

**Journal of the
Quadri Figures Group
2018**

Digital Edition

Chris van Tienhoven et al.

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(jan. 2018 - dec. 2018)

Messages #2800 - #3299

Contents

1	Introduction	2
2	Authors	3
2.1	Author Index	4
2.2	Author Information	6
3	Subjects	17
4	Messages	23
4.1	Message Index	23
4.2	Messages	27
5	Keyword Index	500
6	Colophon	503

1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at <https://groups.io/g/Quadri-and-Poly-Geometry>. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at <https://groups.io/g/Quadri-Figures-Group>.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at <https://www.chrisvantienhoven.nl/>, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

[↔ Table of Contents](#) [↔ Message Index](#)

2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Benedetto Scimemi
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- Eckart Schmidt
- Ngo Quang Duong
- Peter Liepa
- Seiichi Kirikami
- Tran Quang Hung

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.

Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

Antreas Hatzipolakis:

[#2847](#) [#2859](#) [#2861](#) [#2959](#) [#3091](#) [#3106](#) [#3196](#) [#3267](#)

Benedetto Scimemi:

[#2808](#) [#2818](#) [#2820](#) [#2829](#) [#2863](#) [#2866](#) [#2913](#) [#2917](#) [#2946](#) [#2948](#) [#3036](#)
[#3049](#) [#3053](#) [#3056](#) [#3065](#) [#3067](#) [#3071](#) [#3088](#) [#3151](#)

Bernard Keizer:

[#2804](#) [#2826](#) [#2827](#) [#2828](#) [#2831](#) [#2835](#) [#2837](#) [#2839](#) [#2840](#) [#2844](#) [#2870](#)
[#2872](#) [#2877](#) [#2878](#) [#2883](#) [#2885](#) [#2891](#) [#2900](#) [#2918](#) [#2920](#) [#2922](#) [#2925](#)
[#2928](#) [#2936](#) [#2937](#) [#2940](#) [#2941](#) [#2944](#) [#2945](#) [#2951](#) [#2956](#) [#2960](#) [#2967](#)
[#2970](#) [#2976](#) [#2993](#) [#2994](#) [#2995](#) [#2996](#) [#2997](#) [#3001](#) [#3004](#) [#3008](#) [#3012](#)
[#3014](#) [#3016](#) [#3018](#) [#3019](#) [#3040](#) [#3041](#) [#3045](#) [#3061](#) [#3069](#) [#3070](#) [#3073](#)
[#3077](#) [#3078](#) [#3080](#) [#3083](#) [#3084](#) [#3087](#) [#3089](#) [#3090](#) [#3097](#) [#3100](#) [#3101](#)
[#3104](#) [#3109](#) [#3114](#) [#3121](#) [#3123](#) [#3124](#) [#3127](#) [#3129](#) [#3134](#) [#3137](#) [#3142](#)
[#3154](#) [#3158](#) [#3161](#) [#3163](#) [#3166](#) [#3170](#) [#3179](#) [#3180](#) [#3183](#) [#3189](#) [#3192](#)
[#3193](#) [#3195](#) [#3197](#) [#3198](#) [#3199](#) [#3203](#) [#3205](#) [#3209](#) [#3211](#) [#3212](#) [#3214](#)
[#3216](#) [#3220](#) [#3222](#) [#3224](#) [#3232](#) [#3234](#) [#3235](#) [#3237](#) [#3240](#) [#3242](#) [#3248](#)
[#3249](#) [#3251](#) [#3254](#) [#3260](#) [#3266](#) [#3268](#) [#3269](#) [#3271](#) [#3273](#) [#3274](#) [#3280](#)
[#3281](#) [#3284](#) [#3286](#) [#3287](#) [#3288](#) [#3290](#) [#3291](#) [#3293](#) [#3294](#) [#3295](#) [#3297](#)

Chris van Tienhoven:

[#2802](#) [#2806](#) [#2807](#) [#2812](#) [#2815](#) [#2816](#) [#2822](#) [#2823](#) [#2825](#) [#2832](#) [#2833](#)
[#2841](#) [#2849](#) [#2851](#) [#2853](#) [#2857](#) [#2862](#) [#2873](#) [#2874](#) [#2876](#) [#2879](#) [#2880](#)
[#2895](#) [#2898](#) [#2904](#) [#2909](#) [#2911](#) [#2923](#) [#2933](#) [#2934](#) [#2953](#) [#2954](#) [#2958](#)
[#2961](#) [#2962](#) [#2968](#) [#2971](#) [#2973](#) [#2975](#) [#2977](#) [#2978](#) [#2982](#) [#2983](#) [#2986](#)
[#3002](#) [#3003](#) [#3007](#) [#3010](#) [#3011](#) [#3013](#) [#3029](#) [#3032](#) [#3039](#) [#3042](#) [#3043](#)
[#3052](#) [#3058](#) [#3060](#) [#3112](#) [#3136](#) [#3140](#) [#3141](#) [#3143](#) [#3152](#) [#3156](#) [#3165](#)
[#3173](#) [#3177](#) [#3257](#) [#3262](#) [#3263](#) [#3276](#) [#3289](#) [#3298](#)

César Lozada:

[#3105](#)

Eckart Schmidt:

[#2800](#) [#2801](#) [#2803](#) [#2805](#) [#2809](#) [#2810](#) [#2811](#) [#2813](#) [#2814](#) [#2817](#) [#2819](#)
[#2821](#) [#2824](#) [#2830](#) [#2834](#) [#2836](#) [#2838](#) [#2842](#) [#2843](#) [#2845](#) [#2846](#) [#2848](#)
[#2850](#) [#2852](#) [#2854](#) [#2855](#) [#2856](#) [#2858](#) [#2860](#) [#2864](#) [#2865](#) [#2867](#) [#2868](#)
[#2869](#) [#2871](#) [#2875](#) [#2881](#) [#2882](#) [#2884](#) [#2886](#) [#2887](#) [#2888](#) [#2889](#) [#2890](#)
[#2892](#) [#2893](#) [#2894](#) [#2897](#) [#2899](#) [#2901](#) [#2902](#) [#2903](#) [#2905](#) [#2906](#) [#2907](#)
[#2908](#) [#2910](#) [#2912](#) [#2914](#) [#2915](#) [#2916](#) [#2919](#) [#2921](#) [#2924](#) [#2926](#) [#2927](#)
[#2929](#) [#2930](#) [#2931](#) [#2932](#) [#2935](#) [#2938](#) [#2939](#) [#2942](#) [#2943](#) [#2947](#) [#2949](#)
[#2950](#) [#2952](#) [#2955](#) [#2957](#) [#2963](#) [#2964](#) [#2965](#) [#2966](#) [#2969](#) [#2972](#) [#2974](#)
[#2979](#) [#2980](#) [#2981](#) [#2984](#) [#2985](#) [#2988](#) [#2989](#) [#2990](#) [#2991](#) [#2998](#) [#2999](#)
[#3000](#) [#3005](#) [#3006](#) [#3015](#) [#3017](#) [#3020](#) [#3021](#) [#3022](#) [#3024](#) [#3025](#) [#3028](#)

#3030 #3031 #3037 #3038 #3044 #3046 #3047 #3048 #3050 #3051 #3054
#3055 #3057 #3059 #3062 #3063 #3064 #3066 #3068 #3072 #3074 #3075
#3076 #3079 #3081 #3082 #3085 #3086 #3092 #3093 #3094 #3095 #3096
#3098 #3099 #3102 #3103 #3107 #3108 #3110 #3111 #3113 #3115 #3116
#3117 #3118 #3119 #3120 #3122 #3125 #3126 #3128 #3130 #3131 #3132
#3135 #3139 #3144 #3145 #3146 #3148 #3149 #3150 #3153 #3155 #3157
#3159 #3160 #3162 #3164 #3168 #3171 #3172 #3175 #3176 #3178 #3181
#3184 #3185 #3186 #3187 #3188 #3190 #3191 #3194 #3200 #3201 #3202
#3204 #3206 #3207 #3208 #3210 #3213 #3215 #3217 #3218 #3219 #3221
#3223 #3228 #3231 #3236 #3238 #3241 #3244 #3245 #3246 #3247 #3252
#3253 #3256 #3258 #3259 #3261 #3264 #3265 #3270 #3272 #3275 #3277
#3278 #3279 #3282 #3283 #3285 #3292 #3296 #3299

Ngo Quang Duong:

#2896 #3133 #3138 #3147

Peter Liepa:

#2987 #2992 #3009 #3027

Seiichi Kirikami:

#3167 #3169 #3174 #3182

Tran Quang Hung:

#3023 #3026 #3033 #3034 #3035 #3225 #3226 #3227 #3229 #3230 #3233
#3239 #3243 #3250 #3255

[↔ Table of Contents](#) [↔ Message Index](#)

2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location

Lives in Greece.

Year of Birth / Generation

1952.

Short Biography

Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacanthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: <http://www.anthrakitis.blogspot.com/>

Benedetto Scimemi (1938–2023)

Location

Italy

Year of Birth / Generation

1938–2023.

Short Biography

Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.es-teri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter’s 60th birthday (the “A5 Meeting”, named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See [CO-Tf3](#) in EPG.
- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location

Living in the Netherlands.

Year of Birth

1950.

Short Biography

Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. *Journal of Geometry and Graphics*, Vol. 25, No. 1, 2021.

Other Remarks

Website: www.chrisvantienhoven.nl

Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location

Living in Germany.

Year of Birth / Generation

1939.

Short Biography

Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n -gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Ngo Quang Duong

Location

Living in Hanoi, Vietnam.

Year of Birth / Generation

Born in 1998.

Generation Z (approx. 1997–2012).

Short Biography

Ngo Quang Duong studied at the Vietnam National University, where he initially majored in Mathematics before switching to Software Engineering. He has since been working professionally in the software field, while continuing to pursue his interest in Mathematics in his free time. His background combines formal mathematical training with practical experience in computing and problem solving, giving his work a distinctive blend of theoretical insight and computational intuition.

Themes and Interests

- Geometry, Topology, Analysis, and their interactions
- Classical Geometry, especially triangle geometry and quadri-figure geometry
- Contributions to QFG, including n -angle centers and new uses of coordinate systems
- General mathematical exploration and independent study

Selected Publications

- (with T. T. Vu) *A Generalization of the Droz Farny Line Theorem with Orthologic Triangles*, Forum Geometricorum, Volume 16 (2016), 415–418.
- *Generalization of Musselman's theorem. Some Properties of Isogonal Conjugate Points*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 15–29.
- (with O. T. Dao and P. Yiu) *Golden Sections in an Isosceles Triangle and Its Circumcircle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 93–97.
- *Generalizations of Lester Circle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 10 (2021), 49–61.

Additional Remarks

He has not actively returned to Classical Geometry for some time, but remains mathematically engaged through online communities. He is active on Math StackExchange (MSE), where he contributes under the profile: <https://www.math.stackexchange.com/users/821868/duong-ngo>

Seiichi Kirikami (1949?–2023)

Location

Japan.

Year of Birth / Generation

Exact year unknown; passed away on 11 December 2023.

Short Biography

In daily life Seiichi worked as a mechanical engineer in the Thermal Power Division of Hitachi, Ltd. In his free time he enriched the geometry community with original ideas, elegant constructions, and generous participation in many collaborative discussions. From 2013 to 2018, Seiichi contributed intensively to the development of Quadri- and Poly-Geometry within the Quadri-Figures Group. His insights, constructions, and discussions were instrumental in the group's formative years, and his contributions helped define several of the key geometric notions that emerged in those years. Later on he contributed extensively to other groups such as Anopolis, Hyacinthos, Romantics of Geometry, ADGEOM, and the Encyclopedia of Triangle Centers (ETC), where many of his ideas became foundational. His work was characterized by simplicity, depth, and a unique ability to see geometric structures from unexpected angles. He was also known for his humility, kindness, and willingness to help others — qualities remembered fondly by colleagues and friends.

Themes and Interests

- Euclidean and projective geometry
- Triangle geometry and classical configurations
- Geometric problem creation and exploration
- OEIS contributions and combinatorial structures
- Applied mathematics, including epidemiological modelling
- Engineering and turbine-related innovations (patents)

Selected Contributions

Seiichi Kirikami's geometric ideas inspired many theorems, conjectures, and new terminology. Among the most notable:

- The *Kirikami six-circles configuration*, which led to the Hatzipolakis–Moses Theorem.
- His prompting led to the introduction of the term *Cyclologic*, now established in triangle-geometry terminology.
- He suggested naming the line through $X(5)$ perpendicular to the Euler line the *Hatzipolakis axis*.
- Numerous contributions to ETC, Hyacinthos, and other geometry forums.
- 29 OEIS entries associated with his name.

- Several patents in turbine-engine technology.
- Publications in epidemiology, including COVID-related infection-spread modelling.

Community Tributes

Colleagues remembered Seiichi with deep affection:

- “Geometry is the poorer of his death.” — A. P. Hatzipolakis
- “He was a great expert in projective and triangle geometry, and a very kind and helpful person.” — E. Suppa
- “He had a way of seeing things from a different angle and presenting them simply.” — C. van Tienhoven
- “He inspired many of my problems.” — A. Altıntaş
- “He wrote many interesting and innovative messages in ADGEOM.” — F. J. García Capitán
- “He helped me in my beginnings with wise and generous advice.” — C. E. Lozada
- “He contributed to OEIS and had important work in epidemiology.” — P. Moses

Additional Remarks

Seiichi Kirikami is remembered as a gentle, insightful, and generous geometer whose ideas continue to inspire new discoveries. His legacy lives on in the many theorems, concepts, and geometric structures that bear his influence.

Quang Hung Tran

Location

Born and working in Hanoi, Vietnam.

Year of Birth / Generation

Millennial (approx. 1981–1996).

Short Biography

Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdős–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:

www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

subject: 2 Foci and 3 RH's:

[#2996](#)

subject: 3 foci and 4 SC's:

[#2997](#)

subject: 3 foci of Siebeck's sextic:

[#2922](#) [#2928](#) [#2930](#) [#2937](#) [#2938](#) [#2939](#) [#2940](#) [#2941](#) [#2942](#) [#2943](#) [#2944](#)
[#2945](#) [#2949](#) [#2950](#) [#2951](#) [#2979](#) [#2995](#)

subject: 3 QL-versions of QA-Cu7:

[#2968](#) [#2969](#) [#2973](#) [#2974](#) [#2975](#) [#2976](#) [#2980](#) [#2981](#) [#2983](#) [#2984](#) [#2986](#)
[#2988](#) [#2991](#) [#2993](#) [#2998](#) [#3001](#) [#3003](#) [#3004](#) [#3005](#) [#3006](#) [#3008](#) [#3010](#)
[#3012](#) [#3013](#) [#3014](#) [#3015](#) [#3016](#) [#3018](#) [#3019](#) [#3020](#)

subject: 4 circles with 6 concurring radical axes and a new QA-transformation:

[#3032](#) [#3038](#)

subject: 4 circles with a common point on a conic:

[#3182](#) [#3184](#)

subject: 5-line central objects:

[#3151](#) [#3152](#)

subject: 5G-s-P1, 5G-s-P2, 5P-s-P1:

[#2848](#) [#2852](#) [#2853](#)

subject: 5P-s-2Px:

[#2912](#)

subject: 5P-s-Tf3:

[#2805](#) [#2807](#) [#2808](#) [#2809](#) [#2810](#) [#2811](#) [#2812](#) [#2813](#) [#2814](#) [#2817](#) [#2834](#)

subject: 8. Harmonic properties in a Quadrangle:

[#2875](#) [#2876](#) [#2878](#) [#2880](#) [#2881](#)

subject: A property of QA-P4 as the extension of the circular center:

[#3167](#) [#3168](#) [#3169](#) [#3171](#) [#3174](#)

subject: A property of QL-Tf5:

[#2896](#) [#2898](#)

subject: A Quartic for Quadrilaterals:

[#3031](#)

subject: Affinely Regular Components of a 5-gon:

[#2854](#)

subject: Backup for Yahoo Group messages (also extended User Interface):

[#2816](#)

subject: Cayley-Bacharach points on cubics:
[#3219](#) [#3220](#) [#3222](#) [#3223](#) [#3224](#) [#3232](#) [#3234](#) [#3235](#)

subject: Cayley-Bacharach transformation for a QL:
[#2897](#)

subject: Cevian triangles:
[#2835](#) [#2837](#) [#2838](#) [#2839](#) [#2840](#)

subject: Co-geometry with special QA:
[#2921](#)

subject: CO-Tf3 in a separate section on the EQF/EPG-site:
[#2961](#)

subject: Concurrent lines in quadrangle points:
[#3034](#) [#3035](#)

subject: Concurrent Orthotransversal:
[#3227](#) [#3228](#) [#3229](#)

subject: Concurrent Simson lines:
[#3250](#) [#3252](#) [#3255](#)

subject: Construction Cayley-Bacharach Point:
[#3042](#) [#3043](#) [#3046](#) [#3047](#) [#3052](#) [#3054](#) [#3055](#) [#3058](#) [#3059](#) [#3060](#) [#3061](#)

subject: Dual QA / QL:
[#3164](#) [#3165](#)

subject: Errata Corrige. A new edition coming:
[#3067](#) [#3070](#) [#3071](#) [#3072](#) [#3073](#) [#3074](#) [#3075](#) [#3077](#) [#3078](#) [#3079](#) [#3080](#)
[#3082](#) [#3083](#) [#3084](#) [#3085](#)

subject: Excenters/incenter quadrangle A'B'C'D':
[#3116](#)

subject: Fixed points of CB-transformations:
[#3178](#) [#3180](#) [#3183](#) [#3185](#) [#3186](#)

subject: How to construct the inverse of QA-Tf6 (and QA-Tf3). A QA-quartic as:
[#3036](#) [#3048](#) [#3049](#) [#3050](#) [#3051](#) [#3057](#) [#3062](#)

subject: Inscribed QL-conic through QL-P1:
[#2887](#) [#2888](#)

subject: Introduction to Quadrilateral Geometry:
[#2802](#) [#2804](#) [#2806](#) [#2815](#) [#2832](#) [#2841](#) [#2851](#) [#2862](#) [#2863](#) [#2864](#) [#2873](#)
[#2874](#) [#2877](#) [#2879](#) [#2895](#) [#2909](#) [#2923](#) [#2925](#) [#2933](#) [#2936](#) [#2952](#) [#2953](#)
[#2964](#) [#2977](#) [#2978](#)

subject: Isoconjugation for a pentagon:
[#2830](#)

subject: Isoconjugation with CB-points:
[#2869](#) [#2884](#)

subject: Isogonal conjugate points in Quadrigon:
[#3023](#) [#3024](#) [#3025](#) [#3026](#) [#3028](#) [#3030](#) [#3033](#) [#3037](#)

subject: Kantor-Hervey, Serret and generalisation:
[#3121](#) [#3122](#) [#3123](#) [#3124](#) [#3125](#) [#3126](#) [#3127](#) [#3128](#) [#3129](#)

subject: Line $\langle \!-\! \rangle$ point QL-transformation:
[#3017](#) [#3021](#)

subject: Lines is perpendicular to Newton line:
[#3225](#) [#3226](#) [#3230](#) [#3231](#) [#3233](#) [#3239](#) [#3241](#) [#3243](#)

subject: Locus of foci:
[#3041](#) [#3044](#) [#3045](#)

subject: Many beautiful properties:
[#2800](#)

subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point:
[#3143](#) [#3144](#) [#3145](#) [#3146](#) [#3149](#) [#3150](#) [#3153](#) [#3155](#) [#3156](#)

subject: Miquel triangle and Isogonal center:
[#2847](#) [#2849](#) [#2850](#)

subject: More on Co-Tf3:
[#2818](#) [#2819](#) [#2820](#) [#2822](#) [#2826](#) [#2827](#) [#2828](#) [#2829](#) [#2831](#) [#2833](#) [#2836](#)
[#2916](#) [#2918](#) [#2919](#) [#2920](#)

subject: MVP-points for a 5P:
[#2821](#) [#2823](#) [#2824](#) [#2825](#)

subject: New aspect of 5P-s-Tf3:
[#2907](#) [#2908](#) [#2913](#) [#2914](#) [#2917](#)

subject: New aspect of 5P-sTf3:
[#2910](#)

subject: New aspect of QA-P4:
[#2801](#)

subject: New CB-point on QA-Cu1:
[#2868](#) [#2870](#) [#2871](#) [#2872](#)

subject: New concept for QL-cubics:
[#3200](#) [#3201](#) [#3203](#) [#3204](#)

subject: new items in EQF / EPG:
[#3039](#) [#3056](#)

subject: New point for a 5P:
[#3256](#) [#3257](#) [#3258](#) [#3259](#) [#3260](#) [#3263](#) [#3264](#) [#3266](#) [#3267](#) [#3273](#)

subject: New QA-points and lines:
[#3118](#)

subject: New QA-Points wrt QA-P4 and QA-Co3:
[#2803](#)

subject: New QA-Tf, new QA-circumcubics:
[#3175](#)

subject: New QA-transformation?:
[#3110](#) [#3115](#)

subject: New sight of QA-Cu1:
[#2867](#) [#2882](#) [#2883](#) [#2885](#) [#2886](#) [#2889](#) [#2890](#) [#2891](#) [#2894](#)

subject: nL-n-Ci1:
[#2903](#) [#2904](#)

subject: nL-n-Ci2:
[#2905](#) [#2906](#) [#2911](#)

subject: No Subject:
[#3053](#)

subject: Nonpivotal Isocubics for QA/QL:
[#2842](#) [#2846](#)

subject: Old Babylonians used Trigonometry 3700 years ago:
[#3002](#)

subject: Orthogonal chords through a fixed point in a conic:
[#2946](#) [#2947](#) [#2948](#)

subject: Parallelograms inscribed in a quadrigon:
[#3119](#) [#3120](#)

subject: Paralogic n-Line:
[#3133](#) [#3136](#) [#3138](#) [#3140](#) [#3147](#)

subject: Perspective QL-triple triangles:
[#3139](#) [#3141](#)

subject: Ptolemy triangle and QA-Orthopole-circle:
[#2859](#)

subject: QA-Cu7:
[#2965](#) [#3081](#) [#3117](#)

subject: QA-Cu7 as QA-Cu1 of a special quadrigon:
[#3132](#) [#3134](#) [#3135](#) [#3137](#) [#3142](#) [#3148](#) [#3154](#)

subject: QA-Cu7-Geometry:
[#3176](#) [#3177](#) [#3179](#) [#3181](#) [#3187](#) [#3188](#) [#3189](#) [#3190](#) [#3191](#) [#3192](#) [#3193](#)
[#3194](#) [#3195](#) [#3196](#) [#3197](#) [#3198](#) [#3199](#) [#3202](#) [#3205](#) [#3206](#) [#3207](#) [#3208](#)
[#3209](#) [#3210](#) [#3211](#) [#3212](#) [#3213](#) [#3214](#) [#3215](#) [#3216](#) [#3217](#) [#3218](#) [#3221](#)
[#3236](#) [#3237](#) [#3238](#) [#3240](#) [#3242](#) [#3244](#) [#3245](#) [#3246](#) [#3247](#) [#3248](#) [#3249](#)
[#3251](#) [#3253](#) [#3254](#) [#3261](#) [#3262](#) [#3270](#) [#3271](#) [#3272](#) [#3274](#) [#3275](#) [#3280](#)
[#3292](#) [#3293](#) [#3294](#) [#3295](#) [#3296](#) [#3297](#) [#3298](#) [#3299](#)

subject: QA-Cu7-Geometry / new QA-Transformation:
[#3276](#) [#3277](#) [#3278](#) [#3279](#) [#3281](#) [#3282](#) [#3283](#) [#3284](#) [#3285](#) [#3286](#) [#3287](#)
[#3288](#) [#3289](#) [#3290](#) [#3291](#)

subject: QA-Lemniscates:
[#3062](#) [#3063](#)

subject: QA-Orthopole-Circles for a QL:
[#2860](#) [#2861](#)

subject: QA-P4:
[#3064](#)

subject: QA-P4, QA-P2:
[#2915](#)

subject: QA-P4, QL-P1 and QG-P1 for a quadrigon:
[#3131](#)

subject: QA-Qu1 and QA-Qu2:
[#3088](#) [#3093](#) [#3094](#) [#3095](#) [#3096](#)

subject: QA-Tf2:
[#2855](#) [#2856](#) [#2857](#) [#2858](#)

subject: QA-Tf2 in a QL:
[#2982](#) [#2985](#) [#2994](#) [#2999](#) [#3007](#) [#3022](#)

subject: QA/QL-Conic:
[#2843](#) [#2844](#) [#2845](#)

subject: QA: intersection points are on radical axes of certain circle pairs:
[#2987](#) [#2989](#) [#2990](#) [#2992](#) [#3000](#) [#3009](#) [#3011](#) [#3027](#) [#3029](#) [#3040](#)

subject: QG-P18/QG-P19 geometry for a QL:
[#3160](#)

subject: QL, epi- and hypocycloïds and r-angle centers:
[#3166](#) [#3170](#) [#3172](#) [#3173](#)

subject: QL-Ci3: Miquel Circle:
[#3105](#) [#3106](#) [#3107](#) [#3108](#) [#3113](#)

subject: QL-Cox:
[#3068](#) [#3069](#)

subject: QL-Cu7-Geometry:
[#3265](#) [#3268](#) [#3269](#)

subject: QL-P10:
[#3157](#) [#3158](#) [#3159](#) [#3161](#) [#3162](#) [#3163](#)

subject: QL-P17:
[#2893](#)

subject: QL-Qu2:
[#3103](#) [#3112](#)

subject: QL-Strophoid and QL-Ortho-Hyperbola:
[#2902](#)

subject: Relevant line-line transformation for Q:
[#3130](#)

subject: Serret's inconics:
[#3089](#) [#3090](#) [#3091](#) [#3097](#) [#3098](#) [#3099](#) [#3100](#) [#3101](#) [#3102](#) [#3104](#) [#3109](#)
[#3111](#) [#3114](#)

subject: Special CB-points:
[#2892](#)

subject: Square circumscribed a quadrigon:
[#2926](#)

subject: Square inscribed a quadrigon:
[#2924](#) [#2934](#) [#2935](#)

subject: Square perspective a quadrigon:
[#2927](#) [#2929](#) [#2931](#) [#2932](#)

subject: Steiner axes:
[#2899](#) [#2900](#) [#2901](#)

subject: Terms confusion:
[#2865](#) [#2866](#)

subject: Tessellation theory applied in Triangles, Quadrilaterals, Pentalaterals:
[#2954](#) [#2955](#) [#2956](#) [#2957](#) [#2958](#) [#2960](#) [#2962](#) [#2963](#) [#2966](#) [#2967](#) [#2970](#)
[#2971](#) [#2972](#)

subject: The QA-inscribed quartic and a coincidence:
[#3065](#) [#3066](#)

subject: Triangle of bisectors for opposite QL-points:
[#3076](#)

subject: Two cubics for a QA/QL constellation:
[#3086](#) [#3087](#) [#3092](#)

subject: Yahoogroups problems:
[#2959](#)

[↔ Table of Contents](#) [↔ Message Index](#)

4 Messages

4.1 Message Index

Msg #2800: page 27	Msg #2846: page 80	Msg #2892: page 121
Msg #2801: page 28	Msg #2847: page 81	Msg #2893: page 122
Msg #2802: page 29	Msg #2848: page 81	Msg #2894: page 123
Msg #2803: page 30	Msg #2849: page 82	Msg #2895: page 124
Msg #2804: page 33	Msg #2850: page 82	Msg #2896: page 126
Msg #2805: page 34	Msg #2851: page 83	Msg #2897: page 127
Msg #2806: page 35	Msg #2852: page 83	Msg #2898: page 130
Msg #2807: page 36	Msg #2853: page 85	Msg #2899: page 131
Msg #2808: page 36	Msg #2854: page 86	Msg #2900: page 132
Msg #2809: page 37	Msg #2855: page 89	Msg #2901: page 133
Msg #2810: page 38	Msg #2856: page 89	Msg #2902: page 134
Msg #2811: page 39	Msg #2857: page 89	Msg #2903: page 136
Msg #2812: page 40	Msg #2858: page 90	Msg #2904: page 136
Msg #2813: page 41	Msg #2859: page 91	Msg #2905: page 137
Msg #2814: page 41	Msg #2860: page 91	Msg #2906: page 138
Msg #2815: page 44	Msg #2861: page 95	Msg #2907: page 139
Msg #2816: page 45	Msg #2862: page 96	Msg #2908: page 141
Msg #2817: page 46	Msg #2863: page 98	Msg #2909: page 142
Msg #2818: page 47	Msg #2864: page 98	Msg #2910: page 144
Msg #2819: page 48	Msg #2865: page 99	Msg #2911: page 145
Msg #2820: page 49	Msg #2866: page 99	Msg #2912: page 145
Msg #2821: page 50	Msg #2867: page 100	Msg #2913: page 147
Msg #2822: page 51	Msg #2868: page 101	Msg #2914: page 149
Msg #2823: page 52	Msg #2869: page 102	Msg #2915: page 151
Msg #2824: page 53	Msg #2870: page 103	Msg #2916: page 151
Msg #2825: page 53	Msg #2871: page 104	Msg #2917: page 152
Msg #2826: page 54	Msg #2872: page 104	Msg #2918: page 153
Msg #2827: page 55	Msg #2873: page 105	Msg #2919: page 154
Msg #2828: page 55	Msg #2874: page 107	Msg #2920: page 155
Msg #2829: page 56	Msg #2875: page 108	Msg #2921: page 155
Msg #2830: page 56	Msg #2876: page 108	Msg #2922: page 159
Msg #2831: page 59	Msg #2877: page 109	Msg #2923: page 160
Msg #2832: page 60	Msg #2878: page 109	Msg #2924: page 161
Msg #2833: page 64	Msg #2879: page 110	Msg #2925: page 161
Msg #2834: page 65	Msg #2880: page 111	Msg #2926: page 162
Msg #2835: page 66	Msg #2881: page 111	Msg #2927: page 163
Msg #2836: page 67	Msg #2882: page 112	Msg #2928: page 164
Msg #2837: page 70	Msg #2883: page 113	Msg #2929: page 165
Msg #2838: page 71	Msg #2884: page 114	Msg #2930: page 165
Msg #2839: page 72	Msg #2885: page 115	Msg #2931: page 166
Msg #2840: page 72	Msg #2886: page 116	Msg #2932: page 167
Msg #2841: page 73	Msg #2887: page 117	Msg #2933: page 167
Msg #2842: page 74	Msg #2888: page 118	Msg #2934: page 168
Msg #2843: page 77	Msg #2889: page 119	Msg #2935: page 169
Msg #2844: page 78	Msg #2890: page 120	Msg #2936: page 170
Msg #2845: page 79	Msg #2891: page 121	Msg #2937: page 171

Msg #2938: page 172	Msg #2988: page 222	Msg #3038: page 258
Msg #2939: page 174	Msg #2989: page 222	Msg #3039: page 259
Msg #2940: page 176	Msg #2990: page 223	Msg #3040: page 260
Msg #2941: page 177	Msg #2991: page 223	Msg #3041: page 261
Msg #2942: page 178	Msg #2992: page 224	Msg #3042: page 262
Msg #2943: page 178	Msg #2993: page 224	Msg #3043: page 264
Msg #2944: page 179	Msg #2994: page 225	Msg #3044: page 265
Msg #2945: page 180	Msg #2995: page 225	Msg #3045: page 266
Msg #2946: page 181	Msg #2996: page 226	Msg #3046: page 267
Msg #2947: page 183	Msg #2997: page 227	Msg #3047: page 268
Msg #2948: page 184	Msg #2998: page 228	Msg #3048: page 269
Msg #2949: page 185	Msg #2999: page 228	Msg #3049: page 270
Msg #2950: page 185	Msg #3000: page 229	Msg #3050: page 270
Msg #2951: page 186	Msg #3001: page 229	Msg #3051: page 271
Msg #2952: page 187	Msg #3002: page 230	Msg #3052: page 271
Msg #2953: page 188	Msg #3003: page 231	Msg #3053: page 273
Msg #2954: page 190	Msg #3004: page 232	Msg #3054: page 274
Msg #2955: page 190	Msg #3005: page 232	Msg #3055: page 275
Msg #2956: page 192	Msg #3006: page 233	Msg #3056: page 276
Msg #2957: page 193	Msg #3007: page 234	Msg #3057: page 277
Msg #2958: page 194	Msg #3008: page 235	Msg #3058: page 278
Msg #2959: page 195	Msg #3009: page 236	Msg #3059: page 284
Msg #2960: page 196	Msg #3010: page 237	Msg #3060: page 285
Msg #2961: page 197	Msg #3011: page 238	Msg #3061: page 286
Msg #2962: page 199	Msg #3012: page 238	Msg #3062: page 287
Msg #2963: page 199	Msg #3013: page 239	Msg #3063: page 288
Msg #2964: page 200	Msg #3014: page 240	Msg #3064: page 289
Msg #2965: page 202	Msg #3015: page 241	Msg #3065: page 290
Msg #2966: page 203	Msg #3016: page 242	Msg #3066: page 293
Msg #2967: page 204	Msg #3017: page 243	Msg #3067: page 294
Msg #2968: page 204	Msg #3018: page 244	Msg #3068: page 295
Msg #2969: page 205	Msg #3019: page 245	Msg #3069: page 296
Msg #2970: page 205	Msg #3020: page 245	Msg #3070: page 297
Msg #2971: page 206	Msg #3021: page 246	Msg #3071: page 298
Msg #2972: page 207	Msg #3022: page 246	Msg #3072: page 300
Msg #2973: page 208	Msg #3023: page 247	Msg #3073: page 300
Msg #2974: page 209	Msg #3024: page 247	Msg #3074: page 301
Msg #2975: page 209	Msg #3025: page 248	Msg #3075: page 302
Msg #2976: page 210	Msg #3026: page 248	Msg #3076: page 302
Msg #2977: page 210	Msg #3027: page 249	Msg #3077: page 304
Msg #2978: page 211	Msg #3028: page 249	Msg #3078: page 305
Msg #2979: page 213	Msg #3029: page 250	Msg #3079: page 306
Msg #2980: page 214	Msg #3030: page 250	Msg #3080: page 307
Msg #2981: page 215	Msg #3031: page 251	Msg #3081: page 308
Msg #2982: page 216	Msg #3032: page 252	Msg #3082: page 309
Msg #2983: page 217	Msg #3033: page 253	Msg #3083: page 309
Msg #2984: page 218	Msg #3034: page 253	Msg #3084: page 310
Msg #2985: page 219	Msg #3035: page 254	Msg #3085: page 310
Msg #2986: page 220	Msg #3036: page 255	Msg #3086: page 311
Msg #2987: page 221	Msg #3037: page 258	Msg #3087: page 313

Msg #3088: page 314	Msg #3138: page 368	Msg #3188: page 412
Msg #3089: page 324	Msg #3139: page 369	Msg #3189: page 413
Msg #3090: page 325	Msg #3140: page 370	Msg #3190: page 414
Msg #3091: page 325	Msg #3141: page 370	Msg #3191: page 415
Msg #3092: page 326	Msg #3142: page 371	Msg #3192: page 416
Msg #3093: page 327	Msg #3143: page 372	Msg #3193: page 417
Msg #3094: page 327	Msg #3144: page 373	Msg #3194: page 418
Msg #3095: page 328	Msg #3145: page 374	Msg #3195: page 419
Msg #3096: page 329	Msg #3146: page 374	Msg #3196: page 420
Msg #3097: page 330	Msg #3147: page 375	Msg #3197: page 421
Msg #3098: page 331	Msg #3148: page 375	Msg #3198: page 421
Msg #3099: page 332	Msg #3149: page 376	Msg #3199: page 422
Msg #3100: page 333	Msg #3150: page 377	Msg #3200: page 423
Msg #3101: page 333	Msg #3151: page 378	Msg #3201: page 424
Msg #3102: page 334	Msg #3152: page 379	Msg #3202: page 425
Msg #3103: page 335	Msg #3153: page 380	Msg #3203: page 426
Msg #3104: page 335	Msg #3154: page 381	Msg #3204: page 426
Msg #3105: page 336	Msg #3155: page 382	Msg #3205: page 427
Msg #3106: page 337	Msg #3156: page 383	Msg #3206: page 428
Msg #3107: page 338	Msg #3157: page 383	Msg #3207: page 429
Msg #3108: page 339	Msg #3158: page 384	Msg #3208: page 430
Msg #3109: page 340	Msg #3159: page 385	Msg #3209: page 431
Msg #3110: page 341	Msg #3160: page 386	Msg #3210: page 432
Msg #3111: page 342	Msg #3161: page 387	Msg #3211: page 432
Msg #3112: page 342	Msg #3162: page 388	Msg #3212: page 433
Msg #3113: page 343	Msg #3163: page 389	Msg #3213: page 434
Msg #3114: page 344	Msg #3164: page 389	Msg #3214: page 434
Msg #3115: page 345	Msg #3165: page 390	Msg #3215: page 435
Msg #3116: page 346	Msg #3166: page 390	Msg #3216: page 435
Msg #3117: page 347	Msg #3167: page 394	Msg #3217: page 436
Msg #3118: page 348	Msg #3168: page 395	Msg #3218: page 437
Msg #3119: page 349	Msg #3169: page 396	Msg #3219: page 438
Msg #3120: page 350	Msg #3170: page 397	Msg #3220: page 438
Msg #3121: page 351	Msg #3171: page 398	Msg #3221: page 439
Msg #3122: page 352	Msg #3172: page 399	Msg #3222: page 440
Msg #3123: page 353	Msg #3173: page 399	Msg #3223: page 441
Msg #3124: page 354	Msg #3174: page 400	Msg #3224: page 442
Msg #3125: page 355	Msg #3175: page 400	Msg #3225: page 443
Msg #3126: page 356	Msg #3176: page 401	Msg #3226: page 444
Msg #3127: page 356	Msg #3177: page 403	Msg #3227: page 445
Msg #3128: page 357	Msg #3178: page 404	Msg #3228: page 446
Msg #3129: page 357	Msg #3179: page 405	Msg #3229: page 446
Msg #3130: page 358	Msg #3180: page 405	Msg #3230: page 447
Msg #3131: page 359	Msg #3181: page 406	Msg #3231: page 448
Msg #3132: page 361	Msg #3182: page 407	Msg #3232: page 449
Msg #3133: page 363	Msg #3183: page 408	Msg #3233: page 450
Msg #3134: page 365	Msg #3184: page 408	Msg #3234: page 451
Msg #3135: page 366	Msg #3185: page 409	Msg #3235: page 451
Msg #3136: page 367	Msg #3186: page 410	Msg #3236: page 452
Msg #3137: page 367	Msg #3187: page 411	Msg #3237: page 453

Msg #3238: page 454	Msg #3259: page 469	Msg #3280: page 484
Msg #3239: page 455	Msg #3260: page 470	Msg #3281: page 485
Msg #3240: page 456	Msg #3261: page 471	Msg #3282: page 486
Msg #3241: page 457	Msg #3262: page 472	Msg #3283: page 487
Msg #3242: page 457	Msg #3263: page 473	Msg #3284: page 488
Msg #3243: page 458	Msg #3264: page 473	Msg #3285: page 489
Msg #3244: page 458	Msg #3265: page 474	Msg #3286: page 490
Msg #3245: page 459	Msg #3266: page 475	Msg #3287: page 491
Msg #3246: page 459	Msg #3267: page 476	Msg #3288: page 491
Msg #3247: page 460	Msg #3268: page 476	Msg #3289: page 492
Msg #3248: page 461	Msg #3269: page 477	Msg #3290: page 492
Msg #3249: page 462	Msg #3270: page 477	Msg #3291: page 493
Msg #3250: page 463	Msg #3271: page 478	Msg #3292: page 494
Msg #3251: page 463	Msg #3272: page 479	Msg #3293: page 495
Msg #3252: page 464	Msg #3273: page 479	Msg #3294: page 496
Msg #3253: page 464	Msg #3274: page 480	Msg #3295: page 496
Msg #3254: page 465	Msg #3275: page 480	Msg #3296: page 497
Msg #3255: page 466	Msg #3276: page 481	Msg #3297: page 498
Msg #3256: page 467	Msg #3277: page 482	Msg #3298: page 498
Msg #3257: page 468	Msg #3278: page 483	Msg #3299: page 499
Msg #3258: page 468	Msg #3279: page 483	

[↔ Table of Contents](#)

4.2 Messages

Message: #2800

Date: 03/1/2018 4:28:28

From: eckart_schmidt@t-online.de

Subject: many beautiful properties

Dear Bernard, dear Chris,

(a) For a 5P there is a special point P_x on 5P-s-Co1:
The ninth Cayley-Bacharach point
... of P_1, P_2, P_3, P_4, P_5 ,
... .. the foci of 5P-s-Co1
... .. and a point P on 5P-s-Co1
... is a point Q on the main axis of 5P-s-Co1.
... The lines PQ intersect in a common point on 5P-s-Co1.

(b) For a 5L there is a special tangent to 5L-s-Co1:
... The ninth Cayley-Bacharach line
... of L_1, L_2, L_3, L_4, L_5 ,
... .. the axes of 5L-s-Co1
... .. and a tangent L at 5L-s-Co1
... is a line through 5L-s- P_1 , intersecting L in X .
... The points X are collinear on a special tangent of 5L-s-Co1.

Best regards Eckart

[Next →](#) [← Message Index](#) [↑ Subjects](#)

Message: #2801
Date: 06/1/2018 2:36:00
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-P4

Dear Chris,

consider for $QA = P_1P_2P_3P_4$ the six conics $Co(P_i, P_j)$
... with foci P_i, P_j
... and major vertex circle through $QA-P_2$.
Then the conics $Co(P_i, P_j)$, $Co(P_j, P_k)$, $Co(P_k, P_i)$
... have a common tangent
... and these 4 common tangents intersect in $QA-P_4$.
You get these common tangent
... as loci of $QA-P_4$ wrt P_i, P_j, P_k
... and a changing point on the orthogonal hyperbola $QA-Co_2$.

Additional remark:

There are also common tangents
... of $Co(P_i, P_j)$, $Co(P_i, P_k)$, $Co(P_i, P_l)$,
... through $QA-P_2$.
This last property can be generalized
... for n -points,
... replacing $QA-P_2$ by an arbitrary point.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2802

Date: 07/1/2018 10:08:09

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

* 2. What's the difference between a Quadrangle and a Quadrilateral ?*

When googling above question these definitions come forward: A Quadrangle is a shape with four angles and a Quadrilateral is a shape with four sides.

Four angles indicate four points/vertices (from which the angles appear).

So in simple speech a quadrangle is a figure made of 4 points and a quadrilateral is a figure made of 4 lines.

This distinction is not new.

Coxeter and Greitzer wrote in their book "Geometry Revisited" (page 52):

Obviously we should discourage the tendency to call a quadrangle a "quadrilateral". In projective geometry, where the sides are whole lines instead of mere segments, we need both terms with distinct meanings.

What is the fuss about this distinction?

Well,

* A system of 4 lines has 6 intersection points. This configuration is often called a "complete quadrilateral".

* A system of 4 points has 6 connecting lines. This configuration is often called a "complete quadrangle".

These are different things.

The definitions of the words Quadrangle and Quadrilateral as being related to resp. 4 points and 4 lines are distinct definitions that also are used in projective geometry.

So now the popular words Quadrangle and Quadrilateral have gotten a distinct meaning. This distinction is essential for Quadrilateral Geometry.

More information can be found in the Encyclopedia of Quadri-Figures EQF (<https://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia>).

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2803

Date: 07/1/2018 4:10:17

From: eckart_schmidt@t-online.de

Subject: New QA-Points wrt QA-P4 and QA-Co3

Dear Chris,

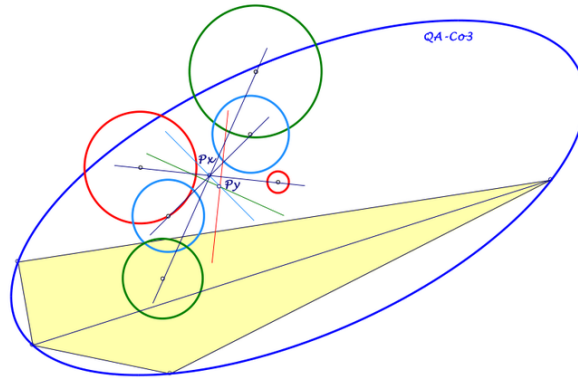
attached another application of QA-P4.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

New QA-Points related to QA-P4 and QA-Co3

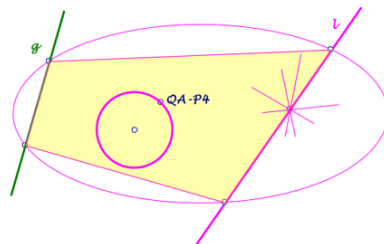
Starting with a special property of QA-P4, mentioned by Roland Stärk (EQF-Ref. [16],16), there are two new QA-points wrt the conic QA-Co3.



Roland Stärk describes the following property of QA-P4, which he named “Tangentialpunkt”:

- **If a conic is intersected by lines l of a line pencil and another line g , the locus of QA-P4 for the intersection quadrangles is a circle.**

(This circle degenerates to a line, if the conic is an orthogonal hyperbola or if the line pencil consists of parallels.)



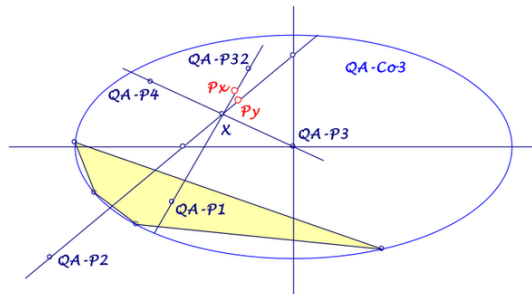
Here we consider the conic QA-Co3 of a quadrangle,
 ... the line pencil of QA-P3, center of QA-Co3
 ... and the six lines of the quadrangle.

In this way we get six circles, three pairs for opposite sides of the quadrangle (see first figure).

- The center lines of the three circle pairs have a common point P_x .
- The radical axes of the three circle pairs have a common point P_y .

These two points have their connections in QA -geometry:
Let X be the intersection of $QA-P3.QA-P4$ and $QA-P1.QA-P32$.

- P_x is the midpoint of $X.QA-P32$.
- P_y is the midpoint of the intersections of the $QA-Co3$ -axes and $X.QA-P2$.



The points P_x, P_y can also be studied, taking any circumconic of the quadrangle and its center.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2018-01-07.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2804

Date: 08/1/2018 3:04:18

From: bernard.keizer

Subject: Introduction to Quadrilateral Geometry

Dear Chris,

These 2 first points were in fact rather elementary for QA/QL experts like the members of the Quadriforum !

I'm waiting now impatiently for the next ones ...

Best regards

Bernard

PS Tsihong Lau made long time ago the suggestion that it would be convenient to merge the dual QA/QL with the same DT and to have in EQF a reference figure like in Kimberling's ETC with the main points, lines, circles, conics, cubics and even quartics (or 2 if we wish to have both cases of monocursal and bicursal QL-Cu1). In other words, we need the coordinates of 4 chosen points as vertices of QA or of 3 vertices of a chosen triangle with a point inside this triangle as one QA vertice ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2805
Date: 08/1/2018 8:38:25
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Benedetto, dear Chris,

5P-s-Tf3 is a conical transformation (see QFG#2791),
it depends only on the circumconic of the 5P.
So we can apply this transformation for quadrangles wrt a
QA-conic, choosing 5 points on the QA-conic.

Here some examples:

- (1) ... wrt any QA-circumconic:
... .. QA-P4 ---> QA-P2
- (2) ... wrt any orthogonal hyperbola all points have the image
in the center.
- (3) ... wrt QA-Co1:
... .. QA-P2 ---> QA-P6, QA-P11 ---> QA-P36, QA-P23 ---> QA-P2,
... .. QA-L1 ---> QA-L4, QA-L4 ---> QA-L1,
... .. QA-L2 ---> parallel to QA-P2.QA-P23 through QA-P6.
- (4) ... wrt QA-Co3:
... .. QA-P4 ---> QA-P2, QA-P6 ---> QA-P23, QA-P12 ---> QA-P30
... .. QA-L1 ---> QA-P3.QA-P4, QA-L2 ---> QA-P2.QA-P23,
... .. QA-L4 ---> parallel QA-L1 through QA-P23

There will be more examples for this Scimemi-transformation for
quadrangles.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2806

Date: 08/1/2018 10:55:55

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

Dear Bernard,

Yes, I am sure first two items about "Introduction to Quadrilateral Geometry" were rather elementary for QA/QL experts.

But actually they aren't really written for QA/QL experts. I noticed there is a very large group who isn't aware of the basic rules of quadrilateral geometry.

Therefore I wrote several items about the subject to spread the news in a larger group, also in other forums.

Of course I do not want to pass our own group.

I start very basically and want to deal with some more interesting applications later on.

After that I want to explain some more items about Polygon Geometry.

But I am pretty sure you will be familiar with most of them.

These subjects are too beautiful not to explain to others.

Maybe you can use the material for interested geometry lovers you know?

Best regards,

Chris

p.s. about your ideas, please create a setup?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2807
Date: 08/1/2018 11:11:07
From: chris.vantienhoven
Subject: 5P-s-Tf3

Dear Eckart,

Very nice examples of the conical Scimemi Transformation.
I already started with a description together with Benedetto. I will add your examples.
I am pretty sure there will be also many examples in Triangle Geometry.
Maybe you can find some. This would be a very nice indication of usability of the transformation.
Also the Inverse of the transformation is very interesting.
It makes me think of the Clawson-Schmidt Conjugate that also proved to be a very useful transformation.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2808
Date: 08/1/2018 11:54:15
From: Benedetto Scimemi
Subject: 5P-s-Tf3

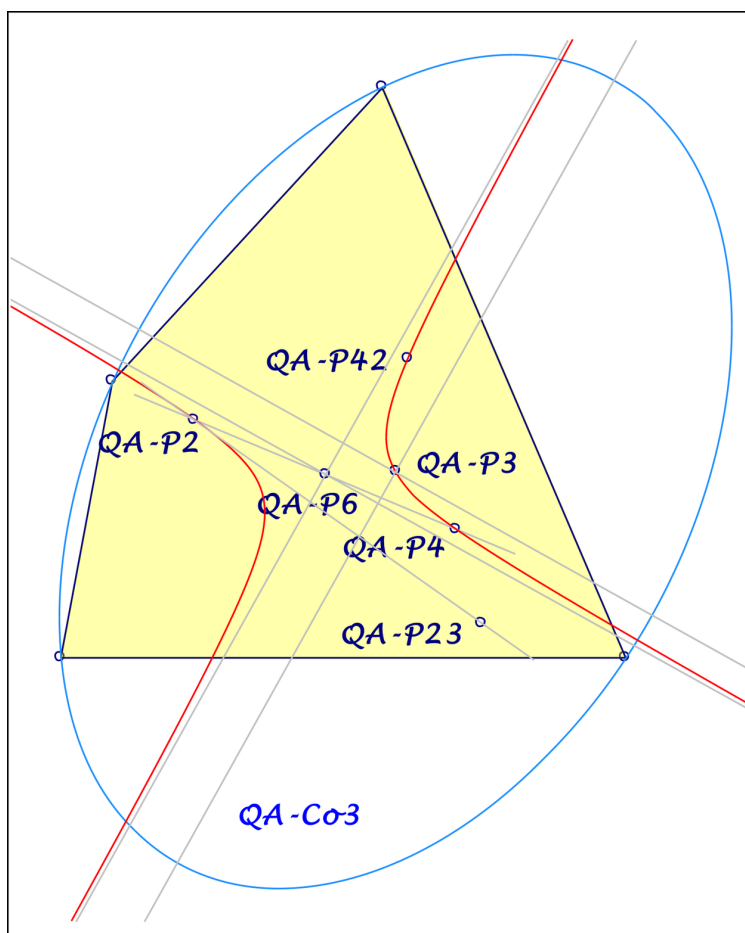
Very good work, Eckart, I am going to help Chris preparing the page, but I am not as fast as both of you are!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2809
Date: 09/1/2018 9:54:00
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Benedetto, dear Chris,
 here a nice application to the Scimemi transformation for
 quadrangles wrt QA-Co3,
 that means: 5P-s-Tf3 for an arbitrary 5P inscribed QA-Co3.
 This leads to a new orthogonal QA-hyperbola, perhaps of
 interest:

The Scimemi transformation maps lines to lines.
 Consider lines L through QA-P4
 and their image-lines,
 ... they intersect on an orthogonal hyperbola,
 ... centered in QA-P6,
 ... bearing QA-P2, QA-P3, QA-P4, QA-P42,
 ... asymptotes parallel to the axes of QA-Co3,
 ... tangents in QA-P2 and QA-P3 through QA-P23.
 Other line pencils give also orthogonal hyperbolas.
 Best regards Eckart



2018-01-09.pdf

Message: #2810
Date: 09/1/2018 10:53:44
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Benedetto, dear Chris,

in addition to my last message 2809:
Consider the Scimemi transformation wrt QA-Co1
... and the line pencil of QA-P23:
... lines and image-lines intersect on an orthogonal hyperbola,
... .. centered in the midpoint of QA-P2.QA-P23,
... .. bearing QA-P1, QA-P2, QA-P23, QA-P33, QA-P42,
... .. tangent in QA-P2 is QA-L1,
 tangent in QA-P23 is parallel QA-L1,
... .. tangent in QA-P42 through QA-P6.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2812
Date: 09/1/2018 9:13:18
From: chris.vantienhoven
Subject: 5P-s-Tf3

Dear Eckart,

Your constructions of 5P-s-Tf3 and 5P-s-Tf4 are very interesting.

I was hoping for such kind of simplification and correlation with pole and polar of a conic.

I think it's now time for even more examples.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2813
Date: 09/1/2018 9:19:07
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Chris,

please apologize profusely my typo in the address of my last message!

Sorry Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2814
Date: 13/1/2018 10:22:59
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Benedetto, dear Chris,

surprising QA-geometry for 5P-s-Tf3 / 5P-s-Tf4 wrt a conic:
Consider a QA and a line L ,
... QA-Tf2 maps points of the line L to a QA-DT circumconic Co ,
... 5P-s-Tf3 / 5P-s-Tf4 wrt this conic Co map the line L
to another line L^\wedge .

The line-line-transformation $L \dashrightarrow L^\wedge$ for a QA
... maps opposite QA-lines to the same sideline of QA-DT,
... maps sidelines of QA-DT to sidelines of its orthic triangle.

The lines L^\wedge for lines L through a point P
... envelope a conic $Co(P)$.

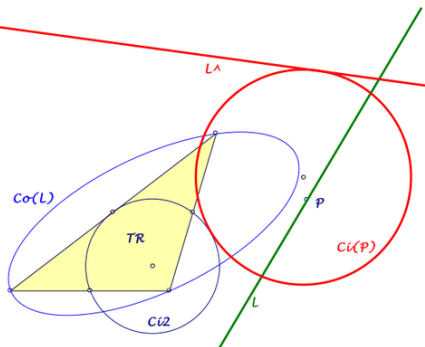
The conic $Co(P_i)$ for a vertex P_i of the QA
... is inscribed the diagonal triangle QA-DT
... with P_i as focus.
Attached the background for this properties.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Triangle Circles and 5P-s-Tf3

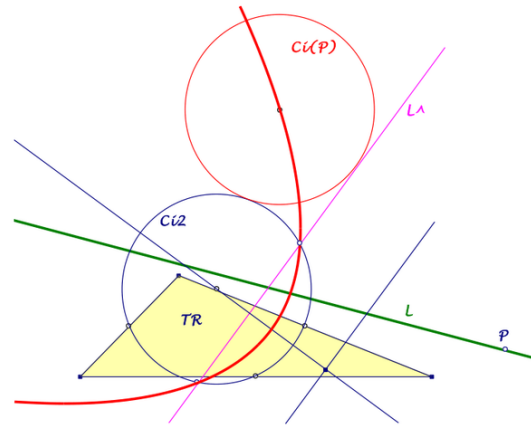
The transformation 5P-s-Tf3 depends only on the circumconic of the 5P (see QFG#2811). So this transformation can be used for triangles wrt a chosen conic. Here a related line-line transformation is researched wrt a conic depending on the line to be transformed.



We start with a triangle TR and a line L ,
 ... the TR -isogonal conjugate of L gives a circumscribed conic $Co(L)$ of TR .
 ... 5P-s-Tf3 wrt the conic $Co(L)$ maps L to another line L^\wedge .
 ... The lines L^\wedge for the line pencil of a point P
 envelope a circle $Ci(P)$.

We study the transformations $L \rightarrow L^\wedge$ and $P \rightarrow Ci(P)$:
 Let Ci_1 be the circumcircle and Ci_2 the nine-point circle of TR .

- Circles $Ci(P)$ contact Ci_2 .
- For points P on circle Ci_1
 ... circles $Ci(P)$ degenerate in points on Ci_2 .
- Circles $Ci(P)$ for points P on a line L
 ... contact L^\wedge and Ci_2 ,
 ... have centers on a parabola $Pb(L)$
 with focus in the center of Ci_2 ,
 bearing the intersections of Ci_2 and L^\wedge ,
 directrix parallel L^\wedge .



- For P as in-/excenter of TR
... circle $Ci(P)$ is the corresponding in-/excircle of TR .
- For L as TR -sideline
... line L^\wedge is a sideline of the orthic triangle of TR .

Final remarks

If we replace the isogonality in the starting procedure by another isoconjugation, we will not get circles $Ci(P)$ but conics $Co(P)$. For the fixed points F of the isoconjugation the conics $Co(F)$ are inscribed TR and have one focus F .

Consequently for quadrangles $P_1P_2P_3P_4$ with diagonal triangle $QA-Tr1 = TR$ and isoconjugation $QA-Tf2$ the vertices P_i have conics $Co(P_i)$ inscribed $QA-Tr1$ with focus P_i .

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2018-01-13.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2815

Date: 14/1/2018 9:30:03

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

3. What is a Quadrigon and how it relates to Quadrangles/Quadrilaterals?

We defined earlier a Quadrangle as a configuration of 4 points and a Quadrilateral as a configuration of 4 lines. Unfortunately these definitions for a quadrangle and a quadrilateral do not match the most popular meaning of the word Quadrilateral as being a bounded figure made up of 4 points as well as 4 lines cyclically connected.

Since we have to deal with the discrepancy of a popular meaning and a literal meaning we can feel free to make our own definitions like we always do in mathematics whenever necessary. Therefore in EQF a new word is introduced: a Quadrigon.

So now we have 3 basic concepts:

1. A figure consisting of 4 points, being called a QUADRANGLE.
2. A figure consisting of 4 lines, being called a QUADRILATERAL.
3. A bounded figure consisting of 4 points as well as 4 line segments (lines), being called a QUADRIGON.

Actually a Quadrigon is a Quadrangle with cyclically connected vertices and it also can be described as a Quadrilateral with cyclically connected lines.

Note that in a Quadrangle we can cyclically connect its vertices in 3 different ways. Therefore a Quadrangle "contains" 3 different Quadrigons, just like it "contains" 4 different Component Triangles. A similar relationship exists between Quadrilaterals and Quadrigons.

A simple application of this in a Quadrangle is:

- * In a Quadrigon we have 2 diagonals intersecting each other in the diagonal crosspoint.
- * In a Quadrangle we have 3 Quadrigons each having a different diagonal crosspoint.
- * The 3 diagonal crosspoints form the diagonal triangle of a Quadrangle.

Similar procedure for a Quadrilateral:

- * In a Quadrigon we have 2 diagonals intersecting each other in the diagonal crosspoint.
- * In a Quadrilateral we have 3 Quadrigons each having a different diagonal crosspoint.

* The 3 diagonal crosspoints form the diagonal triangle of a Quadrilateral.

Note that the diagonal triangle of a Quadrangle differs from the diagonal triangle of a Quadrilateral.

See the pictures in EQF at

* QG-P1

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p1>)

Similarly other Quadrigon points, lines, etc. can be tripled in a Quadrangle or in a Quadrilateral giving interesting triangles, etc. See the more advanced chapter

* QA-Tr-1

(<http://www.chrisvantienhoven.nl/qa-items/qa-triangles/qa-tr-1>)

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2816

Date: 14/1/2018 9:57:16

From: chris.vantienhoven

Subject: Backup for Yahoo Group messages (also extended User Interface)

Dear friends,

If you want to have a backup of the messages and files in the Quadri-Figures Group or any other Yahoo-groups you are a member of, then look at:

<http://www.personalgroupware.com/pgoffline-index.htm>.

It also gives the possibility of an overview of all messages as well as making a selection of this messages. Last but not least, when you found your message you can directly reply to it.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2817
Date: 14/1/2018 11:49:44
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Benedetto, dear Chris,

here another example of 5P-s-Tf3 geometry wrt chosen conics,
perhaps of interest:

Consider the three quadrigons of a quadrangle

... and their circumscribed conics QG-Co2.

... Then 5P-s-Tf3 wrt these conics

gives three images Q_i of a point P .

... The circumcircle of $Q_1Q_2Q_3$ bears QA-P2

... and has a center P^* ,

which shall be the image of P wrt a transformation.

The transformation $P \dashrightarrow P^*$ is QA-Tf3,

... maps points to points:

QA-P4 \dashrightarrow QA-P2, QA-P42 \dashrightarrow QA-P42, ...

... maps lines to lines: QA-L1 \dashrightarrow QA-L4,

QA-L2 \dashrightarrow QA-P2.QA-P23, ...

... maps circles to circles, conics to conics,

... is 5P-s-Tf3 wrt special conics:

... .. centered in QA-P42,

... .. through an arbitrary point X

... .. with a tangent, parallel to

... .. the reflection in the angle bisector of \sphericalangle

... .. of a perpendicular line to QA-P2.QA-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2818
Date: 15/1/2018 6:17:17
From: Benedetto Scimemi
Subject: More on Co-Tf3

Dear Chris, dear Eckart,

I spent some time on Co-Tf3 w.r.t. (both) Steiner ellipses of any triangle and I found Tarry p X98 --> X4 orthoc, as you have indicated in the draft of your page

* Brocard c X182 --> X5 ninepoint c

* Kiepert hyp c X115 --> X6

* Lemoine p X6 --> X115 center of Kiepert hyperbola

* refl- of X115 on X5 --> X3 circumc

Therefore, the Brocard circle is mapped into the nine-point circle.

As lines connecting points at left are mapped into lines connecting corresponding points at right, a different page dedicated to Co-Tf4 seems to me superfluous. Am I wrong?

Can anybody check these results? It is so easy to make mistakes with this matters!

Chris, I wait for your reaction to my last version of the page. Meanwhile I understood that you probably meant X1513 --> X98. I misunderstood, not knowing X1513.

Best regards Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2819
Date: 16/1/2018 11:01:40
From: eckart_schmidt@t-online.de
Subject: More on Co-Tf3

Dear Benedetto, dear Chris,

here some results for 5P-s-Tf3 for a triangle using 5P,
inscribed the Steiner ellipses of a triangle.
I don't know, whether this is your transformation Co-Tf3,
for there are different results:

X1352	---	X3	---	X114,
X6776	---	X4	---	X98,
X182	---	X5	---	X6036,
X2549	---	X6	---	X115,
X3923	---	X10	---	???,
X5091	---	X11	---	???,
X10644	---	X13	---	X6109,
X10653	---	X14	---	X6108.

I hope, I made no mistake!
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2820
Date: 16/1/2018 12:13:28
From: Benedetto Scimemi
Subject: More on Co-Tf3

Dear Eckart, yes, you are applying Co-Tf3 and you found precisely

X3 ---> X114,

X4 ---> X98,

X182 ---> X5

X6 ---> X115,

i.e. my (few!) results. But I cannot do much more than I did.

Dear Chris, take your time.

Thanks Benedetto

2018-01-16 11:01 GMT+01:00, 'eckart_schmidt@t-online.de '

eckart_schmidt@t-online.de [Quadri-Figures-Group]

< Quadri-Figures-Group@yahoogroups.com >:

> > Dear Benedetto, dear Chris,

> > here some results for 5P-s-Tf3 for a triangle

> > using 5P, inscribed the Steiner ellipses of a triangle.

> > I don't know, whether this is your transformation Co-Tf3,

> > for there are different results:

> > X1352 ---> X3 ---> X114,

> > X6776 ---> X4 ---> X98,

> > X182 ---> X5 ---> X6036,

> > X2549 ---> X6 ---> X115,

> > X3923 ---> X10 ---> ???,

> > X5091 ---> X11 ---> ???,

> > X10644 ---> X13 ---> X 6109,

> > X10653 ---> X14 ---> X6108.

> > I hope, I made no mistake!

> > Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2821
Date: 16/1/2018 4:44:30
From: eckart_schmidt@t-online.de
Subject: MVP-points for a 5P

Dear Chris,

if three QA-P x,y,z are collinear with fixed distance ratios,
their MVP-points (see nP-n-Luc1) wrt a 5P are also collinear
with the same distance ratios.
Therefore the seven MVP-points of QA-P1,5,10,20,22,25,26 wrt a
5P are collinear
on a line, perhaps relevant for 5P-geometry.
This line can be leveled up to nP.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2822
Date: 16/1/2018 7:45:42
From: chris.vantienhoven
Subject: More on Co-Tf3

Dear Benedetto, dear Eckart,

I checked the Scimemi Transformations (Co-Tf3) like you did in a triangle wrt 1st/2nd Steiner Ellipse.

It appears we arrive at the same results.

Here are my results with some extra's:

X??? --> X1 --> X5988
X2 --> X2 --> X2
X1352 --> X3 --> X114
X6776 --> X4 --> X98 --> X1513
X182 --> X5 --> X6036 --> X10011
X2549 --> X6 --> X115 --> X230
X??? --> X7 --> X???
X??? --> X8 --> X1281
X??? --> X9 --> X???
X3923 --> X10 --> X???
X5091 --> X11 --> X???
X??? --> X12 --> X???
X10654 --> X13 --> X6109
X10653 --> X14 --> X6108
X??? --> X15 --> X6115
X??? --> X16 --> X6114
X??? --> X17 --> X???
X??? --> X18 --> X???

For the ABC-X2-X6 Hyperbola I found these results:

X??? --> X1 --> X???
X8644 --> X2 --> X???
X??? --> X3 --> X5512?
X??? --> X4 --> X???
X5926 --> X5 --> X???
X??? --> X6 --> X???

The Scimemi Transformation of all points on the Kiepert Hyperbola are identical: X115,
The Scimemi Transformation of all points on the Jerabek Hyperbola are identical: X125.
The Scimemi Transformation of all points on the Feuerbach Hyperbola are identical: X11,
because they are Orthogonal Hyperbolas and so the Scimemi Transformation coincides with the center of the Orthogonal Hyperbola.

Further I noticed that the locus of the Scimemi Transformation of all points on a any hyperbola is a hyperbola with identical asymptotes.

It was nice to do this "dumb" checking work on a day with migraine, when nothing else succeeds.
It distracts me a bit.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2823
Date: 17/1/2018 4:57:12
From: chris.vantienhoven
Subject: MVP-points for a 5P

Dear Eckart,

I agree, not only 3P-central points can be leveled-up to nP, but also 4P-points.

There are not so many 5P-points known and it can be a nice tryout to look for new 5P-points if they relate to 5P-level-up-points.

Did you check with many examples that 4P-points leveled-up into a 5P preserve collinearity?

I did not check it, but leveling-up from 3P to 4P does not preserve collinearity.

For example $X(3)$, $X(6)$ and $X(15)$ are collinear in a 3P. However their MVP-leveled up 4P-versions aren't collinear.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2824
Date: 17/1/2018 9:06:24
From: eckart_schmidt@t-online.de
Subject: MVP-points for a 5P

Dear Chris,
I think, you have to respect " ... collinear with fixed distance ratios ...".
In your example the three triangle points have not a fixed distance ratio.
If the cited property is true, the collinearity can be leveled up. (Have a look on QA-P18 wrt the seven points on QA-L3 in my example.)
Best regards Eckart

PS. Already mentioned? MVP-points for a QA:
X2 ---> QA-P1, X3 ---> QA-P32, X4 ---> QA-P33.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2825
Date: 18/1/2018 12:43:35
From: chris.vantienhoven
Subject: MVP-points for a 5P

Dear Eckart,

Thanks for reminding this extra condition " ... collinear with fixed distance ratios ...".
When three points P1,P2,P3 are collinear with fixed distances, then algebraically P3 can be written as $n \cdot P1 + m \cdot P2$, where m and n are integers.
When consequently leveling up these points they undergo the same operation and then it is not so strange that at one level higher also $P3 = n \cdot P1 + m \cdot P2$.
It is not quite a proof, but at least it makes it plausible.
You will find the level-up relation X2 ---> QA-P1, X3 ---> QA-P32, X4 ---> QA-P33 described in EPG at nP-n-P1 till nP-n-P4.
Stated in another way $nP-n-Luc1(X2) = QA-P1$, $nP-n-Luc1(X3) = QA-P32$, $nP-n-Luc1(X4) = QA-P33$, $nP-n-Luc1(X5) = \text{Midpoint}(QA-P32, QA-P33)$.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2826
Date: 19/1/2018 8:41:29
From: bernard.keizer
Subject: More on Co-Tf3

Dear Benedetto, dear Chris, dear Eckart,
I tried to understand Benedetto's conical transformation, which is indeed a beautiful tool ; let's name it sigma.
It appears, it is the combination of an axial symmetry (main axis of the foci) and an homothety with center the center of the conic and ratio the square of the ratio between the radii of a circle through the foci and the orthoptic circle of the conic (both circles having the same center, which is the center of the conic).

Therefore, the ratio is inverse for the ellipse and the hyperbola ...

This has 2 consequences :

1) the transformed points of the ETC points of a triangle are the ETC points of a similar triangle and the transformation keeps all the colinearities and cocyclicities.

2) the transformation can be reproduced as the combination of an axial symmetry and 2 inversions wrt the 2 mentioned circles.

If we take the Steiner inellipse and remember Benedetto's mu-transformation (also named psi by Bernard Gibert), the transformed point of a point X in the sigma transformation is the inverse in the orthoptic circle of the Steiner inellipse of the transformed point in the mu-transformation.

For example, the mu-transformation swaps X13 and X14 and X15 and X16 and the sigma transformed of X13, X14, X15 and X16 are X6109, X6108, X6115 and X6114, which are the inverses of X14, X13, X16 and X15 in the orthoptic circle of the Steiner inellipse.

The same goes for the QL, as the Cl-S transformation is a mu transformation ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2827
Date: 19/1/2018 9:21:12
From: bernard.keizer
Subject: More on Co-Tf3

Dear Benedetto, dear Chris, dear Eckart,
The last sentence is a little bit elliptic !
All the pivot triangles of QL-Cu1 (the cubic stelloïd) have the same foci of the Steiner inellipse, which are the fixed points of the Cl-S transformation and QL-P1 as X2 ; for any of these ellipses, we may have it's orthoptic circle and apply the preceding construction ...
The main pivot triangle has as the 2 points on the Newton Line (monocursal QL-Cu2) or symmetric wrt the Newton Line (bicursal QL-Cu2) as X15 and X16.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2828
Date: 19/1/2018 3:34:42
From: bernard.keizer
Subject: More on Co-Tf3

Of course, the cubic stelloïd is QL-Cu2 and the Van Rees curve or hessian is QL-Cu1

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2829
Date: 19/1/2018 6:03:47
From: Benedetto Scimemi
Subject: More on Co-Tf3

Dear Bernard, your contribution was essential to put different matters together. I did not know that you were familiar with my F.G. 2010 paper, nor I knew that B.Gibert had studied the same mapping ($\mu=\psi$) before. Apparently I should have cited him on that occasion.

As for Co-Tf3, factorizing the homotheties as product of two circle inversions suggests a substantially unique definition for both Co-Tf3 and its inverse for all conics. A good progress. Thanks.

Too bad I cannot follow you on cubics. Best regards

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2830
Date: 19/1/2018 8:29:27
From: eckart_schmidt@t-online.de
Subject: Isoconjugation for a pentagon

Dear all,

attached some observations in pentagon geometry:
Polarity wrt the inscribed and circumscribed conics of a pentagon leads to an isoconjugation wrt a corresponding reference triangle.

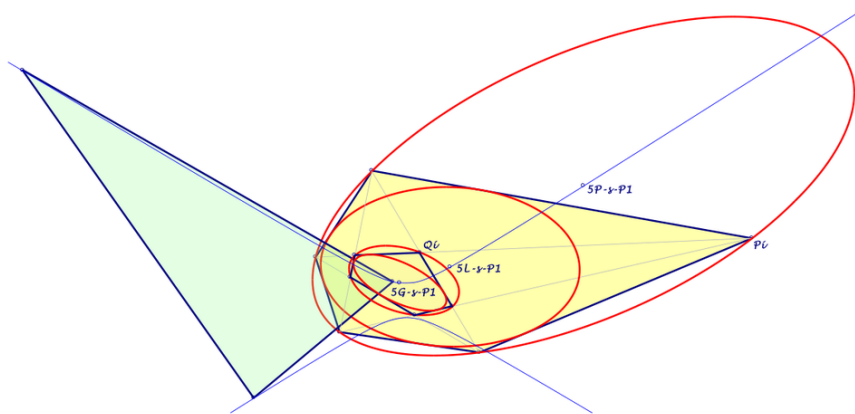
The image conic of the line at infinity bears 5P-s-P1, 5L-s-P1 and 5G-s-P1.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

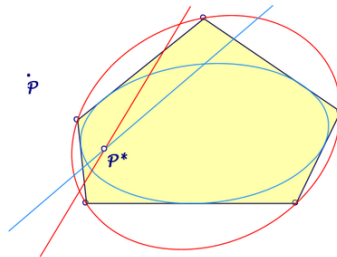
Isoconjugation for a Pentagon

Polarity wrt the inscribed and circumscribed conics of a pentagon lead to an isoconjugation wrt a corresponding reference triangle. The image conic of the line at infinity bears 5P-s-P1, 5L-s-P1 and 5G-s-P1.



The isoconjugation

We start with a pentagon $P_1P_2P_3P_4P_5$ and its circumscribed conic $5P-s-Co1$ and its inscribed conic $5L-s-Co1$ and consider for a point P the intersection P^* of its polars wrt $5L-s-Co1$ and $5P-s-Co1$. The transformation Tf with $P \rightarrow P^*$ shall be researched.



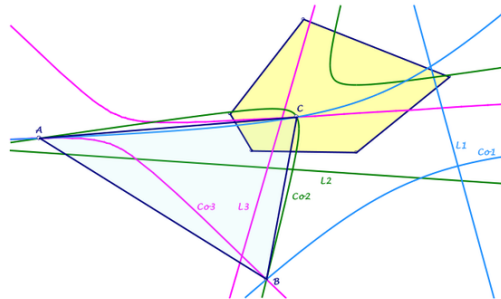
Special Tf -partners are $5L-s-Co1$ contact points P on P_iP_{i+1} and its 4th harmonic point P^* .

- Polars of points at infinity wrt $5P$ -s-Co1 intersect in $5P$ -s-PI.
Polars of points at infinity wrt $5L$ -s-Co1 intersect in $5L$ -s-PI.
- The transformation Tf maps the line at infinity to a conic Co through $5P$ -s-PI, $5L$ -s-PI and $5G$ -s-PI.

The reference triangle

Now we consider a related pentagon $Q_1Q_2Q_3Q_4Q_5$ with $Q_i = P_i.P_{i+2} \cap P_{i+1}.P_{i+3}$ with its inscribed and circumscribed conics.

- Polars of points at infinity wrt the inscribed conic of $Q_1Q_2Q_3Q_4Q_5$ intersect in $5G$ -s-PI.
- The transformation Tf maps lines L to conics $Co(L)$ with three common points A, B, C .



- The four polars of A wrt the inscribed and circumscribed conics of $P_1P_2P_3P_4P_5$ and $Q_1Q_2Q_3Q_4Q_5$ coincide in BC (analog for B, C).
- The transformation Tf is an isoconjugation with reference triangle ABC .
- The conic Co through $5P$ -s-PI, $5L$ -s-PI and $5G$ -s-PI is a circumconic of ABC .

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2018-01-19.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2831
Date: 20/1/2018 9:56:24
From: bernard.keizer
Subject: More on Co-Tf3

Dear Benedetto,
Thanks for your kind answer !
The psi or mu transformation is a Moebius transformation, known long time ago.
The application for the triangle was mentionned by Bernard Gibert in 2003 on the FG (Orthocorrespondance and orthopivotal cubics).
It is also the Cl-S transformation in the QL and is the transformation which associates to any point in the plane the center of the rectangular hyperbola which is the polar conic of the point wrt the cubic stelloid QL-Cu2 and leaves therefore the hessian QL-Cu1 (locus of the points for which the rectangular hyperbola degenerates in 2 perpendicular lines) globally unchanged.
I suppose you read french : if you are interested, I wrote myself several articles on these matters on a personal website www.bernardkeizer.blogspot.fr (<http://www.bernardkeizer.blogspot.fr>).
Your new transformation seems more powerful, as it applies to any conic.
With the Steiner inellipse, the transform of the circumcircle is a circle centered in X114 through X1513, the Newton Line of the transformed triangle is X2X98, it's Brocard diameter is X114X115 (which is, as you mentionned, a diameter of the Euler circle).
May years ago, I spent hours with Steve Sigur's fascinating triangle figures and I think this gives new interesting properties ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2832

Date: 20/1/2018 10:43:41

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

4. Symmetry and preference in Quadrilateral Geometry

Quadrilateral Geometry deals with Quadrilaterals not only in the popular meaning of the word, but also in an extended way. It deals with Quadrangles, Quadrilaterals and Quadrigons. These are systems of random points and / or random lines as defined before.

Important is the notion of "preference".

Preference

Per definition in a quadrilateral we have 4 random lines without preference.

Per definition in a quadrangle we have 4 random points without preference.

However the definition of a quadrigon implies there is a preference of points as well as lines.

Considering the 4 points of a quadrigon it appears that only 4 out of 6 connecting lines are part of the basic figure.

Considering the 4 lines of a quadrigon it appears that only 4 out of 6 intersection points are part of the basic figure.

So, although a quadrigon looks very symmetrically it is an inherently geometrically asymmetric figure because of the preference of points as well as lines.

Symmetry means no preference

A geometrical figure only can be symmetric when it is constructed without preference of reference items.

Symmetry in constructions = symmetry in algebraic expressions

There is a correspondence between geometrically symmetric constructions and the symmetry in corresponding algebraic coordinates and equations, because every single symmetric step in a construction leads to a symmetric algebraic expression for that single step.

Here are 3 examples of a symmetric construction in a quadrangle, a quadrilateral and an apparently symmetric construction in a quadrigon. The (a)symmetry in the constructions can be recognized in the coordinates of the constructed points.

Construction in a Quadrangle (Centroid)

Let the reference points be P1, P2, P3, P4.

1. Choose a set of 2 out of 4 reference points P1 and P2 and denote their midpoint as M12.
Denote the midpoints of the remaining points P3 and P4 as M34.
Let L12-34 be the line connecting M12 and M34.

2. Since we don't want any preference of reference-items we do the same for all other combinations of reference points. There are 2 other combinations delivering lines L13-24 and L14-23.

3. Drawing the figure it appears that L12-34, L13-24 and L14-23 are concurrent. This point is known as the centroid of a quadrangle. See QA-P1

(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p1>). It is constructed in a symmetrical way.

This also can be seen in the algebraical expression of the coordinates of this point.

Let the barycentric coordinates of the reference points be $P1=(1:0:0)$, $P2=(0:1:0)$, $P3=(0:0:1)$, $P4=(p:q:r)$, then the coordinates of this point are $*(2p+q+r : p+2q+r : p+q+2r),*$
Note, there is a symmetry in these coordinates.

Construction in a Quadrilateral (Miquel Point)

Let the reference lines be L1, L2, L3, L4.

1. Choose one reference line L1. Denote the circumcircle of remaining reference lines being L1, L2, L3 as Ci1.

2. Since we don't want any preference of reference-items we do the same for all other reference lines. This delivers 3 other circumcircle Ci2, Ci3, Ci4.

3. Drawing the figure it appears that Ci1, Ci2, Ci3 and Ci4 have a common point. This point is known as the Miquel Point. See QL-P1

(<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p1>). It is constructed in a symmetrical way.

This also can be seen in the algebraical expression of the coordinates of this point.

Let the barycentric expressions of the reference lines be $L1=(1:0:0)$, $L2=(0:1:0)$, $L3=(0:0:1)$, $L4=(l:m:n)$, then the coordinates of this point are $*(a^2 m n (m-n) : b^2 n l (n-l) : c^2 l m (l-m)),*$

Note, there is a symmetry in these coordinates.

Construction in a Quadrilateral (Diagonal Crosspoint)

There are two ways to express reference items in a quadrilateral:

- denote reference lines as L1,L2,L3,L4 and reference points as $L1^{\wedge}L2$, $L2^{\wedge}L3$, $L3^{\wedge}L4$, $L4^{\wedge}L1$,

- or denote reference points as P1,P2,P3,P4 and reference lines as P1.P2, P2.P3, P3.P4, P4.P1.

In next construction we will use last option.

1. Choose a point P1 and connect it with the opposite point P3. Denote the connecting line as L13. This line is also known as a diagonal of the quadrigon.
2. Since we strive for symmetry we do the same for all other combinations. In this case we only get one other possibility, the other diagonal L24.
3. Drawing the figure we end up with the intersection point of the diagonals S. See QG-P1 (<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p1>).

It is constructed in a symmetrical way however the basic figure is inherently asymmetric.

This also can be seen in the algebraical expression of the coordinates of this point.

Let the barycentric coordinates of the reference points be $P1=(1:0:0)$, $P2=(0:1:0)$, $P3=(0:0:1)$, $P4=(p:q:r)$, then the coordinates of this point are $*(p : 0 : r)*$.

Let the barycentric coordinates of the reference lines be $L1=(1:0:0)$, $L2=(0:1:0)$, $L3=(0:0:1)$, $L4=(l:m:n)$, then the coordinates of this point are $*(- m n : + n l : - l m)*$.

Note that in both cases there is **no symmetry** in these coordinates (in the second case there is a difference in sign).

This is because the construction is admittedly symmetrical, however in an **inherently asymmetric basic figure**.

More information about these points (incl. figures) can be found in EQF (<https://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia>) at:

Centroid in a Quadrangle: QA-P1

(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p1>)

Miquel Point in a Quadrilateral: QL-P1

(<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p1>)

Diagonal Crosspoint in a Quadrigon: QG-P1

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p1>)

See also attachment.

Chris van Tienhoven

QUADRANGLE <i>Just four points</i>	QUADRILATERAL <i>Just four lines</i>	QUADRIGON <i>Bounded figure of four points and four lines</i>
<p>P1, P2, P3, P4 are the vertices of the Reference Quadrangle.</p> <ol style="list-style-type: none"> Choose a set of 2 out of 4 reference points e.g. P1 and P2 and denote their midpoint as M12. Denote the midpoints of the remaining points P3 and P4 as M34. Let L12-34 be the line connecting M12 and M34. Since we don't want any preference of reference-items we do the same for all other combinations of reference points. There are two other combinations delivering lines L13-24 and L14-23. It appears that L12-34, L13-24 and L14-23 are concurrent. This point is known as the Quadrangle Centroid. 	<p>L1, L2, L3, L4 are the lines of the Reference Quadrilateral.</p> <ol style="list-style-type: none"> Choose one reference line L1. Denote the circumcircle of remaining reference lines being L1, L2, L3 as C1. Since we don't want any preference of reference-items we do the same for all other reference lines. This delivers 3 other circumcircle C2, C3, C4. It appears that C1, C2, C3 and C4 have a common point. This point is known as the Miquel Point. 	<p>P1,P2,P3,P4 are reference points and P1,P2, P2,P3, P3,P4, P4,P1 are reference lines of the Quadrigon.</p> <ol style="list-style-type: none"> Choose a point P1 and connect it with the opposite point P3. Denote the connecting line as L13. This line is also known as a diagonal of the quadrigon. Draw the other diagonal L24. The intersection point of both diagonals is known as the Diagonal Crosspoint.

QA-QL-QG-Example-02.png

Message: #2833
Date: 21/1/2018 8:28:45
From: chris.vantienhoven
Subject: More on Co-Tf3

Dear Benedetto, Eckart and Bernard,

I am very enthusiastic about our collaboration regarding the Scimemi Transformation.

Like always when we work together on a subject very special properties and insights come forward.

This is QFG at its best.

Not long ago I asked Benedetto the question: what is the real meaning of his transformation?

Sometimes we lose the connection with the real meaning of a transformation and sometimes a simple construction or simple text can clarify it.

Shortly after that Eckart showed up with a construction consisting of a reflection and a polar transformation (#2811). That simplified the transformation a lot.

Thereafter Bernard factorized the homothecies in the Scimemi Transformation as the product of two circle inversions (#2826). As always theoretically well-founded.

Together we found lots of examples in Triangles, Quadrangles as well as Pentangles. So it really is a transformation with lots of applications.

And I think there will be more.

Right now Benedetto and me are making a detailed description of the Scimemi Transformation, which will be included in EPG as Co-Tf3 (Co-Tf1 = Polar of a conic, Co-Tf2 = Pole of a conic).

I decided to start describing also other items not only relating to Quadri-Figures and Poly Figures. So the prefix will be "Co-" as it should be, indicating that this transformation only depends on a conic.

