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2016

Digital Edition

Chris van Tienhoven et al.

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Messages #1403 - #2169

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1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at https://groups.io/g/Quadri-and-Poly-Geometry. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at https://groups.io/g/Quadri-Figures-Group.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Ángel Montesdeoca
- Antreas Hatzipolakis
- Benedetto Scimemi
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- Eckart Schmidt
- Ngo Quang Duong
- Randy Hutson
- Seiichi Kirikami
- Systems Manager
- Tran Quang Hung
- Tsihong Lau

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

Ángel Montesdeoca:
#1440 #1448 #1651 #1660 #1661

Antreas Hatzipolakis:
#1768 #1895 #1896 #1917 #1918 #1933 #2011 #2013 #2052

Benedetto Scimemi:
#1430 #1478 #1553 #1585 #1685 #1902 #1940

Bernard Keizer:
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Chris van Tienhoven:
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#2139 #2147 #2149 #2152 #2153 #2161 #2162 #2163 #2164 #2167 #2169

César Lozada:
#1951 #2012

Eckart Schmidt:
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Ngo Quang Duong:
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Randy Hutson:
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Seiichi Kirikami:
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Systems Manager:
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Tran Quang Hung:
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#2066 #2068 #2073 #2074 #2075 #2078 #2079 #2081 #2082 #2085 #2087
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Tsihong Lau:
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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Benedetto Scimemi (1938–2023)

Location
Italy

Year of Birth / Generation
1938–2023.

Short Biography
Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.esteri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter's 60th birthday (the "A5 Meeting", named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See CO-Tf3 in EPG.

- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Ngo Quang Duong

Location
Living in Hanoi, Vietnam.

Year of Birth / Generation
Born in 1998.
Generation Z (approx. 1997–2012).

Short Biography
Ngo Quang Duong studied at the Vietnam National University, where he initially majored in Mathematics before switching to Software Engineering. He has since been working professionally in the software field, while continuing to pursue his interest in Mathematics in his free time. His background combines formal mathematical training with practical experience in computing and problem solving, giving his work a distinctive blend of theoretical insight and computational intuition.

Themes and Interests

- Geometry, Topology, Analysis, and their interactions
- Classical Geometry, especially triangle geometry and quadri-figure geometry
- Contributions to QFG, including n-angle centers and new uses of coordinate systems
- General mathematical exploration and independent study

Selected Publications

- (with T. T. Vu) *A Generalization of the Droz Farny Line Theorem with Orthologic Triangles*, Forum Geometricorum, Volume 16 (2016), 415–418.
- *Generalization of Musselman's theorem. Some Properties of Isogonal Conjugate Points*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 15–29.
- (with O. T. Dao and P. Yiu) *Golden Sections in an Isosceles Triangle and Its Circumcircle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 93–97.
- *Generalizations of Lester Circle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 10 (2021), 49–61.

Additional Remarks
He has not actively returned to Classical Geometry for some time, but remains mathematically engaged through online communities. He is active on Math StackExchange (MSE), where he contributes under the profile: https://www.math.stackexchange.com/users/821868/duong-ngo

Seiichi Kirikami (1949?–2023)

Location
Japan.

Year of Birth / Generation
Exact year unknown; passed away on 11 December 2023.

Short Biography
In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions
Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes
Colleagues remembered Seiichi with deep affection:

- "Geometry is the poorer of his death." — A. P. Hatzipolakis
- "He was a great expert in projective and triangle geometry, and a very kind and helpful person." — E. Suppa
- "He had a way of seeing things from a different angle and presenting them simply." — C. van Tienhoven
- "He inspired many of my problems." — A. Altıntaş
- "He wrote many interesting and innovative messages in ADGEOM." — F. J. García Capitán
- "He helped me in my beginnings with wise and generous advice." — C. E. Lozada
- "He contributed to OEIS and had important work in epidemiology." — P. Moses

Additional Remarks
Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks
He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

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#2128

subject: 2nd generation of isogonal conjugates:
#2117 #2118 #2119 #2120 #2121 #2122 #2123 #2124 #2125

subject: 3 concurrent ellipses:
#2142 #2143 #2144

subject: 3 Sextics invariant in 3 transformations:
#1403 #1404 #1408 #1410 #1416

subject: 6 conics accompanying Hervey circle and its rectangular hyperbola:
#1550 #1566

subject: A bit QL-geometry:
#1945

subject: A concurrent conics image in quadrangle:
#1491 #1492 #1493 #1496

subject: A family of quadrangles sharing much:
#1685

subject: A generalization of Newton line:
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subject: A line in a cyclic pentangle:
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subject: A lot of cubics of QA/QL:
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subject: A property of cyclic quadrangle with Morley 1st equilateral triangle:
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subject: A question of sum of the squared distances:
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subject: A rectangular hyperbola in Quadrilateral:
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subject: Affine, Möbius and Projective Planes:
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subject: Alternating Sum of Distance Squares:
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subject: An Envelope:
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subject: An involution in pentangle:
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subject: Anticenters in Quadri-Figures Geometry:
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subject: Application of the anticenter in a Pentalateral:
#1904

subject: Begonia point of quadrilateral:
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4.2 Messages

Message: #1403
Date: 03/1/2016 4:58:49
From: bernard.keizer
Subject: 3 Sextics invariant in 3 transformations

Dear Eckart,
Looking again at the quartics invariant in the 3 transformations Cl-S, Cl-Sdiag and Cl-Ster, I've found these 3 sextics, which may interest you.
I consider always the points P1, P17 and Px = Cl-S(P17) = Cl-Sdiag(P1) and the 3 Cl-S transformations Cl-S with center P1, Cl-Sdiag with center P17 and Cl-Ster with center Px.
Cl-S swaps P1 and Px and the line P17Px with the circumcircle through P1, P17 and Px and the same goes for the other transformations.
Co is the parabola with center P1 and directrix P17Px, obviously tangent to the 2 pairs of Steiner axes of the 2 transformations ClSdiag and Cl-Ster.
The CSC of the tangents to this parabola are circles through P1 centered on the circumcircle (P1,P17,Px).
The intersections between the tangents and their CSC circles are CSC partners and give a sextic through the 3 pairs of invariant points of the 3 transformations (QL-2P3a and b) and invariant in the 3 transformations.
The same goes with the 2 other transformations and gives 2 other sextics having the same properties (for the same point on the circumcircle, the corresponding tangents to the 3 parabolas are parallel).
Best regards
Bernard
I hope you will confirm these properties, it was my NewYear's present!

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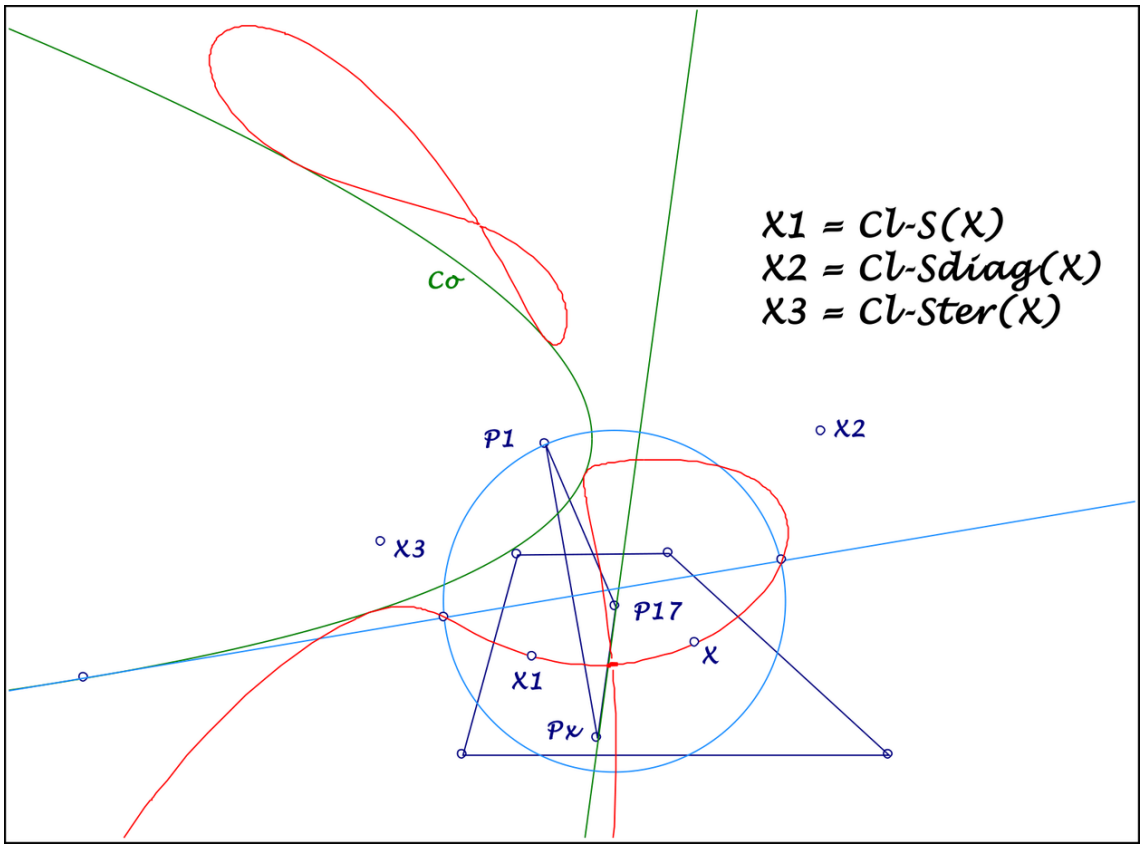
Message: #1404
Date: 04/1/2016 2:32:13
From: eckart_schmidt@t-online.de
Subject: 3 Sextics invariant in 3 transformations

Dear Bernard,
thanks for your message 1403, I have studied the properties of the 3 sexties, but I can´t confirm, that each sextic is invariant wrt the three transformations (see attached 1st file):
The sextic for your parabola Co with focus P1 and directrix P17Px is only invariant wrt the corresponding transformation Cl-S. This holds analog for the two other sextics.
I hope, I have read your message exactly.
I think, such sextics can analog be considered with the same properties replacing P17 and Px through an arbitrary pair P and Cl-S(P).
Best regards Eckart
PS. There will be a typo: Cl-S swaps P17 and Px and directrix P17Px, ...



2016-01-04.docx

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Message: #1405
Date: 04/1/2016 9:07:50
From: eckart_schmidt@t-online.de
Subject: QA-Tf1 analog for QL

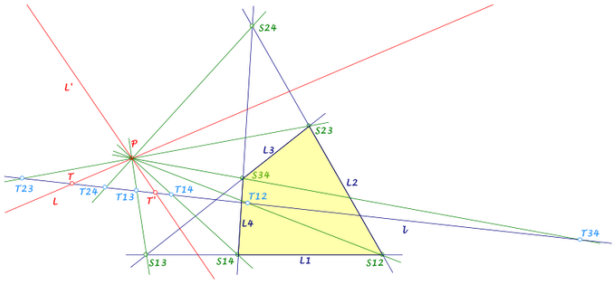
Dear Chris,
for a line L wrt a quadrangle QA there is a unique line involution P --> P´ with P´ as 2nd intersection of L and a QA-circumscribed conic through P. For a point P wrt a QL there is analog a line transformation L --> L´ for lines through P with L´ 2nd tangent from P to a QL-inscribed conic tangent to L. Attached this line transformation is studied, perhaps of interest.
Best regards Eckart

EQF-Note 2016-01-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line Transformation wrt a Point on the Line

*Background for this QL-transformation is **D**esargues **I**nvolution **T**heorem, see QA-Tf1, here used in the dual form for QL-geometry. This leads to a line transformation wrt a point on the line. The transformation maps a line L through a point P into the 2nd tangent from P to the inscribed QL-conic tangent to L.*



The transformation

L through P → L´ through P: DIT(L,P) = L´

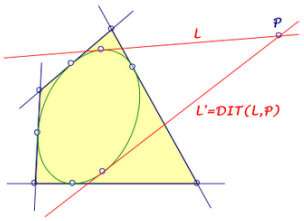
A quadrilateral with its four lines L_i has six intersections $S_{ij} = L_i \cap L_j$. For a point P (not on L_i) there are six lines $P.S_{ij}$, intersecting an arbitrary line l in six points T_{ij}. The three reciprocal pairs T_{ij}, T_{kl} define a line involution on l. Let L be a line through P and T its intersection with l and $T´$ its image wrt the line involution on l. Then the image line of L wrt P is $PT´ = L´$ through P with $DIT(L´,P) = L$.

Example: $DIT(QL\text{-}L9,QL\text{-}P23) = QL\text{-}P8.QL\text{-}P23$.

The point P can also be the point at infinity P_∞ of the line L:

Example: $DIT(QL\text{-}L9, P_\infty) = QL\text{-}P13.P_\infty$.

- ***DIT(L,P)* is the 2nd tangent from *P* at the *QL*-inscribed conic tangent to *L*.**



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Lines *L* through *QL-P1*

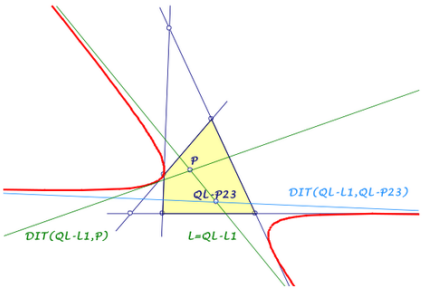
- If *L* is a line through *QL-P1* and *P* = *QL-P1*, the line *L´* is the reflection of *L* in the 1st Steiner axis.
- If *L* is a line through *QL-P1* and *P* another intersection of *L* with *QL-Cu1*, the line *L´* is parallel *QL-L1*.
- For lines *L* = *P.QL-P1* the locus of *P* with *DIT(L,P)* = *L* is the cubic of *QFG*-message 1215.
- In general: For a given point *Q* and *L* = *PQ* the locus of *P* with *DIT(L,P)* = *L* is the cubic of *QFG*-message 1175.

The property *DIT(L,P)* = *L*

- *DIT(L,P)* = *L*,
  … if *L* is tangent in *P* at an *QL*-inscribed conic.
  … if *P* is the intersection of *L* and *QL-Tf2(L)*.
  … if *L* is angle bisector at *P* ∈ *QL-Cu1* wrt two opposite *QL*-points.
  … if *L* = *PQ* and *P, Q, CSC(Q)* collinear on *QL-Cu1*.

Points *P* on a fixed line *L*

- *L* = *QL-L1* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope the conic *QL-Co2* with asymptote *QL-L1* and center *QL-P23*. The 2nd asymptote is *DIT(QL-L1,QL-P23)*.



- *L* = *QL-L2* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope a *QL*-inscribed conic tangent to *QL-L2* with center *QL-P20*.
- *L* = *QL-L3* and *P* ∈ *L*: The lines *L´* = *DIT(L,P)* envelope the inscribed parabola *QL-Co1*.
- In general: Let *L* be a given line and *P* ∈ *L:* The lines *L´* = *DIT(L,P)* envelope a *QL*-inscribed conic tangent to *L*.

Last not least

- For two different points P_1, P_2 with $DIT(P_1P_2,P_1) \cap DIT(P_2P_1,P_2) = Q$ holds: $DIT(QP_1,Q) = P_2Q$ and $DIT(QP_2,Q) = P_1Q$.
- Let P and $P^* = CSC(P)$ be contact points of tangents at *QL-Cu1*, which intersect in Q on *QL-Cu1*:

$$DIT(PP^*,P) = PQ,$$
$$DIT(QP,Q) = QP^*,$$
$$DIT(PQ,P) = PP^* = DIT(P^*Q,P^*).$$

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-01-04.pdf

Message: #1406
Date: 04/1/2016 11:05:13
From: chris.vantienhoven
Subject: QA-Tf1 analog for QL

Dear Eckart,
Very nice Point-on-Line transformation. There is a remarkable relationship with Desargues' Involution Theorem in a Quadrangle indeed.
I wasn't familiar with it.
Did you make calculations? For examples when we have a line (x:y:z) then point (y t : -z - x t : y) is a point on this line.
With only one variable t, what is then the transformation line?
Best regards,
Chris

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Message: #1407
Date: 05/1/2016 3:36:53
From: eckart_schmidt@t-online.de
Subject: QA-Tf1 analog for QL

Dear Chris,
here are the calculations in DT-coordinates:

QA: The line involution QA-Tf1 gives for a point P (u,v,w)
on the line L (v t , -u t - w, v)
the point P´ (t(t u + 2 w) / U - u / W ,
v t² / U + v / W ,
-t² w / U + (2 t u + w) / W) on L
with U = r² v² - q² w²
and W = q² u² - p² v².

QL: The analog line transformation gives for a line L (u,v,w)
through the point P (v t , -u t - w, v)
the line L´ (t(t u + 2 w) / U - u / W ,
v t² / U + v / W ,
-t² w / U + (2 t u + w) / W) through P
with U = n² v² - m² w²
and W = m² u² - l² v².

I hope there is no typo.
Best regards Eckart

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Message: #1408
Date: 07/1/2016 4:10:12
From: bernard.keizer
Subject: 3 Sextics invariant in 3 transformations

Dear Eckart,
Thanks for your correction of the typo and of the property about the Cl-S transformations. You are right !
Nevertheless, it holds that the 3 sextics are through the 3 pairs of invariant points.
By the way, the CSC circles in the 3 transformations of the tangents to the 3 parabolas are centered on the circumcircle of P1, P17 and Px and trough one of these 3 points : they envelop 3 cardioïds, which are the CSC of the parabolas in the transformations.
Best regards
Bernard
PS Any reaction to my message 1401 ?

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Message: #1409
Date: 07/1/2016 4:25:07
From: bernard.keizer
Subject: QA-Tf1 analog for QL

Dear Eckart,
It seems that the involution you describe is the projection on the arbitrary line l of the involutary Cl-S transformation QL-Tf1.
The fixed points should be the projections of the invariant points QL-2P3.
The partners on this line are the projections of CSC partners on QL-Cu1.
(It's always possible to choose as inscribed conic a conic reduced to 2 CSC partners)
For P on Cu1, it can be obtained through a reflexion in the bisector in P ...
Do I interpret correctly your construction ?
Best regards
Bernard

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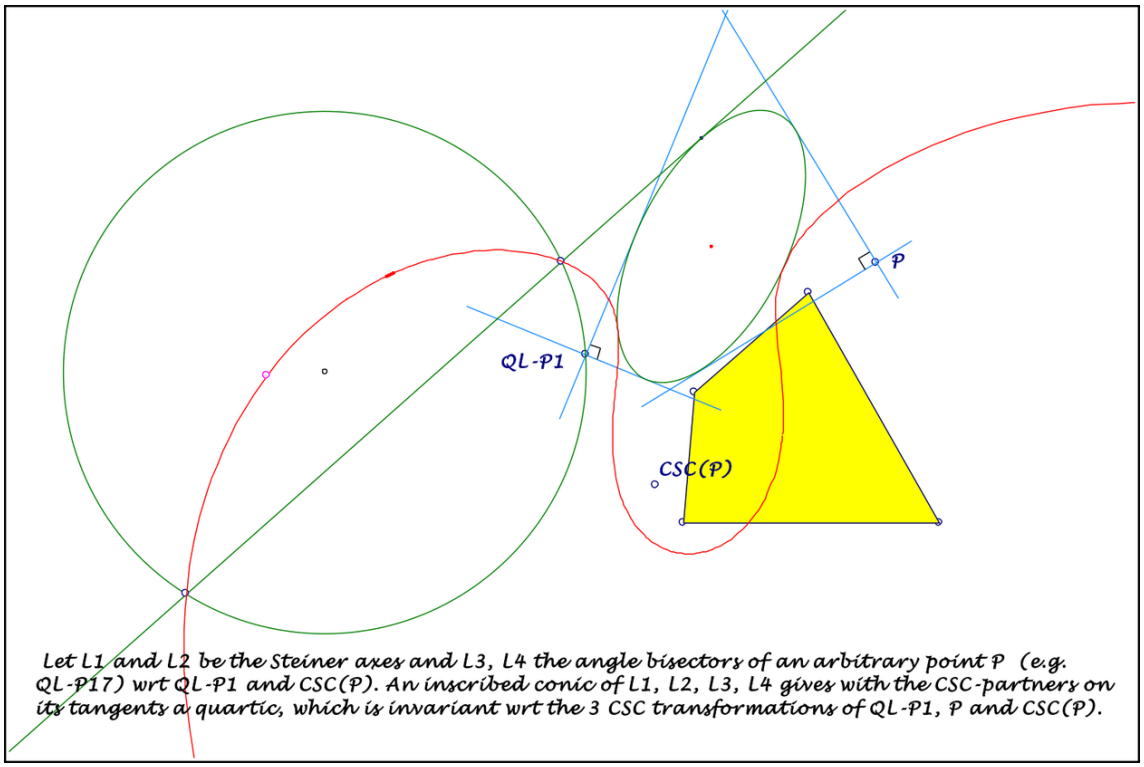
Message: #1410
Date: 08/1/2016 11:03:56
From: eckart_schmidt@t-online.de
Subject: 3 Sextics invariant in 3 transformations

Dear Bernard,
attached a constellation for quartics, invariant wrt 3 transformations, perhaps not already mentioned.
Best regards Eckart



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Message: #1411
Date: 08/1/2016 11:04:11
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as QA-Cu1

```
Dear Bernard,
two short remarks wrt #1401:
... wrt 2) in other words: Let L be a line, intersecting QL-Cu1
in 3 points Pi with CSC(Pi) = Pi*, then L and Pi*Pj* give a QL
on QL-Cu1 with the same QL-P1 and QL-L1 as the reference QL.
... wrt 3) in other words: Let two lines through P on QL-Cu1
intersect QL-Cu1 in Q1, Q2 and R1, R2, then P*, Qi* and Ri* give
a conic tangent in P* to QL-Cu1.
Best regards Eckart
PS. Excuse my late reaction!
```

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Message: #1412
Date: 08/1/2016 12:10:19
From: eckart_schmidt@t-online.de
Subject: QA-Tf1 analog for QL

```
Dear Bernard,
wrt #1409: Your interpretation of the line involution T --> T´
on the arbitrary line l wrt a point P is right for P = QL-P1,
but not in general.
In the 6th line there will be a typo: For T on Cu1, ...
Best regards Eckart
```

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Message: #1413
Date: 09/1/2016 2:14:50
From: eckart_schmidt@t-online.de
Subject: Three QG-Conics wrt a Point

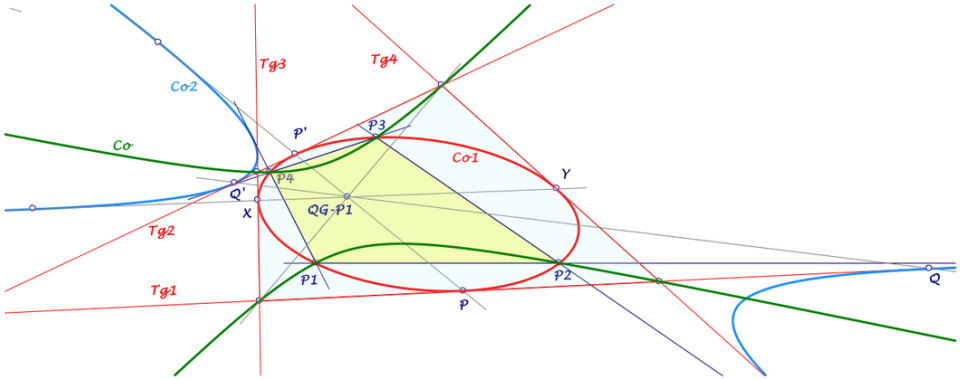
Dear Bernard, dear Chris,
just for fun an excursion in conic-geomerty for quadrigons,
which can be considered as quadrangle and as quadrilateral.
Best regards Eckart

EQF-Note 2016-01-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Three QG-conics wrt a Point

For a given point P there is a QG-circumscribed conic Co_1, and the tangent in P defines an inscribed QG-conic Co_2. The four common tangents of the two conics give a quadrigon, which has a common circumscribed conic Co with the reference quadrigon. There are two QG-circumconics, whose points P give the same conic Co.



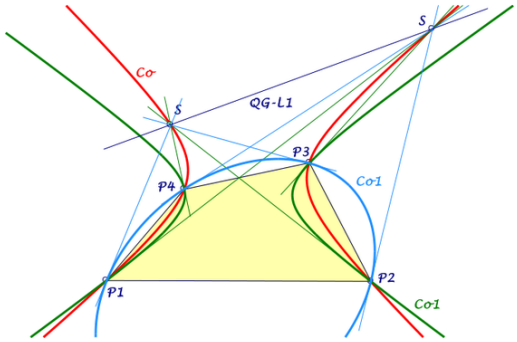
- Let $QG = P_1P_2P_3P_4$ be a quadrigon and P an arbitrary point (not on sidelines and diagonals).
- Let Co_1 be the QG-circumconic through P …
  … and Tg_1 the tangent in P at Co_1.
- Let Co_2 be the QG-inscribed conic tangent to Tg_1 …
  … with contact point Q.
- Let $P´$ be the 2nd intersection of $P.QG\text{-}P1$ and Co_1 …
  … and $Q´$ the 2nd intersection of $Q.QG\text{-}P1$ and Co_2.
- $Tg_1 = PQ$ and $Tg_2 = P´Q´$ are common tangents of Co_1 and Co_2.
- $PP´$ is the 2nd tangent from P to Co_2 …
  … with a contact point as 4th harmonic point of P, $P´$, $QG\text{-}P1$.
- Let Tg be the 2nd tangent from $QG\text{-}P1$ to Co_2 …
  … intersecting Co_1 in X and Y …
  … and contact point on Co_2 as 4th harmonic point of X, Y, $QG\text{-}P1$.
- The tangents Tg_3 and Tg_4 in X and Y at Co_1 are common tangents of Co_1 and Co_2.

- The quadrigon of the common tangents of Co_1 and Co_2 (opposite sides Tg_1, Tg_2 and Tg_3, Tg_4) has the same diagonal crosspoint *QG-P1* and the same 3rd diagonal *QG-L1* as the reference quadrigon.

- **The quadrigon of the common tangents of Co_1 and Co_2 has not only a common inscribed conic Co_2 with the reference quadrigon but also a common circumscribed conic *Co* with the reference quadrigon:**
  Co is a *QG*-circumconic through the poles of the diagonals wrt Co_1.

- **All points *P* on a fixed *QG*-circumscribed conic have the same conic *Co*.**

- **There are two *QG*-circumscribed conics, whose points *P* have the same conic *Co*:**
  Let a given *QG*-circumconic *Co* intersect *QL-L1* in *S*: A first *QG*-circumconic is tangent to $S.P_1$ and $S.P_3$, a second is tangent to $S.P_2$ and $S.P_4$.



- Example: For *Co* = *QG-Co3* the two *QG*-circumconics for the points *P* are centered in the midpoints of the diagonals.
- Example: For *P* on *QG-Co3* the second *QG*-circumconic for *P* with the same *Co* is centered in *QG-P1.QG-P2* ∩ *QG-P12.QG-P14*.

Final remark:
There are analog relationships beginning with a quadrigon and a line, which leads to a *QG*-inscribed conic with contact point …

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-01-09.pdf

Message: #1414
Date: 09/1/2016 9:54:29
From: chris.vantienhoven
Subject: Three QG-Conics wrt a Point

Dear Eckart,
Beautiful features. I like them.
Most of the time I have same additions, however this time I only see the beautiful properties and the beautiful picture.
Thanks,
Chris

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Message: #1415
Date: 11/1/2016 9:16:20
From: eckart_schmidt@t-online.de
Subject: Four Conics Tangent to QL-Cu1

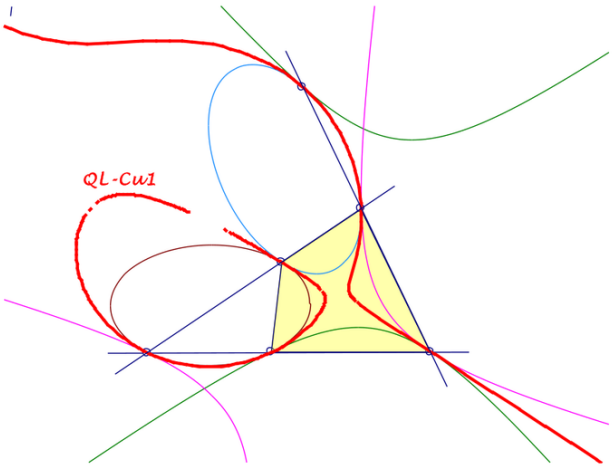
Dear Bernard, dear Chris,
for a quadrilateral there are four circumconics of the triangle components, tangent to the cubic QL-Cu1. I think, this is not mentioned in EQF up to now.
Best regards Eckart

EQF-Note 2016-01-11

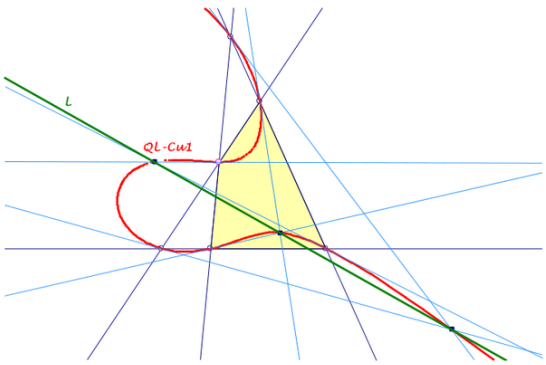
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Four Conics Tangent to QL-Cu1

For a quadrilateral there are four circumconics of the triangle components, tangent to the cubic QL-Cu1 [1].



- **The *QL-Cu1*-tangents in opposite points of a quadrilateral intersect collinear on *QL-Cu1*.**
- **These collinear intersections are the *CSC*-images of the 3rd intersections of the diagonals and *QL-Cu1*.**
- **The corresponding line *L* is the *CSC*-image of the *QL-DT*-medial circle *QL-Ci2*.**

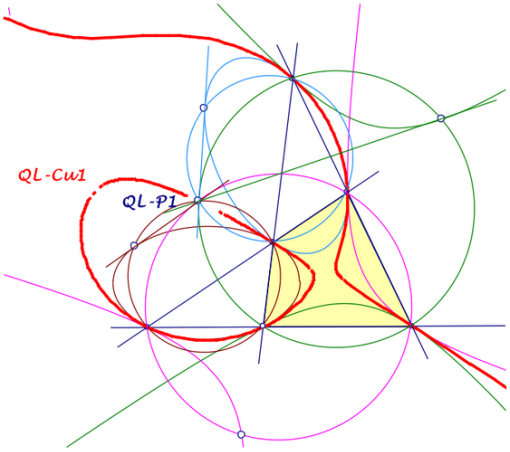


2016-01-11.pdf

- **There are four circumconics Co_i of the triangle components, tangent to the cubic *QL-Cu1*.**

- **These circumconics Co_i are the isogonal conjugates of the lines L_i wrt the remaining triangle $L_jL_kL_l$.**

- **The Co_i-tangents in the 4th intersections of Co_i and the circumcircle of $L_jL_kL_l$ intersect in *QL-P1*.**



References

[1] http://eckartschmidt.de/Zirkul..pdf , 9 (f).

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-01-11.pdf

Message: #1416
Date: 11/1/2016 10:25:48
From: bernard.keizer
Subject: 3 Sextics invariant in 3 transformations

Dear Eckart,
Thanks for this precision.
The most imprtant thing is that this construction holds for any point P and CSC(P) !
Best regards
Bernard

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Message: #1417
Date: 11/1/2016 10:36:26
From: bernard.keizer
Subject: QL-Cu1 as QA-Cu1

Dear Eckart,
wrt 2) not only the same QL-P1 and QL-L1, but also all the same properties I mentionned several times on this forum (same sum of the directions of the 4 lines, same axes of parabola and cardioïd, same direction of the axes of the deltoïd, same cubic stelloïd ...)
wrt 3) it's true, but it's not exactly what I've put: for 2 lines through a point P not on QL-Cu1 which intersect QL-Cu1 in Qi and Ri the 3 Qi and the 3 Ri are coconic
Best regards
Bernard

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Message: #1418
Date: 11/1/2016 10:44:52
From: bernard.keizer
Subject: QA-Tf1 analog for QL

Dear Eckart,
Sorry if I was too quick in the conclusions, but I think this projection from an arbitrary point P on an arbitrary line l desrves more investigation and explanation.
What is it's center and what are the invariant points ?
(It wasn't a typo, I thought the property for a point P on QL-Cu1 is the same as for QL-P1 mentionned on page 2 above in your paper, the bisector being the same for all the copples of conjugate points)
Best regards
Bernard

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Message: #1419
Date: 11/1/2016 11:11:59
From: bernard.keizer
Subject: Four Conics Tangent to QL-Cu1

Dear Eckart,
Interesting construction, indeed !
For 2 CSC partners X and X', the tangential X1 is the CSC-partner of the 3rd intersection : the 2 lines XX' and the perpendicular in X0 form the degenerated polar conic of X1 wrt QL-Cu2 and intersect this curve in points where the tangents pass through X1.
X, X' and X1 are on a circle through QL-P1 (CSC of the line XX')
The line of the 3 tangentials of the opposite vertices form whith the orthic triangle of DT another QL inscribed in QL-Cu1.
As 2 CSC partners are isogonal conjugates wrt the 4 reference triangles and more generally wrt any triangle inscribed in QL-Cu1 and in a circle through QL-P1, the isogonal conjugate of the circumscribed conics are the 4th lines.
The last point of your paper is intriguing ; I don't find an immediate interpretation ! Do you have one ?
Best regards
Bernard

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Message: #1420
Date: 12/1/2016 2:30:50
From: eckart_schmidt@t-online.de
Subject: Four conics Tangent to QL-Cu1

Dear Bernard,
thanks for your interpretation. In addition, perhaps already mentioned, but not in EQF:
The 3 tangentials of opposite QL-points wrt QL-Cu1 are the 3 QG-points QG-P18, which are the CSC-images for the vertices of the orthic triangle of QL-Tr1.
Wrt the last point: The CSC-images of the 4th intersections of circumcircle and circumconic for the QL-triangle components are the contact points of the inscribed parabola and the QL-lines.
Best regards Eckart

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Message: #1421
Date: 12/1/2016 4:00:15
From: bernard.keizer
Subject: Four conics Tangent to QL-Cu1

Dear Eckart,
I didn't find your mentionned reference, could you please complete the title and the year of publication.
For each reference triangle, the circumconic is tritangent to QL-Cu1 in each vertice (perhaps already in your paper ?).
With the last point, in complement to your remark : these points are the contact points between the cardioïd QL-Qu1 and the circumcircles (CSC of the contact points of the 4 lines with the inscribed parabola).
Each 4th point forms with the corresponding reference triangle a QA.
These 4 QA's are similar to the QA of the cicumcenters of the reference triangles.
Best regards
Bernard

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Message: #1422
Date: 13/1/2016 9:24:48
From: eckart_schmidt@t-online.de
Subject: four Conics Tangent to QL-Cu1

Dear Bernard,
wrt the reference:
... in EQF: 15b
... on my homepage: Themen 2005, 05-6 Geometrie auf der Zirkularkurve
I just study my paper oncemore: The "Zirkularkurve" is QA-Cu1 and #1377 shows, that QL-Cu1 can be considered as QA-Cu1. So the properties in my paper can be interpreted as properties of QL-Cu1. Under 9(f) you find the discussed conics. But there are several more properties, three simple examples:
6(a) Three collinear points on QL-Cu1 have collinear tangentials.
6(b) Pairs of collinear triples P1, Q1, R1 and P2, Q2, R2 on QL-Cu1 have collinear 3rd intersections of P1P2, Q1Q2, R1R2 with QL-Cu1.
7(c) For 4 concyclic points Pi on QL-Cu1 the 3rd intersections of PiPj and PkPl give a parallel to QL-L1 (perhaps of interest, if one point is QL-P1).
I shall try to gather these properties in a new paper.
Best regards Eckart

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1423
Date: 13/1/2016 3:45:56
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 and QL-Cu1

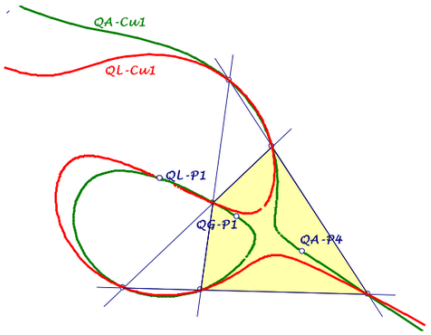
Dear Bernard, dear Chris,
attached a short remark wrt quadrilaterals and quadrangles, whose cubics QL-Cu1 and QA-Cu1 coincide.
Best regards Eckart

EQF-Note 2016-01-13

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Cu1 and QL-Cu1

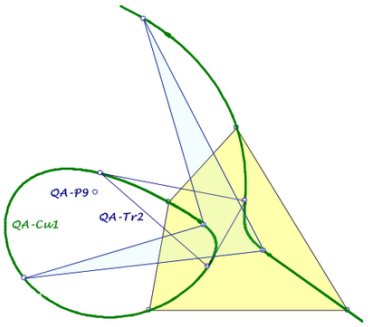
This paper looks for further quadrangles with the same cubic QA-Cu1 and for further quadrilaterals with the same cubic QL-Cu1 and the case, that the cubics coincide.



A quadrigon *QG* can be considered as quadrangle *QA* and as quadrilateral *QL*. The *QA*-cubic *QA-Cu1* and the *QL*-cubic *QL-Cu1* are *QG*-circumcubics, containing the Miquel point *QL-P1* and the intersections of opposite *QG*-sides. The cubic *QA-Cu1* is always bipartite and holds further the *QG*-diagonal crosspoint *QG-P1* and the point *QA-P4*. *QA-Cu1* is isogonal invariant wrt the Miquel triangle *QA-Tr2*. *QL-Cu1* is isogonal invariant wrt the four *QL*-triangle components and invariant wrt *QL-Tf1 = CSC*.

Quadrangles on QA-Cu1

We are looking for further quadrangles on *QA-Cu1* with the same cubic *QA-Cu1*.

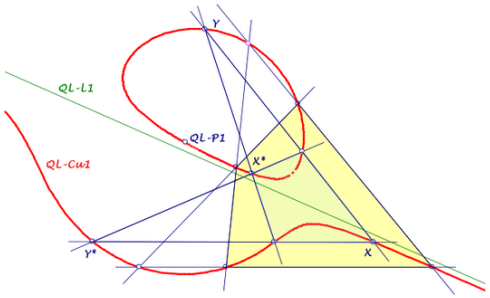


2016-01-13.pdf

QA-Cu1 is invariant wrt *CSC*-analog transformations of the Miquel triangle *QA-Tr2,* which are centered in a vertex and swapping the two other vertices. A point on *QA-Cu1* and its images wrt these three transformations gives a quadrangle with the same cubic *QA-Cu1*. All these quadrangles have the same point *QA-P9*.

Quadrilaterals on QL-Cu1

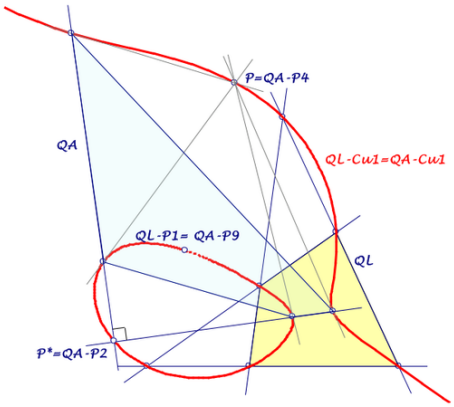
Now we are looking for further quadrilaterals on *QL-Cu1* with the same cubic *QL-Cu1*.



QL-Cu1 is *CSC*-invariant: $P \rightarrow P^*$. Let a line intersect *QL-Cu1* in the points *X* and *Y*, then the quadrilateral *XY, XY*, X*Y, X*Y** has the same cubic *QL-Cu1*. These quadrilaterals have the same Miquel point *QL-P1* and the same Newton line *QL-L1*.

QL-Cu1 as QA-Cu1

Let *QL* be a quadrilateral and *QL-Cu1* its cubic. We look for a quadrangles *QA* on *QL-Cu1*, whose cubic *QA-Cu1* is *QL-Cu1*. This is already described in *QFG*-message 1377. For a point *P* on *QL-Cu1* the contact points of tangents to *QL-Cu1* give a *QA*, whose *QA-Cu1* is *QL-Cu1* of *QL*.



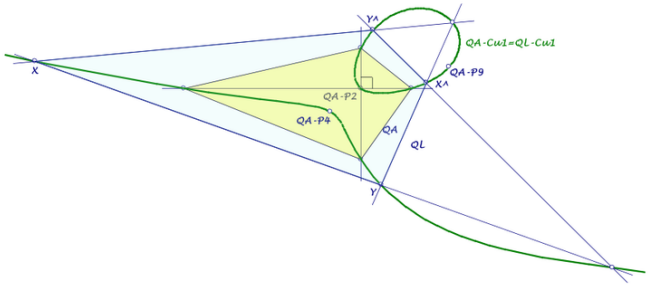
These *QA* are only real, if *QL-Cu1* is bipartite. They are special quadrangles, for the vertices lie on two orthogonal lines, which are the angle bisectors at *P* = QA-P2* wrt opposite points of *QL*. *QA-P4* is the reference point *P* and *QA-P9* is the Miquel point *QL-P1*.

QA-Cu1 as QL-Cu1

Let *QA* be a quadrangle and *QA-Cu1* its cubic. We look for quadrilaterals *QL* on *QA-Cu1*, whose cubic *QL-Cu1* is *QA-Cu1*.

It seems, that this is only possible for quadrangles *QA*, whose vertices lie in pairs on orthogonal lines.

In this case *QA-P2* and *QA-P9* are points of *QA-Cu1*: *QA-P2* is the intersection of the orthogonal lines and *QA-P9* is the same for all *QA* on *QA-Cu1* with this cubic. So we can take an arbitrary *QA* on *QA-Cu1* with this cubic and consider a *CSC*-analog transformation ^, centered in *QA-P9* swapping *QA-P2* and *QA-P4* (see above). Taking a line, which intersects *QA-Cu1* in *X* and *Y*, the lines *XY*, *XY^*, *X^Y*, *X^Y^* give a *QL* with *QL-Cu1* = *QA-Cu1*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #1424
Date: 15/1/2016 11:32:59
From: chris.vantienhoven
Subject: Coaxial system related to QL-Quasi Isogonal Cubic

Dear Eckart and Bernard,
Maybe of interest for you:
http://www.artofproblemsolving.com/community/c6t48f6h1185329_coa⌋
xial_system_related_to_qlquasi_isogonal_cubic
Best regards,
Chris

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Message: #1425
Date: 16/1/2016 11:17:48
From: eckart_schmidt@t-online.de
Subject: Reference Triangle for QL-Cu1

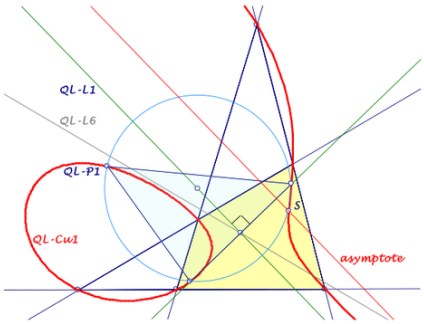
Dear Bernard, dear Chris,
seeing QL-Cu1 in the bipartite case as QA-Cu1, I have translated
QA-Cu1-properties in
http://eckartschmidt.de/Zirkul..pdf
for QL-Cu1. Some of the properties are already mentioned in
earlier messages.
Best regards Eckart

EQF-Note 2016-01-16

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Reference Triangle for QL-Cu1

If the cubic QL-Cu1 for a quadrilateral is bipartite, the cubic can be interpreted as QA-Cu1 for special quadrangles (see QFG-messages 1377, 1423). QA-Cu1 is a pivotal isogonal circular cubic wrt the Miquel triangle QA-Tr2. A corresponding reference triangle for bipartite cubics QL-Cu1 will be researched.



The Reference Triangle

For a quadrilateral *QL* with bipartite cubic *QL-Cu1* we consider a triangle *QL-Trx* with vertices …

> **…in the Miquel point *QL-P1* and the C*SC*-partners on a perpendicular line to the Newton line *QL-L1* in the intersection with *QL-L6*.**

The *CSC*-partners on the mentioned line are the intersections with *QL-Cu1*. If *QL-Cu1* is unipartite, there are no such intersections (but see the final remark).

(1) First properties of *QL-Trx* (see *QFG*-message 1379):
… The circumcircle of *QL-Trx* contains the intersection *S* of *QL-Cu1* and its asymptote diametral *QL-P1*.
… The angle bisectors at *QL-P1* are the Steiner axes.
… The in- and excenters are the intersections of the Steiner axes and *QL-Cu1* (without *QL-P1*).

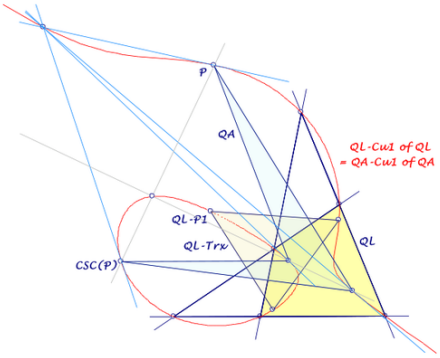
(2) *QL-Trx* as reference triangle for *QL-Cu1*:
… *QL-Cu1* is isogonal invariant wrt *QL-Trx*.

… Isogonal conjugates on *QL-Cu1* lie on parallels to *QL-L1*.
… *QL-Cu1* is a pivotal isogonal cubic with reference triangle *QL-Trx* and pivot in the point at infinity of *QL-L1*.

For the reference triangle *QL-Trx* the cubic *QL-Cu1* is not only isogonal invariant but also invariant wrt the *CSC*-analog transformations, centered in a vertex of the triangle and swapping the other two vertices. If the center is *QL-P1* this is the *CSC*-transformation of the quadrilateral (see *QL-Tf1*).

(3) *QL-Cu1* is invariant wrt the *CSC*-analog transformations for *QL-Trx*.

(4) Points on *QL-Cu1* and their *CSC*-analog images wrt *QL-Trx* have the same tangential on *QL-Cu1*.



(5) A point on *QL-Cu1* and its *CSC*-analog images wrt *QL-Trx* give a quadrangle, whose Miquel triangle *QA-Tr2* is *QL-Trx* and whose cubic *QA-Cu1* is *QL-Cu1*.

These quadrangles on *QL-Cu1* have two pairs of *CSC*-partners as vertices, whose lines intersect on *QL-Cu1* and are the angle bisectors wrt opposite points of the quadrilateral.

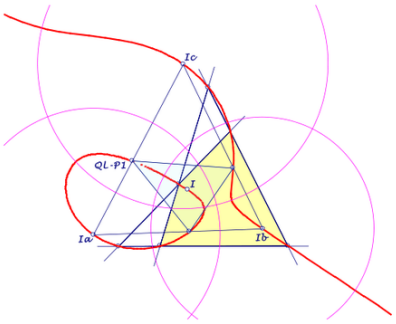
Properties of QA-Cu1 for QL-Cu1

There are a lot of properties for *QA-Cu1* in [1], which now can be translated for bipartite *QL-Cu1*. We shall use the reference triangle *QL-Trx* and the nominations of [1]:

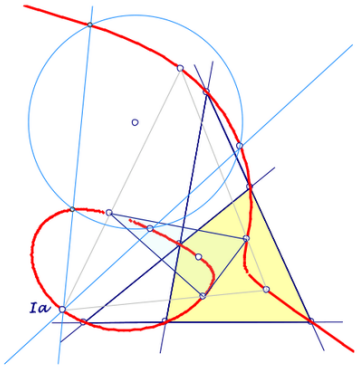
… **Residual point** *res(P,Q)* of two points on *QL-Cu1* is the 3rd intersection of *PQ* and *QL-Cu1*.

… **Corresponding points** on *QL-Cu1* are points with the same tangential.

(6) *QL-Cu1* is an anallagmatic cubic, centers of inversion are the excenters of *QL-Trx*.

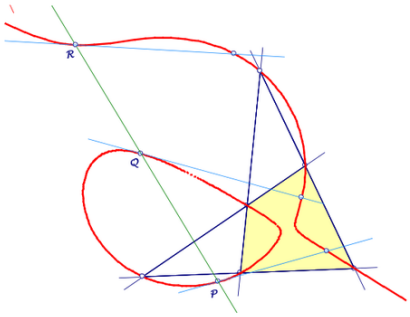


(7) Two lines through an in- or excenter of *QL-Trx* intersect *QL-Cu1* concyclic.

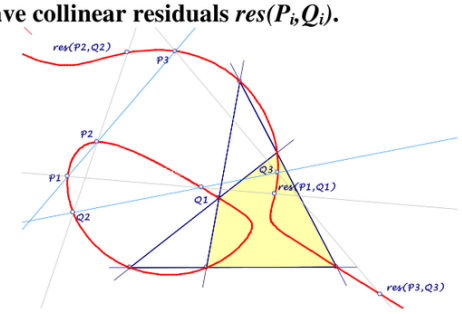


(8) The *QL-Cu1*-tangents in the vertices of *QL-Trx* intersect with the asymptote in *S* on *QL-Cu1*.

(9) Three collinear points on *QL-Cu1* have collinear tangentials.



(10) Two collinear Tripels P_1, P_2, P_3 and Q_1, Q_2, Q_3 on *QL-Cu1* have collinear residuals $res(P_i,Q_i)$.

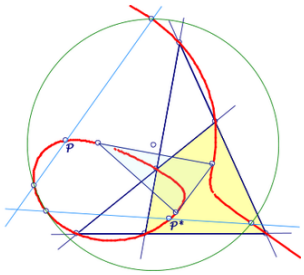


(11) For a point *P* on *QL-Cu1* the corresponding points and the residuals of *P* wrt three collinear points lie on a conic.

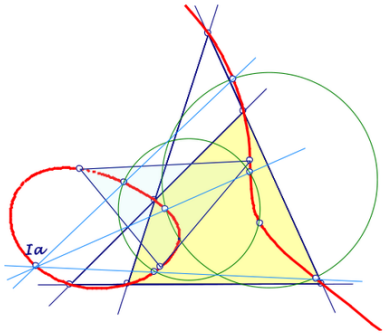
(12) The corresponding quadruples of three collinear points on *QL-Cu1* are perspective in pairs wrt points of the remaining quadruple.

(13) If two of four concyclic points on *QL-Cu1* are isogonal conjugates, the other two are collinear with the point *S*.

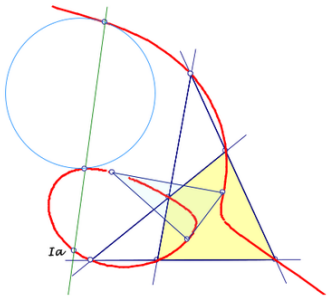
(14) Lines through isogonal points on *QL-Cu1* intersect *QL-Cu1* concyclic.



(15) The residuals of an in- or excenter of *QL-Trx* wrt four concyclic points are also concyclic.

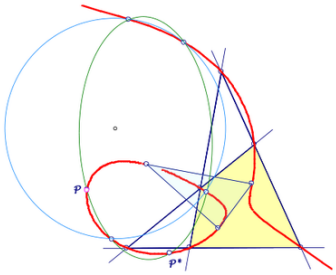


(16) The intersections of a line through an in- or excenter of *QL-Trx* and *QL-Cu1* are contact points of a circle and *QL-Cu1*.

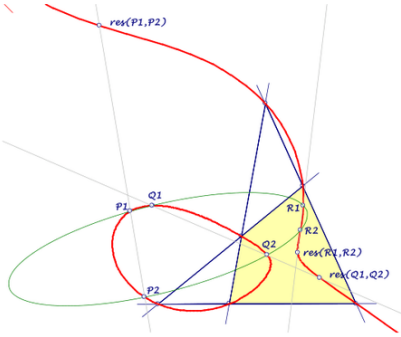


(17) A circumconic of four corresponding points on *QL-Cu1* and the common tangential contacts *QL-Cu1* in this point.

(18) A conic through two isogonal points on *QL-Cu1* has concyclic further intersections with *QL-Cu1*.

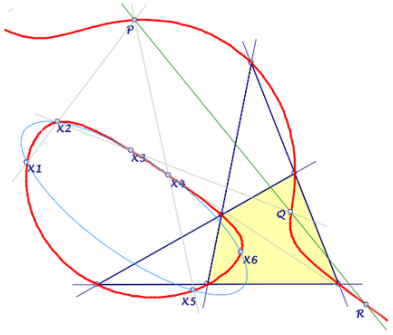


(19) The residuals for three pairs of coconic points on *QL-Cu1* are collinear.



(20) For a point X_1 and three collinear points *P*, *Q*, *R* on *CL-Cu1* the following points are coconic on *QL-Cu1*:

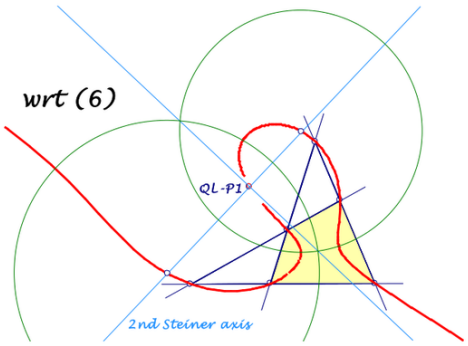
$X_2 = res(X_1,P)$, $X_3 = res(X_2,Q)$, $X_4 = res(X_3,R)$,
$X_5 = res(X_4,P)$, $X_6 = res(X_5,Q)$, $X_1 = res(X_6,R)$.

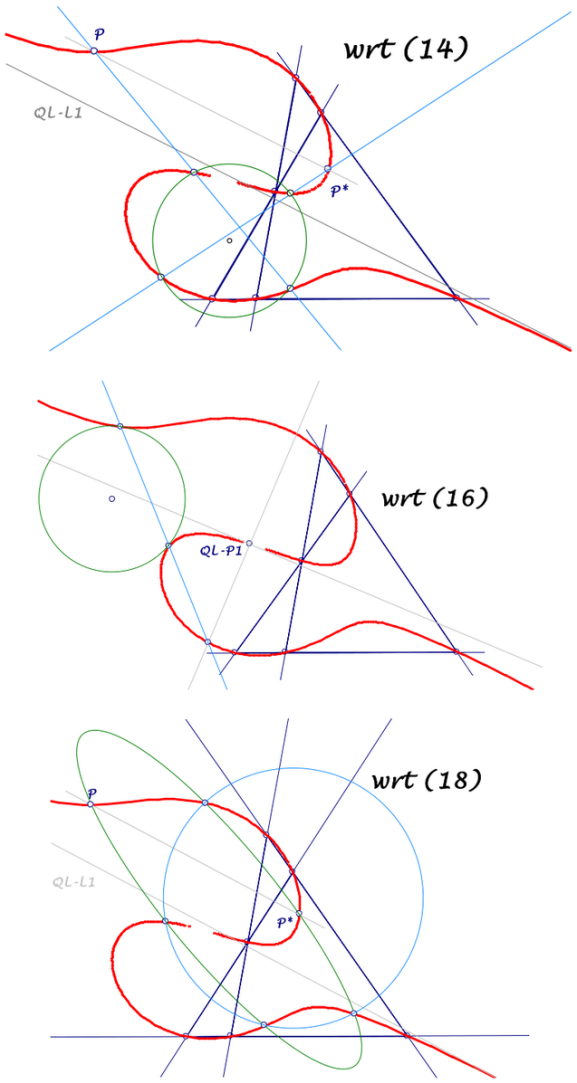


Final Remark

In the unipartite case of *QL-Cu1* there is no reference triangle *QL-Trx*, only the vertex *QL-P1* remains. A point on *QL-Cu1* has only one corresponding partner, which is its *CSC*-partner. Wrt the in- and excenters of *QL-Trx* in the intersections of the Steiner axes and *QL-Cu1* remain only the intersections of the 2nd Steiner axis. The isogonal conjugation wrt *QL-Trx* can be saved as 2nd intersection of a *QL-L1*-parallel and *QL-Cu1*. Wrt this background most of the properties can be hold for the unipartite case of *QL-Cu1* translating the properties in the following way:
… instead of "in- or excenters of *QL-Trx*": **intersections of the 2nd Steiner axis and *QL-Cu1***,
… instead of "isogonal conjugate wrt *QL-Trx* of *P* on *QL-Cu1*": **2nd intersection of *QL-Cu1* and a *QL-L1*-parallel through *P* .**

Examples:





Reference

[1] http://eckartschmidt.de/Zirkul..pdf

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-01-16.pdf

Message: #1426
Date: 16/1/2016 12:03:55
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal Cub ...

```
Dear Chris,

the midpoint of U, V as well as the midpoint of U, CSC(U)=U* lie
on QL-L1, so U*V // QL-L1,
and U*, V are isogonal partners wrt QL-Trx on QL-Cu1 (see
#1425).

Best regards Eckart
```

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Message: #1427
Date: 16/1/2016 12:17:08
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal Cub ...

```
Dear Chris,
forget my last message, it isn't correct!
Excuse Eckart
```

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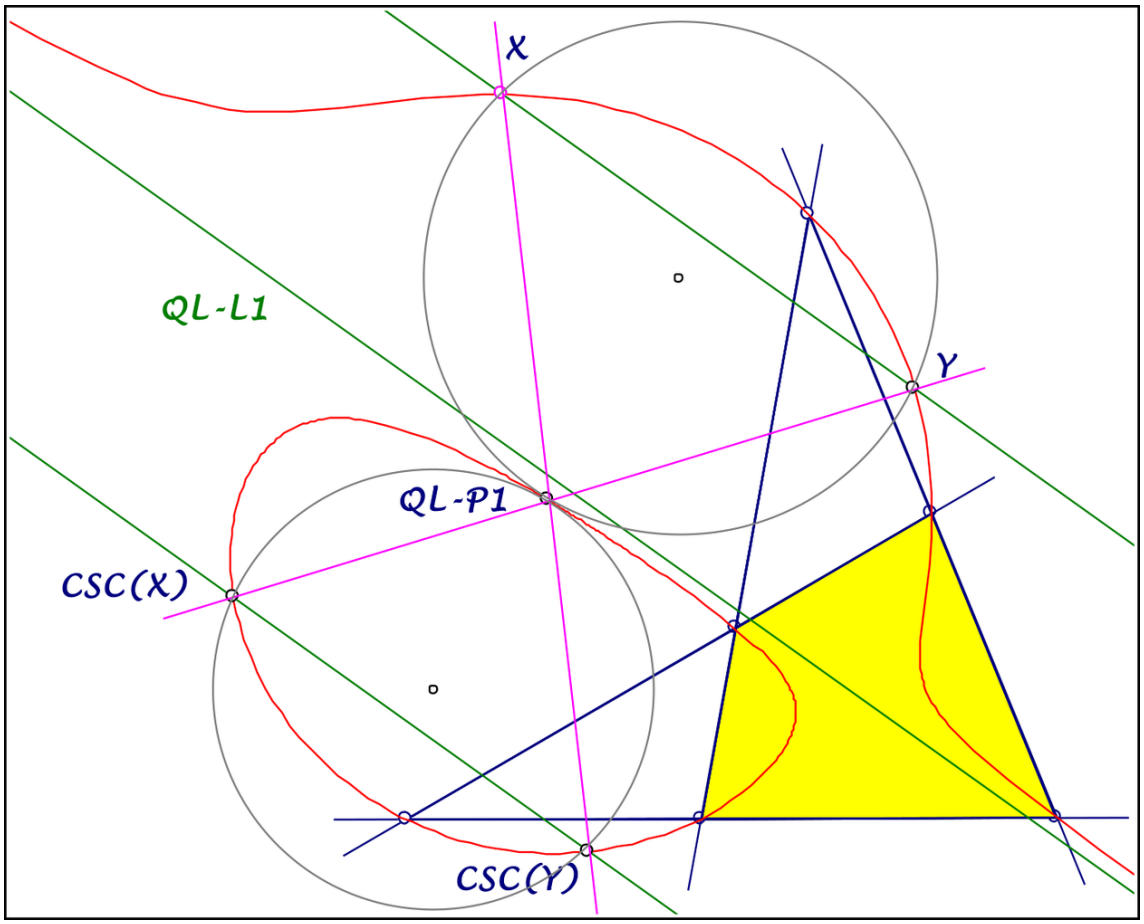
Message: #1428
Date: 17/1/2016 9:47:35
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal ...

Dear Chris, dear Bernard,
background for the property can be the construction method of
QL-Cu1 in message 1273:
Let L and L' be äquidistant parallels of QL-L1 ...
... and X and Y the intersections of L
    and CSC(L') on QL-Cu1, ...
... then CSC(X) and CSC(Y) will be the intersections of L'
    and CSC(L) on QL-Cu1 ...
... with X.CSC(Y) ^ Y.CSC(X) = QL-P1.
For X = U the point V is CSC(Y).
Best regards Eckart



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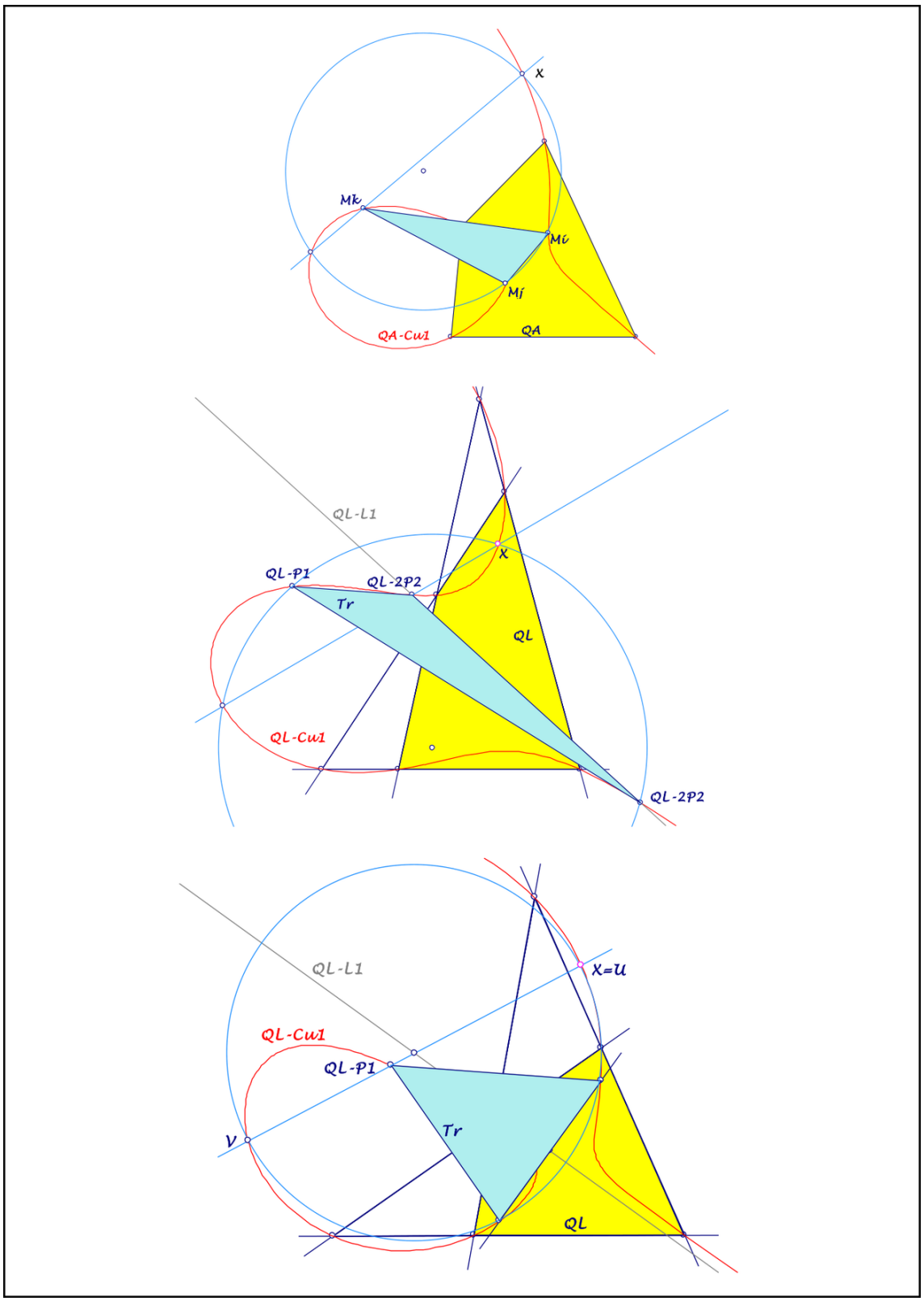
Message: #1429
Date: 18/1/2016 11:07:40
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal ...

Dear Chris, dear Bernard,

for the described QL-property ...
A line through QL-P1 intersects QL-Cu1 with midpoint on QL-L1.
... there is another background,
    unexpected in the following QA-property (see attached file):
Let X be a point on QA-Cu1
... and Ci the circumcircle of X and two Miquel points Mi, Mj,
... then the 2nd intersection of Ci and X.Mk
    is a point on QA-Cu1.
This QA-property can be translated in a QL-property (see
attached file), using the triangle Tr with the vertices
... QL-P1 and QL-2P2, if QL-Cu1 is unipartite,
... or QL-P1 and the CSC-partners on a perpendicular line
    of QL-L1 in the intersection with QL-L6, if QL-Cu1
    bipartite.
Let X be a point on QL-Cu1
... and Ci the circumcircle of X and two Tr-vertices,
... then the 2nd intersection of Ci and the line through X
    and the 3rd Tr-vertex is a point on QL-Cu1.
Taking QL-P1 as 3rd Tr-vertex (see attached file),
there remains the question: Why is the center of Ci on X.QL-P1?

Best regards Eckart



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Message: #1430
Date: 18/1/2016 2:18:38
From: Benedetto Scimemi
Subject: QL-P26

Dear friends, I did not find in EQF the following statement, which I could prove both graphically and analytically:
In a quadrilateral, the three QL-versions of QA-P2 (Euler) and QA-P4 (isogonic) produce inversely similar triangles. The fixed point of this similarity is QL-P26.
Best regards Benedetto

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Message: #1431
Date: 18/1/2016 11:22:07
From: chris.vantienhoven
Subject: QL-P26

Dear Benedetto,
Beautiful result. QL-P26 always has been one of my favorites.
I drew the figures and I can see QL-P26 as the center of similarity. However I don't know how to construct the center of similarity of two inversely similar triangles.
Can you tell me?
Best regards,
Chris

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Message: #1432
Date: 19/1/2016 11:54:05
From: bernard.keizer
Subject: Reference Triangle for QL-Cu1

Dear Eckart,
Many interesting properties, most of them already wellknown, but seldom gathered at the same place !
1) For a point P on bipartite QL-Cu1, the polar conic of P wrt QL-Cu2 is degenerated in 2 perpendicular lines intersecting in P* (id est these 2 lines cut QL-Cu2 in the contact points of the tangents from P to this cubic).
The 4 tangentials of P are on these 2 lines and form 2 pairs of CSC partners ; they are the contact points of the tangents from P to QL-Cu1. The conic through these 4 points and P, tangent in P to QL-Cu1 is the polar conic of P wrt QL-Cu1 and the poloconic of the 1rst degenerated conic.
2) For example, if P is in S, P* is in S*, the foot of the altitude from QL-P1 in the triangle Tr and the polar conic of S is formed by the side opposie to QL-P1 and the corresponding altitude through QL-P1. The contact points of the 4 tangents from S to QL-P1 are the 3 vertices of the triangle Tr and the infinity point of the Newton Line.
3) For unipartite QL-Cu1, I disagree with your formulation that there is no such a triangle ; in fact, as you put yourself in other messages, the triangle is formed by the 3 points QL-P1 and QL-2P2 (this time intersections of the Newton Line and QL-Cu1).
But the properties remain : QL-Cu1 is self isogonal wrt this triangle (2 CSC partners being isogonal points), QL-Cu1 is invariant in the Cl-S trnsformations ...
The same triangle leads to the hessians of the Mac Cay or the Kjp cubics with the same Cl-S transformation, the Newton Line being either the side opposite to QL-P1 or the perpendicular bisector of this side.
Best regards
Bernard

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Message: #1433
Date: 19/1/2016 2:49:24
From: eckart_schmidt@t-online.de
Subject: Reference Triangle for QL-Cu1

Dear Bernard,
thanks for explinations with polar conics.
You are of course right, that QL-Cu1 in the unipartite case is self-isogonal wrt the triangle of QL-P1 and QL-2P2, I just worked out the attached paper. Wrt the reference triangle in the bipartite case it was my ambition to translate properties of QA-Cu1 for QL-Cu1.
I think, that our discussed property is worth to be mentioned generalised in EQF (see especially your #1401):
QL-Cu1 is self-isogonal wrt each triangle ABC
... with A, B different points on QL-Cu1
... and C the CSC-image of the third intersection of AB and QL-Cu1.

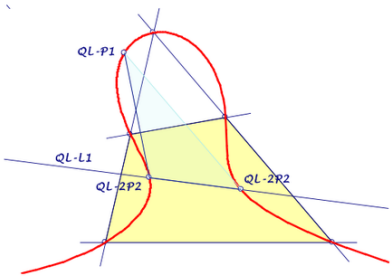
Best regards Eckart

EQF-Note 2016-01-19

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Another Reference Triangle for QL-Cu1

If QL-Cu1 is bipartite, there is already a reference triangle described in QFG-message 1425. Here a corresponding reference triangle for the unipartite case will be researched.



The Reference Triangle

For a quadrilateral *QL* with unipartite cubic *QL-Cu1* we consider a triangle *QL-Try* with vertices …

… in the Miquel point *QL-P1*
and the C*SC*-partners *QL-2P2* on *QL-L1*.

The *CSC*-partners on *QL-L1* are the intersections with *QL-Cu1*.

(1) The angle bisectors of *QL-Try* at *QL-P1* are the Steiner axes.

(2) The circumcircle of *QL-Try* is the *CSC*-image of *QL-L1* contacting *QL-Cu1* in *QL-P1*.

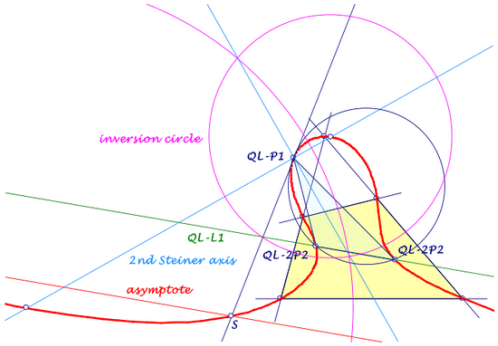
(3) The common tangent in *QL-P1* at the circumcircle of *QL-Try* and *QL-Cu1* contains the intersection *S* of *QL-Cu1* and its asymptote.

(4) *QL-Cu1* is anallagmatic with centers of inversion in the intersections (unequal *QL-P1*) of the 2nd Steiner axis and *QL-Cu1*.

(5) *QL-Cu1* is isogonal invariant wrt *QL-Try*.

(6) The isogonal conjugate of points on *QL-Cu1* is the *CSC*-image.

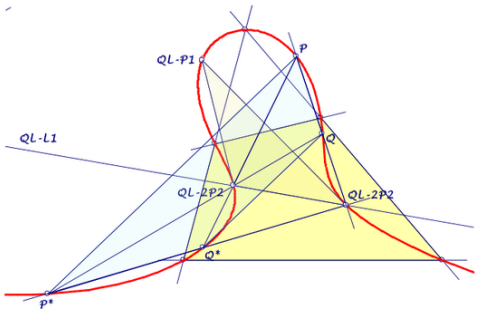
2016-01-19.pdf



(7) *QL-Cu1* is invariant wrt the *CSC*-analog transformations of *QL-Try*.

A point *P* on *QL-Cu1* and its *CSC*-analog images wrt *QL-Try* give a quadrangle on *QL-Cu1* – shortened *P*-quadrangle.

(8) A *P*-quadrangle is a trapezoid of two pairs of *CSC*-partners: *P, P and *Q, Q** with …**
… *PP parallel *QQ** and**
… *PQ* ∩ *P*Q*, PQ ∩ *P*Q* are *QL-2P2*.**



(9) The lines *PQ, P*Q*, PQ*, P*Q* give a quadrilateral with the same *QL-Cu1* as the reference quadrilateral.

(10) *QL-Cu1*-tangents in *P, P and *Q, Q** intersect on *QL-Cu1* collinear with *QL-P1*.**

(11) For the *P*-quadrangle the points *QA-P1, QA-P2, QA-P3* lie on *QL-L1*, *QA-P1* as center of a circle through *QA-P2, QA-P3, QL-P1*.

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2016-01-19.pdf

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Message: #1434
Date: 19/1/2016 4:18:49
From: bernard.keizer
Subject: Coaxial system related to QL-Quasi Isogonal ...

Dear Eckart,
The mentionned property doesn't hold for QL-P1 in the unipartite case !
In this case, the circle through the QL-2P2 and a point X cuts QL-Cu1 in a point Y in line with the tangential of QL-P1 and X and Y are equidistant from QL-P1 (the perpendicular bisector of XY is the line through QL-P1 and the center of the circle through X and the QL-2P2).
The difference is that
1) in the unipartite case, QL-P1 is the tangential of the QL-2P2, whereas
2) in the bipartite case, QL-P1 and the QL-2P2 as well as the infinity point of the Newton Line have the same tangential
In both cases, the tangential of QL-P1 and of the infinity point of the Newton Line is the point where QL-Cu1 cuts it's asymptote.
Using a simple calculation on the curve (Rechnung auf der Zirkularkurve), I call M the Miquel point, S and S* the QL-2P2, I the infinity point of the Newton Line and M1 the tangential of M.
3 points X, Y and Z are on a line if X + Y + Z = 0
4 points X, Y, Z and T are on a circle if X + Y + Z + T = I
the conjugate X* is given by X* - X = X - X* = M - I = I - M
the tangential X1 is given by X1 + 2 X = 0
the complement X0 is given by X + X* + X0 = 0 and it's easy to check that the conjugate of X0 is (X0)* = - X - X* + I - M = - 2X = X1
1) in the unipartite case, S + S* + I = 0 and 2M = 2I = - M1
U and V on a circle through S and S* means U + V + S + S* = I
it follows U + V + M1 = 0 and U, V and M1 are on a line
2) in the bipartite case, 2M = 2S = 2S* = 2I = - M1
U and V on a circle through S and S* means U + V + S + S* = I
it follows this time U + V + M = 0 and U, V and M are on a line
3) in both cases U and V on a circle through M and S means U + V + M + S = I and U + V + S* = 0 and U, V and S* are on a line.
Best regards
Bernard

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Message: #1435
Date: 19/1/2016 9:15:06
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal ...

```
Dear Bernard,
I think, the discussed property holds (see 2nd figur in #1429)!
The calculation "Rechnung auf der Zirkularkurve" is produced for
pivotal isocubics.
QL-Cu1 in the unipartite case isn't pivotal.
Best regards Eckart
```

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Message: #1436
Date: 20/1/2016 8:59:54
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal ...

```
Dear Bernard,
perhaps you mean with "mentioned property"  in #1434 the last
question in my #1429. This question is indeed only correct in
the bipartite case. In the unipartite case X.QL-P1 is tangent to
the circle Ci. But the QA- and QL-property in #1429 - I think -
are correct.
Best regards Eckart
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Message: #1437
Date: 20/1/2016 10:41:53
From: bernard.keizer
Subject: Coaxial system related to QL-Quasi Isogonal ...

Lieber Eckart,
es tut mir wirklich Leid, aber ...
1) I mean exactly what I wrote : the property that any circle through 2 vertices of the triangle Tr cut QL-Cu1 in 2 points in line with the 3rd vertice holds for the 3 points for the bipartite case and for the 2 points QL-2P2 in the unipartite case, but not for QL-P1 as 3rd vertice in this case !
On your figure in the middle in message 1429, if you draw a circle through the point X and the 2 points QL-2P2, it cuts QL-Cu1 in a 4th point Y, which is certainly not on the line XQL-P1.
In fact, the line XY is the polar of QL-P1 wrt the circle and pass through the tangential point of QL-P1 wrt QL-Cu1, which is the intersection of QL-Cu1 with it's asymptote (homothetic of the Newton Line in the homothety with center QL-P1 and ratio 2)
2) the calculation I made is possible for any cubic and not at all reserved for isopivotal cubics ! The calculation is right and so is it's result.
3) the construction of QL-Cu1 unipartite as the locus of the contact points of tangents from QL-P1 to any circle through the 2 points QL-2P2 is classical and you may find it in Bernard Gibert at K048 or in the site mathcurves at circular focal cubic.
(By the way, this construction works also for QL-Cu1 bipartite by replacing the set of circles through the 2 QL-2P2 by the set of circles centered on the Newton Line and orthogonal to the circles of the first set)
I hope this time these explanations will convince you ...
Best regards
Bernard

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Message: #1438
Date: 20/1/2016 1:41:36
From: eckart_schmidt@t-online.de
Subject: Coaxial system related to QL-Quasi Isogonal ...

Dear Bernard,
only wrt 1) of #1437: You write wrt the unipartite case:
"... if you draw a circle through the point X and the 2 points QL-2P2, it cuts QL-Cu1 in a 4th point Y, which is certainly not on the line XQL-P1."
That is right, but not that, what I wrote in #1429:
"Let X be a point on QL-Cu1
... and Ci the circumcircle of X and two Tr-vertices,
... then the 2nd intersection of Ci and the line through X and the 3rd Tr-vertex is a point on QL-Cu1.
If the 3rd Tr-vertex is QL-P1, the 2nd intersection of the line X.QL-P1 and the circle through X, QL-2P2 is also X, for the line is tangent in X to the circle, and X is a point on QL-Cu1.
Best regards Eckart

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Message: #1439
Date: 20/1/2016 7:22:22
From: bernard.keizer
Subject: Coaxial system related to QL-Quasi Isogonal ...

Dear Eckart,
I understand at last !
In fact, your last question in this message 1429 was a little bit misleading ...
But my mistake, I apologise.
Back to this question.
We have to consider 2 orthogonal sets of circles, having these points QL-2P2 as basic points for one set and Poncelet points for the other.
The radical axes of the circles of the 2 sets are the Newton Lines of the 2 curves QL-Cu1, one unicursal and the other bicursal ; both curves intersect in the 3 vertices of the triangle Tr.
Now we have 2 perfectly symmetric constructions : the contact points of the tangents from QL-P1 to the circles of one set on one side and the endpoints of the diameters through QL-P1 of the circles of the other set on the other side give points of one of the curves ; the other curve comes by inverting the 2 sets and keeping the constructions.

Conversely, any line trough QL-P1 intersect the 2 Newton Lines in 2 points, being the centers of 2 circles, one in each set , giving 2 points on each curve as the endpoints of the diameters. Any circle of one set carries 2 points of each curve, 2 as endpoints of the diameter through QL-P1 and 2 as contact points of the tangents from QL-P1.
For example, the circle with diameter the 2 points QL-2P2 belongs to the 1rst set with these points as basic points : the QL-2P2 are the contact points from the tangents from QL-P1 and belong to the unicursal curve and the line through QL-P1 cuts the bicursal curve in 2 points symmetric wrt the middle of the 2 points QL-2P2.
The 2 circles reduced to a point in each of the QL-2P2 belong to the 2nd set and these 2 points belong to the bicursal curve.
Last, the circumcircle of the triangle Tr trough QL-P1 and QL-2P2 belongs to the 1rst set : the contact point of the tangent is QL-P1 on the unicursal curve and the diameter through QL-P1 gives QL-P1 and it's reflexion in the circumcenter as points of the bicursal curve (this reflexion of QL-P1 in the circumcenter is the point where the bicursal curve meets it's asymptote).
I suppose all this is well known, but I never saw it resumed this way.
I hope this will end any misunderstanding between us.
Best regards
Bernard

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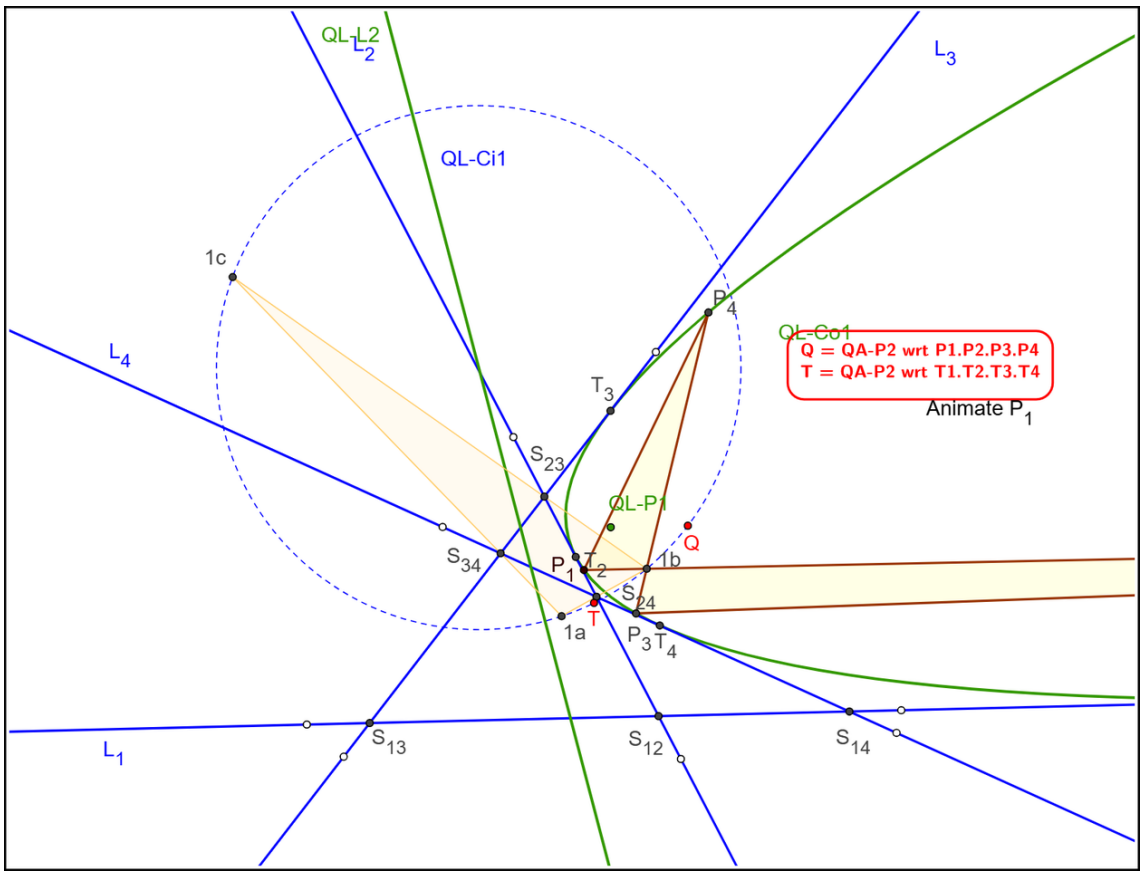
Message: #1440
Date: 20/1/2016 11:22:32
From: Angel Montesdeoca
Subject: QL-Ci1 and QL-Co1

Let L1, L2, L3, L4 be the four lines in a Quadrilateral.
1a, 1b, 1c are vertices od QL-Diagonal-Triangle.
QL-Ci1 is the Circumcircle of QL-Diagonal-Triangle.
QL-Co1 is the unique parabola that can be inscribed within a Quadrilateral.
Let P1 be a point on QL-Co1 and P1P2P3P4 the Quadrangle with 1a1b1c as
QA-Diagonal-Triangle.
Then:
* P1P2P3P4 is inscribed in QL-Co1.
* QA-P2 wrt P1P2P3P4 lies on QL-Ci1.
In particular,
* Let T1, T2, T3, T4 be the points of tangency of QA-Co1 with the basic lines of the Reference Quadrilateral.
1st CT-coordinate of QA-P2 wrt T1T2T3T4:
m n (-a^4 (l-m) (l-n) (-m^4 n^4+l^3 m^2 n^2 (m+n)+2 l m^3 n^3 (m+n)+l^2 m^2 n^2 (m^2-7 m n+n^2)+2 l^4 (m-n)^2 (m^2-3 m n+n^2))-l^2 (m-n)^2 (-3 m n+l (m+n)) (c^4 (l-n) (l m (m-4 n)+2 l^2 n+m^2 n)-2 b^2 c^2 (-5 l^2 m n-m^2 n^2+l^3 (m+n)+2 l m n (m+n))+b^4 (2 l^3 m+4 l m^2 n-m^2 n^2+l^2 (-2 m^2-4 m n+n^2)))+2 a^2 l^2 (m-n) (c^2 (-l+n) (m^3 (2 m-n) n^2+l^2 m n (-5 m^2+16 m n-6 n^2)+l m^2 n (2 m^2-9 m n+3 n^2)+l^3 (m^3-m^2 n-4 m n^2+2 n^3))+b^2 (m^3 (m-2 n) n^3+l m^3 n^2 (-3 m+8 n)+l^4 (2 m^3-4 m^2 n-m n^2+n^3)+l^2 m n (6 m^3-13 m^2 n-4 m n^2+2 n^3)-l^3 m (2 m^3+2 m^2 n-17 m n^2+6 n^3))))

Best regards
Angel M.



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Message: #1441
Date: 21/1/2016 10:53:47
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

```
Dear Angel, dear Eckart,
Very interesting and new property !
The 4 tangents to the parabola in the 4 points Pi form a QL
having the same DT and the same parabola (therefore same Miquel
point QL-P1 and same Steiner Line QL-L2).
If my memory is right, Eckart had studied, some time ago, the
locus of the main points of such QL's ...
Best regards
Bernard
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Message: #1442
Date: 21/1/2016 10:56:58
From: chris.vantienhoven
Subject: QL-Ci1 and QL-Co1

```
Dear Angel,
Interesting property about P1P2P3P4 residing on parabola QL-Co1!
Your point QA-P2 of T1T2T3T4 lies on these lines:
   * line through QL-P16 parallel to the Quasi Ortholine QL-L6,
   * line through QL-P17 parallel to the Newton Line QL-L1,
   * line through QL-Pxx parallel to the Steiner Line QL-L2,
   where QL-Pxx = antipode of QL-P17 on the circumcircle
   of the Diagonal Triangle (QL-Ci1) .
Note that QL-P16, QL-P17, QL-Pxx are all points on the
circumcircle of the Diagonal Triangle (QL-Ci1).
Best regards,
Chris
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Message: #1443
Date: 21/1/2016 11:36:47
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

```
Dear Angel,
that is an interesting method, to get quadrangles in
QL-environment and study the loci of their QA-points. Not only
QA-P2 is on QL-Ci1, but also QA-P30, and on QL-Ci2 we find the
points QA-P29 and QA-P36.
But there are a lot of loci, to be studied:
... lines for QA-P6, 16, 19, 23, 31,
... circles for QA-P2, 4, 28, 29, 35, 36,
... parabolas for QA-P14, 20, 22, 24, 25, 26, 32, 33, 37,
... hyperbolas for QA-P7, 8, 38, 42.
Maybe errors! I shall study it more precisely.
Best regards Eckart
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Message: #1444
Date: 21/1/2016 3:40:49
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

```
Dear Angel,
here are the loci for QA-points of your P1P2P3P4, which are
lines:
... for QA-P6: axis of QL-Co1
... for QA-P16: polar of QL-P8 wrt QL-Co1,
... for QA-P19: parallel polar of QL-P8 and homothetic wrt QL-P8
with factor -2,
... for QA-P23: polar of QL-P10 wrt QL-Co1,
... for QA-P31: parallel polar of QL-P8 and homothetic wrt QL-P8
with factor -1/2.
Best regards Eckart
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Message: #1445
Date: 21/1/2016 8:42:51
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

```
Dear Angel,
here are the loci for QA-points of your P1P2P3P4, which are
circles:
... for QA-P2: QL-Ci1,
... for QA-P4: circle through QL-P10,
    centered in the reflection of QL-P9 in QL-P1,
... for QA-P28: Thales circle about QL-P1.QL-P11,
... for QA-P29: QL-Ci2,
... for QA-P35: center on QL-P9.QL-P10 with ratio 2:3
    and radius 1/5 of the radius of the QL-Tr1-circumcircle,
... for QA-P36: QL-Ci2.
Best regards Eckart
```

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Message: #1446
Date: 22/1/2016 10:59:04
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

Dear Angel, dear Chris, dear Eckart,
QA and QL are as goose and egg, who is first ?
As well known, there are, under certain conditions explained in EQF, 2 parabolas through 4 points and 1 QA and 2 QL's having the same DT.
The QL-points are as well points for the QA and the QA points as well points for the QL's.
Now I have another question.
Given a DT and a parabola inscribed in the medial of DT (and having therefore it's focus on the Euler circle of DT), I consider like Angel the QA's inscribed in the parabola and having the same DT.
I consider then the QL's formed by the tangents to the parabola in these 4 points.
What is the link between all these figures ?
In barycentrics, considering the equation of the parabola leads to something like
l2 - m2(k + 1) + n2k = 0 where k is a constant.
Synthetically, I consider the 4 perspectors between the 4 reference triangles of one QL and DT : they have as coordinates (+-l , +-m , +-n).
The circumcircles of the 4 reference triangles are the cevian circles of these perspectors.
Now, the answer to my question is clear : all the QL's having the same DT and tangent to the same parabola are such as the cevian circles of the 4 perspectors are through the same Miquel point QL-P1.
But in other words, what is the locus of the points such as their cevian circles pass through a given point ?
I tried to draw a figure : it looks like a quartic, passing through 2 vertices of the DT triangle, depending of the position of QL-P1, and trough the centroïd of DT (as QL-P1 is on the Euler circle).
(For example, if QL-P1 is the Kiepert point Ki115 of DT, the locus pass also through the incenter I1 and the orthocenter H4 of DT).
Many thanks in advance for any suggestion, reference or solution to my question.
Best regards
Bernard

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Message: #1447
Date: 22/1/2016 1:53:04
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

Dear Bernard,
if I have followed your construction in the right way, ...
... the four perspectors lie on a circumconic of QA, centered on a circumconic of QL-Tr1 through QL-P8, QL-P13, QL-P24.
There is a further observation:
The cubics QL-Cu1 of the considered QL have common points on QL-Ci2.
Best regards Eckart
PS: Thanks for #1439, no further misunderstandings. I shall study your résumé in the next days.

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Message: #1448
Date: 22/1/2016 2:54:37
From: amontes1949
Subject: QL-Ci1 and QL-Co1

Dear Bernard
I get that: if QL-P1 is the center of hyperbola Kiepert (Ki115) of DT, the locus is the Q066 (http://bernard.gibert.pagesperso-orange.fr/curves/q066.html) = Stammler quartic (together with the Kiepert Hyperbola)
Best regards
Angel M.

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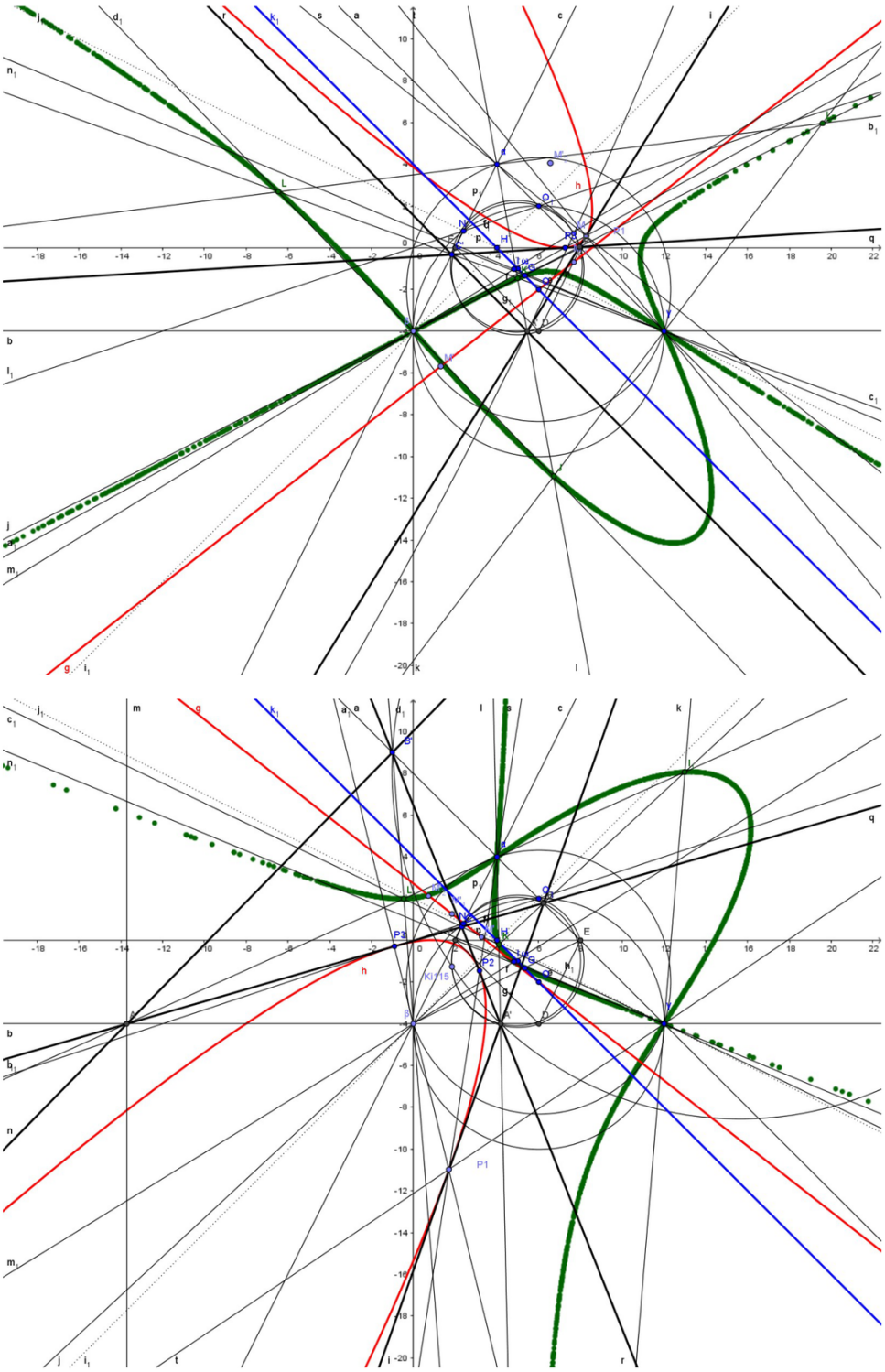
Message: #1449
Date: 22/1/2016 4:56:49
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

Dear Angel, dear Eckart, dear Chris
Thanks a lot for your quick answers !
For Eckart : the construction is following
Once given the DT and the parabola (with focus on the Euler circle of DT) and a point P1 on the parabola, the tangent in P1 to the parabola cuts the 3 sides of DT in A', B and C and the 3 other points A, B' and C' are the harmonic conjugates of A', B and C wrt the corresponding vertices of DT.
The 4 perspectors are I, J, K and L and are themselves by pairs harmonic conjugates wrt corresponding vertices of DT and of the reference triangles of the QL.
For Angel, I'm glad that it confirms what I found and I notice that the Stammler quartic Q066 is also through the excenters of DT.
(In this case, the directrix of the parabola is the Brocard axis of DT)
Are there other well known examples ? (QL-P1 in the Jerabek point Je125 or in other ETC points of DT ...)
I think you've opened a fascinating new field !
For Chris, this quartic (in the general case) is perhaps of interest for EQF ?
Best regards
Bernard

Miquel point QL-P1 any point on the Euler circle of DT (2nd figure QL-P1 in Ki115)



Co1 and DT.docx

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Message: #1450
Date: 23/1/2016 9:03:21
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

Dear Bernard,
I am a bit confused, for I cannot interpret your drawings. So I have tried, to show my results of #1447 (see attached file).
Let Pi give the QA on QL-Co1 in the sence of #1440 and let their tangents Tgi wrt QL-Co1 give the QL. These QLs have beside QL-P8, 9, 10, 11 the same QL-P16 and QL-P24.
The considered perspectors are of the triangles PiPjPk and TgiTgjTgk.
What about the locus of the perspectors with cusps in intersections of QL-Co1 and QL-diagonals?
Best regards Eckart
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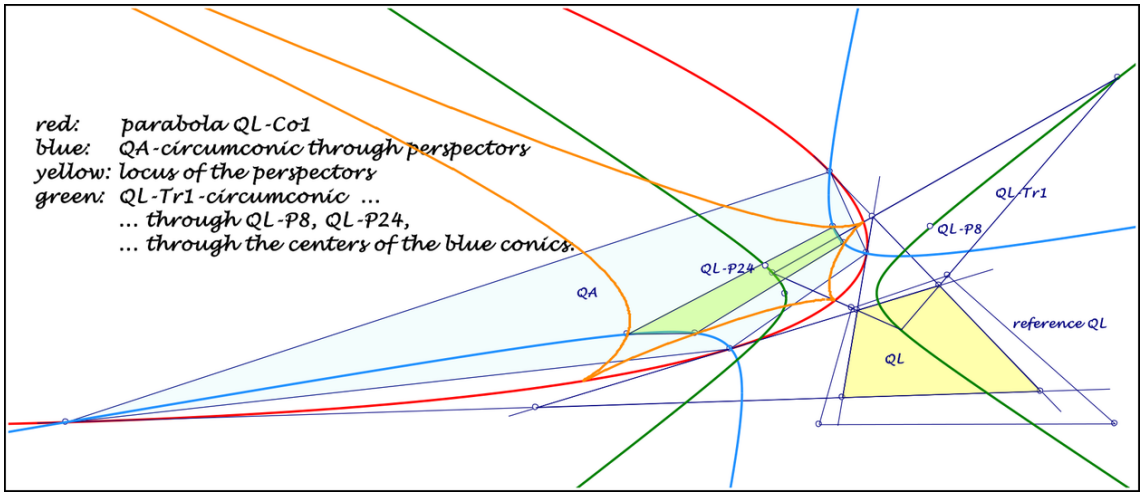

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Message: #1451
Date: 23/1/2016 12:19:16
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

Dear Eckart,
The problem is, I suppose, in the definition of the perspectors.
If we call A and A', B and B' and C and C' the copples of conjugate vertices of a QL and a, b and c the vertices of DT (a is the intersection of BB' and CC' ...), the perspector I is the intersection of aA, bB and cC and so on.
L is the harmonic conjugate of I wrt a and A, K the harmonic conjugate of I wrt b and B and J the harmonic conjugate of I wrt c and C.
J and K are harmonic conjugates wrt a and A' ...
These perspectors have DT-coordinates +-l, +-m, +-n ; they are the duals of the 4 lines.
The 4 refrence triangles of a QL are the cevian triangles of these 4 perspectors.
Your perspectors are other ones, which deserve another study ...
There is in fact a circumconic through the 4 contact points Pi and the 4 perspectors, but not where you put it !
See the message of Angel about Q066 ...
Best regards
Bernard

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Message: #1452
Date: 23/1/2016 1:38:10
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

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Dear Bernard, dear Angel,

here are loci of QL-points for quadrilaterals QL in the sence of
#1440:
for QL-P1, 8, 9, 10, 11, 16, 24: fixed points, equal those of
the reference QL,
for QL-P2, 7:   line QL-L2,
for QL-P3:      line through QL-P3, orthogonal wrt QL-L7,
for QL-P4:      line through QL-P4, orthogonal QL-P9.QL-P16,
for QL-P5:      line through QL-P5, parallel to a QL-Co1-tangent
                in the intersection
                of QL-Co1 and a QL-L1-parallel in QL-P9,
for QL-P6:      line through QL-P6, orthogonal QL-P1.QL-P8,
for QL-P12:     polar of QL-P8 wrt QL-Co1,
for QL-P14, 15, 18: line through QL-P14, 15, 18,
                parallel to the polar of QL-P8 wrt QL-Co1,
for QL-P19:     line QL-L3,
for QL-P20, 21: line through QL-P20, 21,
                parallel to a QL-Co1-tangent in the intersection
                of QL-Co1 and a QL-L1-parallel in QL-P10,
for QL-P22:     line through QL-P22,
                parallel to a QL-Co1-tangent
                in the intersection of QL-Co1 and a
                QL-L1-parallel in QL-P11,
for QL-P17:     circle QL-Ci1,
for QL-P25:     QL-Ci2,
for QL-P13:     circumconic of QL-Tr1 through QL-P8, 13, 24,
for QL-P23:     circumparabola of QL-Tr1 through QL-P23
                with axis parallel QL-L1,
                intersecting QL-Co1 on the polar of QL-P8,
for QL-P27:     circumhyperbola of QL-Tr1
                through QL-P27 and a point,
                which is the image of QL-P17 wrt a
                QL-Tr1-isoconjugation with fixed points
                in the QL-Tr1-tripols of the QL-lines.
Up to QL-P27 only the locus of QL-P26 remains ...

Best regards Eckart
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Message: #1453
Date: 23/1/2016 5:37:19
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

Dear Eckart, dear Angel
In fact, the conic is the same !
Each Pi is a perspector between DT and a reference triangle of the QA, there are 4 perspectors between the reference triangles of the QL and DT and 4 perspectors between the corresponding reference triangles of the QA and of the QL.
(If I'm not wrong, 2 triangles in perspective with a 3rd triangle are themselves in perspective).
The 12 points are on the same conic.
I suppose there is a link between the 3 loci of perspectors (parabola for the perspectors Pi (QA/DT), cubic for your perspectors (QA/QL), quadric for my perspectors (QL/DT).
Best regards
Bernard

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Message: #1454
Date: 23/1/2016 10:22:01
From: chris.vantienhoven
Subject: QL-Tr2 NPC Triangle

Dear Benedetto, Bernard and Eckart,

Recently Benedetto mentioned to me that QL-P2 is the orthocenter of the triangle formed by the common points of the 3 QL-Ninepoint Conics QA-Co1.
He also mentioned that the sides of this triangle are tangent to the inscribed parabola.
These were new items to me.
I remember us talking about these three points in the past.
Because of the recursive interest I registered now this triangle in EQF as QL-Tr2.
It would be fine if you could look at the page and tell me about properties not mentioned yet.
Thanks in advance.

Best regards,
Chris

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Message: #1455
Date: 24/1/2016 12:23:32
From: bernard.keizer
Subject: QL-Ci1 and QL-Co1

Dear Eckart, dear Angel
Please forget the beginning of my last message !
It is not the same conic !
The 12 perspectors are 8 by 8 on 3 different conics.
Eckart, I checked your construction and found the same curve as yours, but I have no idea, what it is !
I hope you will investigate a little more ...
Perhaps Angel will have some idea again ?
Best regards
Bernard

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Message: #1456
Date: 24/1/2016 11:54:43
From: eckart_schmidt@t-online.de
Subject: QL-Tr2 NPC Triangle

Dear Chris,
the triangle QL-Tr2 is already detailed described in # 678 and following messages.
There is a typo in the 1st point of the attached EQF-Note
2014-09-06: QL-P2 and QL-P12 are swapped!
Best regards Eckart

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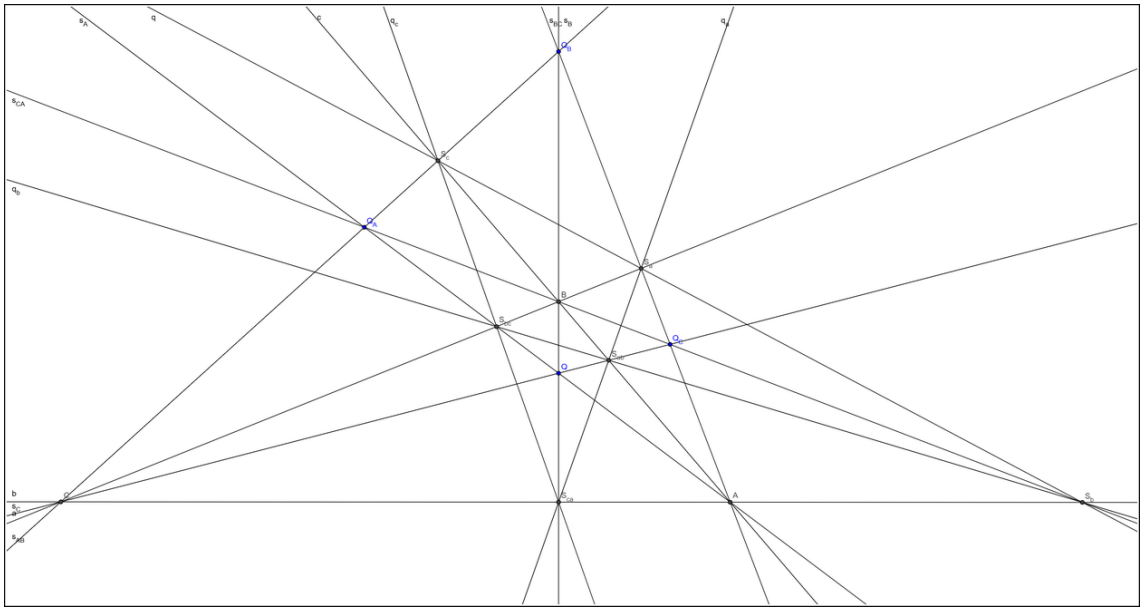
Message: #1457
Date: 24/1/2016 2:42:35
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear all
I have an idea. We should merge quadrangle/quadrilateral figures.
In my page, I show there is a natural one-to-one correspondence between a quadrangle and a quadrilateral.
http://lth.name/geometry/duality.html
All points are in uppercases and all lines are in lowercases.
We can deduce a quadrilateral from a quadrangle and vice versa.
So it is unnecessary to distinguish a quadrangle from a quadrilateral, just like we do not distinguish a triangle from a trilateral. A quadrangle and a quadrilateral are dual figures as a triangle and a trilateral are.
For example, QA-P16 and QL-P13 are the same thing. With DT-coordinates, we get pl=qm=rn=1 and treat quadrangle/quadrilateral figures in the same way. Please refer to the attachment GeoGebra file for more information.
P.S. I coined "diapleural' as a dual of "diagonal".
To be continued...
Best regards,
Tsihong Lau



autopolarity_between_quadrangle_and_quadrilateral.ggb

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Message: #1458
Date: 24/1/2016 7:22:38
From: bernard.keizer
Subject: QL-Tr2 NPC Triangle

Dear Chris, Benedetto and Eckart,
1) I'm very glad that this triangle of the S-points gets at last the place it deserves in EQF
2) I'm rather surprised that you mention these items as new,
a) as these points were already mentionned in EQF at 2 or 3 different places (under QL-Ci6 and under QL-P6 ...)
b) as I mentionned several conics through these points in message 457
c) as Eckart mentionned these properties in message 678
3) Important is following : there are 6 more conics through these 3 S-points
4 are through the vertices of each reference triangle, it's centroïd, it's perspector with DT and the point I called the D-point, 2nd intersection of the line trough QL-P1 and the perspector with the circumcircle (the D-points are on the Dimidium circle)
These 4 conics are the isogonal conjugates of lines through the Lemoine point of each triangle and the isogonal conjugate of it's perspector ; the 4 lines concur in QL-P26, the Lemoine point of the QL.
the 5th is exactly the same description for DT with vertices, centroïd, perspector QL-P13 and D-point QL-P24, which is also the 2nd intersection of QL-P1QL-P13 with the circumcircle of DT and also lies on the Dimidium circle.
The line trough the Lemoine point of DT, the isogonal conjugate of QL-P13 wrt DT also concur in QL-P26
the 6th is through the vertices of DDT (diagonal triangle of DQL) and the 5 perspectors (4 for the reference triangles and QL-P13)
It means that the isogonal conjugates of these 9 circumconics wrt QL-Tr2 are interesting lines ...
Best regards
Bernard
PS It's perhaps also interesting to have a look at the contact points of the 3 sides of QL-Tr2 with the parabola

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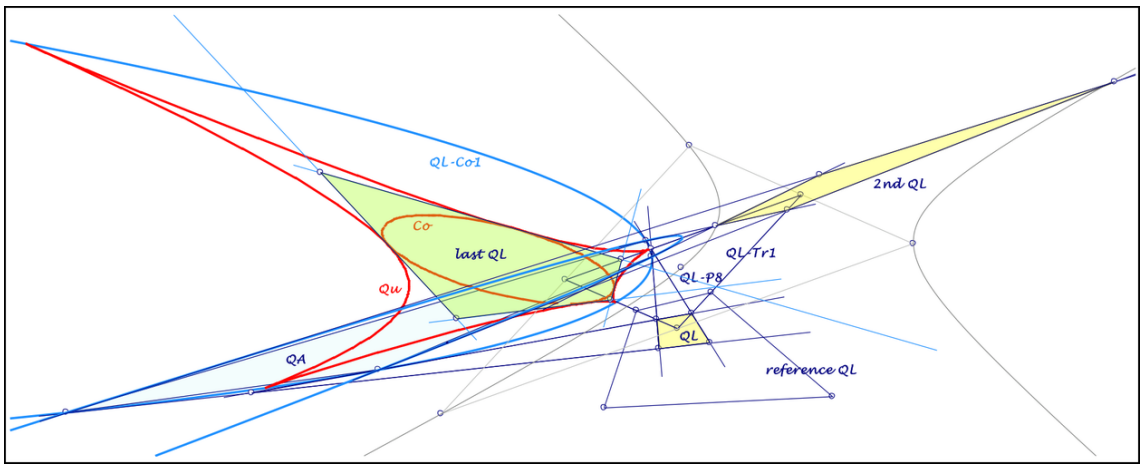
Message: #1459
Date: 25/1/2016 9:18:17
From: eckart_schmidt@t-online.de
Subject: QL-Ci1 and QL-Co1

Dear Bernard,
thanks for clarity wrt the perspectors. Yes, I consider other perspectors:
Let Pi give the QA on QL-Co1 in the sence of #1440, and their tangents give the corresponding QL.
L et Sij be the intersections of the tangents in Pi, Pj wrt QL-Co1, then my perspectors are those of Pi, Pj, Pk and Sjk, Ski, Sij.
In addition to #1447, #1450:
... The perspectors lie on a quartic Qu with four cusps
    in intersections of QL-diagonals and QL-Co1.
... QA has beside QL-Co1 another circumparabola,
... which leads to a second corresponding QL,
... with the same diagonal triangle
... and with four new perspectors,
... whose locus is a conic Co,
... tangent to the quartic Qu.
... Common tangents of Co and Qu are the polars
    wrt QL-Co1 of QL-P8 ant its QL-Tr1-anticevians.
... Co is the envelope of QL-Co1-polars for points on a conic
    through QL-P8 and its QL-Tr1-anticevians,
    tangent in QL-P8 to a QL-Tr1-circumconic
    through QL-P8, QL-P13, QL-P24.
... Consider the QL of the common tangents of Co and Qu
    and its QL-Tf2:
    This transformation gives for tangents at Co lines,
    which envelope Qu.
It is very difficult to make a good drawing, but I tried it attached.
Best regards Eckart



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Message: #1460
Date: 25/1/2016 11:25:33
From: eckart_schmidt@t-online.de
Subject: QL-Tr2 NPC Triangle

Dear Bernard, dear Chris,
wrt the PS of Bernard´s message 1458 I had a look at the contact points of the 3 sides of QL-Tr2 with the parabola, which shall give the triangle QL-Trx:
... There is a perspector of QL-Tr2 and QL-Trx, point Y in #678. This point Y has a lot of interesting properties:
... Point Y is the QL-Tr2-isotomic conjugate of the point at infinity of QL-L1.
... Point Y lies on the QL-Tr2-Steiner ellipse (circumscribed).
... The tangent in Y at the QL-Tr2-Steiner ellipse is Y.QL-P8. (orthogonal QL-L5).
... The point Y is the contact point of two Steiner ellipses: QL-Tr2 circumscribed and QL-Trx-inscribed.
... Point Y is the reflection of point X in #678 in QL-P12.
... Point Y lies on a QL-Tr2-circumconic through QL-P8, QL-P13, QL-P24.
... Point Y is the Brianchon point of QL-Co1 wrt QL-Tr2.
... The point Y is collinear with QL-P7 and the 4th intersection of QL-Co1 and the QL-Trx-circumcircle.
... The trilinear polar of Y wrt QL-Tr2 is ...
... ... also the trilinear polar of Y wrt QL-Trx,
... ... the polar of Y wrt QL-Co1,
... ... the line for the collinear trilinear poles of the QL-lines wrt QL-Tr2,
... ... which is the image of QL-Co1 wrt an isoconjugation of QL-Trx with fixed point Y.
Best regards Eckart

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Message: #1461
Date: 25/1/2016 11:50:26
From: bernard.keizer
Subject: QL-Tr2 NPC Triangle

Dear Eckart, dear Chris
Waouh !
I'm impressed by this rather complicate, but fascinating figur !
By the way, there is typo in my message 1458.
For the 5th circumconic I mention, the D-point is obviously QL-P24, the 2nd intersection of QL-P17QL-P13 with the circumcenter of DT (same definition as the 4 preceding conics, as QL-P17 is the Miquel point of DQL and QL-P13 the perspector between DT and DDT, the DT of DQL.
Best regards
Bernard

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Message: #1462
Date: 26/1/2016 12:17:17
From: seiichikiri
Subject: Degeneration of QL-Cu2

Dear friends,
I have the following questions:
(1) Are there 3 colinear points among 27 centers of QL-Cu2?
(2) Can QL-Cu2 degenerate to the lines and conics because 27=3*9?
An example of degeneration of a conic: See ADGEOM message # 3015.
Best regards, Seiichi.

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Message: #1463
Date: 26/1/2016 10:40:28
From: bernard.keizer
Subject: Degeneration of QL-Cu2

Dear Seiichi,
The 27 centers of cardioïds inscribed in a QL are on the 4*9 = 36 Morley lines of the 4 reference triangles of the QL ; each line carries 3 points and each point is the intersection of 4 lines (one in each set of Morley lines).
QL-Cu2 is either Mac Cay or Kjp cubic stelloïd with 3 asymptotes at 60° intersecting in QL-P1.
Best regards
Bernard

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Message: #1464
Date: 27/1/2016 7:42:36
From: Seiichi Kirikami
Subject: Degeneration of QL-Cu2

Dear Bernard,
Thank you very much for your kind information!
Best regard, Seiichi
On Tuesday, January 26, 2016 6:40 PM, "bernard.keizer@yahoo.com [Quadri-Figures-Group]" wrote:
Dear Seiichi,
The 27 centers of cardioïds inscribed in a QL are on the 4*9 = 36 Morley lines of the 4 reference triangles of the QL ; each line carries 3 points and each point is the intersection of 4 lines (one in each set of Morley lines).
QL-Cu2 is either Mac Cay or Kjp cubic stelloïd with 3 asymptotes at 60° intersecting in QL-P1.
Best regards
Bernard

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Message: #1465
Date: 27/1/2016 8:51:46
From: eckart_schmidt@t-online.de
Subject: QL-Tr2 NPC Triangle

Dear Bernard,

wrt "... 6 more conics through these 3 S-points ... ": I prefer the 5th - shortened Co5 - common circumconic of QL-Tr1 and QL-Tr2 through QL-P8, QL-P12, QL-P24 and point Y (see #1460), centered in the isotomic conjugate of the infinity point of QL-L1 wrt the QL-medial diagonal triangle.
Co5 is the locus for QL-Co1-poles of tangents at the QL-Tr1 inscribed Steiner ellipse ...
... and:
QL-Co1 is the locus for Co5-poles of tangents at the QL-Tr1 inscribed Steiner ellipse.
Co5 is the locus for QL-Tr1-trilinear poles of QL-L1-parallels.
Co5 is the locus for QL-Tr2-trilinear poles of the line pencil ...
... of the isotomic conjugate of the infinity point of QL-P8.QL-P13 wrt the QL-medial diagonal triangle.
Co5 is the QL-Tr1-isotomic conjugate for the QL-Co1-polar of its center (see above).

Best regards Eckart

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Message: #1466
Date: 27/1/2016 11:15:24
From: bernard.keizer
Subject: QL-Tr2 NPC Triangle

Dear Chris, dear Eckart
For Chris, I notice you've made an enormous work of new properties in EQF with calculation (in particular, I'm impressed by the coordinates of the vertices of QL-Tr2, which need an equation of the 3rd degree)
On the last line on QA-Co1, I suppose it is QL-Tr2 and not QL-Tr1
For Eckart, it is QL-P13 and not QL-P12.
For both of you, I like personally the 5 D-points:
Di 2nd intersection of the line through QL-P1 and the perspector with the circumcircle of the reference triangle or 4th intersection between the circumconic QL-Coi with the same circumcircle.
D5 is QL-P24, 2nd intersection of QL-P17QL-P13 with the circumcircle of DT or 4th intersection between the circumconic QL-Co5 with the circumcircle of DT
These 5 D-points are on the Dimidium circle
And I like this simple definition of the 5 circumconics QL-Coi as the isogonal conjugates wrt the corresponding triangles (reference triangle or QL-Tr1) of the line through the symmedian point of the triangle (isogonal of the centroïd) and the isogonal of the perspector (QL-P13 for QL-Tr1).
And for Chris, who likes particulary the point QL-P26 (symmedian point of the QL), these 5 lines concur in QL-P26.
All the equations of the circumconics and of the lines are very easy ...
Best regards
Bernard

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Message: #1467
Date: 27/1/2016 10:37:55
From: chris.vantienhoven
Subject: QL-Tr2 NPC Triangle

Dear Bernard,
Thanks for your appreciation for the work I did wrt QL-Tr2.
The calculation of the coordinates of the vertices of QL-Tr2 took about 10 minutes, but the simplification of the same vertices took me 2 full days. I am glad you saw through this.
With this information I also proved algebraically that these points lie on QL-Ci6 the Dimidium circle.
And I proved algebraically that QL-P12 is the centroid, QL-P2 the orthocenter and QL-P6 the circumcenter of QL-Tr2.
You remarked before that I indicated some items as “new”, whilst I already mentioned them before in EQF. The reason is that I have a pretty bad memory. I even often forget my own discoveries. Unless I reread them. I think that’s why I register so much. When I remember things, then they are relationships, hardly facts. Great advantage of a bad memory is that you start every day with a blank sheet, a “tabula rasa”. No prejudice, just observing and a problem to solve. Fortunately I remember the name of my wife and our wedding day. Otherwise I would have big trouble.
In EQF I changed your remark in QA-Co1 and added the D-points in QL-Ci6.
Thanks for all your remarks, although I not always find time to react.
Best regards,
Chris

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Message: #1468
Date: 28/1/2016 10:10:24
From: chris.vantienhoven
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Tsihong Lau,

I find your approach fascinating, but I still don’t know if it is applicable.
It is always worthwhile investigating new directions because we never know where it ends.

Let me summarize your system in my own way:
1. Two intersecting circle imply two 2 nd degree functions to be solved, implying 4 intersection points. Normally we have two real or imaginary intersection points. The other two intersection points are the two circular points at infinity. They are all the same for each circle.
2. You denote these two circular points at infinity with coordinates P(p:q:r) and U(u:v:w).
3. Since the circular points lie at infinity, the line at infinity is PU with coefficients (qw-rv : ru-pw : pv-qu).
4. Since the centroid of a triangle is the trilinear pole of the line at infinity, the centroid will have coordinates (1/(qw-rv) : 1/(ru-pw) : 1/(pv-qu)).
5. The midpoint of two points X,Y will be the 4 th harmonic point of X,Y and the intersection point of XY and the line at infinity.
6. For a Reference Triangle the coordinates A(1:0:0), B(0:1:0), C(0:0:1) are used.
7. For a Reference Quadrangle the coordinates A(-1:1:1), B(1:-1:1), C(1:1:-1), D(1:1:1) are used, whilst the coordinates of the QA-Diagonal Triangle will be (1:0:0), (0:1:0), (0:0:1).
8. For a Reference Quadrilateral the line coefficients a(-1:1:1), b(1:-1:1), c(1:1:-1), d(1:1:1) are used, whilst the coordinates of the QL-Diagonal Triangle will be (1:0:0), (0:1:0), (0:0:1).

Questions:
a. Can every point constructible with lines, circles, conics, cubics, etc. be described by circular coordinates? More particularly can all ETC-points and EQF-points principally be calculated with circular points?
b. Is there an example of points lines with easier circular coordinates than barycentric/trilinear coordinates?
c. Is there an example of a problem that could be (easier) solved with circular coordinates and not or with more difficulties with other coordinate systems?
d. Using point D=(1:1:1) in a quadrangle system, there is no variation in location of this point wrt ABC. How do you see this?

Best regards,
Chris

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Message: #1469
Date: 29/1/2016 5:11:28
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear van Tienhoven
---In Quadri-Figures-Group@yahoogroups.com, wrote :
...
Questions:
a. Can every point constructible with lines, circles, conics, cubics, etc. be described by circular coordinates? More particularly can all ETC-points and EQF-points principally be calculated with circular points?
b. Is there an example of points lines with easier circular coordinates than barycentric/trilinear coordinates?
c. Is there an example of a problem that could be (easier) solved with circular coordinates and not or with more difficulties with other coordinate systems?
d. Using point D=(1:1:1) in a quadrangle system, there is no variation in location of this point wrt ABC. How do you see this?
...
It is very strange for you to reply this message with questions about circular coordinate system. The first message of this topic does not mention it. I recommended we should merge quadrangle/quadrilateral figures since we can deduce the other from any one. In other words, we should merge QA-P1 to QA-P43 and QL-P1 to QL-P30. In fact, I had said QA-P16 and QL-P13 are the same thing. The other quadrangle/quadrilateral figures should be merged, too.
Even the first message is not related to a coordinate system. We can understand easily a quadrangle and quadrilateral are dual figures from the constructions. You can merge quadrangle quadrilateral figures just making pl=qm=rn=1 in the DT coordinates, no circular points involved!!!
Since you ask some questions here, so I reply you here, too. But you should not follow this topic with circular coordinates. Instead, you should follow the topic in APG.
a. I do not know. I calculate some and post to my website. http://lth.name/geometry/centers_of_triangles_trilaterals.html I think most except transcendental centers can be expressed in circular coordinates.
b. The name "circular coordinate system" misleads! Be careful! Circular coordinates are not exclusive with barycentric/trilinear coordinates. In fact, we can have trilinear circular coordinates. For example, the three trilinear coordinates of the circumcenter X(3) are sidelength - a(-a^2+b^2+c^2):...

angle - cos A:...
circular - p(q^2+r^2):...
The second column from left in the above table is the trilinear circular coordinates.("Orthocentric" means the trilinear poles incenter/excenters are an orthocentric system.)
For X(3), which is simpler?
a(-a^2+b^2+c^2) or
p(q^2+r^2)?
c. I have said in a message of the topic in APG
5. Very general results. For example,
X(2) 1/(qw-rv):... cevamul-
X(3) 1/qw+1/rv:... crossmul
X(4) 1/(qw+rv):... cevamul
X(5) p^2vw+qru^2:... incidence+
X(2),X(3),X(4),X(5) are collinear, in other words, cevalmul-,cevamul,crossmul,incidence+ of the two points/lines P(p:q:r) and U(u:v:w) are collinear/concurrent. The incidence relationship can be derived from circular coordinates.
It is the most important reason to use the circular coordinate system.
The sidelength or angle coordinates do not have this advantage.
d. I do not understand what you mean. The projective base 1:1:1,-1:1:1,1:-1:1,1:1:-1 is more appropriate than 1:1:1,1:0:0,0:1:0,0:0:1.
Best regards,
Tsihong Lau

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Message: #1470
Date: 29/1/2016 7:19:48
From: chris.vantienhoven
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Tsihong Lau,
I just wanted to clarify the background of your question for the QFG-members.
Thanks for your answers.
About your question why not to merge Quadrilaterals figures and Quadrangles figures my answer is simple: because they are different.
Best regards,
Chris

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Message: #1471
Date: 01/2/2016 3:11:07
From: eckart_schmidt@t-online.de
Subject: Stärk´s Sigma of a QA

Dear friends,
in EQF we find the following property for QA-P4:
QA-P4 = Homothetic Center of the 1 st Circumcenter Quadrangle and the 1 st Isogonal Conjugate Quadrangle.
The corresponding ratio is Stärk´s Sigma of a quadrangle in:
Roland Stärk: Eine merkwürdige Zahl des Vierecks.
PM: Praxis der Mathematik in der Schule, 46 (2004) 1, S. 26-31
Stärk´s Sigma of a quadrangle has a lot of interesting properties, some out of the paper:
... Sigma is 0 for cyclic QA, 1 for orthocentric QA, negative for convex QA.
... A quadrigon and the quadrigon of the perpendicular bisectors of its sides have the same Sigma.
... Sigma = $-\cot^2(\text{phi})$, phi = angle of the asymptotes of QA-Co1 (QA convex).
... Sigma = $-(s/p)^2$, p = distance focus-directrix of a circumscribed parabola and s = distance QA-P1 from the parabola axis (QA convex):
... ... consequence: For the circumscribed parabolas QA-2Co1 the focal width and the distances of QA-P1 to the axes have the same ratio.
... For a quadrilateral the Sigmas for the quadrigon components have sum 1.
... For a conic and two circles with 8 intersections: Sigma of four points = Sigma of the other four points .
There are more properties, especially wrt QA-Cu1! Stärk´s comment: "Schade, dass sich niemand auf der Welt für den Begriff der Sigma-Zahl interessiert" (personal letter 20.09.11).
For a QA and wrt DT-coordinates:
$\text{Sigma} = (p^2\ SA + q^2\ SB + r^2\ SC)^2\ /$
$((-p+q+r)(p-q+r)(p+q-r)(p+q+r)S^2)$
Consequence: The vertices of quadrangles with the same diagonal triangle and the same Sigma lie on a quartic. What about these quartic?
Best regards Eckart
PS: There is another paper of Stärk (2014):
http://www.geometria.ch/geometrie/gws.pdf
At 10. he describes QL-Tf1 wrt "gegenwinkelsummengleiche" quadrigons: I try to translate "quadrigons ABCD with <>

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Message: #1472
Date: 02/2/2016 6:48:15
From: minhtenladuong
Subject: New objects in a quadrangle

Dear Geometers

In this message, I introduce some new objects and their geometric properties in a quadrangle.
Given a quadrangle A, B, C, D. AD, BD, CD intersect BC, CA, AB at X,Y,Z
Mx, My, Mz are Miquel points of the quadrilateral (AB,AC,DB,DC), (BC,BA,DC,DA), (CA,CB,DA,DB).
Ibc, Ida, Ica, Idb, Iab, Idc are circumcenters of MxBC, MxDA, MyCA, MyDB, MzAB, MzDC.
Circle (MxIbcIda) intersects Miquel circle of (AB,AC,DB,DC) at X1 and Mx
(MyIcaIdb) intersects Miquel circle of (BC,BA,DC,DA) at Y1 and My
(MzIabIdc) intersects Miquel circle of (CA,CB,DA,DB) at Z1 and Mz.
1) According to the the paper http://forumgeom.fau.edu/FG2014volume14/FG201413index.html , triangle MxMyMz and XYZ are cyclologic. Hence, (MxYZ), (MyZX), (MzXY) are concurrent at a point S.
2) (SMxX), (SMyY), (SMzZ) pass through QA-P4.
3) S lies on QA-Cu1
4) S, X1, Y1, Z1 are concyclic.
5) Ox, Oy, Oz are circumcenter of MxYZ, MyZX, MzXY. Then triangle X1Y1Z1 and OxOyOz are homothetic, the homothetic center is QA-P9.
6) X1Y1Z1 has sidelengths proportional to BC.DA, CA.DB, AB.DC
7) QA-P3 lies on nine-point circle of X1Y1Z1 and Wallace-Simson line of S wrt X1Y1Z1. Or we have a stronger result: QA-P3 is orthopole of the line through circumcenter of X1Y1Z1 and S wrt triangle X1Y1Z1.

Best regard
Ngo Quang Duong

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Message: #1473
Date: 02/2/2016 9:24:32
From: eckart_schmidt@t-online.de
Subject: New objects in a quadrangle

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Dear Ngo Quang Duong,
interesting QA-geometry! The point S is QA-P41.
Best regards Eckart
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Message: #1474
Date: 02/2/2016 10:10:02
From: minhtenladuong
Subject: New objects in a quadrangle

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Dear Eckart Schmidt
Thank you so much.
I didn't check the list of quadrangle centers carefully.
Another remark: QA-P41 is the tangential of QA-P4 on QA-Cu1
Best regard
Ngo Quang Duong
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Message: #1475
Date: 02/2/2016 6:18:35
From: bernard.keizer
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Chris, dear Eckart, dear Tsi Hong Lau
It's not necessary fully appropriate to merge points and lines of QA and QL.
But it would be pityful not to pursue the approach of Tsi Hong Lau ; on a contrary, it's highly promising, but huge work, to study the dual reciprocity.
Let's take a few examples :
1) 4 perspectors and 4 lines are dual
6 vertices of QL and 6 lines of QA are dual
P13 and Newton Line are dual
Any point or line of the QL has a dual line or point wrt DT
2) Any curve through points and curve tangent to the dual lines are dual circumscribed conics and the corresponding inscribed conics are dual
the inscribed conics of a QL, centered on the Newton Line QL-L1, and the circumscribed conics of the QA, centered on the 9 point conic QA-Co1
in particular, the inscribed parabola and the conic through the 4 perspectors and the dual point of QL-L3 or
the inscribed conic QL-Co2 tangent to the Newton Line and my 6th conic in message 1458 , through the 4 perspectors and QL-P13 (as this last conic is through the vertices of QL-Tr2, the dual lines of the S-points are tangent to the conic QL-Co2)
3) What can be said about other curves (cubics QL-Cu1 and QL-Cu2 and quartics QL-Qu1, QL-Qu2 and QL-Qu27Qu1 ?)
What about the cayleyan, tangent to any line XX' and to the perpendicular bisector of XX' (where X and X' are CSC-partners on QL-Cu1) ?
It's easy to draw the dual curve as the locus of the dual points of all these lines ...
Best regards
Bernard

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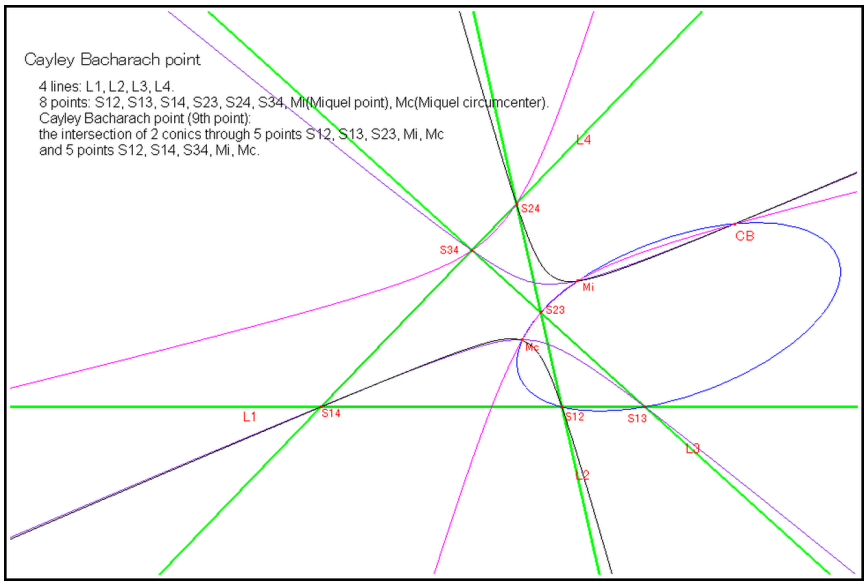
Message: #1476
Date: 03/2/2016 2:06:47
From: seiichikiri
Subject: Cayley Bacharach point

Dear friends,
Let 8 points be on the plane and different from one another.
We suppose that there are 2 different lines which pass 3 points among 8 points respectively.
Cayley Bacharach point (CB point) is the intersection of 2 conics through 5 points except for 3 points on the lines respectively.
Example 1
8 points: the vertices of a quadrangle P1P2P3P4 and the midpoints Mij of PiPj for i, j=1, 2, 3, 4 except for i=j.
2 lines: P1M14P4, P3M34P4.
2 conics: M12P2M23P3M34 which degenerates to the lines P2M23P3 and M12M34,
M14P1M12P2M23 which degenerates to the lines P1M12P2 and M23M14.
CB point is the intersection of M12M34 and M23M14.
Example 2
4 lines: L1, L2, L3, L4.
8 points: 6 intersections Sij of LiLj for i, j=1, 2, 3, 4 except for i=j, Miquel point (Mi), Miquel circumcenter (Mc).
2 lines: S14S34S24, S12S23S24.
2 conics: S12S13S23MiMc, S13S14S34MiMc.
CB point is the intersection of 2 conics above.
See the attached picture.
A similar statement: see ADGEOM message #3036
Best regards, Seiichi.



CB point.png

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Message: #1477
Date: 03/2/2016 10:45:19
From: bernard.keizer
Subject: QL's trilogy

Dear Chris, dear Eckart and dear fellows
I've achieved meanwhile my QL's trilogy and I've recently put on my blog a last article which deals with the links between the triangle, the QL and their cubic stelloïd, the 3 figures defining the same psi or Cl-S transformation.
The address is always http://bernardkeizer.blogspot.fr.
As usual, thanks in advance for any comment or critic.
Best regards
Bernard

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Message: #1478
Date: 03/2/2016 4:52:31
From: Benedetto Scimemi
Subject: Stärk´s Sigma et alia

Dear Eckart and friends, the parameter "sigma" of Stärk's papers (see Eckart's last message) appears as "lambda" in my paper ref.[36] of EQF (3.2.3 to 3.2.6), where its relations with the main QA-P and angles (or conics eccentricities) are reported.
A separate subject.
There is an affinity Ai --> Oi which maps a Q into the Q of circumcenters.
This produces a central point, say Z=QA-?. Here is a list of properties:
1) Z is the fixed point for the affinity: Ai --> Oi
2) Z lies on line QA-P1P4; in fact it is the center of the circle inversion fixing QA-P3 and mapping QA-P1 <--> QA-P4
3) Z lies on the line through QA-P42 parallel to QA-P3P4
4) Z is collinear with the midpoint (QA-P1+ QA-P32)/2 and the reflection of QA-P3 on QA-P32 (Steiner p. of Q of circumcenters).
5) Z is the fixed-point of the inverse similarity between the Q of circumcenters and the reflection on QA-P1 of the Q of ninepoints.
6) Z lies on the rectangular hyperbola S=QA-Co? passing through QA-P1, QA-P3, QA-P32, QA-P42 (S is the locus of centers for the conics pencil generated by QA-Co1 and QA-Co3)
Here is a simple way to construct Z: the axes of QA-Co3 (= the bisectors of angle
N.B. Z plays the role of QA-P42 when the reference Q is replaced by the Q of circumcenters. Thus the similarity in 5) is the analog of QA-Tf3 etc.
Looking forward for corrections and comments, best regard
Benedetto

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Message: #1479
Date: 03/2/2016 5:11:40
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Bernard
I think you and van Tienhoven misunderstood what I mean.
It is very strange. What I mean is very very simple.
Please read my page carefully.
Duality of Triangles/Trilaterals and Quadrangles/Quadrilaterals
(http://lth.name/geometry/duality.html)
Duality of Triangles/Trilaterals and Quadrangles/Quadrilaterals
(http://lth.name/geometry/duality.html)
Duality of Triangle and Trilateral Notation Coordinate or
Equation Construction Notation Coordinate or Equation
Construction Triangle Trilateral A 1:0:0 b∩c a x=0 BC B 0:1:0
c∩a b y=0 CA C 0:0:1 a∩b c
View on lth.name
(http://lth.name/geometry/duality.html)
Preview by Yahoo
The left columns show how to construct a quadrilateral
from a quadrangle.
quadrangle Q,QA,QB,QC
     V
diapleural (sA,sBC),(sB,sCA),(sC,sAB)
(opposite side)
     V
diapleural triangle A,B,C
     V
diagonal trilateral a,b,c
     V
diagonal (Sa,Sbc),(Sb,Sca),(Sc,Sab)
     V
quadrilateral q,qa,qb,qc
The right columns show how to construct a quadrangle from a
quadrilateral.(the reverse order)
quadrilateral q,qa,qb,qc
     V
diagonal (Sa,Sbc),(Sb,Sca),(Sc,Sab)
     V
diagonal trilateral a,b,c
     V
diapleural triangle A,B,C
     V
diapleural (sA,sBC),(sB,sCA),(sC,sAB)
(opposite side)
     V
quadrangle Q,QA,QB,QC

Dual relations
quadrangle Q,QA,QB,QC <->
quadrilateral q,qa,qb,qc
diapleural (sA,sBC),(sB,sCA),(sC,sAB) <->
diagonal (Sa,Sbc),(Sb,Sca),(Sc,Sab)
diapleural triangle A,B,C <->
diagonal trilateral a,b,c
I coin the term "diapleural" as the dual of diagonal.
The trilinear polar relations with respect to the diapleural triangle/
diagonal trilateral.
Pole <-> Polar
Q <-> q
QA <-> qa
QB <-> qb
QC <-> qc
I use the term autopolarity(i.e. self-polarity) to mean that we do not need an extra triangle/trilateral and we can construct the triangle/trilateral from a quadrangle/quadrilateral.
In my page, the quadrangle/quadrilateral have the unit coordinates and equations.

Q	1:1:1	q	x+y+z=0
QA	-1:1:1	qa	-x+y+z=0
QB	1:-1:1	qb	x-y+z=0
QC	1:1:-1	qc	x+y-z=0

(instead of ordinary projective base coordinates 1:1:1, 1:0:0, 0:1:0, 0:0:1)
Using DT coordinates, the quadrangle/quadrilateral have the following coordinates and equations

Q	p:q:r	q	lx+my+nz=0
QA	-p:q:r	qa	-lx+my+nz=0
QB	p:-q:r	qb	lx-my+nz=0
QC	p:q:-r	qc	lx+my-nz=0

Because of the autopolarity, we have pl=qm=rn=1.
For example, the first DT-coordinate of QA-P1 is p^2(-p^2+q^2+r^2).
If we substitute p,q,r with 1/l,1/m,1/n, we get the first DT-coordinate of QA-P1 is (-m^2n^2+n^2l^2+l^2m^2)/l^2.
The first DT-coordinate of QL-P12 is (m+n)(m-n)(-2/l^2+1/m^2+1/n^2).
If we substitute l,m,n with 1/p,1/q,1/r, we get the first DT-coordinate of QL-P12 is p^2(q+r)(q-r)(-2p^2+q^2+r^2).
The first DT coordinates of QA-P16 and QL-P13 are p^2 and 1/l^2. They are the same obviously.

I have given more details and think you should understand what I mean. Sorry! I gave so few details in the first message. I thought it is unnecessary. The most basic rationale is: Since we can construct a quadrilateral from a quadrangle and vice versa, why we distinguish quadrangle figures from quadrilateral figures?
Best regards,
Tsihong Lau

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Message: #1480
Date: 04/2/2016 9:56:32
From: eckart_schmidt@t-online.de
Subject: Stärk´s Sigma et alia

Dear Benedetto,
once more a wunderful excursion in QA-geometry!
In addition: Z lies on the line through QA-P3.QA-P8.
Best regards Eckart
PS. Up to now I try to have a look in your paper EQF-ref.[36],
but I am not so familar with internet references.

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Message: #1481
Date: 04/2/2016 3:05:12
From: eckart_schmidt@t-online.de
Subject: Stärk´s Sigma et alia

Dear Benedetto,
many thanks in advance, here my e-mail address:
eckart_schmidt@t-online.de.
Background of this geometry will be (certainly wellknown), that
...
... QA-P1, QA-P2, QA-P3, QA-P4, QA-P5 , QA-P6, QA-P7 of the 1st
circumcenter quadrangle are ...
... QA-P32, QA-P3,   X, QA-P4, QA-P12,   Y, QA-P8 of the
reference quadrangle (X = reflection of QA-P3 in QA-P32, Y =
midpoint of QA-P3.QA-P4).
The 2nd circumcenter quadrangle is homothetic with the reference
quadrangle, center QA-P4, ratio Sigma.
Best regards Eckart
PS: Wrt 4) QA-P34 is also collinear with these points.

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Message: #1482
Date: 06/2/2016 9:44:52
From: eckart_schmidt@t-online.de
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Tsihong Lau, dear Bernard, dear Chris,
I have difficulties, to follow your discussion. Can it be, that you mean the following correspondence?
The corresponding QL of a reference QA is ...
... the QL of the QA-Tr1-trilinear polars of the QA-vertices.
And:
The corresponding QA of a reference QL is ...
... the QA of the QL-Tr1-trilinear poles of the QL-lines.
Then the following holds, as you mentioned:
QA-P16 is QL-P13 of the coresponding QL and QL-P13 is QA-P16 of the corresponding QA.
Further properties can be described, if QA and QL corresponde in this way.
Some simple examples:
... QA-P32 lies on QL-L2.
... QL-Ci2 is the Thales circle about QA-P29 and QA-P36.
... QL-P8 lies on QA-L3.
... QA-Tf2 of QL-P8 is QL-P13.
... QA-Tf2 transforms QL-L1 into the circumscribed Steiner ellipse of the diagonal triangle.
... QA-Tf2 of QL-P24 lies on QL-P8.QL-P13.
... ...
This is a new sight with new properties. But there is no reason to challenge Chris´ differentiation in QA, QL, QG!
Best regards Eckart

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Message: #1483
Date: 06/2/2016 11:38:44
From: bernard.keizer
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Eckart, dear Chris, dear Tsi Long Hau,
1) I'm glad that you join this new battlefield !
2) As I put in my message 1475
a) I don't find either appropriate to merge QA and QL
b) a copple of dual point and line is mutually trilinear pole and polar or perspector and perspectrix of the cevian triangle ...
For example QL-P13 is the dual of the Newton Line ...
c) dual figurs swap tangential and point properties : a curve is either the locus of the dual points of it's tangents or the envelop of the dual lines of it's points.
For example the inscribed conics of a QL are the dual of the circumscribed conics of the dual QA ...
What are the dual curves of the main curves of the QL or the QA?
For example, what is the locus of the dual points of the lines forming the degenerate polar conics of the conic stelloïd (these lines are the axes of the inscribed conics, either the line through 2 CSC points of the hessian or the perpendicular bisector of the segment joining them)
I'm convinced there are plenty of new properties to discover !
Best regards
Bernard

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Message: #1484
Date: 06/2/2016 5:40:55
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Schmidt
---In Quadri-Figures-Group@yahoogroups.com, wrote :
... The corresponding QL of a reference QA is ...
... the QL of the QA-Tr1-trilinear polars of the QA-vertices.
And:
The corresponding QA of a reference QL is ...
... the QA of the QL-Tr1-trilinear poles of the QL-lines.
Then the following holds, as you mentioned:
QA-P16 is QL-P13 of the corresponding QL and QL-P13 is QA-P16 of the corresponding QA.
...
Yes, you have got my view-point.
...
Further properties can be described, if QA and QL corresponde in this way.
Some simple examples:
... QA-P32 lies on QL-L2.
... QL-Ci2 is the Thales circle about QA-P29 and QA-P36.
... QL-P8 lies on QA-L3.
... QA-Tf2 of QL-P8 is QL-P13.
... QA-Tf2 transforms QL-L1 into the circumscribed Steiner ellipse of the diagonal triangle.
... QA-Tf2 of QL-P24 lies on QL-P8.QL-P13.
... ...
This is a new sight with new properties. But there is no reason to challenge Chris´ differentiation in QA, QL, QG!
...
I do not verify all your results. But I think it is why I recommend to merge quadrangle/quadrilateral figures. If we split both, we will not notice the relations between them. Some figures, such as Newton line, are simpler to describe with a quadrilateral.
But as more complicated a figure, more difficult it is described with a quadrangle or a quadrilateral. Once again:
Since we can construct a quadrilateral from a quadrangle and vice versa, why we distinguish quadrangle figures from quadrilateral figures? Quadrangles are different from quadrilaterals, but the derived figures are not! I do not like the concept of a quadrigon. I will use my own terminologies.
Later!
Best regards, Tsihong Lau

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Message: #1485
Date: 07/2/2016 9:17:32
From: eckart_schmidt@t-online.de
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Tsihong Lau,
you say "I do not like the concept of a quadrigon", but quadrigons, which are as well QA as QL, have its own geometry, not separable in QA- and QL-results.
My interpretation in message 1482 gives for a quadrigon two possibilities for a corresponding quadrigon, here named QGA and QGL, with the following properties for example:
... QG-P13 = QGA-P12 and QG-P12 = QGL-P13,
... QG-Co1 = QGL-Co2 and QG-Co2 = QGA-Co1.
Best regards Eckart

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Message: #1486
Date: 07/2/2016 10:55:41
From: bernard.keizer
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Eckart, dear Chris, dear Tsihong Lau,
As Eckart surely recognised, the last curve I described in my previous message was of course the cayleyan of the cubic stelloïd QL-Cu2.
Drawing plenty of figures, I'm now almost convinced that the dual of the cubic QL-Cu1 is a sextic and the dual of the cayleyan is another cubic
(Remember that QL-Cu1 is a cubic of degree 3 and class 6 and the cayleyan is a sextic of degree 6 and class 3)
I hope you will confirm this and find the dual curves of the 2 quartics (degree 4 and class 3), which should be also cubics ...
Best regards
Bernard

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Message: #1487
Date: 08/2/2016 10:39:10
From: eckart_schmidt@t-online.de
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Bernard,
excuse, but I have difficulties in understanding your messages.
Can you give a definition for "dual figures", especially a
concret example of "... a curve is either the locus of the dual
points of it's tangents or the envelop of the dual lines of it's
points".
Thanks in advance.
Best regards Eckart

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Message: #1488
Date: 08/2/2016 11:55:00
From: bernard.keizer
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Eckart,
Thanks for your interest
Sorry if I wasn't clear enough, but I thought these were well-known properties and I already gave examples in my message 1475.
Basically, taking DT as the reference triangle abc, for any point P with barycentric coordinates (p,q,r), the cevian triangle ABC of P is the 1rst triangle of a QL and the harmonic conjugates A', B' and C' of A wrt b and c, of B wrt a and c and of C wrt a and b are on a line, which is the dual of the point P and has equation lx + my +nz = 0, with pl = qm = rn = 1
BC cuts bc in A', AC cuts ac in B' and AB cuts ab in C'.
Conversely, any line cutting abc in 3 points A', B' and C' has a dual point as intersection of aA, bB and cC, where A, B and C are the harmonic conjugates of A', B' and C'.
There are for points P in a QL, each being the dual point of one of the 4 lines wrt DT (the socalled perspectors) and there are 4 lines in a QA, each being the dual of one of the 4 points wrt DT.
Every inscribed conic of the QL has as dual curve a circumconic of the QA, you only need a 5th line for the QL and a 5th point for the QA (as described in EQF)
The example I gave is the 6th conic mentionned in my message 1458 through the 4 perspectors and QL-P13 as 5th point.
As QL-P13 is the dual of the Newton Line, the dual of this conic is an inscribed conic tangent to the Newton Line : it is described as QL-Co2 or the inscribed midline hyperbola in EQF.
As this 6th conic is through the vertices of QL-Tr2 (the S-points), it follows that the dual lines of these S-points wrt DT are tangent to the conic QL-Co2
The same goes for every curve with a reference triangle : the locus of the dual points of it's tangents or the envelop of the dual lines of it's points give the dual curve wrt this triangle. (The class of a curve being the number of tangents from a point to this curve, the degree of the dual curve is the class of the initial curve : therefore a conic for a conic, a sextic for a cubic of class 6 and a cubic for a sextic of class 3)
I hope this time these explanations will be helpful and I wait for your réflexions ...
Best regards Bernard

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Message: #1489
Date: 08/2/2016 12:12:28
From: minhtenladuong
Subject: Revisit QL-8P1

Dear Geometers,
In the attachment, I re-state the coaxial theorem with some notations under the view of quadrigon for convenient.
With this statement, we can determine which pencil is hyperbolic and which pencil is elliptic.
I also found how these circle relate to Schmidt circle and Schmidt Pair of Points.
Best regard
Ngo Quang Duong

12 angle bisectors of a Quadrilateral

Ngo Quang Duong

February 8, 2016

Proposition 1. *Given 4 points A, B, C, D. $AD \cap BC = I$, $AB \cap CD = J$. ℓ_a, ℓ_b, ℓ_c, ℓ_d are internal angle bisectors of $\angle(AB, AD)$, $\angle(BC, BA)$, $\angle(CD, CB)$, $\angle(DA, DC)$. ℓ_A, ℓ_B, ℓ_C, ℓ_D are external angle bisectors of $\angle(AB, AD)$, $\angle(BC, BA)$, $\angle(CD, CB)$, $\angle(DA, DC)$. Denote intersection of ℓ_x and ℓ_y by P_{xy}. The following groups of points are collinear:*

$$\ell_a : (A, P_{ab}, P_{da}, P_{ac}, P_{aB}, P_{aC}, P_{Da}) \quad \ell_A : (A, P_{Ab}, P_{Ac}, P_{dA}, P_{AB}, P_{AC}, P_{DA})$$

$$\ell_b : (B, P_{bc}, P_{bd}, P_{ab}, P_{bC}, P_{bD}, P_{Ab}) \quad \ell_B : (B, P_{Bc}, P_{Bd}, P_{aB}, P_{BC}, P_{BD}, P_{AB})$$

$$\ell_c : (C, P_{cd}, P_{ac}, P_{bc}, P_{cD}, P_{Ac}, P_{Bc}) \quad \ell_C : (C, P_{Cd}, P_{aC}, P_{bC}, P_{CD}, P_{AC}, P_{BC})$$

$$\ell_d : (D, P_{da}, P_{bd}, P_{cd}, P_{dA}, P_{Bd}, P_{Cd}) \quad \ell_D : (D, P_{Da}, P_{bD}, P_{cD}, P_{DA}, P_{BD}, P_{CD})$$

$$\ell_i : (I, P_{ab}, P_{AB}, P_{cd}, P_{CD}, P_{ij}, P_{iJ}) \quad \ell_j : (J, P_{bc}, P_{BC}, P_{da}, P_{DA}, P_{ij}, P_{Ij})$$

$$\ell_I : (I, P_{aB}, P_{Ab}, P_{cD}, P_{Cd}, P_{Ij}, P_{IJ}) \quad \ell_J : (J, P_{bC}, P_{Bc}, P_{Da}, P_{dA}, P_{iJ}, P_{IJ})$$

$$(P_{ij}, P_{AC}, P_{bd}) \quad (P_{ij}, P_{ac}, P_{BD})$$

$$(P_{IJ}, P_{AC}, P_{BD}) \quad (P_{IJ}, P_{ac}, P_{bd})$$

$$(P_{iJ}, P_{Ac}, P_{bD}) \quad (P_{iJ}, P_{aC}, P_{Bd})$$

$$(P_{Ij}, P_{Ac}, P_{Bd}) \quad (P_{Ij}, P_{aC}, P_{bD})$$

Proposition 2. *The following concyclic points are easily to realize:*

$$\psi_{ab} : (A, B, P_{ab}, P_{AB}) \quad \phi_{ab} : (A, B, P_{Ab}, P_{aB})$$

$$\psi_{bc} : (B, C, P_{bc}, P_{BC}) \quad \phi_{bc} : (B, C, P_{Bc}, P_{bC})$$

$$\psi_{cd} : (C, D, P_{cd}, P_{CD}) \quad \phi_{cd} : (C, D, P_{Cd}, P_{cD})$$

$$\psi_{da} : (D, A, P_{da}, P_{DA}) \quad \phi_{da} : (D, A, P_{Da}, P_{dA})$$

$$\psi_{ac} : (A, C, P_{ac}, P_{AC}) \quad \phi_{ac} : (A, C, P_{Ac}, P_{aC})$$

$$\psi_{bd} : (B, D, P_{bd}, P_{BD}) \quad \phi_{bd} : (B, D, P_{Bd}, P_{bD})$$

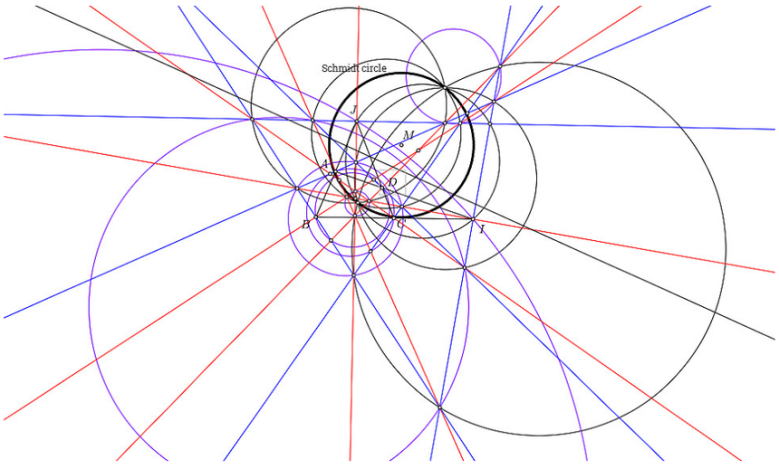
$$\psi_{ij} : (I, J, P_{ij}, P_{IJ}) \quad \phi_{ij} : (I, J, P_{Ij}, P_{iJ})$$

Honsberger found that P_{ab}, P_{bc}, P_{cd}, P_{da}. This interesting result is extended and we obtain 8 quadruples of concyclic points as follow:

12-angle-bisectors.pdf

Proposition 3.

$$O(abcd) : (P_{ab}, P_{bc}, P_{cd}, P_{da}) \quad O(ABCD) : (P_{AB}, P_{BC}, P_{CD}, P_{DA})$$
$$O(aBcD) : (P_{aB}, P_{Bc}, P_{cD}, P_{Da}) \quad O(AbCd) : (P_{Ab}, P_{bC}, P_{Cd}, P_{dA})$$
$$O(abCD) : (P_{ab}, P_{bC}, P_{CD}, P_{Da}) \quad O(ABcd) : (P_{AB}, P_{Bc}, P_{cd}, P_{dA})$$
$$O(aBCd) : (P_{aB}, P_{BC}, P_{Cd}, P_{da}) \quad O(AbcD) : (P_{Ab}, P_{bc}, P_{cD}, P_{DA})$$



Proposition 4 (1)**.** *The following circles belong to a hyperbolic pencil* $\mathcal{H}$*:*

$$O(abcd), O(ABCD), O(aBcD), O(AbCd), \phi_{ac}, \phi_{bd}, \phi_{ij}$$

and the following circles belong to an elliptic pencil $\mathcal{E}$*:*

$$O(abCD), O(ABcd), O(aBCd), O(AbcD), \psi_{ac}, \psi_{bd}, \psi_{ij}$$

$\mathcal{H}$ *is orthogonal to* $\mathcal{E}$*.*

Proposition 5. *Schmidt circle belongs to* $\mathcal{E}$*. Two limit points of* $\mathcal{H}$ *are Schmidt pair of points.*

References

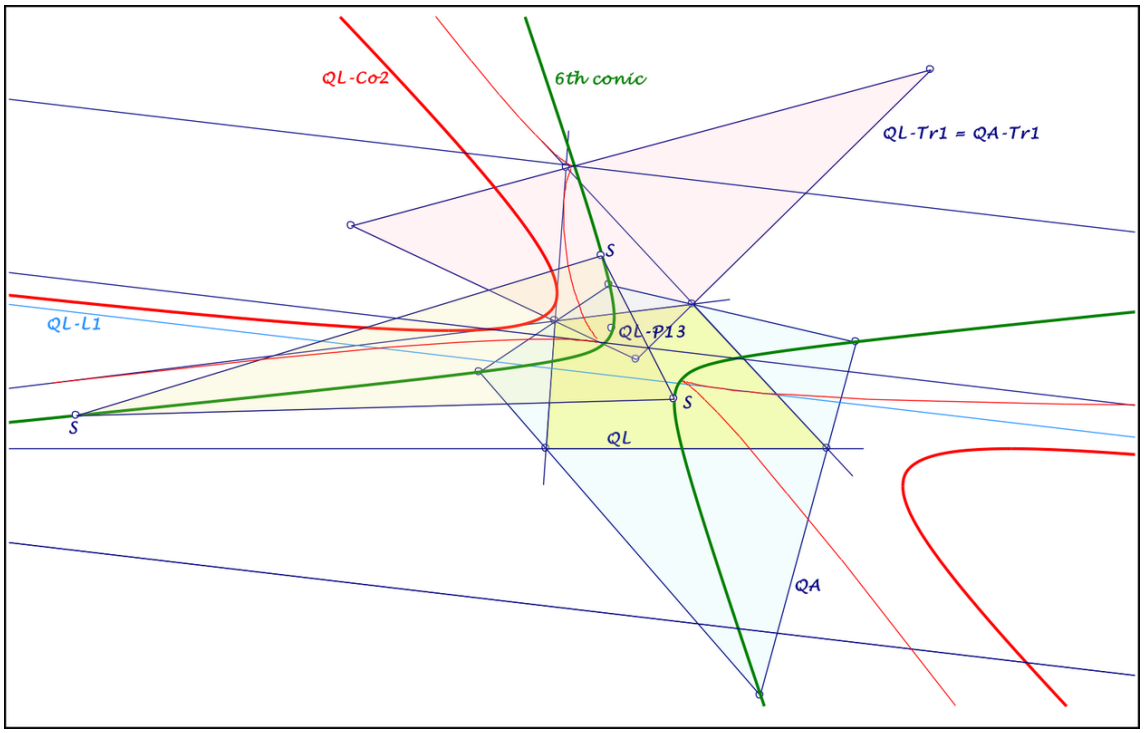
[1] QL-8P1: Steiner Angle Bisector Center Octet
http://chrisvantienhoven.nl/quadrilateral-objects/17-mathematics/quadrilateral-objects/133-ql-8p1.html

Message: #1490
Date: 08/2/2016 3:05:51
From: eckart_schmidt@t-online.de
Subject: Merge Quadrangle/Quadrilateral Figures

Dear Bernard,

it seems, I grow stupid! But first thanks for your explanations.
In the first part of your message you describe trilinear pole and polar wrt a triangle, that´s well-known.
In the second part you give a concret example, see attached file.
But the trilinear polars of the S-points are not tangent to QL-Co2 (they are parallels to QL-L1).
The trilinear polars of points on the 6th conic envelop a curve with 3 cusps.
Where is my error?

Best regards Eckart



2016-02-08.pdf

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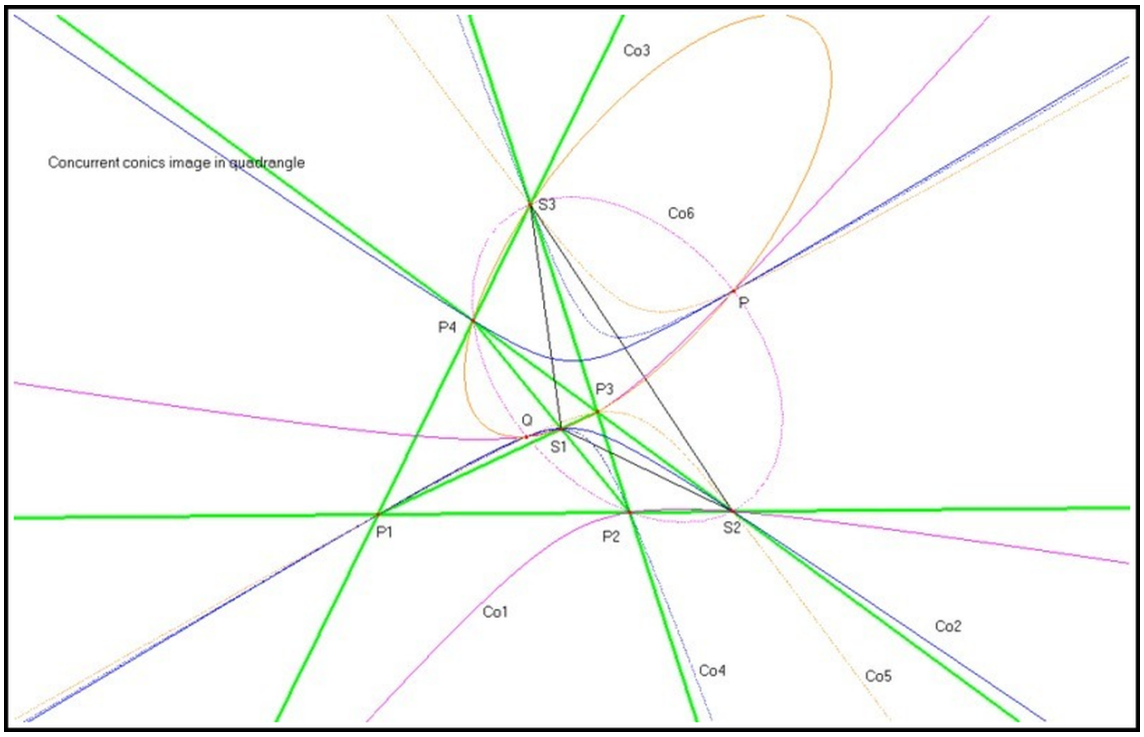
Message: #1491
Date: 09/2/2016 2:22:29
From: seiichikiri
Subject: A concurrent conics image in quadrangle

Dear friends,
Let P1P2P3P4 be a quadrangle, S1S2S3 its diagonal triangle and P a point.
Denote:
Co1= the conic through P2P3S1S2P.
Co2= the conic through P1P4S1S2P.
Co3= the conic through P3P4S1S3P.
Co4= the conic through P1P2S1S3P.
Co5= the conic through P1P3S2S3P.
Co6= the conic through P2P4S2S3P.
Co1, Co2, Co3, Co4, Co5 and Co6 concur at a point Q.
See the attached picture.
Best regards, Seiichi.



ConcurrentConicsImageQF.png

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Message: #1492
Date: 09/2/2016 8:41:18
From: eckart_schmidt@t-online.de
Subject: A concurrent conics image in quadrangle

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Dear Seiichi,
Q = QA-Tf2(P).
Best regards Eckart
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Message: #1493
Date: 09/2/2016 9:06:18
From: seiichikiri
Subject: A concurrent conics image in quadrangle

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Dear Eckart,
Thank you very much for your kind information!
Best regards, Seiichi.
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Message: #1494
Date: 09/2/2016 4:28:58
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear all

Now I present the second message for this topic.
The previous messages I sent were details only, no
new ideas. Please check my updated page carefully!
Duality of Triangles/Trilaterals and Quadrangles/Quadrilaterals
(http://lth.name/geometry/duality.html)

First, look at the third table! It shows the incidence
relationship of the quadrangle and quadrilateral
described in the second table. This table is very like the
table of the projective plane PG(2,3).
Projective plane - Wikipedia, the free encyclopedia
(
https://en.wikipedia.org/wiki/Projective_plane#A_finite_example
)
The vertices of a quadrangle Q,QA,QB,QC lie on three lines only.

The sidelines of a quadrilateral q,qa,qb,qc pass through three
vertices only.
But every point and line in PG(2,3) lies on and passes through
four lines and four points respectively.
If the quadrangle/quadrilateral is on PG(2,3) (or a finite
projective plane with characteristic 3),
Q,QA,QB,QC lie on q,qa,qb,qc respectively.
In other words, the configuration of a quadrangle/quadrilateral
is very like PG(2,3).

Second, go back to the second table.
The basic points and lines of a quadrangle/quadrilateral can be
grouped into
   quadruple - Q,QA,QB,QC and q,qa,qb,qc
    sextuple - Sa,Sbc,Sb,Sca,Sc,Sab
    and sA,sBC,sB,sCA,SC,sAB
      triple - A,B,C and a,b,c
    and (Sa,Sbc),(Sb,Sca),(Sc,Sab)
    and (sA,sBC),(sB,sCA),(sC,sAB)
In fact, many derived figures of a quadrangle/quadrilateral
belong to one of the three categories.
This concept is very essential.

Third, look at the fourth table! Four expoint triangles/exline trilaterals are an example of quadruple and shown in another page of mine.(See below) I will explain them in a later message. Three diagonal quadrangles/diapleural quadrilaterals are an example of triple and not shown in my page yet.
Six diagonal/diapleural conics are an example of sextuple. The equations shown are point-equations not line-equations. If the quadrangle /quadrilateral is on PG(2,3) (or a finite projective plane with characteristic 3), the conics degenerate into two distinct and same lines respectively.
For example,
-x^2+y^2+z^2-y*z=0   -> (+x+y+z)(-x+y+z)=0
-3x^2+4y^2+4z^2+4y*z=0 -> (y-z)^2=0
The three figures are not my invention, the former two appear in the EQF and are under other terminologies.
The last appears in a recent message from Seiichi but was found by me a year ago. I think it must have appeared somewhere long ago.

Fourth, look at another page of mine!
http://lth.name/geometry/centers_of_triangles_trilaterals.html
The caption of the table in it is "Centers of Triangles/Trilaterals Involutary Axes of Quadrangles/Quadrilaterals".
I explain "Involutary Axes" here. I use two coordinate systems - DT barycentric and circular - and give first coordinate/coefficient only. Four transformations are involved:
QA-Tf2: u -> p^2/u(DT) or 1/u(circular) Involutary Conjugate
QL-Tf2: u -> l^2/u(DT) or 1/u(circular) QL-Line Isoconjugate
Trilinear polar or pole: u -> 1/u(DT or circular) (with respect to diapleural triangle/diagonal trilateral)
For example, I explain how to get the involutary axis Newton line corresponding to the centroid of diapleural triangle/diagonal trilateral.

