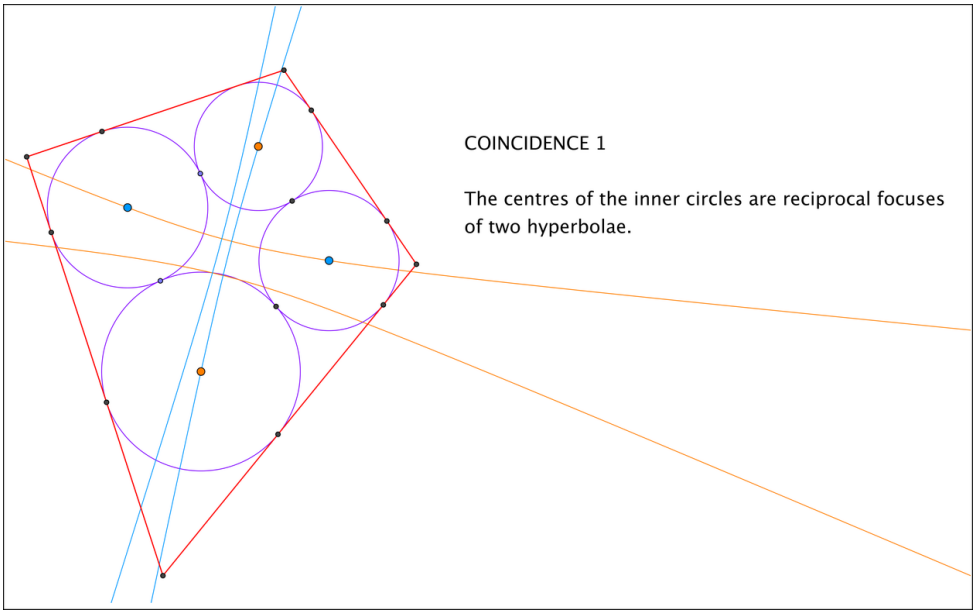
Message: #1257
Date: 25/8/2015 7:15:28
From: chris.vantienhoven
Subject: Malfatti Circles

Dear friends I forward this message from Antonio Sánchez Briones:
HALLO, FRIENDS.
I´VE BEEN PLAYING A LITTLE WITH MY PROBLEM ON MALFATTI FOR QUADS. I DARE NOT SAY I'VE DISCOVERED ANYTHING, BUT JUST SEEN SOME "COINCIDENCES". HERE THEY ARE IN MY 2 ATTACHMENTS.
I SAW WITH MUCH INTEREST THE FILE ABOUT THE NUMBER OF POSSIBLE CIRCLES THAT CAN SOLVE THE PROBLEM.
THANKS A LOT.



Coincidence 2.pdf



Coincidence 1.pdf

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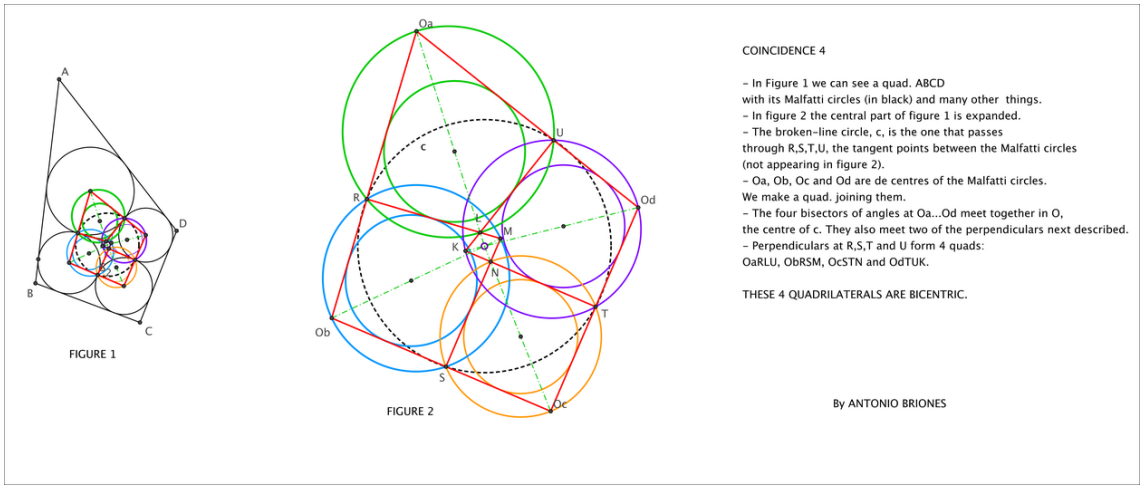
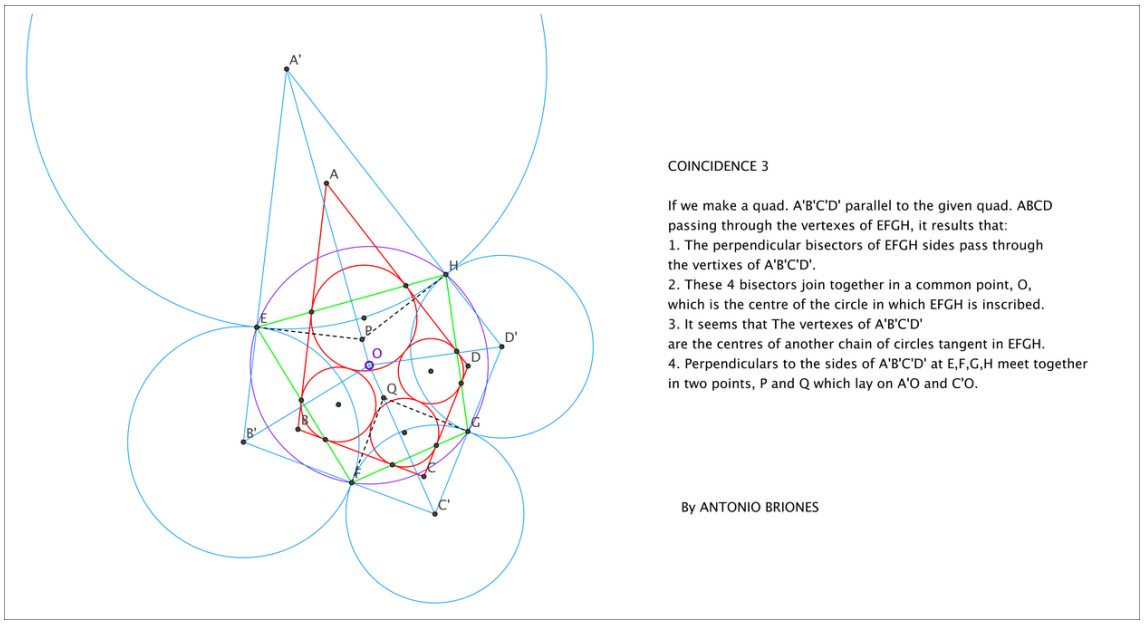
Message: #1258
Date: 26/8/2015 8:13:17
From: chris.vantienhoven
Subject: Malfatti Circles

Another message with attachments from Antonio Sánchez Briones:
THIS SEEMS NOT TO HAVE AN END.
SOME COINCIDENCES ARE REALLY BEAUTIFUL.
REGARDS.
ANTONIO.



COINCIDENCE 4.pdf



COINCIDENCE 3.pdf

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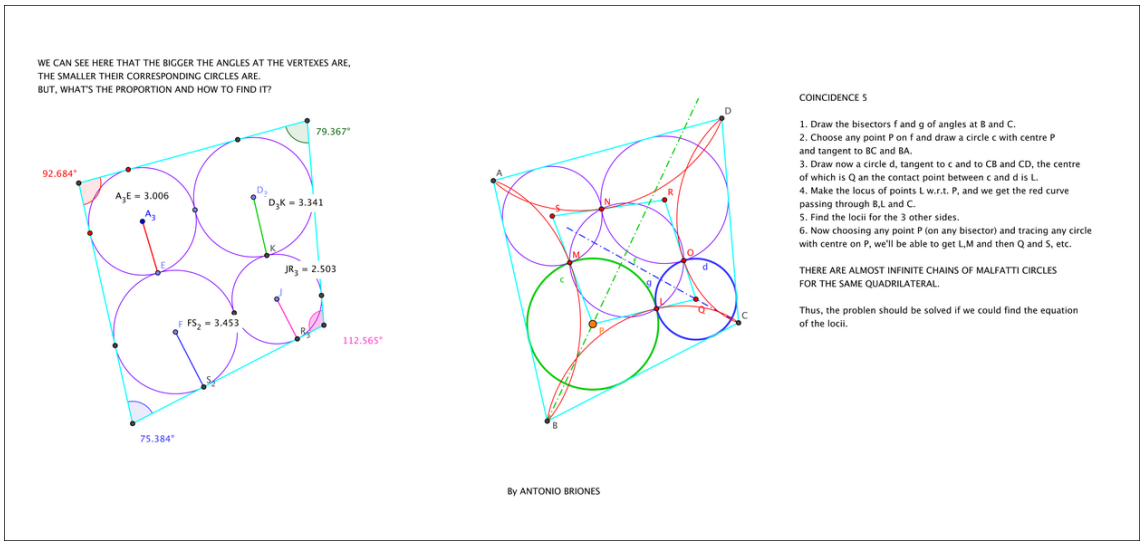
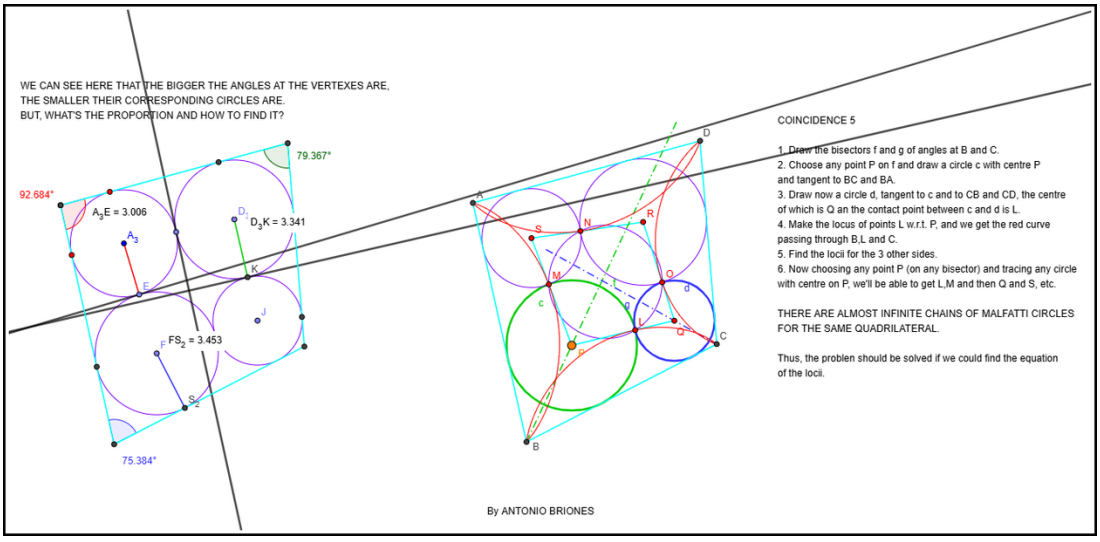
Message: #1259
Date: 27/8/2015 7:46:06
From: chris.vantienhoven
Subject: Malfatti Circles

Another message with attachments from Antonio Sánchez Briones:
I ENCLOSE A NEW STUDY. I THINK THAT A MATHEMATICIAN (WHO'S NOT ME) COULD SOLVE THE PROBLEM HEREAFTER.
I include a .pdf and the .ggb file from which it comes from. ggb is a GeoGebra file, the application I work with. I believe you use Cabri; but GeoGebra is free to underload and very easy.
Using that file you'll be able to see dynamically the conclusion that there are lots of Malfatti chains for every specific quadrilateral.
Bye for now.



COINCIDENCE 5.pdf



COINCIDENCE 5-ggb.png

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Message: #1260
Date: 27/8/2015 9:21:03
From: eckart_schmidt@t-online.de
Subject: Malfatti Circles

Dear Antonio, dear Chris,
I think, Antonios conclusion in message 1259
"THERE ARE ALMOST INFINITE CHAINS OF MALFATTI CIRCLES ..."
can't be right, see my message 1249. Tracing a starting circle there are only 8 Malfatti constellations.
Best regards Eckart
PS. Two additional remarks:
In COINCIDENCE 5 point 3 there are two circles d.
COINCIDENCE 2 also holds, if the circles tangent to adjacent sides don't touch.

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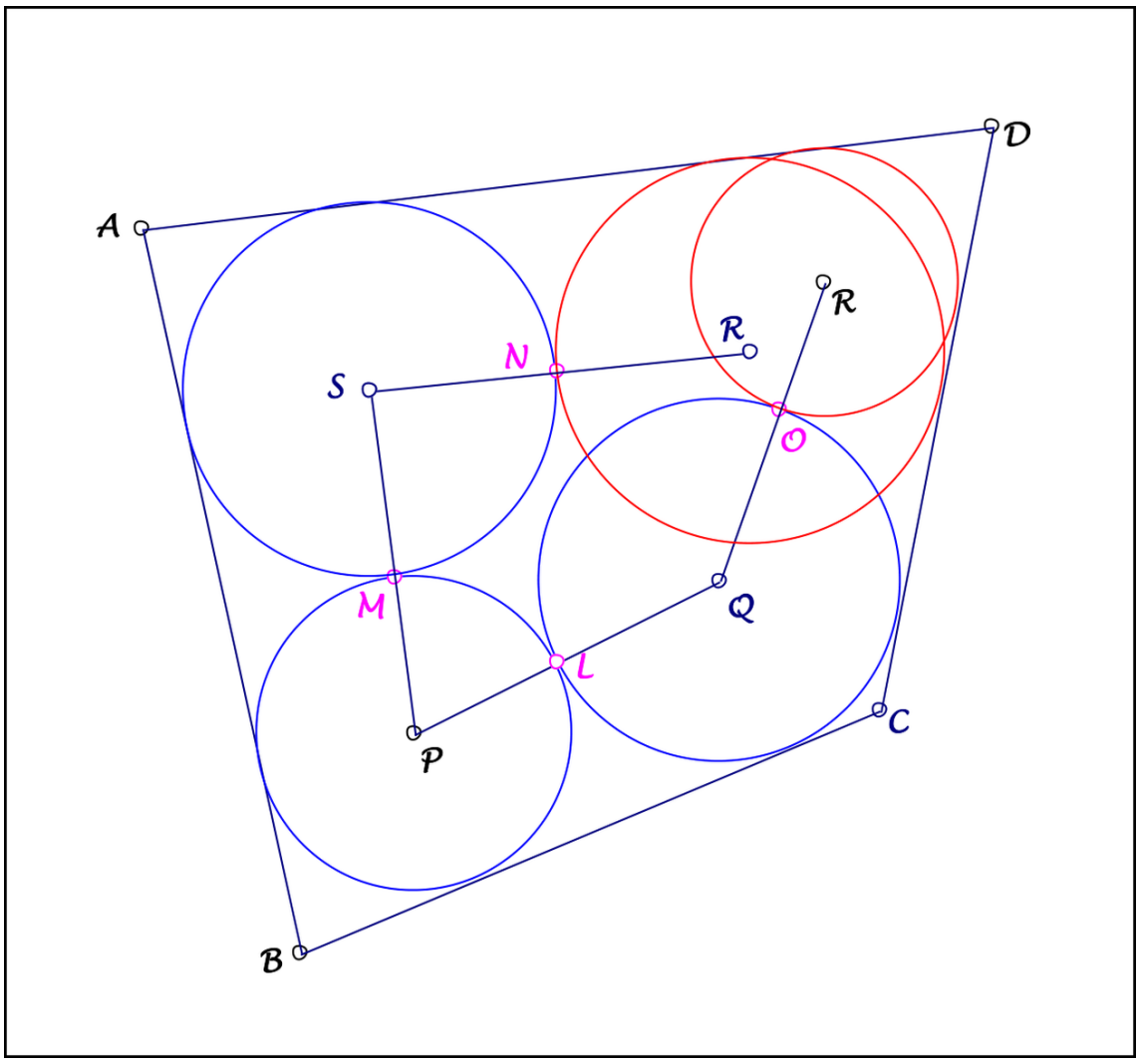
Message: #1261
Date: 28/8/2015 10:44:32
From: eckart_schmidt@t-online.de
Subject: Malfatti Circles

Dear Antonio, dear Chris,
the conclusion in COINCIDENCE 5 is right for tangential quadrigons, but not in general. Antonios drawing seems to show a tangential quadrigon. Following the construction in COINCIDENCE 5, point 6 must not lead to the same circle wrt D, see attached file.
Best regards Eckart
PS: Thanks to Antonio for his explinations in a private e-mail, but there remain doubts. Unfortunately I can't open a .ggb file, for I work with CABRI.



2015-08-28.pdf

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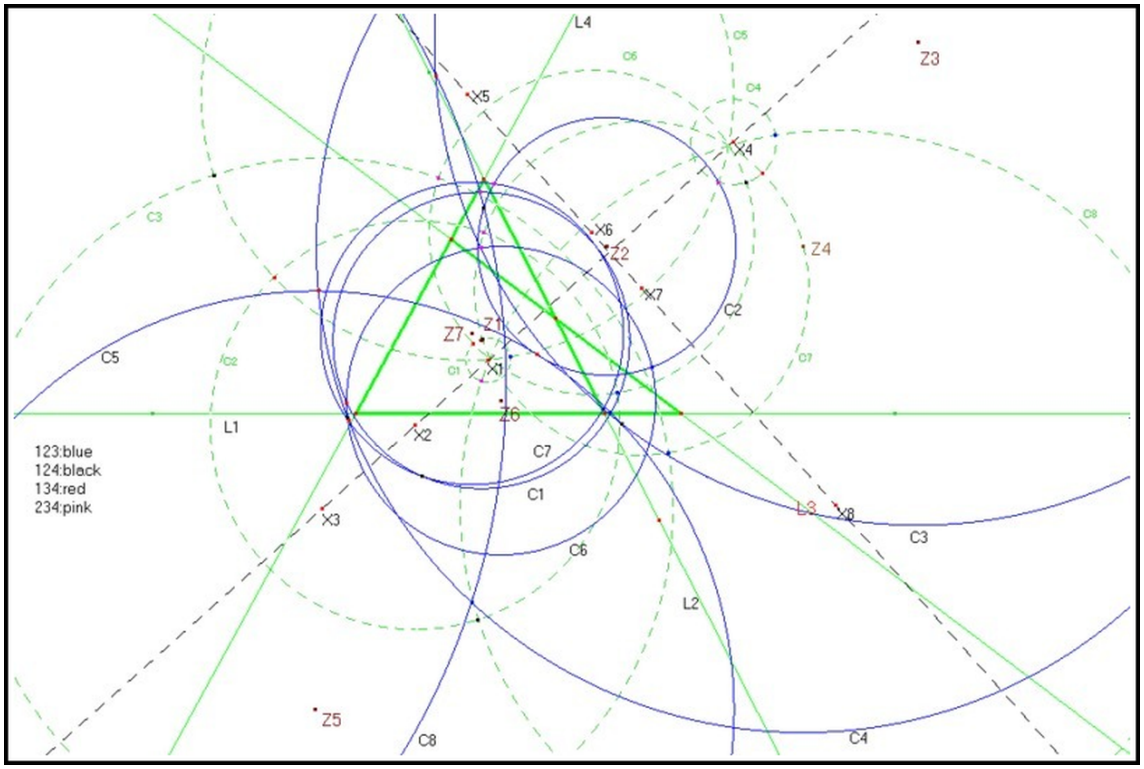
Message: #1262
Date: 01/9/2015 3:43:53
From: seiichikiri
Subject: Antigonal Octet

Dear friends,
Steiner Angle Bisector Octet is constructed by incenters and excenters of trilateral L1L2L3, L1L2L4, L1L3L4 and L2L3L4. They have common circles and their centers denoted by X1, .., X8.
See Encyclopedia of Quadri-Figures, Quadrilateral points and other objects, QL-8P1.
In a similar way, antigonal conjugates of those incenters and excenters have common circles and their centers denoted by Z1, .., Z8.
I could not find any relation among their centers.
See the attachment.
Best regards, Seiichi.



AntigonalOctetZ12345678.png

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Message: #1263
Date: 06/9/2015 4:21:10
From: eckart_schmidt@t-online.de
Subject: Some ”splitter” X

Dear Chris,
here are some additional properties wrt QG-points:
QG-P3 : (pt3) The three QA-versions of QG-P3 are collinear on the tripolar of QA-P16 wrt QA-Tr1.
QG-P5 : (pt11) The three QA-versions of QG-P5 lie on a circle through QA-P3, centered in the reflection of QA-P15 in QA-P1.
QG-P15 : (pt11) .... QA-P3 lies on its circumcircle, which is the reflection of QA-Ci1 in QA-P1.
QG-P18 : (pt3) ... Their centers of perspectivity are collinear, the centers wrt QG-P1/QG-P18 and QG-P1/QG-P19 are QA-Tf1-partners.
QG-P18 and QG-P19 : The QA-versions of the triangles QG-P1.QG-P18.QG-P19 are perspective in pairs. Their centers of perspectivity lie collinear in the intersections of QA-Cu7 and the sidelines of QA-Tr1.
Best regards Eckart

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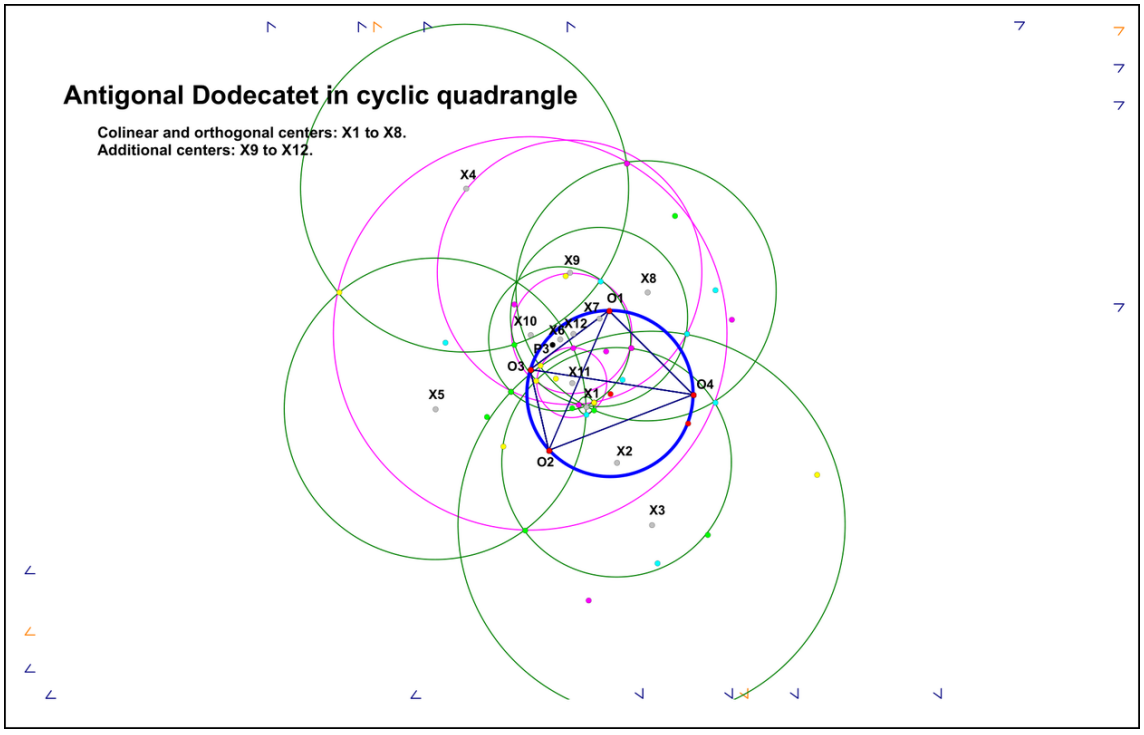
Message: #1264
Date: 07/9/2015 2:14:08
From: seiichikiri
Subject: Antigonal dodecatet in cyclic quadrangle

Dear friends,
Let$O_1O_2O_3O_4$ be a cyclic quadrangle.
(1) The in-ex-centers of its component triangles lie, 4 by 4, on 2 orthogonal sets of 4 lines.
See QL-8P1, Quadrilateral points and other objects, EQF.
(2) The antigonal conjugates of 4 points on a line with regard to their respective component triangles lie on a circle. For one set of 4 lines, there are 4 circles and their centers (X1 to X4), which are collinear. For the other set of 4 lines, there are 4 circles and their centers (X5 to X8), which are collinear. These 2 lines are perpendicular.
(3) If the cyclic quadrangle is one on Miquel circle (QL-Ci3), they intersect at Kantor-Hervey point (QL-P3).
(4) There are 4 additional circles and their centers, which are denoted by X9, X10, X11 and X12 (colored pink).
See the attachement.
G.D. (graphically demonstrated.)
Best regards, Seiichi.



AntigonalDodecatet.docx

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Message: #1265
Date: 08/9/2015 8:11:54
From: chris.vantienhoven
Subject: Some ”splitter” X

Dear Eckart,
I am always very curious for your splitters.
Here are some remarks of mine.
QG-P3 : (pt3) The three QA-versions of QG-P3 are collinear on the tripolar of QA-P16 wrt QA-Tr1.
QG-P5 : (pt11) The three QA-versions of QG-P5 lie on a circle through QA-P3, centered in the reflection of QA-P15 in QA-P1.
QG-P15: (pt11) ... QA-P3 lies on its circumcircle, which is the reflection of QA-Ci1 in QA-P1.
QG-P18: (pt3)   ... Their centers of perspectivity are collinear, the centers wrt QG-P1/QG-P18 and QG-P1/QG-P19 are QA-Tf1-partners.
QG-P18 and QG-P19 : The QA-versions of the triangles QG-P1.QG-P18.QG-P19 are perspective in pairs. Their centers of perspectivity lie collinear in the intersections of QA-Cu7 and the sidelines of QA-Tr1.
In general, what do you mean with (pt3) and (pt11) ?
QG-P3 : (pt3)
The three QA-versions of QG-P3 are collinear on the tripolar of QA-P16 wrt QA-Tr1.
Very nice!
QG-P5 : (pt11) The three QA-versions of QG-P5 lie on a circle through QA-P3, centered in the reflection of QA-P15 in QA-P1.
Very nice the circumcircle of a Triple Triangle being determined.
QG-P15: (pt11)
.... QA-P3 lies on its circumcircle, which is the reflection of QA-Ci1 in QA-P1.
Is actually evident because QA-P2 is on QA-Ci1 and QA-P3 is reflection of QA-P2 in QA-P1.
QG-P18: (pt3) ... Their centers of perspectivity are collinear, the centers wrt QG-P1/QG-P18 and QG-P1/QG-P19 are QA-Tf1-partners.
I do not understand, what is the center of perspectivity of QG-P18?
What are the centers wrt QG-P1/QG-P18 and QG-P1/QG-P19?
QG-P18 and QG-P19 : The QA-versions of the triangles QG-P1.QG-P18.QG-P19 are perspective in pairs. Their centers of perspectivity lie collinear in the intersections of QA-Cu7 and the sidelines of QA-Tr1.
Very nice construction of the intersection points of QA-Cu7 with the 3 diagonals of the Reference Quadrangle.

I will include most of your items in EQF (I collect them and publish later).
Best regards,
Chris

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Message: #1266
Date: 08/9/2015 8:57:13
From: eckart_schmidt@t-online.de
Subject: Some ”splitter” X

Dear Chris,
with (pt3) I mean: point number 3 of the properties in EQF.
Wrt QG-P18: "... center (of perspectivity) wrt QG-P1/QG-P18 ..." means: For a QA there is a QG-P1 triangle and a QG-P18 triangle, which are perspective.
Thanks for interest!
Best regards Eckart

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Message: #1267
Date: 08/9/2015 9:34:03
From: chris.vantienhoven
Subject: Some ”splitter” X

Dear Eckart,
Now I see what you mean.
This also can be found at: QA-Tr-2.
Best regards,
Chris

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Message: #1268
Date: 09/9/2015 12:05:46
From: eckart_schmidt@t-online.de
Subject: Some ”splitter” X

```
Dear Chris,
excuse, I have never studied page QA-Tr-2. I don't know why!
But I can replace some unknown points QA-Px wrt the triple
triangles of ...
... QG-P1  and QG-P16: point at infinity of QA-L4,
... QG-P2  and QG-P8 : point, dividing QA-P1.QA-P10
                       with ratio 3:2,
... QG-P4  and QG-P12: intersection of QA-P5.QA-P16
                       and the polar of QA-P16 wrt the
                       circumconic
                       of QA-Tr1 and QA-P10, QA-P17,
... QG-P12 and QG-P15: intersection QA-P1.QA-P17
                       and QA-P16.QA-P20
                       (mentioned in EQF at QA-Cu2),
... QG-P1  and QL-P12: point,
                       dividing QA-P1.QA-P10 with ratio 3:4,
... QG-P3  and QL-P15: point,
                       dividing QA-P1.QA-P10 with ratio 3:4,
... QG-P7  and QL-P2 : midpoint of QA-P1 and QA-P33,
... QG-P9  and QL-P6 : point,
                       dividing QA-P11.QA-P32 with ratio 2:1,
... QG-P12 and QL-P26: intersection of QA-P8.QA-P23
                       and QA-P2.QA-P7,
... QG-P15 and QL-P12: point,
                       dividing QA-P1.QA-P10 with ratio 3:2.
Please control!
Best regards Eckart
```

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Message: #1269
Date: 09/9/2015 8:36:28
From: eckart_schmidt@t-online.de
Subject: QL-Triple Lines

Dear Chris,
there is an analogon wrt perspectivity of QA-Triple Triangles (QA-Tr-2):
Example: Consider for a QL the triple of lines for QG-L1 and QG-L3 and you have as line of perspectivity QL-Tf2(QL-P8.QL-P13) containing QL-P12.
Perhaps there are further examples.
Best regards Eckart

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Message: #1270
Date: 11/9/2015 8:19:21
From: chris.vantienhoven
Subject: Some ”splitter” X

Dear Eckart,
Very interesting.
I will include these further specifications of QA-Px in EQF.
Best regards,
Chris

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Message: #1271
Date: 11/9/2015 8:41:00
From: chris.vantienhoven
Subject: QL-Triple Lines

Dear Eckart,
Interesting property of the QG-Lines applied in a Quadrilateral.
I also noticed that the perspector of the QG-L1 Triple Triangle and the QG-L3 Triple Triangle is the Infinity Point of the Newton Line QL-L1.
I agree there (most probably) will be more more examples.
it would be very interesting to investigate mutual relationships between Triple Triangles in a Quadrilateral.
I am sure there will be many Perspective, Cyclologic, Orthologic and Parallelologic properties.
Best regards,
Chris
p.s. For those not familiar with Triple Triangles in Quadrilaterals/Quadrangles see QA-Tr-1.

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Message: #1272
Date: 11/9/2015 11:42:34
From: eckart_schmidt@t-online.de
Subject: QA-Cubics as QL-Cu1

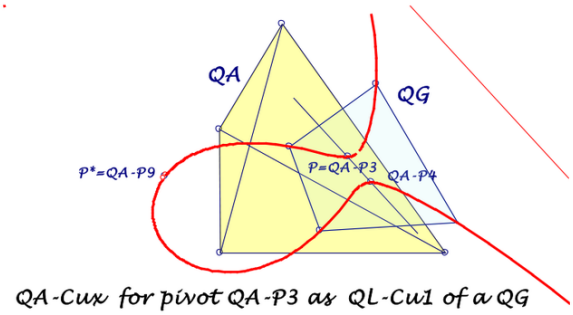
Dear friends,
For a non-cyclic quadrangle a cubic can be defined wrt a pivot P and the QA-Möbius Conjugate QA-Tf4 as transformation. This cubic QA-Cux – dependend on a pivot P – can be interpreted as QL-Quasi Isogonal Cubic QL-Cu1 of special quadrigons. See attached file.
Best regards Eckart

EQF-Note 2015-09-11

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

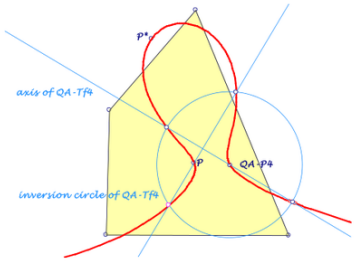
QA-Cubics as QL-Cu1

For a non-cyclic quadrangle a cubic can be defined wrt a pivot P and the QA-Möbius Conjugate QA-Tf4 as transformation. This cubic QA-Cux – dependend on a pivot P – can be interpreted as QL-Quasi Isogonal Cubic QL-Cu1 of special quadrigons. – The results are only Cabri-controlled.



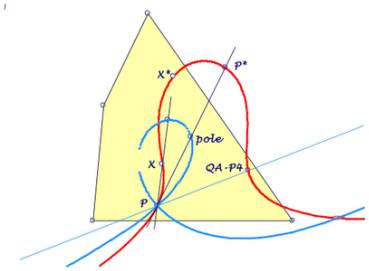
QA-Cux for pivot QA-P3 as QL-Cu1 of a QG

- **Definition of the cubic for non-cyclic quadrangles: The cubic *QA-Cux* is the locus for intersections of lines through a pivot *P* and their *QA-Tf4*-image circles.**
- ***QA-Cux* is invariant wrt *QA-Tf4* (*).**
- **The pivot *P*, its *QA-Tf4*-image *P** and *QA-P4* lie on *QA-Cux*, also the fixed points of *QA-Tf4*.**



- **Further points on *QA-Cux* are the intersections (≠ pivot *P*) of a perpendicular line through *P* wrt the *QA-Tf4*-axis and the inversion circle of *QA-Tf4*.**

- **The asymptote of *QA-Cux* is a parallel to *P.QA-P4* through the reflection of *P** in *P.QA-P4*.**

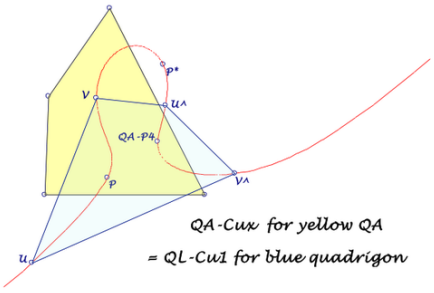


- **The midpoints of *QA-Tf4*-partners on *QA-Cux* give a strophoid of *P.QA-P4* with fixed point *P* and pole in the midpoint of *P.P****.

QA-Tf4 and *QL-Tf1* are Möbius transformations, which are the reflection in an axis followed by an inversion wrt a circle, centered on the axis (see *EQF*). A Möbius transformation is determined by the center *Z* of the inversion circle and two image partners *X, Y*. For *QL-Tf1*: *Z=QL-P1* and *X, Y* e.g. opposite vertices of the *QL*, for *QA-Tf4*: *Z=QA-P4* and *X, Y* e.g. *QA-P3*, *QA-P9* (see *EQF*). Wrt the here discussed *QA-Cux* let us consider …

… a Möbius transformation (^) with center *P** **and image partners *P* and *QA-P4* (or the fixed points of *QA-Tf4*).**

- ***QA-Cux* is invariant wrt this Möbius transformation.**

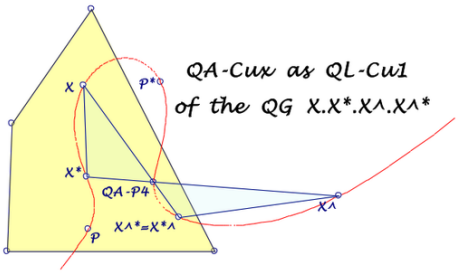


- **For two points *U* and *V* on *QA-Cux* the quadrigon *U.V.U^.V^* has *QA-Cux* as *QL*-Quasi Isogonal Cubic *QL-Cu1*.**

- **For *X* on *QA-Cux* the images *QA-Tf4(X)* wrt the quadrangle and *QL-Tf1(X)* wrt *U.V.U^.V^* are collinear with the pivot *P*.**

- ***QA-Cux* is always unipartite: Pivot *P* and *QA-P4* are the Isogonal conjugated Newton Pair of Points *QL-2P2* of *U.V.U^.V^*.**

- **For points *X* (≠ *P*, *P**, *QA-P4*) on *QA-Cux* the quadrigon *X.X*.X^.X^** (with diagonal crosspoint *QA-P4*) has *QA-Cux* as its *QL-Cu1*.**



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-09-11.pdf

Message: #1273
Date: 11/9/2015 2:06:54
From: eckart_schmidt@t-online.de
Subject: New construction of QL-Cu1?

```
Dear Bernard, dear Chris,
I don't know, whether the following construction of QL-Cu1 is
already mentioned:
QL-Cu1 is the locus for intersections of ...
... parallels wrt QL-L1
... and the CSC-circles of their reflections in QL-L1.
Best regards Eckart
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Message: #1274
Date: 11/9/2015 3:10:11
From: chris.vantienhoven
Subject: New construction of QL-Cu1?