I will let you know when CO-Tf3 will appear in EPG.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2834
Date: 21/1/2018 9:01:59
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3

Dear Bernard, dear Benedetto, dear Chris,

another modification of 5P-s-Tf3 wrt a chosen conic for a triangle ABC:

Let P^* be the image of P for the transformation T_f ,
... using 5P-s-Tf3 for 5P on the circumconic
of A, B, C , $\text{isog}(P)$, $\text{isot}(P)$.

- (1) $X_1 \rightarrow X_{1365}$, $X_3 \rightarrow X_{136}$, $X_4 \rightarrow X_{125}$,
 $X_6 \rightarrow X_{115}$, $X_{110} \rightarrow X_{3258}$, $X_{74} \rightarrow X_{3258}$.
- (2) A^* , B^* , C^* are the pedal points of X_4 .
- (3) The midpoints S_a, b, c of the sides are the images
... of the intersections of the circumcircle and the rays
 S_a, b, c, X_3 .
- (4) T_f maps the circumcircle of ABC and its Jerabek hyperbola
... to the nine-point circle of ABC.
- (5) For points P on the Jerabek hyperbola
... the circumconic of A, B, C , $\text{isog}(P)$, $\text{isot}(P)$ is an
orthogonal hyperbola.
- (6) Diametral points on the circumcircle have the same image.
- (7) Any point Q has three pre-images P with $P^* = Q$:
... X_4 and the intersections of the circumcircle
and the Euler line X_3, X_4
... ... have the image X_{125} (center of the Jerabek hyperbola),
... X_6 and the intersections of the circumcircle
and the line X_3, X_6
... ... have the image X_{115} (center of the Kiepert hyperbola).
- (8) What about the two fixed points of this transformation?

A calculation gives an equation of degree 6 with two real solutions.

These points are not in ETC!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2835
Date: 23/1/2018 11:46:59
From: bernard.keizer
Subject: Cevian triangles

Dear Benedetto, dear Chris, dear Eckart,
Needing a recreation from Benedetto's transformations, I've just revisited some QA/QL's properties.

- 1) Following properties are well-known for a triangle:
 - a) the circle circumscribed to a cevian triangle recuts the 3 sides in the vertices of a 2nd cevian triangle
 - b) the vertices of 2 cevian triangles are coconic
 - c) conversely a conic circumscribed to a cevian triangle recuts the 3 sides in the vertices of a 2nd one
- 2) In the QA/QL, each reference triangle of the QL is the cevian triangle of a QA vertex wrt DT
- 3) We have 3 immediate applications :
 - a) each circle circumscribed to a reference triangle recuts the DT sides in the vertices of a 2nd cevian triangle each circle forms with 4th line a degenerated cubic and the 4 cubics pass through the same 9 points (QL vertices, QL-P1 and circular points)
 - b) the vertices of a cevian triangle of any point wrt DT are coconic with 3 QL vertices ; this conic forms with the 4th line a degenerated cubic and the 4 cubics pass through the same 9 points (QL and triangle vertices)
 - c) the vertices of each reference triangle are coconic with the S-points (vertices of QL-Tr2) ; this conic forms with the 4th line a degenerated cubic which recuts the DT sides in the vertices of a 2nd cevian triangle (the point is the QA-Tf2 of the infinity point of the 4th line). This works also for the medial triangle as cevian of DT centroid, the point of the 2nd cevian triangle being the QA-Tf2 of the infinity point of the Newton Line.

The 4 cubics pass through the same 9 points (QL-vertices and S-points), which form a C-B system.

Any 10th point determines a cubic; for example, there is a cubic through these 9 points and QA-P16 or QL-P13, the line through the 3rd intersections with the sides of QL-Tr2 is the dual of QA-P1 (Newton Line of DQL)

Last, but not least, the QL formed by the sides of QL-Tr2 and this line as 4th line has the same QL-P1 and the same Newton Line as the QL ...

Best regards
Bernard

Message: #2836
Date: 23/1/2018 12:25:41
From: eckart_schmidt@t-online.de
Subject: More on Co-Tf3

Dear Chris,

I tried to study transformations Co-Tf: Here a simple example:
Let P^* be the pedal point of P on the polar of P wrt the
reference conic.

Attached the property, that lines will be mapped to strophoids.
By the way: For a triangle holds wrt the Steiner circumellipse:
 $X1 \rightarrow X5088$, $X4 \rightarrow X5523$, $X10 \rightarrow X5134$.

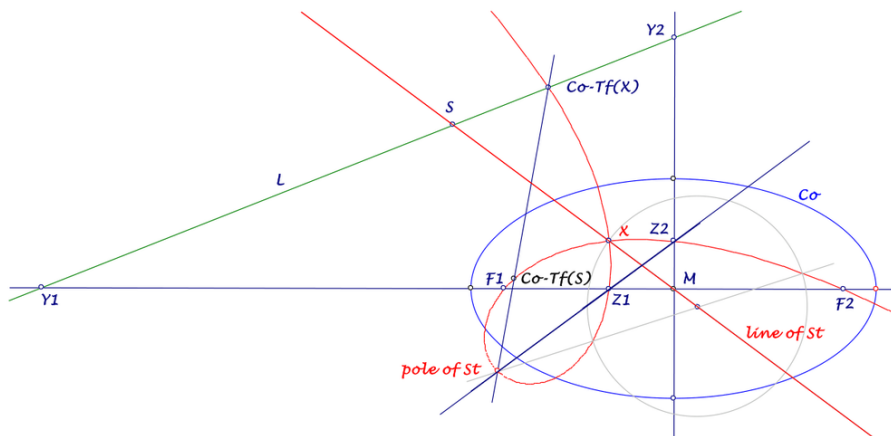
Sorry, I cannot give interesting examples for QA / QL and 5P.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Example for a Conic Transformation

A conic transformation *Co-Tf* is only determined by a conic *Co*. Here a simple example shall be researched, mapping a point to the pedal point on its polar wrt a reference conic *Co*.



Definition: *Co-Tf* is the transformation, which maps a point *P* to the pedal point of *P* on its polar wrt the conic *Co*.

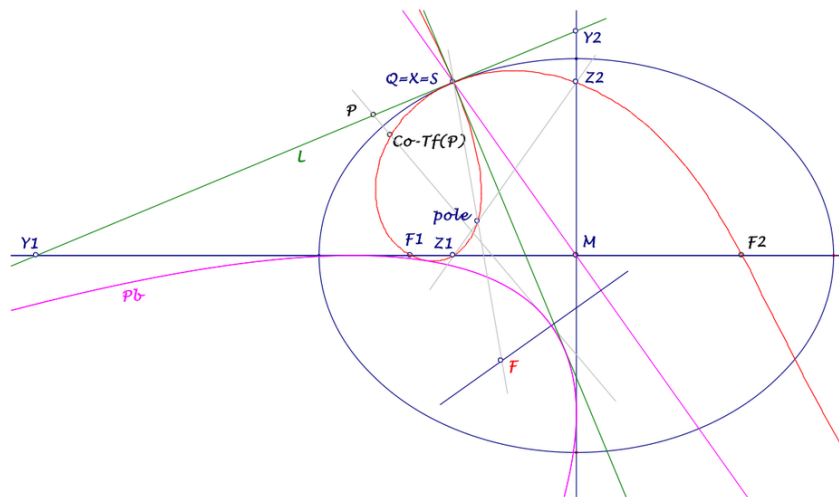
- Points on the conic *Co* are fixed points of *Co-Tf*.

Used points for mapping a line *L*:

- F_i foci of the reference conic *Co*
- M center of *Co*,
- X pole of *L* wrt *Co*,
- S intersection of *L* and *MX*,
- Y_i intersections of the *Co*-axes and *L*,
- Z_i 4th harmonic points of Y_i
 ... wrt the intersections of *Co* and its axes.

- **Lines *L* are mapped by *Co-Tf* to strophoids:**
 ... fixed point of the strophoid is *X*,
 ... line of the strophoid is *MX*,
 ... pole of the strophoid is the intersection
 ... $Z_1Z_2 \cap Co-Tf(X).Co-Tf(S)$.

- The strophoid $Co-Tf(L)$ bears
 - ... beside pole and fixed point
 - ... the foci F_i of Co ,
 - ... the points Z_i
 - ... and $Co-Tf(S)$ and $Co-Tf(X)$.
- The strophoid $Co-Tf(L)$ degenerates:
 - ... For lines through the center of Co
 - to orthogonal hyperbolas
 - centered in the center of Co ,
 - through the foci of Co
 - and the intersections of L and Co .
 - ... For lines parallel to the main axis of Co
 - to circles through the foci and the Co -pole of L .
 - ... For lines orthogonal to the main axis of Co
 - to circles with diameter XU
 - and U 4th harmonic point of X wrt the foci.
- The strophoid $Co-Tf(L)$ of a tangent at Co in $Q=X=S$:
 - ... The lines $P.Co-Tf(P)$ envelope a parabola Pb
 - ... tangent to the axes of Co
 - and to the tangent and normal in Q ,
 - with directrix MQ
 - and focus F in the reflection of Q in the pole.



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2018-01-23.pdf

Message: #2837
Date: 24/1/2018 10:14:55
From: bernard.keizer
Subject: Cevian triangles

Dear Benedetto, dear Chris, dear Eckart
The last sentence of my previous message is in fact not surprising, as the 4 lines of the QL, the 3 sides of QL-Tr2 and the Newton Line of DQL are all tangent to the same parabola QL-Co1 ; the QL formed by the 4 last lines is the dual of the QA formed by the S-points and QA-P1 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2838
Date: 24/1/2018 8:30:12
From: eckart_schmidt@t-online.de
Subject: Cevian triangles

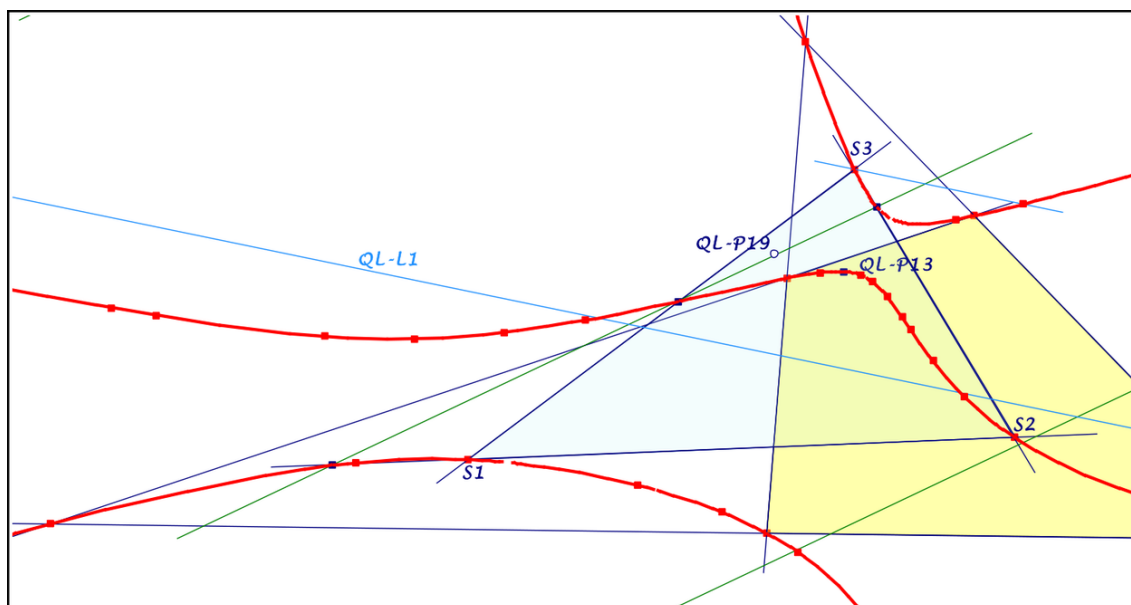
Dear Bernard,

reading your message 2835 I became interested in the cubic through the six QL-points, the three S-points and QL-P13: First I have constructed a lot of Cayley-Bacharach ninth points, then I found a construction (see attached file).

But I don't know, whether this is already mentioned :
The cubic through the six QL-points, the three S-points (QL-Tr2) and QL-P13

- ... is a nonpivotal isocubic:
- ... reference triangle QL-Tr2,
- ... isoconjugation, which swaps opposite vertices of QL
- ... or maps QL-P13 to the infinity point of QL-L1,
- ... and root in the tripole of the dual line of QA-P1,
- ... which is a parallel to QL-L9 through QL-P19.

Best regards Eckart



2018-01-24.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2839
Date: 25/1/2018 10:30:09
From: bernard.keizer
Subject: Cevian triangles

Dear Eckart,
Thanks for your interest and your precisions !
The construction is exactly the same as for the isotomic non pivotal cubic wrt midDT with the same line dual of QA-P1 ; this line is also tangent to the parabola QL-Co1, like the 4 lines of the QL, the sides of QL-Tr2 and the sides of midDT (the contact point with the parabola is the intersection with the Newton Line).
And we have the same kind of construction with the sides of DT, the sides of QL-Tr2 and the Newton Line, which are all tangent to the DQL-Co1, the parabola of DQL, with focus QL-P17.
I suppose, it follows 2 non pivotal cubics wrt DT or QL-Tr2 and root the tripole of the Newton Line wrt these 2 triangles ...
These 2 cubics will pass through the vertices of DT and QL-Tr2.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2840
Date: 25/1/2018 10:49:06
From: bernard.keizer
Subject: Cevian triangles

Dear Eckart,
Please forget the end of my message !
The cubics I mention don't exist ; there is only a degenerated cubic formed by the conic through the 6 vertices of DT and QL-Tr2 and QA-P10 and QA-P16 and the Newton Line.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2841

Date: 27/1/2018 8:59:23

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

5. Three special Quadrangle points

Quadrangle Points are points that are constructed from the reference points without preference of any of them. This can be done by doing the same construction for all possible combinations that are at hand.

There are three special Quadrangle Points that can be constructed using the midpoints of the six line segments spanned between the reference points of the Reference Quadrangle. These six line segments are also called the sides of the Quadrangle. Denote the reference points of the Quadrangle as P_i ($i=1,2,3,4$). Also indices j or k or l can be used with values from $(1,2,3,4)$.

1. There are three pairs of opposite sides $P_i.P_j$ and $P_k.P_l$ in a Quadrangle. The line connecting the midpoints of the opposite sides is called a bimedial. So there are three bimedians. They concur in $QA-P1 =$ Quadrangle Centroid.

2. There are four Component Triangles $P_i.P_j.P_k$. so there are four nine-point circles of these component triangles, each passing through three midpoints of the Quadrangle sides. These circles concur in one point that is called $QA-P2 =$ the Euler-Poncelet Point.

3. Each reference point P_i can be connected with the 3 other reference points P_j, P_k, P_l producing 3 rays from that reference point. A circle can be drawn through the midpoints of these rays.

Since there are four reference points there are four of these circles. These circles concur in one point that is called $QA-P3 =$ the Gergonne-Steiner Point.

These points are constructed using as many components of the quadrangle as existent. Therefore they are real Quadrangle points (or centers).

Special about these three special Quadrangle points is that they are collinear and the Quadrangle Centroid is the midpoint of the other two points.

Look at EQF (<https://www.chrisvantienhoven.nl/index.php/mathematics/encyclopedia>) for the many many other properties of these points:

$QA-P1$

(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p1>),

QA-P2
(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p2>),
QA-P3
(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p3>).

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2842
Date: 28/1/2018 1:45:58
From: eckart_schmidt@t-online.de
Subject: Nonpivotal Isocubics for QA/QL

Dear Bernard,

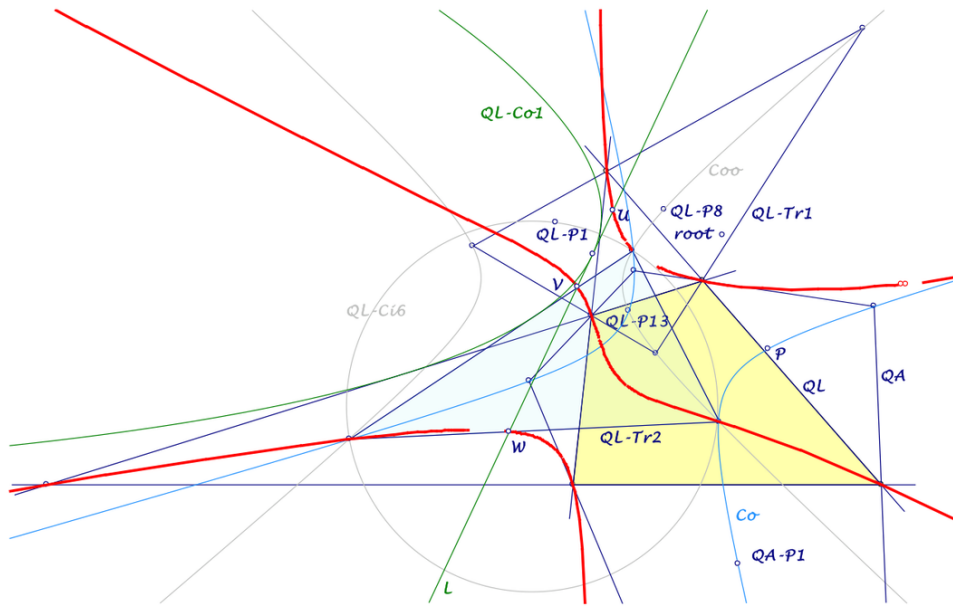
attached a generalization of the cubic in #2835 and #2838,
perhaps of interest.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Nonpivotal Isocubics for QA/QL

For a QA/QL constellation of dual quadrangle and quadrilateral (see EQF, QA-8/QL-8) cubics through the six QL-points and the three vertices of the triangle QL-Tr2 are described as nonpivotal isocubics.



Reference Triangle

The reference triangle is $QL-Tr2$ with vertices in the intersections of the circle $QL-Ci6$ and a circumconic Co_0 of the common diagonal triangle $QL-Tr1 = QA-Tr1$ through $QL-P8$ and $QL-P13$, except $QL-P24$.

Every line L , tangent to the QL -inscribed parabola $QL-Co1$, leads to a nonpivotal isocubic, bearing the six QL -points and the vertices of the triangle $QL-Tr2$. The dual point P lies on a conic Co through $QL-P13$ and $QA-P1$, circumscribed $QL-Tr2$ and also circumscribed the dual quadrangle of the quadrilateral.

Isoconjugation *

The isoconjugation * wrt $QL-Tr2$ shall swap opposite points of the quadrilateral.

The line at infinity will be mapped by this isoconjugation to the conic Co . The Newton line $QL-L1$, which is the tripolar of $QL-P13$ wrt the diagonal triangle $QL-Tr1$, will be mapped to the conic Co_0 , mentioned above. The images of the vertices of $QL-Tr1$ are the intersections of $QL-L1$ and the $QL-Tr1$ -sidelines. For lines L , tangent $QL-Co1$, holds PP^* parallel L .

Root

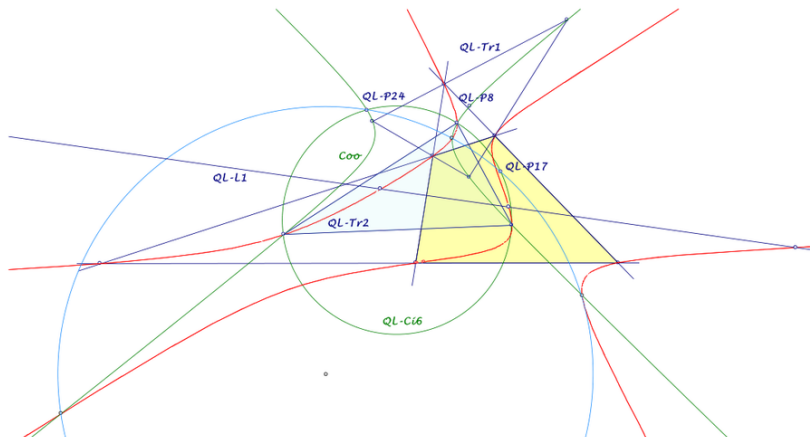
Let the tangent L at $QL-Co1$ intersect the sidelines of $QL-Tr2$ in U, V, W . The root of the searched nonpivotal cubic is the $QL-Tr2$ -tripole of L .

- **The nonpivotal isocubic wrt the described reference triangle, isoconjugation and root bears**
 - ... the six QL -points,
 - ... the three vertices of $QL-Tr2$
 - ... and the points U, V, W .

The cubic for the dual line of $QA-P1$ is already mentioned in $QFG\# 2835$ by Bernard Keizer and in $QFG\# 2838$.

The cubic for the dual line of $QL-P13$, which is the line at infinity, is a further interesting cubic:

The isoconjugation * maps the three intersections of the cubic and the Newton line $QL-L1$ to the three further intersections of the cubic and the conic Co_0 , which lie with $QL-P24$ and $QL-P17$ concyclic.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2018-01-28.pdf

← Previous Next → ↔ Message Index ↑ Subjects

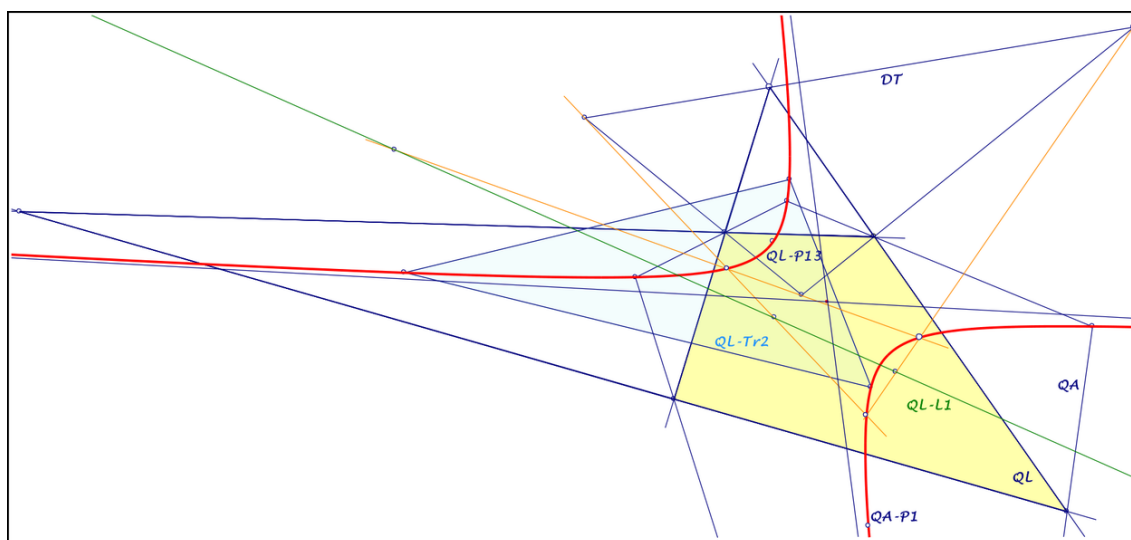
Message: #2843
Date: 28/1/2018 9:14:44
From: eckart_schmidt@t-online.de
Subject: QA/QL-Conic

Dear Chris,

in message 2842 I used a conic Co,
 which seems worth, to be mentioned in EQF.
 Consider a dual QA/QL-configuration and the conic,
 ... circumscribed QL-Tr2,
 ... circumscribed the dual quadrangle,
 ... bearing QA-P1 and QL-P13 = QA-P16,
 ... bearing the intersections of the lines through
 a vertex Si of the diagonal triangle
 and the midpoint of opposite QL-points on SjSk,
 ... locus of the dual points for tangents at the inscribed
 parabola QL-Co1,
 ... image of the line at infinity wrt the QL-Tr2-isoconjugation,
 that maps opposite QL-points,
 ... bearing with a point also its anticevians
 wrt the diagonal triangle,
 ... with equation
 $q^2r^2(q^2-r^2)x^2 + r^2p^2(r^2-p^2)y^2 + p^2q^2(p^2-q^2)z^2 = 0$.

I think, there will be more properties!

Best regards Eckart



2018-01-28a.pdf

Message: #2844
Date: 31/1/2018 3:16:59
From: bernard.keizer
Subject: QA/QL-Conic

Dear Chris, dear Eckart,
Beautiful work of pedagogy !
Thanks to Eckart for showing that for any cubic through the 6 QL vertices and the 3 S-points the line UVW, trilinear polar of the root wrt QL-Tr2 is necessary tangent to the parabola QL-Co1 and therefore dual of a point of the conic Co.
I regret only the definition of the S-points as intersection with QL-Ci6
I prefer to define them as the intersections (other than QA-P16 or QL-P13) between the 2 conics Co and Co0. The duals of these 2 conics are the parabolas QL-Co1 and DQL-Co1 (DT inscribed parabola tangent to the Newton Line) and the triangle QL-Tr2 is self dual, it's sides being the common tangents (other than the infinity line) to the parabolas.
Then, the circumcircle of the S-points is the Dimidium circle, which also bears QL-P1, QL-P17 and QL-P24 ...
All this is well known ...
Last remark : the lines joining a DT vertice to the middle of the segment joining 2 opposite QL vertices on the opposite DT side are precisely the sides of DDT, the DT of DQL and the intersections are the DDT vertices, which are the vertices of the anticevian triangle of QL-P13 wrt DT.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2845
Date: 31/1/2018 3:37:07
From: eckart_schmidt@t-online.de
Subject: QA/QL-Conic

Dear Chris,

further properties of the conic Co, described in QFG#2843:
... The center of the conic Co on QA-Co1
... .. is the QA-Tf2-image of the infinity-point
... .. of a perpendicular to QA-P12.QA-P32,
... .. with coordinates
... .. $(p^2/(q^2-r^2):q^2/(r^2-p^2):r^2/(p^2-q^2))$.
... Infinity points of the asymptotes
... .. are the QA-Tf2-images of the intersections of QA-L3
... .. and QA-Co1.
... The common diagonal triangle DT of dual QA,
... .. QL is selfpolar wrt Co.
... Co intersects the DT-sidelines harmonical.
... The Co-poles of the QL-lines give a quadrangle QA'
... .. with the same diagonal triangle DT,
... .. coconic with the reference quadrangle QA.
... The Co-tangents of the QA-points give a quadrilateral QL'
... .. with the same diagonal triangle DT
... .. and a common inscribed conic with the
... .. referential quadrilateral.
... There is an DT-isoconjugation
... .. mapping the vertices/lines of QA to the vertices of QA',
... .. with fixed points in the intersections of lines
... .. through the DT-vertices and Co-intersections
... .. on the opposite DT-side.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

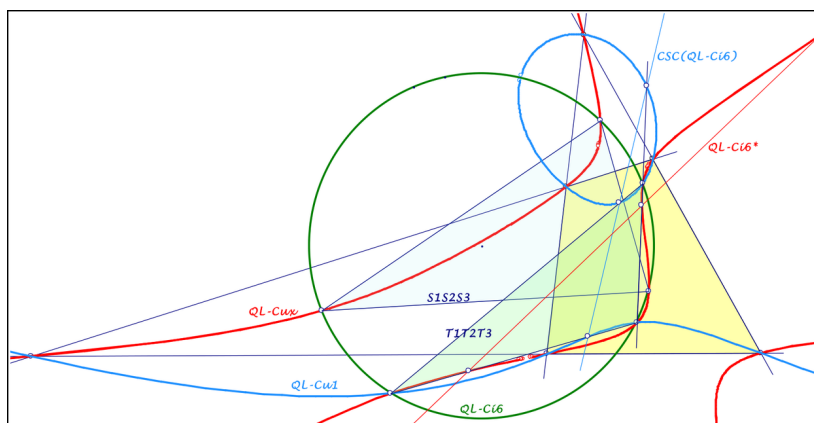
Message: #2846
Date: 01/2/2018 10:43:41
From: eckart_schmidt@t-online.de
Subject: Nonpivotal Isocubics for QA/QL

Dear Bernard,

thanks for lightening the background of cubics through the 6 QL-points and the S-points.
Here some remarkable observations for the example at the end of QFG#2842:

- ... reference triangle QL-Tr2,
- ... isoconjugation *, swapping opposite QL-points,
- ... root QL-P12.
- (1) This cubic QL-Cux intersects the cubic QL-Cu1 in the QL-points
... and one point T on the Dimidium circle QL-Ci6,
... ... if QL-Cu1 unipartite,
... or three points T1, T2, T3 on the Dimidium circle QL-Ci6,
... ... if QL-Cu1 bipartite.
- (2) QL-Cux and QL-Cu1 (bipartite) are invariant
... wrt an T1T2T3- isoconjugation ^,
... ... swapping opposite QL-points.
... For points P on QL-Cu1 holds $P^\wedge = CSC(P)$
- (3) QL-Cu1 (bipartite) is a nonpivotal isocubic:
... reference triangle T1T2T3,
... isoconjugation, swapping opposite QL-points,
... root = tripole of the line CSC(QL-Ci6).
- (4) QL-Cux is a nonpivotal isocubic
... reference triangle T1T2T3,
... isoconjugation, swapping opposite QL-points,
... root = tripole of the line QL-Ci6*.

Best regards Eckart



2018-02-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2847
Date: 01/2/2018 3:06:51
From: Antreas Hatzipolakis
Subject: Miquel triangle and Isogonal center

Maybe of interest

Main result In this section, I'll give a brief introduction to Miquel triangle, Isogonal center and Gergonne-Steiner point of a quadrangle.

Telv Cohl, Miquel triangle and Isogonal center (
<https://artofproblemsolving.com/community/c284651h1584129>)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2848
Date: 01/2/2018 3:46:35
From: eckart_schmidt@t-online.de
Subject: 5G-s-P1, 5G-s-P2, 5L-s-P1

Dear Seiichi, dear Chris,

a short remark wrt Seiichi's points $5G-s-P1 = U$, $5G-s-P2 = V$ and $5L-s-P1 = W$,
which are collinear for a pentagon.

Consider for a pentagon $PG = P1P2P3P4P5$ a partner pentagon $PG' = P1P3P5P2P4$:

... U, V, W, U', V', W' are collinear,
... $W = U', W' = U$,
... V, V' lie harmonically wrt U, U' .

Perhaps evident?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2849
Date: 01/2/2018 4:06:02
From: chris.vantienhoven
Subject: Miquel triangle and Isogonal center

Dear Antreas,

Very nice. Thanks for mentioning!
Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2850
Date: 01/2/2018 9:45:10
From: eckart_schmidt@t-online.de
Subject: Miquel triangle and Isogonal center

Dear Antreas, dear Chris,

with respect for Antreas synthetic proofs
I have reproduced with CABRI the considered properties.
Only a short remark: All considered 20 QA-points
... P1, P2, P3, P4, Mx, My, Mz, X, Y, Z,
 T1, T2, T3, T4, S1, S2, S3, S4, I , G
... are points on the cubic QA-Cu1.
Perhaps the properties 7 and 8 should be mentioned in EQF wrt
QA-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2851

Date: 02/2/2018 8:20:41

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

6. Three special Quadrilateral points

Quadrilateral Points are points that are constructed from the reference lines in a Quadrilateral without preference of any of them. This can be done by doing the same construction for all possible combinations that are at hand.

Next special Quadrilateral Points are constructed using the four combinations of three reference lines (the Component Triangles) that can be made in a system of four reference lines.

QL-P1 (<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p1>)

Miquel Point:

Common point of the circumcircles of the Component Triangles

QL-P2 (<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p2>)

Morley Point

Each Component Triangle is composed of three out of four reference lines leaving one reference line not-used. This not-used reference line can be used for some special operation wrt the Component Triangle at hand. The Morley Point is the common point of the lines through the nine-point centers N_i of the Component Triangles perpendicular to the not-used reference line L_i ($i=1,2,3,4$).

QL-P3 (<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p3>)

Kantor-Hervey Point

Common point of the four perpendicular bisectors $O_i.H_i$

($i=1,2,3,4$), where

O_i = Circumcenter of Component Triangle _{i} and

H_i = Orthocenter of Component Triangle _{i} .

There are many more

Quadrilateral points

(<https://www.chrisvantienhoven.nl/index.php/quadrangle-objects/9-mathematics/encyclopedia-of-quadri-figures/6-quadrilateral-objects>).

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2852
Date: 02/2/2018 11:11:01
From: eckart_schmidt@t-online.de
Subject: 5G-s-P1, 5G-s-P2, 5P-s-P1

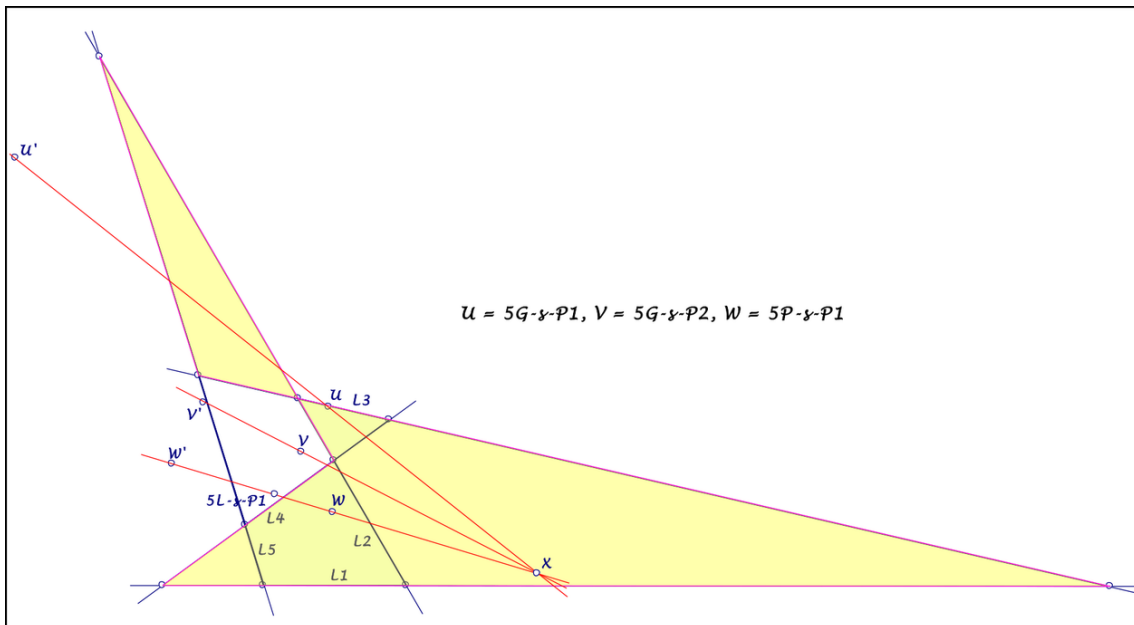
Dear Seiichi, dear Chris,

attached the same procedure as in #2848
... wrt Seiichi's points $5G-s-P1 = U$,
 $5G-s-P2 = V$ and $5P-s-P1 = W$:

Consider for a pentagon $PG = L1L2L3L4L5$ a partner pentagon $PG' = L1L3L5L2L4$:
... The lines UU' , VV' , WW' have a common point X (not in EPG ?),
... which is the 4th harmonic point of $5L-s-P1$ wrt WW' .

Perhaps evident?
Best regards Eckart

PS for Chris: Is it true, that attachments don't appear in QFG,
but in the mails to the members?



2018-02-02.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2853
Date: 03/2/2018 12:31:11
From: chris.vantienhoven
Subject: 5G-s-P1, 5G-s-P2, 5P-s-P1

Dear Eckart,

[ES] PS for Chris: Is it true, that attachments don't appear in QFG, but in the mails to the members?

Yes that's how it appears to be right now. A major omission of Yahoo.
Also the forums of Advanced Plane Geometry and Anopolis have the same problem.
So I suppose it is a problem of all Yahoo Groups.
Unfortunately there is no possibility to communicate with Yahoo about it.

I am still brooding about it. If you know solutions or alternatives please let me know.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2854

Date: 03/2/2018 4:00:29

From: eckart_schmidt@t-online.de

Subject: Affinely Regular Components of a 5-gon

Dear Chris,

after my studies (1968) I became coauthor of my professor for a book "n-Ecke", published 1970 in Germany and later translated in English:

F. Bachmann, E. Schmidt: n-gons,
University of Toronto Press 1975.

Attached I try to describe a very special result of the theory (12.5):

Every 5-gon is uniquely representable as the sum of an affinely regular pentagon and an affinely regular pentagram.

Perhaps of interest.

Best regards Eckart

PS:

I regret highly, that attachments don't no longer appear in QFG. What is geometry without an illustrating figure?

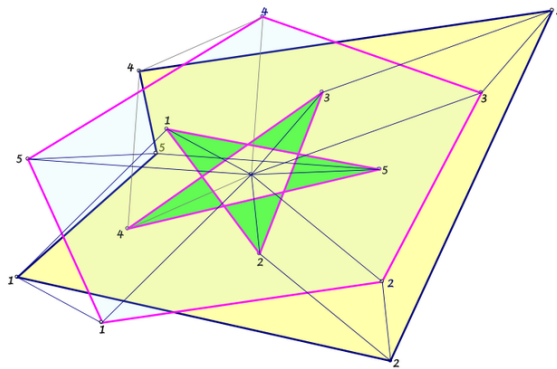
My notes are gathered on my homepage, but it is not always up to date.

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Affinely Regular Components of a 5-Gon

Every 5-gon is uniquely representable as sum of an affinely regular pentagon and an affinely regular pentagram. – This is a special result in the book:

*F.Bachmann / E.Schmidt : n-Ecke
 Hochschultaschenbuch 471/471a
 Bibliographisches Institut AG, Mannheim 1970*



In the cited book n-gons are ordered n-tupel of vectors with an addition and multiplication by real numbers, in this way the set of n-gons can be treated as vector space. Here only n-gons with centroid zero are considered!

Researched are transformations, which are polynomials of the basic transformation

$$\zeta: (a_1, a_2, \dots, a_n) \rightarrow (a_2, a_3, \dots, a_n, a_1).$$

The transformation, which maps a n-gon to the n-gon of the midpoints of the sides, will be

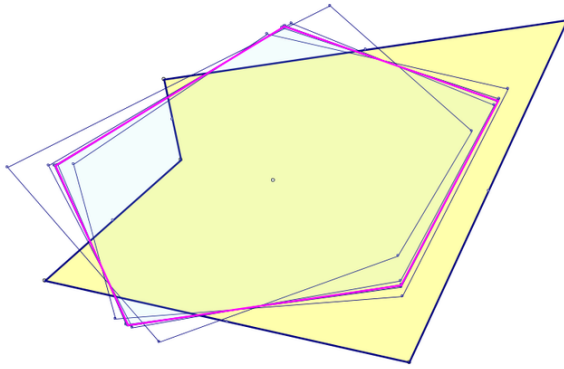
$$\kappa_2 = \frac{1}{2} (I + \zeta).$$

This transformation leads iterated to the n-times counted centroid. But the modified transformation, stretched and shifted indexed,

$$\kappa_2^* = - \frac{1}{\cos \pi/5} \zeta^2 \kappa_2$$

maps affinely regular pentagons to itself. Iterated mapping of κ_2^* for a 5-gon leads to an affinely regular pentagon, which is the image of the reference 5-gon by the transformation

$$\frac{2}{5} (I + \cos 2\pi/5 \zeta + \cos 4\pi/5 \zeta^2 + \cos 4\pi/5 \zeta^3 + \cos 2\pi/5 \zeta^4).$$



The transformation, which maps a n-gon to the n-gon of the 4th parallelogram points of three consecutive vertices

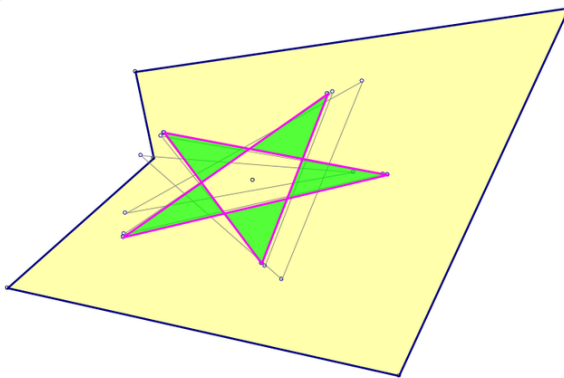
$$\pi_3 = 1 - \zeta + \zeta^2$$

leads iterated used in the modification

$$\pi_3^* = -\frac{\sin\pi/10}{\cos\pi/5} \zeta^4 \pi_3$$

to an affinely regular pentagon, which is the image of the reference 5-gon by the transformation

$$\frac{2}{5}(1 + \cos 4\pi/5 \zeta + \cos 2\pi/5 \zeta^2 + \cos 2\pi/5 \zeta^3 + \cos 4\pi/5 \zeta^4).$$



These pentagon and pentagram are the components of a 5-gon in the sense of the following theorem (see cited book 12.5):

- **Every 5-gon is uniquely representable as a sum of an affinely regular pentagon and an affinely regular pentagram.**

Eckart Schmidt
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eckart_schmidt@t-online.de

2018-02-03.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2855
Date: 05/2/2018 9:18:09
From: eckart_schmidt@t-online.de
Subject: QA-Tf2

Dear Chris,

QA-Tf2(P) is the intersection of the polars of P wrt QA-2Co1.
This was new for me!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2856
Date: 05/2/2018 9:41:01
From: eckart_schmidt@t-online.de
Subject: QA-Tf2

Dear Chris,

sorry wrt my last message:
This is the definition of isoconjugation by Bernard Gibert in
1.2.1 of his "Special Isocubics ..." .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2857
Date: 05/2/2018 10:39:06
From: chris.vantienhoven
Subject: QA-Tf2

Dear Eckart,

Nevertheless a very nice new insight in the Involuntary Conjugate
QA-Tf2.
Thanks!
I will put it in EQF later.

Chris

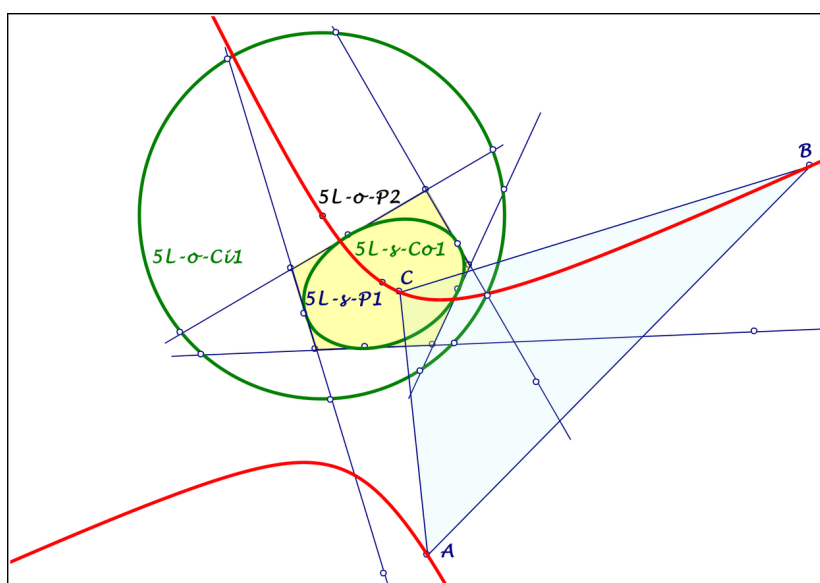
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2858
Date: 06/2/2018 12:13:35
From: eckart_schmidt@t-online.de
Subject: QA-Tf2

Dear Chris,

my remark wrt QA-Tf2 can be generalized:
Two conics give for a point P the intersection P^* of its polars
... and define a transformation $\text{CoCo}(P) = P^*$.
... If the conics have four real intersections A, B, C, D ,
... CoCo is QA-Tf2 for $ABCD$.
This can be generalized for n_P, n_L, n_G
... if there are two special conics for these objects:
... for example see QFG-message 2830.
A further example (see attached file):
 CoCo for a 5L wrt 5L-o-Ci and 5L-s-Co1:
... CoCo maps lines to conics with three common points,
... which give a reference triangle ABC ,
... so that CoCo is an isoconjugation,
... which maps the contact points of 5L-s-Co1 on L_i
... .. to the 4th harmonic point wrt the intersections of L_i
and 5L-o-Ci1.
... The reference triangle ABC has $X_4 = 5L-o-P_2$
... and 5L-o-Ci1 as polar circle.
... The line at infinity will be mapped to an orthogonal
hyperbola
... through $A, B, C, 5L-o-P_2, 5L-s-P_1$.

Best regards Eckart



2018-02-06.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2859
Date: 08/2/2018 4:00:09
From: Antreas Hatzipolakis
Subject: Ptolemy triangle and QA-Orthopole-circle

Telv, Cohl, Ptolemy triangle and QA-Orthopole-circle

<https://artofproblemsolving.com/community/c284651h1587808>

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2860
Date: 09/2/2018 3:48:11
From: eckart_schmidt@t-online.de
Subject: QA-Orthopole-Circles for a QL

Dear Antreas, dear Chris,

studying Antreas paper in #2859, which leads to the QA-orthopole-circle,

I became interested, to use these circles for a QL,

defining a transformation which maps a point

to the radical center of its three QA-orthopole-circles.

Perhaps of interest (see attached file, if possible).

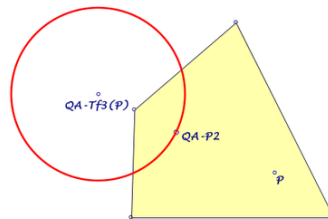
Best regards Eckart

PS wrt Antreas' paper: There will be a typo in Property 1: XYZ
// TXTYTZ.

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

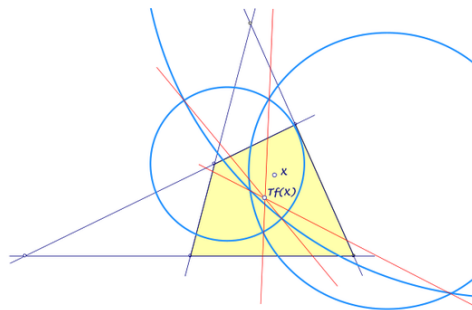
QA-Orthopole-Circles for a QL

The QA-orthopole-circle of a point P is not explicit in EQF, but mentioned under QA-Tf3, found by Antreas P. Hatzipolakis. These circles lead to a QL-transformation, considering the radical center of the three QA-orthopole-circles of a point P wrt the quadrigon-components of the quadrilateral.



The QA-orthopole-circle of a point P is a circle round QA-Tf3(P) through QA-P2. For a quadrilateral we define the following transformation $X \rightarrow Tf(X)$ with

- **Tf(X) is the radical center of the QA-orthopole-circles of X wrt the three quadrigon components of the quadrilateral.**

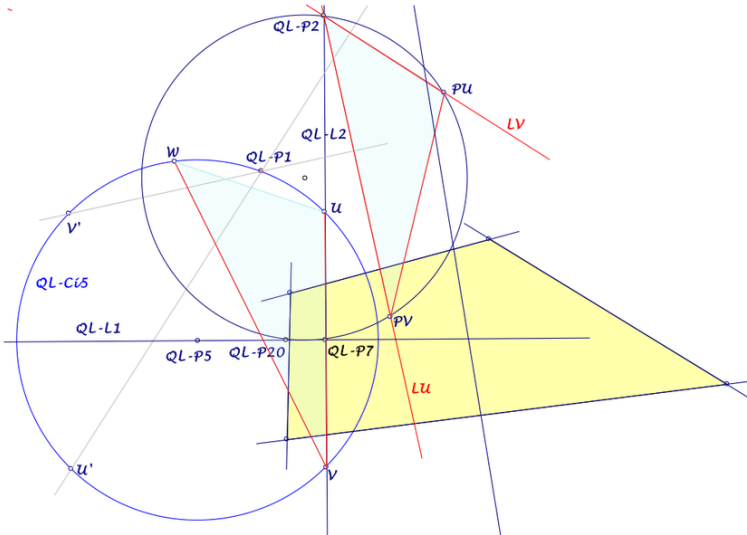


This transformation is not reciprocal, iterated using leads to the point QL-P2, which is a fixed point.

- **For points X on QL-L2: Tf(X) = QL-P2.**

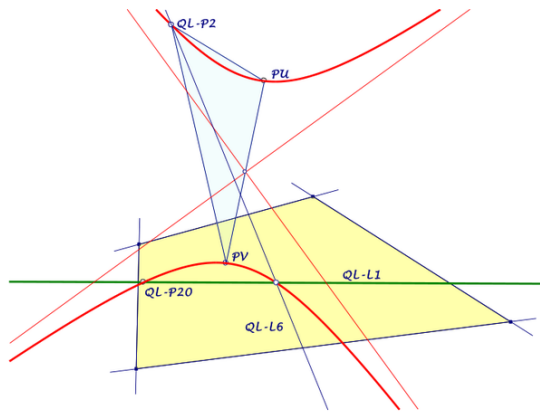
- For points X on the Plücker-circle $QL-Ci5$ $Tf(X)$ are points at infinity.
Diametral points on $QL-Ci5$ give orthogonal directions.

For points on the Plücker-circle the radical axes are parallel:
 ... for $QL-P1$ orthogonal $QL-L1$,
 ... for $QL-P1$, reflected in $QL-P5$, parallel $QL-L1$,
 ... and for the Plücker points U, V ($QL-2P1$) the radical axes coincide and give two lines L_U, L_V through $QL-P2$, orthogonal $V'QL-P1, U'QL-P1$ with U', V' diametral points of U, V on $QL-Ci5$.



Let us consider additional a circle Ci through $QL-P2, QL-P7, QL-P20$, centered in the midpoint of $QL-P1.QL-P20$, with 2nd intersections P_V, P_U on L_U, L_V , further the point $W = CSC(CSC(QL-Ci5) \cap QL-L2)$, which is the 2nd intersection of $QL-Ci5$ and $QL-Ci6$.

- For points X on WU : $Tf(X) = P_V$,
... for points X on WV : $Tf(X) = P_U$.
- The triangles UVW and $P_U P_V QL-P2$ are spiral similar
... with ratio of the radii of $QL-Ci5$ and Ci .
- The transformation Tf maps lines (not $QL-L2, L_U, L_V$) to conics through the points $P_U, P_V, QL-P2$.
- The transformation Tf maps the Newton line $QL-L1$ to an orthogonal hyperbola,
... centered in the midpoint of $P_U P_V$,
... through $P_U, P_V, QL-P2$
... and $QL-P20, QL-L6 \cap QL-L1$.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2018-02-09.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2861
Date: 09/2/2018 3:57:00
From: Antreas Hatzipolakis
Subject: QA-Orthopole-Circles for a QL

Dear Eckart

The author of the paper is Telv Cohl (not me!)

Regards
APH

> On Fri, Feb 9, 2018 at 4:48 PM, 'eckart_schmidt@t-online.de'
eckart_schmidt@t-online.de
> [Quadri-Figures-Group] <Quadri-Figures-Group@yahoogroups.com
>
> wrote:
>> Dear Antreas, dear Chris,
>> studying Antreas paper in #2859, which leads to the
QA-orthopole-circle,
>> I became interested, to use these circles for a QL,
>> defining a transformation which maps a point
>> to the radical center of its three QA-orthopole-circles.
>> Perhaps of interest (see attached file, if possible).
>> Best regards Eckart
>> PS wrt Antreas' paper: There will be a typo in Property 1:
XYZ // TXTYTZ.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2862

Date: 09/2/2018 8:07:00

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

7. Three special Quadrigon points

For readers who missed the earlier paragraphs of this introduction:

A quadrigon is a figure bounded by 4 lines intersecting in 4 points (in ordinary speech just a "quadrilateral").

Here are some points typical for Quadrigons.

Let $P_1.P_2.P_3.P_4$ be the reference Quadrigon.

QG-P10

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p10>)

* 2nd Quasi Orthocenter

Let h_1 be the line through P_1 perpendicular to diagonal $P_2.P_4$.

Let h_2 be the line through P_2 perpendicular to diagonal $P_1.P_3$.

Let h_3 be the line through P_3 perpendicular to diagonal $P_2.P_4$.

Let h_4 be the line through P_4 perpendicular to diagonal $P_1.P_3$.

The lines h_1, h_2, h_3, h_4 form a parallelogram with center QG-P10.

QG-P12

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p12>)

* Inscribed Harmonic Conic Center

Transform the reference Quadrigon into a Square through a projective transformation.

Draw the inscribed circle into this square.

Transform this Square and the inscribed circle back into the Reference Quadrigon. The inscribed circle will be transformed into a conic inscribed in the Reference Quadrigon.

QG-P12 is the center of this conic.

QG-P15

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p15>)

Kirikami Center

Let r_1 be the line through P_1 parallel to diagonal $P_2.P_4$.

Let r_2 be the line through P_2 parallel to diagonal $P_1.P_3$.

Let r_3 be the line through P_3 parallel to diagonal $P_2.P_4$.

Let r_4 be the line through P_4 parallel to diagonal $P_1.P_3$.

The lines r_1, r_2, r_3, r_4 form a parallelogram with center QG-P15.

There are many more Quadrigon points (<https://www.chrisvantienhoven.nl/index.php/quadrangle-objects/9-mathematics/encyclopedia-of-quadri-figures/20-quadrigonobjects>).

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2865
Date: 13/2/2018 9:14:43
From: eckart_schmidt@t-online.de
Subject: Terms confusion

Dear Benedetto,

where is my mistake?

- (1) Qa-Tf3 maps QA-P4 to QA-P2 (see EQF).
- (2) The QA-orthopole-circle of P (see EQF)
... is centered in QA-Tf3(P),
... bearing QA-P2.
- (3) 5P-s-Tf3(P) is the common point of the QA-orthopole-circles
of P
... wrt the four QA of a 5P (see EQF).
- (4) For a conic with inscribed QA and inscribed 5P
... 5P-s-Tf3 maps QA-P4 to QA-P2.

Thanks in advance.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2866
Date: 13/2/2018 2:15:24
From: Benedetto Scimemi
Subject: Terms confusion

Dear Eckart,
your mathematics is obviously OK, but you are using wrong symbols mistake, according to the recent decision of Chris and me about naming the various mappings.
In fact, the present version of EQF must be corrected and I expect Chris will do it soon.
As I mentioned in my last message to the group, after many doubts, we decided -for the future- to call Co-Tf3 and 5P-s-Tf3 the mapping QA-P2 --> QA-P4.
There are pretty good reasons to do it, but, of course, we realized that we would create some problems as yours.
I hope that the final editing of the page will clear up the whole matter.

Best greetings
Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2867
Date: 13/2/2018 8:24:50
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Chris,

there is a further possibility, to describe QA-Cu1 as pivotal isocubic:

Let P be a point on QA-Cu1,
... the circles through P, P_i , P_j and P, P_k , P_l
... have a 2nd intersection Q on QA-Cu1,
... which leads to a Q-triangle.
... The 3rd intersections R of PQ and QA-Cu1
... lead to a R-triangle.

The R-triangle is QA-Tf2 image of QA-Tr2
... as well as the QA-Tr2-isogonal image of QA-Tr1.

QA-Cu1 is a pivotal isocubic
... with the R-triangle as reference triangle,
... an isoconjugation, swapping the vertices of QA-Tr1 and QA-Tr2
... and pivot QA-P3.

QA-Tr1, QA-Tr2 and the R-triangle
... are pairwise perspective reference triangles
... wrt the pivot for the 3rd reference triangle and its isoconjugation.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2868
Date: 14/2/2018 10:58:34
From: eckart_schmidt@t-online.de
Subject: New CB-point on QA-Cu1

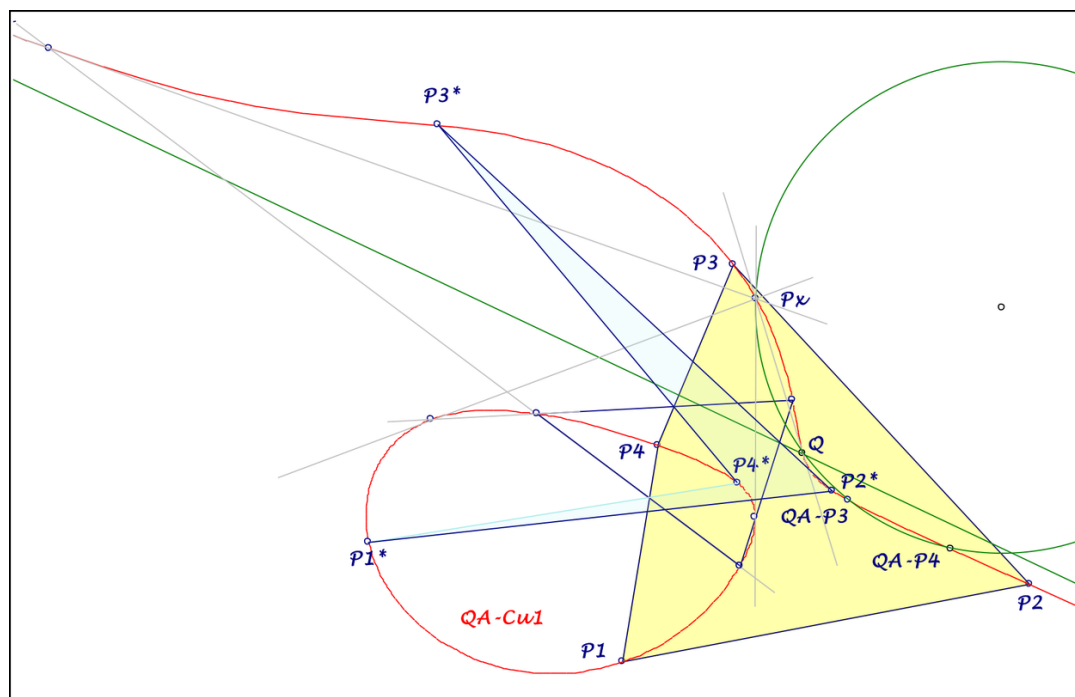
Dear Bernard, dear Chris,
 the Cayley-Bacharach ninth point leads to new interesting points on QA-Cu1.

Examples in QFG#2458 and QFG#2477, here a further one (see attached file):

- ... Px is the QA-Cu1-tangential of the intersection Q of QA-Cu1 and its asymptote.
- ... Px is the tangential of the 3rd intersections of QA-Cu1 and the sides of QA-Tr2.
- ... Px is the 4th intersection of QA-Cu1 and a circle through QA-P3, QA-P4 and Q.
- ... Px is the ninth CB-point of the QA-vertices and their isogonal conjugates wrt QA-Tr2,
- which give a 2nd QA on QA-Cu1 with the same Miquel triangle:
- ... The two QA are perspective wrt the Miquel points and Q
- and have the same Miquel triangle,
- ... so there CB-point is the 3rd intersection of QA-Cu1 and the line of their QA-P41-points.

Best regards Eckart

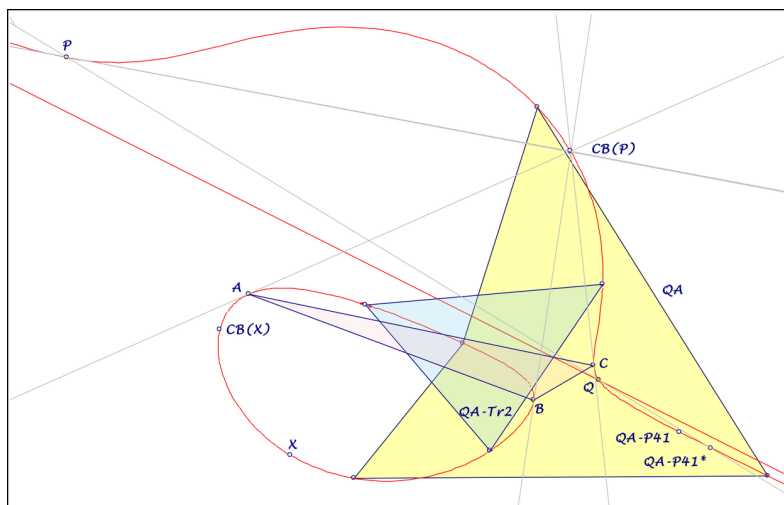
PS for Bernard: I hope, we haven't mentioned Px already last summer.



2018-02-10.pdf

Message: #2869
Date: 15/2/2018 9:09:01
From: eckart_schmidt@t-online.de
Subject: Isoconjugation with CB-points

Dear Bernard, dear Chris,
 here is a further possibility
 for QA-Cu1 as pivotal isocubic,
 ... using Cayley-Bacharach ninth points.
 Let us consider the transformation $X \rightarrow CB(X)$
 ... with $CB(X)$ ninth Cayley-Bacharach point
 ... of X and the vertices of QA and QA-Tr2,
 ... which is again a point on QA-Cu1.
 Let P be the 3rd intersection of Q .QA-P41* and QA-Cu1
 ... with Q intersection of QA-Cu1 and its asymptote
 ... and QA-P41* isogonal conjugate of QA-P41 wrt QA-Tr2.
 The tangential of P is $CB(P)$,
 ... there are three other points A, B, C
 with the same tangential
 ... (constructable with P and Möbius transformations
 wrt QA-Tr2).
 QA-Cu1 is a pivotal isocubic
 ... with reference triangle ABC ,
 ... isoconjugation, swapping QA-P41* and Q
 ... and pivot P .
 This isoconjugation is $X \rightarrow CB(X)$ for X on QA-Cu1.
 There are four points X on QA-Cu1 (not necessary real)
 ... with $X = CB(X)$ and tangential P ,
 ... leading to a new QA with the same Miquel triangle
 ... and diagonal triangle ABC ,
 ... perspective to the Miquel triangle.
 Best regards Eckart



2018-02-15.pdf

Message: #2870
Date: 15/2/2018 11:18:27
From: bernard.keizer
Subject: New CB-point on QA-Cu1

Dear Eckart,
Interesting combinations using the C-B 9th point !
I don't remember we have mentionned these points last summer !
But I will be seventy in June this year and perhaps my memory is
beginning to weaken ...
Alleluiah, the attached files appear again !
Best regards
Bernard
PS Looking again at your different notes about the S-points,
I've found this :
We search a triangle with orthocenter QL-P2, centroid QL-P12 and
circumcenter QL-P6 (these points are the barycenters of the same
elements in each reference triangle of the QL)
The circumcircle is the Dimidium circle QL-Ci6 centered in QL-P6
through QL-P1 and QL-P17
QL-P23 is the contact point between the parabola DQL-Co1
(inscribed DT and tangent to the Newton Line) and the Newton
Line (this simple property is not mentionned in EQF at QL-P23)
The circle through QL-P7, QL-P17 and QL-P23 cuts the Dimidium
circle in a 2nd point X ; the middle Y of the segment QL-P2X is
on the Euler circle of the S-triangle QL-Tr2.
The conic centered in Y through QL-P2, QL-P7, QL-P23 and X is a
rectangular hyperbola which cuts the Dimidium circle in X and
the S-points, vertices of the searched triangle QL-Tr2

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2871
Date: 15/2/2018 3:19:43
From: eckart_schmidt@t-online.de
Subject: New CB-point on QA-Cu1

Dear Bernard,

thanks for reply.

Wrt the S-points, your orthogonal hyperbola was new for me.

There is a further interesting point on the hyperbola:

... intersection of QL-L6 and QL-L9
... or reflection of QL-P17 in QL-L1
... or reflection of QL-P7 in Y
... or X3 of X, QL-P7, QL-P23.

Best regards Eckart

PS: Last Christmas my memory remind me of my 78 birthday.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2872
Date: 16/2/2018 1:15:52
From: bernard.keizer
Subject: New CB-point on QA-Cu1

Dear Eckart,

Thanks for this new point on the rectangular hyperbola !
This QL-Tr2 circumscribed rectangular hypebola can't be totally
new for you, as you gave ourself it's equation in one of your
old notes I mentionned ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2873

Date: 16/2/2018 11:40:15

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

8. Harmonic properties in a Quadrangle

In general a Quadrangle conceals a lot of hidden harmonic properties.

We will first define the notion of "harmonic conjugate".

Given four collinear points W, X, Y, and Z.

Y and Z are harmonic conjugates with respect to W and X if

$$\frac{|WY|}{|YX|} = \frac{|WZ|}{|XZ|}.$$

See Mathworld, *Harmonic Conjugate (

<http://mathworld.wolfram.com/HarmonicConjugate.html>)*.

See ETC, *Harmonic Conjugates (<http://faculty.evansville.edu/ckj/6/encyclopedia/HarmonicConjugates.html>)*.

We will find the hidden harmonic properties of a Quadrangle in 2 steps.

This will be done by just drawing lines by connecting points and drawing points by intersecting lines.

1. Draw all possible lines between Quadrangle vertices

P1,P2,P3,P4.

When we draw all intersection points we find: $S = P1.P3 \wedge P2.P4$, $T = P1.P2 \wedge P3.P4$, $U = P1.P4 \wedge P2.P3$.

These are the vertices of the so called Diagonal Triangle of the Quadrangle P1.P2.P3.P4.

Now we have a network of 6 lines and 7 points. Each line containing 3 points.

2. When we draw again all possible lines between all known points (P1,P2,P3,P4,S,T,U) we get 3 more lines (the sidelines of the Diagonal Triangle).

Drawing the new intersection points with these new lines we find 6 new intersection points, two per sideline of the Diagonal Triangle. Depending on the QA-sideline they intersect, they will be called M12, M13, M14, M23, M24, M34. See attached picture.

Now we have a network of 9 lines and 13 points. Each line containing 4 points.

It is most special that all occurring sets of 4 collinear points are harmonic conjugated.

Example: P1 and P2 are harmonic conjugates with respect to M12 and T, because $\frac{|P1.M12|}{|M12.P2|} = \frac{|P1.T|}{|P2.T|}$.

Desargues expanded this into his Involution Theorem:

"the points of intersection of a line with the three pairs of opposite sides of a complete quadrangle and a conic section circumscribed about the complete quadrangle form the pairs of an involution".

See Heinrich Dörrie: "100 great problems of Elementary Mathematics" page 265.

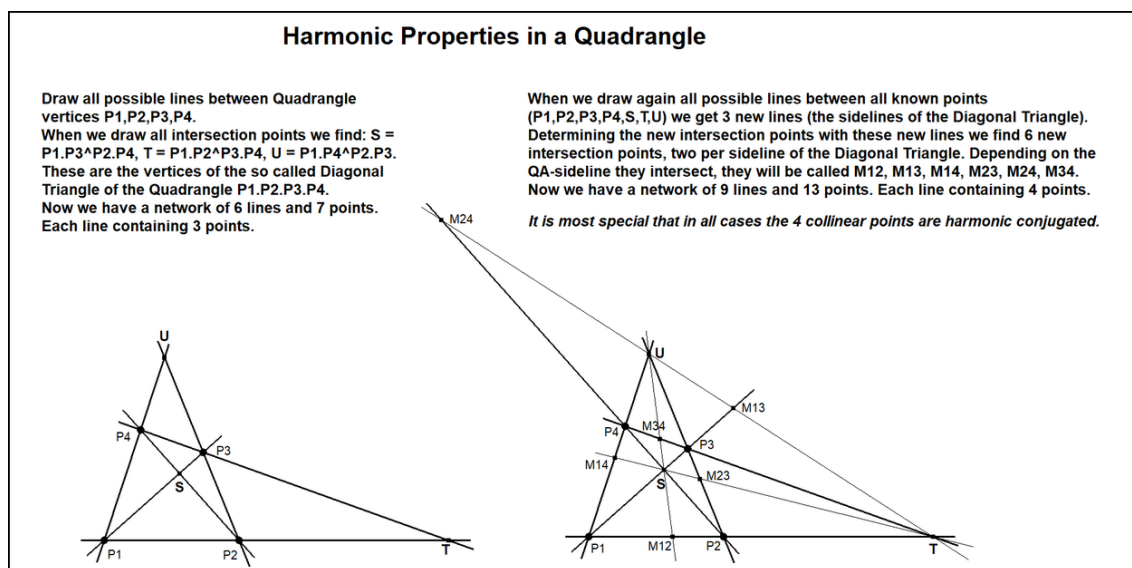
It is also the base for this important Quadrangle Transformation:

QA-Tf2 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf2>): Involutary Conjugate

See also:

QA-Tf1 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf1>): QA-Line Involution Center

Chris van Tienhoven



QA-Harmonic-Properties-01.png

Message: #2874

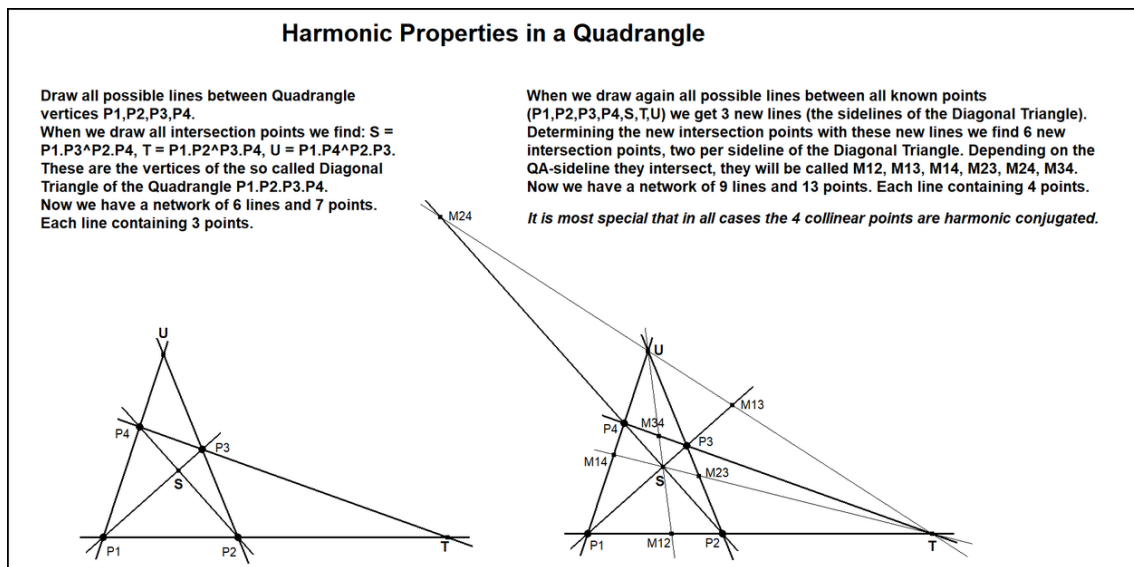
Date: 16/2/2018 11:55:01

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

8. Harmonic properties in a Quadrangle

and here is attached the picture



QA-Harmonic-Properties-01.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2875
Date: 16/2/2018 2:33:13
From: eckart_schmidt@t-online.de
Subject: 8. Harmonic properties in a Quadrangle

Dear Chris,

I regret, that two more relevant aspects of the transformation QA-Tf2 are not mentioned in EQF:

(1) QA-Tf2 is an isoconjugation of the diagonal triangle QA-Tr1 ... with fixed points in the vertices of the quadrangle.

(2) QA-Tf2(X) is the common point for polars of X wrt circumconics of the quadrangle.

I think, this will be more within the meaning of Bernard Gibert (Special Isocubics ..., 1.2.1).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2876
Date: 16/2/2018 3:21:42
From: chris.vantienhoven
Subject: 8. Harmonic properties in a Quadrangle

Dear Eckart,

Thanks for mentioning.

I added them in the list of properties of QA-Tf2.

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2877
Date: 16/2/2018 5:33:26
From: bernard.keizer
Subject: Introduction to Quadrilateral Geometry

Dear Chris,
Your description is unfortunately not complete !
Maybe it is the next lesson, but sorry I can't wait ...
The points you name Mij are obviously the vertices of the dual
QL ; they are on 4 QL lines L_i , which are the duals of the QA
vertices P_i .
By the way, it would be wise to rename your points Mij in M_{kl} in
order to have Mij on P_kP_l as intersection of the QL lines L_i and
 L_j and dual of the QA line P_iP_j ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2878
Date: 16/2/2018 7:03:21
From: bernard.keizer
Subject: 8. Harmonic properties in a Quadrangle

Dear Eckart,
3 of these conics are degenerated in a pair of QA lines through
one DT vertice T
In this case, the polar of P is the 4th harmonic line of TP wrt
the 2 QA lines
QA-Tf2 is obtained as intersection of these 3 4th harmonic lines
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2879
Date: 17/2/2018 9:25:17
From: chris.vantienhoven
Subject: Introduction to Quadrilateral Geometry

Dear Bernard,

You are right the dual QL is included in the figure.
That is quite another subject.
There is so much more that can be said about this figure.
For example the points M_{ij} are the points of tangency of the 3 harmonic conics (see QG-P12) that can be inscribed in the 3 QG-versions that fit into the figure. And that is the base for the QA-Harmonic Center QA-P16 and the QL-Harmonic Center QL-P13. These points are lined up with several other points at the special QG-harmonic line QG-L2.
Then 4 random points as revealed in this chapter define a perspective field in which coordinates can be made up for defining points fitting into this field as displayed in the line network that was described.
It all occurred to me.
But again, it is an introduction and I don't want to feed up newbie-readers with an overload of information. But I can understand and like your impatience. It is all so exciting. It all fits together. Every item evokes several new items and several new insights.
Thanks for your feedback and also for sharing your insight in renaming the M_{ij} into M_{kl} .

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2880
Date: 17/2/2018 10:16:57
From: chris.vantienhoven
Subject: 8. Harmonic properties in a Quadrangle

Dear Bernard,

[BK], 3 of these conics are degenerated in a pair of QA lines through one DT vertice T
In this case, the polar of P is the 4th harmonic line of TP wrt the 2 QA lines
QA-Tf2 is obtained as intersection of these 3 4th harmonic lines
I made note of this property in EQF at QA-Tf2.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2881
Date: 17/2/2018 12:01:43
From: eckart_schmidt@t-online.de
Subject: 8. Harmonic properties in a Quadrangle

Dear Chris,

you can extend your harmonic properties:
I use your nomination.
The 4th harmonic point of Mij wrt Mik, Mjk
... is the intersection Ql of their line with PkPl,
... which is the 4th harmonic point
... of Mk1 wrt Pk, PiPj^PkPl.
Ql and Qk are harmonic conjugates wrt Mk1 and PiPj^PkPl.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2882
Date: 17/2/2018 4:37:48
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Chris,

in addition of #2867 and #2869 a further description of QA-Cu1
... as pivotal isocubic for a quadrigon
QA-Cu1 is a pivotal isocubic wrt a quadrigon:
... reference triangle with vertices QG-2P2 and QA-P4,
... isoconjugation, which swaps opposite QG-vertices,
... pivot QG-P1.

This isoconjugation
... maps QG-P1 to QA-P41,
... maps QL-P1 to QA-P3,
... maps QG-P16 to the infinity point of the asymptote of
QA-Cu1,
... maps the circumcircle of QG-2P2 and QA-P4
... to its chord with midpoint
... QA-Tr2-isogonal conjugate of QA-P41,
... maps QG-L2,3 to circumconics
... of QG-2P2, QA-P4 and the intersections with QA-Cu1
unequal QG-P1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2883
Date: 18/2/2018 11:13:38
From: bernard.keizer
Subject: New sight of QA-Cu1

Dear Eckart,
I found this approach very interesting !
I'm generally not particularly enthousiastic about QG descriptions, as I find them often misleading.
For example, I suppose QL-P1 is one vertice of QA-Tr2, aligned with QA-P3 and one vertice of DT ...
So your isoconjugation swaps 2 QA vertices aligned with one DT vertice and QA-P3 with the corresponding vertice of QA-Tr2 and you describe QA-Cu1 as a pivotal cubic with reference triangle the 2 other DT vertices and QA-P4, the isoconjugation as described and pivot the 1rst DT vertice.
Am I correct in the interpretation of your construction ?
So, there are 3 such constructions, one for each DT vertice.
What are the 4 QA invariant points of these 3 isoconjugations ?
The vertices of QA-Tr2 may be constructed only with circles through the QA vertices and the DT vertices as Miquel points of QL's formed by the 4 QA vertices and 2 DT vertices ; in turn, QA-P4 may be constructed only with circles through QA vertices and QA-Tr2 vertices.
So I suppose there a connexion with the 3 Cl-S transformations centered in the vertices of QA-Tr2 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2884
Date: 18/2/2018 11:43:55
From: eckart_schmidt@t-online.de
Subject: Isoconjugation with CB-points

Dear Bernard, dear Chris,

in QFG#2869 there is a relation
... of isoconjugation and CB-points wrt QA-Cu1.
Here we consider points X on QL-Cu1.

Let CB(X) be the ninth Cayley-Bacharach point
... of X, QL-P1 and the QL-points,
... constructible as 4th intersection
... ... of circumconics for the QL-triangles through X
and QL-P1.

For points X on QL-Cu1
... CB(X) is again a point on QL-Cu1,
... the 3rd intersection of QL-Cu1 and QL-P1.CSC(X).

On the other hand we consider a triangle Tr on QL-Cu1
... with vertices QL-P1 and the foci of an inscribed conic
... ... centered in $QL-L1 \wedge QL-L6$
... with its isogonal conjugation *.

For points X on a bipartite QL-Cu1 holds $X^* = CB(X)$.
For points X on an unipartite QL-Cu1 holds $X^* = CSC(X)$.
For arbitrary points X are CB(X), CSC(X) and QL-P1 collinear.

QL-Cu1 bipartite is a pivotal isocubic:
... reference triangle Tr,
... isogonal conjugation *,
... pivot point at infinity of QL-L1.

QL-Cu1 unipartite is the locus for the intersections
... of conics L^* and circles CSC(L)
... for lines L parallel QL-L1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

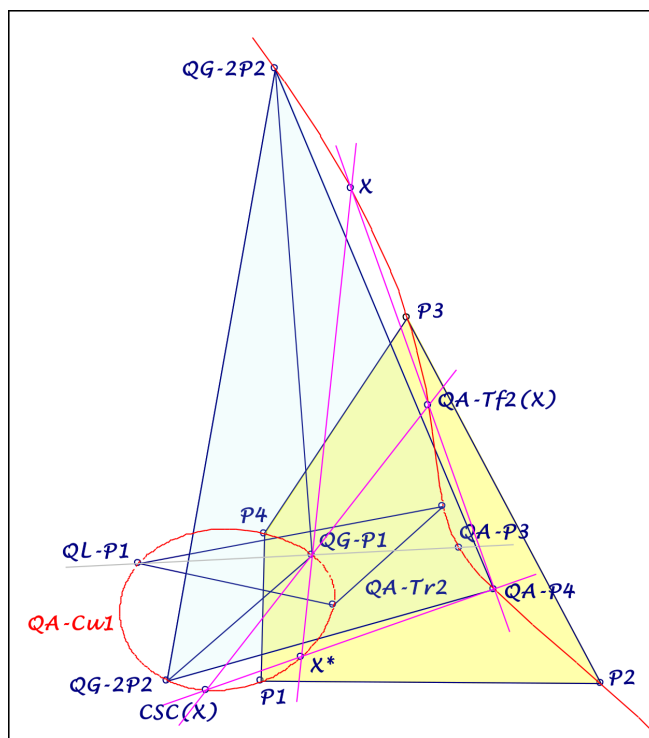
Message: #2885
Date: 18/2/2018 6:17:28
From: bernard.keizer
Subject: New sight of QA-Cu1

Dear Eckart,
I've just discovered the Schmidt point QG-P16, QA-Tf2(vertice of the Miquel triangle) or isogonal(DT vertice) wrt the Miquel triangle QA-Tr2.
QA-P4 is the Cl-S conjugate of each DT vertice in the CL-S transformation centered in the Miquel point aligned with QA-P3 and this DT vertice which swaps the 2 other Miquel points.
So everything is clear now, but I haven't found the fixed points of the isoconjugations wrt triangles QA-P4 and 2 DT vertices ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2886
Date: 19/2/2018 9:09:31
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Bernard,
 thanks for your remarks, your interpretation of my construction is correct.
 But the isoconjugations have no real fixed points to get corresponding QA.
 Your final conjecture wrt a connection with CSC-transformations is of interest:
 For a quadrigon QA-Cu1 is invariant wrt CSC.
 Let $*$ be the isoconjugation
 ... wrt QG-2P2 and QA-P4,
 ... swapping opposite QG-vertices.
 For points X on QA-Cu1 holds:
 ... $X^* = \text{QA-Tf2}(\text{CSC}(X))$,
 $X, \text{QA-Tf2}(X), \text{QA-P4}$ are collinear,
 $X^*, \text{CSC}(X), \text{QA-P4}$ are collinear,
 $X, X^*, \text{QG-P1}$ are collinear,
 $\text{CSC}(X), \text{QA-Tf2}(X), \text{QG-P1}$ are collinear.
 I hope, the attached file will appear in QFG.
 Best regards Eckart
 PS. I tried several times in vain to send this message yesterday, but it did not appear in QFG.



2018-02-18.pdf

Message: #2887
Date: 20/2/2018 8:57:19
From: eckart_schmidt@t-online.de
Subject: Inscribed QL-conic through QL-P1

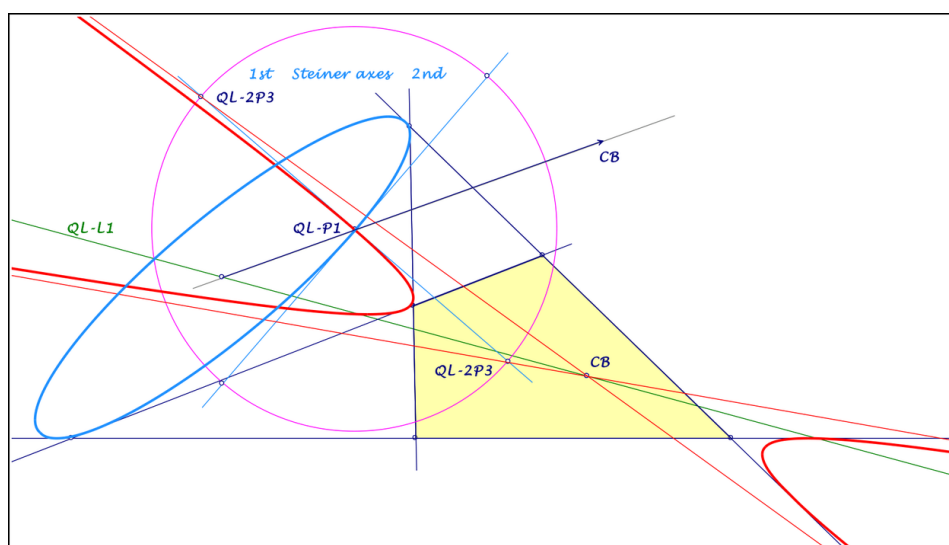
Dear Bernard, dear Chris,

there are two inscribed QL-conics through QL-P1,
... which are 5th line conics QL-Co-1 for the Steiner axes,
... a hyperbola for the 1st Steiner axis
... and an ellipse for the 2nd Steiner ellipse,
... both centered on the Newton line QL-L1.

The hyperbola for the 1st Steiner axis
... is centered on QL-L1 in the Cayley Bacharach ninth point
... .. of the six QL-points and the Schmidt pair
... .. of points QL-2P3,
... .. constructible with circumconics of the QL-triangles
... .. through QL-2P3.

The ellipse for the 2nd Steiner axis
... is centered on QL-L1 in the intersection with the line
... connecting QL-P1 and the Cayley Bacharach ninth point
... .. of the six QL-points and the intersections X, Y
... .. of the 2nd Steiner axis and the Schmidt circle
... .. (see QL-2P3).
... Attention: This CB-point is a point at infinity!
... Lines through X, Y and this infinity point are tangents at
the ellipse.

Best regards Eckart



2018-02-20.pdf

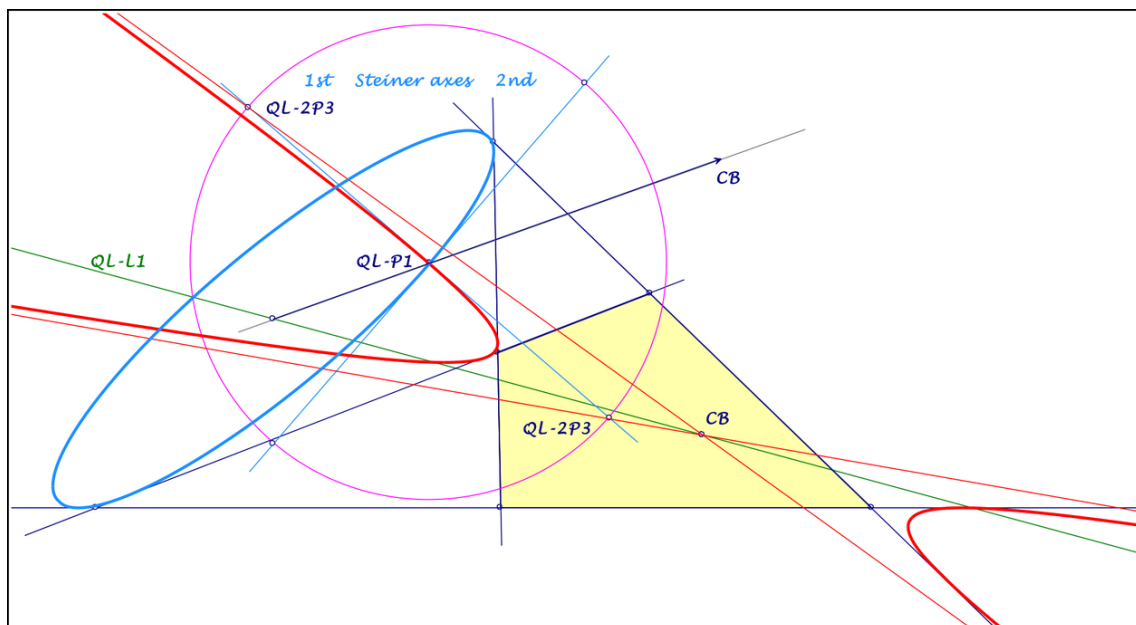
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2888
Date: 20/2/2018 9:11:20
From: eckart_schmidt@t-online.de
Subject: Inscribed QL-conic through QL-P1

Dear Bernard, dear Chris,

the attached file to #2887 doesn't appear in QFG, I try it once more.