```
               coordinate DT - circular
                 centroid  1 - 1/(qw-rv)
                             |
            trilinear polar  v
         line at infinity 1 - qw-rv
                             |
                 QL-Tf2      v
            Newton line l^2 - 1/(qw-rv)

     or
               centroid    1 - 1/(qw-rv)
                             |
                 QA-Tf2      v
QA-P16 or QL-P13 p^2 or 1/l^2 - qw-rv
```

```
                          |
             trilinear polar v
     Newton line 1/p^2 or l^2 - 1/(qw-rv)
```

In other words, every triangle center corresponds the involutary axis of a quadrangle/quadrilateral and the circular coordinate/coefficient is the same!
In the ETC there are thousands of triangle centers listed. (excluding expoints such as X(7), X(8), etc.).
So we have thousands of involutary axes of a quadrangle/quadrilateral already!
Similarly we have thousands of involutary centers of a quadrangle/quadrilateral already!
(using trilinear polars)
(I have not posted the page "Axes of Triangles/Trilaterals=Involutary Centers of Quadrangles/Quadrilaterals" yet!)

Fifth, we can derive an center of a quadrangle/quadrilateral with an center of diapleural triangle/diagonal trilateral and an involutary center of a quadrangle/quadrilateral.
For example, QA-P1 is the QA-P10(centroid of the diapleural triangle/diagonal trilateral)-cevadiv(or ceva conjugate) of QA-P16 (the involutary center of a quadrangle/quadrilateral corresponding to the line at infinity).
QA-P2 is the cevamul(or cevapoint) of X(6) of the diapleural triangle/diagonal trilateral and QA-P16.
Since thousands of centers of triangle/trilateral and involutary centers of quadrangle/quadrilateral are known, we can have hundred thousands centers of quadrangle/quadrilateral!
I do not know if all centers of quadrangle/quadrilateral can be derived in this way.
Similarly, we can have hundred thousands axes of quadrangle/quadrilateral!
To be continued...

Best regards,
Tsihong Lau

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Message: #1495
Date: 09/2/2016 5:24:26
From: bernard.keizer
Subject: Merge Quadrangle/Quadrilateral Figures

Lieber Eckart,
Don't worry, I'm the stupid in this matter !
Thanks for the figures.
It seems I've jumped, as I sometimes do, on a false intuition.
I maintain that the 6th conic through the 4 perspectors and QL-P13 and the inscribed QL-Co1 tangent to the Newton Line are dual.
These 5 points and these 5 lines are dual and mutually trilinear poles and polars wrt DT.
The equations of the 2 conics are easily deductible from one another: they have only terms in x2, y2 and z2 and the coefficient of x2 is l4(m2 - n2) for the 1rst one and l2/(m2 - n2) for the 2nd one ; you get one by replacing l, m and n by their inverses 1/l, 1/m and 1/n in the other and taking the inverse of the result or something like that ...
I suppose also that any inscribed conic of the QL tangent to a 5th line has a dual conic circumscribed to the 4 perspectors and a 5th point, the 5th line and the 5th point being also dual or tripole and tripolar (for example the parabola with L3 as 5th line).
But it is not true for every point of a curve and tangent of the other!
May be you will now find the real relation between the 2 dual curves ...
For the time being, I suggest we both study Tsihong Lau's last message.
Best regards
Bernard

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Message: #1496
Date: 09/2/2016 9:08:10
From: eckart_schmidt@t-online.de
Subject: A concurrent conics image in quadrangle

Dear Seiichi,
the constellation of the six conics is rather curious:
The conics are degenerated pivotal isocubics, that means:
... the conics are QA-Tf2-invariant,
... lines through QA-Tf2-partners have a common point (pivot),
... the pivots are collinear on P.QA-Tf2(P),
... in the intersections with the QA-lines (part of the degenerated isocubic),
... in pairs harmonic with P and QA-Tf2(P).
But what about the centers of the conics?
In addition: There are analog results in QL-geometry, considering inscribed conics of two QL-lines, two sidelines of QL-Tr1 and an additional line L.
The six conics have a common tangent in QL-Tf2(L) ...
Best regards Eckart

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Message: #1497
Date: 10/2/2016 2:28:54
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
there will be the following possibility for "dual curves" wrt corresponding QA and QL in the sence of #1482:
Let Cu be an arbitrary curve:
... The QA-Tf2-images of the DT-trilinear poles of Cu-tangents give the "dual curve".
... The QL-Tf2-images of the DT-trilinear polars of Cu-points give the same "dual curve".
"Dual curve" ...
... of a line will be a point (and vice versa),
... of a DT-circumscribed conic will be a DT-inscribed conic (and vice versa),
... of a QA-circumscribed conic will be a QL-inscribed conic (and vice versa).
... (in general) of a conic will be a conic.
But what about cubics?
Best regards Eckart

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Message: #1498
Date: 11/2/2016 11:38:09
From: bernard.keizer
Subject: Dual curves

Dear Eckart,
Wunderbar !
Thanks to you, I won't die stupid, after all ...
I wasn't so far, but I always forget, each line of the QL is it's own QL-Tf2 and I suppose each point of the QA is it's own QA-Tf2.
The properties you give for conics seem to fit well.
Some questions, in order to check if I understand well :
1) Considering the 5th and 6th conics through the S-points, the QL-Tf2 images of the DT trilinear polars of the S-points are tangent to the duals of these 2 conics (one is QL-Co2, the other is inscribed in DT)
The QL-Tf2 image of the Newton Line is tangent to QL-Co2
2) What is the dual of the inscribed parabola?
What is the QA-Tf2 image of the trilinear pole of QL-L3?
3) What are the duals of the circumcircle and the incircle of DT?
4) What can be said about the duals of all circles through QL-P1? (For example Miquel circle, Plücker circle or Dimidium circle ...) and what is the QL-Tf2 image of the trilinear polar of QL-P1 ...?
I suppose there are plenty of new properties which will arise !
Now for cubics, quartics, quintics or sextics, the construction will be the same and the degree of the dual curve is the class of the curve (sextic for a cubic, cubics for the cardioïds and the deltoïd, cubic again for the cayleyan)
Best regards
Bernard

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Message: #1499
Date: 11/2/2016 8:45:41
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard, dear Chris,

here only a remark wrt Bernards question 2): (remember
QA-P10,11,12,13,16 = QL-P8,9,10,11,13).
The dual of QL-Co1 is an interesting conic Co, I think not
mentioned in EQF:
... Co is a QA-circumscribed hyperbola,
... through QA-P1, tangent to QA-P1.QA-P10,
... through QA-P16, tangent to QA-P16.QA-P10,
... with infinity points in the QA-Tf2-images of the
intersections of QA-Co1 and QA-P1.QA-P10,
... diagonal triangle is self-polar wrt Co,
... Co-polar of QA-P5 is QA-P1.QA-P17,
... Co-polar of QA-P10 is QA-P1.QA-P16,
... Co-polar of QA-P18 is QA-P1.QA-P19,
... Co-polar of QA-P19 is QA-P16.QA-P18,
... Co-polar of QA-P21 is QA-P10.QA-P27,
... Co-polar of QA-P22 is QA-P1.QA-Tf2(QA-P22),
... Co-polar of QA-P23 is QA-P12.QA-P32,
... Co-polar of QA-P25 is QA-P1.QA-Tf2(QA-P25),
... Co-polar of QA-P26 is QA-P1.QA-Tf(QA-P26),
... Co-polar of QA-P31 is QA-P16.QA-Tf2(QA-P31),
... Co-polar of QA-P32 is QA-P23.QA-Tf2(QA-P32),
These results reflect the general properties of QA-circumscribed
conics, that the diagonal triangle is self-polar and the polar
of a point P contains QA-Tf2(P). But there remain several
special properties.

Best regards Eckart

PS:The dual of QA-Co1 is the DT-inscribed Steiner ellipse.

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Message: #1500
Date: 12/2/2016 12:11:39
From: bernard.keizer
Subject: Dual curves

Dear Eckart, dear Chris
I knew it would bring plenty of new properties !
For example :
any 2 conics intersect in 4 points and have 4 common tangents :
the 2 dual conics intersect in 4 points which are the QA-Tf2 images of the trilinear poles of the 4 common tangents and are tangent to 4 lines, which are the QL-Tf2 images of the trilinear polars of the 4 intersection points.
Application:
as QA-P16 = QL-P13 is the trilinear pole of the Newton Line of the QL and belongs to the 2 dual conics of QL-Co1 and QL-Co2, the QL-Tf2 image of the Newton Line is a common tangent to these 2 conics.
Best regards
Bernard

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Message: #1501
Date: 12/2/2016 2:41:34
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
as I hoped: Now QL-Tf2 has its corresponding place in QL-geometry! Moreover a new perspective wrt dual curves!
Here are the duals wrt Bernard´s question 3):
Dual of the DT-circumcircle:
Inscribed DT-conic, which is the envelope for the QL-Tf2-images of lines through the Lemoine point X(6).
Dual of the DT-incircle:
Circumscribed DT-conic, which is the locus for the QA-Tf2-images of points on the trilinear polar of the Gergonne point X(7).
Best egards Eckart

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Message: #1502
Date: 13/2/2016 10:19:47
From: chris.vantienhoven
Subject: Dual curves

Dear Eckart,

It seems that I am missing something.
I try to follow your statements:
1. I construct in a Quadrangle QA the Quadrilateral QL consisting of the QA-Tr1-trilinear polars of the QA-vertices.
2. I draw a random circle Ci.
3. I draw the QA-Tf2-images of the DT-trilinear poles of the Ci-tangents, which produce a conic Co, which should be the "dual curve".
4. I draw the QL-Tf2-images of the DT-trilinear polars of some Ci-point. According to your message #1497 these QL-Tf2-images should be tangent to (envelope) the same “dual curve” Co.
However I do not see this happen in my drawings.
Did I make a mistake in my steps or thoughts?

Best regards,
Chris

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Message: #1503
Date: 13/2/2016 11:01:23
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
wrt point 1) in #1498:
The dual of your 5th conic, which is a DT-circumscribed conic through QL-P8, QL-P13, QL-P24 is ...
... a DT-inscribed parabola
... ... with tangent QL-L1 in QL-P23,
... ... focus QL-P17,
... ... directrix QL-L6.
I have difficulties with your 6th conic. You say, its dual is QL-Co2, but the dual of QL-Co2 doesn´t contain the S-points.
The dual of QL-Co2 is ...
... a QA-circumscribed conic
... through QL-P8,
... with tangent QL-P8.QL-P13,
... centered in the trilinear pole of QL-P8.QL-P13.
Can you give me oncemore a definition of your 6th conic, perhaps I have misunderstood your #1458?
Best regards Eckart

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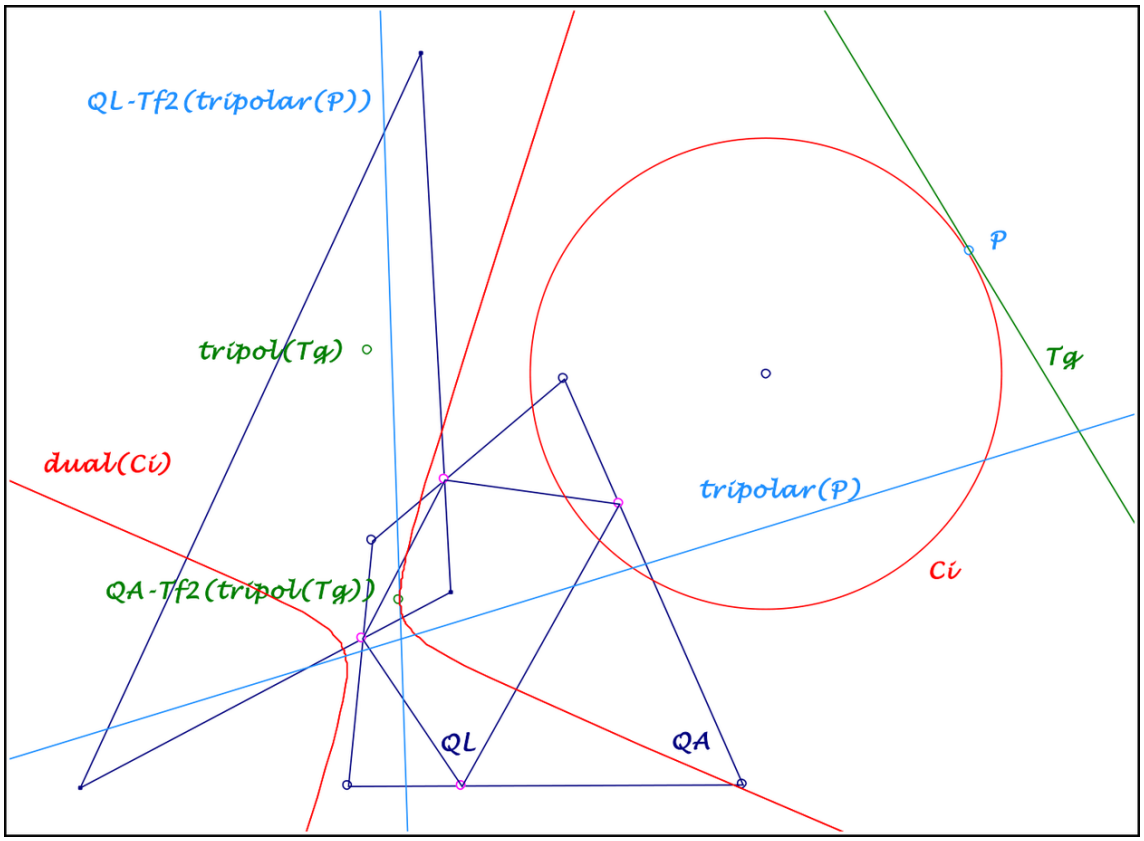
Message: #1504
Date: 13/2/2016 12:14:59
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Chris,
I haven´t found an error (perhaps in construction), attached a drawing.
Best regards Eckart



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Message: #1505
Date: 13/2/2016 7:50:04
From: bernard.keizer
Subject: Dual curves

Lieber Eckart,
Wunderbar again !
1) the dual of the 5th conic is the DQL-Co1, tangent to the 3 sides of DT and to the Newton Line ; it's the anticomplement of QL-Co3 wrt DT.
2) I'm not surprised : as the 6th conic is through the 4 QA vertices and through QL-P13, the dual of this 6th conic is not QL-Co2, as you prove it, but another QL-inscribed conic, tangent to *QL-Tf2 of the Newton Line* , and not to the Newton Line, as I put it (this is QL-Co2).
The 5th and 6th conics are through the S-points and their duals must be tangent to the QL-Tf2 of the trilinear polars of these S-points.
3) Other questions : what is the transformation (point to point) associating the trilinear poles of 2 QL-Tf2 partners and what is the transformation (line to line) associating the trilinear polars of 2 QA-Tf2 partners ?
Best regards
Bernard
PS I suppose we are on the way of finding more points and lines and conics and other curves of higher degree ...

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Message: #1506
Date: 13/2/2016 8:56:03
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
do you mean under 3) of #1505 with ...
... the first point to point transformation chain:
point ---> tripolar ---> QL-Tf2 ---> tripole? This is QA-Tf2.
... the second line to line transformation chain:
line ---> tripole ---> QA-Tf2 ---> tripolar? This is QL-Tf2.
Best regards Eckart

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Message: #1507
Date: 13/2/2016 11:41:49
From: chris.vantienhoven
Subject: Dual curves

Dear Eckart, dear Bernard and all,
The latest items of our Dual-subject give an unexpected possibility for calculating enveloped curves.
In my experience enveloped curves are hard to calculate.
Now an extra possibility pops up:
1. Suppose we have a general expression of enveloping lines L in a Dual QA-/QL-environment.
2. We calculate P = DT-Tripolar(QA-Tf2(DT-Tripole(L))). This point lies on the Dual of the enveloping curve.
3. Knowing the general expression of one point P of the dual curve we can calculate the equation of the locus of the dual curve.
4. Then we can calculate the tangent Tg at P by partial differentiating.
5. Finally DT-Tripole(QL-Tf2(Tg)) is a point on the enveloped curve.
6. From the general expression of this point the equation of the enveloped curve can be calculated.
I didn't try it yet, but in theory it should work.
Best regards,
Chris

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Message: #1508
Date: 14/2/2016 10:32:19
From: bernard.keizer
Subject: Dual curves

Dear Eckart,
Yes, that's what I thought, but I wasn't sure, thank you for this confirmation
Then it is clear that the lines of QL are the only lines to be their own Tf2 and the vertices of QA the only points to be their own QA-Tf2.
Now, any point has a QA-Tf2 and any line has a QL-Tf2 ...
Best regards
Bernard

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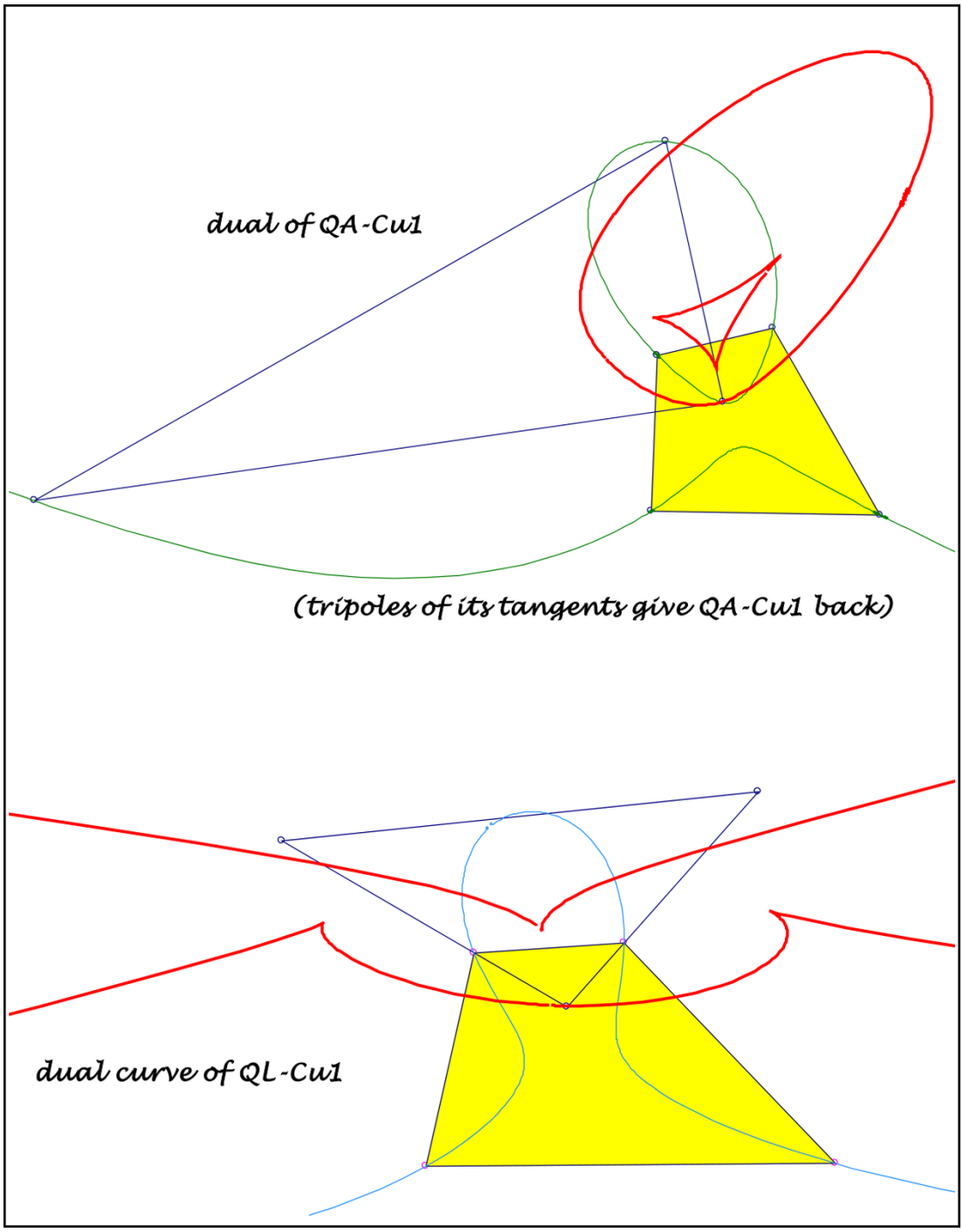
Message: #1509
Date: 14/2/2016 11:55:54
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
only two drawings attached, which show the duals of QA-Cu1 and QL-Cu1 as sextics.
Best regards Eckart



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Message: #1510
Date: 14/2/2016 3:42:05
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear all

I present the third message here. Because I have rename some pages of mine. Please bookmark my home page for "Geometry of Triangle/Trilateral and Quadrangle/Quadrilateral"
http://lth.name/geometry/
Please read the duality page carefully.
Duality of Triangle/Trilateral and Quadrangle/Quadrilateral (http://lth.name/geometry/duality.html)

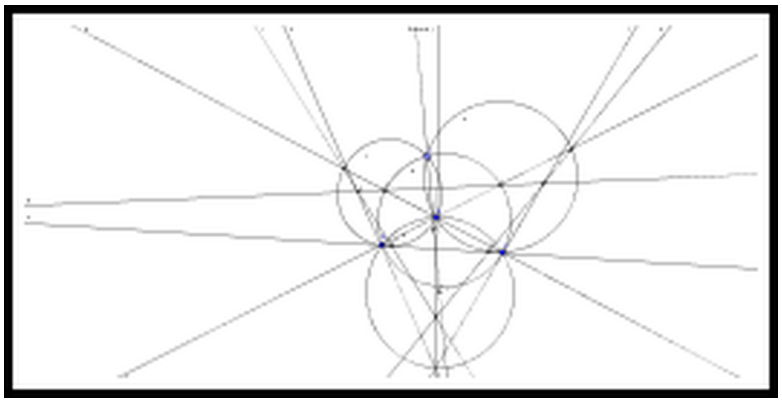
1. A new page is uploaded.
Center/Axis of Quadrangle/Quadrilateral and Central Triangle/Axial Trilateral of Quadrangle/Quadrilateral (http://lth.name/geometry/center_axis_of_quadrangle_quadrilateral.html)
Because of using the circular coordinate system, every row represent a center and axis of Quadrangle/Quadrilateral.
Look at the 3rd to 10th columns. They are "Central Triangle/Axial Trilateral of Quadrangle/Quadrilateral". Since diagonal quadrangle and diapleural quadrilateral are quadrangle and quadrilateral respectively,(Please refer to the duality page for diagonal quadrangle and diapleural quadrilateral.), so every diagonal quadrangle and diapleural quadrilateral has its own center and axis respectively. Because there are three diagonal quadrangles and three diapleural quadrilaterals, so we have a central triangle and an axial trilateral respectively.
So each row represent two central triangles and two axial trilaterals. The last column show how to derive the center/axis. For example, the row "(qw-rv)/(qv-rw)" represent the QA-P2 and the last column "[pu(qw-rv)]cevamul-[qw-rv]" represent QA-P2 is cevamul- of X(6) and QA-P16. Of course this row can represent an axis, then the last column represent it is cevamul- of the involutary axis corresponding to X(6) and the center corresponding to QA-P16.
Please refer to my previous message for more information.
We can derive centers/axes of quadrangle/quadrilateral in such a way. We can derive lots of centers/axes and then study their properties. I will post the meanings of these operations later.

2. The "Center of Triangle/Trilateral" is renamed and updated!

Center of Triangle/Trilateral and Involutary Axis of Quadrangle/Quadrilateral and Central Triangle/Axial Trilateral of Quadrangle/Quadrilateral (http://lth.name/geometry/center_of_triangle_trilateral_and_involutary_axis_of_quadrangle_quadrilateral.html)
Six columns are added to the page. Since the center of triangle/trilatateral and the involutary axis of quadrangle/quadrilateral are special cases of center/axis of quadrangle/quadrilateral, so we can two two central triangles and two axial trilaterals in each row.
But! The diapleural triangle/diagonal trilateral of diagonal quadrangle and diapleural quadrilateral is the same, while the latter two are not autopolar(dual)! So we can get "only one central triangle" but two axial trilaterals in each row! For example, the diapleural triangle /diagonal trilateral of Qa=SbScaScSab and qA=sBsCAsCsAB is the same ASaSbc=asAsBC but the trilinear polars and poles of Qa and qA with respect to the triangle/trilateral are not qA and Qa respectively.

3. The circular page is renamed and updated!
Circular Coordinate System (http://lth.name/geometry/circular.html)
A third table of central triangle/axial trilateral is added. But its meaning is not defined now, I will study the rules of coordinates/coefficients and post later. And a GeoGebra file is attached.
It shows the objects in the polarity page.
Four conics are also shown. "conic" passes through the six tangent points of kbc,kca,kab at qb,qc,qc,qa,qa,qb respectively and its equation is x^2+y^2+z^2+6(yz+zx+xy)=0.
conic,conica,conicb,conicc are an example of conic quadruple.
To be continued...

Best regards,
Tsihong Lau



autopolarity_between_quadrangle_and_quadrilateral-1510.ggb

Message: #1511
Date: 14/2/2016 4:19:20
From: eckart_schmidt@t-online.de
Subject: Dual curves

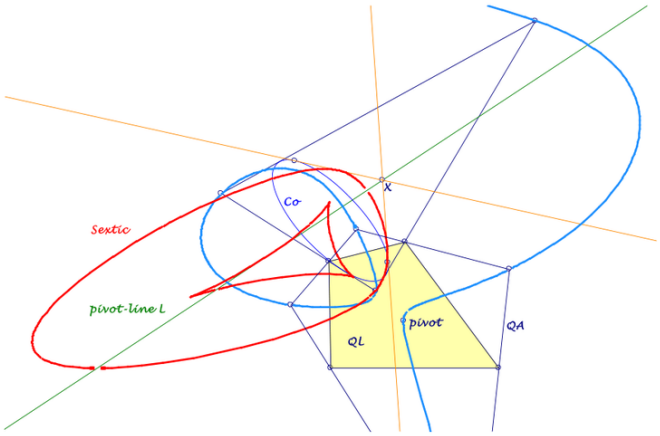
Dear Bernard, dear Chris,
in QA-geometry there are the pivotal isocubics QA-Cu1, 2, 3, 4, 5 wrt the isoconjugation QA-Tf2 for the diagonal triangle QA-Tr1. This can be translated in QL-geometry wrt the isoconjugation for lines QL-Tf2 for the diagonal triangle QL-Tr1 (see attached file).
Best regards Eckart

EQF-Note 2016-02-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-DT Pivotal "Isosextics" wrt Lines

In QA-geometry there are the pivotal isocubics QA-Cu1, 2, 3, 4, 5 wrt the isoconjugation QA-Tf2 for the diagonal triangle QA-Tr1. This can be translated in QL-geometry wrt the isoconjugation for lines QL-Tf2 for the diagonal triangle QL-Tr1.



The mentioned pivotal *QA*-isocubics wrt a pivot *P* and the isoconjugation *QA-Tf2* for the triangle *QA-Tr1* can be constructed in the following way:
Let *l* be lines through the pivot *P* and *Co* the image-conic *QA-Tf2(l)*, then the intersections of *l* and *Co* give the cubic, circumscribed *QA* and *QA-Tr1*.

Translated in *QL*-geometry:

We start with a pivot-line *L* and the line isoconjugation *QL-Tf2* for the diagonal triangle *QL-Tr1*:
… Let *X* be points on the pivot-line *L*.
… The *QL-Tf2*-images of lines through *X* give a *QL-Tr1*-inscribed conic *Co*.
…The tangents from *X* to *Co* envelop a curve, which is a sextic.

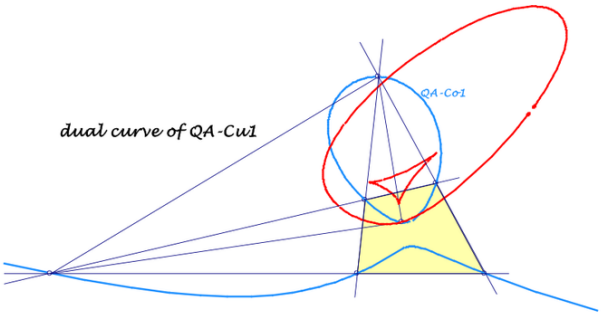
- **This *QL*-sextic is tangent to *QL* and *QL-Tr1* and the pivot-line *L* and *QL-Tf2*-invariant wrt its tangents.**

- **The dual curve (see *EQF*-message 1497) of this *QL*-sextic is a pivotal *QA*-isocubic of the corresponding *QA* (see *EQF*-message 1482) wrt the isoconjugation *QA-Tf2* and a pivot in *QA-Tf2* of the trilinear pole of the pivot-line *L*.**

- **The trilinear poles of tangents at the *QL*-sextic give the dual *QA*-isocubic.**

On the other hand: If we consider a *QA* (with corresponding *QL*) and a *QA*-pivotal isocubic with pivot *P* (as *QA-Cu1, 2, 3, 4, 5*), we get the dual *QL*-sextic for a pivot-line, which is the *QL-Tf2*-image of the trilinear polar of the pivot *P*. These pivot-lines are
… for *QA-Cu3* with pivot *QA-P10* the Newton line *QL-L1*,
… for *QA-Cu5* with pivot *QA-P1* a parallel to *QL-L9* through *QL-P19*.

- **The dual *QL*-sextic of a *QA*-pivotal isocubic (as *QA-Cu1, 2, 3, 4, 5*) is the envelope of trilinear polars of cubic-points.**



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-02-14.pdf

Message: #1512
Date: 15/2/2016 6:58:53
From: bernard.keizer
Subject: Dual curves

Dear Eckart, dear Chris, dear Tsihong Lau
I think I begin to understand a little bit better this question of duality ; in fact, every point, line and curve has 2 duals, one wrt DT, the other wrt QA/QL, which is a generalisation of the triangle to the 4 points/lines.
1) wrt DT, the dual of a point is it's trilinear polar, the dual of a line is it's trilinear pole and the dual of a curve is either the envelop of the trilinear polars of it's points or the locus of the trilinear poles of it's tangents.
2 lines intersect in a point, which is the trilinear pole of the line through their trilinear poles (the vertices of the QA are the duals of the lines of the QL and the vertices of the QL are the duals of the sides of the QA).
My 6th conic is the dual of QL-Co2 wrt DT ...
2) wrt QA/QL, the dual of a point is QL-Tf2 of it's trilinear polar or the trilinear polar of it's QA-Tf2 and the dual of a point is QA-Tf2 of it's trilinear pole or the trinear pole of it's QL-Tf2 and the dual of a curve is either the envelop of the duals of it's points or the locus of the duals of it's tangents.
Obviously, the vertices of QA are their own QA-Tf2 and the lines of QL are their own QL-Tf2.
2 lines intersect the same way in a point, which is the dual of the line through their duals.
For example, 2 QL-Tf2-partner lines intersect in a point, which is the dual of the line through the 2 QA-Tf2 partner points (this point is on 2 QL-inscribed conics, which are the duals of the 2 QA-circumconics through one of the QA-Tf2 partners and the line through the 2 QA-Tf2 partners is tangent to these 2 last 2 conics)
My 6th conic is this time the dual of an inscribed conic tangent to QL-Tf2 of the Newton Line ...
3) What we need now is a systematic drawing or/and calculation of the duals of every point, line and curve of QA/QL.
Eckart has begun this huge work, but it remains a lot to do !
I hope I didn't make this time any basic mistake.
Thanks in advance for confirmation.
Best regards
Bernard

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Message: #1513
Date: 16/2/2016 1:53:20
From: minhtenladuong
Subject: Collinear Orthocorrespondences

Dear geometers,
Given 5 points A,B,C,D,P.
Let Ha,Hb,Hc,Hd be orthocenters of BCD,CDA,DAB,ABC.
A'=H_a/P wrt triangle BCD
B'=H_b/P wrt triangle CDA
C'=H_c/P wrt triangle DAB
D'=H_d/P wrt triangle ABC
Then A',B',C',D' are collinear (L)
Denote the line L by T(P,ABCD), I found some properties:
1) If P varies on a line d, T(P,ABCD) passes through a fixed point.
I don't know how the fixed point relates to the line d ?
2) If P varies on a fixed circle, envelope of T(P,ABCD) is a conic.
3) T(P,ABCD)=T(P,H_aH_bH_cH_d)
Construct the inverse of the transformation:
Given a line L. S varies on L.
H_a/S (wrt BCD), H_b/S(wrt CDA),H_c/S(wrt DAB), H_d/S(wrt ABC) varies on four conics, they are concurrent at P that T(P,ABCD)=L
Best regards
Ngo Quang Duong

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Message: #1514
Date: 16/2/2016 3:17:23
From: Ngo Quang Duong
Subject: Về: Collinear Orthocorrespondences

Dear all,
T(P,ABCD) is polar line of P wrt QA-Co2
Best regards
Ngo Quang Duong
Và o ngà y 19:53 Thá»© Ba, 16 tháng 2 2016,
"minhtenladuong@yahoo.com [Quadri-Figures-Group]" Ä‘ã viáº¿t:
Dear geometers,
Given 5 points A,B,C,D,P.
Let Ha,Hb,Hc,Hd be orthocenters of BCD,CDA,DAB,ABC.
A'=H_a/P wrt triangle BCD
B'=H_b/P wrt triangle CDA
C'=H_c/P wrt triangle DAB
D'=H_d/P wrt triangle ABC
Then A',B',C',D' are collinear (L)
Denote the line L by T(P,ABCD), I found some properties:
1) If P varies on a line d, T(P,ABCD) passes through a fixed point.
I don't know how the fixed point relates to the line d ?
2) If P varies on a fixed circle, envelope of T(P,ABCD) is a conic.
3) T(P,ABCD)=T(P,H_aH_bH_cH_d)
Construct the inverse of the transformation:
Given a line L. S varies on L.
H_a/S (wrt BCD), H_b/S(wrt CDA),H_c/S(wrt DAB), H_d/S(wrt ABC) varies on four conics, they are concurrent at P that T(P,ABCD)=L
Best regards
Ngo Quang Duong

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Message: #1515
Date: 16/2/2016 4:14:46
From: tsihonglau
Subject: Dual curves

Dear all,
Some discussions till now appear in a Paris Pamfilos'
page - Generalized isogonal transformations
http://www.math.uoc.gr/~pamfilos/eGallery/problems/IsogonalGener」
alized.html
His page is somewhat obscure and hard to understand!
Be patient! I summarize the discussions and Pamfilos' idea.
1. Two(in fact three) basic transformations:
x <-> 1/x trilinear pole <-> polar
x <-> 1/x F=t_K(Pamfilos) or QA-Tf2(EQF)
x <-> 1/x F*(Pamfilos) or QL-Tf2(EQF)
We get a point-line duality with composites of the first and the
second or third transformations. The first is
defined with a triangle/trilateral and the second and third with
a quadrangle/quadrilateral. In other words, we get only
trilinear polarity with a triangle/trilateral but duality with a
quadrangle/quadrilateral. And the duality is natural -
coordinates and coefficients are the same! If a point is a
triangle center and then its dual is an involutary axis.
For example, the dual of a triangle centroid is Newton line.
Because coordinates and coefficients are the same, the
projective properties of points and their duals - lines
are the same.(I forgot to mention this in a message of Merge
Quadrangle/Qudrilateral Figures).
For example, triangle centroid,circumcenter,orthocenter,
nine-point center lie on Euler line, so their duals(involutary
axes) concur at "Euler point"! Some points lie on a conic,
their duals lie on another conic.
Some points lie on a point-cubic,
their duals lie on a line-cubic (equivalently a point-sextic!)
2. Four coordinate systems
projective coordinate system(Pamfilos)
1:0:0,0:1:0,0:0:1,1:1:1
barycentric-CT coordinate system(EQF)
1:0:0,0:1:0,0:0:1,p:q:r
barycentric-DT coordinate system(EQF)
-p:q:r,p:-q:r,p:q:-r,p:q:r
circular coordinate system(me)
-1:1:1,1:-1:1,1:1:-1,1:1:1
The former two are similar and the latter two are similar.
In fact, the above are all
projective coordinate systems.
Their projective base (quadrangle) have different coordinates.
 The last is better because it use units and

is symmetric. The former two are not symmetric - the last point have different coordinates. Moreover, the base of the latter two are invariant under QA-Tf2 and QL-TF2. So the last is the most natural and simplest.
3. The relationship of a quadrangle and its autopolar -a quadrilateral is not mentioned in Pamfilos's page and the EQF.
 We can deduce the relationship easily with Pamfilos' idea. But he does not mention it in his page.
I give the relationship and the construction clearly.
I state clearly we should merge quadrangle/quadrilatateral figures! It is very important!
Best regards,
Tsihong Lau

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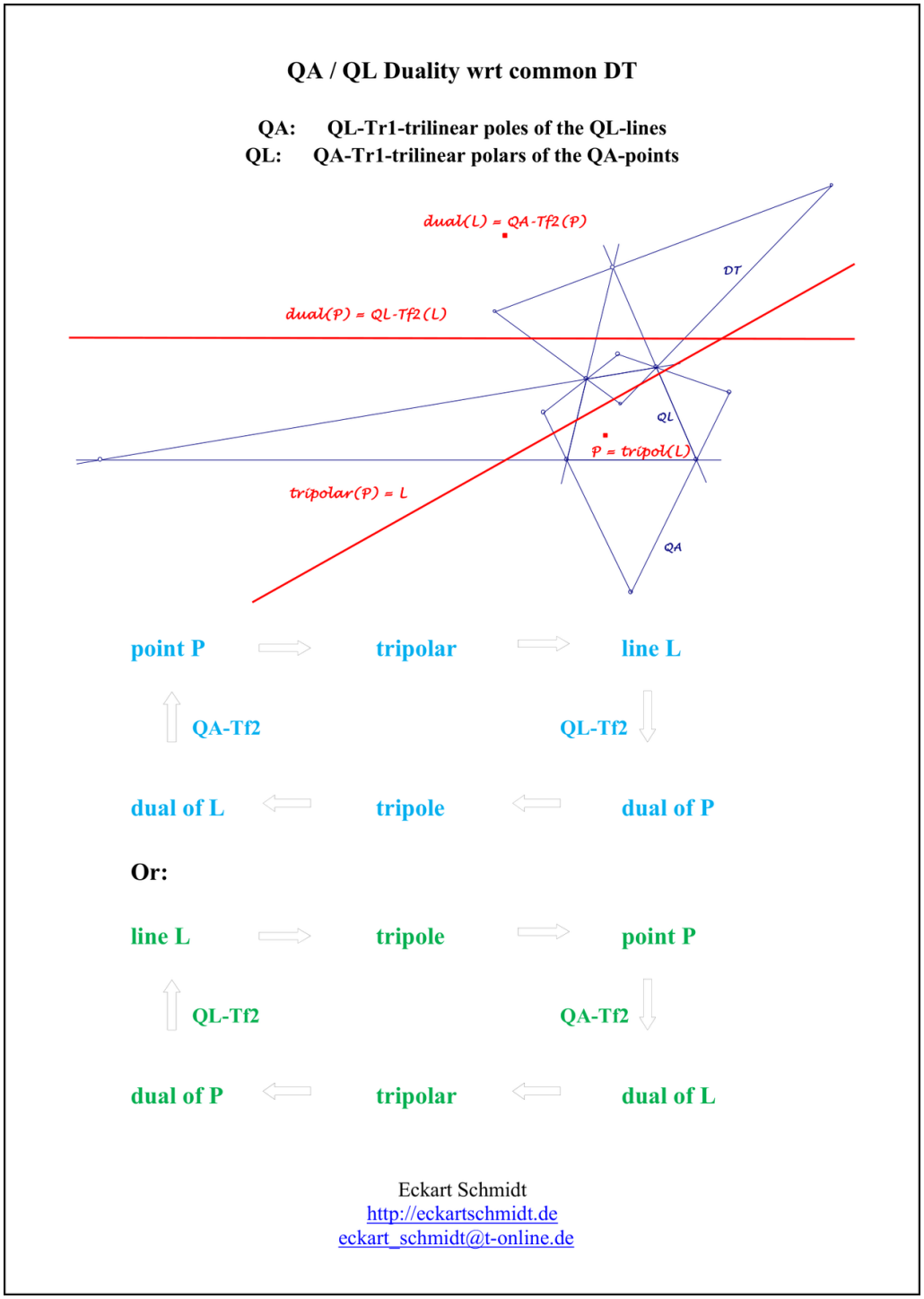
Message: #1516
Date: 16/2/2016 8:29:57
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard, dear Chris,
perhaps helpful for duality, see attached file.
Best regards Eckart



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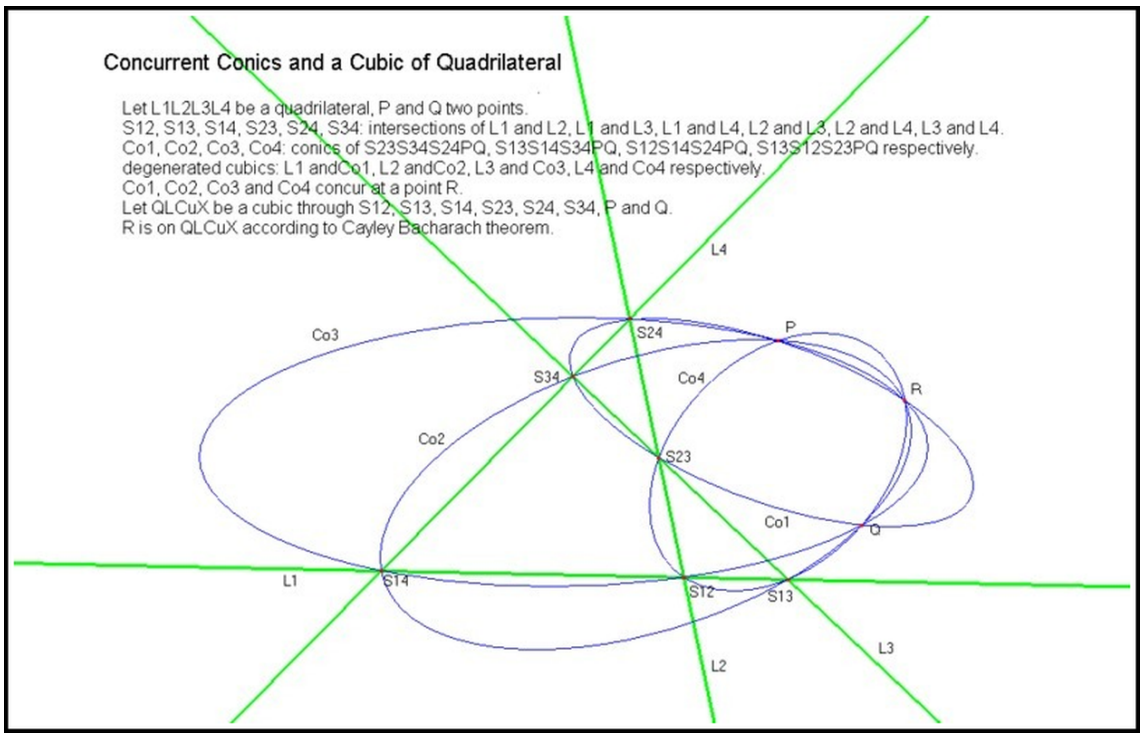
Message: #1517
Date: 17/2/2016 12:32:50
From: seiichikiri
Subject: Concurrent conics and a cubic of quadrilateral

Dear friends,
Let L1L2L3L4 be a quadrilateral, P and Q two points.
Denote:
S12, S13, S14, S23, S24, S34 = the intersections of L1 and L2, L1 and L3, L1 and L4, L2 and L3, L2 and L4, L3 and L4 respectively.
Co1, Co2, Co3 and Co4 = the conics of S23S34S24PQ, S13S14S34PQ, S12S14S24PQ, S13S12S23PQ respectively.
(Degenerated cubics = L1 and Co1, L2 and Co2, L3 and Co3, L4 and Co4.)
Co1, Co2, Co3 and Co4 concur at a point R.
Let QLCuX be a cubic through S12, S13, S14, S23, S24, S34, P and Q.
R is on QLCuX according to Cayley Bacharach theorem.
See the attached picture.
Best regards, Seiichi.



ConcurrentConicsQL.png

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Message: #1518
Date: 17/2/2016 11:55:01
From: eckart_schmidt@t-online.de
Subject: Concurrent conics and a cubic ...

```
Dear Seiichi,
in addition:
... For a QL: If P, Q are CSC-partners on QL-Cu1,
    then their common tangential on QL-Cu1 is R.
... For a QL: If P, Q are QL-Cu1-points
    and PQ parallel QL-L1, then R = QL-P1.
... For a QG: If P, Q are QA-Cu1-points and isogonal conjugates
    wrt the Miquel triangle, then R = QL-P1.
... For a QL: If P, Q are QL-Cu1-points collinear with QL-P1,
    then R is the point at infinity of QL-L1.
... For a QG: If P, Q are QA-Tf2 partners on QA-Cu1,
    then R = QG-P1.
Best regards Eckart
```

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Message: #1519
Date: 17/2/2016 12:11:27
From: bernard.keizer
Subject: Dual curves

Dear Eckart, dear Chris, dear Tsihong Lau

Rereading the messages 1499 and 1503 and 1505 and of course the 2 last important messages 1515 and 1516, I feal a little ashamed, as I didn't recognise obvious properties :
QL-P13 or QA-P16 is QA-Tf2(QL-P8), the Newton Line is QL-Tf2(infinity line), the tripolar of QL-P8 is the infinity line, the tripolar of QL-P13 is the Newton Line, the dual of QL-P8 is the Newton Line and the dual of QL-P13 is the infinity line.
Therefore, the dual of my 5th conic (DT circumconic through QL-P8 and QL-P13) is a DT-inscribed conic, tangent to the Newton Line and to the infinity line : it's the parabola DQL-Co1 (anticomplement of QL-Co3).
The same way, the dual of my 6th conic (QA circumconic through QL-P13)is a QL-inscribed conic, tangent to the infinity line : it's the parabola QL-Co1
As written in the holy book, they have eyes and they don't see ...

Best regards
Bernard

PS I'm always convinced that we won't avoid the systematic calculation of QA-Tf2 of all the points and QL-Tf2 of all the lines as well as the duals of all the points and lines of QA/QL ...

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Message: #1520
Date: 17/2/2016 12:51:48
From: seiichikiri
Subject: Concurrent conics and a cubic ...

Dear Eckart,
Thank you very much for your detailed exposition.
As I think that Cayley Bacharach theorem is one of fundamental tools in geometry, I hope that Chris will include it in QFG explicitly.
Best regards, Seiichi.

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Message: #1521
Date: 17/2/2016 4:56:03
From: bernard.keizer
Subject: Dual curves

Dear Eckart, dear Chris, dear Tsihong Lau
This is a follow-up of my previous message 1519.
The dual lines of the 3 vertices of QL-Tr2 (socalled S-points) are the opposite sides of this triangle ! (this triangle is self-dual ?)
As my 5th and 6th conic are through the S-points and through QL-P13, the 4 common tangents to the dual conics QL-Co1 and DQL-Co1 are the sides of QL-Tr2 and the infinity line !
As the Dimidium circle is through the S-points and through QL-P1, QL-P17 and QL-P24, its dual conic is inscribed in QL-Tr2 and tangent to the dual lines of QL-P1, QL-P17 and QL-P24.
The dual lines of QL-P17 and QL-P24 are also tangent to the dual conic of the circumcircle of DT QL-Ci1 (DT inscribed conic described by Eckart)
Best regards
Bernard
PS May be this conic DQL-Co1 (again anticomplement of QL-Co3, with focus QL-P17 and directrix QL-L6) deserves with this property wrt QL-Tr2 an interest in EQF ?

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Message: #1522
Date: 18/2/2016 10:24:05
From: bernard.keizer
Subject: Dual curves

Dear Eckart, dear Chris, dear Tsihong Lau
Last message with this serie of evidences linked to the QA-Tf2 partners QL-P8 and QL-P13 and their duals, the QL-Tf2 partners Newton Line and infinity line.
Considering this time my 6th conic and the dual of QL-Co2 (QA circumscribed conics through QA-Tf2 partners QL-P8 and QL-P13), the line QL-P8QL-P13 is tangent to these 2 conics.
Considering their duals QL-Co1 and QL-Co2, they intersect in the dual of the line QL-P8QL-P13, which is the intersection of the QL-Tf2 partners Newton Line and infinity line, id est the infinity point of the Newton Line (this line is parallel to the axis of the inscribed quadrilateral parabola and tangent to the inscribed midline hyperbola)
Best regards
Bernard
PS I dream now of a beautiful table with the QA-Tf2 and the dual lines of all the points and the QL-Tf2 and the dual points of all the lines of QA/QL ...

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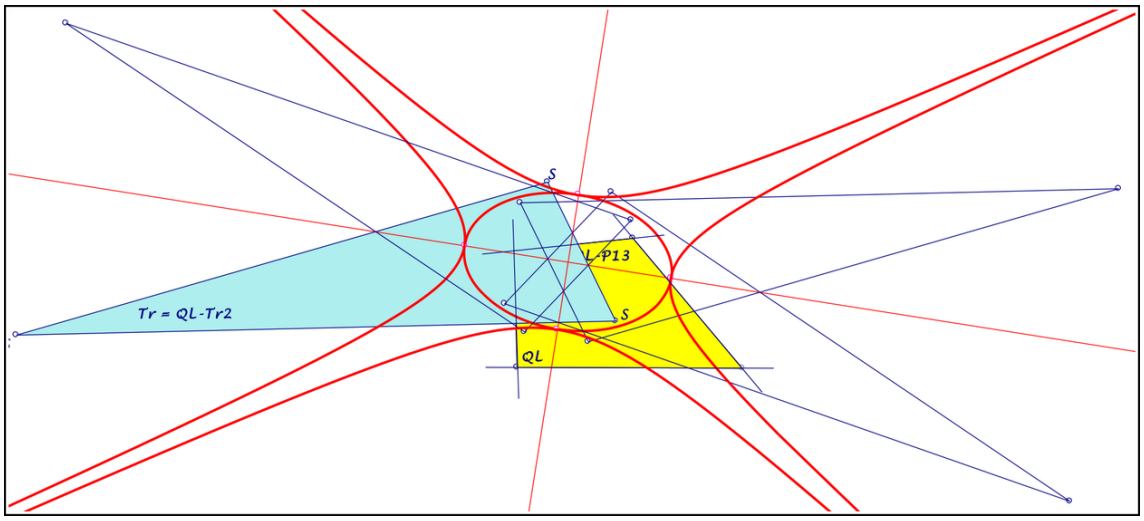
Message: #1523
Date: 18/2/2016 8:49:20
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
there are three self-dual conics,
... all centered in QL-P13,
... an ellipse and two hyperbolas
... with the same axes and the same asymptotes (see attached file).
The conics can be constructed, using the triangle Tr = QL-Tr2 (S-points):
... The common axes are the axes of the dual curve of an arbitrary circle round QL-P13.
... The polars of the S-points wrt the searched ellipse are the reflections of the opposite Tr-side in QL-P13.
... The polars of the S-points wrt a searched hyperbola are the reflections of the opposite Tr-side in one axis.
This allows a construction. I think, these three conics are the only self-dual conics.
Best regards Eckart



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Message: #1524
Date: 20/2/2016 10:51:34
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
in addition to #1523:
... The foci of the self-dual hyperbolas are concyclic
... on the inversion-circle of QL-P17 and QL-P24 wrt QL-P13.
... The common asymptotes are the diagonals of the circumscribed rectangle of the self-dual ellipse.
I have to complete my last message: There are further two self-dual circles. I hope, to describe them in the next message. It seems, that they are centered on the minor axis of the self-dual ellipse, tangent to a corresponding hyperbola.
Best regards Eckart

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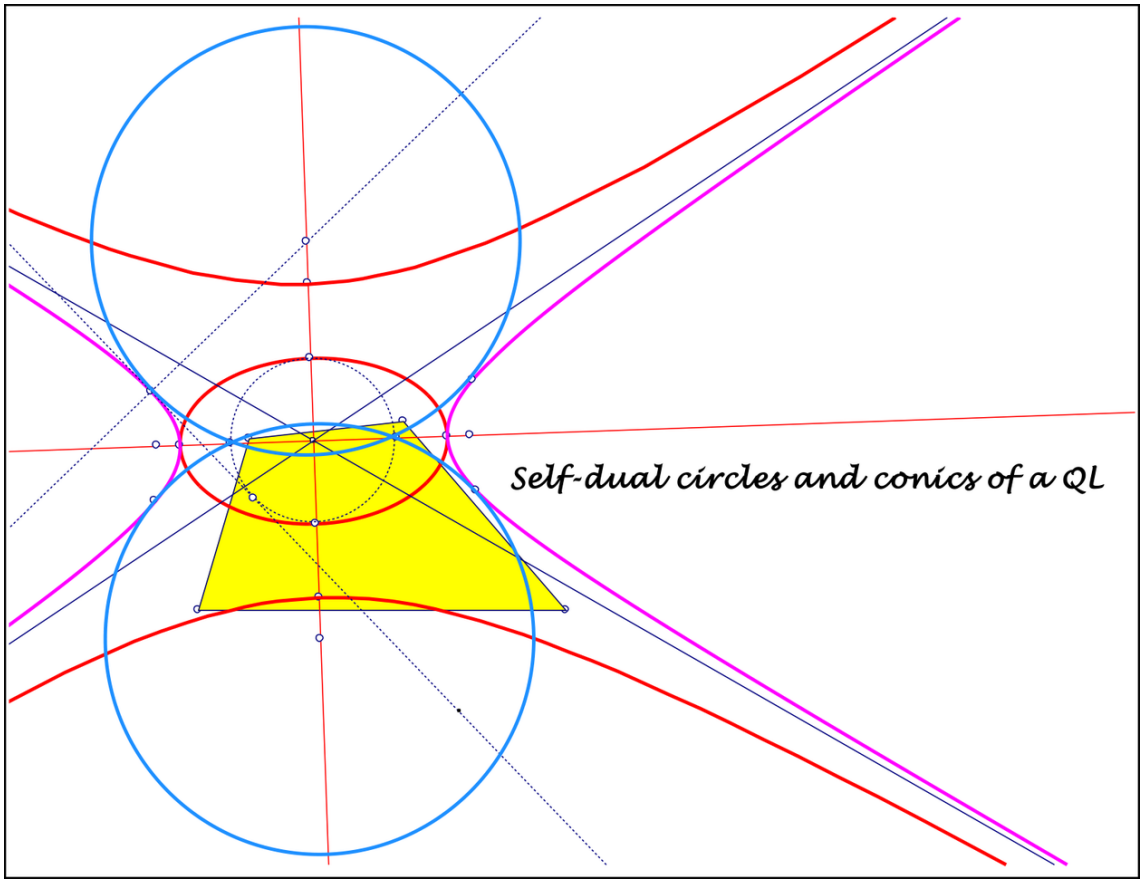
Message: #1525
Date: 20/2/2016 3:12:58
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
in addition to my messages 1523 and 1524:
There are two self-dual circles
(see attached file):
... centered on the minor axis of the self-dual ellipse El,
... tangent to the self-dual hyperbola Hy with the same main axis as El,
... containing the intersections of the inscribed circle of El and the main axis.
Construction:
The Hy-normal in a contact point of a common tangent with El cuts the minor axis of EL in a center of a self-dual circle.
Best regards Eckart



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Message: #1526
Date: 21/2/2016 3:14:37
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

Dear all,
The page is updated.
Center/Axis of Quadrangle/Quadrilateral and Central Triangle/Axial Trilateral of Quadrangle/Quadrilateral (http://l
th.name/geometry/center_axis_of_quadrangle_quadrilateral.html)
The first column shows the first circular coordinate/coefficient of center/axis of quadrangle/quadrilateral. The second column shows the first circular coordinate/coefficient of center/axis of orthocentric quadrangle (i.e. incenter/excenters of diapleural triangle/diagonal trilateral), The last remark column shows how to derive the center/axis of quadrangle/quadrilateral.
Now they are all derived from the operation of a triangle center and a involutary center(or a triangle axis and a involutary axis) with respect to the diapleural triangle/diagonal trilateral. For example, the center/axis [(qv+rw)/(qw+rv)] is the crossmul of [1/(qw+rv)]=X(4) and [1/(pu(qw+rv))]=involutary conjugate of X(3). For an orthocentric quadrangle, it is X(4) of the diapleural triangle/diagonal trilateral.
Never forget, it represent an axis of the same coefficient.
I will upload the triangle axis page and give the definitions of operations concerned later. In the page, we can get some generalizations of triangle centers/axes. For example, [(qv+rw)/(qw+rv)],[(qv+rw)/(qw-rv)],[pu(qv+rw)(qw+rw)] can be treated as X(4),X(2),X(3) respectively. They are collinear for a general quadrangle/quadrilateral, so they lie on a generalization of Euler line. So the special Euler line inherits some properties of this general Euler line. It is not the only generalization of Euler line.
[1/(pu(qv+rw)(qw+rw))]-[1/(pu(qv+rw)(qw-rv))]-[(qw+rv)/(qv+rw)] is another example.
(There are many other generalizations of Euler line.) The special Euler line inherit some properties of the latter general Euler line. In other words, we can analyze the properties of the special Euler line with the geometry of quadrangle/quadrilateral.
To be continued...
Best regards,
Tsihong Lau

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Message: #1527
Date: 21/2/2016 10:26:43
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Bernard,
in #1525 self-dual circles are described: Let r be the radius and d the distance of the center and QL-P13.
There are circles, whose dual is also a circle:
... centered on the minor axis of the self-dual ellipse,
... with the same ratio d / r as the self-dual circle.
A pair of dual circles
... is homothetic wrt QL-P13
... with radius product r1r2 = r^2
... and centers inverse to a circle round QL-P13 through the center of the self-dual circle.
Best regards Eckart

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Message: #1528
Date: 21/2/2016 3:26:49
From: bernard.keizer
Subject: Dual curves

Dear Eckart,
I find all these constructions fascinating and I wonder how you find them !
I tried to calculate the equations of cubics for which the dual of the points are tangents, but I didn't succeed ...
I have plenty of other ideas, but no time to dig them !
(I won't be home next week)
For example, allways the duals of the monocardioïd (it's a cubic) and of the deltoïd and the 27 multi cardioïds (they are cubics through the QA-vertices)
The dual of the cayleyan should also be a cubic ...
More simple : is the dual of QL-P26 a least square line for the QA-vertices ?
QA-P23 is QA-Tf2(QA-P12 or QL-P10) ; it's dual is the ortholine of DT ...
Now, what are the QA-Tf2 of the incenter, the circumcenter, the Euler center, the symmedian point ...of DT ?
Conversely, are the QA-Tf2 of the main points of QA/QL well-known points of DT ?
I suppose, this is the approach of Tsihong Lau, but I'm not sure we need the circular coordinates.
Best regards
Bernard

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Message: #1529
Date: 21/2/2016 4:29:24
From: minhtenladuong
Subject: Locus property of QA-Cu1

Dear Geometers,
I found a nice property of QA-Cu1 as follow:
Given a quadrangle A, B, C, D and a point P.
It is known that for all P, pedal circles of P wrt BCD, CDA, DAB, ABC are concurrent at a point Q.
Then Simson lines of Q wrt pedal triangles of P wrt BCD, CDA, DAB, ABC are concurrent if and only if P lies on QA-Cu1.
Best regards
Ngo Quang Duong

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Message: #1530
Date: 21/2/2016 5:52:23
From: chris.vantienhoven
Subject: Dual curves

Dear Eckart, Bernard and Tsihong Lau,
Eckart, beautiful picture and scheme in message #1516 !
Alas I have limited time to explore the beautiful item of "Quadri-Duals" these days.
Like Bernard commented there is a difference between Triangle-Duals and Quadri-Duals.
The familiar Triangle-Duals actually are:
(TR-)Trilinear Pole (line>point)
(TR-)Trilinear Polar (point>line)
The Quadri-Duals involved are fourfold:
QA-Tf7=QA-Quadrilinear Pole = QA-Tf2[DT-Tripole[{l, m, n}]]
QA-Tf8=QA-Quadrilinear Polar = DT-Tripolar[QA-Tf2[{p, q, r}]]
QL-Tf3=QL-Quadrilinear Pole = DT-Tripole[QL-Tf2[{l, m, n}]]
QL-Tf4=QL-Quadrilinear Polar = QL-Tf2[DT-Tripolar[{p, q, r}]]
Using these transformations we can describe all process steps in the description of Duals in a Quadri-environment preserving the symmetric expressions of QA-objects and QL-objects.
Also in a Quadrangle we can refer to the QA-Trilinear Transformed Quadrilateral and in a Quadrilateral we can refer to the QL-Trilinear Transformed Quadrangle.
In terms of EQF this makes it a lot easier to discuss (I hope).
Best regards,
Chris van Tienhoven

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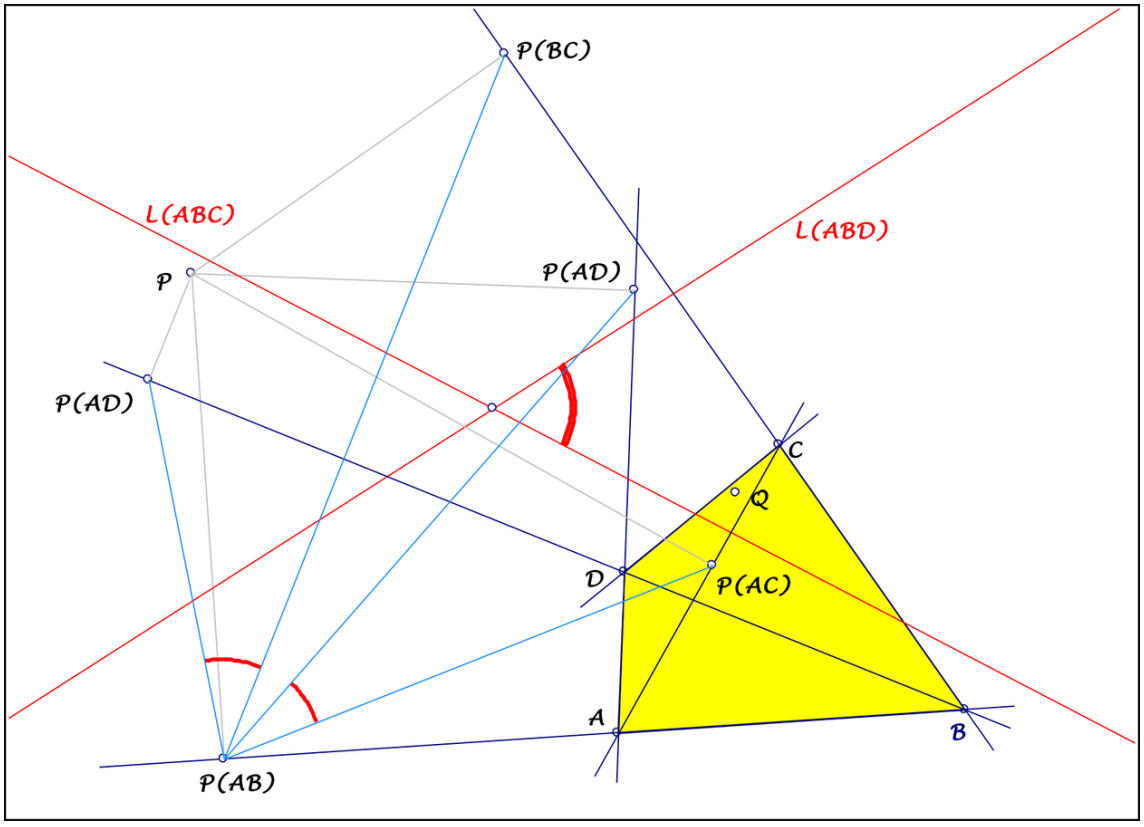
Message: #1531
Date: 22/2/2016 10:51:41
From: eckart_schmidt@t-online.de
Subject: Locus property of QA-Cu1

Dear Ngo Quang Duong,
that is really a nice observation with the following angle property:
The angles between the Simson lines of Q are constant, changing the point P (not only on QA-Cu1).
Let P(XY) be the pedal point of P wrt XY and L(XYZ) the Simson line of Q wrt triangle XYZ:
< (L(ABC), L(ABD)) = <(P(AC), P(AB), P(AD)) + <(P(BC), P(BA), P(BD))
Best regards Eckart



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Message: #1532
Date: 22/2/2016 11:17:31
From: eckart_schmidt@t-online.de
Subject: Dual curves

Dear Chris,
I doubt, whether your nomination will make it easier, for the transformations QA-Tf7 and QL-Tf3 are the same as well as QA-Tf8 and QL-Tf4, if we consider constellations of corresponding QA and QL in the sense of #1482.
Best regards Eckart

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Message: #1533
Date: 22/2/2016 2:37:03
From: eckart_schmidt@t-online.de
Subject: Central points

Dear Benedetto,
I found in your paper (EQF-Ref [36]) very interesting properties and relationships wrt the points
G = QA-P1, H = QA-P2, J = QA-P3, O = QA-P4.
You are interested in a reconstruction of the quadrangle. Not quite in your sense, but:
(1) A quadrangle is completely determined by
... the Miquel triangle
... and O = QA-P4.
The isogonal conjugate of O wrt the Miquel triangle gives J = QA-P3.
QA-Cu1 is an isogonal pivotal cubic of the Miquel triangle wrt a pivot in the point at infinity of OJ = QA-P3.QA-P4.
The contact points of tangents from O = QA-P4 at QA-Cu1 give the vertices of the quadrangle.
(2) A quadrangle is completely determined by
... the diagonal triangle
... and O = QA-P4.
The diagonal triangle D = QA-Tr1 gives the points OD = QA-P11 and HD = QA-P12.
The reflections of O.HD in the D-sides give H = QA-P2 (3.4.4 in your paper).
The reflection of H in OD gives QA-P30.
The midpoint of H and O gives QA-P6.
QA-P6 and QA-P30 define as image partners the isoconjugation QA-Tf2 wrt D.
The fixed points of QA-Tf2 are the vertices of the quadrangle.
So far a first look in your paper.
Best regards Eckart
PS: Perhaps there is an unknown property for you on my homepage: http://eckartschmidt.de/Pktve.pdf

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Message: #1534
Date: 23/2/2016 2:24:32
From: minhtenladuong
Subject: Locus property of QA-Cu1

Dear Eckart Schmidt and Geometers
Thanks for your message.
I have just found a similar locus property and another:
Let Pxy be orthogonal projection of P on XY.
1) Simson lines of P wrt PabPacPad, PbcPbdPba, PcdPcaPcb, PdaPdbPdc are concurrent iff P lies on QA-Cu1. Let the concurrency point be P'
2) P lies on QA-Cu1. Simson lines of Q wrt pedal triangle of P wrt BCD, CDA, DAB, ABC are concurrent at Q'.
It is interesting that PP' || QQ'.
I also have an open question: When P varies on QA-Cu1, which curve that Q varies on?
Best regards
Ngo Quang Duong

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Message: #1535
Date: 02/3/2016 11:47:55
From: bernard.keizer
Subject: QA-Tf2 of the S-points

Dear Eckart, dear Chris
I'm back home and find no activity in the last 7 days, which is rather sad !
1) For Chris : I hope you will find time to put in EQF this simple property (which doesn't need the duality).
The parabola with focus QL-P17 and directrix QL-L6 is
a) the anticomplement of QL-Co3 wrt DT
b) inscribed in QL-Tr1 (already in EQF)
c) *also inscribed in QL-Tr2* (not yet in EQF)
2) For Eckart : you didn't answer my last message with many questions ...
a) I didn't realise that the circle you mention with center QL-P13 swapping QL-P17 and QL-P24 leaves the Dimidium circle globally invariant and swaps also QL-P1 and CSC(QL-P26)
b) considering again the 6th conic through the points Q, Q2, Q3, Q4 and QL-P13, it cuts the Dimidium circle in 4 points : the 3 S-Points and CSC(QL-P26)
c) the QA-Tf2 of the 3 S-points are on the line QL-P8QL-P13, intersections of the tangents in the S-points to the 6th conic and the tangent to QL-P13 to the same conic (which is trough QL-P8, as this point is QA-Tf2 of QL-P13)
Best regards
Bernard
PS I put again my last question : is it possible to calculate a line of QA such as the sum of the squares of the distances from the vertices of QA to this line is a minimum ? This would be the dual property of QL-P26 for the lines of the QL.
(I've tried to draw the dual line of QL-P26, but without success until now)

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Message: #1536
Date: 02/3/2016 3:44:15
From: eckart_schmidt@t-online.de
Subject: Central Points

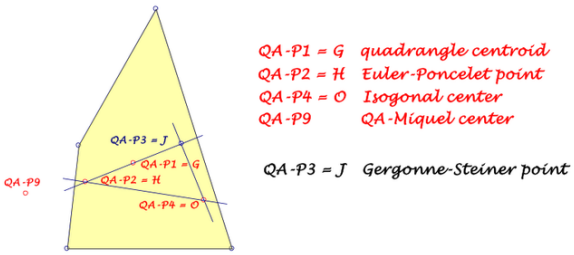
Dear Benedetto,
many thanks for your privat mail with the history of your question wrt quadrangle reconstruction. As we supposed, the four points QA-P1, QA-P2, QA-P4 and QA-P9 will also allow a reconstruction for a quadrangle, see attached file. Unexpected the construction doesn't need the Miquel triangle, it is based on the circumconics QA-Co2 and QA-Co3, independant whether the quadradrangle is convex or not.
Best regards Eckart.

EQF-Note 2016-03-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Reconstruction of a Quadrangle

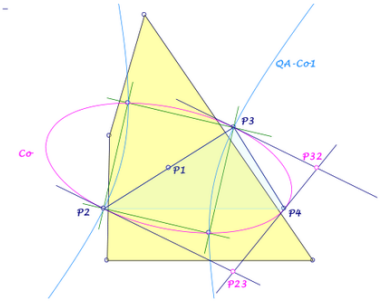
Scimemi researched in his work "Central Points of the Complete Quadrangle" (EQF-Ref.[36]) the question, how to reconstruct a quadrangle with four points. His result: G = QA-P1, H = QA-P2, O = QA-P4 and O_D = QA-P11 are such central points. Here another set is described, replacing the fourth point by the QA-Miquel Center QA-P9.



- **A quadrangle can be reconstructed from the set of central points {*QA-P1, QA-P2, QA-P4, QA-P9*}.**

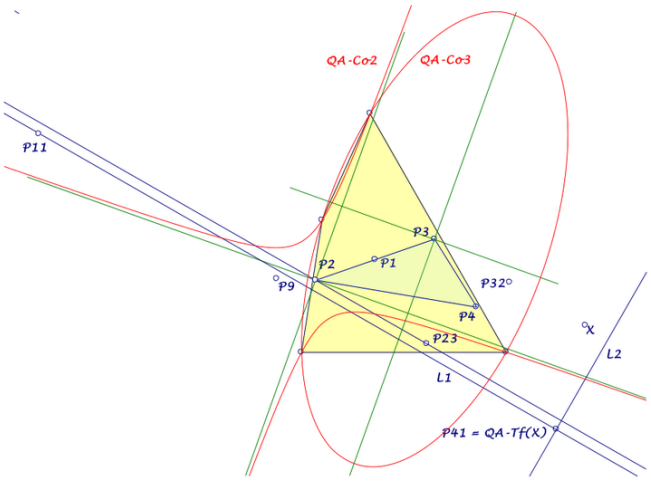
Below we shall omit the prefix *QA* for the points.

Constructions wrt *P1, P2, P4* and *P3* (reflection of *P2* in *P1*):



… The angle bisectors of ∢*P2.P3.P4* are
… … the axes of *QA-Co3* (centered in *P3*),
… … parallel to the asymptotes of *QA-Co2* (centered in *P2*).
… These four lines give a rectangle.
… Let *Co* be a circumscribed conic of the rectangle through *P4*.

… *Co*-tangents in *P2* and *P4* intersect in *P23*.
… *Co*-tangents in *P3* and *P4* intersect in *P32*.
… (The circumscribed conic of the rectangle, which intersects *Co* perpendicular is *QA-Co1*.)



Additional constructions with *P9* and *P23*, *P32* (see above):
… With *P9* the *QA*-Möbius conjugate *QA-Tf4* is defined, centered in *P4* swapping *P3* and *P9*.
… Let X be the reflection of *P3* in *P32*, then *QA-Tf4(X) = P41*.
… Let L_1 be parallel to *P2.P23* through *P41* (containing *P11*).
… Let L_2 be perpendicular to *P2.P23* through *P41*.
… *P4* and L_1 determine as pole-polar-pair *QA-Co2* (asymptotes and center see above).
… *P4* and L_2 determine as pole-polar-pair *QA-Co3* (axes and center see above).
… Intersections of *QA-Co2* and *QA-Co3* give the searched quadrangle.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-03-02.pdf

Message: #1537
Date: 02/3/2016 4:09:15
From: tsihonglau
Subject: Merge Quadrangle/Quadrilateral Figures

```
Dear all,
1. A new page is uploaded.
Class of Objects of Triangle/Trilateral and
Quadrangle/Quadrilateral ( http://lth.name/geometry/class.html )
I give the definitions of many classes of points and lines of
triangle/trilateral and quadrangle/quadrilateral.
2. The page is updated!  Now more than 100 objects.
I will construct them using GeoGebra later.
Center/Axis of Quadrangle/Quadrilateral and Central
Triangle/Axial Trilateral of Quadrangle/Quadrilateral ( http://l
th.name/geometry/center_axis_of_quadrangle_quadrilateral.html )
To be continued...
Best regards,
Tsihong Lau
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Message: #1538
Date: 03/3/2016 12:19:46
From: eckart_schmidt@t-online.de
Subject: Dual Curves

```
Dear Bernard,
excuse, that I not answered in the last time. But I studied
Benedetto's paper about "Central Points", a very interesting
excursion in QA-geometry!
Here only a short answer wrt the dual of QL-P26:
Cabri-observations show, that it is not the least square line
for the QA-vertices.
Best regards Eckart
```

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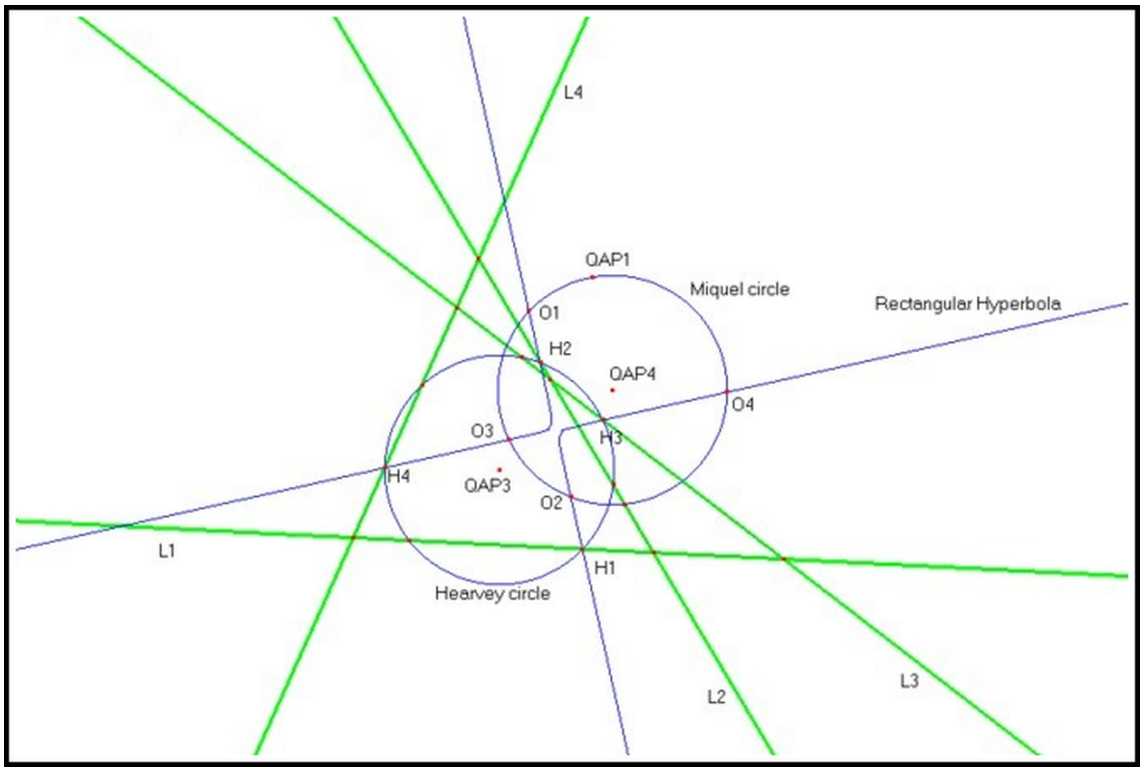
Message: #1539
Date: 04/3/2016 6:46:47
From: seiichikiri
Subject: A rectangular hyperbola in Quadrilateral

Dear Chris,
Is the following already known in quadrilateral?
4 circumcenters which determine Miquel circle and 4 orthocenters which determine Hearvey circle are on a rectangular hyperbola.
See the attached picture.
Best regards, Seiichi.



RectangularHyperbola.png

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Message: #1540
Date: 04/3/2016 9:24:41
From: eckart_schmidt@t-online.de
Subject: A rectangular hyperbola in Quadrilateral

Dear Seiichi,
an interesting for me unknown hyperbola,
... with center QL-P5,
... contacting a QL-inscribed hyperbola with the same center on QL-P1.QL-P5.
There will be more properties.
Best regards Eckart

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Message: #1541
Date: 04/3/2016 11:38:52
From: bernard.keizer
Subject: A rectangular hyperbola in Quadrilateral

Dear Seiichi,
I'm not able to open your file (please, could you use pdf.files)
I suppose nevertheless that your orthocenters are the orthocenters of the triangles formed by the circumcenters of the reference triangles.
In this case, your property is obvious, as the point QL-P5 is the middle of each segment OiHi, Hi being the orthocenter of OjOkOl (this property is in EQF under QL-P5).
By the way, the mentionned rectangular hyperbola is described on figure 12 in the 2nd article on my blog.
Best regards
Bernard

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Message: #1542
Date: 04/3/2016 11:57:24
From: seiichikiri
Subject: A rectangular hyperbola in Quadrilateral

Dear Eckart, dear Bernard,
Thank you very much for the information.
I am now considering an application of the degenerated Cayley Bacharach theorem with Hearvy circle and the rectangular hyperbola. If I will obtain something, I will post it.
B. S. Bernard's interpretation is right.
Best regards, Seiichi.

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Message: #1543
Date: 04/3/2016 2:47:00
From: tsihonglau
Subject: A rectangular hyperbola in Quadrilateral

Dear all,

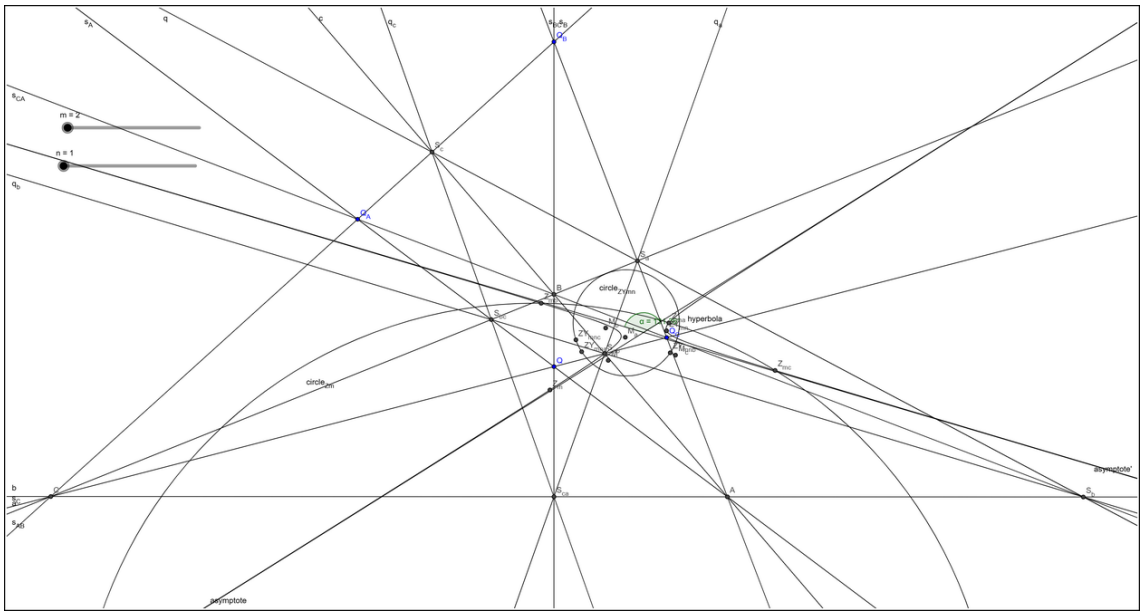
Seiichi, nice job! More observations here.
Given a quadrilateral q,q_a,q_b,q_c, we can get four exline trilaterals(my terminology):
    q_a,q_b,q_c
    q,q_b,q_c
    q_a,q,q_c
    q_a,q_b,q
These four trilaterals have triangle/trilateral centers X(m)'s
    Z_m
    Z_ma
    Z_mb
    Z_mc
These four centers form a quadrangle. Then we get four expoint triangles (my terminology):
    Z_ma,Z_mb,Z_mc
    Z_m,Z_mb,Z_mc
    Z_ma,Z_m,Z_mc
    Z_ma,Z_mb,Z_m
These four triangles have triangle/trilateral centers X(n)'s
    ZY_mn
    ZY_mna
    ZY_mnb

ZY_mnc
If the centers X(n)'s of the four triangles are X(4)'s, these eight centers all lie on the same rectangular hyperbola. Besides this, the centers X(m)'s of the four trilaterals are one of X(3),X(186),X(265),X(5962), then the four segments

Z_m-ZY_mn
Z_ma-ZY_mna
Z_mb-ZY_mnb
Z_mc-ZY_mnc

share the same midpoint, which is the center of hyperbola. Please refer to the attachment GeoGebra file. I do not verify the case of X(5962) bit it should be true. Please refer to the APG topic #3024 for more information. But I think we should follow in this group not APG.

Best regards,
Tsihong Lau



central_circles_of_exline_trilateral.ggb

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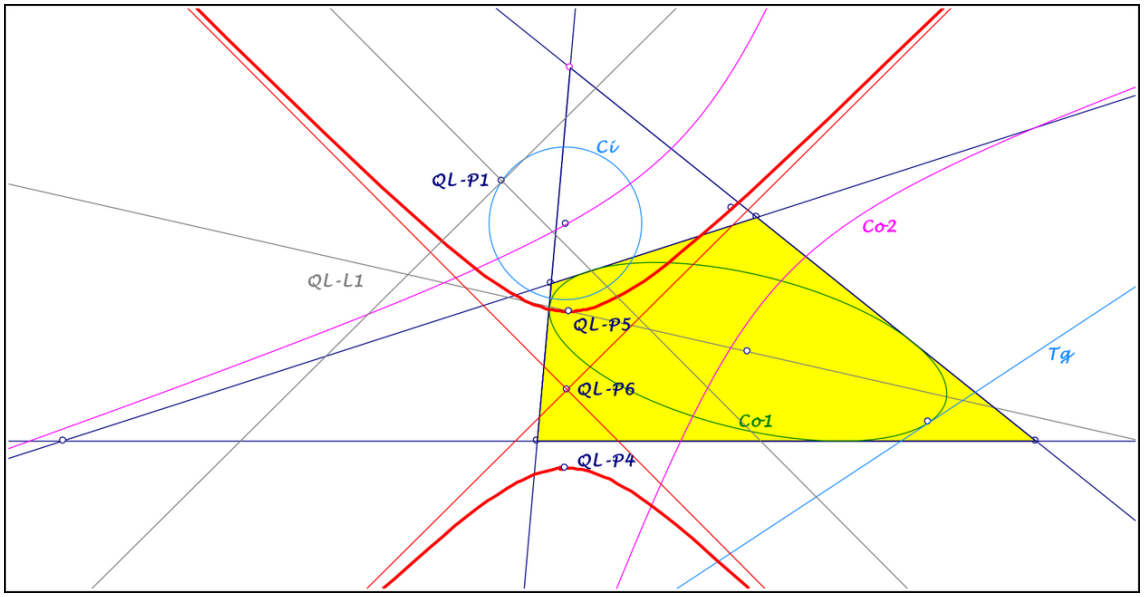
Message: #1544
Date: 04/3/2016 3:05:13
From: eckart_schmidt@t-online.de
Subject: Rectangular hyperbola in Quadrilateral

Dear Seiichi, dear Bernard,
Seiichi studies the rectangular hyperbola QA-Co2 of the O-quadrangle of a quadrilateral.
Circles round points on this hyperbola through QL-P1 give with their CSC-images tangents of a QL-inscribed conic centered in QL-P5.
Vice versa (see attached file):
Let Co1 be an inscribed QL-conic,
... the centers of the CSC-circles Ci for Co1-tangents Tg give a second conic Co2,
... and the centers of Co2 give a third conic,
... which is a rectangular hyperbola,
... ... centered in QL-P6,
... ... through QL-P4 and QL-P5
... ... and asymptotes parallel to the Steiner axes.
I think, this hyperbola isn't mentioned up to now.
Best regards Eckart



2016-03-04.docx

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Message: #1545
Date: 04/3/2016 6:58:39
From: bernard.keizer
Subject: Rectangular hyperbola in Quadrilateral

Dear Eckart,
Your last conic is the QA-Co1 of the O-quadrangle.
Best regards
Bernard

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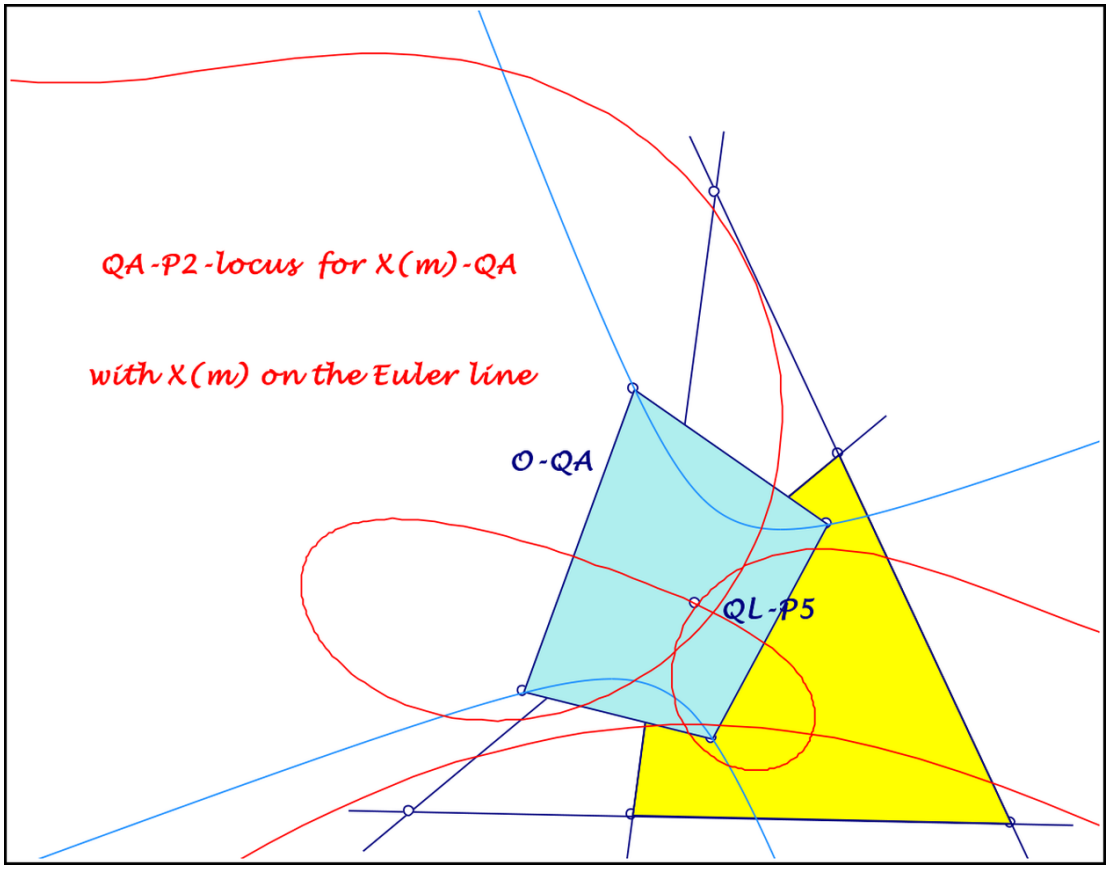
Message: #1546
Date: 05/3/2016 2:15:03
From: eckart_schmidt@t-online.de
Subject: A rectangular hyperbola in Quadrilateral

Dear Tsihong Lau, dear Seiichi, dear Bernard,
studying the message 1543:
(1) I found the following property:
For all ETC-points X(m) on the Euler line the centroids QA-P1 of the X(m)-quadrangles of a quadrilateral lie on QL-P2.QL-P6.
But for QA-P2 the locus is already a complicated curve through QL-P5 (see attached file). Seiichi will recognize his hyperbola.
The loci of QA-P3, 4, 8, 12, 32 contain the point QL-P4, for the X(3)-quadrangle is cyclic with center QL-P4 and the mentioned QA-points coincide with the center.
(2) Geometric background for the properties of X(3), X(186), X(265) is, that these X(m)-quadrangles of a quadrilateral are cyclic, and the X(4)-quadrangle of a cyclic quadrangle is its reflection in QA-P2.
Best regards Eckart
PS for Bernard: Thanks for information wrt my "last" hyperbola, you have the better QL-survey!



2016-03-05.docx

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Message: #1547
Date: 06/3/2016 5:29:13
From: bernard.keizer
Subject: A rectangular hyperbola in Quadrilateral

Lieber Eckart, dear Seiichi, dear Tsilong Hau,
Extraordinary generalisation !
1) The first property is well-known and may be explained with barycentric calculations : considering the 4 reference triangles of the QL, the centroïd of the orthocenters is QL-P2, the centroïd of the circumcenters is QL-P6 and the centroïd of the centroïds is the centroïd itself QL-P12.
The line QL-P2QL-P12 is the real Euler Line of the QL ; by the way, it is the Euler Line of the S-triangle QL-Tr2.
The generalisation for QA-points for any point on the Euler Line is very interesting : curves through QL-P5 for QA-P2, as QA-P2(X(3)quadrangle) is QL-P5 and through QL-P4 for QA-P3, as QA-P3(X(3)quadrangle) is QL-P4 ...
2) I Wonder why the QA is cyclic for X(3), X(186), X(265) and X(5962) ; I had a look in ETC and found only that :
X(3) is the isogonal of X(4)
X(186) is the inverse in the circumcircle of X(4)
X(265) is the isogonal of X(186)
X(5962) is the isogonal of the inverse in the circumcircle of X(265)
For X(4), the QA may be considered as cyclic with circumcenter the infinity point of the Newton Line, as the 4 orthocenters are on the line QL-L2 ...
3) For any QA or QL, there are 4 reference triangles Ti with i = 1 to 4
Taking the same point X(m) in each triangle gives a QA with vertices X(m)i and doing this a 2nd time with another ETC point X(n) gives a 2nd QA with vertices X(mn)i
There is no reason not to pursue the process ; we would find a 3rd QA with vertices X(mnp)i ...
Taking the same line in each reference triangle of the initial QA or QL gives a QL with 4 lines L(m)i and doing this a 2nd time gives a 2nd QL with 4 lines L(mn)i ...
It's also possible to take once the same point and then the same line ...
4) The generalisation to a 5-line doesn't seem possible, as there are 10 reference triangles ...
Best regards
Bernard

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Message: #1548
Date: 07/3/2016 12:24:34
From: bernard.keizer
Subject: Dual Curves

Lieber Eckart,
Thanks for the confirmation that the dual of QL-P26 is not the least square line for the QA-vertices.
I feared, I had made a mistake in the construction !
The mystery remains entire : which line is this least square line ?
Drawing and drawing, I've found an acceptable candidate, the trilinear polar of CSC(QL-P26), but I don't see why !
By the way, the QL-Tf2 of this line, id est the dual of CSC(QL-P26) seems to be tangent to the parabola QL-Co1 ...
Again, I don't know if the property is true and I have no idea of the reason !
I hope you will be able to help me ...
Best regards
Bernard
PS What about my other questions (dual curves of the cardioïd, deltoïd, cubic stelloïd and cayleyan) ?

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Message: #1549
Date: 07/3/2016 10:16:44
From: bernard.keizer
Subject: Dual Curves

Dear Eckart,
The night brings often new ideas !
As I told in a previous message, the point CSC(QL-P26) is the 4th intersection between the Dimidium circle and the socalled 5th conic.
This 5th conic through the QA-vertices and QL-P13 is the dual of the inscribed parabola QL-Co1 ; therefore, the dual line of CSC(QL-P26) or the QL-Tf2 of it's trilinear polar is tangent to QL-Co1.
The intersection point of the trilinear polar and the dual of CSC(QL-P26) is on the parabola QL-Co1 ; it's dual line is the tangent to the 5th conic in CSC(QL-P26), which carries also the QA-Tf2 of this point.
This tangent to the 5th conic in CSC(QL-P26) seems also to be an acceptable candidate for the least square line of the QA-vertices !
Best regards
Bernard

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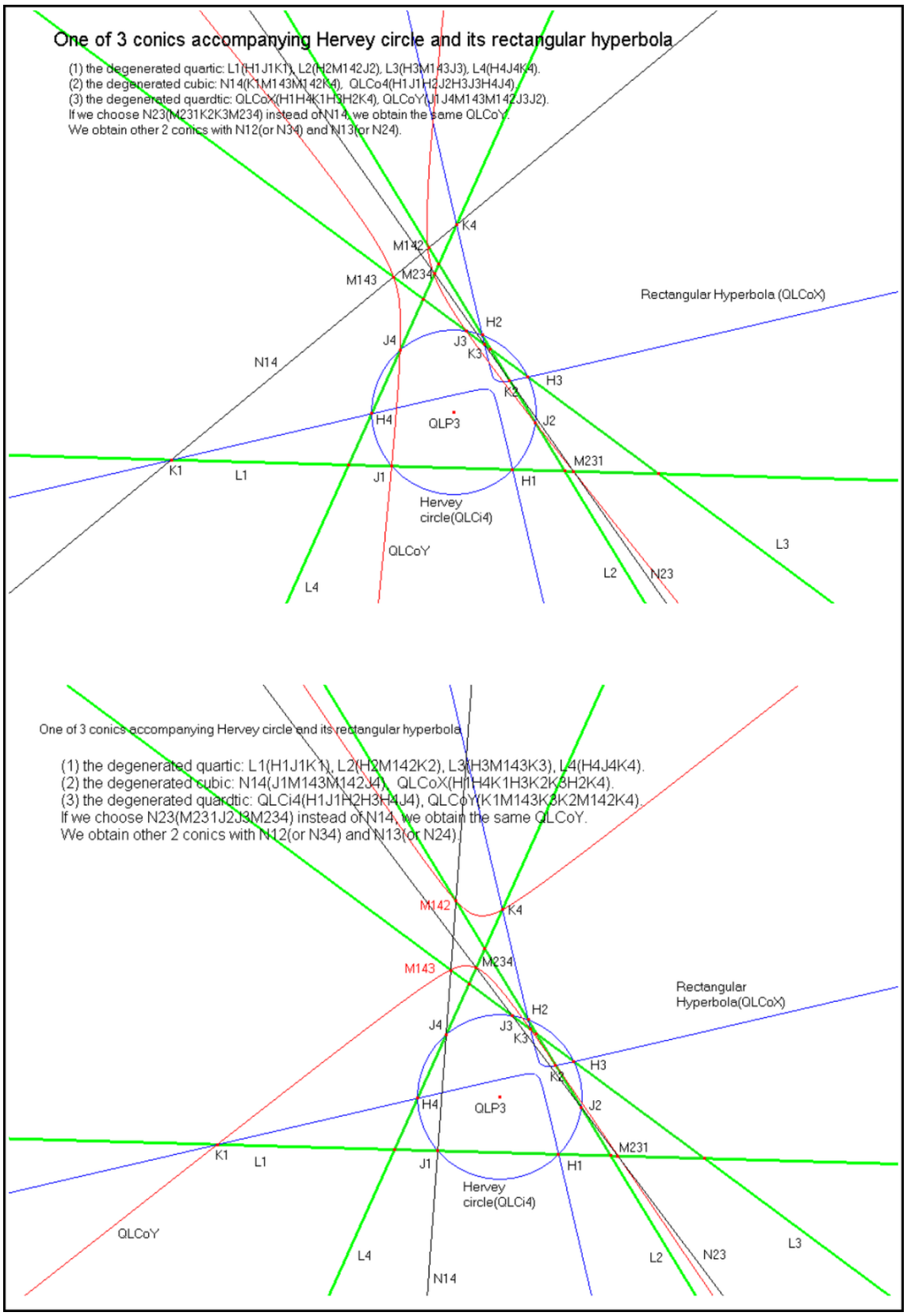
Message: #1550
Date: 09/3/2016 7:25:41
From: seiichikiri
Subject: 6 conics accompanying Hervey circle and its rectangular hyperbola

---

Dear friends,
Let L1L2L3L4 be a quadrilateral and
O1, O2, O3, O4 four circumcenters of 4 QL-Component Triangles.
Denote:
H1, H2, H3, H4= the orthocenters of QA-component quadrangle O1O2O3O4.
QLCi4 (Hervey circle)= the circle through H1, H2, H3, H4.
QLCoX= the rectangular hyperbola through O1, O2, O3, O4 and H1, H2, H3, H4.
J1, J2, J3, J4= the 2 nd intersections of QLCi4 with L1, L2, L3, L4 respectively.
K1, K2, K3, K4=the 2 nd intersections of QLCoX with L1, L2, L3, L4 respectively.
N14= the line through K1 and K4.
M142, M143= the intersections of N14 with L2 and L3 respectively.
J1, J4, M143, J2, J3 and M142 are on a conic (QLCoY).
If we choose N23 through K2 and K3 instead of N14, we obtain the same QLCoY.
There are other 2 conics for N12(or N34) and N13(or N24).
Degenerated curves are as follows:
(1) degenerated quartic: L1(H1J1K1), L2(H2M142J2), L3(H3M143J3), L4(H4J4K4).
(2) degenerated cubic: H14(K1M143M142K4), QLCi4(H1J1H2J2H3J3H4J4).
(3) degenerated quartic: QLCoX(H1H4K1H3H2K4), QLCoY(J1J4M143M142J3J2).
See the upper picture of the attached drawings.
The lower picture of the attached drawings corresponds to the case:
(1) degenerated quartic: L1(H1J1K1), L2(H2M142K2), L3(H3M143K3), L4(H4J4K4).
(2) degenerated cubic: H14(J1M143M142J4), QLCoX(H1H4K1H3K2K3H2K4).
(3) degenerated quartic: QLCi4(H1H2H3H4J4), QLCoY(K1M143K3K2M142K4).
If we choose N23 through J2 and J3 instead of N14, we obtain the same QLCoY.
There are other 2 conics for N12(or N34) and N13(or N24).
Best regards, Seiichi.



six conics.docx

Message: #1551
Date: 09/3/2016 2:19:11
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
Cabri-observations give the guess, that the least square line of the QA-vertices contains QA-P1. But a corresponding Mathematica calculation gives very extensive coefficients.
Best regards Eckart

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Message: #1552
Date: 09/3/2016 2:22:49
From: eckart_schmidt@t-online.de
Subject: QA-P41

Dear Chris,
some remarkable properties of QA-P41, I think not in EQF:
The 5th and 6th intersection of a QA-circumconic and QA-Cu1 are collinear with QA-P41.
Tangents at a QA-circumconic in the 5th and 6th intersection with QA-Cu1 have QA-P4 as common point.
The polars of QA-P4 wrt QA-circumconics have QA-P41 as common point.
Best regards Eckart

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Message: #1553
Date: 09/3/2016 5:31:06
From: Benedetto Scimemi
Subject: QA-P41

Dear Eckart and Chris,

as for the properties of QAP41 just pointed out by Eckart, some light comes from the following remarks (not in EQF):
..... QAP41 is the Miquel point (QAP9) for the q. of the isogonal conjugates
.....The spiral similarity defined by
QAP9 -> QAP41,
QAP2 -> QAP ... = Euler point of the q. of conjugates = reflection of QAP2 on QAP23 (not in EQF) is the product of a rotation around QAP4 of angle
a homothety around QAP4 with scale factor
|P41P4| / |P9P4| = |P3P4| / |P2P4| * |P23/P4| / |P32P4|.

Alias: the central triangle P2P4P9 = Euler- Bennet-Miquel remains directly similar when Q is replaced by its q of isog.conjugates. The same must hold with the q. of circumcenters (whose Miquel point is not in EQF). Various angle equalities follow.

Best regards Benedetto

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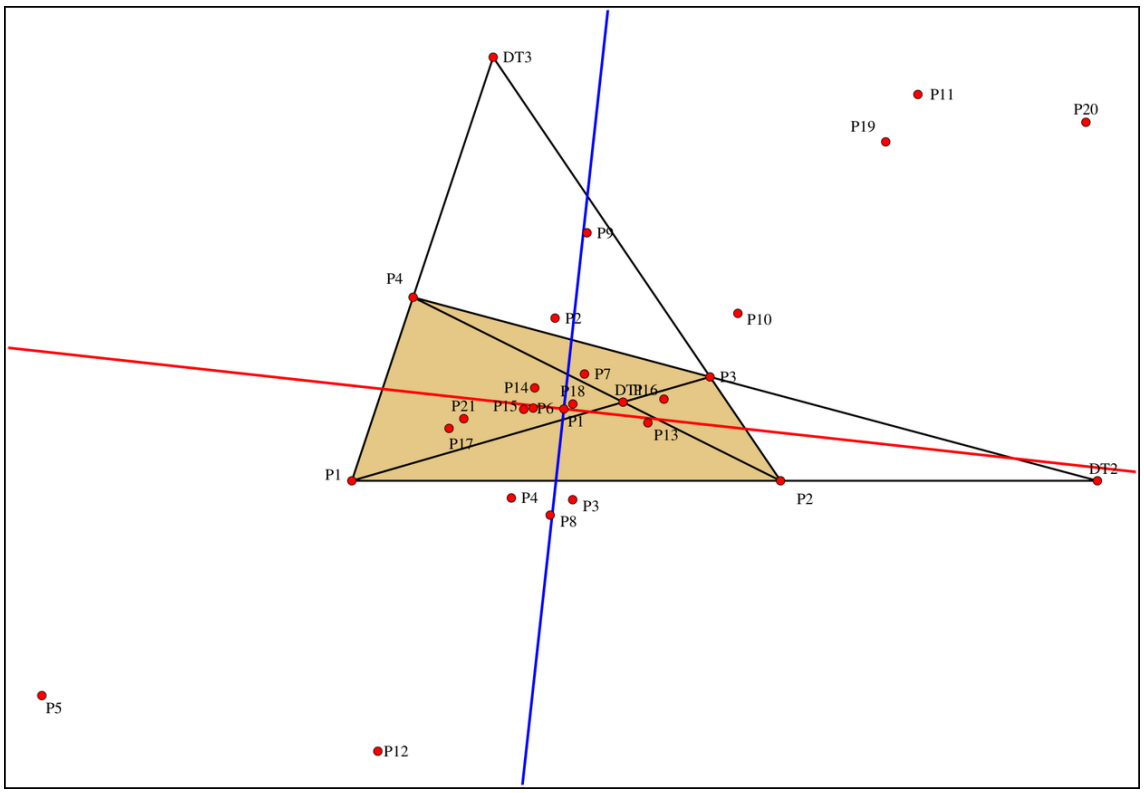
Message: #1554
Date: 09/3/2016 10:39:59
From: chris.vantienhoven
Subject: Dual Curves

Dear Bernard and Eckart,

Bernard, about your question for the least square line in a Quadrangle as the dual for QL-P26.
I thought it would be easy to calculate this line, but it wasn't.
But surprisingly when I saved some intermediate results I encountered a file with some calculation of this line that I performed in the past at January 17, 2012 !
Another proof how bad my memory is!
Fortunately I have a very consistent way of registering my findings. I used the same code and name for registering and so I refound my old results.
By the way the formula for the line is pretty awkward and only QA-P1 lies on it (Eckart also concluded).
Also I found a construction from the same author as I mentioned at QL-P26 in EQF.
"Two geometrical applications of the method of least squares"
J.L. COOLIDGE, Harvard University.
I paraphrase his astonishing simple method:
1. Let P1, P2, P3, P4 be the 4 points of the Reference Quadrangle.
2. Construct a circle with random center O through QA-P1.
3. Let QA-P1.Pi (i=1,2,3,4) meet the circle again in Qi.
4. Let Qg be the Quadrangle Centroid of the points Qi.
5. The line QA-P1.Qg shall meet the circle in R, that is to say, R is the intersection point which lies on the same side of the diameter as does Qg.
6. Then QA-P1.R is the Least Squared Distance line.
Since QL-P26 has a strong relationship with QL-Tf1 maybe this LSD Line also can be transformed to some point/line that gives more incidences. Maybe a new challenge for us now!

Best regards,
Chris

p.s. In 2012 I made a Mathematica picture which I enclose now.
The red line is the LSD line.



QA-L99-LeastSquares Line-03.pdf

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Message: #1555
Date: 09/3/2016 11:46:39
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Bernard and Eckart,
Still some extra considerations regarding the LSD line (found in my notes from 2012):
Consider this
1. Let P1.P2.P3.P4 be a quadrangle.
2. Let P be some random point and Ci some random circle through P with random center M.
3. Draw lines P1.P, P2.P, P3.P, P4.P and let Q1, Q2, Q3, Q4 be the 2nd intersection points of these lines with circle Ci.
4. Let G be quadrangle centroid of Q1.Q2.Q3.Q4.
5. Let S1 and S2 be the intersection points of G.M with circle Ci.
6. Now lines P.S1 and P.S2 are fixed lines independent of the position of circle Ci.
7. When P is the centroid of quadrangle P1.P2.P3.P4 then P.S1 is the Least Squares Line.
8. When P = QL-P1 (Miquel Point) then both lines P.S1 and P.S2 are the Steiner Axes.

Best regards,
Chris

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Message: #1556
Date: 10/3/2016 9:45:25
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Chris,
I have difficulties, to reproduce your constructions for the least squares line :
(1) Can it be, that the first construction is incomplete, for I miss the points Si of the second construction.
(2) Wrt the second construction I can't confirm, that one of the lines QA-P1.Si has the expected property.
May be I'm a bit confused today.
Best regards Eckart

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Message: #1557
Date: 10/3/2016 10:45:51
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris, dear Eckart
Like Eckart, I'm not able to reproduce neither one nor the other construction (both differ anyway QA-P1Qg in the 1rst is different from GM in the 2nd)
I'm very excited, as we are very close ...
Best regards
Bernard

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Message: #1558
Date: 10/3/2016 12:32:56
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris, dear Eckart
This time, i've succeeded in reproducing the 2nd construction for QL-P1, which gives in fact the 2 Steiner axes §
So I did the same with QA-P1 and I suppose you have to correct the 1rst construction as follows :
in point 5) replace QA-P1 by O
the line OQg cuts the circle in 2 points and we have to choose R as the one on the same side of Qg on the diameter OQg.
Congratulations to Chris !
Now we can begin to investigate on this line, it's dual point ...
Best regards
Bernard

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Message: #1559
Date: 10/3/2016 12:59:38
From: eckart_schmidt@t-online.de
Subject: Dual Curves

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Dear Bernard,
sorry, also your correction doesn't give the least squares line.
Best regards Eckart
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Message: #1560
Date: 10/3/2016 1:06:23
From: tsihonglau
Subject: Least Squared Distance Line

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---In Quadri-Figures-Group@yahoogroups.com, wrote :
6. Now lines P.S1 and P.S2 are fixed lines independent of the
position of circle Ci.

I found S1.S2 is a diameter of Ci.

Best regards,
Tsihong Lau
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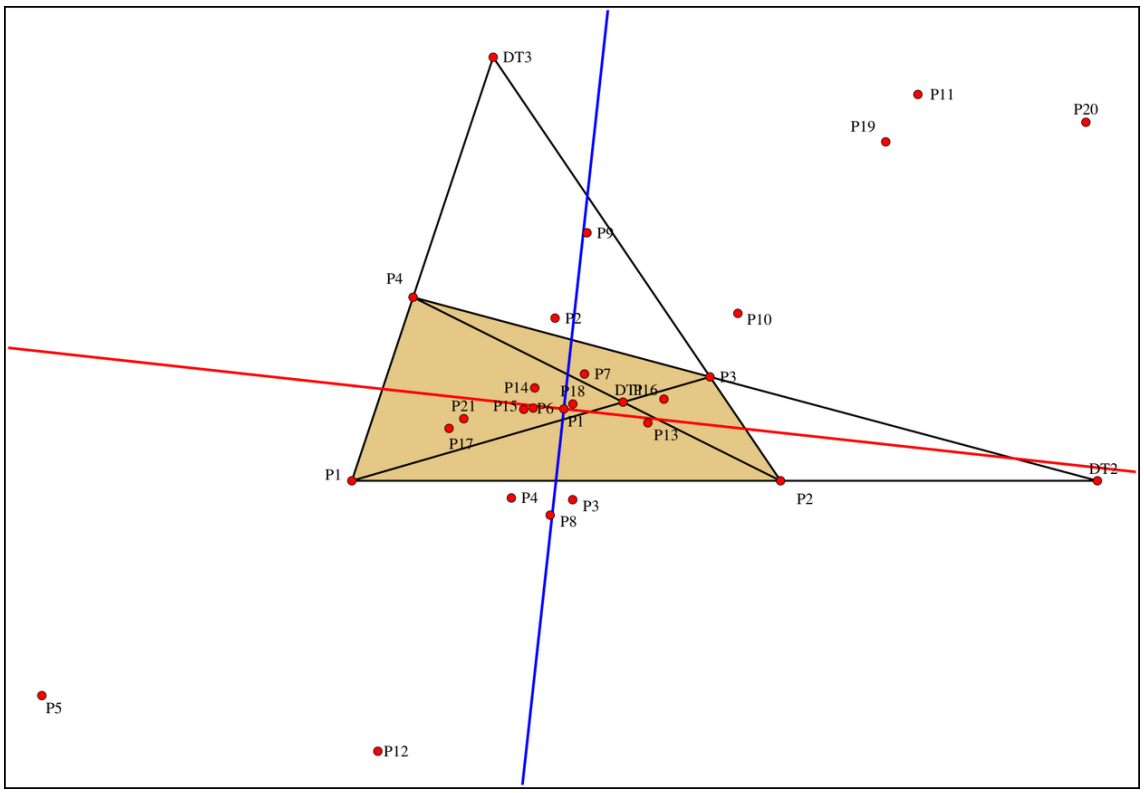
Message: #1561
Date: 10/3/2016 3:46:01
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Bernard,
You are right.
In step 5. QA-P1 should be replaced by O.
I attach the two pictures of both constructions.
I hope Eckart you still can confirm it is the LSD line.
Best regards,
Chris



QA-L99-LeastSquares Line-03-1561.pdf

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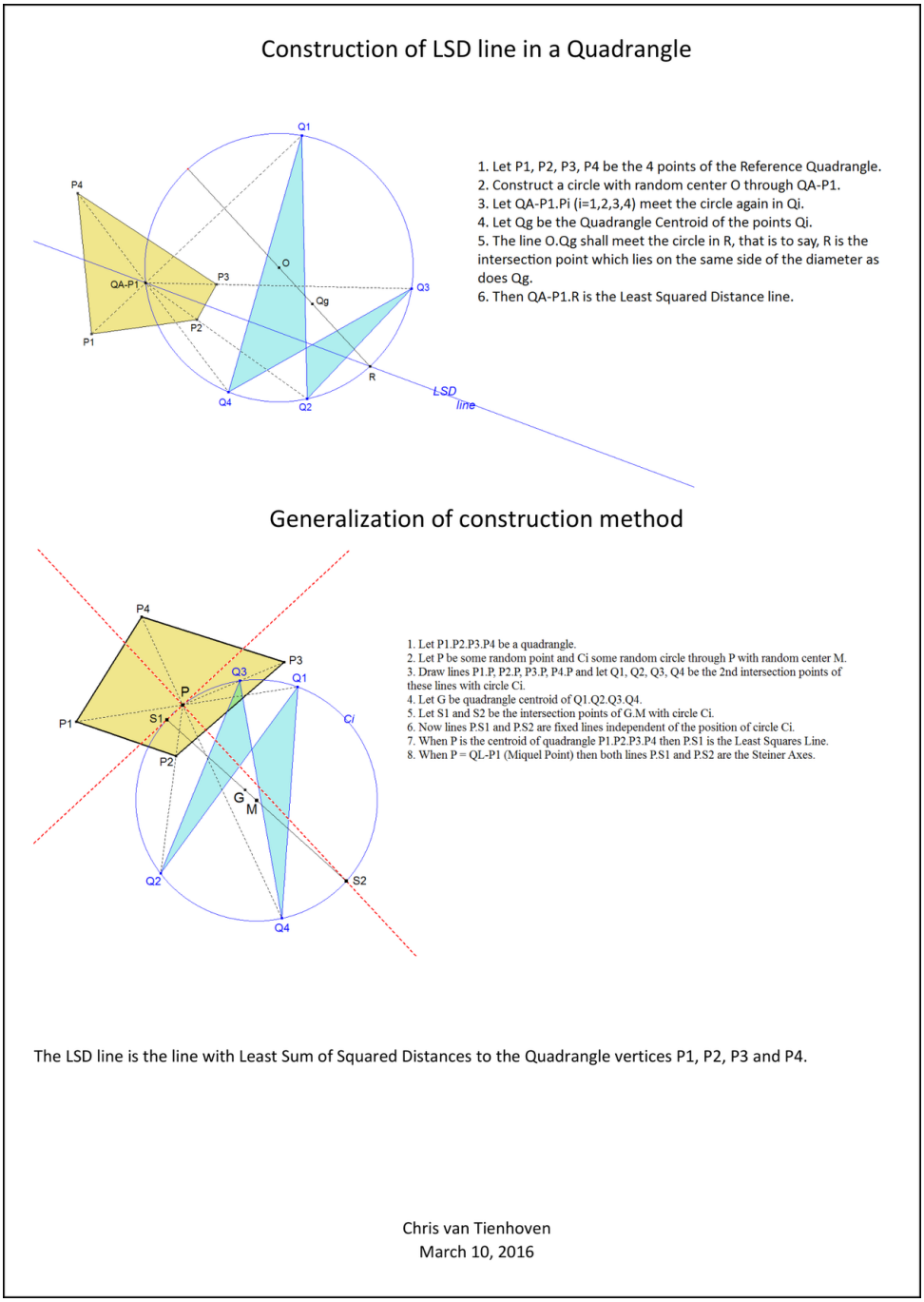
Message: #1562
Date: 10/3/2016 5:10:16
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Bernard, Eckart and Tsihong Lau,
Sorry I attached the wrong file.
I will enclose the right one now.
Best regards,
Chris



QA-L99-LSD Line-00.pdf

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Message: #1563
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Messages

Message number 1563 is not available in Yahoo groups.

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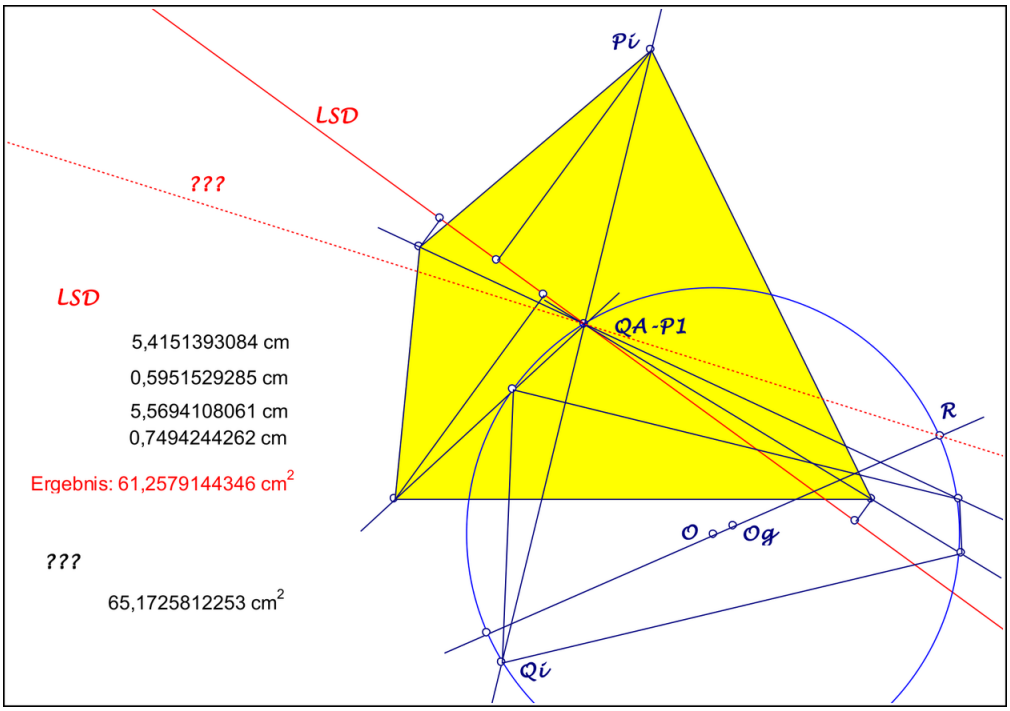
Message: #1564
Date: 11/3/2016 8:51:01
From: eckart_schmidt@t-online.de
Subject: Least Sqared Distance Line

Dear Chris,
several times I have constructed a line in your sense, measured the distances to the vertices, squared and added them, but there are lines through QA-P1 with a lower value.
I don't find my mistake. Attached a file. Can you please check it?
Thanks in advance.
Best regards Eckart
PS: Unfortunately I can't mail a Cabri file.



2016-03-10.docx

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Message: #1565
Date: 11/3/2016 3:56:34
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Eckart,

Thanks for your perseverance.
Alarmed by you I made the construction again and looked by hand for the line through QA-P1 with least sum of squared distances.
And you are right, it is not the constructed line.
So now there are two possibilities;
1. Either I misinterpreted the description of Coolidge
2. Or Coolidge made a misinterpretation in his article.
I checked if I made a misinterpretation in translating his description, but I don't think I did.
So maybe Coolidge made a misinterpretation.
Maybe you and Bernard and Tsihong Lau (and other interested friends) can look with me to the article and look for the cause of discrepancy.
The article can be found at:
[25] J.L. Coolidge, Harvard University, Two geometrical applications of the method of least squares ,
available at: http://www.jstor.org/stable/2973072
Look at pages 189 and 190.
I am curious who will find the flaw.

Best regards,
Chris

p.s. Coolidge deals with 2 problems in his article. The first problem is the LSD-point in a Quadrilateral (and that one is correct). The second problem is the LSD-line in a Quadrangle (here is the flaw).

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Message: #1566
Date: 12/3/2016 1:46:30
From: seiichikiri
Subject: 6 conics accompanying Hervey circle and its rectangular hyperbol

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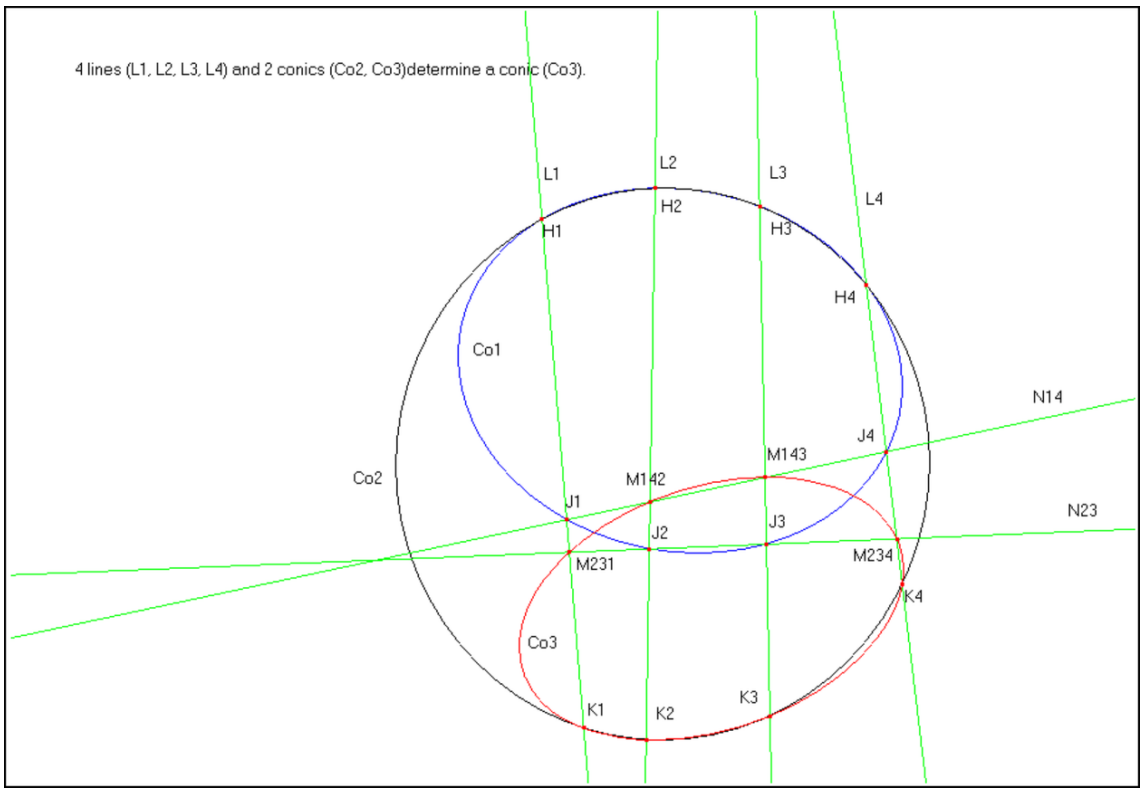
Addition: a configuration easier to understand than one in message #1550.
Let L1, 2, L3, L4 be 4 lines, Co1 and Co2 two conics.
Denote:
H1, H2, H3, H4= the intersections of L1, L2, L3, L4 with Co1 and Co2 respectively.
J1, J2, J3, J4= the 2 nd intersections of L1, L2, L3, L4 with Co1 respectively.
K1, K2, K3, K4= the 2 nd intersections of L1, L2, L3, L4 with Co2 respectively.
N14= the line through J1 and J4.
N23= the line through J2 and J3.
M142, M143= the intersections of N14 with L2, L3 respectively.
M231, M234= the intersections of N23 with L1, L4 respectively.
K1, K2, K3, K4, M142, M143, M231 and M234 are on a conic (Co3).
I did not find any special relation among the centers of 3*2=6 conics.
Best regards, Seiichi.



3Conics.docx

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Message: #1567
Date: 12/3/2016 2:27:47
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Chris,
thanks for the article of Coolidge. I think, your interpretation of the construction is right. The construction is conform with the last formula, but the construction gives not the LSD line. So there will be a misinterpretation in the derivation. Fig.2 is vague wrt the point Q'.
So far, best regards Eckart

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Message: #1568
Date: 12/3/2016 4:21:24
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Eckart,
Here are my interpretations of Coolidge article.
Coolidge first works with a coordinate system with origin O being the Centroid of the reference points. The use of the Centroid arises from his formulas when summarizing the Cartesian x- and y-coordinates of the reference points.
Later he uses the coordinate system with center O' in order to get the double angles that occur in his formulas. By extending the lines through O and Pi (i=1,2,3,4) crossing the unit-circle he gets double angles in the coordinate system with center O'.
I think his approach is sublime but not fully transparent.
Best regards,
Chris

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Message: #1569
Date: 12/3/2016 4:50:29
From: tsihonglau
Subject: Least Squared Distance Line

Dear all
I raise another question. What is the least square point
to the six sidelines of a complete quadrangle? I compute
the squares of distances of any point to the six sidelines.
I found it is not so complicated as this problem. Any idea?
Best regards,
Tsihong Lau

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Message: #1570
Date: 12/3/2016 5:15:21
From: minhtenladuong
Subject: Ptolemy triangle and some related problems

Dear geometers,
Recently, I have been working on a family of similar triangles.
While working on this, I raised some open problems.
All appear in the attachment.
I hope you like this and have answer for these.
Best regards
Sincerely
Ngo Quang Duong

Around the Ptolemy Triangle

Ngo Quang Duong

March 12, 2016

Abstract

We prove the existence of a family of Ptolemy triangles by synthetic methods. In §3, we introduce some related open problems.

1 Introduction

Theorem 1. *(Ptolemy) 4 points A, B, C, D are concyclic if and only if $BC.DA = CA.DB + AB.DC$ or $CA.DB = AB.DC + BC.DA$, or $AB.DC = BC.DA + CA.DB$.*

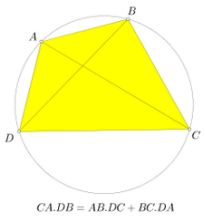

Figure 1. Ptolemy's theorem

Ptolemy's theorem was named after Claudius Ptolemy thousands years ago and it has many extensions. One of the most important extensions is Ptolemy's inequality: *$BC.DA$, $CA.DB$, $AB.DC$ are side lengths of a triangle.* This triangle is degenerated to a line if and only if A, B, C, D are concyclic. Hence, we call a triangle whose sidelengths proportional to $BC.DA$, $CA.DB$, $AB.DC$ is a *Ptolemy triangle* of A, B, C, D. The following result gives us two simple ways to construct a Ptolemy triangle.

Theorem 2. *Given 4 points A, B, C, D.*

1. *B', C', D' lie on AB, AC, AD such that $\overline{AB}.\overline{AB'} = \overline{AC}.\overline{AC'} = \overline{AD}.\overline{AD'}$ then $\triangle B'C'D'$ is a Ptolemy triangle of A, B, C, D.*
2. *Pedal triangle $\triangle H_{AB}H_{AC}H_{AD}$ of A with respect to $\triangle BCD$ is a Ptolemy triangle of A, B, C, D.*

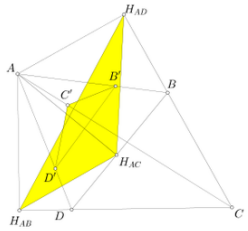

Figure 2. $\triangle B'C'D'$ *and* $\triangle H_{AB}H_{AC}H_{AD}$ *are Ptolemy triangles*

2 A family of Ptolemy triangles

Proposition 3 ([1])**.** *Given 5 points A, B, C, D, P. Let* P_{BC}, P_{DA}, P_{CA}, P_{DB}, P_{AB}, P_{DC} *be orthogonal projections of P on BC, DA, CA, DB, AB, DC. X, Y, Z are midpoints of* $P_{BC}P_{DA}$, $P_{CA}P_{DB}$, $P_{AB}P_{DC}$, *respectively. Then* $\triangle XYZ$ *is a Ptolemy triangle of A, B, C, D.*

This is a complicated but interesting configuration. But to have a synthetic proof for this, we need some lemmas, important results, which contain similarities on the Miquel points.

Proof. Case 1: A, B, C, D are not concyclic.

Lemma 4. *Given 6 points A, B, C, D, I, J. Two complete quadrilaterals* (AB, BC, DC, DA) *and* (AI, IC, JC, AJ) *have a same Miquel point if and only if:*

$$(IA, IB) + (JD, JC) = (DA, BC) \qquad (IC, IB) + (JD, JA) = (DC, AB)$$

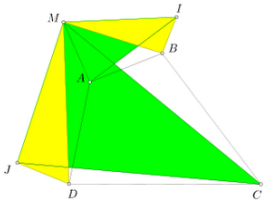

Figure 3. Similarities

Since a given point I has only one corresponding point J, it is suffices to prove these angle equalities when these two complete quadrilateral have a same Miquel point. Notice that Miquel point is the center of spiral similarities. Let

M be the Miquel point.

$$\begin{aligned}(DA, BC) &= (MA, MB) \\ &= (AM, AI) + (IA, IB) + (BI, BM) \\ &= (JM, JC) + (IA, IB) + (JD, JM) \\ &= (IA, IB) + (JD, JC) \\ (DC, AB) &= (MC, MB) \\ &= (CM, CI) + (IC, IB) + (BI, BM) \\ &= (JM, JA) + (IC, IB) + (JD, JM) \\ &= (JD, JA) + (IC, IB)\end{aligned}$$

Lemma 5. *3 following complete quadrilaterals share the same Miquel point*

$$(P_{AB}P_{BC}, P_{BC}P_{DC}, P_{DC}P_{DA}, P_{DA}P_{AB})$$
$$(P_{BC}P_{CA}, P_{CA}P_{DA}, P_{DA}P_{DB}, P_{DB}P_{BC})$$
$$(P_{CA}P_{AB}, P_{AB}P_{DB}, P_{DB}P_{DC}, P_{DC}P_{CA})$$

We only need to show that:

$$(P_{DB}P_{BC}, P_{CA}P_{DA}) = (P_{AB}P_{BC}, P_{AB}P_{CA}) + (P_{DC}P_{DB}, P_{DC}P_{DA})$$

and

$$(P_{DB}P_{DA}, P_{BC}P_{CA}) = (P_{AB}P_{DA}, P_{AB}P_{CA}) + (P_{DC}P_{DB}, P_{DC}P_{BC})$$

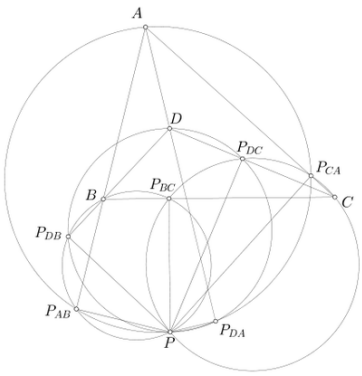

Figure 4

$$\begin{aligned}(P_{DB}P_{BC}, P_{CA}P_{DA}) &= (P_{DB}P_{BC}, P_{DB}P_{DC}) + (P_{DB}P_{DC}, P_{DC}P_{DA}) + (P_{DC}P_{DA}, P_{CA}P_{DA}) \\ &= (BC, BP) + (DP, DC) + (DB, DA) + (DC, DP) + (AP, AC) \\ &= (BC, BP) + (AP, AC) + (DB, DA)\end{aligned}$$

ptolemy-triangle.pdf

$$\begin{aligned}(P_{AB}P_{BC}, P_{AB}P_{CA}) + (P_{DC}P_{DB}, P_{DC}P_{DA}) &= (P_{AB}P_{BC}, P_{AB}P) + (P_{AB}P, P_{AB}P_{CA}) + (DB, DA)\\ &= (BC, BP) + (AP, AC) + (DB, DA)\end{aligned}$$

$$\begin{aligned}(P_{DB}P_{DA}, P_{BC}P_{CA}) &= (P_{DB}P_{DA}, P_{DB}P_{DC}) + (P_{DC}P_{DB}, P_{DC}P_{BC}) + (P_{BC}P_{DC}, P_{BC}P_{CA})\\ &= (DA, DC) + (DB, DP) + (CP, CB) + (CD, CA)\\ &= (AD, AC) + (DB, DP) + (CP, CB)\end{aligned}$$

$$\begin{aligned}(P_{AB}P_{DA}, P_{AB}P_{CA}) + (P_{DC}P_{DB}, P_{DC}P_{BC}) &= (AD, AC) + (P_{DC}P_{DB}, P_{DC}P) + (P_{DC}P, P_{DC}P_{BC})\\ &= (AD, AC) + (DB, DP) + (CP, CB)\end{aligned}$$

Hence, according to the lemma 4, three complete quadrilateral share a same Miquel point.

Lemma 6 ([2])**.** *We denote M_{XY} is midpoint of XY. Let M be Miquel point of (AB, BC, DC, DA), then $\triangle MAB$ and $\triangle M_{AB}M_{DB}M_{CA}$ are directly similar.*

Back to theorem 3. Let E, F be midpoints of $P_{DA}P_{CA}$, $P_{DA}P_{DC}$, M be the Miquel points of 3 complete quadrilaterals above. According to lemma 6,

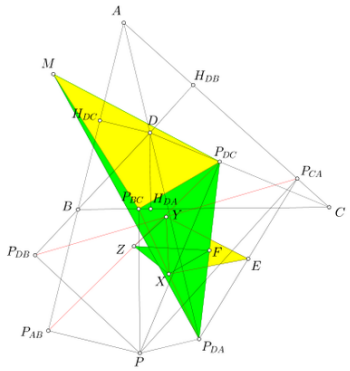

Figure 5

$\triangle EXY$ and $\triangle MP_{BC}P_{DB}$ are directly similar, $\triangle FZX$ and $\triangle MP_{DA}P_{DC}$ are

directly similar.

$$\begin{aligned}(XY, XZ) &= (XY, XE) + (XE, XF) + (XF, XZ)\\ &= (P_{BC}P_{DB}, P_{BC}M) + (P_{BC}P_{CA}, P_{BC}P_{DC}) + (P_{DC}M, P_{DC}P_{DA})\\ &= (P_{BC}P_{CA}, P_{BC}P_{DC}) + (P_{BC}P_{DB}, P_{DC}P_{DA}) + (MP_{DC}, MP_{BC})\\ &= (P_{BC}P_{CA}, P_{BC}P_{DC}) + (P_{BC}P_{DB}, P_{DC}P_{DA}) + (P_{DC}P_{DA}, P_{BC}P_{AB})\\ &= (CA, CD) + (P_{BC}P_{DB}, P_{BC}P_{AB})\\ &= (CA, CD) + (BD, BA)\\ &= (H_{DA}H_{DB}, H_{DA}D) + (H_{DA}D, H_{DA}H_{DC})\\ &= (H_{DA}H_{DB}, H_{DA}H_{DC})\end{aligned}$$

Similarly, we obtain that $(YZ, YX) = (H_{DB}H_{DC}, H_{DB}H_{DA})$. Hence $\triangle H_{DA}H_{DB}H_{DC}$ and $\triangle XYZ$ are directly similar, so $\triangle XYZ$ is a Ptolemy triangle of A, B, C, D.

Case 2. A, B, C, D are concyclic.

We still have:

$$(XY, XZ) = (H_{DA}H_{DB}, H_{DA}H_{DC})$$

When A, B, C, D are concyclic, H_{DA}, H_{DB}, H_{DC} are collinear, the line through these orthogonal projections is known as *Simson line* so X, Y, Z are collinear. Since X, Y, Z and H_{DA}, H_{DB}, H_{DC} are collinear, we can't conclude that YZ, ZX, XY are proportional to $BC.DA$, $CA.DB$, $AB.DC$ directly. Q lies

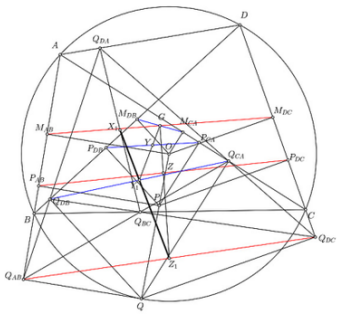

Figure 6

on circumcircle of A, B, C, D such that circumcenter O, P, Q are collinear. Let Q_{BC}, Q_{DA}, Q_{CA}, Q_{DB}, Q_{AB}, Q_{DC} are orthogonal projections of Q on BC, DA, CA, DB, AB, DC and M_{BC}, M_{DA}, M_{CA}, M_{DB}, M_{AB}, M_{DC} are midpoints of BC, DA, CA, DB, AB, DC. G, X_1, Y_1, Z_1 are midpoints of $M_{CA}M_{DB}$, $Q_{BC}Q_{DA}$, $Q_{CA}Q_{DB}$, $Q_{AB}Q_{DC}$. Notice that

$$\frac{OP}{OQ} = \frac{M_{BC}P_{BC}}{M_{BC}Q_{BC}} = \frac{M_{DA}P_{DA}}{M_{DA}Q_{DA}} = \frac{M_{CA}P_{CA}.}{M_{CA}Q_{CA}} = \frac{M_{DB}P_{DB}}{M_{DB}Q_{DB}} = \frac{M_{AB}P_{AB}}{M_{AB}Q_{AB}} = \frac{M_{DC}P_{DC}}{M_{DC}Q_{DC}}$$

According to E.R.I.Q theorem, (G, X, X_1), (G, Y, Y_1), (G, Z, Z_1) are collinear.

$$\frac{GX}{GX_1} = \frac{GY}{GY_1} = \frac{GZ}{GZ_1} = \frac{OP}{OQ}$$

Hence, it suffices to prove $X_1Y_1/X_1Z_1 = CA.DB/AB.DC$. Since Simson lines of Q with respect to $\triangle BCD$, $\triangle CDA$, $\triangle DAB$, $\triangle ABC$ determines a complete quadrilateral, then the circles with diameters $Q_{BC}Q_{DA}$, $Q_{CA}Q_{DB}$, $Q_{AB}Q_{DC}$ are coaxial, therefore:

$$\frac{X_1Y_1}{X_1Z_1} = \frac{|\mathcal{P}_{Q_{BC}/(Q_{CA}Q_{DB})}|}{|\mathcal{P}_{Q_{BC}/(Q_{AB}Q_{DC})}|}$$

$$\begin{aligned}|\mathcal{P}_{Q_{BC}/(Q_{CA}Q_{DB})}| &= |\overrightarrow{Q_{BC}Q_{CA}}.\overrightarrow{Q_{BC}Q_{DB}}| \\ &= Q_{BC}Q_{CA}.Q_{BC}Q_{DB}.|\cos(Q_{BC}Q_{CA}, Q_{BC}Q_{DB})| \\ &= QC.QB.|\sin(CA, CB).\sin(BC, BD).\cos(CD, CA)|\end{aligned}$$

$$\begin{aligned}|\mathcal{P}_{Q_{BC}/(Q_{AB}Q_{DC})}| &= |\overrightarrow{Q_{BC}Q_{AB}}.\overrightarrow{Q_{BC}Q_{DC}}| \\ &= Q_{BC}Q_{AB}.Q_{BC}Q_{DC}.|\cos(Q_{BC}Q_{AB}, Q_{BC}Q_{DC})| \\ &= QB.QC|\sin(BC, BA).\sin(CD, CB).\cos(CD, CA)|\end{aligned}$$

Combine with law of sine

$$\frac{X_1Y_1}{X_1Z_1} = \frac{AB.DC}{CA.DB}$$

□

We also obtain some further results from the configuration of proposition 3.

Proposition 7. *We consider 3 complete quadrilaterals in lemma 2.*
(1) 3 Steiner lines are concurrent.
(2) 3 Miquel circles are coaxial.

Proof. At first, we have a familiar property of Steiner line.

Lemma 8. *Given 4 points A, B, C, D. Then Steiner line of* (AB, BC, DC, DA) *is radical axes of the circles* (CA), (DB). *Note that* (XY) *is the circle with diameter* XY.

According to lemma 8, 3 Steiner lines are radical axes of $(P_{BC}P_{DA})$, $(P_{CA}P_{DB})$, $(P_{AB}P_{DC})$, so they are concurrent.
Note that M is Miquel point of three complete quadrilateral, so the angles (MP_{BC}, MP_{DA}), (MP_{CA}, MP_{DB}), (MP_{AB}, MP_{DC}) share the same bisector ℓ and

$$MP_{BC}.MP_{DA} = MP_{CA}.MP_{DB} = MP_{AB}MP_{DC} = k$$

ptolemy-triangle.pdf

Denote the composition of the inversion $\mathbf{I}_M^k$ and reflection in ℓ by Ψ(also called Clawson-Schmidt transformation)[3]. Then:

$$\Psi : P_{BC}, P_{CA}, P_{AB}, P_{DA}, P_{DB}, P_{DC} \mapsto P_{DA}, P_{DB}, P_{DC}, P_{BC}, P_{CA}, P_{AB}$$

And Ψ swaps Steiner lines to the corresponding Miquel circles, respectively. Hence, three Miquel circles are coaxial. □

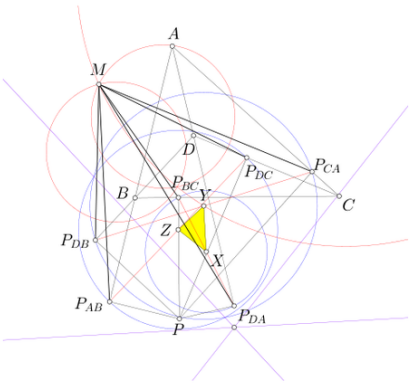

Figure 7. 3 Steiner lines are concurrent and 3 Miquel circles are coaxial

Corollary 9. *3 Steiner lines are parallel, or equivalently, 3 Miquel circles are tangent if and only if A, B, C, D are concyclic.*

Proposition 10 ([4])**.** *Reflection of P in circumcenter of $\triangle XYZ$ is QA-Tf3(P).*

3 Open problems

3.1 Homothety and Rotation

Proposition 11. *Let I be isogonal center of A, B, C, D.*

$$YZ = IP \cdot \frac{BC.DA}{K(A,B,C,D)}$$

The above conclusion generalizes the case of cyclic quadrangle and some checkings with GeoGebra. For any two Ptolemy triangles $\triangle X_1Y_1Z_1$, $\triangle X_2Y_2Z_2$, the ratio Y_1Z_1/Y_2Z_2 equals IP_1/IP_2. When A, B, C, D are concyclic, by law of sine and angle chasing, we have:

$$YZ = OP \cdot \frac{BC.DA}{4R^2}$$

Back to Proposition 10, if we could calculate YZ, IP, DA, BC, then the expression $K(A,B,C,D)$ must be symmetric of A, B, C, D and doesn't depend on P. The circle centered at I and diamter $\sqrt{K(A,B,C,D)}$ will be called *pseudocircumcircle* of A, B, C, D.
With 4 given points A, B, C, D, there exists only one point I such that $H_{BC}H_{DA}$, $H_{CA}H_{DB}$, $H_{AB}H_{DC}$ share the same midpoints T - parabola axes crosspoint QA-P6[5], where H_{BC}, H_{DA}, H_{CA}, H_{DB}, H_{AB}, H_{DC} are orthogonal projections of I on BC, DA, CA, DB, AB, DC. By proof that similar to proof for case 2 of Proposition 3, we easily obtain that if I, P_1, P_2 are collinear, two Ptolemy triangles $\triangle X_1Y_1Z_1$, $\triangle X_2Y_2Z_2$ of P_1, P_2 are homothetic with homothety center T. In general:

Proposition 12.

$$(IP_1, IP_2) + (Y_1Z_1, Y_2Z_2) = 0$$

3.2 Locus and Envelope

Proposition 13.

$$\alpha\overrightarrow{SX} + \beta\overrightarrow{SY} + \gamma\overrightarrow{SZ} = \overrightarrow{0}$$

When P varies on a circle centered at I, S varies on an ellipse centered at T.

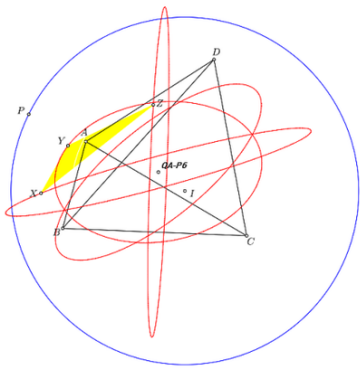

Figure 9. Locus

Proposition 14.

$$\alpha_1\overrightarrow{S_1X} + \beta_1\overrightarrow{S_1Y} + \gamma_1\overrightarrow{S_1Z} = \overrightarrow{0}$$
$$\alpha_2\overrightarrow{S_2X} + \beta_2\overrightarrow{S_2Y} + \gamma_2\overrightarrow{S_2Z} = \overrightarrow{0}$$

When P varies on a circle centered at I, then S_1S_2 is tangent to a central curve centered at T. The envelope is a deformation of astroid.

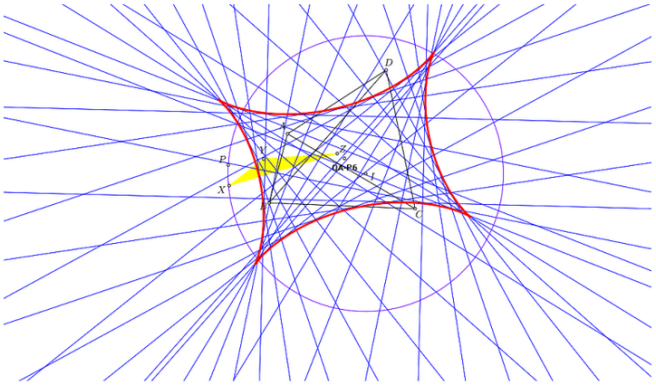

Figure 10. Envelope

3.3 A Transformation point $\mapsto$ point

In the proof of Proposition 3, we proved that 3 quadrilaterals

$$(P_{AB}P_{BC}, P_{BC}P_{DC}, P_{DC}P_{DA}, P_{DA}P_{AB})$$

$$(P_{BC}P_{CA}, P_{CA}P_{DA}, P_{DA}P_{DB}, P_{DB}P_{BC})$$

$$(P_{CA}P_{AB}, P_{AB}P_{DB}, P_{DB}P_{DC}, P_{DC}P_{CA})$$

have a same Miquel point. Denote it by $M(P)$.

Proposition 15. *Properties of $M(P)$*

1. *When A, B, C, D are concyclic, $M(P)$ is midpoint of P and inverse of P in circumcircle.*
2. *$M(P)$ lies at infinity if and only if $P \equiv I$.*
3. *$M(P)$ swaps a line through I to a conic.*
4. *If P, Q are symmetrically through I, then $M(P)$, $M(Q)$ are symmetrically through center of the conic $M(PQ)$. The center is a fixed point K.*

Question. When the line ℓ through I rotates, center K of the conic $M(\ell)$ is fixed, hence it is a quadrangle center. Is this center known?

Proposition 16. *When P varies on a circle centered at I, then $M(P)$ varies on an ellipse centered at K. Two axes of this ellipse is parallel to angle bisectors of $\angle IKE$. Where E is QA-P2.*

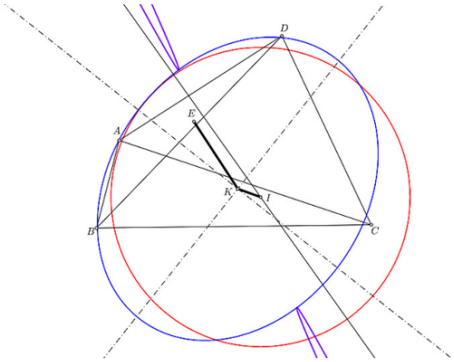

Figure 11

References

[1] Art of Problem Solving `http://artofproblemsolving.com/community/q1h1075966p4700764`

[2] *Michael Rolinek and Le Anh Dung, The Miquel points, pseudocircumcenter, and Euler-Poncelet point of a complete quadrilateral, Forum Geometricorum, 14 (2014) 145-153*

[3] QL-Tf1: Clawson-Schmidt Conjugate `http://www.chrisvantienhoven.nl/other-quadrilateral-objects/17-mathematics/quadrilateral-objects/artikelen-ql/159-ql-tf1.html`

[4] QA-Tf3: QA-Orthopole Transormation `http://www.chrisvantienhoven.nl/other-quadrangle-objects/index.php/15-mathematics/encyclopedia-of-quadri-figures/quadrangle-objects/artikelen-qa/229-qa-tf3.html`

[5] QA-P6: Parabola Axes Crosspoint `http://www.chrisvantienhoven.nl/quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/27-qa-p6.html`

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Email address: tenminhladuong@gmail.com

ptolemy-triangle.pdf

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Message: #1571
Date: 12/3/2016 8:46:49
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Chris,
there is a mistake in the derivation of Coolidge, see attached file.
Best regards Eckart

Remark wrt the following publication:

J.L. Coolidge, Harvard University,
Two geometrical applications of the method of least squares,
available at: http://www.jstor.org/stable/2973072

There is a mistake in the derivation of a formula, which leads to a false construction of the **Least Squared Distance Line** of a quadrangle.

At page 190 (before the substitution) Coolidge gets the following result

$$\tan 2\alpha = \frac{\sum 2 x_i y_i}{\sum (x_i^2 - y_i^2)}.$$

With the following substitution $\frac{y_i}{x_i} = \tan\theta_i$ (here seems to be a typo) the last formula would give

$$\tan 2\alpha = \frac{\sum \frac{2 x_i y_i}{x_i^2 + y_i^2}}{\sum \frac{x_i^2 - y_i^2}{x_i^2 + y_i^2}}.$$

These two terms are of course not the same, for the vertices of the quadrangle have not equal distances from the centroid.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-03-12.docx

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Message: #1572
Date: 12/3/2016 9:48:37
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Eckart,
I saw this discrepancy too, but wasn't sure I overlooked something.
Now you noted it independently we have strong ground for this.
The question is, can we still find a solution in this way of translating algebra to synthetic solutions?
Moreover it still is peculiar that the generalization
of Coolidge's construction method applied to QL-P1 led to the Steiner Axes.
Well, I think it is time for a next step.
Best regards,
Chris

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Message: #1573
Date: 13/3/2016 4:32:06
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Tsihong Lau,
[TL]
I think this is the same point as Seiichi mentioned in QFG-message #13.
I recalculated the point today. Its CT-coordinates are:
{(q - r) (p^2 + 3 q r - 2 p (q + r)), -(p - r) (-2 p q + q^2 + 3 p r - 2 q r), (p - q) (3 p q - 2 p r - 2 q r + r^2)}
Somehow another expression as mentioned in message #13.
The only incidences I found are that it lies on these 2 lines:
1. the line through QA-P10 parallel to QA-P1.QA-P13.
2. The line through QA-P1.QA-Px, where QA-Px is the point (also raised by Seiichi) as described in QFG-messages #593,#594 (this is the LSD-point minimizing the sum of squared distances to 6 QA - vertices & 4 vertices of Pedal – Quadrangle).
Best regards,
Chris

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Message: #1574
Date: 15/3/2016 8:38:56
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Chris,
you say, that the point wrt the following question ...
[TL]
What is the least square point to the six sidelines of a complete quadrangle?
is on the line ...
1. the line through QA-P10 parallel to QA-P1.QA-P13.
But Cabri observations don't confirm this.
Best regards Eckart

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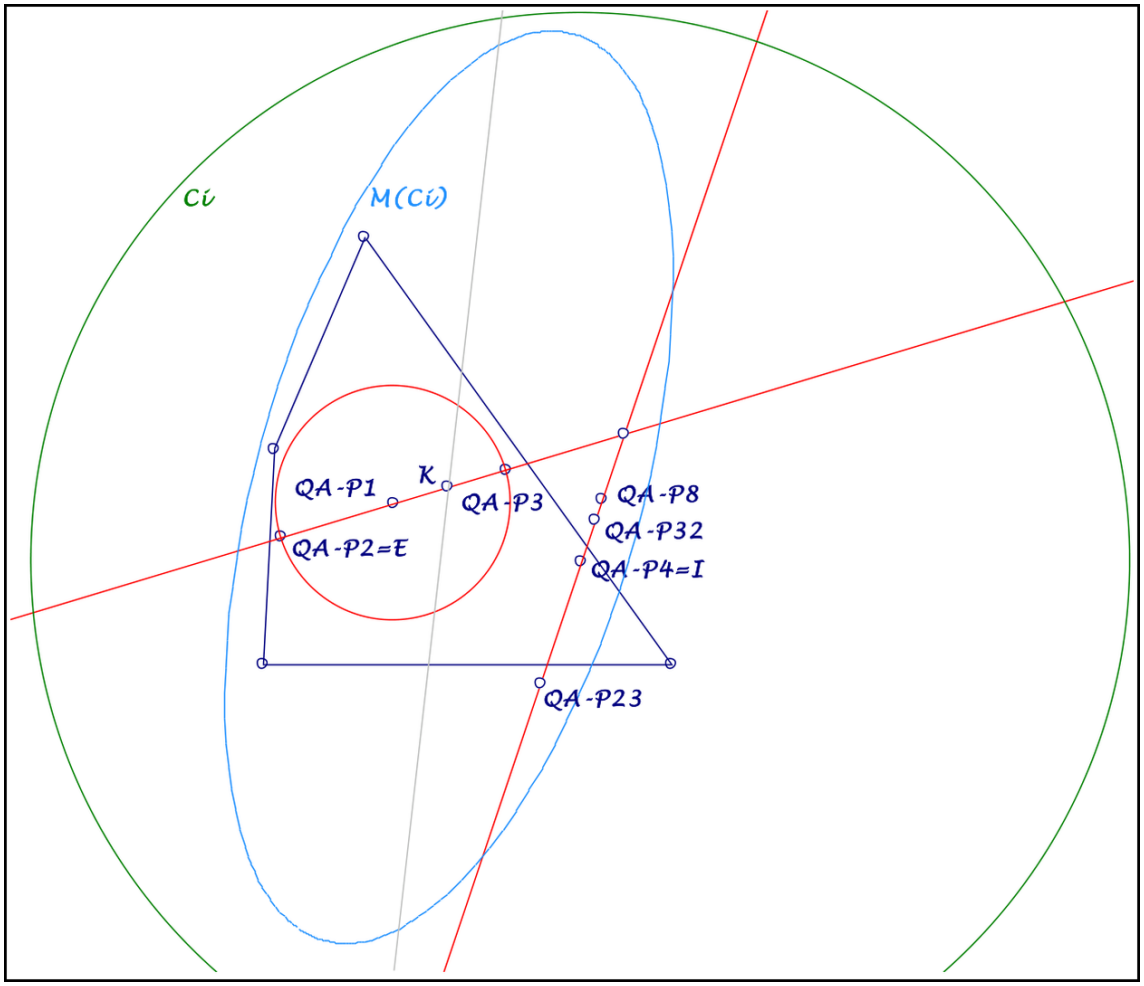
Message: #1575
Date: 15/3/2016 11:50:10
From: eckart_schmidt@t-online.de
Subject: Ptolemy triangle and some related problems

Dear Ngo Quang Duong,
only two short remarks wrt the question on page 9 of your paper "Around the Ptolemy Triangle" (see attached file):
The point K is the inversion ...
... of the intersection QA-P2.QA-P3 ^ QA-P4.QA-P8
... wrt the circle round QA-P1 through QA-P2 and QA-P3
... (up to now not mentioned in EQF).
The property in proposition 16 ...
"Two axes of this ellipse is parallel to angle bisectors of <>
... I cannot confirm with Cabri observations.
I have only tested this question and will continue to study your paper, for the transformation P ---> M(P) seems interesting.
Best regards Eckart



2016-03-15.pdf

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Message: #1576
Date: 15/3/2016 12:54:57
From: tsihonglau
Subject: Least Squared Distance Line

Dear all
We should study the following points and lines and their relationship.
1. The least square point to the three vertices of (diapleural) triangle - triangle centroid X(2)
2. The least square point to the three sidelines of (diagonal) trilateral - symmedian point X(6)
3. The least square point to the four vertices of quadrangle - quadrangle centroid QA-P1
4. The least square point to the four sidelines of quadrilateral - Least Squares Point QL-P26
5. The least square point to the six vertices of quadrilateral - QL-Centroid or Lateral Centroid QL-P12
6. The least square point to the six sidelines of quadrangle - studying
7. The least square line to the three vertices of (diapleural) triangle - I do not know. Can anyone tell me?
8. The least square line to the four vertices of quadrangle - studying
9. The least square line to the six vertices of quadrilateral - studying
And other related objects...
Best regards,
Tsihong Lau

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Message: #1577
Date: 15/3/2016 2:24:16
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Bernard,
Regarding the LSD line in a Quadrangle we still need a construction.
Once you mentioned a construction method of the LSD point in a Quadrilateral from :
[45] M. Léon Ripert, Notes sur le quadrilatère , available at: https://archive.org/stream/compterenduvolu03sciegoog#page/n114/mode/2up
(pages 106-118)
Unfortunately for me it is in French, but fortunately for you.
Could you analyze if there is a dual method to the construction of the LSD line in a Quadrangle?
I already noticed that (analogously to the description at QL-P26.

Let P1.P2.P3 be a reference triangle where 4 th Point P4 making 3 extra triangles with the sides of P1.P2.P3.
LQ4 = P1 ^ P4-Symmedian trace in triangle P2.P3.P4 on (P2^P3)
LR4 = P2 ^ P4-Symmedian trace in triangle P3.P1.P4 on (P3^P1)
LS4 = P3 ^ P4-Symmedian trace in triangle P1.P2.P4 on (P1^P2)
LQ4, LR4, LS4 are concurrent in a point S4 !

For the other 3 Component triangles corresponding lines S1, S2, S3 can be constructed.
Are these 4 Points collinear on QA-L99 ??? Alas NO.
My knowledge of French language is not enough to study for details.
Maybe you can help.

Best regards,
Chris

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Message: #1578
Date: 15/3/2016 3:04:31
From: tsihonglau
Subject: Least Squared Distance Line

Dear all
A question.
X(6) is the centroid of its pedal triangle.
QL-P26 is the centroid of its pedal quadrangle.
Is the least square point to the six sidelines of quadrangle
the centroid of its pedal sexangle?
Best regards
Tsihong Lau

---In Quadri-Figures-Group@yahoogroups.com, wrote :
Dear all
We should study the following points and lines and their
relationship.
1. The least square point to the three vertices of (diapleural)
triangle - triangle centroid X(2)
2. The least square point to the three sidelines of (diagonal)
trilateral - symmedian point X(6)
3. The least square point to the four vertices of quadrangle
- quadrangle centroid QA-P1
4. The least square point to the four sidelines of quad
rilateral - Least Squares Point QL-P26
5. The least square point to the six vertices of quadrilateral -
QL-Centroid or Lateral Centroid QL-P12
6. The least square point to the six sidelines of quadrangle -
studying
7. The least square line to the three vertices of (diapleural)
triangle - I do not know. Can anyone tell me?
8. The least square line to the four vertices of quadrangle -
studying
9. The least square line to the six vertices of quadrilateral -
studying
And other related objects...
Best regards,
Tsihong Lau

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Message: #1579
Date: 15/3/2016 3:15:53
From: minhtenladuong
Subject: Ptolemy triangle and some related problems

Dear Eckart Schmidt
Thanks for your interested. Your construction for the point "K" makes me very suprised!
I am also sorry for my mistake in Proposition 16, two axes of the ellipse is not
parallel to bisector of angle IKE.
But I observed and believe that two axes of the ellipse are invariant when the circle centered at I changes.
About the transformation P--->M(P), I suddenly realized it when I try to find a synthetic proof for Proposition 3.
I can't coordinate the point M(P) yet.
This transformation also raises another question: Does there exist a construction of M(P) that we don't have to construct pedal points?
After finding a simple construction in case of cyclic quadrangle (proposition 15.1), I tried to find a similar construction in general without success. But QA-P4 is the only singular point of the transformation so I think it may relates to the inversion centered at QA-P4.
Best regards
Sincerely
Ngo Quang Duong

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Message: #1580
Date: 15/3/2016 7:09:07
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Eckart and Tsihong Lau,
[ES]
1. the line through QA-P10 parallel to QA-P1.QA-P13.
But Cabri observations don't confirm this.

Please forget my calculation. It was wrong.
I am sorry for the inaccuracy.
Maybe later I will give it a try.
Best regards, Chris

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Message: #1581
Date: 15/3/2016 9:44:17
From: eckart_schmidt@t-online.de
Subject: Least Squared distance Line

Dear Tsihong Lau,
wow!
Cabri observations seem to verify your assumption, that the least square point to the six sidelines of a quadrangle is the centroid of its pedal sexangle.
Best regards Eckart

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Message: #1582
Date: 16/3/2016 9:38:01
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Tsihong Lau,
wrt the least / greatest square line through X(2) to the three vertices of a triangle (7). These lines have the following barycentric coordinates:
(3SA - Sw + 2T, 3SC - Sw - T, 3SB - Sw - T)
with SA, SB, SC, S, Sw Conway nominations and
T = +/-sqr(Sw² - 3S²).
But I can't give a geometric interpretation.
Best regards Eckart

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Message: #1583
Date: 16/3/2016 11:28:15
From: bernard.keizer
Subject: Least Squared distance Line

Dear Eckart, dear Tsihong Lau
If I'm not wrong, the general property, mentionned by Léon Ripert and due to Gauss, is that the least square point of an n-line is the centroïd of it's pedal n-angle.
Best regards
Bernard

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Message: #1584
Date: 16/3/2016 11:50:49
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Tsihong Lau,
the orthogonal least / greatest square lines L through X(2) to the three vertices of a triangle (7) have the following property:
Let L intersect the the triangle sides in Si,
let Li be the perpendiculars in Si to the corresponding sides:
The centroid of the generated triangle is a point on L.
I have proved this result with the equation of L in #1582.
Best regards Eckart

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Message: #1585
Date: 16/3/2016 12:51:09
From: Benedetto Scimemi
Subject: Least Squared Distance Line

Dear friends, as for the line of Least square distance from the vertices of a quadrangle, what follows may be of some interest:
1) When dealing with a triangle T, it is well-known that the LSQ line from its three vertices is the major axis of its Steiner inellipse E.
Less known may be (Marden th., but older result):
let Z1, Z2, Z3 be three complex numbers representing the vertices of T.
Let f(Z)=(Z-Z1)(Z-Z2)(Z-Z3) and take its
Z-derivative f'(Z)=3(Z-F1)(Z-F2).
Then F1,F2 are the foci of E, thus F1F2 is the LSQ line for T.
2) By analogy, one may introduce a fourth point Z4, construct the new minimal polynomial of degree 4, take its Z-derivative = 4(Z-K1)(Z-K2)(Z-K3) and study the triangle K= K1K2K3. Will K have anything to do with the LSQ line for the quadrangle Q=Z1Z2Z3Z4?
Almost trivially, Q and K should have the same centroid. Will anybody like to find more?
Best regards Benedetto

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Message: #1586
Date: 2020-02-21
From: Systems Manager
Subject: Deleted Messages

Message number 1586 is not available in Yahoo groups.

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Message: #1587
Date: 16/3/2016 4:15:00
From: tsihonglau
Subject: Least Squared Distance Line

Dear Benedetto, Bernard, Chris, Eckart
Thanks for your information. I am very interested in "complex projective plane", on which "least" is meaningless. But there exist zero square distance point/line conics. I applied the square distance formula
in this book p.49 onward - http://www.douillet.info/~douillet/triangle/glossary/glossary.pdf
to it and got the following results.(all in barycentrics)
1. The zero square distance point conic to the three vertices of triangle has the equation -
(b^2+c^2)x^2+(-a^2+b^2+c^2)yz+...=0(cyclic)
2. The zero square distance point conic to the three sidelines of trilateral has the equation
(x/a)^2+...=0(cyclic)
3. The zero square distance line conic to the three vertices of triangle has the equation
x^2+...=0(cyclic)
Two questions.
1. Is this approach correct?
2. If so, are there imaginary foci of any well-known conic lying on the above conics?
If anyone is interested, had better reply to a new topic not this.
Best regards,
Tsihong Lau

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Message: #1588
Date: 17/3/2016 11:54:29
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris,
I answered you yesterday, but the message has vanished ...
Unfortunately, I won't help you very much.
1) Léon Ripert doesn't mention the dual properties.
He mentionned the construction 3 in your description of QL-P26, the line you call SL4 being exactly the line described at the end through the symmedian point of the reference triangle and the isogonal conjugate wrt the same triangle of the perspector (trilinear pole and dual of the 4th line)
This line is the isogonal conjugate of the conic through the vertices of the triangle, it's centroïd and the perspector (vertice of the QA). All this is in EQF.
The property works also for DT: the line trough the symmedian point K6 and the isogonal conjugate of QL-P13 is trough QL-P26 ; this line is the isogonal conjugate of the conic circumscribed to DT, through the centroïd QL-P8 and through the point QL-P13.
This conic is the socalled 6th conic, dual of the parabola with focus QL-P17 and directrix QL-L6 which is inscribed in both triangles QL-Tr1 and QL-Tr2. This property is not mentionned by Léon Ripert and is not in EQF.
This 6th conic and the 4 other mentionned are circumscribed to QL-Tr2 and the 4th intersection of the 6th conic with the Dimidium circle is the Cl-S conjugate of QL-P26 (this point lying on QL-P1QL-P13).
2) Wrt Coolidge's calculation, I notice that only the last formula, giving tg 2a is wrong, as well as the deduced construction.
It holds that the searched line is through QA-P1 and that tg 2a is given by the right formula tg 2a = 2 sum of xiyi/ (sum of xi^2 - sum of yi^2).
Taking a pair of orthogonal lines through QA-P1 as axes, it should be easy to calculate the xi and yi (8 values) and to calculate tg 2a and therefore tg a as root of an equation of the 2nd degree and last to draw the searched line. (If this line is an axis, one must verify that sum of xiyi is equal to zero).
I suppose this is easier with Cabri and Mathematica than with Geogebra!
Seeing the searched line as you did among other QA- or QL-points (not forgetting the symmedian point of DT) could perhaps help to find it's properties ...
Best regards Bernard

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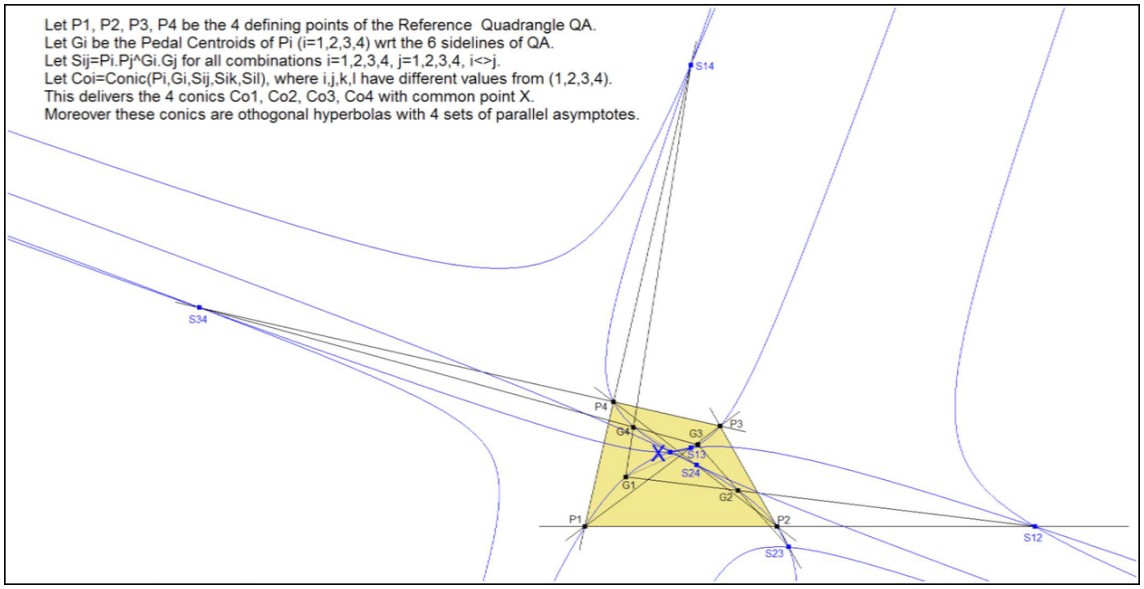
Message: #1589
Date: 17/3/2016 2:33:32
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear friends,
I found a construction for the point X with least sum of squared distances to the 6 sidelines of a quadrangle using conics.

- Let P1, P2, P3, P4 be the 4 defining points of the Reference Quadrangle QA.
- Let Gi be the Pedal Centroids of Pi (i=1,2,3,4) wrt the 6 sidelines of QA.
- Let Sij=Pi.Pj^Gi.Gj for all combinations i=1,2,3,4, j=1,2,3,4, i<>j.
- Let Coi=Conic(Pi,Gi,Sij,Sik,Sil), where i,j,k,l have different values from (1,2,3,4).
- This delivers the 4 conics Co1, Co2, Co3, Co4 with one common point X.
- Moreover these conics are othogonal hyperbolas with corresponding sets of parallel asymptotes.

See attachment.
Eckart, I hope this time you can confirm my construction . . .
Best regards,
Chris



QA-P9060-LSD-point wrt 6 QA-sidelines-12.pdf

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Message: #1590
Date: 17/3/2016 6:45:53
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris, dear Eckart, dear Benedetto
After reading Benedetto's last message, I put my construction of the LSDline of a quadrangle on a more simple way :
taking a random copple of perpendicular lines intersecting in QA-P1 as axes, the complex zi = xi + iyi represent the 4 vertices of the QA;
The centroïd of the vertices is the point QA-P1 where z = 0
The centroïd of the points represented by zi^2 is a 2nd point QA-PZ
Let's draw the line QA-P1QA-PZ (depending of the choosen axes).
The searched LSDline is the bisector of the angle between this line QA-P1QA-PZ and the axis of the xi (this last line is independant of the choosen axes).
Best regards
Bernard

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Message: #1591
Date: 17/3/2016 9:18:34
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Chris,
a remarkable construction! I have checked it with Cabri, no doubts. Congratulation!
Best regards Eckart

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Message: #1592
Date: 18/3/2016 12:57:01
From: tsihonglau
Subject: Least Squared Distance Line

Dear Bernard
I checked your construction. It seems you are right.
In fact, there are two angle bisectors. I wonder the
properties of the other bisector and the construction
not depending on complex numbers.
Best regards,
Tsihong Lau

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Message: #1593
Date: 19/3/2016 12:45:07
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Tsihong Lau,
I'm glad that you could check the construction, which works
obviously for any number of points (for example, the LSDline if
the 6 vertices of a QL is through the centroïd QL-P12 ...)
It's easy to determinate with complex numbers first the centroïd
G of the n points having as complex zi, then the centroïd of the
zi^2 and last the point K, which has as complex the square root
of the complex of this 2nd centroïd.
The LSDline is the line GK through both centroïds.
It's easy also to check that the algebric sum of the rectangles
having as opposites vertices the 1rst centroïd and the n points
is zero. (it means that sum of xiyi = 0 wrt the 2 axes LSDline
and perpendicular)
For the QA, I couldn't unfortunately find other properties wrt
the DT points (centroïd or symmedian point).
The fight goes on !
Best regards
Bernard

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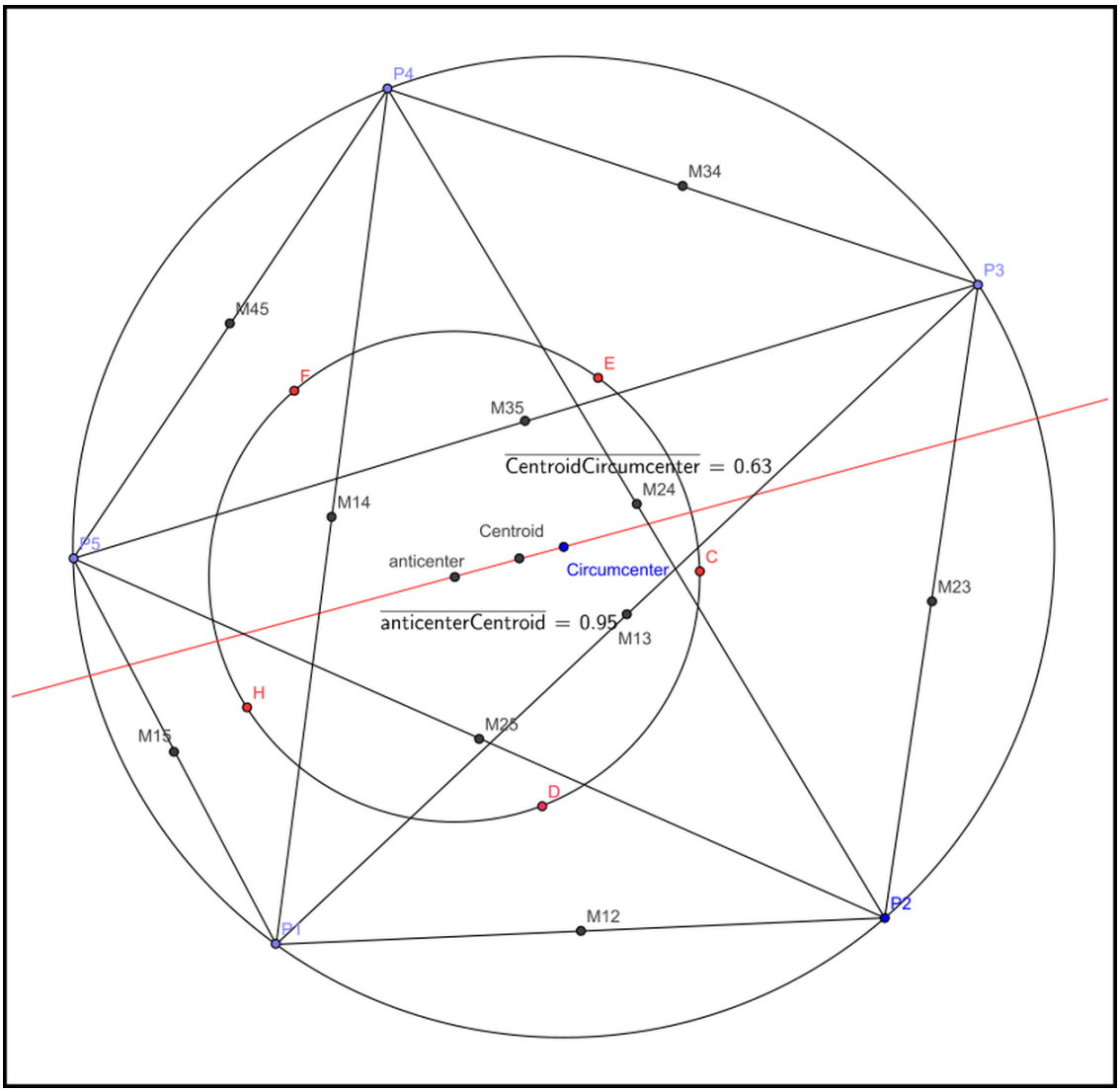
Message: #1594
Date: 20/3/2016 2:06:56
From: seiichikiri
Subject: A line in a cyclic pentangle

Dear friends,
Is the following already known?
Let P1P2P3P4P5 be a cyclic pentangle, its centroid and its circumcenter given.
5 anticenters of quadrangles P1P2P3P4, P2P3P4P5, P3P4P5P1, P4P5P1P2 and P5P1P2P3 are concyclic. See message #1126.
The center of the circumcircle of 5 anticenters, the centroid and the circumcenter of the pentangle are collinear. See the attached pdf.
I hope that this line will be extended to a general pentangle.
Best regards, Seiichi.



OGA.pdf

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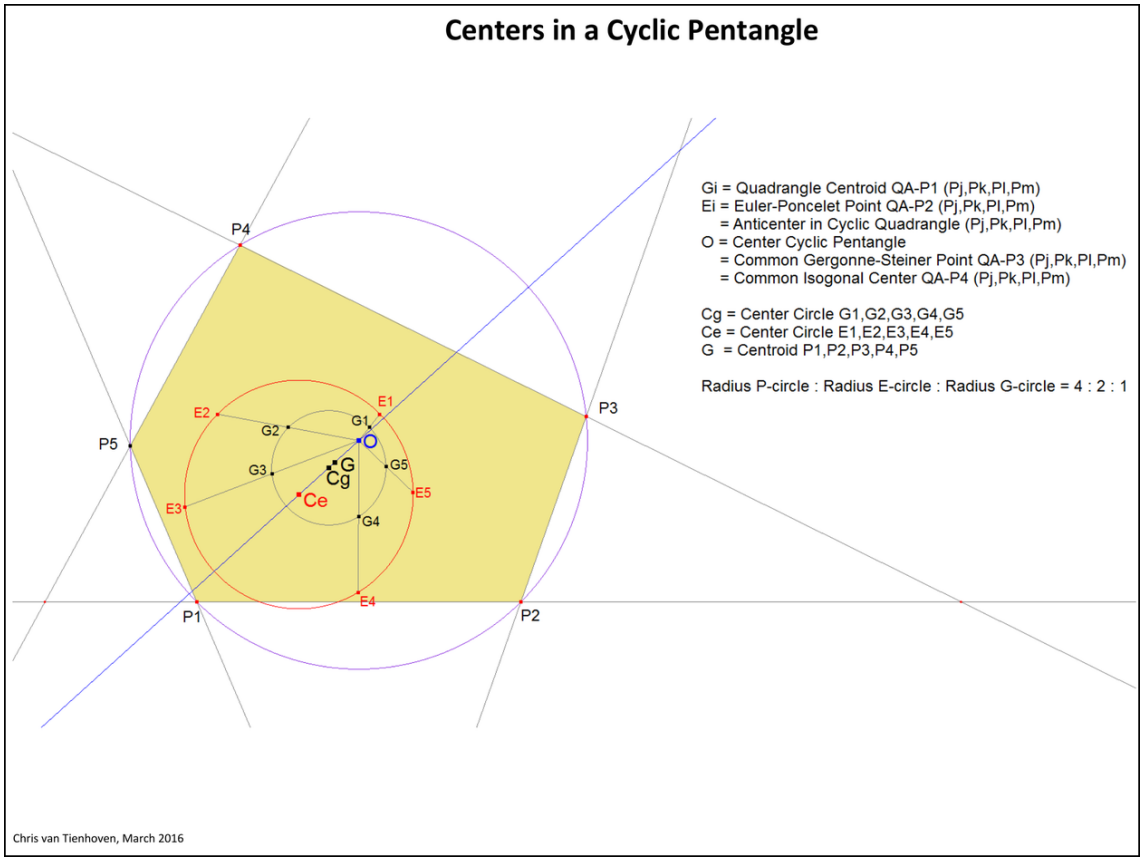
Message: #1595
Date: 20/3/2016 8:14:15
From: chris.vantienhoven
Subject: A line in a cyclic pentangle

Dear Seiichi,
Nice property!
There are properties related to Quadrangles.
Especially with QA-P1, QA-P2, QA-P3, QA-P4 of the Component Quadrangles of the Cyclic Pentangle.
See attached file.
Best regards,
Chris



5P-Cyclic-QA-P1-P2-P3-P4-Centers-01.pdf

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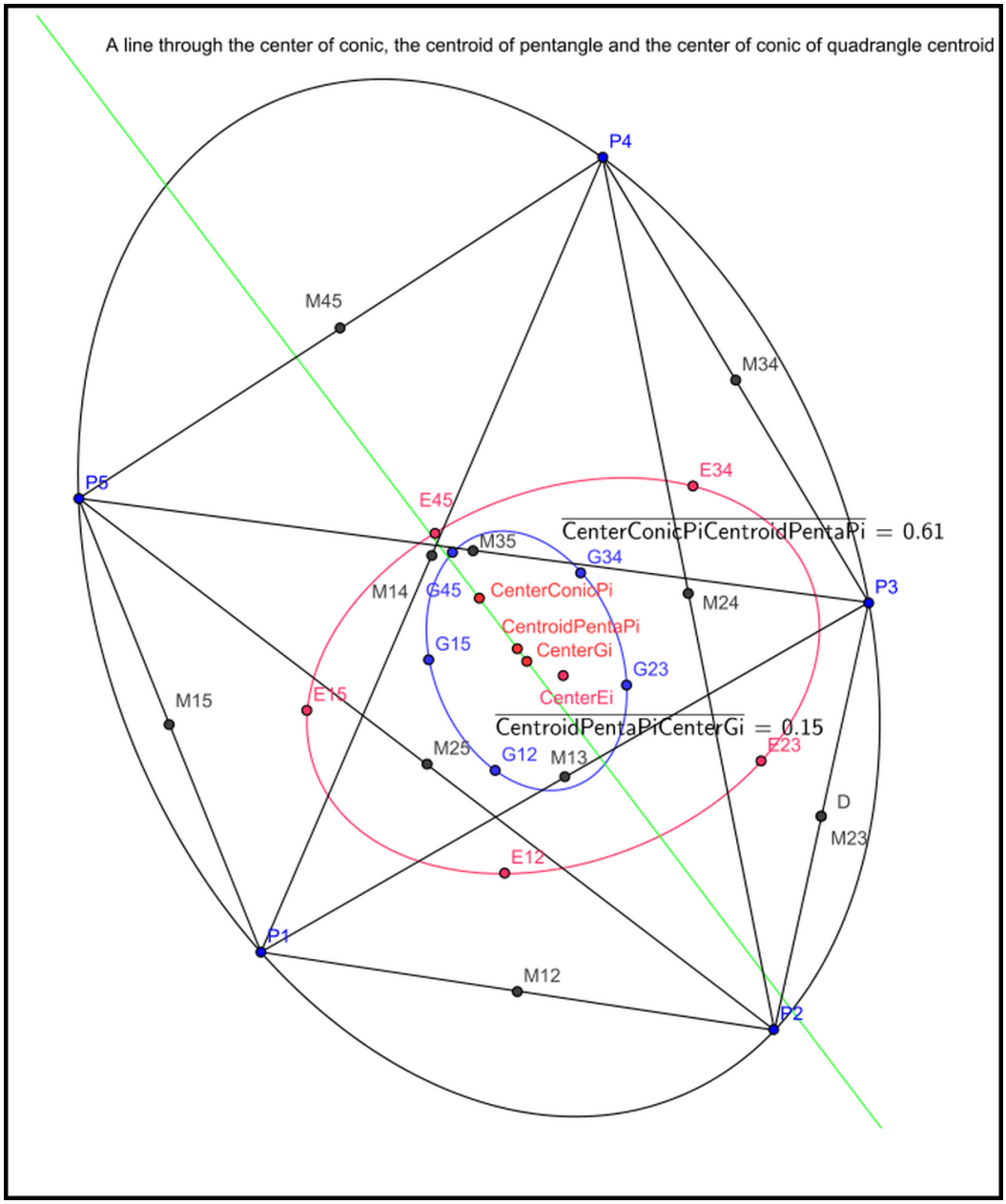
Message: #1596
Date: 21/3/2016 3:17:46
From: seiichikiri
Subject: A line in a cyclic pentangle

Dear Chris,
I drew the attached picture for a general pentangle.
The centroid of P1P2P3P4P5, its conic center and the conic center of the centroids of its quadrangle components are collinear.
This may be already posted before.
Best regards, Seiichi.