```
Dear Eckart,
This construction method of QL-Cu1 is new to me.
Best regards,
Chris
```

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Message: #1275
Date: 12/9/2015 8:41:31
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 self-isogonal-conjugated wrt...

Dear Bernard, dear Chris,
I think, this isn't mentioned in EQF:
QL-Cu1 is self-isogonal-conjugated wrt triangles A.B.QL-P1:
A,B intersections of QL-Cu1 and a parallel of QL-L1 (e.g. A, B = QL-2P2).
For points on QL-Cu1 this isogonal-conjugate is the CSC-transformation QL-Tf1.
Best regards Eckart

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Message: #1276
Date: 14/9/2015 9:10:19
From: minhtenladuong
Subject: Isogonal Center QA-P4

Dear Geometers,
I have found a property of Isogonal Center. Is this known or trivial?
Given 4 point P1P2P3P4. S1S2S3 is the QA-diagonal triangle.
M12,M23,M31,M41,M42,M43 are midpoints of
P1P2,P2P3,P3P1,P4P1,P4P2,P4P3
T1 is isogonal conjugate point of QA-P2,
   with respect to triangle S1M14M23
T2 is isogonal conjugate point of QA-P2,
   with respect to triangle S2M24M31
T3 is isogonal conjugate point of QA-P2,
   with respect to triangle S3M34M12
Then T1,T2,T3 are midpoints of S1QA-P4,S2QA-P4,S3QA-P4
Best regard
Ngo Quang Duong

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Message: #1277
Date: 14/9/2015 9:18:29
From: minhtenladuong
Subject: A Remark on QA-Tf3

Dear Geometers
Let P be an arbitary point, S1S2S3 is QA-Diagonal Triangle.
Orthopole of PH with respect to anticomplement triangle of
S1S2S3 is P*
Where H is QA-P12, orthocenter of S1S2S3.
Then P* lies on the circle that passes through QAP2,
OP1,OP2,OP3,OP4 in http://www.chrisvantienhoven.nl/other-quadran
gle-objects/index.php/15-mathematics/encyclopedia-of-quadri-figu
res/quadrangle-objects/artikelen-qa/229-qa-tf3.html
Reference: I have posted this to AoPS
Community - Art of Problem Solving (
http://artofproblemsolving.com/community/c6h1111858p5071720)
Best regard
Ngo Quang Duong

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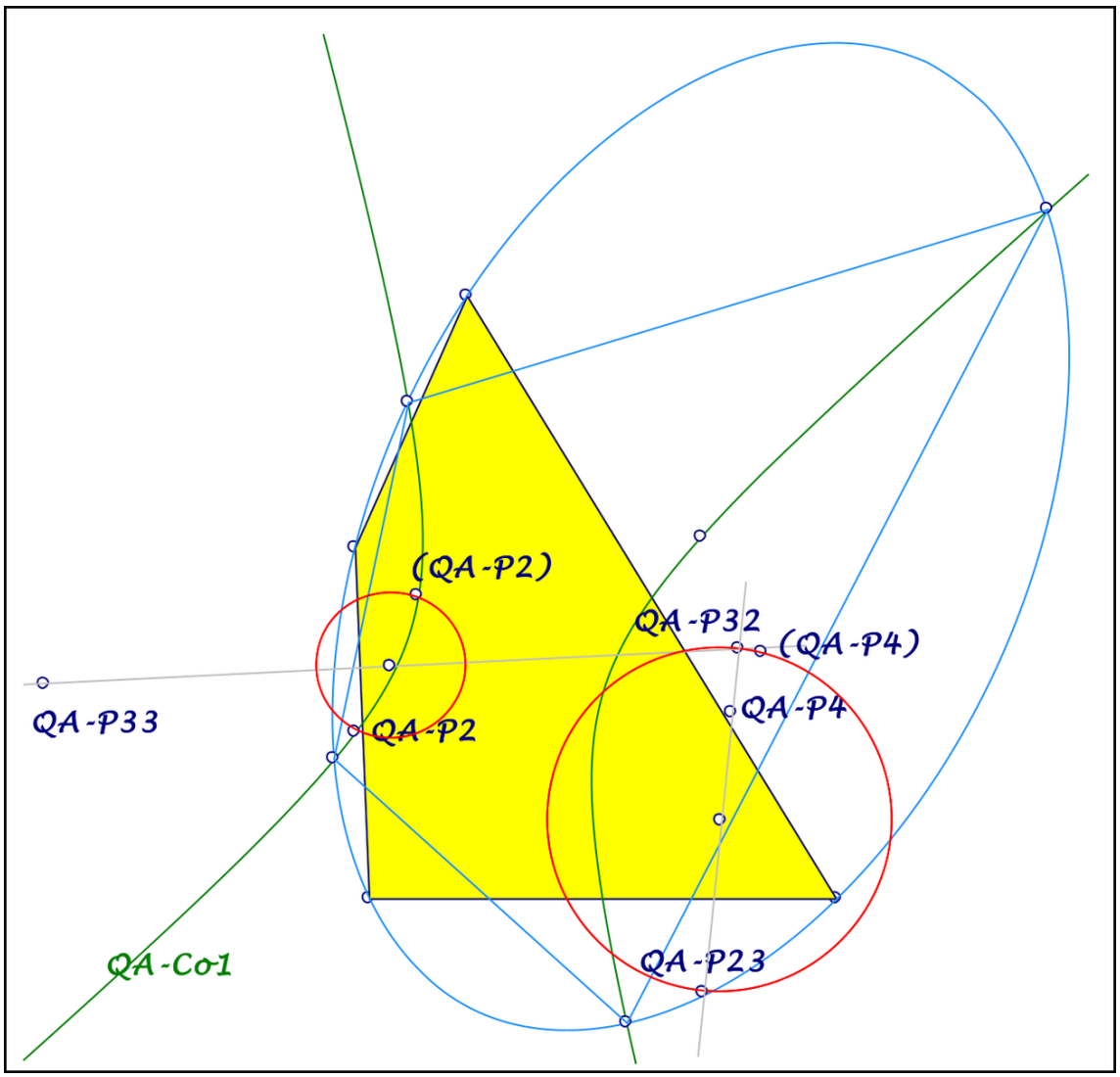
Message: #1278
Date: 15/9/2015 2:34:25
From: eckart_schmidt@t-online.de
Subject: QA-P2 and QA-P4

Dear friends,
once more I had a look in EQF-Ref [16]:
http://geometria.ch/geometrie/empdv.pdf (
http://mathematicus.ch/geometrie/empdv.pdf)
This article is the background for the following properties, perhaps not mentioned in EQF:
wrt 3: QA-P4 is also the Isogonal Center for the circumcenters of the QA-triangles.
wrt 8: The circumconic for the six reflections of QA-P4 in PiPj is centered in QA-P2.
wrt 9(1): QA-P4 is the QA-homothetic center for the circumcenters of the antipedal triangles of Pi wrt PjPkPl.
wrt 9(2): The circumcenters of the pedal triangles of Pi wrt PjPkPl have the Isogonal Center QA-P2.
wrt 9(3): The Simson line of QA-P2 wrt QA-Tr1 is a parallel of QA-P4.QA-P12 through QA-P36.
wrt 9(5): The circumcenters for the pedal triangles of QA-P4 wrt PiPjPk have as Isogonal Center QA-P2.
wrt 9(6): The polar of QA-P4 wrt QA-Co2 is the Steiner Line QL-L2 for the QL of the tangents in Pi at QA-Co2.
wrt 10: The angle bisectors of two opposite QA-lines - intersecting in S - are also the angle bisectors of S.QA-P2 and S.QA-P4.
wrt 11: For a QG the 4 intersections of diagonals and asymptotes of QA-Co2 have the same QA-P2 and QA-P4 as the reference QG.
wrt 12: For a QA-circumconic Co with center M the lines M.QA-P2 and M.QA-P4 are symmetrical wrt the axes of Co.
wrt 14: QA-P2 and QA-P4 for the intersections of QA-Co1 with QA-circumscribed conics lie on two circles (see attached file): circle through QA-P2, centered in the midpoint of QA-P32 and QA-P33 and Thales circle about QA-P23.QA-P32.
I hope there is no error!
Best regards Eckart



2015-09-15.docx

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Message: #1279
Date: 15/9/2015 4:36:39
From: eckart_schmidt@t-online.de
Subject: QA-Tr-1

Dear Chris,
under QA-Tr-1 wrt Perspective Triple-Triangles we find in EQF:
When Px = QG-P12 (http://www.chrisvantienhoven.nl/quadrigon-objects/16-mathematics/encyclopedia-of-quadri-figures/quadrigon-objects/artikelen-qg/177-qg-p12.html) or QL-P1 (http://www.chrisvantienhoven.nl/quadrilateral-objects/17-mathematics/quadrilateral-objects/100-ql-p1.html) , then Triple Triangle K2.K3.K4 is perspective with all 4 component triangles of P1.P2.P3.P4 (probably there will be more Quadri Centers with this property).
I think, this dosn't hold for QG-P12.
Examples you give in QA-Tr-2.
Best regards Eckart

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Message: #1280
Date: 15/9/2015 5:50:27
From: bernard.keizer
Subject: QL-Tf2 and QL-Cu1

Dear Eckart,
Old message, before holiday, but very interesting !
I hope I interprete correctly without figure.
Both axes of every inscribed conic are tangent to the cayleyan.
For the main axis L1 through 2 conjugate points X and X',
QL-Tf2(L1) is the perpendicular to L1 in the 3rd intersection X0 of L1 with the cubic QL-Cu1.
The intersections of L1 (middle of XX') and QL-Tf2(L1) with the Newton Line QL-L1 are symmetric wrt QL-P1. QL-Tf2(L1) is also tangent to the cayleyan and form with L1 the degenerared rectangular hyperbola which is the polar conic of the tangential X1 of X and X' (conjugate of X0).
If I understand you correctly, the same goes for the minor axis L2 ; I suppose this is true when Cu2 is Kjp and Cu1 bicursal, but what if Cu2 is Mac Cay and Cu1 unicursal? The points aren't real?
Thanks in advance for your answer!
Best regards
Bernard

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Message: #1281
Date: 15/9/2015 6:05:20
From: bernard.keizer
Subject: New construction of QL-Cu1?

Dear Eckart, dear Chris,
Another nice construction of QL-Cu1 !
If I refer to my messages 1209, which contained 3 constructons and 1216, which contained a 4th one, I observe that this one gives on a line parallel to the Newton Line Ql-L1 the 2 points I called X and X'1 (conjugate of X1) and on the line symmetric wrt QL-L1 the 2 points X' (conjugate of X) and X1.
The circle with diameter XX1 through QL-P1 is centered on the 1rst Steiner axis and orthogonal to the circle with diameter QL-2P3a and b.
The reason is that X, X', X1 and X'1 form with QL-P1 and the infinity point a QL inscribed in QL-Cu1.
Best regards
Bernard

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Message: #1282
Date: 15/9/2015 6:50:43
From: bernard.keizer
Subject: QL-Cu1 self-isogonal-conjugated wrt...

Dear Eckart, dear Chris
I have in mind and old song from the sixties, which states Sag warum ...
For an ordinary QL, QL-Cu1 is self-isogonal wrt the 4 reference triangles and invariant in the CSC-transformation QL-Tf1, the CSC-conjugates being isogonal (both properties are in EQF).
Now, the property holds for any triangle, of which the circumcircle is through QL-P1 ; the reason is that the conjugates are on a line (property of QL-Cu1) and the 4 lines form a QL inscribed in the same QL-Cu1, the triangle being then one of the 4 reference triangles of this new QL (this property is not in EQF) ...
The property holds also if the triangle has QL-P1 as one vertice, the 2 others being on a parallel to QL-L1, because QL-P1 plays a dopple role.
In this case, the line through QL-P1 parallel to the Newton Line QL-L1 (axis of the parabola) has for CSC the symmetric line wrt the Steiner axes (axis of the cardioïd QL-Qu1).
All the parallels to the Newton Line are tangent in the infinity point and all the circles through QL-P1 and 2 points on a parallel to the Newton Line, which are the CSC-conjugates of these lines, are tangent in QL-P1 to the same line and to the curve QL-Cu1 itself.
In fact, your property is a special case of the general property mentionned.
Best regards
Bernard

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Message: #1283
Date: 16/9/2015 8:38:09
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

Dear Chris,
is this already mentioned?
The intersections of QL-L1 and QL-Ci1 - not always real - are the centers of inscribed orthogonal hyperbolas of a QL.
Best regards Eckart

2015-09-16.docx

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Message: #1284
Date: 16/9/2015 12:08:31
From: chris.vantienhoven
Subject: Inscribed orthogonal QL-hyperbolas

Dear Eckart,
Sometimes something very elementary pops up and I wonder why didn't we see it before . . .
Your double orthogonal hyperbola related to a quadrilateral is one these !
Suddenly I realize:
* A random quadrilateral has only 1 inscribed parabola and 2 inscribed orthogonal hyperbola's.
* A random quadrangle has 2 circumscribed parabola's and 1 circumscribed orthogonal hyperbola.
Questions:
* 1. Any more references for this double orthogonal hyperbola?
* 2. Any proof there are 2 and only 2 orthogonal hyperbolas related to a random quadrilateral?
* 3. What are the equations in DT-format and CT-format?
* 4. Which (simple) construction(s) there are?
Best regards,
Chris

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Message: #1285
Date: 17/9/2015 1:02:31
From: Ken Pledger
Subject: Inscribed orthogonal QL-hyperbolas

>
> > On 16-9-2015, at 10.08 pm, van10hoven@gmail.com [Quadri-Figures-Group] <
> Quadri-Figures-Group@yahoogroups.com > wrote:
> > ….
> > Sometimes something very elementary pops up and I wonder why
> didn't we see it before . . .
> > Your double orthogonal hyperbola related to a quadrilateral is one these!
> > Suddenly I realize:
> > • A random quadrilateral has only 1 inscribed parabola and 2
> inscribed orthogonal hyperbola's.
> > • A random quadrangle has 2 circumscribed parabola's and 1
> circumscribed orthogonal hyperbola….

These follow from properties of pencils and ranges of conics.
All the conics touching the sides of a quadrilateral form a range. All the conics passing through the vertices of a quadrangle form a pencil.
A pencil of conics cuts a general line in pairs of points in involution. The double points of this involution show where just two conics in the pencil touch the line. Applying this to the line at infinity gives your two parabolas in the pencil.
Comparing this involution with the orthogonal involution at infinity, the pair of points common to both involutions lie on your rectangular hyperbola.
A range of conics has the dual properties.
Ken Pledger.

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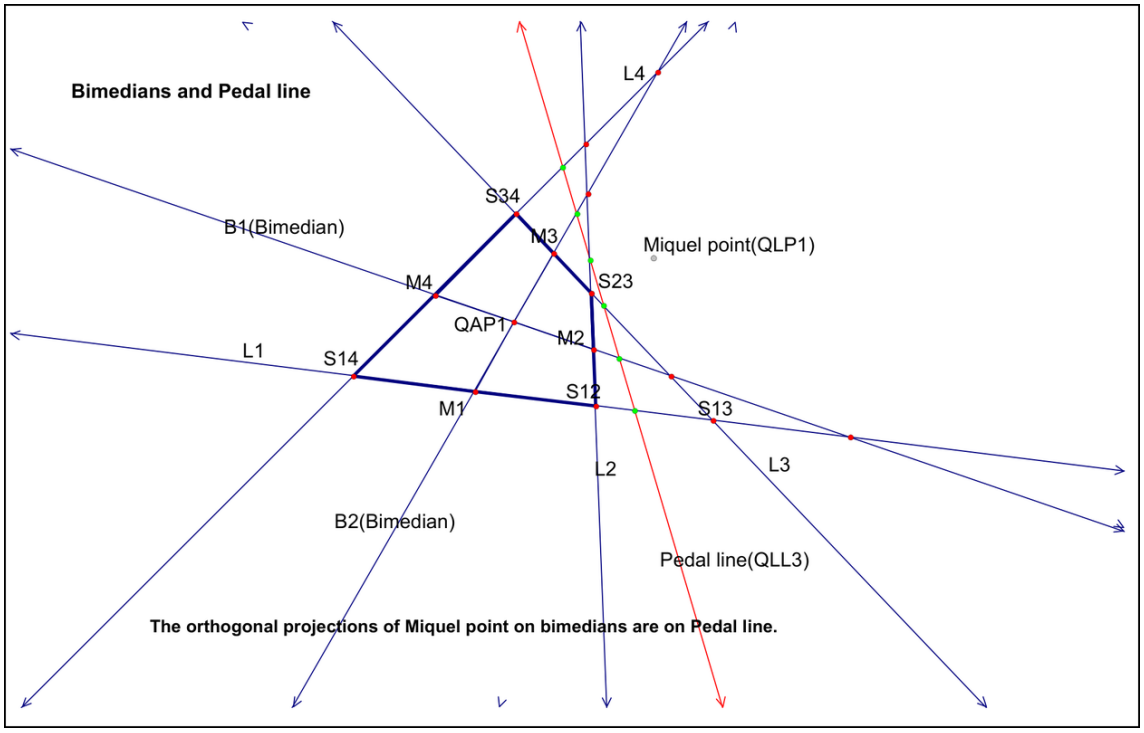
Message: #1286
Date: 17/9/2015 2:38:13
From: seiichikiri
Subject: Bimedians in quadrilateral

Dear friends,
Let L1L2L3L4 be a quadrilateral, QLP1 Miquel point and QLL3 Pedal line.
Denote:
Sij= the intersection of Li and Lj for i, j=1, 2, 3, 4 except for i=j.
B1(Bimedian) = the line through the midpoints of S12S23 and S14S34.
B2(Bimedian) = the line through the midpoints of S12S14 and S23S34.
1) The orthogonal projections of Miquel point (QLP1) on Bimedians (B1 and B2) are on Pedal line (QLL3).
2) Miquel points of L1L2B1B2, L2L3B2B1, L3L4B1B2 and L4L1B2B1 coincide with Miquel point (QLP1) of L1L2L3L4.
See the attachment.
G.D. (graphically demonstrated.)
The above projections are different from those in message #1141.
Best regards, Seiichi.



Bimedians and Pedal line.docx

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Message: #1287
Date: 17/9/2015 8:05:04
From: chris.vantienhoven
Subject: Inscribed orthogonal QL-hyperbolas

Dear Ken,
[KP] Comparing this involution with the orthogonal involution at infinity, the pair of points common to both involutions lie on your rectangular hyperbola.
A range of conics has the dual properties.
Thanks for the clarification!
Some additional questions:
* Why the pair of points common to both involutions lie on the rectangular hyperbola?
* Why the roles of parabolas and rectangular hyperbolas do change in the dual case?
Best regards,
Chris

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Message: #1288
Date: 17/9/2015 12:48:31
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

Dear Chris,
thanks for your remarks, here a CABRI observation:
The intersections of the two inscribed orthogonal QL-hyperbolas give the orthocentric quadrangle of the in-/ex-centers of QL-Tr1.
Best regards Eckart

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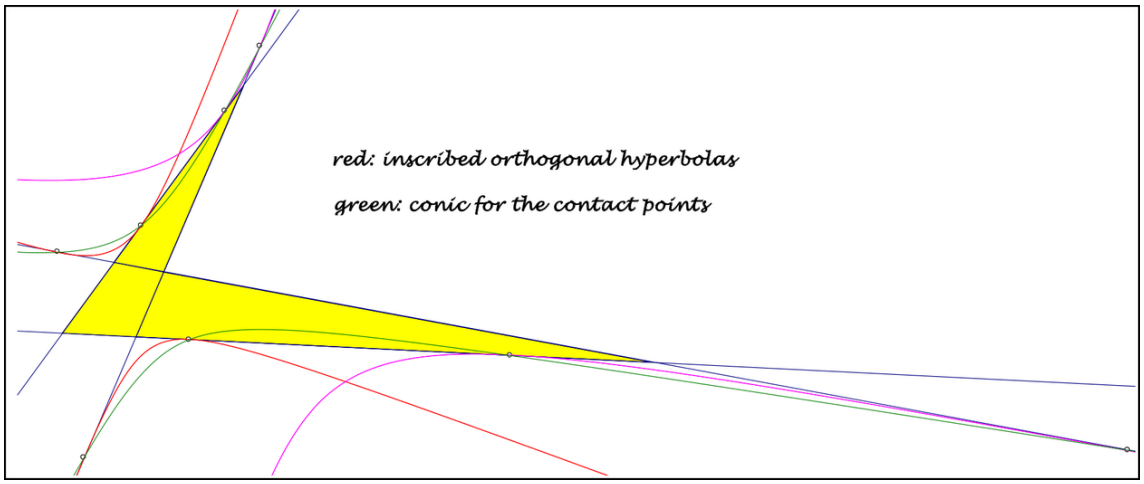
Message: #1289
Date: 18/9/2015 9:38:55
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

Dear Chris,
a further CABRI observation:
The 8 contact points of the two inscribed orthogonal QL-hyperbolas lie on a conic (see attached file), but I can't describe with EQF-objects.
Best regards Eckart



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Message: #1290
Date: 18/9/2015 9:57:18
From: seiichikiri
Subject: Bimedians in quadrilateral

Correction:
The properties of 1) and 2) in message #1286 are not restricted to B1 and B2 (bimedians).
Let L1L2L3L4 be a quadrilateral.
Denote:
Sij= the intersection of Li and Lj for i, j=1, 2, 3, 4 except for i=j.
R1= the line through the points which devide the segment S12S23 and S14S34 internally or externally in the same ratio.
R2= the line through the points which devide the segment S12S14 and S23S34 internally or externally in the same ratio.
1) Pedal line (QLL3) is the locus of the projection of Miquel point (QLP1) on R1.
2) Steiner line (QLL2) is the locus of the reflection of Miquel point (QLP1) in R1.
3) Inscribed quadrilateral parabola (QLCo1) is the envelope of R1.
As for R2, the same statement holds.
I hope Bernard's confirmation in the affirmative or in the negative.
Best regards, Seiichi.

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Message: #1291
Date: 19/9/2015 12:24:41
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

Dear Chris,
another Cabri observation:
The QL-Diagonal Triangle is self-polar wrt the conic for the contact points of the inscribed orthogonal hyperbolas.
Best regards Eckart

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Message: #1292
Date: 20/9/2015 8:35:57
From: seiichikiri
Subject: Inscribed orthogonal QL-hyperbolas

Dear Chris, dear Eckart,
I think that the table which " suddenly you realized" in message #1284 needs the following column.
For quadrangle, a circumscribed conic with least eccentricity, QA-Co3.
For quadrilateral, an inscribed conic with least eccentricity.
Is the last one already solved?
Best regards, Seiichi.

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Message: #1293
Date: 20/9/2015 8:56:03
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

Dear Seiichi,
inscribed orthogonal QL-hyperbolas are not mentioned in EQF, for we haven't searched for, astonishing!
A week ago it was an own experience with Cabri, but today I found a web site:
http://www.math.uoc.gr/~pamfilos/eGallery/problems/RectHyperbolasTangent4Lines.html
... with the result: Only QL with obtuse angled triangle components have inscribed orthogonal hyperboloas.
The polar circles of these triangle components have two common points, which are the intersections of the Newton line QL-L1 and the circumcircle of the QL-diagonal triangle. These two points are the centers of inscribed orthogonal hyperbolas, as I mentioned in message 1283.
Best regards Eckart

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Message: #1294
Date: 21/9/2015 12:43:43
From: Ken Pledger
Subject: Inscribed orthogonal QL-hyperbolas

The asymptotes are the tangents at infinity, obtained by joining the centre to the two points at infinity on the hyperbola. If those two points are a pair in the orthogonal involution, then the asymptotes are perpendicular.
A pencil of conics includes one conic passing through a general point, but two conics touching a general line. A range of conics includes one conic touching a general line, but two conics passing through a general point. The unique conic in the range which touches the line at infinity is a parabola. But I was a bit hasty in just appealing to duality to get two rectangular hyperbolas in the range, so that needs more thought. Sorry! :-(
Ken Pledger.

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Message: #1295
Date: 21/9/2015 5:59:14
From: seiichikiri
Subject: Inscribed orthogonal QL-hyperbolas

Dear Eckart,
Thank you very much for the information. In the picture of the site, I can not discern 2nd inscribed rectangular hyperbola. Can you see it?
I will ask Dr. Pamiflos.
"2 solutions or no solution" is reminiscent of quadratic equation.
Best regards, Seiichi.

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Message: #1296
Date: 21/9/2015 3:45:30
From: eckart_schmidt@t-online.de
Subject: Inscribed orthogonal QL-hyperbolas

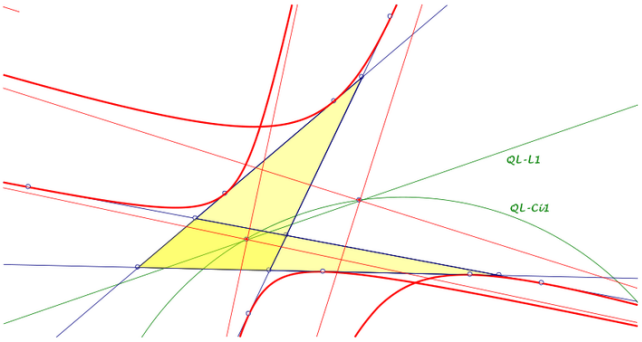
Dear friends,
I have gathered the results of my studies wrt inscribed orthogonal QL-hyperbolas in EQF-geometry, see attached file.
Best regards Eckart
PS. (for Seiichi) The figur on the web site doesn't show the 2nd hyperbola, which has the center O2.

EQF-Note 2015-09-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Inscribed Orthogonal QL-Hyperbolas

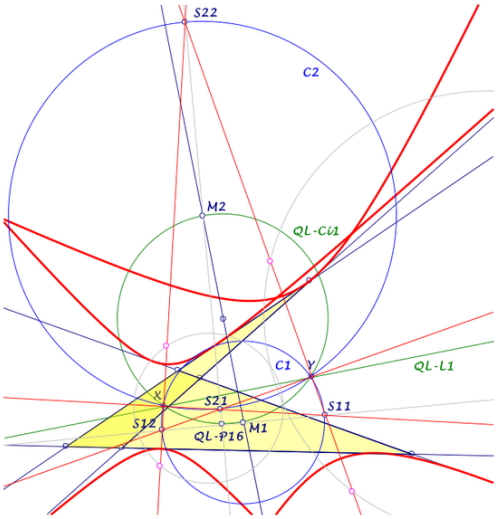
A quadrangle has two circumscribed parabolas and one circumscribed orthogonal hyperbola, a quadrilateral has one inscribed parabola and two inscribed orthogonal hyperbolas (see [1] and QFG-messages sept. 2015). Here the two inscribed orthogonal QL-hyperbolas are described – Cabri controlled – in EQF-geometry.



- **A quadrilateral has only two inscribed orthogonal hyperbolas, if the triangle components are obtuse angled (see [1]).**

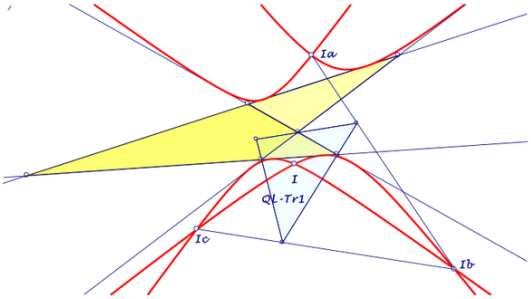
- **The centers *X, Y* of these two hyperbolas are the intersections of the Newton line *QL-L1* and the circumcircle *QL-Ci1* of the *QL*-diagonal triangle (see # 1283).**

- **The axes can be constructed as follows:**
  …Let M_1, M_2 be the intersections of *QL-Ci1* with the perpendicular bisector of the centers *X, Y*.
  … Let C_1 and C_2 be circles round M_1 and M_2 through *X* and *Y*.
  … Let $S_{i,1}$ and $S_{i,2}$ be the intersections of M_i.*QL-P16* and the circle C_i.
  … The lines $S_{i,j}.S_{k,l}$ ($i \neq k$) are two pairs of orthogonal lines through the centers *X, Y*, which are the axes of the hyperbolas.

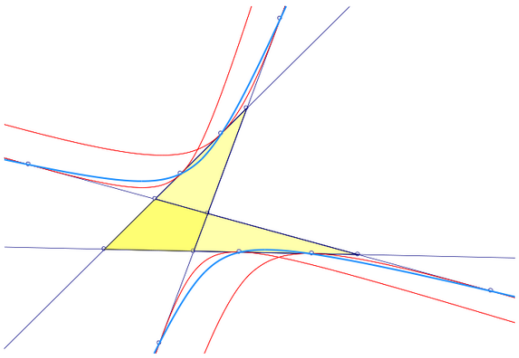


- **The foci are the intersections of axes and their *QL-Tf1* image circle.**

- **The intersections of the two hyperbolas are the in-/ex-centers of the *QL*-diagonal triangle *QL-Tr1* (see #1288).**



- **The 8 contact points of the two hyperbolas lie on a conic (see #1289).**



2015-09-21.pdf

Reference *[1]*:

http://www.math.uoc.gr/~pamfilos/eGallery/problems/RectHyperbolasTangent4Lines.html

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-09-21.pdf

Message: #1297
Date: 22/9/2015 8:43:05
From: eckart_schmidt@t-online.de
Subject: QL-Tf1 and QL-Tf2 wrt QL-Cu1

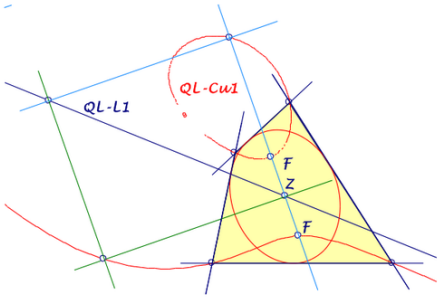
Dear Bernard, dear Chris,
attached a geometry of QL-Tf1 and QL-Tf2 wrt QL-Cu1 and QL-inscribed conics, perhaps some new aspects.
Best regards Eckart
PS for Bernard: Thanks for your messages 1280, 1281 and 1282, I shall answer later on.

EQF-Note 2015-09-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Tf1 and QL-Tf2 wrt QL-Cu1

*The **C**lawson-**S**chmidt **C**onjugate QL-Tf1 is a point to point transformation, shortened **CSC**, the **L**ine **I**so**C**onjugate QL-Tf2 is a line to line transformation, shortened **LIC**. Here QL-inscribed conics are considered. Their foci are CSC-partners on QL-Cu1 and their axes intersect with their LIC-partners orthogonal on QL-Cu1.*



For *QL*-inscribed conics, centered on the Newton line *QL-L1*, holds:

- **Lines through a point *Z* on the Newton line *QL-L1*, which are orthogonal to their *LIC*-image, are axes for the *QL*-inscribed conic, centered in *Z*.**

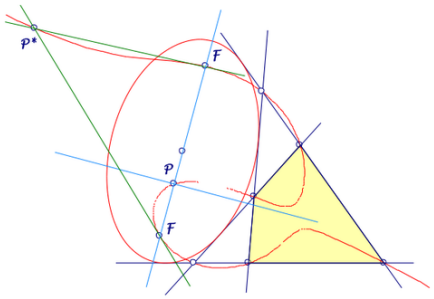
- **Lines through a point *Z* on the Newton line *QL-L1*, which are parallel to their *LIC*-image, are asymptotes for the *QL*-inscribed conic, centered in *Z*.**

QL-Cu1 can be described in several ways with many interesting properties (see *EQF*). Wrt the *CSC*- and *LIC*-transformation here additional shall be mentioned:

- ***QL-Cu1* is the locus for *CSC*-partners *X, X** with *XX** orthogonal *LIC(XX*)*.**

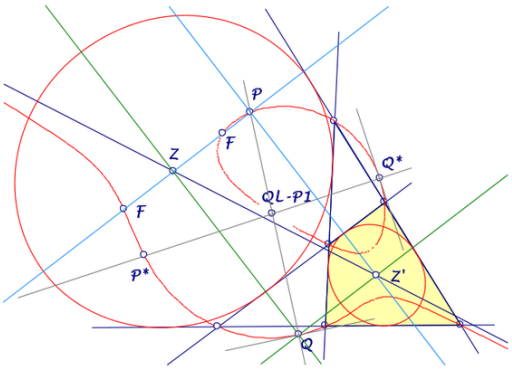
Taking for *X* and *X** the foci of a *QL*-inscribed conic, *XX** is the main axis.

- **The 3rd intersection *P* of *QL-Cu1* and the main axis of a *QL*-inscribed conic is the crosspoint with its orthogonal *LIC*-image.**



- **The *CSC*-image *P**** **is the crosspoint of the tangents in the foci at *QL-Cu1*.**

- **The only intersection *Q* of *QL-Cu1* and the minor axis of a *QL*-inscribed conic is the crosspoint with its orthogonal *LIC*-image.**



- **The intersection of *PQ* and *P*Q**** **is the Miquel point *QL-P1*.**

- **The tangents in *Q* (*Q**) at *QL-Cu1* are orthogonal *PQ* (*P*Q**).**

- **The *LIC*-images of the axes of a *QL*-inscribed conic intersect orthogonal in a point *Z´* on the Newton line *QL-L1*.**

- **The *LIC*-images of the axes of a *QL*-inscribed conic are the axes of a 2nd *QL*-inscribed conic, centered in *Z´*, which has the same points *P* and *Q* as the reference conic.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-09-22.pdf

Message: #1298
Date: 23/9/2015 8:27:48
From: seiichikiri
Subject: QL-Tf1 and QL-Tf2 wrt QL-Cu1

Dear Eckart,
I hope your confirmation of (3) about QL-Cu1.
In EQF:
(1) QL-Cu1 is the locus of all points for which the perpendiculars from P in the 4 quadrilateral lines are concyclic.
(2) QL-Cu1 is the locus of all points for whom the midpoint of P and CSC conjugate P* lie on Newton line QL-L1.
Confirmation:
(3) the circle on which the feet of the perpendiculars from P in the 4 quadrilateral lines lie coincides with that of P* and its center is on Newton line.
P.S. I met the above idea when I saw your attachment (EQF Note 2015-9-22, 1 st figure) in message #1297.
Best regards, Seiichi.

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Message: #1299
Date: 23/9/2015 9:56:57
From: eckart_schmidt@t-online.de
Subject: QL-Tf1 and QL-Tf2 wrt QL-Cu1

Dear Seiichi,
your conclusion (3) is right: CSC-partners on QL-Cu1 have the same circumcircle for their pedal quadrangles.
Additional remark: Let S, T be CSC-partners on QL-Cu1 and Si, Ti their pedal points on the QL-lines, then SiTj and SjTi (i unequal j) intersect on the main axis of the QL-inscribed conic with foci S, T.
Best regards Eckart

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Message: #1300
Date: 23/9/2015 10:13:25
From: bernard.keizer
Subject: QL-Tf1 and QL-Tf2 wrt QL-Cu1

Dear Seiichi, dear Eckart
The properties mentionned (including the last by Eckart) are linked to the fact that the 2 CSC partners are the foci of an inscribed conic.
The CSC partners are isogonal conjugates in each of the 4 reference triangles and the circumcircle of the pedal quadrangles is the principal circle of the inscribed conic ; the center of this circle is the center of the conic and the middle of the segment joining its foci.
QL-Cu1 is Cl-S invariant and self isogonal wrt the 4 reference triangles.
Best regards
Bernard

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Message: #1301
Date: 24/9/2015 3:16:49
From: seiichikiri
Subject: QL-Tf1 and QL-Tf2 wrt QL-Cu1

Dear Eckart, dear Bernard,
Thank you very much for the confirmation and deduction.
Given CSC partners, its midpoint on Newton line and the principal circle,
(1) We can obtain the inscribing conic by the envelope of lines in a manner similar to:
http://www.math.uoc.gr/~pamfilos/eGallery/problems/HyperbolaAsEnvelope.html
(2) The condition of inscribing orthogonal hyperbola is as follows:
the distance between CSC partner and the center of principal circle= Sqrt[2]* the radius of the principal circle.
Best regards, Seiichi.

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Message: #1302
Date: 24/9/2015 4:20:26
From: eckart_schmidt@t-online.de
Subject: QL-Quintic

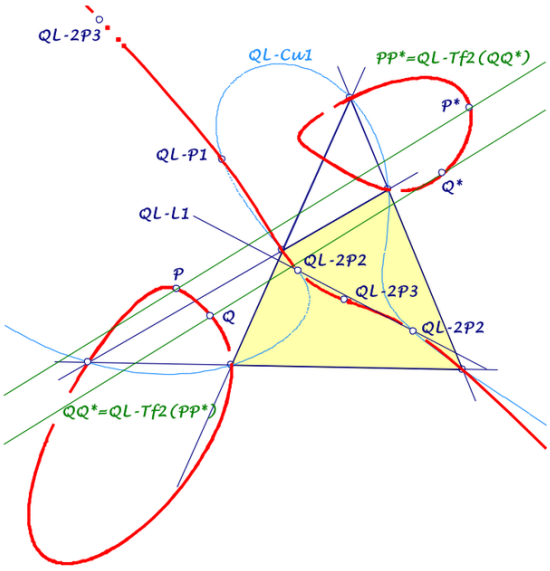
Dear Bernard, dear Chris,
the cubic QL-Cu1 is the locus of QL-Tf1-partners X, X*, with the property: XX* orthogonal QL-Tf2(XX*) (see #1297). What about the locus of QL-Tf1-partners with the property: XX* parallel QL-Tf2(XX*)? See attached file.
Best regards Eckart
PS. The figures are not the best, for my Cabri has its limits.

EQF-Note 2015-09-24

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Quintic

The cubic QL-Cu1 is the locus of QL-Tf1-partners X, X, with the property: XX* orthogonal QL-Tf2(XX*) (see #1297). What about the locus of QL-Tf1-partners with the property: XX* parallel QL-Tf2(XX*)?*

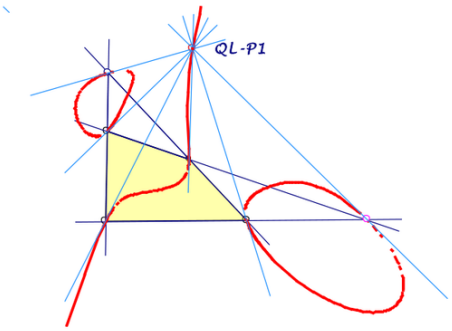


- **The locus of *QL-Tf1*-partners *X*, *X**, with *XX** parallel *QL-Tf2(XX*)* is a quintic, locus of the *QL-Tf1*-partners on the *QL-Tf2*-images of the asymptotes of *QL*-inscribed conics.**

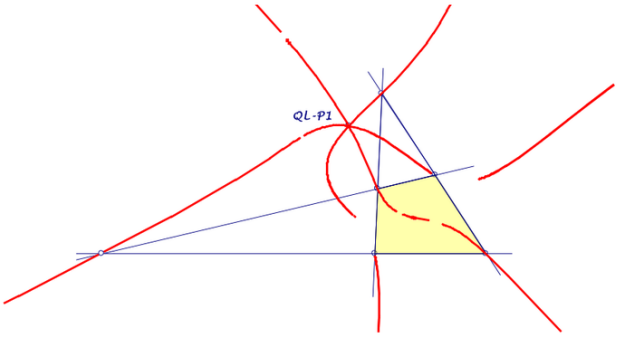
- **The quintic is *QL-Tf1*-invariant** (evident)**.**

- **Following points lie on the quintic:**
  … the 6 intersections of the *QL*-lines,
  … the Miquel point *QL-P1*,
  … the intersections *QL-2P2* of *QL-L1* and *QL-Cu1*,
  … the fixed points *QL-2P3* of *QL-Tf1*,
  … the *QL-Tf1*-partners on the *QL*-lines,
  … the *QL-Tf1*-partners on the sidelines of the anticomplement triangle of *QL-Tr1*.

- **The tangents at the quintic in the 6 intersections of *QL*-lines intersect in the Miquel point *QL-P1*.**



Final remark: *QL-P1* can be a triple point of the quintic.



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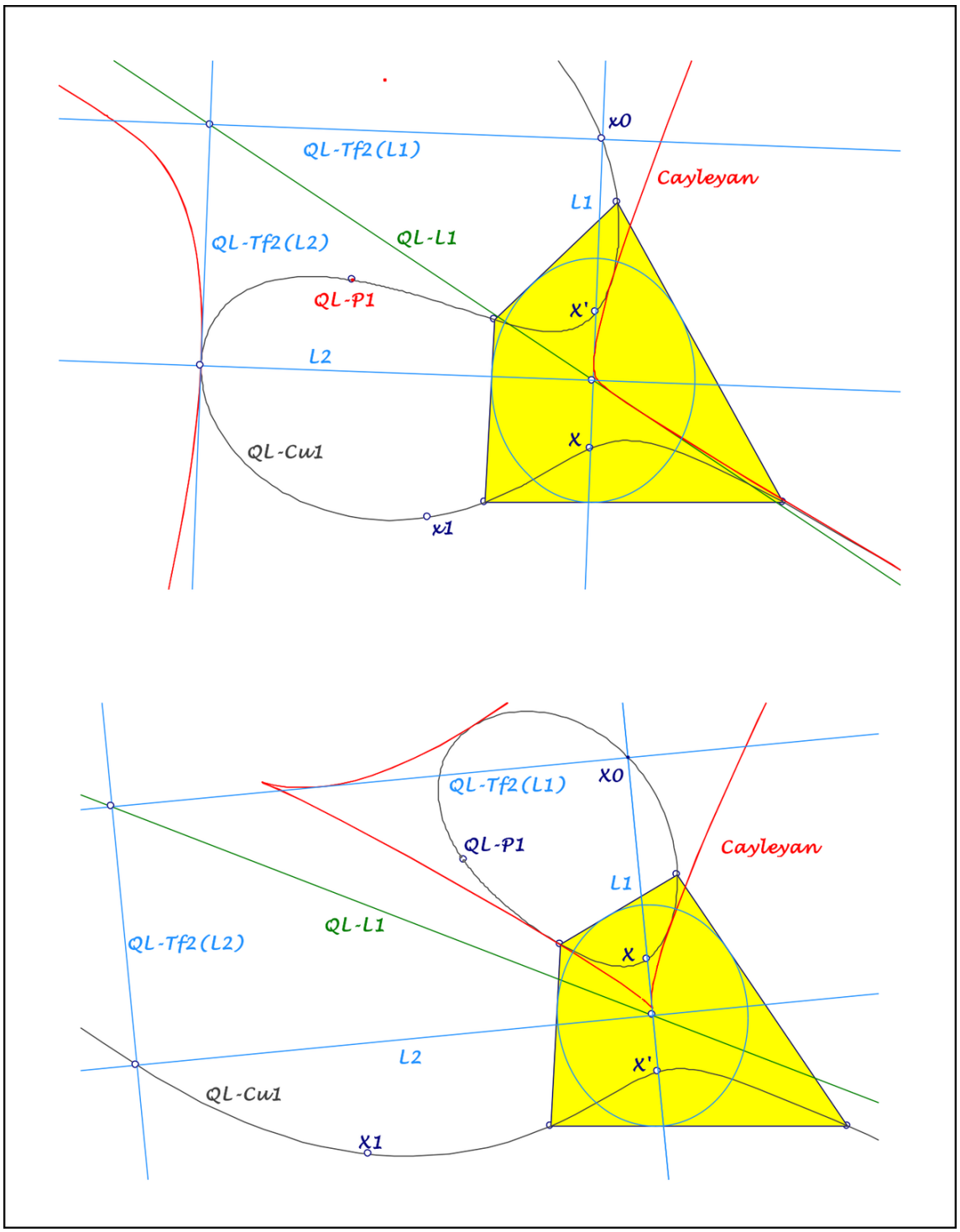
2015-09-24.pdf

Message: #1303
Date: 25/9/2015 12:01:44
From: eckart_schmidt@t-online.de
Subject: QL-Tf2 and QL-Cu1

Dear Bernard,
wrt your message 1280: I have difficulties to understand your following remarks, see attached file.
(1) "Both axes of every inscribed conic are tangent to the cayleyan."
(2) "The intersections of L1 (middle of XX') and QL-Tf2(L1) with the Newton Line QL-L1 are symmetric wrt QL-P1. QL-Tf2(L1) is also tangent to the cayleyan ... "
Perhaps there is a misunderstanding on my side.
Best regards Eckart



2015-09-25.docx

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Message: #1304
Date: 26/9/2015 3:07:25
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

Dear Chris,
I think, it should be mentioned in EQF:
QA-Cu1 is an anallagmatic curve. Circles of inversion have their centers in the excenters of the Miquel Triangle QA-Tr2. The inversion circles intersect orthogonal in pairs, radical center is the incenter of QA-Tr2.
Best regards Eckart

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Message: #1305
Date: 27/9/2015 10:55:54
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

Dear Chris,
I think, the following property of QL-Cu1 in EQF doesn't hold:
"These points lie on QL-Cu1 (
http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-mathematics/quadrilateral-objects/artikelen-ql/156-ql-cu1.html):
...
The Trilinear Poles of Li wrt triangle Lj.Lk.Ll, where (i,j,k,l) ∈ (1,2,3,4)."
Best regards Eckart

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Message: #1306
Date: 27/9/2015 1:08:33
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Bernard, dear Chris,
a further construction for QL-Cu1, perhaps not mentioned:
QL-Cu1 is the locus of intersections ...
... of the isogonal conjugated conics ...
... for parallels to QL-L1 ...
... wrt two QL triangle components.
Best regards Eckart
```