Best regards Eckart



2018-02-20.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2889
Date: 20/2/2018 10:12:26
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Bernard,

thanks for your remarks, your interpretation of my construction is correct.

But the isoconjugations have no real fixed points to get corresponding QA.

Your final conjecture wrt a connection with CSC-transformations is of interest:

For a quadrigon QA-Cu1 is invariant wrt CSC.

Let * be the isoconjugation

... wrt QG-2P2 and QA-P4,

... swapping opposite QG-vertices.

For points X on QA-Cu1 holds:

... $X^* = \text{QA-Tf2}(\text{CSC}(X))$,

X, QA-Tf2(X), QA-P4 are collinear,

X^* , CSC(X), QA-P4 are collinear,

X, X^* , QG-P1 are collinear,

CSC(X), QA-Tf2(X), QG-P1 are collinear.

I hope, the attached file will appear in QFG.

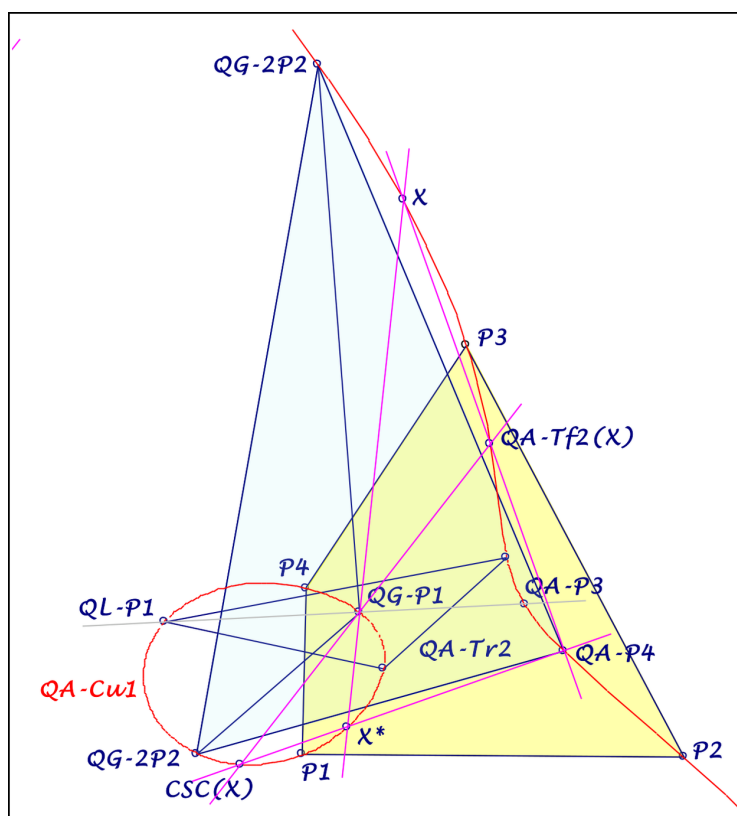
Best regards Eckart

PS. I sent this message five hours ago, but it did not appear in QFG.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2890
Date: 20/2/2018 11:10:55
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Bernard,
 thanks for your remarks, your interpretation of my construction is correct.
 But the isoconjugations will have no real fixed points to get corresponding QA.
 Your final conjecture wrt a connection with CSC-transformations is of interest:
 For a quadrigon QA-Cu1 is invariant wrt CSC.
 Let * be the isoconjugation
 ... wrt QG-2P2 and QA-P4,
 ... swapping opposite QG-vertices.
 For points X on QA-Cu1 holds:
 ... $X^* = \text{QA-Tf2}(\text{CSC}(X))$,
 X, QA-Tf2(X), QA-P4 are collinear,
 X*, CSC(X), QA-P4 are collinear,
 X, X*, QG-P1 are collinear,
 CSC(X), QA-Tf2(X), QG-P1 are collinear.
 I hope, the attached file will appear.
 Best regards Eckart



2018-02-18.pdf

Message: #2891
Date: 21/2/2018 12:12:24
From: bernard.keizer
Subject: New sight of QA-Cu1

Dear Eckart,
Sorry, the answer is in your message !
As QA-P41 is the tangential of the 4 points DT vertices and
QA-P4 and your isoconjugation swaps QA-P41 and the remaining DT
vertice, the fixed points are the points having this DT vertice
as tangential.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2892
Date: 21/2/2018 11:52:04
From: eckart_schmidt@t-online.de
Subject: Special CB-points

Dear Bernard, dear Chris,

here some special Cayley-Bacharach ninth points
... for the six QL-points and a pair of QL-points,
... constructible with circumconics
... .. of the QL-triangles through the pair of QL-points.
Wrt QL-2P1: CB-point = infinity point of QL-L1,
... wrt QL-2P2: CB-point = QL-P1,
... wrt QL-2P3: see QFG#2887,
... wrt QL-2P4: CB-point = QL-P5.

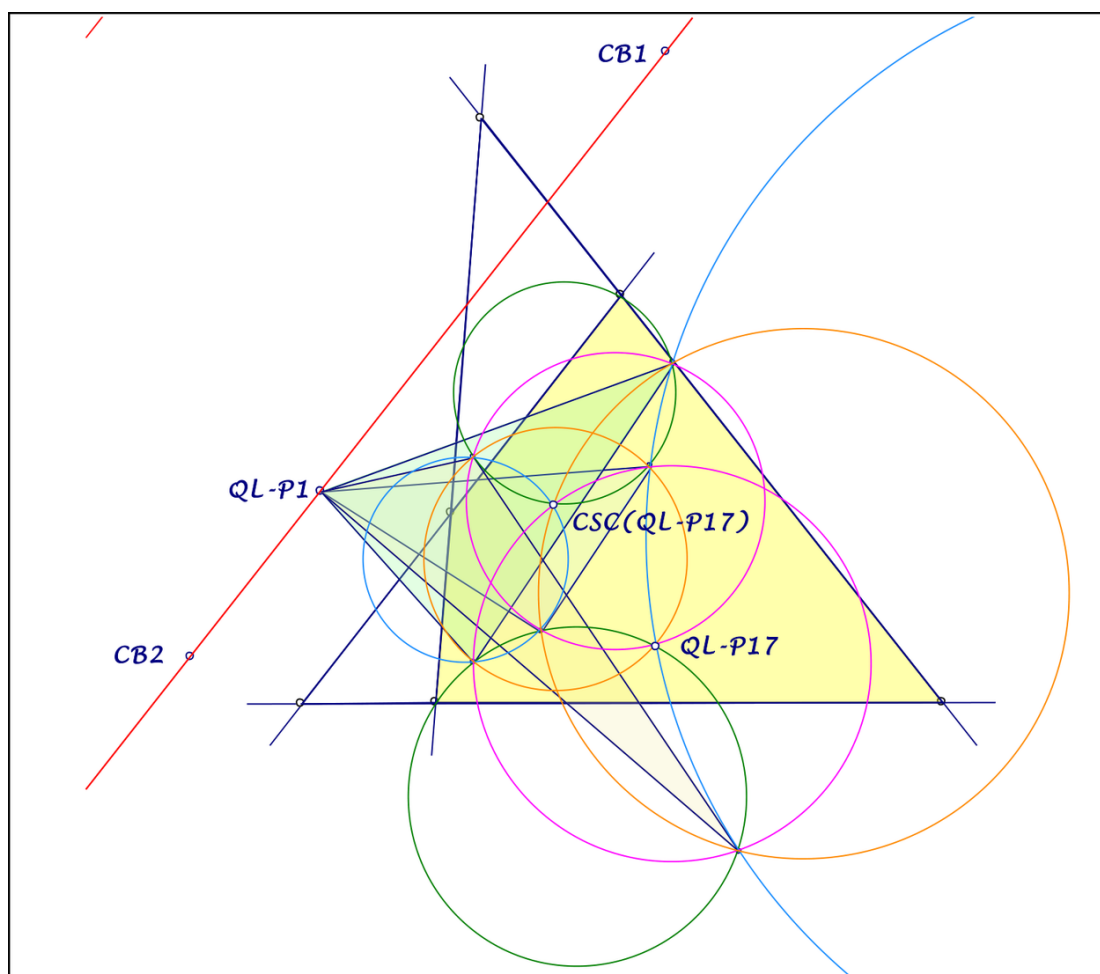
Best regards Eckart

PS: Excuse the three identical messages 2886, 2889, 2890,
but I sent them in reverse order on 18. and 19.02.
I don't know the reason for this confusion nor the missing
attachments.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2893
Date: 22/2/2018 9:40:08
From: eckart_schmidt@t-online.de
Subject: QL-P17

Dear Bernard, dear Chris,
consider a QL with its three QG, interpreted as QA,
... then we get three Miquel triangles QA-Tr2,
... one common vertex QL-P1 and three pairs of CSC-partners.
... The eight circles through one point out of each pair
... are pairwise CSC-images
... with two further quadruple intersections,
... which are QL-P17 and CSC(QL-P17).
The Cayley-Bacharach ninth points
... CB1 of the eight quadruple intersections
... of the eight circles
... and on the other hand CB2 of QL-P17, CSC(QL-P17)
... and the six QL-points
... are collinear with QL-P1 (see attached file, if possible).
Best regards Eckart



2018-02-23.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2894
Date: 23/2/2018 2:44:45
From: eckart_schmidt@t-online.de
Subject: New sight of QA-Cu1

Dear Bernard,

wrt #2891: I think , that QG-P1 on the closed part of QA-Cu1
... cannot be a real tangential of other QA-Cu1-points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2895

Date: 24/2/2018 9:37:59

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

9. Harmonic Centers

In this paragraph we will combine the notions of Quadrilateral, Quadrangle and Quadrigon focusing on harmonic properties.

QG-P12

(<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p12>):
Inscribed Harmonic Conic Center

Transform the reference Quadrigon into a Square through a projective transformation.

Draw the inscribed circle into this square.

Transform this Square and the inscribed circle back into the Reference Quadrigon. The inscribed circle will be transformed into a conic inscribed in the Reference Quadrigon.

QG-P12 is the center of this conic.

For a construction of the conic see QG-Co1

(<http://www.chrisvantienhoven.nl/qg-items/qg-conics/qg-co1>)
Knowing the conic its center easily can be constructed.

QA-P16

(<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p16>):
QA-Harmonic Center

Earlier mentioned was that three Quadrigons can be spanned in a Quadrangle (see paragraph 3 and see the description at QG-P1 (<http://www.chrisvantienhoven.nl/qg-items/qg-points/qg-p1>)). They are called the Component Quadrigons of a Quadrangle (just like there are 4 Component Triangles in a Quadrangle). See also QA-3QG1 (<http://www.chrisvantienhoven.nl/qa-items/qa-components/qa-3qg1>).

When we draw the 3 versions of QG-P12 in these Quadrigons we have a triangle formed of these special points. This triangle is perspective with the Diagonal Triangle of the Reference Quadrangle and the perspector is QA-P16. The coordinates of QA-P16 are surprisingly simple and it has many connections with many other points.

QL-P13

(<http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p13>):
QL-Harmonic Center

In the same way as there are 3 Component Quadrigons in a Quadrangle there also are 3 Component Quadrigons in a Quadrilateral. See QL-3QG1 (<http://www.chrisvantienhoven.nl/ql-items/ql-components/ql-3qg1>).

When we draw the 3 versions of QG-P12 in these Quadrigons we have a triangle formed of these special points. This triangle is perspective with the Diagonal Triangle of the Reference Quadrilateral and the perspector is QL-P13. The coordinates of QL-P13 are surprisingly simple and it has many connections with many other points.*

Combining Quadrangle points and Quadrilateral points in a Quadrangle

We saw that in a Quadrangle as well as in a Quadrilateral three Quadrigons can be spanned.

Conversely a Quadrigon can be treated as a Quadrangle as well as a Quadrilateral. In the other direction there is only one Quadrangle/Quadrilateral that can be laid over the Reference Quadrigon. Therefore we can draw a Quadrangle point as well as a Quadrilateral point in a Quadrigon. As it will appear -in this environment- they even will have special relationships with specific Quadrigon Points.

QG-L2 (<http://www.chrisvantienhoven.nl/qg-items/qg-lines/qg-l2>): The Harmonic Line.

Now that we have found similar "harmonic" points in a Quadrangle as well as in a Quadrilateral we can combine them in a Quadrigon. When we draw both harmonic points in a Quadrigon they are collinear with the Diagonal Crosspoint.

The line that connects these points also is the radical axis of the circumcircles of both the Diagonal Triangle of the Quadrangle as well as the Diagonal Triangle of the Quadrilateral that can be "laid over" the Reference Quadrigon. See picture of QG-L2 for clearing up this item.

There are more points lying on this line and many more interesting properties related to this line. And again the equation of this line is very simple.

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2896
Date: 25/2/2018 10:38:43
From: minhntenladuong
Subject: A property of QL-Tf5

Dear Geometers,

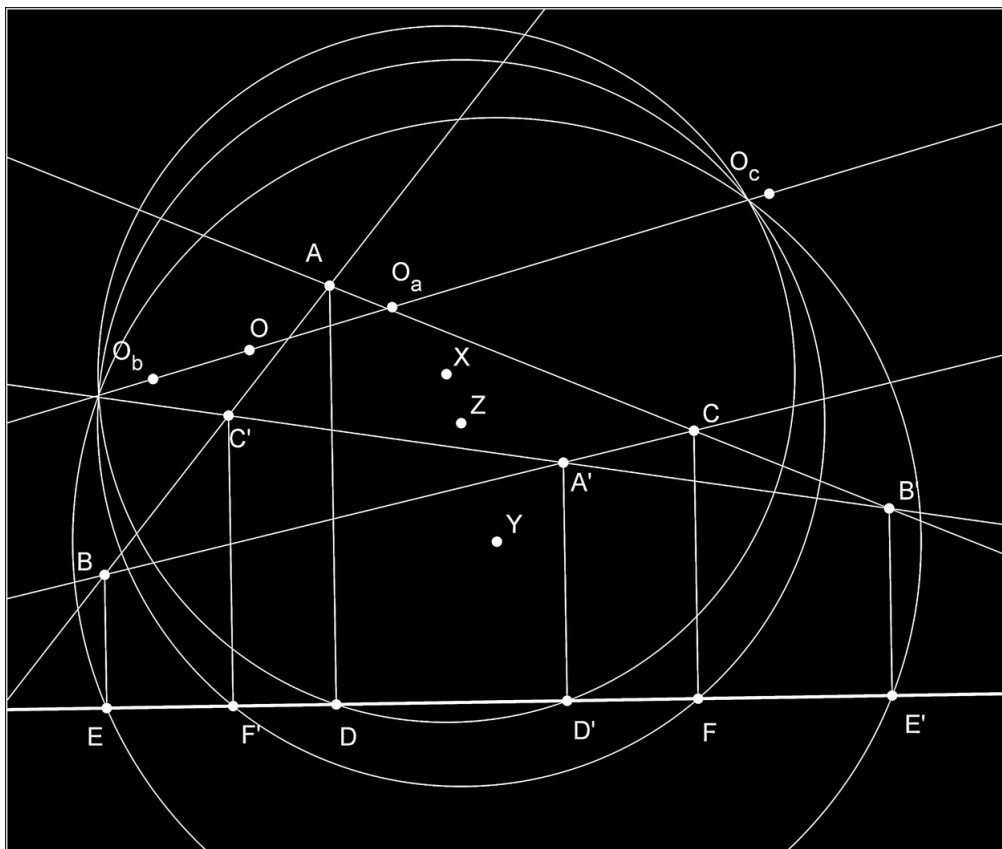
Today I have just remembered a generalization for a property of Steiner that I have found a while before:

Let consider the quadrilateral $(L1, L2, L3, L4)$ and an arbitrary line L .

Three following circles are coaxial: the first is centered at midpoint of $(P12P34)$ and passes through the orthogonal projections of $P12, P34$ on L ; the second is centered at midpoint of $(P23P41)$ and passes through the orthogonal projections of $P23, P41$ on L ; the third is centered at midpoint of $(P24P31)$ and passes through the orthogonal projections of $P24, P31$ on L . And their radical axis is $QL-Tf5(L)$.

When $L = L1$ or $L2$ or $L3$ or $L4$, we obtain the famous Gauss-Bodenmiller's theorem.

Best regards,
Ngo Quang Duong



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Message: #2897

Date: 25/2/2018 3:59:32

From: eckart_schmidt@t-online.de

Subject: Cayley-Bacharach transformation for a QL

Dear Bernard, dear Chris,

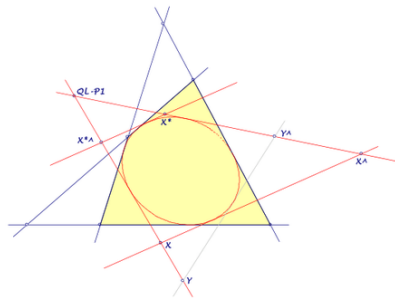
attached another aspect of the Cayley-Bacharach ninth point,
used for a transformation, which is already mentioned in #2884.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

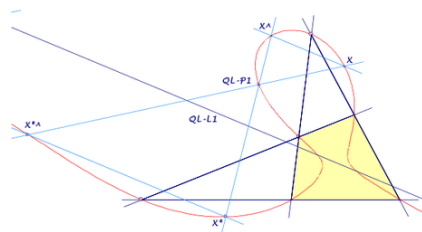
Cayley-Bacharach Transformation for a QL

Here a transformation for quadrilaterals is researched, mapping a point X to the Cayley-Bacharach ninth point of X , $QL-P1$ and the six QL -points.

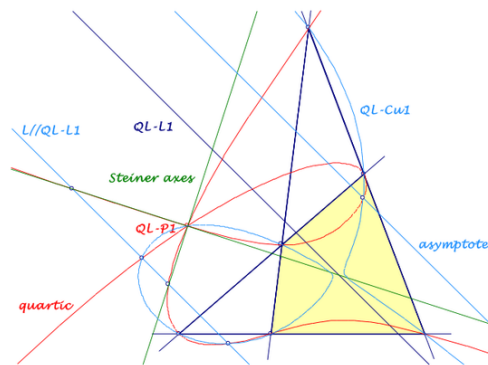


We consider two transformations:
 ... $X \rightarrow X^* = CSC(X) = QL-Tf2(X)$,
 ... $X \rightarrow X^\wedge =$ Cayley-Bacharach ninth point
 ... of X , $QL-P1$ and the six QL -points,
 ... constructible with circumconics
 ... for the QL -triangles through X and $QL-P1$.

- X^* , X^\wedge , $QL-P1$ are collinear,
 X , $X^{*A} = X^{\wedge*}$, $QL-P1$ are collinear,
 $X.X^\wedge$ and $X^*.X^{*A}$ are parallel.
- For X on the Steiner axes (see $QL-Tf1$) holds $X = X^\wedge$,
 ... for the points $QL-2P3$ is $X = X^* = X^\wedge = X^{*A}$.
- For points Y on $X.QL-P1$ the lines $Y.Y^\wedge$ envelope a QL -inscribed conic, tangent also to $X.X^\wedge$, $X^\wedge.X^*$, $X^*.X^{*A}$, $X^{*A}.X$.



- For points X on the cubic $QL-Cu1$
 ... X^* , X^\wedge and $X^{*\wedge}$ also on the cubic,
 ... $X.X^\wedge$ and $X^*.X^{*\wedge}$ equidistant parallel wrt $QL-L1$.
- The CB -transformation $^\wedge$ maps lines L to quartics
 ... with sometimes triple point in $QL-P1$,
 ... through the six QL -points
 ... and the intersections of L and the Steiner axes.
- Parallels L wrt $QL-L1$ intersect its quartic
 ... in two points on the Steiner axes
 ... with the property $X = X^\wedge$,
 ... and two points on $QL-Cu1$,
 ... which are partner wrt $^\wedge$.
 The intersection of $QL-Cu1$ and its asymptote
 ... is a further point of this quartic.



- $QL-Cu1$ is locus for points X with $X.X^\wedge$ parallel $QL-L1$.

Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2018-02-25.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2898
Date: 25/2/2018 5:58:30
From: chris.vantienhoven
Subject: A property of QL-Tf5

Dear Ngo Quang Duong,

Very nice property of QL-Tf5. I made notice of it in EQF at QL-Tf5.

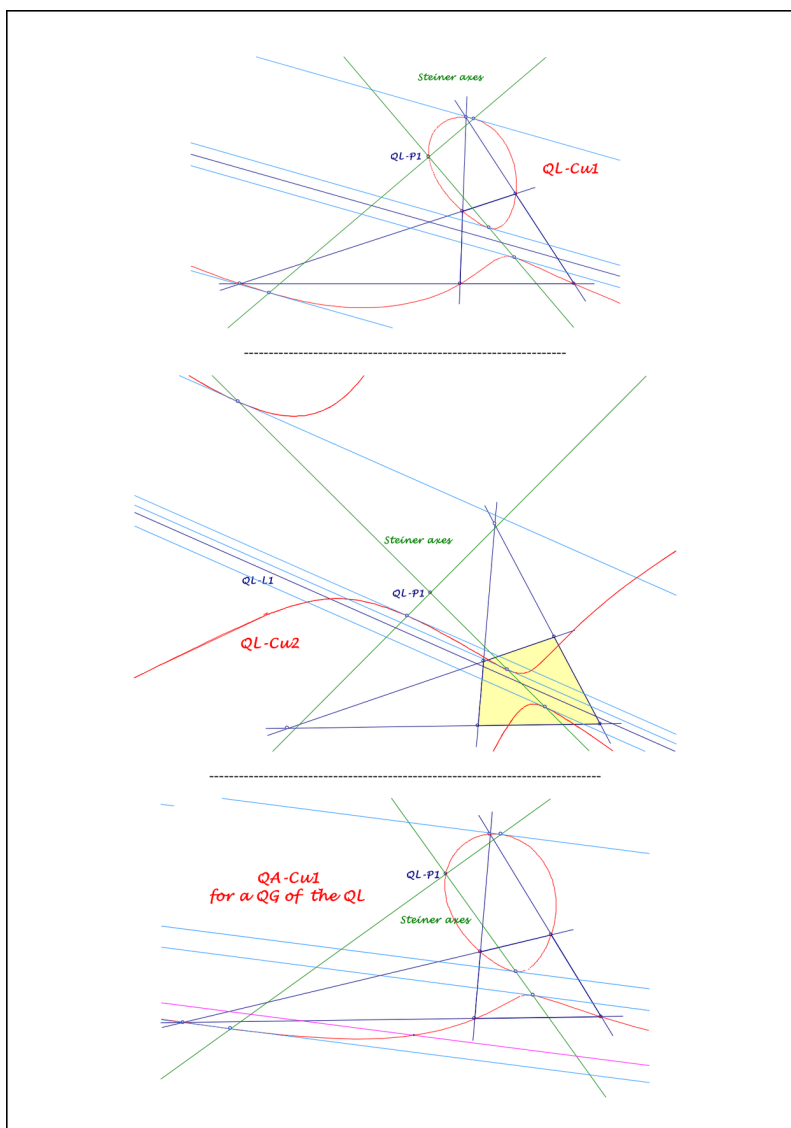
QL-Tf5 is a so far less researched and underestimated QL-Transformation !

Thanks. Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2899
Date: 26/2/2018 8:54:16
From: eckart_schmidt@t-online.de
Subject: Steiner axes

Dear Bernard, dear Chris,
what about this observation (see attached file)?
The Steiner axes intersect the cubics QL-Cu1 and QL-Cu2
... in points with tangents parallel QL-L1.
The Steiner axes intersect the cubics QA-Cu1 for the QG-versions
of QL
... in points with tangents parallel to the asymptote of QA-Cu1.
Perhaps the background: For points X on the Steiner axes
the Cayley-Bacharach ninth point for X, QL-P1 and the six
QL-points is X again.
Best regards Eckart



2018-02-26.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2900
Date: 26/2/2018 11:33:48
From: bernard.keizer
Subject: Steiner axes

Dear Eckart,

This is a special case of a more general property of the cubic stelloïd !

Any line through 2 CSC partners of QL-Cu1 (tangent to the cayleyan) cuts QL-Cu1 in a 3rd point and QL-Cu2 in 3 points (all the points are not necessary real) ; the tangents to QL-Cu2 in these 3 points and the tangents to QL-Cu1 in the 2 CSC partners concur in a point, which is the tangential of these 2 CSC partners on QL-Cu1 and therefore the CSC partner of the 3rd intersection on QL-Cu1.

The 4 points you describe on QL-Cu1 bicursal (there are only 2 real points if QL-Cu1 is monocursal) have the same tangential, which is the CSC partner of QL-P1, id est the infinity point of the Newton Line.

For any point of the hessian QL-Cu1 the polar conic wrt QL-Cu2 is a rectangular hyperbola (like all the polar conics) degenerated in 2 perpendicular lines intersecting in the CSC partner of the point ...

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2901
Date: 26/2/2018 8:49:36
From: eckart_schmidt@t-online.de
Subject: Steiner axes

Dear Bernard,

the argumentation with two CSC-partners is known to me
... and clears the property for QL-Cu1.
New for me (or not present),
... that the tangentials of two CSC-partners X, Y on QL-Cu1
... and the tangentials of the intersections of XY and QL-Cu2
... give the same point on QL-Cu1,
... which is the CSC-partner of the 3rd intersection of XY and
QL-Cu1.

This clears the property for QL-Cu2, thanks!
And wrt QA-Cu1 for a QG of the QL:
All quadrilaterals with the same Miquel triangle
... have the same Steiner axes
... and the same intersections with its QA-Cu1,
... which are the in-/excenters of the Miquel triangle,
... fixed points of the isogonal conjugation.

The tangents in these points
... are parallel to the asymptote of QA-Cu1,
... which has the direction of isogonal conjugated on QA-Cu1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2902
Date: 27/2/2018 9:10:32
From: eckart_schmidt@t-online.de
Subject: QL-Strophoid and QL-Ortho-Hyperbola

Dear Bernard, dear Chris,

if you are interested, I invite you to a short excursion in
QL-geometry:

Let us start with a line L
and its points P (see attached file).
Consider the pedal points of P on $CSC(X).QL-P1$,
... which give a *strophoid* for a line Li ,
... .. tangent in $QL-P1$ to the circle $CSC(L)$
... .. with fixed point $QL-P1$
... .. and pole in the pedal point of $QL-P1$ on L .

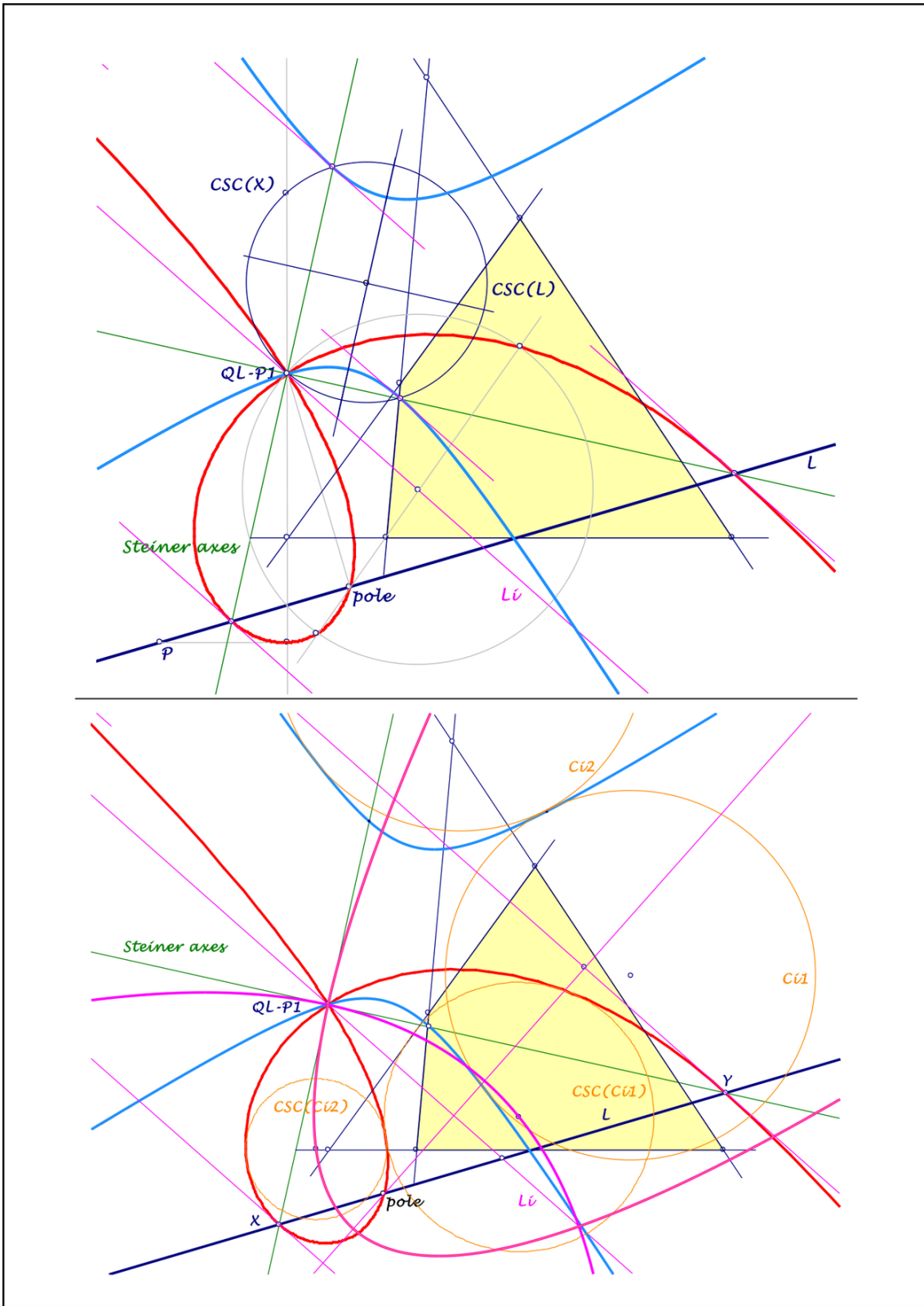
The intersections X, Y of L and the Steiner axes
... are points on the strophoid
... and centers of the inversion circles through $QL-P1$.

The strophoid has orthogonal tangents in $QL-P1$
... and Li -equidistant parallel tangents in the intersections $X,$
 Y with L .

The CSC -image of the strophoid
... is an *orthogonal hyperbola*,
... centered in the center of the circle $CSC(L)$,
... with axes parallel to the Steiner axes,
... bearing $QL-P1$ with tangent parallel L
... and the intersections of the Steiner axes and the circle
 $CSC(L)$
... with tangents parallel to Li .

This configuration can still be studied, for example:
Circles, double tangent to the ortho-hyperbola,
... envelope with their CSC -circles the strophoid.
These CSC -circles are centered on two *parabolas*
... through $QL-P1$ with the Steiner axes as tangents,
... with common focus in the pole of the strophoid,
... common axis orthogonal to Li
... and the tangents at the strophoid in X, Y as directrices.

Best regards Eckart



2018-02-27.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #2903
Date: 28/2/2018 11:56:43
From: eckart_schmidt@t-online.de
Subject: nL-n-Ci1

Dear Chris,

I have difficulties, to read your EPG-chapters:
Wrt nL-n-Ci1 you write:

"When $n=4$, then nL-n-P3 = Quadrilateral Circumcircle QL-Ci1."

I think you mean:

When $n = 4$, then nL-n-Ci1 = Miquel Circle QL-Ci3.

Perhaps you can add:

When $n = 5$, then nL-n-Ci1 = circumcircle of the concyclic five QL-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2904
Date: 28/2/2018 12:19:27
From: chris.vantienhoven
Subject: nL-n-Ci1

Dear Eckart,

Thanks for mentioning.
I made an adjustment right away.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2905
Date: 28/2/2018 3:00:11
From: eckart_schmidt@t-online.de
Subject: nL-n-Ci2

Dear Chris,

I think, there is another error wrt nL-n-Ci2 for $n = 4$:

You wrote:

"When $n=4$, then $nL-n-Ci2 = QL-Ci2$."

The circle 4L-4-Ci2 is centered in QL-P30
and homothetic QL-Ci3 wrt QL-P2, ratio 1:3,
or homothetic QL-Ci3 wrt QL-P20, ratio 1:3.

This circle is not in EQF.

By the way: The three circles 4L-4-Ci2 for a quadrangle
... have a common point, also not in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2906
Date: 01/3/2018 12:10:12
From: eckart_schmidt@t-online.de
Subject: nL-n-Ci2

Dear Chris,

I just noticed, that there is a typo in my last message:
The circle 4L-4-Ci2 is centered in QL-P30
and homothetic QL-Ci3 wrt QL-P2, ratio 1:3,
or homothetic QL-Ci *4*
wrt QL-P20, ratio 1:3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2907
Date: 01/3/2018 3:47:05
From: eckart_schmidt@t-online.de
Subject: New aspect of 5P-s-Tf3

Dear Benedetto, dear Chris,

in EQF-Ref 16, Satz 12 there is a property of QA-P4,
... which leads to a transformation 5P-s-Tfx,
... mapping a line to a point,
... with direct connection to 5P-s-Tf3.

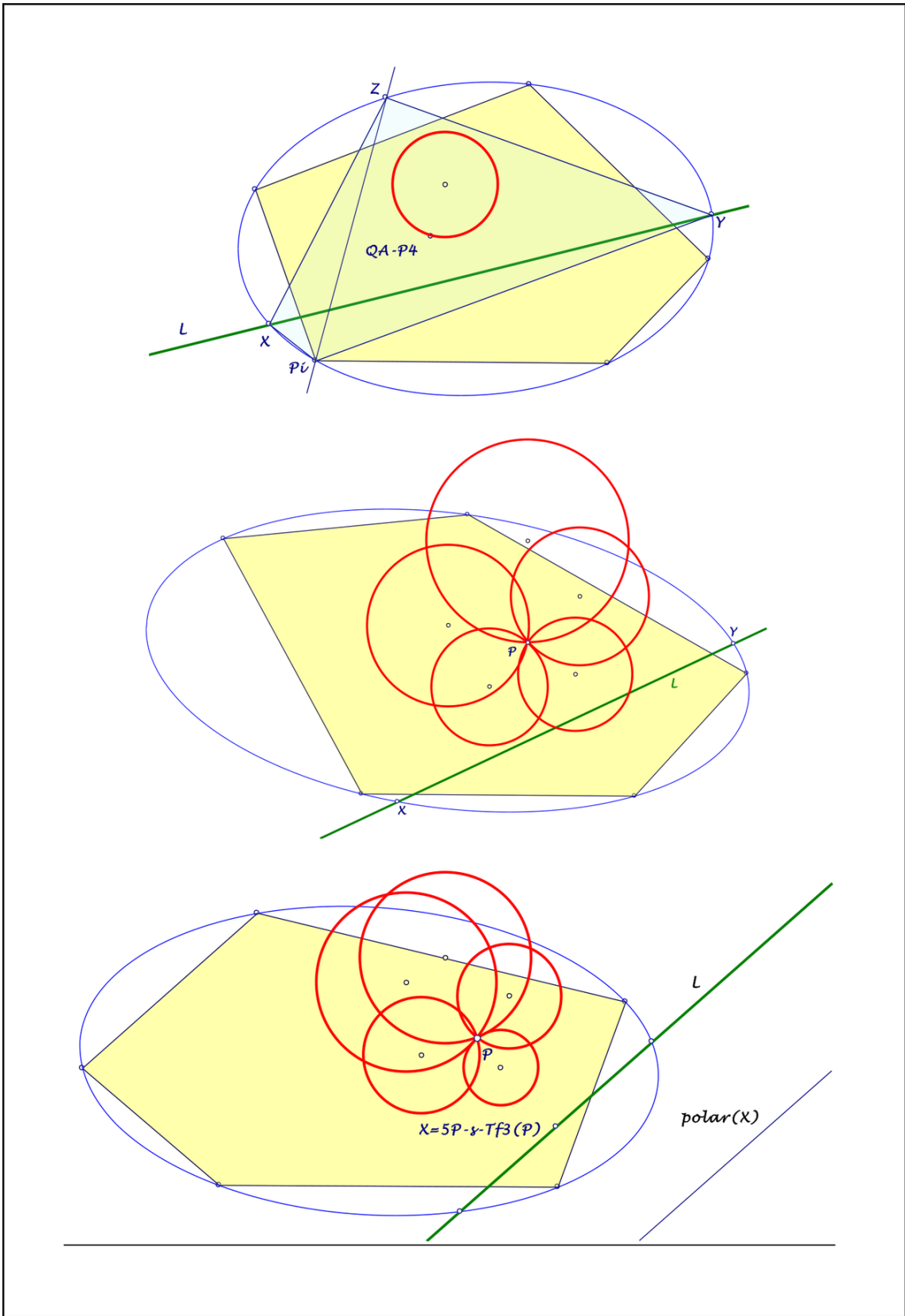
Consider a 5P with vertices P_i and circumconic 5P-s-Co1=Co
... and a line L, intersecting Co in X, Y.
(If there are no intersections, stretch the 5P from 5P-s-P1.)

Take a line pencil of a vertex P_i
... with 2nd Co-point Z of its lines.
The locus for the QA-P4-points of X, Y, Z, P_i is a circle.
There are five of these circles with a common point P,
... which shall be the image of the line L.

Properties of this transformation 5P-s-Tfx: $L \dashrightarrow P$
... shall be researched in another message.
Here the connection with 5P-s-Tf3 (in the first definition):
5P-s-Tf3-inv of a point X
... is 5P-s-Tfx of a parallel to the Co-polar of X through X.

Best regards Eckart

PS: I hope very much, that Chris doesn't rename 5P-s-Tf3 to its
inverse!



2018-03-01.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2908
Date: 02/3/2018 8:47:25
From: eckart_schmidt@t-online.de
Subject: New aspect of 5P-s-Tf3

Dear Benedetto, dear Chris,

I have to correct the cited reference in #2907: EQF-Ref 16, Satz 16.

Best regards Eckart

PS: Dear Benedetto, I am very relieved not renaming 5P-s-Tf3, thanks!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2909

Date: 02/3/2018 1:54:55

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

10. Duals wrt a Quadrangle and Duals wrt a Quadrilateral

There is a configuration where a Quadrangle and a Quadrilateral harmonically fit together.

It was proposed by Tsihong Lau and later on further developed by members of the Yahoo Quadri-Figures Group

(<https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/info>).

Let $P_1.P_2.P_3.P_4$ be the Reference Quadrangle.

Let L_1 be the Trilinear Polar of P_1 wrt $P_2.P_3.P_4$.

Let L_2 be the Trilinear Polar of P_2 wrt $P_3.P_4.P_1$.

Let L_3 be the Trilinear Polar of P_3 wrt $P_4.P_1.P_2$.

Let L_4 be the Trilinear Polar of P_4 wrt $P_1.P_2.P_3$.

In this way Quadrilateral $L_1.L_2.L_3.L_4$ is constructed.

Now it appears that (easy to prove algebraically):

P_1 is the Trilinear Pole of L_1 wrt $L_2.L_3.L_4$.

P_2 is the Trilinear Pole of L_2 wrt $L_3.L_4.L_1$.

P_3 is the Trilinear Pole of L_3 wrt $L_4.L_1.L_2$.

P_4 is the Trilinear Pole of L_4 wrt $L_1.L_2.L_3$.

(For a definition of Trilinear Pole and Trilinear Polar wrt a triangle see Mathworld.

The Trilinear Pole is often called Tripole and the Trilinear Polar is often called Tripolar)

Note that L_i coincides with the line $M_{jk}.M_{kl}.M_{jl}$ of Paragraph 8. (Harmonic properties in a Quadrangle), where (i,j,k,l) are numbers with different values from $(1,2,3,4)$.

This entangled configuration of a Quadrangle and a Quadrilateral has several special properties.

1. Per definition the QA-reference points (vertices) corresponds to a QL-reference line and vice versa.

2. The related Quadrangle and Quadrilateral share the same Harmonic Center, meaning that QA-P16 and QL-P13 (see Paragraph 9) coincide for this entangled configuration

3. The related Quadrangle and Quadrilateral share the same Diagonal Triangle.

4. The correspondence of the QA-reference points and the QL-reference lines is defined by using trilinear poles/polars wrt the Component Triangles. However they can be substituted by using trilinear poles/polars wrt the Common QA-/QL-Diagonal Triangle, because they coincide.

The use of the *Trilinear Poles/Polars* in this configuration lead to the definition of *Quadrilinear Poles/Polars* wrt a Quadrangle

See:

QA-Tf10 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf10>)

and

QA-Tf11 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf11>)

as well as a Quadrilateral

See:

QL-Tf10 (<http://www.chrisvantienhoven.nl/ql-items/ql-transformations/ql-tf10>)

and

QL-Tf11 (<http://www.chrisvantienhoven.nl/ql-items/ql-transformations/ql-tf11>).

Now there is a dual for each point and a dual for each line wrt a Quadrangle as well as a Quadrilateral. For more information on this subject,

See

QA-8 (<http://www.chrisvantienhoven.nl/qa-items/qa-geninf/qa-8>)

QL-8 (<http://www.chrisvantienhoven.nl/qa-items/qa-geninf/qa-8>).

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2910
Date: 02/3/2018 3:34:20
From: eckart_schmidt@t-online.de
Subject: New aspect of 5P-sTf3

Dear Benedetto, dear Chris,

in #2907 I described a transformation 5P-s-Tfx,
... which maps lines L to points P
... and is directly connected with 5P-s-Tf3:
If L has two intersections X, Y with 5P-s-Co1,
... 5P-s-Tfx(L) is 5P-s-Tf3-inv of the midpoint of XY.
If L has no two intersections with 5P-s-Co1,
stretch 5P-s-Co1 from 5P-s-P1, to get two intersections.
Some properties of 5P-s-Tfx:
... Lines L through 5P-s-P1 are mapped to 5P-s-P1.
... 5P-s-Tfx(L) lies on the bisector of XY.
... 5P-s-Tfx(L) for a tangent L at 5P-s-Co1
... ... is 5P-s-Tf3-inv of the contact point.
... 5P-s-Tfx for a line pencil of a point Q
gives a conic Co(Q)
... ... through 5P-s-P1 and 5P-s-Tf3-inv(Q),
... ... centered in the midpoint,
... ... homothetic 5P-s-Co1.

Best regards Eckart

PS: I already mentioned that 5P-s-Tf3 for a stretched 5P (from 5P-s-P1) gives the same image?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

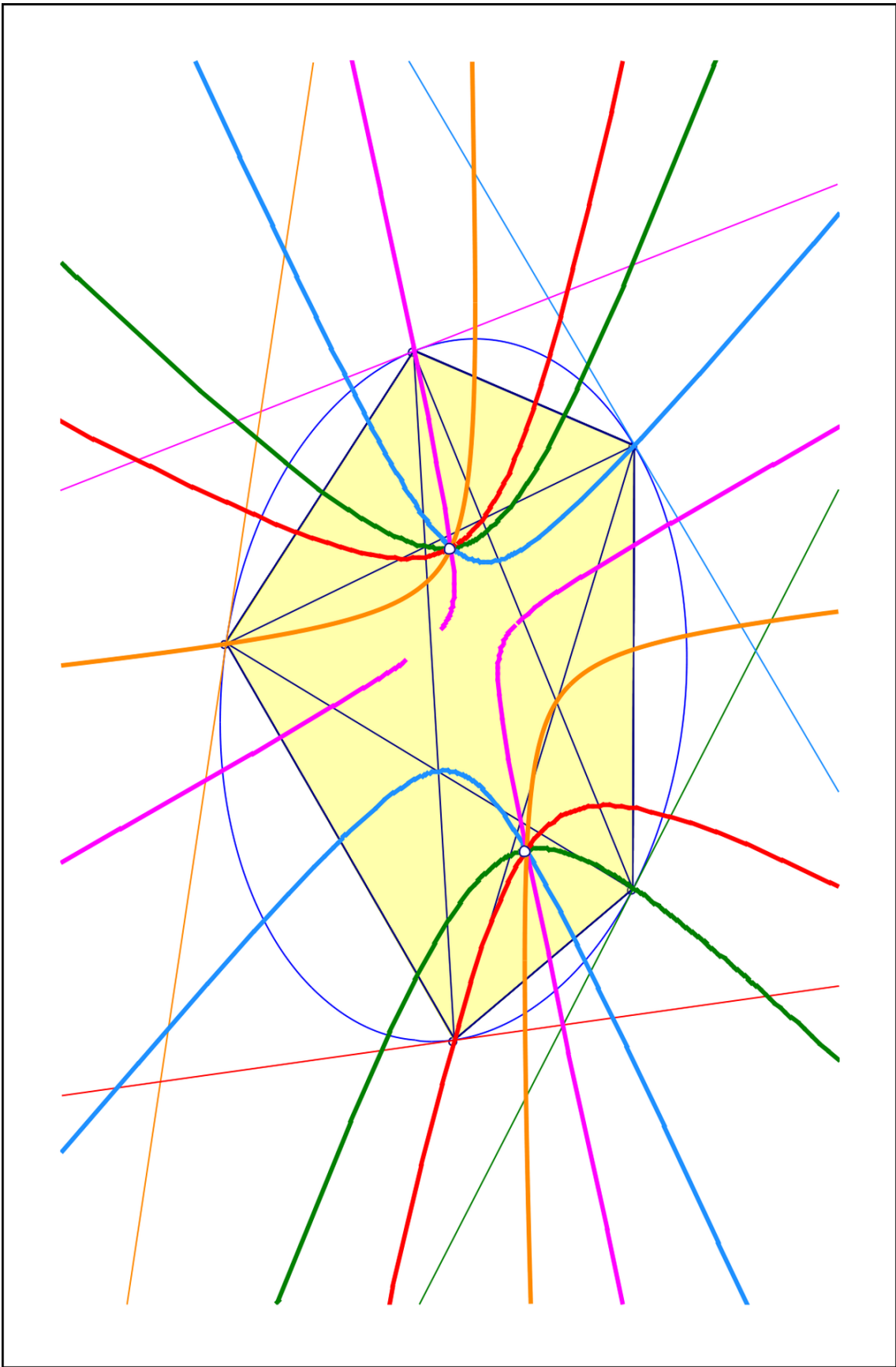
Message: #2911
Date: 03/3/2018 1:19:27
From: chris.vantienhoven
Subject: nL-n-Ci2

Dear Eckart,
Thanks for your correction.
I adjusted the description of nL-n-Ci1 in EQF accordingly.
By the way you denote the circle in a 4-Line with 4L-4-Ci2. This should be 4L-n-Ci2.
The infix -n- is just meant to denote the distinction between n-Points, e-Points, o-Points and s-Points. See the description of the neos-system in the introduction to polygon geometry (<https://www.chrisvantienhoven.nl/20-mathematics/encyclopedia-of-poly-figures/373-epg-1?highlight=WyJuZW9zLXN5c3RlbSJD>). The prefix n- can be substituted with natural numbers indeed.
Then I looked to the common QA-point of the 3 QA-versions of Morley's Second Circles you found. The coordinates are pretty long and there are no incidences with known QA-points and known QA-curves.
Best regards,
Chris
p.s. I am sorry I have so little time to study all your messages.
I am very busy with all kinds of things and wish I had more time to honor them.
Furthermore I probably will be, just like last year, on a long walking holiday in April and part of May this year. Especially then I will hardly have possibilities to study material.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2912
Date: 04/3/2018 9:51:54
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px

Dear all,
what about this pair of points (see attached file)?
Consider tangents T_i at 5P-s-Co1 in the vertices P_i of a 5P
... and the conics QA-Tf2(T_i) wrt the remaining QA.
These five conics have two common points.
Best regards Eckart



2018-03-03.pdf

Message: #2913
Date: 04/3/2018 7:33:50
From: Benedetto Scimemi
Subject: New aspect of 5P-s-Tf3

Dear Eckart and friends,

since Chris is very busy with preparing his new Encyclopedia, I will answer to Eckart's recent messages regarding "New aspects of 5P-s-Tf3". First of all, let me confess that I do not feel at ease when using two different names, 5P-s-Tf3 and $C0-Tf3^{(-1)}$, for what I believe to be the same mapping, and using two very similar names, Co-Tf3 and 5P-s-Tf3, for different mappings! But, after all, this is not important. It seems more interesting to introduce some hierarchy in the great amount of informations regarding this subject. Let me try to do it. I hope the soon coming page on C0-f3 will do the rest.

Here is my re-interpretation of Eckart's construction. I will try to keep his symbols.

- 1) Take any segment XY of a line r and any point P1. Then the locus of QA-P2 of the 4-ng XYP1 Z, when Z moves (at random) in the plane, is the ninepoint circle of the 3-ng XYP1. It passes through the midpoint $M=(X+Y)/2$.
- 2) Replace P1 by P2 and do the same construction. A new circle appears, still passing through $M=(X+Y)/2$. So far, no conic C0, no f3.
- 3) Now take any conic C0 through X, Y, P1, P2. Repeat steps 1) and 2), but let Z run on C0 (rather than at random). You will find the same circles as before, both passing through M.
- 4) Apply the similarity C0-f3 to these circles. You will get two circles passing through $N= C0-f3(M)$. However, by the very special property of C0-f3: QA-P2 \rightarrow QAP4, these circles will be the loci of QA-P4, when Z runs on C0, namely Eckart's circles! In fact, his line-point mapping is $r \rightarrow N$. (central homotheties of C0 may be necessary).

Now my point is: where is the pentagon? why should we collect these facts under a 5P page rather than a C0-page?. But, again, this is not so important. More interesting is Eckart's last comment

5P-s-Tf3-inv of a point X ... is 5P-s-Tfx of a parallel to the Co-polar of X through X.

In order to justify this, I found it useful to recall the following elementary Lemma: Given any conic C_0 centered in Z , for any point P the line ZP cuts in its midpoint the chord XY , intercepted on C_0 by the $*C_0$ -polar* of P . Reality of intercepts can be treated again by applying Z -homotheties of C_0 (this Lemma being invariant for Z -centered hom). I skip details.

What I called the Chord Lemma states precisely: N lies on the perp. bisector of XY . I think I found track of this non trivial fact in Eckart's second message, which I believe to have read quickly yesterday, But today I cannot find it, Also: I cannot recover #2907: EQF-Ref 16, Satz 16. Could you please, Eckart, send both of them to me again? Unfortunately, I am a very poor PC user. I apologize.

Best regards Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2914
Date: 05/3/2018 10:04:45
From: eckart_schmidt@t-online.de
Subject: New aspect of 5P-s-Tf3

Dear Benedetto,

wrt your remark:

Now my point is:

where is the pentagon? why should we collect these facts under a 5P page rather than a C0-page?

You are right, there are aspects and facts, which belong to a separate C0-page.

I used up to now the name 5P-s-Tf3,

... for I got familiar with this transformation wrt 5Ps

... and there was a concret definition in EPG

... useful for someone, to participate the discussion.

A C0-page has its own reasons for a transformation Co-Tf3,

... and a remark, that Co-Tf3 is the inverse of 5P-s-Tf3 for 5Ps, will clear all doubts.

If the new C0-page is published,

... I shall use the new nomination.

With interest I have reproduced your construction,

... which leads to the upper remark.

Am I right, that you finally asked for the messages 2907 and 2910?

Message 2907:

- > Dear Benedetto, dear Chris,
- > in EQF-Ref 16, Satz 12 there is a property of QA-P4,
- > ... which leads to a transformation 5P-s-Tfx,
- > ... mapping a line to a point,
- > ... with direct connection to 5P-s-Tf3.
- > Consider a 5P with vertices P_i and circumconic $5P-s-Co1=Co$
- > ... and a line L , intersecting Co in X, Y .
- > (If there are no intersections, stretch the 5P from $5P-s-P1$.)
- > Take a line pencil of a vertex P_i
- > ... with 2nd Co -point Z of its lines.
- > The locus for the QA-P4-points of X, Y, Z, P_i is a circle.
- > There are five of these circles with a common point P ,
- > ... which shall be the image of the line L .
- > Properties of this transformation 5P-s-Tfx: $L \dashrightarrow P$
- > ... shall be researched in another message.
- > Here the connection with 5P-s-Tf3 (in the first definition):
- > 5P-s-Tf3-inv of a point X
- > ... is 5P-s-Tfx of a parallel to the Co -polar of X through X .
- > Best regards Eckart
- > PS: I hope very much, that Chris doesn't rename 5P-s-Tf3 to its inverse!

> Message 2910
> Dear Benedetto, dear Chris,
> in #2907 I described a transformation $5P-s-Tfx$,
> ... which maps lines L to points P
> ... and is directly connected with $5P-s-Tf3$:
> If L has two intersections X, Y with $5P-s-Co1$,
> ... $5P-s-Tfx(L)$ is $5P-s-Tf3$ -inv of the midpoint of XY .
> If L has no two intersections with $5P-s-Co1$,
> stretch $5P-s-Co1$ from $5P-s-P1$, to get two intersections.
> Some properties of $5P-s-Tfx$:
> ... Lines L through $5P-s-P1$ are mapped to $5P-s-P1$.
> ... $5P-s-Tfx(L)$ lies on the bisector of XY .
> ... $5P-s-Tfx(L)$ for a tangent L at $5P-s-Co1$
> is $5P-s-Tf3$ -inv of the contact point.
> ... $5P-s-Tfx$ for a line pencil of a point Q
> gives a conic $Co(Q)$
> through $5P-s-P1$ and $5P-s-Tf3$ -inv(Q),
> centered in the midpoint,
> homothetic $5P-s-Co1$.
> Best regards Eckart
> PS: I already mentioned that $5P-s-Tf3$ for a stretched $5P$ (from
 $5P-s-P1$) gives the same image?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2915
Date: 05/3/2018 10:54:49
From: eckart_schmidt@t-online.de
Subject: QA-P4, QA-P2

Dear Benedetto, dear Chris,

Stärks paper (EQF-Ref 16, Satz 14, 15)
... is the background for the following properties:
Let Co_1 be the circumconic of QA through a given point P .
Let Co_2 be circumconics of QA-Tr1 and P .

The intersections of Co_1 and Co_2 give QA
... with QA-P4 on a circle Ci_1
... and QA-P2 on a circle Ci_2 ,
... which is the 5P-s-Tf3 image of Ci_1
... wrt the QA-vertices and P .
Varying P on a given QA-circumconic Co_1 ,
... the circles Ci_1 as well as Ci_2 have a common point.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2916
Date: 05/3/2018 12:20:05
From: eckart_schmidt@t-online.de
Subject: More on Co-Tf3

Dear Benedetto,

is the following property a right interpretation?
Let Co be a conic and P any point:
... Co -Tf3(P) is QA-P4 of the intersections
... of Co and two orthogonal lines through P .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2917
Date: 05/3/2018 6:33:01
From: Benedetto Scimemi
Subject: New aspect of 5P-s-Tf3

Dear Eckart, thank you very much for re-sending me your # 2910, which had disappeared.

Let me go back to our recent conversations and try to recollect the main points:

My reaction to your # 2907 was #2913 and yours to the latter was #2914.

As for #2910, the relevant remark is "5P-s-Tfx(L) lies on the bisector of XY" which is equivalent to my Chord Lemma described in #2863. I think the next notes are just consequences of homotheties / similarities properties.

As for #2915 I could not read Stärks but it seems to me that QA-Tr1

plays no role: any other 3 points would give similar results: as I described in #2013 the circles you see are nine-point circles (loci of QA-P2), appearing when three vertices of a QA are fixed and the fourth moves; their C0-f3 images are the corresponding loci of QA-P4.

Finally, I already answered privately to your #2019: yes, you are right.

Thanks again and best regards.
Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2918
Date: 06/3/2018 11:29:59
From: bernard.keizer
Subject: More on Co-Tf3

Dear Eckart,
I hardly follow the Co-Tf3, but I was intrigued by the fact that for a conic Co and a point P QA-P4 of the 4 intersections between Co and 2 perpendicular lines through P is a fixed point. I reproduced your construction, but I cannot give a synthetic proof of your property !
I give only remarks, maybe you will give an interpretation ...
If 2 sides of a QA are perpendicular, the QA-Cu1 of this QA is a QL-Cu1 for the QL formed by the 4 points and the 2 other DT vertices, which lie necessarily on the polar of P wrt Co.
Then P is at the same time the 3rd DT vertice of the QA and it's QA-P2.
QA-P1 and QA-P3 describe 2 conics similar to Co, QA-P4 is the CSC of P for the QL and the fact that it is fixed is remarkable, as QL-P1 and the inversion circle are variable !
Last, the Newton Line turns around the middle of PQA-P4, which is a fixed point and the asymptote of QA-Cu1 of the QA or QL-Cu1 of the QL is QA-P3QA-P4 and turn around the fixed QA-P4.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2919
Date: 06/3/2018 9:54:36
From: eckart_schmidt@t-online.de
Subject: More on Co-Tf3

Dear Bernard,

with interest I reproduced your remarks and agree without the last one.

But first, I think, a simpler view:

For a conic Co and a point P

... with two orthogonal lines through P

... and its intersections with Co

... we get a quadrigon with diagonal crosspoint P ,

... which we can consider as QA and QL .

Then $QA-Cu1$ and $QL-Cu1$ are the same.

Wrt your last remark:

$QA-P3.QA-P4$ is not the asymptote of $QA-Cu1$,

... but a parallel!

Perhaps of interest:

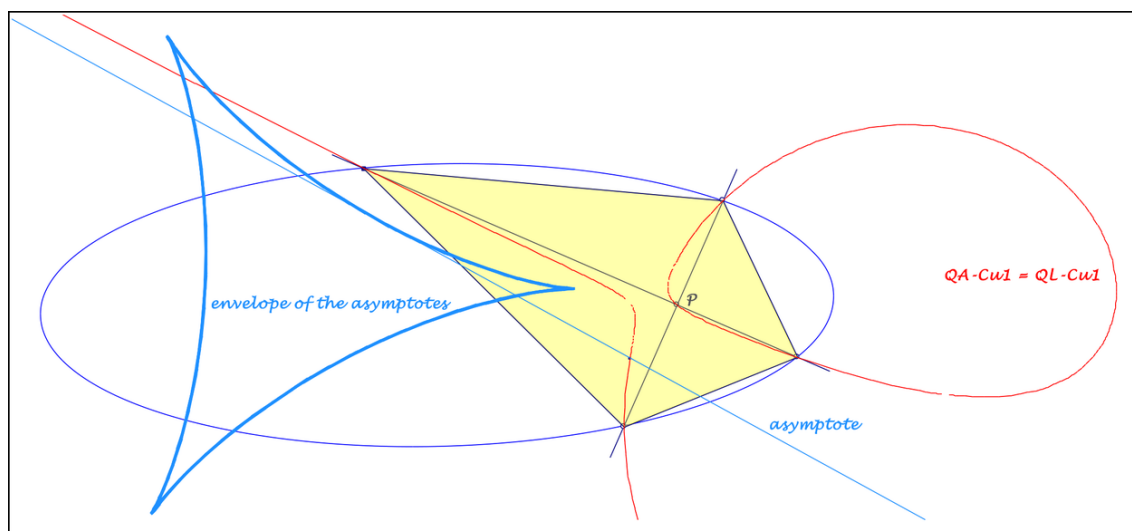
The locus of these asymptotes is a deltoid (see attached file).

I just work out the following aspect:

The considered quadrigons give the possibility for loci

... of all EQF-points for QG , QA , QL .

Best regards Eckart



2018-03-07.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2920
Date: 07/3/2018 11:12:46
From: bernard.keizer
Subject: More on Co-Tf3

Dear Eckart,
Sorry for my mistake !
Of course, you're right, the asymptote is not QA-P3QA-P4, but a parallel.
Interesting, your deltoïd as envelop of the asymptotes ; the axes of the parabolas are the symmetric of the asymptotes wrt the fixed point middle of QA-P2QA-P4 (on the Newton Line) and envelop the symmetric deltoïd.
The conics loci of QA-P1 and QA-P3 seem to be centered on a line through P.
The loci of QL-P1 and the fixed points of the CSC transformation QL-2P3 are 3 circles with aligned centers ; the locus of QL-P7 is also a circle.
For the rest, I will wait for your study !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2921
Date: 08/3/2018 8:55:21
From: eckart_schmidt@t-online.de
Subject: Co-geometry with special QA

Dear Bernard,

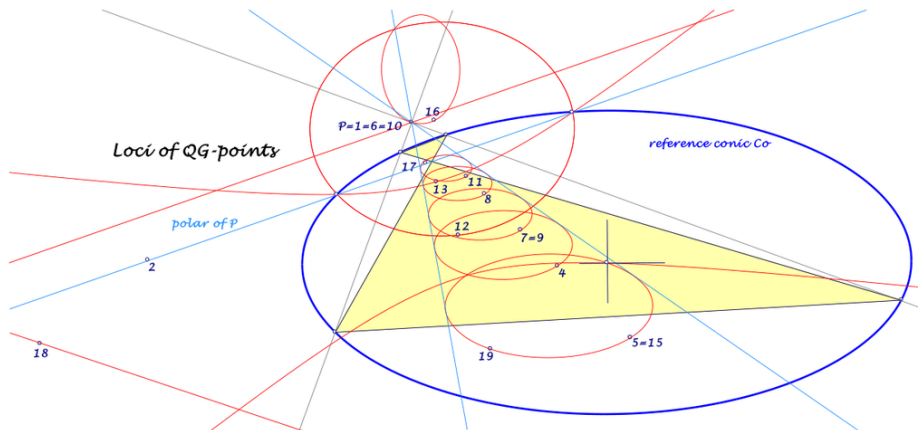
thanks for your examples in #2920.
The circles for QL-2P3 were new for me,
but the aligned center with the circle for QL-L1 cannot be.
Attached a lot of further observations, but without a concret background.

Best regards Eckart

Background for these notes is:
 Chris van Tienhoven:
 Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Conic-Geometry with special Quadrilaterals

Starting with a reference conic Co and a given point P , two orthogonal lines through P intersect Co in the vertices of a quadrilateral, which can also be seen as quadrangle and quadrilateral. The pencil of orthogonal lines through P gives loci for all EQF-points.



Loci of QG-points

The intersections of the reference conic Co and two orthogonal lines through a given point P give a quadrilateral with diagonal crosspoint P . The loci of QG -points for these quadrilaterals shall be researched.

Fixed points are

$$QG-P1 = QG-P6 = QG-P10 = QA-P2 = QL-P10 = P,$$

$$QG-P17 \text{ as pedal point of } P \text{ on its polar,}$$

$$QG-P19 = QA-P4 = Co-Tf3(P).$$

Lines as loci for

$$QG-P2 = QA-P29, QG-P3: \text{ polar of } P,$$

$$QG-P14: \text{ parallel to the polar of } P \text{ through } P,$$

$$QG-P18 = QA-P41: \text{ polar of } Co-Tf3(P).$$

Loci for

$$QG-P4, QG-P5 = QG-P15 = QA-P3,$$

$$QG-P7 = QG-P9 = QA-P1, QG-P8, QG-P11$$

are homothetic conics wrt P , similar to the reference conic. Their common tangents are tangents from P to a circle, which is the locus of $QL-P1$.

Especially for $QG-P5 = QG-P15$ the conic bears diametral the fixed point $QG-P19$ and the center of the reference conic. For $QG-P12, QG-P13$ the loci are conics through the intersections of Co and the polar of P . For $QG-P16$ the locus is a conic through P . Studying the constellation, there will be more properties.

Loci of QA-points

Some QA-points are already mentioned above:

$QA-P1, QA-P2, QA-P3, QA-P4, QA-P29, QA-P41$.

A further fixed point is $QA-P6$, midpoint of $P.QA-P4$.

Here the loci are gathered, which are lines:

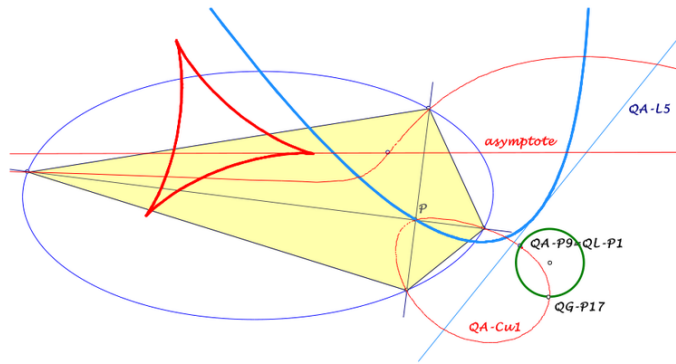
... for $QA-P12, QA-P24, QA-P36$: $P.QA-P4$,

... for $QA-P10, QA-P28, QA-P35$: parallels to the Co -polar of P ,

... for $QA-P11, QA-P13, QA-P23, QA-P30$, ...

The locus of $QA-P9 = QL-P1$ is a circle through $QG-P17$.

The lines $QA-L5$ envelope a parabola and the asymptotes of $QA-Cu1$ envelope a deltoid.



Loci of QL-points

The points $QL-P1$ and $QL-P10$ are already mentioned above.

Some points give lines:

... $QL-P8, QL-P9, QL-P11$: parallels to the polar of P wrt Co .

Some points give circles:

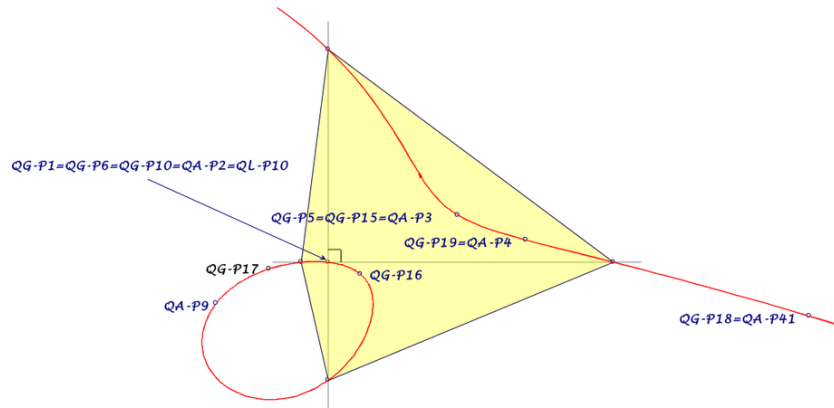
... $QL-P1, QL-P16$: homothetic circles wrt P , ratio 1:2,

... $QL-P7$: circle through $QA-P6$,

... $QL-P24, QL-2P3$ (see #2920), ...

Final remark:

The cubics $QA-Cu1$ and $QL-Cu1$ for the considered quadrilaterals are identical, bearing most of the discussed points. The pictured constellation holds for all quadrilaterals with orthogonal diagonals.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2018-03-08.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2922
Date: 08/3/2018 3:21:20
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,

I tried several ideas wrt the 3 foci of QA's inscribed Siebeck's sextic, dual curve of the QL's non pivotal cubic, isotomic wrt mid DT.

Nothing works and I give up !

The 3 foci are not coconic with the vertices of following triangles : DT, mid DT, S-points, 4 QL's reference.

They don't form a C-B system with the QL's vertices.

I found only this, which could be of interest.

Each dual line of a focus cuts the cubic in only one real point, which is therefore the dual point of a tangent to the sextic through a focus. These 3 tangents concur in a point K and the 3 points on the cubic are aligned on the dual line of this point K.

The exact construction of the foci by solving a complex equation of the 3rd degree allows an exact determination of this point K, but I'm not able to identify this point neither wrt the QA, DT or midDT nor wrt the triangle of the foci itself !

Perhaps you will have an idea ...

Thanks in advance for your attention

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2923

Date: 09/3/2018 8:42:28

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

11. Quadrigons and Squares

For readers who missed earlier paragraphs of this introduction: a quadrigon is in ordinary speech a quadrilateral, a figure bounded by 4 lines.

There are several puzzlers about Quadrigons and Squares.

1. Construct a Square **inscribed** in a random Quadrigon.

2. Construct a Square **circumscribing** a random Quadrigon.

3. Construct a Square **perspective with** a random Quadrigon.

There are solutions for all these questions.

I won't give them right away. I just want to start a discussion here.

Who has a solution for one of these questions?

Chris van Tienhoven

This is the last paragraph in the series of this introduction on Quadrilateral Geometry. Later on I will make another series about Polygon Geometry.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2924
Date: 09/3/2018 4:21:58
From: eckart_schmidt@t-online.de
Subject: Square inscribed a quadrigon

Dear Chris,

wrt:

1. Construct a Square inscribed in a random Quadrigon.
Let L_i be the lines of the quadrigon
... and Q_i square points with Q_i on L_i
... considered for a fixed circulation.
For three lines L_1, L_2, L_3
... the 4th square point Q_4 lies on a line L_4^* ,
... easy constructible in the following way:
(1) Let Q_2 be $L_1 \wedge L_2$ and Q_3 the intersection
... of L_3 and a perpendicular to L_1 in Q_2 , ...
(2) Let Q_2 be $L_3 \wedge L_2$ and Q_1 the intersection
... of L_1 and a perpendicular wrt L_3 in Q_2 , ...
The two Q_4 for (1) and (2) give the line L_4^* .
For four lines the searched square points are $L_i \wedge L_i^*$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2925
Date: 10/3/2018 9:22:55
From: bernard.keizer
Subject: Introduction to Quadrilateral Geometry

Dear Chris, dear Eckart,
For the 2 1rst questions, see C.M. Hebbert The inscribed and
circumscribed squares of a QL ... 1914
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2926
Date: 10/3/2018 10:15:06
From: eckart_schmidt@t-online.de
Subject: Square circumscribed a quadrigon

Dear Chris,
wrt:

2. Construct a Square
circumscribing a random Quadrigon.
Let P_i be the vertices of the quadrigon
and L_i the lines of the square with P_i on L_i .
Any line L_1^* through P_1 leads to a circumscribed rectangle with
lines L_i^* .

Consider the square $L_1^*, L_2^*, L_3^*, L_4^*$.
The 2nd intersection of
... a circle with diameter P_3P_4
... and a circle through $P_3, L_3^* \wedge L_4^*$
... .. and the pedal point of P_2 on P_1P_3
... is $L_3^* \wedge L_4^*$ of the searched square.
Replacing P_2 by P_4 , the construction gives the 2nd square
... with contrary circulation.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2927
Date: 11/3/2018 11:14:08
From: eckart_schmidt@t-online.de
Subject: Square perspective a quadrigon

Dear Chris,

wrt:

3. Construct a Square *perspective with* a random Quadrigon.

Here a special example:

Take the point at infinity of QL-L1 as perspector,

... parallels L_i through P_i

... and squares with vertices Q_i on L_i

... with side lengths $\sqrt{d_{12}^2 + d_{23}^2}$

... with d_{ij} distance L_i, L_j .

Further perspectors lie on two special circumconics of the quadrigon.

Squares on 4 lines L_i

through a common point are only possible

... if the 4th line has the direction of a line L_4^* (see #2924)

... wrt L_1 , a parallel to L_2 and L_3 .

If this is the case, you get squares with diagonals

... parallel to QG-P1.QG-2P2 of a quadrigon

... with lines L_1 , a parallel to L_2, L_3 and the corresponding L_4^* .

Best regards Eckart

PS: What about the QG-circumconics for the perspectors?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2928
Date: 11/3/2018 12:00:38
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
Always involved in the 3 foci, I've just found another property, which might be interesting.
Let's give a triangle with vertices A, B, C, G centroid, F and F' foci of the Steiner inellipse and M middle of BC.
For a vertice A, the product of the distances from A to the intersections R and R' of 2 parallel tangents to the ellipse with the 2 tangents from A is equal to the product of the distances from A to the 2 foci (2nd Siebeck property)
If we take as parallel tangents the side BC and the parallel through the middle of GA, it comes immediately that $AR.AC = AB.AR' = 1/3 AB.AC = AF.AF'$.
Let's give the same way a QA with vertices Pi, centroid Bp and middles of the sides pij. The foci of Siebeck's sextic, calculated by derivating the complex polynom are F1, F2 and F3. For every vertice Pi, we have the same way $1/4 PiPj.PiPk.PiPl = PiF1.PiF2.PiF3$
For QA-P1 or Bp, we have also $1/4 p12p34.p13p24.p14p23 = BpF1.BpF2.BpF3$
Maybe this could help in understanding Siebeck's construction, as this time we don't need to draw parallel tangents in order to find the circles ...
I wonder if this result is general : what happens if we calculate the 4 derived points of a 5P ? Precisely, will we have in each vertice $1/5 PiPj.PiPk.PiPl.PiPm = PiPF1.PiPF2.PiPF3.PiPF4$?
Best regards
Bernard
PS I'm not home and I'm no longer able to access Siebeck's article, could you please send me again the exact reference ?
Thanks in advance

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2929
Date: 11/3/2018 1:17:03
From: eckart_schmidt@t-online.de
Subject: Square perspective a quadrigon

Dear Chris,

the last section of #2927 isn't correct formulated:
If this is the case for 4 lines with common point P,
... you get squares with diagonals Q1Q3
... parallel to P.QG-P1 of a quadrigon
... with lines L1, a parallel to L2, L3
and the corresponding L4*.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2930
Date: 12/3/2018 9:57:24
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

here only the reference for Siebeck's paper:
Siebeck, Ueber eine neue analytische Behandlungsweise der
Brennpunkte, Crelles Journal für die reine und angewandte
Mathematik, Bd. 64, S. 175.

Give me some time for the foci of Siebeck's sextic ,
... for I just look for the squares wrt a quadrigon.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2931
Date: 12/3/2018 3:04:51
From: eckart_schmidt@t-online.de
Subject: Square perspective a quadrigon

Dear Chris,

I have calculated the equations for the two cubics of the perspectors

... for a quadrigon with vertices

$P1(p:q:r), P2(-p:q:r), P3(p:-q:r), P4(p:q:-r).$

$$\begin{aligned} & pry^3(c^2p^2-a^2r^2) + 2pry^2(p^2SAz-r^2SCx) \\ & -pry((c^2q^2+b^2r^2)x^2-(b^2p^2+a^2q^2)z^2) \\ & -2pq^2rxz(SAx-SCz) + q^2S(x+y+z)(r^2x^2-p^2z^2)=0 \end{aligned}$$

Changing the sign of S , you get the other cubic.

The two cubics intersect on the diagonals in

$(p : 2pr(pSA-rSC)/(r^2a^2-p^2c^2) : r)$ on $P1P3$,

$(-p : 2pr(pSA+rSC)/(r^2a^2-p^2c^2) : r)$ on $P2P4$.

These two points are the 3rd intersections of the diagonals and $QL-Cu1$.

Best regards Eckart

PS: What about a construction of these cubics?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2932
Date: 12/3/2018 10:01:17
From: eckart_schmidt@t-online.de
Subject: Square perspective a quadrigon

Dear Chris,

excuse, my memory had a break down.
My results in the last messages are the background of QG-2Cu1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2933
Date: 13/3/2018 4:18:59
From: chris.vantienhoven
Subject: Introduction to Quadrilateral Geometry

Dear Bernard,

Interesting article of C.M. Hebbert.
How do you find these kind of articles?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2934
Date: 13/3/2018 4:25:23
From: chris.vantienhoven
Subject: Square inscribed a quadrigon

Dear Eckart,

You write in QFG#2924 that Q4 for (1) and (2) give the line L4*.

Two questions:

1. How is Q4 obtained in (1) and (2) ?
2. How is L4* obtained ?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

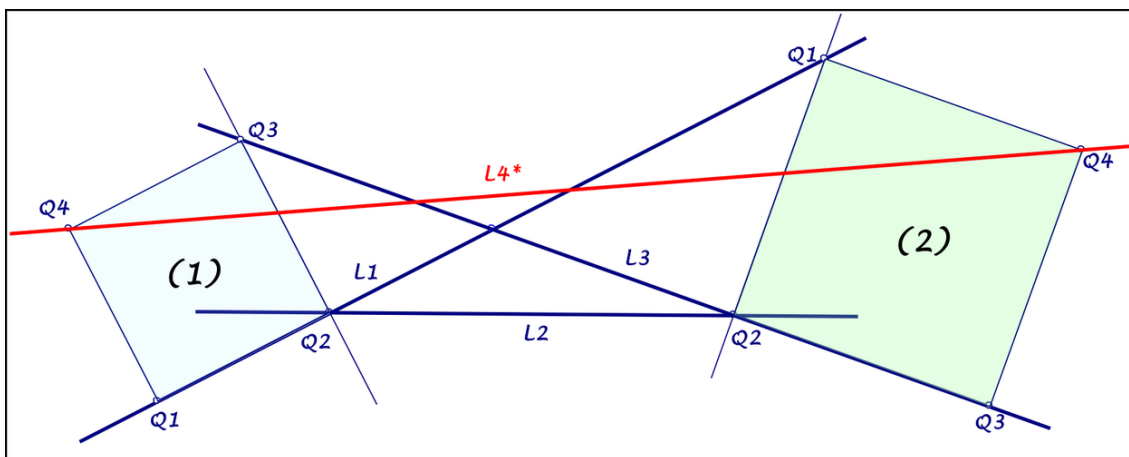
Message: #2935
Date: 13/3/2018 8:41:55
From: eckart_schmidt@t-online.de
Subject: Square inscribed a quadrigon

Dear Chris,

wrt your questions (see attached file):
Starting with three lines $L1, L2, L3$
... and following construction (1)
... you get points $Q2$ and $Q3$,
... which define the points $Q1$ and $Q4$
... for a square wrt a fixed circulation.

Starting once more with the lines $L1, L2, L3$
... and following construction (2)
... you get points $Q2$ and $Q1$,
... which define the points $Q3$ and $Q4$
... for a square wrt the same circulation.
The line, connecting the two $Q4$, is $L4^*$.

Best regards Eckart



2018-03-13.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2936

Date: 14/3/2018 9:59:26

From: bernard.keizer

Subject: Introduction to Quadrilateral Geometry

Dear Chris,

After reading your problem, I've just searched on Google "square inscribed in a quadrilateral" and I found almost immediately Hebbert's article.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2937
Date: 15/3/2018 10:58:18
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,

Thanks for the exact reference, I succeeded in printing and reading again Siebeck's article.

For the properties I mentioned, they are easy to prove simply by writing the equation of the 4th degree and derivating it in order to find the foci and comparing the coefficients of both equations.

Just an example, with O QA-P1, P_i the vertices of QA and F_i the foci, p_{ij} the middle of P_iP_j .

With the origin in O , P_i and F_i are complex numbers, $\sum P_i = \sum f_i = 0$ and $p_{ij} = 1/2(P_i + P_j)$

$Op12.Op13.Op14 = 1/8 (P_1 + P_2)(P_1 + P_3)(P_1 + P_4) = 1/8 \sum P_iP_jP_k$ and

$OF1.OF2.OF3 = F_1.F_2.F_3 = 1/4 \sum P_iP_jP_k$

The product of 2 groups of 3 complex numbers been equal proves at the same time the 1st angle property and the 2nd distance property of Siebeck. Here

1) the mean direction for the 3 tangents in O is the same as for the 3 lines through O and the foci

2) the product of the 3 distances $p_{ij}p_{kl}$ is 4 times the product of the distances from O to the foci.

The same way, we prove for each vertice P_i that $(P_j - P_i)(P_k - P_i)(P_l - P_i) = 4 (F_1 - P_i)(F_2 - P_i)(F_3 - P_i)$.

This proves for each vertice the mean direction and the distance to the 3 other vertices 4 times to the foci.

I haven't succeeded yet in reproducing Siebeck's construction, I hope you will be able to help me !

I have just reproduced step by step the calculation of the cubic roots and it gives the same kind of construction with 3 points C_i on a circle centered in O and 3 points D_i on a 2nd circle : the C_i and the D_i are CSC partners wrt the CSC with fixed points the foci of the Steiner inellipse of the foci F_i (named I and I' by Siebeck), but I can't understand the radius (of course, the product of the 2 radii is the square of OI).

Thanks in advance for your attention and your help !

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2938
Date: 15/3/2018 4:10:41
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

perhaps the following property is helpful for you,
... using Siebeck's nomination (see attached file):
Here is a "construction" of parallel tangents P_1, P_2, P_3
... and tangents L_1, L_2, L_3 from O wrt Siebeck's sextic Se ,
... using the dual cubic Cu of Se .
But we need intersections of lines and conics with cubics,
... possible with Cabri.
We start with a quadrangle QA ,
... the Steiner inellipse St of its diagonal triangle $QA-Tr_1$,
... point $O = QA-P_1, QA-P_{10}$ and $QA-P_{16}$
... and the dual cubic Cu of Siebeck's sextic.
 St intersects Cu
... in the vertices of the medial triangle of $QA-Tr_1$
... and three further points X .
Each line
 $X.QA-P_{16}$ intersects Cu
... in three points $X=Y_1, Y_2, Y_3$
... whose dual lines are parallel tangents P_1, P_2, P_3 of Se .

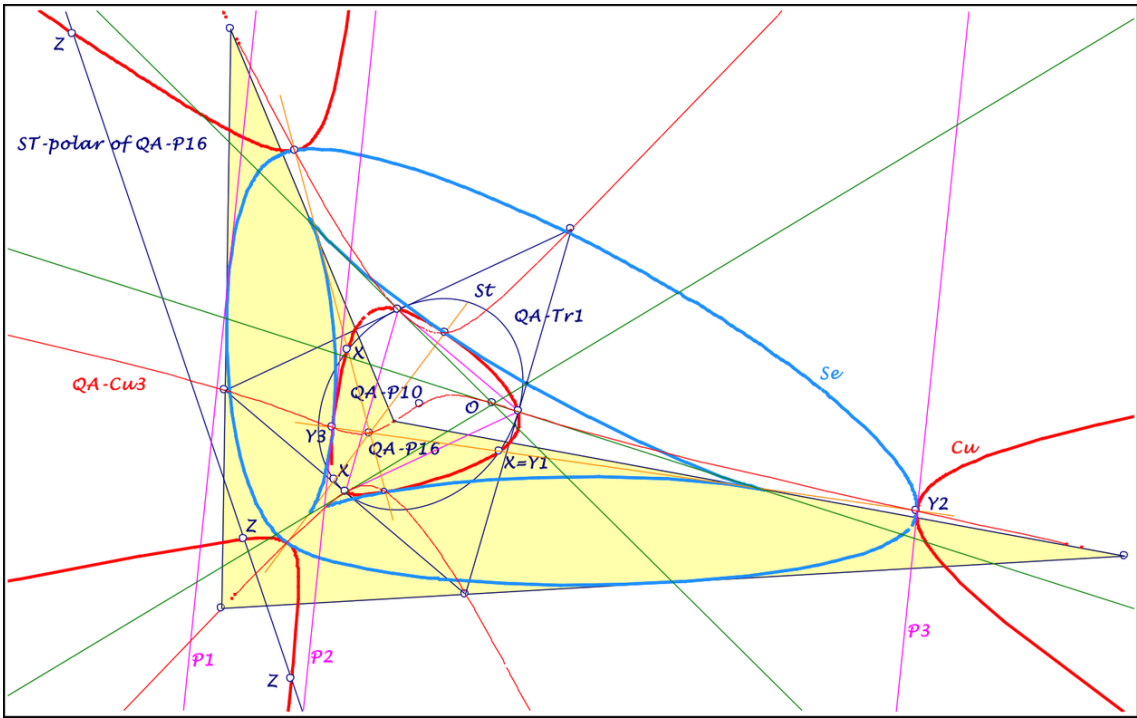
 Y_2 and Y_3 are contact points of Cu and Se ,
... lying also on the cubic $QA-Cu_3$.
The polar of $QA-P_{16}$ wrt St
... intersects Cu in three points Z ,
... whose dual lines are the tangents from O wrt Se :
... or simpler: lines from O to the vertices
of the medial triangle of $QA-Tr_1$.

But what about Siebeck's six asymptotes of the sextic,
... which shall be tangent to the Steiner inellipse of $F_1F_2F_3$?

My configuration doesn't show any asymptotes.

Best regards Eckart

PS. I am still studying your results of the last messages,
thanks!



2018-03-15.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2939
Date: 23/3/2018 9:03:37
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

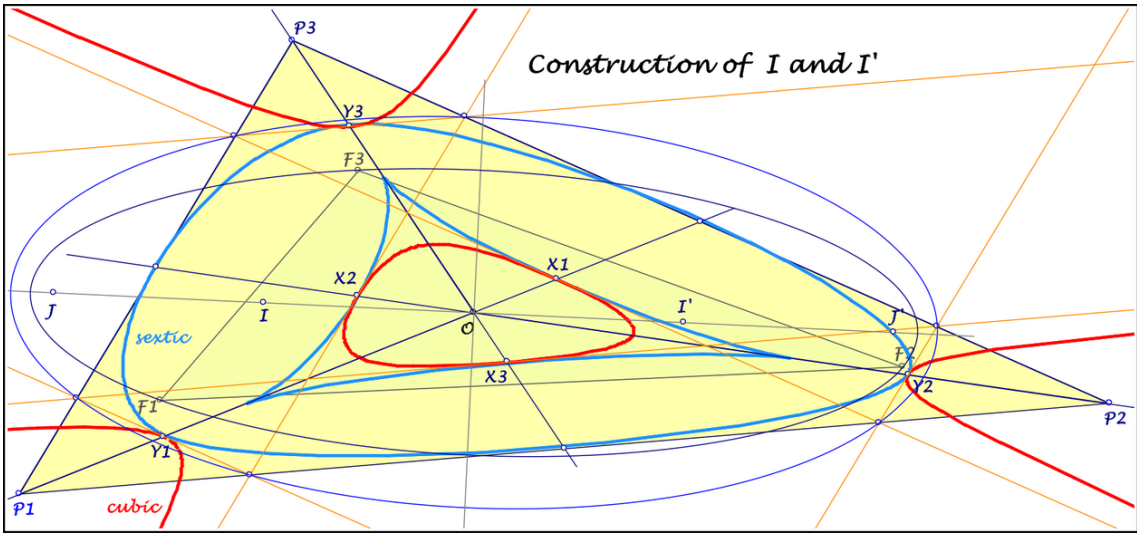
it needs some time, to refresh our discussion wrt Siebeck's "construction".
We finished with the following results in Siebeck's nomination:
... $O = QA-P1$,
... I, I' foci of the Steiner inellipse of $F1F2F3$,
... radius of the C-circle: cubic root of $OA1*OA2*OA3$.

