Journal of the Quadri Figures Group 2017

Digital Edition

Chris van Tienhoven et al.

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Messages #2170 - #2799

Contents

1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at https://groups.io/g/Quadri-and-Poly-Geometry. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at https://groups.io/g/Quadri-Figures-Group.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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- Benedetto Scimemi
- Bernard Gibert
- Bernard Keizer
- Chris van Tienhoven
- Dominique Laurain
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2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Benedetto Scimemi (1938–2023)

Location
Italy

Year of Birth / Generation
1938–2023.

Short Biography
Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.esteri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter's 60th birthday (the "A5 Meeting", named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See CO-Tf3 in EPG.

- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Ngo Quang Duong

Location
Living in Hanoi, Vietnam.

Year of Birth / Generation
Born in 1998.
Generation Z (approx. 1997–2012).

Short Biography
Ngo Quang Duong studied at the Vietnam National University, where he initially majored in Mathematics before switching to Software Engineering. He has since been working professionally in the software field, while continuing to pursue his interest in Mathematics in his free time. His background combines formal mathematical training with practical experience in computing and problem solving, giving his work a distinctive blend of theoretical insight and computational intuition.

Themes and Interests

- Geometry, Topology, Analysis, and their interactions
- Classical Geometry, especially triangle geometry and quadri-figure geometry
- Contributions to QFG, including n-angle centers and new uses of coordinate systems
- General mathematical exploration and independent study

Selected Publications

- (with T. T. Vu) *A Generalization of the Droz Farny Line Theorem with Orthologic Triangles*, Forum Geometricorum, Volume 16 (2016), 415–418.
- *Generalization of Musselman's theorem. Some Properties of Isogonal Conjugate Points*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 15–29.
- (with O. T. Dao and P. Yiu) *Golden Sections in an Isosceles Triangle and Its Circumcircle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 93–97.
- *Generalizations of Lester Circle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 10 (2021), 49–61.

Additional Remarks
He has not actively returned to Classical Geometry for some time, but remains mathematically engaged through online communities. He is active on Math StackExchange (MSE), where he contributes under the profile: https://www.math.stackexchange.com/users/821868/duong-ngo

Seiichi Kirikami (1949?–2023)

Location
Japan.

Year of Birth / Generation
Exact year unknown; passed away on 11 December 2023.

Short Biography
In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions
Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes

Colleagues remembered Seiichi with deep affection:

- "Geometry is the poorer of his death." — A. P. Hatzipolakis
- "He was a great expert in projective and triangle geometry, and a very kind and helpful person." — E. Suppa
- "He had a way of seeing things from a different angle and presenting them simply." — C. van Tienhoven
- "He inspired many of my problems." — A. Altıntaş
- "He wrote many interesting and innovative messages in ADGEOM." — F. J. García Capitán
- "He helped me in my beginnings with wise and generous advice." — C. E. Lozada
- "He contributed to OEIS and had important work in epidemiology." — P. Moses

Additional Remarks

Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

3 Subjects

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subject: Reflexes and Symmetric Objects in DT-Barycentrics:
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subject: Search Keys for Quadrangle/Quadrilateral:
#2401

subject: Selfcentric Centers:
#2234 #2240 #2244 #2248 #2249 #2250

subject: Set of isocubics through the QL-vertices:
#2727 #2729 #2730 #2733 #2734 #2735 #2746 #2747 #2748 #2750 #2751 #2752 #2754

subject: Simpler configurations of focus sharing ellipses and hyperbolas:
#2235 #2238 #2239 #2241 #2243 #2245 #2246

subject: Some QL-Transformation:
#2215 #2217 #2218

subject: Sweeping ETC points from X(1) to X(3053):
#2319 #2320 #2324

subject: Syzegetic pencil:
#2406 #2407 #2411 #2414 #2415 #2417 #2418 #2419 #2422 #2424 #2426 #2432 #2434 #2436 #2437 #2438

subject: The reality of an "Involutary Centerline":
#2180 #2182 #2184 #2186 #2187 #2188 #2189 #2192

subject: Toeplitz' conjecture about inscribed squares:
#2416

subject: Two questions:
#2531

subject: Yahoo attachment problem:
#2728

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4 Messages

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4.2 Messages

Message: #2170
Date: 02/1/2017 9:08:19
From: eckart_schmidt@t-online.de
Subject: Line Perspective QA on QA-Cu1

```
Dear Chris,

with the best wishes for 2017 and your new encyclopedia
here a short additional information wrt # 2166:
The inflection points of QA-Cu1 are perspectors of
line-perspective QA-quadrangles,
whose vertices are perspectors of QA-quadrangles with three
collinear vertices.

Best regards Eckart
```

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Message: #2171
Date: 02/1/2017 9:10:41
From: eckart_schmidt@t-online.de
Subject: Generalization of QL-Tr2

Dear Bernard,

excuse my late answer wrt your messages 2140.

Wrt 1)
The pole of the isoconjugation wrt the S-triangle, which swaps QL-P8 and QL-P23, is not QL-P13. But I cannot describe it simpler, for the characterisation of an isoconjugation with the image of X2 as pole is very special.

Wrt 2)
Perhaps my formulation under "Unexpected" is not clear:
Take a point on QL-L1, then all the considered isoconjugations give the same point on your "5th conic".

Wrt. PS
The circumconic of QL-Tr2 and the middles of the DT-sides is the image of QL-L9 wrt the QL-Tr2-isoconjugation, which swaps QL-P8 and QL-P23.
Its dual conic is inscribed QL-Tr2, tangent to QL-L1 (as asymptote) and QL-L9 (but not tangent to QA-P1.QA-P10).

Best regards Eckart

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Message: #2172
Date: 02/1/2017 9:24:43
From: eckart_schmidt@t-online.de
Subject: Perspectivity of Quadrilaterals

```
Dear Chris,

here a short additional information wrt # 2164:
There is a further description of P(L) and L(P):
... L(P) = P.QA-Tf2(P),
... P(L) = L ^ QL-Tf2(L).

Best regards Eckart
```

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Message: #2173
Date: 02/1/2017 9:36:27
From: chris.vantienhoven
Subject: Perspectivity of Quadrilaterals

```
Hi Eckart,

Yesterday I placed these new items at EQF:
* QA-Tf9 (QA-5th Point Tangent)
* QL-Tf6 (QL-Trilinear Polar)
* QL-Tf7 (QL-5th Line Point of Tangency)
At QA-Tf9 and QL-Tf7 I mentioned the same properties as you did.
We had the same "insight".

Best regards,
Chris
```

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Message: #2174
Date: 03/1/2017 8:27:16
From: eckart_schmidt@t-online.de
Subject: Quasi Point / Line-Perspectivity

Dear Bernard, dear Chris,

for a quadrangle P1P2P3P4 we can consider point-isoconjugations
with fixed point Pi and reference triangle PjPkPl, for a
quadrilateral L1L2L3L4 we can consider line-iosconjugations
with fixed line Li and reference triangle LjLkLl.

For a quadrangle:
Let Qi be the images of a point P wrt the point-isoconjugations,
... then P1P2P3P4 and Q1Q2Q3Q4 are quasi line-perspective in the
following sense:
... the six intersections PiPj ^ QkQl are collinear

Anolog for a quadrilateral:
Let Mi be the images of a line L wrt the line-isoconjugations,
... then L1L2L3L4 and M1M2M3M4 are quasi point-perspective in
the following sense:
... the six lines (Li ^ Lj).(Mk ^ Ml) have a common point.
There are hardly relations to EQF-geometry.

Some remarks for quadrangles, which can be transferred to QLs:
The QA-results above lead to a point-line transformation,
... which maps lines to sextics,
... ... through the DT-vertices
... ... and tangent to the six QA-lines;
... which maps QA-circumconics to other conics ... ...
Perhaps someone can find better properties.

Best regards Eckart

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Message: #2175
Date: 03/1/2017 11:53:10
From: tsihonglau
Subject: Quasi Point / Line-Perspectivity

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>... the six intersections PiPj ^ QkQl are collinear.
>>...
>>... the six lines (Li ^ Lj).(Mk ^ Ml) have a common point.

In my message #2154 paragraph 3,
Given two quadrangles P,Pa,Pb,Pc and U,Ua,Ub,Uc, if the three
points of intersection of PPa,UbUc and PPb,UcUa
and PPc,UaUb are collinear on the line l, then UUa,PbPc and
UUb,PcPa and UUc,PaPb are collinear on it.  Then we
call the two quadrangles parallelologic with respect to l.

The generalized parallelologic quadrangles can be viewed as a
type of perspective quadrangles. Similarly we can get
generailzed parallelologic quadrilaterals.
My message #2158 paragraph 1
Generalized parallelologic quadrangles/quadrilaterals-
  The degree of freedom is 7.
  The Parallelologic quadrangles/quadrilaterals are not dual.
  I guess it is valid on a Pappian plane.
  Discussed in topics #1997, #2106 and #2132

Your definition of Quasi Point/Line-Perspectivity is a special
case of generalized parallelologic quadrangles
/quadrilaterals with the degree of freedom only 2!

PS. Special case of parallelologic quadrilaterals is shown in my
message #2126.

Best regards,
Tsihong Lau
```

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Message: #2176
Date: 04/1/2017 12:39:29
From: eckart_schmidt@t-online.de
Subject: QL-Point-Line-Transformation

```
Dear Bernard, dear Chris,

please consider a quadrilateral QL and its quadrigons QG, which
can be interpreted as quadrangles QA:

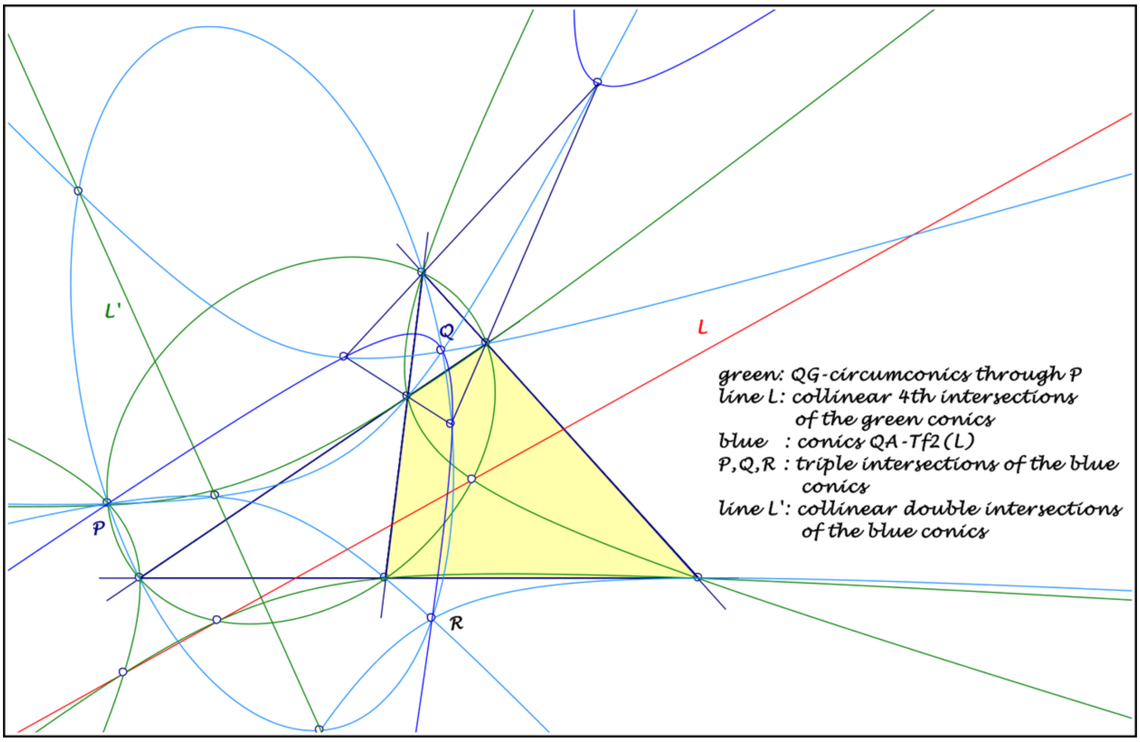
The three QA-Tf2-images of a point P (see attached file)
... are the collinear 4th intersections of the QG-circumconics
    through P.

This gives a QL-point-line-transformation Tf:
... Vertices of QL-Tr1 ---> opposite QL-Tr1-side,
... QL-P1 ---> CSC(QL-Ci6),
... point at infinity of QL-L1 ---> QL-L9,
... QL-P8 ---> line through QL-P23, which is parallel
    to its QL-Tf2-image,
... QL-P23 ---> QL-Tf2(Tf(QL-P8)),
... Tf(QL-P8) // Tf(QL-P23) // Tf(QL-L9 ^ QL-P8.QL-P13)),
... QL-P13 ---> QL-L9-parallel through QL-P19,
... common point of the QG-circumconics
    through QG-P15 ---> line through
... ... intersection of QL-L1 and QL-Co1
... ... and intersection of QL-L9 and a parallel to QL-P8.QL-P13
        through QL-P12,
... S-points (vertices of QL-Tr2) ---> line at infinity.

For a line L there are three points P with Tf(P) = L (see
attached file)
... as triple intersections of the three conics QA-Tf2(L)
... on a QL-Tr1-circumconic.

Example L = QL-L1:
... The three points P with Tf(P) = QL-L1
    are the 3 intersections (unequal QL-P24)
... ...  of the QL-Tr1-circumconic through QL-P8 and QL-P13
... ... and a circle round the reflection of QL-P9 in QL-P6
        through QL-P24.

Best regards Eckart
```



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Message: #2177
Date: 04/1/2017 4:27:13
From: eckart_schmidt@t-online.de
Subject: QL-Point-Line-Transformation

```
Dear Bernard, dear Chris,
I just notice, that the transformation in my message 2176 is
QL-Tf6!
Excuse, that I am not up to date.
Best regards Eckart
```

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Message: #2178
Date: 04/1/2017 4:47:10
From: eckart_schmidt@t-online.de
Subject: QL-Tf7

```
Dear Chris,
I think the following simple description of QL-Tf7 should be
mentioned in EQF:
QL-Tf7(L) = L ^ QL-Tf2(L).
Best regards Eckart
```

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Message: #2179
Date: 04/1/2017 9:25:30
From: eckart_schmidt@t-online.de
Subject: QA-Tf2 / QL-Tf6, QL-Tf2 / QA-Tf7

```
Dear Chris,
are the following relations already mentioned?
QA-Tf2(P) is the common point of the three QL-Tf6(P)
QL-Tf2(L) is the line with the three collinear points QA-Tf7(L).
Best regards Eckart
```

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Message: #2180
Date: 05/1/2017 12:36:46
From: chris.vantienhoven
Subject: The reality of an "Involutary Centerline"

Dear friends,

Desargues' Involution Theorem
A figure consisting of 4 points and their 6 connecting lines is called a (complete) Quadrangle. The 4 points are the vertices. The 6 connecting lines are the sides of the (complete) Quadrangle. Sides in a (complete) Quadrangle that have no vertices in common are called opposite sides.
Desargues' Involution theorem states that the points of intersection of a line with the three pairs of opposite sides of a complete Quadrangle and a conic section circumscribed about the complete quadrangle form the pairs of an involution.
Normally 2 pairs of points describe an involution. Desargues describes in his theorem 4 pairs of points all describing the same involution.
If we restrict ourselves to the 3 pairs of points generated from 3 sets of opposite sides of a Quadrangle, we can deduce that when a line crosses a quadrangle it generates a unique Line Involution with an involution center and 2 double points.

Consequently every line crossing a Quadrangle has an Involution Center and 2 Double Points.
A figure consisting of 4 lines and their 6 connecting points is called a (complete) Quadrilateral.
I wonder if we have a point in this environment, do we have accordingly an "Involutary Centerline" together with 2 "Double Lines".
I made a construction for the 2 double Lines as well as a potential construction for the possible Involutary Centerline.
However I am not sure about the reality of this Involutary Centerline.
In a Quadrangle the Involution Center is the midpoint of the 2 Doublepoints. But what is the position and function of the Involutary Centerline?

Anyone with any ideas?

Best regards,
Chris

Message: #2181
Date: 05/1/2017 2:41:27
From: bernard.keizer
Subject: Happy New Year

```
Dear Chris,

Happy New Year to all the members !
I'm impatient to see your Encyclopedia of Polyfigures ...
In fact, I must confess I'm sometimes a little disappointed by
the lack of reaction to certain items. (The last example is the
generalisation of the Kantor-Hervey theorem with hypocycloïds
with 2n + 1 cusps tangent to 4 lines. Apart of Eckart, noone
seems to have noticed.) But the fight goes on nevertheless ...

Best regards
Bernard
```

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Message: #2182
Date: 05/1/2017 2:51:26
From: tsihonglau
Subject: The reality of an ”Involutary Centerline”

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
> >>Consequently every
> line crossing a Quadrangle has an Involution Center and 2
Double Points.
>>...
>>I made a construction for the 2 double Lines  as well as a
potential construction for the possible Involutary
>>Centerline. However I am not sure about the reality of this
Involutary Centerline.
>>In a Quadrangle the Involution Center is the midpoint of the 2
Doublepoints. But what is the position and
>>function of the Involutary Centerline?

I do not know what your double points are.
I guess you meant fixed points of the involution.
If the line is at infinity, the fixed points are what I call
parabolic points.  Please refer to topic #1939.
You wrote "the Involution Center is the midpoint of the 2
Doublepoints".  The midpoint is only valid on a plane with the
line at infinity.  The dual of the line at infinity is the
vertical point.  Please refer to topic #2106.  Given a vertical
point, the Involutary Centerline is valid.

PS I gave many topics but no replies at all.
Can anyone feedback?

Best regards,
Tsihong Lau
```

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Message: #2183
Date: 05/1/2017 3:28:19
From: tsihonglau
Subject: Happy New Year

```
Dear Bernard, Chris

To Bernard:
I do not reply your message about the generalisation of the
Kantor-Hervey theorem because I am not familiar with high degree
curves.

To Chris:
I have a question about your Encyclopedia of Poly-Figures. How
do you overcome the difficulty of lacking a good coordinate
system for quintangle/quintilateral and sexangle and
sexilateral. As I pointed out in message #1623,
quintangle/quintilateral are dual objects but sexangle and
sexilatateral are not.  But there is no good coordinate system
for them. I dislike the CT coordinate system for
quadrangle/quadrilateral because it lacks of symmetry.  I used
to prefer circulars to DT-barycentrics.  But I think the latter
is more suitable for quadrangle/quadrilateral.
I will post my new ideas about DT-barycentrics.
Circulars may be suitable for quintangle/quintilateral and
sexangle and sexilateral etc.

Best regards,
Tsihong Lau
```

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Message: #2184
Date: 05/1/2017 3:57:55
From: chris.vantienhoven
Subject: The reality of an "Involutary Centerline"

Dear Tsihong Lau,

An explanation of the meaning of Double Points and Involution Center can be found at QA-Tf1.
I don't think they are the same as your parabolic points.
Double Points and Involution Center in a Quadrangle are line specific, whilst the parabolic points are not line specific.
[TL] I gave many topics but no replies at all. Can anyone feedback?
I appreciate the originality of your items.
Somehow your messages are often hard to understand for me.
I often start reading it but often get stuck. Sometimes because of non-standard terminology. Then I start studying it again and still get stuck. Sometimes because your item is rather a field of items than a single item, which tempers my interest. I am sorry about that.

Best regards,
Chris

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Message: #2185
Date: 05/1/2017 6:15:28
From: chris.vantienhoven
Subject: Happy New Year

Dear Tsihong Lau,

[TL] How do you overcome the difficulty of lacking a good coordinate system . . .
That's a question I have been struggling with too.
Therefore in the new Encyclopedia of Poly-Figures there will few coordinates and equations.
DT-coordinates are only useful for quadrangles and quadrilaterals.
CT-coordinates are using 3 basic points as reference and the rest of the points are variables.
For example in case of Pentangles I have used these barycentric coordinates:
P1(1:0:0),P2=(0:1:0),P3=(0:0:1),P4=(p:q:r),P5=(P:Q:R).
This is doable with simple points. All to soon it becomes very complicated. Even Mathematica software gets into errors.

A similar situation occurs with CT-coordinates for Pentalaterals.
Cartesian coordinates have the same problem.
It is all together problematic for n-Lines and n-Points when n>5.
The only partial solution I saw was from Prof. Frank Morley.
He uses complex coordinates and collects in a miraculous way parts of algebraic expressions and combines them with other expressions. Then he jumps to conclusions about the meaning of the resulting expressions being special lines (e.g. a perpendicular bisector), concurring lines, circles, epicycloids, etc.
However he only showed it to be useful working with circles and related curves in n-Lines. I don't know if it is useful in other situations.

About your approach using "circular" coordinates or even "parabolic" coordinates I think there will be the same problems as with CT-coordinates and cartesian coordinates.
But if you can show otherwise I will be glad to hear.

Best regards,
Chris

Message: #2186
Date: 06/1/2017 9:56:28
From: eckart_schmidt@t-online.de
Subject: The reality of an "Involutary Centerline"

```
Dear Chris,

I have constructed the Involutary Centerline for a point P wrt a
quadrilateral QL, using the dual constellation.
Some examples:
... for QL-P1: QL-P1.QL-P26,
... for QL-P8: QL-P8.QL-P23,

In general (see also #1405) :
The Involutary Centerline of a point P
... is the 2nd tangent from P to the inscribed QL-conic
    tangent to P.QL-P13.

Best regards Eckart
```

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Message: #2187
Date: 06/1/2017 11:27:40
From: eckart_schmidt@t-online.de
Subject: The reality of an "Involutary Centerline"

```
Dear Chris,

in addition to my last message:
The Involutary Centerline is the 4th harmonic line of the Double
Lines and P.QL-P13.

Best regards Eckart
```

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Message: #2188
Date: 06/1/2017 11:46:27
From: eckart_schmidt@t-online.de
Subject: The reality of an "Involutary Centerline"

```
Dear Chris,

excuse my splitted messages! Wrt the Double Lines:
There are two QL-inscribed conics through P.
... The tangents in P to these conics are the Double Lines.

Best regards Eckart
```

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Message: #2189
Date: 06/1/2017 11:51:28
From: chris.vantienhoven
Subject: The reality of an "Involutary Centerline"

```
Dear Eckart,

These are remarkable results.
I knew there had to be extra properties to a potential
Involutary Centerline if existing.
You even found it before with another name.
I used the same construction as you did and your complementary
properties fit into this construction.
I will study it later. Now it is family time.

Chris
```

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Message: #2190
Date: 07/1/2017 2:33:24
From: tsihonglau
Subject: Common Harmonics of the Circular and Parabolic Points at Infinity

---

Dear all,

Inspired by Chris' topic #2180 - The reality of an "Involutary Centerline", I think this topic is very important to the quadrangle/quadrilateral geometry.
The parabolic points at infinity are the infinity points of the pair of circumscribed parabola of the reference quadrangle.
Similarly, the circular points at infinity are the infinity points of the pair of the circumscribed
parabolas of the incenter/excenters quadrangle of thediapleural(diagonal) triangle. The parabolic pointsare also the infinity points of the nine-point conic. Similarly the circular points are also the infinity points of the circumcircle of the diapleural triangle = the nine-point conic of the incenter/excenters quadrangle.

In DT-barycentrics, the coordinates of the circular and parabolic points are the same.

```
Circular points:
  -a^2:S_c+S:S_b-S
  -a^2:S_c-S:S_b+S
Parabolic points
  -p^2:S'_c+S':S'_b-S'
  -p^2:S'_c-S':S'_b+S'
where(Conway notation)
  S_c=(a^2+b^2-c^2)/2             S'_c=(p^2+q^2-r^2)/2
  S_b=(a^2-b^2+c^2)/2             S'_b=(p^2-q^2+r^2)/2
  S_ω=(a^2+b^2+c^2)/2             S'_ω=(p^2+q^2+r^2)/2
  S=sqrt(S_ω^2-(a^4+b^4+r^4)/2)
  S'=sqrt(S'_ω^2-(p^4+q^4+r^4)/2)
```

The common harmonics of two pairs (A, B), (C, D) of collinear points are the pair of points (E,F) such that (E,F) are the harmonic conjugates of (A,B) and (C,D) simultaneously.
The question is: What are the common harmonics of the circular and parabolic points?

Best regards,
Tsihong Lau

Message: #2191
Date: 07/1/2017 3:02:53
From: tsihonglau
Subject: Common Harmonics of the Circular and Parabolic Points at Infinit

---

Dear all,

Sorry! The correct formulas are:
In DT-barycentrics, the coordinates of the circular and
parabolic points are the same.
Circular points:
  -a^2:S_c+Si:S_b-Si
  -a^2:S_c-Si:S_b+Si
Parabolic points
  -p^2:S'_c+S'i:S'_b-S'i
  -p^2:S'_c-S'i:S'_b+S'i
where(Conway notation)
  S_c=(a^2+b^2-c^2)/2   S'_c=(p^2+q^2-r^2)/2
  S_b=(a^2-b^2+c^2)/2   S'_b=(p^2-q^2+r^2)/2
  S_ω=(a^2+b^2+c^2)/2   S'_ω=(p^2+q^2+r^2)/2
  S=sqrt(S_ω^2-(a^4+b^4+c^4)/2)
  S'=sqrt(S'_ω^2-(p^4+q^4+r^4)/2)

Best regards,
Tsihong Lau

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Message: #2192
Date: 07/1/2017 9:44:36
From: eckart_schmidt@t-online.de
Subject: The reality of an "Involutary Centerline"

Dear Chris,

there are analog properties for QA-Tf1:
The Involution Center is
... the intersection of L and a QA-circumconic
... ... with asymptote parallel L.
The Double Points are the contact points of L
... and the QA-circumconics tangent to L (see EQF).

Best regards Eckart

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Message: #2193
Date: 08/1/2017 11:45:00
From: chris.vantienhoven
Subject: Common Harmonics of the Circular and Parabolic Points at Infinit

---

Dear Tsihong Lau,

[TL] The common harmonics of two pairs (A, B), (C, D) of collinear points are the pair of points (E,F) such that (E,F) are the harmonic conjugates of (A,B) and (C,D) simultaneously. The question is: What are the common harmonics of the circular and parabolic points?

That's an interesting question.
Your parabolic points are also the infinity points of the Axes of the NinePointConic QA-Co1.
I calculated that the infinity points of the Asymptotes of QA-Co1 are the common harmonics you are looking for. They are actually the double points of the involution created by the circular pair of points and the parabolic pair of points at the line at infinity.

Best regards,
Chris

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Message: #2194
Date: 08/1/2017 4:56:05
From: tsihonglau
Subject: Common Harmonics of the Circular and Parabolic Points at Infinit

---

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
> Your parabolic points are also the
> infinity points of the Axes of the NinePointConic QA-Co1.
> I calculated that the *infinity points of the Asymptotes
> of QA-Co1 are the common harmonics you are >>looking for.
> They are actually the double points of the involution created
> by the circular pair of points and the
> parabolic pair of points at the line at infinity.

Dear Chris,

Thanks a lot!  But I think you had a typo.
The parabolic points are the infinity points of the Asymptotes
of the NinePointConic QA-Co1.
The common harmonics are the infinity points of the Axes of the
NinePointConic QA-Co1.
The latter are also the infinity points of the asymptotes of the
QA-Orthogonal Hyperbola (QA-Co2) and the axes of the
Gergonne-Steiner Conic (QA-Co3) and the reflection axes of the
QA-Orthopole Transformation (QA-Tf3).
This leads to a very important circumconic of the diapleural
triangle.
In DT-barycentrics,
circumcircle: a^2yz+b^2zx+c^2xy=0 through circular points
nine-point-conic: p^2yz+q^2zx+r^2xy=0 through parabolic points
xxx-circumconic:
  (a^2(q^2-r^2)-(b^2-c^2)p^2)yz+
  (b^2(r^2-p^2)-(c^2-a^2)q^2)zx+
  (c^2(p^2-q^2)-(a^2-b^2)r^2)xy=0
If we replace a^2 or p^2 etc with a^2(q^2-r^2)-(b^2-c^2)p^2 etc
in the formulas of circular or parabolic points mentioned
in a previous message, then we get coordinates of the common
harmonics.
I do not know if the perspector of the circumconic has been
studied yet.
But I know it is also very important.
The common harmonics can be viewed as a pair of circular points.

All circumconics through them including QA-Co2 can be viewed as
a new type circle.
Best regards,
Tsihong Lau
```

Message: #2195
Date: 08/1/2017 7:41:00
From: chris.vantienhoven
Subject: Common Harmonics of the Circular and Parabolic Points at Infinit

---

```
Dear Tsihong Lau,

Yes right I interchanged the items.
Best regards,

Chris
```

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Message: #2196
Date: 08/1/2017 9:26:14
From: eckart_schmidt@t-online.de
Subject: Common Harmonics of the Circular and Parabo ...

```
Dear Chris, dear Tsihong Lau,

perhaps of interest: The xxx-circumconic of Tsihong Lau
... is an orthogonal hyperbola,
... bears QA-P3 and QA-P12,
... is the QA-Tf2-image of QA-L4
... and the DT-isogonal conjugate of a line through QA-P11
    parallel QA-P4.QA-P12,
... centered in QA-P29,
... infinity points of the asymptotes are the QA-Tf2-images of
the intersections
    of QL-L4 and QA-Co1.

Best regards Eckart
```

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Message: #2197
Date: 09/1/2017 1:12:50
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,

I'm afraid you are now far from this topic, but I found it very interesting and I continued to work on it and I've found (or refound) some interesting properties which follow your construction.
I hope you will find time for a short comment ...

Best regards
Bernard

PS wrt your last point, I can't understand my mistake (if there is one)
The conic circumscribed to both Tr2 and medial DT is through QA-P1), QA-P10 and the intersection point of the Newton Line (dual of QA-P10) and the DQL Newton Line (dual of QA-P1). Is it through or not?
Therefore it's dual conic should be tangent to both Newton Lines of QL and DQL and to the line QA-P1QA-P10, which is the dual of this last point (the triangle formed by these 3 points on the 1rst conic is self-dual and it's sides should be tangent to the 2nd conic)

S-triangles

1. Properties of a parabola

a. Triangle formed by 3 tangents

Any triangle formed by 3 tangents to a parabola has :

- it's orthocenter on the directrix
- it's circumcircle through the focus

b. Circumcircle through a 2nd fixed point

If the circumcircle is through a 2nd fixed point, we have immediately :

- the circumcenter describes the perpendicular bisector of the segment joining the focus and the 2nd point
- the centroïd describes a 3rd line

2. Isoconjugation

Taking the 4 following elements

- a triangle as a reference triangle
- a circumscribed conic
- a parabola inscribed in the triangle (focus M on the circumcircle)
- a 4th tangent to the parabola (contact point T)

the isoconjugation wrt the triangle which swaps the 2 intersections of the 4th tangent with the circumscribed conic swaps this 4th tangent and the circumscribed conic.
Let's name M the focus, D the 2nd fixed point and D' it's isoconjugate (therefore on the 4th tangent), X and Y the 2 intersection points, mXY the middle of the segment XY, T the contact point of the 4th tangent, G the isoconjugate of T (therefore on the circumscribed conic), P the intersection of the segment MD and the conic and P' it's isoconjugate (therefore on the 4th tangent).
Then the triangle GXY has it's 3 sides tangent to parabola and it's circumcircle passes through the focus M and the 2nd fixed point D.
The 2 triangles form a basis of an infinity of triangles having :

- their vertices on the circumscribed conic
- their sides tangent to the parabola
- their circumcenters on the perpendicular bisector of MD
- their orthocenters on the directrix of the parabola
- their centroids on the line GmXY

S-triangles.pdf

3. **Reverse general construction**

Having the same elements as in point 2, we have following construction :
The circumscribed conic intersects the circumcircle in a 4th point D
The tangent to the parabola intersects the circumscribed conic in 2 points X and Y
The circle through X, Y and D intersect the circumscribed conic in G

4. **Applications to the S-triangle of QA/QL**

The S-triangle is the reference triangle with QL-P6 as circumcenter (Dimidium circle as circumcircle), QL-P12 as centroid and QL-P2 as orthocenter
It is circumcribed to both QL-Co1 and DQL-Co1 (inscribed parabolas of QL and DQL) and the foci QL-P1 and QL-P17 are both on the Dimidium circle)
We may use the construction above to any circumconic of the S-triangle in order to find or refind plenty of interesting properties

a. Parabola DQL-Co1

This parabola is tangent to QL-Tr1 and QL-Tr2 and to the Newton Line QL-L2 ; it has focus QL-P17 and directrix QL-L6 (locus of the orthocenters)
The 4th tangent is the Newton Line and T = QL-P23

- With D = QL-P24 and G = QL-P8, we have mXY = QL-P12 and P = QL-P13

The locus of the circumcenters is the perpendicular bisector of QL-P17QL-P24
The locus of the centroids is QL-P8QL-P12
QL-Tr1, QL-Tr2 and DQL-Tr2 belong to the set of circles (we have DQL-P8 is the middle of QL-P8QL-P12, DQL-P2 is the middle of QL-P2QL-P10 and DQL-P6 is the middle of QL-P6QL-P9)
The conic is circumDT through QL-P8, QL-P13 and QL-P24 (socalled 5th conic)

- With X = QL-P23 and Y = QL-P7, D is the 2nd intersection between the circle through QL-P7, QL-P17 and QL-P23 and the Dimidium circle

The conic is the rectangular hyperbola circumTr2 through QL-P2, QL-P7, QL-P23 and the point D (tangent to the circle in QL-P7)
The locus of the circumcenters is the perpendicular bisector of the segment joining QL-P17 to the point D
The locus of the centroids is the Newton Line

S-triangles.pdf

b. Parabola QL-Co1

This parabola is tangent to the 4 lines and to the Newton Line of DQL ; it has focus QL-P1 and directrix QL-L2

- 4th line is one of the 4 lines Li (dual of one vertice Pi of the QA)

T is the contact point with Li, M is QL-P1, P is the vertice Pi and Di is the intersection between QL-P1Pi with the Dimidium circle

In this case, the conic is circum a reference triangle of the QL taken as a DT and through it's centroid Gi and the vertice of QA Pi (it carries both the s-triangle and the S-triangle)

The locus of the circumcenters is the perpendicular bisector of the segment QL-P1Di

The locus of the centroids is the line GiQL-P12

- 4th line is DQL-Newton Line

  the contact point T is the intersection with the Newton Line
  - Reference triangle is the medial triangle with centroid QL-P8, orthocenter QL-P9 and circumcenter QL-P11
    The locus of the centroids is QL-P8QL-P12 (like in the 1rst example)
    D is the 4th intersection between the conic and the Dimidium circle and the locus of the circumcenters is the perpendicular bisector of the segment QL-P1D
  - D is the point CSC(QL-P26), 2nd intersection of QL-P1QL-P17 with the Dimidium circle
    The locus of the circumcenters is the perpendicular bisector of QL-P1CSC(QL-P26)
    The point G is QA-P1 and the locus of the centroids is QA-P1QL-P12
    The circumconic is the socalled 6th conic through the vertices of the QA and the points QA-P1 and QA-P13 or QL-P17

  (it should be noticed that the DDT triangle is inscribed in the 6th conic, but it's circumcircle is not through QL-P1 and it doesn't belong to the set of S-triangles ; of course, it belongs to the set of DDS-triangles, as it is the DT of DQA/DQL)

S-triangles.pdf

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Message: #2198
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message
2198

```
Message number 2198 is not available in Yahoo groups.
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Message: #2199
Date: 09/1/2017 5:00:28
From: tsihonglau
Subject: Common Harmonics of the Circular and Parabo ...

```
Dear Eckart,

This circumconic is listed as QA-Co4 in EQF.

Best regards,
Tsihong Lau
```

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Message: #2200
Date: 10/1/2017 11:11:10
From: eckart_schmidt@t-online.de
Subject: QA-Inscribed Quartic QA-Tf9(L□)

```
Dear Chris,

this quartic is already mentioned in #141, but under other
aspects and with mistakes.
So I have it worked out once more, perhaps of interest.

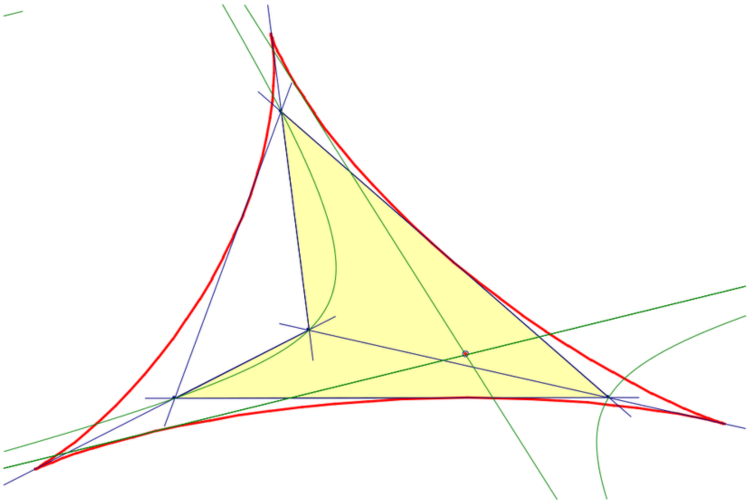
Best regards Eckart
```

EQF-Note 2017-01-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Inscribed Quartic QA-$Tf9(L_\infty)$

The asymptotes of QA-circumscribed hyperbolas envelop a quartic, which is the QA-Tf9-image of the line at infinity. This quartic is already described in QFG-message 141 but wrt other aspects.



Review (see *QFG*-message *141*)

Consider a quadrilateral $L_1L_2L_3L_4$ (with intersections S_{ij}) and a point P. Let L_{ij} be the reflection of the line PS_{ij} in the angle bisector of L_i, L_j. The intersections $L_{12}\cap L_{34}$, $L_{23}\cap L_{41}$, $L_{13}\cap L_{24}$ coincide for points P on the cubic QL-$Cu1$ in the QL-$Tf1$-image of P.
There is an analogon for a quadrangle $P_1P_2P_3P_4$ (with lines L_{ij}) and a line L. Let S_{ij} be the reflections of $L\cap L_{ij}$ in the midpoint of P_iP_j. The lines $S_{12}S_{34}$, $S_{23}S_{41}$, $S_{13}S_{24}$ coincide for lines, which envelop the considered quartic.

Calculations

Here are used barycentric coordinates wrt the QA-diagonal triangle (in *EQF DT*-notation). For lines $L(e,f,g)$ with $S_{12}S_{34}$ = $S_{23}S_{41}$ = $S_{13}S_{24}$ holds

$$\frac{ep^2}{f-g}+\frac{fq^2}{g-e}+\frac{gr^2}{e-f}=0.$$

2017-01-10.pdf

This is valid for the lines P_iP_j of the quadrangle and for example *QA-P3.QA-P4*. For every infinity point $P_{inf}(u:v:w)$ with $u+v+w=0$ there is a line through this point satisfying this condition:

$$L_{uvw} = (u(q^2w^2 - r^2v^2), v(r^2u^2 - p^2w^2), w(p^2v^2 - q^2u^2)).$$

These lines envelop the discussed quartic. Contact points are:

$$(q^2r^2u^4 - p^4v^2w^2 + p^2u(v-w)(r^2v^2 - q^2w^2):$$
$$r^2p^2v^4 - q^4w^2u^2 + q^2v(w-u)(p^2w^2 - r^2u^2):$$
$$p^2q^2w^4 - r^4u^2v^2 + r^2w(u-v)(q^2u^2 - p^2v^2))$$

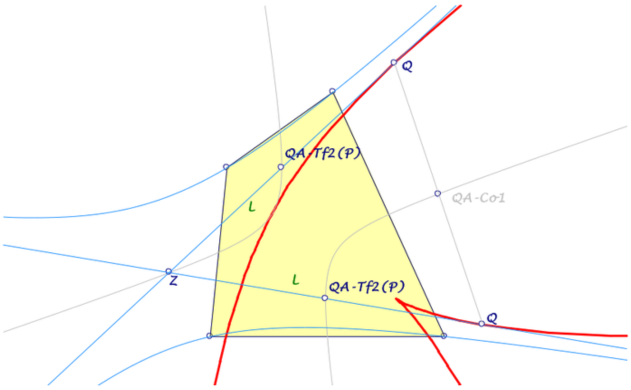
The other two points of intersection with the quartic have midpoint $QA\text{-}Tf2(P_{inf})$ = $(p^2vw : q^2wu : r^2uv)$ on *QA-Co1*.

The very extensive equation of the quartic can be found in *QFG-*message *141*.

Constructions

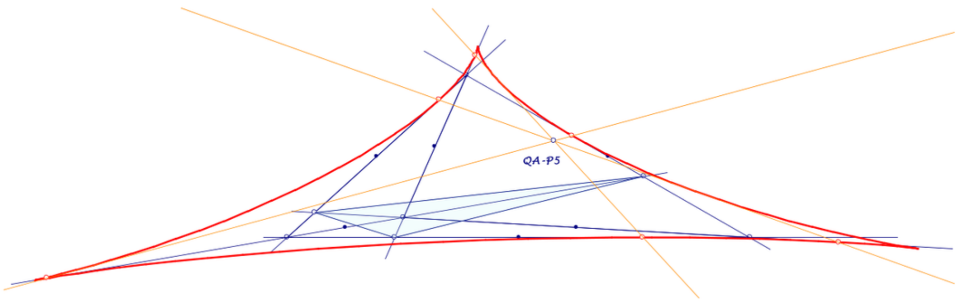
- **The asymptotes of *QA*-circumscribed hyperbolas envelop the quartic.**

Let *L* be an asymptote of a *QA*-circumscribed conic *Co* with center *Z* and infinity point *P*: The contact point *Q* of *L* to the quartic is the reflection of *Z* in *QA-Tf2(P)*.



The construction can be simplified:
Let *X, Y* be two center-symmetric points on *QA-Co1*
… and *Z* the 4^{th} intersection of *QA-Co1* and circle *(X,Y,QA-P3)*,
… *ZX, ZY* are tangents of the quartic
… with contact points in the reflection of *Z* in *X, Y*.

The contact points T_{ij} of the lines P_iP_j and the quartic are the reflections of the *QA-Tr1*-vertices in the midpoints of P_iP_j. The lines $T_{12}T_{34}$, $T_{23}T_{41}$, $T_{13}T_{24}$ are also tangents to the quartic and have a common point in *QA-P5*.



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2017-01-10.pdf

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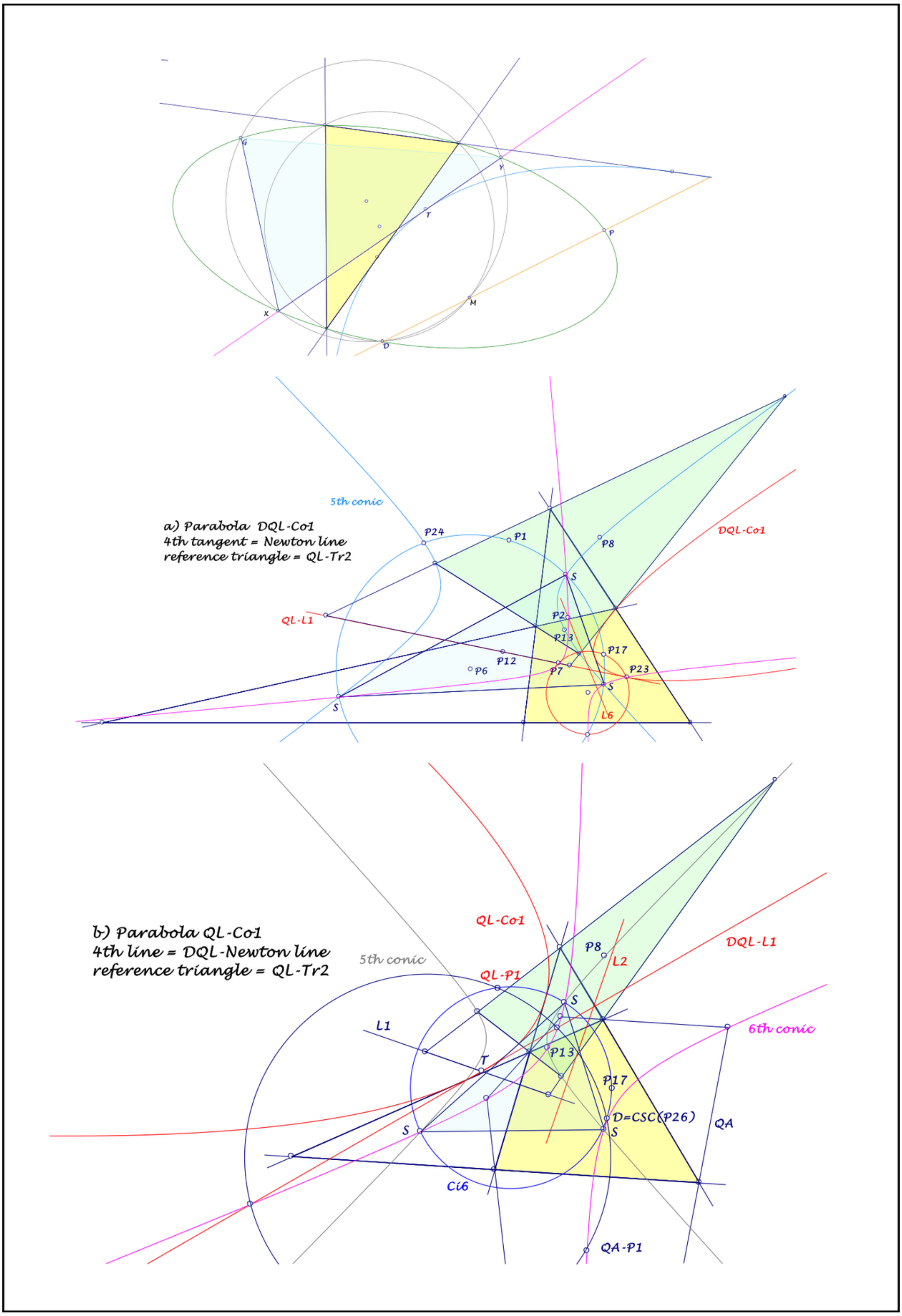
Message: #2201
Date: 10/1/2017 9:29:10
From: eckart_schmidt@t-online.de
Subject: Generalization of QL-Tr2

```
Dear Bernard,

attached you see, that I have reproduced your constructions. It
was a hard work, but interesting.
In the last section will be some typos (corrections in green):
... Reference triangle is the medial triangle
    with centroid QL-P8, orthocenter QL-P9
    and circumcenter QL-P11 / QL-Tr2
... D is the point CSC(QL-P26),
    through the vertices of the QA and the points QA-P1
    and QA-P13 or QL-P17 / QL-P13 or QA-P16.

Best regards Eckart

PS.  Wrt your PS: I think,
     the dual of QA-P1 is not  the DQL Newton line.
```



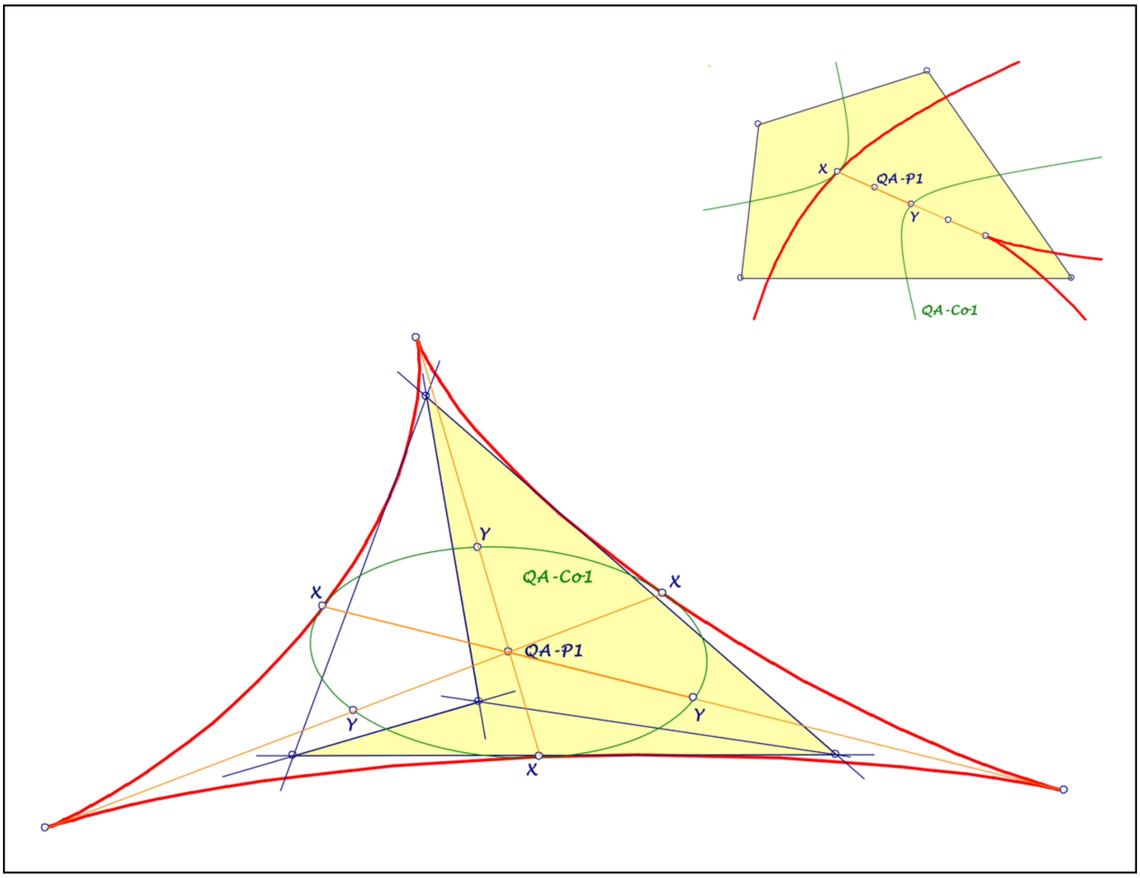
2017-01-10a.pdf

Message: #2202
Date: 11/1/2017 12:34:14
From: eckart_schmidt@t-online.de
Subject: QA-Inscribed Quartic QA-Tf9(L)

Dear Benedetto, dear Chris,

if you study this quartic, perhaps the following
Cabri-observation is of interest (see attached file):
The conic QA-Co1 has one or three contact points X to the
quartic:
... let Y be the reflection of X in the center QA-P1 of QA-Co1,
... then the reflection of X in Y gives a cusp.

Best regards Eckart



2017-01-11.pdf

Message: #2203
Date: 12/1/2017 4:31:15
From: minhtenladuong
Subject: QA-Inscribed Quartic QA-Tf9(L□)

```
Dear Eckart,

It is projective.
That quartic is a generalization of the Steiner deltoid.
When 4 vertices of the quadrangle make an orthocentric system,
we obtain the Steiner deltoid.

Best regards
Ngo Quang Duong
```

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Message: #2204
Date: 12/1/2017 11:41:53
From: eckart_schmidt@t-online.de
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Bernard, dear Benedetto, dear Chris,

perhaps of interest, attached a research for the asymptotes of
QL-inscribed hyperbolas.

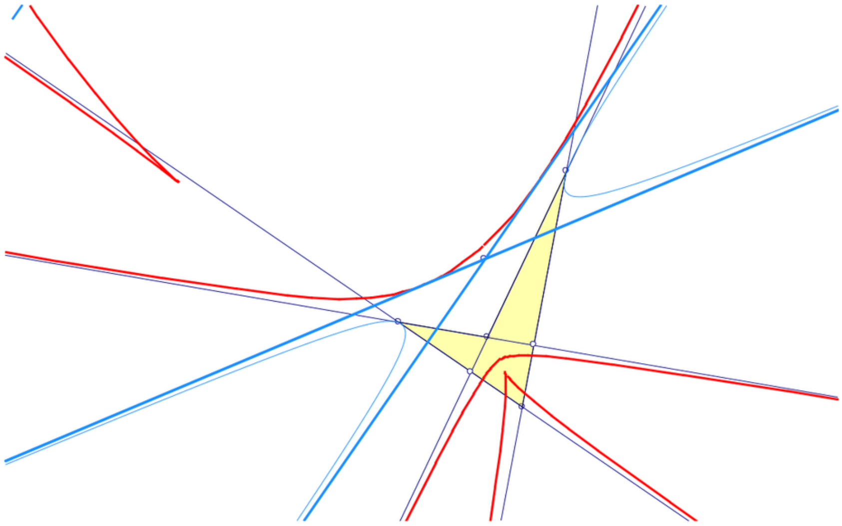
Best regards Eckart
```

EQF-Note 2017-01-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Asymptotes of *QL*-Inscribed Hyperbolas

The asymptotes of QL-iscribed hyperbolas envelop a sextic, which is the envelop of tripolars of points on a pivotal isocubic of the dual quadrangle.

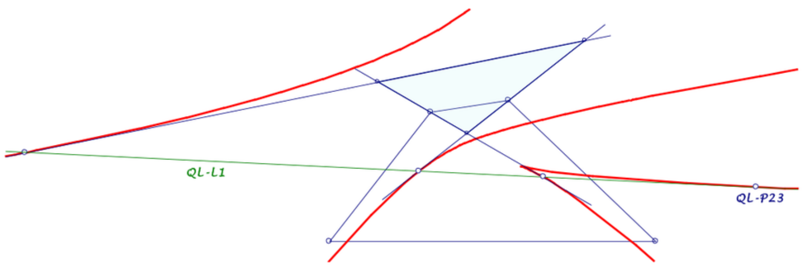


Here some *CABRI*-observations wrt asymptotes of *QL*-inscribed hyperbolas:

- **The asymptotes of *QL*-inscribed hyperbolas envelop a sextic.**

- **The asymptotes of the sextic are the *QL*-lines.**

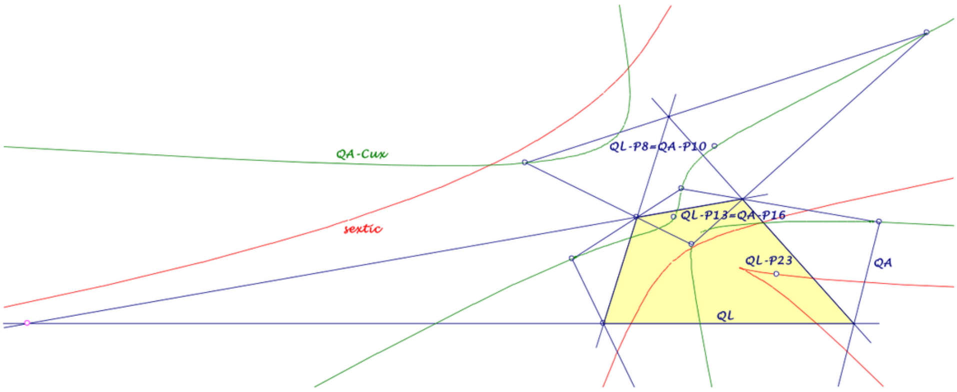
- **The sextic is tangent to the *QL-DT*-sidelines … in the intersections with *QL-L1*.**



- **The sextic is tangent to the Newton line in *QL-P23*.**

- **The tripols of the asymptotes give a pivotal isocubic *QA-Cux* for the dual quadrangle:**
  … reference triangle: *QA/QL* diagonal triangle,
  … isoconjugation: *QA-Tf2*,
  … pivot: *QA-P16* = *QL-P13*.

- **The sextic is the envelop of *QA-DT*-tripolars of points on *QA-Cux*.**



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2017-01-12.pdf

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Message: #2205
Date: 12/1/2017 12:49:46
From: tsihonglau
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Eckart,

>>The tripols of the asymptotes give a pivotal isocubic
>>QA-Cux for the dual quadrangle:
>>… reference triangle: QA/QL diagonal triangle,
>>… isoconjugation: QA-Tf2,
>>… pivot: QA-P16 = QL-P13.

The isocubic is similar to Grebe cubic K102.
(In fact, I will identify both)
Please refer to CTC for more properties.

Best regards,
Tsihong Lau
```

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Message: #2206
Date: 12/1/2017 3:56:39
From: chris.vantienhoven
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Eckart,

Interesting result!
The cubic you described apparently is the QA-DT-P16 cubic.
Do you know a way how to construct a tangent at a given point to
this type of cubic?

Best regards,
Chris
```

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Message: #2207
Date: 12/1/2017 4:35:27
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,

Thank you very much for your answer and the attached figure !
Thanks also for the correction of the typos
Reference triangle is always the unique central S-triangle inscribed in both parabolas QL-Co1 and DQL-Co1
The DT is the 2nd main triangle of the 5th conic in point 1a and the Dpoint is QL-P24, 2nd intersection of the circumcircle of DT with the Dimidium circle.
The same way, the medial triangle is the 2nd main triangle of the conic in point 2c ; therefore the D-point is the 2nd intersection of the circumcircle of the medialtriangle and the Dimidium circle.
For the 6th conic, D is the 2nd intersection of QL-P1QL-P13 with the Dimidium circle and this conic is through QA-P1 and QL-P13 or QA-P16 as you write.
The dual of QA-P1 in the QA/QL figure is the trilinear polar of QA-P20, id est the DQL Newton Line.
Now, new properties appear :
Each of the mentionned conic has a natural parabola QL-Co1 or DQL-Co1
But if we consider the other parabola with the same construction, it simply swaps the points named G and P
5th conic D = QL-P24, DQL-P1 cuts the conic in QL-P8 and DQL-P17 in QL-P13
Is it possible that we could find another QA inscribed in the same conic having QL-P13 as centroïd of the DT and QL-P8 as isoconjugate wrt the DT of this associated QA ?
Rectangular hyperbola D is the intersection of circle through QL-P7, QL-P17 and QL-P23 with the Dimidium circle and DQL-P1 cuts the RH in QL-P7 whereas DQL-P17 cuts the RH in the intersection point of QL-L6 and QL-L9 same question with these 2 points isoconjugates in 2 different DT and the fixed points being on the RH.
Conic circumscribed to the medial triangle of DT D is the 2nd intersection between the circumcircle of the medialDT with the Dimidium circle and DQL-P1 cuts the conic in QL-P8 whereas DQL-P17 cuts the conic in a point P I couldn't identify same question with QL-P8 and this point P as isoconjugates
6th conic D is QL-P24 and QL-P24QL-P1 cuts the 6th conic in QA-P16 or QL-P13 whereas QL-P24QL-P17 cuts the 6th conic in QA-P1 same question with isoconjugate points QL-P13 and QA-P1

It seems we may always associate a 2nd DT to the 1rst and 2 isoconjugate points, which give 4 fixed points and a 2nd QA associated to the 1rst on the same conic (of course different for each conic).
The more I'm working on this construction, the more I find it interesting ! I hope you will help me ...

Best regards
Bernard

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Message: #2208
Date: 12/1/2017 4:57:28
From: Benedetto Scimemi
Subject: QA-Inscribed Quartic QA-Tf9(L□)

Dear Eckart, Dear Ngo Quang Duong,

thanks for the interesting remarks on the inscribed quartics.
One recognizes that these curves are "the negative pedal of a conic w.r.t. one of its points", a family of 3-cusps quartics which generalize, in fact, the Steiner Hypocycloid.
There should be an old literature on this subject.
Does anybody know a reference?

Best regards
Benedetto

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Message: #2209
Date: 12/1/2017 9:03:29
From: eckart_schmidt@t-online.de
Subject: Asymptotes of QL-Inscribed Hyperbolas

Dear Tsihong Lau, dear Benedetto, dear Chris,

the new cubic pK(QA-P16,QA-P16) in Gibert´s nomination and the
sextic are dual curves (dual in the sense of QA-8).
Wrt that pole and pivot are the same, there is a remark in
Gibert´s paper "Special Isocubics" 1.4.2.
So the following property holds:
The cubic pK(QA-P16,QA-P16) wrt QA/QL-DT
... is the locus of points P and QA-Tf2(P), such that their
QA/QL-DT-tripolars are parallel,
... these parallel tripolars are QL-Tf2-partners.

An additional remark: QA-Cu1 and the new cubic intersect in
QA-Tf2-partners on QA-P4.QA-P16.

Best regards Eckart

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Message: #2210
Date: 13/1/2017 12:04:40
From: chris.vantienhoven
Subject: Asymptotes of QL-Inscribed Hyperbolas

Dear Eckart, Tsihong Lau and Benedetto,

I accordance with the sextic of Eckart, being the envelope of the asymptotes of inscribed Hyperbolas in a Quadrilateral I looked for the dual in a Quadrangle.
The asymptotes of QA-circumscribed hyperbolas envelope a higher degree curve QA-Cvi with these properties:
· QA-Cvi passes through the intersection points of all 3 QA-DT-sidelines with all 6 QA-sidelines.
· QA-Cvi is tangent to the 6 QA-sidelines.
· QA-Cvi is tangent to QA-Co1.

The QA-Quadri-poles (QA-Tf2 of the QA-DT-Tripolar) of the asymptotes give a cubic QL-Cux for the dual quadrilateral with these properties:
· QL-Cux has self-intersecting point QL-P13=QA-P16,
· QL-Cux is passing through the 6 intersection points Li^Lj (Li,Lj are basic lines of QL).
· QA-Cvi is enveloped by the QA-Quadri-polars (QA-DT-Tripolar of QA-Tf2) of points on QL-Cux.

Best regards,
Chris

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Message: #2211
Date: 13/1/2017 12:11:03
From: eckart_schmidt@t-online.de
Subject: Asymptotes of QL-Inscribed Hyperbolas

Dear Chris,
you asked for the construction of a tangent at a pivotal isocubic.
Let Tr be the reference triangle, Tf the isoconjugation, P the pivot and X a point of the isocubic:
... the isoconjugate of the 4th harmonic point of P wrt X.Tf(X) gives a point of the tangent.
Best regards Eckart

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Message: #2212
Date: 13/1/2017 2:53:18
From: chris.vantienhoven
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Eckart, Tsihong Lau and Benedetto,
Thanks Eckart for the construction method of the tangent.
This gave me an alternative construction of the sextic you
introduced.
Because the sextic in this construction is point driven and not
an enveloped curve the result in Cabri is much better.
I suppose you already applied the method in the
QA/QL-configuration too.
But I describe it now for anyone not familiar with this method.

QL and dual QA
Let QL be the reference Quadrilateral with lines L1,L2,L3,:L4.
Let QA be the derived dual Quadrangle (Pi=DT-Pole(Li),
i=1,2,3,4)). See QA-8.

Construct Isocubic QA-Cux wrt QA with pivot QL-P13/QA-P16 as
follows:
1. Let V be some point on some circle with center QA-P16.
2. Let QA-Cox be the conical locus of QA-Tf2(V.QA-P16),
   varying V.
3. Let X1 and X2 be the intersection points of QA-Cox
   and V.QA-P16.
They are the variable points, leading to cubical locus QA-Cux,
varying V.

Construct the tangents Tg1 and Tg2 of X1 and X2 at QA-Cux as
follows:
1. Y=QA-Tf2(4th harmonic point of QA-P16 wrt (X1,X2). (Y lies on
QA-Cox)
2. Now Y.X1 and Y.X2 are the tangents (Tg1 and Tg2) of resp. X1
and X2 at QA-Cux.

Construct the sextic QL-Cvx.
1. Construct the Quadri-Poles of the tangents:
T1=QA-Tf2(DT-Tripole(Tg1)).
(DT-Tripole is the Trilinear Pole of some line wrt the Diagonal
Triangle)
2. The locus of T1 and T2 with varying V is the sextic QL-Cvx,
which is the enveloping locus of the asymptotes of the inscribed
QL-Hyperbolas.
Best regards,  Chris
```

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Message: #2213
Date: 14/1/2017 10:21:11
From: eckart_schmidt@t-online.de
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Chris,

wrt #2210:
The "higher degree curve QA-Cvi" is the quartic, I described in
#141 and #2200, #2202.

The "cubic QL-Cux" is a nonpivotal isocubic
... reference triangle = cevian triangle of QL-P13 wrt QL-DT,
... isoconjugation with fixed point QL-P13,
... root = isoconjugate of QL-P8
... ... wrt reference triangle QL-DT
... ... and isocunjugation with fixed point QL-P13.

Best regards Eckart

PS: Wrt "root" see Gibert´s paper "Special Isocubics" 1.5.1.
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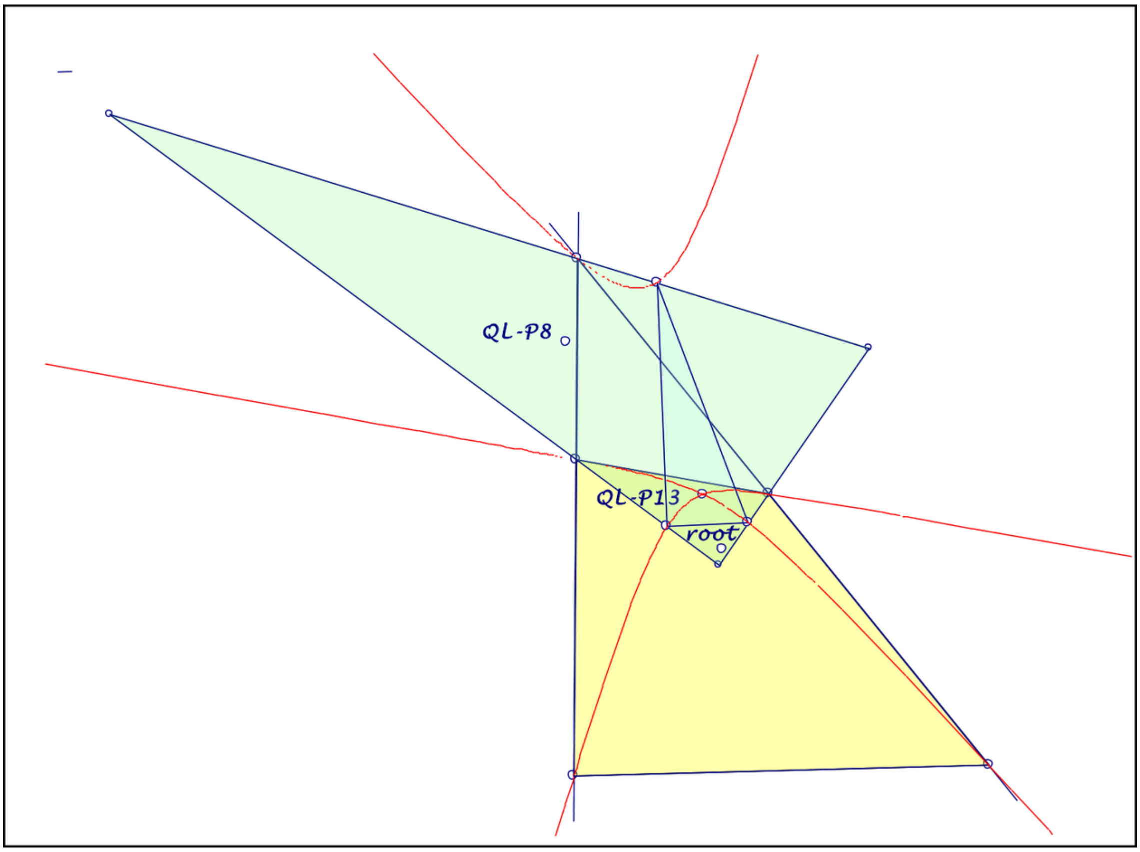

2017-01-14.pdf

Message: #2214
Date: 14/1/2017 11:06:09
From: eckart_schmidt@t-online.de
Subject: Asymptotes of QL-Inscribed Hyperbolas

```
Dear Chris,

I try to summarize, using the dual QA/QL-constellation:
The sextic of the asymptotes of QL-inscribed hyperbolas
is the dual
... of a pivotal isocubic:
... ... reference triangle = QA-DT
... ... isoconjugation = QA-Tf2
... ... pivot = QA-P16.

The quartic of the asymptotes of QA-circumscribed hyperbolas
is the dual
... of a nonpivotal isocubic:
... ... reference triangle = cevian triangle of QL-P13
        wrt QL-DT,
... ... isoconjugation with fixed point QL-P13,
... ... root = isoconjugate of QL-P8
... ... ... wrt reference triangle QL-DT
... ... ... and isocunjugation with fixed point QL-P13.

Best regards Eckart

PS: Duality in the sense of QA-8/QL-8.
```

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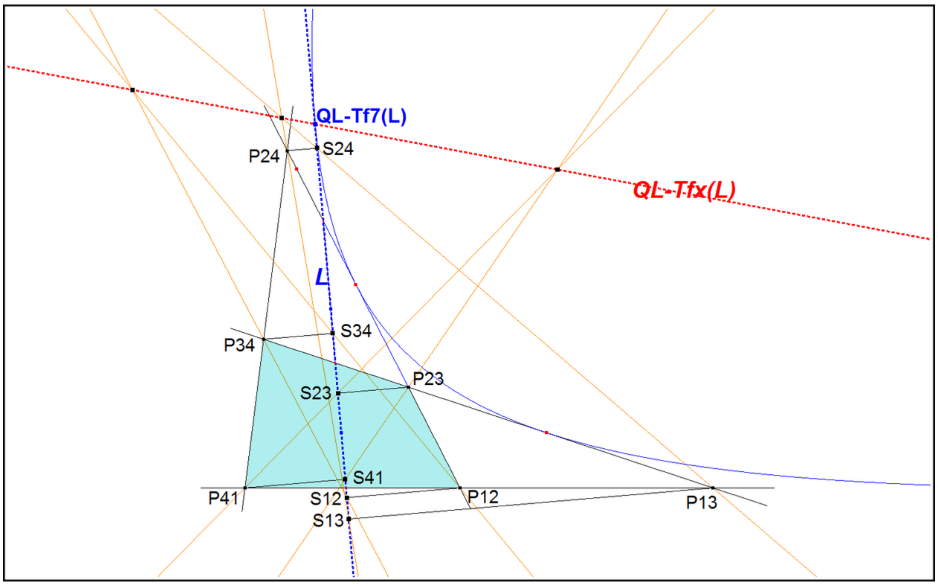
Message: #2215
Date: 14/1/2017 3:12:50
From: chris.vantienhoven
Subject: Some QL-Transformation

```
Dear friends,

Just a transformation QL-Tfx in a Quadrilateral L1.L2.L3.L4
mapping some line L into another line L'.
Construction:
1. Let P12=L1^L2, P13=L1^L3, P14=L1^L4, P23=L2^L3, P24=L2^L4,
   P34=L3^L4.
2. Let S12, S13, S14, S23, S24, S34 be the projection points
   on L from P12, P13, P14, P23, P24, P34.
3. Let L12, L13, L14, L23, L24, L34 be the connecting lines
   of P12 and S34, P13 and S24, P14 and S23, P23 and S14,
   P24 and S13, P34 and S12 respectively.
4. L12^L34, L13^L24 and L14^L23 will be collinear on L'.

1st CT-Coordinate:
   x (a^2 m n x^3 - (a^2 l+SC m) n x^2 y - (a^2 l+SB n) m x^2 z
   - (b^2 m-SA n) l y^2 z - (c^2 n-SA m) l y z^2 + SC l n x y^2
   + SB l m x z^2 + 2 l (SC m+SB n) x y z)
Properties:
· QL-Tfx(L) passes through QL-Tf7(L)
· QL-Tfx(QL-L1) is the line at infinity
There are some similarities with Seiichi's construction of
QA-Tf5(L).

Best regards,  Chris
```



QL-Tf93-line-04.png

Message: #2216
Date: 14/1/2017 4:08:51
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear all,

I posted my basic ideas in message #2440 of topic #2180 of APG.
I think there are main categories in plane geometry.
1. point vs line
2. metric vs projective
3. triangle(trilateral) vs quadrangle(quadrilateral)

I pay attention to line geometry always. My first topic #1457 - Merge Quadrangle/Quadrilateral Figures
- gave the duality of quadrangles/quadrilaterals.
I gave the circular coordinate system so metricscan be defined projectively. I defined expoints of a triangle/trilateral originally in APG topic #2179.
Then I found that we had better define exfigures of a quadrangle/quadrilateral. So I joined QFG.
I think the problems I raised are solved partly.

Now I give two very fundamental ideas - reflexes and symmetric objects. We can define general perpendicularity and circles, etc with a general quadrangle. The idea is not my invention but appeared in Forum Geometricorum Volume 1 (2001) Floor van Lamoen, P_ell-perpendicularity, 151--160
http://forumgeom.fau.edu/FG2001volume1/FG200122.pdf
But the article used CT-notation, while I like DT-notation.
A,B,C,P in it correspond to my notations Q_A,Q_B,Q_C,Q. Line l corresponds to the line at infinity. The fixed points J1,J2 is called the parabolic points at infinity by me.
With the parabolic points we can define pseudo perpendicularity and circles, etc. In fact, the quadrangle/quadrilateral geometry with the parabolic points is the same as the triangle/trilateral geometry with the circular points.
I identify some objects in ETC and EQF in message #2130 of topic #1939. But none replied.

With DT-barycentrics, we exchange a,b,c with p,q,r(I would rather use d,e,f insteadof p,q,r in my website) and get the reflex of the object(point, line, etc).
The following are reflexes of some objects in EQF.
QA-P11 circumcenter - QA-P1 QA-centroid
QA-P3 Gergonne-Steiner point - QA-P30
QA-P12 orthocenter - QA-P20
QA-P13 nine-point center - QA-P22

QA-Ci1 circumcircle - QA-Co1 nine-point conic
circular points - parabolic points
K004 Darboux cubic - QA-Cu2 QA-DT-P5 Cubic
K002 Thomson cubic - QA-Cu3 QA-DT-P10 Cubic
K169 - QA-Cu4 QA-DT-P19 Cubic
K003 McCay cubic - QA-Cu5 QA-DT-P1 Cubic
If the reflex of a object is the same as itself, we call it symmetric object.
The following are some symmetric objects in EQF
the line at infinity
QA-P10 centroid
QA-P2 Euler-Poncelet point
QA-P29
QA-P39
QA-Co2 QA-Orthogonal Hyperbola
QA-Co4 QA-DT-P3-P12 Orthogonal Hyperbola
common harmonics of the circular and parabolic points

The reflex of an object with p,q,r-coordinates or coefficients is an object with a,b,c-coordinates or coefficients.
The latter is usually listed in ETC or CTC, so it is unnecessary to study the former.
Objects with a,b,c,and p,q,r coordinates or coefficients had better be listed with their reflexes.
Since the symmetric objects have symmetric coordinates or coefficients, they should be studied more thoroughly.
For example, the QA-P2 is the cevamul- of X(6)=a^2:b^2:c^2 and QA-P16=p^2:q^2:r^2.
But the results of the other operations-cevamul rossmul-, crossmul, incidence, etc are not listed in EQF.
They should have many nice properties.
To be continued...

Best regards,
Tsihong Lau

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Message: #2217
Date: 14/1/2017 5:01:42
From: tsihonglau
Subject: Some QL-Transformation

```
Dear Chris,

I applied your construction to a quintangle/quintilateral and
found an interesting result.
Given a circumconic conic of quintangle ABCDE and the
quintilateral abcde=tangent lines to conic
on ABCDE respectively, the quintilateral a'b'c'd'e' are QL-Tfx's
of abcde with respecti to bcde,cdea,deab,eabc
abcd respectively.
Quintilaterals abcde and a'b'c'd'e' intersect at the quintangle
ABCDE.

Best regards,
Tsihong Lau
```

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Message: #2218
Date: 14/1/2017 7:07:50
From: chris.vantienhoven
Subject: Some QL-Transformation

```
Dear Tsihong Lau,

Yes it is an interesting result!

It is caused by this property that I mentioned at the end of my
earlier message #2215:
* QL-Tfx(L) passes through QL-Tf7(L)

QL-Tf7(L) is the point of tangency at L, given that L is the 5th
line of the QL-inscribed conic(L1,L2,L3,L4,L).

Best regards,
Chris

p.s. A figure consisting of 5 points is usually named a
Pentangle and a figure of 5 Lines a Pentalateral. And when
points and lines are cyclically connected a Pentagon.
```

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Message: #2219
Date: 15/1/2017 3:22:41
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

```
Dear all,

I give more reflexes of objects and symmetric objects.
  incenter/excenters quadrangle - reference quadrangle
  diapleurals of the first
  diagonals of the first
  dual quadrilaterals of the first
  expoint triangles of the first
  exline trilaterals of the first
  ...

Please refer to http://lth.name/geometry/duality.html
for more information symmetric objects
  diapleural triangle of the first
  diagonal trilateral of the first
  centroid/anticomplementary triangle of the diapleural triangle
  quadrangle
  dual of the above
  diapleurals of the above
  ...

Some concepts are reflexes and some are symmetric
  isogonal conjugation of points and lines - isoconjugation of
  points and lines
  self-isogonal cubic - self-isoconjugation cubic
  isotomic conjugation of points and lines
  midpoint of two points
  reflection point of a point with respect to another point
  complement of a point
  anticomplement of a point
  center of a conic
  centroid of triangle, quadrangle, etc

Be careful, angle bisectors and perpendicular bisector are not
symmetric since there is pseudo perpendicularity.

According to the previous properties, we can deduce the
following results each other easily

reflexes QA-P1 <> QA-P11, QA-P3 <> QA-P30
QA-P1 = Midpoint of QA-P2.QA-P3
QA-P11= Midpoint QA-P2.QA-P30
```

reflexes QA-P1 <>. QA-P11, QA-Co1 <> QA-Ci1
QA-P1 = center of the nine-point conic QA-Co1
QA-P11 = center of the circumcircle QA-Ci1
Centers of two reflex conics are reflexes.
reflexes QA-P1 <>. QA-P11, QA-P20 <> QA-P12
isoconjugate <> isogonal conjugate
QA-P1 = isoconjugate of QA-P20

reflexes
QA-Co1 <> QA-Ci1
QA-P2 lies on the nine-point conic QA-Co1
QA-P2 lies on the circumcircle QA-Ci1
Points of intersection of two reflex curves are symmetric.

reflexes QA-P12 <> QA-P20
QA-P12 lies on QA-Co4
QA-P20 lies on QA-Co4
Two reflex points lie on the same symmetric curve.

reflexes QA-P13 <> QA-P22 , QA-P12 <> QA-P20
QA-P13 = QA-Centroid of the DT-vertices + QA-P12
QA-P22 = QA-Centroid of the DT-vertices + QA-P20
QA-P39 = Midpoint of QA-P12 and QA-P20
Midpoint of two reflex points is symmetric.

To be continued...

Best regards,
Tsihong Lau

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Message: #2220
Date: 16/1/2017 9:26:54
From: chris.vantienhoven
Subject: Concurrent lines in QA with any point and any line

Dear Tran Quang Hung,

Finally I found time to explore your special transformation in #2099,#2100.
I repeat your construction:
1. Let ABCD be the Reference Quadrangle and *d* a random line and P a random point.
2. Line d intersects AB, CD, AC, DB, AD, BC at MAB, MCD, MAC, MDB, MAD, MBC.
3. PMAB, PMCD, PMAC, PMDB, PMAD, PMBC intersect lines CD,AB,BD,AC,BC,AD at NCD, NAB, NBD, NAC, NBC, NAD.
4. Then the lines NABNCD, NBD,NAC, NBCNAD are concurrent in S.
5. The circles (PMABMCD), (PMACMDB), (PMADMBC) have a common point Q other than P.
6. (PMABMCD) cuts AB,CD again at NAB, NCD, (PMBDMCA) cuts BD,AC again at NBD, NCA, (PMBCMAD) cuts BC,AD again at NBC, NAD
7. The lines NABNCD, NBDNCA, NBC NAD at R.

Let S be the point of #2099 and P,Q,R the points as described in #2100.
I found these extra results.
1. The circumcenters of the 3 circles in step 5. are collinear.
2. P, Q, QA-Tf1(P) are collinear
3. P, S, QA-Tf2(P), QA-Tf5(P) are collinear
4. P.S= QA-Tf9(P).
I think it is especially point S that is important.

When P=(x:y:z) and d=(l:m:n), then S has these CT-coordinates:
(x (l q r x^2 - l p r x y - l p q x z - m p q y z - n p r y z) :
 y (-m q r x y + m p r y^2 - l p q x z - n q r x z - m p q y z):
 z (-l p r x y - m q r x y - n q r x z - n p r y z + n p q z^2
 ))
The other points Q and R are not that simple.
Although Eckart did not react on this item I am sure that Eckart can find some extra properties.

Best regards,
Chris

p.s. I found this construction is a bit different from the transformation of Ngo Quang Duong at #1198, although there are some similarities.

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Message: #2221
Date: 16/1/2017 10:17:34
From: eckart_schmidt@t-online.de
Subject: Reflexes and Symmetric Objects in DT-Baryce ...

Dear Tsihong Lau,

what is the geometric relation between a point and its
reflex-point?
For example QA-P3 and QA-P30?
It seems, that it is only a coordinate manipulation,
geometrically interpretable in the case,
that the DT-coordinates are pure in p, q, r.
This is the meaning of your statement:
"The reflex of an object with p,q,r-coordinates or coefficients
is an object with a,b,c-coordinates or coefficients. The latter
is usually listed in ETC or CTC, so it is unnecessary to study
the former."

In this case you get the reflex-point of QA-Px as QA-Px for the
quadrangle of the in- and ex-centers of QA-DT,
which is an ETC-point of QA-DT:
   QA-P1 <> X(3),
   QA-P5 <> X(20),
   QA-P16 <> X(6),
   QA-P17 <> X(64),
   QA-P18 <> X25),
   QA-P19 <> X69),
   QA-P20 <> X(4),
   QA-P21 <> X(1350),
   QA-P25 <> X(10304),
   QA-P26 <> X(3524),
   QA-P27 <> X(3424),
   QA-P31 <> X(141).

If we replace for QA-Px only p, q, r with a, b, c, we get -if
existent- ETC-points of the diagonal triangle.
But it seems, that they are all registered in ETC:
   QA-P1 = P7 = P8 = P11 = P15 = P32 = P33 = P38 = X(3),
   QA-P5 = P37 = X(20),
   QA-P10 = P14 = X(2),
   QA-P12 = P20 = P24 = X(4),
   QA-P13 = P22 = X(5).
If we replace in the QA-Px-coordinates p, q, r with the
coordinates of other ETC-points, we get a lot of new ETC-points:
Example: The Isogonal Center QA-P4 for the quadrangle of X(4)
and its anticevians is not in ETC.

Best regards Eckart

PS. I hope, there is no mistake!

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Message: #2222
Date: 16/1/2017 1:36:59
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,

I've now finished my observations on the construction derived from yours and I would like to know what you think about it.
Shortly, finally every circumconic through the S-points cuts the Dimidium circle in a 4th point which is the D-point of the conic. Now the 2nd intersections between the conic and the line DQL-P1 or DQL-P17 are the points G and P. If the QL-Newton Line and the DQL-Newton Line cuts the conic in 2 points X and Y and X' and Y', then X, Y, D, QL-P17 and G are cocyclic, as well as X', Y', D, QL-P1 and P.
Referring to your note 2013-02-01,
for QA-1 (6th conic), D, G and P are CSC(QL-P26), QA-P16 and QA-P1
for QA-2 (5th conic) QL-P24, QA-P10 and QA-P16
for QA-3, D is the 2nd intersection between Euler circle of DT and Dimidium circle, G is a point I couldn't identify (any idea ?) and P is QA-P10 for QA-4 , the 3 conics are through the S-points, a vertice of DT and the middle of the opposite side of DT, this gives 3 D-points, the corresponding G are the middles of the sides of DT and the P are the vertices of DT (in other words, the triangle of D-points is perspective with medial DT with perspector QL-P1 and with DT with perspector QL-P17) for QL-1 (rectangular hyperbola) , D is the 4th intersection between the hyperbola and the Dimidium circle, G is QL-P7 and P is the intersection of QL-L6 and QL-L9
QL-2 is the same as QA-2
QL-3 is the same as QA-3
for QA-4, the 3 D-points are the Gergonne-Steiner points, the 3 G points are the vertices of DT and the 3 P points are the vertices of DDT (in other words, the triangle of Gergonne-Steiner points in perspective with DT with perspector QL-P1 and with DDT with perspector QL-P17)
I tried another circumconic to check the construction, the Steiner circumellipse of the S-triangle QL-Tr2.
The D-point is the Steiner point of the triangle G seems to be on the 6th conic QA-1 or QL-1 and P on the 5th conic QA-2 or QL-2 ...
Maybe the properties are linked to the fact that DQL-Co1 is the anticomplement of the parabola associated to QL-Co1 (2nd parabola through the 4 contact points between QL-Co1 and the 4 lines of the QL) ...

QL-P1 and QL-P25 are isotomic conjugates wrt DT and QL-P17 is the anticomplement of QL-P25
The Newton Lines of QL and DQL, duals of QA-P10 and QA-P1, are isotomic transversals wrt the S-triangle QL-Tr2
I know you are embedded in many other interesting items (the reflexes ...), but hope you will find a few minutes to have a look on this (I've always found this S-triangle fascinating !)

Best regards
Bernard

PS I think I've found my mistake, the conic QA-3 is not through the intersection point of the 2 Newton Lines ...

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Message: #2223
Date: 17/1/2017 8:30:54
From: Tran Quang Hung
Subject: Concurrent lines in QA with any point and any line

Thank you so much Mr Chris for new remarks for my transformation.
Best regards,
Tran Quang Hung.

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Message: #2224
Date: 17/1/2017 1:08:02
From: eckart_schmidt@t-online.de
Subject: Generalization of QL-Tr2

```
Dear Bernard,

You are right, the geometry of the S-triangle QL-Tr2 is really
fascinating, but it is a very special matter.
I haven´t reproduced all properties, but some remarks:

consider QL-Tr2 circumscribed conis Co and three related points
... D = 4th intersection of Co and QL-Ci6,
... G = 2nd intersection of Co and D.QL-P1,
... P = 2nd intersection of Co and D.QL-P17.
One point implies the others. Wrt your open questions:
... QA-3: G = pole of QA-P8.QA-P13 wrt the 5th conic.
... Steiner circumellipse: G = 4th intersection with your
    5th conic, P 4th intersection with your 6th conic.

Another aspect:
Let G be an arbitrary point,
... D = 2nd intersection of G.QL-P1 and QL-Ci6,
... X, Y intersections of QL-L1 and circle (G,D,QL-P17),
... then D, G, X, Y on a circumconic of the S-triangle.

A question: What are "isotomic transversals" of a triangle?
Starting with this topic in #2138, I wanted to point out the
isoconjugations
... wrt generalized S-triangles (as QL-Tr1, QL-Tr2),
... swapping QL-P8 and QL-P23,
... which map a QL-L1-point to the same point on your 5th conic.
Wrt these isoconjugations the common image of QL-P24 can be an
interesting QL-point on QL-L1.

Best regards Eckart
```

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Message: #2225
Date: 18/1/2017 11:01:56
From: eckart_schmidt@t-online.de
Subject: Four QA-Inconics with Focus QA-P4

```
Dear all,

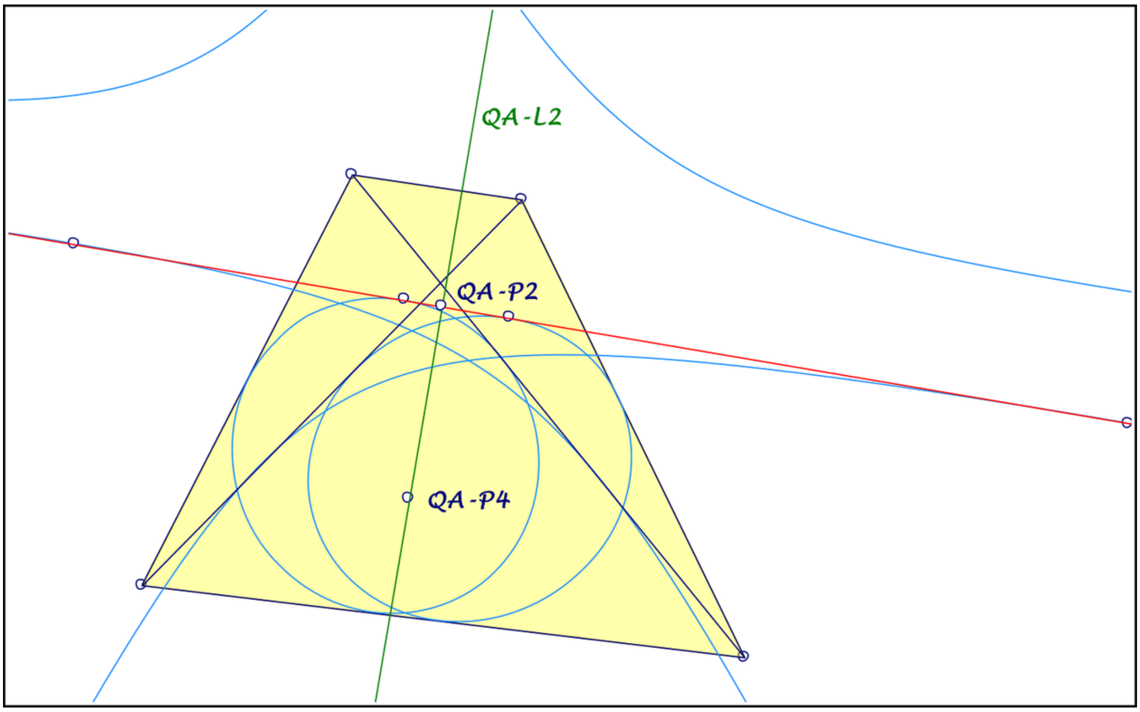
in EQF-Ref [16] 4 Roland Stärk gives a very special property of
QA-P4:
The distance from a QA-vertex to QA-P4
... is the length of the principal axis of a conic
... ... inscribed the remaining triangle with focus QA-P4.

Not this result, but the mentioned conics have an interesting
property (see attached file):

The inscribed conics of the QA-triangles with focus QA-P4 have a
common tangent,
... which is the QA-L2-perpendicular through QA-P2.*

This property can be generalized for other points as QA-P4, but
the common tangent can rarely be identified.
For QA-P3 the common tangent bears QA-P2 ...

Best regards Eckart
```



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Message: #2226
Date: 18/1/2017 1:36:37
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Baryce ...

Dear Eckart,

You have got the point. I would post more later!

Best regards,
Tsihong Lau

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Message: #2227
Date: 18/1/2017 4:13:51
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,

Thank you for your quick answer and your precisions !
I suppose, the last isoconjugation works with every circumconic and can be defined with both parabolas and their tangents as the one wrt every S-triangle which swaps one of the Newton Lines (QL or DQL) with the circumconic ?
Isotomic transversals (see Mathworld) are the same as isotomic points for lines (they cut each side in symmetric points wrt the middles of the sides). For example, the 4 lines of the 2 QL's formed by the tangents to the 2 parabolas through the same 4 points as well as their Newton Lines are transversals wrt the common DT (which is also the DT of the 4 points).

Best regards
Bernard

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Message: #2228
Date: 19/1/2017 9:43:02
From: chris.vantienhoven
Subject: new items in EQF

```
Dear friends,

I placed these new special transformations at EQF:
* QL-Tf8 (QL-Involutary Centerline)
* QL-Tf9 (QL-Involutary Centerline 5th Line Touchpoint)
* QL-Tf10 (QL-QuadriPole)
* QL-Tf11 (QL-QuadriPolar)
* QA-Tf10 (QA-QuadriPole)
* QA-Tf11 (QA-QuadriPolar)
I like them all !

When there are any remarks or typos, please let me know.

Best regards,
Chris
```

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Message: #2229
Date: 20/1/2017 11:13:49
From: eckart_schmidt@t-online.de
Subject: New ETC-Points

```
Dear all,
in # 2221 I described a possibility, to get new ETC-points with
QA-geometry:

QA-Px for the quadrangle of  X(n) and its anticevians.

You can get the barycentric coordinates,
replacing p, q, r in the DT-nomination of QA-Px by the
coordinates of X(n).
Some examples attached (I hope, no error).

Best regards Eckart
```

New ETC-points: QA-Px of the quadrangle with vertices X(n) and its anticevians.

	X(1)	X(2)	X(3)	X(4)	X(5)	X(6)
QA-P1	X(3)	X(2)	X(1147)	X(6523)	X(6663)	X(206)
QA-P2	---	X(99)	X(110)	X(107)	X(476)	X(110)
QA-P3	---	X(671)	X(5504)	NEW	NEW	X(1177)
QA-P4	---	X(316)	X(2071)	NEW	NEW	X(23)
QA-P5	X(20)	X(2)	X(6193)	NEW	NEW	X(5596)
QA-P6	---	X(325)	NEW	NEW	NEW	X(1495)

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-01-20.pdf

Message: #2230
Date: 20/1/2017 12:07:42
From: Seiichi Kirikami
Subject: QG points of focus sharing ellipses and hyperbolas

Dear friends,

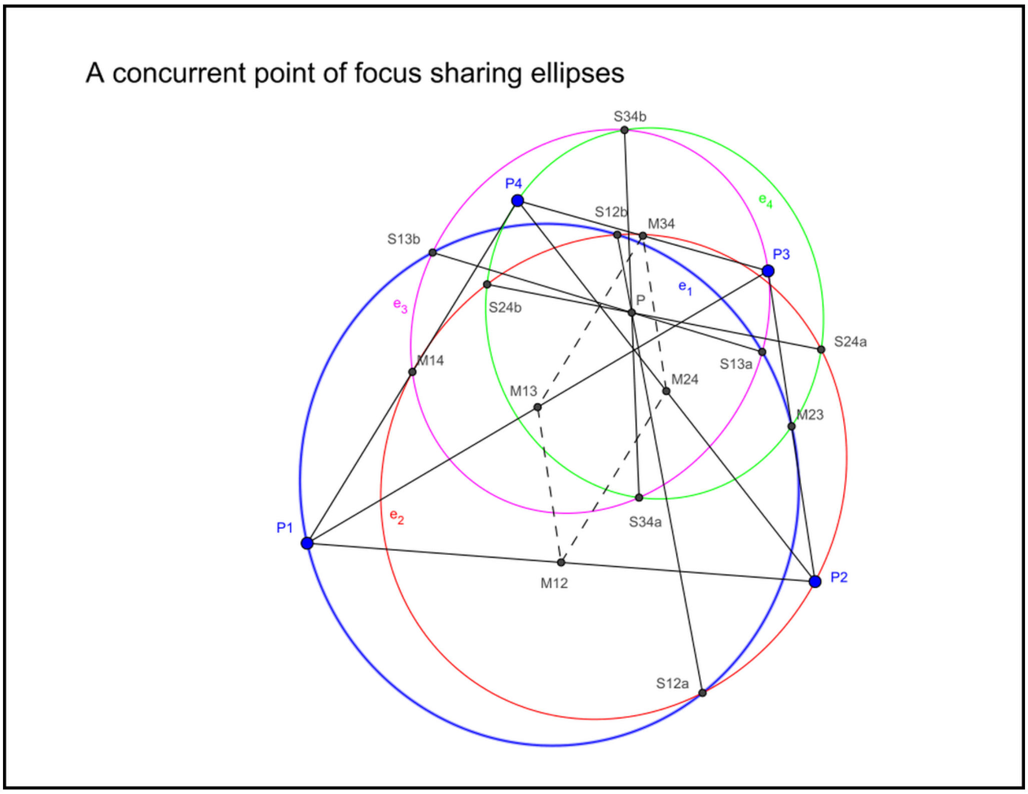
1. Let P1P2P3P4 be a quadrigon.
Mij = midpoints of Mi and Mj.
e1, e2, e4, e3 = ellipses with their foci M12 and M13, M12 and M24, M24 and M34, M34 and M13 through M23, M14, M23, M14 respectively.
Sija, Sijb = intersections of ei and ej.
The lines S12aS12b, S13aS13b, S24aS24b and S34aS34b concur in a point P.

2. Let P1P2P3P4 be a quadrigon.
Mij = midpoints of Mi and Mj.
e1, e2, e4, e3 = ellipses with their foci M12 and M13, M12 and M24, M24 and M34, M34 and M13 through M14, M23, M14, M24 respectively.
Sija, Sijb = intersections of ei and ej.
The lines S12aS12b, S13aS13b, S24aS24b and S34aS34b concur in a point P.

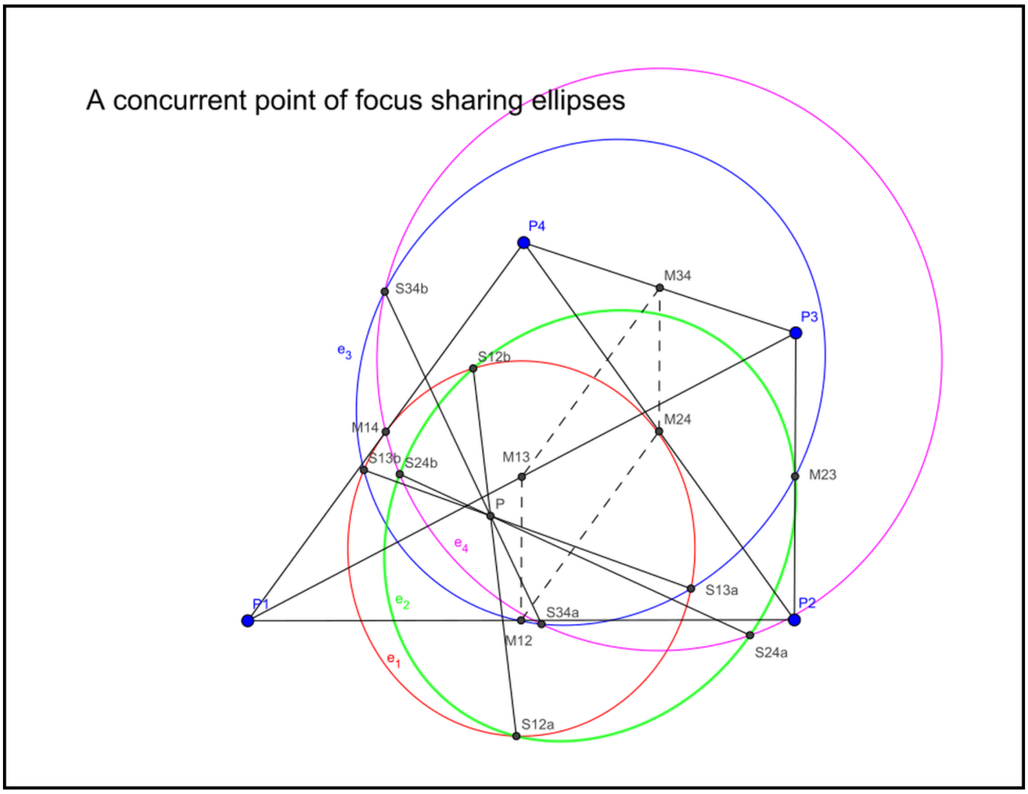
3. Let P1P2P3P4 be a quadrigon and Q12Q23Q34Q14 its van Aubel one.
e1, e2, e3, e4 = ellipses with their foci Q12 and Q14, Q12 and Q23, Q23 and Q34, Q34 and Q14 through P1, P2, P3, P4 respectively.
Sija, Sijb = intersections of ei and ej.
The lines S12aS12b, S23aS23b, S34aS34b and S14aS14b concur in a point P.

4. Let P1P2P3P4 be a quadrigon and Q12Q23Q34Q14 its van Aubel one.
h1, h2, h3, h4 = hyperbolas with their foci Q12 and Q14, q12 and Q23, Q23 and Q34, Q34 and Q14 through P1, P2, P3, P4 respectively.
Sija, Sijb = intersections of hi and hj.
The lines S12aS12b, S23aS23b, S34aS34b and S14aS14b concur in a point P.

Best regards, Seiichi.



FocusSharing.pdf



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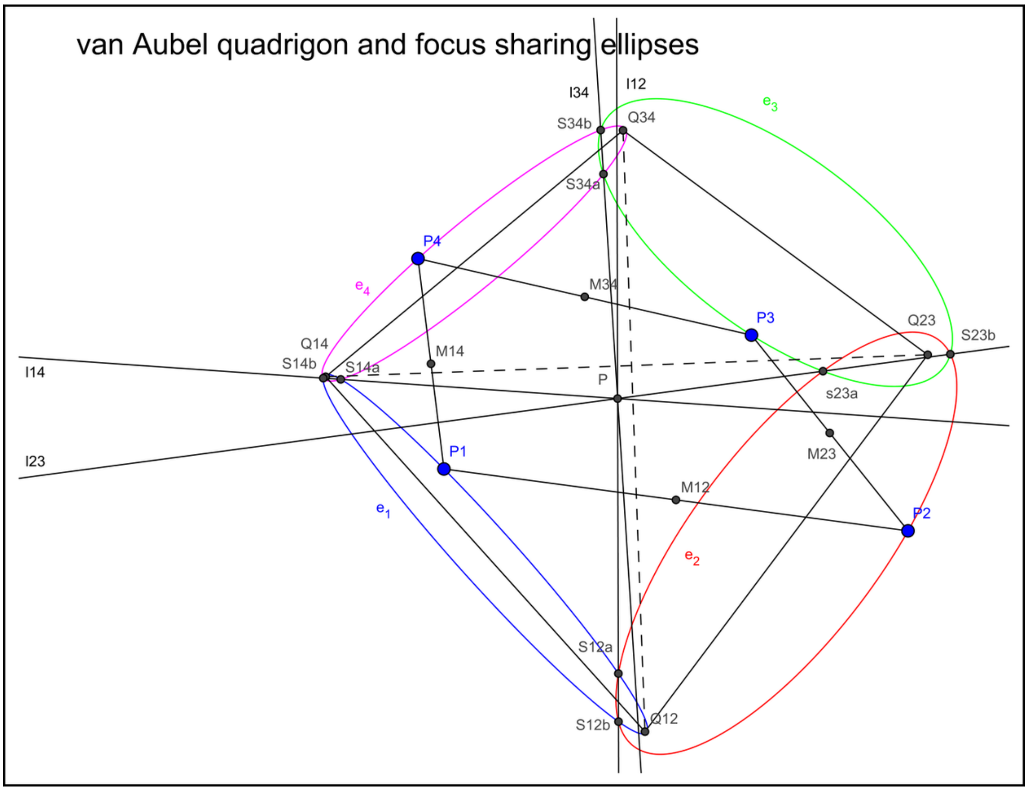

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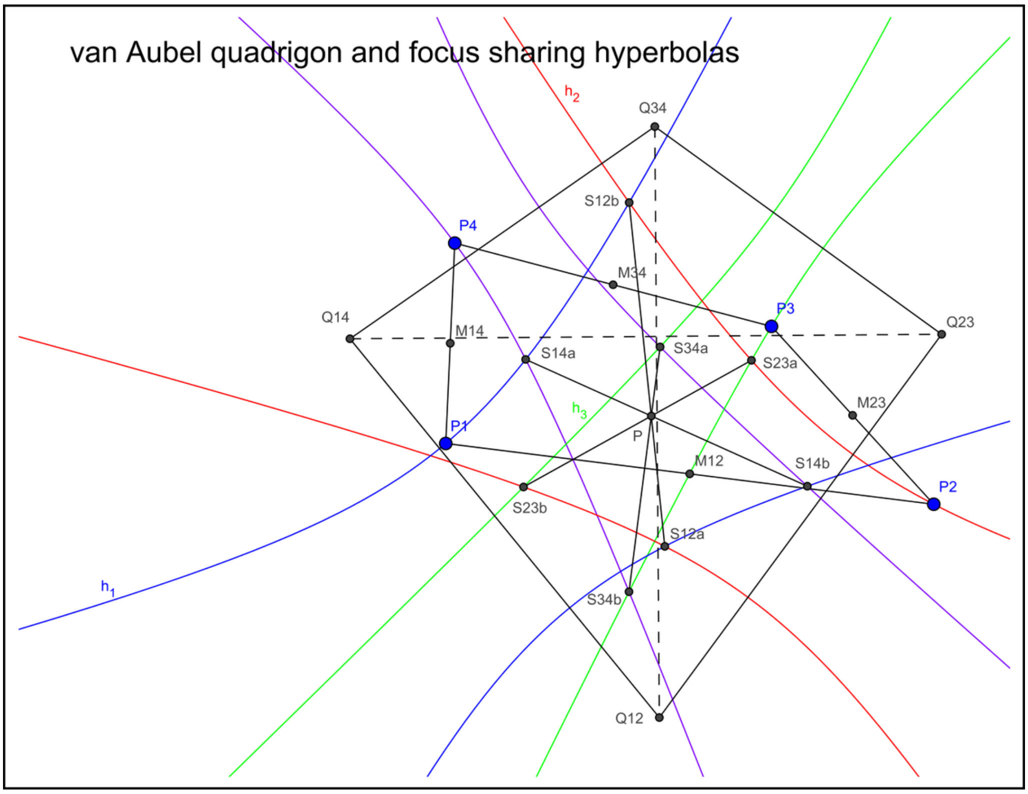

FocusSharing.pdf

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Message: #2231
Date: 20/1/2017 8:57:01
From: eckart_schmidt@t-online.de
Subject: QG points of focus sharing ellipses and hyperbo ...

```
Dear Seiichi,

I think, in construction 2. there is a typo:
"... e1, e2, e4, e3 = ellipses with their foci M12 and M13, M12
and M24, M24 and M34, M34 and M13 through M14, M23, M14, M23
respectively..."

If I change the nomination in 1. in cyclical order,
   I get two double points.
If I change the nomination in 2. in cyclical order,
   I get two double points.
The four points give a parallelogram,
... with center QA-P1.

Best regards Eckart
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Message: #2232
Date: 20/1/2017 11:46:24
From: Seiichi Kirikami
Subject: QG points of focus sharing ellipses and hyperbo ...

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Dear Eckart,

Thank you very much for your correction and comment.
Best regards, Seiichi.
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Message: #2233
Date: 21/1/2017 8:25:49
From: tsihonglau
Subject: New ETC-Points

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Dear Eckart,

An example(in barycentrics):
Two quadrangles with the same diapleural triangle
X1(a:b:c), X1a(-a:b:c), X1b(a:-b:c), X1c(a:b:-c)
X75(1/a:1/b:1/c), X75a(-1/a:1/b:1/c), X75b(1/a:-1/b:1/c),
X75c(1/a:1/b:-1/c)

The centroid of the first quadrangle
   X3(a^2(-a^2+b^2+c^2):...)
The centroid of the second quadrangle
   X6374(b^2c^2(-b^2c^2+c^2a^2+a^2b^2):...)
The pole (in CTC sense) of the first isoconjugation
(isogonal conjugation) is X6.
The pole of the second isoconjugation is X76.
Both centroids and poles are reflexes.
There are many symmetric points,
   which are symmetric operation results of the two poles.

http://lth.name/geometry/class.html
   cevamul-:   X689(1/(a^2(b^4-c^4)):...)
   crossmul-:  X3005(a^2(b^4-c^4)   :...)
   cevamul:    X???(1/(a^2(b^4+c^4)):...)
   crossmul:   X???(a^2(b^4+c^4)    :...)
   incidence:  X???((a^8-b^4c^4)/a^2:...)
   conic:      X???(a^2/(a^8-b^4c^4):...)
   incidence+: X???((a^8+b^4c^4)/a^2:...)
   conic+:     X???(a^2/(a^8+b^4c^4):...)
These points should have many properties.

Best regards,
Tsihong Lau
```

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Message: #2234
Date: 21/1/2017 11:34:20
From: chris.vantienhoven
Subject: Selfcentric Centers

Dear friends,

Let ABC be a triangle.
The orthocenter X(4) has the special property that quadrangle A.B.C.X(4) is orthocentric,
meaning that:
  A is Orthocenter of triangle B.C.X(4)
  B is Orthocenter of triangle C.X(4).A
  C is Orthocenter of triangle X(4).A.B
  X(4) is Orthocenter of triangle A.B.C

A nice example of an orthocentric quadrangle is the quadrangle of Incenter plus Excenters.
Once I read that Randy Hutson made a note that the same is applicable for X(74). So:
  A is Orthocenter of triangle B.C.X(74)
  B is Orthocenter of triangle C.X(74).A
  C is Orthocenter of triangle X(74).A.B
  X(74) is Orthocenter of triangle A.B.C

Apparently there are more of these points having this property.
I propose to call them selfcentric centers.
So I wondered if there are more of these selfcentric centers.
I did some creative calculations with ETC centers X(1)-X(10265) and found this list of selfcentric centers, hoping I made no errors. It may not be complete.
  X(4) = ORTHOCENTER
  X(16) = 2nd ISODYNAMIC POINT
  X(74) = ISOGONAL CONJUGATE OF EULER INFINITY POINT
  X(617) = ANTICOMPLEMENT OF X(14)
  X(1138) = ISOGONAL CONJUGATE OF X(399)
In the reconstruction I had some problems with X(16) and X(617) only selfcentering 3 of the 4 times. Maybe I made errors.

Further it is remarkable that:
1. X(1138) = X(74)-cross conjugate of X(4) and X(4) = X(74)-cross conjugate of X(1138)
2. There are only two points X such that the pedal triangle of X is similar to the cevian triangle of X.
   They are X(4) and X(1138). (Jean-Pierre Ehrmann, January 4, 2003)

(Co)-incidences?

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Any more remarks?
Any examples of selfcentric quadrangles?

Best regards,
Chris van Tienhoven
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Message: #2235
Date: 21/1/2017 12:46:18
From: Seiichi Kirikami
Subject: Simpler configurations of focus sharing ellipses and hyperbolas

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Dear friends,
The following have simpler configurations than 3. and 4. in
message #2230.
1. Let P1P2P3P4 be a quadrigon.
Mij = midpoint of PiPj.
e1, e2, e4, e3 = ellipses with their foci M12 and M13, M12 and
M24, M24 and M34, M34 and M13 through P1, P2, P4, P3
respectively.
Sija, Sijb = intersections of ei and ej.
The lines S12aS12b, S24aS24b, S34aS34b and S13aS13b concur in a
point P.
See the attachment.
I used P1-P2-P4-P3 quadrigon.
Mij = midpoint of PiPj.
h1, h2, h4, h3 = hyperbolas with their foci M12 and M13, M12 and
M24, M24 and M34, M34 and M13 through P1, P2, P4, P3
respectively.
Sija, Sijb = intersections of ei and ej.
The lines S12aS12b, S24aS24b, S34aS34b and S13aS13b concur in a
point P. See the attachment.
Best regards, Seiichi.
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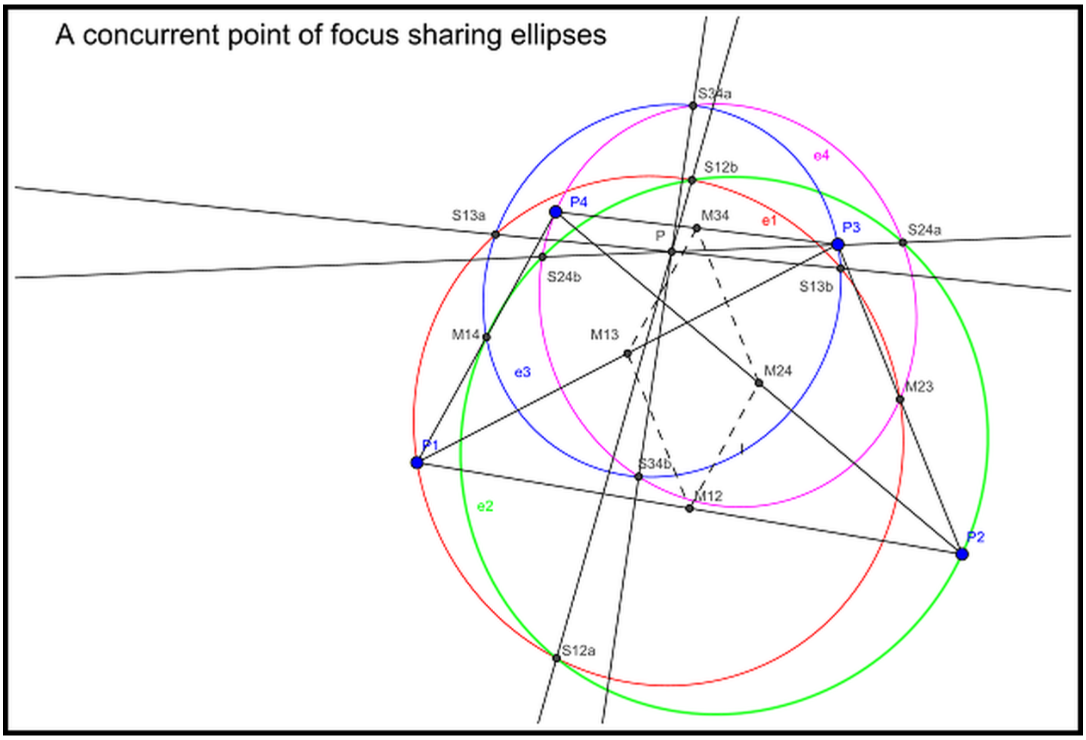

A simpler configuration of focus sharing ellipses.png

Message: #2236
Date: 21/1/2017 1:20:28
From: bernard.keizer
Subject: Generalization of QL-Tr2

Dear Eckart,
Always dealing with the S-triangle QL-Tr2, there are 2 more possible circumconics, the duals wrt QA/QL of the incircle and of the Steinerinellipse of QL-Tr2.
For the dual of the incircle, I didn't find any special property.
For the dual of the Steiner inellipse, it is through QA-P1 (in fact, the Newton Line of DQL, which is the dual of QA-P1 is tangent to the Steiner inellipse) and through the 4th intersection of the Steiner circumellipse and the socalled 5th conic (in fact the dual of this point is tangent to the Steiner inellipse , to the parabola DQL-Co1 and to the dual conic of the Steiner circumellipse).
Best regards
Bernard

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Message: #2237
Date: 21/1/2017 1:41:56
From: bernard.keizer
Subject: New ETC-Points

Dear Eckart, dear Tsihong Lau
The complete QA/QL figure derives from a triangle DT and a unique point QA-P16 or QL-P13.
The QA vertices are the fixed points of the isoconjugation with pole this point.
The QL lines are the trilinear polars of the QA vertices wrt DT.
The Newton Line is the trilinear polar of this point wrt DT.
The sides of DT form with this Newton Line the DQL, which has as DT the socalled DDT.
The vertices of DDT are the vertices of the anticevian triangle of this point wrt DT.
The vertices of the corresponding DQA are the vertices of the anticevian triangle of this point wrt DDT or the trilinear poles of the DT sides wrt DDT ...
If you take an ETC point for this point, not only QA points, but all points, lines, circles, conics, cubics,quartics ... can be seen as ETC elements !
As I told some time ago, as there are already more than 10000 points in ETC, that makes a lot of material !
Best regards
Bernard

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Message: #2238
Date: 22/1/2017 3:47:50
From: Seiichi Kirikami
Subject: Simpler configurations of focus sharing ellipses and hyperbolas

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Dear friends,

Addition of the dual statement of 1. in message #2235.

A collinear line of focus sharing ellipses

Let P1P2P3P4 be a quadrigon.
Mij = midpoint of PiPj.
e1, e2, e4, e3 = ellipses with their foci M12 and M13, M12 and M24, M24 and M34, M34 and M13 through P1, P2, P4, P3 respectively.
Tija, Tijb = tangents of ei and ej.
The intersections of T12aT12b, T24aT24b, T34aT34b and T13aT13b are collinear.
See:
http://www.jcgeometry.org/Articles/Volume1/JCG2012V1pp1-5.pdf

I hope that anyone will teach me how to draw 2 tangents of 2 ellipses determined by their foci and a point.

Best regards, Seiichi.

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Message: #2239
Date: 22/1/2017 9:29:08
From: eckart_schmidt@t-online.de
Subject: Simpler configurations of focus sharing ell ...

Dear Seiichi,

wrt a "construction" of the tangents at two conics with CABRI:
Take the polars of points of one conic wrt the other conic,
... they envelope two conics,
... intersecting the given conics in points,
... which are the contact points of common tangents.

Best regards Eckart

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Message: #2240
Date: 22/1/2017 2:08:48
From: eckart_schmidt@t-online.de
Subject: Selfcentric Centers

Dear Chris,

I think, there is a typo: In the second chapter you have to
replace "Orthocenter " by X(74).
Wrt the selfcentric centers X(16) and X(617) you made no errors.
So we cannot accept these points as selfcentric centers.
Perhaps evident the following observation:
Taking A, B, C and a selfcentric center D of ABC,
... a QA-Px of ABCD
... is the same X(n) for the QA-triangles ABC, BCD, CDA, DAB.

Some Examples:
Selfcentric center X(4): Quadrangle ABCD orthocentic.
QA-P2, 3, 4, 6, 9 and other don´t exist.
QA-P1, 7, 8, 11, 15, 32, 33, 38 give X(5).
QA-P5 gives X(5562),
QA-P10, 14 give X(51),
QA-P12, 20, 24 give X(52),
QA-P13 gives X(143),
QA-P16 gives X(53),
QA-P17 gives X(8905),
QA-P18 gives a new ETC-point,
QA-P19 gives X(6751),
QA-P21 gives a new ETC-point,
...

Selfcentric center X(74): Quadrangle ABCD cyclic.
QA-P1, 6, 36 give a new ETC-point,
QA-P2,37 give X(125),
QA-P3, 4, 8, 12, 32 give X(3),
QA-P5 gives a new ETC-point,
...

Selfcentric center X(1138):
QA-P1 gives a new ETC-point,
QA-P2 gives X(3258),
QA-P3 gives a new ETC-point,
QA-P4, 8, 23, 32 give X(1511),
...

Best regards Eckart
PS.I hope, there are no errors!

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Message: #2241
Date: 22/1/2017 2:23:43
From: chris.vantienhoven
Subject: Simpler configurations of focus sharing ell ...

Dear Eckart,

Very nice method for constructing the 2 or 4 common tangents at
2 conics.
I didn't know this construction.
However there is a slight problem in this method using Cabri.
You start with two conics Co1 & Co2.
Then you take the polar of a variable point V1 on Co1 wrt Co2
which envelopes a conic Co1env.
In the same way the polar of a variable point V2 on Co2 wrt Co1
envelopes a conic Co2env.
In Cabri Co1env and Co2env are loci and not solid
5-point-conics.
I was able to construct these enveloped conics as 5-point-conics
using the new QuadriPolar-transformation QA-Tf10 which stepwise
maps the enveloping lines (tangents actually) into the
touchpoints on the enveloping lines (choosing some random
reference quadrangle).
Having 5-point-conics it was easy to determine the intersection
points in Cabri.
However I wonder if you know a simpler method to construct an
enveloped conic with points (maybe triangular related) ?

Best regards,
Chris

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Message: #2242
Date: 22/1/2017 3:08:42
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear all,

I give two pairs of equivalent geometries.
They are all Pappian planes.
The first pair is a triangle with the circular points and a quadrangle with the line at infinity.
From the former, the line at infinity is the one through the circular points. The incircle and excircles are the four inconics of the triangle through the circular points. The four centers (=poles of the line at infinity with respect to the four circles) of the four circles form the quadrangle. From the latter, the triangle is the diapleural triangle of the quadrangle.
The circular points are the parabolic points (tangent points of the two circumconics of the quadrangle to the line at infinity)

The second pair is a quadrangle with the circular points and two quadrangles with the same diapleural triangle together with the line at infinity. From the former, we get the diapleural triangle of the quadrangle.
According to the above, we get the line at infinity and the incenter /excenters quadrangle from the triangle
and the circular points. The two quadrangle have the same diapleural triangle. From the latter, the circular points are the parabolic points of a quadrangle and the line at infinity.
We get another quadrangle with the circular points. The former uses the circular coordinate system. The latter uses the DT-barycentrics.

Best regards,
Tsihong Lau

Message: #2243
Date: 22/1/2017 4:53:02
From: eckart_schmidt@t-online.de
Subject: Simpler configurations of focus sharing ell ...

```
Dear Chris,

is the following "construction" of the tangents at two conics
with CABRI in your sense?

The poles of 5 tangents at one conic wrt the other conic
... define two 5-point-conics,
... intersecting the given conics in points,
... which are the contact points of common tangents.

Best regards Eckart

PS: Perhaps there is a misunderstanding on my side of your
objection.
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Message: #2244
Date: 22/1/2017 10:56:18
From: chris.vantienhoven
Subject: Selfcentric Centers

Dear Eckart, dear Tsihong Lau,

Eckart, beautiful observation! We often construct quadrangle points using triangle points. Now new triangle points are coming back from quadrangle geometry to triangle geometry.
The whole concept of selfcentric centers is not new.
Like I already noted Randy Hutson mentioned that X(74) has a similar property as X(4) as being orthocentric.
Also Tsihong Lau told me that in ADGEOM #2341 †), he mentioned the concept before. He denoted X(4), X(74), X(1138) as "system centers".

Some extra observations:
Because of their shared properties we could say that that X(15), X(16), X(616), X(617)
are semi-selfcentric centers.
Special is that X(4), X(15), X(16), X(74), X(616), X(617), X(1138) all lie on the Neuberg Cubic.

Eckart noted that
  Taking A, B, C and a selfcentric center D of ABC,
  ... a QA-Px of ABCD
  ... is the same X(n) for the QA-triangles ABC, BCD, CDA, DAB.
The implications of this property are far-reaching!
Not only QA-points represent nested triangle centers, also things made of these QA-points will get a central triangle status. For example lines made of QA-points will become triangle lines,
e.g. the line through QA-P10, QA-P11, QA-P12, QA-P13 being the DT-Eulerline gives rise
  in a X4-centric quadrangle to ETC-line X(51).X(52),
  in a X74-centric quadrangle to ETC-line X(3).X(49),
  in a X1138-centric quadrangle to ETC-line X(30).X(3471).
But also QA-circles, QA-conics and QA-cubics can be transferred to the triangle domain.

Then I tried to find some of the unknown ETC-points.
I did do some calculations and don't pretend to be complete and without errors.
I mention the results. The coordinates are to elaborate to mention here.

X4-centric quadrangle points (QA-points in a ABCX(4)-quadrangle)

* X4-quad-QAP18 lies on X5.X51, X230.X427, X4.X2351, X129.X134, X132.X135, X138.X139, X1899.X2450
* X4-quad-QAP21 = X5.X53 (2:-1) = Reflection of X53 in X5 and lies on X3.X66, X5.X53, X114.X122, X182.X441, X343.X418, X1353.X3284

X74-centric quadrangle points (QA-points in a ABCX(74)-quadrangle)
* X74-quad-QAP5 lies on these lines: X74.X3260, X265.X1531, X1138.X2130
* X74-quad-QAP7 = X3.X125 (2:1) = X74.X5 (2:1) and lies on X3.X125, X5.X74, X67.X182, X110.X140, X13.X1656, X146.X3090, X381.X2777, X498.X3028, X499.X3024, X631.X1511, X1539.X3091

X1138-centric quadrangle points (QA-points in a ABCX(1138)-quadrangle)
* X1138- quad-QAP1 = X1138.X2 (4:-1) and lies on X30.X113, X2.X1138
* X1138- quad-QAP3 = X1138.X476 (1:1) and lies on X30.X113, 476.1138, 3003.3163
* X1138- quad-QAP5 lies on X1138.X1272.X3260
* X1138- quad-QAP6 lies on X30.X113, X140.X523
* X1138- quad-QAP10 lies on X30.X3471, X1495.X3081
* X1138- quad-QAP11 lies on X30.X3471
* X1138- quad-QAP12 lies on X30.X3471, X3.X6, X323.X3470
* X1138- quad-QAP13 = X3471
* X1138- quad-QAP16 lies on X1138.X1989, X1511.X3163, X1918,X3008

Last but not least the same can be done with ETC-lines forming a quadrilateral together with the sidelines of the reference triangle. This quadrilateral should have the property that every reference line is the same type of line wrt the triangle formed by the other 3 reference lines.
And I suppose that also QL-points,
QL-lines, QL-circles, QL-conics and QL-cubics can be transferred to the triangle domain in the same way.
However I did not study it yet.

Best regards,
Chris

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #2245
Date: 23/1/2017 9:11:43
From: chris.vantienhoven
Subject: Simpler configurations of focus sharing ell ...

Dear Eckart,

This solution is simpler indeed for solving the question of the construction of common tangents at two conics and I am glad with that one.

However it is not quite what I had in mind.

My basic question was:
When we have any set of lines enveloping some curve, what is the simplest method to construct this curve using the limited tools of Cabri, but not using the locus-tool.

A nice example is your sextic being enveloped by the asymptotes of QL-inscribed hyperbolas in QFG#2204.
I gave a solution in QFG#2212 using QA-QuadriPoles (QA-Tf10).
I wonder if there is a simpler method (e.g. triangular based and not QA/QL-based).
So far I didn't know any solution for solving this question and I was glad finding the QA/QL-solution last year. Maybe we are restricted to QA/QL-solutions.

Best regards,
Chris

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Message: #2246
Date: 24/1/2017 12:49:56
From: seiichikiri
Subject: Simpler configurations of focus sharing ellipses and hyperbolas

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Dear Eckart, dear Chris,
thank you very much for your information.
Message #2238 does not hold true. Intead of one line, we obtain 4 lines.
Best regards, Seiichi.

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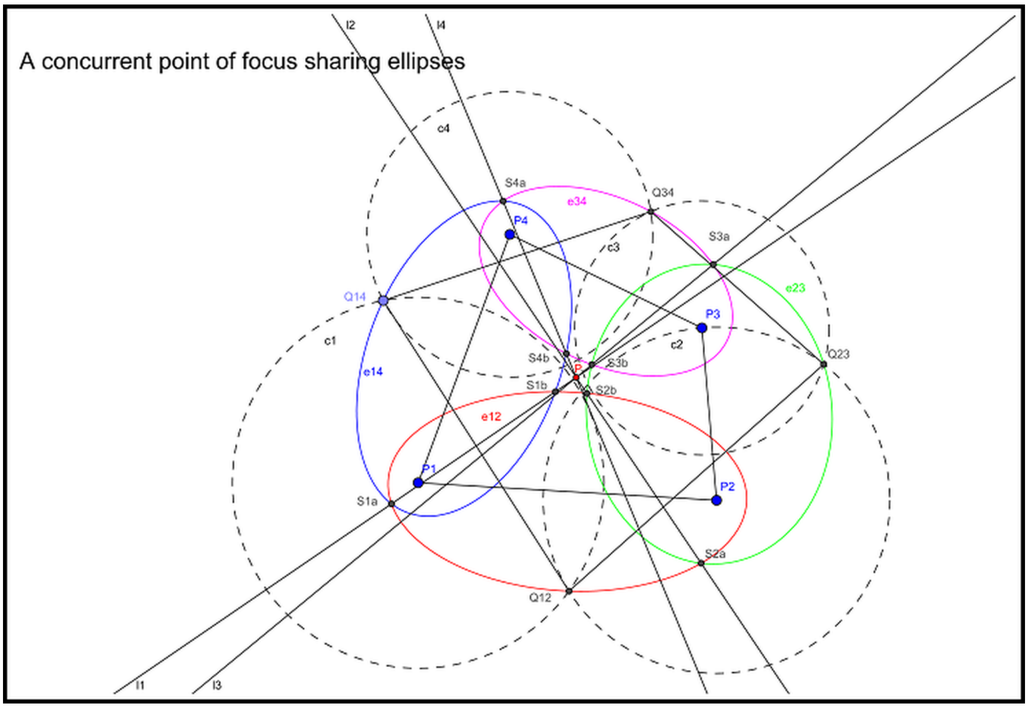
Message: #2247
Date: 24/1/2017 11:37:14
From: Seiichi Kirikami
Subject: Another concurrency of focus sharing ellipses

Dear friends,
Let P1P2P3P4 be a quadrigon.
c1,c2,c3,c4 = circles with their centers P1, P2, P3, P4 respectively.
We suppose that c1 and c2, c2 and c3, c3 and c4, c4 and c1 intersect.
Q12, Q23, Q34, Q14 = external intersections of c1 and c2, c2 and c3, c3 and c4, c4 and c1 respectively.
e12, e23, e34, e14 = ellipses with their foci P1 and P2, P2 and P3, P3 and P4, P4 and P1 through Q12, Q23, Q34, Q14 respectively.
S1a, S1b, S2a, S2b, S3a, S3b, S4a, S4b = external, internal intersections of e14 and e12, e12 and e23, e23 and e34, e34 and e14 respectively.
l1, l2, l3, l4 = lines through S1a and S1b, S2a and S2b, S3a and S3b, S4a and S4b respectively.
l1, l2, l3 and l4 concur in a point P.
This is the elliptical quadri-version of Anopolis message # 4817. †)
Best regards, Seiichi.



ConcurrentEllipses-d.png

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #2248
Date: 24/1/2017 3:14:52
From: tsihonglau
Subject: Selfcentric Centers

Dear Eckart, Chris

The basic problem is what I asked in APG topic #2179 message #2960. I explain it in more details.
Suppose a quadrangle with coordinates(trilinears or barycentrics) p:q:r, -p:q:r, p:-q:r, p:q:-r and its diapleural triangle with sidelength a,b,c. If it is a X(74) system quadrangle. Express p,q,r with a,b,c.
If it is a X(4) system quadrangle, we all know it is the incenter/excenters quadrangle of the diapleural triangle. The isoconjugation induced by the X(4) system quadrangle is the isogonal conjugation.
How about the X(74) system quadrangle?
Topic #2261 shows the geometry of two quadrangles with the same diapleural triangle.
The first quadrangle is usually the X(4) system quadrangle. How about the X(74) system quadrangle?
Similarly, how about the X(1138) system quadrangle?

Best regards,
Tsihong Lau

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Message: #2249
Date: 24/1/2017 3:56:35
From: bernard.keizer
Subject: Selfcentric Centers

Dear Chris, dear Eckart, dear Tsihong Lau
I found this item of selfcentric centers very interesting !
I hope you will find more properties ...
I've noticed in ETC that X1138 is X4 crossconjugate of X74.
Best regards
Bernard

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Message: #2250
Date: 25/1/2017 12:35:33
From: tsihonglau
Subject: Selfcentric Centers

```
Dear Eckart, Chris,

More observations:
System centers: X(4), X(74), X(1138)
X(74) = crossmul(crosspoint) of X(4) and X(1138)
X(74) = cevamul(cevapoint) of X(15) and X(16)
X(30) = cevamul(cevapoint) of X(616) and X(617)
X(30) and X(74) are isogonal conjugates.
X(15), X(16) and X(616), X(617) are bicentric pairs.
The above centers all lie on the Neuberg cubic K001.
X(30) is the pivot of the Neuberg cubic. I guess the solutions
of message #2248 all lie on the
Neuberg cubic

Best regards,
Tsihong Lau
```

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Message: #2251
Date: 27/1/2017 3:58:19
From: Seiichi Kirikami
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear friends,
Let P1P2P3P4 be a quadrangle and P1P2 + P3P4 = P2P3 +P1P4.
The angle bisectors of angles P4P1P2, P1P2P3, P2P3P4 and P3P4P1
are concurrent.
Best regards, Seiichi.
```

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Message: #2252
Date: 27/1/2017 10:54:15
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris, dear Eckart, dear Tsihong Lau
I think I begin to understand the link between the QL, the epi and hypocycloïds tangent to the 4 lines and the n-angle centers.
In fact, the generalised Kantor-Hervey theorem is a special case of a more general property.
For Eckart, it is not surprising that the Kantor-Hervey points do not belong to a particular curve, each corresponding curve is precisely reduced to a point.
For Chris, the use of Hofstadter points is misleading, the adequate points are the n-angle centers ; fortunately, you proved in your message 1887 that both points are the same only for r integer.
Best regards
Bernard

QL, epi- and hypocycloïds and n-angle centers

1) Epi- and hypocycloïds

Let's define an epi- or hypocycloid as the envelop of lines MN, the 2 points M and N describing a circle at different speeds p and q
p and q are integers and mutually prime, same sign for epicycloïds, opposite sign for hypocycloïds

2) Epi- or hypocycloïds tangent to the 4 lines of a QL

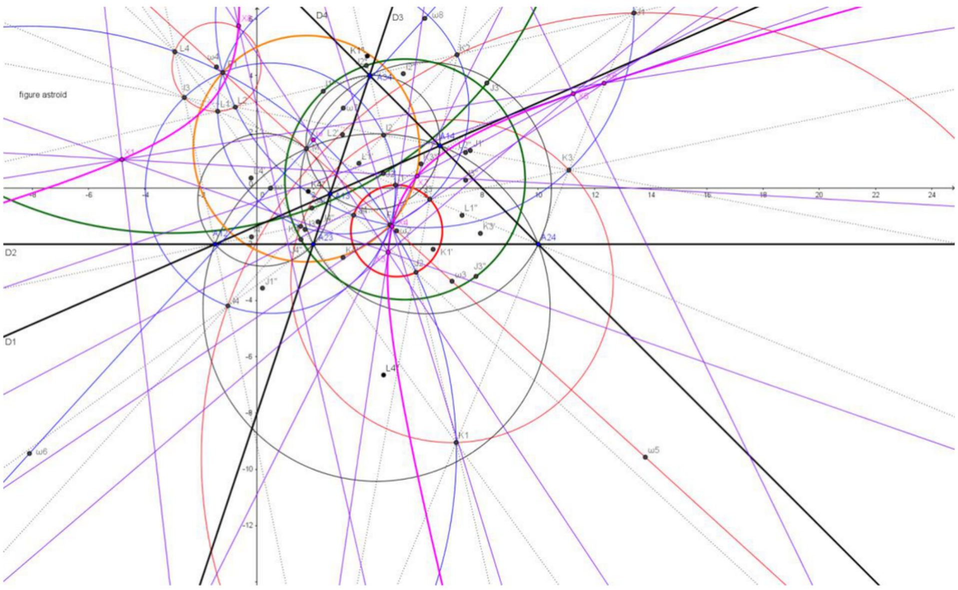
- There are $(p+q)^3$ such curves
- Their centers (centers of the inner circle) are at the intersection of 4 sets of $(p+q)^2$ lines for each reference triangle (p + q centers on each line and each center on 4 lines, one of each set)
- These lines are the perpendicular bisector of copples of isogonal conjugate points wrt the reference triangles from which the sides of the triangle are seen under angles proportional to the opposite angles, the ration being p/(p + q) and q/(p + q)
- These points are the socalled n-angle centers (*) and can be obtained in chains by alterning inversion and isogonality wrt the reference triangle (like Hofstadter, for a point Xr, the inverse is X2-r and the isogonal X1-r)
- There are $(p+q)^2$ such points for each reference triangle and for the QL the $4*(p+q)^2$ points are on $(p+q)^3$ circles of 4 points (one for each triangle) with (p + q) circles through each point
- The circles for isogonal centers are Cl-Sconjugates
- The $(p+q)^3$ centers of the epi-or hypocycloïds are on curves with degree p + q

(*) These point coincide with the Hofstadter points only if p + q = 1

QL, epi and hypocycloïds and n-angle centers.pdf

3) Examples

- For p + q = 1, there are only 4 lines and we find the generalized Kantor-Hervey theorem (the deltoïd is p = 2, q = -1, the other are the H2n+1)
- For p + q = 2, there are 4 points, the beginning of the chain is for p = q = 1 and give the 4 in- and excenters of the reference triangle ; the points are all selfisogonal. We remember the 16 points are on 8 circles of 4 points, each point being on 2 circles. These circles (socalled Steiner circles) are Cl-S invariant.
- The next step in the same chain (inverse and isogonal of the inverse) is for p = 3 and q = -1 and give the 8 pairs of Cl-S conjugate circles ; the centers are the centers of the 8 astroïds tangent to the QL and are on a rectangular hyperbola, which is a conic stelloïd (see figure below)
- For p + q = 3, there are 9 points, the beginning of the chain is with p = 2 and q = 1 and gives the 27 centers of the cardioïds tangent to the QL, which lie on the cubic stelloïd



QL, epi and hypocycloïds and n-angle centers.pdf

Message: #2253
Date: 27/1/2017 3:59:07
From: tsihonglau
Subject: Concurrency with Van Aubel's Theorem

Dear all,

From Mathworld:
Given an arbitrary planar quadrilateral, place a square outwardly on each side, and connect the centers of opposite squares. Then van Aubel's theorem states that the two lines are of equal length and cross at a right angle.