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Message: #1307
Date: 28/9/2015 11:45:08
From: bernard.keizer
Subject: QL-Tf2 and QL-Cu1

Dear Eckart,
Thank you for your answer
Unfortunately, it's exactly what I feared !
Your cayleyan is not complete (remember it is a sextic).
It took me some time to understand and I put in message 840 the exact definition and the way to draw it and I gave in message 843 the reference of my 2nd article on the web where all this matter is explained (unfortunately in french).
(Of course, if you wish, I can send it to you in a pdf version at your personal e-mail adress)
I confirm (1) and 2nd part of (2). In your terms, L1 and L2 as well as QL-Tf2(L1) and QL-Tf2(L2) are tangent to the cayleyan.
And it goes on with the point X2, 3rd intersection of X0X1 with the hessian, the perpendicular in X2 to X0X1 cuts QL-Tf2(L1) in a point on the cayleyan and is tangent itself to the cayleyan in another point...
For the 1rst part of (2), please read the intersections of L1(through XX') and QL-Tf2(L1) with the parallel to QL-L1 through QL-P1 (axis of the parabola) are symmetric wrt QL-P1.
More generally, every rectangular hyperbola which is a polar conic of the stelloïd cuts every major axis XX' of any inscribed conic harmonically wrt X and X'. As the axis of the parabola is such a major axis and your copple L1 and QL-Tf2(L1) is a degenerated rectangular hyperbola, polar conic of the tangential X1, the harmonic division gives the infinity point and the middle of the segment.(You could say as well that QL-P1 is on one diagonal of the rectangle formed by the 4 lines)
I hope these explanations will convince you !
Best regards
Bernard

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Message: #1308
Date: 28/9/2015 4:23:44
From: eckart_schmidt@t-online.de
Subject: QL-Tf1-Related Curves

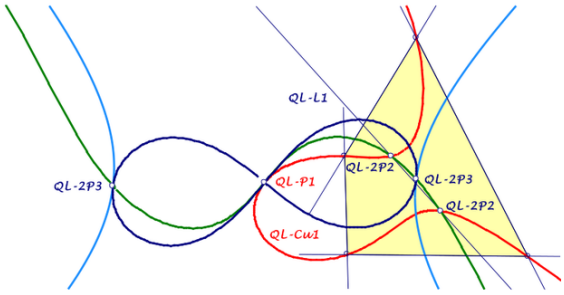
Dear friends,
attached an excursion in CSC-geometry, perhaps of interest
Best regards Eckart

EQF-Note 2015-09-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

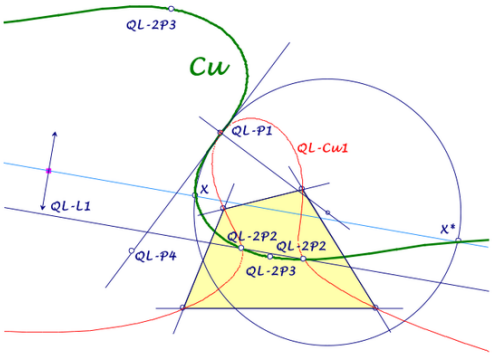
QL-Tf1 Related Curves

Main QL-transformation is QL-Tf1, usual shortened CSC. Central QL-curve is the CSC-invariant cubic QL-Cu1. Changing the construction of QL-Cu1, there is a further CSC-invariant cubic Cu. Its CSC-partners lead to an orthogonal hyperbola Hy, whose CSC-image is a lemniscate Le. Geometric properties are Cabri-tested.



QL-Cu1

QL-Cu1 is the locus for intersections of …
… parallels wrt *QL-L1*
… and the *CSC*-circles of their reflections in *QL-L1*.



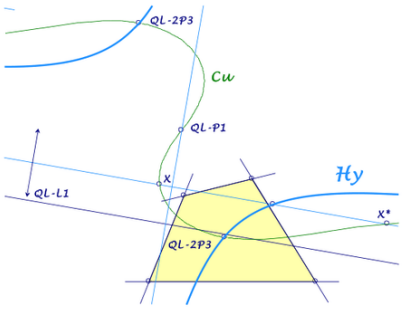
Cu

Reducing the construction of *QL-Cu1* we get a further cubic:
Cu is the locus for intersections of …
… parallels wrt *QL-L1*
… and their *CSC*-circles.

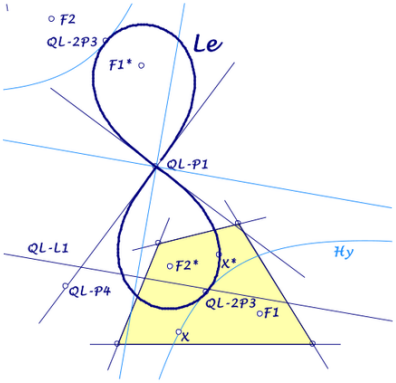
Some properties of the cubic Cu:

… *Cu* is *CSC*-invariant.
… *Cu* is symmetric wrt the Miquel point *QL-P1*.
… *Cu* contains the points *QL-P1, QL-2P2, QL-2P3*.
… Its asymptote is parallel to the Newton line *QL-L1*.
… The tangent in *QL-P1* at *Cu* is *QL-P1.QL-P4*, also the tangent at *QL-Cu1*.

Hy
The midpoints of *CSC*-partners on the cubic *Cu* give an orthogonal hyperbola *Hy*, …
… centered in *QL-P1*,
… through the points *QL-2P3*
… and asymptotes parallel and ortogonal *QL-L1*.
The tangent in a point *X* at *Hy* is *XX**.



Le
The *CSC*-image of the orthogonal hyperbola *Hy* is a lemniscate,
… centered in *QL-P1*,
… tangent in *QL-2P3* to *Hy*,
… tangents in *QL-P1*: *QL-P1.QL-P4* and a perpendicular,
… with foci in the *CSC*-images of the foci of *Hy*.



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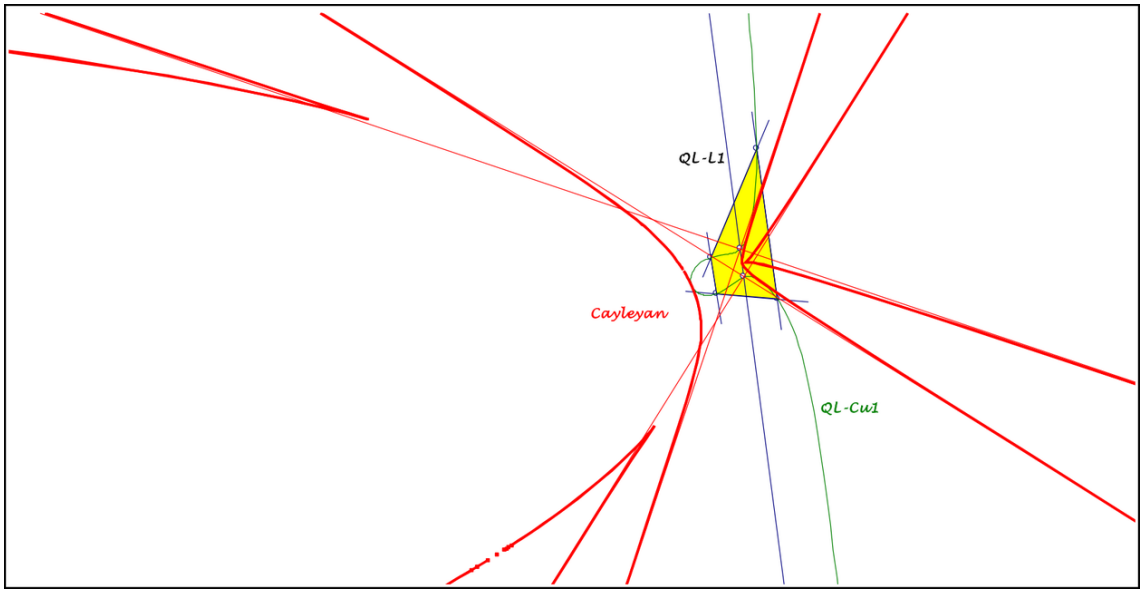
2015-09-28.pdf

Message: #1309
Date: 29/9/2015 1:57:10
From: eckart_schmidt@t-online.de
Subject: QL-Tf2 and QL-Cu1

Dear Bernard,
thank you for the explanations, I had used your first definition of the cayleyan, which you corrected later.
Is the following property already mentioned? I hope, the Cabri observation holds.
... If QL-Cu1 is unipartite, the cayleyan has four asymptotes, which are the axes of inscribed conics, centered in QL-2P2 (see attached file). If QL-Cu1 is bipartite, the caylean has no asymptotes.
Best regards Eckart



2015-09-29.docx

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Message: #1310
Date: 29/9/2015 3:33:01
From: bernard.keizer
Subject: QL-Tf2 and QL-Cu1

Dear Eckart,
This beautiful property is new for me !
On your figure, the axes in each 2P2a or b are perpendicular and form the degenerated rectangular hyperbola, which is the polar conic of the other conjugate wrt the stelloïd !
So, I suppose you're right !
Thank you for this ...
Best regards
Bernard

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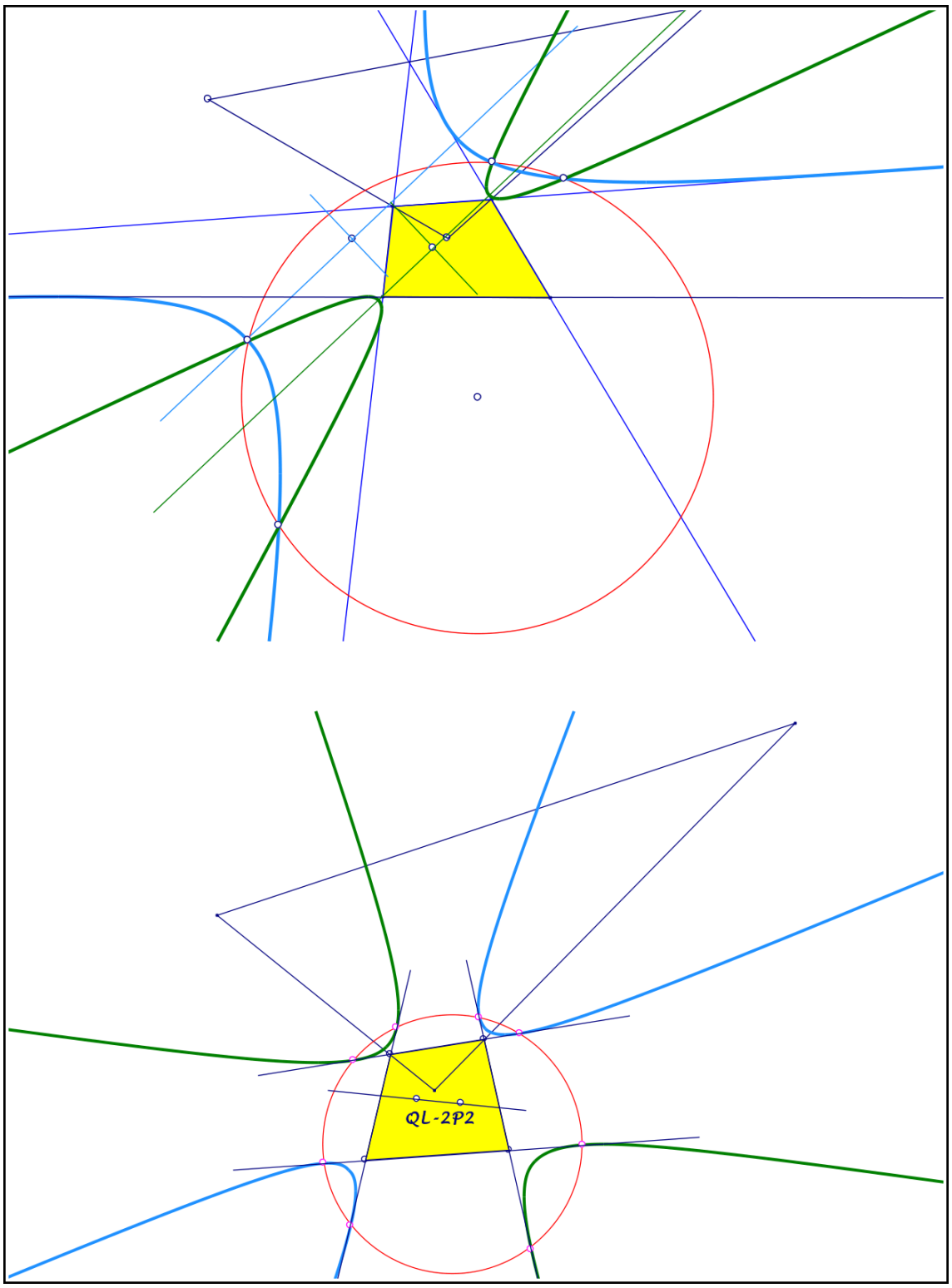
Message: #1311
Date: 30/9/2015 10:56:26
From: eckart_schmidt@t-online.de
Subject: QL-Tr1 Polar Circle

Dear Bernard, dear Chris,
I think, there are two remarkable properties for the polar circle of the QL-diagonal triangle QL-Tr1 wrt QL-inscribed conics:
(1) Two QL-inscribed conics, whose axes are QL-Tf2-partners, intersect on the polar circle of QL-Tr1 (see also # 1248).
(2) The two QL-inscribed conics with centers QL-2P2 have contact points with the QL-lines on the polar circle of QL-Tr1.
See attached file.
Best regards Eckart



2015-09-30.docx

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Message: #1312
Date: 01/10/2015 8:22:28
From: chris.vantienhoven
Subject: Polar Circles

Thank you Eckart for shining your light on polar circles!
The polar circle of a triangle is only defined for obtuse triangles.
This made me think that the diagonal triangles in a quadrilateral as well as a quadrangle always seem to be obtuse!
I never saw a proof of this.
Maybe someone can help me with that?
Moreover because the polar circle of a triangle only defined for obtuse triangles this circle is taylor made for the diagonal triangle in a quadrilateral or a quadrangle. So it becomes likely that there should be more properties.
I found some at the end of the article at Wikipedia:
https://en.wikipedia.org/wiki/Polar_circle_(geometry).
* Any two polar circles of two triangles in an orthocentric system are orthogonal.
* The polar circles of the triangles of a complete quadrilateral form a coaxal system.
* A triangle's circumcircle, its nine-point circle, its polar circle, and the circumcircle of its tangential triangle are coaxal.
But besides your and these properties I think there should be more.
Best regards,
Chris

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Message: #1313
Date: 01/10/2015 9:58:40
From: eckart_schmidt@t-online.de
Subject: Polar circles

Dear Chris,
only a short reaction on your message 1312:
The QA-/QL-diagonal triangles must not be obtuse! See also Ref [4], p.37.
Two further properties of the QL-polar circle:
... The reflection of the Miquel point in the QL-polar circle lies on the circumcircle of the QL-diagonal triangle.
... An QL-inscribed conic cuts the QL-polar circle in a cyclic quadrangle with the same diagonal triangle as the reference quadrilateral.
Best regards Eckart
PS. Time is over, to walk northern the polar circle. But I shall try it with EQF-shoes!

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Message: #1314
Date: 03/10/2015 9:48:46
From: eckart_schmidt@t-online.de
Subject: (QL-Cu1) for a Quadrangle

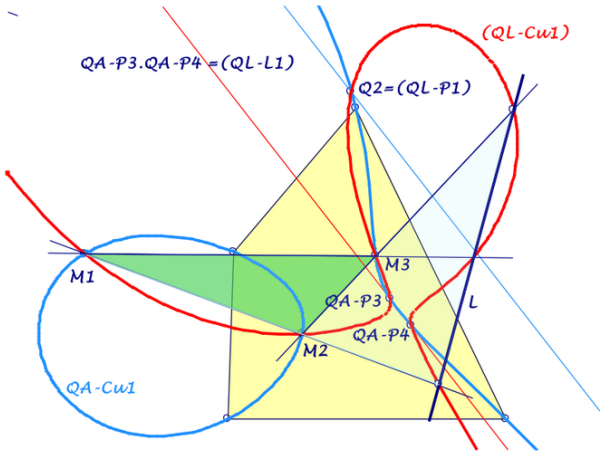
Dear friends,
the constructions of QA-Cu1 and QL-Cu1 lead to two further cubics, one described in #1308, the other in the attached file:
A QL-cubic for a quadrangle.
Best regards Eckart

EQF-Note 2015-10-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

(QL-Cu1) for a Quadrangle

Taking for a quadrangle the lines of the Miquel triangle QA-Tr2 and a special further line L, this quadrilateral has a QL-Cu1 with relevant properties in quadrangle geometry.



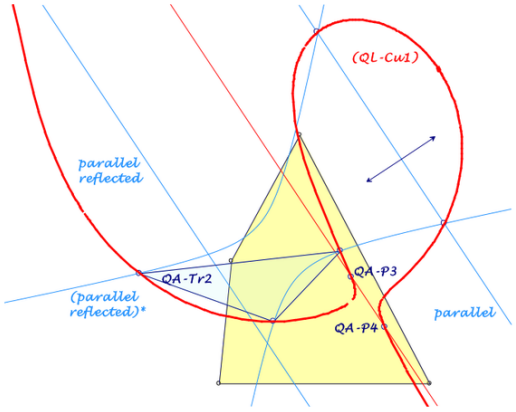
There are similar constructions wrt the following cubics …
… *QL-Cu*, not in *EQF*, but in *QFG*-message 1308,
... *QL-Cu1* for a quadrilateral,
… *QA-Cu1* for a quadrangle
… and a new cubic *(QL-Cu1)* for a quadrangle.

QL-Cu is the locus of intersections of …
… parallels wrt *QL-L1*
… and their *QL-Tf1*-circles.

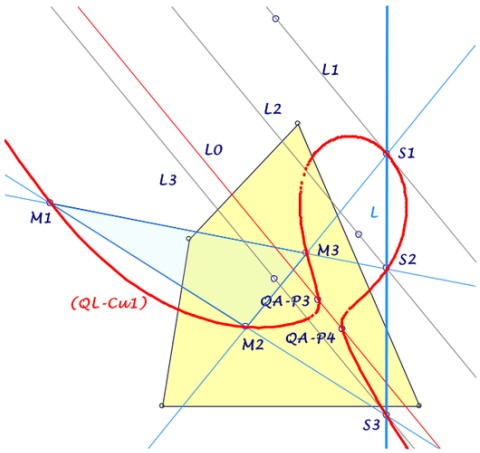
QL-Cu1 is the locus of intersections of …
… parallels wrt *QL-L1*
… and the *QL-Tf1*-circles of their reflections in *QL-L1*.

QA-Cu1 is the locus of intersections of …
… parallels wrt *QA-P3.QA-P4*
… and their *QA-Tr2*-isogonal conjugate conics.

***(QL-Cu1)* is the locus of intersections of …**
… parallels wrt *QA-P3.QA-P4*
… and the *QA-Tr2*-isogonal conjugate conics of their reflections in *QA-P3.QA-P4*.



This cubic *(QL-Cu1)* for a quadrangle is the cubic *QL-Cu1* for a quadrilateral *QL*, which has as three lines the sides of the Miquel triangle *QA-Tr2* and a fourth line *L*, which is the fourth common tangent for all inscribed conics of *QA-Tr2*, which are centered on *QA-P3.QA-P4*. The line *L* can be constructed as follows:
… Let $M_1M_2M_3$ be the Miquel Triangle *QA-Tr2* and L_0 = *QA-P3.QA-P4*.
… Let L_i be a parallel wrt L_0 through the reflection of M_i in L_0.
… Let S_i be the intersection of L_i and M_jM_k.
… $L = S_1S_2S_3$.



- **The cubic *(QL-Cu1)* for a quadrangle is the cubic *QL-Cu1* for the quadrilateral *QL* of the three lines of the Miquel triangle *QA-Tr2* and the line *L*.**

- **The cubic *(QL-Cu1)* for a quadrangle is isogonal invariant wrt the Miquel triangle *QA-Tr2*.**

- **For points on *(QL-Cu1)* the isogonal conjugate wrt the Miquel triangle *QA-Tr2* is the transformation *QL-Tf1* for *QL*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of *QA-Tr2*-isogonal partners with midpoint on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of the foci of all *QA-Tr2*-inscribed conics centered on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is the locus of points, which have *QA-Tr2*-pedal triangles with circumcenter on *QA-P3.QA-P4*.**

- **The cubic *(QL-Cu1)* for a quadrangle is unipartite with *QA-P3.QA-P4* as Newton line of *QL*.**

- **The cubic *(QL-Cu1)* for a quadrangle contains the following points:**
  … the vertices of the Miquel triangle *QA-Tr2*,
  … the intersection *Q2* of *QA-Cu1* and its asymptote (see *EQF*) as Miquel point *QL-P1* of *QL*,
  … the points *QA-P3, QA-P4* as *QL-2P2* of *QL*,
  … the intersection of the *QA-Cu1*-asymptote and the perpendicular bisector of *QA-P3.QA-P4* as point *T* of *QL* (see *EQF*).

- **The cubics *QA-Cu1* and *(QL-Cu1)* have asymptotes parallel and symmetric wrt *QA-P3.QA-P4*.**

- **The tangent wrt *QA-Cu1* at the intersection *Q2* with its asymptote is *QL-L5* of *QL*.**

- **The *QA-Tr2*-circumcircle is tangent to *(QL-Cu1)* in *Q2*.**

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2015-10-03.pdf

Message: #1315
Date: 07/10/2015 11:45:36
From: bernard.keizer
Subject: ETC and EQF

Dear Chris, dear Eckart,
dear Bernard G., dear Benedetto, dear Jean-Louis,
dear Antreas, dear Seiichi and dear members
Conversely, the more I work on the triangle, the better I understand the QL !
In this second message, the reference triangle is the DT triangle of the QL with an interesting (I hope) isotomic conjugation.
Thanks in advance for any comment or critic or complement or reference ...
Best regards
Bernard

Co1, Co3 and Isotomic conjugation wrt DT

A. QL and DQL properties

Let's start with an ordinary QL, called QL1.

It's diagonal QL, called DQL1, is the QL with 4 lines the diagonals and the Newton Line of QL1.

The parabola Co1 with P1 as focus and L2 as directrix is tangent to the 4 lines of the QL and to the sides of the medial triangle of DT (P1 being on the Euler circle of DT).

P1 is the center of a Cl-S which swaps the 4 lines with the 4 circumcircles of the reference triangles and the Miquel circle Ci3 with the Steiner Line L2.

The parabola Γ1 with P17 as focus and L6 as directrix is tangent to the 4 lines of DQL1.

P17 is the center of a Cl-Sdiag which swaps the 2 Plücker points and the circumcircle of DT QL-Ci1 with the Newton Line QL-L1.

The parabola Co3 with P25 as focus and the parallel to L6 in P9 as directrix is the complement wrt the DT (homothetic in the homothety of center P8 and ratio - ½) of the previous parabola ; this parabola is tangent to the 3 sides of the medial triangle of DT and to the complement of the Newton Line of the QL wrt the DT (parallel to L6 through P9).

As it is well known, the 2 parabolas Co1 and Co3 intersect in 4 points, which are the contact points between the parabola Co1 and the 4 lines of the QL1 (the centroïd of the the 4 points is on the Newton Line) ; both parabolas are inscribed in the medial triangle of DT.

The tangents in the 4 contact points to Co3 form a 2nd QL, called QL2, which has the same DT as QL1 ; this DT is also the DT of the QA of the 4 points.

QL2 has the same way a diagonal QL, called DQL2 and the parabola Γ2 is tangent to the 3 sides of DT and to the Newton Line of QL2 ; it has as focus the anticomplement Px of P1 and as directrix the parallel in P10 to the Steiner Line of QL2.

The 2 parabolas Co1 and Co3 have therefore perfectly symmetric properties :

- The 2 parabolas are inscribed in the medial triangle of DT and the2 foci of the parabolas, P1 and P25, lie on the Euler circle of DT QL-Ci2
- The 2 directrices of the parabolas and Steiner Lines of the QLs meet in P9
- The 2 Newton Lines of the QLs meet in the centroïd of the 4 contact points and intersection points of the 2 parabolas
- Each axis of a parabola and each Newton Line of its tangential QL are orthogonal to the lines L5 or P1P7 and L6 of the other QL
- Each directrix of a parabola and Steiner Line of its tangential QL is parallel to the lines P1P7 or L5 and L6 of the other QL

Co1, Co3 and Isotomic conjugation.pdf

The 2 parabolas Γ1 and Γ2 have also symmetric properties :

- The 2 parabolas are inscribed in DT and the 2 foci of the parabolas, P17 and Px, anticomplements of P1 and P25, lie on the circumcircle of DT
- The 2 directrices of the parabolas and Steiner lines of the 2 DQLs meet in P10
- Last, but not least, as it is the most important property, the associated parabola Co3 of one QL is the complement of the parabola Γ1 of it's diagonal QL DQL (formed by the diagonals and the Newton Line of the QL) wrt the DT (homothetic in the homothety with center P8 and ratio - ½) and vice-versa.

B. New property

Let's now consider the 4 perspective points P1 to P4 between the 4 reference triangles of QL1 and DT and the 4 perspective points Q1 to Q4 between the 4 reference triangles of QL2 and DT.
Each reference triangle is the cevian triangle of one perspective point and it's circumcircle is the cevian circle of this point.
The focus P1 is the intersections of the 4 cevian circles of P1 to P4 and the focus P25 is the intersection of the 4 cevian circles of Q1 to Q4 (both also lying on the Euler circle of DT).