If we have the foci I, I' , we can follow Siebeck's "construction".
Here is a possibility, to get X, X' for $O = \text{zero}$, proved with CABRI:

Let $P1P2P3$ be the quadrangle
... with Siebeck's sextic Se and its dual cubic Cu .
... The tangents from O at the sextic Se are OPi .
... The lines OPi contact the sextic and intersect the sextic
... in the midpoint of $PjPk$
... and two points Xi, Yi .
... Xi, Yi are contact points of the sextic Se
and its dual cubic Cu
... with tangents parallel $PjPk$,
... which is also a tangent at the sextic.
... This allows measurements and calculations
... of Siebeck's C-circle and alpha-angles.

The three pairs of parallel tangents in Xi and Yi
... have six intersections with the sidelines of $P1P2P3$,
... which give a conic with foci J and J' .
... This conic is the Steiner circumellipse of $F1F2F3$.
... The midpoints of OJ and OJ' are the searched foci I and I' .

Best regards Eckart



2018-03-23.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2940
Date: 24/3/2018 11:10:45
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
Sorry if I didn't answer your previous message, but I wasn't home this week and had no computer at disposal.
Thank you for the 2 beautiful figures.
In order to avoid too long messages, I answer only this 2nd message and will make another answer for the general case.
Beautiful construction indeed !
Only 2 remarks :
1) The text doesn't fit exactly with the figure. I suppose you mean that the foci J and J' of the 6 intersections are also the foci of the Steiner circumellipse of $F_1F_2F_3$ and the points I and I' are the foci of the Steiner inellipse.
2) Let's name K and K' the 2 foci of the Steiner inellipse of the triangle $P_1P_2P_3$.
By derivating 2 times the complex polynom with roots P_1, P_2, P_3 and 0 and 1 time the complex polynom with roots P_1, P_2 and P_3 and comparing the coefficients, you get immediately $OK = OI \cdot \sqrt{2}$.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2941
Date: 24/3/2018 12:21:03
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,

In the general case, rereading Siebeck pages 180 and 181

1) I and I' are the foci of the Steiner inellipse of F1F2F3 (the 3 points form a degenerated curve of class 3)

I don't understand the 6 asymptotes, maybe they are imaginary

...

2) the tangents from 0 to the sextic are the lines through the middles pij and pkl of 2 opposite sides which pass through the vertices of midDT ; they are the dual lines of the 3 points of the cubic aligned on the dual line of 0 or QA-P1, which is also the ST polar of QA-P16.

3) 3 parallel tangents to the sextic are the duals of 3 points of the cubic aligned with QA-P16.

In fact, you don't need 3 parallel tangents, as you have $p_{12}p_{34}p_{13}p_{24}p_{14}p_{23} = 8 \cdot o_{p12} \cdot o_{p13} \cdot o_{p14} = 4 \cdot o_{F1} \cdot o_{F2} \cdot o_{F3}$ (and in each vertice $P_i P_j \cdot P_i P_k \cdot P_i P_l = 4 \cdot P_i F_1 \cdot P_i F_2 \cdot P_i F_3$...)

The radius of the circle is then $\text{cbroot}(o_{p12} \cdot o_{p13} \cdot o_{p14})$; the coefficient 1/2 in Siebeck's formula is necessary.

The angles are the mean direction of the 3 tangents in 0.

I suppose the problem could be resumed this way :

How to find in the complex plane 3 points knowing the 2 foci of the Steiner inellipse (which gives p) and the product of the complexes of the points (which gives q) : they are the roots of the complex polynom $z^3 + pz + q$.

Best regards

Bernard

PS I would be completely happy if I had succeeded in reproducing exactly Siebeck's construction, but this is not the case !

Perhaps my figures lack of precision ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2942
Date: 24/3/2018 12:49:57
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

thanks for your message.

wrt 1) There is a typo:

"Here is a possibility, to get I, I'
for $O = \text{zero}$, proved with CABRI:"

But the other text - I think - is ok in the sense of your description.

wrt 2) Excuse, I had not parat your result $OK = OI \cdot \sqrt{2}$,
... certainly already mentioned.

So my construction of I and I' in this special case is not necessary.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2943
Date: 27/3/2018 2:00:09
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

thanks for your detailed summary!

After days of confusion here my observation:

If we want follow Siebeck's construction

... and assume $O = QA-P1$ and I, I'

 foci of the Steiner inellipse of $F1F2F3$,

... we need a radius for Siebecks C-circle.

This radius

... is not Siebeck's formula,

... is not my assumption in #2939,

... is not yours in #2941, 3).

The radius is $(a+b)/2$

... with a, b axes of the Steiner circumellipse of $F1F2F3$.

So up to now I see no relations to the points of the quadrangle.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2944
Date: 27/3/2018 7:07:48
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
Thanks for your patience and your courage !
I'm now convinced that Siebeck's construction doesn't hold or that we don't understand it.
The problem seems rather simple : we search the vertices F_1 , F_2 and F_3 of a triangle knowing the centroid in the origin of a complex plane ($F_1+F_2+F_3=0$) and the coefficients $p = F_1*F_2+F_1*F_3+F_2*F_3$ and $q = - F_1*F_2*F_3$.
Then the foci of the Steiner inellipse I and I' are given by $\sqrt{-p/3}$.
The solution is given by searching U and V such as $U+V = - q$ and $U*V = - p*p*p/27$.
Then $u = \text{cbrt}(U)$ and $v = \text{cbrt}(V)$ and $uv = -p/3$; the u and v are 3 couples of CSC conjugates wrt the CSC with fixed points are I and I' (the solutions are u, ju and $j*j u$ and $v, j*j v$ and jv where $1, j$ and $j*j$ are the 3 $\text{cbrt}(1)$).
The points M_1, M_2 and M_3 , middles of u_1v_1, u_2v_2 and u_3v_3 are on the Steiner inellipse and the middles of OF_1, OF_2 and OF_3 . All this is well known and the solution is unique.
I checked dozens of different figures : Siebeck's formula with angle and distance gives the point $\text{cbrt}(-q/2)$, but on this line, whatever the radius, there is no point X such as the middle of XX' , X' being the CSC of X with the same CSC is one of the M_i (the locus of these middles for a given line is an hyperbola ...)
So there is surely a link between the middle of U and V and the 3 middles M_i (see the $\cos 3m$ and $\cos m, \cos(m+2\pi/3)$ and $\cos(m-2\pi/3)$ in Siebeck's formula), but I wasn't able to find it ...
It looks like $(u+v)(ju+j*jv)(j*j u+jv) = U+V$ or $F_1*F_2*F_3 = - q$...
After spending so many hours on this item, I hope sincerely you will find the solution !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2945
Date: 28/3/2018 10:48:44
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
To be complete, with the same notations as the previous message, adding M as middle of UV ($M = -q/2$).
U and V are CSC partners wrt the CSC with fixed points G and G' having the complex coordinates $\pm \sqrt{-p^3/27}$.
It is possible to draw U and V without calculation as we know the middle M and the product $U \cdot V = -p^3/27$.
(U and V on the internal bisector of $\angle G$, center of the circle through U, V, G and G' on the intersection between the external bisector of $\angle G$ and the perpendicular bisector of GG' ...)
Having the U and V gives the 3 u and the 3 v ...
Last, but not least, the 3 circles through u_i, v_i, F and F' have centers O_i and the circle through U, V, G and G' has center O' : the angles $\angle u_i O_i v_i$ and $\angle U O' V$ are $m, m+2\pi/3, m-2\pi/3$ and $3m$!
I hope this time everything is almost clear ...
It works for any curve of class 3 as soon as we have p from I and I' and q from the complex product $OA_1 \cdot OA_2 \cdot OA_3$...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2946

Date: 28/3/2018 7:36:36

From: Benedetto Scimemi

Subject: Orthogonal chords through a fixed point in a conic

Dear Bernard and Eckart, I refer to your #2918 and #2919 of March 6, regarding the QA intercepted on a fixed conic Co by two orthogonal lines through a fixed point P .

At first sight the loci of QA-P1 and QA-P3 look as conics, say Co_1 and Co_3 , homothetic to Co .

However, deeper experiments showed that in some cases Co_1 and Co_3 are not homothetic to Co .

This was particularly intriguing, and it took me some time to understand what was going on.

Here is my explanation.

Of course, there may exist a simpler interpretation.

1) Let Co be any central conic Co of center Z ; let P be any point.

If r is a line through P , let r_{∞} be its point at infinity.

Consider the line p_r , Co-polar of r_{∞} .

p_r is a line through Z (the conjugate diameter of r).

If r cuts a chord on Co , then it is well-known that its midpoint lies on p_r .

2) By construction, $r \rightarrow p_r$ is a projectivity between two pencils of lines: through P and through Z .

When r describes the P -pencil, p_r describes the Z -pencil and their intersection (by Steiner th. on the projective generation of conics), describes a conic, call it Co_{Steiner} .

By construction, Co_{Steiner} passes through P and Z , its center being the mid-point $(P+Z)/2$.

3) When r goes through the points at infinity of Co , then r and p_r are parallel. Therefore Co and Co_{Steiner} have the same points at infinity (same terms of degree 2 in Cartesian equations, same asymptotes for hyperbolas, not always same foci). Hence Co_{Steiner} is homothetic either to Co or to its conjugate Co^* .

It appears that the latter case takes place only when Co is a hyperbola and P lies in the non-convex plane region confined by Co .

4) Now our original problem: fix a conic Co and a point P .

If a line through P cuts on Co a chord AB, let the orthogonal line through P cut the chord CD.
 For the resulting quadrangle $Q=ABCD$, the point P is diagonal and equals QA-P2. Move the pair of orthogonal sides around P.

Since A defines B,C,D, we can consider $Q=Q$ A as a function of A. We want to study the loci of QA-P1, QA-P3 and QA-P4 of QA when A runs on Co.

5) The special properties of the transformation Co-Tf3 (Tf3 for short) will now play their role.

First of all, for all Co-inscribed quadrangles, Tf3 transforms QA-P2 into QA-P4 .

As QA-P2 = P is the same for all A, then QA-P4 is also fixed.

6) Consider the triangle $T=Z.QA-P4.P$ and its (negatively similar) triangle $Tf3^{(-1)}(T)=Z.P.X$, namely set $X=Tf3^{(-1)}(QA-P3)$.

Apply another special property of Tf3 (a direct consequence of the Chord Lemma) : for any point Y, if $X=Tf3(Y)$, then Y is the midpoint of the Co-chord through Y orthogonal to XY.

It follows that Y lies on Co Steiner , which by definition, is the locus for the midpoints of the chords through P.

7) When A moves, Y describes Co Steiner and $X=QA-P3$ describes a (negatively) similar conic $Co\ 3 = Tf3^{-1}(Co\ Steiner)$. However, since the Co-axes are Tf3-fixed, Co 3 looks also homothetic to Co Steiner and Co.

Co 3 passes through Z, QA-P4, QA-P3. Its center is the midpoint $(Z+QA-P4)/2$.

8) Finally, QA-P1 is the midpoint of QA-P2.QA-P3.

As QA-P2 is fixed, the locus Co 1 of QA-P1 is the image of Co 3 under the 1/2 homothety centered in P, hence also homothetic to Co or Co*.

Best regards

Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2947

Date: 29/3/2018 2:28:06

From: eckart_schmidt@t-online.de

Subject: Orthogonal chords through a fixed point in ...

Dear Benedetto,

with interest I have reproduced the properties of Co-Tf3 in #2946.

I think, some of them are already mentioned in #2921, but the conic CoSteiner was new for me.

Wrt 3):

The reference conic in #2921 is an ellipse, wrt your result I have to make a correction at the end of the first page.

Wrt 6) I think, there is a typo "... X=Tf3(-1)(QA-P *4*) ...".

Wrt 7) Perhaps a further typo " ... Co3 = *Tf3* (CoSteiner)..."?

But the properties of Co3 are correct.

In addition: The 2nd intersection of QA-P2.QA-P4 and CoSteiner ... is also a point of Co3

... and the conic for QG-P13 wrt the quadrigon ACBD.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2948

Date: 30/3/2018 1:15:43

From: Benedetto Scimemi

Subject: Orthogonal chords through a fixed point in ...

Thank you, Eckart, for your quick reaction and indicating typos. My explanation was surely confusing: a typo did occur in point 6), although not the one you mention. However, as a result of your alarms, I re-read my message and I found a gap in the argument (I did not prove that XY and PY are orthogonal). Therefore point 6) must be totally rewritten. Thanks.

In any case, the fact that $Tf3(-1)(QA-P3)$ lies on Co-Steiner is true. Therefore the conic Co3 described by QA-P3 is actually the image of Co-Steiner under a similarity. This is why Co3 is homothetic either to Co or Co* (points 1 to 3). And this was for me the most intriguing question.

Best greeting Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2949
Date: 30/3/2018 4:06:58
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

perhaps part of the puzzle:
Consider a triangle UVW (in our discussion F1F2F3),
... its Steiner circumellipse ST with foci F, center O
... and the midpoints I of OF.

Let C_i be a circle round O with radius $r = \sqrt[3]{OU \cdot OV \cdot OW / 2}$
... with a variable point C
... and follow Siebeck's construction C - D - M - X
... with X reflection of O in M.

The loci of X is a new conic, centered in O,
... with the same foci F as the circumscribed Steiner ellipse ST
... and $(a+b)/2 = r$ (a, b axes of ST).
Evident or relevant for Siebeck's construction?

Best regards Eckart

PS: Excuse, up to now I haven't studied your complex calculations.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2950
Date: 30/3/2018 10:02:41
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

excuse, there is a typo in the last result of #2949:
Replace
"... and $(a+b)/2 = r$ (a, b axes of ST)."
by
"... and $(a+b)/2 = r$ (a, b axes of the new conic)."

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2951
Date: 31/3/2018 11:10:06
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
We agree, the loci of M and X are ellipses with foci I and I' and F and F'.
In order to find the right solution, these ellipses have to be the Steiner in- and circumellipses.
So Siebeck's solution doesn't hold, I suppose he has simply forgotten a part of his text ...
You may consider the converse problem : if M is on the Steiner inellipse (and X on the Steiner circumellipse), what are the loci of C and D ?
Again, the C-D transformation of Siebeck is a CSC with II' the 1st Steiner axis and the circle of diameter II' the inversion circle.
So the classical construction works : draw the bisectors of the angle IMI', the outer bisector (tangent to the Steiner inellipse in M) cuts the 2nd Steiner axis in the center of a circle through I, I', C and D and C and D are the intersections between this circle and the inner bisector of the angle IMI'.
C and D describe 2 circles inverse wrt the circle with diameter II'.
Best regards
Bernard
PS In my previous message, I described the correct complete solution ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2952
Date: 03/4/2018 8:51:49
From: eckart_schmidt@t-online.de
Subject: Introduction to Quadrilateral Geometry

Dear Chris,

in QFG-message 2923 you wrote wrt 11. Quadrigons and Squares:
"There are solutions for all these questions.
I won't give them right away..."
Why don't you offer your solutions or your references?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2953

Date: 06/4/2018 10:29:00

From: chris.vantienhoven

Subject: Introduction to Quadrilateral Geometry

Dear Eckart,

Thanks for reminding.

I have nothing to add about the subjects after the discussio

1. Construct a Square **inscribed** in a random Quadrigon.

2. Construct a Square **circumscribing** a random Quadrigon.

after the discussion about this subject in recent messages

#2923-2929, #2931-2936, #2952 and older messages #366, #367,

#370-378, #381, #382, #384-386.

About the 3rd subject (Construct a Square **perspective with** a random Quadrigon)

like you already wrote there is the wonderful

Perspective Squares Double Cubic QG-2Cu1

(<http://www.chrisvantienhoven.nl/qg-items/qg-cubics/qg-2cu1>)

that you once found being the locus of perspectors of the

Reference Quadrigon that are point perspective with a Square.

However there still is the problem of finding a square A1B1C1D1

that is also line perspective with a given Quadrigon ABCD. This

is only the case in very limited instances.

Line perspective means that not only A.A1, B.B1, C.C1, D.D1 are concurrent, but also that the intersection points of the sides $AB \wedge A_1B_1$, $BC \wedge B_1C_1$, $CD \wedge C_1D_1$, $DA \wedge D_1A_1$ are collinear.

For triangles point perspectivity and line perspectivity always go together (Desargues' Theorem) but not so for

"quadrilaterals"□

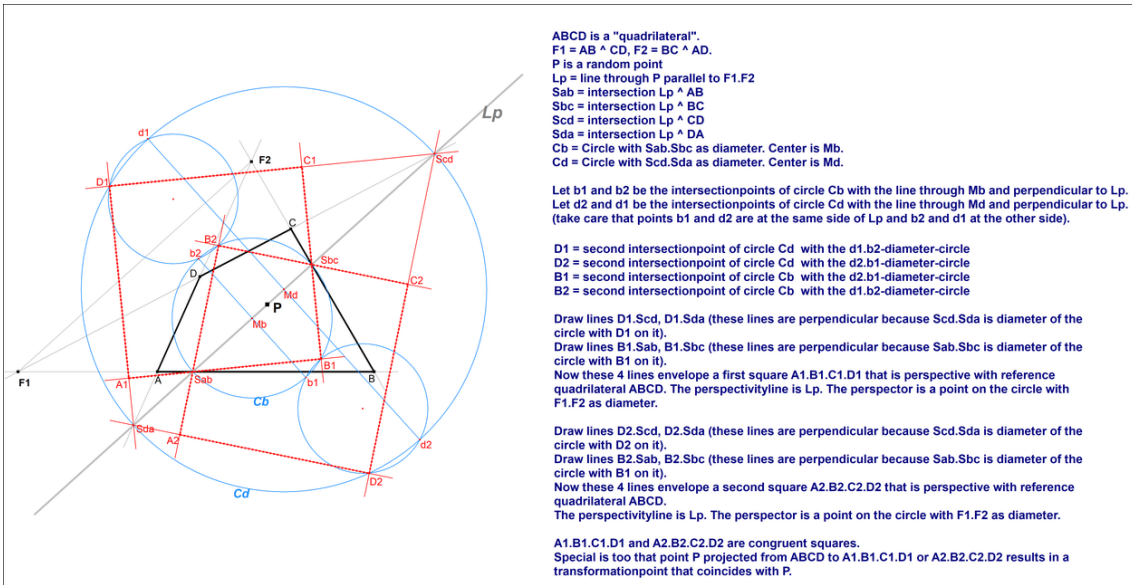
This problem is pretty complicated.

I found in 2009 a solution. Even before EQF existed. You will find it in attached file.

Later I found a solution described in Heinrich Dörrie's book "100 Great Problems of Elementary Mathematics"□, problem 72 at pages 301-303. There are several linkages between our solutions.

Best regards,

Chris



Methode-13-Persp-naar-Rechth-kader-2a.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2954

Date: 07/4/2018 12:12:02

From: chris.vantienhoven

Subject: Tessellation theory applied in Triangles, Quadrilaterals, Pentalater

Dear friends,

I looked at some theory on Tessellations and found applications in Triangles, Quadrilaterals and Pentalaterals.

A special type of Tessellation occurs when a tile is defined as the set of points closest to one of the points in a discrete set of defining points. This is done in a Voronoi Diagram.

In a Voronoi Diagram all kind of convex Polygons are created.

When making a Voronoi Diagram boundary lines between points are used, which are simple constructions of a point wrt two lines.

Therefore I define here:

A Voronoi line of P wrt two crossing lines L1 and L2 is the perpendicular bisector of the reflection points of P in L1 and L2.

A Voronoi line of P wrt L1 and L2 passes through $L1 \wedge L2$.

Application in a triangle (3-Line)

The three P-Voronoi lines in a triangle concur in the P-isogonal conjugate.

Application in a quadrilateral (4-Line)

The locus of all points P for which all Voronoi lines wrt L_i and L_j concur is QL-Cu1.

(i,j different values from (1,2,3,4))

Application in a pentalateral (5-Line)

There are two points for which all Voronoi lines wrt L_i and L_j concur (i,j different values from (1,2,3,4,5)). These points are the foci of the inscribed conic of the Pentilateral. The point of concurrence of one focus is the other focus.

This explains the locus QL-Cu1 in a quadrilateral.

Possibly this is all known with other names, but anyhow it gives a nice insight how things fit together in triangles, quadrilaterals and pentilaterals.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2955

Date: 08/4/2018 9:58:00

From: eckart_schmidt@t-online.de

Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Chris,

the Voronoi line is really a very interesting tool!
Your application in a quadrilateral for points on QL-Cu1 is
QL-Tf1 (CSC).
For any point the six Voronoi lines have four triple
intersections,
... the isogonal conjugated of the point wrt the QL-triangles.

First observation for a quadrangle:
For a QA you can consider the Voronoi line for a point and two
opposite lines
... and the case, that these lines are concurrent.
Then QA-P2 and QA-P4 are partner,
... as well as QA-P41 and the infinity point of perpendiculars
of QA-L2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2956

Date: 09/4/2018 9:54:47

From: bernard.keizer

Subject: Tessellation theory applied in Triangles, Quadrilaterals, Pental

Dear Chris,

I made yesterday a reply, which seems to have disappeared !
Shortly, thanks for this new item, which appears to be promising.

I had never heard about the Voronov line, but it's well known and easy to prove that the isogonal conjugate of a point is the circumcenter of the triangle of the reflexions of the point in the 3 sides.

For a point on QL-Cu1, it's CSC partner is it's isogonal conjugate wrt the 4 reference triangles of any QL inscribed in the curve and the curve is self isogonal wrt any triangle such as it's circumcircle passes through QL-P1.

QL-Cu1 is an isogonal nK wrt any of these triangles and the line through the 3 3rd intersections with it's 3 sides is the CSC line of the circle (for example, the orthic triangle of DT and the CSC line of the Euler circle of DT)

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2957

Date: 09/4/2018 12:36:34

From: eckart_schmidt@t-online.de

Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Chris,

yesterday (08.04.) I sent this message , but it doesn't appear in EQF.

Consider the Voronoi lines wrt opposite lines of a quadrangle
... and points X with concurrent lines in Y,
... then the points X lie on *a pivotal isocubic:*
... ... reference triangle QA-P2.QA-P4.QA-P41,*
... ... isogonal conjugation,*
... ... pivot: infinity point of the bisector of QA-P2.QA-P4.*

The points Y lie also on this cubic,
... image of X wrt a Möbius transformation,
... ... centered in QA-P41, swapping QA-P2 and QA-P4.

Today (09.04.) I can add:
The cubic above is a circumcubic of QA-Tr1,
... invariant wrt QA-Tf2 ...
But there are more properties ...

Best regards Eckart

PS: I don't attach a figure, for the 07.04. I sent a mail
... wrt perspective squares with an attachment,
... but it doesn't appear in EQF.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2958

Date: 09/4/2018 8:53:47

From: chris.vantienhoven

Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Eckart, dear Bernard,

Here another contribution wrt the Voronoi Lines.

Ellipses/Hyperbolas inscribed between QA-opposite lines with QA-P2 and QA-P4 as foci.

Eckart, regarding your remark that QA-P2 and QA-P4 are partner as well as QA-P41 and the infinity point of perpendiculars of QA-L2.

I reasoned that following your property in a 4-Line, combined with the property in a 5-Line (about the foci of inscribed conic being partners), that:

1. there will be a 1st conic (ellipse of hyperbola) tangent to opposite lines P1.P2 as well as P3.P4 with foci QA-P2 and QA-P4.
2. there will be a 2nd conic (ellipse of hyperbola) tangent to opposite lines P1.P3 as well as P2.P4 with foci QA-P2 and QA-P4.
3. there will be a 3rd conic (ellipse of hyperbola) tangent to opposite lines P1.P2 as well as P2.P3 with foci QA-P2 and QA-P4.

Parabolas inscribed between QA-opposite lines with QA-P41 as focus and axis parallel to QA-L9.

Also:

1. there will be a 1st parabola tangent to opposite lines P1.P2 as well as P3.P4 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.
2. there will be a 2nd parabola tangent to opposite lines P1.P3 as well as P2.P4 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.
3. there will be a 3rd parabola tangent to opposite lines P1.P4 as well as P2.P3 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.

I checked it in Cabri and it was confirmed. No synthetic proof yet.

Summary

Then a summary of the application of the Voronoi Lines in a 2-Line, 3-Line, 4-Line, 5-Line.

In a 2-Line the Voronoi Line is a line through $L1 \wedge L2$.

In a 3-Line the 3 Voronoi Lines for every P in the plane concur in the Isogonal Conjugate of P.

In a 4-Line the 6 Voronoi Lines concur only for points on QL-Cu1 in the Clawson-Schmidt Conjugate of P, also lying on QL-Cu1.

In a 5-Line the 10 Voronoi Lines concur only for the 2 foci of the inscribed conic. The concurring point being the other focus.

Best regards,
Chris

p.s. I am really sorry about the bad performance of Yahoo for its groups.
I had the same problems with not appearing of messages.
Other groups have the same problems I heard.
I have been looking for alternatives, but they also have risks.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2959
Date: 09/4/2018 9:06:06
From: Antreas Hatzipolakis
Subject: Yahoogroups problems (was: Tessellation theory applied in Triang

Chris wrote
> p.s. I am really sorry about the bad performance of Yahoo for its groups.
> I had the same problems with not appearing of messages.
> Other groups have the same problems I heard.
> I have been looking for alternatives, but they also have risks.

Dear Chris,

We have the same problem in my yahoogroup Anopolis.
You may check the archive in order to see if your message appeared.
If not, then repost it!
Alternatives is a solution, but what we will do with the archives of the groups?
How could we transfer them in the new group?
I remember, you Chris, transferring message-by-message the messages from your original group in google to your new group in yahoo.
But this is not a good idea for thousands of messages ! :)

Regards
APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2960

Date: 09/4/2018 10:25:43

From: bernard.keizer

Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Chris,

After 2 other lost messages, I try this last one, then I give up!

Your properties are linked to the fact that 2 couples of vertices of QA and 2 corresponding DT vertices form a QL inscribed in QA-Cu7, which is a QL-Cu1 with QA-P2 and QA-P4 or QA-P41 and the infinity point of the perpendicular bisector of QA-P2 and QA-P4 as CSC partners on this cubic.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2961

Date: 09/4/2018 10:26:22

From: chris.vantienhoven

Subject: CO-Tf3 in a separate section on the EQF/EPG-site

Dear friends,

I placed the description of CO-Tf3 on my site in a separate section dedicated to information about conic sections. It can be found at:

<https://www.chrisvantienhoven.nl/mathematics/conic-sections>.

Thanks you all who participated in the construction and description of this special transformation.

This transformation shook all conventions in naming and coding of a new transformation.

First of all basically it is not an n-Point transformation, because in essence it relates to a conic and it can be applied to a triangle, quadrangle, pentangle by relating to their circumscribed conic, but also to a triangle, quadrilateral, pentagonal by relating to their inscribed conic. So it is a conical transformation. Just like a pole or a polar wrt a conic are conical transformations

Therefore its name became CO-Tf3, taking into account that the codes CO-Tf1 and CO-Tf2 should be reserved for the pole and polar wrt a conic.

However we already had an equivalence in the 5-Point (Pentangle) being 5P-s-Tf3. But then it appeared that the coordinates of its inverse transformation were far more easy than the main transformation as it was thought to be. So we (Benedetto and me) decided that we should follow the main road paved by the internal structure of the transformation. Choosing from a transformation and its inverse, the simplest transformation should be the main transformation.

Moreover it appeared that for points on the reference conic the "simplest" transformation in addition is coinciding with Frégier's point.

Then 5P-s-Tf3 should be reconsidered. However 5P-s-Tf3 is a direct result from a related transformation in a Quadrangle, like already many transformations are built up from lower level transformations. This level-up construction is a valid reason for maintaining the 5P-s-Tf3 like it is. Moreover it justifies the existence of a Pentangle Transformation that actually appeared to be a limited version of the conical transformation we are describing, because now it fits into a larger picture. Therefore finally it can be said that

* $5P-s-Tf3^{-1}$ (the inverse of 5P-s-Tf3) is an application of CO-Tf3 in a Pentangle, or

* 5P-s-Tf3 is an application of C0-Tf3 ⁻¹ (the inverse of C0-Tf3) in a Pentangle.
Then we also have 5P-s-Tf4 which is related to 5P-s-Tf3, but also a level-up construction of QA-Tf6.
This justifies its description in a Pentangle.
However I do not see a reason (yet) to appoint a similar Conical Transformation, because it just would be a simple locus of C0-Tf3, mapping all points on a line, which is a triviality.
As yet items will be described in the context they show up.

Right notation of prefix

I use the prefix "C0-" instead of "Co-", because all prefixes (like "QA-", "QL-", "QG-") so far are always written in capitals. Suffixes are usually denoted as "-Co".

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2962
Date: 09/4/2018 11:00:35
From: chris.vantienhoven
Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Bernard,

Finally you succeeded in sending you message. Persistence wins!
As a matter of fact I cannot quite follow your message.
Can you be a bit more specific. For example couples P1,P2 and P3, P4 and then of the QA-Diagonal triangle these points
Which 4 QL-lines can we see then?
Further I noticed that when we draw in the three Component Quadrigons of a QL the cubic QA-Cu7, then there are 4 common points visible of these three versions of QA-Cu7.
Do you happen to know (or Eckart) which 4 QL-points we see?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2963
Date: 10/4/2018 10:29:14
From: eckart_schmidt@t-online.de
Subject: Tessellation theory applied in Triangles, Q ...

Dear Bernard, dear Chris,
I just noticed, that the cubic in my messages 2955 and 2957 is QA-Cu7.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2964
Date: 10/4/2018 11:08:07
From: eckart_schmidt@t-online.de
Subject: WG: Introduction to Quadrilateral Geometry

-----Original-Nachricht-----

Betreff: Re: Introduction to Quadrilateral Geometry
Datum: 2018-04-07T19:07:52+0200
Von: "eckart_schmidt@t-online.de"
An: "-Group, Quadri-Figures"

Dear Chris,

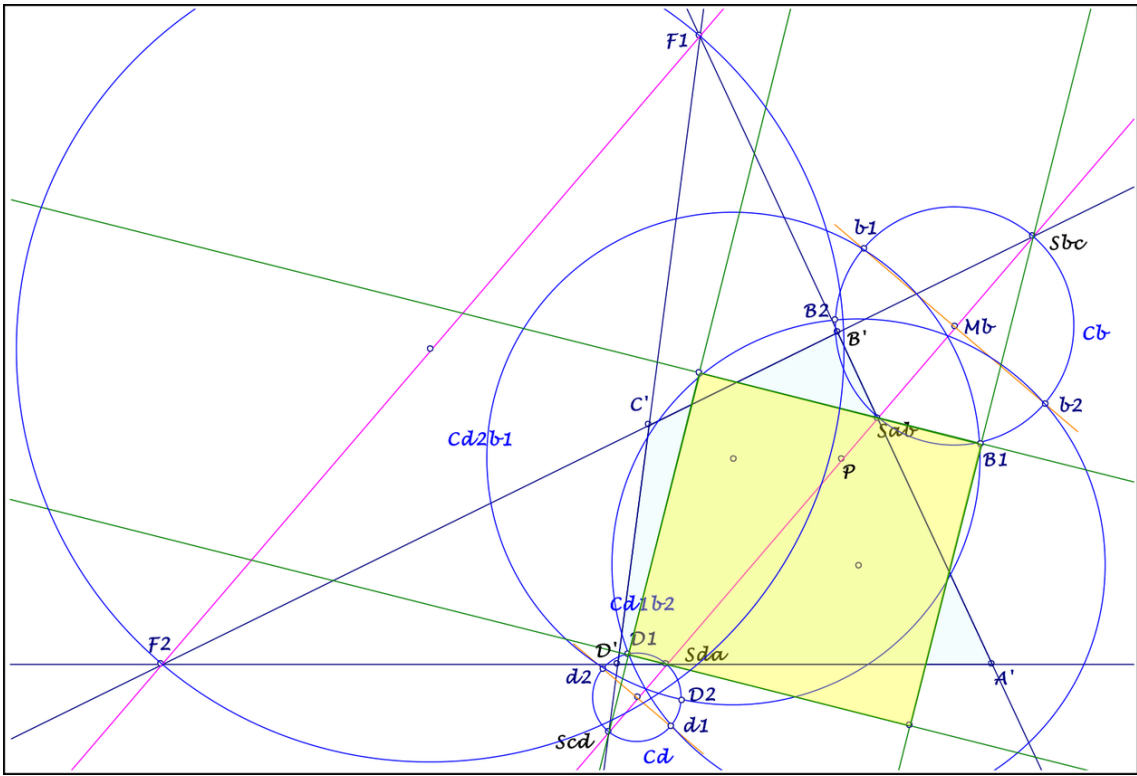
thanks for your detailed resume in #2953
... wrt perspective squares for a quadrigon, but a remark:
In the attached file you construct squares,
... which are as well point as also line perspective.
In a first construction I could confirm your results for a
quadrigon ABCD.

Your construction is orientated at the points B, D.
So I took the same quadrigon but with the nomination $A=D'$, $B=A'$,
 $C=B'$, $D=C'$
... and followed your construction once more, see attached file,
... but it leads not to perspective squares.

Then I controlled your construction *but with b_i , d_i on the same
side of L_p *
... and now I got the perspective squares,
... which are the same as in the first construction.
When have we to use this alternative condition?
Finally some remarks to your construction:

- (1) The perspectors Q_i of $A_iB_iC_iD_i$ on the F_1F_2 -diameter-circle
... have a tangent parallel to the tangent
of B_i at C_b or D_i at C_d .
- (2) The perspectors Q_i lie symmetrical wrt F_1F_2 (see also
QG-2Cu1).
- (3) The radical axe of C_b and C_d is parallel Q_1Q_2 or orthogonal
 L_p .
- (3) The square diagonals of $A_iB_iC_iD_i$
... are parallel to the angle bisectors of $F_1Q_iF_2$.

Best regards Eckart



2018-04-07.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2965
Date: 10/4/2018 3:40:07
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

Dear Bernard, dear Chris,

studying Voronoi lines,
I got new aspects of QA-Cu7,
... which is the locus of points,
... whose Voronoi lines wrt two opposite QA-lines are
concurrent.

(1) In EQF QA-Cu7 is described as
... pivotal isocubic
... ... with reference triangle QA-P2.QA-P41.QA-P4,
... ... isogonal conjugation
... ... and pivot: infinity point of perpendiculars of QA-L2.

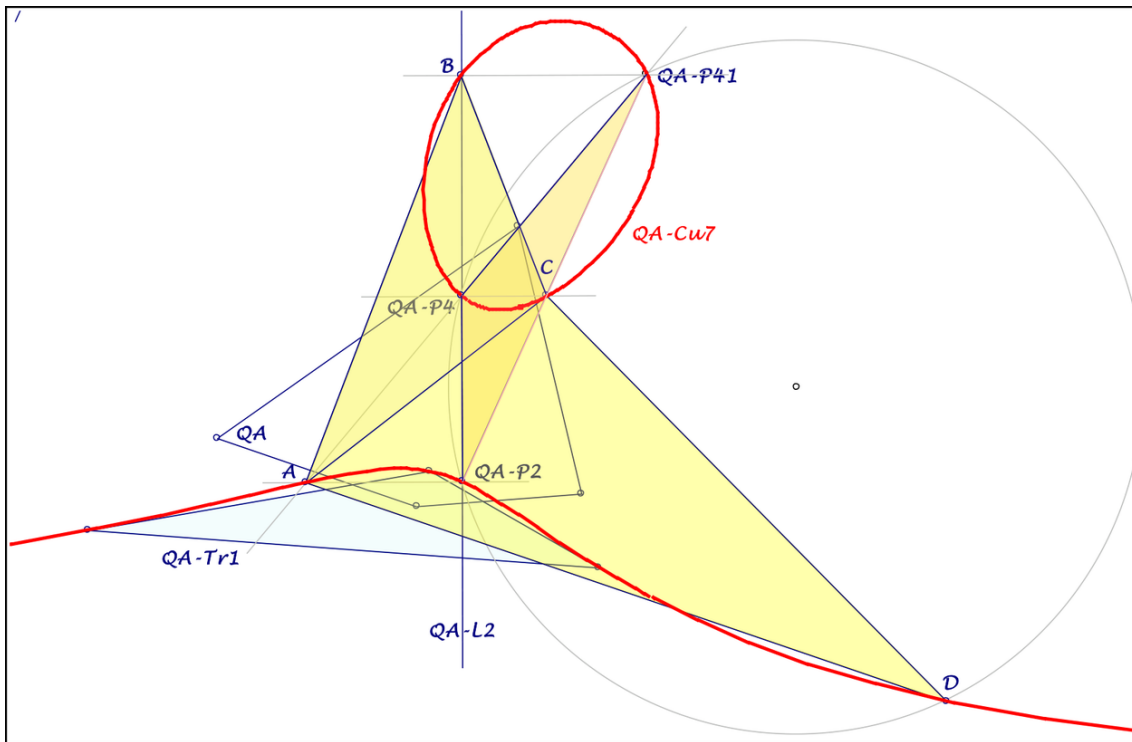
(2) QA-Cu7 can also be described as
... nonpivotal isocubic
... ... with reference triangle QA-Tr1,
... ... isoconjugation QA-Tf2
... ... and root: QA-Tr1 tripole of the collinear intersections
... QG-P1a.QG-P1b^QG-P18a.QG-P18b (on QA-Cu7).

(3) A further description of QA-Cu7 as
... pivotal isocubic
... ... with reference triangle ABC: cevian triangle wrt
QA-P2.QA-P41.QA-P4
... of the infinity point of perpendiculars of QA-L2,
... ... isoconjugation with fixed points QA-P2, QA-P41, QA-P4.
... ... and pivot: reflection of QA-P41 in the circumcenter of
QA-P2.QA-P41.QA-P4.

(4) QA-Cu7 is the cubic QL-Cu1 for a quadrilateral AB, BC, CD,
DA
... with A, B, C vertices of the reference triangle under (3)
... and D = pivot under (3).

I hope, that the attachment will appear.

Best regards Eckart



2018-04-10.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2966

Date: 11/4/2018 2:34:29

From: eckart_schmidt@t-online.de

Subject: WG: Tessellation theory applied in Triangles, Quadr ...

-----Original-Nachricht-----

Betreff: Re: Tessellation theory applied in Triangles, Quadr ...

Datum: 2018-04-11T10:43:45+0200

Von: "eckart_schmidt@t-online.de"

An: "-Group, Quadri-Figures"

Dear Bernard, dear Chris,

wrt Chris' #2962 a
 very curious CABRI-observation:
 The three QA-Cu7 for a QL have 5 common points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

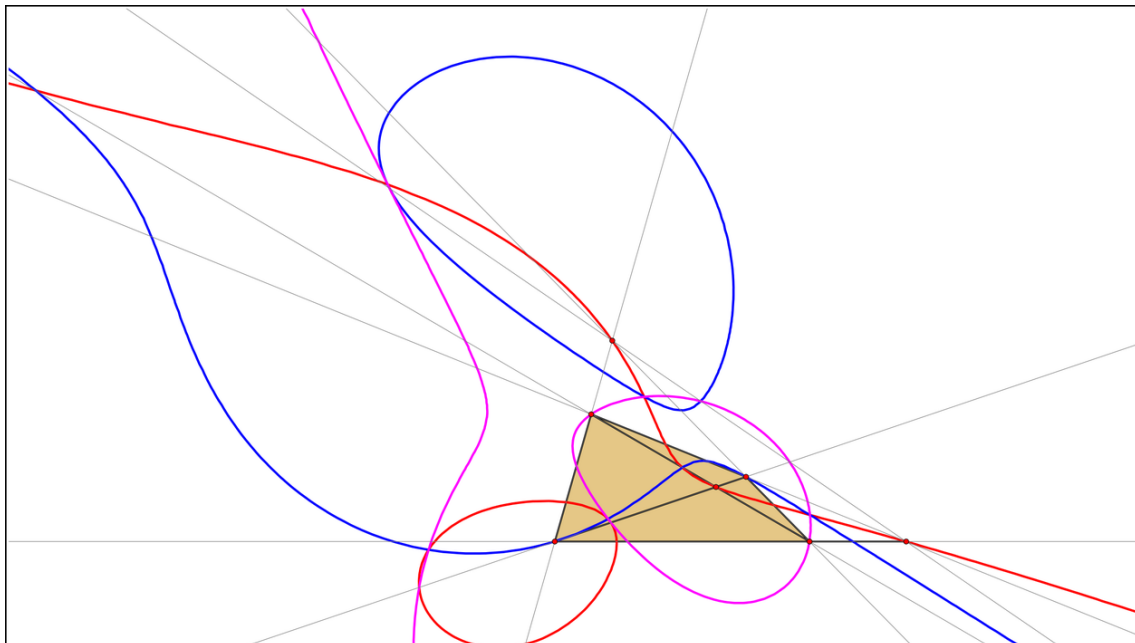
Message: #2967
Date: 11/4/2018 5:18:06
From: bernard.keizer
Subject: WG: Tessellation theory applied in Triangles, Quadr ...

Dear Eckart,
Are the 2 circular points in your 5 points ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2968
Date: 11/4/2018 5:57:11
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Attached a picture about the 5 common points of the 3
QL-versions of QA-Cu7.
Chris



QL-3 versions of QA-Cu7-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2969
Date: 11/4/2018 8:58:30
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

I think Chris' figure answers the question of Bernard.
Up to now I found no properties of the five common points.
But:
The double intersections lie pairwise on the sidelines of
QL-Tr1.
The CSC-images of the vertices of QL-Tr1 are the three QA-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2970
Date: 12/4/2018 3:12:40
From: bernard.keizer
Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Chris, dear Eckart
This cubic is QA-Cu7, the QA-quasi-isogonal cubic
By the way, it is a QL-Cu1, with QL-P1 in QA-P41 and the Newton
Line as the perpendicular bisector of
QA-P2QA-P4 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2971

Date: 12/4/2018 3:34:43

From: chris.vantienhoven

Subject: Tessellation theory applied in Triangles, Quadr ...

Dear Eckart, dear Bernard,

Here another contribution wrt the Voronoi Lines.

Ellipses/Hyperbolas inscribed between QA-opposite lines with QA-P2 and QA-P4 as foci.

Eckart, regarding your remark that QA-P2 and QA-P4 are partner as well as QA-P41 and the infinity point of perpendiculars of QA-L2.

I reasoned that following your property in a 4-Line, combined with the property in a 5-Line (about the foci of inscribed conic being partners), that:

1. there will be a 1 st conic (ellipse of hyperbola) tangent to opposite lines P1.P2 as well as P3.P4 with foci QA-P2 and QA-P4.
2. there will be a 2 nd conic (ellipse of hyperbola) tangent to opposite lines P1.P3 as well as P2.P4 with foci QA-P2 and QA-P4.
3. there will be a 3 rd conic (ellipse of hyperbola) tangent to opposite lines P1.P2 as well as P2.P3 with foci QA-P2 and QA-P4.

Parabolas inscribed between QA-opposite lines with QA-P41 as focus and axis parallel to QA-L9.

Also:

1. there will be a 1 st parabola tangent to opposite lines P1.P2 as well as P3.P4 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.
2. there will be a 2 nd parabola tangent to opposite lines P1.P3 as well as P2.P4 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.
3. there will be a 3 rd parabola tangent to opposite lines P1.P4 as well as P2.P3 with focus QA-P41 and its axis parallel to the perpendicular bisector of line segment QA-P2.QA-P4.

I checked it in Cabri and it was confirmed. No synthetic proof yet.

Summary

Then a summary of the application of the Voronoi Lines in a 2-Line, 3-Line, 4-Line, 5-Line.

In a 2-Line the Voronoi Line is a line through $L1 \wedge L2$.

In a 3-Line the 3 Voronoi Lines for every P in the plane concur in the Isogonal Conjugate of P.

In a 4-Line the 6 Voronoi Lines concur only for points on QL-Cu1 in the Clawson-Schmidt Conjugate of P, also lying on QL-Cu1.

In a 5-Line the 10 Voronoi Lines concur only for the 2 foci of the inscribed conic. The concurring point being the other focus.

Best regards,
Chris

p.s. I am really sorry about the bad performance of Yahoo for its groups.

I have been looking for alternatives, but they too have their own risks.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2972
Date: 12/4/2018 7:24:49
From: eckart_schmidt@t-online.de
Subject: WG: Tessellation theory applied in Triangles, Quadr ...

-----Original-Nachricht-----
Betreff: Re: Tessellation theory applied in Triangles, Quadr ...
Datum: 2018-04-11T10:43:45+0200
Von: "eckart_schmidt@t-online.de"
An: "-Group, Quadri-Figures"

Dear Bernard, dear Chris,

wrt Chris' #2962 a very curious CABRI-observation:

The three QA-Cu7 for a QL have 5 common points.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2973
Date: 13/4/2018 11:08:53
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Eckart, Dear Bernard,

There are 3 versions of QA-Cu7 in a Quadrilateral: Cu7a, Cu7b, Cu7c.

Cu7a, Cu7b, Cu7c each contain points P for which 3 angle-bisector-reflected-lines concur.

We know by sight that there are 5 common points S1, S2, S3, S4, S5 with this concurring-property for all three cubics, however I noticed that for each point Si the concurring points are different !

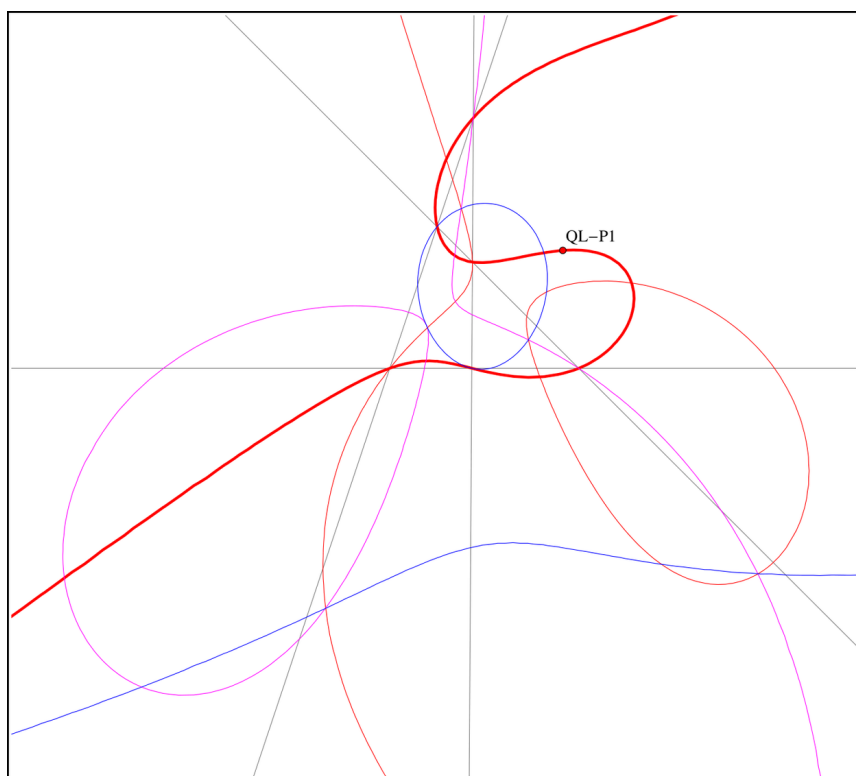
QL-Cu1 has one angle-bisector-reflected-line per cubic of the set Cu7a, Cu7b, Cu7c and produces a quite separate cubic, NOT passing through any of these 5 points S1, S2, S3, S4, S5.

Right now I do not see possibilities to identify these points S1, S2, S3, S4, S5.

Maybe one of you can find a solution?

I enclose another picture with Cu7a, Cu7b, Cu7c and QL-Cu1.

Best regards,
Chris



QL-3 versions of QA-Cu7-02.pdf

Message: #2974
Date: 13/4/2018 3:14:27
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

I searched for the five common points of the three QA-Cu7 for a QL, but I find more properties for the double intersections:

(1) The double intersections lie on the sidelines of QL-Tr1 in the intersections
... with the CSC-circles CI_i of the diameter circles for the sides $S_j S_k$ of QL-Tr1.
(2) The six double intersections are coconic.
(3) The circles CI_i intersect on the cubics QA-Cu7
... in the three QA-P4 and the three collinear QG-P18.
It seems, that the radical center of the $*CI_i*$
... is a point of the circumconic of the five common points.
Can you confirm this?
I have no idea for 5 points wrt a 4-line!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2975
Date: 13/4/2018 5:52:29
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Eckart<

[ES]
It seems, that the radical center of the CI_i
... is a point of the circumconic of the five common points.
Can you confirm this?

Answer: yes, no Cabri-proof because of uncertain five common points, but it looks like it.

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2976
Date: 14/4/2018 11:10:13
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart
No idea for the moment !
I just notice that the 5 real points are in fact 7 with the 2
circular points and define a transformation swapping the 2
double points on the same DT side (harmonic wrt the 2 QL
vertices on this side).
What are the fixed points ? (vertices of the dual QA ?)
Is there a connexion with QA-Cu7 of this dual QA ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2977
Date: 15/4/2018 6:18:26
From: chris.vantienhoven
Subject: WG: Introduction to Quadrilateral Geometry

Dear Eckart,

Thanks for your remarks.
About your question "When have we to use this alternative
condition?", I do not have right away an answer to this.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2978

Date: 15/4/2018 8:38:15

From: chris.vantienhoven

Subject: WG: Introduction to Quadrilateral Geometry

Dear friends,

I received some nice constructions from Nikolaos Dergiades regarding the squares inscribed, circumscribed or perspective with a quadrigon.

Inscribed Square

Construction:

Let ABCD be an arbitrary quadrigon with positive orientation. The rotation of line AD around A by angle -45° gives the line L1.

The rotation of line BC around B by angle 45° gives the line L2. The perpendicular from B to L1 meets the line AD at E and the parallel from E to L1 meets L2 at F.

The perpendicular from A to L2 meets the line BC at H and the parallel from H to L2 meets L1 at G.

The line FG meets the line CD at J.

The parallel from J to L2 meets the line BC at I

and the perpendicular from I to L2 meets the line AB at L.

The perpendicular bisector of JL meets the line AD at K and the line BC at M.

The quadrilateral JKLM is the required square inscribed in ABCD.

Proof:

Let P be an arbitrary point on BC. The perpendicular from P to LP meets the line IJ at Q.

Since the points L, P, I, Q are concyclic

because $\angle LPQ = \angle LIQ = 90^\circ$,

then $\angle PLQ = \angle CIQ = 45^\circ$

$\angle LQP = \angle LIP = 45^\circ$

This means that triangle PLQ is right and isosceles ($PL = PQ$) and if P moves to M then Q moves to J.

Hence triangle LMJ is isosceles with $\angle JML = 90^\circ$.

It is obvious from Thales theorem that the parallel from L to L1 meets

The line AD at I' and the perpendicular from I' to L1 meets CD at J.

Hence similarly as previously the triangle LKJ is isosceles with $\angle LKJ = 90^\circ$ or the quadrigon JKLM is a square.

If the points F, G are on the line CD we have infinite solutions.

e.g. ABCD is a square.

Circumscribed square and perspective square

"I am sending you a trivial solution to problem 3 where the Quadrigon $A_1A_2A_3A_4$ is perspective with the square $S_1S_2S_3S_4$ where $A_1=S_1$.

I am sending also the solution to problem 2 although my solution is no good."

See attachment.

Best regards,

Chris

A quadrigon and a square

Problem 2. Given a quadrigon $A_1A_2A_3A_4$ construct a square $S_1S_2S_3S_4$ circumscribing the quadrigon.

Proof. With diameters $A_1A_2, A_2A_3, A_3A_4, A_4A_1$ we construct the circles C_1, C_2, C_3, C_4 and centers O_1, O_2, O_3, O_4 . We rotate the center O_1 around O_4 by angle $\frac{\pi}{2}$ to get the point O .

If a rectangle $R_1R_2R_3R_4$ is circumscribing the quadrigon then the vertices of the rectangle must lie on the circles C_1, C_2, C_3, C_4 respectively. We rotate the point R_1 around R_4 by angle $\frac{\pi}{2}$ to get the point S .

In order to the rectangle $R_1R_2R_3R_4$ be a square it is sufficient that the point S coincides with R_3 . Since the segments O_1O_4, OO_4 are equally inclined relative to R_1R_4, SR_4 and the orthogonal projection of O_1O_4 on R_1R_4 is $\frac{1}{2}R_1R_4$, the orthogonal projection of OO_4 on SR_4 must be $\frac{1}{2}SR_4$. Hence the point R_3 is the midpoint of SA_4 , that is $OS = OA_4$.

So the locus of S as the point R_4 moves on C_4 is the circle $O(OA_4)$ and since we must have $S \equiv R_3 \equiv S_3$ this point S_3 is found as reflexion of A_4 in OO_3 . The line S_3A_4 gives on the circle C_4 the point S_4 , the line S_4A_1 gives on the circle C_1 the point S_1 and the line S_1A_2 gives on the circle C_2 the point S_2 . $S_1S_2S_3S_4$ is the required square. Similarly we find a second solution if we rotate the center O_1 around O_4 by angle $-\frac{\pi}{2}$.

Problem 3. Given a quadrigon $A_1A_2A_3A_4$ construct a square $S_1S_2S_3S_4$ perspective with the quadrigon.

Proof. (Trivial construction case). With diameter A_1A_4 we construct the semicircle C_4 and diameter A_1A_3 we construct the semicircle C_3 with mid arc M . The line A_4M meets again C_3 at the point S and semicircle C_4 at the point Q .

Since the points A_1, A_3, M, S are concyclic and $\angle QSA_1 = \angle MA_3A_1 = \frac{\pi}{4}$ and the triangle QAS is right at Q we have $QS = QA_1$. If S_3, S_4 are the symmetric of A_1, S relative to Q then the square $S_1S_2S_3S_4$ where $S_1 = A_1$ and $S_2 = S$ is perspective with $A_1A_2A_3A_4$ and perspector S .

Message: #2979
Date: 17/4/2018 11:09:24
From: eckart_schmidt@t-online.de
Subject: 3 foci of Siebeck's sextic

Dear Bernard,

I think, I shall give up. I tried in vain,
to "construct" the Steiner circumellipse St of the foci F_1, F_2, F_3 ,
... for P_1, P_2, P_3 and $P_4 = \text{zero}$,
... using I, I' on the main axis of the Steiner circumellipse of $P_1P_2P_3$
... with $OI = OI'$ as $\sqrt{2}/4$ of the focus width of this circumellipse.

We can get St by Siebeck's method $C - D - M - X$
(X reflection of O in M)
... for special circles round O for C .

But Siebeck's radius for such a circle is not correct!
There are two C -circles for Siebeck's method
... with radii with product OI^2 ,
... their sum is the first axis and their difference the second axis of St .
These properties will be well known to you.

Best regards Eckart

PS: The radii are not only dependent of the Steiner circumellipse of $P_1P_2P_3$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2980
Date: 19/4/2018 9:40:51
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

searching for the five common points of the cubics QA-Cu7 for a QL,

I found no idea for 5 points of a 4-line,
so I had a look on the conic through these five points.
I QFG-message 2974 I mentioned a first point of this conic,
now I can offer three further points (CABRI-observations):

First point R of the conic:

... R = radical center for the CSC-circles
of the diameter circles for the sides of QL-Tr1.

Three further points S of the conic (wrt the quadrigon
versions):

... S = 2nd intersection of QA-P4.QG-P18 and the QA-Tf2-conic
of QA-P4.QG-P19.

Remark:

The points S are the 3rd intersections of QA-P4.QG-P18 and
QA-Cu7.

The lines QA-P4.QG-P18 intersect in the first point R.

What about a 5th point for the conic?

Best regards Eckart

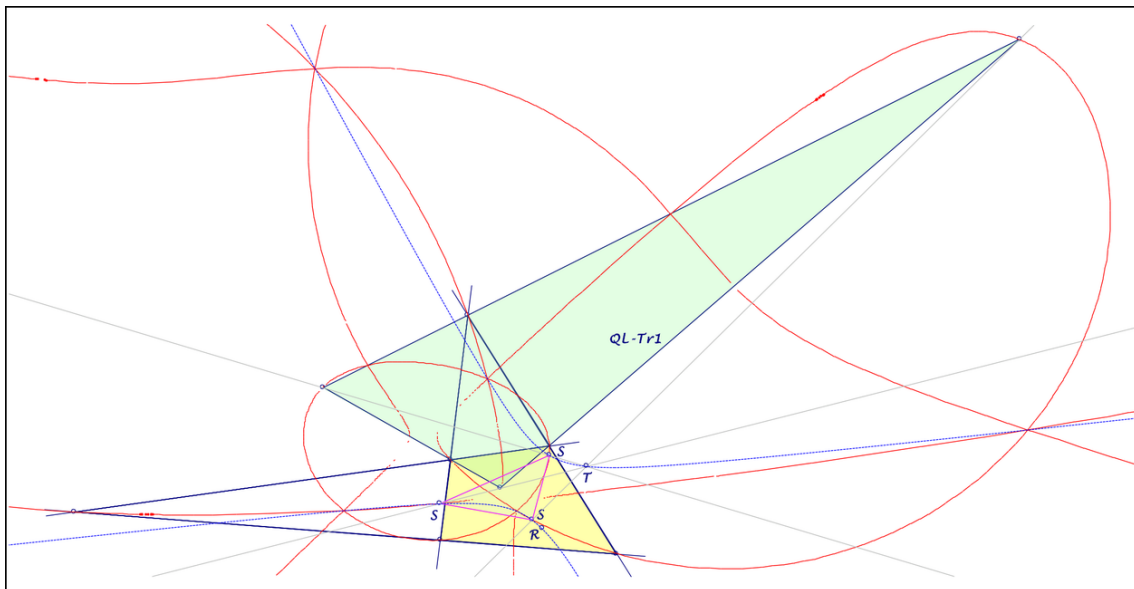
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2981
Date: 19/4/2018 12:00:29
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

I just found a 5th point T (in addition to #2980)
... for the conic of the 5 common points of the QL-versions of
QA-Cu7:
T = perspector of QL-Tr1 and S-triangle.

Best regards Eckart



2018-04-19.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2982
Date: 19/4/2018 11:53:57
From: chris.vantienhoven
Subject: QA-Tf2 in a QL

Dear Friends,

I noticed that the 3 versions of QA-Tf2(P) in a Quadrilateral are collinear.

Hence we have a transformation mapping a point into a QA-Tf2-driven line.

When we transform a point of one of the basic lines of the Reference Quadrilateral it will be mapped into the basic line it is on.

I haven't got time to study it further.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2983
Date: 20/4/2018 12:37:43
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Eckart, dear Bernard,

Eckart, very good findings!
Finally we make progress.

Some additions:

1. The conics can be constructed as the conic through the vertices of the QA-DT of the QL-Quadrignon and QA-P41, QG-P18 of the QL-Quadrignon (obvious when you analyze it).

2. Point R is also the intersection point of the versions in the 3 QL-Quadrignons of QA-P4.QG-P18.

3. Point U (new point) is the intersection point of the versions in the 3 QL-Quadrignons of QA-P41.QG-P18.

4. Sa, Sb, Sc also can be constructed as:

Sa = QA-P41a.QG-P19a ^ QA-P4a.QG-P18a]

Sb = QA-P41b.QG-P19b ^ QA-P4b.QG-P18b]

Sc = QA-P41c.QG-P19c ^ QA-P4c.QG-P18c]

and also as:

Sa = QA-Tf2[QA-P41a.QG-P18a ^ QA-P4a.QG-P19a]

Sb = QA-Tf2[QA-P41b.QG-P18b ^ QA-P4b.QG-P19b]

Sc = QA-Tf2[QA-P41c.QG-P18c ^ QA-P4c.QG-P19c]

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2984
Date: 20/4/2018 1:35:35
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Chris,

thanks for your feedback.

Wrt 1. I think, you mean the conics for my constructing of the points S_a , S_b , S_c .

Wrt 2. The point R has further properties:

... R lies on the circumcircle of the three QA-P4,
which is CSC(QL-Ci1).

... R lies on the perpendicular of the QG-P18-line

... through the center of CSC(QL-Ci1).

... R lies on the polar of QL-P26

... wrt the conic through the double intersections
of the three QA-Cu7.

Wrt 3. Sorry, I cannot reproduce the new point U .

But QA-P41.QG-P18 and QA-P4.QG-P19 intersect on QA-Cu7

... and give a triangle similar to the QA-P4-triangle.

Wrt 4. This construction of the points S_a , S_b , S_c

... seems to be easier as me description.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2985
Date: 20/4/2018 3:50:05
From: eckart_schmidt@t-online.de
Subject: QA-Tf2 in a QL

Dear Chris,

with interest I searched for properties of this new
QL-transformation,
... mapping a point to the line
 of its three collinear QA-Tf2-images,
... but I found only some splitter:

The line for QL-P1 is CSC(QL-Ci6), bearing QL-P26.
The line for QL-P8 bears QL-P23.
... Its QL-Tf2-line is a parallel,
... ... homothetic wrt QL-P8 and ratio $-1/2$.
The line for QL-P13 is orthogonal QL-P1.QL-P7, bearing QL-P19.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2986
Date: 20/4/2018 9:20:54
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Eckart,

Here some additions wrt your last remarks.
You are right, point U should be the common point of the 3 versions of QA-P4.QG-P19.

* This point happens to be CSC(QL-P24).

I also noticed some incidences regarding point R:

* R is collinear with QL-P1, CSC(QL-P10), CSC(QL-P16)

* R is collinear with CSC(QL-P9), CSC(QL-P9005)

* R is collinear with CSC(QL-Px), CSCe(QL-P1.QL-P8)

(QL-Px = Reflection of QL-P1 in QL-P11 = point on Medial Circle)

I have no more ideas yet to find the 5 common points.

Maybe it could be a common Caley-Bacharach point of 8 QA-Cu7-points?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2987

Date: 20/4/2018 10:56:54

From: peter.liepa@gmail.com

Subject: QA: intersection points are on radical axes of certain circle pairs

Hi all,

I just joined this group, after discovering the Encyclopedia of Quadri-Figures. I had innocently been working on a question on math.stackexchange.com when I fell into this world.

On a QA construct the midpoints and intersections. For a given pair of opposite sides, create a circle through the intersection and midpoints corresponding to the pair.

This way we get three circles. I'd like to show that, for any pair of these circles, the intersection that is not on the two circles lies on their radical axis. So far I'm stumped and I was hoping someone here could help.

I've built a Geogebra demonstration at

<https://www.geogebra.org/m/ZJEBQJFq>. Sorry, the labeling there does not conform to the conventions in EQF. The QA is A,B,C,D. The circles are in blue, the radical axes are in green. The intersections are labelled "I" with a subscript indicating the intersecting sides, and the midpoints are labeled M, N, with subscripts indicating the sides they belong to.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2988
Date: 21/4/2018 10:18:51
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Chris,
thanks for the unexpected properties of point R.
The point CSC(QL-P9005) will be a typo, it can be QL-Px in your definition.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2989
Date: 21/4/2018 2:00:22
From: eckart_schmidt@t-online.de
Subject: QA: intersection points are on radical axes of certain circle pa

Dear Peter,
I try, to describe your problem in EQF-geometry:
You consider a quadrangle QA
... with 3 "intersections" S1, S2, S3,
... .. which are the vertices of the diagonal triangle QA-Tr1,
... and three circles wrt Si.

The radical center of the 3 circles is the Gergonne-Steiner point QA-P3.
If you consider a pair of these circles for Si, Sj
... and a quadrigon version QG of QA
... .. with QG-2P2 as Si, Sj,
... the 2nd intersection of the circles
 is the Miquel point QL-P1,
... collinear with Sk = QG-P1 and QA-P3 on the radical axis.
You find these collinear points in EQF under QG-2.
Perhaps there are further informations in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2990

Date: 21/4/2018 2:47:02

From: eckart_schmidt@t-online.de

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Peter,

I try, to describe your problem in EQF-geometry:

You consider a quadrangle QA

... with 3 "intersections" S1, S2, S3,

... .. which are the vertices of the diagonal triangle QA-Tr1,

... and three circles wrt Si.

The radical center of the 3 circles is the Gergonne-Steiner point QA-P3.

If you consider a pair of these circles for Si, Sj

... and a quadrigon version QG of QA

... .. with QG-2P2 as Si, Sj,

... the 2nd intersection of the circles

is the Miquel point QL-P1,

... collinear with Sk = QG-P1 and QA-P3 on the radical axis.

You find these collinear points in EQF under QG-2.

Perhaps there are further informations in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2991

Date: 22/4/2018 12:39:01

From: eckart_schmidt@t-online.de

Subject: 3 QL-versions of QA-Cu7

Dear Chris,

another sight of Sa, Sb, Sc:

Sa, Sb, Sc are the isogonal conjugates of QG-P1 wrt QA-P2,
QA-P4, QA-P41

... for the quadrigon-versions of the quadrilateral.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2992
Date: 22/4/2018 3:56:12
From: peter.liepa@gmail.com
Subject: QA: intersection points are on radical axes of certain circle pa

Dear Eckart,

Thank you for translating into EQF terminology.
Since QG-2 is mainly credited to you, is there a proof on your site (or anywhere) that QL-P1, QG-P1, QA-P3 are collinear? Meanwhile I will try to prove it myself.
Best regards,

Peter

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2993
Date: 23/4/2018 10:00:32
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart
I wasn't home last week, I just notice after coming back that you have made a lot of discoveries. Congratulations !
I try to catch up in the coming days ...
First reaction, I don't understand Eckart's last message.
The DT vertices of the 3 QA's of the QL are the QL vertices and the DT vertices of the QL.
Their isogonal conjugates wrt the 3 triangles QA-P2QA-P4QA-P41 of the 3 QA's belong to the 3 QA-Cu7 and are 9 different points ; they don't belong to the group of 5 triple points and can't be the same 3 points S.
What is my misunderstanding ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2994
Date: 23/4/2018 10:33:44
From: bernard.keizer
Subject: QA-Tf2 in a QL

Dear Chris, dear Eckart
Again, just a first reaction.
Beautiful new QL's transformation mapping a point to the line of the 3 QA-Tf2 wrt the 3 QA's of the QL !
The dual transformation is a QA's transformation mapping a line to the point of concurrence of the 3 QL-Tf2 wrt the 3 QL's of the QA ...
Starting with a dual QA/QL
1) what is the link between a point, it's QA-Tf2 and the dual point of it's line of QA-Tf2 wrt the QL ?
2) what is the link between a line, it's QL-Tf2 and the dual line of the concurring point of it's QL-Tf2 wrt the QA?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2995
Date: 23/4/2018 10:48:12
From: bernard.keizer
Subject: 3 foci of Siebeck's sextic

Dear Eckart,
I realise some of my messages were lost !
For me, there is nothing more to find and all is clear !
1) A few lines are missing in Siebeck's article (he or he's publisher ?)
2) He's construction is exactly the same as the calculation of the cubic roots : with $uv = -p/3$
 u and v , $j*u$ and $j*j*v$, $j*j*u$ and $j*v$ and the 2 circles carrying the u and the v are CSC wrt the circle with diameter II' , which explains the radii (both are correct !) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2996
Date: 23/4/2018 10:57:06
From: bernard.keizer
Subject: 2 Foci and 3 RH's

Dear Chris, dear Eckart

I always fear to loose my messages, which doesn't encourage to make new ones ...

For a triangle, the only points which are symmetric wrt the centroid and isogonal conjugates are the foci of the Steiner inellipse.

Any point X defines with these 2 foci F and F' a rectangular hyperbola centered in their middle G with asymptotes parallel to the bisectors of the angle FXF' ; (the RH passes through the symmetric X' of X wrt G).

This is in particular true for the vertices A , B and C of a triangle having these point F and F' as foci of the Steiner inellipse and we know that the bisectors of the angle FAF' are the bisectors of the triangle in A ...

The foci F and F' are the intersections of 3 rectangular hyperbolas through each vertice with asymptotes parallel to the bisectors in this vertice.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2997
Date: 23/4/2018 11:24:46
From: bernard.keizer
Subject: 3 foci and 4 SC's

Dear Chris, dear Eckart

This is the following of the previous message.

Let's define the "symmetrics" of a point X wrt 2 points F and F' as the 2 points Y and Z such as the triangle has the points F and F' as foci of the Steiner inellipse.

Let's define the trisectors of a point wrt 3 points as the mean directions of the 3 lines passing through the point X and one of the 3 points.

Let's define the "isogonals" of a point X wrt a QA the 2 points such as the trisectors in each vertice wrt the triangle formed by X and the 2 points are the same as the trisectors wrt the triangle formed by the 3 other QA vertices. (The 3 points are the foci of a curve tangent to the 6 QA sides).

The foci of Siebeck's cubic are the only points for which the "symmetrics" and the "isogonals" coincide.

Any point X defines with the 3 points F1, F2 and F3 a cubic stelloid centered in the centroid, passing through the "symmetrics" of the points wrt the foci F and F' of the Steiner inellipse of F1F2F3 and having asymptotes parallel to the trisectors wrt the 3 foci F1, F2 and F3.

This is in particular true for each QA vertice, where the trisectors are the same as wrt the 3 other vertices.

Knowing the 2 foci F and F', the 3 foci of Siebeck's sextic are the intersections of the 4 cubic stelloids passing through a QA vertice and it's 2 "symmetrics" with asymptotes the parallels through the centroid to the trisectors in this vertice.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2998
Date: 23/4/2018 12:16:40
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard,

I try to answer your 5 messages, but give me time!

Wrt #2993:

Take a quadrigon version of the quadrilateral

... and consider it also as quadrangle:

Now the isogonal conjugate of QG-P1 wrt QA-P2.QA-P4.QA-P41

... is one of the S-points (special new QG-point).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2999
Date: 23/4/2018 7:23:56
From: eckart_schmidt@t-online.de
Subject: QA-Tf2 in a QL

Dear Bernard,

wrt #2994 : There are rarely EQF-relevant properties for these new transformations.

The QA-transformation, mapping a line

... to the common point of its QL-Tf2-lines

wrt the QA-quadrigon versions,

... maps a line-pencil to a QA-circumscribed cubic

with double point ...

Wrt 1): Mapping the dual line of a point in the sense above,

... we get a point to point QA-transformation,

... which maps a point P to a point Q

on the QA-circumconic through P

... with QA-Tf2(P) pole of PQ.

There will be an analog result for quadrilaterals.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3000
Date: 23/4/2018 7:24:34
From: eckart_schmidt@t-online.de
Subject: QA: intersection points are on radical axes of certain circle pa

Dear Peter,

sorry, I cannot offer a synthetic proof,
but there will be an analytical method
with coordinates, given in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3001
Date: 23/4/2018 8:08:37
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Eckart,
Thanks for the explanation !
Do I understand correctly that your 3 points S are the 6th
intersections between the mysterious conic through the 5 triple
points and the 3 QA-Cu7 ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3002

Date: 23/4/2018 11:13:46

From: chris.vantienhoven

Subject: Old Babylonians used Trigonometry 3700 years ago

THE EPOCH TIMES (<https://m.theepochtimes.com/>)

3,700-Year-Old Babylonian Stone Tablet Is Translated

A 3,700-year-old Babylonian tablet was translated last August and might rewrite the history of math, suggesting that

trigonometry may have been developed before the ancient Greeks.

The researchers said the tablet proves that the Babylonians

developed trigonometry some 1,500 years prior to the Greeks.

"Our research reveals that Plimpton 322 describes the shapes of

right-angle triangles using a novel kind of trigonometry based

on ratios, not angles and circles," Dr. Daniel Mansfield of the

School of Mathematics and Statistics at the University of New

South Wales Faculty of Science, said in a university news

release

(<https://newsroom.unsw.edu.au/news/science-tech/mathematical-mystery-ancient-clay-tablet-solved>).

"It is a fascinating mathematical work that demonstrates

undoubted genius. The tablet not only contains the world's

oldest trigonometric table; it is also the only completely

accurate trigonometric table, because of the very different

Babylonian approach to arithmetic and geometry."

See https://m.theepochtimes.com/3700-year-old-babylonian-stone-tablet-is-translated_2496112.html

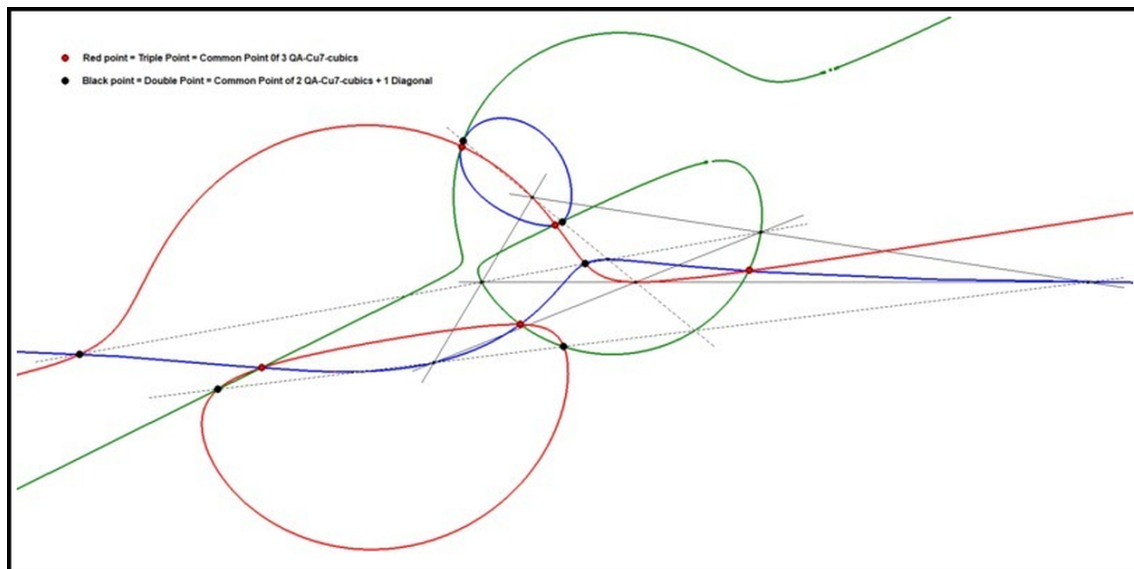
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3003
Date: 24/4/2018 12:01:19
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Eckart, dear Bernard,

1. Attached another picture of the 3 QL-versions of QA-Cu7 with the 5 triple-points and the 6 double-points.
2. The lines QA-P18a.QA-P19a, QA-P18b.QA-P19b, QA-P18c.QA-P19c wrt the 3 QL-Quadrilaterals bound a triangle which is perspective with the QL-Diagonal Triangle with perspector W.

Best regards, Chris



QL-3 QL-versions of QA-Cu7-with-5-common-points-13-DoublePoints.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3004
Date: 24/4/2018 10:55:33
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart
A bicursal QL-Cu1 is determined by the 3 points QL-P1 and QL-2P2a and b ; the CSC is centered in QL-P1 and swaps the 2 QL-2P2, the Newton Line is the perpendicular bisector of QL-2P2a and b ...
For QA-Cu7, QA-P41 is QL-P1 and QA-P2 and QA-P4 are the QL-2P2a and b ; these 2 points play a completely symmetric role and QG-P18 is either the QA-Tf2 with DT which swaps QA-P4 and QA-P41 or with the triangle of the QG-P19 which swaps QA-P2 and QA-P41. All this is well known.
Now, a simple question

Let's give 2 QL-Cu1 : they intersect in 7 real points and in the 2 circular points.
Among the 2 QL-2P2 of each, one is QA-P2 and the other QA-P4 and among the 7 are the 5 searched triple points and 2 double points on a diagonal of DT.
Is it simply possible to identify which is which and to find finally the 3rd QL-Cu1 and the QL, of course with it's own QL-Cu1 (which may be monocursal, as shown in the 1st picture of Chris).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3005
Date: 24/4/2018 11:38:19
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard,

wrt #3001: You understand correctly,
that a S-point is the 6th intersection of a QA-Cu7 and the conic of the 5 triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3006
Date: 24/4/2018 4:15:07
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

further observations wrt QA-Cu7:

Let Tr be a triangle on QA-Cu7

... and X the 4th intersection of QA-Cu7
and the circumcircle of Tr .

- (1) The triangle QA- Tr 1 has the X -point QA-P2.
- (2) The triangle QA-P2.QA-P4.QA-P41 has as X -point
... the reflection of QA-P41 in the center of the circumcircle.
- (3) The QG-P18 triangle has its X -point in QA-P41.
- (4) The QG-P19 triangle has its X -point in QA-P4.

Let P be a point on QA-Cu7 and Q its CB-point wrt ...

... QA- Tr 1, QG-P18 triangle and QA-P2: $P, Q, QA-P41$ collinear.

... QA- Tr 1, QG-P18 triangle and QA-P4: PQ orthogonal

QA-P2.QA-P4.

... QA- Tr 1, QG-P18 triangle and QA-P41: $P, Q, QA-P2$ collinear.

... QA- Tr 1, QG-P19 triangle and QA-P2: $P, Q, QA-P4$ collinear.

... QA- Tr 1, QG-P19 triangle and QA-P4: $P, Q, QA-P2$ collinear.

... QA- Tr 1, QG-P18 triangle and QA-P41: PQ orthogonal

QA-P2.QA-P4.

... QG-P18 and QG-P19 triangle and QA-P2: PQ orthogonal

QA-P2.QA-P4.

... QG-P18 and QG-P19 triangle and QA-P4: $P, Q, QA-P41$
collinear.

... QG-P18 and QG-P19 triangle and QA-P41: $P, Q, QA-P4$
collinear.

QA-Cu7 is isogonal invariant wrt QA-P2.QA-P4.QA-P41:

The midpoints of isogonal points on QA-Cu7

... lie on an orthogonal hyperbola,

... ... centered on the circumcircle of QA-P2.QA-P4.QA-P41

... ... in the 2nd intersection with a parallel to
QA-P2.QA-P4 through QA-P41,

... ... with asymptotes parallel and orthogonal QA-P2.QA-P4,

... bearing QA-P6, the midpoint of QA-P2.QA-P4.

But no link to the 5 triple points!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3007
Date: 24/4/2018 5:26:44
From: chris.vantienhoven
Subject: QA-Tf2 in a QL

Dear Eckart, dear Bernard,

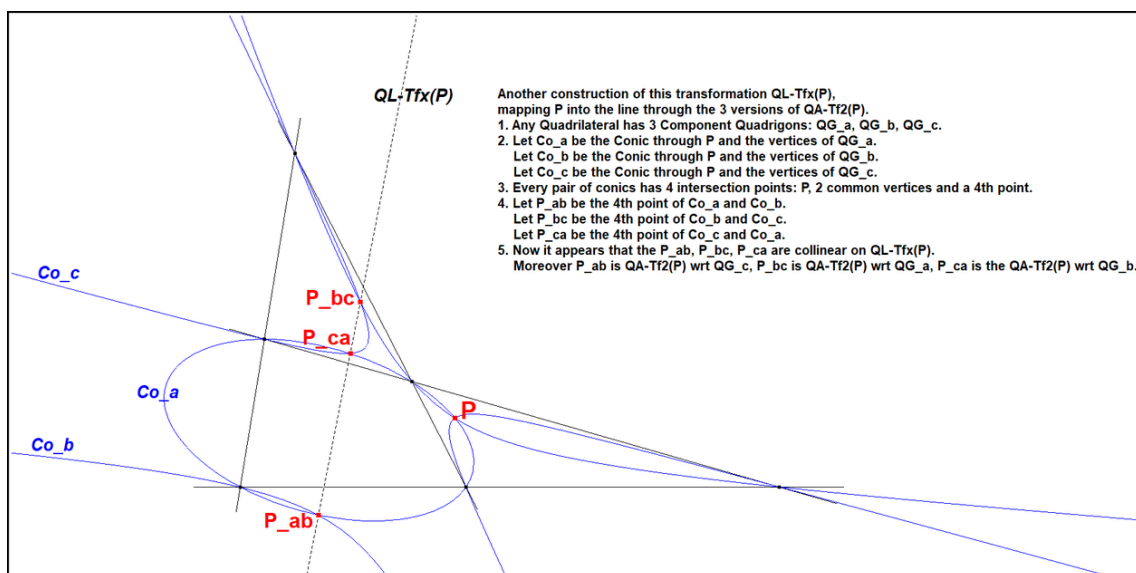
Another construction of this transformation $QL-Tfx(P)$, mapping P into the line through the 3 versions of $QA-Tf2(P)$.

1. Any Quadrilateral has 3 Component Quadrigons:
 QG_a, QG_b, QG_c .
2. Let Co_a be the Conic through P and the vertices of QG_a .
 Let Co_b be the Conic through P and the vertices of QG_b .
 Let Co_c be the Conic through P and the vertices of QG_c .
3. Every pair of these conics has 4 intersection points:
 P , 2 common vertices and a 4th point.
4. Let P_{ab} be the 4th point of Co_a and Co_b .
 Let P_{bc} be the 4th point of Co_b and Co_c .
 Let P_{ca} be the 4th point of Co_c and Co_a .
5. Now it appears that P_{ab}, P_{bc}, P_{ca} are collinear on $QL-Tfx(P)$.

Moreover P_{ab} is $QA-Tf2(P)$ wrt QG_c , P_{bc} is $QA-Tf2(P)$ wrt QG_a , P_{ca} is the $QA-Tf2(P)$ wrt QG_b .

See attachment.

Best regards,
 Chris



QL-Tf92-QA-Tf2-line-10.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3008
Date: 25/4/2018 7:35:24
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear chris, dear Eckart

For any point X on the Van Rees bicursal curve QL-Cu1, defined by the 3 points already explained, it holds that $CSC(isogonal(X)) = isogonal(CSC(X))$; that property explains many of the alignments ...

For any circle through one of the 2 CSC points, the 3 other intersections are DT vertices, the CSC circle cuts the curve in the other point and in the CSC of the DT vertices, id est the QG-P19.

For any couple DT vertice and QG-P19, QG-P18 is the intersection between the lines joining the other point to the DT vertice and the 1rst point to the QG-P19 ; these points QG-P18 are the same whether you choose one or the other point as QA-P2 or QA-P4.

I reproduced without difficulty (aber mit Müh und Not) with Geogebra the properties of points R as intersection of the lines QA-P4isog(Ti), S1, S2 and S3, which are the isogonals of the Ti (or the CSC of the QG-P19) and T as perspector of the Si and the Ti, but I couldn't reproduce the point U.

As there were different versions, could you please confirm me the right one ? Thanks in advance

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3009

Date: 25/4/2018 8:00:45

From: peter.liepa@gmail.com

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Eckart and Chris,

Analytical methods definitely seem like a path of lesser resistance. Although synthetic methods are more satisfying they can be elusive, whereas barycentric methods neither rely on nor build insight.

I've been able to resolve my question about collinearity. I used the Baricentricas package cited in EQF. Barycentric versions of points, lines, and circles are easily transferred into Geogebra as a visual verification. It was fairly easy to prove what I was after, after overcoming initial reservations about using barycentric coordinates.

For anybody who is interested, the result is written up at <https://math.stackexchange.com/a/2753547/1257> (<https://math.stackexchange.com/a/2753547/1257>) , answering an original question posed at <https://math.stackexchange.com/questions/2710487/gf-is-the-radical-axis/2753547#2753547> (<https://math.stackexchange.com/questions/2710487/gf-is-the-radical-axis/2753547#2753547>).

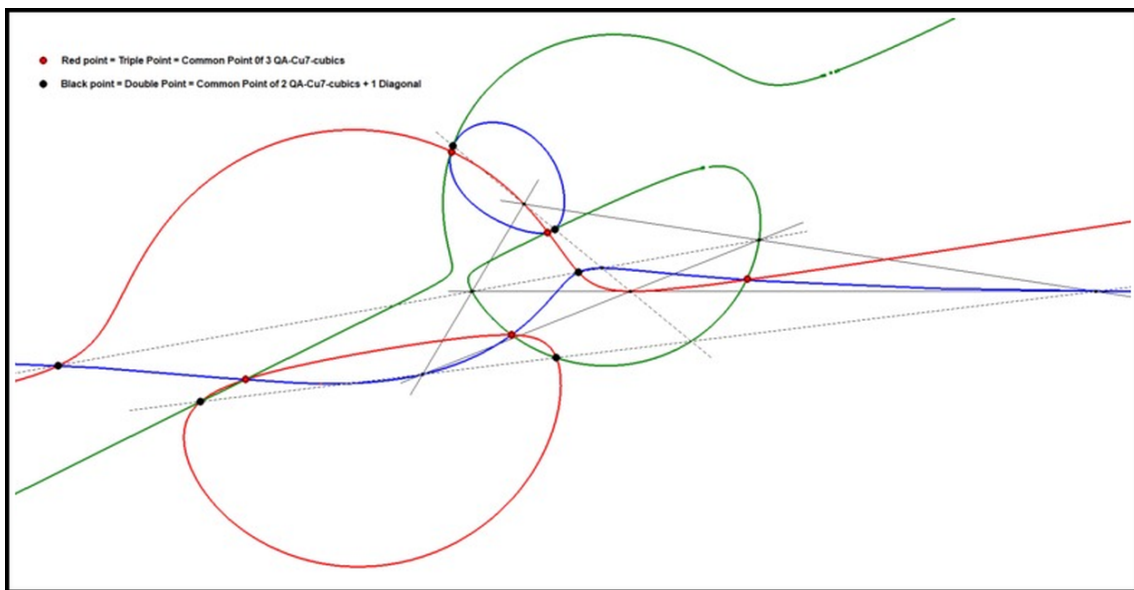
Thank you for your generous help.