OGAgeneral-2.pdf

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Message: #1597
Date: 21/3/2016 7:04:56
From: minhtenladuong
Subject: A line in a cyclic pentangle

Dear Seiichi,
You wrote that " I drew the attached picture for a general pentangle.
The centroid of P1P2P3P4P5, its conic center and the conic center of the centroids of its quadrangle components are collinear. This may be already posted before."
This is simply follows homothety.
Best regards
Sincerely
Ngo Quang Duong

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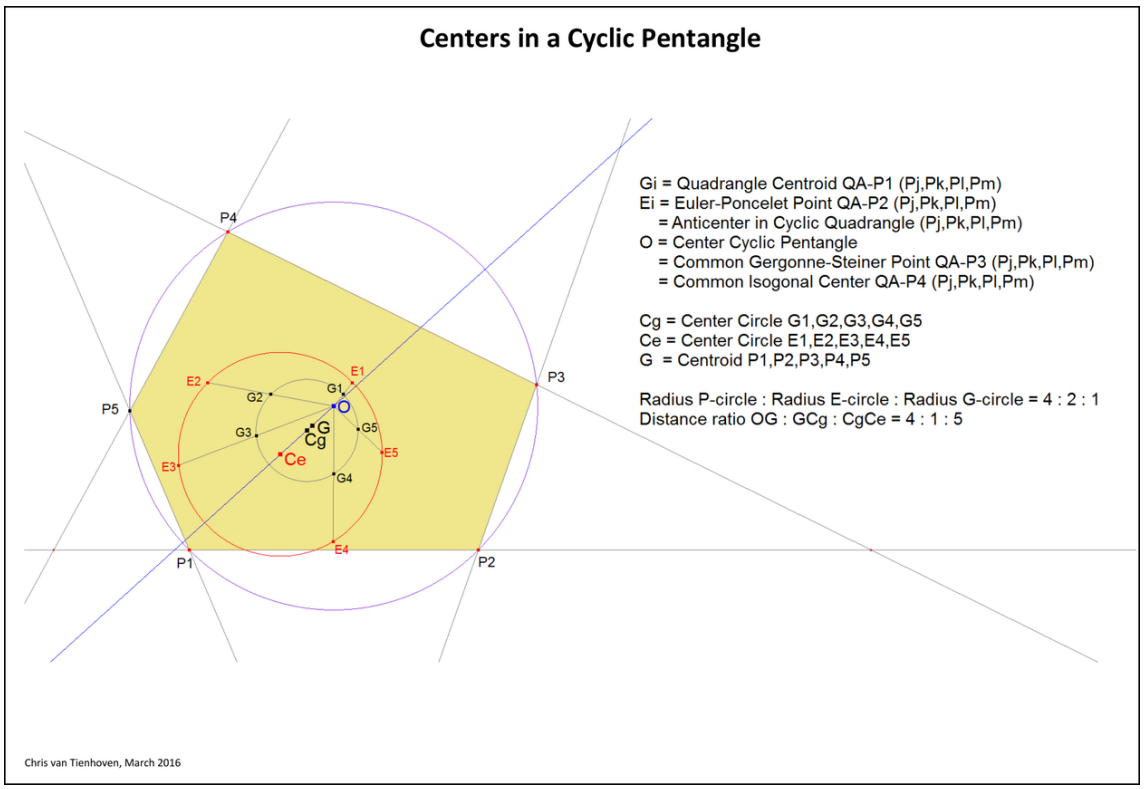
Message: #1598
Date: 21/3/2016 7:10:09
From: chris.vantienhoven
Subject: A line in a cyclic pentangle

Dear Seiichi,
You are right. See QFG#731.
Here is an extra property for the cyclic Pentangle:
distance ratio OG : GCg : CgCe = 4 : 1 : 5
Attached is an updated picture.
Best regards,
Chris



5P-Cyclic-QA-P1-P2-P3-P4-Centers-02.pdf

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Message: #1599
Date: 21/3/2016 8:33:29
From: chris.vantienhoven
Subject: Just a splitter

Dear Eckart and Bernard,
Possibly one of you wrote it before.
In a Quadrangle there are 3 Component Quadrigons representing 3 Quadrilaterals QL1, QL2, QL3 with corresponding Miquel Points MP1, MP2, MP3 forming the Miquel Triangle QA-Tr2
. Let QA-Cix be the circumcircle of QA-Tr2. Let CSC= QL-Tf1
. Now the QL1-CSC mutually transform MP2 <> MP3 and QA-Cix into the sideline MP2.MP3.
  and the QL2-CSC mutually transform MP3 <> MP1 and QA-Cix into the sideline MP3.MP1.
  and the QL3-CSC mutually transform MP1 <> MP2 and QA-Cix into the sideline MP1.MP2.
Just a “splitter”.
Best regards,
Chris

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Message: #1600
Date: 22/3/2016 8:50:11
From: eckart_schmidt@t-online.de
Subject: Just a splitter

Dear Chris,
this splitter is a basic property of CSC for quadrigons (see EQF QL-Tf1, point 3), swapping not only opposite vertices but also the 2nd and 3rd Miquel point. Evident that CSC of the circumcircle of the Miquel triangle is the line, connecting the 2nd and 3rd Miquel point.
Best regards Eckart

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Message: #1601
Date: 22/3/2016 10:07:38
From: chris.vantienhoven
Subject: Just a splitter

Dear Eckart,
There are so many things that are written and discussed before, but let me try this one:
The 3 QA-versions of QL-Tf2 in a Quadrangle of a fixed line Lx are concurrent in some point Px.
The locus of Px of a pencil of lines Lx through some point X is a QA-circumscribed cubic. **
When X=QA-P1 then the cubic passes through QA-P22.
Most probably you wrote something about it.
Best regards,
Chris

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Message: #1602
Date: 23/3/2016 4:00:01
From: eckart_schmidt@t-online.de
Subject: Just a splitter

Dear Chris,
no, I haven't wrote about it, but some remarks:
For your QA-line-point tansformation Lx ---> Px holds
(P1P2P3P4 = QA, S1S2S3 = QA-DT):
... Pi.X ---> Pi,
... Si.QA-P1 ---> Sj.Sk ^ Si.QA-P20,
... Si.QA-P16 ---> Sj.Sk ^ Si.QA-P10,
... Si.X ---> Sj.Sk ^ Si.QA-Tf2(X),
... QA-P1.QA-P16 ---> QA-P22 (your result),
... L ---> pole of L wrt the conic QA-Tf2(L).
The last result is a
remarkable relation between QA-geometry wrt the diagonal
triangle and wrt the Miquel triangle.
Inverse:
... A point Px (unequal Pi) has three lines Lx.
... The vertices of the corresponding triangle give with their
line pencil cubics with double point Px.
Finally another result:
For a QL and a P-line pencil the QL-Tf2 images envelop a QL-DT
inscribed conic.
The corresponding three conics for a QA contact QA-DT in three
collinear points on the trilinear polar of P wrt QA-DT.
I hope, there is no false Cabri observation.
Best regards Eckart

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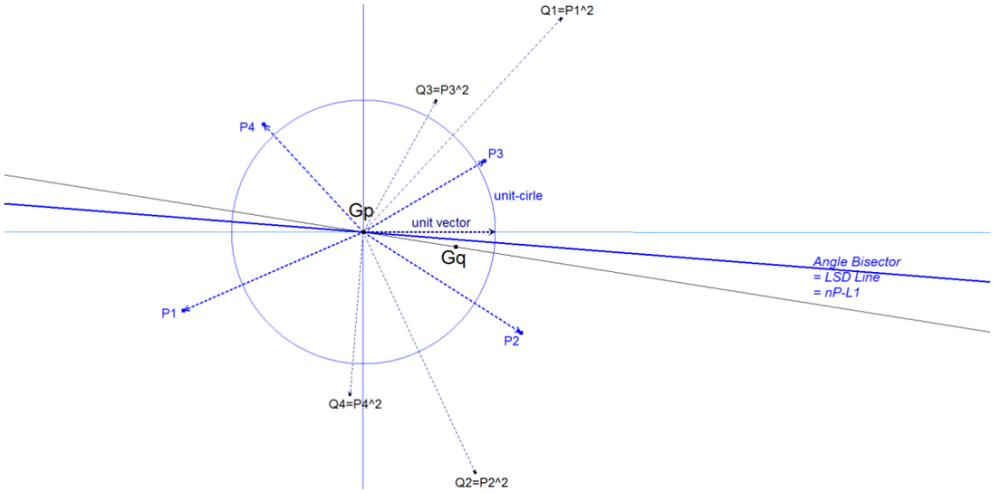
Message: #1603
Date: 23/3/2016 7:38:27
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear friends,
I made a description of the QA-LSD line, being the line in a Quadrangle with the Least Sum of Squared distances from the QA-vertices to this line and naming it QA-L10. See attachment.
I think the line is important because it deals with a for centuries well known problem. Also there is a similar question in a Quadrilateral resulting in a point (QL-P26) with Least Sum of Squared Distances to the QL-sidelines.
With QL-P26 we had from the beginning at least some extra properties.
It seems now that we hardly have extra properties for this QA-LSD line. I found two, see document. I hope we will be able to add some more.
When there are remarks or additions please let me now. Then I will add the item in EQF.
Best regards,
Chris

QA-L10 QA-LSD Line

QA-L10 is the line with the Least Sum of Squared Distances of the QA-vertices to this line.
It is the counterpart of QL-P26 in a Quadrilateral which is the point with Least Sum of Squared Distances from this point to the QL-Lines.
It is described by J.L. Coolidge in [25]. He proves that the QA-Centroid will lie on this line.
Unfortunately his proposed construction doesn't produce the desired result.
Next construction uses properties of the Complex Plane. See [34], QFG-messages #1585, #1590.

1. Draw a random couple of perpendicular lines intersecting in QA-P1 as x-axis and y-axis.
2. Let the 4 vertices of the QA be represented by complex numbers zi = xi + i yi.
3. The Centroid of the QA-vertices is the point QA-P1 where z = 0.
4. Let the QA-Pz = Centroid of the points represented by zi^2 .
5. The searched LSD line is the angle bisector of the x-axis and the line QA-P1.QA-Pz.



CT-Coefficients:

(Ua + (r-q) W : Ub + (p-r) W : Uc + (q-p) W)

where:

$Ua = a^2 (r-q) (p^2+5 p q+4 q^2+5 p r+5 q r+4 r^2)$
$+ b^2 (-2 p^3-4 p^2 q+p q^2+3 q^3-4 p^2 r+2 p q r+6 q^2 r-5 p r^2+3 q r^2)$
$+ c^2 (2 p^3+4 p^2 q+5 p q^2+4 p^2 r-2 p q r-3 q^2 r-p r^2-6 q r^2-3 r^3)$

$Ub = a^2 (-3 p^3-p^2 q+4 p q^2+2 q^3-6 p^2 r-2 p q r+4 q^2 r-3 p r^2+5 q r^2)$
$+ b^2 (p-r) (4 p^2+5 p q+q^2+5 p r+5 q r+4 r^2)$
$+ c^2 (-5 p^2 q-4 p q^2-2 q^3+3 p^2 r+2 p q r-4 q^2 r+6 p r^2+q r^2+3 r^3)$

$Uc = a^2 (3 p^3+6 p^2 q+3 p q^2+p^2 r+2 p q r-5 q^2 r-4 p r^2-4 q r^2-2 r^3)$
$+ b^2 (-3 p^2 q-6 p q^2-3 q^3+5 p^2 r-2 p q r-q^2 r+4 p r^2+4 q r^2+2 r^3)$
$+ c^2 (q-p) (4 p^2+5 p q+4 q^2+5 p r+5 q r+r^2)$

$W = Sqrt[$
$(a^4 (3 p^2+4 p q+4 q^2+6 p r+4 q r+3 r^2) (3 p^2+6 p q+3 q^2+4 p r+4 q r+4 r^2)$
$+ b^4 (4 p^2+4 p q+3 q^2+4 p r+6 q r+3 r^2) (3 p^2+6 p q+3 q^2+4 p r+4 q r+4 r^2)$
$+ c^4 (3 p^2+4 p q+4 q^2+6 p r+4 q r+3 r^2) (4 p^2+4 p q+3 q^2+4 p r+6 q r+3 r^2)$
$- 2 b^2 c^2 (p^2+3 p q+2 q^2+3 p r+q r+2 r^2) (4 p^2+4 p q+3 q^2+4 p r+6 q r+3 r^2)$
$- 2 a^2 c^2 (2 p^2+3 p q+q^2+p r+3 q r+2 r^2) (3 p^2+4 p q+4 q^2+6 p r+4 q r+3 r^2)$

$$- 2\,a^2\,b^2\,(2\,p^2+p\,q+2\,q^2+3\,p\,r+3\,q\,r+r^2)\,(3\,p^2+6\,p\,q+3\,q^2+4\,p\,r+4\,q\,r+4\,r^2))]$$

The algebraic ***least*** distance is:

(A0 (T1 + V1 W)) / (2 (T2 + V2 W))

The algebraic ***largest*** distance for the perpendicular line through QA-P1 is:

(A0 (T1 - V1 W)) / (2 (T2 - V2 W))

where:

$A0 = (a-b-c)\,(a+b-c)\,(a-b+c)\,(a+b+c)$

$V1 = (p^2-p\,q+q^2-p\,r-q\,r+r^2)$

$V2 = +a^2\,(p+q-2\,r)\,(p-2q+r) + b^2\,(p+q-2\,r)\,(-2\,p+q+r) + c^2\,(-2\,p+q+r)\,(p-2q+r)$

T1 =

$$c^2\,(2\,p^4 - 5\,p^3\,q - 5\,p^2\,q^2 - 5\,p\,q^3 + 2\,q^4 + 5\,p^3\,r + p^2\,q\,r + p\,q^2\,r + 5\,q^3\,r + 4\,p^2\,r^2 - 2\,p\,q\,r^2 + 4\,q^2\,r^2 - 2\,p\,r^3 - 2\,q\,r^3 - 3\,r^4)$$
$$+$$
$$a^2\,(-3\,p^4 - 2\,p^3\,q + 4\,p^2\,q^2 + 5\,p\,q^3 + 2\,q^4 - 2\,p^3\,r - 2\,p^2\,q\,r + p\,q^2\,r - 5\,q^3\,r + 4\,p^2\,r^2 + p\,q\,r^2 - 5\,q^2\,r^2 + 5\,p\,r^3 - 5\,q\,r^3 + 2\,r^4)$$
$$+$$
$$b^2\,(2\,p^4 + 5\,p^3\,q + 4\,p^2\,q^2 - 2\,p\,q^3 - 3\,q^4 - 5\,p^3\,r + p^2\,q\,r - 2\,p\,q^2\,r - 2\,q^3\,r - 5\,p^2\,r^2 + p\,q\,r^2 + 4\,q^2\,r^2 - 5\,p\,r^3 + 5\,q\,r^3 + 2\,r^4)$$

T2 =

$$b^4\,(-8\,p^4 - 12\,p^3\,q - 3\,p^2\,q^2 - 2\,p\,q^3 - 3\,q^4 + 4\,p^3\,r + 18\,p^2\,q\,r + 12\,p\,q^2\,r - 2\,q^3\,r - 3\,p^2\,r^2 + 18\,p\,q\,r^2 - 3\,q^2\,r^2 + 4\,p\,r^3 - 12\,q\,r^3 - 8\,r^4) +$$
$$a^4\,(-3\,p^4 - 2\,p^3\,q - 3\,p^2\,q^2 - 12\,p\,q^3 - 8\,q^4 - 2\,p^3\,r + 12\,p^2\,q\,r + 18\,p\,q^2\,r + 4\,q^3\,r - 3\,p^2\,r^2 + 18\,p\,q\,r^2 - 3\,q^2\,r^2 - 12\,p\,r^3 + 4\,q\,r^3 - 8\,r^4) +$$
$$c^4\,(-8\,p^4 + 4\,p^3\,q - 3\,p^2\,q^2 + 4\,p\,q^3 - 8\,q^4 - 12\,p^3\,r + 18\,p^2\,q\,r + 18\,p\,q^2\,r - 12\,q^3\,r - 3\,p^2\,r^2 + 12\,p\,q\,r^2 - 3\,q^2\,r^2 - 2\,p\,r^3 - 2\,q\,r^3 - 3\,r^4) +$$
$$2\,b^2\,c^2\,(4\,p^3\,q + 3\,p^2\,q^2 + 3\,p\,q^3 + 4\,q^4 + 4\,p^3\,r - 18\,p^2\,q\,r - 3\,p\,q^2\,r + q^3\,r + 3\,p^2\,r^2 - 3\,p\,q\,r^2 - 6\,q^2\,r^2 + 3\,p\,r^3 + q\,r^3 + 4\,r^4) +$$
$$2\,a^2\,b^2\,(4\,p^4 + p^3\,q - 6\,p^2\,q^2 + p\,q^3 + 4\,q^4 + 3\,p^3\,r - 3\,p^2\,q\,r - 3\,p\,q^2\,r + 3\,q^3\,r + 3\,p^2\,r^2 - 18\,p\,q\,r^2 + 3\,q^2\,r^2 + 4\,p\,r^3 + 4\,q\,r^3) +$$
$$2\,a^2\,c^2\,(4\,p^4 + 3\,p^3\,q + 3\,p^2\,q^2 + 4\,p\,q^3 + p^3\,r - 3\,p^2\,q\,r - 18\,p\,q^2\,r + 4\,q^3\,r - 6\,p^2\,r^2 - 3\,p\,q\,r^2 + 3\,q^2\,r^2 + p\,r^3 + 3\,q\,r^3 + 4\,r^4)$$

Properties:

- There is no line with largest sum of squared distances. However when we look for the line with this property passing through QA-P1, then QA-L10p being the line through QA-P1 perpendicular to QA-L10 will be the line with largest sum of squared distances.
- In a Quadrilateral the triangle formed by the 3 QL-versions of QA-L10 and the triangle formed by the 3 QL-versions of QA-L10p are perspective in a common point of the circumscribed circles of these triangles.

QAL10-LSD Line-01.pdf

Message: #1604
Date: 24/3/2016 12:10:52
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris,
Congratulation for the calculation !
I was in fact a little bit surprised by your absence of reaction, but I can see now you were working on the item ...
I have 3 remarks, may be of some interest
1) in message 1577, you asked for a synthetic construction
consider the end I of the unit segment (intersection of the unit circle and the unit segment on the x-axis)
the point Ii is the reflexion of the point I in the line GpPi
(the Ii are on the unit circle and on the reflexions of the x-axis in the lines GpPi)
the points Qi are the inverse of the points Ii in the circles with center Gp through the points Pi
the rest is described in your figure (barycenter Gq of the Qi and internal bisector of the angle IGpGq)
2) considering the 4 rectangles with one diagonal being GpPi and 2 sides being QA-L10 and QA-L10p, the algebric sum of their surfaces is equal to zero (meaning that the sum of xiyi wrt the system of axes QA-L10 and QA-L10p is equal to zero)
3) the same construction can be used for any number of points (for example the LSD line of the 6 vertices of a QL, with an arbitrary line through QL-P12 as x-axis and an arbitrary unit circle with center QL-P12 ..)
Best regards
Bernard

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Message: #1605
Date: 25/3/2016 11:13:18
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Chris,
I drew the QL LSD line of the 6 vertices of QL with the same construction as mentionned in the previous message.
Unfortunately, I can not find any interesting property wrt the existing lines L1, L2, L6, nor P3P4 nor P2P6 (QL LSD line intersects Newton Line L1 and P2P6 in QL-P12 ...)
I hope you will be able to find some !
Best regards
Bernard

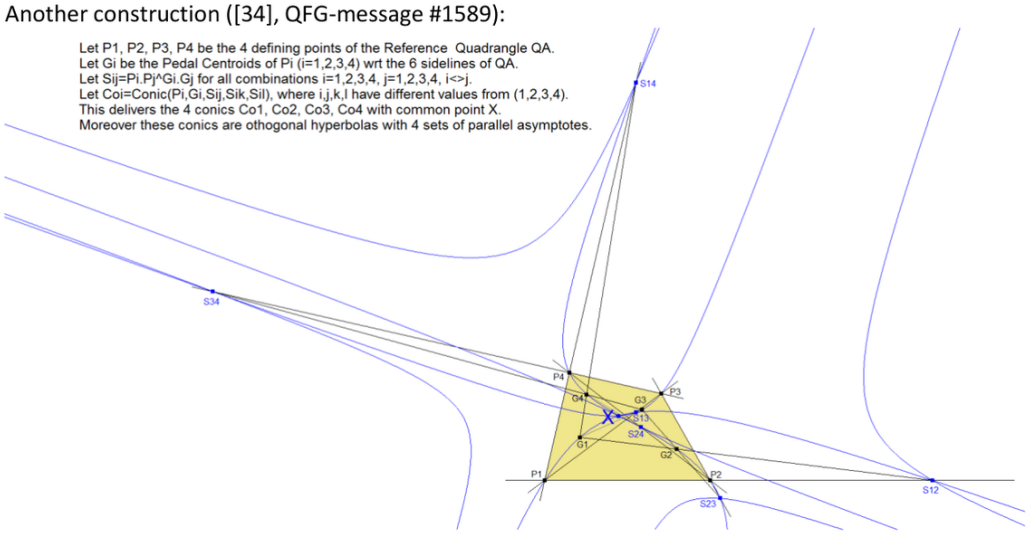
← Previous Next → ↩ Message Index ↑ Subjects

Message: #1606
Date: 25/3/2016 9:34:25
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear friends,
I calculated the coordinates of the LSD point wrt the 6 QA-sidelines.
It was about the same outcome as Seiichi's in QFG=message #13.
I also calculated the Sum of Squared distances of this point.
See attached file.
Unfortunately no incidences at all with other QFG-objects and very few extra properties.
I hope there will come more.
Best regards,
Chris

QA-P44 LSD point wrt the six QA-sidelines

QA-P44 is the point with the Least Sum of Squared Distances to the six sidelines of a Quadrangle.
For a discussion on this point see [34], QFG-messages #13, #1569 and following messages.
The point can be constructed in a similar way as described in the 1st construction of QL-P26 according to [25].
Another construction ([34], QFG-message #1589):



1st CT-Coordinate:

$a^6\,q^2\,(p+q)\,r^2\,(p+r)$
$+c^6\,p^2\,q^2\,(p+r)\,(3\,q+2\,r)$
$+b^6\,p^2\,(p+q)\,r^2\,(2\,q+3\,r)$
$-b^4\,c^2\,p^2\,r\,(4\,p^2\,q-4\,q^3+6\,p^2\,r+9\,p\,q\,r+2\,q^2\,r+6\,p\,r^2+7\,q\,r^2)$
$-b^2\,c^4\,p^2\,q\,(6\,p^2\,q+6\,p\,q^2+4\,p^2\,r+9\,p\,q\,r+7\,q^2\,r+2\,q\,r^2-4\,r^3)$
$+a^4\,c^2\,q^2\,r\,(p+r)\,(2\,p^2-2\,q^2-2\,p\,r-q\,r)$
$+a^4\,b^2\,q\,(p+q)\,r^2\,(2\,p^2-2\,p\,q-q\,r-2\,r^2)$
$-a^2\,c^4\,p\,q^2\,(p+r)\,(3\,p\,q+3\,q^2+4\,p\,r-r^2)$
$-a^2\,b^4\,p\,(p+q)\,r^2\,(4\,p\,q-q^2+3\,p\,r+3\,r^2)$
$+2\,a^2\,b^2\,c^2\,p\,q\,r\,(2\,p^3-p^2\,q-2\,p\,q^2+q^3-p^2\,r-4\,p\,q\,r-3\,q^2\,r-2\,p\,r^2-3\,q\,r^2+r^3)$

The sum of squared distances for QA-P44 is:

$$((-a+b+c)\,(a+b-c)\,(a-b+c)\,(a+b+c)\,(-6\,b^2\,c^4\,p^4\,q^2-3\,a^2\,c^4\,p^3\,q^3-3\,b^2\,c^4\,p^3\,q^3+3\,c^6\,p^3\,q^3-6\,a^2\,c^4\,p^2\,q^4+4\,a^2\,b^2\,c^2\,p^4\,q\,r-4\,b^4\,c^2\,p^4\,q\,r-4\,b^2\,c^4\,p^4\,q\,r+2\,a^4\,c^2\,p^3\,q^2\,r-4\,a^2\,b^2\,c^2\,p^3\,q^2\,r+2\,b^4\,c^2\,p^3\,q^2\,r-4\,a^2\,c^4\,p^3\,q^2\,r-4\,b^2\,c^4\,p^3\,q^2\,r+2\,c^6\,p^3\,q^2\,r+2\,a^4\,c^2\,p^2\,q^3\,r-4\,a^2\,b^2\,c^2\,p^2\,q^3\,r+2\,b^4\,c^2\,p^2\,q^3\,r-4\,a^2\,c^4\,p^2\,q^3\,r-4\,b^2\,c^4\,p^2\,q^3\,r+2\,c^6\,p^2\,q^3\,r-4\,a^4\,c^2\,p\,q^4\,r+4\,a^2\,b^2\,c^2\,p\,q^4\,r-4\,a^2\,c^4\,p\,q^4\,r-6\,b^4\,c^2\,p^4\,r^2+2\,a^4\,b^2\,p^3\,q\,r^2-4\,a^2\,b^4\,p^3\,q\,r^2+2\,b^6\,p^3\,q\,r^2-4\,a^2\,b^2\,c^2\,p^3\,q\,r^2-4\,b^4\,c^2\,p^3\,q\,r^2+2\,b^2\,c^4\,p^3\,q\,r^2+a^6\,p^2\,q^2\,r^2-a^4\,b^2\,p^2\,q^2\,r^2-a^2\,b^4\,p^2\,q^2\,r^2+b^6\,p^2\,q^2\,r^2-a^4\,c^2\,p^2\,q^2\,r^2-12\,a^2\,b^2\,c^2\,p^2\,q^2\,r^2-b^4\,c^2\,p^2\,q^2\,r^2-a^2\,c^4\,p^2\,q^2\,r^2-b^2\,c^4\,p^2\,q^2\,r^2+c^6\,p^2\,q^2\,r^2+2\,a^6\,p\,q^3\,r^2-4\,a^4\,b^2\,p\,q^3\,r^2+2\,a^2\,b^4\,p\,q^3\,r^2-4\,a^4\,c^2\,p\,q^3\,r^2-4\,a^2\,b^2\,c^2\,p\,q^3\,r^2+2\,a^2\,c^4\,p\,q^3\,r^2-6\,a^4\,c^2\,q^4\,r^2-3\,a^2\,b^4\,p^3\,r^3+3\,b^6\,p^3\,r^3-3\,b^4\,c^2\,p^3\,r^3+2\,a^4\,b^2\,p^2\,q\,r^3-4\,a^2\,b^4\,p^2\,q\,r^3+2\,b^6\,p^2\,q\,r^3-4\,a^2\,b^2\,c^2\,p^2\,q\,r^3-4\,b^4\,c^2\,p^2\,q\,r^3+2\,b^2\,c^4\,p^2\,q\,r^3+2\,a^6\,p\,q^2\,r^3-4\,a^4\,b^2\,p\,q^2\,r^3+2\,a^2\,b^4\,p\,q^2\,r^3-4\,a^4\,c^2\,p\,q^2\,r^3+2\,a^2\,c^4\,p\,q^2\,r^3+3\,a^6\,q^3\,r^3-3\,a^4\,b^2\,q^3\,r^3-6\,a^2\,b^4\,p^2\,r^4-4\,a^4\,b^2\,p\,q\,r^4-4\,a^2\,b^4\,p\,q\,r^4+4\,a^2\,b^2\,c^2\,p\,q\,r^4-6\,a^4\,b^2\,q^2\,r^4))\;/$$

$$(4\,(-6\,a^2\,b^2\,c^4\,p^4\,q^2-3\,b^4\,c^4\,p^4\,q^2-2\,b^2\,c^6\,p^4\,q^2-3\,a^4\,c^4\,p^3\,q^3-12\,a^2\,b^2\,c^4\,p^3\,q^3-3\,b^4\,c^4\,p^3\,q^3+2\,a^2\,c^6\,p^3\,q^3+2\,b^2\,c^6\,p^3\,q^3+c^8\,p^3\,q^3-3\,a^4\,c^4\,p^2\,q^4-6\,a^2\,b^2\,c^4\,p^2\,q^4-2\,a^2\,c^6\,p^2\,q^4+4\,a^4\,b^2\,c^2\,p^4\,q\,r-2\,a^2\,b^4\,c^2\,p^4\,q\,r-2\,a^2\,b^2\,c^4\,p^4\,q\,r-6\,b^4\,c^4\,p^4\,q\,r-2\,b^2\,c^6\,p^4\,q\,r+2\,a^6\,c^2\,p^3\,q^2\,r+2\,a^4\,b^2\,c^2\,p^3\,q^2\,r-4\,a^2\,b^4\,c^2\,p^3\,q^2\,r-3\,a^4\,c^4\,p^3\,q^2\,r-22\,a^2\,b^2\,c^4\,p^3\,q^2\,r-3\,b^4\,c^4\,p^3\,q^2\,r+2\,b^2\,c^6\,p^3\,q^2\,r+c^8\,p^3\,q^2\,r-4\,a^4\,b^2\,c^2\,p^2\,q^3\,r+2\,a^2\,b^4\,c^2\,p^2\,q^3\,r+2\,b^6\,c^2\,p^2\,q^3\,r-3\,a^4\,c^4\,p^2\,q^3\,r-22\,a^2\,b^2\,c^4\,p^2\,q^3\,r-3\,b^4\,c^4\,p^2\,q^3\,r+2\,a^2\,c^6\,p^2\,q^3\,r+c^8\,p^2\,q^3\,r-2\,a^6\,c^2\,p\,q^4\,r-2\,a^4\,b^2\,c^2\,p\,q^4\,r+4\,a^2\,b^4\,c^2\,p\,q^4\,r-6\,a^4\,c^4\,p\,q^4\,r-2\,a^2\,b^2\,c^4\,p\,q^4\,r-6\,a^2\,b^4\,c^2\,p^4\,r^2-2\,b^6\,c^2\,p^4\,r^2-3\,b^4\,c^4\,p^4\,r^2+2\,a^6\,b^2\,p^3\,q\,r^2-3\,a^4\,b^4\,p^3\,q\,r^2+b^8\,p^3\,q\,r^2+2\,a^4\,b^2\,c^2\,p^3\,q\,r^2-22\,a^2\,b^4\,c^2\,p^3\,q\,r^2+2\,b^6\,c^2\,p^3\,q\,r^2-4\,a^2\,b^2\,c^4\,p^3\,q\,r^2-3\,b^4\,c^4\,p^3\,q\,r^2+a^8\,p^2\,q^2\,r^2+2\,a^6\,b^2\,p^2\,q^2\,r^2-6\,a^4\,b^4\,p^2\,q^2\,r^2+2\,a^2\,b^6\,p^2\,q^2\,r^2+b^8\,p^2\,q^2\,r^2+2\,a^6\,c^2\,p^2\,q^2\,r^2-12\,a^4\,b^2\,c^2\,p^2\,q^2\,r^2-12\,a^2\,b^4\,c^2\,p^2\,q^2\,r^2+2\,b^6\,c^2\,p^2\,q^2\,r^2-6\,a^4\,c^4\,p^2\,q^2\,r^2-12\,a^2\,b^2\,c^4\,p^2\,q^2\,r^2-6\,b^4\,c^4\,p^2\,q^2\,r^2+2\,a^2\,c^6\,p^2\,q^2\,r^2+2\,b^2\,c^6\,p^2\,q^2\,r^2+c^8\,p^2\,q^2\,r^2+a^8\,p\,q^3\,r^2-3\,a^4\,b^4\,p\,q^3\,r^2+2\,a^2\,b^6\,p\,q^3\,r^2+2\,a^6\,c^2\,p\,q^3\,r^2-22\,a^4\,b^2\,c^2\,p\,q^3\,r^2+2\,a^2\,b^4\,c^2\,p\,q^3\,r^2-3\,a^4\,c^4\,p\,q^3\,r^2-4\,a^2\,b^2\,c^4\,p\,q^3\,r^2-2\,a^6\,c^2\,q^4\,r^2-6\,a^4\,b^2\,c^2\,q^4\,r^2-3\,a^4\,c^4\,q^4\,r^2-3\,a^4\,b^4\,p^3\,r^3+2\,a^2\,b^6\,p^3\,r^3+b^8\,p^3\,r^3-12\,a^2\,b^4\,c^2\,p^3\,r^3+2\,b^6\,c^2\,p^3\,r^3-3\,b^4\,c^4\,p^3\,r^3-3\,a^4\,b^4\,p^2\,q\,r^3+2\,a^2\,b^6\,p^2\,q\,r^3+b^8\,p^2\,q\,r^3-4\,a^4\,b^2\,c^2\,p^2\,q\,r^3-22\,a^2\,b^4\,c^2\,p^2\,q\,r^3+2\,a^2\,b^2\,c^4\,p^2\,q\,r^3-3\,b^4\,c^4\,p^2\,q\,r^3+2\,b^2\,c^6\,p^2\,q\,r^3+a^8\,p\,q^2\,r^3+2\,a^6\,b^2\,p\,q^2\,r^3-3\,a^4\,b^4\,p\,q^2\,r^3-22\,a^4\,b^2\,c^2\,p\,q^2\,r^3-3\,a^4\,c^4\,p\,q^2\,r^3+2\,a^2\,b^2\,c^4\,p\,q^2\,r^3-3\,b^4\,c^4\,p\,q^2\,r^3+2\,a^2\,c^6\,p\,q^2\,r^3+a^8\,q^3\,r^3+2\,a^6\,b^2\,q^3\,r^3-3\,a^4\,b^4\,q^3\,r^3+2\,a^6\,c^2\,q^3\,r^3-12\,a^4\,b^2\,c^2\,q^3\,r^3-3\,a^4\,c^4\,q^3\,r^3-3\,a^4\,b^4\,p^2\,r^4-2\,a^2\,b^6\,p^2\,r^4-6\,a^2\,b^4\,c^2\,p^2\,r^4-2\,a^6\,b^2\,p\,q\,r^4-6\,a^4\,b^4\,p\,q\,r^4-2\,a^2\,b^6\,p\,q\,r^4-2\,a^4\,b^2\,c^2\,p\,q\,r^4-2\,a^2\,b^4\,c^2\,p\,q\,r^4+4\,a^2\,b^2\,c^4\,p\,q\,r^4-2\,a^6\,b^2\,q^2\,r^4-3\,a^4\,b^4\,q^2\,r^4-6\,a^4\,b^2\,c^2\,q^2\,r^4))$$

QA-P44 LSD point wrt six QA-sidelines.pdf

Properties:

- QA-P44 is also the Centroid of the perpendicular projection points of QA-P44 on the six sidelines.

QA-P44 LSD point wrt six QA-sidelines.pdf

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Message: #1607
Date: 26/3/2016 10:33:38
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

```
Dear Bernard, dear Chris,
in addition to your results (sloppy formulated):
if we take points/lines with constant sqared distance sum, we
get conics, whose center/axis give the LSD point/line.
Happy Easter! Best regards Eckart
```

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Message: #1608
Date: 26/3/2016 11:57:25
From: eckart_schmidt@t-online.de
Subject: Just a splitter

```
Dear Chris,
there is an analog transformation in QL-geometry:
Point-line  transformation: Px ---> Lx = polar of Px wrt the
envelop conic of QL-Tf2(L), L lines through Px.
... for Px = QL-P1:  Lx = CSC(QL-Ci6),
... for Px = QL-P8:  Lx = line through QL-P23,
                          parallel to its QL-Tf2-image,
... for Px = QL-P13: Lx = line through QL-P19 with CSC-circle
                          centered in the midpoint
                          of QL-P1.CSC(QL-P19).
Best regards Eckart
```

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Message: #1609
Date: 26/3/2016 2:21:56
From: eckart_schmidt@t-online.de
Subject: Just a splitter

```
Dear Chris,
there are further interesting properties of the
"splitter"-transformations:
A-Tf  (#1602):
... The three image-points for a line L wrt the quadrigons of a
QL are collinear on QL-Tf2(L).
Point to line QL-Tf  (#1608): Px ---> Lx = polar of Px wrt the
envelop conic of QL-Tf2(L), L lines through Px.
... The three image-lines of  a point P wrt the quadrigons of a
QA intersect in QA-Tf2(P).
Best regards Eckart
```

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Message: #1610
Date: 28/3/2016 1:14:39
From: seiichikiri
Subject: Least Squared Distance Line

```
Dear Chris,
Have you confirmed that "LSD" line in QA-L99-LSD line-00.pdf
does not move itself even if the quadrangle is changed to
pentangle and a circle with random center is moved?
Best regards, Seiichi.
```

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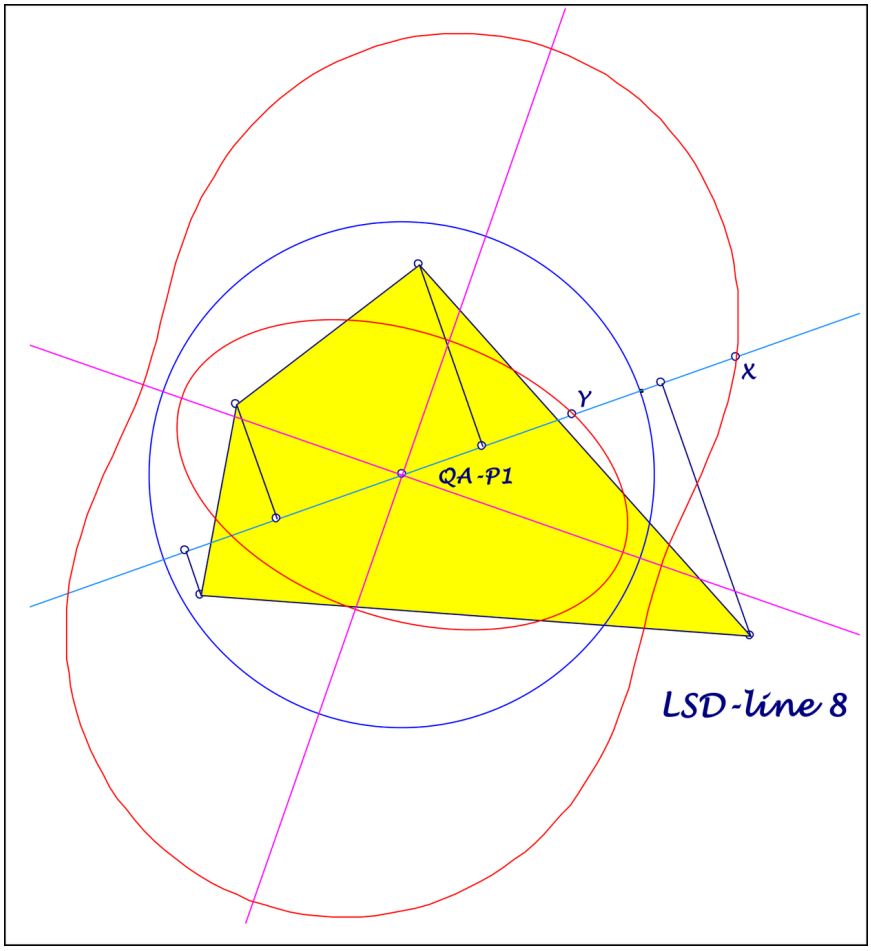
Message: #1611
Date: 28/3/2016 4:47:21
From: eckart_schmidt@t-online.de
Subject: Least Squared Distance Line

Dear Bernard, dear Chris, dear Tsihong Lau,
I think, the following constructions of LSD-line 8 and 9 are more synthetically, here only described for LSD-line 8, but analog applicable for LSD-line 9 and possible for generalization.
8. The least square line to the four vertices of quadrangle.
Let L be a line through QA-P1,
... di the distances to the vertices of the quadrangle.
... Construct a distance sqr(d1^2+d2^2+d3^2+d4^2) with the theorem of Pythagoras
... and a point X on L with this distance.
... Let Y be the inversion of X wrt a fixed circle round QA-P1,
... then the locus of Y is a conic, centered in QA-P1.
... The major axis of the conic is the LSD-line.
Best regards Eckart



2016-03-28.docx

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Message: #1612
Date: 28/3/2016 8:29:36
From: chris.vantienhoven
Subject: QA-related Centers in a configuration of two perspective triangles

---

Dear friends,

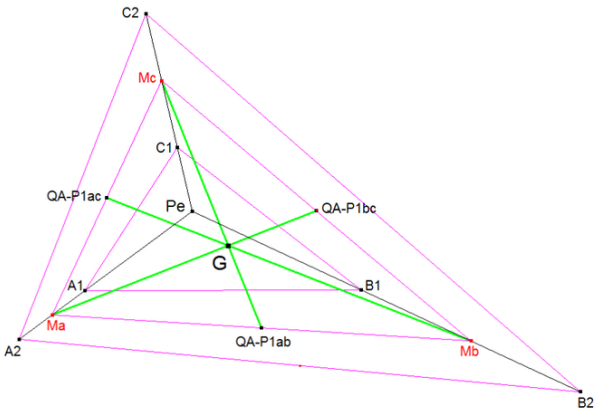
1. Given 2 perspective triangles A1B1C1 and A2B2C2 with perspector Pe.
Let Ma=Midpoint(A1,A2), Mb=Midpoint(B1,B2), Mc=Midpoint(C1,C2).
Let Gab=QA-P1 of A1.A2.B1.B2, Gbc=QA-P1 of B1.B2.C1.C2, Gca=QA-P1 of C1.C2.A1.A2.
Lines Ma.Gbc, Mb.Gca, Mc.Gab are concurrent in a point G.
G is the centroid of the sexangle A1.A2.B1.B2.C1.C2.

2. Given 2 perspective triangles A1B1C1 and A2B2C2 with perspector Pe.
Let Na=HarmonicConjugate of Pe wrt (A1,A2), Nb= HarmonicConjugate of Pe wrt (B1,B2), Nc=HarmonicConjugate of Pe wrt (C1,C2),
Let Hab=QA-P16 of A1.A2.B1.B2, Hbc=QA-P16 of B1.B2.C1.C2, Hca=QA-P16 of C1.C2.A1.A2.
Lines Na.Hbc, Nb.Hca, Nc.Hab are concurrent in a point H.
H is a harmonic center of the sexangle A1.A2.B1.B2.C1.C2.
See attachment.

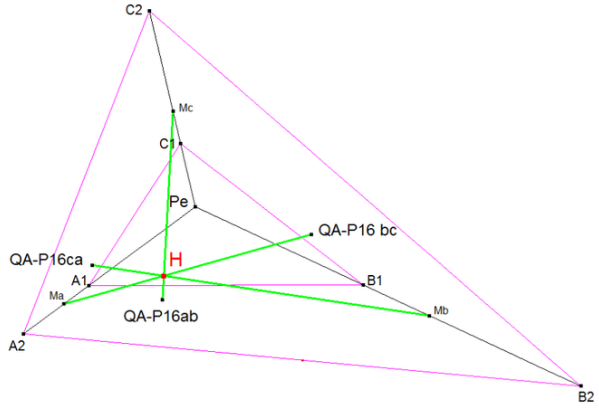
Best regards,
Chris

Centers in a Configuration of two Perspective Triangles

In the configuration of 2 Perspective Triangles there is a Configuration Centroid G.
G is the centroid of the sexangle A1,A2,B1,B2,C1,C2



In the configuration of 2 Perspective Triangles there is a Configuration Centroid G.
G is the centroid of the sexangle A1,A2,B1,B2,C1,C2.
When we work with Perspective Midpoints and QA-P16 instead of QA-P1,
we find another center, the Harmonic Center H.



Chris van Tienhoven, March 2016

PT-Configuration-Centers-01.pdf

Message: #1613
Date: 28/3/2016 9:32:01
From: chris.vantienhoven
Subject: Least Squared Distance Line

Dear Seiichi,
[SK] Have you confirmed that "LSD" line in QA-L99-LSD line-00.pdf does not move itself even if the quadrangle is changed to pentangle and a circle with random center is moved?
Yes I checked that the construction also works for a pentangle and that the constructed line
does not move even if the circle with random center is moved.
Unfortunately this construction (like we discussed in messages #1571, #1572) doesn't produce an LSD-line. But fortunately the dual Coolidge-construction like used with QL-26 does produce the expected LSD point.
Best regards,
Chris

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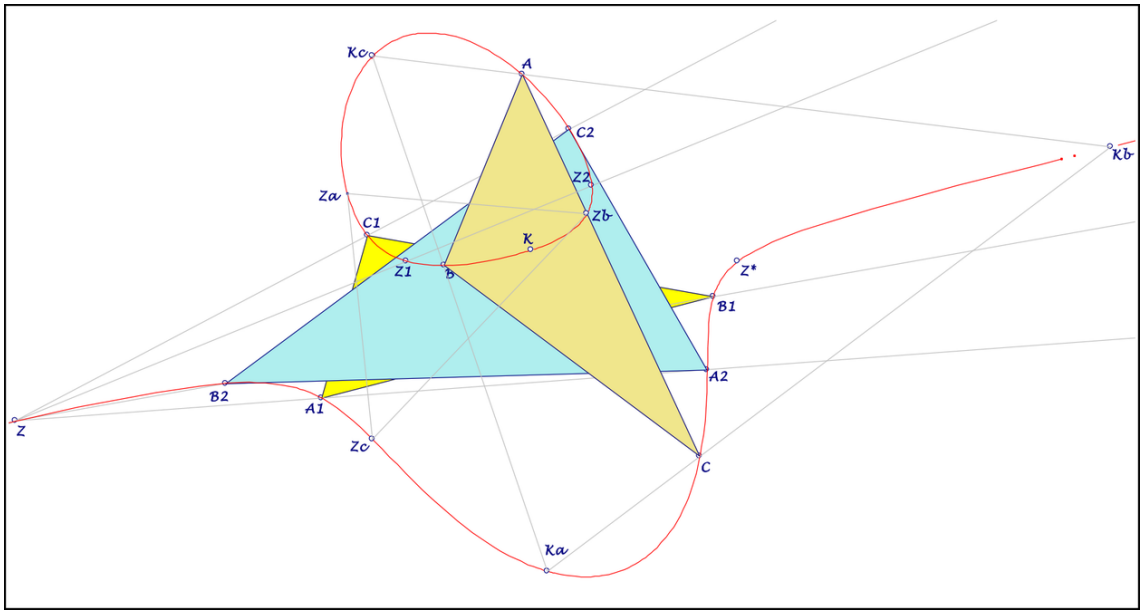
Message: #1614
Date: 29/3/2016 1:44:12
From: eckart_schmidt@t-online.de
Subject: QA-related Centers in a configuration of two perspective triangl

---

Dear Chris,
perhaps of interest: in 2003 I wrote the attached paper about perspective triangles, but in German. Here some remarks (I use the nomination of the paper): Let A1B1C1 and A2B2C2 be perspective triangles with perspector Z,
... let ABC be a reference triangle with
... ... A = B1C2 ^ B2C1, B = A1C2 ^ A2C1, C = A1B2 ^ A2B1.
... ABC is perspective AiBiCi wrt Zi (collinear Z).
There is an interesting pivotal isocubic with 20 adressable points (see attached figure):
Reference triangle ABC,
... isoconjugation, which swaps Z1 and Z2
    (A1 and A2, B1 and B2, C1 and C2),
... pivot Z.
The 20 points are:
... A, B, C,
... A1, A2, A3, B1, B2, B3,
... Z, Z1, Z2,
... the fixed points K, Ka, Kb, Kc of the isoconjugation,
... the vertices Za, Zb, Zc of the cevian triangle of Z wrt ABC
... and the isoconjugate of Z.
Best regards Eckart



2016-03-29.docx

Sechsecke aus perspektiven Tripeln

Eckart Schmidt

Es werden Sechsecke $A_1B_2C_1A_2B_1C_2$ untersucht, deren Teildreiecke $A_1B_1C_1$ und $A_2B_2C_2$ perspektiv zu einem Zentrum Z liegen (Abb. 1).

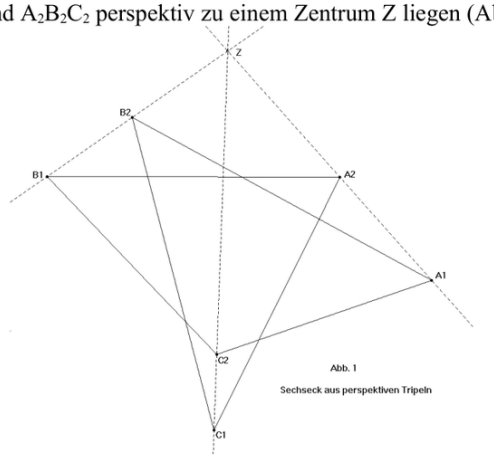


Abb. 1

Sechseck aus perspektiven Tripeln

Zu diesen Sechsecken lässt sich weiterhin das Dreieck ABC aus den Schnittpunkten der Diagonalen betrachten, dabei sei A Schnittpunkt von B_1C_2 und B_2C_1, B Schnittpunkt von C_1A_2 und C_2A_1, C Schnittpunkt von A_1B_2 und A_2B_1. Das Dreieck ABC sei als Schnittdreieck bezeichnet; es liegt perspektiv zu $A_1B_1C_1$ und $A_2B_2C_2$ bzgl. Z_1 und Z_2, wobei die Zentren Z, Z_1, Z_2 kollinear sind (Abb. 2).

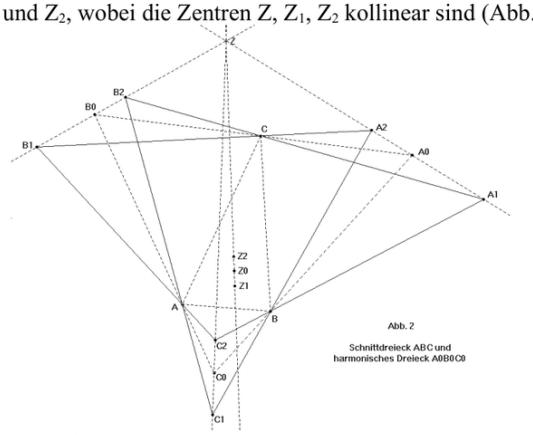


Abb. 2

Schnittdreieck ABC und harmonisches Dreieck A0B0C0

Geht man von dem Schnittdreieck ABC aus, so legt jedes perspektive Dreieck $A_1B_1C_1$ ein Sechseck $A_1B_2C_1A_2B_1C_2$ im Sinne des Themas fest. Beginnt man andererseits einen umbeschriebenen Streckenzug mit beliebigem A_1, wählt B_2 auf A_1C, C_1 auf B_2A, A_2 auf C_1B, so liegen die fehlenden Punkte B_1 und C_2 fest.

Zu zwei perspektiven Tripeln A_1, B_1, C_1 und A_2, B_2, C_2 mit dem Zentrum Z liegt es nahe, die vierten harmonischen Punkte A_o, B_o, C_0 zu betrachten, die die Strecken A_1A_2; B_1B und C_1C_2 betragsmäßig im gleichen Verhältnis wie das Zentrum Z teilen. Das Dreieck $A_oB_oC_o$ – hier als zugehöriges harmonisches Dreieck bezeichnet – ist ein umbeschriebenes perspektives Dreieck zum Schnittdreieck ABC. Das

Zentrum Z_o des harmonischen Dreiecks ist der vierte harmonische Punkt zu Z_1, Z_2, Z.
Ein naheliegendes Beispiel: Spiegelt man einen Punkt A_1 iteriert zweimal um ein Dreieck ABC, so schließt sich die Figur zu einem umbeschriebenen Sechseck mit perspektiven Tripeln; hier ist das Zentrum Z ein Fernpunkt und $A_oB_oC_o$ das Anti-Ceva-Dreieck des Schwerpunktes.

1. Teilverhältnisse

Die angesprochenen Perspektivitäten und Kollinearitäten bestimmen eine Vielzahl von Teilverhältnissen. Geht man vom Zentrum Z das vorerst kein Fernpunkt sei, und dem Dreieck $A_1B_1C_1$ aus, so lässt sich das perspektive Dreieck $A_2B_2C_2$ durch die Teilverhältnisse

$$V(ZA_2A_1) = \alpha,\ V(ZB_2B_1) = \beta,\ V(ZC_2C_1) = \gamma$$

beschreiben; d.h. A_2 teilt ZA_1 im Verhältnis α, B_2 teilt ZB_1 im Verhältnis β und C_2 teilt ZC_1 im Verhältnis γ.
Damit lassen sich die Teilungen der Sechseckseiten durch die Ecken des Schnittdreiecks bestimmen. A teile die Seiten B_1C_2 und B_2C_1 in den Verhältnissen α_1 und α_2; dabei gilt entsprechend auch für die anderen Seiten:

$$V(B_1AC_2) = \alpha_1 = ,\ V(B_2AC_1) = \alpha_2 = ,$$
$$V(C_1BA_2) = \beta_1 = ,\ V(C_2BA_1) = \beta_2 = ,$$
$$V(A_1CB_2) = \gamma_1 = ,\ V(A_2CB_1) = \gamma_2 = .$$

Das Produkt dieser Teilverhältnisse ist offensichtlich 1. Dabei gilt:

$$= \alpha_1\,\beta_2\,\gamma_1,$$

d.h. die ersten drei Teilverhältnisse auf dem umbeschriebenen Streckenzug legen die folgenden fest.
Für das harmonische Dreieck $A_oB_oC_o$ gelten die Teilverhältnisse:

$$V(B_oAC_o) = ,\ V(C_oBA_o) = ,\ V(A_oCB_o) = .$$

Andererseits teilen die Ecken A, B, C die Strecken Z_1A_1, Z_1B_1, Z_1C_1 in den Verhältnissen

$$V(Z_1AA_1) = a_{Z1} = = ,$$
$$V(Z_1BB_1) = \beta_{Z1} = = ,$$
$$V(Z_1CC_1) = \gamma_{Z1} = = .$$

Entsprechend teilen die Ecken A, B, C die Strecken Z_2A_2, Z_2B_2, Z_2C_2 in den Verhältnissen

$$V(Z_2AA_2) = a_{Z2} = = ,$$
$$V(Z_2BB_2) = \beta_{Z2} = = ,$$
$$V(Z_2CC_2) = \gamma_{Z2} = = .$$

Abschließend ergibt sich für das Verhältnis, in dem Z_2 die Strecke ZZ_1 teilt

$$V(ZZ_2Z_1) = = 1 + \alpha + \beta + \gamma.$$

Ein Beispiel:
Teilt man die Strecken ZA_1, ZB_1, ZC_1 im gleichen Verhältnis, d.h. $\alpha = \beta = \gamma = \lambda$ (Abb. 3 für $\lambda = -2$) , so werden die Seiten des Sechsecks abwechselnd in den reziproken Verhältnissen und geteilt. Das Zentrum Z_1 teilt die Strecken A_1A, B_1B, C_1C im Verhältnis ; das Zentrum Z_2 teilt die Strecken A_2A, B_2B, C_2C im Verhältnis ; das Zentrum Z_2 teilt die Strecke ZZ_1 im Verhältnis $1 + 3\lambda$.

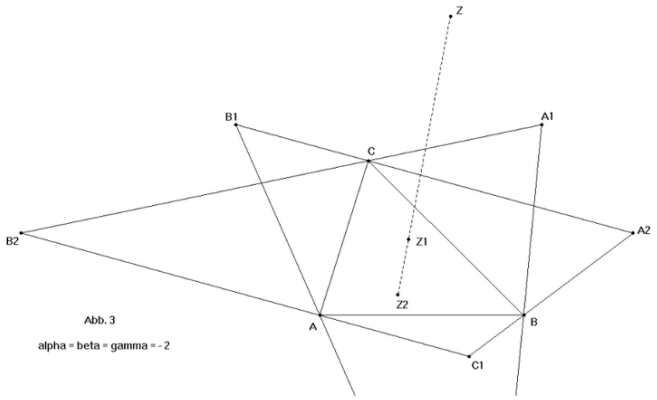


Abb. 3

alpha = beta = gamma = - 2

2. Jacobi-Konstellationen

Zeichnet man über den Seiten eines Dreiecks ABC gleichseitige Aufsatzdreiecke, so ergeben die Spitzen A_1, B_1, C_1 ein zum Ausgangsdreieck perspektives Dreieck mit dem Fermat-Punkt als Zentrum Z_1. Für gleichschenklige Aufsatzdreiecke sind die Ecktransversalen AA_1, BB_1, CC_1 ebenfalls kopunktal. Auch für Jacobi-Konstellationen, in denen jeweils nur noch die Winkel der Aufsatzdreiecke an den Ecken des Ausgangsdreiecks übereinstimmen, bleibt diese Eigenschaft erhalten (Abb. 4).

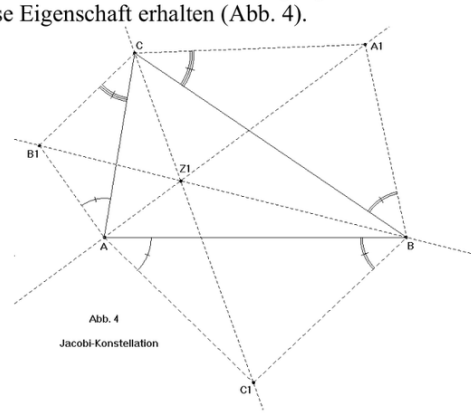


Abb. 4

Jacobi-Konstellation

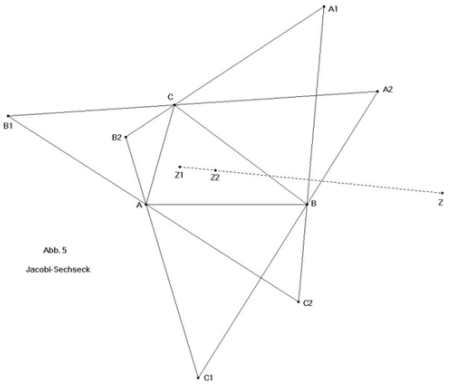


Abb. 5

Jacobi-Sechseck

Ergänzt man eine Jacobi-Konstellation durch die Punkte A_2, B_2, C_2 zu einem umbeschriebenen Sechseck $A_1B_2C_1A_2B_1C_2$, kurz Jacobi-Sechseck genannt, so sind die Ecken A_2, B_2, C_2 und das Zentrum Z_2 die isogonal-konjugierten Bilder von A_1, B_1, C_1 und Z_1 (Abb. 5). Betrachtet man den Sonderfall, dass A_1, B_1, C_1 die Spitzen von gleichschenkligen Aufsatzdreiecken sind, so liegt das Zentrums Z_1 auf der Kiepert-Hyperbel, das Zentrum Z_2 auf der Brocard-Achse und das Zentrum Z auf der Euler- Geraden.

3. Konjugierte Tripel

Die Tatsache, dass bei Jacobi-Sechsecken die perspektiven Tripel auch isogonal-konjugiert sind, ist verallgemeinerungsfähig. Neben der isogonalen Konjugation mit dem Fixpunkt I (Inkreismitte) und der isotomen Konjugation mit dem Fixpunkt S (Schwerpunkt) lässt sich zu jedem Punkt K eine Konjugation bzgl. eines Dreiecks ABC definieren, die den Punkt K als Fixpunkt hat. Zu zwei Punkten X und Y, die nicht durch eine Seitengerade getrennt sind, lässt sich dieser K-Punkt einer Konjugation, die X und Y vertauscht, geometrisch wie folgt ansprechen: Es ist derjenige Punkt, dessen Abstände zu den Seiten sich wie die geometrischen Mittel der Abstände von X und Y zu den Seiten verhalten. Mit K sind auch die Ecken K^a, K^b, K^c des Anti-Ceva-Dreiecks von K Fixpunkte der Konjugation. Ein Kegelschnitt durch diese vier Punkte und das Zentrum Z enthält auch die Ecken A_o, B_o, C_o des harmonischen Dreiecks und sein Zentrum Z_o.
Betrachtet man nun ein Sechseck $A_1B_2C_1A_2B_1C_2$ mit perspektiven Tripeln und dem Schnittdreieck ABC als Bezugsdreieck, so überführt die K-Konjugation, die Z_1 und Z_2 vertauscht, auch die beiden Tripel ineinander.
So lassen sich Sechsecke mit perspektiven Tripeln auch als selbst-konjugierte umbeschriebene Sechsecke eines Dreiecks ansprechen; dabei ist das Schnittdreieck das Bezugsdreieck. Die Konjugation, die Z_1 und Z_2 vertauscht, bildet nicht nur die perspektiven Tripel A_1, B_1, C_1 und A_2, B_2, C_2 aufeinander ab, sondern auch die Seitengeraden A_1B_2 auf A_2B_1, B_1C_2 auf B_2C_1 und C_1A_2 auf C_2A_2. Ein selbst-konjugiertes umbeschriebene Sechseck eines Dreiecks ABC ist bei vorgegebener Konjugation festgelegt durch die Anfangspunkte A_1 und B_2, kollinear mit C.

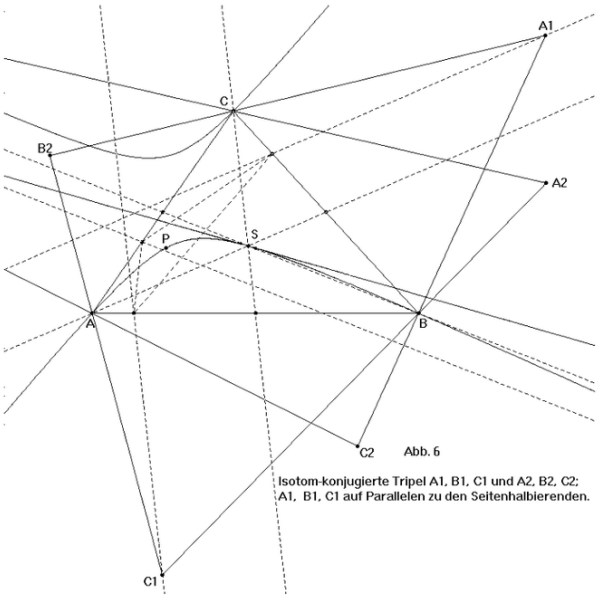


Abb. 6

Isotom-konjugierte Tripel A1, B1, C1 und A2, B2, C2;
A1, B1, C1 auf Parallelen zu den Seitenhalbierenden.

Durchlaufen zu einem Schnittdreieck ABC und vorgegebener K-Konjugation A_1 und B_1 Geraden g_A und g_B, so durchläuft auch C_1 eine Gerade g_c. Diese Geraden schneiden die Seiten des Dreiecks ABC in den Ecken eines Ceva-Dreiecks, der Ceva-Punkt sei P. Die Zentren Z_1 liegen auf einem Umkegelschnitt des Dreiecks ABC durch den K-Punkt und den Ceva-Punkt P; die Zentren Z_2 liegen auf einer Geraden, dem K-konjugierten Bild des Umkegelschnitts. Für Jacobi-Sechsecke wurde ein Sonderfall skizziert; ein weiteres Beispiel ist in Abb. 6 dargestellt: $A_1B_2C_1A_2B_1C_2$ sei ein isotom-selbst-konjugiertes Sechseck; A_1 und B_1 liegen auf Parallelen zu den Seitenhalbierenden, C_1 ebenso.

4. Kurven dritter Ordnung

Geht man von einem Bezugsdreieck ABC und einer vorgegebenen K-Konjugation aus, so existiert zu jedem Punkt P eine Kurve dritter Ordnung – kurz als KP-Kurve angesprochen –, die bzgl. der Konjugation invariant ist und P als Pivot-Punkt hat. Im Pivot-Punkt schneiden sich die Verbindungsgeraden K-konjugierter Kurvenpunkte. Auf dieser KP-Kurve liegen die Ecken des Bezugsdreiecks, die Ecken des Anti-Ceva-Dreiecks von Punkt K und die Ecken des Ceva-Dreiecks von Punkt P. Es stellt sich die Frage, ob die thematisierten Sechsecke durch eine Kurve dritter Ordnung eingefangen werden können. Nimmt man das Schnittdreieck als Bezugsdreieck, einen Fixpunkt der Selbst-Konjugation κ als K-Punkt und das Zentrum Z der perspektiven Tripel als Pivot-Punkt P, so liegen auf der zugehörigen KP-Kurve nicht nur die Ecken des Sechsecks $A_1B_2C_1A_2B_1C_2$ und seines Schnittdreiecks ABC, sondern auch die Zentren Z_1, Z_2 und Z = P, der Fixpunkt K der Konjugation sowie die Ecken seines Anti-Ceva-Dreiecks $K^aK^bK^c$ und der Pivot-Punkt P =Z mit den Ecken seines Ceva-Dreiecks $Z_aZ_bZ_c$. Ergänzt man noch das K-konjugierte Bild Z^κ von Z, so sind das insgesamt 20 unmittelbar ansprechbare Punkte. Man erhält die Kurve als Ortskurve der K-konjugierten Partner auf Geraden durch das Zentrum Z.

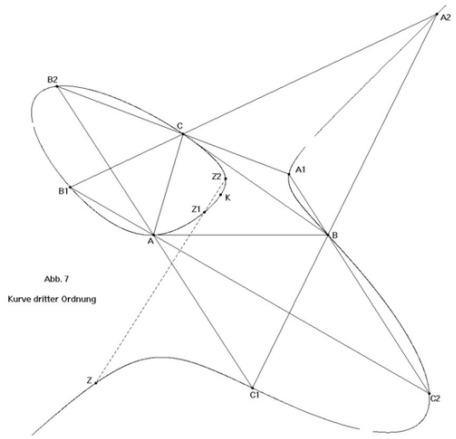


Abb. 7
Kurve dritter Ordnung

Über korrespondierende Punkte gibt die folgende Kurztabelle Auskunft, die alle 20 Punkte enthält:

Punkt	korrespondierende Punkte
Z	A, B, C
Z_1	A_2, B_2, C_2
Z_2	A_1, B_1, C_1
K	K^a, K^b, K^c
Z^κ	Z_a, Z_b, Z_c

Die Verbindungsgeraden K-konjugierter Punkte verlaufen durch den Pivot-Punkt Z; dies sind die Geraden A_1A_2, B_1B_2, C_1C_2, Z_1Z_2; weiterhin die Tangenten K^aZ, K^bZ, K^cZ und ZZ^κ.

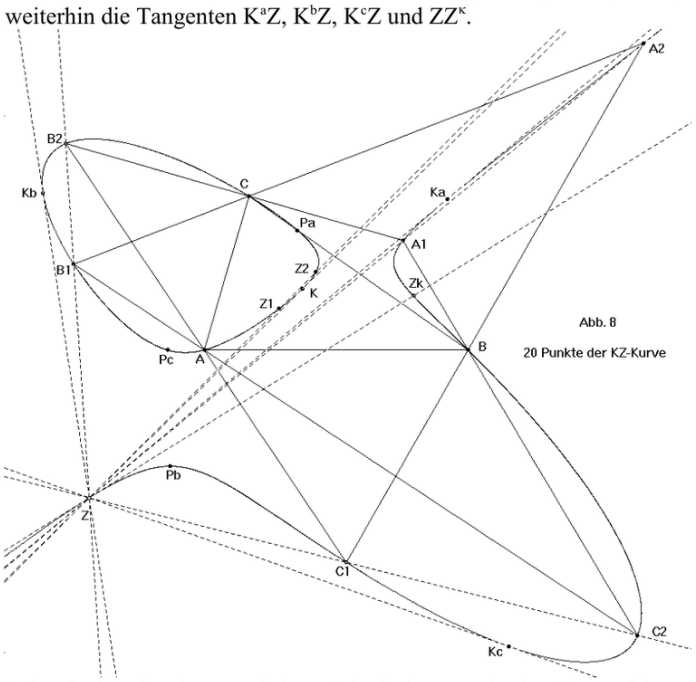


Abb. 8
20 Punkte der KZ-Kurve

Jedes Sechseck mit perspektiven Tripeln legt somit eine Kurve dritter Ordnung fest. Umgekehrt lassen sich die thematisierten Sechsecke als Sechsecke auf Kurven dritter Ordnung studieren. Geht man von einer

KP-Kurve eines Dreiecks ABC aus, so schließt sich jeder umbeschriebene Streckenzug aus Kurvenpunkten zu einem Sechseck mit perspektiven Tripeln; Zentrum ist der Pivot-Punkt P. Oder: Gibt man zwei K-konjugierte Zentren Z_1 und Z_2 auf der Kurve vor, so erhält man die Tripel A_2, B_2, C_2 und A_1, B_1, C_1 als korrespodierende Punkte von Z_1 und Z_2.

5. Einige Berechnungen

Arbeitet man in baryzentrischen Koordinaten eines Bezugsdreiecks ABC mit dem Fixpunkt K(k : l : m) einer Konjugation und dem Pivot-Punkt P(u : v : w), so lautet die Gleichung der zugehörigen KP-Kurve

$$m^2 (v\,x - u\,y)\,x\,y + k^2 (w\,y - v\,z)\,y\,z + l^2 (u\,z - w\,x)\,z\,x = 0 .$$

Geht man von einem Kurvenpunkt X(x : y : z) aus, so ist der dritte kollineare Kurvenpunkt
zu X und A der Punkt (: x y : x z),
zu X und B der Punkt (x y : : y z),
zu X und C der Punkt (x z : y z :).
Wählt man A_1(x : y : z) als Anfangspunkt eines dem Dreieck ABC umbeschriebenen Sechsecks aus Kurvenpunkten, so ergibt sich für die Ecken B_2(x z : y z :),

C_1(k^2 z : : m^2 x) ,
A_2(k^2 y z : l^2 x z : m^2 x y) ,
B_1(k^2 y : l^2 x :)
C_2(x y : : y z).

Entsprechend erhält man die Zentren

Z_1(: x y : x z)
und Z_2(: l^2 z : m^2 y).

Damit werden auch die Teilverhältnisse aus 1 berechenbar, wobei zu berücksichtigen ist, dass die Koordinaten von X = A_1 die Relationsgleichung der Kurve erfüllen.

6. Ein Beispiel

Es wird ein eingangs erwähntes Beispiel wieder aufgegriffen: Spiegelt man einen beliebigen Punkt um ein Dreieck ABC, so schließt sich die Figur zu einem umbeschriebenen Sechseck $A_1B_2C_1A_2B_1C_2$ mit perspektiven Tripeln – kurz Spiegelungs-Sechseck genannt. (Abb. 9). Dabei ist das Zentrum Z ein Fernpunkt P_∞, d.h. die Verbindungsgerade der Zentren und die Diagonalen sind Parallelen. Die entsprechenden Verbindungsstrecken werden durch den Schwerpunkt S und die Ecken seines Anti-Ceva-Dreiecks $S^aS^bS^c$ halbiert. Die zugehörige Kurve dritter Ordnung hat nur eine Asymptote, die dem obigen Parallelenbüschel angehört und die Kurve im K-konjugierten Bild des gemeinsamen Fernpunktes P_∞ schneidet.

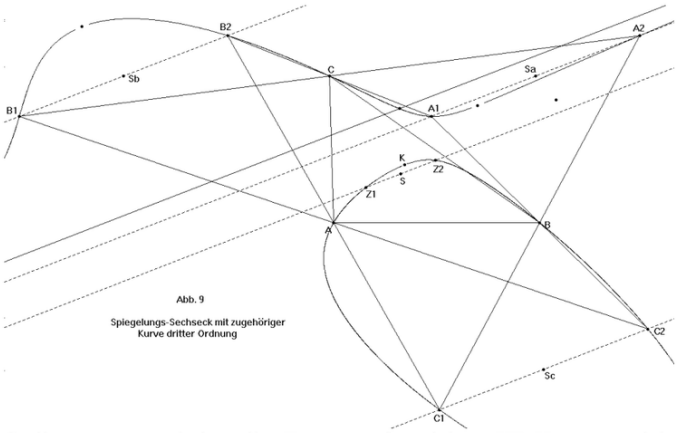


Abb. 9

Spiegelungs-Sechseck mit zugehöriger Kurve dritter Ordnung

Stellt man umgekehrt die Frage, auf welchen KP-Kurven solche Spiegelungs-Sechsecke zu finden sind – das harmonische Dreieck ist das Anti-Ceva-Dreieck des Schwerpunktes –, so muss zumindest der Pivot–Punkt ein Fernpunkt sein, der aber von dem gewählten Konjugations-Punkt K abhängig ist. Geht man von einem Konjugationspunkt K aus, so liegen dieser K-Punkt und die Ecken seines Anti-Ceva-Dreiecks $K^aK^bK^c$ sowie der Schwerpunkt S und die Ecken seines Anti-Ceva-Dreiecks $S^aS^bS^c$ auf einer Hyperbel, die durch das gesuchte Zentrum $Z = P_{\square}$ verläuft. Damit muss $P_{\square}$ einer der Fernpunkte der Asymptoten sein. Verbindet man also den Punkt S^a mit diesem Fernpunkt und betrachtet das K-konjugierte Bild der Verbindungsgeraden $S^aP_{\square}$ – einen Umkegelsschnitt des Dreiecks ABC –, so sind die K-konjugierten Schnittpunkte die Ecken A_1 und A_2 eines Spiegelungs-Sechsecks auf der $KP_{\square}$-Kurve (Abb. 10).

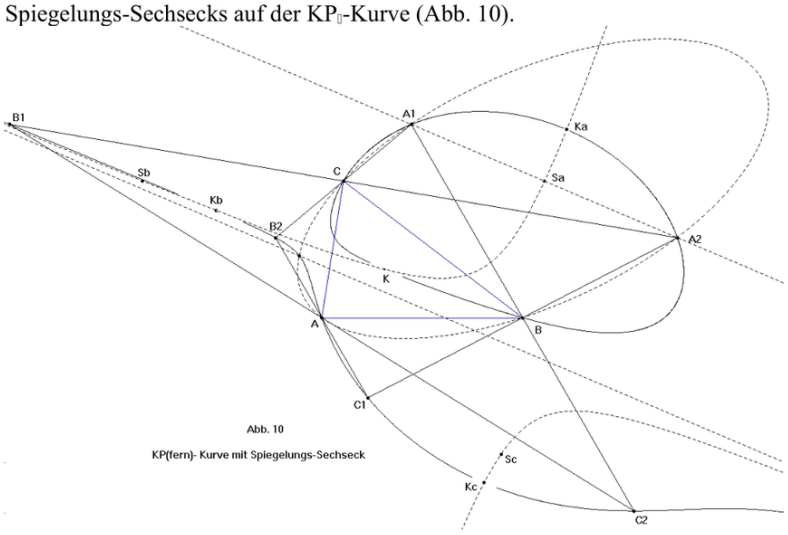


Abb. 10

KP(fern)- Kurve mit Spiegelungs-Sechseck

Eckart Schmidt - Hasenberg 27 24223 Raisdorf
Tel.: 04307/5954 - eckart_schmidt@t-online.de

Sechsecke2.doc

Message: #1615
Date: 30/3/2016 9:56:26
From: bernard.keizer
Subject: Least Squared Distance Line

Dear Eckart,
At first sight, it looks magical !
(What are lines 8 and 9 ?)
The use of Pythagoras is particularly appropriate ...
In fact, it is a direct consequence of Coolidge's calculations !
Let's call xi and yi the coordinates of the 4 vertices Pi in the cartesian system with the LSD line as the x-axis.
For any line L through QA-P1 making an angle t with the LSD line, we have sum of di^2 = sum of (xi cost + yi sint)^2
= cost^2*sum of xi^2 + 2 sin^tcost sum of xiyi + sint^2*sum of yi^2
But sum of xiyi is equal to zero
After the inversion in a circle with radius r, QA-P1X^2 = r^2 / sum of di^2.
This is the equation of an ellipse in polar coordinates.
With this construction, you loose the simple construction of the barycenter of the z^2 which works easily for a greater number of points ...
So I prefer to keep my construction as the main construction and to take yours as an additionnal interesting property, but not for an alternative.
Best regards
Bernard

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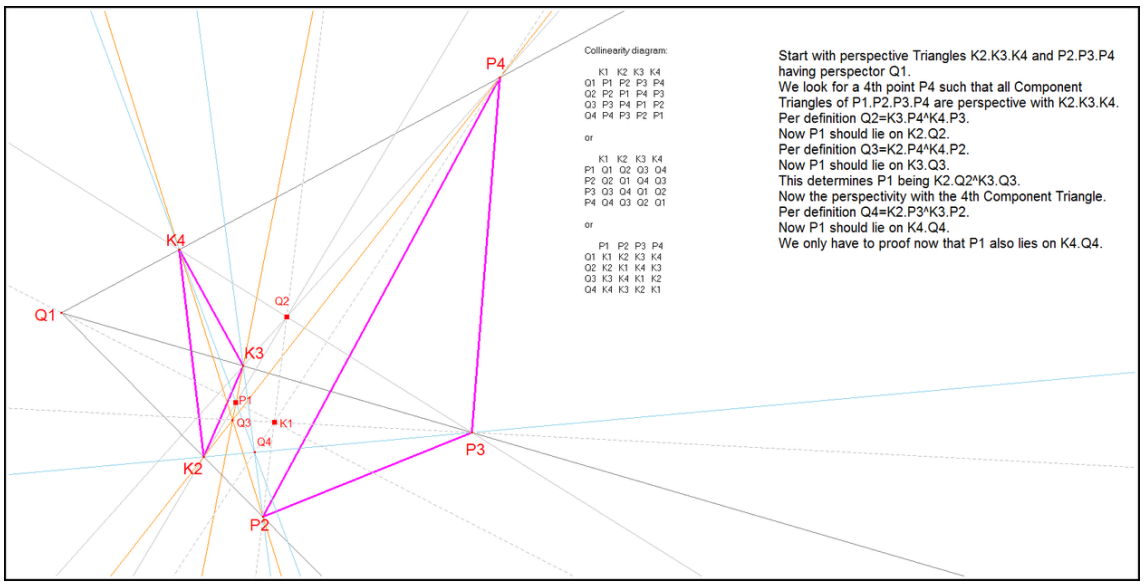
Message: #1616
Date: 30/3/2016 11:39:31
From: chris.vantienhoven
Subject: QA-related Centers in a configuration of two perspective triangl

---

Dear Eckart,
Thank you very much for your document.
Your document shows again that we really share the same interests!
I was familiar with most of it.
For certain subjects I do collect related items so that I can bring them out later in an orderly way.
One of these items is the configuration of 2 perspective triangles. There is so much more to say about this subject than having a perspector and a perspectrix.
Meanwhile I collected:
· two extra derived perspective triangles
· about 12 related central points
· several related central lines
· a cubic (the same you mentioned)
· conics
· related transformations
It is one of the subjects I want to make a description of at later times.
I hope I will have a long life.
About your beautiful paper you discern triangles A1B1C1, A2B2C2 and construct ABC.
I don't know if you are aware that these triangles all three actually are a component triangle of a special Quadrangle. They can be extended to three quadrangles of which each triangle is perspective to each component triangle of one of the other quadrangles. This is a so called Desmic Configuration.
See attached file.
The vertices of these quadrangles lie on a cubic (your mentioned cubic I suppose) also mentioned at Hyacinthos #462 and also #2451. †)
Your triangle ABC is also mentioned as Desmic Mate in Hyacinthos #6293. †)
Desmic Triangles are also mentioned in EQF at QA-Tr-1.
Best regards,
Chris

† Editorial note: The referenced message(s) can be found as listed in EPG-References [11].



QA-Tr-Perspectivity-01.png

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Message: #1617
Date: 01/4/2016 11:04:21
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

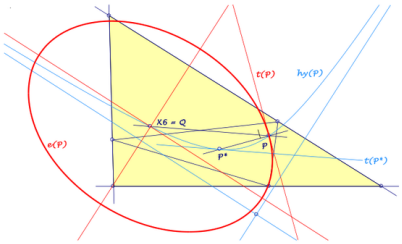
Dear Chris,
if we consider not the least sum of squared distances, but a constant sum, the loci of points with this property wrt a n-line are ellipses.
If the LSD-point is known, these ellipses can be constructed, see attached file.
Best regards Eckart

EQF-Note 2016-04-01

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Constant Sum of Distance Squares

If we consider n lines and for a point the sum of its squared distances to the lines, the locus of points with the same sum is an ellipse. A construction of this ellipse shall be described, if the point with the least sum is known. For a quadrilateral this ellipse is already researched in QFG-message 587 (EQF-Note 2014-06-04).



The construction is here only described for a triangle, but can be generalized for n lines, if the point of least squared distance sum is known. This point is the centroid of its pedal points.

$n = 3$: Lemoine point $X(6)$,
$n = 4$: Least Squares Point QL-$P26$,
$n = 6$: see QFG-message *1589*.

Now we consider a triangle with the Lemoine point $X(6) = Q$:
… Let P^* be the centroid of the pedal triangle of P,
… let $t(P)$ be a perpendicular line to PP^* through P,
… let $t(P^*)$ be a parallel to PQ through P^*,
… let $hy(P)$ be an orthogonal hyperbola through P, P^*, Q, tangent to $t(P^*)$.
… The parallels to the asymptotes of $hy(P)$ through Q are the axes of the searched ellipse $e(P)$,
… which is tangent to $t(P)$.

This allows a construction of the ellipse $e(P)$. Its points have the same sum of squared distances to the triangle sidelines.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-04-01.pdf

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Message: #1618
Date: 01/4/2016 10:57:26
From: tsihonglau
Subject: Constant Sum of Distance Squares

Dear Eckart,
I had considered the special case of zero-sum of distance squares before you post this message.
For the case of triangle,(all in barycentrics)
equation (b^2+c^2)x^2+(b^2+c^2-a^2)yz+...(cyclic)=0
point of intersection with the line at infinity circular points
center 1:1:1
foci ?:?:?
axes ?x+?y+?z=0
asymptotes ?x+?y+?z=0
I think the special case is essential to the general case. Any idea?
Best regards,
Tsihong Lau

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Message: #1619
Date: 02/4/2016 11:44:40
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Tsihong Lau,
excuse, I have a problem in understanding:
What distances do you consider?
How can a sum of distance squares be (not trivial) zero?
Best regards Eckart

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Message: #1620
Date: 02/4/2016 12:32:32
From: tsihonglau
Subject: Constant Sum of Distance Squares

Dear Eckart,
http://www.douillet.info/~douillet/triangle/glossary/glossary.pdf
Section 5.2 of the book "Translation of the Kimberling's
Glossary into barycentrics" covers the "distance square"
(not "distance") of two points in barycentrics. I extend this
formula to circulars and post it to APG #2873. I will post the
results in circulars later. Of course, these conics are all
imaginary. But they are essential to real geometry.
It is somewhat like the imaginary circular points.
That is why I "invent" circular coordinate system.
We can learn more from "imaginary" circular points.
The book above has a good cover on imaginary geometry.
Best regards,
Tsihong Lau

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Message: #1621
Date: 02/4/2016 3:12:05
From: tsihonglau
Subject: Constant Sum of Distance Squares

Dear all,
I post my results for triangles/trilaterals.
Asymptotes are the two lines through the center (pole of the line at infinity with respect to the conic)
and two points of intersection with the line at infinity.
zero-sum of distance squares from a point to a triangle(three points) equation
(b^2+c^2)x^2+(b^2+c^2-a^2)yz+...(cyclic)=0
point of intersection with the line at infinity circular points
center=foci X(2)
asymptotes (a^2-2b^2-2c^2)x^2+(5a^2-b^2-c^2)yz+...(cyclic)=0
perspector X(3424)
axes
imaginary circle
Because this conic passes through the circular points, it is an imaginary circle and center=foci
and no axes exist.
zero-sum of distance squares from a point to a trilateral(three lines)
(x/a)^2+...(cyclic)=0
point of intersection with the line at infinity
a*(c^2+b^2):-b*(c*sqrt(-c^2-b^2-a^2)+a*b):c*(b*sqrt(-c^2-b^2-a^2)-a*c)
a*(c^2+b^2):b*(c*sqrt(-c^2-b^2-a^2)-a*b):-c*(b*sqrt(-c^2-b^2-a^2)+a*c)
center X(6)
asymptotes (b^2+c^2)(x/a)^2-2yz+...(cyclic)=0
perspector self-polar foci ?:?:?
axes ?:?:?
zero-sum of distance squares from a line to a triangle(three points)
equation(point and line) x^2+...(cyclic)=0
point of intersection with the line at infinity
1:ω:ω^2,1:ω^2:ω where ω is an imaginary cubic root of 1
= point of intersection of Steiner ellipses with the line at infinity
center X(2)
asymptotes x^2-yz+...(cyclic)=0
perspector self-polar foci ?:?:?
axes ?:?:?
The polar triangles of the last two conics are
the same as the reference triangle(self-polar).
So no perspectors exist. The last conic is a line-conic in fact.
But the equations for the line and point conics are the same.

The centers of the former two conics are the solutions for the least distance square cases
I hope someone calculate the foci and axes above and the cases of quadrangles/quadrilaterals.
Best regards,
Tsihong Lau

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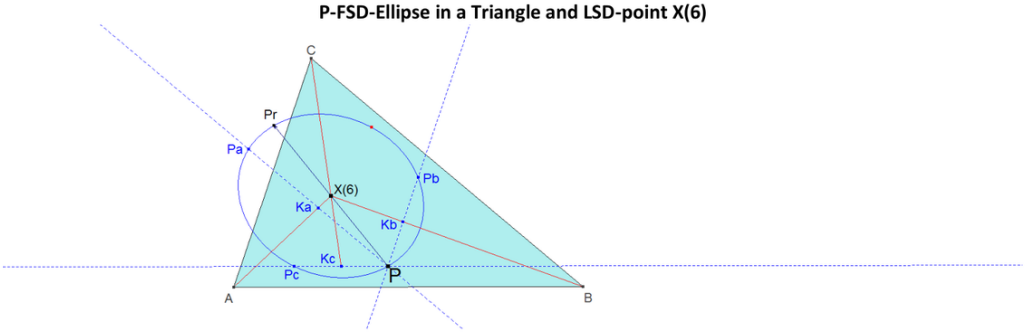
Message: #1622
Date: 02/4/2016 3:54:38
From: chris.vantienhoven
Subject: Constant Sum of Distance Squares

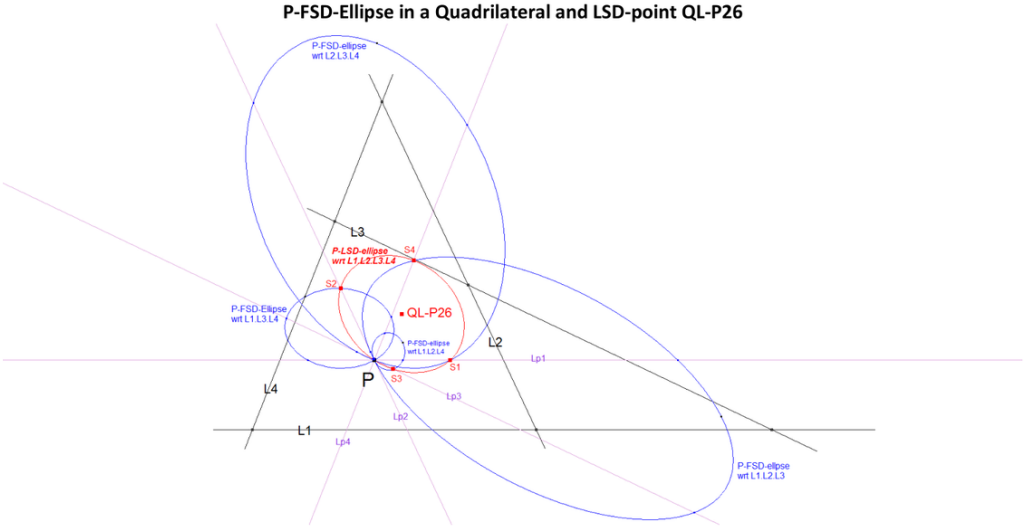
Dear Eckart,
Nice method of determining the ellipses with equal sum of squared distances!
Here some extra ideas.
Let's call this ellipse with a Fixed Sum of Squared Distances an FSD-ellipse and when passing through P it will be called the P-FSD-ellipse.
I think we can use this method for constructing the LSD-point in any n-Line (n>2).
Because when we can find any FSD-ellipse in an n-Line then the center will be the LSD-point of this n-Line, assuming that FSD-ellipses are concentric and equally distributed. Experience will show.
The idea to do so is by drawing lines in an n-Line through P parallel to the n Lines. Now on each of these parallel lines there will be a second point next to P with the same fixed sum of squared distances also lying on the P-FSD-ellipse. When we can find 5 of these second points we have a conic and this conic should be the P-FSD-ellipse. Actually 4 points will be enough for construction because P is per definition also on the conic.
You made a construction for constructing the P-FSD-ellipse in a 3-Line (triangle). Very well!
I found another method using the P-parallel-lines, which easily can be used upwards in an n-Line.
1. Let P be some arbitrary point and K be the Symmedian Point X(6) in a triangle.
2. Draw lines La,Lb,Lc through P parallel to the sidelines a,b,c.
3. Let Ka,Kb,Kc be the intersection points of La,Lb,Lc and the resp. symmedians through A,B,C.
4. Let Pa,Pb,Pc be the reflections of P in Ka,Kb,Kc.
5. Let Pr be the reflection of P in K.
6. The conic through P, Pr, Pa, Pb, Pc will be the P-FSD-ellipse.
In a similar way we can construct a P-FSD-ellipse in a 4-Line (quadrilateral).
1. Let L1,L2,L3,L4 be the 4 defining lines of the 4-Line.
2. Draw lines Lp1,Lp2,Lp3,Lp4 through arbitrary point P parallel to the sidelines L1,L2,L3,L4.

3. We are searching for the second point on Lp1 with same sum of squared distances to L1,L2,L3,L4 as P has. When we vary P on Lp1 at least the distance to L1 is fixed. So we have to find the point with fixed sum of squared distances to L2,L3,L4. This is the FSD-triangle problem like described here before. So construct the P-FSD-ellipse wrt triangle L2.L3.L4. Let S1 be the 2 nd intersection point of this P-FSD-ellipse with Lp1. S1 has the same fixed sum of squared distances to L1,L2,L3,L4 as P.
4. In a similar way we can construct S2, S3, S4.
5. The conic through P, S1, S2, S3, S4 will be the P-FSD-ellipse in a 4-Line (quadrilateral).
6. The center of this ellipse is QL-P26 indeed (graphically confirmed).
In a similar way we can construct now a P-FSD-ellipse in a 5-Line (pentalateral).
1. Let L1,L2,L3,L4.L5 be the 5 defining lines of the 5-Line (pentalateral).
2. Draw lines Lp1,Lp2,Lp3,Lp4.Lp5 through arbitrary point P parallel to the sidelines L1,L2,L3,L4,L5.
3. We are searching for the second point on Lp1 with same sum of squared distances to L1,L2,L3,L4,L5 as P has. When we vary P on Lp1 at least the distance to L1 is fixed. So we have to find the point with fixed sum of squared distances to L2,L3,L4,L5. This is the FSD-triangle-problem for a 4-Line like described here before. So construct the P-FSD-ellipse wrt quadrilateral L2.L3.L4.L5. Let S1 be the 2 nd intersection point of this P-FSD-ellipse with Lp1. S1 has the same fixed sum of squared distances to L1,L2,L3,L4,L5 as P.
4. In a similar way we can construct S2, S3, S4, S5.
5. The conic through P, S1, S2, S3, S4 will be the P-FSD-ellipse in a 4-Line (quadrilateral). It will appear that S5 is also on the conic.
6. The center of this ellipse will be the LSD-point of a 5-Line.
In a recursive way P-FSD-ellipses can be constructed in every n-Line (n>2) and the center of this P-FSD-ellipse will be the LSD-points of the n-Line.
In attached file examples for n=3,4,5.
Best regards,
Chris

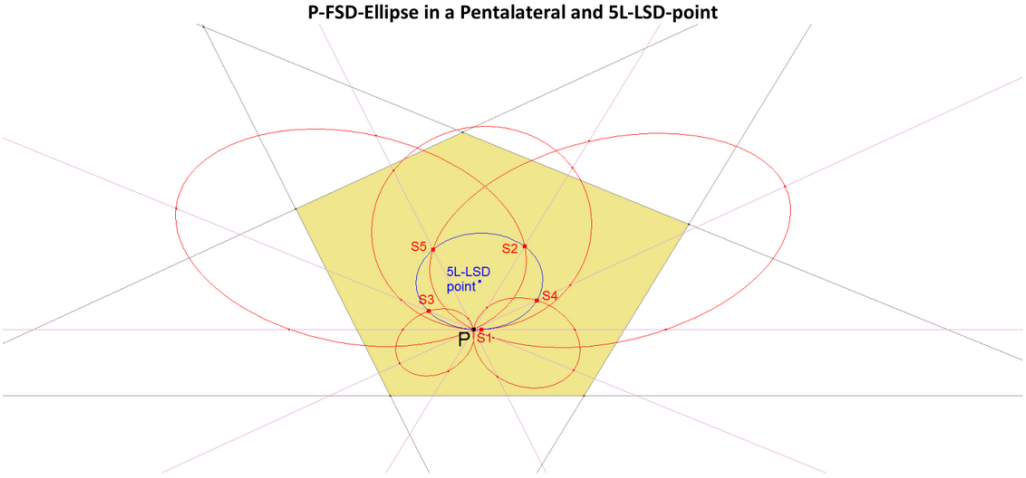
P-FSD-Ellipse in a Triangle and LSD-point X(6)



P-FSD-Ellipse in a Quadrilateral and LSD-point QL-P26

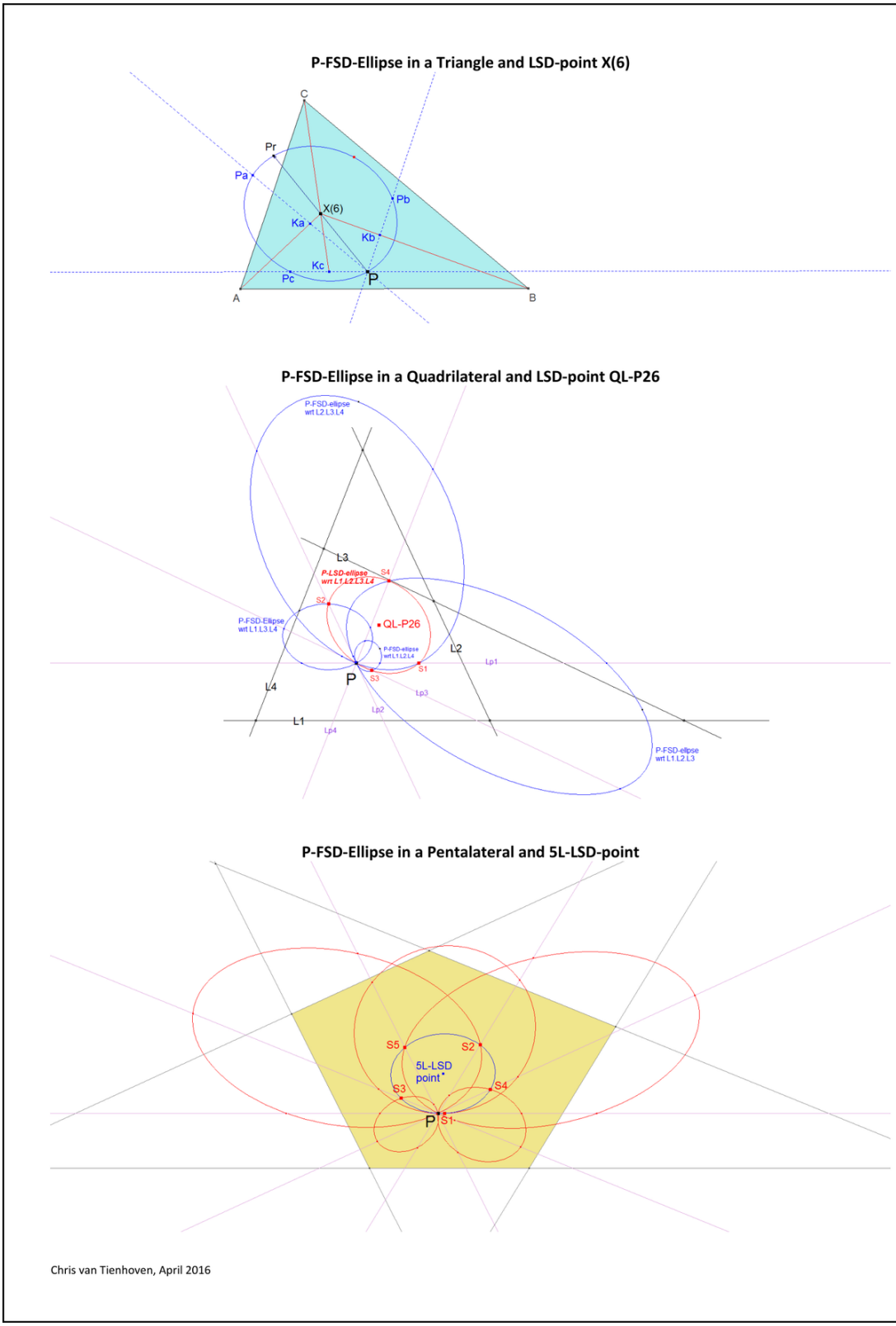


P-FSD-Ellipse in a Pentalateral and 5L-LSD-point



Chris van Tienhoven, April 2016

QL-P26-P-FSD-Ellipses-01.pdf



QL-P26-P-FSD-Ellipses-01.docx

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Message: #1623
Date: 02/4/2016 4:52:03
From: tsihonglau
Subject: Dual Quintangle/Quintilateral

Dear all,
Duality of Triangle/Trilateral and Quadrangle/Quadrilateral (http://lth.name/geometry/duality.html)
My page shows how to derive a trilateral or quadrilateral from a triangle or quadrangle respectively and vice versa. We can call this the duality of triangle/trilateral and quadrangle/quadrilateral.
The duality of quadrangle/quadrilateral can be derived from trilinear polars/poles.
q is the trilinear polar of Q with respect to the triangle QA,QB,QC. Conversely, Q is the trilinear pole of q with respect to the trilateral qa,qb,qc.
Dual points/lines of a quadrangle/quadrilateral are analogous to trilinear polars/poles of a triangle/trilateral.
I wanted to derive the duality of quintangle/quintilateral from dual points/lines of a quadrangle/quadrilateral but failed.
However, there is a very natural duality of quintangle/quintilateral.
Suppose a quintangle P1,P2,P3,P4,P5 (no three points lie on a line) lie on a circumconic.
The tangent lines at P1,P2,P3,P4,P5 to the conic are l1,l2,l3,l4,l5, which form a quintilateral(no three lines pass through a point).
Conversely, this quintilateral has an inconic(line conic).
Obviously, this inconic and the circumconic above are dual and the tangent points are P1,P2,P3,P4,P5.
Hence, the quintangle/quintilateral are dual naturally.
The duality is very easy.
But the problem is: How to establish a coordinate system on it.
On a quadrangle/quadrilateral, the coordinates/equations are shown on my page.
The circular points have coordinates p:q:r,u:v:w.
This unitary circular coordinate system is both symmetric and projective.
Is there a symmetriccoordinate system(not necessarily projective) for a quadrangle/quadrilateral?
If one is found, we can start the study of quadrangle/quadrilateral figures immediately. Any idea?
To be continued...
Best regards,
Tsihong Lau

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Message: #1624
Date: 02/4/2016 9:34:57
From: seiichikiri
Subject: A question of sum of the squared distances

Dear friends,
I have a question:
Given a quadrangle P1P2P3P4, its diagonal triangle S1S2S3 and a point X0, we can compute the cyclic sums of squared distances (X0Pi)^2 for i=1 to 4 and (X0Si)^2 for i=1 to 3 respectively.
We can consider the equations: cyclic sum (XPi)^2 = cyclic sum (X0Pi)^2 and cyclic sum (XSi)^2= cyclic sum (X0Si)^2 for a arbitrary point X.
Do you think that there is the case that the equations have 4 solutions for X?
One solution is X=X0.
Best regards, Seiichi

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Message: #1625
Date: 03/4/2016 10:12:11
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distances Squares

Dear Chris,
my recognition for your idea of P-parallel-lines, searching for a second point with the same sum of distances squares.
I have proved your simple construction for a triangle. The idea with the parallels to the sidelines allows a recursive construction of P-FSD-ellipses. Congratulations!
If we consider lines, which have the same sum of distances squares to n points, they envelop also ellipses, centered in the centroid of the n points. Up to now I cannot give a construction, but Cabri observations show, that these ellipses have the same foci!
Best regards Eckart

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Message: #1626
Date: 03/4/2016 11:21:20
From: eckart_schmidt@t-online.de
Subject: A question of sum of the squared distances

```
Dear Seiichi,
the locus for points X with
cyclic sum (XSi)^2 = cyclic sum (X0Si)^2
is a circle round QA-P10 through X0.
The locus for points X with
cyclic sum (XPi)^2 = cyclic sum (X0Pi)^2
is a circle round QA-P1 through X0.
In general:
The locus for points X with
cyclic sum (XPi)^2 = cyclic sum (X0Pi)^2 wrt n points Pi
is a circle round the centroid through X0.
So there can only be a 2nd solution for your considered
equations.
Best regards Eckart
```

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Message: #1627
Date: 03/4/2016 12:46:22
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

```
Dear Chris,
excuse, there is a typo in my last message: Replace in the last
statement "ellipse" par "conic".
Best regards Eckart
```

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Message: #1628
Date: 03/4/2016 2:25:15
From: seiichikiri
Subject: A question of sum of the squared distances

```
Dear Eckart,
Thanks a lot!
Your information cleared my question fully.
Best regards, Seiichi.
```

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Message: #1629
Date: 03/4/2016 4:55:25
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Chris, dear Eckart
This time, I'm really impressed, not to say totally bluffed !
I feal the same pleasure as the one I had when Eckart showed the first time the stelloïd cubic QL-Cu2 or when we cracked Morley's code with the 64 axes of the the QL.
It's really a remarkable teamwork you achieved; first, the correction of Coolidge by Eckart, then these miraculous conics fallen from the sky, last this amazingly simple recursive construction.
Congratulations to both of you !
I hope you will finally find the same kind of simple recursive construction for the LSD line (I find already the one with complex numbers satisfying).
For these conics fallen from the sky, it's not a miracle, but the result of Coolidge calculations with the normal form of the lines.
For the P FSD ellipses, we have sum of (xcosti + ysinti - pi) ^2 = k^2 and, if we take the origin in the LSD point, where tis sum is the minimum, the terms sum of picosti and sum of pisinti vanish and it remains the equation of an ellipse of center the origin in this LSD point ...
For the L FSD conics, we have sum of (xicost + yisint -p) ^2 = k^2 and, if we take as origin the centroïd and as x-axis the LSD line, where this sum is mimimum, this time the term sum of xiyi vanishes and it should remain the tangential equation of a conic in this system of axes ...
For Eckart, I suppose that the 2 same foci come from the general equation and that it's possible to draw them by calculating sum of zi^2, where zi are the complex numbers of the points Pi in this system of axes and the 2 square roots of this sum of zi^2 (the point sum of zi^2 is on the x-axis).
Last question: by drawing the P FSD ellipses in Chris construction, I have the impression that the axes of the ellipses are always the same. Is it true ? What does that mean ?
Best regards
Bernard

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Message: #1630
Date: 03/4/2016 11:56:06
From: chris.vantienhoven
Subject: Constant Sum of Distance Squares

Dear Bernard and Eckart,
I feel exactly the same satisfaction and enjoy our teamwork very much as well.
It is maybe nice to tell how I found the simple construction and the recursive extension.
I wondered if there wasn't a much simpler construction. So I wondered if there was no possibility to construct a second point on an P-parallel line. With no clues to start with I did some algebraic calculations and transferred them very elaborate in a picture. Finally I succeeded to construct a second point. Then I noticed that when gliding the 1 st point on a parallel line the midpoint of the 1 st and 2 nd point stayed fixed on this line. So I only had to look for the fixed midpoint and this one was found in a minute being the intersection of the corresponding symmedian and the parallel line. According to an earlier idea I worked them out in a recursive way. It was a good feeling! This doesn't happen all the time.
Yes it would be nice to find a recursive method for the LSD-line in an n-Line or n-Point. However Bernard's method (and Benedetto's) is easy enough. But I have to admit a recursive method has a special merit.
Bernard about your question:
"I have the impression that the axes of the ellipses are always the same. Is it true? What does that mean?"
http://mathworld.wolfram.com/ConfocalEllipses.html

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Message: #1631
Date: 04/4/2016 4:33:46
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Bernard, dear Chris,
here is a construction for L-FSD-conics, that means conics, whose tangents have the same sum of distance squares from n points.
These conics must not be ellipses, but they have the same foci.
Beginning with n=2:
Given a line L and two points Pi with distances di from Pi to L.
... Let Ci be a circle round P1 with radius sqr(d1^2+d2^2)
... with tangents T1 and T2 from P2 to Ci (see PS).
... T1, T2, L have the same sum of distance squares wrt Pi.
... The searched L-FSD-conic is centered in the midpoint of P1.P2 and tangent to L,T1, T2.
We use a recursive construction (n>2):
Given a line L and n points Pi
... with centroid Gn,
... LSD-line Ln (construction see #1590 or #1611)
... and L-FSD-conic for n-1 points.
... Let T1 and T2 be tangents from Pn to the L-FSD-conic for the points P1, P2, ..., Pn-1
... and T the reflection of T1 or T2 in the LSD-line Ln.
... Let F1, F2 be the foci of the conic, centered in Gn and tangent to T1, T2, T.
... The searched L-FSD-conic for n points has the same foci and is tangent to L.
Best regards Eckart
PS. There is sometimes a handicap wrt the construction of the L-FSD-conic for n=2. I hope, there are no further unclarities.

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Message: #1632
Date: 05/4/2016 9:23:47
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Bernard, dear Chris,
my construction in message 1631 is not perfect: Handicap: The tangents from Pn can not be constructed, if Pn lies "inside" the conic. If this is the case, you can take another line L, to get the foci of the L-FSD-conic, for they are the same for all lines L.
It is better, to orientate the construction at the foci for all L-FSD-conics.
For n = 2 the foci are the intersections of the Thales circle about P1P2 and its perpendicular bisector.
Best regards Eckart

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Message: #1633
Date: 05/4/2016 11:30:13
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Eckart,
Beautiful construction indeed, always "dual" from the other with symmetry of lines wrt the LSD line instead of symmetry of points wrt the LSD point !
I tried to reproduce your construction at last for the 2 points and the triangle
1) For the 2 points, I don't understand your Thales circle : I found that the 2 fci are simply the intersections between the circle with diameter P1P2 (or is this circle what you call Thales circle ?) and it's perpendicular bisector
2) For the triangle, I don't find an obvious solution (I only know for sure that the 2 foci are not the foci of either the circum- or the inellipse of Steiner ...)
Best regards
Bernard

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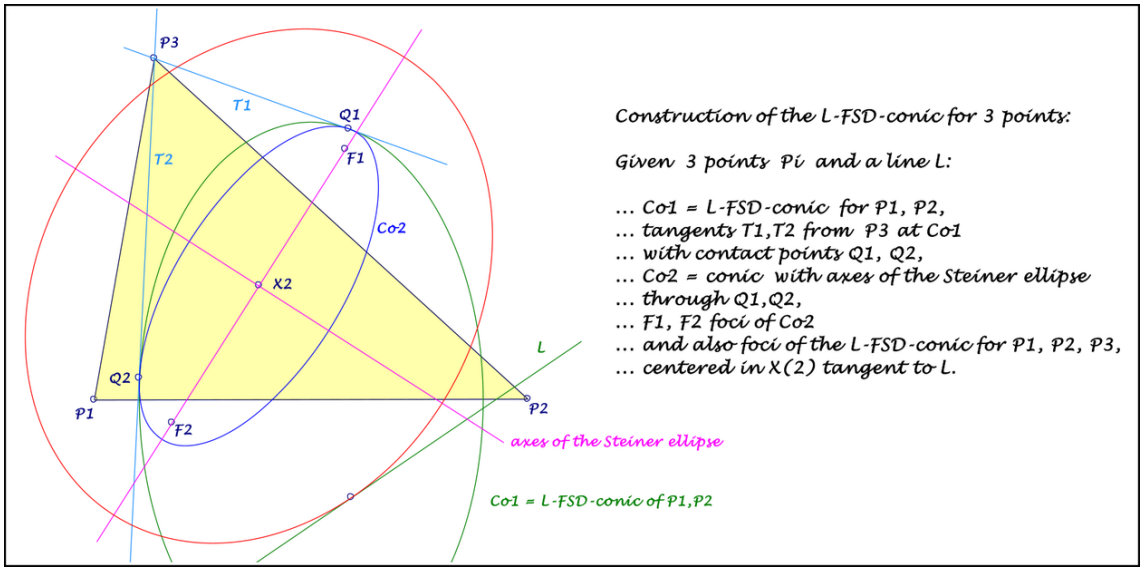
Message: #1634
Date: 05/4/2016 2:05:34
From: eckart_schmidt@t-online.de
Subject: Constant Sum od Distance Squares

Dear Bernard,
attached a construction of the L-FSD-conic for 3 points.
Best regards Eckart
PS: Thales circle about AB means in Germany a circle with diameter AB, as you supposed.



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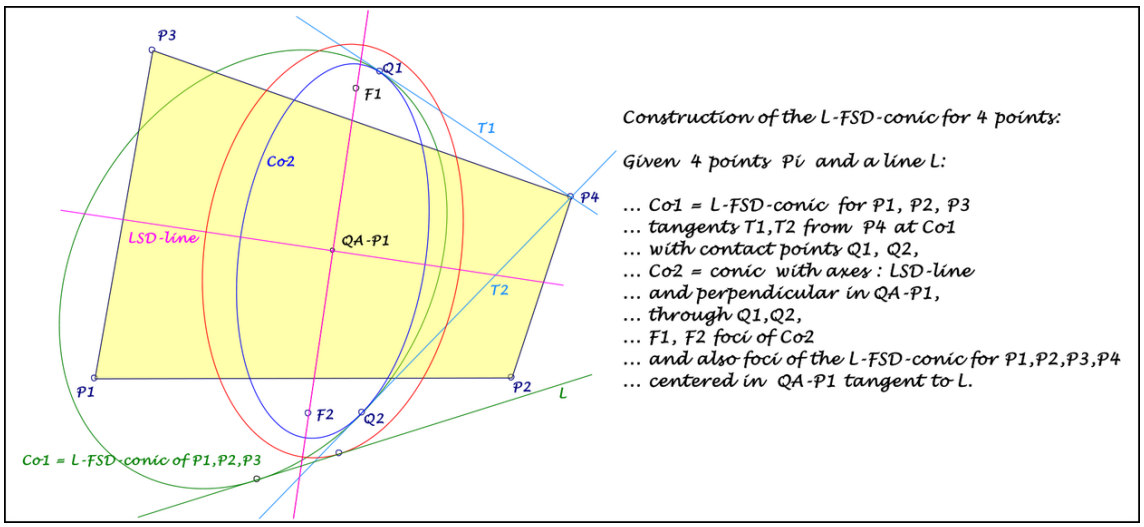
Message: #1635
Date: 05/4/2016 3:15:55
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Bernard,
attached a construction of the L-FSD-conic for 4 points.
Best regards Eckart



2016-04-05a.pdf

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Message: #1636
Date: 05/4/2016 3:49:50
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Eckart,
Thank you for your precision about the Thales circle and for your complete and easy construction.
Checking the whole construction, I find that the LSD line of the 3 points is the minor axis of the Steiner ellipses.
More intriguing (perhaps it's only a coïncidence), if I call S1 and S2 the foci of the Steiner inellipse and , like you, F1 and F2 the foci of the FSD conics and X2 the centroïd, it appears that X2F1 = X2S1 sqrroot(2).
In other terms, X2F1^2 = 2*X2S1^2 and the foci of the Steiner circum- and inellipses are inverse wrt the circle with center X2 through F1 and F2.
What do you think about this ?
Best regards
Bernard

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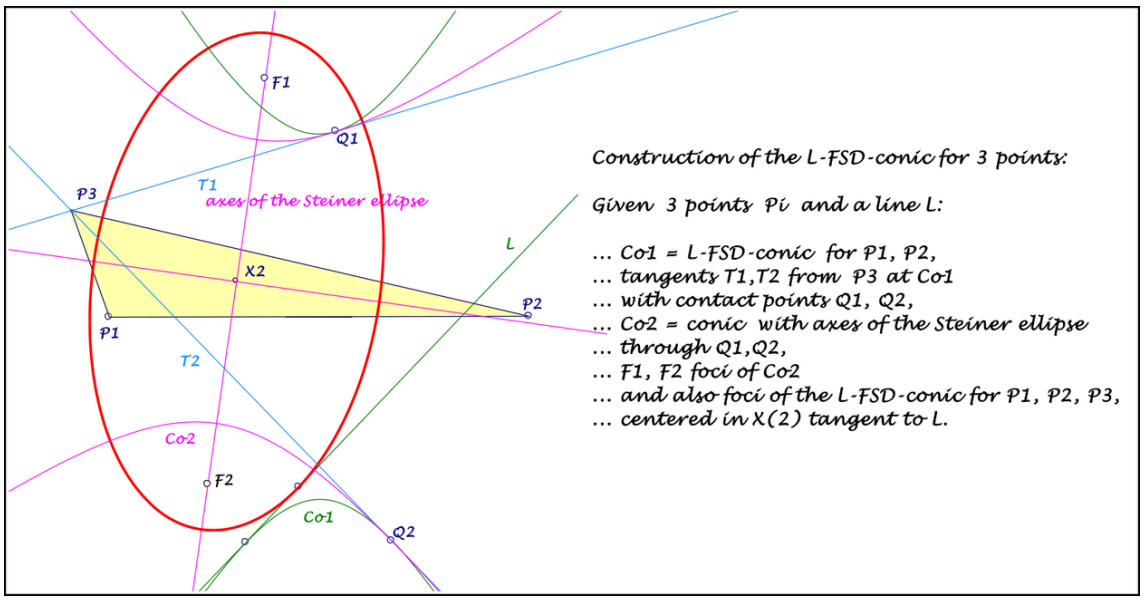
Message: #1637
Date: 05/4/2016 7:20:03
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squiares

Dear Bernard,
please excuse, the construction of the L-FSD-conic for 3 points in message 1634 gives indeed the right result, but doesn't use the L-FSD-conic for P1, P2 wrt L as described.
Attached a new file.
Best regards Eckart



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Message: #1638
Date: 05/4/2016 8:46:28
From: bernard.keizer
Subject: Constant Sum of Distance Squiares

Dear Chris, dear Eckart,
I'm sure now of my last property (message 1636) !
For the triangle, the LSD line is the minor axis of the Steiner ellipses.
Let's S1 and S2 be the foci of the circumscribed Steiner ellipse and T1 and T2 the foci of the Steiner inellipse (T1 is the middle of X2S1 and T2 the middle of X2S2).
Let's draw the circle with diameter S1T1 and the circle with center X2 orthogonal to this 1rst circle ; the 2nd circle intersects the LSD line in the 2 points F1 and F2, which are the 2 foci of the FSD conics of the 3 points.
Best regards
Bernard

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Message: #1639
Date: 05/4/2016 9:10:04
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Bernard,
wrt your message1636: The LSD-line of three points isn't the minor but the major axis of the Steiner ellipses (see Benedetto's #1585), perhaps a typo.
The following is a remarkable result: The foci Fi of the L-FSD-conics for 3 points are points on the minor axis of the inscribed Steiner ellipse with center distance sqr(2) focal length!
So my construction in the last message is unnecessary!
Best regards Eckart

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Message: #1640
Date: 05/4/2016 11:11:23
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Eckart,
Of course, you're right !
Please correct my last message 1638 as follows :
For 2 points, the LSD line is the line through the 2 points and the 2 foci of the FSD conics are on the perpendicular bisector at a distance from the centroïd (middle of the segment) equal to the distance of the points.
For 3 points, the LSD line is the major axis of the Steiner ellipses and the foci of the FSD conics are on the minor axis at a distance sqr(2) the focal lengh of the inscribed Steiner ellipse (or sqr(2)/2 the focal lengh of the Steiner circumellipse).
In fact, your construction for the triangle is no longer necessary, but remains for n points with n>3.
I wonder if there is a possible generalisation : for the 4 lines, the 2 foci of the FSD lines are on the perpendicular in QA-P1 to the LSD line at a distance ... ? (link with the QA-circumconics centered on the LSD line ?)
Best regards
Bernard

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Message: #1641
Date: 06/4/2016 12:08:14
From: chris.vantienhoven
Subject: Constant Sum of Distance Squares

Dear Eckart and Bernard,

I also studied the construction of the L-FSD-conics now. It starts with the case of 2 points P1 and P2 wrt a fixed line L.
We are looking for the conic Co such that all
tangents at Co will have the same sum of squared distances to P1 and P2 as L does.
Initially I encountered the same problems (handicaps) as Eckart did.
I agree with Eckart that the best way to tackle this problem is by determining the foci and then the hyperbola or ellipse.
I found a solution for unambiguously constructing Conic Co.

1. Let P1, P2 be two random points and L be a random line.
   Let Ce = midpoint(P1,P2).
2. Let S1= (line through P2 // L) ^ (line through P1 _|_ L).
   Let S2= (line through P1 // L) ^ (line through P2 _|_ L).
3. Let Q1,Q2 be orthogonal projection points
   resp. of P1,P2 on L.
4. Let Ci0 be circle with P1P2 as diameter.
5. Let Ci1a be circle with center Q1 through S1
   (radius will be d(P2,L)).
   Let Ci2a be circle with center Q2 through S2
   (radius will be d(P1,L)).
6. Let Ci1b be circle with center P1
   through intersection points L^Ci1a.
   Let Ci2b be circle with center P2
   through intersection points L^Ci2a.
7. Let T1 be 2nd intersection point of Ci0 ^ Ci1a.
   Let T2 be 2nd intersection point of Ci0 ^ Ci2a.
8. Let R=S1.T2^S2.T1,
   being the in-/ex-similicenter of Ci1a and Ci2a.
9. Construct the foci F1 and F2 of the conic Co
   we are looking for.
   They are the intersection points of Ci0
   with the perpendicular bisector of P1P2.
10. Finally when d(P1,P2) < d(P1,L) + d(P2,L),
    then Co=Ellipse though R with foci F1,F2,
    else Co=Hyperbola though R with foci F1,F2.

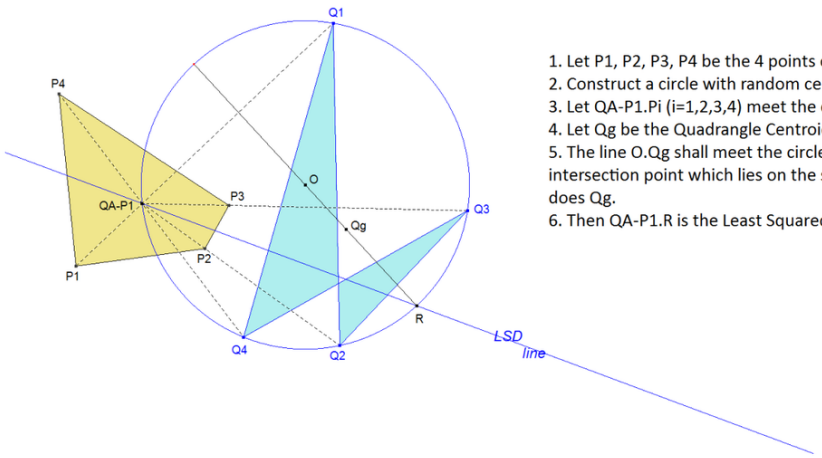
All tangents at Co will have the same sum of squared distances to P1 and P2 as L does.

Note 1: Condition d(P1,P2) < d(P1,L) + d(P2,L) happens
when (P1 and P2 are on equal side of L)
& (Ci1a and Ci2a mutually intersect).

Note 2: When Co is a hyperbola
it also can be constructed as the conic through R
and the intersection points of Ci0^Ci1b and Ci0^Ci2b.
See enclosed files.

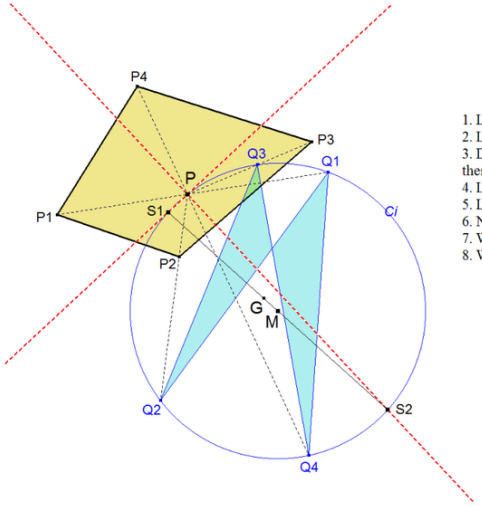
Best regards,
Chris

Construction of LSD line in a Quadrangle



1. Let P1, P2, P3, P4 be the 4 points of the Reference Quadrangle.
2. Construct a circle with random center O through QA-P1.
3. Let QA-P1.Pi (i=1,2,3,4) meet the circle again in Qi.
4. Let Qg be the Quadrangle Centroid of the points Qi.
5. The line O.Qg shall meet the circle in R, that is to say, R is the intersection point which lies on the same side of the diameter as does Qg.
6. Then QA-P1.R is the Least Squared Distance line.

Generalization of construction method



1. Let P1.P2.P3.P4 be a quadrangle.
2. Let P be some random point and Ci some random circle through P with random center M.
3. Draw lines P1.P, P2.P, P3.P, P4.P and let Q1, Q2, Q3, Q4 be the 2nd intersection points of these lines with circle Ci.
4. Let G be quadrangle centroid of Q1.Q2.Q3.Q4.
5. Let S1 and S2 be the intersection points of G.M with circle Ci.
6. Now lines P.S1 and P.S2 are fixed lines independent of the position of circle Ci.
7. When P is the centroid of quadrangle P1.P2.P3.P4 then P.S1 is the Least Squares Line.
8. When P = QL-P1 (Miquel Point) then both lines P.S1 and P.S2 are the Steiner Axes.

The LSD line is the line with Least Sum of Squared Distances to the Quadrangle vertices P1, P2, P3 and P4.

Chris van Tienhoven
March 10, 2016

QA-L10-LSD Line-00.docx

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Message: #1642
Date: 06/4/2016 3:32:14
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Chris, dear Eckart
Looking again at the figures, I think I have foud a simple recursive construction of the 2 foci of the FSD conics, provided we have constructed the LSD line and it's perpendicular in the centroïd.
I use Eckart's construction in message 1635, but without reference to a special line L.
Any conic Co1 suits with the FSD foci for n-1 points, the tangents from the last point to this conic gives the points Q1 and Q2 and the conic Co2 through Q1 and Q2 with axes the LSD line and it's perpendicular in the centroïd of the n points has for foci the 2 searched FSD foci for the n points
In particular, the simplest possible conic Co1 is the segment joining the 2 foci of the FSD conics of n-1 points!
(The tangents to this conic from the last point Pn are the lines through Pn and these 2 foci).
We have then only to find each time a conic knowing it's axes (LSD line and perpendicular in the centroïd) and 2 points (FSD foci of n-1 points).
This method avoids the handicaps encountered by Eckart.
Then, it's possible to find the L-FSD conic for the n points for any line.
Best regards
Bernard
PS for Chris : you put a wrong attached file in your last message ...

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Message: #1643
Date: 06/4/2016 6:27:23
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Chris, dear Eckart
It's even more beautiful than I thought myself !
The 2nd conic is entirely determined, since we know 2 points and their tangents and it's center (which makes by symmetry in this center 4 points and their tangents).
We don't need therefore to know the axes before !
On a contrary, this method allows te determine the LSD axis as perpendicular bisector of the segment joining the 2 foci.
Let's Fn-1 and F'n-1 be the 2 foci of the FSD conics for n-1 points P1 to Pn-1. Then the foci Fn and F'n of the FSD conics for n points P1 to Pn will be the foci of the conic with center the centroïd of the n points passing through the foci Fn-1 and F'n-1 and tangent in these points to the lines PnFn-1 and PnF'n-1.
The LSD line of the n points is the perpendicular bisector of FnF'n-1.
We know already F2 and F'2 for 2 points and F3 and F'3 for a triangle ...
Best regards
Bernard

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Message: #1644
Date: 06/4/2016 9:21:38
From: eckart_schmidt@t-online.de
Subject: Constant Sum of Distance Squares

Dear Bernard,
thanks for explination and simplification of my construction concept. Especially not needing the axes before! You have a better clear view.
Best regards Eckart

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Message: #1645
Date: 06/4/2016 10:05:36
From: chris.vantienhoven
Subject: Constant Sum of Distance Squares

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Dear Bernard and Eckart,
Enclosed the right files I intended to send.
I will study your messages soon.
Best regards,
Chris
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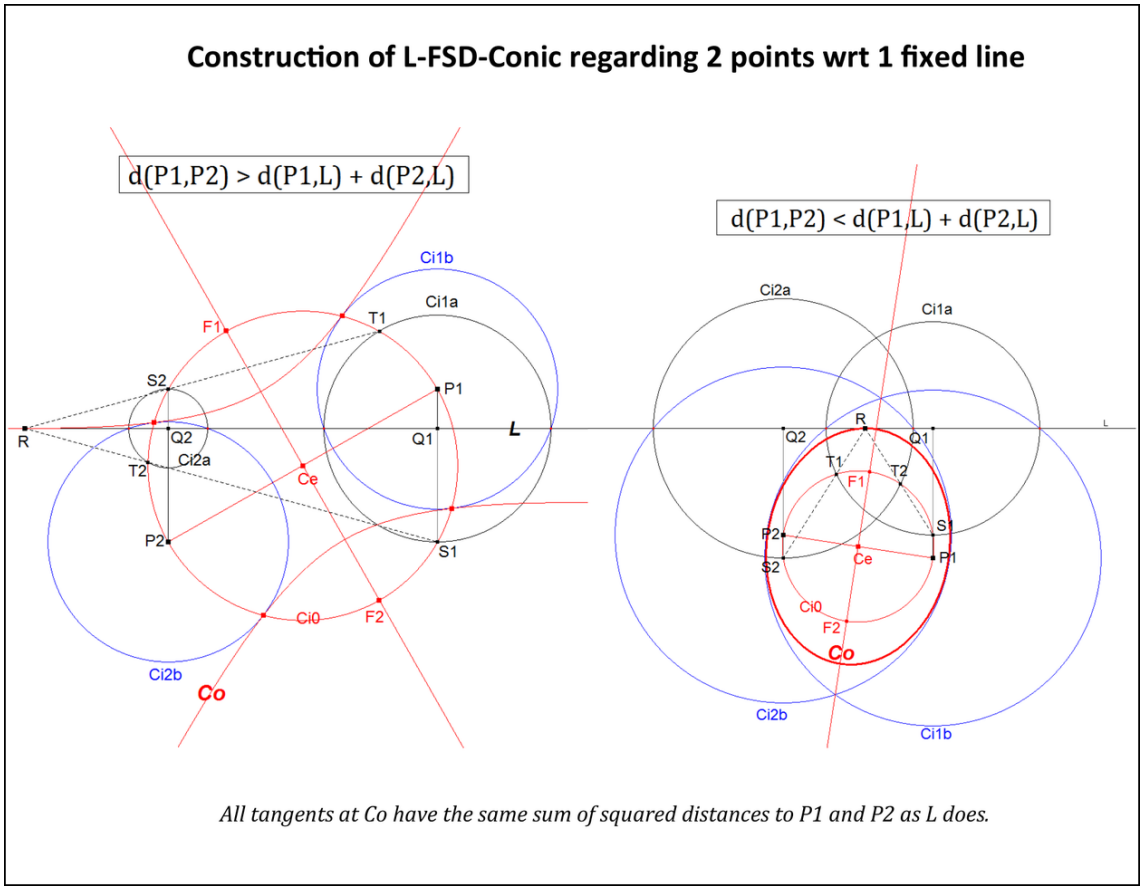

QA-L10-LSD Line-01.docx

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Message: #1646
Date: 07/4/2016 1:15:47
From: eckart_schmidt@t-online.de
Subject: Alternating Sum of Distance Squares

Dear Bernard, dear Chris,
for quadrigons we can consider alternating sums of distance squares. Corresponding results can easier be found with more relationships to EQF-geometry. See attached file.
Best regards Eckart

EQF-Note 2016-04-07

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Alternating Sum of Distance Squares

For quadrigons here are researched the loci for points X or lines L with the following properties:

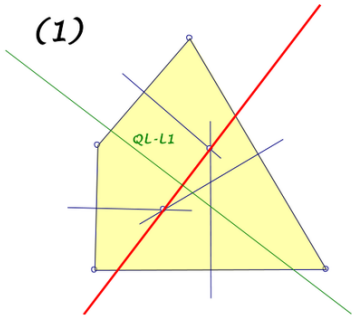
(1) $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = zero\ or\ const.$

(2) $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = zero\ or\ const.$

(3) $LP_1^2 - LP_2^2 + LP_3^2 - LP_4^2 = zero\ or\ const.$

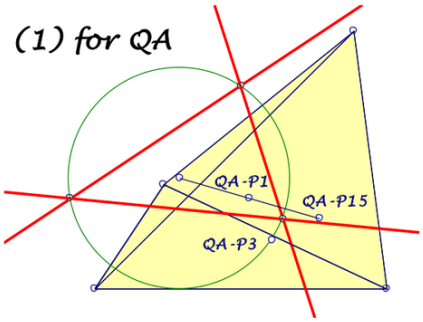
This note is an extension of EQF-Note 2014-01-12 in QFG-message 402.

(1) $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = zero\ or\ const.$



- **The locus for points with alternating sum zero is a perpendicular line to the Newton line *QL-L1*, containing the intersections of the perpendicular bisectors of opposite sides.**

- **The loci for points with constant alternating sum are perpendiculars to the Newton line *QL-L1*.**



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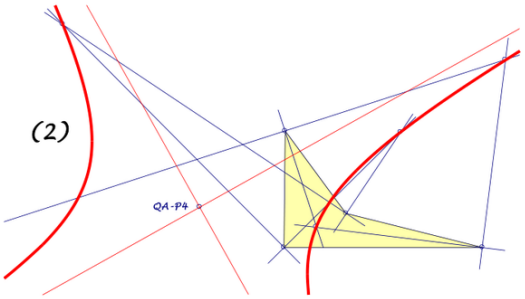
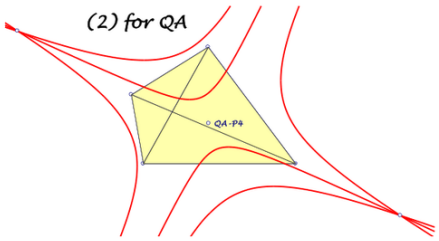
- **For a quadrangle the three lines for alternating sum zero give a triangle with circumcircle through *QA-P3*, centered in the reflection of *QA-P15* in *QA-P1*.**

(2) $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = zero\ or\ const.$

- **The locus of points with alternating sum zero is an orthogonal hyperbola, centered in *QA-P4*, containing the intersections of the inner or outer angle bisectors at opposite vertices.**

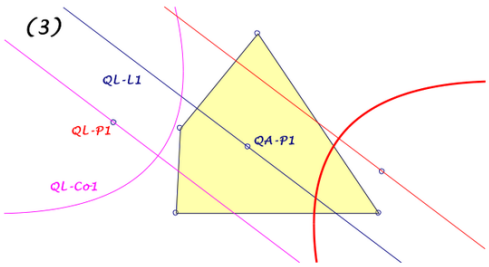


- **The loci of points with constant alternating sum are orthogonal hyperbolas, centered in *QA-P4* with the same asymptotes.**



- **For a quadrangle the three hyperbolas for alternating sum zero have two common points with midpoint *QA-P4*.**

(3) $LP_1^2 - LP_2^2 + LP_3^2 - LP_4^2 = zero\ or\ const.$



- **The lines with alternating sum zero envelop a parabola, which is the reflection of *QL-Co1* in *QA-P1*.**

- **The lines with constant alternating sum envelop parabolas with focus and axis of *QL-Co1* reflected in *QA-P1*.**

- **For quadrilaterals the three parabolas for alternating sum zero have as axis the asymptote of *QL-Cu1*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-04-07.pdf

Message: #1647
Date: 07/4/2016 6:41:05
From: tsihonglau
Subject: Constant Sum of Distance Squares

Dear all,
I follow this topic with the idea - zero-sum of distance squares but only Eckart replied me.
It is very important but seems no one realizes.
I post more results and hope someone replies.
As the topic "Least Squared Distance Line" was post here, I considered the zero-sum of distance square. If we want to study the "real" geometry, we had better study the "complex" geometry first. For example, we had better study "real" circles with the "complex" circular points. So we had better extend the "real" projective plane to the "complex" one. But on the latter, the word "least" is meaningless but "zero" is meaningful.
On a triangle/trilateral, it is very natural to consider the zero-sum of distance square conic.
The results were post to message #1621.
But how about the least-sum of distance squares?
As shown in this topic, there is a least-sum of distance square conic. Fortunately, the former two asymptotes in #1621 are the "least" one.
In other words, although the word "least" is meaningless, the least-sum of distance square conic exists. It is the asymptotes of "zero" one and the only degenerate "fixed" one(I guess, not verified). I hope someone will calculate the last case in message #1621 and quadrangle/quadrilateral cases. Any idea?
Best regards,
Tsihong Lau

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Message: #1648
Date: 08/4/2016 11:18:29
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Tsihong Lau
The FSD conics for n points have 2 common foci Fn and F'n.
There are 2 degenerate conics, which pass through the centroïd of the points : the 1rst one is an hyperbola, the asymptotes and the unic tangent are the LSD line, perpendicular bisector of FnF'n and the 2nd one is an ellipse reduced to the segment FnF'n and the unic tangent is the line FnF'n, which is the MSD (for Maximum square distances)
Best regards
Bernard
PS You annonced once on this forum you would make a figure with Geogebra mixing hundred QA and QL points. Did you do that ?
I would be interested in a corresponding table with points and lines and systematically QA-Tf2 of the points and QL-Tf2 of the the lines and the dual lines of the points and the dual points of the lines.
I know we already began (for example the S-points or the dual of some interesting conics like QL-Co1 and Co2 or the conic with focus QL-P17 and directrix QL-L6), but this lacks of systematic ...

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Message: #1649
Date: 08/4/2016 4:18:01
From: bernard.keizer
Subject: Constant Sum of Distance Squares

Dear Chris, dear Eckart, dear Tsilong Hau
I'm only satisfied when I've found all the possible conséquences of an item, before starting a new one.
I think I've found another interesting property for the FSD lines.
If we consider now among the conics with foci Fn and F'n the particular one reduced to the 2 points Fn and F'n, any line through one of the 2 foci is tangent to this reduced conic.
The sum of the square distances of the n points to any line through one of the 2 foci is therefore always the same, in particular when the line is through the 2 foci !
That means that the LSD line of the n points is unique (perpendicular bisector of the 2 foci), but the lines with MSD (Maximum of square distances) are not unique, but are composed by all the lines through one of the 2 foci (in particular the one through the 2 foci).
I hope I don't make any basic mistake and I would be glad to have your comments on this !
Best regards
Bernard

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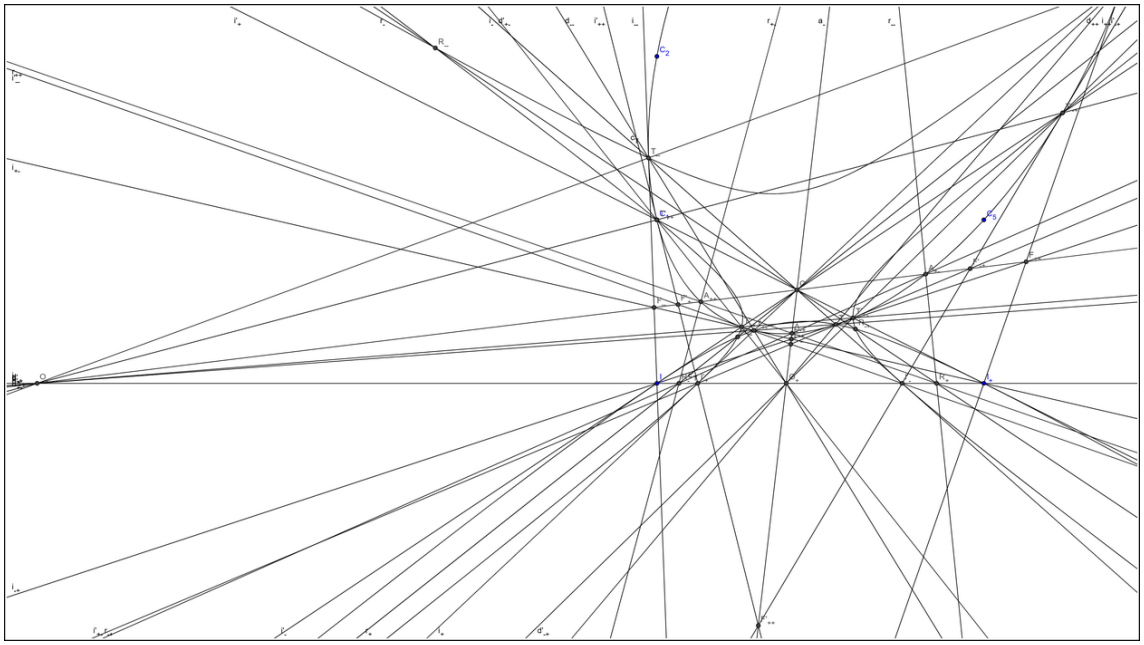
Message: #1650
Date: 08/4/2016 6:31:05
From: tsihonglau
Subject: Constant Sum of Distance Squares

Dear Bernard,
Thanks for your reply. My main approach is
Study first
1. projective plane over a commutative field
(especially algebraic closed one, circular points are arbitrary)
then
2. complex projective plane
(circular points have slopes +i,-i respectively)
last
3. real projective plane
(circular points do not exist!)
The second inherits properties of the first and has more nice properties than the last. For example, on the second any two circles intersect at four points, two of them are circular points and other two lie on the radical line.

>>The FSD conics for n points have 2 common foci Fn and F'n.
>>There are 2 degenerate conics, which pass through the centroïd of the points: the 1rst one is an hyperbola,
>>the asymptotes and the unic tangent are the LSD line, perpendicular bisector of FnF'n and the 2nd one is >>an ellipse reduced to the segment FnF'n and the unic tangent is the line FnF'n, which is the MSD (for
>>Maximum square distances)
On complex projective plane, any "real" conic has two "real" foci and two "imaginary" foci.
"least" and "maximum" is meaningless there but there are "least" square distance conic of a triangle/trilateral, which is shown in my previous message. I composed a GeoGebra file with some objects about conic. Please refer to the attachment file. The filename is quite long and contains the objects concerned. The file contains a definition of eccentricity of a conic.
(using only projective concepts) but no asymptotes. Maybe I include them and post the construction to my page later.
>>PS You annonced once on this forum you would make a figure with Geogebra mixing hundred QA and QL >>points.
Did you do that ?
>>I would be interested in a corresponding table with points and lines and systematically QA-Tf2 of the points >>and QL-Tf2 of the the lines and the dual lines of the points and the dual points of the lines.

>>I know we already began (for example the S-points or the dual of some interesting conics like QL-Co1 and >>Co2 or the conic with focus QL-P17 and directrix QL-L6), but this lacks of systematic ...
No, Sorry! I have not started it. I have calculated more than hundred centers/axes of quadrangle/quadrilateral and post to my page.
http://lth.name/geometry/center_axis_of_quadrangle_quadrilateral.html
They are all in circulars. I have not calculated the barycentrics.(My barycentric WXM file is a mess and I reorganize it now!) Circulars are appropriate to a projective plane over a general commutative field or complex one. If we want to construct with geometry softwares, we had better calculate the barycentrics. But there is no search keys for a quadrangle/quadrilateral.
It is inconvenient for study and search the objects.
I will propose my search keys in another topic.
Best regards,
Tsihong Lau



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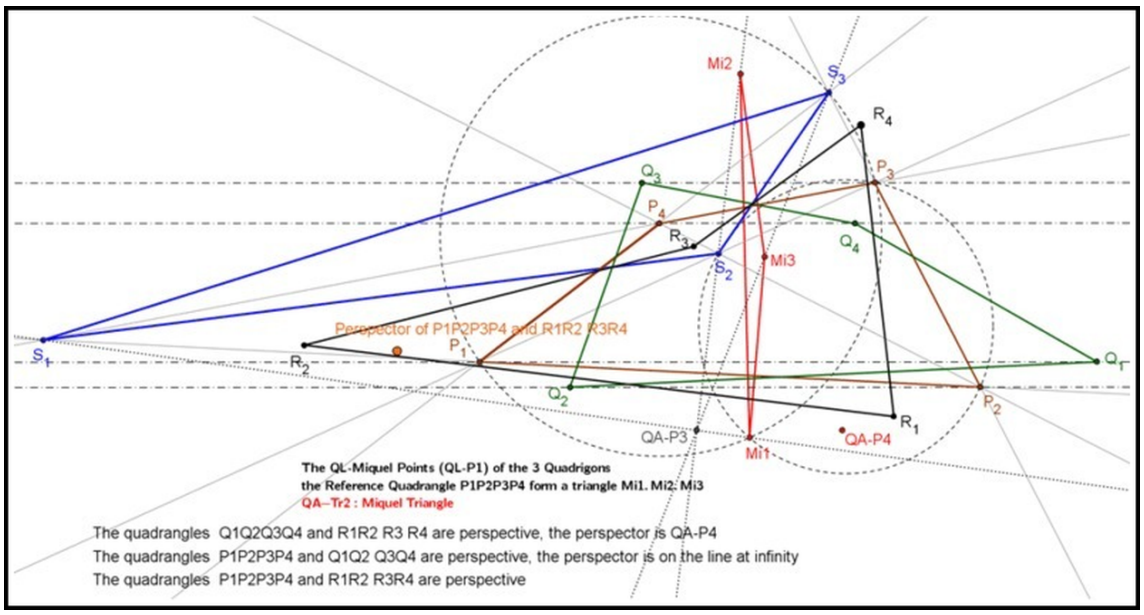
Message: #1651
Date: 08/4/2016 9:57:31
From: Angel Montesdeoca
Subject: Miquel-Triangle and Quadrangle-Centers

Dear all,
In a Quadrangle P1P2P3P4 all Component Triangles are perspective with the Miquel Triangle (QA-Tr2: Mi1Mi2Mi3).
Denote by Q1,Q2, Q3, Q4 the perspector of Mi1Mi2Mi3 and P2P3P4, P1P4P3, P4P1P2, P3P2P1, resp.
In the Quadrangle Q1Q2Q3Q4 all Component Triangles are perspective with the QA-Diagonal Triangle (QA-Tr1: S1S2S3).
Denote by R1, R2, R3, R4 the perspector of S1S2S3 and Q2Q3Q4, Q1Q4Q3, Q4Q1Q2, Q3Q2Q1, resp.
The quadrangles Q1Q2Q3Q4 and R1R2R3R4 are perspective, the perspector is QA-P4.
The quadrangles P1P2P3P4 and Q1Q2Q3Q4 are perspective, the 1st DT-Coordinate of perspector is:
p^2 (p^2 (c^2 q^2+b^2 r^2) + (-2 a^2 +b^2 +c^2)q^2 r^2 - c^2 q^4 - b^2 r^4) is on the line at infinity.
The quadrangles P1P2P3P4 and R1R2R3R4 are perspective, the 1st DT-Coordinate of perspector is:
p^2 (c^6 q^4-b^2 c^2 (-2 a^2+b^2+c^2) q^2 r^2+b^6 r^4) - a^4 q^2 r^2 (c^2 q^2+b^2 r^2)
Best regards,
Angel Montesdeoca



Miquel-Triangle Quadrangle-Centers.gif

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Message: #1652
Date: 09/4/2016 1:49:49
From: tsihonglau
Subject: Search Keys for Quadrangle/Quadrilateral

Dear all
I have got the idea for a long time. We have search keys a=6,b=9,c=13 for a triangle/trilateral. But we have no search keys for a quadrangle/quadrilateral. I propose search keys for it. a=6,b=9,c=13 as triangle/trilateral
(and the objects of the diapleural triangle/diagonal trilateral are the same as triangle/trilateral) and the DT-barycentrics l=11,m=8,n=6
and p,q,r are the reciprocals of l,m,n respectively.
I do not insist on the values of l,m,n,p,q,r and any good ones are acceptable.
If we fixed the cartesian coordinates of the triangle, we can get the cartesian coordinates of a point and
intercepts of a line. I propose A=(sqrt(560)/3,-13/3), B=(0,6), C=(0,0).
Best regards,
Tsihong Lau

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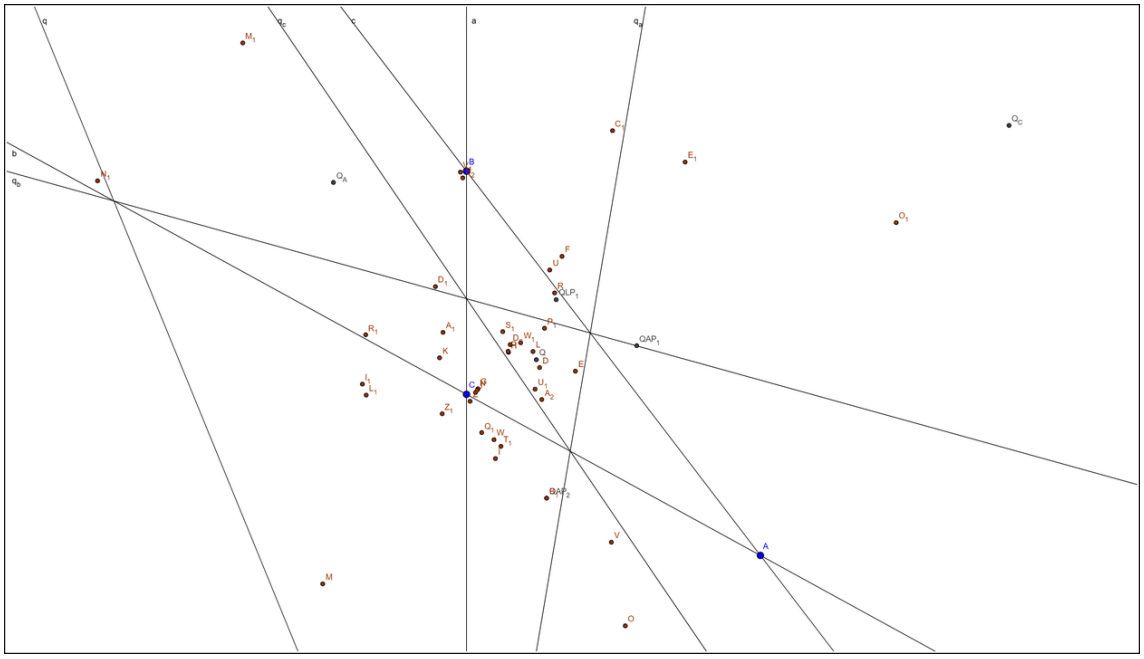
Message: #1653
Date: 09/4/2016 5:29:38
From: tsihonglau
Subject: Search Keys for Quadrangle/Quadrilateral

Dear Bernard,

In the attachment GeoGebra file are some
quadrangle/quadrilateral centers(in brown).
These centers have been in my page already.
Center/Axis of Quadrangle/Quadrilateral and Central
Triangle/Axial Trilateral of Quadrangle/Quadrilateral (http://l
th.name/geometry/center_axis_of_quadrangle_quadrilateral.html)

I did not organize these centers and construct them using the
barycentrics. Maybe I input the circulars of each center later.
We had better design good search keys. Because we have got
thousands of triangle/trilateral centers, we may have million of
quadrangle/quadrilateral centers, which are operation results of
triangle/trilateral centers and quadrangle/quadrilateral
involutary centers.

Best regards, Tsihong Lau



barycentrics5.ggb

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Message: #1654
Date: 09/4/2016 11:23:11
From: bernard.keizer
Subject: Search Keys for Quadrangle/Quadrilateral

Dear Tsihong Lau,
I find it a splendid idea !
(If I'm not wrong, 6, 9 and 13 come from Kimberling in ETC and are used by Douillet in his glossary)
So we could all have a common figure as reference !
I see only a little inconvenient : the hessian QL-Cu1 may be unicursal or bicursal and the cubic stelloïd QL-Cu2 is then Mac Cay or Kjp type.
So it could be wise to have 2 reference figures for both cases ...
To be sure of the beginning, could you please confirm that the cartesian values you give for A, B and C are those of the vertices of DT or QL-Tr1(corresponding to 6, 9 and 13 as lengh of the sides) ang give the cartesian coordinates of one of the 4 vertices of the QA (the inner one corresponding to the p, q and r inverses of 11, 8 and 6 for barycentrics).
We could then search a 2nd point for the other case with the same DT...
The best would be to show your reference figure, so everyone will be sure to reproduce the same with Geogebra or Cabri !
Best regards
Bernard
PS Could you please use word- or pdf files, it's much easier to open or print them than your zipped files. Thanks in advance !

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Message: #1655
Date: 09/4/2016 12:49:29
From: eckart_schmidt@t-online.de
Subject: Miquel-Triangle and Quadrangle-Centers

```
Dear Angel Montesdeoca,
with interest I have reproduced your P-, Q- R-quadrangles. Some
first remarks:
... The perspector of  the P- and Q-quadrangle
    is the point at infinity of QA-L4.
... For the P- and R-quadrangle - I think -
    there are 4 perspectors, a new quadrangle:
... ... perspective to the reference P-quadrangle,
... ... with vertices on the cubic QA-Cu1,
... ... which have a common tangential on QA-Cu1 (not in EQF).
... ... The perspector of the new and the P-quadrangle
        is also on QA-Cu1 (not in EQF).
... ... There will be more interesting properties,
        I shall study the constellation.
Best regards Eckart
```

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Message: #1656
Date: 09/4/2016 4:22:21
From: eckart_schmidt@t-online.de
Subject: Miquel-triangle and Quadrangle-centers

```
Dear Angel Montesdeoca,
in addition to #1655:
The 4 perspectors of the Q- and R-quadrangle
... are QA-P4
... and the vertices of the diagonal triangle.
The 4 perspectors of the P- and Q-quadrangle
... are the point at infinity of QA-L4
... and the vertices of the Miquel triangle.
The 4 perspectors of the P- and R- quadrangle
... are the isogonal conjugates wrt the Miquel triangle
    of QA-P41
... and the isogonal conjugates wrt the Miquel triangle
    of the 3rd intersections of QA-Cu1 and the sidelines
    of the diagonal triangle.
All quadrangles, also the perspector-quadrangles, have their
vertices on the cubic QA-Cu1, which contains also their Isogonal
Center.
Best regards Eckart
```

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Message: #1657
Date: 10/4/2016 9:14:59
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

```
Dear Chris,
QA-Cu1 is a pivotal isocubic (see EQF) wrt
... reference triangle: diagonal triangle,
... isoconjugation with fixed points Pi,
... pivot: QA-P4.
QA-Cu1 is a pivotal isocubic (see EQF) wrt
... reference triangle: Miquel triangle,
... isogonal conjugation,
... pivot: point at infinity of QA-L4.
QA-Cu1 is a pivotal isocubic wrt
... reference triangle:
    3rd intersections of QA-Cu1 and DT-sides,
... isoconjugation with fixed point QA-P4 (and DT-vertices),
... pivot: QA-P41.
In general:
QA-Cu1 is a pivotal isocubic with given pivot P on QA-Cu1:
... reference triangle ABC: 3 CSC-images of P,
... isoconjugation swapping P and the isogonal center of ABCP,
... or:
    isoconjugation with fixed points in the tangentials of P,
... pivot P.
This is a consequence of EQF-Ref [15b], 5.
Best regards Eckart
```

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Message: #1658
Date: 10/4/2016 10:49:11
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

```
Dear Chris,
I just found another description:
QA-Cu1 is a pivotal isocubic wrt
... reference triangle: 3rd intersections of QA-Cu1 and sides of
the Miquel triangle,
... isoconjugation with fixed points in the point at infinity of
QA-L4 and the vertices of the Miquel triangle,
... pivot: intersection of QA-Cu1 and its asymptote.
Best regards Eckart
```

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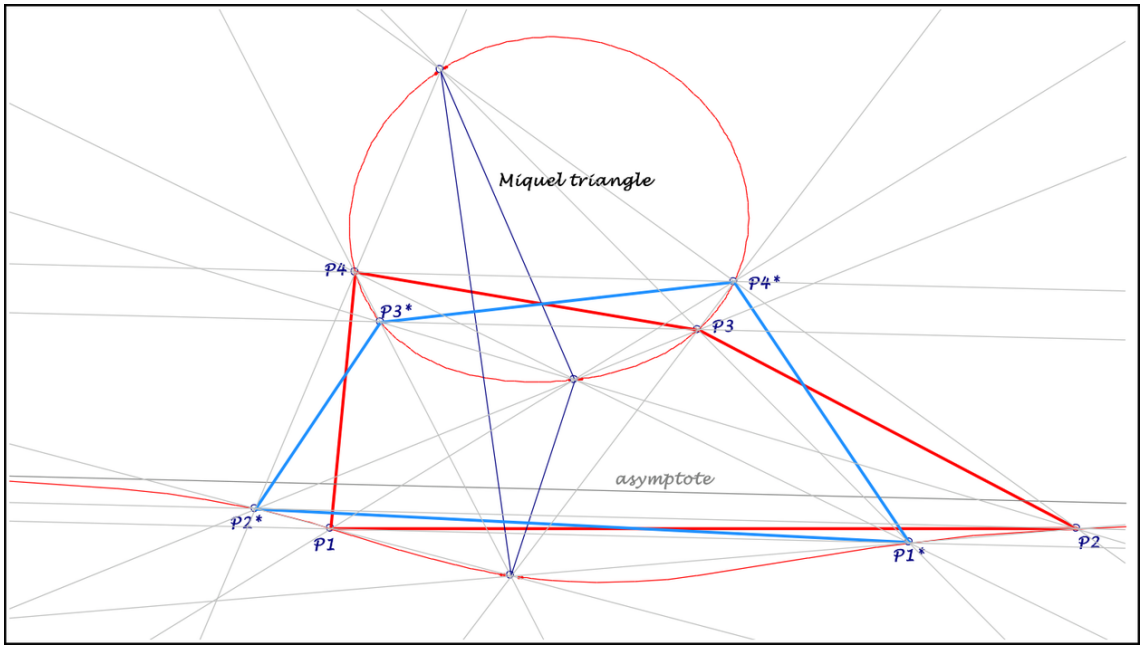
Message: #1659
Date: 10/4/2016 11:30:26
From: eckart_schmidt@t-online.de
Subject: Miquel-Triangle and Quadrangle-Centers

Dear Angel Montesdeoca,
background of the considered QA-perspectivities is the following generalisation:
Let Cu-QA be quadrangles on the cubic QA-Cu1 with Isogonal Center also on the cubic.
You can get a Cu-QA, taking a point P1 on QA-Cu1 and its CSC-images as P2,3,4.
The perspectors of two Cu-QA give a 3rd Cu-QA.
Two Cu-QA of this tripel are perspective wrt the vertices of the 3rd Cu-QA.
The Isogonal Centers of the three Cu-QA are collinear.
Special example (see attached file):
Let * be the isogonal conjugate wrt the Miquel triangle,
... then Pi and Pi* give two Cu-QA,
... the perspector quadrangle consists of the Miquel triangle and the point at infinity of the asymptote.
Best regards Eckart



2016-04-10.pdf

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Message: #1660
Date: 10/4/2016 1:02:52
From: Angel Montesdeoca
Subject: Miquel-Triangle and Quadrangle-Centers

Thanks Eckart.
What surprised me most of these messages is the
relationship with the cubic QA-Cu1
Best regards
Angel

El 2016-04-10 10:30, ' eckart_schmidt@t-online.de '
eckart_schmidt@t-online.de [Quadri-Figures-Group] escribió:
> > [Attachment(s) from eckart_schmidt@t-online.de included
below]
> > Dear Angel Montesdeoca,
> > background of the considered QA-perspectivities is the
following
> > generalisation:
> > Let Cu-QA be quadrangles on the cubic QA-Cu1 with Isogonal
Center also
> > on the cubic.
> > You can get a Cu-QA, taking a point P1 on QA-Cu1 and its
CSC-images as
> > P2,3,4.
> > The perspectors of two Cu-QA give a 3rd Cu-QA.
> > Two Cu-QA of this tripel are perspective wrt the vertices of
the 3rd
> > Cu-QA.
> > The Isogonal Centers of the three Cu-QA are collinear.
> > Special example (see attached file):
> > Let * be the isogonal conjugate wrt the Miquel triangle,
> > ... then Pi and Pi* give two Cu-QA,
> > ... the perspector quadrangle consists of the Miquel
triangle and the
> > point at infinity of the asymptote.
> > Best regards Eckart

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Message: #1661
Date: 10/4/2016 1:49:57
From: Angel Montesdeoca
Subject: QA-Cu1

[Eckart Schmidt]:
The pivot of QA-Cu1 is the intersection of QA-Cu1 and its asymptote.

*** In a Quadrangle P1P2P3P4 all Component Triangles are perspective with the Miquel Triangle (QA-Tr2: Mi1Mi2Mi3). Denote by Q1, Q2, Q3, Q4 the perspector of Mi1Mi2Mi3 and P2P3P4, P1P4P3, P4P1P2, P3P2P1, resp.

The perspector of S1S2S3=QA-Tr1 and QaQbQC=(QA-Tr1 of Quadrangle Q1Q2Q3Q4) is the pivot of pivotal isocubic QA-Cu1.

Angel M.

El 2016-04-10 09:49, ' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de [Quadri-Figures-Group] escribió:
> > Dear Chris,
> > I just found another description:
> > QA-Cu1 is a pivotal isocubic wrt
> > ... reference triangle: 3rd intersections of QA-Cu1 and sides of the
> > Miquel triangle,
> > ... isoconjugation with fixed points in the point at infinity of QA-L4
> > and the vertices of the Miquel triangle,
> > ... pivot: intersection of QA-Cu1 and its asymptote.
> > Best regards Eckart

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Message: #1662
Date: 10/4/2016 3:21:03
From: chris.vantienhoven
Subject: QA-Cu1

Dear Eckart, dear friends,
Thanks for your update on QA-Cu1.
I revised the description of QA-Cu1 in EQF.
When there are inconsistencies please let me know.
I also added these new items:
QA-P44 Least Squared Distance Point of QA-sidelines
QA-L10 QA-LSD Line
Best regards,
Chris

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Message: #1663
Date: 10/4/2016 5:39:43
From: tsihonglau
Subject: Search Keys for Quadrangle/Quadrilateral

Dear Bernard,
>>I see only a little inconvenient: the hessian QL-Cu1 may be unicursal or bicursal and the cubic stelloïd QL->>Cu2 is then Mac Cay or Kjp type.

>>So it could be wise to have 2 reference figures for both cases ...

>>To be sure of the beginning, could you please confirm that the cartesian values you give for A, B and C are >>those of the vertices of DT or QL-Tr1(corresponding to 6, 9 and 13 as lengh of the sides) ang give the >>cartesian coordinates of one of the 4 vertices of the QA (the inner one corresponding to the p, q and r >>inverses of 11, 8 and 6 for barycentrics).

>>We could then search a 2nd point for the other case with the same DT...
I do not think it is a good idea to have two reference quadrangles/quadrilaterals. Not only cubics have different types but also conics. (hyperbolas or ellipse etc.) So how many reference quadrangles/quadrilaterals do we need? Search keys are used to identify points (using coordinates) and lines (using intercepts).
Curves are identified with points on them and tangents at them. We had better not identify curves with search keys directly.

>>PS Could you please use word- or pdf files, it's much easier to open or print them than your zipped files.
I do not upload zip files but Yahoo! Groups zip GeoGebra files automatically. I know not all use GeoGebra but I uploaded the GeoGebra file because it contains barycentrics. Word's and pdf's do not contain barycentrics.
It is very difficult to choose the search keys.
If they are too small, many points and lines coincide. (We may have millions of quadrangle/quadrilateral points and lines) If they are too big, the computer software precision is not enough.
The search keys proposed are not good enough.
I will propose another set.

Another consideration: To use barycentrics or trilinears? EQF use barycentrics. I prefered
barycentrics to trilinears before. But on my page,

Center of Triangle/Trilateral and Involutary Axis of Quadrangle/Quadrilateral and Central Triangle/Axial Trilateral of Quadrangle/Quadrilateral (http://lth.name/geometry/center_of_triangle_trilateral_and_involutary_axis_of_quadrangle_quadrilateral.html)

Orthocentric is trilinear in fact. It is more natural to use trilinear circulars instead of barycentric circulars for triangles/trilaterals, because pu=qv=rw=1 using trilinear circulars. (very simple)
But it is more accurate to construct using barycentrics than trilinears. Because we may construct millions of points and lines, we had better use barycentrics. In other words, on study purpose, we had better use unitary circulars (i.e. p,q,r,u,v,w) for quadrangles/quadrilaterals and orthocentric (trilinear p,q,r only) circulars for
triangles/trilaterals. On construction purpose, we had better use DT-barycentric sidelength coordinates
(i.e. a,b,c,p,q,r,l,m,n) for quadrangles/quadrilaterals and barycentric sidelength coordinates (a,b,c only)
for triangles/trilaterals.

Another propose: I proposed search keys with l,m,n instead of p,q,r. That is because p,q,r coincide with circular point coordinates. But we can use u,v,w instead of l,m,n. For quadrangles/quadrilaterals, pu=qv=rw=1 when using
DT-barycentric sidelength coordinates. For
triangles/trilaterals, pu=qv=rw=1 when using
orthocentric(trilinear) circular coordinates.
The situations do not conflict.

Best regards,
Tsihong Lau

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Message: #1664
Date: 12/4/2016 10:49:48
From: bernard.keizer
Subject: QA-Cu1

Dear Chris,
Congratulations for your efforts for updating EQF regularly !
I found the new complete description of QA-Cu1 very interesting.
Perhaps you could make here a reference to the Desmic configuration, which appears in EQF in QA-Tr1 (see your message 1616) ; if I'm not wrong, QA-Tr1 and QA-Tr2 are only special cases of the general case ...
We have then, for a QA, 4 points with LSD line QA-L10 and 6 lines with LSD point QA-P44 and, for a QL, 4 lines with LSD point QL-P26 and 6 points without LSD line !
Perhaps you could in the future add this LSD line of the 6 vertices of the QL ...
Apart of that, you didn't mention, for the LSD point, your own recursive construction of the FSD ellipses (see message 1622) and the recursive construction of the 2 foci of the FSD conics due to Eckart and me (see messages 1642 and 1643). These 2 foci for 4 lines could be a QL-pair of points and for 6 lines a QA-pair of points ...
Best regards
Bernard

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Message: #1665
Date: 12/4/2016 11:01:56
From: bernard.keizer
Subject: Search Keys for Quadrangle/Quadrilateral

Dear Tsihong Lau
I don't need a long answer !
I'm asking only
1) for the cartesian coordinates of one vertice of the QA
(I'm not familiar with the barycentric properties of Geogebra, I would be pleased and grateful if you could teach me the way of using these triangle commands ...)
2) for a figure I could open and print : this is possible with Cabri, as Chris and Eckart regularly do, and with Geogebra, as I often do myself.
You only have to open a word-file and, when you are in Geogebra, to click on Export and choose Export to the press-files (in french, it is Exporter Graphique vers Press papiers) ; you go back to your word-file and insert the figure which should appear then on the left in the press-files ...
It's then possible, but not necessary, to transform your word-file in a pdf-file.
Best regards
Bernard

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Message: #1666
Date: 12/4/2016 11:05:09
From: eckart_schmidt@t-online.de
Subject: Some Splitter XI

Dear Chris,
I have cleared my desk: Some splitter, perhaps of interest and not already mentioned.
Best regards Eckart

Splitter XI

QA-Co2:	*QA-P11.QA-P41* intersects *QA-Co2* on *QA-Cu1*.
QA-P41:	*QA*-circumconics intersect *QA-Cu1* in two further points collinear with *QA-P41*.
QA-Co2:	… contains with a point also the vertices of its anticevian triangle wrt *QA-Tr1*.
QA-P3:	The *QA-Tf2*-conic of the *QA-Cu1*-asymptote contacts *QA-Cu1* in *QA-P3*.
***QA-P4*:**	A *QA*-circumconic through *QA-P4* contacts *QA-Cu1* in this point.
QA-Tr2:	All *QA-Tr2*-circumconics through the intersection of *QA-Cu1* and its asymptote cut *QA-Cu1* in two isogonal conjugated further points.
QA-Tr2:	All pivotal isogonal isocubics wrt *QA-Tr2* intersect *QA-Cu1* in two isogonal conjugated points collinear with the pivot.
***QA-P9*:**	The reference *QA* and *QA-P4* with *QA-Tr1* have the same *QA*-Miquel Center.
QA-P9:	The Simson line of *QA-P9* wrt *QA-Tr2* is parallel *QA-P3.QA-P4*.
QL-Tf2:	The *QL-Tf2*-image of the *QL-Co1*-polar of a point *P* contains *P*.
QA-P35:	The 1st Penta points of points *P* and its *QA-Tr1*-anticevians lie on a circle through *QA-P35*.
QA-P42:	… collinear with *QA-P1* and the reflection of *QA-P4* in *QA-P3*.
QL-Ci2,6:	2nd intersection of *QL-Ci2* and *QL-Ci6* on *QL-P8.QL-P17.QL-P25*.
QA-Co2:	Let *P* be on *QA-Co2* and *X, Y* its reflections in P_iP_j and P_kP_l, then the circumcircles of XP_iP_j and YP_kP_l intersect in the reflection of *P* in *QA-P2*.
QA-P43:	(wrt a quadrigon) … on a parallel to the polar of *QL-P8* wrt *QL-Co1*, distance 6/7 to *QG-P1*.
QA-P43:	(wrt a quadrigon) … on a hyperbola with asymptotes parallel to the diagonals, centered on *QG-P1.QA-P10* with ratio *6:1*, through points on *QG-P1.QA-P10* with ratio *3:4* or *-9:2*.
???:	The intersections of lines *L* through *P* and the tangents *T* at the *CSC*-circle of *L* in *QL-P1* give an orthogonal hyperbola, centered in the midpoint of *P.QL-P1* with asymptotes parallel to the Steiner axes.

Eckart Schmidt
http://eckartschmidt.de
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Splitter-XI.docx

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Message: #1667
Date: 12/4/2016 3:01:46
From: eckart_schmidt@t-online.de
Subject: QA-L10

Dear Bernard, dear Chris,
I regret the devaluating statement of Bernard at the end of #1615 wrt the construction of QA-L10 in #1611.
The more synthetic concept isn't respected, the construction is nearer to the geometric context compared to follow a complex calculation.
There is no relevant difference in the extensiveness of the constructions.
Best regards Eckart

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Message: #1668
Date: 12/4/2016 3:11:42
From: eckart_schmidt@t-online.de
Subject: QA-Tr1 <–> QA-Tr2 Transformation

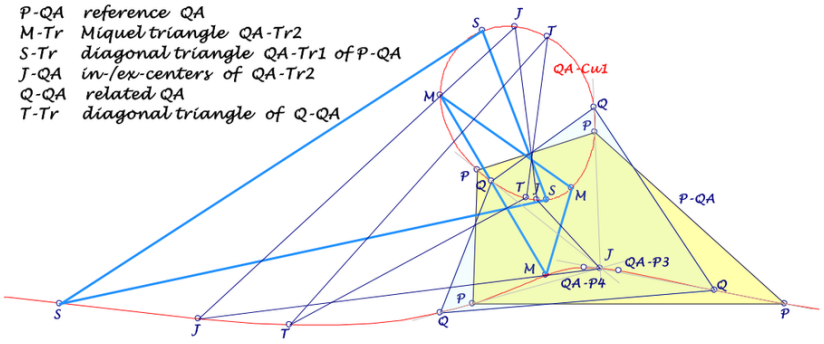
Dear Chris,
QA-geometry can be done wrt the diagonal triangle QA-Tr1 or wrt the Miquel triangle QA-Tr2. Wrt QA-Tr1 one vertex and its anticevian triangle give the QA, wrt the Miquel triangle one vertex and its CSC-images give the QA. Here is a transformation, which swaps the vertices of QA-Tr1 and QA-Tr2. This transformation is the "Involutary Conjugate" of a related QA (see attached file).
Best regards Eckart

EQF-Note 2016-04-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Tr1 ←→ QA-Tr2 Transformation

QA-geometry can be done wrt the diagonal triangle QA-Tr1 or wrt the Miquel triangle QA-Tr2. Wrt QA-Tr1 one vertex and its anticevian triangle give the QA, wrt the Miquel triangle one vertex and its CSC-images give the QA. Here is a transformation, which swaps the vertices of QA-Tr1 and QA-Tr2. This transformation is the "Involutary Conjugate" of a related QA.



Related *QA*: Let *J-QA* be the quadrangle of the in-/ex-centers for the Miquel triangle of the reference quadrangle *P-QA*.

The four perspectors of *J-QA* and *P-QA* give the related quadrangle *Q-QA*.

- **The three perspective quadrangles *P-QA*, *Q-QA*, *J-QA* have their vertices on the cubic *QA-Cu1*.**

- **The related quadrangle *Q-QA***
  … has the same Miquel triangle as *P-QA*,
  … the same cubic *QA-Cu1* as *P-QA*,
  ... its Isogonal Center is *QA-P3* of *P-QA*,
  ... its Gergonne-Steiner Point is *QA-P4* of *P-QA*.

- ***QA-P3* is perspector of the Miquel triangle and the diagonal triangle of *P-QA* (see *EQF*), *QA-P4* is perspector of the Miquel triangle and the diagonal triangle of *Q-QA*, the point at infinity of the *QA-Cu1*-asymptote is perspector of the diagonal triangles of *P-QA* and *Q-QA*.**

- **The *CSC*-Tripel of *QA-P3* gives the diagonal triangle of *Q-QA*.**

- **The sides of the Miquel triangle and the diagonal triangles of *P-QA* and of *Q-QA* have three collinear points on a line *L*:**
  … Wrt the Miquel triangle the line *L* is the trilinear polar of the midpoint of *QA-P4.QA-P3*.
  … Wrt the diagonal triangle of *P-QA* the line *L* is the trilinear polar of the reflection of *QA-P4* in *QA-P3*.
  … Wrt the diagonal triangle of *Q-QA* the line *L* is the trilinear polar of the reflection of *QA-P3* in *QA-P4*.

New *QA*-transformation *QA-Tfx:* Let *QA-Tfx* be the Involutary Conjugate of the related quadrangle *Q-QA*, an isoconjugation wrt the diagonal triangle of *Q-QA* with fixed points in the vertices of *Q-QA*.

- **The Involutary Conjugate of the related quadrangle *Q-QA* swaps the vertices of the diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2*.**

- **The cubic *QA-Cu1* is invariant wrt *QA-Tfx*.**

- **The cubic *QA-Cu1* is a pivotal isocubic wrt**
  … reference triangle: diagonal triangle of *Q-QA*,
  … isoconjugation: *QA-Tfx*,
  … pivot: *QA-P3*.

- ***QA-Tfx(QA-P4)* is the point at infinity of *QA-Cu1*.**

- ***QA-Tfx(QA-P3)* is the 3rd intersection of *QA-Cu1* and the line, connecting *QA-P41* and the intersection of *QA-Cu1* and its asymptote.**

- ***QA-Tfx(Pi)* are the 3rd intersections of *QA-Cu1* and *Pi.QA-P3*.**

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2016-04-12.pdf

Message: #1669
Date: 13/4/2016 8:38:43
From: chris.vantienhoven
Subject: QA-Tr1 <–> QA-Tr2 Transformation

Dear Eckart,
[ES] wrt the Miquel triangle one vertex and its CSC-images give the QA the CSC-images wrt which Quadrilaterals?
Your document is very interesting.
You state: The four perspectors of J-QA and P-QA give the related quadrangle Q-QA.
I think very few will understand this. Maybe you explain a bit more how to do this.
Best regards,
Chris

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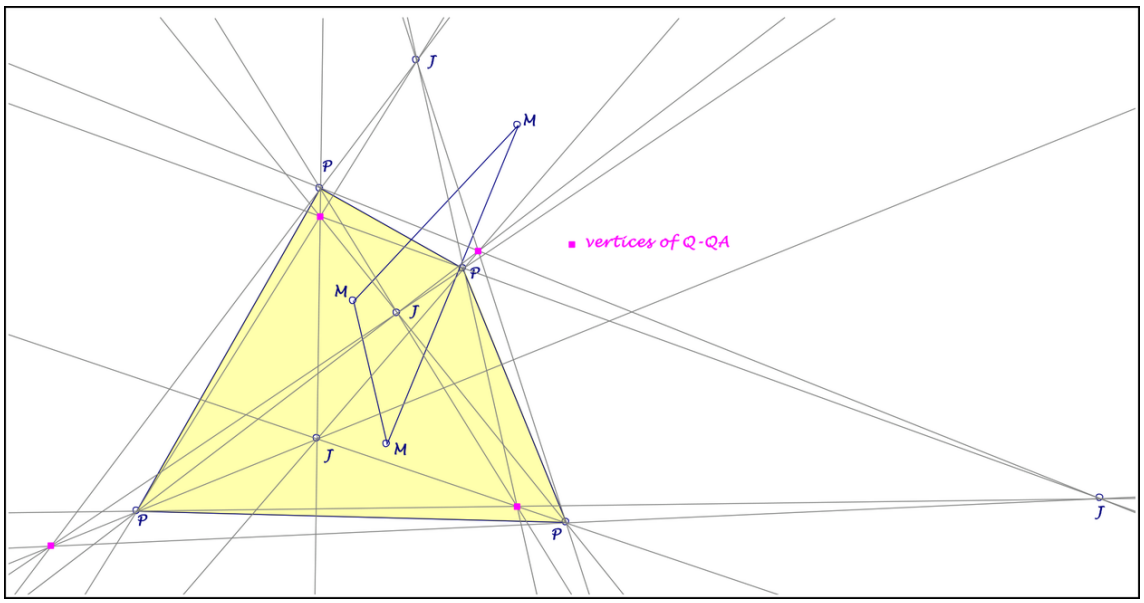
Message: #1670
Date: 13/4/2016 9:37:59
From: eckart_schmidt@t-online.de
Subject: QA-Tr1 <–> QA-Tr2 Transformation

Dear Chris,
let P be the vertices of the reference QA, M the Miquel points, J the in-/ex-centers of the Miquel triangle (see attached file): Draw the 16 lines PJ, there are 4 quad-intersections, which are the vertices of the related Q-QA.
Best regards Eckart



2016-04-13.pdf

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Message: #1671
Date: 13/4/2016 10:56:28
From: eckart_schmidt@t-online.de
Subject: QA-Tr1 <–> QA-Tr2 Transformation

Dear Chris,
excuse, I have not answered your first question:
A quadrangle has three quadrigon versions, which can be considered as quadrilaterals. The CSC-transformations of these quadrilaterals are centered in one Miquel point and swapping the other two. The images of one quadrigon vertex wrt the three CSC-transformations give the remaining vertices of the quadrangle.
Best regards Eckart

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Message: #1672
Date: 13/4/2016 12:51:38
From: bernard.keizer
Subject: QA-L10

Dear Eckart, dear Chris
I regret deeply this misunderstanding between us !
I think I always try to give back to Cesar what belongs to Cesar ...
Please red again my message 1615 and by the way my messages 1629 and 1664.
I didn't put a devaluating statement on your construction, on a contrary !
(I said the use of Pythagoras was particularly appropriate, I could have said clever ...)
I just said it was the same construction as mine with complex calculations, directly derived from Coolidge's calculations, *as corrected by yourself*.
If you consider the sum instead of the barycenter of the 4 points z^2, the orthogonal projection of this new point on the line L gives your point X.
For me, the real alternative constructions are the 2 recursive constructions mentioned in my messages 1629 of congratulations to both of you and repeated in my message 1664 !
(These constructions are due to Chris and you, I've just put a slight improvement in yours)
But of course, if you prefer to call your variant a 2nd construction and your alternative a 3rd one, I would completely agree ...
It remains that I'm fully admirative now for your new and old developments on QA-Cu1 with QA-Tr1 and QA-Tr2, the CSC coming from the Miquel triangle (not forgetting the Desmic configuration mentioned by Chris) ...
I hope the pursuit of our cooperation !
Best regards
Bernard

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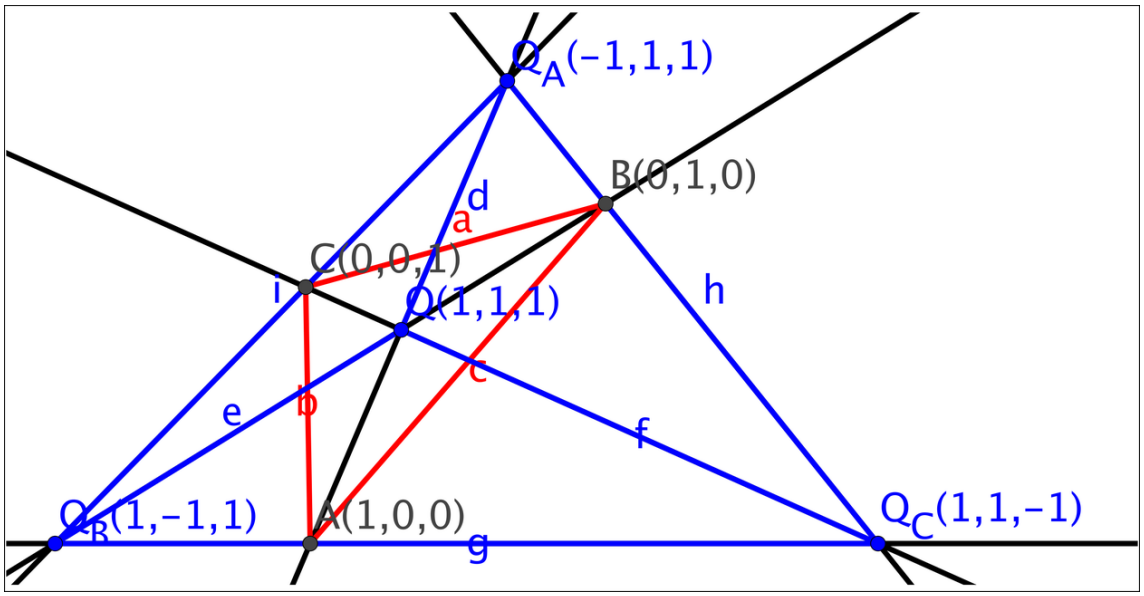
Message: #1673
Date: 13/4/2016 3:16:14
From: tsihonglau
Subject: Search Keys for Quadrangle/Quadrilateral

Dear Bernard,
>>I don't need a long answer !
>>I'm asking only
>>1) for the cartesian coordinates of one vertice of the QA
>>(I'm not familiar with the barycentric properties of Geogebra,
>>I would be pleased and grateful if you could
>>teach me the way of using these triangle commands ...)