Let's call T1 to T4 the 4 contact points or intersection points between Co1 and Co3.
The tangents to the 2 parabolas Co1 and C03 cut the 3 sides of DT in points (vertices of QL1 or QL2) which are symmetric wrt the middles of these sides (for example on the figure, the tangents in T4 to QL-Co1 and QL-Co3 cut the 3 sides in D', E', F' and in Q', R' and S', D' and Q' being symmetric wrt the middle M of BC …) The 6 corresponding vertices of QL1 and QL2 are symmetric by pair wrt the middles of the sides of DT.

In other words, the perspectors Q1 to Q4 are the isotomic conjugates of P1 to P4 wrt DT

The lines L'1 to L'4 forming the 2nd QL QL2 are the isotomic transversal lines of the 4 lines L1 to L4 forming the 1rst QL QL1 ; naturally, the 2 Newton Lines of QL1 and QL2 cut also the 3 sides of DT in 3 points symmetric wrt the middles of the sides and are isotomic transversals

If the equation of a line is lx + my + nz = 0, the equation of it's transversal line is x/l + y/m + z/n = 0.
In fact, it's easy to check that the barycentric coordinates of QL-P25 in DT coordinates are exactly the coordinates of QL-P1 if replacing l, m and n respectively by 1/l, 1/m and 1/n and, for confirmation, that the barycentric coordinates of the 4 contact points T1 to T4 are the same with l, m and n or with 1/l, 1/m and 1/n.

Co1, Co3 and Isotomic conjugation.pdf

C. Application to the triangle

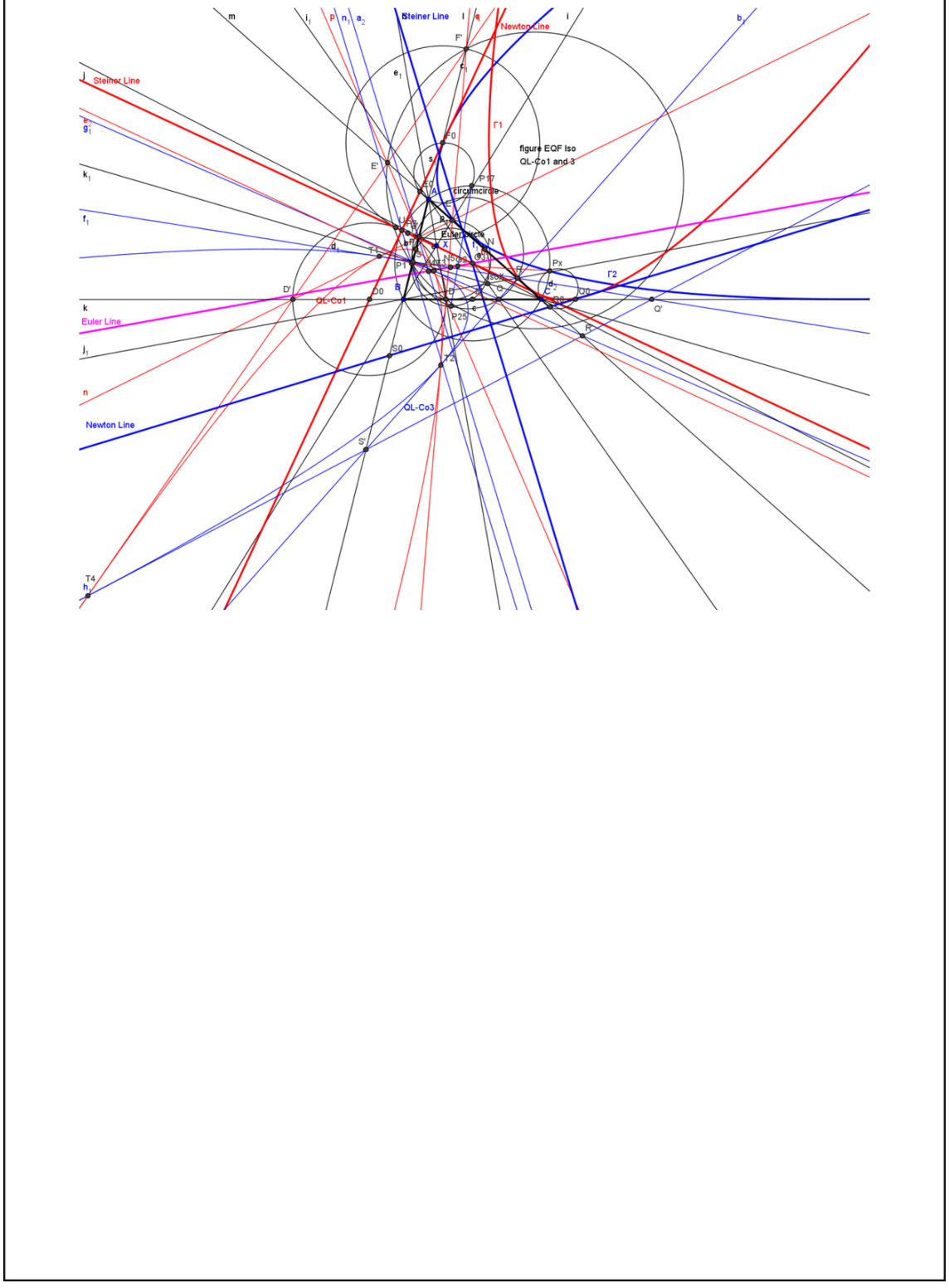
By operating conversely and choosing one point of ETC and it's isotomic conjugate in a triangle given as DT, it's possible to find beautiful constructions.

First example : the incenter I1 and it's isotomic conjugate X75
This figure is particularly beautiful
The 2 Plücker points are the isodynamic points S15 and S'16
The cevian triangle of the incenter is the incentral triangle and it's cevian circle the incentral circle
The parabola Co1 has for focus P1 the point Ki115 (middle of the segment F13F'14 joining the 2 Fermat points, center of the Kiepert hyperbola and 2nd intersection between the Euler circle and the incentral circle, the 1rst being the Feuerbach point Fe11) and for directrix the Brocard axis
The parabola Co3 has for focus the point Je125 (Jerabek point, center of the Jerabek hyperbola) and for directrix the Euler Line (these 2 points come with l, m and n are 1/a, 1/b and 1/c or a, b and c)
The parabola Γ1 has for focus the reflexion point X110, for directrix the Euler Line and is tangent to the Lemoine axis (the Newton Line of QL1) ; it's known as the Kiepert parabola
The parabola Γ2 has for focus the Steiner point St99 (4th intersection between the circumcircle and the Steiner circumellipse), for directrix the line H4IsoH69, IsoCoEuler, parallel to the Brocard axis and is tangent to the Longchamps line (Newton Line of QL2)
The Lemoine axis and de Longchamps line are isotomic transversals

Second example : the Gergonne point Ge7 and it's Isotomic conjugate the Nagel point Na8
In this case, the cevian triangle of the Gergonne point is the intouch triangle, the cevian circle is the incircle and the focus P1 is the Feuerbach point Fe11 ; the cevian triangle of the Nagel point is the extouch triangle and the cevian circle is the Mandart circle with center X1158, the middle of the segment Be40CoBe84 (the point Be40 being the center of the Bevan circle, reflexion of the incenter in the orthocenter and the point CoBe84 being it's isogonal coordinate), the the focus X25 is the 2nd intersection between the Euler circle and this circle (the 1rst being the Feuerbach point Fe11) (Fe11 comes with l , m and n are (b + c – a) …)
Px, the anticomplement of Fe11 is the point X100

Third example : the ortocenter H4 and it's isotomic conjugate IsoH69, which is the anticomplement of the symmedian point K6 ; the cevian triangle of the orthocenter is the orthic triangle and the 4th line of QL1 is the orthic axis. The parabola Co1 is again the same parabola as in the 1rst example with focus Ki115 and directrix the Brocard axis and the parabola Γ2 is it's anticomplement with focus St99 and dirctrix the IsoCoEuler Line H4IsoH69 …Again, Ki115 comes with l, m and n are $b^2 + c^2 – a^2$ …
This parabola with focus Ki115 and directrix the Brocard axis is tangent to the 4 lines of the QLs for I1 and H4 ; nice property, it's interesting to find it simply with the EQF barycentric formulas !

Co1, Co3 and Isotomic conjugation.pdf



Co1, Co3 and Isotomic conjugation.pdf

Message: #1316
Date: 07/10/2015 11:53:58
From: bernard.keizer
Subject: EQF and ETC

Dear Chris, dear Eckart,
dear Bernard G., dear Benedetto, dear Jean-Louis,
dear Antreas, dear Seiichi and dear members,
The more I work on the QL and the better I understand the triangle !
In this first message, the reference triangle is the main pivot triangle of the cubic stelloïd.
Thanks in advance for your comments, critics or references.
Best regards
Bernard
PS This was the first message, but apparently, it didn't work the first time ...

Links between the triangle, the QL and the ψ transformation

A. The ψ transformation and the Brocard triangles

1. The ψ transformation

The ψ transformation is known on a little bit confidential manner ; I found only 2 references, the 1rst by Bernard Gibert, who gives it's barycentric calculation and the 2nd by Benedetto Scicemi, who uses an orthonormal system with origin in the Kiepert point Ki115 and axes parallel to the Steiner axes.

ψ is an involution which associates a symmetry wrt the 1rst Steiner axis and an inversion wrt a circle with diameter the segment joining the 2 foci of the Steiner inellipse (centered in the centroïd G2).

ψ swaps the 2 Fermat points F13 and F'14 as well as the 2 isodynamic points S15 and S'16 ; it swaps also the circumcircle with the Brocard circle and the Parry circle with the Lemoine axis.

Ψ has the same properties as Cl-S (for example, the ψ conjugate of 2 points inverse in a circle are inverse in the ψ conjugate circle ; in particular, the center of a ψcircle is the ψ of the inverse of G2 in the circle or, if the circle is trough G2 and the ψcircle a line, the reflexion of G2 in this line)

2. The Brocard triangles

Let's call ABC a reference triangle, A1B1C1 it's 1rst Brocard triangle, A2B2C2 it's 2nd Brocard triangle and A0B0C0 it's preBrocard triangle, id est the triangle of which ABC is the 1rst Brocard triangle. A1B1C1 and A0B0C0 are homothetic in a homothethy with center the centroïd G2 and are inversely similar to ABC, the similitude being the combination of an axal symmetry wrt the Steiner axes and another homothety with center G2.

For any point X, let's call ψX it's transform under the ψ transformation, Xbr and XBR the corresponding elements of the 1rst –and pre-Brocard triangles and ψXbr and ψXBR their transforms.

- Then X, ψXbr and ψXBR are on a line trough G2 as well as ψX, Xbr and XBR are on the symmetric line wrt the Steiner axes.
- The homotheties being the same , we have systematically $G2X^2$ = G2Xbr*G2XBR as well as $\psi G2X^2$ = ψG2Xbr*ψG2XBR and the circle through X, Xbr and XBR is tangent to G2X in X and the circle through ψX, ψXbr and ψXBR is tangent to G2ψX in ψX
- If 2 points X and Y are on a line trough G2, their ψ conjugates ψX and ψX' are on the symmetric line wrt the Steiner axes and XψX' is parallel to X'ψX

This leads to plenty of properties of alignment, cocyclicity,parallelism, antiparallelism and symmetry under the main triangle elements ; for example :

- For the circumcenter O3, the Euler Line of ABC contains G2, O3, ψOBR1316 (on the Brocard circle) and ψObr23 and the symmetric line wrt the Steiner axes, which is the Euler Line of both A1B1C1 and A0B0C0 contains ψO110, Obr182 and OBR or T98 (Tarry point)
- The circle through O3, Obr182 and T98 is tangent to the Euler Line in O3 and there is a circle through Obr182, T98, H4 and N5…
- N5 being the complement of O3, ψN3448 is the anticomplement of ψO110 …
- H4 being the anticomplement of O3, ψH125 is the complement of ψO110, on line with Hbr1352 and HBR147 …
- The ψEulercircle is centered in the ψ of the inverse of G2 in the Eulercircle, which is the complement X858 of ψObr23, and is therefore Hbr1352, the anticomplement of Obr182 ; as ψH125 lies on the Euler circle, the ψEulercircle is through H4
- The ψincircle (as well as the ψexcircles) are tangent to this ψEuler circle and to the 3 circles G2B2C2, G2A2C2 and G2A2B2, which are the ψ of the 3 sides of ABC
- The 2 isodynamic points S15 and S'16 are ψconjugates and inverse wrt the circumcenter and it's ψ the Brocard circle and the same way the 2 Fermat points F13 and F'14 are ψconjugates and inverse wrt the orthocentroïdal circle and symmetric wrt it's ψline trough ψH125
- As the Fermat axis is parallel through the symmedian point K6 to the ψ EulerLine, the circle trough G2, F13 and F'14 is through ψK111 and tangent to the Euler Line ; this leads to the Lester circle, as JG^2 = JO*JN = JF*JF'
- The Lester circle is trough F13, F'14, O3 and N5 and the ψLestercircle is trough F13, F'14, ψO110 and ψN3448.
- As the line ψO110ψK111 is trough the symmedian point K6, the ψcircle of this line is trough G2, O3, K6 and ψK111.

There are obviously plenty more beautiful properties and I wonder if they have already been systematically studied …

Triangle and QL transformation.pdf

B. The ψ transformation in the triangle and the Cl-S transformation in the QL

1. The ψ transformation

The ψ transformation in the triangle and in the QL is the same, provided the triangle has for points G2, S15 and S'16 the points P1 and 2P2a and b of the QL.

The Newton Line is either the Brocard axis (unicursal curve Cu1) or the Lemoine axis (bicursal Cu1)

The triangle ABC is the main pivot triangle of both cubics Mac Cay or Kjp
The MacCay cubic is the locus of the points M for which (MA,BC) + (MB,AC) + (MC, AB) = π/2
The Kjp cubic is the locus of the points K for which (KA,BC) + (KB,AC) + (KC, AB) = 0

Let's call St, Br and Le the directions of the Steiner, Brocard and Lemoine axis and by S1 to S3 the 3 sides of the triangle and L1 to L4 the 4 lines of the QL.

We have in the triangle Σ (St,Si) = (St,Le) = π/2 + (St,Br) m° π
The Morley axes of the triangle are given by 1/3Σ (St,Si) = 1/3 (St,Le) = π/6 + 1/3(St,Br) m° π/3
The asymptotes of the MacCay cubic trisect the angle between St and Br and are perpendicular to the Morley lines of the triangle ABC and parallel to the Morley lines of the triangle G2S15S'16, whereas the asymptotes of the Kjp cubic are parallel to these Morley lines of the triangle ABC and perpendicular to the Morley lines of the triangle G2S15S'16.

We know that the angle between the axes of the parabola and the cardioïd is the sum of the angles of the lines with the Newton Line and that the Steiner axis bisects this angle (as the parabola and the cardioïd are ψconjugates) and we have Σ (Br,Li) = Σ (Le,Li) = - 2 (St,Br) = - 2 (Le,Br) m° π

This leads to Σ (Br,Si) = π/2 -2 (St,Br) m° π and Σ (Br,Li) = Σ (Le,Li) = π/2 + Σ (Br,Si) = Σ (Le,Si) ; in particular, Σ (St,Li) = 2 (St,Br) = 2 (St,Le)

The sum of the directions of the lines of any QL inscribed in the hessianCu1 is equal to the sum of the directions of the sides of the main pivot triangle (m° a constant depending of the axis taken as origin).

It's interesting to notice that this sum is the same for both unicursal or bicursal hessians.

Triangle and QL transformation.pdf

2. Angle properties

The fundamental property of the QL's inscribed in a given curve Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is a constant given by the cubic stelloïd, more precisely by the sum of the directions of the sides of the main pivot triangle of this curve :

All the QL's share therefore :

- the Miquel point P1, centroïd G2 of the triangle
- the inversion circle, with diameter the segment joining the 2 foci of the Steiner inellipse, invariant points of the ψ transformation
- the Steiner axes, axes of this ellipse and axes of the ψ transformation
- the Newton Line L1, parallel to the asymptote of the hessian Cu1 at half a distance (P1 being the railwaywatcher of the 2 lines)
- the points 2P2a and b, which are the isodynamic points of the triangle S15 and S'16
- therefore the middle of these 2 points where the line L6 cuts the Newton Line
- more generally all the copples of conjugate points as foci of inscribed conics
- the axis of the inscribed parabola (parallel to the Newton Line through the Miquel point)
- the axis of the cardioïd Qu1, tangent to Cu1 in the Miquel point, through the point where the hessian Cu1 cuts it's asymptote and locus of the point P3, center of the Miquel circle
- the direction of the axes of the inscribed deltoïd Qu2, which is the direction of the asymptotes of the cubic stelloïd Cu2 ; these asymptotes trisect the angle between the 1rst Steiner axis and the Newton Line
- the asymptote of the hessian QL-Cu1 is the reflection of the axis of the parabola in the 1rst Steiner axis and is tangent to the inscribed deltoïd
- last, but not least, as discovered by Eckart and studied in details by Bernard G., the centers of the 27 inscribed cardioïds lying on the cubic stelloïd

For a given reference QL, the inscribed conics and their confocals (of which the degenerated ones made of 2 conjugate points) form a tangential set. The QL's inscribed in the hessian are determined by the 4 common tangents to 2 conics of the set of these confocals (for example the 4 lines joining 2 pairs of conjugate points forming 2 conics reduced to the 2 pairs of points)

Triangle and QL transformation.pdf

Message: #1317
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Messages

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Message number 1317 is not available in Yahoo groups.
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← Previous Next → ↩ Message Index ↑ Subjects

Message: #1318
Date: 09/10/2015 11:35:51
From: eckart_schmidt@t-online.de
Subject: ETC and EQF

```
Dear Bernard,
am I right in the following statement wrt #1315?
Let Co1 and Co3 be any two parabolas, tangent to the medial
triangle of a reference triangle, with an intersection T. Then
the tangents at T to the parabolas intersect the sides of the
reference triangle symmetrically to the middle and the
perspectors Pi and Qi are isotomic. So your new property is
independant of QL-geometry.
Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1319
Date: 09/10/2015 2:35:05
From: eckart_schmidt@t-online.de
Subject: Polar Circles

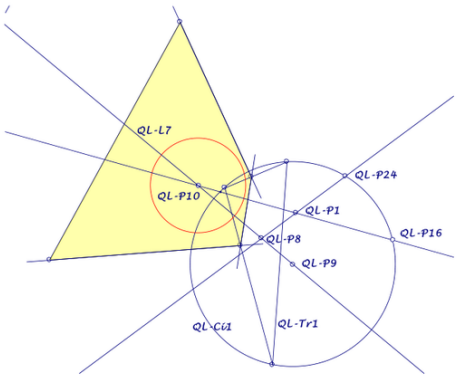
```
Dear Chris,
I have gathered, with some additions, properties of the polar
circle for the QL-diagonal triangle, see attached file.
Best regards Eckart
```

EQF-Note 2015-10-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

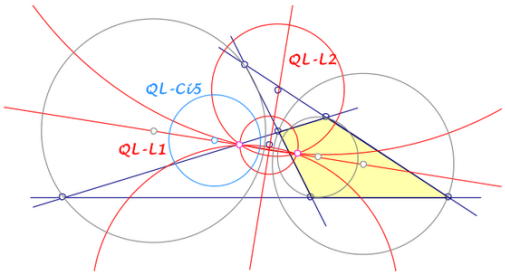
Polar Circle of QL-Tr1

The polar circle for the diagonal triangle QL-Tr1 of a quadrilateral has significant properties in QL-geometry, especially wrt its inversion. Here are gathered some results – Cabri controlled – for later applications.



The polar circle for a triangle is the only circle, to which the triangle is self-polar. Only for obtuse angled triangles the polar circle is real. Wrt a quadrilateral polar circles are mentioned in [1] and [2] with the property:

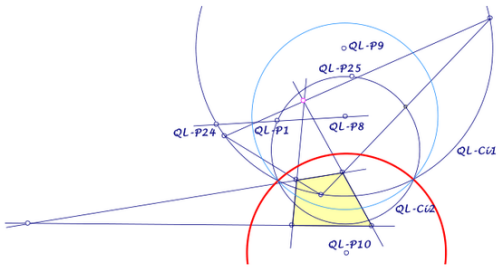
- **The polar circles for the triangle components of a quadrilateral are coaxal, conjugate wrt the circles over the diagonals** (see [1] p.40, [2], p.179).



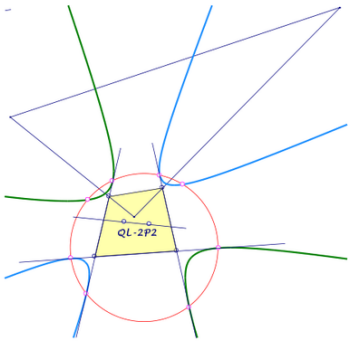
This means in *EQF*-geometry:

... The coaxal line is *QL-L2*, the radical axis is *QL-L1*.
... The polar circle is orthogonal to the Plücker circle *QL-Ci5*.

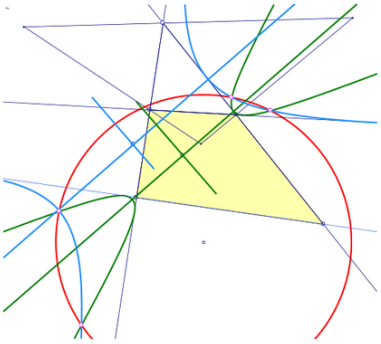
Below the polar circle of the diagonal triangle *QL-Tr1* – not always real – shall be tested.



- **The polar circle is centered in *QL-P10*.**
- **The inversion wrt the polar circle swaps *QL-Ci1* and *QL-Ci2*.**
- **The reflections of *QL-P1* and *QL-P25* in the polar circle lie on the circumcircle *QL-Ci1* of the diagonal triangle.**
- **The inversion circle of *QL-P1* and *QL-P24* wrt *QL-P8* cuts the polar circle orthogonal in the intersections with *QL-Ci1*.**
- **A *QL*-inscribed conic cuts the polar circle in a cyclic quadrangle with the same diagonal triangle as the reference quadrilateral.**



- **The two *QL*-inscribed conics with centers *QL-2P2* have contact points with the *QL*-lines on the polar circle.**
- **Two *QL*-inscribed conics, whose axes are parallel *QL-Tf2*-partners, intersect on the polar circle.**



References

[1] http://forumgeom.fau.edu/FG2004volume4/index.html

[2] R. A. Johnson: Advanced Euclidean Geometry

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-10-09.pdf

Message: #1320
Date: 10/10/2015 10:19:49
From: eckart_schmidt@t-online.de
Subject: New QL-Transformation

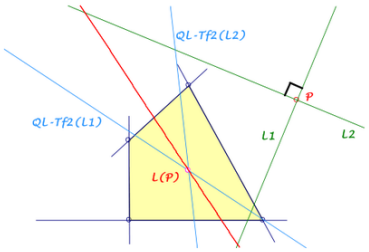
Dear Bernard, dear Chris,
it is remarkable that the only point-to-point QL-transformation QL-Tf1 doesn't map a wellknown QL-point to another wellknown QL-point.
Some considerations:
... First objects in QL-geometry are lines.
... Points can be seen as line-pencils.
... First transformation can be the line isoconjugation QL-Tf2 (analog QA-Tf2 for QA).
These raisons are the background for a new point-to-point QL-transformation P --> P(X):
... a bit complicated in construction (perhaps easier possible),
... no CSC-connection,
... not self-conjugate,
... but multiple linked to QL-EQF points and lines,
... main property: lines will be mapped to lines!!!
If you are interested, see attached file.
Best regards Eckart

EQF-Note 2015-10-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line Orientated QL-Point Transformation

QL-Tf2 is an isoconjugation for lines, images of orthogonal lines through a point P intersect on a line L(P). The corresponding point-to-line transformation leads in a second step to a point-to-point transformation (e.g. QL-P1 → QL-P17), which maps lines to lines (e.g. QL-L1 → QL-L9).



1. Point-to-Line Transformation $P \rightarrow L(P)$

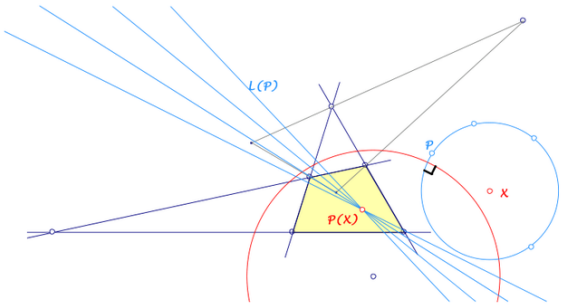
Definition: The image-line *L(P)* for a point *P* is the locus of intersections *QL-Tf2*(L_1) ∩ *QL-Tf2*(L_2) for orthogonal lines L_1 and L_2 through *P*.

This transformation has its own geometry, which shall not be worked out here, but some remarkable properties should be mentioned:

- ***L(QL-P1) = QL-Tf2(QL-L2), L(QL-P10) = QL-L1.***
- ***L(P)* for points *P* on *QL-Cu1* contain the point *P*.**
- **Inverse partners *P* and *Q* wrt the polar circle of *QL-Tr1* have the same line *L(P) = L(Q)*.**
- **For points *P* on a circle, orthogonal to the polar circle of *QL-Tr1*, the lines *L(P)* have a common point.**
- **The locus of all points *P* with line *L(P)* through a fixed point is a circle.**

2. Point-to-Point Transformation $X \rightarrow P(X)$

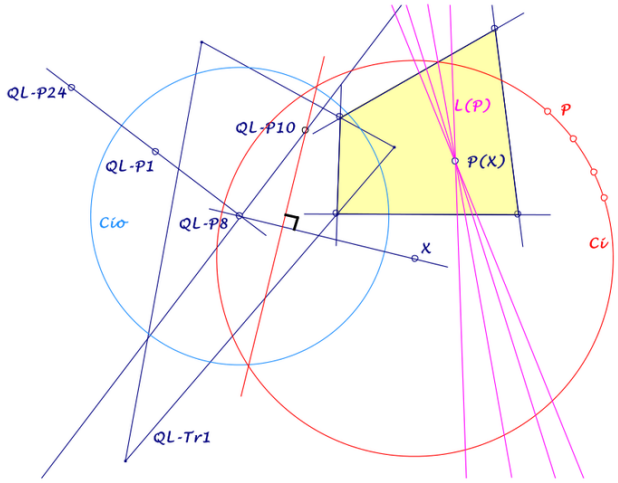
The last two properties lead to a point-to-point transformation. Let X be the center of the circle for the points P and let Y be the common point of the lines $L(P)$, then we have the transformation $X \rightarrow Y = P(X)$.
If the quadrilateral has an obtuse angled diagonal triangle *QL-Tr1*, the circles for the points P are orthogonal to the polar circle of *QL-Tr1*. Only points outside the polar circle have an image.



For a construction in general we need a circle …
… Ci_o inversion circle of *QL-P1* and *QL-P24* wrt *QL-P8*.
The image $P(X)$ then can be constructed as follows:

Construction of $P(X)$:

> **Let *Ci* be a circle round *X*, which has with Ci_o a radical axis through *QL-P10* perpendicular *X.QL-P8*, then the common point of *L(P)* for points *P* on *Ci* is *P(X)*.**



Properties:

- **The transformation $X \to P(X)$ is not self-conjugate, but for iterated applying holds: X_i, X_{i+2}, X_{i+3} are collinear.**

- ***QL-P1* → *QL-P17*, *QL-P8* → *QL-P13*,**
  ***QL-P7* → *QL-L6* ∩ *QL-L9*,**
  ***QL-P19* → point at infinity of *QL-L2*, *QL-L3*.**

- **Points at infinity will be mapped to the Newton line:**
  … of *QL-L1*, *QL-L4* → *QL-P23*,
  … of *QL-L2*, *QL-L3* → *QL-L1* ∩ *QL-L6*,
  … of *QL-L5*, *QL-L6* → midpoint of *QL-P17* and *QL-L6* ∩ *QL-L9*,
  … of *QL-L9* → infinity point of *QL-L1*,
  … of *QL-P1.P8.P24* → *QL-L1* ∩ *QL-P13.P17.P24*.

- **Points on a parallel to *QL-L9* through *QL-P19* will be mapped at infinity.**

If X is the intersection of this line and a line L, then $P(X)$ is the point at infinity of $P(L)$.

- **The transformation $X \to P(X)$ has three fixed points on *QL-Ci6*: S_1, S_2, S_3 (see *EQF*, *QL-Ci6*).**

The triangle $S_1S_2S_3$ is discussed detailed in *QFG*-message 678 and following messages. The vertices are the triple intersections of *QA-Co1* for the three *QG*-components of the quadrilateral. The sidelines are tangent to the *QL*-inscribed parabola *QL-Co1*. The triangle $S_1S_2S_3$ and *QL-Tr1* have a common circumconic through *QL-P8, QL-P13, QL-P24*. Trilinear poles and polars as well as Simson lines and isoconjugations wrt $S_1S_2S_3$ are relevant in *QL*-geometry.

- **For isoconjugations * wrt $S_1S_2S_3$ holds:**
  … The transformation $X \to P(X^*) = P(X)^*$ is self-conjugate.
  … The transformation $X \to P(X^*)^*$ is the inverse of $X \to P(X)$.

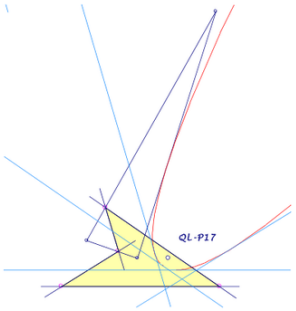
- **Lines will be mapped to lines by $X \to P(X)$:**
  ***QL-L1* → *QL-L9*, *QL-L2* → *QL-L6*,**
  ***QL-L3* → parallel through *QL-L1* ∩ *QL-L6* to *QL-L3*,**
  ***QL-L4* → perpendicular to *QL-P2.P19* through *QL-P23*,**
  ***QL-L5* → parallel to *QL-L2* through *QL-P17*,**
  ***QL-P1.P8.P24* → *QL-P13.P17.P24*,**

QL-P1.P25 → **tangent in** *QL-P17* **at** *QL-Ci1*.

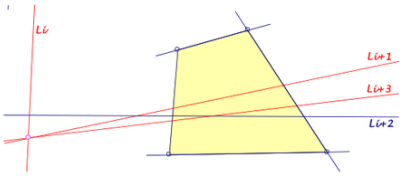
- **Midpoints of the** *QL*-**diagonals will be mapped in the opposite vertex of** *QL-Tr1*.

- **Diagonals will be mapped to lines through the opposite vertex of** *QL-Tr1* **and the intersection of the diagonal with** *QL-L1*.

- *QL*-**lines will be mapped on parallels, tangent to an** *QL-Tr1*-**inscribed parabola with focus** *QL-P17*.



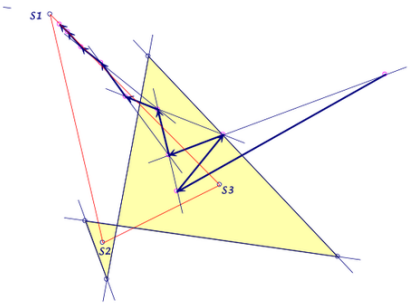
- **For iterated line transformations holds:** L_i, L_{i+1}, L_{i+3} **have a common point.**



- **For lines** *L* **through a point** *P* **the intersections of** *L* **and** *P(L)* **give a circumconic of** $S_1S_2S_3$ **through** *P*:
  … **for** *QL-P1*: **circle** *QL-Ci6*,
  … **for** *QL-P5*: **conic through** S_1, S_2, S_3, *QL-P5* **and** *QL-P23*,
  … **for** *QL-P7*: **orthogonal hyperbola through** S_1, S_2, S_3, *QL-P2, QL-P7, QL-P23, QL-L6* ∩ *QL-L9*,
  … **for** *QL-P8*: **common circumconic for** *QL-Tr1* **and** $S_1S_2S_3$ **through** *QL-P8*, *QL-P13*, *QL-P24*,
  … **for** *QL-P9*: **orthogonal hyperbola through** S_1, S_2, S_3, *QL-P2, QL-P9*,
  … **for** *QL-P12*: **conic through** S_1, S_2, S_3, *QL-P12, QL-P23*,
  … **for** *QL-P20*: **conic through** S_1, S_2, S_3, *QL-P20, QL-P23*,
  … **for** *QL-P22*: **conic through** S_1, S_2, S_3, *QL-P22, QL-P23*.

- **Circles and conics are mapped to conics:**
  … for *QL-Ci6*: circumconic of $S_1S_2S_3$ through *QL-P1* and *QL-P17*.
  … for *QL-Co1*: *QL-Tr1* inscribed parabola with focus *QL-P17*.

- **Iterated applying of $X \rightarrow P(X)$ leads to one of the fixed points S_1, S_2, S_3.**



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-10-10.pdf

Message: #1321
Date: 11/10/2015 10:04:25
From: eckart_schmidt@t-online.de
Subject: New QL-Transformation

Dear Bernard, dear Chris,

only now I saw, that the "new" QL-transformation X --> P(X) is
the same as the transformation QL-Tfy in an early EQF-Note:
http://eckartschmidt.de/2013-03-08.pdf
New simple construction for X --> P(X):
Let Co be a X-centered QL-Tr1-inscribed conic, then P(X) is the
common point of the QL-Tf2-images of its tangents.
There are further results in the EQF-Note wrt quadrigons, e.g.
... QG-P1 --> QG-P13,
... QA-P1 --> QG-P15,
... QG-L1 --> QG-P1.P2,
... QG-L2 --> QG-P3.P13,
... QG-L3 --> QG-P13.P15.

Best regards Eckart

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Message: #1322
Date: 11/10/2015 4:49:48
From: bernard.keizer
Subject: ETC and EQF

Dear Eckart,
I was very surprised, not to say disappointed, by your short answer to my message 1315.
Of course, you are right in your statement that any 2 parabolas inscribed in the medial triangle (with foci on the Euler circle) of a given triangle taken as DT could be the Co1 and Co3 and lead to mentionned property.
This is clearly triangle geometry ! (The applications formed the 2nd part of my message, which was entitled EQF and ETC).
Did you know this property before ?
In this case, have you any reference ?
But this property can be seen from the point of view of the QL.
The fact that Co3 is the complement wrt DT of the parabola of DQL (inscribed in DT and tangent to the Newton Line of the QL) and that the lines forming the tangential QL of Co3 in the intersection points with Co1 as well as the Newton Line of this 2nd QL are the isotomic transversals of the lines and the Newton Line of the 1rst is for me purely quadrilateral geometry !
In fact, I found this property accidentally by drawing the tangents to Co3 and observing the described symmetry wrt the middles of the sides of DT ; then I checked the barycentric DT-coordinates in EQF of P25, of Co1 and Co3 and of their intersection points and found the given explanation.
There are many bridges between triangle and QL-geometries ...
Best regards
Bernard

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Message: #1323
Date: 13/10/2015 8:57:52
From: eckart_schmidt@t-online.de
Subject: ETC and EQF

Dear Bernard,
excuse, if my short message was to rude, but my english isn't the best and often I can't explain my thoughts detailed.
I only wanted to give another sight of your "new property". The background can be generalized as follows:
Any two parabolas give with their intersections a quadrangle,
... tangents in the intersections lead to two quadrilaterals
... and cut the sides of the common diagonal triangle symmetrical to the middle.
This properties were new for me and I can't give any references.
They are independent of your interesting applications in part C.
Best regards Eckart

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Message: #1324
Date: 14/10/2015 11:24:41
From: bernard.keizer
Subject: ETC and EQF

Dear Eckart,
It's not that your message was too rude, but it was wrong in it's conclusion !
That's pure QA- and QL-geometry ...
The 2 parabolas QA-Co1a and b or QL-Co1 and Co3 are the same.
The fact that the 4 lines as well as the Newton Line are copples of isotomic transversals wrt DT is a really beautiful property.
The perspectors as well as the points P13 of the 2 QL's are isotomic wrt DT.
I'm convinced, there are other properties to find in drawing the complete figure of the 2 QL's (the anticomplement of P1 is one intersection of the circumcircle of DT with the Dimidium circle of the other QL ...)
And what about the triangles of the S-points of the 2 QL's ?
Best regards
Bernard

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Message: #1325
Date: 14/10/2015 11:30:20
From: bernard.keizer
Subject: New QL-Transformation

Dear Eckart, dear Chris
The fact that the Cl-S transformation doesn't map well known QL-points doesn't prove necessary that it's not appropriate, but perhaps simply that there are not enough QL-points !
EQF has 30 QL-points, whereas ETC has started with 400 points and counts now several thousands of points !!!
Best regards
Bernard

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Message: #1326
Date: 14/10/2015 2:45:49
From: chris.vantienhoven
Subject: ETC and EQF

```
Dear Bernard,
I placed your new property about the Isotomic relation between
the 2 Quadrilaterals described at QL-Co3 (Inscribed Parabola) in
EQF at QL-Co3.
Best regards,
Chris
```

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Message: #1327
Date: 15/10/2015 8:42:35
From: eckart_schmidt@t-online.de
Subject: ETC and EQF

```
Dear Bernard,
please cancel my remark
"So your new property is independant of QL-geometry."
in message #1318.
Best regards Eckart
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Message: #1328
Date: 15/10/2015 1:22:09
From: Benedetto Scimemi
Subject: Reconstructing a quadrangle from its centroid and diagonal trian

---

Dear friends, encouraged by Chris, who was wondering - in a recent message - about diagonal triangles being obtuse in (most) quadrangles, I hope that you will find of some interest the following construction recipe:

PROBLEM: Given a triangle T and a point G, costruct a quadrangle Q which has T as diagonal triangle and G as centroid.