Best regards,
Peter

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3010
Date: 25/4/2018 9:11:53
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Bernard,
Point U = the common point of the 3 versions of QA-P4.QG-P19.
See attached file.
Best regards,
Chris



QL-3 QL-versions of QA-Cu7-with-5-common-points-13-DoublePoints.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3011

Date: 25/4/2018 9:48:41

From: chris.vantienhoven

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Peter,

I am glad you were able to resolve your problem.

I am impressed that you managed to do so with so much new information for you from EQF as well as the Baricentricas-package.

Hope to meet you soon at the Quadri-Figures Group with another challenge.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3012

Date: 25/4/2018 11:08:25

From: bernard.keizer

Subject: 3 QL-versions of QA-Cu7

Dear Chris,

Thanks for your quick answer §

Unfortunately

1) I cannot open your attached photos; Could you please make it as you usually do as pdf files. Thanks in advance

2) For me, the 3 lines QA-P4QG-P19 intersect in fact in a point U, which lies also on the circle through the 3 QA-P4 and is the diametral point of R on this circle, but the point U doesn't lie on the conic through R, the 3 S and T, which contains the 5 triple points. I don't see where my mistake is ...

Best regards

Bernard

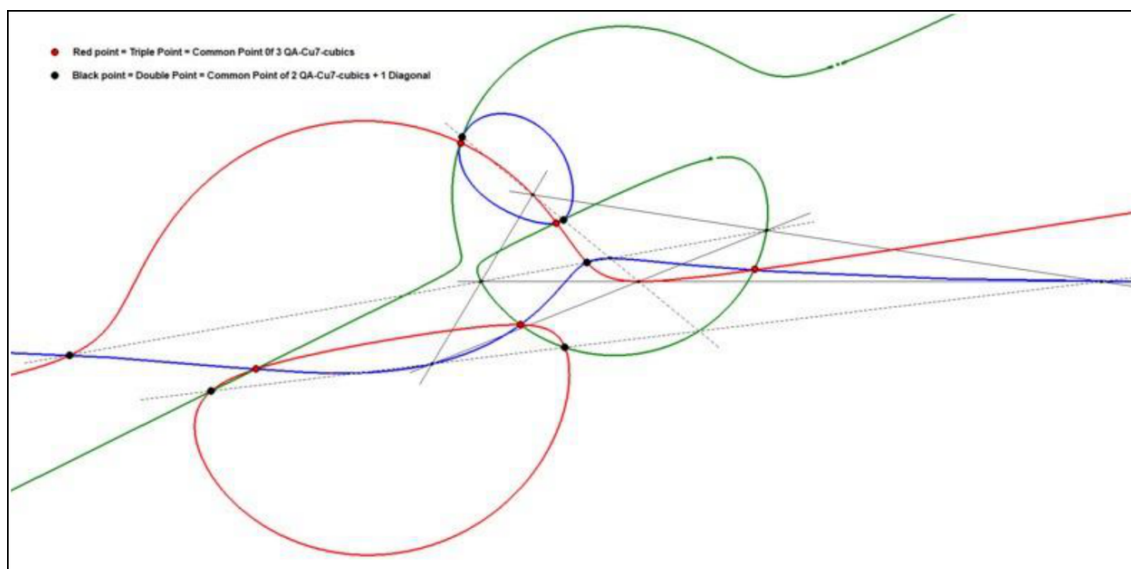
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3013
Date: 26/4/2018 7:52:44
From: chris.vantienhoven
Subject: 3 QL-versions of QA-Cu7

Dear Bernard,

Attached a PDF-file of the picture.
The former picture was a PNG-file. It is possible to import a PNG-file in Microsoft Word. Once imported it also becomes visible. So maybe you still will be able to open it.
For me it is new that U lies on the circumcircle of the 3 QL-versions of QA-P4 and it is diametral to R . Nice finding!
By the way the Clawson-Schmidt Conjugate of this circle is a circle circumscribed to the QL-Diagonal Triangle. I don't know what that means.
And indeed you are right U doesn't lie on the RST-conic (through R , the 3 S and T), which contains the 5 triple points. It was just mentioned as another incidence.
Best regards,

Chris



QL-3 QL-versions of QA-Cu7-with-5-common-points-13-DoublePoints.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3014
Date: 26/4/2018 9:58:30
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris,
Thanks again for your new answer !
I still couldn't open the file, I don't know why ...
But I don't need it now as you precised that U wan't on the conic of the 5 + 5 points !
On a general bicursal Van Rees curve (QA-Cu7 or QL-Cu1), defined by a point QL-P1 and 2 CSC points symmetric wrt the Newton Line, for any point X, there is a circle through X, $\text{isog}(\text{CSC}(X))$, which is also $\text{CSC}(\text{isog}(X))$, as explained in my last message, and these 2 points ; this circle is centered on the Newton Line in the middle of the segment joining X to $\text{isog}(\text{CSC}(X))$.
So the lines joining QA-P2 or QA-P4 to the 2 points X and $\text{isog}(\text{CSC}(X))$ are perpendicular.
This property is true for $\text{isog}(X)$ and $\text{CSC}(X)$, which lie on the curve.
This is true in particular for the isogonals of DT vertices and the CSC of these DT vertices, which are the QG-P19.
For your 2nd remark, if the CSC of the circle through the 3 QA-P4 is the DT circumcircle, containing therefore the QA-P2 of the dual QA, it means that the circle of the 3 QA-P4 contains also the QA-P4 of the dual QA.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3015
Date: 26/4/2018 5:40:20
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard, dear Chris,

related to QA-Cu7, there is another cubic ,
... which leads to a new QL-point,
... perhaps relevant for the 5 common points of QA-Cu7.

The cubic QA-Cu7 for a quadrangle QA
... bears the QG-P1-triangle and is invariant wrt QA-Tf2,
... bears the QG-P18-triangle and is invariant wrt the isogonal
conjugation,
... bears the QG-P19-triangle and is invariant wrt an
isoconjugation,
... .. which swaps QA-P2 and QA-P41.
There is another cubic QA-Cux (see #2017, example 2),
... bearing the same triangles, invariant wrt the same
isoconjugations,
... but additional bearing the QA-vertices and the perspectors
of the triangles.

The 3 QL-versions of QL-Cux have 7 common points,
... six are the QL-vertices, additional a new QL-point.
What about this new QL-point?

Best regards Eckart

PS: Have we already studied the common points
... of the QL-versions of QA-Cu1,2,3,4,5
(except the 6 QL-points)?
For QA-Cu1 it is QL-P1, for QA-Cu2 it is the infinity point of
QL-L1,
... for QA-Cu3 there are up to three points
and a common tangent QL-L1,
... for QA-Cu4,5 I found no further common points.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3016
Date: 27/4/2018 4:33:59
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart

1) Beautiful curve and beautiful properties !

I had completely forgotten the fact that the QG-P18 may be defined as the intersections of the lines through the DT and the QG-P19, as explained in your ancient message 2017 !!!

There are infinities of such QA-Cux for one bicursal QL-Cu1 defined by the 3 points.

Each circle through one of the 2 CSC points gives the DT vertices and the CSC circle gives the other point and the QG-P19 ; QA-Cux bears the fixed points of both isogations with one triangle swapping the 3rd point and one of the 2 points and these 2 QA's are in perspective with perspectors the in and excenters of the QG-P18 triangle ! The 3 QA's form a Desmic system and their DT's, which are the DT triangle, the QG-P19 and the QG-P18 triangles, lie on the same cubic and are also in perspective with aligned perspectors ...

The vertices of the 3 DT, QG-P19 and QG-P18 triangles form a C-B system ...

2) In fact, the intersections of 3 QA-Cux of the same DT or 3 QL-Cux of the same QA deserve a complete study, but some seem more interesting than others ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3017
Date: 27/4/2018 8:53:08
From: eckart_schmidt@t-online.de
Subject: Line \longleftrightarrow point QL-transformation

Dear Bernard, dear Chris,

I found the new QL-point R (see #2974)
... for the conic of the 5 QA-Cu7-intersections
... as radical center of the CSC-circles for the diameter
circles of the vertices of QL-Tr1.
This can be generalized for any three points instead of the
vertices of QL-Tr1,
... so we can map any triple of different points to an image
point wrt a QL.
(1) Every triple of QG-Px can be mapped to a point:
... the collinear QG-P16-triple gives QL-P6,
... the collinear QG-P18-triple gives QL-P11.
(2) Every collinear triple of QA-Tf2(QL-Px) can be mapped to a
point:
... for example: QA-Tf2(QL-P13) give QL-P7.
(3) Equilateral triangles in the same circle give the same
result.
(4) For the four triples of collinear points of the QL
... we get four concyclic points on QL-Ci3.
(5) For the four triples of QL-triangle vertices
... we get four collinear points on CSC(QL-Ci5),
... bearing the center of the QA-P4 circumcircle.
(6) For any three points on a line we get the same point:
... for QL-L1 we get the intersection of the line CSC(QL-Ci5)
... .. and a perpendicular to QL-L1 in the intersection with
QL-L6,
... for QL-L2 we get QL-P4,
... for QL-L3 we get the CSC-image of the pedal point of QL-P1
on QL-L2.
(7) This leads to a QL-transformation Tf, which maps lines to
points.
Wrt its inverse: Tf for a line pencil gives collinear points:
... Tf-1(QL-P26) is a line through QL-P6, orthogonal
QL-P1.QL-P13.
(8) Tf maps tangents at a circle to points on a conic:
... QL-Ci3 gives QL-Co1.
This are first exemplary observations!

Best regards Eckart

Message: #3018
Date: 28/4/2018 11:53:33
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart
Looking again at Eckart's beautiful last (but old) cubic, I realise it is $pK(QA-P16,QA-P41)$, pivotal isocubic wrt DT, transformation QA-Tf2 which swaps QA-P4 and QA-P41 (and QA-P19 and QA-P18) and pivot QA-P41.
Naturally, it is also pivotal isocubic wrt triangle QG-P19, transformation QA-Tf2bis which swaps QA-P2 and QA-P41 (and DT vertices with QG-P18) and pivot the intersection of the lines $TiQG-P18$ (?) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3019
Date: 28/4/2018 2:02:49
From: bernard.keizer
Subject: 3 QL-versions of QA-Cu7

Dear Chris, dear Eckart
Please forget my last message !
The 1rst pivot is not QA-P41, but the intersection of the lines
QG-P19QG-P18 (?)
The rest is correct
Sorry for this mistake !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3020
Date: 28/4/2018 2:43:49
From: eckart_schmidt@t-online.de
Subject: 3 QL-versions of QA-Cu7

Dear Bernard,

wrt #3018, #3019:
The pivots for the three interpretations of the cubic at the end
of #2017
... are already mentioned in that message (found by Chris),
... they are the perspectors
 of the other two reference triangles.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3021
Date: 29/4/2018 11:37:16
From: eckart_schmidt@t-online.de
Subject: Line \longleftrightarrow point QL-transformation

Dear Bernard, dear Chris,

the transformation Tf at the end of #3017 is QL-Tf3!
But I think, the property is new,
... that Tf maps tangents at QL-Ci3 to points of QL-Co1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3022
Date: 29/4/2018 2:46:12
From: eckart_schmidt@t-online.de
Subject: QA-Tf2 in a QL

Dear Bernard, dear Chris,

in #2982 Chris wrote:

"I noticed that the 3 versions of QA-Tf2(P) in a Quadrilateral are collinear."
We have discussed it in several messages in the last time, but if I am not wrong, this is QL-Tf6, see also #2154, #2161, #2176, #2177.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3023
Date: 30/4/2018 8:12:55
From: Tran Quang Hung
Subject: Isogonal conjugate points in Quadrigon

Dear geometers,

Let $A_1A_2A_3A_4$ be a Quadrigon.
 P is any point in plane.
 B_1 is isogonal conjugate of P wrt triangle $A_2A_3A_4$.
 B_2 is isogonal conjugate of P wrt triangle $A_3A_4A_1$.
 B_3 is isogonal conjugate of P wrt triangle $A_4A_1A_2$.
 B_4 is isogonal conjugate of P wrt triangle $A_1A_2A_3$.
We have the following relation of directed angles
 $(A_1A_2, A_1A_4) + (B_1B_2, B_1B_4) =$
 $(A_3A_2, A_3A_4) + (B_3B_2, B_3B_4)$ modulo π .

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3024
Date: 01/5/2018 8:50:20
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugate points in Quadrigon

Dear Tran Quang Hung,

additional remarks to your angle property
... of the quadrangles $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$ in #3023,
... which shall give in a next step $C_1C_2C_3C_4$:
(1) A_iA_j is the bisector of B_kB_l and B_kB_l is the bisector of C_iC_j .
(2) $A_1A_2A_3A_4$ and $C_1C_2C_3C_4$ are homothetic
... wrt $QA-P_4$ and ratio " σ " = $-\cot(\phi)^2$
... with ϕ = angle of the asymptotes of $QA-Co_1$.
The ratio " σ " is an interesting parameter of a quadrangle
(see QFG#1471).
It also appears as " λ " in the paper of Benedetto Scimemi
... EQF-ref. [36], 3.2.3 to 3.2.6.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3025
Date: 03/5/2018 2:44:08
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugate points in Quadrigon

Dear Tran Quang Hung,

excuse, please forget my #3024, I used a wrong construction.
If B_i is the isogonal conjugate of A_i wrt $A_j A_k A_l$, the remarks
will hold.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3026
Date: 03/5/2018 3:07:58
From: Tran Quang Hung
Subject: Isogonal conjugate points in Quadrigon

Thank you so much Mr Eckart for your interest.

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3027

Date: 04/5/2018 12:57:31

From: peter.liepa@gmail.com

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Eckart and Chris,

For anybody who is interested, my original question about collinearity is demonstrated synthetically in Theorem 10 at <http://forumgeom.fau.edu/FG2014volume14/FG201413.pdf> (Rolinek, Michal & Anh Dung, Le. The Miquel points, pseudocircumcenter, and Euler-Poncelet point of a complete quadrilateral. Forum Geometricorum, 14 (2014) 145--153)

Best regards,
Peter

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3028

Date: 04/5/2018 11:05:46

From: eckart_schmidt@t-online.de

Subject: Isogonal conjugate points in Quadrigon

Dear Tran Quang Hung,

if I am not wrong, your angle property in #3023 isn't correct:

The right version will be:

$(A1A2, A1A3) + (A3A4, A3A2) = (B1B2, B1B3) + (B3B4, B3B2)$.

An evident consequence:

If $A1A2A3A4$ has a circumcircle, then also $B1B2B3B4$.

A further aspect:

Your construction maps

... a quadrangle $A1A2A3A4$ wrt a point P

to a quadrangle $B1B2B3B4$.

There is a point Q ,

... which maps the quadrangle $B1B2B3B4$ back to $A1A2A3A4$.

This point Q is the common isogonal conjugate of A_i wrt $B_j B_k B_l$.

We get a curious QA-transformation, which maps P to Q ,

... which lets the orthogonal hyperbola QA-Co2 invariant,

... mapping its points with the reflection in QA-P2.

If $P = QA-P4$, $B1B2B3B4$ is the reflection of $A1A2A3A4$ in QA-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3029

Date: 04/5/2018 11:12:06

From: chris.vantienhoven

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Peter,

Thanks for your reference.

I make an extra note of the collinearity of QG-P1, QL-P1 and QA-P3 in EQF at

QG-2, with reference to involved QFG-messages and the Forum-paper (beautiful approach of the proof).

Like you obviously noted in EQF-terminology:

- P, Q, R are the 3 QA-versions of QG-P1
- Mp, Mq, Mr are the 3 QA-versions of QL-P1.
- the Pseudocircumcenter is QA-P3

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3030

Date: 05/5/2018 2:01:09

From: eckart_schmidt@t-online.de

Subject: Isogonal conjugate points in Quadrigon

Dear Tran Quang Hung,

using your concept for another construction

... we get similar properties:

Starting with a quadrangle $A_1A_2A_3A_4$ and a point P,

... the points $B_i = \text{inv}(P)$ wrt the circumcircle $(A_jA_kA_l)$

... give a quadrangle $B_1B_2B_3B_4$,

... which shows the the same angle property:

... $(A_1A_2, A_1A_3) + (A_3A_4, A_3A_2) = (B_1B_2, B_1B_3) + (B_3B_4, B_3B_2)$.

The point Q, which maps $B_1B_2B_3B_4$ in the same way back to $A_1A_2A_3A_4$,

... is the common point $Q = \text{inv}(A_i)$

wrt the circumcircle $(B_jB_kB_l)$.

The QA-transformation, mapping P to Q,

... lets QA-Cu1 invariant

... with P, Q, QA-P3 collinear for P on QA-Cu1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3031
Date: 06/5/2018 11:17:27
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

Dear all,
before holiday a new aspect of a QL-quartic,
... already detailed described in #364, but without reaction.
Here is a new total other way to this quartic,
... which is invariant wrt CSC and other transformations,
... bearing the six QL-points,
... .. two QL-2P3 and the six Miquel points of the QA-versions,
unequal QL-P1.
First we consider a QA-transformation, described in #3030:
Starting with a quadrangle $A_1A_2A_3A_4$ and a point P ,
... the points $B_i = \text{inv}(P)$ wrt the circumcircle $(A_jA_kA_l)$
... give a quadrangle $B_1B_2B_3B_4$.
The point Q , which maps $B_1B_2B_3B_4$ in the same way back to
 $A_1A_2A_3A_4$,
... is the common point $Q = \text{inv}(A_i)$
 wrt the circumcircle $(B_jB_kB_l)$.
The QA-transformation T_f , mapping P to Q ,
... lets for example QA-Cu1 invariant
... with $P, Q, \text{QA-P}_3$ collinear for P on QA-Cu1.
In a second step we consider a quadrilateral and T_f for its
three QA,
... which give for a point P three Q-points Q_a, Q_b, Q_c .
Example: For $P = \text{QL-P}_1$ the points Q_a, Q_b, Q_c lie on QL-Ci1.

The locus for points P with $Q_a = Q_b = Q_c$ is the quartic in #364.
The quartic contains with a point P also the common point $Q = Q_a = Q_b = Q_c$
... with PQ tangent to a conic
... inscribed QL-Tr1 and tangent to the Steiner axes.
Tangents at this conic intersect their CSC-circle on the
quartic,
... which gives a construction possibility.
A remarkable property of quartic points P :
... circles through P and two opposite QL-points
... intersect in a common point on the quartic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3032

Date: 06/5/2018 4:39:39

From: chris.vantienhoven

Subject: 4 circles with 6 concurring radical axes and a new QA-transformation

Dear friends,

Some new QA-transformation QA-Tfx.

Let P be some random point.

Construct the circumscribed circles of the P-Cevian Triangles of the 4 QA-Component Triangles.

There are 6 radical axes of these 4 circles and they coincide in one point !

Denote this point as QA-Tfx(P).

CT-coordinates are:

$(p u (-r v + q w) (-p r u v - q r u v + p q u w + q r u w + p q v w - p r v w) :$

$q v (r u - p w) (p r u v + q r u v - p q u w + q r u w - p q v w - p r v w) :$

$r w (q u - p v) (-p r u v + q r u v + p q u w + q r u w - p q v w - p r v w))$

Properties:

- All points QA-Tfx(P) lie on the nine-point conic QA-Co1.
- QA-Tfx(QA-P1) = QA-Tfx(QA-P16) = QA-Tf2(InfinityPoint of the line through the 3 QA-versions of QL-P18)
- QA-Tfx(QA-P5) = QA-Tfx(QA-P10)

After discovering this, the surprise was that:

- QA-Tfx(P) = center of the conic(P1,P2,P3,P4,P).

This clarifies that all mapped points lie on the nine-point conic QA-Co1.

Most interesting for me was the configuration of 4 circles with 6 concurrent radical axes.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3033
Date: 06/5/2018 5:07:18
From: Tran Quang Hung
Subject: Isogonal conjugate points in Quadrigon

Thank you so much Mr Eckart for fixing my property. Actually, I saw the cyclic quadrangle before the property of angle. Using your idea about inverse point. Can we consider the transformation with antigonal conjugate mapping?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3034
Date: 07/5/2018 10:35:41
From: Tran Quang Hung
Subject: Concurrent lines in quadrangle points

Dear friends,

We have quadrangle points A_1, A_2, A_3, A_4 and arbitrary line L . L meets A_1A_2 at B_{12} . Let C_{12} be the orthogonal projection of B_{12} on line A_3A_4 .

Define similarly, the points $C_{13}, C_{14}, C_{23}, C_{24}, C_{34}$.

Then the lines (C_{12}, C_{34}) , (C_{23}, C_{14}) and (C_{13}, C_{24}) are concurrent at a point.

Dear friends, is this transform known?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3035
Date: 07/5/2018 10:48:05
From: Tran Quang Hung
Subject: Concurrent lines in quadrangle points

Dear friends,

I have seen this is QA-Tf6 Quang Duong's Transformation.
I only have a remark. When A, B, C, D are on a circle (O) and lines r passes through center O .
Then HAB , HCD , HAC , HDB , HAD , HBC are collinear. This line is known as Dao's generalization of Simson line.

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3036

Date: 07/5/2018 3:43:32

From: Benedetto Scimemi

Subject: How to construct the inverse of QA-Tf6 (and QA-Tf3). A QA-quartic as

Dear friends,

as recent messages of Tran Quang Hung have let QA-Tf6 revive, it may be of some interest the following "point --> line" transformation, which provides a direct construction for its inverse. I did not find it in EQF.

Let QA = A1A2A3A4 be a quadrangle and P any point (\neq QA-P2). Let Sij the (oblique) reflection of P on the side AiAj in the direction orthogonal to the opposite side AhAk.

The midpoints $(S12+S34)/2$, $(S13+S24)/2$, $(S14+S23)/2$ are collinear on a line $s(P)$.

QA-Tf6 maps $s(P)$ into P.

(For $P=QA-P2$ the three midpoints coincide with QA-P4: one can let $s(QA-P2) = QA-P2.QA-P4$).

At page QA-Tf6 of EQF one can already read that:

If $S=S(P)$ is the reflecton of QA-P4 on $s(P)$, then $QA-Tf3(S)=P$.

Thus, if we have $s(P)$ we also have a construction for the inverse of QA-Tf3:

$QA-Tf3^{-1}(X)$ is the reflection of QA-P4 on $s(X)$

Here are some properties of this "point --> line" transformation $P \rightarrow s(P)$.

s maps the points of a *line* r into the tangents of the *parabola* whose focus is QA-P4 and directrix is $QA-Tf3^{-1}(r)$. The orthogonal projection of QA-P2 on r is mapped into the tangent at the parabola vertex.

In particular: s maps the points of a line r through QA-P2 into a pencil of *parallel lines* normal to $QA-Tf3^{-1}(r)$

s maps the points of a *circle* C_i of center C into the *tangents to a conic* with foci QA.P4 and $QA-Tf3^{-1}(C)$. The intersections of C_i with the line $C.QA-P2$ are mapped into to the tangents at the conic vertices.

In particular, s maps the points of a circle centered in QA-P2 into the tangents of a circle centered in QA-P4.

s maps the points of a circle through QA-P2, centered in C , into *the pencil of lines* through $QA-Tf3^{-1}(C)$.

(s maps the points of the circle with diameter QA-P2.QA-P3 into the lines pencil through QA-P3).

This confirms and adds some details to the tables of page QA-Tf6 in EQF. Should Chris eventually find the time, the final list of correspondence of that page could be re-written, to be read in both directions (as he did for CO-Tf3) :

line < --- > point

(N.B. The "orthotransversal" transformation introduced by Ngo (June 2015) maps P into a point of $s(P)$, but I could not locate it on $s(P)$).

(1) as for the *diagonal triangle*, the main properties of $P \rightarrow s(P)$ are already reported in EQF at page QA-Tf6: when P runs on QA-Ci1, then $s(P)$ describes the lines pencil through QA-P12 and contains the reflections of P on the sides of QA-DT; hence its P-1/2-homothetic line is the Simson-Wallace line of P wrt QA-DT.

If $P=X(110)$ of DT, then $s(P)$ is the Euler line of DT .

(2) as for the *ninepoints conic*:

QA-Co1 is the locus of points P which lie on $s(P)$:

in particular

if $P=(A_i+A_j)/2$ is a side- midpoint,

then $s(P)$ is the perp. bisector of A_iA_j

if P is a diagonal point (a DT vertex)

then $s(P)$ is the DT-height at P

if P tends to QA-P2 (on QA.Co1)

then $s(P)$ tends to the line QA-P2.QA-P4

if $P=QA-P3$ then $s(P)$ is normal to QA-P3.QA-P4

There are many QA-lines tangent to this curve:

6 perp bisectors (of QA sides)

3 QA-DT heights

4 QA-central lines, namely QA-L2 = QA-P2.QA-P4 (for $P=QA-P2$)

the normal to QA-P3.QA-P4 (for $P=QA-P3$)

2 bisectrices of (for P at infinity).

Contact points can be constructed. In particular, contact point of central lines give new central points on QA-Qu.

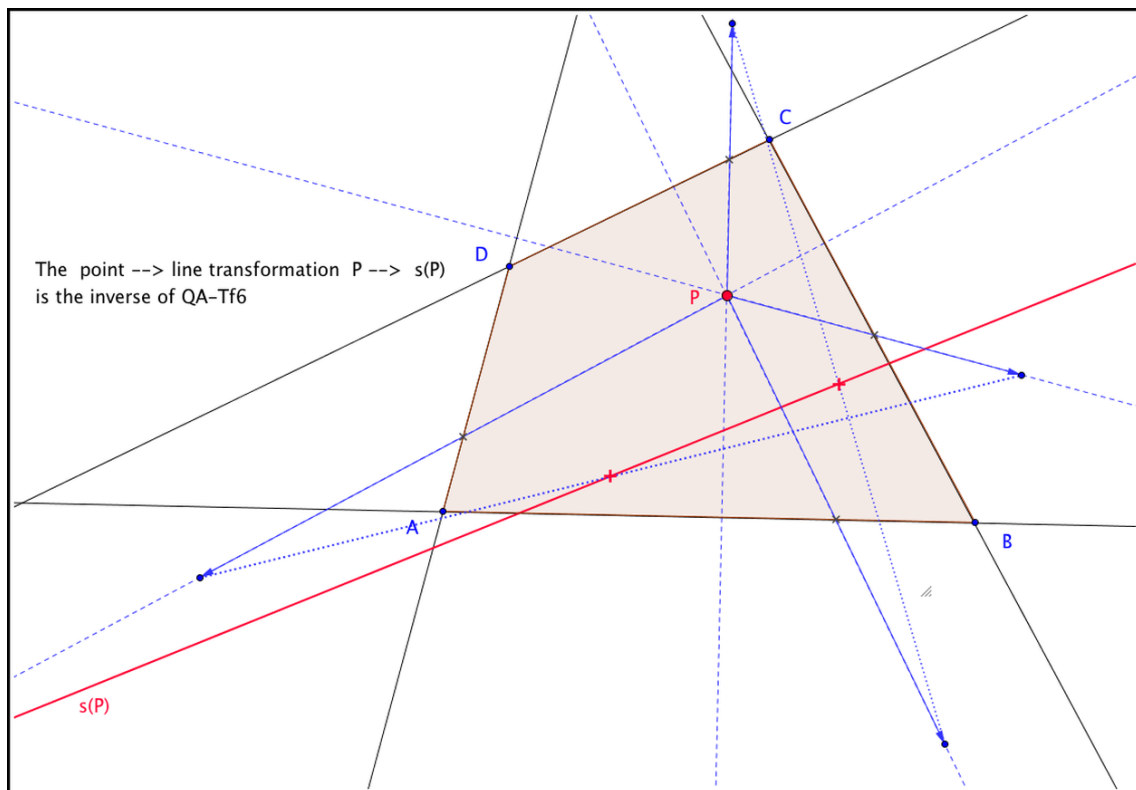
As it happens, P lies on $s(P)$ if and only if P lies on QA-Co1.

This suggests to define a curve, say QA-Qu2, as the envelope of $s(P)$ when P runs on QA-Co1. In fact, this curve is a three cusps quartic (only one cusp is real if QA is convex).

Eckart, years ago (alas, I miss the reference), in a totally independent way introduced and studied another QA-quartic, say QA-Qu1, tangent to the six sides of the QA. The present quartic is obtained when his definition is applied, rather than to QA, to the quadrangle of the circumcenters of QA.

I plan to give more details on both these curves in some next messages.

Best regards
Benedetto



Inverse of QA-Tf6 7-5-18.png

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3037
Date: 08/5/2018 9:57:35
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugate points in Quadrigon

Dear Tran Quang Hung,

if we use antigonial conjugate mapping for your construction in #3023,

... the angle property will be
 $(A1A2, A1A4) + (A3A4, A3A2) + (B1B2, B1B4) + (B3B4, B3B2) \equiv 0 \pmod{2\pi}$.
If $A1A2A3A4$ is cyclic, then $B1B2B3B4$ too.
But I found no point Q for an inverse mapping.

Best regards Eckart

PS: Apologize profusely,
... my formulation of the angle property in #3028
is also not correct.

I try it once more:
 $(A1A2, A1A4) + (A3A4, A3A2) \equiv (B1B2, B1B4) + (B3B4, B3B2) \pmod{\pi}$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3038
Date: 08/5/2018 10:11:33
From: eckart_schmidt@t-online.de
Subject: 4 circles with 6 concurring radical axes and a new QA-transformation

Dear Chris,

last minute before holiday: An additional property of your transformation in #3032:

All points of QA-Co2 will be mapped to QA-P2.

Another similar QA-transformation:

The pedal circles of a point P wrt the QA-triangles

... have a common point Q.

The mapping P to Q

... gives for QA-P4 and points on QA-Co2 the image QA-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3039
Date: 08/5/2018 9:46:08
From: chris.vantienhoven
Subject: new items in EQF / EPG

Dear friends,

I added several new items in EQF and EPG.
Also I adjusted several other items.
They all came from messages in our group.
Doing the job I became very enthusiastic about all these beautiful findings.

For a complete list see Recently Added (
<https://www.chrisvantienhoven.nl/recently-added>).

The main new items are:

- * QA-Tf12: QA-Pedal Point
- * QA-Tf13: QA-AntiPedal Point
- * QA-Tf14: QA-Orthotransversal Point
- * QA-Tf15: P-Circumscribed Conic Center
- * nP-e-Tf1: nP-Pedal Point
- * nP-o-Tf1: nP-Pedal Circle
- * nP-o-Tf2: nP-Pedal Circle Center
- * nP-o-Tf3: nP-AntiPedal Circle
- * nP-o-Tf4: nP-AntiPedal Circle Center
- * 5P-s-P3: 5P-Quang Duong's Point

If there are any comments, please let me know.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3040

Date: 12/5/2018 10:48:33

From: bernard.keizer

Subject: QA: intersection points are on radical axes of certain circle pa

Dear Chris, dear Peter

Here 3 comments after reading the article in reference.

1) The title is misleading as it handles with complete quadrangle and not with complete quadrilateral

And of course, as mentioned already by Chris, the pseudocircumcenter is the Gergonne-Steiner point, which is in fact the perspector of DT and Miquel triangle of the QA.

2) If I'm not wrong, the synthetic given proof of the property that the Euler-Poncelet point lies on the circumcircle of DT is exactly what Jean-Louis Aymé was asking for a few years ago, when he joined the forum.

3) The property of the Feuerbach point mentioned in theorem 2 in connexion with the QA ABCI (I incenter) is also true for the the QA's ABCJ (J being excenters) and leads to 3 other Feuerbach points as contact points between the Euler circle and the excircles.

This leads to the Feuerbach QA of a triangle, which has beautiful properties (it's DT is in perspective with ABC) see The Feuerbach quadrilateral by N.M. Gibbins in Mathematical Gazette vol 22 may 1938 n° 249

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3041
Date: 15/5/2018 12:07:23
From: bernard.keizer
Subject: Locus of foci

Dear Chris, dear Eckart

Let's consider a QA/QL figure.
The conics inscribed in the QL are the duals of conics circumscribed to the QA.
The contact points of the inscribed conics with the 4 lines form a QA with the same DT.
The locus of QA-P2 is the circumcircle of DT
The locus of the centers is the Newton Line QL-L1 and the locus of the foci is the Van Rees curve QL-Cu1.
The tangents to the circumscribed conics form a QL with the same DT.
The locus of QL-P1 is the Euler circle of DT
The locus of the centers is QA-Co1.
The locus of the foci is a strange curve with the vertices of DT as dopple points (quintic or sextic ?) Any idea ?
(The QA of the contact points and the the QL of the tangents for a copple of dual conics are not themselves dual)

Best regards
Bernard

PS For a triangle, the isogonal conjugate of a vertice is not defined : it can be any point on the opposite side.

The same way, the "isogonals" of a vertice of the QA (see previous messages) are not defined : it can be the foci of any conic inscribed in the triangle of the 3 other vertices (following Siebeck's properties)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3042
Date: 15/5/2018 1:44:17
From: chris.vantienhoven
Subject: Construction Cayley-Bacharach Point

Dear Eckart,

Hope you have/had a nice holiday.

I myself had planned another walking holiday this year in Portugal, however because of heel-problems I had to postpone my holiday.

The positive thing about this is that now I found time to look at the construction of A.S. Hart of the Caley-Bacharach point. I liked his method with 6 propositions and his use of anharmonic ratios (it matches with perspective fields).

Only proposition 3 was "open-ended", stating:

Prop. 3.

To find an anharmonic pencil whose ratio is the product of the ratios of two given pencils.

Let the given pencils be cut by two lines at A, B, C, D , A, b, c, d : join Bd , bD , and from their intersection draw lines to C and c : these lines will form the required pencil.

His construction of the product-pencil is bright and clear, but unmentioned is, when lines $P(A-B-C-D)$ are ordered in the mentioned way, which order will have the lines of the product-pencil? Maybe it is clear from the definition of the "product" of two pencils. However I could not find any definition at any place of such a "product" of two pencils. Maybe you know one. I noticed that the ratio of the product-pencil is not equal to the product of the input-pencils. That's ok, he is free to give it a name with not all connotations of the usual meaning.

On the $A-B-C-D$ -line we have product-points B, C, D and a 4th point, let's name it C' .

It is tempting to place it in the order $C'-B-C-D$. But I noticed that it didn't give a right result independent of the order of the 8 basic points.

So I permuted the order and found luckily that the order $C'-C-B-D$ did give a right result independent of the order of the 8 basic points.

As a matter of fact there are 4 orders that give right results: $C'-C-B-D$ or $B-D-C'-C$ or $C-C'-D-B$ or $D-B-C-C'$ (4 out of 24 permutations).

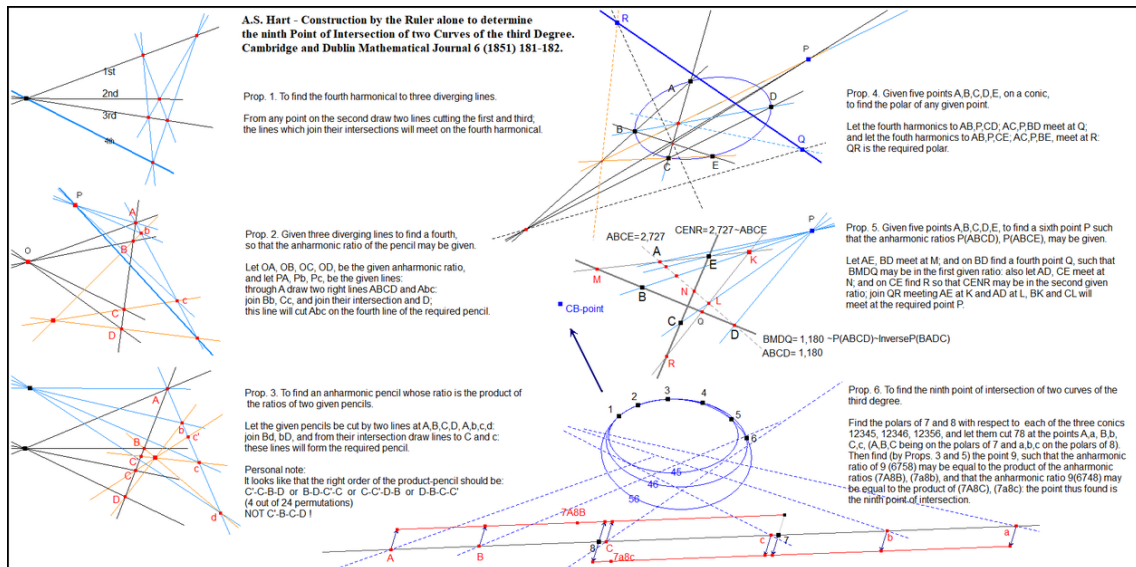
Therefore I suppose that the product of two pencils $P(ABCD)$ and $(Abcd)$ will be $S(C' CBD)$.

Your solution was that the right order was the "inverse of".

Maybe we can compare our solutions.

Not unimportant: I checked my resulting construction-point with your results and they match.
I attach a file with my drawings.

Best regards,
Chris



CU_8P-P1-Cayley-Bacharach Point-Construction-15-Anharmonic Ratios.png

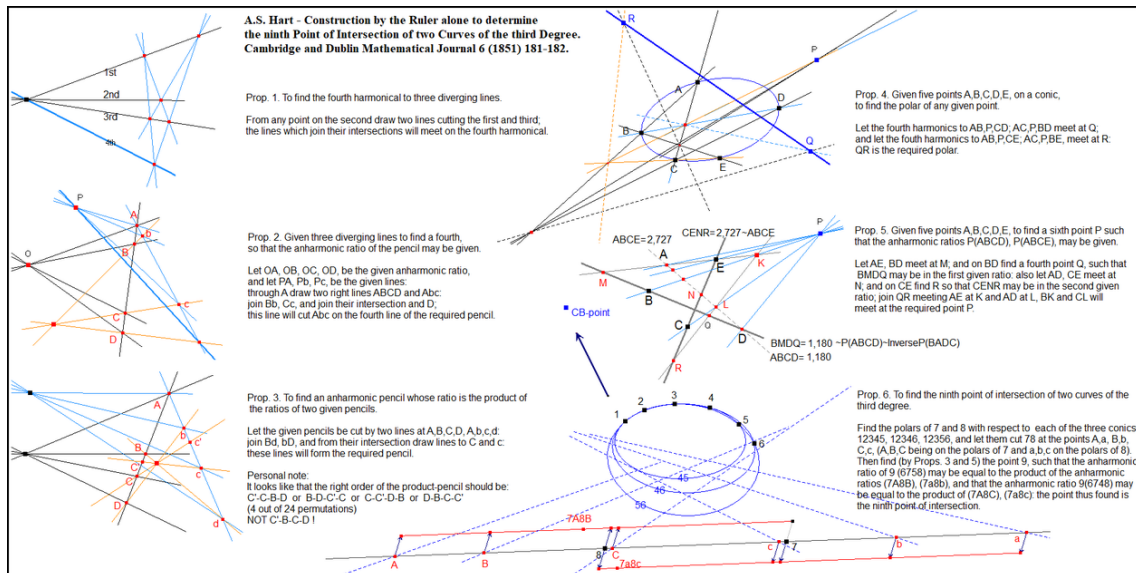
← Previous Next → ↔ Message Index ↑ Subjects

Message: #3043
Date: 15/5/2018 8:10:03
From: chris.vantienhoven
Subject: Construction Cayley-Bacharach Point

Dear Eckart,

I noticed that the attachment of my last message gave problems with downloading.
 Therefore I attach the same file with the drawing again.

Best regards,
 Chris



Cayley-Bacharach Point-Construction-15-Anharmonic Ratios.png

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3044
Date: 18/5/2018 2:46:44
From: eckart_schmidt@t-online.de
Subject: Locus of foci

Dear Bernard,

back from holiday, I studied your message #3041.
The locus of the foci for QA-circumscribed conics
... is a sextic, already described in #496.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3045
Date: 19/5/2018 9:40:37
From: bernard.keizer
Subject: Locus of foci

Vielen Dank, lieber Eckart
Thanks for the curve and it's beautiful properties !
I had drawn it almost accidentally.
My construction is simple : the conic is inscribed in a QL
formed by the 4 tangents in the QA vertices.
The Newton Line is the tangent in Z to QA-Co1, the DT is the
same, the Miquel point is on the Euler circle of DT
(circumscribed to OT) and it defines a Cl-S.
The 2 searched foci are on the QL-Cu1 of this QL (which is an
isogonal nK wrt OT) and their middle is in Z ...
I had the curve, but was not conscious of it's properties.
In fact, I was looking for a curve being the locus of the 3 foci
of the curves of class 3 tangent to the 6 lines of a QA and
therefore duals of cubics through the 6 vertices of the dual QL.
This curve is necessary through the QA vertices (a QA vertice
and any conic inscribed in the triangle of the 3 remaining
vertices), through the DT vertices (the cubic formed by the 3
vertices of DT) and the 3 foci of Siebeck's sextic ; I checked
it is not a cubic ...
So, what could it be? Any idea?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3046
Date: 20/5/2018 8:27:36
From: eckart_schmidt@t-online.de
Subject: Construction Cayley-Bacharach Point

Dear Chris,

I am encouraged,
that someone else examines Hardts construction of the
Cayley-Bacharach point.
You describe all my troubles, I had a year ago with this
problem.
Give me some time, to get once more familiar with the
construction.

For now:

Let us consider the "anharmonic ratio" $R = O(A,B,C,D)$,
... using a line through A,
... intersecting OB in B, OC in C, OD in D,
... with cross-ratio $r = [A, B, C, D] = (A C * BD) / (A D * BC)$:
... For the cyclic order
... ... A,B,C,D or A,D,C,B on L: $R = r-1$,
... ... A,B,D,C or A,C,D,B on L: $R = 1-r$,
... ... A,C,B,D or A,D,B,C on L: $R = r+1$.

Then the "product"-property in prop.3 holds
with the order of the product pencil C'-B-C-D.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3047
Date: 21/5/2018 10:23:29
From: eckart_schmidt@t-online.de
Subject: Construction Cayley-Bacharach Point

Dear Chris,

what is the definition of Hardt's "anharmonic ratio"?
Let us modify my last interpretation wrt "inverse of":
Consider the "anharmonic ratio" $R = O(A,B,C,D)$,
... using a line through A,
... intersecting OB in B, OC in C, OD in D,
... with cross-ratio $r = [A, B, C, D] = (A C * BD) / (A D * BC)$:
... For the cyclic order
... ... A,B,C,D or A,D,C,B on L: $R = 1/(r-1)$,
... ... A,B,D,C or A,C,D,B on L: $R = 1/(1-r)$,
... ... A,C,B,D or A,D,B,C on L: $R = 1/(r+1)$.

Then the "product"-property in prop.3 holds
with the order of the product pencil $C'-B-C-D$,
and the last sentence in Hardt's paper is correct.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3048

Date: 22/5/2018 4:36:22

From: eckart_schmidt@t-online.de

Subject: How to construct the inverse of QA-Tf6 (and QA- ...

Dear Benedetto,

back from holiday I studied your detailed message 3036
... and reproduced with growing interest
the transformation $P \mapsto s(P)$,
... it was a wonderful excursion in QA-geometry,
... full of unexpected results!

A short addition:

The orthogonal hyperbola QA-Co2 will be mapped to a quartic:
... symmetrical to a line through QA-P4,
... parallel

to the main axis of QA-Co2,
... bearing two double points.
What about these two points?

Best regards Eckart

PS: You find my quartic in #141 ("years ago") and #2200.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3049

Date: 22/5/2018 6:22:53

From: Benedetto Scimemi

Subject: How to construct the inverse of QA-Tf6 (and QA- ...

Dear Eckart.

I did recover your messages 141 and 2200.

I also prepared -for our group- a detailed report of my results on your quartic (I provisionally called it QA-Qu1, while my envelope of $s(P)$ could be QA-Qu2), a rich complement of yours. I plan to send it soon.

I also look forward to find the time to consider your new quartic (QA-Qu3 ?) but at the moment I am having guests consuming all my days.

It should be a matter of a week. Thanks.

Best regards

Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3050

Date: 24/5/2018 9:10:54

From: eckart_schmidt@t-online.de

Subject: How to construct the inverse of QA-Tf6 (and ...

Dear Benedetto, dear Chris,

are the following properties already discussed?

(1) The Co-Tf3-images of a point P wrt QA-circumscribed conics ... are collinear on the line QA-Tf6inv(P).

For a 5P these 5 lines have a common point.

(2) The Co-Tf3inv-images of a point P wrt QA-circumscribed conics

... are concyclic on a circle through QA-P2,

... which is the QA-Tf6-image of the line pencil of P .

For a 5P these 5 circles have a common point.

(1) and (2) will lead to (new?) 5P-transformations.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3051
Date: 25/5/2018 9:43:55
From: eckart_schmidt@t-online.de
Subject: How to construct the inverse of QA-Tf6 (and ...

Dear Benedetto, dear Chris,

sorry, the 5P-s-transformations wrt (1) and (2) in #3050 ... are 5P-s-Tf3inv and 5P-s-Tf3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3052
Date: 25/5/2018 6:18:11
From: chris.vantienhoven
Subject: Construction Cayley-Bacharach Point

Dear Eckart,

I do not quite understand what you mean.

You say:

... For the cyclic order
... ... A,B,C,D or A,D,C,B on L: $R = 1/(r-1)$,
... ... A,B,D,C or A,C,D,B on L: $R = 1/(1-r)$,
... ... A,C,B,D or A,D,B,C on L: $R = 1/(r+1)$.

But there are actually 6 main orders:

$(A,B;C,D)=(B,A;D,C)=(C,D;A,B)=(D,C;B,A) = r$
 $(A,B;D,C)=(B,A;C,D)=(C,D;B,A)=(D,C;A,B) = 1 / r$
 $(A,C;B,D)=(B,D;A,C)=(C,A;D,B)=(D,B;C,A) = 1 - r$
 $(A,C;D,B)=(B,D;C,A)=(C,A;B,D)=(D,B;A,C) = 1/(1- r)$
 $(A,D;B,C)=(B,C;A,D)=(C,B;D,A)=(D,A;C,B) = (r-1)/ r$
 $(A,D;C,B)=(B,C;D,A)=(C,B;A,D)=(D,A;B,C) = r/(r-1)$

Then you say:

Then the "product"-property in prop.3 holds

with the order of the product pencil $C'-B-C-D$,

and the last sentence in Hardt's paper is correct.

I noticed that the "product"-property only holds for the order $C'-C-B-D$ or $B-D-C'-C$ or $C-C'-D-B$ or $D-B-C-C'$.

Maybe this is of less importance since we both found a way to construct the Cayley-Bacharach point.

But I also have the question if the "product"-property gave a real product of anharmonic ratios. Did you check that?
Then another question popped up doing the "Hart-construction".
The Hart-construction is completely according the principles of projective geometry.
So we should be able to interchange the notions of "point" and "line", doing a dual construction.
Then we suddenly have a construction of a Cayley-Bacharach Line instead of a Cayley-Bacharach Point for a Cubic (degree 3)
instead of Curve of class 3.
However I am not quite sure if the dual of a cubic is another cubic or a curve of class 3.
Nevertheless I tried to do this dual construction, but did not succeed in finding a consistent Cayley-Bacharach Line for a cubic. Maybe I made an error or maybe it is not possible at all, because of the difference of a degree-3-curve and a class-3-curve.
Do you have any idea?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3053
Date: 26/5/2018 7:36:58
From: Benedetto Scimemi
Subject: No Subject

Dear Eckart, I gave a look to the curve you pointed out on May 22, the envelope of the lines $s(P)$ (= inverse of QA-Tf6) when P runs on QA-Co2. I found that it is not a quartic but a sextic, symmetrical w.r.t. the lines through QA-P4, parallel to both the main axes of QA-Co2. The two (QA-P4-symmetrical) double points lie on one of these axes. The points at infinity of these two lines are triple. But the most beautiful fact about this curve - in my opinion - is that its *shape* is the same for all QA, as they have the same equation (in a convenient xy-frame). The Cartesian equation is very simple, as well as the coordinates of the singular points. I could not recognize the curve among the classic example of the literature. Perhaps somebody has already met it. If you are interested, I can send you these details and you may be able to make some progress. For the time being I am attaching two examples, regarding a convex and a non-convex QA. Best regards Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3054
Date: 27/5/2018 2:50:41
From: eckart_schmidt@t-online.de
Subject: Construction Cayley-Bacharach Point

Dear Chris,

please forget my last message #3047, the last line doesn't hold ... and return to #3046.

Wrt my formulation:

... For the cyclic order

... ... A,B,C,D or A,D,C,B on $L:R = (r-1)$, ...

means: If you go along the line in any direction, you meet A, B, C, D

... in the order ABCD or BCDA or CDAB or DABC

... ...or ADCB or DCBA or CBAD or BADC.

In this interpretation of an "anharmonic ratio" R

... the product of the ratios of two given pencils (A,B,C,D) , (A,a,b,c) in prop.3

... is the ratio $(C'BCD)$ of the constructed pencil.

But then the last sentence in Hardt's paper is not correct,

... we have to replace $9(6758)$ by $9(6857)$ and $9(6748)$ by $9(6847)$.

You can check this in the attached CABRI-files.

What is your definition of "anharmonic ratio" $R = (A,B,C,D)$?

You find my interpretation of a Cayley-Bacharach line in #2511.

My experiences are, that the dual of a cubic is a curve of class 3.

Best regards

Eckart

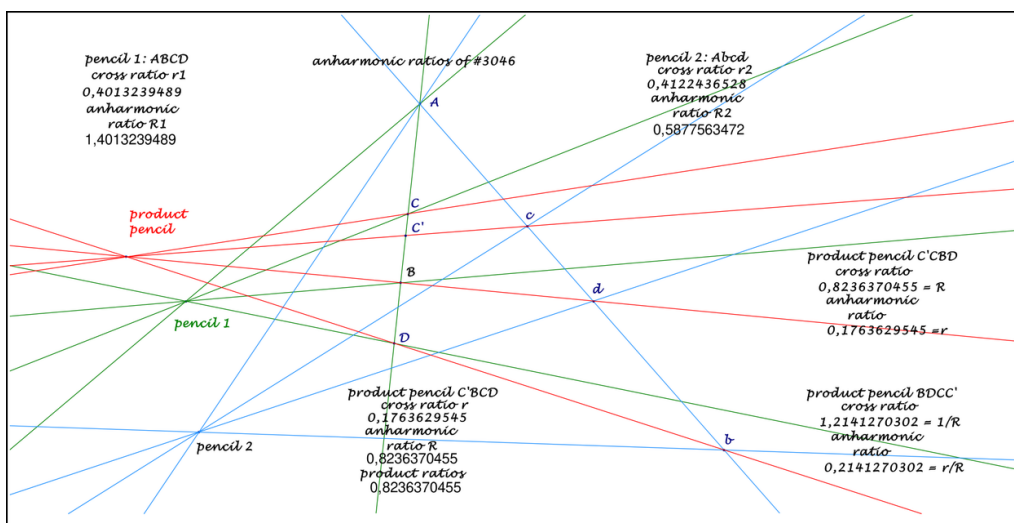
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3055
Date: 27/5/2018 3:16:42
From: eckart_schmidt@t-online.de
Subject: Construction Cayley-Bacharach Point

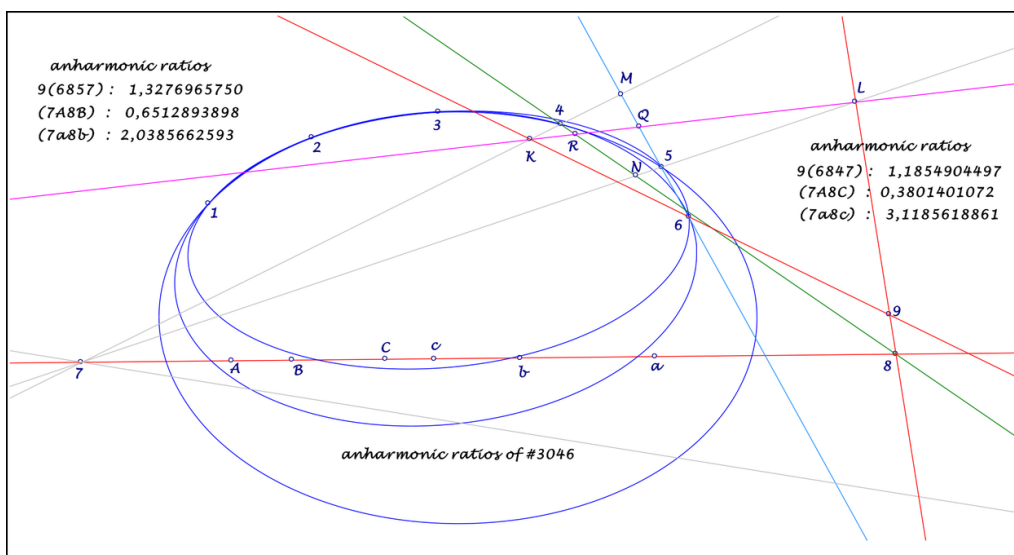
Dear Chris,

unfortunately the CABRI-files are not transmitted,
so I can only try it with pdf-files.

Best regards Eckart



2018-05-26a.pdf



2018-05-26b.pdf

Message: #3056
Date: 27/5/2018 11:21:14
From: Benedetto Scimemi
Subject: new items in EQF / EPG

Following the message I sent yesterday -to avoid different software problems- I am attaching new pdf versions for the two example of (similar) sextic curves, envelopes of the mapping $QA-Tf6^{(-1)}$: point $P \rightarrow$ line $s(P)$ for P running on $QA-Co2$.

Best regards
Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

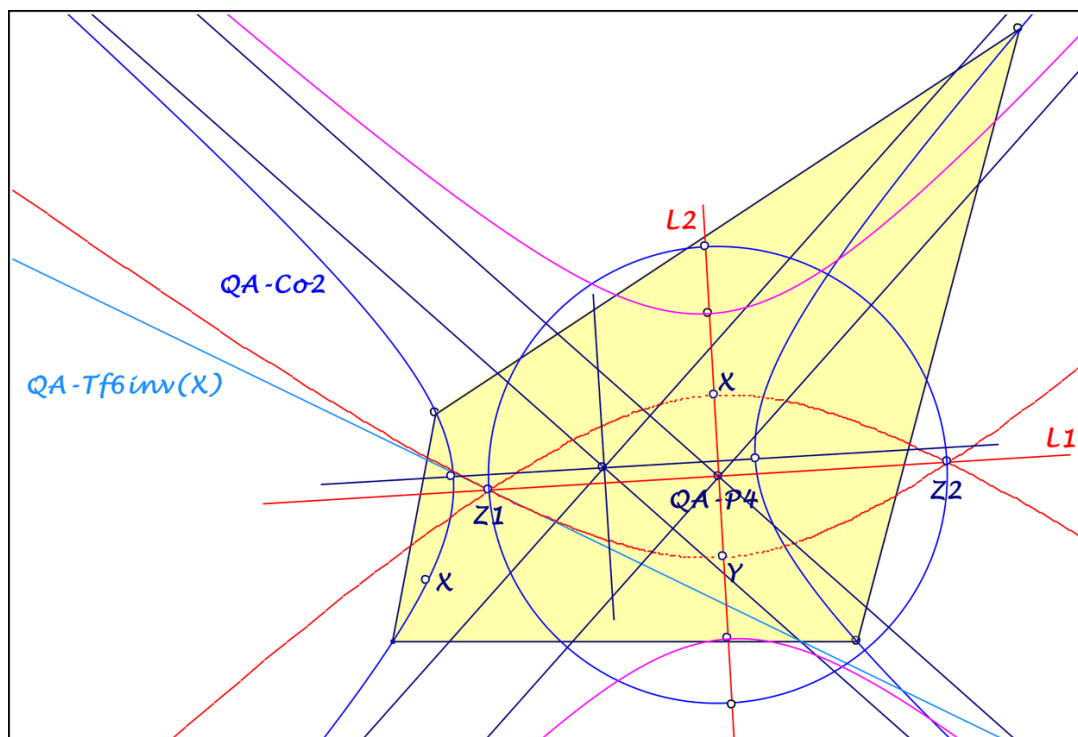
Message: #3057
Date: 28/5/2018 1:24:27
From: eckart_schmidt@t-online.de
Subject: How to construct the inverse of QA-Tf6 (and ...

Dear Benedetto,

sorry, I cannot open your attached files in #3053 and #3056.
Wrt the envelope of lines QA-Tf6inv of points on QA-Co2:
I still think, the curve is a quartic:
... with the same infinity points as QA-Co2,
... symmetrical wrt L1 and L2,
... ... parallels to the main and minor axis of QA-Co2 through QA-P4,
... with X, Y on L2: QA-Tf6inv of the vertices of QA-Co2,
... with two double points Z1, Z2 on L1:
... ... intersections of L1 and a circle, centered in QA-P4,
... ... through the foci of an orthogonal hyperbola,
... ... centered in QA-P4, axes parallel to axes of QA-Co2,
... ... through the reflections of QA-P4 in X, Y,
... $Z1Z2/XY = 2 \sqrt{2}$.
The last property is your observation, really unexpected!

Best regards Eckart

PS: I hope, the attached files will be appear!



2018-05-28.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3058
Date: 29/5/2018 10:04:55
From: chris.vantienhoven
Subject: Construction Cayley-Bacharach Point

Dear Eckart,

1.

[ES]: If you go along the line in any direction, you meet A, B, C, D

... in the order ABCD or BCDA or CDAB or DABC

... .. or ADCB or DCBA or CBAD or BADC.

[ES] What is your definition of "anharmonic ratio" $R = (A,B,C,D)$?

For me an anharmonic ratio is the double ratio of 4 points:

$(A,B;C,D) = (AC/BC) / (BC/BD) = (AC/BD) / (BC/AD)$

See: <https://en.wikipedia.org/wiki/Cross-ratio>.

It is not dependent of the "visual/actual" order of the involved points.

But when you mention an anharmonic ratio you have to define for which points in which order and measuring lengths as signed lengths.

Having defined the anharmonic ratio this way, of course you can reorder the points and look what the consequences are for the result.

2.

[ES] In this interpretation of an "anharmonic ratio" R

... the product of the ratios of two given pencils (A,B,C,D) ,

(A,a,b,c) in prop.3

... is the ratio $(C'BCD)$ of the constructed pencil.

But then the last sentence in Hardt's paper is not correct,

... we have to replace 9(6758) by 9(6857) and 9(6748) by 9(6847).

You can check this in the attached CABRI-files.

I don't know about an official definition of the product of two 4-line-pencils.

I only noticed that when we have two given pencils (A,B,C,D) ,

(A,a,b,c) in prop.3, the outcome is a set of 4 lines $S(C'BCD)$,

of which the order not is defined by Mr. Hart. Therefore I

suppose that the order not automatically should be $C'-B-C-D$, but

better the order $C'-C-B-D$ or $B-D-C'-C$ or $C-C'-D-B$ or

$D-B-C-C'$, because these orders give the right result.

3.

[ES] My experiences are, that the dual of a cubic is a curve of class 3.

I am still wondering having any cubic (curve of degree 3) and 8 tangent lines isn't there always a ninth tangent line as an equivalent of the Cayley-Bacharach point?

4.

Another interesting Article (see attached file) is:

Thomas Weddle - On the construction of the ninth point of intersection of two curves of the third degree, when the other eight points are given.

Cambridge and Dublin Mathematical Journal 6 (1851) 83-86.

It describes the Cayley-Bacharach point as the 4th intersection point of two conics. These conics are each defined by four of the 8 points and the 5th point can be constructed in relationship with the other points.

Page 85/86:

Hence we may find a fifth point on the conic (3) in the following manner:

Join 8 and 1, and 7 and 2, by straight lines intersecting in P; and find a,b, the other points of intersection of these lines with the conic passing through the points 12345;

also find the other points a',b' of intersection of the same lines with the conic passing through 12346.

Through 8 and 7 draw any parallel lines, the former line meeting the lines (57) and (67) in c and d, and the latter meeting (58) and (68) in e and f.

To 8c, 8a, and Pa' find a fourth proportional p ;

to 8d, 8a', and Pa, a fourth proportional q ;

to 7f, 7b', and Pb, a fourth proportional r ;

to 7e, 7b, and Pb', a fourth proportional s ;

and finally, to q, p, and r, find a fourth proportional t.

Divide cd in k, so that $kc : kd :: s : t$,

then k will be a point on the conic (3).

Note conic(3) = conic (5678k)

Nevertheless I found problems with this construction.

I didn't find a conic k.5.6.7.8 passing through 8P-s-P1.

Moreover s/t isn't constant, when varying points.

So I wonder if my interpretation of the text is right, or maybe there are typos in Weddle's text.

However I found $(s/t) / (kc/kd)$ is a constant when varying the direction of the parallel lines.

The matter is pretty specialized.

Best regards,

Chris

Thomas Weddle – On the construction of the ninth point of intersection of two curves of the third degree, when the other eight points are given.
 Cambridge and Dublin Mathematical Journal 6 (1851) 83-86.

ON THE CONSTRUCTION OF THE NINTH POINT OF INTERSECTION OF TWO CURVES OF THE THIRD DEGREE, WHEN THE OTHER EIGHT POINTS ARE GIVEN.

By THOMAS WEDDLE, F.R.A.S., Royal Military College, Sandhurst.

DENOTE the eight given points by 1, 2, 3 8; also let $(12) = 0$ be the equation to the straight line passing through the two points 1, 2; and $(12345) = 0$,* the equation to the conic passing through the five points 1, 2, 3, 4, 5.

If the constants λ and μ be properly taken, the equations to two particular curves of the third degree passing through the eight given points, will be

$$(12345).(67) = \lambda.(12346).(57) \dots\dots (1),$$

and $(12345).(68) = \mu.(12346).(58) \dots\dots (2).$

Divide the former equation by the latter, and reduce, therefore $\lambda.(57).(68) = \mu.(58).(67) \dots\dots (3),$

which is evidently the equation to the conic passing through the points 5, 6, 7, 8, and the "ninth" point sought.

Again, let us interchange the points 1 and 8 in all that precedes. The equations to two particular curves of the third degree passing through the eight given points will thus be

$$(82345).(67) = \rho.(82346).(57) \dots\dots (4),$$

and $(82345).(61) = \sigma.(82346).(51) \dots\dots (5),$

where ρ is such a constant as to make (4) pass through the point 1, and σ such as to make (5) pass through 7.

* Of course, (12345) will = $(12).(34) + k(14).(23)$, where the constant k is determined so that the conic shall pass through the point 5.

Divide (4) by (5); therefore

$$\rho \cdot (57) \cdot (61) = \sigma \cdot (51) \cdot (67) \dots \dots \dots (6),$$

which is the equation of the conic passing through the points 1, 5, 6, 7, and the "ninth" point.

Hence, the point sought will be the fourth point of intersection of the two conics (3) and (6), three of the points 5, 6, 7, of intersection being already known.

The method of finding this point algebraically, by eliminating x or y from the equations (3) and (6), (always bearing in mind that we know three roots of the resulting biquadratic), seems therefore evident enough, and I have only to shew how the point may be found geometrically.

Before doing so, I must however be permitted to state that I lay no claim to the preceding analysis. It was communicated to me by the Rev. T. P. Kirkman, in a letter dated April 16th, 1850; and I immediately inferred the construction about to be given. It would seem however that Mr. K. was not the first to solve this important problem, for in a subsequent letter to me he said, "I have heard from Dublin that Mr. Hart had made known in the University the method of finding the ninth point that I have given you, about a month before I hit upon it. The proof he gives is pretty much the same as mine. Let

$$(94) \cdot (23567) - (92) \cdot (43567) = 0$$

be the curve through six points 234567. If this contains 1, 8, we have $\left(\frac{94}{92}\right)_1 = r$, a given ratio, and $\left(\frac{94}{92}\right)_8 = r$, a given ratio, so that the anharmonic ratio of the points 2481 is given, and 9 is on a given conic."

To facilitate the subsequent investigation, I shall suppose that, in the preceding analysis, the origin of coordinates has been taken at P the intersection of the lines (81) and (72); that the absolute term in each of the equations (12345) = 0, (12346) = 0, (82345) = 0, and (82346) = 0, is + 1; and that (67), (57), (68), (58), (61), and (51) have been multiplied by such constants that the values which these symbols take at any point (xy) are equal to the portions of the straight line drawn through (xy) parallel to a line Z (assumed at pleasure), and intercepted between (xy) and the lines (67), (57), (68), (58), (61), and (51) respectively. Also by the notation 12, (without the parenthesis), I mean the *distance* between the points 1 and 2, by $P2$ the distance between P and 2, &c.

In the conic (3), we know the four points 5, 6, 7, 8, and I shall now shew how a fifth point may be found.

Find the other points a and a^* in which the conics (12345) and (12346) are intersected by the straight line (81), and the other points b, b' , in which the same conics are intersected by the straight line (72). Now, (1), the equation to determine λ is

$$\lambda = \frac{(12345)_8 \cdot (67)_8}{(12346)_8 \cdot (57)_8},$$

where $(12345)_8$, &c. mean the values which (12345), &c. take when the coordinates of the point 8 are substituted for x and y .

Now (*Mathematician*, vol. II. p. 74), $(12345)_8 = \frac{81 \cdot 8a}{P1 \cdot Pa}$, and $(12346)_8 = \frac{81 \cdot 8a'}{P1 \cdot Pa'}$; also $(67)_8$ and $(57)_8$ are the portions of the line drawn through the point 8 parallel to Z , and intercepted between 8 and the lines (67) and (57) respectively; and they are therefore known lines: therefore

$$\lambda = \frac{8a \cdot Pa' \cdot (67)_8}{8a' \cdot Pa \cdot (57)_8}.$$

Similarly,
$$\mu = \frac{7b \cdot Pb' \cdot (68)_7}{7b' \cdot Pb \cdot (58)_7};$$

therefore
$$\frac{\lambda}{\mu} = \frac{8a \cdot Pa' \cdot 7b' \cdot Pb \cdot (67)_8 \cdot (58)_7}{8a' \cdot Pa \cdot 7b \cdot Pb' \cdot (57)_8 \cdot (68)_7} \dots\dots\dots(7),$$

and this determines the ratio $\lambda : \mu$.

Through the point 8 draw a line parallel to Z , meeting the line (57) in c , and (67) in d ; also suppose k to be the other point in which this line intersects the conic (3). At the point k we have

$$(58) = k8, (68) = k8, (57) = kc, \text{ and } (67) = kd;$$

therefore (3),
$$\lambda \cdot kc = \mu \cdot kd,$$

or,
$$kc : kd :: \mu : \lambda \dots\dots\dots(8).$$

Hence we may find a fifth point on the conic (3) in the following manner.

Join 8 and 1, and 7 and 2, by straight lines intersecting in P ; and find a, b , the other points of intersection of these lines with the conic passing through the points 12345; also find the other points a', b' of intersection of the same lines with the conic passing through 12346. Through 8 and 7 draw any parallel lines, the former line meeting the lines

* This can, of course, be done in several ways, one of which immediately follows from Pascal's theorem.

(57) and (67) in e and d , and the latter meeting (58) and (68) in e and f . To $8c$, $8a$, and Pa' find a fourth proportional p ; to $8d$, $8a'$, and Pa , a fourth proportional q ; to $7f$, $7b'$, and Pb , a fourth proportional r ; to $7e$, $7b$, and Pb' , a fourth proportional s ; and, finally, to q , p , and r , find a fourth proportional t . Divide cd in k , so that

$$kc : kd :: s : t,$$

then k will be a point on the conic (3).

In like manner a point k' may be found on the conic (6); and having thus found a fifth point on each conic, we may now proceed as follows.

Having given five points 5 , 6 , 7 , 8 , k on one conic, draw the tangents to the conic at the points 6 and 7 , and let them intersect in L ; also having given five points 1 , 5 , 6 , 7 , k' on the other conic, draw tangents at the points 6 and 7 , intersecting in M ; join LM ; join also the points 5 , 6 and the points 5 , 7 by straight lines intersecting LM in A and B respectively; join A , 7 and B , 6 by straight lines intersecting in 9 ; then 9 is the fourth point of intersection of the two conics (3) and (6), and therefore the required ninth point of intersection of the two curves of the third degree.

The truth of this construction is easily seen, for the two conics (3) and (6) having the same inscribed quadrilateral 5697 , the intersection of each pair of tangents at the opposite angles 6 and 7 , must lie on the line joining the intersections of the opposite sides of the quadrilateral.

It will be observed, that if we multiply (2) by an arbitrary constant θ , and add the result to (1), we have

$$(12345) \cdot \{(67) + \theta \cdot (68)\} = (12346) \cdot \{\lambda(57) + \theta\mu \cdot (58)\}$$

for the general equation to curves of the third degree passing through the eight given points.

From the last equation we also infer that the equation to a curve of the third degree may be written

$$C \cdot s = C' \cdot s'$$

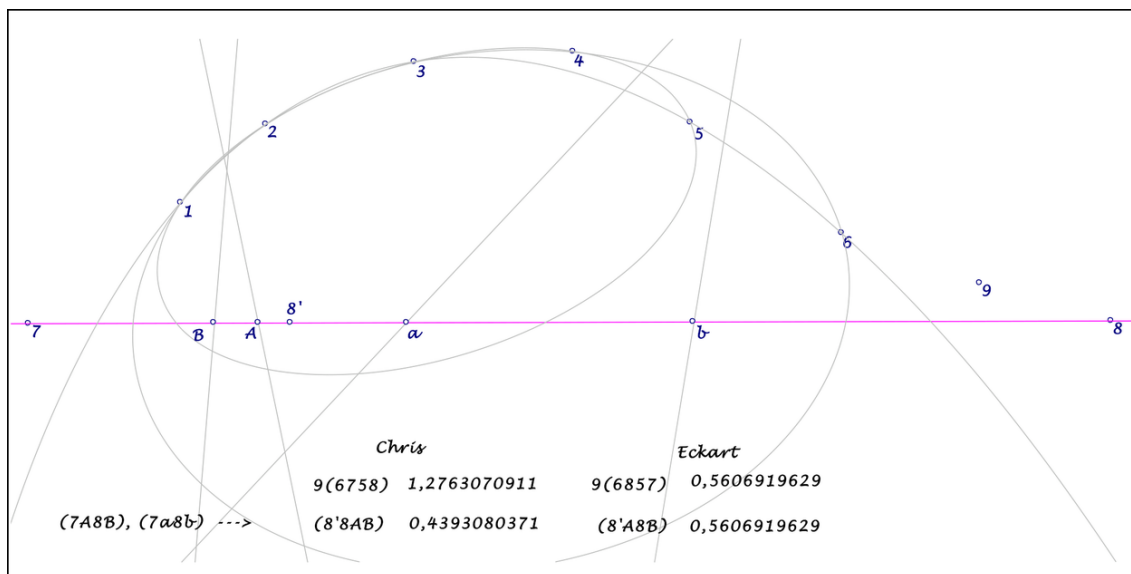
in an infinite number of ways; where C and C' are of the second degree, and s and s' of the first. This equation seems to have some analogy to the equation of a conic circumscribed about a quadrilateral, and several interesting inferences may be drawn from it.

Nov. 22, 1850.

Message: #3059
Date: 29/5/2018 4:18:35
From: eckart_schmidt@t-online.de
Subject: Construction Cayley-Bacharach Point

Dear Chris,
 thanks for your detailed reply!
 Wrt 1): The anharmonic ratio in Wikipedia is $(A,B,C,D = (AC*BD)/(BC*AD)$,
 but that isn't your double ratio. I suppose typos.
 Wrt 2): I have tried with my interpretation of Hardt's anharmonic ratio,
 ... to find a value, so that prop.3 and the last sentence of Hardt's paper hold.
 I succeeded wrt prop.3, but his last sentence needs a correction.
 Wrt your sight, that the anharmonic ratio is the cross ratio:
 ... Let us replace "product" by "construction prop.3"
 ... with your order C'CBD for the constructed pencil:
 Also in this case Hardt's last sentence isn't correct!
 If we use my order C'BCD
 ... and replace 9(6758) by 9(6857) and 9(6748) by 9(6847),
 ... the last sentence is correct.
 Perhaps the attached file is helpful.
 Wrt. 3) Sorry, I can't help.
 Wrt. 4) I studied the paper of Weddle last summer, but gave up.

Best regards Eckart



2018-05-29.pdf

Message: #3060
Date: 29/5/2018 5:15:03
From: chris.vantienhoven
Subject: Construction Cayley-Bacharach Point

Dear Eckart,

wrt 1) typos indeed, thanks for mentioning.

wrt 2) your description helps.

As a matter of fact I replaced 9(6758) by 9(6785) and 9(6748) by 9(6784). When I correct that in the anharmonic-product-order, then indeed reordering of the most obvious order C'BCD is needed.

I concluded that that apparently the most obvious order C'BCD wasn't the right one.

You figured that the most obvious order C'BCD was the right one and the anharmonic-product-order apparently had to be changed. For me it is clear now that we encountered the same problem and each found a different way of describing it.

wrt 3) Maybe Bernard Keizer can have a look ?

wrt 4) Good to know that you gave up too.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3061
Date: 29/5/2018 5:20:51
From: bernard.keizer
Subject: Construction Cayley-Bacharach Point

Dear Chris, dear Eckart
Wrt 3) I don't think there is generally a 9th tangent to a cubic for 8 given tangents !
We know the duality wrt a triangle and wrt a QA/QL, I don't know a 5P/5L havinf the same property.
The degree of a curve is the number of points of intersection with a line, the class of a curve is the number of tangents which can be drawn from a point. The class of a curve is the degree of it's dual and vice-versa (the dual of a curve of class 3 is by definition a cubic).
2 cubics intersect in 9 points ; therefore if 2 cubics intersect in 8 points, they necessary intersect in a 9th point, named the C-B point of the 8 points.
2 curves of class 3 have in common 9 tangents (duals of the 9 intersection points of the 2 dual cubics) ; therefore if 2 curves of class 3 have in common 8 tangents, they are also tangent to a 9th line (dual of the 9th C-B point of the 8 dual points of the 8 tangents).
For example in the QA/QL world, the deltoïd tangent to the 4 lines is a quartic QL-Qu1 of class 3 and it's dual is a cubic of class 4, but the cayleyan, envelop of the lines through 2 CSC partners on the hessian QL-Cu1 is a sextic also of class 3 (according to Siebeck, it's foci are the 2 invariant points of the Cl-S transformation and the infinity point of the perpendicular to the Newton Line) and it's dual is a cubic of class 6 ; Siebeck's curve tangent to the 6 sides of a QA in their middles is also a sextic of class 3 and it's dual is a cubic of class 6 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

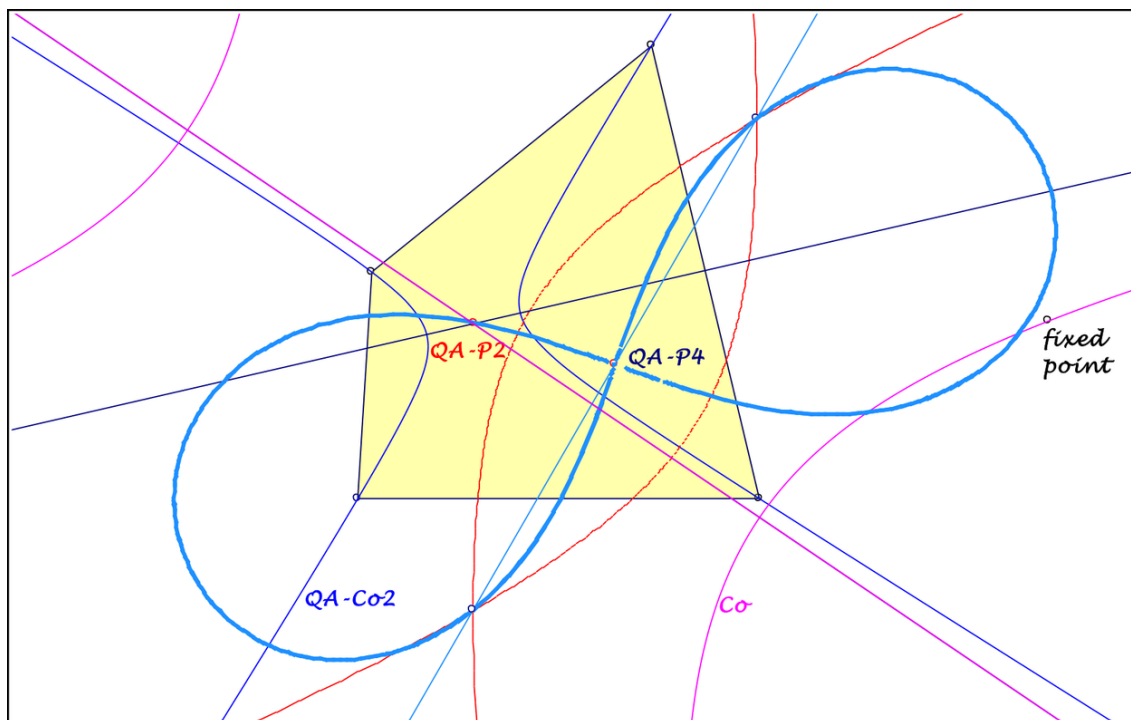
Message: #3062
Date: 30/5/2018 10:19:05
From: eckart_schmidt@t-online.de
Subject: How to construct the inverse of QA-Tf6 (and ...

Dear Benedetto,

in addition to the envelope of lines QA-Tf6inv of points on QA-Co2:

Let us replace QA-Co2 by orthogonal hyperbolas Co,
... centered in QA-P2 through a fixed point P,
... and we get curves with the same shape as for QA-Co2,
... whose double points lie on a *lemniscate*.
I hope, the attached file will be transmitted.

Best regards Eckart



2018-05-30.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

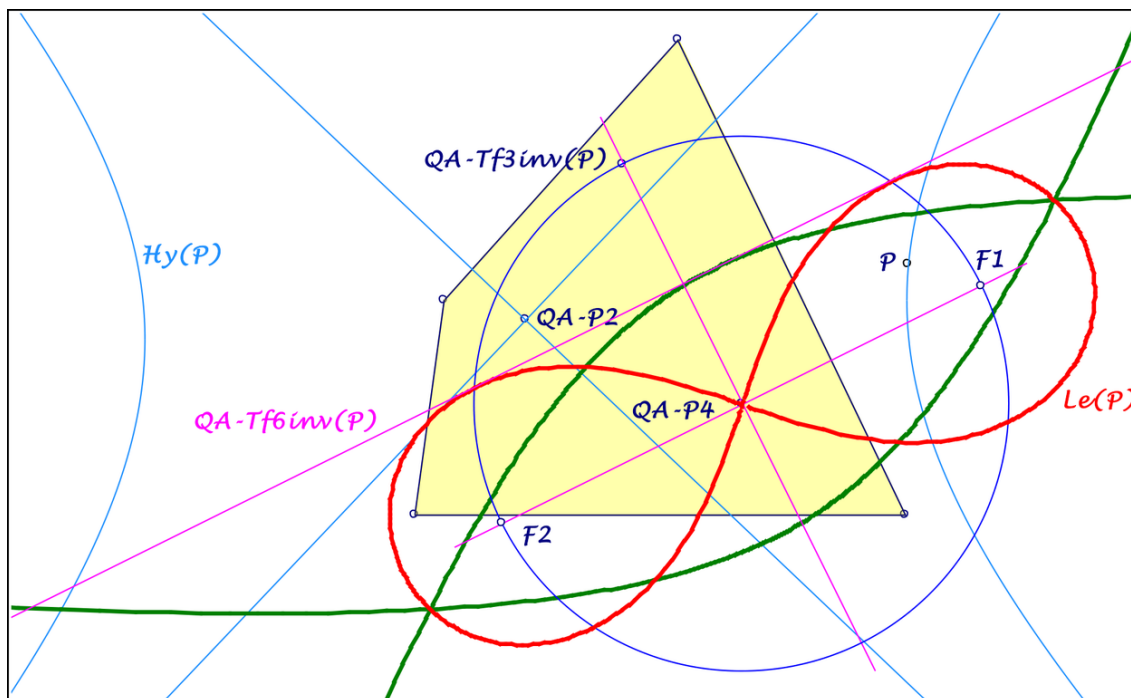
Message: #3063
Date: 31/5/2018 4:27:47
From: eckart_schmidt@t-online.de
Subject: QA-Lemniscates

Dear Benedetto,

in #3062 I mentioned QA-lemniscates, here some results:
Let $Hy(P)$ be orthogonal hyperbolas,
... centered in QA-P2,
... bearing a fixed point P .
... The QA-Tf6inv-lines of points on $Hy(P)$
... envelope a curve with two double points (example in #3057).
... The locus for these double points is a lemniscate $Le(P)$:
... centered in QA-P4,
... main axis parallel QA-Tf6(P),
... QA-Tf6(P) is one double tangent of the lemniscate,
... minor axis QA-P4.QA-Tf3inv(P),
... foci on a circle round QA-P4 through QA-Tf3inv(P).
Special example for $P = QA-P4$
... with minor axis parallel to QA-P2.QA-P23.

Best regards Eckart

PS. I hope, the attached file will be transmitted.



2018-05-31.pdf

Message: #3064
Date: 01/6/2018 10:20:07
From: eckart_schmidt@t-online.de
Subject: QA-P4

Dear Chris,

I couldn't find the following property of QA-P4 in EQF:
The six pedal points of QA-P4 wrt the QA-lines are coconic.

Or:

The reflections of QA-P4 in the six QA-lines are coconic.
Let us have a look on the corresponding conic in the second case:

- ... centered in QA-P2,
- ... with axes parallel to the asymptotes of QA-Co2,
- ... bearing QA-P4,
- ... tangent in QA-P4 orthogonal QA-L4.

Further remark:

The intersections of QA-L4 and the asymptotes of QA-Co2
... lie on QA-Co1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3065

Date: 01/6/2018 4:41:32

From: Benedetto Scimemi

Subject: The QA-inscribed quartic and a coincidence

Dear Eckart, dear Chris and other friends, in a past message I described how to construct a point \rightarrow line QA-transformation, $P \rightarrow s(P)$, the inverse of QA-Tf6.

The envelope of $s(P)$ when P runs on QA-Co1 is a quartic which I provisionally called QA-Qu2, as I realized an analogy with the "inscribed quartic", say QA-Qu1, that Eckart studied, in a totally independent context, first in his old message #141, then again in a EQF-Note of 2017/01/10.

In fact, if Eckart's definition for QA-Qu1 is applied to the 4-ng of QA circumcenters (in the role of QA), one finds QA-Qu2. Both curves are examples of "negative pedals of a conic w.r.t. one of its points", a problem I had studied in the past, outside quadrangles matters.

A curious coincidence is the following: the conic and the point involved above are precisely the conic that *Eckart* describes in his *today message* , and the point on it is QA-P4.

I had recently studied both QA-Qu1 and QA-Qu2 in detail. As you will see, the most interesting properties regard non-convex quadrangles (then QA-Co1 is an ellipse). In the present message I list my results for QA-Qu1. QA-Qu2 - the envelope of $s(P)$ - will be the subject of another message.

All results were proved analytically. I apologize in advance for possible typos.

Best regards

Benedetto

* QA-Qu1 inscribed quartic

Eckart defined it (#141, Note of 2017/01/10) in several ways:

as an envelope of a family of lines satisfying a convenient analytical condition (#141)

as QA-Tf9 image of the line at infinity

as an envelope of the asymptotes of QA-circumscribed hyperbolas.

He calculated the curve equation and the coordinates for tangent lines and contact points.

He also described a "simple construction" of QA-Qu1 *as an envelope* of a family of *lines* (2017 Note)

The following construction of QA-Q1 *as a locus of points* seems simpler and somehow more natural:

1) The reflections of QA-P4 on the 6 QA sides lie on a conic, say QA-CoE (similar to QA-Co1 or to its conjugate). QA-CoE passes through QA-P4; its center is QA-P2. (These facts are also in Eckart's today message)

2) For a point E of QA-CoE, let $t(E)$ be perpendicular bisector of segment E.QA-P4.

Then QA-Qu1 is the envelope of $t(E)$, when E runs on QA-CoE.

3) The intersection $Q(E)$ of $t(E)$ with the line normal to QA-CoE in E gives the contact point $Q(E)$ of $t(E)$ on QA-Qu1. Thus $Q(E)$ describes QA-Qu1 as a *point locus* , when E runs on QA-Co1.

The above construction defines mappings

E --> $t(E)$ ---> $Q(E)$

point on QA-CoE --> tangent to QA-Qu1 --> point on QA-Qu1
 (= perp.bis. of E.QA-P4)

In particular, the following 10 cases deserve to be pointed out:

(1 to 6). $E = (A_i + A_j)/2$, a side midpoint. By construction, $t(E)$ is the side $A_i A_j$ of QA. The contact point $Q(E) = V_{ij}$ is the reflection of the diagonal point $A_{ij}.hk$ on E.

The lines $V_{ij}.V_{hk}$ meet in QA-P5 (This is already in Eckart's previous messages)

In order to find further tangents, we construct three circles passing through Q-P4:

- Ci1-4, centered in QA-P1;
- Ci2-4, centered in QA-P2;
- Ci3-4, centered in QA-P3.