GeoGebra's barycentrics syntax is very simple.
For example, given a reference triangle ABC and its sidelengths a,b,c, some famous centers are described as follows:
Centroid Barycenter[{A,B,C},{1,1,1}]
Incenter Barycenter[{A,B,C},{a,b,c}]
Orthocenter Barycenter[{A,B,C},
{1/(-a^2+b^2+c^2),1/(a^2-b^2+c^2),1/(a^2+b^2-c^2)}]
...
Barycenter applies to any number of vertices.
Centroid of a quadrangle ABCD is given by
Barycenter[{A,B,C,D},{1,1,1,1}]
I propose another set of search keys a=6,b=9,c=13,p=11,q=14,r=15
And if the diapleural(diagonal) triangle is
A(4*sqrt(35)/3,-13/3),B(0,6),C(0,0),
then the quadrangle vertices are Q((11*sqrt(35))/30,109/120)
= Barycenter[{A,B,C},{p,q,r}]
Q_A(-(22*sqrt(35))/27,395/54)
=Barycenter[{A,B,C},{-p,q,r}]
Q_B((11*sqrt(35))/9,-395/36)
=Barycenter[{A,B,C},{p,-q,r}]
Q_C((22*sqrt(35))/15,109/30)
=Barycenter[{A,B,C},{p,q,-r}]
The new search keys may be better than the previous ones. But who knows? We have millions of centers/axes then know which is better.
Because the search keys uses p,q,r instead of l,m,n, I have to recalculate the formulas of quadrangle/quadrilateral centers.
>>2) for a figure I could open and print : this is possible with Cabri, as Chris and Eckart regularly do, and with >>Geogebra, as I often do myself.
>>You only have to open a word-file and, when you are in Geogebra, to click on Export and choose Export to >>the press-files (in french, it is Exporter Graphique vers Press papiers) ; you go back to your word-file >>and insert the figure which should appear then on the left in the press-files ...

>>It's then possible, but not necessary, to transform your word-file in a pdf-file.
Someone posts Word files but I have no MS Word. I open them with LibreOffice but my Mac Mini shows them a mess! I propose another unitary quadrangle/quadrilateral projective coordinate system. But I know it does not have unique formulas. It may be a geometry topic. Please refer to APG #3124 for defining a unitary quadrangle/quadrilateral
projective coordinate system. This is called unitary sidelength coordinate system.
Let d=QQ_A,e=QQ_B,f=QQ_C,g=Q_BQ_C,
h=Q_CQ_A,i=Q_CQ_A. I made a pdf for demonstration. Because d,e,f,g,h,i are a solution of some equation, they are not independent.
So formulas of quadrangle/quadrilateral centers/axes are not unique.
Best regards
Tsihong Lau



unitay_sidelength_coordinate.pdf

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Message: #1674
Date: 13/4/2016 10:59:14
From: chris.vantienhoven
Subject: Some Splitter XI regarding QA-P35

Dear Eckart,
Thanks for the beautiful splitter and thanks for giving them so organised by mentioning the EQF-obect involved.
I will include most of them in EQF.
I have not finished checking them all, but there is one item that struck me.
“The 1 st Penta points of points P and its QA-Tr1-anticevians lie on a circle through QA-P35.”
At first sight I was pleased because QA-P35 is a special point that doesn’t have many special properties.
Then my analyses:
It is well known that when we have the Diagonal Triangle of a Quadrangle QA-DT and one vertice P1 of the Quadrangle we can find the other QA-vertices being the vertices of the P1-Anticevian wrt QA-DT. The consequence of your QA-P35-statement is that there is a circle where QA-P35 lies on whatever location P1 or other QA-vertices have, as long as they have QA-DT as Diagonal Triangle. So actually it is a Central Triangle Circle with a Central Triangle Circle Center with no dependency of P1(p:q:r) at all.
In Mathematica I calculated for the circle this equation:

```
   2 ( -2 a^2 + 3 b^2 + 3 c^2) x^2
   + (-13 a^2 + 2 b^2 + 2 c^2) y^z
   + 2 (3 a^2 - 2 b^2 + 3 c^2) y^2
   + ( 2 a^2 - 13 b^2 + 2 c^2) x z
   + 2 (3 a^2 + 3 b^2 - 2 c^2) z^2
   + ( 2 a^2 + 2 b^2 - 13 c^2) x y
```

In a quick search I didn’t find any ETC-points lying on this circle.
For the the circle center I found this 1 st barycentric coordinate:

```
   a^4 + 2 (b^2 - c^2)^2 - 3 a^2 (b^2 + c^2)
```

This point happens to be X(1656),
which is the INTERSECTION OF EULER LINE AND LINE X(17)X(18).
It seems to me that this is a very special result for QA-P35!
I wonder if there are more of these EQF-points with these kind of properties.
For me it shows the more that often your splitter are jewels!
Best regards,
Chris

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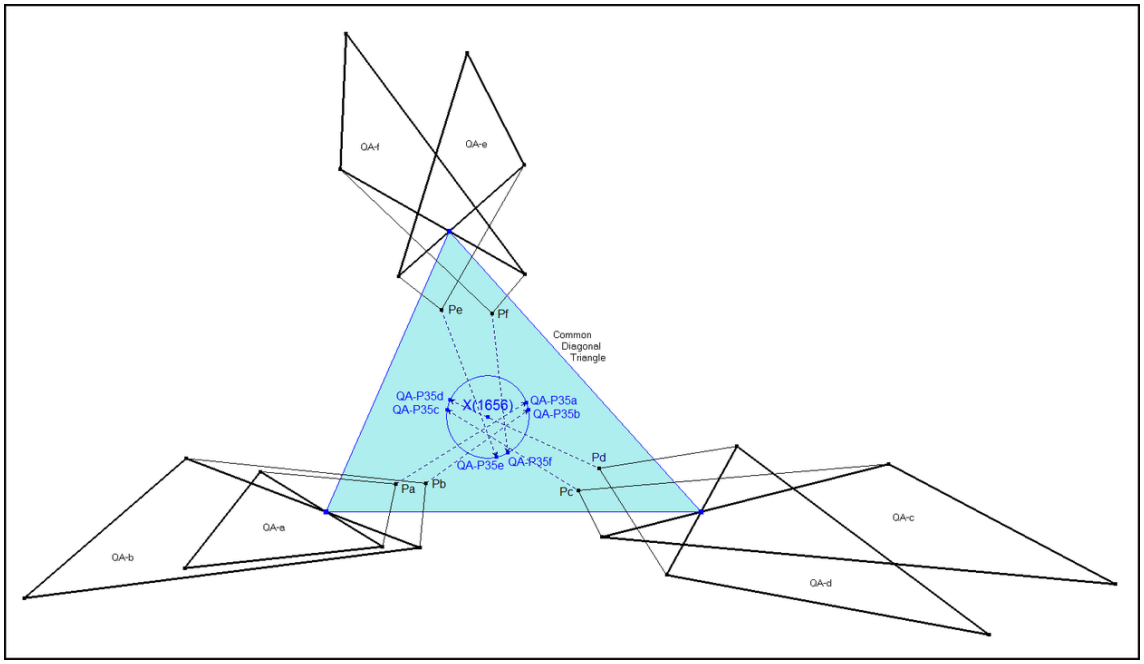
Message: #1675
Date: 14/4/2016 10:16:41
From: chris.vantienhoven
Subject: Some Splitter XI

Dear Eckart, dear friends,

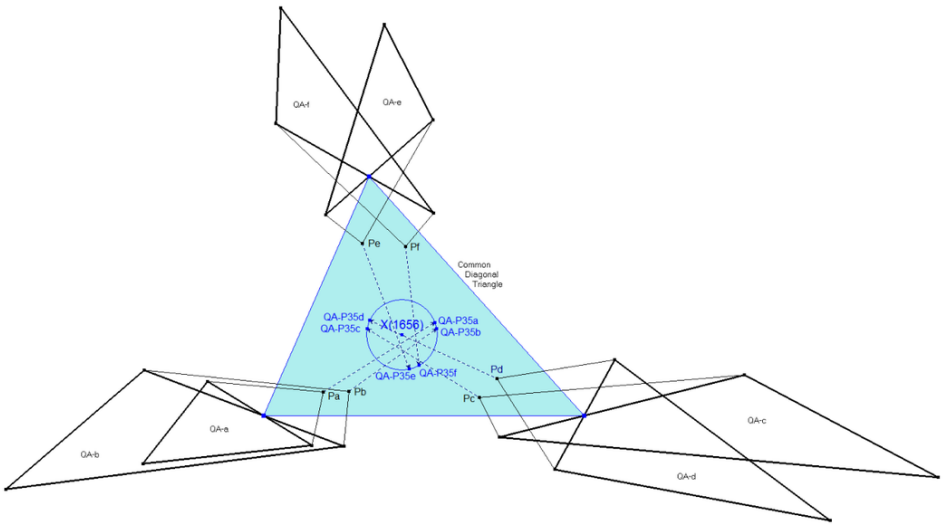
I always try to visualize beautiful properties like
happening with QA-P35 now.
See attached pictures.

Best regards,
Chris

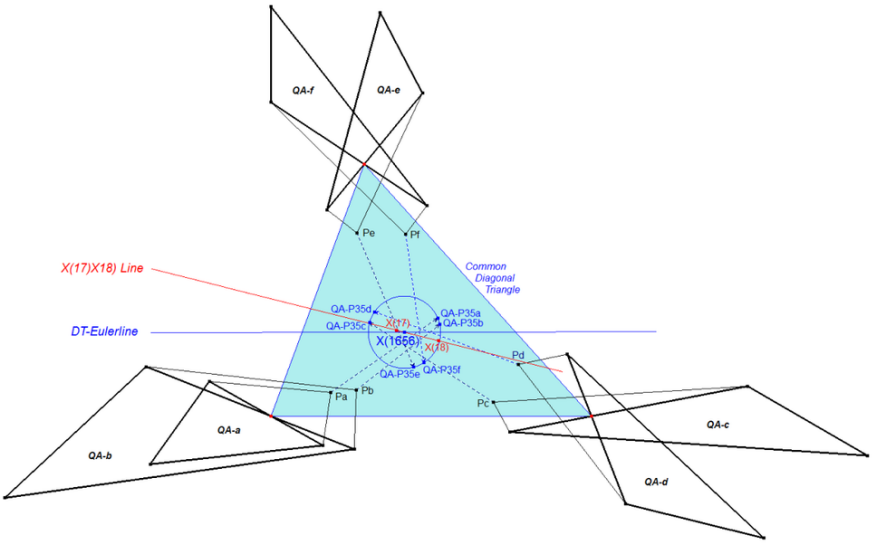


QA-P35-PentaPoint-10–X1656-circle.png

QA-P35 (1st Penta Point) lies on a fixed QA-DT-circle with center X(1656) for all Quadrangles with the same Diagonal Triangle QA-DT



X(1656) is intersection point of the Euler line and line X(17)X(18)



Chris van Tienhoven, April 2016

QA-P35-PentaPoint-10–X1656-circle.pdf

Message: #1676
Date: 14/4/2016 11:18:06
From: eckart_schmidt@t-online.de
Subject: Some Splitter XI

Dear Chris,
you are right wrt the splitter
“The 1st Penta points of points P and its QA-Tr1-anticevians lie on a circle through QA-P35.”
The circle is a special triangle circle of the diagonal triangle,
centered on QA-L5, dividing QA-P11.QA-P12 with ratio 2:3
... with radius 1/5 of the QA-Tr1-radius.
It seems, that there are only the less interesting further cases for QA-P10 - QA-P13, QA-P2 and QA-P29, QA-P36.
Best regards Eckart

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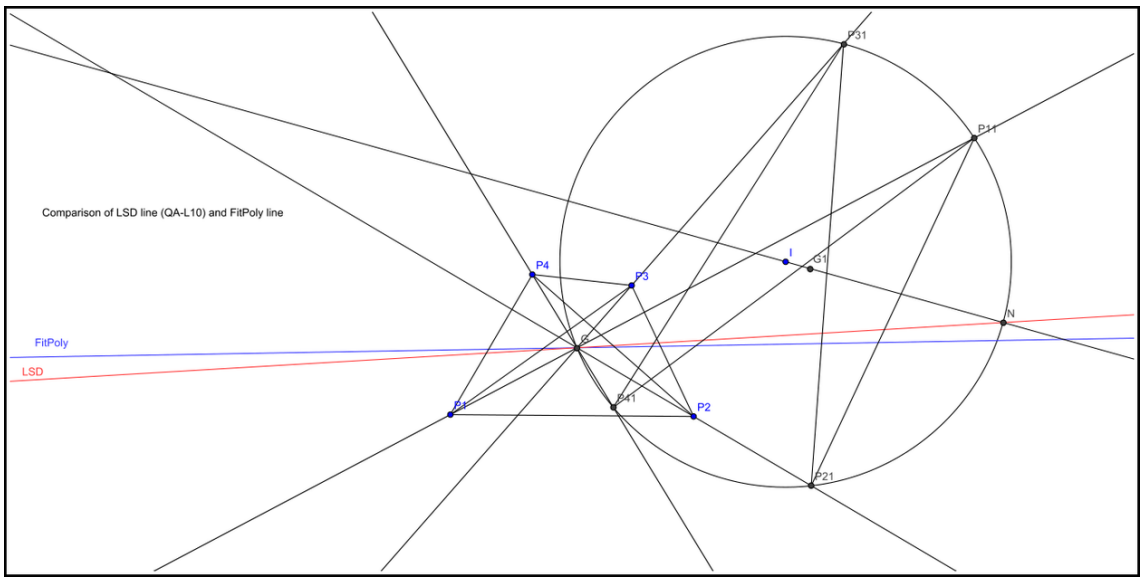
Message: #1677
Date: 15/4/2016 2:08:29
From: seiichikiri
Subject: LSD line and FitPoly line

Dear Chris,

I compared LSD line(QA-L10) and FitPoly line of Geogebra.
I think that they are about in the same direction in the
attached pdf, but do not coincide with each other.
My conclusion is that there is another line of optimality.
Geogebra command: FitPoly[{P1,P2,P3,P4},1]

Best regards, Seiichi.



FitPoly.pdf

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Message: #1678
Date: 15/4/2016 9:30:09
From: chris.vantienhoven
Subject: LSD line and FitPoly line

Dear Seiichi,

When I look to your drawing I think you used the Coolidge-method which (like we discussed) doesn't produce right results.
I think when you use Benedetto Scimemi /Bernard Keizer's method (message #1590) or Eckart's method (message #1611) the results will be right. These results are Cabri-checked.
When you still find a discrepancy with Geogebra please let me know.

Best regards,
Chris

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Message: #1679
Date: 15/4/2016 1:35:24
From: eckart_schmidt@t-online.de
Subject: Qa-Reproduction with QA-Tr1, QA-Tr2

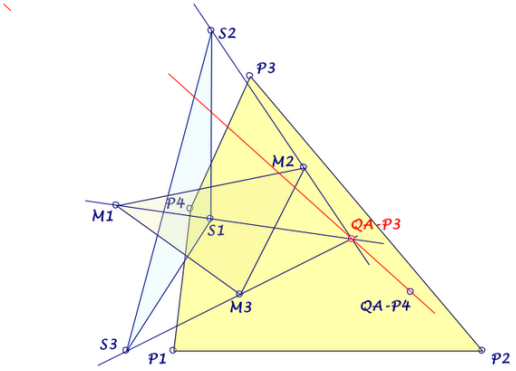
Dear Benedetto, dear Chris,
Benedetto researched in his work "Central Points of the Complete Quadrangle" (EQF-Ref.[36]) the question, how to reconstruct a quadrangle with four points. Attached the question is modified in the sense, how to reconstruct a quadrangle with the diagonal triangle and the Miquel triangle.
Best regards Eckart

EQF-Note 2016-04-15

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Reproduction with QA-Tr1 and QA-Tr2

Scimemi researched in his work "Central Points of the Complete Quadrangle" (EQF-Ref.[36]) the question, how to reconstruct a quadrangle with four points. Here the question is modified in the sense, how to reconstruct a quadrangle with the diagonal triangle and the Miquel triangle.

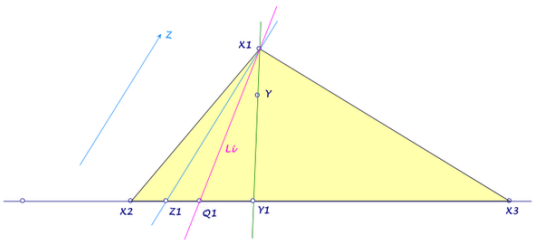


Preliminary remarks (see *EQF*):

- The diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2* are perspective wrt *QA-P3*.
- The isogonal conjugate of *QA-P3* wrt the Miquel triangle is *QA-P4*.
- The involutary conjugate *QA-Tf2* of *QA-P3* wrt the diagonal triangle is the point at infinity of *QA-P3.QA-P4*.
- The fixed points of *QA-Tf2* are the vertices of the quadrangle.

Construction

The idea of the construction is, to find the fixed points of an isoconjugation wrt a reference triangle, knowing a pair of conjugated points. Here:
… reference triangle *QA-Tr1* = $X_1X_2X_3$,
… conjugated points: *QA-P3* = *Y* and point at infinity of *QA-P3.QA-P4* = *Z*.



Let Y_i be the intersection of X_iY and X_jX_k,
…let Z_i be the intersection of X_iZ and X_jX_k,
… let Q_i be a double point of the X_jX_k-line involution (see *QA-Tf1*) wrt X_j, X_k and Y_i, Z_i.
… The lines $L_i = X_iQ_i$ intersect in a point P,
… which gives with its anticevians wrt $X_1X_2X_3$ the fixed points of the isoconjugation.

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2016-04-15.pdf

Message: #1680
Date: 15/4/2016 1:36:38
From: seiichikiri
Subject: LSD line and FitPoly line

```
Dear Chris,
I drew a rectangle: the ratio of the lengths of its horizontal
and vertical segments = 1 : 3.
LSD line (QA-L10) is vertical.
FitPoly line is horizontal.
Sorry for posting an improper message.
Best regards, Seiichi.
```

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Message: #1681
Date: 15/4/2016 8:54:17
From: eckart_schmidt@t-online.de
Subject: QA-L10

```
Dear Bernard,
there is no reason, not to continue our cooperation. But at the
moment, I am engaged in QA-geometry.
Best regards Eckart
```

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Message: #1682
Date: 16/4/2016 11:08:15
From: chris.vantienhoven
Subject: New items in EQF

Dear friends,

I placed these new items in EQF:
· QL-L10 QL-LSD Line
· QL-2Co1 Pair of Inscribed Parabolas
· QL-Tf3 CSCe Transformation
· Ref-   addition new references
Especially the items QL-2Co1 and QL-Tf3 were long on my list.
Thanks to the great help of Angel Montesdeoca I was able to
interpret and indicate the very special properties of QL-Tf3.
When there are additions or remarks please let me know.

Best regards,
Chris

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Message: #1683
Date: 17/4/2016 2:57:02
From: eckart_schmidt@t-online.de
Subject: QA-Sigma-Quartic

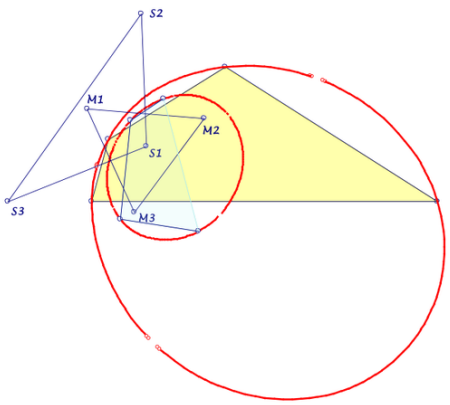
Dear Benedetto, dear Chris,
in message1471 I described Stärk´s parameter Sigma for a
quadrangle and mentioned a quartic for quadrangles with the same
diagonal triangle and the same Sigma. Attached you find a
construction of this quartic.
Best regards Eckart

EQF-Note 2016-04-17

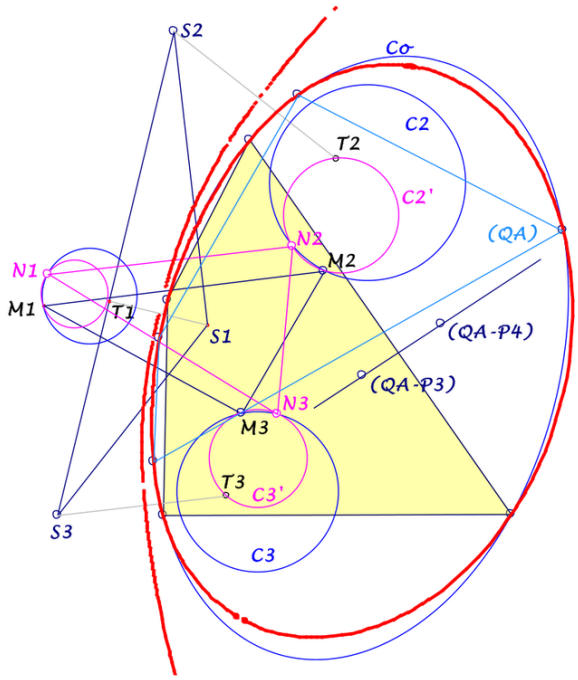
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Sigma-Quartic

QA-P4 is the homothetic center of the 1st circumcenter quadrangle and the 1st isogonal conjugate quadrangle. The corresponding ratio is Stärk´s Sigma of a quadrangle in:
Roland Stärk: Eine merkwürdige Zahl des Vierecks. PM: Praxis der Mathematik in der Schule, 46 (2004) 1, S. 26-31.
This parameter also appears as "lambda" in the paper of Benedetto Scimemi (EQF-ref.[36], 3.2.3 to 3.2.6).
Stärk's Sigma of a quadrangle has a lot of interesting properties, see also EQF-message 1471. In this message a quartic is mentioned for the vertices of quadrangles with the same diagonal triangle and the same Sigma. Here for this quartic a construction is described.



For a reference quadrangle *QA*
… let $S_1S_2S_3$ be the diagonal triangle *QA-Tr1*,
… $M_1M_2M_3$ the Miquel triangle *QA-Tr2* and
… $T_1T_2T_3$ the triangle of the pedal points of S_i wrt S_jS_k.
… Let C_i be the circles of Apollonius for M_i wrt *SiTi*.
… Let *Co* be a circumconic of *QA* and *P* points on *Co*,
… *P* and its *QA-Tr1*-anticevians give a *P-QA* on *Co* (not in the figure).
… The Miquel points of these *P-QA* give circles C_i´
… through M_i and T_i
… with 2nd intersections N_i with C_i.



Wrt the searched quadrangle brackets are used: The triangle $N_1N_2N_3$ will be the Miquel triangle *(QA-Tr2)* of the searched quadrangle *(QA)*. For this quadrangle we know its diagonal triangle *QA-Tr1* and its Miquel triangle *(QA-Tr2)*. The perspector of these triangles is *(QA-P3)*. The isogonal conjugate of *(QA-P3)* wrt *(QA-Tr2)* is *(QA-P4)*. The involutary conjugate *(QA-Tf2)* of *(QA-P3)* is the point at infinity of *(QA-P3).(QA-P4)*. The fixed points of the isoconjugation *(QA-Tf2)* are the vertices of *(QA)*. The construction of a quadrangle, using the diagonal triangle and the Miquel triangle and knowing a pair of points wrt the involutary conjugate, is described in *QFG*-message 1679.
Changing the *QA*-circumconic *Co*, we will get the searched quartic in the vertices of *(QA)*.

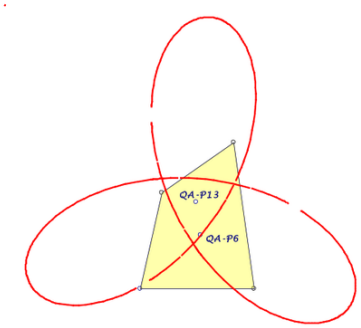
Some properties:

- The quartic is *QA*-circumscribed.
- The quartic contains with a point also its *QA-Tr1*-anticevians.
- Let a *QA*-line P_iP_j intersect the quartic also in Q_1,Q_2, then the line involution wrt these pairs of points has double points in a *QA-Tr1*-vertex and in the intersection with the opposite *QA-Tr1*-sideline.
- A line S_iS_j cuts each part of the quartic harmonically.
- A line S_iT_i cuts each part of the quartic harmonically.

2016-04-17.pdf

It is interesting, to study the loci of *EQF*-points for quadrangles *(QA)* on the quartic. Some examples beside the evident constellations wrt the common diagonal triangle:

Wrt ***QA-P4*** : Circle (through *QA-P4*) round *QA-P12*.
… ***QA-P6*** : See final figure.
… ***QA-P8*** : Conic (through *QA-P8*), centered on *QA-L5* …
… ***QA-P23***: Conic (through *QA-P23*), centered in *QA-P11* …
… ***QA-P28***: Circle (through *QA-P28*) round *QA-P13*.
… ***QA-P35***: Circle (through *QA-P35*), centered in a point, dividing *QA-P11.QA-P12* with ratio 2:3 and radius 1/5 of the *QA-Tr1*-circumcircle. This circle is the locus of all 1st Penta Points for an arbitrary point and its *QA-Tr1*-anticevians.

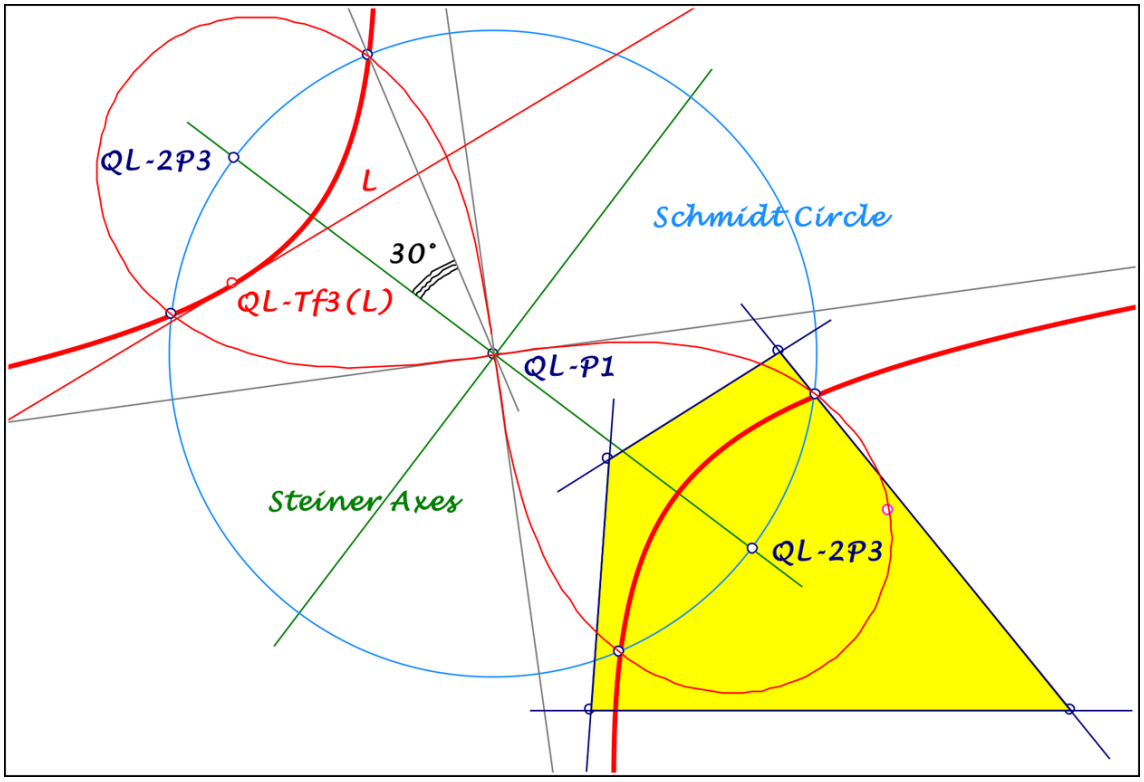


Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-04-17.pdf

Message: #1684
Date: 18/4/2016 8:46:39
From: eckart_schmidt@t-online.de
Subject: QL-Tf3

Dear Bernard, dear Chris,
I appreciate that CSCe appears as QL-Tf3 in EQF. There are several messages with further properties. But is the following property already mentioned (see attached file):
The envelope for lines, which bear their QL-Tf3-image,
... is an orthogonal hyperbola
... with the Steiner axes as axes,
... foci in QL-2P3
... and special points X on the Schmidt circle
... ... with
The inverse of the hyperbola wrt the Schmidt circle
... or the CSC-image of the hyperbola
... give a lemniscate with foci QL-2P3.
Best regards Eckart



2016-04-18.docx

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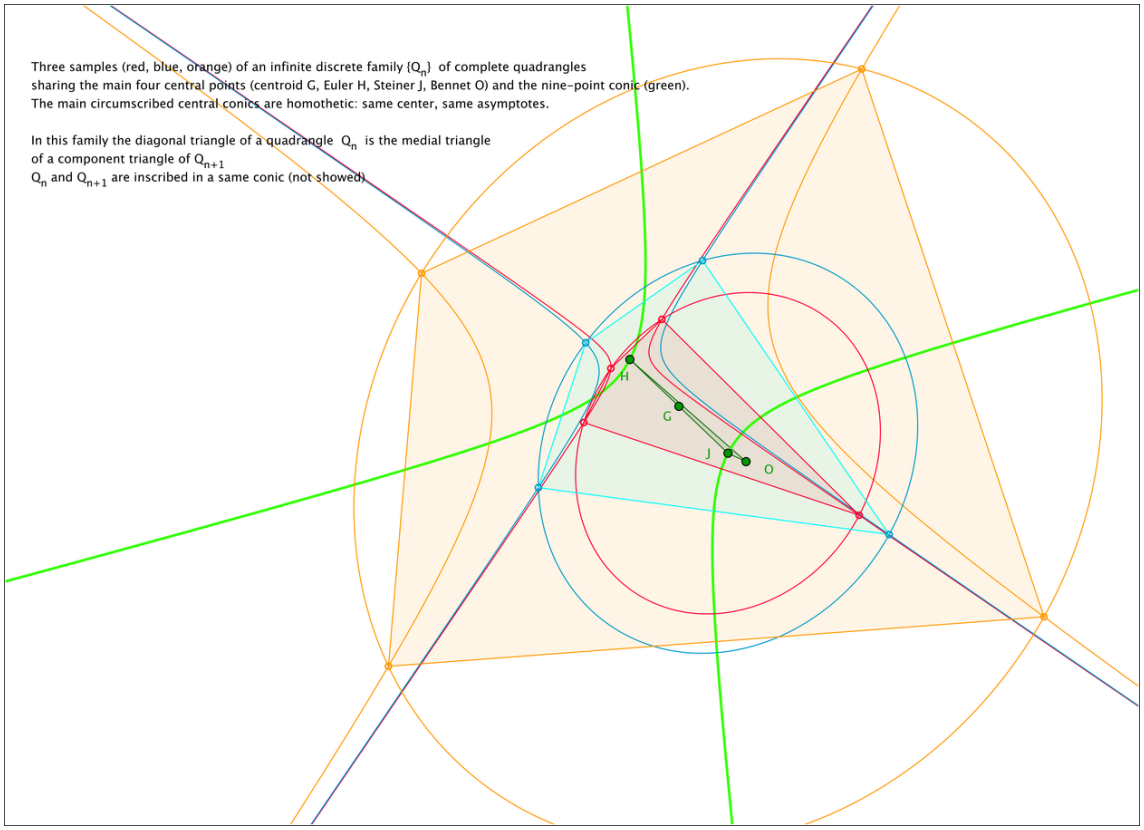
Message: #1685
Date: 18/4/2016 5:14:38
From: Benedetto Scimemi
Subject: A family of quadrangles sharing much

Dear Eckart, I have greatly appreciated your pages on QA-Sigma-Quartic (I would have difficulties in dealing with cubics and quartics).
Just by analogy, let me attach here the illustration (as recovered from my old archiv, thus not using QA-symbols) of a family of quadrangles all having the same Sigma (my Lambda) and strictly related diagonal triangles. Best regards Benedetto



4ng sharing JOHG etc.pdf

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Message: #1686
Date: 18/4/2016 7:37:57
From: tsihonglau
Subject: Circumconics through Circular Points

Dear all,
The circular(isotropic) points at infinity are very important and every center/axis of quadrangle/quadrilateral can be derived from them.(I think) That is why I study the circular coordinate system. I calculated some circular coordinates/coefficients of centers/axes of quadrangle/quadrilateral recently. I found two centers QA-P4 and QA-P42 can be constructed easily from the circumconics through the circular points.
Suppose c_P and c_U are the circumconics of a quadrangle and the circular points P and U respectively.
(Despite of concyclic quadrangles, there are no circumconics through both circular points.)
The tangent lines of c_P and c_U at P and U respectively intersect at QA-P4.
The polars of P and U with respect to c_U and c_P respectively intersect at QA-P42. Suppose P=p:q:r and U=u:v:w. Then the first circular coordinates of QA-P4 and QA-P42 are respectively
(qw-rv)(p^2u^2-qvrw)-(qv-rw)(p^2vw-qru^2)
qw(p+q)(p-q)(w+u)(w-u)-rv(r+p)(r-p)(u+v)(u-v)
I think these two circumconics deserve study.
Because of the "imaginary" circular points,
I did not provide images.
Best regards,
Tsihong Lau

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Message: #1687
Date: 19/4/2016 4:28:00
From: eckart_schmidt@t-online.de
Subject: QL-Tf3

```
Dear Chris,

wrt the properties of QL-Tf3: Most of the properties are
examples of the following generalizations:
... If lines have a common point,
    their CSCe-images are collinear (example 6, 7).
... If two lines intersect in P,
    their CSCe-images and the CSC-images of points on a circle
    round P through QL-P1 are collinear (example 8).
... ... special: If two lines are parallel, their CSCe-images
        and QL-P1 are collinear (example 3, 4).
... CSC-images of two points and the CSCe-image of lines through
    the circumcenter of the two points and QL-P1 are collinear
    (example 1, 2, 5).

Best regards Eckart

PS: CSCe(QL-P3.QL-P4) is the intersection of QL-L2 and
CSC(QL-Ci6).
```

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Message: #1688
Date: 20/4/2016 11:01:57
From: chris.vantienhoven
Subject: Circumconics through Circular Points

Dear Tsihong Lau,

Interesting that there are two circumscribed QA-conics each through one circular point at infinity.
I also calculated just like you the polar at the circular point of infinity wrt both conics and found them crossing again at QA-P4 and not at QA-P42 like you did. I hope I didn't make errors. Maybe you can recalculate.
I tried to make a picture of these conics but it looks like their image is imaginary except for their common points P1,P2,P3,P4.
Another example of circular points at infinity of double curves are
QG-2Cu1: Perspective Squares Double Cubic.
QG-2Cu1a contains one circular point at infinity and QG-2Cu1b contains the other one.
Although these curves also pass through P1,P2,P3,P4 and per curve through one circular point at infinity their images are real.

Best regards,
Chris

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Message: #1689
Date: 21/4/2016 8:50:58
From: chris.vantienhoven
Subject: QL-Tf3

```
Dear Eckart and Bernard,

Thanks Eckart for the generalized CSC(e) properties!
I also found some simple properties of CSC I didn't know.
1. Let Q1 be a random point.
   Let Q2 be the midpoint of QL-P1 and Q1.
   Now CSC[Q1) will be the midpoint of QL-P1 and CSC(Q2).
2. Let Q1 be a random point.
   Let Q2 be a point collinear with QL-P1 and Q1.
   (consequently QL-P1, CSC(Q1) and CSC(Q2) will be collinear
   too)
   Now Q1.CSC[Q2) will be parallel to Q2.CSC(Q1).
Did you know these ones?

Best regards,
Chris
```

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Message: #1690
Date: 21/4/2016 10:24:36
From: bernard.keizer
Subject: QL-Tf3

```
Dear Chris, dear Eckart
Both examples are a direct application of the Thales theorem to
the definition of the Clawson-Schmidt transformation (named psi
by Bernard Gibert or mu by Benedetto Scimemi) ...
These and other properties could be found
1) in the article of Bernard Gibert Orthocorrespondance and
Orthopivotal Cubics Forum Geometricorum 2003
2) in the article of Benedetto Scimemi Simple relations
regarding the Steiner Inellipse of a Triangle Forum
Geometricorum 2010
3) in my last article Le Triangle, le Quadrilatère Complet, la
Stelloïde Cubique et la transformation psi 2015 at
http://bernardkeizer.blogspot.fr
Best regards
Bernard
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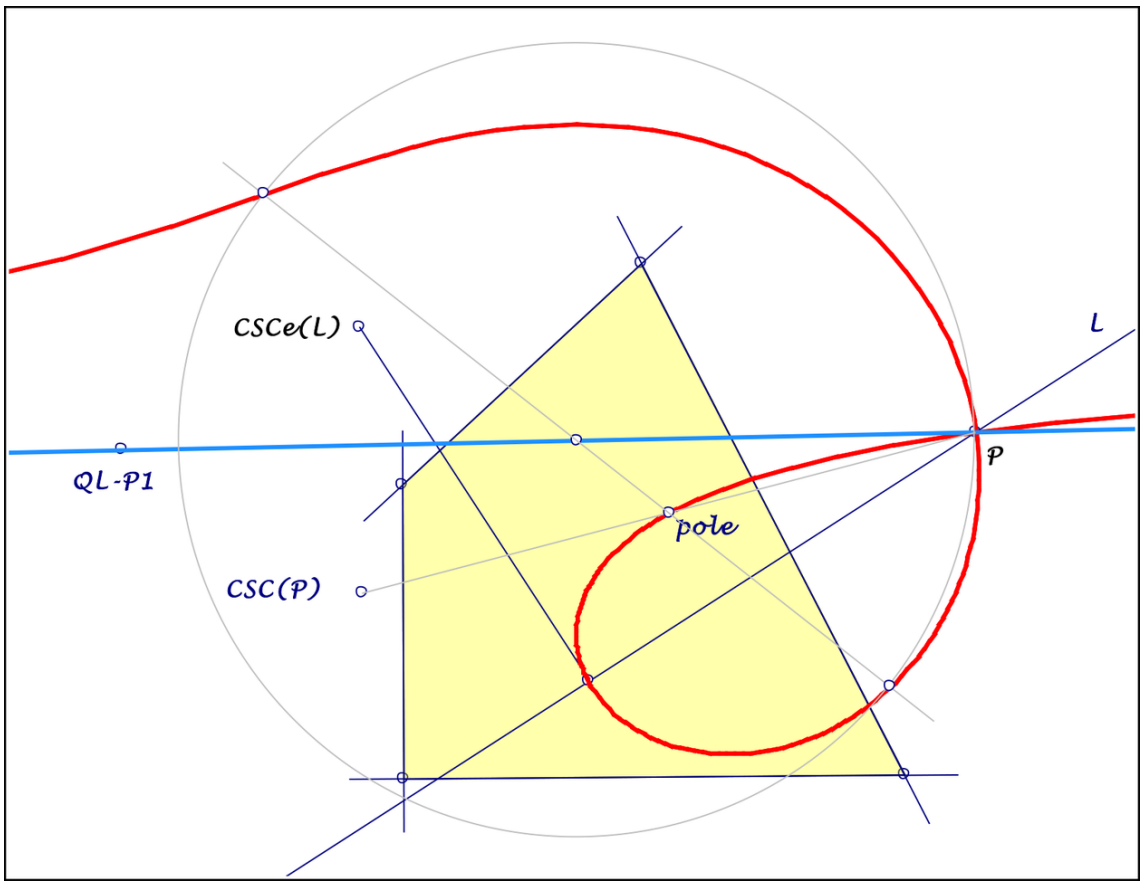
Message: #1691
Date: 21/4/2016 11:31:52
From: eckart_schmidt@t-online.de
Subject: QL-Tf3

Dear Chris, dear Bernard,
Chris' CSC-properties are evident, for
... CSC(Q1).CSC(Q2) is the reflection of Q1.Q2 in the 1st Steiner axis
... and QL-P1. Qi * QL-P1.CSC(Qi) is the square of the radius of the Schmidt circle.
Some further remarks wrt QL-Tf3 = CSCe:
(1) I think, that the following interpretation of CSCe should be mentioned:
... for L not bearing QL-P1: CSCe(L) is the CSC-image of the reflection of QL-P1 in L.
... for L bearing QL-P1: CSCe(L) is the infinity point of a perpendicular to the reflection of L in the 1st Steiner axis.
(2) Further examples:
... CSCe of the QG-P16-line is QL-P6.
... CSCe of the QG-P18-line is QL-P11.
... The locus for the CSCe-images of QL-Co1-tangents is QL-Ci3.
... CSCe(QL-L1) lies on a QL-L1-perpendicular through the intersection of QL-L1 and QL-L6.
(3) QL-Tf3 for quadrangles, considering the three lines of a point wrt QL-Tf3R:
... For points on the circumcircle of the Miquel triangle the three QL-Tf3R-lines are parallel
... ... to the Simson line of the point wrt QA-Tr2.
... Each parallel envelopes a parabola,
... ... with a Miquel point as focus
... ... and the opposite QA-Tr2-sideline as directrix.
... These parabolas are the loci of the QL-Tf3-points for tangents at the QA-Tr2-circumcircle.
(4) The locus for the pedal points of CSCe(L) on L for a line pencil of a point P give a strophoid (see attached file)
... of the line P.QL-P1
... with fixed point P
... and pole: midpoint of P and CSC(P).
(5) Further properties perhaps in my messages 453, 454, 456, 465, 470, 471, 480, 489, 492.
Best regards Eckart



2016-04-21.pdf

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Message: #1692
Date: 21/4/2016 2:16:34
From: chris.vantienhoven
Subject: QL-Tf3

Dear Eckart and Bernard,
[ES]
Chris' CSC-properties are evident
Eckart, when I considered it to be evident I wouldn't have mentioned it. So I must have overlooked something.
Of course I considered the defining CSC-rules too.
Still I think it is not evident especially the case that lines Q1.CSC[Q2) and Q2.CSC(Q1) are parallel.
Can you explain to me in max 2 simple steps why this is the case?
Of course I have to agree that at the end everything in mathematics is obvious (in 2+ steps).
I will study your further remarks later. Thanks!
Bernard, thank you for your explanation!
There is a world behind us!
Best regards,
Chris

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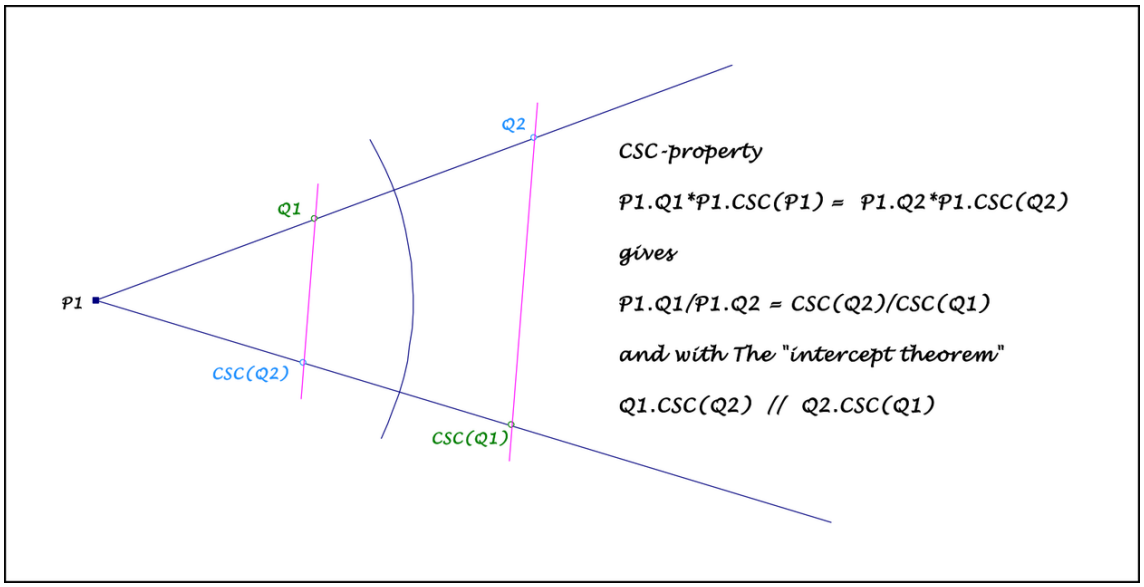
Message: #1693
Date: 21/4/2016 4:07:30
From: eckart_schmidt@t-online.de
Subject: QL-Tf3

Dear Chris,
please excuse, if I was too hard in my formulation, but for me it was a direct consequence of the "intercept theorem" (in German "Strahlensatz"), see attached CABRI-file.
I'm sorry! Eckart



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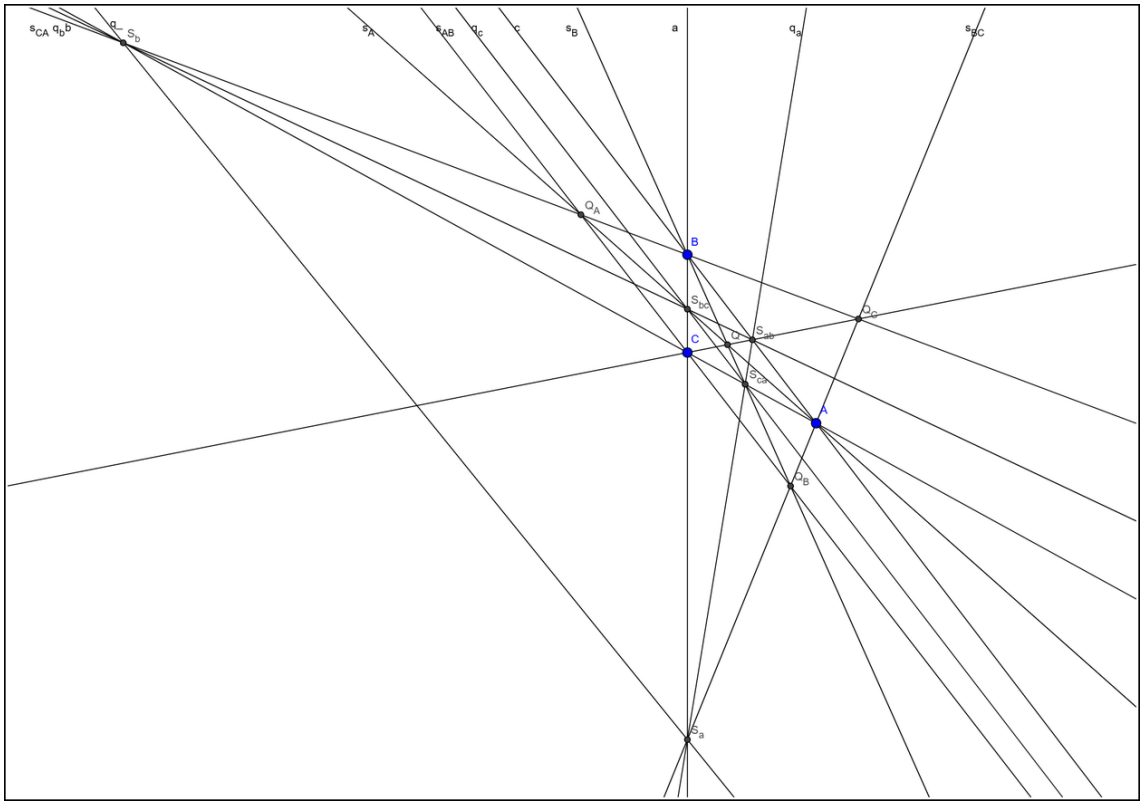
Message: #1694
Date: 21/4/2016 5:24:14
From: tsihonglau
Subject: Circumconics through Circular Points

Dear Chris,

>>I also calculated just like you the polar at the circular point of infinity wrt both conics and found them crossing again at QA-P4 and not at QA-P42 like you did. I hope I didn't make errors. Maybe you can recalculate.
My statement was "The polars of P and U with respect to c_U and c_P respectively intersect at QA-P42." You should calculate "The polars of P and U with respect to c_P and c_U respectively intersect at QA-P42."
You interchanged!
The polars of P and U with respect to c_P and c_U are the tangent lines at P and U respectively. So they intersect at QA-P4 again.
Now I give some new figures.
Point conics (made of tangent points) can be treated as line conics(made of tangent lines).
The circumconics c_P and c_U can be treated as line conics. Just like two point conics pass through four common points, two line conics are tangent at four tangent lines.
The first coefficients of four common tangent lines are:
+-sqrt((q^2-r^2)(v^2-w^2)) - circular
+-p*sqrt(b^2*r^2+(-c^2-b^2+a^2)*q*r+c^2*q^2)*sqrt(b^2*r^2+(c^2+b^2-a^2)*q*r+c^2*q^2) - DT barycentric
I do not know if they are all "real" lines!
c_P and c_U have their own centers - poles of the line at infinity.
The first coefficients of the line through the two centers are:
(q^2-r^2)(v^2-w^2)/(qw-rv) - circular
p^2*(b^2*r^2+(-c^2-b^2+a^2)*q*r+c^2*q^2)(b^2*r^2+(c^2+b^2-a^2)*q*r+c^2*q^2) - DT barycentric
The four lines above are sqrtmul of the line at infinity and the above line. If P(p:q:r) and U(u:v:w) are two points(or lines), the sqrtmul of them are (+-sqrt(pu):+-sqrt(qv):+-sqrt(rw)).
They become a quadrangle (or quadrilateral).
If P and U are the circular points, the sqrtmul of them are incenter/excenters of the reference triangle.
I think the figures above should have many nice properties.
I made a GeoGebra file with the four lines (q_,q_a,q_b,q_c) and the search keys (a=6,b=9,c=13,p=11,q=14,r=15) but without the last line since I do not know any quadrangle quadrilateral points lie on it.

Best regards,
Tsihong Lau



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Message: #1695
Date: 21/4/2016 5:43:20
From: bernard.keizer
Subject: QL-Tf3

Dear Chris, dear Eckart
I don't know the intercept theorem, but it seems to me that the egality P1Q1*P1CSC(Q1) = P1Q2*P1CSC(Q2) is equivalent to the following P1Q1/P1Q2 = P1CSC(Q2)/P1CSC(Q1); the Thales theorem states then that Q1CSC(Q2) and Q2CSC(Q1) are parallel.
I hope my 3 references will be useful, as the QL-Tf1 or Clawson-Schmidt transformation is the same as the psi transformation in the triangle with fixed points the foci of the Steiner inellipse and the transformation which associates to any point the center of the polar conic wrt the cubic stelloïd of the triangle. (The triangle is the main pivot triangle of the Mac Cay cubic and has QL-P1 as centroïd G2 and QL-2P2a and b as isodynamic points S15 and S'16).
On page 2 of my articel is an abstract in english as well as eine Zusammenfassung in german.
Best regards
Bernard

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Message: #1696
Date: 23/4/2016 12:31:37
From: seiichikiri
Subject: Cayley Bacharach formulas (arXiv 1405.6438v2)

Dear friends,
For information only:
According to the above paper, we can determine the 9 th point of Cayley Bacharach theorem.
See http://arxiv.org/pdf/1405.6438v2.pdf
Problems: messages #732, #733, #736 and #737.
Best regards, Seiichi.

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Message: #1697
Date: 25/4/2016 9:27:09
From: eckart_schmidt@t-online.de
Subject: QL-Tf3

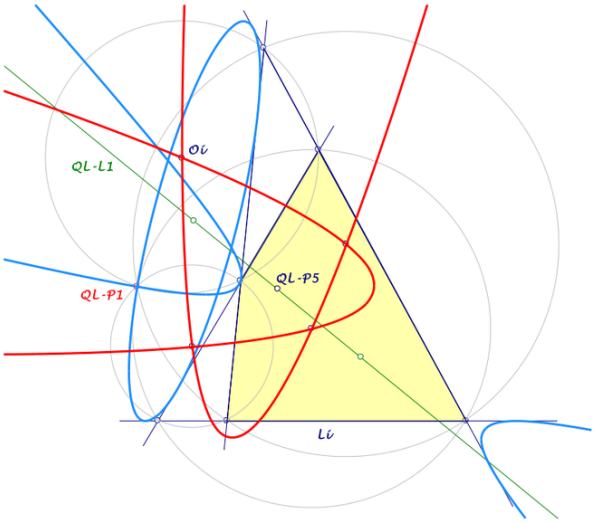
Dear Bernard, dear Chris,
before holiday a last message wrt QL-Tf3 for QL-inscribed conics (see attached file).
Best regards Eckart
PS. for Bernard: LEO gives the following translation for "Strahlensatz": "théorême d´intersection".

EQF-Note 2015-04-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Tf3 for inscribed conics

QL-Tf3, shortened CSCe, of tangents to QL-inscribed conics gives conics through the circumcenters O_i of the QL-triangle components. QL-inscribed conics, centered symmetrically to QL-P5, lead in a special case to two conics, which give with the CSCe-images of their tangents the circumscribed parabolas of the Oi-quadrangle.



CSCe is a *QL*-transformation, which maps a line to the *CSC*-image of the reflection of *QL-P1* in the line.

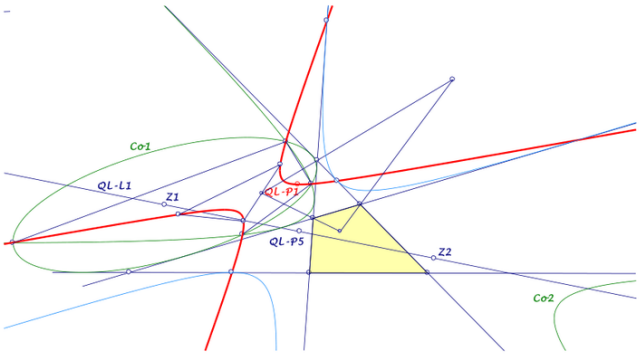
- ***CSCe* of a *QL*-line is the circumcenter for the triangle of the remaining *QL*-lines.**

- ***CSCe* of tangents to a *QL*-inscribed conic gives a conic through the circumcenters O_i of the *QL*-triangles:**
  ... *QL-Co1* gives *QL-Ci3* (circumcircle of O_i).
  … *QL*-inscribed conic with center *QL-P5* gives the orthogonal hyperbola of O_i.

Now we consider pairs of *QL*-inscribed conics Co_1 and Co_2, centered symmetrically to *QL-P5*.

- **Co_1 and Co_2 intersect on a conic**
  … bearing *QL-P1* and its *QL-Tr1*-anticevians,
  … bearing the contact points of the inscribed conic centered in *QL-P5*,
  … with a tangent in *QL-P1* perpendicular *QL-P3.P4.P5.P6*.

- **The intersection quadrangle of Co_1 and Co_2 has as diagonal triangle *QL-Tr1*.**



Now we consider the special case, that the distance of the conic centers to *QL-P5* is the *QL-Ci3*-radius (see first figure).

- **Two *QL*-inscribed conics, centered symmetrically to *QL-P5* in a distance of the *QL-Ci3*-radius**
  … intersect in *QL-P1*
  … orthogonal
  … with the Steiner axes as tangents.

- **The *CSCe*-images of these two conics give the circumscribed parabolas for the quadrangle of the circumcenters O_i of the *QL*-triangles.**

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2016-04-25.pdf

Message: #1698
Date: 27/4/2016 1:17:55
From: seiichikiri
Subject: An involution in pentangle

Dear friends,
Let P1, P2, P3, P4, P5, P6, P7 be 7 points and P8 be an arbitrary point. Cayley-Bacharach theorem determines P9. This transformation is an involution.
(1) Let P1P2P3P4 be a quadrangle and P5P6P7 its diagonal triangle. Then an arbitrary point P8 determines P9. This is an involution (QA-Tf2). In this case we can construct P9 with conic concurrency or other methods.
(2) In (1), if we use the vertices of Miquel triangle instead of those of its diagonal triangle, we obtain another involution. We can construct P9 with cubic concurrency.
(3) Let P1P2P3P4P5 be a pentangle and P6P7 a line. Then an arbitrary point P8 determines P9. This is an involution. We can construct P9 with cubic concurrency.
P6 and P7 might be the foci of the conic through P1P2P3P4P5.
It seems that if P8 is on P6P7, then P9 is on the conic through P1P2P3P4P5.
P.S. In Cayley-Bacharach Formulas, p12, arXiv1405.6438v2, “The corresponding Geiser involution is the map G that takes P8 to the Cayley-Bacharach point P9.”
Best regards, Seiichi.

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Message: #1699
Date: 27/4/2016 9:55:11
From: chris.vantienhoven
Subject: An involution in pentangle

Dear Seiichi,

I share your interest for the Caley-Bacharach Theorem.
Thanks for the reference to
http://arxiv.org/pdf/1405.6438v2.pdf.
I implemented the method (2) of the writers of this paper in Mathematica:
P9 = CxDyDz·P1 + DxCyDz·P2 + DxDyCz·P3 (2)
Then I calculated the Caley-Bacharach point belonging to this set of 8 points:
   (1: 0: 0), (0: 1: 0), (0: 0: 1),
   (p: q: r), (-p: q: r), (p: -q: r), (p: q: -r),
   (x: y: z).

The resulting point happened to be (p^2 y z, q^2 x z, r^2 x y), which is the Involutary Conjugate of (x: y: z) in the Quadrangle with:
* Diagonal Triangle vertices (1: 0: 0), (0: 1: 0), (0: 0: 1) and
* Quadrangle vertices (p: q: r), (-p: q: r), (p: -q: r), (p: q: -r),
which actually represents the QA-DT-coordinate system.
So this confirms your remark “Let P1, P2, P3, P4, P5, P6, P7 be 7 points and P8 be an arbitrary point. Cayley-Bacharach theorem determines P9. This transformation is an involution.”
And this explains why every QA-DT-Px Cubic passes though QA-Tf2(Px).
See:
   QA-Cu-1,
   QA-Cu1,
   QA-Cu2,
   etc.

Best regards,
Chris

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Message: #1700
Date: 27/4/2016 12:55:16
From: seiichikiri
Subject: An involution in pentangle

Dear Chris,
Thank you very much for your confirmation.
I am very pleased that EQF is connected with an item of algebraic geometry, even though it is very classical. At the same time I regret that when I was young, I did not learn anything about classical or modern one.
Best regards, Seiichi.

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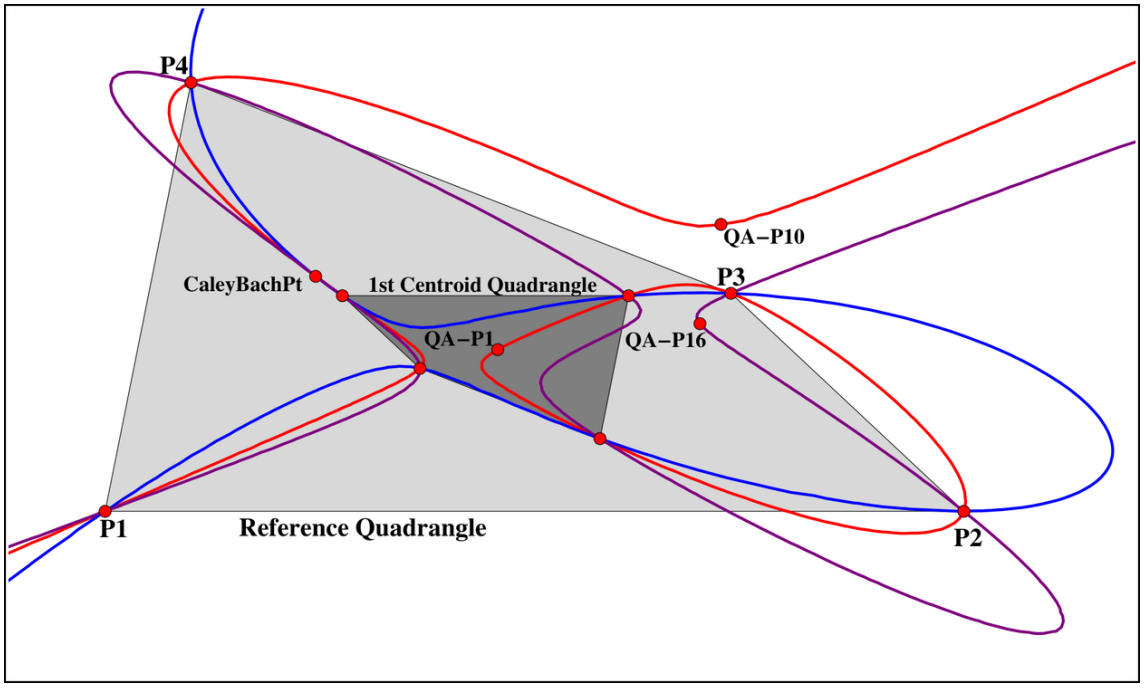
Message: #1701
Date: 27/4/2016 7:48:11
From: chris.vantienhoven
Subject: An involution in pentangle

Dear Seiichi,
The Caley Bacharach point of these 8 points:
· the 4 vertices of the QA-Reference Quadrangle
· the centroids of the 4 QA-Component Triangles
produces a Caley Bacharach point with 1 st CT-coordinate:
(q + r) (2 p + q + r) / (p^3 + p^2 q + p^2 r + p q^2 + p r^2 + 2 q^2 r + 2 q r^2 + 3 p q r)
All calculated with the method of the paper you referred to.
This point lies on these lines:
· line QA-P5.QA-P27
· line through QA-P10 parallel to QA-P1.QA-P16
· line through QA-P43 and QA-Tf1(QA-P1.QA-P16) (=Involution Center)
I made a picture in Mathematica with 3 cubics passing through the given 8 points and with common point the newly found point.
See attachment.
Best regards,
Chris



Caley-Bacharach-Solution-01.pdf

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Message: #1702
Date: 28/4/2016 2:00:44
From: seiichikiri
Subject: An involution in pentangle

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Dear Chris,
A beautiful result !!!  I am very pleased and satisfied.
Best regards, Seiichi.
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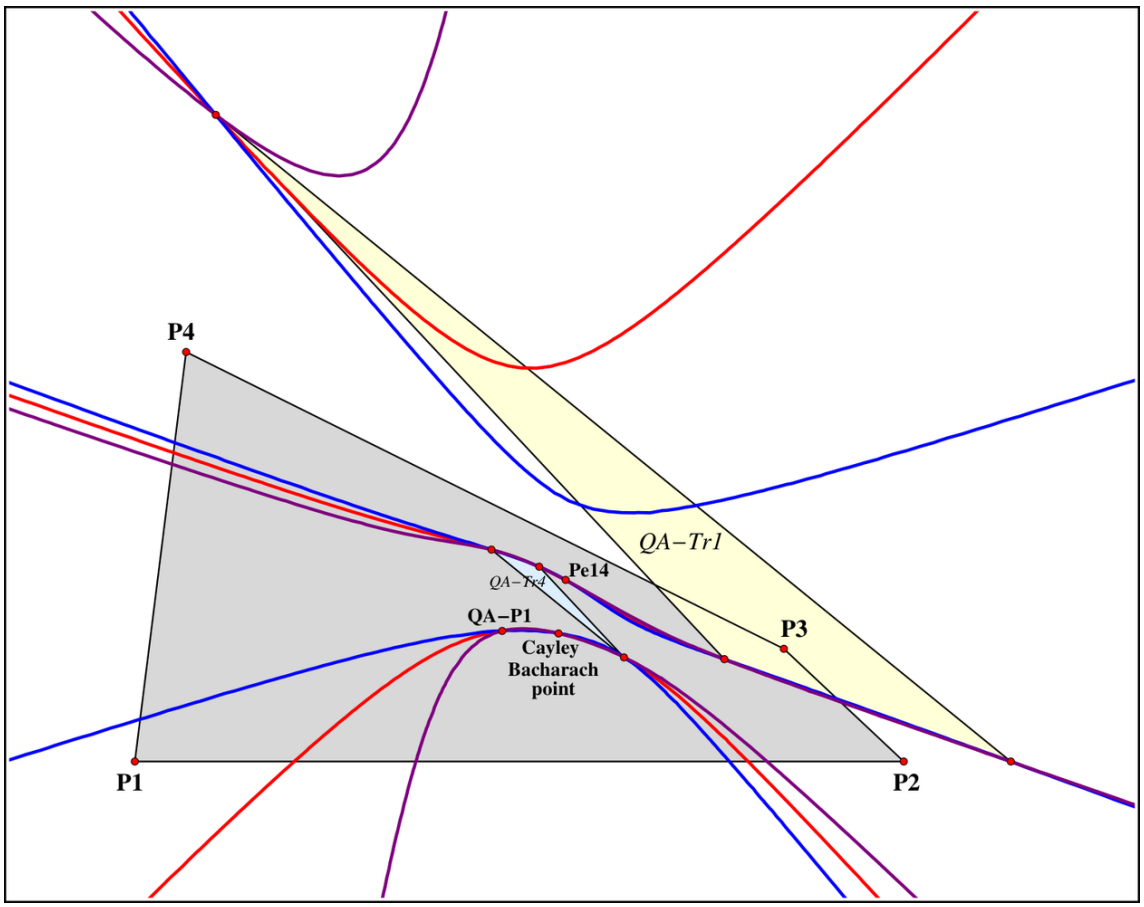
Message: #1703
Date: 29/4/2016 1:36:36
From: chris.vantienhoven
Subject: An involution in pentangle

Dear Seiichi,
Following on your
ADGEOM message #3174:
“Let A, B, C, D, E, F, G be 7 points, A, D, G collinear, B, E, G collinear and C, F, G collinear. Let P be an arbitrary point.
Denote:
Co1= the conic through B, E, C, F and P.
Co2= the conic through C, F, A, D and P.
Co3= the conic through A, D, B, E and P.
Co1, Co2 and Co3 concur in a point Q.”.
Of course this is an application of the Cayley Bacharach Theorem.
It can be restated as:
“Let ABC and DEF be two perspective triangles with perspector G and let P be an arbitrary point,
then the conics (B,E,C,F,P), (C,F,A,D,P) and (A,D,B,E,P) have a common point Q.”
In Cayley Bacharach terms: “All cubics through A,B,C,D,E,F,G,P have a common point Q”.
I translated it to perspective triangles QA-Tr1 (Diagonal Triangle) and QA-Tr4 (QL-P12 Triple Triangle) with perspector Pe14 (=QA-P5.QA-P10 (6 : 1)).
All cubics through the vertices of QA-Tr1 (A,B,C) and QA-Tr4 (D,E,F) and Pe14 (G) and QA-P1 (P) have a common point Q.
I calculated this point Q in this special case in CT-coordinates. The 1 st CT-coordinate is:
(2 p + q + r) (-3 p^2 - 3 p q - 3 p r + 2 q r) (-p^2 q^2 - p q^3 + 3 p^2 q r + 2 p q^2 r + q^3 r - p^2 r^2 + 2 p q r^2 + 2 q^2 r^2 - p r^3 + q r^3)
This point actually lies on QA-Co5.
I enclose a picture of 3 arbitrary cubics though the vertices of QA-Tr1 and QA-Tr4 and through Pe14 and QA-P1 and the inevitable common Cayley Bacharach point Q.
Of course similar cases can be done for other sets of perspective QA-triangles combined with another point.
Best regards,
Chris



QA-Tr1-Tr4-03.pdf

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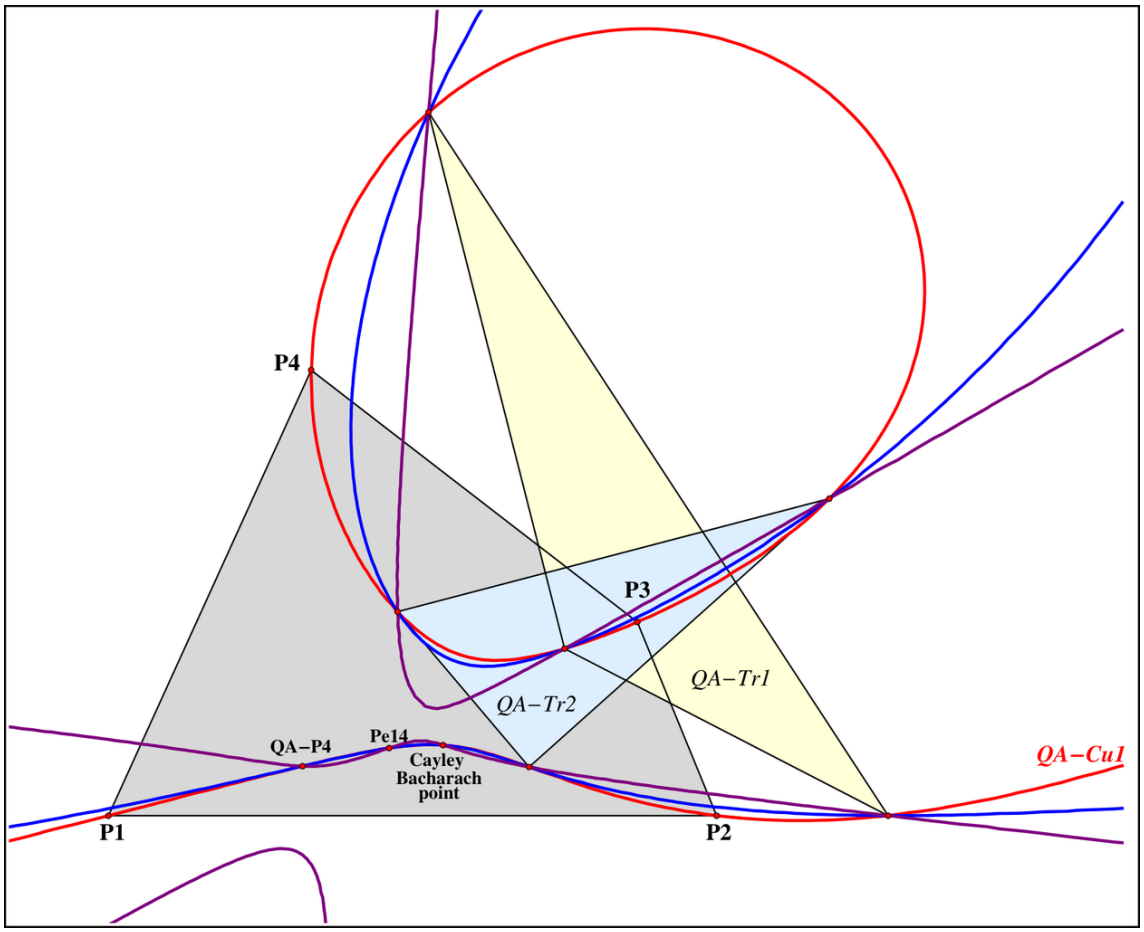
Message: #1704
Date: 29/4/2016 9:00:46
From: chris.vantienhoven
Subject: An involution in pentangle

Dear Seiichi,
This time I enclose a picture of 3 cubics though
the vertices of QA-Tr1 and QA-Tr2 and perspector Pe12 (=QA-P3)
and QA-P4 (total 8 points) and the resulting common Cayley
Bacharach point Q. The red cubic is QA-Cu1.
The 1 st CT-coordinate of Q is pretty long:
....
I noticed that almost all of these Caley Bacharach points
have very long coordinates.
Even much longer than above one.
After all they are the 9 th intersection point of two 3 rd
degree equations.
Best regards,
Chris



QA-Tr1-Tr2-04.pdf

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Message: #1705
Date: 30/4/2016 2:56:51
From: seiichikiri
Subject: An involution in pentangle

Dear Chris,
It is really long!. But it is not a bad thing to think that QFG has taken a step forward.
Incidentally, in the cases (lines and conics) of degenerate CB theorem, we can always consider their dual statements. I could not find the dual statement of general CB theorem in the web. If it would be possible, it might be able to enrich QFG again.
Best regards, Seiichi.

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Message: #1706
Date: 04/5/2016 9:13:05
From: bernard.keizer
Subject: QL-Tf3

Dear Eckart, dear Chris
Nice properties, this locus of CSCe of tangents to an inscribed conic of a QL being a conic through the circumcenters !
It follows that the CSC of the tangents are circles through QL-P1 centered on this circumconic ; the envelop of these circles is the CSC of the inscribed conic, which is a quartic tangent to the 4 circumcircles of the reference triangles. (For the parabola, it's the monocardioïd of the QL).
PS for Eckart: in fact, we were talking from the same theorem, without knowing; it is explained in Wikipédia that the English and the German name intercept theorem what the French name Thales theorem, whereas the Thales theorem is another one for the Germans (right angle inscribed in a circle for a diameter ...)

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Message: #1707
Date: 04/5/2016 9:34:49
From: bernard.keizer
Subject: QA-Cu1 and QL-Tr2

Dear Chris, dear Eckart
First of all, many thanks for both of you for the new presentation of QA-Cu1 in EQF ; I took a great interest in readind it and Bernard Gibert's Isocubics ...
1) P9 is the singular focus of QA-Cu1 (it's polar conic is a circle) ; the diametral point on the QA-Miquel circumcircle is the point Q, isogonal conjugate of the infinity point of the asymptote and the 2nd intersection of the asymptote with the circle is a point E, which is QA-P2 of the J-triangle, center of the rectangular hyperbola through the in/excenters.
2) The Cu-QA with a pivot P and it's 3 CSC conjugates A, B and C P* is the isopivot and is the inverse in DT circumcircle of the isogonal conjugate of P wrt DT ; the property is mentionned in EQF for the point P4, but holds for the point P41, which is igP4 wrt DT.
For every such Cu-QA, P* is the QA-P4 of ABCP and the the isogonal conjugate wrt the Miquel triangle is QA-P3 of ABCP ; QA-P3QA-P4 is parallel to the asymptote and the inverse in the circumcircle of DT of the complement of P wrt DT lies on the asymptote.
The pivot of DT is the isopivot of the QA and
the polar conic of a point P wrt QA-Cu1 can be seen either as a diagonal conic through the 4 fixed points of the isoconjugation and the isopivot P or as a circumconic through the vertices of DT, the pivot P and the isopivot. (For example, for PjPkPl, Pi is the pivot and QA-P4 the isopivot and for DT, QA-P4 is the pivot and QA-P41 the isopivot ...)
3) We have defined a QA and a QL, which are duals and share the same DT.
QL-DT is a triangle formed by the 3 diagonals of the QL ; DQL is a QL formed by DT and the Newton Line and QL-DDT is it's diagonal triangle.
QA-DT is the QA diagonal triangle ; DQA is the QA formed by QA-DT and the point QA-P4 and QA-DDT is it's diagonal triangle.
QA-DT is the same triangle as QL-DT, but QL-DDT and QA-DDT are different.
In the QL environment, the S-points or vertices of QL-Tr2 belong to 9 conics (3 nine-point conics of the QA's formed by 2 copples of CSC vertices of the QL, 4 through vertices, centroïd and perspector of the reference triangles, 1 through the vertices of DT, the centroïd and the last perspector QL-P13 and 1 through the 5 perspectors and the vertices of QL-DDT)

The 2 last mentionned conics belong also to the QA-environment, as the perspectors are the Pi and QL-P13 = QA-P16.
I've found a 10th belonging to the QA-environment through the vertices of QA-DDT (3rd intersection of the sides of QA-DT with QA-Cu1).
With this simple definition, I suppose all the followers QA-DDDT, QA-DDDDT and so on will share the same property, that their vertices are coconic with the S-points or vertices of QL-Tr2 ...
Any idea ?
Best regards
Bernard

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Message: #1708
Date: 05/5/2016 11:40:29
From: bernard.keizer
Subject: QA-Cu1 and QL-Tr2

Dear Chris, dear Eckart
Please cancel the last paragraph of my previous message.
The property doesn't hold for QA-DDDT.
It remains in the QA-environment the 3 mentionned conics
1) through the QA vertices, QA-P16 and the QL-DDT vertices
2) through the QA-DT vertices, QA-P10 and QA-P16
   (these 2 belong also to the QL-environment)
3) through the QA-DDT vertices (DT of the QA formed by QA-P4 and the QA-DT vertices)
The next one QA-DDDT is the DT of the QA formed by QA-P41 and the QA-DDT vertices: the vertices are of course on QA-Cu1, but aren't coconic with the S-points or vertices of QL-Tr2.
Best regards
Bernard

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Message: #1709
Date: 07/5/2016 4:07:33
From: bernard.keizer
Subject: QA-Cu1 and QL-Tr2

Dear Chris, dear Eckart
Last remark, before I stop for a while.
The 2 1rst mentionned conics in the QA environment are the polar conics of QA-P16 and QA-P10 wrt the DT cubic pK(P16, P16), where pole = pivot.
The tangents in the Pi concur in P16 and the tangents in Si and in P16 concur in P10 (for the Si, it's the medians of DT).
These 2 conics intersect in 4 points P16 and the S-points or vertices of QL-Tr2.
The duals of these 2 conics are the parabola QL-Co1 (the Li are the duals of the Pi and the infinity line is the dual of QA-P16) and the parabola Co1 of DQL, with focus QL-P17 and directrix QL-L6 (the sides of DT are the duals of the vertices, the dual of QA-P10 is the Newton Line).
The 4 common tangents to these 2 parabolas are the sides of QL-Tr2 (this triangle is self dual) and the infinity line (as they are both parabolas).
All the other mentionned conics through the S-points have as duals conics inscribed in QL-Tr2. This is also true for the Dimidium circle !
Best regards
Bernard

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Message: #1710
Date: 09/5/2016 10:01:10
From: bernard.keizer
Subject: QA-Cu1 and QL-Tr2

Dear Chris, dear Eckart
Without reaction from your side, I try another copple of remarks about these 2 conics.
The 1rst one, through the Pi and QA-P16, is also through QA-P1 (property in EQF) and the tangents in QA-P1 and QA-P16 intersect in QA-P10.
This conic is the dual of QL-Co1.
The dual line of QA-P1 is tangent to QL-Co1 in the point where the Newton Line cuts the parabola ; this line is the Newton Line of DQL, perpendicular to QL-L6. The dual line of QA-P16 is the infinity line, tangent to QL-Co1 in the infinity point of the Newton Line.
The 4th intersection with the Dimidium circle is the CSC of QL-P26 and this point is in line with QL-P1 and QL-P13 = QA-P16; the dual lines of QL-P1 and CSC(QL-P26) are parallel.
The 2nd one, through the vertices of DT, QA-P10 and QA-P16 is the dual of DQL-Co1 (focus QL-P17, directrix QL-L6, tangent to the sides of DT and to the Newton Line QL-L1).
The dual line of the contact point of DQL-Co1 with the Newton Line is the tangent to the 2nd conic in QA-P10.
The 4th intersection of this 2nd conic with the Dimidium circle is the point QL-P24, in line with QL-P17 and QL-P13 = QA-P16; the dual lines of QL-P17 and QL-P24 are parallel.
Best regards
Bernard
PS As already mentionned by Eckart, the dual of the circumcircle of DT is it's Steiner inellipse. The dual of the Dimidium circle is not the Steiner inellipse of QL-Tr2 and also not the one of the triangle QL-P1QL-P17QL-P24; it remains the 3 Gergonne-Steiner points ?

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Message: #1711
Date: 09/5/2016 2:56:36
From: tsihonglau
Subject: QA-Cu1 and QL-Tr2

Dear Bernard,
Sorry! I did not read your messages. But I have an old idea called pivot chain. Please read APG topic #2364 carefully for more information.
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/topics/2364 †)
I guess pivot chain is correct and applies to not only triangle/trilateral but quadrangle/quadrilateral.
sqrtdiv should be replaced with involutary conjugate.
I do not know what cevadiv should be replaced with.
Any idea? I will check your messages and share my idea.
Best regards,
Tsihong Lau

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Message: #1712
Date: 10/5/2016 12:03:11
From: Randy Hutson
Subject: maltitude quadrangle

Dear friends,
I have been away from quadri-figures for awhile, but a recent new center in ETC has a property that involves what I am proposing to call the maltitude quadrangle of a reference quadrangle, and thought EQF would be an appropriate place to have this defined:
Given a reference quadrangle P1, P2, P3, P4, define the maltitude ('midpoint altitude') Mij as the perpendicular to line PkPl from the midpoint of Pi and Pj. The 6 maltitudes intersect in 4 triple points, which form the maltitude quadrangle of P1P2P3P4.
Best regards,
Randy

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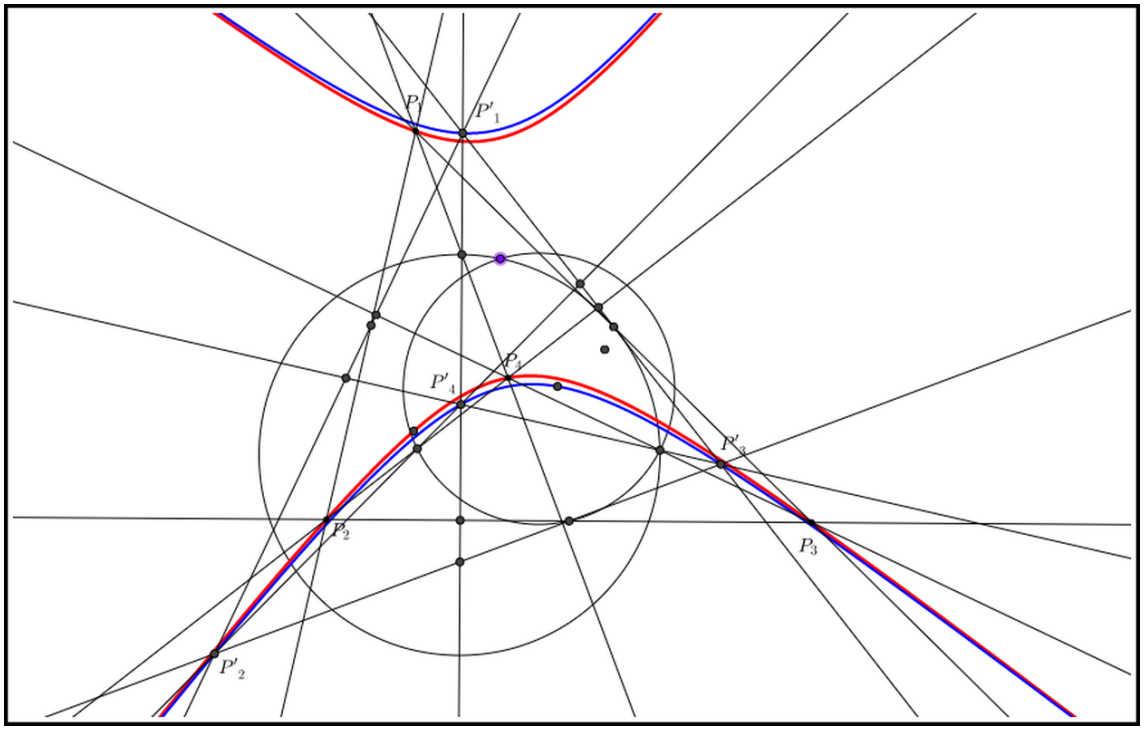
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #1713
Date: 10/5/2016 6:27:42
From: minhtenladuong
Subject: maltitude quadrangle

Dear Randy Hutson,
This maltitude quadrangle is very interesting.
I have observed that:
1) The QA-Co2 of the maltitude quadrangle P'1P'2P'3P'4 is homothetic image of the QA-Co2 of P1P2P3P4. The homothetic center is QA-P2.
2) P1P'1, P2P'2, P3P'3, P4P'4 are tangent to QA-Co2 of P1'P'2P'3P'4 at P'1, P'2,P'3,P'4.
3) Newton line of the quadrilateral (P1P'1, P2P'2, P3P'3, P4P'4) contains QA-P2.
Best regards
Sincerely
Ngo Quang Duong



maltitudes.png

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Message: #1714
Date: 10/5/2016 9:12:28
From: eckart_schmidt@t-online.de
Subject: maltitude quadrangle

Dear Randy Hutson,
this maltitude quadrangle is really interesting. Already testing its QA-Co1 gives some properties:
... The QA-Co1 of the reference and the maltitude quadrangle have orthogonal asymptotes.
... QA-Co1 of the maltitude quadrangle has center QA-P32.
... QA-P3 is a common point for the QA-Co1 of the reference and the maltitude quadrangle.
... The circumconic of the maltitude quadrangle, centered in QA-P3, is its orthogonal hyperbola.
So far a first feedback, I shall look for more properties.
Best regards Eckart

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Message: #1715
Date: 10/5/2016 10:15:01
From: eckart_schmidt@t-online.de
Subject: maltitude quadrangle

```
Dear Randy Hudson,
I just noticed, that the vertices of the maltiutude quadrangle
are the QA-Tf4-images of the vertices of the reference
quadrangle.
Best regards Eckart
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Message: #1716
Date: 10/5/2016 3:12:24
From: eckart_schmidt@t-online.de
Subject: maltitude quadrangle

```
Dear Randy Hudson,
please forget my messages 1714, 1715. Excuse, I have tested the
quadrangle of the four circumcenters of the triangle components.
Best regards Eckart
```

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Message: #1717
Date: 10/5/2016 3:59:40
From: eckart_schmidt@t-online.de
Subject: maltitude quadrangle

Dear Randy Hudson,
the properties in message 1714 and message 1715 are not valid for the maltitude quadrangle, but for the O-quadrangle.
The maltitude quadrangle is the reflection of the O-quadrangle in QA-P1, so the properties in 1714 and 1715 can be transferred for the maltitude quadrangle.
In the same way the following points of the O-quadrangle (in brackets) can be interpreted for the maltitude quadrangle:
(QA-P1) = QA-P32,
(QA-P2) = QA-P3,
(QA-P3) = reflection of QA-P3 in QA-P32,
(QA-P4) = QA-P4,
(QA-P5) = QA-P12,
(QA-P6) = midpoint QA-P3.QA-P4,
(QA-P7) = QA-P8,
(QA-P8) = intersection of QA-P1.QA-P4 and a parellel to QA-L4 through QA-P8,
(QA-P9) = QA-Tf4(QA-P2),
(QA-P10) divides QA-P12.QA-P32 with ratio -4:1,
(QA-P11) = reflection of QA-P15 in QA-P1,
(QA-P12) lies on QA-P4.QA-P5 ...
(QA-P22) divides QA-P12.QA-P32 with ratio -3:1
(QA-P23) = QA-P1,
(QA-P26) divides QA-P12.QA-P32 with ratio -10:1,
(QA-P32) = intersection of QA-P1.QA-P4 and a parellel to QA-L4 through QA-P32,
(QA-P33) = intersection of QA-P3.QA-P23 and a parallel to QA-P3.QA-P4 through QA-P32,
(QA-P37) lies on QA-P3.QA-P20 ...
(QA-P41) = QA-P9,
(QA-P42) = intersection of QA-P3.QA-P8 and QA-P1.QA-P4.
Excuse my confusion!
Best regards Eckart

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Message: #1718
Date: 10/5/2016 5:12:58
From: Randy Hutson
Subject: maltitude quadrangle

Thank you, Eckart Schmidt and Ngo Quang Duong, for your interesting observations! Another property I found is that the 2nd generation maltitude quadrangle is homothetic to the reference quadrangle at QA-P2. This probably follows from Eckart's observation that the maltitude quadrangle is the reflection of the O-quadrangle in QA-P1.
Another question: Is there anything interesting about the centroid of the 6 maltitude feet?
Best regards,
Randy

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Message: #1719
Date: 11/5/2016 6:29:16
From: minhtenladuong
Subject: maltitude quadrangle

Dear Randy Hutson and Eckart Schmidt
I didn't find anything about centroid of 6 maltitude feet, so I considered three midpoints.
In detail:
N12, N23, N34, N41,N13, N24 are 6 maltitude feet.
Let A, B, C be midpoints of N12N34, N23N41, N31N24.
1) ABC is Ptolemy triangle - its sidelengths are proportional to P1P2.P3P4, P2P3.P1P4, P3P1.P2P4.
This leads to the corollary: A, B, C are collinear iff the quadrangle is cyclic.
You can see topic 1570, where I mentioned the midpoint-triangle.
This midpoint-triangle is similar to ABC, directly.
2) Circumcircle of ABC contains QA-P2.
Best regards
Sincerely
Ngo Quang Duong

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Message: #1720
Date: 12/5/2016 12:53:11
From: chris.vantienhoven
Subject: maltitude quadrangle

```
Dear Randy,
I am curious: for which ETC point you can use the Maltitude
Quadrangle?
Best regards,
Chris
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Message: #1721
Date: 12/5/2016 7:11:11
From: Randy Hutson
Subject: maltitude quadrangle

```
Hi Chris,
A few examples:
X(51)   = centroid of maltitude feet of quadrangle ABCX(4)
X(373)  = centroid of maltitude feet of quadrangle ABCX(2)
X(381)  = centroid of maltitude quadrangle of quadrangle ABCX(2)
X(9955) = centroid of maltitude quadrangle of quadrangle
ABCX(1), which consists of X(946) and the extraversions of X(10)
Best regards,
Randy
```

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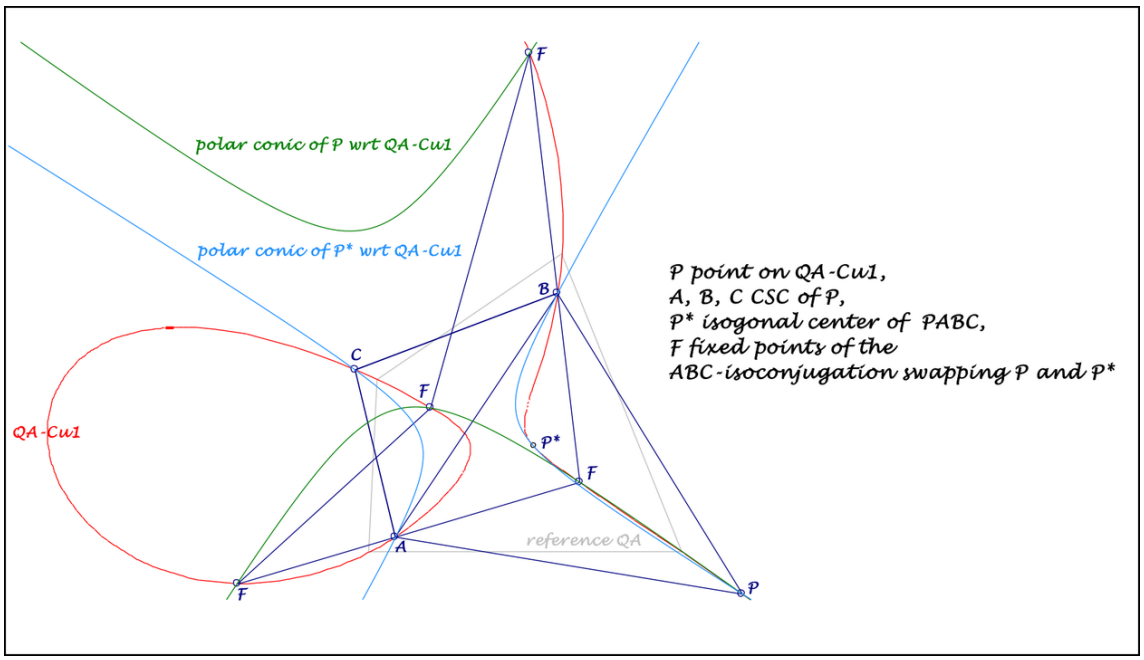
Message: #1722
Date: 12/5/2016 11:15:24
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 and QL-Tr2

Dear Bernard,
back from holiday, I try to read your message 1707, but there are questions:
wrt 1) " ... a point E, which is QA-P2 of the J-triangle, center of the rectangular hyperbola through the in/excenters."
I think "J-triangle" is a typo; but if it is the quadrangle of the in/excenters of the Miquel triangle, this is an orthocentric QA and every point can be the center of a rectangular circumscribed hyperbola through the in/excenters. The rectangular hyperbola with center E has E.Q (QA-Cu1-asymptote) and E.QA-P9 as asymptotes.
wrt 2) I think, you derscribe the general interpretation of QA-Cu1 in #1657, but: What is the definition of "isopivot" ? What is the meaning of * for P*?. What property do you mean, mentioned in EQF for the point QA-P4? By the way: QA-P41 isn't igP4 wrt DT, but QA-Tf2(P4) and I can't reproduce, that "... the inverse in the circumcircle of DT of the complement of P wrt DT lies on the asymptote".
In the 2nd line of the last passage of 2) there will be a typo: I think, you mean pivot P (see attached file). The 2nd description of the polar conic of P can't be right: It is the polar conic of P* (QA-P4 of PABC), if you take as DT the diagonal triangle ABC of the quadrangle of the fixed points of the ABC-isoconjugation, which swaps P and P*.
wrt 3) I shall answer in the next days.
Please excuse all the questions, perhaps my misunderstandings, but I am interested in your sight of QA-Cu1.
Best regards Eckart



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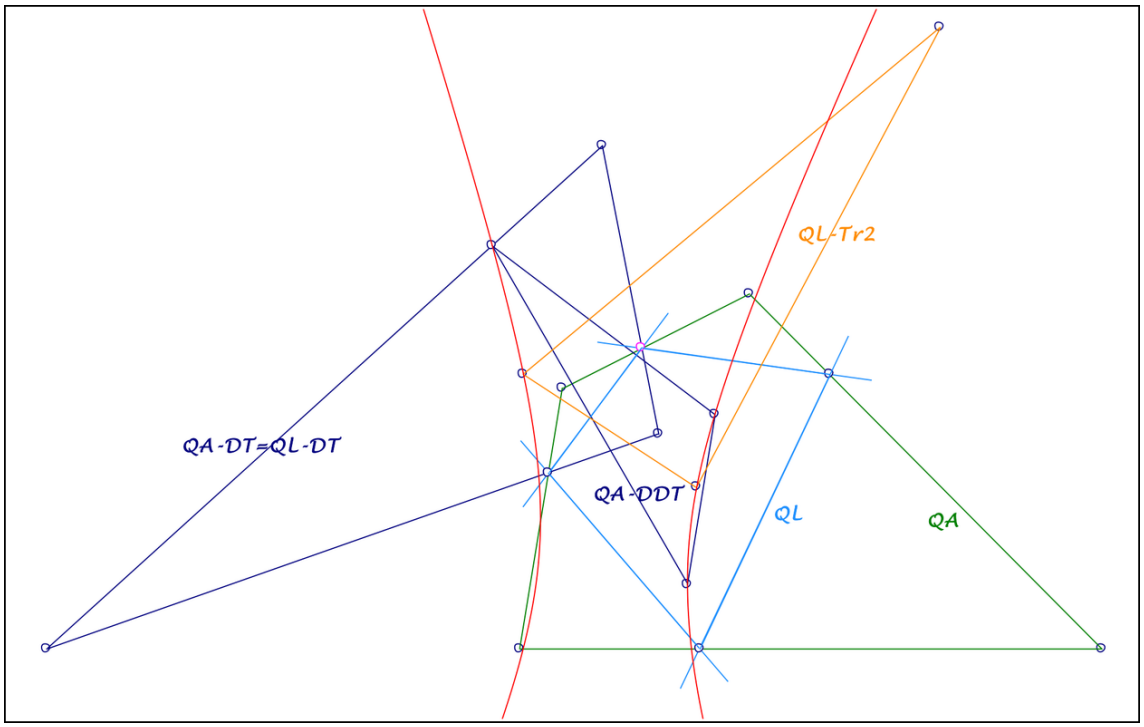
Message: #1723
Date: 12/5/2016 4:28:48
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 and QL-Tr2

Dear Bernard,
sorry, I can't reproduce, that the vertices of QA-DDT and the 3 S-points are coconic (see attached file).
Best regards Eckart



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Message: #1724
Date: 12/5/2016 10:52:19
From: bernard.keizer
Subject: QA-Cu1 and QL-Tr2

Dear Eckart
In fact, you are right !
The DT triangle depends on the choosen cubic.
The vertices of QL-Tr2 aren't coconic with the vertices of QA-DDT neither for QA-Cu1 = pK(QA-P16,QA-P4) nor for QA-Cux = pK(QA-P16,QA-P16).
So please forget about my 10th conic !
Best regards
Bernard

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Message: #1725
Date: 16/5/2016 9:48:58
From: eckart_schmidt@t-online.de
Subject: Cubic derived from QL-Cu1 with QL-Tf3

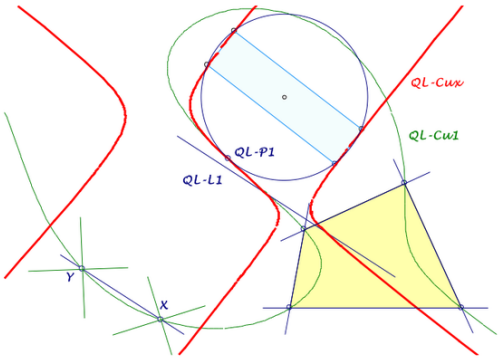
Dear Bernard, dear Chris,
attached a curious QL-cubic with unexpected properties (e.g. with reference to the asymptotes of QL-Cu2).
I hope, there will be no mistakes, for it's only CABRI-controlled.
Best regards Eckart

EQF-Note 2016-05-16

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubic derived from QL-Cu1 wrt QL-Tf3

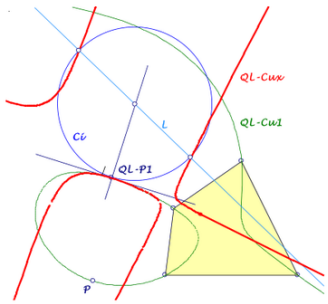
The angle bisectors for points on the cubic QL-Cu1 wrt opposite QL-points have QL-Tf3-images on a cubic, which shall be tested in this note. The properties are only CABRI-controlled.



Let *X*, *Y* be points on *QL-Cu1* with *XY* parallel *QL-L1*.
… take the angle bisectors at *X* and *Y* wrt opposite points of the quadrilateral:
… Their *QL-Tf3*-images give a rectangle:
… with sides parallel to the Steiner axes,
… with circumcircle through *QL-P1*,
… centered on a perpendicular to *QL-P1.QL-P4* through *QL-P1*.
… **The locus of the rectangle vertices is a cubic *QL-Cux***.

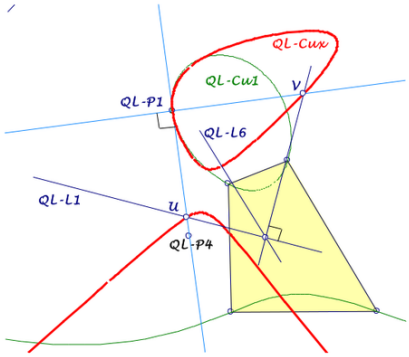
The construction can be simplified, taking the 2nd intersections of the angle bisectors at *X* and *Y* with the circumcircle of *X, Y, QL-P1*.

There is a further alternative construction of the cubic *QL-Cux*:

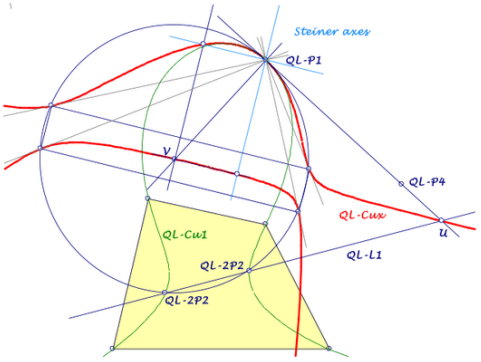


… Let *P* be point on *QL-Cu1* and *L* the line with *QL-Tf3(L)=P*.
… Let *Ci* be a circle through *QL-P1*, centered in the intersection of *L* and a perpendicular to *QL-P1.QL-P4* in *QL-P1*.
… The intersections of *L* and *Ci* give the cubic *QL-Cux*.

- ***QL-Cu1* and *QL-Cux* have a common tangent in *QL-P1* (*QL-P1.QL-P4*).**
- **The intersection *U* of the common tangent in *QL-P1* and *QL-L1* is a point of the cubic *QL-Cux*.**
- **The intersection *V* of the common normal in *QL-P1* and a *QL-L1*-perpendicular line in the intersection with *QL-L6* is a point of the cubic *QL-Cux*.**
- ***V* = *QL-Tf3(QL-L1)*.**
- ***UV* is tangent to *QL-Cux* in *V*.**
- ***QL-Tf3(UV)* = *QL-L1* ∩ *QL-L6*.**

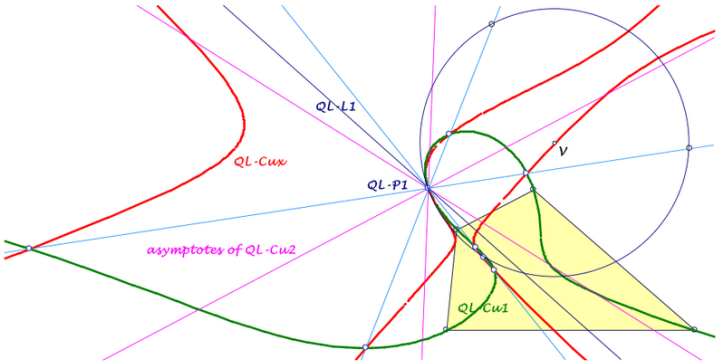


- **If *QL-Cu1* is unipartite, *V* is the center of the circumcircle *Ci* of *QL-P1* and *QL-2P2*, which intersects *QL-Cux* in rectangle-points, whose tangents bear *QL-P1*.**
- **Parallels to the Steiner axes through *V* intersect the Steiner axes on the cubic *QL-Cux*.**

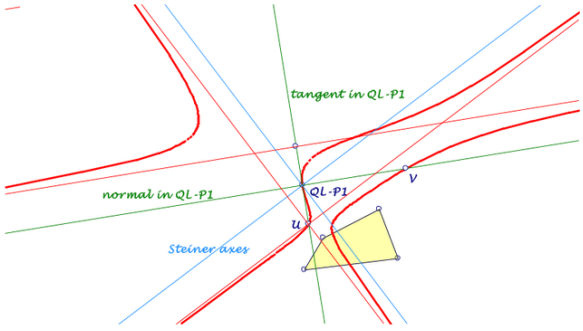


2016-05-16.pdf

- **Six further intersections of *QL-Cu1* and *QL-Cux* lie in pairs collinear with *QL-P1*.**
- **The lines of these collinearities are the reflections of the asymptotes of *QL-Cu2* in the first Steiner axis (intersecting with 60°).**
- **The 4th harmonic points of *QL-P1* wrt the pairs of intersection of *QL-Cu1* and *QL-Cux* lie on a circle round *V* through *QL-P1*.**



- **The asymptotes of *QL-Cux* are**
  … parallels to the Steiner axes through *U*
  … and a perpendicular line to *U.QL-P1* through the reflection of *U* in *QL-P1*.
- **The triangle of the asymptotes is rectangular:**
  … altitude *QL-P1.QL-P4*,
  … Euler line parallel *QL-L2*,
  … Apollonius circle through *U* centered on *QL-L1*.



- **On lines *L* through *QL-P1* the intersections with *QL-Cu1* and *QL-Cux* define a line involution with center in the pedal point of *V* on *L* (one double point is *QL-P1*).**

Eckart Schmidt
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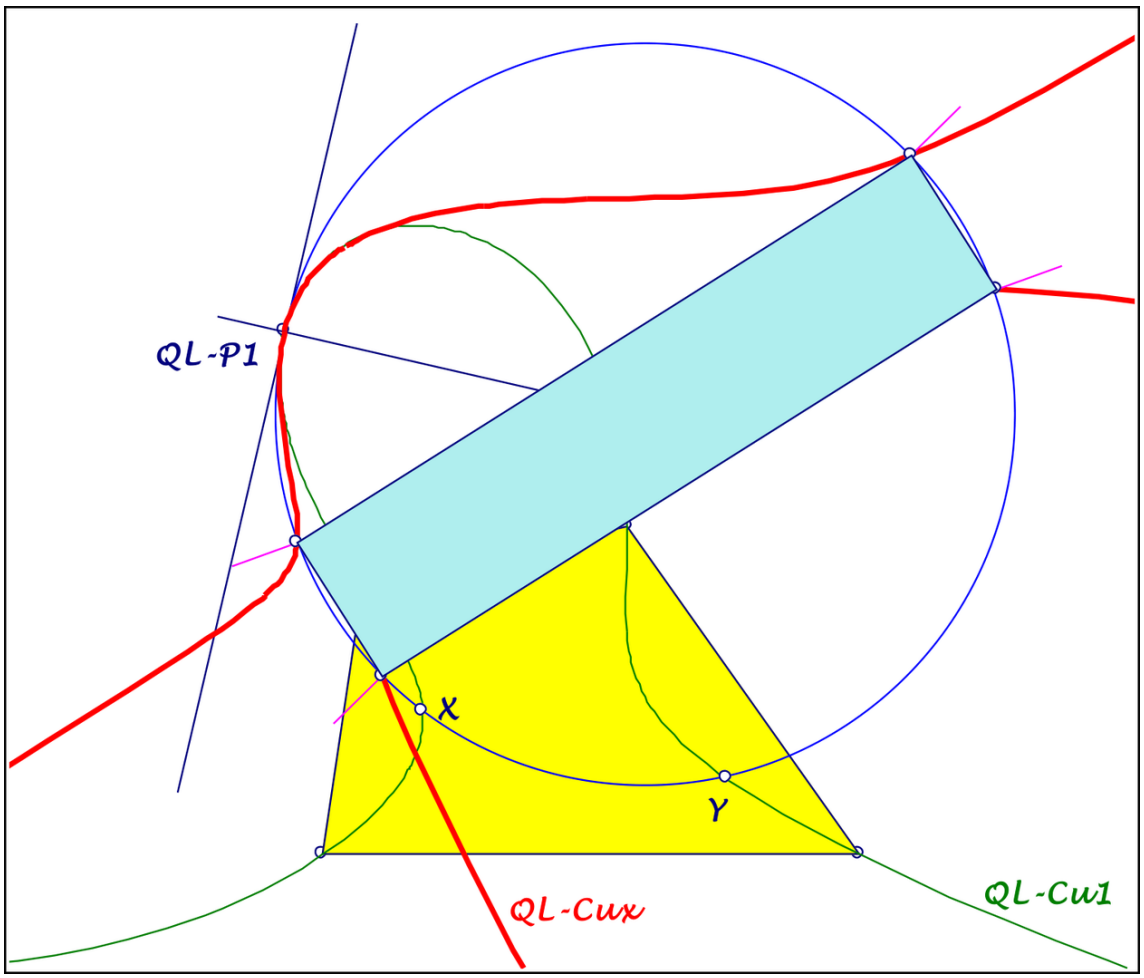
2016-05-16.pdf

Message: #1726
Date: 17/5/2016 10:50:38
From: eckart_schmidt@t-online.de
Subject: Cubic derived from QL-Cu1 wrt QL-Tf3

Dear Bernard, dear Chris,
in addition to #1725 I would name the new cubic "rectangle cubic", for every circle through QL-P1, centered on the normal of QL-P1 wrt QL-Cu1, intersects this cubic in the vertices of a rectangle.
There is a description of the new cubic independent of CSC and QL-Tf3 (see attached file):
Let X, Y be the further intersections of QL-Cu1 and a circle Ci through QL-P1, centered on the normal of QL-P1 wrt QL-Cu1.
The perpendicular bisectors of X.QL-P1 and Y.QL-P1 intersect Ci in the vertices of a rectangle on the new cubic.
Best regards Eckart
PS. My EQF-Notes are gathered in
http://eckartschmidt.de/EQF-Notes.html.

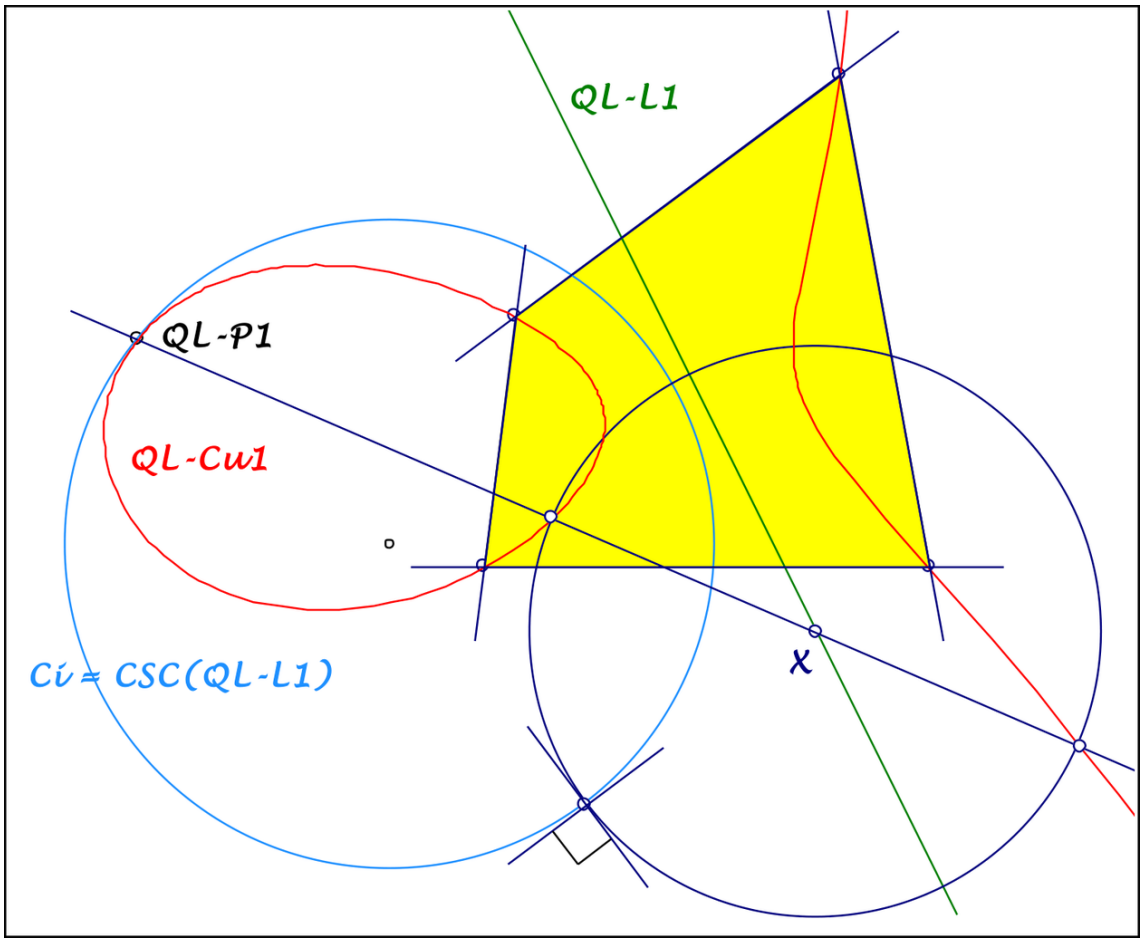


2016-05-17.docx

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1727
Date: 18/5/2016 8:32:11
From: eckart_schmidt@t-online.de
Subject: Construction of QL-Cu1

Dear Bernard, dear Chris,
attached a simple construction of QL-Cu1, perhaps already mentioned:
Let X be a point on QL-L1,
... let Ci be the circle centered in X, orthogonal intersecting the circle CSC(QL-L1),
... the intersections of Ci and X.QL-P1 give QL-Cu1.
Best regards Eckart

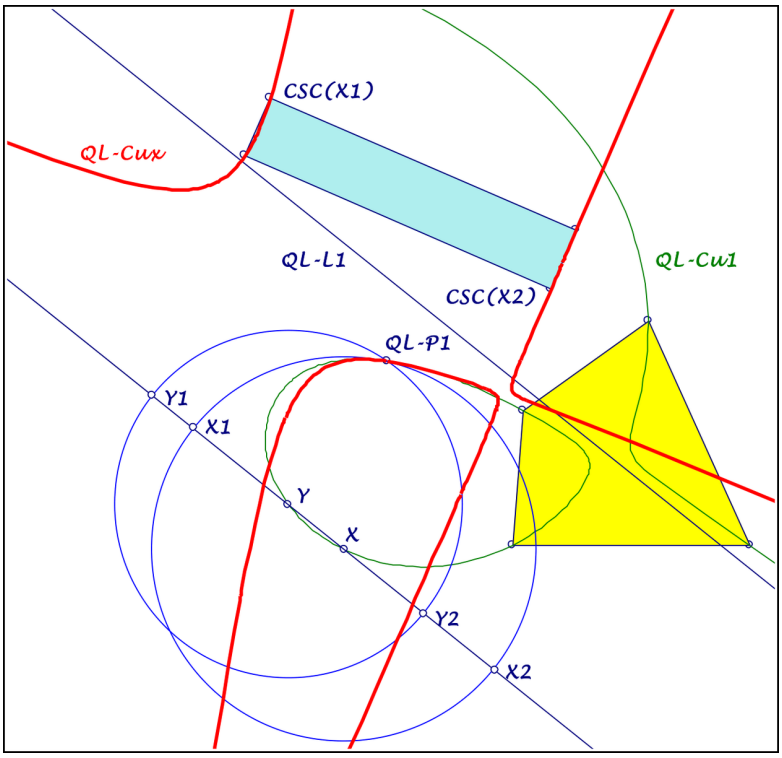


2016-05-18.docx

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1728
Date: 20/5/2016 4:02:31
From: eckart_schmidt@t-online.de
Subject: Cubic derived from QL-Cu1 wrt QL-Tf3

Dear Bernard, dear Chris,
in addition to my messages #1725, #1726 attached a very simple construction of the cubic QL-Cux:
Let X, Y be two points on QL-Cu1 with XY parallel QL-L1:
... let Xi be the intersections of XY with a circle round X
    through QL-P1, analog Yi .
... CSC(Xi) and CSC(Yi) give a rectangle on the cubic.
Some further properties of QL-Cux:
... QL-Cux contains the 12 QL-TF3-images of the angle bisectors
    of the QL-lines.
... The polar conic of QL-P1 wrt QL-Cux
    is the circle CSC(QL-L1).
... The polar conic of U wrt QL-Cux is an orthogonal hyperbola,
... ... through U, V and QL-P1
... ... with asymptotes parallel to the Steiner axes.
... The polar conic of V wrt QL-Cux is a conic through V,
... ... centered in QL-P1
... ... with the Steiner axes as axes,
... ... tangent to UV.
Best regards Eckart



2016-05-19.docx

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Message: #1729
Date: 20/5/2016 4:24:44
From: eckart_schmidt@t-online.de
Subject: Nonpivotal Isocubic wrt QA / QL

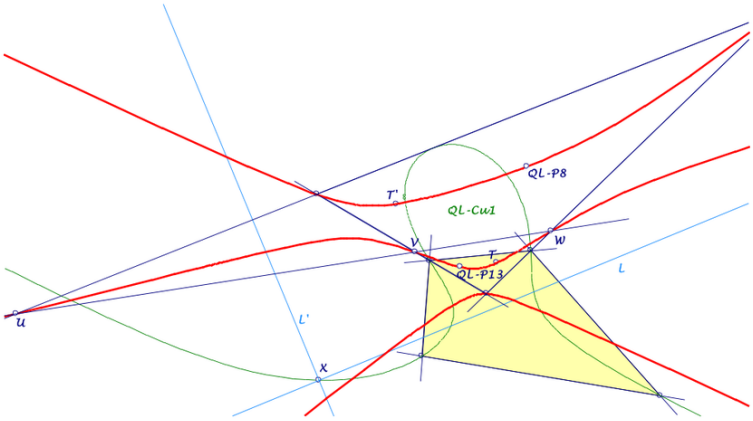
Dear Bernard, dear Chris,
the QL-DT-trilinear poles for angle bisectors of QL-Cu1-points wrt opposite QL-points lead to a nonpivotal isocubic, which can also be interpreted wrt the dual quadrangle (see attached file).
Best regards Eckart

EQF-Note 2016-05-20

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Nonpivotal Isocubic wrt QA/QL

The angle bisectors at points on the cubic QL-Cu1 wrt two opposite QL-points have trilinear poles wrt QL-DT on a nonpivotal isocubic, which shall be tested for the reference quadrilateral and its dual quadrangle.



Construction of *QL-Cuy*

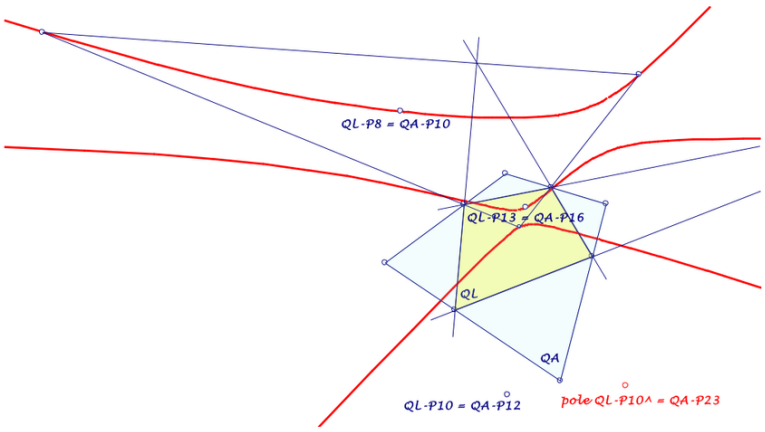
Let *X* be a point on the cubic *QL-Cu1*,
… *L, L´* its angle bisectors wrt opposite *QL*-points,
… *T, T´* trilinear poles of *L, L´* wrt *QL-DT*.
… The locus of *T, T´* is the cubic *QL-Cuy*.

- ***QL-Cuy* is a circumcubic of *QL-DT*,**
  … bearing *QL-P8*, *QL-P13*,
  … intersecting the *QL-DT*-sides collinear in *U, V, W*.

QL-Cuy* as nonpivotal isocubic of *QL

- ***QL-Cuy* is invariant wrt the *QL-DT*-isoconjugation ^,**
  swapping *QL-P8* and *QL-P13*.
- **Fixed points of the isoconjugation ^ are the trilinear poles of the *QL*-lines wrt *QL-DT*.**
- ***QL-P10*^ is the trilinear pole of *UVW*.**
- ***QL-Cuy* is a nonpivotal isocubic**
  … wrt *QL-DT*
  … the isoconjugation ^
  … and pole *QL-P10*^.

2016-05-20.pdf



QL-Cuy* as nonpivotal isocubic of the dual *QA

The dual quadrangle *QA* of a quadrilateral *QL* is the quadrangle of the trilinear poles for the quadrilateral lines wrt the *QL*-diagonal triangle, which is also the *QA*-diagonal triangle.

- **Wrt the common diagonal triangle the isoconjugation ^ is the isoconjugation *QA-Tf2*.**

- ***QL-Cuy* is a nonpivotal isocubic for the dual quadrangle**
  … wrt the common diagonal triangle,
  … the isoconjugation *QA-Tf2*
  … and the pole *QA-P23*.

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2016-05-20.pdf

Message: #1730
Date: 22/5/2016 8:53:22
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as Nonpivotal Isogonal Cubic

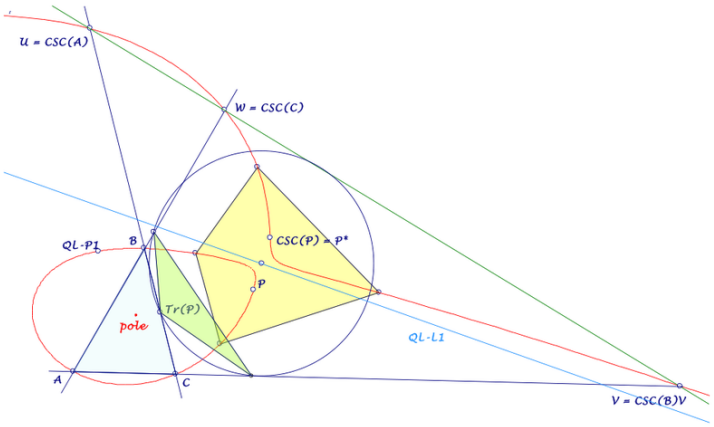
Dear Chris, dear Bernard,
QL-Cu1 is not only a pivotal isogonal cubic, but can also be interpreted as nonpivotal isogonal cubic wrt a suitable reference triangle. I think, that this aspect isn´t mentioned up to now (see attached file).
Best regards Eckart

EQF-Note 2016-05-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Cu1 as Nonpivotal Isogonal Cubic

QL-Cu1 is not only self-isogonal wrt the QL-triangle components. Here a new aspect shall be researched for reference triangles ABC with vertices in the CSC-images of three collinear points of QL-Cu1.



Preliminary remark: Let F_i be the foci of a *QL*-inscribed conic, centered in *QL-L1* ∩ *QL-L6,* which are in the unipartite case of *QL-Cu1* the points *QL-2P2*.
In *QFG*-messages 1425 and 1433 we find for the reference triangle $F_1.F_2.QL\text{-}P1$:

- ***QL-Cu1* is isogonal invariant wrt $F_1.F_2.QL\text{-}P1$.**
- **In the unipartite case of *QL-Cu1* the *CSC*-image of *QL-Cu1*-points is the isogonal conjugated wrt $F_1.F_2.QL\text{-}P1$.**
- **In the bipartite case of *QL-Cu1* the cubic is a pivotal isogonal cubic of $F_1.F_2.QL\text{-}P1$ with pivot in the point at infinity of *QL-L1*.**

Special generalization:

- ***QL-Cu1* is isogonal invariant wrt each triangle *ABC*, whose vertices are the *CSC*-images of three different collinear points *U, V, W* on *QL-Cu1*.**

If one of the collinear points is *QL-P1*, the triangle degenerates.

2016-05-22.pdf

- **Wrt *ABC* the isogonal conjugate of *QL-Cu1*-points is the *CSC*-image.**

- **Wrt *ABC* the cubic *QL-Cu1* is**
  … a nonpivotal isogonal cubic
  … with pole in the *ABC*-trilinear pole of *UVW*.

- ***QL-Cu1* is the locus of points, whose pedal circle wrt *ABC* is centered on *QL-L1*.**

The last property is a consequence of *EQF-Ref [17b] 1.5.5*.

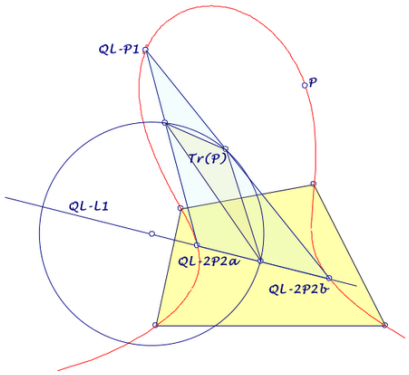
The pedal circle of *QL-P1* wrt *ABC* degenerates to the Simson line, orthogonal *QL-L1*. The pedal circles of *CSC*-partners coincide.

Special triangles *ABC*

If one of the collinear points *U, V, W* is the point at infinity of the asymptote, we get a triangle *ABC* with one side parallel *QL-L1* as reflection of *UVW* in *QL-L1* and the opposite vertex *QL-P1*.
If *U* and *V* of the collinear points are *CSC*-partners, we get the triangle *ABC = U.V.CSC(W).*

Example for *QL-Cu1* unipartite: For *U, V* in *QL-2P2* and *W* in the point at infinity of *QL-L1* we get *ABC = QL-2P2a.QL-2P2b.QL-P1*.

- **In the unipartite case of *QL-Cu1* the cubic is the locus of points whose pedal circle wrt *QL-2P2a.QL-2P2b.QL-P1* is centered on *QL-L1*.**



Eckart Schmidt
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eckart_schmidt@t-online.de

2016-05-22.pdf

Message: #1731
Date: 25/5/2016 12:17:53
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard, dear Chris,
in message 1523 I have described self-dual conics with the statement "... these three conics are the only self-dual conics".
This is not correct, there are more, see attached file.
There is another aspect
of Bernard´s QL-DDT.
Best regards Eckart

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Message: #1732
Date: 25/5/2016 1:16:54
From: eckart_schmidt@t-online.de
Subject: Dual Curves

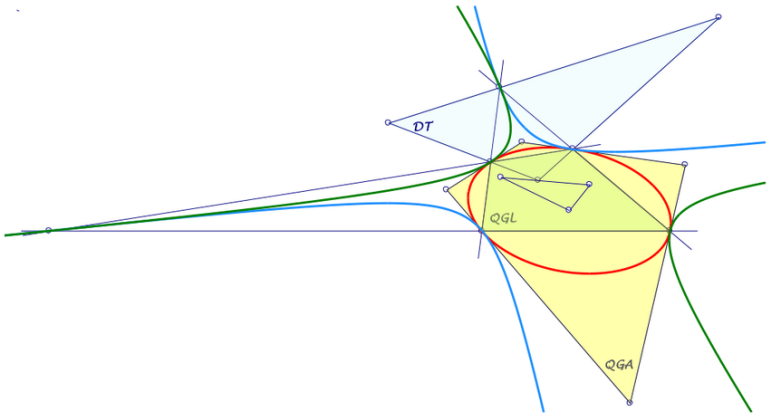
Dear Bernard, dear Chris,
it seems, that the attached file in #1731 isn´t transmitted. I try it once more.
Excuse Eckart

EQF-Note 2016-05-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Three Self-dual Conics

Let QL be a quadrilateral and QA its dual quadrangle with common diagonal triangle DT (see QFG-message 1516). For corresponding quadrigons QGL and QGA there are three self-dual conics, circumscribed QGL and inscribed QGA.



For a reference *QL* we consider a quadrigon component *QGL*. The *DT*-trilinear poles of the *QGL*-lines give the dual quadrigon *QGA*.

- **For the quadrigons *QGL* and *QGA* there is a conic *Co*, circumscribed *QGL* (*QG-Co2* of *QGL*) and inscribed *QGA* (*QG-Co1* of *QGA*).**

In this constellation duality means:
… for lines *L* : dual(*L*) = *QA-Tf2*(*DT*-tripole of *L*),
… for points *P*: dual(*P*) = *QL-Tf2*(*DT*-tripolar of *P*).

- **The three conics *Co* are self-dual.**

- **The conics *Co* are centered**
  … in *QG-P13* of the quadrigons *QGL*,
  which are
  … the duals for the sides of the *DT*-medial triangle
  and give
  … the *DT*-anticevian triangle of
  ***QL-P13* = *QA-P16* = *DT*-tripole of *QL-L1*.**

This triangle is already mentioned as *QL-DDT* by Bernard Keizer (*EQF*-message 1458) with the property (6^{th} conic in *EQF*-messages 1488, 1535, 1707):

- **The triangle of the *Co*-centers and *QL-Tr2* have a common circumconic through *QL-P13*.**

- **The circumcircle of the *Co*-centers is centered on *QL-L6* (see *EQF*).**

- **The poles of an arbitrary line *L* wrt the three conics *Co* give the …**
  … *DT*-anticevian triangle of *DT*-tripole (*QL-Tf2(L)*).

- **The polars of an arbitrary point *P* wrt the three conics *Co* give the …**
  … *DT*-cevian triangle of *QA-Tf2(P)*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-05-25.pdf

Message: #1733
Date: 25/5/2016 2:08:28
From: seiichikiri
Subject: Confirmation of general Cayley Bacharach theorem by Geogebra

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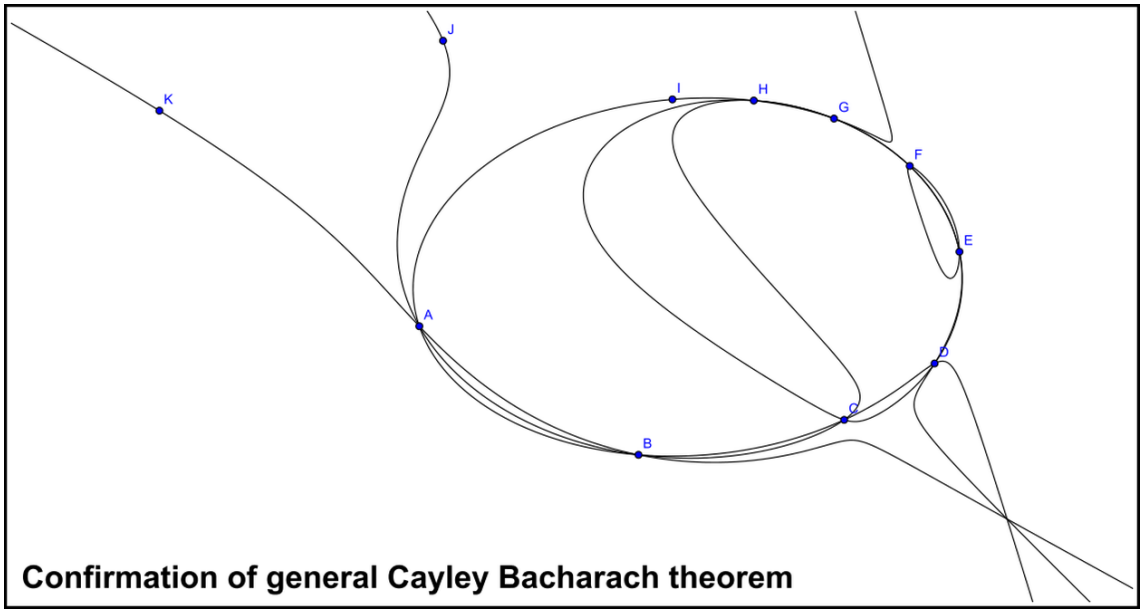
Dear friends,
For information only:
Attached is the picture of general Cayley Bacharach theorem by Geogebra.
3 commands:
ImplicitCurve[A,B,C,D,E,F,G,H,I]
ImplicitCurve[A,B,C,D,E,F,G,H,J]
ImplicitCurve[A,B,C,D,E,F,G,H,K]
The common intersection at the right-under location.
Best regards, Seiichi.



Cayley-Bachrach theorem.pdf

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Message: #1734
Date: 26/5/2016 3:17:37
From: chris.vantienhoven
Subject: Confirmation of general Cayley Bacharach theorem by Geogebra

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Dear Seiichi,
Beautiful picture!
Best regards,
Chris
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Message: #1735
Date: 28/5/2016 1:02:25
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
(1) You often cite your 5th and 6th conic. I have gathered your main properties for my own, perhaps of interest:
5th conic:
... common circumconic of QL-DT and QL-Tr2,
... ... through QL-P8, QL-P13 = QA-P16, QL-P24,
... ... through QA-P18, QA-Tf2(QA-P31),
... dual curve of a parabola,
... ... focus QL-P17, directrix QL-L6,
... ... tangent to QL-L1 in QL-P23.
6th conic:
... common circumconic of the dual QA and QL-Tr2,
... ... through QA-P1, QA-P16 = QL-P13, CSC(QL-P26),
... ... through the QG-P13-tripel for the QG-components
of QL (QL-DDT),
... ... through the QG-P12-tripel for the QG-components
of QA (QL-DDT),
... dual curve of QL-Co1.
(2) Wrt #1710, PS: You write:
"As already mentionned by Eckart, the dual of the circumcircle of DT is it's Steiner inellipse."
I doubt, that I have mentioned this. It is not correct, for I wrote in #1501:
"Dual of the DT-circumcircle: Inscribed DT-conic, which is the envelope for the QL-Tf2-images of lines through the Lemoine point X(6)."
In addition: The center of this conic is the DT-complement of the DT-isotomic image of the Brianchon point, which is the DT-tripole of a perpendicular line to QA-L2 through QA-P6.
The dual of the DT-Steiner inellipse is QA-Co1.
Best regards Eckart

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Message: #1736
Date: 29/5/2016 2:48:48
From: tsihonglau
Subject: Unit Conic and Dual Triangle/Trilateral and Perspector Quadr

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Dear all,
Chasles's Polar Triangle Theorem states that a triangle ABC and its polar triangle A'B'C' with respect to a conic c are perspective.
APG topic #3242 discusses it: https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/topics/3242 †)
we can get the (imaginary) unit conic, i.e. x^2+y^2+z^2=0.
Then the dual of a point/line is the same as the polar/pole of a point/line with respect to the unit conic. (Using the unitary coordinate system, the dual of the point p:q:r is the line px+qy+rz=0).
Given a triangle L(l:m:n),P(p:q:r),U(u:v:w), the perspector of LPU and the dual (polar) triangle of LPU is: (first coordinate)
-(m^2*q*r*w^2+l*m*p*r*w^2-m*n*q^2*w^2-l*n*p*q*w^2-m^2*r^2*v*w+n^2*q^2*v*w-l*m*r^2*u*w-l*m*q^2*u*w+n^2*p*q*u*w+m^2*p*q*u*w-l^2*p*q*u*w+l*m*p^2*u*w+m*n*r^2*v^2-n^2*q*r*v^2+l*m*p*r*v^2-l*n*p*q*v^2+l*n*r^2*u*v-n^2*p*r*u*v-m^2*p*r*u*v+l^2*p*r*u*v+l*n*q^2*u*v-l*n*p^2*u*v-l*m*p*r*u^2+l*n*p*q*u^2)
But in fact, there are other three perspectors.
That is because we can redefine the coordinate system.
The ordinary base of a unitary coordinate system is
    Q(1:1:1),QA(-1:1:1),QB(1:-1:1),QC(1:1:-1).
We can redefine is as
    Q(-1:1:1),QA(1:1:1),QB(1:1:-1),QC(1:-1:1).
(i.e. Q,QA and QB,QC interchanged)
Then the first coordinate of the perspector above becomes the additive inverse and the other two coordinates are unchanged.
We can get the other two perspectors similarly. To sum up, the three perspectors are the vertices of the ancticevian triangle of the perspector above. The four perspectors form a quadrangle and its diapleural(diagonal) triangle is the reference triangle.
Given the incenter/excenter quadrangle and a central triangle, we can get a triangle center and its anticevian triangle(four perspectors). For example, given the anticomplementary triangle, we can get the perspector X(194) and its anticevian triangle (four perspectors). Because of the principle of duality, the duals of the above statements hold, too. We can study the perspectors derived from QA-Trx and QL-Trx.
Best regards, Tsihong Lau

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #1737
Date: 29/5/2016 3:59:15
From: seiichikiri
Subject: Anticenters in Quadri-Figures Geometry

Dear friends,
The following paper almost proves the statement of message #1119, 2)
See https://www.awesomemath.org/wp-pdf-files/math-reflections/mr-2014-01/newton_and_midpoints.pdf
See Theorem 3.
This paper is shown by Ngo Quang Duong.
Best regards, Seiichi.

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Message: #1738
Date: 30/5/2016 6:26:30
From: bernard.keizer
Subject: Dual Curves

Dear Eckart,
As I told you, I wasn't home in the last 2 weeks.
1) Back to QA-Cu1, which is the only circular pivotal isocubic wrt DT.
The 4 points Pi give the isotransformation QA-Tf2 wrt DT.
The pole is the isoconjugate of the centroïd of DT, id est the point QA-P16.
The isopivot for each pivot is simply the isoconjugate of the pivot.
The singular focus of QA-Cu1 is the point QA-P9.
The point QA-P41 is of course QA-Tf2(QA-P4), but it is also ig (inverse in circumDT of the isogonal conjugate wrt DT) of QA-P4.
This opens a chain like mentionned by Tsihong Lau, as the next will be igQA-P41 wrt QA-DDT ...
By the way, QA-P4 is igPi wrt the triangle PjPkPl and also the reflexion of QA-P2 in the isoconjugate of the circumcircle of DT (line which passes through the point QA-P6, the QA-Tf2 of QA-P6 being the diametral point of QA-P2 on the circumcircle of DT).
2) Nothing to add to this summary of my 5th and 6th conics (which mentions the QL-DDT).
The 1rst parabola is obviously DT-inscribed.
The dual of QA-P1 is a tangent to QL-Co1 in the point where the Newton Line cuts the parabola ; it is the Newton Line of QL-DDT.
For me, the main merit of these 2 conics issued from the QA environment is that their intersections gives the vertices of QL-Tr2, the Dimidium circle and the points QL-P1, QL-P17, QL-P24 and CSC(QL-P26).
These 2 conics are the polar conics of the pivot QA-P16 and the isopivot QA-P10 of the pivotal isocubic pK(QA-P16, QA-P16) where pivot = pole.
This cubic is through QA-P10 and tangent to the medians of DT in the vertices.
We have probably another chain, the isopivot wrt DT being always the pivot wrt DDT and so on ...
3) Of course, you're right, the dual of QA-Co1 is the Steiner inellipse of DT.
I suppose, I read your messages 1499 and 1501 too quickly ...
Best regards
Bernard

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Message: #1739
Date: 31/5/2016 2:40:00
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
thanks for your explinations wrt QA-Cu1: Reason for my misunderstandings: I interpreted "ig" as "isogonal" and not in the sence of Bernard Gibert as "inverse of isogonal", please excuse!
My interest grows in duality geometrie, studying QA and QL in a common constellation. For example: Your 5th and 6th conic have as well properties in QA- as in QL-environment. I regret, that I can't follow the analytic aspect of Tsihon Lau. My results are only Cabri controlled with real constructions.
Is the following already mentioned?
(1) For dual QA and QL the isoconjugation wrt the common DT and fixed point QL-P13 swaps QL-L1 and QA-Co1.
(2) The discussed duality can also be decribed wrt QL-Tr2 and its isoconjugation Tf, which swaps QL-P12 and QL-P13:
... for a point P: tripolar of the Tf-image of P,
... for a line L: Tf-image of the tripol of L.
Tf swaps also QL-P8 and the tripol X of QL-L1 (on QL-L9).
The Tf-image of the 5th conic is the line X.QL-P12.
The Tf-image of the 6th conic is the line Y.QL-P12,
... with Y pole of X.QL-P12 wrt QL-Co1.
Finally a curious connection of the 5th and the 6th conic:
5th (6th) conic Co
... L = Tf of Co,
... P = pole of L wrt QL-Co1,
... Tf of P.QL-P12
6th (5th) conic.
Best regards Eckart

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Message: #1740
Date: 31/5/2016 9:51:18
From: bernard.keizer
Subject: Dual Curves

Dear Eckart,
Thanks for your quick answer !
1) I'm not sure I understand your formulation " wrt DT and with fixed point QL-P13 " ...
Could you precise either a copple of isoconjugate points or 4 fixed points.
Your isoconjugation swaps QA-Tf2(infinity line), which is QA-Co1 and QL-Tf2(infinity line), which is the Newton Line.
Maybe it works generally and swaps any DT-circumconic with QL-Tf2 of it's QA-Tf2 (QA-Tf2 of the DT circumcircle is for example the perpendicular bisector of QA-P2QA-P4 and it's QL-Tf2 should be the isoconjugate of the DT-circumcircle ...)
2) Brillant and very promising idea to take QL-Tf2 as the reference triangle with the given isoconjugation swapping QL-P12 and QL-P13 !
Maybe it will be possible to explain all the conics through the vertices of QL-Tr2 (the 4 through one reference triangle of QL, it's centroïd and the Pi dual of the 4th line, the 5th and 6th, the 3 QA-Co1 of 3 QA's of the QL and of course the Dimidium circle itself, which makes 10 conics !)
Best regards
Bernard

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Message: #1741
Date: 01/6/2016 12:52:30
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
wrt 1) of your #1740: The four fixed points of the
DT-isoconjugation Tf are QL-P13 and the vertices of your QL-DDT
(QG-P13-Triple).
Wrt the pole of this isoconjugation I only found: Its
DT-tripolar bears QL-P23 and its dual is parallel QL-L9.
This isoconjugation doesn´t work generally swapping any
DT-circumconic with QL-Tf2 of its QA-Tf2.
wrt 2) There will be more interesting circumconics of QL-Tr2,
for example:
QL-Tr2-circumconic through QL-P12, QL-P13:
... locus of QL-Tr2-tripols of parallels to QL-L1,
... through the 2nd intersection of QL-P1.QL-P12 and QL-Ci6,
... Tf-image of QL-P8.QL-P13.
Best regards Eckart

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Message: #1742
Date: 01/6/2016 11:49:53
From: bernard.keizer
Subject: Dual Curves

Dear Eckart
wrt 2) the pole of this 2nd Tf isoconjugation is then again
QL-P13 = QA-P16 like for QA-Tf2 !
Have you found the 4 fixed points ?
Your last conic is quasi per definition the image in this
isoconjugation of the line QL-P12QL-P13 ...
Intriguing this 2nd intersection of QL-P1QL-P12 with the
Dimidium circle ; it reminds the D-points (2nd intersection of
the segments joining QL-P1 to the vertices of DT, the same with
QL-P24, 2nd intersection of QL-P17QL-P13) and 2nd intersection
of QL-P1CSC(QL-P26) ...
I'm sure you're right and there are more interesting
circumconics of QL-Tr2.
Thanks in advance for your answer !
Best regards
Bernard

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Message: #1743
Date: 02/6/2016 11:22:26
From: eckart_schmidt@t-online.de
Subject: Dual Curves

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Dear Bernard,
excuse, there is a typo in #1741: In 2) the circumconic of
QL-Tr2 bears QL-P12 and QL-P23!
The QL-Tr2-circumconic through QL-P12, QL-P13 is of course
Tf(QL-P12.QL-P13),
... with parallel QL-Tr2-tripolars,
... and a QL-Tr2-inscribed parabola as dual curve.
Best regards Eckart
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Message: #1744
Date: 02/6/2016 1:56:17
From: tsihonglau
Subject: QA-Cu1 and QL-Tr2

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Dear all
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>I guess pivot chain is correct and applies to not
>>only triangle/trilateral but quadrangle/quadrilateral.
>>sqrtdiv should be replaced with i nvolutary conjugate.
>>I do not know what cevadiv should be replaced with.
cevadiv remains cevadiv
Best regards,
Tsihong Lau
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Message: #1745
Date: 02/6/2016 6:22:28
From: bernard.keizer
Subject: Dual Curves

Dear Eckart,
Many, many beautiful properties at the same time !
I believe, I need a little rest ...
Did you find the 4 fixed points of your last Tf wrt QL-Tr2 ?
(Starting as you do with a copple of isoconjugates, is there a simple way to find them synthetically ?)
And what about this 4th intersection of this last conic with the Dimidium circle, in line with QL-P1 and QL-P12 ?
Best regards
Bernard

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Message: #1746
Date: 04/6/2016 12:13:51
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
wrt the fixed points of an isoconjugation, knowing a copple of isoconjugates, see my homepage 06-5 (15).
The fixed points of Tf (QL-Tr2-isoconjugation, which swaps QL-P12, QL-P13)
... lie on QA-Co1,
... their DT-tripolars bear QL-P13.
Further observations wrt dual curves:
... The dual curve of a parabola bears QL-P13.
... The dual curves of QL-Tr1-inscribed parabolas (focus F on the circumcircle QL-Ci1)
... ... are QL-Tr1-circumconics
... ... through QL-P13,
... ... through the 2nd intersection of QL-Ci1 and F.QL-P13.
... The dual curves of QL-Tr2-inscribed parabolas (focus F on the circumcircle QL-Ci6)
... ... are QL-Tr2-circumconics
... ... through QL-P13,
... ... through the 2nd intersection of QL-Ci6 and F.QL-P13.
Final remark wrt the QL-Tr2-inscribed parabolas:
... directrices bear QL-P2,
... axes envelop a deltoid, circumscribed QL-Ci6 with axes parallel to the QL-Cu2-asymptotes.
Best regards Eckart

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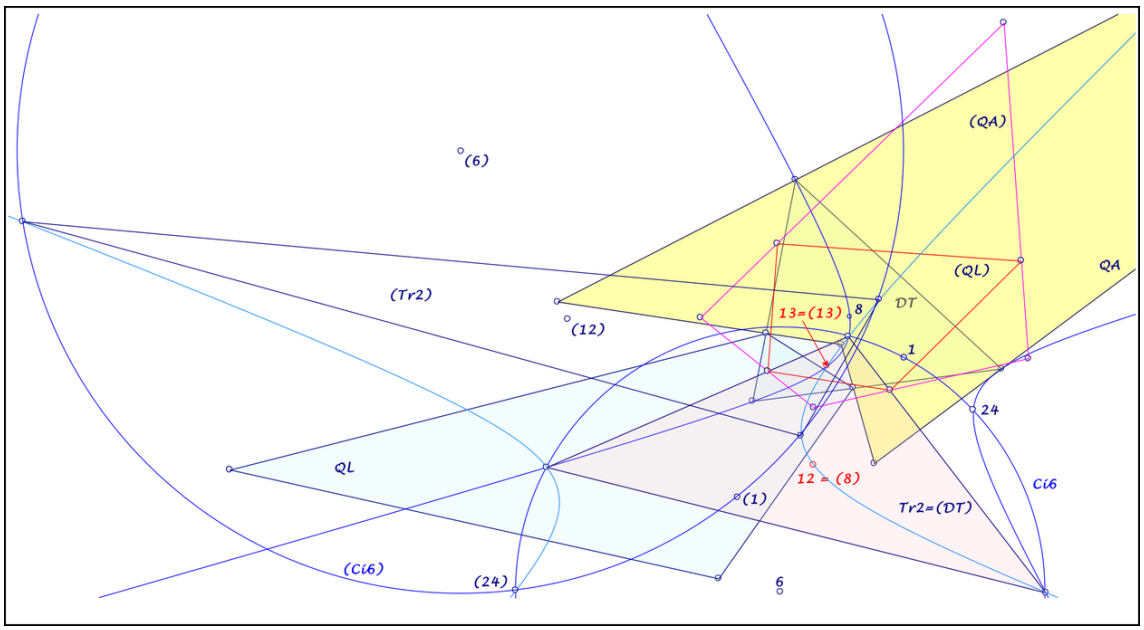
Message: #1747
Date: 06/6/2016 9:37:21
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard,
there are further observations wrt QL-Tr2 ...
... with its isoconjugation Tf,
... ... which swaps QL-P13 and QL-P12,
... ... which maps the tripolar of QL-P13 to the circumscribed Steiner-ellipse,
... and its line-isoconjugation Tf*,
... ... which swaps the tripolars of QL-P13 and QL-P12,
... ... which maps the QL-P13-line-pencil to the inscribed Steiner ellipse.
If we consider dual QA and QL with common diagonal triangle DT, there is a 2nd generation of this constellation (in brackets):
(QA) quadrangle of the fixed points of Tf ,
(QL) quadrilateral of the fixed lines of Tf*,
(QA) and (QL) are dual with common (DT) = QL-Tr2.

Properties:
... (QL-P13) = QL-P13,
... (QL-P12) = dual of the line, connecting QL-P12 with the QL-Tr2-tripol of QL-P12.QL-P13,
... (QL-P8) = QL-P12,
... (QL-L1) = tripolar of QL-P13,
... (QL-Tr2) and QL-Tr2 have a common circumconic Co through QL-P12 and QL-P13 (conic in #1741),
... ... this is the (QL-P8)-(QL-P13)-(QL-P24) circumconic of (QL-Tr2),
... ... intersecting QL-Ci6 and (QL-Ci6) in (QL-P24).
There will be more properties.
Finally: I think, the following descriptions for duality are remarkable:
(1) Wrt QL-Tr2 and its isoconjugation Tf, which swaps QL-P12 and QL-P13
(see #1739):
... for a point P: tripolar of the Tf-image of P,
... for a line L: Tf-image of the tripole of L.
(2) Wrt QL-Tr2 and its line-isoconjugation Tf*, which swaps the tripolars of QL-P12 and QL-P13:
... for a point P: Tf*-image of the tripolar of P,
... for a line L: Tripole of the Tf*-image of L.

Best regards Eckart



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Message: #1748
Date: 06/6/2016 10:53:07
From: bernard.keizer
Subject: Dual Curves

Lieber Eckart,
Erst mal vielen Dank für diese neue Entwicklungen !
Thanks to you, I feel a little less stupid about these S-points, vertices of QL-Tr2, for which I have a great interest since a long time.
Any group of 4 points Pi leads to a DT and to a isoconjugation Tf, which swaps the centroïd G of DT with the pole Q of the transformation and any point P taken as pivot with the isopivot P* and QxG = Q = PxP*.
The polar conic of P wrt the cubic pK(Q,P) is the diagonal conic through the 4 points Pi and P and the polar conic of P* is DT-the circumconic through P and P*.
A key property is that the diagonal conic bears the vertices of the anticevian triangle of P and more generally, the vertices of the anticevian triangles of any of it's points.
If we take P = Q, it means pivot = pole and isopivot = centroïd G of DT, the polar conics of Q and G wrt the cubic pK(Q,Q) intersect in 4 points, one is the pole Q by definition and the 3 others give the vertices of QL-Tr2.
Now, Q = QA-P13 = QL-P16 for DT and for Tr2 and G = QL-P8 for DT and QL-P12 for Tr2 ...
1) The fact that my 6th conic through the Pi and QL-P13 bears the vertices of DDT is obvious, as DDT is precisely the anticevian triangle of QL-P16.
2) The fact that QA-Co1 is a diagonal conic through the fixed points of the isoconjugation wrt Tr2 is absolutely not obvious and deserves a description in EQF ; this conic contains therefore the vertices of the anticevian triangles of all it's points wrt Tr2 (for example QA-P2).
3) DT and Tr2 are directly linked and it appears possible
a) to take Tr2 as a DT and to find it's QA, QL and Tr2 as you do
b) to take DT as a Tr2 and to find it's originel QA, QL and DT
...
This opens again an infinite chain in both directions !
Best regards
Bernard

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Message: #1749
Date: 07/6/2016 10:42:14
From: bernard.keizer
Subject: Dual Curves

Dear Eckart,
As you say yourself, there are plenty more properties, directly reproduced from the QA/QL and it's DT and Tr2.
For example, for QL-Tr2 = (DT),
(QL-P17) as the 2nd intersection between Ci6 = (Ci1) and (Ci6)
(QL-P1) as one intersection between (Ci6) and circumcircle of the medial triangle of Tr2 = (QL-Ci2)
the common circumconic of QL-Tr2 and (QL-Tr2) is the socalled 5th conic for (QL) ; by the way, it is not the conic described in message 1741, as you corrected yourself a typo in message 1743 ...
I haven't started yet the reverse construction with DT = (QL-Tr2) ...
Best regards
Bernard
PS The socalled 6th conic trough the QA-vertices and QL-P13 = QA-P16 bears, as we saw, the vertices of DDT, as DDT is the anticevian triangle of QL-P13 wrt DT ; this conic bears also QA-P1 and therefore the vertices of it's anticevian triangle !

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Message: #1750
Date: 07/6/2016 3:52:52
From: tsihonglau
Subject: Dual Curves

Dear Bernard,
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>The point QA-P41 is of course QA-Tf2(QA-P4), but it is also ig (inverse in circumDT of the isogonal conjugate wrt DT) of QA-P4.
>>This opens a chain like mentionned by Tsihong Lau, as the next will be igQA-P41 wrt QA-DDT ...

I do not know if your statements are true or not.
I present my assertion here. Given a P-pivotal isocubic(i.e. fixed under involutary conjugation QA-Tf2).
Suffices _i and _c represent involutary respectively. Then we can get a pivot chain (not verified).
steps conjugates->pivots X, X_i->P
1. X, X_c->P_i
2. X, ((X_i)_c)_i->(P_i)_c
3. X, ((X_c)_i)_c->((P_i)_c)_i
4. X, ((((X_i)_c)_i)_c)_i->(((P_i)_c)_i)_c
5. X, ((((X_c)_i)_c)_i)_c->((((P_i)_c)_i)_c)_i
...
Odd step, swap _i and _c in the conjugate and add _i to the pivot in the previous step,
Even step, circumfix the conjugate with two _i's and add _c to the pivot in the previous step.
The above algorithm is an adaptation of this one mentioned in APG topic #2364
View on groups.yahoo.com
(https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/topics/2364)
Preview by Yahoo
I guess it is true and cannot prove it!
Best regards,
Tsihong Lau

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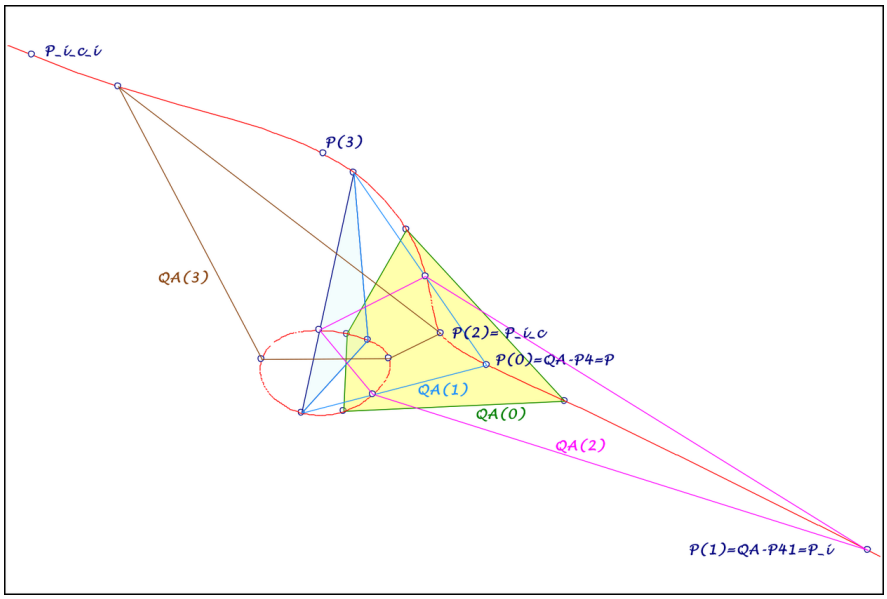
Message: #1751
Date: 08/6/2016 12:32:19
From: eckart_schmidt@t-online.de
Subject: Dual Curves

Dear Bernard, dear Tsihong Lau,
I try, to understand your discussion wrt a "pivot chain".
If I am right, Bernard considers a chain of quadrangles QA(n),
... beginning with a reference quadrangle QA(0),
... and recursive:
    QA(n+1) = diagonal triangle of QA(n) and QA-P4 of QA(n).
These quadrangles have the same cubic QA-Cu1, isocubic wrt the
diagonal triangle of QA(n), pivot P(n) in QA-P4 of QA(n) and an
isoconjugation with fixed points in the vertices of QA(n):
Pivot chain: P(0) = QA-P4, P(1) = QA-P41, ...
Now I try to reproduce Tsihong Laus pivot chain:
... beginning with the diagonal triangle QA-Tr1
    of a reference quadrangle,
... an isoconjugation _i, which is QA-Tf2,
... and a pivot P = P(0) = QA-P4.
Question wrt "1. X,X_c->P_i": I can't interpret X,X_c as
isoconjugation.
Using _c as P-ceva conjugate wrt QA-Tr1 we get the ...
... pivot chain:
    P = P(0) = QA-P4, P_i = P(1) = QA-P41, (P_i)_c = P(2), ...
... but ((P_i)_c)_i unequal P(3) (see attached file).
Were are my misunderstandings? Thanks in advance.
Best regards Eckart
PS: P, (P_i)_c, ((P_i)_c)_i are collinear.



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Message: #1752
Date: 08/6/2016 4:22:46
From: tsihonglau
Subject: Dual Curves

Dear Eckart,

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>Now I try to reproduce Tsihong Laus pivot chain:
>>... beginning with the diagonal triangle QA-Tr1 of a reference quadrangle,
>>... an isoconjugation _i, which is QA-Tf2,
>>... and a pivot P = P(0) = QA-P4.
>>Question wrt "1. X,X_c->P_i": I can't interpret X,X_c as isoconjugation.
>>Using _c as P-ceva conjugate wrt QA-Tr1 we get the ...
>>... pivot chain: P = P(0) = QA-P4, P_i = P(1) = QA-P41, (P_i)_c = P(2), ...
>>... but ((P_i)_c)_i unequal P(3) (see attached file).

If we use unitary coordinate system - i.e. the coordinates of the reference quadrangle = 1:1:1,-1:1:1,1:-1:1,1:1:-1, then the involutary conjugate QA-Tf2 of X(x:y:z) = 1/X(1/x:1/y:1/z). And P-cevadiv(ceva conjugate) of X is as usual x(-qrx+rpy+pqz):...

Now I present the first coordinates of the pivot chain steps conjugates->pivots X(x),X_i(1/x)->P(p)
1. X(x),X_c(x(-qrx+rpy+pqz))->P_i(1/p)
2. X(x),((X_i)_c)_i(x/(-qryz+rpzx+pqxy))->(P_i)_c((-q^2r^2+r^2p^2+p^2q^2)/p)
3. X(x),((X_c)_i)_c((p^2*q^3*r*y*z^3-p^3*q^2*r*x*z^3-2*p^2*q^2*r^2*y^2*z^2+p^3*q^3*x*y*z^2+2*p^2*q^2*r^2*x^2*z^2+p^2*q*r^3*y^3*z+p^3*r^3*x*y^2*z-q^3*r^3*x^2*y*z-p*q^2*r^3*x^3*z-p^3*q*r^2*x*y^3+2*p^2*q^2*r^2*x^2*y^2-p*q^3*r^2*x^3*y)/(x*(p*q*z+p*r*y-q*r*x)))->((P_i)_c)_i(p/(-q^2r^2+r^2p^2+p^2q^2))
...
I have verified the above steps but the following are not verified. I guess it is true.
If so, not only pivot chain but conjugate chain exists.

Best regards,
Tsihong Lau

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Message: #1753
Date: 09/6/2016 10:54:55
From: bernard.keizer
Subject: Chain of Tr2

Dear Eckart,
Starting with a QA of 4 points Pi, it's DT and the corresponding QL, the 3 S-points, vertices of QL-Tr2 are determined as the 3 intersections other than QL-P13 of the socalled 5th and 6th conic : Co5 is circumDT through QL-P8 and QL-P13 and Co6 is through the 4 Pi, QA-P1 and QL-P13 = QA-P16.
QL-P13 is the pole, isoconjugate of the centroïd of DT, of the isoconjugation Tf with fixed points the 4 Pi.
Let's name DQL the QL formed by the diagonals and the Newton Line and DX the corresponding element X of DQL ; the same way, we could name dQL one QL of which the DQL is QL (there are 4 dQL's for one QL) and dX the corresponding element of dQL.
Co6 is through the vertices of the anticevian triangle of QL-P13, which is DDT.
Let's now consider the reference triangle Ti of QL, formed by the lines Lj,Lk and Ll. It's vertices are Pjk, Pjl and Pkl. The isoconjugation dTf swapping the centroïd and the point Pi has for fixed points 4 points dPi.
dCo5 is through the vertices of the reference triangle Ti, it's centroïd and Pi ; dCo6 is through the 4 dPi, their centroïd and Pi and the vertices of it's anticevian triangle wrt Ti, which is DT.
So we have dTr2 and it's circumcenter the dDimidium circle through dQL-P1, QL-P1(dQL-P17) and the 2nd intersection of the circumcircle of Ti or dcircumDT with the dDimidium circle ; this point is dD24 or the D-point mentionned in EQF.
The key-property is that this 5th conic of dQL is also through the vertices of QL-Tr2 ; the dDimidium circle and the Dimidium circle intersect in the same 2 points QL-P1 or dQL-P17 and the D-point or dQL-P24.
It's easy to go on : the DPi are the point QL-P13 and the vertices of it's anticevian triangle wrt DDT (like the point Pi and the points Pj, Pk and Pl, which are the vertices of the anticevian triangle of Pi wrt DT).
We find the DTr2 as intersection of DCo5 and DCo6 ; the DDimidiumcircle, the Dimidium circle and the DT circumcircle intersect in the 2 points QL-P17 and QL-P24 ...
The chain of the 5th conics Co5 and of the Tr2 seems interesting : every Co5 is through 2 consecutive QL-Tr2 and the S-points or vertices of every QL-Tr2 are on 2 consecutive Co5.
The same way, one Dimidium circle bears 2 consecutive points QL-P1 or P17 and 2 consecutive points QL-P24 and each of these points belongs to 2 consecutive Dimidium circles.

I hope this could be of interest ...
Best regards
Bernard
PS I haven't identified the 4th point of intersection of
2 consecutive Co5.

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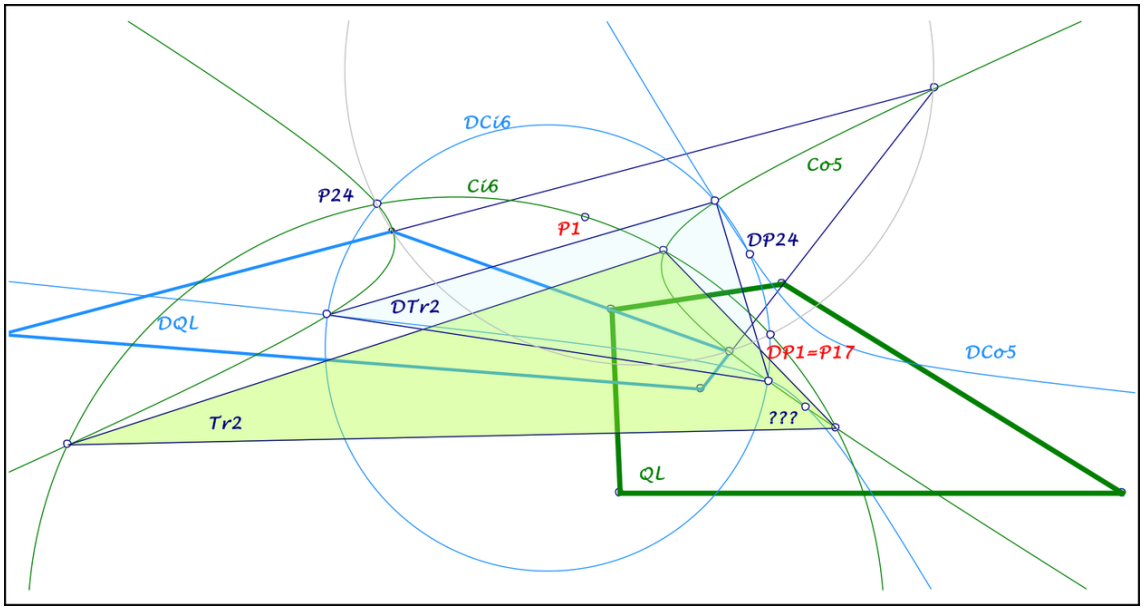
Message: #1754
Date: 09/6/2016 8:59:27
From: eckart_schmidt@t-online.de
Subject: Chain of QL-Tr2

Dear Bernard,
Thanks for your interesting detailed message. I have made a consruction (see attached file, perhaps helpfully).
Wrt the 4th intersection of Co5 and DCo5, here a possible construction:
... Take the line QA-P1.QA-P10
... and the DT-tripolars of its points,
... which envelop a parabola.
... Consider a tangent parallel QL-L1,
... its DT-tripol is the searched point (on QA-P1.QA-P10).
Best regards Eckart
PS: There will be typos in your #1748:
"Now, Q = QA-P16 = QL-P13
for DT and for Tr2 and G = QL-P8 for DT and QL-P12 for Tr2 ...
1) The fact that my 6th conic through the Pi and QL-P13 bears the vertices of DDT is obvious, as DDT is precisely the anticevian triangle of QA-P16."



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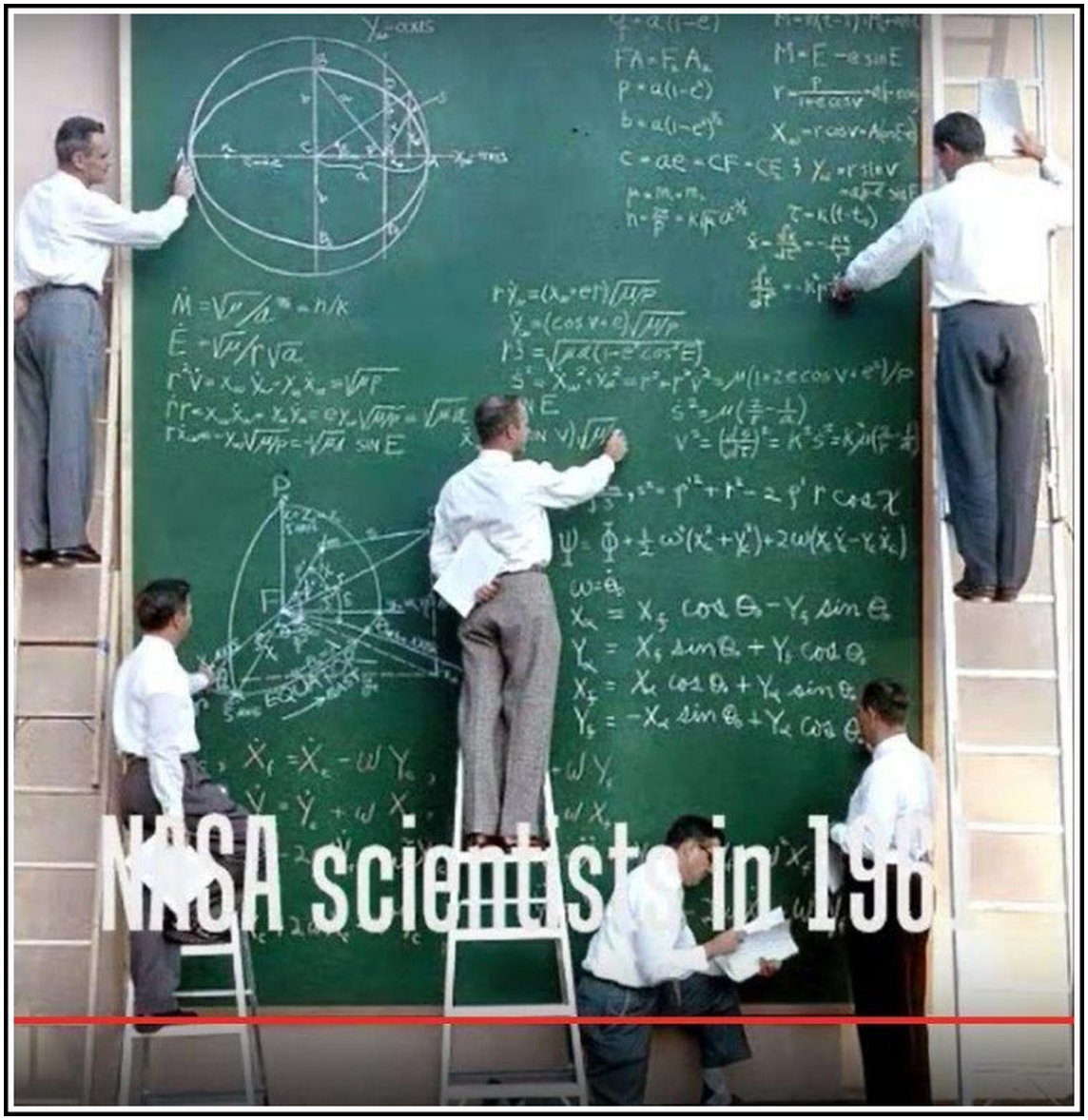
Message: #1755
Date: 10/6/2016 2:22:44
From: chris.vantienhoven
Subject: NASA geometry 1963

Geometry is of alle times.
See attached picture about NASA geometry in 1963.
Best regards,
Chris



nasa wiskunde 1963.jpg

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Message: #1756
Date: 12/6/2016 8:21:00
From: seiichikiri
Subject: NASA geometry 1963

Dear Chris,
I was fifteen in 1963. Then I learned factorization of polynomial, Pythagorean theorem and so on. Naturally I did not think a bit of numerical or symbolic computations. Now we are entering in the stage of automated discovery and proof. Are we happy because there remain things to discover and prove yet?
Best regards, Seiichi.

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Message: #1757
Date: 13/6/2016 10:40:20
From: chris.vantienhoven
Subject: NASA geometry 1963

Dear Seiichi,
In 1963 I was 13 years old.
At boring Sundays I started to make geometrical drawings like a regular 64-gon with all its diagonals. That gave beautiful pictures. Also I made drawings of Cardioids using expanding circles.
My father encouraged me to show them to the director of the school en he was impressed.
At secondary school I read in a student paper (named Pythagoras) about the trisection of an angle. I learned then that it was impossible to perform the trisection with ruler and compass. However with a self-made tool named “tomahawk” it should be possible to perform the trisection. Of course I made this tomahawk of cardboard and I still conserve it together with a compass and a wooden triangle in an empty metal cigar box of my father.
When I was 16 or 17 years old I studied the Brocard angles. Also I learned about Van Aubels Theorem and proved it when challenged by a friend. For the first time I was impressed about a Quadri-Figure!
Just like you good memories about geometry.
And yes I am very happy there remain so many things to discover and prove yet.
Are there more memories of QFG-members from 1963 (or later)?
Best regards,
Chris

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Message: #1758
Date: 15/6/2016 12:20:53
From: eckart_schmidt@t-online.de
Subject: Dual Cubics and Quartics

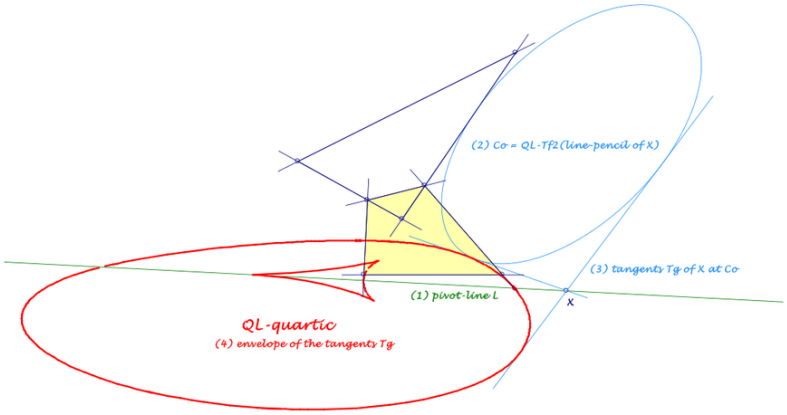
Dear Bernard, dear Chris,
attached another aspect of duality: Duals for pivotal
QA-isocubics of the type QA-Cu1,2,3,4,5, which give QL-quartics.
Something will be obvious, but I have worked it out, for it was
new for me.
Best regards Eckart

EQF-Note 2016-06-15

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Dual Cubics and Quartics

The construction concept for points of pivotal QA-isocubics can be used for lines, to get QL-quartics, whose duals are pivotal QA-isocubics of a well-known type.



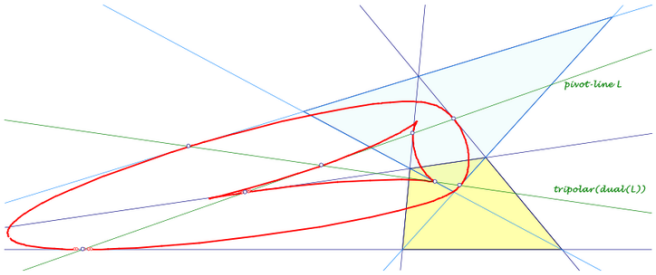
Pivotal QA-Cubics

QA-Cu1, 2, 3, 4, 5 are pivotal isocubics wrt the diagonal triangle *DT = QA-Tr1* and the isoconjugation *QA-Tf2*. A construction concept can be described as follows:
Let *P* be the pivot-point,
… take lines *L* through *P*
… and their *QA-Tf2* images, which give *DT*-circumconics *Co*.
… The intersections of *L* and *Co* give the pivotal *QA*-isocubic.

Pivotal QL-Quartics

This construction concept can be translated in *QL*-geometry, using the diagonal triangle *DT = QL-Tr1* and the line isoconjugate *QL-Tf2* (see figure above):
Let *L* be a pivot-line,
… take points *X* on *L*,
… and the *QL-Tf2* images for the line pencil of *X*, which envelop a *DT*-inscribed conic *Co*.
… The tangents of *X* at *Co* envelop the pivotal *QL*-quartic.

- **Pivotal *QL*-quartics are**
  … tangent to the four *QL*- lines
  … … with contact points on the pivot-line,
  … tangent to the three *DT*-lines
  … … with contact points on the tripolar(dual(*L*)),
  … tangent to the pivot-line
  … … with contact point also on tripolar(dual(*L*)).

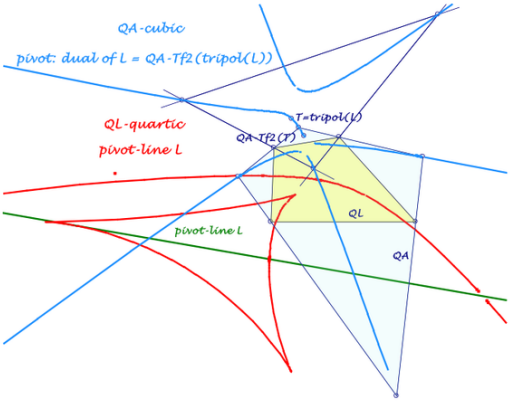


If the pivot-line bears a *QL*-point *S*, the *QL*-quartic degenerates in the point *S* and a conic, tangent to two *QL*-lines and two diagonals, not containing *S*.

- **Pivotal *QL*-quartics are *QL-Tf2*-invariant wrt their tangents (*QL-Tf2* swaps tangents from *X* at *Co*).**

Dual QL-Quartics and QA-Cubics

Now we consider a constellation of dual *QA* and *QL* with common diagonal triangle *DT* (in the sense of *QFG*-message 1516).



- **The dual of a pivotal *QL*-quartic with pivot-line *L* is a pivotal *QA*-isocubic**
  … wrt the common diagonal triangle *DT*,
  … the isoconjugation *QA-Tf2*
  … and a pivot, which is the dual of the pivot-line *L*, the *QA-Tf2* image of the *DT*-tripol of *L*.

On the other hand:

- **The dual of a pivotal *QA-Tf2*-isocubic with pivot *P* is a *QL*-quartic with pivot-line dual(*P*), the *QL-Tf2*-image of the *DT*-tripolar of *P*.**

Examples

- ***QA-Cu1*: dual of a *QL*-quartic with pivot-line tripolar(*QA-P41*).**
- ***QA-Cu2*: dual of a *QL*-quartic with pivot-line tripolar(*QA-P17*).**
- ***QA-Cu3*: dual of a *QL*-quartic with pivot-line *QL-L1*.**
- ***QA-Cu4*: dual of a *QL*-quartic with pivot-line tripolar(*QA-P18*).**
- ***QA-Cu5:* dual of a *QL*-quartic with pivot-line tripolar(*QA-P20*) (orthogonal *QL-L6* through *QL-P19*).**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-06-15.pdf

Message: #1759
Date: 15/6/2016 3:48:44
From: eckart_schmidt@t-online.de
Subject: Dual Cubics and Quartics

Dear Bernard, dear Chris,
excuse, forget my last message! The quartics will be sextics, already described in # 1511.
I'm sorry Eckart

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Message: #1760
Date: 16/6/2016 10:27:17
From: bernard.keizer
Subject: Dual Cubics and Quartics

Dear Eckart,
This gave me the opportunity of reading again your very interesting message 1511 !
As the duality swaps points of a curve and tangents to it's dual curve (and vice-versa), the degree of the dual curve is necessary the class (number of tangents to the curve from any point) of the initial curve (and vice-versa).
That's why conics are duals of conics and sextics duals of cubics.
For quartics, it depends.
The duals of the QL-deltoïd and the 27 multi-cardioïds are QA circumscribed cubics ; the dual of the monocardioïd is also a cubic ...
The duals of QL-Cu1 and QL-Cu2 are also sextics.
The cayleyan is a sextic, envelop of the main and minor axes of the QL-inscribed conics or of any line XX' with X and X' CSC-conjugates on QL-Cu1 ; the tangents in X and X' to QL-Cu1 intersect in the tangential of X and X' on QL-Cu1. The dual will be a cubic with special properties ...
I drew part of these dual curves, but not all ; maybe you already did ...
Best regards
Bernard

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Message: #1761
Date: 20/6/2016 10:51:52
From: eckart_schmidt@t-online.de
Subject: Dual of QL-Cu1

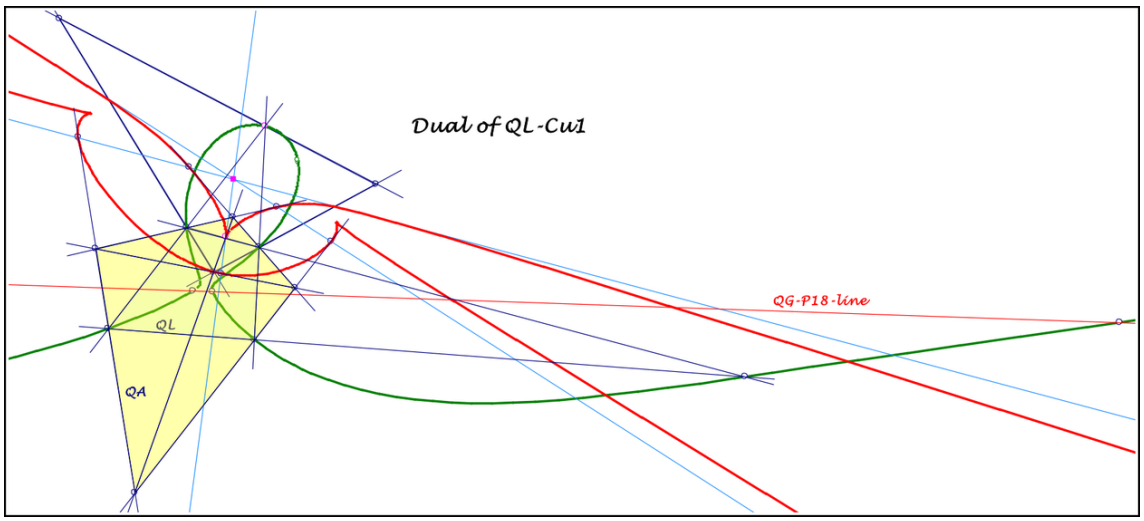
Dear Bernard,

thanks for your message 1760, you have a better overview. It was a good summary for me.
Here some observations for the dual of QL-Cu1, most of them will be obvious wrt the duality:

```
... The dual of QL-Cu1 is a sextic.
... The sextic contacts the QA-lines (see attached file).
... ... The contact points are the duals of the QL-Cu1-tangents
        in the QL-points.
... ... The three lines, connecting the contact points
        on opposite QA-lines,
... ... ... are the duals of the QG-P18-points of the QL,
... ... ... are tangent to the sextic,
... ... ... have a common point,
... ... ... which is the dual of the line,
            connecting the collinear QG- P18 of the QL.
... ... (The 4th harmonic points of the contact points
         on the QA-lines give a QL ...)
... QL-P8.QL-P13 is tangent to the sextic
    (dual of the point at infinity of QL-Cu1)
... The cusps are the duals of the inflection tangents.
... The duals of the QG-P17-points
    of the QL (DT-orthic triangle) are the 3rd tangents
    from the DT-vertices at the sextic.
... The duals of QL-2P2 in the bipartite case of QL-Cu1
    are tangents from QL-P8 at the sextic.
... The duals of the three intersections of QL-P8.QL-P13
    and QL-Cu1 are tangents to the sextic parallel QL-L1.
... In general: The duals of the intersections of QL-Cu1
    and a line L are the tangents from dual(L) to the sextic.
```

By the way, is already mentioned, that for a QG the lines QG-P18.QG-2P2 are the QL-Cu1-tangents in QG-2P2?

Best regards Eckart



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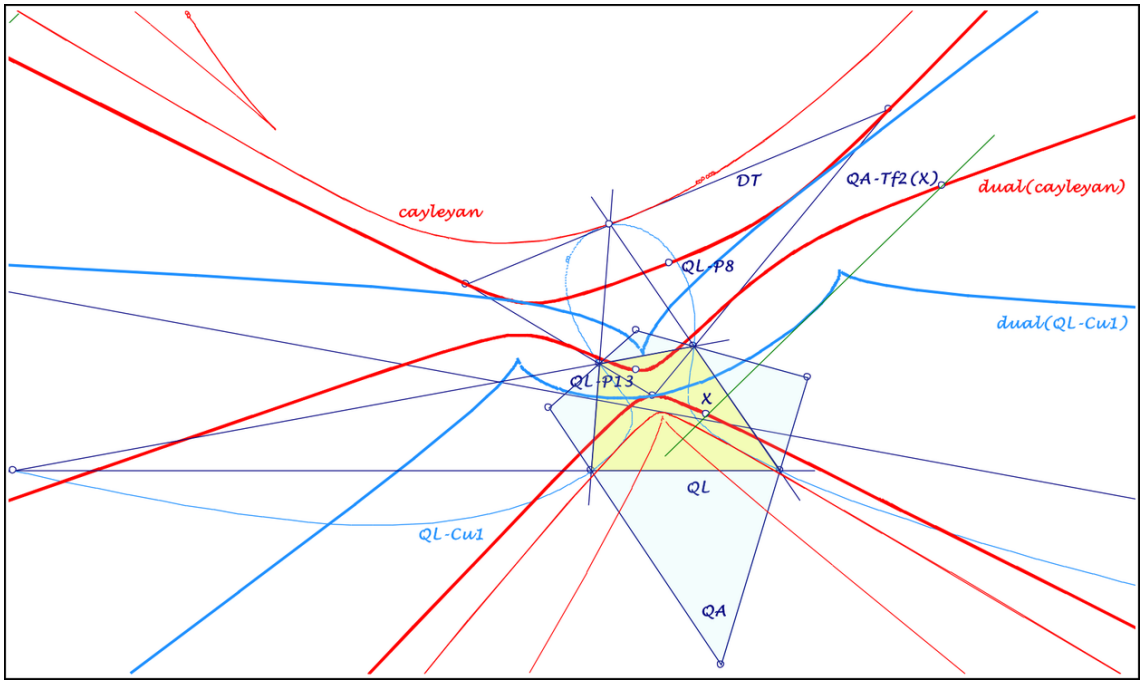
Message: #1762
Date: 25/6/2016 11:12:18
From: eckart_schmidt@t-online.de
Subject: Dual of the Cayleyan

Dear Bernard,
I don´t know whether the following is already mentioned (see attached file):
... The dual of the cayleyan is a nonpivotal isocubic
... ... reference triangle DT,
... ... isoconjugation QA-Tf2,
... ... pole QA-Tf2(QA-P12) = QA-Tf2(QL-P10),
... bearing QL-P8 and QL-P13,
... X.QA-Tf2(X) tangent to dual(QL-Cu1) for X on dual(cayleyan).
Best regards Eckart



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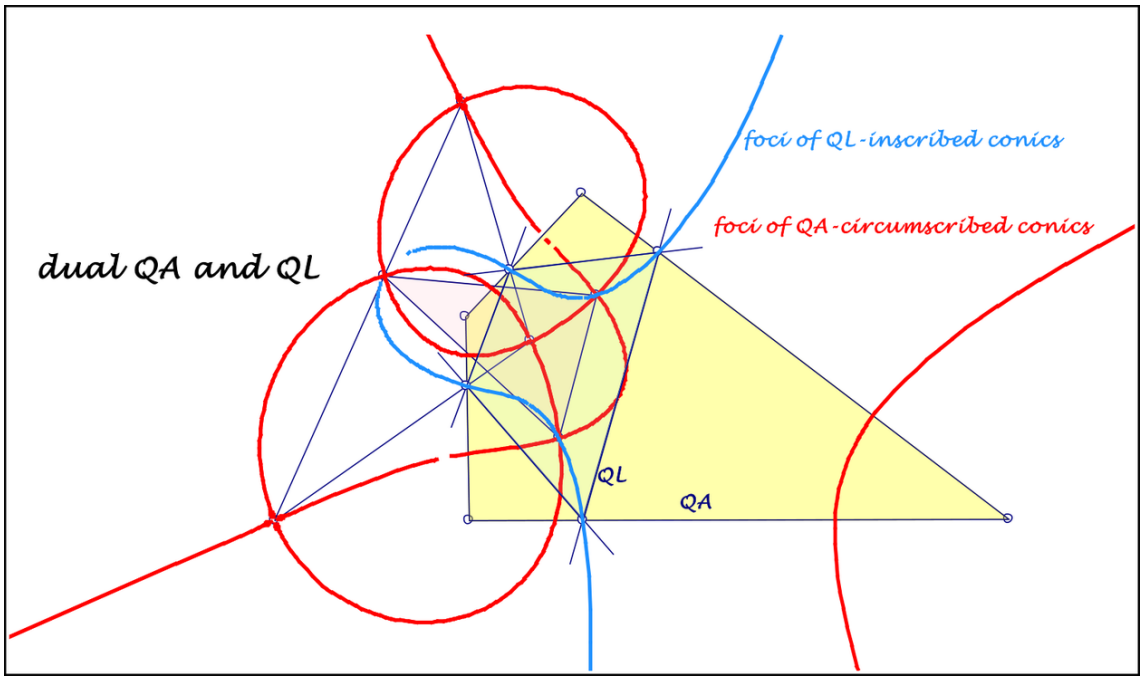
Message: #1763
Date: 26/6/2016 3:46:37
From: eckart_schmidt@t-online.de
Subject: QG-P17 as focus

Dear Bernard, dear Chris,
QG-17 is a focus of an inscribed QG-conic, for QG-P17 is a point on QL-Cu1, 2nd focus is QG-P18 (see EQF).
QG-P17 is focus of two circumscribed QG-conics, for QG-P17 is a knot of the sextic in #496.
2nd foci are the intersections of (see 1st attached file)
... a homothetic conic wrt QA-Co1, center QG-P17 and ratio 2:1,
... and the opposite side of QG-P17 wrt the orthic triangle of QA-Tr1.
For dual QA and QL the vertices of the DT-orthic triangle are the only points, which are focus of two QA-circumscribed conics and focus of a QL-inscribed conic (see 2nd attached file).
Best regards Eckart



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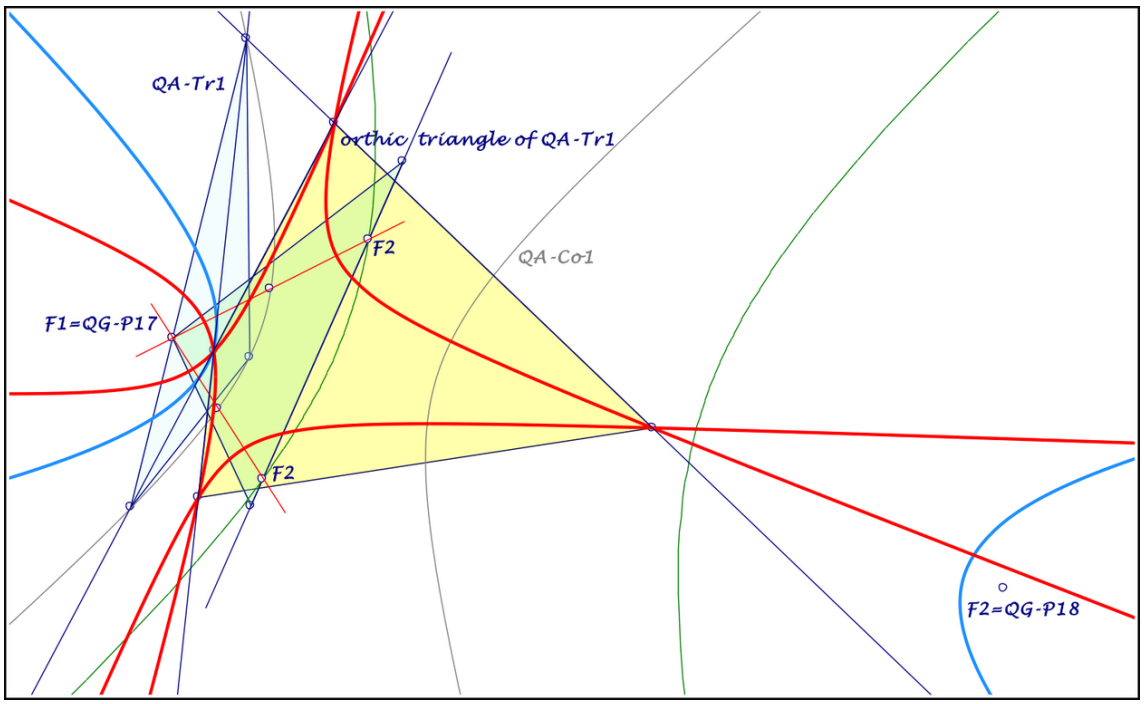

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Message: #1764
Date: 27/6/2016 8:33:33
From: eckart_schmidt@t-online.de
Subject: Dual of the Cayleyan

```
Dear Bernard,
excuse , there is a typo in #1762:
... The dual of the cayleyan is a nonpivotal isocubic
... ...
... ... *root* QA-Tf2(QA-P12) = QA-Tf2(QL-P10).
Best regards Eckart
```

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Message: #1765
Date: 27/6/2016 8:36:32
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Chris,
in EQF only the first two properties of QL-Cu1 are mentioned:
QL-Cu1 is a nonpivotal isocubic
... reference triangle: orthic triangle of QL-Tr1,
... isoconjugation: isogonal conjugate,
... root: trilinear pole of the QG-P18-line.
The fixed circle wrt Bernard Gibert`s theorem 1.5.5 degenerates
into the line at infinity and QL-L1.
Best regards Eckart
```

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Message: #1766
Date: 29/6/2016 11:04:35
From: bernard.keizer
Subject: QL-Cu1

Dear Eckart,
Many thanks for your beautiful figurs of the duals of QL-Cu1 and the cayleyan and for the mentionned properties.
For QL-Cu1, if I'm not wrong, it's always a circular focal isocubic (the curve contains the singular focus and the 2 infinite points).
But it's non pivotal only in the unicursal case (MacKay's hessian).
In the bicursal case (Kjp's hessian), it is pivotal (the reference triangle is the triangle QL-P1QL2P2a and b and the pivot is the infinite point of the perpendicular bisector of QL2P2a and b).
And if I'm not wrong again, in the non pivotal case, any triangle such as it's circumcircle is through QL-P1 may be a reference triangle ...
The orthic triangle is a special case (the circumcircle of the medial triangle of DT is through QL-P1 and the parabola QL-Co1 is inscribed in this medial Triangle of DT).
Best regards
Bernard

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Message: #1767
Date: 29/6/2016 11:23:00
From: bernard.keizer
Subject: Harmonic conjugates

Dear Chris, dear Eckart
A friend of mine sent me recently this property (I don't know if it's already known) with a dutch reference for Chris !
The harmonic conjugates of a point wrt the 3 pairs of opposite vertices of a QL lie on a straight line or a circle which passes through the symmetric of the point wrt the Miquel point. (see Nieuw tijdschrift voor wiskunde, 35th year, 1947-48, p.151)
Best regards
Bernard

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Message: #1768
Date: 30/6/2016 12:00:28
From: Antreas Hatzipolakis
Subject: O, midpoints, perp. bisectors, orthologic, locus

```
[APH]:
Let ABC be a triangle and P a point.
Denote:
Oa, Ob, Oc = the circumcenters of PBC, PCA, PAB, resp.
Bc, Cb = the midpoints of BOc, COb, resp.
Ca, Ac = the midpoints of COa, AOc, resp.
Ab, Ba = the midpoints of AOb, BOa, resp.
Ma, Mb, Mc = the midpoints of BcCb, CaAc, AbBa, resp.
Which is the locus of P such that:
1. MaMbMc, ABC are orthologic?
[.....]
[César Lozada]:
Let P=u:v:w (trilinears)
> Which is the locus of P such that:
> 1. MaMbMc, ABC are orthologic?
Locus = all the plane
Za(P)=orthologic center (ABC,MaMbMc) =
= u/(a*(a*(a^2-b^2-c^2)*u^2+b*(a^2-b^2)*u*v+c*(a^2-c^2)*w*u+a*b*
c*v*w)) : :
= antigonal-conjugate-of-P
Zm(P)=orthologic center (MaMbMc, ABC) =
= u^2*b*c*(SB*v^2+SC*w^2)-a^2*b*c*v^2*w^2+(5*S^2-3*SB*SC)*w*v*u^
2+3*SA*a*u*v*w*(b*v+c*w) : :
= QA-P32 : Centroid of the Circumcenter Quadrangle ?
(confirmation needed)
Zm(P on circumcircle) = O
ETC's (P,Zm(P)):
(2,3), (4,5), (13,140), (14,140), (30,2777), (80,9956),
(249,1511), (262,8359), (514,952), (519,952), (523,542),
(524,542), (525,2777), (671,5), (1138,1511), (3413,30),
(3414,30), (5627,125)
[.....]
César Lozada
**************
PS The entire post :
https://groups.yahoo.com/neo/groups/hyacinthos/conversations/mes
sages/23659  †)
```

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† Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #1769
Date: 30/6/2016 12:36:32
From: eckart_schmidt@t-online.de
Subject: O, midpoints, perp. bisectors, orthologic, ...