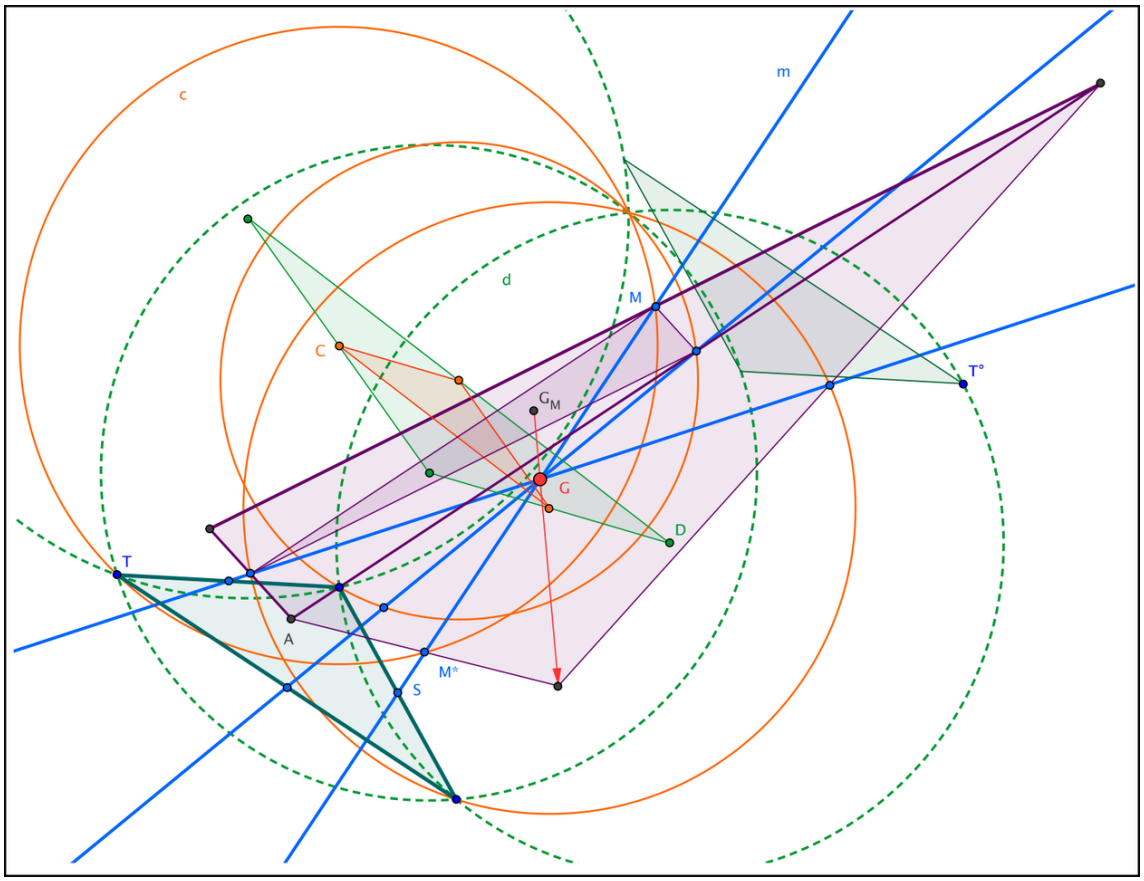
SOLUTION (by ruler and compass; see attached figure)
1) construct triangle S =S1S2S3 , medial of T =T1T2T3
2) c. (the bimedian) lines mi = GSi
3) c. triangle T ° = T1 ° T2 ° T3 ° by reflecting T on G
4) c. circles d i = (Ti Tj Th °) and their centers Di
(Their common point will be QA-P3)
5) c. triangle C1C2C3 , medial of D1D2D3
6) c. circles ci , centered in Ci through Ti
(these will be QG-ci4)
7) intersect ci with mi to get three pairs
(of side midpoints) Mi , Mi * (see below for existence)
8) c. triangle ABC antimedial of M1 M2 M3
9) c. point D = 4G - A - B - C
Then quadrangl e Q = ABCD has G as centroid and T as diagonal triangle.

A LOCUS FOR G, GIVEN T (see attached figure)
The procedure above is successful iff in step 7 the lines and circles actually meet. To see when this happens, consider the six lines T i Tj and S i Sj , which separate the plane into 16 regions. Now look at where G is located: 7 of these regions are "G -good", 9 are "G -bad". Each pair of parallel sides T i Tj , S i Sj produces a stripe. The points of the good regions are either inside or outsides all three stripes. When G lies on the borders, the quadrangle degenerates.
Cabri and Geogebra users may enjoy watching the quadrangle Q appear and disappear when G moves across the sides of T and S.

SPECIAL CASE: CYCLIC QUADRANGLES
In general, a quadrangle Q is cyclic if and only if its centroid G lies on the ninepoint circle of the diagonal triangle T.
Thus, for cyclic quadrangles, we find a G -good and a G- bad arc on this circle. And one sees that good arcs exist (i.e. are non-empty) iff T is obtuse (see the figure below).
In conclusion:

The diagonal triangle of a cyclic quadrangle is obtuse; and any obtuse triangle is the diagonal triangle of (an infinite number of) cyclic quadrangles.
N.B. As we have the tendency to conceive quadrangles not far from being cyclic, the good arc turns out to be ... short, and this explains why "most" of our diagonal triangles are obtuse.

A RELATED MATTER : A CHAIN OF QUADRANGLES SHARING A LOT
Given a quadrangle Q , consider a quadrangle R obtained from Q as folllows: three vertices from the antimedial of the diagonal triangle, plus the isotomic point (QA-P5). Then Q and R share the central points QAP1,QAP2, QAP3, QAP4 (!), the ninepoint conic QACo1 and much more.
In particular, Q is cyclic iff R is cyclic.
By iterating, one gets a chain of quadrangles which share a lot of features.
The construction above is in fact a way to produce Q starting from R.
Best regards
Benedetto



4ng from DT, G .pdf

Message: #1329
Date: 15/10/2015 1:37:43
From: minhtenladuong
Subject: A Transformation

Dear Geometers
Today I found a transformation point---> line as follow:
Given a quadrangle ABCD and a point P
A',B',C',D' are isogonal conjugate of P WRT BCD,CDA,DAB,ABC, respectively.
Then midpoints of AA',BB',CC',DD' are collinear.
Sincerely
Ngo Quang Duong

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Message: #1330
Date: 16/10/2015 10:39:05
From: eckart_schmidt@t-online.de
Subject: A Transformation

Dear Ngo Quang Duong,
a very interesting transformation, first properties:
(1) Some single results:
... image line of QA-P2 through QA-P23,
... image line of QA-P3 through QA-P1,
... image line of QA-P4 does not exist, for the midpoints of AA', BB', CC', DD' are QA-P2,
... image line of QA-P8 through QA-P7,
... image line of QA-P12 through QA-P11,
... image line of QA-P17 parallel QA-P2.QA-P11.
(2) Points on a line of the quadrangle give lines through the midpoint of the opposite line.
(3) The Miquel points give lines through QA-P1.
(4) Points on the cubic QA-Cu1 give lines through QA-P1.
Wrt this remarkable property let us consider the "inverse" of the transformation:
(5) There are always two points, which give the same line.
These points can be constructed as follows:
... Let L be a line, A' the reflection of A in L,
... L' a parallel to L through A',
... CoA the isogonal image conic of L' wrt BCD,
... analog CoB, CoC, CoD
... with two common points, which give the line L wrt your transformation.
(6) These inverse point pairs for lines through QA-P1 give the conic QA-Co1.
(7) These inverse point pairs for lines through other QA-points give interesting QA-circumcubics through QA-P4.
Best regards Eckart

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Message: #1331
Date: 16/10/2015 2:31:15
From: minhtenladuong
Subject: A Transformation

```
Dear mr Eckart Schmidt ,
They are very interesting properties.
Thank you for your interest.
Best regard
Ngo Quang Duong
```

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Message: #1332
Date: 16/10/2015 5:44:19
From: Benedetto Scimemi
Subject: A Transformation

```
Dear
Dear Ngo Quang Duong,
about your interesting last transformation:  P -->  r(P) :
QA-Tf3 (orthopole)  takes  P into a point of r(P)
This generalizes the "single results" of Eckart.
Best regards
Benedetto
```

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Message: #1333
Date: 17/10/2015 9:55:20
From: eckart_schmidt@t-online.de
Subject: A Transformation

Dear Ngo Quang Duong,
in my message 1330 there is a mistake: Replace (6) by
(6) These inverse point pairs for lines through QA-P1 give the cubic QA-Cu1.
In addition another property:
Let L be lines through a point P
... and L' the line of the inverse pair of points for L
... and X the intersection L^L'.
... The locus of the points X is an orthogonal hyperbola
... through point P and the common point of L'
... with asymptotes parallel to the asymptotes of QA-Co2
... and center in the midpoint of P and the common point of L'.
As Benedetto Scimemi (#1332) noticed: P is the QA-Tf3-image of the common point of L'.
Best regards Eckart

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Message: #1334
Date: 17/10/2015 11:15:20
From: chris.vantienhoven
Subject: A Transformation

Dear Ngo Quang Duong, Eckart and Benedetto,
Thanks Ngo Quang Duong for your nice transformation (Tf1), transforming a point into a line!
Like Eckart commented there is an inverse transformation (Tf2), transforming a line into 2 points.
When we perform these transformations consecutively (first Tf1, then Tf2), we transform one point Q1 into two points Q1 & Q2. So actually Q1 produces a new point Q2. When we call this combined transformation Tf3 we actually can speak of a conjugate because Tf3(Q1)=Q2 and Tf3(Q2)=Q1.
The coordinates of Tf3 are very nasty and very long.
I found these incidences:
* Tf3(QA-P3) = point on QA-Cu1
* Tf3(QA-P7) = point on QA-P33.QA-Px
(QA-Px = perspector mentioned by Seiichi, see QFG # 324)
* Tf3(QA-P9) = point on QA-P2.QA-P9
* Tf3(QA-P41) = point on QA-P3.QA-P41
* Tf3(QA-P41) = point on QA-Cu1
Numerically determined. There can be precision errors.
About Tf1 already mentioned by Eckart:
* Tf1(QA-P2) = line through QA-P23
* Tf1(QA-P3) = line through QA-P1
* Tf1(QA-P8) = line through QA-P7
* Tf1(QA-P12) = line through QA-P11
* Tf1(QA-P17) = line parallel to QA-P2.QA-P11
Not mentioned by Eckart:
* Tf1(QA-P9) = line QA-P2.QA-P9
* Tf1(QA-P32) = line through QA-P2.QA-P7 ^ QA-P1.QA-P32
* Tf1(QA-P41) = line through QA-P1
* Tf1(QA-P42) = line through QA-P42
Numerically determined. There can be precision errors.
Best regards,
Chris

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Message: #1335
Date: 18/10/2015 9:59:37
From: eckart_schmidt@t-online.de
Subject: ETC and EQF

Dear Bernard,
perhaps something new for the constellation you discuss, already known properties are not always mentioned.
Let QA be a reference quadrangle and Pb1 and Pb2 the circumscribed QA-parabolas (QA-2Co1a,b) with foci F1 and F2.
... The line F1F2 is parallel to the Simson line of QA-P36 wrt the QA-medial DT.
... The parabolas Pb1 and Pb2 are tangent to the QA-medial DT.
... The Brianchon points B1, B2 are the isotomic conjugates (wrt QA-medial DT) of the infinity points of the parabola axes
... or the QA-DT complements of the isotomic conjugates (wrt QA-DT) of the infinity points of the parabola axes.
... The lines FiBi intersect on QA-Ci2 in the 4th intersection with the Steiner circumellipse of the QA-medial DT.
... The tangents to the parabolas in the vertices of QA give two quadrilaterals QL1, QL2
... with common diagonal triangle in QA-DT,
Miquel points F1, F2 and inscribed parabolas Pb1 and Pb2.
... Points and lines wrt the common QL-DT = QA-DT coincide for the two QL.
... The two QL have the same QL-L8.
... The lines QL-L5 of the two QL intersect in QA-P36.
... The Dimidium circles of QL1, QL2, which contain Fi, have a radical axis through QA-P10.
... The triangles S1S2S3 (see EQF QL-Ci6) on these Dimidium circles are tangent to the parabolas Pb1, Pb2
... with Brianchon points C1, C2 on QA-Co1.
... For the triangles S1S2S3 the centroids lie symmetric to QA-P10, the orthocenters to QA-P13 and the circumcenters to the midpoint of QA-P11.QA-P13.
... Tangents in the Brianchon points C1, C2 at the Steiner circumellipse of S1S2S3 intersect in QA-P10.
... The 4th nintersection of the circumcircle and the Steiner circumellipse of S1S2S3 are collinear with Fi and Ci.
... The 6 contact points of the parabolas to the triangles S1S2S3 lie on the conic QA-Co1.
... The triangles S1S2S3 have also a common circumconic.
Best regards Eckart

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Message: #1336
Date: 18/10/2015 6:07:53
From: eckart_schmidt@t-online.de
Subject: A Transformation

Dear Chris, Ngo Quang Duong and Benedetto,
Chris has proposed in #1334 a further transformation Tf3, this is an interesting aspect. In the following remarks I shall use his nominations Tf1 (point to line), Tf2 (line to pair of points) and Tf3 (point to point).

... Tf3 maps the diagonal triangle QA-DT
    to the Miquel triangle (perspector QA-P3).
... The line pencil of QA-P3 gives wrt Tf2 pair of points
    on a QA-circumconic through QA-P4,
... which divides the sides of QA-DT harmonically.
... Points on this conic will be mapped
    by Tf3 in points at infinity.
... The line pencil of QA-P1 gives wrt Tf2 pair of points
    on the cubic QA-Cu1 (see #1330, corrected in #1333).
... QA-Cu1 is Tf3 invariant
... with pivot QA-P3 (that means Tf3 partners on QA-Cu1
    are collinear with QA-P3).
... There are four fixed points of Tf3 on QA-Cu1,
    whose tangents intersect in QA-P3.
... The quadrangle of these four points has the same QA-Cu1
    as the reference quadrangle.
... In general the fixed points of Tf3 lie on a circumquartic
    of the reference quadrangle through QA-P4.
... The line pencil of a point P (unequal QA-P3) gives wrt Tf2
    pair of points on a QA-circumcubic through QA-P4
... and a pivot on the cubic,
... containing four fixed points of Tf3,
    whose tangents intersect in the pivot.

Best regards Eckart

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Message: #1337
Date: 19/10/2015 8:14:53
From: chris.vantienhoven
Subject: A Transformation

Dear Eckart,
Beautiful and surprising properties of Tf3 you describe.
Especially that Tf3 maps the diagonal triangle QA-DT to the Miquel triangle (perspector QA-P3),
and
QA-Cu1 is Tf3 invariant with pivot QA-P3.
We already knew that cubic QA-Cu1 is QA-Tf2 invariant with pivot QA-P4.
Best regards,
Chris
p.s. Tf3 should not be confused with QA-Tf3.
Tf3 is meant to be a general indication for a new transformation.

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Message: #1338
Date: 20/10/2015 11:05:48
From: bernard.keizer
Subject: ETC and EQF

Wunderbar, lieber Eckart !
Thanks a lot, it's exactly the kind of answer I hoped ...
The properties with the S-points are particularly interesting !
Best regards
Bernard

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Message: #1339
Date: 20/10/2015 3:26:13
From: eckart_schmidt@t-online.de
Subject: A Transformation

```
Dear Chris,
thanks for your feedback, there is a further transformation
possible:
Tf4 maps a line L to the line of the point pair wrt Tf2(L).
Some first properties wrt this line-to-line transformation:
... Pi.Pj --> Pk.Pl for a quadrangle P1P2P3P4.
... QA-L1 --> QA-P3.4, QA-L4 --> QA-L1.
... Line pencil of P --> line pencil of QA-Tf3inv(P).
... Tf4(Tf4(L)) // L.
... QA-Tf3(L) // Tf4(L).
... Lines parallel to an axis of QA-Co2
    have a parallel Tf4-image line.
... Parallel (orthogonal) lines
    have parallel (orthogonal) image lines wrt Tf4.
... In general: Tf4 is angle-true.
... Tf4 of the axis of a circumscribed parabola
    is parallel to the other axis.
... For a line pecil wrt a point P the intersections L ^ Tf4(L)
    give an orthogonal hyperbola (see #1333):
... ... through point P and QA-Tf3inv(P) = Q
... ... with asymptotes parallel to the asymptotes of QA-Co2
... ... and center in the midpoint of P.Q
... ... and tangent P.QA-Tf3(P) in P
... ... and tangent Tf4(P.Q) in Q.
... For a quadrilateral the three Tf4-images of QL-L1
    are the lines from QL-P1 to the QL-DT-vertices.
Best regards Eckart

PS: I am not so familar with QA-Tf3, perhaps something is
already mentioned or evident.
```

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Message: #1340
Date: 20/10/2015 9:33:32
From: chris.vantienhoven
Subject: A Transformation

```
Dear Eckart,
I have a short holiday now.
So I cannot verify everything.
However the transformation you described is (when I am not
wrong) the inverse of QA-Tf3.
I used this property when calculating the coordinates of TF3
(P).
Again QA-Tf3 and Tf3 are not to be confused.
When I look to your properties it looks they belong to the
inverse of QA-Tf3 indeed.
Maybe you can check it fully.
Best regards,
Chris
```

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Message: #1341
Date: 21/10/2015 6:55:22
From: eckart_schmidt@t-online.de
Subject: A Transformation

```
Dear Chris,
you are right, QA-Tf3inv maps a line as Tf4! Thanks for
clarification!
Best regards Eckart
```

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Message: #1342
Date: 21/10/2015 7:07:52
From: chris.vantienhoven
Subject: A Transformation

Dear Eckart, Benedetto and Ngo Quang Duong,
Like Benedetto commented in message #1332:
QA-Tf3(Q1) is a point on the line L=Tf1(Q1), where Tf1 = the transformation Ngo Quang Duong initiated in message #1329.
Since this line has another point Q2 being transformed by Tf1 into the same line L,
we can say that QA-Tf3(Q2) also is on the same line Tf1(Q2)=Tf1(Q1)=L.
So QA-Tf3(Q1) as well as QA-Tf3(Q2) lie on L and since QA-Tf3 is a linear transformation it has to be that the inverse of QA-Tf3 (your Tf4) transforms line L into line Q1.Q2.
Best regards,
Chris

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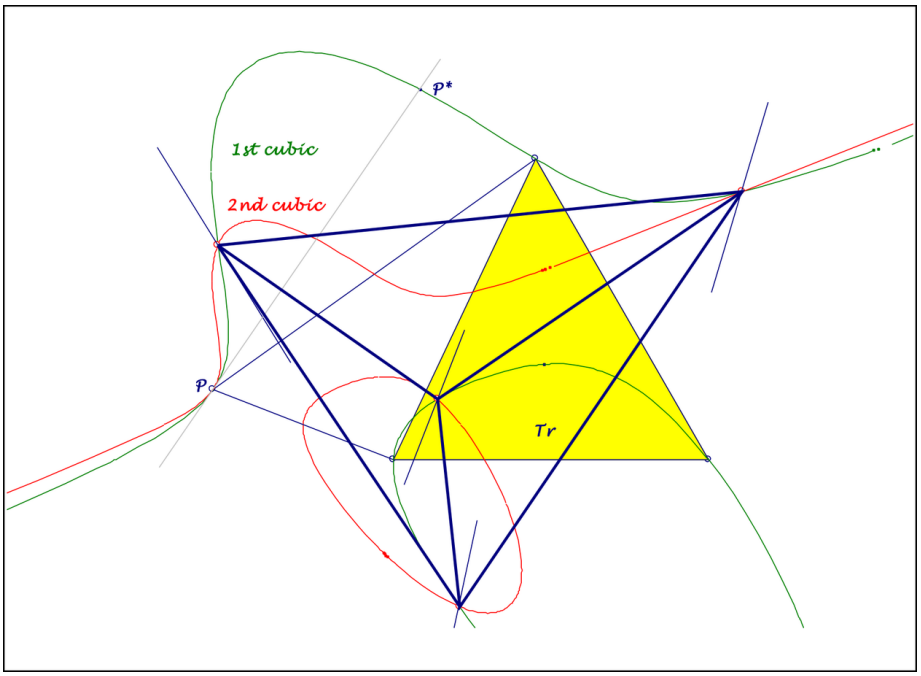
Message: #1343
Date: 24/10/2015 11:16:20
From: eckart_schmidt@t-online.de
Subject: A Transformation

Dear Chris, Benedetto and Ngo Quang Duong,
wrt the primary transformation of Quang Duong
(for Chris Tf1) I have looked for points X, which lie on their image line Tf1(X):
... The locus is a quartic,
... circumscribed the quadrangle,
... through QA-P4, QA-P42,
... with asymptotes parallel to the asymptotes of QA-Co2.
These properties are Mathematica controlled. I failed at constructing the quartic.
Special points with X on Tf1(X) are the fixed points of the initial mappings wrt a triangle component Tr:
"isogonal conjugate and midpoint to the remaining point P".
For each triangle component Tr there are 4 such points, forming an orthocentric quadrangle.
These 4 points are intersections of two QA-circumcubics, contacting in P with tangent PP* (see attached file):
... 1st cubic Cu: isogonal isocubic wrt Tr and pivot P.
... 2nd cubic: Locus for the midpoints of P and points on Cu.
The last properties are only Cabri controlled and Cabri has its limits!
Best regards Eckart



2015-10-24.docx

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Message: #1344
Date: 31/10/2015 4:48:10
From: eckart_schmidt@t-online.de
Subject: Some QL-loci

```
Dear Chris,
here are gathered some loci for QA-points of the contact-QAs of
QL-inscribed conics:
The contact-QA has the same diagonal triangle as the reference
QL, so the QA-points QA-P10 til QA-P13 are the QL-points QL-P8
til QL-P11.
Locus of QA-P2: QL-Ci1,
... QA-P16: QL-DT inscribed conic with Brianchon point
            QL-P13 or envelope of the QL-Tf2-images
            of lines through QL-P8,
... QA-P19: homothetic wrt the locus of QA-P16,
            center QL-P8, factor -2,
... QA-P23: QL-DT inscribed conic, centered on QL-L2:
            envelope of the QL-Tf2-images of lines through
            QL-P10,
... QA-P29: QL-Ci2,
... QA-P30: QL-Ci1,
... QA-P31: homothetic wrt the locus of QA-P16,
            center QL-P8, factor -1/2,
... QA-P35: circle: center divides QL-P9.P10 with ratio 2/3,
            radius 1/5 of the radius of QL-Ci1,
... QA-P36: QL-Ci2.
Perhaps some EQF-properties.
Best regards Eckart
```

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Message: #1345
Date: 01/11/2015 11:59:20
From: chris.vantienhoven
Subject: Some QL-loci

```
Thanks Eckart,
Nice observations!
Best regards,
Chris
```

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Message: #1346
Date: 01/11/2015 3:20:05
From: Benedetto Scimemi
Subject: Isogonal conjugates of points at infinity

Dear friends, the following observations may be well-known, but I did not find them in EQF. By the way, I think that in EQF it would be nice to have pages dedicated to central points at infinity.
Given a quadrangle A=A1A2A3A4 and a direction d (= any point at infinity), the points Di, isogonal conjugates of d on the component triangles AjAhAk, give a quadrangle D=D1D2D3D4 with the following properties:
(For any d) D is similar to O=O1O2O3O4, the quadrangle of circumcenters. The mapping Di -> Oi is a spiral similarity (depending on d) centered in QAP4. When d rotates of an angle alpha, Di moves on the circumcircle (AjAhAk), rotating in the opposite direction of angle - 2 alpha around Oi. Central points of D also describe circles, centered in corresponding central points of O.
Special choices for d:
As a function of d, the quadrangle D has minimal and maximal size when d is a "principal" direction of A , i.e. a bisector of the angle Oi is a homothety. The geometric mean of the areas of Dmax and Dmin is the area of A.
Best regards
Benedetto

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Message: #1347
Date: 03/11/2015 9:35:02
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugates of points at infinity

Dear Benedetto,
in addition to your observation:
Let A1A2A3A4 be a quadrangle,
O1O2O3O4 the quadrangle of circumcenters,
d any point at infinity,
Ei isogonal conjugates of d on the component triangles OjOkOl.
E1E2E3E4 is similar to the reference quadrangle A1A2A3A4.
The mapping Ai -> Ei is a spiral similarity (depending on d) centered in QA-P4.
Background: The 2nd quadrangle of orthocenters is similar to the reference quadrangle wrt a spiral similarity centered in QA-P4 (see EQF Ref [23]).
This holds not only for the 2nd O-quadrangle, but also for the 2nd D-quadrangle and the 2nd E-quadrangle.
For the direction d of the angle bisector of
Best regards Eckart

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Message: #1348
Date: 03/11/2015 10:50:32
From: Benedetto Scimemi
Subject: Isogonal conjugates of points at infinity

Dear Eckart,
yes (about your last remark) and the same holds if the q. of circumcenters is replaced by the q. of isogonal conjugates C1C2C3C4 and any other q. in the chain .. --> Ci --> Ai --> Oi ->... where the (principal) central points at infinity are conserved and a double step implies a QA-P4 homothety.
Benedetto

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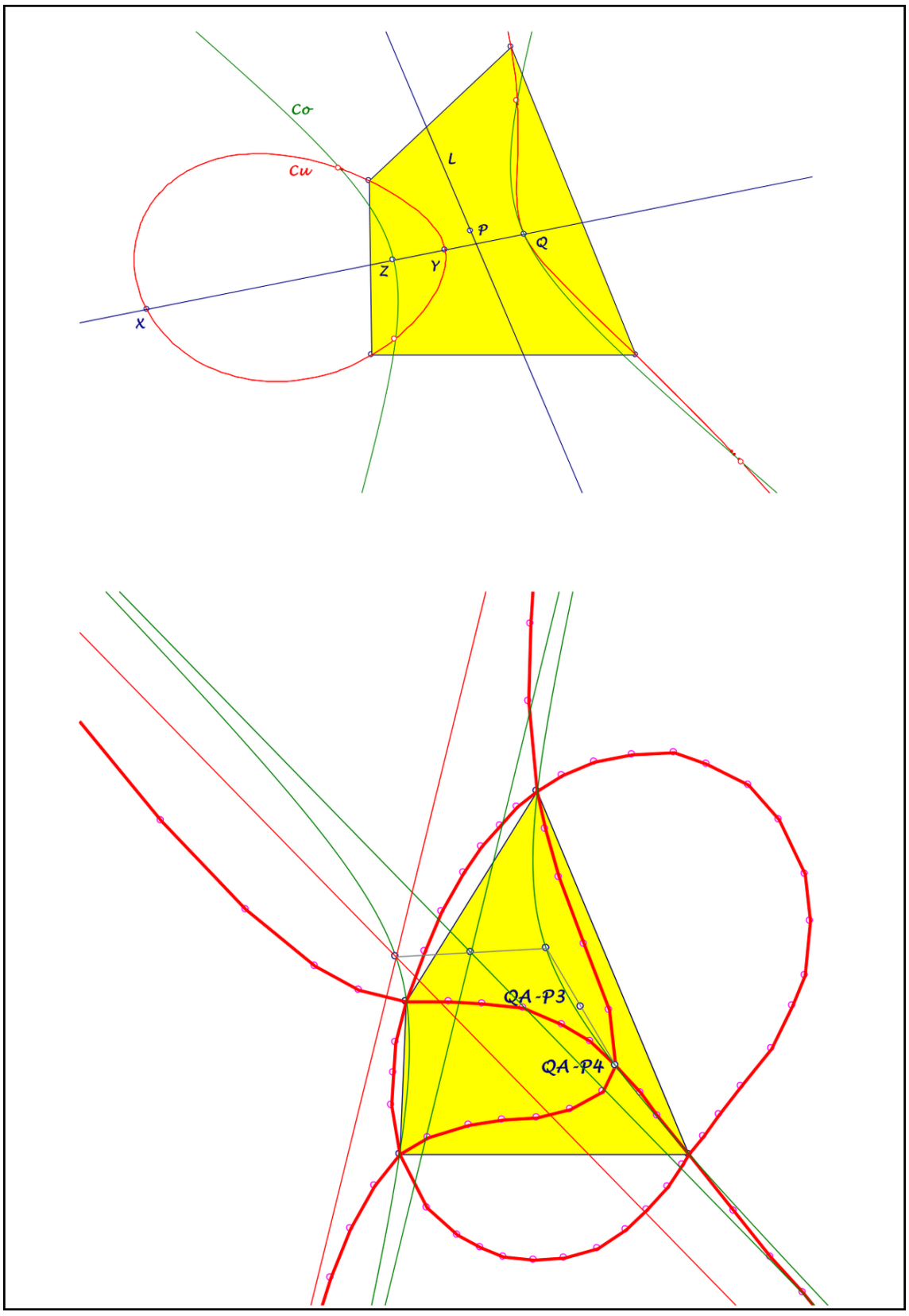
Message: #1349
Date: 03/11/2015 4:00:08
From: eckart_schmidt@t-online.de
Subject: Fixed points of Tf3

Dear Chris, Benedetto and Ngo Quang Duong,
some remarks wrt the fixed points of Tf3 (definition see # 1333):
(1) Fixed points on QA-Cu1:
Lines through a QA-vertex and an in-/ex-center of the Miquel triangle QA-Tr2 intersect four times on QA-Cu1. These four points are fixed points of Tf3 and give a new quadrangle with the same Miquel triangle and QA-Cu1, four times perspective to the reference quadrangle wrt an in-/ex-center of the Miquel triangle. - These fixed points of Tf3 are the contact points of tangents from QA-P3 at QA-Cu1.
(2) Generalisation (see attached file):
Let L be lines through an arbitrary point P, then Tf2(L) give pair of points X, Y,
... which lead to a QA-circumscribed cubic Cu
    (for P=QA-P1 this cubic is QA-Cu1).
... The lines XY intersect in a pivot Q on Cu: Q=QA-Tf3inv(P).
... The contact points of tangents from Q at Cu
    are fixed points of Tf3.
... or:
... The locus of the 4th harmonic points Z of Q wrt X, Y
    give a conic Co.
... The intersections of the conic Co and the cubic Cu
    (without contact point Q) give the same fixed points of Tf3.
... The four fixed points of Tf3 on Cu (not always real)
    have Cu as QA-Cu1.
(3) The "curve" for the fixed points of Tf3 seem to be two QA-circumscribed quartics (see sketch in the attached file).
(4) Points on the QA-circumconic through QA-P4 are mapped by Tf3 in points at infinity, This conic and the "curve" have parallel asymptotes.
(5) Intersection of the "curve"-asymptotes will be the following point: Reflect QA-P4 in QA-P3, then reflect this point in the center of the conic (4).
I failed at constructing the curve.
Best regards Eckart



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Message: #1350
Date: 09/11/2015 8:38:38
From: eckart_schmidt@t-online.de
Subject: Some QA-loci

```
Dear Chris,
here are gathered some loci of QL-points for QA-circumscribed
quadrilaterals with sidelines tangent to a QA-circumscribed
conic.
The circumscribed QL has the same diagonal triangle as the
reference QA, so the QL-points QL-P8 til QL-P11 are the
QA-points QA-P10 til QA-P13.
Locus of QL-P1:  circle QA-Ci2,
... QL-P16: circle round QA-P11 through QA-P2 and QA-P30
            (diametral),
... QL-P17: circle round QA-P11 through QA-P2 and QA-P30
            (diametral),
... QL-P24: circle QA-Ci1,
... QL-P25: circle QA-Ci2,
... QL-L1:  envelope of QA-Co1,
... QL-L2:  line pencil of QA-P11,
... QL-L6:  line pencil of QA-P12,
... QL-L7:  fixed in QA-L5,
... QL-L8:  line pencil of QA-P10.
Perhaps some EQF-properties.
Best regards Eckart
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Message: #1351
Date: 09/11/2015 9:23:50
From: chris.vantienhoven
Subject: Some QA-loci