(7,8) $E =$ an intersection of C2-4 with the line through QA-P2 parallel to QA-P3.QA.P4;

$t(E)$ is an axis of QA-CoE (parallel to those of QA-Co2).

$Q(E)$ is an intersections of Ci3-4 with the line QA-P3.QA.P4.

These 2 points Q form a central pair of points on QA-Qu1.

(9) $E = QA-P4$ (as a limit). $t(E) =$ line QA-P3.QA-P4.

To construct $Q(E)$, first intersect in M (a central point on QA-Co1) the line QA-P3.QA-P4 with the line through QA-P2 normal to QA-P2.QA-P4, then reflect M on QA-P3.

This point $Q(QA-P4)$ is a (new?) QA-central point lying on QA-Qu1.

(10). $E = QA-P?$, the refl. of QA-P4 on QA-P2. $t(E)$ is the line through QA-P2 normal to QA-P2.QA-P4. For $Q(E)$, reflect the point M above on QA-P2.

This is another (new?) QA-central point lying on QA-Qu1.

Construction of cusps (one or three).

This problem has algebraic degree 3 and cannot be solved by ruler and compass beginning from the reference QA. In this case, however, the classical method of intersecting two conics makes the job:

Intersect QA-CoE with Ci1-4. Besides QA-P4, one finds either one real point E1 (when QA is convex) or three real points E1, E2, E3 (when QA is non-convex). Then $t(E_i)$ is tangent to QA-Qu1 in a cusp $C_{ui}=Q(E_i)$ of QA-Qu1.

Repetition: a cusp C_{ui} is constructed as intersection of the perpendicular bisector of QA-P4.Ei (a line through QA-P1) with the normal to QA-CoE in line Ei.

* Tangency with QA-Co1

The same line $t(E_i)$ cuts QA-Cu1 in another point $T(E_i) \neq C_{ui}$ such that $T(E_i) : QA-P1 : Q(E_i) = 1 : 3$. (This was noticed by Eckart in previous messages; by CABRI?).

QA-Co1 is tangent to QA-Qu1 in $T(E_i)$.

(What follows regards only non-convex QA, when the real cusps are three)

* Properties of the triangle $E = E_1E_2E_3$

For the triangle E

the orthocenter of E is the refl. of QA-P3 in QA-P2. The height of E in Ei is $t(E_i)$.

the circumcenter is QA-P1.

the centroid is QA-P2.

the Steiner circumellipse is QA-CoE, similar and orthogonal to QA-Co1

the Steiner point is QA-P4.

* Properties of the cusps triangle $Cu = Cu_1Cu_2Cu_3$

For the triangle Cu

the centroid is QA-P2. The median in Ci is $T(E_i)$

the Steiner point is the refl. of QA-P3 in QA-P2.

the Steiner circumellipse is QA-P1-homothetic to QA-Co1, factor -3

* Properties of the contact points with QA-Co1, triangle $T =$

$T_1T_2T_3$

The triangle $T = T_1T_2T_3$ is QA-P1-homothetic to $Cu = Cu_1Cu_2Cu_3$, factor -3

Message: #3066
Date: 02/6/2018 12:13:54
From: eckart_schmidt@t-online.de
Subject: The QA-inscribed quartic and a coincidence

Dear Benedetto,

once more a wonderful excursion in QA-geometry,
... full of unexpected results, especially wrt E1E2E3 and
Cu1Cu2Cu3, congratulation!
I was complete surprised, that the conic for the reflections of
QA-P4 in PiPj
... leads to a simple construction of the quartic QA-Qu1!

With great interest I reproduced your results, some remarks:
(1) Could you have a look on (7,8), I cannot follow the
construction.
(2) I think, two typos wrt Cu1Cu2Cu3:
... the centroid will be QA-P1
... and the Steiner point the reflection of QA-P2 in QA-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3067
Date: 03/6/2018 12:08:53
From: Benedetto Scimemi
Subject: Errata Corrige. A new edition coming

Thanks, Eckart, for your kind message. I am glad you have appreciated the construction of cusps by intersecting conics. Your final remarks are perfectly right. The corrections of the two typos are in fact:

The centroid of the cusps triangle is QA-P1 (not QA-P2).

The Steiner point of the cusp triangle is the reflection of QA-P2 in QA-P3 (not of QA-P3 in QA-P2)

A few lines before these typos I discovered one more:

The height of E in E_i is the normal to QA-CoE (in E_i). (not the tangent $t(E_i)$)

As for (7) and (8), of course you could not follow me! Those two lines are just wrong, not a typo. They should start as:

(7,8) E = reflections of QA-P4 on the axes of QA-CoE..... etc.

I know how to correct also the rest, but at this point I believe it is better to prepare a NEW EDITION of the whole QA-Qu1 message. I plan to do it -more carefully, not in a rush- in a few days.

Best regards Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3068
Date: 04/6/2018 9:07:45
From: eckart_schmidt@t-online.de
Subject: QL-Cox

Dear Bernard,

in our discussion of QL-geometry, we often used a conic QL-Cox,
... which is not in EQF, but well known
with interesting properties.

QL-Cox

... is inscribed QL-Tr1 and tangent to the Steiner axes,
... envelope of QL-Tf2-lines for the QL-P1-pencil,
... or envelope of polars for QL-P1 wrt QL-inscribed conics.
There are further properties in QFG-message 481
... and you will of course be able to add more.
This conic leads to QL-quartics,
... as we discussed in November 2015 (also recently in #3031).

Perhaps the following quartic isn't mentioned:

Reflecting QL-P1 in tangents at QA-Cox,
... we get a quartic with double point QL-P1,
... which is the CSC-image of an orthogonal hyperbola,
... centered in QL-P6 with asymptotes parallel to the Steiner
axes,
... bearing QL-P4 and QL-P5.

I think, it would be a good present to your 70th birthday,
... if Chris published QL-Cox in EQF.
Congratulations and best wishes!

Eckart

PS. Perhaps the attached photo will be transmitted.
(note editor: this photo 2018-06-040001.jpg was lost because of
Yahoo Groups-problems)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3069
Date: 06/6/2018 3:46:30
From: bernard.keizer
Subject: QL-Cox

Dear Eckart,
Thanks a lot for this unexpected birthday present !
I was already more than happy with your sextic in messages 3044
(and 496) as locus of the foci of QA-circumscribed conics.
For both items, you give the definition, the construction, the
properties and the equation !
What else could we need ?
Now we have in fact to hope and wait until Chris publishes them
in EQF ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3070
Date: 06/6/2018 5:34:25
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Benedetto, dear Chris, dear Eckart,

Wonderful curve discovered long time ago by Eckart and renewed beautifully by Benedetto.

Here 3 comments before the new edition (I hope I come in time)

1) There are 3 other typos in the message 3065

points 1 to 6 : according to the definition, the points E are the reflexions of QA-P4 in the QA-sides (not the middles), but that doesn't change the Q(E)

points 7 and 8 : the axes of QA-CoE are parallel to the axes of QA-Co3 (and not QA-Co2)

the points Q(E) with E reflexions of QA-P4 in both axes are on QA-P3QA-P4 the intersections with the circle with center QA-P3 through QA-P2 (and not QA-P4)

2) The described quartic for QA non convex is a deformed deltoïd and we have exactly the same properties of a deltoïd with inner curve QA-Co1 instead of an inner circle.

Starting from one of the 3 contact points T1, T2 or T3, 2 points describe the inner curve in the same direction, one N (the secondary point) at a speed double from the other M (the primary point). But here, we deal with lengths on the ellipse (or the hyperbola) instead of on the circle and the arcs of conics are not (as for the circle) proportionnal to the angles). Note that the 3 arcs T1T2, T2T3 and T3T1 are equal, as QA-Co1 is the Steiner circumellipse of the T-triangle. The lines MN through the 2 points envelop the deltoïd and the contact point is the reflexion of the secondary point in the primary point (which explains the given examples)

In each point of QA-Co1, there are 3 tangents to the deltoïd, 2 with the point as secondary point and one as primary point. For E and it's reflexion E' in QA-P2, the perpendicular bisectors of QA-P4E and QA-P4E' intersect in a point N as secondary point, the second intersections of the 2 tangents with QA-Co1 are 2 points M and M' symmetric wrt QA-P1, Q and Q' are the reflexions of N in M and M' and QQ' is tangent to the deltoïd and passes through N', the reflexion of N in QA-P1 (N and N' are this time primary points) ...

3) the curve with 1 or 3 cusps is a quartic of class 3 and it's dual curve is a cubic circumscribed to the QL-vertices and through the dual points of the tangents (QA-P3QA-P4 or for example the 3 lines VijVkl through QA-P5 have 3 dual points aligned on the dual line of QA-P5 which passes through the intersection of the Newton Line, dual of QA-P10 and the Newton Line of DQL, dual of QA-P1)

It is a NK wrt the cevian triangle wrt DT of a point I cannot identify ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3071

Date: 06/6/2018 6:57:09

From: Benedetto Scimemi

Subject: Errata Corrige. A new edition coming

Dear Bernard, thank you for taking active and precious part in this subject. I am attaching a second edition as it was yesterday. You will see that some of your corrections were already made, other were not. Thanks. Also I hope my attached pictures will be readable.

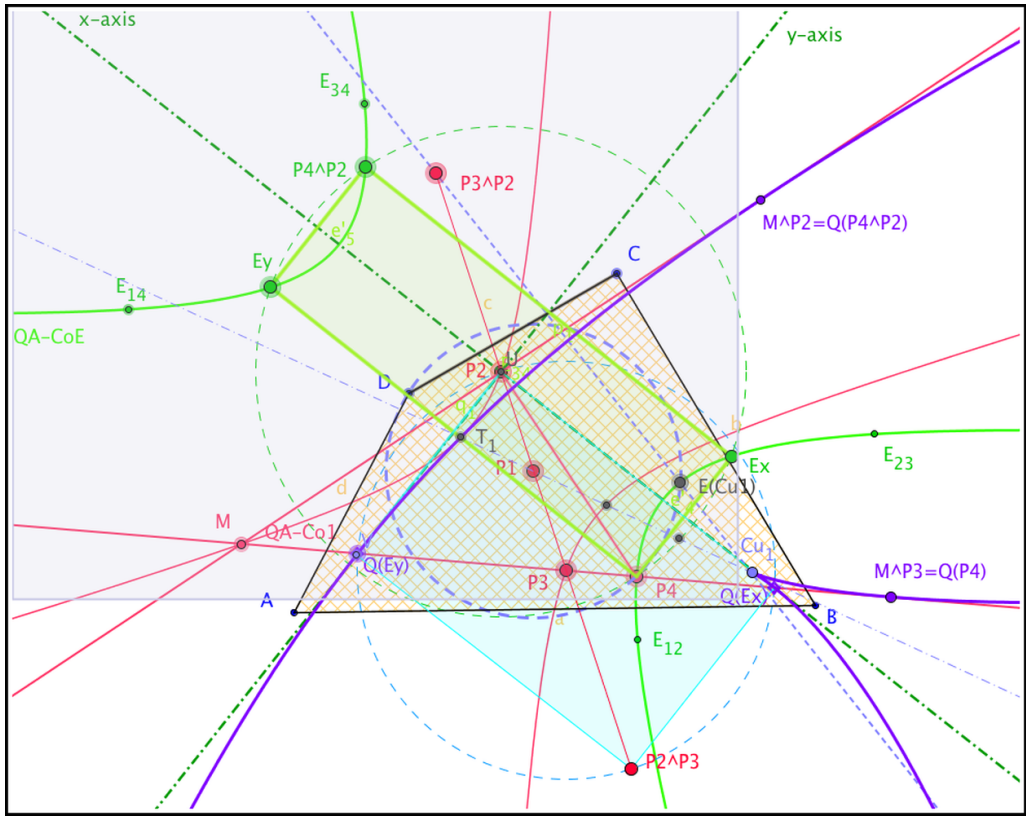
As for the stretched deltoid, this is the subject of a paper I never published. It was precisely when studying the negative pedals that to discover an application to QA, i.e QA-Qu1 and QA-Qu2. Since you were so patient to read carefully # 3065, let me attach also the last draft of this paper. I wonder if I should eventually publish it (but I do not know where and I am old and lazy anyway). You will find there an analytic proof of the angular relations for the so-called ex-centric anomalies, that you mention. The last section must be revised anyway; it regards the Steiner triangle and finally the quadrangle. Tell me what you think of it.

Helas, I could not follow the very last part of your message, as I am not familiar with duals and cubics. Too bad.

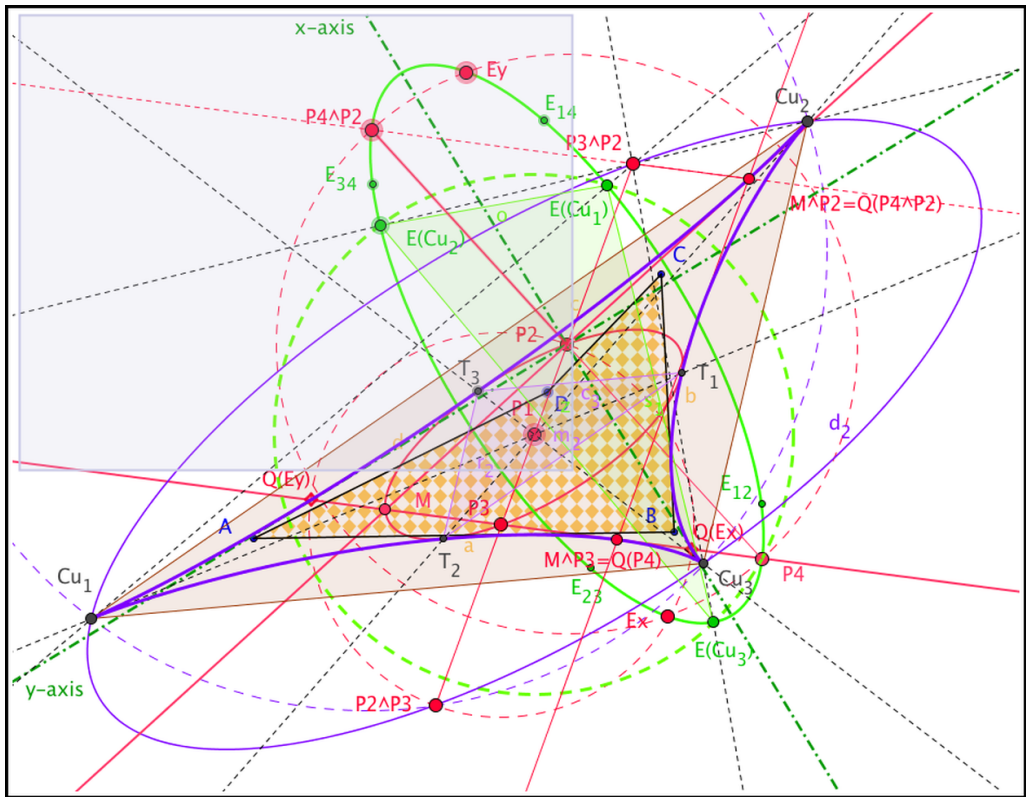
Convex case

Non-convex case

Best regards Benedetto



QA-Q1 convesso per il gruppo.png



QA-Qu1 concavoper il gruppo.png

Message: #3072
Date: 07/6/2018 10:15:02
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Bernard, dear Benedetto,

wrt Bernard's mentioned typo in #3065 wrt (1 to 6)
I think, Benedetto means really E midpoint of A_iA_j ,
... only the 2nd sentence isn't correct,
... for $t(E)$ is the perpendicular bisector of $E.QA-P4$.
... Without the 2nd sentence the properties hold.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3073
Date: 07/6/2018 12:29:25
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Benedetto, dear Eckart

Thanks for Benedetto's answer, I could print the message with the 2 figures, but I cannot open the attachments, which I deeply regret.

Perhaps you could send them as pdf.files at my personal e-mail adress ?

For Eckart, the middles cannot be E points as they aren't on the conic $QA-CoE$. (Anyhow, it's clearly the points named E_{12} , E_{14} , E_{23} and E_{34} on Benedetto's figures, E_{13} and E_{24} are omitted). Have you tried to draw the dual of the curve $QA-Qu1$? In this case, are you able to identify the DT point I mention ? The dual cubic (and I suppose the quartic $QA-Cu1$) are invariant wrt the cevian triangle of this point wrt DT which has this point and the DT vertices as fixed points.

Best regards

Bernard

PS the construction holds for any QA and it's $QA-P4$, for example $QA-P41$ and the DT formed by the DT-vertices and $QA-P4$ or for the 3 QA's of a QL (what about the intersections of the 3 curves $QA-Qu1$?)

There is no limit to geometric games with QA and QL ... (Only time and I'm getting old and lazy like Benedetto)

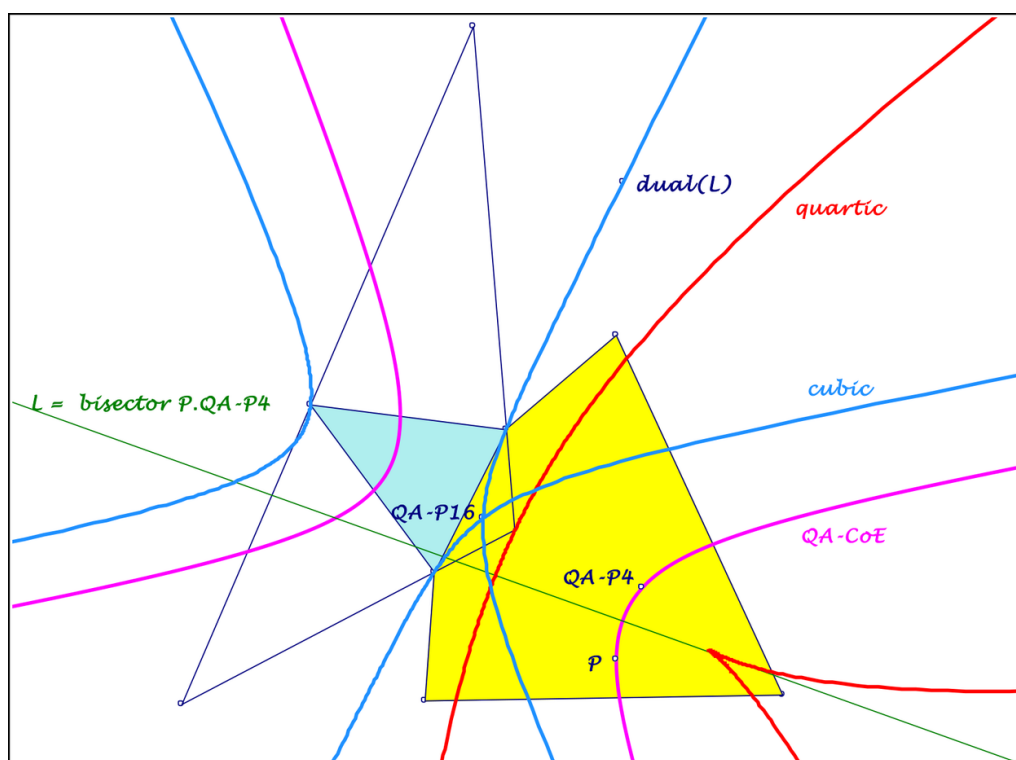
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3074
Date: 07/6/2018 2:12:41
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Bernard, dear Benedetto,

wrt Bernard's comment 3) in #3070:
Bernard, you describe the dual of the quartic
... as nonpivotal isocubic wrt a cevian triangle of QA-DT.
Here a precision:
... reference triangle: cevian triangle of a QA-vertex P_i
wrt QA-DT,
... isoconjugation with fixed point QA-P16,
... root P_i .
There is a simple construction for the conic:
Take the intersections
... of tangents at the QA-DT-inscribed Steiner ellipse
... and their isoconjugate conic wrt the isoconjugation above.

Best regards Eckart
PS: Perhaps the attached file appears.



2018-06-07.docx

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3075
Date: 08/6/2018 9:09:57
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Bernard,

there is a typo in #3074, last remark:
"There is a simple construction for the *cubic*."

Excuse, Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3076
Date: 08/6/2018 10:14:15
From: eckart_schmidt@t-online.de
Subject: Triangle of bisectors for opposite QL-points

Dear all,

this is a research of the triangle QL-Trx,
... whose sidelines are the bisectors of opposite QL-points
... with vertices in the three quadrigon points QG-P5
... and connected with a lot of collinear and concyclic QL-points.

QL-Trx is similar to QL-Tr1
... with orthogonal corresponding sides
... and perspective wrt QL-P16.

The circumcircles of QL-Trx and QL-Tr1
... intersect orthogonal in QL-P16 and QL-P17.

The Simson line
... of QL-P16 wrt QL-Trx is parallel QL-L1,
... of QL-P17 wrt QL-Trx is parallel QL-L9.

The Euler lines of QL-Tr1 and QL-Trx
... intersect orthogonal in a special point X,
... which is the intersection of QL-L7
... and a parallel to QL-P5.QL-P7 through QL-P17.

Let G, O, H be the centroid, circumcenter, orthocenter of QL-Trx.

... $G = QL-P16.QL-P24 \wedge QL-P6.QL-P10$, H on $QL-L9$
 and O on the tangent in $QL-P17$ at $QL-Ci1$.

The circle $Ci(G)$ with diameter in the centroids
 ... bears G , $QL-P8$, $QL-P17$, $QL-P24$, X .

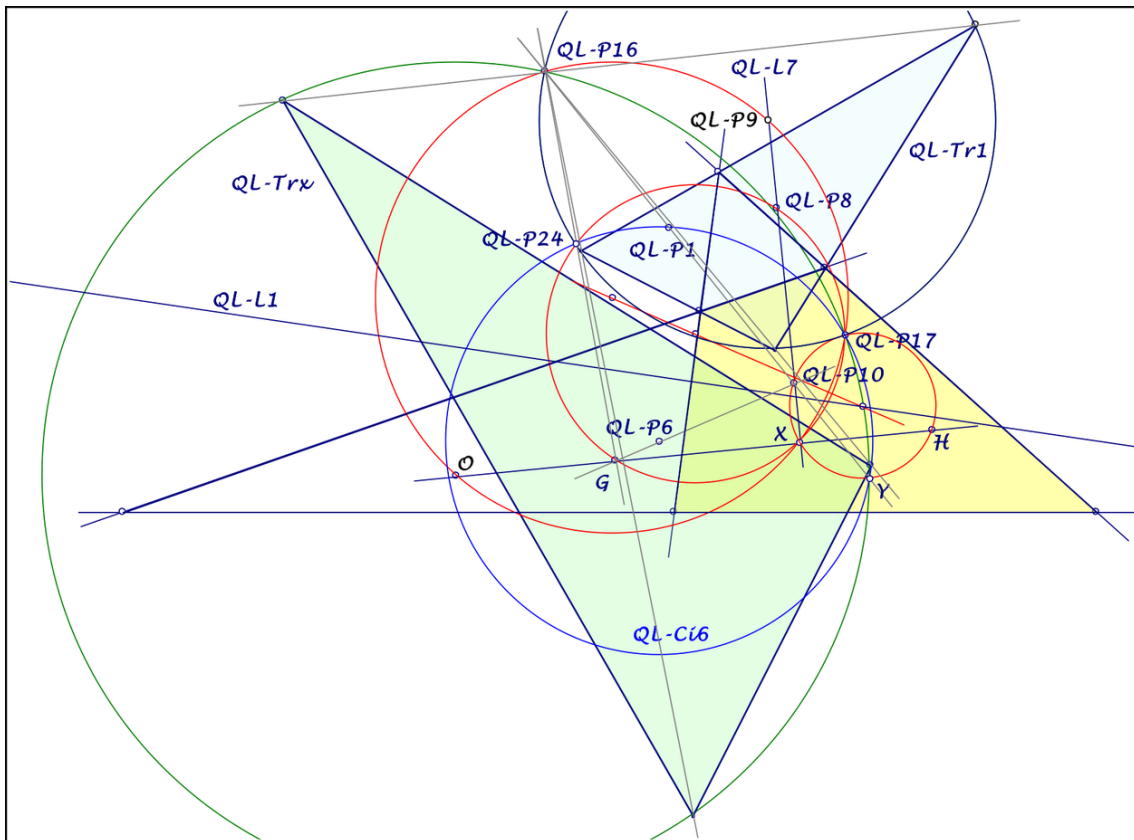
The circle $Ci(O)$ with diameter in the circumcenters
 ... bears O , $QL-P9$, $QL-P16$, $QL-P17$, X .

The circle $Ci(H)$ with diameter in the orthocenters
 ... bears H , $QL-P10$, $QL-P17$, X and a point Y ,
 ... which is the 2nd intersection of $QL-P1.QL-P10$ and $QL-Ci6$
 on the circumcircle of $QL-Trx$.

The circles $Ci(G)$, $Ci(O)$, $Ci(H)$
 ... have common points $QL-P17$ and X
 ... and collinear centers on the bisector of $QL-P17$ and X ,
 ... which intersects $QL-L1$ in the center of $Ci(H)$.

Best regards Eckart

PS. Perhaps the attached file will be transmitted.



2018-06-08.pdf

Message: #3077
Date: 08/6/2018 11:20:01
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Eckart,

Of course, my searched point is QA-P16, thanks a lot !

You give 4 transformations (the cevian of P_i wrt DT is a reference triangle of the QL and the line which defines the nK is the 4th line of the QL hence the root is P_i)

There is a 5th one with the cevian triangle of QA-P16 wrt DT as reference and the transformation wrt this triangle with fixed points in QA-P16 and the vertices of DT which swaps opposite QL vertices.

Amusing that the sides of this cevian triangle of QA-P16 wrt DT pass through the middles of the segments joining 2 opposite QL vertices.

The line defining the nK through the intersections of the cubic with the sides of this triangle is the dual line of QA-P5 which passes through the intersections of the Newton Line, dual of QA-P10 and the Newton Line of the diagonal QL, dual of QA-P1.

Best regards

Bernard

PS For the points 1 to 6, I give up ! I don't understand what's worrying you : on your own figure, which you sent me, forget the point E, as it precisely defines 2 different points, just name E_{ij} the reflexion of QA-P4 in the sides A_iA_j and p_{ij} the middle of A_iA_j .

V_{ij} is the intersection between the side A_iA_j and the normal in E_{ij} to QA-CoE and it happens that it is also the reflexion of $A_{ij}.h_k$ in p_{ij} (not in E_{ij}) ; that's exactly Benedetto's formulation. The same goes for V_{hk} .

As I tried to explain in my answer to Bedetto, this is a general property of the deltoïd (stretched or regular) ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3078
Date: 09/6/2018 11:01:48
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Eckart,
Rereading the different messages about this stretched deltoïd, I found that everything was said in your message 2000 (note 2017/01/10) with the construction given at the end of the 2nd page !
I checked that it works with any conic (apart of parabola) and any point on this conic.
If we have the conic as QA-Co1, it's center as QA-P1 and a point as QA-P2, we get QA-P3 and QA-P4 (new construction not in EQF) and the curve with one cusp for hyperbola and 3 cusps for ellipse (in this case, the curve is a stretched deltoïd tangent in 3 points to QA-Co1, which is the Steiner ellipse of the triangle of the contact points and QA-P2 is the Steiner point).
I suppose any circle through QA-P2 will cut the conic in 3 other points being the DT vertices of a suitable QA...
I just wanted to give back to Cesar what belongs to Cesar !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3079
Date: 09/6/2018 7:23:01
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Benard,

wrt #3077: You are right, the cubic dual(QA-Qu1) is also a nK with
... reference triangle: cevian triangle of QA-P16 wrt QA-DT,
... isoconjugation with fixed point QA-P16,
... root QA-P5.

You can construct the cubic
... with the intersections of tangents at the QA-DT-inscribed Steiner ellipse
... and their isoconjugate conic wrt the isoconjugation above.
F or points on the cubic this isoconjugation and that in #3074 give the same image.

Additional remark:

Wrt nonpivotal isocubics Bernard Gibert describes a circle
... in 1.5.3 of his "Special Isocubics ...":
"Theorem: Any nK can be considered as the locus of point M such that the points M and M* are conjugated with respect to a fixed circle."

This circle for the here discussed nK is easy to be constructed:
... centered on QL-L2,
... through QA-P16 = QL-P13
... and orthogonal QL-Ci5.
Perhaps a new QL-circle?

Best regards Eckart

PS: Wrt #3078: My note 2017-01-10 was in #2200, not 2000.

I shall answer in a further message.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3080
Date: 10/6/2018 7:53:03
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Benedetto, dear Eckart
The more I look on this wonderful item of stretched deltoïd, the more I get excited about it !
Just a conic and a point on it !
We may consider it as a QA-Co1 centered in QA-P1 and the point in QA-P2 and we get QA-P3 and QA-P4 and the QA-CoE centered in QA-P2 through QA-P4 ; Benedetto's method applied to QA-CoE or Eckart's method applied to QA-Co1 give the same result. (If we start with QA-CoE centered in QA-P2 and the point QA-P4, we get obviously QA-Co1 centered in QA-P1 and QA-P3 ...)
But we could as well take the conic QA-CoE as a QA-Co1 with the point QA-P4 as a point QA-P2 and would find a new quartic !!!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3081
Date: 11/6/2018 8:57:23
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

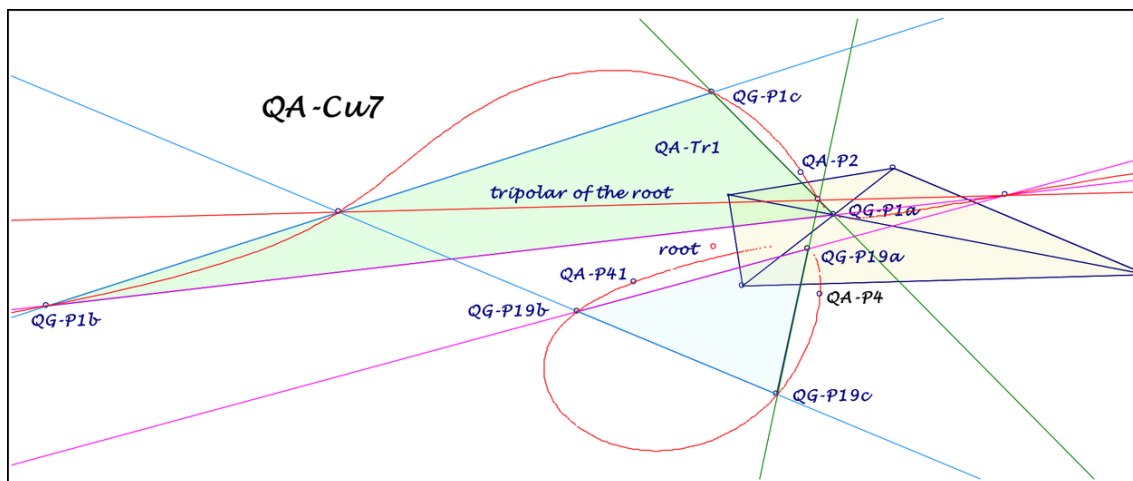
Dear Chris,

the cubic QA-Cu7 is finally in EQF described
... as pivotal isocubic with
... .. reference triangle QA-P2.QA-P4.QA-P41,
... .. isoconjugation: isogonal conjugacy,
... .. pivot: infinity point of perpendiculars of QA-L2.

The cubic QA-Cu7 can also be described (see attached file)
... as nonpivotal isocubic with
... .. reference triangle QA-Tr1,
... .. isoconjugation QA-Tf2,
... .. root: trilinear pole for the line L
... .. of the collinear intersections
QG-P1i.QG-P1j^QG-P19i.QG-P19j.

QA-Tf6inv maps points of L to lines,
... which envelope a parabola with focus QA-P4.
What about this parabola?

Best regards Eckart



2018-06-11.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3082
Date: 11/6/2018 9:22:24
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Bernard,
sorry, I can't follow in #3078 and 3080:
How to construct QA-P4 with QA-Co1, QA-P1, QA-P2, QA-P3?
What is the "new construction not in EQF"?
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3083
Date: 11/6/2018 11:30:37
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Eckart,
Sorry for being too elliptic !
In the general configuration, we have 2 conics in correspondance
(we don't need the QA) :
1) one is Co1 with center I and 2 particular points X_0 and Y_0
symmetric wrt I
2) the other is Co2 centered in X_0 and a point E_0
For X and Y on Co1 symmetric wrt I, the circle through X, Y and
 Y_0 cuts the conic Co1 in a 4th point Z and the reflexions of E_0
in ZX and ZY are 2 points E and E' on Co2
Conversely, for E and E' on Co2 symmetric wrt X_0 , the
perpendicular bisectors of X_0E , X_0E' and EE' intersect in a
point Z on Co1.
ZX and ZY envelop the quartic and the current points are the
reflexions of Z in X and Y.
If the 2 conics are in correspondance, in the limit case the
circle through X_0 and Y_0 and tangent to Co1 in Y_0 cuts Co1 in a
4th point Z_0 ; the point E_0 is the intersection between the line
 Z_0Y_0 and the perpendicular line in X_0 to Z_0X_0 .
If Co1 is QA-Co1 with I as QA-P1 and X_0 as QA-P2, Y_0 is QA-P3 and
 E_0 is QA-P4.
Then the vertices of a possible DT are on QA-Co1 the 3 other
intersections with any circle through QA-P2 ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3084
Date: 11/6/2018 3:52:40
From: bernard.keizer
Subject: Errata Corrige. A new edition coming

Dear Eckart,
Applying twice your transformation to the 1st conic, you get first Z for X and Y symmetric wrt the center I, then W for Z and Z' symmetric wrt the center I : the reflexions of W in X and Y describe a strange curve (sextic with 3 locks) which is also tangent to the conic in the same points T1, T2 and T3 and passes through the 3 cusps Cu1, Cu2 and Cu3 ...
Fascinating, isn't it ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3085
Date: 12/6/2018 8:31:58
From: eckart_schmidt@t-online.de
Subject: Errata Corrige. A new edition coming

Dear Bernard,

thanks for the interesting construction
... of QA-P4 with QA-P1, QA-P2, (QA-P3) and QA-Co1.

You construct QA-P4 as intersection
... of Z_0 .QA-P3 and a perpendicular to Z_0 .QA-P2 in QA-P2,
... with $Z_0 = QA-Tf2$ of the infinity point of the tangent at QA-Co1 in QA-P3.

As you proposed: I took the conic QA-CoE
... as QA-Co1 with QA-P4 as a point QA-P2,
... but I found no relevant properties.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3086
Date: 13/6/2018 10:00:11
From: eckart_schmidt@t-online.de
Subject: Two cubics for a QA/QL constellation

Dear Bernard, dear Benedetto, dear Chris,

this is a comparison orientated at the asymptotes
... of QA-circumscribed hyperbolas
... and QL-inscribed hyperbolas,
... first mentioned in #2200 and #2204.

Let us consider the constellation of a QA and its dual QL:

Asymptotes of QA-circumscribed hyperbolas
... envelope a quartic with a dual cubic QA-Cu,
... recently discussed (#3074 and #3077).

Asymptotes of QL-inscribed hyperbolas
... envelope a quartic with a dual cubic QL-Cu.

QA-Cu bears the QL-points,
... QL-Cu bears the QA-points and the vertices of the common DT.

Both cubics have a common point QA-P16 = QL-P13
... and bear the cevians of this point wrt DT.

The 9th Cayley-Bacharach point on QA-Cu
... of the 6 QL-points, QA-P16 = QL-P13 and any point on QA-Cu
... is QA-P16 = QL-P13, double point of QA-Cu.

The 9th Cayley-Bacharach point on QL-Cu
... of the 4 QA-points, the DT-vertices and QA-P16 = QL-P13
... is QA-P10 = QL-P8.

Both cubics are invariant for the isoconjugation *
... wrt the cevian triangle of QA-P16 = QL-P13
... with fixed point QA-P16 = QL-P13.

QA-Cu is a nonpivotal isocubic with
... reference triangle: cevian triangle of QA-P16 = QL-P13,
... isoconjugation with fixed point QA-P16 = QL-P13
... root QA-P5.

QL-Cu is a pivotal isocubic with
... the same reference triangle
... and the same isoconjugation,
... but pivot QA-P10 = QL-P8.

QL-Cu can also be described as pivotal isocubic with

... reference triangle DT,
... isoconjugation QA-Tf2,
... pivot QA-P16 = QL-P13.

Let Co be the a conic
... circumscribed the cevian triangle of QA-P16 = QL-P13
... and centered in a point
... ... dividing QA-P10.QA-P16 with ratio 1:3.

The isoconjugation * (see above)
maps the line at infinity to Co,
... which bears the six isoconjugated of the infinity points of
QA-Cu and QL-Cu.

Best regards Eckart

PS:

The 9th Cayley-Bacharach point
of the 6 QL-points, QL-P8 and QL-P13
... is the intersection of QL-P8.QL-P23
and a parallel to QL-L9 through QL-P13.
The dual of QL-P23 is tangent in QA-P10 to QL-Cu.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3087
Date: 13/6/2018 4:26:10
From: bernard.keizer
Subject: Two cubics for a QA/QL constellation

Dear Eckart,
Waouh ! Very amazing !!!
It's the 3rd time we find this configuration.
For any point P, the pivotal isocubic $pK(QA-P16,P)$ is also pivotal wrt the cevian triangle $T(P)$ of P wrt DT with transformation with fixed points P and the vertices of DT and pivot the isoconjugate of P.
The vertices of the dual QL and the vertices of $T(P)$ form a C-B system and there is an infinity of nK's wrt $T(P)$ with the same transformation swapping the opposite QL-vertices ; each one is defined by it's root.
For $P=QA-P10$ or $QL-P8$, the QA-Cu is $pK(QA-P16,QA-P10)$ or QA-Cu3 in EQF and one QL-Cu is Siebeck's cubic, dual of Siebeck's sextic.
For $P=QA-P16$ or $QL-P13$, the QA-cubic is $pK(QA-P16,QA-P16)$ and one of the QL-Cu is here described with root QA-P5, dual of the quartic named stretched deltoid.
For $P=QA-P12$ or $QL-P10$, the QA is $pK(QA-P16,QA-P12)$ and one of the QL-Cu is QL-Cu1 in EQF, which is a nK wrt the orthic triangle of DT.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3088
Date: 17/6/2018 10:42:45
From: Benedetto Scimemi
Subject: QA-Qu1 and QA-Qu2

Dear friends, as it is not clear to me if you could read the attachments of my message #3071, I will copy again here the (promised 2nd edition of) a summary of properties of the QA-inscribed quartic, first introduced by Eckart.

My approach is different from his. Some of the following properties are already in Eckart's #141 and 2200. On the other hand, some results of his do not appear here. In the second section you will find a similar summary regarding another quartic, QA-Qu2 (announced in #3036). I had first met QA-Qu2 in a different context and only later realized the connection with QA-Qu1. The last figure shows both QA-Qu1 and QA-Qu2.
I apologize in advance for typos and possible mistakes.
Best regards Benedetto

QA-Qu1 inscribed quartic
(2nd edition)
QA-Qu1 is a quartic with one (QA convex) or three (QA non-convex) real cusps; QA-Qu1 is tangent to all the sides of QA.

(attached picture-1)

Eckart defined it (#141 and the Note attached to #2200) in several ways:
as an envelope of a family of lines satisfying a convenient analytical condition (#141)
as QA-Tf9 image of the line at infinity
as an envelope of the asymptotes of the QA-circumscribed hyperbolas
He calculated the curve equation and the coordinates for tangent lines and contact points.
He also described some constructions (end of Note in #2200)
The following alternative construction permits perhaps simpler proofs: it corresponds to the notion of "negative pedal of a curve w.r.t. a point" (Weisstein, Eric W., MathWorld -- <http://mathworld.wolfram.com/NegativePedalCurve.html>)

- 1) The reflections of QA-P4 on the 6 QA sides lie on a conic, say QA-CoE, similar to QA-Co1 or its conjugate. QA-CoE passes through QA-P4; its center is QA-P2; its symmetry axes are parallel to those of QA-Co1.
- 2) For a point E of QA-CoE, let $t(E)$ be perpendicular bisector of the segment E.QA-P4. Then QA-Qu1 is the envelope of $t(E)$, when E runs on QA-CoE.
- 3) The intersection $Q(E)$ of $t(E)$ with the normal to QA-CoE in E is the contact point $Q(E)$ of $t(E)$ on QA-Qu1. Thus $Q(E)$ describes QA-Qu1 as a point locus, when E runs on QA-CoE.

The above construction defines mappings

E	-->	$t(E)$	-->	Q(E)
point on QA-CoE	-->	tangent to QA-Qu1 (= perp.bis. of E.QA-P4)	-->	point on QA-Qu1

In particular, the following 10 cases of tangents deserve to be pointed out:

(1 to 6). $E = E_{ij}$ the reflexion of QA-P4 on the QA side A_iA_j . By construction, $t(E)$ is the side A_iA_j of QA. The contact point $Q(E) = V_{ij}$ is the reflection of the diagonal point $A_{ij}.hk$ on E_{ij} . The lines $V_{ij}.V_{hk}$ meet in QA-P5 (Eckart's remarks).

(7,8) $E = E_x, E_y$ the reflexions of QA-P4 on the axes of QA-CoE; by construction, $t(E)$ is an axis of QA-CoE. $Q(E)$ is an intersection of the line QA-P3.QA.P4 with the circle C_{i3-2} centered in QA-P3 through QA-P2. These 2 points Q form a central pair of points lying on QA-Qu1.

(9) $E = QA-P4$ (as a limit).
 $t(E) = \text{line } QA-P3.QA-P^*$.
 To construct $Q(E)$, first intersect in M (a central point on QA-Co1) the line QA-P3.QA-P4 with the line through QA-P2 normal to QA-P2.QA-P4, then reflect M on QA-P3.
 This point $Q(QA-P4)$ is a QA-central point lying on QA-Qu1.

(10). $E = P4^{\wedge}P2$, the reflection of QA-P4 on QA-P2. $t(E)$ is the line through QA-P2 normal to QA-P2.QA-P4. For $Q(E)$, reflect the point M above on QA-P2. This is another *QA-central point lying on QA-Qu1.*
 Points E from (7) to (10) form a rectangle inscribed in QA-CoE.

Points at infinity

When QA is convex, QA-Qu1 has two real points at infinity, parallel to the asymptotes of QA-Co1, orthogonal to those of QA.CoE. These points are double for QA-Qu1, hence

(11) For QA convex, the line at infinity is tangent to QA-Qu1 in two real points.
 When QA is non-convex, QA-Qu1 has no real points at infinity.

Construction of cusps (one or three).
 This problem, of algebraic degree 3, cannot be solved by ruler and compass beginning from QA. In this case, however, the classical method of intersecting two conics makes the job: Intersect QA-CoE with the circle Ci1-4, centered in QA-P1 through QA-P4. Besides QA-P4, one finds either one real point Ecu1 (when QA is convex) or three real points Ecu1, Ecu2, Ecu3 (when QA is non-convex). Then t(Ecui) is tangent to QA-Qu1 in a cusp.
 The cusp Cui = Q(Ecui) is thus constructed as intersection of the perpendicular bisector of QA-P4.Ecui with the normal to QA-CoE in Ecui.
 The tangent in a cusp passes through QA-P1.

Tangency with QA-Co1
 The same line t(Ecui) cuts QA-Qu1 in another point Ti such that Ti : QA-P1 : Cui = 1 : 3.
 QA-Co1 is tangent to QA-Qu1 in Ti.

(attached picture-2)

(What follows regards only non-convex QA, when the real cusps are three)

Properties of the triangle Ecu = Ecu1Ecu2Ecu3 (inscribed in QA-CoE)

For the triangle Ecu:
 the orthocenter is the refl. of QA-P3 in QA-P2. The height of Ecu in Ecui is normal to QA-CoE.
 the circumcenter is QA-P1.
 the centroid is QA-P2.
 the Steiner circumellipse is QA-CoE, similar and orthogonal to QA-Co1
 the Steiner point is QA-P4.

Properties of the QA-Qu1 cusps triangle Cu = Cu1Cu2Cu3
 the centroid is QA-P1. The median in Ci is t(Ei)
 the Steiner point is the refl. of QA-P2 in QA-P3.
 the Steiner circumellipse is QA-P1-homothetic to QA-Co1, factor -3.

Properties of the contact points with QA-Co1, triangle $T = T_1T_2T_3^*$

The triangle $T = T_1T_2T_3$ is QA-P1-homothetic to $Cu = Cu_1Cu_2Cu_3$, factor -3

QA-Co1 is the Steiner-circumellipse of T and QA-P2 is its Steiner point.

The QA-P1 centered homothety of coeff. -3 maps T into Cu and QA-Co1 into the Steiner circumellipse of Cu .

QA-Qu2 (quartic of the nine tangents ?)

QA-Qu2 is a quartic with one (QA convex) or three (QA non-convex) real cusps; QA-Qu2 is tangent to nine lines passing through the nine points of QA.

Although it appeared in a different context, involving QA-f3 and QA-Tf 6, the quartic QA.Cu2 turns out to be the same as QA-Qu1, provided the role of QA is played by the quadrangle of its circumcenters

(attached picture-3)

The point-line transformation $R \rightarrow s(R)$ is the inverse of QA-Tf6 (see #3036).

QA-Co1 is the locus of points R such that R lies on $s(R)$.

When R runs on QA-Co1, $s(R)$ describes a family of lines whose envelope is, by definition, QA-Qu2.

Consider the action of the similarity $QA-f3^{(-1)}$: $R \rightarrow R^*$

QA-Co1 \rightarrow QA-CoF (definition)

QA-P1 \rightarrow QA-P3

QA-P2 \rightarrow QA-P4

The center of QA-CoF is QA-P3; the symm. lines of QA.CoF are the bisectors

of the angle $\angle QA-P2 \cdot QA-P3 \cdot QA-P4$, parallel to the axes of QA-Co1

QA-CoF passes through QA-P4 and the reflection of QA-P4 in QA-P3;

the normal to QA-CoF in QA-P4 is QA-P4. QA-P2.

QA-CoF passes through the reflections F_{ij} of QA-P4 in the perp. bisect. of the QA-side A_iA_j

The above properties ensure that QA-Qu2 is the same as QA-Qu1 if the quadrangle

of circumcenters plays the role of QA. Thus most of its properties can be derived, by analogy, from the QA-Qu1 page. Other properties, however, are not so derived

We may call QA-Qu2 the quartic of the nine tangents of QA because

if R is one of the classical QA-nine points (defining QA-Co1) then $s(R)$ is :

- (1 to 6) the perpendicular bisector of a QA-side, if R is the side midpoint;
- (7 to 9) a height of the diagonal triangle, if R is a diagonal point.

N.B. These 9 points R are NOT the contacts of $s(R)$ with QA-Qu2. The following central lines are also tangent to QA-Qu2:

- (10) QA-P2.QA-P4 (R=QA-P2)
- (11) the normal to QA-P3.QA-P4 in QA-P3 (R=QA-P3)
- (12, 13) the axes of QA-CoF (R = Rx , Ry , intersections (\neq QA-P3) of the QA-CoF axes with QA-Co1.

Let N be the intersection of (10) and (11). Then
 the contact of (10) is the reflection of N on QA-P3^QA-P32
 the contact of (11) is the reflection of N on QA-P3
 the contact of (12,13) are the intersections (\neq QA-P3) of the QA-CoF axes with a circle through QA-P3 centered in P3^P32
 These four contacts are QA-central points lying on QA-Qu2:

QA-Qu2 is the negative pedal of QA-CoF w.r.t. its point QA-P4. i.e.: when R runs on QA-Co1 and $R^*=F$ runs on QA-CoF then QA-Qu2 is the locus of the intersection $S(F)$ of
 the normal $m(F)$ to QA-CoF at F with
 the perpendicular bisector $s(F)$ of the segment F. QA-P4
 (the last line is the image of F by the inverse of QA-Tf6) .

Points at infinity

When QA is convex,

QA-Qu2 has two real points at infinity, the same as QA-CoE, orthogonal to those of QA-Co1 and QA-Qu1.

These points are double, hence

- (14) For QA convex, the line at infinity is tangent to QA-Qu2 in two points.

When QA is non-convex, QA-Qu2 has no real points at infinity.

Construction of the cusps

Let Ci32-4 be the circle centered in QA-P32 passing through QA-P4.

Ci32-4 and QA-CoF meet in 1 or 3 points (\neq QA-P4) say D1, (D2, D3),

according to QA being convex or non-convex.

The point $S(D_i)$ is a cusp D_{ui} of $QA-Qu_2$; the tangent $s(D_i)$, the tangent to $QA-Qu_2$ at D_{ui} , passes through $QA-P_{32}$.
 Repetition: The cusp D_{ui} is the intersection of the perpendicular bisector of Di . $QA-P_4$. (which passes through $QA-P_{32}$) and the line normal to $QA-CoF$ in Di (which passes through $(P_3 \wedge P_{32}) \wedge P_3$).

For some families of QA , $QA-Qu_2$ is tangent to $QA-Co_1$, in one or 3 points (even for QA convex).
 The last figure shows an example.

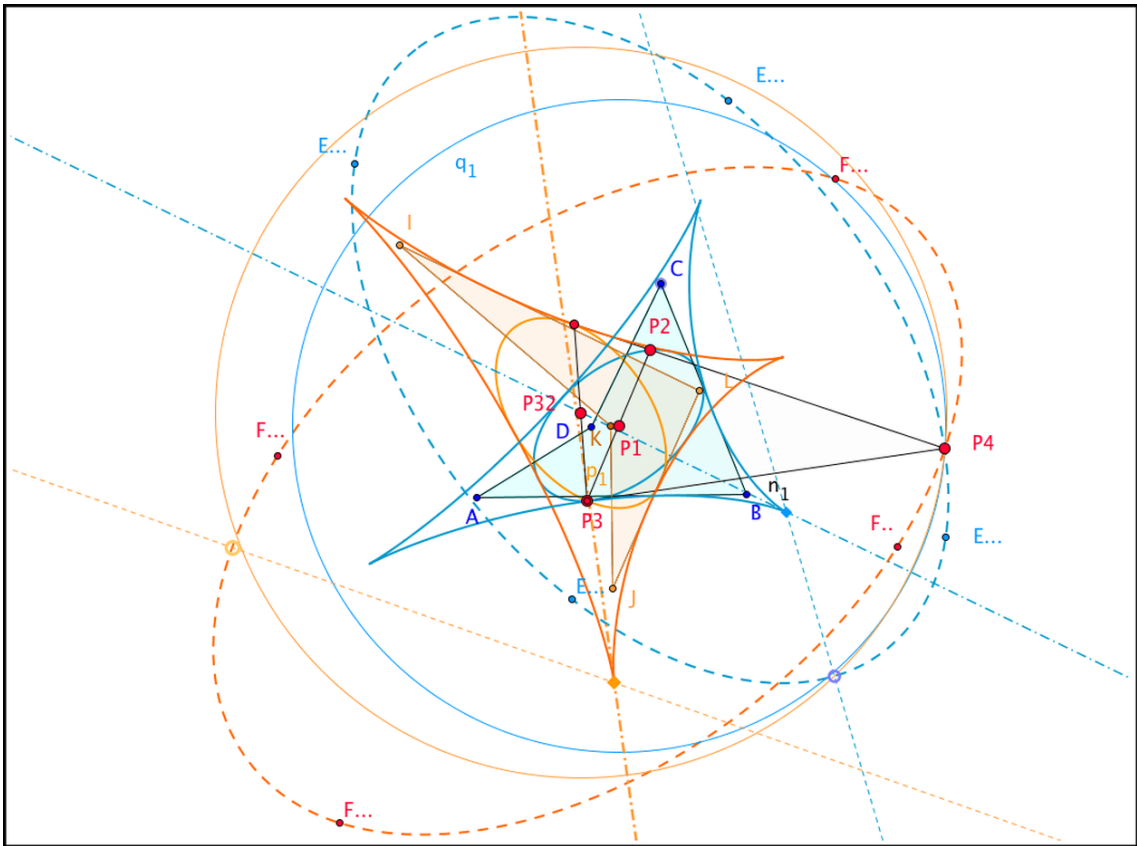
(What follows regards only non-convex QA , when the real cusps are three)

*Properties of the triangle $F_{cu} = F_{cu1}F_{cu2}F_{cu3}$
 the circumcenter is $QA-P_{32}$.
 the centroid is $QA-P_3$.
 the orthocenter is $(P_3 \wedge P_{32}) \wedge P_3$.
 the Steiner circum-ellipse is $QA-CoF$
 the Steiner point is $QA-P_4$.

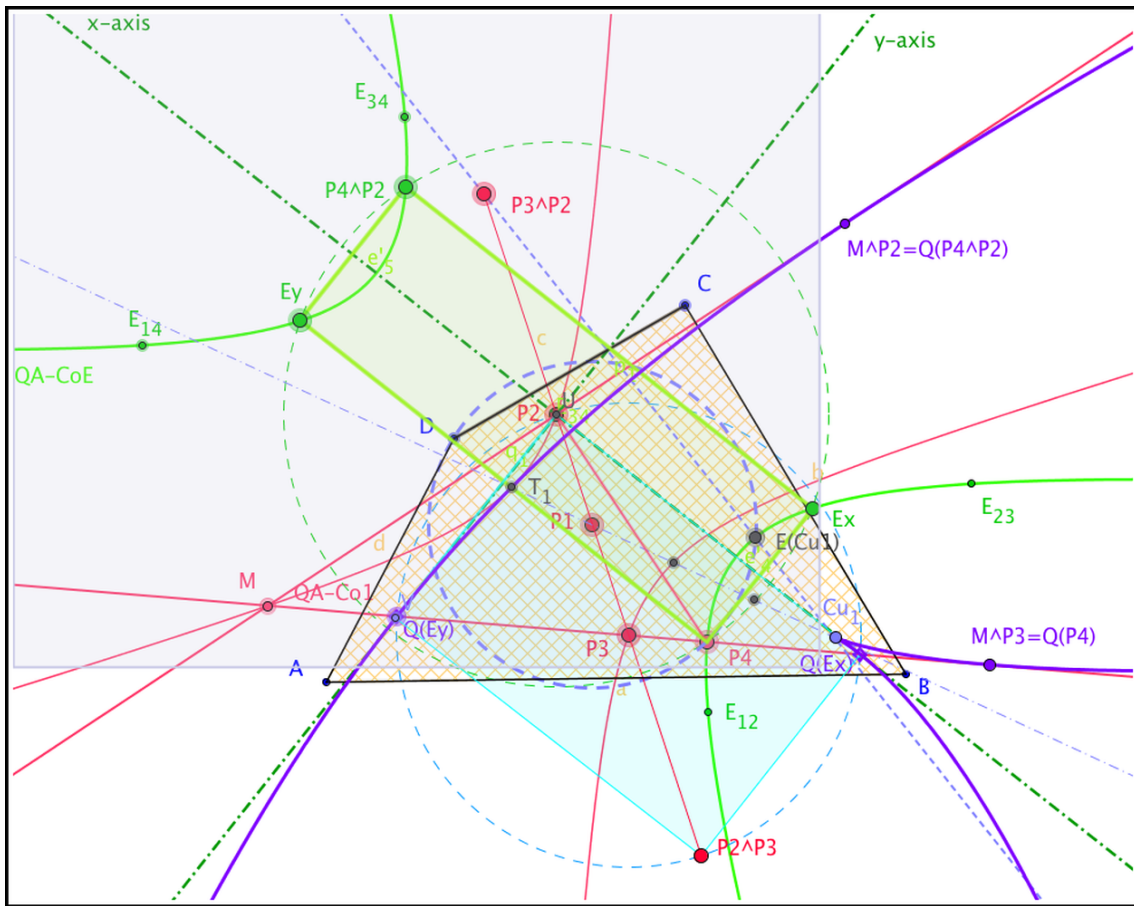
Properties of the cusps triangle $D_u = D_{u1}D_{u2}D_{u3}$
 the centroid is $QA-P_{32}$. The median in C_{ui} is $s(F_i)$
 the Steiner point is $P_3 \wedge (P_3 \wedge P_{32})$.
 the Steiner circum-ellipse is similar and orthogonal to $QA-Co_1$

(attached picture-4)

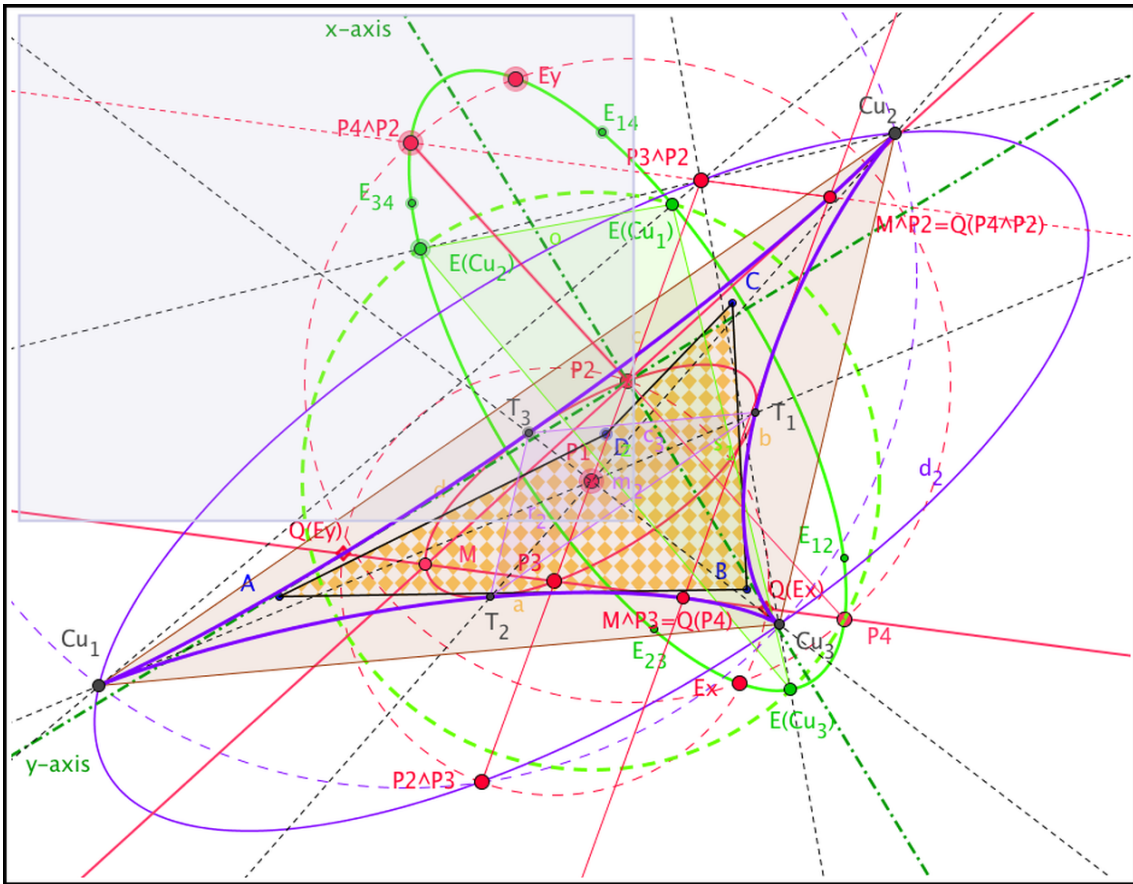
For non-convex quadrangles, both $QA-Cu_1$ and $QA-Cu_2$ reduce to a standard deltoid by the action of a stretch that takes an ellipse into a tangent circle, say $x \rightarrow \lambda x$, $y \rightarrow y$, where x, y are the ellipse axes and $\lambda = b/a$ is the ratio of their lengths. From this fact many of the above properties can be derived, regarding tangency, collinearity (of cusps and contacts), lengths and areas ratios.
 As an example, if V_{ij} are the contacts of $QA-Qu_1$ with the QA -sides, then the four triangles $V_{23}V_{34}V_{24}$, $V_{13}V_{34}V_{14}$, etc. and the diagonal triangle have the same area.



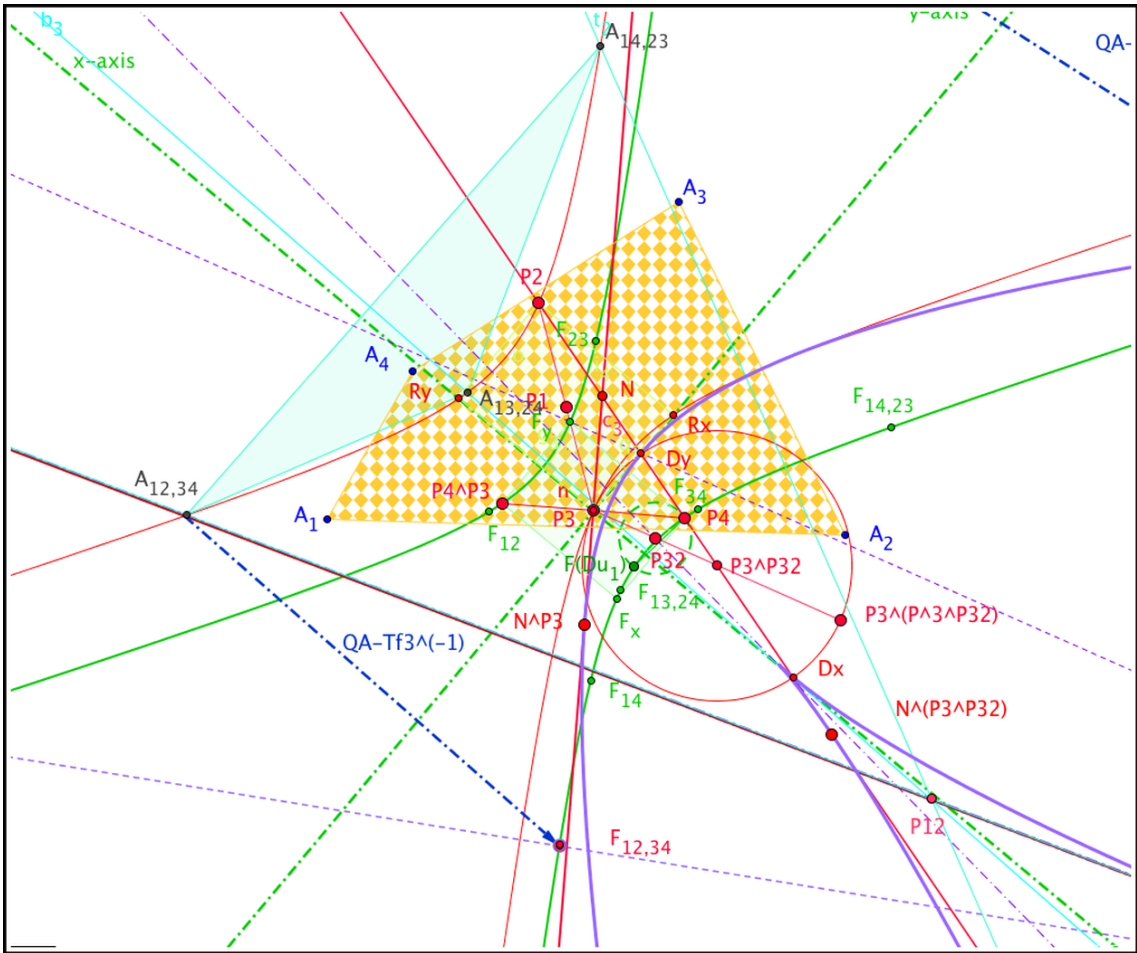
Confronto-QA-Qu1 e QA-Qu2.png



QA-Q1 convesso per il gruppo.png



QA-Qu1 concavoper il gruppo.png



QA-Qu2 convesso.png

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3089
Date: 18/6/2018 12:23:03
From: bernard.keizer
Subject: Serret's inconics

Dear Benedetto, dear Chris, dear Eckart
Thanks to Benedetto for his pedagogical presentation of QA-Qu1 and QA-Qu2 as negative pedals of 2 conics QA-CoE and QA-CoF wrt QA-P4.
Here another application of the same construction for the QL and it's regular deltoïd QL-Qu2.
Considering a triangle, the negative pedal of the circumcircle wrt the orthocenter gives the traditionnally but wrongly named Mac Beath inconic, which was in fact earlier studied by Serret (see Math World).
This conic envelops the perpendicular bisectors of the segments joining the orthocenter to a point of the circumcircle ; it envelops also the inner circles of the deltoïds tangent to the 3 sides of the triangle (in particular the Euler circle, which is the inner circle of the Steiner deltoïd tangent to the 3 sides and the 3 heights).
Conversely, for the QL, the Hervey circle QL-Ci4, inner circle of the deltoïd QL-Qu2 tangent to the 4 lines, is tangent to the 4 Serret's inconics of the 4 reference triangles of the QL.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3090
Date: 18/6/2018 7:10:15
From: bernard.keizer
Subject: Serret's iniconics

Dear friends,
The Serret's iniconic has as center the center of the nine-points circle (X5 in ETC), as foci the circumcenter X3 and the ortocenter X4 and is tangent to the nine-points circle in 2 points of the Euler Line, X1312 and X1313, symmetric wrt X5.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3091
Date: 18/6/2018 8:59:04
From: Antreas Hatzipolakis
Subject: Serret's iniconics

Dear Bernard
It is also known as MacBeath Inconic
<http://mathworld.wolfram.com/MacBeathInconic.html>
APH

> On Mon, Jun 18, 2018 at 8:04 PM, Bernard Keizer wrote:
>> Dear friends,
>> The Serret's iniconic has as center the center of the
nine-points circle
>> (X5 in ETC), as foci the circumcenter X3 and the ortocenter
X4 and is
>> tangent to the nine-points circle in 2 points of the Euler
Line, X1312 and
>> X1313, symmetric wrt X5.
>> Best regards
>> Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3092
Date: 19/6/2018 12:12:22
From: eckart_schmidt@t-online.de
Subject: Two cubics for a QA/QL constellation II

Dear Bernard, dear Benedetto, dear Chris,

the asymptotes of
... circumscribed QA-conics and inscribed QL-conics
... envelope quartics,
... with dual cubics as described in #3086.
This concept can also be used for
... the axes of QL-inscribed conics,
... which envelope a quartic,
... whose dual cubic QL-Cu is a nonpivotal isocubic
... through QL-P8 = QA-P10 and QL-P13 = QA-P16:
... reference triangle QL-DT = QA-DT,
... isoconjugation QA-Tf2,
... root: tripole of the QL-Tf2-image
... of the radical axis of QL-Ci1 and QL-Ci2.
The duals of the axes for a QL-inscribed conic
... lie collinear with QL-P8 = QA-P10
... and coconic with QL-P13 = QA-P16 and the DT-vertices.

The duals of the axes for a QA circumscribed conic
... lie on a tangent at the DT-inscribed Steiner ellipse
... and coconic with QA-P16 = QL-P13 and the DT-vertices.
They envelope a quartic,
... whose dual cubic QA-Cu is a nonpivotal isocubic
... through the dual point of QA-L2,
... for any reference triangle with vertices on QA-Cu,
... which are the contact points of a conic Co (???
construction ???),
... isoconjugation with fixed point QL-P13 = QA-P16,
... root: tripole of the isoconjugated line of Co.
The six axes of the three two-line-degenerated QA-circumconics
... have coconic duals on QA-Cu.
Both cubics QL-Cu and QA-Cu have common points
... in the intersections of DT and the tripolar of QA-P23.

Best regards Eckart

PS: Thanks to Benedetto for his detailed paper wrt QA-Qu1 and
QA-Qu2,
but it will take some time to study the seven pages.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3093
Date: 19/6/2018 1:18:34
From: eckart_schmidt@t-online.de
Subject: QA-Qu1 and QA-Qu2

Dear Benedetto,

if I am not wrong after a first look at your paper:
QA-Qu2 is the envelope
... for the axes of QA-circumscribed conics.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3094
Date: 20/6/2018 9:40:16
From: eckart_schmidt@t-online.de
Subject: QA-Qu1 and QA-Qu2

Dear Benedetto,

now we can consider the quartics
... QA-Qu1 as envelope of the asymptotes of QA-circumscribed
conics,
... QA-Qu2 as envelope of the axes of QA-circumscribed conics.

Let QA be the reference quadrangle
... and QA-01 the first quadrangle of QA-triangle circumcenters,
... then your theorem holds:

QA-Qu2 for QA is QA-Qu1 for QA-01.

QA-Qu1 for QA is homothetic QA-Qu2 for QA-01,
... perspector QA-P4, ratio your "lambda".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3095
Date: 22/6/2018 10:35:04
From: eckart_schmidt@t-online.de
Subject: QA-Qu1 and QA-Qu2

Dear Benedetto, dear Bernard,

Benedetto's paper is full of interesting results and properties, one was very helpful for me:

> "QA-Qu2 is the *negative pedal* of *QA-CoF* w.r.t. its point *QA-P4*.
> i.e.: when R runs on QA-Co1 and $R^* = F$ runs on QA-CoF
> then QA-Qu2 is the locus of the intersection $S(F)$ of
> the normal $m(F)$ to QA-CoF at F with
> the perpendicular bisector $s(F)$ of the segment F. QA-P4
> (the last line is the image of F by the inverse of QA-Tf6)."

By the way there will be a typo in the last sentence:
"(the last line is the image of R by the inverse of QA-Tf6)."

In message 3092 I researched the dual cubic of QA-Qu2 as nonpivotal isocubic
... but without a concrete construction for a reference triangle.

Your property above gives the possibility,
... to construct points and its tangents for QA-Qu2 and dual for the cubic:
... .. Take a point R on QA-Cu1 with $F = \text{QA-Tf3inv}(R)$ on QA-CoF,
... .. then the bisector of F.QA-P4 = $\text{QA-Tf6inv}(R)$ is tangent to the quartic
... .. and the intersection with the normal in F wrt QA-Co1 gives the contact point.

Construction of a reference triangle TR :
... Let R be the reflection of a QA-DT-vertex in QA-P1,
... then F.QA-P4 = $\text{QA-Tf6inv}(R)$ is tangent to the quartic
... and its dual is a point of the cubic as vertex of TR.

Now the dual of QA-Qu2 is a nonpivotal isocubic (see also #3092):
... through the dual of QA-L2,
... reference triangle TR (see above),
... isoconjugation with fixed point QL-P13 = QA-P16,
... root: TR-tripole of the QA-DT-tripolar of QA-P23.

Best regards Eckart

(1) If I am not wrong, you use another definition as MathWorld

... for negative pedal of a curve C wrt a point O :
... instead of "
... then for a point P on C , draw a line perpendicular to OP ... "
... you use "... then for a point P on C , draw a bisector of OP ... ".

(2) There will be a typo in the last sentence:
" (the last line is the image of $*R*$ by the inverse of QA-Tf6)."

(3)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3096
Date: 22/6/2018 10:45:31
From: eckart_schmidt@t-online.de
Subject: QA-Qu1 and QA-Qu2

Dear Benedetto, dear Bernard,

I very much apologize for the not canceled remarks
... at the end of my last message,
... remained of a first version.

Sorry Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3097
Date: 26/6/2018 11:46:04
From: bernard.keizer
Subject: Serret's inconics

Dear Eckart,

Back from a short week in Berlin, I'm rather surprised that you didn't answer my messages 3087 and 3089.

Did you already know the property that the Hervey circle of the QL is tangent in 2 points to each Serret (or Mac Beath) conic of the 4 reference triangles ?

You described the properties of the quartics enveloping the asymptotes of QA circumscribed and QL inscribed hyperbolas and their dual cubics and the connexion between these 2 cubics. Then you described the properties of the quartic of class 3 enveloping the axes of QA circumscribed conics and it's dual cubic.

We knew already the sextic enveloping the axes of QL inscribed conics : it's the cayleyan of the cubic stelloïd QL-Cu2, which is tangent to the hessian Van Rees curve QL-Cu1 ; this curve is of class 3 and DT inscribed and it's dual is a DT circumscribed cubic. Have you already investigated this last cubic and is there a connexion with the preceding one ?

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3098
Date: 26/6/2018 8:54:13
From: eckart_schmidt@t-online.de
Subject: Serret's inconics

Dear Bernard,

excuse, that I haven't read your messages carefully,
but there was another headline and I studied Benedettos paper.

In message #3087 you give examples
... starting with a pivotal QA-isocubic:
... .. reference triangle QA-DT,
... .. isocunjugation QA-Tf2,
... .. pivot P,
... or
... .. reference triangle: cevian triangle of P wrt QA-DT,
... .. isoconjugation with fixed point P,
... .. pivot QA-Tf2(P),
... to get nonpivotal isocubics for the dual QL:
... .. reference triangle:
cevian triangle of P wrt QA-DT = QL-DT,
... .. isoconjugation swapping opposite QL-points
... .. and any chosen root.

In message #3089 you consider Serret-conics f
or the QL-triangles,
... but CABRI-observations don't confirm,
... that QL-Ci4 is always tangent to all four
Serret-conics!

Wrt message #3097:
Excuse, that I haven't noticed,
... that the envelope for axes of QL-inscribed conics is the
cayleyan of QL-Cu2.
But what are the two cubics, you ask for in the last sentence?
Is there a typo in the sentence bevor?
The dual cubic of the cayleyan is described in #3092.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3099
Date: 26/6/2018 9:46:00
From: eckart_schmidt@t-online.de
Subject: Serret's inconics

Dear Bernard,

I just noticed, that the minor axes of the Serret-conics for the
QL-triangles
... have the common point QL-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3100
Date: 27/6/2018 11:40:19
From: bernard.keizer
Subject: Serret's inconics

Dear Eckart,

Thanks for your answer !

Everything is clear now for the curves QA-Qu1 and QA-Qu2 and their dual cubics as well as for the equivalent curves for QL and their dual cubics : the dual cubics for QA-Qu1 and QL equivalent have a strong connexion, but the dual cubics for QA-Qu2 and the QL equivalent (cayleyan) have not the same connexion.

All is explained in fact in your message 3092 and the complements in 3095 and I should have read it more carefully. My apologise for this lack of attention !

For the Serret's inconic, your answer surprises me. Could you please check it once more.

For any triangle, the Serret's inconic is at the same time the envelop of the perpendicular bisectors of the segments joining the orthocenter to any point on the circumcircle (that's exactly Benedetto's definition) and the envelop of the inner circles of all deltoïds tangent to the 3 sides (in particular the Euler circle, inner circle of the Steiner deltoïd). This Serret's inconic is an ellipse only if the triangle is acutangle and an hyperbola if not.

In the QL, the 4 lines are tangent to the deltoïd, as well as the asymptote of the hessian QL-Cu1.

This gives 4 Serret's inconics of the reference triangles formed by 3 of the 4 lines and 6 for the triangles formed by 2 of the 4 lines and the asymptote ; these 10 Serret's inconics are tangent to the Hervey circle QL-Ci4.

Perhaps also interesting : take one QL vertice and the 2 lines intersecting in this point ; on these lines the reflexions wrt QL-P5 of the 2 circumcenters of the reference triangles through this point. The reflexions of the considered QL vertice in these 2 points form with this QL vertice a triangle and the Hervey circle is the Euler circle of this triangle, the deltoïd being it's Steiner deltoïd. There are 6 such triangles, one for each QL vertice and of course again 6 new Serret's inconics bitangent to the Hervey circle.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

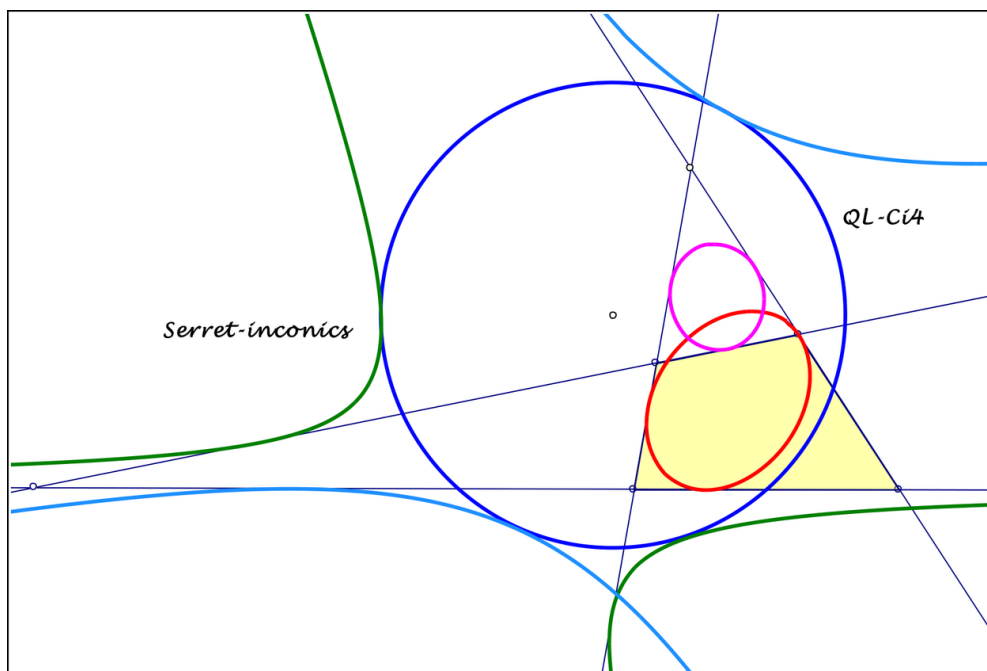
Message: #3101
Date: 27/6/2018 11:42:27
From: bernard.keizer
Subject: Serret's iniconics

Dear Eckart,
Sorry, I forgot to comment your remark, it's precisely the Kantor-Hervey theorem !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3102
Date: 27/6/2018 3:20:37
From: eckart_schmidt@t-online.de
Subject: Serret's iniconics

Dear Bernard,
wrt contacts of the Serret-conics and QL-Ci4,
I don't find my errors (see attached drawing).
Best regards Eckart



2018-06-27.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3103
Date: 28/6/2018 8:18:37
From: eckart_schmidt@t-online.de
Subject: QL-Qu2

Dear Chris,

in EQF wrt QL-Qu2 under "Another construction ... 2.
... there is to add "P.QL-P1".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3104
Date: 29/6/2018 11:32:36
From: bernard.keizer
Subject: Serret's inconics

Dear Eckart,

There is no error in your drawing.

1) I found years ago somewhere an analytical proof of my property

Using cartesian coordinates for a triangle with origin in X_5 , the Euler Line as x axis, the perpendicular bisector of X_3X_4 as y axis, the equation of the Hervey circle is $x^2 + (y - atg t)^2 = b^2 (1 + (tgt)^2)$, where $2a$ is the distance X_3X_4 and $2b$ is the radius of the circumcircle (for $t = 0$, we have the Euler circle).

By derivating in t and eliminating $tg t$ between the 2 expressions, we come to the equation $x^2/b^2 + y^2/(b^2 - a^2) = 1$, which is the equation of the Serret's conic.

In fact, if the conic is an hyperbola, it is bitangent to the Hervey circle, if it is an ellipse, there is no contact.

Perhaps you will be able to solve this mystery, I'm now too busy with my grand-children (I have seven, exactly one per decade) and the summer will be hot !

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3105
Date: 30/6/2018 1:49:19
From: César E. Lozada
Subject: QL-Ci3: Miquel Circle

Dear all,

My knowledge about quadrifigures is very, very limited, therefore I beg you for apologizes if this subject is enough known.

Definition of QL-Ci3 is given at <https://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3> and follows:

The circumcenters ($=X(3)$) of the 4 Component Triangles of a Quadrilateral are concyclic and lie on QL-Ci3, the Miquel Circle. QL-P4 is its center.

Playing with Geogebra, I found that there are several other ETC centers $X(n)$, apart from the circumcenters $X(3)$, such that $X(n)$ of the 4 component triangles are concyclic. The list of such centers includes n in the following set: {3, 4, 19, 53, 186, 265, 286, 378, 440, 464, 1136, 1176, 1444, 1593, 1597, 1598, 1798, 1839, 1842, 1867, 1880, 1890, 1973, 2226, 2333, 2354, 2718, 2884} ($n \leq 3054$) (For $n=4$, centers are collinear).

Is this locus described anywhere in EQF? If so, where is it?
Thanks in advance,

César Lozada

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3106
Date: 30/6/2018 9:01:31
From: Antreas Hatzipolakis
Subject: QL-Ci3: Miquel Circle

[César Lozada]:

> Dear all,
> My knowledge about quadrifigures is very, very limited,
therefore I beg you for apologizes if this subject is enough
known.
> Definition of QL-Ci3 is given at
<https://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3> and
follows:
> The circumcenters (=X(3)) of the 4 Component Triangles of a
Quadrilateral are concyclic
> and lie on QL-Ci3, the Miquel Circle. QL-P4 is its center
> Playing with Geogebra, I found that there are several other
ETC centers X(n), apart from the circumcenters X(3), such that
X(n) of the 4 component triangles are concyclic. The list of
such centers includes n in the following set: {3, 4, 19, 53,
186, 265, 286, 378, 440, 464, 1136, 1176, 1444, 1593, 1597,
1598, 1798, 1839, 1842, 1867, 1880, 1890, 1973, 2226, 2333,
2354, 2718, 2884} (n<=3054) (For n=4, centers are collinear).
> Is this locus described anywhere in EQF? If so, where is it?
> Thanks in advance,
> César Lozada

Dear César,

I think that your locus is a locus of Triangle Geometry rather
than Quadri Geometry:
Let ABC be a triangle and L a line intersecting BC, CA, AB at
A', B', C', resp.
Let P, Pa, Pb, Pc be same points wrt triangles ABC, AB'C',
BC'A', CA'B', resp.
Which is the locus of P such that P, Pa, Pb, Pc are concyclic?
The locus is independent of the line L.
The locus of the circumcenters of PPaPbPc (as P moves on the
locus) depends on the line L.
If L = the Euler line, which is it ? (the locus of the
circumcenters of PPaPbPc)

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3107
Date: 30/6/2018 11:59:58
From: eckart_schmidt@t-online.de
Subject: QL-Ci3: Miquel Circle

Dear Cesar Lozada,

wrt concyclic ETC-points for the QL-triangles:

If I am not wrong, your property doesn't hold for X(19), X(53), X(286) and X(378).

I have only checked the first eight points of yours.

But there are further examples:

... the four Hofstadter points X(5961), X(5962), X(5963), X(5964).

Perhaps you can have a look at #1546 (2) and #1847

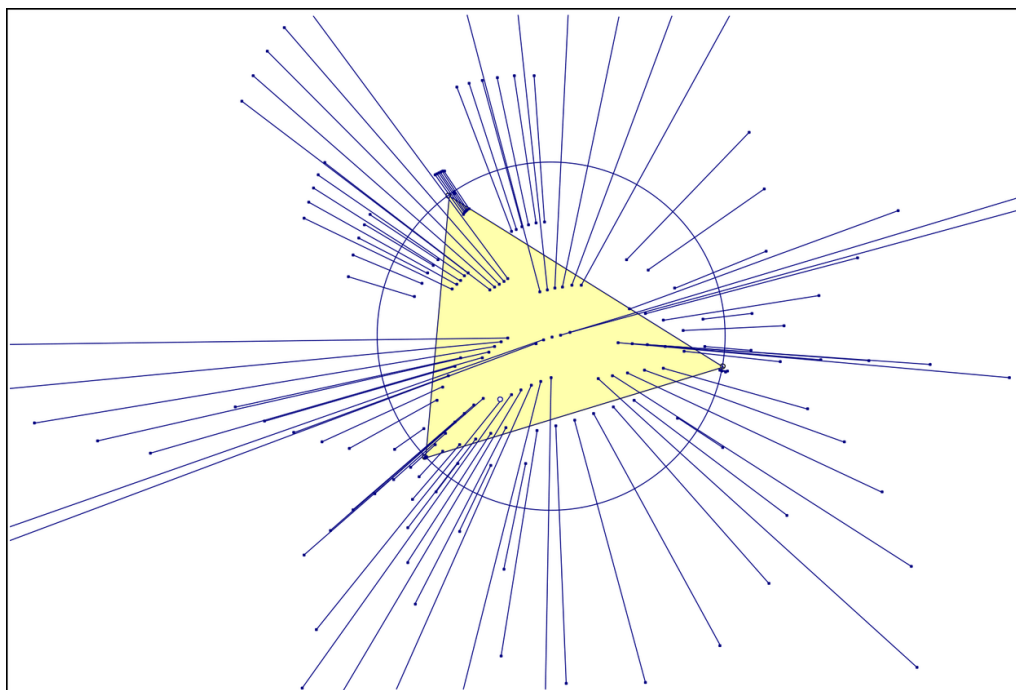
... and the discussion of "n-angle centers" in Aug/Sep 2016.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3108
Date: 01/7/2018 9:46:53
From: eckart_schmidt@t-online.de
Subject: QL-Ci3: Miquel Circle

Dear Cesar Lozada,
you can get a preview for the locus of triangle points,
... which are concyclic for the QL-triangles:
Let Tr be a reference triangle
... "isg" its isogonal conjugated,
... "inv" the reflection in the circumcircle:
... $P_0 = X(3)$,
 $Q_i = \text{isg}(P_i)$, $P_{i+1} = \text{inv}(Q_i)$.
The series $P_0, Q_0, P_1, Q_1, P_2, Q_2, P_3, Q_3, \dots$
... gives points with the required property, starting with
 $X(3), X(4), X(186), X(265), X(5961), X(5962), X(5963), X(5964),$
...
The locus is invariant wrt
... the isogonal conjugate and inversion wrt the circumcircle:
... have a look at the attached file with the first 100 pairs
 $Q_i P_{i+1}$.
Best regards Eckart
PS. If the attached file doesn't appear in QFG,
use my privat address: eckart_schmidt@t-online.de.