More precisely:
Given an arbitrary planar quadrigon, place a square clockwise(or anticlockwise) on each side, and connect the centers of opposite squares. Then van Aubel's theorem states that the two lines are of equal length and cross at a right angle.
So according to it, a quadrigon has two pairs of perpendicular lines(= a quadrilateral). The quadrilateral has a right-angled diagonal trilateral.
We construct the circumcircle through the points of intersection of the two pairs of perpendicular lines and the vertex of the right angle. Any quadrangle has three component quadrigons. The three corresponding circumcircles concur at a point. Is this point known already?

Best regards,
Tsihong Lau

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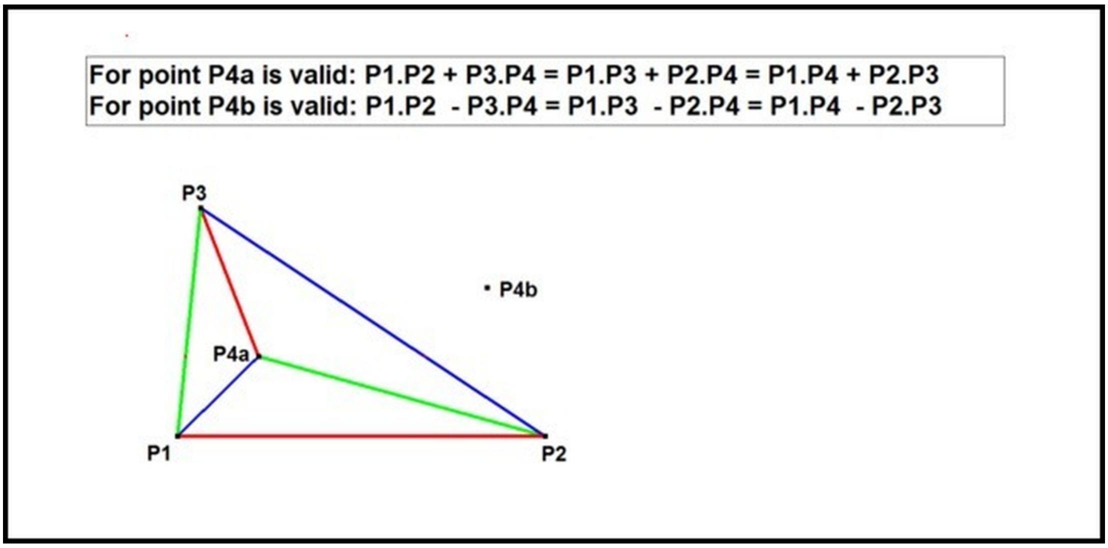
Message: #2254
Date: 28/1/2017 12:16:00
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

Dear Seiichi,
Very nice observation!
I wondered in a *triangle P1.P2.P3:*
· on which locus P4 will lie so that P1.P2+P3.P4=P2.P3+P4.P1.
· on which locus P4 will lie so that P1.P3+P2.P4=P2.P3+P4.P1.
· on which locus P4 will lie so that P1.P3+P2.P4=P1.P2+P3.P4.
(in fact we are looking for the 3 quadrigons within a quadrangle)
For point P4a is valid:
  P1.P2 + P3.P4 = P1.P3 + P2.P4 = P1.P4 + P2.P3
For point P4b is valid:
  P1.P2 - P3.P4 = P1.P3 - P2.P4 = P1.P4 - P2.P3
See attached file.
It appears these points are X(175) and X(176).
So with these points we can apply your property:
· The 4 Angle Bisectors in the cycle P1.P2.P3.P4a concur.
· The 4 Angle Bisectors in the cycle P1.P3.P2.P4a concur.
· The 4 Angle Bisectors in the cycle P1.P4.P2.P3a concur.
· The 4 Angle (External) Bisectors in the cycle P1.P2.P3.P4b concur.
· The 4 Angle (External) Bisectors in the cycle P1.P3.P2.P4b concur.
· The 4 Angle (External) Bisectors in the cycle P1.P4.P2.P3b concur.
Best regards,
Chris



Sum-Difference-P1-P4 points-00.png

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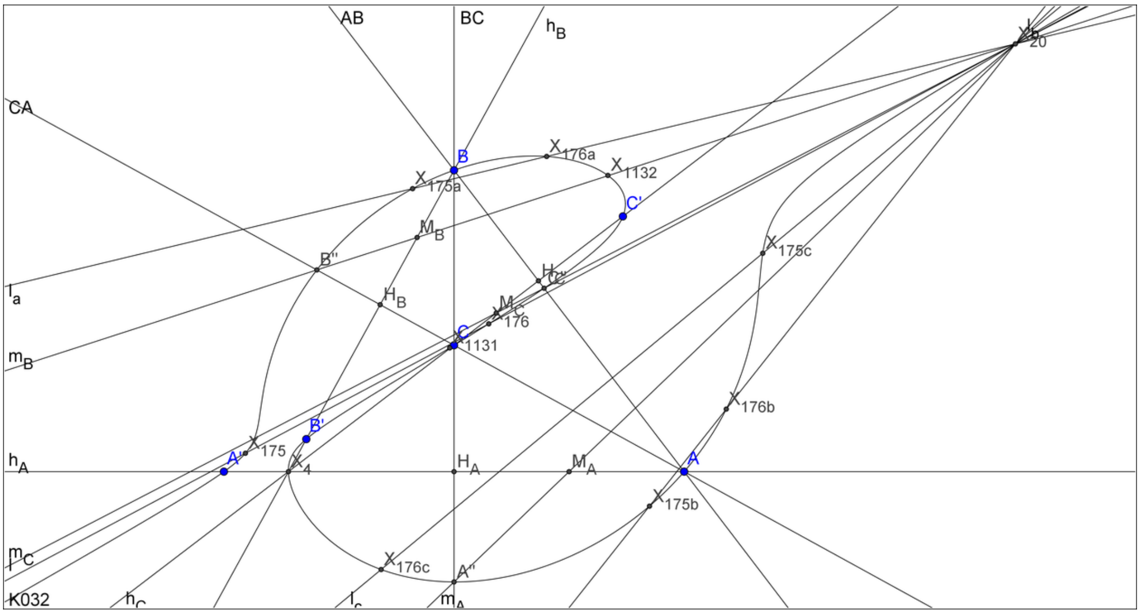
Message: #2255
Date: 28/1/2017 4:51:34
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Chris,
There are typo's.  Please check them.
X(175) and X(176) are not only two points but eight expoints(aka
extraversions), which lie on K032. Please check CTC.
Best regards,
Tsihong Lau
```



K032.png

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Message: #2256
Date: 28/1/2017 9:24:51
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Tsihong Lau,

I do not see the extraversions of X175 and X176 having the same
properties.
Algebraically they do not show up.
Can you explain or make a picture with one extraversion having
the same sum-property?
By the way do you know a general method for constructing the
extraversions of a random point wrt some reference triangle?

Best regards,
Chris
```

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Message: #2257
Date: 28/1/2017 11:10:17
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Chris,

You have typo's not corrected!
The 4 Angle Bisectors in the cycle P1.P2.P3.P4a concur.
The 4 Angle Bisectors in the cycle P1.P3.P2.P4a concur.
The 4 Angle Bisectors in the cycle P1.P4.P2.P3a concur. <- What
P3a?
The 4 Angle (External) Bisectors in the cycle P1.P2.P3.P4b
concur.
The 4 Angle (External) Bisectors in the cycle P1.P3.P2.P4b
concur.
The 4 Angle (External) Bisectors in the cycle P1.P4.P2.P3b
concur. <- What P3b?

In fact, I do not understand what you mean.
I just showed the multiplicity of X(175) and X(176).
You said the expoints have no the sum properties.
Acoording to my theory, there are always distance squares, no
distances themselves. There are always
area squares, no areas themselves.  If we insist on distances
and areas themselves, they always have
two values - positive and negative(in fact, they can be complex
numbers).  That is why we get expoints,
exlines, bicentric pairs(Fermat points etc) etc.
Please check my page - http://lth.name/geometry/coordinate.html
and APG message #2666 for more information.
Because of negative measures, we cannot check sum or differences
of distances and internal or external angle bisectors
respectively only.  We must check all combinations of sum or
differences of distances and internal or
external angle bisectors.
Please try it yourself.  It is not difficult.
Expoints, exlines etc are closely related to message #2242. I
will explain them later!

Best regards,
Tsihong Lau
```

Message: #2258
Date: 28/1/2017 11:30:38
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Tsihong Lau,

Now I understand what you mean.
Yes there are typos: P3a should be P4a, P3b should be P4b.
I thought you found errors wrt number of points (by mentioning
expoints) having the sum/difference property and that is
obviously not what you meant. The expoints actually have no
direct relationship with the subject.

Best regards,
Chris
```

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Message: #2259
Date: 28/1/2017 11:33:13
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Tsihong Lau,

The right expression should be:
· The 4 Angle Bisectors in the cycle P1.P2.P3.P4a concur.
· The 4 Angle Bisectors in the cycle P1.P3.P2.P4a concur.
· The 4 Angle Bisectors in the cycle P1.P4a.P2.P3 concur.
· The 4 Angle (External) Bisectors in the cycle P1.P2.P3.P4b
  concur.
· The 4 Angle (External) Bisectors in the cycle P1.P3.P2.P4b
  concur.
· The 4 Angle (External) Bisectors in the cycle P1.P4b.P2.P3
  concur.

Best regards,
Chris
```

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Message: #2260
Date: 28/1/2017 12:53:05
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>The expoints actually have no direct relationship with the subject.

Dear Chris,

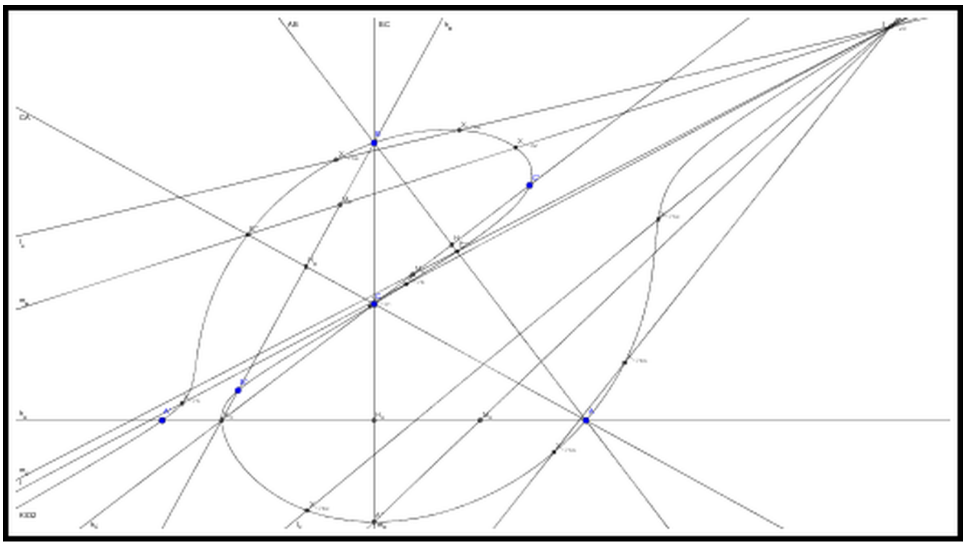
No, the eight expoints of X(175) and X(176) are very essential to this subject. I hoped originally you check them on yourself and find the properties of expoints and negative measures. I checked the sum or differences of distances and post them now.

```
a=6,b=9,c=13 assumed
 X175 .A+B.C = X175 .B+C.A =  X175 .C+A.B
-X175a.A+B.C = X175a.B-C.A =  X175a.C-A.B
 X175b.A-B.C =-X175b.B+C.A =  X175b.C-A.B
 X175c.A-B.C = X175c.B-C.A = -X175c.C+A.B
```

If we replace 175's with 176's, then the above equations are valid!

Please check combinations of internal and external angle bisectors on yourself!
P.S. We usually consider differ ences of distances.
But they are negative distances essentially!

Best regards,
Tsihong Lau



K032-2260.ggb

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Message: #2261
Date: 28/1/2017 7:16:48
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris, dear Eckart, dear Tsihong Lau
I've found a supplementary clue.
The rectangular hyperbola, which bears the 8 centers of astroïds tangent to the 4 lines cuts one diagonal of the QL in 2 harmonic points wrt the vertices and the parallel to the Newton Line through QL-P1 (axis of the parabola) in 2 points symmetric wrt QL-P1, id est harmonic wrt QL-P1 and the infinity point of the Newton Line.
Therefore, this rectangular hyperbola is a polar conic of the cubic stelloïd.
Best regards
Bernard

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Message: #2262
Date: 28/1/2017 8:14:51
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

DearTsihong Lau,

Let X175a, X175b, X175c be the extraversions of X175.
Let X176a, X176b, X176c be the extraversions of X176.
I calculated an example of X175, X175a, X175b, X175c, X176, X176a, X17b, X176c.

This gave these results:

- Wrt X175 the outcome of (A.X175a - B.C), (B.X175a - C.A), (C.X175a - A.B) is identical.
- Wrt X175a the outcome of (A.X175a - B.C), (B.X175a - C.A), (C.X175a - A.B) is identical, with this restriction that the sign of the 1st outcome is different from the other two.
- Wrt X175b the outcome of (A.X175b - B.C), (B.X175b - C.A), (C.X175b - A.B) is identical, with this restriction that the sign of the 2nd outcome is different from the other two.
- Wrt X175c the outcome of (A.X175c - B.C), (B.X175c - C.A), (C.X175c - A.B) is identical, with this restriction that the sign of the 3rd outcome is different from the other two.

- Wrt X176 the outcome of (A.X176a + B.C), (B.X176a + C.A), (C.X176a + A.B) is identical.
- Wrt X176a the outcome of (A.X176a - B.C), (B.X176a - C.A), (C.X176a - A.B) is identical, with this restriction that the sign of the 1st outcome is different from the other two.
- Wrt X176b the outcome of (A.X176b - B.C), (B.X176b - C.A), (C.X176b - A.B) is identical, with this restriction that the sign of the 2nd outcome is different from the other two.
- Wrt X176c the outcome of (A.X176c - B.C), (B.X176c - C.A), (C.X176c - A.B) is identical, with this restriction that the sign of the 3rd outcome is different from the other two.

It was relatively easy to calculate.
However my earlier question stays relevant:
1. Is there a general method to construct the extraversions of some point?
And my second question to you as expoints-expert is:
2. Why the sum-difference-property is valid for the extraversions of X175 and X176?

Best regards,
Chris

p.s.
For those who are not familiar with extraversions (expoints),
the coordinates of the a-/b-/c-extraversions of some point X are
derived by resp. substituting a -> -a, b -> -b, c -> -c within
the coordinates of X.
See also in ETC the preamble of X(7001) and diverse writings of
Tsihong Lau.

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Message: #2263
Date: 28/1/2017 11:43:10
From: chris.vantienhoven
Subject: Concurrency with Van Aubel's Theorem

Dear Tsihong Lau,

I try to examine your Van Aubel figure.
Do you mean with "two pairs of perpendicular lines" the pairs
consisting of inner and outer perpendicular lines of equal
length?
If so, I cannot see the 3 QA-versions of the circumscribing
circle concur at a point.
Could you explain and send a picture?
Thanks in advance.

Best regards,
Chris

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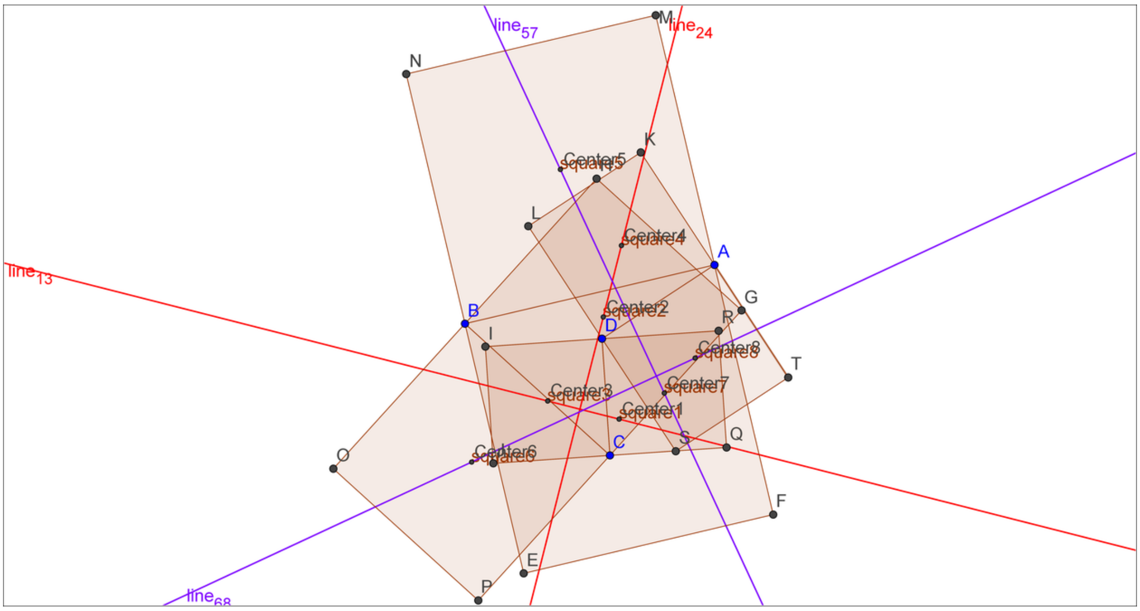
Message: #2264
Date: 29/1/2017 2:47:40
From: tsihonglau
Subject: Concurrency with Van Aubel's Theorem

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>...
>>Do you mean with "two pairs of perpendicular lines" the pairs
consisting of inner and outer perpendicular lines of equal
length?

Dear Chris,

The more accurate terms are "clockwise" and "counterclockwise",
since the quadrigon may even self-intersect.  The point
mentioned is simply the centroid QA-P1.  I will post the whole
graph later.  Please refer to the attachment picture of the
quadrigon.

Best regards,
Tsihong Lau
```



van_aubel_theorem.png

Message: #2265
Date: 29/1/2017 6:26:41
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

--In Quadri-Figures-Group@yahoogroups.com, wrote :
>>...
>>1. Is there a general method to construct the extraversions of some point?
>>And my second question to you as expoints-expert is:
>>2. Why the sum-difference-property is valid for the extraversions of X175 and X176?
>>...

Dear Chris,

I am not an expoints-expert. I am a pioneer.
However, we are all newbies.
My answers for the two questions:

1. If a figure is derived from the quadrangle A,B,C,and incenter a:b:c, we replace the incenter with excenters -a:b:c,a:-b:c,a:b:-c and get the corresponding exfigures. For example, the intouch triangle is the cevian triangle of X(7). The intouch triangle is the three perpendicular feet on the three sides of the incenter. Hence the exfigures of the intouch triangle are the cevian triangles of the expoints of X(7). The exfigures of the intouch triangle are the perpendicular feet on the three sides of the excenters.. In other words, we can derive the expoints of X(7) from the excenters.

2. As I wrote in the previous message -
We usually consider differences of distances.
But they are negative distances essentially!
A paragraph from APG message #2666
The incenters of the 8 triangle with these 8 combinations are described as follows:
   (+a, +b, +c) coincides with (-a, -b, -c)
   (+a, +b, -c) coincides with (-a, -b, +c)
   (+a, -b, +c) coincides with (-a, +b, -c)
   (+a, -b, -c) coincides with (-a, +b, +c)
The first is called incenter in ordinary terminology,
The three below are called excenters (the extraversions of the incenter). In other words, the excenters are the incenters of triangles with "negative" sidelengths.
The idea can be applied to the area of a triangle.
The famous Fermat points, isodynamic points, Napoleon points are pairs of points with positive and negative triangle area.

(See the ETC for more information)
The radius of a circle may be positive or negative.
For example, the radius r of the incircle is +-Î”/(+-a+-b+-c).
It seems that r has 16 possible values.
But in fact, only 8 values are possible.

```
    r  = +Δ/(+a+b+c) = -Δ/(-a-b-c)
   -r  = -Δ/(+a+b+c) = +Δ/(-a-b-c)
    ra = +Δ/(-a+b+c) = -Δ/(+a-b-c)
   -ra = -Δ/(-a+b+c) = +Δ/(+a-b-c)
    rb = +Δ/(+a-b+c) = -Δ/(-a+b-c)
   -rb = -Δ/(+a-b+c) = +Δ/(-a+b-c)
    rc = +Δ/(+a+b-c) = -Δ/(-a-b+c)
   -rc = -Δ/(+a+b-c) = +Δ/(-a-b+c)
```

The two above are positive and negative radii of the incircle.
The six below are of the excircles.
The differences of distances are the sum of a positive and a negative distances indeed. In fact, if we consider
a Pappian plane over a finite commutative field, the positivity and negativity are meaningless on the field.
For example on finite field Z7, 2^2=5^2=4. So 5=-2.5 on Z7 is neither positive and negative. The equations (if distances=square roots of distance squares, exist on the filed) mentioned by you and me are still valid on the plane. The positive distances are meaningless on the plane. At first glance, negative measures are unacceptable. But they are more reasonable.

Best regards,
Tsihong Lau

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Message: #2266
Date: 29/1/2017 10:13:08
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Tsihong Lau,

Thank you very much for your stepwise explanation.
This time I understand you much better.

Some new questions/remarks:

1. I restate your first statement as follows in my own words
(when I am wrong please tell me):
· Infigure = pedal triangle of X1-incenter
           = cevian triangle of X7-incenter
· exfigure = pedal triangle of X1-excenter
           = cevian triangle of X7-excenter
When I draw the pedal triangle of some X1-excenter then this
triangle isn't perspective with the reference triangle, so the
corresponding X7-excenter cannot be obtained this way. What went
wrong?

2. You explain how to construct the expoints of specific point
X7. For me the question stays how to construct the expoints of a
random point in general and of X175 & X176 in particular?
When I read Clark Kimberlings notes on extraversions at the
preamble of X(7001) in ETC, I gather there is no standard
construction known yet. However there are some specific methods,
unfortunately not including X175 and X176.

3. I am familiar with "signed distances". They pop-up with
several algebraic elaborations and all the time they have a
practical explanation/interpretation.

Best regards,
Chris
```

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Message: #2267
Date: 29/1/2017 10:55:28
From: Seiichi Kirikami
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Chris, dear Tsihong,

What I proposed in message # 2251 is well known.
For example, Mathworld " Tangential quadrilateral".
I think that the connexion with X(175) and X(176) is new.

Thanks for unfolding.

Best regards, Seiichi.
```

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Message: #2268
Date: 29/1/2017 2:26:29
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> ...
>> 1. I restate your first statement as follows in my own words (when I am wrong please tell me):
>> · Infigure = pedal triangle of X1-incenter = cevian triangle of X7-incenter
>> . exfigure = pedal triangle of X1-excenter = cevian triangle of X7-excenter
>> When I draw the pedal triangle of some X1-excenter then this triangle isn't perspective with the reference >>triangle, so the corresponding X7-excenter cannot be obtained this way. What went wrong?

The construction does not go wrong. You are infamiliar with exfigures. I construct one and please check it.

>> 2. You explain how to construct the expoints of specific point X7. For me the question stays how to
>> construct the expoints of a random point in general and of
>> X175 & X176 in particular?
>> When I read Clark Kimberlings notes on extraversions at the preamble of X(7001) in ETC, I gather there is >>no standard construction known yet. However there are some specific methods, unfortunately not including >>X175 and X176.

As I wrote
If a figure is derived from the quadrangle A,B,C,and incenter a:b:c, we replace the incenter with excenters -
a:b:c,a:-b:c,a:b:-c and get the corresponding exfigures. If we want to get the expoints of X(175), we have to get X(175) from A,B,C,and the incenter. Then we replace the incenter with the excenters and get the expoints. If there is no way to construct X(175) from A,B,C,and the incenter, there is also no way to construct the expoints. I do not think there exist a general way to get the exfigures directly from A,B,C and a figure.

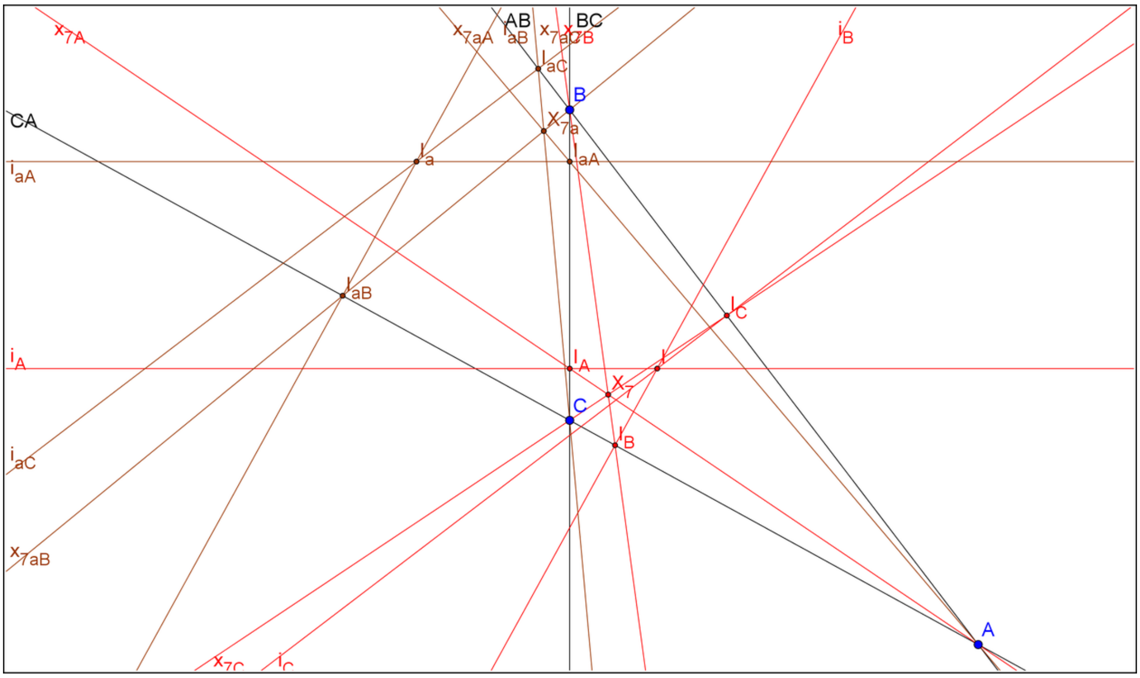
>> 3. I am familiar with "signed distances". They pop-up with several algebraic elaborations and all the time >>they have a practical explanation/interpretation.

As I wrote:
The positivity and negativity are meaningless on a finite field. Distances and areas have always two values.

Squares of both have only one. Exfigures can classified at
least two categories - distance and area. The former is related
to the quadrangle. The latter is related to the parabolic
points. I will post the relationship to topic #2216.

P.S. I have a hidden unfinished page in Chinese on exfigures -
http://lth.name/club/expoint.html
I think you cannot read Chinese. But there are many terms in
English. Exfigures is a huge topic.
But I give up the page. As mentioned above, I will interpret
exfigures with the quadrangles and the parabolic points. New
interpretation is more reasonable and general. Originally I
focused on exfigures, and then I found they should belong to the
quadrangle/quadrilateral geometry. So I joined this group.

Best regards,
Tsihong Lau



X(7).png

Message: #2269
Date: 29/1/2017 11:59:20
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Seiichi and Tsihong Lau,

Seiichi, your remark about tangential quadrilaterals brought me
to new ideas.
When the sum-property is identical to the
tangential-quadrilateral-property only X(175) and (X176) (and
extraversions) have the property of producing tangential
quadrilaterals ABCX(x).
Stated in another way:
Let X(n) be the ETC-point with index n.
When we consider quadrilaterals ABCX(n), BCAX(n) and CABX(n),
then only for n=175 and for n=176 these quadrilaterals will be
tangential. The same is valid for the extraversions of X(175)
and X(176).
(basically combinations ACBX(n), CBAX(n) and BACX(n) are also
possible, but they produce the same quadrilaterals)
Because of the tangential quadrilaterals I studied the incircles
of these quadrilaterals.
This resulted in 8 new triangle points.

Preliminary remarks:
The Radical Axis of 2 circles is actually the perpendicular at
the connecting line of the circle centers in the InsimiliCenter.
We could name this the Internal Radical Axis. In accordance we
could name the corresponding perpendicular at the ExsimiliCenter
the External Radical Axis.

About X(175)
1. Construct the incircle CiA
   of Tangential Quadrilateral X(175).C.A.B with center A1.
   Construct the incircle CiB
   of Tangential Quadrilateral X(175).A.B.C with center B1.
   Construct the incircle CiC
   of Tangential Quadrilateral X(175).B.C.A with center C1.
2. The Internal Radical Axes of Cia, CiB, CiC concur
   in point P1a.
3. ABC is perspective with A1B1C1 with perspector X(1).
4. Let A3,B3,C3 be the ExsimiliCenters of the pairs
   of CiA, CiB, CiC.
   They lie resp. on BC,CA,AB and moreover they are collinear.
5. Let the mutual intersection points
   of the External Radical Axes be A2, B2, C2.
6. ABC is perspective with A2B2C2 with perspectrix line A3.B3.C3
```

and perspector P2a.
7. A1B1C1 is also perspective with A2B2C2
with the same perspectrix line A3.B3.C3 and another
perspector P3a.
8. Let A4B4C4 be the triangle with sidelines A.A3, B.B3, C.C3.
9. A1B1C1 and A4B4C4 are perspective in X(175).
10. The circumcircles (A,B,C4), (B,C,A4), (C,A,B4)
are concurrent in a point P4a.

Finally we found 4 new triangle points P1a,P2a,P3a,P4a, all
related to X(175).
It looks like they are all non-ETC-points.

About X(176)
The same procedure can be followed for X(176) instead of X(175).
Consequently again 4 new triangle points P1b,P2b,P3b,P4b can be
found all related to X(176).
It looks like they are all non-ETC-points.

It is very late now so I stop.
I hope I did not make to many mistakes.

Best regards,
Chris

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Message: #2270
Date: 30/1/2017 11:29:32
From: bernard.keizer
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Chris, dear Seiichi and dear Tsihong Lau,

I try to follow your exchange, but I can't see the connexion
between the property described for a quadrangle
ABCX(175) or ABCX(176) and the property described for the
tangential quadrilateral (socalled Urquhart's theorem).
I woundn't like to die stupid, so could someone help me?

Best regards
Bernard

PS I understand it's difficult to follow several items at the
same time, but I'm surprised that noone reacted to my message
2252. Is it lack of time, lack of interest or is the item too
complicate ? Personnally, I was glad to find at last a link
between the Morley's theorem in the triangle and the
Kantor-Hervey theorem in the QL ...
```

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Message: #2271
Date: 30/1/2017 3:42:14
From: Antreas Hatzipolakis
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Bernard,

Tangential quadrilateral theorem is different from Urquhart's
theorem.

Tangential Theorem (Inscriptible Quadrilateral):
http://www.cut-the-knot.org/proofs/InscriptibleQuadrilateral.sht
ml

Urquhart's theorem
http://www.cut-the-knot.org/pythagoras/UrquhartsTheorem.shtml

APH

[Bernard]:
I try to follow your exchange, but I can't see the connexion
between the property described for a quadrangle
ABCX(175) or ABCX(176) and the property described for the
tangential quadrilateral
(socalled Urquhart's theorem).
```

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Message: #2272
Date: 30/1/2017 5:17:39
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear Antreas,
No, I do not think so.  They should be unified.

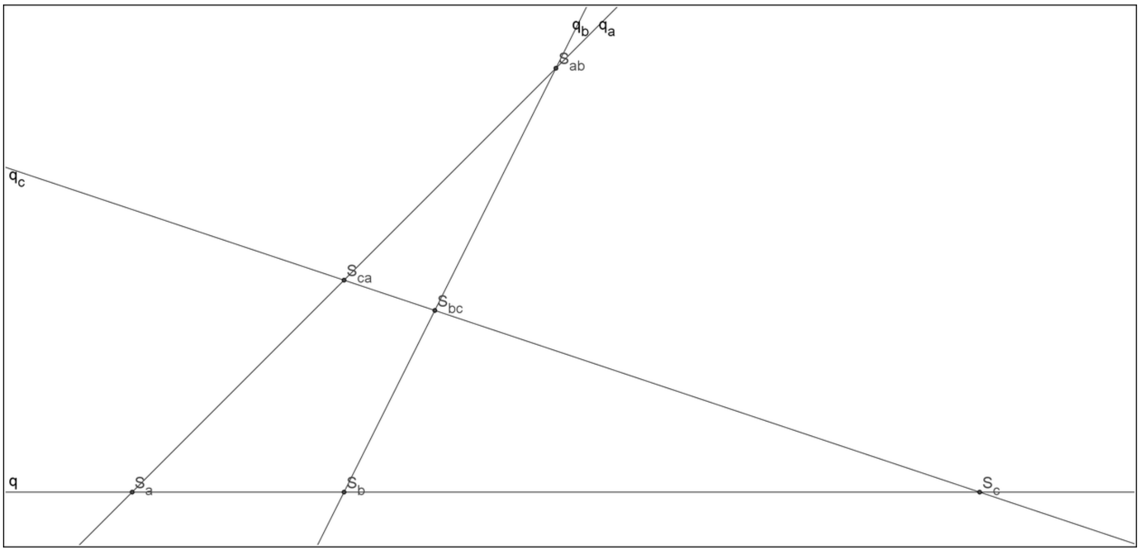
In my notations:

Urquhart's Theorem, given the critical diagonal SaSbc

SaSb+SbcSca=ScaSa+SbSbc <=> SaSab+SbcSc=ScSa+SabSbc <=>
SbSc-ScaSab=ScSca-SabSb <- inscribed circle

SaSb-SbcSca=ScaSa-SbSbc <=> SaSab-SbcSc=ScSa-SabSbc <=>
SbSc+ScaSab=ScSca+SabSb <- escribed circle
If we consider negative distances, they are the same.

Best regards,
Tsihong Lau
```



quadrilateral.png

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Message: #2273
Date: 30/1/2017 5:18:02
From: bernard.keizer
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

Dear Antreas,
Thank you for your quick reaction.
If I'm not wrong, in the Urquhart figure the 4 lines of the QL are tangent to the excircle of the angle in A (it is an extangential quadrilateral).
The excenter is center and double focus of the excircle, which is an inscribed conic and lies therefore on the Newton Line and on the circular focal Van Rees cubic of the QL ...
Best regards
Bernard

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Message: #2274
Date: 31/1/2017 7:46:34
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

Dear Seiichi, Tsihong Lau, Bernard and Antreas,
Thanks Antreas for the references to Bogomolny's site!
Bernard, I hope with this analyses also to answer some of your questions.

Tangential-theorem:
When in a figure ABCD is circumscribed around a circle we deal with a closed figure (4 points connected by 4 line segments) which is called a Quadrigon in EQF.
If a Quadrigon ABCD is circumscribed around a circle (situation called tangential), then AB + CD = BC + AD.
Any Quadrigon ABCD that satisfies AB + CD = BC + AD is tangential. See proof Bogomolny for the convex case in http://www.cut-the-knot.org/proofs/InscriptibleQuadrilateral.shtml#anotherProof.
An extension on this theorem is that the lengths of the line segments also can be considered as negative. This was mentioned by Tsihong Lau and relates the Tangential Quadrilateral Theorem with Urquhart's Theorem.

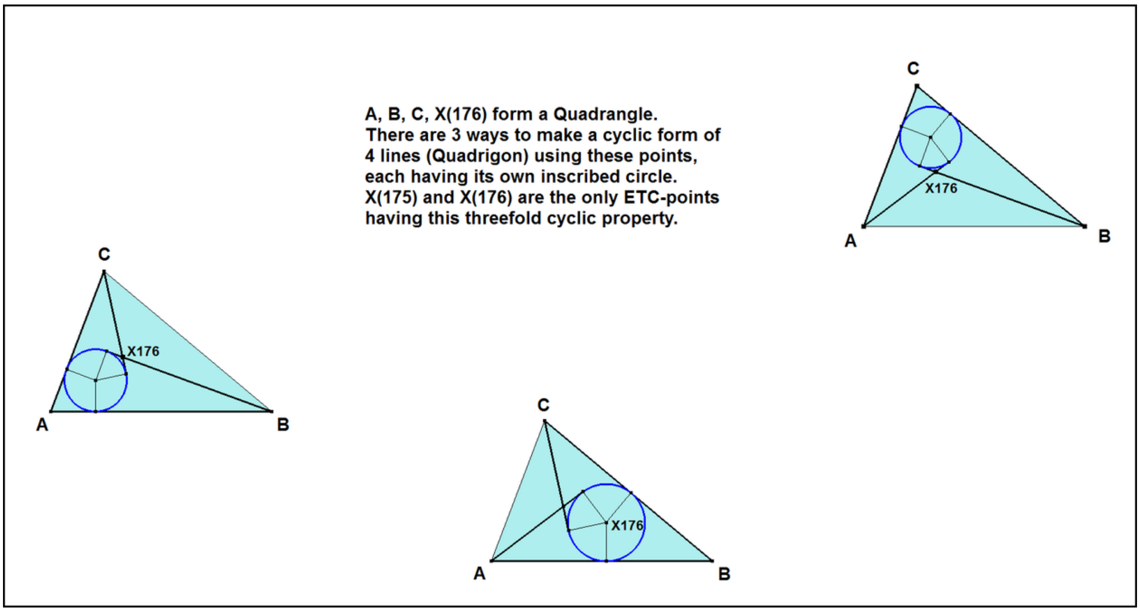
Urquhart's Theorem

Urquhart’s Theorem states that if ABB' and AC'C are straight lines with BC and B'C' intersecting in D and if AB + BD = AC' + C'D, then AB' + B'D = AC + CD.
See
http://www.cut-the-knot.org/pythagoras/UrquhartsTheorem.shtml.
Analyzing this theorem it is clear that we have 4 lines AB, BC, CA, B’C’ with 6 mutual intersection points A, B, C, D, B’, C’.
So Urquhart’s Theorem describes the configuration of a Complete Quadrilateral, namely 4 lines intersecting in 6 points.
The two equalities in Urquhart’s Theorem both represent one Quadrigon belonging to the quadrilateral.
Because a Quadrilateral is containing three Quadrigons (see QL-3QG in EQF) each will have its own equality and so there is one missing: BC + BB' = B'C' + CC'.
This extra equality is an extension to Urquhart's Theorem and was not mentioned by him. See attached file.

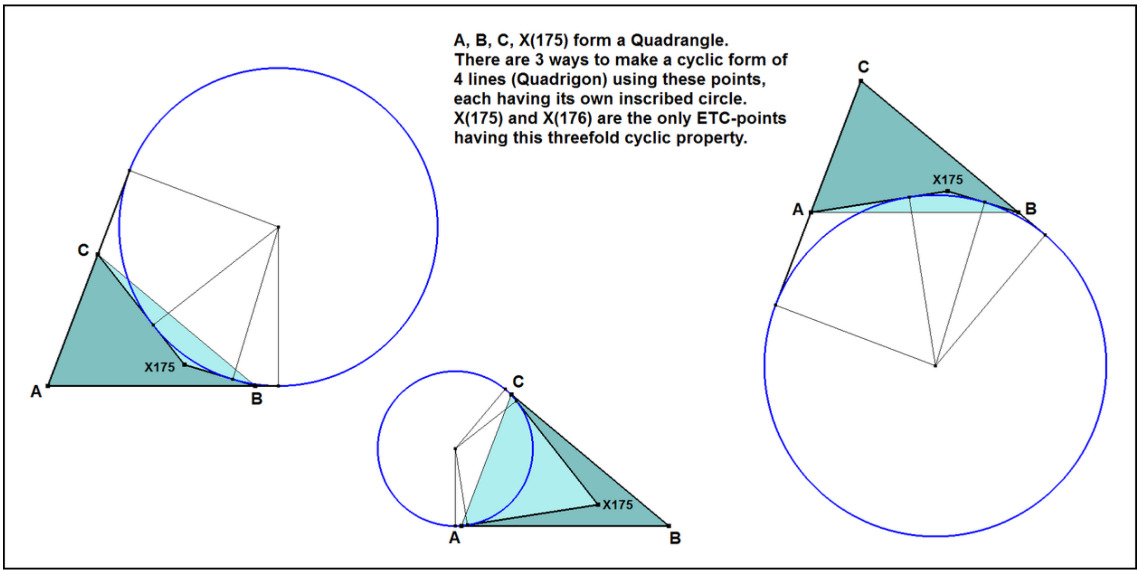
The Tangential-theorem in a Quadrangle
A Quadrangle (system of 4 random points) also contains 3 Quadrigons.
When in one Quadrigon of the Quadrangle the distance property is valid, it is not automatically valid for the 3 other Quadrigons.
The only exceptions are the Quadrangles A.B.C.X(n), where X(n)=X(175 or X(176) or one of their extraversions. Only here the distance-property is valid for all three Quadrigons and as a consequence in these cases there are 3 inscribed circles with many special properties as mentioned in messages #2254, #2255, #2269. See attached files.

Extra properties for the A.B.C.X(175) / A.B.C.X(176)-Quadrangle
Apart from the properties in message #2269 there are these extra properties:
Wrt A.B.C.X176:
· P1a is ETC Center X(5) of A1.B1.C1.
· The Newton Lines of X175.C.A.B, X175.A.B.C and X175.B.C.A are concurrent in P5a.
The Newton lines pass resp. through A1, B1, C1.
Wrt A.B.C.X176:
· P1b is ETC Center X(5) of A1.B1.C1.
· The Newton Lines of X175.C.A.B, X175.A.B.C and X175.B.C.A are concurrent in P5b.
The Newton lines pass resp. through A1, B1, C1.

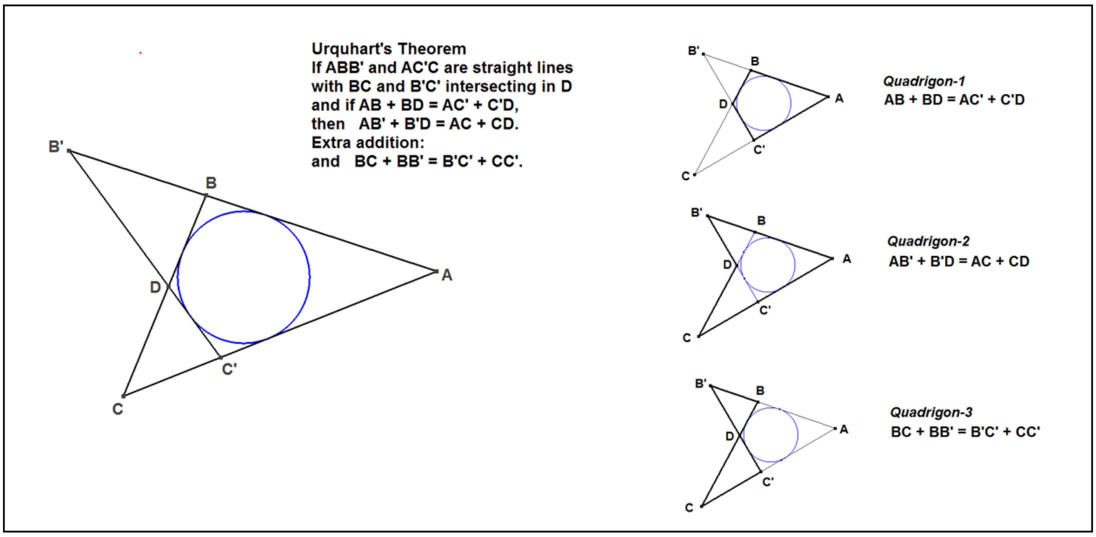
Best regards,
Chris



X176-Tangential-Quadrilateral-Property-01.png



X175-Tangential-Quadrilateral-Property-01.png



QL-Urquhart's Theorem-01.png

Message: #2275
Date: 01/2/2017 1:36:28
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

Dear all,

There were errors in my previous message.
In my notations:
Urquhart's Theorem, given the critical diagonal SaSbc
SaSb+SbcSca=ScaSa+SbSbc <=> SaSab+SbcSc=ScSa+SabSbc <=>
SbSc-ScaSab=SabSb-ScSca <- inscribed circle
SaSb-SbcSca=ScaSa-SbSbc <=> SaSab-SbcSc=ScSa-SabSbc <=>
SbSc-ScaSab=SabSb-ScSca <- escribed circle
There is a common equation for inscribed and escribed circles
SbSc-ScaSab=SabSb-ScSca

Best regards,
Tsihong Lau

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Message: #2276
Date: 01/2/2017 7:33:12
From: chris.vantienhoven
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear friends,
There were some typos in the last part of my last message #2274.
It should be:

Extra properties for the A.B.C.X(175) / A.B.C.X(176)-Quadrangle

Apart from the properties in message #2269 there are these extra
properties:
Wrt A.B.C.X(175):
· P1a is ETC Center X(5) of A1.B1.C1.
· The Newton Lines of X(175).C.A.B,  X(175).A.B.C and
  X(175).B.C.A are concurrent in *5a.
  The Newton lines pass resp. through A1, B1, C1.
Wrt A.B.C.X(176):
· P1b is ETC Center X(5) of A1.B1.C1.
· The Newton Lines of X(176).C.A.B,  X(176).A.B.C and
  X(176).B.C.A are concurrent in *P5b*.
  The Newton lines pass resp. through A1, B1, C1.

Best regards,
Chris
```

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Message: #2277
Date: 01/2/2017 11:57:31
From: bernard.keizer
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

Dear Chris,
Thanks a lot for these complete explanations !
I knew the Urquhart theorem, but not the points X(175) and X(176).
I understand it's quite the same thing.
If I'm not wrong, on your figure of Urquhart's theorem, D is necessary X(175) or X(176) of ABC.
The points you name A1B1C1 are the centers of circles inscribed in a QL.
As I put in my answer to Antreas, these circles are inscribed conics for the QL's.
Therefore, these points are as centers on the Newton Line (as you notice) and, as dobble focus on the circular focal Van Rees curves (QL-Cu1) ; they are knots of these curves, which are strophoïds and necessary one of the 2 points QL-2P3a or b as well as the 2 points QL-2P2a and b.
Best regards
Bernard
PS I'm always waiting desperatly on some remarks from you or Eckart to my last messages 2252 and 2261

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Message: #2278
Date: 02/2/2017 9:06:17
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Bernard,

When I read your message #2252 I had very little clues to arouse my interest.
Here are my questions:

QFG#2252
“epi”, do you mean Morley’s pi-points or epi-cycloids?
“hypocycloïds” I did read your messages about it but not thoroughly
“link between …”, ok what is the link?
“Kantor-Hervey theorem”, do you mean construction method QL-P3 Kantor-Hervey Point?
“the use of Hofstadter points is misleading”, why?

QFG#2261
“The rectangular hyperbola, which bears the 8 centers of astroïds”, did you mention it before?
“astroids”, I think this item is new in our messages, do you mean the epicycloid with 4 cusps?
“this rectangular hyperbola is a polar conic of the cubic stelloïd”, which cubic, I think I missed several items about this and I never got a good grasp on polar conics and stelloids.

I think you really went very deep in the matter which is very well of course, but I did not follow all your steps and could not find a summary of earlier conclusions.
If you want to, maybe you can build your conclusions stepwise from the beginning in simple words so that even I can understand them (“I am just a poor boy”, see lyrics The boxer from Simon and Garfunkel with also these remarkable words: “Still, a man hears what he wants to hear and disregards the rest”).
I am looking forward.

Best regards,
Chris

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Message: #2279
Date: 02/2/2017 4:38:29
From: tsihonglau
Subject: A Circular Quartic of Quadrangle

```
Dear all,

I think a circular quartic should have many nice properties. I
give circular curves from degree one to four together with their
isoconjugates.
All coordinates are in circulars.  Circular points
P(p:q:r) and U(u:v:w) and the reference quadrangle
(1:1:1),(-1:1:1),(1:-1:1),(1:1:-1)

1. Line through P and U = line at infinity
(qw-rv)x+... = 0
isoconjugate = QA-Co1 nine-point conic
(qw-rv)yz+...= 0

2. Conic through P and U and diapleural triangle
= QA-Ci1 circumcircle of diapleural triangle
pu(qw-rv)yz+...= 0
isoconjugate   = QA-L9
pu(qw-rv)x+... = 0

3. Cubic through P and U and diapleural triangle
and reference quadrangle = QA-Cu1 QA-DT-P4 Cubic
((qw-rv)(p^2u^2-qvrw)-(qv-rw)(p^2vw-qru^2))x(y+z)(y-z)+...=0
isoconjugate = QA-Cu1 pivotal isocubic

4. Quartic through P and U and diapleural triangle
and reference quadrangle and (0:1:1),(0:1:-1),
(1:0:1),(-1:0:1),(1:1:0),(1:-1:0)
pu(qw(p+q)(p-q)(w+u)(w-u)-rv(r+p)(r-p)(u+v)(u-v))yz(y+z)(y-z)+..
= 0
isoconjugate
pu(qw(p+q)(p-q)(w+u)(w-u)-rv(r+p)(r-p)(u+v)(u-v))x^3(y+z)(y-z)+.
. = 0

In fact, the circular points above can be replaced with any two
points to get more general results.

Best regards,
Tsihong Lau
```

Message: #2280
Date: 03/2/2017 9:22:03
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

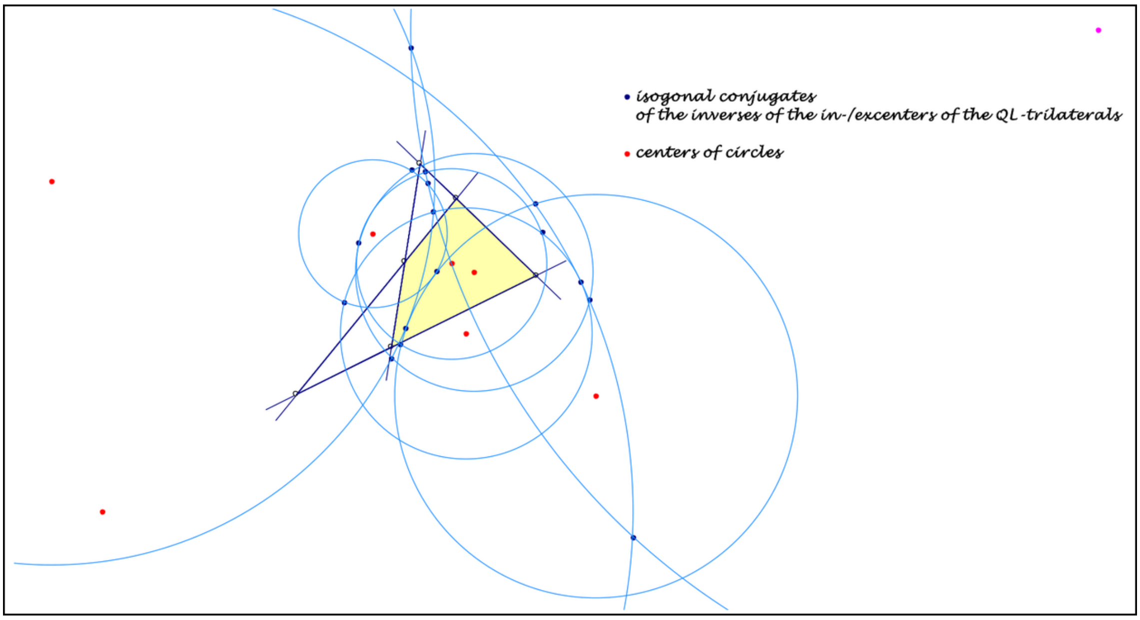
```
Dear Bernard, dear Chris,

50 years married, there were some holidays, so I could not
follow your discussion.
Trying in vain, to understand Bernard´s constructions, I found
the following property, perhaps well-known?

Consider for the in-/excenters of a triangle
... the isogonal conjugates of their inverses
    wrt the circumcircle.
These 4 points give for the 4 QL-trilaterals 16 points
... on 8 circles
(4 points on a circle, 2 circles through each point).

Perhaps a consequence of the Steiner circles, but I found no
properties for the centers of these circles (see attached file).

Best regards Eckart
```



2017-02-03.pdf

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Message: #2281
Date: 03/2/2017 10:14:46
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris, dear Eckart
I'm glad you're both back, I began really to desperate !
First congratulations to Eckart for his wedding birthday, 50 years is half a century !!!
Then I fear there is a misunderstanding between us, as Chris questions as well as Eckart's message show that you have certainly not read the file attached to my message (the title is mentionned in the list of attachments, but the file has apparently disappeared).
So I cut it in 2 pieces, one of text, the other with a figure.
For Chris, I hope your questions will find answers (the cubic stelloïd is QL-Cu2 and it's properties were developped by Bernard Gibert - see cardioïds and Eckart'scubic)
For Eckart, the isogonal of your points are also on 8 circles, Cl-S conjugates of your 8 circles and then it goes on with the inverse and the isogonals ...
I now need support and help to continue, the drawing of such epi -or hypocycloïds takes too much time !
Best regards
Bernard

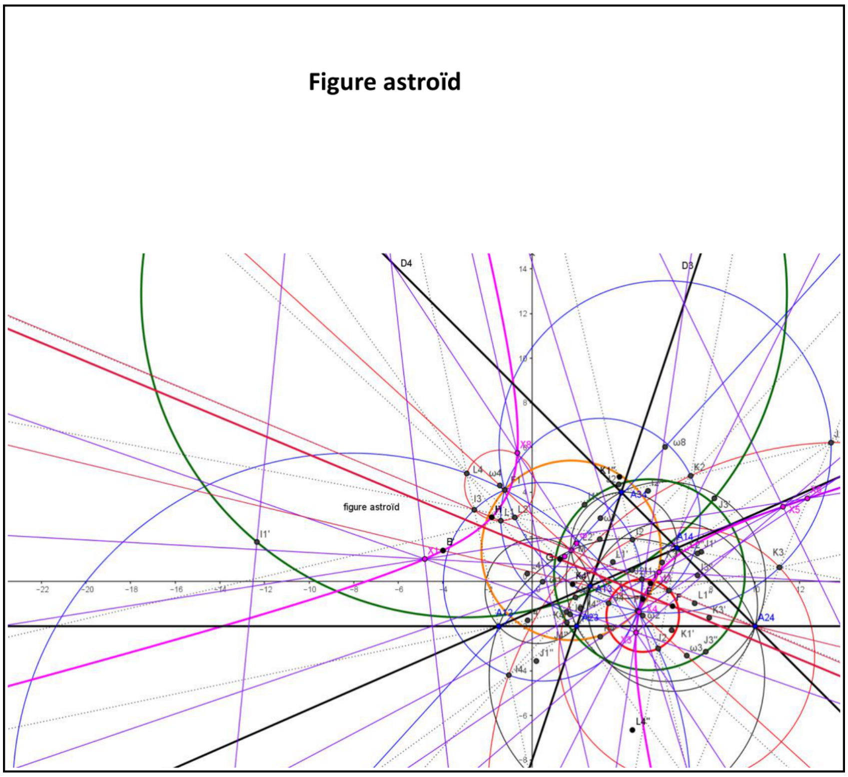

figure astroïd.pdf

QL, epi- and hypocycloïds and n-angle centers

1) Epi- and hypocycloïds

Let’s define an epi- or hypocycloid as the envelop of lines MN, the 2 points M and N describing a circle at different speeds p and q
Let’s choose p and q integers and mutually prime, same sign for epicycloïds, opposite sign for hypocycloïds

2) Epi- or hypocycloïds tangent to the 4 lines of a QL

- There are $(p+ q)^3$ such curves
- Their centers (centers of the inner circle) are at the intersection of 4 sets of $(p+ q)^2$ lines for each reference triangle (p + q centers on each line and each center on 4 lines, one of each set)
- These lines are the perpendicular bisector of copples of isogonal conjugate points wrt the reference triangles from which the sides of the triangle are seen under angles proportional to the opposite angles, the ratio being p/(p + q) and q/(p + q)
- These points are the socalled n-angle centers (*) and can be obtained in chains by alterning inversion and isogonality wrt the reference triangle (like Hofstadter, for a point Xr, the inverse is X2-r and the isogonal X1-r)
- There are $(p + q)^2$ such points for each reference triangle and for the QL the $4*(p + q)^2$ points are on $(p+ q)^3$ circles of 4 points (one for each triangle) with (p + q) circles through each point
- The circles for isogonal centers are Cl-S conjugates
- The $(p+ q)^3$ centers of the epi-or hypocycloïds are on curves (stelloïds ?) with degree p + q

(*) These point coincide with the Hofstadter points only if p + q = 1

QL, E, H and n-angle.pdf

3) Examples

- For p + q = 1, there are only 4 lines and we find the generalized Kantor-Hervey theorem (the deltoïd is p = 2, q = -1, the other are the H2n+1)
- For p + q = 2, there are 4 points, the beginning of the chain is for p = q = 1 and give the 4 in- and excenters of the reference triangle ; the points are all selfisogonal. We remember the 16 points are on 8 circles of 4 points, each point being on 2 circles. These circles (socalled Steiner circles) are Cl-S invariant.
- The next step in the same chain (inverse and isogonal of the inverse) is for p = 3 and q = -1 and give the 8 pairs of Cl-S conjugate circles ; the centers are the centers of the 8 astroïds tangent to the QL and are on a rectangular hyperbola, which is a conic stelloïd (see figure below)
- For p + q = 3, there are 9 points, the beginning of the chain is with p = 2 and q = 1 and gives the 27 centers of the cardioïds tangent to the QL, which lie on the cubic stelloïd.

Message: #2282
Date: 03/2/2017 10:30:13
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Chris, dear Eckart
Again, the 2nd piece has disappeared!
(But it is in the list of mails)
I try again with only this one.
I don't know if you had the figure of Kantor-Hervey complete
generalisation (it is sometimes ago)
Best regards
Bernard
```

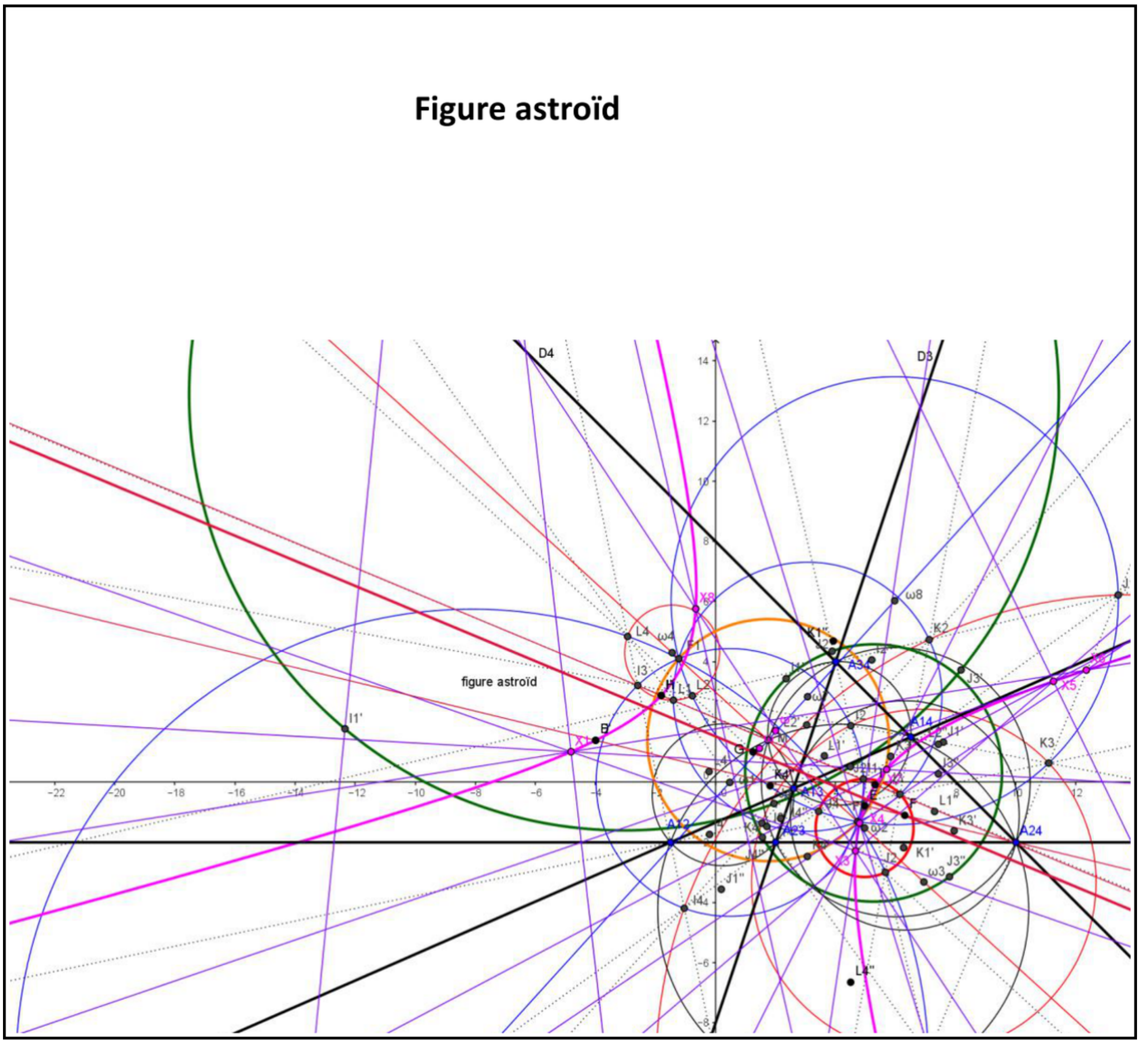

figure astroïd.pdf

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Message: #2283
Date: 03/2/2017 11:24:50
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,

I checked if I had received your attachment(s).
And after all I found in the initial e-mail there was an
attachment and clicking on it appeared.
Also your latest messages contain the attached files you
described.
The problem is that when looking back to some message-# you do
not see there is an attached file.
So don't you worry your attachments come across.
We only have to be aware that on the Yahoo-website it is not
always shown when looking back.

Best regards,
Chris
```

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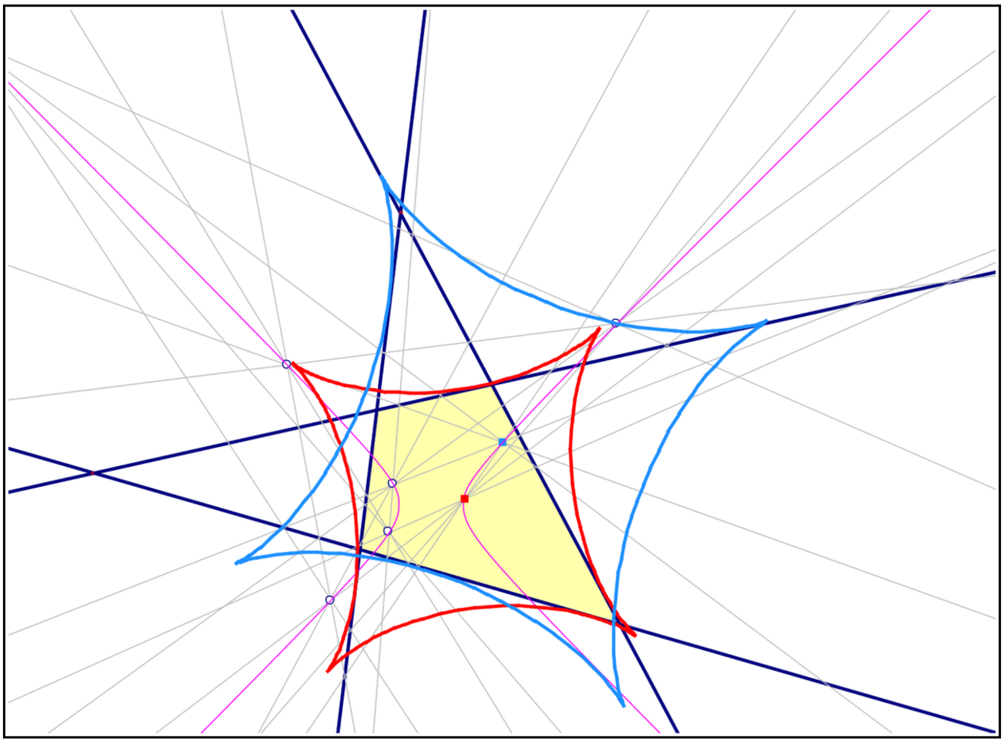
Message: #2284
Date: 03/2/2017 9:55:32
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,
over and over I read your paper attached in #2252,
to get the inscribed astroids of a quadrilateral for p=3 and
q=-1.
Then I found the solution in #1873:
The n-angle centers are
... P(3/2) = inverses of the in-/excenters wrt the circumcircle,
... P(-1/2) = isogonal conjugate of P(3/2).
... The 4 perpendicular bisectors of P(3/2).P(-1/2)
... ... give for the four QL-trilaterals 16 lines
... ... with 8 four-times-intersections,
... ... which are the centers of the inscribed astroids
        of the QL,
... ... which lie on an orthogonal hyperbola.
Attached a construction of the centers of the astroids and the
corresponding hyperbola,
drawn (not constructed) two of the astroids.
Final question: Are these astroids already anywhere mentioned?
I think, your paper is remarkable!
Best regards Eckart
PS: My observation in #2280 is already mentioned in #1873.
```



2017-02-04.pdf

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Message: #2285
Date: 04/2/2017 11:26:09
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,

the center of the orthogonal hyperbola for the centers of the
QL-inscribed astroids
... is the reflection of the 2nd intersection of QL-Ci5 and
QL-Ci6 in QL-P5 (on QL-Ci4 and QL-Ci5).

Best regards Eckart
```

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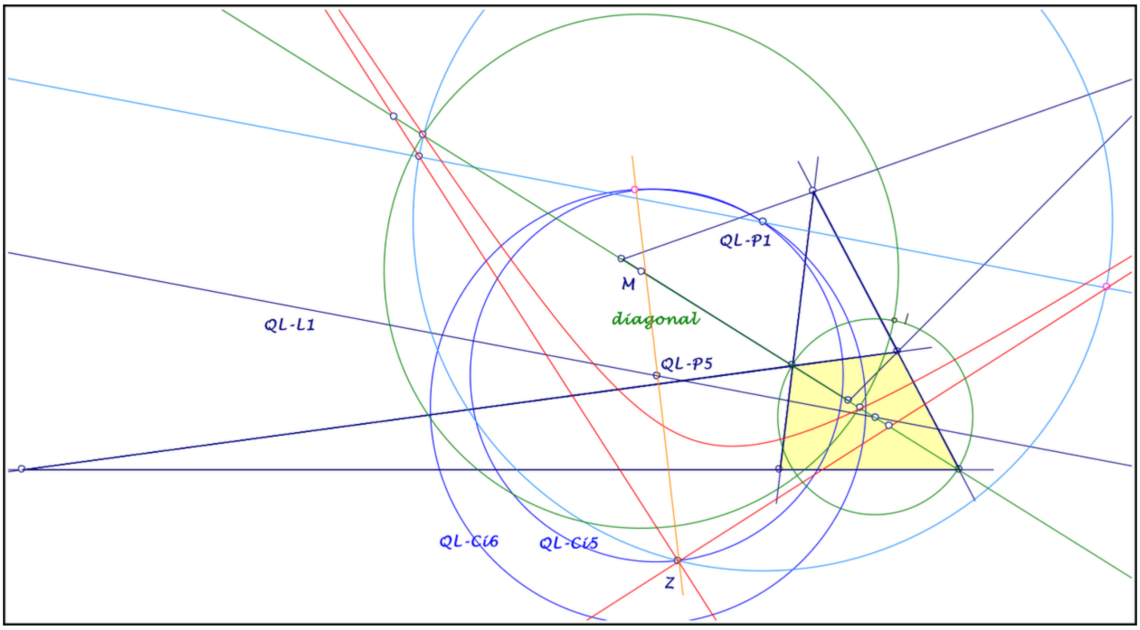
Message: #2286
Date: 04/2/2017 2:11:03
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,
attached a construction of the orthogonal hyperbola for the
centers of the QL-inscribed astroids,
using your properties:
... The center Z is the reflection of the 2nd intersection of
QL-Ci5 and QL-Ci6 in QL-P5.
... A circle round QL-P1 through Z intersects a QL-L1-parallel
through QL-P1 on the asymptotes.
... Let a diagonal intersect the asymptotes in two points with
midpoint M,
... ... then a circle round M orthogonal to the circle with the
diagonal as diameter
... ... intersects the diagonal in points of the hyperbola.
Best regards Eckart
```



2017-02-04a.pdf

Message: #2287
Date: 05/2/2017 4:18:55
From: tsihonglau
Subject: A property of a special quadrangle (P1P2 + P3P4 = P2P3 +P1P4)

```
Dear all,

Some properties of ellipses and hyperbolas with focal triangle
are interesting.  Given triangle ABC, hA,hB,hC
are hyperbolas with foci BC,CA,AB and through A,B,C
respectively. eA,eB,eC are ellipses with foci BC,CA,AB and
through A,B,C respectively.

The concurrences are:
hA,hB,hC - X(175),X(176)
hA,eB,eC - X(175)a,X(176)a
eA,hB,eC - X(175)b,X(176)b
eA,eB,hC - X(175)c,X(176)c

The Soddy line are the line through X(175) and X(176).
Similarly we get exlines of the Soddy line.
The four exlines concur at X(20).
 hB and hC concur at two other points (real or imaginary) than
X(175),X(176).
l_A is the line through the two points.
We get l_B,l_C similarly.
l_A,l_B,l_C concur at X(8).
We get the expoints of X(8) similarly.

Best regards,
Tsihong Lau
```

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Message: #2288
Date: 05/2/2017 11:01:54
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Bernard,

Finally I found time to read your analyses.
It is a beautiful synopsis of many items we discussed before.
It seems they all fit together.
Morley mentions Cardioids C 3 in a 4-line and generalizes by developing his EnnaCardioids C n-1 in n-Lines (n>4). You stick to the subject and generalize the Cardioids(p=2,q=1) by developing epi-/hypo-cycloids(p,q) in 4-Lines.
You define epi-/hypo-cycloids as an envelope of lines. Usually they are defined by tracing a chosen point on the edge of a circle of radius r2 rolling on the outside of a circle of radius r1? Why did you chose this definition? It is not quite clear to me how your defining circles are situated. Are they tangent also rolling on each other?
How do (p,q) relate to (r1, r2)?
I also see connections with the Nephroid we discussed before in November 2014.
What are the (p,q) for the Nephroid?
Further questions:
Is it possible to construct some epi-/hypo-cycloid in a 5-Line?
Is it possible to construct epi-/hypo-cycloid(s) circumscribing a Quadrangle?

Best regards,
Chris

p.s. Eckart, congratulions for you and your wife being married for 50 years !

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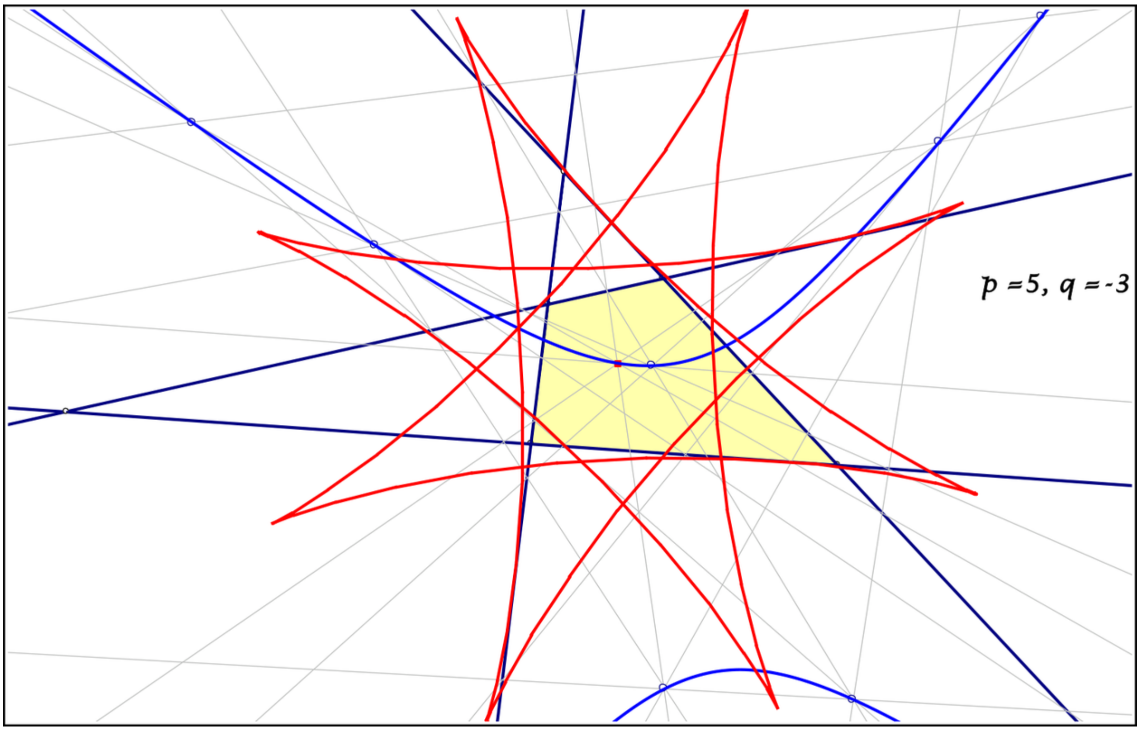
Message: #2289
Date: 05/2/2017 11:42:21
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard, dear Chris,
attached a drawing for a hypocycloid wrt p = 5 and q = -3,
constructing their 8 centers, also on an orthogonal hyperbola.
Best regards Eckart

PS: Thanks for your congratulations.
```



2017-02-05.pdf

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Message: #2290
Date: 05/2/2017 1:25:34
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Eckart,

Beautiful!
Now a question :
Is this rectangular hyperbola also a polar conic of the cubic stelloïd ?
(In other words, does it cut the diagonals as well as the axis of the parabola or any line XX' with X and X' Cl-S conjugates on QL-Cu1 harmonically ?)
The polar conic wrt a cubic is the conic through the 6 contact points of the tangents from the point to the cubic, it has center the conjugate of the point in the transformation which leaves the hessian invariant.
For QL-Cu2, the hessian is QL-Cu1, the polar conic of a pointare is a rectangular hyperbola with center the Cl-S conjugate of the point. (The 1rst rectangular hyperbola is the polar conic of the Cl-S conjugate of the point Z)
For p+q = 1, each curve is reduced to a point, as we saw already.
For p+q = 2, each curve should be a rectangular hyperbola ...
Next step is 7 and -5
May be there are other properties concerning the centers of these rectangular hyperbolas and their Cl-S ?
Then come, for p+q = 3, the 1rst step is for cardioïds with p=2 and q=1, the 2nd is for the hypocycloïd with 5 cusps with p=4 and q=-1

Best regards
Bernard

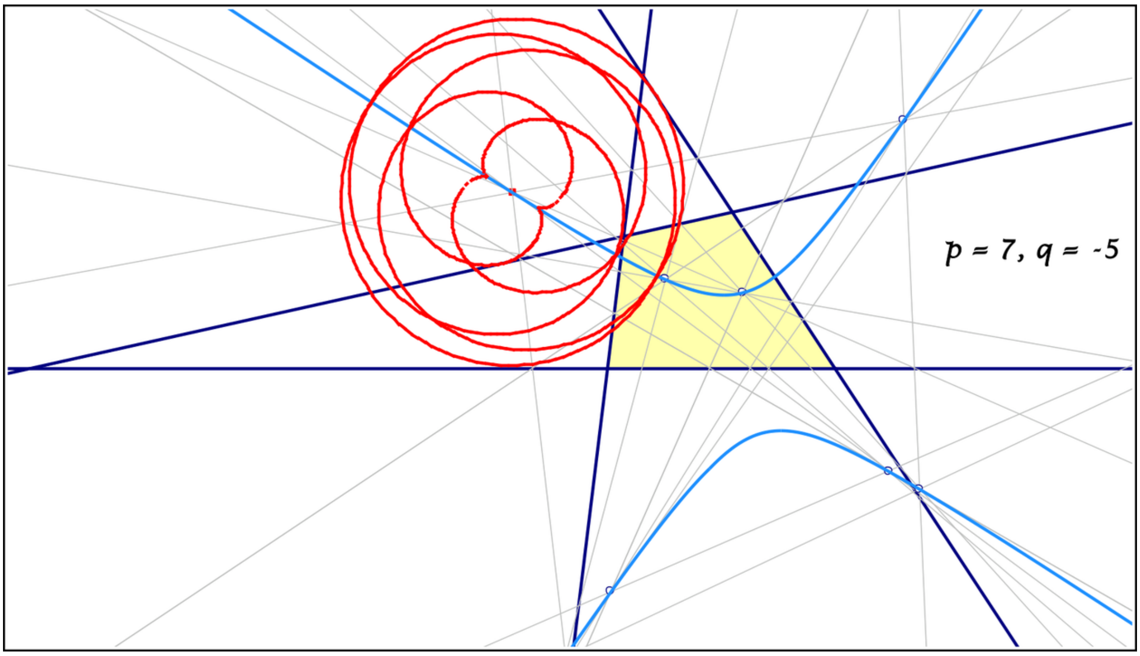
PS We certainly lack of a good comprehension of the stelloïds or Hyperbeln of nter Ordnung
PPS Wrt your remark in previous messages, we deal always with chains of isogonal and inverse and precisely QA-P4 (A,B,C,X) is the inverse wrt the circumcircle of ABC of the isogonal of X wrt the triangle ABC ...

Message: #2291
Date: 05/2/2017 4:46:19
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Bernard, dear Chris,

Cabri- observations for (p=3,q=-1), (p=5,q=-3), (p=7,q=-5) - the last attached - verify, that the 8 centers of the inscribed hypocycloids lie on an orthogonal hyperbola, which is a polar conic of the cubic stelloïd with the properties ...
... it cut the diagonals as well as the axis of the parabola or any line XX' with X and X' Cl-S conjugates on QL-Cu1 harmonically.

Best regards Eckart



2017-02-05a.pdf

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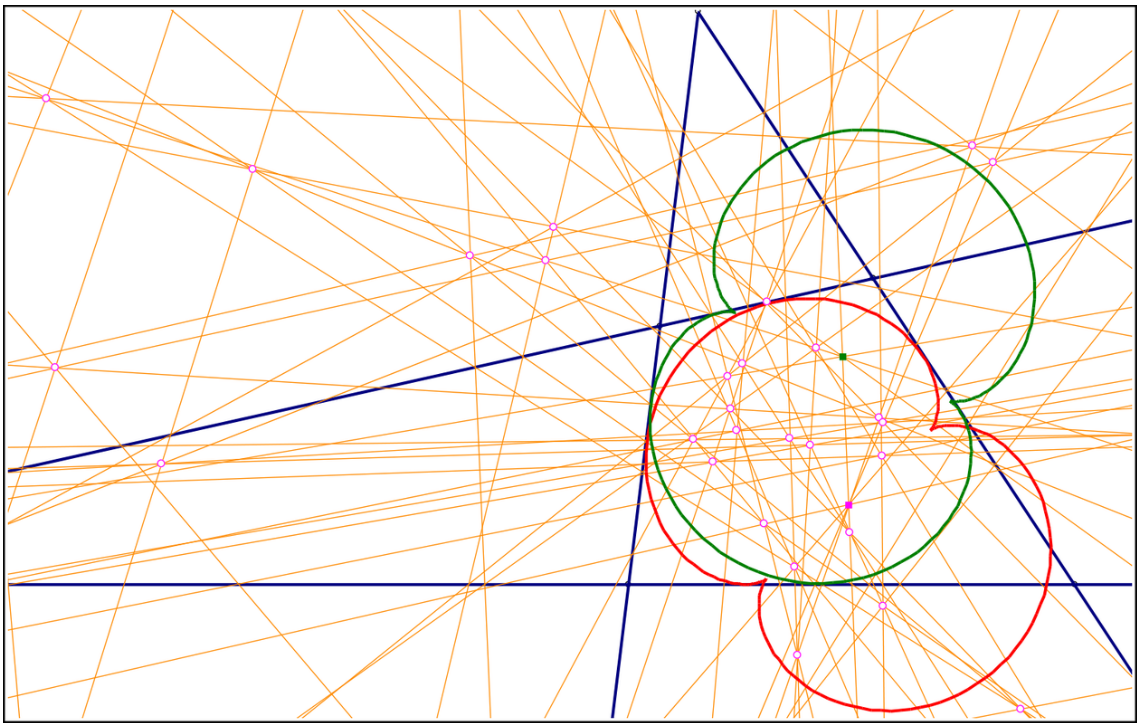
Message: #2292
Date: 05/2/2017 9:36:42
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard, dear Chris,
attached the drawing of two of the 64 inscribed nephroids of a
QL, whose centers can be constructed for p = 3 and q = 1.
Best regards Eckart
```



2017-02-05b.pdf

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Message: #2293
Date: 06/2/2017 10:08:46
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Eckart,
Wunderbar !
If there is a curve through the 64 centers (quartic stelloïd), it's not simply visible !
As the properties of the curves depend only from the 4 lines, I suppose that the rectangular hyperbolas are all polar conics of all cubic stelloïds and the same way the cubic stelloïds are all polar curves for all quartic stelloïds ...
The last step is the polar line of the same point (Cl-S conjugate of the center of the hyperbola) wrt the rectangular hyperbola.
As you have (I suppose) 3 examples of rectangular hyperbolas with the same QL, you have the 3 centers and their Cl-S conjugates ; what can be said about the 3 polar lines or about the circle through the 3 centers ...
I suppose the polar line cuts the hessian QL-Cu1 in 2 Cl-S conjugate points and the cubic stelloïd in 3 points having a particular property wrt the 2 Cl-S conjugate points.
But again, I lack of a good knowledge of the properties of the stelloïds ...
Best regards
Bernard

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Message: #2294
Date: 06/2/2017 10:50:31
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris, dear Eckart
Thanks for your interest
There are many questions in Chris message, so it took me time to answer. In fact, it is like a puzzle where at the end miracleously every peace finds it's place !
Morley generalised the cardioïd envelopping the 4 circles and invented the ennacardioïd envelopping the 5 cardioïds and then the 256 ennacardioïds tangent to the 5 lines ...
The ennacardioïd looks like a smashed nephroïd and looks rather ugly.
The epi- or hypocycloïds are beautiful curves with many axes of symmetry.
There are several definitions of these curves, one is in fact with one circle rolling on another inside or outside with 2 possibilities each time (socalled dobble generation of Cremona) ; see for example the site Mathcurves.
Another is the tangential definition I used ; I took precisely this one because the 4 lines are 4 tangents !
The 4 lines define all the E or H and there is generally noone of these curves tangent to 5 lines.
The explanation is simple : the angle between 2 tangents can take only a finite number of values ; so we have to inscribe triangles of primary or secundary points in a reference triangle of the QL or quadrangles of primary or secundary points in a QL of 4 lines (again, each line is a tangent).
It is known since Lalesco that the locus of the circumcenters of similar triangles of a given shape inscribed in a triangle is a line, perpendicular bisector of 2 isogonal conjugate points ...
And there are only a finite number of shapes for the QA's inscribed in the 4 lines.
Last remark : every curve has a degree (equation in cartesian, barycentrics or comples coordinates) and a class (number of tangents drawn from a point).
For example, the cardioïd and the deltoïd are both quartics but the cardioïd is of class 3, the deltoïd of class 1.
For a point definition counts the degree and there are infinities of E or H trough 4 points.
For a tangential definition counts the class and p+q is also the class of the E or H, hence the finite number of similar elements for each type.
Best regards Bernard

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Message: #2295
Date: 07/2/2017 4:44:23
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,
I have constructed 6 orthogonal hyperbolas for p + q = 2, but
found no properties for their centers.
These orthogonal hyperbolas are determined only by their
centers, the construction is described in # 2286.
Best regards Eckart
```

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Message: #2296
Date: 08/2/2017 11:48:09
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Eckart,
Thanks a lot for your efforts !
This time, I don't know what to say or what to do (apart of course of studying the properties of the stelloïds).
You have a certain number of Kantor-Hervey points (p+q=1), 6 rectangular hyperbolas (p+q=2), which intersect harmonically the lines XX' (Cl-S conjugates on QL-Cu1) and are therefore polar conics of the Cl-S of their centers wrt the cubic stelloïd QL-Cu2 (p+q=3).
I wonder what the polar lines of the Cl-S of the Kantor-Hervey points and of the Cl-S of the centers of the RH wrt the RH would give ...
I've just found that the polar lines of the Cl-S of the 1rst Kantor-Hervey point and of the center of the 1rst RH wrt this RH have directions symmetric wrt the Steiner axes, but I haven't the slightest idea what it could mean !
In the same way, are the next curves for p+q=3 also cubic stelloïds ? In this case, are the RH polar conics of all the cubic stelloïds ?
As you've drown an example for p+q=4, do you know a way of drawing a quartic through the 64 points ? In this case, is it a quartic stelloïd ?
As see, I have far more questions as ways of answering them ...
Best regards
Bernard

PS I've found on the web old articles of W. Walton on rhizic curves, of G. Fouret on the stelloïds and of E. Kasner on the algebraic potential curves, but unfortunately it doesn't help !
Do you know by any chance what is understood under apolar group of points wrt a copple of points on a line ?

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Message: #2297
Date: 09/2/2017 8:41:42
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Bernard,

I still got some questions about p and q from your first message.
You say “the ratio n being p/(p+q) and q/(p+q)”.
You are mentioning the example of p=3 and q=-1, then the ratio n between angles in a triangle will be 3/2 and -1/2 or 3:-1 and we will have (p+q)^2=4 points.
I thought that when p=3 and q=-1, that we actually have an n-Angle Center P(3/(-1)), which is a
Hofstadter point H(-3) and so we have only 1 point.
Where did I go wrong?

Best regards,
Chris

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Message: #2298
Date: 09/2/2017 10:27:34
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris,

If p=3 and q=-1, p+q=2 and the points are 3/2 and -1/2 4 points for each
If p=3 and q=-2 p+q=1 and the points are H(3) and H(-2) 1point for each
If p=4 and q=-3 p+q=1 and the points are H(4) and H(-3) 1point for each

Best regards
Bernard

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Message: #2299
Date: 10/2/2017 10:38:58
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Eckart,
Alternatively drawing and thinking, I'm now convinced that the
next curves for p+q=3 cannot be cubic stelloïds.
The Cl-S transformation given by the QL defines QL-Cu1 and
QL-Cu2 and these curves are unique !
I'm not even sure that the curves are cubics ...
And of course, I'm almost convinced that the curve you found for
p+q=4 is not a quartic stelloïd, perhaps not even a quartic ...
The remaining property is that the curves found for p+q=2 are
all rectangular hyperbolas and polar conics of the cubic
stelloïd, which after all is not so bad !
We could even accept for p=q=1 the union of 4 circles (one in-
or excircle in each triangle) as a particular cycloïd ; there
are 8 different curves like this. In this case, we could accept
as locus of the centers the 6 pairs of bisectors in each
vertice, which are degenerated rectangular hyperbolas and the
polar conics of the 6 opposite vertices wrt the cubic stelloïd
(the 6 vertices are Cl-S conjugate points of the Van Rees curve,
which is the hessian of the cubic stelloïd).
What do you think of all this ?
Best regards
Bernard
```

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Message: #2300
Date: 10/2/2017 1:41:56
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Bernard and Eckart,

Could you tell me which method(s) you use for constructing the
epi-/hypo-cyloids in a given Quadrilateral and having
constructed the centers?
Best regards,

Chris

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Message: #2301
Date: 10/2/2017 4:55:11
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris,
I try, to describe my construction of the centers for p = 3 and
q = -1,
based on Bernards paper in #2252,
using the four n-angle centers for P(3/2) and for P(-1/2)
(pairwise isogonal conjugate):
The four bisectors of P(3/2) and its isogonal conjugate P(-1/2)
give for the four triangle components of a QL 16 lines,
4-times-intersecting in 6 points, which are the centers of the
astroids.
Also my construction of the astroid is based on Bernard´s paper:
Take a circle Ci round P through Q,
... and consider the ray PQ and a variable ray PX (X on Ci),
... further a ray PM (M on Ci) with <>
... and a ray PN (N on Ci) with <>
... The ebvelope of MN, changing X on Ci, gives the astroid.
*The drawn astroids in messages #2284 for the centers above are
"hand-made" fit into place, not constructed!*
Best regards Eckart

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Message: #2302
Date: 12/2/2017 3:58:19
From: tsihonglau
Subject: Orthocentric/Orthoaxial System

Dear all,

We all know the orthocentric system - a quadrangle such that the orthocenter of any expoint triangle (component triangle) is the fourth vertex. We almost do not know the orthoaxial system - a quadrilateral is the dual of a orthocentric system. Given an orthocentric system Q,Qa,Qb,Qc, we know the identifies
QQa^2+QbQc^2 = QQb^2+QcQa^2 = QQc^2+QaQb^2

Given an orthoaxial system q,qA,qB,qC, let iA=q∩qA, iB=q∩qB,
iC=q∩qC, iBC=qB∩qC, iCA=qC∩qA, iAB=qA∩qB
and a,b,c are the three sidelengths of the diagonal triangle.
We get the identities
(iAiBC*(b^2-c^2))^2 = (iBiCA*(c^2-a^2))^2 = (iCiAB*(a^2-b^2))^2

Are there simpler identities?
Given three lines qA,qB,qC, how do we construct a fourth line q such that q,aA,qB,qC
is an orthoaixal system.

Best regards,
Tsihong Lau

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Message: #2303
Date: 12/2/2017 10:35:15
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

Dear Chris, dear Eckart
We always need a center P, a circle and a point Q on this circle.
I only draw the hypocycloïds H2n+1 with p=q+1.
Only one point P
The points pi and qi on the 4 lines are determined using Eckart's method (circles through each vertice and 2 of the points of the same rang in the chain).
This gives 8 points on a circle with center P.
Then we have to find a point Q which divides the arcs of circle piqi in the same ratio p/p+1. (Of course, there are 2n+1 points for the hypocycloïd with 2n+1 cusps)
For the deltoïd, I used a method of trisection of the angle, explained long time ago.
For the others, I measured carefully the angles in order to determine one point Q.
Then the method is explained by Eckart ...
I suppose Eckart did the same for the astroïds (in this case, the quadrisection of the angles is easier !)
Best regards
Bernard
PS Dear Eckart, any reaction to my last messages ?
PPS For Chris : when the 4 lines are tangent to a circle, the point P is the center of the circle, the circle through Q is the circle itself , Q is any point of the circle and the epi- or hypocycloïd is degenerated in the circle ...

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Message: #2304
Date: 12/2/2017 2:33:05
From: eckart_schmidt@t-online.de
Subject: Orthocentric/Orthoaxial System

```
Dear Tsihong Lau,

wrt your question:
"Given three lines qA,qB,qC,
how do we construct a fourth line q such that q,aA,qB,qC is an
orthoaixal system."

Wrt the given trilateral qAqBqC the line q is the tripolar of a
special point P,
... which is the incenter of its anticevian triangle.
Calculations show, that this point isn´t an ETC-point of qAqBqC.

Best regards Eckart
```

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Message: #2305
Date: 12/2/2017 2:34:41
From: eckart_schmidt@t-online.de
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Bernard,
excuse, if I do not participate better in your reflections wrt
this topic, for I am not familar with the background.
My hobby is, to get geometric properties with constructions,
followed by - if possible - confirming calculations.
But the background or history has often to be lightened by
another.
Best regards Eckart
```

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Message: #2306
Date: 12/2/2017 2:59:45
From: tsihonglau
Subject: Orthocentric/Orthoaxial System

Dear Eckart,

Yes! I tried X(1) up to X(3053) - upper limit of TriangleCenter
function of GeoGebra but failed.
Please refer to APG topic #2179 message #3134 for more
information. This question is fundamental.
The isoconjugation with respect to an incenter/excenters
quadrangle is the isogonal conjugacy of points. The
isoconjugation with respect to an antiorthic axis/excentral
trilateral quadrilateral is the isogonal conjugacy
of lines. Please refer to topic #2104.

Best regards,
Tsihong Lau

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Message: #2307
Date: 13/2/2017 3:52:42
From: chris.vantienhoven
Subject: Orthocentric/Orthoaxial System

Dear Eckart,
[ES] Wrt the given trilateral qAqBqC the line q is the tripolar
of a special point P,
... which is the incenter of its anticevian triangle.
Calculations show, that this point isn´t an ETC-point of qAqBqC.
Can you give some background to this point, which is the
incenter of its Anticevian triangle?
When I try to draw such a point in Cabri I can't make a random
point P coincide with the Incenter of its Anticevian triangle.
So I wonder if there is a real occurrence of this point. Neither
do I succeed in calculating its coordinates.
Further I wonder why the tripolar of this point has selfcentric
properties.
Best regards,
Chris

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Message: #2308
Date: 14/2/2017 12:36:19
From: seiichikiri
Subject: Orthocentric/Orthoaxial System

```
Dear Tsihong,

I have the following questions about your message and Geogebra.
1) What is APG topic #2197 message #3134?
2) Is there any command (or commands) of Geogebra to run from
X(1) to X(3053) automatically in a special picture?
3) If Geogebra hits what you want, does it stop computation?
4) How many minutes did it take for you to run from x(1) to
X(3053) in your problem?

Best regards, Seiichi.
```

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Message: #2309
Date: 14/2/2017 11:15:54
From: eckart_schmidt@t-online.de
Subject: Orthocentric/Orthoaxial System

Dear Chris, dear Tsihong Lau,

you are right Chris, there is something wrong in my message
#2304, it should be:
Wrt the given trilateral qAqBqC the line q is the tripolar of
special points P,
... which are in- or excenter of its anticevian triangle.

Cabri-observations give two solutions
(e.g. a = 3, b = 4, c = 5)
... or four solutions
(e.g. a = 5, b = 5, c = 9),
... confirmed by calculation.

For a = 6 , b = 9, c = 13 are these points:
{1.0125056071396716,1.9820004988400033,1.},
{-0.35341703247925393,-0.5608435170848592,1.}
which are not in ETC.

I calculated these points as follows:
Asking with MATHEMATICA, which point is its isogonal conjugate
wrt its anticevian triangle,
you get the following equations for the searched points {u,v,1}:
c^2 u^2 v - 2 SA u^2 +2 SC u - a^2 v = 0
c^2 u v^2 - 2 SB v^2 -b^2 u + 2 SC v = 0,
with two or four real solutions (horrible coordinates!).

Best regards Eckart

Message: #2310
Date: 14/2/2017 1:29:47
From: tsihonglau
Subject: Orthocentric/Orthoaxial System

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>1) What is APG topic #2197 message #3134?

The most notable orthocentric system is the incenter/excentral
triangle quadrangle.  So its dual quadrilateral antiorthic
axis/incentral trilateral is an axis system.  The following are
coordinates/coefficients of the quadrangle/quadrilateral.

barycentrics
Q a:b:c incenter - q x/a+y/b+z/c=0 antiorthic axis
Q_A -a:b:c excentral triangle - q_a -x/a+y/b+z/c=0 incentral
trilateral
Q_B a:-b:c excentral triangle - q_b x/a-y/b+z/c=0 incentral
trilateral
Q_C a:b:-c excentral triangle - q_c x/a+y/b-z/c=0 incentral
trilateral

trilinears
Q 1:1:1 incenter - q x+y+z=0 antiorthic axis
Q_A -1:1:1 excentral triangle -q_a -x+y+z=0 incentral trilateral
Q_B 1:-1:1 excentral triangle - q_b x-y+z=0 incentral trilateral
Q_C 1:1:-1 excentral triangle - q_c x+y-z=0 incentral trilateral

The problem is:
Given any three of the four lines q,q_a,q_b,q_c, how to
construct the fourth not knowing the diagonal triangle ABC?

>>2) Is there any command (or commands) of Geogebra
     to run from X(1) to X(3053) automatically in a special
     picture?
>>3) If Geogebra hits what you want, does it stop computation?
>>4) How many minutes did it take for you to run
     from x(1) to X(3053) in your problem?

I just press my keyboard more than 3000 times!
I know it could be done automatically using GeoGebra!
I was too lazy to dig out GeoGebra usage.!

Best regards,
Tsihong Lau
```

Message: #2311
Date: 14/2/2017 1:35:28
From: tsihonglau
Subject: Orthocentric/Orthoaxial System

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>I calculated these points as follows:
>>Asking with MATHEMATICA, which point is its isogonal conjugate
wrt its anticevian triangle,
>>you get the following equations for the searched points
{u,v,1}:
>>c^2 u^2 v - 2 SA u^2 +2 SC u - a^2 v = 0
>>c^2 u v^2 - 2 SB v^2 -b^2 u + 2 SC v = 0,
>>with two or four real solutions (horrible coordinates!).

Dear Eckart,

The orthoaxial system
is the dual of an orthocentric system.
Could the orthoaxial system be complex not real?
The fourth point of an orthocentric system is the orthocenter of
the triangle formed with the other three points.
The fourth line of an orthoaxial system is what axis of the
trilateral formed with the other three lines.
Could you give the coefficients of the fourth line in a,b,c -
the three sidelengths of the trilateral formed with the other
three lines.
Could you confirm that the fourth line is a system axis like an
orthocenter.

Best regards,
Tsihong Lau
```

Message: #2312
Date: 14/2/2017 8:23:08
From: bernard.keizer
Subject: Orthocentric/Orthoaxial System

```
Dear Seiichi,
You may use in Geogebra the command (to enter below)
trianglecenter [A,B,C,n], where n is the ETC number.
(See help or manual ...)
As I use it in french, it is trianglecentre, Aide or Manuel
Best regards
Bernard
```

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Message: #2313
Date: 15/2/2017 1:47:20
From: seiichikiri
Subject: Orthocentric/Orthoaxial System

```
Dear Tsihong,

Thank you very much for your information.(Your handling of
quadri-figures is very interesting.)
I think that pressing key board more than 3000 times is
absolutely right because it shows your strong will to obtain
result.

Best regards, Seiichi.
```

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Message: #2314
Date: 15/2/2017 3:25:37
From: seiichikiri
Subject: Orthocentric/Orthoaxial System

```
Dear Bernard,

Thank you very much for the information.
According to my experience, Geogebra has more various built-in
functions than Sketchpad or Cabri. One of them is to construct
ellipse or hyperbola with 2 given foci through a given point. I
made some pictures of focus sharing conics. But it seems that
there are very few who have interest in them.

Best regards, Seiichi.
```

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Message: #2315
Date: 15/2/2017 9:12:33
From: eckart_schmidt@t-online.de
Subject: Orthocentric/Orthoaxial System

```
Dear Chris, dear Tsihong Lau,

the triangle points, which are in- or excenter of its anticevian
triangle lie on a cubic with the DT-equation

     x((2SB+c^2)y^2-(2SC+b^2)z^2) + ...

This is a consequence of the equations in #2309.
The cubic is unfortunately no isocubic.
The cubic contains the points X(2) and X(5).
I am shure, you will find more ETC-points.

Best regards Eckart
```

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Message: #2316
Date: 15/2/2017 9:15:51
From: seiichikiri
Subject: An inverse problem of quadrilateral

Dear friends,

Is the following solvable or already solved?
Let (O1), (O2), (O3) and (O4) be 4 circles with their centers
O1, O2, O3 and O4 respectively.
Let O1, O2, O3 and O4 be on a circle and (O1), (O2), (O3) and
(O4) have a common point.

Q12 = external similitude center of O1 and O2.
Q13 = external similitude center of O1 and O3.
Q14 = external similitude center of O1 and O4.
Q23 = external similitude center of O2 and O3.
Q24 = external similitude center of O2 and O4.
Q34 = external similitude center of O3 and O4.

l1 = line through Q23, Q24 and Q34.
l2 = line through Q13, Q14 and Q34.
l3 = line through Q12, Q14 and Q24.
l4 = line through Q23, Q12 and Q13.

Inversely, Given 11, l2, l3 and l4, are there 4 circles (O1),
(O2), (O3) and (O4) such that O1, O2, O3 and O4 are on a circle
and (O1), (O2), (O3) and (O4) have a common point?

Best regards, Seiichi.

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Message: #2317
Date: 15/2/2017 9:23:07
From: Bernard Gibert
Subject: Orthocentric/Orthoaxial System

```
Dear Eckart,

>> the triangle points, which are in- or excenter of its
anticevian triangle
>> lie on a cubic with the DT-equation
>> x((2SB+c^2)y^2-(2SC+b^2)z^2) + ...

This is K124. See the related Table 23:
http://bernard.gibert.pagesperso-orange.fr/Tables/table23.html

Best regards
Bernard
```

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Message: #2318
Date: 15/2/2017 10:20:59
From: eckart_schmidt@t-online.de
Subject: Orthocentric/Orthoaxial System

```
Dear Bernard Gibert,

thanks for the identification of the cubic.
Now the searched points, which are in- or excenter of its
anticevian triangle, can be studied as "Ix-anticevian points" in
your papers.

Best regards Eckart
```

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Message: #2319
Date: 15/2/2017 12:32:24
From: seiichikiri
Subject: Sweeping ETC points from X(1) to X(3053)

```
Dear friends,

This message is the continuation of message # 2311 and 2313.
With a- slider with {min:1, max: 3053, increase:1 } and
TriangleCenter[A, B, C, a], we can sweep ETC points from X(1) to
X(3053).

Best regards, Seiichi.
```

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Message: #2320
Date: 15/2/2017 12:41:25
From: tsihonglau
Subject: Sweeping ETC points from X(1) to X(3053)

```
> >>---In Quadri-Figures-Group@yahoogroups.com, wrote :
> >>Dear friends,
> >>This message is the continuation of message # 2311 and 2313.
> >>With a- slider with {min:1, max: 3053, increase:1 } and
> TriangleCenter[A, B, C, a], we can sweep ETC points
> from X(1) to X(3053).

Dear Seiichi,

Yes! I always slide with pressing keyboard.
But we can do it automatically with script!
I have not digged it out!

Best regards,
Tsihong Lau
```

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Message: #2321
Date: 15/2/2017 1:00:13
From: tsihonglau
Subject: Orthocentric/Orthoaxial System

Dear all,

Here are what I understand. Are they wrong?
Given three lines q_A,q_B_C, the fourth line q of an orthoaxial system(dual of an orthocentric system) is the tripolar of Ix-anticevian points of triangle/trilateral q_A,q_B,q_C. Four of all these points are real, or two of them are real and two other are imaginary. Three questions are raised:
1. What are the dual of the four Ix-anticevian points (either real or imaginary)?
2. What are the isoconjugations with respect to Ix-anticevian points and their dual?
3. What are the DT's of the four Ix-anticevian points and their dual?

Best regards,
Tsihong Lau

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Message: #2322
Date: 15/2/2017 8:55:24
From: eckart_schmidt@t-online.de
Subject: An inverse problem of quadrilateral

Dear Seiichi,

perhaps a puzzle property for your inverse problem:
The common point lies on the cubic QL-Cu1,
... also its inverse wrt the Oi-circle.

Best regards Eckart

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Message: #2323
Date: 16/2/2017 1:33:27
From: seiichikiri
Subject: An inverse problem of quadrilateral

```
Dear Eckart,

Thank you very much for your response. The properties are very
interesting!
Is there any point in EQF which corresponds to the common point
of (O1), (O2), (O3) and (O4)?

Best regards, Seiichi.
```

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Message: #2324
Date: 16/2/2017 1:37:39
From: seiichikiri
Subject: Sweeping ETC points from X(1) to X(3053)

```
Dear Tsihong,

Thank you very much for your information. I will post about it
to GeoGebra User Forum for the best method.

Best regards, Seiichi.
```

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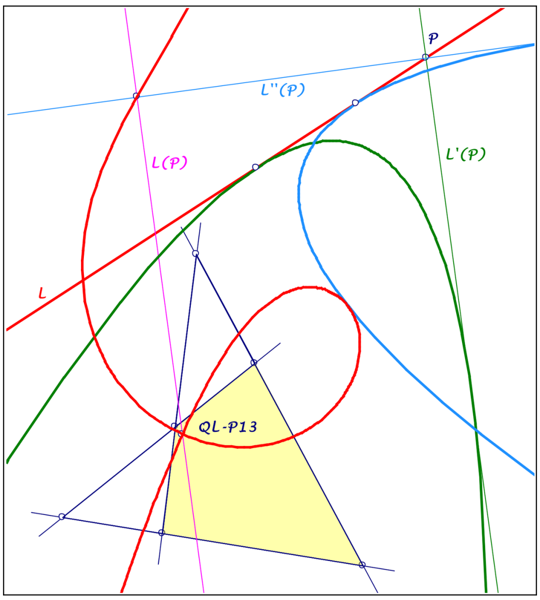
Message: #2325
Date: 16/2/2017 9:16:11
From: eckart_schmidt@t-online.de
Subject: "Orthic" points

```
Dear Bernard, dear Chris,

is the following idea already researched?
Lines can be orthogonal - an analogon for points doesn´t exist.
Duality gives a possibility:
Let "orthic" points have orthogonal dual lines.
... Then the "orthic" points of a point P lie on a line L(P)
through QL-P13 = QA-P16
... and the orthic point of P on a line L is L ^ L(P).
Geometry (see attached file):
Let L´(P) be a parallel to L(P) through P,
... then the transformation P --> L´(P) maps points
    of a line L to lines,
    ... which envelop a parabola tangent to L.
Let L´´(P) be a perpendicular line to L(P) through P,
... then the transformation P --> L´´(P) maps points
    of a line L to lines,
    ... which envelop another parabola tangent to L.
The transformation P --> L(P) ^ L´´(P)
... maps a line L to a strophoid.

Best regards Eckart
PS: Further properties of these loci here are placed back.
Analog the parallelism can be treated.
```



2017-02-14.pdf

Message: #2326
Date: 16/2/2017 11:02:11
From: bernard.keizer
Subject: An inverse problem of quadrilateral

```
Dear Seiichi,
I haven't found yet an explanation, but your little problem is
very interesting !
The figur is full of inversions ...
For example, as the Qij are the external homothety centers of
the copples of circles, they are also inversion centers.
Considering 3 circles, the 3 points which swap 2 of them are
necessary aligned ...
Best regards
Bernard
```

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Message: #2327
Date: 17/2/2017 9:02:56
From: eckart_schmidt@t-online.de
Subject: An inverse problem of quadrilateral

```
Dear Seiichi,

If there is a solution of your inverse problem,
... let O be the center of the circle Ci for the Oi, P the
common point and Q its inverse wrt Ci.

Some remarks:
Cabri observations show
(1) ... that P and Q lie on QL-Cu1 of the reference QL (#2322).
(2) ... that there are further solutions:
(3) ... replacing P by Q, you get also the reference QL,
(4) ... replacing Ci by another circle Ci´,
        so that P and Q are inverse
... ... and replacing Oi, Oj by intersections of OiOj and Ci´,
... ... you get also the reference QL.
If your inverse problem has a solution, it is not unique.

Best regards Eckart
```

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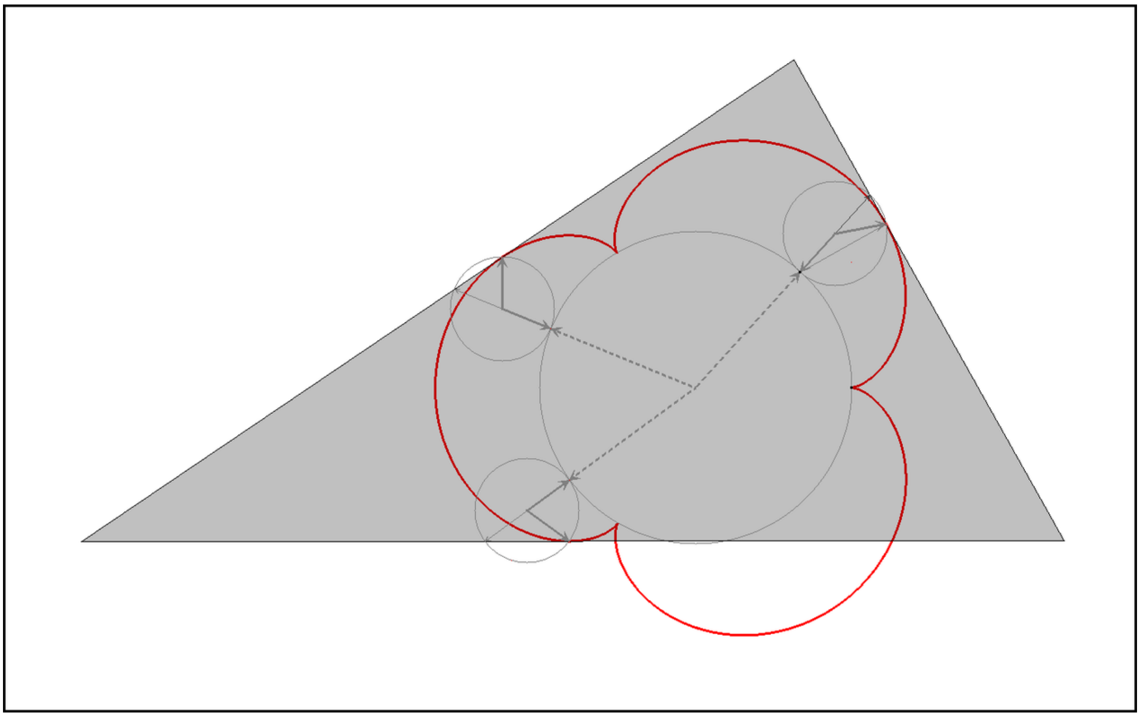
Message: #2328
Date: 17/2/2017 10:53:35
From: chris.vantienhoven
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear friends,

Just a beautiful picture of an epicycloid with 3 cusps inscribed
in a triangle (see attached file).
Bernard and Eckart, I am still busy to find a general method for
constructing epi-/hypocycloids with n cusps inscribed in a
Quadrilateral. And I have good hopes.

Best regards,
Chris
```



QL-iCv99-Epi-Hypo-Cycloids-10-in-Triangle.png

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Message: #2329
Date: 17/2/2017 1:30:40
From: bernard.keizer
Subject: QL, epi and hypocycloïds and n-angle centers

```
Dear Chris,

I would be very happy if you found a general construction !

Best wishes
Bernard
```

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Message: #2330
Date: 17/2/2017 1:33:47
From: bernard.keizer
Subject: An inverse problem of quadrilateral

```
Dear Eckart, dear Seiichi,

This problem is very fascinating !
Finally, there are perhaps an infinity of solutions ...
Eckart, I like when you searched simple synthetic proofs or
explanations or constructions ...

Best regards
Bernard
```

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Message: #2331
Date: 17/2/2017 1:41:30
From: bernard.keizer
Subject: ”Orthic” points

Dear Eckart, dear Chris

Very beautiful properties!
parallel lines intersect the same point on the infinity line and
their dual points are on the dual line of this point.
orthogonal lines intersect in 2 points of the infinity line
harmonic wrt the 2 cyclic points
And this strophoïd falling from the sky!
(Strophoïd is the limit position of QL-Cu1 between mono- and
bicursal when the node is one of the 2 invariant points on the
Steiner axis; in this case, it is also on the Newton Line. But
where is the QL ?)

Best regards
Bernard

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Message: #2332
Date: 17/2/2017 3:39:09
From: tsihonglau
Subject: Cyclocevian Conjugates

```
Dear all,

Two message # 2405 and #2218 in APG.
Cyclocevian Conjugates of Line!?
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/messages/2405  †)
Generalization of Cyclocevian Conjugacy
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/messages/2218  †)
Any ideas?

Best regards,
Tsihong Lau
```

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Message: #2333
Date: 18/2/2017 2:07:24
From: seiichikiri
Subject: An inverse problem of quadrilateral

```
Dear Bernard, dear Eckart,
Thank you very much for the valueable informations.
As for me, I think that the uniqueness is necessary to solve.
I will restate the problem for its uniquness.
Best regards, Seiichi.
```

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #2334
Date: 18/2/2017 4:38:27
From: Antreas Hatzipolakis
Subject: Circles related to Poncelet point

Let ABC be a triangle, AhBhCh the pedal triangle of H and O' the
Poncelet point of ABCO [Po = X125]
(ie O' is the point the NPCs of ABC, OBC, OCA, OAB are
concurrent at).

Denote:
hA = the second intersection of AhO' and the NPC of OBC [other
than O']
hB = the second intersection of BhO' and the NPC of OCA
hC = the second intersection of ChO' and the NPC of OAB
O', hA, hB, hC are concyclic.
Which point is the center of the circle?

GENERALIZATION (Conjecture):
Let ABC be a triangle, D, D* two isogonal conjugate points,
AdBdCd the pedal triangle of D and D' the Poncelet point of
ABCD*
(ie D' is the point the NPCs of ABC, D*BC, D*CA, D*AB are
concurrent at).

Denote:
dA = the second intersection of AdD' and the NPC of D*BC [other
than D']
dB = the second intersection of BdD' and the NPC of D*CA
dC = the second intersection of CdD' and the NPC of D*AB
D', dA, dB, dC are concyclic.
Call this circle (ABC, D). Which is its center in terms (of the
coordinates) of D?

Now, in a quadrangle:
Let ABCD be a quadrangle and A*, B*, C*, D* the isogonal
conjugates of A,B,C,D wrt triangles BCD, CDA, DAB, ABC, resp.
We have two tetrads of circles:
(ABC, D), (BCD, A), (CDA, B), (DAB, C)
(ABC, D*), (BCD, A*), (CDA, B*), (DAB, C*)
Which properties have these circles?
If no one can be found, then at least Cayley-Bacharach theorem
can be applied:
The cubics passing through the eight centers of the circles pass
through a fixed point.

APH

Message: #2335
Date: 18/2/2017 8:25:59
From: Antreas Hatzipolakis
Subject: Circles related to Poncelet point

[APH]:
GENERALIZATION (Conjecture):
Let ABC be a triangle, D, D* two isogonal conjugate points,
AdBdCd the pedal triangle of D and D' the Poncelet point of
ABCD*
(ie D' is the point the NPCs of ABC, D*BC, D*CA, D*AB are
concurrent at).

Denote:
dA = the second intersection of AdD' and the NPC of D*BC [other
than D']
dB = the second intersection of BdD' and the NPC of D*CA
dC = the second intersection of CdD' and the NPC of D*AB
D', dA, dB, dC are concyclic.
Call this circle (ABC, D). Which is its center in terms (of the
coordinates) of D?

****

Let Na, Nb, Nc be the centers of the NPCs of D*BC, D*CA, D*AB,
resp.
The center of the circle in question is the orthocenter of
NaNbNc

APH

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Message: #2336
Date: 18/2/2017 11:49:20
From: eckart_schmidt@t-online.de
Subject: Circles related to Poncelet point

```
Dear Antreas,

CABRI-observations show, that the center of O´, hA, hB, hC is
not an ETC-point
and the last four circles (ABC,D*), ... intersect in QA-P2.

Best regards Eckart
```

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Message: #2337
Date: 18/2/2017 2:24:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

```
Dear Chris,

in an old message (30.03.2013) I found the following property:
    QA-Cu1 is the locus of points P,
    whose CSC-images lie perspective with QA-DT.
    The perspector is QA-Tf2(P).
Perhaps worth, to be mentioned in EQF?

Best regards Eckart
```

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Message: #2338
Date: 18/2/2017 6:39:15
From: amontes1949
Subject: Circles related to Poncelet point

Dear Antreas and Eckart,

The center of the circle passing through O',.b4, hA, hB, hC is
W = 3 X(3) + X(155)

W = (a^2 (a^2-b^2-c^2) (2 a^6+2 a^2 b^2 c^2-3 a^4 (b^2+c^2)+(b^2-c^2)^2 (b^2+c^2)) : ... : ...),
with (6-9-13)-search numbers
(5.02069206557260, 3.50252376074738, -1.10140215195116).

W is the midpoint of X(i) and X(j) for these {i,j}: {3,1147}, {155,7689}, {156,11250}.

W is the reflection of X(i) in X(j) for these {i,j}: {5448,9820}, {5449,140}.

W on lines: {2, 9927}, {3, 49}, {4, 11449}, {5, 1511}, {20, 5654}, {24, 5446}, {26, 11202}, {30, 5448}, {52, 186}, {54, 5504}, {68, 631}, {74, 9705}, {110, 3520}, {140, 5449}, {156, 6000}, {182, 8548}, {378, 10539}, {382, 1495}, {511, 1658}, {539, 549}, {541, 5894}, {550, 5944}, {567, 2931}, {569, 5892}, {578, 5462}, {1069, 5217}, {1152, 8909}, {1614, 2071}, {3043, 11562}, {3157, 5204}, {3523, 6193}, {3524, 11411}, {3530, 3564}, {3576, 9928}, {3855, 10546}, {5010, 6238}, {5646, 7393}, {5657, 9933}, {5663, 10226}, {5890, 9545}, {6146, 10257}, {6200, 10666}, {6241, 9544}, {6396, 10665}, {6418,8912}, {6642, 11425}, {6689, 7399}, {6699, 10116}, {7280, 7352}, {7488, 10625}, {7503, 10170}, {7506, 11424}, {7514, 9938}, {7526, 9306}, {7575, 10263}, {8546, 8681}, {9707, 11413}, {10020, 10182}, {10298, 11412}, {10540, 11381}, {10645, 10662}, {10646, 10661}.

Angel Montesdeoca

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Message: #2339
Date: 18/2/2017 10:08:09
From: Antreas Hatzipolakis
Subject: Circles related to Poncelet point

[APH]
Let ABC be a triangle, AhBhCh the pedal triangle of H and O' the Poncelet point of ABCO [O' = X125]
(ie O' is the point the NPCs of ABC, OBC, OCA, OAB are concurrent at).

Denote:
hA = the second intersection of AhO' and the NPC of OBC [other than O']
hB = the second intersection of BhO' and the NPC of OCA
hC = the second intersection of ChO' and the NPC of OAB
O', hA, hB, hC are concyclic
Which point is the center of the circle?

GENERALIZATION (Conjecture):
Let ABC be a triangle, D, D* two isogonal conjugate points, AdBdCd the pedal triangle of D and D' the Poncelet point of ABCD*
(ie D' is the point the NPCs of ABC, D*BC, D*CA, D*AB are concurrent at).

Denote:
dA = the second intersection of AdD' and the NPC of D*BC [other than D']
dB = the second intersection of BdD' and the NPC of D*CA
dC = the second intersection of CdD' and the NPC of D*AB
D', dA, dB, dC are concyclic.
Call this circle (ABC, D). Which is its center in terms (of the coordinates) of D?
Now, in a quadrangle:
Let ABCD be a quadrangle and A*, B*, C*, D* the isogonal conjugates of A,B,C,D wrt triangles BCD, CDA, DAB, ABC, resp.
We have two tetrads of circles:
(ABC, D), (BCD, A), (CDA, B), (DAB, C)
(ABC, D*), (BCD, A*), (CDA, B*), (DAB, C*)
Which properties have these circles?
If no one can be found, then at least Cayley-Bacharach theorem can be applied:
The cubics passing through the eight centers of the circles pass through a fixed point.

[César Lozada]:
> GENERALIZATION (Conjecture):

> D', dA, dB, dC are concyclic.

Let D=u:v:w (trilinears). Then the center Z(D) of the circle
{D’, dA, dB, dC} is:
Z(D) = (v^2+w^2)*a*b*c*(b*c*u^2 - SA*w*v) +
2*a*(S^2-SA^2)*v^2*w^2 + b*c*u*(SB*b*w^3 + SC*c*v^3) +
u*v*w*((3*S^2+SA*SC)*c*v + (3*S^2+SA*SB)*b*w + 2*a*b*c*SA*u) : :

If D is on the circumcircle, Z(D)=isogonal(D)
If D is in the infinity, Z(D) = Midpoint(0, isogonal(P)), so
Z(D) is on the circle (O, R/2). ETC pair (D, Z(D)): (523, 1511)
Other ETC pairs (D, Z(D)) : (1,1), (3,5), (15,5459), (16,5460),
(36,11)

Some non-ETC :
Z(G) = midpoint of X(6) and X(8542)
= (2*a^6-3*(b^2+c^2)*a^4-2*(b^4+ b^2*c^2+c^4)*a^2+(b^2-3*c^2)*(
3*b^2-c^2)*(b^2+c^2))*a : : (trilinears)
= X(6)+X(8542) = 3*X(182)-X(8547) = X(8547)+9*X(9813)
= on lines: {6,373}, {39,9145}, {182,2393}, {193,7605},
{523,7804}, {524,547}, {575,2854}, {576,10170}, {597,5972},
{1843,2916}, {3618,5486}, {5092,8705}, {5650,10510},
{9730,11579}, {11003,11188}
= midpoint of X(6) and X(8542)
= [2.377903015171476, 0.84411325894432, 1.958784680251474]

Z(H) = midpoint of X(3) and X(1147)
= (2*a^6-3*(b^2+c^2)*a^4+2*b^2* c^2*a^2+(b^4-c^4)*(b^2-c^2))*(
-a^2+b^2+c^2) *a : : (trilinears)
= cos(A)*(1+2*cos(2*A)+cos(2*B)+ cos(2*C)) : : (trilinears)

= 3*X(2)-X(9927) = 3*X(3)+X(155) = X(3)+X(1147) =
5*X(3)+3*X(3167) = 3*X(3)-X(7689) = X(20)+3*X(5654) =
X(155)-3*X(1147) = 5*X(155)-9*X(3167) = X(155)+X(7689) =
5*X(1147)-3*X(3167)

= On lines: {2,9927}, {3,49}, {4,11449}, {5,1511}, {20,5654}, {24,5446}, {26,11202}, {30,5448}, {52,186}, {54,5504}, {68,631}, {74,9705}, {110,3520}, {140,5449}, {156,6000}, {182,8548}, {378,10539}, {382,1495}, {511,1658}, {539,549}, {541,5894}, {550,5944}, {567,2931}, {569,5892}, {575,12006}, {578,5462}, {858,11750}, {1069,5217}, {1152,8909}, {1614,2071}, {3043,11562}, {3157,5204}, {3517,12002}, {3523,6193}, {3524,11411}, {3530,3564}, {3576,9928}, {3855,10546}, {5010,6238}, {5646,7393}, {5657,9933}, {5663,10226}, {5890,9545}, {6146,10257}, {6200,10666}, {6241,9544}, {6396,10665}, {6418,8912}, {6642,11425}, {6689,7399}, {6699,10116}, {7280,7352}, {7488,10625}, {7503,10170}, {7506,11424}, {7514,9938}, {7526,9306}, {7575,10263}, {8546,8681}, {9707,11413}, {10020,10182}, {10298,11412}, {10540,11381}, {10645,10662}, {10646,10661}
= midpoint of X(i) and X(j) for these {i,j}: {3,1147}, {155,7689}, {156,11250}
= reflection of X(i) in X(j) for these (i,j): (5448,9820), (5449,140)
= complement of X(9927)
= {X(i),X(j)}-Harmonic conjugate of X(k) for these (i,j,k): (3,49,185), (3,155,7689), (3,1092,1216), (578,6644,5462), (1147,7689,155), (1614,2071,10575)
= [5.020692065572603, 3.50252376074738, -1.101402151951162]

Z(K) = midpoint of X(2) and X(11165)
= (8*a^4-17*(b^2+c^2)*a^2-2*b^2* c^2+5*c^4+5*b^4)/a : : (trilinears)
= 5*X(2)-X(5485) = 7*X(2)+X(11148) = X(2)+X(11165) = X(5)-2*X(9771) = 2*X(140)-X(7610) = 3*X(549)-2*X(5569) = X(549)-2*X(7622) = 7*X(5485)+5*X(11148) = X(5485)+5*X(11165) = X(5569)-3*X(7622) = X(7618)+X(11184) = X(11148)-7

*X(11165)
= on lines: {2,2418}, {3,9770}, {5,543}, {30,7618}, {39,9167}, {83,5503}, {99,3363}, {140,7610}, {182,524}, {538,7619}, {547,7615}, {550,7775}, {597,620}, {631,9740}, {1007,5077}, {2482,3815}, {2549,8355}, {3845,8176}, {3849,8703}, {5013,8360}, {5055,7620}, {5215,5306}, {7763,8359}, {7769,9166}, {7777,8598}, {7870,8362}, {8182,9766}, {8667,11812}
= midpoint of X(i) and X(j) for these {i,j}: {2,11165}, {3,9770}, {7615,8716}, {7618,11184}, {8182,9766}
= reflection of X(i) in X(j) for these (i,j): (5,9771), (549,7622), (3845,8176), (7610,140), (7615,547)
= [-0.273892582550908, 3.02194816081169, 1.674958485599784]

Z(X(30)) = midpoint of X(3) and X(74)

= (2*a^8-3*(b^2+c^2)*a^6-3*(b^4- 4*b^2*c^2+c^4)*a^4+(b^2+c^2)*(
7*b^4-15*b^2*c^2+7*c^4)*a^2-(3*b^4+7*b^2*c^2+3*c^4)*(b^2-c^
2)^2)*a : : (trilinears)
= (3*cos(2*A)+7/2)*cos(B-C)-6* cos(A)-cos(3*A) : : (trilinears)
= 3*X(2)-X(7728) = X(3)+X(74) = 3*X(3)-X(110) = 5*X(3)-X(399) =
2*X(3)-X(1511) = 4*X(3)-X(5609) = 3*X(3)+X(10620) =
2*X(5)-X(1539) = X(5)-2*X(6699) = 3*X(5)-4*X(6723) =
3*X(74)+X(110) = 5*X(74)+X(399) = X(1539)-4*X(6699) = 3

*X(1539)-8*X(6723) = 3*X(6699)-2*X(6723)
= on lines: {2,7728}, {3,74}, {5,1539}, {20,265}, {30,125},
{35,3028}, {55,10081}, {56,10065}, {64,9934}, {113,140},
{146,631}, {182,2781}, {185,10226}, {376,3448}, {378,1112},
{381,10721}, {511,11806}, {517,11709}, {541,549}, {542,8703},
{550,10264}, {567,1986}, {974,1204}, {1154,2071}, {1350,5621},
{1351,5622}, {1657,10733}, {2420,3269}, {2771,9943},
{2780,9208}, {2854,3098}, {2935,7526}, {3521,6143}, {3524,5655},
{3530,10272}, {3532,5504}, {3534,9140}, {3576,9904},
{3581,7464}, {3627,7687}, {3818,6698}, {5050,10752},
{5054,10706}, {5085,9970}, {5092,6593}, {5204,10091},
{5217,10088}, {5462,11807}, {5544,9818}, {6101,7689},
{6409,10819}, {6410,10820}, {6642,9919}, {6644,10117},
{6689,11805}, {7280,7727}, {7502,8717}, {7583,8994},
{7722,11003}, {7731,10574}, {7978,10246}, {8718,11559},
{9729,11557}, {10610,10628}, {11438,11746}
= midpoint of X(i) and X(j) for these {i,j}: {3,74}, {20,265},
{64,9934}, {110,10620}, {113,10990}, {550,10264}, {1350,11579},
{1657,10733}, {3534,9140}, {3581,7464}, {8718,11559}
= reflection of X(i) in X(j) for these (i,j): (5,6699),
(113,140), (1511,3), (1539,5), (3627,7687), (3818,6698),
(5609,1511), (6102,974), (6593,5092), (10113,125), (10272,3530),
(11557,9729), (11702,10610), (11805,6689), (11807,5462)
= complement of X(7728)
= circumcircle-inverse-of-X(10620)
= [9.719762854243672, 9.29279598626787, -7.278854056698148]

Z(X(511)) = midpoint of X(3) and X(98)
= (2*a^8-3*(b^2+c^2)*a^6+3*(b^4+ c^4)*a^4-(b^2+c^2)*(2*b^4-3*b^
2*c^2+2*c^4)*a^2-(b^2-c^2)^2* b^2*c^2)/a : :)trilinears)
= 3*X(2)-X(6033) = X(3)+X(98) = 3*X(3)-X(99) = X(5)-2*X(6036) =
3*X(5)-4*X(6722) = X(20)+X(6321) = 3*X(98)+X(99) =
X(114)-2*X(140) = X(114)+X(10991) = X(115)-3*X(6055) =
2*X(140)+X(10991) = 3*X(6036)-2*X(6722)

= on lines: {2,5191}, {3,76}, {5,2794}, {20,6321}, {30,115}, {32,2023}, {35,3027}, {36,3023}, {55,10069}, {56,10053}, {114,140}, {141,542}, {147,631}, {148,376}, {157,1605}, {182,10007}, {262,11842}, {378,5186}, {381,3972}, {404,5985}, {517,11710}, {543,8703}, {550,11623}, {632,6721}, {671,3534}, {1657,10723}, {1916,7793}, {2080,5999}, {2784,6684}, {3095,7766}, {3098,5969}, {3111,5663}, {3329,3398}, {3523,5984}, {3524,8289}, {3576,9860}, {3830,9166}, {3845,5461}, {4027,7824}, {5027,11176}, {5050,10753}, {5054,6054}, {5149,7815}, {5182,12017}, {5204,10089}, {5217,10086}, {5569,9830}, {5961,7502}, {5986,7485}, {5987,7496}, {6642,9861}, {6671,6771}, {6672,6774}, {7583,8980}, {7776,8781}, {7798,9737}, {7857,9873}, {7970,10246}, {8667,9888}, {8725,11606}, {9167,11812}, {10352,11285}

= midpoint of X(i) and X(j) for these {i,j}: {3,98}, {20,6321}, {114,10991}, {376,11632}, {671,3534}, {1657,10723}, {1916,9821}, {2080,5999}, {6033,9862}, {6295,6582}, {8667,9888}, {8724,11177}, {8725,11606}

= reflection of X(i) in X(j) for these (i,j): (5,6036), (114,140), (3845,5461), (5026,5092)

= complement of X(6033)

= {X(i),X(j)}-Harmonic conjugate of X(k) for these (i,j,k): (2,9862,6033), (1078,5152,5976), (3524,11177,8724)

= [8.454179614470462, 9.60084291120245, -6.908001970988301]

César Lozada

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Message: #2340
Date: 20/2/2017 10:19:33
From: bernard.keizer
Subject: QA-Cu1

```
Dear Eckart, dear Chris

This property is the direct consequence of the definition of the
isopivotal cubic with pivot QA-P4, id est QA-Cu1.
As P and QA-P2(P) are aligned with the pivot QA-P4, these 3
points form with their CSC conjugates 3 Cu-QA's in a Desmic
configuration on QA-Cu1 (the CSC of QA-P4 are the DT vertices).
If P is the infinity point on QA-P3QA-P4, the CSC of P are the
vertices of the Miquel triangle and QA-Tf2(P)  is QA-P3. If P is
QA-P4, QA-Tf2(QA-P4) is QA-P41, the tangential point of QA-P4
and the DT vertices.

Best regards
Bernard
```

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Message: #2341
Date: 21/2/2017 10:00:41
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Bernard, dear Chris,

thanks to Bernard for his explinations.
There is another property, already mentioned in #2016, but only
in parts
in EQF:
QL-Cu1 is a nonpivotal isogonal circular isocubic wrt the orthic
DT-triangle
... and root (see EQF-Ref.[17b],1.5): trilinear pole
    of the QG-P18-line.