```
Dear Eckart,
Very nice. Thank you!
Best regards,
Chris
```

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Message: #1352
Date: 10/11/2015 10:56:05
From: eckart_schmidt@t-online.de
Subject: Foci of QA-Circumscribed Conics

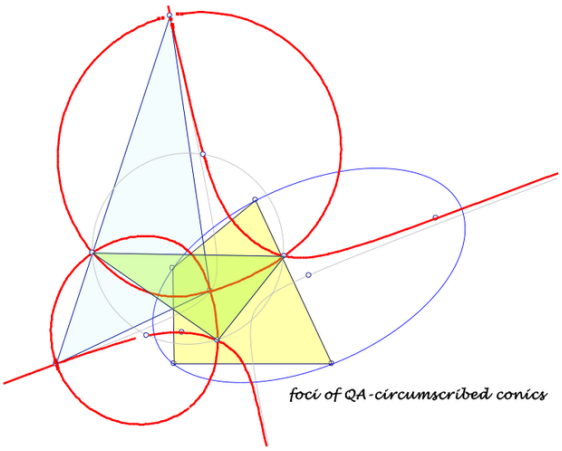
Dear Bernard, dear Chris,
for a quadrilateral the locus of foci for inscribed conics is the well known cubic QL-Cu1. What about the foci of QA-circumscribed conics?
I don't know, whether the following property is already published:
* The foci of QA -circumscribed conics are isogonal conjugates wrt the orthic triangle of the diagonal triangle QA-Tr1.
First Cabri-observations in the attached file.
Best regards Eckart

EQF-Note 2015-11-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Foci of QA-Circumscribed Conics

For a quadrilateral the locus of foci for inscribed conics is the well known cubic QL-Cu1. What about the foci of QA-circumscribed conics? Here are gathered first Cabri-observations.

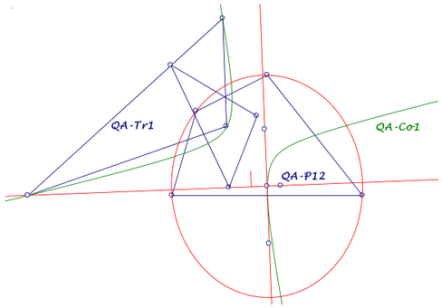


- The locus for the foci of *QA*-circumscribed conics seems to be a sextic.
- This sextic has six nodal points in the vertices of the diagonal triangle *QA-Tr1* and its orthic triangle.
- The further intersections of the circle *QA-Ci2* and the sextic are the foci of the two *QA*-circumscribed parabolas *QA-2Co1*.
- The two asymptotes of the sextic are parallel to the axes of the paraboloas *QA-2Co1*, intersecting in the reflection of *QA-P6* in *QA-P1*.
- The 2nd foci wrt the vertices of the orthic triangle of *QA-Tr1* are the further intersections of the opposite side and the sextic.
- **The foci of *QA*-circumscribed conics are isogonal conjugates wrt the orthic triangle of the diagonal triangle *QA-Tr1*.**

2015-11-10.pdf

- The sextic is isogonal invariant wrt the orthic triangle of *QA-Tr1*.

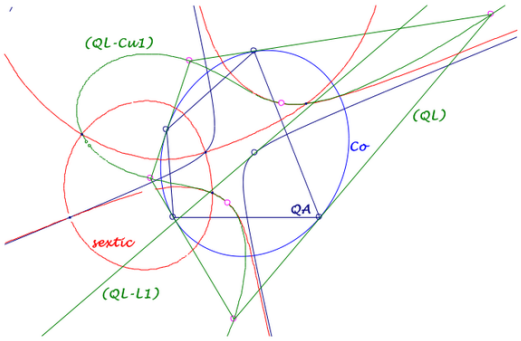
- The sextic is the locus of isogonal conjugates wrt the orthic triangle of *QA-Tr1*, whose midpoints lie on the conic *QA-Co1*.



- For the 2nd intersection of an *QA-Tr1*-altitude and the conic *QA-Co1* as center of an *QA*-circumscribed conic the main axis is orthogonal to the altitude.

Now we consider for a quadrangle *QA* a circumscribed quadrilateral *QL* with sidelines tangent in the vertices to a circumscribed conic *Co*.

- The Newton line of the circumscribed quadrilateral *QL* is the tangent to *QA-Co1* at the center of the conic *Co*.



- The cubic *QL-Cu1* of the circumscribed quadrilateral *QL* is tangent to the sextic in the foci of the circumscribed conic *Co*. The intersections are the vertices of the orthic triangle of *QA-Tr1*.

Eckart Schmidt
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eckart_schmidt@t-online.de

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Message: #1353
Date: 10/11/2015 12:40:33
From: eckart_schmidt@t-online.de
Subject: Foci of QA-Circumscribed Conics

Dear Bernard, dear Chris,
sorry, I just noticed, that there is already an EQF-Note 2014-04-02 wrt this theme in #496 with some further properties.
Excuse Eckart

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Message: #1354
Date: 13/11/2015 8:34:05
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

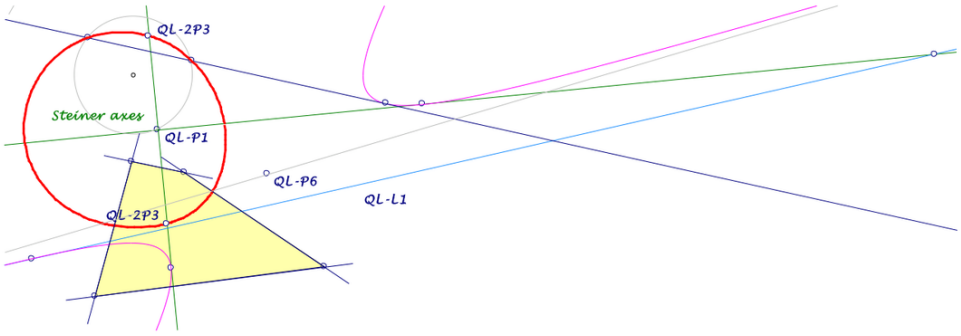
Dear Bernard, dear Chris,
CSC-partners on tangents of a conic give a quartic, CSC-invariant through the CSC-fixed points QL-2P3. In # 364 such a QL-quartic is described, attached another example. Perhaps of interest.
Best regards Eckart

EQF-Note 2015-11-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A Quartic for Quadrilaterals II

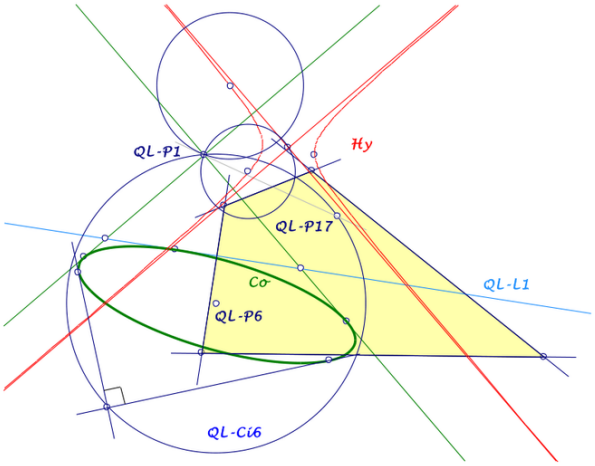
Image partners wrt QL-Tf1 – shortened CSC – on tangents of a conic give a quartic, CSC-invariant through the CSC-fixed points QL-2P3. An example can be found in QFG-message 364. Here another quartic is described with a geometric interpretation orientated at Apollonius circles.



A special conic

Let *Co* be a conic, centered in *QL-P6* and tangent to the Steiner axes and the Newton line *QL-L1*.

- Points on the Dimidium circle *QL-Ci6* have orthogonal tangents wrt the conic *Co*.



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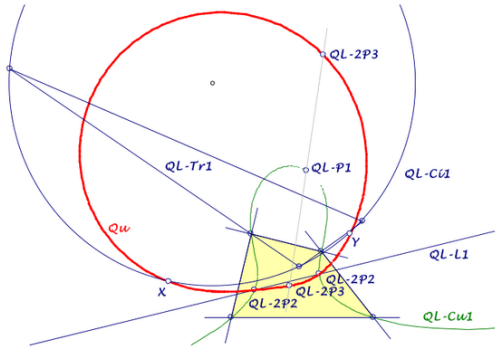
- The *CSC*-circles of tangents at *Co* are centered on an orthogonal hyperbola *Hy* ...
  … through the center of the *CSC*-circle of *QL-L1*,
  … centered in the pole of *QL-P1.QL-P17* wrt *QL-Ci6*,
  … with asymptotes parallel to the Steiner axes,

The Quartic

- *CSC*-partners on tangents of *Co* give a *CSC*-invariant quartic *Qu* through the *CSC*-fixed points *QL-2P3*.

- The quartic *Qu* is also the locus of *CSC*-partners on circles through *QL-P1* centered on the orthogonal hyperbola *Hy*.

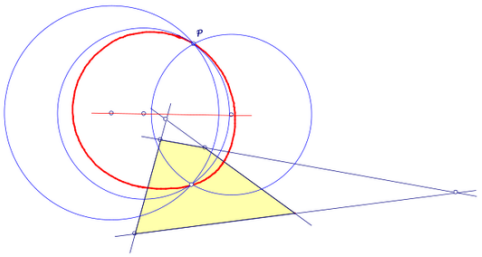
- If the cubic *QL-Cu1* is unipartite, the quartic *Qu* contains the *CSC* conjugated intersections *QL-2P2* of the Newton line and the cubic *QL-Cu1*.



- If the cubic *QL-Cu1* is unipartite, the cubic *Qu* contains the *CSC*-partners *X, Y* on the circumcircle *QL-Ci1* of the diagonal triangle.

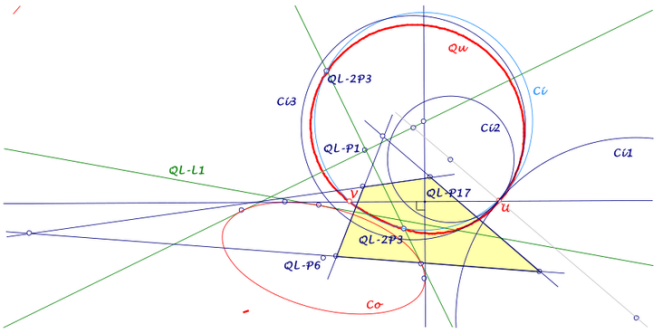
Geometric background

For two opposite points of the quadrilateral we can take the Apollonius circle wrt a given point *P*. There are three such Apollonius circles for a point *P*.



- **The quartic *Qu* is the locus for points *P* with coaxal Apollonius circles.**

Final remark: There are two special points *U, V* on the quartic, which are the contact points of their Apollonius circles Ci_1, Ci_2, Ci_3 with a common tangent. These points *U, V* can be found as follows: Take the orthogonal tangents from *QL-P17* at the conic *Co*. The *CSC*-partners on one tangent are *U, V*. The other tangent has no *CSC*-partners, but cuts the 2nd Steiner axis in the center of a circle *Ci* through *QL-2P3* and *U, V*. The common tangent of the Apollonius circles is also tangent to *Co*.



Eckart Schmidt
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eckart_schmidt@t-online.de

2015-11-13.pdf

Message: #1355
Date: 15/11/2015 11:12:47
From: bernard.keizer
Subject: QL-Quartic II

Dear Eckart, dear Chris
The nightmare goes on and it is again horror and terror in Paris!
Your property with quartics is really interesting.
I tried to compare the 2 already studied : in message 364 with conic inscribed in DT or QL-Tr1 and tangent to the 2 Steiner axes and to the Steiner line QL-L2 and in message 1354 with conic tangent to the 2 Steiner axes and to the Newton Line QL-L1.
Now I use following notations : DQL is the QL formed by the diagonals or sides of QL-Tr1 and the Newton Line QL-L1, QL-P17 is the Miquel point of DQL, Cl-Sdiag is the Cl-S associated to DQL with invariant points DQL-2P3 symmetric wrt QL-P17 on the 1rst Steiner axis of DQL.
We have following properties : Cl-S(QL-P17) = Cl-Sdiag(QL-P1) (same point), DQL-2P3 are CSC-partners and QL-2P1 (Plücker points) as well as QL-2P3 are CSdiagC-partners.
The 2 quartics are not only CSC-invariant and through QL-2P3, but also CSdiagC-invariant and through DQL-2P3.
I suppose it's a general property of these set of quartics, each tangent to a given conic (as well as it's CSC-circle and it's CSdiagC-circle ?) cutting the quartic in 4 points, 2 being CSC-partners and 2 being CSdiagC-partners.
In the message 364, the Plücker points QL-2P1 were on the quartic, QL-L2 being a tangent to the conic.
In the last message, the points X and Y on page 2 are the CSdiagC of the points QL-2P2. The points U and V on page 3 are the points DQL-2P3 on the 1rst Steiner axis of DQL ; as they are CSC-partners, they are on a circle through the 2 points QL-2P3 (socalled generalised Steiner circles) and conversely, this proves that the 2 points QL-2P3 are CSdiagC-partners. The center of the circle through the 4 points is the intersection of the 2 2nd Steiner axes of QL and DQL.
Best regards
Bernard

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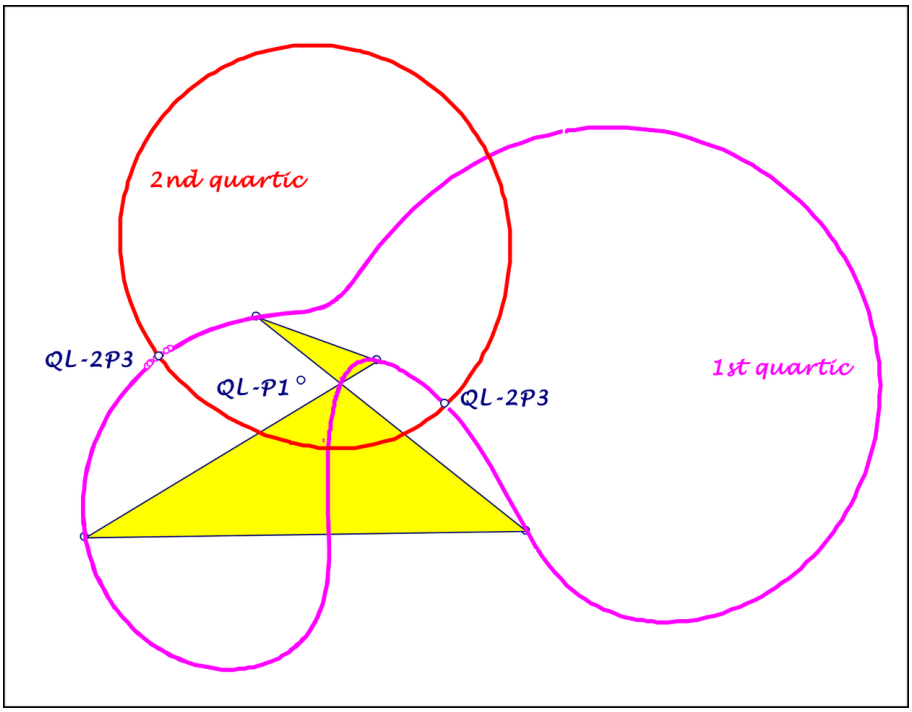
Message: #1356
Date: 17/11/2015 9:56:43
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

Cher Bernard,

les conflits d'Orient sont arrivés en Europe et les institutions européennes n'ont pas tellement de concepts. Ce n'est pas "guerre", c'est "terreur" sans vainqueurs mais beaucoup de morts innocents et réfugiés désespérés. Mon Français modeste ne peut pas exprimer ma consternation.

Back to geometry:
Thanks for the interesting comparison of the two quartics wrt CSC and CSdiagC. I have reproduced your properties, but I can't follow your conjecture:
"I suppose it's a general property of these set of quartics, each tangent to a given conic (as well as it's CSC-circle and it's CSdiagC-circle ?) cutting the quartic in 4 points, 2 being CSC-partners and 2 being CSdiagC-partners."
Is it possible, to explain a bit?
Attached a figure with the two quartics. I suppose, they intersect orthogonal.

Best regards Eckart



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Message: #1357
Date: 17/11/2015 3:37:32
From: eckart_schmidt@t-online.de
Subject: Triple of QG-circles

Dear friends,
here are considered two properties for QG-points:
Let CCx be the circumcircle of QG-2P2 and a point QG-Px
and ACx the Apollonius circle of QG-2P2 and a point QG-Px.
(1) For a quadrangle there are 3 circles CCx,
    which can have a common 2nd intersection.
This is true for (QG-P1), QG-P12, QG-P15, QG-P16, QG-P18,
QG-P19.
For QG-P19 this point is QA-P4 (on QA-Cu1),
... for QG-P18 this point is QA-Tf2(QA-P4) = QA-P41 (on QA-Cu1),
... for QG-P16 this point is the 3rd intersection of QA-Cu1 and
Q2.QA-P4, which is QA-Tf2(Q2),
... for QG-P15 this point is QA-P3 (on QA-Cu1),
... for QG-P12 this point is ??? (not on QA-Cu1).
(2) For a quadrangle there are three circles ACx,
    wich can have ...
... two common points,
... which are collinear with QA-P11
... and inverse wrt QA-Ci1.
This is true for (QG-P1), QG-P12, QG-P15, QG-P16, QG-P18,
QG-P19.
For QG-P1 the circles CC1 are the same and the circles AC1 give
a well known triangle property.
What is the background, that these two properties only hold for
the same QG-points?
Best regards Eckart
PS: QA-P41 is the 3rd intersection of the tangent in QA-P4 at
QA-Cu1 (not in EQF).

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Message: #1358
Date: 19/11/2015 11:08:33
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

Dear Bernard,
wrt your transformation CSdiagC (perhaps evident?):
There are two points Pi with CSC(Pi) = CSdiagC(Pi):
These ar the intersections of the circle round M = CSC(QL-P17)
with radius sqrt(M.QL-P1 * M.QL-P17) and its CSC-circle.
Properties:
... midpoint of Pi is CSC(QL-P17)
... Pi are concyclic with QL-P1, QL-P17 (see above)
... and concyclic with the CSC-fixed points QL-2P3
... and concyclic with the CSdiagC-fixed points.
Best regards Eckart

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Message: #1359
Date: 19/11/2015 5:52:58
From: bernard.keizer
Subject: QL-Quartic II

Lieber Eckart,
Vielen Dank für Deine Antwort auf französisch, es hat mich tief gerührt!
For the QL-quartics, I don't want to delay my answer too long, but there are plenty of interesting properties, which deserve a complete study!
1) For any conic, the intersections between the tangents and the CSC-circles give a pair of CSC-partners on a quartic through the points 2P-3 and CSC-invariant. (If you consider the 2 tangents in each of the points 2P3, each point 2P3 is a necessary a dobble point of the quartic).
2) In the case of your 2 special quartics, the center of the conic and the center of the hyperbola are the Dimidium point P6 and the pole of P1P17 wrt the Dimidium circle and vice-versa.
If you consider the QL formed by the 2 pairs of Steiner axes of QL and DQL (or Cl-S and Cl-Sdiag), it's Newton line is precisely the perpendicular bisector of P1P17 (intersection points and right angles in P1 and P17).
It means that the 2 conics are tangent to the 2 pairs of Steiner axes and that the orthoptic circles of the conics are 2 orthogonal circles with centers the 2 points described above.
The 2 quartics intersect orthogonally in the 2 points 2P3 of both QL and DQL (the 4 points are concyclic, as the QL-2P3 are CS-diagC-partners and the DQL-2P3 are CSC-partners). On the 1rst Steiner axis, the points QL-2P3 are counted as dopple points (see above) and as CSdiagC-partners and the same goes for the DQL-2P3; thats why the 4 points are the only intersections of the 2 quartics.
For any other tangent, there are 4 points, 2 are CSC-partners and the 2 others CSdiagC-partners ; for example, on L2 in your first quartic (message 364-465-774), the Plücker points are the CSdiagC-partners and the intersections with the Miquel circle are the CSC-partners.
3) I suppose any copple of points on the perpendicular bisector of P1P7 choosen identically will bring the same way 2 quartics intersecting orthogonally in the same 4 points QL-2P3 and DQL-2P3, but I didn't have time to check it. (For example, what happens if you take as 5th tangent to the conic the line through your 2 points, which are CSC-partners and CSdiagC-partners ?)
I hope you will be interested in pursuing the study ...
Best regards Bernard

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Message: #1360
Date: 19/11/2015 7:35:35
From: bernard.keizer
Subject: QL-Quartic II

Dear Eckart,
Each 2P3 is a dopple point on each tangent, id est a quadruple point for your 2 quartics !
Another interesting point on the perpendicular bisector of P1P17 is the center X of the circle through P1, P17 and the point Y = CSC(P17), which is also CSdiagC(P1) ; CSC(X) is the reflexion of P1 in the line YP17, which is the CSC of this circle and CSdiagC(X) is the reflexion of P17 in the line YP1, which is the CSdiagC of the same circle ...
Best regards
Bernard

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Message: #1361
Date: 20/11/2015 11:06:35
From: bernard.keizer
Subject: QL-Quartic II

Dear Eckart,
I realise I didn't answer exactly to your question !
Each line L to a conic tangent to the 2 pairs of Steiner axes of QL and DQL and centered on the perpendicular bisector of P1P17 has a CSC-circle centered on one hyperbola and through P1 and a CSdiagC-circle centered on another hyperbola and through P17.
I suppose the 2 hyperbolas have the same center (?) on the perpendicular bisector of P1P17 with axes parallel to the 2 pairs of Steiner axes.
Let's take the same example as in my previous message.
The line L1 is the Steiner Line of the QL, tangent to the conic Co of your 1rst message 364, and cuts the quartic in 4 points P and P', the Plücker points and 2 points on the Miquel circle R and R' ; the 2nd line L2 will be QQ', where Q and Q' are the CSC-partners of P and P' and are also CSdiagC-partners, is also tangent to the conic Co and cuts it's CSC-circle, which is the Plücker circle in 2 other points S and S', which are CSC-partners.
The 8 points are on the quartic and form 4 pairs of CSC-partners P and Q, P' and Q', R and R' and S and S' and 4 pairs of CSdiagC-partners P and P', Q and Q', R and S and R' and S'. L1 has a CSC-circle, which is the Miquel circle through P1, Q and Q' and R and R' and a CSdiagC-circle, through P17, P and P' and R and R'.
L2 has a CSC-circle, which is the Plücker circle through P1, P and P' and S and S' and a CSdiagC-circle, through P17, Q and Q' and S and S'.
It appears that the same 8 points form also 4 different cocyclic quartets.
I hope it's now a little bit clearer and I will wait for your comments ...
Best regards
Bernard

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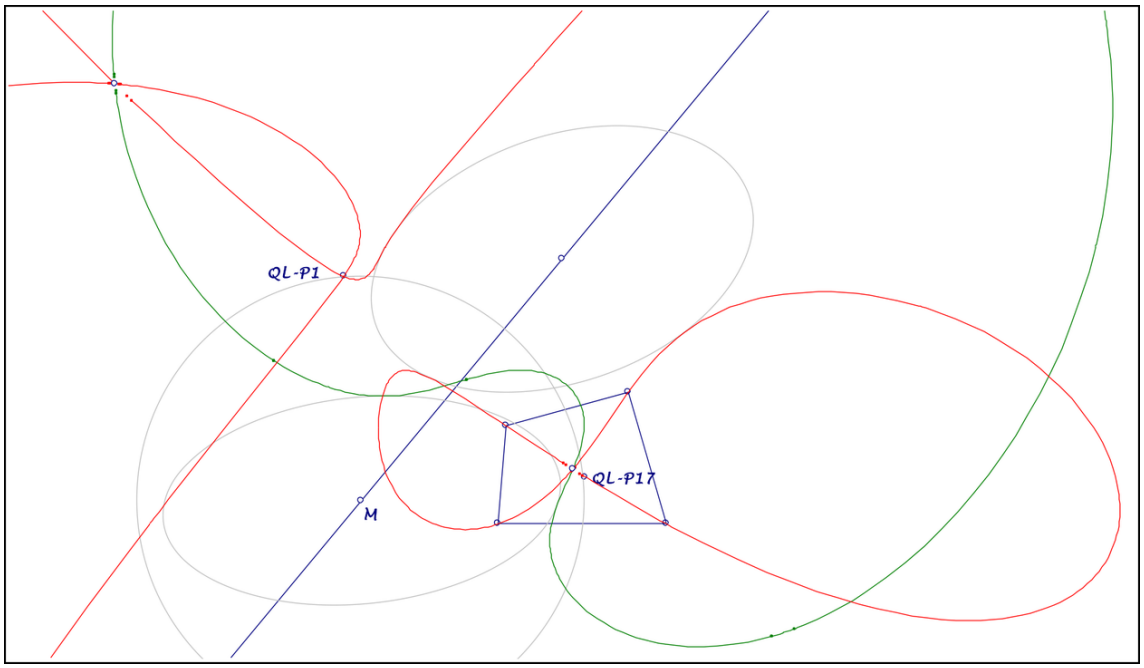
Message: #1362
Date: 20/11/2015 11:09:21
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

Dear Bernard,
thanks for your detailed explinations, especially under 2).
wrt 1): In general the CSC-partners on tangents at a conic must not be a quartic (see attached file)!
wrt 3): I failed in verifying your conjecture with Cabri, taking the following conics (see attached file):
... centered in M on the perpendicular bisector of QL-P1 and QL-P17, tangent to the Steiner axes and QL-L1,
... centered in the pole of QL-P1.QL.P17 wrt a circle round M through QL-P1 and QL-P17.
The CSC-partners on the tangents of these conics give two curves, intersecting in QL-2P3, but not orthogonal. The other two or four intersections depend on M.
Maybe I haven't interpreted your exlinations correct.
Best regards Eckart



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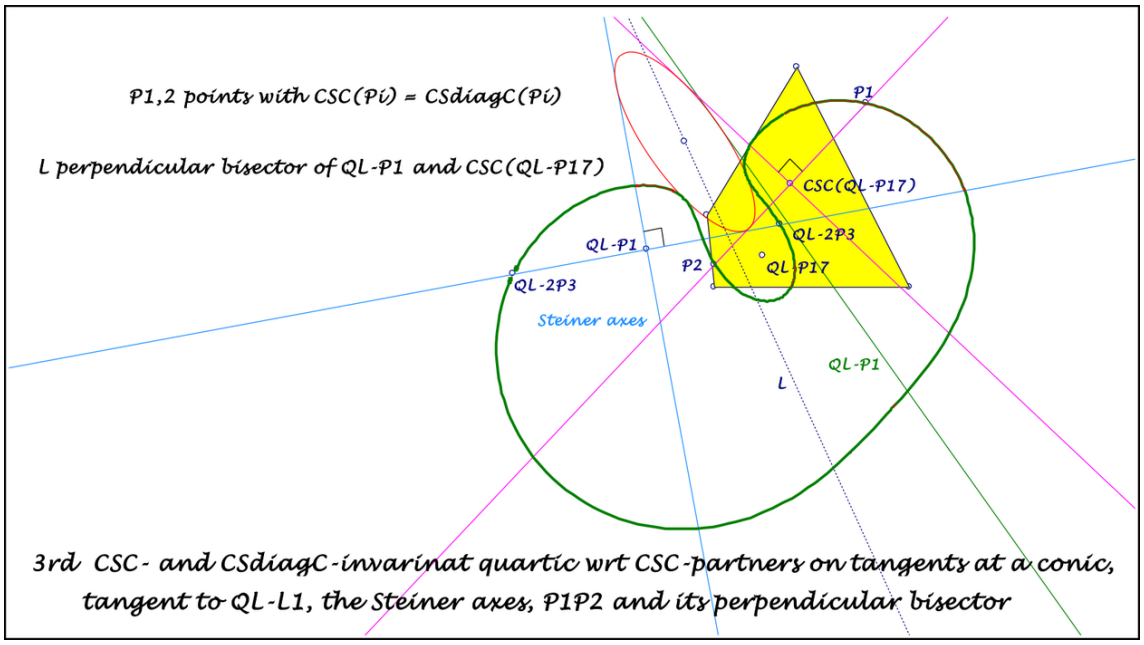
Message: #1363
Date: 21/11/2015 9:45:50
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

Dear Bernard,
attached a 3rd quartic, CSC- and CSdiagC-invariant. I suppose, there will be no more of this type.
Best regards Eckart



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Message: #1364
Date: 21/11/2015 11:36:13
From: bernard.keizer
Subject: QL-Quartic II

Dear Eckart,
This time, it's my turn, I do not understand your message !
If you consider the 4 Steiner axes of CSC and CSdiagC, there are
1) 2 conics Co centered in P6 and in the pole of P1P17 wrt the Dimidium circle ; their axes seem to be parallel (main axes perpendicular)
a) the 1rst one (in message 364) is tangent to the 4 Steiner axes, to the sides of DT, to the Steiner Line L2 and to the line QQ' (see my previous message)
b) the 2nd one (in message 1354) is tangent to the 4 Steiner axes and to the Newton Line L1
2) 4 rectangular hyperbolas, each conic having one locus of CSCe and one locus of CSdiagCe of the tangents
Each conic seems to be tangent to the 2 rectangular hyperbolas associated to the other (?)
3) 2 quartics CSC and CSdiagC invariant intersecting orthogonally in the 2 pairs of 2P3 points ; each quartic seems to be tangent to it's conic (it's sure for the 1rst, less sure for the 2nd ?)
As I already told you, Cabri is much powerful than Geogebra and all these constructions need time and efforts ...
Perhaps, you could complete your beautiful figure in message 1356 with the 2 conics, the 2 orthoptic circles (one is the Dimidium circle) and the 4 hyperbolas ...
Best regards
Bernard

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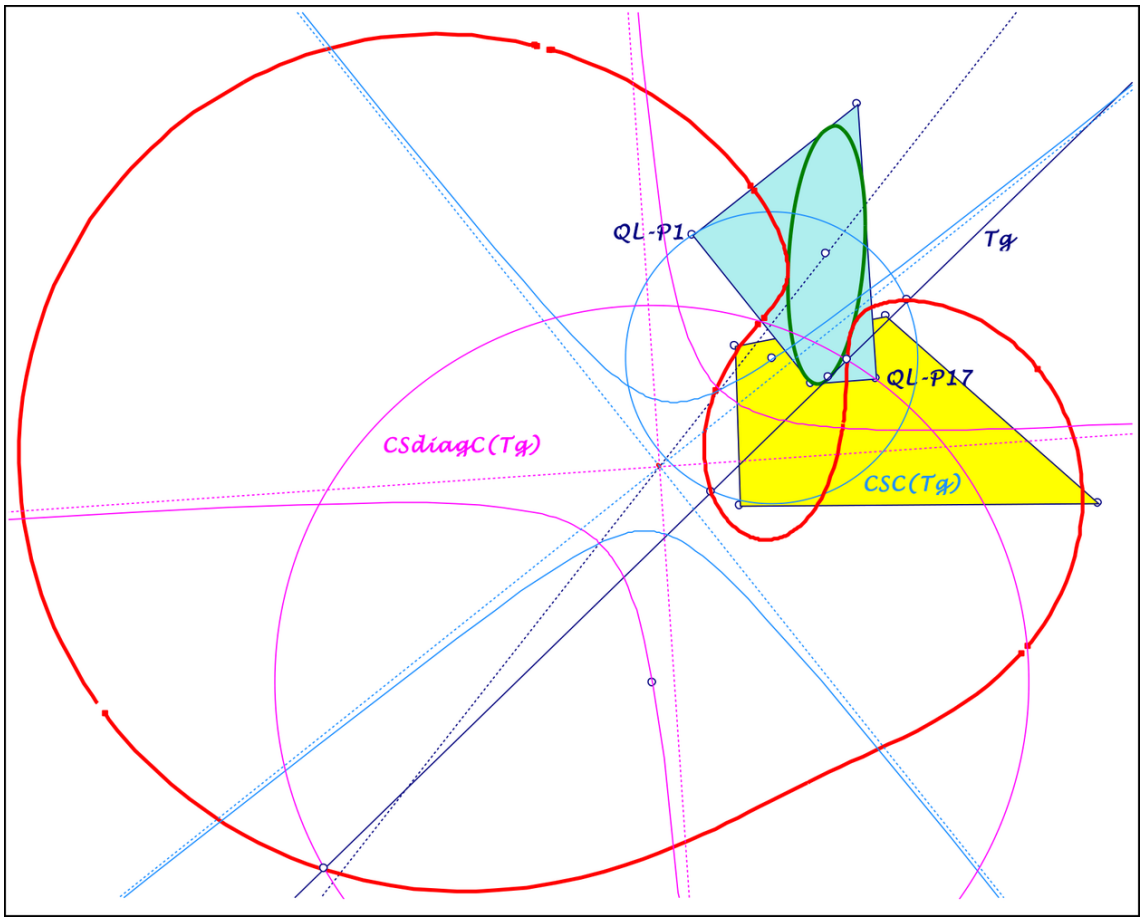
Message: #1365
Date: 21/11/2015 1:53:02
From: eckart_schmidt@t-online.de
Subject: QL-Quartics II

Dear Bernard,
your sight of the quartics is the best! I had just finished my paper (attached), considering special conics tangent to QL-L1 and the Steiner axes, but that is now out of discussion.
There is much more geometry in your conics tangent to the Steiner axes of QL and DQL, centered on the perpendicular bisector of QL-P1.QL-P17. They give CSC- and CSdiagC-invariant quartics.
The 2nd attached file shows all your described properties will be right. Also the last: Taking two tangents to the conic, the CSC-partners are concyclic as well as the CSdiagC-partners.
In addition: These quartics are anallagmatic. Let Si be the CSC-fixed points and Ti the CSdiagC-fixed points, then the centers of the inversion circles are S1T1^S2T2 and S1T2^S2T1.
Best regards Eckart



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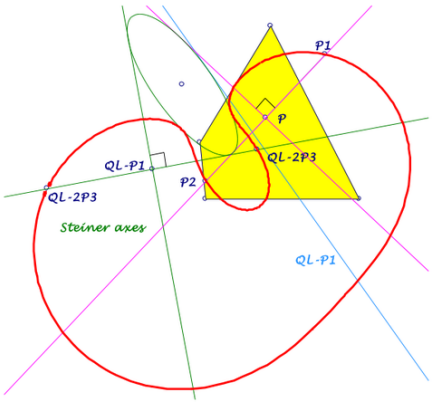
EQF-Note 2015-11-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Quartics

Here is a generalization of the quartic in QFG-message 1354. These generalized quartics are the loci of CSC-partners on tangents of special conics, depending on a point P. They are CSC-invariant and anallagmatic.

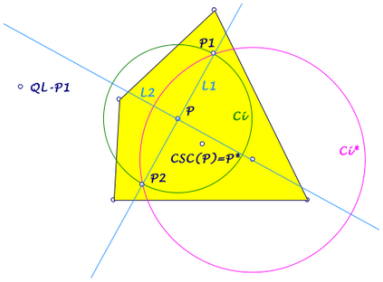


Construction of a P-Quartic

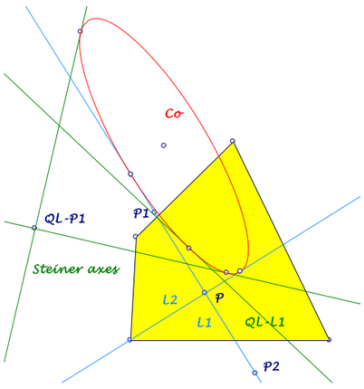
Let *P* be the defining point and *P** its image wrt *QL-Tf1*, shortened *CSC*. Take a circle *Ci* round *P* with radius

$$\sqrt{P.P*\,i\,P.QL-P1}$$

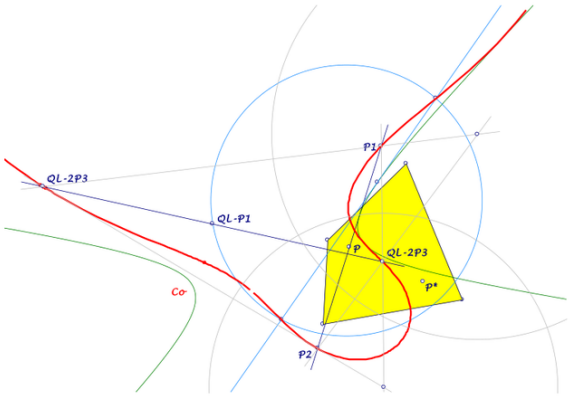
and the intersections P_1, P_2 with its *CSC*-circle *Ci** with the lines $L_1 = P_1P_2$ and L_2 perpendicular bisector of P_1P_2.



Let *Co* be the defining conic tangent to …
… the Newton line *QL-L1*,
… the Steiner axes
… and the lines L_1 and L_2.



The *CSC*-partners on tangents at this conic give the *P*-quartic.



Properties:

- **The *P*-quartic contains …**
  … the *CSC*-fixed points *QL-2P3*,
  … the points P_1, P_2 (see above).

- **The *P*-quartic is *CSC*-invariant (evident).**

- **The *P*-quartic is invariant wrt a combination of …**
  … an inversion wrt the circle *Ci* (see above)
  … and a reflection in the angle bisector of *QL-P1.P.P*.*

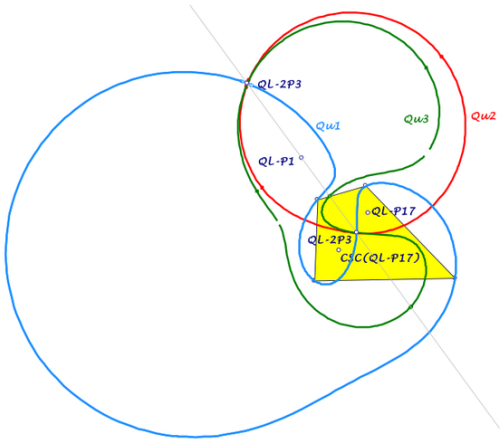
- **The *P*-quartic is invariant wrt a combination of …**
  … an inversion wrt a circle round *P with radius**
  $$\sqrt{P*,P \times P*,QL-P1}$$
  … and a reflection in the angle bisector of *QL-P1.P*.P.*

- **The *P*-quartic is anallagmatic, centers for the inversion circles are P_1.*QL-2P3a* ∩ P_2.*QL-2P3b* and P_2.*QL-2P3a* ∩ P_1.*QL-2P3b*.**

Final remarks

Qu1: This quartic – **not a *P*-quartic** – is described in *QFG*-message 364. Its conic is tangent to the *QL*-diagonals, the Steiner axes and *QL-L2* (with orthogonal tangents to *QL-P17*).
Qu2: For *P* = *QL-P17* the *P*-quartic is described in *QFG*-message 1354.
Qu3: For *P* = *CSC(17)* the *P*-quartic is described in *QFG*-message 1363.

The quartics *Qu2* and *Qu3* are tangent in *QL-2P3,* they have orthogonal intersections with *Qu1*.
Bernard Keizer considers a related quadrilateral *DQL* wrt the diagonals and *QL-L1* with the corresponding *CSC*-transformation *CSdiagC* (see *EQF*-message *1355*). He found, that *Qu1* and *Qu2* are not only *CSC*-invariant wrt the reference *QL*, they are also *CSdiagC*-invariant. **This holds also for *Qu3***.

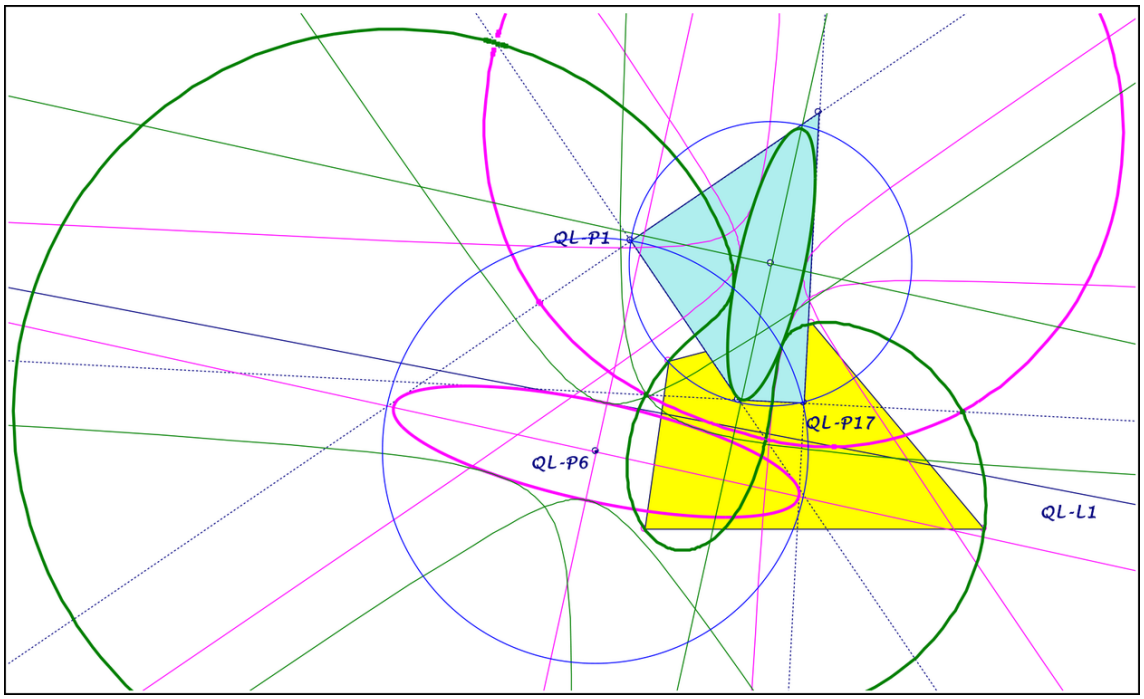


Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2015-11-21.docx

Message: #1366
Date: 21/11/2015 3:52:59
From: eckart_schmidt@t-online.de
Subject: QL-Quartic II

Dear Bernard,
I just noticed your message 1364. Perhaps the attached Cabri file can clear your questions.
Best regards Eckart



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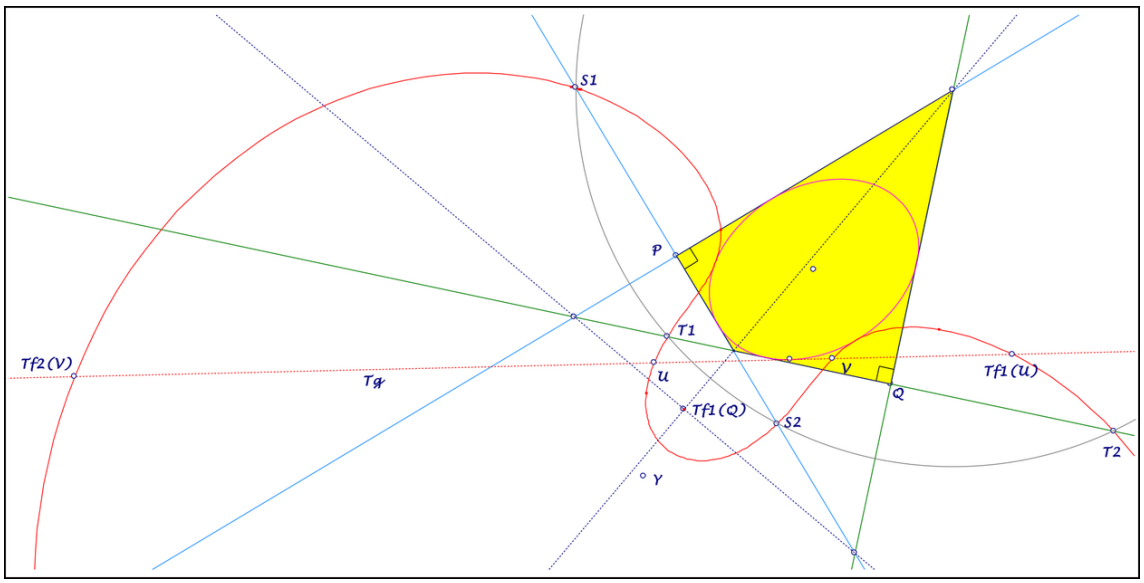
Message: #1367
Date: 23/11/2015 9:08:57
From: eckart_schmidt@t-online.de
Subject: QL-Quartics II

Dear Bernard,

the CSC- and CSdiagC-geometry can be generalized for two
CSC-analog transformations independant of a reference
quadrilateral:
Tf1: centered in P, mapping X to Y,
Tf2: centered in Q, mapping P to Tf1(Q).
Every conic tangent to the four axes of the two transformations
give with their Tf1- and Tf2-partners on the tangents a quartic,
of course Tf1- and Tf2-invariant (see attached file).
... The fixed points of Tf1 and Tf2 are concyclic on the
quartic,
    center of the circle is the intersection
    of the two 2nd axes of Tf1 and Tf2.
... Tf1-partners as well as Tf2-partners on two tangents
    of the conic are concyclic.
... The quartic is also invariant wrt the CSC transformation
    for the QL of the axes of Tf1,Tf2
    with Miquel point Tf1(Q) = Tf2(P).
... The quartic is anallagmatic.
    If Si and Ti are the fixed points of Tf1 and Tf2,
    the center of the inversion circles
    are S1T1^S2T2 and S1T2^S2T1.
For the quartics in #364 #1354: P=QL-P1, X, Y two opposite
QL-vertices, Q=QL-P17. The quartics are CSC- and
CSdiagC-invariant, but also wrt CSC for the QL of the axes of
CSC and CSdiagC.
For the quartic in #1363: P=QL-P1, X, Y two opposite
QL-vertices, Q=CSC(QL-P17). The quartic is Tf1- that means
CSC-invariant and Tf2-invariant (see above), but also wrt CSC
for the QL of the axes of CSC and Tf2, which is CSdiagC.

Best regards
Eckart



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Message: #1368
Date: 23/11/2015 11:10:25
From: bernard.keizer
Subject: QL-Quartics II

Wunderbar, lieber Eckart !
I came yesterday evering to the same kind of consideration and was preparing a longer answer, but it's no longer needed. Let's put it on a simpler way.
For any triangle ABC, let's consider the 3 CL-S, centered in one vertice and swapping the 2 others: Tf1 centered in A and swapping B and C, Tf2 centered in B and swapping A and C and Tf3 centered in C and swapping A and B.
You may define a table of composition of the 3 transformations. Tf1*Tf2 = Tf3, Tf1*Tf3 = Tf2 and Tf2*Tf3 = Tf1 and Tf1*Tf2*Tf3 = Tf1*Tf1 = Tf2*Tf2 = Tf3Tf3 = Identity
The 3 1rst Steiner axes are the internal bisectors of the triangle and intersect in the incenter and the 2nd Steiner axes are the external bisectors and form the triangle of excenters (ABC is it's orthic triangle).
The 2 invariant points of one of the 3 transformations are CSC-partners of the 2 others and are on 3 circles, one through the 2 centers of these 2 other transformations (CSC-partners of the 1rst one) and 2 through each pair of invariant points of the 2 others (as they are themselves CSC-partners in these 2 transformations). In our problem, A is QL-P1 and Tf1 is QL-Tf1, B is QL-P17 and Tf2 is DQL-Tf1 and C is CSC(QL-P17) or CSdiagC(QL-P1) and Tf3 is the last transformation you described. The invariant points of the last one are, as you put it, on a circle through QL-P1 and QL-P17 and on 2 other circles with the points QL-2P3 and DQL-2P3.
As I put already in my old message 774, Qu1 (as well as Qu2) are CSC-and CSdiagC-invariant, but also CSC*CSdiagC-invariant! At this time, I didn't know exactly why, but it's clear now ...
Your last quartic Qu3 gave me the key with the 3rd transformation!
If you had the courage, I would be interested in seeing 6 quartics on the same picture with conics tangent to 2 pairs of the Steiner axes, the 5th line being the Newton Line QL-L1 or the Steiner Line QL-L2 (3 are already known). I suppose there will be 3 pairs of quartics through 3 pairs of points with 2 pairs of quartics through each point and 2 pairs of points on each quartic and the contacts being either tangential or orthogonal ...
Best regards Bernard
PS Its a real pleasure to make pure geometry together like this!

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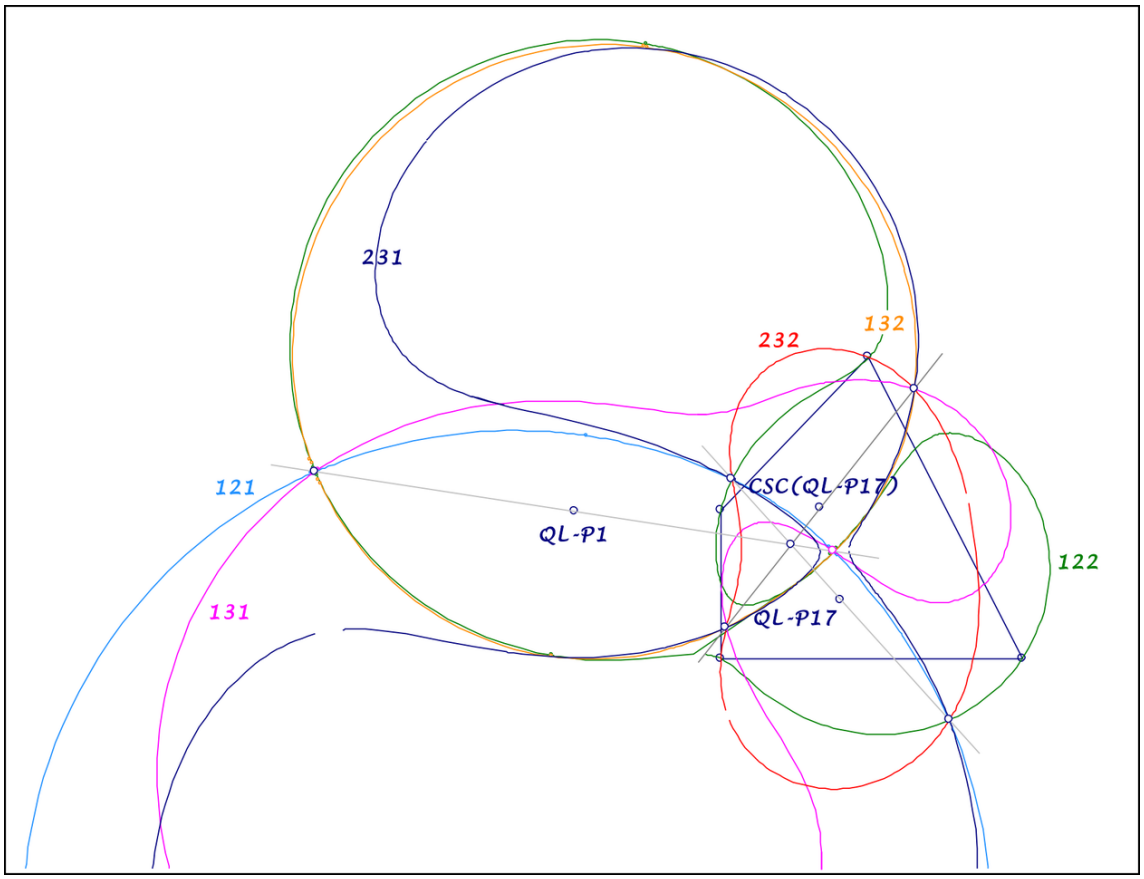
Message: #1369
Date: 23/11/2015 2:45:15
From: eckart_schmidt@t-online.de
Subject: QL-Quartics II

Dear Bernard,
attached the six quartics. Wrt the numeration:
1st and 2nd number: pairs of Steiner axes (1 CSC, 2 CSdiagC, 3 ...),
3rd number: 5th line (1 QL-L1, 2 QL-L2).
Best regards Eckart



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Message: #1370
Date: 23/11/2015 7:16:42
From: bernard.keizer
Subject: QL-Quartics II

Dear Eckart, thanks a lot !
I will study the figure later.
Meanwhile some obvious properties :
1) in the triangle ABC
a) each Tf centered in a vertice swaps the opposite side and the circumcircle
b) the centers of the circles through 2 pairs of invariant points are the excenters
2) in the QL
a) Cl-Ster swaps the CSC-circle of the Newton Line and the DTcircumcircle
b) Cl-Ster swaps the Miquel circle and the circle through P17 and the 2 Plücker points
c)Cl-Ster swaps the Plücker circle and the circle through P17 and the 2 CSC-partners of the Plücker points
Best regards
Bernard

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Message: #1371
Date: 24/11/2015 5:12:15
From: Benedetto Scimemi
Subject: A quadrangle of conics intersections

Dear friends,
1) If (*) in a quadrangle the conics QA-Co1 (ninepoint) and QA-Co3 (Steiner) meet in four (real) points Si, they create a cyclic quadrangle *S* = S1 S2 S3 S4.
2) the circumcenter of *S* is QA-P32, centroid of the circumcenters Q
3) the centroid of *S* is the midpoint between QAP1 and QAP3
4) the Euler point of *S* (reflection of 2 on 3) lies on line QA-P2 QA-P7; it is the reflection of QA-P2 on the centroid of the ninepoints Q (not in EQF)
5) the ninepoints conic of *S* contains QA-P1, QA-P3 and QA-P32 (as centers of circumscribed conics) but also the orthopole center QA-P42. Its asymptotes are parallel to the symmetry axes of QA-Co1 and QA-Co3.
(*) These statements were proved analytically, but an intriguing problem is open: for which quadrangles the intersections of the conics above are 4 real points?
By graphic experiments, being convex seems sufficient but not necessary.
Can anybody help?
Best regards
Benedetto Scimemi

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Message: #1372
Date: 28/11/2015 5:38:47
From: seiichikiri
Subject: A quadrangle of conics intersections

Dear Benedetto,
I considered a bit generalized problem of intersections in DT coordinates.
Co1: p^2yz+q^2zx+r^2xy=0.
CoX: ux^2+vy^2+wz^2=0 where up^2+vq^2+wr^2=0.
Barycentric relation: x+y+z=S.
I eliminated variables y and z in the above equations. For the quartic equation of variable x,
Delta = 432 p^4 S^12 u v w (p^4 u (v - w)^2 + (q^2 - r^2) (v + w)^2 (q^2 v - r^2 w))^2 (p^12 u^3 + 3 p^8 u^2 (q^4 v + r^4 w) + (q^4 v + r^4 w)^3 + 3 p^4 u (q^8 v^2 - 7 q^4 r^4 v w + r^8 w^2)).
Symmetric portion = u v w(p^12 u^3 + 3 p^8 u^2 (q^4 v + r^4 w) + (q^4 v + r^4 w)^3 + 3 p^4 u (q^8 v^2 - 7 q^4 r^4 v w + r^8 w^2)).
The latter determines the boundary between 2 real roots and 4 real roots or 4 imaginary roots.
See http://homepages.warwick.ac.uk/~masgaj/papers/r34jcm.pdf, proposition 7.
H and Q are too complicated to distinguish 4 real roots and 4 imaginary roots. So I used the following reason.
The assumption: 3 diagonal points (S1, S2 andS3) on Co1 are on the same branch of it.
Case 1: 4 connected points (P1, P2, P3 and P4) form the ellipse (CoX). S1 is in I-area. S2 and S3 are in O-area. Since S1, S2 and S3 are on the same branch of Co1, Co1 and CoX have intersection.
Case 2: 3 connected points (P2, P3 and P4) and 1 separated point (P1) form the hyperbola (CoX). S1 and S2 are in M-area. S3 is in R-area. Co1 and CoX have intersection.
Case 3: 2 connected points (P1 and P4) and 2 connected points (P2 and P3) form the hyperbola (CoX). S1 and S3 are in M-area. S2 is in R-area. Co1 and CoX have intersection.
See the attachement.
Owing to algebraic consideration, there are at least 2 real roots and no imaginary root.
Concerning the above assumption, I hope comments or counterexamples.
Best regards, Seiichi.

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Message: #1373
Date: 29/11/2015 12:28:29
From: seiichikiri
Subject: A quadrangle of conics intersections