2018-07-01.pdf

Message: #3109
Date: 02/7/2018 10:01:29
From: bernard.keizer
Subject: Serret's iniconics

Dear Eckart,

With the given calculation, you get easily the coordinates of the 2 double contact points :

$y = - (b^2 - a^2) \operatorname{tgt} / a$ and $x = \pm b^2 / a^2 * (a^2 (1 + \operatorname{tgt}^2) - b^2 \operatorname{tgt}^2)$

The 2 points are symmetric wrt the perpendicular bisector of X_3X_4 (which is obviously an axis of symmetry for both the Serret's conic and the Hervey circle.

They are real only if $\operatorname{tgt}^2 / (1 + \operatorname{tgt}^2) \leq a^2 / b^2$, which is the case for all hyperbolas, as $a/b > 1$ and for some ellipses (for example, as already mentioned, the Euler circle as Hervey circle for the Steiner deltoid, $\operatorname{tgt} = 0$).

My reference was an old article in french by Bickart, which didn't mention this restriction.

(You find it under the reference (12) in the bibliography given in my first article on my blog)

I hope I didn't make any mistake and you will confirm these calculations/observations.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3110
Date: 04/7/2018 1:36:21
From: eckart_schmidt@t-online.de
Subject: New QA-transformation?

Dear Tsihong Lau, dear all,

in message 4768 of Advanced Plane Geometry
... Tsihong Lau describes a QA-transformation,
... here shortened TF with further properties:

TF has four fixed points
... in the 3rd intersections of QA-Cu1
... .. and lines from the QA-vertices to the same in- or excenter
of QA-Tr2,
... which give a new quadrangle
... with the same Miquel triangle
... and common tangential QA-P3.
TF is QA-Tf2 for the new quadrangle,
... with pivot QA-P3 we get QA-Cu1.

Wrt quadrigons:

TF swaps Miquel point QL-P1 and diagonal crosspoint QG-P1.
TF maps the diagonals PiPk of a quadrigon
... to circles through Pi, Pk, QL-P1 and QA-P4.
I think, there will be more interesting properties.

Best regards Eckart

PS. Tsihong Lau says, that message 4768 was posted to QFG long ago,
... but I don't remember.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3111
Date: 05/7/2018 2:03:11
From: eckart_schmidt@t-online.de
Subject: Serret's inconic

Dear Bernard,

thanks for clearness wrt the contact points of Serret-conics and QL-Ci4.

Can you check the last passage in your message #3100?

I cannot reproduce " ... and the Hervey circle is the Euler circle of this triangle, ... ".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3112
Date: 05/7/2018 6:44:43
From: chris.vantienhoven
Subject: QL-Qu2

Dear Eckart,

Thanks for mentioning the typo in QL-Qu2.

I corrected it in EQF.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3113
Date: 06/7/2018 9:01:26
From: eckart_schmidt@t-online.de
Subject: QL-Ci3: Miquel Circle

Dear Cesar Lozada,

am I right, that only the points $X(3)$, $X(4)$, $X(186)$, $X(265)$ of yours in #3105

... have the concyclic property wrt the QL-triangles?

There are further triangle points with this property,

... but dependent of the reference quadrilateral:

Example:

... On the Euler line there is a point

... which divides $X(2).X(4)$ in a special ratio,

... depending on the reference quadrilateral.

I calculated this ratio in DT-notation (see EQF):

$$\frac{(2LMN(SaSb+SbSc+ScSa)(l^2L^2Sa+m^2M^2Sb+n^2N^2Sc)}{(3(L^2Sa+M^2Sb+N^2Sc)(l^2L^3Sa^2+m^2M^3Sb^2+n^2N^3Sc^2+LMN(l^2SbSc+m^2ScSa+n^2S_$$

$$aSb))$$

with $L=m^2-n^2$, $M=n^2-l^2$, $N=l^2-m^2$.

What about the corresponding new QL-circle,

... which divides the Euler-lines of the QL-triangles in the same ratio?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3114
Date: 06/7/2018 11:48:22
From: bernard.keizer
Subject: Serret's inconic

Dear Eckart,

In fact, a part of the sentence is missing !

Consider A_{ij} , QL vertice intersection of L_i and L_j .

L_i cuts the Hervey circle in 2 points, h_i reflexion of O_i in $QL-P_5$ and q_i , the contact point of the deltoïd with L_i is Q_i , reflexion of q_i in h_i ; the same goes for L_j .

The reflexions of A_{ij} in q_i and q_j (and not h_i and h_j) give 2 points B_{ij} and C_{ij} and a triangle $A_{ij}B_{ij}C_{ij}$ with circumcenter O_{ij} and orthocenter H_{ij} .

The 4 triangles $A_{ij}B_{ij}C_{ij}$, $H_{ij}B_{ij}C_{ij}$, $H_{ij}A_{ij}B_{ij}$ and $H_{ij}A_{ij}C_{ij}$ share the same Euler circle, which is the Hervey circle and the same Steiner deltoïd, which is the deltoïd $QL-Qu_2$.

There are $6 \cdot 4 = 24$ such triangles.

The important thing is that their Serret's inconics (ellipses or hyperbolas) are all bitangent to the Hervey circle in 2 points on the corresponding Euler Lines symmetric wrt $QL-P_3$.

My purpose was only to show that there are examples of Serret's inconics as ellipses which touch at least one inner circle of an inscribed deltoïd.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3115
Date: 07/7/2018 9:36:30
From: eckart_schmidt@t-online.de
Subject: New QA-transformation?

Dear Tsihong Lau,

in Advanced Plane Geometry #4768 you gave a QA-transformation,
... which I interpreted in QFG #3110,
... but that doesn't hold
for orthocentric quadrangles.
In this case there is another sight,
... which leads to a computation of the image point,
... you asked for in your private messages.
Let TR be a reference triangle
... and QA the quadrangle of the in-/excenters.
... with TR as diagonal as well as Miquel triangle.
Let P be any point,
... P* its TR-isogonal conjugate,
... Q the point at infinity of PP*,
... CU the pivotal isocubic
... .. with reference triangle TR,
... .. isoconjugation: isogonal conjugate,
... .. pivot: infinity point Q of PP*,
... R the 4th common point of CU and the TR-circumcircle:
... then the 3rd intersection of PR and CU is the image P' of P.

With this background I computed the TR-coordinates of P' for
P(u:v:w):

... first barycentric coordinate:
$$a^2 a^2 v^2 w^2 (u+2v+2w) - b^2 b^2 u^3 w^2 - c^2 c^2 u^3 v^2$$
$$- a^2 b^2 u w^2 ((u+w)^2 + 2uv - v^2) - a^2 c^2 u v^2 ((u+v)^2 + 2uw - w^2)$$
$$+ b^2 c^2 u^3 ((u+v+w)^2 - 2vw).$$

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3116
Date: 08/7/2018 2:20:45
From: eckart_schmidt@t-online.de
Subject: Excenters/incenter quadrangle A'B'C'D'

Dear Tsihong Lau, dear all,

in Advanced Plane Geometry #4789 is constructed
... an excenters/incenter quadrangle A'B'C'D'
for a reference quadrangle ABCD,
... which can shortened be described
... as QA-perspective quadrangle on the cubic QA-Cu1,
... perspector in the excenters and incenter of the Miquel
triangle QA-Tr2,
... with vertices in the QA-Cu1-tangentials of QA-P3.
ABCD and A'B'C'D' have the same Miquel triangle and cubic QA-Cu1
... and are invariant wrt the isogonal conjugate of QA-Tr2.
QA-P4 of ABCD is QA-P3 of A'B'C'D' and vice versa.
QA-Tf2 of ABCD maps QA-Tr2 to QA-Tr1 of A'B'C'D' and vice versa.
The Cayley-Bacharach point of the two quadrangles is
... the intersection of QA-Cu1 and its asymptote.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3117
Date: 12/7/2018 9:28:26
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

Dear Chris,

QA-Cu7 is a nonpivotal isocubic:

... reference triangle QA-Tr1,
... isoconjugation QA-Tf2,
... .. swapping QA-P4 and QA-P41 on QA-Cu7,
... root: QA-Tr1-tripole for the line
... .. of the perspector of the triangles QG-P1.P18.P19.

QA-Cu7 is a nonpivotal isocubic

... reference triangle QG-Tr3 of a quadrigon version,
... isoconjugation QG-Tr3-isogonal conjugate or QG-Tf2,
... .. swapping QG-P1 and QG-P19 on QA-Cu7,
... root: QG-Tr3-tripole of a line L ...

Construction of the line L:

... The three versions of QG-P1.19 have a common point Q,
... let $X = QG-2P2a.Q \wedge QG-2P2b.QG-P18$
... and $Y = QG-2P2b.Q \wedge QG-2P2a.QG-P18$,
... then $L = XY$.

Finally:

QA-Cu7 is the locus of points,
... whose images wrt the three QG-Tf2 coincide.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3118
Date: 13/7/2018 8:26:30
From: eckart_schmidt@t-online.de
Subject: New QA-points and lines

Dear Chris,

QG-lines for the three QG-versions of a quadrangle
... often have a common QA-point (see QG-2).
The sidelines of the QG-triangle QG-P1.P18.P19
... give in this way three new collinear QA-points.
As mentioned in QG-P18
... the triangles QG-P1.P18.P19 are pairwise perspective
... with collinear perspectors on QA-Cu7.
The QA-Tr1-tripole of this perspector-line
... lies on the line of the new QA-points
... as 2nd intersection with
... the QA-circumconic through the common point of QG-P18.P19
... and is the root for the nonpivotal isocubic QA-Cu7
... wrt QA-Tr1 and QA-Tf2 (see #3117).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3119
Date: 14/7/2018 9:08:37
From: eckart_schmidt@t-online.de
Subject: Parallelograms inscribed in a quadrigon

Dear all,

in Advanced Plane Geometry #4816 and 4820
... "Parallelograms inscribed in a quadrilateral" are discussed.
Perhaps there are further informations in
<http://eckartschmidt.de/Pgive.pdf> (sorry in German).
For calculations in barycentric coordinates (EQF DT)
... wrt the QA-diagonal triangle of a quadrigon P1P2P3P4:
... P1(-p:q:r), P2(p:-q:r), P3(p:q:-r), P4 (p:q:r),
... and a parallelogram Q12Q23Q34Q41 with Qij on PiPj,
... with diagonal crosspoint P(u:v:w) of the parallelogram,
holds:

$$\begin{aligned} Q12 &= (p:-q:(uq^2+vp^2+wpq)/(uq-vp), \\ Q23 &= ((uqr+vr^2+wq^2)/(vr-wq):q:-r), \\ Q34 &= (p:q:(-uq^2-vp^2+wpq)/(uq+vp)), \\ Q41 &= ((uqr-vr^2-wq^2)/(vr+wq):q:r). \end{aligned}$$

Examples:

The inscribed parallelograms degenerate for P on QA-Co1.
For P = QA-P6 we get the pedal-parallelogram of QA-P4.
For calculations in barycentric coordinates (EQF DT)
... wrt the QL-diagonal triangle of a quadrangle L1L2L3L4:
... L1(-l,m,n), L2(l,-m,n), L3(l,m,-n), L4 (l,m,n),
... and a parallelogram Q1Q2Q3Q4 with Qi on Li,
... with diagonal crosspoint P(u:v:w) of the parallelogram,
holds:

$$\begin{aligned} Q1 &= (-lmu+lnu-mnu-m^2v-u^2w:-(l+n)(lu+mv-nw): \\ & -lmw-lnw-mnw+l^2u+m^2v), \\ Q2 &= (-lmw+lnw+mnw+l^2u+m^2v), \\ Q3 &= (+lmw+lnw+mnw+l^2u+m^2v), \\ Q4 &= (-lmw-lnw+mnw-l^2u-m^2v). \end{aligned}$$

Examples:

For P on the Newton line QL-L1
... the parallelogram sides are parallel to the QG-diagonals.
For P on QG-L1
... the parallelogram diagonals are parallel to QG-P1.QG-2P2.
These barycentric coordinates allow the calculation for loci of
P,
... if the parallelogram is a rhombus or rectangle (see cited
reference).
The nomination in the cited reference of 2011

... is not the here used EQF-DT nomination,
EQF started 2013.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3120
Date: 15/7/2018 9:27:44
From: eckart_schmidt@t-online.de
Subject: Parallelograms inscribed a quadrigon

Dear all,

there is a typo in #3119:
"For calculations in barycentric coordinates (EQF DT)
... wrt the QL-diagonal triangle of a *quadrigon* L1L2L3L4:"

Sorry Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3121
Date: 20/7/2018 9:49:16
From: bernard.keizer
Subject: Kantor-Hervey, Serret and generalisation

Dear Chris, dear Eckart

As I mentioned earlier, Bickart asked a century ago for the locus of the center of the deltoïds or H3 inscribed in a triangle and for the envelop of their inner circles ; the locus of the centers is the perpendicular bisector of the segment joining the circumcenter and the orthocenter and the envelop of the inner circles is the Serret's or MacBeath inconic with foci the circumcenter and the orthocenter.

For a QL, it leads to the Kantor-Hervey theorem and to the fact that the Hervey circle is envelopped by the 4 Serret's inconics of the reference triangles (as noticed by Eckart, either no intersection or bitangency, the 2 points being symmetric wrt the perpendicular bisector of the segment joining the circumcenter and the orthocenter). This property could or should be mentioned in EQF.

If we consider now the followers, inverse of the orthocenter wrt the circumcircle and it's isogonal conjugate (iHi and $giHi$ in the notations of Bernard Gibert) and so on with the Hofstadter points, the properties hold.

The iHi are cocyclic, as well as the $giHi$, the 2 circles being Cl-S partners, the 4 segments joining iHi and $giHi$ concur in the center of the unique hypocycloid with 5 cusps tangent to the 4 lines and the inner circle of this H5 is envelopped by the 4 inconics with foci iHi and $giHi$ (with the same meaning as for the deltoïd or H3 either no intersection or bitangency).

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3122
Date: 21/7/2018 3:19:50
From: eckart_schmidt@t-online.de
Subject: Kantor-Hervey, Serret and generalisation

Dear Bernard,

that is really a relevant property:
The bisectors of the following pairs (P_i, Q_i) of isogonal triangle points,
... with $P_0 = X_3$, $Q_i = gP_i$, $P_{i+1} = iQ_i$:
... (X_3, X_4) , (X_{186}, X_{265}) , (X_{5961}, X_{5962}) , (X_{5963}, X_{5964}) , ...
... used for the triangles of a quadrilateral,
... have a common QL-point:
... for the first pair QL-P3,
but what about the other new QL-points?

By the way: In the last passage of your message 3121
... you have to replace "... segments joining ..."
by "... bisectors of ...".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3123

Date: 21/7/2018 7:45:52

From: bernard.keizer

Subject: Kantor-Hervey, Serret and generalisation

Dear Eckart,

Thanks for your answer and for the typo.

Searching properties of the Miquel circles (the 1st one is QL-Ci3), the Steiner circles (the 1st one is the Steiner line QL-L2) and the Hervey circles (the 1st one is QL-Ci4), I have found only this one :

the 2nd Miquel circle passes through QL-P4 and therefore the 2nd Steiner circle is through the Cl-S partner of QL-P4, id est the reflexion of QL-P1 in the Steiner Line QL-L2 ; more intriguing, the center of this circle is the reflexion of QL-P3 in QL-L2 and it's radius is equal to the distance of QL-P1 to QL-P3 and it's. In other words, the 2nd Steiner circle is the reflexion in QL-L2 of the circle with center QL-P3 through QL-P1.

I don't know if this property may be generalised ...

I haven't found properties of the 2nd Kantor-Hervey point (the 1st one being QL-P3).

But you didn't answer the 2nd part of my message with the link between the following Hervey circles and the following Serret's inconics ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3124

Date: 22/7/2018 4:50:44

From: bernard.keizer

Subject: Kantor-Hervey, Serret and generalisation

Dear Eckart,

I started the item in the message 2133 with 1 attachment, which went lost and repeated in message 2281 with 2 separated attachments (text + figure); after that, Chris told the attachments had been founded, but they do not appear in the list of attachments.

The discussion about this item ended with a message from Chris telling he had almost found a method to draw epi- or hypocycloïds tangent to 4 lines, but I suppose he didn't success !

The loosing of attached pieces is a little bit discouraging, that's the reason I no longer try to send figures ...

Do you have this attachment (text + figure) ? If not, I'll send it on your personal e-mail.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

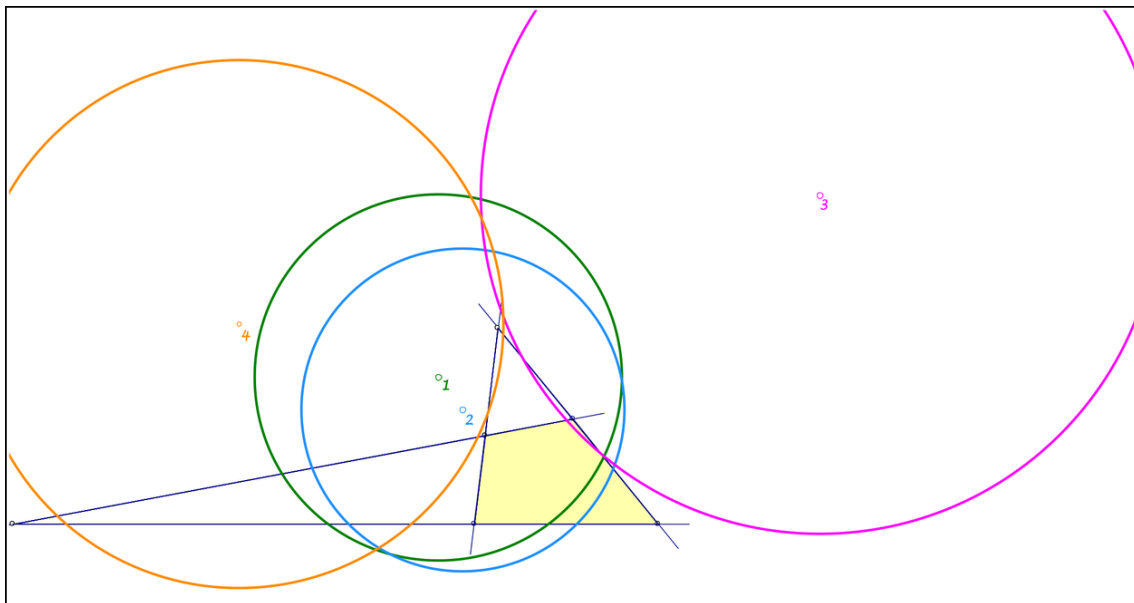
Message: #3125
Date: 22/7/2018 9:41:05
From: eckart_schmidt@t-online.de
Subject: Kantor-Hervey, Serret and generalisation

Dear Bernard,

it was exciting and interesting, to construct
... the first four Kantor-Hervey points
... and the corresponding Hervey circles
... with their bitangency to the Serret-inconics,
... but I found no EQF-properties!
Perhaps the attached file does appear.

Best regards Eckart

PS: Wrt #3124: I had copied the attachment of message 2281
... and shall study it once more.



2018-07-23.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3126
Date: 24/7/2018 10:03:32
From: eckart_schmidt@t-online.de
Subject: Kantor-Hervey, Serret and generalisation

Dear Bernard,

wrt #3121: I cannot confirm the hypocycloid,
... you describe in the last passage:
The 4 bisectors of X186 and X265 concur
... in the center of the unique hypocycloid with 5 cusps
... tangent to the 4 lines and the inner circle
... enveloped by the 4 inconics with foci X186 and X265.
Perhaps a helpful further property, found in #1787:
The cyclic quadrangles of X186 and X265
... have diagonal triangles perspective QL-DT.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3127
Date: 24/7/2018 3:18:50
From: bernard.keizer
Subject: Kantor-Hervey, Serret and generalisation

Dear Eckart,
This time I no longer understand !
You just said in the previous message it had been exciting to
construct the Kantor-Hervey points and the corresponding Hervey
circles bitangent to the Serret's inconics ?
Then you don't confirm the construction of the hypocycloid with
5 cusps.
All my constructions were explained in the previous message I
mentioned with 2 figures (I've drawn the 3 hypocycloids with 3,
5 and 7 cusps tangent to the 4 lines). You told me you had saved
this message and it's attachment.
So where is the trouble ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3128
Date: 25/7/2018 10:17:45
From: eckart_schmidt@t-online.de
Subject: Kantor-Hervey, Serret and generalisation

Dear Bernard,

please forget the first passage of my last message
... and excuse my insecurity in hypocycloids!
I constructed the hypocycloid with 5 cusps
... wrt X186 and X265
... with $p=4$ and $q=-1$ instead of $p=3$ and $q=-2$.
I had copied the attachment of #2281:
"QL, epi- and hypocycloids and n-angle centers".
Is that the attachment, you cited in the last messages?
If not, can you send this cited attachment at my private
address?
Thanks in advance.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3129
Date: 26/7/2018 9:44:22
From: bernard.keizer
Subject: Kantor-Hervey, Serret and generalisation

Dear Eckart,
I prefer this !
By the way, $p=4$ and $q=-1$ give also an hypocycloid with 5 cusps,
but as $p+q=3$, there are 27 like this (like the cardioïds), but
nothing to do with X186 and X265 ...
It was indeed my attachment, but the initial one had a text and
2 figures (the 2nd figur is also page 31 in the 4th article
january 2017 on my blog)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3130
Date: 27/7/2018 10:29:31
From: eckart_schmidt@t-online.de
Subject: Relevant line-line transformation for Q

Dear Benedetto, dear Bernard, dear Chris,

this is a plea for a line-line QL-transformation,
... to be offered in EQF, shortened LLT,
... already mentioned in QFG-message 1205,
... which leads in a simple way to two detailed discussed
quartics:
... (1) Kantor-Hervey Deltoid QL-Qu2 (see #1205),
... (2) Benedetto's QA-Qu1 (see #3088, #3065, #2200, #141).

The transformation $L \rightarrow LLT(L)$:
Let Li be the lines of the QL with intersections Sij ,
... L any line with $L \wedge Li = Si$,
... circumcircles (Si, Sj, Sij) with 4 concyclic triple
intersections
... on a circle through QL-P1,
... whose QL-Tf1-image is the image $LLT(L)$.

Properties of this transformation are gathered in #1205,
... inclusive result (1):
... The line pencil of QL-P1 gives QL-Qu2.

New is the property, that in general for a line pencil
... the LLT-images envelope a quartic, tangent to the QL-lines,
... whose dual is a circumcubic of the dual QA.

Wrt result (2):

Consider for a quadrangle a quadrigon version
... and the line pencil of QG-P1:
... The LLT-images wrt the quadrigon, interpreted as
quadrilateral,
... envelope Benedetto's QA-Qu1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3131
Date: 31/7/2018 10:05:26
From: eckart_schmidt@t-online.de
Subject: QA-P4, QL-P1 and QG-P1 for a quadrigon

Dear all,

in QFG-message 2014 there is an interesting
... QG-circumcubic through QA-P4,

QL-P1 and QG-P1,
... very specific points of quadrangle, quadrilateral and
quadrigon.
... This leads to the question:

What about the geometry of these three points?
Let us start with any three points A_4 , L_1 , G_1 ,
... looking for a quadrigon with
... $QA-P_4 = A_4$, $QL-P_1 = L_1$ and $QG-P_1 = G_1$,

(1) The angle bisector at L_1 gives the Steiner axis,
... a circle round L_1 with radius $\text{sqr}(L_1.A_4 \cdot L_1.G_1)$
... is the Schmidt circle (see EQF QL-Tf1),
... so we have the transformation $CSC = QL-Tf1$
... with fixed points F_1 , F_2 as QL-2P3 of the searched QG.

(2) If we consider the line pencil of G_1 ,
... the lines intersect their CSC-circles on a cubic Cu ,
... circumscribed the searched QG, bearing A_4 , L_1 , G_1 , F_1 , F_2 ,
... detailed described in #2014.

(3) Tangents at the cubic Cu :
... in G_1 and L_1 through A_4 , in F_1 , F_2 through G_1 ,
... the asymptote is parallel to $G_1.L_1$ through
... the reflection of A_4 in $G_1.L_1$ (see(9)).

(4) The cubic is CSC-invariant (see(2)),
... also invariant for the isogonal conjugation wrt $A_4L_1 G_1$
... and invariant for the Möbius transformations of $A_4L_1 G_1$
(see QA-Tf4).

For points P on the cubic $CSC(P)$ is
... the Möbius transformation wrt L_1 , swapping G_1 and A_4 ,
... or the 2nd intersection of $P.G_1$ and the circumcircle of P , A_4 ,
 L_1 .

(5) For a chosen point P on the cubic
... the Möbius transformations give a P -quadrangle on Cu ,
... with two isogonal conjugated pairs of points

as well as two pairs of CSC-partners,
... two parallel lines, two lines through G1, two lines through L1,
... with isogonal center in A4.

(6) The trapezoid

version of this P-quadrangle with two sides through L1
... is a special searched quadrigon, but there are more:
... Let P1, P2 be two points on the cubic and P3 = CSC(P1), P4 = CSC(P2),
... then P1P2P3P4 is a quadrigon with QA-P4 = A4, QL-P1 = L1 and QG-P1 = G1.

(7) The mid-parallel of these P-trapezoids

... envelope a parabola with focus A4 and directrix G1.L1,
... tangent to the Steiner axes (see(1)).

(8) Some loci for QG-points of the P-trapezoids, changing P on Cu:

... QG-P4, QG-P8, QG-P13, QG-P15: line G1.L1,
... QG-P5: perpendicular to G1.A4 in A4,
... QG-P6: orthogonal hyperbola through G1,
... .. centered on the 2nd Steiner axis,
... .. one asymptote parallel G1L1, 3 times distance to A4,
... QG-P9: bisector G1.A4,
... QG-P10: orthogonal hyperbola through G1,
... .. centered in the reflection of A4 in G1.L1,
... .. with asymptotes parallel and orthogonal G1.L1,
... QG-P16, QG-P19: line A4.L1,
... QG-P18: perpendicular to A4.L1 through A4.

(9) The cubic is a QL-Cu1 for quadrilaterals

... with Newton line G1L1 and Miquel point A4,
... Steiner axis: angle bisector at A4,
... Schmidt-circle round A4 with radius $\text{sqr}(A4.G1*A4.L1)$.

Final remark :

If we use A4L1G1

as reference triangle for barycentric calculations,

... we get very simple terms

... for the CSC-mapping and the equation of the cubic (see end of #2014).

Best regards Eckart

Message: #3132
Date: 03/8/2018 2:14:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Bernard, dear Chris,

perhaps the following sight of QA-Cu7 can help
... to understand the five common points
... of the three QA-Cu7 for a quadrilateral.

Let us start with a quadrigon, interpreted as quadrangle,
... and its cubic QA-Cu7, bearing the points:
... QG-P1, QG-P18, QG-P19, QA-P2, QA-P4, QA-P41.

The lines QG-P1.QA-P2, QG-P18.QA-P41, QG-P19.QA-P4
... intersect in K on QA-Cu7, not in EQF.

Let us consider a reference triangle TR = QA-P2.QA-P4.QA-P41,
... QA-Cu7 is invariant wrt its isogonal conjugacy *,
... also invariant wrt the Möbius transformations of TR.

Now we construct a special quadrigon on QA-Cu7:
... P1 = QG-P1*: This is the point S, discussed in #2980, #2983,
#2991,
... P2 = QG-P18*, P3 = QG-P19*, P4 = K.

QA-Cu7 is QA-Cu1 for this special quadrigon P1P2P3P4.
... The reference triangle TR is the Miquel triangle of P1P2P3P4.

Let L be the bisector of QA-P2.QA-P4,
... parallel to the asymptote of QA-Cu7,
... parallel to PP* for points P on QA-Cu7.

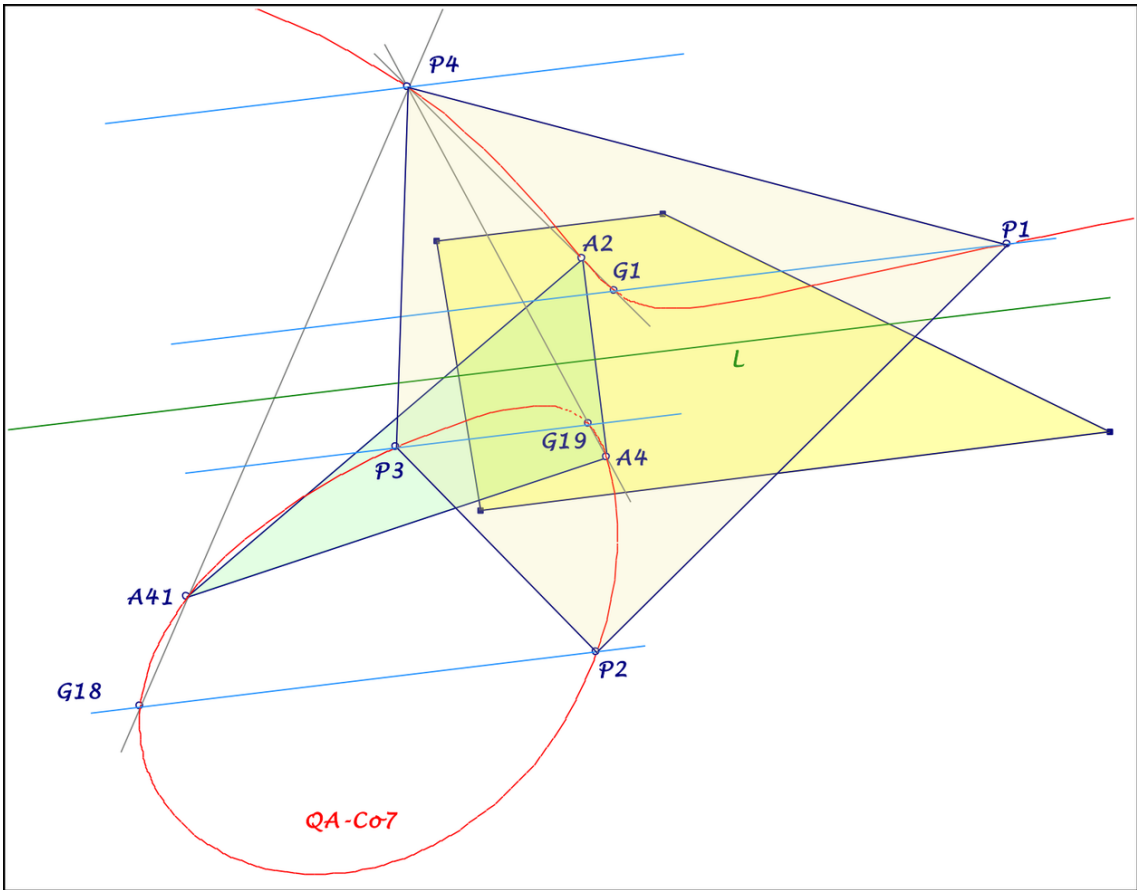
The parallels P1P1*, P3P3* as well as P2P2*, P4P4* are symmetric
wrt L.

Why special quadrigon?

... Its diagonal crosspoint is its Euler-Poncelet point.
... Its Miquel point is its QA-Miquel center,
... which is QA-P41 of the reference quadrigon.

For a quadrilateral there are three of this constellations ...

Best regards Eckart



2018-08-03.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3133
Date: 05/8/2018 4:39:09
From: Ngo Quang Duong
Subject: Paralogic n-Line

Dear Geometers,
Few days ago, I came up with an idea of a generalization of paralogic triangle.
I generalized the paralogic triangle to "paralogic" n-Line.

First of all, let me write down here the definition and geometric properties of the paralogic triangle:
Definition. Given a triangle ABC and a line L that intersects BC, CA, AB at D, E, F, respectively.
Then the lines through D, E, F and perpendicular to BC, CA, AB at D, E, F formed a triangle A'B'C' - which is called the paralogic triangle of the triangle ABC with respect to the line L.

- *- Property 1. If A'B'C' is the paralogic triangle of the triangle ABC wrt the line L, then ABC is also the paralogic triangle of the triangle A'B'C' wrt the line L.
- *- Property 2. The circumcircle of the triangles ABC and A'B'C' are orthogonal.
- *- Property 3. Two intersections of the circumcircle of the triangle ABC and A'B'C' are QL-P1 of (BC, CA, AB, L) (also identical to QL-P1 of (B'C', C'A', A'B', L)) and the perspector of ABC and A'B'C'.
- *- Property 4. (BC,CA,AB,L), (B'C',C'A',A'B',L) share the same QL-Ci3. This circle, together with the circumcircles of ABC and A'B'C', are coaxial.

References: paralogic triangles (
<http://mathworld.wolfram.com/ParalogicTriangles.html>)

And this is my definition for the paralogic n-Line:
Definition. Given n lines $\{L_1, L_2, L_3, \dots, L_n\}$ and an arbitrary line L such that L intersects L_i at P_i ($1 \leq i \leq n$).
 L'_i is the line that passes through P_i and perpendicular to L_i . Then $\{L'_1, L'_2, L'_3, \dots, L'_n\}$ is the paralogic n-Line of $\{L_1, L_2, L_3, \dots, L_n\}$ with respect to the line L.
Note that I uses curly bracket to indicate sets, I think that is convenient to demonstrate one of the following properties.
Followed by this definition are these properties:

*Property a. (L_1, L_2, \dots, L_n) is also the paralogic n -Line of $(L'_1, L'_2, \dots, L'_n)$ with respect to the line L .
Property b. For $n = 2k$. $(L_1, L_2, \dots, L_n, L)$ and $(L'_1, L'_2, \dots, L'_n, L)$ share the same Clifford point.
Property c. For $n = 2k+1$, the Clifford circles of (L_1, L_2, \dots, L_n) and $(L'_1, L'_2, \dots, L'_n)$ are orthogonal. One of the intersections is the common of n Clifford circles of $\{L, L_1, L_2, \dots, L_n\} \setminus \{L_i\}$.

Until now, I don't know if the property 4 can be generalized by this way.

I also have no idea of the second intersection.

These ideas and properties are quite promising, I guess.

But our knowledge about the polygon geometry are still so little.

Best regards,
Ngo Quang Duong

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3134

Date: 06/8/2018 12:28:36

From: bernard.keizer

Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Eckart,

I'm glad to see that you didn't abandon definitively this intriguing item !

Your construction is very interesting.

If I'm not wrong, P_1 , P_2 , P_3 and K are the isogonals of $QG-P_1$, $QG-P_{18}$, $QG-P_{19}$ and $QG-P_{17}$.

As $QG-P_{19}$ and $QG-P_{18}$ are the CSC of QGP_1 and $QG-P_{17}$, P_1 and P_2 and P_3 and P_4 are CSC.

Of course, P_2P_4 and P_1P_3 are orthogonal and QA-Cu7 of the initial QA is the QA-Cu1 of the QA $P_1P_2P_3P_4$ as well as the QL-Cu1 of the 4 lines P_1P_2 , P_1P_4 , P_2P_3 and P_3P_4 (the 2 other vertices of the QL being P_5 and P_6 intersections of P_1P_4 and P_2P_3 and of P_1P_2 and P_3P_4 also CSC).

But I still don't see the connexion with the 7 intersections of the 3 QA-Cu7 of a QL (5 real points and the 2 circular points)

...

The fight goes on !

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3135
Date: 06/8/2018 11:09:32
From: eckart_schmidt@t-online.de
Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Bernard,

the property "P2P4 orthogonal P1P3" was new for me,
... as well as "QA-Cu1 = QL-Cu1 for the quadrigon P1P2P3P4",
thanks!

But some remarks: I started with a QG,
interpretable as QA and QL.

(1) P1, P2, P3 are the TR-isogonals * of QG-P1, QG-P18, QG-P19,
... but K* is the 3rd intersection of QG-L1 and QA-Cu7.

(2) Wrt the reference QG:
... QG-P18 is the CSC of QG-P17, but the CSC of QG-P1 is QA-P4.

Wrt the QG P1P2P3P4:
... QG-P19 is CSC of QG-P1, but the CSC of QG-P18 is K*.

It is a bit confusing, but QA-Cu7 is invariant
... wrt QA-Tf2 for QA-Tr1,
... wrt isogonality for QA-P2.4.41,
... wrt QG-Tf2 for QG-Tr3,
... wrt CSC for the QG P1P2P3P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3136
Date: 06/8/2018 1:03:23
From: chris.vantienhoven
Subject: Paralogic n-Line

Dear Ngo Quang Duong,

The paralogic triangle adds an extra line to the triangle.
It's an interesting approach from you adding more extra lines to the formed n-Lines.

When you substitute in the 4 paralogic triangle properties the items:

- "triangle" into "n-Line"
- "circumcircle" as well as "QL-Ci3" into "nL-CenterCircle (nL-n-Ci1)"
- "QL-P1" into "nL-Centric Focus (nL-n-P1)"

then most of the properties will stay valid.

Property 1 stays like you mentioned completely valid

Property 2 the CenterCircles are not orthogonal for $n > 3$

Property 3b not valid (there will be no perspector for $n > 3$)

Property 4b not valid (not coaxial for $n > 3$),

Property 5. The centers of the CenterCircles of property 2 lie diametrically on the CenterCircle of property 4.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3137
Date: 06/8/2018 3:35:32
From: bernard.keizer
Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Eckart,

My apologise !

In fact K^* is not QG-P17

QG-P1 and QG-P19 and QG-P18 and K^* are CSC only wrt the QA
P1P2P3P4

QG-P1 and QA-P4 and QG-P17 and QG-P18 are CSC wrt the initial QA
Thanks for the precision

Now, what about the 5 real points ?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3138
Date: 07/8/2018 11:59:57
From: minhntenladuong
Subject: Paralogic n-Line

Dear Chris,

Actually, in my substitution, the circumcircle of a triangle becomes the Clifford circle of n -lines (n is odd). When n is odd, the Clifford circles of $\{L_1, L_2, \dots, L_n\}$ and $\{L'_1, L'_2, \dots, L'_n\}$ are orthogonal.

Best regards,
Ngo Quang Duong

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3139
Date: 08/8/2018 9:33:56
From: eckart_schmidt@t-online.de
Subject: Perspective QL-triple triangles

Dear Chris,

in EQF we find perspective QA-triple triangles under QA-Tr-2,
... but no information about perspective QL-triple triangles,
... here some examples:

- (1) QA-P41-Tr perspective QG-P1-Tr,
- (2) QG-P1-Tr perspective QG-P19-Tr,
- (3) QG-P19-Tr perspective QA-P4-Tr,
- (4) QA-P4-Tr perspective QG-P18-Tr.

This row of perspective QL-triple triangles plays a role
... wrt the five common points of the three QA-Cu7 of a QL.

If we accept the QG-point S (see #2980),
... which is the isogonal of QG-P1 wrt QA-P2.4.41:

- (5) S-Tr perspective QG-P1-Tr,
- (6) S-Tr perspective QA-P4-Tr.

Some perspectors:

... wrt (3) point U, see #2986,
... wrt (4) point R, see #2983,
... wrt (5) point T, see #2981, wrt (6) point R (see #2983).

The perspectors (3) and (4) lie diametral

... on the QA-P4-circumcircle CSC(QL-Ci1),
... perpendicular to the QG-P18-line.

The perspector (1) lies on the QA-P41-circumcircle.

I think, you can calculate more.

Example: QA-P4-Tr perspective QG-P16-Tr wrt QL-P1.

Best regards Eckart

PS: QG-P18-Tr and QG-P16-Tr are collinear degenerated.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3140
Date: 09/8/2018 8:55:46
From: chris.vantienhoven
Subject: Paralogic n-Line

Dear Ngo Quang Duong,
You are right.
The Clifford Circle gives an extra property for the odd case.
Maybe there are even other properties too?
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3141
Date: 09/8/2018 12:19:45
From: chris.vantienhoven
Subject: Perspective QL-triple triangles

Dear Eckart,

After our research for QA-Triple Triangles finally QL-Triple Triangles are researched.
Very well. Thank you!
I am sure this is just a start.
There will be many more perspective QL-Triple Triangles with many incidences.
But I assume there will be also Desmic, Orthologic, Cyclologic, Parallelologic and Eulerologic QL-Triple Triangles.
A new world to enter.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3142

Date: 10/8/2018 10:37:10

From: bernard.keizer

Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Eckart,

Drawing a new figure for the 3 QA-Cu7 of a QL, I find this property.

I don't remember we have mentioned it before, but you know it perhaps already ...

The CSC circle of a DT side is of course through the 2 QL vertices on this side, which are CSC, through QL-P1 and through 2 QA-P4, which are the CSC of 2 DT vertices, but also through the 3rd QA-P41.

This means that the CSC of the QA-P41 are the intersections of the line CSC of the circle of the QA-P41 with the diagonals or DT sides.

Any idea about this line or it's QL-Tf2 ?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3143

Date: 11/8/2018 8:27:14

From: chris.vantienhoven

Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear friends,

I found some old research of mine about related transformations for 2-Point-, 3-Point and 4-Point-Configurations.

Mid-Reflect Transformation 2P-s-Tfx in a 2-Point configuration

Let $P1.P2$ be a Reference 2-Point and L a random line.

Then 2P-s-Tf1 is the transformation that maps L into the reflection of the intersection point $L \wedge P1.P2$ about the midpoint of line segment $P1.P2$.

Mid-Reflect Transformation 3P-s-Tfx in a 3-Point configuration (triangle)

Let $P1.P2.P3$ be a Reference 3-Point (triangle) and L a random line

Transform L with 2P-s-Tfx wrt the Component 2-Points into points $R12, R23, R31$.

$R12, R23, R31$ are collinear on a line which is the 3P-Mid-Reflect Transformation $3P-s-Tfx(L)$.

Line $(x : y : z)$ is being transformed into line $(y z : z x : x y)$.

I suppose this transformation is already known for a long time with another name.

Mid-Reflect Transformation 4P-s-Tfx in a 4-Point configuration (quadrangle)

Let $P1.P2.P3.P4$ be a Reference 4-Point (quadrangle) and L a random line.

Transform L with 3P-s-Tfx wrt the Component Triangles into lines $L1, L2, L3, L4$.

Let $S12, S13, S14, S23, S24, S34$ be the intersection points of $L1, L2, L3, L4$.

Note that $S12$ lies on $P3.P4$, $S13$ lies on $P2.P4$ (opposite sides), etc.

Now the triangle bounded by the lines $S12.S34, S13.S24$ and $S14.S23$ (which is the QL-Diagonal Triangle of $L1.L2.L3.L4$) is perspective with the QA-Diagonal Triangle of the Reference Quadrangle.

Its perspector is being called the QA-Mid-Reflect Transformation $4P-s-Tfx_a(L)$ of L .

Its perspectrix is being called the QA-Mid-Reflect Transformation $4P-s-Tfx_b(L)$ of L .

Especially the mapping of QA-L3 (QA-P1.QA-P5) is interesting.

4P-s-Tfx_a transforms QA-L3 into the intersection point of these lines:

- QA-P1.InvoluntaryConjugate[InfinityPoint[QA-P1, QA-P16]]
- QA-P5.InvoluntaryConjugate[InfinityPoint[QA-P5, QA-P17]]
- QA-P10.InvoluntaryConjugate[InfinityPoint[QA-P10, QA-P16]]
- QA-P19.InvoluntaryConjugate[InfinityPoint[QA-P1, QA-P5]]

with these CT-coordinates:

$(p(q-r)^2(q+r)(2p+q+r)(p^2+pq+q^2+pr+3qr+r^2) :$

$q(p-r)^2(p+r)(p+2q+r)(p^2+pq+q^2+3pr+qr+r^2) :$

$r(p-q)^2(p+q)(p+q+2r)(p^2+3pq+q^2+pr+qr+r^2))$

4P-s-Tfx_b transforms QA-L3 into this line: QA-P10.QA-P16

with these CT-coordinates:

$(qr(q-r)(p+2q+r)(p+q+2r) :$

$rp(r-p)(q+2r+p)(q+r+2p) :$

$pq(p-q)(r+2p+q)(p+r+2q))$

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3144

Date: 11/8/2018 9:55:48

From: eckart_schmidt@t-online.de

Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

3P-s-Tfx(L) is the trilinear polar

... of the isotomic conjugate

... of the trilinear pole of L.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3145
Date: 11/8/2018 11:29:22
From: eckart_schmidt@t-online.de
Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

reproducing your message 3143, here another property:
4P-s-Tfxa of a line pencil
... gives a circumconic of the diagonal triangle,
... example: DT-Steiner circumellipse for QA-P5.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3146
Date: 11/8/2018 2:06:52
From: eckart_schmidt@t-online.de
Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

excuse my splitter messages, here another property:
4P-s-Tfa of the line pencil of QA-P1
... gives the conic QA-Co1.
QA-Co4 and QA-Co5
... are the images of line pencils wrt infinity points.
QA-Ci1 can also be considered as image of a line pencil.
I try to find the points for these line pencils.

Best regards Eckart

PS: What about the inverses of 4P-s-Tfxa,b?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3147
Date: 12/8/2018 8:08:13
From: minhntenladuong
Subject: Paralogic n-Line

Dear Chris,
As you know, for even case, n-Line has a Clifford point. The new property is quite different.
When n is even, the Clifford circles of $\{L_1, L_2, \dots, L_n, L\}$ and $\{L'_1, L'_2, \dots, L'_n, L\}$ are identical.
Best regards,
Ngo Quang Duong

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3148
Date: 12/8/2018 10:42:37
From: eckart_schmidt@t-online.de
Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Bernard,

thanks for the property in #3142,
... but I cannot add any new observation.
The constellation is full of geometry
... without any link to the 5 points, very extraordinary.

For example:
The CSC-circle of the circle with diameter QG-P1a,b bears
... QA-P4a,b and QG-P18a,b
... and the double intersections of QA-Cu7 on QG-L1c.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3149
Date: 13/8/2018 11:59:25
From: eckart_schmidt@t-online.de
Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

your line-to-point QA-transformation 4P-s-Tfxa -here shortened TF-

... maps line pencils to DT-circumscribed conics (see #3145).

For QA-Co4 and QA-Co5 we need line pencils of infinity points.

Both conics bear $QA-P20 = QA-Tf2(QA-P1)$, so we shall consider

... DT-circumscribed conics $C0$ through $QA-P20$:

Let L be the line through $QA-P1$, bearing the intersections of $C0$ and the cubic $QA-Cu1$.

... Consider a DT-circumscribed conic through $QA-P20$

and $TF(L)$ on $QA-Co1$,

... $QA-Tf2$ of this conic gives a line L' through $QA-P1$,

... and TF of parallels to the line L' will generate the starting conic $C0$.

By the way: $TF(L')$ is the 4th intersection of $C0$ and $QA-Co1$.

What about the line pencil, that generates $QA-Ci1$?

... The pencil point lies on a line L' through $QA-P1$

... with $TF(L') = 4th$ intersection of $QA-Ci1$ and $QA-Co1$.

... ..

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3150

Date: 13/8/2018 12:45:06

From: eckart_schmidt@t-online.de

Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

excuse, there is a typo in #3149:

Replace QA-Cu1 by QA-Cu5, cubic with pivot QA-P1.

Sorry Eckart

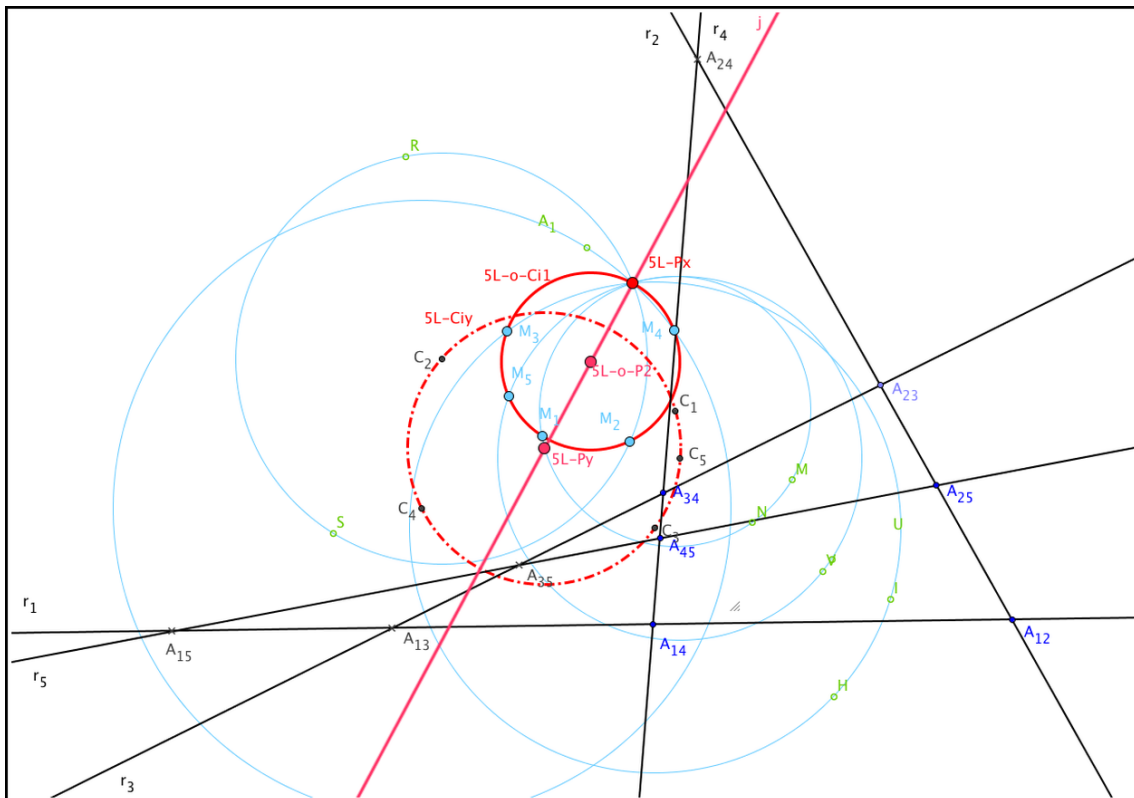
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3151
Date: 13/8/2018 6:46:22
From: Benedetto Scimemi
Subject: 5-line central objects

Dear friends, is any the following observations new ?
 In a 5-line the Miquel points $4L-P1_i$ of the complementary 4-lateral lie on a (Clifford) circle $5L-o-Ci1$ whose center is $5L-o-P2$ (# 1999)
 The 5 (Miquel) circles $4L-Ci4_i$ of the complementary 4-lateral have a common point $5L-Px$ which lies on $5L-o-Ci1$
 The 5 centers of these circles $4L-Ci4_i$ lie on a circle $5L-Ciy$ whose center is a central point $5L-Py$
 $5L-Px$, $5L-Py$, $5L-Po-C1$ are collinear; indeed $5L-o-P1$ and $5L-Px$ are *inverse* points on $5L-Ciy$

Dear Eckart, I read with some delay your message of July 27 about your LTL transformation.
 I found it very interesting and eventually I plan to study it analytically. Did you?

Best regards
 Benedetto



Schermata 2018-08-13 a 18.41.38.png

Message: #3152
Date: 13/8/2018 11:58:41
From: chris.vantienhoven
Subject: 5-line central objects

Dear Benedetto,

Items in a Pentalateral are at least as fascinating as items in a Quadrilateral.
Because some items are valid for all n and others only for $n=\text{odd}$ and others only for $n=5$, the coding of the items is a bit complicated. But infix $-n-$ means "for all n " and infix $-o-$ means for " $n=\text{odd}$ " and infix $-s-$ means for " $n=\text{special number}$ " like $n=5$.

Note that in a 5-Line $5L-o-Ci1 = 5L-n-Ci1$ and that $5L-o-P2 = 5L-n-P3$. For $n>5$ these items start to diverge.
Most points you describe can be found at $nL-n-P1$
In EPG terminology your $5L-Px = 5L-n-P1$ and $5L-Py = 5L-n-P3$.
See $nL-n-P1$ and $nL-n-P3$.
Your $4L-Ci4 = QL-Ci3 = 4L-n-Ci1$ and $5L-Ciy = 5L-n-Ci1$.
See $nL-n-Ci1$.

The collinearity of $5L-Px$, $5L-Py$, $5L-o-P2$ is described at $5L-s-L1$.

Special is that just like the 4 circles $QL-Ci3$ are circumscribed by a Cardioid $QL-Qu1$, all n circles $nL-n-Ci1$ are circumscribed by a so-called EnnaCardioid $nL-n-Cv1$ (one of the many findings by Morley).

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3153

Date: 14/8/2018 2:56:40

From: eckart_schmidt@t-online.de

Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

wrt QA-Tfxa, here shortened TF, a first step to its inverse:
For points Q on QA-Co1 here is a construction
... for the line L with $TF(L) = Q$:

Let C0 be any DT-circumconic through Q,
... consider parallels Li to the line QA-Tf2(C0),
... then $TF(Li)$ give a second DT-circumconic,
... whose QA-Tf2-image is the line L with $TF(L) = Q$.

So for a DT-circumconic C0 (see#3149),
... intersecting QA-Co1 further in Q,
... we get a line L through QA-P1,
... bearing the point P, whose line pencil generates C0.

This holds also for the circle QA-Ci1 with $Q = QA-P2$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3154

Date: 15/8/2018 10:31:53

From: bernard.keizer

Subject: QA-Cu7 as QA-Cu1 of a special quadrigon

Dear Eckart,

Thanks for your answer and this new property !

I begin to ask me if we are not "searching midday at 14 h" (a french expression).

2 bicursal Van Rees focal circular cubics defined by the 3 couples of points $P_{2a,b}$, $P_{4a,b}$ and $P_{41a,b}$ intersect in 7 points (not necessary real) and the 2 circular points. The 9 points form a C-B system.

Taking 2 such curves with 7 real points and choosing any couple of 2 points as 2 points on a diagonal and the 5 others as points on the conic gives immediately the isogonals $g_{Ta,b}$ of the 2 vertices T_a, T_b as 6th intersections between the conic and the 2 VR's and the vertices on the 2 VR's T_a, b .

It happens that these 2 vertices are on line with the 2 points, which gives 21 possibilities ...

More interesting, your point T, perspector of the points T_a, b and $g_{Ta,b}$ lies also on the conic.

But the point R, intersection of the lines $P_{4g_{Ta,b}}$ and b doesn't necessary !

I suppose there is for a given VR only other VR such as the point R is also on the conic.

In this case, knowing the 2 couples $P_{4a,b}$ and T_a, b gives QL-P1 and P41c as intersection of the 2 circles through the $P_{41a,b}$ and the $P_{4a,b}$.

I suppose it's possible to achieve the construction in order to find the 3rd VR ...

Intuition is not proof and analytical proof could be found by using the classical complex equation of a VR

$(z-a)(z-a') = t(z-b)(z-b')$, with t real, where a and a' , b and b' are 2 couples of conjugate points.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3155

Date: 15/8/2018 12:06:41

From: eckart_schmidt@t-online.de

Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Chris,

your line-to-point transformation 4P-s-Tfxa, here shortened TF,
... maps line pencils to DT-circumscribed conics,
... examples in previous messages.

Here is the inverse construction in general,
... as well as the inverse of TF in general:

(1) Let C_0 be a DT-circumconic,
... consider $Q = QA-Tf_2(TF(QA-Tf_2(C_0)))$,
... and the line pencil of Q will generate C_0 .

(2) Let P be a point
... and C_01, C_02 two DT-circumconics through P ,
... which give two points Q_1, Q_2 (see(1)),
... then $L = Q_1Q_2$ has the image $TF(L) = P$.

In this way we can find the line pencil,
... which generates the DT-circumcircle $QA-Ci1$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3156
Date: 16/8/2018 6:24:45
From: chris.vantienhoven
Subject: Mid-Reflect Transformation in a 2-Point, 3-Point, 4-Point

Dear Eckart,

Thanks for all your remarks.
When I worked on the transformations 2P-s-Tfx, 3P-s-Tfx and 4P-s-Tfx_a, 4P-s-Tfx_b I liked especially the development of the transformations from a 2-Point, to a 3-Point, to a 4-Point-configuration.
It did do some calculations but then it stopped.
Thanks to you it appears that the transformation 4P-s-Tfx_a is much richer than I thought.
Very nice findings about the construction of the inverse transformation of 4P-s-Tfx_a !
I also think that examples of triangle line transformation 3P-s-Tfx could be very interesting.
I exercised only three examples:
* 3P-s-Tfx[Eulerline] = line X122-X125
* 3P-s-Tfx[Brocard Axis] = line X115-X125
* 3P-s-Tfx[X2-X6-axis] = line X115-X127

Best regards,
Chris

p.s. Very nice your clarification:
3P-s-Tfx(L) is the trilinear polar
... of the isotomic conjugate
... of the trilinear pole of L.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3157
Date: 18/8/2018 3:36:53
From: eckart_schmidt@t-online.de
Subject: QL-P10

Dear Chris,
only a QL-splitter:
QL-P10 is the DT-trilinear pole
... for the three collinear QG-P18 points of the QL.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3158
Date: 18/8/2018 5:49:51
From: bernard.keizer
Subject: QL-P10

Dear Eckart,
I have followed so far all your properties in this 3 QA-Cu7 problem !
But I cannot reproduce this one, that the QG-P18 line is the DT orthic axis.
It should be perpendicular to the DT Euler Line ?
Is it true that each QG-P18 is the intersection on each QA-Cu7 of the lines $A_{ij}CSC(A_{kl})$ and $A_{kl}CSC(A_{ij})$ with A_{ij} and A_{kl} 2 vertices of the QL?
Is it also true that each QG-P18 is on the line through your point R, QA-P2 and $isog(T)$?
I don't understand my mistake ? Best regards
Bernard
PS If you make easily such complicated barycentric calculations, it should be possible to calculate the equations of the 3 QA-Cu7 and of the conic. The intersection must give an equation of the 6th degree with a different solution (gTi) in each case ; the remaining equation of the 5th degree must be the same ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3159
Date: 19/8/2018 10:09:51
From: eckart_schmidt@t-online.de
Subject: QL-P10

Dear Bernard, dear Chris,

thanks to Bernard,
please forget my last message 3157!
It is a false overhasty observation,
confusing two lines.

In my next message QL-P10 will appear
... as DT-trilinear pole of a line,
whose QL-Tf2-image
... is the DT-trilinear polar of a new QL-point.

Wrt Bernard's #3158:
I don't understand "lines AijCSC(Ak1)",
for me holds $CSC(Ak1) = Aij$.

Further I cannot reproduce
... "QG-P18 on the line through R, QA-P2 and isog(T)".
Are there misunderstandings on my side?
Perhaps helpful:
... QG-point S: isogonal of QG-P1 wrt QA-P2,4,41,
... QL-point R: perspector of the S-, the QA-P4- and the
QG-P18-triangle,
... QL-point T: perspector of the S-triangle and the diagonal
triangle QL-Tr1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3160
Date: 20/8/2018 9:21:29
From: eckart_schmidt@t-online.de
Subject: QG-P18/QG-P19 geometry for a QL

Dear Bernard, dear Chris,

perhaps these observations are helpful for the 3 QA-Cu7 for a QL:

Let us start with a QL
... and its three quadrilaterals QGa, QGb, QGc,
... they can be interpreted as quadrangles,
... which lead to nine quadrilaterals:
... QGaa=QGa, QGab, QGac, QGba, QGbb=QGb, QGbc, QGca, QGcb,
QGcc=QGc.

The three QG-P18 of QGa, QGb, QGc are collinear.
The three QG-P19 of QGa, QGb, QGc give a triangle,
... perspective to the QA-P4-triangle wrt point U (see #2986),
... perspective to QL-DT wrt a new QL-point V.
The three QG-P18-points lie on the sidelines of the
QG-P19-triangle.

Now we consider the QG-P18- and QG-P19-points for the nine
quadrilaterals:
... The six lines
... P19a.P18ab, P19a.P18ac, P19b.P18ba, P19b.P18bc, P19c.P18ca,
P19c.P18cb
... are the lines of a *quadrangle QA* ,
... perspective to the dual QA of the initial QL
... with perspector in a new QL-point W,
which is QA-Tf2(V).

The diagonal triangle of QA is the QG-P19-triangle of QL,
... the Involutory Conjugate of QA swaps V and W,
... as well as the double intersections of the three QA-Cu7.

The diagonal triangle of the dual QA is QL-DT,
... the Involutory Conjugate of the dual QA swaps QL-P8 and QL-P13
... as well as QL-P10 and W:
... $W = \text{DT-pole}(\text{QL-Tf2}(\text{DT-polar}(\text{QL-P10})))$.

So far, there will be more properties,
... perhaps of interest

the nonpivotal isocubic
... with reference triangle QG-P19a,b,c,

... isoconjugation swapping V and W
... and root in the tripole of the QG-P18-line of the reference
QL.

This cubic bears the 3 QG-P19, the 3 QG-P18 and V, W.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3161
Date: 22/8/2018 11:20:50
From: bernard.keizer
Subject: QL-P10

Dear Eckart,
My previous message went lost !
I send it again.
1) There are 4 CSC, the main QL-CSC and the 3 minor csc for each
QA-Cu7 (center P41, swapping P2 and P4) Then, my property holds
...
2) I made a confusion, it's P4 and not P2 : then the property is
the same as yours about R
What about the line of the QG-P18 wrt DT ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3162
Date: 23/8/2018 3:39:27
From: eckart_schmidt@t-online.de
Subject: QL-P10

Dear Bernard,

I have further problems with your observations
... in #3158, corrected in #3161:

Wrt 1): My sight: The Möbius transformation,
... centered in *QG-P18* , swapping QA-P2 and QA-P4,
... maps A_{ij} to a point, collinear with A_{k1} and QG-P18.
Wrt 2) QG-P18, R and QA-P4 are collinear,
... but what is the reference triangle for the isogonal of T?

Wrt your last question: CSC of the QG-P18-line is QL-Ci2 (not in EQF).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3163
Date: 23/8/2018 4:43:50
From: bernard.keizer
Subject: QL-P10

Dear Eckart,
It's perhaps a question of definition or naming of points, I must confess I'm not really familiar with the QG points
I have the same points QG-P18 and P19 as you, as I find the QG-P18 line as CSC(Ci2)
Beautiful QL formed by the 6 points indeed !
Isog(T) are your points S (the T are the DT vertices or QG-P1)
Then, for each csc with center QA-P41 and swapping P2 and P4, the intersection of Aijcsc(Akl and Aklcsc(Aij) is the point QG-P18 on the corresponding QA-Cu7 (with DT AijAklQG-P1)
Best regards
Bernard
PS I tried to draw the QL-Cu1 of the new QL
Then I tried to draw curves through the 5 triple points V 1 to 5, the 6 double points U 1 to 6 and the vertices of this QL.
Strange quartics (?) appear ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3164
Date: 05/9/2018 11:25:15
From: eckart_schmidt@t-online.de
Subject: Dual QA / QL

Dear Chris,
perhaps worth, to be mentioned in EQF under QA-8/QL-8:
(A QL has three QG-versions, which can be interpreted as QAs.)
The dual QA of a QL has as vertices
... the triple intersections of QA-DT sidelines
... unequal the QL-DT vertices.
(A QA has three QG-versions, which can be interpreted as QLs.)
The dual QL of a QA has as sides
... the lines of 3 collinear QL-DT vertices
... unequal the QA-DT sidelines.
Best regards Eckart

PS: If I am not wrong,
... QL-Tf11(P) is the dual of P,
... QA-Tf10(L) is the dual of L.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3165
Date: 09/9/2018 4:41:10
From: chris.vantienhoven
Subject: Dual QA / QL

Dear Eckart,

About your first remarks I leave it the reader for discovering it themselves looking at the picture.

(a picture tells more than a thousand of words)

Nevertheless you are quite right and thanks for noting.

About the duals, QL-Tf11(P) is the dual of P and QA-Tf10(L) is the dual of L indeed.

I added this in EQF at QA-8/QL-8.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3166
Date: 11/9/2018 3:44:18
From: bernard.keizer
Subject: QL, epi- and hypocycloïds and r-angle centers

Dear Chris, dear Eckart, dear NgoQuang Duong

Thanks to Chris, I've revisited the item of epi- and hypocycloïds tangent to 4 lines with the n-angle centers and I have found that it is perfectly possible to deal with real numbers.

I hope I didn't make basic mistakes and I thank you in advance for any comment or critic.

Best regards
Bernard

QL, epi- and hypocycloids and r-angle centers

1) Epi- and hypocycloids E and H

Let's define an epicycloid E or hypocycloid H as the envelop of lines MN, the 2 points M (primary point) and N (secondary point) describing a circle with center O at different speeds p and q, same sign for epicycloids, opposite sign for hypocycloids.

q/p can be an irreducible fraction ; in this case, p and q are integers, mutually prime and the epi- or hypocycloids have a finite number of points of tangency with the inner circle, of cusps and of axes of symmetry.

But q/p can also be real, we better choose - 1 and r for hypocycloids and 1 and r for epicycloids ; in these cases, the curves have an infinite number of points of tangency with the inner circle, of cusps and of axes of symmetry.

2) Properties of 4 tangents to an epi- or hypocycloid

Starting with a circle with center O and origin on the circle I, we may choose 4 points M_i ($i = 1$ to 4) on the circle as primary points.

We have 4 secondary points N_i such as the angles $\angle O N_i$ are equal to $r \cdot \angle O M_i$

Then we have 4 tangents $M_i N_i$ to the curve forming a QL.

We have following well-known properties :

- For each reference triangle $A_{ij}A_{ik}A_{jk}$, we have 4 points X_i , the 4th point X_i being the intersection of the 3 circles $A_{ij}M_iM_j$, $A_{ik}M_iM_k$ and $A_{jk}M_jM_k$ and 4 points X'_i , X'_i being the intersection of the 3 circles $A_{ij}N_iN_j$, $A_{ik}N_iN_k$ and $A_{jk}N_jN_k$ (each time 6 circles intersecting in 4 points with 2 points on each circle and 3 circles through each point)
- The X_i and X'_i are isogonal conjugates wrt the reference triangle $A_{ij}A_{ik}A_{jk}$
- The 4 couples of points X_i and X'_i are the so called r-angle centers
- The perpendicular bisectors of $X_i X'_i$ intersect in the circumcenter O

- The 4 X_i are concyclic as well as the 4 X'_i and the 2 circles are CSC in the transformation QL-Tf1 (as a consequence, the ratio of the radii is equal to the ratio of the distances from the circumcenters to QL-P1)
- The angles of the reference triangles, of the triangles $M_iM_jM_k$, $N_iN_jN_k$ and the angles under which we see the sides of the reference triangles from the r-angle centers X_i and X'_i are proportional
- The explanation is that the angle between 2 perpendicular bisectors of M_iN_i and M_jN_j is equal to the angle between the 2 lines M_iN_i and M_jN_j ; if $m_i = \angle IOM_i$ and n_i is $\angle ION_i$, the perpendicular bisector is $\frac{1}{2}(m_i + n_i)$ with $n_i = r \cdot m_i$ and we have the angle $a_{ij} = \frac{1}{2}(m_j - m_i)(1 + r)$.
- For the triangle $M_iM_jM_k$, the angle in M_k is $\frac{1}{2}(m_j - m_i) = a_{ij} / (1 + r)$; for the triangle $N_iN_jN_k$, the angle in N_k is $\frac{1}{2}(n_j - n_i) = a_{ij} \cdot r / (1 + r) \dots$

3) Epi- or hypocycloïds tangent to 4 lines

Having the 4 couples of points X_i and X'_i allows easily to find the 8 concyclic points M_i and N_i ; then we have to find one of the points I on the circle.

1. p and q integers

- For $p + q = 1$, there are only 4 lines and we find the generalized Kantor-Hervey theorem (the deltoïd is $p = 2$, $q = -1$, the other are the H_{2n+1})
- For $p + q = 2$, there are 4 points, the beginning of the chain is for $p = q = 1$ and give the 4 in- and excenters of the reference triangle ; the points are all selfisogonal. We remember the 16 points are on 8 circles of 4 points, each point being on 2 circles. These circles (socalled Steiner circles) are Cl-S invariant.
- The next step in the same chain (inverse and isogonal of the inverse) is for $p = 3$ and $q = -1$ and give the 8 pairs of Cl-S conjugate circles ; the centers are the centers of the 8 astroïds tangent to the QL and are on a rectangular hyperbola, which is a conic stelloïd

- For $p + q = 3$, there are 9 points, the beginning of the chain is with $p = 2$ and $q = 1$ and gives the 27 centers of the cardioïds tangent to the QL, which lie on the cubic stelloïd.

2. r real

This case is more a curiosity, as the curves are rather complicate !

The difficulty is to start chains ...

References : messages 1843 from Ngo Quang Duong and 1872 from Chris

QL, E, H and r-angle.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3167

Date: 17/9/2018 8:52:05

From: seiichikiri

Subject: A property of QAP4 as the extension of the circular center.

Dear friends,

Did the following already appear?

Let a quadrangle be $P_1P_2P_3P_4$ and QAP_4 its isogonal center.

Denote:

N_{12} = the projection of QAP_4 on P_1P_2 ,

N_{13} = the projection of QAP_4 on P_1P_3 ,

N_{14} = the projection of QAP_4 on P_1P_4 ,

N_{23} = the projection of QAP_4 on P_2P_3 ,

N_{24} = the projection of QAP_4 on P_2P_4 ,

N_{34} = the projection of QAP_4 on P_3P_4 ,

QAP_4 , N_{12} , N_{13} , N_{14} , N_{23} , N_{24} and N_{34} are on a conic.

N_{12R} = the intersection of $N_{12}QAP_4$
rotated by the fixed angle with P_1P_2 ,

N_{13R} = the intersection of $N_{13}QAP_4$
rotated by the fixed angle with P_1P_3 ,

N_{14R} = the intersection of $N_{14}QAP_4$
rotated by the fixed angle with P_1P_4 ,

N_{23R} = the intersection of $N_{23}QAP_4$
rotated by the fixed angle with P_2P_3 ,

N_{24R} = the intersection of $N_{24}QAP_4$
rotated by the fixed angle with P_2P_4 ,

N_{34R} = the intersection of $N_{34}QAP_4$
rotated by the fixed angle with P_3P_4 ,

QAP_4 , N_{12R} , N_{13R} , N_{14R} , N_{23R} , N_{24R} and N_{34R} are on another conic.

In this example, we might also consider this property of QAP_4 as the extension of the circular center.

<https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/1007>

Best regards, Seiichi.

Message: #3169

Date: 19/9/2018 6:31:19

From: seiichikiri

Subject: A property of QAP4 as the extension of the circular center.

Dear Eckart,

Sorry for incompleteness of the expression.

Correction:

N12R = the intersection of the line N12QAP4 rotated around QAP4 by the fixed angle with P1P2.

Other notations should be interpreted in the similar manner.

Best regards, Seiichi.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3170

Date: 20/9/2018 10:48:21

From: bernard.keizer

Subject: QL, epi- and hypocycloïds and r-angle centers

Dear Chris, dear Eckart,

I'm really surprised that I got no answer to this message !

Chris, it's exactly the answer to your questions with a synthetic proof by consideration of angles ...

Eckart, it's the generalisation for a real number of definition of n-angle of a fractional number ...

Let's resume it again with obvious notations for 3 lines L_i with $i = 1$ to 3:

1) Any triangle $M_iM_jM_k$ inscribed in a triangle $A_{ij}A_{ik}A_{jk}$ leads to a 2nd triangle $N_iN_jN_k$ and 2 points X and X' isogonal wrt $A_{ij}A_{ik}A_{jk}$ and we see the side $A_{ij}A_{ik}$ from X and X' under the angles X_i and X'_i

We have $A_{jk} + M_i + N_i = \pi$, $X_i = \pi - N_i$, $X'_i = \pi - M_i$ and $X_i + X'_i = \pi + A_{jk}$

2) If on the circumcircle of M_i, M_j, M_k, N_i, N_j and N_k $ION_i = r \cdot ION_i$ with r a real number,

we have $N_i = r \cdot M_i$ modulo $\pi \cdot r$, $X_i = r / (1+r) A_{jk}$ modulo $\pi / (1+r)$ and $X'_i = 1 / (1+r) A_{jk}$ modulo $r \cdot \pi / (1+r)$

X and X' are the $r / (1+r)$ and $1 / (1+r)$ angle centers

3) If $r = p/q$ is a fraction, X and X' are the $p / (p+q)$ and $q / (p+q)$ angle centers (as Eckart mentionned, there are $q \cdot q$ such points), which are constructible in chains with isogonality and inversion ...

4) It's always possible to approximate a real number by a fraction with the wanted precision.

With one decimal, there would be 100 such points, with 2 decimals 10000 ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3171

Date: 20/9/2018 12:13:25

From: eckart_schmidt@t-online.de

Subject: A property of QAP4 as the extension of the circular center.

Dear Seiichi,

thanks for explanations, me sight:

Consider a quadrigon $P_1P_2P_3P_4$

... with the pedal parallelogram $N_{12}N_{23}N_{34}N_{41}$ of QA-P4.

... Rotating the rays QA-P4.Nij with the same angle round QA-P4,

... the intersections N_{ijR} of QA-P4.Nij and P_iP_j

... give always a parallelogram,

... centered in Z on the perpendicular of QA-L2 in QA-P6.

... The angle of rotation is

This property for the quadrigons $P_1P_2P_3P_4$, $P_1P_2P_4P_3$, $P_1P_4P_2P_3$

... leads to your described conics.

Best regards Eckart.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3172
Date: 20/9/2018 9:10:41
From: eckart_schmidt@t-online.de
Subject: QL, epi- and hypocycloïds and r-angle centers

Dear Bernard,

excuse, if at present I don't participate in the discussion of this topic.

I tried in vain, to reproduce your constructions, I am currently not familiar with our detailed extensive earlier discussion.

Sorry, give me some time.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3173
Date: 20/9/2018 11:10:10
From: chris.vantienhoven
Subject: QL, epi- and hypocycloïds and r-angle centers

Dear Bernard,

I am sorry. I was glad with your response and I noticed that you answered my questions indeed.

However I am extremely busy right now and there is no light at the end of the tunnel yet.

I only spend one hour studying your message and then other facts of life took over.

Be sure I will go on studying your message. It is at the top of the pile as soon as I find time.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3174
Date: 21/9/2018 2:00:21
From: seiichikiri
Subject: A property of QAP4 as the extension of the circular center.

Dear Eckart,

Thank you very much for your explanation with parallelogram!

Best regards, Seiichi.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

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Message: #3175
Date: 21/9/2018 3:52:46
From: eckart_schmidt@t-online.de
Subject: New QA-Tf, new QA-circumcubics

Dear all,

without QA-Cu6 all QA-cubics in EQF are QA-Tr1-circumscribed,
... here a new concept for QA-circumscribed cubics
... without this property, but QA-Cu6 as special example:
Let us consider a line pencil of a point P
... and the QA-Tf2-conics of its lines
... with polars of P wrt these conics,
... which have a common point Q.
Let TF be the QA-transformation $P \dashrightarrow Q$:
... QA- and QA-Tr1-vertices are fixed points of TF,
... TF maps QA-P16 to QA-P1,
... TF is not self-inverse,
... TF(P) is a point on the QA-circumconic through P,
... TF maps lines to QA-circumscribed cubics,
... especially the line at infinity to QA-Cu6.
The TF-cubic of a line L has a double point R,
... which is the pole of L wrt the conic QA-Tf2(L).
... The QA-circumconic through R
... intersects L in two points with TF-image R.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3176
Date: 26/9/2018 9:03:04
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

this is an essay, to gather main properties of QA-Cu7,
perhaps helpful,
... to lighten the geometry of the 5 common points for its three
versions wrt a QL.

Let us consider a quadrangle QA:

(1) QA-Cu7 as isocubic (see #2965)

... is invariant wrt QA-Tf2,

... is invariant wrt the isogonal conjugation

... .. of the triangle TR = QA-P2.QA-P4.QA-P41,

... is invariant wrt the three Möbius transformations of TR.

(2) QA-Cu7-points (see EQF):

... vertices of diagonal triangle QA-Tr1,

... vertices QA-P2, QA-P4, QA-P41 of TR,

... in- and excenters of TR,

... intersection of QA-Cu7 and its asymptote,

... .. diametral to QA-P41 on the circumcircle of TR,

... and isogonal conjugated as well as Möbius transformed wrt TR.

Let us consider a quadrigon QG, interpretable as QA:

(3) Further QA-Cu7-points:

... QG-P1, QG-2P2, QG-P18, QG-P19,

... QG-point S not in EQF (see #2980, #2983, #2991),

... .. which is the isogonal conjugated of QG-P1 wrt TR.

Let us consider a quadrilateral QL

... with its three quadrigon versions QG,

... interpretable as quadrangles QA.

So we get triples of QG- and QA-points on the three QA-Cu7
versions

... with several perspectors, which give new QL-points.

(4) Perspectors of triangles (see #3139):

... QL-point R

not in EQF (see #2974, #2983, #2984, #2986):

... .. perspector of S- and QA-P4-triangle,

... as well as perspector of QA-P4- and QG-P18-triangle,

... QL-point T

not in EQF (see #2981)

... perspector of S- and QG-P1-triangle,

... QL-point U

not in EQF (see #2986, #3012)

... perspector of QG-P19- and QA-P4-triangle,

... QL-point V not in EQF:
... perspector of QG-P1- and QG-P19-triangle,
... QL-point W not in EQF:
... perspector of QG-P1- and QA-P41-triangle.

(5) Some properties:

... U = CSC(QL-P24) on the QG-P16-line, which is QL-Tf1(QL-Ci6).
... R and U lie diametral on the circumcircle of the
QA-P4-triangle (CSC(QL-L1)),
... with RU orthogonal to the QG-P18-line.
... W lies on the circumcircle of the QA-P41-triangle.
... For a QG the points S, QG-P19, QA-P2, QA-P4 are concyclic
... with diameter S.QG-P19.
... For a QG the points S, R, QG-P18, QA-P4 are collinear.
... For a QG the points S, QG-P19, QA-P41 are collinear.
... 8 concyclic points on the circle CSC(QG-P1b.QG-P1c):
... QL-P1, QG-P18a, QA-P41a, QA-P4b,c, two opposite QL-points
... and QG-P1b.QA-P4b ^ QG-P1c.QA-P4c (on QG-P1a.QA-P41a).

Finally:

(6) The three QA-Cu7 for a quadrilateral
... have six double intersections,
... in pairs on the QL-Tr1-sidelines as intersections
... with the CSC-circles of the circles with QL-Tr1-sides as
diameter.

(7) The three QA-Cu7 for a quadrilateral
... have five common points,
... which define a conic, bearing *Sa, Sb, Sc, R, T*
(see #2981).

Best regards Eckart

PS: What about the geometry of the 5 common points in (7)?
I found only the conic, but no relations to the properties
above.

Several tests with CB-points (proposal of Chris) without
success.

Perhaps the perspectors (4) lighten the background.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3177
Date: 26/9/2018 9:48:34
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart,

Very good summary.
Thank you!

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3178
Date: 27/9/2018 10:46:23
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformations

Dear Bernard, dear Chris,

seven points define a CB-transformation:

... the image of a point X is the Cayley-Bacharach 9th point

... 8P-s-P1

of the defining seven points and X.

What about the fixed points of such transformations

... and their geometric background?

Here some CABRI-observations, using our discussed construction:

In general these fixed points seem to give a curve of degree 5,

... attached an inexact drawing.

If we consider a CB-transformation for the six QL-points

... and a 7th point P on the cubic QL-Cu1,

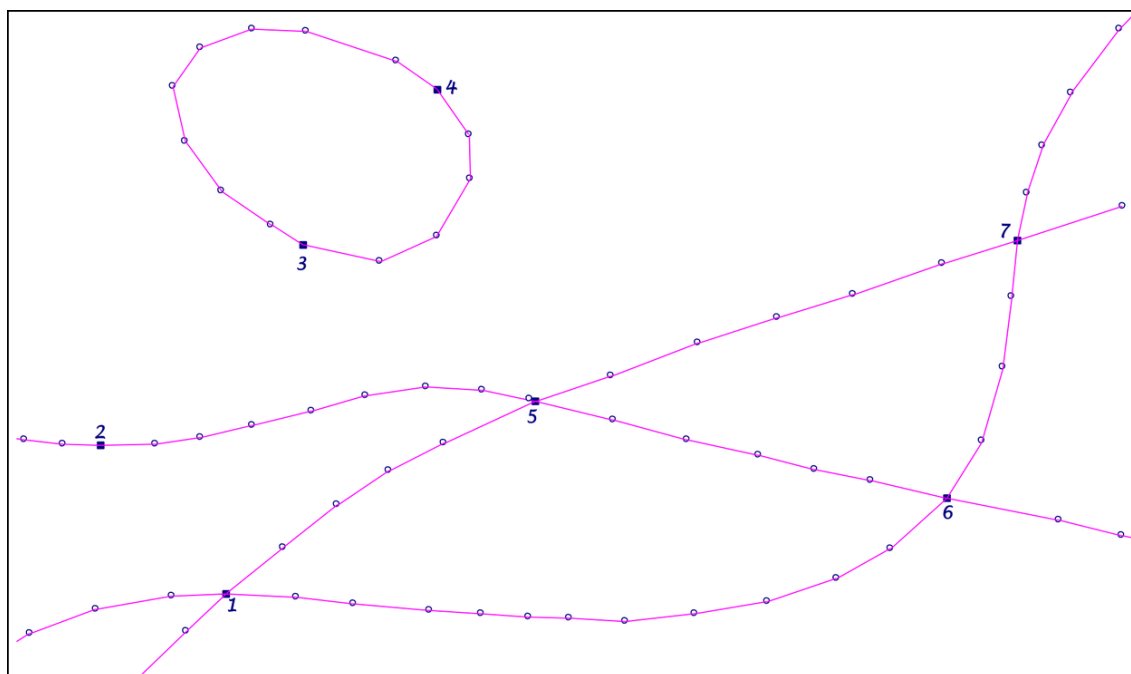
... the loci of the fixed points are two orthogonal lines
through P,

... angle bisectors at P wrt two opposite QL-points,

... which are QL-Tf2-partners (see QL-Cu1 last but two in EQF) .

... Example: Steiner axes.

Best regards Eckart



2018-09-26.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3179
Date: 28/9/2018 7:09:03
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Very beautiful and useful summary indeed !
Point (6) If I'm not wrong, the 6 double intersections are coconic
Last question : the 5 points form with the 2 circular points a system of 7 points and defines a CB transformation
these 7 points form with each couple of double points a CB system (as we have each time 2 cubics through the 9 points)
What about the fixed points of this CB transformation ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3180
Date: 28/9/2018 7:23:13
From: bernard.keizer
Subject: Fixed points of CB-transformations

Dear Eckart,
How do you define and construct the 4 fixed points ? Wrt which triangle ?
Sorry if my question is naïve, but I don't understand how you draw your quintic ?
The 2 lines through P you describe form the degenerated polar conic of CSC(P) wrt the cubic stelloïd QL-Cu2.
For the interpretation, there are an infinity of cubics through 9 points forming a CB system ; if there is for 7 points a transformation with 4 fixed points, I suppose there are an infinity of cubics through the 7 points and tangent in one fixed point to the same line ?
For a bicursal QL-Cu1, the 2 Steiner axes intersect the curve in 4 points in which the tangent is parallel to the asymptote of the curve (parallel to the Newton Line) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3181
Date: 29/9/2018 2:36:57
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

you are right wrt (6), the 6 double intersections of QA-Cu7 are coconic (see #2974).

I forgot these property, thank you.

The fact, that the 5 common QA-Cu7-points, the two circular points

... and two double QA-Cu7-intersections give a CB-system,

... is an interesting aspect,

... but I can't handle it for real constructions.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3182

Date: 30/9/2018 5:52:02

From: seiichikiri

Subject: 4 circles with a common point on a conic

Dear friends,

Let $P_1P_2P_3P_4$ be a quadrangle and QAP_4 its isogonal center.

Denote:

N_{ij} = the projection of QAP_4 on the line P_iP_j for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ such that $i < j$.

H_{ijR} = the intersection of the line $N_{ij}QAP_4$ rotated around QAP_4 by an arbitrary angle and P_iP_j for $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$ such that $i < j$.

ce_1 = the circle through N_{23R} , N_{24R} and N_{34R} .

ce_2 = the circle through N_{13R} , N_{14R} and N_{34R} .

ce_3 = the circle through N_{12R} , N_{14R} and N_{24R} .

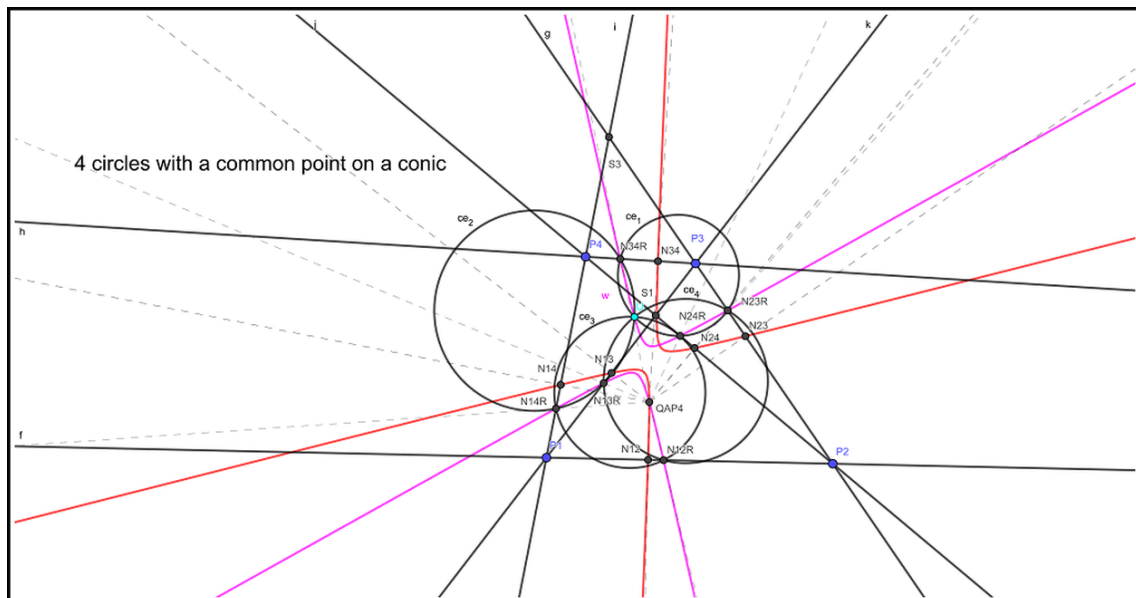
ce_4 = the circle through N_{12R} , N_{13R} and N_{23R} .