Dear Antreas,
let P1P2P3P4 be a quadrilateral, then Cabri-observations confirm,
... that Zm(Pi) wrt PjPkPl is QA-P32,
... that Za(Pi) wrt PjPkPl give a quadrangle, which is the reflection of the P1P2P3P4 in the Euler-Poncelet Point QA-P2.
Best regards Eckart

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Message: #1770
Date: 30/6/2016 2:11:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

Dear Bernard, dear Chris,
QA-Cu1 can be described in different ways as pivotal isocubic (see EQF):
... pivot: P on QA-Cu1 with tangential Q,
... reference triangle: points (unequal P) with tangential Q,
... isoconjugation, which swaps P and Q,
The vertices of the reference triangle are the QL-Tf1 images of P wrt the QA-quadrigons
and the tangential of P is the Involutary Conjugate (QA-P4) of P and the reference triangle (see EQF).
Wrt the pivots QA-P4, point at infinity of QA-L4, QA-P41, intersection of QA-Cu1 and its asymptote see EQF QA-Cu1,
here are two further possibilities:
QA-Cu1 is a pivotal isocubic wrt
... pivot: QA- P3,
... reference triangle: QG-P16 of the QA-quadrigons,
... isoconjugation, which swaps corresponding vertices of QA-Tr1 and QA-Tr2.
The tangential of QA-P3 is the 3rd intersection of QA-Cu1 and a line through QA-P41 and the intersection of QA-Cu1 and its asymptote.
QA-Cu1 is a pivotal isocubic wrt
... pivot: P = QA-Tf1(QA-L9) (3rd intersection of QA-Cu1 and a parrallel to the asymptote through QA-P41)
... reference triangle: QA-Tr2-isogonal conjugates of the vertices of the QA-Tr1-cevian triangle of QA-P4,
... isoconjugation, which swaps corresponding vertices of QA-Tr2 and the QA-Tr1-cevian triangle of QA-P4.
The tangential of P = QA-Tf1(QA-L9) is the 2nd common point of the circumcircles of P and corresponding vertices of QA-Tr2 and the reference triangle.
Best regards Eckart

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Message: #1771
Date: 30/6/2016 3:32:33
From: tsihonglau
Subject: Harmonic conjugates

Dear Bernard,
I cannot understand what you wrote.
I have some observations. (my notations)
Given three pairs of opposite vertices of a quadrilateral (Sa,Sbc) (Sb,Sca) (Sc,Sab) and a point X. Three circles (Sa,Sbc,X) (Sb,Sca,X) (Sc,Sab,X) do not concur at a point necessarily. If they do at a point X', then Miquel point is the midpoint of XX'. I am not sure it is true. If so, I wonder the locus of X,X' - what is the central curve?
(center - Miquel point)

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Message: #1772
Date: 01/7/2016 9:55:03
From: bernard.keizer
Subject: Harmonic conjugates

Dear Tsihong Lau,
Thanks for your interest
1) I hadn't checked the property myself, I've just copied a given text.
2) The text doesn't say that the 3 circles you describe must concur, but only that the 3 harmonic conjugates of X wrt 2 opposite vertices of the QL on these circles are themselves on a circle or a line through the reflexion of X in the Miquel point.
3) In fact, it seems that the locus of the harmonic conjugates of X wrt any copple of Cl-S conjugate points of a given Van Rees curve is a circle or a line through the reflexion of X in the Miquel point.
4) For X being one of the 2 fixed points of the Cl-S transformation, the circle is reduced to the other fixed point (the middle of the segment joining the 2 fixed points being the Miquel point).
For any copple of Cl-S conjugate points, the fixed points are on a circle through these 2 conjugate points and the division formed on this circle by the 4 points is harmonic.
Best regards
Bernard

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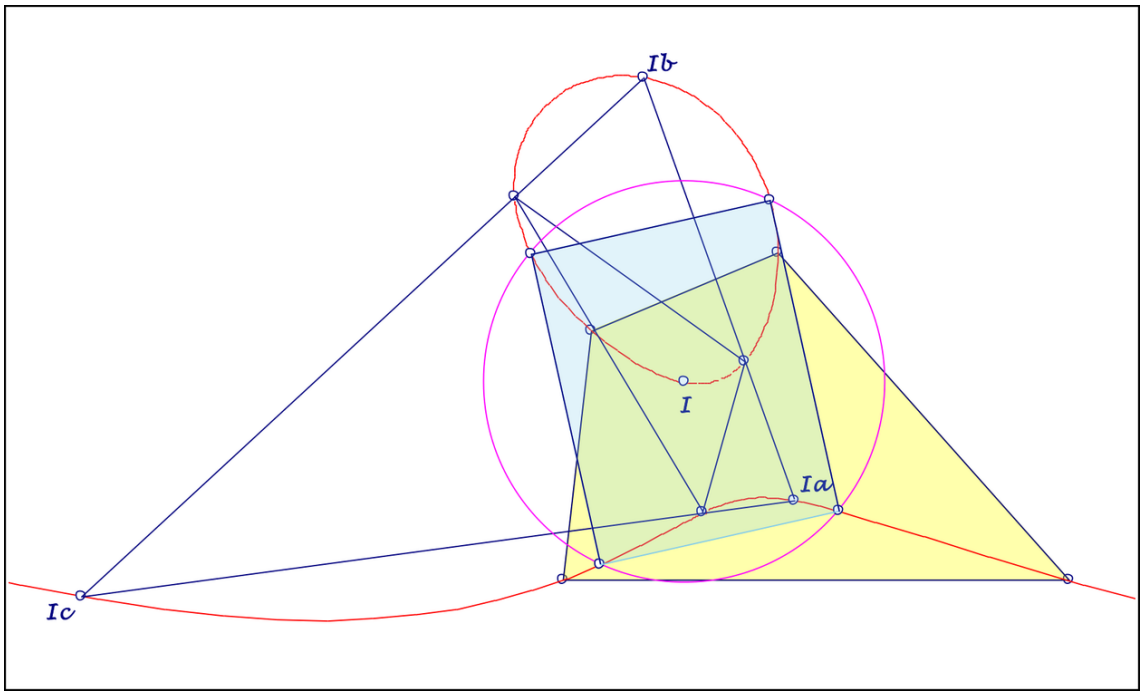
Message: #1773
Date: 01/7/2016 3:42:32
From: eckart_schmidt@t-online.de
Subject: Rectangle in QA-Cu1

Dear Bernard, dear Chris,
if we consider a triangle, there is a circle round the incenter, whose inversion swaps a vertex and the reflection of the corresponding excenter in the incenter. Perhaps this is a well known circle in triangle geometry.
Now let us consider this circle for the Miquel triangle of a quadrangle (see attached file):
... Its inersections with the cubic QA-Cu1 give a rectangle.
... The inversion of QA-Cu1 wrt this circle is the reflection in the incenter of the Miquel triangle.
Best regards Eckart



2016-07-01.pdf

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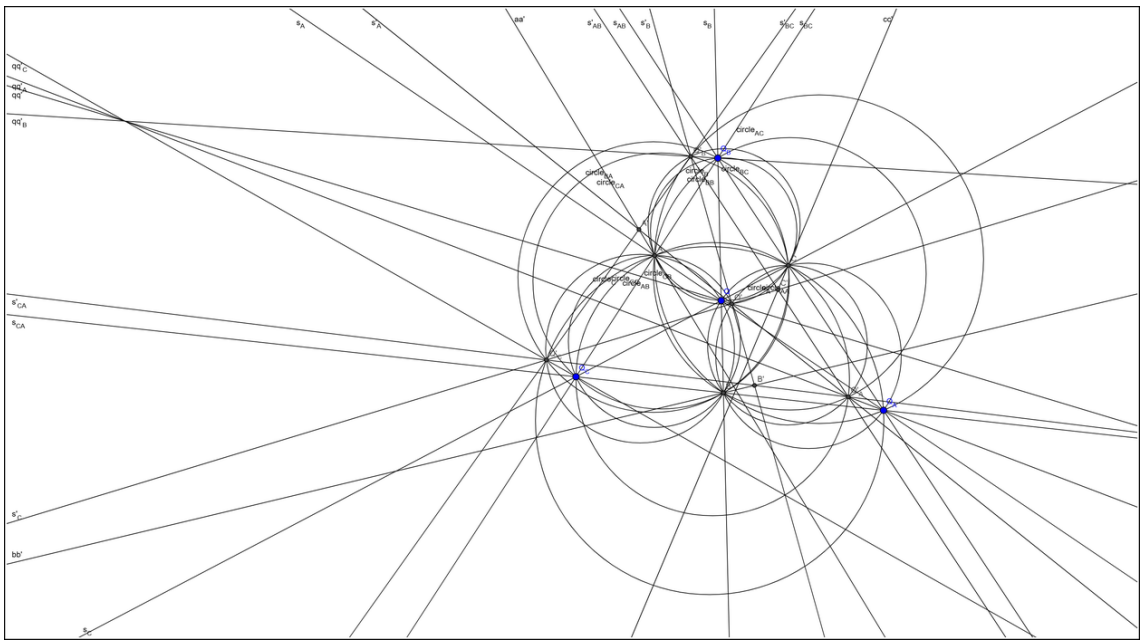
Message: #1774
Date: 01/7/2016 5:44:58
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

Dear all,
Inspired by topic #1767
Yahoo! Groups (https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/topics/1767)
I found two perspectors related to "Miquel Quadrangle". Given a triangle ABC and three points A'B'C' other than ABC on the three sidelines, the three circles AB'C',BC'A',CA'B' concur at the Miquel point according to Miquel's theorem.
I use my own notations here-
http://lth.name/geometry/duality.html
Given a quadrangle Q,Q_A,Q_B,Q_C and its diapleural(diagonal) triangle ABC(=ceva triangle of Q with respect to Q_AQ_BQ_C), the three circles Q_ABC,Q_BCA,Q_CAB concur at the Miquel point Q' according to Miquel's theorem. Q' is derived from Q and an expoint(component) triangle T=Q_AQ_BQ_C.
Similarly, we can get the other three Miquel points Q'_A,Q'_B,Q'_C. Q'Q'_AQ'_BQ'_C can be called a Miquel quadrangle, which is perspective with the reference quadrangle QQ_QQ_BQ_C.
The first coordinates of the perspector:
circular
(p^2*r^3*v^2*w^3-p^2*q^2*r*v^2*w^3-2*p*q*r^3*u*v*w^3+2*p*q^3*r*u*v*w^3+q^2*r^3*u^2*w^3-p^2*q^2*r*u^2*w^3-p^2*q*r^2*v^3*w^2+p^2*q^3*v^3*w^2-q^3*r^2*u^2*v*w^2+2*p^2*q*r^2*u^2*v*w^2-p^2*q^3*u^2*v*w^2+2*p*q*r^3*u*v^3*w-2*p*q^3*r*u*v^3*w-q^2*r^3*u^2*v^2*w-p^2*r^3*u^2*v^2*w+2*p^2*q^2*r*u^2*v^2*w+q^3*r^2*u^2*v^3-p^2*q*r^2*u^2*v^3)/(q*w-r*v)
infinity sidelength
l*(a^4*b^2*m^2*n^4-b^6*l^2*n^4+a^4*c^2*m^4*n^2+b^2*c^4*l^2*m^2*n^2+b^4*c^2*l^2*m^2*n^2-2*a^2*b^2*c^2*l^2*m^2*n^2-c^6*l^2*m^4)
DT-barycentric
p^2*(a^4*b^2*q^2*r^4-b^6*p^2*r^4+a^4*c^2*q^4*r^2+b^2*c^4*p^2*q^2*r^2+b^4*c^2*p^2*q^2*r^2-2*a^2*b^2*c^2*p^2*q^2*r^2-c^6*p^2*q^4)
The diapleural(diagonal) triangles ABC and A'B'C' of the reference and Miquel quadrangles are perspective, too. I do not know if the two perspectors have been studied already or not.
Best regards,
Tsihong Lau



miquel_quadrangle_perspector.ggb

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Message: #1775
Date: 01/7/2016 9:01:27
From: eckart_schmidt@t-online.de
Subject: Miquel Quadrangle and Perspector

```
Dear Tsihong Lau,
the perspector is QA-Tf1(QA-L9) on QA-Cu1.
Best regards Eckart
```

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Message: #1776
Date: 01/7/2016 9:20:51
From: eckart_schmidt@t-online.de
Subject: Miquel Quadrangle and Perspector

```
Dear Tsihong Lau,
the perspector for the diagonal triangles of the reference
quadrangle and your "Miquel Quadrangle" is
... the tangential of QA-Tf1(QA-L9) wrt QA-Cu1.
Best regards Eckart
```

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Message: #1777
Date: 02/7/2016 1:13:32
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

Dear Eckart,
Thanks for your information. Can you check or give the coordinates of the two perspectors? We better dig their more properties now!
Best regards,
Tsihong Lau

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Message: #1778
Date: 02/7/2016 4:30:51
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

```
Dear all,
Two points of concurrency found!
Four circumconics
   (QQ'ABC),
   (Q_AQ'_AABC),
   (Q_BQ'_BABC),
   (Q_CQ'_CABC) concur
at the fourth point.
So do four circumconics
   (QQ'A'B'C'),
   (Q_AQ'_AA'B'C'),
   (Q_BQ'_BA'B'C'),
   (Q_CQ'_CA'B'C').
Best regards,
Tsihong Lau
```

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Message: #1779
Date: 02/7/2016 10:33:40
From: eckart_schmidt@t-online.de
Subject: Miquel Quadrangle and Perspector

```
Dear Tsihong Lau,
the four circumconics
   (QQ'ABC),
   (Q_AQ'_AABC),
   (Q_BQ'_BABC),
   (Q_CQ'_CABC) concur in QA-P3.
Best regards Eckart
PS: I haven´t calculated the coordinates of the perpectors
QA-Tf1(QA-L9) and its tangential wrt QA-Cu1.
But these points are also mentioned in #1770.
```

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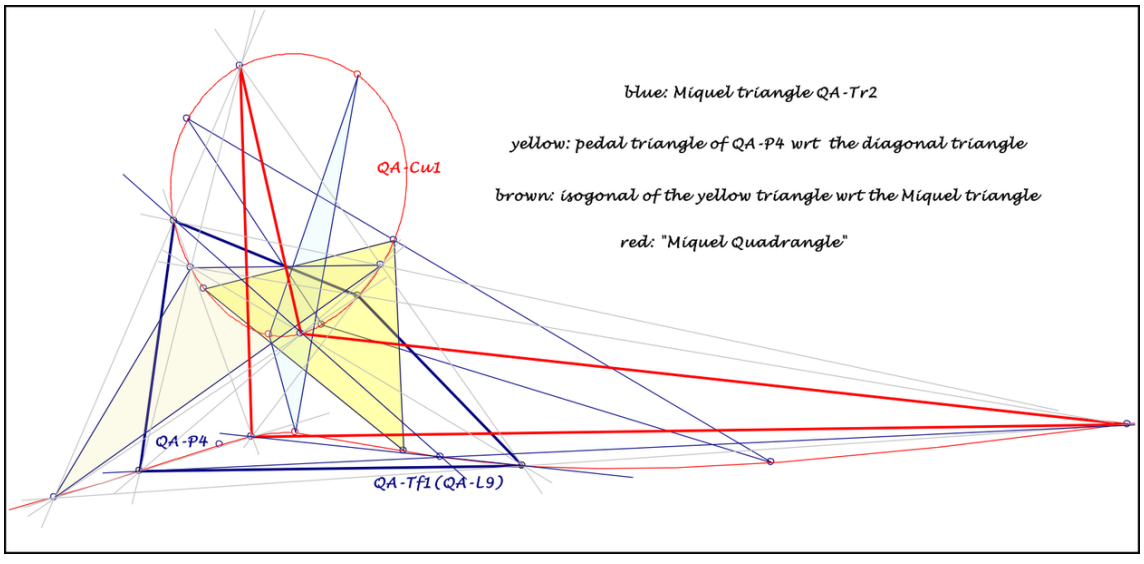
Message: #1780
Date: 02/7/2016 3:13:58
From: eckart_schmidt@t-online.de
Subject: Miquel Quadrangle and Perspectors

Dear Tsihong Lau,
we can get your "Miquel Quadrangle" also in the following way
with the Miquel Triangle QA-Tr2 (see attached file):
Let TR be the triangle with vertices in the QA-Tr2-isogonal
conjugates of the pedal triangle of QA-P4 wrt QA-Tr2.
The four perspectors of TR and the QA-triangles give your
"Miquel Quadrangle".
"Miquel Quadrangle" and reference quadrangle are also
perspective wrt the vertices of TR.
If we consider
... the reference quadrangle,
... the "Miquel Quadrangle"
... and the quadrangle of TR with QA-Tf1(QA-L9)
    (your perspector of the first two),
... then two of them are perspective
    wrt each vertice of the third
... and their Isogonal Centers (QA-P4) are collinear.
By the way: The Isogonal Center of the 3rd quadrangle is your
perspector of the diagonal triangles of the first two.
All considered points lie on the cubic QA-Cu1.
Best regards Eckart



2016-07-02.pdf

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Message: #1781
Date: 03/7/2016 5:34:11
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

Dear Eckart,
Thanks for your replies. "Miquel Quadrangle" does not belong to a class in my classification of quadrangle/quadrilateral objects.
Geometry of Triangle/Trilateral and Quadrangle/Quadrilateral
Duality of Triangle/Trilateral and Quadrangle/Quadrilateral
Coordinate System
View on lth.name (http://lth.name/geometry/)
Preview by Yahoo
In QL-P4, O1,O2,O3,O4 are circumcenters of exline (component) trilaterals.
In QA-P7, Q1,Q2,Q3,Q4 are nine-point-centers of expoint (component) triangles.
The above two objects belong to two classes-centers of exline trilaterals and expoint triangles.
In QA-P4 and QA-P5, Q1,Q2,Q3,Q4 are the isogonal and isotomic conjugates of vertices with respect to expoint triangles. These two quadrangles and Miquel quadrangle belong to a class different from the above two classes. The above are objects of exline trilaterals and expoint triangles (duality cannot be applied). The below is an object of quadrangle (hence quadrilateral due to duality).
In QA-P2, C1,C2,C3,C4 are nine-point-circles of expoint triangles.
In QA-P3, C1,C2,C3,C4 are objects of quadrangle.
I will add a new class of these objects to my website.
Best regards,
Tsihong Lau

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Message: #1782
Date: 04/7/2016 10:26:36
From: bernard.keizer
Subject: S-points conics

Dear Eckart,
A friend of mine found 3 conics perhaps of interest, centered in the 3 vertices of the anticevian triangle of QA-P16 wrt DT, which I named DDT; these conics are 2 by 2 bitangent in the QL vertices and DT is self-polar wrt the 3 conics.
This gave me the idea of rereading one old jewel of yours with the equations in DT coordinates of many QL-Tr2 circumscribed conics.
I made several observations and comments and I hope it could be useful.
Best regards
Bernard

Some remarks about the QL-Tr2 circumconics

(reference EQF Note 2013-02-01 by Eckart Schmidt)

As well known, QA-DT = QL-DT, QA-P10 = QL-P8 and QA-P16 = QL-P13

For a point (p,q,r), the trilinear polar is (l,m,n) and pl = qm = rn ; for example, the trilinear polar of the point (p^2,q^2,r^2) is the Newton Line.

QA-1 is the socalled 6th conic, better named diagonal conic, through the 4 Pi, their centroïd QA-P1, QA-P16 and the vertices of it's anticevian triangle wrt DT, the socalled DDT, DT of the DQL formed by the diagonals and the Newton Line. The 4th intersection with the Dimidium circle is in line with QL-P1 and QA-P16 (this point is the CSC of the Lemoine point of the QL QL-P26).

QA-2 is the socalled 5th conic, better named circumconic, through the vertices of DT, it's centroïd and QA-P16 (and QA-P18) : it's the QA-Tf2 of the line QA-P10QA-P16 (which contains the point QA-P19, anticomplement of QA-P16 wrt DT and QA-Tf2(QA-P18)). The 4th intersection with the Dimidium circle is QL-P24, in line with QL-P17 and QA-P16 .

QA-3 circumscribed conic of the medial DT through QA-P1 and QA-P10 (here is a typo : it's not QA-P16 as mentionned)

This conic is the isotomic conjugate of the line QA-P10QA-P16 wrt medial DT (QA-P1 is the isotomic conjugate of QA-P16 wrt medial DT). In medial DT coordinates, QA-P16 has coordinates $(- p^2 + q^2 + r^2, \ldots)$ and QA-P1 $(1/(- p^2 + q^2 + r^2)\ldots)$

The equation of this conic is exactly the sum of the equations of the 3 conics in QA-4

The 4th intersections of the 3 conics QA-1, QA-2 and QA-3 taken 2 by 2 are the 3 points QA-P1, QA-P10 and QA-P16

There are 4 other QL-Tr2 circumconics circumscribed to the medial triangles of the reference triangles of the QL, but this time not trough the centroïds.

QA-4 3 conics

QL-1 orthogonal hyperbola through QL-P2, QL-P7, QL-P23 (contact point between the Newton Line and DQL-Co1) and the intersection between QL-L6 and QL-L9. The 4th intersection of this conic with the Dimidium circle is in line with QL-P1and QL-P7.

QL-2 is the same as QA-2

The equation of this conic is exactly the sum of the equations of the 3 conics in QL-4 (nine-points conics)

I've always found intriguing these combinations of equations : for example, the 3 in QA-4 and the 3 in QL-4 lead to the following egalities : $l^2x /(y+z-x) = m^2y /(x+z-y) = n^2z /(x+y-z) = (l^2x+m^2y) / 2z = (m^2y+n^2z) / 2x = (l^2x+n^2z) / 2y = (l^2x+m^2y+n^2z) / (x+y+z)$.

The equations of these 3 conics in CT coordinates are the same with replacing l^2 , m^2 , n^2 by l, m , n ; by adding these 3 equations the same way, we get the equation of a conic circumscribed to a reference triangle of the QL, through it's centroïd and the corresponding QA-vertice

As well-known, there are 4 conics like this, one for each reference triangle

QL-3 is the same as QA-3

QL-4 3 nine-point conics QA-Co1 for the 2 pairs of opposite QL-vertices

Each nine-point conic intersect the conic QA-1 in one of the 3 vertices of the anticevian triangle of QL-P13 (DDT triangle)

This anticevian triangle is perspective with the medial triangle of DT, the perspector being QA-P1

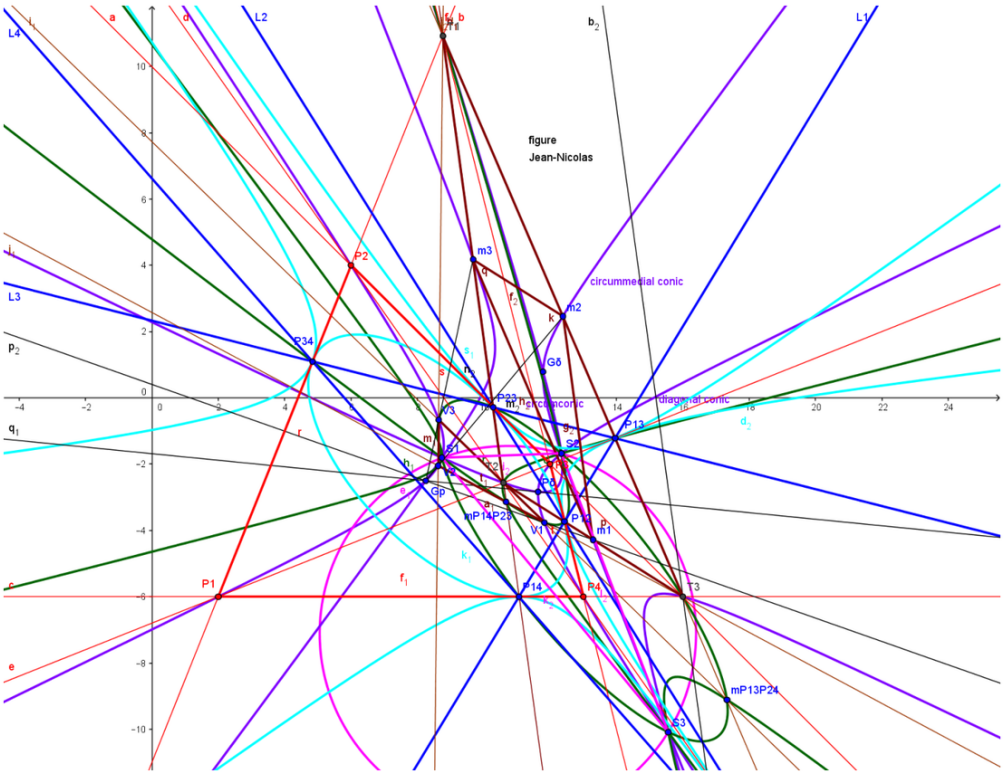
The 3 conics centered in these 3 DDT vertices through the 2 pairs of corresponding QL vertices are 2 by 2 bitangent in the common pair of QL vertices. The DT triangle is self-polar wrt these 3 conics (the common tangents to 2 conics are the lines through a vertice of DT and 2 opposite vertices of the QL (property found by a friend of mine, who doesn't use a computer and made all the calculations by hand).

The equation of these 3 conics are easily in the form $l^2x^2 - m^2y^2 - n^2z^2 = 0$ ...

QL-5 is the Dimidium circle through QL-P1, the D-points (QL-P1 and each D-point being the intersections with the circumcircle of a reference triangle of the QL), QL-P17 and QL-P24 (these 2 points being the intersections with the DT circumcircle).

If I'm not wrong, this makes all together 19 different QL-Tr2 circumconics (10 QA conics and 11 QL conics, of which 2 are counted twice)

QL-Tr2 circumconics.pdf



Obviously, Gp is the centroïd QA-P1, Gδ is the centroïd of DT QA-P10 = QL-P8 and Pδ is QA-P16 = QL-P13 ...

V1V2V3 is the anticevian triangle of QA-P16 wrt DT, which I named DDT, as it is the DT of DQL

Diagonal conic, circumconic and circummedial conic are in purple

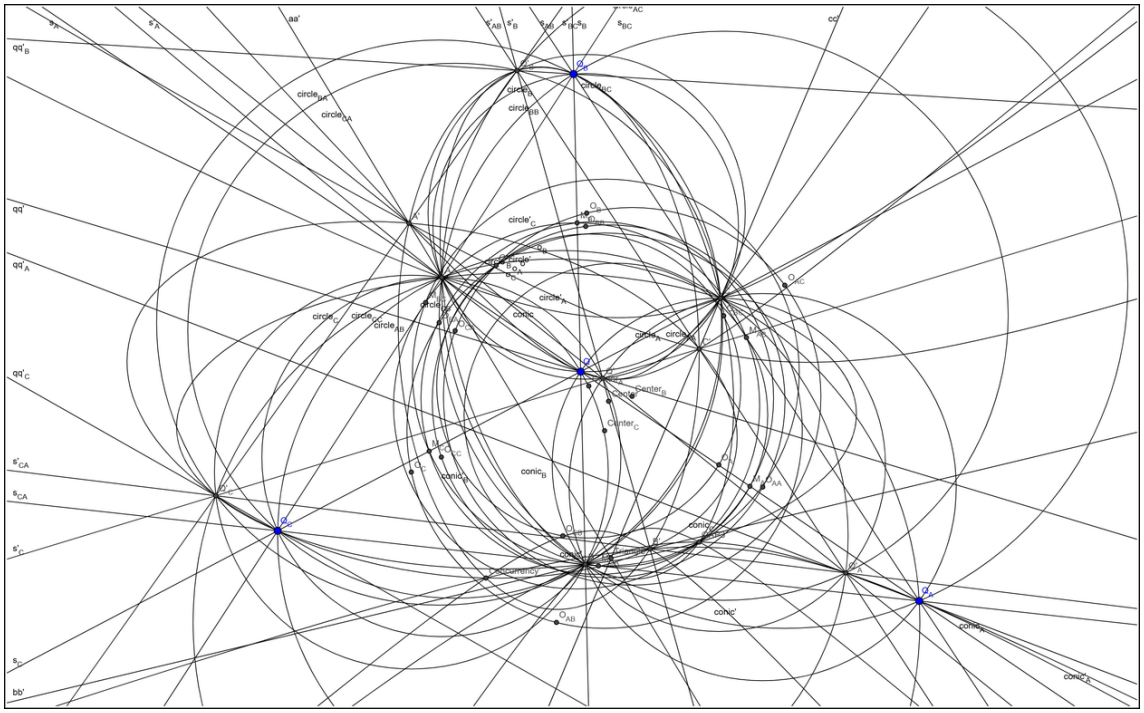
9 points conics are in green

Conics through the 2 pairs of opposite QL-vertices centered in V1, V2 and V3 are in blue

QL-Tr2 circumconics.pdf

Message: #1783
Date: 04/7/2016 4:04:33
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

Dear all,
Another point of concurency found!
Using the same notations before, the three circles Q_ABC,Q_BCA,Q_CAB concur at the Miquel point Q' according to Miquel's theorem.
Let o be the circumcircle of the centers of the three circles above.
Similarly, we can get o_A, o_B, o_C. The four circles o, o_A, o_B, o_C concur at a point!
Best regards,
Tsihong Lau



miquel_quadrangle_perspector_concurrency.ggb

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Message: #1784
Date: 04/7/2016 5:22:12
From: bernard.keizer
Subject: Miquel Quadrangle and Perspectors

Dear Tsihong Lau,
This seems interesting, but
1) As I told you before, I'm not able to open your zipped files; if you want people share your discoveries, why don't you use word. or pdf.files ?
2) Sorry if I missed one step, but could you explain your notations again ; what are Q_A, Q_B and Q_C and the same way o, o_A, o_B and o_C ?
I suppose ABC is a triangle, Q a point, are the other points the vertices of the cevian or anticevian triangles ?
Thanks in advance for your answer
Best regards
Bernard

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Message: #1785
Date: 04/7/2016 7:15:16
From: tsihonglau
Subject: Miquel Quadrangle and Perspectors

Dear Bernard,
>>This seems interesting, but
>>1) As I told you before, I'm not able to open your zipped files ; if you want people share your discoveries,
>>why don't you use word. or pdf.files ?
I said zipped files are made by Yahoo! Groups automatically.
I dislike word, pdf, image files. No construction in them!
You can study GGB files and find out the construction.
(even adapt it), In the last attachment GGB file, there are 88 objects in it.
It is too complicated to put in an image file.
You can study some objects with other objects hidden in a GGB file. GGB files are smaller and more useful than the other files. I post my construction in the messages always, so the attachments are unnecessary. If you do not use GGB, just ignore it and construct yourself.
>>) Sorry if I missed one step, but could you explain your notations again ; what are Q_A, Q_B and Q_C and >>the same way o, o_A, o_B and o_C ?
>>I suppose ABC is a triangle, Q a point, are the other points the vertices of the cevian or anticevian triangles ?
In the first message of this topic, I said I use my own notations
http://lth.name/geometry/duality.html
I dislike Chris' notations and use my own.
Now I write down the construction of five perspectors or points of concurrency in details. The constructions are described before.
I use the word "Similarly". If you cannot understand easily, I describe more details.
Given a quadrangle Q,Q_A,Q_B,Q_C, we can get the diapleural (diagonal) triangle ABC. In my page, I distinguish diapleural triangle
ABC and diagonal trilateral abc.
Miquel quadrangle Q',Q'_A,Q'_B,Q'_C
Miquel points - circumcircles
Q' - (Q_A,B,C),
     (Q_B,C,A),
     (Q_C,A,B)
Q'_A -
     (Q,B,C),
     (Q_B,B,A),
     (Q_C,C,A)

```
Q'_B -
     (Q_A,A,B),
     (Q,C,A),
     (Q_C,C,B)
Q'_C -
     (Q_A,A,C),
     (Q_B,B,C),
     (Q,A,B)
A'B'C' is the diapleural triangle of
Q',Q'_A,Q'_B,Q'_C.
The two perspectors are of (Q,Q_A,Q_B,Q_C-Q',Q'_A,Q'_B,Q'_C) and
(ABC-A'B'C').
circumcircles of centers of the Miquel circles
circle - the circumcircle of the centers of the Miquel circles
o - (Q_A,B,C),
    (Q_B,C,A),
    (Q_C,A,B)
o_A -
    (Q,B,C),
    (Q_B,B,A),
    (Q_C,C,A)
o_B -
    (Q_A,A,B),
    (Q,C,A),
    (Q_C,C,B)
o_C -
    (Q_A,A,C),
    (Q_B,B,C),
    (Q,A,B)
o,o_A,o_B,o_C concur at a point.
Not only the above quadrangles and triangles are perspective but
the two set of circumconics concur at two fourth points.
ABC and QQ',
Q_AQ'_A,
Q_BQ'_B,
Q_CQ'_C respectively - QA-P3 by Eckart
A'B'C; and QQ',
Q_AQ'_A,
Q_BQ'_B,
Q_CQ'_C respectively
Never forget Eckart have more properties.
Best regards,
Tsihong Lau
```

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Message: #1786
Date: 04/7/2016 9:00:55
From: eckart_schmidt@t-online.de
Subject: Miquel Quadrangle and Perspectors

```
Dear Tsihong Lau,
the common point of the four circles o, o_A, o_B, o_C is QA-P9.
Best regards Eckart
```

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Message: #1787
Date: 05/7/2016 3:42:48
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilateral

---

```
Dear all,
Given a quadrilateral, we can construct triangle centers
X(n)'s of its exline trilaterals(component triangles).
For example, n=3,186,265,5962, then these X(n)'s lie
on a circle.  We can construct the diapleural(diagonal)
triangle of these X(n)'s.
If n = 3,19,24,25,48,49,63,68,69,92,93,184,186,264,265,
       304,305,317,328,340,378,562,563,847...
then
the diapleural(diagonal) triangles of X(n)'s and the
quadrilateral are perspective. Please notice some of the above
triangle centers X(n)'s are isotomic conjugates.
For example, X(19) and X(304) are isotomic conjugates.
Best regards,
Tsihong Lau
```

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Message: #1788
Date: 06/7/2016 10:34:58
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

```
Dear Tsihong Lau,

let Ci(n) be the  X(n)-circle of a quadrilateral with midpoint
M(n):
n = 3  : Ci(3)   = QL-Ci3,
         M(3)    = QL-P4 (already wellknown),
n = 186: Ci(186) = QL-Tf1(Ci(256)) through QL-P4,
         M(186)  = QL-Tf1 of the reflection of QL-P1 in Ci(256),
n = 256: Ci(256)   through QL-Tf1(QL-P4),
         M(256)  = reflection of QL-P3 in QL-P2.

Best regards Eckart
```

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Message: #1789
Date: 06/7/2016 10:58:11
From: eckart_schmidt@t-online.de
Subject: S-points conics

```
Dear Bernard,
thanks for the interesting interpretation of EQF-Note
2013-02-01, thanks also for additional remarks and correction of
the typo.
I shall study your paper furthermore. Background for
corresponding properties is duality, here some remarks with the
following concept:
Let Co be a QL-Tr2 circumscribed conic,
... Tf  the QL-Tr2-isoconjugation, which swaps QL-P12 and QL-P13
    (see #1747),
... line L = Tf(Co),
... T1 = QL-Tr1-tripol of L,
... T2 = QL-Tr2-tripol of L,
... dual(L) = QA-Tf2(T1) = Tf(T2)
With the nomination of your paper:
QA-1 = Co:
... L = QL-Tf2 of a parallel to QL-L1 through QL-P13,
... T1 = point at infinity of QL-P8.QL-P13,
... T2 = 2nd intersection of QA-2
    and a parallel to QL-L1 through QL-P13,
... dual(L) = 2nd intersection of T2.QL-P12 and QA-2.
QA-2 = QL-2 = Co:
... L = T1T2 of QA-1,
... T1 = QA-Tf2 of dual(L),
... T2 = Tf of dual(L), point on QA-1,
... dual(L) = contact point of a QA-3-tangent
    parallel QL-P8.QL-P13 (unequal QL-P8).
So far with best regards Eckart
```

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Message: #1790
Date: 06/7/2016 11:20:40
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

```
Dear Tsihong Lau,
excuse there is a typo in my last message: Replace 256 with 265!
Sorry, Eckart
```

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Message: #1791
Date: 07/7/2016 11:12:27
From: bernard.keizer
Subject: Miquel Quadrangle and Perspectors

Dear Tsihong Lau, dear Eckart
Thanks to Tsihong Lau for his pedagogical effort, although I regret the figure with 88 objects (by the way, all my own figures are made with Geogebra)
Thanks to Eckart for his different messages about this item
As all teachers know, pupils understand quickly, provided they have sufficient explanations !
It all deals with QA-Cu1 and Cu-QA's of this curve (a point and it's 3 CSC)
Given an ordinary QA of 4 Pi, the 2nd " Miquel quadrangle" is formed by the 4 Qi, each Qi constructed as the intersection of 3 circles ...
If we name DQA the QA formed by the point QA-P4 and the vertices of DT and DDQA the QA formed by QA-P41 and the vertices of DDT (cevian triangle of QA-P4 wrt DT), we start a chain of pivots and isopivots ...
The 3rd QA is formed by the isogonal conjugates wrt the Miquel triangle of the vertices of DDQA (the point QA-Tf1(QA-L9) is the isogonal conjugate of QA-P41 and the QA-P3 of the DQA).
These 3 QA's are Cu-QA's of QL-Cu1 and form a Desmic configuration.
Their DQA's form also a Desmic configuration and the QA-P4 of the 3 QA's are aligned ; the QA-P3 are coconic with the vertices of the Miquel triangle.
The perspector of the 2 DT triangles of the 2 1rst QA's is the QA-P3 of the DDQA (QA-P41 and the vertices of DDT)
As I said before, I found this construction very interesting !
By drawing the figures, I found that QA-P1 of the original QA lies on the circumDT of the 2nd and that QA-P3 of the 2nd (perspector of it's DT and the Miquel triangle) lies on the circumDT of the 1rst ...
Best regards
Bernard

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Message: #1792
Date: 08/7/2016 12:43:44
From: eckart_schmidt@t-online.de
Subject: Miquel Triangles and Perspectors

Dear Bernard,
you consider Cu-QA´s, taking a defining point Q on QA-Cu1 and its tangentials, all Cu-QA have the same Miquel triangle and QA-P9.
If U, V, W are collinear defining points on QA-Cu1, you get a tripel of Cu-QA, which are perspective in pairs wrt the vertices of the 3rd.
For these QA-tripel holds:
... Their QA-P4 are collinear on QA-Cu1.
... Their QA-P3 are coconic with the vertices
    of the Miquel triangle.
Your three QA in # 1791 are of this type (nomination of Tsihong Lau):
... U = Q of the reference QA,
... V = Q´,
... W = 3rd intersection of QQ´ and QA-Cu1.
Best regards Eckart

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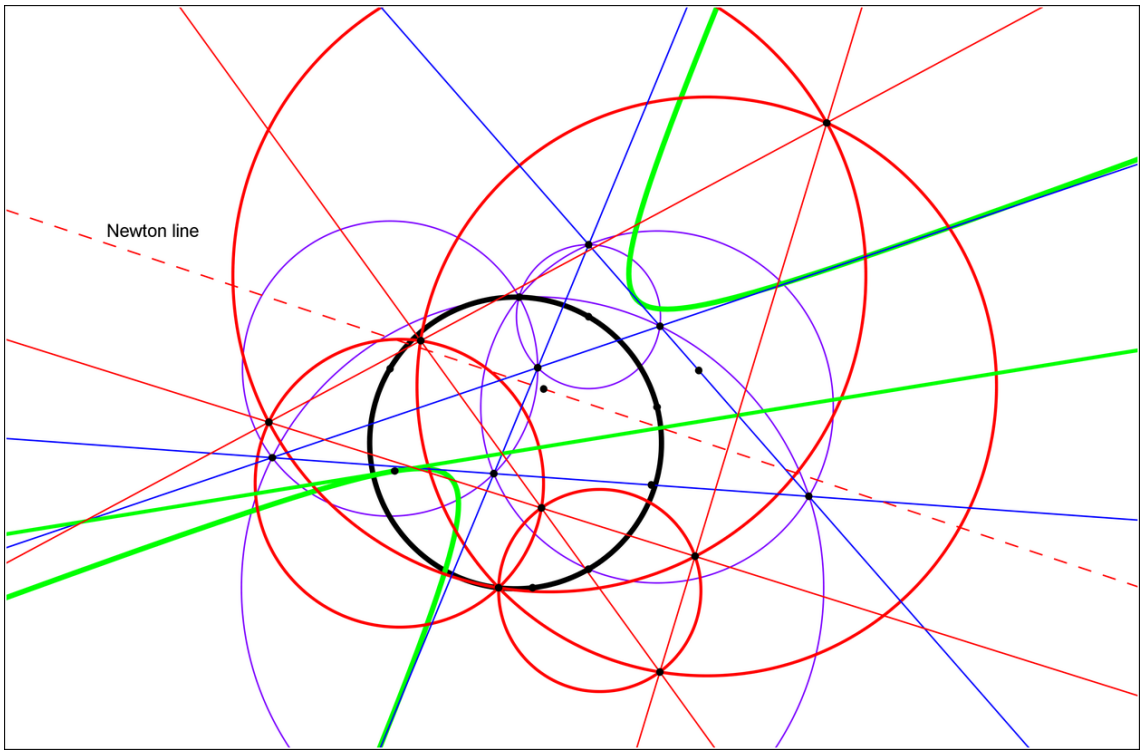
Message: #1793
Date: 09/7/2016 9:55:50
From: minhtenladuong
Subject: An Envelope

Dear Geometers
I posed an envelope problem as follow: L1, L2, L3, L4 are 4 lines of a quadrilateral. For which L, the reflections of L in L1, L2, L3, L4 form a quadrilateral Q such that Miquel point of Q lies on Miquel circle of the quadrilateral (L1,L2,L3,L4)?
I haven't calculated yet. But L1, L2, L3, L4 are also tangent to this envelope.
I think this envelope is just a conic, which can be defined as follow:
Miquel circle of (L1,L2,L3,L4) intersects the circumcircles of the component triangles at M1,M2,M3,M4, respectively. D1, D2, D3, D4 are Steiner lines of M1, M2, M3, M4 with respect to the corresponding component triangles. Then there exists a conic that tangent to 8 lines L1, L2, L3, L4, D1, D2, D3, D4.
Furthermore, its center lies on the Newton line.
Is this known?
Best regards
Sincerely
Ngo Quang Duong



envelope.png

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Message: #1794
Date: 09/7/2016 11:02:33
From: eckart_schmidt@t-online.de
Subject: En Envelope

```
Dear Ngo Quang Duong,
The envelope ...
... "L1, L2, L3, L4 are 4 lines of a quadrilateral. For which L,
the reflections of L in L1, L2, L3, L4 form a quadrilateral Q
such that Miquel point of Q lies on Miquel circle of  the
quadrilateral (L1,L2,L3,L4)?"
... is an QL-inscribed conic, centered in QL-P20 on QL-L1
(Cabri-controlled).
Best regards Eckart
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Message: #1795
Date: 09/7/2016 12:50:35
From: minhtenladuong
Subject: En Envelope

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Dear Eckart Schmidt,
Thank you so much for showing its center.
Best regards
Sincerely
Ngo Quang Duong
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Message: #1796
Date: 09/7/2016 9:21:42
From: eckart_schmidt@t-online.de
Subject: Harmonic conjugates

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Dear Bernard, dear Tsihong Lau,
in #1771 Tsihong Lau had formulated a locus problem. The
solution is a quartic, described in #364.
Best regards Eckart
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Message: #1797
Date: 09/7/2016 9:34:23
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
it seems, that I´m a bit stupid: In # 1787 you wrote:
"... then the diapleural (diagonal) triangles of X(n)'s and the quadrilateral are perspective."
How can a triangle and a quadrilateral be perspective ?
Best regards Eckart

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Message: #1798
Date: 10/7/2016 3:35:21
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

---

Dear Eckart,
I wrote "the diapleural(diagonal) triangles of X(n)'s and the quadrilateral are perspective." - "triangles" is in the plural form.
It meant the diapleural(diagonal) triangle of the quadrangle X(n)'s and the diapleural(diagonal) triangle of the reference quadrilateral are perspective.
A quadrangle and a triangle are sometimes perspective in the following sense. Given a quadrangle Q,Q_Q,Q_B,Q_C and a triangle D,E,F, then the perspective quadrangle is described as follows:
perspector of expoint(component) triangle and DEF
Q' - Q_A,Q_B,Q_C and D,E,F
Q'_A - Q,Q_B,Q_C and D,E,F
Q'_B - Q_A,Q,Q_C and D,E,F
Q'_C - Q_A,Q_B,Q and D,E,F
Similarly, we can get a perspective quadrilateral.
I have not studied this topic. you can try it yourself.
P.S. You better not adjust the topic of your reply, otherwise your reply and the original message cannot be put in the same topic by Yahoo! Groups automatically.
Best regards,
Tsihong Lau

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Message: #1799
Date: 10/7/2016 4:24:43
From: eckart_schmidt@t-online.de
Subject: Harmonic conjugates

Dear Tsihong Lau,
thanks for your explanations wrt the property
"the diapleural(diagonal) triangles of X(n)'s and the quadrilateral are perspective."
First reading this property, I had it interpreted in your sence, but a construction mistake led to my question, excuse!
Here a conjecture for points X(n) with this property:
For points X(n) with the property above
... their isogonal conjugate * and their isotomic conjugate ^ have also this property
... except X(3)*, for the X(4)-QA degenerates collinear.
So there are chains of points with this property (it = isotomic, ig = isogonal conjugated):
... X(305) it X(25) ig X(69)
[it X(4) ig]
X(3) it X(264) ig X(184) it X(184)^ ig X(184)^* it X(184)^*^ ...

```
... X(1973)^  it  X(1973)  ig  X(304)  it  X(19)  ig  X(63)
              it  X(92)    ig  X(48)   it  X(1969)  ...
... X(49)^    it  X(49)    ig  X(93)   ...
... X(317)    it  X(68)    ig  X(24)   ...
... X(328)    it  X(186)   ig  X(265)  it  X(340) ...
... X(378)^   it  X(378)   ig  X(4846) ...
... X(847)^   it  X(847)   ig  X(1147) ...
... X(5962)^  it  X(5962)  ig  X(5961) ...
```

There are points, not mentioned in ETC. As far as here noticed the points are CABRI-controlled (I hope without error).
Perhaps relevant:
... X(3), X(24), X(25), X(186), X(378) collinear on the Euler line,
... [X(4)], X(69), X(264), X(317), X(340) collinear,
... [X(4)], X(68), X(847) collinear,
... X(69), X(265), X(328) collinear.
Have you more points X(n) as mentioned in # 1787?
Best regards Eckart

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Message: #1800
Date: 10/7/2016 6:35:03
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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```
Dear Eckart,
Good observations!  I checked it with GeoGebra and found
n = 3,19,24,25,48,49,63,68,69,92,93,184,186,264,265,304,305,
    317,328,340,378,562,563,847,1092,1093,1102,1136,1137,
    1147,1593,1597,1598,1748,1820,1969,1973,1974,2351,3043
1 up 3053 (maybe incorrect)
We better check the rules of coordinates of these triangle
centers.
P.S. Your replied with a wrong topic!
Best regards,
Tsihong Lau
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Message: #1801
Date: 10/7/2016 7:54:57
From: bernard.keizer
Subject: Harmonic conjugates

```
Dear Eckart, dear Tsihong Lau
This is the answer to Tsihong Lau's locus in message 1771, the
quartic of generalised Plücker points.
But this wasn't my question (see my messages 1770 and 1772) !
I was asking for the locus, for a given point, of the harmonic
conjugates of this point wrt all pairs of conjugate points on
QL-Cu1 ...
It seems, it is a circle through the reflexion of the point wrt
QL-P1.
The question is why ?
Best regards
Bernard
```

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Message: #1802
Date: 12/7/2016 11:11:44
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau, dear Bernard, dear Chris,

thanks to Tsihong Lau for # 1800 and the correction, that my # 1799 belongs to this topic.
Already in earlier QFG-messages the following triangle points are discussed
X(3), X(186), X(265), X(5962)
with the property, that the X(n)´s for the trilaterals of a quadrilateral lie on a circle.
I found the following chain for further points with this property (ig = isogonal conjugated, iv =inversion in the circumcircle)
X(3) ig [X(4)] iv X(186) ig X(265) iv X(5961) ig X(5962) iv X(5963) ig X(5964) iv X ig Y iv Z ig ... iv ...
Without X(4) all these points have the property:
... the four mentioned above,
... further the ETC points X(5961), X(5963), X(5964)
... and X, Y, Z not in ETC (but CABRI-controlled).
I suppose, the chain can be continued at infinity and contains all points with this property.

Best regards Eckart

PS: Perhaps relevant for this chain: iv(ig(P)) = QA-P4 of PABC.

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Message: #1803
Date: 13/7/2016 7:45:07
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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Dear Eckart,
Wonderful observations! More observations here.
The Steiner line QL-L2 and the line at infinity can be viewed as a degenerate conic passing through the circular points at infinity as a circle.
So the case of X(4) applies, too.
QL-Ci3 Miquel circle(X(3)) and QL-L2 Steiner line(X(4)) are QL-Tf1 Clawson-Schmidt conjugates.
Circle through X(186)'s and X(265)'s of exline trilaterals are QL-Tf1 Clawson-Schmidt conjugates, too.
I wonder if the cases of X(5961) and X(5962) and other centers apply.
The 5 versions of the QL-X(186) circle in a 5-Line (Pentalateral) all pass through a common point.
The 5 versions of the QL-X(265) circle in a 5-Line (Pentalateral) all pass through a common point.
The 5 versions of the QL-X(3) circle in a 5-Line (Pentalateral) all pass through a common point.
(not mentioned in EQF)
The 5 versions of the QL-X(4) Steiner line in a 5-Line (Pentalateral) do not pass through a common point.
But the Steiner lines together with the line at infinity all pass through at least 3 points at infinity.
I wonder if the cases of X(5961) and X(5962) and other centers apply.
QL-Ci4 Hervey circle = the circumcircle through X(4)'s of the expoint triangles of the quadrangle formed by X(3)'s of the exline trilaterals of the quadrilateral.
QL-CiX XXX circle = the circumcircle through X(265)'s of the expoint triangles of the quadrangle formed by X(186)'s of the exline trilaterals of the quadrilateral.
QL-CiY YYY circle = the circumcircle through X(186)'s of the expoint triangles of the quadrangle formed by X(265)'s of the exline trilaterals of the quadrilateral.
In fact, we can interchange X(3)'s and X(4)'s roles in Hervey circle considering the line at infinity.
I wonder if the cases of X(5961) and X(5962) and other centers apply.
Best regards,
Tsihong Lau

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Message: #1804
Date: 14/7/2016 9:20:19
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
fascinating addition, that isognal conjugated X(n) in the chain of # 1802 lead to CSC-conjugated X(n)-circles for a quadrilateral!
I have controlled this with CABRI for X(5961 and X(5962) as well as for X(5963) and X(5964), so I´m shure, that it will hold also for further isogonal partners of the chain.
The common point of the X(n)-circles for the tetralaterals of a 5-line are already mentioned in # 710. I have CABRI-controlled this for X5961), X(5962), X(5963), X(5964).
So far, best regards Eckart

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Message: #1805
Date: 14/7/2016 12:02:32
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
what about the circles at the end of your #1803?
There is a further observation:
For X(n) of the chain in # 1802
... the X(n)´s for the expoint triangles of a cyclic quadrangle lie on circle.
Best regards Eckart

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Message: #1806
Date: 14/7/2016 11:54:31
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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Dear Eckart,
For the following n, X(n)'s of the expoint triangles of a cyclic quadrangle are concyclic.
n= 1,2,3,4,5,13,14,15,16,20,23,26,36,40,74,80,98,99,100,101,102,103,104,105,106,107,108,109,110,111,112,125,140,165,186,265,376,381,382,399,476,477,546.547,548,549,550,631,632,675,681,689,691,697,699,701,703,705,707,709,711,713,715,717,719,721,723,725,727,729,731,733,735,737,739,741,743,745,747,753,755,759,761,767,769,773,777,779,781,783,785,787,789,791,793,795,797,803,805,807,809,813,815,817,819,825,827,831,833,835,839,840,841,842,843,898,901,907,915,917,919,925,927,930,931,932,933,934,935,953,972,1113,1114,1141,1286,1287,1288,1289,1290,1291,1292,1293,1294,1295,1296,1297,1298,1299,1300,1301,1302,1303,1304,1305,1306,1307,1308,1309,1310,1311,1380,1381,1382,1385,1477,1482,1511,1656,1657,2041,2042,2043,2044,2045,2046,2070,2071,2072,2077,2222,2249,2291,2365,2366,2367,2368,2369,2370,2371,2372,2373,2374,2375,2376,2377,2378,2379,2380,2381,2382,2383,2384,2675,1676,2687,2688,2689,2690,2691,2692,2693,2694,2695,2696,2697,2698,2699,2700,2701,2702,2703,2704,2705,2706,2707,2708,2709,2710,2711,2712,2713,2714,2715,2716,2717,2718,2719,2720,2721,2722,2723,2724,2725,2726,2727,2728,2729,2730,2731,2732,2733,2734,2735,2736,2737,2738,2739,2740,2741,2742,2743,2744,2745,2746,2747,2748,2749,2750,2751,2752,2753,2754,2755,2756,2757,2758,2759,2760,2761,2762,2763,2764,2765,2766,2767,2768,2769,2770,2855,2856,2857,2858,2859,2860,2861,2862,2863,2864,2865,2866,2867,2868,
Many of them lie on the circumcircle.
For the following n, X(n)'s of the expoint triangles of a cyclic quadrangle are perspective with the reference quadrangle.
n=2,3,4,5,140,376.381,382,546.547,548,549,550,631,632,1656,1657,2041,2042,2043,2044,2045,2046,2675,2676,
They all lie on the Euler line and part of the above.
Best regards,
Tsihong Lau

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Message: #1807
Date: 15/7/2016 11:44:31
From: bernard.keizer
Subject: Centers of Exline Trilaterals

Dear Eckart, dear Tsihong Lau
Congratulations ! ; you two made a really fascinating discovery:
1) with the chain of n such as the X(n)'s of the 4 reference triangles are concyclic as an alternance of isogonal conjugation and inversion in the circumcircle
2) with the property that the 2 circles of 2 following isogonal points are CSC conjugates
I make one remark and formulate one question :
1) The chain of points is the chain of Hofstadter points Hr with r integer
(the description can be found in ETC at X360)
Using the simple notation of Bernard Gibert, g is the isogonal conjugate, i the inverse in the circumcircle and ig the inverse of the circumcircle of the isogonal conjugate.
For r not 0, 1 or 2, if X = Hr, gX = H1-r and iX = H2-r ; therefore, igX = H(2-(1-r)) = Hr+1 and giX = H(1-(2-r)) = r-1.The antigonal image of Hr is H-r.
Starting with the circumcenter X3 = H2, we find X4 = gX3 = H-1, X186 = igX3 = H3, X 265 = gigX3 = H-2 (antigonal of X3), X5961 = igigX3 = H4, X5962 = gigigX3 = H-3 (antigonal of X186), X5963 = igigig = H5, X5964 = gigigigX3 = H-4 (antigonal of X5961) ; and last
X = igigigigX3 = H6, Y = gigigigigX3 = H-5 (antigonal of X5963) and Z = igigigigigX3 = H7
2) It's well known that the 9 points conic of the 4 points X3 is a rectangular hyperbola centered in QL-P6 with axes parallel to the Steiner axes.
It's also well-known that the 4perpendiculars of the 4 segments X3X4 of the reference triangles concur in QL-P3 and that the 4 perpendiculars in the points X5 to the 4th line concur in QL-P2.
I wonder if the following copples of isogonal points(X186 and X265 ...) share similar properties.
Best regards
Bernard
PS As you surely heard, the terror goes on and I'm not in the mood of drawing figures ...

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Message: #1808
Date: 15/7/2016 4:25:51
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
thanks for the results of your calculations. Evident, that your list has to contain all ETC-points on the circumcircle.
Perhaps also evident:
For X(n) with concyclic quadrupel for the expoint triangles of a cyclic quadrangle holds:
... The inversion (iv) of X(n) wrt the circumcircle has the same property.
... The reflection (re) of X(n) in X(3) has the same property.
The combination of these transformations give an involution, so there are for each point in your list quadrupels of the form:
X(2077) - iv - X(40) - re - X(1) - iv - X(36) - re - X(2077,
X(20) - re - X(4) - iv - X(186) - re - X(2071) - iv - X(20),
Best regards Eckart

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Message: #1809
Date: 16/7/2016 9:30:54
From: bernard.keizer
Subject: Centers of Exline Trilaterals

Dear Eckart, dear Tsihong Lau
Finally, I drew my own figure with the X3's, the X4's,the X186's and the X265's.
It appears
1) that the 9 points conics for
X3's give a rectangular hyperbola (property of concyclic QA's) centered in QL-P6 through QL-P4 and QL-P6 with asymptotes parallel to the Steiner axes
X186's give a rectangular hyperbola through the circumcenter of the X186's (the circumcircle is one of the circumconics) and through QL-P4
X265's give a rectangular hyperbola through the circumcenter of the X265's
The asymptotes are not parallel to the Steiner axes
2) the perpendicular bisectors of the segments X3X4 concur in QL-P3 (Kantor-Hervey theorem) and the perpendiculars in the points X5 to the 4th line concur in QL-P2
the perpendicular bisectors of the segments X186X265 concur in a point (generalization of the Kantor-Hervey theorem), but the perpendiculars through the middles of the segments X186X265 to the 4th line apparently do not concur
So far for today
Best regards
Bernard

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Message: #1810
Date: 16/7/2016 9:42:52
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Bernard,
wrt 1): I had read the name Hofstadter point in ETC, but was not familar with these points, thanks for explanation.
wrt 2) CABRI-controlled in examples:
QA-Co1 of a cyclic QA is always a rectangular hyperbola.
The four bisectors of isogonal X(n) in the chain of # 1802 have a common point for a quadrilateral.
But the perpendiculars from the midpoints of the isogonal X(n) to the 4th line don´t concur in general.
Best regards Eckart
PS. Flags fly at half-mast also in Kiel!

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Message: #1811
Date: 16/7/2016 11:50:59
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Bernard,
excuse, I hadn´t read your last message with the same observations.
Best regards Eckart

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Message: #1812
Date: 17/7/2016 6:33:16
From: tsihonglau
Subject: Line(Inscribed) Parabola of Diapleural Quadrilateral

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Dear all,
I use my own notations -
Duality of Triangle/Trilateral and Quadrangle/Quadrilateral (
http://lth.name/geometry/duality.html)
Diagonal Conic Diapleural Conic COA COBC QQASbScaScSab
QBQCSbScaScSab -x2+y2+z2-yz=0 -x2+y2+z2+yz=0 coa cobc
qqasBsCAsCsAB qbqcsBsCAsCsAB -3x2+4y2+4z2+4yz=0
-3x2+4y2+4z2-4yz=0 COB COCA QQBScSabSaSbc
View on lth.name
(http://lth.name/geometry/duality.html)
Preview by Yahoo
Given a reference quadrangle Q,Q_A,Q_B,Q_C,
we can get the six diapleurals(lines) as follows:
   s_A - QQ_A
   s_BC - Q_BQ_C
   s_B - QQ_B
   s_CA - Q_CQ_A
   s_C - QQ_C
   s_AB - Q_AQ_B
The three diapleural quadrilaterals are:
   q_A - s_B,s_CA,s_C,s_AB
   q_B - s_C,s_AB,s_A,s_BC
   q_C - s_A,s_BC,s_B,s_CA
There is only one line(inscribed) parabola of a quadrilateral.
So the above three quadrilaterals can determine the three
diapleural parabolas. Each pair of two line(inscribed) conics
have four lines of intersection(common tangents).
Each pair of the above parabolas have three trivial lines of
intersection(common tangents). For example, the diapleural
parabolas of q_B,a_C have three lines of intersection(common
tangents) s_A,S_BC,line at infinity.
Each pair has the fourth line of intersection(common tangent).
There are three pairs of parabolas, so there are three fourth
lines of intersection(common angents). Surprisingly, the three
lines concur at the centroid QA-P1 of the reference quadrangle.
Best regards,
Tsihong Lau

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Message: #1813
Date: 17/7/2016 7:02:45
From: tsihonglau
Subject: Circumcircle of Diagonal Triangle

Dear all,
I use my own notations -
http://lth.name/geometry/duality.html
Continuing #1812,
Yahoo! Groups (https://groups.yahoo.com/neo/groups/Quadri-Figur⌋
es-Group/conversations/messages/1812)
The three diapleural trilaterals are the diagonal trilaterals
of the three diapleural quadrilaterals:
   t_A - a=BC,s_A,s_BC
   t_B - b=CA,s_B,s_CA
   t_C - c=AB,s_C,s_AB
   A = s_A∩s_BC
   B = s_B∩s_CA
   C = s_C∩s_AB
The three diagonal triangles are the diaplueral triangles
of the three diapleural quadrilaterals:
   T_a - A,S_a=a∩s_BC,S_bc=a∩s_A
   T_b - B,S_b=b∩s_CA,S_ca=b∩s_B
   T_b - C,S_c=c∩s_AB,S_ab=c∩s_C
There are three circumcircles of the three diagonal triangles.
The three radical lines of each pair of the circumcircles concur
at a point. However, I cannot compute its coordinates and
identify it.
Best regards,
Tsihong Lau

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Message: #1814
Date: 17/7/2016 10:11:39
From: eckart_schmidt@t-online.de
Subject: QL-2P1 / QL-2P4

Dear Bernard, dear Chris,
I think, the following property of the Plücker Points is worth to be mentioned:
A Plücker Point QL-2P1 has with the vertices of the four QL-trilaterals the same Isogonal Centers QA-P4.
The two Isogonal Centers are CSC(QL-2P1) = QL-2P4.
Best regards Eckart

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Message: #1815
Date: 17/7/2016 12:05:00
From: eckart_schmidt@t-online.de
Subject: Circumcircle of Diagonal Triangle

Dear Tsihong Lau,
I tried to reproduce your constructions, but your notation needs getting used to me. If you ask at the end of #1813 for the redical center of the circumcircles of the diagonal triangles of the quadrilaterals q-A, q-B, q-C, this will be QA-P16 of the reference quadrangle.
Best regards Eckart

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Message: #1816
Date: 17/7/2016 2:39:34
From: eckart_schmidt@t-online.de
Subject: Circumcircle of Diagonal Triangle

```
Dear Tsihong Lau,
am I right, describing your constructions in 1812 / 1813 as
follows?
Consider a quadrilateral
... with its three quadrigons,
... which can be interpreted as quadrilaterals.
... Their QL-diagonal triangles
... have circumcircles
... with radical center QA-P16.
Best regards Eckart
```

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Message: #1817
Date: 17/7/2016 3:03:17
From: eckart_schmidt@t-online.de
Subject: Circumcircle of Diagonal Triangle

```
Dear Tsihong Lau,
excuse, there is a typo in my last message, which has to begin:
Consider a quadrangle ...
Here is an analogon:
Consider a quadrilateral
... with its three quadrigons,
... which can be interpreted as quadrangles.
... Their QA-diagonal triangles
... have circumcircles
... with radical center QL-P13.
Best regards Eckart
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Message: #1818
Date: 17/7/2016 4:19:49
From: tsihonglau
Subject: Circumcircle of Diagonal Triangle

Dear Eckart,
Thanks for your remarks! You are right!
I dislike the concept of quadrigon. I use only triangle/trilateral and quadrangle/quadrilateral.
I use my own notations - http://lth.name/geometry/duality.html
Given a quadrangle Q,Q_A,Q_B,Q_C, we can get six diapleurals(opposite sides) s_A/s_BC,s_B/s_CA,s_C/s_AB and then the diapleural triangle ABC.
Given a quadrilateral q,q_a,q_b,q_c, we can get six diagonals(opposite angles) S_a/S_bc,S_b/S_ca,S_c/S_ab and then the diagonal trilateral abc.
Because of duality, from a quadrangle/quadrilateral, we can get the six diapleurals/diagonals and then the diapleural triangle/diagonal trilateral and then the diagonal trilateral/diapleural triangle and then the six diagonals/diapleurals and then the quadrilateral/quadrangle.
>From the six diapleurals/diagonals, we can get the three diapleural quadrilaterals=q_A,q_B,q_C/diagonal quadrangles=Q_a,Q_b,Q_c and then their three diagonal trilaterals(=diapleural trilaterals = t_A,t_B,t_C)/diapleural triangles(=diagonaltriangles=T_a,T_b,T_c) respectively.
Please notice the diapleural trilaterals t_A,t_B,t_C and the diagonal triangles T_a,T_b,T_c are dual respectively but the diapleural quadrilaterals q_A,q_B,q_C and the diagonal quadrangles Q_a,Q_b,Q_c are not.
We can get the three centers/axes of the three diagonal triangles=diapleural trilaterals. We can get also the three centers/axes of the three diagonal quadrangles and the three centers/axes of the three diapleural quadrilaterals.
Best regards,
Tsihong Lau

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Message: #1819
Date: 17/7/2016 6:12:55
From: tsihonglau
Subject: Circumcircle of Diagonal Triangle

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Dear all,
There is a center related to three circles.
I do not know what it is called in the bibliography.
Given three circles with centers D,E,F, the three poles of
EF,FD,DE with respect to circles D,E,F are D',E',F'
respectively.
Then lines DD',EE',FF' concur at a center.
I wonder what is the center for the three circumcircles of
diagonal triangles.
Best regards,
Tsihong Lau
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Message: #1820
Date: 21/7/2016 3:59:37
From: tsihonglau
Subject: Circumcircle of Diagonal Triangle

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Dear all,

Sorry! The center is the orthocenter of the circumcenters of
diagonal triangles(or diapleural trilaterals). I have computed
its circular coordinates for more than three days.
First coordinate:
(q*w-r*v)*(p^6*q^6*r*v^2*w^11-p^10*q^2*r*v^2*w^11+p^5*q^7*r*u*v*
w^11-4*p^7*q^5*r*u*v*w^11+3*p^9*q^3*r*u*v*w^11-p^6*q^5*r^2*v^3*w
^10-p^8*q^3*r^2*v^3*w^10-15*p^5*q^6*r^2*u*v^2*w^10+11*p^7*q^4*r^
2*u*v^2*w^10+6*p^9*q^2*r^2*u*v^2*w^10+p^5*q^8*u*v^2*w^10-3*p^7*q
^6*u*v^2*w^10+3*p^9*q^4*u*v^2*
.................
(editor: omitted because of very long expression)

Best regards,
Tsihong Lau
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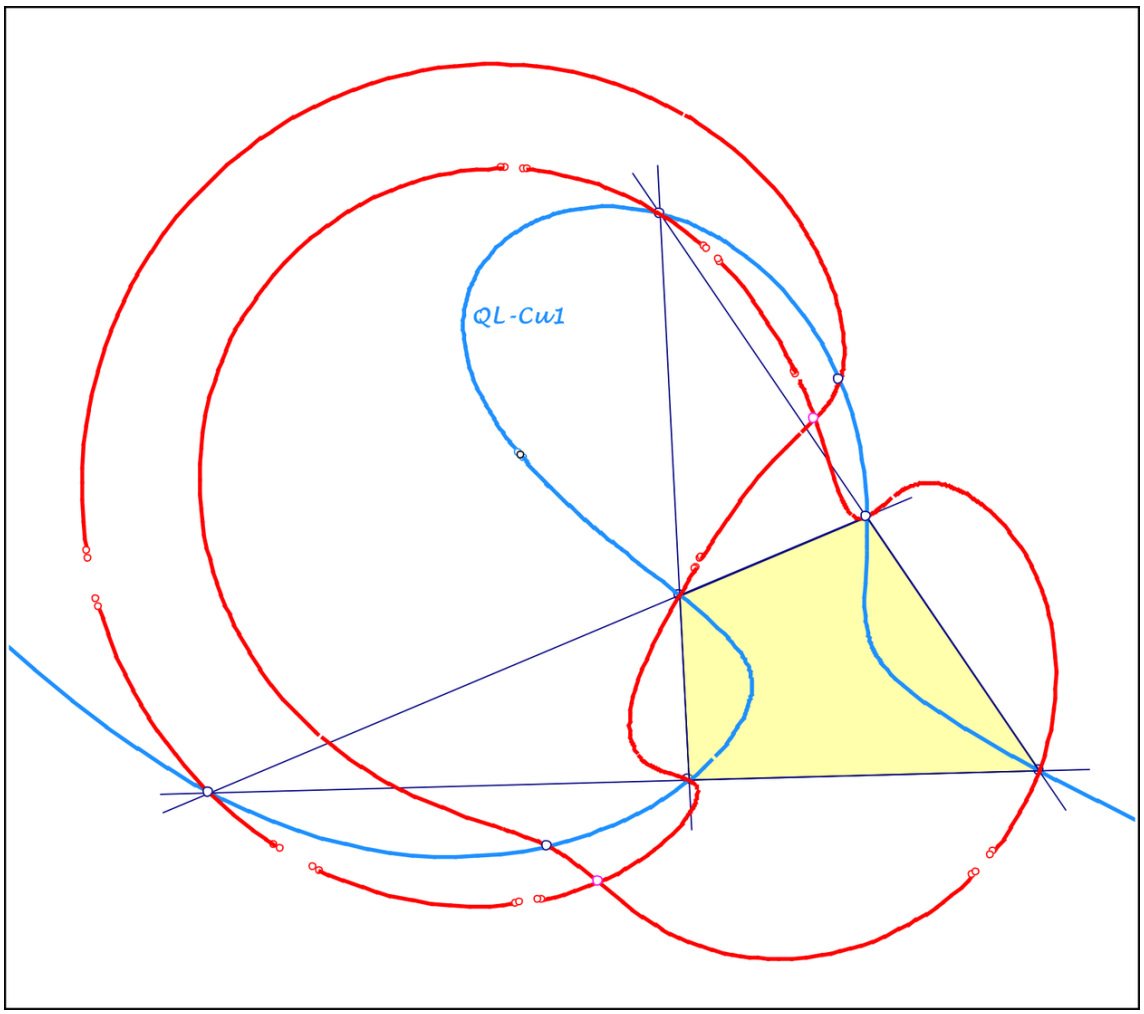
Message: #1821
Date: 22/7/2016 9:12:01
From: eckart_schmidt@t-online.de
Subject: Circles for QL-Cu1-Points

Dear Bernard, dear Chris,
this property is well-known:
QA-P4 is the inversion of the isogonal conjugate of a vertex wrt the remaining QA-triangle (see EQF).
This is the background for following circles wrt points X on QL-Cu1 (see attached file):
Ci1(X): The reflections of X in the QL-lines lie on a circle (see EQF),
... centered in CSC(X) on QL-Cu1.
... The corresponding circle for CSC(X)
(see Ci4) has the same radius.
Ci2(x): The reflections of X in the circumcircles of the QL-trilaterals lie on a circle (see EQF),
... centered in the 2nd intersection of QL-Cu1
and a QL-L1-parallel through X.
Ci3(X): The Isogonal Centers QA-P4 of X and vertices of the QL-trilaterals lie on a circle,
... which is CSC(Ci1(X)) = Ci2(CSC(X)).
Ci4(X): The isogonal conjugates of the inversions of X wrt the QL-trilaterals lie on a circle,
... which is CSC(Ci2(X)) = Ci1(CSC(X)).
The intersections of Ci1(X), Ci2(X) or Ci3(X), Ci4(X)) give a curve of degree 8 (see attached file)
... through the 6 points of the quadrilateral,
... fragmented in two curves,
which are CSC-images of each other *),
... with CSC-partners in possible knots
... and CSC-partners in possible further intersections
with QL-Cu1.
*) Curious observation: One curve bears the vertices of a special QL-trilateral, the other bears the collinear CSC-partners on the remaining line. What property determines this pointed QL-trilateral?
Finally: For points X on QL-Cu1 and a QL-trilateral holds:
CSC - inversion wrt circumcircle - CSC = reflection in the 4th line of the quadrilateral.
Best regards Eckart

PS: I haven´t found in EQF, but well-known, that for points on QL-Cu1 the transformation QL-Tf1 is the isogonal conjugated wrt a QL-trilateral. This would be an easier construction as described under "Easy construction method of Clawson-Schmidt conjugated points on QL-Cu1".



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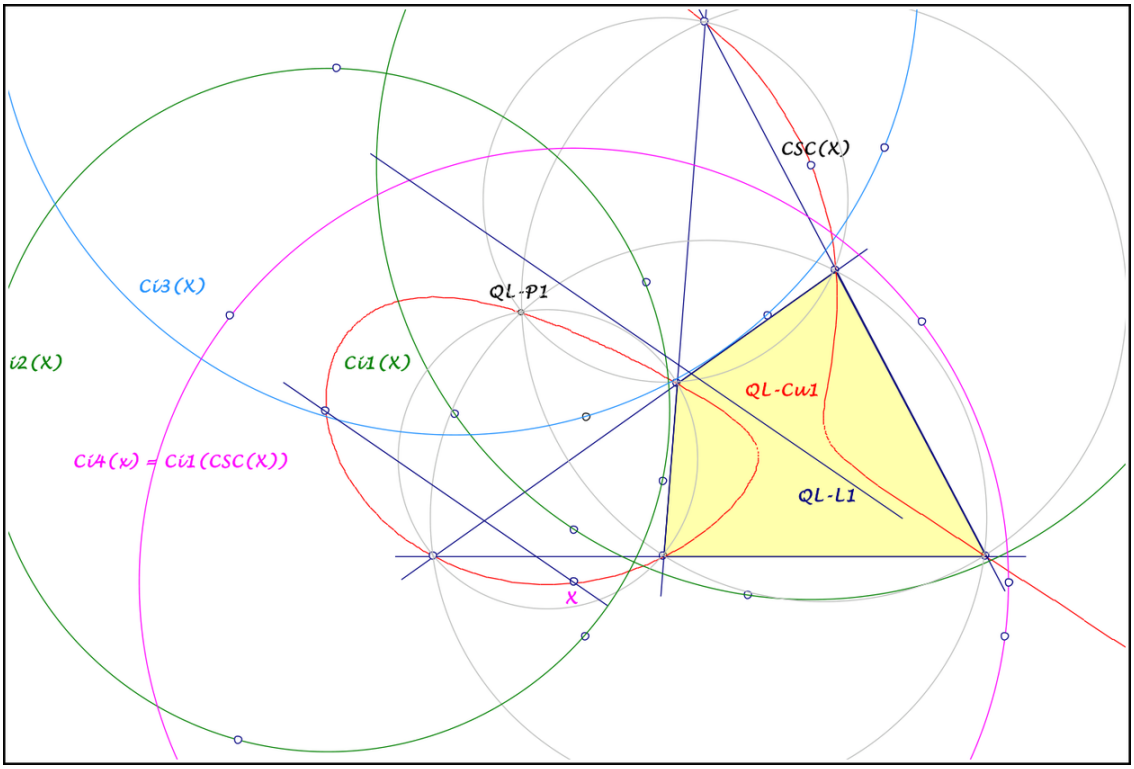

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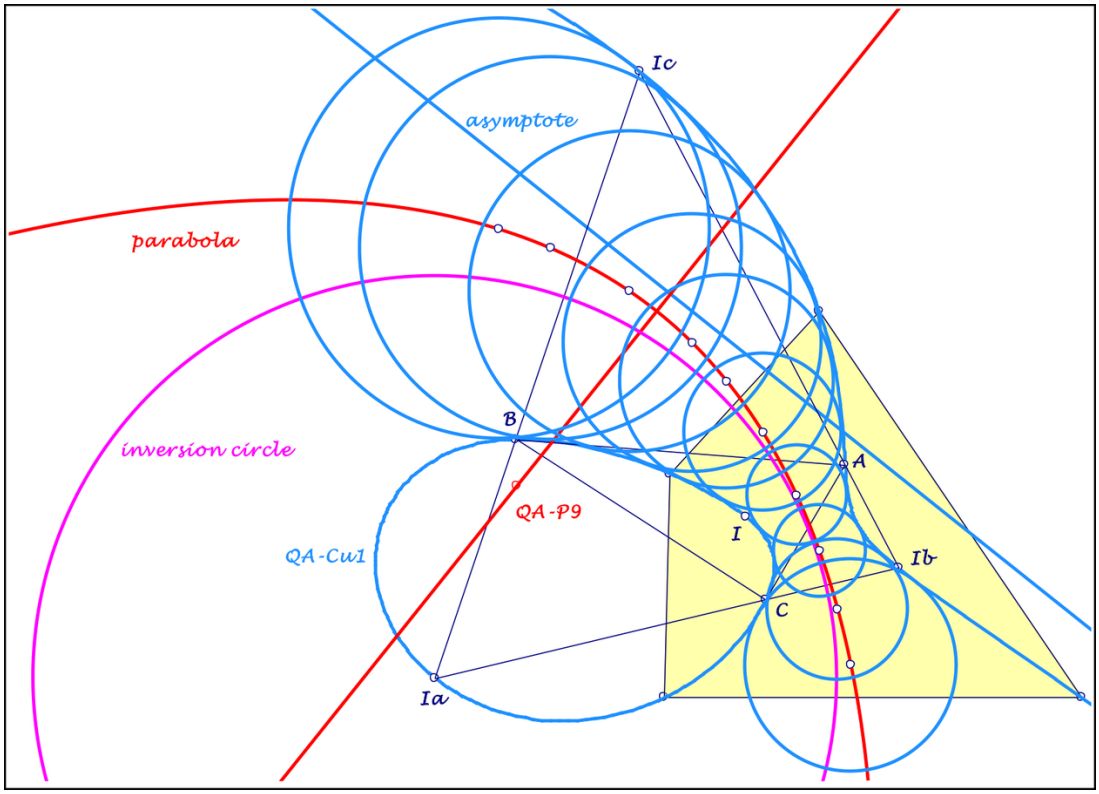
Message: #1822
Date: 22/7/2016 2:48:26
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 as Anallagmatic Curve

Dear Bernard, dear Chris,
QA-Cu1 is already described in EQF as anallagmatic curve, but there is a further possibility:
(1) In EQF: QA-Cu1 has 4 inversion circles
... centered in the in-/ex-centers I, Ia, Ib, Ic
    of the Miquel triangle QA-Tr2 = ABC,
... the inversion of a circle round Ia swaps B, Ic and C, Ib,
... the inversion of a circle round I swaps
    e.g. Ia and the reflection of A in I.
(2) QA-Cu1 is four times the envelope of circles (see attached file)
... orthogonal to one of the inversion circles (see above),
... centered on a parabola,
... ... with focus QA-P9,
... ... with an axis orthogonal to the asymptote of QA-Cu1,
... ... and tangents in the perpendicular bisectors
        for further intersections of QA-Cu1 and lines
        through the center of the inversion circle.
Best regards Eckart



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Message: #1823
Date: 22/7/2016 6:50:02
From: bernard.keizer
Subject: Desmic, QA's and QL's

Dear Eckart,
As you seem always interested in QA-Cu1 and QL-Cu1, I've found this with the Desmic configuration, I hope this will please you.
QA-Cu1 is a pivotal circular cubic
QL-Cu1 in it's bicursal mode, is a pivotal circular focal cubic
QL-Cu1 is therefore a special QA-Cu1
It has a Miquel triangle, QL-P1 and QL-2P2a and b and the focus QA-P9 is in QL-P1 ; the Newton Line is the perpendicular bisector of QL-P2a and b.
A Cu-QA is formed, for any point X on the curve by X and the 3 CSC conjugates A' (Cl-S with center QL-P1), A'' and A'''; A''' and A'' are also CSC conjugates and QA-P1 is on the Newton Line.
2 of the 6 sides of this QA are perpendicular ; they intersect in a point on QL-Cu1, which is at the same time QA-P2 and a vertice of DT and the CSC of this point is QA-P4.
For example, the QA formed by the infinity point of the Newton Line, QL-P1 and the QL-2P2a and b is a Cu-QA ; QA-P4 is the point where the asymptote cuts the curve (opposite to QL-P1 on the circumcircle of the Miquel triangle).
Now taking 3 aligned points A, B and C on QL-Cu1, the 3 CSC circles give the points A'B'C' (on a circle through QL-P1, A"B"C" (on a circle through QL-2P2a) and A"', B"' and C"' (on a circle through QL-2P2b) ; the 3
corresponding QA's are in a Desmic configuration, the QA-P4 are aligned and the QA-P2 on the CSC circle.(through QL-P1).
This configuration leads to 12 inscribed QL's ; 4 of these QL's have the same Miquel point and Newton Line. The 8 others are in 2 groups of 4 ; their Newton Lines form 2 other QL's with the same Newton Line as the 4 of the 1rst group.
The same configuration with an ordinary non-focal QA-cu1 doesn't give this result.
Best regards
Bernard

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Message: #1824
Date: 23/7/2016 11:23:22
From: bernard.keizer
Subject: QA-Cu1 as Anallagmatic Curve

Dear Eckart,
This envelop is very beautiful !
If my memory is not bad, there was already such a construction for QL-Cu1, with QL-P1 instead of QA-P9 and Newton Line instead of asymptote.
Logical, as QL-Cu1 is a special case of QA-Cu1 with focus in QL-P1 on the curve and asymptote parallel to the Newton Line ...
Best regards
Bernard
PS If I remember, there were 2 parabolas in the case of QL-Cu1 ?

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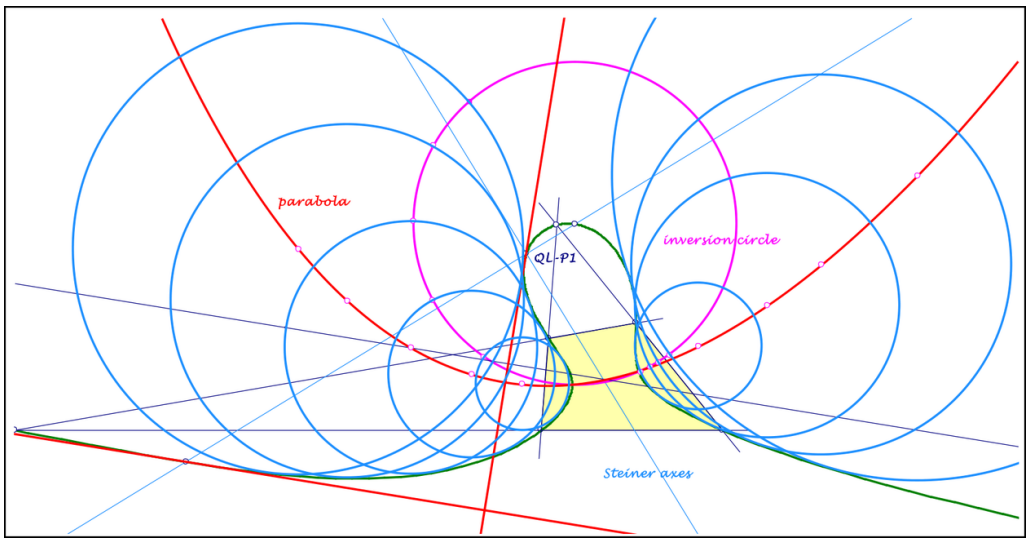
Message: #1825
Date: 23/7/2016 3:26:36
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as Anallagmatic Curve

Dear Bernard, dear Chris,
the fact, that QL-Cu1 is anallagmatic, is already mentioned on my homepage (see 11.1) and in several QFG-messages of Bernard and me. I think, it´s worth, to be mentioned in EQF,
... for the centers of inversion circles lie in intersections of the Steiner axes and QL-Cu1.
So I have tried to give a short description (see attached files).
... In the unipartite case of QL-Cu1 there are two inversion circles, centered in the intersections of QL-Cu1 and the 2nd Steiner axis, bearing QL-2P3.
... In the bipartite case of QL-Cu1 there are three inversions, circles centered in the excenters of the triangle QL-P1, QL-2P2, swapping incenter and vertex of this triangle.
Further:
In the bipartite case QL-Cu1 can be considered as QA-Cu1 (see #1377) and described as envelope of circles (see #1822).
In the unipartite case QL-Cu1 is two times the envelope of circles (see attached file)
... orthogonal to one of the inversion circles,
... centered on a parabola,
... ... with focus QL-P1,
... ... with an axis orthogonal to the asymptote of QL-Cu1
... ... and a directrix through the center of the other inversion circle.
Best regards Eckart
PS. Bernard, I just read your last message: Your memory was right!



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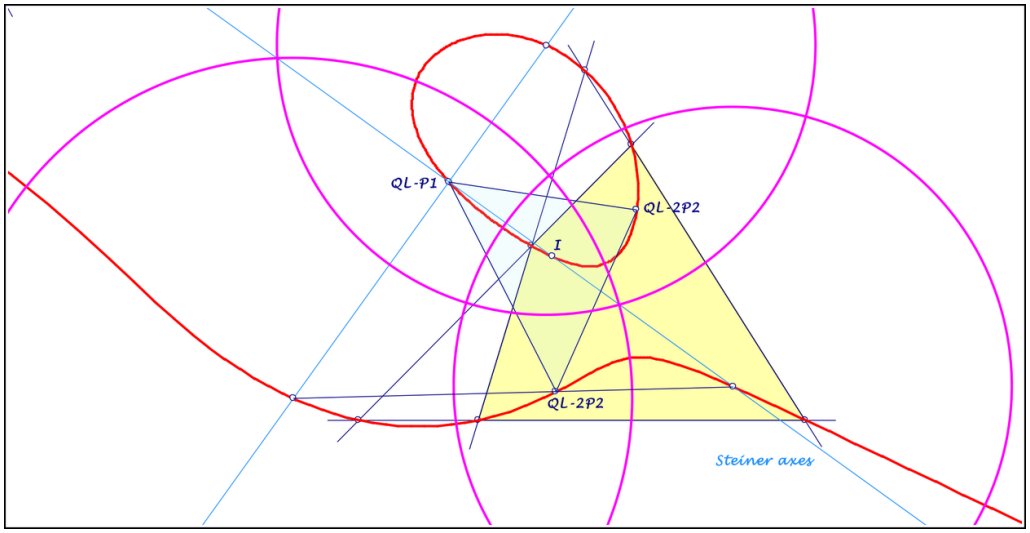

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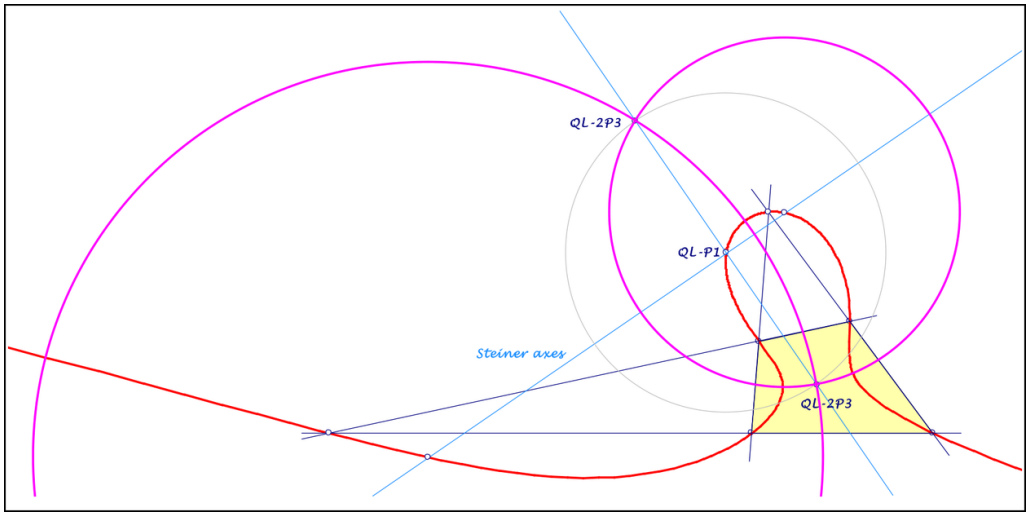

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Message: #1826
Date: 24/7/2016 11:23:01
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

Dear Bernard,
I´m not familar with Desmic configurations, but I had a look in "Desmic Mate" from Wolfram MathWorld. Am I right, that QA-Cu1 is the final isocubic described there, using ABC = QA-Tr1 and A´B´C´= QA-Tr2 with perspector D´´ = QA-P3?
Best regards Eckart

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Message: #1827
Date: 25/7/2016 10:38:20
From: bernard.keizer
Subject: QA-Cu1

Dear Eckart,
I'm not a specialist of the Desmic configuration either, but I suppose you are right !
In fact, it is exactly the given definition. The point D is necessary the point QA-P4 and ABCD forms a Cu-QA with tangential QA-P41, QA-P4 is the pivot for the DT triangle QA-Tr1, the point D' is for the same reason the infinity point of the asymptote and A'B'C'D' forms a 2nd Cu-QA with tangential the point where the curve cuts it's asymptote, this infinity point is the pivot for the Miquel triangle QA-Tr2 (the isoconjugation being then the isogonal transformation).
D" is QA-P3 and the 3 other points A", B" and C" are the Cl-S conjugates wrt the vertices of Tr2 ; the tangential of the QA A"B"C"D" is on a parallel to QA-P3QA-P4 and to the asymptote through QA-P41.
Best regards
Bernard
PS The definition given for the QL-2P2a and b in EQF is misleading.
In the monocursal curve, the 2 points are the intersection of the curve with the Newton Line and QL-P1 is their tangential ; each of the points is the tangential of 2 points symmetric wrt the other and these 4 points are the fixed points of the Cl-S conjugations ...
In the bicursal curve, the 2 points are symmetric wrt the Newton Line and share with QL-P1 and the infinity point of the Newton Line a common tangential (point where the curve cuts it's asymptote) .
Maybe we have to consider 4 points of which only 2 are real in each case. These 4 points would be the foci of the same inscribed conic ...
What do you think of that ?

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Message: #1828
Date: 25/7/2016 1:24:03
From: bernard.keizer
Subject: QA-Cu1

Dear Eckart,
There'is a slight mistake in my last message ; the tangential of the QA with QA-P3 is aligned with QA-P41 and the opposite point of QA-P9 on the Miquel circle, it cannot be on the parallel through the asymptote through QA-P41.
I suppose you had corrected by yourself, but anyway ...
Best regards
Bernard

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Message: #1829
Date: 26/7/2016 9:24:44
From: eckart_schmidt@t-online.de
Subject: QL-2P2

Dear Bernard,
wrt QL-2P2 in PS of #1827, you are right: "The definition of QL-2P2 in EQF is misleading." But this an old discussion with Chris, see messages in sept.13.
In EQF the definition gives only real points for the unipartite case of QL-Cu1. But we often need the corresponding points for the bipartite case, which are not mentioned in EQF. I think, it would be better, to define QL-2P2 as the foci of an QL-inscribed conic, centered in the intersection of QL-L1 and QL-L6 (see #254). But then the name Newton in its description would be improper.
Best regards Eckart

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Message: #1830
Date: 26/7/2016 10:38:49
From: chris.vantienhoven
Subject: QL-2P2

Dear Bernard and Eckart,
I hope I understand you right.
According to my understanding QL-2P2 is the pair of Isogonal Conjugated points on the Newton Line common for all 4 QL-Component Triangles. QL-2P2 can be a real pair of points as well an imaginary pair of points. That is how QL-2P2 is defined. And with this definition QL-2P2 lies on QL-Cu1.
When this is not true I would like to hear it.
When I understand it right in the bipartite case of QL-Cu1 the foci of the QL-inscribed conics take over some function for QL-Cu1. However these points have another location. So they should have another registration. True?
Since both of you are QL-Cu1 experts and when you both want me to register this new pair please exactly mail me a definition and description of these points, a drawing (JPG- or PNG- or GIF-format), CT- and DT-coordinates and a list of properties, then I can register it in EQF under QL-2P5.
Eventually also additional mutations for QL-2P2.
Best regards,
Chris

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Message: #1831
Date: 26/7/2016 4:07:05
From: eckart_schmidt@t-online.de
Subject: QL-2P2

Dear Chris,
I am not interested to start a new discussion wrt QL-2P2. I shall avoid this nomination. It is not good, to change an EQF-definition, but we shall say clearly, that QL-2P2 are real only in the unipartite case of QL-Cu1.
Best regards Eckart

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Message: #1832
Date: 26/7/2016 5:01:46
From: minhtenladuong
Subject: Isogonal conjugate points on QL-Cu1

Dear all
I found the isogonal conjugate pairs on QL-Cu1 are very interesting.
Here are some observations:
1. Let (A,A*), (B,B*) be the isogonal conjugate pairs of the reference quadrilateral.
It is well-known that (C*=AB ∩ A*B*, C=AB* ∩ BA*) are also isogonal conjugate pairs.
According to QL-Tf1, the reference quadrilateral and the quadrilateral (AB,A*B*,AB*,BA*) share the same Miquel point.
Combine with these two quadrilateral share the same QL-Cu1 ----> They also share the same Newton line.
2. Let (X,X*), (Y,Y*), (Z,Z*) be the pairs of opposite vertices of the reference quadrilateral.
Pedal circles of (A,A*), (B,B*), (C,C*) with respect to the reference quadrilateral and pedal circles of (X,X*),(Y,Y*),(Z,Z*) with respect to the quadrilateral (AB,A*B*,AB*,A*B) are coaxial !

_____

I have only proved that Pedal circles of (A,A*), (B,B*), (C,C*) with respect to the reference quadrilateral are coaxial (see attach file).
I am looking forward to hearing from you.
Best regards
Sincerely
Ngo Quang Duong

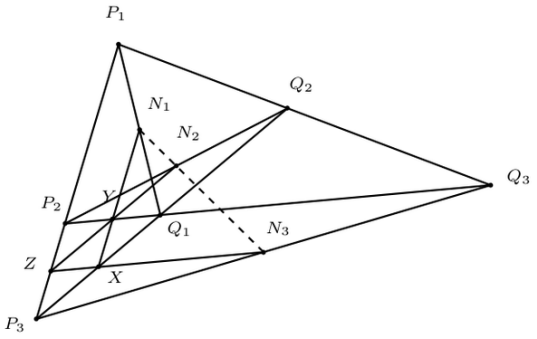
Coaxial pedal circles

Ngo Quang Duong

July 26, 2016

Theorem (Two pairs 1)**.** *(P_1, Q_1) and (P_2, Q_2) are two isogonal pairs WRT $\triangle ABC$. $P_1P_2 \cap Q_1Q_2 = P_3$, $P_1Q_2 \cap P_2Q_1 = Q_3$.*
It is well-known that P_3, Q_3 are also isogonal conjugate points.
Then pedal circles of (P_1, Q_1), (P_2, Q_2), (P_3, Q_3) are coaxial.

Proof. Let D_i, X_i be orthogonal projections of P_i, Q_i on BC; N_i be midpoint of P_iQ_i.



According to Newton line theorem, N_1, N_2, N_3 are collinear. To prove these pedal circles are coaxial, we will use ratio of power.[1]

$$\begin{aligned}\frac{\mathcal{P}_{D_3/(N_1)}}{\mathcal{P}_{D_3/(N_2)}} &= \frac{\overline{D_3D_1}\cdot\overline{D_3X_1}}{\overline{D_3D_2}\cdot\overline{D_3X_2}}\\ &= \frac{\overline{D_3D_1}}{\overline{D_3D_2}}\cdot\frac{\overline{D_3X_1}}{\overline{D_3X_2}}\\ &= \frac{\overline{P_3P_1}}{\overline{P_3P_2}}\cdot\frac{\overline{P_3Q_1}}{\overline{P_3Q_2}}\end{aligned}$$

[1] $\overline{AB}$ is signed length of segment AB. $\mathcal{P}_{M/(O_1)}$ is power of M to the circle (O_1).

Let X, Y, Z be midpoints of P_3Q_1, P_2Q_1, P_2P_3. By Menelaus's theorem

$$\begin{aligned}\frac{\overline{N_3N_1}}{\overline{N_3N_2}} &= \frac{\overline{XN_1}}{\overline{XY}} \cdot \frac{\overline{ZY}}{\overline{ZN_2}} \\ &= \frac{\overline{P_3P_1}}{\overline{P_3P_2}} \cdot \frac{\overline{P_3Q_1}}{\overline{P_3Q_2}} \\ &= \frac{\mathcal{P}_{D_3/(N_1)}}{\mathcal{P}_{D_3/(N_2)}}\end{aligned}$$

Similarly, $\dfrac{\mathcal{P}_{D_3/(N_1)}}{\mathcal{P}_{D_3/(N_2)}} = \dfrac{\mathcal{P}_{X_3/(N_1)}}{\mathcal{P}_{X_3/(N_2)}}$

Combine with the collinearity of N_1, N_2, N_3 we conclude that (N_1), (N_2), (N_3) are coaxial. □

coaxial-pedal-circles.pdf

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Message: #1833
Date: 27/7/2016 8:43:05
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugated points on QL-Cu1

Dear Ngo Quang Duong,
wrt # 1832 1): This is already mentioned in # 1432.
wrt # 1832 2): In addition of the last statement: "... are coaxial on the Newton Line QL-L1".
Best regards Eckart

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Message: #1834
Date: 27/7/2016 2:55:55
From: tsihonglau
Subject: QL-2P2

Dear all,
I do not think we should redefine QL-2P2.
The Newton line QL-L1 is the involutary conjugate QL-Tf2 of the line at infinity.
The circular points are the isogonal conjugates on the line at infinity.
QL-2P2 are the isogonal conjugates on the Newton line.
Generally speaking, there is only one pair of isogonal conjugates on a given line.
But isogonal conjugates are a metric concept.
In the world of quadrangle/quadrilateral there are only involutary conjugates QA-Tf2/QL-Tf2.
So there is only one pair of involutary conjugates on/through a given line/point.
We can study the pairs of involutary conjugates on/through a given line/point- QA-P1,QA-P2... QL-L1,QL-L2... etc.
We can construct the lines through pairs of involutary conjugates on an isocubic (point cubic).
If it is pivotal, these lines pass through the pivot.
If it is non-pivotal, these lines form a line curve, which is the envelope of a point curve.
If the line curve or point curve is cubic, then it is isocubic?

For example, QA-Cu7 is a non-pivotal isocubic.
The lines through pairs of involutary conjugates form a line curve.
What are the equations for the line curve and its envelope?
Any of them is isocubic? Any idea will be appreciated.
Best regards,
Tsihong Lau

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Message: #1835
Date: 27/7/2016 5:48:50
From: minhtenladuong
Subject: Isogonal conjugated points on QL-Cu1

Dear Eckart Schmidt,
Thank you very much.
Sincerely
Ngo Quang Duong

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Message: #1836
Date: 27/7/2016 9:36:28
From: chris.vantienhoven
Subject: QL-2P2

Dear Eckart and Bernard,
I removed in EQF at QL-2P2 the remarks about the foci of the inscribed conic with center QL-L1^QL-L6. I also added your remark about QL-Cu1 in bi-partite state.
Thank you for your remarks and sorry for the confusion. When still something is wrong please mail me.
Best regards,
Chris

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Message: #1837
Date: 28/7/2016 10:39:35
From: bernard.keizer
Subject: QL-2P2

Dear Chris, dear Eckart, dear Tsihong Lau
I'm glad we all agree!
When I asked Eckart, I didn't think of changing the definition of QL-2P2, but of giving precisions about these points in both cases.
In fact, defining these points as the foci of the inscribed conic centered in the intersection of QL-L1 and QL-L6 avoids any discussion, as these foci are always real.
Then, we have different properties in the mono- or bicursal curve:
in the monocursal, QL-P1 is the tangential of the QL-2P2 and the intersection point of the curve and it's asymptote is the tangential of QL-P1 and the infinity point of the Newton Line
in the bicursal, the intersection point of the curve and it's asymptote is the common tangential of QL-P1, the QL-2P2 and the infinity point.
In both cases, the QL-2P2 are centers of a Cl-S transformation, the curve QL-Cu1 is invariant in the 3 Cl-S transformations and the triangle QL-P1QL-2P2a and b is a Miquel triangle.
QL-Cu1 is anallagmatic, there are 2 real centers of inversion in the monocursal curve (on the 2nd Steiner axis) and 4 in the bicursal (in-and excenters of the Miquel triangle).
Until recently, I drew the mono- and bicursal curves with a unique Miquel triangle and the same Cl-S transformation ; the difference was the Newton Line, the opposite side of QL-P1 in the monocursal, the perpendicular bisector of the opposite side in the bicursal.
The bicursal is a pivotal isocubic with the pivot as the infinity point of the Newton Line as perpendicular bisector (hessian of the Kjp cubic).
But it's possible to have a different approach with a unique Miquel point QL-P1, the same Newton Line and the same Schmidt (inversion) circle ; the difference is then the Steiner axes, the 1rst and the 2nd being swapped. With this approch, the Cl-S are also different and we have in fact 4 points, 2 on the Newton Line on the monocursal curve and 2 symmetric wrt this same line on the bicursal curve!
My question to Eckart was to find the link (if there is one) between these 2 copples of points ...
In both cases, we have also QA's with 2 perpendicular cords, but one of the cords is imaginary in the monocursal curve.
Best regards
Bernard

PS I remember Tsihong Lau had once spoken about 4 foci of an inscribed conic with 2 real and 2 imaginary, but I must confess I hadn't understood ...

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Message: #1838
Date: 28/7/2016 4:09:13
From: eckart_schmidt@t-online.de
Subject: Rectangles on QA-Cu1 and QL-Cu1

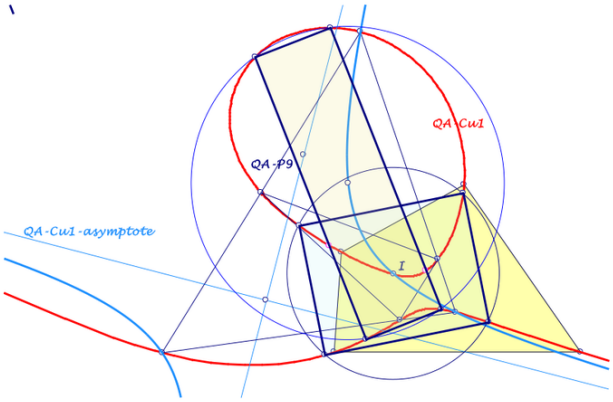
Dear Bernard, dear Chris,
in QFG-message 1773 a special rectangle on the cubic QA-Cu1 is described, but there is a set of inscribed rectangles, also for the cubic QL-Cu1. The centers of these rectangles lie on orthogonal hyperbolas (see attached file).
Best regards Eckart

EQF-Note 2016-07-28

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Rectangles on QA-Cu1 and QL-Cu1

In QFG-message 1773 a special rectangle on the cubic QA-Cu1 of a quadrangle is described, but there is a set of inscribed rectangles, also for the cubic QL-Cu1 of a quadrilateral. Here the loci for the centers of these rectangles are described, which are orthogonal hyperbolas. The results are CABRI-observations, a general construction for the circumcircles of the rectangles cannot be given.



Rectangles on QA-Cu1

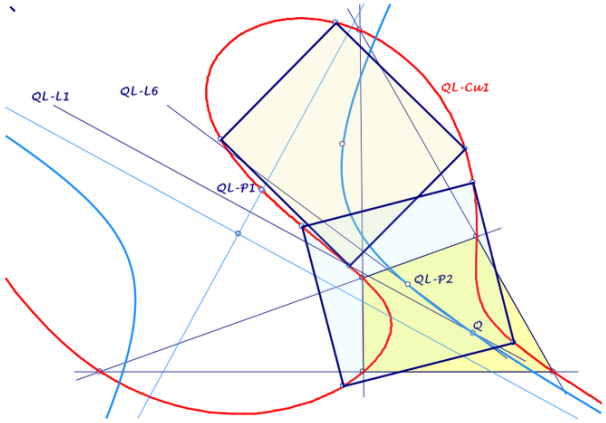
For a triangle there is a circle round the incenter *I*, whose inversion swaps a vertex and the reflection of the corresponding excenter in the incenter. If we consider this circle for the Miquel triangle of a quadrangle, its intersections with the cubic *QA-Cu1* give a rectangle (see *QFG*-message 1773). The inversion of *QA-Cu1* wrt this circle is the reflection in the incenter of the Miquel triangle.
But there are more rectangles inscribed *QA-Cu1* (see figure above):

- **The centers of rectangles on *QA-Cu1* lie on an orthogonal hyperbola**
  … through the in-/ex-centers of the Miquel triangle *QA-Tr2*,
  … with center in the pedal point of *QA-P9* on the asymptote of *QA-Cu1*,
  … which is also asymptote of the hyperbola.

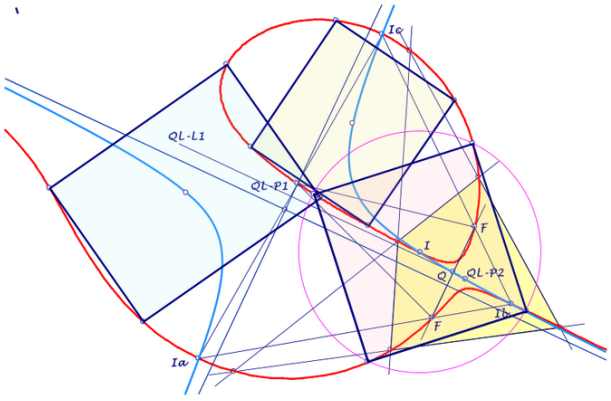
Rectangles on QL-Cu1

If *QL-Cu1* of a quadrilateral is bipartite, it can be considered as *QA-Cu1* of a special quadrangle (see *QFG*-message 1377) and the observations above hold for *QL-Cu1* in a corresponding way. But we will give a description in *QL*-geometry.

- **The centers of rectangles on *QL-Cu1* lie on an orthogonal hyperbola,**
  … through *QL-P2* and the intersection *Q* of *QL-L1* and *QL-L6*,
  … centered in the reflection of *QL-P1* in the Newton line *QL-L1*,
  … with an axis parallel *QL-L1*.



- **In the bipartite case of *QL-Cu1* the orthogonal hyperbola of the rectangle centers**
  … bears the in-/ex-centers of the triangle with vertices *QL-P1* and the foci *F* of a *QL*-inscribed conic centered in *Q* = *QL-L1*∩*QL-L6*.



If we consider for the described triangle the circle, mentioned at the beginning, the intersections with *QL-Cu1* give the corresponding rectangle. Analog: The inversion wrt this circle gives for *QL-Cu1* the reflection in *I*.

On the orthogonal hyperbolas there are only special regions for the rectangle centers. A general construction of the circumcircles of the rectangles cannot be given.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-07-28.pdf

Message: #1839
Date: 29/7/2016 9:53:05
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

Dear Tsihong Lau,
wrt QA-Cu7 in #1834:
The lines through pairs of involutary conjugates give a sextic, tangent to the 6 QA-lines.
So the dual of this sextic is a cubic through the vertices of the dual QL
... and the duals of QA-P4.QA-P41 and a perpendicular wrt QA-L2 in QA-P2.
But I couldn´t it identify as isocubic.
Best regards Eckart

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Message: #1840
Date: 29/7/2016 10:57:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

Dear Chris, dear Tsihong Lau,
in EQF wrt QA-Cu7 Chris used the ending "ic", I think for "involutary conjugate". Perhaps this can be avoided:
"QA-P4ic" is QA-P41.
"Sic" = T is the intersection of QA-P4.QA-P41 and a perpendicular line in QA-P2 wrt QA-L2.
S is the intersection of QA-Cu7 and its asymptote
... or the involutary conjugate of T
... or the intersection of the tangents in QA-P2, QA-P4, QA-P41
... or the diametral point of QA-P41 on the circumcircle of QA-P2, QA-P4, QA-P41
... with Simson line QA-L2.
I think, some properties are new, especially the following:
QA-Cu7 is a pivotal isogonal cubic wrt
... the reference triangle QA-P2.QA-P4.QA-P41,
... the isogonal conjugacy
... and pivot in the point at infinity of a perpendicular of QA-L2 = QA-P2.QA-P4.
Best regards Eckart

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Message: #1841
Date: 30/7/2016 6:21:42
From: minhtenladuong
Subject: Rectangles on QA-Cu1 and QL-Cu1

Dear Eckart Schmidt,
You always have very nice observation!
Can you show how to construct these rectangles?
And could you check my observations, because my tools are not "strong" enough to verify them: the rectangular hyperbola of QA-Cu1 is polar conic of the infinity point(real) of QA-Cu1 with respect to QA-Cu1 and the rectangular hyperbola of QL-Cu1 is polar conic of the infinity point of Newton line with respect to QL-Cu1.
Sincerely
Ngo Quang Duong

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Message: #1842
Date: 30/7/2016 9:03:36
From: eckart_schmidt@t-online.de
Subject: Rectangles on QA-Cu1 and QL-Cu1

Dear Ngo Quang Duong,
you are right, the rectangular hyperbolas for the centers of the rectangles are the polar conics of the infinity point of QA-Cu1 wrt QA-Cu1 and of the infinity point of QL-Cu1 wrt QL-Cu1.
These rectangular hyperbolas can easily be constructed, but I cannot give a general construction for the rectangles, as I pointed out in my paper. The rectangles are drawn manually. In vain I tried to find a construction for the circumcircles of these rectangles.
Thanks for interest.
Best regards Eckart

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Message: #1843
Date: 04/8/2016 6:12:31
From: minhtenladuong
Subject: Four n-angle centers

Dear all,
Before giving the generalization, I introduce a type of triangle centers:
Pn is "n-angle center" of triangle ABC if (PB,PC)=n(AB,AC)(mod pi); (PC,PA)=n(BC,BA)(mod pi) then of course we have (PA,PB)=n(CA,CB)(mod pi)
Particularly: X4 is -1-angle center; X3 is 2-angle center, X186 is 3-angle center, X265 is -2-angle center, X5962 is -3-angle center.
Thus I conjectured that: n-angle centers of 4 component triangles of a quadrilateral are concyclic. (let the circle be C(n))
I also generalized some others:
- C(n) is QL-Tf1 image of C(1-n).
- Let 4 component lines of the quadrilateral be a,b,c,d and n-angle center of triangle bcd be P(na).
We have a generalization of Kantor-Hervey point: perpendicular bisector of P(na)P((1-n)a), P(nb)P((1-n)b), P(nc)P((1-n)c), P(nd)P((1-n)d) are concurrent.
- 5 C(n) circles of 5 component quadrilaterals of a pentalateral are concurrent.
Sincerely
Ngo Quang Duong

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Message: #1844
Date: 04/8/2016 1:00:45
From: tsihonglau
Subject: Four n-angle centers

Dear Ngo Quang Duong,
Your ideas have appeared in topic #1787 & #1788.
These two topics should be the same topic.
But Eckart replied with a new topic, so they were spiltted.
Best regards,
Tsihong Lau

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Message: #1845
Date: 04/8/2016 3:44:25
From: minhtenladuong
Subject: Four n-angle centers

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Dear Tsihong Lau
Thank you for showing the messages.
Sincerely
Ngo Quang Duong
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Message: #1846
Date: 04/8/2016 9:12:15
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

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Dear Ngo Quang Duong,
excuse, there are sign mistakes in my last message: Please
replace the additional property by:
The isogonal conjugated of this chain give P1-n.
That gives:
   X3*    =  X4    is a P-1,
   X186*  =  X265  is a P-2,
   X5961* =  X5962 is a P-3,
   X5963* =  X5964 is a P-4, ...
Best regards Eckart
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Message: #1847
Date: 05/8/2016 12:22:11
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear Ngo Quang Duong,
there is a recursion for your "n-angle center" in an earlier paper (2003) on my homepage (sorry in German)
http://eckartschmidt.de/Triple.pdf
Under (15) you find ("n-angle center" in my nomination "Vielfach-Winkel-Punkt"):
Starting with P2 = X3,
... Pn+1 = QA-P4(A,B,C,Pn) is a "n+1-angle point".
That gives: X3 is a P2, X186 is a P3, X5961 is a P4, X5963 is a P5, ...
In addition:
The isogonal conjugated of this chain gives a Pn-1.
That gives: X186* = X265 is a P2, X5961* = X5962 is a P3, X5963* = X5964 is a P4, ...
So your conjecture will be right.
Best regards Eckart

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Message: #1848
Date: 05/8/2016 7:31:25
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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Dear Eckart, Bernard, Ngo Quang Duong,

I would like post this message weeks ago but did not.
Due to the join of Ngo Quang Duong, I post it now.

1. Can Anyone prove the assertions in topics #1787 and #1788?

2. I found these centers are related to Hofstadter points and posted the idea to APG topic #3024 before I joined this group. After that topic, Chris invited me to this group. In that topic I gave X(3),X(186),X(265),X(5962) are Hofstadter 2,3,-2,-3 points. Thanks for Eckart and Bernard, we have the center chain-H2=X(3) isogonal H(1-2)=H-1=X(4) inverse H(2-(-1)=H3=X(186) isogonal H(1-3)=H-2=X(265) inverse H(2-(-2))=H4=X(5961)...etc.
I give three other chains:
H-1=X(4) isogonal H(1-(-1))=H2=X(3) antigonal H-2=X(265) isogonal H(1-(-2))=H3=X(186) antigonal H-3=X(5962)...etc.
H-1=X(4) inverse H(2-(-1))=H3=X(186) antigonal H-3=X(5962) inverse H(2-(-3))=H5=X(5963) antigonal H-5=X(?)...etc. <- odd
H2=X(3) antigonal H-2=X(265) inverse H(2-(-2))=H4=X(5961) antigonal H-4=X(5964) inverse H(2-(-4))=H6=X(?)...etc. <- even
Please notice Hofstadter 0 and 1 points are transcendental and not in the chains.

3. Isgogonal centers are related to CSC conjugates. What are antigonal and inverse centers related to?

Best regards,
Tsihong Lau

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Message: #1849
Date: 07/8/2016 5:50:58
From: tsihonglau
Subject: Classification of Triangle/Trilateral and Quadrangle/Quadrilateral C

Dear all,
This topic is as important as the topic #1457 -
Merge Quadrangle/Quadrilateral Figures
I think the present classification of triangle/trilateral and quadrangle/quadrilateral centers/axes in ETC and EQF are not good enough. I would like to introduce a new classification last year. Now I present it. Please refer to my website for more info.
Geometry of Triangle/Trilateral and Quadrangle/Quadrilateral (http://lth.name/geometry/)
Geometry of Triangle/Trilateral and Quadrangle/Quadrilateral Duality of Triangle/Trilateral and Quadrangle/Quadrilateral Coordinate System
View on lth.name (http://lth.name/geometry/)
Preview by Yahoo
Given two points P(p:q:r) and U(u:v:w) and a function of P and U, we have the following properties.
1. Cyclic Defined
f(p:q:r,u:v:w):f(q:r:p,v:w:u):f(r:p:q,w:u:v)
2. s,t-Homogenous(Quadrangle/Quadrilateral Well-Defined)
f(kp:kq:kr,u:v:w)=ksf(p:q:r,u:v:w)
f(p:q:r,ku:kv:kw)=ktf(p:q:r,u:v:w)
3. +1,-1-Homogenous(Triangle/Trilateral Well-Defined)
f(kp:q:r,ku:v:w)=klf(p:q:r,u:v:w)
f(p:kq:r,u:kv:w)=kmf(p:q:r,u:v:w)
f(p:q:kr,u:v:kw)=knf(p:q:r,u:v:w)
+1-Homogenous l=m+1=n+1
-1-Homogenous l=m-1=n-1
4. P,U-Symmetric(Commutative)
f(p:q:r,u:v:w)=f(u:v:w,p:q:r)
5. qv,rw-Symmetric(Triangle/Trilateral Vertex/Side Symmetric)
f(p:q:r,u:v:w)=f(p:r:q,u:w:v)
6. P,U,qv,rw-Symmetric
f(p:q:r,u:v:w)=f(u:w:v,p:r:q)
A point/line which satisfies Cyclic Defined, s,t-Homogenous, +1-Homogenous, P,U-Symmetric, qv,rw-Symmetric, P,U,qv,rw-Symmetric is called Symmetric Point/Involutary Line Operation respectively.
If P and U are the circular points, the point/line is called (triangle/trilateral)Center/Involutary Axis.
For example, 1/(qw-rv):1/(ru-pw):1/(pv-qu) is the centroid of triangle or the newton line of quadrilateral.
A point/line which satisfies Cyclic Defined, s,t-Homogenous,

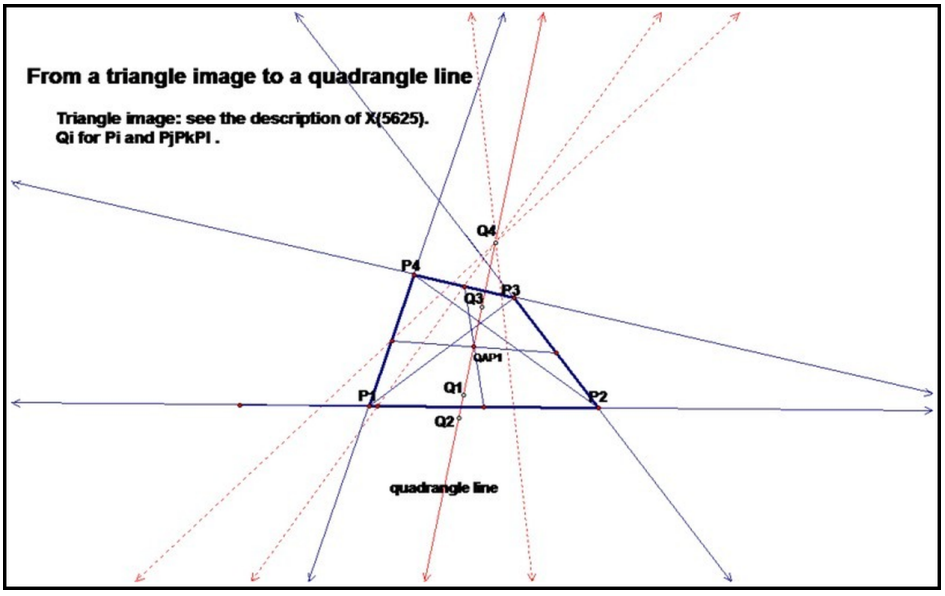
-1-Homogenous, P,U-Symmetric, qv,rw-Symmetric,
P,U,qv,rw-Symmetric is called Symmetric Line/Involutary Point
Operation respectively.
If P and U are the circular points, the point/line is called
(triangle/trilateral) Axis/Involutary Center.
For example,
(qw-rv):(ru-pw):(pv-qu) is the line are infinity or the harmonic
center of quadrangle/quadrilateral.
A point/line which satisfies Cyclic Defined, s,t-Homogenous,
+1-Homogenous, qv,rw-Symmetric is called Point /Involutary
Line Operation respectively.
A point/line which satisfies Cyclic Defined, s,t-Homogenous,
-1-Homogenous, qv,rw-Symmetric is called Line /Involutary
Point Operation respectively.
A point/line which satisfies Cyclic Defined, s,t-Homogenous
P,U-Symmetric, qv,rw-Symmetric, P,U,qv,rw-Symmetric is
called Symmetric Point/Line respectively.
If P and U are the circular points, the point/line is called
(quadrangle/quadrilateral) Center/Axis..

Given a unitary coordinate system -
Q   1: 1: 1 q   x+y+z=0
QA -1: 1: 1 qa -x+y+z=0
QB 1:-1: 1 qb   x-y+z=0
QC 1: 1:-1 qc   x+y-z=0
and a (triangle/trilateral)Center/Axis L(l:m:n)
and its involutary conjugate 1/L(1/l,1/m,1/n),
a point/line operation of L and 1/L is called a
(quadrangle/quadrilateral) Unicenter/Uniaxis derived from L.
For example, QA-P1 is cevadiv of 1/L and L, where L is the line
at infinity.
And QA-P38 is a cevadiv of X(6) and 1/X(6), where X(6) is the
symmedian point of diapleural triangle/diagonal trilateral. A
point/line operation of two Unicenters/Uniaxes is called
Bicenter/Biaxis.
For example, QA-P2 is cevamul- of X(6) and 1/X(2).

I raise two questions:
1. Are the sets of unicenters/uniaxes derived from two
triangle/trilateral centers/axes disjoint?
For example, are the sets of unicenters/uniaxes derived from the
line at infinity and X(6) disjoint?
2. Are all quadrangle/quadrilateral centers/axes
bicenters/biaxes?
To be continued...
Best regards,
Tsihong Lau

Message: #1850
Date: 09/8/2016 12:37:56
From: seiichikiri
Subject: From a triangle image to a quadrangle line

Dear friends,
Is the following already known?
Let P1P2P3P4 be a quadrangle.
Denote:
L12, L23, L31 = the lines through P4 parallel to P1P2, P2P3, P3P1 respectively.
P13, P23 = the intersections of L12 and P1P3, L12 and P2P3 respectively.
P21, P31 = the intersections of L23 and P1P2, L23 and P1P3 respectively.
P32, P12 = the intersections of L13 and P2P3, L13 and P1P2 respectively.
m12, m23, m31 = the midpoints of P12P21, P23P32, P31P13 respectively.
n12, n23, n31 = the midpoints of P31P32, P13P12, P23P21 respectively.
The lines through m12 and n12, m23 and n23, m31 and n31 concur in a point Q4.
In a similar manner we obtain Q1, Q2 and Q3.
Q1, Q2, Q3 and Q4 are collinear through QA-P1.
See the attachment.
Best regards, Seiichi.
P.S. as for the triangle image, see the description of X(5625) in ETC.



imagetoquad.png

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Message: #1851
Date: 09/8/2016 1:47:44
From: eckart_schmidt@t-online.de
Subject: From a triangle image to a quadrangle line

Dear Seiichi,
the collinear points Qi lie on QA-P1.QA-P16.QA-P21.
Best regards Eckart

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Message: #1852
Date: 09/8/2016 3:36:21
From: eckart_schmidt@t-online.de
Subject: QA-hyperbolas wrt QA-Tf1

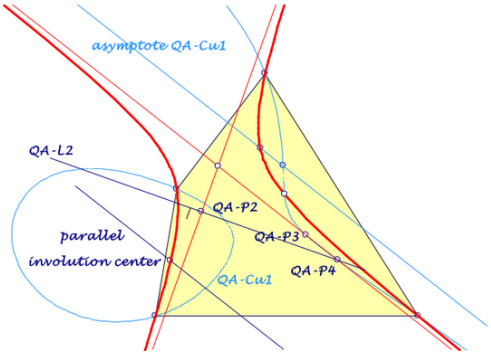
Dear friends,
the QA-Line Involution QA-Tf1 marks on a line an involution center (see EQF). For a line pencil this transformation gives a curve, for example QA-Cu6 for the pencil of QA-P1. If we take the line pencil of a point at infinity P ∞ , that means a set of parallels, the locus of the involution centers is a QA-circumscribed hyperbola (see attached file).
Best regards Eckart

EQF-Note 2016-08-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Circumscribed Hyperbolas wrt QA-Tf1

The QA-Line Involution QA-Tf1 marks on a line an involution center (see EQF). For a line pencil this transformation gives a curve, for example QA-Cu6 for the pencil of QA-P1. If we take the line pencil of a point at infinity P_∞, that means a set of parallels, the locus of the involution centers is a QA-circumscribed hyperbola.



First example

Let P_∞ be the point at infinity of the line L = *QA-P3.QA-P4*, which is a parallel to the asymptote of *QA-Cu1*.

- **The involution centers of parallels to the line *QA-P3.QA-P4* give a**
  … *QA*-circumscribed hyperbola,
  … centered in the *QA-Tf2*-image of the point at infinity of a perpendicular to *QA-P2.QA-P23*,
  … one asymptote *QA-P3.QA-P4*,
  … the other asymptote a perpendicular to *QA-L2* through *QA-P2*,
  … intersecting *QA-Cu1* in the isogonal conjugate of *QA-P41* wrt the Miquel triangle *QA-Tr2*.

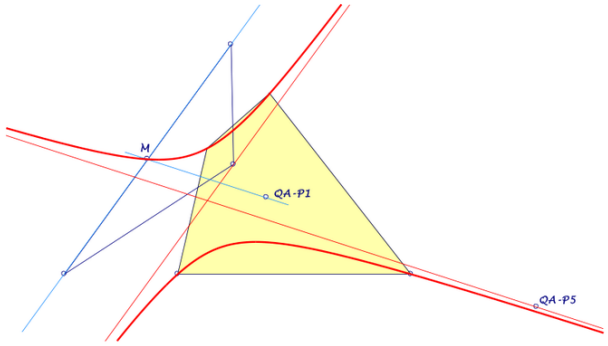
The equation of this hyperbola in *DT*-notation is:

$$q^2 r^2 \left(q^2 (c^2 p^2 - a^2 r^2)^2 - r^2 (b^2 p^2 - a^2 q^2)^2\right) x^2$$
$$+ r^2 p^2 \left(r^2 (b^2 p^2 - a^2 q^2)^2 - p^2 (c^2 q^2 - b^2 r^2)^2\right) y^2$$
$$+ p^2 q^2 \left(p^2 (c^2 q^2 - b^2 r^2)^2 - q^2 (c^2 p^2 - a^2 r^2)^2\right) z^2 = 0$$

- **In general: The point at infinity of the parallels is the point at infinity of one asymptote of the involution center hyperbola. Parallels to the other asymptote give the same involution center hyperbola.**

Second example

- **Parallels to a side of the diagonal triangle *QA-Tr1* give an involution center hyperbola**
  … circumscribed the quadrangle through the midpoint *M* of the *QA-Tr1*-side,
  … one asymptote parallel to the *QA-Tr1*-side through the opposite *QA-Tr1*-vertex,
  … the other asymptote parallel to *M.QA-P1* through *QA-P5*.



Final remark

- **The infinity points of the axes of the two quadrangle parabolas give *QA-Co1* as involution center hyperbola, for these axes are parallel to the asymptotes of *QA-Co1*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #1853
Date: 09/8/2016 4:18:28
From: tsihonglau
Subject: From a triangle image to a quadrangle line

Dear Seiichi, Eckart,
This line is cevacom- of the newton line and the line at infinity. For cevacom- and other operations,
please refer to my webpage-
Class of Objects of Triangle/Trilateral and Quadrangle/Quadrilateral (http://lth.name/geometry/class.html)

Class of Involutary Point/Line Operations and Centers/Axes of Quadrangle/Quadrilateral
I1- Symmetric Involutary Point Involutary Center + + -1 + + +
I1+ Symmetric Involutary Line Involutary Axis + +
View on lth.name
(http://lth.name/geometry/class.html)
Preview by Yahoo

Best regards,
Tsihong Lau

---In Quadri-Figures-Group@yahoogroups.com, wrote :
> Dear Seiichi,
> the collinear points Qi lie on QA-P1.QA-P16.QA-P21.
> Best regards Eckart

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Message: #1854
Date: 10/8/2016 12:53:30
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-Tf1

Dear Ngo Quang Duong,

in your note "Complete quadrilateral" you define in problem 1
... for a triangle ABC, a point P and a line d
... three collinear points in the following way:

Problem 1. Triangle ABC, a point P and a line d intersects
BC;CA;AB at A´;B´;C´.
Pa is a point that PaB´//PC and PaC´//PB
Pb is a point that PbC´//PA and PbA´//PC
Pc is a point that PcA´//PB and PcB´//PA
Prove that APa;BPb;CPc are parallel and P; Pa; Pb; Pc are
collinear.
Let me use the nomination L(ABC;P;d) for the line P.Pa.Pb.Pc.

In problem 2, 3, 4, 5 you give interesting consequences for
quadrilateral geometry.
But there is also an interesting aspect of this line in
quadrangle geometry:
Let P1P2P3P4 be a quadrangle and L a fixed line:
... The lines Li = L(PjPkPl;Pi;L) intersect in a point on L,
... which is the line involution center of L (see QA-Tf1).
For lines L = P.QA-Tf2(P) the line involution center is the
midpoint of P.QA-Tf2(P).

Best regards Eckart

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Message: #1855
Date: 10/8/2016 3:47:44
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

---

Dear all,
In APG topic #2179,
I propose a class of triangle centers - system centers.
Given a triangle center X(n) and D is the X(n) of ABC,
if A is the X(n) of DBC, B of ADC, C of ABD, then we call X(n) a system center.
ABCD is called an X(n) center system.
There are three known system centers- X(4), X(74), X(1138).
The dual quadrilateral of a center system(quadrangle) is axis system.
If Q,Q_A,Q_B,Q_C is an X(4) center system(the incenter/excenters of the diaplerual triangle ABC), then its dual q,q_a,q_b,q_c is an axis system.
Then we get the X(n) of exline trilaterals - X_n,X_na,X_nb,X_nc.

If n=4, then they all lie on the Steiner line and Q,Q_A,Q_B,Q_C and X_n,X_na,X_nb,X_nc are perspective at X(30)(with respect to ABC) = isogonal of X(74).
If n=3, then they all lie on Miquel circle and the two quadrangles are perspective at X(399) = isogonal of X(1138).
If n=265, then they are concyclic and the two quadrangles are perspective at a point, not listed in ETC.
If n=186, then they are concyclic and the two quadrangles are not perspective.
I wonder the results of the cases n=5962,5961,...etc.
I tried the cases X(74),X(1138) center system. The two quadrangles are not perspective, unfortunately.
Best regards,
Tsihong Lau

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Message: #1856
Date: 11/8/2016 1:27:33
From: chris.vantienhoven
Subject: Four n-angle centers

Dear Ngo Quang Duong,

After my holiday I read very interestedly your message #1843 about “Four n-angle centers”.
Here are some remarks:
1. I like the concept of n-angle centers. It is different from the concept of Hofstadter points.
I tried to prove it, but it is not simple.
2. I think your remark that C(n) = QL-Tf1 image of C(1-n) is conditionally true:
C(n) = QL-Tf1(C(1-n)) when n=2,4,6,….
Actually C(1) and C(2) are mutual QL-Tf1-mappings, C(3) and C(4) are mutual QL-Tf1-mappings, probably and so on.
3. Your remark that perpendicular bisector of P(na)P((1-n)a), P(nb)P((1-n)b), P(nc)P((1-n)c), P(nd)P((1-n)d) are concurrent I think is also conditionally true:
Here also perpendicular bisectors of P(na)P((1-n)a), P(nb)P((1-n)b), P(nc)P((1-n)c), P(nd)P((1-n)d) are only concurrent when n=2,4,6,…
The case of the common point of the perpendicular bisectors P(1a)P(2a), P(1b)P(2b), P(1c)P(2c), P(1d)P(2d) delivers the well-known construction of Kantor-Hervey Point (QL-P3).
The case of the common point of the perpendicular bisectors P(3a)P(4a), P(3b)P(4b), P(3c)P(4c), P(3d)P(4d) delivers a new point with pretty long coordinates (as well as its QL-Tf1-transformed point) and has no connections with familiar QL-points.
4. Your remark that 5 C(n) circles of 5 component quadrilaterals of a pentalateral are concurrent.
I checked them earlier to be true for C(2), C(3), C(4). Only the initial case for C(1) when the circle is a line (Steiner Line) appears not to be true. I didn't check the cases C(i) with i>4.

Best regards,
Chris

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Message: #1857
Date: 11/8/2016 4:10:50
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear Chris,
wrt #1856, 3):
You write:
" Here also perpendicular bisectors of P(na)P((1-n)a), P(nb)P((1-n)b), P(nc)P((1-n)c), P(nd)P((1-n)d) are only concurrent when n=2,4,6,…",
but I think, this is also valid for n=3,5,...
Further:
" The case of the common point of the perpendicular bisectors P(3a)P(4a), P(3b)P(4b), P(3c)P(4c), P(3d)P(4d) delivers a new point with ... ",
but I think, there is no common point.
Perhaps there is a mistake in my points:
P-4 = X5964, P-3 = X5962, P-2 = X265, ... , P2 = X3, P3 = X186, P4 =X5961.
Best regards Eckart

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Message: #1858
Date: 11/8/2016 7:22:30
From: chris.vantienhoven
Subject: Four n-angle centers

Dear Eckart,
Thanks for your remarks.
I made some interpretation, so I will have to re-interprete things.
I will come back to this item.
Best regards,
Chris
p.s. "The case of the common point of the perpendicular bisectors P(3a)P(4a), P(3b)P(4b), P(3c)P(4c), P(3d)P(4d) delivers a new point with ... ", related to the perpendicular bisectors of the four QL-versions of X(186) and X(265) and then we have a common point.

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Message: #1859
Date: 12/8/2016 3:20:43
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear Ngo Quang Duong, dear Chris,

let Pn be the n-angle centers, defined by Ngo Quang Duong in #1856:
... , P-4 = X5964, P-3 = X5962, P-2 = X265, P-1 = X4, ... , P2 = X3, P3 = X186, P4 = X5961, P5 = X5963.
With the following properties:
Pn and P-n+2 (n>2)
... are collinear with P2 = X3,
... are inverse wrt the circumcircle.

Already mentioned in the swapped messages 1846 and 1847 (I don´t know why!):
Starting with P2 = X3,
... Pn+1 = QA-P4(A,B,C,Pn).
... The isogonal conjugated of Pn is P-n+1.

Best regards Eckart

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Message: #1860
Date: 12/8/2016 4:53:03
From: chris.vantienhoven
Subject: Four n-angle centers

Dear Eckart, Dear Ngo Quang Duong,

Eckart these are startling properties! And you described them already in 2003!
I read in the preamble of ETC wrt Hofstadter points (with similar properties):

at X(359)
Conjecture and corollary (Randy Hutson, August 10, 2014)
If r is an integer other than 0, 1, or 2, then the inverse-in-circumcircle of the Hofstadter r point is the Hofstadter (2-r) point;
thus, since the isogonal conjugate of the Hofstadter r point is the Hofstadter (1-r) point, if r is not -1, 0 or 1,
then the antigonal image of the Hofstadter r point is the Hofstadter -r point.

at X(5961)
X(5961) is the Hofstadter 4 point and the antigonal image of X(5964), in accord with the following conjecture and corollary:
If r is an integer other than 0, 1, or 2, then the inverse-in-circumcircle of the Hofstadter r point is the Hofstadter (2-r) point; thus, since the isogonal conjugate of the Hofstadter r point is the Hofstadter (1-r) point, if r is not -1, 0 or 1, then the antigonal image of the Hofstadter r point is the Hofstadter -r point. (Randy Hutson, August 10, 2014)

So it appears that also the Antigonal Conjugate plays a role!
And it appears that these Triangle properties have their effects in Quadrilaterals as well as Pentalaterals and maybe even higher-level n-Lines.

Best regards,
Chris

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Message: #1861
Date: 13/8/2016 5:03:57
From: seiichikiri
Subject: Four n-angle centers

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Dear friends,

is the following already pointed?
In tripolar coordinates, given a triangle ABC and a point P,
  a^2=y^2+z^2-2y z Cos[AA],
  b^2=z^2+x^2-2z x Cos[BB],
  c^2=x^2+y^2-2x y Cos[CC],
where AA, BB, CC are the angles around P.
There are 2 solutions which are antigonal to each other.

Best regards, Seiichi.
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Message: #1862
Date: 13/8/2016 11:53:33
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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Dear Bernard, Chris, Eckart, Ngo Quang Duong, Seiichi,
The same topic now splits into three topics - #1787, #1788, #1843.
I raised this topic #1787 and then Eckart replied me in another topic #1788 and then Ngo Quang Duong did not know our discussions and raised the same questions in another topic #1843. We had better agree to discuss in some topic.

We return to the Hofstadter points Hn.
The first orthocentric (trilinear) coordinate of Hn is sin(-nA)/sin((1-n)A). I have proved the first quadrangle/quadrilateral circular coordinate is ((qw)^-n-(rv)^-n)/((qw)^(1-n)-(rv)^(1-n)) with integer n.
Obviously, n is not equal to 0,1.
H0 and H1 is the two transcendental points. We can prove easily Hn and H(1-n) is isogonal.

A similar series arises. The first orthocentric(trilinear) coordinate of H'n is cos(-nA)/cos((1-n)A).
I have proved the first quadrangle/quadrilateral circular coordinate is ((qw)^-n+(rv)^-n)/((qw)^(1-n)+(rv)^(1-n)) with integer n.
Obviously, n can be equal to 0,1.
H'0 and H'1 are X4 and X3 respectively.
H'-1 and H'2 X68 and X24 respectively.
Other points are not listed in ETC.
We can prove easily H'n and H'(1-n) is isogonal.
I do not know the geometric meaning of this series and if any transcendental points exist.

I give the circular coordinates of Hofstadter points because Hn's of exline trilaterals are concyclic. In other words, these four Hn's and circular points are conconic.
The circular points are fixed.
So I hope the circular coordinates will help the proof of concyclicity.

I think the above n's can be any real number but I cannot prove it.
Best regards,
Tsihong Lau

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Message: #1863
Date: 14/8/2016 5:31:45
From: tsihonglau
Subject: Sexangle from Eckart's Construction of Clawson-Schmidt Conjugate

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Dear all,

Because of the Hofstadter points of exline trilaterals, I studied Eckart's Construction of Clawson-Schmidt conjugate and found an interesting sexangle.

I use my notations -
http://lth.name/geometry/duality.html
Given a point P and a quadrilateral q,q_a,q_b,q_c and their points of intersection:
S_a=q∩q_a
S_bc=q_b∩q_c
S_b=q∩q_b
S_ca=q_c∩q_a
S_c=q∩q_c
S_ab=q_a∩q_b
(A B C) denotes the circumcircle of A,B,C.
Eckart's Construction of Clawson-Schmidt Conjugate is described as follows:
Fourth(including circular points) points of intersection
P_a=(P S_b S_c)∩(P S_ca S_ab)
P_b=(P S_c S_a)∩(P S_ab S_bc)
P_c=(P S_a S_b)∩(P S_bc S_ca)
P_bc=(P S_b S_ab)∩(P S_c S_ca)
P_ca=(P S_c S_bc)∩(P S_a S_ab)
P_ab=(P S_a S_ca)∩(P S_b S_bc)
The six points P_a,P_b,P_c,P_bc,P_ca,P_ab form a sexangle but not conconic.
The Clawson-Schmidt conjugate P' of P is the fourth point of intersection
=(P_a S_b S_ab)∩(P_a S_c S_ca)
=(P_b S_c S_bc)∩(P_b S_a S_ab)
=(P_c S_a S_ca)∩(P_c S_b S_bc)
=(P_bc S_b S_c)∩(P_bc S_ca S_ab)
=(P_ca S_c S_a)∩(P_ca S_ab S_bc)
=(P_ab S_a S_b)∩(P_ab S_bc S_ca)
We can reverse the construction to get P from P'.

We found the fourth points of intersection R,R' of the eight circumcircles.
R=(P_a P_b P_c)∩(P_a P_ca P_ab)
∩(P_bc P_b P_ab)∩(P_bc P_ca P_c)
R'=(P_bc P_ca P_ab)∩(P_bc P_b P_c)

∩(P_a P_ca P_c)∩(P_a P_b P_ab)
Very surprisingly! R and R' are Clawson-Schmidt conjugates!

Although I have not proved the above assertions!
Best regards,
Tsihong Lau

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Message: #1864
Date: 14/8/2016 10:11:20
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear all,
wrt the four n-angle centers P(n) for a quadrilateral:
Well known:
... The four n-angle centers are concyclic on a circle Ci(n)
... with Ci(n) = QL-Tf1(Ci(1-n)).
Perhaps new:
... The center of Ci(n) is the QL-Tf1-image of the inverse of QL-P1 wrt the circle QL-Tf1(Ci(n)) = Ci(1-n).
Best regards Eckart
PS: Thanks to Chris and Bernard for the background wrt the Hofstadter points!

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Message: #1865
Date: 14/8/2016 12:00:20
From: eckart_schmidt@t-online.de
Subject: Sexangle from Eckart´s Construction of Clawson-Schmidt Conjugat

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Dear Tsihong Lau,
I have reproduced your constructions, some remarks:
>From Sbc = CSC(Sa), Sca = CSC(Sb), Sab = CSC(Sc) follows:
... The sexangle consists of two tripels, which are CSC-partners: Pbc = CSC(Pa), Pca = CSC(Pb), Pab = CSC(Pc).
... Consequently the circles (PaPbPc) and (PbcPcaPab) , ... are CSC-partners and so R´ = CSC(R).
Best regards Eckart

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Message: #1866
Date: 14/8/2016 12:10:28
From: minhtenladuong
Subject: Four n-angle centers

Dear everyone

Thank you so much for your great interests with my ideas!
And I'm so sorry for not checking the mail recently. I was busy to prepare for my 1st year in university.
I have checked it for n=3 and the result Cn=QL-Tf1(C(1-n)) and the others are still true. I think it must be true for all integers.
If C(2n)=QL-Tf1(C(1-2n)) then C(1-2n)=QL-Tf1(C(2n)), since QL-Tf1 is an involution. Hence it is also true for odd integers.
In case n=1, 1-angle center is undefined cause every point on circumcircle is a 1-angle center.
But I have the following construction: Consider the quadrilateral (BC,CA,AB,L). L meets BC, CA, AB at A', B', C'. P is a point on Pa, Pb, Pc. Now follow my old notes "Complete quadrilateral"
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2647 †) we construct Pa, Pb, Pc. Then P, Pa, Pb, Pc are collinear and Pa lies on the circle AB'C', Pb lies on the circle BC'A', Pc lies on the circle CA'B' - obviously, since (PaB',PaC')=(PC,PB)=(AC,AB)=(AB',AC')(mod pi).
About the antigonal conjugate, I pose a conjecture: C(n) is degenerated to a line iff C(-n) is degenerated to a line.
The conjecture is based on the following: P, Pa, Pb, Pc are collinear iff the antigonal conjugate of P, Pa, Pb, Pc with respect to triangle ABC, AB'C', BC'A', CA'B' are collinear
- where P, Pa, Pb, Pc have already defined in my notes "Complete quadrilateral".

Sincerely
Ngo Quang Duong

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #1867
Date: 14/8/2016 1:37:51
From: tsihonglau
Subject: Sexangle from Eckart´s Constru ction of Clawson-Schmidt Conjuga

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Dear Eckart,
Thanks for your reply!
Best regards,
Tsihong Lau

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Message: #1868
Date: 15/8/2016 3:50:52
From: seiichikiri
Subject: Four n-angle centers

Dear Ngo Quang Duong,
I think that you are a very young man, though you did many things to Euclidean geometry. So I would like to hear your area of study and your aimes in your university.
As I am an old man of age 67, it would refresh me to think of your hopeful career since now.
Best regards, Seiichi.

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Message: #1869
Date: 15/8/2016 4:05:21
From: minhtenladuong
Subject: Four n-angle centers

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Dear Seiichi,

I have already chosen the faculty of mathematics. That's my
choice because I want to improve my knowlegde in mathematics, to
do better and deeper in researching. In university, I will also
learn English, programming and some others computer skills.
I want to contribute to geometry and education(be a teacher).
Anyway, I will keep my interests with Euclidean geometry.

Sincerely
Ngo Quang Duong
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Message: #1870
Date: 16/8/2016 1:07:45
From: seiichikiri
Subject: Four n-angle centers

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Dear Ngo Quang Duong,
I think that after graduation you should better study abroad
because there are many attractive places and universities in the
world.
Best regards, Seiichi.
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Message: #1871
Date: 16/8/2016 8:48:58
From: minhtenladuong
Subject: Four n-angle centers

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Dear Seiichi,
Thank you for your advice, I will consider it.
Sincerely
Ngo Quang Duong
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Message: #1872
Date: 16/8/2016 2:40:14
From: chris.vantienhoven
Subject: Four n-angle centers

Dear friends,
I made a summary of all properties of the Hofstadter-points/n-Angle Centers occurring in a Triangle/Quadrilateral/Pentalateral as discussed in our group.
See attachment.
If you find omissions or errors, please let me know.
I also found one new item:
The ratio of the radii of Ci(n) and gCi(n) is the same as the ratio of the distances of QL-P1 to their Centers. (see attachment)
Best regards,
Chris van Tienhoven

Hofstadter Points in a Triangle, Quadrilateral and Pentalateral

After a discussion in 2016 at the Yahoo Quadri-Forum QFG by Eckart Schmidt, Tsihong Lau, Bernard Keizer, Ngo Quang Duong, Seiichi Kirikami and Chris van Tienhoven these results regarding the Hofstadter points popped up.

Hofstadter points vs n-Angle points in a triangle

Hofstadter points are defined in ETC at the preamble of X(360):

> Let r denote a real number, but not 0 or 1. Using vertex B as a pivot, swing line BC toward vertex A through angle rB and swing line BC about C through angle rC. Let A(r) be the point in which the two swung lines meet. Obtain B(r) and C(r) cyclically. Triangle A(r)B(r)C(r) is the r-Hofstadter triangle; its perspector with ABC is called the Hofstadter H(r) point.

n-Angle Centers are defined by Ngo Quang Duong in QFG#1843:

> Pn is "n-angle center" of triangle ABC if (PB,PC)=n(AB,AC)(mod pi); (PC,PA)=n(BC,BA)(mod pi) then of course we have (PA,PB)=n(CA,CB)(mod pi).

Hofstadter points H(r) are points defined for r=real number.
n-Angle Centers P(n) are points defined for n=integer <> 0 and 1.

Hofstadter Points H(n) in a Triangle:

H(-4) = X(5964)
H(-3) = X(5962)
H(-2) = X(265)
H(-2/3) = X(5458)
H(-1/2) = X(79)
H(-1/3) = X(5457)
H(-1) = X(4)
H(0) = X(359)
H(1/4) = X(1127)
H(1/3) = X(357)
H(1/2) = X(1)
H(2/3) = X(358)
H(3/4) = X(1129)
H(1) = X(360)
H(3/2) = X(35)
H(2) = X(3)
H(3) = X(186)
H(4) = X(5961)

n-Angle Centers P(n) in a Triangle:

P(-4) = X(5964)
P(-3) = X(5962)
P(-2) = X(265) producing QL-Center QL-P29
P(-1) = X(4)
P(0) = undefined
P(+1) = undefined
P(+2) = X(3) producing QL-Center QL-P4
P(+3) = X(186) producing QL-Center QL-P28
P(+4) = X(5961)
P(+5) = X(5963)

QL-P-1-Hostadter-Points.pdf

It appears that the n-Angle Centers P(n) match with the Hofstadter points H(n) provided that n = integer <> 0 and 1.

Relationships between P(n) and H(n) points:

Let: g = isogonal conjugate
i = inverse in circumcircle
a = g.i.g = antigonal image
q = i.g = QA-P4

For n <> 0, 1 or 2 now we have:

- g.P(n) = P(1-n)
- i.P(n) = P(2-n)
- g.i.P(n) = P(n-1)
- i.g.P(n) = P(n+1) = q.P(n)
- g.i.g.P(n) = P(-n) = a.P(n)

Other properties:

In a Triangle

- P(n) and i.P(n) are inverse wrt the circumcircle (per definition)
- P(n) and i.P(n) and X(3) are collinear (consequently)

In a Quadrilateral

Let Ci(n) be the circle through the 4 QL-versions of P(n) in a Quadrilateral.
Let gCi(n) be the circle through the 4 QL-versions of g.P(n) in a Quadrilateral.

- The 4 QL-versions of P(n) are concyclic on circle Ci(n)
- The ratio of the radii of Ci(n) and gCi(n) is the same as the ratio of the distances of QL-P1 to their Centers.
- The 4 perpendicular bisectors of P(n) and g.P(n) concur
- Ci(n) = QL-Tf1 (gCi(n)) and vice versa gCi(n) = QL-Tf1(Ci(n))
- QL-Tf1(Center Ci(n)) = inverse of QL-P1 wrt gCi(n)

In a Cyclic Quadrangle

- The 4 QA-versions of P(n) in a cyclic Quadrangle are concyclic

In a Pentalateral

- 5 Ci(n) circles of 5 component quadrilaterals of a pentalateral are concurrent.

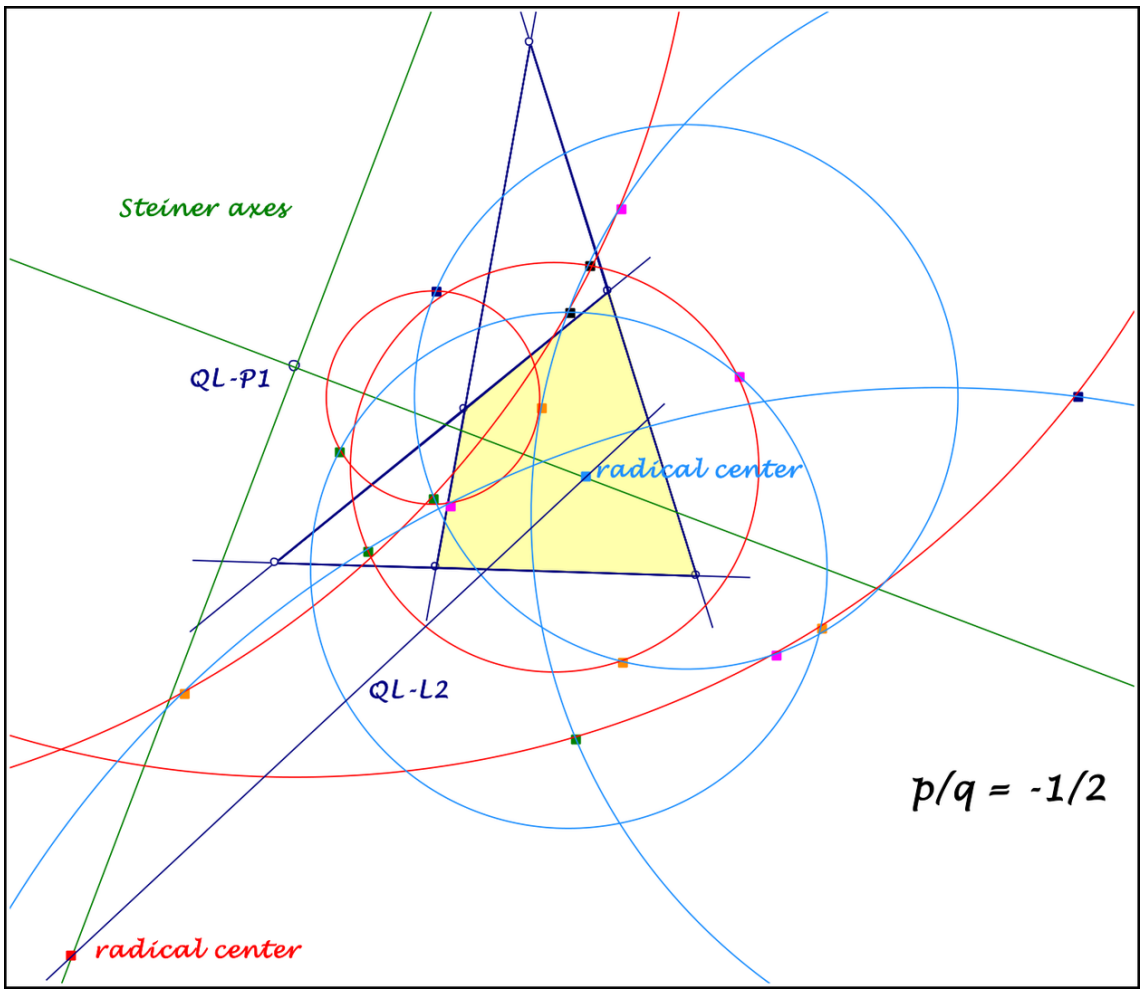
QL-P-1-Hostadter-Points.pdf

Message: #1873
Date: 17/8/2016 10:54:10
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear all,
I think, there is a generalization of "n-angle centers" for a triangle wrt n = p/q (rational, reduced), using
... P = P(p/q) with q*<APB = p*<ACB mod Pi, q*<BPC = p*<BAC mod Pi, q*<CPA = p*<CBA mod Pi.
... There will be q^2 such centers,
... with the recursion P(1+p/q) = QA-P4(A,B,C,P(p/q))
... and isogonal conjugate of P(p/q) is P(1-p/q).
Cabri observations confirm this conjecture. Are these points already discussed anywhere?
Well known: For a quadrilateral the n-angle centers (n integer) are concyclic.
New the p/q-angle centers for a quadrilateral :
The four P(1/2) of a trilateral are the in- and excenters,
... which give for a quadrilateral 4 x 4 points,
... in quadruples on the 12 angle bisectors of the QL-lines,
... in quadruples on the eight Steiner circles of a complete quadrilateral.

The four P(-1/2) of a trilateral are the isogonal conjugated for the inverse of the in- and excenters wrt the circumcircle (see attached file),
... which give for a quadrilateral 4 x 4 points,
... in quadruples on eight circles,
... two quadruples of circles, each with common radical center
... ... on a Steiner axis in the intersection with QL-L2,
... ...

So far, there will be more properties in these constellations for q = 2!
Best regards Eckart



2016-08-17.pdf

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Message: #1874
Date: 18/8/2016 8:40:46
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear all,
in #1873 I gave a generalization of P(n) to P(p/q), especially for q = 2 (p unequal zero)
In this case a triangle has four P(p/2),
... so there are 4x4 P(p/2) four a quadrilateral,
... in quadruples on eight circles,
... two quadruples of circles, each with a common radical center.
For these circles holds also the known recursion Ci(n) = QL-Tf1(Ci(1-n)) in the following way:
The eight P(p/2) circles of a quadrilateral are the QL-Tf1-images of the eight P(1-p/2) circles.
Cabri-observations verify this assumption (controlled for p = 3).
A simple example for p =1: The eight Steiner circles of a complete quadrilateral are CSC-invariant.
Best regards Eckart

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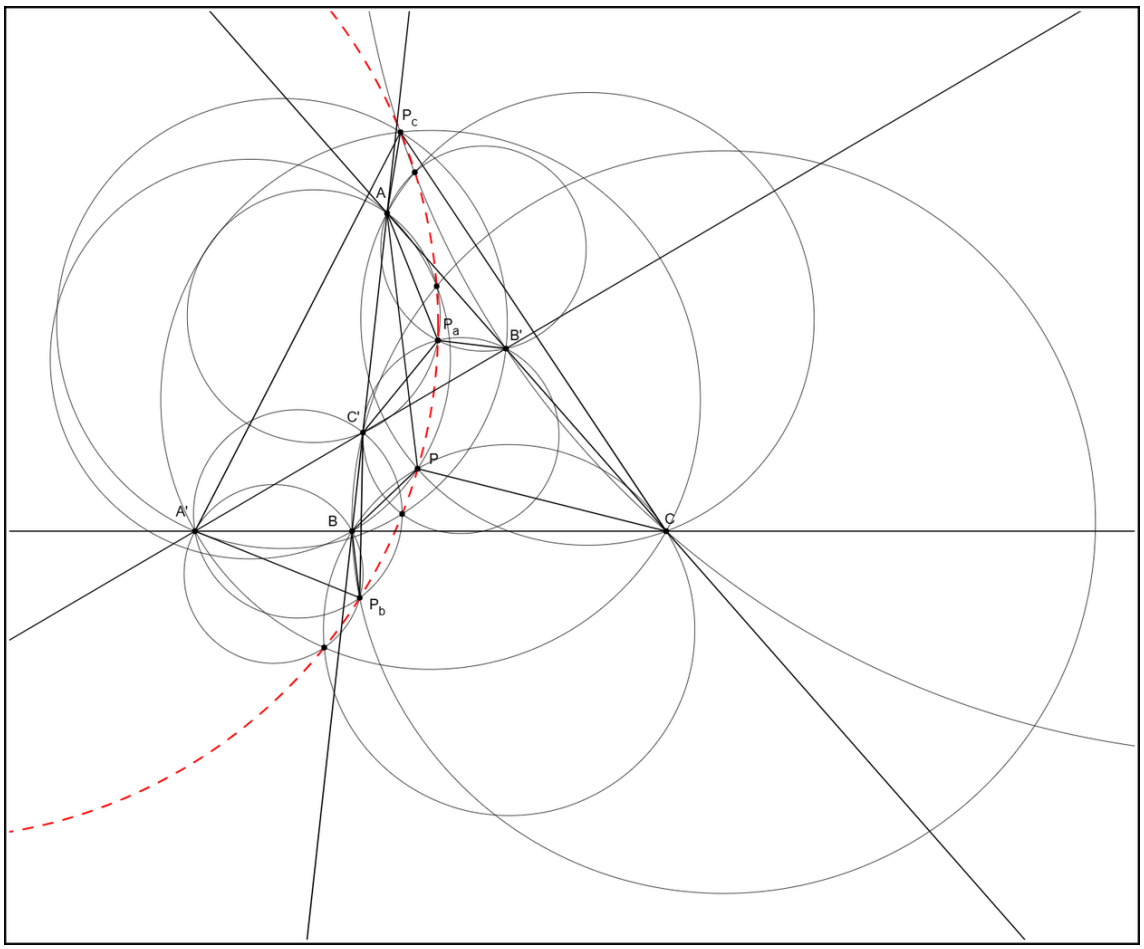
Message: #1875
Date: 18/8/2016 12:05:57
From: minhtenladuong
Subject: Four n-angle centers

Dear everyone,
Eckart has extended the integer number center to rational number center in QFG message #1873, 1874. That was an important development. But we can extend it to real number center, it suffices to define the irrational case.
Theorem. r is an irrational number. The limit of the rational sequence (r_n): [n.r]/n equals r; where [x] is the floor function.
Thus, we have defined the r-angle center of a triangle for r is real number different from 0 and 1.

Now I would like to introduce another generalization, which I think can't be more general and does not involve to numbers.
Generalization. Consider the quadrilateral which formed by triangle ABC and a line L intersects BC, CA, AB at D, E, F.
P, Pa, Pb, Pc satisfy the following conditions:
(PB,PC)=(P_aF,P_aE)(mod pi); (PC,PA)=(P_bD,P_bF)(mod pi); (PA,PB)=(P_cE,P_cD)(mod pi)
(P_aA,P_aE)=(P_bB,P_bD)(mod pi); (P_cC,P_cD)=(P_aA,P_aF)(mod pi); (P_bB,P_bF)=(P_cC,P_cE)(mod pi)
The circles PaEF, PbFD, PcDE are concurrent at Q.
The circles PBC, PcCD, PbBD are concurrent at Q_a.
The circles PCA, PaAE, PcCE are concurrent at Q_b.
The circles PAB, PbBF, PbAF are concurrent at Q_c.
Then 8 points P, Pa, Pb, Pc, Q, Qa, Qb, Qc are concyclic(this can be proved by directed angle chasing)
And I have another remarkable observation: QL-Tf1 swaps Q, Qa, Qb, Qc to P*, P*a, P*b, P*c, which are the isogonal conjugate points of P, Pa, Pb, Pc with respect to triangles ABC, AEF, BFD, CDE, respectively.