```
Addition: algebraic proof of the existence of intersections
   Co1: p^2yz+q^2zx+r^2xy.
   CoX: ux^2+vy^2+wz^2
where up^2+vq^2+wr^2=0.
At the midpoints of the diagonal triangle,
f(0,1,1)=p^2>0, f(1,0,1)=q^2>0, f(1,1,0)=r^2>0.
Hence the vertices (S1, S2 and S3) of the diagonal triangle are
on the same branch of Co1.
At the vertices (S1, S2 and S3) of the diagonal triangle,
g(1,0,0)=u, g(0,1,0)=v, g(0,0,1)=w.
Since up^2+vq^2+wr^2=0, u, v and w can not have the same sign.
That is, the vertices
(S1, S2 and S3) are in the different areas bounded by g(x, y,
z)=0.
f(x, y, z)=0 and g(x, y, z)=0 have intersections.
Best regards, Seiichi.
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Message: #1374
Date: 29/11/2015 1:03:55
From: Benedetto Scimemi
Subject: A quadrangle of conics intersections

```
Thanks, Seiichi, for your prompt contribution.
I will use your ideas to check, by applying my coordinates (not
trilinear), the existence of real intersections of the
ninepoints conic with the Steiner conic of any quadrangle. By
the way, when they are only two, they turn out to be still
equidistant from QA-P32.  Best regards  Benedetto
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Message: #1375
Date: 30/11/2015 4:01:36
From: seiichikiri
Subject: A quadrangle of conics intersections

Dear Benedetto,
I think that there is a possibility that 2 other imaginary solutions are formally on the circle with its center QA-P32 and the radius of QA-P32 and real solution.
Best regards, Seiichi.

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Message: #1376
Date: 03/12/2015 1:05:14
From: eckart_schmidt@t-online.de
Subject: 3rd quartic for a QG

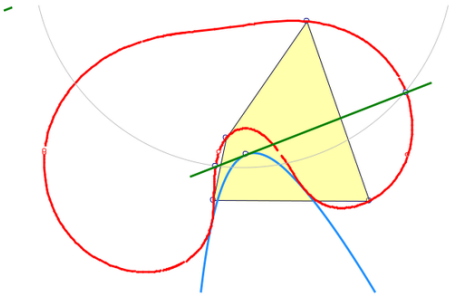
Dear Bernard, dear Chris,
perhaps of interest: A QG-circumquartic related to a special conic wrt CSC-partners on its tangents.
The conic is tangent to
... the Steiner axes,
... the diagonals
... and the Newton line.
Best regards Eckart

EQF-Note 2015-12-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A Third Quartic for a Quadrigon

In QFG-messages 362 and 363 two circumquartics of a quadrigon are described. Here is the geometry of a third quartic, related to a conic wrt CSC-partners on its tangents. This quartic is anallagmatic and invariant wrt several transformations.



Preliminary

The *CSC*-partners on tangents of a given conic give a quartic:

- For a *QL*-conic, tangent …
  … to the Steiner axes,
  the quartic is unipartite.

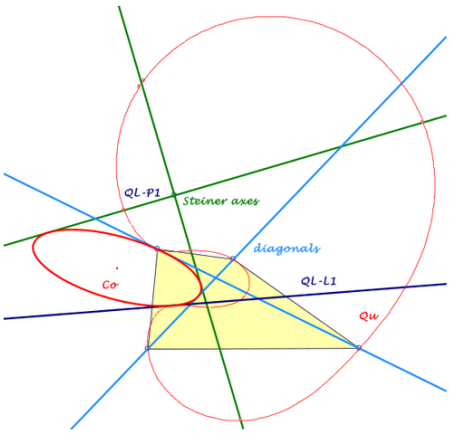
- For a *QL*-conic, tangent …
  … to the Steiner axes
  … and the Newton line *QL-L1*,
  the quartic is a *P*-quartic (see #1365 attached).

- For a *QG*-conic, tangent …
  … to the Steiner axes
  … and the diagonals,
  the quartic is a *QG*-circumquartic.

- **For a *QG*-conic, tangent …**
  … to the Steiner axes
  … and the diagonals
  … and the Newton line *QL-L1*,
  there is a special *QG*-circumquartic.

Here only the last quartic – shortened ***Qu*** – shall be treated, related to the special conic – shortened ***Co***.

The Quartic *Qu*



- The quartic *Qu* is an unipartite circumquartic of the quadrigon.

- The quartic *Qu* contains …
  … the *CSC*-fixed points *QL-2P3*
  … and the points *QL-2P2* on the cubic *QL-Cu1*.

- If there are conic-points, whose *CSC*-partner lies on its tangent, the conic *Co* contacts the quartic *Qu* in these points.

- The quartic *Qu* is invariant wrt the *CSC*-transformation $X \to X^*$ (evident).

Beside the *CSC*-transformation $X \to X^*$, there is a *QG*-transformation $X \to X^\wedge$, described in #362: The image $X^\wedge$ is the second intersection of the two circles through X and opposite *QG*-vertices.

- The quartic *Qu* is invariant wrt the *QG*-transformation $X \to X^\wedge$.

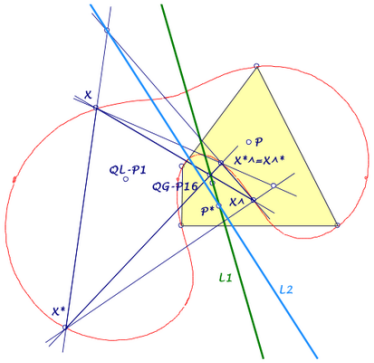
The Image Points, *X*, X^, X*^ = X^

The geometry of the four points X, X^*, $X^\wedge$, $X^{*\wedge} = X^{\wedge *}$ is worth to be mentioned:

- For points X on the quartic *Qu* …
  … $XX^\wedge$ and $X^*X^{*\wedge}$ intersect on a line L_1 through *QG-P16* of the reference quadrigon,
  … XX^* and $X^\wedge X^{\wedge *}$ intersect on a second line L_2.

1. Consider the quadrigons *X.X^.X*.X*^*:

- These quadrigons have the same Miquel point *QL-P1* as the reference quadrigon.
- The points *QG-P16* of these quadrigons lie on the line L_1 collinear with *QG-P16* of the reference quadrigon.



2. Consider the quadrigons *X.X*.X^.X^*:*

- These quadrigons have the same Miquel point in
$$P = CSC(L_1 \cap L_2).$$
- The points *QG-P16* of these quadrigons lie on the line L_2.

3. Consider the quadrigons *X.X*.X*^.X^:*

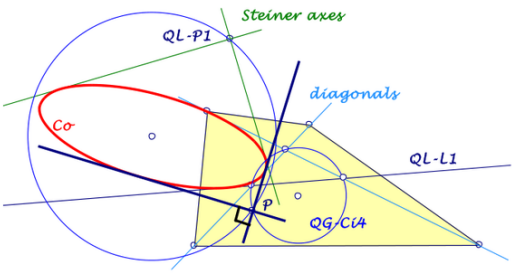
- These quadrigons have the same Miquel point in
$$P^* = L_1 \cap L_2 .$$
- The points *QG-P16* of these quadrigons lie on a circle, which is the *CSC*-image of the line L_2.

The Point $P = CSC(L_1 \cap L_2)$

The point *P* has a special geometry wrt the defining conic *Co* and the quartic *Qu*:

- *P* is a point on the quadrigon circle *QG-Ci4*.

This circle contains the diagonal crosspoint *QG-P1* and the diagonal midpoints on the Newton line *QL-L1*.



- P is a point on the circle round the center of the conic Co through the Miquel point $QL\text{-}P1$.

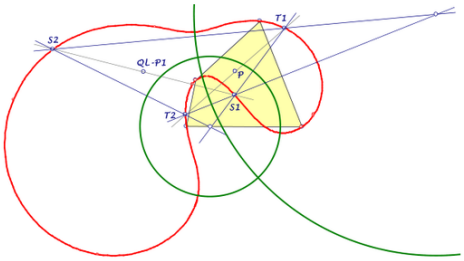
This circle is the locus of points with orthogonal tangents to the conic Co. The 2nd intersection with $QG\text{-}Ci4$ is a Plücker point for the quadrilateral of the Steiner axes and the diagonals.

- The quartic is invariant wrt a CSC-analog transformation $X \to X'$, centered in P, which swaps P^* and $QL\text{-}P1$.
- The quartic Qu contains the fixed points of $X \to X'$.
- For points on the quartic Qu the transformation $X \to X'$ is the same as $X \to X^\wedge$.
- The quartic is invariant wrt a CSC-analog transformation $X \to X''$, centered in P^*, which swaps P and $QL\text{-}P1$.
- For points on the quartic Qu the transformation $X \to X''$ is the same as $X \to X^{\wedge *}$.

The transformations $X \to X^*$, $X \to X'$, $X \to X''$ are the CSC-analog transformations of the triangle $QL\text{-}P1.P.P^*$, centered in a vertex swapping the other two vertices.

- The quartic Qu is anallagmatic.

Let S_i be the CSC-fixed points $QL\text{-}2P3$ and T_i the fixed points of $X \to X'$, then the centers of inversion are $S_1T_1 \cap S_2T_2$ and $S_1T_2 \cap S_2T_1$.



- The quartic is a P-quartic (see #1365 attached).

In this way the defining conic Co is tangent to the Newton line of the reference quadrigon and inscribed the cyclic quadrigon of the axes of $X \to X^*$ and $X \to X'$.

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2015-12-03.pdf

Message: #1377
Date: 04/12/2015 12:42:36
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

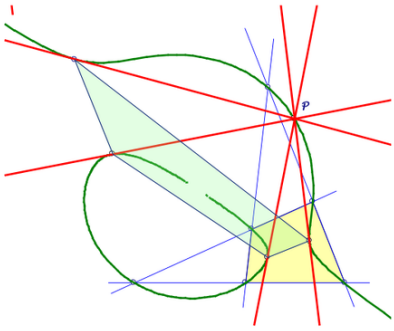
Dear Bernard, dear Chris,
I think, this are interesting observations (see attached file):
A simple construction of the four tangents from a point on
QL-Cu1 at QL-Cu1 gives the quadangle of the contact points,
whose QA-Cu1 is the QL-Cu1 of the reference quadrilateral.
Best regards Eckart

EQF-Note 2015-12-04

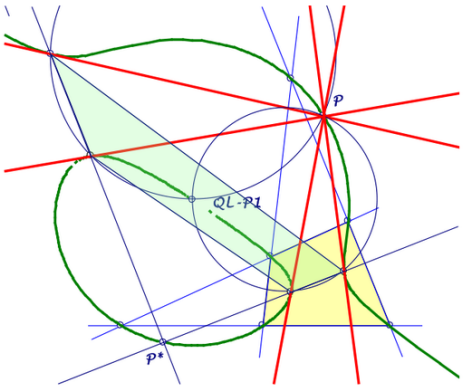
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Cu1 as QA-Cu1

What about the four possible tangents from a point on the cubic QL-Cu1 at QL-Cu1? This leads to a quadrangle of contact points (not always real), whose cubic QA-Cu1 is QL-Cu1 of the reference quadrilateral.



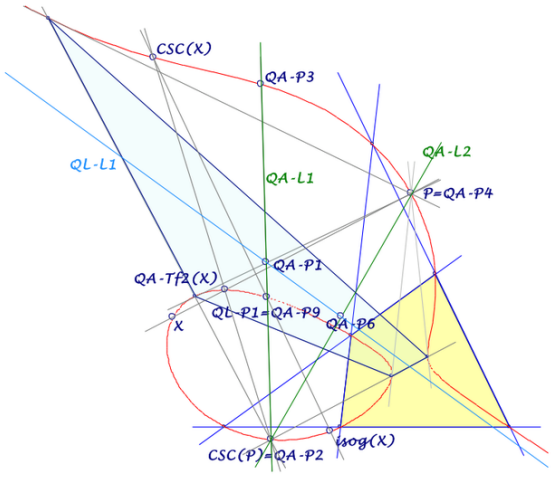
The central curve for a quadrilateral *QL* is the *QL*-Quasi Isogonal Cubic *QL-Cu1*, invariant wrt the transformation *QL-Tf1* – shortened *CSC*. In a first step a construction is given for the tangents from a point *P* on *QL-Cu1* at *QL-Cu1*:
… Let *P** be the *CSC*-image of *P*.
… Let L_1 and L_2 be the orthogonal angle bisectors in *P** wrt two opposite points of the quadrilateral.
… The intersections of L_i with their *CSC*-circle $CSC(L_i)$ give four points (not always real).
… These four points are the contact points of the tangents from *P* at *QL-Cu1*.



2015-12-04.pdf

In a second step we study the quadrangle of the four contact points depending on a chosen point *P* on *QL-Cu1*. **It is a special quadrangle, for the vertices lie on two orthogonal lines.**
Nomination: The first letters *QA* refer to such a quadrangle, *QL* to the reference quadrilateral.
The main result:

- **The Cubic *QA-Cu1* is the Cubic *QL-Cu1*.**
- One vertex of the diagonal triangle *QA-Tr1* is *CSC(P)*.
- The vertices of the diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2* lie on *QL-Cu1*.
- *QL-Cu1* is invariant wrt *QA-Tf2* and isogonal invariant wrt the Miquel triangle *QA-Tr2*.



- For points *X* on *QL-Cu1* holds
  … *X, QA-Tf2(X), P* are collinear,
  … *CSC(X), QA-Tf2(X), CSC(P)* are collinear,
  … *CSC(X), isog(X), QL-P1* are collinear,
- Identical points:
  QA-P4 = P, QA-P2 = CSC(P), QA-P9 = QL-P1.
- Identical lines:
  QA-P1.QA-P6 = QL-L1,
  QA-L1 = QA-L6 = CSC(P).QL-P1
  = QA-P1.QA-P2.QA-P3.QA-P9,
  QA-L2 = P.CSC(P) = QA-P2.QA-P4.QA-P6.
- Loci of *QA*-points, changing *P* on *QL-Cu1*:
  QA-P1, QA-P6, QA-P28, QA-P29 on *QL-L1*,
  QA-P3, QA-P4 on *QL-Cu1*.

Finally we study the *QG*-component of *QA*, which has not an angle bisector at *P** as sideline (see L_1, L_2 above).

- Identical points:
  QG-P19 = QA-P4 = P,
  QG-P1 = QG-P6 = QA-P2 = CSC(P).

- Loci of *QG*-points, changing *P* on *QL-Cu1*:
  QG-P2 = QA-P29, QG-P7 = QG-P9 = QA-P1, QG-P12
  on *QL-L1*,
  QG-P5 = QG-P15 = QA-P3, QG-P16, QG-P18,
  on *QL-Cu1*.

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2015-12-04.pdf

Message: #1378
Date: 05/12/2015 11:30:03
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
These are indeed very interesting properties !
I'm glad you start this new field and I hope you will be able to help me.
For any cubic,
1) the polar conic of a point P is through the contact points of the tangents from this point to the cubic (6 in general)
2) the hessian is the locus of the points for which the polar conic degenerates in 2 lines
3) the transformation which associates to the point P the center of it's polar conic leaves the hessian globally invariant
4) the polar conics of a point P wrt the cubic and it's hessian are mutually poloconics and cut the hessian in points conjugates in the transformation.
For the stelloïd cubic, the polar conics are rectangular hyperbolas, which degenerate for the points on the hessian in 2 orthogonal lines.
On your figure, the 2 points X and X' on L1 and Y and Y' on L2 are 2 copples of Cl-S partners ; XX' and YY' intersect in P*, XY' and X'Y in Z and XY and X'Y' in Z' : P* is the Cl-S Partner of P, therefore on QL-Cu1 and Z and Z' are a 3rd copple of CSC-partners, also on QL-Cu1 (property of the curve). The vertices of the QA-Tr1 are on QL-Cu1. The QL formed by the 3 copples of CSC partners is inscribed in QL-Cu1 and has QL-P1 as Miquel point (one of the 3 vertices of the Miquel triangle) ...
The important is that the 2 lines L1 and L2 form the degenerated polar conic of P wrt the stelloïd, through the contact points X1, X2 and X3 and Y1, Y2 and Y3 with the tangents fom P to the stelloïd and X, X', Y, Y' and P are on a conic, which is the polar conic of P wrt the hessian.
These 2 conics are mutually poloconics. and cut the hessian in Cl-S partners.
Now my 2 questions :
1) For an ordinary point P, it's possible to draw the 2*6 tangents to both curves (stelloïd and hessian) and the 2 conics of the tangential points (3rd point on each tangent). Are there links between these 4 conics ? (2 are already poloconics and cut the hessian in Cl-S partners)
2) For a point P on the hessian, the polar conic wrt the stelloïd is formed by the 2 perpendicular lines in P* and the conic of the tangentials wrt the hessian is reduced to the point P. Are there links between the 5 points X, X', X1, X2 and X3 and Y, Y', Y1, Y2 and Y3?

Naturally, you could put it in another way: any line through 2 CSC partners of the hessian cut the stelloïd in 3 points and the tangents to the stelloïd in these 3 points concur on the hessian in the tangential point of the 2 CSC partners.
Best regards
Bernard
PS Of course, the 4 points are real only for the bicursal hessian

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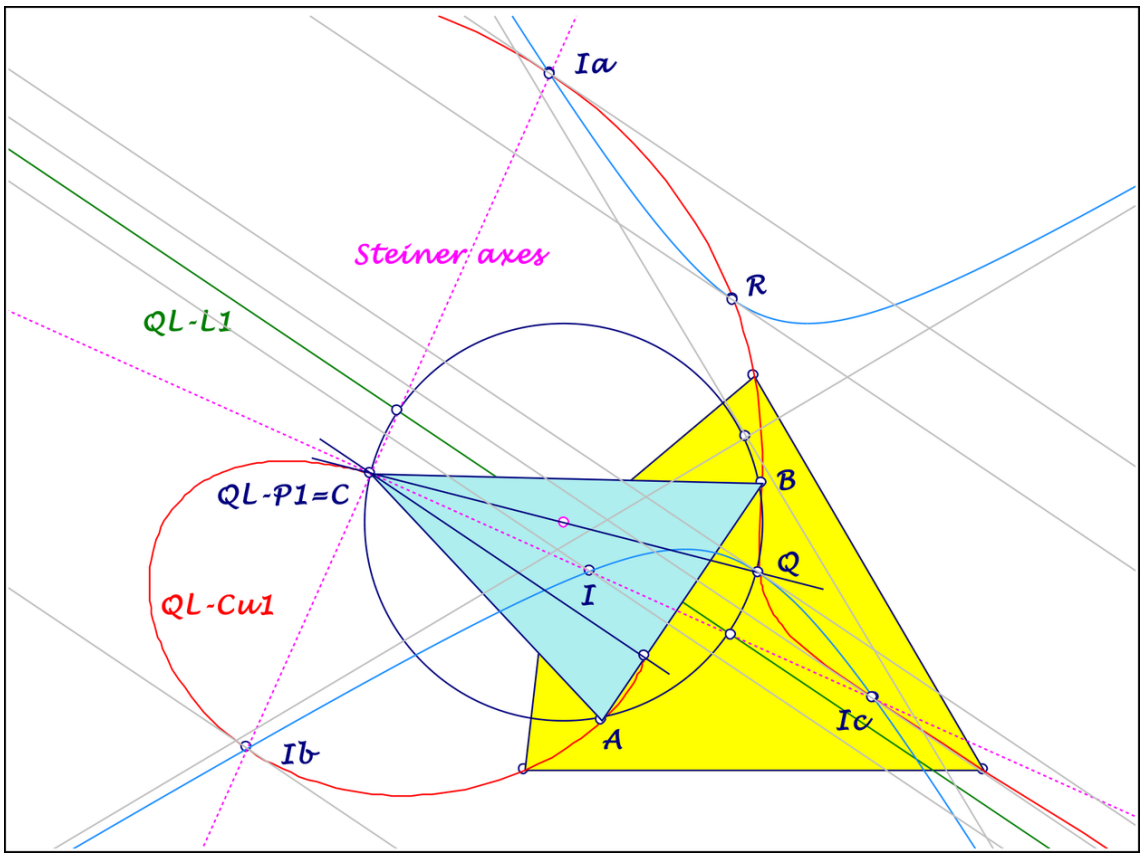
Message: #1379
Date: 09/12/2015 9:41:18
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard, dear Chris,
as consequence of this sight:
A bipartite QL-Cu1
    is an isogonal pivotal circular isocubic wrt ...
... a triangle with vertices in QL-P1 and the CSC-partners on
    a perpendicular line to QL-L1 in the intersection with
    QL-L6,
... and pivot in the point at infinity of QL-L1.
In this way all properties of isogonal pivotal circular
isocubics can be used for QL-Cu1.
The reference triangle is already mentioned on my homepage under
11.1. Here some observations (see attached file):
... The pedal point of QL-P1 on the opposite side
    is a point on QL-Cu1.
... The circumcircle cuts -diametral to QL-P1- the cubic QL-Cu1
    in the intersection Q with its asymptote.
... The angle bisectors at QL-P1 are the Steiner axes.
... The circumcircle cuts the Steiner axes
    on the Newton line QL-L1.
... The in- and ex-centers are the intersections
    of the cubic QL-Cu1 and the Steiner axes (without QL-P1).
... The cubic QL-Cu1 is anallagmatic:
    The excenters are the centers of the inversion circles,
    swapping vertices and excenters.
... The in- and ex-centers and Q lie on an orthogonal hyperbola,
    centered in R on the circumcircle.
... The 4th intersection of this hyperbola and QL-Cu1 is the
    reflection of Q in the center R of the hyperbola.
... The tangents in Q and R at the hyperbola
    are parallel to QL-L1.
... The reference triangle is the Miquel triangle
    of all contact-QA for tangents
    from a point on QL-Cu1 to QL-Cu1.
Best regards Eckart
PS: Thanks to Bernard for his detailed comparison of QL-Cu1 and
QL-Cu2.
    Wrt his questions no results up to now.



2015-12-09.docx

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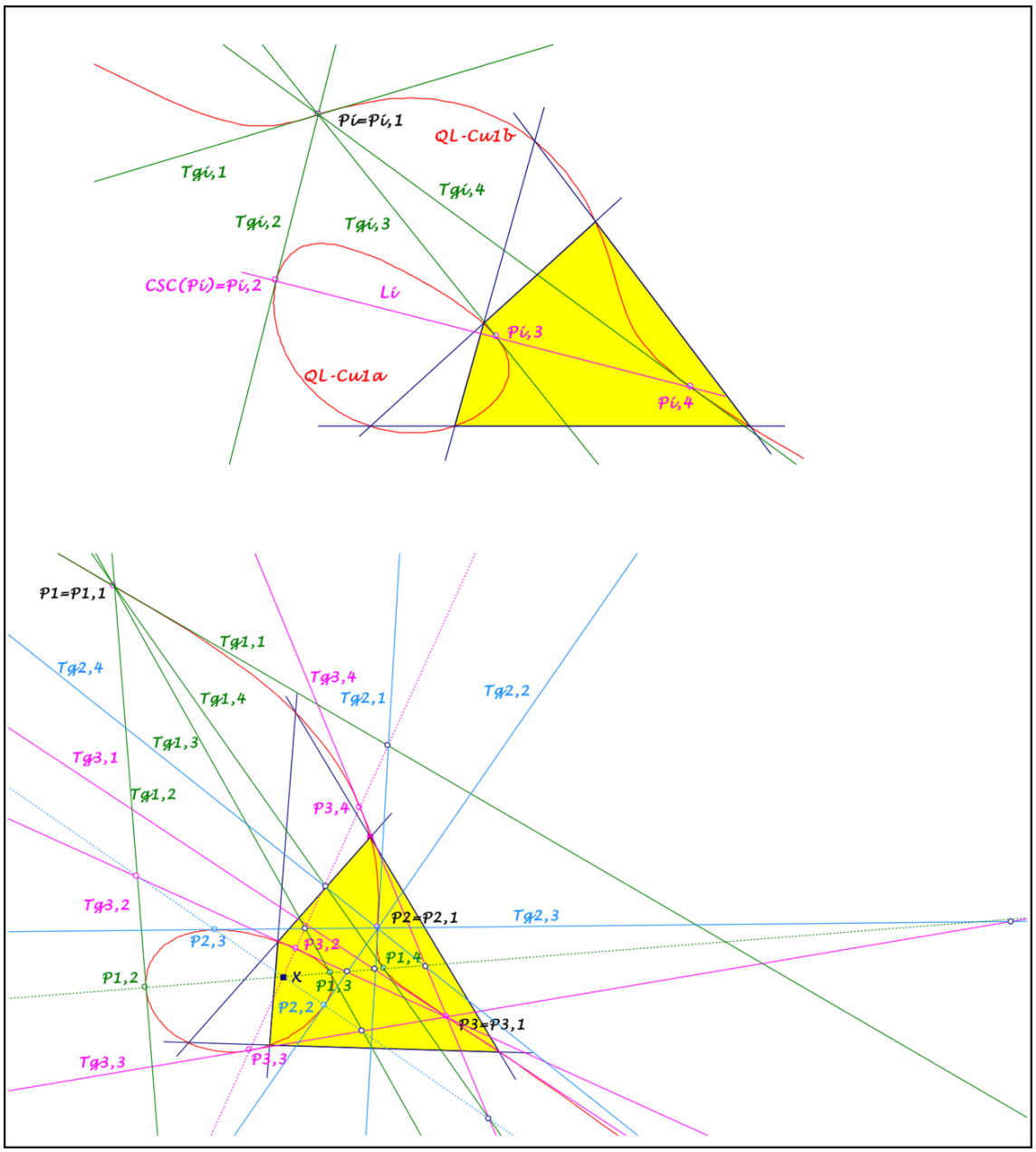
Message: #1380
Date: 10/12/2015 2:19:45
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 inflection points

Dear Bernard,
up to now there is no construction of the QL-Cu1- inflection points, but here are perhaps some new observations:
Let QL-Cu1 be bipartite with QL-Cu1a and QL-Ci1b. There are three collinear inflection points Pi, intersections with QL-Cu2.
Let us consider the tangents from Pi at QL-Cu1 (see attached file):
... Tgi,1 inflection tangent with contact point Pi = Pi,1,
... Tgi,2 tangent with contact point CSC(Pi) = Pi,2,
... Tgi,3 tangent with contact point Pi,3 on QL-Cu1a,
... Tgi,4 tangent with contact point CSC(Pi,3) = Pi,4 on QL-Cu1b.
The contact points Pi,2, Pi,3, Pi,4 are collinear on a line Li.
The lines L1, L2, L3 have a common point X.
This point X is the 4th harmonic point of
... Tgj,1 ^ Tgk,1 wrt Pi,2 and Pi,3,
... Tgj,2 ^ Tgk,2 wrt Pi,3 and Pi,4,
... Tgj,3 ^ Tgk,3 wrt Pi,2 and Pi,4,
... Tgj,4 ^ Tgk,4 wrt Pi,2 and Pi,3.
In this sight X is collinear with 7 points on L1:
P1 / P1,2 / P1,3 / Tg2,1 ^ Tg3,1 / Tg2,2 ^ Tg3,2 / Tg2,3 ^ Tg3,3 / Tg2,4 ^ Tg3,4
What about this point X in QL-geometry?
Best regards Eckart



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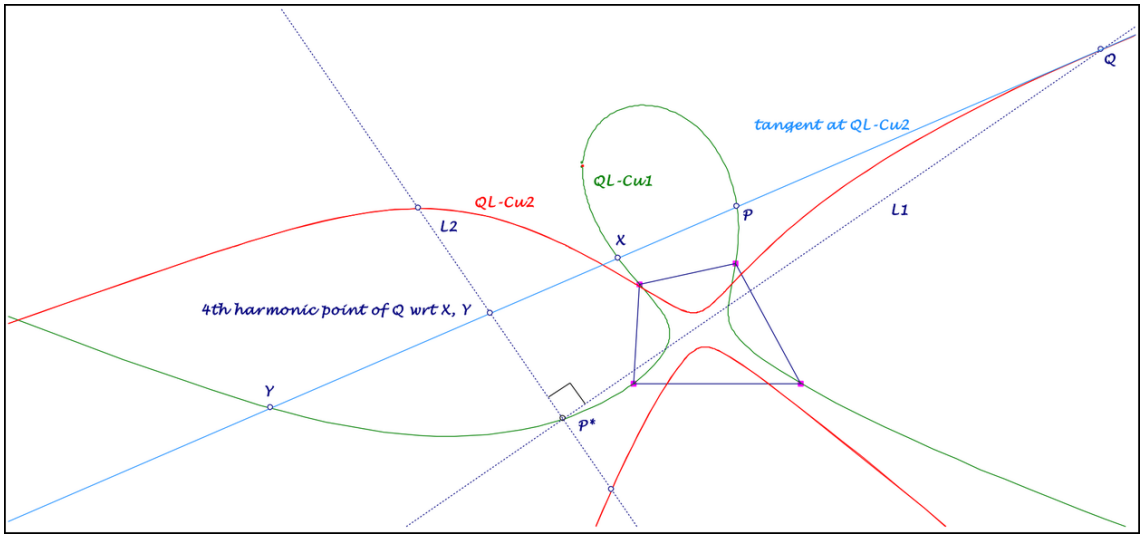
Message: #1381
Date: 11/12/2015 10:18:59
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard,
wrt your questions in #1378 here an observation (see attached file):
Let P be a point on QL-Cu1,
... P* its CSC-image,
... L1, L2 angle bisectors at P* wrt opposite QL-points,
... Q intersection of L1 with QL-Cu2,
... which is a contact point of a tangent from P to QL-Cu2,
... X, Y further intersections of the tangent PQ with QL-Cu1:
... The 4th harmonic point of Q wrt X, Y
    is the intersection of PQ and L2.
Best regards Eckart



2015-12-11.docx

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Message: #1382
Date: 11/12/2015 5:04:51
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
All these properties are indeed very interesting, but almost all already well-known !
The bipartite QL-Cu1 is the hessian of the Kjp cubic, referenced K193 by B. Gibert.
The new (at last for me) property is the last one in red : I already told you in my message 1378 that one of the vertices was QL-P1, but I don't see why the 2 others are the QL-2P3 !
Best regards
Bernard

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Message: #1383
Date: 11/12/2015 5:18:22
From: bernard.keizer
Subject: QL-Cu1 inflection points

Dear Eckart,
I always appreciate your beautiful figures !
The construction is very interesting.
The point X is not an early beard (see your own message 584)
The triangle P1,2P2,2P3,2 is the orthic triangle of the triangle formed by the 3 common tangents to the hessian and the cayleyan and X is the incenter of the orthic triangle and the orthocenter of the triangle of the tangents. It is the concurrence point of the normals in the 3 points to the hessian, which are the tangents in the inflection points of the cayleyan.
The line through the inflection points of both the stelloïd and the hessian is the polar line of X wrt the circumcircle of the orthic triangle (Euler circle of the triangle of the tangents).
Last, but not least, I suppose this point X is a special point in the triangle QL-P1QL-2P3 (excenter ?)
Best regards
Bernard

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Message: #1384
Date: 11/12/2015 5:31:04
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Wunderbar, lieber Eckart
This is very promising indeed !
The same goes obviously for one of the 2 points where L2 cuts QL-Cu2.
I notice immediately that this gives a new way of drawing QL-Cu1, providing you have already QL-Cu.
I'm convinced that there is more to find with this harmonic division !
Best regards
Bernard

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Message: #1385
Date: 13/12/2015 4:40:38
From: seiichikiri
Subject: An example of Cayley Bacharach theorem

Dear friends,
Cubics of QA-Cu2, Cu3 and Cu5 have 9 common points of the vertices of the reference quadrangle, those of the diagonal triangle, QA-P1, QA-P20 on them.
QA-Cu5 is expressed as the linear combination of QA-Cu2 and Cu3.
In CT coordinates,
QA-Cu5=(pqr QA-Cu2+(p+q+r)QA-Cu3)/(p+q)/(q+r)/(r+p).
Best regards, Seiichi.

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Message: #1386
Date: 13/12/2015 8:06:53
From: chris.vantienhoven
Subject: An example of Cayley Bacharach theorem

Dear Seiichi,
Yes the Caylay Bacharach Theorem is very intriguing.
I never studied it with the QA-cubics. Thank you!
By the way QA-Cu2 is also in another way a very special cubic.
It probably is the most symmetric central cubic that can be found in a Quadrangle.
It passes through the vertices of the Reference Quadrangle and the Diagonal Triangle.
Moreover it passes through the centroid of the Reference Quadrangle QA-P1, which is a real center of the cubic.
All of the cubic is symmetric around QA-P1.
Last but not least the asymptotes coincide with the three bimedians of the Reference Quadrangle.
Best regards,
Chris

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Message: #1387
Date: 13/12/2015 1:12:47
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard,
you have a better knowledge of QL-Cu1 and QL-Cu2, thanks for your messages 1382 and 1384.
But my message 1381 isn't so promising as I noticed now:
The property holds not only for PQ, but for any line through P, independent of QL-Cu2:
Best regards Eckart

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Message: #1388
Date: 13/12/2015 1:13:05
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 inflection points

Dear Bernard,
thank you to remind me on #584. A question wrt your remark
"The line through the inflection points of both the stelloïd and the hessian is the polar line of X wrt the circumcircle of the orthic triangle (Euler circle of the triangle of the tangents)."
I think, the following is true:
"The line through the inflection points of both the stelloïd and the hessian is the trilinear polar of X wrt the triangle of the tangents."
Best regards Eckart

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Message: #1389
Date: 14/12/2015 12:05:12
From: seiichikiri
Subject: An example of Cayley Bacharach theorem

Dear Chris,
Message #1385 is my interpretation of Cayley Bacharach theorem. It may be different from what the theorem really means, but can produce the relations among cubics.
At the same time I realized that there was no geometric content in my interpretation.
As for QA-Cu2, I think so. " Bimedian"□ is a very pleasing word for me.
Best regards, Seiichi.

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Message: #1390
Date: 14/12/2015 10:13:38
From: bernard.keizer
Subject: QL-Cu1 inflection points

Dear Eckart,
Sorry for my confusion about the polar of X
What about a relationship between X and the triangle QL-P1QL-2P3?
Best regards
Bernard
PS If I'm not wrong, in the unipartite QL-Cu1, X becomes the concurrence point of the 3 common tangents.

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Message: #1391
Date: 14/12/2015 11:17:42
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Lieber Eckart,
When I said promising, I didn't mean easy ! It's just a beginning ...
Let's complete your figure with the 3rd part of QL-Cu2 (above on the left).
1) Any line through a fixed point P cuts QL-Cu1 in X and Y. P*X cuts QL-Cu1 in Y*, the CSC Partner of Y and P*Y cuts QL-Cu1 in X*, the CSC Partner of X ; X*, Y* are on a line trough P. The 4 lines form an inscribed QL of QL-Cu1 and the sum of their directions is a constant given by the stelloïd curve QL-Cu2 ; the constant is obviously 2 * (sum of the directions of the bisectors in P and P*) for any P. If you take the 2 points QL-2P3, the constant is equal to 2 * (sum of the Steiner axis and the Newton Line).
In fact, the triangle QL-P1QL-2P3 can be seen as the limit of a QL with one vertice in QL-P1 formed by 2 lines symmetric wrt the 1rst Steiner axis and you find the same result.
2) There are 4 particular positions of the line as tangent to QL-Cu-2, which give Q1 on L1 and Q2, Q3 and Q4 on L2 (I suppose,in the bipartite QL-Cu1, it would be 6 real points ?). Each point Q has a tangential point T.
The tangents in P to QL-Cu1 give 2 points R and R', which are CSC partners on the line L1 (P* is their complement and P their tangential).
I'm convinced that there is a relationship between all these points X, Y, Q and T on the 4 lines and P, P*, R and R*.
3) Last, but not least, the parallel in P to the asymptotes of QL-Cu2 should cut the stelloïd in 6 points lying on a rectangular hyperbola centered in the middle of PP* and with asymptotes parallel to L1 and L2.
I hope these considerations will help ...
The fight goes on!
(A complete figure with QL-Cu1 and QL-Cu2 with QL-P1, Newton Line and asymptotes and the 4 tangents in a point P would help me greatly!)
Best regards
Bernard

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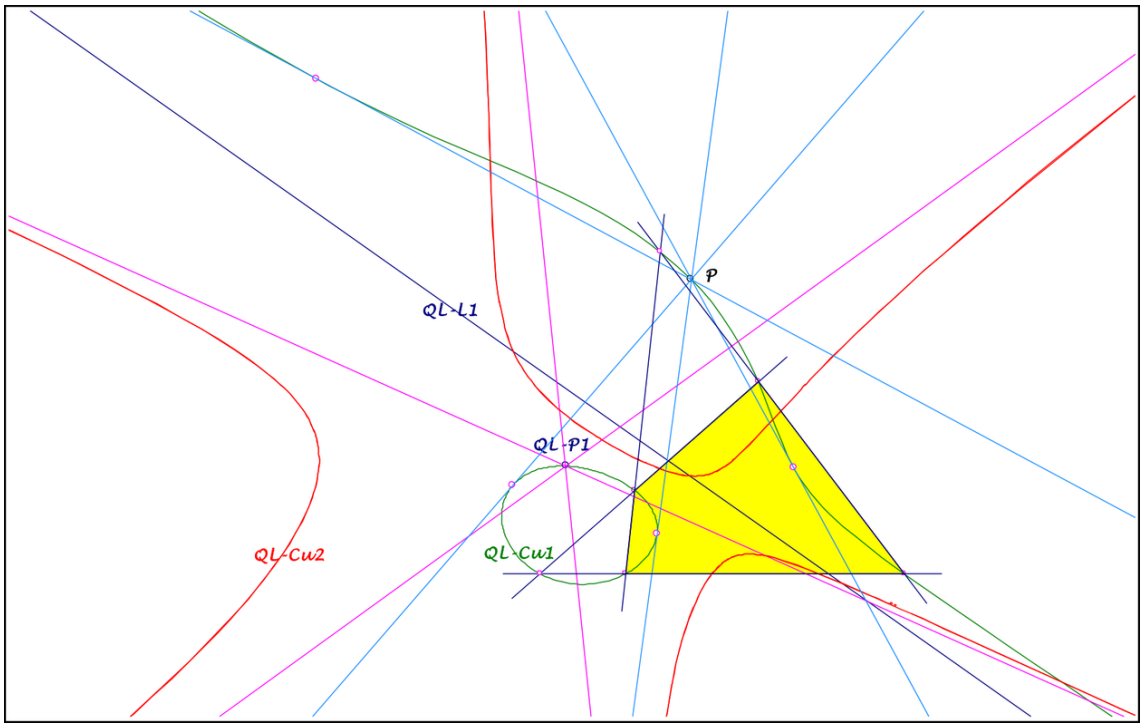
Message: #1392
Date: 14/12/2015 6:58:24
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard,
before studying your message 1391, attached the figure you asked for.
Best regards Eckart



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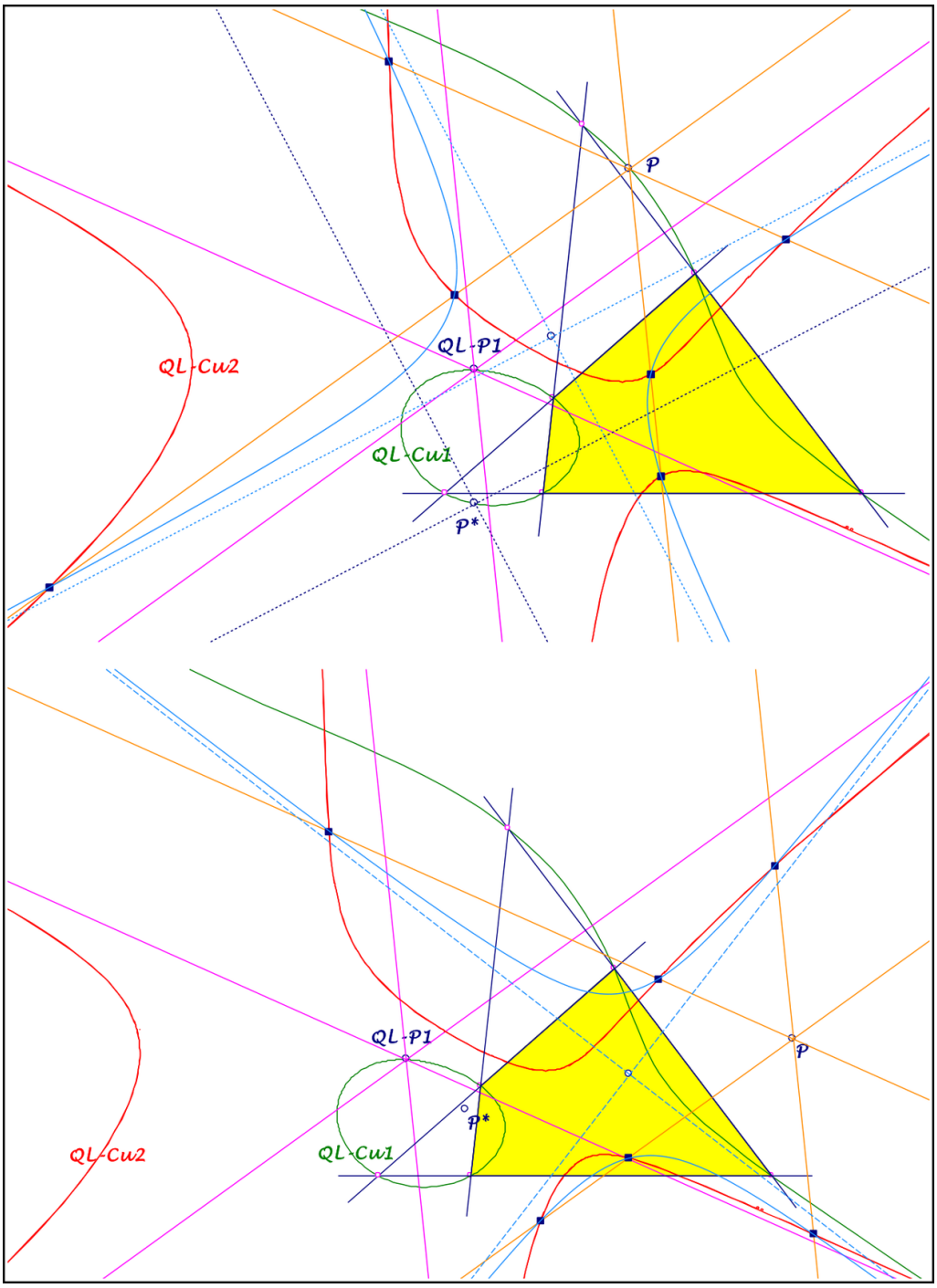
Message: #1393
Date: 15/12/2015 11:52:10
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard,
I study your message 1391 from the end: Attached two figures wrt 3), 1st P on QL-Cu1, 2nd P not on QL-Cu1.
Best regards Eckart



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Message: #1394
Date: 15/12/2015 1:23:11
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
Thanks a lot for the figure !
>From P, there are 4 real tangents to Cu1 and 2 real tangents to Cu2.
>From P*, there are 0 real tangent to Cu1 and 4 real tangents to Cu2.
This gives 10 real contact points: 2 pairs of CSC partners on Cu1, R1 and R1* and R2 and R2* and 6 points Q1 to Q6 on Cu2.
P is the common tangential of R1, R1*, R2 and R2* on Cu1.
Each Qi has a tangential Ti on Cu2.
If we draw the bisectors in P and P*, we find the alignments on these 4 lines : R1, R1*, P* and Q1, R2, R2*, P* and Q2, Q3, Q4 and P and Q5, Q6 and P.
The Cl-S conjugates of these bisectors are 2 pairs of orthogonal circles through P and QL-P1 and R1 and R2 or R3 and R4 and through P* and QL-P1.
The polar conic of P wrt Cu2 is formed by the 2 bisectors in P* ; it's poloconic is the polar conic of P wrt Cu1 through R1, R1*, R2, R2* and P (tangent to Cu1 in P).
The polar conic of P* wrt Cu2 is formed by the 2 bisectors in P (with no other intersection with Cu1) ; it's poloconic is through P* (?)
Best regards
Bernard

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Message: #1395
Date: 15/12/2015 1:27:16
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
Thanks again for this 2nd figure !
In both cases, the asymptotes of the polar conic of P, centered in P* and the conic centered in the middle of PP* are parallel (homothetic in the homothety P, 1/2)
Best regards
Bernard

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Message: #1396
Date: 16/12/2015 9:25:35
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