Best regards Eckart
```

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Message: #2342
Date: 21/2/2017 1:38:59
From: tsihonglau
Subject: Conic and Quartic Tangent at Four Points

Dear all,

This topic is inspired by a conic together with the circular points and the four foci.
P and U are the circular points and c is a conic.
p+,p- are the isotropic(=through P) tangent line of c, while u+,u- are the isotropic(through U) tangent line of c.
We get four foci F++=p+∩u+, F--=p-∩u-, F+-=p+∩u-, F-+=p-∩u+.
If c is a real conic, the two pairs of foci F++,F-- and F+-,F-+ are real and imaginary respectively, or vice versa.
In another perspective, the circular points P,U and the four foci F++.F--,F+-,F-+ are the vertices of the quadrilateral - the tangent lines p+,p-,u+,u-.

Now a question is raised. If we replace the above quadrilateral with a general quartic, what about the geometry? In other words, a conic and a quartic (better non-degenerate) are tangent at four points, what about the geometry of this configuration?

Best regards,
Tsihong Lau

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Message: #2343
Date: 21/2/2017 2:59:51
From: bernard.keizer
Subject: QL-Cu1

```
Dear Eckart, dear Chris

Interesting property !
In fact, the QG-P18 are the tangentials of the copples of Cl-S
conjugate vertices of the QL.
The line of these tangentials form with the orthic triangle of
DT another QL inscribed in QL-Cu1.
Therefore, your property is easy to generalise.
Any line cuts QL-Cu1 in 3 points ; the Cl-S conjugates of these
points form a triangle inscribed in a circle through QL-P1, Cl-S
conjugate of the line.
QL-Cu1 is a non-pivotal circular isocubic wrt any triangle
inscribed in a circle through QL-P1 with root the trilinear pole
of the Cl-S conjugate line of the circle wrt the triangle.

Best regards
Bernard
```

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Message: #2344
Date: 22/2/2017 11:43:18
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Bernard,

nice generalization! My interpretation:
QL-Cu1 as nonpivotal circular isocubic
... wrt a line L, intersecting QL-Cu1 in U, V, W:
... reference triangle: CSC of U, V, W,
... isoconjugation: isogonal conjugate,
... root: tripole of L,
... point: QL-P1.

Best regards Eckart

PS: Perhaps this is relevant for a construction of the collinear
inflection points of QL-Cu1, if possible.
```

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Message: #2345
Date: 23/2/2017 1:41:01
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

```
Dear all,

The first barycentric coefficient is
d( 2*a^2*c^4*e^4*f^4-4*a^2*b^2*c^2*e^4*f^4-a^4*c^2*e^4*f^4+2*a^2
*b^4*e^4*f^4-a^4*b^2*e^4*f^4-a^6*e^4*f^4-c^6*d^2*e^2*f^4+3*b^2*c
^4*d^2*e^2*f^4+3*a^2*c^4*d^2*e^2*f^4-3*b^4*c^2*d^2*e^2*f^4-4*a^2
*b^2*c^2*d^2*e^2*f^4-3*a^4*c^2*d^2*e^2*f^4+b^6*d^2*e^2*f^4-3*a^2
*b^4*d^2*e^2*f^4+a^4*b^2*d^2*e^2*f^4+a^6*d^2*e^2*
f^4-b^4*c^2*d^4*f^4-b^6*d^4*f^4+a^2*b^4*d^4*f^4+c^6*d^2*e^4*f^2-
3*b^2*c^4*d^2*e^4*f^2-3*a^2*c^4*d^2*e^4*f^2+3*b^4*c^2*d^2*e^4*f^
2-4*a^2*b^2*c^2*d^2*e^4*f^2+a^4*c^2*d^2*e^4*f^2-b^6*d^2*e^4*f^2+
3*a^2*b^4*d^2*e^4*f^2-3*a^4*b^2*d^2*e^4*f^2+a^6*d^2*e^4*f^2+c^6*
d^4*e^2*f^2+b^2*c^4*d^4*e^2*f^2-3*a^2*c^4*d^4*e^2*f^2
+b^4*c^2*d^4*e^2*f^2-4*a^2*b^2*c^2*d^4*e^2*f^2+3*a^4*c^2*d^4*e^2
*f^2+b^6*d^4*e^2*f^2-3*a^2*b^4*d^4*e^2*f^2+3*a^4*b^2*d^4*e^2*f^2
-a^6*d^4*e^2*f^2-c^6*d^4*e^4-b^2*c^4*d^4*e^4+a^2*c^4*d^4*e^4)

I use d,e,f instead of p,q,r.

Best regards,
Tsihong Lau
```

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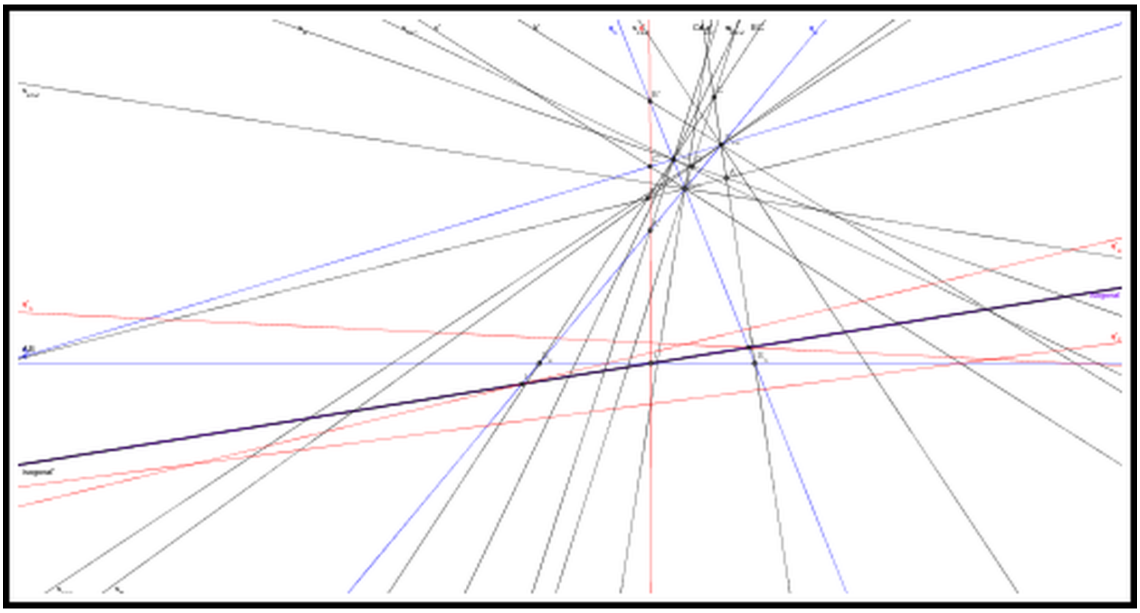
Message: #2346
Date: 24/2/2017 1:39:40
From: tsihonglau
Subject: Isogonal and Isotomic Conjugacies of Lines

```
Dear all,

Oops! The correct first barycentric coefficient of the isogonal
axis is:
2*a^2*c^4*e^4*f^4-4*a^2*b^2*c^2*e^4*f^4-a^4*c^2*e^4*f^4+2*a^2*b^
4*e^4*f^4-a^4*b^2*e^4*f^4-a^6*e^4*f^4-c^6*d^2*e^2*f^4+3*b^2*c^4*
d^2*e^2*f^4+3*a^2*c^4*d^2*e^2*f^4-3*b^4*c^2*d^2*e^2*f^4-4*a^2*b^
2*c^2*d^2*e^2*f^4-3*a^4*c^2*d^2*e^2*f^4+b^6*d^2*e^2*f^4-3*a^2*b^
4*d^2*e^2*f^4+a^4*b^2*d^2*e^2*f^4+a^6*d^2*e^2*
f^4-b^4*c^2*d^4*f^4-b^6*d^4*f^4+a^2*b^4*d^4*f^4+c^6*d^2*e^4*f^2-
3*b^2*c^4*d^2*e^4*f^2-3*a^2*c^4*d^2*e^4*f^2+3*b^4*c^2*d^2*e^4*f^
2-4*a^2*b^2*c^2*d^2*e^4*f^2+a^4*c^2*d^2*e^4*f^2-b^6*d^2*e^4*f^2+
3*a^2*b^4*d^2*e^4*f^2-3*a^4*b^2*d^2*e^4*f^2+a^6*d^2*e^4*f^2+c^6*
d^4*e^2*f^2+b^2*c^4*d^4*e^2*f^2-3*a^2*c^4*d^4*e^2*f^2
+b^4*c^2*d^4*e^2*f^2-4*a^2*b^2*c^2*d^4*e^2*f^2+3*a^4*c^2*d^4*e^2
*f^2+b^6*d^4*e^2*f^2-3*a^2*b^4*d^4*e^2*f^2+3*a^4*b^2*d^4*e^2*f^2
-a^6*d^4*e^2*f^2-c^6*d^4*e^4-b^2*c^4*d^4*e^4+a^2*c^4*d^4*e^4
I use d,e,f,instead of p,q,r
I hope more properties to be found!

Best regards,
Tsihong Lau
```



isogonal_axis-2346.ggb

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Message: #2347
Date: 24/2/2017 2:49:51
From: eckart_schmidt@t-online.de
Subject: Four Circles for a QA

```
Dear Bernard, dear Chris,

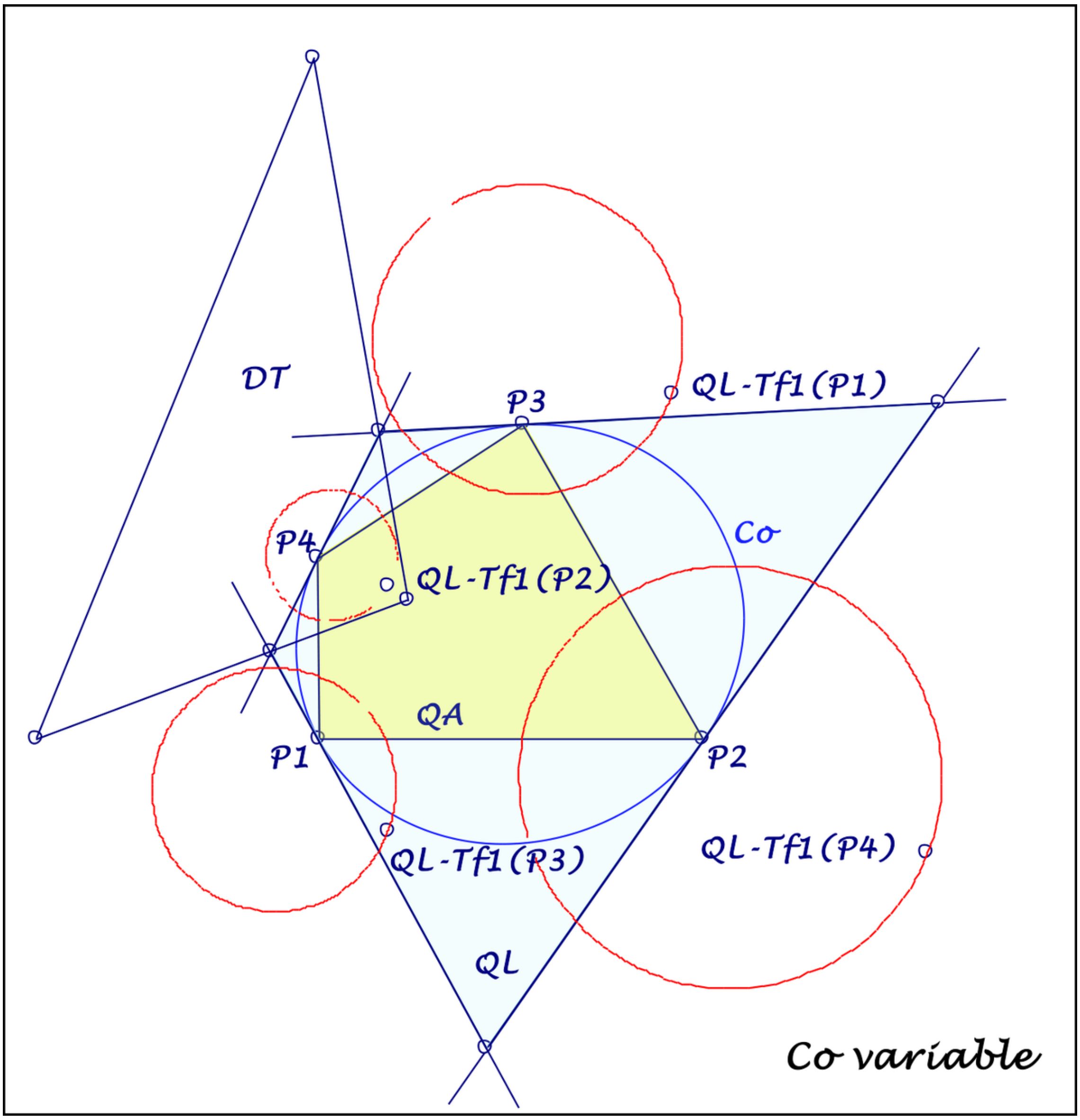
this is a new aspect, to use QL-Tf1 for quadrangles (see
attached file):

Consider a quadrangle QA with vertices Pi,
... a variable circumconic Co
... with the corresponding tangential quadrilateral QL
... and the QL-Tf1-images of the QA-vertices Pi,
... which give four circles Ci(Pi).

What about these four circles?
(1) The radical axes of Ci(Pi) and the circumcircles
    of Pj, Pk, Pl
... give a quadrilateral with the same DT as QA.
(2) Wrt the 12 similitude centers of the 4 circles:
... 6 lie on the QA-diagonals
... ... pairwise in harmonic position
... ... and isogonal conjugated wrt the orthic triangle
        of QA-DT,
... ... defining a further quadrilateral with the same DT as QA.
... 6 lie pairwise collinear with a DT-vertex and QA-P12,
... ... in harmonic position
... ... and isogonal conjugated wrt the orthic triangle
        of QA-DT.

I think, there will be more interesting properties.

Best regards Eckart
```



2017-02-24.pdf

Message: #2348
Date: 24/2/2017 5:22:53
From: chris.vantienhoven
Subject: QA-Cu1

Dear Eckart,

I included your property in EQF at QA-Cu1.

Best regards,
Chris

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Message: #2349
Date: 26/2/2017 11:19:42
From: bernard.keizer
Subject: Four Circles for a QA

Dear Eckart, dear Chris

In a QA/QL figur, the diagonal conics (circumvertices of QA) are
the duals of the inscribed conics.
You may consider as well the tangential QL of the
QA-circumscribed conics, with QL-Tf1 of the QA vertices, as you
do, as the QA of the contact points of the QL-inscribed conics,
with QA-Tf2 of these contact points ...
You may study the locus of the main QL-points in the 1rst case
and of the main QA-points in the 2nd case !
I suppose you will find plenty of interesting properties ...

Best regards
Bernard

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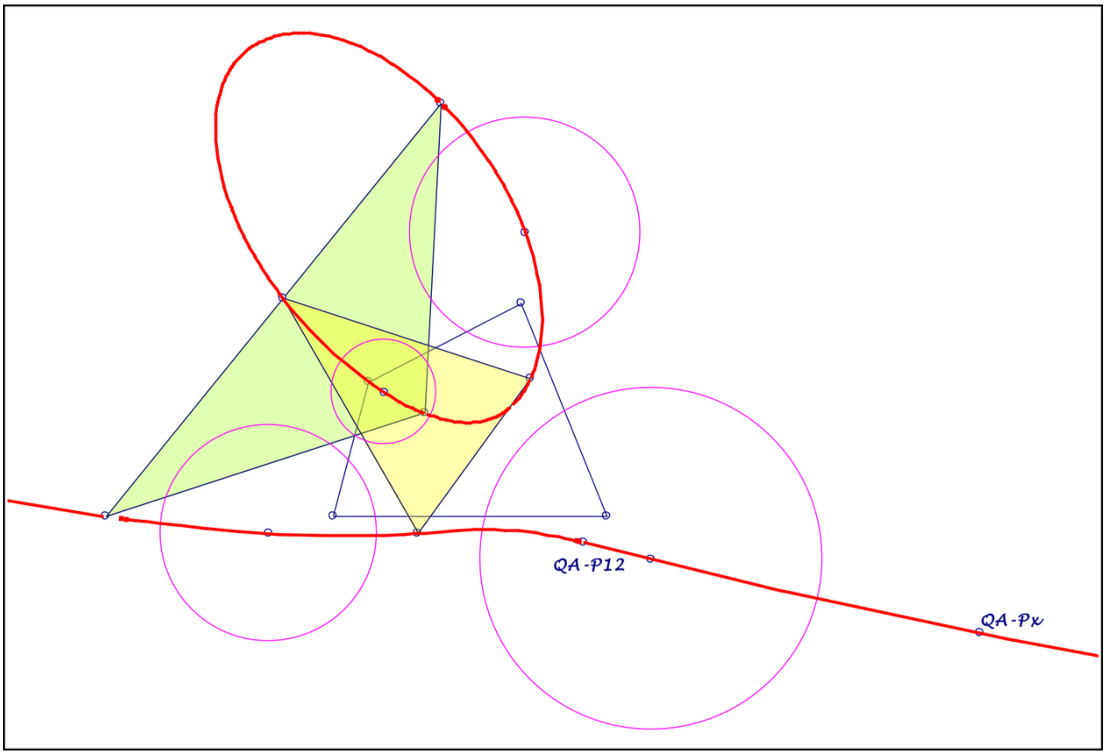
Message: #2350
Date: 26/2/2017 12:11:04
From: eckart_schmidt@t-online.de
Subject: Four Circles for a QA

```
Dear Bernard, dear Chris,

attached a further property of the four circles for a quadrangle
(see #2347):
The centers of the four circles lie on a pivotal isocubic:
... reference triangle: orthic triangle of QA-Tr1,
... isoconjugation: isogonal,
... pivot: new QA-point QA-Px.
  Or:
The centers of the four circles lie on a pivotal isocubic:
... reference triangle: QA-Tr1,
... isoconjugation: swaps QA-Px and QA-P12
... pivot: QA-P12.
What about QA-Px?

Construction:
Common point of the lines, connecting the centers of the circles
with their isogonal conjugate wrt the orthic triangle of QA-Tr1.

Best regards Eckart
```



2017-02-26.pdf

Message: #2351
Date: 26/2/2017 1:51:29
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

```
Dear all,

I found a very important cubic about cyclologic quadrangles.
On a circular Pappian plane, for two cyclologic quadrangles
(i.e. Miquel's six circle theorem holds),
there is a circular (i.e. through the circular points)
circumcubic of the eight vertices of both quadrangles.
I do not know if this cubic was known before.
I wonder its more properties.
Such a cubic does not exist for parallelologic quadrangles,
since they do not relate to the circular points.
I wonder if such a cubic exists for orthologic quadrangles.

The recent messages about cyclologic triangles/quadrangles are:
  Special n-angle Points as Cyclologic Centers: message #1941
  Generalized Cyclologic Center Chain in
  Quadrangle/Quadrilateral Geometry: message #968
  QA-Tr4: message #1971
  Re: QA-Tr-4: message #1983
  Re: Generalized Cyclologic Center Chain in Quadrang ...:
  message #1984
  Re: Generalized Cyclologic Center Chain in ...: message #1988
  Cubics for Cyclologic QA-Triple Triangles: message #1990
  Points on QA-Co2: message #2003
  Re: Cubics for Cyclologic QL-Triple Triangles: message #2009
  Re: Generalized Cyclologic Center Chain in Quad ...: message
  #2019
  Cyclologic Triangles on QA-Cu1: message #2023
  Similar Constructions of QL-Cu1 and QA-Cu1: message #2048
  Cubics of Generalized Cyclologic Centers: message #2067
  QL-Triple Triangles of QG-P1 and QA-P4: message #2096
  Vertical Parabolic Pappian Plane: message #2106
  new items in EQF: : message #2132
  Perspectivity of Quadrilaterals: message #2146

Best regards,
Tsihong Lau
```

Message: #2352
Date: 27/2/2017 1:03:19
From: tsihonglau
Subject: Affine, Möbius and Projective Planes

Dear all,

This circular cubic was found by Eckart and described in message #2067. It had better be called the cyclologic cubic and is very essential to the study of the relation between a circular Pappian plane and a Miquelian Möbius plane. According to message #2090, there is a similar cubic on a (vertical) parabolic Pappian plane.
This cubic is very essential to the study of the relation between a parabolic Pappian plane and a Miquelian Laguerre plane.

The point at infinity on the asymptote (also found by Eckart) is the point of intersection of many generalized lines at infinity. (lines through the genralized circular points).
It is also described in message #2027.
I guess the point at infinity is the ninth point predicted by Cayley-Bacharach theorem.
(This point was searched in message #2026)

I hope someone will summarize all the study of cyclologic triangles/quadrangles in the previous messages.
I hope also someone will focus on quadrangles not triangles.

Best regards,
Tsihong Lau

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Message: #2353
Date: 27/2/2017 4:46:44
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

Dear all,

Generalized cyclologic center chains were proposed by me in topic #1968. They lead to the cyclologic cubics found by Eckart in topic #2067 and called by me in topic #1997. But no orthologic and parallelologic center chains exist. However, the orthologic and parallelologic triangle chains exist.
I use notations in #1997.
Given two orthologic triangles PaPbPc and UaUbUc, thats is, there exist two points P and U such that the following six pairs of lines are perpendicular.

   PPa _|_ UbUc - UUa _|_ PbPc
   PPb _|_ UcUa - UUb _|_ PcPa
   PPc _|_ UaUb - UUc _|_ PaPb

Then we get the diapleural triangles P'aP'bP'c and U'aU'bU'c of quadrangles PPaPbPc and UUaUbUc respectively:

   P'a = PPa ∩ PbPc - U'a = UUa ∩ UbUc
   P'b = PPb ∩ PcPa - U'b = UUb ∩ UcUa
   P'c = PPc ∩ PaPb - U'c = UUc ∩ UaUb

Surprisingly, triangles P'aP'bP'c and U'aU'bU'c are orthologic. Thats is, there exist two points P' and U' such that the following six pairs of lines are perpendicular.

   P'P'a _|_ U'bU'c - U'U'a _|_ P'bP'c
   P'P'b _|_ U'cU'a - U'U'b _|_ P'cP'a
   P'P'c _|_ U'aU'b - U'U'c _|_ P'aP'b

And so on.

We get the orthologic triangle and center chains:

   PaPbPc - UaUbUc > P'aP'bP'c - U'aU'bU'c > P''aP''bP''c - U''aU''bU''c > ...
   P - U > P' - U' > P'' - U'' > ...

If we replace orthologic with parallelologic, the chains are valid, too. I wonder their application to the triangle/trilateral and quadrangle/quadrilateralgeometries.
There are no correspondent cyclologic triangle chains.

Best regards,
Tsihong Lau

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Message: #2354
Date: 04/3/2017 1:47:58
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Tsihong Lau, dear Chris,

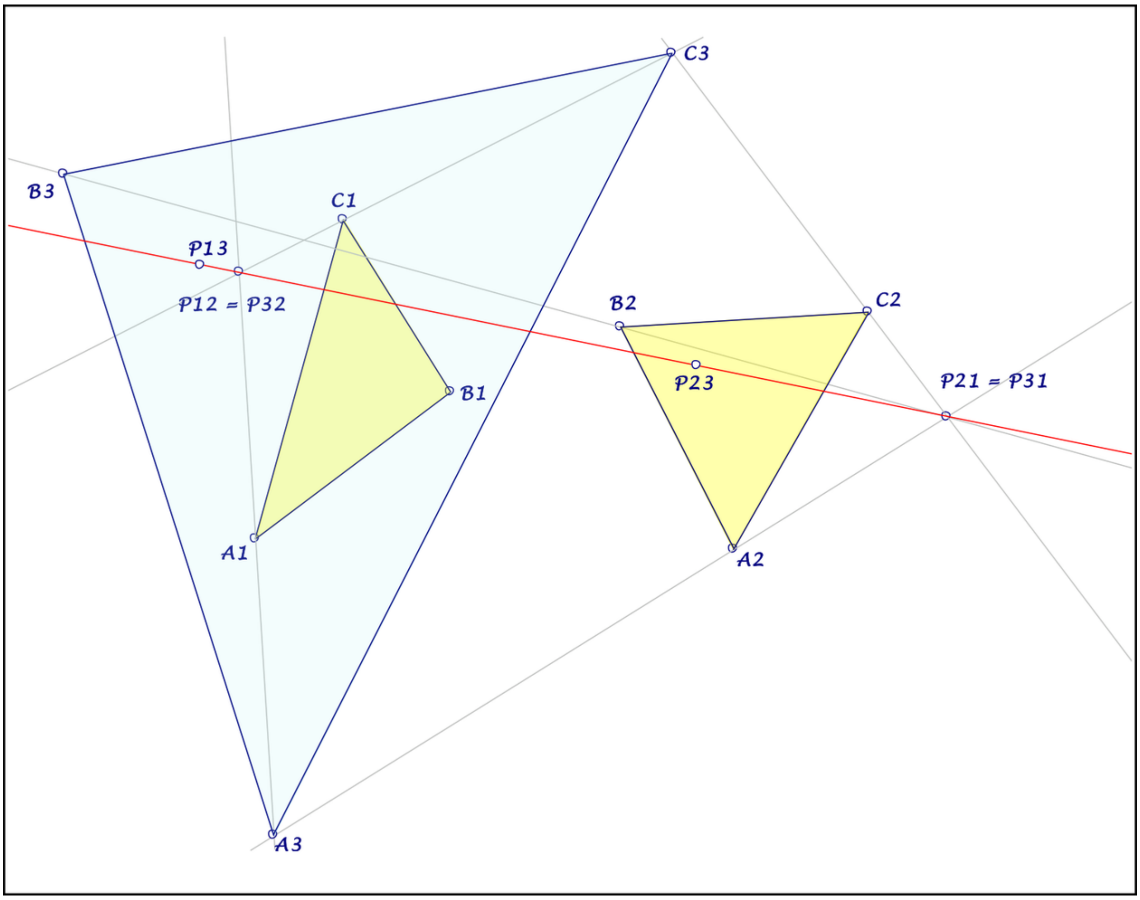
I just study orthologic triangles. Are the following properties
wellknown?
Let A1B1C1 and A2B2C2 orthologic triangles with their centers
P12 and P21.
There is a 3rd triangle A3B3C3, orthologic to A1B1C1 and A2B2C2
(see attached file),
... P12-perspective wrt A1B1C1 and P21-perspective wrt A2B2C2,
... with vertices in the intersections of perpendiculars
... ... from A1 on B2C2 and from A2 on B1C1,
... ... from B1 on A2C2 and from B2 on A1C1,
... ... from C1 on A2B2 and from C2 on A1B1.

For the orthologic centers holds:
P12 = P32, P21 = P31, P13, P23 collinear.

Further 3rd orthologic triangles can be constructed, pairwise
orthologic.
An example wrt orthologic QA-triple triangles (see QA-Tr-3):
Let A1B1C1 and A2B2C2 be the orthologic QA-triple triangles of
QG-P1 and QG-P6
... with P12 = QA-Px = QA-Tf2 of the reflection
    of QA-P23 in QA-P1
... and P21 = QA-P24.
... The 3rd orthologic triangle is the QA-triple triangle
    of the QG-point
... ... in the intersection of QG-P1.QA-Px and QG-P6.QA-P24.

I think, this can be generalized as follows:
Let A1B1C1 and A2B2C2 be two orthologic QA-triple triangles
... for the points Q1 and Q2 with centers P12 and P21,
... then the 3rd orthologic triangle
    is the QA-triple triangle of
... the QG-point in the intersection Q1.P12 and Q2.P21.
    (I proved the last for the QA-triple triangles
     of QG-P5 and QL-P4
... with centers P12 = QA-P9 and P21 = the reflection
    of QA-P15 in QA-P1.)

Best regards Eckart
```



2017-03-04.pdf

Message: #2355
Date: 04/3/2017 3:08:59
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Eckart,

Nice work!  I applied your construction to parallelologic
triangles and found similar results.  P12 = P32, P21 = P31 but
unfortunately, they are not colinear with P13, P23.
There are two main types of xxxlogic triangles:
    perspective and cyclologic
    orthologic and parallelologic
Each type share the same properties. I think we can find more
other xxxlogic triangles of these two types.
These two types are "quadrangle" xxxlogic in fact.
APG message #3646 "Parallelogramologic Triangle" is "triangle"
xxxlogic in fact.
So I post it to APG not here.

Best regards,
Tsihong Lau
```

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Message: #2356
Date: 05/3/2017 10:37:54
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

Dear Tsihong Lau, dear Chris,

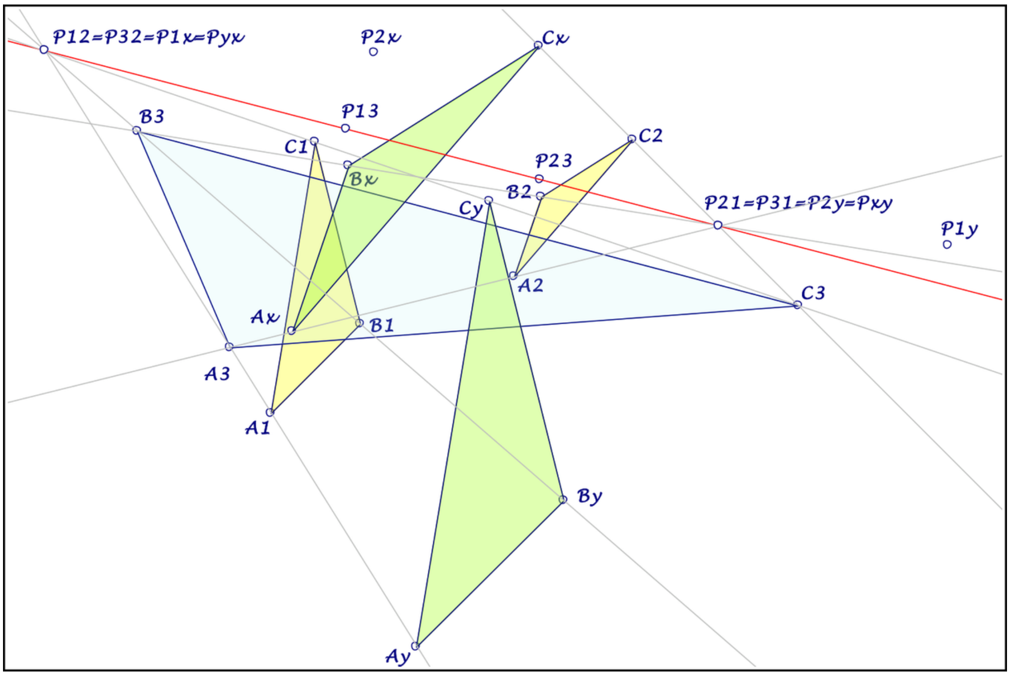
in #2354 I described a 3rd orthologic triangle perspective to the reference two ortholigic triangles wrt their orthologic centers.

Starting with two orthologic triangles Tr1 and Tr2
... this leads to the attached constellation of only 5 pairwise orthologic triangles
... Tr1,Tr2 -->Tr3; Tr1,Tr3 --> Trx; Tr2,Tr3 --> Try;
... and Trx,Try --> Tr3.
... Tr1, Tr3, Try are perspective, Tr3, Try yet homothetic,
... Tr2, Tr3, Trx are perspective, Tr3, Trx yet homothetic.

For the corresponding orthologic centers holds
... P12 = P32 = P1x = Pyx, P21 = P31 = P2y = Pxy, P13, P23
    collinear.

I shall study this constellation for orthologic QA-triple triangles.

Best regards Eckart



2017-03-05.pdf

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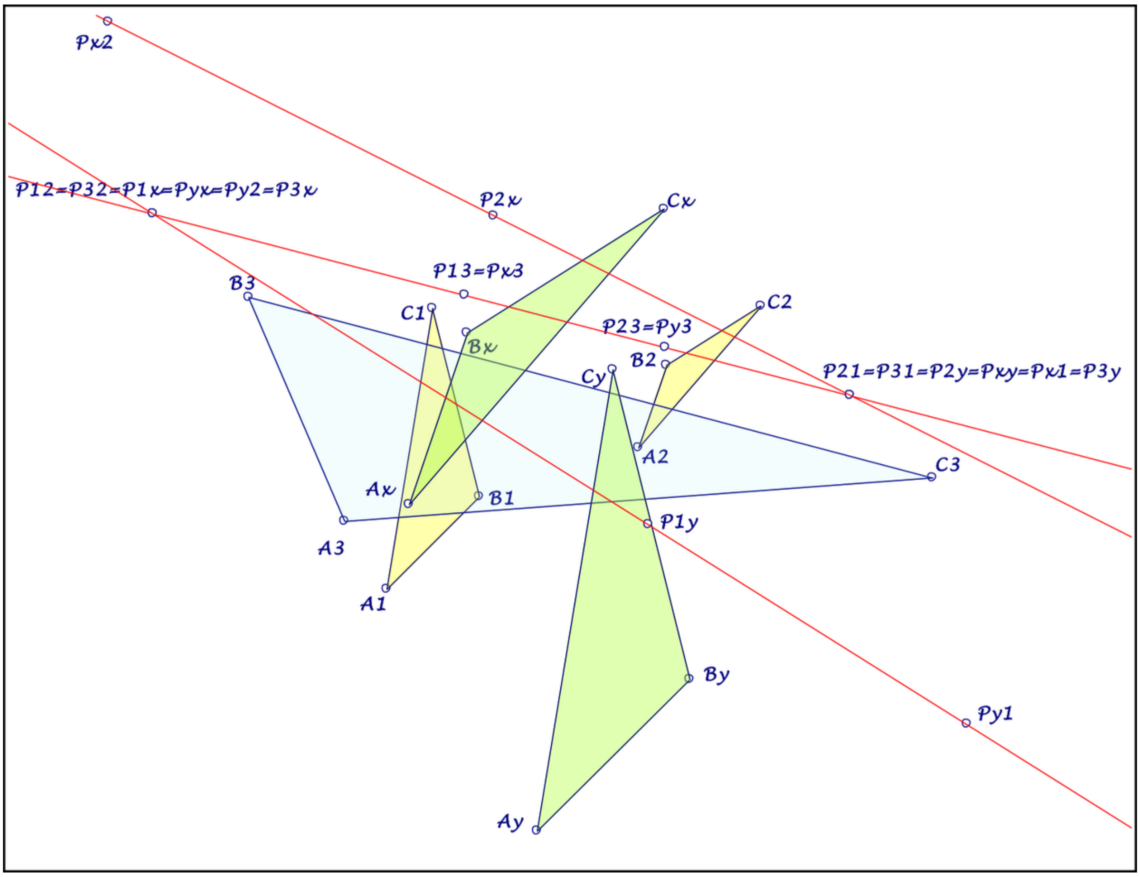
Message: #2357
Date: 05/3/2017 11:33:39
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

Dear Tsihong Lau, dear Chris,

excuse the description of the orthologic centers in #2356 is
incomplete (see also attached file):
... P12 = P32 = P1x = Pyx = Py2 = P3x,
... P21 = P31 = P2y = Pxy = Px1 = P3y,
... P13 = Px3,
... P23 = Py3,
... ... these 4 points are collinear.

Further:
... P12, P1y, Py1 are collinear and P21, P2x, Px2 are collinear.

Best regards Eckart



2017-03-05a.pdf

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Message: #2358
Date: 05/3/2017 2:28:05
From: Tran Quang Hung
Subject: Another generalization of Newton line

Sorry I mean ABCD is quadrigon
Let ABCD be a quadrigon and let P, Q, R, S be points on the segments AB, BC, CD, and DA, respectively. It is given that the segments PR and QS dissect ABCD into four quadrilaterals, each of which has perpendicular diagonals.
a) Then the points P, Q, R, S lie on circle (K). (RMM 2017)
b) Let M,N be midpoints of segment AC,BD then M,K,N are collinear.

Best regards,
Tran Quang Hung.

2017-03-05 20:26 GMT+07:00 Tran Quang Hung <analgeomatica@gmail.com>:
>> Dear geometers,
>> From problem of Nikolai Beluhov in RMM 2017
>> https://artofproblemsolving.com/community/q1h1389640p7773917
>> I see a property that consider as a generalization of Newton line
>> Let ABCD be any convex quadrilateral and let P, Q, R, S be points on the
>> segments AB, BC, CD, and DA, respectively. It is given that the segments
>> PR and QS dissect ABCD into four quadrilaterals, each of which has
>> perpendicular diagonals.
>> a) Then the points P, Q, R, S lie on circle (K). (RMM 2017)
>> b) Let M,N be midpoints of segment AC,BD then M,K,N are collinear.
>> Is this result known before ?
>> Best regards,
>> Tran Quang Hung.

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Message: #2359
Date: 05/3/2017 2:47:02
From: Tran Quang Hung
Subject: Another generalization of Newton line

Dear geometers,

>From problem of Nikolai Beluhov in RMM 2017
https://artofproblemsolving.com/community/q1h1389640p7773917

I see a property that consider as a generalization of Newton line
Let ABCD be any convex quadrilateral and let P, Q, R, S be points on the segments AB, BC, CD, and DA, respectively.
It is given that the segments PR and QS dissect ABCD into four quadrilaterals, each of which has perpendicular diagonals.
a) Then the points P, Q, R, S lie on circle (K). (RMM 2017)
b) Let M,N be midpoints of segment AC,BD then M,K,N are collinear.
Is this result known before ?

Best regards,
Tran Quang Hung.

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Message: #2360
Date: 05/3/2017 3:15:26
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

Dear Eckart,
Nice work! Your message gave the dual of orthologic triangles:
A1B1C1,A2B2C2 <> AyByCy,AxBxCx
In other words, given AyByCy,AxBxCx, your construction produces A1B1C1,A2B2C2.
12 orthologic centers(in fact only 8) lie on three lines (a degenerate cubic).
I wonder if the 15 vertices of A1B1C1, A2B2C2, A3B3C3, AyByCy, AxBxCx lie on a quartic! I also wonder if the dual of parallelologic triangles exist!
Best regards,
Tsihong Lau

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Message: #2361
Date: 06/3/2017 8:46:23
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Tsihong Lau, dear Chris,

in addition to the #2356 and #2357 wrt the five orthologic
triangles Tr1, Tr2, Tr3, Trx, Try, deduced from two reference
orthologic triangles Tr1 and Tr2 (see attached file in # 2357):
    X(4) of Tr1 is P1y,
    X(4) of Tr2 is P2x,
    X(4) of Tr3 is P12.Px2 ^ P21.P1y,
    X(4) of Trx is Px2,
    X(4) of Try is Py1.

The following sets of points lie on orthogonal hyperbolas with
parallel asymptotes:
... A1, B1, C1, P12, P13, P1y;
... A2, B2, C2, P21, P23, P2x;
... Ax, Bx, Cx, Px1, Px2, Px3;
... Ay, By, Cy, Py1, Py2, Py3;
... A3, B3, C3, P12, P21.

Best regards Eckart
```

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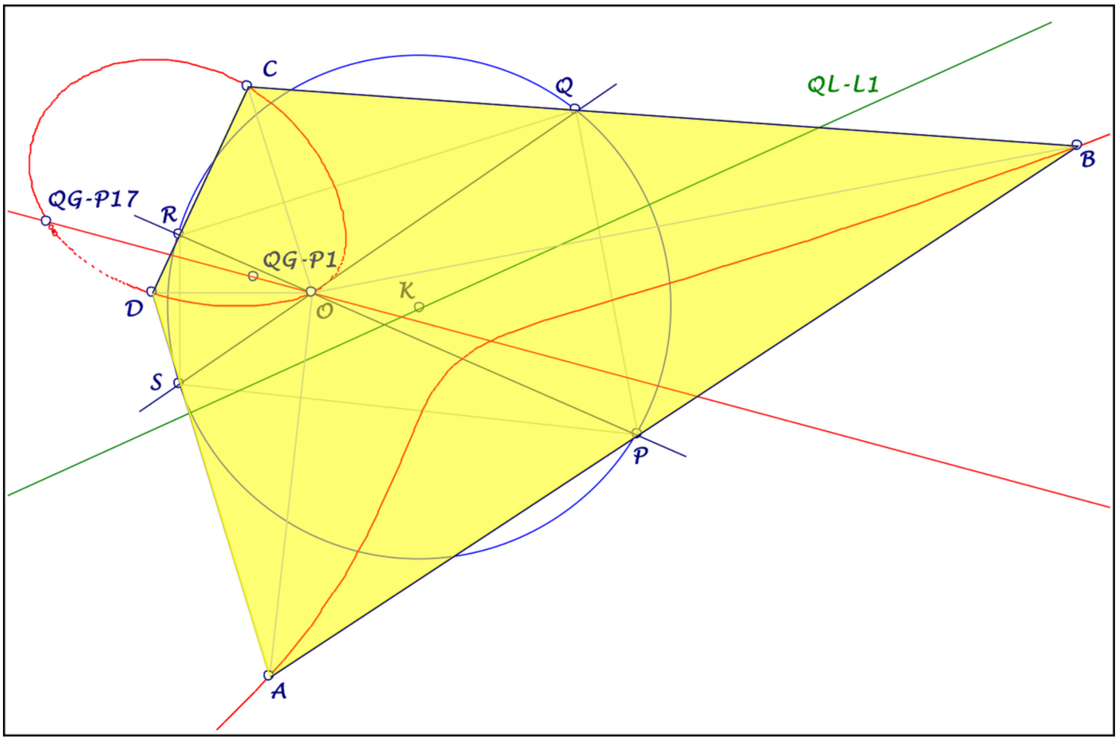
Message: #2362
Date: 07/3/2017 2:38:27
From: eckart_schmidt@t-online.de
Subject: Another generalization of Newton line

```
Dear Tran Quang Hung,

some remarks wrt the constellation described in #2358, #2359:
(1) The center K of the circle lies on QL-L1 of the quadrigon
(already mentioned).
(2) CABRI-observations lead to the assumption, that the
constellation exists only,
... if the cubic QL-Cu1 of the quadrigon is bipartite (see
attached file).
(3) In this case the point O = PR ^ QS*
... is the 2nd intersection of QG-P1.QG-P17 and the closed part
of the cubic QL-Cu1.
(4) I suppose, all constellations for a quadrigon have the same
point O:
... Let P be a point on AB,
... Q = intersection of BC and a perpendicular through P wrt OB,
... R = intersection of CD and a perpendicular through Q wrt OC,
... S = intersection of DA and a perpendicular through R wrt OD.
I hope, someone can confirm these results.

Best regards Eckart
```



2017-03-07.pdf

Message: #2363
Date: 07/3/2017 3:14:57
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

Dear Eckart,

I checked your construction and found some errors!
>> X(4) of Tr3 is P12.Px2 ^ P21.P1y, ??? Please check it.
>> ... Ax, Bx, Cx, Px1, Px2, Px3;
>> ... Ay, By, Cy, Py1, Py2, Py3;

Should be Ax,Bx,Cx,P21,P13,Px2
and Ay,By,Cy,P12,P23,Py1

Suppose the circumcenter of Tr3 is H,
The following lines and conics concur:
(Ay,By,Cy,P12,P23,Py1) - (A2,B2,C2,P21,P23,P2x) - (H,P23) - (A3,B3,C3,H,P12,P21)
(Ax,Bx,Cx,P21,P13,Px2) - (A1,B1,C1,P12,P13,P1y) - (H,P13) - (A3,B3,C3,H,P12,P21)

I also found the dual of parallelologic triangles do not exist.
(A1B1C1,A2B2C2 <> AyByCy,AxBxCx)
We should study the dual orthologic triangles of quadrangles/quadrilaterals.

Best regards,
Tsihong Lau

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Message: #2364
Date: 08/3/2017 9:54:06
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Tsihong Lau,

you are right, X(4) of Tr3 isn´t P12.Px2 ^ P21.P1y. Thanks!

X(4) of Tr3 is the intersection of
... the line through P12.Px2 ^ P21.P1y and P13
... and the line through P12.P2x ^ P21.Py1 and P23.

But P12.Px2 ^ P21.P1y lies on the orthogonal hyperbola through
Ax, Bx, Cx, Px1, Px2, Px3
... as P12.P2x ^ P21.Py1 lies on the orthogonal hyperbola
    through Ay, By, Cy, Py1, Py2, Py3;
... both points lie on the orthogonal hyperbola
    through A3, B3, C3, P12, P21.

Your further corrections are not necessary, for the point sets
are the same.
Wrt the point H = circumcenter of Tr3: I don´t see, that the
mentioned conics and lines concur.

Best regards Eckart
```

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Message: #2365
Date: 08/3/2017 3:36:05
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Eckart,
>>Wrt the point H = circumcenter of Tr3: I don´t see, that the
mentioned conics and lines concur.
You showed the points of concurrence.
P12.Px2 ^ P21.P1y
P12.P2x ^ P21.Py1
Best regards,
Tsihong Lau
```

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Message: #2366
Date: 08/3/2017 9:12:28
From: eckart_schmidt@t-online.de
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Tsihong Lau,
I think, the reason for our misunderstanding is a typo in your
#2363:
H should be the orthocenter of Tr3 and not the circumcenter.
Best regards Eckart
```

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Message: #2367
Date: 08/3/2017 11:25:25
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear Eckart,
>> I think, the reason for our misunderstanding is a typo in
your
   #2363:
>> H should be the orthocenter of Tr3 and not the circumcenter.
Thanks for your correction!
Best regards,
Tsihong Lau
```

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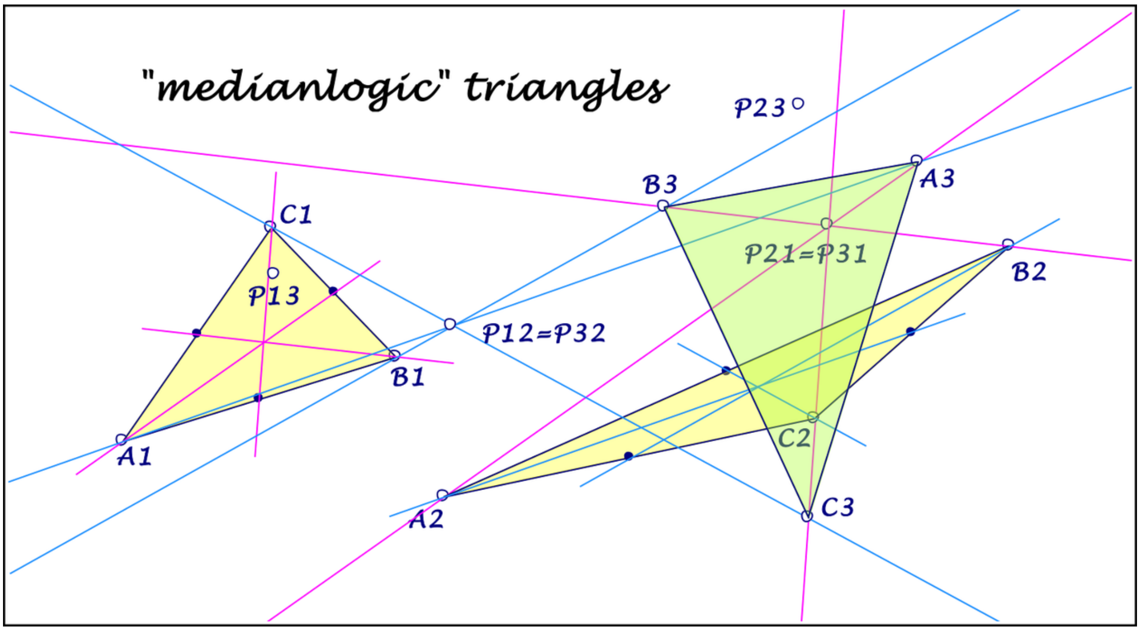
Message: #2368
Date: 09/3/2017 10:54:33
From: eckart_schmidt@t-online.de
Subject: "Medianlogic" Triangles

```
Dear Tsihong Lau, dear Chris,

is the following modification of parallelologic triangles
already researched?
A1B1C1 shall be "medianlogic" wrt A2B2C2, if the parallels
through A1, B1, C1 wrt the medians of B2C2, C2A2, A2B2 are
concurrent.
... If A1B1C1 is medianlogic wrt A2B2C2,
    then also A2B2C2 wrt A1B1C1.
... Examples:
... ... The QA-triple triangles of QG-P1 and QG-P4
        are medianlogic with centers QA-P10 and QA-P25.
... ... The QA-triple triangles of QG-P1 and QG-P8
        are medianlogic with centers QA-P10 and QA-P26.
... ... (I think, Chris can calculate more examples.)
... There is also a 3rd medianlogic triangle wrt A1B1C1
    and A2B2C2
... ... with vertices in the intersections of corresponding
        parallels to the medians of A1B1C1 and A2B2C2.

Best regards Eckart
```



2017-03-09.pdf

Message: #2369
Date: 09/3/2017 3:13:02
From: tsihonglau
Subject: ”Medianlogic” Triangles

```
Dear Eckart,

Your assertion can be generalized further!
I use my notations.
Given two triangles PaPbPc and UaUbUc together with their
triangle centers Pn and Un respectively, suppose there exists a
point P such that PPa || UaUn and PPb || UbUn and PPc || UcUn
and then there exists a point U
such that UU a || PaPn and UUb || PbPn and UUc || PcPn.  If both
Pn and Un are centroids or orthocenters, then PaPbPc and UaUbUc
are madianlogic or orthologic respectively.

Suppose there exists a point P such that PPa _|_ UaUn and PPb
_|_ UbUn and PPc _|_ UcUn and then there exists a point U such
that UUa _|_ PaPn and UUb _|_ PbPn and UUc _|_ PcPn.

If both Pn and Un are centroids or orthocenters, then PaPbPc and
UaUbUc are xxx logic or parallelologic respectively. The latter
case cover some triangle bicentric pairs such as Fermat and
Vecten points.
I think my assertion can be generalized further!

Best regards,
Tsihong Lau
```

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Message: #2370
Date: 09/3/2017 9:12:17
From: chris.vantienhoven
Subject: "Medianlogic" Triangles

```
Dear Eckart,

The concept of medianlogic is new to me.
The example that you give is rather obvious because the QG-P1
Triple Triangle and the QG-P4 Triple Triangle are homothetic and
so their medians will be parallel. Even all X(i)-cevians will be
parallel and therefore "X(i)-cevian-logic".
Basically I think that it is possible that non-homothetic
triangles are medianlogic, but I don't think it will
occur "easily".

Best regards,
Chris
```

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Message: #2371
Date: 11/3/2017 1:08:53
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

```
Dear all,

Another chains are found!
Given two orthologic quadrangles PPaPbPc and UUaUbUc, we get the
orthocenters of expoint triangles of both: orthocenter - expoint
triangle
   P4  - PaPbPc : U4  - UaUbUc
   P4a - PPbPc  : U4a - UUbUc
   P4b - PaPPc  : U4b - UaUUc
   P4c - PaPbP  : U4c - UaUbU
Then P4P4aP4bP4c and U4U4aU4bU4c are also orthologic
quadrangles!
If we replace orthologic with parallelologic, the above property
is also valid!

Best regards,
Tsihong Lau
```

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Message: #2372
Date: 11/3/2017 3:46:04
From: tsihonglau
Subject: ”Medianlogic” Triangles

Dear Eckart, Chris

I generalize the results further in message #2369.
Given two triangles PaPbPc and UaUbUc in general position, there exist two moving points P' and U' such that P'Pa || U'Ua and P'Pb || U'Ub and P'Pc || U'Uc.
Then the loci of P' and U' are circumconics of PaPbPc and UaUbUc respectively.
The conics intersect at two points at infinity (i.e. They have the parallel asymptotes.) and two other points.
The results can be generalized further.
The points of intersection of P'Pa,U'Ua and P'Pb,U'Ub and P'Pc,U'Uc lie on a line which two points of intersection of the conics lie on also.

If we apply the above statements to two orthologic quadrangles PPaPbPc and UUaUbUc, we getwonderful results. Please refer to topic #2353.
The two circumconics(rectangular hyperbolas) pass through PPaPbPcP4P4aP4bP4cP13 UUaUbUcU4U4aU4bU4cP23 respectively.
When P' moves on the former circumconic, U' moves on the second circumconic.

The correspondence of P' and U':

```
P' - U'
P - U4
Pa - U4a
Pb - U4b
Pc - U4c
P4 - U
P4a - Ua
P4b - Ub
P4c - Uc
P13 - P23
```

Best regards,
Tsihong Lau

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Message: #2373
Date: 12/3/2017 3:48:00
From: tsihonglau
Subject: "Medianlogic" Triangles

Dear Eckart, Chris

I generalize the results further in message #2369.
Given two triangles PaPbPc and UaUbUc in general position, there exist two moving points P' and U' such that P'Pa _|_ U'Ua and P'Pb _|_ U'Ub and P'Pc _|_ U'Uc. Then the loci of P' and U' are circumconics of PaPbPc and UaUbUc respectively.
The conics intersect at two points at infinity (i.e.They have the parallel asymptotes.) and two other points.
The results can be generalized further. We can choose any two points as the generalized circular points and the line through them as the generalized line at infinity
Then we can define generalized perpendicularity and two points of intersection of the conics lie on the generalized line at infinity.

If we apply the above statements to two parallelologic quadrangles PPaPbPc and UUaUbUc, we get wonderful results.
Please refer to topic #2353.
The two circumconics(rectangular hyperbolas) pass through PPaPbPcP4P4aP4bP4c but no P13 UUaUbUcU4U4aU4bU4c but no P23 respectively.
When P' moves on the former circumconic, U' moves on the second circumconic.

The correspondence of P' and U':

```
P'  - U'
P   - U4
Pa  - U4a
Pb  - U4b
Pc  - U4c
P4  - U
P4a - Ua
P4b - Ub
P4c - Uc
```

If the two classes of conics mentioned in message #2372 and this are not studied before, I would like to call them as parallel and perpendicular perspective triangle conics respectively.

Best regards,
Tsihong Lau

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Message: #2374
Date: 12/3/2017 5:23:48
From: tsihonglau
Subject: ”Medianlogic” Triangles

Dear Eckart, Chris,

Sorry! Some errors in the previous message!
>>The conics intersect at two points at infinity(i.e.They
>>have the parallel asymptotes.) and two other points.
It should be "The conics intersect at four points"

>>Then we can define generalized perpendicularity and
>>two points of intersection of the conics lie on the
>>generalized line at infinity.
Four points of intersection of the conics are not necessary to lie on the g eneralized line at infinity. But when applying to parallelologic quadrangles, the two conics(rectangular hyperbolas) have parallel asymptotes and intersect at two points at infinity.

Best regards,
Tsihong Lau

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Message: #2375
Date: 13/3/2017 4:30:29
From: tsihonglau
Subject: Orthologic and Parallelologic Triangle Chains

Dear all,

The chains mentioned in message #2371 are valid for not only orthocenters but also triangle centers on the Euler line with constant Shinagawa coefficients.

Best regards,
Tsihong Lau

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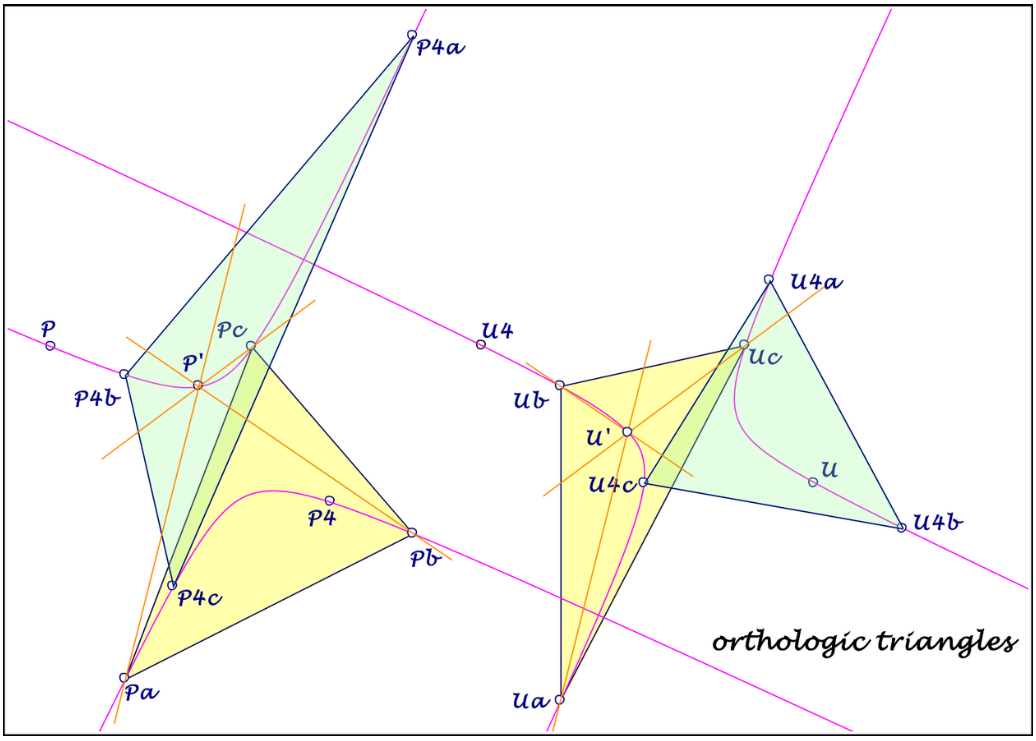
Message: #2376
Date: 14/3/2017 10:48:52
From: eckart_schmidt@t-online.de
Subject: "Medianlogic" Triangles

Dear Tsihong Lau,

I have reproduced your constellation in #2372 with CABRI.
My observation: The two conics - loci for P' and U' are
... either two homothetic ellipses (with parallel axes)
... or two hyperbolas with parallel asymptotes.
So the conics can have only two or zero real intersections, if they aren´t tangent.

With great interest I have done the application for orthologic quadrangles PPaPbPc, UUaUbUc
with orthogonal hyperbolas as loci for P' and U' (see attached file).
I think P4 and U4 shall be X(4) for the triangles PaPbPc and UaUbUc, then P4aP4bP4c, U4aU4bU4c are orthologic triangles
... with centers P4 and U4
... and X(4) in P and U.

In this way you get a chain of pairs of orthologic triangles on the two conics.
Best regards Eckart



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Message: #2377
Date: 14/3/2017 11:38:04
From: eckart_schmidt@t-online.de
Subject: ”Medianlogic” Triangles

```
Dear Tsihong Lau,
in addition to my last message wrt orthologic triangles PaPbPc
and UaUbUc:
Corresponding triangles of PPaPbPc, P4P4aP4bP4c (or UUaUbUc,
U4U4aU4bU4c)
... have the same area.
Best regards Eckart
```

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Message: #2378
Date: 15/3/2017 4:03:30
From: eckart_schmidt@t-online.de
Subject: Intersection of QL-L1 and QL-P1.QG-P1

```
Dear Chris,

I think, this is an interesting QG-point: QG-Px = QL-L1 ^
QL-P1.QG-P1.
... The QA-triple triangle of QG-Px is
... ... the 3rd orthologic triangle (see #2354)
        of the QA-triple triangles of QG-P1 and QG-P9.
... The QA-triple triangle of QA-Px is orthologic
    with the QA-triple triangles of
... ... QG-P1, -P2, -P4, -P5, -P6, -P7, -P8, -P9,
               -P10, -P11, -P15
... ... ... (except -P2 all QG-points on QG-L3, -L4, -L5),
... ... QL-P6, -P12.

CABRI-observations lead to the following generalization:
Let QG-Py and QG-Pz have orthologic QA-triple triangles,
... then every point dividing QG-Py.QG-Pz in a fixed ratio
... leads to a QA-triple triangle,
... orthologic to the first two QA-triple triangles.

Best regards Eckart
```

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Message: #2379
Date: 16/3/2017 12:27:16
From: chris.vantienhoven
Subject: examples of quadrilaterals in real life?

```
Dear friends,

I encountered this question:
***********************************************************
What are some examples of quadrilaterals used in real life?
***********************************************************
It would be nice if we could answer it.
Anyone can help me to answer this?

Best regards,
Chris
```

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Message: #2380
Date: 17/3/2017 11:55:18
From: eckart_schmidt@t-online.de
Subject: Orthologic triangles

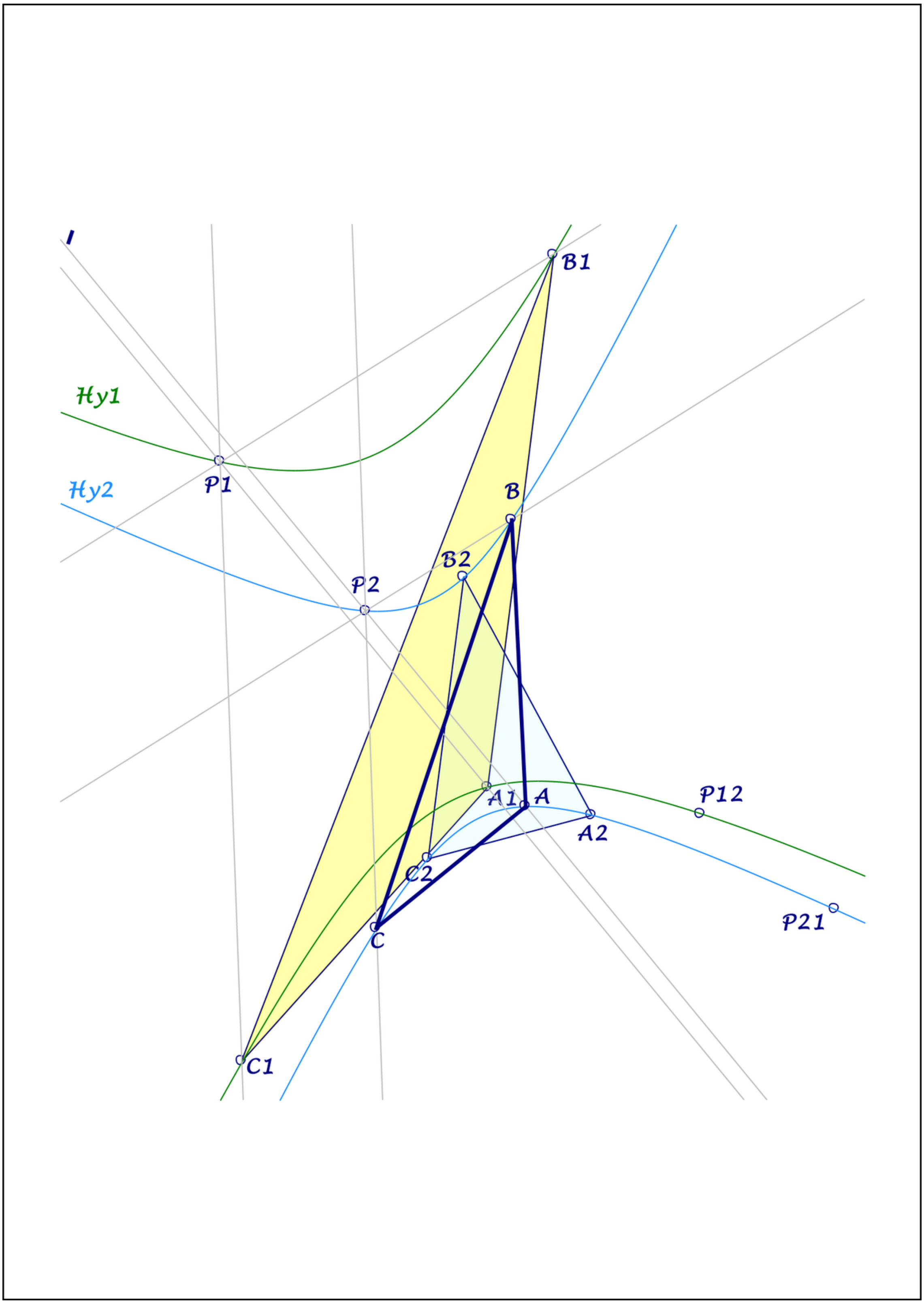
Dear Tsihong Lau,

let A1B1C1 and A2B2C2 be two orthologic triangles with centers
P12 and P21.
Two arbitrary points Ao and Bo can be complemented by a point Co
to a third triangle AoBoCo,
orthologic to A1B1C1 and A2B2C2:
... Let S1 be the intersection of the perpendiculars
    from Ao to B1C1 and Bo to C1A1,
... let S2 be the intersection of the perpendiculars
    from Ao to B2C2 and Bo to C2A2,
... then the intersection of the perpendiculars
    from S1 to A1B1 and S2 to A2B2
... give the third vertex Co of the triangle AoBoCo,
    orthologic to A1B1C1 and A2B2C2.
The loci for the orthologic centers are
... for P10: orthogonal hyperbola of A1B1C1 through P12,
... for P20: orthogonal hyperbola of A2B2C2 through P21.

These are your conics in #2372.
Result:
If a triangle AoBoCo is orthologic wrt two orthologic triangles
A1B1C1 and A2B2C2,
... the orthogonal center P1o lies on an orthogonal hyperbola
    of A1B1C1 through P12,
... the orthogonal center P2o lies on an orthogonal hyperbola
    of A2B2C2 through P21.

Inverse (see attached file):
Let A1B1C1 and A2B2C2 be two orthologic triangles with centers
P12 and P21
... and P1 and P2 points on the orthogonal hyperbolas
... Hy1 of A1B1C1 through P12 and Hy2 of A2B2C2 through P21.
... Let the parallel to P1.A1 through P2 intersect Hy2 in A,
... let the parallel to P1.B1 through P2 intersect Hy2 in B,
... let the parallel to P1.C1 through P2 intersect Hy2 in C.
The triangle ABC on Hy2 is orthologic to A1B1C1 and A2B2C2,
... with the same area as A2B2C2 (independant of P1 and P2!).
(Analog a triangle on Hy1.)

Best regards Eckart



2017-03-17.pdf

Message: #2381
Date: 17/3/2017 4:49:08
From: tsihonglau
Subject: ”Medianlogic” Triangles

Dear Eckart,

These two conics are very essential to what I called ordered triangle pair geometry. I will post my ideas to another topic. Because they belong to ordered triangle pair geometry, I use new notations. I present parallel perspective triangle conics only. Perpendicular perspective triangle conics are similar. Given two triangles
ABC and DEF in general position, there exist two moving points P and U such that PA || UD and PB || UE and PC || UF. Then the loci of Pand U are circumconics of ABC and DEF respectively. I call them parallel perspective
triangle conics of ABC and DEF.
The correspondence table of the vertices of ABC and DEF are:

```
   P  - U
   A  - A'
   B  - B'
   C  - C'
   D' - D
   E' - E
   F' - F
```

Then parallel perspective triangle conics of ABC-DEF and D'E'F'-A'B'C' are the same!
Moreover, areas of triangle pairs are the same -ABC=D'E'F' and DEF=A'B'C'. In other words, we get chains of triangle pairs with the same areas on the same parallel perspective triangle conics! Moreover, suppose triangles A''B''C'' and D''E''F'' are the triangles bounded by AD',BE',CF' and DA',EB',FC' respectively.
The ratios of areas are equal: ABC:DEF=A''B''C'':D''E''F''.
The above properties of parallel perspective triangle conics apply to perpendicular perspective triangle conics, too. If we apply parallel perspective triangle conics to orthologic quadrangles.
The correspondence table:

```
   P   - U4
   Pa  - U4a
   Pb  - U4b
   Pc  - U4c
   P4  - U
   P4a - Ua
   P4b - Ub
   P4c - Uc
```

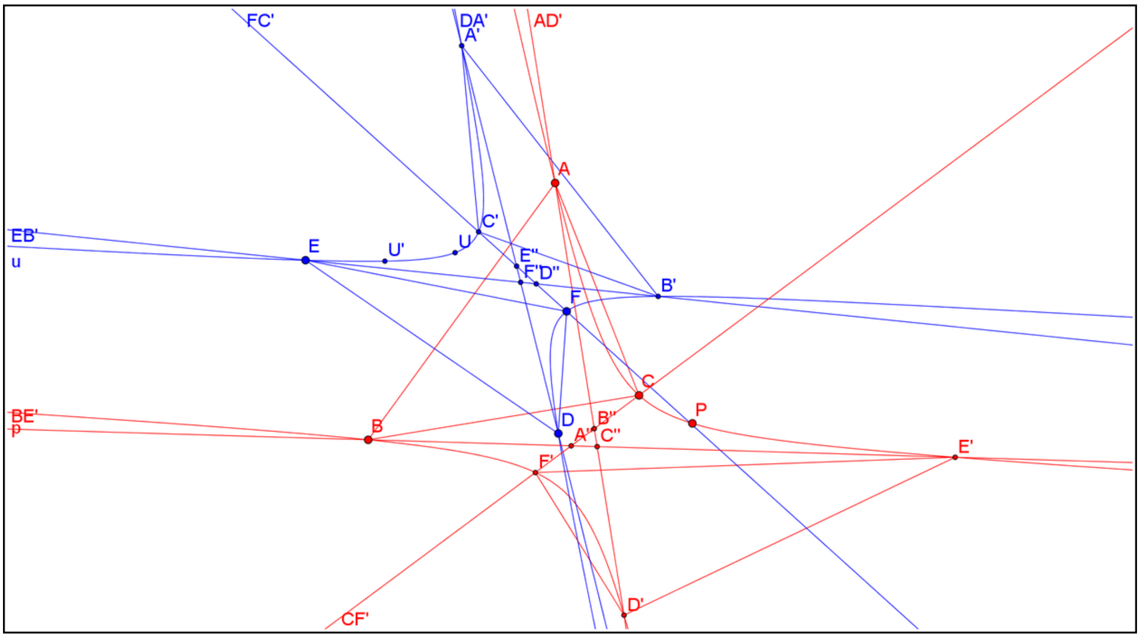
We can apply perpendicular perspective triangle conics to parallelologic quadrangles.
The correspondence table is the same.

>> My observation: The two conics - loci for P' and U' are
>> ... either two homothetic ellipses (with parallel axes)
>> ... or two hyperbolas with parallel asymptotes.

In the case of homothetic ellipses, the two parallel perspective triangle conics intersect at two imaginary points at infinity.
In the case of two hyperbolas with parallel asymptotes, they intersect at two real points at infinity.
They even intersect at the circular points at infinity when they are two circles.

In another perspective, the two points of intersection at infinity can viewed as generalized circular points at infinity.
Then two parallel perspective triangle conics can be viewed as two circumcircles of triangles!
Given two generalized circular points and two triangles, we can define two metric systems (lengths,areas,angles).
That is, another new geometry is born!
I will post my ideas later.

Best regards,
Tsihong Lau



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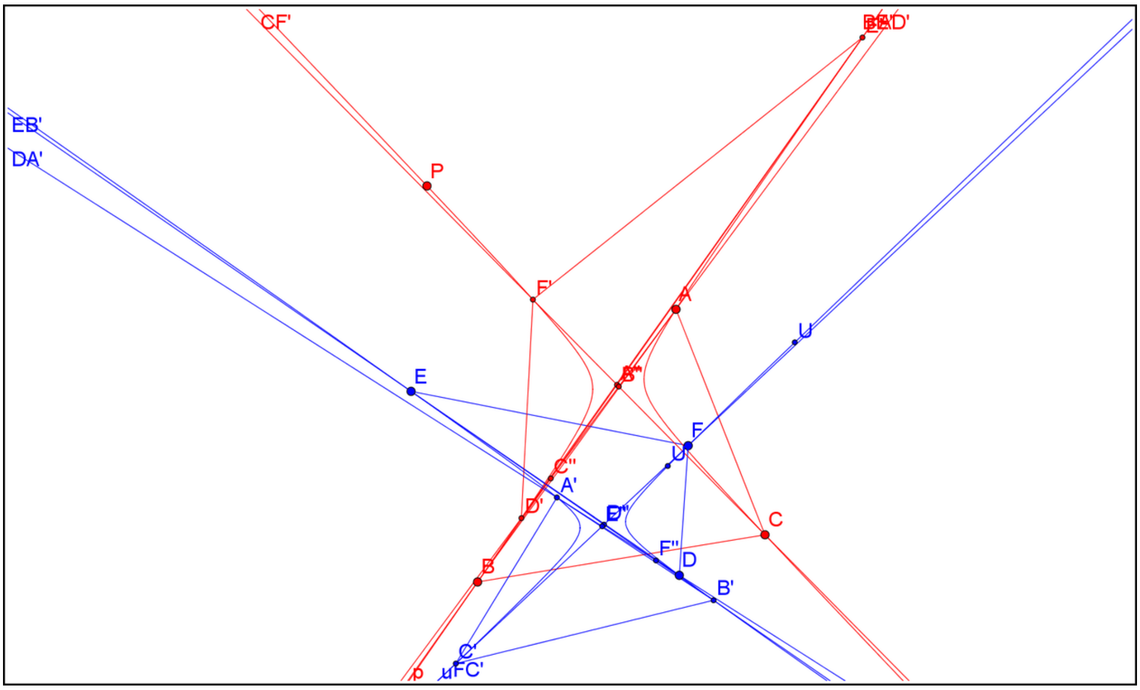

perpendicualr_perspective_triangle_conic.png

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Message: #2382
Date: 21/3/2017 4:01:52
From: eckart_schmidt@t-online.de
Subject: QG-Circumcubic wrt QG-P1, QL-P1, QA-P4

Dear all,

attached a cooperation of QG-, QL- and QA-geometry:
A CSC- invariant circumcubic of a quadrigon
... through the points QG-P1, QL-P1, QA-P4
... and the CSC-fixed points QG-2P3.

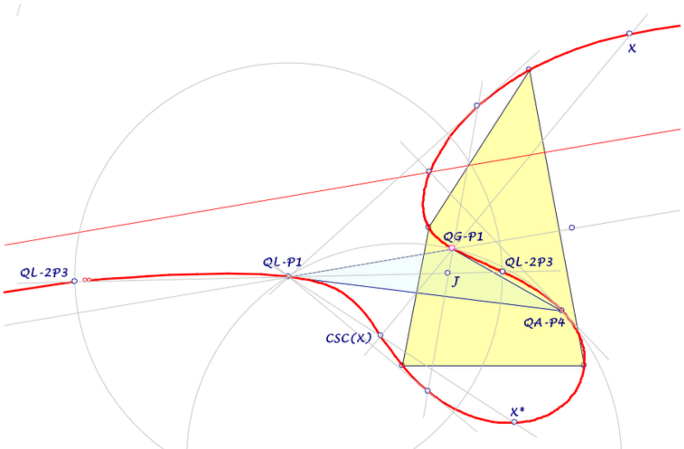
Perhaps of interest.
Best regards Eckart

EQF-Note 2017-03-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

_QG_-Circumcubic wrt _QG-P1, QL-P1, QA-P4_

This is a cooperation of QG-, QL- and QA-geometry: A CSC- invariant circumcubic of a quadrigon through the points QG-P1, QL-P1, QA-P4 and the CSC-fixed points QG-2P3.



In *QFG*-message *1237* pivotal *CSC*-cubics for quadrilaterals are described:

- **A pivotal _CSC_-cubic is the locus for intersections of lines _L_ through a pivot _P_ and the circles _CSC(L)_.**

Here the special *CSC*-cubic *QG-Cu* is researched for a quadrigon (see figure above)

… with pivot _QG-P1_.

- **_QG-Cu_ is a _CSC_-invariant _QG_-circumcubic bearing**
  … _QG-P1, QL-P1, QA-P4 = CSC(QG-P1)_
  … and the fixed points _QL-2P3_ of _CSC_.

- **_QG-Cu_ is isogonal (*) invariant wrt the reference triangle _QG-P1.QL-P1.QA-P4_.**

- **For _X_ on _QG-Cu_ holds**
  … _X, CSC(X), QG-P1_ are collinear,
  … _X*, CSC(X), QL-P1_ are collinear,
  … midpoints of _X.X*_ lie on _QG-P1.QL-P1_.

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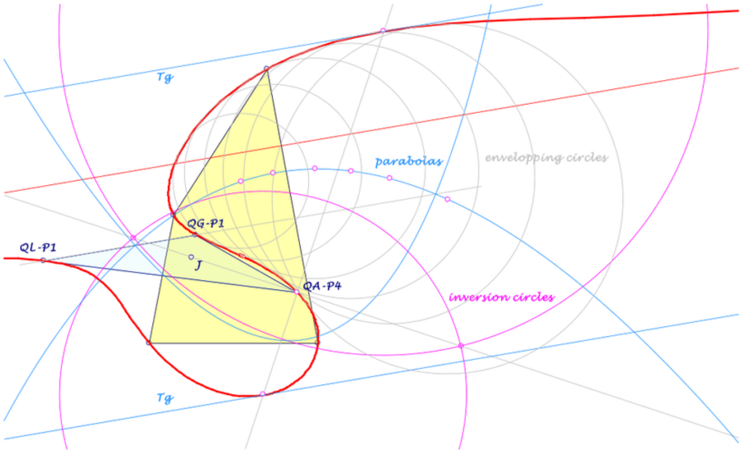
- **Tangent in *QG-P1* to *QG-Cu* is *QG-P1.QA-P4*,**
  … tangent in *QL-P1* to *QG-Cu* is *QL-P1.QA-P4*,
  … tangent in *QA-P4* to *QG-Cu* is the tangent in *QA-P4* at the circumcircle of the reference triangle,
  … tangents in *QL-2P3* at *QG-Cu* intersect in *QG-P1*.

- **Asymptote of *QG-Cu* is a parallel of *QG-P1.QL-P1***
  … through the reflection of *QA-P4* in *QG-P1.QL-P1,*
  … bearing the tangential of *QA-P4*.

- ***CSC*-partner on *J.QG-P1* (*J* incenter of the reference triangle) are points on *QG-Cu***
  … with tangential *QL-P1*.

- ***QG-Cu* is a nonpivotal isocubic**
  … with reference triangle *QG-P1.QL-P1.QA-P4,*
  … isogonal isoconjugation
  … and root in the midpoint of *QG-P1.QL-P1*.

- ***QG-Cu* is anallagmatic …**
  … with two inversion circles,
  … one *CSC*-image of the other,
  … intersecting orthogonal on the angle bisector at *QA-P4*,
  … centered on *QG-Cu* in the intersections with the ex-angle bisector at *QA-P4*,
  ... which have tangents *Tg* parallel *QG-P1.QL-P1*.



- **As anallagmatic curve *QG-Cu* is twice the envelope of circles,**
  … orthogonal wrt an inversion circle,
  … centered on a parabola
  … … with focus *QA-P4*
  … … and directrice *Tg* (see above).

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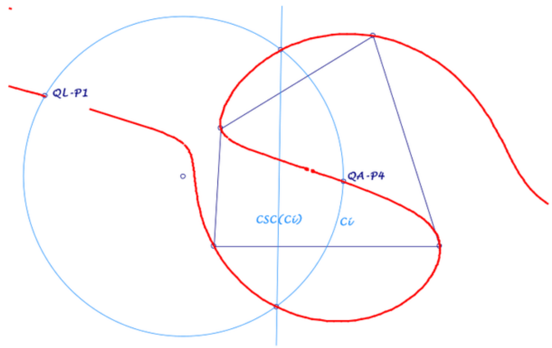
- ***QG-Cu* is the locus for intersections of circles *Ci* through *QL-P1* and *QA-P4* and the lines *CSC(Ci)*.**



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Message: #2383
Date: 22/3/2017 4:46:32
From: eckart_schmidt@t-online.de
Subject: Orthologic QA-Triple Triangles

Dear Chris,

attached a note of two aspects
-already mentioned in earlier messages-
... wrt further orthologic QA-triple triangles for two
orthologic QA-triple triangles.

The 2nd aspect leads to an error in EQF, QA-Tr-2:
QL-P15 and QL-P18 can´t generate orthologic triple triangles!

If QL-P15, and QL-P18 generate orthologic triple triangles,
the collinear points QL-P8, QL-P12, QL-P14 on QL-L8
with fixed distance ratios wrt QL-P15 and QL-P18
would generate orthologic QA-triple triangles: But that doesn´t
hold.

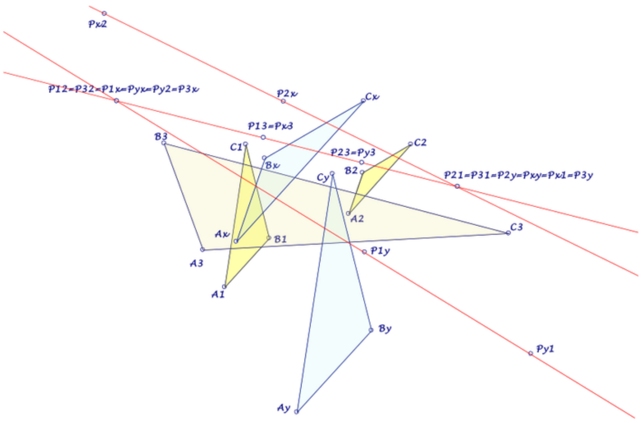
Best regards Eckart

EQF-Note 2017-03-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Orthologic QA-Triple Triangles

Two orthologic QA-triple triangles have a lot of further orthologic QA-triple triangles. Here two aspects are researched. Of interest are the generating QG-points and the orthologic centers of these triangles.



(1) Third Orthologic Triangle

Let $A_1B_1C_1$ and $A_2B_2C_2$ be two orthologic triangles with centers P_{12} and P_{21} (see *QFG*-message *2354*):

- **The 3rd orthologic triangle $A_3B_3C_3$ of two orthologic triangles $A_1B_1C_1$ and $A_2B_2C_2$ is P_{12}-perpective wrt $A_1B_1C_1$ and P_{21}-perspective wrt $A_2B_2C_2$.**
- **The three triangles are pairwise orthologic and for the centers holds**
  $P_{12} = P_{32}$, $P_{21} = P_{31}$, P_{13}, P_{23} are collinear.
- **For two orthologic *QA*-triple triangles wrt the points Q_1 and Q_2 the 3rd orthologic triangle is the *QA*-triple triangle of $Q_3 = Q_1.P_{12} \cap Q_2.P_{21}$.**

Example: Let Q_1 = *QG-P*$_1$, Q_2 = *QG-P9*
… Q_3 = *QL-L*$_1$ ∩ *QL-P*$_1$.*QG-P*$_1$,
… $P_{12} = P_{32}$ = *QA-P3*, $P_{21} = P_{31}$ = *QA-P32*,
… P_{13} = 2nd intersection of *QA-P3.QA-P32* and *QA-Co4*,
… P_{23} = 2nd intersection of *QA-P3.QA-P32* and the orthogonal hyperbola for the *QG-P9*-triple triangle through *QG-P32*.

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There are two further triangles, orthologic to $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$:
… $A_xB_xC_x$ = 3rd orthologic triangle of $A_1B_1C_1$ and $A_3B_3C_3$,
… $A_yB_yC_y$ = 3rd orthologic triangle of $A_2B_2C_2$ and $A_3B_3C_3$.
The 3rd orthologic triangle of $A_xB_xC_x$ and $A_yB_yC_y$ is $A_3B_3C_3$.

- **The five triangles $A_1B_1C_1$, $A_2B_2C_2$, $A_3B_3C_3$, $A_xB_xC_x$, $A_yB_yC_y$ are pairwise orthologic.**

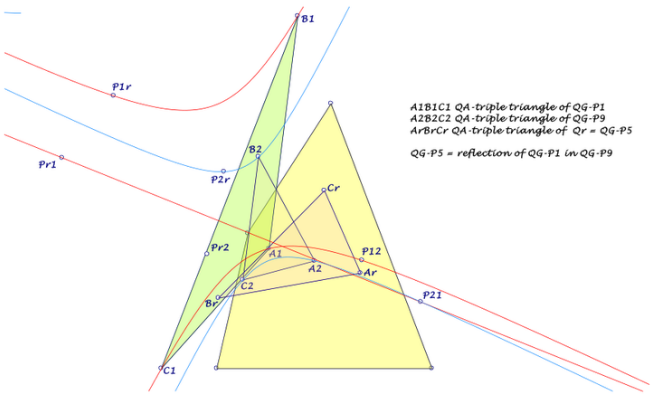
Wrt the orthologic centers see the figure above. Further properties can be found in *QFG*-message *2356*, *2357*, *2361* with correction in message *2364*.

(2) Orthogonal *QA*-Triple Triangles for collinear points

In *QFG*-message *2378* there is another possibility, to get further orthologic *QA*-triple triangles:

Let Q_1 and Q_2 be *QG*- or *QL*-points with orthologic *QA*-triple triangles $A_1B_1C_1$ and $A_2B_2C_2$ (see *QA-Tr-3*) and Q_r a point, dividing Q_1Q_2 with ratio r (independent of the quadrangle) and generating a *QA*-triple triangle $A_rB_rC_r$.

- **The *QA*-triple triangles of Q_1, $Q2$, Q_r are pairwise orthologic.**



- **The loci for the orthologic centers are:**
  … for P_{1r}: orthogonal circumhyperbola of $A_1B_1C_1$ through P_{12},
  … for P_{r1}: line through P_{21} and $X(4)$ of $A_1B_1C_1$,
  … for P_{2r}: orthogonal circumhyperbola of $A_2B_2C_2$ through P_{21},
  … for P_{r2}: line through P_{12} and $X(4)$ of $A_2B_2C_2$.

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2017-03-22.pdf

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Message: #2384
Date: 22/3/2017 10:09:33
From: chris.vantienhoven
Subject: Orthologic QA-Triple Triangles

Dear Eckart,

[ES] QL-P15 and QL-P18 can´t generate orthologic triple triangles!
If I am not wrong QL-P15 and QL-P18 generate flat triangles, meaning that the vertices of the Triple Triangles are collinear.
As a consequence the perpendiculars will be perpendicular to the same line and so they will be parallel and consequently concurrent in their Infinity point.
Is there a mistake in my reasoning?

Best regards,
Chris

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Message: #2385
Date: 23/3/2017 12:15:27
From: eckart_schmidt@t-online.de
Subject: Orthologic QA-Triple Triangles

Dear Chris,

You are right!
My macro for QL-P15 shows for an overturned QG an incorrect point
(surprisingly discovery after over 5 years of our cooperation!).

So I have to limit my result to Q1 and Q2 with not degenerated QA-triple triangles.
But I think, Steiners theorem in the preface of QA-Tr-3 has also to be limited in this sense.

Best regards Eckart

PS: What about Orthology Center in WolframMathWorld: "The points are isogonal conjugates
of each other".

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Message: #2386
Date: 26/3/2017 3:42:56
From: seiichikiri
Subject: A problem: quadrigon and triangle

Dear friends,

Given a quartic with complex coefficients and its derivative, we have 4 roots and 3 roots respectively in Gaussian plane.
Can we explain the connection of 4 points of the quartic and 3 points of its derivative in the context of the present quadri-figures geometry?
For example, a quadrigon has triple triangles.
A property: if 4 points of a quadrigon are collinear, 3 points are between their successive points.
The case of 3 points and 2 points is Marden's theorem.

Best regards, Seiichi.

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Message: #2387
Date: 26/3/2017 12:36:43
From: Benedetto Scimemi
Subject: A problem: quadrigon and triangle

Dear Seichi,

I once tried to generalize Marden theorem with scarse success.
The only property I found regards the coincidence of centroids of the quadrangle and the triangle of the derivative roots, an easy consequence of linearity. This also explains the collinearity you mentioned in your message.

Best regards Benedetto

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Message: #2388
Date: 27/3/2017 11:13:22
From: eckart_schmidt@t-online.de
Subject: Orthologic QA-Triple Triangles

```
Dear Chris,

what is the exact definition of orthologic triangles?
If we accept orthology wrt collinear degenerated triangles,
any triangle will be orthologic wrt a collinear degenerated
triangle and the cited result of Steiner in the preface of
QA-Tr-3 doesn´t hold.

Best regards Eckart
```

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Message: #2389
Date: 29/3/2017 4:56:28
From: eckart_schmidt@t-online.de
Subject: Four Circles for a QA

```
Dear all,

in #2347  I described four circles for a QA, using QL-geometry
for quadrangles.
Attached more observations, perhaps an interesting QA-cubic.

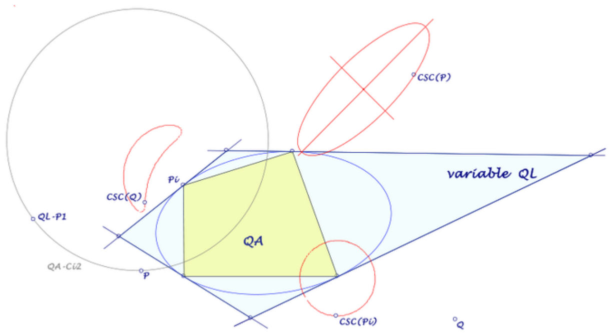
Best regards Eckart
```

EQF-Note 2017-03-29

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Four Circles for a Quadrangle

Tangential quadrilaterals of a quadrangle and their transformation QL-Tf1 (CSC) lead to a lot of curves. CSC-images of the QA-vertices give four circles (see #2347) and CSC-images of points on QA-Ci2 give special conics.



This is a new aspect, to use the *CSC*-transformation for quadrangle geometry:

Consider a quadrangle *QA* with vertices P_i,
... a variable circumconic *Co*
... with the corresponding tangential quadrilateral *QL*
... and its transformation *CSC*.

- **The reference quadrangle and the tangential quadrilaterals have the same diagonal triangle *DT*.**

(1) *CSC*-Traces of the *QA*-Vertices

Normally the *CSC*-trace of a point is a quartic. Here two special cases are researched:

- **The *CSC*-traces of the *QA*-vertices P_i are circles $Ci(P_i)$.**

These four circles are already described in #2347, here a new aspect:

- **The centers of *Ci(Pi)* give a new quadrangle**
  … with a diagonal triangle perspective *DT*
  … and the same *QA-P1*.

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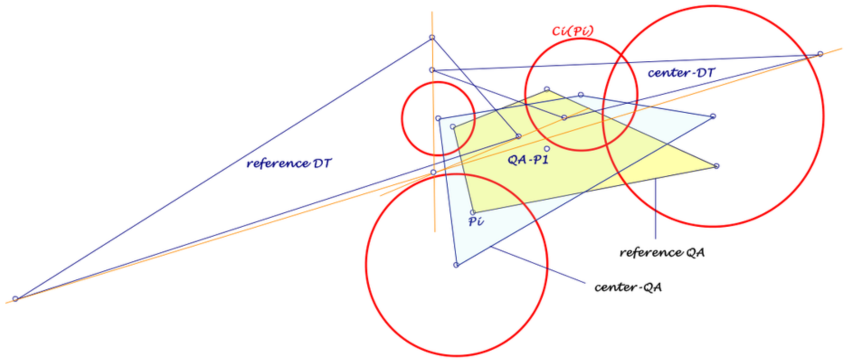
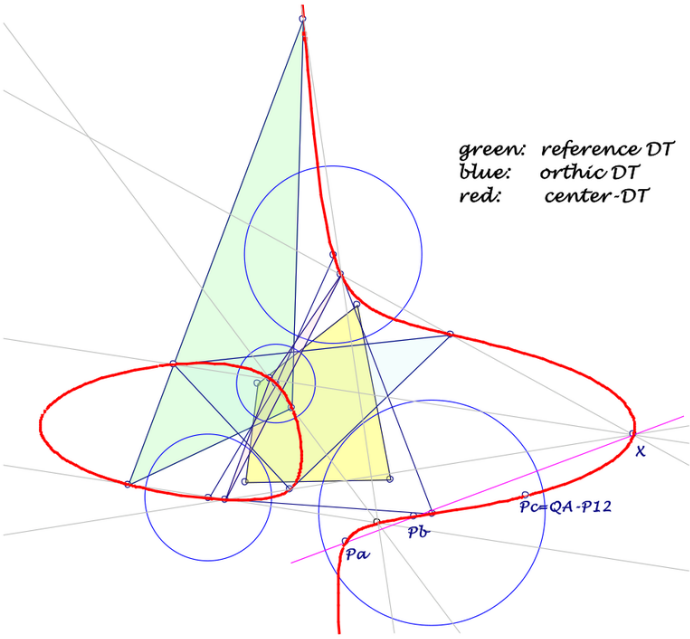


- **The centers of the four circles lie on a cubic bearing the following 16 points:**
  … vertices of the center-*QA*,
  … vertices of the reference *DT*,
  … vertices of the orthic triangle of the reference *DT*,
  … vertices of the center-*DT*,
  … *QA-P12* of the reference *QA*,
  … perspector of reference *DT* and center-*DT*,
  … perspector *X* of orthic triangle and center-*DT*.



- **The cubic is a pivotal isocubic …**
  (a) wrt the orthic triangle of the reference *DT*,
  … isoconjugation: isogonal conjugated,
  … pivot P_a: *QA-Tf2* of *X* wrt the center-*QA*.

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(b) wrt the center-*DT*,
… isoconjugation: *QA-Tf2* of the center-*QA*,
… pivot P_b: isogonal conjugated of *X* wrt the orthic triangle.
(c) wrt the reference *DT*,
… isoconjugation: swaps *QA-P12* and P_a,
… pivot P_c: *QA-P12*.

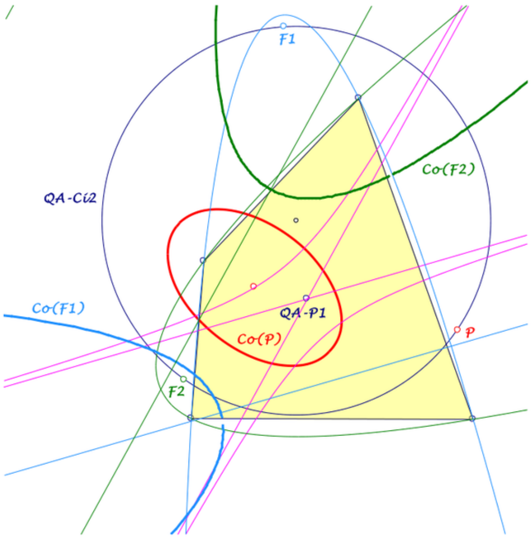
- The common tangential for the vertices …
… of the reference *DT* and *QA-P12* is P_a,
… of the center-*QA* is P_b,
… of the orthic triangle and P_a is the isogonal conjugated of P_a wrt the orthic triangle.

(2) *CSC*-Traces of *QA-Ci2*-Points

- **The *CSC*-traces of points *P* on *QA-Ci2* are conics *Co(P)*,**
… centered on a hyperbola *Hy*
… with *Hy*-center *QA-P1*.

- The *CSC*-traces for the vertices of the orthic triangle (on *QA-Ci2*) degenerate collinear on the opposite side.

- The *CSC*-traces for the foci of *QA-2Co1* (on *QA-Ci2*, not always real) are parabolas again
… with axes parallel to the asymptotes of *Hy*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

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Message: #2390
Date: 31/3/2017 9:58:48
From: eckart_schmidt@t-online.de
Subject: Construction 4 for QL-P26

```
Dear Chris,

QL-P26 is the radical center of the 3 circles
... through two opposite QL-points
... and CSC of the non-collinear QL-DT vertex.

Or:

QL-P26 is the radical center of the 3 QG-circles
... through QG-2P2 and QG-P19.

Best regards Eckart

PS:
QL-P1   is the common point of the 3 QG-circles through QG-2P2
and QG-P18.
QL-P17  is the common point of the 3 QG-circles through QG-2P2
and QG-P15.
```

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Message: #2391
Date: 31/3/2017 11:16:13
From: bernard.keizer
Subject: A problem: quadrigon and triangle

Dear Seiichi, dear Benedetto,
Since Benedetto's message 1585, I also tried to calculate, but without result ...
Here some reflexions
We look for 3 points which resume the 4 points like the 2 points resume the 3 points.
A stelloïd is given by a number of points called pivots and a fixed direction sum of the directions of the lines joining a point of the curve to the n points ; a given stelloïd has an infinity of polygons of the same number of pivots, but always the same fixed direction.
In the case of the Marden theorem, for a given cubic stelloïd, we have following properties :
1) the 2 points are the foci of the Steiner inellipse of every pivot triangle (with necessary the same centroïd)
2) the pivot triangles have therefore the same LSD line
3) the polar curves of the points on the infinity line form a pencil of rectangular hyperbolas, centered in the centroïd and through the 2 points
4) one is degenerated in 2 perpendicular lines, one through the 2 points and the other it's perpendicular bisector
Now, back to your question :
Let's have a quadrangle, it's centroïd and it's LSD line.
If we had a quartic stelloïd through the vertices of this QA with 4 asymptotes intersecting at pi/4 in the centroïd, we would have the same properties for the quadrangles of pivots, which would share the same triangle of 3 points as basis of a pencil of cubic stelloïds formed by the polar curves of the points of the infinity line and the same 2 points on the LSD line of the QA and of the triangle !
I hope these reflexions will give you new ideas ...
Best regards
Bernard

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Message: #2392
Date: 31/3/2017 3:15:41
From: eckart_schmidt@t-online.de
Subject: Circumcircles of QG-2P2 and QG-Px

```
Dear Chris,

in addition to my last message:
QL-P1   is the common point of the 3 QG-circles
        through QG-2P2 and QG-P18.
QL-P17  is the common point of the 3 QG-circles
        through QG-2P2 and QG-P15.
QA-P3   is the common point of the 3 QG-circles
        through QG-2P2 and QG-P15.
QA-P4   is the common point of the 3 QG-circles
        through QG-2P2 and QG-P19.
QA-P41  is the common point of the 3 QG-circles
        through QG-2P2 and QG-P18.
Another aspect:
QA-P16  is the radical center of 3 QL-Ci1.
QA-P9   is the common point of 3 QL-Ci3.
QA-P3   is the common point of 3 QL-Ci6.
QL-P13  is the radical center of 3 QA-Ci1.

I think, there are some new properties for EQF.

Best regards Eckart
```

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Message: #2393
Date: 01/4/2017 10:27:42
From: bernard.keizer
Subject: QL-2P3

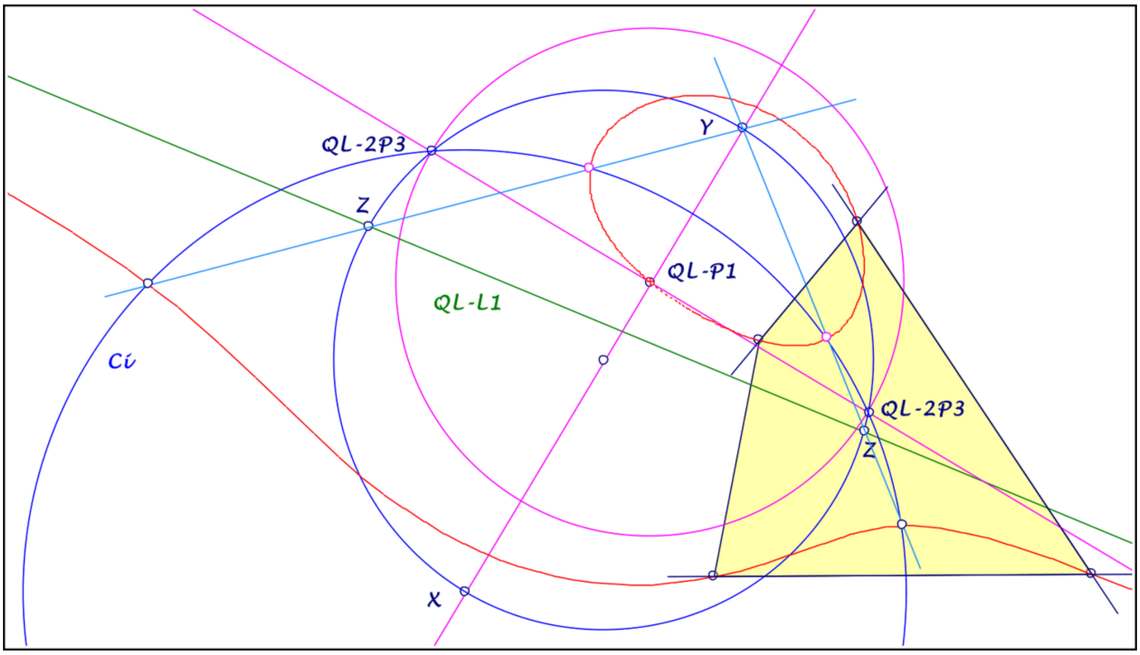
Dear Seiichi, dear Benedetto, dear Bernard G., dear Chris, dear Eckart,
QL-Cu2 is the QL's conjugated cubic stelloïd, it's hessian is the circular focal Van Rees cubic QL-Cu1.
Wrt the 1rst Steiner axis, the sum of the directions of the 3 sides of the pivot triangles of QL-Cu2 and the sum of the directions of the 4 lines of the QL's inscribed in QL-Cu1 are fixed and equal to the direction of QL-Cu2.
The Cl-S transformation of the QL's is the same as the Möbius transformations of the pivot triangles named mu by Benedetto and psi by Bernard G.
The main pivot triangle has the points QL-2P2a and b as the isodynamic points X15 and X16. (Therefore, the Newton Line is the Brocard axis in the unicursal case of QL-Cu1 and the Lemoine axis in the bicursal case).
The 2 points QL-2P3a and b are the foci of the Steiner inscribed inellipses of all pivot triangles and the fixed points of the Cl-S or mu or psi transformations.
These 2 points have several properties, not in EQF :
1) the points are harmonically conjugates wrt the axes of all inscribed conics
2) more precisely, if X and X' are 2 CSC points on QL-Cu1 and m is their middle, XX' is the external bisector of the angle QL-2P3amQL-2P3b and it's perpendicular bisector is the internal bisector of the angle
3) the points X and X' are on 2 orthogonal circles, one centered on the 2nd Steiner axis through QL-2P3a and b and the other centered on the 1rst Steiner axis. These 2 circles are Cl-S invariant and generalise the 8 socalled Steiner circles.
4) the points are harmonically conjugates wrt X and X' on the 1rst circle
Best regards
Bernard

Message: #2394
Date: 01/4/2017 9:05:15
From: eckart_schmidt@t-online.de
Subject: QL-2P3

```
Dear Bernard,

thanks for new aspects of QL-2P3.
Attached a new construction of QL-Cu1, using QL-2P3 without CSC:
Consider circles Ci through QL-2P3
... centered in X on the 2nd Steiner axis.
... Let Y be the inverse of QL-P1 wrt Ci.
... The circle with diameter XY (through QL-2P3)
... intersects QL-L1 in two points Z.
... The intersections of YZ and Ci give points of QL-Cu1.

Best regards Eckart
```



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Message: #2395
Date: 02/4/2017 8:30:23
From: bernard.keizer
Subject: QL-2P3

Dear Eckart,
thanks for your interest
Many thanks in particular for your figure
1) it gives me the opportunity of correcting a mistake in my 2nd property : the line XX' through 2 Cl-S conjugate points is the internal (and not external) bisector of the angle QL-2P3amQL-2P3b2P and it's perpendicular bisector is the external (and not internal) bisector
2) I couldn't dream a better illustration of my properties
Please complete the figure as following : XZ cuts the 1rst Steiner axis through the QL-2P3 in a point U and let's name V and V' the 2 points where one line YZ intersects the circle Ci ; let's name C'i the circle with center U through V and V' (C'i is orthogonal to Ci)
Then remark that you use the Cl-S transformation without saying, as Y, the inverse of QL-P1 wrt Ci is precisely the Cl-S conjugate of X and remember that the Cl-S conjugate of a 1rst circle not through QL-P1 is a 2nd circle centered in the CL-S conjugate of the inverse of QL-P1 in the 1rst circle.
Your construction is in fact my favorite construction, but with a different presentation.
ZY is clearly the internal bisector of QL-2P3ZQL-2P3 and V and V' with middle in Z are 2 Cl-S conjugate points on QL-Cu1 (I start with a variable point Z on QL-L1 and draw the bisectors in order to determine X and U as well as the circles Ci and C'i and the result is the same figur).
The properties 1 and 2 are now clear
Ci and C'i are Cl-S invariant (generalisation of the socalled Steiner circles) : property 3
On the circle Ci, it's visible thatV and V' are harmonic conjugates wrt the QL-2P3a and b
Best regards
Bernard
PS for Chris These simple properties of the QL-2P3 as well as the fact that they are the foci of the Steiner inellipses of all pivot triangles of QL-Cu2 may perhaps be of interest for EQF ...

Message: #2396
Date: 02/4/2017 4:00:00
From: eckart_schmidt@t-online.de
Subject: QL-QA-QL-Circle Geometry

```
Dear all,

what about this observation?

Take a circle
... centered in QL-Px
... through QL-P1.
... Consider these 3 circles for a QA
... and a circle orthogonal to these 3 circles.
... Take this QA-circle 3-times for a QL,
... their radical center will be QL-Px.

Is this evident?

Best regards Eckart
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Message: #2397
Date: 03/4/2017 11:44:12
From: seiichikiri
Subject: A problem: quadrigon and triangle

Dear Benedetto, dear Bernard,

Let z1, z2, z3 and z4 be complex numbers and form a quadrifigure.
They satisfy the equation x^4-(z1+z2+z3+z4)x^3+(z1z2 z1z3+z1z4+z2z3+z2z4+z3z4)x^2-(z1z2z3+z1z2z4+z1z3z4+z2z3z4)x+z1z2z3z4 = 0.
Its 1st differentiated equation is 4x^3-3(z1+z2+z3+z4)x^2+2(z1z2 z1z3+z1z4+z2z3+z2z4+z3z4)x-(z1z2z3+z1z2z4+z1z3z4+z2z3z4) = 0.
Its 3 roots are expressed by xi = (z1+z2+z3+z4)/4+fi (z1-z2,z1-z3,z1-z4,z2-z3,z2-z4,z3-z4) for i = 1, 2 and 3.
Its 2 nd differentiated equation is 12y^2-6(z1+z2+z3+z4)y+2(z1z2 z1z3+z1z4+z2z3+z2z4+z3z4) = 0.
Its 2 roots are expressed by y1, y2 = (z1+z2+z3+z4)/4+-g (z1-z2,z1-z3,z1-z4,z2-z3,z2-z4,z3-z4), where f1, f2, f3 and g are homogeneous equations of degree 1.
x1, x2, x3 and y1, y2 are independent from translation (zj -> zj+k), rotation (zj exp(itheta)) and changes z3->z4, z4->z3 and z2->z3, z3->z2.
So 3 roots and 2 roots of differentiations are quadrangle points.

Best regards, Seiichi.

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Message: #2398
Date: 04/4/2017 10:40:43
From: bernard.keizer
Subject: A problem: quadrigon and triangle

Dear Seiichi,
the formulas of cubic or square roots of equations may in fact allow to solve the equations, but they are not simple and use i and j, the square and cubic roots of the unity ...
You are perfectly right, the 5 points are QA points and the triangle has centroïd QA-P1 and the 2 roots are symmetric wrt QA-P1, but this doesn't say where the 5 points are ...
Best regards
Bernard

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Message: #2399
Date: 04/4/2017 11:03:38
From: bernard.keizer
Subject: Apolarity

Dear Chris, dear Eckart,
2 other simple properties of the stelloïds apply to the cubic stelloïd QL-Cu2 and it's hessian QL-Cu1
The cubic stelloïd is apolar to any inscribed conic, in particular to any pair of conjugate points of it's hessian.
It's polar conics are also apolar to these pair of points.
Every line XX', where X and X' are Cl-S points on QL-Cu1 (the line is tangent to the cayleyan), cuts any polar conic of the cubic stelloïd QL-Cu2 in 2 points harmonic conjugates wrt X and X' (well known property)
The line XX' cuts the stelloïd in 3 points P1, P2 and P3
1) the group of 3 points is apolar to the 2 points, which means that the 3 anharmonic ratios are equal :
(X, X', P1, P2) = (X, X', P2, P3) = (X, X', P3, P1)
2) the tangents to the stelloïd QL-Cu2 in the 3 points and the tangents to the hessian QL-Cu1 in X and X' concur in the tangential X1 of X and X' on the hessian (XX' is one of the 2 perpendicular lines forming the degenerated polar conic of this tangential point X1 wrt the cubic stelloïd ; this polar conic is centered in X0, the Cl-S conjugate point of X1 and the 3rd intersection of XX' with the hessian QL-Cu1).
These 2 properties may be of interest for EQF ...
Best regards
Bernard

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Message: #2400
Date: 04/4/2017 2:46:57
From: eckart_schmidt@t-online.de
Subject: Apolarity

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Dear Bernard,

am I right, that only one of the points P1, P2, P3 is real?

Best regards Eckart
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Message: #2401
Date: 04/4/2017 3:21:49
From: tsihonglau
Subject: Search Keys for Quadrangle/Quadrilateral

```
Dear all,
I propose another search keys for triangle/trilateral and
quadrangle/quadrilateral.
a=5,b=29,c=30 and d=28,e=15,f=11 (In EQF p=28,q=15,r=11 and
l=1/28,m=1/15,n=1/11)
The rationale is the two areas of (diapleural) triangles are
72(a,b,c) and 72i(d,e,f) respectively.
Imaginary area is necessary for two real
circumparabolas(QA-2Co1).
We had better fix (diapleural) triangle vertices- A(0,0)
B(24,18) C(20,21).
For a finite point, we search the x,y-coordinates.
For an infinite point, we search the slope.
For a line, we search the x,y-interceps.
I do not if there is any coincidence for ETC centers.
I will try the search keys.

Best regards
Tsihong Lau
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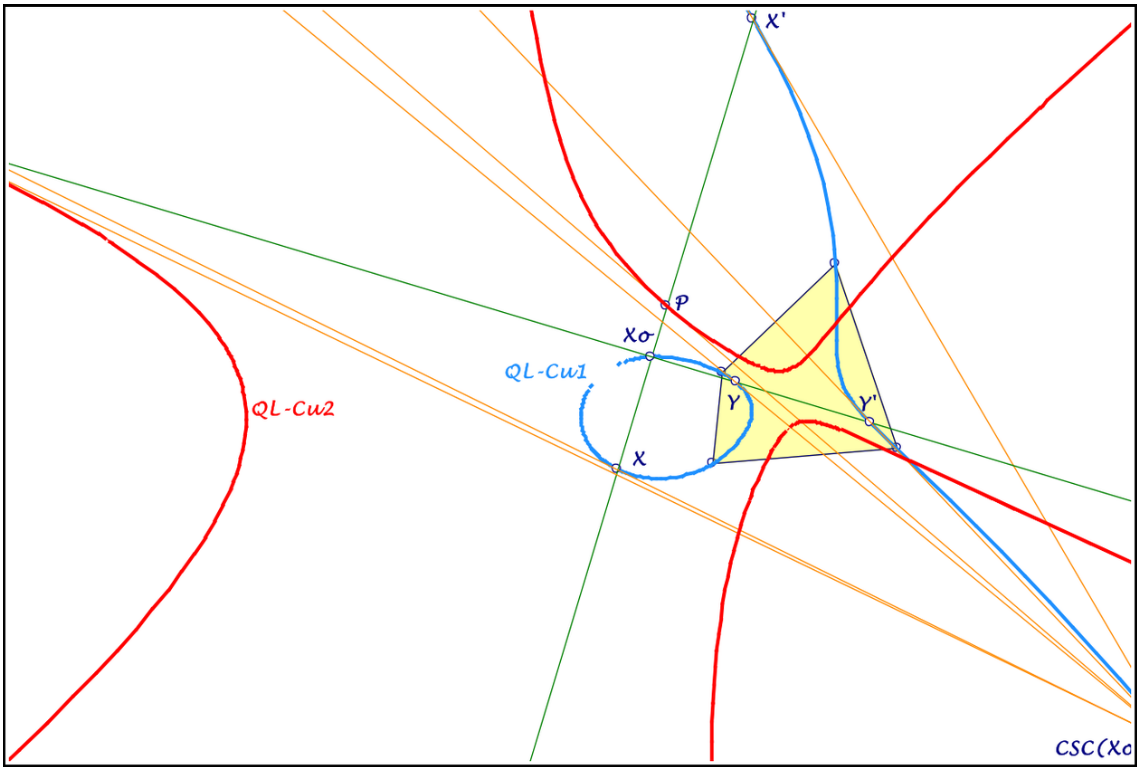
Message: #2402
Date: 05/4/2017 9:26:30
From: eckart_schmidt@t-online.de
Subject: QL-Cu1, QL-Cu2

Dear Bernard,

I try to describe the constellation in your message 2399 with
two attached figures.
Let X, X' be CSC-partner on QL-Cu1,
... X0 3rd intersection of XX' and QL-Cu1,
... L perpendicular in X0 to XX',
... P intersection of XX' and QL-Cu2.
In the unipartite case of QL-Cu1
... has the line L no further intersection with QL-Cu1,
... has the line L three intersections S1, S2, S3 with QL-Cu2.
... The tangents in X, X' at QL-Cu1
... and the tangents in P, S1, S2, S3 at QL-Cu2
... have a common point in CSC(Xo) on QL-Cu1.
In the bipartite case of QL-Cu1
... has the line L two CSC-partner Y, Y'
    as intersections with QL-Cu1,
... has the line L one intersection Q with QL-Cu2.
... The tangents in X, X', Y, Y' at QL-Cu1
... and the tangents in P, Q at QL-Cu2
... have a common point in CSC(Xo) on QL-Cu1.

Best regards Eckart



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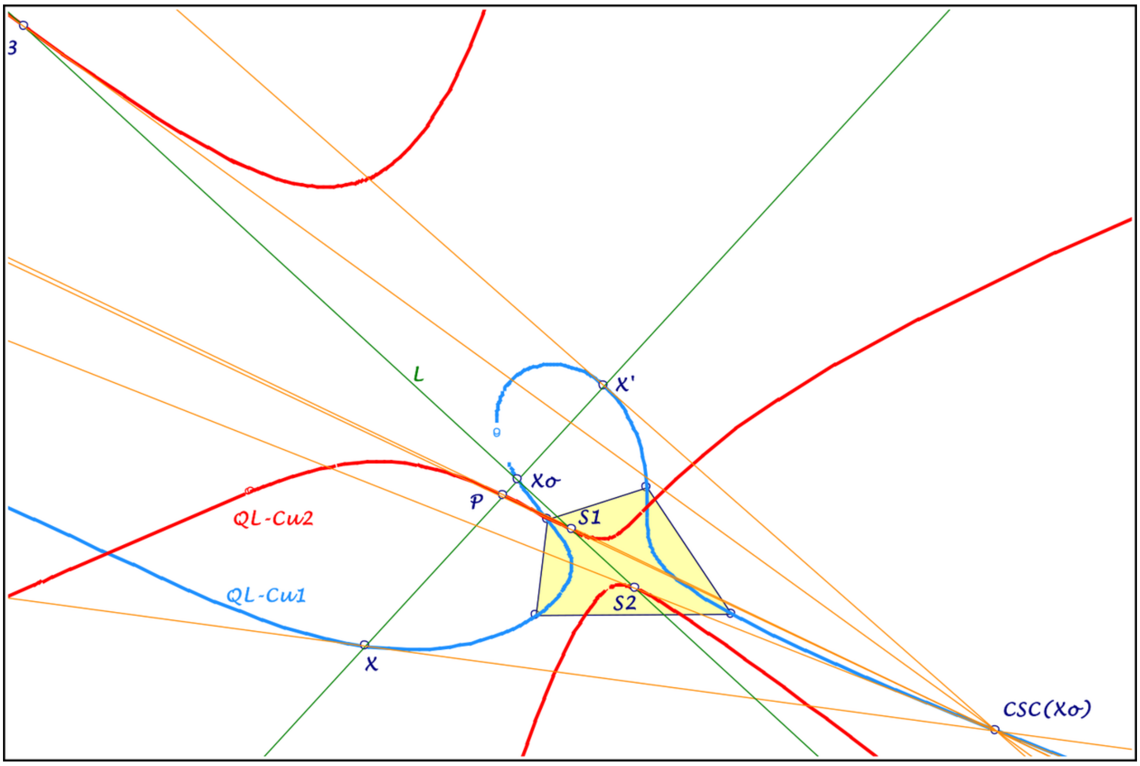

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Message: #2403
Date: 05/4/2017 3:33:14
From: bernard.keizer
Subject: QL-Cu1, QL-Cu2

```
Dear Eckart,

I tried in vain to find a counter-example.
So it seems you're right and we have never 5 real points
(curiously, it's always 3 real and 3 imaginary)!
Of course, I don't know what the anharmonic ratios are with
imaginary points ...

Best regards
Bernard
```

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Message: #2404
Date: 05/4/2017 4:02:06
From: eckart_schmidt@t-online.de
Subject: 3rd cut wrt CSC-partner on QL-Cu1

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Dear Bernard, dear Chris,

for CSC-partner X, X' on QL-Cu1 the 3rd intersection Xo of XX'
and QL-Cu1 can easily be constructed:
Xo is the intersection of the perpendiculars XX' and
QL-Tf2(XX').
CSC(Xo) is the common tangential of X and X'.
This gives an easy construction of the tangent in X at QL-Cu1.
If QL-Cu1 is bipartite:
... Let X, X' be CSC-partner on QL-Cu1
... and Y, Y' CSC-partner on QL-Tf2(XX')
    (intersections with QL-Cu1),
... then is QL-Cu1 the cubic QA-Cu1 for XX'YY'.

Best regards Eckart
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Message: #2405
Date: 06/4/2017 9:27:50
From: bernard.keizer
Subject: 3rd cut wrt CSC-partner on QL-Cu1

Dear Eckart,
As XX' and Tf2(XX') form the polar conic of X1 = CSC(X0), the 2 lines intersect every other line joining 2 CSC points in 2 harmonic points wrt these 2 CSC points.
If we take the parallel to the Newton Line through QL-P1 (axis of the parabola), the 2 harmonic points wrt QL-P1 and the infinity points are 2 points symmetric wrt QL-P1.
Long explanation, but simple construction : QL-Tf2(XX') is the perpendicular to XX' through the symmetric wrt QL-P1 of the intersection of XX' with the axis of the parabola ; this gives X0 as intersection of XX' and QL-Tf2(XX').
Your last remark is very important as it leads to plenty of well-known interesting properties : for example
° vertices of variable DT on QL-Cu1
° fixed Miquel triangle on QL-Cu1 : QL-P1 and the QL-2P2
° point QA-P4 in X1 and QA-P2 in X0, both on QL-Cu1
° QA-P1 is the middle of the segment joining the middles of X and X' and of Y and Y', therefore on QL-L1
° perspector of DT and Miquel triangle is QA-P3, symmetric of QA-P2 in QA-P1 : the line QA-P2QA-P3 passes through QL-P1 and QA-P3QA-P4 is parallel to the asymptote of QL-P1 and to the Newton Line.
It could be interesting to study the locus of the QA-points when X and X' describe QL-Cu1 ...
Best regards
Bernard

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Message: #2406
Date: 06/4/2017 3:46:21
From: bernard.keizer
Subject: Syzegetic pencil

Dear Eckart,
As you seem to revisit the cubics QL-Cu1 and QL-Cu2, this construction may perhaps interest you. Let's start with a copple of CSC points X and X' on QL-Cu1, with tangential X1 and 3rd intersection X0. (The curve QL-Cu1 can be unicursal or bicursal). Let's draw the polar conics of X and of X' wrt QL-Cu2; each is degenerated in 2 perpendicular lines intersecting in the other point.
The 4 lines intersect in 4 points Y and Y', Z and Z'.
1) Y, Y', Z and Z' form an orthocentric QA and the common Euler circle is through X, X', X1 and QL-P1
2) Y, Y', Z and Z' are the poles of the line XX' wrt QL-Cu2, it means that the polar conic of each point of XX' passes through the 4 points ; in X0, it degenerates in 2 perpendicular lines.
3) YY' and ZZ' intersect in X1, XX' and YY' in Z1 and XX' and ZZ' in Y1
Y1 and Z1 are harmonic conjugates wrt X and X', X1 and Y1 wrt Z and Z' and X1 and Z1 wrt Y and Y'
4) The perpendicular to XX' in X0 intersect YY' in Y0 and ZZ' in Z0
5) As the 4 lines as well as XX' and the perpendicular to XX' in X0 are tangents to the cayleyan, it follows that from each of the 12 points X, X', X0 and X1, Y, Y', Y0 and Y1 and Z, Z', Z0 and Z1 we may draw 2 tangents to the cayleyan (obviously not in the same order)
6) X, X', X0 and X1 are on the hessian QL-Cu1 of stelloïd QL-Cu2
Y, Y', Y0 and Y1 are on the hessian of a 2nd cubic
Z, Z', Z0 and Z1 are on the hessian of a 3rd cubic
7) QL-Cu2, the 2nd and 3rd cubic have the same cayleyan
8) These 3 cubics (only QL-Cu2 is known) as well as their hessians have the same real inflexion points on a line and form a syzegetic pencil of cubics.
9) One of the circles through X, X', Y and Y' is through the 2 points QL-2P3 and the other through X, X', Z and Z' is orthogonal to the 1rst and centered on the 1rst Steiner axis.
Both circles are the generalisation of the socalled Steiner circles.
For the vertices of the QL, this construction gives the in- and excenters of the 4 reference triangles.
Best regards
Bernard

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Message: #2407
Date: 08/4/2017 10:34:16
From: bernard.keizer
Subject: Syzegetic pencil

Dear Eckart,
Waiting impatiently for your comments, I wish to add some commentars on my last comment about the QL vertices.
In fact, there are 2 Steiner circles for each copple X and X' of CSC points on QL-Cu1, and again for X0 and X1.
For the QL, there are the 8 historical Steiner circles, with in- and excenters, the 6 Steiner circles for the 3 copples of CSC vertices and the 6 Steiner circles for the copples of CSC formed by the tangentials of these 3 copples and the 3rd intersections of the 3 diagonals (the polar conics of the tangentials are formed by the sides and altitudes of DT) ...
But conversely, every circle through the points QL-2P3 cuts the hessian QL-Cu1 in 3 copples of CSC points, one being formed by the 2 circular points ; and of the 2 remaining copples of points, one may be imaginary. Again, any circle through 2 CSC points and centered on the 1rst Steiner axis cuts the hessian QL-Cu1 in 2 other CSC points.
This leads then to plenty of other socalled Steiner circles ...
Best regards
Bernard
PS there was a typo in my message 2403 : it is 3 real points and 2 imaginary (the number of imaginary points must be even)

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Message: #2408
Date: 08/4/2017 11:07:37
From: bernard.keizer
Subject: QA and apolarity

Dear Chris, dear Eckart, dear Bernard G.
In his article about Cardioïds and Eckart's cubic, Bernard G; mentionned an article by Jules Marchand Géométrie du quadrilatère complet.
Jules Marchand described the QL's conjugated cubic stelloïd as having as set of polar conics the rectangular hyperbolars intersecting the 3 diagonals of the QL in points harmonically conjugated to the vertices of the QL and as set of apolar conics the confocals of the QL's inscribed conics.
As well known, the cubic stelloïd is QL-Cu2 and it's hessian, locus of the foci of the inscribed conics is QL-Cu1.
At the end of this article, Jules Marchand adds, it would be interesting to find the same way a QA's conjugated cubic apolar to all QA's circumscribed (Bernard G. names them diagonal) conics.
In this set of conics are 3 degenerated in 3 copples of lines (it's the dual property of the 3 copples of opposite vertices of the QL).
To get the cubic stelloïd, it was necessary to add another copple of points, the circular points.
The same way, to get the searched QA's cubic, we have to add another copple of lines forming a 4th degenerated apolar conic.
I don't know if Jules Marchand has written another article, I haven't found any other reference.
I don't know either if Bernard G. has already studied such a cubic wrt a QA and not a triangle.
I would be highly grateful for any reference or reflexion about this new item.
(For example, what is the locus of the foci of these diagonal conics?)
Many thanks in advance.
Best regards
Bernard

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Message: #2409
Date: 08/4/2017 11:33:26
From: Bernard Gibert
Subject: QA and apolarity

```
Dear Bernard K.

>> I don't know if Jules Marchand has written another article,
>> I haven't found any other reference
> See:
> http://www.numdam.org/item?id=NAM_1927_6_2__320_1
> http://www.unige.ch/math/EnsMath/EM_fr/welcome.html
> 1930, p.289
> I don't know either if Bernard G. has already studied
> such a cubic wrt a QA and not a triangle.