Actually, ALL results that Chris have summarized in QFG message #1872 are still available with this generalization(except the pentalateral and cyclic quadrangle). I am very optimistic about the synthetic proofs for these results, since there are so many angle relations and groups of concyclic points.
Please see the figure in my geogebra file.
Sincerely
Ngo Quang Duong



Miquelcircle.ggb

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Message: #1876
Date: 18/8/2016 12:12:39
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

```
Dear all,
in #1873 I gave a generalization of P(n) to P(p/q),
in #1874 I described first observations for q = 2 (p unequal
zero),
here are further properties:
A triangle has four points P(p/2),
... so there are 4x4 points P(p/2) four a quadrilateral,
... in quadruples on eight circles Ci(p/2),
... two quadruples of circles,
    each with a common radical center R(p/2).
... The eight circles Ci(p/2) are the CSC-images
    of the eight circles Ci(1-p/2).
new:
... The lines from the radical centers to QL-P1
    are perpendicular,
... these lines for p/2 and 1-p/2 are CSC-partner.
... Each radical center R has a circle Ci(R) intersecting
    orthogonal the quadruple of circles.
... This circle is the inversion circle round R(p/2)
    for QL-P1 and R(1-p/2).
... So the CSC-image of the circle Ci(R)
    for p/2 is the the corresponding circle for 1-p/2.
Best regards Eckart
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Message: #1877
Date: 19/8/2016 10:03:06
From: bernard.keizer
Subject: Four n-angle centers

Dear Chris, dear Eckart, dear Tsihong Lau, dear Ngo Quang Duong
Back from 3 weeks away from home, I'm impressed by the great number of messages exchanged and I try to catch up !
If I understand correctly, the Hofstadter points and the n-Angle centers are now both defined for n real and have each their own properties; I suppose Chris summary has to be rewritten.
These 2 series of points coïncide only for n integer and in this case, we have 2 interesting properties :
a) the 4 same points for the 4 reference triangles of a QL are concyclic as well as their isogonal points and the 2 circles are CSC partners.
b) the perpendicular bisectors of the segments joining a point and it's isogonal conjugates concur ; this property generalises the Kantor-Hervey theorem.
I just have 2 remarks at this step :
1) it's very easy to prove 2 properties enonced by Eckart and Chris about 2 CSC circles C1 with center O1 at a distance d1 from QL-P1 and radius r1 and C2 with center O2 at a distance d2 from QL-P1and radius r2.
Let's consider the circle C'1 reflexion of C1 in the 1rst Steiner axis: QL-P1 is per definition an inversion center of C'1 and C2 and therefore also an homothety center and d2/d1 = r2/r1.
The inversion circle of the Cl-S transformation centered in QL-P1 has as radius k1k2 with k1^2 + r1^2 = d1^2 and k2^2 + r2^2 = d2^2 and we have k2/k1 = r2/r1 = d2/d1.
The inverse of QL-P1 in the circle C2 is at a distance r2^2/d2 from O2 and the Cl-S of O1 is on QL-P1O2 at a distance k1k2/d1 from QL-P1 and the 2 points coïncide, as k1k2/d1 = k2^2/d2 and k2^2/d2 + r2^2/d2 = d2.
2) whereas the item n-Angle center has made hugh progresses thanks to Eckart and Ngo Quang Duong, the item generalisation of the Kantor-Hervey theorem has not moved since the messages 1787, 1788 by Tsihong Lau and Eckart and 1807, 1809 and 1810 by Eckart and me!
My intuition is that these points of concurrence are linked to the family of hypocycloïds tangent to the 4 lines (as the Kantor-Hervey point was the circle of the Hervey circle or inner circle of the deltoïd) ...
Best regards
Bernard

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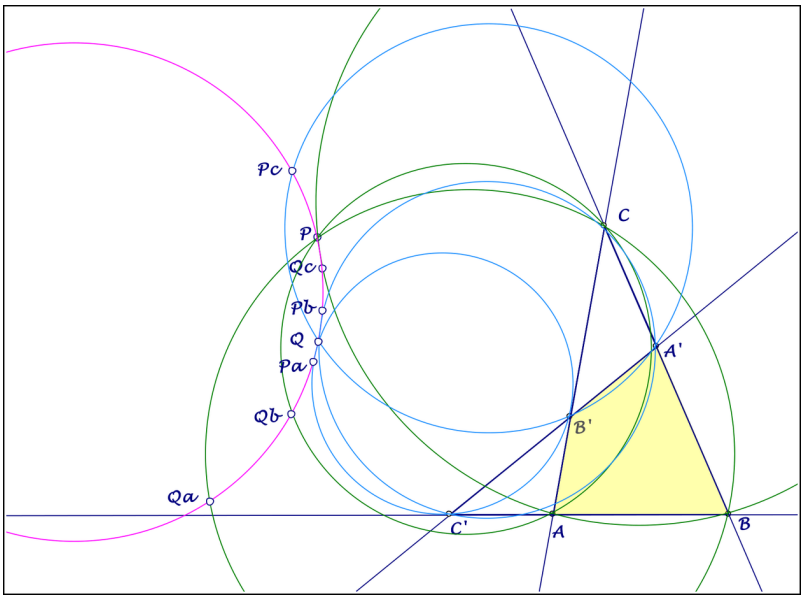
Message: #1878
Date: 19/8/2016 2:39:30
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear Duong,
it needs some time, to understand the constellation in #1875.
To show, that I have studied it: May I describe it in another way without angles:
Consider a quadrilateral with opposite points A, A´, B, B´, C, C´.
... Let P be an arbitrary point
... and Pc a point on the circle through A, B, P.
... Let CSC(P*) = Q (* means isogonal conjugate wrt ABC) :
... 2nd intersections of Ci(P,Q,Qc)
... ... with Ci(A´,Q, B´) = Pc,
... ... with Ci(B,P, C) = Qa and with Ci(B´,Q, C´) = Pa,
... ... with Ci(C,P, A) = Qb and with Ci(C´,Q, A´) = Pb,
... Then P, Pa, Pb, Pc, Q, Qa, Qb, Qc are concyclic
... with concyclic quadrupels
... ... (P,Qa,B,C), (P,A,Qb,C), (P,A,B,Qc),
... ... (Q,Pa,B´,C´), (Q,A´,Pb,C´), (Q,A´,B´,Pc),
... ... (A,Pa,B´,Qb), (A,Pa,C´,Qc),
... ... (B,Pb,A´,Qa), (B,Pb,C´,Qc),
... ... (C,Pc,A´,Qa), (C,Pc, B´,Qb).
Best regards Eckart

PS. There will be a typo: " The circles PAB, PbBF, PaAF are concurrent at Q_c.



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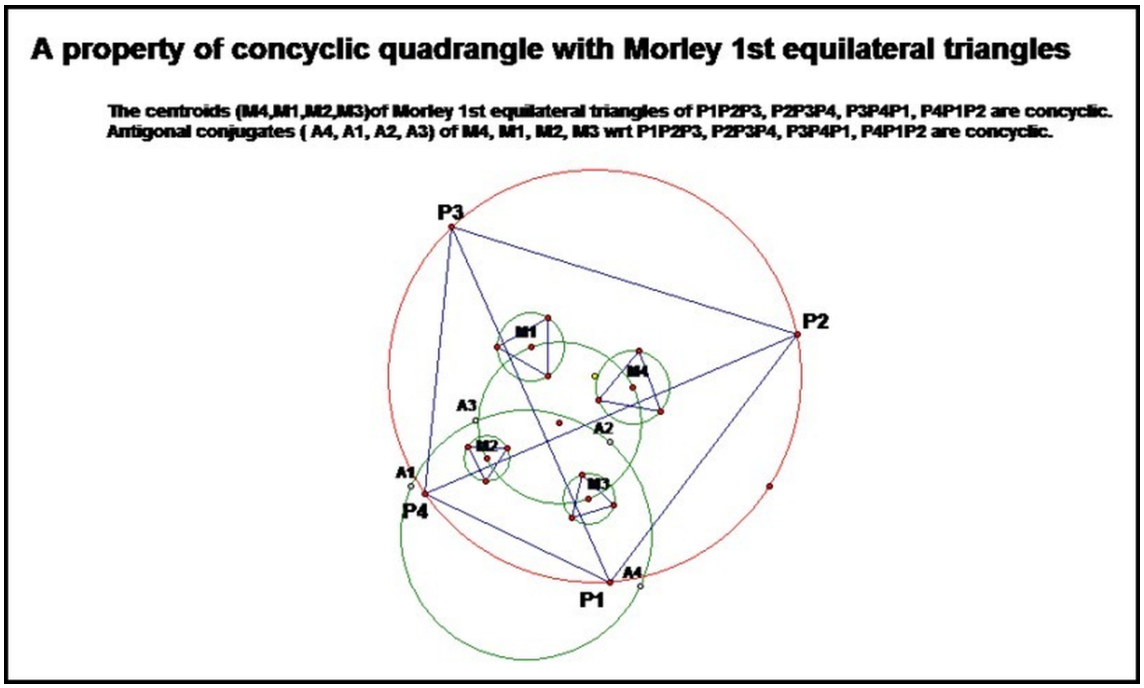
Message: #1879
Date: 20/8/2016 4:55:03
From: seiichikiri
Subject: A property of cyclic quadrangle with Morley 1st equilateral triangle

---

Dear friends,
The following seems true.
Let P1P2P3P4 be a cyclic quadrangle.
The centroids (M4, M1, M2, M3) of Morley 1 st equilateral triangles of the component triangles P1P2P3, P2P3P4, P3P4P1, P4P1P2 are concyclic.
The antigonal conjugates of M4, M1, M2, M3 wrt P1P2P3, P2P3P4, P3P4P1, P4P1P2 are concyclic.
See the attachment.
Best regards, Seiichi.



concyclic quadrangle.png

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Message: #1880
Date: 20/8/2016 12:07:55
From: chris.vantienhoven
Subject: Four n-angle centers

Dear friends,

Going back to the roots of Eckart's p/q-Angle Center points,

I checked the situation for p/q = 1/2 giving 4 points where:
   angle-BPC=angle-BAC/2 + i*90°, where i=0,1
   angle-CPA=angle-CBA/2 + i*90°, where i=0,1
   angle-APB=angle-ACB/2 + i*90°, where i=0,1
This delivered 4=2^2 points.
6 circles were needed to construct these 4 points.
2 points lying on each circle, 3 circles passing through each point.
One of these points is an ETC-point, namely X(1)-Incenter.
The other points are the excenters.

I checked the situation for p/q = 1/3 giving 9 points where:
   angle-BPC=angle-BAC/3 + i*60°, where i=0,1,2
   angle-CPA=angle-CBA/3 + i*60°, where i=0,1,2
   angle-APB=angle-ACB/3 + i*60°, where i=0,1,2
This delivered 9=3^2 points.
9 circles were needed to construct these 9 points.
3 points lying on each circle, 3 circles passing through each point.
None of the 9 points is an ETC point, neither their centroid.

I checked the situation for p/q = 1/4 giving 16 points where:
   angle-BPC=angle-BAC/4 + i*45°, where i=0,1,2,3
   angle-CPA=angle-CBA/4 + i*45°, where i=0,1,2,3
   angle-APB=angle-ACB/4 + i*45°, where i=0,1,2,3
This delivered 16=4^2 points.
12 circles were needed to construct these 16 points.
4 points lying on each circle, 3 circles passing through each point.
None of the 16 points is an ETC point.

When we compare this with the Hofstadter Points we see:
· Hofstadter Point H(1/2)=X(1).
p/q-Angle Center P(1/2) is a set of 4 points, one of them being X(1).
· Hofstadter Point H(1/3)=X(356).
p/q-Angle Center P(1/3) is a set of 9 points, none of them being X(356).
· Hofstadter Point H(1/4)=X(1127).

p/q-Angle Center P(1/4) is a set of 16 points, none of them being X(1127).

So it appears that the p/q-Angle Centers do not generally match with the Hofstadter Points H(p/q) for q > 2.
Having methodically checked that I feel inclined to search for other properties.

Best regards,
Chris

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Message: #1881
Date: 20/8/2016 12:25:50
From: seiichikiri
Subject: Four n-angle centers

Dear Eckart,
You refer to 4*4 points in messages
1873 and 1874.
You mean that there are another configurations similar or corresponding to Steiner Angle Bisector Octet. You confirmed it for p=3.
Would you kindly let us see the picture for p=3?
Best regards, Seiichi.
P. S. I wrote a message #1264. I would like you to see it from the viewpoint of Four n-angle centers.

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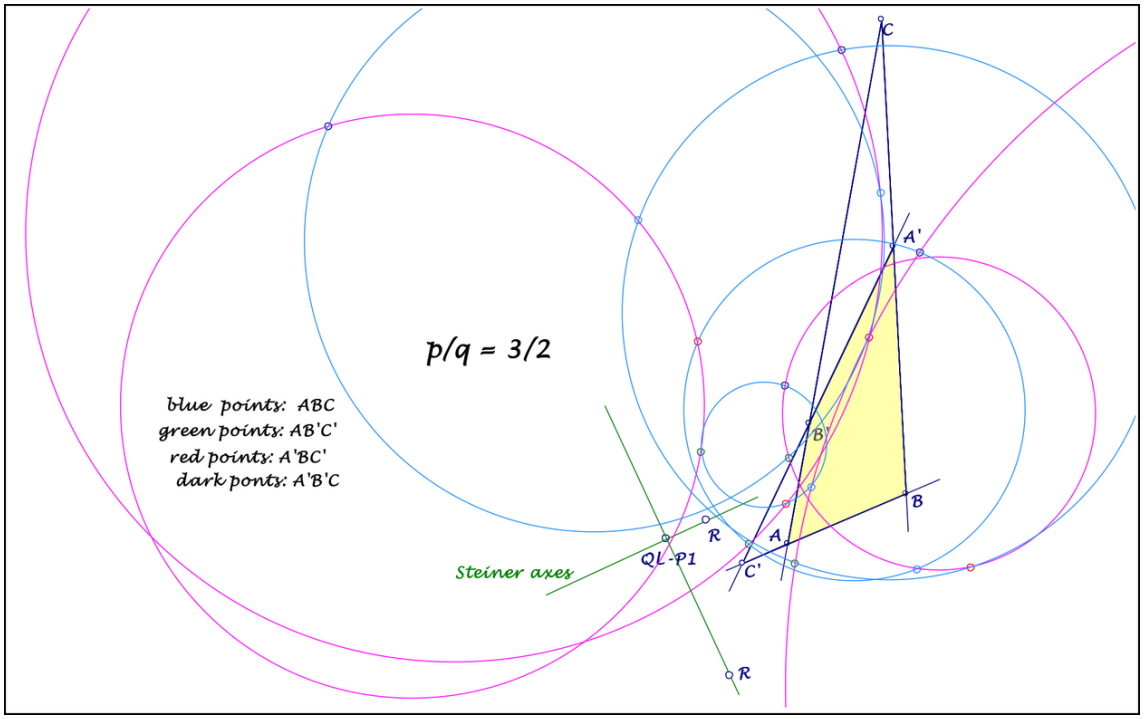
Message: #1882
Date: 20/8/2016 4:14:05
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear Seiichi,
it is a bit difficult to make a good drawing of the
constellation for p/q = 3/2, but perhaps it is sufficient.
By the way: The line of the radical centers for p/q = 3/2 is the
perpendicular bisector of QL-P1 and CSC(QL-P3).
Best regards Eckart
PS: I shall have a look in #1264.



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Message: #1883
Date: 21/8/2016 3:35:21
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

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Dear all,
The topics #1787, #1788, #1843 started from #1878.
But the original message #1787 has not discussed
further. I now return to it. I revised it and found
up to 3053. If n=3,19,24,25,48,49,63,68,69,92,93,
184,186,264,265,304,305,317,328,340,378,562,563,847,
,1092,1093,1102,1136,1137,1147,1593,1597,1598,1748,
1820,1969.1973,1974,2351,3043,... then the diapleural
(diagonal) triangles of X(n)'s and the quadrilateral are
perspective. X(n)'s are the triangle centers of its exline
trilaterals(component triangles). I did not find the rules
for these triangle centers. But they include all integral
Hofstadter points(except 0 and 1) and all integral another
series mentioned in message 1862. I verified H-1 to H-30
and H2 to H31 and H'0 to H'-29 and H1 to H'30. I think
we should try to find the rules for all X(n)'s.
Best regards,
Tsihong Lau

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Message: #1884
Date: 21/8/2016 10:02:54
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear all,
I am not so familar with Hofstadter points, so I don´t know,
whether the following observation is already mentioned:
Consider n-angle points P(p/2) and the recursion formulas in
#1873,
... beginning with the excenters Ia, Ib, Ic for p = 1
... you get the n-angle centers P(p/2)a, P(p/2)b, P(p/2)c.
... The lines A.P(p/2)a, B.P(p/2)b, C.P(p/2)c intersect in the
Hofstadter point H(p/2).
Cabri observations confirm this assumption for p = -1, p = 1, p
= 3, p = 5.
Best regards Eckart

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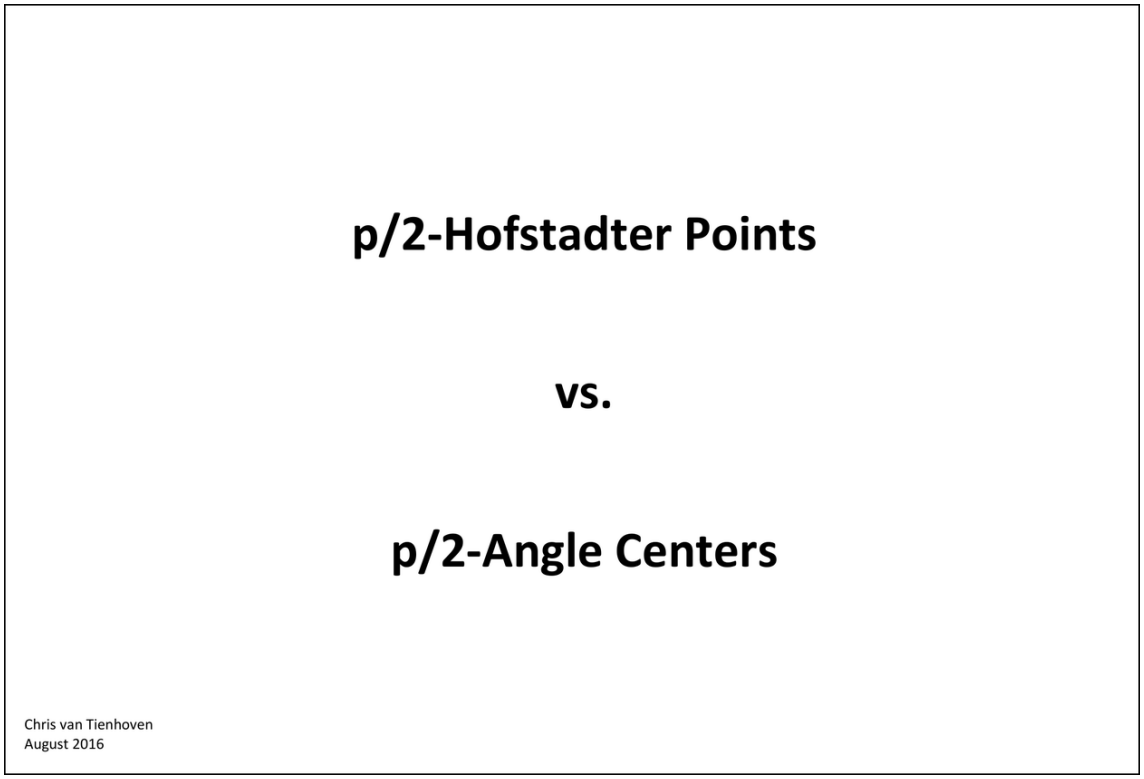
Message: #1885
Date: 21/8/2016 6:03:19
From: chris.vantienhoven
Subject: Four n-angle centers

Dear Eckart,
We walked the same track today!
Indeed there is a remarkable relationship for p/2-Angle Centers and p/2-Hofstadter Points.
For example 3/2-Hofstadter Point = X(35) but the 3/2-Angle Center = X(36).
Note that X(35) and X(36) are mutually inverse wrt the circumcircle.
I made a picture of this all and proved their relationship. See attachment.
This leaves open the question how things work for p/q when q>2. Then there are many more "extraversions"□. But I think that all the time one of these extraversions will be a regular Central Point, whether or not occurring in ETC. Like of the set of In-/Excenters there is only one ETC-point X(1).
Best regards,
Chris



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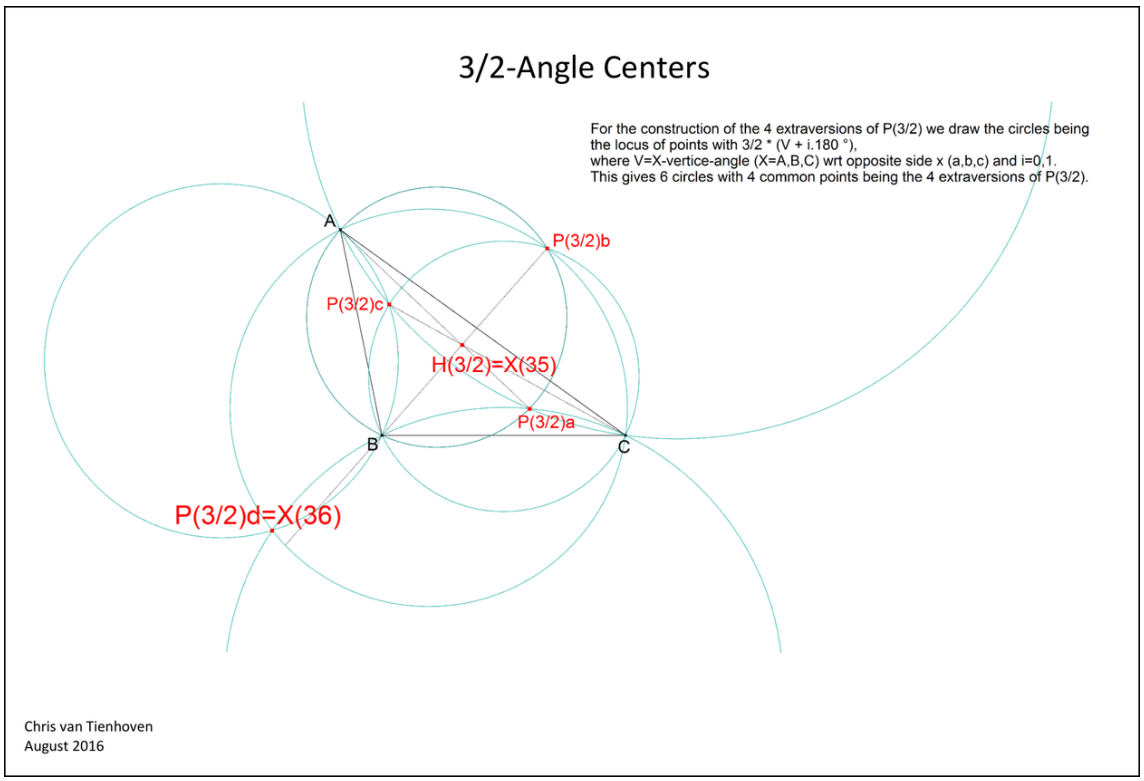

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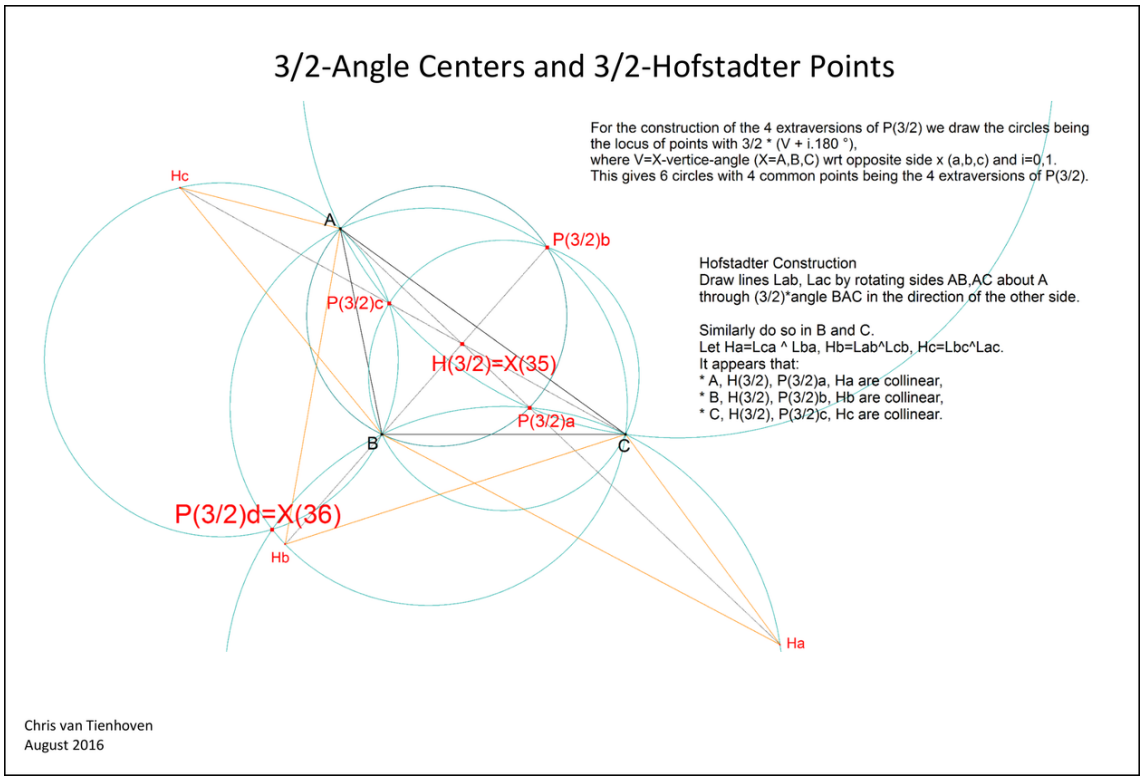

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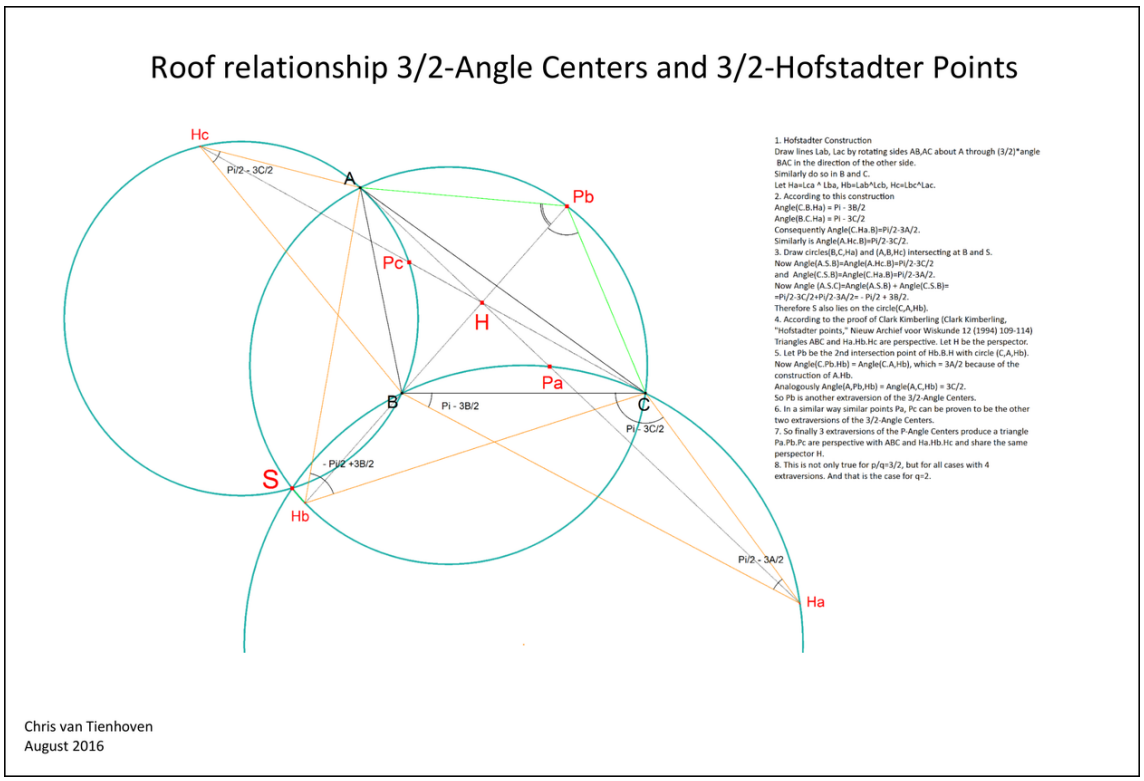

pq-Hofstadter-Points-01.pdf

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Message: #1886
Date: 21/8/2016 7:19:07
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
you have listed X(n), whose quadrangle for a quadrilateral has a perspective diagonal triangle wrt the diagonal triangle of the quadrilateral, and you ask for rules.
Perhaps an interesting observation: There are curious chains
(* means isogonal and ^ isotomic conjugate):
... X(1974) * X(305) ^ X(25) * X(69) ^ (X(4)) * X(3) ^
X(264) * X(184) ...
... X(1973) * X(304) ^ X(19) * X(63) ^ X(92) * X(48) ^
X(1969) ...
... X(24) * X(68) ^ X(317) * X(2351) ...
... X(340) ^ X(265) * X(186) ^ X(328) ...
It seems, that these chains can be continued. I have proved the next points on both sides, which are not ETC-points. Each pair of isogonal conjugated Hofstadter points H(n), H(1-n) will start such a chain, for example:
... X(5963)* X(5964) .... (next points are Cabri controlled).
Perhaps these chains are connected.
Best regards Eckart

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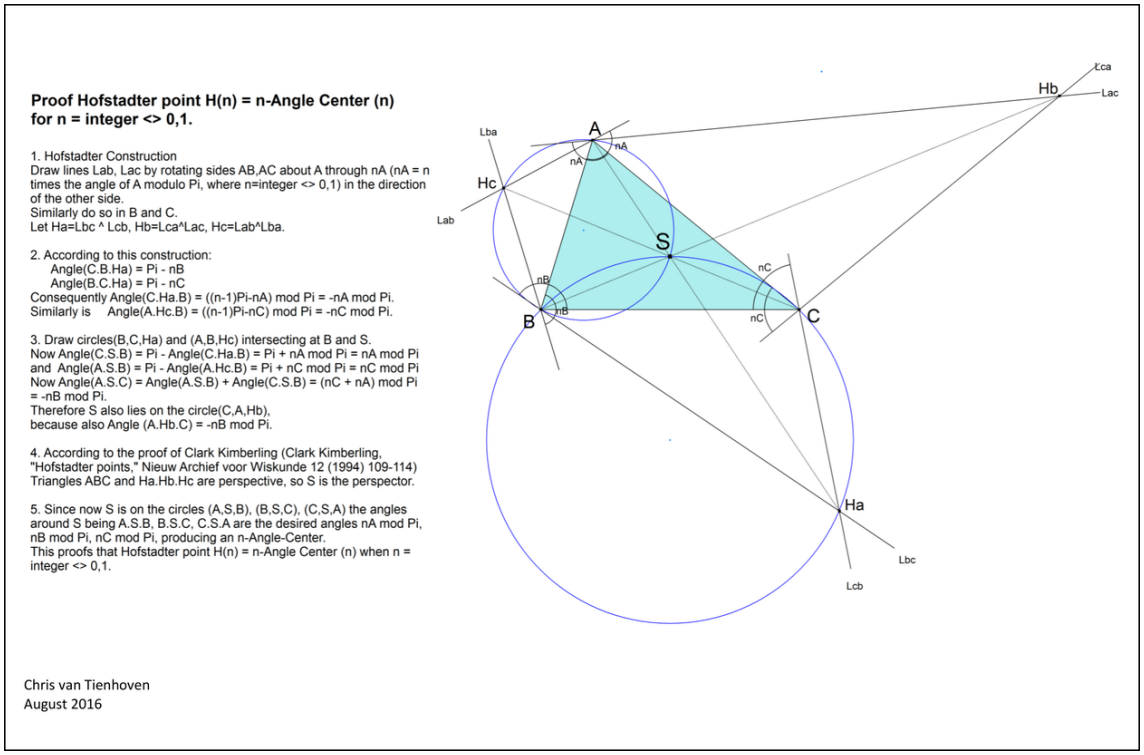
Message: #1887
Date: 22/8/2016 2:02:00
From: chris.vantienhoven
Subject: Four n-angle centers

Dear Ngo Quang Duong, Eckart, Bernard, Tsihong Lau, Seiichi,

Yesterday I forwarded the proof that p/2-Hofstadter Point = p/2-Angle Center (p is integer <>0,1).
I realized this morning this actually is a more complex case of n-Hofstadter Point = n-Angle Center (n is integer <>0,1).
So attached a proof of the latter one.

Best regards,
Chris



N-P-Angle-Points-01-Proof.fig.pdf

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Message: #1888
Date: 23/8/2016 1:58:33
From: seiichikiri
Subject: QA version of n- angle centers

Dear friends,
I propose the following problems.
(1) Let P1P2P3P4 be a quadrangle and P a point.
Angle (PiPPj) = n(angle (PiPlPj)+angle (PiPkPj), (mod pi),
where {i, j, k, l} is cyclic of {1,2,3,4}.
For n=1, P = QA-P4.
(2) Let P1P2P3P4 be a quadrangle and P a point.
Angle (PiPPj) = n(angle (PiPlP)+angle (PPkPj), (mod pi), where
{i, j, k, l} is cyclic of {1,2,3,4}.
For n=1, P = 2 intersections of Schmidt circle and 1 st Steiner
axis.
I hope that the above problems have meaning.
Best regards, Seiichi.

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Message: #1889
Date: 23/8/2016 9:21:34
From: eckart_schmidt@t-online.de
Subject: QA version of n- angle centers

Dear Seiichi,
Cabri observations show, that wrt (1) and (2) there are no such
points for n = 2?
Wrt the result for n =1 in (1) it is not necessary to demand
"... where {i, j, k, l} is cyclic of {1,2,3,4}".
Wrt the result for n =1 in (2) you consider a quadrigon
P1P2P3P4.
Best regards Eckart

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Message: #1890
Date: 23/8/2016 11:36:52
From: seiichikiri
Subject: QA version of n- angle centers

Dear Eckart,
Thank you very much for your research.
Best regards, Seiichi.

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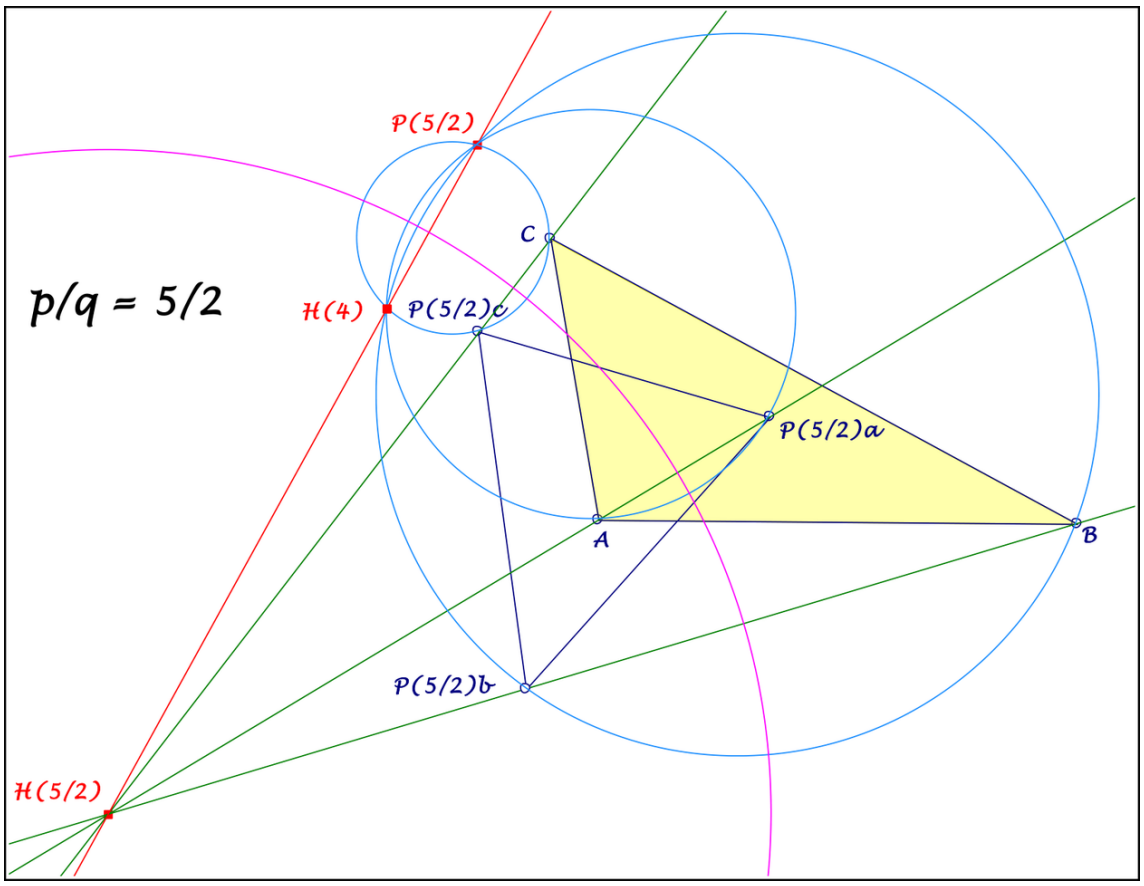
Message: #1891
Date: 23/8/2016 11:40:21
From: eckart_schmidt@t-online.de
Subject: Four n-angle centers

Dear all,
for n integer (unequal o,1) the Hofstadter points H(n) are the n-angle centers P(n).
For n = p/2 (p unequal 0) there are
... four p/2-angle centers P(p/2), P(p/2)a, P(p/2)b, P(p/2)c ,
... related to the in-/excenters I, Ia, Ib, Ic of the triangle ABC for p = 1
... with the recursion formulas in # 1873.
In # 1884 there is a first relationship of these p/2-angle centers and the Hofstadter point H(p/2):
... The lines A.P(p/2)a, B.P(p/2)b, C.P(p/2)c intersect in the Hofstadter point H(p/2).
Here is the property for the 4th p/2-angle center P(p/2), which is related to the incenter I (see attached file):
... The line P(p/2).H(p/2) bears H(p-1),
... H(p-1) is the 2nd common point of the circles (A,P(p/2),P(p/2)a), (B,P(p/2),P(p/2)b), (C,P(p/2),P(p/2)c),
... H(p-1) is the inversion of P(p/2) in a circle round H(p/2) orthogonal to the three circles above.
Best regards Eckart



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Message: #1892
Date: 23/8/2016 4:38:15
From: tsihonglau
Subject: Expoint Quadrangle/Exline Quadrilateral

Dear all,
In my webpage - http://lth.name/geometry/duality.html
there were two very important objects missing -
Expoint Quadrangle/Exline Quadrilateral
I have updated it.
Given a quadrangle Q,Q_A,Q_B,Q_C and its dual
quadrilateral q,q_a,q_b,q_c, there are four sets
of objects - Expoint Triangle/Exline Trilateral
Expoint Quadrangle/Exline Quadrilateral
Each set have four objects.

Expoint Triangle		Exline Trilateral	
T	QA,QB,QC	t	qa,qb,qc
TA	Q,QB,QC	ta	q,qb,qc
TB	QA,Q,QC	tb	qa,q,qc
TC	QA,QB,Q	tc	qa,qb,q

Expoint Quadrangle		Exline Quadrilateral	
E	ABC,Q	e	abc,q
EA	ABC,QA	ea	abc,qa
EB	ABC,QB	eb	abc,qb
EC	ABC,QC	ec	abc,qc

The diagonal trilaterals of expoint quadrangles
E,EA,EB,EC are exline trilate rals t,ta,tb,tc respectively.
The diapleural triangles of exlinequadrilaterals
e,ea,eb,ec are expoint triangles T,TA,TB,TC respectively.

We have run across centers of exline trilaterals
in topic #1787. I give two examples of centers
of exline quadrilaterals and expoint quadrangles.
Let M,MA,MB,MC are QL-P1 miquel points of e,ea,eb,ec
respectively and AM,BM,CM are the diapleural triangle
of quadrangle M,MA,MB,MC. ABC and AM,BM,CM
are perspective at QL-P17.
Let P,PA,PB,PC are QA-P2 Euler-Poncelet points
of E,EA,EB,EC respectively and AP,BP,CP are
the diapleural triangle of quadrangle P,PA,PB,PC.
ABC and AP,BP,CP are perspective at
p^2(qSB+rSC)(qSB-rSC)/a^2 (DT-barycentric)
I think many objects can be derived from Expoint
Quadrangle/Exline Quadrilateral.
Best regards,
Tsihong Lau

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Message: #1893
Date: 26/8/2016 12:13:12
From: chris.vantienhoven
Subject: Four n-angle centers

Dear friends,

Eckart, beautiful properties you found in #1891.
It is quite unsuspected and unexpected that line P(p/2).H(p/2) is collinear H(p-1).
I cannot see any connection in this.
In between I delved more into the QL-Hofstadter circles now.
I found some special properties:
Let L1,L2,L3,L4 be the defining lines of a quadrilateral.
Let P1,P2,P3,P4 be the 4 QL-versions of Hofstadter point H(h), where h = integer <>-1,0,1.
Let Ci(h) be the corresponding QL-Hofstadter Circle.

1. Triangles (Pj,Pk,Pl) are perspective with corresponding triangles (Lj,Lk,Ll).
Let Ri be their perspector, where (i,j,k,l) are different numbers from (1,2,3,4).

2. Perspectors R1,R2,R3,R4 lie on circle Ci(h).

3. Each Ri.Rj-intercepted inscribed Ci(h)-angle = h.Angle(Li,Lj) mod Pi.
Note that intercepted inscribed angles in a circle are twofold: a and Pi - a.
When taken mod Pi the angles are a and -a. Anyway the circle is the locus of points which form inscribed angles (mod Pi) with a line segment, +a when occurring on one side of the line segment and -a when occurring on the other side.
The same is true for angles between two intersecting lines. They are twofold and when taken mod Pi they are a and -a.
A similar property is also valid in Ci(2) (= QL-Ci3) for the points O1,O2,O3,O4.

4. Each Oi.Oj-intercepted inscribed QL-Ci3-angle = Angle(Li,Lj) mod Pi.

5. This property is recursive in an n-Line.
A 3-Line has a circumcircle with a circumcenter C3.
A 4-Line has 4 Component 3-Lines producing 4 concyclic circumcenters C3 with new circumcenter C4.
A 5-Line has 5 Component 4-Lines producing 5 concyclic circumcenters C4 with new circumcenter C5.
These “circumcircles” with n Points in an n-Line are often called CenterCircles.

Etc.
The special property on the n-Line Circumcircle is:
Each Oi.Oj-intercepted inscribed nL-Ci-angle = Angle(Li,Lj) mod Pi,
   where (i,j) are different numbers from (1, … ,n)
   and nL-Ci-circle = nL-CenterCircle.
I find especially last property amazing: n points on a circle mutually giving the same angles as occuring between the defining lines of an n-Line.
Finally and maybe least interesting:

6. In a Quadrilateral circles (Pj,Sik,Sil), (Pk,Sij,Sil), (Pl,Sij,Sik) are concurrent in a point Qi on Ci(h). By interchanging (i,j,k,l) this will produce 4 new points on Ci(h).

Best regards,
Chris

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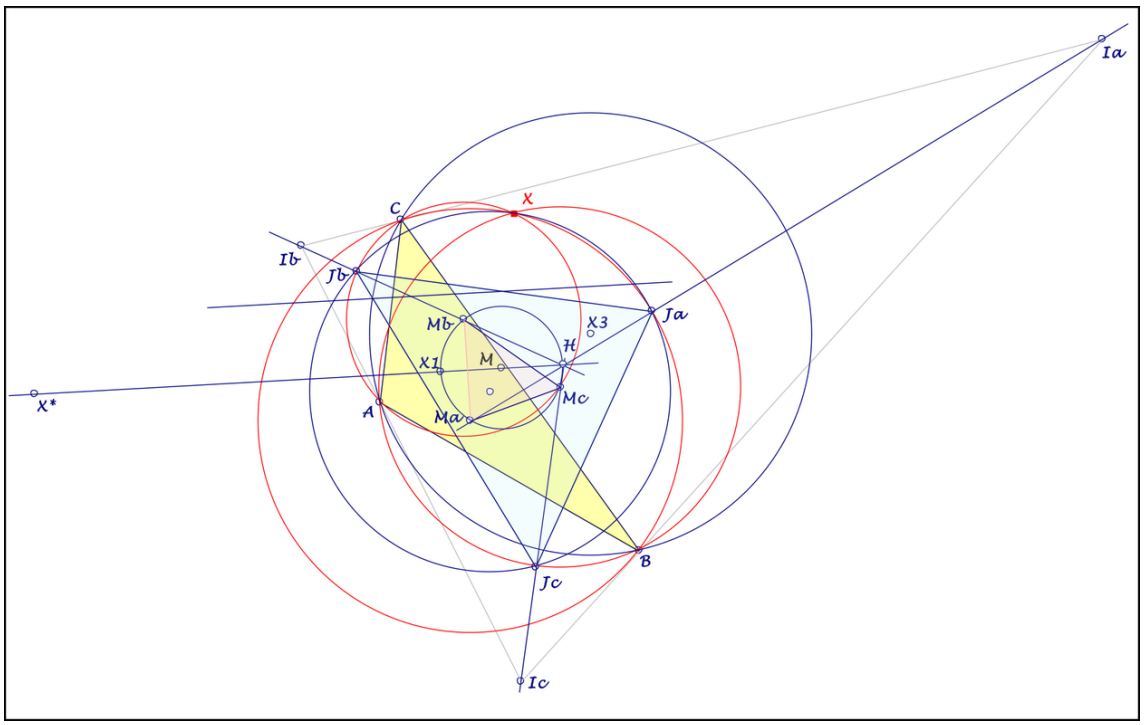
Message: #1894
Date: 26/8/2016 1:22:06
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

Dear all,

constructing the 16 n-angle points P(1/4) for a triangle ABC,
there is a special point X (see attached file):
Let Ia, Ib, Ic be the excenters of ABC,
... let Ja, Jb, Jc be the incenters of Ia,B,C and A,Ib,C
    and A,B,Ic,
... the circumcircles of Ja,B,C and A,Jb,C and A,B,Jc
    have a common point X.
Properties:
... X is a point of the JaJbJc-circumcircle,
... Let Ma, Mb, Mc be the circumcenters of Ja,B,C
    and A,Jb,C and A,B,Jc,
... X is the MaMbMc-isogonal conjugate of the ABC-circumcenter,
... X is the intersection of parallels to MiMj through Jk,
... let M be the MaMbMc-circumcenter
    and H the JaJbJc-orthocenter (= IaIbIc-incenter),
... the ABC-incenter I and the JaJbJc-orthocenter H
    lie diametral on the circumcircle of MaMbMc,
... the triples Ia, Ja, Ma and Ib, Jb, Mb and Ic, Jc, Mc
    are collinear with the orthocenter H of JaJbJc,
... the ABC-isogonal conjugate of X is a point X*
    on the line I.M.H,
... the Simson line of X wrt JaJbJc
    is a parallel to I.M.H in half the distance,
... the inversion of X in the MaMbMc-circumcircle
    is the inversion of X* in the ABC-circumcircle.
The point X is not in ETC!

Best regards Eckart

PS:
... let Ja´, Jb´, Jc´
    be the incenters of I,B,C and A,I,C and A,B,I,
... the intersection of AJa´, BJb´, CJc´
    is the Hofstadter point H(1/4).



2016-08-26.pdf

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Message: #1895
Date: 26/8/2016 1:38:44
From: Antreas Hatzipolakis
Subject: New ETC-point?

```
>> Let Ia, Ib, Ic be the excenters of ABC,
>> ... let Ja, Jb, Jc be the incenters of Ia,B,C and A,Ib,C and
A,B,Ic,
>> ... the circumcircles of Ja,B,C and A,Jb,C and A,B,Jc have a
common point X.

Dear Eckart,

And by the "main cyclologic theorem" the circumcircles of AJbJc,
BJcJa, CJaJb are concurrent (at a point on the circumcircle of
ABC)

APH
```

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Message: #1896
Date: 26/8/2016 1:48:14
From: Antreas Hatzipolakis
Subject: New ETC-point?

```
>>> Let Ia, Ib, Ic be the excenters of ABC,
>>> ... let Ja, Jb, Jc be the incenters of Ia,B,C and A,Ib,C and
A,B,Ic,
>>> ... the circumcircles of Ja,B,C and A,Jb,C and A,B,Jc have a
common point X.

>> Dear Eckart,
>> And by the "main cyclologic theorem" the circumcircles of
AJbJc, BJcJa,
>> CJaJb are concurrent (at a point on the circumcircle of ABC)
>> APH

The same is true if we replace the incenters Ja, Jb, Jc with
circumcenters
Oa, Ob, Oc or orthocenters Ha, Hb, Hc of IaBC, IbCA, IcAB, resp.

APH
```

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Message: #1897
Date: 26/8/2016 3:42:58
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

```
Dear Antreas,

thanks for your informations.
The circumcircles of AJbJc, BJcJa, CJaJb intersect in X(3659),
the points Oa, Ob, Oc lie on the circumcircle of ABC,
the circumcircles of ABHc, BCHa, CAHb intersect in X(80).
But I haven´t found the considered special n-angle point X in
ETC.
This point has a further interesting property:
X is the Hofstadter point H(-3) of the triangle MaMbMc.

Best regards Eckart
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Message: #1898
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Messages

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Message number 1898 is not available in Yahoo groups.
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Message: #1899
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Messages

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Message number 1899 is not available in Yahoo groups.
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Message: #1900
Date: 27/8/2016 8:10:52
From: chris.vantienhoven
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear friends,
I wondered if there was a way to reconstruct a Quadrilateral when the circumcenters O1, O2, O3, O4 of its component triangles are known.
Surprisingly I found that there is a whole range of Quadrilaterals that fit that condition.
See attached file for the construction.
Best regards,
Chris

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Message: #1901
Date: 27/8/2016 12:38:35
From: seiichikiri
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Chris,
I can not find the attached file.
Best regards, Seiichi.

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Message: #1902
Date: 27/8/2016 2:54:28
From: Benedetto Scimemi
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Chris,

by a dimensional argument, it is not surprising that an infinity of quadrilaterals exist with the same circumcenters Oi for component trilaterals.
In fact the four Oi must lie on a circle, hence the "real dimension" of the system is 7 (3 free points O1 O2 O3, i.e. 6 real numbers, for defining a circle;
then O4 must lie on it and depends on just one real parameter).
But a quadrilateral has dimension 8.
To fill the missing information you can, for example, choose any fifth point M on the circle and give it the role of Miquel point.
Then the 6 reflections of M on the lines OiOj give the vertices of your quadrilateral.

Best regards Benedetto

2016-08-27 12:38 GMT+02:00 seiichikiri@yahoo.com [Quadri-Figures-Group] <Quadri-Figures-Group@yahoogroups.com>:
>> Dear Chris,
>> I can not find the attached file.
>> Best regards, Seiichi.

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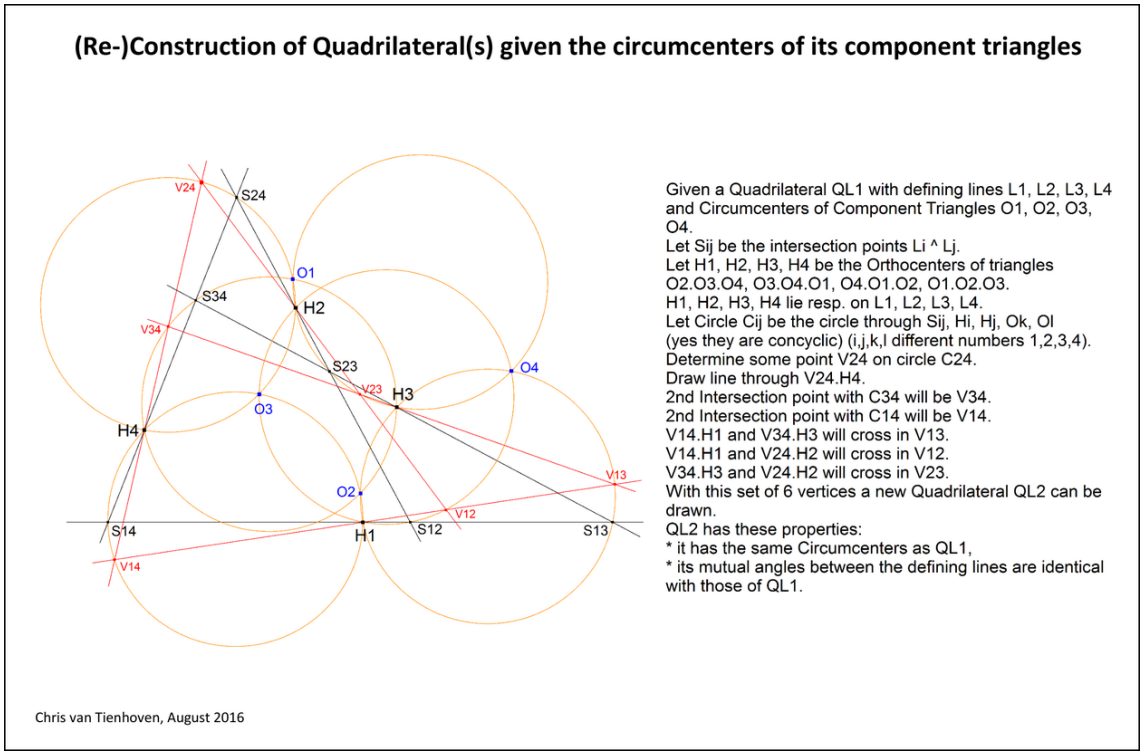
Message: #1903
Date: 27/8/2016 3:18:54
From: chris.vantienhoven
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Seiichi, dear Benedetto,
Thanks for your messages.
Attached the forgotten file.
Best regards,
Chris



QL-Ci3-Reconstruction-QL-from-QL-Ci3.pdf

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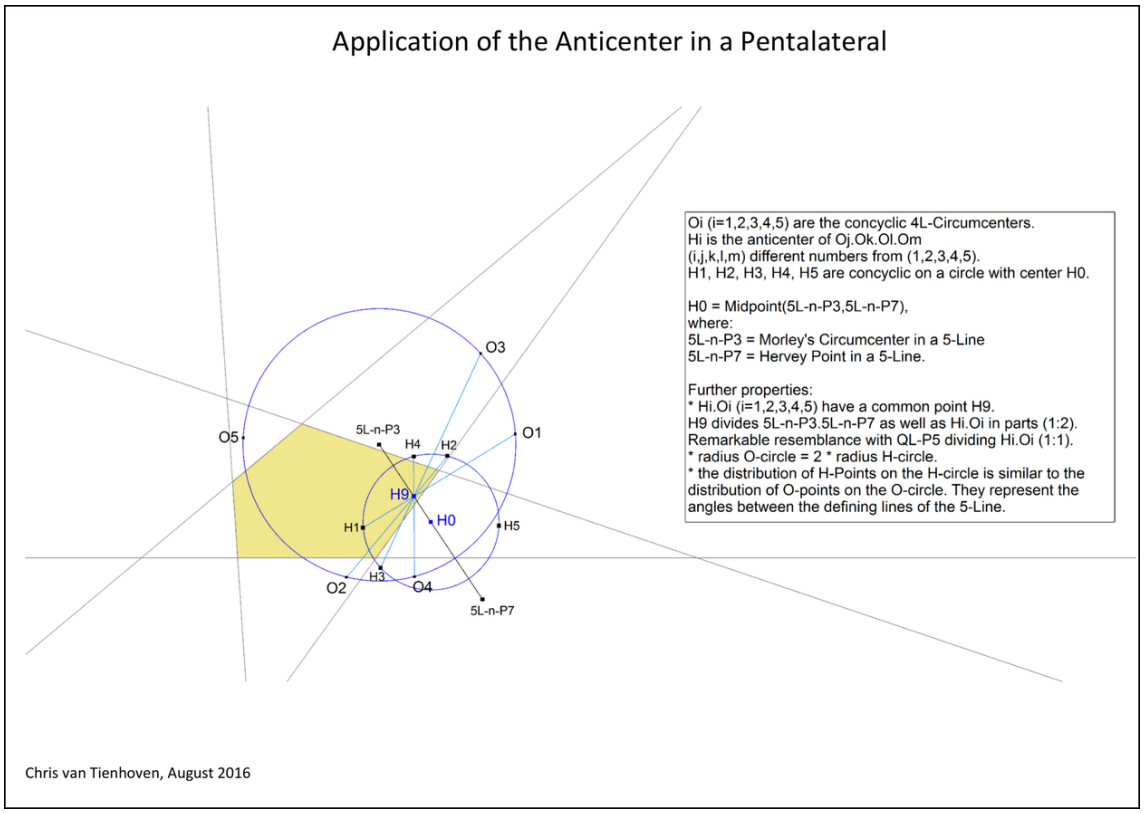
Message: #1904
Date: 27/8/2016 11:44:15
From: chris.vantienhoven
Subject: Application of the anticenter in a Pentalateral

Dear friends,
I noticed that in a Pentalateral (5-Line) the Anticenters of the 5 Component Quadrilaterals (4-Lines) are concyclic.
The distribution of the Anticenters on the circle is identical to the distribution of the 4L-Circumcenters on the 5L-Centric Circle.
They represent the mutual angles of the defining lines of the 5-Line (cf. message#1893).
See attachment.
Best regards,
Chris



5L-QL-P2-CircumCenter-02.pdf

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Message: #1905
Date: 28/8/2016 7:06:56
From: minhtenladuong
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear everyone
I have another construction as follow:
Let M be an arbitrary point on the circle through O1, O2, O3, O4.
Then 6 intersections other than M of the circles (O1,MO1); (O2,MO2); (O3,MO3); (O4,MO4) are 6 vertices of a quadrilateral.
where (O1,MO1) is the circle centered at O1, radii MO1.
Furthermore, I have observed some more properties of the family of quadrilaterals:
- The Newton line passes through a fixed point.
- The Steiner line is tangent to a fixed circle.
Sincerely
Ngo Quang Duong

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Message: #1906
Date: 28/8/2016 9:06:15
From: chris.vantienhoven
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Ngo Quang Duong,
Thank you for the alternative construction of the variable QL-constellation!
I have some additions to your remarks:
* The fixed point on the Newton Line = QL-P5.
* The fixed circle tangent to the Steiner Line is the circle with center QL-P3 through QL-P2.
* M will be the Miquel Point QL-P1 of the new QL-constellation.
* The locus of QL-P28 (X(186)-point) is a fixed circle.
* The locus of QL-P29 (X(265)-point) is a fixed circle centered in QL-P3.
* The locus of QL-P30 is a fixed circle centered in the reflection of QL-P27 in QL-P5.
List of fixed items:
* O1, O2, O3, O4, H1, H2, H3, H4
* QL-Ci3, QL-Ci4
* QL-P3, QL-P4, QL-P5, QL-P6, QL-P27
* QL-L4, Line(QL-P3,QL-P4)
Best regards, Chris

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Message: #1907
Date: 28/8/2016 3:40:10
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

---

Dear Chris and Eckart and Other,
This message is a reply to Eckart's #1886 and Chris' #1893.
Reply to Eckart:
There are some X(n)'s in #1883 missing in #1886.
These X(n)'s trilinears are very difficult to analyse.
For example, X(1136)'s first trilinear is cos(A/3 + 4π/3) and X(1593)'s is cos A + sec A.
Besides these, I found four sets of triangle centers satisfying #1787 before I posted #1883.
Given m and n are positive integer or 0, there are four sets triangle centers with first trilinears cos mA/cos nA, cos mA/sin nA, sin mA/cos nA, sin mA/sin nA.
If one of m and n is odd and the other is even, the first and fourth sets satisfy #1787.
If both m and n are odd or even, the second and third satisfy #1787.
These sets are a generalization of two series mentioned in #1862.
But they unlike the two series are not well-defined.
I will explain it in topics #1849 and #1892 later.
Reply to Chris:
Wonderful observations in #1893.
I apply your construction to X(n)'s in #1883 and the above four sets and get general results.
Given a quadrilateral q,qa,qb,qc, we can get triangle centers X(n)'s of exline trilaterals X_n,X_na,X_nb,X_nc.
The four exline trilaterals and four expoint triangles are perspective.

Perspectors	-	Exline Trilaterals	-	Expoint Triangles
P	-	qa,qb,qc	-	X_na,X_nb,X_nc
Pa	-	q,qb,qc	-	X_n,X_nb,X_nc
Pb	-	qa, q,qc	-	X_na,X_n,X_nc
Pc	-	qa,qb,q	-	X_na,X_nb,X_n

The eight points X_n,X_na,X_nb,X_nc and P,Pa,Pb,Pc are conconic.
If X(n) is Hofstadter integral points, the conic becomes a circle!
In other words, the X(n)'s satisfying #1787 and the above condition seem to be the same!
I think we should study #1787 and #1893 further!
Best regards,
Tsihong Lau

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Message: #1908
Date: 29/8/2016 10:45:45
From: minhtenladuong
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Chris van Tienhoven

That's great. When M varies on the circumcircle through O1, O2, O3, O4, we would obtain many loci, fixed points and special envelopes as you have shown.
You have wrote that "The locus of QL-P28 (X(186)-point) is a fixed circle."
I have also tried for some n-angle centers, then I realized that: The center of the n-angle circle of the quadrilateral of the family of quadrilaterals varies on a fixed circle, too!
Inspired from your reconstruction problem, I pose an open question: Given 4 concyclic points A,B,C,D. How to construct the quadrilateral formed by 4 lines a,b,c,d such that A, B, C, D are n-angle center of the triangles bcd, cda, dab, abc, respectively?

Sincerely
Ngo Quang Duong

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Message: #1909
Date: 30/8/2016 10:43:22
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,

here is another aspect for the set of X(n), whose quadrangle for a quadrilateral has a perspective diagonal triangle wrt the diagonal triangle of the quadrilateral.
My orientation: Including X(4), the set of X(n) is isogonal and isotomic invariant (see #1886).
This property can perhaps be generalized:
Including X(4) the set of X(n) is invariant wrt isoconjugations with fixed point (a^r : b^r : c^r) in barycentric coordinates.
Example for r = 3/2, fixed point X(365), pole X(31): This isoconjugation swaps the points of the two first chains in #1886, also X(24) <--> X(1820) and X(563) <--> X(847).
Further examples:

```
r =    0, fixed point X(2):   X(3) <--> X(264), X(4) <--> X(69),
r =  1/2, fixed point X(366): X(3) <--> X(92), X(4) <--> X(63),
r = -1/2, fixed point not in ETC:
                              X(3) <--> X(1969),
                              X(4) <--> X(304),
r =    1, fixed point X(1):   X(3) <--> X(4), X(4) <--> X(3),
r =   -1, fixed point X(75),  X(3) <--> not in ETC,
                              X(4) <--> X(305),
r =  3/2, fixed point X(365): X(3) <--> X(19), X(4) <--> X(48),
r =    2, fixed point X(6):   X(3) <--> X(25), X(4) <--> X(184),
r = 5 /2, fixed point not in ETC:
                              X(3) <--> X(1973),
                              X(4) <--> X(9247),
r =    3, fixed point X(31):  X(3) <--> X(1974),
                              X(4) <--> not in ETC.
```

Perhaps r had to be n or n/2 (n integer), so that the pole of the isoconjugation is (a^m : b^m : c^m) m integer.
Perhaps all points of the set can be developed with these isoconjugations from the Hofstadter points H(n).

Best regards Eckart

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Message: #1910
Date: 30/8/2016 9:47:02
From: chris.vantienhoven
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Ngo Quang Duong,
About your question:
I figured out that it can be done in exactly the same way as reconstructing the quadrilateral from O1, O2, O3, O4 as mentioned in message #1900.
The only difference is that the “pivot-points” H1, H2, H3, H4 on L1,L2,L3,L4 are altered.
I made several constructions about how it should be but wasn’t able to determine the altered H1, H2, H3, H4.
Maybe you or someone else know a way of construction.
Best regards,
Chris

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Message: #1911
Date: 30/8/2016 9:56:26
From: chris.vantienhoven
Subject: special property tangents at QL-Co1

Dear Bernard,
I know you have a special interest for QL-Co1.
I found a special property for tangents at the inscribed parabola QL-Co1.
Let L1, L2, L3, L4 be the defining lines of the Reference Quadrilateral.
Let Sij be intersection points Li^Lj.
Let Lt be some tangent at QL-Co1.
Let T1, T2, T3, T4 be the intersection points of Lt with resp. L1, L2, L3, L4.
Now Ti, Tj, Sij and QL-P1 are concyclic for all sets (i,j) with values from (1,2,3,4).
Can you explain this?
Best regards,
Chris

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Message: #1912
Date: 31/8/2016 9:07:15
From: eckart_schmidt@t-online.de
Subject: special property tangents at QL-Co1

Dear Chris,
a remark wrt the considered constellation:
The six midpoints of the described circles define a quadrilateral,
... turned around the Miquel point with the angle between QL-L2 and the tangent Lt,
... stretched with the ratio of the distances QL-P1 to Lt and QL-P1 to QL-L2.
Best regards Eckart

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Message: #1913
Date: 31/8/2016 1:00:51
From: minhtenladuong
Subject: special property tangents at QL-Co1

Dear Chris van Tienhoven
Actually it is a property of parabola: focus of a parabola lies on the circumcircle of the triangle that bounded by three tangent lines of the parabola.
Sincerely
Ngo Quang Duong

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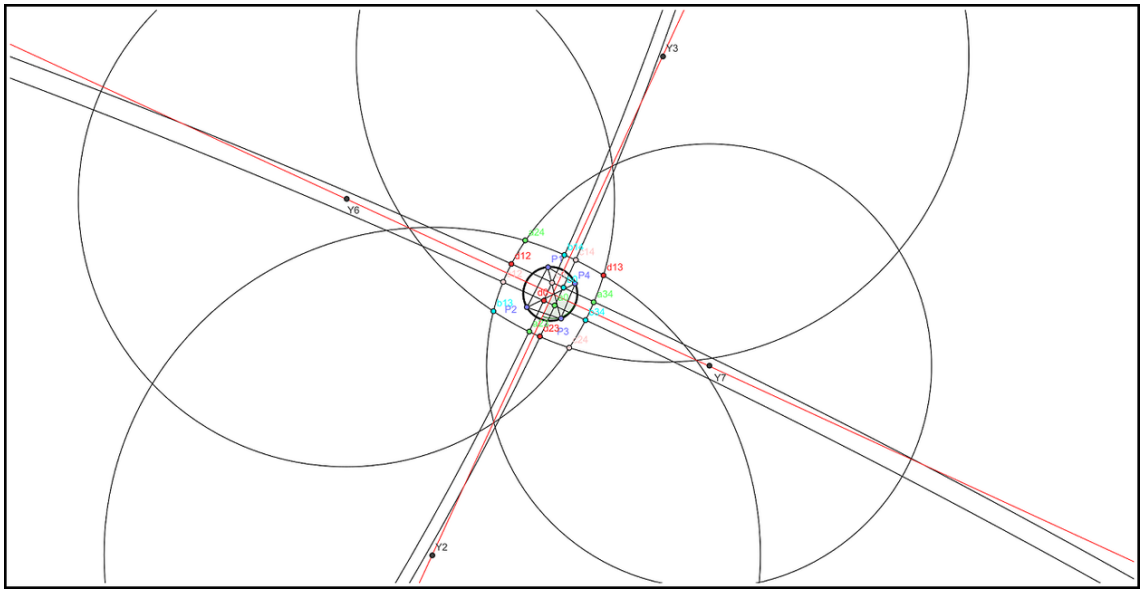
Message: #1914
Date: 31/8/2016 1:18:14
From: seiichikiri
Subject: A property of cyclic quadrangle with Morley 1st equilateral tria

---

Addition:
We can construct QL-8P1 (X1 to X8) from the incenters and excenters of the component triangles of the concyclic circumcenters (O1, O2, O3, O4) of the reference quadrilateral and obtain 4*4 colinear points as shown in the last lines of Properties of QL-8P1 of EQF.
In a similar manner, from a concyclic quadrangle P1P2P3P4, we can construct 4*4 circular points with 1 st Morley centers and their extraversions. If we denote the centers of circular points by Y1 to Y8, the lines through Y1 to Y4 and Y5 to Y8 are perpendicular.
See the attachment.
We denote the component triangles P2P3P4, P1P3P4, P1P2P4, P1P2P3 by a, b, c, d and 1 st Morley center of P2P3P4 by a0. We denote its extraversions across P2P3, P3P4, P2P4 by a23, a34,a24 respectively.
I applied this construction to quadrilateral. But I could not obtain any conspicuous figure.
Best regards, Seiichi.



CyclicQuad.png

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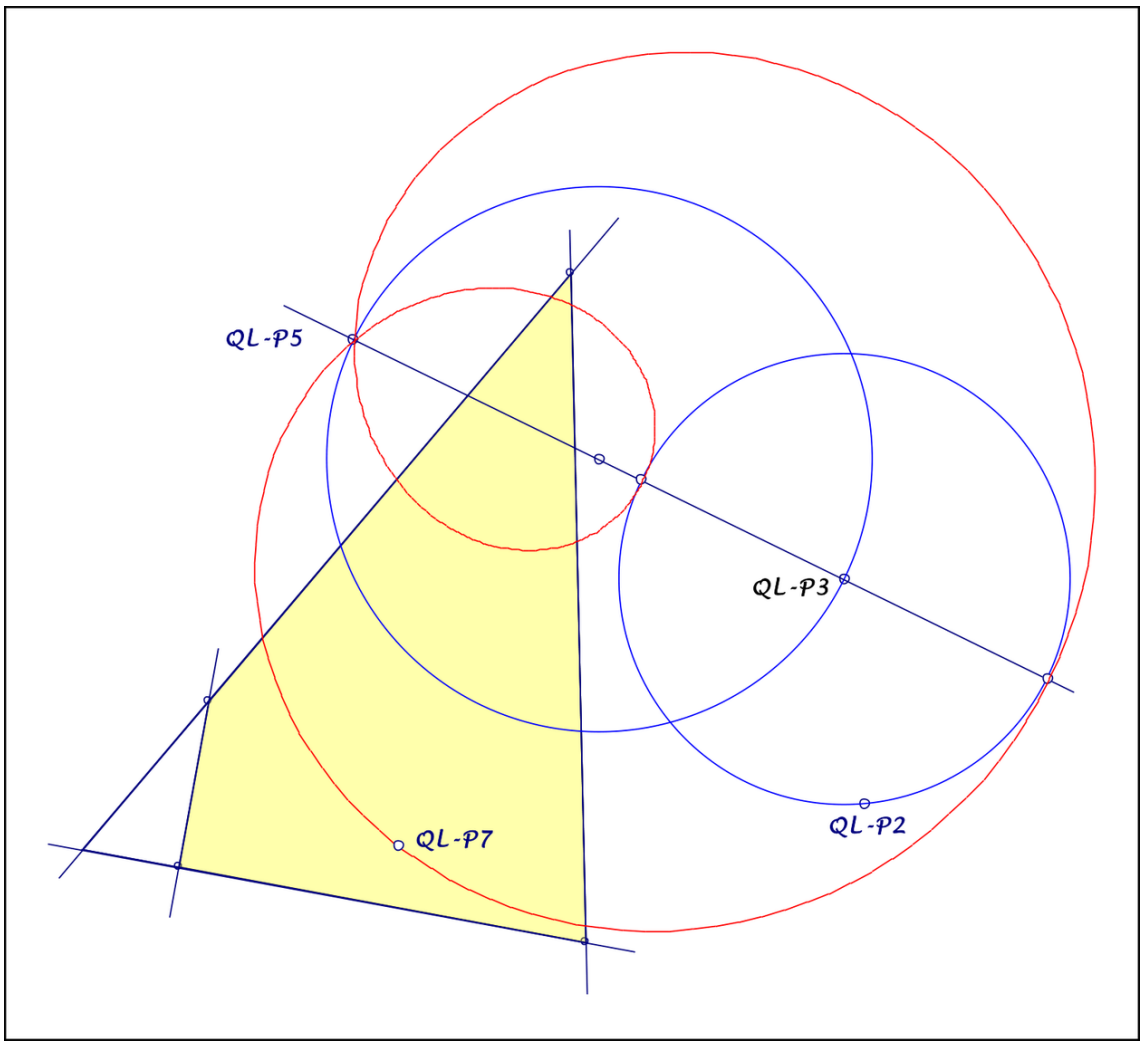
Message: #1915
Date: 02/9/2016 1:39:02
From: eckart_schmidt@t-online.de
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Chris, dear Duong,

the reconstruction of quadrilaterals, given the circumcenters of its trilaterals, leads to a family of related quadrilaterals, depending of a point M on QL-Ci3 (well-known). Fixed elements of these related quadrilaterals are already mentioned. Here are some loci of their points and lines.
... QL-L1 gives the line pencil of QL-P5.
... The Steiner axes give parallels to the Steiner axes of the reference quadrangle through M.
... The lines QL-L2 envelope a circle through QL-P2, centered in QL-P3.
... QL-L4 gives the line pencil of QL-P3.
... QL-Ci5/6 give circles round QL-P5/6 through M.
... QL-P2 gives a circle round QL-P3 (through QL-P2).
... QL-P12 gives a circle round QL-P5 (through QL-P12).
... QL-P20 gives a circle round QL-P5 (through QL-P20).
... QL-P22 gives a circle round QL-P5 (through QL-P22).
... QL-P7 gives a limacon wrt a circle
    with diameter QL-P5.QL-P3, knot QL-P5,
    cusps on a circle Ci round QL-P3 through QL-P2
    (see attached file).
... ... This limacon is an envelope of circles through QL-P5,
        centered on a circle homothetic to the circle
        round QL-P3 through QL-P2 wrt QL-P5 and ratio 1:2.
... QL-P21 gives a conic, centered in QL-P3 with axes
    parallel to the Steiner axes (through QL-P21) ...
... X(186)-circles bear QL-P4, they envelope a limacon,
    their centers give a circle, ...
... X(265)-centers give a circle round QL-P3
    through the reflection of QL-P2 in QL-P3.

Best regards Eckart



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Message: #1916
Date: 02/9/2016 7:04:57
From: eckart_schmidt@t-online.de
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Chris, dear Duong,

modifying Chris´ construction we get a reconstruction of
quadrilateral, when the n-angle points P(n)i = Ni of the
trilaterals are given:
Consider the circumcircles of Sij, Nk, Nl,
... with the 2nd intersections Hi, Hj on Li, Lj,
... rotating the lines Li around Hi with the same angle
... you get a quadrilateral with the same n-angle points.

Best regards Eckart

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Message: #1917
Date: 03/9/2016 1:20:08
From: Antreas Hatzipolakis
Subject: Cyclologic centers

[APH]:
By Eckart Schmidt (see messages to quadri figures group below (*):
Let ABC be a triangle and IaIbIc the excentral triangle.
Denote:
Ja, Jb, Jc = the incenters of IaBC, IbCA, IcAB, resp.
The triangles ABC, JaJbJc are cyclologic
The cyclologic center (JaJbJc, ABC) lies on the circumcircle of JaJbJc.
The cyclologic center (ABC, JaJbJc) is X(3659), on the circumcircle of ABC.
Which are the coordinates of the cyclologic center (JaJbJc, ABC)? Is it in ETC ?
APH
(*)
https://beta.groups.yahoo.com/ neo/groups/Quadri-Figures-Group/conversations/messages/ 1894
https://beta.groups.yahoo.com/ neo/groups/Quadri-Figures-Group/conversations/messages/ 1895
https://beta.groups.yahoo.com/ neo/groups/Quadri-Figures-Group/conversations/messages/ 1897

****
I think that it is true for the antipedal triangle of any point P, not just the antipedal triangle of I (excentral triangle).
That is:
Let ABC be a triangle, P a point and A'B'C' the antipedal triangle of P.
Denote:
Ja, Jb, Jc = the incenters of JaBC, JbCA, JcAB, resp.
The triangles ABC, JaJbJc are cyclologic.
The cyclologic center (JaJbJc, ABC) lies on the circumcircle of JaJbJc.
The cyclologic center (ABC, JaJbJc) lies on the circumcircle of ABC.

APH

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Message: #1918
Date: 03/9/2016 9:19:48
From: Antreas Hatzipolakis
Subject: Cyclologic centers

[APH]:
By Eckart Schmidt (see messages to quadri figures group below (*)):
Let ABC be a triangle and IaIbIc the excentral triangle.
Denote:
Ja, Jb, Jc = the incenters of IaBC, IbCA, IcAB, resp.
The triangles ABC, JaJbJc are cyclologic
The cyclologic center (JaJbJc, ABC) lies on the circumcircle of JaJbJc.
The cyclologic center (ABC, JaJbJc) is X(3659), on the circumcircle of ABC.
Which are the coordinates of the cyclologic center (JaJbJc, ABC)? Is it in ETC ?
Orthic triangle version:
Let ABC be a triangle and A'B'C' the pedal triangle of H.
Denote:
Ja, Jb, Jc = the incenters of AB'C', BC'A', CA'B', resp.
The triangles A'B'C', JaJbJc are cyclologic.
The cyclologic center (JaJbJc, A'B'C') lies on the circumcircle of JaJbJc.
The cyclologic center (A'B'C', JaJbJc) lies on the circumcircle of A'B'C' (= NPC of ABC)
Which points are they ?

[César Lozada]:
>Which are the coordinates of the cyclologic center (JaJbJc, ABC)? Is it in ETC ?
It is not in ETC.
Zj = (b*sin(C/2)+c*sin(B/2)-(b+c-a)/2)/(2*a*sin(B/2)*sin(C/2)+(b- a)*sin(B/2)+(c-a)*sin(C/2)-(b+c-a)/2) : : (trilinears)
= on lines: (1, 6724), (10, 188), (80, 1128), (177, 1130), (505, 8078), (1488, 8241)
= [4.679680507921859, 7.63551339563622, -3.805312718727705]
> Orthic triangle version:
The cyclologic center (JaJbJc, A'B'C')
is X(1845)
The cyclologic center (A'B'C', JaJbJc) is X(11)=FEUERBACH POINT !!!
César Lozada
**************************** ****

>> The cyclologic center (A'B'C', JaJbJc) is X(11)=FEUERBACH POINT !!!

The natural question is: Since the NPC is the common circumcircle of the pedal triangles of H and O, is the Feuerbach point the cyclologic center of (A'B'C', JaJbJc), where A'B'C' is the pedal triangle of O ?
It is!
We have:
Let ABC be a triangle and A1B1C1, A2B2C2 the pedal triangles of O, H, resp. (medial, orthic, resp.)
Denote:
J11, J12, J13 = the incenters of AB1C1, BC1A1, CA1B1, resp.
J21, J22, J23 = the incenters of AB2C2, BC2A2, CA2B2, resp.
The triangles:
1. A1B1C1, J11J12J13
2. A2B2C2,J21J22J23
are cyclologic.
Cyclologic center (A1B1C1,J11J12J13) = Cycllogic center (A2B2C2, J21J22J23) = the Feuerbach Point.
This is not a general property for two isogonal conjugate points.
PS: In the case of the pedal triangle of O, which is the other cyclologic center (J11J12J13, A1B1C1) lying on the circumcircle of J11J12J13 ?
APH

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Message: #1919
Date: 03/9/2016 11:28:26
From: bernard.keizer
Subject: special property tangents at QL-Co1

Dear Chris,
For a given parabola, the reflexions of the focus are on the directrix.
It follows that for any triangle formed by 3 tangents,
a) the focus is on the circumcenter
b) the orthocenter is on the directrix, which is the Steiner Line of the focus wrt the triangle
Best regards
Bernard

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Message: #1920
Date: 03/9/2016 11:51:03
From: bernard.keizer
Subject: Reconstruction of Quadrilateral when O1, O2, O3, O4 are known

---

Dear Chris,
For a QA, you need 4 points, but for a QL you need 5 points (for example the vertices of a reference triangle and 2 other points which determine the 4th line)
If you have the 4 circumcenters Oi on a circle, a 5h point on this circle will be QL-P1.
If the Oi are fixed points, the orthocenters hi of the triangles OjOkOl are also fixed points : QL-P4, QL-P3 and therefore QL-P5 and QL-P6 are also fixed points.
It's also promising to notice that all the ETC points for the triangles OjOkOl are also fixed points.
The Newton Lines are through QL-P5 ...
The 4 circles with centers Oi through QL-P1 are tangent to the cardioïd with center QL-P3 and summit QL-P1 (all the cardioïds have the same inner and outer circles).
The 6 2nd intersections of the circles (other than QL-P1) are the vertices of the QL and lie 3 by 3 on the 4 lines of the QL (a point is on 2 lines).
Particularly interesting is the fact that the reference triangles are similar to the triangles OjOkOl and the 4 lines of all QL's make the same angles 2 by 2.
The center of similitude is QL-P1.
The ETC points of the reference triangles can be deduced from the fixed ETC points of the triangles OjOkOl in the 4 same similitudes ...
Best regards
Bernard

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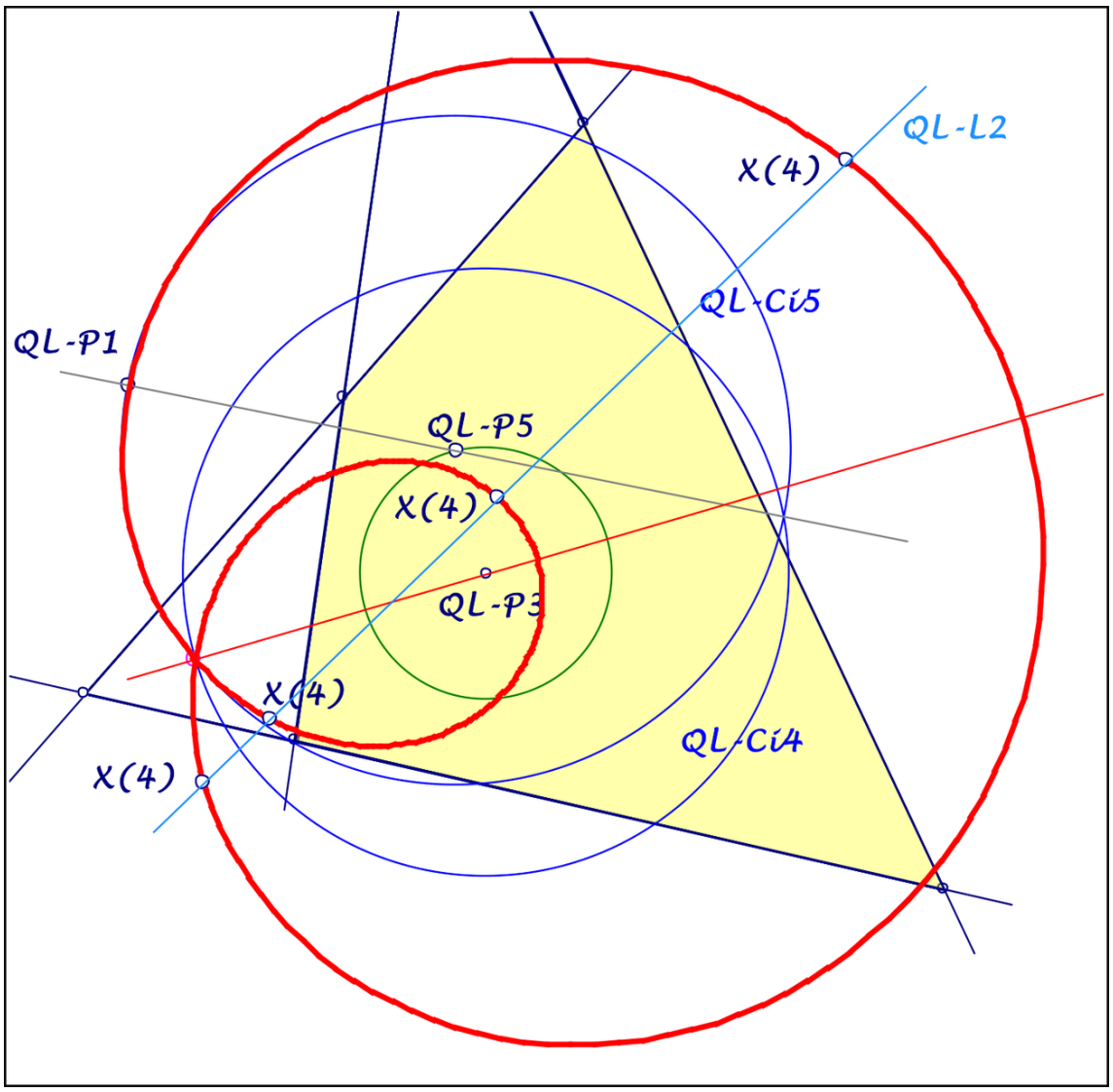
Message: #1921
Date: 03/9/2016 1:33:41
From: eckart_schmidt@t-online.de
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Chris, dear Duong, dear Bernard,
wrt the reconstruction of quadrilateral, when X(4) of the trilaterals are given: Here some remarks for the family of quadrilaterals with the same collinear X(4)-points:

```
... QL-P1 give a limacon
    (see below and attached),
... QL-P2,3 fixed,
... QL-P4,5,6 give circles round QL-P3 (through QL-P4,5,6),
... QL-P7,9 give QL-L2,
... QL-P12 gives a circle (through QL-P12) centered in a point,
    dividing QL-P2.QL-P3 with ratio 2:1,
... QL-P20 gives a circle round QL-P2 (through QL-P20),
... QL-P21 gives a circle (through QL-P21),
    centered in the reflection of QL-P3 in QL-P2,
... QL-P22 gives a circle (through QL-P22),
    centered in the midpoint of QL-P3 in QL-P2,
... QL-P26 gives a circle (through QL-P26) ...
... QL-L1,3 give parallels to QL-L1,3 of the
    reference quadrilateral,
... QL-L2,4 fixed,
... QL-L6 line pencil of QL-P2,
... QL-Ci1 are circles, centered on QL-L2,
... QL-L4 fixed,
... QL-Ci5 envelope a limacon,
... ... which is the limacon for QL-P1 (see above and attached),
... ... bearing QL-P1 and the X(4) of the trilaterals,
... ... envelope of circles,
        centered on a circle round QL-P3 through QL-P5,
... ... ... through the intersection of QL-Ci4 and QL-Ci5,
            not collinear with QL-P1 and QL-P5.
```

Best regards Eckart

PS. The last intersection is perhaps a relevant EQF-point.



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Message: #1922
Date: 04/9/2016 2:17:45
From: tsihonglau
Subject: Classification of Triangle/Trilateral and Quadrangle/Quadrilater

---

Dear all,
I would follow this topic long ago but missed two very important objects Expoint Quadrangle/Exline Quadrilateral and posted them in #1892.
Yahoo! Groups (https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1892)
So I delayed this reply till now. I would post different ideas originally but I found a very important idea missing in current research on triangle/trilateral and quadrangle/quadrilateral.

Please refer to a message from APG #2666 †)
Yahoo! Groups (https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2666)
A very important idea - Is distance on a plane positive or negative or both? A mainstream answer is "positive".
But if we consider an important theorem - pythagorean theorem - a^2+b^2=c^2. a,b,c may be positive or negative.

```
     3^2 +  4^2 =  5^2
   (-3)^2+(-4)^2=(-5)^2
   (-3)^2+  4^2 =  5^2
     3^2 +(-4)^2=(-5)^2
     3^2 +(-4)^2=  5^2
   (-3)^2+  4^2 =(-5)^2
     3^2 +  4^2 =(-5)^2
   (-3)^2+(-4)^2=  5^2
```

So there is a set of 8 solutions.
This set resembles the incenter/excenters barycentrics
a:b:c -a:b:c a:-b:c a:b:-c
If we say the three sidelengths may be positive or negative, then the incenter/excenters is a set not four centers. Because of positivity or negativity expoints and exlines emerge.
These objects are important to invariant cubics and discussed in APG #2434. †)
Yahoo! Groups (https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/topics/2434)
We consider the triangle area square - Heron's formula
(a+b+c)(-a+b+c)(a-b+c)(a+b-c)/16
   =(2(b^2c^2+c^2a^2+a^2b^2)-(a^4+b^4+c^4))/16
It is invariant whether three sidelengths a,b,c are positive or negative.
Please notice Heron's formula resembles the above incenter/excenters barycentrics.

But the triangle area is the square root of the above formula so it may be positive or negative.
We consider sines and cosines of trilateral angles.
There is no difference between an internal angle and an external angle.
As the above the sines and cosines of an angle may be positive or negative.
The sum of internal angles equals to Pi.

```
sin(A+B+C)=sin Pi=0
sin A cos B cos C
+sin B cos C cos A
+sin C cos A cos B
=sin A sin B sin C
```

Because of the positivity or negativity of sines and cosines, there are three other identities.

```
sin A sin B sin C
+sin B cos C cos A
+sin C cos A cos B
=sin A cos B cos C
sin A cos B cos C
+sin B sin C sin A
+sin C cos A cos B
=sin A cos B cos C
sin A cos B cos C
+sin B cos C cos A
+sin C sin A sin B
=sin A cos B cos C
```

In fact, all metrics(distances, areas, sines, cosines) may be positive or negative.
So we had better consider metric squares(only positive).
The negative metrics are related to quadrangle/quadrilaterals.
I will post the relationship later.
To be continued...
Best regards,
Tsihong Lau

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #1923
Date: 04/9/2016 2:30:17
From: tsihonglau
Subject: Centers of Exline Trilaterals(Component Triangles) of a Qudrilat

---

Dear Eckart,

This message is a reply to #1909
Yahoo! Groups (https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1909)
(https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1909)

The two X(n)'s X(1136) and X(1593) mentioned in #1907 are still missing in #1909.
Moreover, Some X(n)'s are not well-defined because of negative metrics.
Please refer to #1922 for more information.
The message explains why there are four n/2 angle centers or Hofstadter n/2 points.

Best regards,
Tsihong Lau

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Message: #1924
Date: 04/9/2016 10:41:06
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
the property in #1909 holds for X(1593) for r = 0, r = 1, r = 2, r = 3/2 (Cabri-controlled), but the images are not in ETC.
If we reduce r to n/2, I don´t see problems with negative metrics.
Best regards Eckart

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Message: #1925
Date: 04/9/2016 11:40:36
From: bernard.keizer
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Eckart,
Could you please be a little more detailed in your construction! It's obvious that QL-P2 is fixed, as it is the barycenter of the orthocenters of the O-triangles, but I can't see immediately why QL-P3 is also fixed ...
In fact, through all these constructions, I'm always hoping for a proof (synthetic or analytic)
a) of the concyclicity of the Hofstadter points for r integer
b) of the concurrency of the perpendicular bisectors of 2 isogonal conjugate points in this Hofstadter list (generalisation of Kantor-Hervey)
Best regards
Bernard
PS Your last point is the reflexion of QL-P1 in the line QL-P3,4,5 and 6, it's the 2nd intersection of the Miquel, Plücker and Dimidium circles and the CSC of the 3 lines CSC of these circles (Steiner Line, line through the CSC of the Plücker points and line through QL-P26)

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Message: #1926
Date: 04/9/2016 4:49:02
From: eckart_schmidt@t-online.de
Subject: Expoint Quadrangle / Exline Quadrilateral

Dear Tsihong Lau,
this seems an interesting aspect, to get new EQF-points. In my words:
For a quadrangle P1P2P3P4 you consider
... the quadrangles QA-Tr1 with Pi (Expoint Quadrangles)
... the quadrangle of their
QA-Px points,
... its diagonal triangle
... and the perspector wrt QA-Tr1.
You gave an example for QA-P2: The resulting perspector is not in EQF, here my construction:
... Take the line QA-P12.QA-P23,
... the infinity point of the perpendicular lines,
... its QA-Tr1-isogonal image
... and finally the QA-Tf2-image.
(By the way, this point is on the perpendicular line wrt QA-L2 through QA-P6.)
For a quadrilateral L1L2L3L4 you consider
... the quadrilaterals QL-Tr1 with Li (Exline Quadrilaterals)
... the quadrangle of their QL-Px points,
... its QA-diagonal triangle
... and the perspector wrt QL-Tr1.
You gave an example for QL-P1 with the resulting perspector QL-P17.
Alternatively I had expexted:
For a quadrilateral L1L2L3L4 you consider
... the quadrilaterals QL-Tr1 with Li (Exline Quadrilaterals)
... the quadrilateral of their QL-Lx lines,
... its QL-diagonal triangle
... and the perspector wrt QL-Tr1.
Example: For QL-L1 the resulting perspector is QL-P8.
Best regards Eckart

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Message: #1927
Date: 04/9/2016 4:50:46
From: seiichikiri
Subject: QA-P2 and QA-P3

Dear Chris,
Are the following already described in EQF?
Let P1P2P3P4 be the reference quadrangle. As for QA-P2, we denote the circle through the midpoints of Pj.Pk, Pk.Pl, Pl.Pj by Ci for i=1,2,3,4. The centers of Ci form 1 st quadrangle. We construct 2 nd quadrangle from 1 st one. The reference quadrangle and 2 nd one are homothetic.
As for QA-P3, the reference quadrangle and 2 nd one are homothetic.
Best regards, Seiichi.

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Message: #1928
Date: 05/9/2016 4:23:32
From: seiichikiri
Subject: QA-P2 and QA-P3

Sorry.
It is in EQF, QA-P7 and P8.
Best regards, Seiichi

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Message: #1929
Date: 05/9/2016 11:29:53
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

```
DearTsihong Lau,
in #1886 I wrote: Including X(4), the set of X(n) is isogonal
and isotomic invariant,
in #1909 I gave a generalization: Including X(4) the set of X(n)
is invariant wrt isoconjugations with fixed point (a^r:b^r:c^r).
I think, I have to reduce r to n/2 (n integer),
... then the isoconjugation is
    (x:y:z) --> (a^n y z : b^n z x : c^n x y),
... which can be generalized
    with the isogonal and isotomic invariance to
... ... (x:y:z) --> (a^n x : b^n y : c^n z),
... ... (x:y:z) --> (b^n c^n x : c^n a^n y : a^n b^n z).
Example: For n = 3 and X(3) these three isoconjugations give
X(19), X(9247), X(304) of the set.
Best regards Eckart
```

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Message: #1930
Date: 05/9/2016 2:57:19
From: eckart_schmidt@t-online.de
Subject: Reconstruction of Quadrilateral when O1, O2 ...

```
Dear Bernard,
I cannot reproduce the PS of your #1925. The last point in my
#1921 is, as I described:
... ... ...  the intersection of QL-Ci4 and QL-Ci5, not
collinear with QL-P1 and QL-P5.
You get this point, reflecting QL-P1 in QL-P3.QL-P4 and then
reflecting in QL-P5.
I think, you describe only the first reflection.
You get my point, reflecting QL-P1 in a perpendicular line to
QL-P3.QL-P4 through QL-P5.
Best regards Eckart
```

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Message: #1931
Date: 06/9/2016 9:30:41
From: bernard.keizer
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Eckart,
My apologise, QL-Ci4 is the Hervey circle and not the Miquel circle.
I can understand the limaçon as the envelop of circles centered on a fixed circle and through a fixed point not on this circle.
But I don't see why QL-P3 is a fixed point and why the distances QL-P3,4,5,6 are constant. Can you explain that?
Best regards
Bernard

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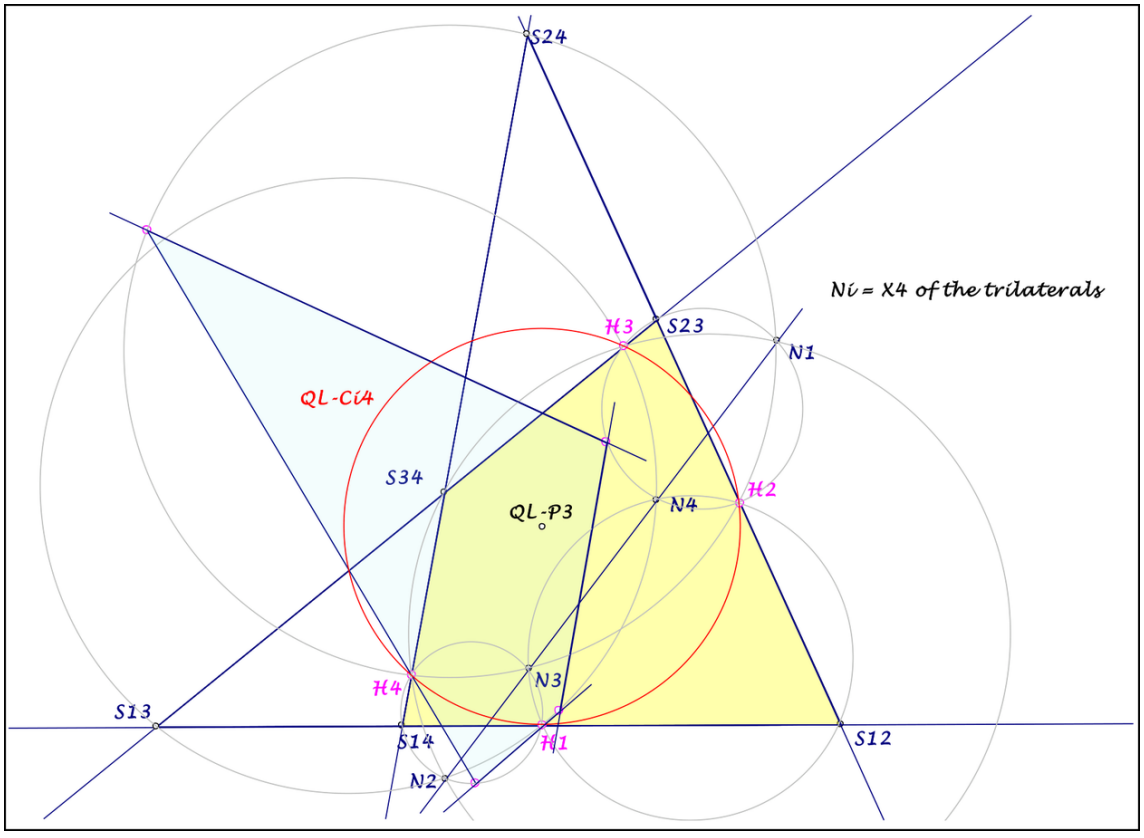
Message: #1932
Date: 06/9/2016 4:13:03
From: eckart_schmidt@t-online.de
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Bernard,
perhaps the attached file for the construction is helpful, to see why QL-P3 is a fixed point:
The 2nd intersections Hi, Hj of the circumcircles of Sij, Nk, Nl with the lines Li, Lj lie on the Hervey circle QL-Ci4 of the reference quadrangle. These points are the same for all related quadrilaterals, so QL-Ci4 and QL-P3 are fixed elements.
Wrt the limacon for QL-Ci5: It is the envelope of circles,
... centered on a circle round QL-P3 through QL-P5
... with a fixed point in the intersection of QL-Ci4 and QL-Ci5, not collinear with QL-P1, QL-P5.
By the way:
The circles QL-Ci3 are centered on a circle round QL-P3 through QL-P4 with the same radius as QL-Ci3 of the reference quadrilateral. They envelope two concentric circles round QL-P3.
Best regards Eckart



2016-09-06.pdf

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Message: #1933
Date: 07/9/2016 11:27:19
From: xpolakis
Subject: New ETC-point?

[Eckart Schmidt] :
Dear all,
Let Ia, Ib, Ic be the excenters of ABC,
... let Ja, Jb, Jc be the incenters of Ia,B,C and A,Ib,C and A,B,Ic,
... the circumcircles of Ja,B,C and A,Jb,C and A,B,Jc have a common point X.
[...]
Dear Eckart
The point is now in ETC
X(10215) = SCHMIDT-LOZADA CYCLOLOGIC CENTER
http://faculty.evansville.edu/ck6/encyclopedia/ETCPart6.html#X10215
Antreas

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Message: #1934
Date: 08/9/2016 9:31:46
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

Dear Antreas,
thanks for information, never thought, that this could be possible.
There are further cyclologic centers, X(10215) is the 2nd in a row of n-angle points, beginning with the incenter X(1). I shall describe it in the next days.
Best regards Eckart

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Message: #1935
Date: 08/9/2016 10:20:45
From: bernard.keizer
Subject: Reconstruction of Quadrilateral when O1, O2 ...

Dear Eckart,
1) First many thanks for your explanations.
I recognize a figure I already knew.
The 4 fixed points for the QL's having the same circumcenters X3 are, as well known, the orthocenters X4 of the triangles of circumcenters.
The 4 fixed points for the QL's having the same orthocenters X4 are the 2nd intersections of the 4 lines Li of the QL with the Hervey circle.
In the construction of the deltoïd, the 4 1rst points are named primary points and the 4 others secondary points : from 3 fixed points on the inner circle of the deltoïd, the secondary point describe the circle at a speed dobble of the primary and the deltoïd envelop the lines trough the 2 points, the contact point being the reflexion of the secondary point in the primary.
All the QL's having either the same X3 or X4 are similar and share the same Hervey circle, the deltoïds being simply rotated.
Of course, the Kantor-Hervey theorem states the concurrence of the perpendicular bisectors of the segments X3X4.
2) Applying the same construction (already described in your message 1916) to the points X186 and X265 (next Hofstadter points Hr with r integer), you find the same way 2 groups of 4 fixed points ; the 8 points are on the same circle with center the point of concurrency of the perpendicular bisectors of the segments X186X265 (generalisation of the Kantor-Hervey theorem).
The construction looks furiously the same and I hoped I would find easily another hypocycloïd tangent to the 4 lines, but I haven't succeeded yet!
The fight goes on ...
Best regards
Bernard
PS Congratulations for your discovery of new ETC points!

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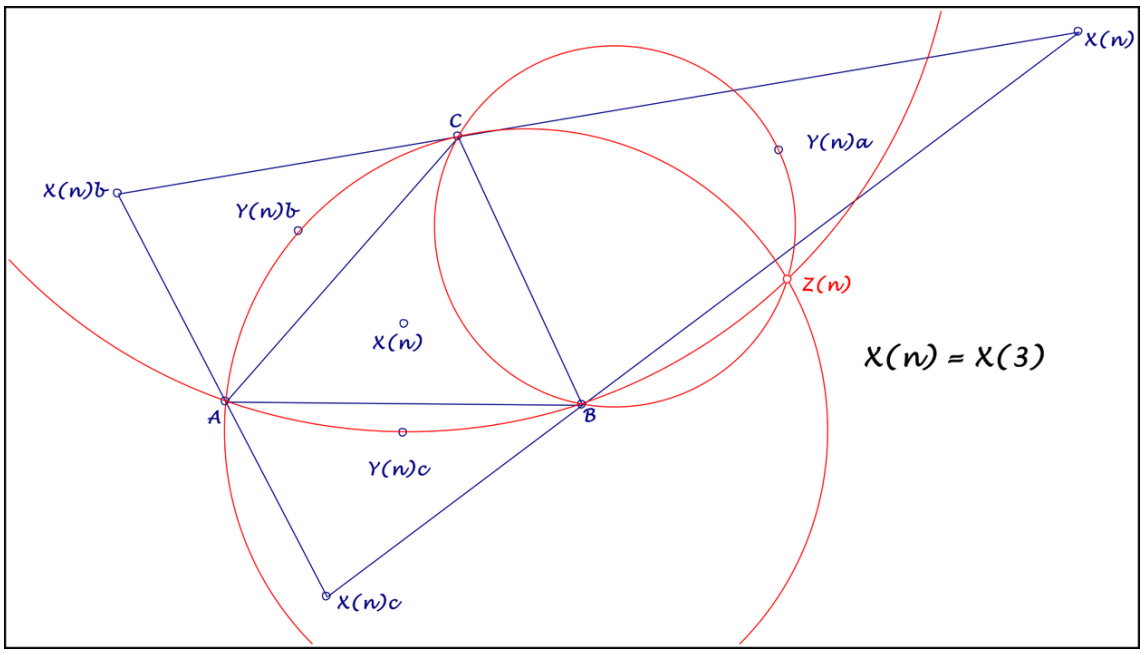
Message: #1936
Date: 09/9/2016 9:56:36
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

Dear Antreas,
This is not the announced message, but another possibility, to get cyclologic centers analog to the construction of X(10215) (see attached file):
Consider an ETC-point X(n),
... its anticevian triangle X(n)a,b,c,
... the points Y(n)a,b,c,
    which are the X(n) of ABY(n)c, BCY(n)a, CAY(n)b,
... the cyclologic center Z(n) = (Y(n)aY(n)bY(n)c, ABC).
This center doesn´t exist for all X(n):
Z(1) = X(10215), Z(2) = X(671), Z(3) not in ETC,
Z(4) = X(10152), Z(5) and Z(6) doesn´t exist ...
But there are Z(n) for the Hofstadter points, Cabri-proved for X(3), X(4), X(35), X(79), X(186), X(265), X(5961), X(5962), X(5963), X(5964). But the corresponding Z(n) are not in ETC.
Question: Are there beside the Hofstadter points and X(2) other points with a center Z(n)?
Best regards Eckart
PS: Perhaps Z(3) for ETC. First barycentric coordinate:



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Message: #1937
Date: 10/9/2016 7:57:04
From: chris.vantienhoven
Subject: Homothetic Center Reference QL and 2nd Perpendicular QL

Dear friends,

I encountered a structural construction for QL-points of which I don’t know if it is general.
However there are some indications.
Let X be some Central Triangle Point, let’s say an ETC-point.
Let Ref be a Reference Quadrilateral of lines L-1,L-2,L-3,L-4.
Let P-i = ETC-point Px of triangle (Lj.Lk.Ll), where (i,j,k,l) are different numbers from (1,2,3,4).
Let Lp-i be the perpendiculars from P-i on L-i (i=1,2,3,4).
Now we have a 1 st generation perpendicular quadrilateral Per1.
By doing the same construction on Per1 (instead of on Ref) we get a 2nd generation perpendicular quadrilateral Per2.
Now I checked with several ETC-points that all the time Ref is homothetic with Per2 (except for cases with extremities). As a consequence there will be for all these ETC-points Px a QL-homothetic center (HC) QL-Px.

Some Results:
Hofstadter Points & n-Angle Centers
For Px = X(186), X(265), X(5961), X(5962), X(5963), X(5964), (all regular Hofstadter Points / n-Angle Centers)
Ref <homothetic> Per2 -> HC= Unknown QL-Point
For Px = X(3), Ref = Per2 -> Homothetic Center is undefined.
For Px = X(4), Ref = Per2 -> HC = Infinity Point

Hofstadter Point/Non n-Angle Center:
For Px = X(1), Ref = Per2 -> HC = Unknown QL-Point

Other points:
For Px = X(2), Ref = Per2 -> HC = Unknown QL-Point
For Px = X(5), Per2 is undefined because all
1st gen. perpendiculars coincide in QL-P2.
For Px = X(6), Ref = Per2 -> HC = Unknown QL-Point
For Px = X(7), Ref = Per2 -> HC = Unknown QL-Point
For Px = X(11), Ref = Per2 -> HC = Unknown QL-Point
For Px = X(370), Ref = Per2 -> HC = Unknown QL-Point

Example unknown QL-point for X(2) is:
Midpoint QL - P5.QL - P29
Midpoint QL - P2.QL - P20
Reflection of QL - P6 in QL - P22

So far I didn't find examples of ETC-points not producing Ref-Per2-Homothetic Centers in a quadrilateral (except for cases of extremities).
I wonder if the Reference Quadrilateral and the 2nd Perpendicular Quadrilateral wrt some Central Triangle Point always will be homothetic and have a Homothetic Center.

Best regards,
Chris

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Message: #1938
Date: 11/9/2016 1:46:42
From: eckart_schmidt@t-online.de
Subject: Homothetic Center Reference QL and 2nd Perpendi ...

Dear Chris,
with interest I have reproduced your HC for X(n), here a short remark:
Let X(r) be a point on the Euler line, dividing X(3).X(4) with ratio r,
... then HC is a point on the line QL-P2.QL-P20.

```
r = 0,       X(r) = X(3),   HC = QL-P20,
r =infinite, X(r) = X(4),   HC = infinity point of QL-P2.QL-P20,
r = 1,       X(r) = X(5),   HC = QL-P2,
r = -1,      X(r) = X(3),   HC = reflection of QL-P2 in QL-P20,
r = 1:2,     X(r) = X(2),   HC = midpoint QL-P2.QL-P20
                                        (see your message),
r = -1:2,    X(r) = X(20),  HC dividing QL-P2.QL-P20
                             with ratio -3:1,
r = 2,       X(r) = X(381), HC = reflection of QL-P20 in QL-P2,
r = -2,      X(r) = X(382), HC = dividing QL-P2.QL-P20
                             with ratio -3:2,
r = 3,       X(r) = X(3627),HC = dividing QL-P2.QL-P20
                             with ratio -2:3,
```

...
I have tried some other lines in the same manner, but the HC were not collinear, but on a conic.
Best regards Eckart

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Message: #1939
Date: 11/9/2016 3:57:41
From: tsihonglau
Subject: Parabolic Points at Infinity

Dear all,
http://lth.name/geometry/coordinate.html
We know every circle passes through the circular points at infinity. We also know the pair of circumscribed QA-Parabolas QA-2Co1 and the nine-point conic QA-Co1 share the same infinity points.
We call these two points the parabolic points at infinity, which are analogous to the circular points at infinity.
Given a parabolic point with unitary coordinates p:q:r, the unitary coordinates of the other are 1/p:1/q:1/r.
Like the circular coordinate systems for the triangle/trilateral and the quadrangle/quadrilateral, many quadrangle/quadrilateral objects can be expressed with the parabolic coordinate system.
For example, the parabolic coordinates of the centroid of quadrangle QA-P1 are p(q^2+r^2):q (r^2+p^2):r (p^2+q^2), which are the same as the circular coordinates of the circumcenter X3 of triangle.
I will include the parabolic coordinate system in topic #1849 and #1892.
This topic should focus on the parabolic points at infinity not the parabolic coordinate system.
A question: What properties do the two quadrangles with the same parabolic points have?
We know all triangles/trilaterals and quadrangles/quadrilaterals share the same circular points.
Best regards,
Tsihong Lau

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Message: #1940
Date: 11/9/2016 7:46:44
From: Benedetto Scimemi
Subject: Parabolic Points at Infinity

Dear Tsi Hong Lau,
the "parabolic points" of your to-day message are studied in par. 2.7 of my old paper (ref 36 in EQF), where, for example, the cosinus of their angle is described as a function of the distances between other central points.
In fact, these directions are just a central pair-of-points (a bi-center?) at infinity.
The name "parabolic", however, is perhaps not convenient, as they exist as real points even in non-convex quadrangles, when circumscribed parabolas disappear (from the real plane); then they appear, for example, as points at infinity of QA-Co3 and in other objects. Perhaps "asymptotic points" (of the quadrangle) would be a more comprehensive name.
Your problem about quadrangles sharing (only) this pair of points would leave space to very large families (a similar question would regard quadrangles sharing, for example, QA-P2 and/or QA-P1).
Incidentally: I already suggested to Chris to add - if and when possible - central points at infinity in his list of QA-P.. in EQF; he could also add other bi-centers at finite, as conics foci, and denote them as QA-2P...
Best regards
Benedetto

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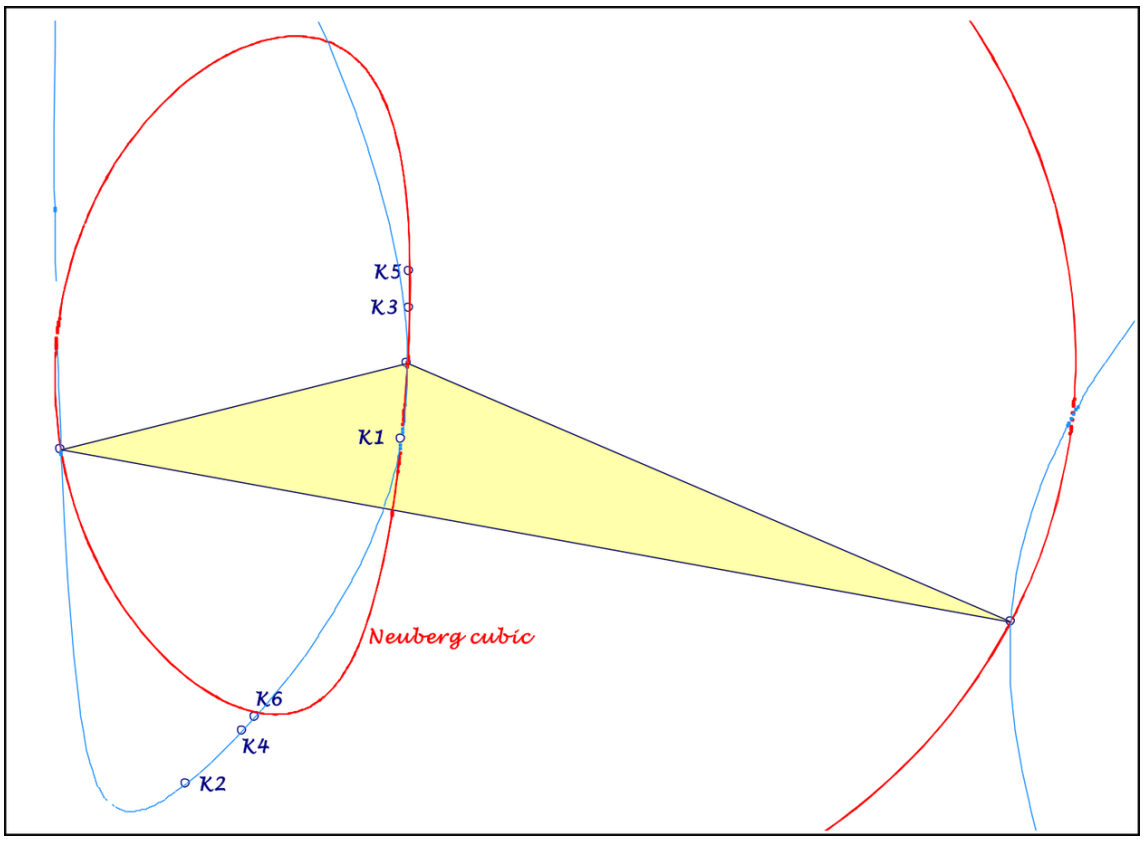
Message: #1941
Date: 12/9/2016 9:17:29
From: eckart_schmidt@t-online.de
Subject: Special n-angle Points as Cyclologic Centers

Dear all,

in #1856 n-angle points of a triangle are defined.
For n integer (unequal 0, 1) these points are the Hofstadter points.
In #1873 the definition is generalized for rational n = p/q (unequal 0, 1, canceled).
Now there are q^2 n-angle points.
Here the case n = 1/2^m (m = 1, 2, 3, ...) is considered.
Now there are 2^(2m) n-angle points,
... but one is special: Km !
Let J1a,b,c be the excenters, J2a,b,c the incenters of BCJ1a, CAJ1b, ABJ1c, ... and recursive so on.
... m = 1: The four n-angle points
           are the in- and excenters, K1 = X(1),
... ...   cyclologic center (J1aJ1bJ1c,ABC)
... m = 2: The special n-angle center is K2 = X(10215),
... ... cyclologic center (J2aJ2bJ2c,ABC)
For m = 3 there is no cyclologic center (J3aJ3bJ3c,ABC).
The circumcircles of ABJ3c, BCJ3a, ABJ3c have three 2nd intersections Sa, Sb, Sc with a cyclologic center K3 = (SaSbSc,ABC), not in ETC.
For m = 4 there are two such interim triangles SaSbSc necessary, to get a cyclologic center, in general m-2 such interim triangles.

Attached a construction of the special n-angle points Km for m = 1, 2, 3, 4, 5, 6.
There is a conjecture, but I am not sure, whether the accuracy of Cabri is sufficient, to judge it, for there are iterating constructions:
    K1, K3, K5, ... lie on the Neuberg cubic,
    K2, K4, K6, ... on another pivotal isogonal cubic.
Perhaps someone can decide this.

Best regards Eckart



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Message: #1942
Date: 12/9/2016 10:36:40
From: eckart_schmidt@t-online.de
Subject: Parabolic Points at Infinity

Dear Tsihong Lau, dear Benedetto,
perhaps of interest, some remarks wrt infinity points:
http://eckartschmidt.de/2013-04-05.pdf
Best regards Eckart

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Message: #1943
Date: 15/9/2016 4:08:09
From: tsihonglau
Subject: Parabolic Points at Infinity

Dear Benedetto and Eckart
To Benedetto:
I have not read your paper. I have a different approach.
I try to study triangle/trilateral and quadrangle/quadrilateral objects with projective geometry.
In my page Coordinate System (
http://lth.name/geometry/coordinate.html)
Coordinate System (http://lth.name/geometry/coordinate.html)

P(p;q;r) and U(u:v:w) can be any points on the plane not only the circular points.
A triangle/trilateral or quadrangle/quadrilateral object can be viewed as an operation result of the circular points or any other bi-centric pair.
I also want to unify ETC, EQF and CTC. They should not be divided into three websites.
So I study the coordinate systems for and classification of objects of triangle/trilateral and quadrangle/quadrilateral.
I found some objects in EQF can be expressed in parabolic or infinity coordinate system and other cannot.
I will explain these two classes in topic #1849.
The parabolic points are analogous to the circular ones.
In fact, given a orthocentric quadrangle, the former coincide with the latter.
So the former can be viewed as a generalization of the latter.
Circular points are also called isotropic points, which are better but not for a beginner.
So "circular" points are more common.
The same situation happens to the "parabolic" points.
Please notice circle and parabola are examples of conic sections.
To Eckart:
Thanks for your information. But parabolic points are bi-centric and different from objects in your paper.
Best regards,
Tsihong Lau

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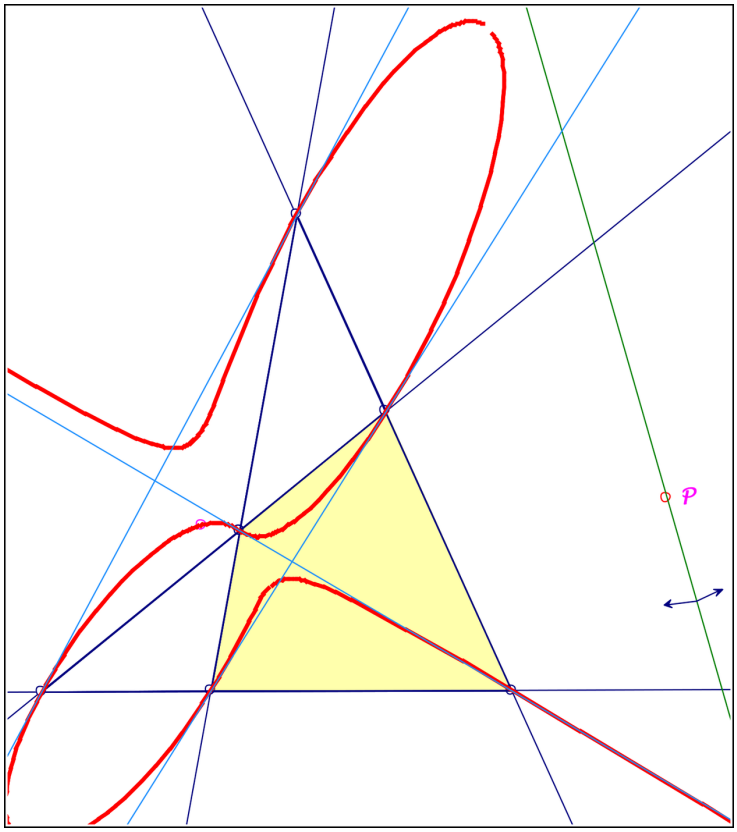
Message: #1944
Date: 17/9/2016 11:54:22
From: eckart_schmidt@t-online.de
Subject: Further QL-perspectors

Dear Bernard, dear Chris,
(1) Consider for the trilaterals of a QL
... the incenters
... and their trilinear polars Li',
... which give a new QL,
... perspective to the reference QL (L1L2L3L4 perspective L3'L4'L1'L2').
Perhaps a new QL-point? Are there other X(n) with this property?
(2) Consider for the trilaterals of a QL
... the line isoconjugations wrt the the remaining line,
... which give for a given line a new QL L1'L2'L3'L4',
... perspective to the reference QL (L1L2L3L4 perspective L3'L4'L1'L2').
The perspectors of a line pencil give a cubic,
... double tangent to the diagonals in the QL-vertices (see attached file).
Unexpected I found no connections to other QL-elements.
Best regards Eckart



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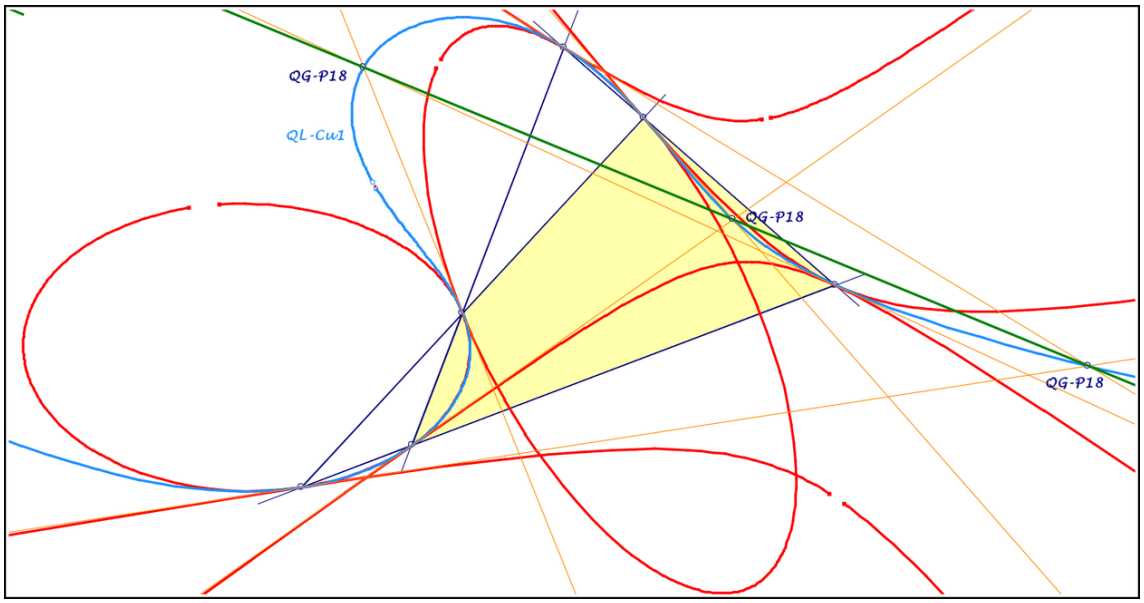
Message: #1945
Date: 18/9/2016 10:17:12
From: eckart_schmidt@t-online.de
Subject: A bit QL-geometry

Dear Bernard, dear Chris,
here is a short excursion in QL-geometry (see attached file):
Consider the isogonal conjugate wrt the trilaterals of a quadrilateral:
... The images of the remaining lines are circumconics of the trilaterals,
... tangent in the QL-points to the cubic QL-Cu1.
... The common tangents in opposite QL-points intersect on QL-Cu1
... in the collinear QG-P18-points of the three quadrigons
... with CSC-images on QL-Ci2.
Best regards Eckart



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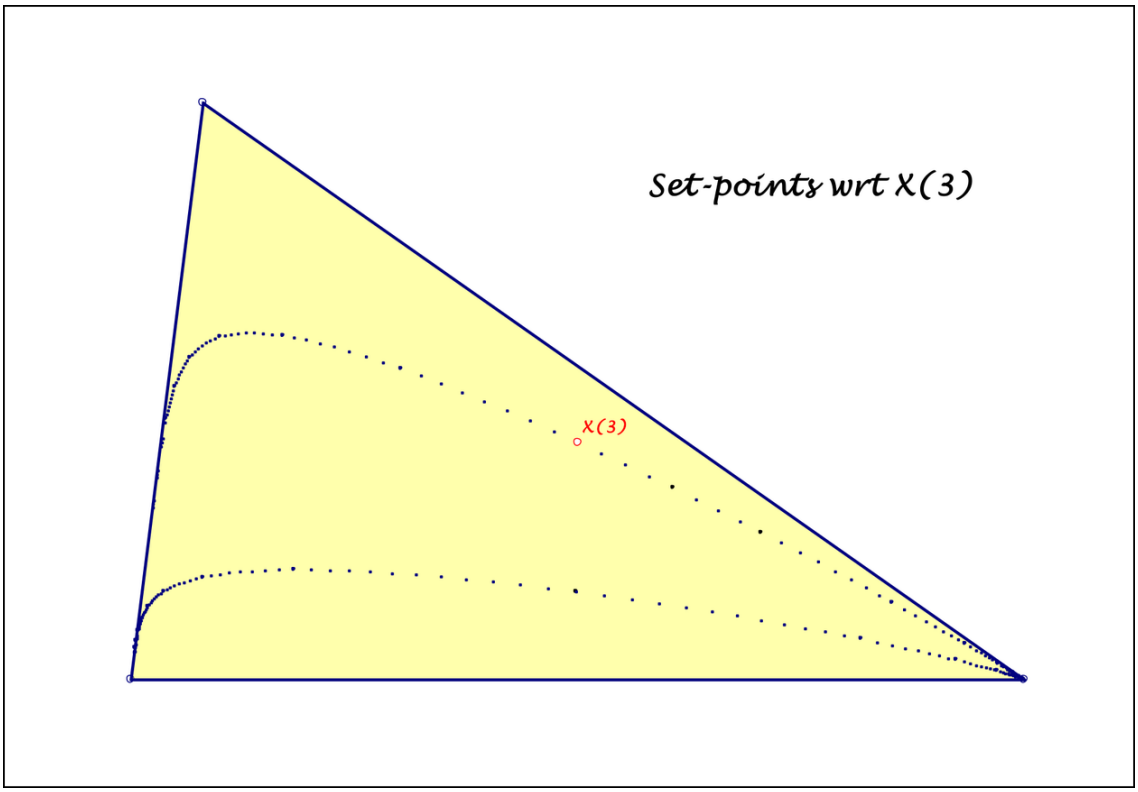
Message: #1946
Date: 19/9/2016 9:59:38
From: eckart_schmidt@t-online.de
Subject: Centers of Exline Trilaterals

Dear Tsihong Lau,
wrt your set of X(n), whose quadrangle for a quadrilateral has a perspective diagonal triangle with the diagonal triangle of the reference quadrilateral.
In #1886 I described chains of set-points (* means isogonal and ^ isotomic conjugate):
... * ...^ X(n) * ... ^ ...
These chains can be splitted in two rows:
... *^ ... *^ X(n) *^ ... *^ ... and
... ^* ... ^* X(m) ^* ... ^* ... with X(m) = X(n)^.
Each row can be refined iterated with transformations, described in #1929:
... the first row with (x:y:z) --> (x/a:y/b:z/c),
... the second row with (x:y:z) --> (x*a:y*b:z*c).
For each point X(n) of the set there are these two rows of set-points.
Attached a figure for X(3), which contains the first two chains in #1886.
Best regards Eckart



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Message: #1947
Date: 21/9/2016 10:47:52
From: bernard.keizer
Subject: Hidden Desmic

Dear Chris, dear Eckart
Like trains, a Desmic may hide another one !
I consider a Desmic system of 3 QA's the vertices of one being the perspectors of the 2 others, or a Reye configuration, the 12 points being on 16 lines.
Chris, I refer to your figure in message 1616.
It appears that the lines Q1Q2 and K1K2 intersect on the line P3P4, giving a point X1.
Generalising this observation, we find 12 points in a Reye configuration, the 3 QA's forming a 2nd Desmic system being X1 and it's harmonic conjugates wrt the 3 copples of points Q1, Q2, K1, K2 and P3, P4 and the same with the other points.
By doing the same construction starting with the 2nd group of QA's, we would find the 12 points of the 1rst group.
Each side of the 1rst group of QA's (18 lines) caries is also a side of the 2nd group, the 2 pairs being harmonic conjugates.
Each QA of the 2nd group has 2 common sides with each of the 3 QA's of the 1rst ; it follows that the DT's of the QA's of the 2nd group have one common vertice with each of the 3 QA's of the 1rst.
The 1rst group of QA's leads to a pivotal isocubic, the 2nd group to another pivotal isocubic and the 2 cubics intersect in the 9 common DT vertices.
I don't know if this is obvious or already well-known, but I found it interesting ...
Best regards
Bernard

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Message: #1948
Date: 21/9/2016 11:38:02
From: bernard.keizer
Subject: QL-2P2 again

Dear Chris, dear Eckart
I know a lot has been already written on this subject (I refer in particular to Eckart's messages 1425 and 1431), but I think it's worth trying a comparison between mono-and bicursal QL-Cu1.
In order to avoid any confusion, I name M the Miquel point QL-P1 and S and S' or V and V' the 2 real foci of the inscribed conic centered in the intersection of QL-L6 with QL-L1.
N is the point where QL-Cu1 crosses it's asymptote.
N is the common tangential of M, V and V' and the infinity point of the Newton Line.
M is the tangential of S and S' and the infinity point is the tangential of the in- and excenters of MVV'.
V and V' are 2 points on QL-Cu1 symmetric wrt the Newton Line, whereas S and S' are the 2 intersections of QL-Cu1 with the Newton Line.
S and S' and V and V' are not the same points and cannot be real at the same time.
The triangle MVV' is the Miquel triangle of QL-Cu1 and the 3 Cl-S centered in one vertice and swapping the 2 others leave the curve globally invariant, the 3 Cl-S conjugate of a
point forming with this point a Cu-QA with perpendicular cords, QA-P1 is on the Newton Line, QA-P4 is the tangential, CL-S of the intersection of the 2 cords, wich is QA-P2 (the cords are the degenerated rectangular hyperbola polar conic of QA-P4 wrt the stelloïd).
QA-P3 is the perspector of DT QA-Tr1 and the Miquel
triangle QA-Tr2, QA-P3QA-P4 is parallel to the the Newton Line ...
The triangle MSS' is another remarkable triangle and the 3 CL-S defined the same way leave also the curve QL-Cu1 globally invariant, but lead this time to 2 parallel cords instead of perpendicular ! (It cannot be the Miquel triangle).
When QL-Cu1 is monocursal, S and S' are real, V and V' are imaginary and there is for a point QA-P4 on the curve only 2 real pretangentials, the 2 intersections of the perpendicular cord with QL-Cu1 in QA-P2 are imaginary (but their middle as well as QA-P1 and QA-P3 are real).
We can have Desmic systems with 2 real vertices for each QA ...
When QL-Cu1 is bicursal , V and V' are real, S and S' are imaginary ; the QA's have 4 real points (for example, M, V, V' and the infinity point are a Cu-QA, with leads to Desmic configurations ...)

Obviously, QL-Cu1 bipartite is a pivotal isocubic, the pivot being QA-P4 and the reference triangle the DT of the corresponding QA with perpendicular cords in QA-P2 and QA-P3QA-P4 parallel to the Newton Line.
But it seems possible to consider it also true for QL-Cu1 monocursal (the pivot is real, although the reference DT triangle is imaginary).
What do you think of this appproach ?
Best regards
Bernard

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Message: #1949
Date: 23/9/2016 11:02:13
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

Dear Antreas, Bernard, Chris,
there was no reaction on #1936, but two of the described points are now in ETC: X(10230 and X(10231).
Attached another concept for new ETC-points.
Best regards Eckart

Concept for new *ETC*-points:

For an *ETC*-point $X(n)$

… and its anticevian triangle $X(n)_a\, X(n)_b\, X(n)_c$

… consider the quadrangle $X(n)\, X(n)_a\, X(n)_b\, X(n)_c$

… and take the *QA*-points as triangle centers.

Examples:

	QA-P1	QA-P2	QA-P3	QA-P4
X(1)	X(3)	not ex.	not ex.	not ex.
X(2)	X(3)	X(99)	X(671)	X(316)
X(3)	X(1147)	X(110)	X(5504)	X(2071)
X(4)	X(6523)	X(107)	not ETC	not ETC
X(5)	X(6663)	X(476)	not ETC	not ETC

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-09-23.docx

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Message: #1950
Date: 23/9/2016 5:54:54
From: chris.vantienhoven
Subject: New ETC-point?

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Dear Eckart,
Very nice concept!
Best regards,
Chris
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Message: #1951
Date: 23/9/2016 8:39:32
From: César Lozada
Subject: New ETC-point?

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Dear Eckart,
Let's denote QAP_acev(k,n) the QA-P(k) point of the quadrangle
{X(n),vertices-of-the-anticevian-of-X(n)}
Then:
QAP_acev(3,4) =
= sec(A)^2/((5*cos(2*A)+7)*cos(B-C)-cos(A)*cos(2*(B-C))-10*cos(A
)-cos(3*A)) :: (trilinears)
= Trilinear pole of the line {393,6587}
= reflection of X(i) in X(j) for these (i,j): (107,6523),
(3346,122)
= On lines: {20,107}, {122,3346}, {133,1249}, {2777,3183}
= [ -2.158256348287418, -2.68134960274278, 6.493101752246650 ]
QAP_acev(3,5) =
= 1/((4*cos(2*A)+2*cos(4*A)+3)*cos(B-C)+2*cos(A)*cos(2*(B-C))-7*
cos(A)-3*cos(3*A)-cos(5*A)) : : (trilinears)
= antigonal conjugate of X(6662)
= reflection of X(i) in X(j) for these (i,j): (476,6663),
(6662,3258)
= On lines: {476,6663}, {3258,6662}, {9214,10201}
= [ -0.053793793378817, -0.08040278306128, 3.721155851740104 ]
QAP_acev(4,4) =
= ((2*cos(2*A)+3)*cos(B-C)-6*cos(A)-cos(3*A))*sec(A)^2 : :
(trilinears)
= antigonal conjugate of X(2071)
= polar circle-inverse-of-X(185)
= midpoint of X(i),X(j) for these {i,j}: {4,6761}
= reflection of X(i) in X(j) for these (i,j): (1304,403),
(6760,5)
```

= on cubic K025 and these lines:
{4,51}, {5,6760}, {20,6526}, {30,107}, {316,6528}, {403,1300},
{1503,1559}, {1596,1629}, {3146,6523}, {3543,6525},
{3839,10002}, {5523,6529}, {6530,10151}
= [-2.719974554883476, -3.27985730468963, 7.166707795100341]
QAP_acev(4,5) =
= (6*cos(2*A)+2*cos(4*A)+7)*cos(B-C)+(-6*cos(A)-2*cos(3*A))*cos(
2*(B-C))+(2*cos(2*A)+2)*cos(3*(B-C))-cos(5*A)-7*cos(A)-cos(3*A)
: : (trilinears)
= (2*SW^2-(17*R^2-2*SA)*SW+(39*R^2-7*SA)*R^2)*S^2-(18*R^4-11*R^2
*SW+2*SW^2)*(SB+SC)*SA : : (barycentrics)
= polar circle-inverse-of-X(6746)
= On lines: {4,94}, {110,2888}, {9143,10201}
= [-0.110950800319549, -0.09306259735188, 3.756300495606244]
Regards,
César Lozada

El Viernes 23 de septiembre de 2016 11:02,
"'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group]" escribió:
>[Attachment(s) (https://es-mg42.mail.yahoo.com/neo/launch?.ra
nd=e5enkqns4svlv#TopText) from eckart_schmidt@t-online.de
included below]
>Dear Antreas, Bernard, Chris,
>there was no reaction on #1936, but two of the described points
are now in ETC: X(10230 and X(10231).
>Attached another concept for new ETC-points.
>Best regards Eckart

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Message: #1952
Date: 24/9/2016 8:42:54
From: chris.vantienhoven
Subject: New ETC-point?

Dear Eckart, dear Cesar,

Cesar, very nice analyses!
I think also
QAP_acev(k,n) for
k = 5,10 (evt. 11,12,13),16 and
n = 2,3,4,5
can be very interesting.
And what about Q *LP_acev(k,n) for*
k = 1 (Miquel Point to start with), 12, 13, 26 and
n = 2 (Euler line), 3 (Brocard Axis)
Where Q *LP_acev(k,n) is the point defined by:*
For an ETC-central-line L(n)
· and its antiLINEcevian triangle L(n) a L(n) b L(n) c
(Let antiLINEcevian triangle = ABC-circumscribed triangle that
is perspective with ABC having perspectrix L(n))
· consider the quadrilateral L(n) L(n) a L(n) b L(n) c
· and take the QL-points as triangle centers.

Best regards,
Chris

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Message: #1953
Date: 24/9/2016 1:38:52
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

Dear César Lozada, dear Chris, dear Bernard,
I am glad, you are interested in the concept for new ETC-points, and I am fascinated of the possibilities in Cesars calculations!
I had only calculated the 1st barycentric coordinate for QAP_acev(4,4) = (see picture)

There are some evident results, for the reference triangle is the diagonal triangle of X(n)X(n)aX(n)bX(n)c:
QAP_avec(10,n) = X(2), QAP_avec(11,n) = X(3), QAP_avec(12,n) = X(4), QAP_avec(13,n) = X(5) for arbitrary n.
In addition to Chris´ generalisations this concept for new ETC-points can also be modified wrt the dual QL of X(n)X(n)aX(n)bX(n)c, taking QL-points for triangle centers. Here some examples wrt wellknown X(n):
... X(1), QL-P1 --> X(115),
... X(1), QL-P2 --> X(3),
... X(1), QL-P5 --> X(6140),
... X(1), QL-P7 --> X(187),
... X(1), QL-P8 --> X(2),
... X(1), QL-P12 --> X(351),
... X(1), QL-P13 --> X(6),
... X(3), QL-P13 --> X(577),
... X(4), QL-P2 --> X(389),
... X(4), QL-P1 --> X(115),
... X(4), QL-P2 --> X(389),
... X(4), QL-P13 --> X(393),
... X(6), QL-P13 --> X(32),
... and the evident cases for X(2), X(3), X(4), X(5) and QL-P8, QL-P9, QL-P10, QL-P11.
Best regards Eckart
PS: This modification was also mentioned in a private mail of Bernard.

$$(SB^2\ SC^2\ (SA^3\ a^2 + SA^2\ (SB^2 - 3\ SB\ SC + SC^2) - SB^2\ SC^2) : \ :)$$

hpc3YPMK.png

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Message: #1954
Date: 24/9/2016 4:22:36
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

```
Dear Cesar Lozada, dear Chris, dear Bernard,
there is a further modification of my concept, to get new
ETC-points:
Take for a reference triangle ABC
... the quadrangle QA with one vertex X(n)
    and ABC as Miquel triangle QA-Tr2.
(For X(1) this gives the anticevian case.)
Then QA-Px  can be interpreted as triangle center, for example:
... X(3), QA-P3 --> X(1263),
... X(3), QA-P4 --> X(1157),
... X(4), QA-P3 --> X(8439),
... X(4), QA-P4 --> X(3484).
Best regards Eckart
```

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Message: #1955
Date: 24/9/2016 7:11:01
From: bernard.keizer
Subject: New ETC-point?

Dear Eckart,
As Bob Dylan said : The times, they are changing !
I remember an old time, when I was interested in properties for QA vertices isotomic wrt DT (the result swaps QL-P1 and QL-P25), your short comment was: That's not QA or QL geometry, that's triangle geometry!
Anyhow, a short comment to these results.
It appears that QL-P1 is X115 for the incenter X1 and the orthocenter X4; in fact, it is also X115 for X2.
For I1, QL-P1 = X115, QL-P25 is X125 and QL-P17 is X110.
For X75 = isotomic of X1, QL-P17 = X99 ...
Conversely, the locus of the points for which QL-P1 is X115 is the Stammler quartic Q066 in Bernard Gibert (isogonal of the Stammler rectangular hyperbola), which passes in fact through X1, X2 and X4 and some other points. This property was mentionned by Angel.
More generally, for a given point on the Euler circle as focus, the parabola QL-Co1 is given (inscribed in the medial triangle). The locus of the QA vertices corresponding to a QL formed by 4 tangents to the parabola and having the same DT is a curve with similar form ; the isotomic of these vertices give a 2nd QA and the dual QL is formed by the tangents to QL-Co3.
Best regards
Bernard

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Message: #1956
Date: 25/9/2016 4:52:49
From: bernard.keizer
Subject: New ETC-point?

Dear Eckart,
It doesn't matter if it is QA/QL- or triangle geometry, the important thing is that it leads to a wide range of reflexions!
Only a detail more on your table in message 1949 : you say for X(1) that QA-P2, P3 and P4 do not exist.
It would be more precise to say for P2 and P3 undetermined on the circumcircle and for P4 undtermined on the infinity line.
Any point on the infinity line leads to an isogonal circular cubic, for example K001 = pK(X6,X30) or K021 = pK(X6,X512).
Best regards
Bernard
PS For X7, QL-P1 = X11
I'm awaiting impatiently for a bigger table with plenty of ETC and EQF points ...

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Message: #1957
Date: 25/9/2016 9:13:31
From: eckart_schmidt@t-online.de
Subject: New ETC-point?

```
Dear Bernard,
thanks for additional examples wrt # 1953.
Wrt X(2) we have to respect, that the dual QL degenerates.
With great interest I have constructed the Stammler quartic as
locus of the points for which QL-P1 is X(115),
thanks for information.
It took a long time, to understand and do your last
construction:
... QL1 reference quadrilateral,
... QA1 dual of QL1,
... QA2 isotomic conjugate of QA1 wrt the common DT,
... QL2 dual of QA2,
... with the result,
... ... that QL-P1 of  QL2 is QL-P25 of  QL1
... ... and QL1 and QL2 have the same contacts
        with their inscribed parabolas.
So I have to cancel my comment - excuse!
But I don´t see the reference to the results in the middle of
the message.
Best regards Eckart
```

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Message: #1958
Date: 26/9/2016 10:53:59
From: bernard.keizer
Subject: New ETC-point?

```
Dear Eckart,
Angel's message was 1448, my construction was explained in 1449
and 1451.
Best regards
Bernard
```

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Message: #1959
Date: 27/9/2016 11:46:27
From: bernard.keizer
Subject: New ETC-point?

Dear Eckart,
I was desperate as I couldn't find my initial messages !
But it's now done : they were in messages 1315 and 1316.
The property was also explained in EQF at QL-Co3.
QL-P1 is necessary one of the 2 intersections between the Euler circle (circummedial circle) and the cevian triangles of the vertices of QA. That's the way I found X115 for X1, X11 for X7.
And of course, for X2, the point is undetermined on the Euler circle ...
This time, I hope I was complete.
Best regards
Bernard

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Message: #1960
Date: 27/9/2016 12:45:34
From: eckart_schmidt@t-online.de
Subject: Conics in the dual QA/QL-constellation

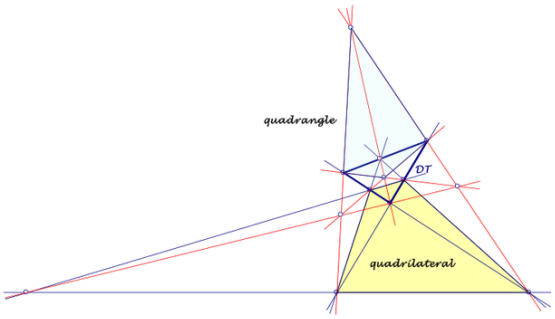
Dear Bernard, dear Chris,
incidence geometry wrt a quadrilateral and its dual quadrangle leads to two triples of unexpected conics (see attached file).
I tried to describe, but there will be a better sight. More conics are hidden in the constellation.
Best regards Eckart

EQF-Note 2016-09-27

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

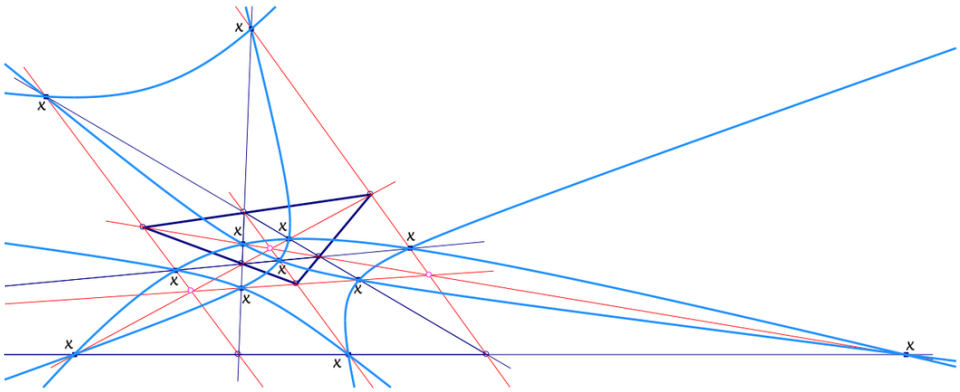
Conics in the dual QA/QL-constellation

Incidence geometry wrt a quadrilateral and its dual quadrangle leads to two triples of unexpected conics.



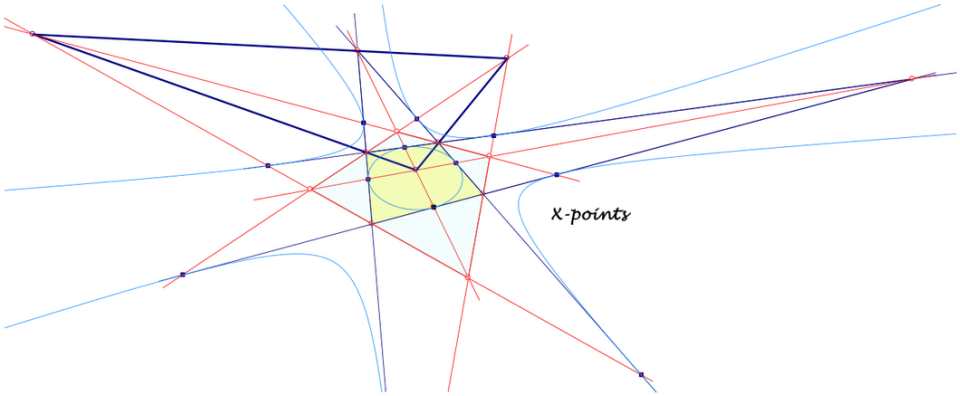
Consider a quadrilateral *QL* and its diagonal triangle *DT*. The lines from each *DT*-vertex to opposite *QL*-points on the opposite *DT*-sideline give the lines of the dual quadrangle *QA*.

- **The *CSC*-images of *DT*-vertices wrt four of the *QA*-lines, not bearing the *DT*-vertex, coincide in *QA-P4* of the dual quadrangle.**

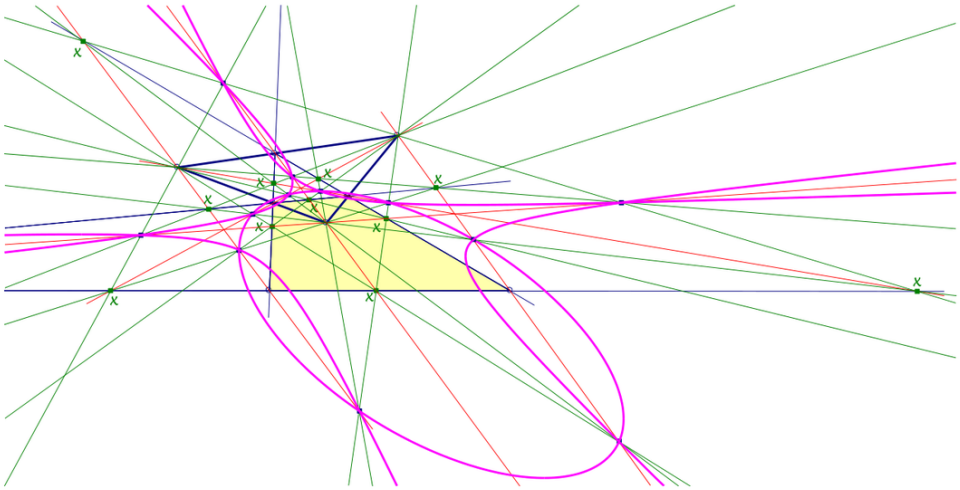


- **The six *QA*-lines intersect the *QL*-lines in 12 new points *X***
  … on three conics, each bearing eight points,
  ... intersecting in the *X*-points on the *QL*-lines,
  … which are the contact points of the conics *QG-Co1* for the *QL*-quadrigons .

2016-09-27.pdf



- **Each *DT*-vertex is – beside the *QA*-lines – four times collinear with two conic intersections *X*,**
  ... corresponding lines intersect the *QA*-lines in 12 new points *Y*,
  … on three conics, each bearing eight points,
  … intersecting in the *Y*-points on the *QA*-lines,
  … which are the contact points of inscribed conics of *QA*-quadrigons tangent to two *QL*-lines intersecting on a *QA*-quadrigon diagonal.



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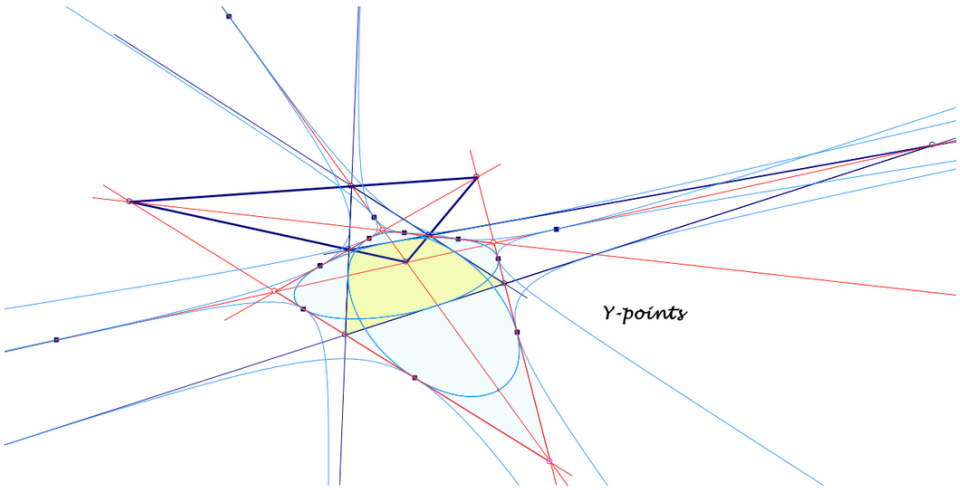

For both conic triples the following properties hold:

- **The dual of one conic is a conic tangent to the other two conics.**
- **The polars of a *DT*-vertex wrt the three conics coincide in the opposite *DT*-side.**

For a *DT*-vertex there are two opposite *QL*-points on the opposite *DT*-side and two opposite *QA*-lines bearing the *DT*-vertex. This constellation has a corresponding conic in both conic triples.

- **The polars of opposite *QL*-points are opposite *QA*-lines wrt the corresponding conics.**

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2016-09-27.pdf

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Message: #1961
Date: 28/9/2016 11:12:07
From: bernard.keizer
Subject: Conics in the dual QA/QL-constellation

Dear Eckart,
I find it very interesting (that's this time without any doubt pure QA/QL geometry !)
I react immediately, but I will study the figure more completely later.
I suppose there are a lot of properties more or less linked with the QA Miquel triangle.
First, that the CSC images of the DT vertices are QA-P4 is well-known as DT and QA-P4 are precisely what you name a Cu-QA for the pivotal isocubic with pivot QA-P4 QA-Cu1.
Then, if you consider directly the dual lines of your X points, they are the 12 lines PiPij and PjPij through a QL vertice Pij intersection of 2 lines of the QL Li and Lj and the 2 QA vertices Pi and Pj duals of these lines. These 12 lines form 3 QL's and each of the 3 dual conics of your conics are inscribed in 2 of the 3 QL's, meaning the foci are the intersections of the 2 corresponding Van Rees curves QL-Cu1.
I wouldn't be surprised (but I haven't checked it yet) if the 3 Cl-S were precisely the 3 CL-S of the QA with the Miquel triangle ...
Best regards
Bernard

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Message: #1962
Date: 30/9/2016 10:45:34
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 Invariant Transformation

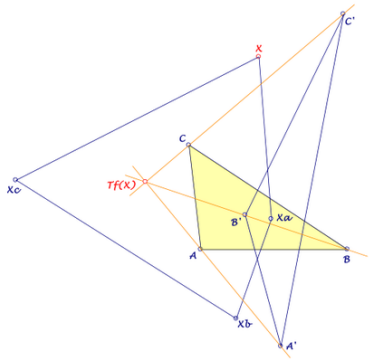
Dear all,
perhaps of interest (see attached file):
Hidden in Bernard Gibert´s paper “Perspective Triangles Inscribed in the Neuberg Cubic”
http://bernard.gibert.pagesperso-orange.fr/files/Resources/NeubergTriangles.pdf
there is a triangle transformation, not involutary, but mapping the Neuberg Cubic to itself. This transformation can be developed with QA-geometry and maps – wrt the Miquel triangle – the cubic QA-Cu1 to itself.
Best regards Eckart

EQF-Note 2016-09-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Cu1 Invariant Transformation

Hidden in Bernard Gibert´s paper
"Perspective Triangles
Inscribed in the Neuberg Cubic"
there is a triangle transformation, not involutary, but mapping the Neuberg Cubic to itself. This transformation can be developed with QA-geometry and maps – wrt the Miquel triangle – the cubic QA-Cu1 to itself.



Consider for a reference triangle *ABC* and a point *X*
… the images X_a, X_b, X_c of *X* wrt
…the Möbius transformations, centered in a vertex and swapping the other two vertices (see *QA-Tf4*).
…The diagonal triangle of the quadrangle $XX_aX_bX_c$ is perspective to the reference triangle in *Tf(X)*.
The first coordinate in barycentric coordinates is:

$$(c^2 x^2 y + b^2 x^2 z + 2 a^2 x y z + a^2 y^2 z + a^2 y z^2) /$$
$$(b^2 c^2 x^4 + 2 b^2 c^2 x^3 y - a^2 c^2 x^2 y^2 + b^2 c^2 x^2 y^2 + c^4 x^2 y^2 -$$
$$2 a^2 c^2 x y^3 - a^2 c^2 y^4 + 2 b^2 c^2 x^3 z + 4 b^2 c^2 x^2 y z -$$
$$2 a^2 c^2 y^3 z - a^2 b^2 x^2 z^2 + b^4 x^2 z^2 + b^2 c^2 x^2 z^2 + a^4 y^2 z^2 -$$
$$a^2 b^2 y^2 z^2 - a^2 c^2 y^2 z^2 - 2 a^2 b^2 x z^3 - 2 a^2 b^2 y z^3 - a^2 b^2 z^4)$$

This is the transformation $M \rightarrow M\sim^*$ in 2.2 of Bernard Gibert's paper, only considered for the Neuberg cubic. Properties should be read there. Not mentioned a generalization:

- **All pivotal isogonal circular cubics are invariant wrt *Tf*.**

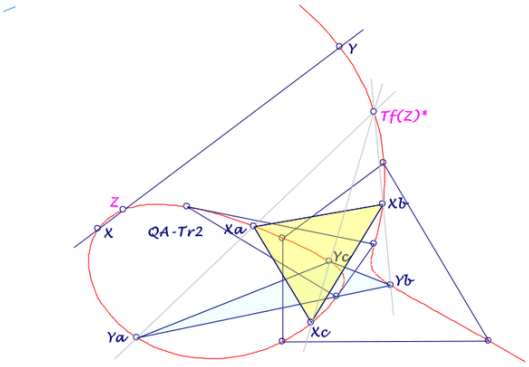
The cubic *QA-Cu1* is a pivotal isogonal circular cubic wrt the Miquel triangle *QA-Tr2*:

- ***QA-Cu1* is invariant wrt *Tf* for the Miquel triangle.**

There are always four points on *QA-Cu1* with the same *Tf*-image:

- **The *Tf*-images of the quadrangle vertices are *QA-P3*.**
- **The *Tf*-images of the vertices of the diagonal triangle and *QA-P4* are the isogonal conjugate of *QA-P41*.**
- **The *Tf*-images for the in- and excenters of *QA-Tr2* are the intersection of *QA-Cu1* and its asymptote.**
- **The *Tf*-images of *QA-Cu1*-points with the same tangential are the isogonal conjugate of the tangential.**

The last property gives a simple construction for the tangential *Tf(X)** of a *QA-Cu1*-point *X*.
The following property is an analogon to chapter 3 in Bernard Gibert's paper:



- **For two *QA-Cu1*-points *X, Y***
  … the triangles $X_aX_bX_c$ and $Y_aY_bY_c$ are perspective
  … at the tangential of the third intersection *Z*: *Tf(Z)**.**

Example: Let *X* = *QA-P3* and *Y* = *QA-P4*: The third intersection *Z* is the point at infinity of *QA-Cu1* with tangential in the intersection of *QA-Cu1* and its asymptote, which is the perspector of $X_aX_bX_c$ and $Y_aY_bY_c$.

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2016-09-30.pdf

Message: #1963
Date: 30/9/2016 12:47:17
From: bernard.keizer
Subject: Conics in the dual QA/QL-constellation

Dear Eckart,
I think I have understood your mysterious conics.
The figure is full of harmonic divisions.
The points X on the line Li of the QL are the harmonic conjugates of one of the 3 points Pij wrt the 2 others Pik and Pil.
The points Y on the same line are the harmonic conjugates of Pik wrt the 2 points Xi and Pil.
Let's change the notations and name Xi, Yi and Zi the X points on the line Li.
The other Xpoints are the vertices of the anticevian triangles of Xi, Yi and Zi wrt DT.
A conic through the 4 points Xi with the same DT is a diagonal conic in the sense of Bernard Gibert ; it carries the vertices of the anticevian triangles of each of it's points. That explains the 3 conics with each 8 points, the 4 points of 2 of the 3 groups.
The same goes for the Ypoints.
I tried to calculate the DT barycentric coordinates of these points.
For the QA vertices being +/- p, +/-q and +/-r, I found for the Xpoints +/- 2p, +/-q, +/-r, +/-p, +/ 2q, +/-r and +/-p, +/-q, +/-2r and for the Ypoints the same with 3 instead of 2.
I hope I didn't make an error in these calculations ; if it's true, there is no reason to stop the process ...
Best regards
Bernard

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Message: #1964
Date: 30/9/2016 5:29:45
From: bernard.keizer
Subject: Conics in the dual QA/QL-constellation

Dear Eckart,
I make a correction for the Ypoints : as they are on each line Li the harmonic conjugates of Pik wrt Xi and Pil, it can also be another point harmonic conjugate of Pil wrt Xi and Pik.
In other words, there are 6 points of this 2nd generation on each line Li, meaning alltogether 24 points and not 12.
The other points are the vertices of the anticevian triangles of these 6 points.
DT coordinates are +/-3p, +/- 2q, +/-r ; +/-3p, +/-q, +/-2r ; +/- 2p, +/-3q, +/-r ; +/- p, +/-3q, +/- 2r ; +/- 2p, +/-q, +/-3r ; +/-p, +/- 2q, +/-3r
There are many conics of 8 points ...
Best regards
Bernard

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Message: #1965
Date: 01/10/2016 10:53:36
From: bernard.keizer
Subject: QA-Cu1 Invariant Transformation

```
Dear Eckart,
Please, I need your help !
The properties given at the beginning are obvious as it is the
definition of QA-P3 for a Cu-QA of the curve QA-Cu1.
The last figure seems far more interesting, but I can't
understand it !
I tried to read Bernard Gibert's paper, specially the part 3.
X, Xa, Xb and Xc should have the same tangential, as well as Y,
Ya, Yb and Yc but this isn't the case on your figure.
XaXbXc should be perspective to QA-Tr2,with perspector X*, as
well as YaYbYc with perspector Y* which seems true, but XX*
should be parallel to YY*, both parallel to the asymptote and
that doesn't seem true ...
The only thing which feets is Tf(Z)* = tangential of Z !
I've spent a long time on your figure, but I give up. Could you
please explain a little more the construction ?
Thanks in advance, the property seems to be worth a little
effort ...
Best regards
Bernard
PS 1) I suppose this could lead to a Desmic system
   2) This applies to QL-Cu1 bicursal
```

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Message: #1966
Date: 01/10/2016 12:58:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu1-Invariant Transformation

```
Dear Bernard,
sorry there is a mistake in my #1962!
Replace the line
"... The triangles XaXbXc and YaYbYc are perspective."
     by
"... The diagonal triangles of XXaXbXc and YYaYbYc are
perspective."
Thanks for your precise review!
Best regards Eckart
```

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Message: #1967
Date: 01/10/2016 6:44:45
From: bernard.keizer
Subject: QA-Cu1-Invariant Transformation

Dear Eckart,
Thanks a lot for your quick answer : in fact, I understand better now !
But if I'm not wrong, it is not Bernard Gibert's construction. For any point X on the curve, you defined a Cu-QA X, Xa, Xb and Xc and XaXbXc is perspective with QA-Tr2 with perspector X*, the isogonal conjugate of X wrt QA-Tr2 (this appears clearly on your 1rst figur). For QA-P4, the corresponding triangle is DT, which is in perspective with QA-Tr2 with perspector the isogonal conjugate QA-P3 of QA-P4 wrt QA-Tr2
If X, Y and Z are on a line, so are their tangentials and the 3 QA's form a Desmic system (the line XYZ is one of the 16 lines of the system). In particular, the triangles XaXbXc and YaYbYc are in perspective with perspector Z (and the same way XaXbXc and ZaZbZc with perspector Y and YaYbYc and ZaZbZc with perspector X). (Bernard Gibert's construction)
If 3 QA's form a Desmic system, the QA's formed by the 3 tangentials and the corresponding DT's form also a Desmic system (the line of the tangentials being one of the 16 lines of the system). In particular, the DT's for X and Y are in perspective with perspector the tangential of Z (your construction in the 2nd figur).
Best regards
Bernard

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Message: #1968
Date: 02/10/2016 5:02:49
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral Geom

---

```
Dear all,
I posted the APG message #3275 in May
- Generalized Cyclologic Center Chain
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver
sations/messages/3275  †)
I got no reply but I think it is true.
I wonder its application to quadrangle/quadrilateral
geometry.  QA-Tr-4: Cyclologic pairs of Triple Triangles
summarizes a few examples of cyclologic centers.
We can extend them into cyclologic center chains.
Any idea?
Best regards,
Tsihong Lau
```

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Message: #1969
Date: 02/10/2016 9:07:59
From: eckart_schmidt@t-online.de
Subject: QA-Cu1-Invariant Transformation

```
Dear Bernard,
you are right, Bernard Gibert takes the perspectivity for XaXbXc
and YaYbYc in the 3rd intersection Z of XY and QA-Cu1, I take
the perspectivity for the diagonal triangles of XXaXbXc and
YYaYbYc in Tf(Z)*.
Best regards Eckart
```

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #1970
Date: 02/10/2016 10:07:16
From: eckart_schmidt@t-online.de
Subject: Dual Conics in the QA/QL-constellation

Dear Bernard,
I´m not sure, whether the conics in the attached file are already discussed, but perhaps their centers are new.
Best regards Eckart
PS. Excuse, that I didn´t answer your messages #1961, #1963, #1964, there was no time, I shall make up leeway.

EQF-Note 2016-10-02

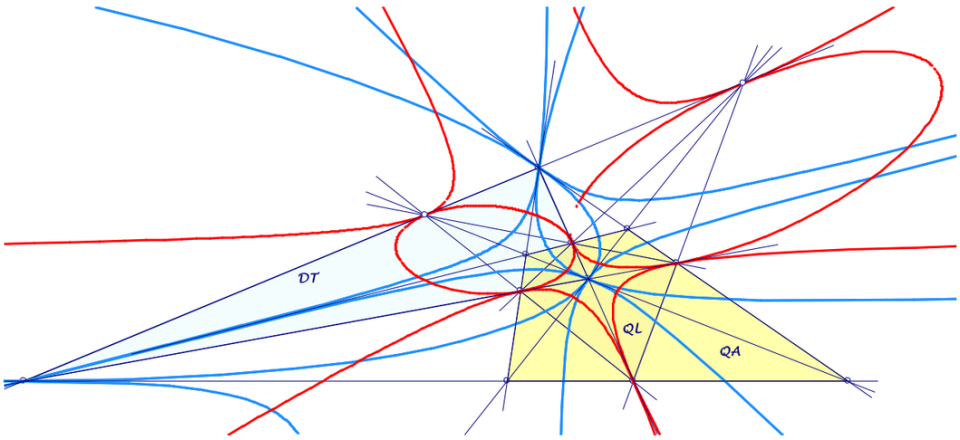
Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Dual Conics in the QA/QL-constellation

In addition to EQF-Note 2016-09-22 here are described 2 x 4 dual conics in the QA/QL-constellation, using point and line isoconjugations:

(1) Conics inscribed the diagonal triangle DT and circumscribed a QL-triangle.

(2) Conics circumscribed the diagonal triangle DT and inscribed a QA-triangle.



Starting with a quadrangle *QA* and its diagonal triangle *DT*, the *DT*-trilinear polars of the *QA*-vertices give the dual *QL* with the same diagonal triangle.

- **The *DT*-trilinear polars of points on a *QL*-line envelope a conic of type (1).**
- **The line isoconjugation *QL-Tf2* for the line pencil of a *QA*-vertex gives a conic of type (1).**
- **The isoconjugate of a *QL*-line wrt an isoconjugation for the remaining *QL*-trilateral with fixed point in the corresponding *QA*-vertex is a conic of type (1).**

The dual interpretation:

- **The *DT*-trilinear poles of lines through a *QA*-vertex give a conic of type (2).**

- **The isoconjugation *QA-Tf2* for a *QL*-line gives a conic of type (2).**

- **The line isoconjugate of the line pencil of a *QA*-vertex wrt a line isoconjugation for the remaining *QA*-triangle with fixed line in the corresponding *QL*-line gives a conic of type (2).**

Conclusion:

- **The conics of type (1) are the duals of the conics of type (2).**

Additional remarks:

- **The centers of the four conics of type (1) lie on a *DT*-circumscribed conic through *QL-P8, QL-P13, QL-P24.***

- **The centers of the four conics of type (1) are the duals of parallels to the *QL*-lines two thirds distance to the corresponding *QA*-vertex.**

- **The centers of the four conics of type (2) are the duals of parallels to the *QL*-lines one third distance to the corresponding *QA*-vertex.**

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2016-10-02.pdf

Message: #1971
Date: 03/10/2016 9:29:25
From: eckart_schmidt@t-online.de
Subject: QA-Tr4

```
Dear Chris,

here are some centers for (Quadri-)Cyclologic QA-Triple
Triangles:
... wrt QG-P1 and QG-P16: center 2 = QA-Tf2
                          of the intersection of QA-Cu1 and its
                          asymptote,
... wrt QG-P1 and QG-P18: center 1 = QA-P2,
... wrt QG-P2 and QL-P1:  center 1 = QA-Tr2-isogonal conjugate
                          of QA-P41,
... wrt QG-P4 and QG-P8:  center 2 divides QA-P1.QA-P2
                          with ratio 1:2,
... wrt QG-P9 and QL-P1:  center 2 = QA-P9,
... wrt QG-P9 and QL-P4:  center 1 on QA-P1.QA-P4,
... wrt QG-P16 and QL-P1: center 2 = third intersection
                          of QA-Cu1
... ... and the line through QA-P3 and the intersection
        of QA-Cu1 and its asymptote,
... wrt QL-P1 and  QL-P17: center 1 on circumcircle
    of the Miquel triangle.

Best regards Eckart
```

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Message: #1972
Date: 03/10/2016 12:44:38
From: chris.vantienhoven
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

---

Dear Tsihong Lau,
I like the concept of the Generalized Cyclologic Center Chain.
I made a picture of it myself with the QA-Triple Triangle TT1 of QL-P1 wrt all QA-Component Triangles.
Let TT1 be the Triple Triangle with vertices:
QL-P1_2 = QL-P1 of P1.P3.P2.P4
QL-P1_3 = QL-P1 of P1.P2.P3.P4
QL-P1_4 = QL-P1 of P1.P2.P4.P3
Let CT1 be Component Triangle P2.P3.P4.
TT1 is cyclologic wrt CT1 giving Cyclologic Centers P'=QA-P4 (common for all Component Triangles) and a 2nd unknown center U'.
According to your conjecture
Conics:
(P2,P3,QL-P1_4,P,U),
(P3,P4,QL-P1_2,P,U),
(P4,P2,QL-P1_3,P,U) have a common point P'' being QA-vertex P1.
Conics:
(QL-P1_P2,QL-P1_P3,P4,P,U),
(QL-P1_P3,QL-P1_P4,P2,P,U),
(QL-P1_P4,QL-P1_P2,P3,P,U) have a common point U'' being QA-P3.
For the other Component Triangles we find corresponding points P'' being QA-vertices P2,P3,P4 and U'' being the common point QA-P3.
It appears QA-P4, QA-P3 and the QA-vertices have a special role in the initial steps of your chain for the QL-P1 Triple Triangle.
Best regards, Chris

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Message: #1973
Date: 03/10/2016 12:48:24
From: chris.vantienhoven
Subject: QA-Tr4

Dear Eckart,
Thanks, I will mention them in EQF at QA-Tr-4.
Best regards, Chris

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Message: #1974
Date: 03/10/2016 2:47:31
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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```
Dear Chris,
Thanks for your reply.
I give the important problems to be solved.
1. Prove the generalized cyclologic center chain.
2. How many reverse generalized cyclologic centers
are there?  In other words, how many P U pairs
correspond to the P' U' pair?  There should be at least
two since P' U' are interchangeable.
3. Given two triangles in nondegenerate position, what
is the locus of P U pair such that the two triangles are
generalized cyclologic?
Best regards,
Tsihong Lau
```

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Message: #1975
Date: 03/10/2016 3:20:28
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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```
Dear Chris,
the unknown center U´ in #1972 is the QA-Tr2-isogonal conjugate
of QA-vertex P1.
Best regards Eckart
```

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Message: #1976
Date: 03/10/2016 8:58:47
From: eckart_schmidt@t-online.de
Subject: QA-Tr4

```
Dear Chris,
your example in #1972 can also be seen under a general aspect:
Let ABC be the Miquel triangle
... and A', B', C' three points on QA-Cu1
    with common tangential T,
... then ABC and A'B'C' are cyclologic
... with center 1 = T
...  and center 2 = isogonal conjugate of the 4th point
    with tangential T.
Example: T = QA-P41, A'B'C' = QA-Tr1, 4th point QA-P4 with
isogonal conjugate QA-P3.
Best regards Eckart
```

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Message: #1977
Date: 04/10/2016 7:44:47
From: chris.vantienhoven
Subject: QA-Tr4

```
Dear Eckart,
It seems at the end somehow it all fits together :)
Best regards,
Chris
```

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Message: #1978
Date: 04/10/2016 9:55:24
From: bernard.keizer
Subject: QA/QL's QA-Px and QL-Px QAs

Dear Chris, dear Eckart
Sometimes you deal with QA's QL-Px triangles named QA-Tr-1 in EQF (examples Miquel triangle, Morley triangle or QL-P12 triangle) and sometimes with QL's QA-Px triangles not named in EQF (example Gergonne-Steiner triangle inscribed in the Dimidium circle).
If we consider a QA/QL (dual figurs with same DT), we could deal directly with QA's or QL's formed by the same point or line in the QA and in the 3 QL's QAs or in the QL and in the 3 QA's QLs. All the points of these QAs and QLs will be points of the QA/QL ...
Best regards
Bernard

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Message: #1979
Date: 04/10/2016 4:20:49
From: tsihonglau
Subject: QA/QL's QA-Px and QL-Px QAs

Dear Bernard,
What you propose is the same as my fisrt QFG message
#1457 - Merge Quadrangle/Quadrilateral Figures.
Here is more information. Please refer to my page -
Duality of Triangle/Trilateral and Quadrangle/Quadrilateral (
http://lth.name/geometry/duality.html)
and message #1892 - Expoint Quadrangle/Exline Quadrilateral
The diagonal trilaterals of expoint quadrangles E,EA,EB,EC are
exline trilaterals t,ta,tb,tc respectively.
The diapleural triangles of exline quadrilaterals e,ea,eb,ec are
expoint triangles T,TA,TB,TC respectively.
Although expoint triangles and exline trilaterals are not dual
and expoint quadrangles and exline quadrilaterals are not dual.
Expoint quadrangle E is called CT-coordinates in EQF.
Exline quadrilateral e is called CT-coordinates in EQF.
The diapleural triangles of diagonal quadrangles Qa,Qb,Qc
are diagonal triangles Ta,Tb,Tc respectively.
The diagonal trilaterals of diapleural quadrilateral qA,qB,qC
are diapleural trilaterals tA,tB,tC respectively.
Diagonal triangles and diapleural trilaterals are dual although
diagonal quadrangle s and diapleural quadrilaterals are not.
Diagonal quadrangles are called QA's QLs in EQF.
Diapleural quadrilaterals are called QL's QAs in EQF.
I dislike the concept of quadrigon. It should be replaced with
quadrangle/quadrilateral.
Best regards.
Tsihong Lau

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Message: #1980
Date: 04/10/2016 5:16:32
From: chris.vantienhoven
Subject: QA/QL's QA-Px and QL-Px QAs

Dear Bernard, Tsihong Lau and Eckart,
First of all it is quite true that when we consider a dual QA/QL-configuration we can deal directly with QA's or QL's.
It's all about perspective: "what do we observe from which point of view". And then: "which figure you choose to make your reference figure".
There can be reasons to choose a QA as reference-QA or to choose a QL as a reference-QL.
Just like we can choose in a cartesian coordinate system to replace the origin and the x-/y-axes. We do so when it is more convenient and when it gives easier calculations.
When our basic figure is composed of 4 lines we naturally will choose for a reference-QL and consequently get symmetric QL-coordinates. When in this case we would have chosen for a reference-QA the calculations would be much more complicated and the results would not be symmetric. Of course you can say there is a dual QA-figure, but to what end?
Corresponding statements can be said when our basic figure is composed of 4 points.
Nevertheless it can be interesting to consider the dual QA/QL-configuration like we did before in earlier messages. It helps us to understand the dual properties and not the specific QA-properties or QL-properties.
About the Quadrigon that's quite another subject. In a Quadrangle or a Quadrilateral we deal with 4 random objects (points or lines). In a Quadrigon we connect these points/lines in a fixed order and this creates quite another field of geometry. A Quadrigon is actually another level of Quadri objects.
>From a system of 4 random points 3 cyclic orders can be made up and so a Quadrangle has 3 Component Quadrigons. Correspondingly from a system of 4 random lines 3 cyclic orders can be made up and so a Quadrilateral also has 3 Component Quadrigons.
So Quadrilaterals and Quadrangles meet at the Quadrigon level just as Quadrilaterals and Quadrangles do meet in a dual QA/QL-configuration. We can choose which relationships are relevant and do our reasoning and calculations in accordance.
It's all about perspective and finally it all fits together in a mysterious and very consistent way.
I am open for all angles of view.
Best regards, Chris

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Message: #1981
Date: 04/10/2016 8:16:34
From: bernard.keizer
Subject: Dual Conics in the QA/QL-constellation

```
Dear Eckart,
Thanks for these new conics, I will add them to my collection of
dual and self dual conics.
Best regards
Bernard
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Message: #1982
Date: 07/10/2016 4:54:14
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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Dear all,
Another chain related is found and not proved yet.
Clifford circle theorem states that four circles q,qa,qb,qc (conics passing through the circular points P,U) concur at the point P'.
The six points Sa,Sb,Sc,Sbc,Sca,Sab are the fourth points of intersection of the following pairs of circles:
   Sa - q,qa
   Sb - q,qb
   Sc - q,qc
   Sbc - qb,qc
   Sca - qc,qa
   Sab - qa,qb
Then the four circumcircles
   q' - Sbc,Sca,Sab
   q'a - Sbc,Sb,Sc
   q'b - Sa,Sca,Sc
   q'c - Sa,Sb,Sab
concur at the point U'.
Then the four circumconics
   r - Sa,Sb,Sc,P',U'
   ra - Sa,Sca,Sab,P',U'
   rb - Sbc,Sb,Sab,P',U'
   rc - Sbc,Sca,Sc,P',U'
concur at the point P''.
Then the four circumconics
   r' - Sbc,Sca,Sab,P',U'
   r'a - Sbc,Sb,Sc,P',U'
   r'b - Sa,Sca,Sc,P',U'
   r'c - Sa,Sb,Sab,P',U'
concur at the point U''.
In a similar way, we can construct an infinite generalized Clifford point of concurrency chain - P,U -> P',U' -> P'',U'' -> P''',U''' -> ...
I think the two chains are valid on a Möbius plane.
Möbius plane - Wikipedia, the free encyclopedia (https://en.wikipedia.org/wiki/M%C3%B6bius_plane)
I wonder their application to quadrangle/quadrilateral geometry.
Any idea?
Best regards,
Tsihong Lau

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Message: #1983
Date: 08/10/2016 11:29:28
From: eckart_schmidt@t-online.de
Subject: QA-Tr-4

Dear Chris,
attached there are corrections (also wrt my results) and additions for the table in QA-Tr-4 (in red). Please control!
Cyclologic QA-Triple Triangles can be perspective, so I have added a further column.
If QG-points are symmetrical wrt QA-P1, also their QA-Triple Triangles and their cyclologic centers, which are the 4th intersections of their circumcircles with the conic through their six vertices.
Sometimes the perspector is a cyclologic center, perhaps of interest the last line .
Best regards Eckart
PS. Are there informations, whether the table is complete wrt the EQF-points up to now?