```
Dear Bernard,
I think, the following observation holds:
Let P be a point on QL-Cu1 with its 2 x 4 possible tangents at
QL-Cu1 and QL-Cu2.
... Six of these 2 x 4 tangents are real.
... The contact points lie in triples on the angle bisectors
    at CSC(P) wrt opposite QL-points.
... If QL-Cu1 is unipartite:
... ... 2 tangents at QL-Cu1 with contact points as CSC-partners
        on a 1st bisector,
... ... 4 tangents at QL-Cu2 with one contact point
        on the 1st bisector and 3 on the 2nd.
... If QL-Cu1 is bipartite and P
    on the part without infinity point:
... ... 0 tangent at QL-Cu1,
... ... 6 tangents at QL-Cu2 with 3 contact points
        on each bisector.
... If QL-Cu1 is bipartite
    and P on the part with infinity point:
... ... 4 tangents at QL-Cu1 with 2 contact points
        as CSC-partners on each bisector,
... ... 2 tangents at QL-Cu2 with contact points
        on different bisectors.
Some of these properties you have already mentioned.
Perhaps the following property is relevant:
... If QL-Cu1 is bipartite and P on the part
    with infinity point, the 4 real tangents from P to QL-Cu1
    are symmetric wrt the angle bisectors at P
    wrt opposite QL-points.
... In this case the cross ratio of the 4 tangents
    is constant for points P on QL-Cu1.
Best regards Eckart
```

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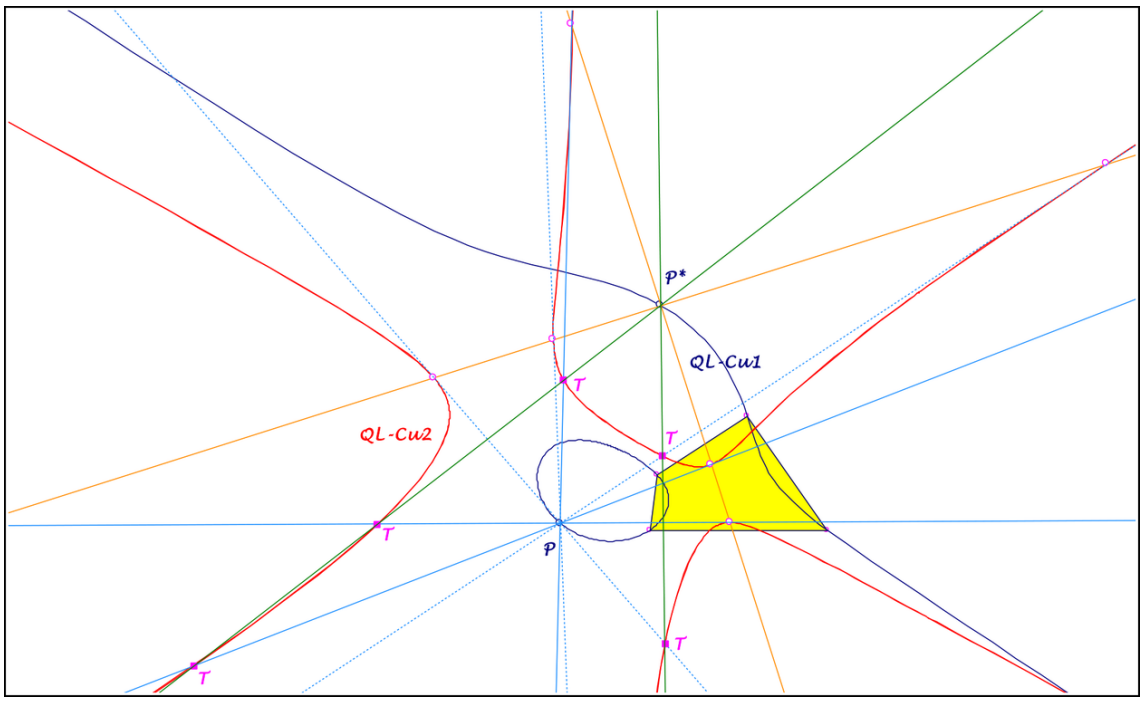
Message: #1397
Date: 16/12/2015 11:37:35
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

Dear Bernard,
wrt the points T in 2) of #1391 (see attached file):
Let QL-Cu1 be bipartite and P a point on the part with infinity point. Then there are 6 tangents at QL-Cu2 with 2 x 3 contact points Q on the angle bisectors of CSC(P) wrt opposite QL-points.
The QL-Cu2-tangentials T for 3 contact points Q on the same angle bisector are collinear with CSC(P).
Best regards Eckart



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Message: #1398
Date: 16/12/2015 6:03:10
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
It's interesting to find that for a point on Cu1 the conic of the tangentials degenerates the same way as the polar conic wrt Cu2.
We have now an almost complete image of the different possible cases :
random P
P on Cu1
P on Cu2
P on the Newton Line (in this case, the polar conic wrt Cu1 is a rectangular hyperbola)
and the result in polar conics wrt Cu1 and Cu2 (which are poloconics and cut the hessian in CSC partners) as well as conic of tangentials
Thanks again for your figures !
I won't be home until next year.
I wish you Merry Christmas and Happy New Year and I hope we will pursue our cooperation !
Best regards
Bernard

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Message: #1399
Date: 23/12/2015 9:33:21
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

```
Dear Bernard, dear Chris,
is the following observation already mentioned?
Let A, B, C, QL-P1 be concyclic points on QL-Cu1,
... then QL-Cu1 is invariant wrt the ABC-isogonal conjugate.
Or:
Let L be a line, not containing QL-P1 but with 3 intersections
with QL-Cu1,
... then the CSC-images of the intersections give a triangle ABC
... and QL-Cu1 is ABC-isogonal invariant.
Example: Take the three collinear QG-P18 on QL-Cu1, whose
CSC-images give the orthic triangle of QL-Tr1.
First properties:
... For points P on QL-Cu1 holds wrt ABC  isog(P) = CSC(P).
... isog(L) is an  ABC-circumconic tangent to QL-Cu1 in A, B, C.
... The triangle ABC and the orthic triangle of QL-Tr1
    have a common circumconic.
... The ABC-isogonal conjugate of this common circumconic
    is the line of the QG-P18-points.
... The isogonal conjugate wrt the orthic triangle of QL-Tr1
    of the common circumconic is the line L.
Merry Christmas and a Happy New Year!
Best regards Eckart
```

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Message: #1400
Date: 27/12/2015 8:42:24
From: chris.vantienhoven
Subject: QL-Cu1 as QA-Cu1

```
Dear Eckart,
Very nice properties!
I like especially the first property because it is so simple.
I wish you and all other QFG-members a very special and happy
New Year.
Best regards,
Chris
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Message: #1401
Date: 28/12/2015 10:31:57
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart, dear Chris
Of course, these are nice properties !
They are basic properties of QL-Cu1, partly already in EQF, perharps not totally explained.
1) Any QL has 4 reference triangles.
Two CSC partners on QL-Cu1 are the foci of an inscribed conic ; the foci of a conic inscribed in a triangle are isogonal conjugates wrt this triangle.
As well-known, QL-Cu1 is CSC-invariant and self-isogonal wrt each of the 4 reference triangles
2) conversely, each triangle ABC on a circle through QL-P1 is one reference triangle of another QL, also inscribed in the same QL-Cu1.
This is in particular true for the orthic triangle of DT.The vertices of this triangle are the 3 3rd intersections of the diagonals with QL-Cu1 and their CSC are the 3 tangentials of the 3 copples of conjugate vertices of the QL. These tangentials are on a line, which is the CSC of the Euler circle of DT.
The Miquel point QL-P1 is on this circle, as the parabola is inscribed in the medial triangle of DT.
This is also true for the triangle QL-P1QL-2P3, QL-P1 playing a dopple role and the circumcircle is tangent to QL-Cu1 in this point.
As I put it several times on this forum, this triangle may also be seen as a limit QL, with twice the same line through the QL-2P3.
3) Last, but not least, 2 lines through a point P form a degenerate conic. Each line cut QL-Cu1 in 3 points ; each group of 3 points have for CSC partners 3 points on a circle trough QL-P1 and the 2 circles intersect in QL-P1 and in the CSC of P.
Now, the 2 groups of 3 CSC partners are coconic ; the conic is the poloconic of the degenerated conic formed by the 2 lines. (If one of the 2 lines is the infinity line, the poloconic is the CSC circle of the other line).
This is true for any 2 groups of 3 points on 2 circles through QL-P1 (reference triangles of the QL, orthic triangle, triangle QL-P1QL-2P3, triangle through 2 conjugate vertices and their tangential ...)
Geometry is really beautiful !
Happy New Year for both of you
Best regards Bernard

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Message: #1402
Date: 28/12/2015 10:41:19
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

```
Dear Eckart, dear Chris
Sorry for the mistake, it's not QL-2P3, but QL-2P2 !
(I suppose, you would have corrected by yourself)
Best regards
Bernard
```

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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

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- Volume 3 (2015), messages #917–#1402
- Volume 4 (2016), messages #1403–#2169
- Volume 5 (2017), messages #2170–#2799
- Volume 6 (2018), messages #2780–#3299
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- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60
- Volume 2 (2020), messages #61–#630
- Volume 3 (2021), messages #631–#1294
- Volume 4 (2022), messages #1295–#1544
- Volume 5 (2023), messages #1545–#2051
- Volume 6 (2024), messages #2052–#2559
- Volume 7 (2025), messages #2560–#2897

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