Actually, I haven’t

Best regards
Bernard
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Message: #2410
Date: 08/4/2017 7:07:33
From: bernard.keizer
Subject: QA and apolarity

```
Dear Bernard G.

Thanks a lot for your quick answer!
I knew the 1rst article, but not the 2nd.
I've also found on Amazon Etude géométrique des courbes
apolaires à la paire ombilicale simple ou multiple, which is the
print of his thesis in Lausanne.
But nothing about a QA's conjugated cubic apolar to all the
diagonal conics ...

Best regards
Bernard
```

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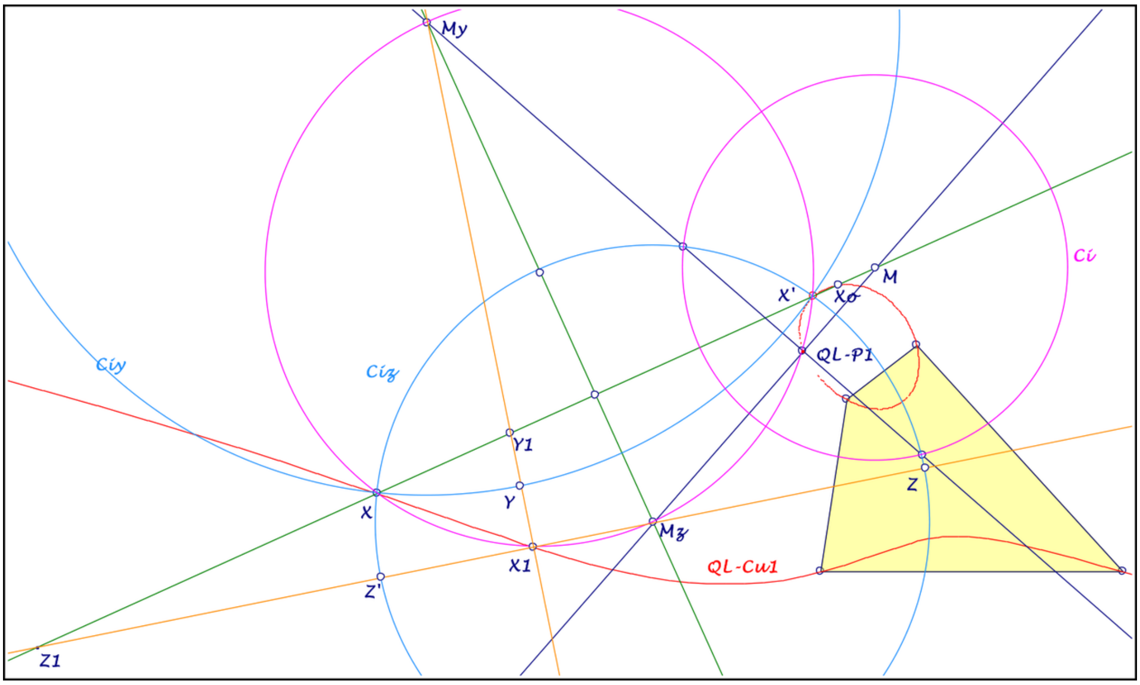
Message: #2411
Date: 08/4/2017 8:56:17
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

```
Dear Bernard,

thanks for the detailed construction sequence in #2406.
My handycap: I am not familar with polar conics.
So I made a modified construction of the 12 points (see attached
file):
Let X, X' be CSC-partner on QL-Cu1
... with common tangential X1 and 3rd intersection Xo.
The bisector of XX'
... intersects the 1st Steiner axis in My
... and the 2nd Steiner axis in Mz.
Consider the CSC-invariant circles
... Ciy centered in My through X and X',
... Ciz centered in Mz through X and X',
... Ci orthogonal to Ciy and Ciz,
... ... centered in the intersection M of XX'
        and the 2nd Steiner axis (inverse of QL-P1 wrt Ciz).
CSC-partner
... on Ciy are collinear with the intersection of XX'
    and the 1st Steiner axis,
... on Ciz are collinear with the center M of Ci,
... on Ci are collinear with the center Mz of Ciz.
The points QL-2P3
... lie on Ciz and Ci
... and are inverse wrt Ciy.
The points X, X' and QL-P1, Mz lie inverse wrt Ci.
Let Z1 be the inverse of Xo wrt Ci
... let Y1 be the 4th harmonic point of Z1 wrt XX'.
Let Y, Y' be the intersections of Ciy and X1My,
... let Z, Z' be the intersections of Ciz and X1Mz.
The points X, X', My, Mz, X1, QL-P1
... lie on a circle,
... which is the CSC-image of XX'
... and the locus for the centers of orthogonal circumhyperbolas
... ... of the orthocentric quadrangle YY'ZZ'.

Perhaps some new aspects.
Best regards Eckart
```



2017-04-08.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2412
Date: 09/4/2017 5:55:22
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear all,

I update the list.
reflexes
incenter/excenters quadrangle - reference quadrangle
diapleurals of the two quadrangles
diagonals of the two quadrangles
dual quadrilaterals of the two quadrangles
expoint triangles of the two quadrangles
exline trilaterals of the two quadrangles
the circular and parabolic points at infinity

symmetric objects
diapleural triangle of the two quadrangles
diagonal trilateral of the two quadrangles
centroid/anticomplementary triangle of the diapleural triangle quadrangle
dual quadrilateral of the above quadrangle
diapleurals of the above qudrangle
diagonals of the above quadrangle
the line at infinity
common harmonics of the circular and parabolic points at infinity

reflex concepts
isogonal conjugations of points and lines - isoconjugations of points and lines
self-isogonal cubic - self-isoconjugation cubic

symmetric concepts
isotomic conjugations of points and lines
midpoint of two points
reflection point of a point with respect to another point
complement of a point
anticomplement of a point
center of a conic
centroid of triangle, quadrangle, etc
common harmonic isoconjugations
of points and lines(See below)
Be careful, angle bisectors and perpendicular
bisector are not symmetric since there is pseudo
perpendicularity.

symmetric objects
QA-P10=X(2) centroid

symmetric objects derived from common harmonic isoconjugations
QA-P2 Euler-Poncelet point
QA-P29
QA-P35 1st Penta Point
QA-P39
QA-Co2 QA-Orthogonal Hyperbola
QA-Co4 QA-DT-P3-P12 Orthogonal Hyperbola

reflex objects
triangle/trilateral objects - quadrangle/quadrilateral objects
X(20) de Longchamps Point - QA-P5 Isotomic Center
QA-P11=X(3) Circumcenter - QA-P1 QA-Centroid
QA-P12=X(4) Orthocenter - QA-P20
QA-P13=X(5) Nine-point center - QA-P22
X(6) Symmedian Point - QA-P16 QA-Harmonic Center =
QL-P13 QL-Harmonic Center
X(64) - QA-P17
X(25) - QA-P18
X(69) - QA-P19
X(1350) - QA-P21
X(10304) - QA-P25 1st QA-Quasi Centroid
X(3524) - QA-P26 2nd QA-Quasi Centroid
X(3424) - QA-P27 M3D Center
X(141) - QA-P31
X(3523) - QA-P43 Least Squared Dist. Point QA and QA-DT
X(351) Center of the Parry Circle - QL-P12 QL-Centroid or
Lateral Centroid
X(?) - QL-P14 1st QL-Quasi Centroid
X(?) - QL-P15 2nd QL-Quasi Centroid
X(?) - QL-P18
X(669) - QL-P23 Center of the Inscribed Midline Hyperbola
QA-L5=Euler line - QA-L3 QA-Centroids Line
Jerabek Hyperbola - QA-Co5
K004 Darboux cubic - QA-Cu2 QA-DT-P5 Cubic
K002 Thomson cubic - QA-Cu3 QA-DT-P10 Cubic
K169 - QA-Cu4 QA-DT-P19 Cubic
K003 McCay cubic - QA-Cu5 QA-DT-P1 Cubic
K258 - QA-Cu6 QA-P1-Involution Center Cubic

quadrangle/quadrilateral objects - quadrangle/quadrilateral
objects
QA-P3 Gergonne-Steiner point - QA-P30
QA-L8 - QA-L9
QA-Ci1 Circumcircle - QA-Co1 Nine-Point Conic

The equation for QA-Co4 is

```
(a^2(q^2-r^2)-p^2(b^2-c^2))yz+(b^2(r^2-p^2)-q^2(c^2-a^2))zx+(c^2
(p^2-q^2)-r^2(a^2-b^2))xy=0
```

The common harmonics of the circular and parabolic points at infinity are the points at infinity of QA-Co4.

The transformation of points is called the common harmonic isoconjugation of points:

```
x:y:z -> (a^2(q^2-r^2)-p^2(b^2-c^2))yz:(b^2(r^2-p^2)-q^2(c^2-a^2
))zx:(c^2(p^2-q^2)-r^2(a^2-b^2))xy
```

The QA-Co4 is the common harmonic isoconjugation of the line at infinity.

The transformation of lines is called the common harmonic isoconjugation of lines:

```
ux+vy+wz=0 -> ux/(a^2(q^2-r^2)-p^2(b^2-c^2))+vy/(b^2(r^2-p^2)-q^
2(c^2-a^2))+wz/(c^2(p^2-q^2)-r^2(a^2-b^2))=0
```

The common harmonic isoconjugations are much like isogonal conjugations and isoconjugations of points and lines. We should study their properties.

Please refer to topics #2190 and #2196 for more information.

Best regards,
Tsihong Lau

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Message: #2413
Date: 09/4/2017 4:05:59
From: tsihonglau
Subject: A Circular Quartic of Quadrangle

Dear all,

I studied the lines and curves mentioned in the previous message and got another view point.

1. Given two points P(p:q:r) and U(u:v:w), the line through P and U is:
(qw-rv)x+...=0 or det([x,y,z],[p,q,r],[u,v,w])=0
If P and U are the circular points, it is the line at infinity.
The circumconic of the reference triangle ABC and P and U is: pu(qw-rv)yz+...=0 or det(1/[x,y,z],1/[p,q,r],1/[u,v,w])=0
If P and U are the circular points, it is the circumcircle of ABC.

2. Given two quadrangles (a:b:c),(-a:b:c),(a:-b:c),(a:b:-c) and (d:e:f),(-d:e:f),(d:-e:f),(d:e:-f), the circumconic of the two quadrangles is: det([x,y,z]^2,[a,b,c]^2,[d,e,f]^2)=0
If the former is the incenter/excenters quadrangle and the latter is the reference quadrangle, it is QA-Co2 of the latter.
Now we can get a circumquartic of both quadrangles and the diapleural triangle ABC:
det(1/[x,y,z]^2,1/[a,b,c]^2,1/[d,e,f]^2)=0
Has the quartic been studied before?
I wonder more properties of it.

Best regards,
Tsihong Lau

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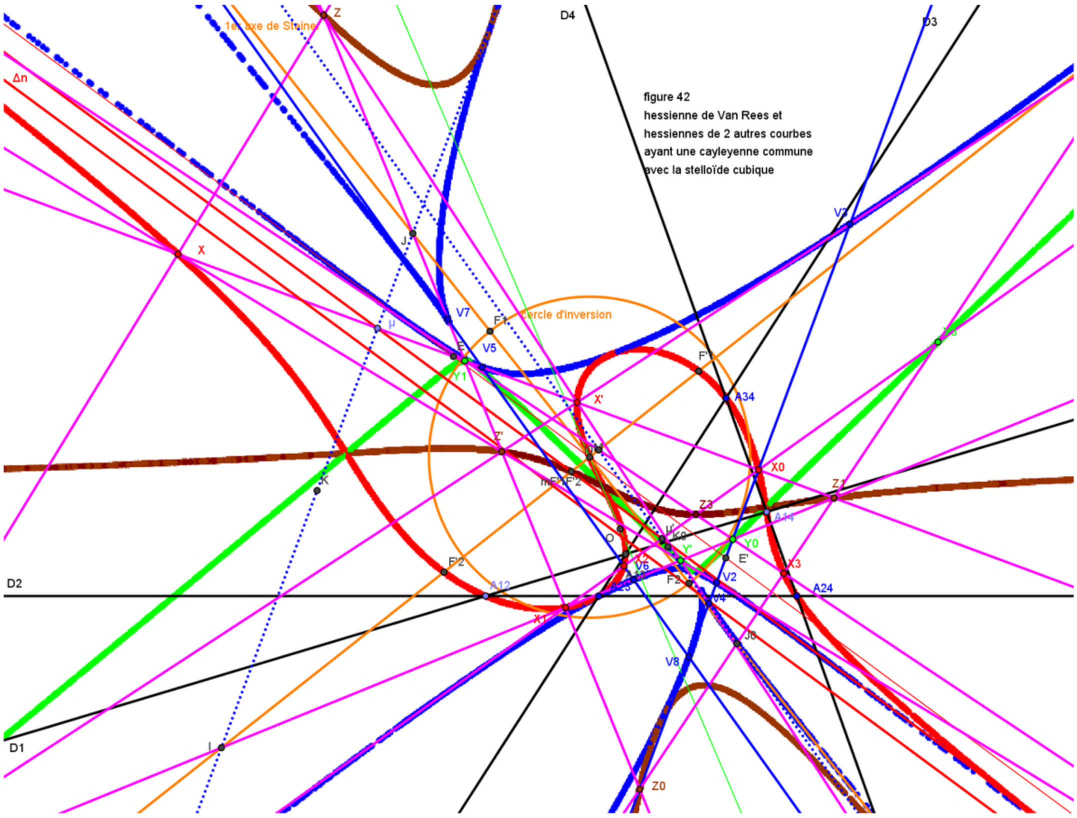
Message: #2414
Date: 09/4/2017 6:46:52
From: bernard.keizer
Subject: Syzegetic pencil

Dear Eckart,
Thanks for your reaction !
The way you get the 12 points is secundary, the important thing is that we have a common figure to discuss.
Personnally, I find more simple to say that YZ and Y'Z' are the bisectors of the angle X0XX1 and Y'Z and YZ' the bisectors of the angle X0X'X1 (well-known property of QL-Cu1, as X0 and X1 are CSC partners).
The rectangular circumhyperbolas of YY'ZZ' are the polar conics of the points of XX' wrt QL-Cu2.
You have only 10 points ; let's add Y0 and Z0 as intersections between the perpendicular to XX' in X0 and YY' and ZZ'. Let's add 3 new points, U, V and W as harmonic conjugates of X0, Y0 and Z0 wrt XX', YY' and ZZ'.
Now the important thing is following : while X and X' (and X0 and X1) describe QL-Cu1, Y and Y' (and Y0 and Y1) on one side and Z and Z' (and Z0 and Z1) on the other side describe the hessians of the 2 other cubics having the same cayleyan as QL-Cu2 and U, V and W are the contact points of XX', YY' and ZZ' with this cayleyan.
The 3 cubics (QL-Cu2 and 2 other unknown) and the 3 hessians (one of them is QL-Cu1) belong to the same syzegetic pencil of cubics determined by QL-Cu2 and it's hessian QL-Cu1 and having the same inflexion points (see Bernard Gibert Inscribed cardioïds and Eckart's cubic page 18).
I always prefer your figurs, but I join one figure I made some times ago (the notations are a little bit different, but I suppose, it will be clear; QL-P1 is M and QL-2P3 are F1 and F2 ...)
I hope this will retain your attention.
Best regards
Bernard

3 hessians of the 3 cubics

having the same cayleyan



3 hessians and cayleyan.pdf

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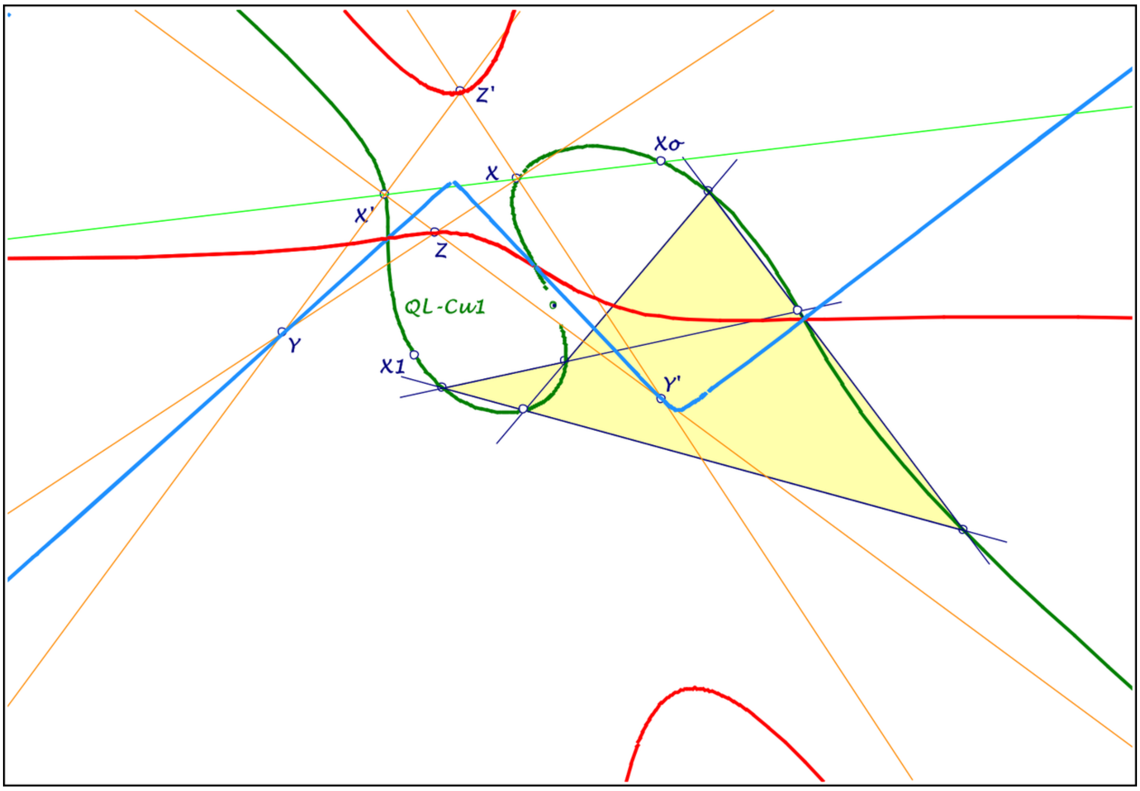
Message: #2415
Date: 10/4/2017 9:43:13
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

Dear Bernard,

thanks for explanations! Perhaps helpful, attached a construction for the three hessians.

Best regards Eckart



2017-04-10.pdf

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Message: #2416
Date: 10/4/2017 6:33:41
From: chris.vantienhoven
Subject: Toeplitz' conjecture about inscribed squares

```
Dear friends,

The inscribed square problem, also known as the square peg
problem or the Toeplitz' conjecture, is an unsolved question in
geometry: Does every plane simple closed curve contain all four
vertices of some square? This is true if the curve is convex or
piecewise smooth and in other special cases.
This stunning conjecture is described here:
https://en.wikipedia.org/wiki/Inscribed_square_problem (
https://en.wikipedia.org/wiki/Inscribed_square_problem )

Best regards,
Chris
```

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Message: #2417
Date: 11/4/2017 11:51:23
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

Dear Bernard,

I try to get familar with polar conics:
Please correct, if my interpretation of your results isn´t right
(see attached files).

Let X, X' be CSC-partner on QL-Cu1,
... Xo 3rd intersection of XX' and QL-Cu1
... and X1 = CSC(Xo).

The polar conic of a point X on QL-Cu1 wrt QL-Cu2
... is degenerated in two orthogonal lines,
... which are the inner/outer angle bisectors of <>

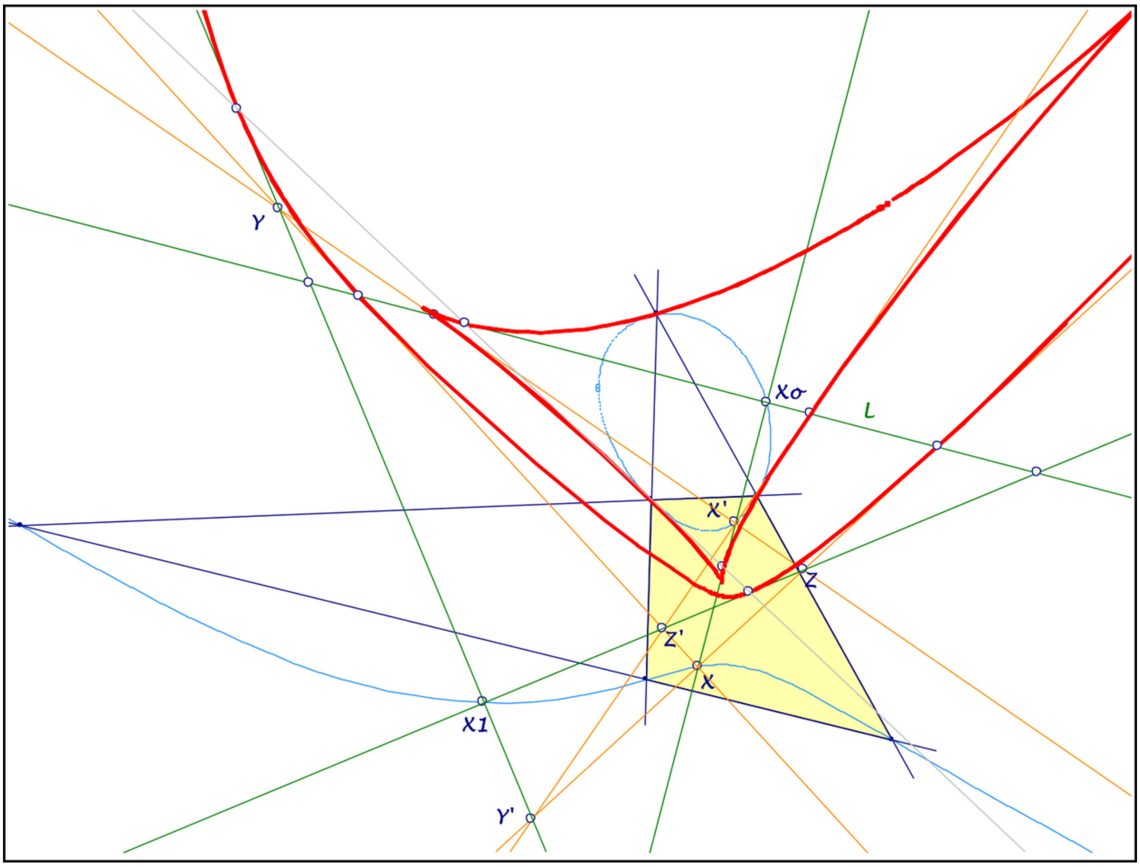
Intersections of the angle bisectors of <>
... Y = intersection of the inner/inner bisectors,
... Y' = intersection of the outer/outer bisectors,
... Z = intersection of the outer/inner bisectors,
... Z' = intersection of the inner/outer bisectors.

Polar conics
... of X = YZ and Y'Z', of X' = YZ' and Y'Z,
... of Xo = YY' and ZZ' , of X1 = XX' and the
    perpendicular L in Xo wrt XX'.

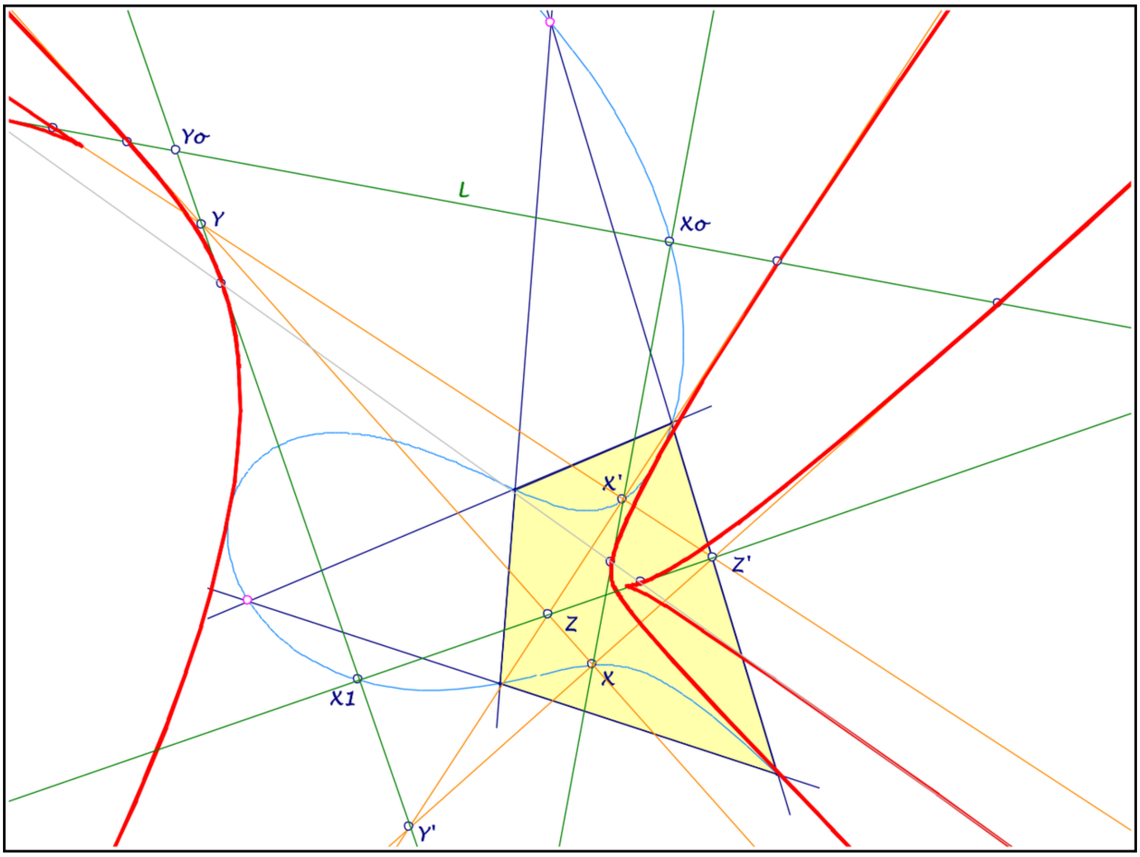
The polar conics of X, X', Xo, X1
... are tangent to the cayleyan
... with following contacts:
... ... YZ, Y'Z', YZ', Y'Z: intersections
        with the perpendicular L in Xo wrt XX',
... ... YY', ZZ': 4th harmonic points
        of Yo = YY'^L, Zo = ZZ'^L,
... ... XX': 4th harmonic point of Xo,
... ... L: intersection with the line
        through the contacts of XX', YY' and ZZ'.

The loci of Y, Y', Yo and Z, Z', Zo give the 2nd and 3rd
hessian.

Best regards Eckart



2017-04-11b.pdf



2017-04-11a.pdf

Message: #2418
Date: 11/4/2017 4:17:02
From: bernard.keizer
Subject: Syzegetic pencil

Dear Eckart,
It's a real pleasure to have this discussion with you.
In fact, all this seems perfect and I don't see what I could add!
Maybe a few remarks :
QL-Cu1 and QL-Cu2 intersect in 9 points (3 real and 6 imaginary), which are aligned and correspond to the 3 cusps of the cayleyan ; you could perhaps make 2 beautiful figures synthetising the 2402, the 2415 and the 2417 with the cubic stelloïd, the 3 hessians and the cayleyan in both unicursal and bicursal cases.
The polar conic of a point X is formed by the 2 bisectors of all the angles UX'U', where U and U' are any other CSC partners on QL-Cu1 (opposite vertices of the QL, points QL-2P2, therefore X0 and X1 ...).
The 3 copples of points X and X', Y and Y' and Z and Z' play a symmetrical role ; XX' and YY' intersect in Z1 and XX' and ZZ' in Y1. T he polar conic of Y wrt the 2nd unknown cubic is made of the 2 lines X'Z and XZ' (not perpendicular, as this 2nd cubic is not a cubic stelloïd), the polar conic of Y' is XZ and X'Z', the one of Y0 is XX' and YY', the one of Y1 is XX' and your line L. the same goes for Z, Z', Z0 and Z1.
X, X', Z and Z' are the 4 poles of the line YY', meaning that the polar conic of every point of the line is a circumconic of the 4 points ; and one is the socalled Steiner circle.
The same goes for the line ZZ' ; the polar conic of every point of the line is a circumconic of the 4 points X, X', Y and Y' ; and one is a socalled Steiner circle, through the 2 points QL-2P3. (the 2 circles are orthogonal).
The beauty of this construction is that it makes clear why 3 cubics have the same cayleyan : the 4 lines are the same, but in a different order (for example, XX', YY', ZZ' and your line L are the degenerated polar conics of X0 and X1 wrt the cubic stelloïd, but also of Y0 and Y1 wrt the 2nd unknown cubic and of Z0 and Z1 wrt the 3rd unknown cubic).
The last step would be to identify these 2 unknown cubics, but that's another story ...
Best regards
Bernard

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Message: #2419
Date: 11/4/2017 6:23:30
From: bernard.keizer
Subject: Syzegetic pencil

```
Dear Eckart,
Sorry, I forgot a last detail
The perpendicular bisectors of XX' and X0X1 are also tangent to
the cayleyan (which is the envelop of both axes of the inscribed
conics)
Best regards
Bernard
```

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Message: #2420
Date: 12/4/2017 8:24:40
From: chris.vantienhoven
Subject: Holiday

```
Dear friends,

Next saterday I will be going for a 6 week long walk from St.
Jean-Pied de Port (France) to Santiago de Compostella (Spain).
As a consequence I will not be able to do any geometry. Because
of all kind of circumstances and because of the preparation of
my journey I wasn't able to study any QFG-messages last two
months. I don't think I can catch this up later. It is a pity. I
saw passing all kind of interesting subjects. Have a good time
you all!

Best regards,
Chris
```

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Message: #2421
Date: 12/4/2017 10:40:51
From: bernard.keizer
Subject: Holiday

```
Dear Chris,

I knew the Flying Dutchman, but not the Walking Dutchman!
Have a nice trip

Best regards
Bernard
```

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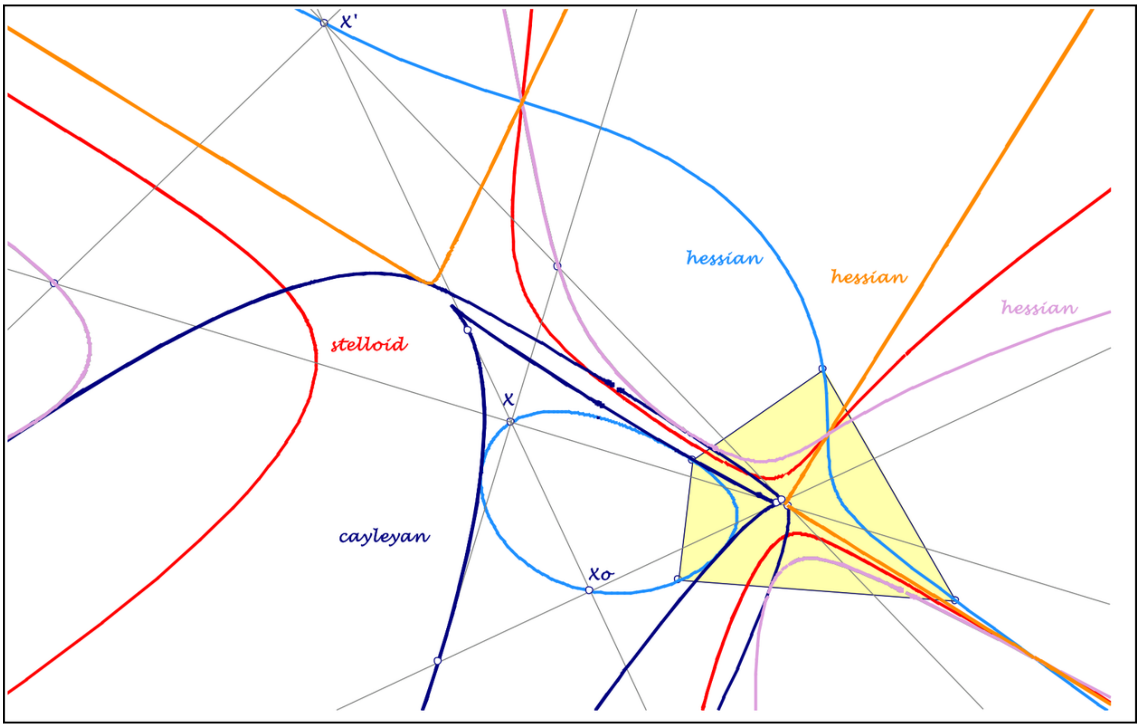
Message: #2422
Date: 12/4/2017 11:40:26
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

Dear Bernard,

I tried, to make a construction of the four cubics and the cayleyan in the unipartite and bipartite case of QL-Cu1, but it is difficult to do this clearly arranged.
I have only shown the polar conics of X, X', Xo.

Best regards Eckart



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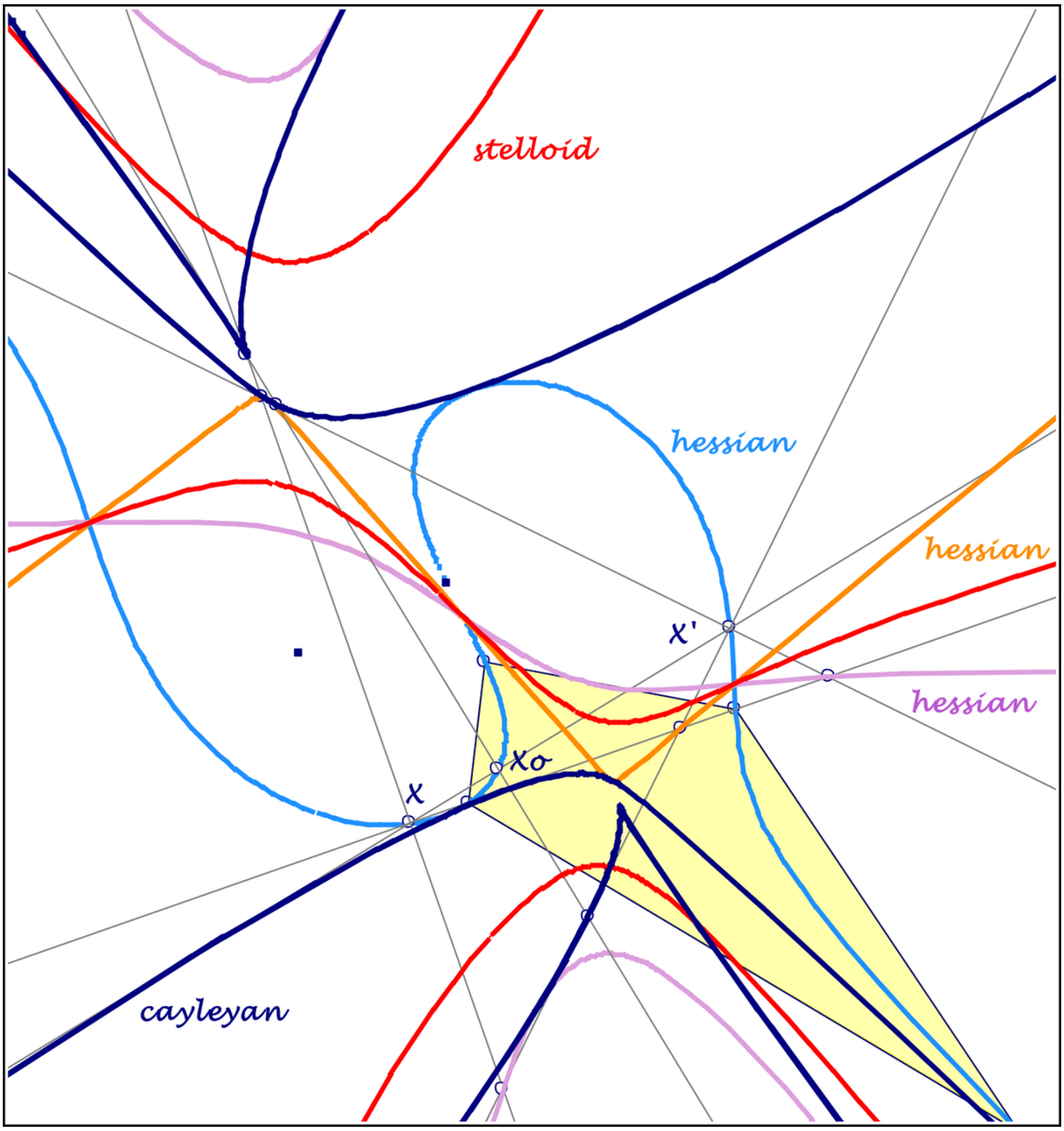

2017-04-12a.pdf

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Message: #2423
Date: 12/4/2017 11:48:26
From: eckart_schmidt@t-online.de
Subject: Holiday

```
Dear Chris,
I missed your participation in QFG-messages in the last months.
Have a nice trip - enviable!
Best regards Eckart
```

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Message: #2424
Date: 12/4/2017 3:19:10
From: bernard.keizer
Subject: Syzegetic pencil

```
Dear Eckart,
The figurs are beautiful and we understand eachother perfectly !
What do you want more ? Perhaps the identification of the 2
missing cubics, any idea ?
Best regards
Bernard
```

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Message: #2425
Date: 13/4/2017 4:26:26
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear all,

There are two pairs of points of intersection of QA-Co2 and QA-Co4. The first pair is the common harmonics of the circular and parabolic points at infinity. The second pair lies on the line through X(6) of the diapleural triangle(DT) and QA-P16 of the quadrangle. QA-Co2, QA-Co4, the line at infinity and the line through X(6) and QA-P16 and the two pairs of points of intersection are all symmetric objects. I think these objects have many nice properties.

Best regards,
Tsihong Lau

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Message: #2426
Date: 14/4/2017 2:43:15
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

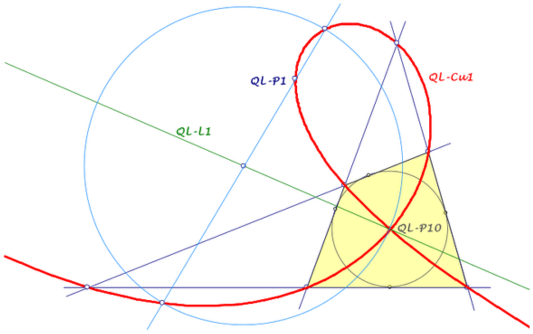
Dear Bernard,
Thanks for the friendly comments.
I try, to find a construction for QL-Cu2, using QL-Cu1 and its properties.
But I only succeeded for quadrilaterals, which have an inscribed circle (see attached file).
Best regards Eckart

EQF-Note 2017-04-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QL-Cu1 as Strophoide of a Tangential QL

For a quadrilateral with inscribed circle the cubic QL-Cu1 is a strophoid as Bernard Gibert describes in [1]. The corresponding cubic QL-Cu2 can be constructed in an alternative way using QL-Cu1 and its properties. The results are only CABRI-controlled.



- **The cubic *QL-Cu1* of a quadrilateral with inscribed circle is a strophoid [1]**
  … of the line *QL-L1*,
  … with pole *QL-P1*
  … and fixed point *QL-P10*.

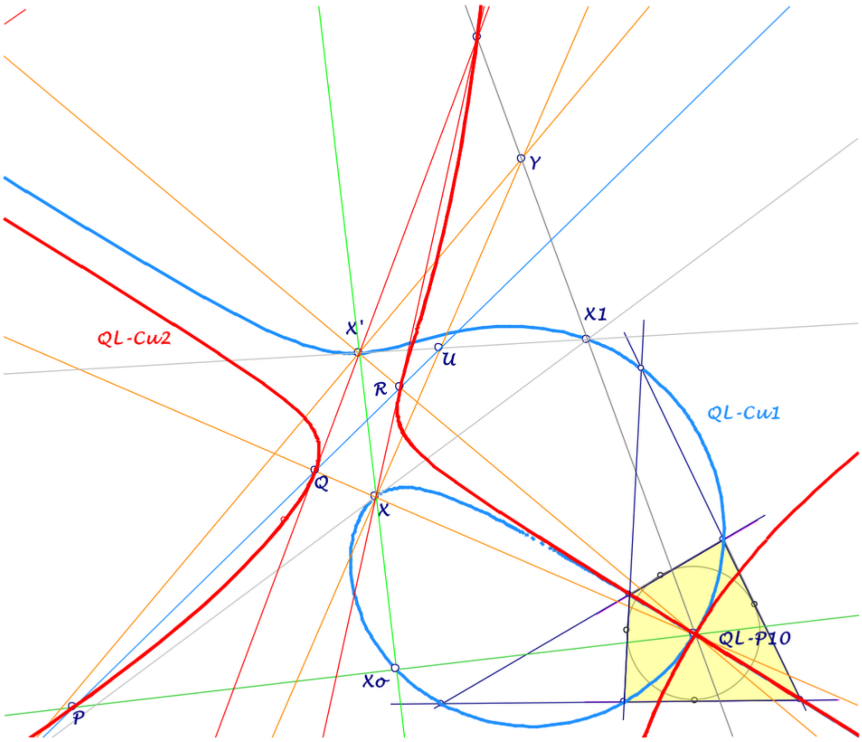
This cubic *QL-Cu1* and its properties now shall be used for a construction of the corresponding cubic *QL-Cu2*, which is a nodal stelloid with node *QL-P10*.

Let X and X' be two *QL-Tf2*-partner on *QL-Cu1*,
… X_0 the 3rd intersection of XX' and *QL-Cu1*
… and X_1 the *QL-Tf1*-image of X_0.

For points on *QL-Cu1* the polar conics wrt *QL-Cu2* are degenerated orthogonal hyperbolas:
… for X' the lines X.*QL-P10* and the perpendicular in X,
… for X the lines X'.*QL-P10* and the perpendicular in X'.
Let Y be the intersection of the two perpendiculars.

Consider the following points:
… $U = XY \cap X'X_1$, $V = X'Y \cap XX_1$,
… P = intersection of UV and perpendicular in X_0 to XX',
… $Q = UV \cap X.QL\text{-}P10$, $R = UV \cap X'.QL\text{-}P10$.

2017-04-14.pdf



- ***P, Q, R* are points on *QL-Cu2*.**
- ***PX, QX´, RX* are tangents of *QL-Cu2* in *P, Q, R*.**
- **The tangents *QX´, RX* at *QL-Cu2* and the line *X1.QL-P10* intersect on *QL-Cu2*.**

References:

[1] http://bernard.gibert.pagesperso-orange.fr/files/Resources/eckart.pdf

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-04-14.pdf

Message: #2427
Date: 14/4/2017 3:56:45
From: tsihonglau
Subject: Pivotal Waw-Conjugate Cubic on Quadrangle/Quadrilateral Plane

---

```
Dear all,

I gave the notion of "Pivotal Waw-Conjugate Cubic"
in APG but got no reply.  I wonder its application
to quadrangle/quadrilateral geometry.
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver
sations/topics/3257  †)
The notion is related to pivot chain
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver
sations/topics/2364  †)
and daleth conjugate.
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver
sations/topics/3173  †)

Best regards,
Tsihong Lau
```

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #2428
Date: 18/4/2017 11:52:24
From: eckart_schmidt@t-online.de
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear Tsihong Lau,

I am not familar with "reflexes and symmetic objects",
but some observations wrt the mentioned constellation
of the conics QA-Co2, QA-Co4 and the line X(6).QA-P16.
The conics QA-Co2 and QA-Co4 are
... homothetic orthogonal conics
... with centers QA-P2 and QA-P29.
Let L = X(6).QA-P16 and Lc = QA-P2.QA-P29.
Circles round L^Lc through the centers of QA-Co2 and QA-Co4
intersect the asymptotes on Lc.
The line through the intersections of the asymptotes is parallel
L.
The tangents to the conics in the intersections with Lc are
parallel to L.
The polars of points P on Lc wrt QA-Co2 are parallel L,
... bearing QA-Tf2(P) and the QA-DT-isogonal conjugate of P.
The line L is the polar of QA-P10 wrt QA-Co2.
The two centers of similitude of QA-Co2 and QA-Co4
... lie harmonic wrt QA-P2 and QA-P29
... and have polars symmetric L.
The two centers of similitude are not mentioned in EQF.
Their coordinates are with different signs for the squareroot
        (see picture-1 below)
with
        (see picture-2 below)

Best regards Eckart

$$\{Q\left((R-P)\sqrt{PQR(P+Q)(P+R)(-Q-R)}+PR(P+Q)(Q+R)\right),\ P\left((R-Q)\sqrt{PQR(P+Q)(P+R)(-Q-R)}+QR(P+Q)(P+R)\right),\ PQ(P+Q)(PQ+R^2)\}$$

dlzqoRMv.png

$$P := b^2 r^2 - c^2 q^2;\ Q := c^2 p^2 - a^2 r^2;\ R := a^2 q^2 - b^2 p^2;$$

ZOlUwYKQ.png

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Message: #2429
Date: 18/4/2017 12:21:01
From: eckart_schmidt@t-online.de
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

```
Dear Tsihong Lau,

it seems, that the coordinates of the centers of similitude are
not transmitted:
The two centers of similitude are not mentioned in EQF.
Their coordinates are with different signs for the squareroot
    (PR(P+Q)(Q+R)+(R-P)sqr(-PQR(P+Q)(P+R)(Q+R)),
     QR(P+Q)(P+R)+(R-Q)sqr(-PQR(P+Q)(P+R)(Q+R)),
     PQ(P+Q)(PQ+R^2))
with
P =
b^2 r^2 - c^2 q^2, Q = c^2 p^2 - a^2 r^2, R = a^2 q^2 - b^2 p^2

Best regards Eckart
```

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Message: #2430
Date: 18/4/2017 3:18:41
From: tsihonglau
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

```
Dear Eckart,

Thanks for your observations!  But I have difficulty to
understand. Can you give your graph?
I think you have difficulty to understand reflexes and symmetric
objects.
I will explain them with more details.

Best regards,
Tsihong Lau
```

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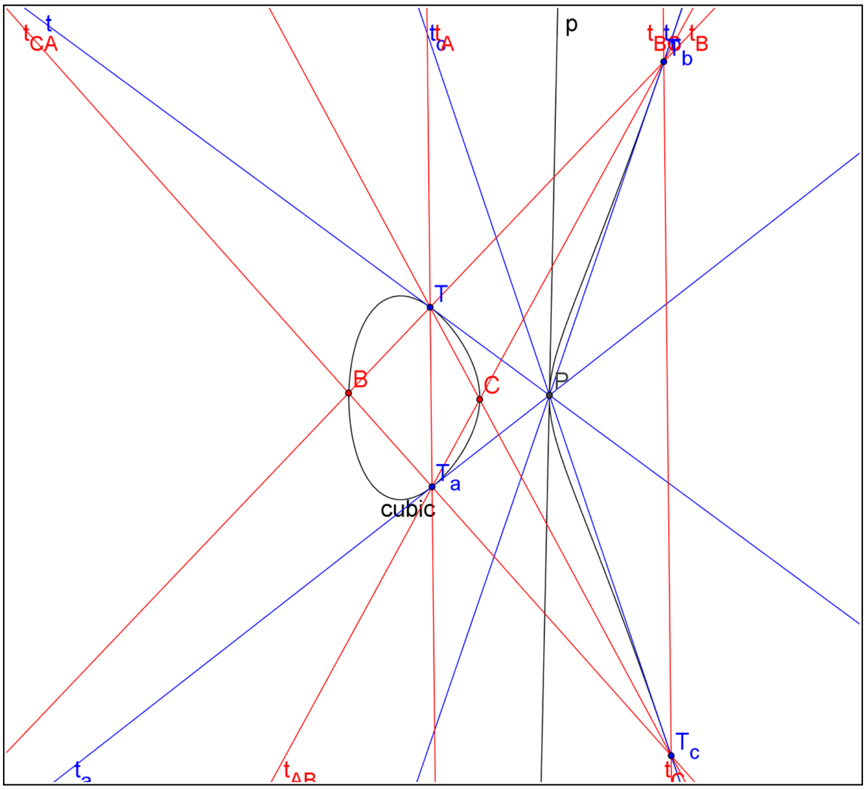
Message: #2431
Date: 18/4/2017 4:04:31
From: tsihonglau
Subject: Any Point on Cubic as Pivot

```
Dear all,
The polar conic(real or imaginary) of a point with respect to a
non-degenerate cubic is the conic through the six tangent (real
or imaginary) points on tangent lines of cubic through the given
point.
If the given point lies on the cubic, the six tangent points
become five(real or imaginary) - one is the given point and the
other four become a quadrangle.
Then the cubic becomes a circumcubic of the quadrangle.
Who can prove(or disprove) this - the diapleural(diagonal)
triangle(real or imaginary) of the quadrangle lies
on the cubic. In other words, the cubic is a circumcubic of the
diapleural triangle, too. If so, given the  quadrangle, we get
QA-Tf2(Involutary Conjugate) from it.  the given point is the
common point of tangent lines through the quadrangle.
The cubic become an isocubic with the givenpoint as pivot.
That is, any point on a cubic as a pivot!
Best regards,
Tsihong Lau
```



any_point_on_cubic_as_pivot.png

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Message: #2432
Date: 18/4/2017 6:31:17
From: bernard.keizer
Subject: Syzegetic pencil

Dear Eckart,
I found this construction very interesting, but I searched vainely a link with the construction given by Bernard Gibert on the pages 11 and 12 of his paper.
So your own construction remains magical for me !
Could you please help me and explain it with a little more details ...
Thanks in advance
Best regards
Bernard

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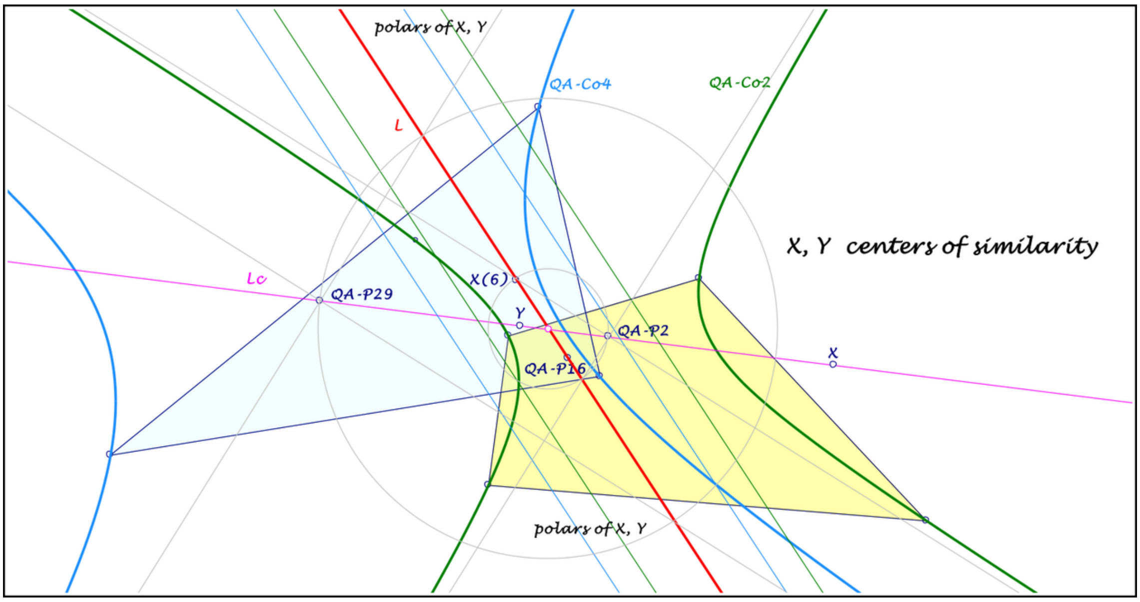
Message: #2433
Date: 19/4/2017 8:39:18
From: eckart_schmidt@t-online.de
Subject: Reflexes and Symmetric Objects in DT-Barycentrics

Dear Tsihong Lau,
attached a figur of the constellation in #2428, #2429.
Attention: The centers of similitude are "handmade", not constructed.
I tried in vain, to find a construction.
Best regards Eckart



2017-04-19.pdf

Message: #2434
Date: 19/4/2017 9:53:58
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

```
Dear Bernard,

I discovered the new construction of QL-Cu2 for a tangential QL
by studying your construction sequence in #2406,
adding two further points U and V,
which lead to the points P, Q, R on QL-Cu2.
There are no explanatory statements, only CABRI-observations.
I cannot see a link to Bernard Gibert´s construction,
for he works with triangle geometry wrt an additional line
and I use QL-geometry, especially properties of QL-Cu1.
I hope, there is a generalization, to get QL-Cu2 with QL-Cu1.

Best regards Eckart
```

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Message: #2435
Date: 20/4/2017 11:25:07
From: eckart_schmidt@t-online.de
Subject: Any Point on Cubic as Pivot

```
Dear Tsihong Lau,

only a perhaps evident remark:
Let Cu be a pivotal isocubic with reference triangle Tr,
isoconjugation Tf and pivot P.
Any point P' on Cu can be pivot
... wrt reference triangle on Cu
... ... perspective Tr wrt Tf(P')
... and isoconjugation swapping P and Tf(P').

Examples:
QA-Cu1, 2, 3, 4, 5 are wellknown pivotal isocubics.
QL-Cu1 is a pivotal isocubic in the bipartite case (see #1425).

Best regards Eckart
```

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Message: #2436
Date: 21/4/2017 10:18:00
From: bernard.keizer
Subject: Syzegetic pencil

```
Dear Eckart,
Thanks for the explanations.
I hope sincerely that you will find a general construction of
QL-Cu1 using only the properties of QL-Cu1.
It would be great !
In this case, you no longer need a particular QL or a particular
triangle, but only the point QL-P1 and the 2 points QL-2P3 (foci
of the Steinerinellipse of any pivot triangle) and the Newton
Line (which gives the 2 points QL-2P2) or conversely QL-P1 and
the 2 points QL-2P2 (which gives the 2 points QL-2P3).
Best regards
Bernard
```

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Message: #2437
Date: 23/4/2017 4:23:31
From: eckart_schmidt@t-online.de
Subject: Syzegetic pencil

Dear Bernard,

wrt #2426 here is a similar, but perhaps simpler construction of QL-Cu2 for a QL with incircle,
using QL-Cu1 and its properties (see 1st attached file):

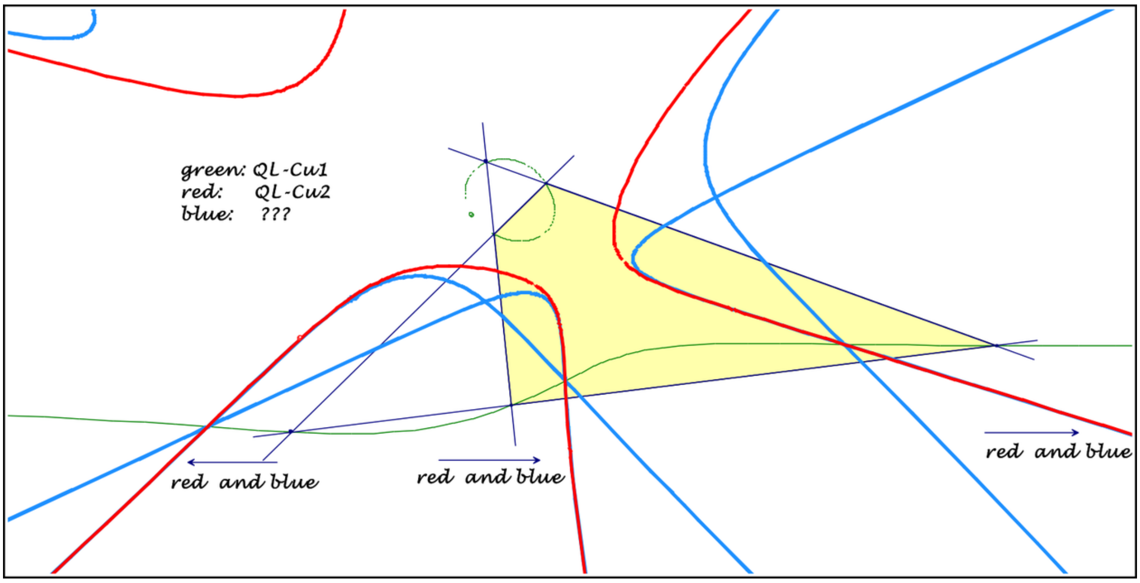
Let X and X' be two CSC-partner on QL-Cu1,
... X0 the 3rd intersection of XX' and QL-Cu1
... and X1 the CSC-partner of X0.
Let Y be the intersection of perpendiculars
... in X to X.QL-P10 and in X' to X'.QL-P10.
Let Z be the intersection of XX' and Y.QL-P10.
The 4th harmonic point P of X1 wrt YZ lies on QL-Cu2.

Generalizing this construction for arbitrary QL,
... replacing QL-P10 by the intersection Y' of the angle bisectors of <>
... and adding the 4th harmonic points Q of X1 wrt Y'Z,
we get a curious constellation of curves (blue in 2nd and 3rd attached file),
... fractional identical QL-Cu2 ???

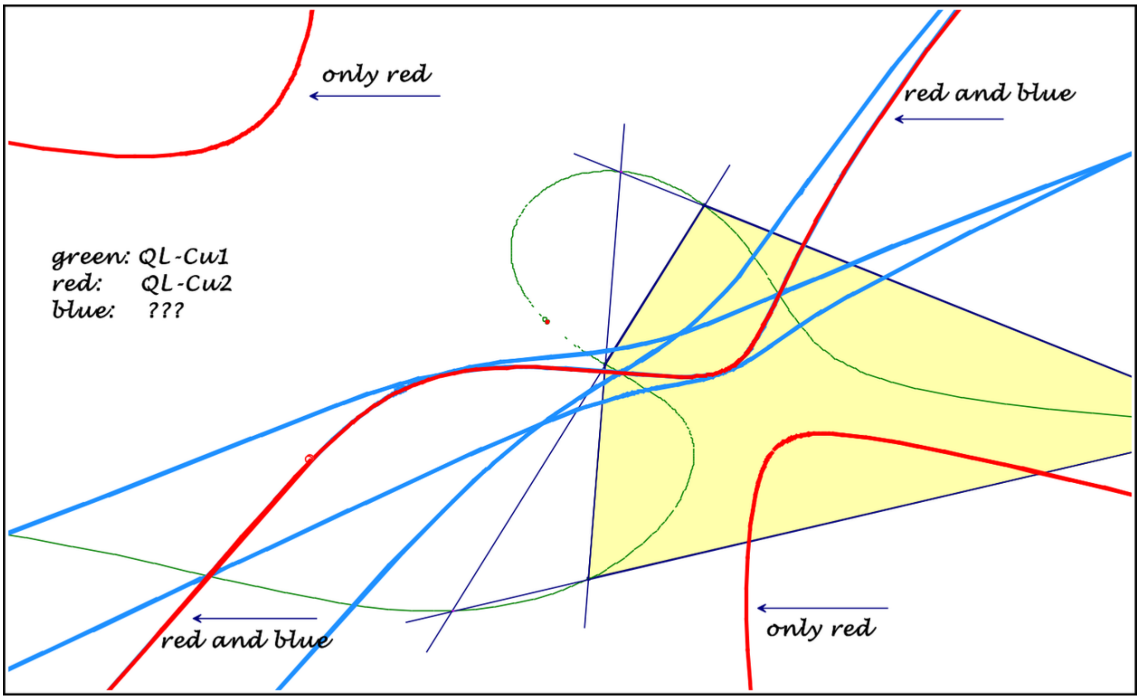
What about this constellation?
In the bipartite case perhaps two cubics
... with the same inflection points as QL-Cu1,2.
In the unipartite case
... there are intersections in the inflection points of QL-Cu1,2,
... but there are further inflection points in the intersections with QL-Cu1 ...

Have you a conclusive interpretation of these curves?

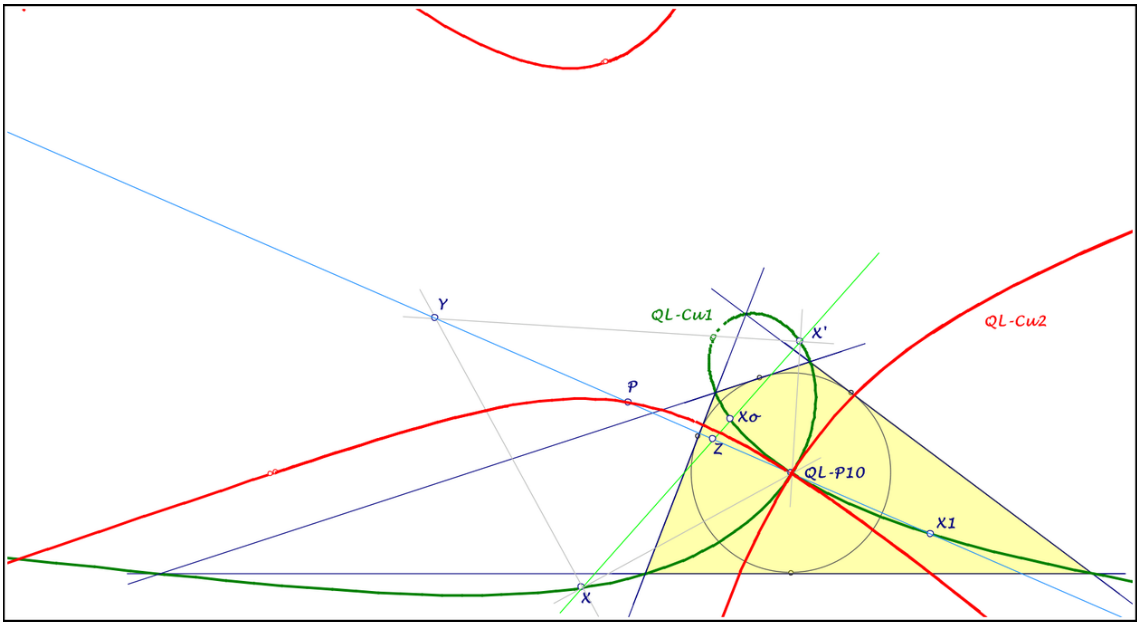
Best regards Eckart



2017-04-23-2.pdf



2017-04-23-1.pdf



2017-04-23-0.pdf

Message: #2438
Date: 25/4/2017 4:24:24
From: bernard.keizer
Subject: Syzegetic pencil

```
Dear Eckart,
I reproduced your construction with a triangle, it's Mac Cay
cubic and it's hessian.
It seems you are right, but I'm unable to give an
interpretation.
As Victor Hugo said : " Ces choses-là sont rudes, il faut, pour
les comprendre, avoir fait ses études ! "
But I hope you will continue to dig this item ...
Best regards
Bernard
```

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Message: #2439
Date: 30/4/2017 3:12:36
From: tsihonglau
Subject: An involution in pentangle

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>(3) Let P1P2P3P4P5 be a pentangle and P6P7 a line. Then an
arbitrary point P8 determines P9.
>>This is an involution. We can construct P9 with cubic
concurrency.
>>P6 and P7 might be the foci of the conic through P1P2P3P4P5.
>>It seems that if P8 is on P6P7, then P9 is on the conic
through P1P2P3P4P5.

Dear Seiichi

A subcase of this is that P6,P7 are the circular points at
infinity.
This involution should be very fundamental in
quintangle/quintilateral geometry.

Best regards,
Tsihong Lau
```

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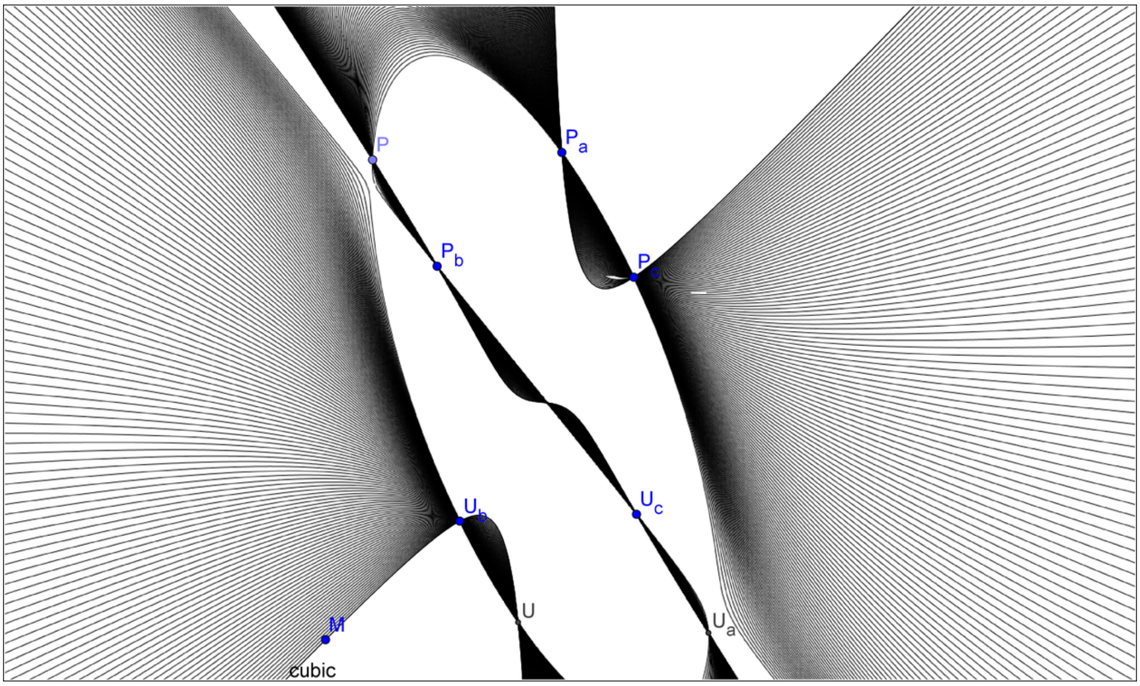
Message: #2440
Date: 30/4/2017 4:09:36
From: tsihonglau
Subject: Cayley-Bacharach Ninth

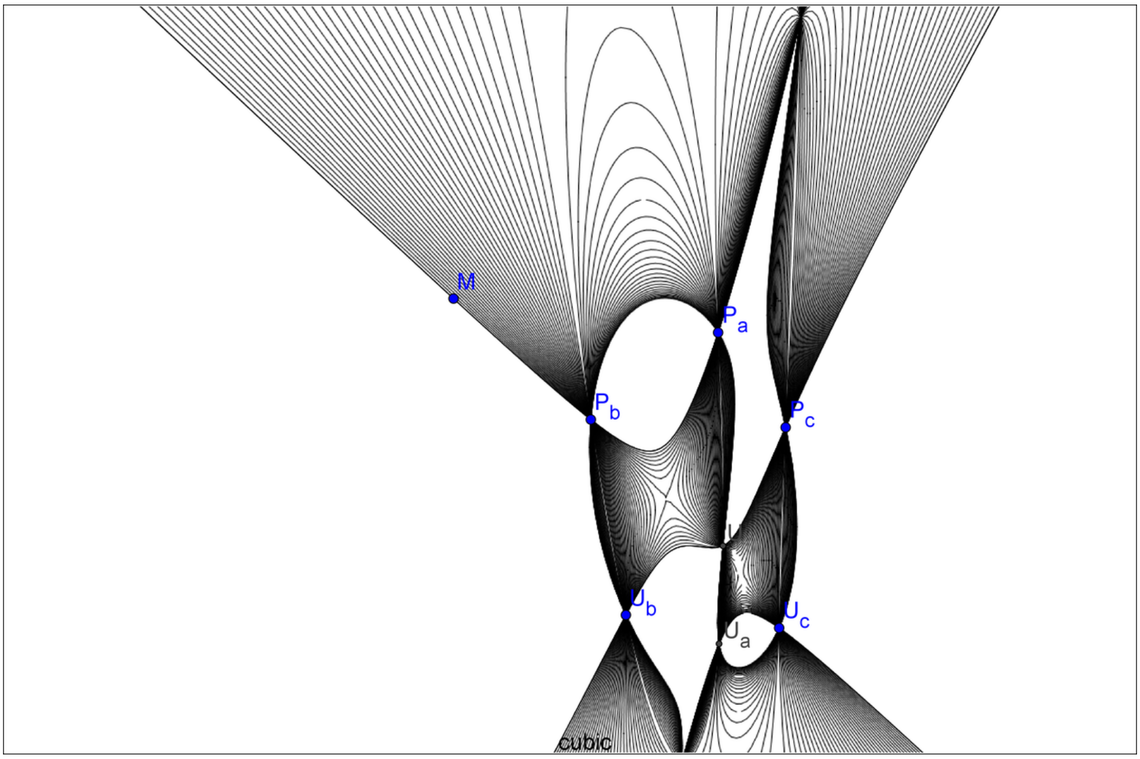
Dear all,

Cayley-Bacharach theorem states that:
Every cubic curve C1 on an algebraically closed field that passes through a given set of eight points P1, ..., P8 also passes through a certain (fixed) ninth point P9, counting multiplicities.
Usually we discuss the non-degenerat cases:
If no seven points out of P1, ..., P8 lie on a non-degenerate conic, and no four points out of P1, ..., P8 lie on a line.
A common case is that the eight points are two quadrangles.
There are three common subcases: cyclologic, orthologic and parallelologic.
The attachments show the three. Can anyone give the constructions of these Cayley-Bacharach ninth points?
To be continued...

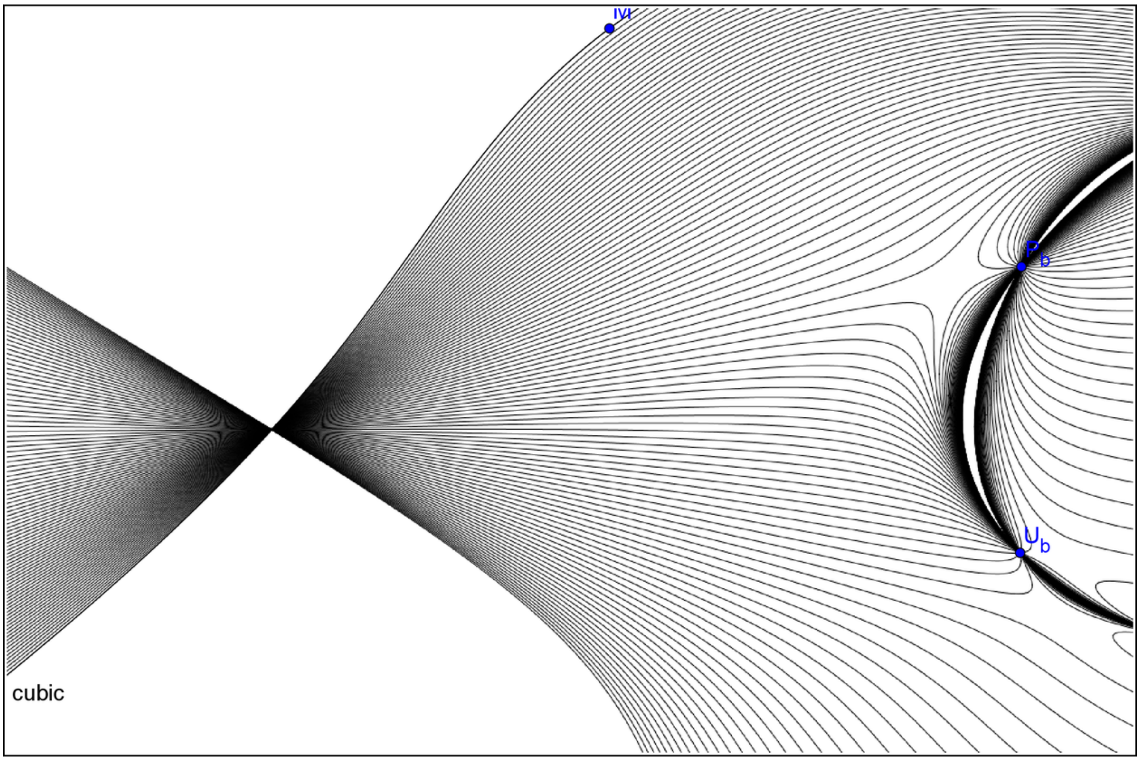
Best regards,
Tsihong Lau



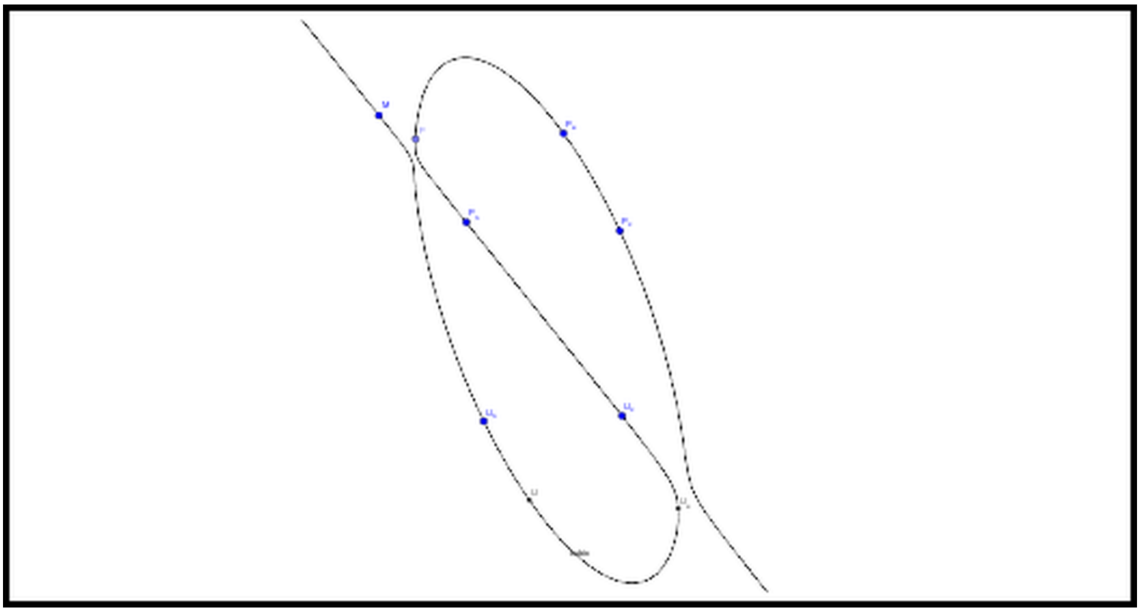
parallelologic__quadrangle__cayley__bacharach__theorem.png



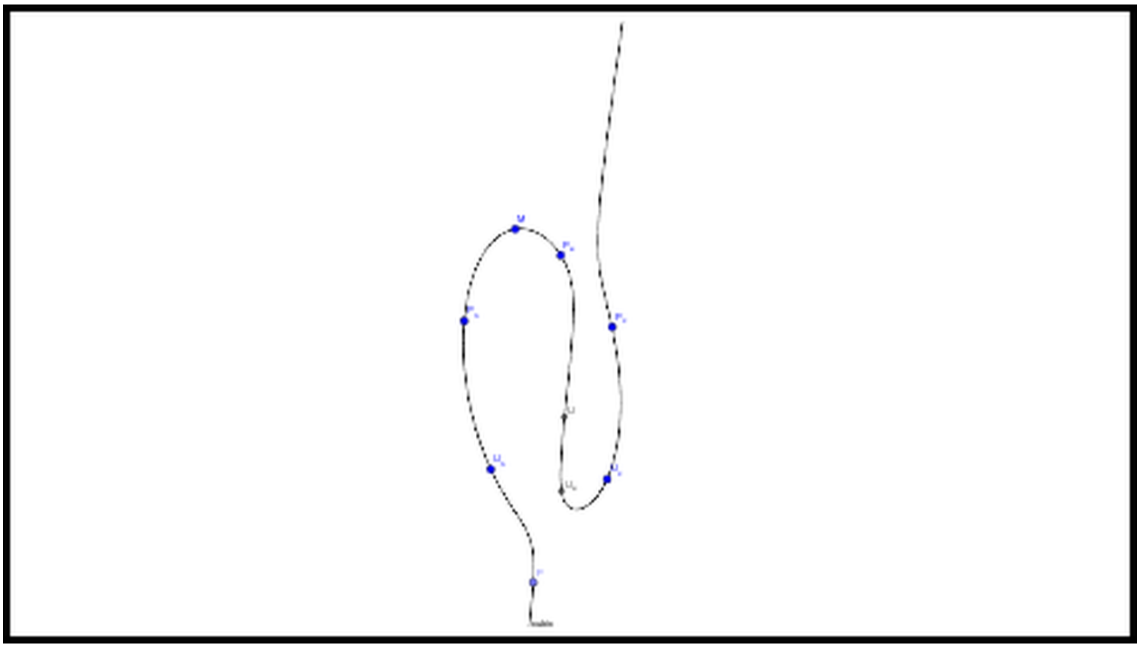
orthologic_quadrangle_cayley_bacharach_theorem.png



cyclologic_quadrangle_cayley_bacharach_theorem.png



parallelologic_quadrangle_cayley_bacharach_theorem.ggb



orthologic_quadrangle_cayley_bacharach_theorem.ggb

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Message: #2441
Date: 02/5/2017 4:08:04
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear all,

Given the incenter/excenters quadrangles, we can construct the
expoint triangle center quadrangles.
That is, given the incenter/excenters I,Ia,Ib,Ic, we can
construct triangle centers Xn,Xna,Xnb,Xnc of IaIbIc and IIbIc
and  IaIIc and IaIbI respectively.
The two quadrangles IIaIbIc and XnXnaXnbXnc may have the
Cayley-Bacharach ninth point.
triangle center of IaIbIc=triangle center of ABC -
Cayley-Bacharach ninth
X(2)=X(165) - X(6194)

The above states triangle center X(2) of IaIbIc is the same as
triangle center X(165) of ABC.
And the expoints of X(165)(a qaudrangle) and IIaIbIc(another
quadrangle) determine the Cayley-Bacharach ninth triangle center
X(6194) of ABC.

more examples
X(3)=X(40) - X(3)
X(6)=X(9) - X(1073)
X(20)=X(7991) - X(194)
X(54)=X(191) - X(399)
X(68)=X(1490) - X(3348)

If there exist(sometimes does not exist) a cubic through IIaIbIc
and XnXnaXnbXnc and ABC(diapleural triangle
of IIaIbIc), it seems the cubic also passes through the
diapleural triangle of XnXnaXnbXnc.

cubic - triangle center of IaIbIc = triangle center of ABC -
Cayley-Bacharach ninth
K002 - X(6)  =X(9)    - X(1073)
K002 - X(25) =X(57)   - X(?)
K004 - X(3)  =X(40)   - X(3)
K004 - X(68) =X(1490) - X(3348)
K006 - X(24) =X(46)   - X(?)
X(?)'s mean triangle centers not listed in ETC.

more unknown Cayley-Bacharach ninth triangle center of
IaIbIc=triangle center of ABC
X(23)=X(5536)
```

```
X(26)=X(5709)
X(32)=X(169)
X(39)=X(3730)
X(64)=X(2136)
X(66)=X(3174)
X(67)=X(5528)
X(69)=X(2951)
X(74)=X(5541)
X(76)=X(170)
X(95)=X(2938)
X(96)=X(2939)
X(97)=X(2941)
X(98)=X(1282)
X(107)=X(1054)
X(110)=X(1768)
```

I think Cayley-Bacharach ninth is very essential to
triangle/trilateral cubics.
To be continued...

Best regards,
Tsihong Lau

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Message: #2442
Date: 13/5/2017 2:36:56
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Tsihong Lau,

back from holidays at the seaside, not only drawing quadrangles
in the sand ...

I am not familar with the Cayley-Bacharach ninth point, so the
following remark will shurely already be mentioned:
QA-Tf2(P) is the ninth Cayley-Bacharach point of
... the 4 vertices of the quadrangle QA,
... the 3 vertices of the diagonal triangle QA-Tr1
... and the point P.

Best regards Eckart

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Message: #2443
Date: 14/5/2017 2:53:58
From: tsihonglau
Subject: Cayley-Bacharach Ninth

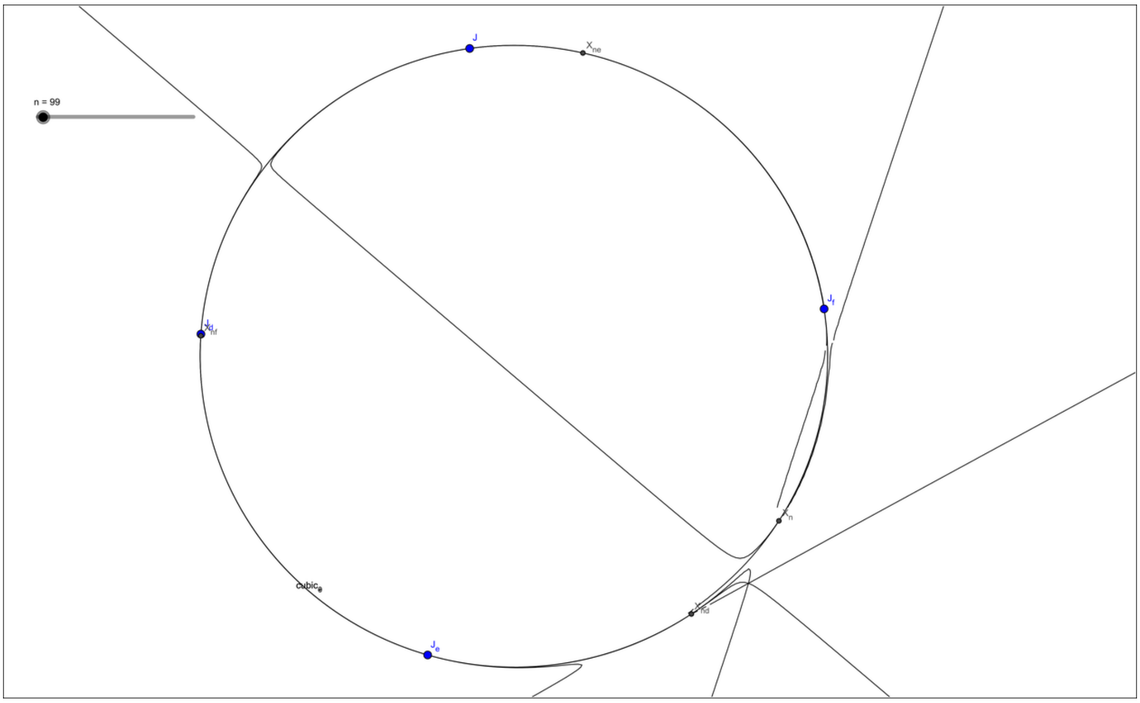
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>QA-Tf2(P) is the ninth Cayley-Bacharach point of
>>... the 4 vertices of the quadrangle QA,
>>... the 3 vertices of the diagonal triangle QA-Tr1
>>... and the point P.

Dear Eckart,

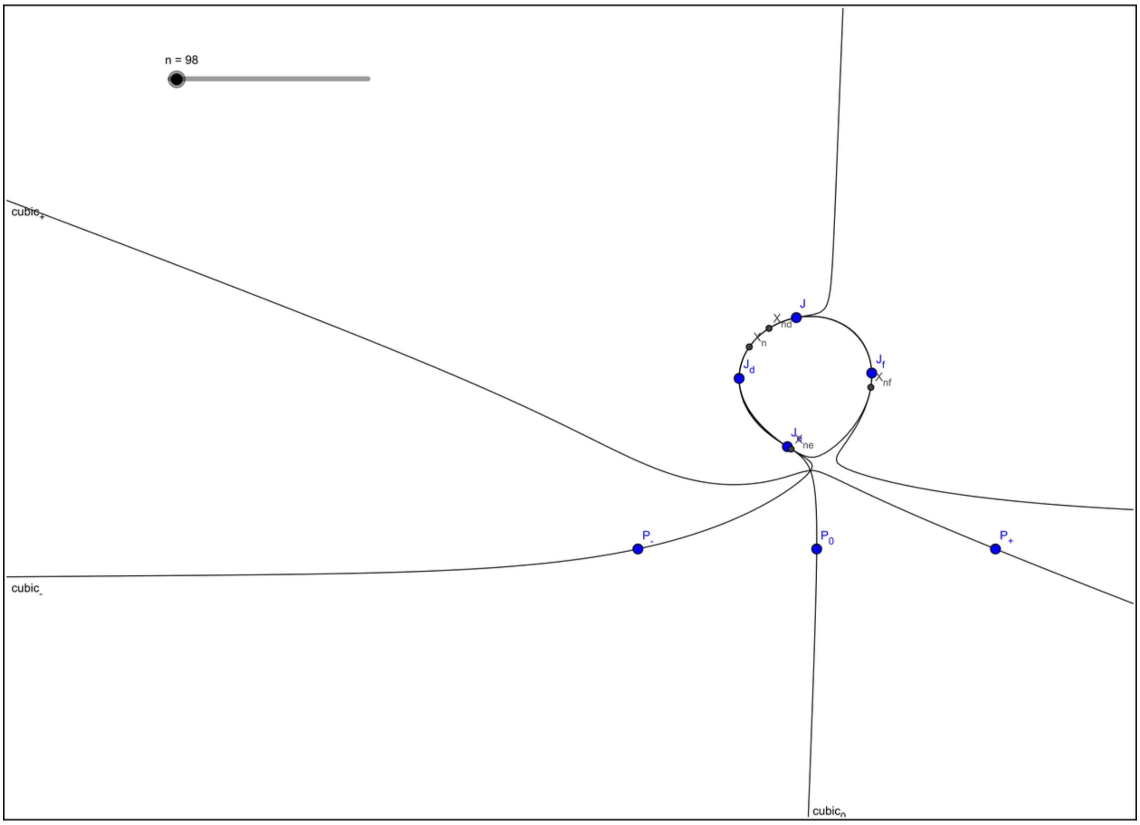
I knew it about two years ago.
It has been mentioned by Seiich!
Here are more generalized results:
Given a quadrangle J,Jd,Je,Jf, we can get the expoint triangle centers Xn,Xnd,Xne,Xnf of JdJeJf and J,JeJf and JdJJf and JdJeJ.

There may(or may not) exist Cayley-Bacharach ninth points for these pairs of quadrangles
J,Jd,Je,Jf and Xn,Xnd,Xne,Xnf.
Please refer to the attachments for more information. In the previous message
are the special cases.

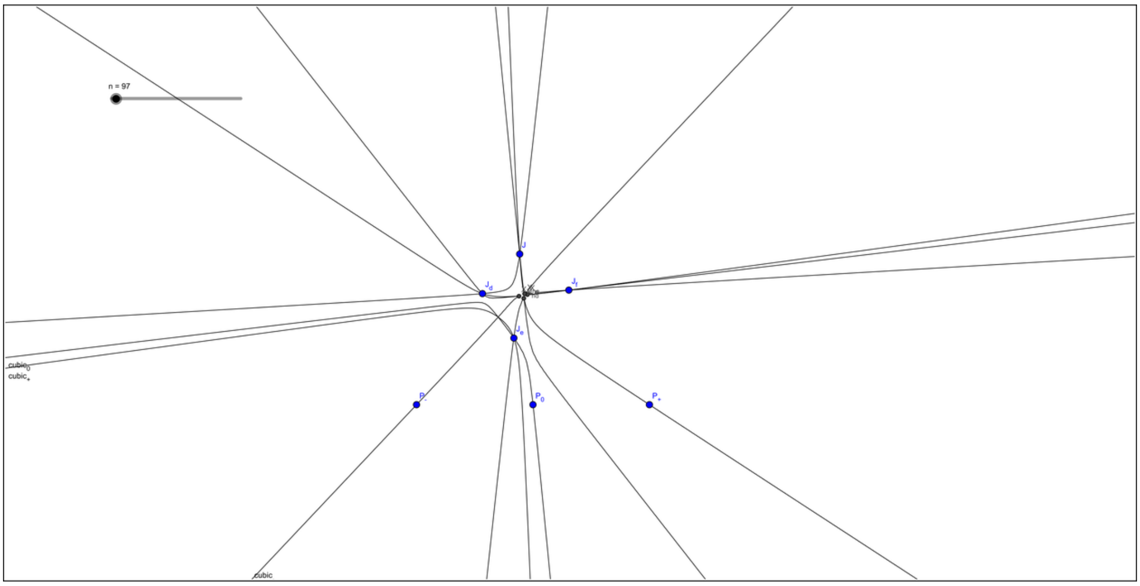
Best regards,
Tsihong Lau



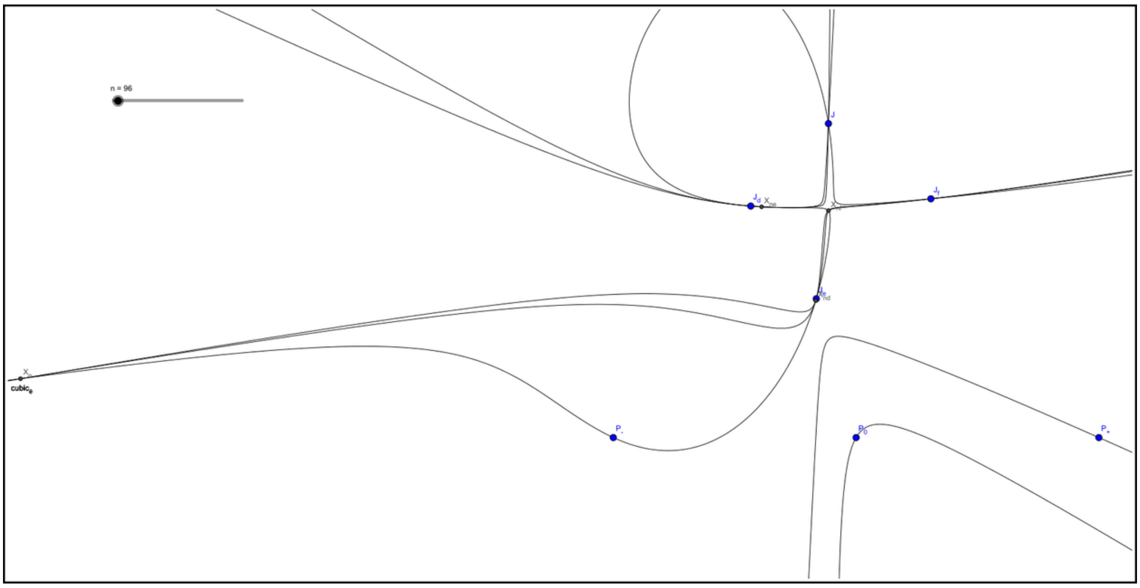
expoint_triangle_center_cayley_bacharach_theoremX(99).png



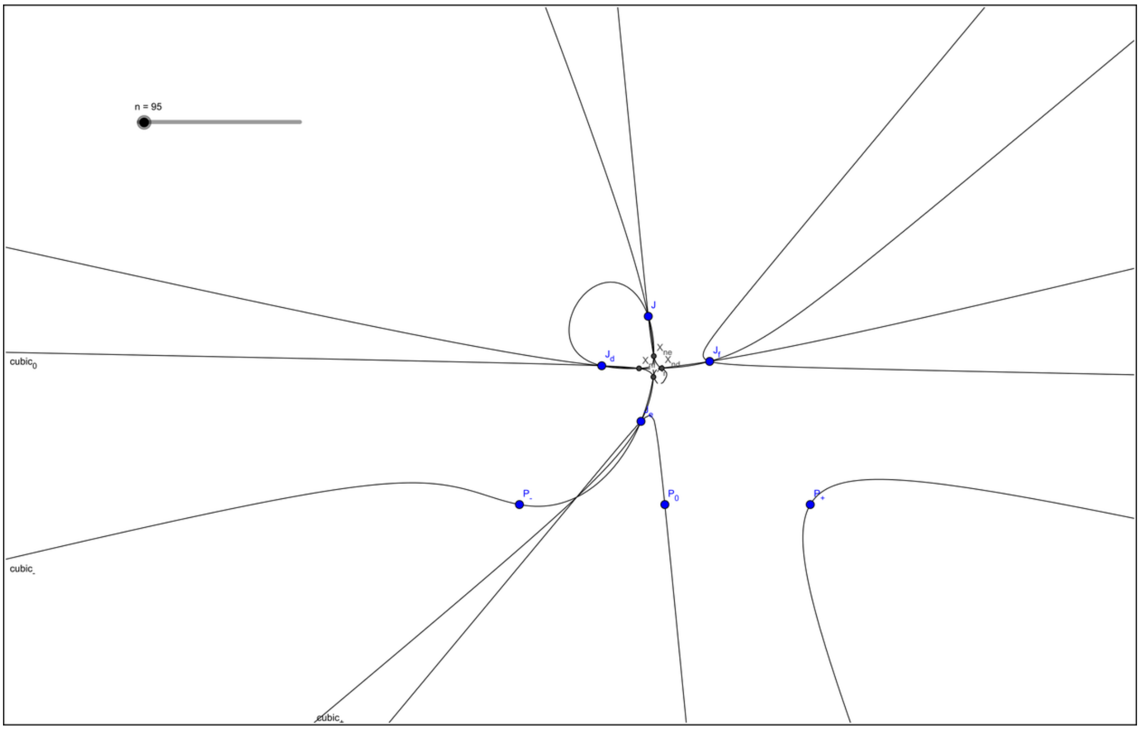
expoint_triangle_center_cayley_bacharach_theoremX(98).png



expoint_triangle_center_cayley_bacharach_theoremX(97).png



expoint_triangle_center_cayley_bacharach_theoremX(96).png



expoint_triangle_center_cayley_bacharach_theoremX(95).png

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Message: #2444
Date: 15/5/2017 11:04:25
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau, dear Seiichi,

in Seiichi's message 1476 I found property (1).
Here some special applications, perhaps of interest,
if not already mentioned (* denotes QL-Tf1):

(1) Consider the following 8 points:
... 6 points of the QL,
... P, Q arbitrary points,
... CB-pt = common point of the circumconics of the QL-triangles
    through P,Q.

(2) Consider the following 8 points:
... 6 points of the QL,
... P and P* on QL-Cu1.
... CB-pt = common tangential of P and P* wrt QL-Cu1.

(3) Consider the following 8 points:
... 6 points of the QL,
... P, Q arbitrary points on QL-Cu1.
... CB-pt = PQ* ^ P*Q.

(4) Consider the following 8 points:
... 6 points of the QL,
... P arbitrary point and P*.
... CB-pt = (PP* ^ QL-Tf2(PP*)*.

(5) Consider the following 8 points:
... 6 points of the QL,
... P = QL-P1, Q = arbitrary point on the Steiner axes.
... CB-pt = Q.

(6) Consider the following 8 points:
... 6 points of the QL,
... QL-P1,
... X variable point on a fixed line L through QL-P1.
... Locus for CB-pts: L*.

Best regards Eckart

PS: Excuse to Seiichi, that I hadn´t parat his results.
```

Message: #2445
Date: 15/5/2017 4:41:18
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear Eckart,

Thanks for your reply! Let's consider more general cases.

On a general Pappian plane(neither line nor circular points at infinity)
1. Eight points(octangle) P1P2P3P4P5P6P7P8 determine a point P
2. Seven points(septangle) P1P2P3P4P5P6P7 determine an involution P<->P'
3. Six points(sexangle) P1P2P3P4P5P6 determine an triple operation P op Q=R and Q op R=P and R op P=Q
Never forget Cayley-Bacharach theorem applies to line geometry.
"Points" in the above statements can be replaced with "lines"

On a Pappian plane with line at infinity
4. Seven lines(septilateral) l1l2l3l4l5l6l7 determine a line l.
5. Six lines(sexilateral) l1l2l3l4l5l6 determine an involution l<->l'
6. Five lines(quintilateral) l1l2l3l4l5 determine an triple operation l op m=n and m op n=l and l op m=n.

On a Pappian plane with circular points at infinity
7. Six points(sexangle) P1P2P3P4P5P6 determine a point P
8. Five points(quintangle) P1P2P3P4P5 determine an involution P<->P'
9. Four points(quadrangle) P1P2P3P4 determine an triple operation P op Q=R and Q op R=P and R op P=Q
Given a quadrangle on Pappian plane with line at infinity, we can get two parabolic points at infinity.
So the case 9 applies to it!

Eckart's case (1) is a special case of my case 3. Can someone study my case 9?
In message #1696, Seiichi mention a formula for Cayley-Bacharach ninth points.
Can someone compute them in this topic with it?
https://arxiv.org/pdf/1405.6438v2.pdf

Best regards,
Tsihong Lau

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Message: #2446
Date: 21/5/2017 6:09:58
From: tsihonglau
Subject: geometry=incidence+metric

Dear all,

I would like to post this message long ago.
This topic include my many fundamental ideas.
I present some ideas now!

1. geometry=incidence+metric
In plane(or solid)geometry, incidence means that objects lie on or pass through other objects.
Points lie on lines or point curves or cycles(see below).
Lines pass through points or lie on line curves or cycles.
Metric means lengths and areas and angles, etc.
Geometry consists of these two main notions.

2. fundamental object(0-dimensional)=point+line+cycle
There are three main classes of objects in plane geometries - points, lines and cycles.
Cycles are lines or circles or parabolas or rectangular hyperbolas.
All other objects are made of these classes of objects.
Points curves are made of points, while line curves are made of lines.
I think there are "cycle" curves, which made of cycles.

3. curve(1-dimensional)=point+line+cycle
As mentioned above, there are point or line or even cycle curve(1 dimensional object).
Any curve has a degree. Degree 2 are called conics and degree 3 are called cubics, etc.
Bézout's theorem may apply-
A degree m curve and a degree n curve intersect at most mn fundamental objects.

4. configuration=set of ordered or unordered objects with certain relation Desargues configuration is made of 10 points and 10 lines,
Pappus configuration is made of 9 points and 9 lines
Reye configuration is made of 12 points and 16 lines.
Complete quadrangle/quadrilateral configuration is made of 13 points and 13 lines.
It is unnecessary to have the same number of points on lines or lines through points for my term "configuration". It can even include "cycles"

So there exists Miquel configuration.
Even there exists order of objects for it.
The relation can be incidence or other such as perpendicularity.

So orthologic quadrangles are treated as configuration.

5. multiangle=set of unordered points,
multilateral=set of unordered lines,
polygon=set of cyclic ordered points or lines
A triangle is a set of 3 unordered points.
A trilateral is a set of 3 unordered lines.
Both are dual configurations.
A quadrangle is a set of 4 unordered points.
A quadrilateral is a set of 4 unordered lines.
Both are dual configuration on a Desarguesian plane.
A tetragon is a set of 4 cyclic ordered points or lines.
Given 4 points P1P2P3P4 or 4 lines l1l2l3l4, where l1=P1P2,
l2=P2P3, l3=P3P4, l4=P4P1 and P1=l4∩l1, P2=l1∩l2, P3=l2∩l3,
P4=l3∩l4
4 points or 4 lines configurations are dual.

6. configuration=variable+fixed objects
Some objects are variable while other are fixed.
The most common fixed objects are the line and the circular
points at infinity.
The most common variable objects are the reference
triangle/trilateral and quadrangle/quadrilateral.
We can discuss two reference triangles/trilaterals but the line
and the circular points at infinity remain the same.
The basic distinction is very fundamental.

7. metric=function of objects indifferent to variable objects
relative to fixed objects
As mentioned above, there are metrics such as lengths, angles
and areas etc.
They are functions of objects (two or more).
They depend on the choice of the fixed objects (the line at
infinity, etc).
If we choose other fixed objects, they become different
functions.
They do not depend on the choice of the variable objects (the
reference triangle etc).
If we choose other variable objects, they remain the same.
That is why we must distinguish fixed and variable objects.

8. incidence theorem
There are many incidence theorems.
They apply to different configurations.
Desargues' theorem applies to Desargues configuration.

Pappus's theorem applies to Pappus configuration.
Miquel's theorem applies toMiquel configuration.
Newton line exists on a quadrilateral together with the line at infinity.

9. classification of geometry with fixed objects and incidence theorem
I have classify some geometries with them.
I give two common geometries - vertical parabolic Pappian plane and circular Pappian plane.
The former is a Pappian plane with the vertical point on the line at infinity.
The latter is a Pappian plane with the circular points on the line at infinity.
There is another common plane - called affine plane, which has the line at infinity.
There are affine Desarguesian or Pappian planes.
Pappian plane is a plane on which Pappus's theorem holds.
Desarguesian plane is a plane on which Desargues' theorem holds.
There is also Miquelian plane on which Miquel's theorem holds.

Best regards,
Tsihong Lau

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Message: #2447
Date: 28/5/2017 3:53:11
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Seiichi, dear Tsihong Lau,

at last I succeeded reproducing a construction of the
Cayley-Bacharach ninth point with a CABRI-macro.
Only 2 pages of reference, but the "anharmonic ratio" is a
malicious tool:

A.S.Hart:
Construction by the Ruler alone to determine the ninth Point of
Intersection of two Curves of the third Degree.
Cambridge and Dublin Mathematical Journal 6 (1851) 181-182.

But it seems, that there is a relevant mistake in the last
sentence. It has to be:
"... such that the anharmonic ratio of 9(6758) may be equal
to the inverse of the product of the anharmonic ratios (7A8B),
(7a8b),
and that the anharmonic ratio of 9(6748) may be equal
to the inverse of the product of the anharmonic ratios (7A8C),
(7a8c): ..."

Perhaps an interesting application:
Consider the QA-transformation Tr,
... which maps a point X to the Cayley-Bacharach ninth point of
... ... the 4 QA-vertices, the 3 QA-Tr2-vertices and X.
This transformation maps
... QA-sides PiPj to QA-Tr2-circumconics through PkPl,
... QA-Tr2-sides to QA-circumconics through the opposite
QA-Tr2-vertex,
... QA-circumconics to quartics through the QA-vertices and the
QA-Tr2 vertices as double points.

Unexpected:
The cubic QA-Cu1 is invariant wrt this transformation Tr.
X and Tr(X) on QA-Cu1 have a fixed 3rd collinear point on
QA-Cu1.

This fixed point on QA-Cu1 is a new QA-point:
... 3rd QA-Cu1-intersection of the line connecting
... ... QA-Tr2-isogonal conjugated of QA-P41
... ... and intersection of QA-Cu1 and its asymptote.

Best regards Eckart

Message: #2448
Date: 28/5/2017 10:51:04
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>>The cubic QA-Cu1 is invariant wrt this transformation Tr.
>>X and Tr(X) on QA-Cu1 have a fixed 3rd collinear point on
QA-Cu1.
>>This fixed point on QA-Cu1 is a new QA-point:
>>... 3rd QA-Cu1-intersection of the line connecting
... ... QA-Tr2-isogonal conjugated of QA-P41
... ... and intersection of QA-Cu1 and its asymptote.

Dear Eckart,
Thanks for your reply!  I guess the transformation Tr is QA-Tf2
of a certain quadrangle.
Please refer to topic #2431
"Any Point on Cubic as Pivot" for more information.

Best regards,
Tsihong Lau
```

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Message: #2449
Date: 29/5/2017 10:28:15
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,

you are right:
Let Tr be the transformation of #2447, X points on QA-Cu1 and S
the fixed 3rd intersection of QA-Cu1 and the lines X.Tr(X),
then is Tr(X) = QA-Tf2(X) for the quadrangle of the contact
points of tangents from S to QA-Cu1.

Best regards Eckart
```

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Message: #2450
Date: 29/5/2017 3:26:57
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Chris, Seiichi and Tsihong Lau,

a CABRI-macro for the Cayley-Bacharach ninth point (C-B-point)
gives the possibility for some remarks:
(1) In #1704 Chris considers the C-B-point of
... the vertices of QA-Tr1, QA-Tr2
    and the points QA-P3 and QA-P4.
This point is
... the QA-Tf2-image of the 3rd QA-Cu1-intersection of the line
... connecting QA-P41 and the intersection of QA-Cu1
    and its asymptote.

(2) In #2445 Tsihong Lau states, that seven points determine a
transformation wrt the C-B-point.
In #2447 I defined such a transformation for the vertices of a
quadrangle and the vertices of QA-Tr2.
In addition: This transformation maps the intersection of QA-Cu1
and its asymptote
to the intersection of QA-Cu1 and a parallel to the asymptote
through QA-P41.

(3) The six QL-points and QL-P1 determine another transformation
Tf wrt the C-B-point:
Let Tf(X) be the C-B-point of the six QL-points, QL-P1 and X:
... For points on the Steiner axes holds X = Tf(X).
... For the QL-lines holds Tf(Li) = Li.
... A diagonal is mapped to the conic
    through the remaining 5 points.
... The cubic QL-Cu1 is invariant wrt this transformation Tf.
... For points X, Tf(X) on QL-Cu1 the line X.Tf(X)
    is parallel to QL-L1.

(4) In #737 Antreas asked for the C-B-point of the 6 QL-points,
QL-P1 and QL-P4.
This point will be the point at infinity of QL-L1.

Best regards Eckart

PS: The transformation in (3) can easily be constructed,
using degenerated cubics of a line through 3 collinear points
and the conic through the remaining 5 points.
```

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Message: #2451
Date: 29/5/2017 6:02:08
From: chris.vantienhoven
Subject: Back home

Dear friends,

I am back from my journey (the Camino-Frances) to Santiago de Compostela and Fisterra.
See: https://en.wikipedia.org/wiki/French_Way
I walked about 900 kilometers in 37 days.
I loved the journey but I am glad to be back home.
As soon as I recovered from the (culture-)shock I hope to restart my geometric contributions.
As a first start I enclose a picture of the logo of the journey, being the contours of a shell having a nice geometric background. Also I enclose a picture of my arrival at Fisterra.
Good to be back home also with my geometrical friends.

Best regards,
Chris

IMG_20170523_111734.jpg

IMG_20170514_111623.jpg

Message: #2452
Date: 30/5/2017 4:49:17
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> (4) In #737 Antreas asked for the C-B-point
>> of the 6 QL-points, QL-P1 and QL-P4.
>> This point will be the point at infinity of QL-L1.

Dear Eckart,

In message #2445, I gave three cases of triple operations.
There are two famous triple operations in triangle/trilateral
geometry- incidence and conic triples.  Given two points/lines
P(p:q:r) and U(u:v:w) incidence triple(aka Hirst inverse in ETC)
is p^2vw-qru^2:...(cyclic defined), while conic triple is
pu/(p^2-vw-qru^2):...(cyclic defined).
If P and U are the circular points at infinity, the two triples
are X(523) and X(110) respectively. There is a special case for
case 3 in message #2445.
3. Six points(sexangle) P1P2P3P4P5P6 determine an triple
operation P op Q=R and Q op R=P and R op P=Q
Given a quadrilateral, the six points are the diagonal points.
We can derive a triple operation from them.  Given two points
P(p:q:r) and U(u:v:w) and the six diagonal points
with unit coordinates 0:1:-1 ~ 0:1:1 ~ -1:0:1 ~ 1:0:0 ~ 1:-1:0 ~
1:1:0.
P op U = ((ru-pw)+(pv-qu))((ru-pw)-(pv-qu))((-p^2+q^2+r^2)vw-qr(
-u^2+v^2+w^2)):...
(cyclic defined).

If P and U are the circular points at infinity, we can get the
result QL-P1 the Miquel point easily.
In other words, the circular points and the Miquel point form a
triple for a quadrangle/quadrilateral.
The Antreas' case is a special case of the triple.
Never forget Cayley-Bacharach theorem applies to line geometry.
"Points" in the above statements can be replaced with "lines".
I hopethe triple operations mentioned in message #2445 will be
studied thoroughly.
Please refer to my pages for more information-
  http://lth.name/geometry/duality.html
  http://lth.name/geometry/class.html
Best regards,
Tsihong Lau
```

Message: #2453
Date: 30/5/2017 9:59:53
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Chris,

welcome back home and in QFG. Admirable activity!

I think, the Cayley-Bacharach ninth point leads to an
interesting QL-transformation Tf (see #2450 (3)).
Example: Tf maps the QL-Tr1-vertices to collinear points with
QL-P26.

A bit geometry: Consider an inscribed QL-conic Co and the
tangents from QL-P1 to Co.
Every other tangent to Co intersects the QL-P1-tangents in
Tf-partners,
that means these intersections are Cayley-Bacharach ninth points
of another wrt the QL-vertices and QL-P1.

Best regards Eckart

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Message: #2454
Date: 30/5/2017 3:40:36
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Tsihong Lau,

only a short remark to the triple operation wrt the six
QL-vertices.
Let X, Y be CSC-partners and Z the C-B-point of X, Y and the six
QL-vertices:
... X, Y, Z concyclic with QL-P1 on CSC(X.Y)
... For X, Y as CSC-partners on QL-Cu1 is Z the common
tangential.

Best regards Eckart

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Message: #2455
Date: 01/6/2017 5:14:15
From: tsihonglau
Subject: Focus of QA-Cu1

```
Dear all,

Both QA-Cu1 and QL-Cu1 are circular cubics.
The focus of QL-Cu1 is QL-P1 on QL-Cu1 itself.
So QL-Cu1 is a focal cubic.  How about QA-Cu1?
```

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Message: #2456
Date: 01/6/2017 10:29:04
From: bernard.keizer
Subject: Focus of QA-Cu1

```
Dear Tsihong Lau,

QA-Cu1 is circular, but not focal.
If I'm not wrong, it's focus is QA-P9.

Best regards
Bernard
```

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Message: #2457
Date: 02/6/2017 11:02:21
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Chris, dear Seiichi, dear Tsihong Lau, dear Eckart,
First, wecome back to the forum for Chris !
Then some remarks about this last item of the C-B ninth point, which is very promising :
1) the idea is that a cubic is determined by 9 points in a general position ; if 2 cubics intersect in 9 points, these points are in a particular position and determine a pencil of cubics (the same occurs for quartics ...)
Hence, 8 points give the ninth, 7 points give an involution and 6 give triples ...
The idea of applying this to the QA and the QL was brilliant !
2) The QA vertices determine with any triangle a transformation
It's QA-Tf2 for QA-Tr1 and the QA-Tf2 of another QA for QA-Tr2 ; did someone try with the triangle of the S-points (3 intersections other than QA-P16 between the conics through QA vertices, QA-P1 and QA-P16 and through DT vertices, QA-P10 and QA-P16)
But of course it can be 2 triangles and any point. For example, as the vertices of DT and the S-triangle are coconic, the transformation gives 2 points aligned with the 7th point ?
3) The QL vertices and any point give a transformation
If this point is QL-P1, the transformation swaps the 2 circular points and the pencil contains QL-Cu1 and the 4 degenerated cubics formed by a line through 3 points and the circle through QL-P1 and the 3 others.
(This confirms Eckart's property as QL-P1 is the tangential of the 2 circular points, which are CSC partners.
It appears that the line QL-P1TfX and QL-P1X are symmetric wrt the Steiner axes.
For the 3 vertices Ti of DT, TfTi is the CSC of the 2nd intersection of QL-P1Ti with the Dimidium circle and the 3 points TfTi are aligned on the CSC of this circle, which passes through QL-P26, as mentionned by Eckart.
I couldn't reproduce the property that TfQL-P4 is the infinity point of QL-L1, but I may have made mistake ...
I hope you will find plenty of other properties
Best regards
Bernard

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Message: #2458
Date: 02/6/2017 2:08:56
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Chris, dear Seiichi, dear Tsihong Lau, dear Bernard,

attached a research wrt the Cayley-Bacharach ninth point for
... the vertices of QA-Tr1 and QA-Tr2
... and two QA-Cu1-points,
... ... which are QA-Tr2-isogonal conjugated or QA-Tf2-partners.

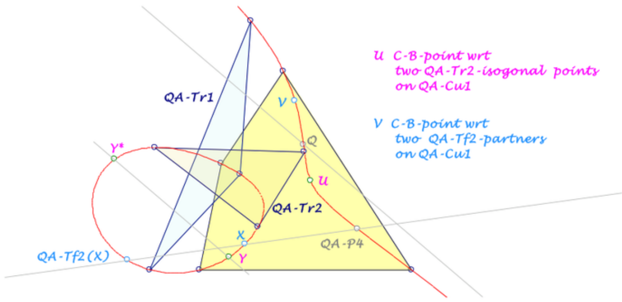
The results lead to several interesting points on the cubic.
Best regards Eckart
```

EQF-Note 2017-06-02

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cayley-Bacharach Ninth Point on QA-Cu1

The Cayley-Bacharach ninth point – here shortened C-B-point – is the ninth common point of cubics through eight given points. A.S. Hart gives a construction of the C-B-point in [1]; a mistake is corrected in QFG-message 2447. – With a CABRI-macro here are researched the C-B-points for the vertices of the diagonal triangle QA-Tr1, the vertices of the Miquel triangle QA-Tr2 and two points on the cubic QA-Cu1, which are QA-Tr2-isogonal conjugated or QA-Tf2-partner.

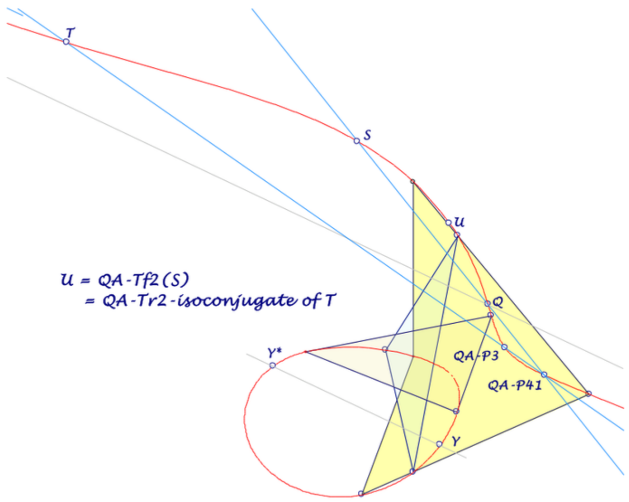


The cubic *QA-Cu1* is invariant to the isogonal conjugation wrt the Miquel triangle *QA-Tr2* and the transformation *QA-Tf2*. Let *Q* be the intersection of *QA-Cu1* and its asymptote. Two *QA-Tr2*-isogonal conjugated points on *QA-Cu1* lie on a parallel to the asymptote. Two *QA-Tf2*-partners on *QA-Cu1* are collinear with *QA-P4*.

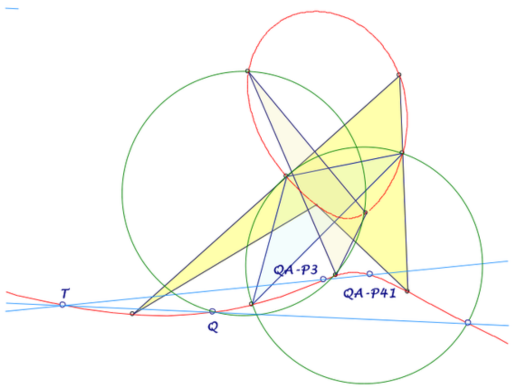
(1) Two QA-Tr2-isogonal points on QA-Cu1

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tr2*-isogonal points on *QA-Cu1*
  … is a fixed point *U* on the cubic *QA-Cu1*.

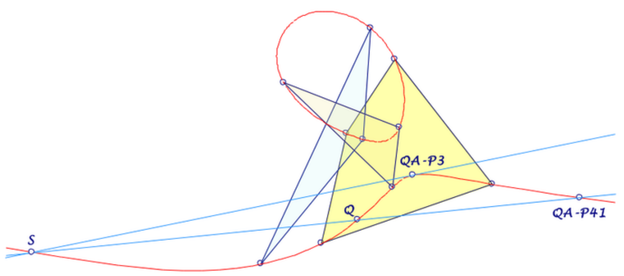
- **The *C-B*-point *U* is**
  … the *QA-Tr2*-isogonal conjugated of the third intersection *T* of *QA-Cu1* and *QA-P3.QA-P41*,
  as well as
  … the *QA-Tf2*-image of the third intersection *S* of *QA-Cu1* and *Q.QA-P41*.



- **The point *T* on *QA-Cu1* is the intersection**
  … of *QA-P3.QA-P41* and the line,
  … connecting the fourth intersections of *QA-Cu1* and the circumcircles of *QA-Tr1* and *QA-Tr2*.



- **The point *S* on *QA-Cu1* is the intersection**
  … of *Q.QA-P41* and the tangent in *QA-P3* at *QA-Cu1*.



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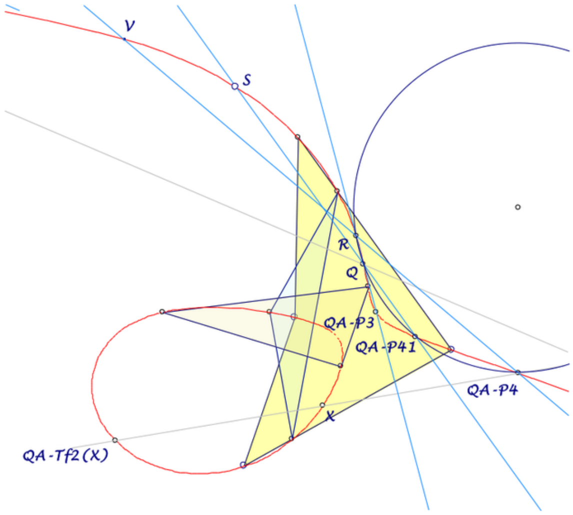
(2) Two QA-Tf2-partners on QA-Cu1

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tf2*-partners on *QA-Cu1*
  … is a fixed point *V* on the cubic *QA-Cu1*.

- **The *C-B*-point *V* is**
  … the *QA-Tr2*-isogonal conjugated of the third intersection *S* (see above) of *QA-Cu1* and *Q.QA-P41*,
  as well as
  … the *QA-Tf2*-image of the third intersection *R* of *QA-Cu1* and *Q.QA-P3*.

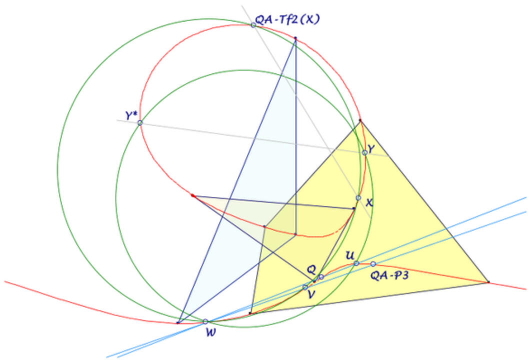


- **The point *R* is the second intersection**
  … of the line *Q.QA-P3*
  … and the circumcircle of *Q, QA-P4, QA-P41*.

Finally:

- **Circumcircles**
  … of two *QA-Tr2*-isogonal points on *QA-Cu1* and *U*
  as well as
  … of two *QA-Tf2*-partner on *QA-Cu1* and *V*
  … have a fixed fourth intersection *W* on *QA-Cu1*.

- **The point *W* is the intersection of *U.Q* and *V.QA-P3*.**



[1] A. S. Hart: Construction by Ruler alone to determine the ninth Point of Intersection of two Curves of the third degree. Cambridge and Dublin Mathematical Journal 6 (1851) 181-182.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-06-02.pdf

Message: #2459
Date: 02/6/2017 6:11:28
From: tsihonglau
Subject: Cayley-Bacharach Ninth

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> attached a research wrt the Cayley-Bacharach ninth point for
>> ... the vertices of QA-Tr1 and QA-Tr2
>> ... and two QA-Cu1-points,
>> ... ... which are QA-Tr2-isogonal conjugated or
QA-Tf2-partners.
>> The results lead to several interesting points on the cubic.

Dear Eckart,

According to topic #2431 "Any Point on Cubic as Pivot",
(though I cannot prove it), we can generalize your results.
Given a point P on a cubic, we can get five tangent lines
through P.
Four tangent lines pass through tangent points T,Ta,Tb,Tc, whose
diapleural triangle is ABC.
One tangent line is tangent to the cubic on P and intersect it
at I, which is the point of intersection of three tangent lines
to it on A.B,C.
The cubic becomes a pivotal isocubic with the pivot P and the
reference quadrangle is T,Ta,Tb,Tc.
Any QA-Tf2 conjugates are collinear with the pivot P.
P and I are conjugates.
Any circumconic through the reference triangle ABC and
conjugates passes through I, too.
Given another point P' on the cubic, we can get another
reference quadrangle T',T'a,T'b,T'c and triangle A'B'C' and
conjugate I'.
Your results can be derive from P=QA-P4 and P'=the point at
infinity of the asymptote of QA-Cu1.
The quadrangles T,Ta,Tb,Tc=the reference quadrangle,
T',T'a,T'b,T'c=the incenter/excenters of the Miquel triangle.
ABC=the reference triangle and A'B'C'=the Miquel triangle.
The isoconjugations are QA-Tf2 and isogonal conjugation with
respect to the Miquel triangle.
I=QA-P41 and I'=Q.

Best regards,
Tsihong Lau

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Message: #2460
Date: 02/6/2017 6:35:42
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
I found the answer by reading again your many messages about the
C-B ninth point !
I suppose it needs now a pedagogical summary ...
QL-P1 and the infinity point on QL-L1 are CSC partners on
QL-Cu1; therefore the ninth C-B point is their commun
tangential, the point where QL-Cu1 cuts it's asymptote (parallel
to QL-L1) and not QL-P4 ...
The C-B ninth point of QL-P1 and QL-P4 cannot be the infinity
point on QL-L1 ; it is on the symmetric line of QL-P1QL-P4 wrt
the Steiner axes, which is parallel to QL-L1 and the axis of the
parabola inscribed in the QL.
If Tf defines the transformation of the 6 QL-vertices and QL-P1
and X is any point on QL-Cu1, TfX is the CSC of the point X' on
QL-P1X such as the middle of XX' is on QL-L1 and XTfX is
parallel to QL-L1.
Best regards
Bernard
```

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Message: #2461
Date: 03/6/2017 9:42:49
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

thanks for your attention, I have to cancel the special
observation (4) in #2450.
I used a wrong constellation without further control, sorry!
The C-B ninth point of the QL-points, QL-P1 and QL-P4 is the
second intersection
... of the QA-Co1-axis
... and a circumconic of a QL-trilateral
    through QL-P1 and QL-P4.
The C-B ninth point of the QL-points, QL-P1 and a point X on
QL-Cu1
... is the 3rd intersection of QL-Cu1 and the line QL-P1.CSC(X).

Best regards Eckart
```

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Message: #2462
Date: 03/6/2017 11:49:47
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,
If I follow you, any cubic is in any of it's points a pivotal
isocubic wrt a particular triangle, which is the DT (diapleural)
of the QA formed by the contact points of the 4 tangents (other
than the tangent in the point).
(The polar conic of the point wrt the cubic is tangent in the
point to the cubic)
If we take 3 aligned points on the cubic, your construction
leads to 3 QA's in a Desmic configuration.
Best regards
Bernard
```

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Message: #2463
Date: 03/6/2017 11:56:47
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I found this note very interesting !
It's possible to jump with the QA-Tf2 and the isogonal transformation from QA-P3 to the infinity point of the asymptote and to QA-P4 and then to the point Q and the point QA-P41 ...
Have you other ideas on such items ?
For example, what about DT and S-triangle ? (the 6 vertices are coconic)
What about DT and the triangle QA-P2QA-P4QA-P41 ?The 2 circular cubics QA-Cu1 and QA-Cu7 intersect in 8 points ; what is the ninth C-B point ? What is the transformation for the 6 points and this C-B point ?
I'm sure there are plenty of other possible constructions ...
Best regards
Bernard

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Message: #2464
Date: 03/6/2017 3:37:14
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I'm sorry, it doesn't work so easily, as QA-P2 doesn't lie on QA-Cu1.
But QA-P1 and QA-P7 have in commun 7 points vertices of DT, QA-P4, QA-P41 and the 2 circular points ; hence a transformation with 2 other points on QA-Cu1 and QA-Cu7 ...
The same way, the 7 points vertices of the Miquel triangle QA-Tr2, QA-P3, QA-P4 and the 2 circular points define another transformation with 2 points on QA-Cu1 and on another circular cubic with triangle QA-P3QA-P4QA-P9 (which is a QL-Cu1 like QA-Cu7).
Best regards
Bernard

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Message: #2465
Date: 03/6/2017 7:29:37
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
Last remark, for the moment!
Considering a QL, the cubics through the vertices, QL-P1 and the
2 circular points form a pencil.
As already mentionned, QL-Cu1 and the 4 degenerated cubics
formed by a line of the QL through 3 vertices and the
circumcircle through the 3 others and QL-P1 belong to this
pencil.
But the 3 QA-Cu1 of the 3 QA's of the QL (formed by 2 copples of
vertices/CSC partners) belong also to this pencil.
The same isn't true for the QA, as the 3 QL-Cu1 of the the 3
QL's of the QA don't form a pencil with QA-Cu1.
Best regards
Bernard
```

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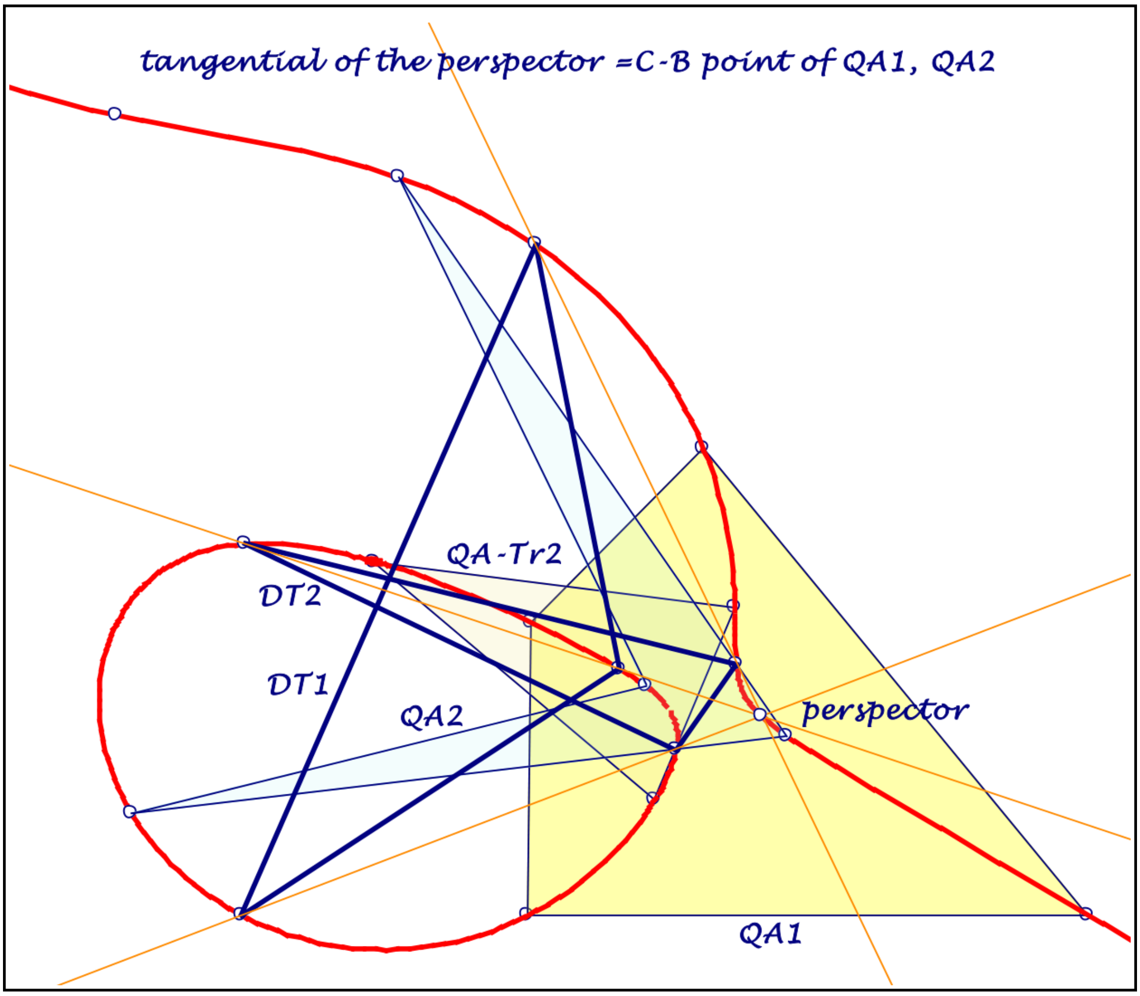
Message: #2466
Date: 04/6/2017 9:56:18
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard, dear Tsihong Lau,
not yet an answer to your messages, but a further aspect:
Consider a quadrangle QA1,
... the Möbius conjugates wrt QA-Tr2 of a point on QA-Cu1
... lead to a quadrangle QA2 (on QA-Cu1)
... with the same Miquel triangle
... and perspective diagonal triangle (on QA-Cu1)
... with perspector on QA-Cu1,
... whose tangential is the C-B point of QA1 and QA2.
Best regards Eckart
```



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Message: #2467
Date: 04/6/2017 10:40:52
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear Eckart,

A proposal! Try to summarize the results mentioned
in #2459, #2462, #2466!

Best regards,
Tsihong Lau

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Message: #2468
Date: 04/6/2017 7:07:57
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
If I'm not wrong and if I interpret your construction correctly,
you could say it this way :
3 Cu-QAs in a Desmic configuration on QA-Cu1, the 3 tangentials
QA-P4 are aligned and form with the 3 DTs 3 other QAs in a
Desmic configuration.
Then the perspector of the DTs of 2 QAs is the point QA-P4 of
the 3rd and the ninth C-B point of the vertices of the 2 QAs is
the point QA-P41 of the 3rd QA.
Best regards
Bernard

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Message: #2469
Date: 06/6/2017 11:20:36
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard, dear Tsihong Lau,

the Dimidium circle QL-Ci6 = Ci
... and the circumconic Co of the diagonal triangle QL-Tr1
    through QL-P8 and QL-P13
... intersect in QL-P24 and three points, which are the vertices
    of the triangle QL-Tr2.
For the six vertices of QL-Tr1 and QL-Tr2 there are triples of
points,
... so that the C-B point of the six vertices and two points
    of a triple is the third of the triple.

Here are such triples even collinear:
(1) QL-P24, QL-P1, QL-P8,
(2) QL-P24, QL-P13, QL-P17,
(3) QL-P24 and an arbitrary point Q and the intersection
    of Q.QL-P24 and Co.
(4) P on Co and an arbitrary point Q not on Co
    and the intersection of P.Q and Co.
Background for this interesting result:
The C-B point of six points on a conic (here QA-Tr1 and QA-Tr2
on Co)
... and two arbitrary points P, Q*
    (not both on the conic)
... is collinear with P and Q.
If P is a fixed point on QL-Ci6 and Q a variable point on QL-Ci6
... then the locus for the C-B points is a circumconic of QL-Tr2
... through P and the intersection of P.QL-P24 and Co.

Examples:
If P = QL-P24 and Q a variable point on QL-Ci6
... then the locus for the C-B points is Co.
If P = QL-P1 and Q a variable point on QL-Ci6
... then the locus for the C-B points is a QA-Tr2-circumconic
    through QL-P1 and QL-P8.
If P = QL-P17 and Q a variable point on QL-Ci6
... then the locus for the C-B points is a QA-Tr2-circumconic
    through QL-P17 and QL-P13.
If P = QL-P10 and Q a variable point on QL-Ci6
... then the locus for the C-B points is a QA-Tr2-circumconic
... ... through QL-P10 and the intersection of QL-P10.QL-P24
        and Co.
It would be good, if someone could confirm these observations!
```

Best regards Eckart

PS for Bernard: I think, your interpretation of the construction in #2466 is right, thanks for interest.