Triple Triangle-1 formed by 3 QA-versions of:	**Triple Triangle-2** formed by 3 QA-versions of:	**Cyclologic Center-1**	**Cyclologic Center-2**	**Persp. Center of Tr1 Tr2 ^^)**
QG-P1	QG-P15	QA-P2	QA-P3	QA-P1
QG-P1	QG-P16	3rd intersection of QA-Cu1 and the line through QA-P3 and the intersection of QA-Cu1 and its asymptote	QA-Tf2(QA-Cu1 ^ asymptote) *)	infinity point of QA-Cu1-asymptote
QG-P1	QG-P18	QA-P2 *)	QA-P41	QA-Tf2(X)
QG-P1	QG-P19	QA-P2	QA-P4	X on a QA-L2-parallel through QA-P12
QG-P1	QL-P1	QA-P3	QA-P41	QA-P3
QG-P1	QL-P17	QA-Px	QA-P3	no
QG-P2	QL-P1	QA-P41*	QA-P1	no
QG-P3	QL-P17	QA-Px	QA-Px	no
QG-P4	QG-P8	QA-P34	point on QA-P1.QA-P2 with ratio 1:2 *)	QA-P1
QG-P5	QG-P10	QA-Px	QA-Px	QA-P1
QG-P5	QL-P1	QA-P3	QA-P9	yes, but ???
QG-P5	QL-P4	QA-Px	QA-Px	no
QG-P6	QL-P21	QA-Px	QA-Px	no
QG-P7	QG-P9	QA-Px	QA-Px	QA-P1
QG-P9	QL-P1	QA-P1	QA-P9 *)	no
QG-P9	QL-P4	point on QA-P1.QA-P4 *)	QA-Px	no
QG-P9	QL-P5	QA-Px	QA-Px	no
QG-P12	QL-P1	QA-Px	QA-Px	no
QG-P16	QL-P1	QA-P4	tangential of QA-P3 wrt QA-Cu1	QA-P4
QG-P18	QG-P19	QA-P41	QA-P4	on X.QA-Tf2(X)
QL-P1	QL-P17	point on circumcircle of QA-Tr2 *)	QA-P3	no
QL-P1	all Component Triangles (QA-4Tr1)	common Quadri-Cyclologic point QA-P4	4 different ^) Quadri-Cyclologic points	^)

QA-Px is a QA-point not registered in EQF.
*) these points were identified by Eckart Schmidt. See Ref-34, QFG#1971.

^) perspective centers of QA-Tr2 and the QA-component triangles: These points lie on QA-Cu1 with common tangential in the 3rd intersection of QA-Cu1 and the line through QA-P4 and the intersection of QA-Cu1 and its asymptote.

^^) If the perspector is QA-P1, the Triple Triangles are symmetric wrt QA-P1, also their cyclologic centers, which are the 4th intersections of the triangle circumcircles and the conic through their 6 vertices.

2016-10-06.docx

Message: #1984
Date: 09/10/2016 2:28:09
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in Quadrang ...

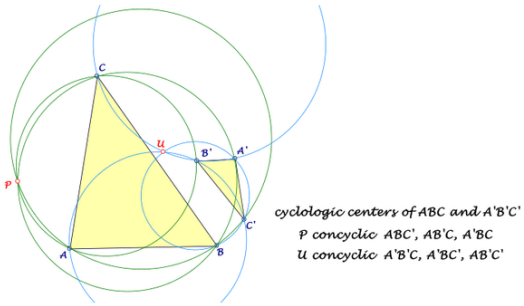
Dear Tsihong Lau, dear Chris,
wrt "generalized cyclologic center chains" described in APN #3275:
Attached some remarks for the starting constellation of two cyclologic triangles and their cyclologic centers. Background are cyclologic QA-triple triangles in QA-Tr-4.
Best regards Eckart

EQF-Note 2016-10-09

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Generalized Cyclologic centers

This is an idea of Tsihong Lau
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/3275
here reduced to a starting constellation of two cyclologic triangles and their cyclologic centers. Background are cyclologic QA-triple triangles (see QA-Tr-4).

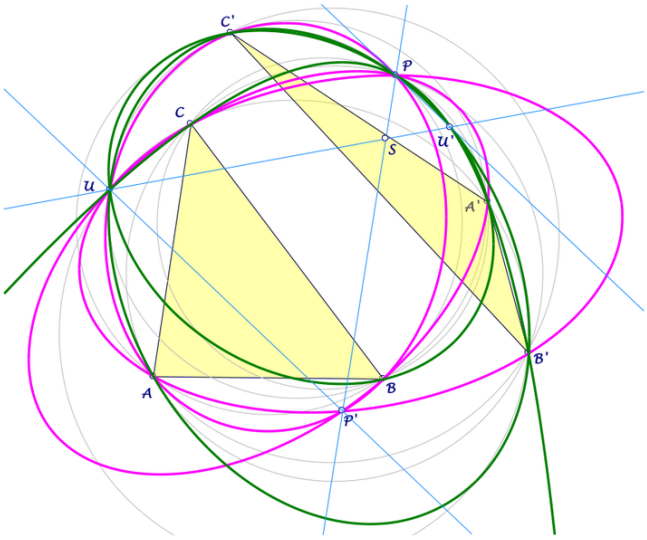


Let *ABC* and *A´B´C´* be two cyclologic triangles with centers *P* and *U* as in the figure above.
Tsihong Lau consideres **6 conics**
… circumscribed *ABC´, AB´C, A´BC, A´B´C, A´BC´, AB´C´*,
… through *P* and *U*
… with a further intersection in *P´* for the first 3 conics
… and a further intersection in *U´* for the last 3 conics.
The points *P´* and *U´* shall here be named the first generation of generalized cyclologic centers.
For triangles, which are symmetric wrt a point and therefore cyclologic, the first generation of generalized cyclologic centers doesn´t exist. The six conics are identical.
For the cyclologic centers *P*, *U* and the generalized cyclologic centers *P´*, *U´* of the first generation holds

- ***PU´* and *P´U* are parallel.**
- **The Möbius transformation centered in *PP´∩UU´* swapping *P* and *U* swaps also *P´* and *U´*.**

Normally this cannot be generalized for further generations. But let us consider cyclologic *QA*-triple triangles as described in *QA-Tr-4*. For *QA-P1*-symmetric pairs *QG-P1/QG-P15, QG-P4/QG-P8, QG-P5/QG-P10, QG-P7/QG-P9* the *QA*-triple

triangles as well as their cyclologic centers are symmetric wrt *QA-P1*, so there are no generalized cyclologic centers.

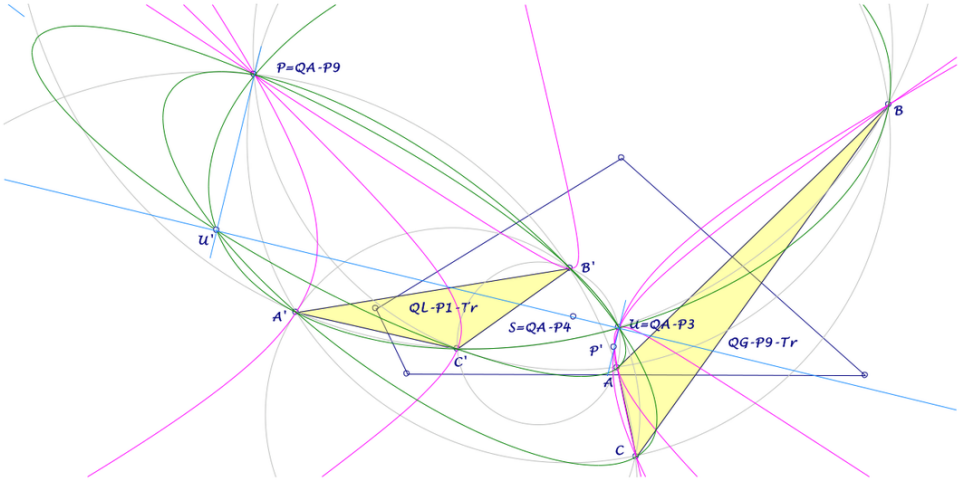


Example 1: Let
… *ABC* be the Miquel triangle *QA-Tr2*,
… *A´B´C´* a component triangle $P_iP_jP_k$
… with *P* = perspector of $P_iP_jP_k$ and *QA-Tr2*, *U* = *QA-P4*,
… … *P´* = *QA-P3*, *U´* = P_l.

Example 2: Let wrt ***QG-P1/QL-P1***
… *ABC* be the diagonal triangle *QA-Tr1*,
… *A´B´C´* the Miquel triangle *QA-Tr2*
… with *P* = *QA-P41*, *U* = *QA-P3*, *P´* = *QA-P4*, *U´* = *QA-P41** (*QA-Tr2*-isogonal conjugate *).

Example 3: Let wrt ***QG-P5/QL-P1***
… *ABC* be the *QG-P5*-triangle,
… *A´B´C´* the Miquel triangle *QA-Tr2*
… with *P* = *QA-P9*, *U* = *QA-P3*, *S* = *QA-P4*,
… … *P´* = intersection of *QA-P4.QA-P9* and a perpendicular to *QA-P3.QA-P4* in *QA-P3*,
… … *U´* = intersection of *QA-P4.QA-P3* and a perpendicular through *QA-P9*.

- **For the cyclologic *QA*-triple triangles of *QG-P5* and *QL-P1* the transformation *QA-Tf4* swaps the cyclologic centers *P* = *QA-P9* and *U* = *QA-P3* as well as *P´* and *U´*, *P´´* and *U´´*, …**



There are other examples, that the Möbius transformation swaps also the generalized cyclologic centers of further generations:

- **For the cyclologic *QA*-triple triangles wrt**
  QG-P1/QG-P18, QG-P1/QG-P19, QG-P1/QL-P1, QG-P1/QL-P17, QG-P5/QL-P1, QG-P5/QL-P4, QG-P9/QL-P1, QG-P9/QL-P4, QG-P18/QG-P19, QL-P1/QL-P17
  the Möbius transformation, centered in *PP´∩UU´* and swapping the cyclologic centers *P, U,* swaps not only the generalized cyclologic centers *P´, U´* of the first generation, but also those of further generations.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-10-09.pdf

Message: #1985
Date: 10/10/2016 6:31:03
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

---

Dear Eckart,
Wonderful results! But I am not familiar with Möbius transformation.
So I focus another result- PU´ and P´U are parallel.
I use the same notations for two chains.
Start points P0,U0-
There are eight circumconics p1,p1a,p1b,p1c and u1,u1a,u1b,u1c of the two triangles PaPbPc and UaUbUc and P0,U0:

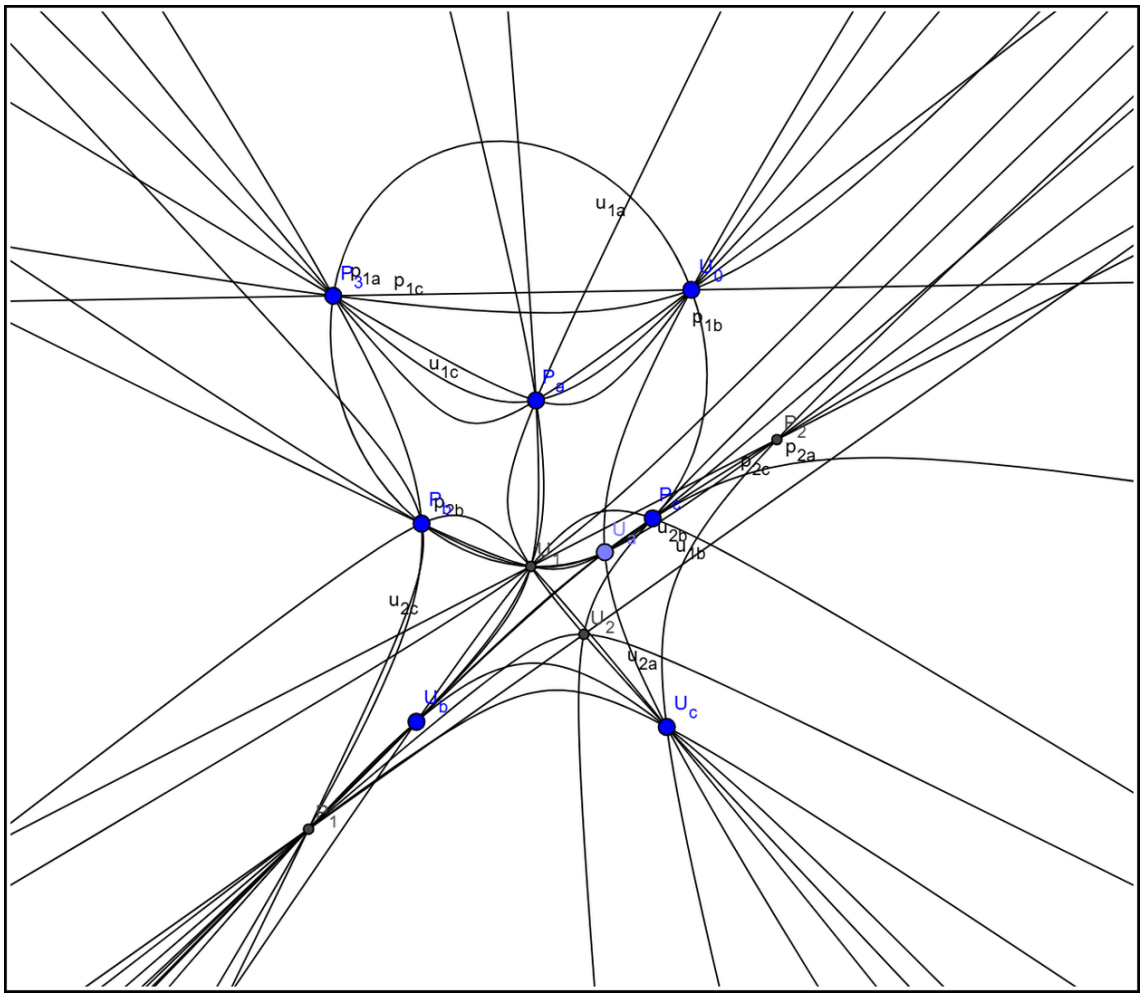
```
   p1  - Pa,Pb,Pc,P0,U0
   p1a - Pa,Ub,Uc,P0,U0
   p1b - Ua,Pb,Uc,P0,U0
   p1c - Ua,Ub,Pc,P0,U0
   u1  - Ua,Ub,Uc,P0,U0
   u1a - Ua,Pb,Pc,P0,U0
   u1b - Pa,Ub,Pc,P0,U0
   u1c - Pa,Pb,Uc,P0,U0
```

The two triangles P aPbPc and UaUbUc are generalized cyclologic if the three conics p1a,p1b,p1c concur at P1. If so, the three conics u1a,u1b,u1c concur at U1.
The two triangles PaPbPc and UaUbUc are generalized Clifford if the four conics p1,p1a,p1b,p1c concur at P1. If so, the four conics u1,u1a,u1b,u1c concur at U1. P1 and U1 are called the cyclologic centers of the two triangles PaPbPc and UaUbUc.
First generation P1,U1-
There are eight circumconics p2,p2a,p2b,p2c and u2,u2a,u2b,u2c of the two triangles PaPbPc and UaUbUc and P1,U1 :

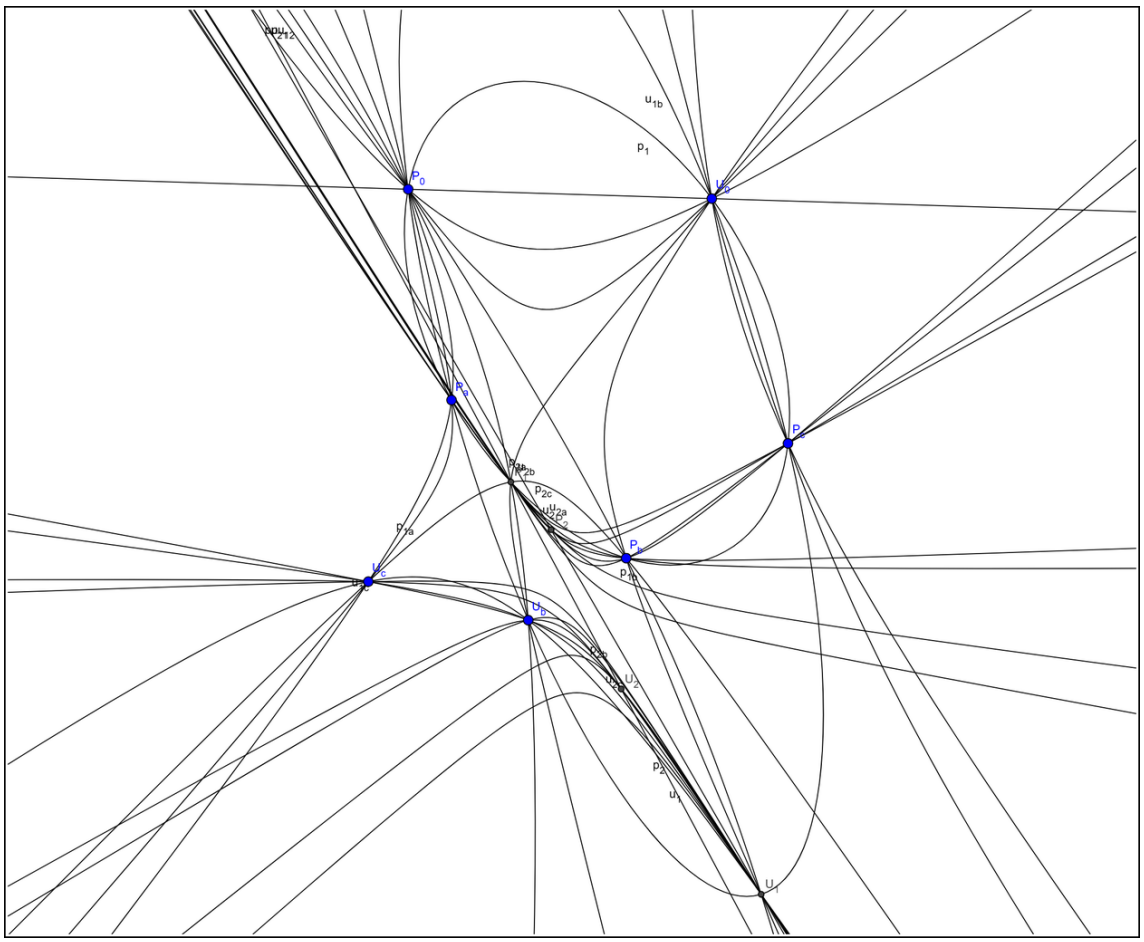
```
    p2  - Pa,Pb,Pc,P1,U1
    p2a - Pa,Ub,Uc,P1,U1
    p2b - Ua,Pb,Uc,P1,U1
    p2c - Ua,Ub,Pc,P1,U1
    u2  - Ua,Ub,Uc,P1,U1
    u2a - Ua,Pb,Pc,P1,U1
    u2b - Pa,Ub,Pc,P1,U1
    u2c - Pa,Pb,Uc,P1,U1
```

If the two triangles PaPbPc and UaUbUc are generalized cyclologic, the three conics p2a,p2b,p2c concur at P2 and the three conics u2a,u2b,u2c concur at U2.
If the two triangles PaPbPc and UaUbUc are generalized Clifford, the four conics p2,p2a,p2b,p2c concur at P2 and the four conics u2,u2a,u2b,u2c concur at U2.
Second generation P2,U2- and so on...
We can construct an infinite generalized cyclologic center chain- P0,U0 -> P1,U1 -> P2,U2 -> P3,U3 -> ...

Your result states that the three lines P0U0,P1U2 P2U1 are concurrent. The result can be applied to next generations obviously. I think the Möbius transformation can be applied to next generations, too.
The first result answer partly the problem I raised.
3. Given two triangles in nondegenerate position, what is the locus of P U pair such that the two triangles are generalized cyclologic?
How about the two other problems?
You adapted the title of message, so your reply belongs to another topic!
Best regards,
Tsihong Lau



cyclologic_generalized.ggb



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Message: #1986
Date: 11/10/2016 9:15:40
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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Dear Tsihong Lau,
thanks for your detailed answer. Sorry I can´t open the attached files, but I have reproduced your construction in the general case. I haven´t seen the conclusion of my result for the general case.
Is the following property evident?
If two triangles are generalized cyclologic wrt Po and Uo, they are also generalized cyclologic
... wrt Po´ = Po and Uo´ = P1 with P1´ = Uo and U1´ on the circumconic UaUbUcPoU1.
Analog:
... wrt Po´ = U1 and Uo´ = Uo with U1´ = Po and P1´ on the circumconic PaPbPcP1Uo.
This can be generalized to further generations.
Best regards Eckart

PS. The application of Möbius transformations holds only for the special case, that Uo and Po are the cyclologic centers.

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Message: #1987
Date: 11/10/2016 4:59:39
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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Dear Eckart,

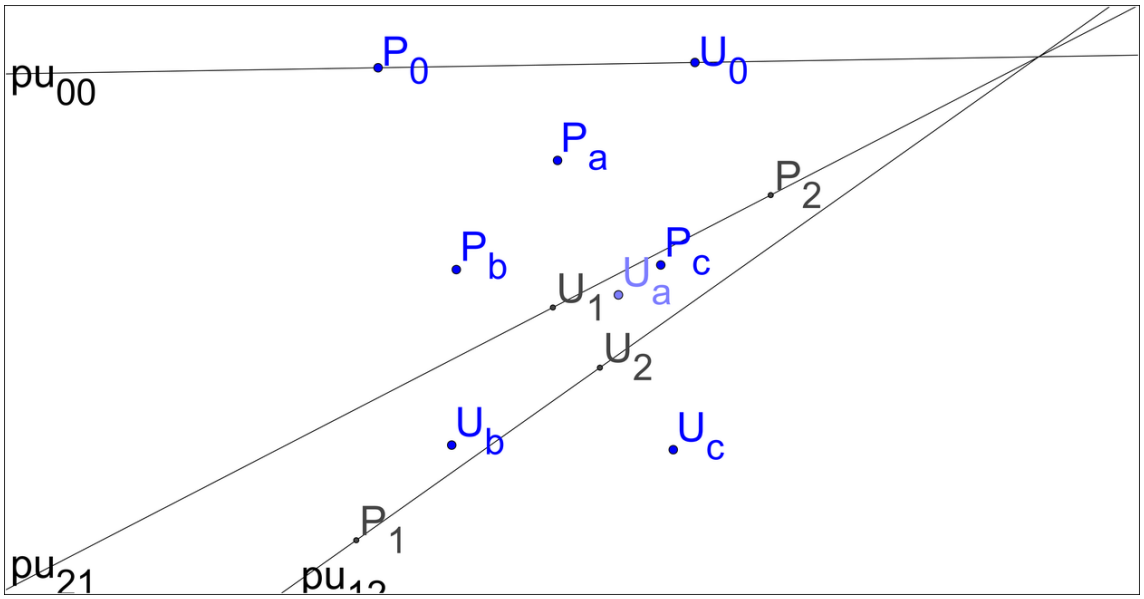
>>---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>...
>>thanks for your detailed answer. Sorry I can´t open the attached files, but I have reproduced your
>>construction in the general case. I haven´t seen the conclusion of my result for the general case.
>>...
>>PS. The application of Möbius transformations holds only for the special case, that Uo and Po are the
>>cyclologic centers.

Please refer to the attachment file. P0U0, P1U2, P2U1 are concurrent indeed. Sorry!
P3 should be P0 in a previous attachment file. Indeed, P0U0, P1U1, P2U2 and so on are generalized circular points.
What are circular points? Any pair of points in nondegenerate position are! What are generalized circles? Any conics passing through generalized circular points are! What is the generalized line at infinity? The line passing through generalized circular points is! What is a generalized parallel pair of lines?
The pair and the generalized line at infinity are concurrent.
 In other words, we can define all geometrical terminologies with circular points.(That is why I invent circular coordinate system) So you may
ignore my construction. If your results are true, my construction is also true. The converse holds!
My construction just shows generalized pairs of circular points.
 Your results:
1. PU´ and P´U are parallel.
2. The Möbius transformation centered in PP´∩UU´ swapping P and U swaps also P´ and U´.
If 1 holds, 2 holds, too. My construction shows P0U0, P1U2, P2U1 are concurrent indeed. So 2 holds. The Möbius transformation can be defined using circular points. But it is somewhat complicated.
Maybe I give a try!

>>Is the following property evident?
>>If two triangles are generalized cyclologic wrt Po and Uo, they are also generalized cyclologic

>>... wrt Po´ = Po and Uo´ = P1 with P1´ = Uo and U1´ on the circumconic UaUbUcPoU1.
>>Analog:
>>... wrt Po´ = U1 and Uo´ = Uo with U1´ = Po and P1´ on the circumconic PaPbPcP1Uo.
>>This can be generalized to further generations.
Wonderful results! I found it leads to an infinite chain of generalized pairs of circular points! I will summarize my results and post later!
P.S. The most important problem- Prove the cyclologic center chain. I tried to prove it brute force with Maxima but failed!

Best regards,
Tsihong Lau



cyclologic_generalized.png

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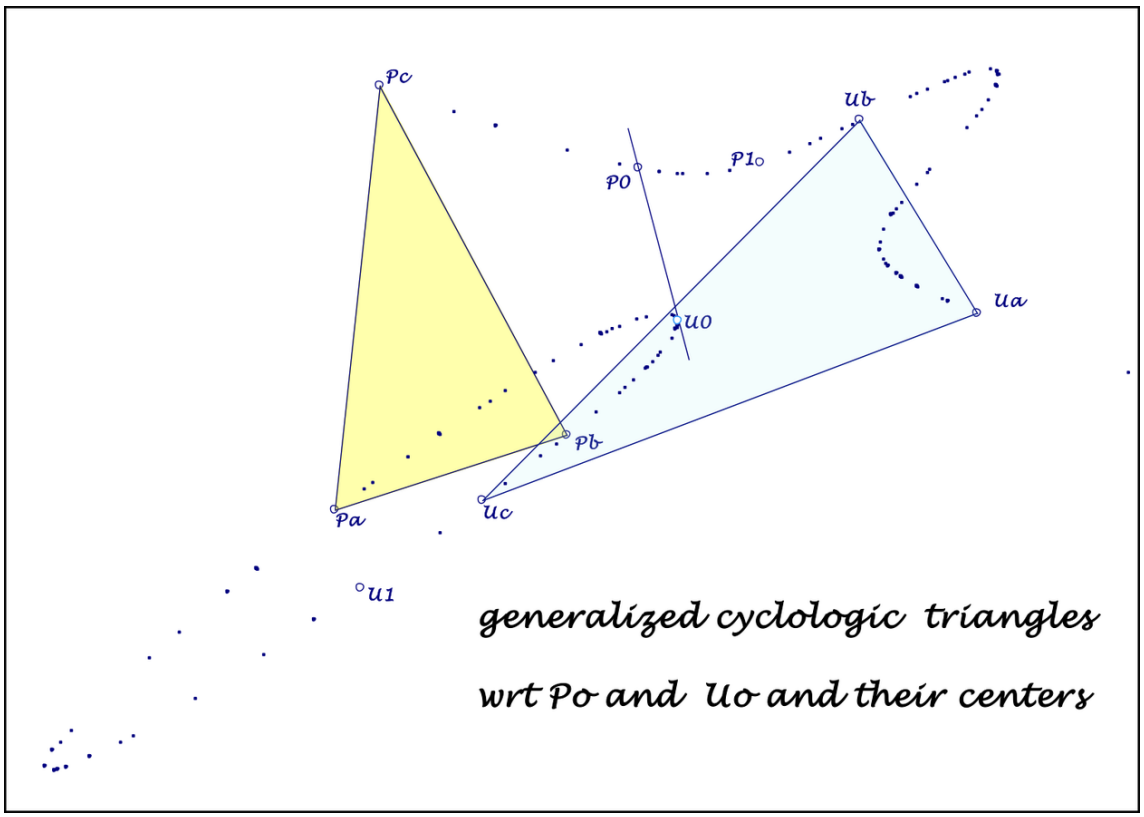
Message: #1988
Date: 12/10/2016 9:21:00
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in ...

Dear Tsihong Lau, dear Chris,
Cabri observations (see attached file) lead to the following theorem, using the nomination of #1985:
Two triangles - generalized cyclologic wrt P0 and U0 -
... are generalized cyclologic wrt *any two* points of a cubic
... through the vertices of the triangles and P0, U0, P1, U1.
Examples in QA-geometry (see QA-Tr-4):
The Triple Triangles of QG-P1 and QL-P1 (QA-Diagonal and Miquel Triangle) are cyclologic whith center P0 = QA-P3 and U0 = QA-P41, P1 = QA-P4 and U1 = QA-Tr2-isogonal conjugate of QA-P41.
They are generalized cyclologic wrt any two points of QA-Cu1.
Also generalized cyclologic wrt any two points on QA-Cu1 are
... the Triple Triangles of QG-P1 and QG-P16,
... the Triple Triangles of QG-P16 and QL-P1,
... the Triple Triangles of QL-P1 and QG-P1.
Best regards Eckart



2016-10-12.pdf

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Message: #1989
Date: 13/10/2016 11:07:32
From: bernard.keizer
Subject: Generalized Cyclologic Center Chain in ...

Dear Eckart, dear Chris, dear Tsihong Lau,
I hardly but with greatest interest try to follow the developments of these generalized cyclologic triangles.
The mentionned property is very beautiful !
I only notice that the given example is a property of a Desmic system on QA-Cu1.
The 3 Cu-QA's formed by QA-P3, QA-P4 and the infinity point of QA-P3QA-P4 and their 3 CSC conjugates, respectively QG-P16 (isogonal conjugates of vertices of DT), vertices of DT and vertices of the Miquel triangle form a Desmic system.
QA-P3, QA-P4 and the infinity point are aligned.
QA-P3 is the perspector between the vertices of DT and Miquel triangles (well known property), QA-P4 is the perspector of the triangle of the 3 QG-P16 and the Miquel triangle and the infinity point is the perspector between the vertices of DT and the QG-P16 (which are their isogonal conjugates).
The tangentials of the 3 QA's are also in line : the tangential of QA-P3 is the 3rd intersection on QA-Cu1 of the line joining QA-P41 and the intersection of the curve with it's asymptote.
Best regards
Bernard.

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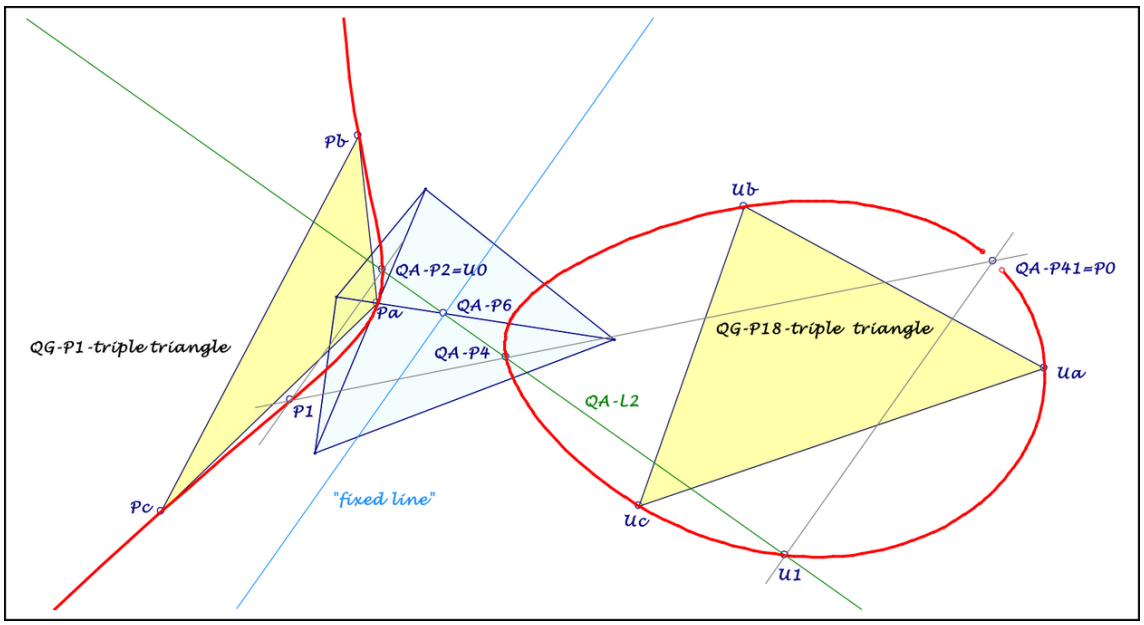
Message: #1990
Date: 13/10/2016 8:48:15
From: eckart_schmidt@t-online.de
Subject: Cubics for Cyclologic QA-Triple Triangles

Dear Tsihong Lau, dear Bernard, dear Chris,
cyclologic QA-triple triangles, not perspective QA-P1, have cubics for their generalized cyclologic centers (see #1984, #1988).
Here an interesting example
... for the QA-triple triangles of QG-P1 and QG-P18
    (see attached file)
... with cyclologic centers P0 = QA-P41 and U0 = QA-P2,
... with the generalized cyclologic centers
    of the 1st generation
... ... P1 = intersection of QA-P4.QA-P41
        and a perpendicular to QA-L2 in QA-P2,
... ... U1 = intersection of QA-L2
        and a perpendicular through QA-P41.
... The cubic for the generalized cyclologic centers
... ... bears the vertices of the QA-triple triangles
        and QA-P2, QA-P4, QA-P41,
... ... is invariant wrt QA-Tf2,
... ... is isogonal invariant wrt the QA-triple triangle
        of QG-P18,
... ... is a non-pivotal isogonal circular cubic
        for the QA-triple triangle of QG-P18
... ... with a perpendicular to QA-L2 in QA-P6
            as degenerated "fixed circle",
... ... with an asymptote orthogonal QA-L2 through the
            reflection of QA-P41 in the "fixed circle",
... ... is the cubic QL-Cu1 for the quadrilateral.
... ... of the QA-triple triangle of QG-P18
             and the trilinear polar of the "root".
Wrt "fixed circle", "root" and a construction see EQF-Ref [17b], 1.5.5 and 1.5.6 of Bernard Gibert.
Perhaps this cubic can also be described wrt the diagonal triangle.
I think other examples in QA-Tr-4 will give further interesting cubics.
Best regards Eckart



2016-10-13.pdf

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Message: #1991
Date: 13/10/2016 9:40:17
From: eckart_schmidt@t-online.de
Subject: Cubics for Cyclologic QA-Triple Triangles

```
Dear Tsihong Lau, dear Bernard, dear Chris,
in addition to my last message #1990 wrt the cubic for the
generalized cyclologic centers of the QA-triple triangles of
QG-P1 and QG-P18:
... This cubic bears also the vertices of the QA-triple triangle
    of QG-P19,
... The cubic is invariant wrt an isoconjugation
    of this triple triangle
... ... swapping QA-P2 and QA-P4,
... ... swapping QA-P4 and the point at infinity of QA-L2,
... ... swapping the vertices of the QA-triple triangles
        of QG-P1 and QG-P18.
Analog: The vertices of the QA-triple triangles of QG-P1 and
QG-P19 are isogonal conjugated wrt the QG-P18-triple triangle.
Best regards Eckart
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Message: #1992
Date: 14/10/2016 11:11:21
From: bernard.keizer
Subject: Cubics for Cyclologic QA-Triple Triangles

```
Dear Eckart, dear Chris, dear Tsihong Lau
Eckart knows these things much better than me, but it seems it
is again the properties of a Desmic system of 3 QA's formed by
the 3 triangles QG-P1, QG-P18 and QG-P19 and their 3 perspectors
2 by 2.
The described curve is QA-Cu7, which is a non pivotal isocubic
wrt DT with root QA-P4 or a QL-Cu1, id est a pivotal isogonal
cubic wrt triangle QA-P2QA-P4QA-P41 and pivot the infinity point
of the perpendicular bisector of QA-P2QA-P4 (this line is the
isoconjugate of the circumcircle of DT).
Best regards
Bernard
```

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Message: #1993
Date: 14/10/2016 1:15:32
From: eckart_schmidt@t-online.de
Subject: Cubics for Cyclologic QA-Triple Triangles

Dear Bernard,
I know, you have a better overview! Wonderful identification of the common Cubic for the pairwise Cyclologic QA-Triple Triangles of QG-P1, QG-P18 and QG-P19, thanks! So the properties of the cubic will be well known.
Best regards Eckart

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Message: #1994
Date: 16/10/2016 8:38:34
From: eckart_schmidt@t-online.de
Subject: Cubics for Cyclologic QA-Triple Triangles

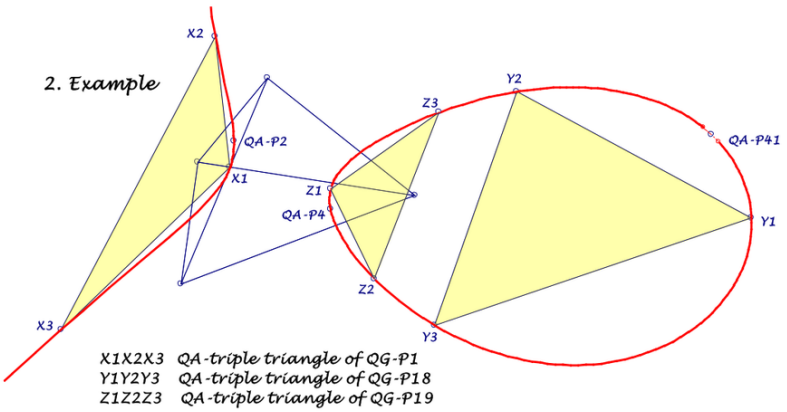
Dear Bernard,
if we define a new QG-point, there is a third Desmic system for three pairwise perspective QA-triple triangles, which leads to a new nonpivotal isogonal circular QA-cubic (see attached file).
Best regards Eckart

EQF-Note 2016-10-16

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubics of Generalized Cyclologic Centers

Generalized cyclologic centers are defined by Tsihong Lau
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/3275
here reduced to a starting constellation of two cyclologic triangles and their cyclologic centers. Then the generalized cyclologic centers give a cubic (see #1988), here reduced to perspective cyclologic QA-triple triangles (see QA-Tr-4). There are triples of pairwise perspective QA-triple triangles with the same cubic of generalized cyclologic centers. Two examples are already described in #1988 and #1990, here a third one is researched.



The cubic of generalized cyclologic centers shall be shortened by ***CGCC***.

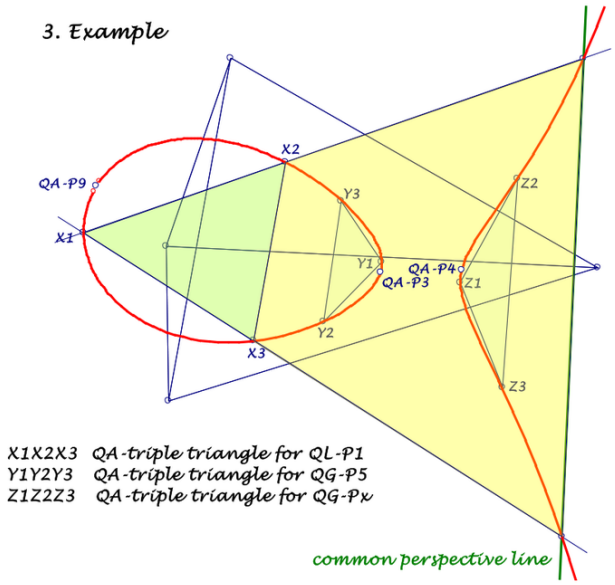
1. Example (see #1988 and #1989)
   The *QA*-triple triangles of *QG-P1, QL-P1, QG-P16*
   … have pairwise the same cubic *QA-Cu1* as *CGCC*.

2. Example (see #1990, #1991 and #1992)
   The *QA*-triple triangles of *QG-P1, QG-P18, QG-P19*
   … have pairwise the same cubic *QA-Cu7* as *CGCC*.
   This cubic is *QL-Cu1* for the quadrilateral of the *QA*-triple triangle of *QG-P18* and the perspective line of the three *QA*-triple triangles.

3. Example
In the table *QA-Tr-4* there remains only one pair of perspective cyclologic *QA*-triple triangles wrt *QG-P5* and *QL-P1*.
If we define a new *QG*-point *QG-Px*, we can consider a third *QA*-triple triangle:

Let *QG-Px* be the image of the Miquel point *QL-P1* wrt the Möbius transformation, centered in *QA-P3* and swapping *QA-P4* and *QA-P9*.

The *QA*-triple triangles of *QG-P5*, *QL-P1*, *QG-Px* … have pairwise the same cubic *QA-Cux* as *CGCC*. This *CGCC* is *QL-Cu1* for the quadrilateral of the Miquel triangle *QA-Tr2* and the perspective line of the three *QA*-triple triangles.



This *CGCC* is a nonpivotal isogonal circular cubic wrt the Miquel triangle.

Final remark:
The three *QG-Px* for a quadrilateral are collinear. The line bears *CSC(QL-P17)* and is orthogonal to *QL-P26.CSC(QL-P17)*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-10-16.pdf

Message: #1995
Date: 19/10/2016 1:02:28
From: bernard.keizer
Subject: Cubics for Cyclologic QA-Triple Triangles

Dear Eckart,
I don't feel very comfortable with the QG-Px points !
It takes me always a little time to identify the QG-P16 as the isogonals of the vertices of DT and so on ...
If I understand well, 3 QA in a Desmic system give a QA-Cu1 with a Miquel triangle and each of the possible DT's leads to a QA-Cu7.
Any QA and the isogonals of it's vertices form a Desmic system with the last QA formed by the Miquel triangle and the infinity point of QA-Cu1.
Your example 1 with QA-P4 and DT, QA-P3 and the QG-P16 and the Miquel triangle and infinity point.
Your example 2 is on QA-Cu7 with QA-P4 and DT, the QG-P19 or quasi-isogonals of the vertices of DT and QA-P2 and the QG-P18 and QA-P41. QA-Cu7 is a QL-Cu1 and has in turn the triangle QA-P2QA-P4QA-P41 as Miquel triangle.
Your example 3 is with the vertices of the Miquel triangle and the infinity point, the QG-P5 and the QG-Px and perspectors and leads to a new cubic QA-Cux ; this cubic is also a QL-Cu1 and has QA-P3QA-P4QA-P9 as Miquel triangle.
Again if my understanding is correct
1) there must be a 3rd cubic QL-Cu1 defined with the QG-P16 as DT and root QA-P3, which has in turn ??? as Miquel triangle.
2) Each of these 3 QL-Cu1 (which are QA-Cu1 with 2 perpendicular cords) leads in turn to 3 other cubics ...
3) Is there, by any chance, a connexion with my hidden Desmic in my message 1947 ?
Best regards
Bernard

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Message: #1996
Date: 20/10/2016 1:40:27
From: chris.vantienhoven
Subject: QA-Tr-4

Dear Eckart,
I added your comments in of message #1983 in QA-Tr-4.
I wonder
* Are there corresponding properties for Orthologic and Parallelologic Triple Triangles?
* What about Triple Triangles wrt QA- and QG-points in the QL-environment? I think there will a new realm of very interesting properties.

About your question “Are there informations, whether the table is complete wrt the EQF-points up to now?”.
The base of the items/tables QA-Tr-1 till QA-Tr-5 was a program in Mathematica including triple triangles of all known QG- and QL-points that I made at April/May 2015. Given that no more new points were introduced since then, I think the table should be complete. However there always can be inaccuracies due to programming failures.
Best regards,
Chris

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Message: #1997
Date: 20/10/2016 5:39:50
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

Dear all,
I focused on projective planes originally. I want to establish triangle/trilateral and quadrangle/quadrilateral geometry on a projective planes.
But I repost the generalized cyclological center as #1968 and gave another chain in #1982.
In #1982, I raised a question these chains should be valid on a Möbius plane. I thought the relation of these chains and Möbius plane and found these chains are not valid on the latter.
But the latter is a very important subject missing in the current quadrangle/quadrilateral geometry.
There are two main approaches in plane geometry.
One is to study the whole plane and another is to study the objects on it. Triangle/trilateral and quadrangle/quadrilateral geometries belong to the latter. But what planes are the objects on? The most common is the Euclidean plane - a kind of affine plane. But affine plane is not enough.
Usually it is extended to other planes by adding some objects.
Together with an point at infinity, it becomes a Möbius plane. Lines and circles become cycles and not distinguished between them on it.
Together with an line at infinity and points at infinity on it, it becomes a projective plane. Points and lines are dual objects. The principle of duality applies.
Be careful! Cycles and points are not dual objects on a Möbius plane. Together with circular points at inifinity, a projective plane owns circles and becomes a Möbius plane. A question arised:
Can a Möbius plane become a projective plane such that cycles have dual objects?
There are two important projective planes:
Pappian and Desarguesian planes.
Pappus' theorem holds on the former which is a proective plane over a commutative field.
Desargues' theorem holds on the latter which is a projective plane over a division ring.
There are two important Möbius planes:
Miquelian and ovoidal planes.
Miquel's six circle theorem holds on the former which is a Möbius plane over a commutative field.
Bundle theorem holds on the latter which is a Möbius plane over a division ring.

There is analogy between Pappian to Desarguesian and Miquelian to ovoidal planes.
Miquel's six circle theorem is closely related to cyclologic triangles in fact.
The theorem states:
circle - points : circle - points
pa - P,Pa,Ub,Uc : ua - U,Ua,Pb,Pc
pb - P,Ua,Pb,Uc : ub - U,Pa,Ub,Pc
pc - P,Ua,Ub,Pc : uc - U,Pa,Pb,Uc
Any five of the six circles exist, then the last exists, too.
cyclologic centers - triangles
 P,U - PaPbPc,UaUbUc
Pa,Ua - PPbPc ,UUbUc
Pb,Ub - PaPPc ,UaUUc
Pc,Uc - PaPbP ,UaUbU
In other words, cyclologic centers belong to triangle geometry.
But Miquel's six circle theorem belongs to quadrangle geometry.
 We should study the pairs of quadrangles on six circles. though I have not found an example.
Another aspect from circles:
point - circles : point - circles
P - pa,pb,pc : U - ua,ub,uc
Pa - pa,ub,uc : Ua - ua,pb,pc
Pb - ua,pb,uc : Ub - pa,ub,pc
Pc - ua,ub,pc : Uc - pa,pb,uc
Any seven of eight points of concurrency exist, then the last exists, too. We should study the pairs of triple circles concurrent at eight points, though I have not found an example.
P.S. PU,PaUa,PbUb,PcUc should be treated as an unordered quadruple of ordered pairs or an unordered pair of ordered quadruples.
paua,pbub,pcuc should be treated as an unordered triple of unordered pair.
If three circles concur at two points, they are called coaxial circles.
Bundle theorem states:
circle - radical lines : circle - radical lines
    pa - PU,PaUa   :   ua - PbUb,PcUc
    pb - PU,PbUb   :   ub - PcUc,PaUa
    pc - PU,PcUc   :   uc - PaUa,PbUb
Any five of six circles exist, the last exists, too.
Since the points of concurrency on the radical lines are interchangeable, PU,PaUa,PbUb,PcUc should be treated as an unordered quadruple of unordered pairs.
I have not found an example.
Another aspect from circles:
radical line - circles
     PU   - pa,pb,pc
    PaUa   - pa,ub,uc

```
    PbUb     - ua,pb,uc
     PcUc    - ua,ub,pc
Any three of four concurrent radical lines exist, then the last
exists, too. Since the triples papbpc and uaubuc are not
interchangeable.  they should be treated unordered triple of
ordered pairs.
P.S. the above systems of objects are called configurations.  I
will discuss them in another topic.
Links:
Affine plane (incidence geometry) - Wikipedia (
https://en.wikipedia.org/wiki/Affine_plane_(incidence_geometry)
)
(
https://en.wikipedia.org/wiki/Affine_plane_(incidence_geometry)
)

Möbius plane - Wikipedia (
https://en.wikipedia.org/wiki/M%C3%B6bius_plane )
( https://en.wikipedia.org/wiki/M%C3%B6bius_plane )

Projective plane - Wikipedia (
https://en.wikipedia.org/wiki/Projective_plane )
( https://en.wikipedia.org/wiki/Projective_plane )

Miquel's theorem - Wikipedia (
https://en.wikipedia.org/wiki/Miquel%27s_theorem )
( https://en.wikipedia.org/wiki/Miquel%27s_theorem )

Bundle theorem - Wikipedia (
https://en.wikipedia.org/wiki/Bundle_theorem )
( https://en.wikipedia.org/wiki/Bundle_theorem )

An article on affine, Möbius and projective Planes:
http://www.mathematik.tu-darmstadt.de/~ehartmann/circlegeom.pdf
It is difficult but good.
I hope this topic focus on various classes of planes, not the
objects on them.
There are many other classes- Laguerre and Minkowski planes,
translation planes, etc.  I hope other planes will be discussed
in this topic!
Best regards,
Tsihong Lau
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Message: #1998
Date: 20/10/2016 7:50:43
From: bernard.keizer
Subject: Cubics for Cyclologic QA-Triple Triangles

Dear Eckart, dear Chris, dear Tsihong Lau
In fact, you don't need a Desmic system to describe your examples.
For any Cu-QA on a cubic QA-Cu1, it's DT and it's QA-P4, you may define a quasi-isogonality and you get a QA-Cu7, which is a QL-Cu1 !
The Miquel point QL-P1 of this QL-Cu1 is the isoconjugate of one of the 2 QL-2P2 and the 2 QL-2P2 points are quasi-isogonal.
There are an infinity of QA-Cu7 for a given QA-Cu1 and they are all QL-Cu1's...
The question is : for a given QL-Cu1 taken as a QA-Cu7, what is the corresponding initial QA-Cu1 (is it unique ?)
Best regards
Bernard

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Message: #1999
Date: 21/10/2016 11:21:53
From: chris.vantienhoven
Subject: Clifford's Chain Theorem and Miquels Pentagon Theorem

---

Dear friends,
Morley described *Clifford's Chain Theorem* in [Metric geometry of the plane n-Line, page 103] and [Extensions of Clifford's Chain-Theorem].
It is best to understand with numbers:
· Clifford's Circle in a 3-Line is supposed to be the circumscribed circle of the triangle.
· 4 Clifford's Circles of a 4-Line (Circumscribed circles of the component triangles) intersect at a common point 4L-e-P2: Clifford's Point, which is QL-P1 (Miquel Point) in the case of a 4-Line.
· 5 Clifford's Points of a 5-Line (Miquel Points of the 5 component 4-Lines) lie on a circle 5L-o-Ci1: Clifford's Circle with center 5L-o-P2: 5L-Clifford's Circle Center.
· 6 Clifford's Circles of a 6-Line (5L-o-Ci1 of the 6 component 5-Lines) intersect at a common point 6L-e-P2: 6L-Clifford's Point.
· 7 Clifford's Points of a 7-Line (6L-e-P2 of the 7 component 6-Lines) lie on a circle 7L-o-Ci1: 7L-Clifford's Circle with center 7L-o-P2: 7L-Clifford's Circle Center.
· 8 Clifford's Circles of a 8-Line (7L-o-Ci1 of the 8 component 7-Lines) intersect at a common point 8L-e-P2: 8L-Clifford's Point.
. 9 Clifford's Points of a 9-Line (8L-e-P2 of the 9 component 8-Lines) lie on a circle 9L-o-Ci1: 9L-Clifford's Circle with center 9L-o-P2: 9L-Clifford's Circle Center.
· etc.

There is also Miquels Pentagon Theorem
Let ABCDE be a convex pentagon. Extend all sides until they meet in five points F,G,H,I,K and draw the circumcircles of the five triangles CFD, DGE, EHA, AIB and BKC. Then the second intersection points (other than A,B,C,D,E), namely the new points M,N,P,R and Q are concyclic (lie on a circle).
(see: https://en.wikipedia.org/wiki/Miquel%27s_theorem)

So Clifford's Chain Theorem is valid for systems of n Lines.
Miquels Pentagon Theorem is valid for 5-Gons (system of 5 sequential points sequentially connected by 5 lines).
I noticed constructing Miquels Pentagon Circle for the different Pentagons in a 5-Line that the different Miquels Pentagon Circles are identical. Moreover they coincide with Clifford's Circle in a 5-Line.

So actually Miquels Pentagon Theorem isn't really a Pentagon Theorem but a Pentalateral Theorem.
Best regards,
Chris

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Message: #2000
Date: 22/10/2016 8:59:37
From: eckart_schmidt@t-online.de
Subject: QA-Tr-4

Dear Chris,
thanks for your interest and informations. I have made no efforts wrt Orthologic and Parallelologic Triple Triangles, for I am rather confused wrt cyclologic and generalized cyclologic centers and their cubic.
In the table QA-Tr-4 there are now 3 empty positions, printed out they disappear but show greater text size. Perhaps a problem of my computer?
By the way: The new point QG-Px in #1994 is
... the image of QL-P1 wrt the Möbius transformation centered in QA-P3 and swapping QA-P4 and QA-P9.
... the CSC-image of the intersection
... ... of QG-P5.QL-P1 and a perpendicular through QG-P1.
... The QL-Triple Triangle of QG-Px degenerates collinear
... ... on the CSC-image of the circle through QL-P1, QL-P17 and the 2nd intersection of QL-P1.QL-P16 and QL-Ci1.
... The QA-Triple Triagles of QL-P1, QG-P5, QG-Px are pairwise cyclologic and perspective.
Best regards Eckart

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Message: #2001
Date: 22/10/2016 6:41:46
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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Dear all,

Message #1997 mentions affine, möbius and projective planes.
So the plane in this message is Pappian planes with circular
points.
In other words, conics and circles are meaningful.
Given two quadrangles PPaPbPc,UUaUbUc, the expoint triangles are
all cyclologic.
Their generalized cyclologic centers are:
cyclologic centers - circular points - expoint triangles
       P1,U1       -       P,U       -   PaPbPc,UaUbUc
      P1a,U1a      -      Pa,Ua      -   PPbPc,UUbUc
      P1b,U1b      -      Pb,Ub      -   PaPPc,UaUUc
      P1c,U1c      -      Pc,Uc      -   PaPbP,UaUbU
Please check and prove the two quadrangles
P1P1aP1bP1c,U1U1aU1bU1c have the same properties as
PPaPbPc,UUaUbUc.
In other words, the expoint triangles of P1P1aP1bP1c,
U1U1aU1bU1c are all cyclologic.
Their generalized cyclologic centers are:
cyclologic centers - circular points - expoint triangles
      P2,U2        -      P1,U1      - P1aP1bP1c,U1aU1bU1c
     P2a,U2a       -     P1a,U1a     - P1P1bP1c,U1U1bU1c
     P2b,U2b       -     P1b,U1b     - P1aP1P1c,U1aU1U1c
     P2c,U2c       -     P1c,U1c     - P1aP1bP1,U1aU1bU1
and so on...
We get a chain of quadrangles: PPaPbPc,UUaUbUc
-> P1P1aP1bP1c,U1U1aU1bU1c
-> PcPcaPcbPcc,UcUcaUcbUcc
-> and so on.
This chain is a quadrangle not triangle chain!

Best regards
Tsihong Lau
```

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Message: #2002
Date: 23/10/2016 11:49:51
From: tsihonglau
Subject: Desargues' Theorem and Perspective Quadrangle/Quadrilateral

---

Dear all,
Desargues' theorem states that two triangles
ABC and A'B'C' are perspective at a point
then the points
of intersection BC,B'C' and
CA,C'A' and AB.A'B' are
collinear on a
perspective
axis. Dually,
two trilaterals
abc and a'b'c' are perspective on a line
then the lines through b ∩ c, b' ∩ c'
and c ∩ a, c' ∩ a'
and a ∩ b, a' ∩ b' concur at a perspective
center.
But AA',BB' and CC' are interchangeable, so there are four
perspective axes=perspective quadrilateral.
Dually, aa',bb',cc' are interchangeable, so there are four
perspective centers=perspective quadrangle.
Especially, the radical line of two circles is the line through
two points of intersection(real or imaginary).
The three radical lines of three circles concur at the radical
center.
The two points of intersection of two circles are
interchangeable.
So three circles can determine a perspective quadrilateral.
Because there are many triples of circles for a
quadrangle/quadrilateral, we can get many perspective
quadrilaterals.
Best regards,
Tsihong Lau

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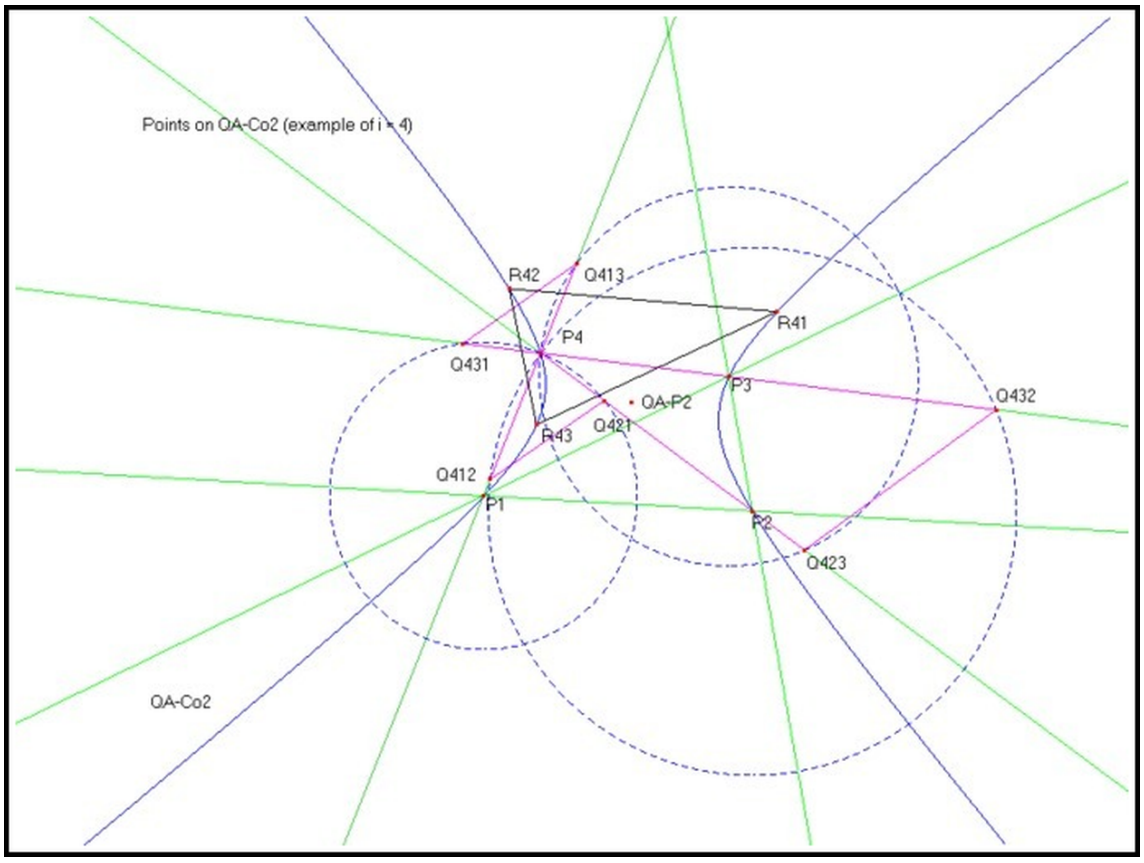
Message: #2003
Date: 24/10/2016 2:41:01
From: Seiichi Kirikami
Subject: Points on QA-Co2

Dear friends,
QA-Co2 is an orthogonal hyperbola with its center QA-P2 and through the reference quadrangle P1P2P3P4.
In the following, (i, j, k, l) means all combinations of (1,2,3,4).
Denote:
Qijk = 2nd intersection of PiPj and the circle with its center Pk and radius PiPk.
Ril = circumcenter of PQijkQikj.
Ril is on QA-Co2.
See the attachment.
Best regards, Seiichi.



QACo2.png

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Message: #2004
Date: 24/10/2016 8:28:13
From: chris.vantienhoven
Subject: Points on QA-Co2

Dear Seiichi,
A remarkable construction!
At first I could not understand this coincidence.
But later it dawned.
I figured out an example:
Q123 = 2nd intersection of P1P2 and the circle with its center P3 and radius P1P3.
Q132 = 2nd intersection of P1P3 and the circle with its center P2 and radius P1P2.
R14 = circumcenter of P1Q123Q132.
R14 is on QA-Co2 indeed.
R14 is only dependent on P1, P2, P3, so it actually is a triangle point.
After observation it appears that R14 = H4 = Orthocenter P1.P2.P3 and H4 lies on QA-Co2 indeed.
So it can be concluded that Ril = Orthocenter of Pi.Pj.Pk and consequently lies on QA-Co2.
This clarifies it all.
Best regards,
Chris

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Message: #2005
Date: 24/10/2016 2:32:45
From: Seiichi Kirikami
Subject: Points on QA-Co2

Dear Chris,
Let P1P2P3P4 be a quadrangle.
Denote:
H1, H2, H3, H4 = orthocenters of P2P3P4, P1P3P4, P1P2P4, P2P3P4 respectively.
They are on QA-Co2.
PiPjPk and HiHjHk are orthologic. They are on QA-Co2.
(HiHjHk, PiPjPk)-orthologic center coincides with P1, P2, P3 or P4.
PiPjPk and HiHjHk are cyclologic. They are on QA-Co2.
Best regards, Seiichi.

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Message: #2006
Date: 24/10/2016 5:28:24
From: tsihonglau
Subject: Points on QA-Co2

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>Let P1P2P3P4 be a quadrangle.
>>Denote:
>>H1, H2, H3, H4 = orthocenters of P2P3P4, P1P3P4, P1P2P4,
P2P3P4 respectively.
>>They are on QA-Co2.
>>PiPjPk and HiHjHk are orthologic. They are on QA-Co2.
>>(HiHjHk, PiPjPk)-orthologic center coincides with P1, P2, P3
or P4.
>>PiPjPk and HiHjHk are cyclologic. They are on QA-Co2.

Dear Seiichi
Wonderful results!  You found an example a chain of quadrangles
mentioned in #2001.
Please refer to it and the attachment files.
I do not check if there is an orthologic quadrangle chain.
Best regards,
Tsihong Lau
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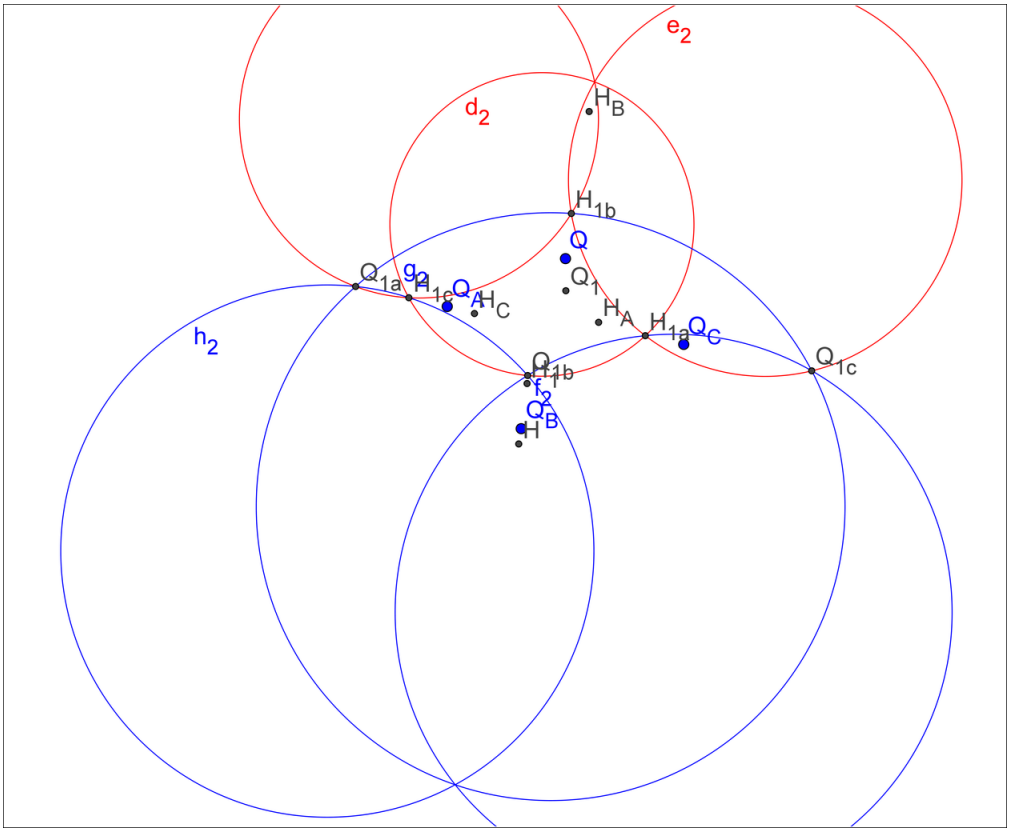

QA_Co2_cyclologic_center_chain_quadrangle.png

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Message: #2007
Date: 24/10/2016 6:22:55
From: tsihonglau
Subject: Points on QA-Co2

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>Wonderful results!  You found an example a chain
>>of quadrangles mentioned in #2001.  Please refer
>>to it and the attachment files.  I do not check if
>>there is an orthologic quadrangle chain.
Dear Seiichi,
Oops! I am sorry. Because these points lie on QA-Co2.
So it is different from the chain mentioned in #2001.
Given two quadrangles QQaQbQc,HHaHbHc, (=P1P2P3P4,H1H2H3H4)you
found.  Their cyclologic centers are:
cyclologic centers - expoint triangles
       Q1,H1       -   QaQbQc,HaHbHc
      Q1a,H1a      -    QQbQc,HHbHc
      Q1b,H1b      -    QaQQc,HaHHc
      Q1c,H1c      -    QaQbQ,HaHbH
The expoint triangles of QQaQbQc,HHaHbHc, are all cyclologic, so
do Q1Q1aQ1bQ1c, H1H1aH1bH1c.
I guess we can get Q2 Q2aQ2bQ2c, H2H2aH2bH2c and so on
similarly.
Best regards,
Tsihong Lau
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Message: #2008
Date: 24/10/2016 7:08:56
From: tsihonglau
Subject: Points on QA-Co2

Dear Seiichi,
If we replace X(4)'s with X(74)'s but not X(1138)'s.
The cyclologic center quadrangles seem to be valid!
Best regards,
Tsihong Lau

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Message: #2009
Date: 24/10/2016 8:54:14
From: eckart_schmidt@t-online.de
Subject: Cubics for Cyclologic QL-Triple Triangles

Dear Bernard, dear Chris,
the following observation leads to more cyclologic triple triangles:
Two triangles - one Möbius transformation of the other - are cyclologic.
The cyclologic centers lie on the circumcircles of the triangles.
For quadrangles there is the Möbius transformation QA-Tf4,
... swapping for example QG-P5 and QL-P1 (see QA-Tr-4).
For quadrilaterals there is the Möbius transformation QL-Tf1,
... swapping QG-P16 and QA-P3,
... or QG-P1 and QA-P4,
... or QA-P41 and the intersection of QG-P1.QA-P4 and QG-L1,
... or QG-P17 and QG-P18.
This leads to cyclologic QL-triple triangles and their cubics for generalized cyclologic centers:
Example: QL-triple triangles of QG-P16 and QA-P3:
... One cyclologic center lies on QL-Ci6, the other on its CSC-line, they are CSC-partners.
... The cubic for the generalized cyclologic centers degenerates in QL-Ci6 ant its CSC-line.
But: For QL-triple triangles of QG-P17 and QG-P18 the cyclologic constellation degenerates.
Best regards Eckart

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Message: #2010
Date: 25/10/2016 1:24:04
From: Seiichi Kirikami
Subject: Points on QA-Co2

```
Dear Tsihong Lau,
Antreas has defined "Eulerologic Triangles" in Anopolis #3841.
†)
The component triangles of P1P2P3P4 and H1H2H3H4 are
eulerologic.
But regrettably their centers coincide with their orthologic
centers.
Best regards, Seiichi.
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Message: #2011
Date: 25/10/2016 3:51:05
From: Antreas Hatzipolakis
Subject: Points on QA-Co2

```
>> Dear Tsihong Lau,
>> Antreas has defined " Eulerologic Triangles" in Anopolis
#3841.
>> The component triangles of P1P2P3P4 and H1H2H3H4 are
eulerologic. But
>> regrettably their centers coincide with their orthologic
centers.
>> Best regards, Seiichi.

Dear Seiichi,
Anopolis #3841 = Hyacinthos # 24415  †)
( https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/m
essages/24415 )
And you and I have defined the term "cyclologic":
Hyacinthos # 22721  †)
( https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/m
essages/22721 )
Antreas
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† Editorial note: The referenced messages can be found as listed in EPG-References [11].

Message: #2012
Date: 25/10/2016 4:36:54
From: César E. Lozada
Subject: Points on QA-Co2

Dear Antreas and Seiichi
>>And you and I have defined the term "cyclologic":
>>Hyacinthos #22721
>>(https://groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/22721)

Don´t forget the definition of “circumcyclologic”, in
https://groups.yahoo.com/neo/groups/Anopolis/conversations/messages/2447 †)
Regards,
César

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Message: #2013
Date: 25/10/2016 4:46:53
From: Antreas Hatzipolakis
Subject: Points on QA-Co2

Dear César,
Thank you for reminding it !

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #2014
Date: 26/10/2016 12:02:27
From: tsihonglau
Subject: Points on QA-Co2

Dear all,
If the expoint triangles of two quadrangles PPaPbPc
and UUaUbUc on a rectangular hyperbola are all cyclologic:
cyclologic centers - expoint triangles
P1,U1 - PaPbPc,UaUbUc
P1a,U1a - PPbPc,UUbUc
P1b,U1b - PaPPc,UaUUc
P1c,U1c - PaPbP,UaUbU
then it seems the expoint triangles of two quadrangles
P1P1aP1bP1c and U1U1aU1bU1c on the same rectangular hyperbola
are all cyclologic:
cyclologic centers - expoint triangles
P2,U2 - P1aP1bP1c,U1aU1bU1c
P2a,U2a - P1P1bP1c,U1U1bU1c
P2b,U2b - P1aP1P1c,U1aU1U1c
P2c,U2c - P1aP1bP1,U1aU1bU1
and so on, we get a chain of cyclologic quadrangles.
If the expoint triangles of two quadrangles PPaPbPc and UUaUbUc
on a rectangular hyperbola are all cyclologic and the fourth
points of intersection of the circumcircles of expoint triangles
with the hyperbola:
fourth points of intersection - expoint triangles
P1,U1 - PaPbPc,UaUbUc
P1a,U1a - PPbPc,UUbUc
P1b,U1b - PaPPc,UaUUc
P1c,U1c - PaPbP,UaUbU
then it seems the expoint triangles of two quadrangles
P1P1aP1bP1c and U1U1aU1bU1c on the same rectangular hyperbola
are all cyclologic:
fourth points of intersection - expoint triangles
P2,U2 - P1aP1bP1c,U1aU1bU1c
P2a,U2a - P1P1bP1c,U1U1bU1c
P2b,U2b - P1aP1P1c,U1aU1U1c
P2c,U2c - P1aP1bP1,U1aU1bU1
and so on, we get a chain of cyclologic quadrangles.
Best regards,
Tsihong Lau

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Message: #2015
Date: 26/10/2016 9:14:50
From: chris.vantienhoven
Subject: Eulerologic Triangles in a Quadrangle

Dear Seiichi, Antreas, Cesar and all,
I studied the Eulerologic relationship for Triple Triangles (see QA-Tr-1) in a Quadrangle.
I also encountered an example of " Brocardologic "Triangles occurring when the Brocard Axes of involved triangles concur.
Gratefully I used the property from Seiichi that also 3 collinear points form a triangle with an Eulerline (Anapolis #4178).
Last but not least -special for Quadrangles- I encountered an example where a special triangle appeared to be Eulerologic with all 4 Component Triangles of the Reference Quadrangle.
See attachment.
When there are remarks/extensions/improvements please let me know.
Best regards,
Chris

QA-Tr-6: Eulerologic QA-Triple Triangles

Eulerologic pairs of Triple Triangles

The notion of two triangles having an Eulerologic relation was introduced by Antreas Hatzipolakis. See [11], Hyacinthos #24415.
Triangle A1B1C1 is Eulerologic wrt A2B2C2 if the Eulerlines of triangles A1B2C2, A2B1C2, A2B2C1 are concurrent. The point of concurrence is known as the Eulerologic Center of A1B1C1 with respect to A2B2C2.
In constrast to the orthologic, cyclologic and parallelologic relationship this Eulerologic relationship isn't always reciprocal. However there are cases of reciprocity.

Here is a list of Eulerologic pairs of Triple Triangles in a Quadrangle.

Nr	Triple Triangle-1 formed by 3 QA-versions of:	Triple Triangle-2 formed by 3 QA-versions of:	Eulerologic Center-1	Eulerologic Center-2
1	QG-P1	QG-P2	InfinityPoint(QA-L5) =X(30) of QA-DT	QA-P11 = X(3) of QA-DT
2	QG-P1	QG-P17	X(125) of QA-DT *)	QA-P10 = X(2) of QA-DT *)
3	QG-P2	QG-P17	QA-P13 = X(5) of QA-DT = X(3) of QG-P17TT	QA-P13 = X(5) of QA-DT = X(3) of QG-P2TT
4	QG-P5	QL-P4	none	QA-P1.QA-P15 (-1:2)
5	QG-P5	All Component Triangles QA-CTi	4 points E1,E2,E3,E4, where QG-P5 TT is the DT of E1.E2.E3.E4. Ei=X(3) of QA-CTi.	none
6	QG-P8	QL-P12	InfinityPoint(QA-L5) =X(30) of QA-DT	X(3) of QG-P8TT
7	QG-P9	QL-P1	X(3) of QL-P1TT	none
8	QG-P9	QL-P6	Infinitypoint of the Eulerline of QL-P6TT	X(3) of QG-P9TT
9	QG-P10	QL-P2	Infinitypoint of the Eulerline of QL-P2TT	X(3) of QG-P10TT
10	QG-P10	QL-P3	none	QA-P15
11	QG-P10	QL-P29	none	QA-P15

QA-Px stands for some QA-point not registered in EQF.
QG-PiTT/QL-PiTT stands for Triple Triangle of QG-Pi/QL-Pi (i=serial number of point).
QA-DT stands for the Diagonal Triangle of the Reference Quadrangle.
QA-CTi stands for the Component Triangle-i (i=1,2,3,4) of the Reference Quadrangle.

*) see [33], property Seiichi Kirikami at Anapolis #4178

Summary:

- The first 3 mentioned instances of Eulerologic Centers all fully relate to the QA-Diagonal Triangle and therefore are Eulerologic examples wrt a Reference Triangle.
- In instance 2 there is not only an Eulerologic relationship but also a "Brocardologic" relationship because also the Brocard Axes of involved triangles concur. They even concur with the Eulerlines because involved triangles are isosceles.
- Instances 1, 6, 8, 9 relate to triangles with their medial triangle giving parallel Eulerlines, meeting in their Infinitypoint, and Eulerlines meeting in the circumcenter of the larger triangle.
- Instance 5 is an example of a non-recursive *Quadri-Eulerologic* relationship where a Triple Triangle is Eulerologic related to all QA-Component Triangles. The result is 4 Eulerologic Centers forming a quadrangle with the Triple Triangle as Diagonal Triangle.
- The other instances are specific non-recursive Eulerologic relationships with only one Eulerologic Center.
- It is remarkable that often the Eulerologic Center is the circumcenter of one of the Triple Triangles.

QA-Tr-6.pdf

Message: #2016
Date: 27/10/2016 1:15:18
From: Seiichi Kirikami
Subject: Eulerologic Triangles in a Quadrangle

Dear Chris,
I am very happy that Eulerologic triangles and centers have taken root in Quadrilateral Geometry.
Best regards, Seiichi.

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Message: #2017
Date: 27/10/2016 12:03:14
From: eckart_schmidt@t-online.de
Subject: Cubics for Perspective QA-Triple Triangles

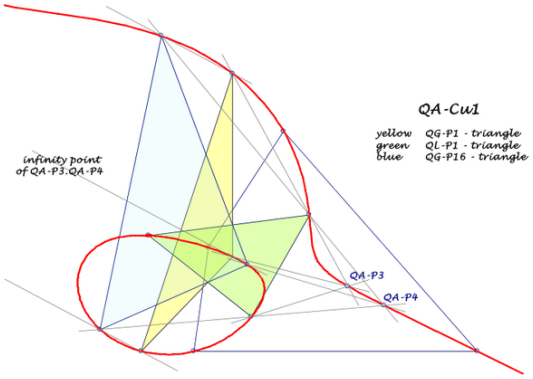
Dear Bernard, dear Chris,
starting with two perspective QA-triple triangles there is a third QA-triple triangle, perspective with both and a cubic, bearing the 3 x 3 vertices of the perspective triangles and their perspectors. The cubic QA-Cu1 is of this type, attached analog a further interesting cubic (example 2).
Best regards Eckart

EQF-Note 2016-10-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubics for Perspective QA-Triple Triangles

Starting with two perspective QA-triple triangles (see QA-Tr-2) there is a third QA-triple triangle, perspective with both and a cubic, bearing the 3 x 3 vertices of the perspective triangles and their perspectors. The cubic QA-Cu1 is of this type.



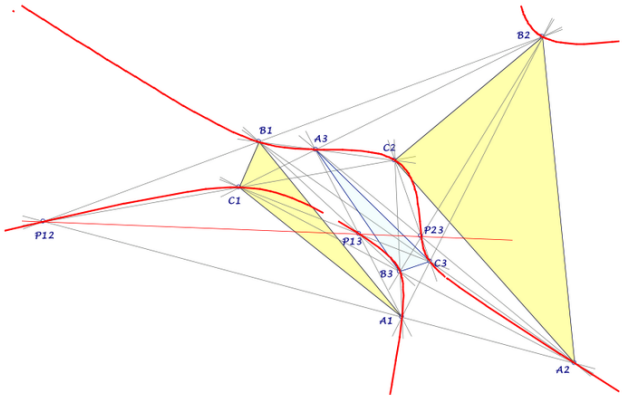
Wellknown Example QA-Cu1

The diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2* are perspective wrt *QA-P3*. The *QG-P16*-triangle is perspective to *QA-Tr1* wrt the infinity point of *QA-P3.QA-P4* and perspective to *QA-Tr2* wrt *QA-P4*.
The isoconjugation *QA-Tf2* wrt *QA-Tr1* swaps the vertices of *QA-Tr2* and the *QG-P16*-triangle. The isogonal conjugate wrt *QA-Tr2* swaps the vertices of *QA-Tr1* and the *QG-P16*-triangle. There is a third isoconjugation wrt the *QG-P16*-triangle, which swaps the vertices of *QA-Tr1* and *QA-Tr2*. Fixed points of this isoconjugation are the points of the cubic *QA-Cu1* with tangential *QA-P3*.
Summary: The three triangles are pairwise perspective and pairwise isoconjugated wrt the third triangle. Their vertices and the perspectors lie on a cubic. This can be generalized.

General Observations

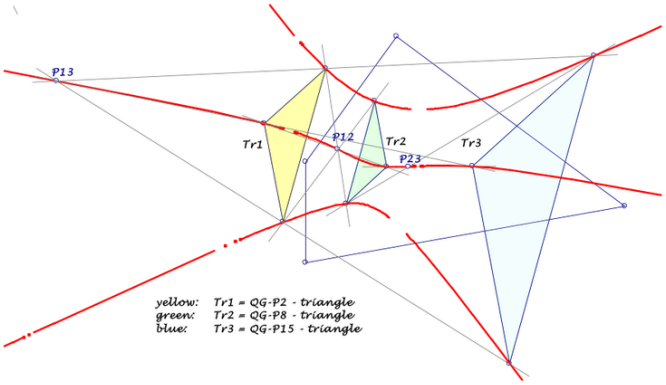
Two perspective triangles $A_1B_1C_1$, $A_2B_2C_2$ have a third triangle $A_3B_3C_3$
… with $A_3 = B_1C_2 \cap B_2C_1$, $B_3 = A_1C_2 \cap A_2C_1$, $C_3 = B_1A_2 \cap B_2A_1$,
… perspective to $A_1B_1C_1$ in $P_{13} = A_1A_3 \cap B_1B_3 \cap C_1C_3$,

… perspective to $A_2B_2C_2$ in $P_{23} = A_2A_3 \cap B_2B_3 \cap C_2C_3$,
… with collinear perspectors,
… with a pivotal isocubic:
… … one triangle as reference triangle,
… … isoconjugation swapping the vertices of the other two triangles,
… … or: isoconjugation swapping the perspectors with the other two triangles,
… … pivot in the perspector of the other two triangles,
… … bearing the vertices of the three triangles
… … and their perspectors.

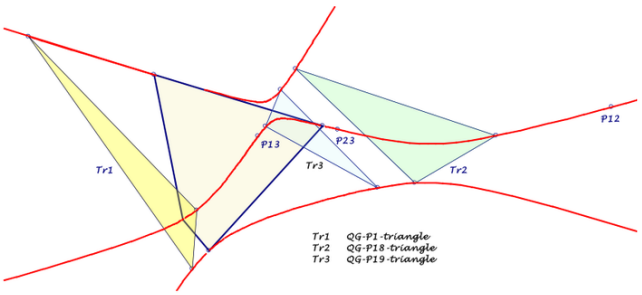


Generalization for perspective QA-triple triangles

In *QA-Tr-2* perspective *QA*-triple triangles are listened, which can be considered in the described manner.



Example 1:
The *QA*-triple triangles *Tr1* of *QG-P2* and *Tr2* of *QG-P8*
… have as perspector a point P_{12}, which divides *QA-P1.QA-P10* with ratio *3:2*.
... The third triangle *Tr3* is the *QA*-triple triangle of *QG-P15*,
… perspector P_{13} = *QA- P20*, perspector P_{23} = *QA-P1*.



Example 2:
This is a very special example, for the cubic contains also the vertices of the quadrangle.
The *QA*-triple triangles *Tr1* of *QG-P1, Tr2* of *QG-P18* and *Tr3* of *QG-P19* have the perspectors (found by Chris van Tienhoven)
… P_{12} = *QA-Tf2*(P_{13}),
… P_{13} = *QA-P12.QA-P24*∩*QA-P6.QA-Tf2*(*QA-P11*),
… P_{23} = P_{12}.P_{13}∩*QA-P4.QA-Tf4*(*QA-P12*).

The corresponding cubic
… contains a lot of points:
… … 4 vertices of the quadrangle,
… … 3x3 vertices of the three perspective triangles,
… … 3 perspectors (pivots),
… … 3x3 cevian points of the perspectors wrt the third triangle,
… is invariant wrt isoconjugations for the *QA*-triple triangles
… … of *QG-P1*: *QA-Tf2*,
… … of *QG-P18*: isogonal conjugation,
… … of *QG-P19*: swapping *QA-P2* and *QA-P41*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-10-27.pdf

Message: #2018
Date: 29/10/2016 5:21:12
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

Dear all,
I found two theorems are closely related to orthologic and parallelologic triangles in fact.
The first is Miquel's six perpendicular line theorem. It states:
perpendicular lines : perpendicular lines
pa=PPa,ubc=UbUc : ua=UUa,pbc=PbPc
pb=PPa,uca=UcUa : ub=UUb,pca=PcPa
pc=PPa,uab=UaUb : uc=UUc,pab=PaPb
Any five of the six pairs of perpendicular lines exist, then the last exists, too. It is the aspect of quadrangle of orthologic triangles. The orthologic centers of two triangles PaPbPc and UaUbUc are P and U. The other are:
orthologic centers - triangles
P,U - PaPbPc,UaUbUc
Pa,Ua - PPbPc,UUbUc
Pb,Ub - PaPPc,UaUUc
Pc,Uc - PaPbP,UaUbU
The theorem is very like Miquel's six circle theorem except the former is concerned about unordered pairs of unordered pairs of points (quadrigon) the latter is concerned about unordered quadruples(quadrangle). The
perpendicular lines pa=PPa,ubc=UbUc are made of an unordered pair of unordered pair of points.
The circumcircle pa=PPaUbUc are made of an unordered quadruple of points. However, the two theorems are both concerned about unordered pair of ordered quadruple PPaPbPc,UUaUbUc or unordered quadruple of ordered pairs
PU,PaUa,PbUb,PcUc. But the diapleural triangles of the two quadrangles PPaPbPc and UUaUbUc are orthologic triangles in the former.
They are not cyclologic triangles in the latter.
If we replace "perpendicular" with "parallel", then the above statements are true also.
We get Miquel's six parallel line theorem closely related to parallelologic triangles.
We should formulate the geometries with the two theorems.
The geometries are quadrigon (unordered pairs of unordered pairs of points) geometries.
Such a geometry is studied so less before.
Best regards,
Tsihong Lau

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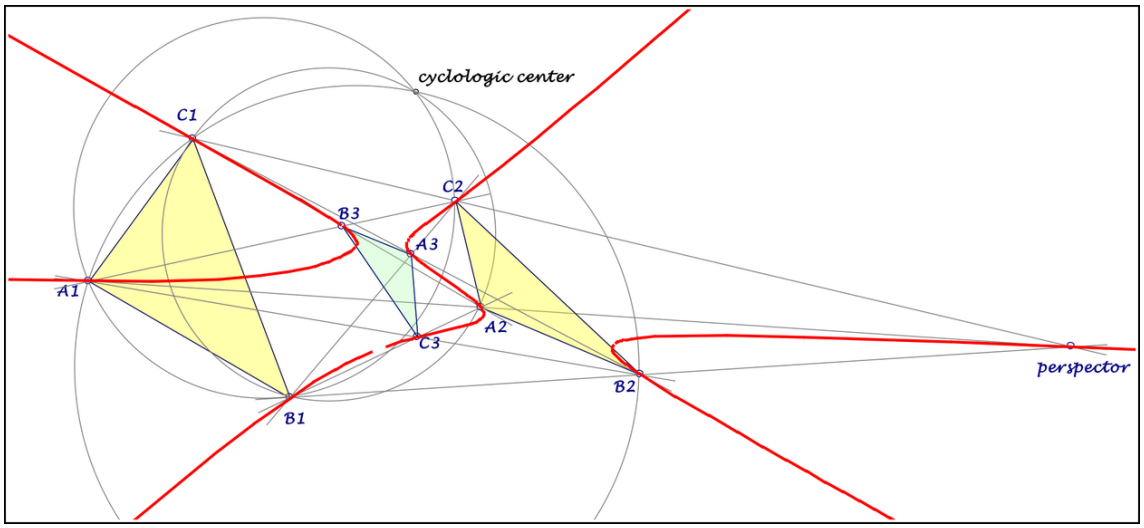
Message: #2019
Date: 29/10/2016 8:46:26
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in Quad ...

Dear Tsihong Lau, dear Bernard, dear Chris,
Tsihong Lau has defined generalized cyclologic centers and asked for their loci. Here an answer for
... perspective cyclologic triangles
A1B1C1, A2B2C2:
The locus for generalized cyclologic centers
... is a pivotal isocubic (see attached file)
... wrt a triangle
    with vertices A3=B1C2^B2C1, B3=A1C2^A2C1, C3=A1B2^A2B1,
... an isoconjugation,
    which swaps the vertices of A1B1C1 and A2B2C2
... and a pivot, which is the perspector of A1B1C1 and A2B2C2.
Perspective cyclologic triangles are generalized cyclologic wrt any two points P and Q on the described cubic, so that the conics through A1,B1,C2,P,Q and A1,B2,C1,P,Q and A2,B1,C1,P,Q have a common third point.
Best regards Eckart



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Message: #2020
Date: 29/10/2016 3:20:05
From: tsihonglau
Subject: Desargues Configuration

```
Dear all,
This topic is different from the wikipage -
Desargues configuration - Wikipedia (
https://en.wikipedia.org/wiki/Desargues_configuration )

This page is about the ordered pair of ordered
quintuples(quintigons) of points and lines on a Desarguesian
plane.  Given two ordered quintuples of points -
P0P1P2P3P4,U0U2U4U1U3 and two ordered quintuples of lines -
p0p1p2p3p4, u0u2u4u1u3, the incidence tables:
line - points    : line -  points
 p0  - U0,P2,P3  :  u0  - P0,U1,U4
 p1  - U1,P3,P4  :  u1  - P1,U2,U0
 p2  - U2,P4,P0  :  u2  - P2,U3,U1
 p3  - U3,P0,P1  :  u3  - P3,U4,U2
 p4  - U4,P1,P2  :  u4  - P4,U0,U3

point - lines    : point - lines
 P0   - u0,p2,p3 :  U0   - p0,u1,u4
 P1   - u1,p3,p4 :  U1   - p1,u2,u0
 P2   - u2,p4,p0 :  U2   - p2,u3,u1
 P3   - u3,p0,p1 :  U3   - p3,u4,u2
 P4   - u4,p1,p2 :  U4   - p4,u0,u3
Please notice the order of P,U quintuples of points and lines
are different.
So they are not interchangeable.  The perspective centers and
axes:
center - axis - triangles=trilaterals
 P0    -  p0  -   U1U3P4 =u4p1u2,U4P1U2=u1u3p4
 P1    -  p1  -   U2U4P0 =u0p2u3,U0P2U3=u2u4p0
 P2    -  p2  -   U3U0P1 =u1p3u4,U1P3U4=u3u0p1
 P3    -  p3  -   U4U1P2 =u2p4u0,U2P4U0=u4u1p2
 P4    -  p4  -   U0U2P3 =u3p0u1,U3P0U1=u0u2p3
 U0    -  u0  -   P2U3P1 =p3p4u2,P3P4U2=p2u3p1
 U1    -  u1  -   P3U4P2 =p4p0u3,P4P0U3=p3u4p2
 U2    -  u2  -   P4U0P3 =p0p1u4,P0P1U4=p4u0p3
 U3    -  u3  -   P0U1P4 =p1p2u0,P1P2U0=p0u1p4
 U4    -  u4  -   P1U2P0 =p2p3u1,P2P3U1=p1u2p0
I think we should study this configuration thoroughly.
Best regards,
Tsihong Lau
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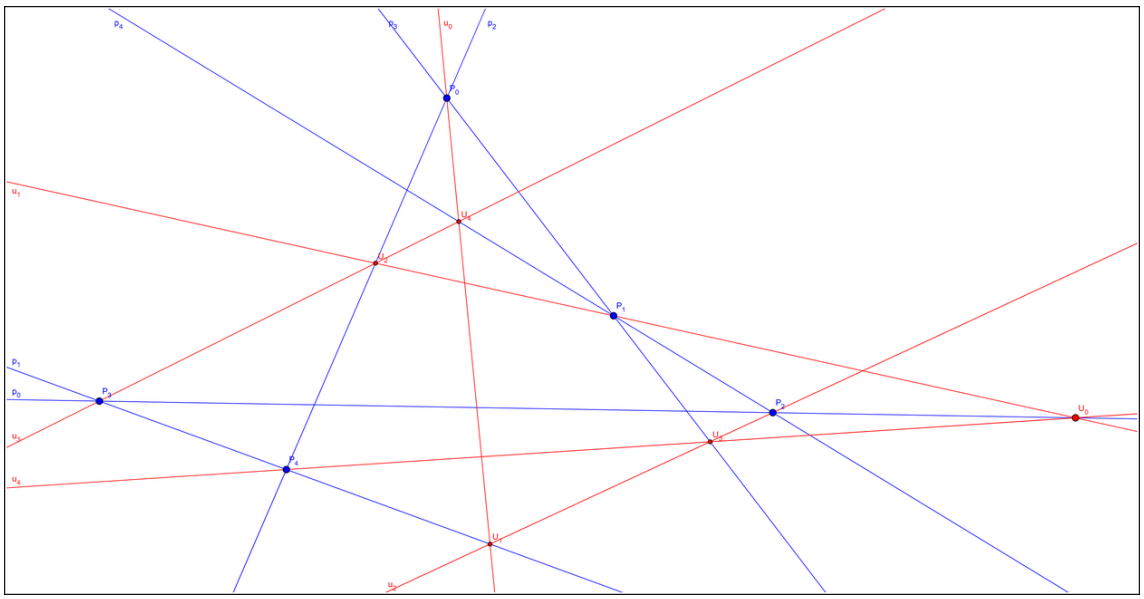

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Message: #2021
Date: 30/10/2016 12:54:55
From: chris.vantienhoven
Subject: Cubics for Perspective QA-Triple Triangles

Dear Eckart,
I made a reference to the two Desmic triples of Triple Triangles in QA-Tr-1 and QA-Tr-2 in EQF.
Best regards,
Chris

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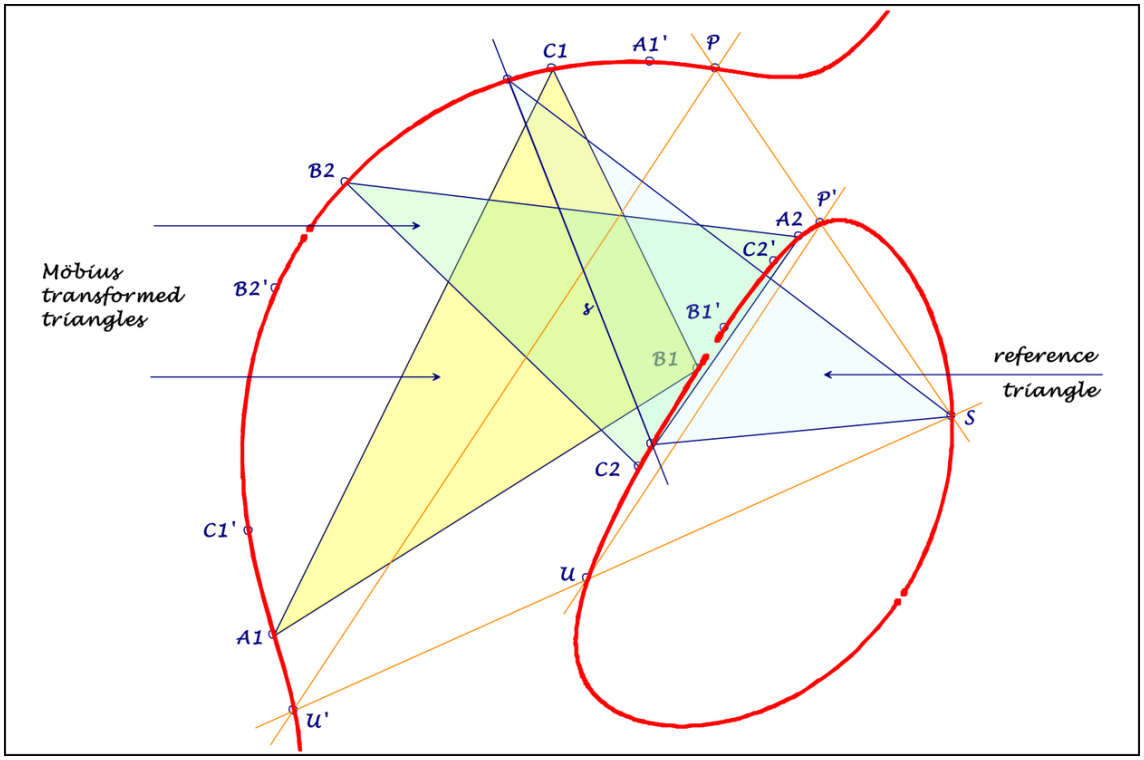
Message: #2022
Date: 30/10/2016 7:29:09
From: eckart_schmidt@t-online.de
Subject: Generalized Cyclologic Center Chain in Quad ...

Dear Tsihong Lau, dear Bernard, dear Chris,
Tsihong Lau has defined generalized cyclologic centers and asked for their loci. Here an answer for
... Möbius transformed triangles
A1B1C1, A2B2C2, which are always cyclologic (see #2009).
The locus for generalized cyclologic centers
... is a pivotal isogonal circular cubic
(see attached file).
Let P and U the cyclologic centers of A1B1C1 and A2B2C2,
... and P´ and U´ the generalized cyclologic centers
    of the 1st generation (see #1984),
... then PU´ and UP´ are parallel
... ... and give with their point at infinity
        the pivot of the cubic.
... PP´ and UU´ intersect in the center S
    of the Möbius transformation Tf,
... ... which swaps P and U as well as P´ and U´
        as well as the vertices of the two triangles.
... Define for a triangle vertex Ai a partner Ai´,
... ... which is the intersection of SAj
        and a parallel to PU´ through Ai,
... ... analog there are pairs (A1, A1´), (A2,A2´),
        (B1,B1´), (B2,B2´), (C1,C1´), (C2,C2´),
... ... all points of the cubic.
... Consider these pairs as isogonal conjugated
    wrt a reference triangle with vertex S,
... ... then the triangle is constructable and gives the
reference triangle for the cubic.
... ... (Construction: A1A2^A1´A2´, B1B2^B1´B2´, C1C2^C1´C2´
         are points of the opposite side s of S and the vertices
         are the intersections of s and Tf(s).)
... The cubic contains
... ... the vertices of triangles A1B1C1 and A2B2C2,
... ... the center S of the Möbius transformation,
... ... A1´,A2´,B1´,B2´,C1´,C2´,
... ... A1B2^A2B1, A1C2^A2C1, B1C2^B2C1,
... ... A1B1^A2B2, A1C1^A2C2, B1C1^B2C2,
... ... A1P^A2U, B1P^B2U, C1P^C2U, A1U^A2P, B1U^B2P, C1U^C2P.
Two Möbius transformed triangles are generalized cyclologic wrt any two points on this cubic.
Final remark wrt cyclologic QA-triple triangles (see QA-Tr-4):
There are Möbius transformed QA-Triple triangles wrt:
... QG-P1 / QL-P17, QG-P3 / QL-P17, QG-P5 / QL-P4,

... QG-P9 / QL-P1, QG-P9 / QL-P4, QL-P1 / QL-P17.
Best regards Eckart



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Message: #2023
Date: 01/11/2016 11:22:57
From: eckart_schmidt@t-online.de
Subject: Cyclologic Triangles on QA-Cu1

```
Dear Tsihong Lau, dear Bernard, dear Chris,
take two points X and Y on QA-Cu1
... and their triangles XaXbXc and YaYbYc wrt the Möbius
transformations of the Miquel triangle QA-Tr2.
... XaXbXc and YaYbYc are cyclologic.
... The cyclologic centers P and U lie on QA-Cu1,
... as well as the generalized cyclologic centers
    of the 1st generation P' and U'
... with PU' // P'U // asymptote of QA-Cu1.
... P and U' as well as P' and U are isogonal conjugated
    wrt QA-Tr2.
... QA-Cu1 is the locus
    for the generalized cyclologic centers of XaXbXc and YaYbYc.
... If X = QA-P4, then XaXbXc = QA-Tr1.
... If X = point at infinity of the asymptote,
    then XaXbXc = QA-Tr2.
... If X = QA-P3, then XaXbXc = QA-triple triangle of QG-P16.
... If X = QA-P41, then XaXbXc = QA-triple triangle
... ... of a new QG-point: QG-L1 ^ QG-P1.QA-P4.
... If X = Q = intersection of QA-Cu1 and its asymptote,
... ... then XaXbXc  = QA-triple triangle
        of a new QG-point: CSC(Q).
Best regards Eckart
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Message: #2024
Date: 03/11/2016 11:33:54
From: bernard.keizer
Subject: Cubics for Perspective QA-Triple Triangles

Dear Eckart, dear Chris
I'm always trying to follow the developments of the cubics of generalized cyclologic centers and of perspective QA-triple triangles.
The properties seem to be the same ...
1) I regret that noone of you answered my message 1947 about the 2nd hidden Desmic (the 2 cubics interesecting in the 9 DT vertices). I would be pleased if you could have a look and tell me if it is obvious or already well-known (in this case, a reference would be appreciated). The link between the 2 cubics seem reciprocical (starting with one give the other) and there are plenty of harmonic divisions on the figur ...
2) Looking again at the messages 1994 and 2017 from Eckart, I find a contradiction between the 2nd example in 1947 and the 2nd in 2017, both from Eckart : QA-Cu7 doesn't contain the QA-vertices, so I suppose it's another cubic ...
3) Summarising the circular cubics, I find :
a) for any triangle taken as Miquel triangle QA-Tr2 3 QL-Cu1 with Miquel point in a vertice and Newton Line the perpendicular bisector of the opposite side (the 3 Newton Lines concur in the circumcenter of the Miquel triangle).
b) with a point QA-P9 as focus on the circumcircle of this triangle 1 QA-Cu1 (cutting the asymptote in the diametral point of the focus)
c) with any point on QA-Cu1 taken as QA-P4 : it's Möbius transform give the DT and the 4 points having QA-P4 as tangential are the fixed points of the isoconjugation. There are 3 QL-Cu1 with Miquel point in a vertice of the Miquel triangle and Newton Line the line through the middles of 2 opposite sides of the QA and the corresponding side of DT (the 3 Newton Lines concur in QA-P1).
There are 2 more QL-Cu1, one with QA-P41 as Miquel point and the perpendicular bisector of QA-P2QA-P4 as Newton Line (this line is the isoconjugate of the circumcircle of DT) .THis cubic is non pivotal quasiisogonal wrt DT and pivotal isogonal wrt QA-P2QA-P4QA-P41. The other QL-Cu1 is non pivotal isogonal wrt the Miquel triangle and pivital isogonal wrt QA-P3QA-P4QA-P9.
My question in message 1995 also remains : for a QA-Cu1, there are an infinity of the 2 last QL-Cu1 (one of each type for every point of QA-Cu1 taken as QA-P4) ; conversely, for a given QL-Cu1, how could we find the original QA-Cu1 ?
Did I miss something and are there other interesting cubics to be considered ?

Thanks in advance for your attention and your help !
Best regards
Bernard

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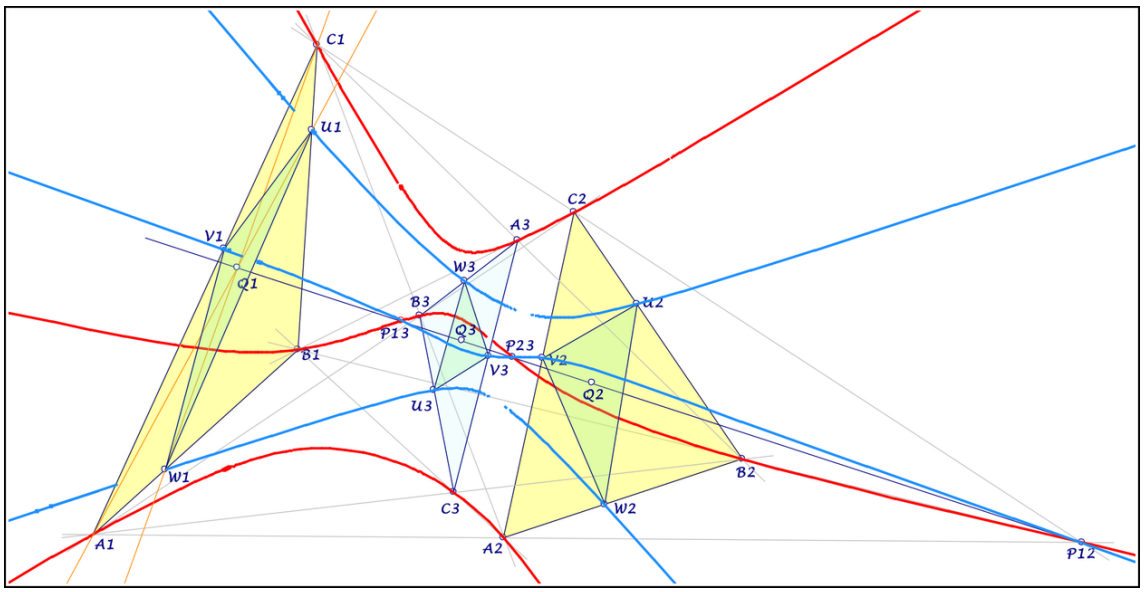
Message: #2025
Date: 04/11/2016 11:35:24
From: eckart_schmidt@t-online.de
Subject: Cubics for Perspective QA-Triple Triangles

Dear Bernard, dear Chris,
first an excuse to Bernard, that I have not answered several messages, but I was fascinated of cyclologic centers.
Wrt Bernard´s #2024:
2) The 2nd example in #2017 has no relationship to cyclologic triangles. It is another cubic as in #1994, but it bears also the vertices of the QA-triple triangles of QG-P1, QA-P18, QA-P19.
1) Wrt the 2nd hidden Desmic (see attached file): I am not familar with Desmic systems, so I try to describe the constellation in Bernard´s #1947 and Chris´ #1616 in my nomination - please excuse:
Starting with two perspective triangles A1B1C1 and A2B2C2 with perspector P12,
... there is a 3rd triangle A3B3C3
... ... with A3=B1C2^B2C1, B3=A1C2^A2C1, C3=A1B2^A2B1,
... ... perspective with A1B1C1 in P13=A1A3^B1B3^C1C3,
... ... perspective with A2B2C2 in P23=A2A3^B2B3^C2C3,
... with collinear perspectors
... and a pivotal isocubic:
... ... one triangle as reference triangle,
... ... isoconjugation,
        swapping the vertices of the other two triangles,
... ... pivot in the perspector of the other two triangles,
... ... bearing the vertices of the three triangles
        and their perspectors.
I think, the three QA in Bernard´s Desmic system are A1B1C1P23, A2B2C2P13, A3B3C3P12.
I hope, I describe the "hidden Desmic system" in a right way:
Let Qk be the 4th harmonic conjugate of Pij wrt Pik and Pkj
... and UkVkWk the cevian triangle of Qk wrt AkBkCk,
... then U1V1W1, U2V2W2 and U3V3W3 are pairwise perspective
... with the same perspectors,
... leading to three QA of the hidden Desmic system:
    U1V1W1P23, U2V2W2P13, U3V3W3P12
... and analog to a 2nd cubic
... intersecting the 1st cubic in the perspectors.
Final remark: UiViWi, UjVjWj and AkBkCk have as common perspector Pij.
Best regards Eckart
PS: Reading once more Bernard´s #1947, there remain differences wrt the properties, perhaps my interpretation is wrong.



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Message: #2026
Date: 04/11/2016 12:03:20
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

Dear all,
There are two quadrangles = eight points in
Miquel's six circle and perpendicular line and
parallel line theorems. According to Cayley-
Bacharach theorem
Cayley-Bacharach theorem - Wikipedia (
https://en.wikipedia.org/wiki/Cayley%E2%80%93Bacharach_theorem)
Every cubic curve C1 on an algebraically closed field that
passes through a given set of eight points P1, ..., P8 also
passes through a certain (fixed) ninth point P9, counting
multiplicities
These eight points determine a fixed ninth point.
Any idea?
Best regards,
Tsihong Lau

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Message: #2027
Date: 05/11/2016 3:33:03
From: tsihonglau
Subject: Generalized Cyclologic Center Chain in Quadrangle/Quadrilateral

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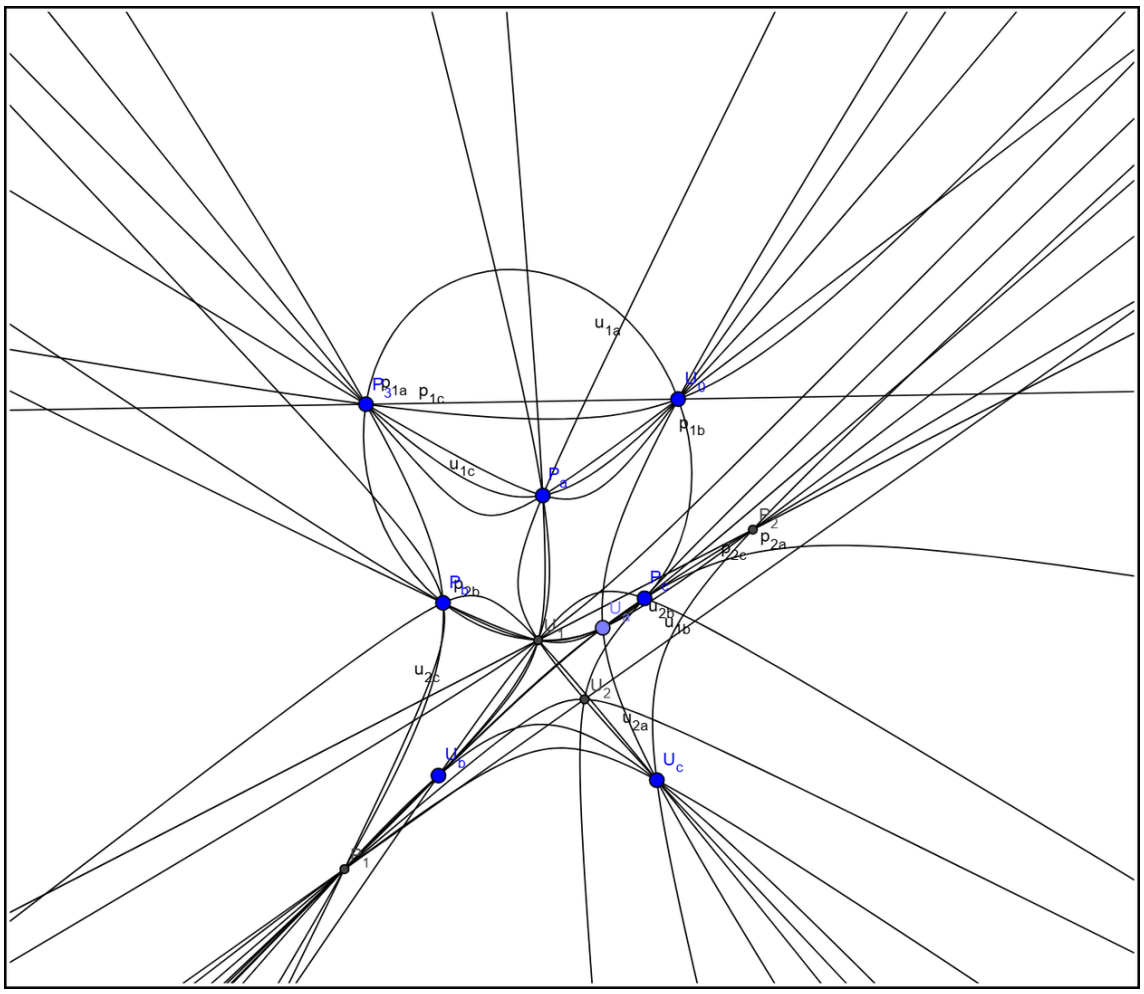
Dear all,
This message is a reply to Eckart's #1986.
Sorry! I would like to post this long ago.
Given a pair of generalized circular points (P0+0,U0+0) and a a pair of generalized cyclologic triangles PaPbPc and UaUbUc, the pair of cyclologic centers is (P1,U1).
In other words, conics PaUbUcP0+0U0+0,U aPbUcP0+0U0+0, UaUbPcP0+0U0+0 concur at P1
and
   UaPbPcP0+0U0+0,
   PaUbPcP0+0U0+0,
   PaPbUcP0+0U0+0 concur at U1.
Given two ordered pairs of generalized circular points (Q,R) and (S,T) and their same corresponding pair (P1,U1)
of cyclologic centers, conics PaUbUcU1Q,UaPbUcU1Q,UaUbPcU1Q concur at S and conics UaPbPcP1R,PaUbPcP1R,PaPbUcP1R concur at T.
(Q,R) and (S,T) are called a dual pair of generalized circular points with respect to the same pair (P1,U1) of
generalized cyclologic centers. It is easy to check (Q,R) and (S,T) are interchangeable.
With respect to the generalized cyclologic center pair (P1,U1), we can construct a chain of dual pairs of generalized circular points.
dual <->
   (P0+0,U0+0)<->(P0-0,U0-0)
   (U0+0,P0+0)<->(P0+1,U0+1)
   (U0+1,P0+1)<->(P0+2,U0+2)
and so on...
   (U0-0,P0-0)<->(P0-1,U0-1)
   (U0-1,P0-1)<->(P0-2,U0-2)
and so on...
The generalized cyclologic center pair (P2,U2) corresponds to the generalized circular point pair (P1,U1).
Then lines P1U2,P2U1,P0+0U0+0,P0-0U0-0,P0+1U0+1,P0-1,U0-1,P0+2U0+2,P0-2U0-2, and so on... concur.
I tried to apply the above construction to parallelogic triangles but failed!
Best regards,
Tsihong Lau



cyclologic_generalized.ggb

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Message: #2028
Date: 05/11/2016 4:05:02
From: bernard.keizer
Subject: Cubics for Perspective QA-Triple Triangles

Dear Eckart,
thanks this time you your quick answer !
Wrt the cubics in 1994 and 2017, they intersect in the 9 points QG-P1, QG-P18 and QG-P19 ; the one in 1994 is the non-pivotal circular QA-Cu7 wrt DTand the other is a pivotal (non circular) isocubic wrt DT with pivot the point described as P23.
By the way, I've found the answer to my question about QA-Cu7 : it has many QA-Cu1 with QA-P9 in the Miquel point and the vertices of the Miquel triangle the 3 other intersections of QL-Cu1 with any circle through QL-P1 ; the points QA-P4 and QA-P3 are any copple of Cl-S conjugate points (isogonal wrt the Miquel triangle).
For the hidden Desmic, your simple and beautiful figur helped me to understand my own construction (I wonder how you draw cubics so easily, my own figurs are most of the time rather ugly !)
First, you may complete your figur with 3 points U4 as intersection of B1C1, B2C2, B3C3, V1W1, V2W2 and V3W3, V4 as intersection of A1C1, A2C2, A3C3, U1W1, U2W2 and U3W3 and W4 as intersection of A1B1, A2B2, A3B3, U1V1, U2V2 and U3V3.
Then you could adopt a more symmetric notation by naming P12 D3, P23 D1 and P13 D2.
The 3 QA's ABCD 1 to 3 form a Desmic system (12 points on 16 lines, 3 points on a line and 4 lines through each point).
There is only one red cubic through the 3*4 = 12 points ABCD 1 to 3 and through the 3*3 = 9 vertices of their DT's.
There are 16 blue cubics like yours, each through the 3 points on one of the 16 lines of the Desmic system and 9 of the 12 points UVW 1 to 4.
The 12 points UVW 1 to 4 form a 2nd Desmic system, the 3 QA's being U 1 to 4, V 1 to 4 and W 1 to 4.
And that is my 2nd cubic !
The particularity is that the 2 sets of 3 QA's have the same DT vertices (one DT of a QA in the 2nd set is formed by 3 vertices of the 3 DT's of the 1rst with one of each of the 3). The 2 cubics intersect in the 9 DT vertices.
(For example, P12 and C3 are harmonic wrt W1 and W2 and the line through the 4 points cuts A3B3 and W3W4 in the same point, which is one of the 9 DT vertices of the 2 sets of QA's).
Conversely, if you start with the 2nd Desmic system, the construction leads the the 1rst system, so both systems and cubics are reciprocical !

I hope everything is clear now ! I had already noticed that these harmonic properties play a fundamental role in the QA/QL geometry.
Thanks anyway for your attention ...
Best regards
Bernard

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Message: #2029
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Messages

Message number 2029 is not available in Yahoo groups.

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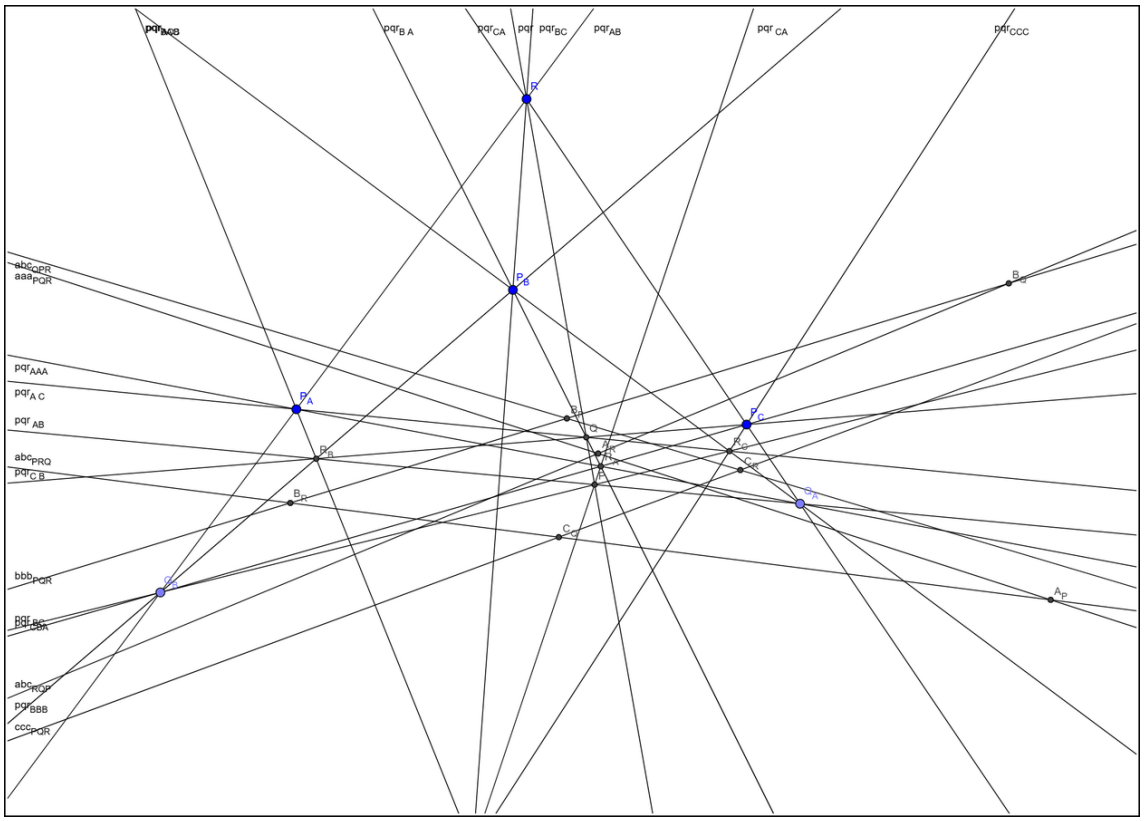
Message: #2030
Date: 05/11/2016 5:50:38
From: tsihonglau
Subject: Reye Configuration

Dear all,
I have difficulty in understanding the Desmic system on Chris' webpage.
So I try on myself and figure out a quadrangle aspect of Reye configuration.
Reye configuration - Wikipedia (https://en.wikipedia.org/wiki/Reye_configuration)

I use my notation. Given three quadrangles = 12 points PPaPbPc,QQaQbQc,RRaRbRc(in attachment PP_AP_BP_C, QQ_AQ_BQ_CRR_AR_BR_C instead), the incidence table:
16 lines
   PQR
   PQbRc
   PQcRa
   PQaRb
   PaQaRa
   PaQRc
   PaQbR
   PaQcRb
   PbQbRb
   PbQRa
   PbQaRc
   PbQcR
   PcQcRc
   PcQbRa
   PcQRb
   PcQaR
Then we say the three quadrangles and 16 lines form a Reye configuration.
The diapleural(diagonal) triangles of the three quadrangles are ApBpCp,AqBqCq,ArBrCr respectively(in attachment A_PB_PC_P,A_QB_QC_Q,A_RB_RC_R instead), the incidence table:
6 lines
   ApAqAr
   BpBqBr
   CpCqCr
   ApBrCq
   AqBpCr
   ArBqCp
In other words, the three diapleural triangles = 9 points are in Cayley-Bacharach theorem positions.
Cayley-Bacharach theorem - Wikipedia (https://en.wikipedia.org/wiki/Cayley%E2%80%93Bacharach_theorem)

(https://en.wikipedia.org/wiki/Cayley%E2%80%93Bacharach_theorem)

We should study the relationship between the Reye configuration and the Desmic system.
Best regards,
Tsihong Lau



reye_configuration.ggb

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Message: #2031
Date: 05/11/2016 6:06:55
From: bernard.keizer
Subject: Cubics for Perspective QA-Triple Triangles

Dear Eckart,
Sorry for the mistake, it's not QA-Cu7, but your 3rd example in message which leads to a Miquel circle through the Miquel point QL-P1.
For QA-Cu7, there are also an infinity of QA-Cu1 which have a QL-Cu1 as QA-Cu7 : the point QA-P41 is QL-P1, one of the QL2P2a and b is QA-P2, the other QA-P4 ; any circle through QA-P2 cuts QL-P1 in 3 other points giving the vertices of DT and the isoconjugation which swaps QA-P4 and QA-P41 wrt this DT give the cubic QA-Cu1 ...
Best regards
Bernard

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Message: #2032
Date: 05/11/2016 6:23:02
From: bernard.keizer
Subject: Reye Configuration

Dear Tsihong Lau,
For my understanding, the Desmic system and the Reye configuration are the same object !
Desmic system applies to 3 QA's which belong to the same isopivotal cubic ; in this case, the 3 tangentials are aligned and form with the 3 DT's another Desmic system on the same cubic.
Reye configuration applies to 12 points on 16 lines, 3 points on a line and 4 lines through a point. The 16 lines form 12 QL's, each point belonging to 6 different QL's.
Both are the plane projection of a space configuration with tetrahedrons instead of QA's.
In the space, the 12 points could be the 8 vertices of a cube, it's center and the 3 infinity points of the directions of it's sides.
Best regards
Bernard

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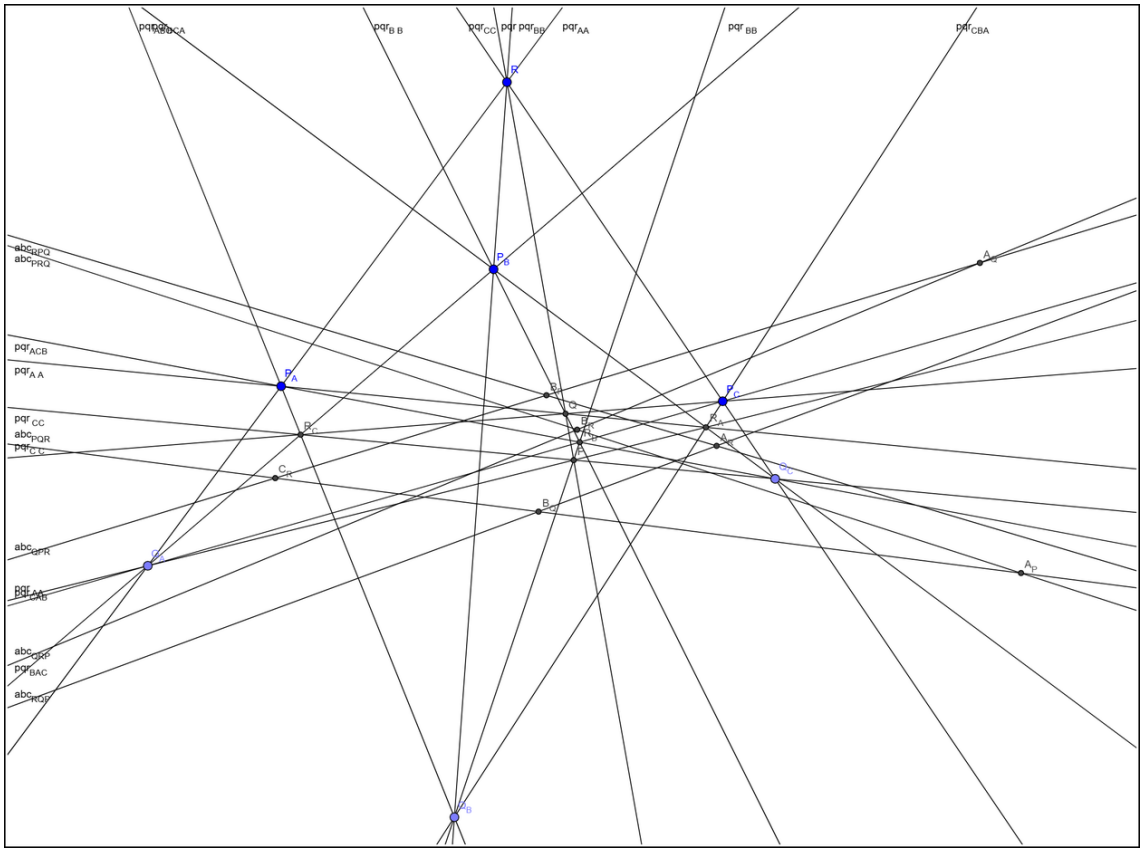
Message: #2033
Date: 06/11/2016 2:47:24
From: tsihonglau
Subject: Reye Configuration

Dear all,
I do not think what I gave and Desmic system is the same thing. My topic is misleading. Reye configuration is unordered. I gave notations for points and lines. So they are ordered. Desmic system is ordered, too. I cannot access EQF right now! I think Desmic system is:
16 lines
   PQR
   PQaRa
   PaQRa
   PaQaR
   PQbRb
   PbQRb
   PbQbR
   PQcRc
   PcQRc
   PcQcR
   PaQbRc
   PaQcRb
   PbQcRa
   PbQaRc
   PcQaRb
   PcQbRa
6 lines
   ApBqCr
   ApBrCq
   AqBrCp
   AqBpCr
   ArBpCq
   ArBqCp
In my system,
   PQR
   PaQaRa
   PbQbRb
   PcQcRc
not only PQR are collinear but the other three triples are collinear, too. So I said "I try on myself and figure out a quadrangle aspect of Reye configuration". On the other hand, Desmic system describe three pairwise perspective triangles. The perspectors arecollinear, ie PQR. To sum up, my system is a triple of ordered quadrangles on Reye configuration and Desmic system is a triple of pairwise perspective triangles on Reye configuration.
Best regards,

Tsihong Lau



desmic_system.ggb

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Message: #2034
Date: 06/11/2016 11:32:39
From: bernard.keizer
Subject: Cubics for Perspective QA-Triple Triangles

Dear Eckart,
Last idea this morning !
I forgot to precise that U4, V4 and W4 are aligned, but that's obvious ...
As the 2 systems of ABCD 1 to 3 and UVW 1 to 4 are mutually reciprocal, you have also 16 cubics through 3 aligned points UVW and 9 corresponding points of the 12 BCD (one is for example through the ABC 1 to 3 and through the aligned points U4V4W4).
Best regards
Bernard

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Message: #2035
Date: 07/11/2016 10:39:24
From: bernard.keizer
Subject: Desmic, Reye and duals

Dear Chris, dear Eckart, dear Tsihong Lau
Here some considerations about Desmic, Reye and their duals
1) Desmic is a system of 3 pairwise 4 times perspective QA's (not triangles !), the vertices of one QA being the 4 perspectors of the 2 others. The 12 vertices belong to the same isopivotal cubic. What can be said about the 18 sides of the QA's ?
2) A QA is 4 times perspective with itself : the perspectors are the 3 vertices of DT and the tangential of it's 4 vertices (when the vertices are in the same position). That's why the 3 QA's formed by the DT's and the tangentials form also a Desmic system on the same cubic.
3) the 12 vertices of 3 QA's forming a Desmic system are in a Reye configuration : they are on 16 lines, 3 points on a line and 4 lines trough a point and the lines form 12 QL's, 6 QL's for each point, 3 QL's for each line and 6 points and 4 lines for each QL.
4) If the cubic is ordinary, the 3 QA's have different Miquel triangles and the QL's have different Miquel points. If the cubic is circular, the 3 QA's have the same Miquel triangle and the 12 QL's have the vertices of this Miquel triangle as Miquel points (4 QL's for each vertice).
5) The dual of a Desmic system is a system of 3 QL's ; I don't know it's properties, in particular the 18 vertices, but it's worth studying !
6) The dual of the Reye configuration is formed by the 12 lines of these 3 QL's ; they intersect in 16 points, 4 points on a line and 3 lines through each point and the points form 12 QA's, 3 QA's for each point, 6 QA's for each line and 4 points and 6 lines for each QA.
I don't know either the properties of these QA's. Do they form several Desmic systems ? Do the 16 points belong to the same quartic ?
Is there a particular property if the 3 initial QA's are on a circular cubic ?
I would be very happy if someone could give answers to these or part of these questions ...
Thanks anyway for your attention !
Best regards
Bernard

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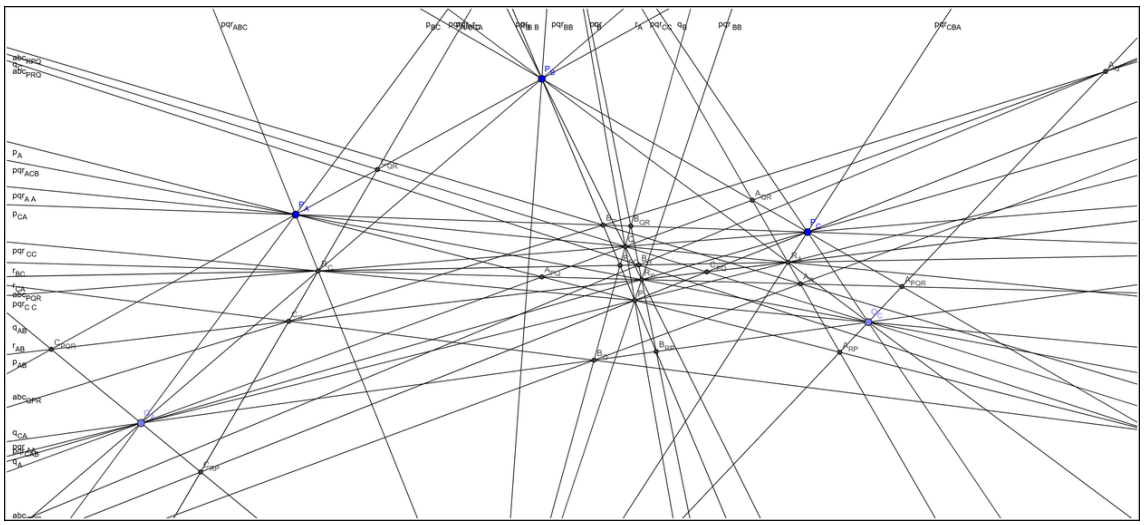
Message: #2036
Date: 07/11/2016 3:37:49
From: tsihonglau
Subject: Reye Configuration

Dear all.
I think I found a transpose Reye configuration.
It may be found already! But I think it is as important as the dual quadrangle/quadrilateral in my webpage.
http://lth.name/geometry/duality.html
I use the notations in #2033
Given quadrangles PPaPbPc,QQaQbQc,RRaRbRc
concurrency - lines
    Apqr   - PbPc,QbQc,RbRc
    Bpqr   - PcPa,QcQa,RcRa
    Cpqr   - PaPb,QaQb,RaRb
    Aqr   - PbPc,QQa,RRa
    Bqr   - PcPa,QQb,RRb
    Cqr   - PaPb,QQc,RRc
    Arp   - PPa,QbQc,RRa
    Brp   - PPb,QcQa,RRb
    Crp   - PPc,QaQb,RRc
    Apq   - PPa,QQa,RbRc
    Bpq   - PPb,QQb,RcRa
    Cpq   - PPc,QQc,RaRb
quadrangles ApqrAqrArpApq, BpqrBqrBrpBpq, CpqrCqrCrpCpq form a transpose Reye configuration.
The diapleural triangles of PPaPbPc,QQaQbQc,RRaRbRc are ApBpCp,AqBqCq,ArBrCr respectively.
The diapleural triangles of ApqrAqrArpApq,BpqrBqrBrpBpq,CpqrCqrCrpCpq are ApAqAr,BpBqBr,CpCqCr respectively.
That's why ApqrAqrArpApq, BpqrBqrBrpBpq, CpqrCqrCrpCpq is called the transpose Reye configuration of PPaPbPc,QQaQbQc,RRaRbRc.
Like a transpose matrix, the latter is the transpose Reye configuration of the former.
To sum up, the transpose Reye configurations have distinct points and lines but share the same(transpose) diapleural triangles. We should study them very much!
Best regards,
Tsihong Lau



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Message: #2037
Date: 07/11/2016 4:12:59
From: eckart_schmidt@t-online.de
Subject: Cubics for Perspective QA-Triple Triangle

```
Dear Bernard,
I am quite confused, trying to answer the last question of your
#2024.
Here only my simplified sight up to now.
Starting with a triangle Tr and the background of QA-Cu1 and
QL-Cu1, I see the following cubics:
... Type 1: three QA-Cu1 with Miquel triangle Tr,
... ... isogonal conjugation,
... ... pivot: infinity point of one triangle side.
... Type 2: three QA-Cu1 with Miquel triangle Tr,
... ... isogonal conjugation,
... ... pivot: infinity point of the perpendicular bisector
              of one triangle side.
... Type 3: three unipartite QL-Cu1 with Miquel point M
            in a vertex of Tr,
... ... bearing the other two vertices,
... ... Newton line: opposite sideline of M.
... Type 4: three bipartite QL-Cu1 with Miquel point M
            in a vertex of Tr,
... ... bearing the other two vertices,
... ... Newton line: perpendicular bisector
        of the opposite side of M.
Properties:
... Type 1 and 4 give the same cubics.
... Type 1/4 are tangent to type 3.
... Type 1/4 intersects type 2 orthogonal
... ... in the in- and excenters of Tr.
Best regards Eckart
PS. Thanks for your detailed explanations in #2028 and 2031.
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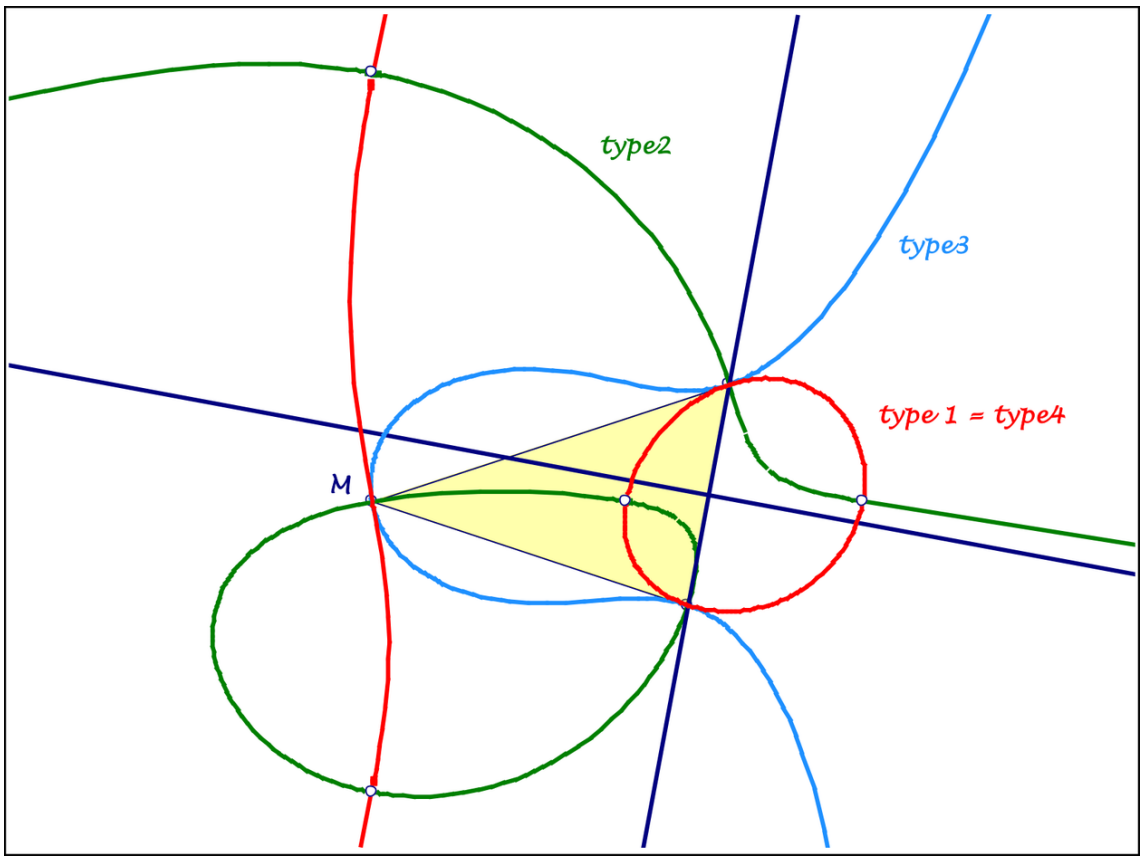

2016-11-05.pdf

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Message: #2038
Date: 07/11/2016 6:05:53
From: bernard.keizer
Subject: Reye Configuration

Dear Tsihong Lau,
As said earlier, I'm not able to open your zipped Geogebra files!
But if I'm not wrong, your construction is exactly what I described in message 1947 as hidden Desmic.
Please read the messages 1947 and 2024, Eckart's answer (with figure) in 2025 and my last reply in 2028.
When I say Desmic and Reye are the same object (Desmic describes 3 QA's and Reye considers the 12 points)!
Best regards
Bernard

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Message: #2039
Date: 07/11/2016 11:03:00
From: tsihonglau
Subject: Reye Configuration

---In Quadri-Figures-Group@yahoogroups.com, wrote :
> >>But if I'm not wrong, your construction is exactly what I described in
> message 1947 as hidden Desmic.
> >>Please read the messages 1947 and 2024, Eckart's answer (with figure) in
> 2025 and my last reply in 2028.
> >>When I say Desmic and Reye are the same object (Desmic describes 3 QA's
> and Reye considers the 12 >>points) !

1. Sorry I did not study your 1947 before. Yes!
Your hidden Desmic and my transpose Reye configuration is the same thing.
Eckart's 2024 is another. But as I say, Chris' Desmic system is from the aspect of three pairwise perspective triangles.
So the term "hidden Desmic" is not appropriate.
"Transpose Reye" is better. These two triples
of quadrangles have the 3 "tranpose" diapleural triangles.
2. I found the harmonic conjugates as yours but did not mention them.
3. I guessed the two cubics intersect at the three transpose triangles but I have no tool to construct
the cubics.
4. I derived the "transpose Reye" from 3 dimensional Reye configuration. It is much like a plane in 3 dimensional projective plane must be Desarguesian! I do not know if Reye configuration exists in a Pappian or Desarguesian plane. Please refer to topic #1997.
5. I conjecture a "line quartic" (quartic made of tangent lines) passes(tangent) through 16 lines of a Reye configuration since each point lie on 4 lines. But this "line quartic" should be a point curve with degree 12! So We had better study the dual Reye configuration. A point quartic passes through the 16 points!
Best ragards,
Tsihong Lau

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Message: #2040
Date: 07/11/2016 11:43:25
From: bernard.keizer
Subject: Reye Configuration

Dear Tsihong Lau,
I'm glad we begin to converge!
Please read completely the Wikipedia definition of Desmic system and of Reye configuration !
Then read my message 2035 (specially the 1rst point)!
Perhaps you should also reread Chris definition of Desmic : the construction starts with 2 triangles and a perspector, but the complete figure ends with 3 pairwise 4 times perspective QA's, the vertices of one QA being the 4 perspectors between the 2 others!
I've explained this now several times and Chris himself did it perfectly clearly in the message 1616 in answer to the message 1614 from Eckart ...
By the way, the 2 cubics intersect in the 9 vertices of the 3 DT's !
It's possible to draw an acceptable cubic with Geogebra: if you are interested, I will be pleased to write the method (using the animation of a variable point and the trace of other linked points).
Best regards
Bernard
PS for Eckart: did you succeed in drawing my 2nd cubic after my reply in message 2028 ? Maybe it could convince Tsihong Lau ...

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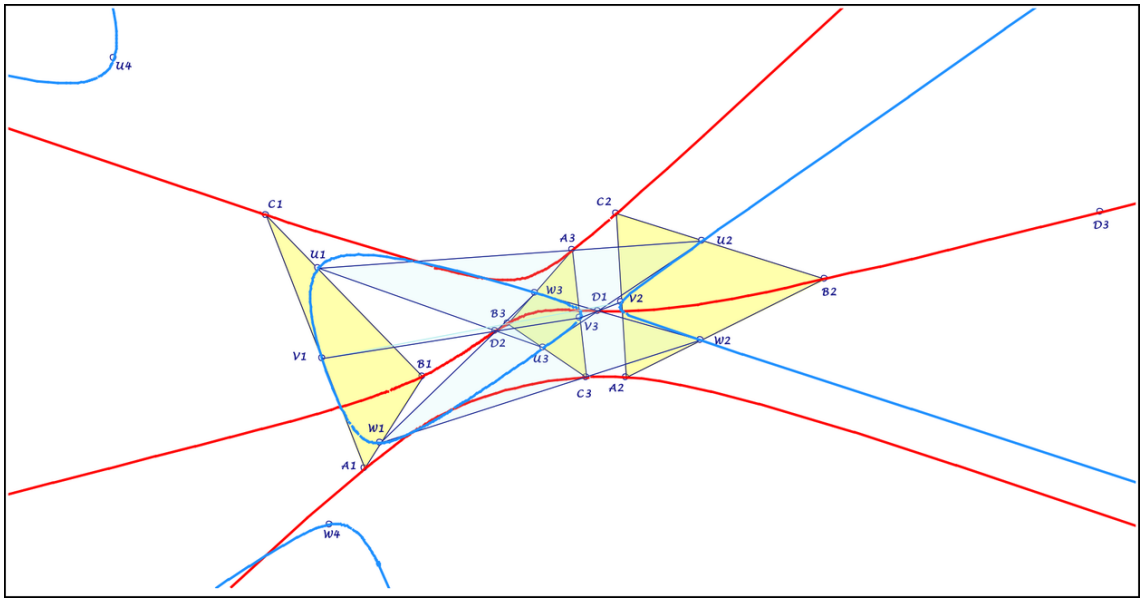
Message: #2041
Date: 08/11/2016 1:42:54
From: eckart_schmidt@t-online.de
Subject: Reye Configuration

Dear Bernard, dear Tsihong Lau,
I try to follow your discussion wrt Desmic systems and Reye configurations.
I hope, there are the right cubics in the attached file, described in my nomination:
Let AiBiCi (i=1,2,3) be pairwise perspective triangles with perspectors D1, D2, D3 (the three QA are AiBiCiDi).
Let UiViWi (i=1,2,3) be the cevian triangles wrt AiBiCi for the 4th harmonic point of Di wrt Dj, Dk.
Then U1U2U3, V1V2V3, W1W2W3 are three pairwise perspective triangles with perspectors U4, V4, W4,
which lead to a 2nd Desmic system / Reye configuration.
The corresponding cubics can be constructed as follows:
(1) reference trangle e.g. A1B1C1; isoconjugation swaps A2,A3 or B2,B3 or C2,C3; pivot D1.
(2) reference triangle e.g. U1U2U3; isoconjugation swaps V1,W1, or V2,W2 or V3,W3; pivot U4.
Best regards Eckart



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Message: #2042
Date: 08/11/2016 3:20:30
From: bernard.keizer
Subject: Reye Configuration

Dear Eckart, dear Tsihong Lau,
These are in fact the 2 right cubics, which intersect in the 9 DT vertices, common to the 2 Desmic systems.
A DT from a system has a common vertice with each of the 3 DT of the other and vice-versa.
Obviously, the 2 cubics are also isopivotal cubics wrt each DT with correspondonding pivots and the DT form with the corresponding pivots 2 other Desmic systems. The 3 pivots on each cubic are aligned ...
Many thanks to Eckart for the figure !
Best regards
Bernard

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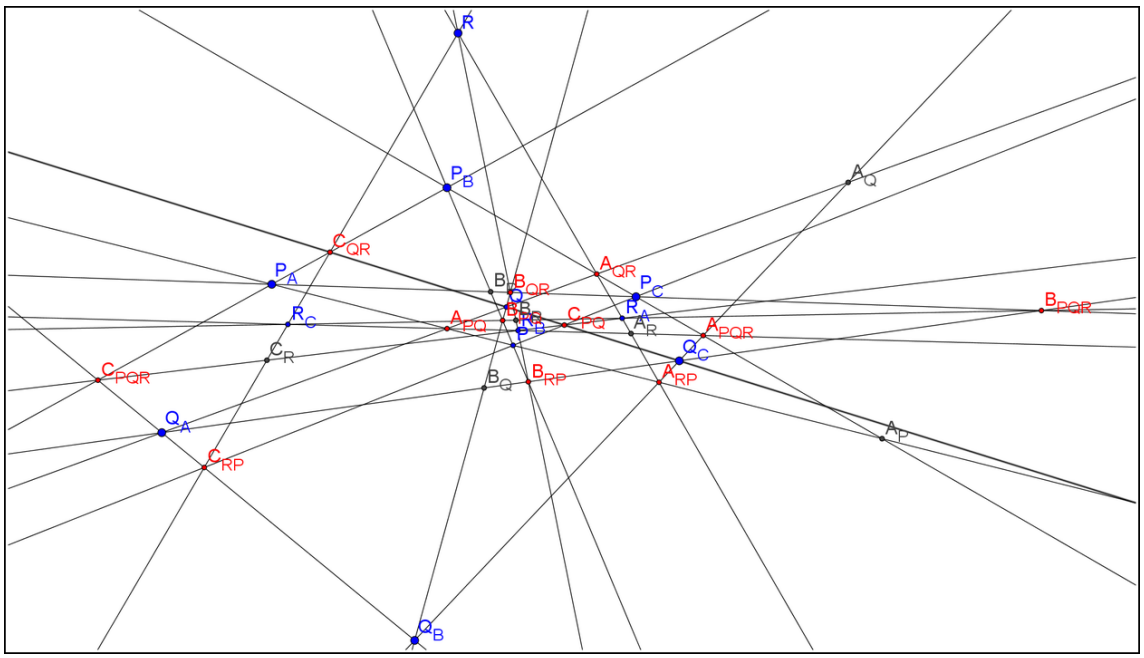
Message: #2043
Date: 08/11/2016 3:28:30
From: tsihonglau
Subject: Reye Configuration

Dear Eckart,
Your system is not "hidden Desmic"="transpose Reye".
My notations in #2036.
In attachment, uppercase footnotes..
Blue - first Desmic(or Reye) quadrangles PPaPbPc,QQaQbQc,RRaRbRc
Black - diapleural(diagonal)triangles ApBpCp,AqBqCq,ArBrCr
Red - second Desmic(or Reye) quadrangles
ApqrAqrArpApq,BpqrBqrBrpBpq,CpqrCqrCrpCpq
Black - diapleural(diagonal)triangles ApAqAr,BpBqBr,CpCqCr
You see diapleural triangles are transpose!
Black lines = diapleurals(diagonals) of two triples of
quadrangles.
The two "hidden Desmic" = "transpose Reye" share the same
diapleurals and diapleural vertices(ie transpose diapleural
triangles).
The two pairs of quadrangle vertices on the same line are
harmonic conjugates.
For example, PPa and ArpApq.
The above statements are not related to Desmic.
The following are:
The perspectors of the triple of pairwise perspective triangles
PaPbPc,QaQbQc,RaRbRc are P,Q,R.
Similarly the perspectors of the triple of pairwise perspective
triangles AqrArpApq,BqrBrpBpq,CqrCrpCpq
are Apqr,Bpqr,Cpqr.
Your system is important, too. But as I said before, "hidden
Desmic"="transpose Reye" is as important
as dual quadrangle/quadrilateral in quadrangle/quadrilateral
geometry.
Best regards,
Tsihong Lau



desmic_system.png

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Message: #2044
Date: 08/11/2016 4:02:35
From: tsihonglau
Subject: Reye Configuration

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---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> Your system is not "hidden Desmic" = "transpose Reye".
Sorry Eckart!  Your system is!  You did not show diapleural
vertices, I miscomprehended your drawing!
The transpose diapleural vertices are very important.
They are related to Pappus configuration!  I will post a new
topic about it!
Best regards,
Tsihong Lau
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Message: #2045
Date: 10/11/2016 9:22:36
From: Tran Quang Hung
Subject: Begonia point of quadrilateral

Dear geometers,
Let ABC be a triangle. A'B'C' is cevian triangle of a point P. A'',B'',C'' are reflections of P through B'C',C'A',A'B'. Then AA'',BB'',CC'' are concurrent at point B(P) which was called by Begonia point. This is from Darij in his web http://web.mit.edu/~darij/www/ Begonia.zip (http://web.mit.edu/~darij/www/Begonia.zip)
Let ABCD be a quadrilateral.
B(A) is Begonia point of A wrt triangle BCD.
B(B) is Begonia point of B wrt triangle CDA.
B(C) is Begonia point of C wrt triangle DAB.
B(D) is Begonia point of D wrt triangle ABC.
Then the lines AB(A), BB(B),CB(C),DB(D) are concurrent at point Be. I call this point by Begonia point of ABCD.
Another property with Begonia point
Prove that if ABCD is inscribed in circle (O) and AC cuts BD at P. Then lines AB(B),BB(A),CB(D),DB(C) are concurrent at a point on circle diameter OP.
Best regards,
Tran Quang Hung.

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Message: #2046
Date: 10/11/2016 9:58:46
From: eckart_schmidt@t-online.de
Subject: Begonia point of quadrilateral

Dear Tran Quang Hung,
the described Begonia point of a quadrangle is QA-P23.
Best regards Eckart

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Message: #2047
Date: 10/11/2016 11:49:42
From: Tran Quang Hung
Subject: Begonia point of quadrilateral

Thank you very much Mr Eckart for notice me the property QA-P23.
Best regards,
Tran Quang Hung.

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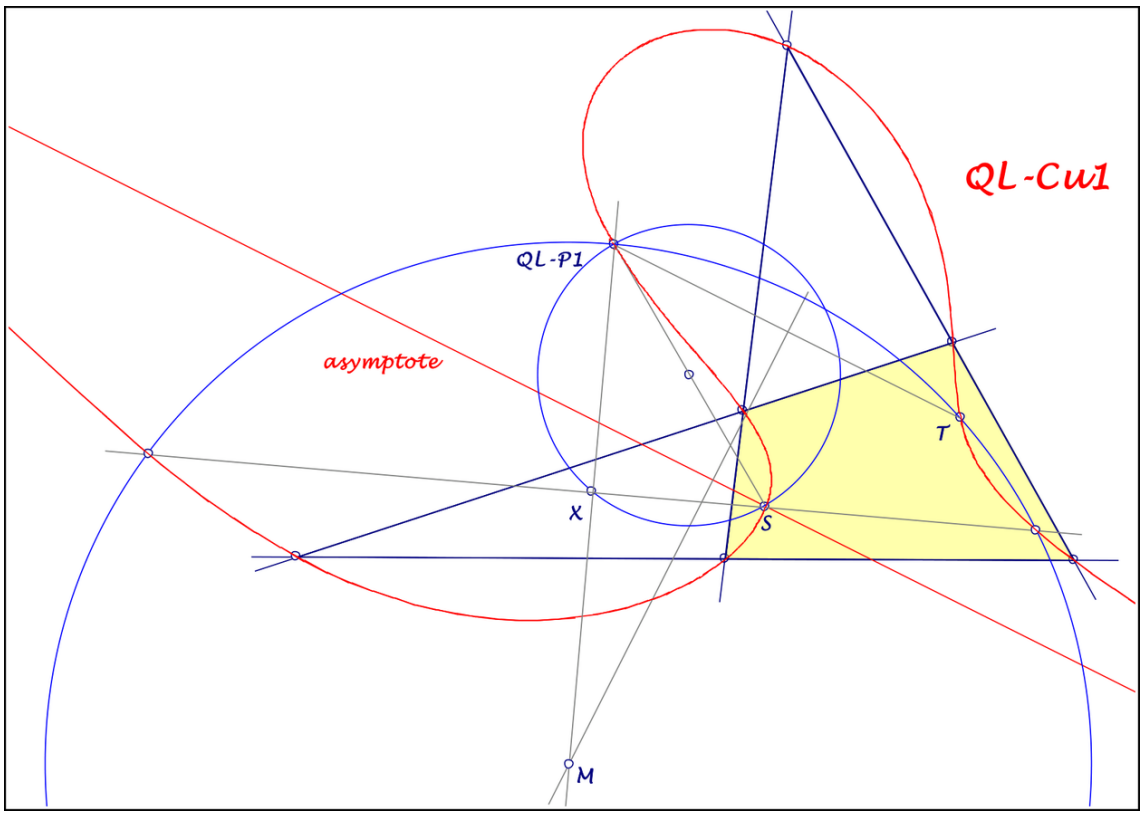
Message: #2048
Date: 13/11/2016 2:45:40
From: eckart_schmidt@t-online.de
Subject: Similar Constructions of QL-Cu1 and QA-Cu1

Dear Bernard, dear Chris,
here are similar constructions for QL-Cu1 and QA-Cu1 (see attached files):
QL-Cu1:
Let S be the intersection of QL-Cu1 and its asymptote and T its CSC-image (see EQF QL-Cu1).
... Let X be a point on the circle with diameter S.QL-P1
... and M the intersection of X.QL-P1
    and the perpendicular bisector of T.QL-P1.
... The circle round M through QL-P1 intersects SX on QL-Cu1.

QA-Cu1:
Let S be the intersection of QA-Cu1 and its asymptote,
... diametral point of QA-P9 on the circumcircle
    of the Miquel triangle QA-Tr2.
... Let X be a point on the circle with diameter S.QA-P9.
... Consider a perpendicular line L through QA-P9
    wrt the asymptote
... ... and its QA-Tr2-isogonal conic Co
... ... with 2nd intersection Y with X.QA-P9.
... The circle with diameter Y.QA-P9 intersects SX on QA-Cu1.
These constructions will be helpful for the cubic of generalized cyclologic centers of two cyclologic triangles.
Best regards Eckart



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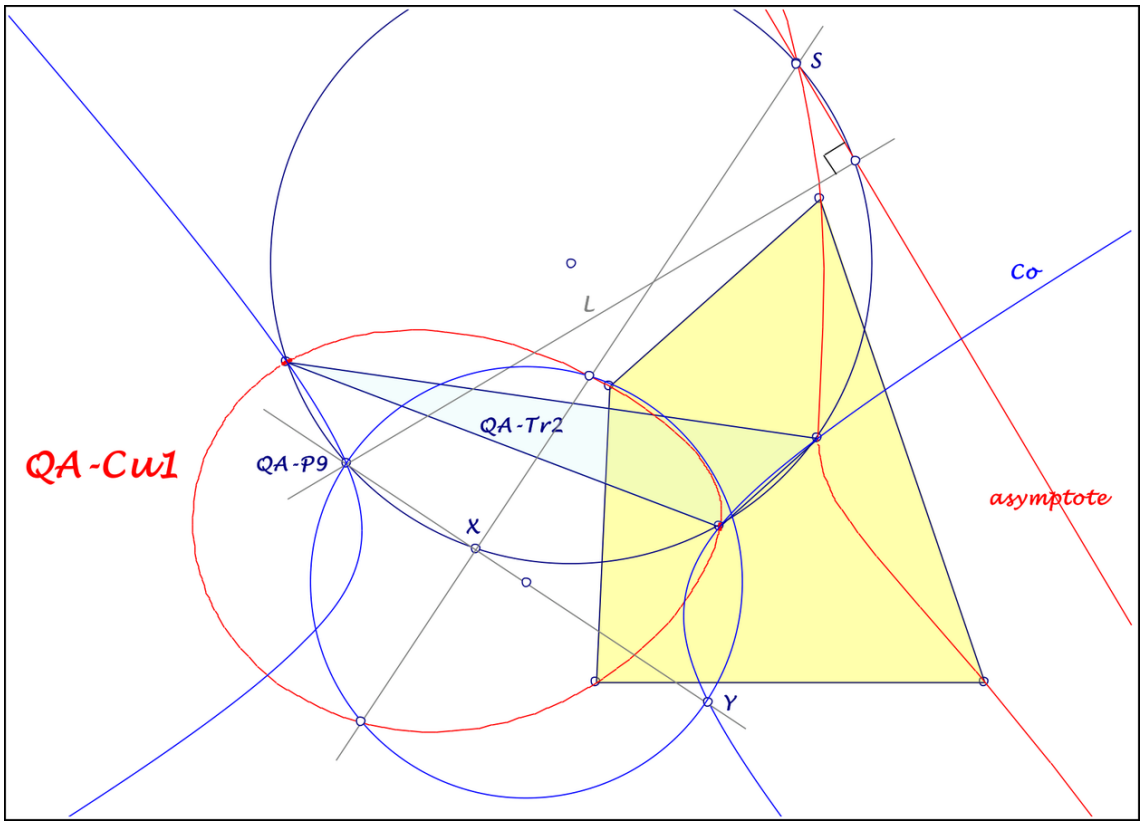

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Message: #2049
Date: 14/11/2016 10:32:14
From: bernard.keizer
Subject: New QA-Cux wrt DT

Dear Eckart,
It's well-known that the 3 Gergonne-Steiner points QA-P3 of the 3 QA's of a QL are on the Dimidium circle QL-Ci6 of the QLand dually that the 3 Dimidium circles QL-Ci6 of the 3 QL's of a QA intersect in the Gergonne-Steiner point QA-P3 of the QA. (properties in EQF, due to you).
Considering now the triangle of these 3 Gergonne-Steiner points, it appears that it is perspective with the diagonal triangle DT QL-Tr1 with perspector QL-P1 (this is obvious with the fact that each Gergonne-Steiner point is the perspector of the Miquel triangle and DT and that the Miquel point QL-P1 is each time one of the 3 vertices of the Miquel triangle with corresponding DT vertice one vertice of DT).
It follows that these 2 triangles (Gergonne-Steiner and DT) with perspector QL-P1 are the beginning of a Desmic system and of a new pivotal isocubic.
I cannot identify the pivots, in particular the one wrt DT.
Maybe you will
Best regards
Bernard

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Message: #2050
Date: 14/11/2016 7:33:00
From: Tran Quang Hung
Subject: Newton line passes through Euler-Poncelet point of cyclic quadrilate

---

Dear geometers,
Let ABCD be a cyclic quadrilateral with circumcircle (O).
P is a point on (O).
Let sa,sb,sc,sc be the Steiner lines of P wrt triangle BCD,CDA,DAB,ABC.
Then Newton line of (sa,sb,sc,sd) passes through Euler-Poncelet point of ABCD.
Best regards,
Tran Quang Hung.

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Message: #2051
Date: 14/11/2016 7:53:10
From: Tran Quang Hung
Subject: Circumcircles are concurrent in quadrilateral

Dear geometers,
Let ABCD be a quadrilateral.
AB cuts CD at E.
AD cuts BC at F.
AC cuts BD at G.
M,N are midpoints of AB,CD.
P,Q are midpoints of AD,BC.
S,T are midpoints of AC,BD.
Then the circles (EMN),(FPQ),(GST) are concurrent.
Best regards
Tran Quang Hung.

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Message: #2052
Date: 14/11/2016 8:09:38
From: Antreas Hatzipolakis
Subject: Circumcircles are concurrent in quadrilateral

Dear Tran
I did not draw a figure, but I see that the point of concurrence corresponds to vertex A of the quadrangle.
Similarly we get 3 more, corresponding to three other vertices.
How about the four points? Are they concyclic? Or what properties have they?
APH
http://anopolis72000.blogspot.com/

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Message: #2053
Date: 14/11/2016 8:47:40
From: Tran Quang Hung
Subject: Circumcircles are concurrent in quadrilateral

Dear Mr Antreas,
I attached the figure.
Best regards,
Tran Quang Hung.

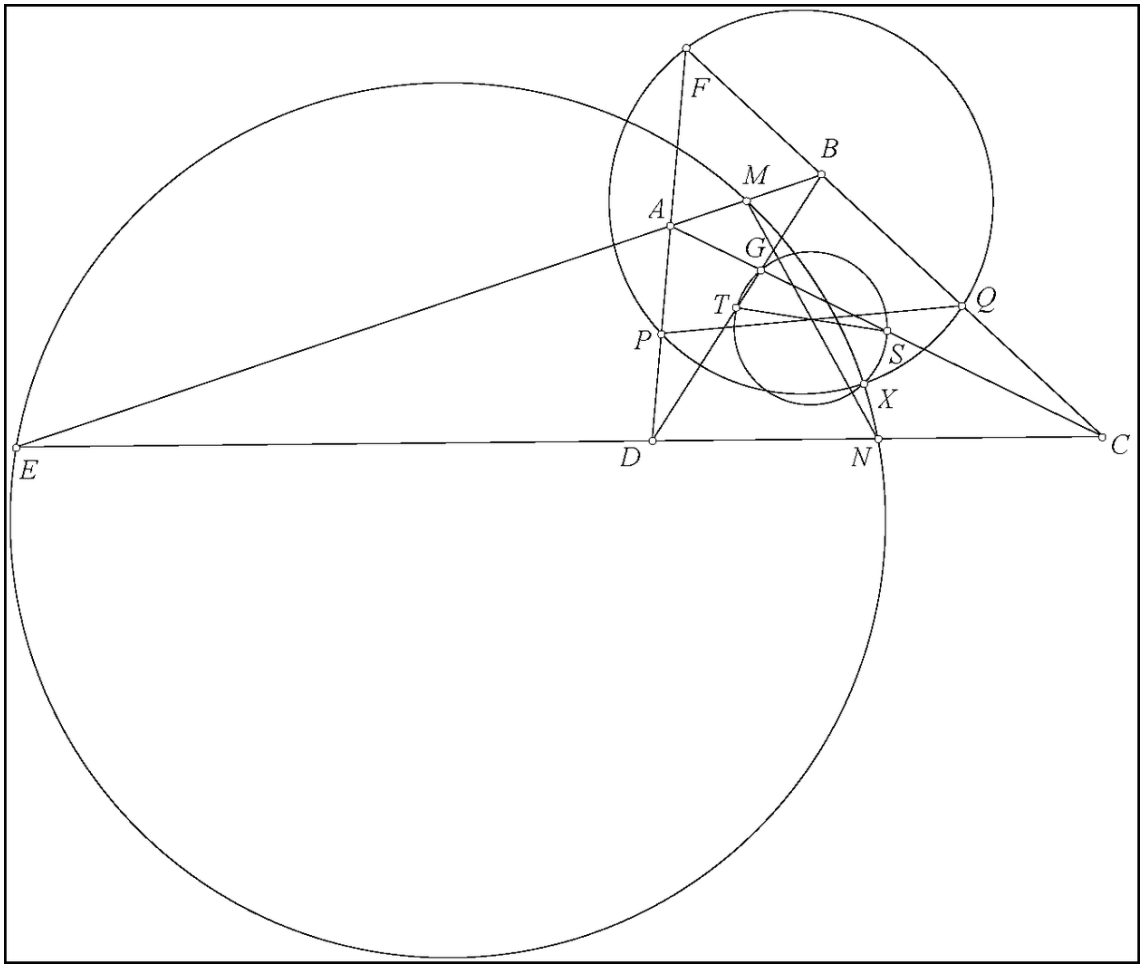

Figure4134.png

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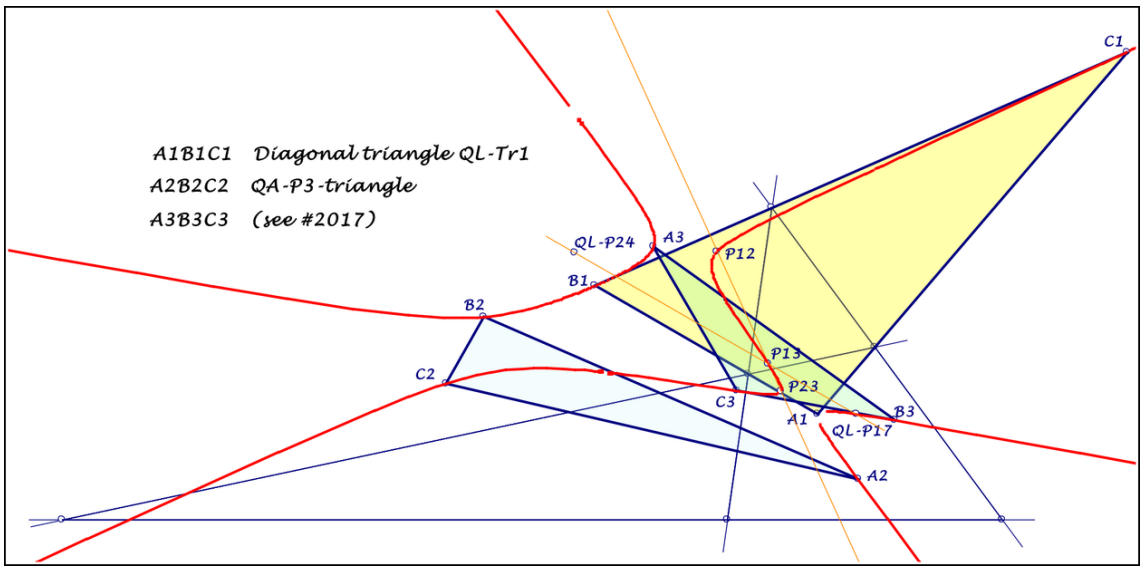
Message: #2054
Date: 14/11/2016 9:25:54
From: eckart_schmidt@t-online.de
Subject: New QA-Cux wrt DT

Dear Bernard,
I only found the following properties (see attached file,
nomination of #2017):
... perspector P13 lies on QL-P13.17.24,
... QL-P17 is a point of the cubic.
Best regards Eckart
PS: I think it is a QL-Cux.



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Message: #2055
Date: 14/11/2016 10:27:00
From: eckart_schmidt@t-online.de
Subject: Circumcircles are concurrent in quadrilater ...

```
Dear Tran, dear Antreas,
Tran Quang Hung wrote:
Dear geometers,
Let ABCD be a quadrilateral.
AB cuts CD at E.
AD cuts BC at F.
AC cuts BD at G.
M,N are midpoints of AB,CD.
P,Q are midpoints of AD,BC.
S,T are midpoints of AC,BD.
Then the circles (EMN),(FPQ),(GST) are concurrent.
Best regards
Tran Quang Hung.
I think, you consider a quadrigon ABCD,
... then the common point of the circles is the intersection  of
... ... QG-P1.QL-P1 and a perpendicular line through QG-P5.
For the quadrigons of a quadrilateral
... these three points lie on the Dimidium circle QL-Ci6.
Best regards Eckart
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Message: #2056
Date: 15/11/2016 4:18:48
From: Tran Quang Hung
Subject: Circumcircles are concurrent in quadrilater ...

```
Thank you very much Mr Eckart and Mr Antreas for you interest.
Best regards,
Tran Quang Hung.
```

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Message: #2057
Date: 15/11/2016 9:37:38
From: eckart_schmidt@t-online.de
Subject: Circumcircles are concurrent in quadril ...

```
Dear Tran,
the common point of the three circles is the QA-point QA-P3.
Best regards Eckart
```

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Message: #2058
Date: 15/11/2016 12:55:17
From: Tran Quang Hung
Subject: Circumcircles are concurrent in quadril ...