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Message: #2470
Date: 07/6/2017 11:09:13
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I don't intend to check in detail your observations, as they seem all correct.
I've the impression we have already discussed many of these properties of the mysterious S-triangle.
The vertices of this S-triangle QL-Tr2 (not QA-Tr2 as put at the end of your message) are coconic with many triangles on more than a dozent of identified conics (DT, DDT, Euler DT ...) and of course an infinity of unidentified (at the end, I've checked even the Steiner circumellise of QL-Tr2 and the duals of the incircle or of the Steiner inellipse of this triangle ...)
QL-Tr2 belongs as well to the QA environment as to the QL environment.
For example, it can be described with the vertices as intersections other than QA-P16 of the 2 conics through the QA vertices, QA-P1 and QA-P16 and through the DT vertices, QA-P10 and QA-P16 or with the sides as common tangents other than the infinity line to the inscribed parabolas of both QL and DQL (diagonals and Newton Line), which are the duals of these 2 circumconics.
Any circumconic to QL-Tr2 cuts the Dimidium circle in a 4th point D ; DQL-P1 and DQL-P17 recut the circumconic in G and P respectively. The Newton Lines of QL and DQL cut the same conic respectively in X and X' and Y and Y' ; D, X, X', QL-P17 and G are cocyclic, as well as D, Y, Y', QL-P1 and P.
For the circumconic through the vertices of DT, D is QL-P24, G is QL-P8 and P is QL-P13. Hence your triples in (1) and (2) QL-P24, QL-P1 and QL-P8 and QL-P24, QL-P13 and QL-P17.
For the circumconic through the vertices of the QA, QA-P1, QA-P16 and the vertices of DDT (anticevian triangle of QA-P13 wrt DT), D is the CSC of QL-P26, G is QA-P10 and P is QA-P1.
I suppose your triples of aligned points will be CSC(QL-P26), QL-P1 and QA-P16 and CSC(QL-P26), QA-P1 and QL-P17 ...
All this needs a generalisation and a systematisation (perhaps an explanation, but I'm not able to give one ...)
I hope you will soon make a complete recension of all these aligned triples ...
Best regards
Bernard
PS Can you provide me a simple construction of the ninth C-B point ? Thanks in advance

---

Message: #2471
Date: 08/6/2017 10:18:06
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

you ask for a simple construction of the ninth C-B point,
but I only can try to describe the construction of Hart in
#2447, respecting the observed mistake:

Let the 8 points be 1, 2, 3, A, B, C, D, E.
Find the polars of A and C with respect to each of the three
conics 123DE, 123BE, 123BD.
Let the conics cut AC at the points U, u, V, v, W, w
U, V, W being on the polars of A and u, v, w on the polars of C.

Consider the anharmonic ratios (see Hart´s prop.3),
(1) ... which is the inverse of the product
        of the anharmonic ratios (AUCV), (AuCv),
(2) ... which is the inverse of the product
        of the anharmonic ratios (AUCW), (AuCw).

Let AE, BD meet at M; and on BD find a fourth point Q,
such that (BMDQ) is (1),
let AD, CE meet at N; and on CE find a fourth point R,
such that (CENR) is (2).

Let K be QR ^ AE and L be QR ^ AD,
then BK ^ CL will be the ninth Cayley-Bacharach point.

Best regards Eckart

PS: I used the nomination of Hart,
    but replaced 4 = E, 5 = D, 6 = B, 7 = A, 8 = C
    and in Hart´s last passage A = U, B = V, C = W,
    a = u, b = v, c = w, to avoid confusion.
```

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Message: #2472
Date: 08/6/2017 6:02:23
From: Antreas Hatzipolakis
Subject: Isogonal Pivotal Cubics properties

I am fwding this, since there is an interest on cubics in this group.

[Telv Cohl]:
I found (with proof) the following property of isogonal pivotal cubic and I am finding the generalization of it.
Given a triangle ABC and a isogonal pivotal cubic [T] of ABC. P is a point on [T], a line through P cuts [T] at U, V.
Let U', V' be the isotomic conjugate of U, V WRT ABC, respectively, then
(1) U'V' passes through a fixed point.
(2) the intersection of UV, U'V' lies on a fixed conic.

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Message: #2473
Date: 10/6/2017 10:31:45
From: eckart_schmidt@t-online.de
Subject: Isogonal Pivotal Cubics properties

[AH]
> Given a triangle ABC and a isogonal pivotal cubic [T] of ABC.
> P is a point on [T], a line through P cuts [T] at U, V.
> Let U', V' be the isotomic conjugate of U, V WRT ABC,
> respectively, then
> (1) U'V' passes through a fixed point.
> (2) the intersection of UV, U'V' lies on a fixed conic.

Dear Antreas,

studying your constellation above, some remarks,
using the following nominations:
... pivot Q, fixed point F, fixed conic Co,
... res(x,y) = 3rd intersection of XY and [T],
... isogonal conjugate *, isotomic conjugate ^.

wrt (1): F = res(P,Q*)*^

Wrt (2): The points P and F lie on Co.
... You can get further points on Co, using points X of [T]: P.X
^ F.res(P,X)^.
... Such points on [T] are for example:
... ... in-/excenters of ABC, P*, Q, Q*, F^, F^*.

Best regards Eckart

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Message: #2474
Date: 10/6/2017 10:46:41
From: Antreas Hatzipolakis
Subject: Isogonal Pivotal Cubics properties

Dear Eckart,
Thank you!
I will FWD your results to Telv Cohl,
the proposer of the problem.
Best Regards
APH

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Message: #2475
Date: 10/6/2017 3:10:56
From: tsihonglau
Subject: Generalization of a Dao's Conjecture

```
Dear all,

In the paper http://vixra.org/pdf/1507.0218v2.pdf,
Dao Thanh Oai gave the following conjecture:
Let P be a point on the Neuberg cubic. Let P_A be the
reflection of P in line BC, and define P_B and P_C cyclically.
It is known that the lines AP_A, BP_B, CP_C concur. Let Q(P) be
the point of concurrence. Then two Fermat points, P, Q(P) lie on
a circle.
When P = X(3), it is well-know that Q(P) = Q(X(3)) = X(5), the
conjecture becomes Lester theorem.
The above are due to Dao.  Moreover, The circle above intersects
the Neuberg cubic at the
five points- two Fermat points, two circular points and P.
Let P' be the sixth point of intersection.  Then P, P' and
X(1138) is collinear.  X(1138) becomes another
pivot of the Neuberg cubic besides X(30).
The problem is who can prove the above and generalize it to
quadrangle/quadrilateral geometry!
The Neuberg cubic K001 is very similar to QA-Cu1.  If the
quadrangle is the incenter/excenters quadrangle, QA-Cu1
becomes K001.  The following are my ideas.  In message #1022 of
topic #1018, Seiichi gave a property of QA-Cu1:
given a quadrangle P1P2P3P4 and a point P, we denote the
reflections of P in P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 by P12,
P13, P14, P23, P24, P34 respectively. P12P34, P13P24, P14P23 are
concurrent.
If the quadrangle is the incenter/excenters quadrangle, the
point of intersection Q1(P) lies on K001 and P, Q(P)
and Q1(P) are collinear.  Moreover, Seiichi gave two other
points of intersection Q2(P) and Q3(P) in topic #1018.
P,Q1(P),Q2(P),Q3(P) are collinear and equidistant.  Kn=K060 is
the locus of the perspector Q(P).  K001 and K060 intersect at
the 9 points triangle ABC, X(4), two circular points, two Fermat
points and X(30)=the pivot of K001.

Best regards,
Tsihong Lau
```

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Message: #2476
Date: 10/6/2017 10:55:30
From: Antreas Hatzipolakis
Subject: Generalization of a Dao's Conjecture

Dear Tsihong Lau

A Lester-like circle by Alexander Skutin:
Let F1. F2 be the first and second Fermat points of ABC and
A'B'C' the cevian triangle of F1 (or F2).
The circumcenter and the orthocenter of A'B'C' and F1, F2 are
concyclic.
https://beta.groups.yahoo.com/neo/groups/Hyacinthos/conversations/messages/26125 †)

aph

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Message: #2477
Date: 12/6/2017 9:00:50
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard, dear Tsihong Lau,

for geometry on the cubic QA-Cu1 the Cayley-Bacharach point is a
good tool,
not only for two points wrt QA-Tr1 and QA-Tr2, but also for four
points wrt the QA.
Beginning with:
The C-B-point wrt QA-Tr1 and QA-Tr2
… and two points X, Y on QA-Cu1
… is the fourth intersection of QA-Cu1 and the circumcircle of
X, Y and a fixed point W.
Ending with:
The C-B-point of two quadrangles on QA-Cu1 with the same Miquel
triangle
… is the third intersection of QA-Cu1 and the line through their
QA-P41-points.

This is the content of the attached file, a continuation of
#2458.
That are CABRI-observations and I hope for no mistakes.

Best regards Eckart

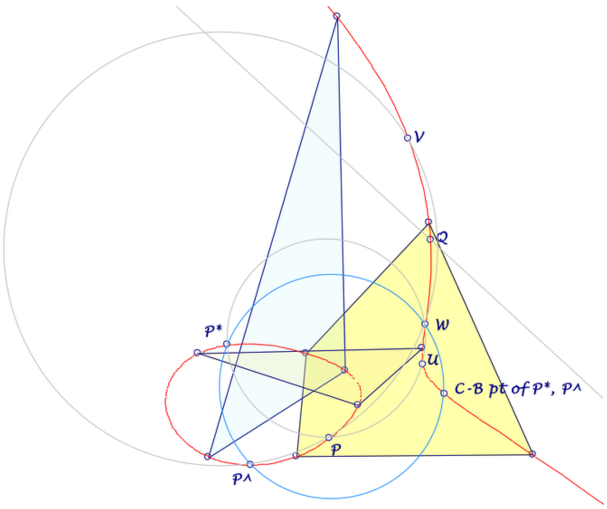
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

EQF-Note 2017-06-12

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cayley-Bacharach Ninth Point on QA-Cu1 (II)

This note is a direct continuation of EQF-Note 2017-06-02 in QFG-message 2458, where Cayley-Bacharach ninth points are discussed wrt QA-Tr1, QA-Tr2 and two points on QA-Cu1. Here finally also C-B-points of the reference QA and four points on QA-Cu1 are researched.



Remember (1) and (2) of the previous note:

(1) Two QA-Tr2-isogonal points on QA-Cu1

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tr2*-isogonal points on *QA-Cu1*
  … is a fixed point *U* on the cubic *QA-Cu1*.
- **The *C-B*-point *U* is**
  … the *QA-Tr2*-isogonal conjugated of the third intersection *T* of *QA-Cu1* and *QA-P3.QA-P41,*
  as well as
  … the *QA-Tf2*-image of the third intersection *S* of *QA-Cu1* and *Q.QA-P41*.

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(2) Two QA-Tf2-partners on QA-Cu1

- **The Cayley-Bacharach point of**
  … the vertices of *QA-Tr1* and *QA-Tr2*
  … and two *QA-Tf2*-partners on *QA-Cu1*
  … is a fixed point *V* on the cubic *QA-Cu1*.

- **The *C-B*-point *V* is**
  … the *QA-Tr2*-isogonal conjugated of the third intersection *S* (see above) of *QA-Cu1* and *Q.QA-P41,*
  as well as
  … the *QA-Tf2*-image of the third intersection *R* of *QA-Cu1* and *Q.QA-P3*.

- **Circumcircles**
  … of two *QA-Tr2*-isogonal points on *QA-Cu1* and *U*
  as well as
  … of two *QA-Tf2*-partner on *QA-Cu1* and *V*
  … have a fixed fourth intersection *W* on *QA-Cu1*.

- **The point *W* is the intersection of *U.Q* and *V.QA-P3*.**

New:

(3) Two or Four points on *QA-Cu1*

Nominations: Let *Q* be the intersection of the cubic *QA-Cu1* and its asymptote. Let *P** be the isogonal conjugate of *P* wrt the Miquel triangle *QA-Tr2* and *P^* the *QA-Tf2*-image of *P*. For *P* on *QA-Cu1* these points *P** and *P^* are also on *QA-Cu1*. For two points *X, Y* on *QA-Cu1* let *res(X,Y)* be the third intersection of *QA-Cu1* and *XY*. Finally: *X´* shall be the tangential of *X*.

The properties above can be generalized:

> **The *C-B*-point wrt *QA-Tr1* and *QA-Tr2***
> **… and two points *X, Y* on *QA-Cu1***
> **… is the fourth intersection of *QA-Cu1* and the circumcircle of *X, Y, W*.**

- **Points *X, Y* on *QA-Cu1***
  … with the same 3rd intersection *P* of *XY* and *QA-Cu1*
  … have the same *C-B*-point wrt *QA-Tr1* and *QA-Tr2*,
  … which is the 3rd intersection of *QA-Cu1* and *WP.**

> **The *C-B*-point wrt *QA-Tr1* and *QA-Tr2***
> **… of *X, Y* on *QA-Cu1***
> **is the *C-B*-point wrt the reference *QA***
> **… of *P = res(X,Y), P*, P^* and tangential *U´* of *U*.**

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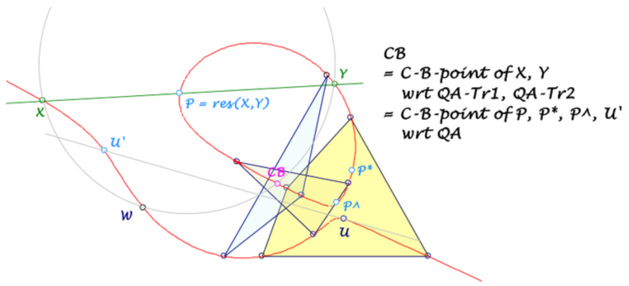

Final remarks

The results lead to new points *U*, *V*, their tangentials *U´*, *V´* and *W* on *QA-Cu1*. Point *W* can be taken as basic point in the following sense:

- ***U* = *res*(*W*,*Q*),**
  ***V* = *res*(*W*, *QA-P3*),**
  ***U´* = *res*(*W^*, *QA-P41*),**
  ***V´* = *res*(*W**, *QA-P3*).**

The *C-B*-point wrt *QA* of *U*, *V*, *U´*, *V´* is *W.**

Further *C-B*-points:

- **The *C-B*-point wrt *QA-Tr1*, *QA-Tr2* of *U*, *V* is *V**.**
- **The *C-B*-point wrt *QA-Tr1*, *QA-Tr2* of *W*, *W** is *U*.**
- **The *C-B*-point wrt *QA-Tr1*, *QA-Tr2* of *W*, *W^* is *V*.**
- **The *C-B*-point wrt *QA-Tr1*, *QA-Tr2* of *W**, *W^* is *V´*.**

- **Four points on *QA-Cu1* with *res*(*A*,*B*) = *res*(*C*,*D*) = *P***
  … have *C-B*-point wrt *QA*
  … in *res*(*P´*,*QA-P41*).

A perspective quadrangle on *QA-Cu1*
… with perspector *P* on *QA-Cu1*
… has a *C-B*-point wrt the reference *QA*
… in the tangential of the tangential of *P*.

- **The *C-B*-point of the *QA*-vertices and their *QA-Tr2*-isogonal conjugates**
  … is the tangential of *Q*.

The *C-B*-point of two quadrangles on *QA-Cu1* with the same Miquel triangle
… is the third intersection of *QA-Cu1* and the line through their *QA-P41*-points.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-06-12.pdf

Message: #2478
Date: 12/6/2017 4:43:07
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Very interesting , these 2 notes 2458 and 2477 !
Some properties are linked with the Desmic configuration (the 3rd intersection with QA-Cu1 of the line through the 2 QA-P41 is the P41 of the 3rd QA) ...
For example, the QA formed by the DT and QA-P4 and the QA formed by the Miquel triangle QA-Tr2 and the infinity point of the asymptote are in a Desmic configuration with a 3rd QA formed by the isogonal conjugates of the DT vertices wrt QA-Tr2 and the isogonal conjugate of QA-P4, which is QA-P3 (as well known perspector of QA-Tr1 and QA-Tr2). Therefore, the tangential of the 3rd QA is in line with QA-P41 and Q, the tangentials of the 2 others ; it is the point you named S and the point you named U is the tangential of this point S ...
I'm sure other such examples could explain other points like R, T and V, W ...
I'm awaiting now impatiently such a note about QL-Cu1, which is a particular case of QA-Cu1 with 2 opposite sides perpendicular, generalising your 1rst conclusions in the note 2469 !
Best regards
Bernard

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Message: #2479
Date: 15/6/2017 3:09:43
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard,

thanks for your certain comments, lightening the background of my observations.
Excuse, if I can´t complete them, but they give motivation for new tests.

You ask for observations wrt QL-Cu1:
The results of my last note can be taken for QL-Cu1 in the bipartite case for QA on QL-Cu1, whose vertices have a common tangential on QL-Cu1.

Let these QA be named as T-QA on QL-Cu1:
... The vertices of a T-QA have a common tangential on QL-Cu1.
... QA-Cu1 of a T-QA is QL-Cu1.
... Two T-QA are perspective with perspector on QL-Cu1.
... T-QA have the same Miquel triangle QA-Tr2.
... T-QA can be constructed with the Möbius transformations
    wrt the common QA-Tr2
... (the vertices of the common QA-Tr2 are QL-P1
... ... and the CSC-partner on a QL-L1-perpendicular
        in the intersection with QL-L6).

This will be wellknown. Perhaps new:

The C-B-point of two T-QA on QL-Cu1 is
... the tangential of the tangential of the perspector,
... or
... the 3rd intersection of QL-Cu1 and the line connecting
    the QA-P41 of the T-QA.

Finally attached a more basic observation:

Consider a pivotal isocubic Cu
... with referent triangle ABC,
... isoconjugation *,
... pivot Q;
further four points X1, Y1, X2, Y2 on Cu
... with res(X1,Y1) = res(X2,Y2) = P.
The C-B-point of A, B, C, P, X1, Y1, X2, Y2
... is independent of X1, Y1, X2, Y2
... the 6th intersection of Cu and a cubic
    through A, B, C, Q, P*

```
... (easily constructable as 4th intersection
    of the conics (A,B,C,X1,Y1), (A,B.C,X2,Y2).

Best regards Eckart

PS: Was #2471 helpful, to get a macro for the C-B-point?
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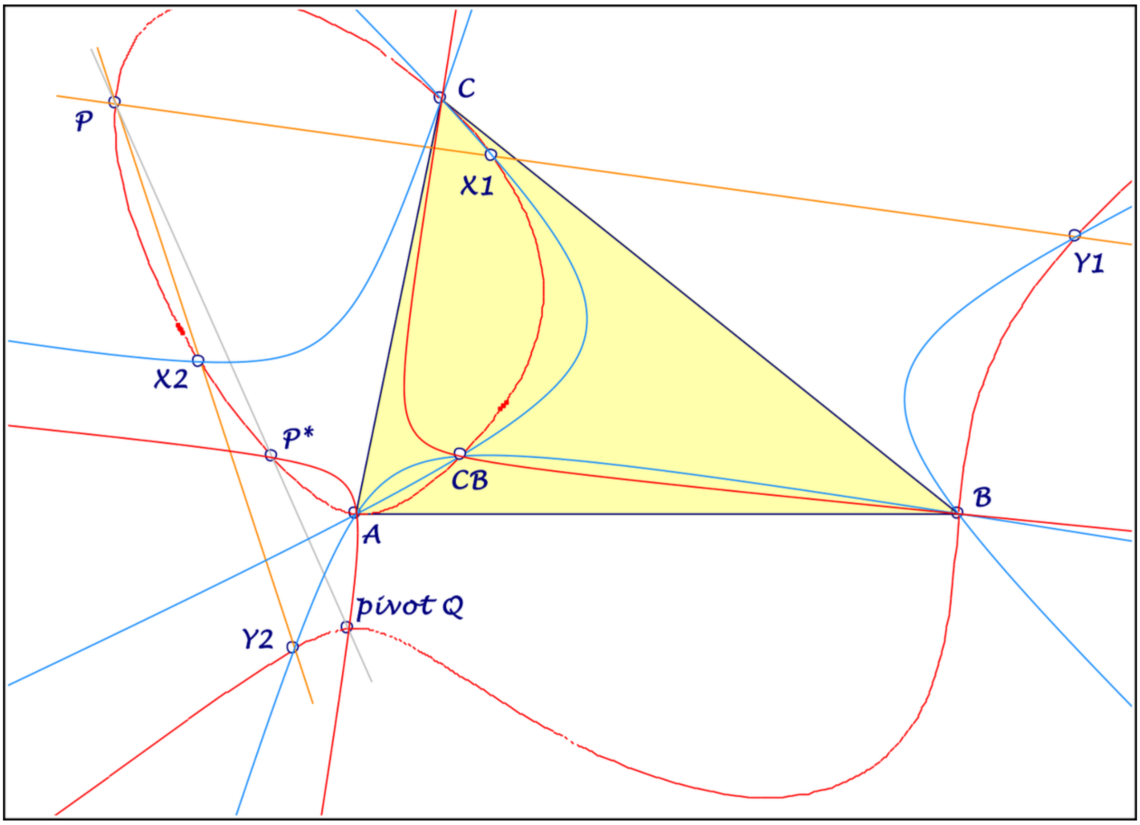

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Message: #2480
Date: 16/6/2017 9:09:02
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

I am not familar with Desmic configurations,
but I hope my interpretation of your remarks in #2478, #2468,
#2462 is right:

Consider a quadrangle and its QA-Cu1
... and three collinear points Pi (i = 1,2,3)
    on the open part of QA-Cu1
... with the quadrangles QAi of the contact points
    of their tangents at QA-Cu1
... and the diagonal triangles Tri of QAi.

The quadrangles QAi give a Desmic configuration;
... the C-B-point of two QAi is QA-P41 of the third;
... the points QA-P41 of QAi lie collinear on QA-Cu1.

The quadrangles QUi of the Tri-vertices and Pi give another
Desmic configuration;
... the C-B-point of QUi, QUj is the tangential of QA-P41
    of QAk;
... the tangentials of QA-P41 of QAi lie collinear on QA-Cu1.

Best regards Eckart
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Message: #2481
Date: 17/6/2017 8:23:08
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard,

I have found a reference for generalisation of my last result in #2477:

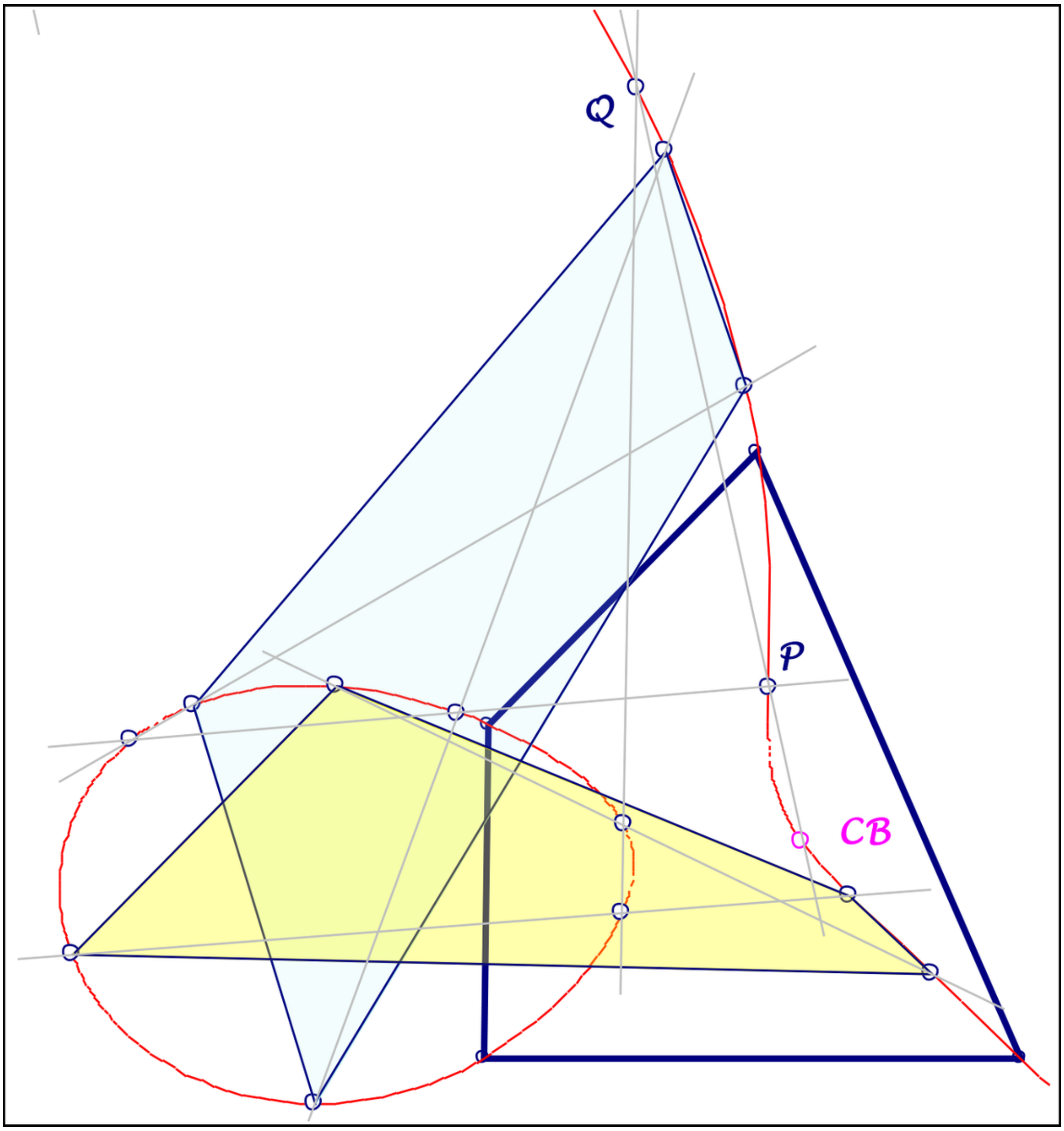
Thomas Cotterill
A Geometrical Property of Curves of the third Order
The Cambridge and Dublin Mathematical Journal
Vol VII, p.14, 1851

The C-B-point of 8 points on a cubic:

Let P1,2,3,4 and Q1,2,3,4 be 8 points on a cubic Cu.
... Consider a circumconic of P1, P2, P3, P4,
... ... intersecting Cu in two further points P5, P6
... ... with res(P5,P6) = P.
... Analog for Q1, Q2, Q3, Q4 the point Q.
... The C-B-point is res(P,Q).

For a construction wrt QA-Cu1 (see attached file):
... consider for circumconics line pairs
... for example P1P3, P2P4 and Q1Q3, Q2Q4,
... then P = res(res(P1,P3),res(P2,P4))
 and Q = res(res(Q1,Q3),res(Q2,Q4)).

Best regards Eckart



2017-06-17.pdf

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Message: #2482
Date: 18/6/2017 1:45:04
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,

Nice!  Please apply this construction to the three classes
of xxxlogical quadrangles in the first message #2440
and find more properties of Cayley-Bacharach ninth points!

Best regards,
Tsihong Lau

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>  The C-B-point of 8 points on a cubic:
>  Let P1,2,3,4 and Q1,2,3,4 be 8 points on a cubic Cu.
>  ... Consider a circumconic of P1, P2, P3, P4,
>  ... ... intersecting Cu in two further points P5, P6
>  ... ... with res(P5,P6) = P.
>  ... Analog for Q1, Q2, Q3, Q4 the point Q.
>  ... The C-B-point is res(P,Q).
```

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Message: #2483
Date: 18/6/2017 11:23:55
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,

the C-B-point of two cyclologic triangles and their cyclologic
centers
... is the intersection S of the cubic
    of generalized cyclologic centers and its asymptote,
... detailed described in my note in #2067.

Please help, I don´t remember:
What are cyclologic quadrangles?

Best regards Eckart
```

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Message: #2484
Date: 18/6/2017 2:48:13
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,

>> the C-B-point of two cyclologic triangles and their
cyclologic centers
>> ... is the intersection S of the cubic of generalized
cyclologic centers and its asymptote,
>> ... detailed described in my note in #2067.
Thanks!  I will check it if I am not so busy.

>> Please help, I don´t remember:
>> What are cyclologic quadrangles?
Message #2351 summarized many topics about cyclologic
triangles/quadrangles.  Please check it!  Topic #1997 focuses
on quadrangles while topics #1968 and #2353 focus on triangles.
We can get triangle chains or center chain only in triangle
geometry.

Best regards,
Tsihong Lau
```

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Message: #2485
Date: 18/6/2017 4:01:17
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Wunderbar ! This short article by Thomas Cotterill is very interesting and intriguing.
Thanks for your messages 2479, 2480 and 2481 !
It gives now many examples and properties with a synthetic explanation.
1) I couldn't use your 1rst construction of the 9nth C-B point, as I don't use macros with Geogebra, but I read in the mentionned article page 15 below that abler geometers could give a construction for determining the 9nth point by purely descriptive linear methods. That's precisely what I hoped !
Maybe you will find such a method ...
2) With the article, maybe it will be clear, as QA-P3 is the perspector of DT (QA-Tr1) and Miquel (QA-Tr2) vertices, why the point you named U is precisely the tangential of the tangential of QA-P3 ...
3) Last idea : We perfectly agree on the 3 QA's forming a Desmic system (or the 12 points a Reye configuration) and the DT's and the tangentials forming again a 2nd Desmic system.
I had described in the message 1947 what I named a hidden Desmic: each side of one of the 3 QA's cuts each side of the 2nd on a side of the 3rd ; it happens that the 12 new points form a 2nd Reye configuration on a 2nd cubic. The 2 Desmic systems of 3 QA's lead to the same DT vertices, the vertices of a DT of a system being one of each DT of the other and vice-versa.
The 2 cubics are different and intersect in the 9 DT vertices ; the 9 points form a C-B system ...
In other words, taking a Desmic system of 3 QA's on a cubic ; you may withdraw 3 points on a line (one of each QA). The remaining 9 points form a C-B system, where each group of vertices is a triple for the 6 other points.
(Again, QA-Tr1, QA-Tr2 and the triangle having as vertices the isogonal conjugates of QA-Tr1 wrt QA-Tr2 form such a system)
Best regards
Bernard

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Message: #2486
Date: 19/6/2017 4:02:25
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard, dear Tsihong Lau,
thanks for your comments, here some further C-B-splitter:

(1) C-B-point on QA-Cu7
... of the 3 QA-Tr1-vertices, the 3 QG-P18, QA-P2, QA-P4
... is res(QA-P2,QA-P4),
... which is the 2nd intersection of QA-P2.QA-P4
... ... and a QA-Tr1-circumconic through QA-P4 and the infinity point QA-Tf2(QA-P2).

(2) The C-B-point of 4 pairs of points Xi, Yi on QA-Cu1 with res(Xi,Yi) = P
... is the tangential of the tangential of P.

(3) The C-B-point of the QL-Cu1-intersections of parallels to the QL-lines through QL-P1
... is the tangential of the intersection of QL-Cu1 and its asymptote.

(4) Two circles, intersecting QA-Cu1/QL-Cu1 in 2x4 points
... give a C-B-point in the intersection of the cubic and its asymptote.

(5) The QA-vertices
... and 4 intersections of a circle and QA-Cu1
... have a C-B-point isogonal conjugated to QA-P41 wrt QA-Tr2.

(6) The 4 intersections of a circle and QA-Cu1
... and their tangentials wrt QA-Cu1
... have a C-B-point in the the point at infinity of the asymptote.

(7) The circumcircles of the QA-triangle components
... intersect QA-Cu1 in 4 further points,
... adding the 4 QA-vertices
... gives a C-B-point res(W, QA-P4)
... ... (wrt point W see #2477).

I hope, someone can confirm these results.

Best regards Eckart

Message: #2487
Date: 19/6/2017 4:17:58
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Reading again this short article by Thomas Cotterill, I find it really fascinating !
For any triangle on a cubic, the transformation which associates to a 4th point the focus of the 4 points seems very interesting.
For QA-Tr1, I suppose it will be QA-Tf2 and for QA-Tr2, it will be the isogonal transformation.
Did you already study this note completely ?
For example, the conic through DT vertices and 2 QA-Tf2 partners is QA-Tf2 of a line through the 2 partners ; it has a fixed points which is QA-Tf2(QA-P4), id est QA-P41.
The same way, the conic through Miquel vertices and 2 isogonal partners is the isogonal of a line through the 2 partners is contains the isogonal of the infinity point, id est the point Q where QA-Cu1 meets it's asymptote.
In both cases, the line and the conic form a degenerated cubic ...
It seems many well-known properties are only special cases of a more general configuration !
I hope you will find plenty more properties ...
Best regards
Bernard

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Message: #2488
Date: 20/6/2017 1:29:21
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach-Ninth

```
Dear Bernard,

I think, your assumption:

"For any triangle on a cubic, the transformation which
associates to a 4th point the focus of the 4 points seems very
interesting. For QA-Tr1, I suppose it will be QA-Tf2 and for
QA-Tr2, it will be the isogonal transformation."

... has to be:

The focus of a point P on QA-Cu1
... wrt QA-Tr1 is res(QA-Tf2(P),QA-P41),
... wrt QA-Tr2 is res(P*,Q)
... ... (* isogonal conjugated wrt QA-Tr2,
         Q intersection of QA-Cu1 and its asymptote).

Perhaps of interest, C-B-points for a special Desmic
configuration:
... Consider a quadrilateral with bipartite QL-Cu1
... with a QL-line L4, intersecting the open part of QL-Cu1
    in 3 points S14, S24, S34.
... The contact points of their tangents at QL-Cu1 give 3 QA
    in Desmic configuration.
... The C-B-points of two QA give the QG-P18-points
    of the QG-versions of the quadrilateral.

Best regards Eckart
```

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Message: #2489
Date: 20/6/2017 5:38:59
From: bernard.keizer
Subject: Cayley-Bacharach-Ninth

Dear Eckert,
Thank you very much for this correction !
I suppose this leads to plenty of new properties, alignments or coconicity ...
I tried for example to associate to the vertices of Tr1 and Tr2 to 2 of the 3 points I (infinity point of the asymptote), P3 and P4. If I'm not wrong somewhere, the foci are
for Tr1 and I, P3 and P4 respectively T, Isg(P41) and tg(P41) and
for Tr2 and the same points tgQ, Tf2(Q) and R
This means that U for Tr1, Tr2, P3 and P4 is res(R,Isg(P41)) and res(tg(P41),Tf2(Q)),
V for Tr1, Tr2, P3 and I is res(TTf2(Q)) and res(tgQ,Isg(P41))
Z for Tr1, Tr2, P4 and I is res(R,T) and res(tg(P41),tgQ)
This confirms the fact that the 9nth C-B point for the 4 copples of points having the same perspector P3, id est the vertices of Tr1 and Tr2 and P4 and I is the tangential of the tangential of P3.
I, P3 and P4 being aligned, their tangentials Q, S and P41 are also aligned, as well as the tangentials of their tangentials ; therefore res(tg(P41),tgQ)) is tgS = tgtgP3.
And we have many copples of conics circumTr1 through one point or circumTr2 through the other and the focus of the other QA which intersect in the 9nth C-B point.
Best regards
Bernard
PS Thanks to you, I begin to understand your first notes on the subject, which was a remarkable pioneer work, as long as you didn't know Cotterill's paper ...

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Message: #2490
Date: 21/6/2017 2:55:26
From: bernard.keizer
Subject: Cayley-Bacharach-Ninth

Dear Eckart,
Reading again Special Isocubics in the Triangle Plane by J.P. Ehrmann and B. Gibert, specially on page 8, I find this :
For a triangle ABC let's consider a pivotal isocubic with isoconjugation * and pivot P.
The isocubic is the locus of the points M for which M and M* are collinear with P.
P*, M and P/M are also collinear, as well as P*, M* and P/M*.
P* is the isopivot or secondary pivot and P/M is the P-ceva conjugate of M or cevian quotient : it is the perspector of the cevian triangle of P and the anticevian triangle of M wrt ABC.
The focus of the 4 points A, B, C and M on the isocubic is the point res(M*,P*), which is P/M*.
For QA-Cu1, for Tr1, P and P* are P4 and P41 and for Tr2, P and P* are Q and tgQ ...
Do you agree with this ?
The interest is that P/M is constructible only with lines !
Best regards
Bernard

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Message: #2491
Date: 21/6/2017 3:01:43
From: bernard.keizer
Subject: Cayley-Bacharach-Ninth

Dear Eckart,
Sorry, I forgot half of the properties !
We have also P/P*, M* and (P/M)* collinear, as well as P/P*, M and (P/M*)*, where P/P* is tgP* ...
Best regards
Bernard

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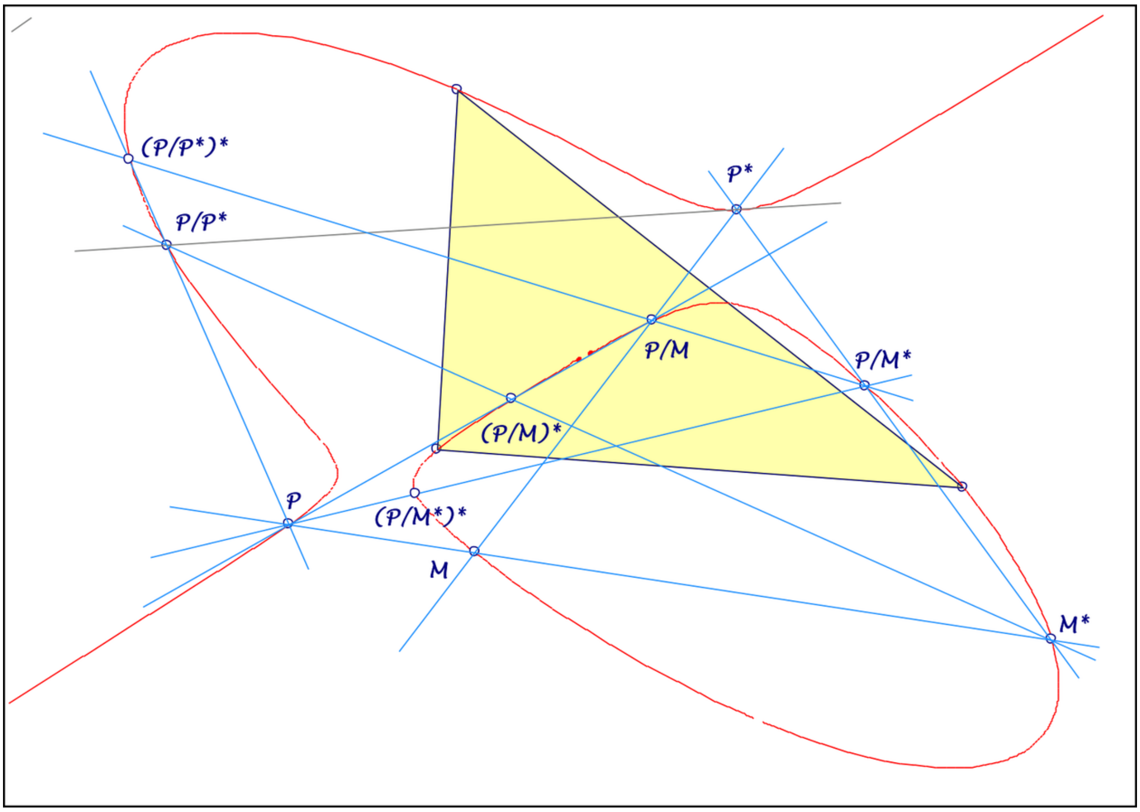
Message: #2492
Date: 21/6/2017 4:58:20
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard,

I am glad, that someone studies my results in such a thorough investigation!
Your #2489 was an interesting excursion in QA-Cu1-geometry!
I have reproduced your results for the new C-B-point Z.

Wrt # 2490/#2491, using the ceva-conjugate, I agree with your remarks (see attached file):
But:
"For QA-Cu1, for Tr1, P and P* are P4 and P41 and for Tr2, P and P* are the point at infinity of QA-Cu1 and Q."

Best regards Eckart



2017-06-21.pdf

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Message: #2493
Date: 21/6/2017 5:59:40
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
it's a real pleasure !
Thanks for the figure.
Of course, you're right for Tr2, I meant I and tgI ...
Best regards
Bernard
```

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Message: #2494
Date: 21/6/2017 6:45:04
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
One collinearity is missing in the attachment!
P/P*-M-(P/M*)*
Best regards,
Tsihong Lau!

---In Quadri-Figures-Group@yahoogroups.com, wrote :
> Dear Bernard,
> I am glad, that someone studies my results in such a thorough
investigation!
> Your #2489 was an interesting excursion in QA-Cu1-geometry!
> I have reproduced your results for the new C-B-point Z.
> Wrt # 2490/#2491, using the ceva-conjugate, I agree with your
remarks (see attached file):
> But:
> "For QA-Cu1, for Tr1, P and P* are P4 and P41 and for Tr2, P
and P* are the point at infinity of QA-Cu1 and Q."
> Best regards Eckart
```

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Message: #2495
Date: 22/6/2017 3:01:33
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard, dear Tsihong Lau,

another observation:

Consider the Desmic configuration
... of the collinear inflection points of QL-Cu1 in the
bipartite case:
... which leads to three degenerated QA,
... consisting in one inflection point
... ... and three collinear contact points of its tangents at
QL-Cu1.
... The C-B-point of two of these QA is the third inflection
point
... (which is QA-P41 of the 3rd QA).

Best regards Eckart
```

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Message: #2496
Date: 23/6/2017 10:03:32
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
If I'm not wrong, this configuration holds for both unipartite
and bipartite and for any 3 aligned points of QL-Cu1, taken as
QA-P4; the QA-P41 are also in line.
The beauty of your construction is that each inflexion point is
at the same time it's own tangential ?
Best regards
Bernard
```

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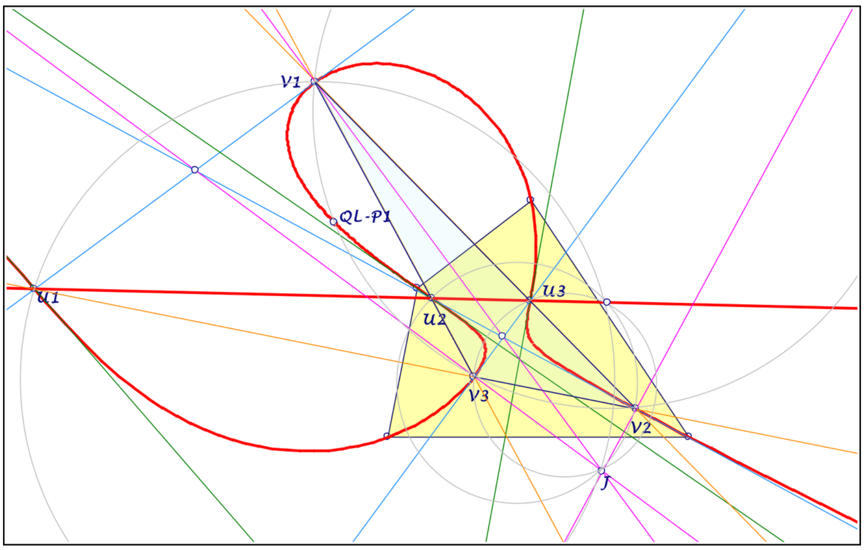
Message: #2497
Date: 23/6/2017 8:30:17
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 inflection points

```
Dear Bernard,

If QL-Cu1 is bipartite,
... there are 4 tangents through an inflection point Ui:
... one is the inflection tangent,
... one contacts QL-Cu1 in Vi = CSC(Ui),
... two are CSC-partner on a perpendicular to UiVi in Vi.
If QL-Cu1 is unipartite, there are only two tangents through an
inflection point Ui,
... one is the inflection tangent,
... the other contacts QL-Cu1 in Vi = CSC(Ui).
In general holds (see attached file):
... Vi, Vj, Uk are collinear,
... the circumcircle of V1V2V3 bears QL-P1,
... CSC-partner  on QL-Cu1 are isogonal conjugated
    wrt the the triangle V1V2V3,
... the tripol J of U1U2U3 wrt V1V2V3
    is an in-/excenter of V1V2V3,
... UiVi and ViJ are orthogonal angle bisectors of V1V2V3,
... inflection tangents in Ui, Uj intersect on VkJ,
... tangents UiVi and UjVj intersect on VkJ,
... circumcircles of UiViJ have a common 2nd intersection
    on U1U2U3.
But: What about a construction of the inflection points?

Best regards Eckart
```



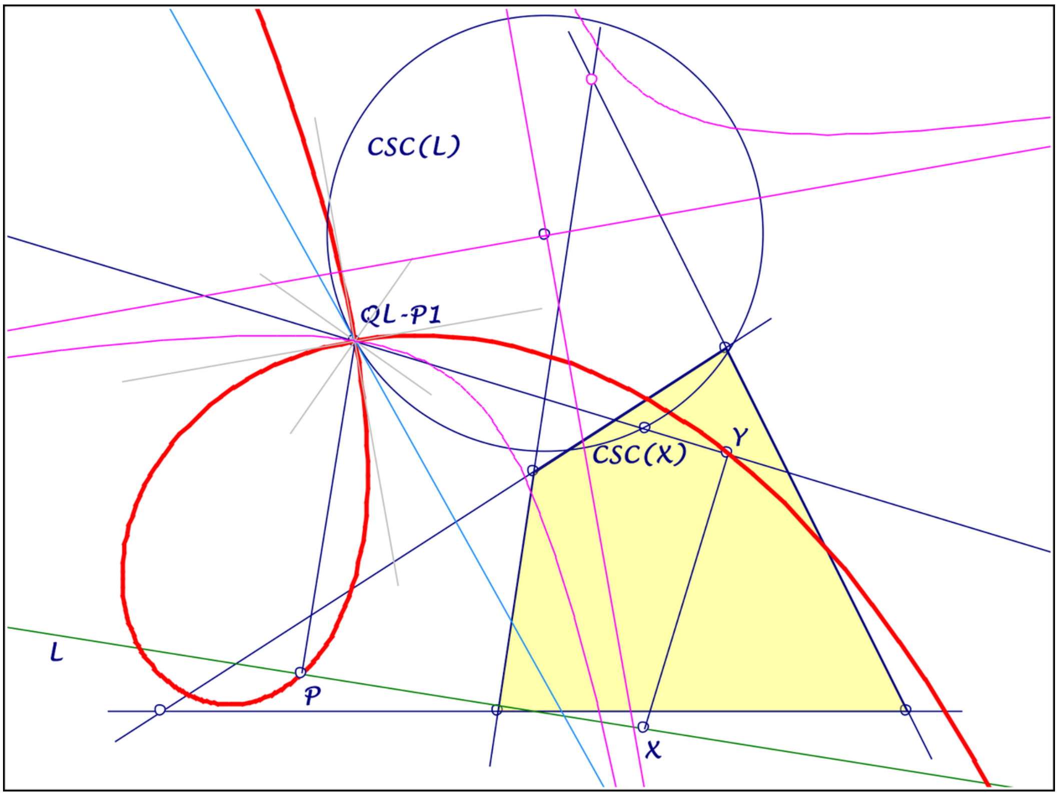
2017-06-23.pdf

Message: #2498
Date: 24/6/2017 9:10:17
From: eckart_schmidt@t-online.de
Subject: QL-Strophoids

```
Dear Bernard, dear Chris,

a short excursion in QL-geometry:
Consider a quadrilateral (see attached file)
... and a line L with points X:
... The pedal points Y of X wrt QL-P1.CSC(X)
... give a strophoid:
... ... line: tangent in QL-P1 at CSC(L),
... ... pole P: pedal point of QL-P1 on L,
... ... fixed point: QL-P1,
... with orthogonal tangents in QL-P1,
... ... which are the angle bisectors of the Steiner axes.
The CSC-image of the strophoid
... is an orthogonal hyperbola
... ... through QL-P1,
... ... centered in the middle of the circle CSC(L)
... ... with asymptotes parallel to the Steiner axes.

Best regards Eckart
```



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Message: #2499
Date: 24/6/2017 9:01:25
From: Antreas Hatzipolakis
Subject: Miquel point

```
I guess the following theorem (by Kerasarides (*)) is old !
Let M be the Miquel point of a quadrilateral and A*B*C* the
triangle bounded by its diagonals.
1. M lies on the NPC of A*B*C*
2. M is the orthopole of the Steinel line of the quadrilateral
wrt triangle A*B*C*.
(*)
Romantics of Geometry # 900 ( https://www.facebook.com/photo.php
?fbid=10211507042086441&set=gm.1354270578019978&type=3&theater )
aph
```

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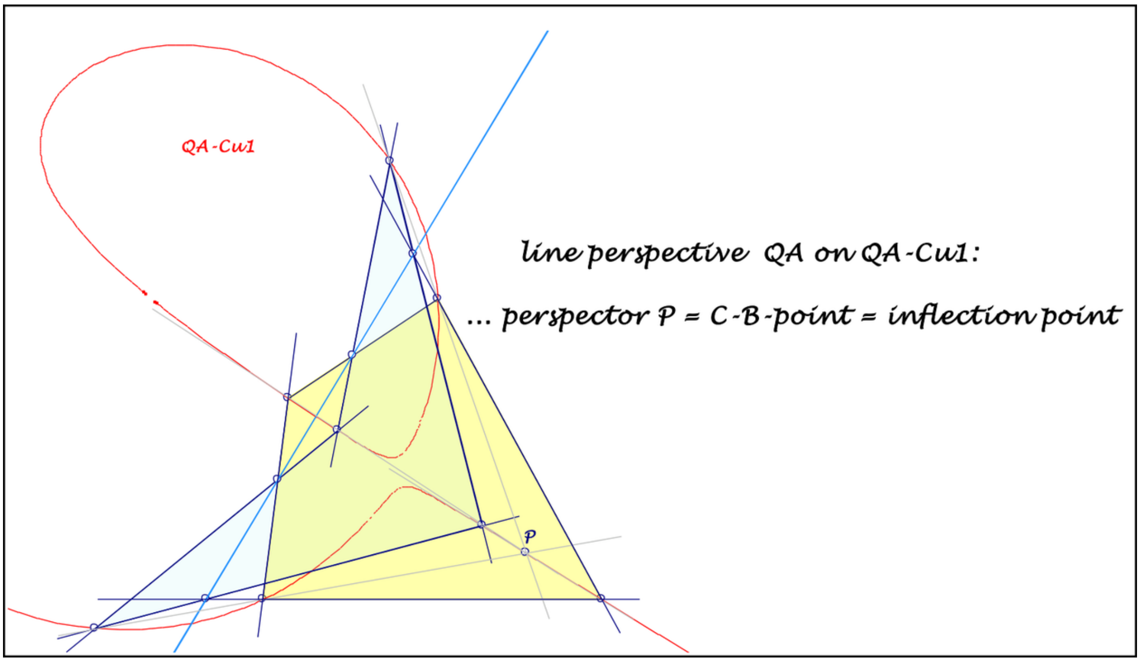
Message: #2500
Date: 27/6/2017 12:06:53
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard, dear Chris,

point-perspective quadrangles must not be line-perspective (see Chris´ #2139),
... but line-perspective quadrangles are point-perspective.

There are three quadrangles QAi on QA-Cu1,
... line-perspective with the reference quadrangle QA:
... the C-B-points of QA and QAi are the perspectors of QA and QAi,
... the perspectors are the inflection points of QA-Cu1,
... the perspectrix is the line
... ... bearing the contact points of tangents from the perspector at QA-Cu1.

Best regards Eckart



2017-06-27.pdf

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Message: #2501
Date: 27/6/2017 6:38:36
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear all,

I gave the topic "Cayley-Bacharach Ninth" not "Cayley-Bacharach
Ninth Point" deliberately.
That is because we can have "Cayley-Bacharach Ninth Line".
In #2445, I gave six cases(1 to 6) of "Cayley-Bacharach Ninth
Line".
But no one reply!
I hope someone will study them and post the results!

Best regards,
Tsihong Lau
```

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Message: #2502
Date: 30/6/2017 10:40:42
From: bernard.keizer
Subject: QL-Cu1 inflection points

Dear Eckart,
Thanks for all these precisions !
If I remember an old message by Bernard Gibert, there isn't a simple construction of the 3 inflexion points.
These 3 points are the only real intersections of the cubics belonging to the socalled syzygetic pencil (the 6 others are imaginary).
Therefore, the 3 points may be constructed as intersections between QL-Cu1 and QL-Cu2 or between QL-Cu1 and the 2 other hessians of the 2 cubics having the same cayleyan as QL-Cu2.
We discussed this topic before, these 2 other hessians are the locus of the points Y and Y' one one part and Z and Z' on the other, where Y, Y', Z and Z' are the 4 poles of a line XX' joining 2 CSC partners on QL-Cu1 ...
Best regards
Bernard

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Message: #2503
Date: 30/6/2017 10:51:30
From: bernard.keizer
Subject: QL-Strophoids

Dear Eckart,
Very beautiful construction of a new family of strophoïds !
You may define it directly with the rectangular hyperbolas : for any point X, the rectangular hyperbola is centered in CSC(X), has asymptotes parallel to the Steiner axes and passes through QL-P1 ; the strophoïd is the CSC of this Rectangular hyperbola ...
Best regards
Bernard

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Message: #2504
Date: 30/6/2017 11:05:29
From: bernard.keizer
Subject: Miquel point

Dear Antreas,
The 1rst property is already in EQF :
the Miquel point QL-P1 is the focus of the inscribed parabola QL-Co1, which is also inscribed in QL-Tr2, the medial triangle of DT or QL-NPC triangle.
The 2nd property is not in EQF :
it is explained in Mathworld that the orthopole of any line through the circumcenter of a triangle lies on the NPC triangle and on the Simson Line which is perpendicular to the line ; here in fact, the Steiner Line QL-L2 passes through the circumcenter of DT, QL-P9 and QL-P1 lies on the NPC of DT and also on the axis of the inscribed parabola, which is parallel to the Newton Line QL-L1 and perpendicular to the Steiner Line QL-L2 and is the Simson Line of the point QL-P16 (reflexion of the ortocenter of DT QL-P10 in the Miquel point QL-P1).
Best regards
Bernard

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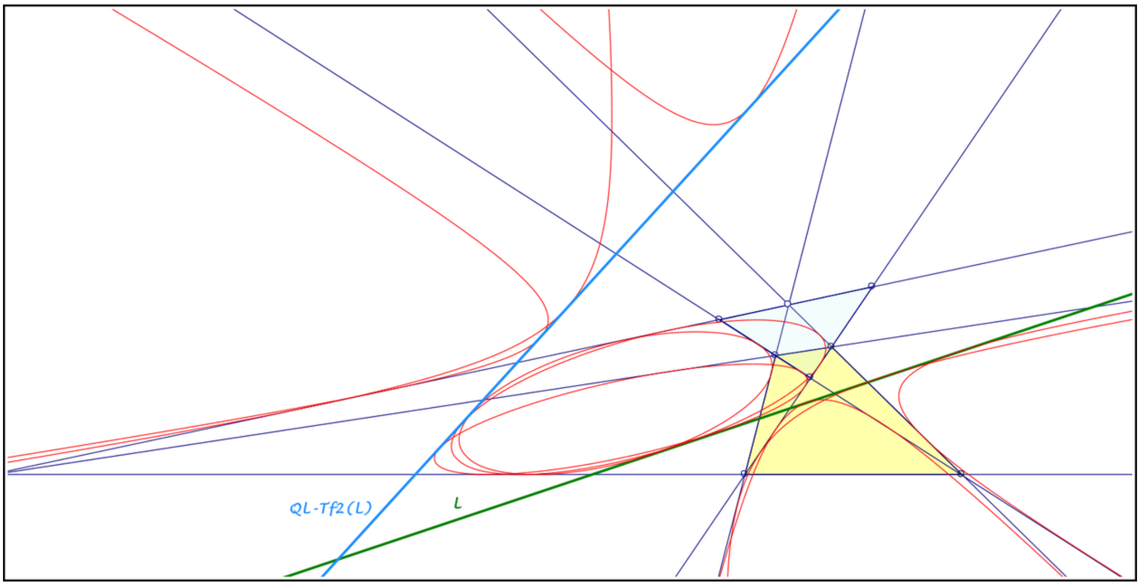
Message: #2505
Date: 01/7/2017 9:16:04
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau, dear Bernard,
perhaps on the way to a Cayley-Bacharach ninth line (see
attached file):
Consider for a QL the following 8 lines:
... 4 QL-lines,
... 3 side lines of the diagonal triangle QL-Tr1
... and an arbitrary line L.
Let S be the intersection of two QL-lines and a QL-Tr1-sideline:
The conics tangent to the remaining 5 lines
... have a common tangent in QL-Tf2(L).
Best regards Eckart
```



2017-07-01.pdf

Message: #2506
Date: 02/7/2017 4:16:33
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear all,

To Eckart, QL-Tf2 is the dual of QA-Tf2.
Both are special cases of Cayley-Bacharach Theorem!
In #2452, I gave triple operations of points/lines.
I give a special case of case 5 in #2445.
Given a quadrangle(or quadrilateral), we have six diagonal lines
together with the line at infinity, according to
Cayley-Bacharach theorem we have a line involution.

Using DT-barycentrics in EQF, the involution is
ux+vy+wz -> (q^2(u-v)^2-r^2(w-u)^2)(-p^2u^2+q^2v^2+r^2w^2-vw(-p^
2+q^2+r^2))x+...(cyclic defined)
This involution is missing in EQF.

If we replace p,q,r with a,b,c respectively, we get a line
involution
of triangle/trilateral.
This involution is missing in ETC, too.

Best regards,
Tsihong Lau
```

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Message: #2507
Date: 02/7/2017 11:08:09
From: eckart_schmidt@t-online.de
Subject: QG-P18

```
Dear Bernard, dear Chris,

perhaps not mentioned in EQF:
The 3 QL-versions of QG-P18 are collinear points on QL-Cu1 (see
EQF),
... they are the common tangentials of opposite QL-points wrt
QL-Cu1.

Best regards Eckart
```

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Message: #2508
Date: 02/7/2017 11:15:48
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

you have studied the paper of Cotterill, there we find:
"Every conic touching four tangents to a curve of the third
class will be touched again by two of its tangents, and ... "
Can it be, that this doesn´t hold? I can´t confirm this with
construction.

Best regards Eckart
```

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Message: #2509
Date: 02/7/2017 2:48:58
From: bernard.keizer
Subject: QG-P18

```
Dear Eckart,
>From the 6 vertices of QL, 3 are on line and the 3 others are
on a circle through QL-P1.
The 3 tangentials of the 3 copples of opposite points are
aligned as tangentials of 3 points themselves aligned !
Best regards
Bernard
```

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Message: #2510
Date: 02/7/2017 4:28:14
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I didn't try to reproduce this property.
But I've noticed that the the curve mentionned just below the sentence you quote is the envelop of the Simson Lines or Steiner deltoïd, which is not a cubic, but a quartic !
Best geometers can make mistakes ...
Best regards
Bernard

PS My answer to your preceding message was perhaps not complete : if you name A and A', B and B' and C and C' the 3 copples of Cl-S conjugate vertices of the QL and A1 the tangential of A and A', the Cl-S of A1 is A0, the 3rd intersection of AA' with QL-Cu1. Then, the bisector of the angles BAB', CAC' and A0AA1 is the same as well as the bisector of the angles BA'B', CA'C' and A0A'A1 (property of QL-Cu1 in each of it's points), which explains your construction ...

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Message: #2511
Date: 02/7/2017 4:45:22
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Tsihong Lau, dear Bernard,

I suppose, this will be the Cayley-Bacharach ninth line:
Construction:
Consider eight lines and take four for a reference QL and its dual QA,
... duality gives eight points for the eight lines,
... take their Cayley-Bacharach ninth point,
... its dual will give the Cayley-Bacharach ninth line.
The construction is independent of the choosen reference QL!!!

Best regards Eckart

PS: Thanks for your last mails, I shall answer later.

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Message: #2512
Date: 02/7/2017 8:47:22
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

thank you very much for the remark wrt the Steiner deltoid!
Perhaps we have to replace cubics by quartics wrt C-B-lines.
My observation:
The C-B-line, constructed as in #2511, of eight Simson lines
... is again a tangent to the Steiner deltoid.

Best regards Eckart
```

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Message: #2513
Date: 02/7/2017 9:15:36
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

I am excited:
The Cayley-Bacharach ninth line - in the sense of #2511 -
... for eight tangents to a cardioid
... is the double tangent.
I think, there will be more interesting results.

Best regards Eckart
```

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Message: #2514
Date: 02/7/2017 9:27:45
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Just in order to check if I understand you well,
let's take as 8 lines the 4 lines of the QL, the sides of DT and the Newton Line
then the duals will be the vertices of the dual QA, the vertices of DT and the centroïd of DT
the ninth point is QA-P16 or QL-P13 and the ninth line is the infinity line ...
Best regards
Bernard
PS What if we take the 4 lines of the QL and the sides of QL-Tr2 ? The duals are the vertices of QA and the vertices of QL-Tr2, which is self-dual like DT ...

Message: #2515
Date: 03/7/2017 10:27:48
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Wunderbar !
The degree of the dual of a curve is given by the number of tangents drawn from a point to the curve.
Duals of conics are therefore always conics.
If I'm not wrong, duals of cubics are generally sextics, but duals of cardioïds or deltoïds are also cubics !
In your cardioïd, what is the dual of the double tangent ? (You said, it's independant of the 4 lines choosen as QL among the 8 tangents ?)
Best regards
Bernard
PS I understand finally why the 4 QA vertices and the 3 DT vertices give the involution QA-Tf2 ; for any point P, there are 2 pivotal isocubics through the 7 vertices, P and P*=QA-Tf2(P), one has P as pivot (and P* as isopivot) and the other P* as pivot and P as isopivot.
The same goes with the 4 lines of a QL and the 3 sides of DT, wich give the transformation QL-Tf2.

Message: #2516
Date: 03/7/2017 2:41:21
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard,

sorry, please correct my #2513:
The Cayley-Bacharach ninth line - in the sense of #2511 -
... for eight tangents to a cardioid
... is also tangent to the cardioid.

Once more thanks for your comment: Your remark ...
"The degree of the dual of a curve is given by the number of
tangents drawn from a point to the curve."
... was new for me.

The corrected property above for a cardioid doesn´t hold for
quartics as limacon and lemniscate!
Now understandable, for the degree of their duals is not three.
So I think, wrt C-B-lines we have to consider curves, whose
duals have degree three.

Best regards Eckart

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Message: #2517
Date: 03/7/2017 4:05:13
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I'm very glad about your correction, as I couldn't understand why the ninth tangent to a cardioïd had to be precisely the dobble tangent !
To be more precise, any curve has a degree (degree of the equation) and a class (number of tangents).
The degree of the dual curve is the class of the curve and vice-versa.
So I understand better your construction of the ninth tangent with curves of class 3, as they are equivalent by duality to the quest of the ninth point for cubics.
For example, another quartic of the QL is of class 3, it's the deltoïd QL-Qu1, tangent to the 4 lines of the QL and to the asymptote of QL-Cu1. I suppose then it will hold that the ninth tangent for 8 tangents to a deltoïd is another tangent to the deltoïd ...
And for the Steiner deltoïd of a triangle, you have already 6 tangents : the 3 sides and the 3 altitudes ...
(Of course, any deltoïd is the Steiner deltoïd of an infinity of triangles).
Best regards
Bernard

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Message: #2518
Date: 03/7/2017 7:46:27
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Of course, the QL deltoïd is QL-Qu2 and not Qu1 ; it's dual cubic should be through the QA vertices, I don't know what the dual of the asymptote of QL-Cu1 is ...
The dual of QL-Qu1 is probably also a cubic, but I don't know the dual of the line through QL-P1 and QL-P4 or the dual of the dobble tangent ...
A nd the QL-27Qu1 are 27 cardioïds tangent to the 4 lines ; their duals shoud all pass through the 4 QA vertices I hope you will soon confirm all these intuitions !
Best regards
Bernard

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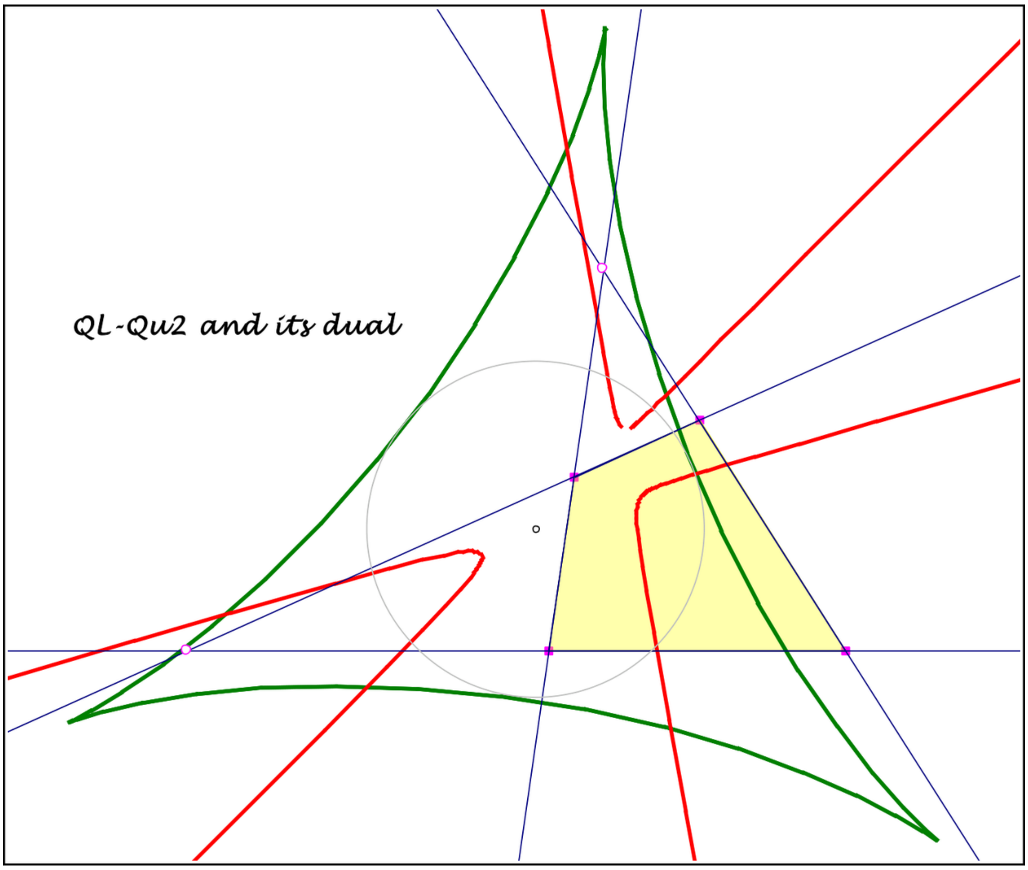
Message: #2519
Date: 04/7/2017 2:48:17
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

perhaps the following conjecture holds:
For curves of class 3 eight tangents determine a ninth
Cayley-Bacharach tangent in the sense of #2511.
For example the duals of QA-Cu1,2,3,4,5 have this property
(described in #1511, #1758 and #1759).
Wrt your #2518:
"Of course, the QL deltoïd is QL-Qu2 and not Qu1 ; it's dual
cubic should be through the QA vertices, I don't know what the
dual of the asymptote of QL-Cu1 is ..."
The dual cubic of QL-Qu2 (see attached file) can´t bear the
QA-vertices, for QL-Qu2 is not tangent to the QL-lines.
The dual of the asymptote of QL-Cu1 is a point on QL-P8.QL-P13
...

Best regards Eckart
```



2017-07-04.pdf

Message: #2520
Date: 04/7/2017 3:19:05
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
I agree with your conjecture.
But QL-Qu2 is in fact tangent to the 4 QL-lines ! (see EQF)
Best regards
Bernard
```

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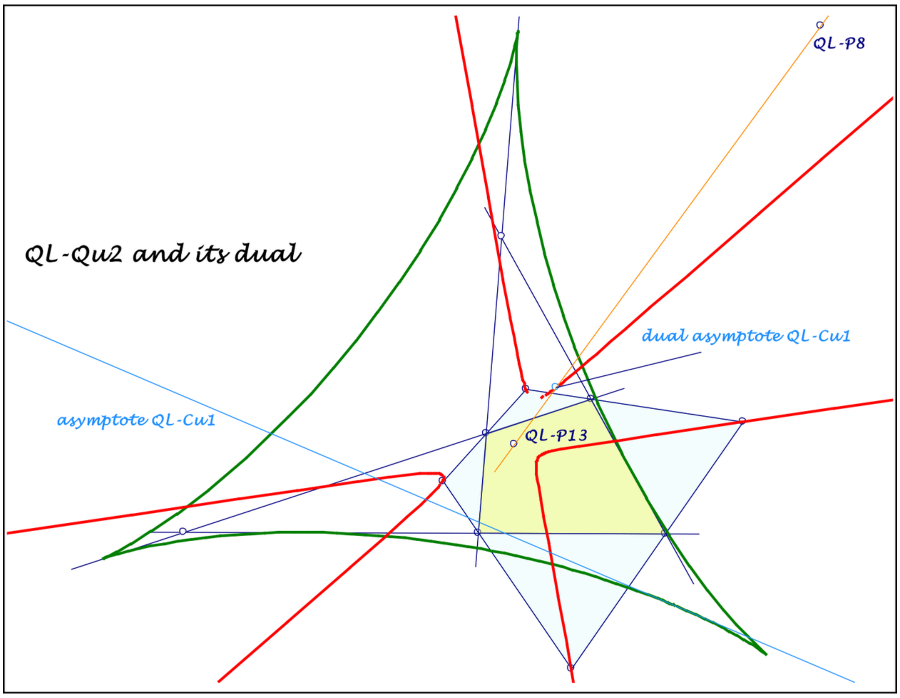
Message: #2521
Date: 04/7/2017 4:50:03
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,
excuse very much, I used not the right macro!
Attached - I hope - a right figure.
Best regards Eckart
```



2017-07-04a.pdf

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Message: #2522
Date: 05/7/2017 3:04:07
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

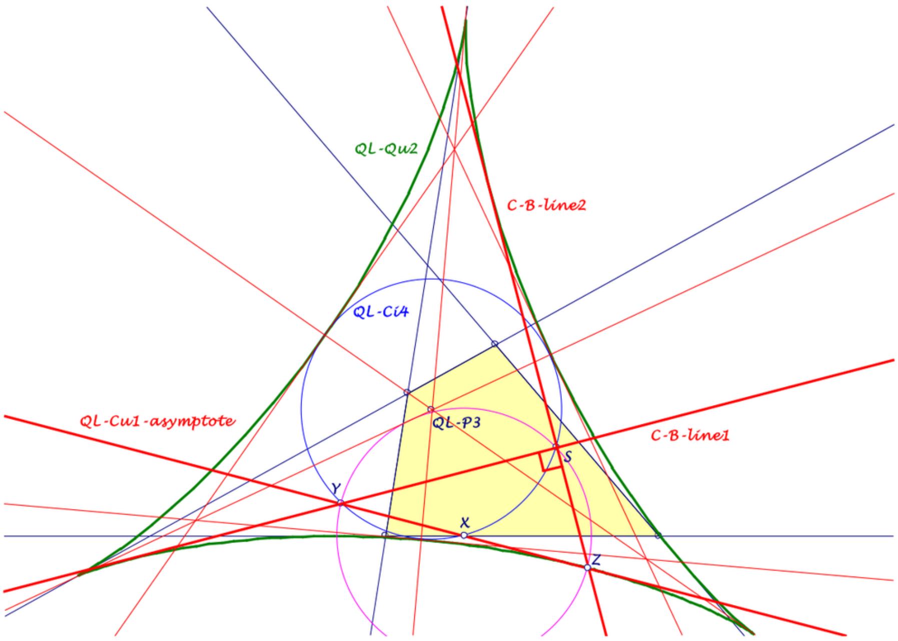
```
Dear Bernard,
attached a note wrt two C-B-lines for QL-Qu2, perhaps of
interest.
Best regards Eckart
```

EQF-Note 2017-07-05

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cayley-Bacharach Lines for QL-Qu2

Two orthogonal ninth Cayley-Bacharach tangents for the deltoïd QL-Qu2 of a quadrilateral are researched, considering the four lines of the quadrilateral, the asymptote of QL-Cu1 and the three common tangents of QL-Qu2 and QL-Ci4 on the one hand or the three axes of symmetry for QL-Qu2 on the other hand.



In *QFG*-message *2511* a Cayley-Bacharach ninth line is defined:
Consider eight lines,
… four lines for a reference *QL* and its dual *QA*,
... duality gives eight points for the eight lines,
... take their Cayley-Bacharach ninth point,
... its dual will give the Cayley-Bacharach ninth line.
For duality see *EQF*-message *1516*, a construction for the Cayley-Bacharach ninth point is described in *EQF*-message *2471*.

The deltoid *QL-Qu2* is a quartic of class three, so eight tangents will determine a ninth Cayley-Bacharach tangent.

2017-07-05.pdf

There are the following well known tangents of *QL-Qu2*:

1. four *QL*-lines,
2. the *QL-Cu1*-asymptote,
3. three common tangents of *QL-Qu2* and *QL-Ci4*,
4. three axes of symmetry for *QL-Qu2*.

The lines 1, 2, 3 will give the *C-B*-line 1, the lines 1, 2, 4 will give the *C-B*-line 2.

- **The *C-B*-lines 1, 2 are orthogonal tangents at *QL-Qu2* …with intersection *S* on *QL-Ci4*.**

Alternatively the two lines can be constructed as follows:
… The *QL-Cu1*-asymptote cuts *QL-Ci4* in the points *X* and *Y*,
… *X* shall be the middle of the asymptote as chord of *QL-Qu2*.
… A circle *Ci* round *X* through *Y*
… cuts the asymptote once more in *Z*,
… cuts the circle *QL-Ci4* once more in *S*.
… *S* is the intersection of the *C-B*-lines,
… *Y* and *Z* their intersections with the asymptote.

Further properties of the configuration:

Let *Tr* be the equilateral triangle of the contact points of *QL-Qu2* and *QL-Ci4*:
The Simson line of *X* wrt *Tr*
… is orthogonal to the asymptote
… … through the middle of *X.QL-P3*.
The Simson line of *Y* wrt *Tr*
… is parallel *X.QL-P3*,
… … half of the distance
… is parallel *C-B*-line 2,
… … fourth part of the distance.
The Simson line of *S* wrt *Tr*
… is parallel *Y.QL-P3*
… … half of the distance.

Eckart Schmidt
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eckart_schmidt@t-online.de

Message: #2523
Date: 05/7/2017 4:41:17
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
I find your development about QL-Qu2 very interesting !
Now, your construction is general for any point of QL-Ci4.
If you consider the point X' reflexion of X in QL-P3, the line orthogonal to the asymptote through Y is also tangent to the deltoïd, the contact point Y' being the reflexion of Y in X'.
The circle centered in X' through Y and Y' gives the same point S.
The 2 C-B-lines for the perpendicular tangent to the asymptote and 1 and 3 or 1 and 4 are the same, just swapped.
This goes for any tangent to the deltoïd and it's perpendicular tangent (both intersect on QL-Ci6, which is the orthoptic curve of the deltoïd) taken as 8th line with 1 and 3 or with 1 and 4; the ninth are 2 perpendicular tangents to the deltoïd ...
Best regards
Bernard

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Message: #2524
Date: 06/7/2017 12:39:28
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard,
thanks for additional remarks wrt QL-Qu2.
Perhaps a typo in your last passage: not QL-Ci6, but QL-Ci4.
A further observation wrt the C-B-lines of QL-Qu2:
C-B-line 1 and C-B-line 2 are orthogonal and parallel to the tangent QL-P1.QL-P4 of QL-Cu1 in QL-P1.
Best regards Eckart

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Message: #2525
Date: 08/7/2017 4:29:56
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

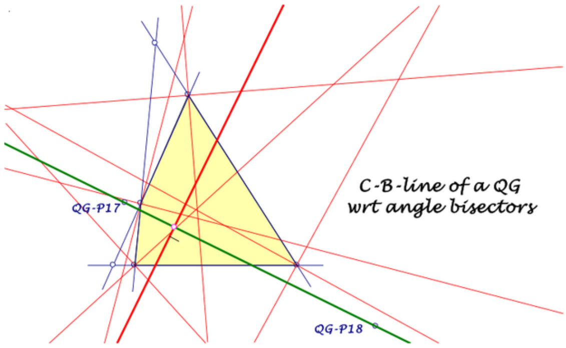
```
Dear Bernard, dear Chris,
the attached geometric excursion shows,
that the Cayley-Bacharach ninth line is a relevant tool in
EQF-geometry.
Perhaps Chris can find some properties of the point P,
the DT-coordinates are given.
Best regards Eckart
```

EQF-Note 2017-07-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cayley-Bacharach Lines for a Quadrilateral

This geometric excursion shows, that the Cayley-Bacharach ninth line (see QFG-message 2511) is a relevant tool in EQF-geometry.



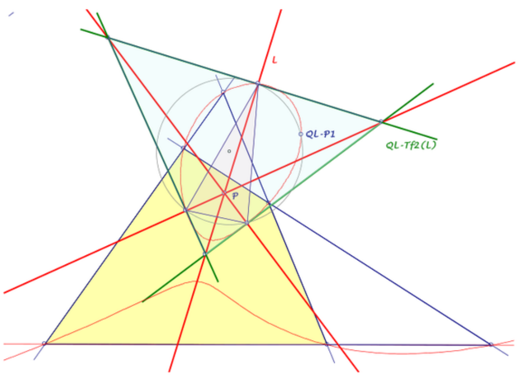
Let us start with a quadrigon and consider the eight angle bisectors.

- ***C-B*-line *L* of the eight angle bisectors of a quadrigon is *QL-Tf2(QG-P17.QG-P18).***

- **The *C-B*-line *L* of a *QG* and its *QL-Tf2*-image *QG-P17.QG-P18* are orthogonal.**

Now consider a quadrilateral and its three *QG*-versions:

- **The three *C-B*-lines for the *QG*-versions of a *QL* have a common point *P*.**



2017-07-08.pdf

- **The point *P* is not in *EQF*: *1st DT*-coordinate**
  $m^2 n^2 Sa^2 (l^2 Sb Sc a^2 + m^2 Sc S^2 + n^2 Sb S^2)$.

- **The trilateral *Tr* of the three lines *QG-P17.QG-P18* for a *QL* has the three *C-B*-lines *L* as altitudes.**

- **The nine-point circle of *Tr* bears *QL-P1*,**
  … its Simson line wrt the orthic triangle of *Tr* is parallel *QL-L2*.

- **The vertices of the orthic triangle of *Tr* are points on the cubic *QL-Cu1*,**
  … their *CSC*-partners are collinear on *QL-Cu1*.

- ***QL-Cu1* is isogonal invariant wrt the orthic triangle of *Tr*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-07-08.pdf

Message: #2526
Date: 09/7/2017 2:48:07
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear all,
In APG message #2440, I gave some of my ideas.
Please refer to it for more information!
I think there are main categories in plane geometry.
1. point vs line
2. metric vs projective
3. triangle(trilateral) vs quadrangle(quadrilateral)
We concentrate on the former three more than the latter.
Point conics are equivalent to line conics.
But point cubics are not equivalent to line cubics, although
their complexities are the same.
We have known pivotal point isocubics.
We should study pivotal line isocubics.
Pivotal point isocubics(i.e. isoconjugation QA-Tf2) are studied
much.
But pivotal line isocubics(i.e. isoconjugation QL-Tf2) are
little known.
QA-Tf2 is derived from Cayley-Bacharach (point) theorem as
QL-Tf2 is derived from Cayley-Bacharach (line) theorem.
CTC contains a lot of pivotal line isocubics but no pivotal line
isocubics.  Line cubics are usual point sextics.
But some point sextics had better be studied as line cubics for
reduction of complexity.
Best regards,
Tsihong Lau
```

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Message: #2527
Date: 09/7/2017 3:49:14
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,
you will find informations for "pivotal line isocubics" in #1511
and #1759,
in the last message wrongly nominated (point) quartics instead
of sextics.
Best regards Eckart
```

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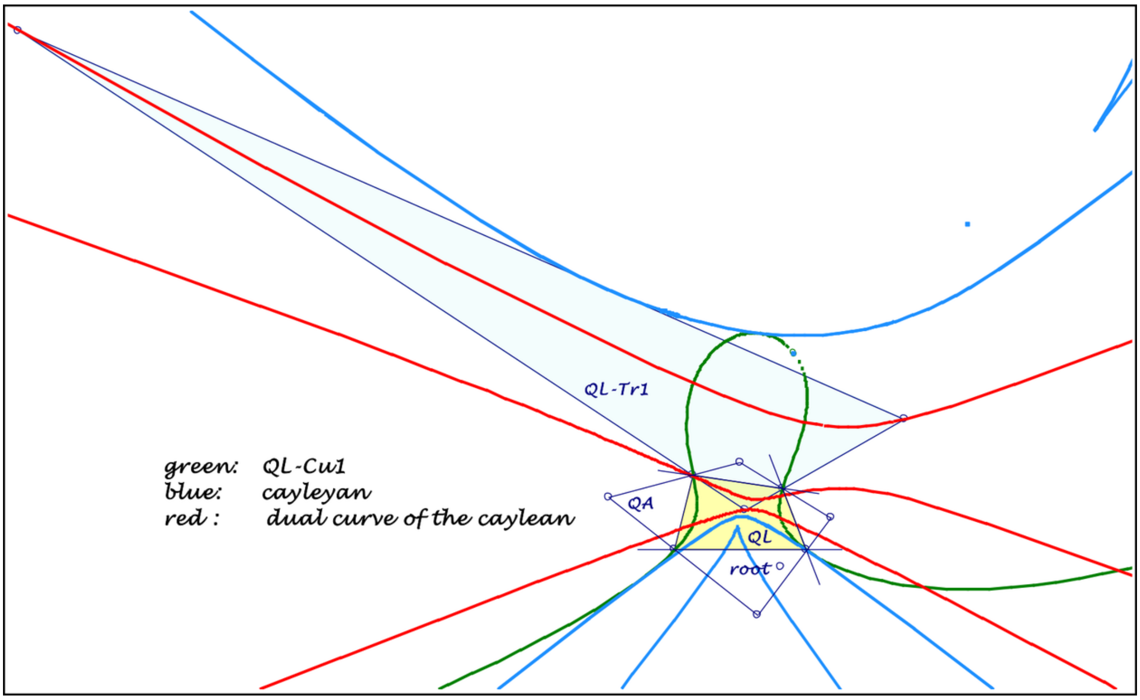
Message: #2528
Date: 11/7/2017 2:50:27
From: eckart_schmidt@t-online.de
Subject: Caylean of QL-Cu1

```
Dear Bernard,

I don´t know, whether this is already wellknown:
The dual curve of the cayleyan
... is a nonpivotal isocubic wrt
... reference triangle: QL-Tr1,
... isoconjugation: QA-Tf2
... root: tripole of QL-Tf2 of the tripolar of QL-P10,
... points: QL-P8, QL-P13, ...

Best regards Eckart
```



2017-07-11.pdf

Message: #2529
Date: 12/7/2017 8:32:30
From: eckart_schmidt@t-online.de
Subject: Cayleyan of QL-Cu1

Dear Bernard,
the dual cubic of the cayleyan is already described in #1729.
Best regards Eckart

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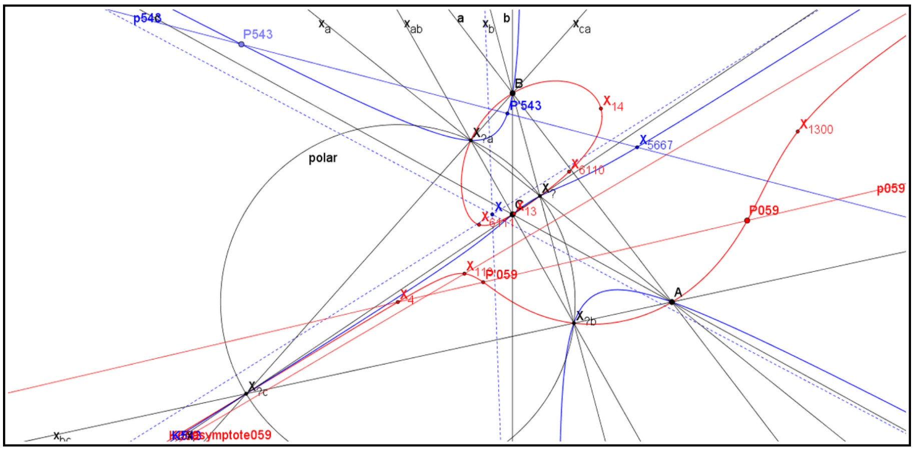
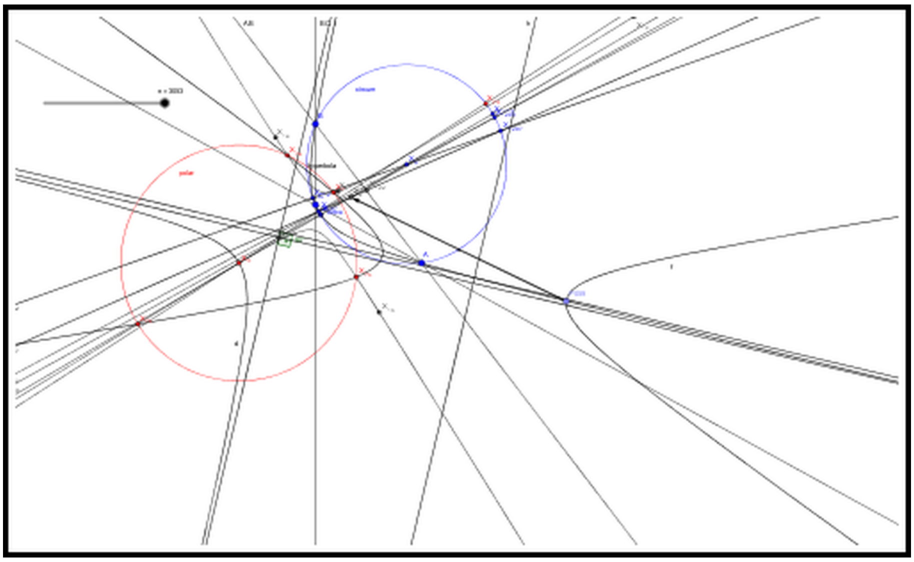
Message: #2530
Date: 14/7/2017 5:32:06
From: tsihonglau
Subject: Isoxxx Conjugate

```
Dear all,
I post this to APG about a month ago but got no reply.
I hope someone will study it using quadrangle/quadrilateral
perspective.
I defined the term system centers in message #2314
and found the third one X(1138) besides X(4) and X(74).
There are several topics about system centers.
Which center is the Feuerbach point?
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/2179  †)
A property of X1138
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/2458  †)
Concurrent lines with Neuberg cubic
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/2731  †)
Circular(Isotropic) Coordinate System
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/2873  †)
Selfcentric Centers
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/3597  †)
X(74)
https://groups.yahoo.com/neo/groups/AdvancedPlaneGeometry/conver⌋
sations/topics/3627  †)
Incenter/excenters X1,X1a,X1b,X1c form an X(4) system.
We can deduce isogonal conjugation from them.
In message #3816, I got the X(74) system X?,X?a,X?b,X?c.
Their barycentrics:(using Conway's notation)
X? - sqrt(a^2-2S_B S_C/S_A):sqrt(b^2-2S_C S_A/S_B):sqrt(c^2-2S_A
S_B/S_C)
X?a - -sqrt(a^2-2S_B S_C/S_A):sqrt(b^2-2S_C
S_A/S_B):sqrt(c^2-2S_A S_B/S_C)
X?b - sqrt(a^2-2S_B S_C/S_A):-sqrt(b^2-2S_C
S_A/S_B):sqrt(c^2-2S_A S_B/S_C)
X?c - sqrt(a^2-2S_B S_C/S_A):sqrt(b^2-2S_C
S_A/S_B):-sqrt(c^2-2S_A S_B/S_C)
They all lie on the polar circle.
http://mathworld.wolfram.com/PolarCircle.html
As the polar circle, they may be imaginary.
They are missing in the ETC.
We can deduce isoxxx conjugation from them.
x:y:z -> (a^2-2S_B S_C/S_A)/x:(b^2-2S_C S_A/S_B)/y:(c^2-2S_A
S_B/S_C)/z
```

Surpringly! It has appeared in CTC already.
There are two pivotal isocubics K059=pK(X1990,X4)
and K543=pK(X1990,X5667) and two non-pivotal isocubics
K393=nK0+(X1990,X4) and X628=nK+(X1990,X27,X1).
The poles in CTC term are all X1990. Please refer to
the attchment file for K059 and K543.
I hope this conjugation to be applied to more cubics.
Because they lie on the polar circle, I recommend the term
isopolar conjugate.
To be continued...
Best regards,
Tsihong Lau



K059K543.png



K059K543.ggb

†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #2531
Date: 21/7/2017 12:51:18
From: jeanlouisayme
Subject: Two questions

Dear Geometers,
1. Where can I find a photo of Charles Julien Brianchon which seems to be inaccessible on the Web?
2. I am actually working on the Brianchon-Poncelet circle and I come on the Bennett point...
Who is Bennett? G.T. Bennett ? any reference for this result : this point is the symmetric of the Poncelet's point wrt a the median point of a quadrilateral...
Very sincerely
Jean-Louis

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Message: #2532
Date: 23/7/2017 10:02:08
From: bernard.keizer
Subject: Cayleyan of QL-Cu1

Dear Eckart,
I wasn't home for a few weeks, so I didn't immediately reply your 2 messages.
Your dobble approach of the dual of the cayleyan is very interesting !
In fact, the cayleyan is a sextic of class 3 ; therefore the dual is a cubic.
>From a point X on QL-Cu1, the 3 tangents to the cayleyan are the lines named L and L' in message 1729 and the line XX', X' being the CSC of X.
L and L' form the degenerated polar conic of X' wrt QL-Cu2 and are QL-Tf2 partners ; their duals are T' and T, which are QA-Tf2 partners. The dual of XX' is the 3rd intersection of the line TT' with your cubic ...
Best regards
Bernard

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Message: #2533
Date: 23/7/2017 10:37:22
From: bernard.keizer
Subject: A problem: quadrigon and triangle

Dear Benedetto, dear Eckart, dear Seiichi,
The problem of the generalisation of the Marden theorem
continues to fascinate me !
The calculation by derivating 2 times shows that the QA, the
triangle and the bipoint share the same centroïd (middle of the
2 points) and the same LSD line (through the 2 points).
I'm convinced that the QA's sharing the same triangle are the
pivots on a quartic stelloïd the same way as the triangles
sharing the same bipoint (foci of the Steiner inellipse) are the
pivots on a cubic stelloïd (see Bernard Gibert).
A stelloïd is a generalisation of the rectangular hyperbola and
can be defined as the focus of a variable point M wrt fixed
points Ai (called pivots) such as the sum of the directions of
MAi wrt a fixed line D is constant.
Could some one help me in providing an exact figure with a QA, a
triangle (in calculating the 3 roots of an equation of the 3rd
degree) and the bipoint.
I suppose the searched triangle is then one of the pivot
triangles of a cubic stelloïd, probably the main pivot triangle
obtained in dawing the tangent to the cubic stelloïd from the
centroïd ...
Is it possible to draw the cubic stelloïd having only the
bipoint ?
I hope this problem will be solved some day !
Best regards
Bernard

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Message: #2534
Date: 23/7/2017 7:34:58
From: bernard.keizer
Subject: A problem: quadrigon and triangle

Dear Benedetto, dear Eckart, dear Seiichi,
Considering a pencil of rectangular hyperbolas through a bipoint, the polar lines of the points of the infinity line wrt the RH are a pencil of lines through the midpoint.
Considering a pencil of cubic stelloïds through 3 points, the polar conics of the points of the infinity line wrt the CS are a pencil of RH through the bipoint (foci of the Steiner inellipse of the triangle).
Considering a pencil of quartic stelloïds through 4 points, the polar curves of the points of the infinity line wrt the QS are a pencil of CS through the 3 searched points of the QA.
Does that make sense ?
(Of course and unfortunately, that doesn't give a construction of the 3 points, but this could give ideas ...)
Best regards
Bernard

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Message: #2535
Date: 25/7/2017 5:51:10
From: tsihonglau
Subject: Isoxxx Conjugate

Dear all,

I give the reflex of polar circle here - polar conic.
Please refer to topic #2216 for more information about reflex objects.
The following coordinates are all DT-barycentrics.

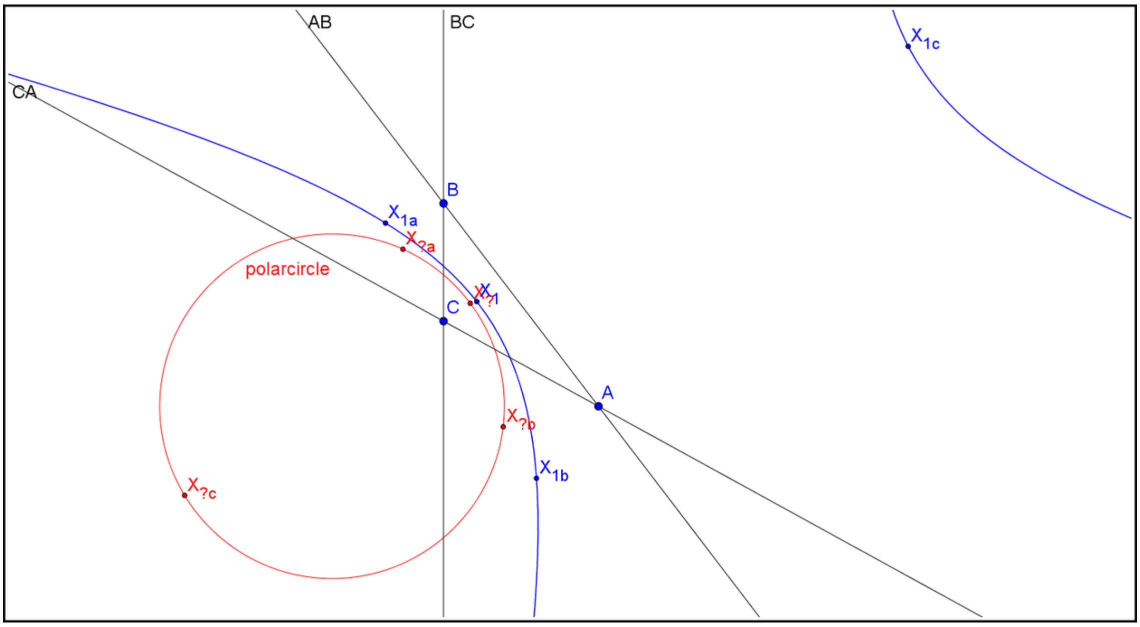
1. QA-Ci1 circumcircle of ABC=QA-Co1 of incenter/excenters quadrangle
a^2yz+b^2zx+c^2xy=0
QA-Co1 nine-point conic
d^2yz+e^2zx+f^2xy=0 (d,e,f,here instead of p,q,r in EQF)

2. circular points at infinity=infinity points of QA-Ci1=
parabolic points of incenter/excenters quadrangle
-2a^2:(a^2+b^2-c^2)+sqrt(-(a+b+c)(-a+b+c)(a-b+c)(a+b-c)):
(a^2-b^2+c^2)-sqrt(-(a+b+c)(-a+b+c)(a-b+c)(a+b-c))
-2a^2:(a^2+b^2-c^2)-sqrt(-(a+b+c)(-a+b+c)(a-b+c)(a+b-c)):
(a^2-b^2+c^2)+sqrt(-(a+b+c)(-a+b+c)(a-b+c)(a+b-c))
parabolic points at infinity=infinity points of QA-Co1
-2d^2:(d^2+e^2-f^2)+sqrt(-(d+e+f)(-d+e+f)(d-e+f)(d+e-f)):
(d^2-e^2+f^2)-sqrt(-(d+e+f)(-d+e+f)(d-e+f)(d+e-f))
-2d^2:(d^2+e^2-f^2)-sqrt(-(d+e+f)(-d+e+f)(d-e+f)(d+e-f)):
(d^2-e^2+f^2)+sqrt(-(d+e+f)(-d+e+f)(d-e+f)(d+e-f))

3. polar circle=polar conic of incenter/excenters quadrangle
(-a^2+b^2+c^2)x^2+ (a^2-b^2+c^2)y^2+ (a^2+b^2-c^2)z^2=0
polar conic
(-d^2+e^2+f^2)x^2+ (d^2-e^2+f^2)y^2+ (d^2+e^2-f^2)z^2=0
Given two quadrangles with the same diapleural triangle, it is obvious that one quadrangle lies on the polar conic of the other if and only if the latter lie on the polar conic of the former.

For example, X?,X?a,X?b,X?c lie on the polar conic of X1,X1a,X1b,X1c, so the latter lie on the polar conic of the former.
Please refer to the attachments for more information.
One question raised: Given two quadrangles with the same diapleural triangle, one quadrangle lies on the polar conic of the other if and only if the two pairs of parabolic points are the harmonic conjugates on the line at infinity. Can anyone prove or disprove it?

```
Best regards,
Tsihong Lau
```



X(4)X(74).png

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Message: #2536
Date: 25/7/2017 9:11:59
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard, dear Chris,

I found an interesting paper:
Siebeck, Jörg (1864): Ueber eine neue analytische
Behandlungsweise der Brennpunkte.
Journal für die reine und angewandte Mathematik, 64: 175–182

In I.4) and 5) we find a complete description of the
CSC-transformation (more than 50 years before Clawson).

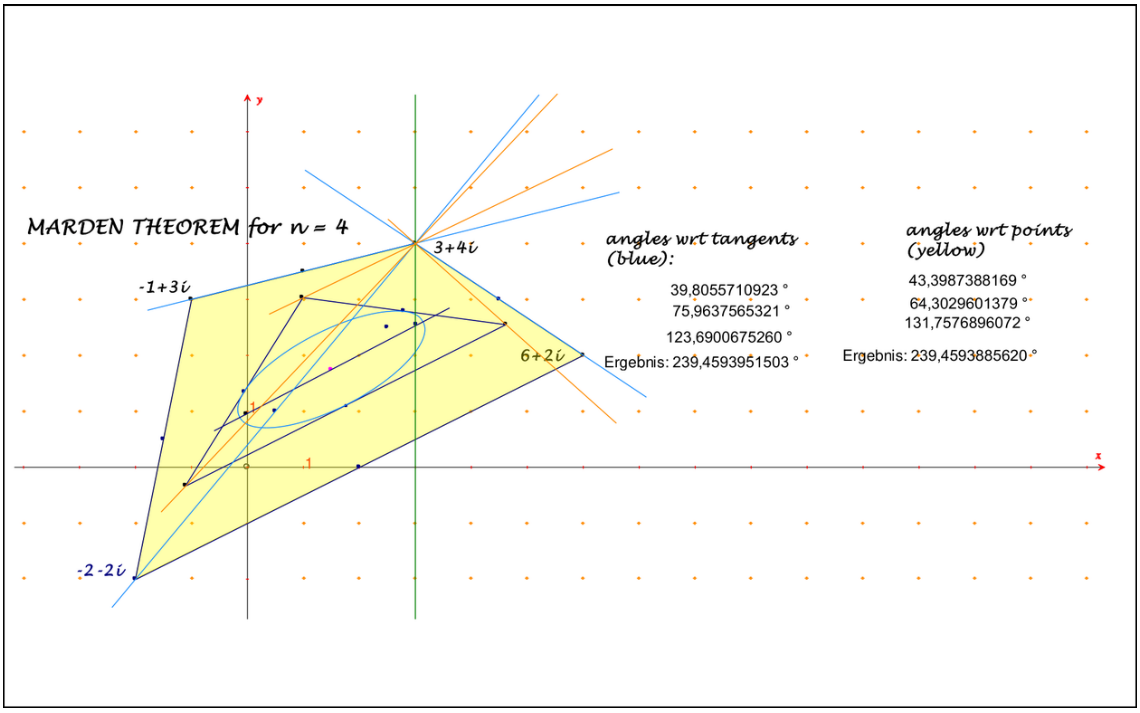
In I.6) the cayleyan is described.

In II. we found "Brennpunkte algebraischer Curven", that was new
for me.

In II.3) there is an interesting aspect wrt the Marden theorem:
"Sind in einer Ebene n beliebige Punkte P1, P2, ... Pn gegeben,
so giebt es immer eine Curve (n-1)ter Classe, welche die
n(n-1)/2 Verbindungslinien jener Punkte berührt und zwar so,
dass die Berührpunkte sämmtlich in die Mittelpunkte der Linien
fallen. Die n-1 rellen Brennpunkte ... "
... are the vertices of the triangle in the Marden theorem for
n=4.

This would be in the sence of a generalization.
I have confirmed the angle condition in II.1) for the
"Brennpunkte"
by CABRI with an accuracy of 6 digits (see attached file).
I think, this will be right also for higher n.
Perhaps someone can construct these curve of class 3, tangent to
the six lines of a quadrangle in their midpoints.

Best regards Eckart



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Message: #2537
Date: 26/7/2017 8:43:37
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

```
Dear Bernard, dear Chris,

excuse, my observations in the last message wrt Marden´s theorem
are already mentioned in Wikipedia:
James L. Parish: On the Derivative of a Vertex Polynomial
Forum Geometricorum, Volume 6 (2006) 285-288.

Best  regards Eckart
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Message: #2538
Date: 28/7/2017 11:11:32
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Wunderbar, lieber Eckart !
Thanks a million time for this new and precious reference !!!
It contains many old properties of the QL, brings new elements
and answers a lot of questions.
1) QL's old properties like QL-2P3 points harmonisch on a circle
wrt any copple of CSC partners on QL-Cu1 (page 177). I've put
this property (unfortunately not in EQF) in the forum in point
4) on my message 2393.
As you mention, there are also a construction of QL-Cu1 using
the CSC transformation, the special case of the strophoïd, the
cayleyan ...
2) New also for me were the foci for a curve of beliebiger class
...
The example mentionned page 180 in I6 for the 3 foci of the
cayleyan is very interesting !
3) The sum of angles mentionned in II1 is exactly the definition
of the stelloïds, which seems to confirm my conjectures in my
last message 2534.
The article mentions also the 1rst polars of the points of the
infinity line ( bottom of page 175 and point II4)
The product of distances mentionned in II2 is exactly the
definition of the cassinians.
It's not surprising, as stelloïds and cassinians are linked by
the same complex polynoms ...
There is a lot to study and to work.
I'm now to busy with my grand-children and have little time (my
holidays begin when the grand-children go to school !), but I'm
very excited about all this new material !
Thanks also for your figure, I'll try to work on the same in
order to converge.
Last remark : the seached curve of class 3 is the dual of a
cubic through the 6 vertices of the dual QL tangent to the dual
lines of the 6 midpoints ...
Best regards
Bernard
```

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Message: #2539
Date: 30/7/2017 4:23:54
From: tsihonglau
Subject: Isoxxx Conjugate

```
Dear all,

Topic #2216 contains more information about reflex and symmetric
objects.
Message #2412 mentioned "The common harmonics of the circular
and
parabolic points at infinity are the points at infinity of
QA-Co4."
Centers and axes, etc derived from the pair of points are all
symmetric.
I give the analogy of some objects.(all in DT-barycentrics
d,e,f,instead of p,q,r)

1. circular points - parabolic points - common harmonics

2. QA-Ci1 circumcircle a^2yz+b^2zx+c^2xy=0
- QA-Co1 nine-point conic d^2yz+e^2zx+f^2xy=0
- QA-Co4 QA-DT-P3-P12 orthogonal hyperbola
(a^2(e^2-f^2)-d^2(b^2-c^2))yz+(b^2(f^2-d^2)-e^2(c^2-a^2))zx+(c^2
(d^2-e^2)-f^2(a^2-b^2))xy=0

3. isogonal conjugate x:y:z -> a^2yz:b^2zx:c^2xy
- QA-Tf2 isoconjugate x:y:z -> d^2yz:e^2zx:f^2xy
- common harmonic isoconjugate x:y:z ->
(a^2(e^2-f^2)-d^2(b^2-c^2))yz:(b^2(f^2-d^2)-e^2(c^2-a^2))zx:(c^2
(d^2-e^2)-f^2(a^2-b^2))xy
QA-Ci1, QA-Co1, QA-Co4 are the isogonal conjugate, isoconjugate,
common harmonic isoconjugate of the line at infinity
respectively.

4. polar circle
(-a^2+b^2+c^2)x^2+(a^2-b^2+c^2)y^2+(a^2+b^2-c^2)z^2=0
- polar conic
(-d^2+e^2+f^2)x^2+(d^2-e^2+f^2)y^2+(d^2+e^2-f^2)z^2=0
- QA-Co2 QA-orthogonal hyperbola
(b^2f^2-c^2e^2)x^2+(c^2d^2-a^2f^2)y^2+(a^2e^2-b^2d^2)z^2=0

5. QA-P11 circumcenter=center of QA-Ci1
a^2(-a^2+b^2+c^2):b^2(a^2-b^2+c^2):c^2(a^2+b^2-c^2)
- QA-P1 QA-centroid = center of QA-Co1
d^2(-d^2+e^2+f^2):e^2(d^2-e^2+f^2):f^2(d^2+e^2-f^2)
```

- QA-P29 complement of QA-P2 = center of QA-Co4
(b^2f^2-c^2e^2)(a^2(e^2-f^2)-d^2(b^2-c^2)):(c^2d^2-a^2f^2)(b^2(f^2-d^2)-e^2(c^2-a^2)):(a^2e^2-b^2d^2)(c^2(d^2-e^2)-f^2(a^2-b^2))

6. QA-P12 orthocenter=center of polar circle
1/(-a^2+b^2+c^2):1/(a^2-b^2+c^2):1/(a^2+b^2-c^2)
- QA-P20 reflection of QA-P5 in QA-P1=center of polar conic
1/(-d^2+e^2+f^2):1/(d^2-e^2+f^2):1/(d^2+e^2-f^2)
- QA-P2 Euler-Poncelet Point=center of QA-Co2
1/(b^2f^2-c^2e^2):1/(c^2d^2-a^2f^2):1/(a^2e^2-b^2d^2)
QA-P11, QA-P1, QA-P29 are complements of QA-P12, QA-P20, QA-P2 respectively.

7. Orthic axis (-a^2+b^2+c^2)x+(a^2-b^2+c^2)y+(a^2+b^2-c^2)z=0
- Pseudo-orthic axis
(-d^2+e^2+f^2)x+(d^2-e^2+f^2)y+(d^2+e^2-f^2)z=0
- Pseudo-orthic axis
(b^2f^2-c^2e^2)x+(c^2d^2-a^2f^2)y+(a^2e^2-b^2d^2)z=0
These three versions of orthic axes are the radical lines of QA-Ci1 and polar circle, QA-Co1 and polar conic, QA-Co4 and QA-Co2 respectively and also the trilinear polars of QA-P12, QA-P20, QA-P2 respectively.

8. QA-L5 Euler line (b^2-c^2)(-a^2+b^2+c^2)x+(c^2-a^2)(a^2-b^2+c^2)y+(a^2-b^2)(a^2+b^2-c^2)z=0
- QA-L3 QA-centroids-line (e^2-f^2)(-d^2+e^2+f^2)x+(f^2-d^2)(d^2-e^2+f^2)y+(d^2-e^2)(d^2+e^2-f^2)z=0
- pseudo-Euler line (b^2f^2-c^2e^2)(a^2(e^2+f^2)-d^2(b^2+c^2))x+(c^2d^2-a^2f^2)(b^2(f^2+d^2)-e^2(c^2+a^2))y+(a^2e^2-b^2d^2)(c^2(d^2+e^2)-f^2(a^2+b^2))z=0
These three versions of Euler lines are the lines through QA-P11 and QA-P12, QA-P1 and QA-P20, and QA-P29 and QA-P2 respectively.
There are three versions of isoxxx conjugates
x:y:z -> (a^2-(a^2-b^2+c^2)(a^2+b^2-c^2)/(-a^2+b^2+c^2))yz:...
(cyclically defined)
x:y:z -> (d^2-(d^2-e^2+f^2)(d^2+e^2-f^2)/(-d^2+e^2+f^2))yz:...
(cyclically defined)
x:y:z -> ((a^2*b^2*f^4-a^2*c^2*e^2*f^2-a^2*b^2*e^2*f^2-2*a^4*e^2*f^2-b^2*c^2*d^2*f^2+b^4*d^2*f^2+2*a^2*b^2*d^2*f^2+a^2*c^2*e^4+c^4*d^2*e^2-b^2*c^2*d^2*e^2+2*a^2*c^2*d^2*e^2-2*b^2*c^2*d^4)/((b*f-c*e)*(b*f+c*e)))yz:...(cyclically defined)

I wonder the properties of latter two.

Best regards,
Tsihong Lau

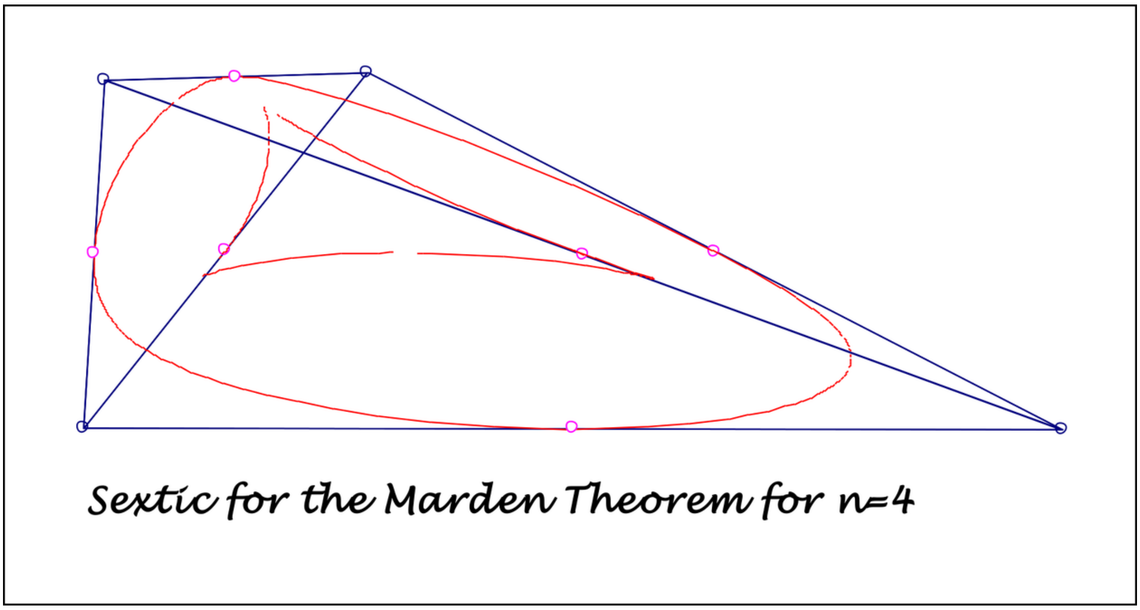
Message: #2540
Date: 30/7/2017 9:51:56
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard,

I have researched your last remark in #2538:
Let QA be the quadrangle of the zeros of a complex equation of degree four,
... Tr the triangle of the zeros of the derivation,
... whose vertices are the foci (in the sence of Siebeck) of a curve (sextic) of class 3,
... tangent to the six lines of the QA in their midpoints.
... The dual of this sextic is a cubic Cu,
... through the six points of the dual QL,
... tangent to the dual lines of the QA-midpoints.
... This cubic Cu is a nonpivotal isocubic (see attached file):
... reference triangle is the medial diagonal triangle,
... isoconjugation is the isotomic conjugate,
... root is the tripole of the line
... ...through the three collinear QL-versions of the midpoints X of QG-P1.QG-P2.
... The dual of the cubic gives the sextic (see attached file),
... whose foci are the zeros of the derivation of the initial equation.

Best regards Eckart

PS: How to construct the foci?



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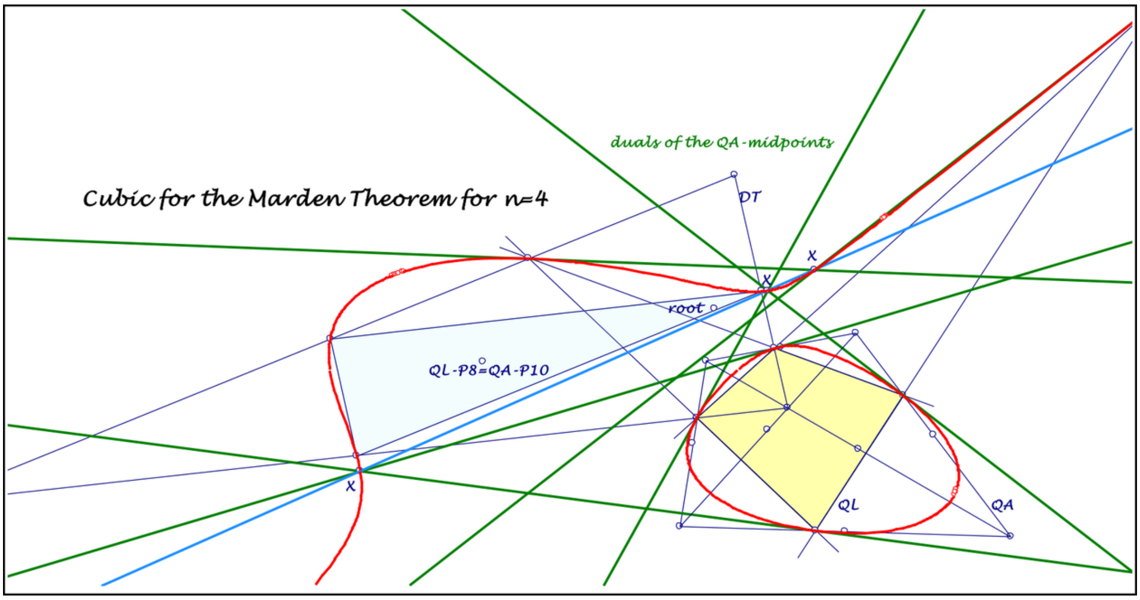

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Message: #2541
Date: 31/7/2017 9:57:50
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Dear Eckart,
Noch schöner !
Unfortunately, I don't know how to construct the 3 foci ...
I can't understand the construction given by Siebebeck in II4
(What is der Durchschnittspunkt ?)
2 remarks on your 2 marvellous curves, perhaps of interest for
you
1) the midpoints of DT (vertices of midDT) being on the cubic,
their dual lines are also tangent to the sextic.
I suppose the 3 points form a triple of the 6 vertices of the
QL, as well as their duals form a triple with the 6 sides of the
QA. Another C-B system ...
2) the points X are the intersections of the duals of 2 opposite
midpoints of the QA ; the dual of each X is the line through 2
opposite midpoints of the QA, which is therefore also tangent to
the sextic.
The line through the X is therefore the dual of QA-P1
Best regards
Bernard
```

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Message: #2542
Date: 31/7/2017 11:07:20
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Dear Eckart,
It follows of course directly from your definition with QG-P1
and QG-P2 that this line XXX, dual of QA-P1, is the Newton Line
of DQL ...
Best regards
Bernard
```

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Message: #2543
Date: 31/7/2017 9:43:16
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

```
Dear Bernard,

thanks for your messages, espacially for the observation, that
the XXX-line is the dual of QA-P1!
Attached the Marden Theorem for n = 4
... with sextic (constructed)
... and the foci (calculated and pointed out).
Perhaps helpful for consideration.

CABRI-observations give the careful assumption,
... that the tangents from the foci to the sextic
    have a common point.
What about this point?
As you I cannot reproduce the "construction" of the foci in the
sense of Siebeck.

Best regards Eckart
PS: "Durchschnittspunkt" will be intersection.
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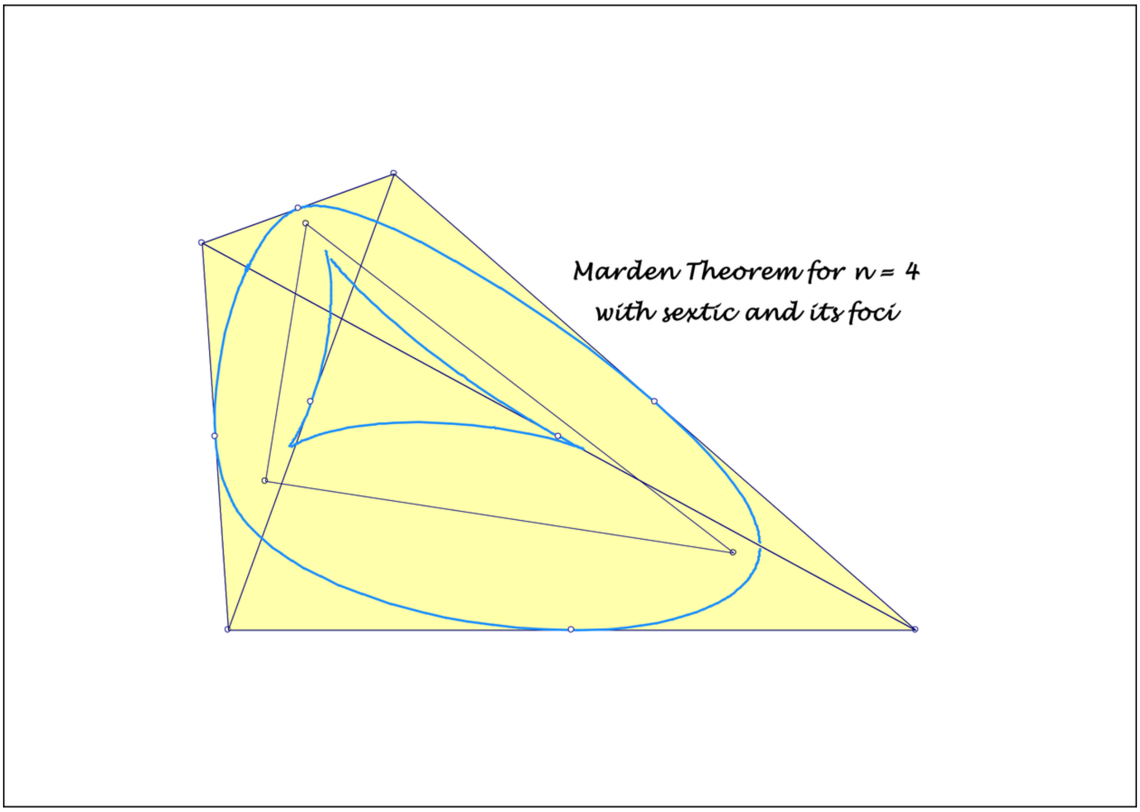

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Message: #2544
Date: 02/8/2017 10:35:32
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
First, congratulations for the result sextic of class 3 + 3 foci.
Perhaps, it would be useful to draw on the same figur the Steiner inellipse of the triangle with it's 2 foci, the LSD line for the QA, for the triangle and for the bipoint being the same.
I would like also the 3 segments joining the opposite midpoints intersecting in QA-P1and the tangents from the foci to sextic intersecting in a point I.
Then a few remarks :
1) if the root of your dual cubic is the trilinear pole (I suppose wrt DT ?) of the dual of QA-P1, it must be QA-P5
2) if my idea that the 3 mentionned segments are tangent to the sextic, it must be possible to verify the angle property with the point QA-P1.
3) this angle property with the point I proves your careful assumption (the intersection point I on 2 tangents from 2 foci to the sextic lies necessary on the tangent from the last focus to the sextic).
4) as I am curious, if the transformation of your cubic is the isotomic wrt midDT, did you try to build the QL from the 6 isotomic wrt midDT of the vertices of the QL ? MidDT is the same, but not DT and it leads to another QA having the same sextic with the same 3 foci, the same LSD line and the same QA-P1 and QA-P5 ...
Best regards
Bernard

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Message: #2545
Date: 02/8/2017 10:46:49
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
Until now, I don't see any contradiction with my assumption in message 2534.
Considering the pencil of quartic stelloïds through the 4 points, the 3 points may be the fixed points of a pencil of cubic stelloïds being the 1rst polar curves of the points of the infinity line wrt the QS of the pencil.
It is exactly the same as the 2 fixed points of the pencil of the rectangular hyperbolas being the polar conics of the points of the infinity line wrt the cubic stelloIds of a pencil through 3 fixed points.
The sectics are all different but have the same 3 foci exactly like the Steiner inellipses are all different but have the same 2 foci (we may speak of confocal sextics of class 3 as of confocal Steiner inellipses).
I know, it doen't bring any construction, but I would like to know what you think of this idea ?
Thanks in advance for any advice ...
Best regards
Bernard

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Message: #2546
Date: 02/8/2017 2:39:32
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
I have to correct the end of the last point :
If the 2nd QA/QL is not the same, the duality will change and the 2nd dual sextic is not the same !
I wonder if the 3 foci are the same ...
Best regards
Bernard

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Message: #2547
Date: 02/8/2017 2:46:02
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
I suppose the assumption contained in my message 2533 holds too
:
For a given quartic stelloïd, of the pencil, the different QA's
of pivots lead to confocal sextics of class 3 !
They have the same QA-P1, the same LSD line ...
It is the same as the different pivot triangles of a given cubic
stelloïd leading to confocal Steiner ellipses, with the same
centroïd, the same LSD line ...
Best regards
Bernard

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Message: #2548
Date: 03/8/2017 10:19:36
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard,

before answering your last messages, two further references for the foci of algebraic curves:

(1) Tadahiko Kubota (1930): Einige Bemerkungen zur Theorie der algebraischen Kurven.
The "Chasleysche Mittelpunkt" for the sextic in Marden´s Theorem for n = 4 will be QA-P1.

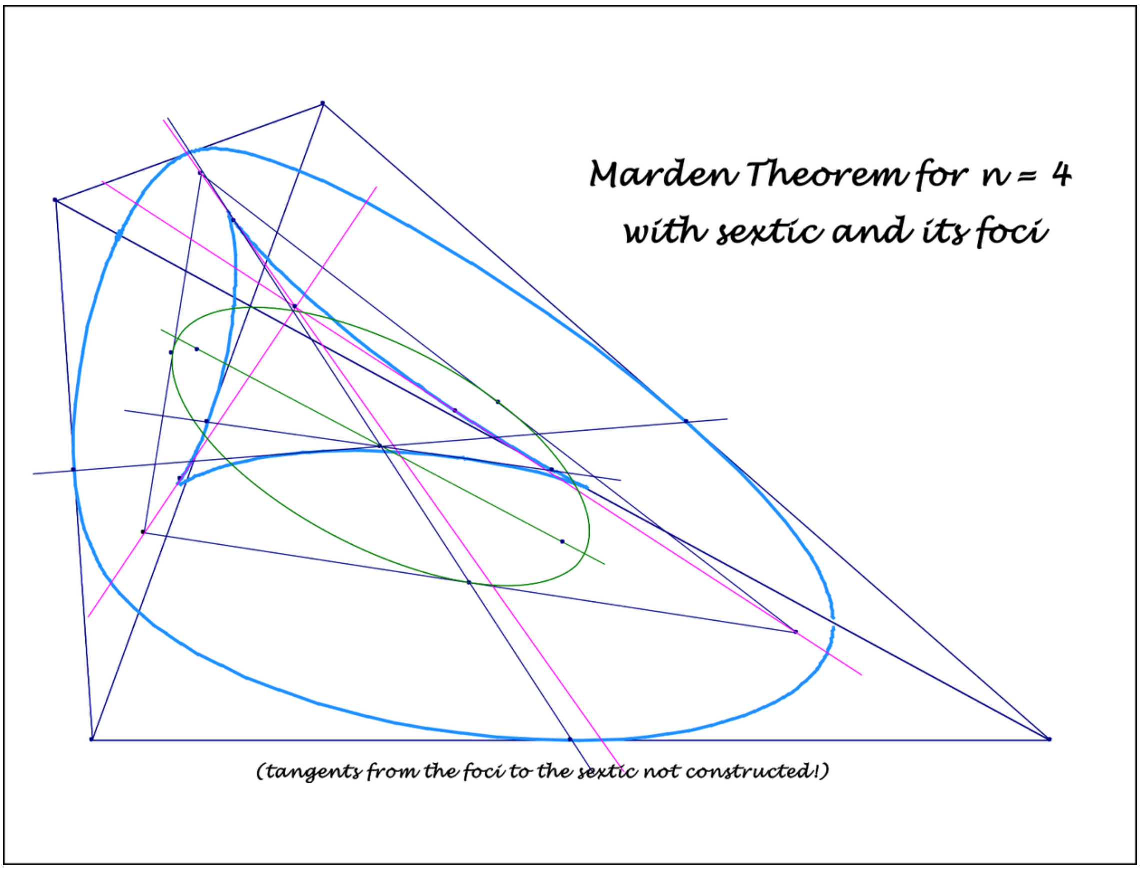
(2) Marcel Grossmann (1925): Das vollständige Fokalsystem einer ebenen algebraischen Kurve.
The "metrisch assoziierte Curve" C(n-2) for the sextic C(n) with n = 3
... will be the common point of the tangents from the foci at the sextic.

With respect I skim over these texts, but I cannot follow!

Your message 2544:
I completed my figure in your sense (see attached), but now it is very confusing, so I resigned the nomination.
Wrt 1): The reference triangle is midDT, so the root is the tripole wrt midDT of the dual of QA-P1 .
Wrt 2): Excuse, that I overlooked your property. Of course the angle property holds!
3) The isotomic wrt midDT swaps opposite points of the QL.

Best regards Eckart

PS. Please give me some time, to study your messages 2545 and 2546.



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Message: #2549
Date: 03/8/2017 3:26:59
From: tsihonglau
Subject: Common QA-Tf2 Conjugates of Two Quadrangles

Dear all,

We know that there are two circumconics through a given quadrangle tangent to a given line.
So we get two tangent points (real or imaginary) (or QA-Tf2 conjugates)
Given two quadrangles in general position, does there exist a line such that they share the same pair of tangent points? (or the same pair of QA-Tf2 conjugates)
If there exist a line, how many do there exist such lines (real or imaginary)?

Best regards,
Tsihong Lau

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Message: #2550
Date: 03/8/2017 4:06:17
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
Thank you for these 2 new references, for the figure and for the answers to my different questions !
(The fact that the opposite QL vertices are isotomic conjugates wrt midDT is new for me)
For the last messages, take all the time you will, your comment (precious for me) will be welcome anytime.
I've just tried to generalise Bernard Gibert's commentars on pages 13-17 in his note Inscribed Cardioïds and Eckart Cubics
...
Best regards
Bernard

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Message: #2551
Date: 04/8/2017 2:55:49
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard,

the sextic wrt Marden´s theorem for n = 4 is the dual of a cubic
with the DT-equation
$q^2r^2(-x+y+z)x^2+r^2p^2(x-y+z)y^2+p^2q^2(x+y-z)z^2 = 0$.

Best regards Eckart

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Message: #2552
Date: 06/8/2017 12:05:27
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
Some questions before going to bed :
1) the duals of the vertices of midDT are the line through QG-P1
and QG-P2, which are therefore also tangent to the sextic
2) the cubic and QL-Cu1 intersect in 9 points, the vertices of
QL and a C-Btriple : what about these 3 points ?
3) there is a curve of class 6 tangent to the 15 segments
joining the 6 vertices of QL in their middles : any idea about
this curve (it could be a cubic, but I have some doubts ...)
Best regards
Bernard

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Message: #2553
Date: 06/8/2017 6:34:58
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear all,

Two questions raised:
1. Nine points A,B,C,D,E,F,G,H,I lie on a cubic only.
We can get nine other Cayley-Bacharach ninth points-

```
A' - B,C,D,E,F,G,H,I
B' - A,  C,D,E,F,G,H,I
C' - A,B,  D,E,F,G,H,I
D' - A,B,C,  E,F,G,H,I
E' - A,B,C,D,  F,G,H,I
F' - A,B,C,D,E,  G,H,I
G' - A,B,C,D,E,F,  H,I
H' - A,B,C,D,E,F,G, I
I' - A,B,C,D,E,F,G,H
```

It is apparent that A',B',C',D',E',F',G',H',I' lie on the cubic through
A,B,C,D,E,F,G,H,I.
But do A',B',C',D',E',F',G',H',I' lie on the cubic only?
Are the nine lines AA',BB',CC',DD',EE',FF',GG',HH',II' tangent to a line cubic only?

2. We get a unique Cayley-Bacharach ninth point from two quadrangles.
We get a unique Cayley-Bacharach ninth line from the two dual quadrilaterals.
What is the relationship between the point and the line?
Does the point lie on the line?

Best regards,
Tsihong Lau

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Message: #2554
Date: 06/8/2017 7:08:56
From: tsihonglau
Subject: Common QA-Tf2 Conjugates of Two Quadrangles

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
> If there exist a line,
> how many do there exist such lines(real or imaginary)?
> The question is equivalent to:
> Given two triangles A_I,B_I,C_I and A_J,B_J,C_J,
> how many common isogonal conjugates(real or imaginary)
> besides the circular points of them?
> Let I,Ia,Ib,Ic and J,Ja,Jb,Jc be the incenters/excenters
> quadrangles of A_I,B_I,C_I and A_J,B_J,C_J respectively.
> The common QA-Tf2 conjugates of the former are the
> common isogonal conjugates of the latter.

Best regards,
Tsihong Lau
```

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Message: #2555
Date: 06/8/2017 8:21:47
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Dear Eckart,
Please, forget the last point ; as the 6 vertices of the QL are
3 by 3 on 4 lines, I'm not sure the curve exists (apart of that,
it would be of course be of class 5 and not 6 and certainly not
a cubic).
Best regards
Bernard
```

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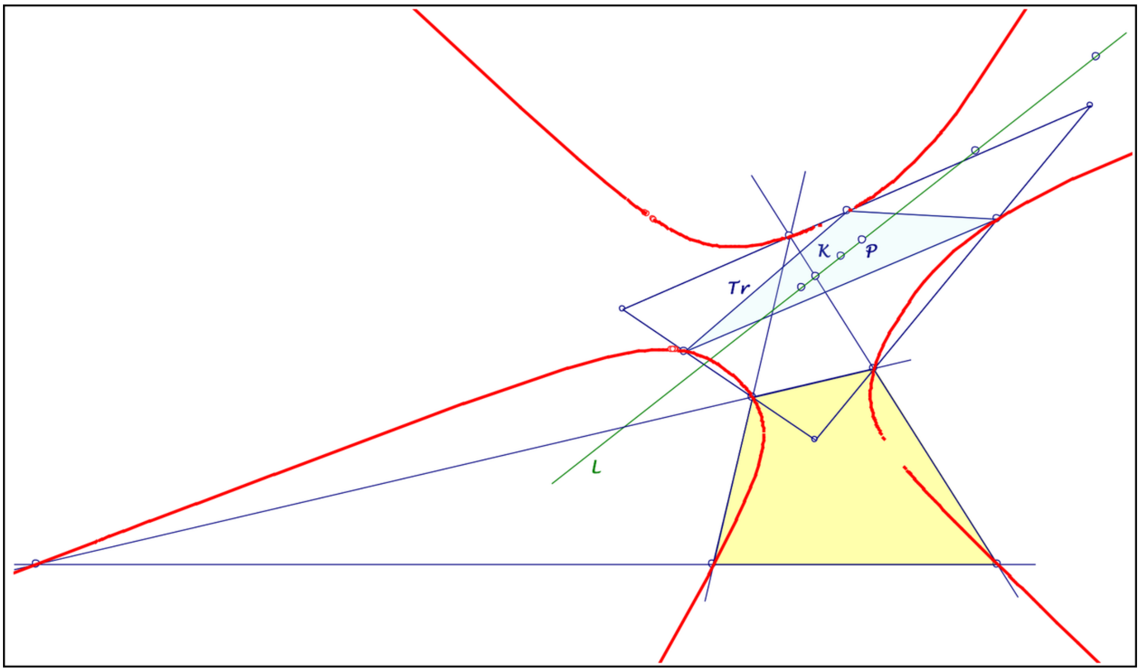
Message: #2556
Date: 06/8/2017 12:15:44
From: eckart_schmidt@t-online.de
Subject: Further QL-Cubics

```
Dear Bernard, dear Chris,
this observation was new for me (see attached file):
Consider the diagonal triangle QL-DT
... with the cecian triangle Tr of a point K
... and an Tr-isoconjugation with fixed point K,
... then opposite QL-points are conjugated.
Further:
The Tr-tripoles of the QL-lines are collinear on a line L
through K.
Then nonpivotal isocubics
... with reference triangle Tr,
... isoconjugation with fixed point K
... and root P on L
... bear the 6 QL-points (and the Tr-vertices).
For K = QL-P10 and P = Tr-tripole of the QG-P18-line we get
QL-Cu1 (see #1765).
For K = QL-P8 and P = Tr-tripole of the collinear midpoints of
QG-P1.QG-P2
... we get the cubic of the Marden theorem for n = 4
    (see #2540).
Best regards Eckart
```



2017-08-06.pdf

Message: #2557
Date: 07/8/2017 10:14:56
From: bernard.keizer
Subject: Further QL-Cubics

```
Dear Eckart,
It's highly interesting ! That's pure QA/QL geometry ...
There are as many isocubics through the 6 vertices of QL and of
dual sextics of class 3 tangent to the 6 sides of QA as DT
points ...
Best regards
Bernard

PS My question about the 3 other intersection points of the 2
isocubics for K = QL-P8 or QL-P10 becomes of course less
interesting
I've printed without difficulty your first article (Marcel
Grossmann) and I understand the curve Cn-2 giving a point for n
= 3, but I'm not able to find the 2nd (Tadahiko Kurota), I have
only foud a complete book ...
I've also foud Roger Böttcher Einführung in die Theorie des
algebraischen Kurven ...
```

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Message: #2558
Date: 07/8/2017 10:39:48
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

```
Dear Bernard,
wrt the paper of Kubota:
https://www.jstage.jst.go.jp/article/tmj1911/33/0/33_0_252/_pdf
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2559
Date: 07/8/2017 3:22:39
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard,

a short excursion in the geometry of the cubic Cu and the sextic
Se wrt Marden´s theorem for n = 4.