```
Thank you very much Mr Eckart.
Sincerely,
Tran Quang Hung.
```

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Message: #2059
Date: 16/11/2016 11:02:22
From: bernard.keizer
Subject: New QA-Cux wrt DT

Dear Eckart,
Thank you very much for your quick answer !
I checked your property wrt P13 on the line QL-P13.17.24 (but it's not QL-P13 !)
P12 is QL-P1 on the Dimidium circle.
Any other idea about P13 and P23 ?
You're right, it's rather a QL-Cux (but it's neither through the QA nor the QL vertices) ...
Do you know other such cubics ?
Best regards
Bernard

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Message: #2060
Date: 16/11/2016 11:19:02
From: Tran Quang Hung
Subject: Generalization of QA-Tf6

Let ABCD be the Reference Quadrangle and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Mab, Mcd, Mac, Mdb, Mad, Mbc.
Line p interesect perpendicular bisector of the segments MabMcd, MacMdb, MadMbc at X,Y,Z.
Circle (X) center X passes through Mab, Mcd.
Circle (Y) center Y passes through Mac, Mdb.
Circle (Z) center Z passes through Mad, Mbc.
We see circles (X),(Y),(Z) are coaxal.
Circle (X) intersects lines AB,CD again at Nab,Ncd.
Circle (Y) intersects lines AC,BD again at Nac,Nbd.
Circle (Z) intersects lines AD,BC again at Nad,Nbc.
Then the lines NabNcd, NacNbd, NacNbd are concurrent at point Mpr.
When (X),(Y),(Z) are circles diameter MabMcd, MacMdb, MadMbc then we get QA-Tf6.
Best regards,
Tran Quang Hung.

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Message: #2061
Date: 16/11/2016 11:48:40
From: Tran Quang Hung
Subject: A generalization of Newton line

Dear geometers,
Let L1,L2,L3,L4 be four lines.
L1,L2 intersect at P12. Similarly, we have the vertex
P13,P14,P23,P24,P34.
Line L intersect line P12P34 at X.
Line L intersect line P13P24 at Y.
Line L intersect line P24P14 at Z.
And X',Y',Z' lies on theses lines P12P34, P13P24, P24P14 such
that the cross ratio are harmonic
(P12,P34;X,X')=(P13,P24;Y,Y')=(P24,P14;Z,Z')=-1.
Then X',Y',Z' are collinear on line L'.
When L is infinity line then L' is Newton line.
Best regards,
Tran Quang Hung.

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Message: #2062
Date: 16/11/2016 7:06:03
From: Tran Quang Hung
Subject: Orthopole in quadrilateral

Dear geometer,
Let L1,L2,L3,L4 be a quadrilateral and L is any line.
Let X1 is orthopole of L wrt triangle (L2,L3,L4).
Y1 is the projection of X1 on L.
d1 is the line passing through Y1 and is perpendicular to L1.
Similarly, we have the lines d2,d3,d4.
Then d1,d2,d3,d4 are concurrent at a point P.
I call P by the orthopole of L wrt (L1,L2,L3,L4)
Best regards,
Tran Quang Hung.

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Message: #2063
Date: 16/11/2016 11:23:16
From: bernard.keizer
Subject: A generalization of Newton line

Dear Tran Quang Hong,
This property is already in EQF !
L' is QL-Tf2(L) and the dual of QL-P8 is the Newton Line ...
Best regards
Bernard

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Message: #2064
Date: 16/11/2016 11:26:13
From: chris.vantienhoven
Subject: Orthopole in quadrilateral

Dear Tran,
Very nice this QL-Orthopole. I like it!
This is a completion to the QL-Orthopolar line from Seiichi.
June 2012 Seiichi Kirikami attended me with:
“Given a Quadrangle ABCD and a line, then the orthopoles of ABC, BCD, ACD and ABD have a common line called the orthopolar line.“
In your example this means that X1, X2, X3, X4 are collinear on an orthopolar line. Also it appears that P lies on line X1.X2.X3.X4.
Then we discovered that in a Quadrilateral every orthopolar is perpendicular to the Newton Line QL-L1 and consequently parallel to the Steiner Line QL-L2.
At that time there was no Quadri-Figures-Group, so I made notice of it at Hyacinthos #21070 †) mentioning some extra properties.
By the way there is also an Orthopolar line in a Quadrangle (the line through the Orthopoles of a given line wrt the QA-Component Triangles).
Best regards,
Chris

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† Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #2065
Date: 17/11/2016 2:03:18
From: Tran Quang Hung
Subject: Orthopole in quadrilateral

Thank you so much Mr Chris van Tienhoven for your interest.
I really hope, I can participate to contribute the EQF.
Best regards
Tran Quang Hung.

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Message: #2066
Date: 17/11/2016 2:04:16
From: Tran Quang Hung
Subject: A generalization of Newton line

Thank you Mr Bernard for point out me the old result.
Best regards,
Tran Quang Hung.

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Message: #2067
Date: 17/11/2016 11:47:07
From: eckart_schmidt@t-online.de
Subject: Cubics of Generalized Cyclologic Centers

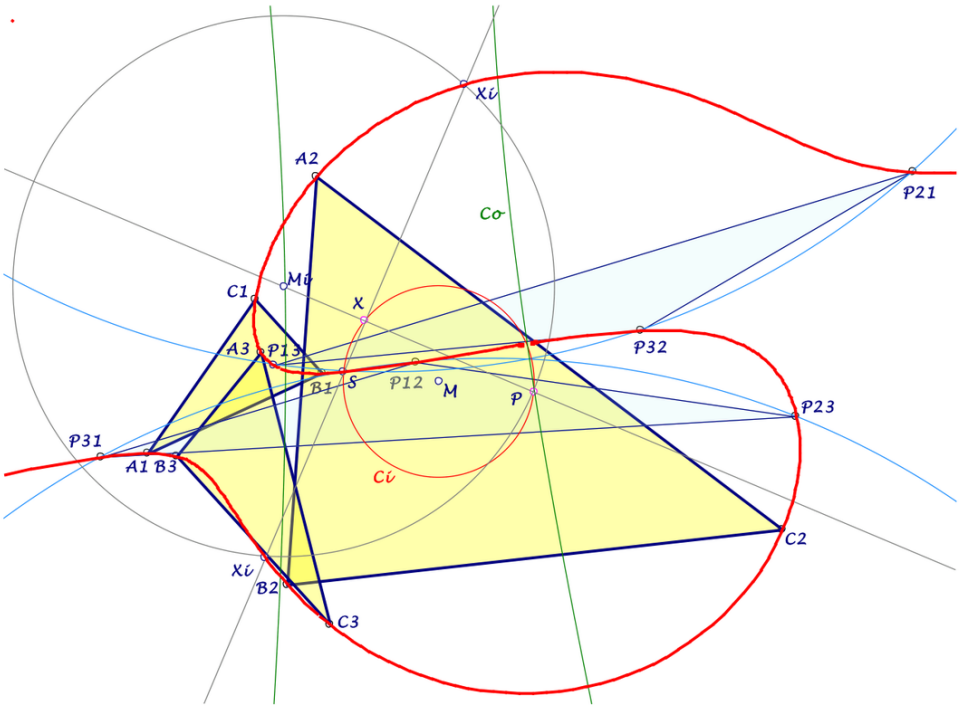
Dear Tsihong Lau, dear Bernard, dear Chris,
Tsihong Lau has defined generalized cyclologic centers for two cyclologic triangles and asked for their locus.
In several messages I have mentioned a cubic in this regard, but there was no reaction.
Now I can offer a general construction (see attached file).
Best regards Eckart

EQF-Note 2016-11-17

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubics of Generalized Cyclologic Centers II

Generalized cyclologic centers are defined by Tsihong Lau [1]. Starting with two cyclologic triangles and their cyclologic centers the generalized cyclologic centers give a cubic (see #1988, #1990 and #1994). Here a construction of this cubic is given for the general case.



Two triangles $A_1B_1C_1$ and $A_2B_2C_2$ are cyclologic, if the circles $(A_1B_1C_2)$, $(B_1C_1A_2)$, $(C_1A_1B_2)$ have a common point, the cyclologic center P_{12}. The circles $(A_2B_2C_1)$, $(B_2C_2A_1)$, $(C_2A_2B_1)$ then have also a common point, the cyclologic center P_{21}.
There are generations of generalized cyclologic centers, the first generation is:
$P_{12}{}'$ = 3rd common point of the conics $(A_1B_1C_2P_{12}P_{21})$, $(B_1C_1A_2P_{12}P_{21})$, $(C_1A_1B_2P_{12}P_{21})$,
$P_{21}{}'$ = 3rd common point of the conics $(A_2B_2C_1P_{12}P_{21})$, $(B_2C_2A_1P_{12}P_{21})$, $(C_2A_2B_1P_{12}P_{21})$.

Cabri observations show, that the locus of these generalized cyclologic centers is a cubic, which now shall be constructed.

For two cyclologic triangles there is a third triangle $A_3B_3C_3$ with the vertices

$A_3 = B_1C_2 \cap B_2C_1,\ B_3 = C_1A_2 \cap C_2A_1,\ C_3 = A_1B_2 \cap A_2B_1.$

These three triangles are pairwise cyclologic, so we get over all six cyclologic centers P_{12}, P_{21}, P_{13}, P_{31}, P_{23}, P_{32}.

- **The triangles $P_{12}P_{23}P_{31}$ and $P_{32}P_{13}P_{21}$ are perspective and similar.**

Let
… S be the perspector, which is one intersection of the circumcircles of the two triangles $P_{12}P_{23}P_{31}$ and $P_{32}P_{13}P_{21}$,
… M the midpoint of the circumcenters of the two triangles
… and Ci the circle round M through S,
… P the reflection of S in M,
… X a point on Ci.

For cyclologic triangles on the cubic *QA-Cu1* is
… point S = intersection of *QA-Cu1* and its asymptote,
… point P = *QA-P9*,
… circle Ci = the circumcircle of the Miquel triangle *QA-Tr2*.

For cyclologic triangles on the cubic *QL-Cu1* is
… point S = the intersection of *QL-Cu1* and its asymptote,
… point P = *QL-P1*,
… circle Ci = the *CSC*-image of a *QL-L1*-perpendicular line through the intersection with *QL-L6*.

The searched cubic bears
… the vertices of the triangles $A_1B_1C_1$, $A_2B_2C_2$ and $A_3B_3C_3$,
… the cyclologic centers P_{12}, P_{21}, P_{13}, P_{31}, P_{23}, P_{32}
… and the perspector S.

- **The asymptote of the cubic is a parallel to $P_{12}P_{21}'$ or $P_{21}P_{12}'$ through S.**

Let X be a point on Ci and X_1 and X_2 the further intersections of XS and the cubic. Cabri observations show:

- **A line XS intersects the cubic symmetrically to X.**
- **Centers of circles through X_1, X_2, P lie on a conic Co through P.**

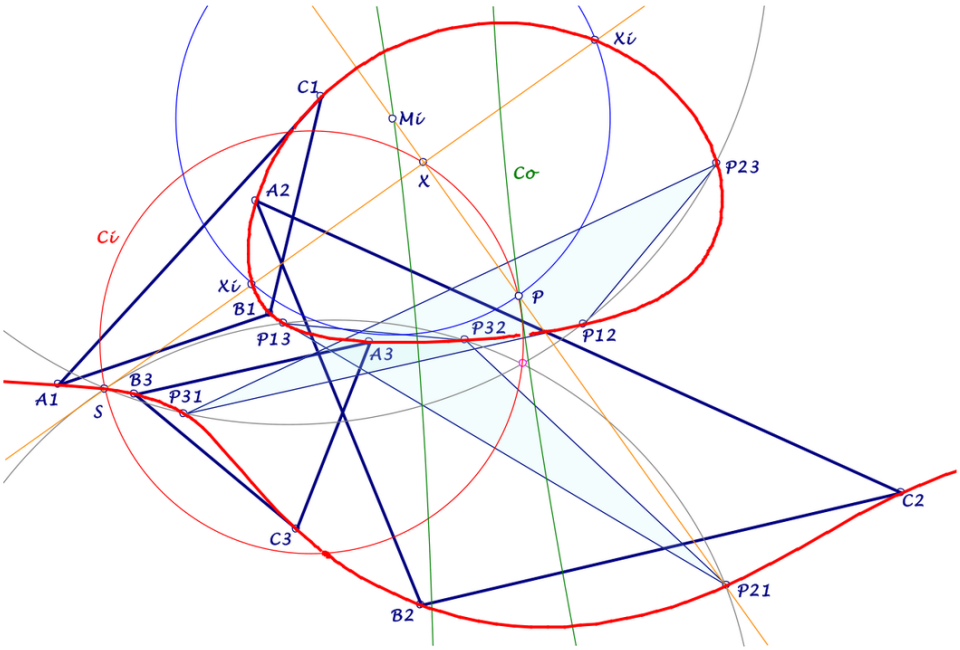
This conic Co can be constructed as follows:
… Let X_i be an already known point of the cubic (see above),
… let X be the 2nd intersection of X_iS and Ci,
… let M_i be the intersection of XP and the perpendicular bisector of X_iP,
… then 4 points M_i and the point P define the conic Co.

Now the cubic can be constructed:
… Let X be a variable point on Ci,
… M_i the 2nd intersection of XP and Co,
… then the circle round M_i through P intersects XS in points X_i of the cubic.

The cubic can be tested with the following property:

- **Pairs of the triangles $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$ are generalized cyclologic wrt any two points of the constructed cubic.**

Final remark: The cubic can be unipartite or bipartite. In the bipartite case the three intersections – unequal S – of the cubic and the circle Ci give a reference triangle, so that the cubic is an isogonal pivotal isocubic with pivot in the infinity point of the asymptote.



Ref [1]
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/3275

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-11-17.pdf

Message: #2068
Date: 17/11/2016 12:07:13
From: Tran Quang Hung
Subject: Tripolar tranversals

Dear geometers,
Let L1,L2,L3,L4 be a quadrilateral.
P is any point.
Tripolar of P wrt triangle (L2,L3,L4) intersect L1 at X1.
The line passes through P and is parallel to tripolar of P wrt triangle (L2,L3,L4) intersect L1 at Y1.
Similarly, we have X2,X3,X4 and Y2,Y3,Y4.
Then X1,X2,X3,X4 are collinear.
Y1,Y2,Y3,Y4 are collinear.
Are they new tranversals of P?
Best regards,
Tran Quang Hung.

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Message: #2069
Date: 17/11/2016 1:28:04
From: chris.vantienhoven
Subject: Orthopole in quadrilateral

Dear Tran,
You are very welcome.
I did do some calculations on the QL-Orthopoles and QL-Orthopolars wrt the known QL-Lines:

QL-Orthopolars:
* The QL-Orthopolars of QL-L1, QL-L4, QL-L5 are identical being the Pedal Line QL-L3
* The QL-Orthopolar of QL-L3 is the Steiner Line QL-L2

QL-Orthopoles:
* The QL-Orthopoles of QL-L1, QL-L4 and QL-L5 are collinear on the Pedal Line QL-L3
* The QL-Orthopoles of QL-L1 and QL-L3 are collinear with QL-P1
* The QL-Orthopoles of QL-L2 and QL-L3 are collinear with QL-P21 on a line parallel with QL-P2.QL-P3
* The QL-Orthopole of QL-L1 lies on the Newton Line
* The QL-Orthopole of QL-L3 lies on QL-P2.QL-P7
* The QL-Orthopole of QL-L4 lies on QL-P2.QL-P3

* There are no QL-Orthopoles lying on known circles, conics & cubics.

Last observation:
* The QL-Orthopole of QL-L1 has shortest coordinates.
1 st CT-coordinate:
(a^2 l m - b^2 l m + c^2 l m + a^2 l n + b^2 l n - c^2 l n - 2 a^2 m n) (-c^2 l^3 m^3 + c^2 l^3 m^2 n + 3 c^2 l^2 m^3 n + b^2 l^3 m n^2 + a^2 l^2 m^2 n^2 - 3 b^2 l^2 m^2 n^2 - 3 c^2 l^2 m^2 n^2 - a^2 l m^3 n^2 + 2 b^2 l m^3 n^2 - 2 c^2 l m^3 n^2 - b^2 l^3 n^3 + 3 b^2 l^2 m n^3 - a^2 l m^2 n^3 - 2 b^2 l m^2 n^3 + 2 c^2 l m^2 n^3 + a^2 m^3 n^3)
QL-Orthopolars:
* The QL-Orthopolars of QL-L1, QL-L4, QL-L5 are identical being the Pedal Line QL-L3
* The QL-Orthopolar of QL-L3 is the Steiner Line QL-L2
QL-Orthopoles:
* The QL-Orthopoles of QL-L1, QL-L4 and QL-L5 are collinear on the Pedal Line QL-L3
* The QL-Orthopoles of QL-L1 and QL-L3 are collinear with QL-P1
* The QL-Orthopoles of QL-L2 and QL-L3 are collinear with QL-P21 on a line parallel with QL-P2.QL-P3
* The QL-Orthopole of QL-L1 lies on the Newton Line
* The QL-Orthopole of QL-L3 lies on QL-P2.QL-P7
* The QL-Orthopole of QL-L4 lies on QL-P2.QL-P3
* There are no QL-Orthopoles lying on known circles, conics & cubics.

Best regards,
Chris van Tienhoven

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Message: #2070
Date: 17/11/2016 2:36:59
From: minhtenladuong
Subject: Orthopole in quadrilateral

Dear Mr Tran Quang Hung
I really like this transformation.
Here I have two simple geometric properties:
- d is a line that perpendicular to the Newton line. (P,d) is the parabola of which foci is QL-P1 and directrix is d.
L is tangent to (P,d). Let T be the orthopole of L with respect to the quadrilateral (L1,L2,L3,L4).
As L varies, T varies on a fixed line L', which is also perpendicular to the Newton line.
Particularly, when (P,d) is the inscribed parabola, T lies on the Steiner line.
- L is the reflection of the Steiner line in L'.
Best regards
Sincerely
Ngo Quang Duong

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Message: #2071
Date: 17/11/2016 3:15:43
From: eckart_schmidt@t-online.de
Subject: Tripolar tranversals

Dear Tran,
only a first observation, testing your #2068:
For P = QL-P1 the collinear Xi give a line, parallel QL-L2 through QL-P26.
Best regards Eckart

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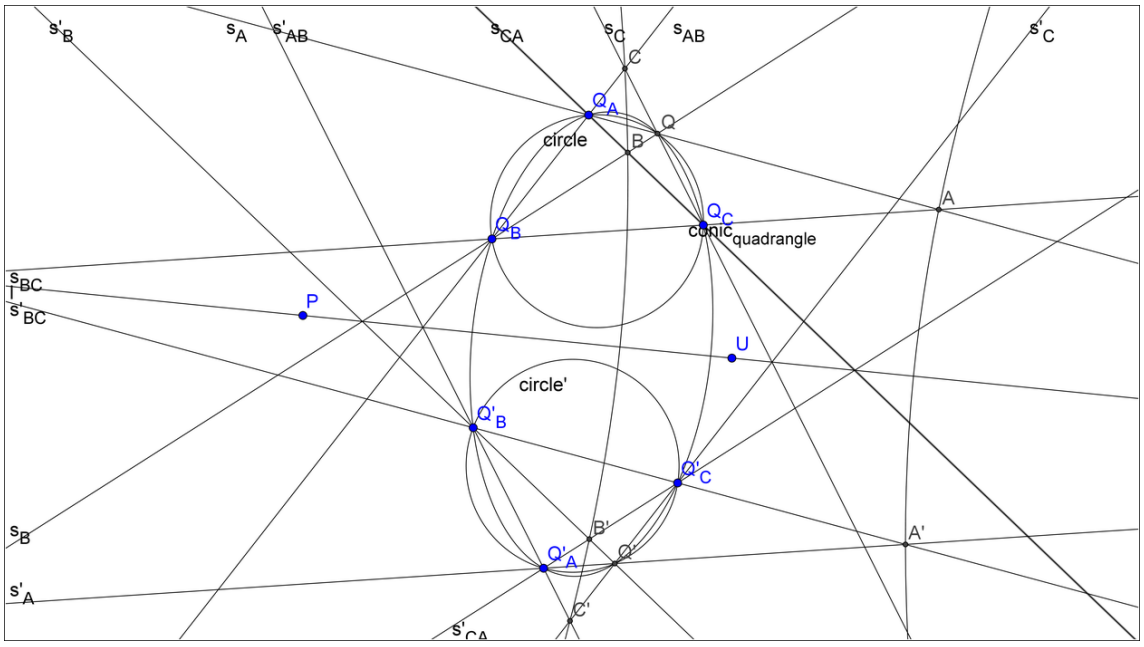
Message: #2072
Date: 17/11/2016 3:24:16
From: tsihonglau
Subject: Cyclic Mirror Quadrangle

Dear all
I think it must have been known already.
But I do not know what it was called.
In attachment, footnotes are in uppercases.
Given a triangle QaQbQc and a line l, the reflection to l of the triangle is Q'aQ'bQ'c.
It is easy to show that QaQbQc and Q'aQ'bQ'c lie on a conic.
The fourth points of intersection of the conic and the circumcircles of QaQbQc and Q'aQ'bQ'c are Q,Q' respectively.
With similar construction, we can get Qa,Qb,Qc from QQbQc,QaQQc,QaQbQ and l respectively.
We call QQaQbQc and Q'Q'aQ'bQ'c the cyclic mirror quadrangles.
The following are pairs of parallel lines:
QQa || Q'bQ'c - QbQc || Q'Q'a
QQb || Q'cQ'a - QcQa || Q'Q'b
QQc || Q'aQ'b - QaQb || Q'Q'c
A question: Given a cyclic quadrangle, does there exist a line such that the quadrangle and its reflection are the cyclic mirror quadrangle?
Best regards,
Tsihong Lau



cyclic_mirror_quadrangle.png

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Message: #2073
Date: 17/11/2016 5:03:42
From: Tran Quang Hung
Subject: Orthopole in quadrilateral

```
Thank you very much Mr Chris van Tienhoven, I'll do my best.
Thank you Duong for your contribution.
Best regard
Tran Quang Hung.
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Message: #2074
Date: 17/11/2016 5:23:56
From: Tran Quang Hung
Subject: Tripolar tranversals

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Thank you very much Mr Eckart,
Are these transformation appeared on EQF ?
Best regards
Tran Quang Hung.
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Message: #2075
Date: 17/11/2016 6:12:32
From: Tran Quang Hung
Subject: Concyclic points in quadrilateral

```
Dear geometers,
Let (L1,L2,L3,L4) be a quadrilateral.
H1 is orthocenter of triangle (L2,L3,L4)
K1 is orthopole of L1 wrt triangle
(L2,L3,L4)
X1 lies on L1 such that X1K1 is perpendicular to K1H1.
Similarly, we have X2,X3,X4.
Then X1,X2,X3,X4 lie on a circle.
Which are properties of this circle ?
Best regards,
Tran Quang Hung.
```

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Message: #2076
Date: 17/11/2016 9:45:43
From: eckart_schmidt@t-online.de
Subject: Concyclic points in quadrilateral

Dear Tran,
the circle in #2075 is QL-Ci4.
Best regards Eckart

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Message: #2077
Date: 18/11/2016 12:23:59
From: bernard.keizer
Subject: Cubics of Generalized Cyclologic Centers

Dear Eckart,
A little bit complicate, but really fascinating construction of this circular cubic !
Very clever, this idea of searching a 3rd triangle A3B3C3 with the same construction as in the Desmic construction and to find then the 6 cyclologic centers ...
(There is a cubic obtained with the 2 perspective triangles of these 6 points with perspector S, but it's not the same)
Best regards
Bernard

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Message: #2078
Date: 18/11/2016 1:03:49
From: Tran Quang Hung
Subject: Tripole transform

Dear geometers,
Let ABCD be a quadrangle.
L is any line.
Let A' be tripole of L wrt BCD. Similarly, we have B',C',D'.
Then AA',BB',CC',DD' are concurrent.
Is this transform known ?
Best regards,
Tran Quang Hung.

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Message: #2079
Date: 18/11/2016 1:04:40
From: Tran Quang Hung
Subject: Concyclic points in quadrilateral

Thank you very much Mr Eckart for pointing out me the
QL-Ci4.
Best regards,
Tran Quang Hung.

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Message: #2080
Date: 18/11/2016 3:22:15
From: tsihonglau
Subject: Tripole transform

Dear Tran Quang Hung
Using unitary coordinates -
http://lth.name/geometry/coordinate.html
The transformation is L(lx+my+nz) -> point of concurrency
l(-l^2+m^2+n^2):m(l^2-m^2+n^2):n(l^2+m^2-n^2).
If the line is the line at infinity, the point is the centroid
of quadrangle.
Best regards
Tsihong Lau

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Message: #2081
Date: 18/11/2016 5:18:04
From: Tran Quang Hung
Subject: Concyclic point in QL with reflections of orthopole

Dear geometers,
Let quadrilateral (L1,L2,L3,L4)
Let X1 be the reflections of orthopole of L1 wrt triangle (L2,L3,L4) through L1.
Similarly, we have X2,X3,X4.
Then X1,X2,X3,X4 are concyclic on a circle and center of this circle lies on QL-L1 Newton line.
Which is this circle ?
Best regards,
Tran Quang Hung.

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Message: #2082
Date: 18/11/2016 5:21:36
From: Tran Quang Hung
Subject: Tripole transform

Thank you Tsihong Lau for nice remark.
Best regards
Tran Quang Hung.

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Message: #2083
Date: 18/11/2016 8:34:38
From: eckart_schmidt@t-online.de
Subject: Concyclic point in QL with reflections of ortho ...

Dear Tran,
the described circle is the Plücker circle QL-Ci5.
Best regards Eckart

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Message: #2084
Date: 19/11/2016 11:25:20
From: bernard.keizer
Subject: A lot of cubics of QA/QL

Dear Eckart,
I'm using only 2 well-known properties :
a) 2 triangles perspective with a 3rd one are themselves perspective
b) if the 3 perspectors are aligned, the form with the 3 triangles 3 QA in a Desmic system
Combining the vertices of QA,QL and DT, Miquel and Gergonne-Steiner triangles, there are a lot of perspective triangles and of cubics on the QA/QL figure
1) each reference triangle of QA is perspective with DT
   each reference triangle of QL is perspective with DT
   each reference triangle of QA is perspective with each reference triangle of QL
If I'm not wrong, this gives 12 different cubics, one in each case for each QA vertice
2) in QA, DT and Miquel triangle are perspective with perspector QA-P3
   in QL, DT and QA-P3 triangle are perspective with perspector QL-P1
(these 2 properties are dual)
therefore, QA Miquel triangle and QL QA-P3 triangle are perspective this gives again 3 different cubics (the 1rst one is QA-Cu1).
But these 2 triangles are perspective with every reference triangle of both QA and QL ...
3) For each of the 3 QA's of QL, there is a cubic QA-Cu1 through the 6 QL vertices, Miquel point and 1 DT
For each of the 3 QL's of QA, there is a cubic through the 4 QA vertices, 2 DT and 1 Miquel vertices
This gives 6 other different cubics.
Best regards
Bernard

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Message: #2085
Date: 19/11/2016 5:00:28
From: Tran Quang Hung
Subject: Concyclic point in QL with reflections of ortho ...

Thank you so much Mr Eckart for pointing out the circle QL Ci5.
Best regards,
Tran Quang Hung.

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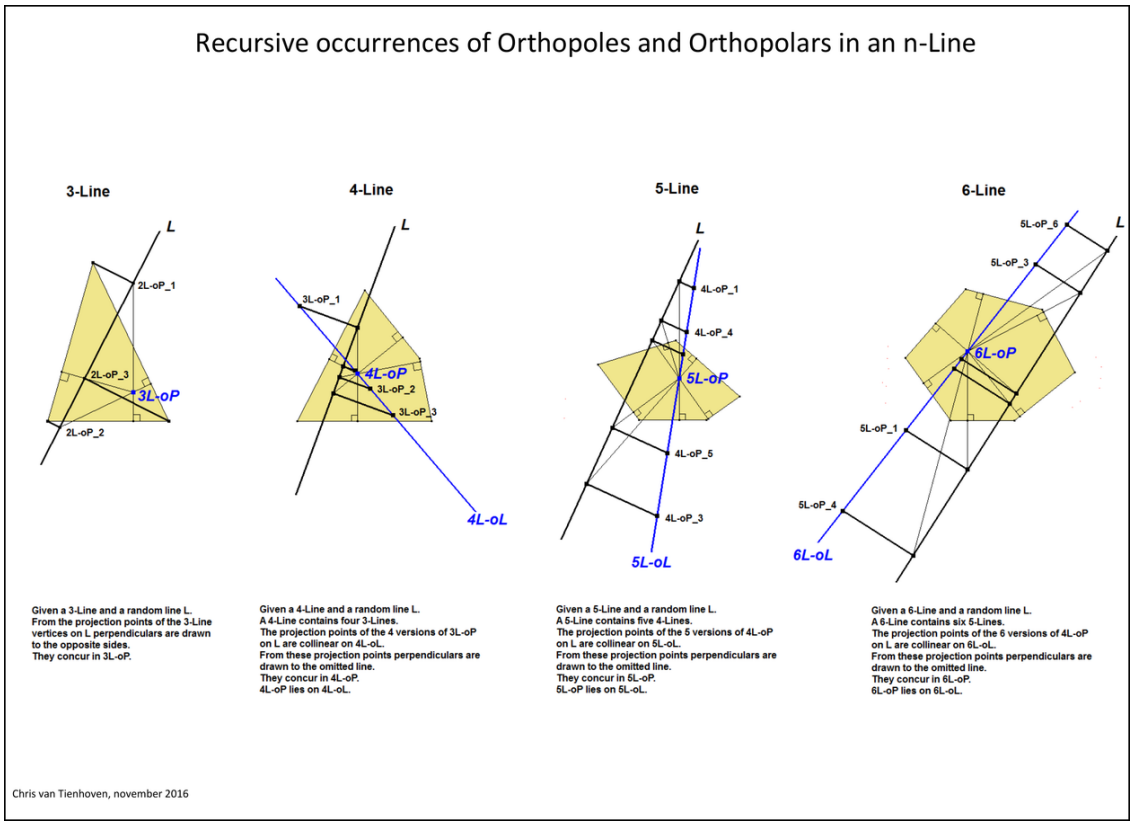
Message: #2086
Date: 19/11/2016 10:10:01
From: chris.vantienhoven
Subject: Orthopole in quadrilateral

Dear all,
The method for constructing an Orthopole and Orthopolars can be made general in a recursive process for an n-Line.
This means that a 3-Line, a 4-Line, a 5-Line, a 6-Line, etc., all have their own Orthopole and Orthopolar.
Only the 3-Line has an Orthopole without Orthopolar.
See attachment.
Best regards,
Chris



nL-n-Tfx-Orthopole and Orthopolar-01.pdf

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Message: #2087
Date: 20/11/2016 1:39:26
From: Tran Quang Hung
Subject: Orthopole in quadrilateral

Thank you very much Mr Chris van Tienhoven, your idea about n-line is so nice.
Best regards,
Tran Quang Hung.

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Message: #2088
Date: 20/11/2016 4:09:40
From: tsihonglau
Subject: Cyclic Mirror Quadrangle

Dear all,
The answer is any line parallel or perpendicular to an angle bisector of diagonal trilateral (triangle).
Best regards,
Tsihong Lau

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Message: #2089
Date: 20/11/2016 4:14:42
From: eckart_schmidt@t-online.de
Subject: Orthopole in quadrilateral

```
Dear Duong, dear Tran, dear Chris,

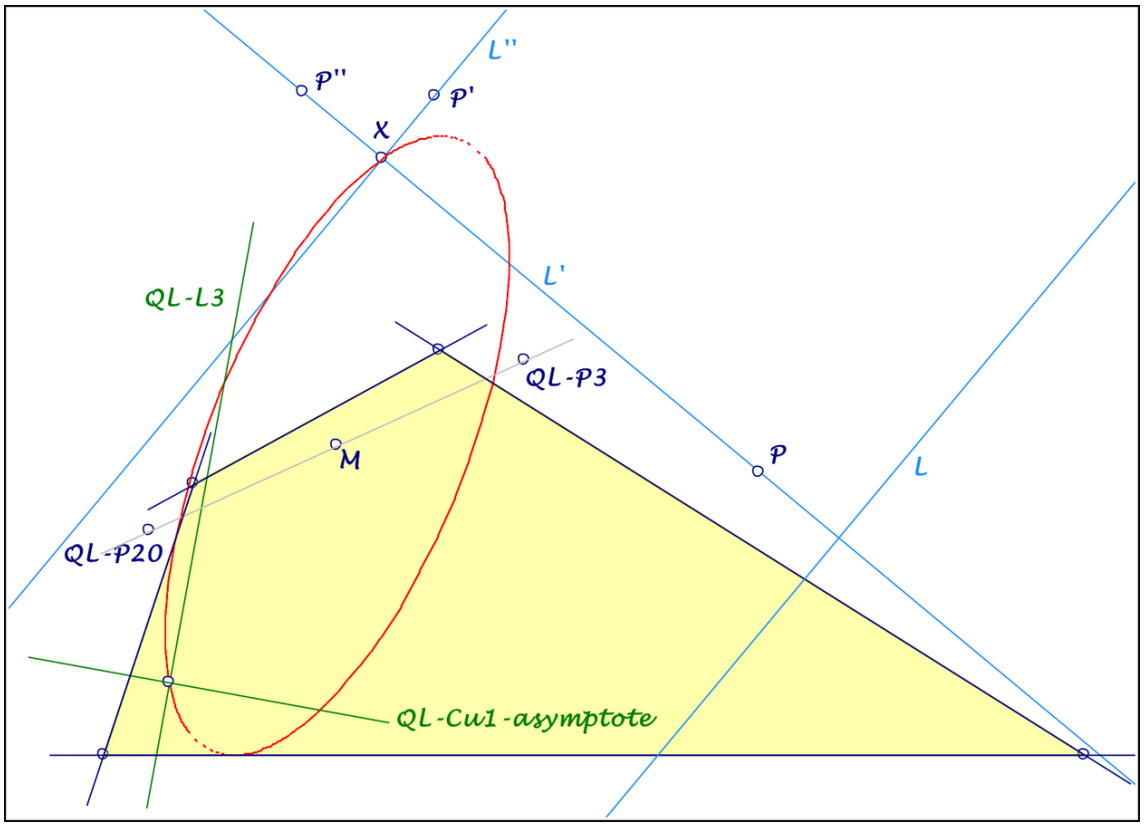
in addition to #2069 and #2070 of Chris and Duong, some
properties already mentioned:
... The QL-orthopolar of a line L is orthogonal to QL-L1
... ... through the intersection of L and a tangent to QL-Co1
        perpendicular L.
... The QL-orthopolar of tangents at QL-Co1 is QL-L2.
... All lines with the same QL-orthopolar L
    (perpendicular to QL-L1)
... ... are tangents of a parabola
        with the same focus and axis as QL-Co1
... ... intersecting QL-Co1 orthogonal in the intersections
        with L.
... The QL-orthopoles of lines through QL-P1 lie on QL-L3.
... The QL-orthopoles of tangents at QL-Co1 lie on QL-L2.
... The QL-orthopoles of QL-L1-parallels lie on QL-L3.
... The QL-orthopoles of QL-L1-perpendiculars
    lie on the QL-Cu1-asymptote.
... All lines with the same QL-orthopole P
... ... have the same QL-orthopolar,
... ... which is the QL-L1-perpendicular through P (see above).
... The QL-orthopoles of parallels L to a fixed line Lo
... ... lie on a perpendicular line L´
        in a constant orientated distance d.

Examples:
... Lo = QL-L1, L´ = QL-L3, d = 0.
... Lo = QL-L2, L´ = QL-Cu1-asymptote,
                d = distance of QL-L3 and QL-L2.

Finally a short excursion (see attached file):
Let L be any line,
... P = QL-orthopole of L,
... L´ = perpendicular line to L through P,
... P´ = QL-orthopole of L´,
... L´´ = perpendicular line to L´ through P´,
... X = intersection of L´ and L´´,
... then the locus of X is an ellipse,
... ... centered in the midpoint M of QL-P3.QL-P20,
... ... bearing the intersection of QL-L3
        and the QL-Cu1-asymptote
... ... and the pedal points on the QL-lines
        for the QL-orthopoles of their parallels.
```

```
... Simple construction of the ellipse:
... ... Take an arbitrary line pencil
... ... with orthogonal lines L1, L2
... ... and their QL-orthopoles P1, P2.
... ... The intersections
        of the perpendiculars to Li through Pi give the ellipse.

Best regards Eckart
```



2016-11-20.pdf

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Message: #2090
Date: 20/11/2016 4:28:56
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

Dear all,
I hope someone will reply this topic.
I give two subclasses of Pappian planes.
The first is the circular Pappian plane.
We choose any two points as the circular points.
The line at infinity is the line through them.
A circle is a conic through them.
Then we can define perpendicularity and many other geometric objects.
This geometry is the most familiar one.
The first is the (vertical) parabolic Pappian plane.
We choose any point on any line.
They are called the vertical point and the infinite line respectively.
A vertical parabola is a conic tangent to the line at the point.
Then we can define many other geometric objects. This geometry is new.
We should study it thoroughly.
Best regards,
Tsihong Lau

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Message: #2091
Date: 23/11/2016 11:57:22
From: eckart_schmidt@t-online.de
Subject: A lot of cubics of QA/QL

```
Dear Bernard,
thank you for the different possibilities, to get new QA- and
QL-cubics. In addition a further aspect:
A QG-point X, a QL-point Y and a QA-point Z,
... which are collinear (several examples in QG-2),
... lead to a QA- and a QL-cubic as follows:
QA-cubic (see #2017):
... The QA-triple triangles of X and Y are perspective
... ... with perspector Z,
... ... and give a 3rd triangle in the sense of #2017,
... ... which is perspective to the first two triangles.
... Then there is a pivotal isocubic:
... ... reference triangle is the 3rd triangle,
... ... pivot is the perspector of the first two triangles,
... ... the isoconjugation swaps the vertices
        of the first two triangles.
... This cubic bears the vertices of the three triangles
    and their perspectors.
QL-cubic (new)
... The QL-triple triangles of X and Z are perspective
... ... with perspector Y,
... ... (see above)
This gives for a quadrigon two cubics, intersecting in X, Y, Z.
Best regards Eckart
```

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Message: #2092
Date: 28/11/2016 12:07:42
From: bernard.keizer
Subject: A lot of cubics of QA/QL

Dear Eckart,
Sorry for being late in answering your message.
First thanks for your interest and many thanks for the reference QG-2 in EQF.
I see my cubics with QA-P3 and QL-P1 were only an example of many other interesting aligments.
Now I need your help !
Is it true or not that 2 triangles perspective with a 3rd one are themselves perspective?
I tried to check your property again with QG-P1 (vertices of DT), QL-P1 (vertices of Miquel triangle for QA perspective to DT with perspector QA-P3) and QA-P3 (vertices of a triangle for QL on the Dimidium circle perspective to DT with perspector QL-P1).
I can't find that the triple triangles of QL-P1 and QA-P3 are perspective.
Then I have 3 points X (vertices of DT) , but 4 points QL-P1 and 4 points QA-P3.
What do you mean exactly by the 2 cubics intersecting in X, Y and Z?
The 1rst cubic in the same example is through the vertices of DT and of the Miquel triangle and through the 4th point QA-P3 (is it QA-Cu1?)
The 2nd cubic is through the vertices of DT and of the 3 QA-P3 and through the 4th Miquel point QL-P1.
The intersections between the 2 cubics are the vertices of DT and ???
Thanks in advance for your help (as you see, I'm a little lost ...)
Best regards
Bernard
PS I understand the definition of QG-P13 in EQF as center of a diagonal conic, but not the property as " intersection of lines through intersection of a diagonal with a 3rd one and middle of the last one " ...

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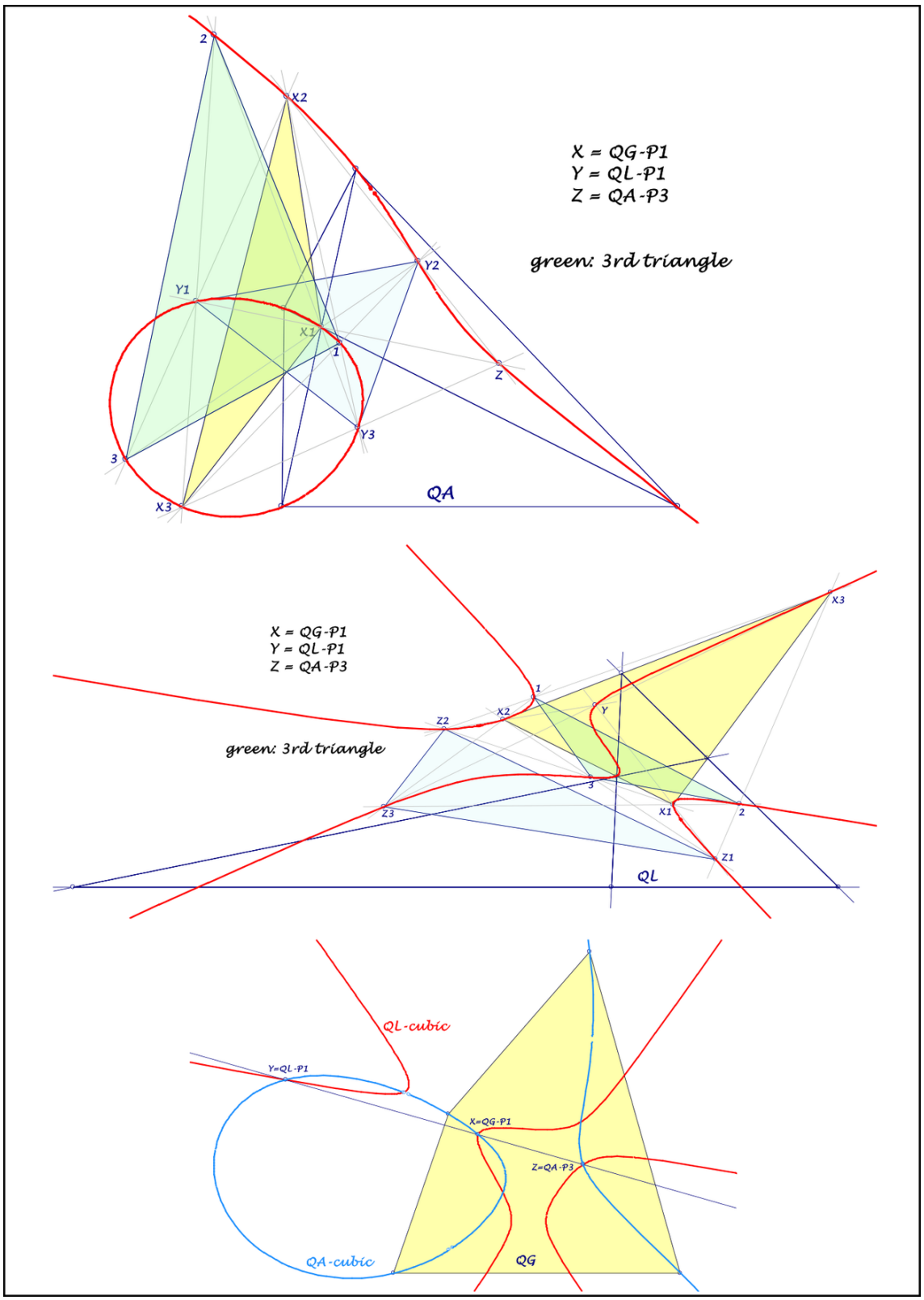
Message: #2093
Date: 28/11/2016 9:07:53
From: eckart_schmidt@t-online.de
Subject: A lot of cubics of QA/QL

Dear Bernard,
thanks for your interest.
You are right: Two triangles perspective with a third must not be perspective themselves.
But that isn´t used anywhere as well as ...
"... that the triple triangles of QL-P1 and QA-P3 are perspective."
We can consider QL-triple triangles for QA- and QG-points or QA-triple triangles for QL- and QG-points.
You have researched the collinear points QG-P1, QL-P1 and QA-P4 for a quadrigon,
so I made attached the corresponding constructions in the sense of #2091,
perhaps helpful.
Best regards Eckart
PS: My description of QG-P13 in EQF isn´t the best:
Let P1P2P3P4 be a quadrigon,
... QG-P13 is the intersection of lines,
... connecting Pi.Pi+2 ^ QG-L1 with the midpoint of Pi+1.Pi+3.



2016-11-28.pdf

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Message: #2094
Date: 30/11/2016 9:13:57
From: chris.vantienhoven
Subject: Tripole transform

Dear Tran Quang Hung,
I looked to this QA-Tripolar Transformation QA-Tfx.
Very interesting.
I noticed that there is a correlation with other transformations:
QA-Tfx = ConicPerspector[QA-Tf2(L)].
For definition of ConicPerspector see QA-Co-1.
QA-Tf2 is the Involutary Conjugate. The Involutaty Conjugate of a line is a conic circumscribing the Diagonal Triangle.
For definition of ConicPerspector see QA-Co-1.
Best regards,
Chris

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Message: #2095
Date: 30/11/2016 10:49:28
From: bernard.keizer
Subject: A lot of cubics of QA/QL

Dear Eckart,
Thanks a lot for your patient explanations and specially for the figures.
I think I've understood now.
1) Your figure above with the QA-cubic is QA-Cu1 and the green triangle is the triangle of the QG-P16 with the other alignment on the same cubic QG-P16 QA-P4 QL-P1
2) Your figure in the middle with the QL-cubic is the one already shown in your message 2054 : the point Y is QL-P1 on the Dimidium circle circumscribed to Z1Z2Z3 and QL-P17 is another intersection between the cubic and the circle.
It could be interesting to compare this cubic with the one obtained with the 2 perspective triangles QG-P13 and QA-P3 with perspector QL-P17 and with the one obtained with 2 perspective triangles QG-P1 and QG-P13 with perspector QL-P13 (on the line QL-P17QL-P24, but it is not the point P13 on your figure) ...
3) Your fire below shows the intersection of the 2 cubics for a quadrigon; you have necessary 3 copples of cubics, one for each alignment QG-P1 QL-P1 QA-P3 (QA-P3 is the same for the 3 copples of cubics)
I was dealing with a dual QA/QL with the same DT; hence I have only 2 cubics intersecting in the 3 vertices of DT.
Best regards
Bernard
Do you keep a library of the different cubics you've already shown ? It took me time to find the references of your past messages. My memory is not so bad after all, but I'm afraid it won't last for ever ...

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Message: #2096
Date: 30/11/2016 11:37:19
From: eckart_schmidt@t-online.de
Subject: QL-Triple Triangles of QG-P1 and QA-P4

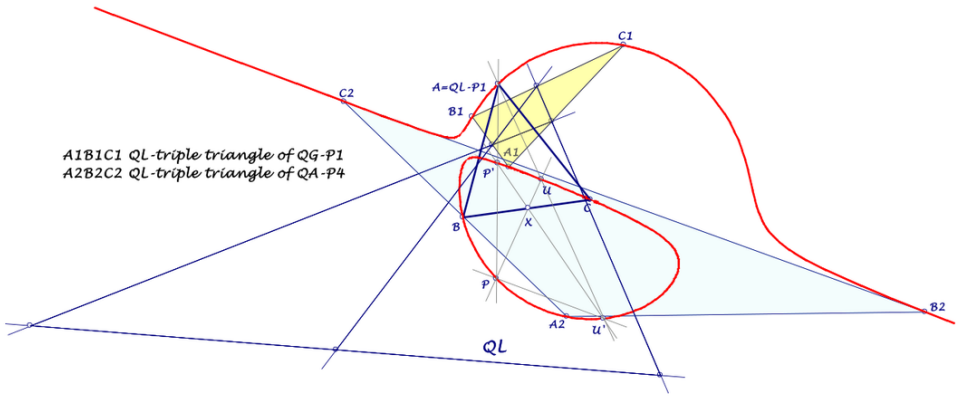
Dear Tsihong Lau, dear Bernard, dear Chris,
attached an addition to #2009 with a concret example for the cubic of generalized cyclologic centers, perhaps of interest.
Best regards Eckart

EQF-Note 2016-11-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Triple Triangles for QG-P1 and QA-P4

The transformation QL-Tf1 = CSC swaps QG-P1 and QA-P4 for a quadrigon. So we have for a quadrilateral cyclologic QL-triple triangles for these two points (QFG #2009). Here the corresponding cubic for the generalized cyclologic centers (QFG #2022) shall be researched.



Let $A_1B_1C_1$ be the *QL*-triple triangle of *QG-P1*, which is the diagonal triangle *QL-Tr1* of the quadrilateral.
Let $A_2B_2C_2$ be the *CSC*-image of $A_1B_1C_1$, which is the *QL*-triple triangle of *QA-P4*.

- **The *QL*-triple triangles of *QG-P1* and *QA-P4* are cyclologic.**

Let P and U be the cyclologic centers:
P common point of the circles $(A_1B_1C_2)$, $(B_1C_1A_2)$, $(C_1A_1B_2)$.
U common point of the circles $(A_2B_2C_1)$, $(B_2C_2A_1)$, $(C_2A_2B_1)$.
Let P' and U' be the generalized cyclologic centers of the first generation:
P' common point of the conics $(A_1B_1C_2PU)$, $(B_1C_1A_2PU)$, $(C_1A_1B_2PU)$.
U' common point of the conics $(A_2B_2C_1PU)$, $(B_2C_2A_1PU)$, $(C_2A_2B_1PU)$.

- ***P, P'* and *U, U'* are collinear with *QL-P1*.**
- ***PU'* and *P'U* are parallel.**
- ***P, U* and *P', U'* are *CSC*-partners.**
- ***P* lies on the circle $(A_2B_2C_2)$, *U* on the circle $(A_1B_1C_1)$.**

Further we consider the triangles $A_3B_3C_3$ and $A_4B_4C_4$ with …
… $A_3 = B_1C_1 \cap B_2C_2$, $B_3 = C_1A_1 \cap C_2A_2$, $C_3 = A_1B_1 \cap A_2B_2$,
… $A_4 = B_1C_2 \cap B_2C_1$, $B_4 = C_1A_2 \cap C_2A_1$, $C_4 = A_1B_2 \cap A_2B_1$.

- **$A_3B_3C_3$ is the cevian triangle of *QL-P26* wrt $A_2B_2C_2$.**
- **$A_4B_4C_4$ is the *CSC*-image of $A_3B_3C_3$.**
- **$A_3B_3C_3$ and $A_4B_4C_4$ are cyclologic triangles.**

- **The cubic for the generalized cyclologic centers bears**
  … the vertices of the four triangles,
  … the cyclologic centers *P, U, P´, U´*
  … and the Miquel point *QL-P1*.

- **The cubic for the generalized cyclologic centers**
  … is *CSC*-invariant
  … with an asymptote parallel *PU´* or *P´U*.

The construction of the cubic is already described in *QFG*-message 2022, here another construction:
… Let *X* be the intersection of *PU* and *P´U´*,
… let *Y* be the corresponding point for the cyclologic triangles $A_3B_3C_3$ and $A_4B_4C_4$
… and *B, C* the intersections of *XY* and its *CSC*-circle,
… additional *A* = *QL-P1* gives a reference triangle *ABC*.

- **The cubic for the generalized cyclologic centers of the *QL*-triple triangles for *QG-P1* and *QA-P4* is**
  … a pivotal isogonal circular cubic
  … with reference triangle *ABC*
  … and pivot in the point at infinity of *PU´* or *P´U*.

In this way the cubic can be considered as *QA-Cu1* for a quadrangle, taking a point of the cubic and its three Möbius transformations wrt *ABC*. Final result:

- **Points *D* of the cubic and the *ABC*-isogonal conjugate of *CSC(D)* are collinear with *QL-P1*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2016-11-30.pdf

Message: #2097
Date: 30/11/2016 5:24:47
From: Tran Quang Hung
Subject: Tripole transform

Thank you so much Mr Chris van Tienhoven for your interest and your remark.
Best regards,
Tran Quang Hung.

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Message: #2098
Date: 01/12/2016 6:10:09
From: bernard.keizer
Subject: A lot of cubics of QA/QL

Dear Eckart,
Thanks also for the PS in message 2093 about QG-P13
1) The definition for a QA/QL is perfectly clear
2) For the quadrigon, it gives in fact the DDT vertices (anticevian triangle of QA-P16 or QL-P13 wrt DT)
I could reproduce the property that this triangle is perspective with DT with perspector QA-P16 (as well as the triangle of QG-P13 of the QA and both triangles are in perspective with the same perspector) and with the triangle of the QA-P3 of the 3 quadrigons with perspector QL-P17
As the QA-P3 triangle is also perspective with DT with perspector QL-P1, the same construction for the diagonal QA/QL gives a triangle of DQA-P3 perspective with DDT with perspector QA-P17, which is DQL-P1.
Hence, the 2 triangles of QA-P3 on the Dimidium circle with QL-P1 and QL-P17 and of DQA-P3 on the DDimidium circle, Dimidium circle of DQA/DQL, with QL-P17 are also perspective with the same perspector QL-P17 ...
Best regards
Bernard

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Message: #2099
Date: 01/12/2016 7:44:43
From: Tran Quang Hung
Subject: Concurrent lines in QA with any point and any line

Dear geometers,
Let ABCD be the Reference Quadrangle and d a random line.
Line d intersects AB, CD, AC, DB, AD, BC at MAB, MCD, MAC, MDB, MAD, MBC.
P is any point.
PMAB, PMCD, PMAC, PMDB, PMAD, PMBC intersect lines CD,AB,BD,AC,BC,AD at NcD, NAB, NBD, NAC, NBC, NAD.
Then the lines NABNCD, NBDNAC,NBCNAD are concurrent.
Best regards,
Tran Quang Hung.

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Message: #2100
Date: 01/12/2016 8:42:16
From: Tran Quang Hung
Subject: Coaxal circles in QA

Dear geometers,
Let ABCD be the Reference Quadrangle and d a random line.
Line d intersects AB, CD, AC, DB, AD, BC at M AB, M CD, M AC, M DB, M AD, M BC.
P is any point.
The circles (PMABMCD),(PMACMDB),(PMADMBC) have a common point Q other than P.
(PMABMCD) cuts AB,CD again at NAB,NCD
(PMBDMCA) cuts BD,AC again at NBD,NCA
(PMBCMAD) cuts BC,AD again at NBC,NAD
The lines NABNCD,NBDNCA,NBCNAD at R.
Are these transformations new ?
Best regards,
Tran Quang Hung.

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Message: #2101
Date: 02/12/2016 4:31:51
From: tsihonglau
Subject: Concurrent lines in QA with any point and any line

```
Dear Tran Quang Hung,
It is very interesting.
I guess the following statements are true.
I use my own notations-
http://lth.name/geometry/duality.html
reference quadrangle Q,Q_A,Q_B,Q_C
points on diapleural(diagonal)
    QQ_A - P_A,U_A
    QQ_B - P_B,U_B
    QQ_C - P_C,U_C
    Q_BQ_C - P_BC,U_BC
    Q_CQ_A - P_CA,U_CA
    Q_AQ_B - P_AB,U_AB
concurrency
    P_AP_BC,P_BP_CA,P_CP_AB concur at P.
    U_AU_BC,U_BU_CA,U_CU_AB concur at U.
    P_AU_BC,P_BU_CA,P_CU_AB
   U_AP_BC,U_BP_CA,U_CP_AB
concur at X.
Then P,U,X are collinear.
Best regards,
Tsihong Lau
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Message: #2102
Date: 02/12/2016 8:46:28
From: eckart_schmidt@t-online.de
Subject: Tripole Transform

```
Dear Tran Quang Hung, dear Chris,
the transformation of Tran in #2075
... maps a line L to its pole wrt the conic QA-Tf2(L),
the Conic-Perspector QA-Co-1 of a circumconic Co of QA-DT
... is the pole of the line QA-Tf2(Co) wrt Co.
Best regards Eckart
```

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Message: #2103
Date: 02/12/2016 8:48:15
From: eckart_schmidt@t-online.de
Subject: Concurrent lines in QA with any point and any l ...

```
Dear Tran Quang Hung,
I followed your construction in #2099, where you describe a
transformation, which maps a line d and a point P to a new point
U.
Observation: For a fixed point P and variable line d
... the points U lie on a line through P.
... So we get a new transformation Tf,
    which maps a point P to a line L = Tf(P) = PU.
If we consider points P on a line QPi for a quadrangle P1P2P3P4
... and have a look for the envelope of the lines Tf(P),
... we get a conic
... ... inscribed the triangle PjPkPl
... ... and tangent to QPi.
If we consider points P on a line QSij (Sij vertex of QA-DT)
... and have a look for the envelope of the lines Tf(P),
... we get a conic
... ... tangent to PiPk, PiPl, PjPk, PjPl
... ... and tangent to QSij.
If we consider points P on a line SijSik
... the lines Tf(P) give the line pencil of Sjk.
Best regards Eckart
```

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Message: #2104
Date: 03/12/2016 4:46:32
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

Dear all,
This topic is not about the usual isogonal and isotomic conjugacies of points.
This topic is related to my message #2090, and my APG topics #2180 and #2873.
We say two lines are isogonal and isotomic conjugates if their trilinear poles(=points) are isogonal and isotomic conjugates respectively.
I present the constructions of the isogonal and isotomic conjugates of lines.
Given a line l and the points A,B,C of intersection of l and three lines of triangle, A',B',C' are the point reflections of A,B,C with respect to the midpoints of triangle respectively.
A',B',C' are collinear on a line called the isotomic conjugate of l.
Given a line l and the points A,B,C of intersection of l and three lines of triangle DEF, DA',EB',FC' are the reflections of DA,DB,DC with respect to the angle bisectors of DEF respectively and A',B',C' lie on EF,FD,DE respectively.
A',B',C' are collinear on a line called the isogonal conjugate of l.
I give two examples. QA-L9 and QL-L1 are the isotomic and isogonal conjugates of QA-L8 respectively.
If P,U are the circular points, then the isogonal conjugate of a point is the PU-conjugate of it.
If p,u are the trilinear poles of the circular points, then the isogonal conjugate of a line is the pu-conjugate of it. Given circular points P(p:q:r) and U(u:v:w), the isogonal conjugates of a point (l:m:n) and a line (lx+my+nz=0) are (pu/l:qv/m:rw/n) and (x/lpu+y/mqv+z/nrw=0) respectively.
If L is the line at infinity, then the isotomic conjugate of a line is the L sqrtdiv of it.
If l is the trilinear pole (=centroid) of the line at infinity, then the isotomic conjugate of a point is the l sqrtdiv of it.
Given the line at infinity L(lx+my+nz=0), the isotomic conjugates of a line (px+qy+rz=0) and a point(p:q:r) are (l^2x/p+m^2y/q+n^2z/r=0) and (1/l^2p:1/m^2q:1/n^2r) respectively.
On a circular Pappian plane, we have the circular points and the line at infinity.
Hence we have isogonal and isotomic conjugates of points and lines.

On a (vertical) parabolic Pappian plane, the circular points become one.
Hence the isogonal and isotomic conjugacies are dual concepts!
The isogonal and isotomic conjugates of all points on a line are two point conics.
The isogonal and isotomic conjugates of all lines through a points are two line conics.
Never forget point and line conics are equivalent!
Best regards,
Tsihong Lau

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Message: #2105
Date: 03/12/2016 5:05:52
From: minhtenladuong
Subject: Coaxal circles in QA

Dear Mr Tran Quang Hung
I have already posted it here Yahoo! Groups (https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1198)
Sincerely
Ngo Quang Duong

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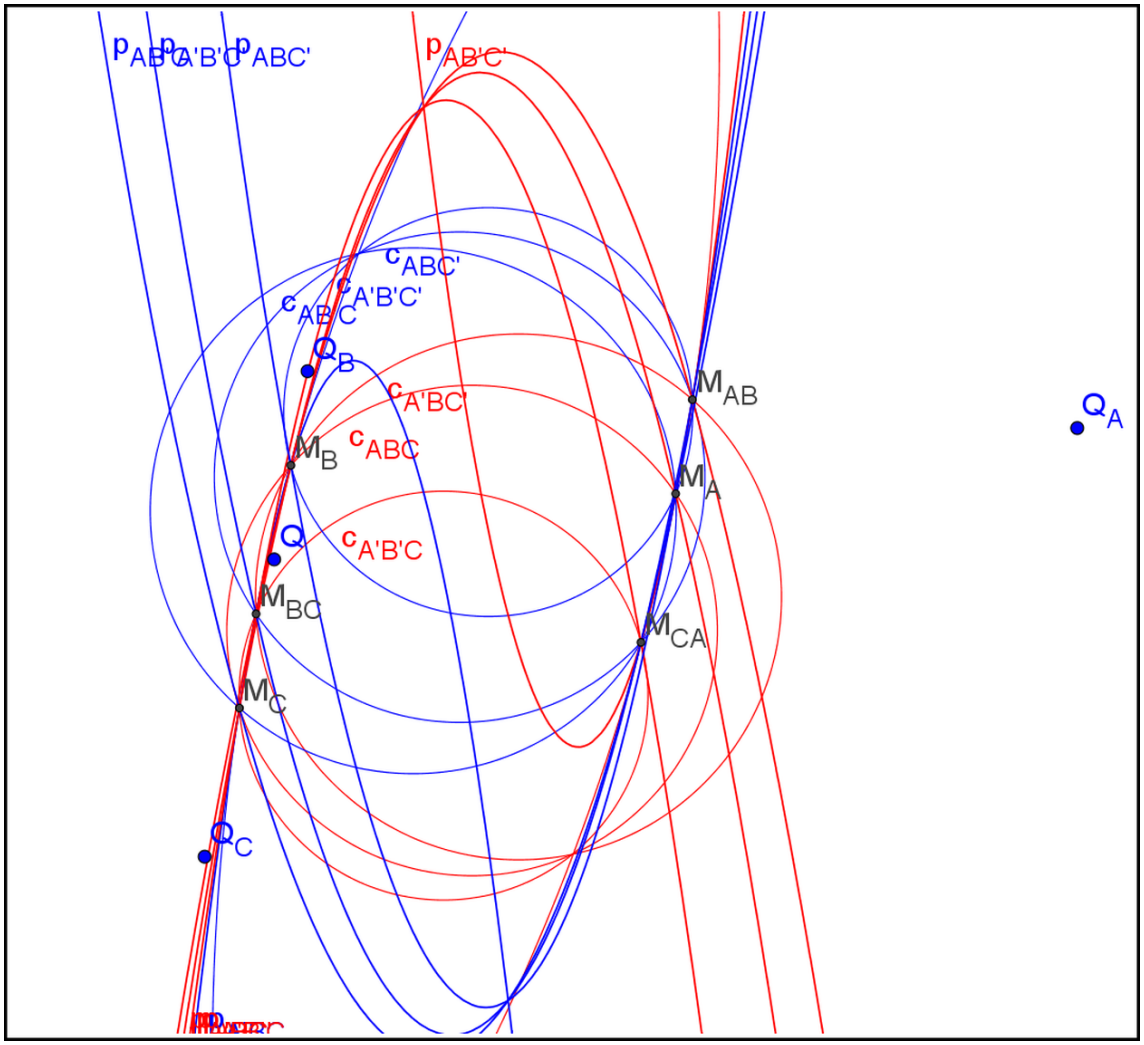
Message: #2106
Date: 04/12/2016 6:32:32
From: tsihonglau
Subject: Vertical Parabolic Pappian Plane

Dear all,
This topic is a derivative of message #2090.
We study circular Pappian plane very much.
But we study vertical parabolic Pappian planes so little. One of the latter is so called the classical real Laguerre plane.
Laguerre plane - Wikipedia (
https://en.wikipedia.org/wiki/Laguerre_plane)
(https://en.wikipedia.org/wiki/Laguerre_plane)
I give two objects of a quadrangle on a vertical parabolic Pappian plane- Euler-Poncelet and Gergonne-Steiner points.
I use my notations - http://lth.name/geometry/duality.html
Given a quadrangle Q,Q_A,Q_B,Q_C, the midpoints of diapleurals are:
   M_A - Q,Q_A
   M_B - Q,Q_B
   M_C - Q,Q_C
   M_BC - Q_B,Q_C
   M_CA - Q_C,Q_A
   M_AB - Q_A,Q_B
The Euler-Poncelet point on a circular Pappian plane is the point of concurrency of the circumcircles of the following triangles.
   M_BC,M_CA,M_AB
   M_BC,M_B,M_C
   M_A,M_CA,M_C
   M_A,M_B,M_AB
The Gergonne-Steiner point on a circular Pappian plane is the point of concurrency of the circumcircles of the following triangles.
   M_A,M_B,M_C
   M_A,M_CA,M_AB
   M_BC,M_B,M_AB
   M_BC,M_CA,M_C
On a vertical parabolic Pappian plane, the "circumcircles" are replaced with vertical "circumparabolas".
The two points exists, too.
As on a circular Pappian plane, the centroid of the quadrangle is the midpoint of the two.
Please see the attachments.
Unlike a circular Pappian plane, a vertical parabolic plane is dual.
In other words, if we replace "quadrangle" with "quadrilateral" and vertical "circumparabola" with vertical

"inparabola" in the above statements, the Euler-Poncelet and
Gergonne-Steiner points become Euler-Poncelet and
Gergonne-Steiner lines.
To be continued...
Best regards,
Tsihong Lau



QA-P2QA-P3.png

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Message: #2107
Date: 04/12/2016 11:55:20
From: tsihonglau
Subject: Vertical Parabolic Pappian Plane

```
Dear all,
I did not give the basic terminology in the previous message.
Now I give some terms of a vertical parabolic Pappian plane.

point, line     - basic elements
vertical point  - the fixed point on the infinite line
infinite line   - the fixed line through the vertical point
oblique point   - a not vertical point
                  either on the infinite line or not
finite line     - a not infinite line
                  either through the vertical point or not
infinite point  - a point on the infinite line
                  including the vertical point
vertical line   - a line through the vertical point
                  including the infinite line
finite point    - a point not on the infinite line
oblique line    - a line not through the vertical point
parallel points - two or more oblique points on a vertical line
parallel lines  - two or more finite lines
                  through an infinite point
vertical point parabola -
                  point conic intersecting the infinite line
                  at the vertical point only
vertical line parabola -
                  line conic intersecting the vertical point
                  at the infinite line only
tangent point   - the only point of intersection of a point
conic
                  and a line or two point conics
tangent line    - the only line of intersection of a line conic
                  and a point or two line conics

Because the point and line conics on a Pappian plane are
equivalent, so tangent points/lines are valid
for both point and line conics and vertical point and line
parabolas are equivalent.
Any three points/lines (any two of them are not parallel)
determine only one vertical point/line parabola
respectively.

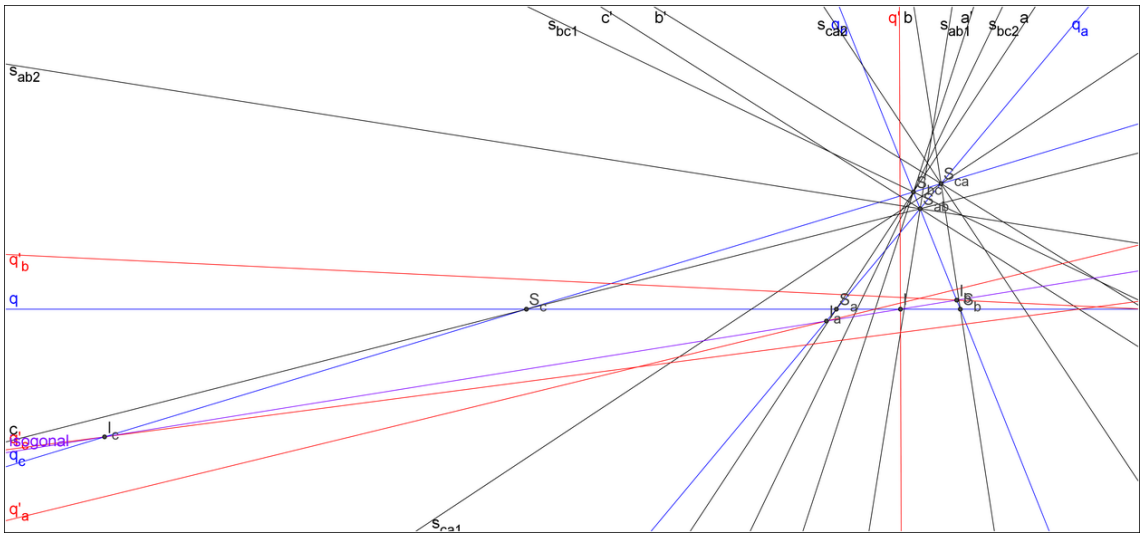
To be continued...
Best regards,
Tsihong Lau
```

Message: #2108
Date: 05/12/2016 3:21:51
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

Dear all,
I found a paper studying the same topic.
http://www.mccme.ru/~akopyan/papers/conjugation_en.pdf
The definitions are the same.
I also found a line of a quadrilateral.
Given a quadrilateral q,q_a,q_b,q_c, q',q'_a,q'_b,q'_c are the isogonal conjugates of q,q_a,q_b,q_c with respect to the exline trilaterals(component triangle) respectively.

   q' - q_a,q_b,q_c
   q'_a - q,q_b,q_c
   q'_b - q_a,q,q_c
   q'_c - q_a,q_b,q

Then the points of intersection of q,q' and q_a,q'_a and q_b,q'_b and q_c,q'_c are collinear.
The line maybe called the isogonal axis.
The fist circular coordinate is
q*r^2*w^3-q^3*w^3+p^2*q*w^3+r^3*v*w^2-q^2*r*v*w^2-p^2*r*v*w^2+2*p*q*r*u*w^2-q*r^2*v^2*w+q^3*v^2*w-p^2*q*v^2*w+2*p*r^2*u*v*w+2*p*q^2*u*v*w-2*p^3*u*v*w-q*r^2*u^2*w+q^3*u^2*w-p^2*q*u^2*w-r^3*v^3+q^2*r*v^3+p^2*r*v^3+2*p*q*r*u*v^2+r^3*u^2*v-q^2*r*u^2*v-p^2*r*u^2*v-2*p*q*r*u^3
I do not know if it has been studied before.
Best regards,
Tsihong Lau



isogonal_axis.png

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Message: #2109
Date: 05/12/2016 8:50:18
From: eckart_schmidt@t-online.de
Subject: Isogonal and Isotomic Conjugacies of Lines ...

Dear Tsihong Lau
your isogonal axis in #2108 of a quadrilateral bears
QL-Tf1(QL-P25)
and is the QL-Tf1-image of a circle round QL-P11 through QL-P1.
Best regards Eckart

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Message: #2110
Date: 06/12/2016 2:21:29
From: eckart_schmidt@t-online.de
Subject: Conic-Perspector QA-Co-1

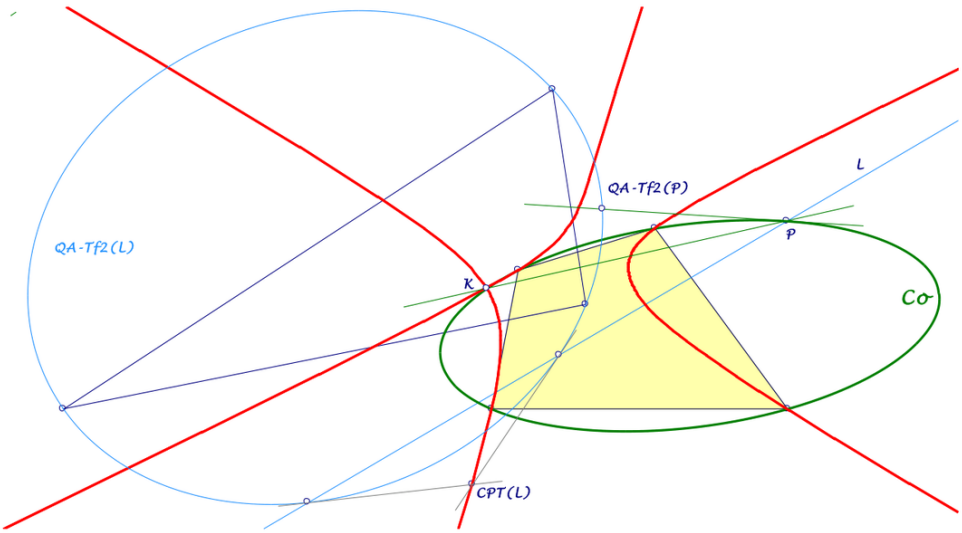
Dear Tran Quang Hung, dear Chris,
in EQF we find a conic-perspector QA-Co-1, defined for
circumconics Co of the QA-diagonal triangle. These conics can be
considered as QA-Tf2-image of a line L , so we get a
transformation CPT, which maps a line to a point. This
transformation is also described by Tran Quang Hung in #2078.
Attached special cubics for a line pencil wrt this
transformation are researched, perhaps of interest.
Best regards Eckart

EQF-Note 2016-12-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Conic-Perspector QA-Co-1

In EQF we find a conic-perspector QA-Co-1, defined for circumconics Co of the QA-diagonal triangle. These conics can be considered as QA-Tf2-image of a line L, so we get a transformation CPT, which maps a line to a point. This transformation is also described by Tran Quang Hung in QFG-message 2078. Here special cubics for a line pencil wrt this transformation are researched.



Tran Quang Hung describes his transformation as follows:
Let $P_1P_2P_3P_4$ be a quadrangle,
… *L* any line,
… *Pi´* the tripole of *L* wrt *PjPkPl*,
… then the lines *PiPi´* are concurrent in *CPT(L)*.
The nomination *CPT* shall be a shortcut of "Conic-Perspector of the *QA-Tf2*-Transformed line".

- ***CPT(L)* is the pole of *L* wrt the conic *QA-Tf2(L)*.**

Examples (see also *EQF*):
(1) *Co* = *QA-Ci1*,
... *L* = polar of *QA-P38* wrt *QA-Ci1*,
…*CPT(L)* = *QA-P38*.

(2) *Co* = *QA-Co1*,
… *L* = line at infinity,
… *CPT(L)* = *QA-P1*.
(3) *Co* = *QA-Co4*,
… *L* = *QA-L4*,
… *CPT(L)* = pole of *QA-L4* wrt *QA-Co4*.
(4) *Co* = *QA-Co5*,
… *L* = *QA-L3*,
… *CPT(L)* = pole of *QA-L3* wrt *QA-Co5*.

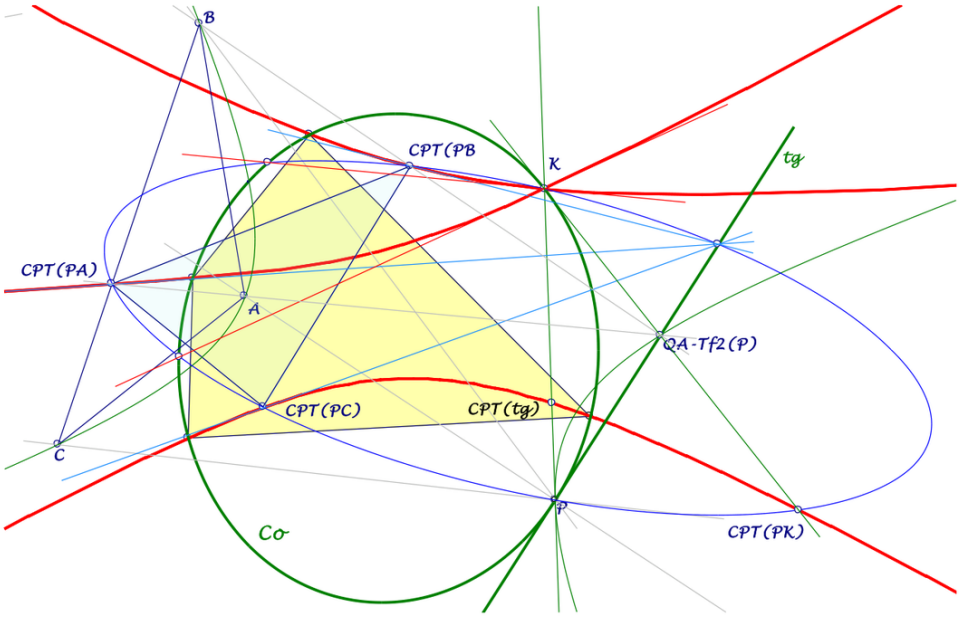
Now we consider the line pencil of a point *P*.

- **The *CPT*-images of lines *L* through a point *P* give a *QA*-circumcubic**
  … with knot *K*
  … … on the *QA*-circumconic *Co* through *P*
  … … as intersection – unequal *P* – of *Co* and the *Co*-polar of *QA-Tf2(P)*.

- **The 3rd intersection of *PK* and the cubic is the *CPT* of the tangent *tg* in *P* at *Co*.**
- ***CPT(PK)*, *QA-Tf2(P)* and *K* are collinear.**

Let *ABC* be the *QA*-diagonal triangle, then *CPT(PA)*, *CPT(PB)*, *CPT(PC)* give a special triangle on the cubic.

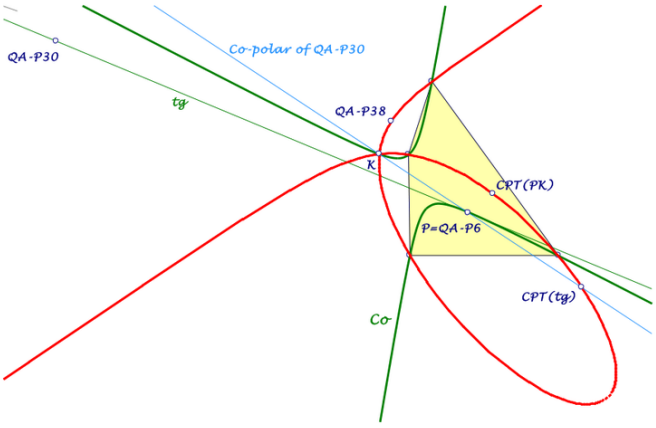
- **For the *QA*-diagonal triangle *ABC* the *CPT*-images of the lines *PA*, *PB*, *PC* give the *ABC*-cevian triangle of *QA-Tf2(P)*.**



- **The tangents in *CPT(PA), CPT(PB), CPT(PC)* at the cubic have a common point**
  … on the tangent in *P* at *Co*
  … in the 2nd intersection with the circumconic of *CPT(PA), CPT(PB), CPT(PC), P* and *K*.

- **The tangents in *K* at the cubic are *KX* and *KY* with *X, Y* 3rd and 4th intersection of the conics**
  … *Co* through the *QA*-vertices and *P*
  … and the circumconic of *CPT(PA), CPT(PB), CPT(PC), P* and *K*.

Example for *P* = *QA-P6* with *QA*-circumconic *Co* through *P*:
… *CPT* of a perpendicular in *QA-P6* wrt *QA-L2* is *QA-P38*,
… tangent in *P* at *Co* is *QA-P6.QA-P30*,
… knot *K* is the 2nd intersection of *Co* and the polar of *QA-P30*,
… *CPT(PK)* is collinear with *K* and *QA-P30*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #2111
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Messages

```
Message number 2111 is not available in Yahoo groups.
```

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Message: #2112
Date: 06/12/2016 3:03:16
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

```
Dear Eckart,
Thanks for your information. But can you compute the coordinates
of the isogonal axis?
I computed the circulars only!
If we replace "isogonal" with "isotomic" in #2108,
The corresponding isotomic axis does not exist!
But q',q'_a,q'_b,q'_c concur at the infinity point of the
inscribed parabola!
This point is very important to a vertical parabolic Pappian
plane.
Please refer to the topic #2106.
Best regards,
Tsihong Lau
```

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Message: #2113
Date: 06/12/2016 3:41:16
From: Tran Quang Hung
Subject: Conic-Perspector QA-Co-1

```
Thank you very much dear Mr Eckart for nice contributation.
Best regards,
Tran Quang Hung.
```

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Message: #2114
Date: 06/12/2016 3:42:42
From: Tran Quang Hung
Subject: Concurrent lines in QA with any point and any line

```
Thank you Mr Tsihong Lau for your interest.
Best regards,
Tran Quang Hung.
```

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Message: #2115
Date: 06/12/2016 3:53:56
From: tsihonglau
Subject: Vertical Parabolic Pappian Plane

Dear all,
Given a quadrilateral and the infinite line.
we can construct the inscribed parabola.
The tangent point on the infinite line can be viewed as the vertical point.
Then we get a vertical parabolic Pappian plane.
There is a quadrangle dual to the quadrilateral.
Given a quadrangle and the infinite line, we get the parabolic points on the infinite line. (Topic #1939)
The parabolic points can be viewed as the circular points.
Then we get the isogonal and isotomic conjugations on Pappian plane with the infinite line. (topic #2104)
Best regards,
Tsihong Lau

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Message: #2116
Date: 07/12/2016 10:52:35
From: eckart_schmidt@t-online.de
Subject: Isogonal and Isotomic Conjugacies of Lines ...

Dear Tsihong Lau,

here the equation of your isogonal axis in DT-nomination (see EQF):
(see picture)

Best regards Eckart

$$\left(SA\,(m^2\,b^2+n^2\,c^2+l^2\,a^2)^2-4\,m^2\,n^2\,SA^3+4\,l^2\,S^2\,(n^2\,SB+m^2\,SC)+4\,l^4\,SB\,SC\,a^2\right)x+$$
$$\left(SB\,(m^2\,b^2+n^2\,c^2+l^2\,a^2)^2-4\,n^2\,l^2\,SB^3+4\,m^2\,S^2\,(l^2\,SC+n^2\,SA)+4\,m^4\,SC\,SA\,b^2\right)y+$$
$$\left(SC\,(n^2\,c^2+l^2\,a^2+m^2\,b^2)^2-4\,l^2\,m^2\,SC^3+4\,n^2\,S^2\,(m^2\,SA+l^2\,SB)+4\,n^4\,SA\,SB\,c^2\right)z=0$$

iNDO8U7b.png

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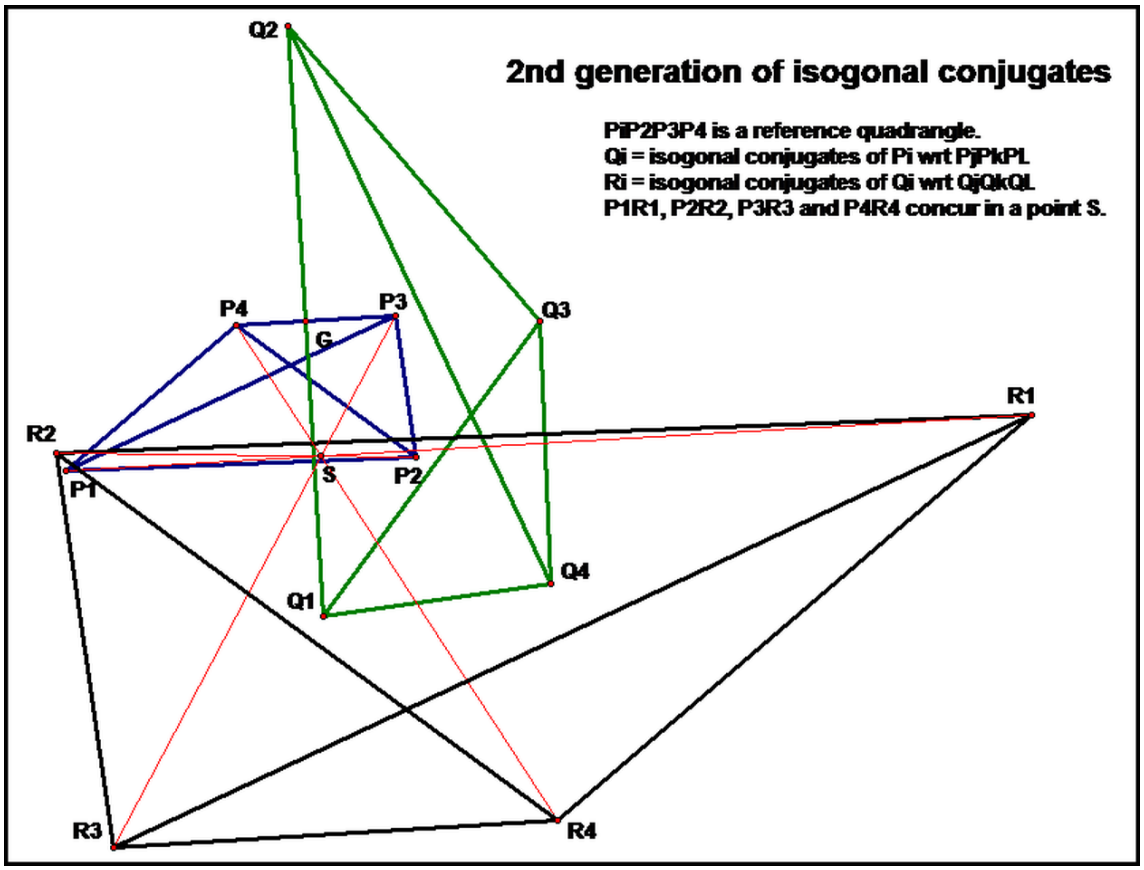
Message: #2117
Date: 09/12/2016 5:01:39
From: Seiichi Kirikami
Subject: 2nd generation of isogonal conjugates

Dear friends,
“transformed quadrangles” in QA-1 of EQF describes about 2nd generation of T quadrangle.
Let P1P2P3P4 be a reference quadrangle.
Denote:
Qi = isogonal conjugate of Pi wrt PjPkPl.
Ri = isogonal of Qi wrt QjQkQl.
P1R1, P2R2, P3R3 and P4R4 concur in a point S.
See the attachment.
Does this point have its corresponding one in EQF?
Though I replaced “isogonal” in the above by “isotomic”, I could not obtain any concurrent point.

(The reference quadrangle and 1st generation quadrangle determine the isotomic center.)
Best regards, Seiichi.



2ndIsogonal.png

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Message: #2118
Date: 09/12/2016 8:50:44
From: eckart_schmidt@t-online.de
Subject: 2nd generation of isogonal conjugates

Dear Seiichi,
the point S will be QA-P4.
Best regards Eckart

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Message: #2119
Date: 09/12/2016 8:52:52
From: chris.vantienhoven
Subject: 2nd generation of isogonal conjugates

Dear Seiichi,
The 2nd generation Isogonal Homothetic Center is QA-P4.
The 1st generation Isotomic Perspector is -like you noted- QA-P5.
The 2nd generation Isotomic Perspector is rather complicated.
I studied it before. There are no incidences with known points/lines/curves.
These are questions from the very beginning of EQF.
See Hyacinthos #19635, #19636, #19649. †)
At that time (January 2011) I discovered that QA-P4 is the Inverse
in the circumcircle of PiPjPk of the isogonal conjugate of Pi wrt PjPkPl, for each combination (i,j,k,l) from (1,2,3,4).
This special symmetric property inspired me for finding more of these points.
Best regards,
Chris

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #2120
Date: 09/12/2016 10:34:27
From: eckart_schmidt@t-online.de
Subject: 2nd generation of isogonal conjugates

```
Dear Chris,
as Seiichi already mentioned, I also cannot obtain "the 2nd
generation Isotomic Perspector".
Best regards Eckart
PS. The mentioned property of QA-P4 is already published 2000
    (see EQF, Ref-18).
```

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Message: #2121
Date: 09/12/2016 11:53:58
From: chris.vantienhoven
Subject: 2 nd generation of isogonal conjugates

```
Dear Eckart,
Thanks for your reaction.
You are right in both remarks.
What I described with
"the 2nd generation Isotomic Perspector" doesn't have the same
construction as "the 2nd generation Isogonal Homothetic Center".
I am sorry for my misunderstanding. What I described was the
Isotomic Center of the "the 1st generation Isotomic Quadrangle".
And you are absolutely right that QA-P4 already was already
published 2000 (see EQF, Ref-18).
Best regards,
Chris
```

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Message: #2122
Date: 09/12/2016 11:56:21
From: tsihonglau
Subject: 2nd generation of isogonal conjugates

Dear all,
The second generation of perspectrix of isogonal conjugates of lines for a quadrilateral does not exist, either.
See topic #2104
Best regards,
Tsihong Lau

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Message: #2123
Date: 09/12/2016 12:01:51
From: eckart_schmidt@t-online.de
Subject: 2 nd generation of isogonal conjugates

Dear Seiichi, dear Chris,
here a generalization of the wellknown property of QA-P4:
... Pi ---> PjPkPl- isogonal conjugate ---> inversion wrt the circumcircle of PjPkPl ---> QA-P4.
Generalization for the isotomic conjugation:
... Pi ---> PlPkPl-isotomic conjugate ---> "inversion" wrt the Steiner PjPkPl-ellipse ---> Qi.
... Q1Q2Q3Q4 is perspective P1P2P3P4
... ... with perspector: 4th harmonic point of QA-P21 wrt QA-P5, QA-P17.
Best regards Eckart
PS: "Inversion" wrt the Steiner ellipse means: 4th harmonic point of X wrt the intersections of the Steiner ellipse and a line, connecting X and the center of the Steiner ellipse.

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Message: #2124
Date: 09/12/2016 2:49:59
From: tsihonglau
Subject: 2nd generation of isogonal conjugates

Dear Eckart,
The generalization may be generalized further.
circumcircle = isogonal conjugate image of the line at infinity
Steiner ellipse = isotomic conjugate image of the line at infinity
center = pole of the line at infinity with respect to a conic
isogonal isotomic conjugate are examples of isoconjugation.
If the generalization hold, the dual(quadrilateral) should be true, too.
Best regards,
Tsihong Lau

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Message: #2125
Date: 10/12/2016 2:57:12
From: Seiichi Kirikami
Subject: 2nd generation of isogonal conjugates

Dear Eckart, dear Chris, Dear Tsihong,
Thank you very much.
I also tried in vain to apply “hyperbolic conjugates” to a quadrangle.
(My intention was to introduce a new point in EQF.)
See:
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2976 †)
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2977 †)
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2979 †)
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2980 †)
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conversations/messages/2992 †)
Best regards, Seiichi.

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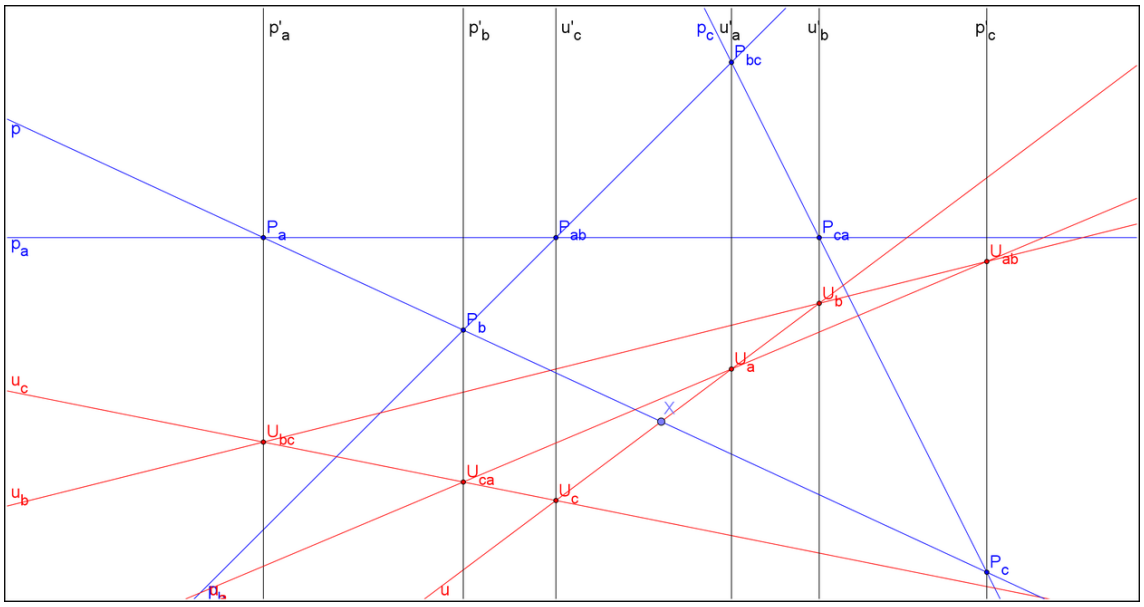
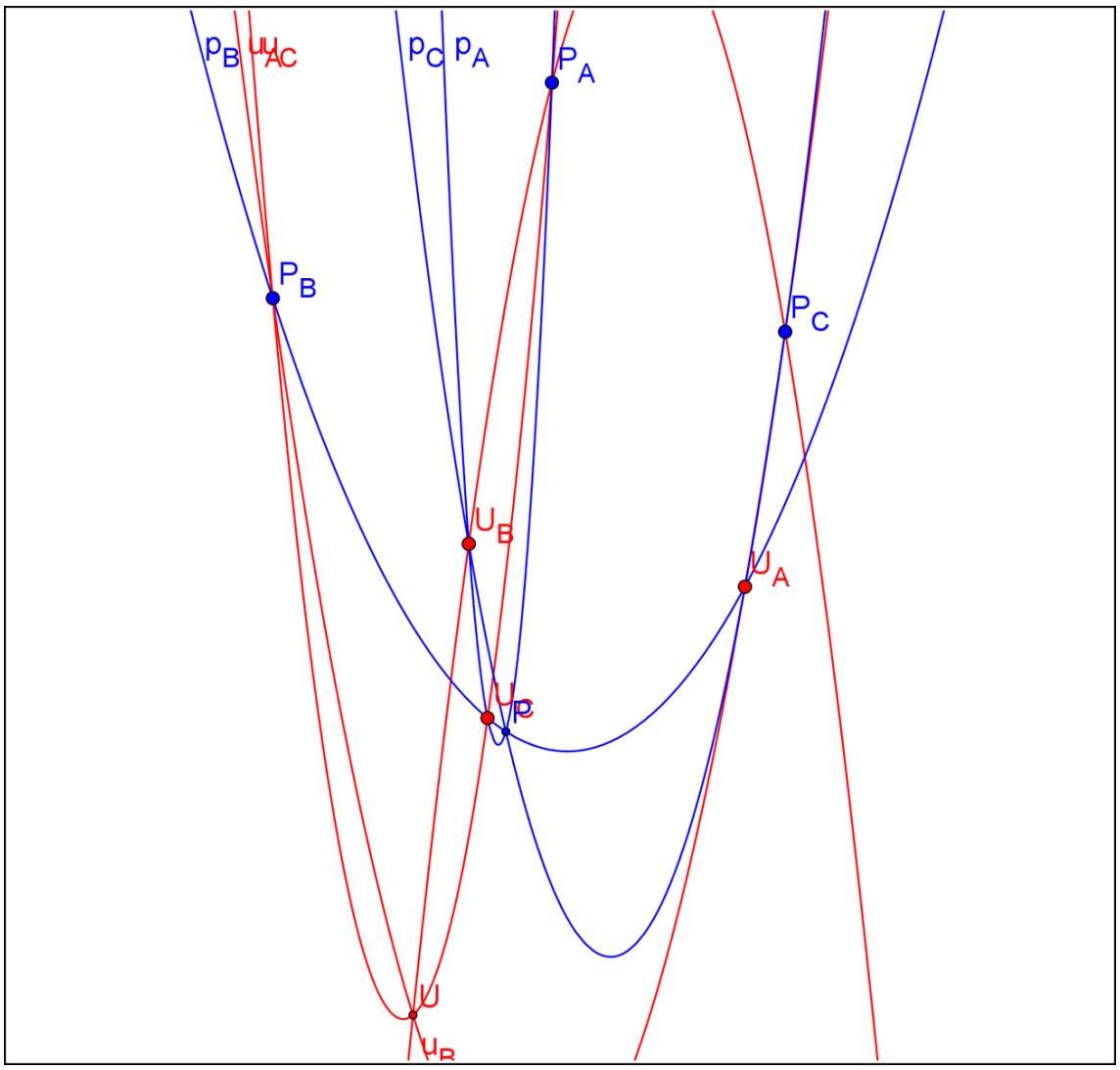
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #2126
Date: 11/12/2016 2:55:32
From: tsihonglau
Subject: Vertical Parabolic Pappian Plane

Dear all,
On a vertical parabolic Pappian plane, there is dual of parallelologic quadrangles, maybe called parallelologic(or verticalogic) quadrilaterals.
We can replace the infinite line with the vertical point.
Please refer to the attachment files.
Moreover, if we replace circles with vertical parabolas, we can get cyclologic quadrangles for vertical parabolas.
There is dual of cyclologic quadrangles, maybe called cyclologic quadrilaterals.
Please refer to the attachment files.
According to the previous message, there is a vertical point on a quadrangle/quadrilateralPappian plane with the infinite line.
The above three types of quadrangles/quadrilaterals canbe applied also to a circular Pappian plane.
Best regards,
Tsihong Lau



parallelologic_quadrilateral.png



cyclologic_quadrangle_parabola.png

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Message: #2127
Date: 13/12/2016 9:58:22
From: eckart_schmidt@t-online.de
Subject: QA-L-1

Dear Chris,
wrt QL-L-1 there are only two examples of the 5th point tangent,
a general property is
QA-L-1(P) = P.QA-Tf2(P).
Another aspect:
If we consider isoconjugations
... with fixed point P
... and reference triangle PjPkPl,
... the images of Pi are collinear on QA-L-1(P).
Best regards Eckart

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Message: #2128
Date: 13/12/2016 8:56:42
From: eckart_schmidt@t-online.de
Subject: "QA-Euler Lines" wrt QA-Points

Dear Chris,
in my 17. "Review" (25.02.2012) to you I described Euler lines
for quadrangles wrt special QA-points: If we consider the X(2)-,
X(3)-, X(4)-quadrangles of a reference QA
... and their QA-Px points,
... we often get three collinear points
... in Euler constellation.

Examples for:
QA-P1, -P2, -P3, -P5, -P10, -P16, -P17 , -P18, -P19, -P20, -P21,
-P22, -P25, -P26, -P27, -P29, -P31, -P34, -P35.
CABRI- observations confirm the assumptions:
For any two of these points every point, dividing the segment in
a fixed ratio, leads also to a QA-Euler line.
For any two lines through these points the intersection leads to
a QA-Euler line.
Iterating this gives a net of infinite QA-points, generating an
Euler line, but the not mentioned QA-points don´t belong to this
net.
Perhaps someone can better interpret these results.
Best regards Eckart

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Message: #2129
Date: 14/12/2016 3:59:19
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

Dear all,
The formulas for isogonal and isotomic conjugates
of lines are the same as of points. That is,
Using trilinear coordinates, the isogonal conjugates
of a point (p:q:r) and a line (lx+my+nz=0) are
(1/p:1/q:1/r) and (x/l+y/m+z/n=) respectively.
Using barycentric coordinates, the isotomic conjugates
of a point (p:q:r) and a line (lx+my+nz=0) are
(1/p:1/q:1/r) and (x/l+y/m+z/n=) respectively.
Best regards,
Tsihong Lau

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Message: #2130
Date: 15/12/2016 3:06:41
From: tsihonglau
Subject: Parabolic Points at Infinity

```
Dear all,
I would post this message long ago!
If we replace the circular points on a triangle/trilateral
Pappian plane with the parabolic points on a quadrangle
/quadrilateral Pappian plane, we can get a lot of
quadrangle /quadrilateral centers from the present ETC
immediately. The correspondence table is:
ETC                         - EQF
X(3) circumcenter           - QA-P1 QA-centroid
X(20) de Longchamps point   - QA-P5 isotomic center
X(6) symmedian point        - QA-P16 QA-harmonic center and
                              QL-P13 QL-harmonic center
X(64)                       - QA-P17
X(25)                       - QA-P18
X(69)                       - QA-P19
X(4) orthocenter            - QA-P20
X(1350)                     - QA-P21
X(5) nine-point center      - QA-P22
X(?)                        - QA-P25
X(?)                        - QA-P26
X(?)                        - QA-P27
X(141)                      - QA-P31
X(?)                        - QA-P43
X(351) center of the Parry circle - QL-P12
X(?)                        - QL-P14
X(?)                        - QL-P15
X(?)                        - QL-P18
X(669)                      - QL-P23
Euler Line                  - QA-L3
?                           - QA-L7
?                           - QL-L1
?                           - QL-L8
?                           - QL-L9
The transformation is easy - a=p=1/l, b=q=1/m, c=r=1/n
in barycentric (DT) coordiantes.
There are thousands of ETC centers, so we get thousands of
quadrangle/quadrilateral centers immediately.
Moreover, we can define pseudo-distances and pseudo-angles with
the parabolic points.
I will explain them in next message.
To be continued...
Best regards,  Tsihong Lau
```

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Message: #2131
Date: 15/12/2016 4:14:28
From: eckart_schmidt@t-online.de
Subject: QG-P2, QG-P15, QG-P18

```
Dear Chris,
The three QL-versions of QG-P2
are collinear on QL-L1 (see EQF),
... the three QG-circumconics through QG-P2
... ... have a common point on QL-L1
... ... in the infinity point.
The three QL-versions of QG-P15 are collinear on QL-L9
(see EQF),
... the three QG-circumconics through QG-P15
... ... have a common point on QL-L9
... ... in the intersection with QL-P8.QL-P13.
The three QL-versions of QG-P18 are collinear (see EQF)
on L = CSC(QL-Ci2),
... the three QG-circumconics through QG-P18
... ... have a common point on L
... ... in the intersection with the radical axis
        of QL-Ci1 and QL-Ci2.
Best regards Eckart
```

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Message: #2132
Date: 16/12/2016 7:34:16
From: chris.vantienhoven
Subject: new items in EQF

```
Dear friends,
I placed these new items in EQF:
   QA-8/QL-8 The dual QA/QL-Configuration
   QA-Tf7    QA-Trilinear Pole
   QA-Tf8    QA-Orthopolar Line
   QL-Tf4    QL-Orthopole
   QL-Tf5    QL-Orthopolar Line
If you have any additions or remarks please let me know.
Best regards,
Chris
```

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Message: #2133
Date: 17/12/2016 10:39:46
From: bernard.keizer
Subject: QL and hypocycloïds with 2n+1 cusps

Dear Chris, dear Eckart, dear Tsihong Lau
As well known, Kantor referred explicitely to the tricuspid hypocycloïd when he proved his theorem, known as Kantor-Hervey theorem.
Here is a complete generalisation of this theorem to hypocycloïds with 2n+1 cusps using the Hofstadter points of the 4 reference triangles of the QL with r integer.
The barycentric calculation or calculation with complex coordinates seems possible as the reference triangles are similar to the reference triangles of the X3 QA (see attached file).
I suppose it could also interest Jean-Pierre Ehrmann or Jean-Louis Aymé as well as other geometers ...
Best regards
Bernard

QL and hypocycloïds with 2n + 1 cusps

or

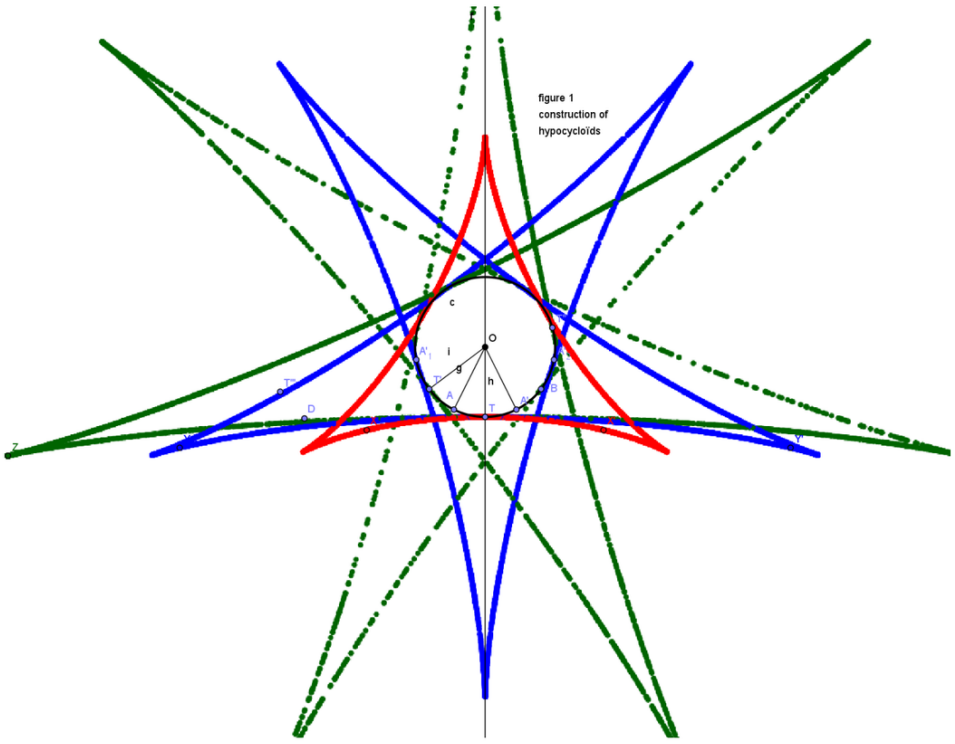
generalisation of the Kantor-Hervey theorem

1) Hypocycloïds with 2n + 1 cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio n+1/n (precisely the first point A being the primary point and the other the secondary point B at a speed n+1/n the speed of the first).

The lines through the 2 points envelop a hypocycloid with 2n + 1 cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio n+1/n.(we have TB/n+1 = TA/n = AB)

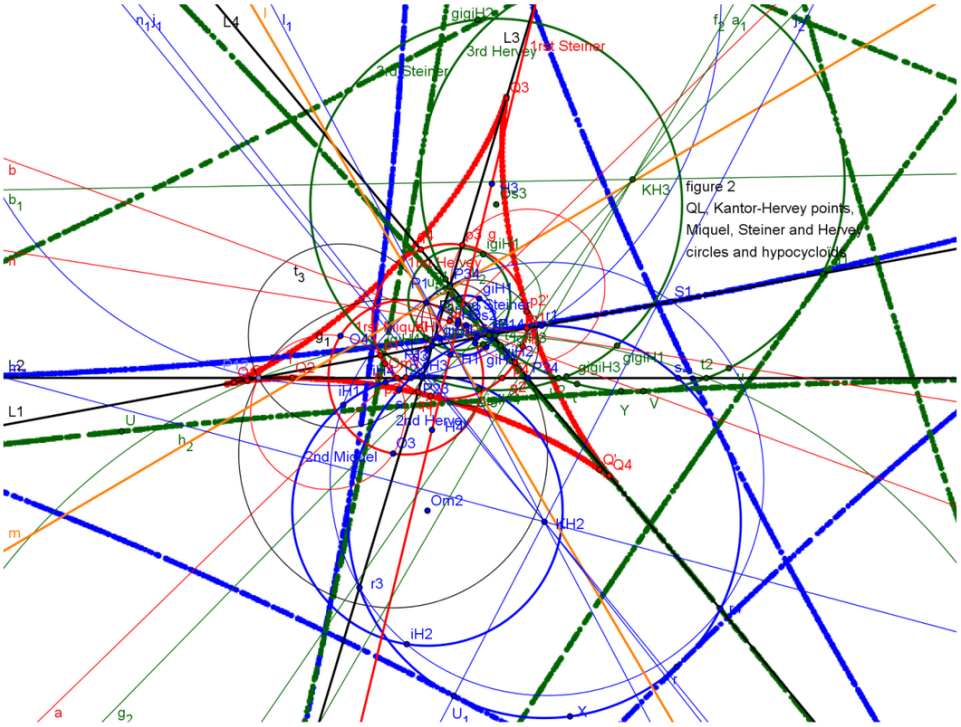
For example, if n=1, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For n=2 or 3, it's a hypocycloid with 5 or 7 cusps …



- The construction in part 1 gives from any of these points a hyperbola with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hyperbolas with 2n + 1 cusps and the inner circles of these hyperbolas are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



- The construction in part 1 gives from any of these points a hyperbola with 2n + 1 cusps tangent to the QL

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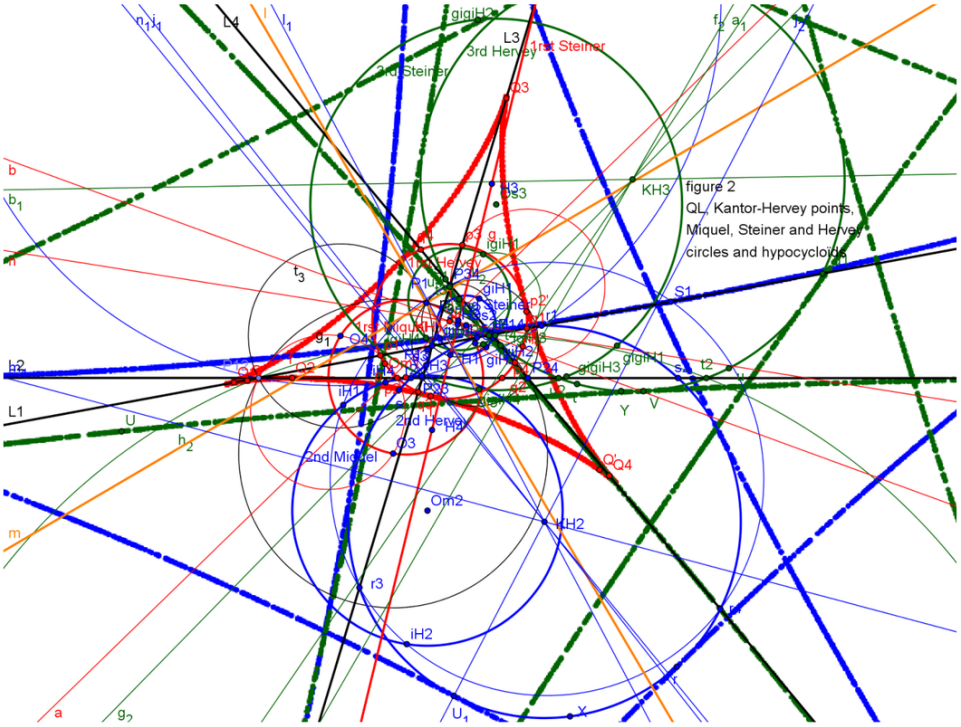
This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocycloids.docx

Message: #2134
Date: 18/12/2016 11:00:23
From: tsihonglau
Subject: Parabolic Points at Infinity

Dear all,
I knew I violated my hope that this topic should not cover the coordinate system.
But the parabolic points are as important as the circular points.
If we replace the circular points on a triangle/trilateral Pappian plane with the parabolic points on a quadrangle/quadrilateral Pappian plane, we identify the geometry of one with the geometry of the other.

line at infinity - line at infinity
circular points - parabolic points
reference triangle/trilateral - diapleural triangle/diagonal trilateral
incenter/excenters quadrangle - reference quadrangle
circumcircle - nine-point conic
...
The distance formula on the triangle/trilateral plane is the same as the pseudo-distance formula on the quadrangle/quadrilateral.
Please refer to my page - http://lth.name/geometry/coordinate.html
Moreover, we can define pseudo-perpendicaularity of two lines on the quadrangle/quadrilateral plane.
The formulas of both are the same.
The incenter/excenters quadrangle is orthocentric as the reference quadrangle is pseudo-orthocentric.
Even we can define pseudo-angles on the quadrangle/quadrilateral plane.
In other words, we cannot distinguish both geometries.
The objects of ETC are also the objects of a quadrangle/quadrilateral.
The objects of CTC are, too.
We have already thousands of quadrangle/quadrilateral centers and hundreds of cubics!
It is unnecessary to include the objects of quadrangle/quadrilateral with coordinates /coefficients in p,q,r or l,m,n only in EQF since most of them are in ETC or CTC already.
We should only include objects with coordinates/coefficients in a,b,c and p,q,r or l,m,n since they are defined with real circular points and pseudo-circular points (parabolic points).
The quadrangle/quadrilateral geometry

with only pseudo-distance, pseudo-perpendicularity,
pseudo-angle, etc should be distinguished from the geometry
with real distance, real perpendicularity, real angle etc.
Best regards,
Tsihong Lau

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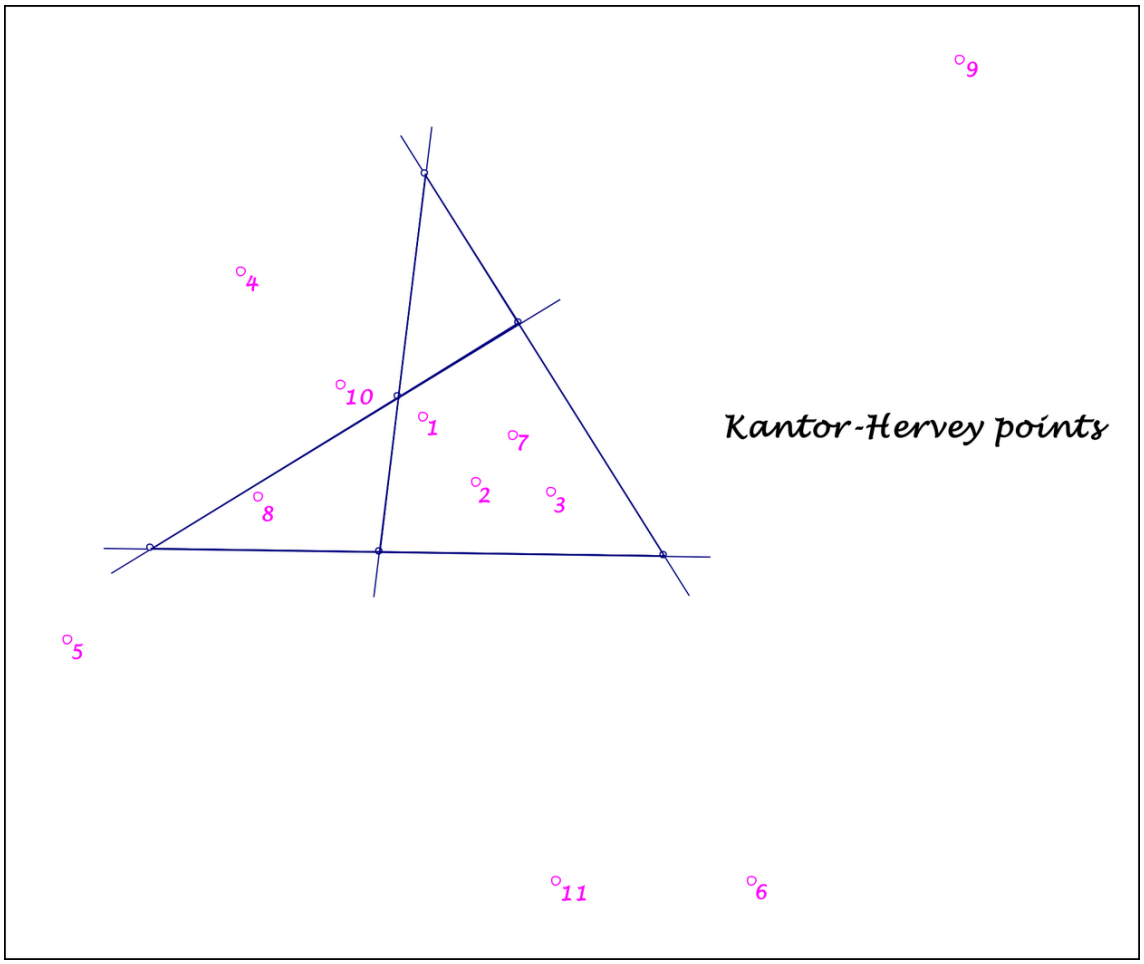
Message: #2135
Date: 19/12/2016 4:40:24
From: eckart_schmidt@t-online.de
Subject: QL and hypocycloïds with 2n+1 cusps

Dear Bernard,
I just study your generalisation of the Kantor-Hervey theorem.
Am I right, that there are some typos?
Page 2: (point 3 and 5): ... QL-P4 / QL-P3
Page 3: ... hyperbola / hypocycloid.
I tried, to find the locus for the Kantor-Hervey points, but in vain (see attached file).
Best regards Eckart



2016-12-19 .pdf

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Message: #2136
Date: 19/12/2016 9:45:32
From: bernard.keizer
Subject: QL and hypocycloïds with 2n+1 cusps

Dear Eckart,
Thanks for your interest !
I had this intuition long time ago, but never succeeded until yet in drawing the figures ...
Thanks also for correcting the typos.
I hope you will be able to calculate the barycentric coordinates of all these points by using the similitude between the reference triangles of the QL and of the X3-QA on the Miquel circle.
Best regards
Bernard
PS I've put my file after correcting the typos under in a pdf-file and I attach it again

QL and hypocycloïds with 2n + 1 cusps

or

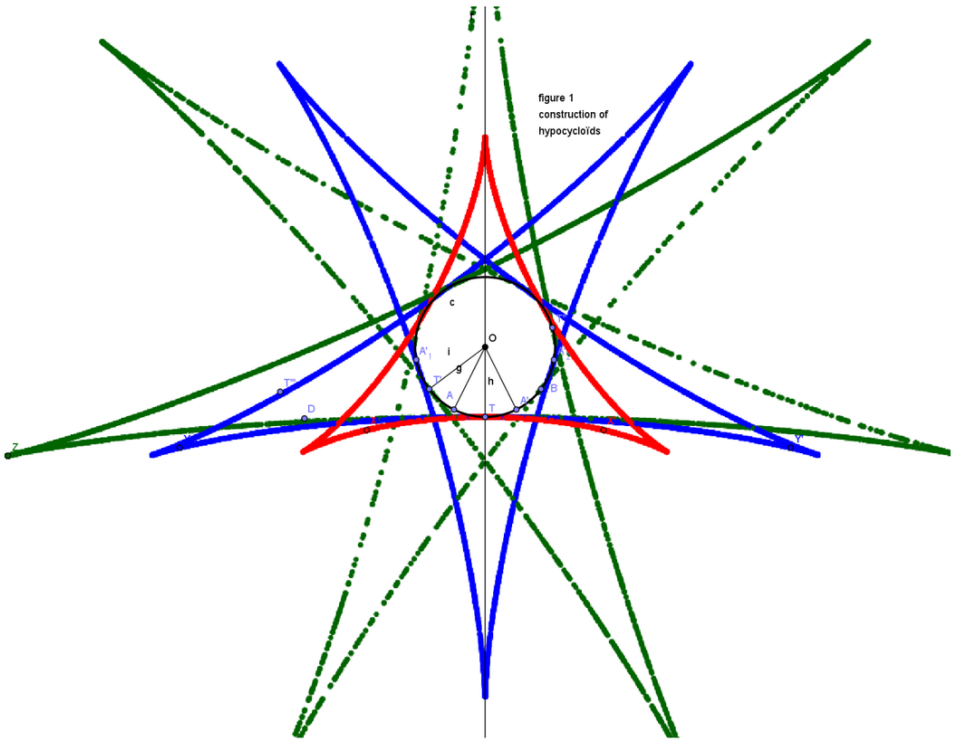
generalization of the Kantor-Hervey theorem

1) Hypocycloïds with 2n + 1 cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio n+1/n (precisely the first point A being the primary point and the other the secondary point B at a speed n+1/n the speed of the first).

The lines through the 2 points envelop a hypocycloid with 2n + 1 cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio n+1/n.(we have TB/n+1 = TA/n = AB)

For example, if n=1, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For n=2 or 3, it's a hypocycloid with 5 or 7 cusps …

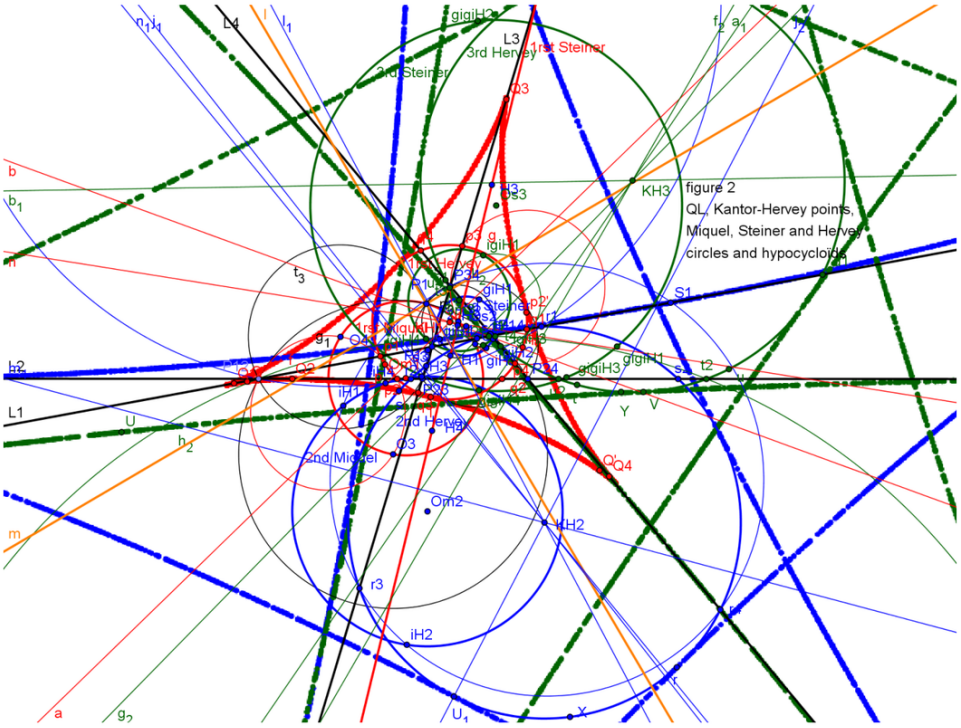


QL and hypocycloids.pdf

- The construction in part 1 gives from any of these points a hypocycloïd with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloïds with 2n + 1 cusps and the inner circles of these hypocycloïds are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.

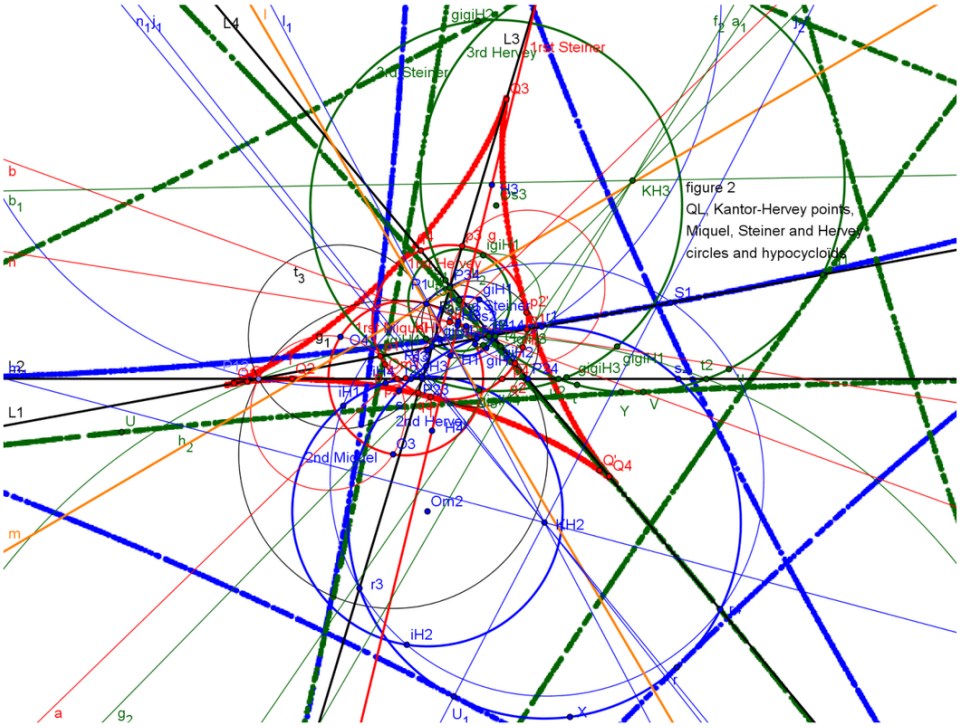


QL and hypocycloids.pdf

- The construction in part 1 gives from any of these points a hypocycloïd with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloïds with 2n + 1 cusps and the inner circles of these hypocycloïds are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocycloids.pdf

Message: #2137
Date: 20/12/2016 10:32:16
From: bernard.keizer
Subject: Berlin

```
Lieber Eckart,
Heute sind wir alle Berliner
Bernard
```

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Message: #2138
Date: 21/12/2016 4:06:21
From: eckart_schmidt@t-online.de
Subject: Generalization of QL-Tr2

```
Dear Bernard,
if I am not wrong the circumconic of QL-DT through QL-P8,
QL-P13, QL-P24 is your often discussed "5th conic".
This conic intersects the Dimidium circle in the socalled
S-points (unequal QL-P24), which give QL-Tr2.
I don´t know whether the following properties are already
mentioned:
... The isoconjugation wrt the S-triangle,
    which swaps QL-P8 and QL-P23,
... maps the Newton line QL-L1 to your "5th conic".
This can be generalized for the following constellation:
... replacing the Dimidium circle
    by any circle through QL-P17 and QL-P24
... and the S-points by the intersections
    (unequal QL-P24) with your "5th conic".
Example for QL-Ci1:
The isoconjugation wrt QL-DT with pole QL-P23 (swapping QL-P8
and QL-P23)
... maps QL-P13 to the point at infinity of QL-L1,
... maps QL-L1 to your "5th conic",
... maps QL-P8.QL-P13 to a parabola, circumscribed QL-DT
... ... through QL-P23
... ... and an axis parallel QL-L1.
Unexpected: All these isoconjugations map a fixed QL-L1-point to
the same point on your "5th conic".
Best regards Eckart
PS. Wrt "Berlin": Thanks for your sympathies.
I can only once again cite Albert Schweizer:
"Aus der Ehrfurcht vor dem Leben erwächst der Welt Frieden".
But how to explain it to terrorists?
```

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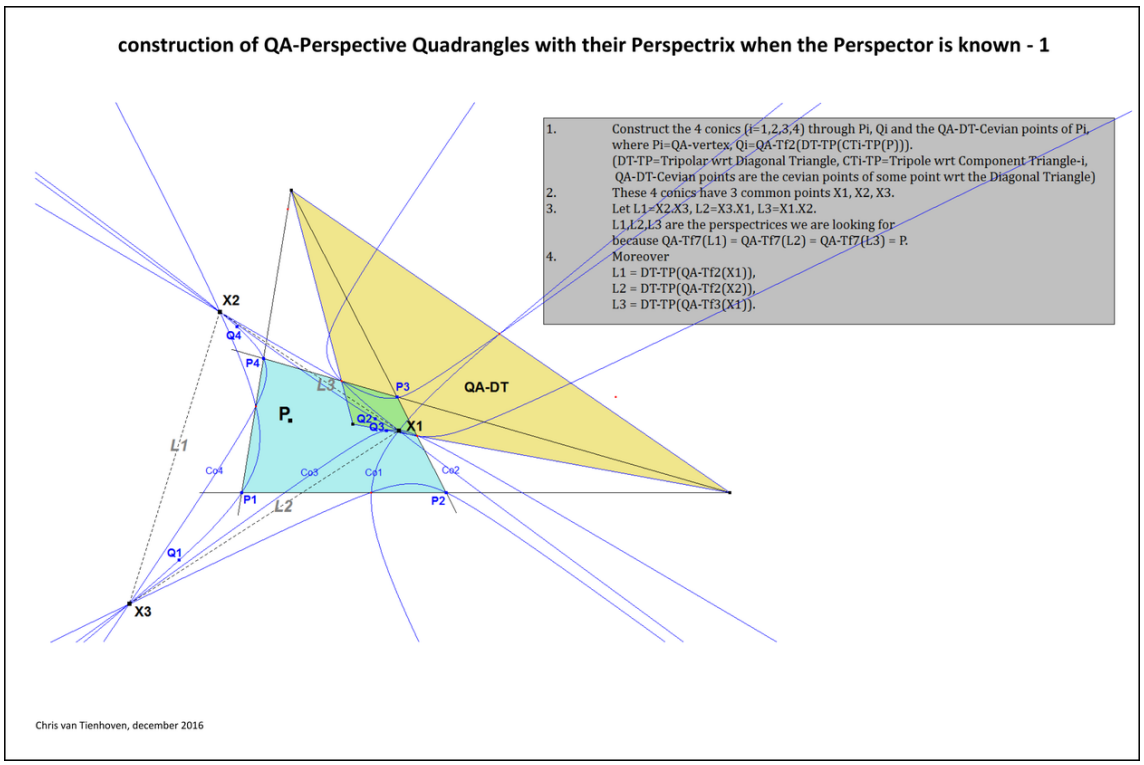
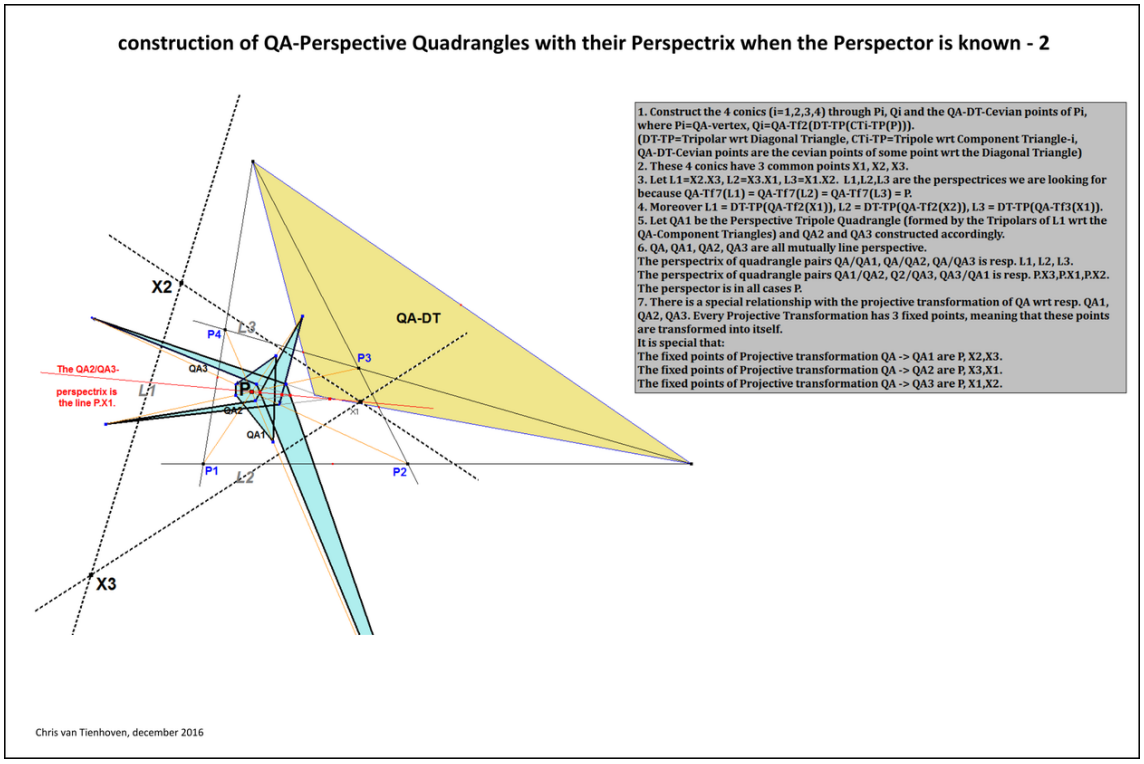
Message: #2139
Date: 21/12/2016 7:52:10
From: chris.vantienhoven
Subject: Perspectivity of Quadrangles

Dear friends,
The properties of perspectivity wrt Quadrangles and Triangles are different.
Definition:
· Two quadrangles are point perspective when the lines through corresponding vertices concur.
· Two quadrangles are line perspective when the intersection points of corresponding sidelines are collinear.
When two triangles are point perspective they also are line perspective and vice versa.
Special for quadrangles is that when two quadrangles are point perspective they not automatically will be line perspective (as a matter of fact they usually are not). However when two quadrangles are line perspective they also are point perspective.
There is a special question for two point perspective Quadrangles to determine under which circumstances they are line perspective.
This very question arises with the QA-Trilinear Pole brought up by Tran Quang Hung at QFG message #2084.
Given Reference Quadrangle QA and a random line L, a Quadrangle QA0 can be created (from the Trilinear Poles of L wrt the Component Triangles) being line perspective with QA, having perspectrix L and perspector QA-Tf7(L).
Now another question arises:
Given a Reference Quadrangle QA and a random point P, how to create another Quadrangle QA0 with perspector P such that QA and QA0 are line perspective. What is the corresponding perspectrix?
This is actually the reversed transformation of QA-Tf7.
Since QA-Tf7 is a nonlinear transformation there can be none or more solutions.
In this case there are 3 solutions.
There are many surprises in this configuration.
See attachment.
Best regards,
Chris
p.s. There is a related question on how to create a Square SQ that will be point and line perspective with a given quadrangle QA. See remarks at QG-2Cu1. There are 2 solutions per QA-Quadrigon. For the construction see attachment at QFG#1240.



QA-Tf7-TripolarTransformation-20.pdf



QA-Tf7-TripolarTransformation-20.pdf

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Message: #2140
Date: 21/12/2016 9:35:17
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,
It seems very interesting !
As I'm leaving for more than a week, I will study it later.
Already 2 questions :
1) what is the pole of your 1rst isoconjugation wrt S-triangle? Could it be QL-P13?
2) what are the fixed points on QL-L1 and on the socalled 5th conic?
By the way, the 5th conic bears also the vertices of the S-points of the DQL, and the Dimidium circle of DQL is through QL-P17, which is DQL-P1 and through DQL-P17 and DQL-P24, intersections with the DDT circumcircle, and through QL-P24 (see your message 1754)...
Merry Christmas and Happy Newyear
Best regards
Bernard

PS Another question : the 3rd QA- or QL-conic through the S-points is through the middles of the DT sides, through QA-P1, QA-P10 and through the point U, intersection of the Newton Lines of both QL and DQL and intersection of QL-L1 with QL-Co1 or contact point of DQL-L1 with QL-Co1.
It's dual conic is therefore inscribed in QL-Tr2 (wich is self-dual), inscribed in the DDT triangle (it's sides are the duals of the middles of the DT sides) and inscribed in the triangle formed by the 2 Newton Lines (duals of QA-P10 and QA-P1) and the line QA-P1QA-P10 (dual of the point U)
Have you by any chance already studied this conic?

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Message: #2141
Date: 22/12/2016 10:20:08
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,
Before leaving, a few ideas without conclusion
QL-P23 is the contact point between the Newton Line, dual of QL-P8 and DQL-Co1, dual of the 5th conic.
It is on the rectangular hyperbola circumscribed to QL-Tr2 and through QL-P2 (orthocenter of the S-triangle), QL-P7 and the intersection of QL-L6 (directrix of DQL-Co1 and Steiner Line of DQL) and QL-L9 (perpendicular to QL-L6 and parallel to the Newton Line of DQL in QL-P23 or in the reflexion of QL-P8 in QL-P12).
I suppose every S-triangle is inscribed in the 5th conic and circumscribed to DQL-Co1.
The 2 intersection points of the Newton Line and the 5th conic are isoconjugate wrt every isoconjugation of triangle S-triangle which swaps QL-P8 and QL-P23 and the Newton Line and the 5th conic, but why?
The 5th conic through the vertices of DT and it's centroïd is centered on the Steiner inellipse of DT.
Best regards
Bernard

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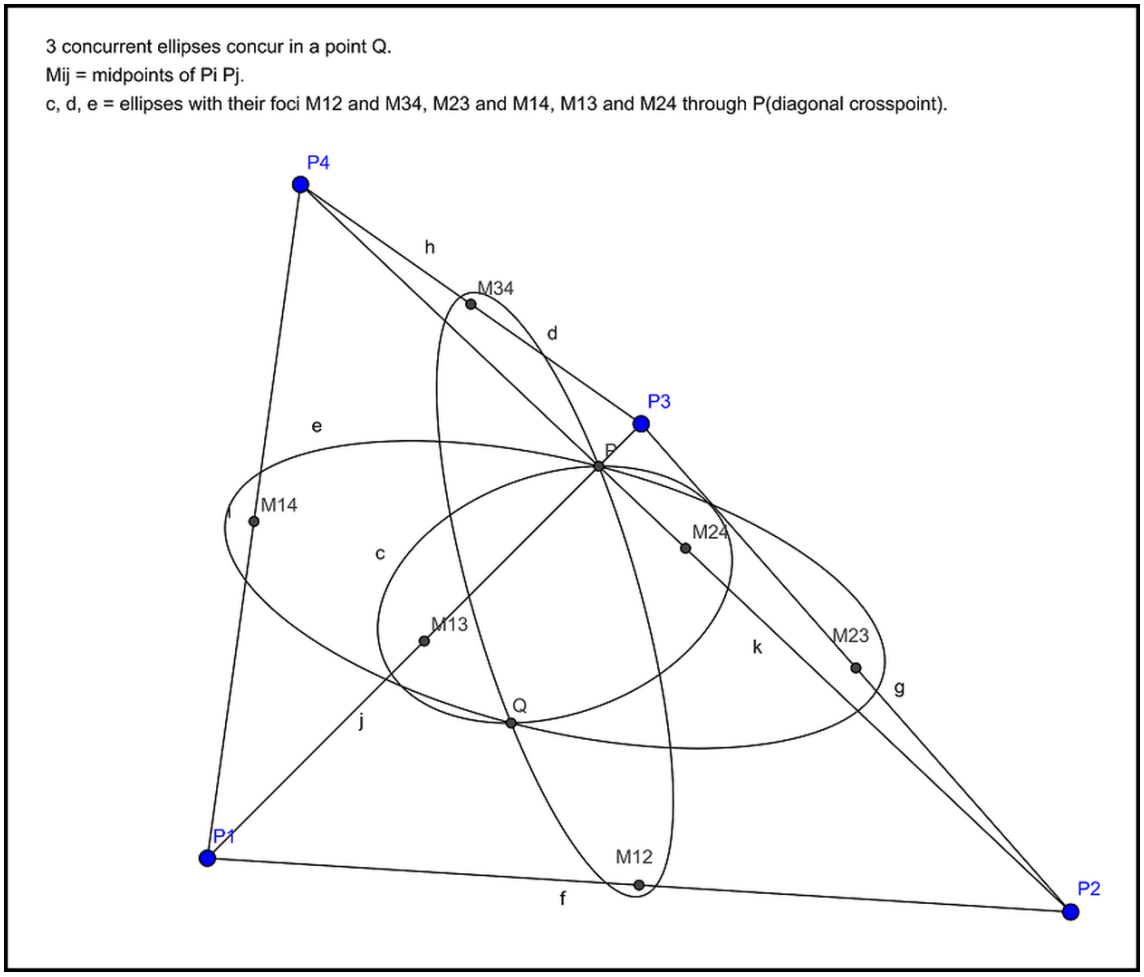
Message: #2142
Date: 23/12/2016 6:29:43
From: Seiichi Kirikami
Subject: 3 concurrent ellipses

Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
Mij = midpoint of PiPj.
c, d, e = ellipses with their foci M12 and M34, M23 and M14, M13 and M24 through QG-P1 respectively.
c, d and e concur in a point Q.
Best regards, Seiichi.



3 concurrent ellipses.png

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Message: #2143
Date: 23/12/2016 7:27:29
From: Seiichi Kirikami
Subject: 3 concurrent ellipses

We can replace QG-P1 by an arbitrary point.
Best regards, Seiichi.
On Friday, December 23, 2016 2:29 PM, "Seiichi Kirikami seiichikiri@yahoo.com [Quadri-Figures-Group]" wrote:
[Attachment(s) (https://mg.mail.yahoo.com/neo/launch?.rand=8midmqfdoor51#TopText) from Seiichi Kirikami included below]
Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
Mij = midpoint of PiPj.
c, d, e = ellipses with their foci M12 and M34, M23 and M14, M13 and M24 through QG-P1 respectively.
c, d and e concur in a point Q.
Best regards, Seiichi.

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Message: #2144
Date: 23/12/2016 8:14:07
From: Seiichi Kirikami
Subject: 3 concurrent ellipses

Sorry for the following.
Conclusion: this concurrency is not a bit interesting because it is trivial.
Best regards, Seiichi.
On Friday, December 23, 2016 3:27 PM, "Seiichi Kirikami seiichikiri@yahoo.com [Quadri-Figures-Group]" wrote:
We can replace QG-P1 by an arbitrary point.
Best regards, Seiichi.
On Friday, December 23, 2016 2:29 PM, "Seiichi Kirikami seiichikiri@yahoo.com [Quadri-Figures-Group]" wrote:
[Attachment(s) (https://mg.mail.yahoo.com/neo/launch?.rand=8midmqfdoor51#TopText) from Seiichi Kirikami included below]
Dear friends,
Let P1P2P3P4 be a quadrangle.
Denote:
Mij = midpoint of PiPj.
c, d, e = ellipses with their foci M12 and M34, M23 and M14, M13 and M24 through QG-P1 respectively.
c, d and e concur in a point Q.
Best regards, Seiichi.

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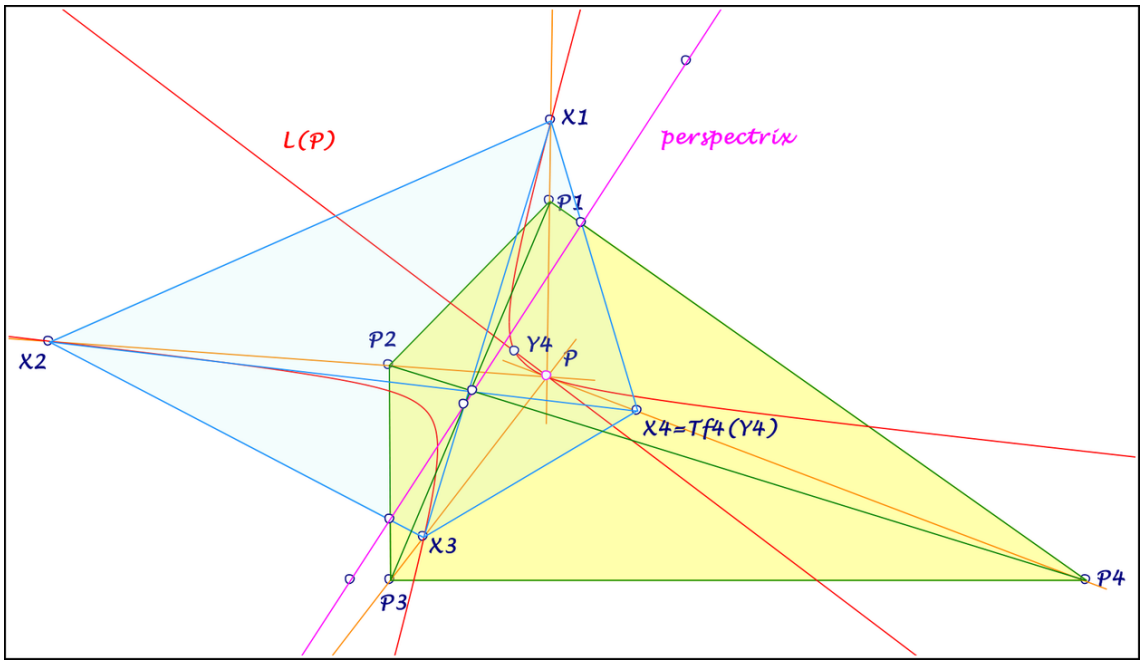
Message: #2145
Date: 23/12/2016 11:20:32
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrangles

Dear Chris,
with interest I have reproduced your construction of line perspective QAs, if the perspector is given.
Perhaps there are several typos:
"DT-TP= Tripolar/ Tripole
wrt Diagonal Triangle, CTi-TP= Tripole/ Tripolar
wrt Component Triangle-i".
Wrt your first question:
Two P-perspective quadrangles are line perspective, if the following QA-lines L(P) coincide.

Wrt L(P):
The isoconjugations Tfi
... with reference triangle PjPkPl
... and fixed point P
... give for the Pi
... four collinear points on L(P).
Wrt your second question:
I think, for a quadrangle P1P2P3P4 there are more than three line perspective quadrangles X1X2X3X4, if the perspector P is known:
Three points Xi can be any points on PPi, e.g. X1, X2, X3 (see attached file):
Then Tf4 (see above) wrt X1X2X3 maps PP4
... to a circumconic of X1X2X3 through P
... with 2nd intersection Y4 with L(P),
... whose Tf4-image is X4.
... X1X2X3X4 is line perspective to P1P2P3P4.
For a reference quadrangle and a random point P there are infinite line perspective quadrangles with perspector P.
Best regards Eckart.



2016-12-23.pdf

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Message: #2146
Date: 23/12/2016 11:00:13
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrilaterals

```
Dear Chris,
my results for the perspectivity of quadrangles can analog be
transferred to quadrilaterals.
I shall work it out after the festive days.
Merry Christmas and Happy Newyear.
Best regards Eckart.
```

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Message: #2147
Date: 24/12/2016 10:15:00
From: chris.vantienhoven
Subject: Perspectivity of Quadrangles

```
Dear Eckart,
Probably evident for you.
Which construction do you use for
isoconjugation Tfi ?
Best regards,
Chris
```

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Message: #2148
Date: 24/12/2016 12:43:11
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrangles

Dear Chris,
you can construct the isoconjugate
... wrt a reference triangle ABC
... and fixed point P
as follows:
Let Pa, Pb, Pc be the anticevians of P wrt ABC,
then QA-Tf2 wrt PPaPbPc gives the isoconjugation.
Best regards Eckart

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Message: #2149
Date: 24/12/2016 2:22:04
From: chris.vantienhoven
Subject: Perspectivity of Quadrangles

Dear Eckart,
Thanks for your quick answer.
Your construction gives another point than as defined in the glossary of ETC.
In the glossary it says:
Let P = p : q : r and U = u : v : w be points, neither on a sideline of ABC.
The P-isoconjugate of U is the point (
qrvw : rpwu : pquv)
These coordinates do not match with your construction.
The same definition is used at Wolfram-keyword: Isoconjugation.
Are there maybe more isoconjugates with the same name?
What are your sources?
Best regards,
Chris
p.s. In the meantime wrt the perspectivity of quadrangles I shall use your definition.

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Message: #2150
Date: 24/12/2016 3:27:43
From: tsihonglau
Subject: Perspectivity of Quadrangles

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>...
>>Your construction gives another point than as defined in the glossary of ETC.
>>In the glossary it says:
>>Let P = p : q : r and U = u : v : w be points, neither on a sideline of ABC.
>>The P-isoconjugate of U is the point (qrvw : rpwu : pquv)
>>...
No! Eckart's construction is projective. No circular points are involved! The isoconjugate in ETC is not.
The formula is only valid for trilinears. The circular points are involved!
I used to prefer barycentrics to trilinears. But now I reverse the preference.
If we replace the circular points on a triangle/trilateral Pappian plane with the parabolic points on a quadrangle/quadrilateral Pappian plane, we identify the geometry of one with the geometry of the other.
Trilinears are identified with unitary coordinates.
Please refer to topic #1939 and my two pages -
Many classes of objects on triangle/trilateral and quadrangle/quadrilateral planes. (In circulars)
The page does not include well known axes yet!
The first coefficient of Euler line of diapleural triangle/diagonal trilateral is (qw+rv)(qw-rv)(p^2vw-qru^2)
http://lth.name/geometry/class.html
Explanation for several coordinate systems
http://lth.name/geometry/coordinate.html
I dislike the notion of isoconjugate in ETC.
(Valid only in trilinears)
Isoconjugate should be only projective notion. I also dislike the notion of poles of isoconjugation in CTC. (Valid only in barycentrics)
The isoconjugation is a purely quadrangle/quadrilateral notion.
I applied it to pivotal isocubic and found pivotal chains.
But no one reply! Please refer to APG topics #2364 and #2434.
Best regards,
Tsihong Lau

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Message: #2151
Date: 24/12/2016 3:34:45
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrangles

```
Dear Chris,
in my construction is P(p:q:r) a fixed point of the
isoconjugation
     (x:y:z) ---> (p^2 y z : q^2 z x : r^2 x y).
I think in the sense of Bernard Gibert my construction will
succeed.
Best regards Eckart
```

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Message: #2152
Date: 24/12/2016 4:38:13
From: chris.vantienhoven
Subject: Perspectivity of Quadrangles

```
Dear Eckart,
I realized that there probably would be more quadrangles that
are line perspective with the Reference Quadrangle. However I
limited myself by searching for the quadrangles belonging to the
Reverse of QA-Tf7 (the vertices of the line perspective
quadrangle should be trilinear poles of the perspectrix). I
hadn't come yet to generalizing it.
You broadened are scope by looking for a general method for even
more line perspective quadrangles. Very well!
Best regards,
Chris
```

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Message: #2153
Date: 24/12/2016 4:42:21
From: chris.vantienhoven
Subject: Perspectivity of Quadrangles

Dear Eckart, dear Tsihong Lau,
I see there is a schism in the use of the word Isoconjugates. I never realized that.
However I think it is important we know what we mean with what we state.
For me it is clear now.
Best regards,
Chris

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Message: #2154
Date: 25/12/2016 4:45:51
From: tsihonglau
Subject: new items in EQF

Dear Chris,

1. The dual QA/QL-Configuration should be valid on a
Desarguesian plane with characteristic not equal to 2.

2. There is a dual of QA-Trilinear Pole - QL-Trilinear-Polar.
The coefficients of the latter are the same as the coordinates
of the former.
But in fact, this transformation should not be included in EQF.
As I pointed out in #2080 - The transformation is L(lx+my+nz) ->
point of concurrency
l(-l^2+m^2+n^2):m(l^2-m^2+n^2):n(l^2+m^2-n^2).
If the line is the line at infinity, the point is the centroid
of quadrangle.
the transformation has been listed as QA-P1.
Moreover, as I pointed out in topic #1939 -
If we replace the circular points on a triangle/trilateral
Pappian plane with the parabolic points on a
quadrangle/quadrilateral Pappian plane, we identify the geometry
of one with the geometry of the other.
QA-P1 has been listed in ETC as X(3).
It is unnecessary to list it again in EQF.
In other words, the objects listed in ETC are also objects of
quadrangle/quadrilateral automatically.
They are transformations of both geometries automatically.
In my page - http://lth.name/geometry/class.html 1/(qw+rv):...
represents not only cevamul of P and U but X(4) of diapleural
triangle/diagonal trilateral. (Given circular points P and U).
We express the objects and operations of triangle/trilateral or
quadrangle/quadrilateral in the same formulas.
We distinguish both usages upon the context.
In similar ways, the objects of triangle/trilateral can be
viewed as transformation result of the line at infinity on a
quadrangle/quadrilateral plane.
To sum up, objects and transformations can be identified.

3. As I pointed out in topic #1997. Parallelologic, orthologic
and cyclologic are quadrangle notions instead
of triangle notions.

I applied triangle aspect to cyclologic and got topic #1968
- Generalized Cyclologic Center Chain in
Quadrangle/Quadrilateral Geometry. But I did not mention that I failed to generalize it to parallelologic and orthologic. As I pointed out in topic #2106, we replace the line at infinity with the vertical point, we get the dual of parallelologic quadrangles - parallelologic quadrilaterals.
We should study and apply it to quadrangle/quadrilateral geometry.
There should be duals of orthologic and cyclologic quadrangles - orthologic and cyclologic quadrilaterals.
But we usually have circular points but no circular lines.
So the latter two notions are not so meaningful. The line at infinity in the parallelologic quadrangles can
be replaced with any line.
So the parallelologic quadrangles can be generalized as follows:
Given two quadrangles P,Pa,Pb,Pc and U,Ua,Ub,Uc, if the three points of intersection of PPa,UbUc and PPb,UcUa and PPc,UaUb are collinear on the line l, then UUa,PbPc and UUb,PcPa and UUc,PaPb are collinear on it.
Then we call the two quadrangles parallelologic with respect to l.
The generalized parallelologic quadrangles can be viewed as a type of perspective quadrangles.
Similarly we can get generailzed parallelologic quadrilaterals.
I have checked that the dual quadrilaterals of two generalized parallelologic quadrangles are not generalized parallelologic.
Given generalized circular points(any two points), we can define orthologic or cyclologic quadrangles.
Given any two quadrangles, we can find a pair of generalized circular points such that the two quadrangles are orthologic or cyclologic.
I do not know the locus of these pairs of generalized circular points.
I also do not know if generalized parallelologic, orthologic and cyclologic notions are valid on a Desarguesian
or Pappian plane.
I guess all Pappian.

Best regards,
Tsihong Lau

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Message: #2155
Date: 26/12/2016 1:54:40
From: Seiichi Kirikami
Subject: An application of “Coolidge theorem”

Dear Chris,
I found “Coolidge theorem” in a book:
Let P1P2P3P4 be a cyclic quadrangle with the radius R.
Ni = nine-point center of PjPkPl
Then N1, N2, N3 and N4 are on a circle with its radius R/2.
If we apply this theorem to O1, O2, O3 and O4 on Miquel circle (QL-Ci3), we obtain “ Coolidge circle”□ with its center Clawson point (QL-P5).
I could not find “Coolidge theorem” in Internet.
J. L. Coolidge published this theorem in Ann. of Math. 13 (1910-1911), according to the above book.
This theorem holds for any cyclic polygon of n>=4.
Best regards, Seiichi.

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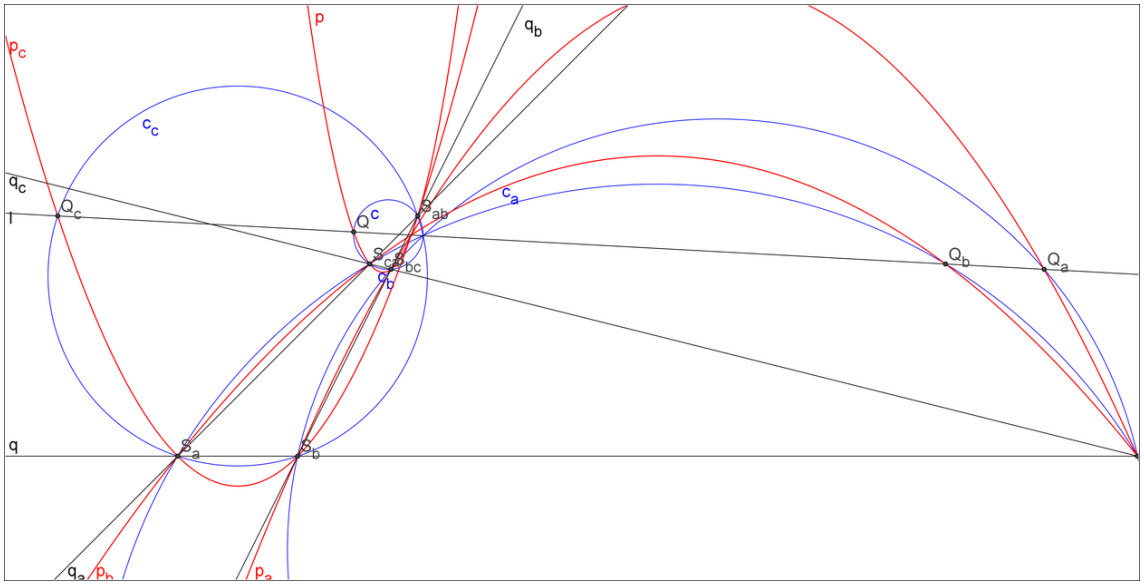
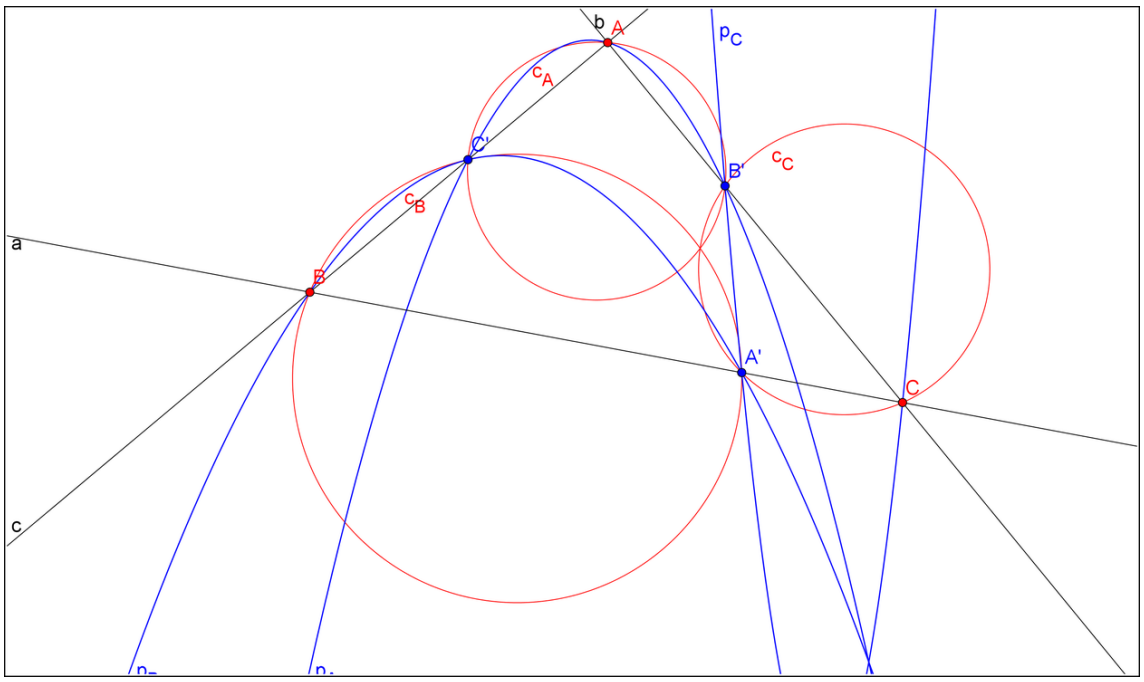
Message: #2156
Date: 26/12/2016 4:51:30
From: tsihonglau
Subject: Vertical Parabolic Pappian Plane

Dear all,
There is a correspondence to the Miquel's three
circle theorem in a vertical parabolic Pappian plane
if we replace the circles with the vertical parabolas.
Similarly, there is also a correspondence to the QL-P1
in a vertical parabolic Pappian plane. Moreover the
fourth points of intersection of the correspondent
circles and parabolas are collinear on the line through
the two QL-P1's. Please refer to the attachments!
Never forget there are duals of the above objects
on a vertical parabolic trilateral or quadrangle plane!
Best regards,
Tsihong Lau



QL-P1.png



miquel's_theorem-2156.png

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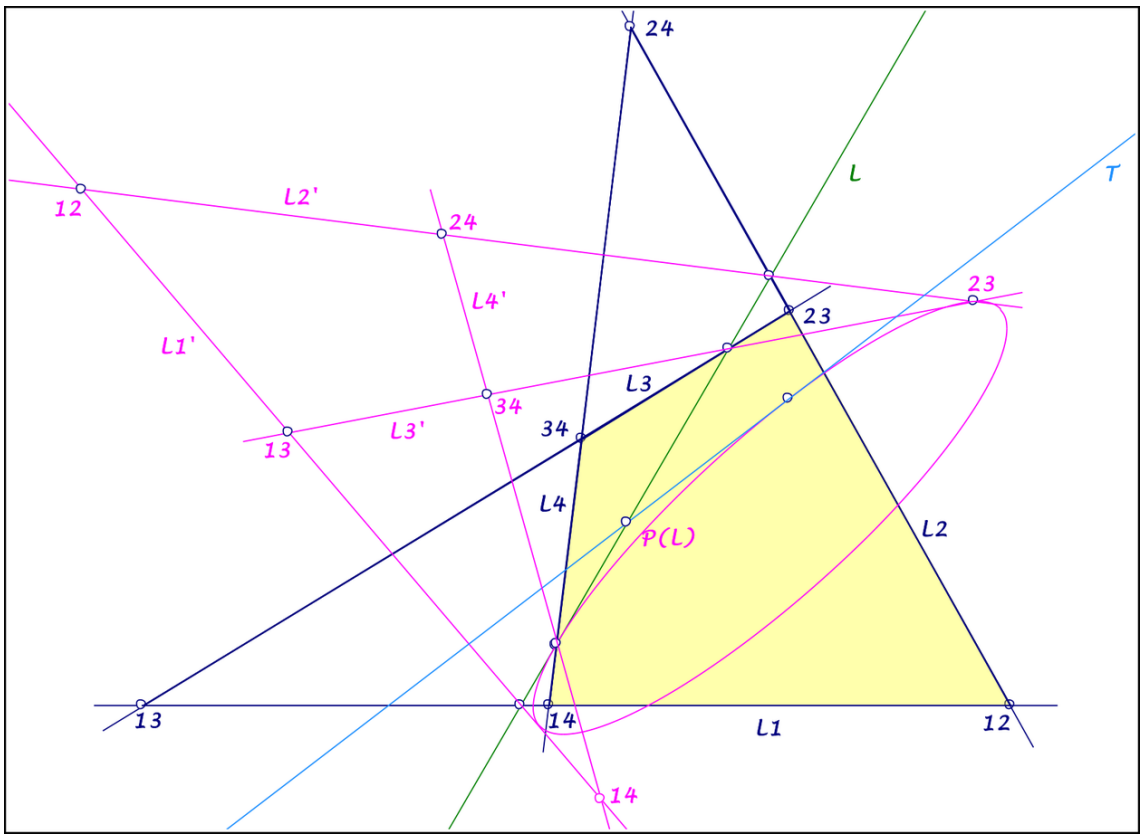
Message: #2157
Date: 26/12/2016 11:28:29
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrilaterals

Dear Chris,
your definitions of point / line perspectivity of quadrangles
can be transferred to quadrilaterals:
... Two quadrilaterals are line perspective
    when the intersections of corresponding lines are collinear.
... Two quadrilaterals are point perspective
    when the lines of corresponding points concur.
... If two quadrilaterals are point perspective,
    they are line perspective.
... Two L-perspective quadrilaterals are point perspectivve, if
the following QL-points P(L) coincide.

Wrt. P(L) :
The isoconjugation for lines Tfi
... with reference trilateral LjLkLl
... and fixed line L
... give for the Li
... four lines with common point P(L).
For a reference quadrilateral L1L2L3L4 and a random line L there
are infinite point perspective quadrilaterals L1´L2´L3´L4´ with
perspectrix L.
Three lines through the intersections of L and Li can be chosen,
e.g. L1´, L2´, L3´ (see attached file):
Then Tf4 (see above) wrt L1´L2´L3´ maps lines through L^L4
... to an inscribed conic of L1´L2´L3´
... with 2nd tangent T through P(L),
... whose Tf4-image is the 4th line L4´.
Remember (see QL-Tf2):
You can construct the line isoconjugate
... wrt a reference trilateral L1L2L3
... and fixed line L
as follows:
Consider the quadrilateral with diagonal triangle L1L2L3 and one
line L, then QL-Tf2 wrt this quadrilateral gives the line
isoconjugation.
Best regards Eckart



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Message: #2158
Date: 26/12/2016 1:47:36
From: tsihonglau
Subject: Perspectivity of Quadrilaterals

Dear Eckart and Chris
This topic is very fundamental.
There are several types of perspectivity.
Perspectivity is basic in projective geometry in which the principle of duality applies.
So perspectivity of quadrilateral is only the dual of perspectivity of quadrangle.
The most basic theorems of perspectivity are Desargues' and Pappus' theorems.
They distinguish two classes of projective planes - Desarguesian and Pappian planes.
The duality of quadrangle/quadrilateral is valid on the former and algebraic (point or line) curves(including conics) usually defined on the latter.
There is only one theorem about triangle/trilateral perspectivity - Desargues' theorem, which is a consequence of Pappus' theorem, which is a special case of Cayley-Bacharach theorem.
Every cubic curve C1 on an algebraically closed field that passes through a given set of eight points P1, ..., P8 also passes through a certain (fixed) ninth point P9, counting multiplicities.
There are several types of perspectivity of quadrangle/quadrilateral:
Given two quadrangles, the former is fixed and we account for the degree of freedom of the latter.

1. Generalized parallelologic quadrangles/quadrilaterals-
The degree of freedom is 7. The parallelologic quadrangles/quadrilaterals are not dual.
I guess it is valid on a Pappian plane.
Discussed in topics #1997, #2106 and #2132

2. Reye configuration of triple quadrangles/quadrilaterals
The degree of freedom is 3. I do not know if the configurations of triple quadrangles/quadrilaterals are dual or not. I do not know if it is valid on a Pappian or Desarguesian plane.
 Discussed in topics #986, #1063, #1823, #1947, #2030, and #2035, etc.

3. Point/line perspectivity of quadrangles/quadrilaterals-
The degree of freedom is 6.
Point/line perspective quadrangles/quadrilaterals are not dual.

I guess it is valid on a Desarguesian plane. Discussed in topics #2139 and #2146.

4. Line /point perspectivity of quadrangles/quadrilaterals-
I do not know the degree of freedom. I do not know if line/point perspective quadrangles/quadrilaterals are
dual or not. I guess it is valid on a Desarguesian plane.
Discussed in topics #2139 and #2146.

5. Generalized orthologic and cyclologic quadrangles -
Generalized orthologic and cyclologic quadrilaterals are not so meaningful because generalized circular
lines are not studied.
They can be viewed as generalized perspectivity. The degree of freedom is 9, which means the locus of the generalized circular points is in degree of freedom 1.
I guess it is valid on a Pappian plane. Discussed in topics #1997 and #2132.

6. Generalized cyclologic quadrangles/quadrilaterals on a vertical Pappian plane -
The degree of freedom is 8, which means we can determine a pair(or several) of vertical point and infinite line given two quadrangles in general position.
I do not know if cyclologic quadrangles/quadrilaterals are dual or not. Discussed in topics #1997 and #2106.
In topic #1997, I raised a question - Given above types of two perspective quadrangles/quadrilaterals (eight
points/lines) on a Pappian plane, we can determine the ninth point/line according to Cayley-Bacharach theorem.
I think there are many other types of perspectivity of quadrangles/quadrilaterals.

Best regards,
Tsihong Lau

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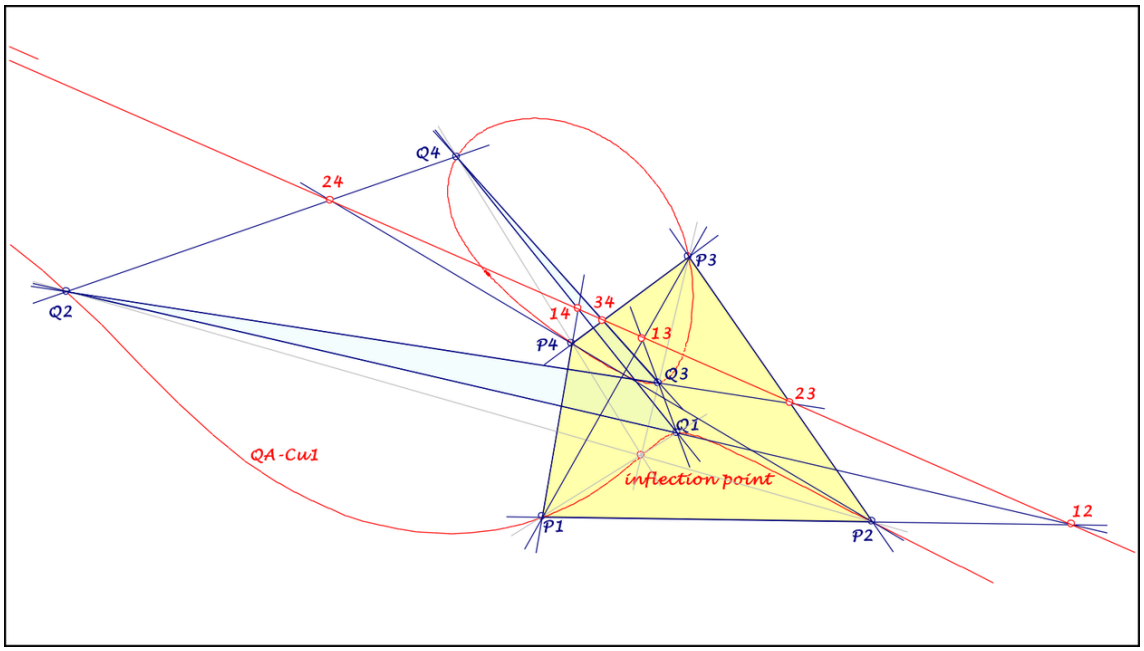
Message: #2159
Date: 28/12/2016 12:50:39
From: eckart_schmidt@t-online.de
Subject: Line Perspective QA on QA-Cu1

Dear Chris,
if we consider a reference quadrangle QA, its cubic QA-Cu1 and a point P on the cubic as perspector,
there is a point perspective quadrangle QA´ on the cubic.
QA and QA´ are line perspective, if P is an inflection point.
This can be proved with point-calculation on the cubic (see http://eckartschmidt.de/Zirkul..pdf).
The inflection point in the attached file isn´t constructed, only an aproximation.
Best regards Eckart



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Message: #2160
Date: 28/12/2016 4:37:20
From: eckart_schmidt@t-online.de
Subject: Line Perspective QA on QA-Cu1

Dear Chris,
Cabri observations verify, that the property of #2159 is valid for all pivotal isocubics
... with reference triangle QA-Tr1
... and an isoconjugation with fixed points in the QA-vertices;
... that means for QA-Cu2, 3, 4, 5.
Best regards Eckart

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Message: #2161
Date: 28/12/2016 10:02:58
From: chris.vantienhoven
Subject: new items in EQF

Dear Tsihong Lau,
You are right, there is a dual of QA-Trilinear Pole, the QL-Trilinear-Polar.
This is one of these special items that can be found in QA as well as QL.
Another example is QA-P16 and QL-P13.
Stuff related to Trilinear Poles and Polars have their mutual counterpart in QA and QL because of dual properties.
QA-P16 and QL-P13 have their mutual counterpart in QA and QL because of projective properties.
About the QL-Trilinear-Polar I found these properties:
• The QL-Trilinear Polar of QL-P1 is:
  - the line through these points: QL-P26, QL-Tf1(QL-P17), QL-Tf1(QL-P27), QL-Tf3(QL-P2.QL-P6), QL-Tf3(QL-P3.QL-P4),
  - also being the perpendicular bisector of QL-P1.QL-Tf1(QL-P6),
  - also being QL-Tf1(QL-Ci6).
• The QL-Trilinear Polar of QL-P8 is a line through QL-P23 and is the 2nd asymptote of QL-Co2 and is parallel to the QL-Trilinear Polar of QL-P23.
• The QL-Trilinear Polar of QL-P13 is a line through QL-P19.
Best regards,
Chris van Tienhoven
p.s. I know your opinion that all QA- and QL-objects and preferably also Triangle objects should be integrated. I wish you all the best doing so.
I prefer the QA-/QL-/QG-classification.

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Message: #2162
Date: 28/12/2016 10:10:54
From: chris.vantienhoven
Subject: Line Perspective QA on QA-Cu1

```
Dear Eckart,
Very nice property for inflection points on QA-Cu1 - QA-Cu5.
Is it that your line perspective construction can be used for
constructing the inflection points on the curves?
Best regards,
Chris
```

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Message: #2163
Date: 28/12/2016 10:12:06
From: chris.vantienhoven
Subject: Line Perspective QA on QA-Cu1

```
Dear Eckart,
Very nice property for inflection points on QA-Cu1 - QA-Cu5.
Is it that your line-perspective-construction can be used for
constructing the inflection points on these curves?
Best regards,
Chris
```

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Message: #2164
Date: 30/12/2016 11:29:37
From: chris.vantienhoven
Subject: Perspectivity of Quadrilaterals

```
Dear Eckart,
I summarize two items L(P) and P(L) of your messages #2145,
#2148, #2157.
```

```
------------------------------------------------------------------
QFG#2145
Wrt L(P):
The isoconjugations Tfi
... with reference triangle PjPkPl
... and fixed point P
... give for the Pi
... four collinear points on L(P).
------------------------------------------------------------------
QFG#2148
you can construct the isoconjugate
... wrt a reference triangle ABC
... and fixed point P
as follows:
Let Pa, Pb, Pc be the anticevians of P wrt ABC,
then QA-Tf2 wrt PPaPbPc gives the isoconjugation.
------------------------------------------------------------------
QFG#2157
Wrt. P(L):
The isoconjugation for lines Tfi
... with reference trilateral LjLkLl
... and fixed line L
... give for the Li
... four lines with common point P(L).
Remember (see QL-Tf2):
You can construct the line isoconjugate
... wrt a reference trilateral L1L2L3
... and fixed line L
as follows:
Consider the quadrilateral with diagonal triangle L1L2L3 and one
line L, then QL-Tf2 wrt this quadrilateral gives the line
isoconjugation.
------------------------------------------------------------------

I noticed this meaning of these transformations:
• In a quadrangle a point P is transformed into the 5th point
  tangent. (the tangent to the conic at P through P1,P2,P3,P4,P)
• In a quadrilateral a line L is transformed into the 5th line
  point of tangency. (the point of tangency from L at the
  inscribed conic of L1,L2,L3,L4,L)
It is not quite unexpected since conics and
isoconjugation/involution go hand in hand.
A very nice set of dual transformations.
Possibly you knew it, but I think it is worthwhile mentioning
it.
Best regards,  Chris
```

Message: #2165
Date: 30/12/2016 3:01:10
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

Dear Bernard, dear Chris,
I don´t know, whether the following property is already mentioned:
Let X be a point on QA-Cu1
... and XaXbXc the triangle on QA-Cu1 wrt the Möbius transformations of QA-Tr2.
... Then QA-Cu1 is a pivotal isocubic
... ... with reference triangle XaXbXc,
... ... with pivot X
... ... and an isoconjugation with fixed points on QA-Cu1, whose tangential is X.
Best regards Eckart

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Message: #2166
Date: 30/12/2016 9:25:27
From: eckart_schmidt@t-online.de
Subject: Line Perspective QA on QA-Cu1

Dear Chris,
I suppose my properties will not lead to a construction of the inflection points of QA-Cu1. But there are other observations:
Consider a reference quadrangle P1P2P3P4,
... its cubic QA-Cu1,
... perspectors P on QA-Cu1
... and P-perspective quadrangles Q1Q2Q3Q4 on QA-Cu1.
There are 12 perspectors P for Q-quadrangles with 3 collinear vertices,
... whose tangents give three 4-intersections on QA-Cu1
... with QA-Tf2-images in the inflection points.
The 12 perspectors are an interesting constellation on QA-Cu1, without construction, but further relevant properties.
Best regards Eckart
PS for Chris wrt #2164: You are right with your meanings of P(L) and L(P), they are simpler in construction. I haven´t seen it.

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Message: #2167
Date: 31/12/2016 9:13:06
From: chris.vantienhoven
Subject: Happy New Year

Dear friends,
Our QFG-forum has grown a lot this year. My impression is that in 2016 many more subjects have been brought to light.
Maybe not always people do react but it is my impression that always several members take notice.
I am counting my blessings to have so many friends who bring joy to my life.
A new year is like a blank book, and the pen is in our hands. It is our chance to write a beautiful book with beautiful intentions, new thoughts, ideas and insights.
Happy New Year!
Chris

By the way it is my intention to release a new Encyclopedia in the new year.
It will be named the Encyclopedia of Poly-Figures and is a sequel to the Encyclopedia of Quadri-Figures.
Like we have central points and related objects in Triangles and Quadri-Figures, they also exist in figures of n (poly) points or n lines.
It is a fascinating story with a prominent place for Morley and his papers as prepared in 2015 by Bernard Keizer, Eckart Schmidt and me at this forum.
It was a challenge to me to translate this somewhat complicate matter to the most simple form.
But that's not all of it.
There is much more to tell about all these intricate points and lines and curves ... !

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Message: #2168
Date: 31/12/2016 9:20:32
From: Tran Quang Hung
Subject: Happy New Year

Thank you very much Mr Chris van Tienhoven and all friends in QFG for your intersest and your helping.
I am a new member of QFG but I like QFG very much, I will try the best in the problems of QFG.
Happy New Year 2017.
Best regards,
Tran Quang Hung.

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Message: #2169
Date: 31/12/2016 9:23:15
From: chris.vantienhoven
Subject: Happy New Year !

Dear friends,
QFG has grown a lot this year. My impression is that in 2016 many more subjects have been brought to light.
Maybe not always people do react but it is my impression that always several members take notice.
I am counting my blessings to have so many friends who bring joy to my life.
A new year is like a blank book, and the pen is in our hands. It is our chance to write a beautiful book with beautiful intentions, new thoughts, ideas and insights.
Happy New Year!
Chris

By the way it is my intention to release a new Encyclopedia in 2017.
It will be named the Encyclopedia of Poly-Figures and it is a sequel to the Encyclopedia of Quadri-Figures.
Like we have central points and related objects in Triangles and Quadri-Figures, they also exist in figures of n (poly) points or n lines.
It is a fascinating story with a prominent place for Prof. Frank Morley and his papers as prepared in 2015 by Bernard Keizer, Eckart Schmidt and me at this forum. I felt challenged to state this somewhat complicated matter in the most simple version. But that is only part of it. It appears there is much more to tell about all these intricate points and lines and curves ....!

Happy NewYear 2017.JPG

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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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Cayley-Bacharach, CB:
#1472 #1476 #1698 #1699 #1705 #1768 #1843 #2045 #2158

Hypocycloid:
#2135

Isoconjugation, isoconjugate:
#1452 #1460 #1494 #1511 #1533 #1614 #1657 #1658 #1661 #1707 #1722 #1738 #1739 #1740 #1741 #1742 #1746 #1747 #1748 #1751 #1752 #1753 #1762 #1765 #1770 #1789 #1827 #1909 #1929 #1991 #1992 #1998 #2019 #2024 #2025 #2031 #2041 #2091 #2124 #2125 #2138 #2140 #2141 #2147 #2148 #2149 #2150 #2151 #2157 #2160 #2164 #2165

Isocubics:
#1435 #1496 #1511 #1707 #1758 #2160

Miquel-point, Miquel Point:
#1429 #1434 #1441 #1446 #1461 #1472 #1476 #1553 #1555 #1599 #1600 #1670 #1671 #1691 #1767 #1771 #1772 #1774 #1783 #1785 #1793 #1794 #1823 #1832 #1837 #1892 #1902 #1906 #1912 #1948 #1952 #1998 #1999 #2024 #2028 #2031 #2035 #2037 #2049 #2084 #2092

Miquel-triangle, Miquel Triangle:
#1518 #1533 #1536 #1599 #1600 #1602 #1651 #1656 #1657 #1658 #1659 #1660 #1661 #1668 #1669 #1670 #1672 #1679 #1691 #1698 #1707 #1722 #1773 #1780 #1791 #1792 #1822 #1823 #1827 #1837 #1948 #1954 #1961 #1962 #1971 #1976 #1978 #1988 #1989 #1995 #2023 #2024 #2028 #2035 #2037 #2048 #2049 #2084 #2092

Newton Line:
#1432 #1434 #1437 #1439 #1442 #1475 #1483 #1484 #1488 #1494 #1495 #1498 #1500 #1505 #1512 #1515 #1519 #1522 #1547 #1605 #1707 #1709 #1710 #1713 #1738 #1740 #1753 #1793 #1823 #1824 #1827 #1830 #1832 #1833 #1834 #1837 #1841 #1849 #1853 #1905 #1906 #1920 #1948 #2024 #2037 #2050 #2061 #2063 #2064 #2065 #2066 #2069 #2070 #2073 #2081 #2138 #2140 #2141

Pivotal Isocubic:
#1435 #1496 #1511 #1614 #1657 #1658 #1661 #1729 #1738 #1762 #1764 #1765 #1770 #1834 #1837 #1947 #1948 #1961 #1992 #2019 #2025 #2049 #2091 #2150 #2160 #2165

QA-Cu1:
#1422 #1423 #1425 #1429 #1433 #1471 #1472 #1474 #1509 #1511 #1518 #1529 #1531 #1533 #1534 #1552 #1655 #1656 #1657 #1658 #1659 #1660 #1661 #1662 #1664 #1672 #1699 #1704 #1707 #1708 #1722 #1724 #1738 #1739 #1751 #1758 #1770 #1773 #1775 #1776 #1779 #1780 #1791 #1792 #1822 #1823 #1824 #1825 #1826 #1838 #1841 #1842 #1961 #1962 #1965 #1969 #1971 #1976 #1988 #1989 #1995 #1998 #2017 #2023 #2024 #2028 #2031 #2037 #2048 #2084 #2092 #2095 #2159 #2162 #2163 #2165 #2166

QA-Cu7:
#1834 #1839 #1840 #1992 #1995 #1998 #2024 #2028 #2031

QA-P3:
#1472 #1478 #1480 #1481 #1533 #1546 #1547 #1575 #1595 #1704 #1707 #1714 #1717 #1770 #1779 #1781 #1785 #1791 #1792 #1826 #1827 #1828 #1927 #1948 #1954 #1965 #1967 #1971 #1972 #1976 #1988 #1989 #1995 #2000 #2009 #2023 #2028 #2049 #2057 #2058 #2084 #2092 #2093 #2095 #2098

QA-P4:
#1430 #1445 #1471 #1472 #1474 #1478 #1481 #1533 #1536 #1552 #1575 #1579 #1595 #1651 #1656 #1657 #1686 #1688 #1694 #1704 #1707 #1708 #1717 #1722 #1724 #1738 #1750 #1751 #1752 #1770 #1780 #1781 #1791 #1792 #1802 #1814 #1821 #1823 #1827 #1839 #1840 #1847 #1859 #1873 #1888 #1948 #1954 #1961 #1967 #1971 #1972 #1976 #1988 #1989 #1990 #1991 #1992 #1995 #1998 #2000 #2009 #2023 #2024 #2028 #2031 #2093 #2095 #2118 #2119 #2120 #2121 #2123 #2124 #2125

QA-P41:
#1473 #1474 #1552 #1656 #1657 #1707 #1708 #1717 #1722 #1738 #1750 #1751 #1752 #1770 #1791 #1827 #1828 #1839 #1840 #1971 #1976 #1988 #1989 #1990 #1995 #2009 #2023 #2024 #2031

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#1915

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Van Rees:
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circular cubic:
#1823 #1956 #1990 #2022 #2024 #2035 #2077

circular points:
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Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
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