(1) Cu is a nonpivotal isocubic
... with reference triangle midDT,
... isotomic isoconjugation *,
... root: tripole of the dual of QA-P1.

(2) Se and Cu are dual curves.

(3) Isotomic conjugates of Cu-points lie on their dual line,
... which is tangent to Se.

(4) Let X be an intersection of Cu
    and the circumscribed Steiner ellipse of midDT,
... unequal midDT-vertices, then three X are possible.
... X* is a point at infinity of an Cu-asymptote.

(5) There are three Cu-tangents parallel XX*:
... one common tangent of Cu and the Steiner ellipse,
... two common tangents with Cu and Se
... ... with the same contact points,
... ... which are isotom conjugates
... ... and collinear with X and QL-P13 = QA-P16.

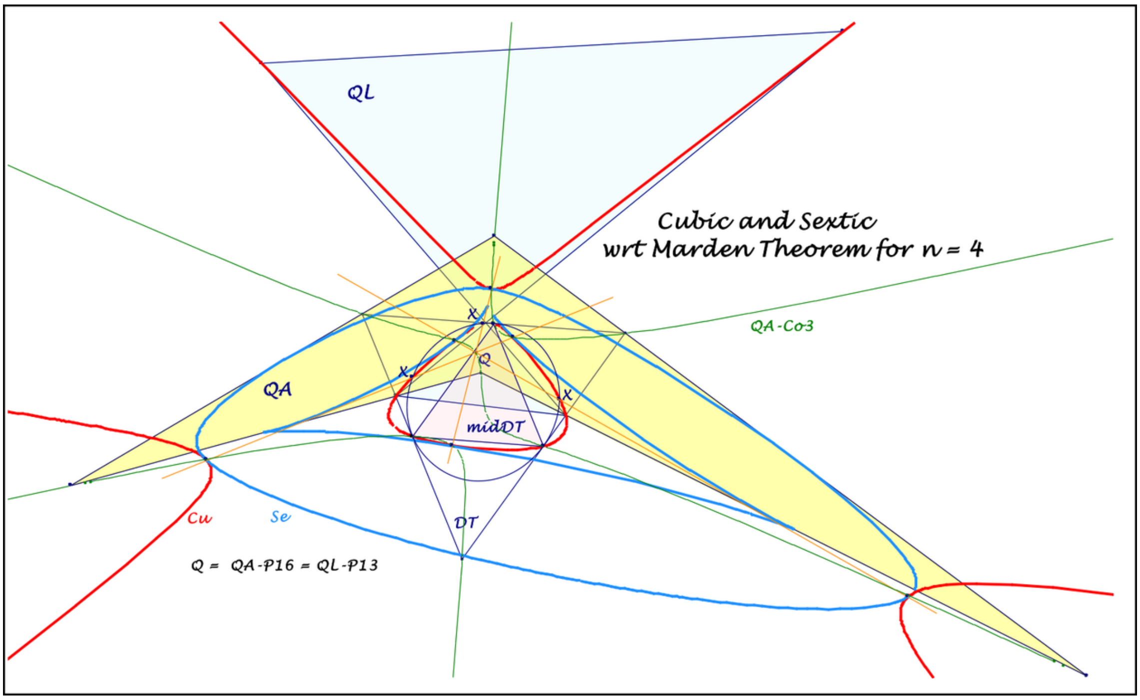
6) Tangents from QL-P13 = QA-P16 at the sextic Se bear
... two contact points with Cu
... and one intersection of Cu and the Steiner ellipse.

(7) The six possible contact points of Cu and Se lie on QA-Cu3.

(8) Tangents in isotomic conjugates Y and Y*
    on Cu intersect on Cu,
... in the isotomic conjugate of the 3rd intersection
    of Cu and YY*.

Attached a figure for further properties.

Best regards Eckart



2017-08-07.pdf

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Message: #2560
Date: 08/8/2017 11:27:41
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Dear Eckart,
Amazing properties !
The only one I understand easily is the last one, as it is a
general property of the isocubics ...
The one I prefer is the one with QA-Cu3 (as in the text, not Co3
as on the figur) !
Did you abandon definitively the possibility of constructing the
3 foci of Se ?
Did you identify the point  Cn-2 of Grossmann for n = 3 ?
Best regards
Bernard
PS Any comment to my conjectures with stelloïds ?
```

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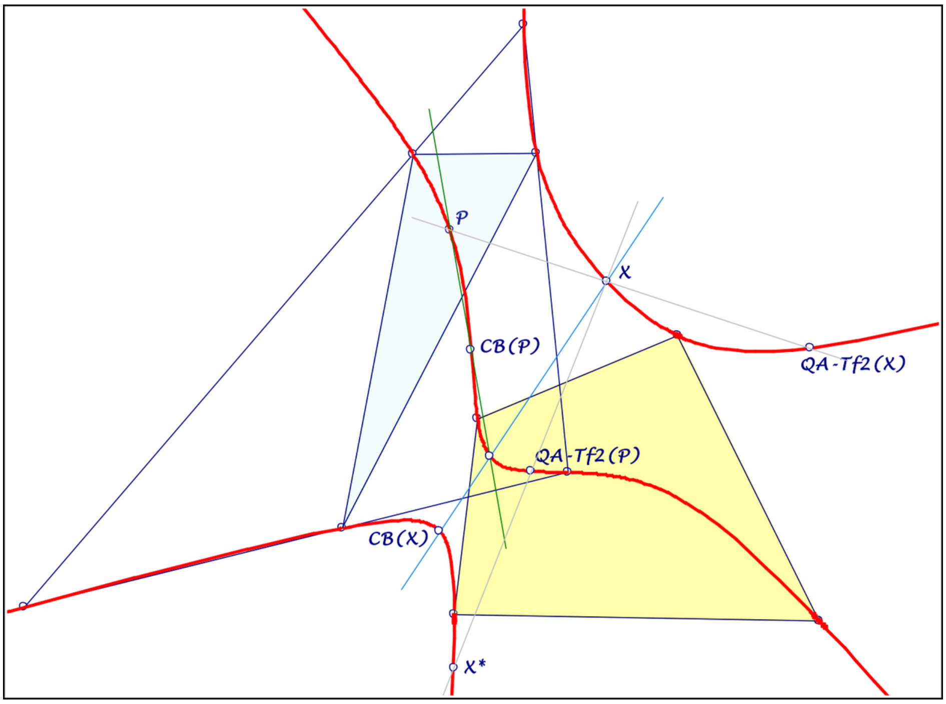
Message: #2561
Date: 08/8/2017 4:19:20
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

Dear Bernard, dear Tsihong Lau,

seven points give a transformation, mapping a point X to the
C-B-point of X and the seven points.
Wellknown is, that QA-Tf2 is such a transformation wrt the 4
QA-vertices and the 3 QA-Tr1-vertices.
Here is a generalization (see attached file):
QA-Cu1,2,3,4,5 are pivotal isocubics wrt:
... reference triangle QA-DT,
... isoconjugation QA-Tf2 with QA-vertices as fixed points
... and a pivot P.
These cubics can also be described as pivotal isocubics wrt:
... reference triangle: DT-cevian triangle of P,
... isoconjugation * with fixed point P,
... pivot = QA-Tf2(P).
These cubics are invariant wrt the transformation X-->CB(X),
... mapping X to the C-B-point of
... X and the 4 QA-vertices and the 3 DT-cevians of P.
Third intersection of X.CB(X) and the cubic is a fixed point:
... the third intersection of P.CB(P) and the cubic.

Best regards Eckart



2017-08-08.pdf

Message: #2562
Date: 08/8/2017 5:31:29
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear all,
I explain my post #2553 in more details.
In fact, there are four questions in it.
1. Do A',B',C',D',E',F',G',H',I' lie on the cubic only?
If so, from A,B,C,D,E,F,G,H,I we get A',B',C',D',E',F',G',H',I'.
From A',B',C',D',E',F',G',H',I', we
get A'',B'',C'',D'',E'',F'',G'',H'',I''.
And so on, we can get infinite nine-point chain!
2. Are the nine lines AA',BB',CC',DD',EE',FF',GG',HH',II'
tangent to a line cubic only?
If so, let
a=AA',b=BB',c=CC',d=DD',e=EE',f=FF',g=GG',h=HH',i=II'.
According to the principle of duality, from a,b,c,d,e,f,g,h,i,
we get nine points A*,B*,C*,D*,E*,F*,G*,H*,I*.
Moreover, what is the relationship between
A',B',C',D',E',F',G',H',I' and A*,B*,C*,D*,E*,F*,G*,H*,I*?
3. Does the point lie on the line?
If so, any two quadrangles/quadrilaterals in general position
can be viewed as on a vertical parabolic pappian plane.
The Cayley-Bacharach ninth point or line is the infinite point
or line respectively.
Please refer to topic #2106 for more information.
4. What is the relationship between the point and the line?
If the above is not true, the Cayley-Bacharach ninth point or
line is the infinite point or line respectively, too.
But the properties are not so nice as the above.
However, the configuration of two quadrangles/quadrilaterals
contradict to point 6 "configuration=variable+fixed objects" in
the message #2446 "geometry=incidence+metric.
The two quadrangles/quadrilaterals are variable and no fixed
objects exist!
A question more is raised:
In 3 and 4, the Cayley-Bacharach ninth line is the infinite
line.
There are two pairs of parabolic points(infinite points of
QA-Co1) for the two dual quadrangles.
What is the relationship between the two pairs?
If they form harmonic conjugates, the properties will be very
nice!
Best regards,
Tsihong Lau

Message: #2563
Date: 09/8/2017 3:54:38
From: bernard.keizer
Subject: Cayley-Bacharach Ninth

Dear Eckart,
Another interesting property !
The first definition is exactly the one of the pivotal isocubics with DT, QA-Tf2, fixed points and pivot P
The second follows immediately with the cevian triangle of P as DT, the transformation X to X* is the cevian quotient P/X, the fixed points are P and the vertices of DT, the pivot is QA-Tf2(P) therefore named isopivot
I suppose the last one with pivot the 3rd intersection Q of PCB(P) with the isocubic defines a 3rd transformation swapping P and CB(P) with fixed points the 4 points having this pivot Q as tangential and reference triangle the DT of this QA, the tangential of Q being the isopivot ...
Best regards
Bernard

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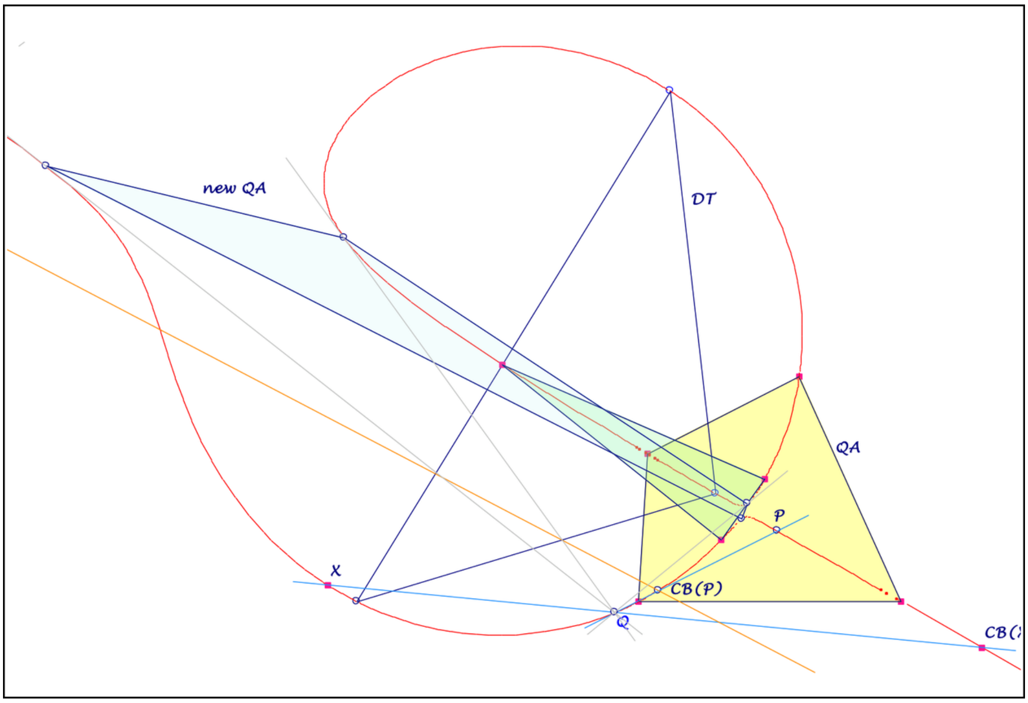
Message: #2564
Date: 10/8/2017 4:38:55
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Bernard,

your last conjecture is right, thanks!
CABRI-observations confirm your new result:
Let X be a point on a pivotal QA-isocubic:
... reference triangle QA-Tr1 = DT,
... isoconjugation QA-Tf2
... and pivot P.
The C-B-point of X, the four QA- and the three DT-vertices is
QA-Tf2(X).
The C-B-point of X, the four QA-vertices and the three
DT-cevians of P
... is QA-Tf2(X) wrt a new QA:
... vertices in the contact points of tangents from a point Q
    at the isocubic
... with Q = 3rd intersection of the cubic and P.CB(P).

Best regards Eckart
```



2017-08-10.pdf

Message: #2565
Date: 10/8/2017 8:25:21
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

Dear Bernard,
thanks for correction.Wrt Marden´s theorem for n=4 no progress
on my side.
Only the property, mentioned by Kubota:
The mean distance of a foci to three parallel sextic tangents
... is the distance of QA-P1 to a parallel through the foci.
But that holds for any point.
Another observation:
The three QA-versions of the midpoints of QG-P1.QG-P3
... are points of the cubic Cu;
... their duals are tangent to the sextic Se,
... connecting opposite QA-midpoints.
Best regards Eckart
PS: Excuse, I cannot judge your conjectures, for I am not
familar enough with stelloids.

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Message: #2566
Date: 10/8/2017 11:15:34
From: bernard.keizer
Subject: Generalization Marden Theorem

Dear Eckart,
The property you mention is not new : the middles of QG-P1QG-P3
are precisely your points X of your first figure (but no longer
of the last one) and that's why the line XXX is the dual of
QA-P1, as QA-P1 is the middle of the 3 segments joining opposite
QA-midpoints, which are the duals of the X ...
By the way, if I'm not wrong, the duals of QG-P2 (vertices of
midDT) are the lines QG-P1QG-P3, which are also tangent to the
sextic ...
Best regards
Bernard
PS Of course, as you may imagine, I regret deeply that you can't
judge my conjectures !
Again, it is only a generalisation of Bernard Gibert's
properties wrt the cubic stelloïd ...

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Message: #2567
Date: 11/8/2017 12:05:33
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Dear Eckart,
It helps also to remember that the line through 2 opposite
QA-midpoints passes also through the corresponding QA-P2.
The points X in your message 2540 are not only the middles of
the QG-P1QG-P3, but also the 3rd intersections of the sides of
midDT with your cubic ; as midDT is the reference triangle of
the cubic, the root is the trilinear pole of the XXX line wrt
this triangle (general property of the non pivotal isocubics).
Best regards
Bernard
```

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Message: #2568
Date: 11/8/2017 8:27:52
From: bernard.keizer
Subject: Generalization Marden Theorem

```
Please read QG-P2 and not QA-P2 !
```

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Message: #2569
Date: 11/8/2017 4:43:10
From: bernard.keizer
Subject: Marden, Siebeck and polygons of pivots of stelloïds and cassinians

---

Dear Eckart,
As I'm very disappointed by your last answer, I try to put it another way.
The stelloïds are defined by a polygon of pivots Pi as locus of points M for which the sum of the directions of MPi wrt an arbitrary line keeps a constant direction.
The cassinians are defined by a polygon of points Pi as the locus of points M such as the product of the distances MPi remains constant.
For n = 2, you have the rectangular hyperbolas and the ovals of Cassini, which have both the middle of the segment joining the 2 points as center of symmetry.
For n = 3, all the pivot triangles of the set of cubic stelloïds or cassinians defined by the 3 points have the same 2 foci of their Steiner inellipses ; all these Steiner inellipses are confocal, the 2 foci forming themselves a degenerated confocal ellipse (curve of class 2).
For n = 4, all the pivot quadrangles of the set of quartic stelloïds or cassinians defined by the 4 points have the same 3 foci of their sextics of class 3 ; all these sextics of class 3 are confocal, the 3 points forming themselves a degenerated confocal curve of class3.
Does that seem correct ?
Best regards
Bernard
PS I found the fact that the foci form themselves a degenerated confocal curve of class n-1 in Kubota and it explains the properties with the tangents mentionned by Siebeck ...

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Message: #2570
Date: 11/8/2017 4:56:22
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,

wrt #2553 2):
CABRI-observations show, that the C-B-point of two quadrangles
doesn´t lie on the C-B-line of the dual quadrilaterals.

Best regards Eckart
```

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Message: #2571
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message #2571

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Message number 2571 is not available in Yahoo groups.
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Message: #2572
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message #2572

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Message number 2572 is not available in Yahoo groups.
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Message: #2573
Date: 11/8/2017 5:37:10
From: tsihonglau
Subject: Cayley-Bacharach Ninth

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> wrt #2553 2):
>> CABRI-observations show, that the C-B-point of two quadrangles
>> doesn´t lie on the C-B-line of the dual >>quadrilaterals.

Dear Eckart,

I gave more details in #2562.
You answered questions 3 and 4.
I hope the answers of the other questions.
More questions raised:
There are dualities of triangle/trilateral and quadrangle/quadrilateral and quintangle/quintilateral.
The duality of triangle/trilateral is very trivial.
The duality of quadrangle/quadrilateral is QA-8.
The duality of quintangle/quintilateral is the tangent points/lines of a conic.
Are there more dualities of multangle/multilateral?
Can the duality of novemangle/novemlateral be derived from Cayley-Bacharach theorem?

Best regards,
Tsihong Lau

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Message: #2574
Date: 13/8/2017 12:16:17
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Eckart,
I don't know what to add in order to convince you totally !
Perhaps this : the same n points define 2 sets of orthogonal curves, the cassinians and the stelloïds.
Each stelloïd has an infinity of polygons of pivots and each polygon gives again 2 different sets of orthogonal curves. All the polygons share the same property, the curves defined by Siebeck are confocal.
Best regards
Bernard
PS I'm not a specialist neither of stelloïds nor of cassinians, all I know comes from Bernard Gibert or Robert Ferréol in Mathcurves

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Message: #2575
Date: 13/8/2017 4:21:55
From: tsihonglau
Subject: Clockwise Counterclockwise Rotation Perspective Triangle Conic

---

Dear all

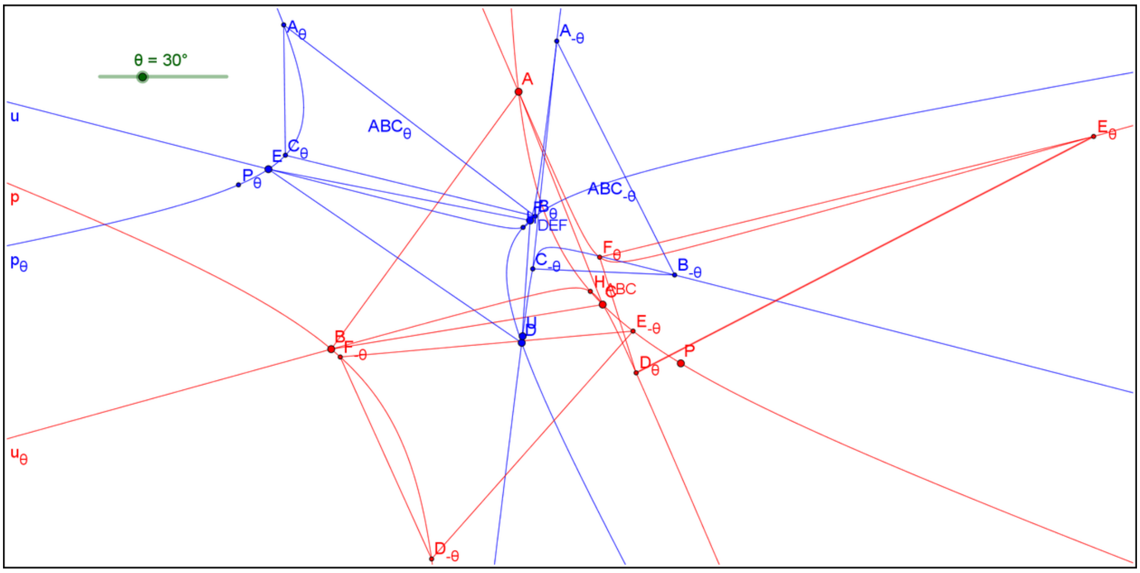
I would post this message several months ago!
I post it so late!
This message introduces a new geometry of two ordered triangles.
It contains a pair of points as orthocenters for two triangles.

Given two triangles ABC and DEF and an angle θ,
if P is a point and lines p_A,p_B,p_C are lines AP,BP,CP rotated by θ(counterclockwise),
suppose parallel lines to p_A,p_B,p_C through D,E,F respectively concur at another point P_θ.
The loci of P and P_θ are circumconics p and p_θ of triangles ABC and DEF respectively.
As P moves along p through A,B,C, P_θ moves along p_θ through A_θ, B_θ,C_θ.
As P_θ moves along p_θ through D,E,F, P moves along p through D_-θ,E_-θ,F_-θ.
Similarly, we get circumconics u and u_θ of D,E,F,A_-θ,B_-θ,C_-θ and A,B,C,D_θ,E_θ,F_θ respectively.
Circumonic p and u_θ of triangle ABC concur at the fourth point H_ABC.
Circumonic u and p_θ of triangle DEF concur at the fourth point H_DEF.
Points H_ABC and H_DEF remain fixed as θ varies.
Moreover, H_ABC and H_DEF behave like orthocenters of triangles.
That is, the fourth points of triangles H_ABC,B,C and H_DEF,E,F are A and D respectively.
the fourth points of triangles A,H_ABC,C and D,H_DEF,F are B and E respectively.
the fourth points of triangles A,B,H_ABC and D,E,H_DEF are C and F respectively.
Areas of three triangles ABC, D_θE_θF_θ, D_-θE_-θF_-θ are equal.
Areas of three triangles DEF, A_θB_θC_θ, A_-θB_-θC_-θ are equal.
If θ=0 or π/2, triangles A_θB_θC_θ and A_-θB_-θC_-θ coincide and D_θE_θF_θ and D_-θE_-θF_-θ do, too.

Suppose barycentrics of ABC and DEF are
A(1:0:0) B(0:1:0) C(0:0:1) D(dx:dy:dz) E(ex:ey:ez) F(fx:fy:fz)
a b c are "squares" of sidelengths BC,CA,AB.

First barcentric of H_ABC is (dx + dy + dz) / (-c (fx dy - dx fy) (dx ey - ex dy) + (a - b) (fx dy - dx fy) (dy ez - ey dz) + (-a + b + c) (fx dy - dx fy) (dz ex - ez dx) + c (fy dz - dy fz) (dx ey - ex dy) - a (fy dz - dy fz) (dy ez - ey dz) - (c - a) (fy dz - dy fz) (dz ex - ez dx) + b (dy ez - ey dz) (fz dx - dz fx) - b (dz ex - ez dx) (fz dx - dz fx))
To be continued...

Best regards,
Tsihong Lau



clockwise_counterclockwise_rotation_perspective_triangle_conic.png

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Message: #2576
Date: 14/8/2017 11:25:52
From: eckart_schmidt@t-online.de
Subject: Generalization Marden Theorem

```
Dear Bernard,

wrt #2569 many thanks for your efforts, to make me familar with
stelloids.

Now I understand your conjecture in #2533:
"I'm convinced that the QA's sharing the same triangle are the
pivots on a quartic stelloïd the same way as the triangles
sharing the same bipoint (foci of the Steiner inellipse) are the
pivots on a cubic stelloïd (see Bernard Gibert)."

But CABRI-observations show, that this holds only for two QAs
sharing the same triangle, but not for three:
I made a figure with three QAs P1P2P3P4, Q1Q2Q3Q4, R1R2R3R4,
sharing the same triangle,
... let Q1 define a stelloid wrt the pivots P1, P2, P3, P4,
... which bears also Q2, Q3, Q4,
... but not Ri.

This holds also for triangles, sharing the same bipoint.
Are these reflections correct?

Best regards Eckart
```

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Message: #2577
Date: 14/8/2017 5:17:28
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Endlich, lieber Eckart !
I think we begin to understand eachother perfectly !
Your observation is perfectly correct. 2 polygons of pivots sharing the same foci of the curve defined by Siebeck define a unique stelloïd. There is no reason that a 3rd polygon lies on the same stelloïd (but it defines 2 other stelloïds with each of the 2 1rst polygons). That's true in particular for 2 QA's or 2 triangles.
You should start with the rectangular hyperbola, which is a conic stelloïd. 2 bipoints sharing the same middle define a unique RH (and 2 ovals of Cassini orthogonal to it). Any other bipoint sharing the same middle doesn't lie necessary on the same RH, but defines 2 other RH's with each of the 1rst bipoints (and 4 new ovals of Cassini orthogonal to them). This way, bipoints sharing the same middle define infinities of RH's and of orthogonal ovals of Cassini, all centered in the middle ...
The same goes for triangles sharing the same bipoint, which define an infinity of cubic stelloïds and for QA's sharing the same triangle, which define an infinity of quartic stelloïds.
I've finally understood this only recently, that's why I wanted so desperatly convince you.
I hope I've succeeded now and I thank you, as you helped me to clarify my original intuitions.
Best regards
Bernard

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Message: #2578
Date: 18/8/2017 9:57:33
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cass

---

```
Dear Eckart,
While waiting for your conclusions, perhaps this little problem.
The polynoms of the 4rst degree having the same 3 roots of the
derived polynom differ only by the last term.
If this term is equal to zero, one of the 4 points is the
barycenter of the 3 others and of the QA.
Do the curves Cu and Se exist in this case ?
DT is the medial triangle of the triangle and midDT is the
medial of the medial, but the vertices of the QL are on the
infinity line ?
Best regards
Bernard
```

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Message: #2579
Date: 19/8/2017 8:49:16
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

```
Dear Bernard,
what do you mean with "barycenter" in #2578, it cannot be the
centroid.
Cu and Se exist without problem, the QA isn´t a special one in
this case.
Is there a misunderstanding on my side?
Best regards Eckart
```

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Message: #2580
Date: 20/8/2017 4:42:08
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

```
Dear Eckart,
Thank you for your answer, sorry if I wasn't clear enough !
I'm interested in the equations of the quadrangles taking the
centroïd QA-P1 as origin.
Then the 1rst term of the equation vanishes : sum of z = 0, sum
of z2 = 2 sum of zizj ... and only the last term or constant
differs from one to the other quadrangle.
It is the same for bipoints and triangles.
For example, for a bipoint, if one point is in the origin
(middle of the bipoints), the other gives the same point.
For a triangle, if one point is the centroïd, the 2 others are
symmetric wrt the origin, the triangle is flat and the Steiner
inellipse is reduced to a segment.
For a quadrangle, if one point is in the origin and at the same
time QA-P1, it is the centroïd of the triangle of the 3 others.
I wonder if the degenerated sextic formed by the 3 Steiner
inellipses of the 3 triangles joining this centroïd to the
vertices would be acceptable (this curve is also tangent to the
3 segments joining opposit sides, which are in this case the
medians of the triangle). But the curve should also have the
same 3 foci ...
What do you think of that ?
Best regards
Bernard
PS As you may imagine, I'm a little sad that you didn't comment
my last message ...
```

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Message: #2581
Date: 22/8/2017 9:22:23
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

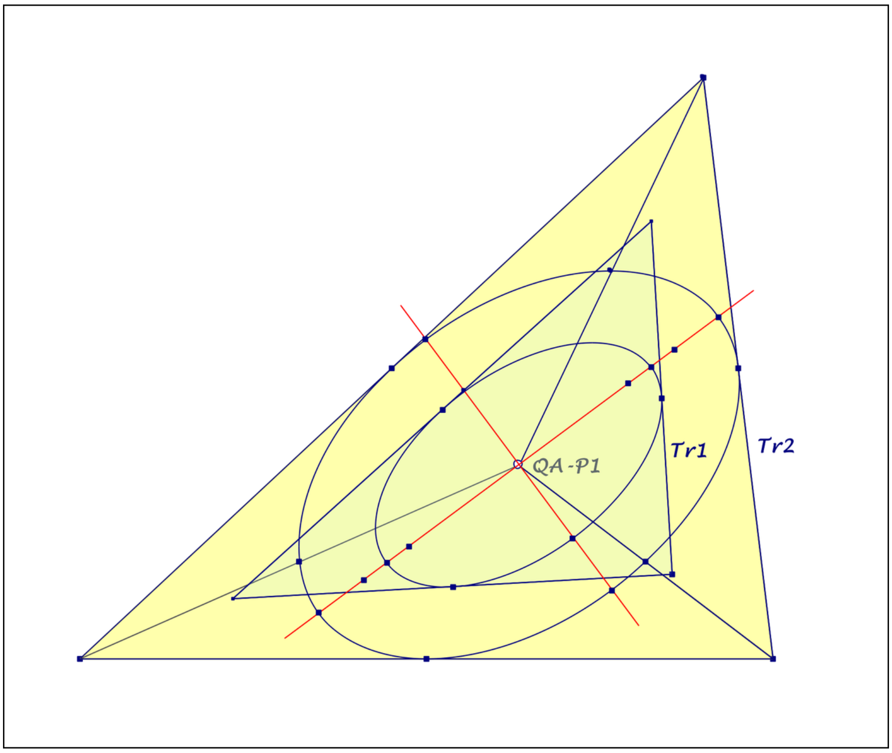
---

```
Dear Bernard,

thanks for detailed information in #2580, my misunderstanding: I
have only considered "term zero".
If additional "QA-P1 origin", I think, your three Steiner
inellipses cannot be the searched sextic.
Their duals had to give a cubic, but that doesn´t hold, as
CABRI-observations show.

Another aspect:
Among the zero-quadrangles with the same
zero-derivation-triangle Tr1
... there is one with a vertex QA-P1
    and a remaining triangle Tr2 (your example).
... Both triangles have inscribed Steiner ellipses
... with the same axes.
Is this evident? Several CABRI figures show this property
(calculated and drawn with accuracy of 6 digits).

Best regards Eckart
```



2017-08-23.pdf

Message: #2582
Date: 23/8/2017 9:59:33
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

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```
Dear Eckart,
Yes, it's evident !
By derivating 2 times the equation of the 4th degree in z, you
get 12 z2 + 2 S2 = 0.
The 1rst term S1 is equal to 0, the 2nd term is S2 (sum of
zizj), the 3rd is S3 (sum of zizjzk), the 4th is S4 (product of
the zi).
All the QA's have the same LSD line, which is also the LSD line
of the triangle Tr1, through the 2 foci of the Steiner inellipse
of Tr1.
The particular QA formed by QA-P1 and the vertices of Tr2 has
the last term equal to 0 ; the vertices of Tr2 verify a
different equation and by derivating this equation, you get 3 z2
+ S2 = 0.
The distance from QA-P1 to the foci of the Steiner inellipse of
Tr2 and the distance from QA-P1 to the foci of the Steiner
inellipse of Tr1 are therefore in the ratio sqr(2), which is
easy to check on your figure.
Perhaps there are other properties : on you figure, Tr1 and Tr2
seem to be perspective, like their midtriangles
Best regards
Bernard
```

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Message: #2583
Date: 23/8/2017 4:13:58
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

```
Dear Bernard,
thanks for verifying and explanation.
Excuse my ignorance: What is the LSD line of a QA or triangle?
Wrt perspectivity of Tr1 and Tr2: Cabri observations don´t
confirm this.
Best regards Eckart
```

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Message: #2584
Date: 23/8/2017 4:57:28
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Dear Eckart,
This time, I'm a little bit surprised, as you had greatly contributed to this item, founding an error in Coolidge's calculations ...
The QA-LSD line is QA-L10, the line with least sum of squared distances from the vertices (counterpart of QL-P26 for the QL).
The LSD line for the triangle is the line through the 2 foci of the Steiner inellipse and the LSD line for a bipoint is obviously the line through the 2 points.
All this is obtained by derivating 2 times the equation of the 4th degree in z.
As sum of z = 0, you get sum of z2 = 2 sum of zizj for the QA and for the triangle and the bipoint.
The centroïds of the z2 of the 3 figures are aligned (hence the construction given in EQF)
Best regards
Bernard

Message: #2585
Date: 26/8/2017 4:51:28
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Bernard,
thanks for your reply. 1000 messages ago, I had forgotten the shortcut for QA-L10, for I studied this line, but never used it. Excuse!
In message 2578 you asked for the cubic and the sextic, if one vertex of the QA is QA-P1.
My interpretation leads to a discrepance, perhaps you can find the error.
CABRI observations show:
The cubic degenerates in the three sidelines of the medial diagonal triangle.
The sextic degenerates in the three vertices of QA unequal QA-P1.
But then Siebeck´s angle condition II.1) for the foci doesn´t hold for an arbitrary point.
Best regards Eckart

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Message: #2586
Date: 27/8/2017 3:53:59
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Wunderbar, lieber Eckart !
As I told you before, I'm very grateful for your help, in particular your comments, figures and reactions.
I'm convinced we begin to understand many things, which were stil mysterious.
At the beginning, my little interrogation was more a curiosity.
But thanks to you, it becomes key of my comprehension.
In fact, the medial triangle of Tr2 is the DT of the QA formed by the vertices of Tr2 and QA-P1 ; this DT is self-dual wrt the QA/QL. If the cubic degenerates in the 3 sidelines of the medial triangle of Tr2, the sextic degenerates in the 3 vertices of the same triangle and not in the vertices of Tr2.
The property of Siebeck in II1 can be verified for Tr1 and midTr2 and the reason is that both triangles share the same foci of the Steinerinellipse.
(see my previous message about the ratio of sqr(2), it's precisely the ratio for the distance from the center to the foci of the Steiner inellipse and circumellipse of midTr2).
It's possible to generalise to QA's having confocal Siebeck's inscribed sextics : for any point in the space,they have the same sum of angles of lines drawn from this point to the 4 vertices.
Best regards
Bernard

Message: #2587
Date: 27/8/2017 8:57:15
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

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```
Dear Bernard,

perhaps an interesting observation:

Starting with a triangle Tr1 and its centroid
... we get a quadrangle QA1 with one vertex QA-P1,
... which leads to a derivation-zero triangle Tr2,
... adding QA-P1, we get a quadrangle QA2
... which leads to a derivation-zero triangle Tr3,
... adding QA-P1, we get a quadrangle QA3,
... and so on.

The vertices of the triangles Tr1, Tr2, ...
... lie on three conics
... through QA-P1 (see attached file).
What about these special conics of a triangle?

Best regards Eckart
PS. I shall answer your last message later.
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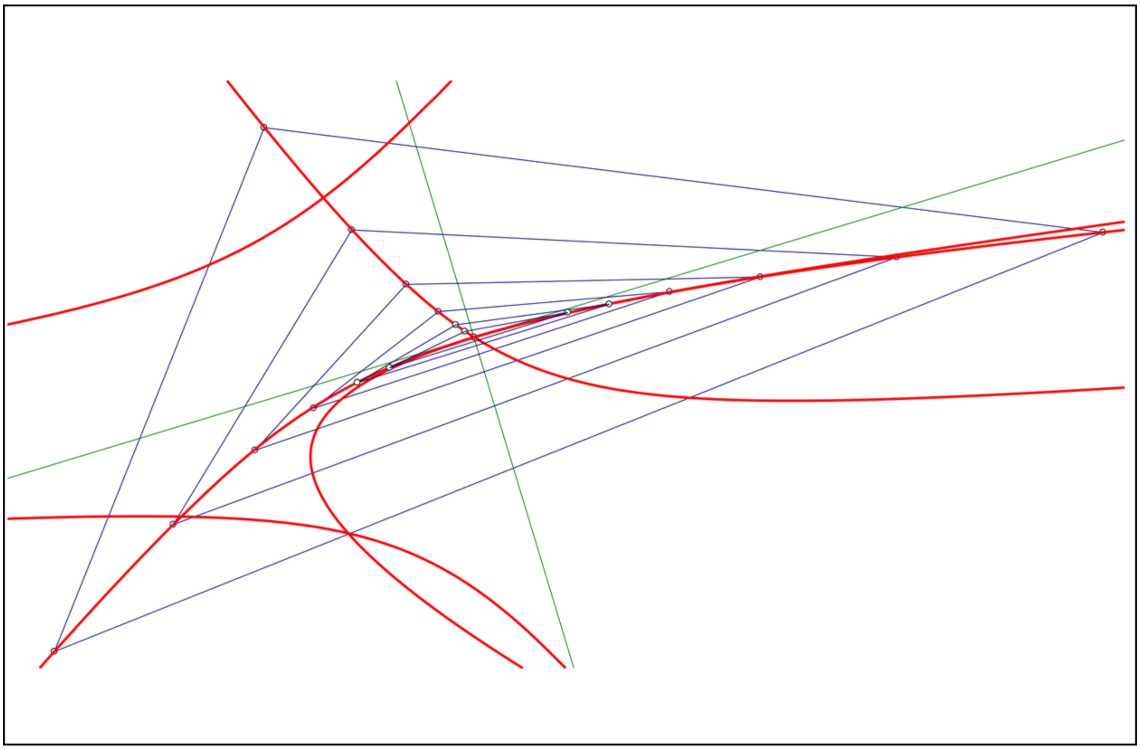

2017-08-27.pdf

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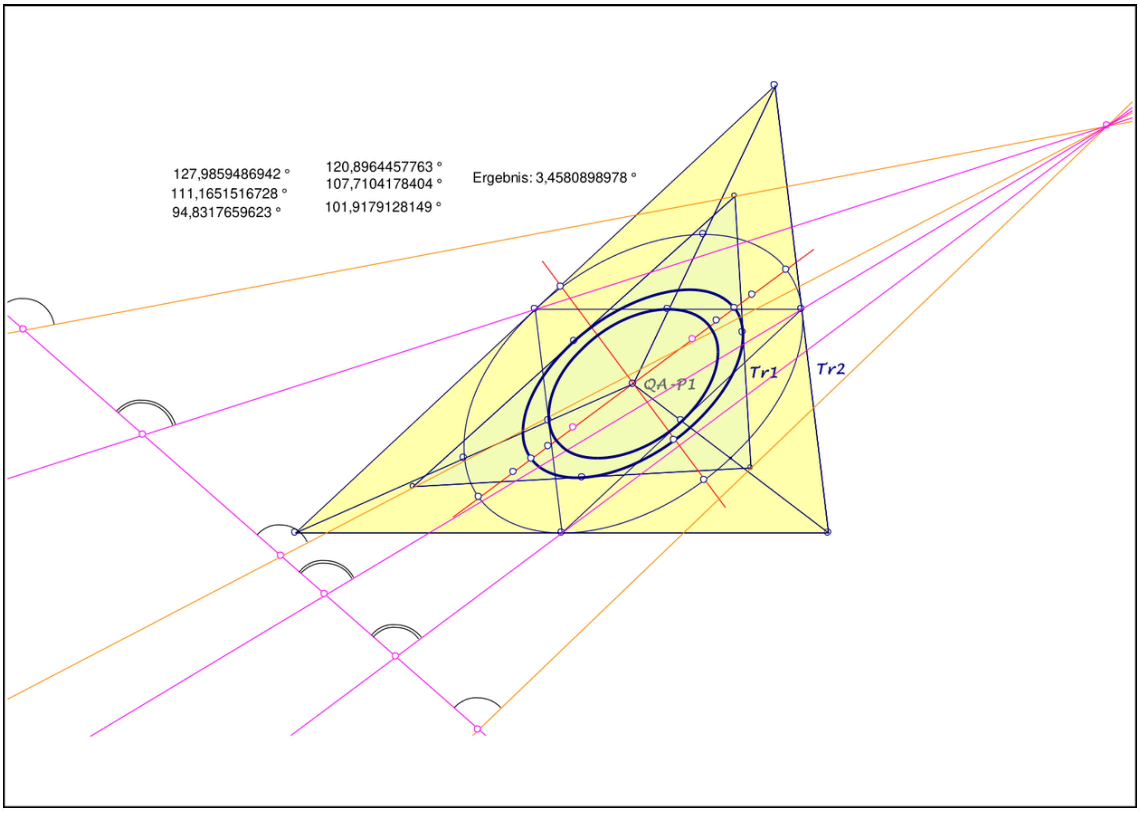
Message: #2588
Date: 28/8/2017 9:44:45
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Bernard,

wrt your message #2586:
1) "If the cubic degenerates in the 3 sidelines of the medial triangle of Tr2, the sextic degenerates in the 3 vertices of the same triangle and not in the vertices of Tr2."
I think, the cubic degenerates in the sidelines of the medial diagonal triangle (see #2540),
... which is the medial triangle of the medial triangle of Tr2.
2) "The property of Siebeck in II1 can be verified for Tr1 and midTr2 and the reason is that both triangles share the same foci of the Steinerinellipse."
I think, this doesn´t hold (see attached file).
Is there a misunderstanding on my side?

Best regards Eckart



2017-08-28.pdf

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Message: #2589
Date: 29/8/2017 5:16:51
From: amontes1949
Subject: Clockwise Counterclockwise Rotation Perspective Triangle Conic

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Dear Tsihong Lau

A particular case: DEF=FaFbFc=Feuerbach triangle

H_ABC = X(5620) (http://faculty.evansville.edu/ck6/encyclopedia/ETCPart4.html#X5620)

H_DEF = (2r+R)(rR(2r+5R)-2(2r+R)s^2)) X(12) -
(2R(2r^3+7rR(r+R)-(2r+R)s^2) X(5620)

H_DEF = ((b+c)(-a^14 (b+c)
-a^13 (b^2+c^2)
+a^12 (5 b^3+7 b^2 c+7 b c^2+5 c^3)
+2 a^11 (2 b^4+3 b^3 c+6 b^2 c^2+3 b c^3+2 c^4)
-a^10 (11 b^5+15 b^4 c+16 b^3 c^2+16 b^2 c^3+15 b c^4+11 c^5)
-a^9 (5 b^6+19 b^5 c+35 b^4 c^2+30 b^3 c^3+35 b^2 c^4+19 b c^5+5 c^6)
+a^8 (15 b^7+18 b^6 c+2 b^5 c^2-3 b^4 c^3-3 b^3 c^4+2 b^2 c^5+18 b c^6+15 c^7)
+a^7 b c (24 b^6+43 b^5 c+26 b^4 c^2+22 b^3 c^3+26 b^2 c^4+43 b c^5+24 c^6)
+a^6 (-15 b^9-19 b^8 c+14 b^7 c^2+26 b^6 c^3+13 b^5 c^4+13 b^4 c^5+26 b^3 c^6+14 b^2 c^7-19 b c^8-15 c^9)
+a^5 (5 b^10-18 b^9 c-40 b^8 c^2-5 b^7 c^3+12 b^6 c^4+2 b^5 c^5+12 b^4 c^6-5 b^3 c^7-40 b^2 c^8-18 b c^9+5 c^10)
+a^4 (11 b^11+15 b^10 c-19 b^9 c^2-38 b^8 c^3-10 b^7 c^4+21 b^6 c^5+21 b^5 c^6-10 b^4 c^7-38 b^3 c^8-19 b^2 c^9+15 b c^10+11 c^11)
-a^3 (b^2-c^2)^2 (4 b^8-10 b^7 c-21 b^6 c^2-7 b^5 c^3+5 b^4 c^4-7 b^3 c^5-21 b^2 c^6-10 b c^7+4 c^8)
-a^2 (b-c)^2 (b+c)^3 (5 b^8-8 b^6 c^2-7 b^5 c^3+16 b^4 c^4-7 b^3 c^5-8 b^2 c^6+5 c^8)
+a (b^2-c^2)^4 (b^6-3 b^5 c-4 b^4 c^2+4 b^3 c^3-4 b^2 c^4-3 b c^5+c^6)
+(b-c)^6 (b+c)^7 (b^2-b c+c^2)) : ... : ...),

with (6 - 9 - 13) - search numbers (0.181640280798688, -4.92623466068555, 6.96730065585960).
H_DEF lies on lines X(i)X(j) for these {i, j}: {12, 5620}, {101, 5949}, {442, 5953}.

Image and more information in:
http://amontes.webs.ull.es/otrashtm/HGT2017.htm#HG290817

```
Best regards,
Angel Montesdeoca
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Message: #2590
Date: 29/8/2017 6:03:23
From: tsihonglau
Subject: Clockwise Counterclockwise Rotation Perspective Triangle Conic

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```
Dear Montesdeoca,
Thanks for your reply!
Best regards,
Tsihong Lau
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Message: #2591
Date: 30/8/2017 1:01:09
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

```
Dear Bernard,
excuse, please forget message 2587!
Control calculations doesn´t confirm the observation with
acceptable accuracy!
Best regards Eckart
```

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Message: #2592
Date: 31/8/2017 12:10:37
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

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```
Dear Eckart,
My apologise !
In fact, the distances from the foci of the Steiner circum- and
inellipses are in a ratio 2/1 and not sqr(2) !
Wrt your point 1), what are finally the dual points of the sides
of medialDT (midmidTr2) ?
It's neither the vertices of Tr2 (your message 2585) nor the
vertices of midTr2 (your message 2588) ...
Best regards
Bernard
PS I begin to be no longer sure of anything !
If the vertices of 2 triangles verify Siebeck's property II1, do
they have necessary the same foci of their Steiner inellipses ?
The converse is certainly not true !
```

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Message: #2593
Date: 31/8/2017 10:16:09
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cass

---

Dear Eckart,
For a normal QA, the dual lines of the vertices of midDT are the lines through one vertice of DT and the middle of the opposite QL vertices on the opposite side of DT (if I'm not wrong, you name it QG-P1QG-P2).
The dual points of the sides of midDT are therefore the 3 vertices of the triangle formed by these 3 lines.
What becomes of these 3 points when the QA is formed by the vertices of Tr2 and it's centroïd QA-P1 ?
Best regards
Bernard
PS The points you named X in your 1rst message 2540 being the dual points of the segments joining 2 opposite sides of QA are aligned on the dual line of QA-P1 ; they are also the middles of the segments joining the QG-P1 and QG-P2 mentionned above. In the special QA, they become the infinity points of the sides of midDT, id est midmidTr2 and of Tr2 itself.

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Message: #2594
Date: 31/8/2017 10:57:00
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cas

---

Dear Eckart,
I realise I come precisely to the same points as yours in the 1rst message, id est the vertices of Tr2 !
Something is wrong somewhere ...
Best regards
Bernard

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Message: #2595
Date: 31/8/2017 12:21:03
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

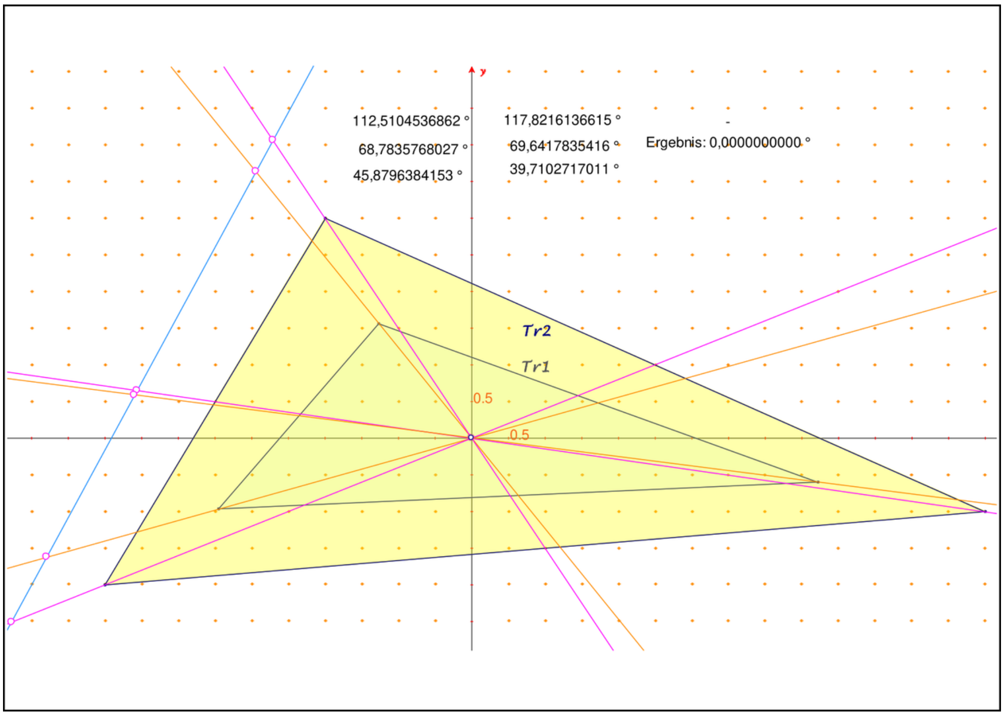
---

Dear Bernard,

wrt my point 1) in #2588:
 Several construction controls show, that the dual points of the
 sides of medialDT
... are the vertices of Tr2 (see my #2585).
In #2588 I don´t mention, that this are the vertices of midTr2.

What do you mean with "If the vertices of 2 triangles verify
Siebeck's property II1, ..." ?
Siebeck considers
... the lines from a point to the vertices of a triangle
... and the tangents from the point to a curve.
If the curve degenerates in the vertices of a 2nd triangle,
... we get the case above,
... and the tangents to the curve are the lines to the vertices
    of the 2nd triangle.
Then Siebeck´s property holds for QA-P1 (see attached file) but
not for any point in the plane.

Best regards Eckart



2017-08-31.pdf

Message: #2596
Date: 02/9/2017 11:17:34
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Bernard,

a discussion of the equation x(-x^2+y^2+z^2)+... cycl. = 0
for the cubic with p=1, q=1, r=1 (see #2551) leads to a
solution.

The cubic isn´t degenerated (see attached file):

Let P1P2P3P4 be a quadrangle
... with P4 = QA-P1
... and a remaining triangle P1P2P3 with centroid G=P4=QA-P1.
... Let Q1Q2Q3 be the QA-diagonal triangle, which is midP1P2P3
... and R1R2R3 the medial triangle of Q1Q2Q3,
    which is midmidP1P2P3.

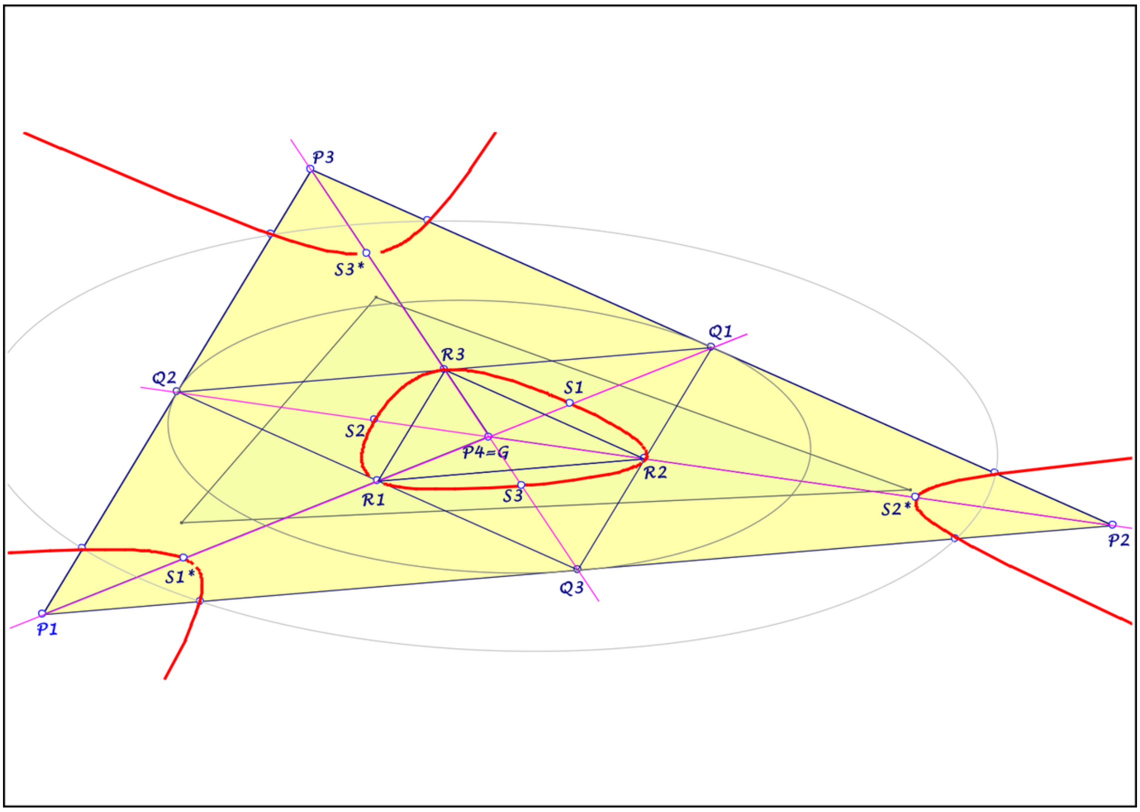
The cubic is a nonpivotal isocubic
... reference triangle: R1R2R3,
... isoconjugation: isotomic conjugate wrt R1R2R3,
... special points: Si, dividing Qi.G with ratio sqr3.
... Further points:
... ... points at infinity of PiPj,
... ... R1R2R3-isotomic conjugated of Si on PiQiRiG
... ... points dividing PiPj with ratio 3+2sqr3 and 3-2sqr3.

I tested the dual sextic of the cubic wrt Siebecks angle
condition for the foci,
... taking three concurrent dual lines of cubic points,
    which are tangents to the sextic.
I shall try, to give a drawing of the sextic.

Best regards Eckart

PS. The reason for my incorrect CABRI-observations was,
... that I took not the right special point
    for the nonpivotal cubic.

Your doubts were with good cause, excuse!



2017-09-02.pdf

Message: #2597
Date: 02/9/2017 2:05:17
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

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```
Dear Bernard,

attached a figure with cubic and sextic and an example of
Siebeck´s condition.
Perhaps helpful for further observations.

Best regards Eckart
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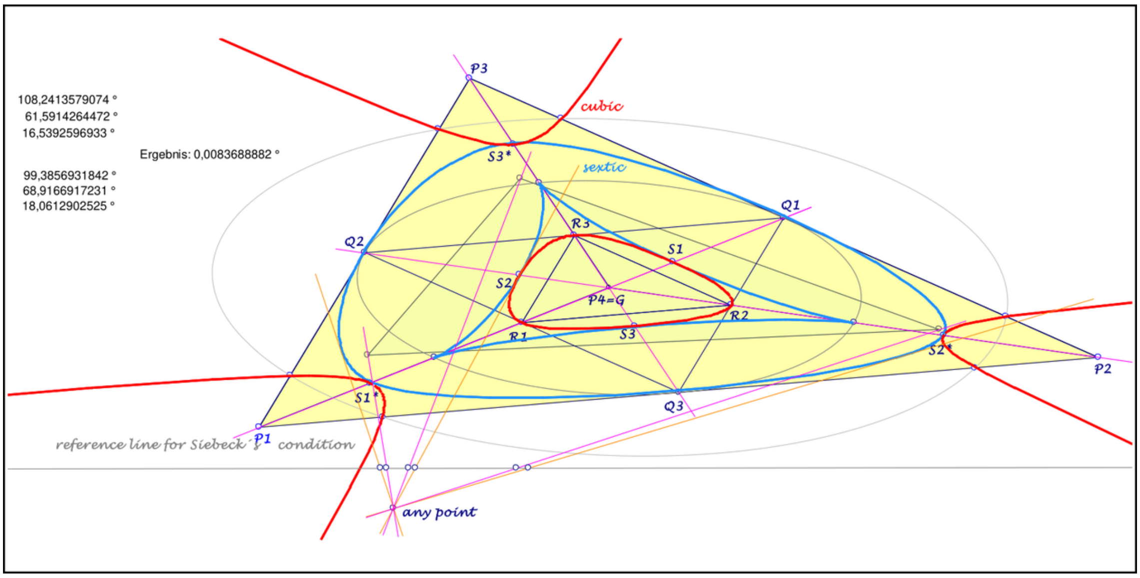

2017-09-2a.pdf

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Message: #2598
Date: 03/9/2017 12:19:52
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

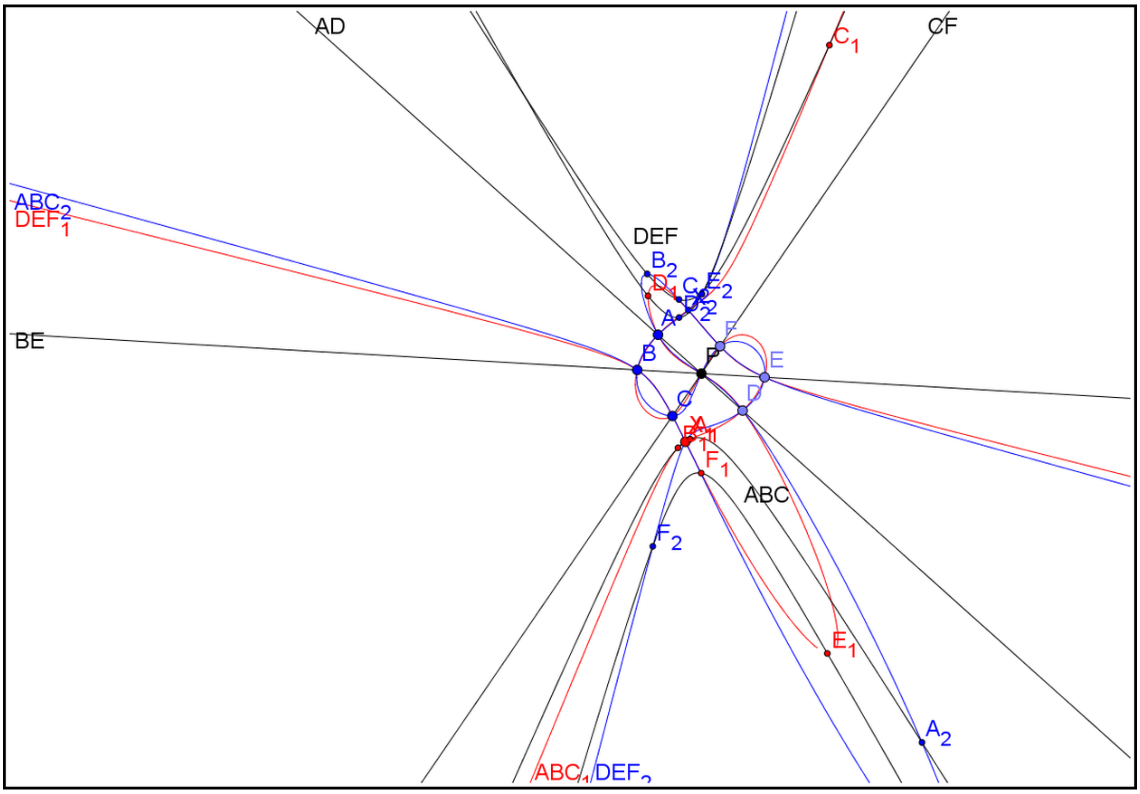
Dear Eckart,
I first answer to this message.
Sorry for being so slow, but this time I wanted to check carefully before answering !
I was convinced of course that the dual points of the sides of midmidTr2 were the vertices of Tr2.
The dual QL of the QA formed by the vertices of Tr2 and it's centroïd is formed by the 3 sides of midmidTr2 and the infinity line and the 6 vertices are the vertices of midmidTr2 and the infinity points of the sides of midmidTr2, which are also the infinity points of the sides of Tr2 itself.
But I was not convinced that the sextic was degenerated in the vertices of Tr2 and therefore not that the cubic was degenerated in the 3 sides of midmidTr2.
The reason is simple : it cannot be 2 different triangles with the vertices verifying Siebeck's property.
A point and the vertices of a triangle define a cubic stelloïd and Siebeck's property holds only for points on this cubic stelloïd and cannot hold for any point of the plane !
I understood after reading your message several times, specially your last remark that the property holds for QA-P1 wrt the vertices of Tr2.
Any point in the plane define a QA having the same foci of it's Siebeck's curve (if the point has complex z0, writing that the product of the z-za equals to product of the z0-za gives 3 other roots ...).
Then for each of the vertices of this QA Siebeck's property holds, the sum of the directions of the 3 sides passing through this vertice equals the sum of the directions of the lines from this vertice to the 3 foci.
The QA formed by the vertices of Tr2 and it's centroïd QA-P1 is exactly like the others : the property holds for QA-P1 wrt the vertices of Tr2, but also for one vertice of Tr2 wrt QA-P1 and the 2 other vertices of Tr2.
We don't need to identify exactly the sextic, we only need to know that it is tangent to the 6 sides of this QA ...
Best regards
Bernard

Message: #2599
Date: 03/9/2017 1:03:37
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear all,

This message was inspired by APG message #4034.
Message #2445 stated "Seven points(septangle) P1P2P3P4P5P6P7 determine an involution P<->P'"
We consider a subclass of it.
Two non-degenerate(=not conconic) triangles ABC and DEF are perspective at P.
Given a point X1, we define six points A1,B1,C1,D1,E1,F1 as follows:
A1=BC∩DX1 - D1=EF∩AX1
B1=CA∩EX1 - E1=FD∩BX1
C1=AB∩FX1 - F1=DE∩CX1
Then the two circumcubics of A1B1C1ABCDEF and D1E1F1ABCDEF pass through P and X1 and concur at the ninth point X2.
It is apparent that the transformation X1<->X2 is an involution.
Similarly, we define six points A2,B1,C2,D2,E2,F2 as follows:
A2=BC∩DX2 - D2=EF∩AX2
B2=CA∩EX2 - E2=FD∩BX2
C2=AB∩FX2 - F2=DE∩CX2
Then the two circumcubics of A2B2C2ABCDEF and D2E2F2ABCDEF pass through P and X2 and concur at the ninth point X1.
Surprisingly, the two sexangles A1B1C1A2B2C2 and D1E1F1D2E2F2 are conconic!
If X1 and ABCDEF are conconic, then A1,B1,C1,P are collinear and D1,E1,F1,P are collinear. X2 does not exist!
This case is the Generalization 2 of the Simson line listed in the Wikipedia.
https://en.wikipedia.org/wiki/Simson_line#Generalization_2
According to Desargues' theorem, perspective triangles = perspective trilaterals.
Trilaterals ABC and DEF induce the line involution similar to the point involution.
I wonder the relationship between the point and line involutions.
I hope someone apply both involutions to triangle/trilateral and quadrangle/quadrilateral geometries.

Best regards
Tsihong Lau



cayley_bacharach_ninth_perspective_triangle_involution.png

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Message: #2600
Date: 03/9/2017 4:59:41
From: tsihonglau
Subject: Clockwise Counterclockwise Rotation Perspective Triangle Conic

---

Dear Montesdeoca,

I reorganize the formulas.

Suppose barycentrics of ABC and DEF are
A(1:0:0) B(0:1:0) C(0:0:1) D(dx:dy:dz) E(ex:ey:ez) F(fx:fy:fz)
a b c are "squares" of sidelengths BC,CA,AB.
or
A(ax:ay:az) B(bx:by:bz) C(cx:cy:cz) D(1:0:0) E(0:1:0) F(0:0:1)
d e f are "squares" of sidelengths EF,FD,DE.

The following variables are defined as:
bcyz=by cz - bz cy ~ bczx=bz cx - bx cz ~ bcxy=bx cy - by cx
cayz=cy az - cz ay ~ cazx=cz ax - cx az ~ caxy=cx ay - cy ax
abyz=ay bz - az by ~ abzx=az bx - ax bz ~ abxy=ax by - ay bx
bcx=bczx - bcxy ~ bcy=bcxy - bcyz ~ bcz=bcyz - bczx
cax=cazx - caxy ~ cay=caxy - cayz ~ caz=cayz - cazx
abx=abzx - abxy ~ aby=abxy - abyz ~ abz=abyz - abzx
as=ax+ay+az=bs=bx+by+bz=cs=cx+cy+cz

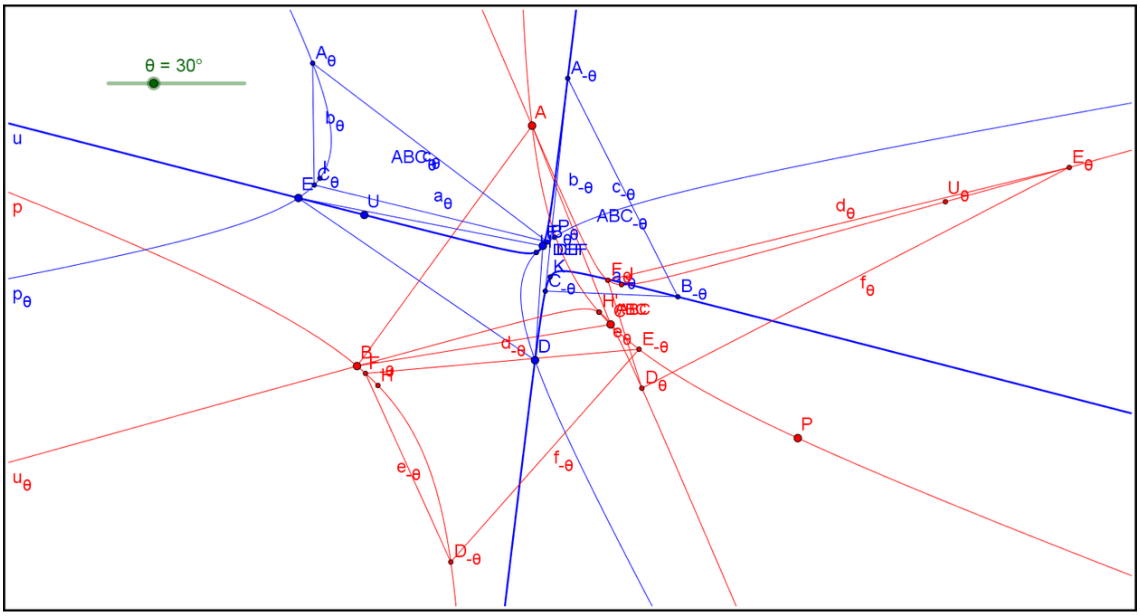
Similarly, we can define
efyz,efzx,efxy,efx,efy,efz
fdyz,fdzx,fdxy,fdx,fdy,fdz
deyz,dezx,dexy,dex,dey,dez
ds,es,fs

The barycentric equation of p with respect to ABC is
(S_{ABC} efx cos(θ¸) + (S_B efy - S_C efz) sin(θ¸)) y z +
(S_{ABC} fdy cos(θ¸) + (S_C fdz - S_A fdx) sin(Î¸)) z x +
(S_{ABC} dez cos(θ¸) + (S_A dex - S_B dey) sin(θ¸)) x y = 0
The barycentric equation of u_Î¸ with respect to ABC is
(S_{ABC} efx cos(θ¸) - (S_B efy - S_C efz) sin(θ¸)) y z +
(S_{ABC} fdy cos(θ¸) - (S_C fdz - S_A fdx) sin(Î¸)) z x +
(S_{ABC} dez cos(θ¸) - (S_A dex - S_B dey) sin(θ¸)) x y = 0
where S_A,S_B,S_C are Conway's notations, S_{ABC}(contrast with S_{DEF}) is Conway's S.

the barycentrics of H_ABC with respect to ABC is
1 / (a fdy dez + b fdx dez + c fdy dex):1 / (a dez efy + b dez efx + c dey efx):1 / (a efz fdy + b efx fdz + c efx fdy)
or
1 / (d cay abz + e cax abz + f cay abx):1 / (d abz bcy + e abz bcx + f aby bcx):1 / (d bcz cay + e bcx caz + f bcx cay)

The formulas of d,e,f in a,b,c are:
d=-(a efy efz+b efz efx+c efx efy)/(es fs)^2
e=-(a fdy efz+b fdz fdx+c fdx fdy)/(fs ds)^2
f=-(a dey efz+b dez dex+c dex dey)/(ds es)^2

Best regards,
Tsihong Lau



clockwise_counterclockwise_rotation_perspective_triangle_conic-2600.png

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Message: #2601
Date: 04/9/2017 11:43:23
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Dear Eckart,
I'm really glad that you finally succeeded in finding the real cubic and sextic in this case with a QA formed by a tiangle and it's centroïd. That's fine work, congratulations !
I'm particulary glad that these curves are not a degenerated and that this QA is normal from the point of vue of Siebeck's properties.
I tried to have a look in Bernard Gibert : if I'm not wrong, the cubic is a Tücker cubic T(P) with pole and root in G or nK(G,G,P), where P is a point defining the curve (here one Si for example).
Now, I suppose the game is over !
I have one last interrogation and one last final remark.
Did you identify, in the general case or in the special case, the point of concurrence of the 3 tangents to the sextic through the 3 foci ? What is it's dual line, carrying the 3 dual points of the tangents and what are the dual lines of the foci ?
As the stelloïds generalise the rectangular hyperbola (conic stelloïd), the 1rst polar curves of the points of the infinity lines wrt any stelloïd through n points form a pencil of stelloïds through the n-1 foci of the Siebeck's curves (as well known, the 1rst polar curve of a point wrt a curve carries all contact points of the tangents from the point to the curve).
Let say it more simply :
for any rectangular hyperbola through 2 points, 2 parallel tangents touch the RH in 2 points symmetric wrt the center (middle of the bipoint)
for any cubic stelloïd through 3 points, parallel tangents touch the SC in points on RHs through the 2 foci of the Steiner inellipse of the triangle formed by the 3 points
for any quartic stelloïd through 4 points, parallel tangents touch the QS in points on SCs through the 3 foci of the Siebeck's sextic of the QA formed by the 4 points
It's easy to draw a specific RH through 2 points or a specific SC through 3 points (Mac Kay or Kjp for example) and to have 2 sets of parallel tangents ...
I don't know how to draw a QS through 4 points. Any idea ?
Best regards
Bernard

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Message: #2602
Date: 05/9/2017 1:22:09
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

```
Dear Bernard,

I am not familar wrt Tucker cubics,
... but there seems relations to the cubic in #2596:
The areas of the cevian triangles of cubic points are equal
... wrt triangle R1R2R3
... as well as wrt S1S2S3
... as well as wrt S1*S2*S3*.
The areas of the cevian triangles of cubic points wrt R1R2R3
... are twice the area of the cevian triangle
    of QA-P1 wrt R1R2R3.

Best regards Eckart
```

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Message: #2603
Date: 06/9/2017 10:59:30
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

```
Dear Eckart,
I'm not either familiar with anything !
I discovered the Tücker cubic a few days ago ...
But I'm curious and when you send me a message, specially with a
beautiful picture, I always try to find something referring to
it.
You may find an article by Bernard Gibert on his website under
the file Downloads : Tücker cubics and Bicentric Cubics ...
This was for me only an interesting detail explaining or
completing your own description.
Much more important is for me your answer to my questions or
messages starting with 2569 and followings until 2601.
Best regards
Bernard
```

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Message: #2604
Date: 07/9/2017 9:10:07
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

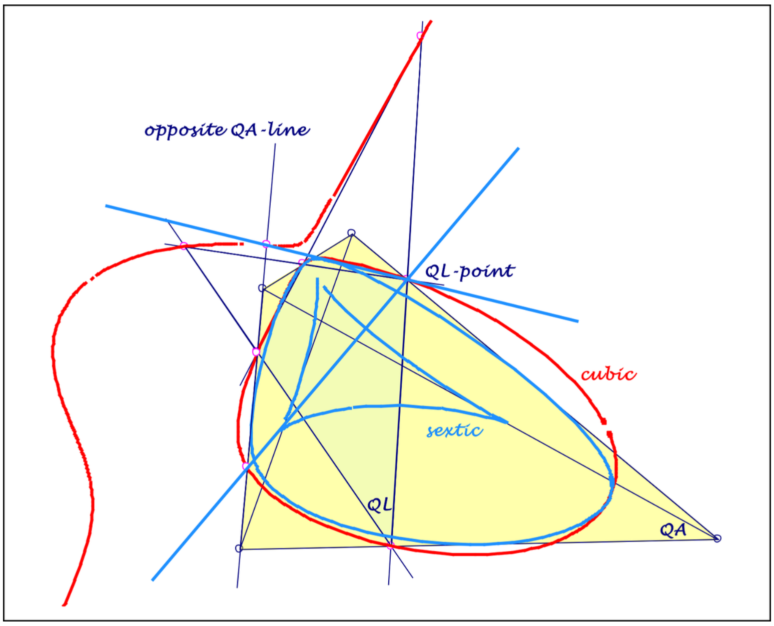
---

```
Dear Bernard,

the equation for DT-coordinates (x,y,z) of points on Siebeck´s
cubic for a QA is
... q^2 r^2 x^2 (-x+y+z) + ... cycl. = 0 (see #2551);
... the equation for DT-coefficients (k,l,m)
    of tangents at Siebeck´s sextic for a QA is
... k^3 p^4 - k l (k+l) p^2 q^2 + ... cycl. = 0.
So we can easy calculate
... the three intersections of a QA-line and the cubic,
... ... one intersection is a point of the dual QL,
... and the three tangents from a QL-point at the sextic,
... ... one tangent is a line of the dual QA.
In this way we get 12 new tangents at the sextic
    (see attached file),
... which are the two lines from each QL-point on a QA-line
... to the intersections of the cubic
    and the opposite QA-line unequal its QL-point.

Best regards Eckart

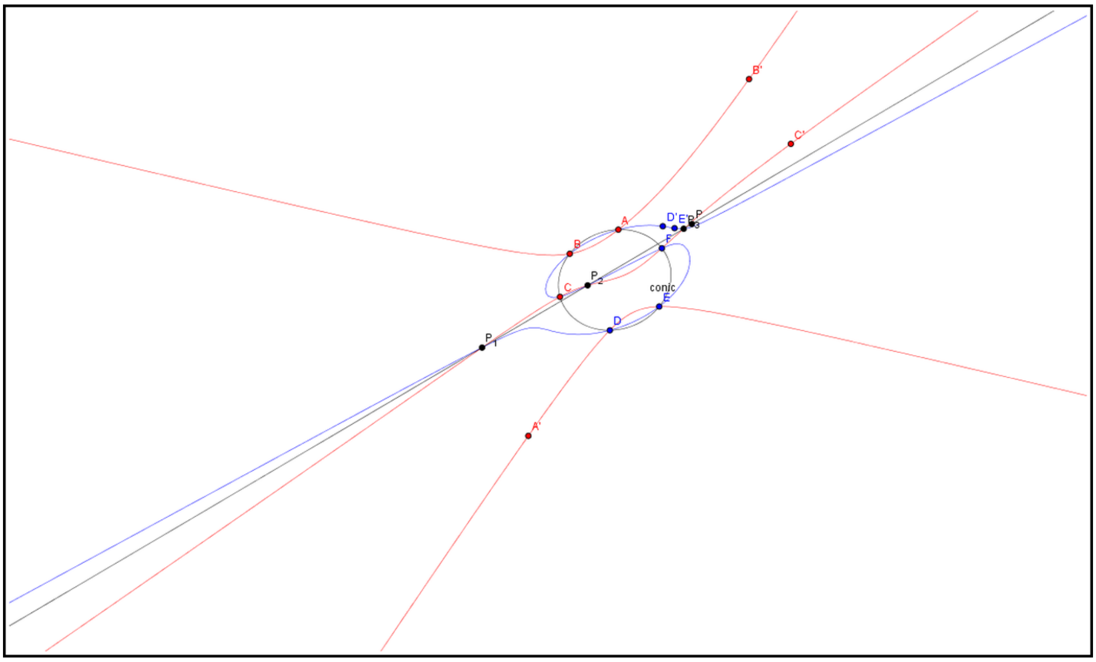
PS: Wrt your last message:
    If I had answers to your questions in #2569 to #2601, I
    would have informed you.
```



2017-09-07.pdf

Message: #2605
Date: 09/9/2017 5:06:55
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear all,
This message was a following message of message #2599.
Given two conconic triangles ABC and DEF and a point P, we
define six points A',B',C',D',E',F' as follows:
   A'=BC∩DP - D'=EF∩AP
   B'=CA∩EP - E'=FD∩BP
   C'=AB∩FP - F'=DE∩CP
Then the two circumcubics of A'B'C'ABCDEF and D'E'F'ABCDEF pass
through three other points P1,P2,P3.
Since six points ABCDEF are conconic, P1,P2,P3 lie on a line p.
Surprisingly, P lies on p, too!
In other words, a point P determines a unique line p passing
through it.
Suppose abc and def are the tangent lines of ABC and DEF to the
conic.
The two trilaterals abc and def are conconic.
Using the dual of the above construction, the line p determines
a unique point P' lying on it.
What is the relationship of P and P'?
I hope someone apply both point-line transformations to
triangle/trilateral and quadrangle/quadrilateral geometries.
Best regards
Tsihong Lau



cayley_bacharach_ninth_conconic_triangle_point_line.png

Message: #2606
Date: 12/9/2017 10:40:31
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Bernard,

there are a lot of connections between Siebeck´s cubic and the sextic for n = 4.
Here only some remarks, leading to a very curious property wrt the foci-triangle of the sextic.

Let QA be a nonconvex starting quadrangle
... with diagonal triangle DT and its midDT,
... isotomic conjugate wrt midDT,
... El Steiner inellipse of DT
... and Siebeck´s cubic Cu and sextic Se
(see previous messages).

(1) El intersects Cu in the vertices of midDT
... and 3 further points with isotomic conjugates
in the infinity points of Cu.

(2) El intersects QA-Cu3 in the vertices of mid-DT
and 3 further points
... with El-tangents parallel to the Cu-asymptotes.

(3) QA-Cu3 intersects Cu in the vertices of mid-DT
... and 6 contact points of Cu and Se with common tangents,
... ... pairwise parallel to the asymptotes,
... ... contact points pairwise collinear with QA-P16
... ... and isotomic conjugates wrt midDT,
... ... with connection lines tangent to Se,
... ... bearing the further intersections of El and Cu.

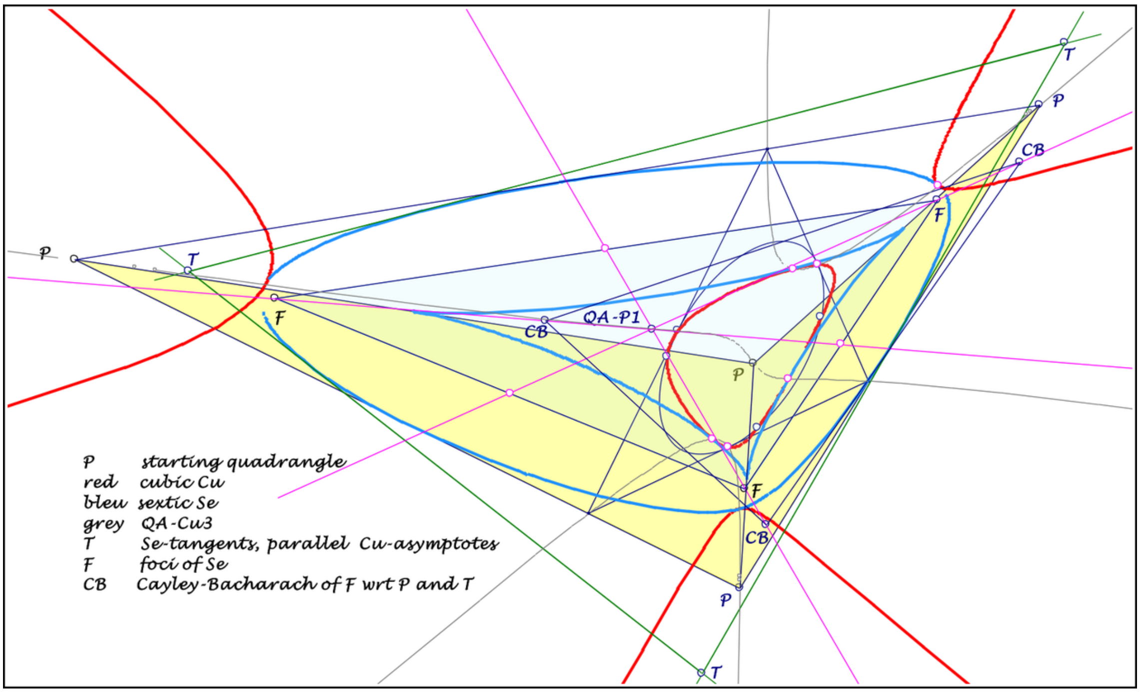
(4) The dual lines of the further intersections of El and Cu
... give 3 parallels to the asymptotes,
... ... tangent to Se,
... ... intersecting on the lines through QL-P16 (see (3)),
... ... leading to a triangle *Tr*
with centroid QA-P1 (see attached file).

(5) Consider a Cayley-Bacharach transformation
... wrt the 4 QA-points and the vertices of Tr :
... The triangle of the foci of Se
... ... and the triangle of its CB-images
... ... are perspective wrt QA-P1.

I tested this property for several nonconvex QA with CABRI.

In the convex case the triangle Tr isn´t real.
I hope, someone can confirm this curious property,
for the CB-transformation was unexpected.

Best regards Eckart



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Message: #2607
Date: 13/9/2017 4:27:03
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

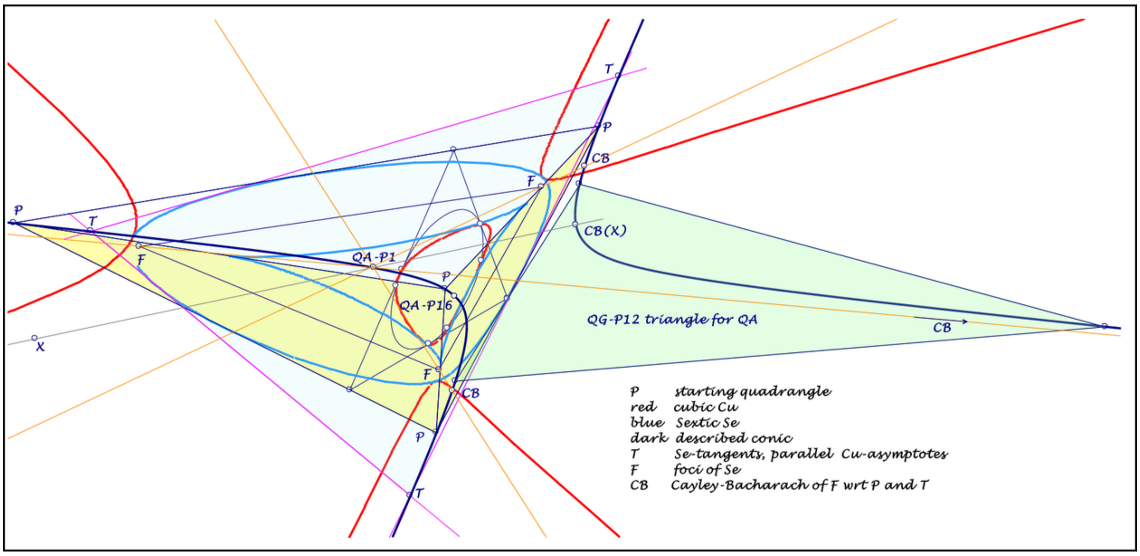
---

```
Dear Bernard,

the last property in my previous message isn´t special for the
foci-triangle:
Any point and its CB-image in the sence of #2606 lie collinear
with QA-P1.
There is a QA-circumconic, not in EQF (see attached file):
... through QA-P1 and QA-P16,
... the vertices of triangle Tr (see #2606),
... and the QA-versions of QG-P12.
Wrt the QG-P12 triangle of QA
... this conic is the isotomic image of QA-P16.QA-P20.
The dual of this conic is a parabola
... tangent to the midDT.
The CB-images of the Se-foci (see #2606) lie on this conic.
It seems, that the CB-image of any point X
... is the 2nd intersection of X.QA-P1 and the described conic.
I hope, my observations are correct, but even CABRI has its
limits in accuracy.

Best regards Eckart

PS. There is a typo in my previous message:
Please replace under (4) QL-P16 by QA-P16,
furthermore one CB-point isn´t correct, excuse.
```



2017-09-13.pdf

Message: #2608
Date: 14/9/2017 10:47:07
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Dear Eckart,
1) The CB transformation wrt 7 points is an involution, but if the 7 points are coconic, which is here the case, the cubics through the 9 points are degenerated and split between the conic and a line.
The CB point of CB(X) is any point on the line QA-P1CB(X) ; for QA-P1, it's any point on the tangent in QA-P1 to the conic.
I can't explain why this line is always through QA-P1, I just remark that QA-P1 is the centroïd of the 7 points.
2) this conic is the socalled 6th conic, it bears also the famous S-points, vertices of QL-Tr2, and the vertices of the anticevian triangle of any of it's points wrt DT (in particular, the anticevian triangle of QA-P16 or QL-P13 is the DDT triangle) the dual of the conic is a parabola inscribed in midDT and in QL-Tr2 (this triangle is self dual wrt QA/QL), it's the QL-inscribed parabola.
Maybe you will in the end find a link between the triangle of the foci and the QL-Tr2 triangle ...
Best regards
Bernard

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Message: #2609
Date: 15/9/2017 11:50:32
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

```
Dear Bernard,

thanks for clarification, you have a better overview!

1) You are right, it doesn´t hold, that CB(X) for seven coconic
points is a clearly point on X.QA-P1.
I was a victim of the accuracy of CABRI.
If the complicated constructions give one point T, not exact on
the conic,
... the CB-image CB(X) is collinear with X and T.

2) I felt a bit sheepish about it, that I doesn´t recognize the
conic as your 6th QL-conic.
It seems, that the CB-transformation doesn´t lead to the
foci-triangle.
Please forget my last two messages!

Best regards Eckart
```

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Message: #2610
Date: 22/9/2017 12:46:31
From: eckart_schmidt@t-online.de
Subject: New QL-circumscribed Cubics

```
Dear all,

There is only the cubic QL-Cu1 in EQF, bearing the six
QL-points.
Attached further nonpivotal isocubics with this property,
.... using the Cayley-Bacharach ninth point.
Perhaps of interest.

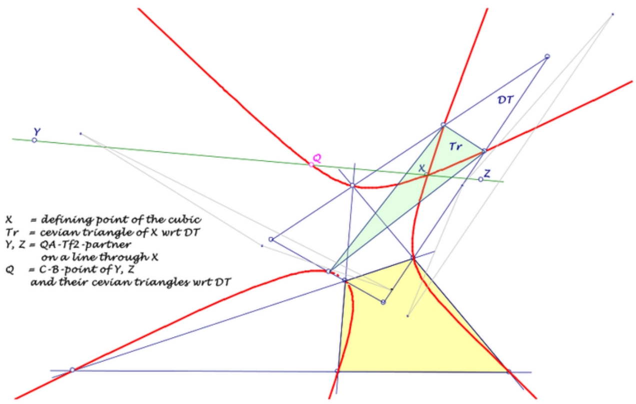
Best regards Eckart
```

EQF-Note 2017-09-22

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

New *QL*-circumscribed Cubics

There is only the cubic QL-Cu1 in EQF, bearing the six QL-points. Here further nonpivotal isocubics with this property are researched, using the Cayley-Bacharach ninth point.

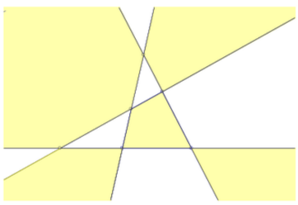


Definition of the cubic:

Consider a dual *QA/QL*-configuration with common diagonal triangle *DT*
… with a defining point *X* for the cubic,
… lines *L* through *X* with *QA-Tf2*-partners *Y* and *Z*
… and the *DT*-cevian triangles of *Y* and *Z*.
… The cubic is the locus of the Cayley-Bacharach ninth points *Q* of *X*, *Y* and the vertices of their cevian triangles.

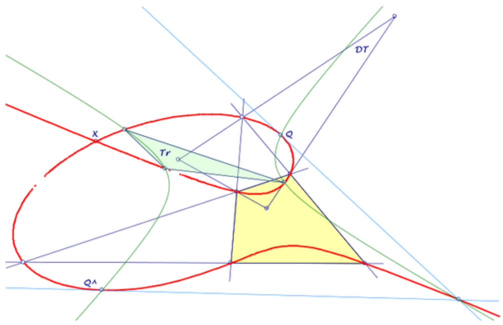
Properties:

- **The cubic is *QL*-circumscribed through the six *QL*-points.**
- **The cubic has a double point in *X*, if *X* is a point in the marked *QL*-regions:**



2017-09-22.pdf

- **The cubic is circumscribed the cevian triangle *Tr* of *X* wrt *DT*.**
- **The cubic is invariant wrt the *Tr*-isoconjugation ^ with fixed point *X*.**
- **The *Tr*-isoconjugation ^**
  … has further fixed points in the *DT*-vertices
  … and swaps opposite *QL*-points.
- **A special point Q_0 on the cubic is the *CB*-point of *X*, *QA-Tf2(X)* and the vertices of their *DT*-cevian triangles.**
- **Isoconjugated points *Q* and *Q*^ on the cubic and the six *QL*-points have their *CB*-point on the cubic**
  … in the intersection of their tangents,
  … which is the 6th intersection of the cubic and a *Tr*-circumscribed conic through *Q* and *Q*^.



- **The cubic intersects the *Tr*-sidelines in the common tangentials of opposite *QL*-points.**
- **The cubic is a nonpivotal isocubic**
  … with the *DT*-cevian triangle of the defining point *X* as reference triangle
  … and the isoconjugation ^ with fixed point *X*.

The **root** of this nonpivotal isocubic doesn´t lie on the cubic, it can be constructed with the circle Γ as described by Bernard Gibert (*EQF*: *Ref [17b], 1.5.6*).
The circle Γ bears the defining point *X*
… and is centered on *QL-L2* in the radical center
… … of the circle with diameter $Q_0.Q_0$^
… … and circles with diametral points in opposite *QL*-points.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-09-22.pdf

Message: #2611
Date: 25/9/2017 8:36:08
From: eckart_schmidt@t-online.de
Subject: New QL-circumscribed Cubics

Dear all,

in my last message new QL-circumscribed cubics were described
using the Cayley-Bacharach ninth point.
Here alternative a simpler construction is offered and further
properties are mentioned,
if the defining point lies on QL-Cu1.

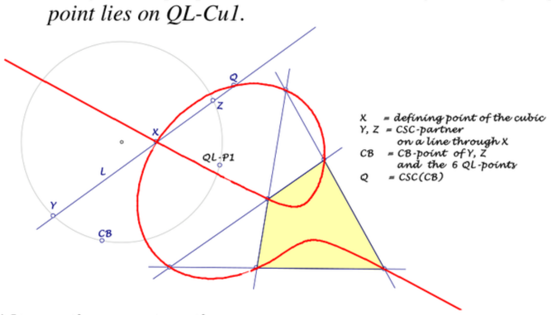
Best regards Eckart

EQF-Note 2017-09-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

New QL-circumscribed Cubics II

In EQF-Note 2017-09-22 new QL-circumscribed cubics were described using the Cayley-Bacharach ninth point. Here alternative a simpler construction is offered and further properties are mentioned, if the defining point lies on QL-Cu1.



Alternative construction

There is an interesting property in dual geometry. The following two constructions give the same point:

(1) For a line *L*
… the ***QA-Tf2***-partner on *L*
… and their *DT*-cevian triangles
… give a Cayley-Bacharach ninth point *Q* on *L*.

(2) For a line *L*
… the ***QL-Tf1***-partner on *L*
… and the six *QL*-points
… give a Cayley-Bacharach ninth point *CB*,
… with *QL-Tf1(CB)* = *Q* on *L*.

The construction of the *CB*-point in (2) is comparatively simple.

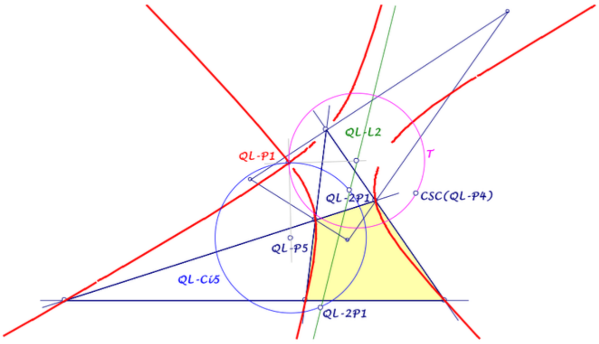
The points *Q* for a line pencil wrt a defining point *X* give a *QL*-circumscribed cubic *QL-Cux*, which can be described as nonpivotal isocubic (see *QFG*-message 2610):
… reference triangle ***Tr*** is the cevian triangle of *X* wrt the *QL*-diagonal triangle,
… isoconjugation **^** with fixed point *X*
… and Bernard Gibert´s circle *Γ* (*EQF*: *Ref [17b], 1.5.6*).
Then *QL-Cux* is the locus of points *M* such that *M* and *M^* are conjugated wrt the circle *Γ*.

2017-09-25.pdf

QL-Cux* with *X* = *QL-P1:

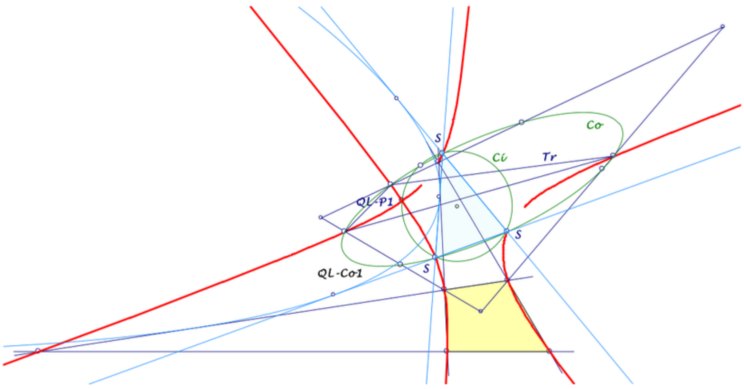
A construction of this cubic is only possible with method (1).

- **The defining point *QL-P1* is double point of the cubic with orthogonal tangents, which are the Steiner axes.**



- **The circle *Γ* for *X* = *QL-P1***
  … bears *QL-P1* and *CSC(QL-P4)*,
  … is centered on *QL-L2*,
  … tangent to *QL-P1.QL-P5*,
  … orthogonal to *QL-Ci5*,
  … with inverse Plücker points *QL-2P1*.

Two helpful curves:
… conic ***Co*** circumscribed the cevian triangle *Tr* of *QL-P1* through the midpoints of the diagonal triangle (^isoconjugate of the line at infinity),
… circle ***Ci*** through *QL-P1*, which is the *CSC*-image of a *QL-L1*-parallel through *QL-P21*.
One of the four intersections of *Co* and *Ci* lies on the circumcircle of *Tr*.

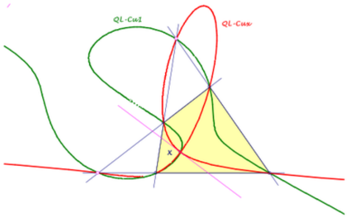


- **Three intersections *S* of *Co* and *Ci* lie on the cubic *QL-Cux*:**
  … their ^isoconjugates are the points at infinity of the asymptotes,
  … the sidelines of their triangle are parallels to the asymptotes
  … and tangent to the inscribed parabola *QL-Co1*.

Cubics *QL-Cux* with $X \neq$ *QL-P1* on *QL-Cu1*:

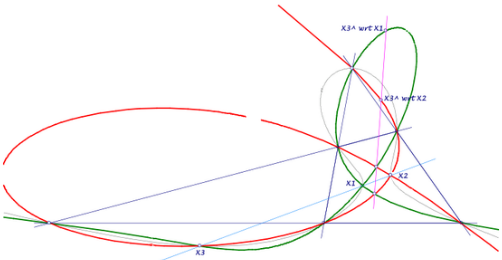
- **For *X* on *QL-Cu1*, the cubic *QL-Cux* bears always the defining point *X*.**

- **The defining point *X* as double point has orthogonal *QL-Cux*-tangents,**
  … which are the angle bisectors at *X* wrt two *CSC*-partner on *QL-Cu1*.



For *X* in an intersection of *QL-Cu1* and *QL-L1* these tangents are the axes of *QL*-inscribed conics. In general: For *X* on *QL-Cu1* these orthogonal tangents are the degenerated polar conics of *QL-Cu2*.

- **The cubics *QL-Cux* for *CSC*-partner X_1 and X_2 on *QL-Cu1* intersect**
  … in the 3rd intersection X_3 of X_1X_2 and *QL-Cu1*
  … and two other points
  … collinear with the ^isoconjugated of X_3 wrt X_1, X_2.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-09-25.pdf

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Message: #2612
Date: 26/9/2017 12:49:26
From: eckart_schmidt@t-online.de
Subject: New QL-circumscribed Conics

Dear all,

in message # 2611 I described a cubic QL-Cux for a defining
point X = QL-P1.
This cubic is the locus for intersections of lines through QL-P1
and their QL-Tf2-image
(see # 467).

This cubic is already researched in # 466
... as locus of contact points for tangents from QL-P1 at
QL-inscribed conics.

Best regards Eckart

PS: Attention, there are mistakes in #466 wrt the equation of
conic Co and its contact point to QL-L2.

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Message: #2613
Date: 28/9/2017 11:02:11
From: bernard.keizer
Subject: New QL-circumscribed Cubics

Dear Eckart,
I had answered your messages, but my message is lost and I haven't the courage of rewrite it completely !
1) The vertices of the cevian triangles of Y and Z are coconic and I could use Cotterill's construction.
X in QA-P10 gives the Cu dual of Se and X in QA-P12 gives the Van Rees curve QL-Cu1
The root is the trilinear pole of the line through the tangentials of the QL vertices
2) Very interesting property of the dual geometry, why is this construction more simple ?
X in QL-P1 : Co is through QA-P1 and QA-P10, Ci is the Dimidium circle and the points S are the S-points ...
3) The duals of the QLcircumscribed cubics are QA inscribed curves of 3rd class
Siebeck's property 1 gives a generalisation for the QA of the isogonality for the triangle
The mean direction of the 3 lines joining a QA vertice to the 3 foci of any of the inscribed curves is the same, id est the mean direction of the 3 QA sides in this vertice.
Best regards
Bernard

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Message: #2614
Date: 30/9/2017 11:14:24
From: eckart_schmidt@t-online.de
Subject: New QL-circumscribed Cubics

Dear Bernard,

thanks for your answer!

You are right, the alternative constructions are of the same level of dificulty.
I didn´t respect, that the cevian triangles of Y and Z are coconic.

Sorry, I can´t reproduce (see attached file) ...
"X in QA-P10 gives the Cu dual of Se and X in QA-P12 gives the Van Rees
curve QL-Cu1".

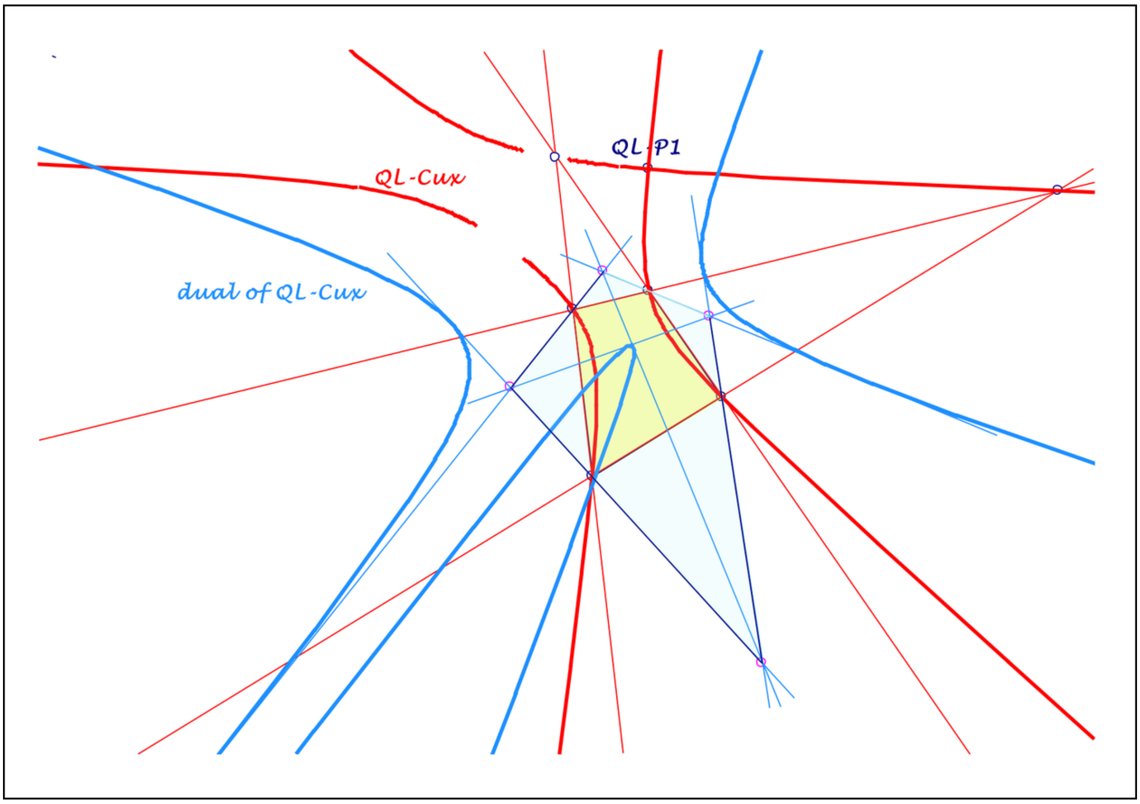
The root can also be described wrt the DT-cevian triangle of X ... as tripol of the line through the intersections of the cubic and the sides .

Further I can´t confirm ...
"X in QL-P1 : Co is through QA-P1 and QA-P10, Ci is the Dimidium circle and the points S are the S-points ..."

Wrt 3) I can only construct an example (see attached file), but I have no ideas wrt the foci.
But what do you mean with ...
" Siebeck's property 1 gives a generalisation for the QA of the isogonality for the triangle."
Can you please give some explanations?

Please excuse, that my message contains only questions and doubts.

Best regards Eckart



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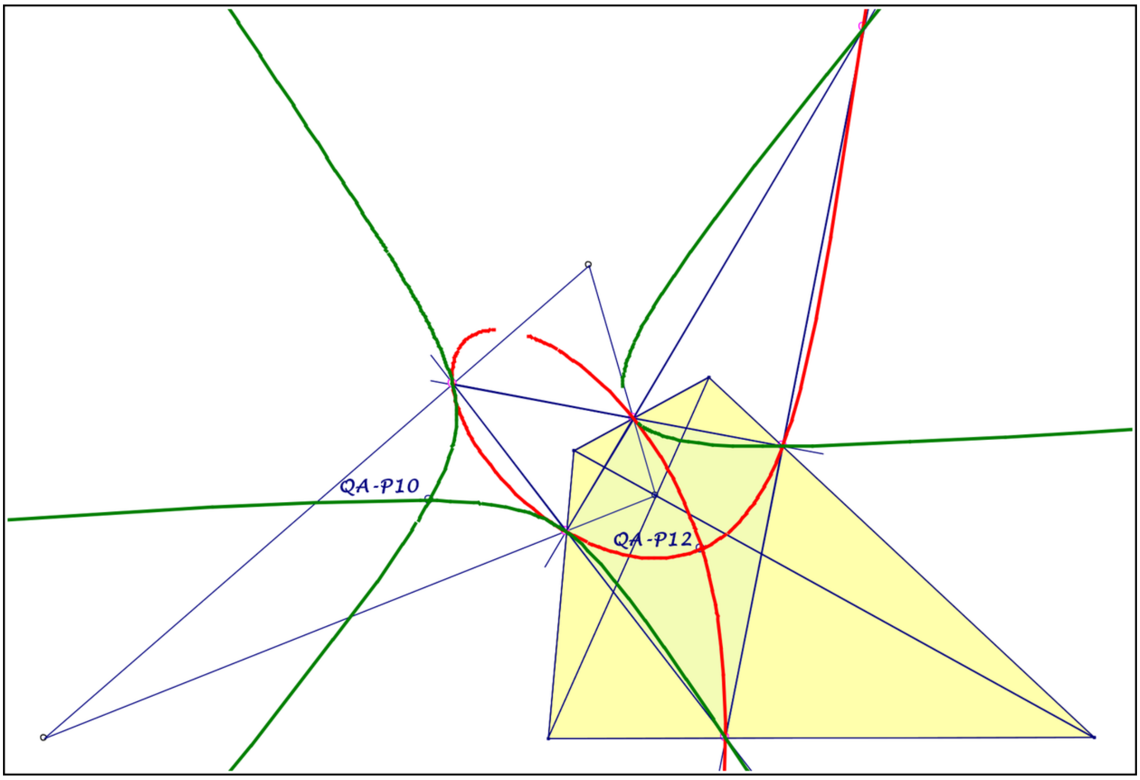

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Message: #2615
Date: 03/10/2017 9:39:01
From: eckart_schmidt@t-online.de
Subject: New QG-point

Dear Chris,

there is an interesting new QG-point X on QG-L2 (see attached file):

Consider a quadrigon
... as QL there are four lines Li,
... as QA there is a dual QL with four lines Ki.
... There are 8 further intersections of Li and Ki
... ... (without points of the dual QL),
... in pairs collinear with QG-P1 and QG-2P2.
... These 8 points lie on a conic,
... centered in the new point X on QG-L2,
... ... (collinear with QG-P1, QG-P12, QG-P13, QA-P16, QL-P13 and T = QG-L1^QG-L2),
... X is 4th harmonic point of QA-P16 wrt QG-P13 and T,
... X is 4th harmonic point of QG-P12 wrt QG-P13 and QA-P16
... with DT-coordinates (-5p^2 : q^2 : -5r^2).

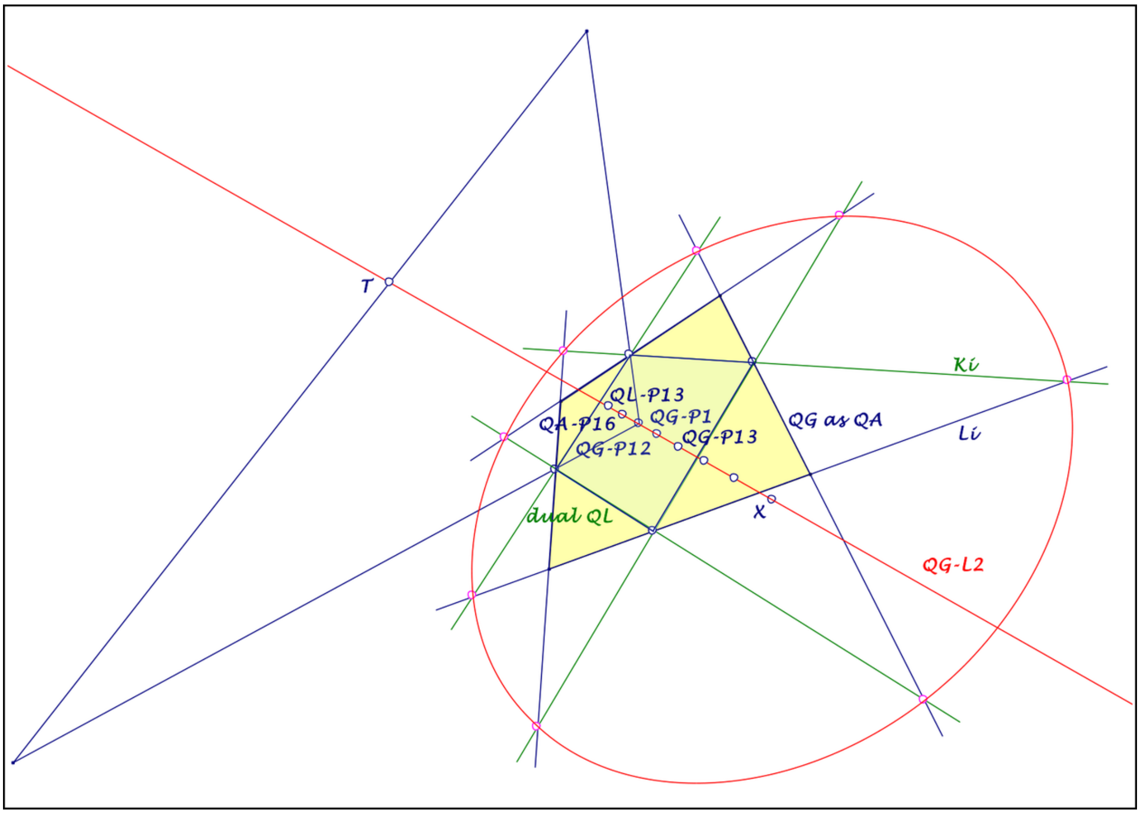
X is a point of the "consecutive perspective row", mentioned in EQF wrt QG-L2:
... X1 = QL-p13, X2 = QA-P16,
... Xn+1 = 4th harmonic point of Xn-1 wrt T and Xn:
... X3 = QG-P1, X4 = QG-P12, X5 = QG-P13, ... X8 = X.

For a QA the X-triangle and QA-DT are perspective wrt QA-P16,
for a QL the X-triangle and QL-DT are perspective wrt QL-P13.

Wrt the conic through the 8 points:
... DT-equation: q^2 r^2 x^2 - 5 p^2 r^2 y^2 + p^2 q^2 z^2 = 0,
... the polar of T is a parallel to QG-L1 through QG-P1,
... the conic contains with a point also the vertices of the anticevian triangle
... ... wrt the QA- or QL-diagonal triangle,
... QA-DT and QL-DT of the quadrigon are selfpolar
    wrt the conic.

Best regards Eckart

PS: Six years ago you asked for a review of EQF,
followed by years of interesting cooperation,
but times have changed ...



2017-10-03.pdf

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Message: #2616
Date: 03/10/2017 11:01:03
From: bernard.keizer
Subject: New QL-circumscribed Cubics

Dear Eckart,
Many thanks for your patience !
This time, it's my turn to feel a little bit sheepish ...
1) For a point X and it's DT cevian triangle Tr, there is only
but one pivotal isocubic wrt DT and fixed points of the
isoconjugation the QA vertices with pivot X, which is also a
pivotal isocubic wrt Tr and fixed points of the isoconjugation X
and the DT vertices with pivot QA-Tf2(X) = X*.
But there are an infinity of non pivotal isocubics wrt Tr with
the same isoconjugation and different roots.
In fact, for X = QA-P10, Cu and your cubic are 2 different non
pivotal isotomic cubics wrt midDT
and, for X = QA-P12, QL-P1 and your cubic are 2 different non
pivotal isogonal cubics wrt orthicDT.
Do you know, by any chance, a construction of Cu or QL-P1 using
the CB 9th point ?
2) It seems your construction of the points S and the 4th point
are general and not reserved for QL-P1.
Let's put it this way :
point X, cevian triangle Tr, non pivotal isocubic wrt Tr Cu
Co conic through Tr and midDT vertices
Cx circumcircle of Tr, Ce circumcircle od midDT points S 3
intersections other than Tr vertices between Cu and Co
Ci circumcircle of triangle of the points S
4th point = intersection of Co, Cx, Ce and Ci
Here 2 perhaps interesting observations :
a) if we consider the 2 cubics with X in QA-P10 and QA-P12 (see
your figur), the conic Co and the circles Cx, Ce and Ci coïncide
and the 4th point doesn't exist ; the points S exist for both
cubics and it seems also that the 2 cubics are tangent in 3 of
the 6 QL vertices ...
b) if we do the reverse construction with Co the conic through
midDT, QA-P1 and QA-P10, we find the vertices of Tr as
intersections with DT sides other than the middles and a point X
having this triangle Tr as cevian triangle ; curiously, X and
the vertices of Tr lie on the 9 points conic QA-Co1 !
This time, the points S are the S-points and Ci is the Dimidium
circle and the 4th point is the intersection of the Dimidium
circle and the Eulercircle of DT, also on Co and Cx and,
curiously again, on the 9 points conic ...
Best regards Bernard

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Message: #2617
Date: 03/10/2017 11:37:58
From: bernard.keizer
Subject: New QL-circumscribed Cubics

Dear Eckart,
I hope I didn't make this time any basic mistake !
As I fear now that too long messages are getting lost, I prefer to make a separate answer for the last point.
Siebeck's properties 1) and 2) hold for any curve of class n with n-1 foci and may be rewritten this way :
1) for any point in the plane, the mean direction of the n-1 tangents to the curve is constant and equal to the mean direction of the lines joinig the point to the n-1 foci ; the mean direction is m° pi/(n-1).
2) for any point in the plane and it's n-1 tangents and any system of n-1 parallel tangents (tangents from an infinity point in any direction), let's consider the intersections of the 2 groups of lines (we have to take one intersection on each line of the 2 groups and there are several possibilities of doing this with different results) ; then the mean distance from the point to the n-1 intersections remain constant with varying directions of the parallel tangents and equal to the mean distance from the point to the n-1 foci.
If we consider a system of n points, we may consider the tangents in each vertice of the figur and the variable foci of the curves of class 3 tangent to the n(n-1) sides, but not necessary in their middles.
For the triangle, the foci F1 and F2 of the inscribed conics are isogonal conjugates wrt the triangle, which means that the bisectors of the angles F1PiF2 are the bisectors of the angles in the vertices Pi.
For the QA, the same way, the foci F1, F2 and F3 of any inscribed curve of class 3 verify that the mean direction of the lines joining a vertice Pi to these points is equal to the mean direction of the 3 tangents in this vertice, which are precisely the 3 sides of the QA through this vertice.
In fact, Siebeck's properties may be read with a fixed curve and variable tangents from a point of the plane or with a fixed triangle or QA and variable inscribed curves of class 2 (id est conics) or of class 3 ...
Sorry for having been so allusive in my 1rst message, I hope it's clearer now ...
Best regards Bernard
PS Of course, I would be more satisfied if I could reproduce Siebeck's construction of the foci ...

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Message: #2618
Date: 03/10/2017 3:39:35
From: bernard.keizer
Subject: New QL-circumscribed Cubics

```
Dear Eckart,
I was too quick again : in the cases 2a, the conic Co and the
circles Cx and Ce coincide, but not Ci and the 4th point exist !
Best regards
Bernard
```

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Message: #2619
Date: 04/10/2017 10:56:31
From: chris.vantienhoven
Subject: New QG-point

```
Dear Eckart,

Thanks for the special new QG-conic and QG-point!
When you take the harmonic conics QG-Co1 and QG-Co2 of the QA
and QL of the QA/QL-Dual configuration you get 3 conics growing
in size to the conic you discovered.
Also from this point of view their 4 centers form a perspective
row on QG-L2.

[ES] PS: Six years ago you asked for a review of EQF,
followed by years of interesting cooperation,
but times have changed ...
Yes I am still grateful for these beautiful years of
cooperation.
For me this period certainly has not ended yet.
Admittedly I do not participate regularly right now at the
Quadri-Forum but I am finishing now the description of the
results we had regarding the works of Morley wrt n-Lines.
I placed them in a new Encyclopedia for points, lines, conics,
curves, transformations, etc. wrt n-Lines, n-Points and n-Gons.
It is almost finished now on my website.

Best regards,
Chris
```

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Message: #2620
Date: 04/10/2017 4:43:40
From: eckart_schmidt@t-online.de
Subject: New QL-circumscribed Cubics

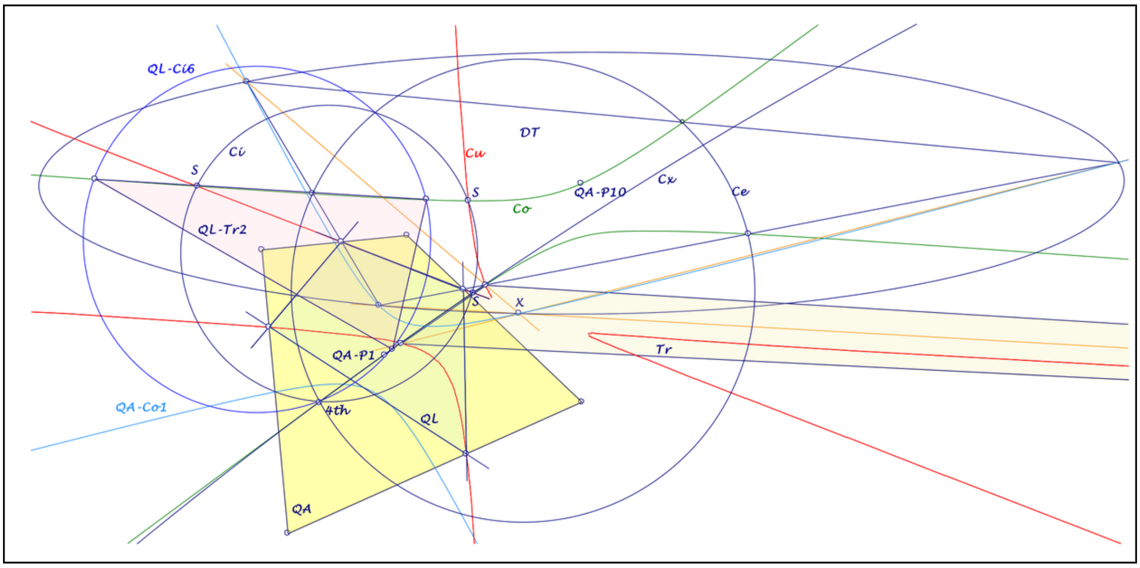
Dear Bernard,

thanks for your detailed remarks! Here only an answer to #2616:

Wrt 2a) I cannot confirm your assumption ...
"... it seems also that the 2 cubics are tangent in 3 of the 6
QL vertices."

Wrt 2b) You discuss a special cubic in reverse construction.
The point X is the 4th intersection of the DT-circumscribed
Steiner ellipse and QA-Co1.
I cannot reproduce, that the vertices of Tr lie on QA-Co1.
The points S of QL-Tr2 and my points S lie on your special conic
Co, but are not the same.
I tried to make a construction to think about, but it is
difficult, to get a good attached figur.
Hopefully there are no mistakes.

Best regards Eckart



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Message: #2621
Date: 04/10/2017 5:59:27
From: tsihonglau
Subject: Pappus Configuration

Dear all,

I post my most important message #2446
"geometry=incidence+metric" in these two years but got no reply.
It mentioned four configurations in paragraph 4:
Desargues configuration is made of 10 points and 10 lines. ->
topic #2020
Pappus configuration is made of 9 points and 9 lines. -> this
topic
Reye configuration is made of 12 points and 16 lines. -> topic
#2030
Complete quadrangle/quadrilateral configuration is made of 13
points and 13 lines. -> topic #1457

Usually Pappus theorem(or configuration) refers to three points
each on three lines(totally 9 points and 9 lines).
But we can refer to three tri-homological triangles, please
refer to Ch.3 of "THE GEOMETRY OF HOMOLOGICAL TRIANGLES" by
FLORENTIN SMARANDACHE and ION PÄ‚TRAŠžCU
https://arxiv.org/ftp/arxiv/papers/1204/1204.1585.pdf
Please refer to the attachment image file.

Given three triangles/trilaterals in three colors.
triangle - triangle - homological center
1. A1B1C1 - A2C2B2 - A3
2. A1B1C1 - C2B2A2 - B3
3. A1B1C1 - B2A2C2 - C3

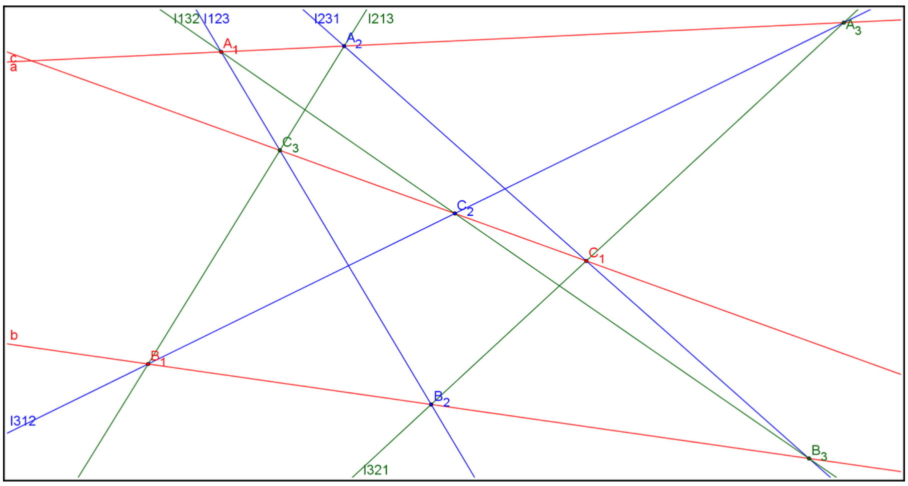
The two triangles A1B1C1 and A2B2C2 are (reverse) bi-homological
if any two of the three conditions are true.
The two triangles A1B1C1 and A2B2C2 are (reverse)
tri-homological if all three conditions are true.
Pappus theorem states that (reverse) bi-homological is
equivalent to (reverse) tri-homological.

Moreover, A2B2C2 and A3B3C3 are (reverse) tri-homological:
1. A2B2C2 - A3C3B3 - A1
2. A2B2C2 - C3B3A3 - B1
3. A2B2C2 - B3A3C3 - C1
A3B3C3 and A1B1C1 are (reverse) tri-homological:
1. A3B3C3 - A1C1B1 - A2
2. A3B3C3 - C1B1A1 - B2
3. A3B3C3 - B1A1C1 - C2

Due to the principle of duality and self-dualily of Pappus
configuration, the three trilaterals are (reverse)
tri-homological pairwise and the homological axes form the third
trilateral.
Please check them yourself.

I found a cubic derived from Pappus configuration and do not
know if it was studied before.
The three trilaterals form three triangles T1,T2,T3.
There is a unique cubic through nine vertices of them.
They are (reverse) tri-homological pairwise but they do not form
Pappus configuration.
homological center triangle - (reverse) tri-homological
triangles
T'1 - T2 and T3
T'2 - T3 and T1
T'3 - T1 and T2
The cubic passes through the nine vertices of T'1,T'2,T'3, too.
There are three more pairs of (reverse) tri-homological
triangles.
homological center triangle - (reverse) tri-homological
triangles
T''1 - T1 and T'1
T''2 - T2 and T'2
T''3 - T3 and T'3
The cubic passes through the nine vertices of T''1,T''2,T''3,
too.
Due to the principle of duality, there is a "line cubic" derived
from Pappus configuration.

Best regards,
Tsihong Lau



pappus_configuration.png

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Message: #2622
Date: 05/10/2017 9:40:34
From: bernard.keizer
Subject: New QL-circumscribed Cubics

Dear Eckart,
Another lost message, I will soon give up !
Anyhow, 3 points :
1) I checked on my own figure, I have the same point X (and the vertices of Tr as well as the 4th point aren't on the conic QA-Co1)
2) Therefore, there isn't in your family of cubics a cubic through the QL vertices and the 3 S-points
3) Is there a cubic through these 9 points or even a set of cubics if the 3 points are C-B triple wrt the QL vertices
Best regards
Bernard
PS Any comment to my message 2617, which was in fact the most important ?

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Message: #2623
Date: 06/10/2017 8:28:45
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

---

Dear Bernard,

thanks for the explinations in #2617, now I understand your link wrt the isogonality for triangles.
Drawing oncemore the constellation for n = 4, I made two observations, perhaps already mentioned or evident conclusion:

a) Let Pi be the vertices of a quadrangle and Fi the foci of its sextic: PiPj . PiPk . PiPl = 4 PiF1 . PiF2. PiF3

b) Let X, Y be two opposite intersections
... of two tangents at the sextic parallel PiPj
... and the QA-lines PiPk and PiPl: PiPk . PiPl = 2 PiX . PiY

Best regards Eckart

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Message: #2624
Date: 09/10/2017 6:45:08
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassi

---

Dear Eckart,
The observations have not been already mentionned, are not in Siebeck's article and are not at all evident !
And last, but not least, they are really interesting ...
The 2nd one is a particular case of the 1rst : when one the 3 parallel tangents approaches the position of PiPj, the point Z, intersection of this parallel with PiPj, is in the middle of PiPj.
We have PiPj.PiPk.PiPl = 4 PiF1.PiF2.PiF3 = 4 PiX.PiY.PiZ with PiPj = 2 PiZ hence the conclusion.
Encouraged by your example, I tried to measure distances on your figure in message 2548, but it lacks of precision ...
I understand for example why the product of the distances OAi is the same for the 6 possible triple of points Ai, as there are with the parallel tangents plenty of similar triangles ...
By the way, I tried again to understand Siebeck's construction.
I suppose the point named O is QA-P1 and the points named I and I', "Brennpunkte des von den sechs Asymptoten der Curve berührten Kegelschnitts" are the foci of the Steinerinellipse of the triangle of the foci, which may allow to reproduce the construction ...
Best regards
Bernard

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Message: #2625
Date: 09/10/2017 9:01:27
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cassin

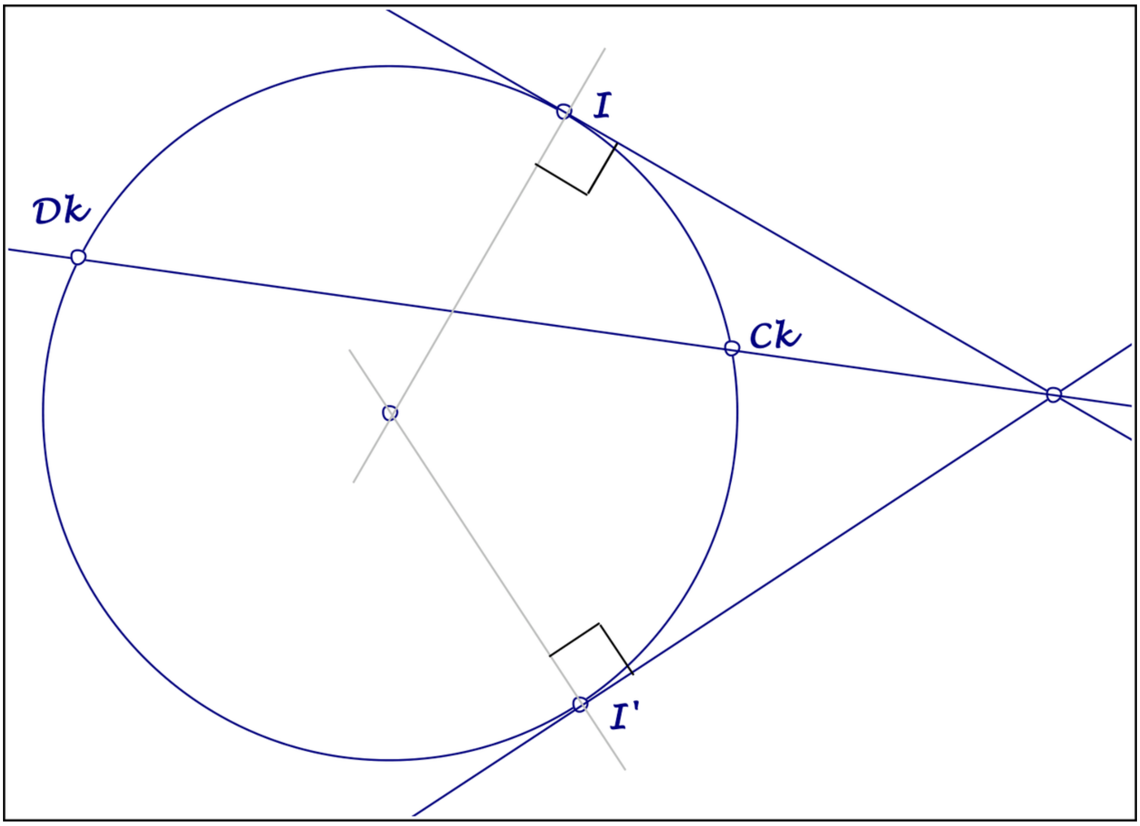
---

```
Dear Bernard,
already some days I try to understand Siebeck's construction of
the foci in your sense:
... O = QA-P1 and I,
    I' foci of the Steiner inellipse of the foci triangle,
... using a calculated constellation.

But in vain!!!
Siebeck uses "... dass Dk zu I, I' und Ck vierter harmonischer
Kreispunkt ist...".

Is this the constellation, attached?

Best regards Eckart
```



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Message: #2626
Date: 10/10/2017 10:16:00
From: bernard.keizer
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and cass

---

Dear Eckart,
I think I've understood the property during the night !
In complex numbers, we may form the polynom f(z)/(z-za) - f'(z)
where f(z) = (z-za)(z-zb)(z-zc)(z-zd), A, B, C and D are the
vertices of the QA and E, F and G the foci of the curve, roots
of f'(z).
It's not too difficult to prove that the value of this polynom
is zero for za.
So we have (zb-za)(zc-za)(zd-za) = 4 (ze-za)(zf-za)(zg-za).
This proves at the same time both Siebeck's properties in each
vertice :
1) the mean direction of the 2 groups of 3 points (3 other
vertices and 3 foci) is the same
2) the product of the distances to the 2 groups of points is the
same.
This is particularly interesting, as it is a general property :
1) for n= 3, in each vertice, the 2 groups of 2 points are the 2
other vertices and the 2the foci of the Steiner inellipse and
the property hold with 3 instead of 4
2) for a n-angle, in each vertice the 2 groups of n-1 points are
the n-1 other vertices and the n-1 foci and the property holds
with n instead of 4
Best regards
Bernard
PS You didn't attach a figure in your last message

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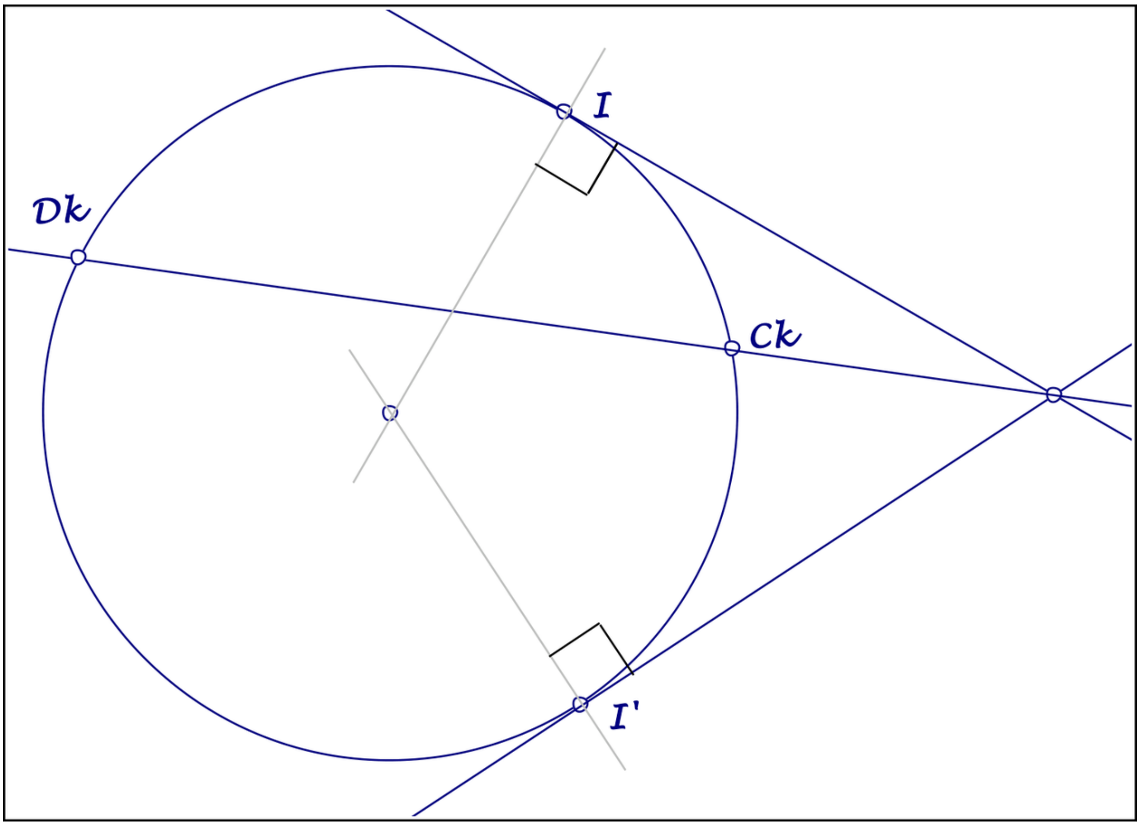
Message: #2627
Date: 10/10/2017 12:35:23
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygons of p ivots of stelloïds and

---

Dear Bernard,

wrt the lost figure in #2625, I try it once more.
In words: Dk is the 2nd intersection of the circle (Ck,I,I')
and the line connecting Ck and the pole of II'.
Thank you very much for the background of my observations.

Best regards Eckart



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Message: #2628
Date: 10/10/2017 3:06:19
From: bernard.keizer
Subject: Marden, Siebeck and polygo ns of p ivots of stelloïds and

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```
Dear Eckart,
Then Mk, middle of CkDk and of OFk is such as CkDk through the
pole of II' is the internal bisector of IMkI'.
We have already met this property with QL-Cu1 : 2 conjugate
points X and X' of the curve are the same way harmonic
conjugates wrt the 2 fixed points F and F' of Cl-S on the 1rst
Steiner axis, the pole of this axis wrt the circle is on XX' as
intersection with the 2nd Steiner axis and the middle m of XX',
wich describes the Newton Line, is such as XX' is the internal
bisector of the angle FmF'.
Best regards
Bernard
```

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Message: #2629
Date: 11/10/2017 11:38:52
From: bernard.keizer
Subject: Marden, Siebeck and polygo ns of p ivots of stelloïds and

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```
Dear Eckart,
I'm not able to reproduce your figure with QA and foci.
But I've checked following construction :
Given 2 points I and I' with middle O and a point C1.
On the circle with center O through C1, I draw the points C2 and
C3 forming an equilateral triangle.
Then I draw the points D1, D2, D3, M1, M2, M3, F1, F2 and F3
following Siebeck's indications.
D1D2D3 form also an equilateral triangle inscribed in a circle
centered in O.
Then the ellipse centered in O with foci I and I' through M1, M2
and M3 is the Steinerinellipse of the triangle F1F2F3.
I feel now very excited and I hope you will soon produce another
of your marvellous figures ...
Best regards
Bernard
```

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Message: #2630
Date: 11/10/2017 2:29:55
From: eckart_schmidt@t-online.de
Subject: QL-2P2

```
Dear Bernard, dear Chris,

if the cubic QL-Cu1 is unipartite,
... QL-2P2 are two CSC-partner on QL-L1,
... centers of two QL-inscribed conics:
(1) With four concyclic foci
... as harmonic points on the circle,
... ... centered on the 1st Steiner axis,
... ... orthogonal to the Schmidt circle,
... ... CSC-invariant.
(2) With eight contact points on a circle,
... polar circle of the DT-triangle,
... centered in QL-P10,
... with one point containing its DT-anticevians.
Perhaps something worth to be mentioned in EQF?

Best regards Eckart
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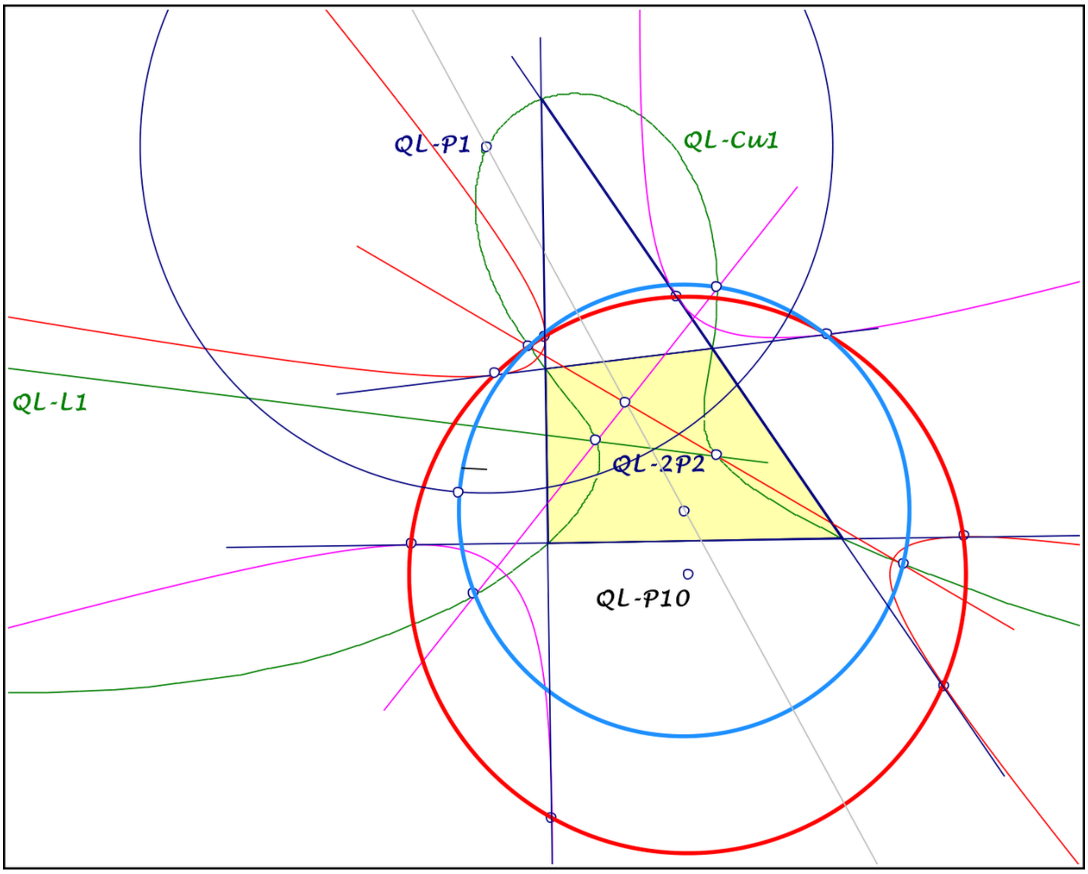

2017-10-11.pdf

Message: #2631
Date: 11/10/2017 3:40:43
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygo ns of p ivots of stelloïds and

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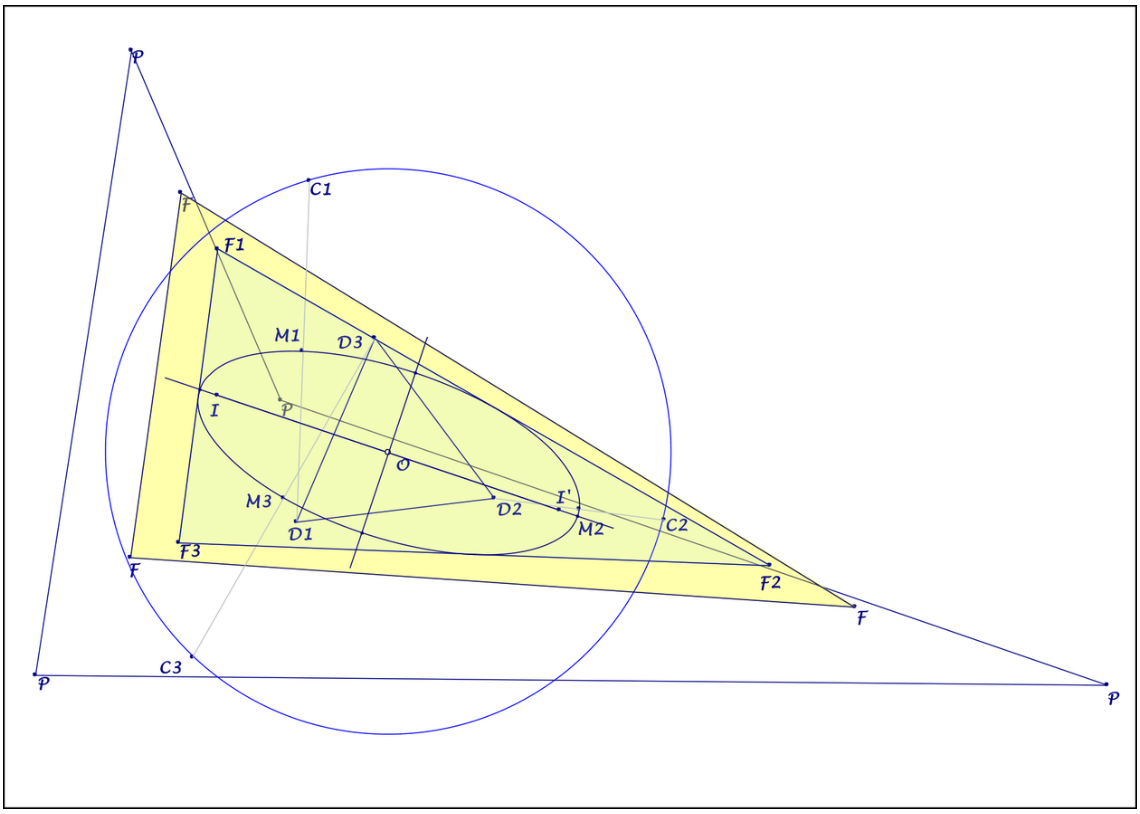
```
Dear Bernard,

that is a good observation!
For more thoughts attached a figure wrt your construction:
(a) O = QA-P1
(a) The circle has Siebeck`s radius.
(b) C1 variable point on the circle.
(c) I variable point.
But in vain I tried to get the calculated foci triangle,
changing C1 and I.

Best regards Eckart
```



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Message: #2632
Date: 12/10/2017 12:39:56
From: bernard.keizer
Subject: QL-2P2

Dear Eckart,
This figure has many other properties and we have already discussed part of them some years ago !
If you consider the triangle formed by QL-P1 and the the 2 QL-2P2, the points you describe are the fixed points of the 2 Cl-S with centers one QL-2P2 and swapping the other with QL-P1 ; they lie on the 2 internal bisectors of the triangle in the points QL-2P2.
The circle is centered in the QL-P1 excenter of the triangle.
The 2 copples of points are symmetric wrt one QL-2P2 and have the other as tangential.
Adding now the 2 points having the infinity point of the Newton Line as tangential (on the 2nd Steiner axis), the 3 copples of Cl-S conjugate points form a QL inscribed in QL-Cu1.
The points QL-2P2 and the infinity point of the Newton Line are aligned by definition and the QA's having these 3 points as tangentials have their vertices on the 2 perpendicular lines forming the polar conics of these 3 points wrt QL-Cu2 : on each QA, 2 vertices are real and are one of the 3 copples of points and the 2 other are imaginary ...
Best regards
Bernard

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Message: #2633
Date: 12/10/2017 12:49:06
From: bernard.keizer
Subject: Marden, Siebeck and polygo ns of p ivots of stelloïds and

---

```
Dear Eckart,
Although my figures are not as precise as yours, as I told you,
I'm now convinced : Siebeck's construction holds with QA-P1 as O
and the foci of the Steiner inellipse as I and I', but the
formula of the radius is wrong !
The radius is the cubic root of the product OA1.OA2.OA3 without
the coefficient 1/2.
The formula of the angle is correct.
By the way, it seems that the lines OCi and ODi are symmetric
wrt II' (IOCi = - IODi) ...
Best regards
Bernard
```

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Message: #2634
Date: 12/10/2017 2:03:39
From: eckart_schmidt@t-online.de
Subject: Marden, Siebeck and polygo ns of p ivots of stelloïds and

---

```
Dear Bernard,

I think, you are right,
for I just finished the third "construction" in Siebeck´s sense,
using the corrected radius,
which was also my assumption.
CABRI shows only very small deviations
in consequence of nessessary calculations and transfers of
measurements.
CABRI confirms also your observation,
that the lines OCi and ODi are symmetric wrt II'.

Best regards Eckart
```

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Message: #2635
Date: 15/10/2017 11:08:40
From: tsihonglau
Subject: Locus of Foci of Circumconic of Quadrangle

Dear all,

This message is inspired by Q108 listed in Higher Degree Related Curves.
http://bernard.gibert.pagesperso-orange.fr/curves/q108.html
Given a quadrangle, the locus of centers of circumconic is
QA-Co1 Nine-point Conic.
The locus of foci is a tricircular (The circular points at
infinity are triple points) sextic.
The vertices of the diapleural(diagonal) triangle and the orthic
triangle of it are double points of the sextic.
Could anyone compute the equation and dig out more properties of
the sextic?

Best regards,
Tsihong Lau

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Message: #2636
Date: 15/10/2017 11:30:59
From: eckart_schmidt@t-online.de
Subject: Locus of Foci of Circumconic of Quadrangle

Dear Tsihong Lau,

please have a look at QFG-message 496.

Best regards Eckart

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Message: #2637
Date: 15/10/2017 2:46:19
From: tsihonglau
Subject: Foci of QA-Circumscribed Conics

```
Dear Eckart,
Thanks for your information!
I reply your topic instead of mine!
If your equation listed in 2014-04-02 note for the sextic is
right, my assertion that it is tricircular is wrong.
It passes through the circular points at infinity and the
infinity points of both QA-Co1 and QA-2Co1.
Please notice the infinity points and centers of QA-2Co1 are the
same.
Now the sextic is circular or bicircular?
I guess it is bicircular, that is it passes through the infinity
points(may be imaginary) of QA-Co1 once!
However, if the quadrangle is orthocentric, then the circular
points at infinity and the infinity points(may be imaginary) of
QA-Co1 coincide.
The sextic becomes tricircular!
If it is circular, it has only one focus.
If it is bicircular, it has four foci=quadrangle.
Never forget any conic has four foci
Your observation consider only real foci.
How about imaginary foci?
I think imaginary foci lie on the sextic, too.
Last question: what is the envolope of real and imaginary
axes(lines through real and imaginary foci)?
Best regards,
Tsihong Lau
```

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Message: #2638
Date: 17/10/2017 9:05:39
From: eckart_schmidt@t-online.de
Subject: Foci of QA-Circumscribed Conics

```
Dear Tsihong Lau,
perhaps of interest for you:
QFG-message 2200 wrt the asymptotes of QA-circumscribed
hyperbolas.
But I cannot give informations wrt the axes.
Best regards Eckart
```

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Message: #2639
Date: 18/10/2017 3:42:19
From: eckart_schmidt@t-online.de
Subject: Center loci of special QA-conics

```
Dear all,

in QA-geometry we have two reference triangles
... the diagonal triangle QA-Tr1 and the Miquel triangle QA-Tr2
... each with an isoconjugation:
... QA-Tf2 for QA-Tr1 and isogonal conjugate for QA-Tr2
... and a common isocubic QA-Cu1.
Let us consider circumconics
... of the reference triangles QA-Tr1 and QA-Tr2
... through two isoconjugated points on the cubic QA-Cu1.
(1) Circumconics of QA-Tr1
... through two QA-Tf2-partner on QA-Cu1
... have a common point QA-P41
... and centers on a common circumconic of mid-QA-Tr1
... ... and the QA-Tr1-cevian triangle of QA-P41.
(2) Circumconics of QA-Tr1
... through two isogonal conjugate points on QA-Cu1
... have a common point U = QA-Tf2(isog(QA-P41))
... and centers on a common orthogonal hyperbola of mid-QA-Tr1
... ... and QA-Tr1-cevian triangle of U.
(3) Circumconics  of QA-Tr2
... through QA-Tr2-isogonal partner on QA-Cu1
... have a common point Q (intersection of QA-Cu1
    and its asymptote)
... and centers on a common orthogonal hyperbola of mid-QA-Tr2
... ... and QA-Tr2-cecian triangle of Q.
(4) Circumconics of QA-Tr2
... through QA-Tf2-partner on QA-Cu1
... have a common point V = isog(QA-Tf2(Q))
... and centers on a common circumconic of mid-QA-Tr2
... ... and QA-Tr2-cevian triangle of V.
The QA-Tf2-images of U and V are collinear with Q,
... the isogonal images of U and V are collinear with QA-P41.

Best regards Eckart
```

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Message: #2640
Date: 19/10/2017 9:14:13
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear all,

this summer we discussed a lot of properties of the Cayley
Bacharach point 8P-s-P1,
but I miss results in EQF and EPG.
Here further properties, perhaps not mentioned up to now:
(1) Consider the vertices of a quadrangle and their complements
wrt the remaining triangle.
... The CB-point of these 8 points is QA-P1.
(2) Consider the vertices of a quadrangle and their
anticomplements wrt the remaining triangle.
... The CB-point of these 8 points is a new QA-point
... ... on a QA-circumscribed conic through QA-P10 and QA-P5
... ... and a parallel to QA-P1.QA-P16 through QA-P5,
... ... with tangent through QA-P16.
(3) Consider the vertices of a quadrangle and the in-/ex-centers
of the Miquel triangle QA-Tr2.
... The CB-point of these 8 points
... ... is the 3rd intersection of QA-Cu1 and Q.QA-P41
... ... with Q = intersetion of QA-Cu1 and its asymptote
        (see EQF).
(4) Consider the vertices of a quadrangle and their isogonal
conjugates wrt the Miquel triangle,
... The CB-point of these 8 points is the tangential
    of Q wrt QA-Cu1.
(5) Consider the vertices of a quadangle, its mid-DT and QA-P10.
... The CB-point of these 8 points
... ... is the 2nd intersection of QA-P16.QA-P20 and the
QA-circumconic through QA-P20.
(6) Consider the vertices of a quadrigon
... and the 4th parallelogram points
    of three consecutive vertices.
... The CB-point of these 8 points is
    the reflection of QG-P1 in QG-P15.
    (The QA-triangle of these CB-points is homothetic
     to QA-Tr1 wrt QA-P1 and factor -3.)
(7) Consider the vertices of QA-Tr1 and QA-Tr2 on QA-Cu1
... with Q and QA-P3: CB-point = QA-Tf2(res(QA-P4,QA-P41)) =
QA-P41,
... with Q and QA-P4: CB-point = QA-Tf2(res(QA-P3,QA-P41)),
... with QA-P3 and QA-P4: CB-point = QA-Tf2(res(Q,QA-P41),
... with QA-P4 and QA-P41:CB-point = QA-Tf2(res(Q,QA-P3)).
Or: If you take the vertices of QA-Tr1 and QA-Tr2
```

```
... and two of the points Q, QA-P3, QA-P4, QA-P41,
... you get the CB-point as QA-Tf2-image
... ... of the 3rd intersection of QA-Cu1
        and the line trough the other two points.
Finally:
(8) Consider the vertices of a quadrangle and Q, QA-P3, QA-P4,
QA-P41.
... The CB-point of these 8 points
    is res(Q,QA-Tf2(res(QA-P3,QA-P41))).

Best regards Eckart

PS: res(X, Y) = 3rd intersection of QA-Cu1 and XY.
```

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Message: #2641
Date: 20/10/2017 12:20:50
From: chris.vantienhoven
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,

Obviously you found the new Encyclopedia of Polygon Geometry
http://www.chrisvantienhoven.nl/index.php?option=com_content&vie
w=article&id=285:epg-options&catid=7:mathematics

It is not quite finished yet but already very readable.
Especially the part with Morley's Points we studied together
will be very interesting for you and Bernard.
Thanks for your summary of EQF-related properties of the
CB-point.
This summary is just what I needed for EPG.
I included it in the description of
8P-s-P1 (http://www.chrisvantienhoven.nl/en/np-items/8p-obj/8p-p
ts/8p-s-p1).

Best regards,
Chris
```

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Message: #2642
Date: 20/10/2017 1:18:19
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear Chris,

I cannot access EQF for a long time.
The error message is ERR_TOO_MANY_REDIRECTS.
What happen?

Best regards,
Tsihong Lau
```

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Message: #2643
Date: 20/10/2017 1:48:59
From: chris.vantienhoven
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,
When you make access via www.chrisvantienhoven.nl
(http://www.chrisvantienhoven.nl) and then go to "Mathematics"
etc., do you still have the same problem?
Which browser do you use?
Best regards,
Chris
```

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Message: #2644
Date: 20/10/2017 2:53:56
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Ninth

```
Dear Chris,

The last 8 properties of CB-points were no summary!
In the discussion with Bernard Keizer and Tsihong Lau there are
a good deal more,
most of them more important than the last.
My ambition was to underline the all-round aspect of this point.
It would be good, if someone could find a simpler construction.

Best regards Eckart
```

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Message: #2645
Date: 20/10/2017 3:00:42
From: tsihonglau
Subject: Cayley-Bacharach Ninth

```
Dear Chris,

I cannot access either landline or mobile network regardless of
IE or Edge or Chrome!
The error message is the same.

Best regards,
Tsihong Lau

>>When you make access via www.chrisvantienhoven.nl (
http://www.chrisvantienhoven.nl ) and then go to "Mathematics"
etc., do you still have >>the same problem?
>>Which browser do you use?
```

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Message: #2646
Date: 20/10/2017 3:35:43
From: chris.vantienhoven
Subject: Cayley-Bacharach Ninth

```
Dear Tsihong Lau,

Thanks for this info.
1. Clean the cache of your browser and tell if the problem is
gone.
2. If not give me one link that doesn't work.

Chris
```

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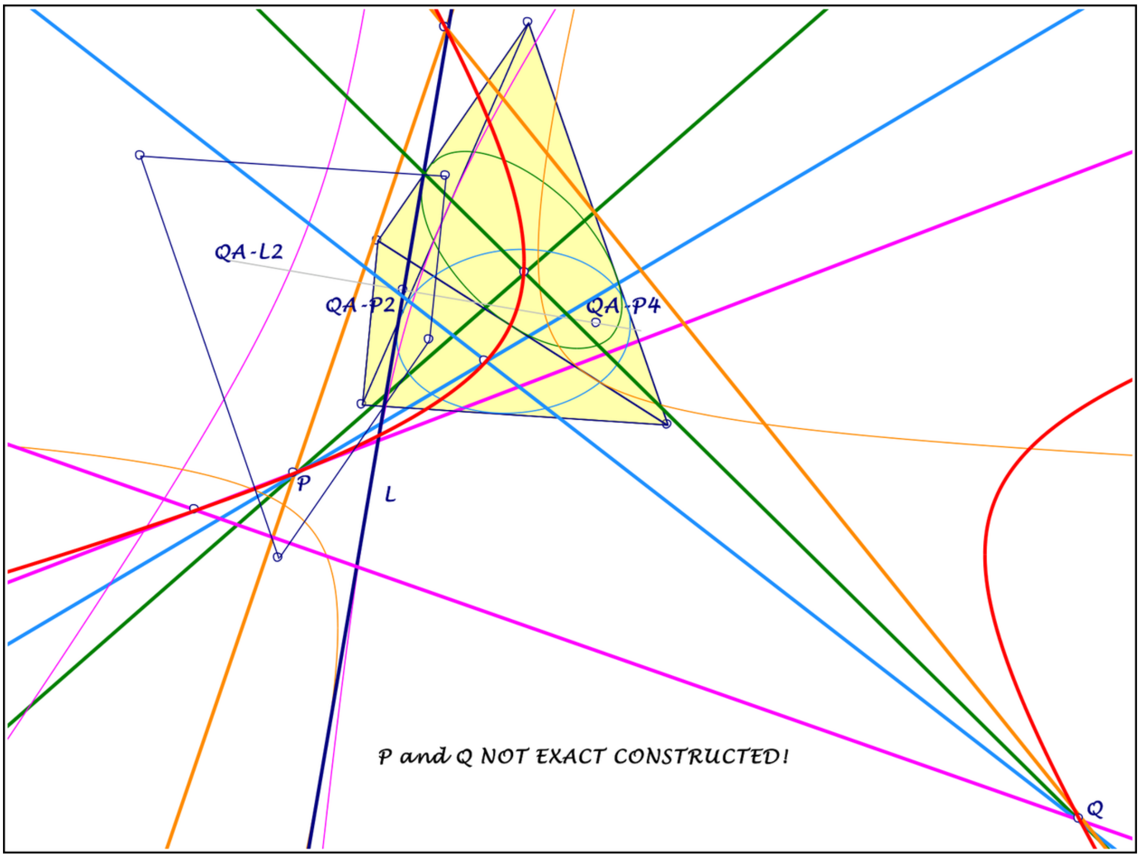
Message: #2647
Date: 20/10/2017 4:08:59
From: eckart_schmidt@t-online.de
Subject: Four QA-Triangle Inconics with Focus QA-P4

```
Dear all,

attached perhaps an interesting configuration:
The four inscribed conics Co of the QA-triangles with focus
QA-P4
... have a common tangent L,
... which is the QA-L2-perpendicular through QA-P2.
There are two points P, Q,
... whose polars wrt these conics Co intersect
    in the other point.
... P, Q and the four intersections of corresponding polars
... lie on a conic, whose polar wrt QA-P4 is L.
The 2nd foci give a QA, reflection of the reference QA in QA-P2.
What about these two points P and Q?

Best regards Eckart
```



2017-10-20.pdf

Message: #2648
Date: 20/10/2017 9:33:08
From: tsihonglau
Subject: Cayley-Bacharach Ninth

Dear Chris,

Thanks!
APG topics #4138 discuss geometry on complex plane!
I think why not studying objects on complex plane.
It is unnecessary to invent a coordinate system for every
n-points or lines!
I am working on this!
I give some objects!

1. For a septangle(7 points), there is a unique circular
circumcubic with its singular focus.
There is a unique K60 circumcubic.
There is the involution between a point and the Cayley-Bacharach
ninth point derived from it.

2. For a sexangle(6 points), there is the Cayley-Bacharach ninth
point derived from circular circumcubics.
There is a unique K60+ circumcubic with the point of concurrency
and its polar conic.
(It should be generalized)

3. For a quintangle(5 points), there is the involution between a
point and the Cayley-Bacharach ninth point for circular
circumcubics
derived from it.

4. For a octilateral(8 lines), there is the Cayley-Bacharach
ninth line.

5. For a septilateal(7 lines), there is the involution between a
line and the Cayley-Bacharach ninth line derived from it.

Best regards,
Tsihong Lau

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Message: #2649
Date: 21/10/2017 9:53:24
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

Dear Tsihong Lau,

I will answer your question in QFG-message #2648 in this new thread now.
Thanks for your ideas in that message.
I agree with you that studying objects in the complex plane can be a functional way to calculate in n-Points or n-Lines.
That's what Frank Morley did and that's how he found all these EPG-described items:
* nL-n-P1 till nL-n-P5,
* nL-n-P7,
* nL-n-L1,
* nL-n-iL1,
* nL-n-Ci1,
* nL-n-Ci2,
* nL-n-Cv1,
* nL-n-iCv1,
* nL-n-Cv2,
* nL-e-P1,
* nL-e-P2
and other nL-e-Objects and nL-o-Objects,

Calculating in the complex plane is particularly useful when working with circles.
I am looking forward to your progress in this.
About the objects you mentioned they look very interesting. I would be pleased to have some more detailed information about them. A simple general description, a construction method, a simple picture, relationships with ETC/EQF, references to earlier research, etc.
With this information I could include them in EPG.
About the Octalateral (8-Line), I am not sure we have enough information to conclude there is a Cayley-Bacharach ninth line.

Best regards,
Chris

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Message: #2650
Date: 21/10/2017 10:19:43
From: chris.vantienhoven
Subject: Cayley-Bacharach Ninth

```
Dear Eckart,
Regarding your message #2644.
[ES] The last 8 properties of CB-points were no summary!
Thanks for the correction.
[ES] In the discussion with Bernard Keizer and Tsihong Lau there
are a good deal more,
most of them more important than the last.
I would like to have a simple list of them.
[ES] It would be good, if someone could find a simpler
construction.
I do agree.
(wrt the Encyclopedia I also aim to keep things as simple as
possible)
Best regards,
Chris
```

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Message: #2651
Date: 21/10/2017 10:36:53
From: chris.vantienhoven
Subject: Four QA-Triangle Inconics with Focus QA-P4

```
Dear Eckart,
Very nice new QA-tangent line and other properties.
Best regards,
Chris
```

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Message: #2652
Date: 22/10/2017 10:29:26
From: bernard.keizer
Subject: n-Lines, n-Points and n-Gons

Dear Chris,
I had just a glance at EPG, but I find it amazing !
It's an enourmous work and I can better understand your
disparition from the Quadriforum.
Congratulations for having achieved this giant's work.
I will try to read it completely, but it surely will take time !
For the beginning, I have only one remark and one question :
1) For the n-Points, there are n-1 points which deserve a
description : the foci of the curve of class n-1, tangent to the
n(n-1) segments joining 2 vertices in their middle ("Siebeck's
curve") see messages from Eckart and me in EQF and reference
Siebeck's article mentionned by Eckart generalising Marden's
theorem for the triangle and description of the curve given by
Eckart
2) Is there a duality swapping the n-Points and the n-Lines as
the QA an the QL having the same DT ?
Best regards
Bernard

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Message: #2653
Date: 22/10/2017 11:07:34
From: tsihonglau
Subject: n-Lines, n-Points and n-Gons

Dear Bernard,

> 2) Is there a duality swapping the n-Points and the n-Lines
> as the QA an the QL having the same DT ?
> There is simple duality between quintangle and quintilateral.
> Construct the circumconic of the quintangle
> and the five tangent lines at
> vertices of the quintangle form the quintilateral desired.
> As triangle/trilateral and quadrangle/quadrilateral,
> we should merge quintangle and quintilateral objects.
> I have no idea about multangles and multilaterals above 5.

Best regards,
Tsihong Lau

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Message: #2654
Date: 22/10/2017 11:38:13
From: eckart_schmidt@t-online.de
Subject: n-Lines, n-Points and n-Gons

Dear Bernard, dear Chris, dear Tsihong Lau,

in #2649 Chris mentioned missing informations wrt the
Cayley-Bacharach ninth line.
In #2511 I gave a construction of my interpretation,
... see also following correspondence with Bernard.
But we have to replace cubics by curves, whose dual has degree
three (see #2516).
I tested also the sextic in Marden´s theorem:
Eight tangents at the sextic give a ninth tangent to the sextic.

Best regards Eckart

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Message: #2655
Date: 22/10/2017 4:11:33
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

```
Dear Eckart,

Regarding your message #2654:
I did read your writings.
But maybe I missed some clue.
Suppose we have an 8-Line 8L.
Let’s dual-transform 8L into an 8-Point 8P.
The basic question is: How can we find any curve through the
eight reference points of 8P that will be re-dual-transformed
into a cubic?
When we can’t find unambiguously such a curve we can’t be sure
there will be a ninth tangent in an 8-Line.
But maybe you know some method to do so?

Best regards,
Chris
```

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Message: #2656
Date: 22/10/2017 4:35:07
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

Dear Bernard,

Thanks you for enthusiasm. It really spoke to my heart.

>> [BK, #2652]
>> 1) For the n-Points, there are n-1 points which deserve a description :
>> the foci of the curve of class n-1, tangent to the n(n-1) segments joining
>> 2 vertices in their middle ("Siebeck's curve")
>> see messages from Eckart and me in EQF and reference Siebeck's article
>> mentioned by Eckart generalizing Marden's theorem for the triangle and
>> description of the curve given by Eckart

Because of my many other investigations I didn't find time to read all these articles.
But maybe you can make a short and simple description with picture readable for a layman that could be placed in EPG.
I myself do not quite understand what are the foci of some curve of class n-1. I only know the foci of a conic.

>> [BK, #2652]
>> 2) Is there a duality swapping the n-Points and the n-Lines as the QA and
>> the QL having the same DT?

Well I saw Tsihong Lau already answering this question. He defines a nice duality between a 5-Line and a 5-Point. I don't know either any other forms of duality swappings.

Best regards,
Chris

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Message: #2657
Date: 22/10/2017 4:52:02
From: tsihonglau
Subject: n-Lines, n-Points and n-Gons

Dear Chris,

My most important message to QFG in two years is #2446
"geometry=incidence+metric".
I explain my fundamental ideas in it.
Please refer to it carefully.
Point six states "configuration=variable+fixed objects".
Point seven states "metric=function of objects indifferent to
variable objects relative to fixed objects".
So we must distinguish configurations with and without fixed
objects.
The common fixed objects are the line and the circular points at
infinity(projective plane) and the point at infinity(Möbius
plane) etc.
With fixed objects, we have the metrics(length, angle, areas
etc.)
Without them, we have only incidence relations.
We also must distinguish ideas or objects concerning and not
concerning fixed objects.
For example, QA-Tf2 and QL-Tf2, the isoconjugations of
quadrangle and quadrilateral are involutions not concerning
fixed objects.
They must be distinguished from the transformations such as
QA-Tf3 and QL-Tf1 concerning the circular points at infinity.
We also must distinguish ideas or objects concerning different
fixed objects.
For example, QA-P1 can be derived from the quadrangle and the
line at infinity.
QA-P3 must be derived from the quadrangle and the circular
points at infinity.
In quadrangle/quadrilateral geometry, we can distinguish
symmetric and asymmetric objects.
Please refer to topic #2216 for more information.
Now we discuss EPG.
5P-s-Co1, 5P-s-P2, 5L-s-Co1,
5L-s-L2, 5P-s-Tf1, 5P-s-Tf2 are objects not concerning fixed
objects.
5P-s-P1 and 5L-s-P1 are objects concerning the line at infinity.
As said before, we should merge 5P-s-Co1 with 5L-s-Co1
and 5P-s-P1 with 5L-s-P1.
The duals of 5P-s-Tf1, 5P-s-Tf2 are not listed in EPG.
5L-s-L1 is an object concerning the circular points at infinity.
8P-s-P1 Cayley Bacharach Point

is an object not concerning fixed objects.
As said before, from 7 points/lines we can deduce point/line involutions and from
6 points/lines point/line triple operations.
>From a septilateral and the line at infinity, we can deduce a line.
>From a sexilateral and the line at infinity, we can deduce a line involution.
>From a sexangle and the circular points at infinity, we can deduce a point.
>From 6 diagonal points/diapleural lines of quadrilateral/quadrangle, we can deduce triple operations.
>From 6 diagonal points of quadrilateral, the result of triple operation of the circular points at infinity is QL-P1.
From 6 diapleural lines of quadrangle and the line at infinity, we can deduce an line involution.
>From a quintangle and the circular points at infinity, we can deduce a point involution.
>From a quintilateral and the line at infinity, we can deduce a line triple operation.
>From a quadrangle and the circular points at infinity, we can deduce a point triple operation.
The objects mentioned in message #2648 are being studied now!
I will post my results to APG later!
I have some other configurations in other posts.
Topic #2575 "Clockwise Counterclockwise Rotation Perspective Triangle Conic" discusses two ordered triangles and the circular points at infinity.
Message #2621 discusses Pappus configuration.
Message #2020 discusses Desargues configuration.
Message #2156 discusses some configurations.
Topic #2106 discusses an dual fixed object (an infinite line and a vertical point on it) geometry - Vertical Parabolic Pappian Plane (equivalent to Laguerre plane).
Message #2242 discusses the equivalences of two configurations (=variable+fixed objects).
The posts are very important but most got no replies.
I hope someone will be interested!

Best regards,
Tsihong Lau

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Message: #2658
Date: 23/10/2017 9:27:55
From: bernard.keizer
Subject: n-Lines, n-Points and n-Gons

```
Dear Tsihong Lau,
Thanks for your answer.
Why not ?
But you only define the dual quintangle and quintilateral.
You need then to define the duality, id est the dual line of a
random point in the plane !
(like the polar line in the triangle or the mixed of QA-Tf1 and
QL-Tf2 in the QA/QL)
Best regards
Bernard
PS I completely agree with you that we should merge objects of
dual figurs.
For the QA/QL, apart of DT points and QA-P16 or QL-P13, the best
example is for me the S-triangle QL-Tr2, which belongs as well
to the QA- as to the QL-environment.
```

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Message: #2659
Date: 23/10/2017 9:34:58
From: bernard.keizer
Subject: n-Lines, n-Points and n-Gons

```
Dear Chris,
Your message has been cut in EQF, but I could read it in the
mailbox.
Eckart discovered Siebeck's article and made an exact definition
of the sextic Se.
But I'm not sure that a few lines could resume easily dozens of
messages ...
Best regards
Bernard
```

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Message: #2660
Date: 23/10/2017 9:53:10
From: bernard.keizer
Subject: n-Lines, n-Points and n-Gons

Dear Eckart,
I read again your message 2511 ; beautiful idea indeed, as you are always able to find the 9nth point, provided you have a cubic through the 8 points by applying Coolidge's method with the focus of 4 points ...
It works for the Siebeck's curve, which is a sextic of class3.
Does it work for the cayleyan, which is also a sextic of class3?
If my memory is not wrong, you have tested it also for the deltoïd, which is a quartic of class 3.
Does it work also for the cardioïd, which also a quartic of class 3?
Best regards
Bernard

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Message: #2661
Date: 23/10/2017 11:05:49
From: eckart_schmidt@t-online.de
Subject: n-Lines, n-Points and n-Gons

```
Dear Chris,

it needs some efforts to become a fan of EPQ,
too manifold are objects, their nomination and their properties!
But highest respect for your immense work!!!
Really my own results got foreign to me.

Wrt 8L-s-P1: Is the following conclusion wrong?

Consider curves of class three
... with 8 common tangents,
... four for a reference QL.
... The duals of the curves have degree three,
... are cubics with 8 common points
    and a unique ninth common CB-point.
... The dual of this ninth CB-point
... is a unique common tangent of the initial curves.

Dear Bernard, my interpretation of a ninth CB-line works
... for Siebeck´s sextic, the cayleyan,
    the deltoid and the cardioid,
... all curves of class 3.

Best regards Eckart

PS: I found no explination of "Par2 ..." in nL-n-Luc5e.
```

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Message: #2662
Date: 23/10/2017 7:53:55
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

Dear friends,
I will give some more explanation about the new Encyclopedia of Polygon Geometry (http://www.chrisvantienhoven.nl/index.php?option=com_content&view=article&id=285:epg-options&catid=7:mathematics).

When I started earlier with EQF, I had to make special arrangements for identifying and coding new points. That's why I introduced the prefixes QA- (for Quadrangles), QL-(for Quadrilaterals) and QG- (for Quadrigons). The structure of Quadri Geometry evoked the new coding.
In the same way I noticed that when dealing with Poly Figures we had new structures that had to be filled in with new codes and new names.

Frank Morley introduced the name of an n-Line, being a configuration of n reference lines.
Thereafter the names n-Point and n-Gon were easily derived.
Frank Morley also introduced points that occurred for 3-Lines as well as 4-Lines as well as 5-Lines etc.. So they were present for n-Lines where n = natural number > 2.
But he also introduced points in n-Lines where n = an odd number or where n = an even number.
Then there were specific points in 5-Lines. So these points only existed for n=5.

How to distinguish these different types of points?
That's why I made this coding for points in n-Lines: xL- y-P z, where

x = n (when this point occurs for all natural numbers n > 2)
x = 3,4,5,6,7,etc. (when this point occurs for some specific fixed value of n > 2)

y = n (when this point occurs for all natural n > 2)
y = e (when this point occurs for all even n > 2)
y = o (when this point occurs for all odd n > 2)
y = s (when this point occurs for only specific fixed values of n > 2)

z = serial number of the point in the range it is defined

For example, 5L-n-P1, 5L-o-P1 and 5L-s-P1 are different points.

5L-n-P1 is a special case of the generic point nL-n-P1, namely
the case where n=5.
5L-o-P1 is a special case of the generic point nL-o-P1, namely
the case where n=5.
5L-s-P1 is a point in a 5-Line that only exists in a 5-Line.
5L-e-P1 does not exist, because an 'even' point can't exist in a
5-Line.

Then I introduced a special type of construction, the Level-up
Construction (Luc).
It took me some time to get it straight for myself that
constructions of points in an (n+1)-Line using similar points in
an n-Line deserved to be described as a special type.
A Level-up Construction is a construction that develops objects
in an n-Point or n-Line to similar objects in an (n+1)-Point or
(n+1)-Line.
Again Frank Morley gave very special examples of these type of
constructions. For example see nL-n-P4 (http://www.chrisvantienhoven.nl/en/nl-items/nl-obj/nl-pts/nl-n-p4).

Also the Ref/Par/Per-constructions
popped up to be Level-up constructions. See nL-n-Luc5
(http://www.chrisvantienhoven.nl/en/nl-items/nl-luc/nl-n-luc5)

I hope this will clarify some unfamiliar items.
I have to admit, I had to get used to it too.

Best regards,
Chris

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Message: #2663
Date: 24/10/2017 7:39:51
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

```
Dear Eckart,

>> [ES,QFG#2661]
>> Wrt 8L-s-P1:  Is the following conclusion wrong?
>> Consider curves of class three
>> ... with 8 common tangents,
>> ... four for a reference QL.
>> ... The duals of the curves have degree three,
>> ... are cubics with 8 common points
>>     and a unique ninth common CB-point.
>> ... The dual of this ninth CB-point
>> ... is a unique common tangent of the initial curves.

Your reasoning is quite all right.
It leads to this conclusion:
    Given an 8-Line, then all inscribed curves (if any)
    of class three have a unique common ninth tangent line.

Some remarks:
1. This is not quite Cayley-Bacharach style, because there are
no cubics
wrt the 8-Line involved.

2. The conclusion itself is worthwhile enough, when it can be
proven that
there is at least one curve of class three tangent to any 8
lines. Maybe
it is obvious, I don't know.

3. Is there a proof that the duals of curves concurrent in P
will be
tangent at common line dual(P) and vice versa?

Best regards,
Chris
```

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Message: #2664
Date: 25/10/2017 9:33:31
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

```
Dear Eckart,

[ES#2661] PS: I found no explination of "Par2 ..." in
nL-n-Luc5e.
I added some explanation at the item nL-n-Luc5e
(http://www.chrisvantienhoven.nl/en/nl-items/nl-luc/nl-n-luc5e).

Best regards,
Chris
```

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Message: #2665
Date: 25/10/2017 4:06:06
From: eckart_schmidt@t-online.de
Subject: Orthogonal 5L-Hyperbola

```
Dear Chris,

to get familar with the new nomination,
I reproduced somewhat 5L-geometry.
Perhaps of interest, see attached file.

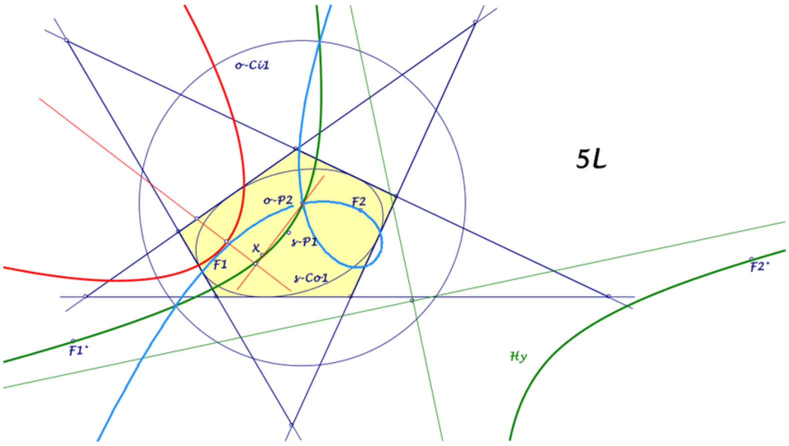
Best regards Eckart
```

EQF-Note 2017-10-25

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Polygon Geometry
http://www.chrisvantienhoven.nl/

Orthogonal 5L-Hyperbola

This hyperbola is already mentioned in QFG#762, 769, 780, 790, also in EPG under 5L-s-Tf1. Wrt special transformations it leads to a 5L-parabola.



Starting with the *5L*-inscribed conic *5L-s-Co1* and Clifford´s circle *5L-o-Ci1*, we get an interesting point

$$X = F_1F_2° \cap F_1°F_2.$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
$F_1°$ and $F_2°$ inverses of F_1 and F_2 wrt *5L-o-Ci1*.

The transformation *5L-s-Tf1* maps *X* to the center of Clifford´s circle *5L-o-P2*, but this transformation isn´t reciprocal.

- **The intersections**
  … of lines through *X* and their *5L-s-Tf1*-image lines
  … give an orthogonal hyperbola *Hy* (*QFG#762*).

- **The orthogonal hyperbola *Hy***
  … is centered in the midpoint of $F_1°F_2°$,
  … bears $F_1°$, $F_2°$, *5L-o-P2*, *X*
  … and the fixed points of *5L-s-Tf1*,
  … is tangent to *5L-s-P1.5L-o-P2* and *5L-s-P1.X*.
  … Polars of F_1,F_2 intersect in $F_1°.F_2° \cap$ *X.5L-o-P2*.

In *QFG#780* there is a transformation *Tf2*,
... that maps a line *L* (or circle *Ci*) to the common point
... of all radical axes wrt the 5 *CSC*-circles of *L* (or *Ci*).

- **The *Tf2*-image of the line pencil of *X* is a strophoid,**
  … for a line connecting *5L-o-P2* and the *Hy*-center,
  … fixed point *5L-o-P2*
  … and pole in the *5L-o-Ci1*-inverse of the reflection of *5L-o-P2* in the center *5L-s-P1* of the inscribed conic.

- **The strophoid is the inverse of the hyperbola *Hy* wrt *5L-o-Ci1*.**

In *QFG#790* there is are reciprocal transformations *Tf3* and *Tf4*:
... *Tf3* maps a point to the radical axis of its *CSC*-circle and *5L-o-Ci1*,
… *Tf4* maps a line to the common point of the radical axes of the *CSC*-circles of its points.

- ***Tf4* maps the *Hy*-tangents to a parabola**
  … with directrix *X.5L-o-P2*
  … and focus in *Tf4* of the perpendicular line of *Z.5L-o-P2* in Z (Z *Hy*-center).

Then *Hy* is the envelope of the *Tf3*-image lines of points on this parabola.

Eckart Schmidt
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Message: #2666
Date: 26/10/2017 11:01:07
From: chris.vantienhoven
Subject: Orthogonal 5L-Hyperbola

Dear Eckart,

Very nice to see your beautiful old results regarding
5L-geometry in a new context
. Thank you!
Because of the many Morley-related stuff I had to make a choice
of the main items there were.
In the process of developing EPG I also read your analyses about
Hodgson's Directed n-Lines.
They also gave very impessive results.

Best regards,
Chris

p.s. Could you please check your construction in nL-n-Cv1 (http://www.chrisvantienhoven.nl/en/nl-items/nl-obj/nl-cv/nl-n-cv1) of
the 5L-n-Cv1 EnnaCardioid to be sure I made no mistakes in the
transcription.

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Message: #2667
Date: 26/10/2017 2:59:22
From: eckart_schmidt@t-online.de
Subject: Orthogonal 5L-Hyperbola

Dear Chris,
I reproduced the construction of 5L-n-Cv1 and the translation
into the new nomination.
I think it´s right.
Perhaps you would change "QL-P1-circle" in "5L-o-Ci1".
It is difficult for me, to handle the new nomination:
A question: Is the following 5L-line in EPG?
In QFG#790 there is a 5L-transformations Tf,
… which maps a line/circle to the common point of the radical
axes of the CSC -circles of its points.
The Tf-images of the 5 QL-Ci3 are collinear.
Best regards Eckart

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Message: #2668
Date: 26/10/2017 11:09:35
From: chris.vantienhoven
Subject: Orthogonal 5L-Hyperbola

Dear Eckart,

[ES, QFG#2667]
Perhaps you would change "QL-P1-circle" in "5L-o-Ci1".
Done
[ES, QFG#2667]
A question: Is the following 5L-line in EPG?
In QFG#790 there is a 5L-transformations Tf,
… which maps a line/circle to the common point of the radical axes of the CSC-circles of its points.
The Tf-images of the 5 QL-Ci3 are collinear.
This is an interesting 5L-line indeed. It is not in EPG.
I drew it again in Cabri and noticed that this line is tangent at 5L-s-Co1, the inscribed 5L-conic.
I noticed it also can be constructed by drawing 2 tangents from 5L-n-P1 at 5L-s-Co1.
Since 5L-n-P1 lies on 5L-o-Ci1 these 2 tangents both have a 2nd intersection point with 5L-o-Ci1.
The line through these two 2nd intersection points is your interesting 5L-line.
You probably mentioned it one of your papers about the subject.
I was brought on the idea by the surrounding triangle of tangents around 5L-s-Co1 with vertices on 5L-o-Ci1 that I found in one of your papers.
I also read in your paper about the foci F1 and F2 of the inscribed 5L-conic.
I noticed that not only 5L-s-Tf1(F1) = F2 and 5L-s-Tf1(F2) = F1 (like I found in your description),
but also that QL-Tf1(F1) = F2 and QL-Tf1(F2) = F1.
That means that for all 4-Lines contained in the reference 5-Line the Clawson-Schmidt Conjugate QL-Tf1 for these points do coincide. And that actually is very special.
But I think, most probably you described this too.

Best regards,
Chris
p.s. Since I still am very busy with all kind of things I am not in the position to make many additions or changes in EPG right now.

Message: #2669
Date: 28/10/2017 11:54:16
From: eckart_schmidt@t-online.de
Subject: CSC-related 5L-Transformation

Dear Chris,

in EPG there is already a CSC-related transformation 5L-s-Tf1,
but it is not reciprocal. Every point has two pre-images.
Taking these pre-images as image-partner,
we get a new transformation,
which is reciprocal,
mapping circles to circles as CSC wrt quadrilaterals.
I shall look for further properties,
but I am not yet familar with all 5L-objects.

Best regards Eckart

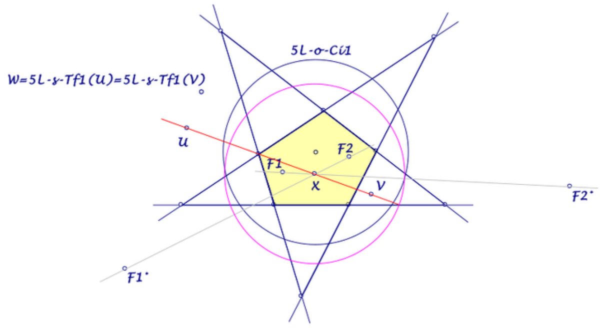
PS: Thanks for additional remarks wrt #2668.

EQF-Note 2017-10-28

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-related 5L-Transformation

In EPG there is already a CSC-related transformation 5L-s-Tf1, but it is not reciprocal. Every point has two pre-images. Taking these pre-images as image-partner, we get a new transformation, which is reciprocal, mapping circles to circles as CSC wrt quadrilaterals.



Starting with the *5L*-inscribed conic *5L-s-Co1* and Clifford´s circle *5L-o-Ci1*, we get an interesting point

$$X = F_1F_2^\circ \cap F_1^\circ F_2.$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
F_1° and F_2° inverses of F_1 and F_2 wrt *5L-o-Ci1*.

The transformation *5L-s-Tf1* maps X to the center of Clifford´s circle *5L-o-P2*, but this transformation isn´t reciprocal.
Let us further consider a circle *Ci(X)*
…centered in X with radius $\sqrt{X.F_1 \times X.F_2^o} = \sqrt{X.F_1^o \times X.F_2}$.

Definition of the transformation *Tf*:

- **Reflection in *X*, followed by inversion wrt *Ci(X)* give the *Tf*-image of a point.**

Main properties will be:

- ***Tf* is reciprocal: *Tf(Tf(P))* = *P*.**
- ***Tf* maps lines and circles to circles, … specially lines to circles.**
- ***P* and *Tf(P)* have the same *5L-s-Tf1*-image.**

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2017-10-28.pdf

Message: #2670
Date: 29/10/2017 2:34:57
From: tsihonglau
Subject: Concurrency with Van Aubel's Theorem

Dear Chris,

More properties!
Given an arbitrary planar quadrigon, place a square clockwise(or anticlockwise) on each side, and connect the centers of opposite squares.
Van Aubel's theorem states that the two lines are of equal length and cross at a right angle.
So a quadrigon has two pairs of perpendicular lines(= a quadrilateral).
This quadrilateral can form an orthocentric system.
The circumcircle of the diagonal triangle of the orthocentric system passes through QA-P1,
which is the midpoint of two vertices of the orthocentric system.
Since a quadrangle has three quadrigons, so we have three circumcircles through QA-P1 and three pairs of vertices of orthocentric systems with QA-P1 as midpoints.
So a circumconic pass the three pairs of vertices.
Is the circumconic known?

Best regards,
Tsihong Lau

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Message: #2671
Date: 29/10/2017 3:39:11
From: tsihonglau
Subject: Dual Reye Configuration from Incenters/Excenters of Quadrilateral

---

Dear all,

Messages #1947, #2035 and topic #2030 and other discuss Reye configuration.
Given a quadrilateral with six vertices, we can construct six pairs of angle bisectors through the six vertices.
We can get four orthocentric systems of incenters/excenters as the points of intersection of the angle bisectors.
It is easy to check that the 16 incenters/excenters and 12 angle bisectors form a dual Reye configuration.
So there exists a "line cubic tangent to the 12 angle bisectors"!

Best regards,
Tsihong Lau

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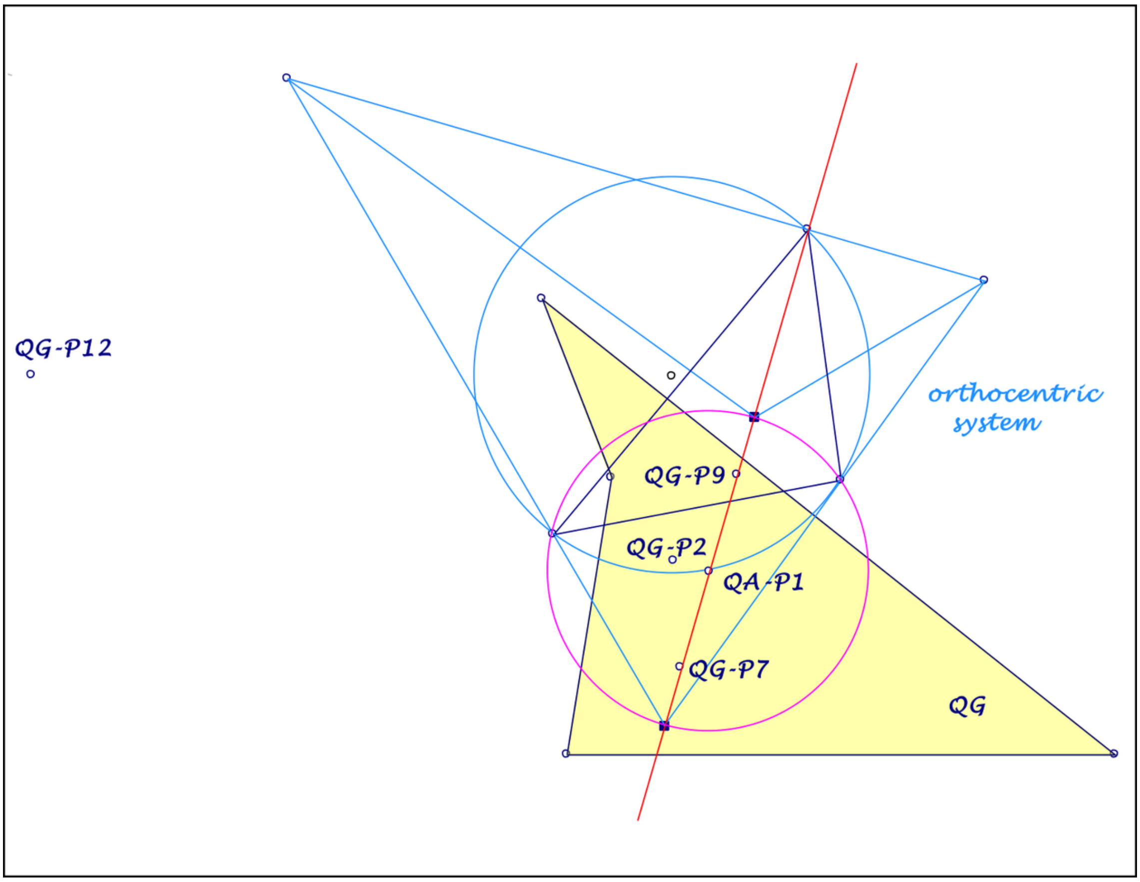
Message: #2672
Date: 30/10/2017 11:42:56
From: eckart_schmidt@t-online.de
Subject: Concurrency with Van Aubel's Theorem

```
Dear Tsihong Lau,

the last QA-circumconic in #2670 is not in EQF.
Wrt the considered quadrigons of the QA (see attached file):
The two vertices of the orthocentric system with midpoint QA-P1
... are the intersections of QG-P7.QG-P9
... and the inversion circle round QA-P1 wrt QG-P2 and QG-P12.

Best regards Eckart
```



2017-10-30.pdf

Message: #2673
Date: 30/10/2017 12:08:15
From: tsihonglau
Subject: Concurrency with Van Aubel's Theorem

```
Dear Eckart,
Thanks!
Could you give the equation of the circumconic?
Best regards,
Tsihong Lau
```

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Message: #2674
Date: 30/10/2017 9:23:41
From: bernard.keizer
Subject: Dual Reye Configuration from Incenters/Excenters of Quadrilatera

---

```
Dear Tsihong Lau,
It is well known that the 16 points lie on 8 circles, socalled
Steiner circles, centered on the Steiner axes (4 points on a
circle, each point belonging to 2 circles).
If you consider the 2nd intersections of each of the 4 circles
of the 1rst group with the 4 circles of the 2nd, the 16 new
points are also by definition on the 8 same circles (4 on each
circle and 2 circles through a point).
Less known, but also true, they lie on 12 other circles (4
points on each circle and each point belonging to 3 circles) ;
these new circles are all through QL-P1 and form 2 sets of
orthogonal circles, the 2nd intersection of 2 circles of
different sets being one QL vertice.
For each vertice, the 2 bisectors in the opposite vertice form
it's degenerated polar conic wrt QL-Cu2 ; the 12 bisectors are
tangent to the cayleyan of the cubic stelloïd QL-Cu2, with is a
sextic of class 3.
The dual points wrt the QA/QL of these 12 bisectors lie
therefore, like the dual points of every line joining 2 CSC
partners on QL-Cu1, on a cubic.
Best regards
Bernard
```

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Message: #2675
Date: 30/10/2017 10:29:47
From: chris.vantienhoven
Subject: n-Lines, n-Points and n-Gons

Dear friends,

Pentalaterals
It looks like that the Pentalateral (5-Line) has many
interesting properties just like the Quadrilateral (4-Line).
We now have 25 items 5L-n-Xx (Points, Lines, Circles, etc.) in a
Pentalateral (5-Line) that are valid for all n.
We have 6 items 5L-o-Xx for n=odd, so also for n=5.
And then we have 15 items 5L-s-Xx for n=specifically 5.
That makes together 46 5L-items!
They all can be found in EPG.
These 5L-items also show the layering in EPG, difference between
nL/nP/nG (prefix) and difference between n/e/o/s (infix). See
nL-1 http://www.chrisvantienhoven.nl/en/nl-items/nl-geninf/nl-1
Not very long ago we didn't know anything about a Pentalateral.
Thanks to Morley and our cooperation in QFG we found them all !

Application in Triangle Geometry
These items in a Pentalateral (5-Line) can be used in Triangle
Geometry by combining the 3 sidelines of the Reference Triangle
with two other lines delivering new ETC-points.
Similarly the results in a Pentangle (5-Point) can be used in
Triangle Geometry by combining the 3 vertices of the Reference
Triangle with two other points (e.g. a bicentric pair)
delivering new ETC-points.
Best regards,
Chris

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Message: #2676
Date: 30/10/2017 10:31:06
From: chris.vantienhoven
Subject: CSC-related 5L-Transformation

Dear Eckart,
Eckart, I am looking forward to your analyses of your new
5L-transformation.
Best regards,
Chris

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Message: #2677
Date: 31/10/2017 9:33:29
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Lines for QG/QL

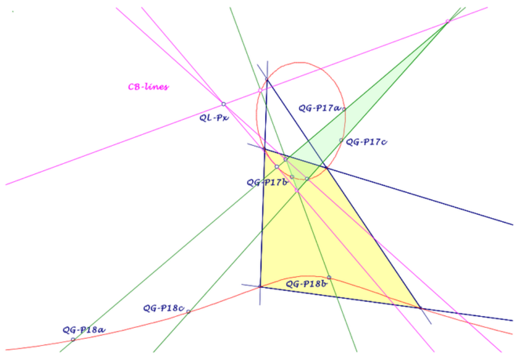
Dear all,
the eight angle bisectors of a quadrigon have a Cayley-Bacharach ninth line.
These CB-lines for the quadrigons of a quadrilateral have a common point.
Attached a simple construction and EQF-DT-coordinates.
Best regards Eckart

EQF-Note 2017-10-31

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Cayley-Bacharach Lines for *QG/QL*

The eight angle bisectors of a quadrigon have a Cayley-Bacharach ninth line. These CB-lines for the quadrigons of a quadrilateral have a common point. A simple construction and coordinates of this point can be given.



We start with a quadrilateral *QL*
…and the lines *QG-P17.QG-P18* for its three quadrigons,
…which give a triangle,
…whose altitudes are the *QL-Tf2*-images of its sidelines
… with pedal points on *QL-Cu1*.

- **The *QL-Tf2*-image of *QG-P17.QG-P18* is the ninth *CB*-line wrt the eight angle bisectors of a quadrigon.**
- **These three *CB*-lines for the quadrigons of a quadrilateral have a common point**

… with *EQF-DT*-coordinates:
$(m^2 n^2 SA^2(l^2 SB\, SC\, a^2 + m^2 SC\, S^2 + n^2 SB\, S^2) : cycl)$.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-10-31.pdf

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Message: #2678
Date: 01/11/2017 7:05:56
From: chris.vantienhoven
Subject: Cayley-Bacharach Lines for QG/QL

Dear Eckart,

It looks like a nice result.
But it also leaves me with lots of question marks.
1. I suppose you constructed the QuadriPoles of the angle bisectors.
2. Then you determined somehow a cubic through these 8 QuadriPoles? How?
3. Then you determined somehow the CB-point. How?
4. Then you transformed the CB-point into a CB-line by constructing the QuadriPolar and you noticed it was QG-P17.QG-P18?
5. Did you also transform the cubic though the 8 QuadriPoles in a curve of class 3? If so, was it a cubic? Otherwise we could discuss the name of a CB-Line (see my remarks/questions in message #2663).
I am curious to hear your answers.

Best regards,
Chris

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Message: #2679
Date: 02/11/2017 12:40:29
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Lines for QG/QL

Dear Chris,

my position is independent of curves of degree 3 or class 3,
but orientated at 8 points or 8 lines.

a) I use a construction of the Cayley-Bacharach ninth point for
8 points as described by A.S. Hart (see QFG#2447):

b) I use my interpretation of the Cayley-Bacharach ninth line as
described in QFG#2511.

c) I constructed these CB-line for the angle bisectors of a
quadrigon
... and alternative the QL-Tf2-image line of QG-P17.QG-P18
... and found - Cabri controlled - no difference.

I cannot prove the Cayley-Bacharach theorem for curves of class
three with eight common tangents,
but control my interpretations in a lot of examples.

In this case:
Starting with a QG and its 8 bisectors,
... whose duals give 8 points and its CB-point:
... the dual of this CB-point gives also the CB-line
    of the bisectors in the sense of QFG#2511.

By the way: Wrt the construction in QFG#2511 you can chose any
QA or QL,
... independent of the 8 lines!

Best regards Eckart

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Message: #2680
Date: 03/11/2017 2:43:45
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilateral

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Dear all

I hope someone reply my message #2246.
To Bernard: This message is a reply to #2658
To Chris: This message is a reply to some of your messages.

There are many definitions of conic sections.
One of these is Von Staudt conic:
https://en.wikipedia.org/wiki/Von_Staudt_conic
In projective geometry, a von Staudt conic is the point set
defined by all the absolute points of a polarity that has
absolute points.
A polarity, π, of a projective plane, P, is an involutory (i.e.,
of order two) bijection between the points and the lines of P
that preserves the incidence relation. Thus, a polarity relates
a point Q with a line q and, following Gergonne, q is called the
polar of Q and Q the pole of q.[1] An absolute point (line) of a
polarity is one which is incident with its polar (pole).
A polarity may or may not have absolute points. A polarity with
absolute points is called a hyperbolic polarity and one without
absolute points is called an elliptic polarity.[4] In the
complex projective plane all polarities are hyperbolic but in
the real projective plane only some are.

Now we return to QA-8/QL-8:The Dual QA-QL-Configuration.
This duality is a polarity.
What is the conic?
If we use unitary coordinate system.
That is, four vertices of the quadrangle are 1:1:1, -1:1:1,
1:-1:1, 1:1:-1.
Four edges of the quadrilateral are x+y+z=0, -x+y+z=0, x-y+z=0,
x+y-z=0.
Then the (point or line) conic equation is x^2+y^2+z^2=0.
I knew this conic more than one year ago.
But last week I understood its relationship to von Staudt conic.
This conic does not exist on the real projective plane!
But It is a natural object on the complex projective plane.
It intersects the diagonal trilateral at 0:1:i, 0:1:-i, i:0:1,
-i:0:1, 1:i:0, 1:-i:0<-absolute points
It intersects the diapleural triangle at y+iz=0, y-iz=0, ix+z=0,
-ix+z=0, x+iy=0 x-iy=0<- absolute lines
Like the circular points at infinity, these objects exist on the
complex projective plane only but the are very essential!

We must study them thoroughly!
Since a conic induces a polarity.
A polarity is a duality itself.
We have to study conics as dualities!
Now we discuss 5P-s-P2,5L-s-L2
involutary conjugate - point - quadrangle
Q1 - P1, P2,P3,P4,P5
Q2 - P2, P1,P3,P4,P5
Q3 - P3, P1,P2,P4,P5
Q4 - P4, P1,P2,P3,P5
Q5 - P5, P1,P2,P3,P4
It seems that P1,P2,P3,P4,P5 and Q1,Q2,Q3,Q4,Q5 lie on a cubic!
L1,L2,L3,L4,L5 are tangent lines to the circumconic of
P1,P2,P3,P4,P5 at P1,P2,P3,P4,P5.

Now what is the relationship of the 5P-s-P2 of P1,P2,P3,P4,P5
and 5L-s-L2 of L1,L2,L3,L4,L5?
dual(QA-8/QL-8) - point - quadrangle
L1 - P1, P2,P3,P4,P5
L2 - P2, P1,P3,P4,P5
L3 - P3, P1,P2,P4,P5
L4 - P4, P1,P2,P3,P5
L5 - P5, P1,P2,P3,P4
Q1 - L1, L2,L3,L4,L5
Q2 - L2, L1,L3,L4,L5
Q3 - L3, L1,L2,L4,L5
Q4 - L4, L1,L2,L3,L5
Q5 - L5, L1,L2,L3,L4
What is the relationship of P1,P2,P3,P4,P5 and Q1,Q2,Q3,Q4,Q5?
To be continued...

PS. To Chris: My colleague - a high school math teacher - cannot
access your website.

Best regards,
Tsihong Lau

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Message: #2681
Date: 04/11/2017 9:55:15
From: eckart_schmidt@t-online.de
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Dear Tsihong Lau,

wrt QFG#2680:
"...
involutary conjugate - point - quadrangle Q1 - P1, P2,P3,P4,P5
..."

The lines PiQi are the tangents Li in Pi at the circumconic of
P1P2P3P4P5.
5L-s-L2 of L1L2L3L4L5 is the polar of 5P-s-P2 wrt the
circumconic of P1P2P3P4.

Perhaps evident?

Best regards Eckart
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Message: #2682
Date: 04/11/2017 1:46:41
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Dear Eckart,

>>The lines PiQi are the tangents Li in Pi at the circumconic of P1P2P3P4P5.
>>5L-s-L2 of L1L2L3L4L5 is the polar of 5P-s-P2 wrt the circumconic of P1P2P3P4.

1. First generation - L1,L2,L3,L4,L5 are tangent lines to the circumconic of P1,P2,P3,P4,P5 at P1,P2,P3,P4,P5.(polarity from Quintangle/Quintilateral)

2. Second generation -
involutary conjugate - point - quadrangle
Q1 - P1, P2,P3,P4,P5
Q2 - P2, P1,P3,P4,P5
Q3 - P3, P1,P2,P4,P5
Q4 - P4, P1,P2,P3,P5
Q5 - P5, P1,P2,P3,P4
involutary conjugate - line - quadrilateral
M1 - L1, L2,L3,L4,L5
M2 - L2, L1,L3,L4,L5
M3 - L3, L1,L2,L4,L5
M4 - L4, L1,L2,L3,L5
M5 - L5, L1,L2,L3,L4
incidence relation
point - line
P1,Q1 - L1
P2,Q2 - L2
P3,Q3 - L3
P4,Q4 - L4
P5,Q5 - L5
P1 - L1,M1
P2 - L2,M2
P3 - L3,M3
P4 - L4,M4
P5 - L5,M5
It seems that P1,P2,P3,P4,P5 and Q1,Q2,Q3,Q4,Q5 lie on a point cubic!
And L1,L2,L3,L4,L5 and M1,M2,M3,M4,M5 lie on a line cubic

3. Third generation -
involutary conjugate - point - quadrangle
R1 - Q1, Q2,Q3,Q4,Q5

```
R2 - Q2, Q1,Q3,Q4,Q5
R3 - Q3, Q1,Q2,Q4,Q5
R4 - Q4, Q1,Q2,Q3,Q5
R5 - Q5, Q1,Q2,Q3,Q4
involutary conjugate - line - quadrilateral
N1 - M1, M2,M3,M4,M5
N2 - M2, M1,M3,M4,M5
N3 - M3, M1,M2,M4,M5
N4 - M4, M1,M2,M3,M5
N5 - M5, M1,M2,M3,M4
P1,P2,P3,P4,P5 and R1,R2,R3,R4,R5 perspective at 5P-s-P2 of
P1,P2,P3,P4,P5.
L1,L2,L3,L4,L5 and N1,N2,N3,N4,N5 perspective at 5L-s-L2 of
L1,L2,L3,L4,L5.
You mean that 5P-s-P2 of P1,P2,P3,P4,P5 and 5L-s-L2 of
L1,L2,L3,L4,L5 are in polarity relation with respect to the
circumconic of P1,P2,P3,P4,P5=inscribed conic of L1,L2,L3,L4,L5?
How about the relations of all objects mentioned above?

The are the circumconic of Q1,Q2,Q3,Q4,Q5 and the inscribed
conic of M1,M2,M3,M4,M5.
What are their relations?
The are 4 common points(= a quadrangle) of the circumconics of
P1,P2,P3,P4,P5 and Q1,Q2,Q3,Q4,Q5 and 4 common tangent lines(= a
quadrilateral) of the inscribed conics of L1,L2,L3,L4,L5 and
M1,M2,M3,M4,M5.
What are their relations?
Is it possible to establish a coordinate system using them?

Best regards,
Tsihong Lau
```

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Message: #2683
Date: 04/11/2017 3:07:12
From: eckart_schmidt@t-online.de
Subject: CSC-related 5L-Transformation

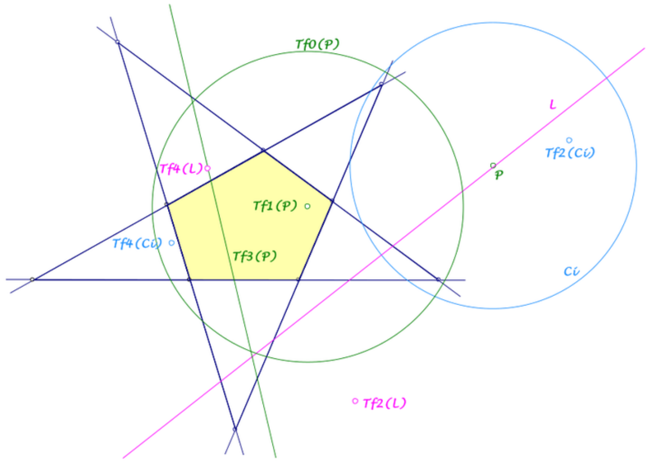
```
Dear Chris,
wrt the CSC-Transformation for quadrilaterals several
transformations for 5-lines can be considered.
Attached a try of survey for such transformations, mentioned in
earlier messages.
Best regards Eckart
```

EQF-Note 2017-11-04

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-related 5L-Transformation II

Wrt the CSC-Transformation QL-Tf1 for quadrilaterals several transformations for 5-lines can be considered.



Transformations

Tf0 ***point → circle***: The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).

Tf2 ***line/circle → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line/circle → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Inverse Transformations

$Tf0_{inv}$ *Tf0(P)* are special circles, *Tf2* and *Tf4* of these circles give *P* again.

$Tf1_{inv}$ … does not exist. Every point has two pre-images, which are partner wrt the transformation *Tf* in *QFG# 2669*.

$Tf2_{inv}$ … does not exist. *Tf2(L)* is a point, which has three pre-images. Let *Tf2(Lx)* = *Tf2(Ly)* = *Tf2(Lz)*, then

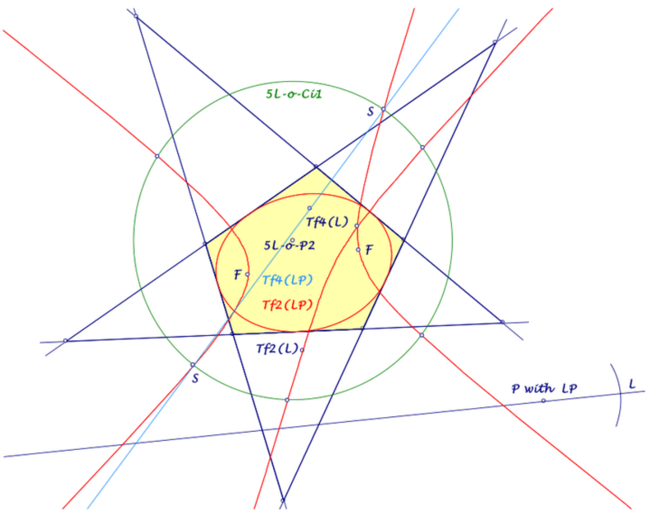
2017-11-04.pdf

Tf4(Lx)=Ly∩Lz, Tf4(Ly)=Lx∩Lz, Tf4(Lz)=Lx∩Ly.
$Tf3_{inv}$ *Tf3(P)* is a line and *Tf4* of this line is *P* again.
$Tf4_{inv}$ *Tf4(L)* is a point and *Tf3* of this point is *L* again,
Tf4(Ci) is a point and *Tf0* of this point is *Ci* again.

Tf*-Geometry for *5L-o-Ci1* and *5L-s-Co1

For properties of the transformations see the cited messages. Here finally the *Tf*-geometry of the Clifford circle *5L-o-Ci1* and the inscribed conic *5L-s-Co1* shall be researched.

- **For points *X* on *5L-o-Ci1* the degenerated circles *Tf0(X) = Tf3(X)* are tangents of *5L-s-Co1*.**
- **For points *X* on *5L-o-Ci1* the points *Tf1(X)* are points at infinity with direction orthogonal *Tf3(X)*.**
- **For points *X* on *5L-s-Co1* the circles *Tf0(X)* contact *5L-o-Ci1*.**
- **For tangents *L* at *5L-o-Ci1* the points *Tf4(L)* lie on *5L-s-Co1*.**
- **For tangents *L* at *5L-s-Co1* the points *Tf2(L) = Tf4(L)* lie on *5L-o-Ci1*.**
- **For points *X* on a line *L* the points *Tf1(X)* lie on a conic through *5L-o-P2*.**
- **For points *X* on a line *L* the lines *Tf3(X)* give a line pencil of *Tf4(L)*.**



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- ***Tf2* maps a line pencil *LP* to a cubic**
  … through the foci F_1, F_2 of *5L-s-Co1*
  … with $Tf2(PF_i) = F_j$.

- ***Tf4* maps a line pencil *LP* to a line,**
  … intersecting *5L-o-Ci* on the cubic in S_1, S_2
  … with $Tf0(S_i)$ tangents from *P* at *5L-s-Co1*,
  … … $Tf3(S_i) = PS_j$ tangent at *5L-s-Co1*.
  … … $Tf2(S_1S_2)$ is a double point of the cubic,
  … … $Tf4(S_1S_2)$ is the point *P* again,
  … … $Tf2(Tf0(S_i)) = Tf4(Tf0(S_i)) = S_j$.

- **The *Tf2*-cubic of a line pencil *PL* intersects *5L-o-Ci* in S_1, S_2 and four further points *T* with *T* on *Tf2(PT)*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-11-04.pdf

Message: #2684
Date: 04/11/2017 10:41:51
From: chris.vantienhoven
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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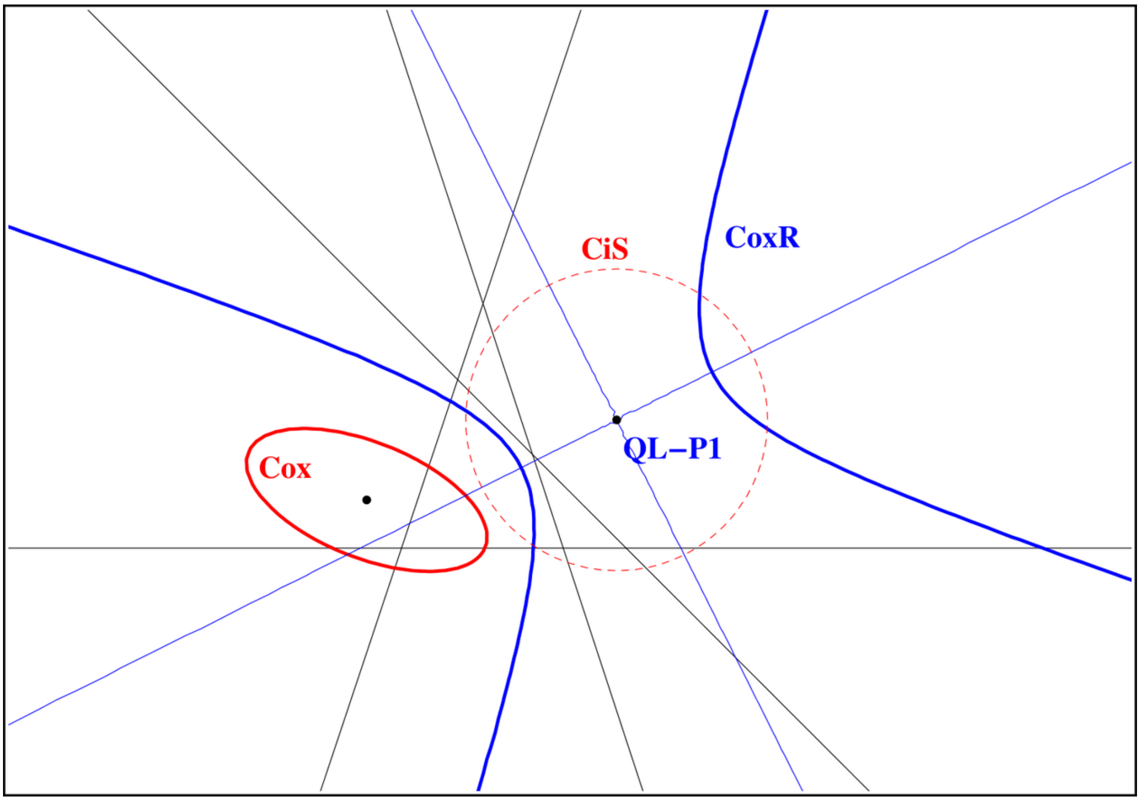
```
Dear Tsihong Lau,

Another example in EQF of a polarity is QL-Tf3.
The Von Staudt Conic (Cox) wrt QL-Tf3 =
The Center of the Von Staudt Conic wrt QL-Tf3 =
The Von Staudt Conic (CoxR) wrt QL-Tf3R =
This is an Orthogonal Hyperbola with these properties:
· Center is QL-P1.
· The axes of the Orthogonal Hyperbola are the Steiner Axes
· The foci lie on the Schmidt Circle ?

See attached picture.

Best regards,
Chris
```



QL-Tf3-CSCe-Transformation-10-Polarity Properties.pdf

Message: #2685
Date: 04/11/2017 10:46:05
From: chris.vantienhoven
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Maybe this message is better readable than former message.
Dear Tsihong Lau,

Another example in EQF of a polarity is QL-Tf3.

The Von Staudt Conic Cox wrt QL-Tf3 =
```
-a^2 mn(a^4 l^2-a^2 b^2 l^2-a^2 c^2 l^2-a^4 lm+a^2 b^2 lm+2a^2
c^2 lm+b^2 c^2 lm-c^4 lm-a^4 ln+2a^2 b^2 ln-b^4 ln+a^2 c^2
ln+b^2 c^2 ln+a^4 mn-2a^2 b^2 mn+b^4 mn-2a^2 c^2 mn-2b^2 c^2
mn+c^4 mn)x^2+2a^2 b^2 lmn(a^2 l-b^2 l+c^2 l-a^2 m+b^2 m+c^2
m-2c^2 n)xy-b^2 ln(a^2 b^2 lm-b^4 lm+a^2 c^2 lm+2b^2 c^2 lm-c^4
lm-a^2 b^2 m^2+b^4 m^2-b^2 c^2 m^2+a^4 ln-2a^2 b^2 ln+b^4
ln-2a^2 c^2 ln-2b^2 c^2 ln+c^4 ln-a^4 mn+2a^2 b^2 mn-b^4 mn+a^2
c^2 mn+b^2 c^2 mn)y^2+2a^2 c^2 lmn(a^2 l+b^2 l-c^2 l-2b^2 m-a^2
n+b^2 n+c^2 n)xz+2b^2 c^2 lmn(-2a^2 l+a^2 m+b^2 m-c^2 m+a^2
n-b^2 n+c^2 n)yz-c^2 lm(a^4 lm-2a^2 b^2 lm+b^4 lm-2a^2 c^2
lm-2b^2 c^2 lm+c^4 lm+a^2 b^2 ln-b^4 ln+a^2 c^2 ln+2b^2 c^2
ln-c^4 ln-a^4 mn+a^2 b^2 mn+2a^2 c^2 mn+b^2 c^2 mn-c^4 mn-a^2
c^2 n^2-b^2 c^2 n^2+c^4 n^2)z^2
```

The Center of the Von Staudt Conic wrt QL-Tf3 =

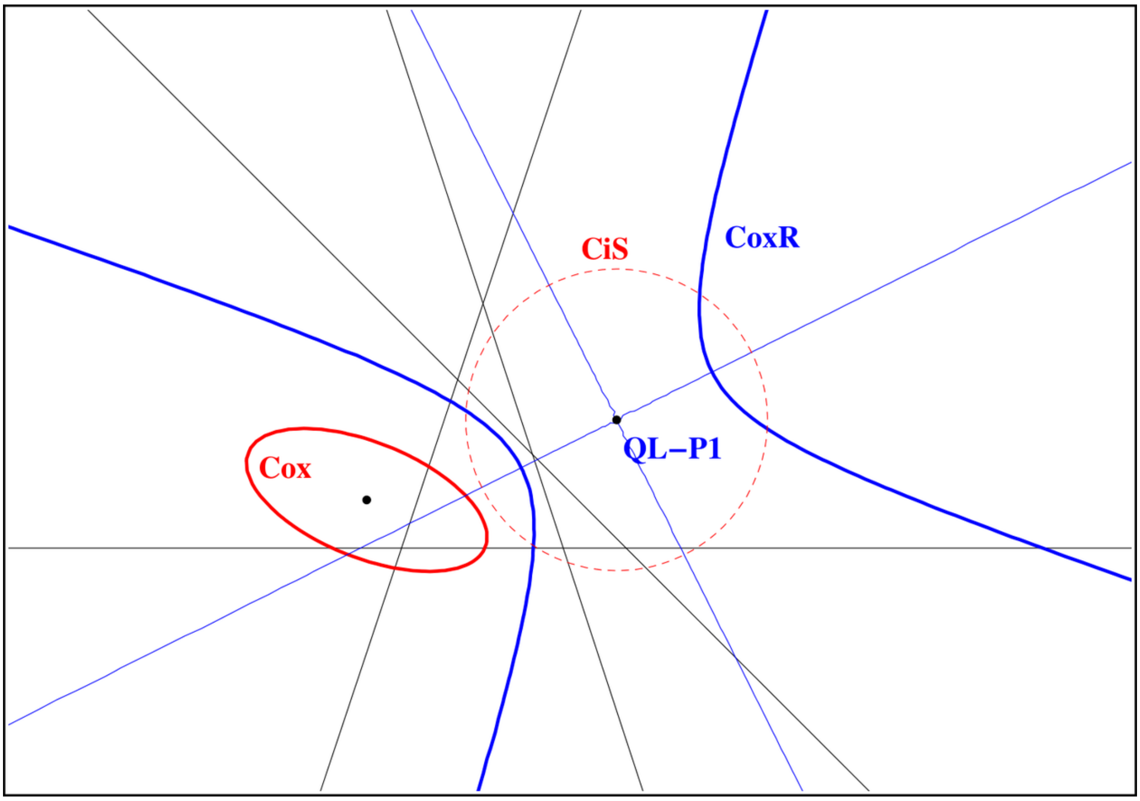
{-b^2 c^2 l(-a^4 l^2 m^2+b^4 l^2 m^2-2a^2 c^2 l^2 m^2-2b^2 c^2 l^2 m^2+c^4 l^2 m^2+a^4 lm^3-b^4 lm^3-a^2 c^2 lm^3+b^2 c^2 lm^3-a^4 l^2 mn+2a^2 b^2 l^2 mn-2b^4 l^2 mn+2a^2 c^2 l^2 mn+4b^2 c^2 l^2 mn-2c^4 l^2 mn+2a^4 lm^2 n-2a^2 b^2 lm^2 n+2b^4 lm^2 n+3a^2 c^2 lm^2 n-b^2 c^2 lm^2 n-c^4 lm^2 n-a^4 m^3 n+a^2 c^2 m^3 n-a^4 l^2 n^2-2a^2 b^2 l^2 n^2+b^4 l^2 n^2-2b^2 c^2 l^2 n^2+c^4 l^2 n^2+2a^4 lmn^2+3a^2 b^2 lmn^2-b^4 lmn^2-2a^2 c^2 lmn^2-b^2 c^2 lmn^2+2c^4 lmn^2-a^4 m^2 n^2-a^2 b^2 m^2 n^2-a^2 c^2 m^2 n^2+a^4 ln^3-a^2 b^2 ln^3+b^2 c^2 ln^3-c^4 ln^3-a^4 mn^3+a^2 b^2 mn^3),-a^2 c^2 m(-a^4 l^3 m+b^4 l^3 m+a^2 c^2 l^3 m-b^2 c^2 l^3 m+a^4 l^2 m^2-b^4 l^2 m^2-2a^2 c^2 l^2 m^2-2b^2 c^2 l^2 m^2+c^4 l^2 m^2-b^4 l^3 n+b^2 c^2 l^3 n+2a^4 l^2 mn-2a^2 b^2 l^2 mn+2b^4 l^2 mn-a^2 c^2 l^2 mn+3b^2 c^2 l^2 mn-c^4 l^2 mn-2a^4 lm^2 n+2a^2 b^2 lm^2 n-b^4 lm^2 n+4a^2 c^2 lm^2 n+2b^2 c^2 lm^2 n-2c^4 lm^2 n-a^2 b^2 l^2 n^2-b^4 l^2 n^2-b^2 c^2 l^2 n^2-a^4 lmn^2+3a^2 b^2 lmn^2+2b^4 lmn^2-a^2 c^2 lmn^2-2b^2 c^2 lmn^2+2c^4 lmn^2+a^4 m^2 n^2-2a^2 b^2 m^2 n^2-b^4 m^2 n^2-2a^2 c^2 m^2 n^2+c^4 m^2 n^2+a^2 b^2 ln^3-b^4 ln^3-a^2 b^2 mn^3+b^4 mn^3+a^2 c^2 mn^3-c^4 mn^3),-a^2 b^2 n(b^2 c^2 l^3 m-c^4 l^3 m-a^2 c^2 l^2 m^2-b^2 c^2 l^2 m^2-c^4 l^2 m^2+a^2 c^2 lm^3-c^4 lm^3-a^4 l^3 n+a^2 b^2 l^3 n-b^2 c^2 l^3 n+c^4 l^3 n+2a^4 l^2 mn-a^2 b^2 l^2 mn-b^4 l^2 mn-2a^2 c^2 l^2 mn+3b^2 c^2 l^2 mn+2c^4 l^2 mn-a^4 lm^2 n-a^2 b^2 lm^2 n+2b^4 lm^2 n+3a^2 c^2 lm^2 n-2b^2 c^2 lm^2 n+2c^4 lm^2 n+a^2 b^2 m^3 n-b^4 m^3 n-a^2 c^2 m^3 n+c^4 m^3 n+a^4 l^2 n^2-2a^2 b^2 l^2 n^2+b^4 l^2 n^2-2b^2 c^2 l^2 n^2-c^4 l^2 n^2-2a^4 lmn^2+4a^2 b^2 lmn^2-2b^4 lmn^2+2a^2 c^2 lmn^2+2b^2 c^2 lmn^2-c^4 lmn^2+a^4 m^2 n^2-2a^2 b^2 m^2 n^2+b^4 m^2 n^2-2a^2 c^2 m^2 n^2-c^4 m^2 n^2)}

The Von Staudt Conic CoxR wrt QL-Tf3R (reversed transformation) =

b^2 c^2 l(-a^2 b^2 l^2 m^2+b^4 l^2 m^2-a^2 c^2 l^2 m^2-2b^2 c^2 l^2 m^2+c^4 l^2 m^2+a^2 b^2 lm^3-b^4 lm^3+b^2 c^2 lm^3-a^4 l^2 mn+2a^2 b^2 l^2 mn-2b^4 l^2 mn+2a^2 c^2 l^2 mn+4b^2 c^2 l^2 mn-2c^4 l^2 mn+a^4 lm^2 n-a^2 b^2 lm^2 n+2b^4 lm^2 n-b^2 c^2 lm^2 n-c^4 lm^2 n-a^2 b^2 m^3 n-a^2 b^2 l^2 n^2+b^4 l^2 n^2-a^2 c^2 l^2 n^2-2b^2 c^2 l^2 n^2+c^4 l^2 n^2+a^4 lmn^2-b^4 lmn^2-a^2 c^2 lmn^2-b^2 c^2 lmn^2+2c^4 lmn^2-a^4 m^2 n^2+a^2 b^2 m^2 n^2+a^2 c^2 m^2 n^2+a^2 c^2 ln^3+b^2 c^2 ln^3-c^4 ln^3-a^2 c^2 mn^3)x^2-2a^2 b^2 c^2 lm(a^2 l^2 m-b^2 l^2 m+c^2 l^2 m-a^2 lm^2+b^2 lm^2+c^2 lm^2+b^2 l^2 n-c^2 l^2 n-a^2 lmn-b^2 lmn-3c^2 lmn+a^2 m^2 n-c^2 m^2 n+2c^2 ln^2+2c^2 mn^2-c^2 n^3)xy+a^2 c^2 m(-a^4 l^3 m+a^2 b^2 l^3 m+a^2 c^2 l^3 m+a^4 l^2 m^2-a^2 b^2 l^2 m^2-2a^2 c^2 l^2 m^2-b^2 c^2 l^2 m^2+c^4 l^2 m^2-a^2 b^2 l^3 n+2a^4 l^2 mn-a^2 b^2 l^2 mn+b^4 l^2 mn-a^2 c^2 l^2 mn-c^4 l^2 mn-2a^4 lm^2 n+2a^2 b^2 lm^2 n-b^4 lm^2 n+4a^2 c^2 lm^2 n+2b^2 c^2 lm^2 n-2c^4 lm^2 n+a^2 b^2 l^2 n^2-b^4 l^2 n^2+b^2 c^2 l^2 n^2-a^4 lmn^2+b^4 lmn^2-a^2 c^2 lmn^2-b^2 c^2 lmn^2+2c^4 lmn^2+a^4 m^2 n^2-a^2 b^2 m^2 n^2-2a^2 c^2 m^2 n^2-b^2 c^2 m^2 n^2+c^4 m^2 n^2-b^2 c^2 ln^3+a^2 c^2 mn^3+b^2 c^2 mn^3-c^4 mn^3)y^2-2a^2 b^2 c^2 ln(-b^2 l^2 m+c^2 l^2 m+2b^2 lm^2-b^2 m^3+a^2 l^2 n+b^2 l^2 n-c^2 l^2 n-a^2 lmn-3b^2 lmn-c^2 lmn+2b^2 m^2 n-a^2 ln^2+b^2 ln^2+c^2 ln^2+a^2 mn^2-b^2 mn^2)xz-2a^2 b^2 c^2 mn(-a^2 l^3+2a^2 l^2 m-a^2 lm^2+c^2 lm^2+2a^2 l^2 n-3a^2 lmn-b^2 lmn-c^2 lmn+a^2 m^2 n+b^2 m^2 n-c^2 m^2 n-a^2 ln^2+b^2 ln^2+a^2 mn^2-b^2 mn^2+c^2 mn^2)yz+a^2 b^2 n(-a^2 c^2 l^3 m+a^2 c^2 l^2 m^2+b^2 c^2 l^2 m^2-c^4 l^2 m^2-b^2 c^2 lm^3-a^4 l^3 n+a^2 b^2 l^3 n+a^2 c^2 l^3 n+2a^4 l^2 mn-a^2 b^2 l^2 mn-b^4 l^2 mn-a^2 c^2 l^2 mn+c^4 l^2 mn-a^4 lm^2 n-a^2 b^2 lm^2 n+2b^4 lm^2 n-b^2 c^2 lm^2 n+c^4 lm^2 n+a^2 b^2 m^3 n-b^4 m^3 n+b^2 c^2 m^3 n+a^4 l^2 n^2-2a^2 b^2 l^2 n^2+b^4 l^2 n^2-a^2 c^2 l^2 n^2-b^2 c^2 l^2 n^2-2a^4 lmn^2+4a^2 b^2 lmn^2-2b^4 lmn^2+2a^2 c^2 lmn^2+2b^2 c^2 lmn^2-c^4 lmn^2+a^4 m^2 n^2-2a^2 b^2 m^2 n^2+b^4 m^2 n^2-a^2 c^2 m^2 n^2-b^2 c^2 m^2 n^2)z^2

This is an Orthogonal Hyperbola with these properties:
* Its center is QL-P1.
* The axes of the Orthogonal Hyperbola are the Steiner Axes
* The foci lie on the Schmidt Circle ?

Best regards,
Chris



QL-Tf3-CSCe-Transformation-10-Polarity Properties-2685.pdf

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Message: #2686
Date: 05/11/2017 12:40:14
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Dear Chris,

Thanks for your reply!
I hope someone would reply my message #2246.
According to Von Staudt, a conic corresponds to a polarity.
We can get as many polarities as conics.
Message #2680 discusses the conics/polarities from general quadrangle/quadrilateral and quintangle/quintilateral, no fixed objects involved.
You give the conic for QL-Tf3 polarity, fixed objects(circular points at infinity) involved.
It is a real conic, not imaginary.
But long ago, I give several essential imaginary conics in topic #1617 message #1621.
I repost it with more information here!

a,b,c indicate barycentric sidelengths
p,q,r,indicate trilinear circulars (circular points p:q:r 1/p:1/q:1/r)

1.zero-sum of distance squares from a point to a triangle(three points)
- equation
(b^2+c^2)x^2+(b^2+c^2-a^2)yz+...(cyclic)=0
p^2*(r-q)^2*(r+q)^2*(q^2*r^4+q^4*r^2-4*p^2*q^2*r^2+p^4*r^2+p^4*q^2)*x^2-q*(q-p)^2*(q+p)^2*r*(r-p)^2*(r+p)^2*(r^2+q^2)*y*z+...(cyclic)=0
- point of intersection with the line at infinity
circular points
- center=foci
X(2)
- asymptotes
(a^2-2b^2-2c^2)x^2+(5a^2-b^2-c^2)yz+...(cyclic)=0
(2*q^2*r^3*z-2*p^2*r^3*z-q^4*r*z+p^4*r*z+q*r^4*y-2*q^3*r^2*y+2*p^2*q^3*y-p^4*q*y-p*r^4*x+2*p^3*r^2*x+p*q^4*x-2*p^3*q^2*x)*(q^4*r^3*z-p^4*r^3*z-2*p^2*q^4*r*z+2*p^4*q^2*r*z-q^3*r^4*y+2*p^2*q*r^4*y-2*p^4*q*r^2*y+p^4*q^3*y-2*p*q^2*r^4*x+p^3*r^4*x+2*p*q^4*r^2*x-p^3*q^4*x)=0
- perspector
X(3424)
- axes
imaginary circle
Because this conic passes through the circular points, it is an imaginary circle and center=foci and no axes exist.

- The polarity transforms X(523) into Euler's line.
We had better dig out more properties!

2.zero-sum of distance squares from a point to a trilateral(three lines)
- equation
(x/a)^2+...(cyclic)=0
(trilinear)x^2+y^2+z^2=0<- indicates the conic is derived from incenter/excenters quadrangle described in #2680
- point of intersection with the line at infinity
a*(c^2+b^2):-b*(c*sqrt(-c^2-b^2-a^2)+a*b):c*(b*sqrt(-c^2-b^2-a^2)-a*c)
a*(c^2+b^2):b*(c*sqrt(-c^2-b^2-a^2)-a*b):-c*(b*sqrt(-c^2-b^2-a^2)+a*c)
- center
X(6)
- asymptotes
(b^2+c^2)(x/a)^2-2yz+...(cyclic)=0
- perspector
self-polar
- foci
?:?:?
- axes
?:?:?
- The polarity transforms a point into the trilinear polar of isogonal conjugate of it.
zero-sum of distance squares from a line to a triangle(three points)
- equation
(point and line)x^2+...(cyclic)=0<- indicates the conic is derived from centroid/anticomplementary triangle quadrangle described in #2680
(trilinear circular, point)(p*(q+r)*(q-r)x)^2+...(cyclic)=0
- point of intersection with the line at infinity
1:ω:ω^2,1:ω^2:ω where ω is an imaginary cubic root of 1
= point of intersection of Steiner ellipses with the line at infinity
- center
X(2)
- asymptotes
x^2-yz+...(cyclic)=0
- perspector
self-polar
- foci
?:?:?
- axes
?:?:?
- The polarity transforms a point into the trilinear polar of isotomic conjugate of it.

The conics are all imaginary.
But centers, perspector and polarities are all real.
The locus of constant sum of distance squares described in topic #1617 is not an algebraic curve!
Only zero sum of distance squares is meaningful.
I also compute the equation of zero-sum of distance squares from a point to a quadrangle.
But it is imaginary circle and somewhat complicated.
To be continued...
Best regards,
Tsihong Lau

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Message: #2687
Date: 05/11/2017 4:23:41
From: chris.vantienhoven
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

Dear Tsihong Lau,

[TL] QFG#2686
According to Von Staudt, a conic corresponds to a polarity.
We can get as many polarities as conics.

Given a conic
a11 x^2 + 2 a12 x y + a22 y^2 + 2 a13 x z + 2 a23 y z + a33 z^2,
the polarity will be this transformation:
{x,y,z} -> {a11 x + a12 y + a13 z,a12 x + a22 y + a23 z,a13 x + a23 y + a33 z}
where {x,y,z} can be a point being transformed into a line,
and {x,y,z} can be a line being transformed into a point.
So there actually are 2 polarities giving the Von Staudt Conic:
a point->line polarity and a line->point polarity.

Best regards,
Chris

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Message: #2688
Date: 05/11/2017 4:36:18
From: chris.vantienhoven
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Dear Tsihong Lau,

[TL] To Chris: My colleague - a high school math teacher -
cannot access your website.

1. It often helps clearing the cache of your browser.
2. To enter EQF this link is the right one:
http://www.chrisvantienhoven.nl/mathematics/encyclopedia
3. To enter EPG this link is the right one: http://www.chrisvant
ienhoven.nl/mathematics/encyclopedia-of-poly-geometry
4. Sometimes it helps to change to another browser: Internet
Explorer, Chrome, Firefox.

Let me know if he still can't enter my site and which browser he
uses.

Best regards,
Chris
```

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Message: #2689
Date: 05/11/2017 4:47:52
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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Dear all,

We return to polarities without fixed objects(circular points
etc).
Given two point conics with four points of intersection(=a
quadrangle) and a point P, the point of intersection of two
polars of P with respect to the two conics is the isoconjugate
of P with respect to the quadrangle.
The construction 2 of QA-Tf2 listed in EQF seems to be a special
case of the above.
According to principle of duality, there is dual of the above.
Now given two conics(as both point and line conics) with four
common points(=a quadrangle) and four common lines(=a
quadrilateral), we can get a point and a line isoconjugations.
What is the relation of both isoconjugations?
What is the relation of the quadrangle and the
quadrilateral(maybe imaginary)?

Best regards,
Tsihong Lau
```

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Message: #2690
Date: 06/11/2017 12:10:17
From: eckart_schmidt@t-online.de
Subject: 5L-Example for Polarity with Conic

```
Dear Tsihong Lau, dear Chris,

you discuss polarities and corresponding conics.
Attached an example for the 5L-transformations Tf3(P) and Tf4(L)
in QFG# 2683.

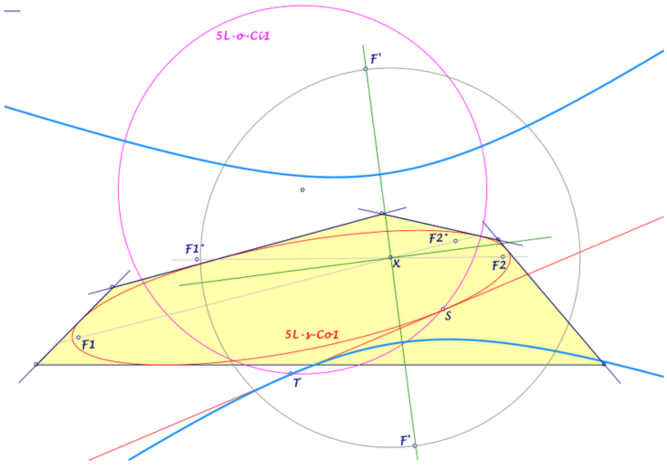
Best regards Eckart
```

EQF-Note 2017-11-06

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

5L-Example for Polarity with Conic

"According to Von Staudt, a conic corresponds to a polarity": This is discussed in QFG, started by Tsihong Lau QFG#2680. In QFG#2683 there are two 5L-transformations Tf3(P), Tf4(L), which define a polarity. Here a construction of the corresponding conic is described.



If we have a look in *QFG#2683* and take the transformation …

Tf0 ***point → circle***: The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

… we get two further transformations

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line → point:*** Common point of the radical axes for the *Tf0*-circles of the points of the line (see *QFG#790*).

… which give a bijection between points and lines, that preserves the incidence relation. The corresponding conic can be constructed as follows:

- Center of the conic is the point X (see *QFG#2669*)
  $$X = F_1F_2° \cap F_1°F_2.$$
  F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
  $F_1°$ and $F_2°$ inverses of F_1 and F_2 wrt *5L-o-Ci1*.

- Main axis is the angle bisector of $\sphericalangle F_1XF_2$.

- Foci F' are the intersections of the main axis and a circle round X with radius $\sqrt{X.F_1 \times X.F_2}$.
- Special points T of the hyperbola (not always real):
  … Let S be an intersection of *5L-s-Co1* and *5L-o-Ci1,*
  … Tg the tangent in S at *5L-s-Co1,*
  … T second intersection of Tg and *5L-o-Ci1*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-11-06.pdf

Message: #2691
Date: 07/11/2017 6:50:56
From: chris.vantienhoven
Subject: Cayley-Bacharach Lines for QG/QL

Dear Eckart,

Before I go on a short 5-day holiday this short response to your message #2679.

Dual Transformation
We have 3 items:
1. Cv_1 is a curve of degree n (n=1,2,3,…)
2. Pt_1 is a point of tangency at the curve Cv_1
3. Lt_1 is a line tangent to the curve Cv_1 at Pt_1
We have two mutually reverse transformations Tf1 and Tf1R with these properties:
Tf1 transforms Cv_1 into Cv_2, Pt_1 into Lt-2, Lt_1 into Pt_2 such that:
1. Cv_2 is a curve of class n (n=1,2,3,…)
2. Pt_2 is a point of tangency at the curve Cv2
3. Lt_2 is a line tangent to the curve Cv_2 at Pt_2
Tf1R transforms reversely Cv_2 into Cv_1, Pt_2 into Lt-1, Lt_2 into Pt_1.
In EQF-terminology:
Tf1/Tf1R = QA-Tf11/QA-Tf10 when we work in a Quadrangle
Tf1/TF1R = QL-Tf11/QL-Tf10 when we work in a Quadrilateral
The very special thing is that these dual transformations transform a point at a curve to a transformed tangent to a transformed curve and vice versa.

(Dual) Cayley-Bacharach Theorem
With these definitions we can define a dual of the Cayley-Bacharach Theorem.
The Cayley-Bacharach Theorem
states that given 8 random planar points there will be a unique 9th point such that every cubic through these 8 points also will pass through this unique 9th point.
The conjecture for the Dual Cayley-Bacharach Theorem
will be that given 8 random planar lines, there will be a unique 9th line such that every curve of class 3 tangent to these 8 lines also will be tangent to this unique 9th line.

We have to realize that this is a conjecture for the dual transformation in its properties of tangency as well as the dual Caley-Bacharach Theorem.
It is good to know that the conjectures were confirmed by lots of/several examples from you and me and probably also Bernard.

Many credits go to you for your insight and inventions regarding the dual transformation as well as the Dual Cayley-Bacharach Theorem. Very well !

Can you agree with my summary?

Best regards,
Chris

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Message: #2692
Date: 07/11/2017 10:19:14
From: tsihonglau
Subject: Curves of Quintangle from Generalized Circular Points

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Dear all,

Any two points can be treated as generalized circular points!
>From a quintangle, we can choose any two points as generalized circular points and a triangle remains!
>From a triangle and generalized circular points, we can get generalized triangle centers.
For example, A(1:0:0),B(0:1:0),C(0:0:1) is the reference triangle and D(p:q;r), E(u:v:w) are circular points.
The generalized orthocenter is 1/(qw+rv):1/(ru+pw):1/(pv+qu) = cevamul(aka ceva point) of circular points.
The generalized X(523) is
(p^2vw-qru^2:q^2wu-rpv^2:r^2uv-pqw^2)=Hirst inverse of circular points.
Any general quintangle has 10 generalized orthocenters and X(523)'s as the above, respectively.
The 10 orthocenters and 10 X(523)'s lie on two circumquartics of the quintangle, respectively.
The 10 orthocenters lie on a cubic.

Best regards,
Tsihong Lau

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Message: #2693
Date: 08/11/2017 8:32:54
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach Lines for QG/QL

```
Dear Chris,

no objections wrt your summary. In addition:

In EPG-terminology :
your Tf1 / Tf1R = my Tf3 / Tf4 when we work in a 5L.
(wrt my Tf2, Tf3, Tf4 see QFG#2683)

Attached two examples wrt cubics,
... which are the Tf2-image of a line pencil.
1st drawing, studying your summary,
2nd construction:
... 8 arbitrary points Pi on the cubic
... and their CB-point, always the double point,
... further the 8 lines Li = Tf3(Pi)
... and their CB-line,
... which is the Tf3-image of the double point
... and the double tangent of the Tf3-image of the cubic.

Best regards Eckart

PS: Thanks for your accepting remarks. Have recreating holidays.
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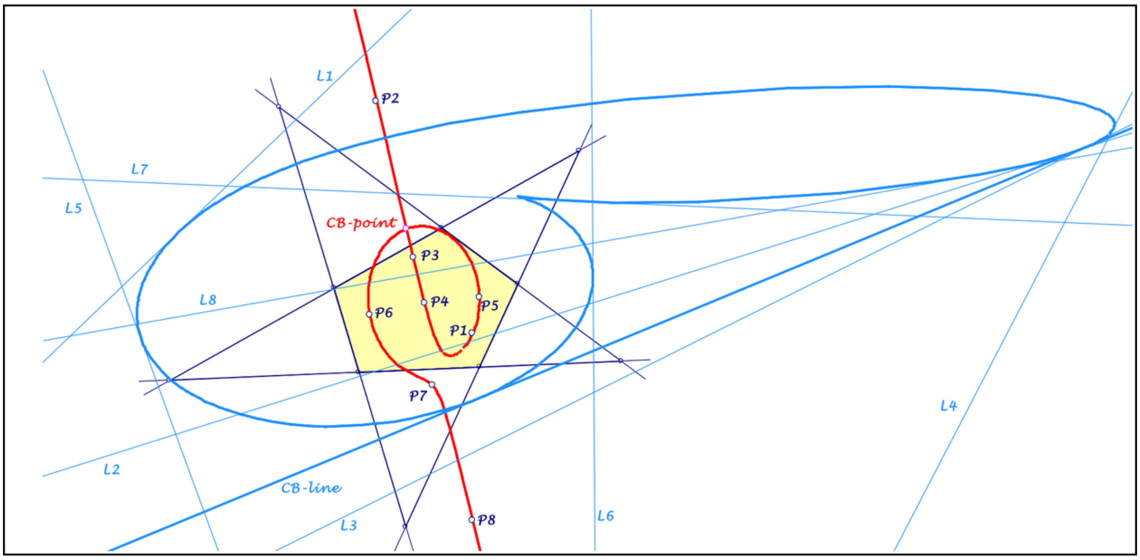

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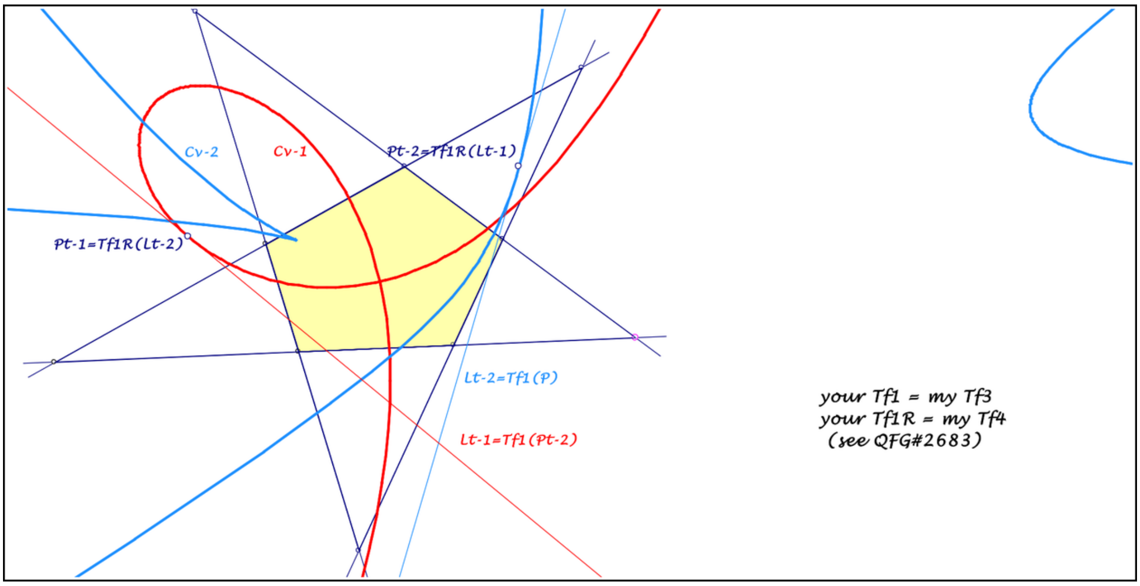

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Message: #2694
Date: 09/11/2017 1:08:49
From: tsihonglau
Subject: Foci of QA-Circumscribed Conics

```
Dear Eckart,

I have checked the sextic(=locus of foci of QA-Circumscribed
conics) is bicircular!
But I have no idea how to calculate the four foci of it.

Best regards,
Tsihong Lau
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Message: #2695
Date: 09/11/2017 3:38:57
From: eckart_schmidt@t-online.de
Subject: Foci of QA-Circumscribed Conics

```
Dear Tsihong Lau,
if I am not wrong this sextic is of class 6 and has 6 foci.
Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2696
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message
2696

```
Message number 2696 is not available in Yahoo groups.
```

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Message: #2697
Date: 15/11/2017 8:49:44
From: eckart_schmidt@t-online.de
Subject: 5L-analogon for 5P-s-Tf2

Dear Chris,

for a 5P the transformation 5P-s-Tf2 is the polarity wrt the
circumscribed conic 5P-s-Co1.
I haven´t found the analogon for a 5L in EPG:
... Consider a 5L and a line L
... with its 5 QL-Tf2 image lines,
... which have a common point.
This transformation maps a line to a point
... and is the polarity wrt the inscribed conic 5L-s-Co1.

Best regards Eckart

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #2698
Date: 15/11/2017 3:33:44
From: eckart_schmidt@t-online.de
Subject: CSC-related 5L-transformations

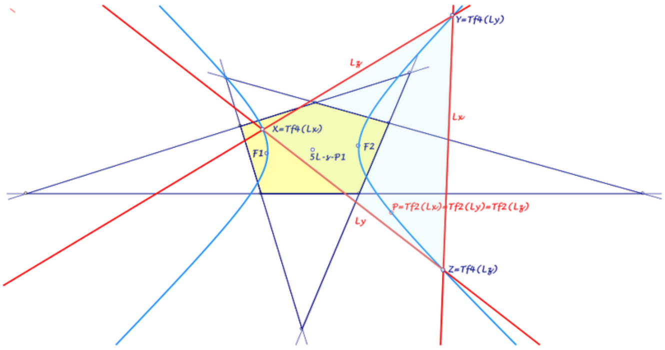
Dear Chris,
the 5L-transformation Tf2 in QFG#2683 maps a line to a point,
but there are two other lines with the same image.
Attached a construction of these further lines
with some properties of the corresponding triangles.
Two questions:
(1) How to construct the three lines,
... only knowing the common Tf2-image?
(2) What about the ninth Cayley-Bacharach line
... of the five 5L-lines and the three lines with the same
Tf2-image?
Best regards Eckart

EQF-Note 2017-11-15

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-related *5L*-Transformation III

The 5L-transformation Tf2 in QFG#2683 maps a line to a point, but there are two other lines with the same image. Here a construction of these further lines is given.



In *QFG#2683* several *5L*-transformations are described:

Tf0 ***point → circle:*** The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).

Tf2 ***line → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Tf3 and *Tf4* are inverse transformations. But wrt *Tf1* every point has two pre-images, which are partner wrt the transformation *Tf* in *QFG#2669*. *Tf2* has the curious property, that there are three lines with the same image point.

For a line L_x and its image point $Tf2(L_x)$ there are two other lines L_y, L_z with $Tf2(L_x) = Tf2(L_y) = Tf2(L_z)$.

2017-11-15.pdf

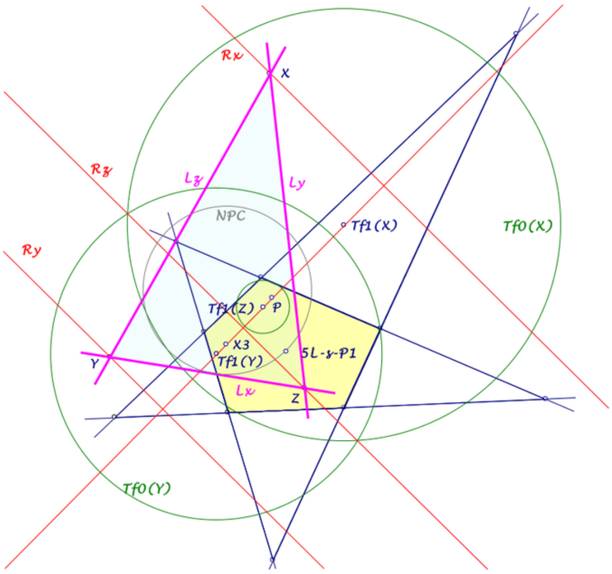
Construction:

We start with a line $L_{x,}$
… its *Tf2*-point *P*, its *Tf4*-point *X* and the foci F_1,F_2 of the inscribed conic *5L-s-Co1*
… and construct the orthogonal hyperbola *Hy*,
… … centered in *5L-s-P1,*
… … through *P, X,* F_1, F_2.
… The intersections *Y* and *Z* of L_x and *Hy*
… give the lines $L_y = XZ$ and $L_z = XY$.

Properties:

- **For points *X, Y, Z* with $L_x = YZ$, $L_y = ZX$, $L_z = XY$ … and $Tf2(L_x) = Tf2(L_y) = Tf2(L_z)$ holds:**
  $Tf4(L_x) = X$, $Tf4(L_y) = Y$, $Tf4(L_z) = Z$,
  $Tf3(X) = L_x$, $Tf3(Y) = L_y$, $Tf3(Z) = L_z$.

- **The points *X, Y, Z,* F_1, F_2 and $P = Tf2(L_{x,y,z})$ lie on an orthogonal hyperbola, centered in *5L-s-P1*.**



- **The nine-point circle of *XYZ* bears *5L-s-P1*.**

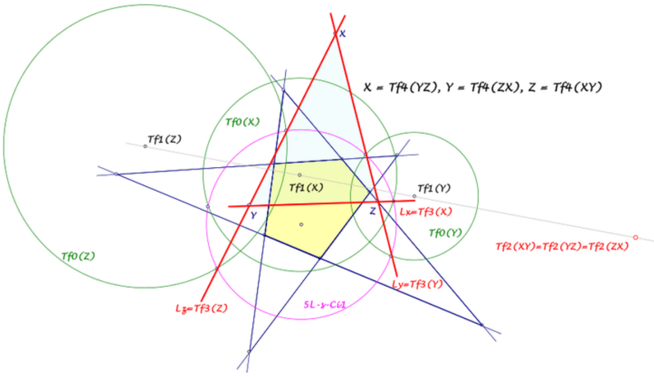
- **The *Tf1*-images of *X, Y, Z* are collinear with $P = Tf2(L_{x,y,z})$ and the orthocenter *X3* of *XYZ*.**

- **The radical axis R_z of the circles *Tf0(X)* and *Tf0(Y)* bears *Z* and is parallel to R_x and R_y …**

- **The radical axis of *Tf0(X)* and *5L-s-Ci1* is *YZ*, …**

2017-11-15.pdf

Finally a figure with all transformations for a triangle *XYZ* with
Tf2(YZ) = Tf2(ZX) = Tf2(XY).



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-11-15.pdf

Message: #2699
Date: 18/11/2017 1:57:24
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> 1. First generation - L1,L2,L3,L4,L5 are tangent lines to the circumconic of P1,P2,P3,P4,P5 at >>P1,P2,P3,P4,P5.(polarity from Quintangle/Quintilateral)
>> 2. Second generation -
>> involutary conjugate - point - quadrangle
>> Q1 - P1, P2,P3,P4,P5
>> Q2 - P2, P1,P3,P4,P5
>> Q3 - P3, P1,P2,P4,P5
>> Q4 - P4, P1,P2,P3,P5
>> Q5 - P5, P1,P2,P3,P4
>> It seems that P1,P2,P3,P4,P5 and Q1,Q2,Q3,Q4,Q5 lie on a point cubic!
>> And L1,L2,L3,L4,L5 and M1,M2,M3,M4,M5 lie on a line cubic

Suppose the coordinates of the first-generation quintangle are
P1(1:1:1)
P2(-1:1:1)
P3(1:-1:1)
P4(1:1:-1)
P5(p:q:z)
Then the coordinates of the second-generation quintangle are
Q1(r^2-q*r-p*r+q^2-p*q-3*p^2:r^2-q*r-p*r-3*q^2-p*q+p^2:-3*r^2-q*r-p*r+q^2-p*q+p^2)
Q2(r^2-q*r+p*r+q^2+p*q-3*p^2:r^2-q*r+p*r-3*q^2+p*q+p^2:-3*r^2-q*r+p*r+q^2+p*q+p^2)
Q3(r^2+q*r-p*r+q^2+p*q-3*p^2:r^2+q*r-p*r-3*q^2+p*q+p^2:-3*r^2+q*r-p*r+q^2+p*q+p^2)
Q4(r^2+q*r+p*r+q^2-p*q-3*p^2:r^2+q*r+p*r-3*q^2-p*q+p^2:-3*r^2+q*r+p*r+q^2-p*q+p^2)
Q5(1/p:1/q:1/r)
The equation of the circumcubic of both quintangles is:
(q+r)*(q-r)*x^2*(p*(q+r)*(q-r)*x+(r-p)*(p^2+q*(p-q+r))*y+(p-q)*(p^2+r*(p+q-r))*z)+...=0(cyclic)
We better dig out more properties of this cubic!
Please notice that this circumcubic and the circumconic of the quintangle and curves mentioned in #2692 are not related to any metrics.
That is, neither the line nor the circular points at infinity are involved. Please refer to #2246 for more information!
Best regards,
Tsihong Lau
```

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Message: #2700
Date: 18/11/2017 2:02:26
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

```
Dear all
Oops!  Please refer to my message #2446(not #2246) for more
information!
Best regards,
Tsihong Lau
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2701
Date: 18/11/2017 3:14:39
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> Now given two conics(as both point and line conics)
>> with four common points(=a quadrangle) and four
>> common lines(=a quadrilateral), we can
>> get a point and a line isoconjugations.
>> What is the relation of both isoconjugations?
>> What is the relation of the quadrangle and the
>> quadrilateral(maybe imaginary)?

The quadrangle and the quadrilateral are not dual!
But I found an interesting property.
There are 8 tangent points on the quadrilateral to the two conics.
They lie on a conic.
The diapleural(diagonal) triangle of the quadrangle is self-polar with respect to the last conic!
According to the principle of duality, there are 8 tangent lines through the quadrangle to the two conics.
They lie on a conic.
The diagonal trilateral(triangle) of the quadrilateral is self-polar with respect to the last conic!
What is the relation of the two conics and the two last conics?

Best regards,
Tsihong Lau

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2702
Date: 19/11/2017 2:16:25
From: tsihonglau
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

```
---In Quadri-Figures-Group@yahoogroups.com, wrote :
>> The quadrangle and the quadrilateral are not dual!
>> But I found an interesting property.
>> There are 8 tangent points on the quadrilateral
>> to the two conics.
>> They lie on a conic.
>> The diapleural(diagonal) triangle of the quadrangle
>> is self-polar with respect to the last conic!
>> According to the principle of duality, there are 8 tangent
>> lines through the quadrangle to the two conics.
>> They lie on a conic.
>> The diagonal trilateral(triangle) of the quadrilateral
>> is self-polar with respect to the last conic!

In fact, the quadrangle and the quadrilateral have the
same diapleural triangle/diagonal trilateral but are not dual.
So the properties are consequences of the above fact.

>> What is the relation of the two conics
>> and the two last conics?

I have no idea now!

Best regards,
Tsihong Lau
```

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Message: #2703
Date: 19/11/2017 3:34:37
From: tsihonglau
Subject: Operations and Involutions from Intersection of Algebraic Curves

---

```
Dear all,
We have known the famous Desargues involution theorem:
The two points of intersection of a line and circumconics of a
quadrangle form an involution.
http://users.math.uoc.gr/~pamfilos/eGallery/problems/DesarguesIn
volution.html
Let's consider higher degree algebraic curves.
1. The three points of intersection of a line and circumcubics
of a septangle(7-angle) form a 2 to 1 triple operation:
We can get P3 from (P1,P2).
Then we can get P1 from (P2,P3) and P2 from (P3,P1).
2. The three points of intersection of a line and circumcubics
of a octangle(8-angle) form a 1 to 2 triple operation:
We can get (P2,P3) from P1.
Then we can get (P3,P1) from P2 and (P1,P2) from P3.
3. The four points of intersection of a line and circumquartics
of a undecangle(11-angle) form 3 to 1 quadruple operation :
We can get P4 from (P1,P2,P3).
Then we can get P1 from (P2,P3,P4) and P2 from (P3,P4,P1) and P3
from (P4,P1,P2).
4. The four points of intersection of a line and circumquartics
of a duodecangle(12-angle) form a double pair involution :
(P1,P2) and (P3,P4) are double pairs. That is, we can get
(P1,P2) from (P3,P4) and vice versa.
Then (P1,P3) and (P2,P4) are double pairs, (P1,P4) and (P2,P3)
are, too.
5. The four points of intersection of a line and circumquartics
of a tredecangle(13-angle) form 1 to 3 quadruple operation :
We can get (P2,P3,P4) from P1.
Then we can get (P3,P4,P1) from P2 and (P4,P1,P2) from P3 and
(P1,P2,P3) from P4.
and so on...
According to the principle of duality, the duals of the above
operations are valid!
To be continued...
Best regards,
Tsihong Lau
```

Message: #2704
Date: 19/11/2017 7:25:11
From: eckart_schmidt@t-online.de
Subject: CSC-Circles for Points wrt a 5L

Dear Chris,

the CSC-images of a point wrt the 4L of a 5L are concyclic.
These CSC-circles are special circles:
For a given center there are two circles, which are CSC-circles.
Attached a construction.

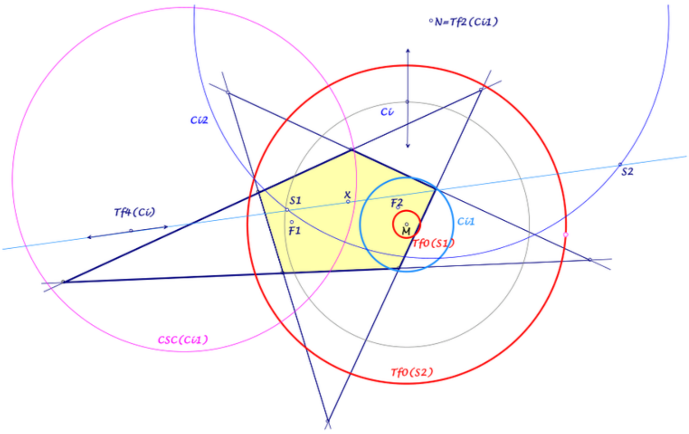
Best regards Eckart

EQF-Note 2017-11-19

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-Circles for Points wrt a 5L

The CSC-images of a point wrt the 4L of a 5L are concyclic. These CSC-circles are special circles: For a given center there are two circles, which are CSC-circles. Here a construction is given.



Transformations

For a construction several transformations are used, already mentioned in earlier *QFG*-messages, up to now not in *EPG* (except *Tf1* = *5L-s-Tf1*):

Tf0 ***point → circle:*** The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).

Tf2 ***line/circle → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line/circle → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Further a point *X*:

$$X = F_1F_2^\circ \cap F_1^\circ F_2.$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
F_1° and F_2° inverses of F_1 and F_2 wrt *5L-o-Ci1*.

2017-11-19.pdf

Construction

point *M*: Given center of the searched *CSC*-circles.

circle Ci_1: The two *CSC*-circles, centered in the given point *M* are inverse wrt a circle Ci_1 round *M* with radius $\sqrt{M.F_1 \times M.F_2}$.

line *L*: The *Tf4*-images of circles *Ci* round *M* give a line *L*, bearing point *X*.

circle Ci_2: Circle round $N = Tf2(Ci_1)$ orthogonal intersecting the circle $CSC(Ci_1)$ wrt any *4L* of the *5L*.

points S_1, S_2: Intersections of Ci_2 and *L*. $Tf0(S_1)$ and $Tf0(S_2)$ give the searched *CSC*-circles.

Background for the construction is the property of *CSC*-circles, that their *Tf2*- and *Tf4*-image points coincide.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-11-19.pdf

Message: #2705
Date: 24/11/2017 12:47:53
From: bernard.keizer
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

---

Dear Tsihong Lau,
This is indeed a beautiful property of the QA or the QL, which I didn't know ! (If I'm not wrong, it's not in EQF).
Let say it another way.
Any 2 conics inscribed in a QL (having therefore center on the Newton Line and foci on the Van Rees curve QL-Cu1) intersect in 4 points forming a QA ; the DT's of all these QA's are the same as the DT of the QL.
Neither of the QA's is the dual QA, as in this case a conic can apparently not be at the same time circumQA and QLinscribed.
Any 2 conics circumscribing a QA (having therefore the center on the 9points conic QA-Co1) have 4 common tangents forming a QL ; the DT's of all these QL's are the same as the DT of the QA.
For the same reason, neither of the QL's is the dual QL.
Best regards
Bernard

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Message: #2706
Date: 24/11/2017 1:18:35
From: eckart_schmidt@t-online.de
Subject: CSC-Circles for Points wrt a 5L

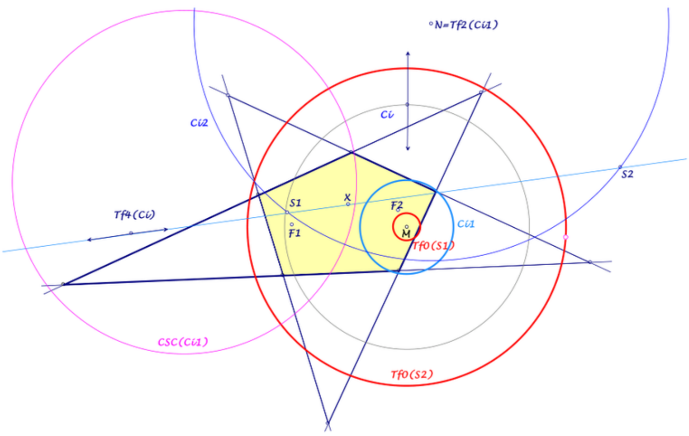
This message was postet 19.11 with an attachment, which doesn´t appear in QFG.
I try it once more. Two further messages sended 21.11 and 22.11. I haven´t found in QFG.
Dear Chris,
the CSC-images of a point wrt the 4L of a 5L are concyclic.
These CSC-circles are special circles:
For a given center there are two circles, which are CSC-circles.
Attached a construction.
Best regards Eckart

EQF-Note 2017-11-19

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-Circles for Points wrt a 5L

The CSC-images of a point wrt the 4L of a 5L are concyclic. These CSC-circles are special circles: For a given center there are two circles, which are CSC-circles. Here a construction is given.



Transformations

For a construction several transformations are used, already mentioned in earlier *QFG*-messages, up to now not in *EPG* (except *Tf1* = *5L-s-Tf1*):

Tf0 ***point → circle:*** The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.

Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).

Tf2 ***line/circle → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).

Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).

Tf4 ***line/circle → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Further a point *X*:

$$\boldsymbol{X = F_1F_2° \cap F_1°F_2.}$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
$F_1°$ and $F_2°$ inverses of F_1 and F_2 wrt *5L-o-Ci1*.

Construction

point *M*: Given center of the searched *CSC*-circles.

circle Ci_1: The two *CSC*-circles, centered in the given point *M* are inverse wrt a circle Ci_1 round *M* with radius $\sqrt{M.F_1 \times M.F_2}$.

line *L*: The *Tf4*-images of circles *Ci* round *M* give a line *L*, bearing point *X*.

circle Ci_2: Circle round $N = Tf2(Ci_1)$ orthogonal intersecting the circle $CSC(Ci_1)$ wrt any *4L* of the *5L*.

points S_1, S_2: Intersections of Ci_2 and *L*. $Tf0(S_1)$ and $Tf0(S_2)$ give the searched *CSC*-circles.

Background for the construction is the property of *CSC*-circles, that their *Tf2*- and *Tf4*-image points coincide.

Eckart Schmidt
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eckart_schmidt@t-online.de

2017-11-19.pdf

Message: #2707
Date: 24/11/2017 1:19:54
From: eckart_schmidt@t-online.de
Subject: 5L-X(...) Hofstadter points

```
Dear Chris,

excuse, I have difficulties to find observed properties in EPG:
Let us consider isogonal conjugated Hofstadter pair of points X
and Y
... as X(186) and X(265), X(5961) and X(5962), X(5963)
    and X(X5964)
... and the corresponding 5L-Hofstadter points 5L-X and 5L-Y
    as described wrt 5L-s-P2 and 5L-s-P3.
5L-X lies on the circle of the 5 CSC-images of 5L-Y.
Perhaps evident or already mentioned?

Best regards Eckart
```

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Message: #2708
Date: 24/11/2017 1:35:26
From: bernard.keizer
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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```
Dear Tsihong Lau,
This is indeed a beautiful property of the QA or QL, which I
didn't know (If I'm not wrong, it's not in EQF) !
Any 2 conics inscribed in a given  QL (with center on the Newton
Line and foci on the Van Rees cubic QL-Cu1) intersect in 4
points forming a QA ; the DT's of these QA's are the same as the
DT of the QL.
Any 2 conics circumscribed to a given QA (with center on the 9
points conic QA-Co1) have 4 common tangents forming a QL ; the
DT's of these QL's are the same as the DT of the QA.
These QA's or QL's are not the duals of the QL or the QA, as
apparently a conic cannot be at the same time circum-QA and
QL-inscribed if the QA and the QL are duals.
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2709
Date: 24/11/2017 1:57:09
From: eckart_schmidt@t-online.de
Subject: Polarity from Quadrangle/Qiadrilateral and Quintangle/Quintilate

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```
Dear Tsihong Lau,
that is an interesting configuration!
Starting with two inscribed conics of a QL.
... Their intersections give a QA.
... The 8 contact points on the QL-lines lie on a conic CoL,
... the 8 tangents in the QA-points envelop a conic CoA.

My observations:
(1) QA-Tf2 swaps the contact points on the QL-lines.
(2) QL-Tf2 swaps the tangents in the QA-points.
(3) For intersections P of CoA and a starting conic holds
... QA-Tf2(S) lies on CoL,
... on the tangent in S at the starting conic.
(4) (analog) For common tangents L of CoL
    and a starting conic holds
... QL-Tf2(L) is tangent to CoA,
... bearing the contact point of L and the starting conic.
(5) The center of CoA is the pole of QL-L1 wrt QA-Co1,
... ...  or: the QA-Tf2-image of the pole of QL-L1 wrt CoL.
... The center of CoL is the QA-Tf2-image of the pole
    of QL-L1 wrt CoA.
(6) If the centers of the starting conics
    are the points QL-2P2,
... then CoL is the polar circle of the diagonal triangle,
... intersecting QA-Co1 on QL-Cu1.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2710
Date: 25/11/2017 12:15:07
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,

I asked someone to reply my most important message #2446 but got no reply!
I post 6 messages to topic #1997 "Affine, Möbius and Projective Planes" but got no reply, either.
I will concentrate on Möbius plane in this topic.
Please refer to Wikipedia page for basic information! Especially miquelian Möbius plane!
https://en.wikipedia.org/wiki/M%C3%B6bius_plane
After EPG appears, I found many objects in EQF and EPG had better be studied on a Möbius plane.
I will give some examples in this and next messages of this topic.
Given a configuration of four points and four circles, we can get a conjugation from it related to QA-P4.
incidence relation
point - cycles ; cycle - points
A - b,c,d ; a - B,C,D
B - c,d,a ; b - C,D,A
C - d,a,b ; c - D,A,B
D - a,b,c ; d - A,B,C
The conjugate of a point P with respect to the above configuration can be constructed as follows:
point - inversion of P with respect to cycle
Pa - a
Pb - b
Pc - c
Pd - d
cycle - three points
pa - PbPcPd
pb - PcPdPa
pc - PdPaPb
pd - PaPbPc
point - inversion of P with respect to cycle
P'a - pa
P'b - pb
P'c - pc
P'd - pd
cycle - three points
p'a - P,A,P'a
p'b - P,B,P'b
p'c - P,C,P'c

p'd - P,D,P'd
The last four cycles concur at a point U, which is the conjugate of P.
We can construct the conjugate of U similarly as follows:
point - inversion of U with respect to cycle
Ua - a
Ub - b
Uc - c
Ud - d
cycle - three points
ua - UbUcUd
ub - UcUdUa
uc - UdUaUb
ud - UaUbUc
point - inversion of U with respect to cycle
U'a - ua
U'b - ub
U'c - uc
U'd - ud
cycle - three points
u'a - U,A,U'a
u'b - U,B,U'b
u'c - U,C,U'c
u'd - U,D,U'd
The last four cycles u'a,u'b,u'c,u'd coincide with the above four cycles p'a,p'b,p'c,p'd respectively and concur at P, which is the conjugate of U.
If P is the point at infinity, then Pa,Pb,Pc,Pd and P'a,P'b,P'c,P'd become the first and second generations of circumcenter quadrangles of the reference quadrangle ABCD and U becomes the homothetic center of ABCD and P'a,P'b,P'c,P'd.
In other words, U is QA-P4 the isogonal center of ABCD.
Ua,Ub,Uc,Ud is the isogonal conjugate quadrangle of ABCD.
I will explain how to construct isogonal conjugates on Möbius plane in next message.
To be continued...
Best regards,
Tsihong Lau

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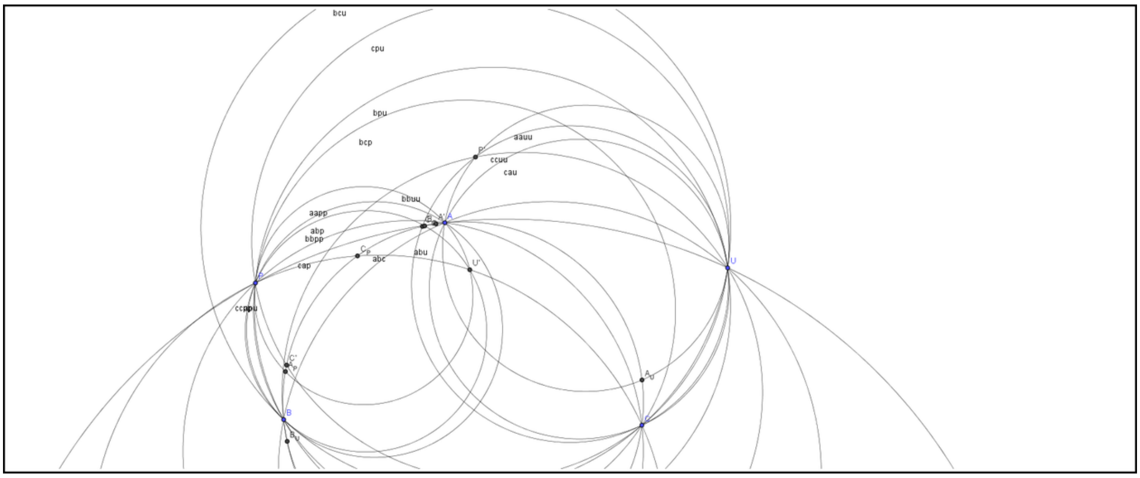
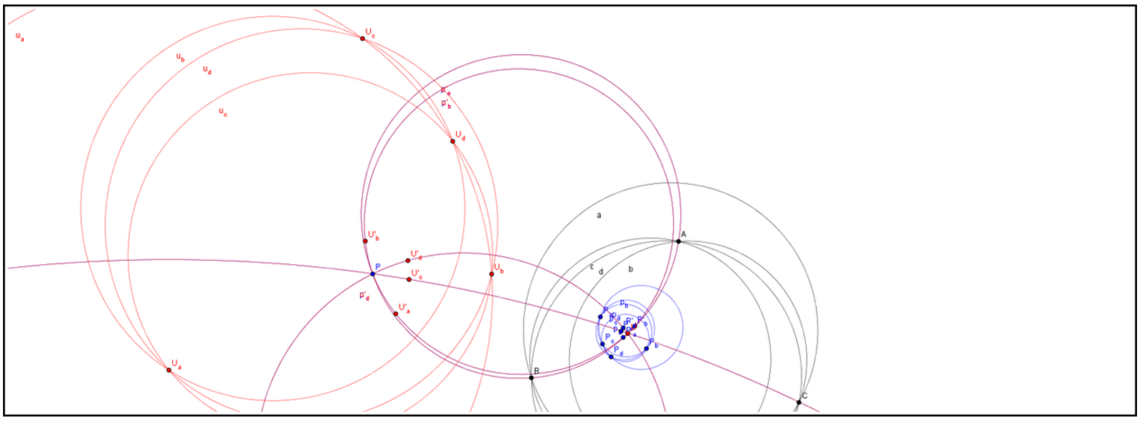
Message: #2711
Date: 25/11/2017 12:50:46
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,
I explain how to construct the isogonal conjugate of a point P with respect to a reference triangle ABC on a Möbius plane.
First, we must have the point at infinity U to define a "line" on a Möbius plane.
Any cycle through U is called a line!
Two lines tangent at U(intersecting only at U) are parallel!
Let A',B',C' be the points of intersection of the lines AP,BP,CP and the cycle ABC respectively.
Let A_U,B_U,C_U be the second points of intersection of the parallel lines through A',B',C' to three sidelines BC,CA,AB repsectively and the cycle ABC.
Then the three lines AA_U,BB_u,CC_U concur at P', which is the isogonal conjugate of P with respect to ABC.
The above construction is the same as on a Euclidean plane!
Please notice there is no point at infinity(a fixed object in message #2446) in the previous message!
If we interchange the roles of P and U, we can get the isogonal conjugate U' of U with respect to ABC.
The P' and U' are inverses with respect to cycle ABC!
To be continued...
Best regards,
Tsihong Lau



mutual_isogonal.png



conjugate_four_point_four_cycle.png

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Message: #2712
Date: 25/11/2017 12:55:54
From: eckart_schmidt@t-online.de
Subject: CSC-Circles for Points wrt a 5L

```
This message was posted 19.11 and 23.11 with an attachment,
which doesn´t appear in QFG.
I try it once more.

Dear Chris,

the CSC-images of a point wrt the 4L of a 5L are concyclic.
These CSC-circles are special circles:
For a given center there are two circles, which are CSC-circles.
Attached a construction.

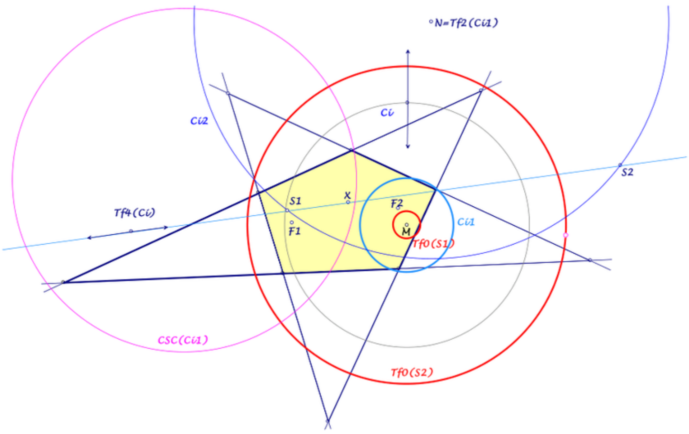
Best regards Eckart
```

EQF-Note 2017-11-19

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

CSC-Circles for Points wrt a 5L

The CSC-images of a point wrt the 4L of a 5L are concyclic. These CSC-circles are special circles: For a given center there are two circles, which are CSC-circles. Here a construction is given.



Transformations

For a construction several transformations are used, already mentioned in earlier *QFG*-messages, up to now not in *EPG* (except *Tf1* = *5L-s-Tf1*):

Tf0 ***point → circle:*** The 5 *CSC*-images of a point *P* wrt the *4L* of a *5L* are concyclic on the circle *Tf0(P)*.
Tf1 ***point → point:*** *Tf1(P)* is the center of the circle *Tf0(P)* (see *5L-s-Tf1* in *EPG*).
Tf2 ***line/circle → point:*** Radical axes for the 5 *CSC*-images of a line/circle have a common point (see *QFG#780*).
Tf3 ***point → line:*** *Tf3(P)* is the radical axis of *Tf0(P)* and *5L-o-Ci1* (see *QFG#790*).
Tf4 ***line/circle → point:*** Radical axes for the *Tf0*-circles of the points of a line/circle have a common point (see *QFG#790*).

Further a point *X*:

$$X = F_1F_2^\circ \cap F_1^\circ F_2.$$

F_1 and F_2 foci of the inscribed conic *5L-s-Co1*,
F_1° and F_2° inverses of F_1 and F_2 wrt *5L-o-Ci1*.

Construction

point *M*: Given center of the searched *CSC*-circles.

circle Ci_1: The two *CSC*-circles, centered in the given point *M* are inverse wrt a circle Ci_1 round *M* with radius $\sqrt{M.F_1 \times M.F_2}$.

line *L*: The *Tf4*-images of circles *Ci* round *M* give a line *L*, bearing point *X*.

circle Ci_2: Circle round $N = Tf2(Ci_1)$ orthogonal intersecting the circle $CSC(Ci_1)$ wrt any *4L* of the *5L*.

points S_1, S_2: Intersections of Ci_2 and *L*. $Tf0(S_1)$ and $Tf0(S_2)$ give the searched *CSC*-circles.

Background for the construction is the property of *CSC*-circles, that their *Tf2*- and *Tf4*-image points coincide.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-11-19.pdf

Message: #2713
Date: 25/11/2017 8:56:48
From: eckart_schmidt@t-online.de
Subject: Pivotal isocubic for a 5L

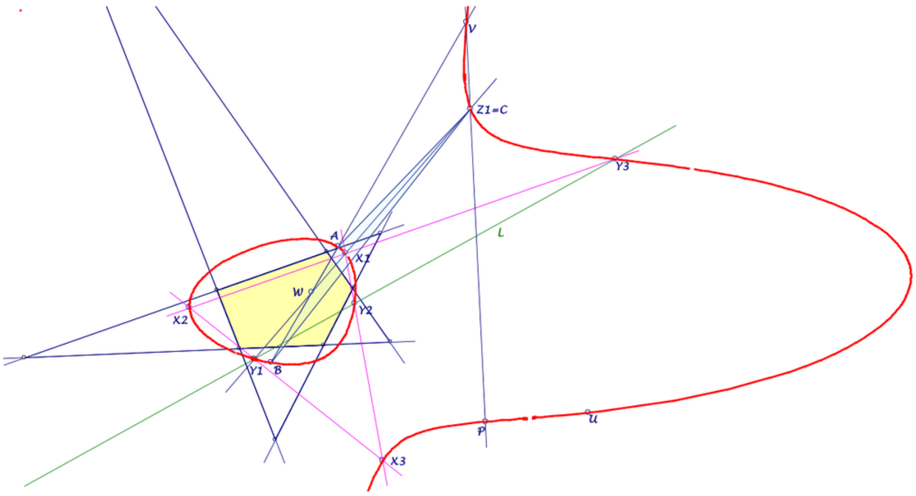
Dear Chris,
a point X has wrt the five quadrilaterals of a 5L five dual lines,
which can have a common point Y.
Points X with this property give a pivotal isocubic,
which bears also the common points in a reciprocal relation.
Attached a research as pdf and word file.
I hope, the attachments appear in QFG.
There will be an analog result for 5Ps.
Best regards Eckart

EQF-Note 2017-11-25

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Pivotal Isocubic for a 5L

A point X has wrt the five quadrilaterals of a 5L - in the sense of QFG#1516 - five dual lines, which can have a common point Y. Points X with this property Φ give a pivotal isocubic, which bears also the common points Y in a reciprocal relation.



We start with a *5L* and an arbitrary line *L*
… and points on *L* with their 5 dual lines wrt the *4L* of the *5L*.
The loci of intersections of the five dual lines
… are conics with three common points X_1, X_2, X_3,
… which have the property Φ
… as well as the common points Y_1, Y_2, Y_3,
… with $Y_i = L \cap X_jX_k$.

The points X_i and Y_i for a line pencil give a construction for the curve of the points with property Φ.

- **Points with the property Φ give a pivotal isocubic.**

Now we repeat the procedure for one line $X_iY_i = L$
… and get three *X*-points on the cubic: X_i, Y_i and a new point *P*,
… which is the tangential of X_i and Y_i wrt the cubic,
… and will be the pivot.

2017-11-25.pdf

The common point of the dual lines of P is a new point Z_i,
… which is the 3rd intersection of X_iY_i and the cubic,
… and will be one vertex for a reference triangle.

Once more we repeat the procedure for the line $PZ_i = L$
… and get three X-points on the cubic: P, Z_i and a new point U,
… which is the tangential of P and Z_i wrt the cubic.
The common point of the dual lines of U is a new point V,
… which is the 3rd intersection of PZ_i and the cubic.

Let W be the 4th harmonic point of Z_i wrt X_i and Y_i.

Once more we repeat the procedure for the line $VW= L$
… and get three X-points: V and two new points A and B,
… which give with $C = Z_i$ the reference triangle.

In this way the cubic is a pivotal isocubic
… with reference triangle ABC,
… an isoconjugation with fixed points X_i and Y_i
… and the pivot P.

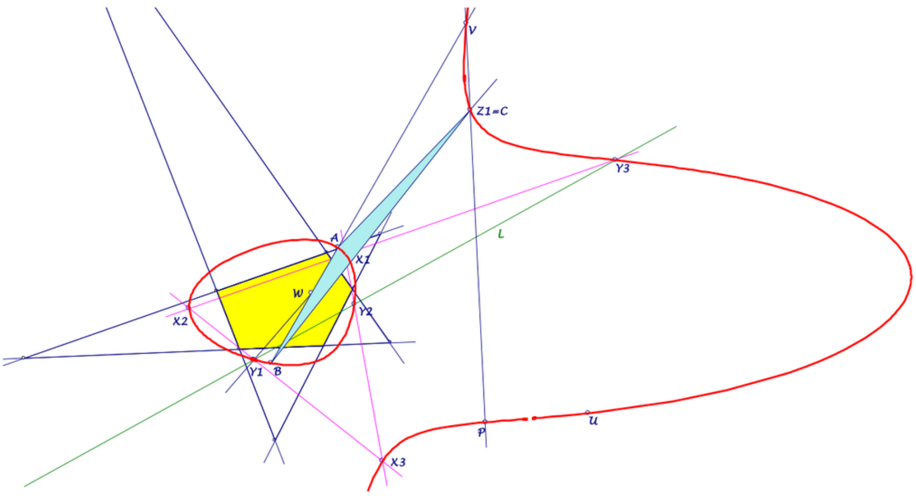
Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

EQF-Note 2017-11-25

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Pivotal Isocubic for a 5L

A point X has wrt the five quadrilaterals of a 5L - in the sense of QFG#1516 - five dual lines, which can have a common point Y. Points X with this property give a pivotal isocubic, which bears also the common points Y in a reciprocal relation.



We start with a *5L* and an arbitrary line *L*
… and points on *L* with their 5 dual lines wrt the *4L* of the *5L*.
The loci of intersections of the five dual lines
… are conics with three common points X_1, X_2, X_3,
… which have the property
… as well as the common points Y_1, Y_2, Y_3,
… with $Y_i = L \cap X_jX_k$.

The points X_i and Y_i for a line pencil give a construction for the curve of the points with property .

- **Points with the property give a pivotal isocubic.**

Now we repeat the procedure for one line $X_iY_i = L$
… and get three *X*-points on the cubic: X_i, Y_i and a new point *P*,
… which is the tangential of X_i and Y_i wrt the cubic,
… and will be the pivot.

The common point of the dual lines of P is a new point Z_i,
… which is the 3rd intersection of X_iY_i and the cubic,
… and will be one vertex for a reference triangle.

Once more we repeat the procedure for the line $PZ_i = L$
… and get three X-points on the cubic: P, Z_i and a new point U,
… which is the tangential of P and Z_i wrt the cubic.
The common point of the dual lines of U is a new point V,
… which is the 3rd intersection of PZ_i and the cubic.

Let W be the 4th harmonic point of Z_i wrt X_i and Y_i.

Once more we repeat the procedure for the line $VW= L$
… and get three X-points: V and two new points A and B,
… which give with $C = Z_i$ the reference triangle.

In this way the cubic is a pivotal isocubic
… with reference triangle ABC,
… an isoconjugation with fixed points X_i and Y_i
… and the pivot P.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

Message: #2714
Date: 26/11/2017 5:02:41
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,

The previous two messages discuss the dual 4 point and 4 cycle configuration.
This message discusses the dual 8 point and 8 cycle configuration.
The former is much like a quadrangle, while the latter is much like a quadrilateral.
Please notice points and cycles are not dual on a Möbius plane.
These two configurations are the most important on a miquelian Möbius plane, I think.

incidence relation

```
point - cycle   ; cycle - point
P  - p,pa,pb,pc ; p - P,Pa,Pb,Pc
Pa - p,pa,ub,uc ; pa - P,Pa,Ub,Uc
Pb - p,pb,uc,ua ; pb - P,Pb,Uc,Ua
Pc - p,pc,ua,ub ; pc - P,Pc,Ua,Ub
U  - u,ua,ub,uc ; u - U,Ua,Ub,Uc
Ua - u,ua,pb,pc ; ua - U,Ua,Pb,Pc
Ub - u,ub,pc,pa ; ub - U,Ub,Pc,Pa
Uc - u,uc,pa,pb ; uc - U,Uc,Pa,Pb
```

This configuration is the one of the initial Clifford's circle theorem.
https://en.wikipedia.org/wiki/Clifford%27s_circle_theorems
(Pa,Ua),(Pb,Ub),(Pc,Uc)
The 4 pairs of points (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc) share no common no cycles.
The 4 pairs of cycles (p,u),(pa,ua),(pb,ub),(pc,uc) share no common no points.
If P is the point at infinity, then p,pa,pb,pc become the reference quadrilateral and u,ua,ub,uc become the circumcircles of the 4 exline(component) trilaterals and concur at U, which is QL-P1 the Miquel point of the reference quadrilateral.
The same property applies to the other 3 pairs of points (Pa,Ua),(Pb,Ub),(Pc,Uc).

No we discuss the Clawson-Schmidt conjugation of it.
Given a point X in general position, we choose two pairs of points, for example, (Pb,Ub) and (Pc,Uc).

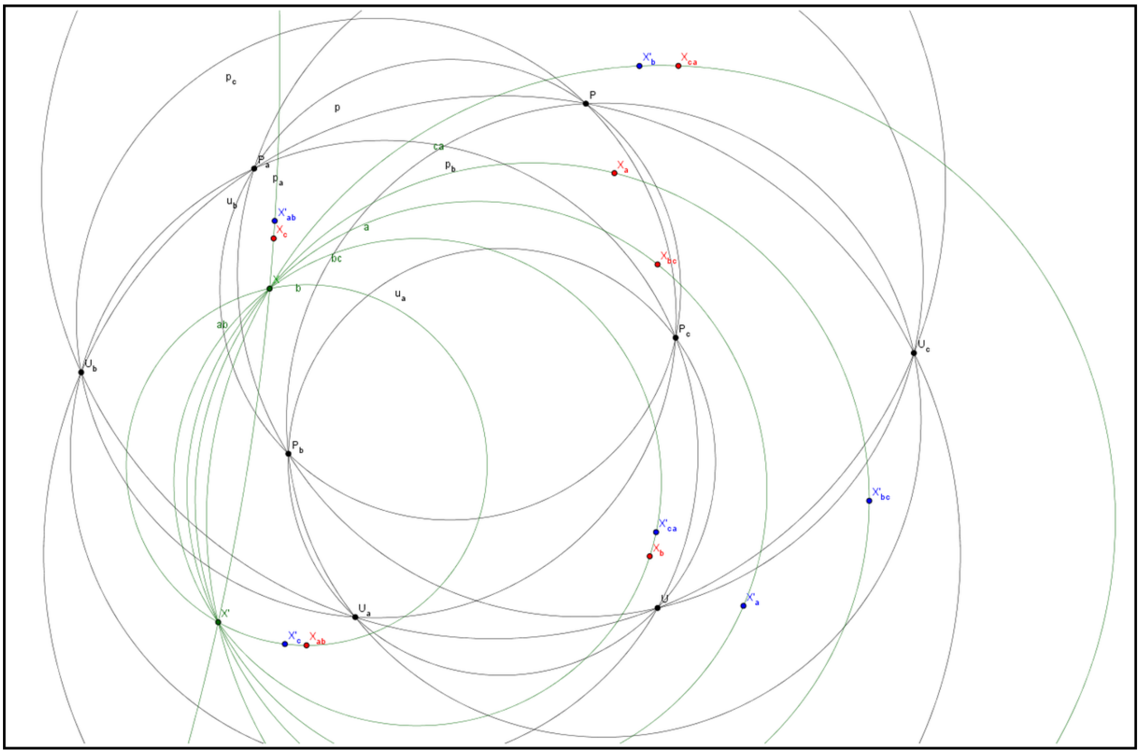
We define two points Xbc and X'bc as the second points of intersection of two cycle pairs ((Pb,Pc,X),(Ub,Uc,X)) and ((Pb,Uc,X),(Ub,Pc,X)) respectively.
Then the four cycles
(Pb,Pc,X'bc),(Ub,Uc,X'bc),(Pb,Uc,Xbc),(Ub,Pc,Xbc) concur at the Clawson-Schmidt conjugate X' of X with respect to the configuration.
We can choose any two pairs of points and get the same Clawson-Schmidt conjugate X'.

```
points   - two pairs of points
Xa ,X'a  - (P,U),(Pa,Ua)
Xb ,X'b  - (P,U),(Pb,Ub)
Xc ,X'c  - (P,U),(Pc,Uc)
Xbc,X'bc - (Pb,Ub),(Pc,Uc)
Xca,X'ca - (Pc,Uc),(Pa,Ua)
Xab,X'ab - (Pa,Ua),(Pb,Ub)
```

Moreover, there exist six cycles (Xa,X'bc,X,X'),(Xb,X'ca,X,X'),(Xc,X'ab,X,X'),(Xbc,X'a,X,X'),(Xca,X'b,X,X'),(Xab,X'c,X,X').
The 4 pairs of points (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc) are Clawson-Schmidt conjugates!

We can even define QL-Cu1 on a miquelian Möbius plane as the locus of X such that X and its Clawson-Schmidt conjugate X' and the 8 points lie on a cubic.
In other words, QL-Cu1 is Clawson-Schmidt conjugate invariant.
In the perspective of Möbius plane, we can conceive easily the pair properties.
For example, the 5 and 9 properties listed in EQF:
5. QL-Cu1 is the locus of all points P for which the reflections of P in the 4 quadrilateral lines are concyclic (Seiichi Kirikami in Ref-34, EQF message #1093).
9. QL-Cu1 is the locus of points, whose reflections in the circumcircles of the QL-Component Triangles are concyclic (Eckart Schmidt in Ref-34, EQF message #403).
It seems the pair property of 6 is not listed in EQF.
6. QL-Cu1 is the locus of all points P for which the feet of the perpendiculars from P to the 4 quadrilateral lines are concyclic (all lie on a circle).

Best regards,
Tsihong Lau



clawson_schmidt_conjugate_eight_point_eight_cycle.png

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Message: #2715
Date: 26/11/2017 9:55:43
From: chris.vantienhoven
Subject: Möbius Plane Geometry

Dear Tsihong Lau,

I admire your perseverance. I read your message #2446 again.
I did not react earlier because the matter was quite new to me and I did not (and still have not) the time to study it fully.
Nevertheless I share your opinion that the Desarguesian Projective Plane, the Pappian Plane, the Möbius Plane and the Miquelian Möbius Plane have lots of potential for Quadrilateral Geometry.
I like your systematic approach in message #2446. Especially "4. configuration=set of ordered or unordered objects with certain relation" I find it very interesting. I myself also was wondering about the many special point/lines-network-configurations we encounter in our studies.
And now you enlisted some of them.
The initial Clifford's circle theorem you mentioned in your last message I find amazing.
I am sure you will go on with your search in this matter and hope for more results.

Best regards,
Chris

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Message: #2716
Date: 26/11/2017 4:11:01
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,

I continue my previous message #2714.
Clawson-Schmidt conjugate table
P,Pa,Pb,Pc <-> U,Ua,Ub,Uc
p,pa,pb,pc <-> u,ua,ub,uc
cycles through P,Pa,Pb,Pc <-> cycles through U,Ua,Ub,Uc
cycles through (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc) <-> cycles through (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc)
cycles such that (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc) are inverses <-> cycles such that (P,U),(Pa,Ua),(Pb,Ub),(Pc,Uc) are inverses

We can define QL-Ci3 the Miquel circle and QL-L2 the Steiner line with respect to the pair of points (P,U) as:
The inverses(=circumcenters) of P with respect to u,ua,ub,uc are concyclic with U.
The cycle is equivalent to QL-Ci3.
The inverses(=reflections) of U with respect to p,pa,pb,pc are concyclic with P.
The cycle is equivalent to QL-L2.
We can choose other pairs of points and get other Miquel circles and Steiner lines.
They are all Clawson-Schmidt conjugates.
The orthocenters(=isogonal conjugates of circumcenters with respect to the point at infinity P) of u,ua,ub,uc are concyclic with P.
The cycle is also equivalent to QL-L2.
The Orthocenters of the 4 component triangles of a Quadrilateral are collinear. The line through these 4 Orthocenters is the Steiner Line.
The reflections of QL-P1 in the 4 basic lines of the Reference Quadrilateral lie on QL-L2. See Ref-34, Seiichi Kirikami, QFG message # 1091.
It seems that the pair property the orthocenters(=isogonal conjugates of reflections with respect to QL-P1 the Miquel point U) of p,pa,pb,pc are concyclic with U is not listed in EQF.

In topic #1788, especially message #1802, Eckart found a triangle chain:
I found the following chain for further points with this property (ig = isogonal conjugated, iv =inversion in the circumcircle)

X(3) ig [X(4)] iv X(186) ig X(265) iv X(5961) ig X(5962) iv
X(5963) ig X(5964) iv X ig Y iv Z ig ... iv ..
I think it had better be studied using this configuration on a
Möbius plane.
Isogonal conjugation and inversion are very basic involutions on
it!

Best regards,
Tsihong Lau

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Message: #2717
Date: 27/11/2017 11:33:26
From: bernard.keizer
Subject: Möbius Plane Geometry

Dear Tsihong Lau,
I continue to admire your efforts in order to fine new
properties in old figures wih alternative geometries !
Only a remark.
The chain you describe at the end is known as Hofstadter chain,
explained in ETC at X360.
The Hofstadter points coincide with the socalled n-angle centers
Hr only if r is integer.
As I explained in old messages, you may find a link with epi- or
hypocycloïds tangent to 4 lines ...
Best regards
Bernard

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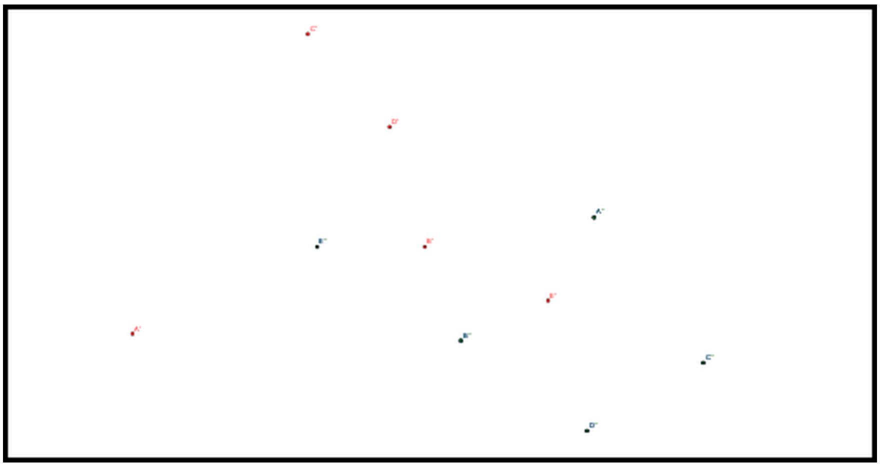
Message: #2718
Date: 28/11/2017 12:25:49
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,
I continued to study the conjugation mentioned in the first
message #2710 in the topic today and found a wonderful
conjugation of quintangles!
Given five points A,B,C,D,E(=a quintangle) in general position
on a miquelian(I think) Möbius plane, we can construct the
conjugates A',B',C',D',E' of A,B,C,D,E:
conjugates - with respect to
A,A' - B,C,D,E
B,B' - C,D,E,A
C,C' - D,E,A,B
D,D' - E,A,B,C
E,E' - A,B,C,D
We can construct the conjugates A'',B'',C'',D'',E'' of
A',B',C',D',E'
conjugates - with respect to
A',A'' - B',C',D',E'
B',B'' - C',D',E',A'
C',C'' - D',E',A',B'
D',D'' - E',A',B',C'
E',E'' - A',B',C',D'
Surpringly! Quintangles ABCDE and A''B''C''D''E'' coincide!
In other words, quintangles ABCDE and A'B'C'D'E' are conjugates!
We must study this conjugation thoroughly!
For example, given ABCDE on a complex plane, we compute
A'B'C'D'E'.
Best regards,
Tsihong Lau



conjugate_quintangle_four_point_four_cycle.ggb

Message: #2719
Date: 28/11/2017 4:37:21
From: tsihonglau
Subject: Möbius Plane Geometry

Dear all,
I gave two conjugations in messages #2710 and #2714.
I raise two questions:
1. How many fixed points do these two conjugations have?
I observed that the former have at least four fixed points and the latter have at least two.
2. What is the locus of P such that P and its conjugate P'(former) are conconic with the quadrangle?
I define QL-Cu1 QL-Quasi Isogonal Cubic be the locus of P such that P and its conjugate P'(latter) are concubic with the octangle.
Best regards,
Tsihong Lau

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Message: #2720
Date: 28/11/2017 5:13:19
From: eckart_schmidt@t-online.de
Subject: Möbius Plane Geometry

Dear Tsihong Lau,
some observations wrt the conjugation in QFG#2710 for a QA:
(1) The conjugation maps the diagonal triangle to the Miquel triangle.
(2) Image of QA-P4 is any point at infinity.
(3) QA-Cu1 is invariant wrt the conjugation.
(4) The conjugation has four fixed points
... on QA-Cu1 with tangential QA-P3.
(5) The quadrangle of the fixed points
... has the same Miquel triangle
... and the same QA-Cu1,
... its QA-P3 is the initial QA-P4,
... its QA-P4 is the initial QA-P3.
(6) The conjugation maps lines to sextics
... QA-circumscribed
... through QA-P4.
(7) The Cayley Bacharach ninth point of the two quadrangles
... is the intersection of QA-Cu1 and its asymptote.
Best regards Eckart

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Message: #2721
Date: 28/11/2017 6:17:55
From: tsihonglau
Subject: Möbius Plane Geometry

Dear Eckart,

You think in projective plane not Möbius plane.
This topic discusses Möbius plane geometry.
In message #2711, I gave " First, we must have the point at infinity U to define a "line" on a Möbius plane.
Any cycle through U is called a line!"
The conjugation in message #2710 is not concerned about the point at infinity!
Please refer to my message #2446:
"6. configuration=variable+fixed objects
Some objects are variable while other are fixed.
The most common fixed objects are the line and the circular points at infinity. The most common variable objects are the reference triangle/trilateral and quadrangle/quadrilateral. We can discuss two reference triangles/trilaterals but the line and the circular points at infinity remain the same. The basic distinction is very fundamental."
The point at infinity is "the only fixed object" on a Möbius plane.
What I ask is not concerned about it!
I guessed the locus is the Möbius plane version of QA-Cu1 but I cannot construct it!
I cannot construct the Möbius plane version of QL-Cu1, either.
Although I defined it!

Best regards,
Tsihong Lau

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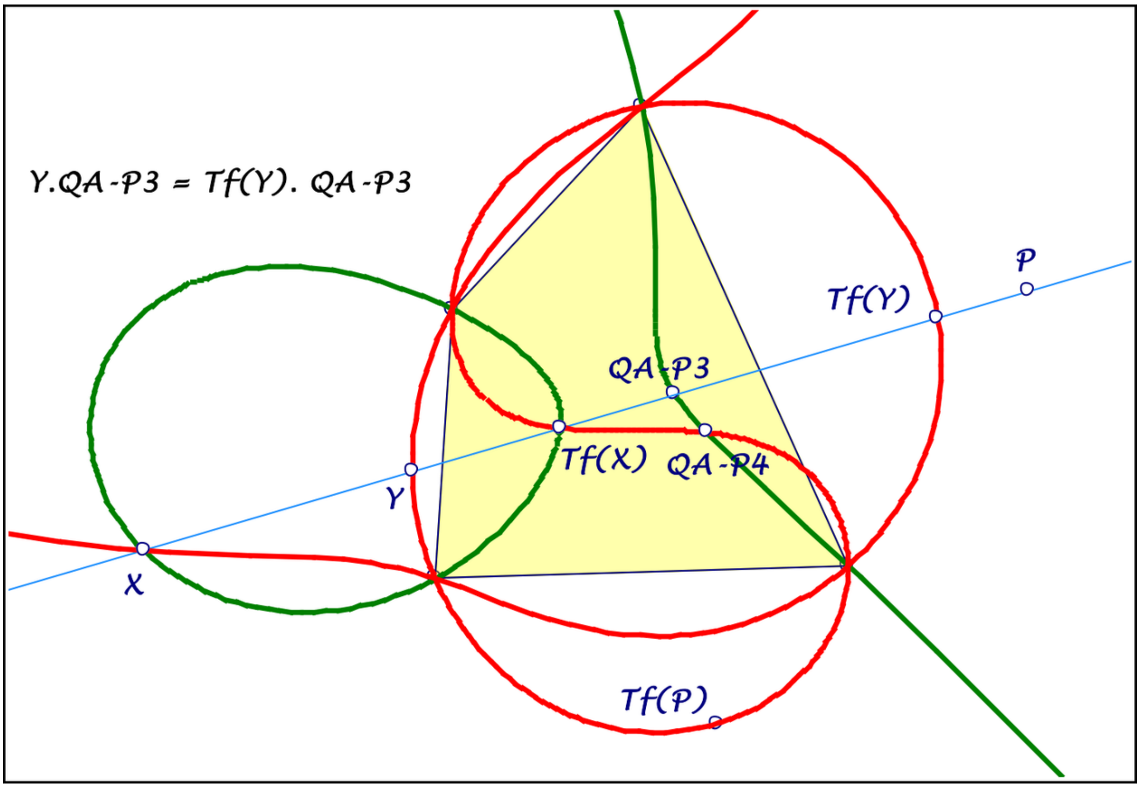
Message: #2722
Date: 29/11/2017 12:01:51
From: eckart_schmidt@t-online.de
Subject: Möbius Plane Geometry

Dear Tsihong Lau,

I think, it can be of interest, to study the conjugation Tf in QFG#2710 also for a projective plane.
For example:
QA-Cu1 is the locus for not QA-P3-equidistant Tf-partners on lines through QA-P3.

Best regards Eckart



2017-11-29.pdf

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Message: #2723
Date: 30/11/2017 3:19:36
From: tsihonglau
Subject: Möbius Plane Geometry

```
Dear all,

This message is a sequel to messages #2710 and #2711.
If P is the point at infinity, then Pa,Pb,Pc,Pd and
P'a,P'b,P'c,P'd become the first and second generations of
circumcenter quadrangles of the reference quadrangle ABCD and
Ua,Ub,Uc,Ud and U'a,U'b,U'c,U'd become the first and second
generations of isogonal conjugate quadrangles of it.
Moreover, we get the following concyclic points with P and U.
   Pa,Ua
   Pb,Ub
   Pc,Uc
   Pd,Ud
   A,P'a,U'a
   B,P'b,U'b
   C,P'c,U'c
   D,P'd,U'd
Because P and U are conjugates, we can interchange the roles of
P and U.
So the circumcenter and isogonal conjugate quadrangles are
objects derived from this conjugation!
We comprehend the essence of QA-P4 the isogonal center from the
perspective on a Möbius plane.

Best regards,
Tsihong Lau
```

Message: #2724
Date: 02/12/2017 5:02:31
From: tsihonglau
Subject: Möbius Plane Geometry

```
Dear all,

This message is a sequel to messages #2723.
If P is the point at infinity, let A_P,B_P,C_P be the diapleural
triangle of the reference quadrangle ABCD.
Similarly we can get A_U,B_U,C_U as U is the point at infinity.
The latter is the QA-Tr2 Miquel triangle of the reference
quadrangle ABCD with respect to the point at infinity P in fact.
Because P and U are conjugates, we can interchange the roles of
P and U.
So the diapleural and Miquel triangles are objects derived from
this conjugation!
We comprehend the essence of QA-P4 the isogonal center from the
perspective on a Möbius plane.
However, unlike on a projective plane,
ABCD,A_P,B_P,C_P,A_U,B_U,C_U do not lie on a cubic on a Möbius
plane.
In other words, it seems there is no QA-Cu1 on the latter.

Best regards,
Tsihong Lau
```

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Message: #2725
Date: 08/12/2017 11:06:23
From: chris.vantienhoven
Subject: 5L-X(...) Hofstadter points

```
Dear Eckart,

This beautiful feature is new to me.
It appears that the Hofstadter points in 3L, 4L and 5L still
produce new properties.
I don't know if you noticed that also the 5 CSC-images of 5L-Y
each lie on the corresponding 4L-X-circle.
I made notice of your beautiful property in EPG at the pages of
5L-s-P2 and 5L-s-P3.

Best regards,
Chris
```

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Message: #2726
Date: 08/12/2017 11:07:58
From: chris.vantienhoven
Subject: In case of no access to EQF or QPG please let me know

---

```
Dear friends,
In case of no web-access to EQF or QPG please let me know.
Best regards,
Chris
```

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Message: #2727
Date: 09/12/2017 10:58:58
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
Trying to reproduce Bernard Gibert's construction of the nK named Cu and it's dual Siebeck's sextic of class 3 named Se, I come to following remarks and questions and I hope you will be able to help me.
For a given dual QA/QC and it's diagonal triangle DT, you already mentionned that for any point P there is a nK through the 6 QL-vertices having the cevian triangle of P wrt DT as reference triangle and the transformation swapping the opposite QL-vertices as isoconjugation. The root R of the nK is the trilinear pole of the line L through the 3rd intersections of tne sides of the cevian triangle of P wrt DT with the nK. Let's name Q the dual point of L.
We know perfectly 2 examples of these nK's :
For P being the centroïd of DT, L is the Newton Line of DQL (QL formed by the diagonals, sides of DT and the Newton Line), Q is QA-P1 and R is the trilinear pole of L wrt mid DT (cevian triangle of the centroïd). The nK is Cu and it's dual is Se. You made a complete study of these 2 curves.
For P being the orthocenter of DT, the cevian triangle is the orthic triangle of DT, the transformation is the isoconjugation and the nK is the isogonal circular focal cubic of Van Rees QL-Cu1. What are L, Q and R ?
The dual of this nK is a sextic tangent to the QA-sides like Se, but not in their middles.
More generally, what is the link between the 3 points P, Q and R ?
It's remarkable that we have a set of cubics through QL-vertices and of sextics of class 3 tangents to the QA-sides ; the 3 foci of all these sextics verify Siebeck's properties : in each QA vertice, the mean direction of the 3 lines through this vertice and a focus is the same and the product of the distances from this vertice to the foci is the same.
Best regards
Bernard

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Message: #2728
Date: 10/12/2017 9:55:26
From: chris.vantienhoven
Subject: Yahoo attachment problem

```
Dear friends,
It seems there are problems with sending messages with an
attachment.
I have the same problem too.
It looks like it that this is occurring for all Yahoo Groups.
Most of these problems take some time to be solved by Yahoo.
At least that's what I hope. So have some patience.
Of course there is also
the possibility to share a file separately  by clicking on the
button "files" and then the button "upload".
Best regards,
Chris
```

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Message: #2729
Date: 10/12/2017 12:45:29
From: eckart_schmidt@t-online.de
Subject: Set of isocubics through the QL-vertices

```
Dear Bernard,
you trie to describe QL-Cu1 in the sense of QFG#2610, QFG#2611,
using P = QL-P10, orthocenter of the diagonal triangle.
Then P defines a nK QL-Cux completely:
... reference triangle Tr is the orthic triangle of QL-Tr1,
... isoconjugation is isogonal conjugate wrt Tr,
... but root R and line L are not the same as for QL-Cu1,
... see attached file.
For QL-Cu1
... with the same reference triangle and isoconjugation
... the line L bears the collinear CSC-images
... ... of the vertices of the orthic triangle.
For QL-Cux I cannot give a relation between P and L.
Best regards Eckart
PS: I hope the attached file will be transmitted.
```

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Message: #2730
Date: 11/12/2017 10:51:53
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

```
Dear Eckart,
Thanks a lot for your quick and kind answer, as well as for the
figure !
In fact, for a given QA/QL with DT, the vertices of the cevian
triangle of any point P form a triple of points in the sense of
the C-B property and the 9 points define a set of nK's.
The cevian triangle of P defines an isoconjugation with fixed
points P and the vertices of DT, which swaps the opposite
vertices of QL and all the nK's of the set are invariant in this
isoconjugation.
Shortly, the points P and R or L are independant and define
together a unique QL-circumscribed nK.
Best regards
Bernard
Reading again your messages 2610-2611, I agree with you and
regret that so many beautiful properties of the QA/QL are not in
EQF ...
```

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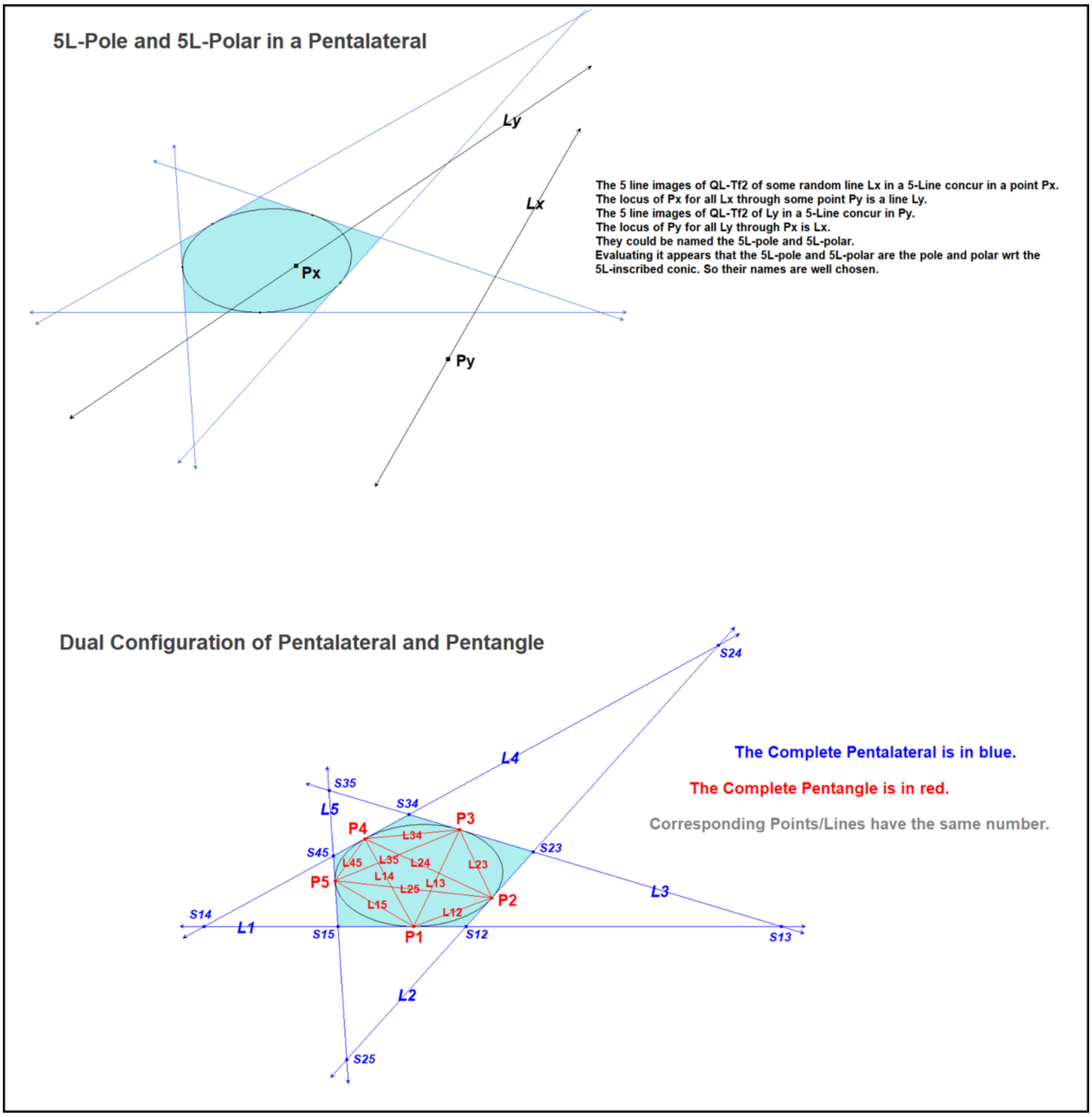
Message: #2731
Date: 11/12/2017 12:16:05
From: chris.vantienhoven
Subject: 5L-analogon for 5P-s-Tf2

Dear Eckart, Tsihong Lau and Bernard,
Eckart, thank you for your description of the 5L-analogon for 5P-s-Tf2 in your message #2697.
This gives us the possibility of describing a Dual Pentalateral/Pentangle Configuration like we discussed before.
I made a picture and description of this Dual Pentalateral/Pentangle Configuration.
See attachment.
Best regards,
Chris



DualPenta.pdf

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Message: #2732
Date: 11/12/2017 12:22:01
From: chris.vantienhoven
Subject: 4 random points on a sphere

```
Here a nice problem:
Four points are chosen uniformly at random on the surface of a
sphere.
What is the probability that the center of the sphere lies
inside the tetrahedron whose vertices are at the four points?
For an answer see:
https://www.quora.com/Four-points-are-chosen-uniformly-at-random
-on-the-surface-of-a-sphere-What-is-the-probability-that-the-cen
ter-of-the-sphere-lies-inside-the-tetrahedron-whose-vertices-are
-at-the-four-points

Best regards,
Chris
```

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Message: #2733
Date: 11/12/2017 4:16:03
From: eckart_schmidt@t-online.de
Subject: Set of isocubics through the QL-vertices

Dear Bernard,

here is a construction
... for the roots of the nonpivotal cubics described in QFG#2610 and QFG#2611
... without using Cayley-Bacharach points,
... but using Bernard Gibert´s circle T in 1.5.3 of his "Special Isocubics".

Let X be the defining point of the cubic,
... Tr its cevian triangle wrt the diagonal triangle.
... T is a circle through X
... ... centered on QL-L2,
... ... orthogonal QL-Ci5.

The circle T can be constructed as follows (perhaps simpler):
... Let Y be the pole of QL-L2 wrt QL-Ci5
... and Z the common point of the radical axes
... ... for QL-Ci5 and circles through X centered on QL-L2.
... YZ is the radical axis of QL-Ci5 and the circle T
... ... centered on QL-L2.

Polars of Tr-vertices wrt T
... intersect the opposite Tr-line in a point of the line L,
... whose trilinear pole wrt Tr gives the root R of the cubic.

The considered cubic is a nK
... with reference triangle Tr
... and root R
... through QL-points.

Best regards Eckart

PS. Because of the problems with attachments I added none.

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Message: #2734
Date: 12/12/2017 11:21:21
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
If the defining point X is on the circle T orthogonal to any circle through the 2 Plücker points, the center of T is on the ortholine QL-L2 at the intersection with the line perpendicular in X to the line through X and the middle of the segment joining the 2 Plücker points.
I suppose the fact that the circle T is through X is a defining property of your set of cubics described in 2610 ?
I suppose also that in the general case any circle centered on QL-L2 leads with the same construction to the line L and to the root R ?
Last, you showed me already that QL-Cu1 is an isogonal nK wrt the orthic triangle of DT, circumscribed to the QL-vertices, but doesn't belong to your set of cubics.
I'm almost convinced that the cubic you named Cu, dual of Se, is an isotomic nK wrt midDT, circumscribed to the QL-vertices, but doesn't belong either to you set of cubics.
If you had the courage, I would be very happy if you sent me (on my personal e-mail) a beautiful picture with the centroïd of DT, the cevian triangle midDT and the 2 cubics, one being Cu and the other ythe cubic of your set with X being the centroïd.
Many thanks in advance
Best regards
Bernard

Message: #2735
Date: 12/12/2017 12:23:52
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
Sorry for a mistake at the beginning of my message !
You have to consider the circle through X and the 2 Plücker points ; the center of the circle T is on QL-L2 at the intersection with the tangent in X to this circle ...
Best regards
Bernard
PS It remains that the dual curves of these nK's are sextics of third class tangent to the 6 QA-sides and that their 3 foci verify Siebeck's properties in each QA-vertice (generalisation of isogonality wrt a triangle)

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Message: #2736
Date: 12/12/2017 4:26:58
From: chris.vantienhoven
Subject: Pivotal isocubic for a 5L

Dear Eckart,
Wrt the construction of your pivotal isocubic for a 5L (message # 2713) I got these questions:
1. Do you have a construction method for selecting unambiguously X1, X2, X3 out of the 4 intersection points of the conics, which are the loci of intersections of the five dual lines?
2. This part I do not understand:
"Now we repeat the procedure for one line XiYi = L … and get three X-points on the cubic: Xi, Yi and a new point P, … which is the tangential of Xi and Yi wrt the cubic, … and will be the pivot."
* Are Xi, Yi the points mentioned above?
* What do you mean with the tangential of Xi and Yi wrt the cubic?
When I will understand this, I cannot guarantee that I will understand what is coming next, but I do it step by step.
Best regards,
Chris

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Message: #2737
Date: 12/12/2017 4:55:35
From: chris.vantienhoven
Subject: many beautiful properties

```
Dear Bernard and dear Eckart,

[BK, QFG#2730] Reading again your messages 2610-2611, I agree
with you and regret that so many beautiful properties of the
QA/QL are not in EQF ...

I totally agree with you. I wish I had more time to incorporate
all this wonderful items that come forward in the Quadri-Figures
Group. Alas every item takes a lot of time to put it on the web.
As a consequence I have hardly time left to do my own research
that I love so much.

Bernard, also your item about the epi- or hypocycloïds tangent
to 4 lines I love it very much and it is still on the list of
describing it in EQF. I was halfway of finding construction
methods for these epi- / hypo-cycloids, but I had to choose
priorities.

So I cannot give guarantees other than that I do my very best of
honoring as many items as I can handle.

Nevertheless I think we have a wonderful time together. We are
doing wonderful research in mathematical fields never seen
before. It is just great!

Hmm I think this is a nice Christmas message. I better stop now.

Best regards,
Chris
```

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Message: #2738
Date: 13/12/2017 11:56:16
From: eckart_schmidt@t-online.de
Subject: Pivotal isocubic for a 5L

Dear Chris,

I have done my construction once more, which is really very
extensive.
I think, someone will find an easier version.

Wrt your questions:

1. Sorry, you have to control,
whether an intersection has the property
and is a so called X-point.
But you need only one pair Xi,Yi - e.g. X1,Y1 - for the further
construction.

2. X1 and Y1 remain the same,
new X- and Y-points have another name.
The tangents in X1, Y1 at the cubic intersect in P on the cubic.

There is an unessential typo in the passage next to the last:
"... and get three X-points: U and two new points A and B, ..."

I hope, you can complete the construction and draw the cubic as
pivotal isocubic.

Best regards Eckart

Message: #2739
Date: 14/12/2017 9:54:52
From: chris.vantienhoven
Subject: Pivotal isocubic for a 5L

Dear Eckart,

[ES, QFG#2736] "you have to control, whether an intersection has the property and is a so called X-point. But you need only one pair Xi,Yi - e.g. X1,Y1 - for the further construction."

I am still puzzled by your remark.
We are looking for the triple that is common for all conics.
However intersecting two conics produces 4 intersection points.
I hoped you had a solution for finding the common triple out of these 4 points.
Because choosing one pair Xi,Yi still has the risk of choosing a "wrong" intersection point.
I know of other examples where several conics meet and have a common triple.
Most of the times the 4th point has a special function. The other 3 will form the common triplet.
Then there is this very nice construction method:
1. The 4 intersection points form a quadrangle P1.P2.P3.P4 with diagonal triangle DT.
2. Let P1 be the intersection point with special function.
3. The vertices of the Anticevian triangle of P1 wrt DT will be the triple we were looking for.
For example see 7L-s-3P1 last construction step. Also QL-Tr2 can be constructed this way.
In this way in our drawing programs like Cabri we can uniquely determine the common triplet.
Therefore I was looking for the special function of the 4th point in your example. I hoped you could indicate that.
But maybe you still can tell me after this explanation?

Best regards,
Chris

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Message: #2740
Date: 14/12/2017 11:44:10
From: eckart_schmidt@t-online.de
Subject: Pivotal isocubic for a 5L

Dear Chris,

a simple control, whether a common intersection of the conics is an X-point:
The dual lines wrt the 4L intersect in a common point on L.

Best regards Eckart

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Message: #2741
Date: 14/12/2017 1:22:11
From: chris.vantienhoven
Subject: Pivotal isocubic for a 5L

Dear Eckart,

Your message #2740: “a simple control, whether a common intersection of the conics is an X-point: The dual lines wrt the 4L intersect in a common point on L.”

Thanks for this solution. That’s remarkable indeed. However I think when changing the shape of the 5-line or the position of line L, then still intersection points mutually can interchange. But I think I found a method now of uniquely determining the 3 intersection points.

Your initial remarks at message #2736 were: “We start with a 5L and an arbitrary line L … and points on L with their 5 dual lines wrt the 4L of the 5L. The loci of intersections of the five dual lines … are conics with three common points X1, X2, X3, …..“
In your description the “dual lines wrt the 4L of the 5L” are actually QL-Tf11(P) where P is any point on L. The loci of intersections of the five dual lines are conics. These conics are based on intersection points of two dual lines based on two different QL’s. Denote these QL’s with QL1, QL2, QL3, QL4, QL5 (produced by omitting resp. L1,L2,L3,L4,L5).

The dual lines could be named correspondingly Lt1, Lt2, Lt3, Lt4, Lt5.
Denote the locus of Lt1^Lt2 as conic Co12.
Denote the locus of Lt1^Lt3 as conic Co13.
Co12 and Co13 have 4 intersection points of which 3 points are shared with other conics Co23, Co45, etc.
Now the 4th intersection point Co12^Co13 is QL-Tf10(L1).
In general the 4th intersection point Coij^Coik is QL-Tf10(Li).
*QL-Tf10 (http://www.chrisvantienhoven.nl/ql-items/ql-transformations/ql-tf10)
* (L) is the Dual Pole / QuadriPole.
Since we have 10 possible intersection points of Lt1, Lt2, Lt3, Lt4, Lt5, there will be 10 conics. They all share the same 3 common intersection points. Only of one pair of conics we have to determine the 4th point in order to know the common 3 points.
Like I described in former message the way to construct them is:
1. The 4 intersection points form a quadrangle P1.P2.P3.P4 with diagonal triangle DT.
2. Let P1 be the intersection point of special function.
3. The vertices of the Anticevian triangle of P1 wrt DT will be the triple we were looking for.
So far the solution of only a small part of your construction.
Now I can look to the rest.

Best regards,
Chris

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Message: #2742
Date: 14/12/2017 3:18:39
From: eckart_schmidt@t-online.de
Subject: Pivotal isocubic for a 5L

Dear Chris,

thanks for your detailed description. Here a further observation:
If we consider all 10 possible conics, there are only up to three common real intersections,
all with the considered property.
For L = 5L-s-L2 I found only one common intersection,

Best regards Eckart

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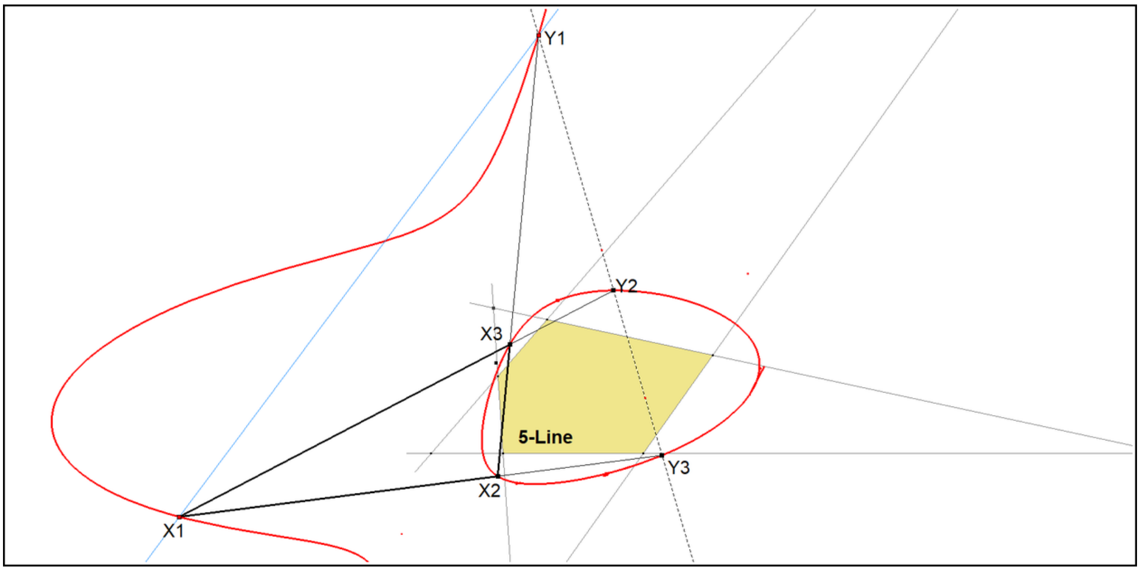
Message: #2743
Date: 14/12/2017 8:16:40
From: chris.vantienhoven
Subject: Pivotal isocubic for a 5L

Dear Eckart,

I managed to construct the cubic now as a locus of X1, X2, X3 varying L.
I needed the certainty of the triple of common points we discussed before.
See attachment.
I skipped the procedure in your initial description after constructing P.
Very complicated for me and I don't know what I am doing and why I am doing it.
But that is maybe because cubics are not my main interest. So forgive me.
Anyway I am glad being able to construct the cubical locus of points whose 5 4L-dual lines (QL-Tf11 Quadri-Polars) concur.
It is the first described cubic I know about in a Pentalateral/5-Line. Congratulations!

Best regards,
Chris



5L-s-Cu1-Schmidt Cubic-21.png

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Message: #2744
Date: 15/12/2017 11:37:19
From: eckart_schmidt@t-online.de
Subject: Pivotal isocubic for a 5L

Dear Chris,

I be pleased about your construction of the cubic.
You skipped the construction successful,
so you will be interested in a new view of the cubic:
The cubic is (nominations see QFG#2713)
... a nonpivotal isocubic
... with reference triangle X1X2X3,
... isoconjugation, which swaps P and Zi
... and root in the trilinear pole of L wrt X1X2X3.

Best regards Eckart

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Message: #2745
Date: 15/12/2017 2:47:55
From: minhtenladuong
Subject: About QA-Tf6

Dear geometers,
Yesterday, I came back with some good news but my message didn't appear in the group, so
Chris wanted me to post it to QFG. It is all about QA-Tf6:
*Let consider a straight line L and the quadrangle P1P2P3P4. M is an arbitrary point on L.
*Let Pij be the reflection of M in PiPj. X = midpoint of P12P34, Y = midpoint of P13P24, Z = midpoint of P14P23. *Then QA-Tf6(L) always lies on the circumcircle of triangle XYZ.
We can consider it as a geometric property. But, a fun fact, this, together with a result in my old paper(Ptolemy triangle) that I have posted here (https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1570;_ylc=X3oDMTJxc2NtcmMzBF9TAzk3MzU5NzE0BGdycElkAzg3Njg2NjExBGdycHNwSWQDMTcwNTA4MzM4NgRtc2dJZAMxNTcwBHNlYwNmdHIEc2xrA3JwbHkEc3RpbWUDMTQ1Nzc5OTMyMg--?act=reply&messageNum=1570&soc_src=mail&soc_trk=ma) leads to a synthetic proof for QA-Tf6.
Best regards,
Ngo Quang Duong

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Message: #2746
Date: 15/12/2017 3:00:02
From: eckart_schmidt@t-online.de
Subject: Set of isocubics through the QL-vertices

Dear Bernard,
excuse my late reaction on your message 2734,
but I sent a picture with the two inquired cubics at your private adress.
The construction of the center for the circle T was new to me, thanks.
But excuse, what do you mean with
"...I suppose also that in the general case any circle centered on QL-L2 leads with the same construction to the line L and to the root R ?"
Thanks in advance for an explanation.
Best regards Eckart

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Message: #2747
Date: 15/12/2017 5:05:30
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
Many thanks for your figure, I got on my private adress.
Looking at it, I found something I didn't realise before.
The root of the socalled Siebeck's cubic Cu is simply QA-P5, the reflexion in QA-P1 of the anti complement QA-P20 of QA-P1 wrt DT and the root of the cubic of your set is the reflexion of QA-P5 in QA-P20.
(It must be possible to find it with the barycentric coordinates wrt DT and midDT).
My mysterious sentence was only following idea.
In the general case, meaning a point X and it's cevian triangle T wrt DT, there is an isoconjugation wrt T which swaps the opposite QL-vertices and defines a set of cubics through the 6 vertices and the vertices of T. For a given root R and it's trilinear polar L wrt T, there is only one cubic and the circle orthogonal to any circle with diameter 2 conjugate points, in particular the 3 with diameters the opposite QL-vertices through the 2 Plücker points. This circle is therefore centered on QL-L2.
I suppose the converse is true ; any circle centered on QL-L2 (real or imaginary if the center is outside or inside the segment of the Plücker points) leads to a line L and it's trilinear pole R wrt T.
Best regards
Bernard

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Message: #2748
Date: 16/12/2017 1:47:00
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
Please forget my last message !
I checked with my own figures.
The root of the Marden cubic is certainly not QAP5.
Other properties are coming ...
Best regards
Bernard

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Message: #2749
Date: 16/12/2017 11:19:15
From: chris.vantienhoven
Subject: About QA-Tf6

Dear Ngo Quang Duong,

I like this property of QA-Tf6.
Here are some extra properties I noticed:
1. When you define point M as a random point, then the circumcircle of XYZ will coincide with the circle in the construction of QA-Tf3 with center QA-Tf3(M). This circle also passes through QA-P2.
2. The triangle XYZ also was mentioned by Benedetto Scimemi for a preliminary version of the QA-Tf3-transformation. See remarks at QA-Tf3.
3. Your transformation QA-Tf6 has found usage in QA-Tf8 as well as in the Encyclopedia of Polygon Geometry: 5P-s-Tf3 and 5P-s-Tf4.
I am looking forward to more specials coming from you.

Best regards,
Chris

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Message: #2750
Date: 17/12/2017 5:11:52
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
This time, I checked carefully as well the barycentric calculations as my own figures !
The root R (red on your figure) of the Siebeck's cubis is either the complement of the isotomic conjugate wrt DT of QL-P13 or QA-P16 or the isotomic conjugate wrt midDT of the complement of QL-P13 or QA-P16.
This means that the root of the cubic of your set (green on your figure) is QL-P13 or QA-P16.
If you consider now the isotomic transversal of the Newton Line wrt DT, it is the anticomplement of the Newton Line of DQL (QL formed by the 3 diagonals and the Newton Line) !
As well known, the Newton Line of DQL is the dual of QA-P1 wrt QA/QL.
And of course, the isotomic transversal of the Newton Line of DQL wrt midDT is the complement of the Newton Line !!
For a line cutting the 3 sides of a triangle in 3 points, the isotomic transversal is the line through the reflexions of the 3 points in the 3 middles of the sides ; the trilinear poles of 2 isotomic transversals wrt a triangle are themselves isotomic conjugates wrt this triangle.
The trilinear pole of the Newton Line wrt DT is QL-P13 or QA-P16 and the trilinear polar of it's isotomic conjugate is the anticomplement of the Newton Line of DQL.
The trilinear pole of the Newton Line of DQL is the root of the Siebeck's cubic and the trilinear polar of it's isotomic conjugate wrt midDT is the complement of the Newton Line.
Last property : the root R of the Siebeck's cubic is aligned with QA-P1 and QA-P16, which are themselves isotomic conjugates wrt midDT and the isotomic conjugate of R wrt midDT is on the conic through the vertices of midDT and QA-P1 and QA-P16.
If you consider the QL formed by the 3 sides of midDT and the complement of the Newton Line, it is the complement of DQL ; it's Miquel point is QL-P25 and the parabola tangent to the 4 lines of this QL is QL-Co3, 2nd parabola tangent to the 4 lines of the QL.
All this form a link between Siebeck's cubic and old properties of QL and DQL ...
Best regards
Bernard

Message: #2751
Date: 17/12/2017 5:51:00
From: eckart_schmidt@t-online.de
Subject: Set of isocubics through the QL-vertices

Dear Bernard,
That is a good result, respect!
I haven´t proved all resulting properties,
but they show its place in QA/QL-geometry!
In the second sentence there will be a typo:
The root of the cubic of my set is *the DT-isotomic of*
QL-P13.
Best regards Eckart

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Message: #2752
Date: 17/12/2017 9:44:56
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
Thanks for your comment !
Sorry for the typo, of course R green is the anticomplement of R
red, id est the DT isotomic of QA-P16 or QL-P13 ...
Best regards
Bernard

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Message: #2753
Date: 18/12/2017 2:37:00
From: minhtenladuong
Subject: About QA-Tf6

Dear Chris,
Thank you for your reply.
I'm so glad to see the applications of QA-Tf6.
I will keep looking for new ideas and contact to QFG later.
Best regards,
Ngo Quang Duong

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Message: #2754
Date: 18/12/2017 7:11:34
From: bernard.keizer
Subject: Set of isocubics through the QL-vertices

Dear Eckart,
There is another typo !
The parabola QL-Co3 is the 2nd parabola not tangent to the 4 lines (there is only one !), but circumscribed to the QA of the contact points of the tangents with QL-Co1 (which has the same DT as the QA and the QL).
The tangents to the parabola QL-Co3 in the same points are the isotomic transversals of the 4 lines wrt DT ; the QL formed by these 4 lines has also the same DT, it's vertices are the isotomic conjugates of the QL-vertices and the vertices of the dual QA's of the 2 QL's are also isotomic conjugates wrt DT.
The point QL-P25 is the isotomic conjugate of QL-P1 wrt DT ...
I suppose the isotomic non-pivotal cubic with midDT as reference triangle and root the isotomic conjugate of the root R of the Siebeck's cubic will be the isotomic of the 1rst one, will pass through the vertices of the 2nd QL and it's dual wrt it's QA/QL will be it's Siebeck's sextic, tangent in their middles to the sides of the 2nd QA.
I had put some properties of QL-Co1, QL-Co3 and the isotomic transformation in an old message, but of course I didn't know then the properties of the Siebeck's cubic !
I hope you will be interested in continuing the drawings ; I wonder in particular in which points (other than the midDT vertices) these 2 cubics will intersect ...
Best regards
Bernard

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Message: #2755
Date: 19/12/2017 10:18:48
From: bernard.keizer
Subject: many beautiful properties

Dear Chris, dear Eckart,
I haven't answered this nice message yet, but as you say, it is soon Christmas and it's wishes !
So I express my main regrets :
* the epi- and hypocycloïds tangent to 4 lines, in connexion with the n-angle centers and the Hofstadter points (you mention it yourself)
* focus of a QA on any circumcubic (Cotterill's construction) leading to the construction of the C-B point
* non-pivotal isocubics through the 6 QL-vertices and it's dual tangent to the 6 QA-sides (in particular, QL-Cu1 is an isogonal non-pivotal focal circular cubic wrt the orthic triangle of DT, as mentionned in EQF)
* the 3 foci of the QA (generalisation of Marden's theorem) and Siebeck's cubic (isotomic non-pivotal cubic wrt midDT) and it's dual the sextic of class 3 tangent in their middles to the 6 QA-sides.
But don't worry, Chris, yes we had a wonderful time together !
Merry Christmas and Happy New Year to both of you and to all members of the Quadriforum
Best regards
Bernard

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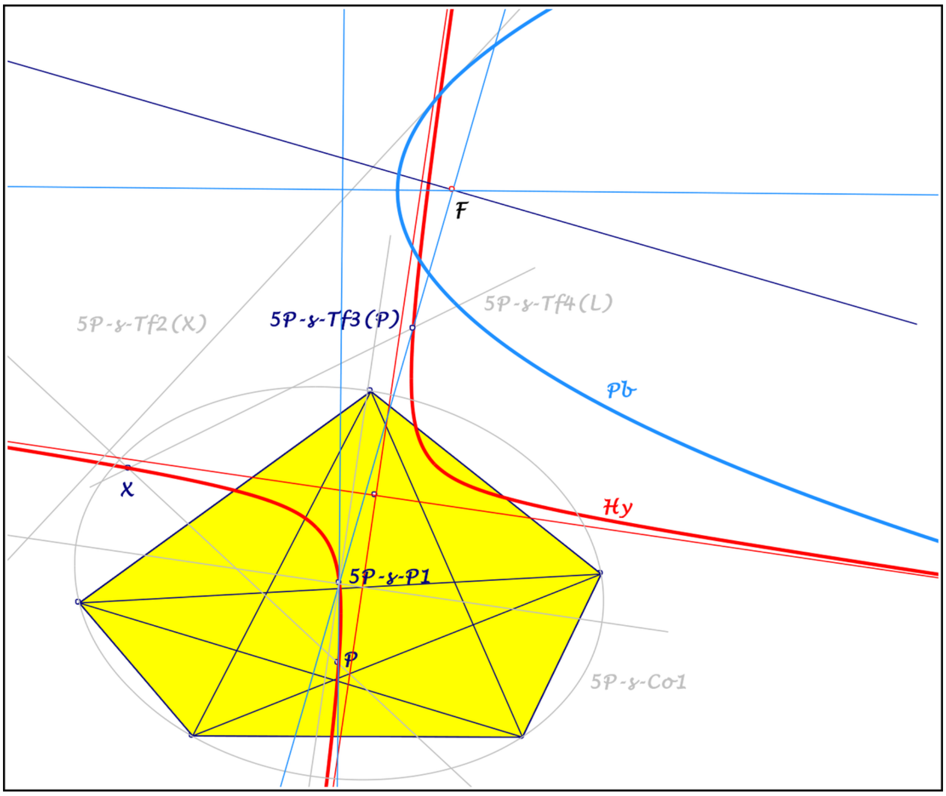
Message: #2756
Date: 19/12/2017 9:15:36
From: eckart_schmidt@t-online.de
Subject: About QA-Tf6

```
Dear Ngo Quang Duong, dear Chris,

up to now I am not so familiar with 5P-s-geometry,
but here some observations wrt 5P-s-Tf2, 5P-s-Tf3,  5P-s-Tf4:
Let us start with lines L of a line pencil for a point P:
... the intersections X = L ^ 5P-s-Tf4(L)
... give an orthogonal hyperbola Hy
... ... with center in the midpoint of P and 5P-s-Tf3(P),
... ... axes parallel to the axes
        of the circumscribed conic 5P-s-Co1
... ... and points P, 5P-s-Tf3(P), 5P-s-P1.
... The 5P-s-Tf2-image of the hyperbola Hy is a parabola
... with directrix  P.5P-s-P1
... and focus in the intersection of 5P-s-P1.5P-s-Tf3(P)
...   ... and the 5P-s-Co1-polar of the Hy-center.
I think, there will be more properties.

Best regards Eckart
PS. The attached pictures are the same in pdf- and word-file.
```



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Message: #2757
Date: 20/12/2017 8:24:45
From: chris.vantienhoven
Subject: About QA-Tf6

```
Dear Eckart,

When I conceptualized EPG I was wondering if EPG could deliver
so many new items as EQF did.
Well I think we made a very good start.
It seems that especially the configurations of a Pentangle (5P)
and Pentalateral (5L) are subjects of investigation.
I think there will be many items like in a Quadrangle and a
Quadrilateral.
As soon as I find time I will incorporate several of them in
EPG.

I like your P-OrthogonalHyperbola Hy and P-Parabola.
Two comments:
1. Not Hy-axes are parallel to the axes of the circumscribed
conic 5P-s-Co1,
but Hy-asymptotes are parallel to the axes of 5P-s-Co1.
2. 5P-s-P1.5P-s-Tf3(P) and the 5P-s-Co1-polar of the Hy-center
(lines through Parabola Focus) are perpendicular.

Best regards,
Chris
```

Message: #2758
Date: 20/12/2017 4:15:53
From: eckart_schmidt@t-online.de
Subject: Inverse Transformations of 5P-s-Tf1,2,3,4

```
Dear Chris,

the transformations 5P-s-Tf1,2,3,4 are not reciprocal,
so their inverses are of interest.
Here are constructions for these mappings:
(I leave the prefix 5P-s- and pole/polar are wrt 5P-s-Co1.)

Tf1.
... Let L be a line
... with pole X
... and Y = L ^ XP1.
... The circle round Y, so that X and P1 are inverse,
... intersects the distance XP1 in P
... with Tf1-image L.

Tf2
... Tf2 and its inverse are the polarity with Co1.

Tf3
... Let P be a point.
... The perpendicular through P wrt the polar of P
... intersects Tf4(PP1) in Q
... with Tf3-image P.

Tf4
... Let L be a line
... with pole X.
... The polar of Tf3(X)
... has the Tf4-image L.

Best regards Eckart

PS. I omit attachments, but I regret very much, that they don´t
appear in QFG!
```

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Message: #2759
Date: 20/12/2017 6:11:43
From: minhtenladuong
Subject: Applications of Clifford's circle theorems

Dear geometers,

Recently, I study the transformation which I have posted here
(https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1218)
It is a point to point transfomation in quadrangle: Let P1P2P3P4 be a quadrangle and P be an arbitrary point, then the pedal circles of P with respect to 4 component triangles are concurrent.
In the above link, Chris also wrote that this result is still true for antipedal circles.
I denote this point by 4-pedal(P).
This can be extended for n-point by the Clifford's circle theorems as follow:
- Consider 5 points P1P2P3P4P5 and a point P. Then 4-pedal(P) of 5 component quadrangles are concyclic.
I denote this circle by 5-pedal(P).
- Consider 6 points P1P2P3P4P5P6 and a point P. Then 5-pedal(P) of 6 component pentagles are concurrent.
- etc...

In fact, these are just the consequences of the Clifford's circle theorems.
Let me explain:
- For 4 points. Let consider the inversion with center P, power k (I(P,k)). Pij is the orthogonal projection of P on PiPj. I(P,k)(Pi)=P'i. Since P12, P13, P14, P are concyclic(they lie on the circle of which diameter is PP1) then P'12, P'13, P'14 are collinear. Similarly, P'21, P'23, P'24 are collinear,... Hence after the inversion, it reduces to Miquel's theorem (Clifford's theorem for n=4)
- For 5 points, we consider the same inversion. Since P, P12, P13, P14, P15 are concyclic then P'12, P'13, P'14, P'15 are collinear. Now, it reduces to Clifford's theorem for n=5.
- etc...
And about the antipedal circles, we can apply the Clifford's theorem directly.

Best regards,
Ngo Quang Duong

Message: #2760
Date: 21/12/2017 7:11:17
From: chris.vantienhoven
Subject: many beautiful properties

Dear Bernard,
Thanks for your answer and very nice to read your “wishlist”.
Unfortunately I didn’t follow all discussions about the last three of your subjects because I was very busy with EPG.
But of course you (and everybody) can always propose one or more new items. What I need then is:

- Short explanation and description of the item.
- A simple picture.
- If possible a construction of the item.
- If possible coordinates or an equation.
- Finally a list of properties relating to other items.
- References should be included.

All together fitting on about one A4-paper.
It’s my aim to be generalistic and not too specialistic.
For specialistic items readers can look up references.
Send it personally to my email-address.
Merry Christmas and Happy New Year to you and to all members of the Quadriforum.
Best regards,
Chris

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Message: #2761
Date: 22/12/2017 8:38:32
From: chris.vantienhoven
Subject: Applications of Clifford’s circle theorems

Dear Ngo Quang Duong,
I like your pedal and anti-pedal circles, centers and points very much.
This is a very nice contribution to EPG.
They will be the first odd and even items in an n-Point.
I will add them as soon as possible in EPG.
Please go on finding beautiful features.
Best regards,
Chris

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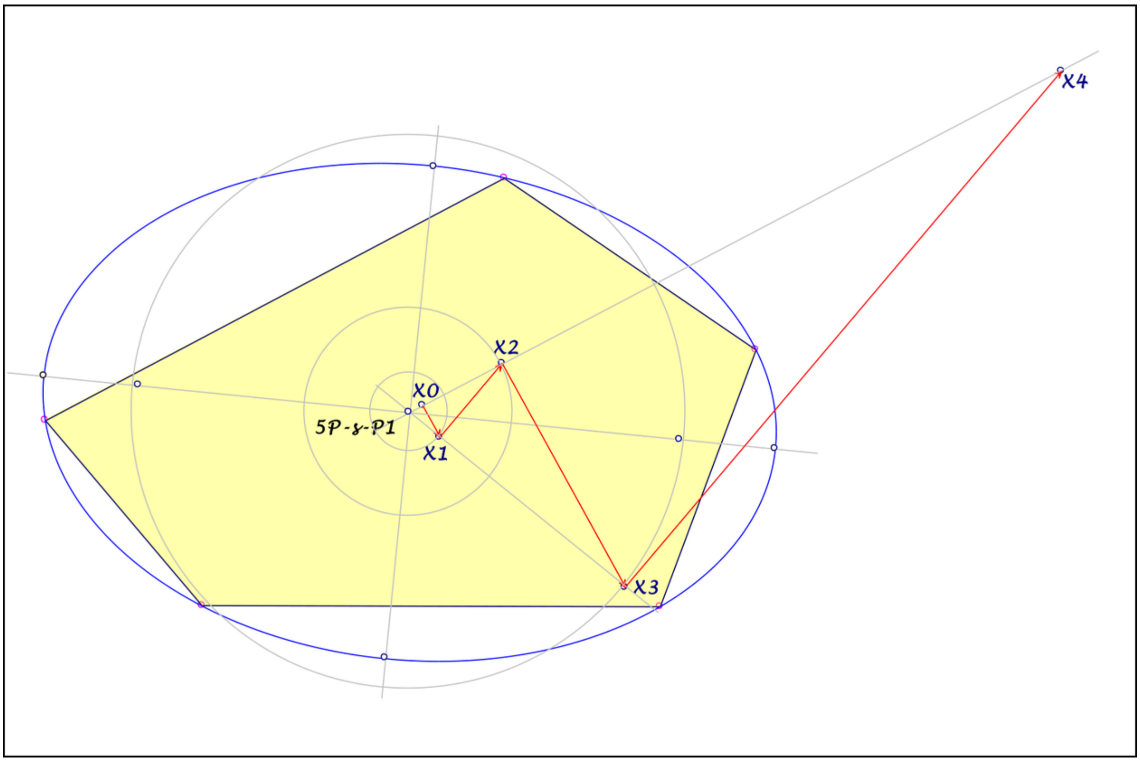
Message: #2762
Date: 22/12/2017 1:51:24
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

```
Dear Chris,
the transformation 5P-s-Tf3 is not reciprocal,
so we get iterating a sequence of points starting with X0 and
Xn+1 = 5P-s-Tf3(Xn) with following properties (see attached
file):

(1) Xn+2 is the inverse of Xn wrt a circle round 5P-s-P1 through
Xn+1.
(2) Xn+1 lies on the reflection of 5P-s-P1.Xn in the main axis
of 5P-s-Co1.
(3) Xn.Xn+1 is parallel Xn+2.Xn+3.
(4) 5P-s-P1.Xn / 5P-s-P1.Xn+1 = const.

This constant depends only of the circumscribed conic,
independent of the starting point X0
and the chosen 5P on the cubic.
What about this characteristic constant of a conic?

Best regards Eckart
```



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Message: #2763
Date: 22/12/2017 4:39:33
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf4

Dear Chris,

There are analog properties for 5P-s-Tf4:
T he transformation 5P-s-Tf4 is not reciprocal,
so we get iterating a sequence of lines
starting with L0 and Ln+1 = 5P-s-Tf4(Ln)
with following properties:

(1) The product of the distances of 5P-s-P1 to Ln and Ln+2
is the square of the distance of 5P-s-P1 and Ln+1.
(2) The intersections of L2n and L2n+1 are collinear with
5P-s-P1.
The intersections of L2n and L2n-1 are collinear with 5P-s-P1.
These two lines are symmetric wrt the main axis of 5P-s-Co1.
(3) Ln is parallel Ln+2.
(4) The quotient of the distances of 5P-s-P1 to Ln and Ln+1 is
const.

This constant depends only of the circumscribed conic,
independent of the starting line L0
and the chosen 5P on the cubic.

This is the same constant as for 5P-s-Tf3.

Once more: What about this characteristic constant of a conic?

Best regards Eckart

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Message: #2764
Date: 22/12/2017 7:50:00
From: minhtenladuong
Subject: 5P-s-Tf3

Dear Eckart,

Very nice observation! Follow your constructions, 5P-s-Tf3 is a similarity.
As you wrote, these transformation depends on the circumscribed conic only.
I have drawn and tried many times with GeoGebra(it has Cartesian coordinates) and I have found out what that constant is. I would like to rewrite your construction:

Given a conic (C), which has center S and a point P. Let Q be the reflection of P in the main axis of (C). The homothetic H(S,k) maps Q to P'. P -> P' is the construction (k = 2/(e^2)-1 and e is the eccentricity of (C)).

And the constant that you have mentioned is equal to 1/k.

Best regards,
Ngo Quang Duong

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Message: #2765
Date: 22/12/2017 10:03:23
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Ngo Quang Duong,

sorry, I cannot confirm your constant.
Meanwhile I found the constant as (a^2-b^2)/(a^2+b^2)
for a and b axes of the circumconic.

Best regards Eckart

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Message: #2766
Date: 23/12/2017 7:26:55
From: minhtenladuong
Subject: 5P-s-Tf3

```
Dear Eckart,
They are the same.
In definition, the eccentricity e = sqrt(a^2-b^2)/a. You can see
https://en.wikipedia.org/wiki/Conic_section
k = 2/(e^2) - 1 = 2a^2/(a^2-b^2) - 1 = (a^2+b^2)/(a^2-b^2)
Best regards,
Ngo Quang Duong
```

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Message: #2767
Date: 23/12/2017 8:42:21
From: chris.vantienhoven
Subject: 5P-s-Tf3

```
Dear Eckart, Dear Ngo Quang Duong,
Wrt QA-Tf3 we have a similar feature. See QA-P42 for its
description.
I wonder if there is a QA-conic that describes with its
eccentricity (or a and b) the constant factor.
Benedetto Scimemi wrote something about it.
I ain't got no time to explore it. Maybe you can help me.
Best regards,
Chris
```

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Message: #2768
Date: 23/12/2017 10:26:49
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

```
Dear Ngo Quang Duong,
please excuse many times my insecurity wrt "eccentricity", I
used a false formula!
Best regards Eckart
```

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Message: #2769
Date: 23/12/2017 9:19:14
From: eckart_schmidt@t-online.de
Subject: many beautiful properties

```
Dear Bernard, dear Chris,
what about the point 5L-s-Px,
... which is the center of the circumconic of its pedal points?
Merry Christmas and Happy New Year to you and your families.
Best regards Eckart
```

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Message: #2770
Date: 24/12/2017 4:36:13
From: bernard.keizer
Subject: many beautiful properties

```
Dear Eckart,
I don't get your point.
Do you mean the transformation, which associates to a point P
the center Px of the circumconic of it's pedal points ? If P =
Px, it is the point 5L-s-P1, belonging to the 5 Newton Lines of
the 5 QL's.
By the way, the 2 foci of the inscribed conic may also be
considered, they belong to the 5 QL-Cu1.
Merry Christmas and Happy New Year
Bernard
```

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Message: #2771
Date: 24/12/2017 7:06:18
From: Antreas Hatzipolakis
Subject: A conconicity criterion

```
A new (?) criterion for six points on a conic:
Let P 0 …P 5 be six distinct points in general position. Let Uk
, Vk (k=0…2) be the points given by:
  Uk = P 2*k P 2*k+2 ∩ P 2*k+1 P 2*k+5
  Vk = P 2*k+2 P 2*k+4 ∩ P 2*k+3 P 2*k+5 (all indexes modulus
  6).
Then, the six points P0 …P5 lie on a conic if and only if the
lines U0V0, U1V1 and U2V2 concur.
César Lozada

Hyacinthos 26965 (https://groups.yahoo.com/neo/groups/Hyacinthos
/conversations/messages/26965)  †)
***********************************************

Questions:
1. Is it new ? If no, references?
2. Which is its dual ?

Season's Greetings
APH
```

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Message: #2772
Date: 25/12/2017 12:53:51
From: bernard.keizer
Subject: many beautiful properties

```
Dear Eckart,
The 2 foci of the inscribed conic in the 5L are also the 2 only
points for which the conic of pedal points is a circle.
Best regards
Bernard
```

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [11].

Message: #2773
Date: 25/12/2017 4:26:18
From: minhtenladuong
Subject: 5 concyclic points

Dear Geometers,

At first, new year is coming, I wish you and your families a merry christmas and happy new year!
Beside, in this mail, I would like to share my newest discovery.
While I was trying to "extend" a quadrangle center to pentangle center, I found this curious result:
Given a pentangle P1P2P3P4P5. I1, I2, I3, I4, I5 are QA-P4 of 5 component quadrangles P2P3P4P5, P3P4P5P1, P4P5P1P2, P5P1P2P3, P1P2P3P4. Then the midpoints of I1P1, I2P2, I3P3, I4P4, I5P5 are concyclic.

Best regards,
Ngo Quang Duong

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Message: #2774
Date: 25/12/2017 5:52:40
From: chris.vantienhoven
Subject: 5 concyclic points

Dear Ngo Quang Duong,

That's a remarkable circle indeed.
I don't know about any circle in a 5P-configuration yet.
So far I can't see any relation with other 5P-items.
I hope this construction opens the gate for more of these circles.

Best regards,
Chris

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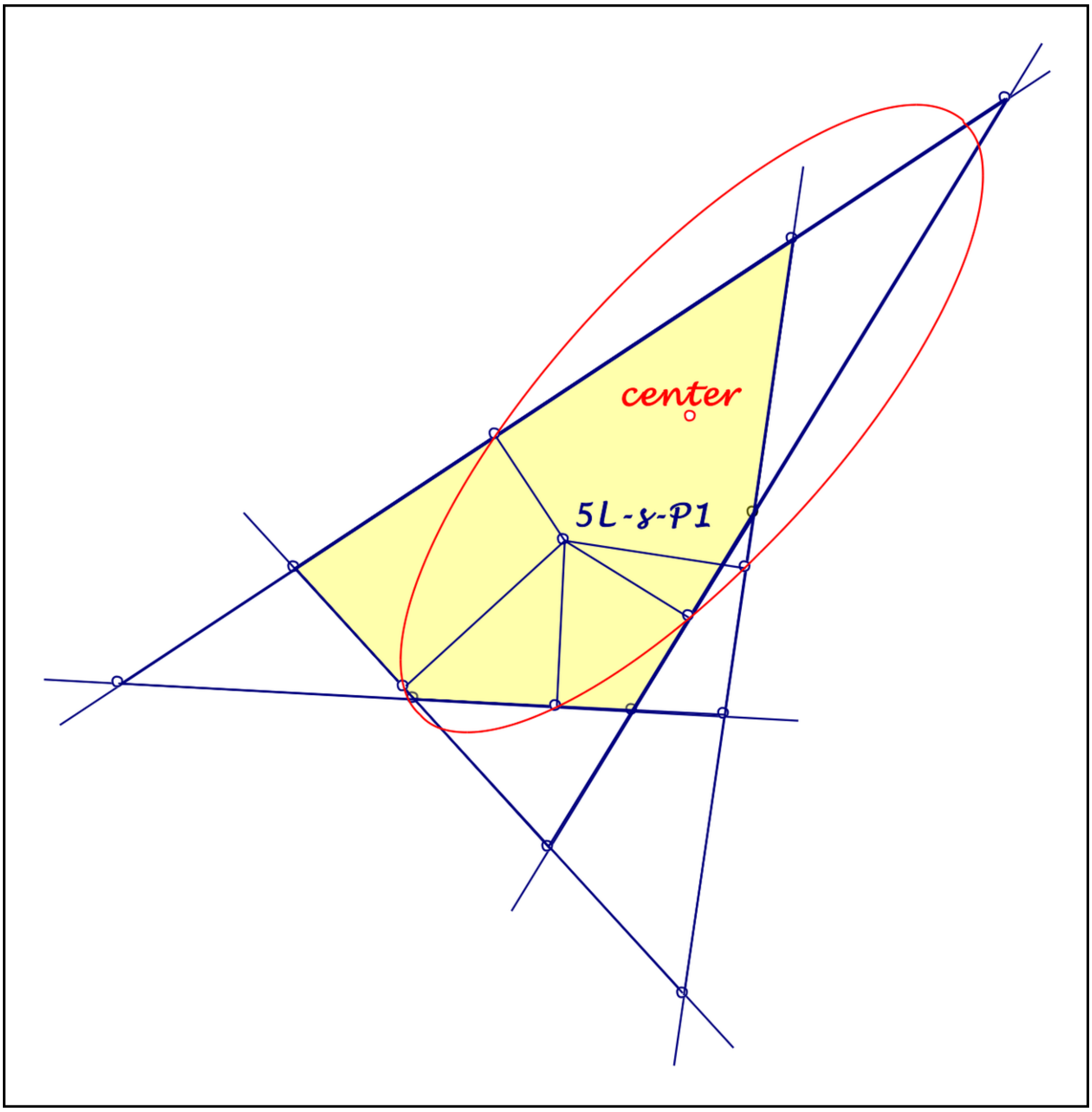
Message: #2775
Date: 26/12/2017 10:46:38
From: eckart_schmidt@t-online.de
Subject: many beautiful properties

Dear Bernard,

there will be any misunderstanding,
but 5L-s-P1 can not be the center of the circumconic of its
pedal points (see attached file).

Best regards
Eckart



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Message: #2776
Date: 26/12/2017 4:06:42
From: bernard.keizer
Subject: many beautiful properties

```
Dear Eckart,
In fact, I didn't understand your simple definition: a point
which is the center of the conic of it's pedal points.
Of course, it cannot be 5L-s-P1 (without attached file !)
My apologise!
Best regards
Bernard
```

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Message: #2777
Date: 26/12/2017 4:14:09
From: bernard.keizer
Subject: 5 concyclic points

```
Dear Ngo Quang Duong,
I checked your construction, it's a very beautiful and simple
property !
I suppose, it is linked to the fact that the definition of QA-P3
involves only isogonal conjugation (the isogonal of a point wrt
a triangle is the center of the circle of the pedal points) and
inversion.
QA-P3 is in a QA 4 times the inverse wrt the circumcircle of 3
vertices of the isogonal of the 4th vertice wrt the triangle of
these 3 vertices.
I suppose also that your property may be proved by barycentric
calculations, but I'm not able to do it.
Congratulations and happy new year
Best regards
Bernard
```

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Message: #2778
Date: 26/12/2017 4:39:39
From: bernard.keizer
Subject: many beautiful properties

```
Dear Chris, dear Eckart,
Reading again your answer, I have the feeling that the 3 last
items I mention were already almost completely described by
Eckart in some old messages (focus of a QA on a particular cubic
or Cotterill's construction, set of nK's through the 6 vertices
of a QL wrt the cevian triangle of a given point wrt DT with
examples QL-Cu1 as isogonal nK wrt the orthic triangle of DT or
Siebeck's cubic as isotomic nK wrt midDT and it's dual sextic of
class 3 tangent in their middles to the 4 sides of the dual QA,
it's 3 foci being found by derivating an equation of the 4th
degree in complex coordinates in connexion with the calculation
of the LSD line of the QA or set of particular cubics mentionned
by Eckart in connexion with the C-B construction).
The only point for which we didn't succeed is a simple synthetic
construction of these 3 foci ...
Best regards
Bernard
```

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Message: #2779
Date: 27/12/2017 6:48:01
From: chris.vantienhoven
Subject: 5P-s-Tf3

```
Dear Ngo Quang Duong and dear Eckart,

The invariant (a^2-b^2)/(a^2+b^2) was being mentioned by
Bendetto Scimemi in his message #932.
Fortunately he helped me tracking this message.

Best regards,
Chris van Tienhoven
```

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Message: #2780
Date: 28/12/2017 10:42:12
From: bernard.keizer
Subject: 5 concyclic points

```
Dear Ngo Quang Duong,
Of course, it's QA-P4 and not QA-P3 !
Sorry for the mistake ...
Best regards
Bernard
```

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Message: #2781
Date: 28/12/2017 8:20:56
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

```
Dear Chris, dear Benedetto, dear Ngo Quang Duong,
the invariant (a^2-b^2)/(a^2+b^2) is also already mentioned by
Roland Stärk, Ref. [16], 12.
Best regards Eckart
```

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Message: #2782
Date: 28/12/2017 9:32:24
From: chris.vantienhoven
Subject: Introduction to Quadrilateral Geometry

Dear friends,

I intend to post in the coming period every week a subject about quadrilateral geometry.
It is a new and thrilling branch of geometry and it has its own rules.
Knowing these simple rules a new world opens.
I will post them at the Yahoo Groups: Advanced Plane Geometry, Anopolis and The Quadri-Figures Group. If you know about other forums that might be interested, please let me know and I will post them there too with their permission. You also can copy them to another forum. Please let me know if you do.
I will start pretty simple.
Here is my first contribution.
How many vertices does a quadrilateral have?
The answers I hear most are 4 and 6.
The answer to this question depends on the definition of the word “quadrilateral”.
When I google the word quadrilateral I am finding many different definitions.
• Wat I find most: It is a polygon with four sides.
• But I also find: It is a four-sided shape.
These are different concepts:
• a polygon with four sides implicates a bounded figure with 4 sides and 4 vertices.
So the answer is that a quadrilateral has 4 vertices.
• a four-sided shape implicates just a figure of four lines.
Now the answer is that a quadrilateral has 4 lines implicating 6 intersection points and so there are 6 vertices!
(this figure is also called a “complete quadrilateral”)
Although this seems very obvious, this distinction is essential.
When we deal with a polygon of four sides and four vertices we deliberately omit two intersection points (vertices) of the four sides.
Geometry is built upon general principles and omitting two vertices has to be done purposely and consistently and combinatorics teaches us that there are several ways to omit two vertices out of six.
Next question is now: “how many four-sided polygons can be drawn in a system of four lines”.
We need some combinatorics to find the answer. The answer is 3, there are three different four-sided polygons that can be drawn in a system of four lines.

These simple principles are fundamental for Quadrilateral Geometry.
More information can be found in the Encyclopedia of Quadri-Figures EQF.

Chris van Tienhoven

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Message: #2783
Date: 29/12/2017 9:56:23
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Chris, dear Ngo Quang Duong, dear Bernard,
some remarks wrt 5P-s-Tf3, perhaps not already mentioned:
(1) 5P-s-Tf3 maps conics to conics, the centers are also 5P-s-Tf3-partner!

(2) For every QA with vertices on 5P-s-Co1
5P-s-Tf3 maps QA-P4 in QA-P2.

(3) For three vertices Pi, Pj, Pk of a 5P
and a variable point P on 5P-s-Co1
(a) ... the QA-P4 points lie on a circle Ci1
through the QA-P4 points of PiPjPkPl and PiPjPkPm
and the center of the circumcircle of PiPjPk.
(b) ... the QA-P2 points lie on a circle Ci2,
which is the 5P-s-Tf3 image of Ci1,
through the QA-P2 points of PiPjPkPl and PiPjPkPm
and the 3 midpoints of Pi, Pj, Pk.
(c) The perspectors of Ci1 and Ci2 lie on the axes of 5P-s-Co1.
For a 5P there are 10 Ci1 and 10 Ci2,
but I found no links to the mysterious circle of Ngo Quang Duong.

Best regards Eckart

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Message: #2784
Date: 29/12/2017 3:06:23
From: eckart_schmidt@t-online.de
Subject: many beautiful properties

```
Dear Bernard, dear Chris,
what about the point 5P-s-Px,
... which is the center of the inscribed conic of its 5
perpendicular bisectors wrt the 5P-vertices?
Best regards Eckart
```

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Message: #2785
Date: 29/12/2017 5:23:54
From: chris.vantienhoven
Subject: many beautiful properties

```
Dear Eckart,

[ES, QFG#2784]
what about the point 5P-s-Px,
... which is the center of the inscribed conic of its 5
perpendicular bisectors wrt the 5P-vertices?
Aren't there 10 perpendicular bisectors in a 5-Point
configuration?
Or did you mean the point 5G-s-Px ?

Best regards,
Chris
```

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Message: #2786
Date: 29/12/2017 5:44:17
From: Benedetto Scimemi
Subject: 5P-s-Tf3

```
It seems to me that 5P-f3 is exactly the similarity I described
in my paper attached to the messages recently recovered by
Chris. Am I wrong?
```

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Message: #2787
Date: 29/12/2017 5:59:01
From: Ngo Quang Duong
Subject: Về: Introduction to Quadrilateral Geometry

Dear Chris,
I really like your idea. Most of people think a quadrilateral has 4 vertices. It is time to spread the idea of EQF.
May I share your posts to this facebook group
BTH-LGD (https://www.facebook.com/groups/Loicenter/) ?
This is a mathematics group in Vietnam. If I share these, I will translate them.
Best regards,
Ngo Quang Duong

BÀI TOà�N HAY - Lá»œI GIáº¢I Ä�áº¸P - Ä�AM MÊ TOà�N Há»ŒC

Tiêu chí cá»§a chúng ta là Ä‘Æ°a các váº¥n Ä‘á»� toán há»�c khó, cao cáº¥p, trá»«u tÆ°á»£ng thà nh nhá»¯ng tri thá»©c gáº§n cuá»™c sá»‘ng, dá»…....
(https://www.facebook.com/groups/Loicenter/)
Và o ngà y 4:15 Thá»© Sáu, 29 tháng 12 2017, "van10hoven@gmail.com [Quadri-Figures-Group]" Ä‘ã viáº¿t:
Dear friends,
I intend to post in the coming period every week a subject about quadrilateral geometry.
It is a new and thrilling branch of geometry and it has its own rules.
Knowing these simple rules a new world opens.
I will post them at the Yahoo Groups: Advanced Plane Geometry, Anopolis and The Quadri-Figures Group. If you know about other forums that might be interested, please let me know and I will post them there too with their permission. You also can copy them to another forum. Please let me know if you do.
I will start pretty simple.
Here is my first contribution.
How many vertices does a quadrilateral have?
The answers I hear most are 4 and 6.
The answer to this question depends on the definition of the word “quadrilateral”�.
When I google the word quadrilateral I am finding many different definitions.
• Wat I find most: It is a polygon with four sides.
• But I also find: It is a four-sided shape.
These are different concepts:
• a polygon with four sides implicates a bounded figure with 4 sides and 4 vertices.
So the answer is that a quadrilateral has 4 vertices.

• a four-sided shape implicates just a figure of four lines.
Now the answer is that a quadrilateral has 4 lines implicating 6 intersection points and so there are 6 vertices!
(this figure is also called a "complete quadrilateral"□)
Although this seems very obvious, this distinction is essential.
When we deal with a polygon of four sides and four vertices we deliberately omit two intersection points (vertices) of the four sides.
Geometry is built upon general principles and omitting two vertices has to be done purposely and consistently and combinatorics teaches us that there are several ways to omit two vertices out of six.
Next question is now: "how many four-sided polygons can be drawn in a system of four lines"□.
We need some combinatorics to find the answer. The answer is 3, there are three different four-sided polygons that can be drawn in a system of four lines.
These simple principles are fundamental for Quadrilateral Geometry.
More information can be found in the Encyclopedia of Quadri-Figures EQF (https://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia).
Chris van Tienhoven

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Message: #2788
Date: 29/12/2017 6:57:33
From: chris.vantienhoven
Subject: Introduction to Quadrilateral Geometry

Dear Ngo Quang Duong,
I will be honored if you could translate my messages for the mathematics group in Vietnam.
When there are remarks it would be nice to transfer them to me.
Give them my regards,
Chris

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Message: #2789
Date: 29/12/2017 8:47:51
From: eckart_schmidt@t-online.de
Subject: many beautiful properties

```
Dear Chris,

perhaps my question was bad formulated:
What about the point 5P-s-Px,
... which is the center of the inscribed conic 5L-s-Co1
    of the 5 perpendicular bisectors of 5P-s-Px.Pi?

Best regards Eckart
```

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Message: #2790
Date: 29/12/2017 11:16:17
From: chris.vantienhoven
Subject: 5P-s-Tf3

Dear Benedetto,

Permit me to describe your transformation ζ from QFG-message #932 in a slightly modified way:
1. Given a pentangle with circumscribed conic 5P-s-Co1 with center 5P-s-P1 and major axis length a and minor axis length b.
2. Let ζ be a transformation acting upon some point P such that ζ is the product of
* the reflection on the major axis of the circumscribed conic 5P-s-Co1 and
* the homothety centered in 5P-s-P1 with scale factor (a^2 − b^2)/(a^2 + b^2)^(±1), (exponent +1 when ellipse, -1 when hyperbola)
I hope you can recognize your transformation in this description.
I checked it with the Orthopole transformation 5L-s-Tf3 and indeed it satisfies this description.
Only I am not quite sure if the exponent (±1) has to be the other way around, -1 when ellipse and + 1 when hyperbola.
Maybe you can check it.
You told me once that you already presented this transformation at Feb. 2005 in Bloomington as a hommage to Doug Hofstadter for his 60° birthday.
I am very glad that your visionary transformation now has been researched further and was complemented with the Orthopole construction.
I will surely make notice of it in the new EPG-encyclopedia at item 5P-s-Tf3.

Best regards and best wishes for the new year.
Chris

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Message: #2791
Date: 30/12/2017 9:15:31
From: chris.vantienhoven
Subject: 5P-s-Tf3

Dear Benedetto, dear friends

Benedetto, I am still fascinated by your transformation mapping QA-P4 into QA-P2 and vice versa.
It occurred to me that it actually isn’t a 5P-transformation. It is a conical transformation.
It can be applied in all situations where a conic is involved.
Let’s call this “Scimemi-transformation” Co-Tfx, mapping a point into another point.
It is defined by the rules you described, shortened:
1. reflection of a point about the major axis of the conic and
2. then a homothecy with the conical center as homothetic center and with ratio r.

There are 2 versions:
- Co-Tfx1 with ratio r = (a^2 – b^2) / (a^2 + b^2)
- Co-Tfx2 with ratio r = (a^2 + b^2) / (a^2 – b^2)

where a = length major axis of the conic and b = length minor axis of the conic.
Probably you have a better definition. I would like to hear that from you.
With limited research I found quickly these applications in different environments:
· Co-Tfx1[X(98)] = X(4), with reference conic = Steiner Inellipse or Steiner Circumellipse
Co-Tfx2[X(4)] = X(98), with reference conic = Steiner Inellipse or Steiner Circumellipse
· Co-Tfx2[X(98)] = X(1513) , with reference conic = Steiner Inellipse or Steiner Circumellipse
Co-Tfx1[X(1513)] = X(98) , with reference conic = Steiner Inellipse or Steiner Circumellipse
· Co-Tfx1[QA-P2] = QA-P4, with reference conic = any QA-circumconic
Co-Tfx2[QA-P4] = QA-P2, with reference conic = any QA-circumconic
· Co-Tfx1[5P-s-P1] = 5P-s-P1, with reference conic = circumscribed conic 5P-s-Co1
Co-Tfx2[5P-s-P1] = 5P-s-P1, with reference conic = circumscribed conic 5P-s-Co1
· Co-Tfx1[5P-s-Px] = 5P-s-Py, with reference conic = circumscribed conic 5P-s-Co1
Co-Tfx2[5P-s-Py] = 5P-s-Px, with reference conic = circumscribed conic 5P-s-Co1,

```
where 5P-s-Px = Conical center of five 5P-versions of QA-P2
where 5P-s-Py = Conical center of five 5P-versions of QA-P4
I think there will be a multitude of other applications in other
environments.
Maybe someone can find out some other appealing applications.

Best regards,
Chris
```

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Message: #2792
Date: 30/12/2017 11:04:23
From: eckart_schmidt@t-online.de
Subject: Loci wrt 5P-s-Tf1,2,3,4

```
Dear all,

attached some interesting loci wrt 5P-s-Tf1,2,3,4 for
... points on a line,
... lines through a point.

Best regards Eckart
```

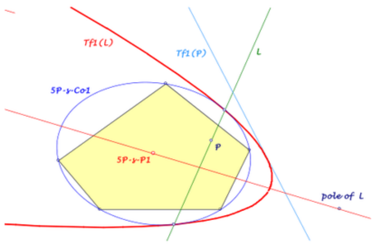
EQF-Note 2017-12-30

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Loci wrt 5P-s-Tf1,2,3,4

Tf1,2 map points to lines, Tf3 maps points to points and Tf4 maps lines to lines. So there are several possibilities, to study loci wrt points on a line or lines through a point.

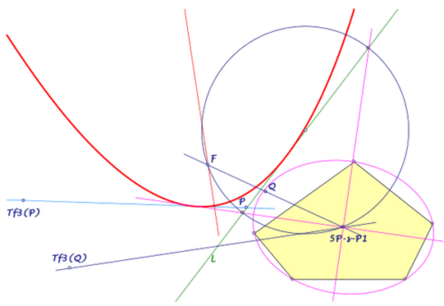
***Tf1* of points *P* on a line *L* give a parabola,**
… tangent to *5P-s-Co1* in the intersections with *L*,
… with axis through *5P-s-P1* and pole of *L* wrt *5P-s-Co1*.



***Tf2* of points *P* on a line *L* give a line pencil**
… for the pole of *L* wrt *5P-s-Co1*.

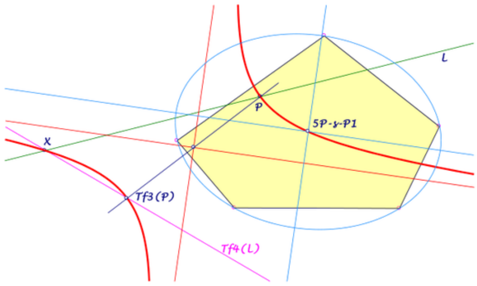
***Tf3* of points *P* on a line *L* give the line *Tf4(L)*.**

Tf4* of lines *L* through a point *P
… give the line pencil of *Tf3(P)*.

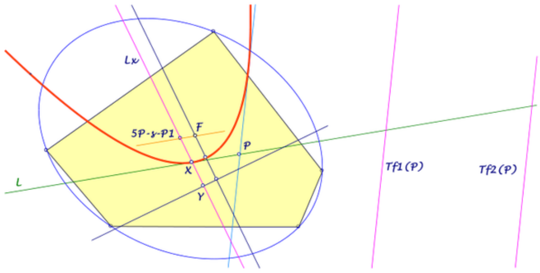


Lines *P.Tf3(P)* for points *P* on a line *L* give a parabola
… tangent to *L* and the axes of *5P-s-Co1*
… directrix *5P-s-P1.Tf3(Q)* with *Q* = pole of *L* wrt *5P-s-Co1*
… and focus in the 2nd intersection of *Q.5P-s-P1*
… … and the circumcircle of the triangle
… … with sidelines *L* and the axes of *5P-s-Co1*.

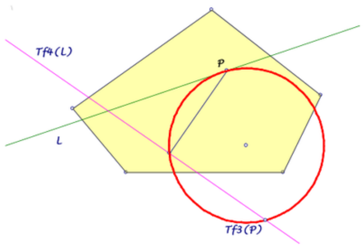
Points *L* ∩ *Tf4(L)* for a line pencil of point *P*
… give an orthogonal hyperbola,
… through *5P-s-P1*, *P* and *Tf3(P)*,
… centered in the midpoint of *P* and *Tf3(P)*
… with asymptotes parallel to the axes of *5P-s-Co1*.



Parallels to *Tf1,2(P)* through *P* for points *P* on a line *L*
… give a parabola,
… tangent to *L* in *X*, the intersection with the line L_x
… … through *5P-s-P1* and pole of *L* wrt *5P-s-Co1*,
… with directrix orthogonal L_x
… … through *Y*, the reflection of *5P-s-P1* in *X*,
… with focus *F* in the reflection of *Y* in *L*.



Pedal points of *P* on *Tf4(L)* for lines *L* through *P*
… give a circle with diameter *P.Tf3(P)*.

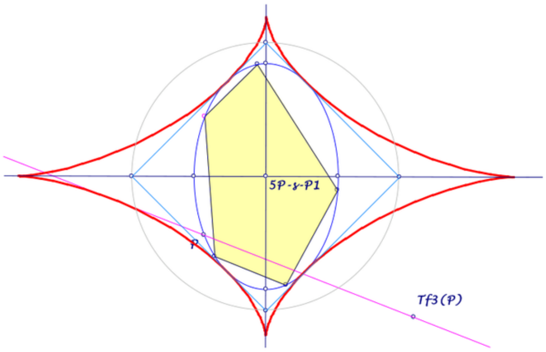


Final remarks

- ***Tf3* maps a conic to a conic, also the centers of the conics.**

2017-12-30.pdf

Lines *P.Tf3(P)* for points *P* on *5P-s-Co1* give an astroid
… with the same axes as *5P-s-Co1*
… and common tangents with *5P-s-Co1*,
… … which form a square,
… and cusps inverse to the apexes of *5P-s-Co1* wrt the circumcircle of the square.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-12-30.pdf

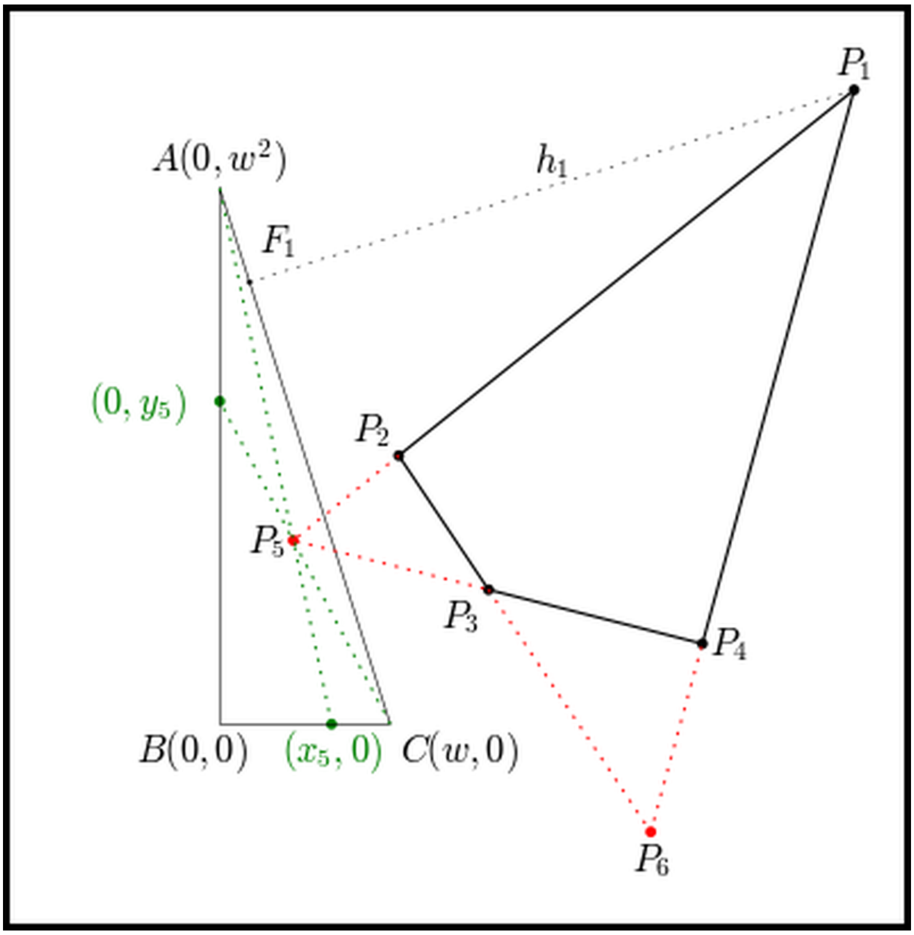
Message: #2793
Date: 30/12/2017 11:08:56
From: dominique.laurain31@orange.fr
Subject: Introduction to Quadrilateral Geometry

How many vertices does a quadrilateral have?

Hello Chris and geometers,

I like the idea to define a quadrilateral by four vertices (not three aligned) and two extra vertices for the missing intersection of couple of edges lines.
On the picture, the quad is defined by the vertices Pi(xi,yi) for i =1,2,3,4 with cartesian coordinates xi,yi in a subfield K of field of reals R.
We add the two extra vertices P5(x5,y5) and P6(x6,y6).
The quad can be defined too in trilinear coordinates using a reference triangle ABC with coordinates A(0,w^2), B(0,0) C(w,0) and slope w = AB / BC a real not in K.
Because no vertice Pi can be on BC line (otherwise w would be in K) we can set the second trilinear coordinates to 1, and have trilinear coordinates Xi : 1 : Zi so again the quad is defined by the eight values Xi and Zi.
If all Pi are in the first cartesian quadrant (x > 0, y > 0) we can set w large enough to have all Pi inside ABC, and trilinear coordinates Xi,Zi are positive not 0. In that case, we can also define the quad using the four cevian cuts ((x,y) for pictured point P5).
One example is : cartesian coordinates in the "golden field" K = QQ(sqrt(5)) and w = sqrt(3)
Another example: vertices on the lattice K = Z(sqrt(5)) and w = sqrt(3)
Another: K is quadratic extension of QQ and w = pi

Dominique



NewDefinitionQuad.png

Message: #2794
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message #2794

```
Message number 2794 is not available in Yahoo groups.
```

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Message: #2795
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message #2795

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Message number 2795 is not available in Yahoo groups.
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Message: #2796
Date: 2020-02-22
From: Systems Manager
Subject: Deleted Message #2796

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Message number 2796 is not available in Yahoo groups.
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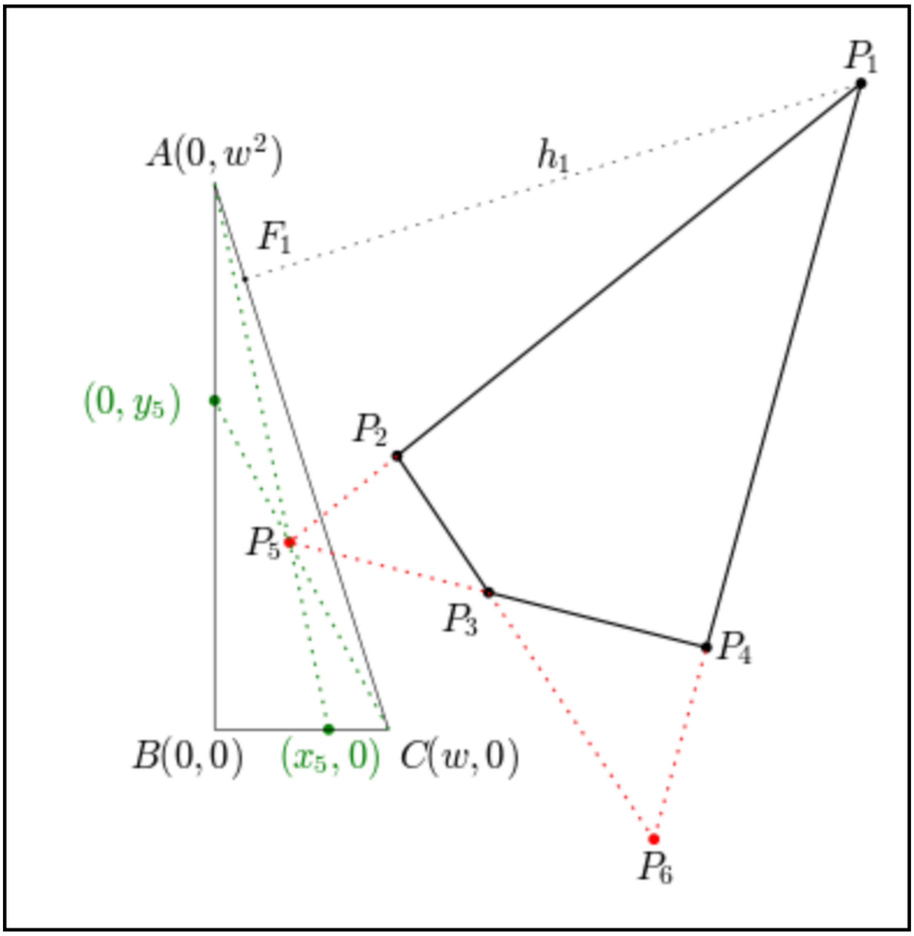
Message: #2797
Date: 30/12/2017 11:35:21
From: dominique.laurain31@orange.fr
Subject: Introduction to Quadrilateral Geometry

```
Hello Chris,

I give up (again): cannot attach picture file (PNG,JPEG,..) in
this forum.
Nevertheless: better than before, when attaching the file seems
to be attached...but in fact it is not

Dominique.
```



NewDefinitionQuad.pdf

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Message: #2798
Date: 30/12/2017 11:21:48
From: chris.vantienhoven
Subject: Introduction to Quadrilateral Geometry

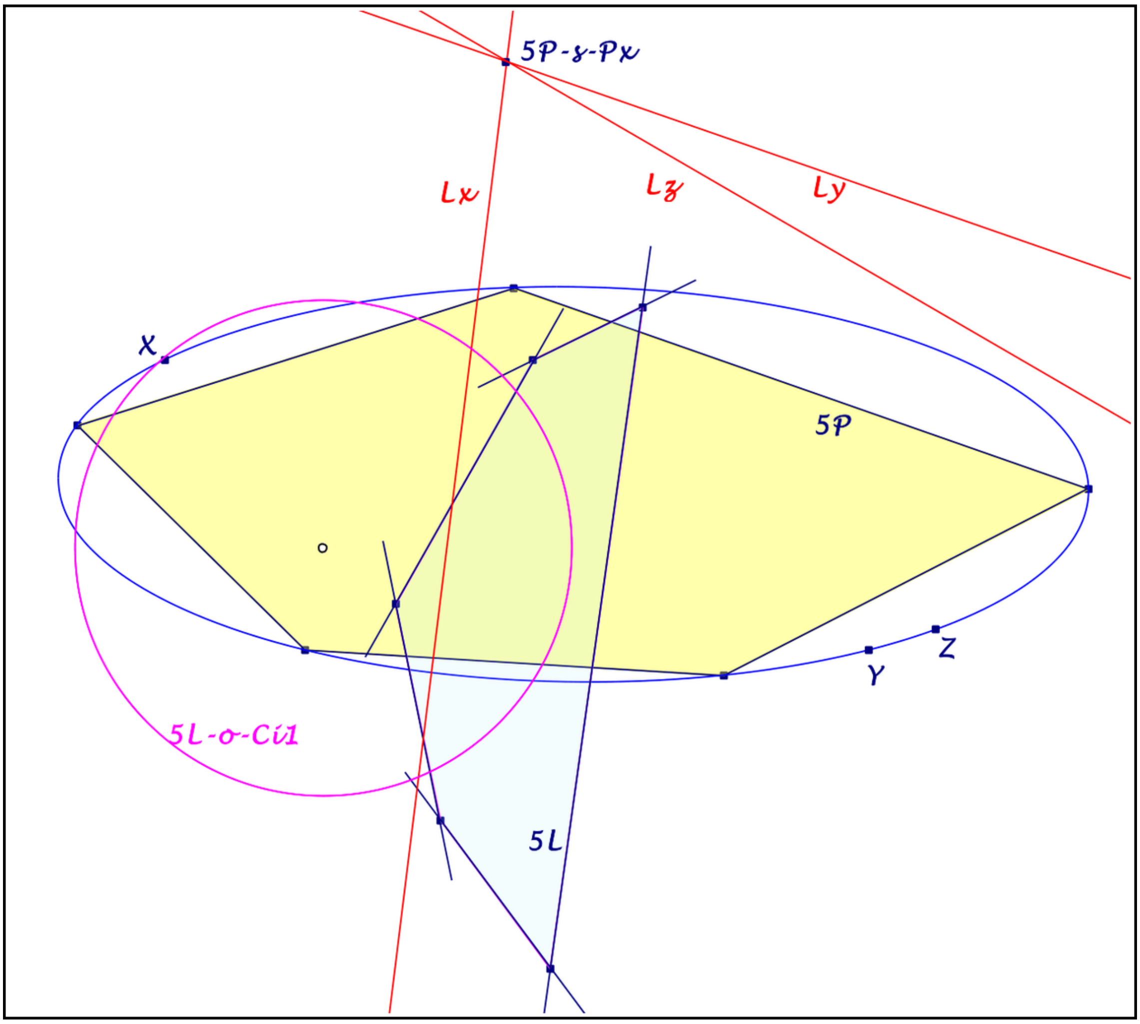
```
Dear Dominique,
Thanks for trying to attach PNG- and JPG-files.
But don't worry, your first 4 messages all had an attachment.
I could read the PNG-file and also the JPG-file.
However the PDF-file wasn't readable for me.
Best regards,
Chris
```

Message: #2799
Date: 31/12/2017 10:41:00
From: eckart_schmidt@t-online.de
Subject: many beautiful properties

```
Dear Bernard, dear Chris,

what about the following point 5P-s-Px?
Let X be a variable point on 5P-s-Co1:
... Consider the 5L of the five bisectors of X.Pi
... and its 5L-o-Ci1,
... bearing X,
... so that the CSC-circle of X wrt the 5L is a line Lx.
These lines Lx have a common point!

Best regards and Happy New Year
Eckart
```



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5 Keyword Index

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QA-Cu1:
#2170 #2209 #2279 #2337 #2340 #2348 #2351 #2404 #2435 #2447 #2448 #2449 #2450 #2455 #2456 #2458 #2459 #2463 #2464 #2465 #2466 #2468 #2475 #2477 #2478 #2479 #2480 #2481 #2486 #2487 #2488 #2490 #2492 #2494 #2500 #2519 #2561 #2639 #2640 #2720 #2721 #2722 #2724

QA-Cu7:
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QA-P3:
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QA-P4:
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QA-P41:
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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393