QAP_4 , N_{12R} , N_{13R} , N_{14R} , N_{23R} , N_{24R} , N_{34R} are on a conic (w).

ce_1 , ce_2 , ce_3 , ce_4 have a common point U on w

Related topics: Yahoo Group - Quadrilateral Geometry & Polygon Geometry #126 and #3167

Best regards, Seiichi.



ConicQuadrangle U.png

Message: #3183
Date: 30/9/2018 10:01:19
From: bernard.keizer
Subject: Fixed points of CB-transformations

Dear Eckart,
You didn't answer my 2nd message.
I suppose you use Bernard Gibert's construction in determining the 4 fixed points of an isoconjugation wrt a triangle knowing 2 conjugate points.
If this is the case, you take different triangles in order to get a locus of the fixed points knowing 2 conjugate points.
Thank you for confirming my understanding of your rather laconic message.
But now, in the 3 QA-Cu7 problem, knowing the 3 couples of conjugate points (the so-called double points) should allow to find the 4 fixed points of a transformation wrt any triangle (DT for example).
The fact that 2 of the 7 points are circular doesn't change the result
If the 3 couples of points are conjugate in the same transformation, defined by 7 points (5 triple points and 2 circular points), we would have an explanation ...
Perhaps the locus of the 4 fixed points could be interesting too (as your loci are through the 7 points, this time the 2 circular points will play a particular role) ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3184
Date: 30/9/2018 10:43:24
From: eckart_schmidt@t-online.de
Subject: 4 circles with a common point on a conic

Dear Seiichi,
the common point U of the circles ce_1, ce_2, ce_3, ce_4
... is the reflection of QA-P4 in the center of conic w .
The 6 double intersections of the circles lie also on the conic w ,
... in pairs reflections in the center of the conic w .
The loci of the centers of the circles are four lines ...
Best regards Eckart
PS: There will be a typo: $HijR = NijR$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3185
Date: 30/9/2018 10:54:46
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformations

Dear Bernard,

is there a misunderstanding wrt your first two questions?
There is an infinity of fixed points for a CB-transformation,
... defined by seven points.
In the attached file of #3178 I used Hardt's construction (see
#2471),
... to find by approximation some fixed points,
... to give a vague image of the locus.
Now I can give you a real construction for at most 42 fixed
points:
... Take two of the 7 points for a line and the other 5 for a
conic,
... the two intersections will be fixed points.

Am I right in the following conclusion?
Starting with seven points
... and a fixed point P of the corresponding CB-transformation,
... then all cubics through these 8 points
... have a common tangent in P.
But I have difficulties to argument with circular points,
excuse.

Best regards Eckart

PS: This is an answer to #3180, not having studied your #3183, I
just noticed.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3186
Date: 30/9/2018 6:39:39
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformations

Dear Bernard,

there is a very interesting aspect in your #3183:
You search a triangle for an isoconjugation
... with isoconjugated double QA-Cu7-intersections
... on the QL-DT-sidelines.
Unexpected the QG-P19-triangle has this property!!!
An isoconjugation must not have fixed points,
... but this isoconjugation has four fixed points
... in the perspectors of the QG-P19-triangle and the four
QL-triangles.
Thanks for this idea!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3187
Date: 02/10/2018 10:12:29
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

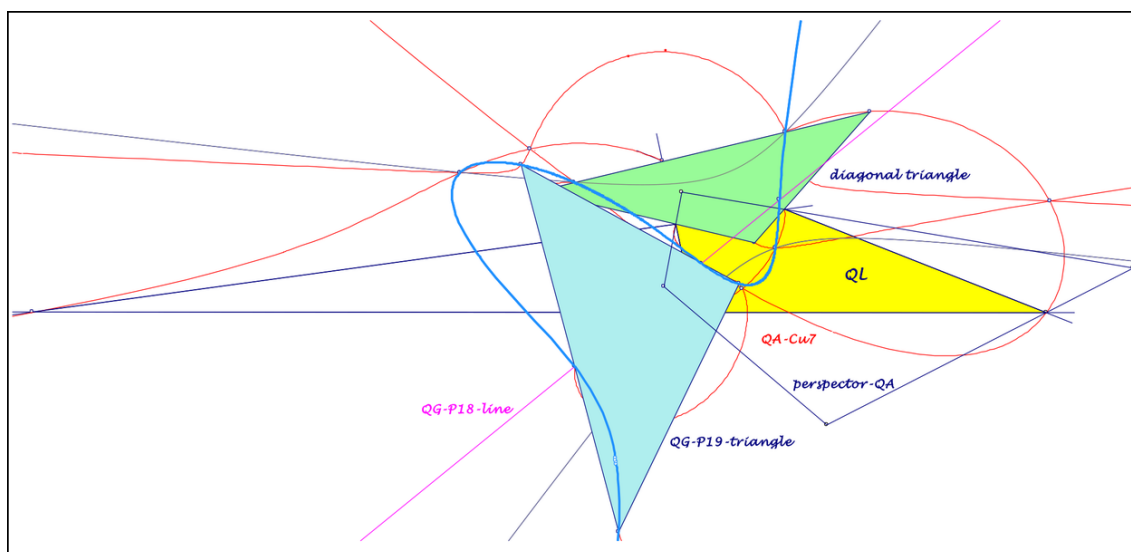
Dear Bernard,

there is an interesting QL-cubic (see attached file),
... bearing the double intersections of the three QA-Cu7:
Consider the nonpivotal QL-isocubic with
... reference triangle $Tr = QG-P19$ -triangle of QL,
... isoconjugation with fixed points
... .. in the perspectors of Tr and the QL-triangles,
... root = Tr -tripole of the QG-P18-line,
... bearing the following points:
... .. three QG-P19, three QG-P18
... .. and six double intersections of the three QA-Cu7.

Or:

... The four perspectors give a QA
... with the QG-P19-triangle as diagonal triangle
... and its QA-Tf2 as isoconjugation.
The isoconjugate of the QG-P18-line gives a conic
... contacting the cubic in the vertices of the QG-P19-triangle.
The 3rd intersections of the cubic
... and the QL- $Tr1$ -sidelines are collinear.

Best regards Eckart



2018-10-02.docx

Message: #3188
Date: 07/10/2018 10:18:00
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

wrt the QA-Cu7-property, cited:

Any line through P_i, P_j (P_i, P_j are two of the four points P_1, P_2, P_3, P_4) intersects QA-Cu7 in one of the QA-DT-vertices and a pair of points (T_1, T_2) .

Now (T_1, T_2) are harmonic conjugated with (P_i, P_j) .

Let (P_k, P_l) be the other pair of points of the Quadrangle.

Again $P_k.P_l$ intersects QA-Cu7 in one of the QA-DT-vertices and a pair of points (U_1, U_2) . Similarly (U_1, U_2) are harmonic conjugated with (P_k, P_l) .

Now $T_1.U_1 \wedge T_2.U_2$ as well as $T_1.U_2 \wedge T_2.U_1$ are points lying on QA-Cu7 (

<http://www.chrisvantienhoven.nl/qa-items/qa-cubics/qa-cu7>).

Consider a quadrangle with diagonals $P_i P_j$ and $P_k P_l$ as QA,

... then the last two points on QA-Cu7 are QG-P18 and QG-P19,

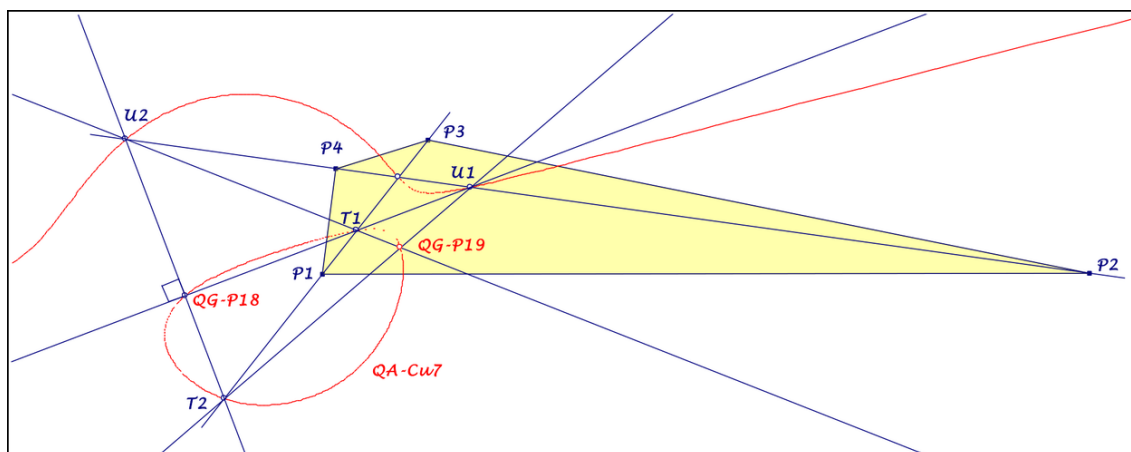
... with orthogonal intersection in QG-P18.

For a QL with its 3 QG, interpreted as QA

... there are 3 quadruples of T_1, T_2, U_1, U_2 ,

... which are the twice counted 6 double intersections of the 3 QA-Cu7.

Best regards Eckart



2018-10-06.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3189
Date: 09/10/2018 10:33:49
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Sorry if I could not answer your messages earlier, but I wasn't home last week.

You made a beautiful work with your complements and I think the puzzle is now almost complete !

1) wrt your message 3185, the CB transformation with 7 points is an isoconjugation. There are 2 ways of finding fixed points, the Hardt's method with intersection line/conic which gives your 42 points and Bernard Gibert's construction, which gives 4 points (not necessary real) for each triangle.

With 2 circular points among the 7, your 1rst method applies following : a circle through 3 of the 5 real points contains also the 2 circular points and the intersections of the circle with the line through the 2 remaining real points are fixed points ; this method gives 20 points.

2) wrt your message 3186, the fact that the CB transformation of the 5+2 points gives an isoconjugation in which the 3 copples of dobbel points are isoconjugates explains (or proofs ?) that the 3 QA-Cu7 intersect in the same 5 real points and in the 2 circular points. This gives already 4 other fixed points of the CB transformation.

3) wrt your message 3188, T1, T2, U1 and U2 form a QA with 2 orthogonal sides with DT QG-P1, QG-P18 and QG-P19 ; each QA-Cu7 is the QA-Cu1 of this 2nd QA (which is a QL-Cu1 as 2 sides are orthogonal).

T1 and U2 and T2 and U1 are csc partners in the csc centered in QA-P41 swapping QA-P2 and QA-P4.

(If I'm not wrong, QA-P2, QA-P4 and QA-P41 are the vertices of the Miquel triangle of the 2nd QA)

Last, but not least, the fact that the 6 points are coconic proves that the 3 points QG-P18 are aligned (it's Pascal's mystic hexagram !)

That's all for today ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3190
Date: 10/10/2018 8:59:32
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

This message was worked out before receiving your #3189.
Some further observations

... wrt the 6 double intersections of the 3 QA-Cu7 for a QL,
... which lie coconic in pairs on the QL-DT-sides.

(1) The double intersections of QA-Cu7
... are the intersections on the QL-DT-sides
... of angle bisectors at QG-P18 wrt opposite QG-points.
Or:

(2) The double intersections wrt QA-Cu7a,b
... are the intersections of the QL-DT-side QG-L1c,
... and a circle centered on this side through QG-P18a,b.

(3) Further intersections of these circles are the points
QA-P4a,b,c,
... their radical center is the CSC-image
... of the intersection of QL-P1.QL-P10 and QL-Ci1, unequal
QL-P16.

(4) The six angle bisectors in (1) give two new triangles,
... similar and perspective wrt QL-DT
... with perspector QL-P10,
... point on the circumcircles of the two new triangles.

(5) Circumconics of the two new triangles,
... bearing two opposite QL-points,
... have a common point in QL-P10.

(6) The CB-transformation
... wrt the vertices of the two new triangles and QL-P10
... swaps the double intersections of QA-Cu7 on the QL-DT-sides,
... with fixed points in R (see #3176), QG-P18a,b,c
... and the 2nd intersection of the circles in (4) ...

(7) This CB-transformation
... maps lines to curves of degree five
... through 3 pairs of fixed points on QL-P10.QG-P1i,
... which are intersections of
the bisectors at QG-P18j and QG-P18k.

(8) The CB-transformation wrt the six points in (7) and QL-P10

... is the same transformation as in (6),
... helpful for an easier construction for this transformation.

But further no connection with the 5 triple intersections of the
3 QA-Cu7!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3191
Date: 10/10/2018 9:01:24
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your interesting answer #3189,
... especially wrt the CB-transformation
... for the five QA-Cu7 points and two circular points.

Wrt 1) and 2):

A question wrt your statement
... " the CB transformation of 7 points gives an isoconjugation".
Is this Gibert's construction of 4 fixed points?
Can you explain or give references,
... for I don't know, how to find for 7 points
... a reference triangle and an isoconjugation,
... which gives the same image as the CB-transformation?

Thanks in advance.

Wrt 3) Your additional properties are correct.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3192
Date: 10/10/2018 11:41:35
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Thanks to you, I progress slowly, but surely !

The CB transformation for 7 points has, as you mentioned, $42 + 7 = 49$ fixed points.

The transformation swaps each line through 2 points and the conic through the 5 other points. More precisely, if X is a point on the line, the CB transformed point lies on the conic. For X being one of the 2 intersection points, X' is also in X and all the cubics through the 7 points and X are tangent to the tangent in this point X to the conic.

For X being one of the 2 points among the 7, there is a point X' on the conic and all the cubics through the 7 points and X' are tangent in the chosen point among the 2 to the line through the 2 points (for one point among the 7, there are 6 lines and conics).

Therefore, the transformation is certainly not an isoconjugation, as the reference triangle should belong to the 21 conics.

The same way, the CB transformation for 5 real points and the 2 circular points has $20 + 5$ fixed points and contains the 2 circular points.

The transformation swaps this time each line through 2 points and the circle through the 3 other points (which contains the 2 circular points).

For X being on the line, X' is on the circle.

For X being one of the 2 intersections, X' is in X and all the circular cubics through the 5 points and X are tangent in X to the circle.

For X being one of the 2 points, X' is on the circle and all the circular cubics through the 5 points and X' are tangent in X to the line (for one point chosen among the 5, there are 4 lines and conics).

Therefore, the CB transformation is not a Cl-S transformation, as there isn't a point belonging to the 10 circles.

Sorry for the false conclusion in the message 3189.

Best regards Bernard

PS But I don't agree with your conclusion that there is no link between the 6 double points and the 5 real triple points ; your preceding messages show plenty of interesting properties linking the 2 groups. The 5 points are the real intersections of the 3 QA-Cu1 of the 3 pairs of double points taken 2 by 2 ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

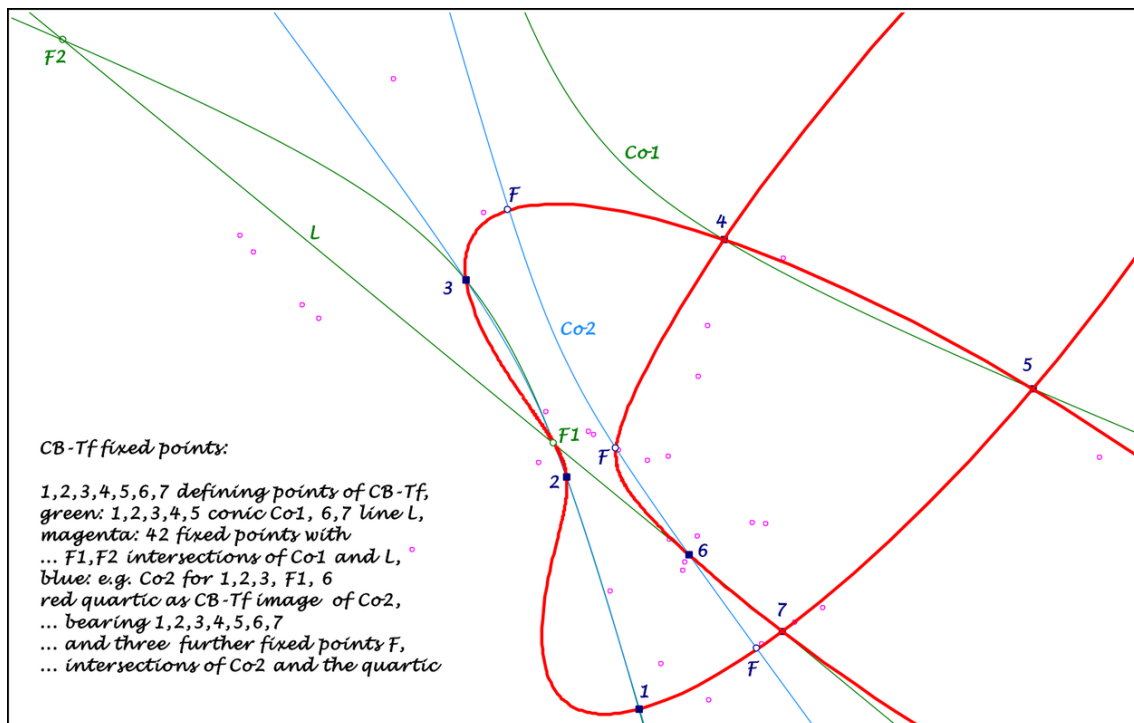
Message: #3193
Date: 11/10/2018 10:24:17
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
In fact, it is at last 49 points !
The 7 points count 6 times and there are 84 points on 21 lines.
As the curve seems to be a quintic, there is on each line a 5th
point. Any idea about it's signification ?
Geogebra allows to draw a curve as ImplicitCurve (a cubic with 9
points, a quartic with 14 points and a quintic with 20 points).
Is it the same with Cabri ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3194
Date: 13/10/2018 9:27:04
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,
thanks for your detailed remarks in #3192.
In addition to #3193:
We agree for a CB-transformation, defined by 7 points,
... in 42 fixed points (not necessary real), ...
constructed by line/conic intersections (see #3185).
We can get further fixed points as described in the attached
file.
My conclusion in #3178, based on the attached drawing,
that the locus for the fixed points will be a quintic,
doesn't hold,
for there are double points in some defining points,
whose connection line bears two further fixed points.
Best regards Eckart
PS: I have no possibility, to draw with CABRI a cubic with 9
points, a quartic with 14 points and a quintic with 20 points.



2018-10-13.docx

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3195
Date: 13/10/2018 11:39:02
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
The CB transformation is really powerful !
Taking your notations, the 6 double points are T_i and U_i for $i = 1$ to 3 .
 $T_i U_i$ and $T_j U_j$ intersect in $QG-P1k$, $T_i T_j$ and $U_i U_j$ in $QG-P19k$ and $T_i U_j$ and $T_j U_i$ orthogonally in $QG-P18k$.
If you consider the 2 triangles having for sides $T_1 U_2, T_2 U_3$ and $T_3 U_1$ and $T_2 U_1, T_3 U_2$ and $T_1 U_3$, they form 2 degenerated cubics through the 6 points and the 3 $QG-P18$, which form therefore a CB system.
As the 6 points are coconic, the 3 other points are aligned, as the conic and the line form also a degenerated cubic through the 9 points.
The Cayley-Bacharach theorem proves the Pascal theorem for 6 points on a conic.
Swapping one T_i with one U_i , you prove the same way that $QG-Pk$ is aligned with $QG-P19i$ and $QG-P19j$.
The 6 points form with the 3 groups of 3 points 3 other CB systems.
(The 12 points are cocubic as you showed in your message 3187).
But there are other possible alignments and CB systems with the 6 points with the same construction (for example swapping one T_i and one T_j gives a line through $QG-P1k$...)
Best regards
Bernard
PS It doesn't help to find a construction of the 5 points not intersecting 3 QA-Cu1, but I found it too beautiful ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3196
Date: 14/10/2018 12:13:12
From: Antreas Hatzipolakis
Subject: QA-Cu7-Geometry

[Bernard Keizer]:

> The Cayley-Bacharach theorem proves the Pascal theorem for 6 points on a conic.

Dear Bernard,

C-B Theorem is used as "a bazooka to kill a fly" in theorems with conics :)

Another example: *The three conics theorem:

If three conics pass through two given points, then the lines joining the other two intersections of each pair of the conics are concurrent.*

Let S_1, S_2, S_3 be three conics passing through two points I and J , the remaining intersections of S_2 and S_3 being P_1 and Q_1 , while those of S_3 and S_1 are P_2 and Q_2 and those of S_1 and S_2 are P_3 and Q_3 .

A less elementary proof of the theorem may be given by making use of the theorem of *nine associated points* , namely, that *any cubic curve that passes through eight of the nine intersections of two given cubic curves automatically passes through their ninth intersection.*

Thus, taking the given cubics to be the reducible curves consisting of S_2 and P_2Q_2 and S_3 and P_3Q_3 we see that the reducible cubic consisting of S_1 and P_1Q_1 passes through eight of their nine intersections (namely $I, J, P_1, Q_1, P_2, Q_2, P_3, Q_3$) and so passes through the ninth (which is where P_2Q_2 meets P_3Q_3). As this ninth point plainly does not lie on S_1 , it must lie on P_1Q_1 and we have that P_1Q_1, P_2Q_2 and P_3Q_3 are concurrent at the ninth point.

>From the book:

C. J. A. Evelyn, G. B. Money - Coutts, J. A. Tyrrell: The Seven Circles Theorem and other new theorems. London 1974, p. 15

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[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3197
Date: 14/10/2018 8:07:22
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Antreas,
Thank you for your comment !
I've appreciated the bazooka and the fly
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3198
Date: 15/10/2018 8:25:46
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Revisiting once again this question of the 3 QA-Cu7 of a QL, I found following idea.
In order to get a QL, we need always 5 points (for example vertices of a triangle and 2 points on 2 sides defining the 4th line).
I tried to find 6 points in the configuration of the 6 double points : it's possible by giving the 3 QG-P18 on a line and 2 of the 6 points T1 and T2 for example ; using all alignments and cocyclicities, we get easily U1 and U2 and finally U3 and T3. Then it's possible to reconstitute the complete figure, including the 6 vertices of the QL.
(I used also the property that 2 lines T_iU_j and T_jU_i intersect in QG-P18k, which is QA-P2 of the QA $T_iT_jU_iU_j$ and that the QA-P3 is the perspector of the triangles QA-Tr1 (DT) and QA-Tr2 (Miquel), DT being QA-P1QA-P18QA-P19 and Miquel QA-P2QA-P4QA-P41 of the QA form by 2 couples of opposite QL vertices).
The 3 QA-Cu7 are in fact 3 QA-Cu1 (and QL-Cu1) of the 3 QAs like $T_iT_jU_iU_j$ and intersect in the 5 triple points (and the 2 circular points).
Is it possible to start with these 5 triple points ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

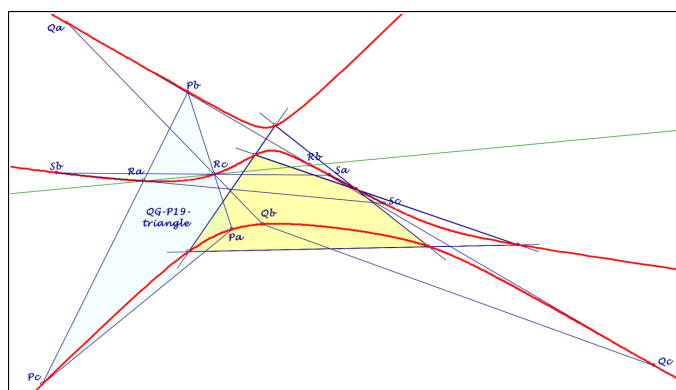
Message: #3199
Date: 16/10/2018 12:44:41
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Of course I forgot the initial condition that the 2 points T1 and T2 must be chosen on 2 perpendicular lines in the corresponding QG-P18.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3200
Date: 17/10/2018 2:16:09
From: eckart_schmidt@t-online.de
Subject: New concept for QL-cubics

Dear Bernard, dear Chris,
in general nine points define a cubic:
Let us take the six points of a quadrilateral QL
... and three vertices of a QG-Px-triangle.
Attached the cubic for the QG-P19-triangle is researched:
Let QG-P19a,b,c be shortened Pa,b,c,
... with Qi the 9th Cayley-Bacharach point
... .. of Pj, Pk and the six QL-points:
... QAQbQc is a cevian triangle of QL-DT (see also #2610),
... Qa,b,c and the 6 QL-points give a CB-system,
... the CB-point of 8 points is the remaining point.
... Let Ri be the intersection of PjQj and PkQk,
... Ra,b,c are collinear on a line L.
... Let Si be the CB-point of Pj, Rk or Rk, Pj and the 6 QL-points
... and consider for the reference triangle of QG-P19
... .. the isoconjugation Tf, which swaps Qi and Si.
The cubic,
... defined by the 6 QL-points and the vertices
... of the QG-P19-triangle
... is a nonpivotal isocubic,
... reference triangle: QG-P19-triangle,
... isoconjugation Tf, swapping Qi, Si,
... root: tripol of the line L = Ra.Rb.Rc wrt the QG-P19-triangle.
All mentioned points lie on the cubic,
... any 8 points on the cubic have their CB-point on the cubic.
There will be generalizations, for example for QG-P13,
... but several QG-Px-triangles are degenerated lines.
Best regards Eckart
PS: This is a first observation,
... please have a look at the next message.



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Message: #3201
Date: 17/10/2018 9:17:37
From: eckart_schmidt@t-online.de
Subject: New concept for QL-cubics

Dear Bernard, dear Chris,

in general nine points define a cubic:
Let us take the six points of a quadrilateral QL
... and three vertices of a QG-Px-triangle.
In message #3200 there is a construction
... for the QL-circumcubic through the vertices of the
QG-P19-triangle.
In the same way the cubic for the QG-P13-triangle can be
constructed.
F or QG-P10, QG-P12, QG-P15, QG-P16, QG-P18
... the triangles degenerate collinear.
If we know for a QL-circumcubic three collinear points,
... the cubic can be constructed as follows:
Let PaPbPc be three collinear points on a line L,
... with Qi the 9th Cayley-Bacharach point
... of Pj,Pk and the six QL-points.
The QL-circumcubic through collinear Pa,Pb,Pc
... is a nonpivotal isocubic,
... reference triangle: *QaQbQc,*
... isoconjugation, swapping opposite QL-points,
... root: QaQbQc-tripol of the line L = Pa.Pb.Pc.
If QaQbQc is a QL-DT-cevian triangle,
... have also a look at #2610.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3202
Date: 19/10/2018 9:50:38
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

perhaps a relevant observation
... wrt the 5 common points of the QA-Cu7 for a QL:
The locus for the fixed points of a CB-transformation
... for the 6 QL-points and an arbitrary point P
... are two lines through P, *not always real* ,
... which are QL-Tf2-partner.

CABRI-observations give the conjecture,
... that only two of the 5 QL-Cu7-points
... give real lines for the fixed points above.

Best regards Eckart

PS: I very much apologize for not yet have answered your last two messages.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3203
Date: 19/10/2018 6:50:30
From: bernard.keizer
Subject: New concept for QL-cubics

Dear Eckart,

I'm admiring your ability of finding always new interesting constructions !

Only 2 reactions at this stage :

1) The 6 vertices of 2 cevian triangles of a triangle are always coconic.

As each reference triangle of the QL is the cevian triangle of a vertex of the dual QA, you may form 4 degenerated cubics through the 6 vertices of the QL and the vertices of any cevian triangle (each conic associated to the line through the 3 other QL vertices) and the 6 QL vertices and the 3 points Qa,b and c form a CB system.

2) On your figure, it seems that the points Qa, Sb and Pc, Qb, Sc and Pa and Qc, Sa and Pb are aligned ?

Best regards

Bernard

PS The converse of 1) is not true : for example, the S-points are coconic with the vertices of the 4 reference triangles, but the S-triangle is not a cevian triangle of DT.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3204
Date: 19/10/2018 9:01:16
From: eckart_schmidt@t-online.de
Subject: New concept for QL-cubics

Dear Bernard,

thanks for your #3203,

... you are right with your observation 2).

The construction of the points Si in #3200

... can be done simple as intersection $PkQl \wedge PlQk$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3205
Date: 20/10/2018 10:48:13
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I try in vain to follow this construction of the fixed points (Hart's construction ?)
You said once that for 7 points the intersections between lines through 2 points and conics through the 5 other points were among the fixed points (42 points on 21 lines) ; the 7 points were also fixed points (each counted 6 times).
For the 6 QL vertices and a point P, if I'm not wrong, 6 lines are through P and each of the 6 vertices and the corresponding conics through the 5 other QL vertices are degenerated in 2 lines. The intersections between the line through P and the 2 lines of the conic are not on 2 perpendicular lines ?
For example for QL-P1, it should be the Steiner axes ? But I cannot reproduce this property ...
Where is my mistake ?
Thanks in advance for your patience !
Best regards
Bernard
PS Could you please give me the exact reference of Hart's article

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3206
Date: 21/10/2018 10:44:04
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt your message 3205:

You find an exact reference for Hart's article in QFG-message 2447.

Hart's construction of the 9th Cayley-Bacharach point is very complex,

... the description contains a mistake, see also #2447.

But wrt the six points of a QL and two points P, Q

... you find the CB-point as further intersection

... of two line/conic cubics.

For a CB-transformation wrt the 6 QL-points and a point P

... you find the image of a point X in the 4th intersection

... of conics through P, X and the vertices of a QL-triangle.

The locus for fixed points of such a CB-transformation

... are two lines through P,

... which are QL-Tf2-partner.

For a fixed point X of such a CB-transformation

... the conics through P, X and the vertices of a QL-triangle

... have a common tangent in X.

Best regards Eckart

PS. You are right: The construction of special fixed points

... as described in #3185 fails for this QL-constellation.

... Sorry, I don't know reason!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3207
Date: 22/10/2018 4:23:19
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

two further observations wrt common points of the 3 QA-Cu7 for a QL:

- (1) Two pairs of double points on QG-P1a.b and QA-P1a.c
... give a quadrangle, whose QA-Cu1 is QA-Cu7a.
- (2) The CB-transformation wrt the 5 triple intersections of the QA-Cu7
... and two double intersections of QA-Cu7a and QA-Cu7b
... maps QA-Cu7a as well as QA-Cu7b to itself.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3208
Date: 23/10/2018 11:33:48
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse, there is a typo in #3207:

(1) Two pairs of double points on QG-P1a.b and QG-P1a.c
... give a quadrangle, whose QA-Cu1 is QA-Cu7a.

In addition to this observation:

... QA-Cu7 can be described as QA-Cu1 as well as QL-Cu1.

If we consider a quadrilateral,

... the three pairs of QA-Cu7 double intersections X_i , Y_i
... on the QL-DT sidelines can be constructed (#3176, (6)).

Now let us start with a quadrigon,

... which can be used as quadrangle as well as a quadrilateral,

... with X_a , Y_a on QG-P1.QG-2P3a and X_b , Y_b on QG-P1.QG-2P3b.

... QA-Cu7 is QA-Cu1 for the quadrangle of X_a , Y_a , X_b , Y_b ,

... with pairs of vertices on orthogonal lines

... and the Miquel triangle QA-P2. QA-P4.QA-P41.

As described in #1423, last passage,

... QA-Cu1 for a quadrangle QA with pairs of vertices on
orthogonal lines

... can be considered as QL-Cu1:

... QA-Cu7 is QL-Cu1 for the quadrilateral

... of the QA-lines except the orthogonal lines.

Finally:

For points on QA-Cu7 the isogonal conjugate wrt QA-P2.

QA-P4.QA-P41

... is the CB-transformed wrt the 5 QA-Cu7 triple intersections
and X_a , Y_a or X_b , Y_b .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3209
Date: 23/10/2018 11:57:29
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Thanks for your explanations ! I think I've finally understood your construction.

In fact, it's the bazooka theorem of conics and CB.

If 3 points P, Q, R are coconic with the 3 vertices of each reference triangle of the QL, we may form 4 degenerated cubics with the conic and the 4th line of the QL. (for example any cevian triangle of DT or the S-triangle ...). Therefore, the QL vertices form with P, Q and R a CB system.

Any point P defines with the 6 QL vertices a CB transformation which swaps lines through 2 points and conics through the 5 other points.

There are 4 lines of the QL (each counted 3 times), 3 diagonals and 6 lines joining P to a vertice (21 lines).

In the 1st case, P is the fixed point.

In the 2nd case, the intersections of a diagonal and the conic through P and the 4 other points gives 2 points harmonically conjugates wrt the 2 vertices on this diagonal ; they are 2 fixed points. The same goes for the 2 other diagonals and this gives an easy construction of 2 QL-Tf2 lines through a point P. It happens that every point on these 2 QL-Tf2 lines through P is also a fixed point, I don't see why, but it works.

In the 3rd case, we can't accept a priori the intersections between the line through P and a vertice with the conic formed by the 2 lines of the QL through the 5 other vertices, as there would be 4 points for a cubic on these 2 last lines. But I'm finally not so sure, as the triangle having these 3 lines as sides is a degenerated cubic through the 7 points and the fixed point. What do you think of this interpretation ?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3210
Date: 24/10/2018 9:20:23
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your remarks in #3209, which are correct.
The line/conic construction for fixed points
... of a CB-transformation, defined by seven points,
... fails, if the conic degenerates into two lines.
Wrt the degenerated cubic in the last passage:
The CB-transformation, defined by the 6 QL-points and a point P,
... maps a line through P and a QL-vertex to the opposite vertex,
... maps the QL-lines through the opposite vertex to itself
... and the conic through the remaining QL-vertices to a
QL-diagonal.

But what do you mean with "... 7 points and the fixed point"
... in the next to last sentence?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3211
Date: 24/10/2018 6:03:01
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
It was only a questioning !
The line through P and a vertice and the 2 QL lines through the
opposite vertice form a triangle, which is a degenerated cubic ;
the intersections between the 1rst line and the 2 others forming
a degenerated conic correspond to the definition of fixed points
and are on the triangle ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3212
Date: 24/10/2018 6:10:30
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Wrt 3207 and 3208, I don't understand your last formulation.
Of course, the CB 9th point of 8 points on a cubic is by definition on this cubic.
But I don't find the connexion with the isogonality !
For example, with your notations, the 5 triple points and X_a and Y_a define a CB transformation in which the circular points are CB partners as well as X_b and Y_b ; but X_b and Y_b are not isogonals wrt QA-P2,P4 and P41.
The CB transformation would swap the line X_aY_a and the conic through the 5 points whereas the isogonal transformation swaps the line X_aY_a and a conic through QA-P2,P4 and P41 and through the isogonals of X_a and Y_a (which are of course on QA-Cu7).
It's true that the QG-P1, intersection of X_aY_a and X_bY_b , has it's isogonal on the conic of the 5 points (your points S).
Where is my misunderstanding or my mistake ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3213
Date: 25/10/2018 2:29:04
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse my inexact formulation, more precisely:
For points X on QA-Cu7c the isogonal conjugate wrt QA-P2c.
QA-P4c.QA-P41c
... is the CB-transformed of X wrt the 5 QA-Cu7 triple
intersections and X_a, Y_a or X_b, Y_b .
The points X_a, Y_a or X_b, Y_b lie not on the cubic QA-Cu7,
... so I cannot follow your argumentation.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3214
Date: 25/10/2018 5:00:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Sorry, but I'm completely lost !
If X_a and Y_a, X_b and Y_b and X_c and Y_c are the 3 double
intersections and the QA-Cu7c is the QA-Cu1 of the QA formed by
 X_a, Y_a, X_b and Y_b , then X_a, Y_a, X_b and Y_b lie on QA-Cu7c !
 X_a and Y_a are the double intersections of QA-Cu7b and QA-Cu7c
and X_b and Y_b are the double intersections of QA-Cu7a and
QA-Cu7c.
What do I miss ?

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3215
Date: 26/10/2018 8:55:27
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse my overhasty evident false remark,
... you are right: X_a, Y_a, X_b, Y_b lie on QA-Cu7c.
But the property at the end of #3208, revised in #3213, will hold,
... it is only valid for points on QA-Cu7c,
... not for any point on $X_a Y_a$, as you describe.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3216
Date: 26/10/2018 1:54:19
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
My apologise in return !
In fact, your beautiful property holds for the 3 cubics QA-Cu7a,b and c.
The 5 triple points and 2 double points X_a and Y_a define a CB transformation $CB(X)$; for any point X on QA-Cu7, $CB(X) = \text{isog}(X)$ wrt QA-P2, 4 and 41.
For X in X_a or Y_a , the set of cubics have the same tangent in this point.
But $CB(X_a) = \text{isog}(X_a)$ is not X_a and $CB(Y_a) = \text{isog}(Y_a)$ is not Y_a ! (Hence my mystake)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3217
Date: 26/10/2018 3:42:58
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

a new sight of QA-Cu7, using the CB-transformation TF
... defined by the 6 double intersections and a point Q on QA-Cu7.
For a QL there is only one of the 3 QA-Cu7 with the property,
... that TF maps QA-Cu7 to itself (see PS).
Let us consider this QA-Cu7,
... the corresponding QG-version of the QL and a point Q on
QA-Cu7:
For points X on QA-Cu7 the lines X.TF(X)
... have a common point P on QA-Cu7.

This point P can be interpreted as pivot for QA-Cu7 as pivotal
isocubic:

... reference triangle ABC with A = Q,
... .. B 3rd intersection of QA-Cu7 and Y.QA-P2
... .. with Y = Q.QA-P4^P.QA-P41 on QA-Cu7,
... .. C 3rd intersection of QA-Cu7 and Z.QA-P2
... .. with Z = Q.QA-P41^P.QA-P4 on QA-Cu7,
... isoconjugation swapping 2 points on QA-Cu7 collinear with P,
... .. as QA-P41 and Y or QA-P4 and Z.
Perhaps helpful for the 5 triple intersections of QA-Cu7,
... taking for Q the point S (see #3176).

Best regards Eckart

PS. This is a curious property, which of the 3 QA-Cu7 is it?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3218
Date: 27/10/2018 10:19:30
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

in addition to #3217:

Let us consider a QL

... with its 3 QA-Cu7 and their 5 triple and 6 double intersections.

Let TF be a CB-transformation,

... defined by the 6 double intersections and a point Q,

... which gives for the triple intersections 5 image points.

For any point Q on QA-Cu7 there is only a special QA-Cu7

... which bears with Q the 5 image points of the triple intersections.

If Q is one of the triple intersections,

... the image points of the other triple intersections

... lie on this special QA-Cu7.

There is a further point Q with the property,

... that the 5 image points lie on this special QA-Cu7,

... this is the point S (see #3176) coconic with the triple intersections.

The question remains:

Which of the three QA-Cu7 is the special QA-Cu7?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3219
Date: 28/10/2018 8:24:12
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach points on cubics

Dear Bernard, dear Chris,
consider a cubic CU and
... a CB-transformation TF defined by 7 CU-points, which maps CU
to itself:
Lines $X.TF(X)$ have a common point Y on CU,
... and the image $Z = TF(Y)$ is the tangential of Y.
Is this well known or evident?
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3220
Date: 29/10/2018 11:12:06
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
It's neither well-known nor evident (at last for me), but it's
beautiful !
I suppose it's linked with cotterill's constructions (focus of 4
points on a cubic ...)
As an immediate application, as the CB transformation swaps the
line through 2 points with the conic through the 5 other, the
3rd intersection with the line and the 6th intersection with the
conic are CB partners ; this gives 21 lines through the same
point !
For the 3 QA-Cu7, this point is the infinity point of the cubic
(on the perpendicular bisector of QA-P2 and QA-P4 and it's
tangential is the corresponding point S or isog(QG-P1) ; the CB
transformation of the 5 triple points and the 2 circular points
swaps the conic through the 5 points and the infinity line
through the 2 circular points.
The transformation swaps also lines through 2 of the 5 points
and circles through the 3 others (and the 2 circular points) ;
all this opens a lot of possibilities ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3221
Date: 29/10/2018 11:14:23
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

if I am not wrong, I have to cancel my messages #3217 and #3218,
... for my construction for the 9th CB-point
... doesn't hold, if 6 of the 8 points lie on a conic,
... but the 6 double points are coconic!

I apologize (<https://dict.leo.org/englisch-deutsch/apologize>)
profusely (<https://dict.leo.org/englisch-deutsch/profusely>) my
error!

It seems, that none of the three QA-Cu7
... is invariant wrt a CB-transformation TF,
... defined by the 6 double points and a point Q on QA-Cu7.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3222
Date: 29/10/2018 7:16:04
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
I'm very excited by your last discovery !
In general for 7 points P_1 to P_7 , the CB transformation swaps X and $CB(X)$.
If (P_1, P_2) is the 3rd intersection of the line P_1P_2 with the cubic, the focus F of P_1, P_2, P_3 and P_4 is given by $[(P_1, P_2), (P_3, P_4)]$ and the same way the focus F' of P_5, P_6, P_7 and X is $[(P_5, P_6), (P_7, X)]$.
Then $CB(X) = (F, F')$ and $[X, CB(X)]$ is a fixed point on the cubic depending on the 7 points.
Is it true for any cubic ? It looks like your Kalkulation auf der Zirkularkurve ...
If the cubic is a pivotal isocubic, the CB transformation is an isoconjugation wrt a certain triangle with fixed point in the pivot.
For the 5 triple points and one couple of double points on it, it's the isogonal conjugation wrt P_2, P_4, P_4 with pivot the infinity point on the perpendicular bisector of P_2P_4 .
For the 5 triple points and the 2 circular points, as the 2 couples of double points are CB partners, the corresponding QG-P1 point is the pivot. It's easy to check that the circle through 3 triple points (and the circular points) cuts the cubic in a 4th point and the line through the 2 other triple points in a 3rd point ; as the CB transformation swaps the circle and the line, these 2 points are CB partners and the lines through the 2 points pass through the point QG-P1 (there are 10 such lines).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3223
Date: 30/10/2018 8:55:13
From: eckart_schmidt@t-online.de
Subject: Cayley-Bacharach points on cubics

Dear Bernard,

very interesting message 3222, the last passage was new for me!
Is my following interpretation correct?
Consider QA-Cu7 for a quadrigon and a CB-transformation,
... defined by the 5 triple points and the two circular points,
... which maps QA-Cu7 to itself,
... with CB-partner collinear with QG-P1 (see your message).
For points on QA-Cu7
... this CB-transformation is the isoconjugation QA-Tf2 for a QA
... with vertices with common tangential QG-P1.
There will be generalizations.

Best regards Eckart

PS. Your statement wrt "... fixed point in the pivot..."
in the middle of your message cannot be correct.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3224
Date: 31/10/2018 4:47:26
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
I have the feeling that we are rediscovering old well-known properties !
But anyhow, I take pleasure in it.
For 7 points on a cubic, each conic through 5 points and line through the 2 other intersect the cubic in 2 points, one on the conic and one on the line which are necessary CB partners (as we have 2 cubics, one normal, the other degenerated, through the 7 initial points and the 2 new ones)
The 3rd intersection between the line through the 2 last CB partners is a fixed point on the cubic.
We have already 21 couples of CB partners through this fixed point (for symmetry reasons, it has to be the same for all the couples conic/line).
The 4 points on the cubic, which have the fixed points as tangential form a QA ; the QA-Tf2 of this QA wrt it's DT is the CB transformation on the cubic.
I think therefore that the cubic is a pivotal isocubic wrt DT with fixed points in the 4 QA vertices with pivot the fixed points. (Of course, the points are not necessary real ...)
If the cubic is circular, the fixed point and pivot is QA-P4 of the QA.
Do you agree with this ?
Best regards
Bernard
PS I agree with your interpretation for QA-Cu7 and the QG-P1 must be the QA-P4 of it's tangential QA

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3225
Date: 01/11/2018 9:53:35
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Dear geometers,

Consider quadrilateral (d_1, d_2, d_3, d_4) .
Intersection of pair lines (d_1, d_2) , (d_1, d_3) , (d_1, d_4) , (d_2, d_3) ,
 (d_2, d_4) and (d_3, d_4) are A_{12} , A_{13} , A_{14} , A_{23} , A_{24} and A_{34} .
We consider orthocorrespondent of A_{12} wrt triangles $A_{13}A_{24}A_{34}$
and $A_{23}A_{34}A_{14}$ be two lines $d(13, 24)$ and $d(23, 14)$.
Two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ pass through A_{12} and are
perpendicular to lines $d(13, 24)$ and $d(23, 14)$ respectively.
Thus,
There are two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ through A_{12}
There are two lines $k(23)(34, 12)$ and $k(23)(13, 24)$ through A_{23}
There are two lines $k(34)(23, 14)$ and $k(34)(24, 13)$ through A_{34}
There are two lines $k(13)(12, 34)$ and $k(13)(23, 14)$ through A_{13}
There are two lines $k(24)(23, 14)$ and $k(24)(34, 12)$ through A_{24}
There are two lines $k(14)(12, 34)$ and $k(14)(24, 13)$ through A_{14}
We see that
the lines $k(12)(13, 24)$, $k(23)(13, 24)$, $k(34)(24, 13)$, $k(14)(24, 13)$
are concurrent at X .
the lines $k(12)(23, 14)$, $k(34)(23, 14)$, $k(13)(23, 14)$,
 $k(24)(23, 14)$ are concurrent at Y .
the lines $k(23)(34, 12)$, $k(13)(12, 34)$, $k(24)(34, 12)$, $k(14)(12, 34)$
are concurrent at Z .
Then X , Y , Z are collinear on the line which is perpendicular to
Newton line of (d_1, d_2, d_3, d_4) .
Which is the line (X, Y, Z) ?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3226
Date: 01/11/2018 5:12:51
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Dear geometers, sorry for wrong term, I mean Orthotransversal

Consider quadrilateral (d_1, d_2, d_3, d_4) .
Intersection of pair lines (d_1, d_2) , (d_1, d_3) , (d_1, d_4) , (d_2, d_3) ,
 (d_2, d_4) and (d_3, d_4) are A_{12} , A_{13} , A_{14} , A_{23} , A_{24} and A_{34} .
We consider Orthotransversal of A_{12} wrt triangles $A_{13}A_{24}A_{34}$ and
 $A_{23}A_{34}A_{14}$ be two lines $d(13, 24)$ and $d(23, 14)$.
Two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ pass through A_{12} and are
perpendicular to lines $d(13, 24)$ and $d(23, 14)$ respectively.
Thus,
There are two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ through A_{12}
There are two lines $k(23)(34, 12)$ and $k(23)(13, 24)$ through A_{23}
There are two lines $k(34)(23, 14)$ and $k(34)(24, 13)$ through A_{34}
There are two lines $k(13)(12, 34)$ and $k(13)(23, 14)$ through A_{13}
There are two lines $k(24)(23, 14)$ and $k(24)(34, 12)$ through A_{24}
There are two lines $k(14)(12, 34)$ and $k(14)(24, 13)$ through A_{14}
We see that
the lines $k(12)(13, 24)$, $k(23)(13, 24)$, $k(34)(24, 13)$, $k(14)(24, 13)$
are concurrent at X .
the lines $k(12)(23, 14)$, $k(34)(23, 14)$, $k(13)(23, 14)$,
 $k(24)(23, 14)$ are concurrent at Y .
the lines $k(23)(34, 12)$, $k(13)(12, 34)$, $k(24)(34, 12)$, $k(14)(12, 34)$
are concurrent at Z .
Then X , Y , Z are collinear on the line which is perpendicular to
Newton line of (d_1, d_2, d_3, d_4) .
Which is the line (X, Y, Z) ?

Best regards,
Tran Quang Hung.

Vào Th 5, 1 thg 11, 2018 vào lúc 15:53 Tran Quang Hung <
analgeomatica@gmail.com > đã viết:
>> Dear geometers,
>> Consider quadrilateral (d_1, d_2, d_3, d_4) .
>> Intersection of pair lines (d_1, d_2) , (d_1, d_3) , (d_1, d_4) ,
 (d_2, d_3) , (d_2, d_4) and
>> (d_3, d_4) are A_{12} , A_{13} , A_{14} , A_{23} , A_{24} and A_{34} .
>> We consider orthocorrespondent of A_{12} wrt triangles $A_{13}A_{24}A_{34}$
and
>> $A_{23}A_{34}A_{14}$ be two lines $d(13, 24)$ and $d(23, 14)$.
>> Two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ pass through A_{12} and
are
>> perpendicular to lines $d(13, 24)$ and $d(23, 14)$ respectively.

>> Thus,
>> There are two lines $k(12)(13,24)$ and $k(12)(23,14)$ through A12
>> There are two lines $k(23)(34,12)$ and $k(23)(13,24)$ through A23
>> There are two lines $k(34)(23,14)$ and $k(34)(24,13)$ through A34
>> There are two lines $k(13)(12,34)$ and $k(13)(23,14)$ through A13
>> There are two lines $k(24)(23,14)$ and $k(24)(34,12)$ through A24
>> There are two lines $k(14)(12,34)$ and $k(14)(24,13)$ through A14
>> We see that
>> the lines $k(12)(13,24)$, $k(23)(13,24)$, $k(34)(24,13)$,
 $k(14)(24,13)$ are
>> concurrent at X.
>> the lines $k(12)(23,14)$, $k(34)(23,14)$, $k(13)(23,14)$,
 $k(24)(23,14)$ are
>> concurrent at Y.
>> the lines $k(23)(34,12)$, $k(13)(12,34)$, $k(24)(34,12)$,
 $k(14)(12,34)$ are
>> concurrent at Z.
>> Then X, Y, Z are collinear on the line which is perpendicular
to Newton
>> line of $(d1,d2,d3,d4)$.
>> Which is the line (X,Y,Z) ?
>> Best regards,
>> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3227
Date: 01/11/2018 5:15:40
From: Tran Quang Hung
Subject: Concurrent Orthotransversal

Dear geometers,

Let $(A1,A2,A3,A4)$ be Quadrangle.
Euler-Poncelet point of this Quadrangle is E.
Then Orthotransversal of E wrt triangles $A1A2A3$, $A2A3A4$, $A3A4A1$
and, $A4A1A2$ are concurrent.
Which is this concurrent point?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3228
Date: 01/11/2018 8:27:01
From: eckart_schmidt@t-online.de
Subject: Concurrent Orthotransversal

Dear Tran Quang Hung,

wrt #3227: The concurrent point is QA-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3229
Date: 02/11/2018 1:40:04
From: Tran Quang Hung
Subject: Concurrent Orthotransversal

Thank you very much Mr Eckart.

Best regards,
Tran Quang Hung.

Vào Th 6, 2 thg 11, 2018 vào lúc 04:28 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

>> Dear Tran Quang Hung,
>> wrt #3227: The concurrent point is QA-P4.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3230
Date: 02/11/2018 1:45:47
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Dear Mr Eckart and geometers,

I guess this problem is true but I can't check because it is complicate. Can you please help me to check it? Is it true?

Best regards,
Tran Quang Hung.

Vào Th 5, 1 thg 11, 2018 vào lúc 23:12 Tran Quang Hung < analgeomatrica@gmail.com > đã viết:
>> Dear geometers, sorry for wrong term, I mean Orthotransversal
>> Consider quadrilateral (d_1, d_2, d_3, d_4) .
>> Intersection of pair lines (d_1, d_2) , (d_1, d_3) , (d_1, d_4) ,
 (d_2, d_3) , (d_2, d_4) and
>> (d_3, d_4) are A_{12} , A_{13} , A_{14} , A_{23} , A_{24} and A_{34} .
>>

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3231
Date: 02/11/2018 8:47:15
From: eckart_schmidt@t-online.de
Subject: Lines is perpendicular to Newton line

Dear Tran Quang Hung,

I tried in vain, to verify the concurrent points X, Y, Z with CABRI.

Is my understanding correct?

$k(12)(13,24)$ is the line through A_{12} ,

... orthogonal to the orthotransversal of A_{12} wrt $A_{34}A_{13}A_{24}$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3232
Date: 02/11/2018 10:46:31
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
I'm waiting desperately for your confirmation !
I've found another perhaps obvious property.
One of the 3 QA-Cu7 carries among other points one QG-P1, one QG-P18, 2 couples of double points, 2 vertices of the QL and the points QA-P2, P4 and P41 of the QA formed by the 4 remaining QL vertices.
The cubic is either QA-Cu7 of the QA of the 4 QL vertices or the QA-Cu1 of the QA of the 4 double points.
The last intersection between the cubic and the opposite DT side to QG-P1 (through the 2 QL vertices) is the CSC of QG-P18 in the csc with center QA-P41 swapping QA-P2 and QA-P4.
This point is the QA-P4 of the QA of the 4 double points as QG-P18 is the QA-P2.
QG-P1 is QA-P4 of it's tangential QA (as the cubic is circular).
The QA-P4 of the 2 QA's (4 double points and tangential QA of QG-P1), id est $\text{csc}(\text{QG-P18})$ and QG-P1 have the same tangential on the cubic, which is their commun QA-P41.
Best regards
Bernard
PS After all these considerations, I don't think there is a possible construction of 7 points on a cubic knowing a CB transformation on this cubic ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3233
Date: 02/11/2018 11:44:39
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Dear Mr Eckart,

You are right, $k(12)(13,24)$ is the lines passing through A_{12} and is perpendicular to orthotransversal of A_{12} wrt triangle $A_3A_4A_{13}A_{24}$.

Thank you very much
Best regards
Tran Quang Hung.

Vào Th 6, 2 thg 11, 2018 lúc 16:08 ' eckart_schmidt@t-online.de
' eckart_schmidt@t-online.de [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Tran Quang Hung,
>> I tried in vain, to verify the concurrent points X, Y, Z with
CABRI.
>> Is my understanding correct?
>> $k(12)(13,24)$ is the line through A_{12} ,
>> ... orthogonal to the orthotransversal of A_{12} wrt $A_3A_4A_{13}A_{24}$.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3234
Date: 02/11/2018 3:09:03
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
In fact, I suppose it was obvious.
The QA-P41 described is also the tangential of QG-P18 and of QG-P19 (which is CSC of QG-P1)
The tangential QA of this point is formed by QG-P1 and QG-P19 and QG-P18 and it's CSC (perhaps a well-known point ?) and the 2 sides QG-P1QG-P19 and QG-P18CSC(QG-P18) intersect orthogonally in the CSC of the point (respectively QA-P2 and QA-P4 of this last QA) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3235
Date: 02/11/2018 5:37:43
From: bernard.keizer
Subject: Cayley-Bacharach points on cubics

Dear Eckart,
Indeed, it was obvious !
The QA of the 2 copples of double points has for DT QG-P1QG-P19QG-P18 and for QA-P4 the CSC of QG-P18 ; then the QA-P41 is the isopivot and the tangential of DT vertices as well as of the pivot QA-P4.
I'm I correct by assuming that the DT of the tangential QA of QG-P1 is QG-P19QG-P18CSC(QG-P18) ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3236
Date: 03/11/2018 10:18:30
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

it took some time, to test all the observations in your last 4 messages.

Now there is a chaos of single properties in my mind,
... I try some remarks:

(1) Wrt the messages 3224 and 3232:

I think, the results are correct.

Background for several properties in #3232 will be,

... that a tangential QA wrt QA-Cu7

... has one QG-version with orthogonal diagonals,

... intersecting on QA-Cu7.

(2) Wrt message 3234 and 3235:

The QA of the 4 double points has DT QA-P1,18,19,

... .. but QG-P19 = QG-Tf2(QG-P1) not CSC(QG-P1),

... its QA-P4 is the 3rd intersection X

of QA-Cu7 and QG-L1,

... .. but not CSC(QG-P18) = QG-P17;

... its QA-P41 is the tangential of QG-P1,18,19 and X.

The tangential QA of QG-P1 has as DT: QG-P18,19 and X.

(3) Finally really new observations wrt the 5 triple QA-Cu7-points:

Consider a line L, connecting any two of the 5 points,

... and the circumcircle C_i of the remaining three.

The circle CSC(L) intersects C_i orthogonal.

The circle CSC(C_i) is centered on L

... as inversion circle of the two points on L.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3237
Date: 04/11/2018 1:39:18
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Thanks for your message !

1) Wrt (1) At last something on which we agree
2) Wrt (2) First I was desperate, then I tried to understand I'm lost with your barbarian notations mixing permanently QL, which I understand the best, QA, which I begin to understand better and QG, with which I slowly get accustomed ...

We deal with the 3 QA-Cu7 for 2 copples of opposite vertices of QL, which are also QA-Cu1 for 2 copples of double points and QL-Cu1 for the QL formed by these 2 copples and QG-P1 and QG-P19.

Therefore, we deal with 4 different CSC transformations, one I name CSC for the original QL, centered in QL-P1 and swapping the opposite QL vertices and 3, one on each QA-Cu7 I name csc, centered in QA-P41 and swapping QA-P2 and QA-P4.

Then, of course, the QA-P4 are the CSC of the QG-P1 (vertices of DT) and the QG-P18 are the CSC of the QG-P17 (vertices of the orthic triangle of DT).

But on each QA-Cu7, 2 copples of double points are 2 copples of csc partners, the 2 last vertices of the inscribed QL are QG-P1 and QG-P19, which are csc partners and the QA-P4 of the QA formed by these 4 double points is the csc of QA-P2 (which is the last vertice of the DT of this QA)

3) Wrt (3) Which CSC do you consider (CSC or csc) ?

Your 2 properties are the same, as CSC conserves the orthogonality : a circle and a diameter have as CSC 2 orthogonal circles.

Interesting new property, I don't see an explanation ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3239
Date: 04/11/2018 4:06:25
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Dear Mr Eckart and geometers,

I am very sorry about my mistake. I hope I can repost this problem.

Consider quadrilateral (d_1, d_2, d_3, d_4) .

Intersection of pair lines (d_1, d_2) , (d_1, d_3) , (d_1, d_4) , (d_2, d_3) , (d_2, d_4) and (d_3, d_4) are A_{12} , A_{13} , A_{14} , A_{23} , A_{24} and A_{34} respectively.

We consider orthotransversal of A_{12} wrt triangles $A_{13}A_{24}A_{34}$ and $A_{23}A_{34}A_{14}$ be two lines $d(13, 24)$ and $d(23, 14)$ respectively.

$B(13, 24)$ and $B(23, 14)$ are orthopoles of two lines $d(13, 24)$ and $d(23, 14)$ wrt $A_{13}A_{24}A_{34}$ and $A_{23}A_{34}A_{14}$ respectively.

Two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ pass through A_{12} and $B(13, 24)$ and $B(23, 14)$ respectively, i.e

$k(12)(13, 24) = A_{12}B(13, 24)$ and $k(12)(23, 14) = A_{12}B(23, 14)$.

Thus,

There are two lines $k(12)(13, 24)$ and $k(12)(23, 14)$ through A_{12} .

There are two lines $k(23)(34, 12)$ and $k(23)(13, 24)$ through A_{23} .

There are two lines $k(34)(23, 14)$ and $k(34)(24, 13)$ through A_{34} .

There are two lines $k(13)(12, 34)$ and $k(13)(23, 14)$ through A_{13} .

There are two lines $k(24)(23, 14)$ and $k(24)(34, 12)$ through A_{24} .

There are two lines $k(14)(12, 34)$ and $k(14)(24, 13)$ through A_{14} .

We see that

The lines $k(12)(13, 24)$, $k(23)(13, 24)$, $k(34)(24, 13)$, $k(14)(24, 13)$ are concurrent at X .

The lines $k(12)(23, 14)$, $k(34)(23, 14)$, $k(13)(23, 14)$, $k(24)(23, 14)$ are concurrent at Y .

The lines $k(23)(34, 12)$, $k(13)(12, 34)$, $k(24)(34, 12)$, $k(14)(12, 34)$ are concurrent at Z .

Then X , Y , Z are collinear on the line which is perpendicular to Newton line of (d_1, d_2, d_3, d_4) .

I hope Mr Eckart will help me to check this problem, I'm so sorry for bothering you.

Best regards,

Tran Quang Hung.

Vào Th 6, 2 thg 11, 2018 vào lúc 17:44 Tran Quang Hung <analgeomatrica@gmail.com> đã viết:

>> Dear Mr Eckart,

>> You are right, $k(12)(13, 24)$ is the lines passing through A_{12} and is

>> perpendicular to orthotransversal of A_{12} wrt triangle $A_{34}A_{13}A_{24}$.

>> Thank you very much
>> Best regards
>> Tran Quang Hung.
>>
>> Vào Th 6, 2 thg 11, 2018 lúc 16:08 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
>> [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com
> đã viết:
>>> Dear Tran Quang Hung,
>>> I tried in vain, to verify the concurrent points X, Y, Z
with CABRI.
>>> Is my understanding correct?
>>> $k(12)(13,24)$ is the line through A12,
>>> ... orthogonal to the orthotransversal of A12 wrt A34A13A24.
>>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3240
Date: 04/11/2018 6:50:17
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Wunderbar !
I have my answer, it is the CSC defined by the original QL. So
there is at last a property between the QL and the 5 triple
points !!!
I checked immediately that the CSC of the circle through 3
points is centered on the line through the 2 others (and the
circle through 3 points is centered on the line through the CSC
of the 2 others)
It seems very promising !
Just let me time ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3241
Date: 04/11/2018 6:53:26
From: eckart_schmidt@t-online.de
Subject: Lines is perpendicular to Newton line

Dear Tran Quang Hung,

now I could reproduce your constructions:
The line XYZ is QL-L2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3242
Date: 04/11/2018 9:16:24
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

As the CB transformation defined by the 5 triple points and the 2 circular points swaps the conic through the 5 points and the infinity line through the 2 circular points and the circle through 3 points and the circular points and the line through the 2 other points, the 4th point on the circle is the CB partner of the infinity point on the line.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3243
Date: 05/11/2018 1:08:33
From: Tran Quang Hung
Subject: Lines is perpendicular to Newton line

Thank you very much Mr Eckart.

Best regards,
Tran Quang Hung.

Vào Th 2, 5 thg 11, 2018 vào lúc 01:11 ' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Tran Quang Hung,
>> now I could reproduce your constructions:
>> The line XYZ is QL-L2.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3244
Date: 05/11/2018 10:09:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

this seems to be a remarkable result
... of your knowledge of CB-transformations wrt circular points:
The CB-transformation wrt the 5 triple points and the circular points
... swaps the infinity point of the line, connecting two triple points,
... to the 4th intersection of the circle through the remaining 3 triple points
... and the conic of the five triple points.
Thank you very much!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3245
Date: 05/11/2018 10:53:37
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

using a further of your results, we get:
Two triple points define the other three.
Take any two of the five triple points: P_1, P_2 ,
... the line P_1P_2 and the circle $CSC(P_1P_2)$.
Take the lines La, b, c , connecting $QG-P_{1a, b, c}$
... with the 3rd intersection of P_1P_2 and $QA-Cu7_{a, b, c}$,
... which have a common point Q .
Consider the circle C_i through Q
... centered on $CSC(P_1).CSC(P_2)$
... and orthogonal intersecting the circle $CSC(P_1P_2)$.
 C_i bears Q and the three remaining triple points P_3, P_4, P_5
... in intersections with the conic through the five triple
points,
... which is constructible independently (see#3176).
Wrt the 4th intersection of C_i and the conic of the five triple
points
... see #3238, #3242 and #3244.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3246
Date: 05/11/2018 4:42:53
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

an additional remark to #3245:
The line from Q to the 4th intersection
... of the circumcircle of P_3, P_4, P_5 and the conic of the 5
triple points
... is parallel P_1P_2 .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3247
Date: 05/11/2018 9:36:55
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

for the CB-transformation,
... defined by the 5 QA-Cu7-triple points and the two circular
points,
... messages 3238, 3242 and 3244 allow
... to construct the CB-partner of the infinity point
... .. of a line through two triple points.

The connecting lines of these CB-partners
... have the common point *T*
- see #3176 -
... on the conic of the 5 triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3248
Date: 05/11/2018 9:40:09
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I think everything is much clearer now !
7 points define a CB transformation swapping 21 couples of lines/conics.
On a cubic, 7 points define a fixed point or pivot of the transformation (isoconjugation with fixed points the vertices of the tangential QA of the pivot wrt the DT and pivot the fixed point)
2 cubics intersecting in the 7 points and 2 others define a couple of CB partners and the line through these 2 points intersect each of the 2 cubics in it's pivot !
For the 5 triple points + 2 circular points, there are :
3 cubics QA-Cu7, 10 degenerated cubics line + circle and 1 cubic conic + infinity line.
2 QA-Cu-7 intersect in 2 double points U_i and V_i and U_iV_i cuts the 2 cubics in the pivots QG-P1
2 degenerated line/circle cubics intersect in the the 5 + 2 points and in 2 others (2nd intersection of each line with the other circle) ; the line through these 2 points cuts each circle in the pivot of the cubic (your point Q)
1 QA-Cu7 intersect each line/circle cubic in 2 points, 1 on the line and 1 on the circle ; the line through the 2 points cuts the QA-Cu7 in QG-P1 and the line/conic in the point Q
1 QA-Cu7 intersects the infinity line/conic in 2 points, 1 on the conic (ypur point S) and 1 on the infinity line ; the line through the 2 points cuts QA-Cu7 in QG-P1 and the infinity line/conic in the perspector of the points S and the QG-P1 (your point T)
Last, one of the line/circle and the infinity line/conic interset in 2 points, 1 the infinity point of the line and the 4th intersection of the circle and the conic ; the line through the 2 points (parallel to the line through the 4th point) intersects the circle in Q and the conic in T
I hope I didn't make any basic mistake ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3249
Date: 06/11/2018 11:18:11
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
It seems our 2 last messages have crossed each other !
I wrote my last message without checking, as my figure is now so full that I can't draw anything new on it !
My head is also full with all these properties ...
I'm glad to read in your message the confirmation that the point T is the pivot for the degenerated cubic linfinity ine/5 points conic wrt the CB transformation (the CB partner of T is the tangential of T, id est the infinity point on the tangent in T to the conic).
That means that for any point X on the conic, the CB partner of X is the infinity point on TX and there are an infinity of circular cubics through the 5 points and these 2 points (each cubic has asymptote TX and cuts the asymptote in X) ?
I was so excited that I didn't congratulate you for you last discovery that 2 points define the 3 others.
Really remarkable and amazing indeed !
What do the Miquel point QL-P1 and the CSC represent for the 5P of the 5 triple points ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3250
Date: 08/11/2018 4:32:08
From: Tran Quang Hung
Subject: Concurrent Simson lines

Dear geometers,

Let P be Euler-Poncelet of quadrangle (A_1, A_2, A_3, A_4) .
Then Simson lines of P wrt medial triangles of $A_1A_2A_3$, $A_2A_3A_4$,
 $A_3A_4A_1$ and $A_4A_1A_2$ are concurrent.
Which is this point?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3251
Date: 08/11/2018 7:02:15
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Drawing the points Q , I find this perhaps obvious property
Naming Q_{ij} the point Q on the circle through the 3 other points,
on the parallel through T to X_iX_j , the 3 points $Q_{ij}Q_{ik}Q_{jk}$ are
concylic with T .
The points Q and the point T depend only of the 5 points $X_i \dots$
What about the rest of the construction ? (in particular $QL-P_1$
or the point R)
In other words, I'm wondering if there is a unique QL having
these 5 points as intersection of the 3 $QA-Cu_7$?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3252
Date: 09/11/2018 9:43:23
From: eckart_schmidt@t-online.de
Subject: Concurrent Simson lines

Dear Tran Quang Hung,

wrt #3250: The described point is QA-P6.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3253
Date: 09/11/2018 10:00:55
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

Thanks for the detailed explanations and summary in #3248, #3249.

But further no tip, how to get 5 points for 4 lines with 3 cubics.

The considerations wrt CB-transformations show,
... that there are connections between the five triple points,
... but I cannot add new properties.

What about the following idea:

The double points are the intersections
... of a known conic and the three QL-DT-lines,
... which can be interpreted as degenerated QL-DT-circumcubic.
The triple points and T may be the intersections
... of a known conic and a special QL-DT-circumcubic,
... which I try to find.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3254
Date: 10/11/2018 3:34:45
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for your answer.
I fear, it has become a real addiction, as I look always, like you apparently, for new properties.
But it is timeconsuming !
For example, your 6 points are also intersection between their own conic and the cubic you described in your message3187 and the same way, the 5 points and T could be seen as intersection between their conic and a cubic through the QG-P19 ...
I also began to look for the 3 QA-Cu7 of the 3 QA's formed by 2 copples of double points (9 new cubics and 3 new groups of 5 triple points and associated points T).
It seemed promising, as QA-P41 is the tangential of QG-P1, P18 and P19, QG-P18 is QA-P2 as well as DT vertice and QA-P4 is the 4th intersection of QG-L1 with the QA-Cu7, but I didn't achieve the constructions.
So I rest for a while !
Best regards
Bernard
PS T is certainly determined from the 5 triple points (a new point for the 5-P)
I remain convinced that QL-P1 and the Cl-S are also determined by the 5 points (maybe another one ?)
I have the intuition, there are several QL's sharing these 5 triple points with different points R.
As I often noticed, convictions and intuitions are not proofs
...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3255
Date: 10/11/2018 3:41:27
From: Tran Quang Hung
Subject: Concurrent Simson lines

Thank you very much Mr Eckart.

Best regards,
Tran Quang Hung.

Vào Th 6, 9 thg 11, 2018 vào lúc 15:43 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt #3250: The described point is QA-P6.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3256
Date: 12/11/2018 8:27:46
From: eckart_schmidt@t-online.de
Subject: New point for a 5P

Dear Bernard,

as you mentioned in #3254
... our discussion wrt QA-Cu7-geometry leads to a new 5P-point:
Consider a 5P with circumconic Co
... and a line L connecting two 5P-vertices
... and the circumcircle Ci of the remaining vertices.
... Let X be the 4th intersection of Ci and Co ,
... let L' be a parallel to L through X ,
... intersecting Co further in Y ,
... which will be a new 5P-point on the circumconic,
... independent of the choice of vertices.

Obvious a new QA-transformation,
... mapping a point P to the new 5P-point Y
of the QA-vertices and P .

But I found no properties wrt the QA-points,
... lines are mapped to quintics
with double points in the QA-vertices.

However:
For a point P on QA-Co3
the image Y lies diametral.

Finally:
Consider lines through QA-P4,
... intersecting their QA-Tf2-conic in U and V .
... The new 5P-points of the 3 QA-DT-vertices and U , V
... lie on a line through QA-P41, parallel QA-P3.QA-P4.

Best regards Eckart

PS. I think, Chris will read this message.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

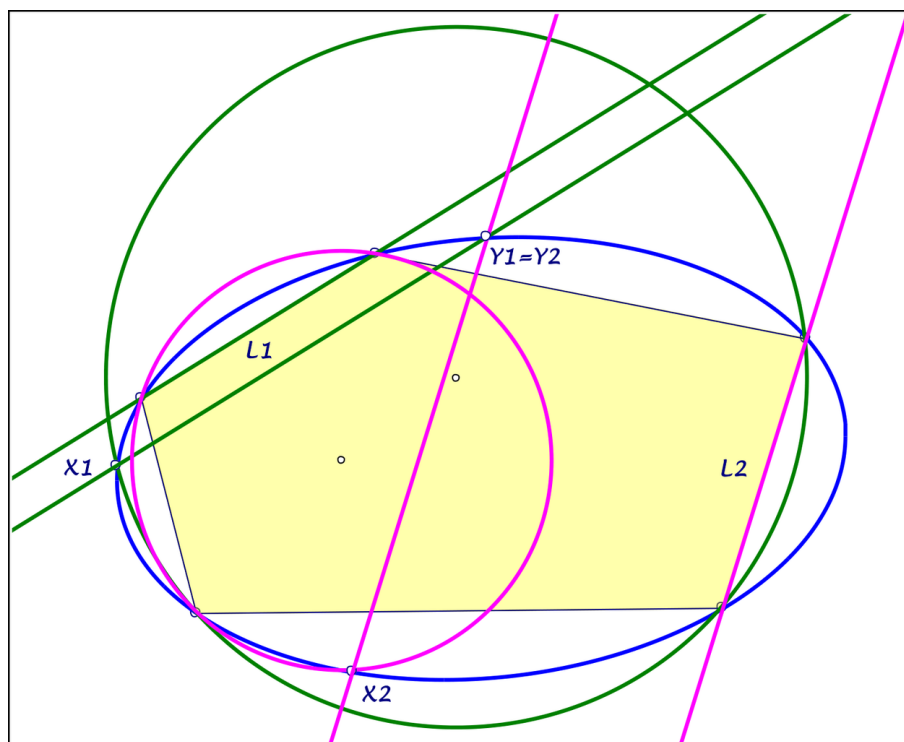
Message: #3257
Date: 13/11/2018 4:17:50
From: chris.vantienhoven
Subject: New point for a 5P

Dear Eckart,
I checked your construction of the 5P-point, but I couldn't find a fixed point on the 5P-conic when changing L and Ci over the 5 reference points.
Maybe you can make a picture of two instances of (L1,Ci1) and (L2,Ci2) producing an identical 5P-point on Co.
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3258
Date: 13/11/2018 9:16:51
From: eckart_schmidt@t-online.de
Subject: New point for a 5P

Dear Chris,
attached a construction for the new 5P-point.
Best regards Eckart

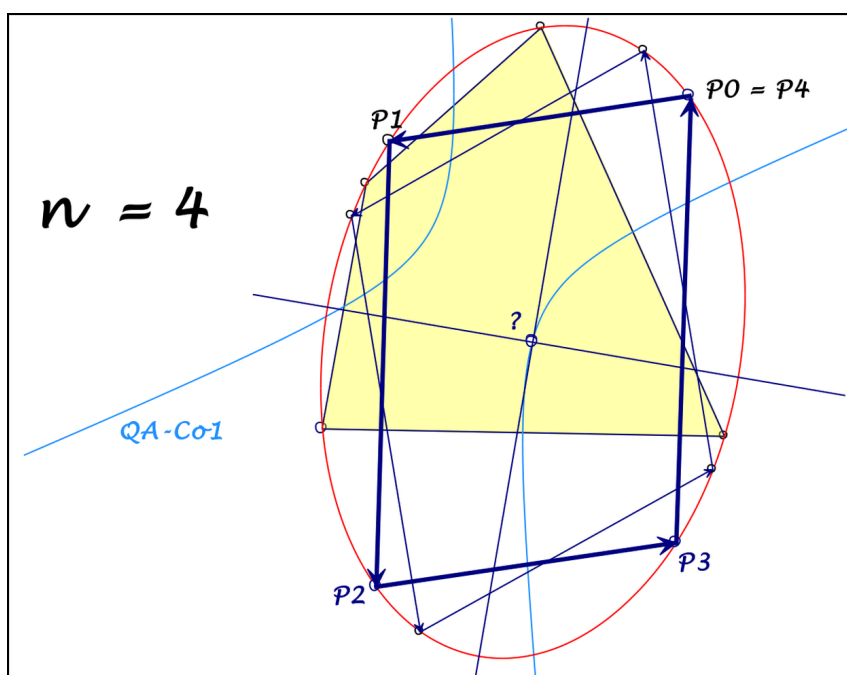


2018-11-13.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3259
Date: 13/11/2018 8:30:27
From: eckart_schmidt@t-online.de
Subject: New point for a 5P

Dear Chris, dear Bernard,
in #3256 a new 5P-point is described
... with a QA-transformation TF, which maps
... a point P to the new 5P-point of P and the vertices of QA.
Starting with a point P_0 and defining $P_n = TF(P_{n-1})$,
... let us consider the case $P_0 = P_n$:
For $n = 1$ we get the DT-vertices as limit case,
... for $n = 2$ the locus for P_0 is QA-Co3,
... for $n > 2$ the loci for P_0^*
are $n-1$ QA-circumconics
... with affinely regular n -gons $P_1P_2P_3\dots P_n$,
... which have the same area for each conic.
Example (see attached file):
For $n = 4$ we get QA-Co3 (see above for $n = 2$)
... and two QA-circumconics
... with parallelograms $P_1P_2P_3P_4$ of same area;
... the axes of the conics
 divide the parallelogram sides in the same ratio.
What about the $n-1$ QA-circumconics
with special centers on QA-Co1?
Best regards Eckart
PS: Can someone confirm these CABRI-observations?



2018-11-13a.pdf

Message: #3260
Date: 14/11/2018 7:12:52
From: bernard.keizer
Subject: New point for a 5P

Dear Eckart,
Looking again at your construction in message 3238, I'm more and more convinced that the Cl-S transformation is determined by the 5 triple points.
Having 3 points X_1 , X_2 and X_3 and the Cl-S, you may find X_4 and X_5 without using the conic as intersection of the 3 circles centered on the line $CSC(X_1)CSC(X_2)$, orthogonal to any circle through these 2 points (let's take for example the circle through the 3 CSC of X_1 , X_2 and X_3) and passing through X_3 (the 2 other circles are the same by permutation).
All together, we have 5 points X_i , $i=1$ to 5, and a Cl-S such as 2 points are inverse wrt the circle through the CSC of the 3 others ; the converse is true and the Cl-S of 2 points are inverse wrt the circle through the 3 others. The 2 groups of points are partners with the same commun Cl-S !
The 5 points have a point T on their circumconic and the same way the 5 CSC have thir own point T.
But despite all my efforts, I'm not able to find a construction of the Cl-S ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3261
Date: 15/11/2018 8:56:57
From: eckart_schmidt@t-online.de
Subject: Qa-Cu7-Geometry

Dear Bernard,

it took some time, to understand your message 3260,
... but it was a great surprise for me!
Your observation wrt the 5 triple QA-Cu7-intersections for a QL:

Two triple points
... are inverse wrt the circumcircle
... of the CSC-images of the other three.
As well as:
The CSC-images of two triple points
... are inverse wrt the circumcircle
... of the remaining three triple points.
I shall research your conjecture,
... that the 5 triple points determine the CSC.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3262
Date: 15/11/2018 12:59:48
From: chris.vantienhoven
Subject: Qa-Cu7-Geometry

Dear Eckart, dear Bernard,

[BK,ES]

Two triple points
... are inverse wrt the circumcircle
... of the CSC-images of the other three.

Here are some resembling situations:

* QL-P1 and CSC(QL-P29) are inverse wrt the circumcircle of the CSC-images of three of the X(265)-points of the QL-Component Triangles.

* QL-P1 and CSC(QL-P28) are inverse wrt the circumcircle of the CSC-images of three of the X(186)-points of the QL-Component Triangles.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3263
Date: 15/11/2018 7:32:58
From: chris.vantienhoven
Subject: New point for a 5P

Dear Eckart, Dear Bernard,

I included the new 5P-point in the Encyclopedia of Polygon Geometry.

It really was a beautiful coproduction of the two of you.

See 5P-s-P4

When we call the derived QA-Transformation QA-Tfx, I figured out that:

- $QA-Tfx(QA-P4) = QA-P1.QA-P42 \wedge QA-P3.QA-P4 \wedge QA-P6.QA-P34.$
- $QA-Tfx(QA-P41)$ lies on $QA-P3.QA-P41.$

Best regards,
Chris

p.s. I also included another coproduction of the two of you, that was on my list for a long time.

See 8L-s-L1.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3264
Date: 15/11/2018 9:47:19
From: eckart_schmidt@t-online.de
Subject: New point for a 5P

Dear Chris, dear Bernard,

I am very glad, that Chris has respected results ... of Bernard's and my QFG-discussion in EQF/EPG.

Special thanks to Bernard, who has always ... commented my observations with new aspects.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3265
Date: 16/11/2018 8:24:50
From: eckart_schmidt@t-online.de
Subject: QL-Cu7-Geometry

Dear Bernard,

I think, you are right,
... that the 5 triple points of QA-Cu7 for a QL
... determine their CSC-images
... independent of the reference QL.

My CABRI-observation:

Let P_i be the triple points and Q_1 an arbitrary point:
... Q_2 the inverse of Q_1 wrt (P_3, P_4, P_5) , Q_3 the inverse of Q_1 wrt
 (P_2, P_4, P_5) ,
... Q_4 the inverse of Q_1 wrt (P_2, P_3, P_5) , Q_5 the inverse of Q_1 wrt
 (P_2, P_3, P_4) ,
... then there is only one point Q_1
with P_1 inverse P_2 wrt (Q_3, Q_4, Q_5) ,
... in this case holds $Q_i = \text{CSC}(P_i)$ for $i=1,2,3,4,5$.
What about a construction?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3266
Date: 16/11/2018 3:46:16
From: bernard.keizer
Subject: New point for a 5P

Dear Chris, dear Eckart,
I'm glad and honored to see our beautiful coproduction mentioned in EQF !

As I already told Eckart, this search around the 3 triple points of the 3 QA-Cu7 has become a real addiction, but it has allowed me to understand much better the power of this Cayley-Bacharach property.

Many thanks to Eckart to whom I due so many new ideas and with whom I share so many moments of happiness ...

And I'm sure we haven't finished to discover new points and transformations ...

1) 8 points define a CB 9th point (in EQF)

(For Chris, the degenerated cubic is a conic and a line, not 2 lines ; it could be also 3 lines)

2) 7 points define a CB transformation (not in EQF)

For 7 points on a cubic, it gives 21 copples of line trough 2 points/conic through the 5 others ; the transformation is an isoconjugation with a pivot on the cubic as intersection of 21 lines joining the intersection of a line with the conic of another cople and vice-versa. The 4 fixed points of the isoconjugation are the vertices of the tangential QA of the pivot and the reference triangle is the DT of this tangential QA.

The cubic is a pivotal isocubic wrt the DT with fixed points the QA vertices and pivot as defined above.

3) 6 points and the circular points define a CB 9th point (special case of point 1)

4) 5 points and the circular points define a CB transformation (special case of 2)

For 5 points on a circular cubic, it gives 10 copples of line through 2 points/circle trough the 3 others and the 2 circular points ; in particular, for the cubic formed by the infinity line and the 5 points conic, the pivot is the point 5P-s-P4 with the construction given in EQF.

5) Last, but not least, there is a beautiful 5P C1-S transformation according to which 2 points are inverse wrt the circle through the CSC of the 3 others and vice-versa. We haven't identified the construction yet, but I have no doubt we will soon succeed ; the fight goes on !

Best regards

Bernard

Message: #3267
Date: 16/11/2018 4:01:45
From: Antreas Hatzipolakis
Subject: New point for a 5P

Regarding this beautiful point:
See 5P-s-P4.
How about if the pentagle is cyclic (if the circumconic is circle) ?

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3268
Date: 16/11/2018 4:03:05
From: bernard.keizer
Subject: QL-Cu7-Geometry

Dear Eckart,
I'm really glad I have convinced you !
First I tried to find if by any chance $CSC(P_i)$ was a QA-point for the QA of the 4 other points ; but it doesn't work
Last idea: with your notations, the locus of the points Q_1 such as Q_2 is the inverse of P_1 wrt $(P_3,4,5)$, Q_3 is the inverse of P_1 wrt $P(2,4,5)$ and at the same time Q_3 is the inverse of Q_2 wrt $(P_1,4,5)$
is a curve passing through $CSC(P_1)$, P_4 and P_5 (it's certainly not a line and apparently not a circle, but perhaps a conic).
If we could identify this curve, it would be possible to do the same with each triple $Q_1,2,4 \dots$ and $CSC(P_1)$ would be the intersection of these 10 curves...
I hope you will find a solution!
Best regards
Bernard
PS Of course, then remains my last interrogation: is there a unique QL having these 5 triple points as intersection of 2 QA-Cu7 or are there an infinity?
Normally, 5 independant points define a unique QL ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3269
Date: 16/11/2018 5:54:18
From: bernard.keizer
Subject: QL-Cu7-Geometry

Dear Eckart,
My apologise,
There are 4 such curves and not ten and these curves are finally
circles through P4,P5 and Q1 and the like
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3270
Date: 16/11/2018 9:13:09
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your encouraging messages,
... the good cooperation goes on:
Wrt your observation,
... that the 5 triple points of QA-Cu7 for a QL
... determine their CSC-images
... independent of the reference QL.
Here a construction, starting with the triple points
P1,P2,P3,P4,P5:
Let Q1 be P1
... Q2 the inverse of P1 wrt (P3,P4,P5), Q3 the inverse of P1 wrt
(P2,P4,P5),
... Q4 the inverse of P1 wrt (P2,P3,P5), Q5 the inverse of P1 wrt
(P2,P3,P4),
... then CSC(P1) is
... .. the inverse of P2 wrt the circle (Q3,Q4,Q5),
... .. the inverse of P3 wrt the circle (Q2,Q4,Q5),
... .. the inverse of P4 wrt the circle (Q2,Q3,Q5),
... .. the inverse of P5 wrt the circle (Q2,Q3,Q4).

Best regards Eckart

PS: Wrt #3266, #3268, #3269 I shall answer later on.

Message: #3271
Date: 17/11/2018 10:39:47
From: bernard.keizer
Subject: QA-Cu7-Geometry

Alleluiah !

Dear Eckart,

Many thanks and congratulations for your simple and beautiful construction !

There is, as announced in a previous message, a Cl-S transformation for a 5P such as 2 points are inverse wrt the circle of their CSC and vice-versa ; this leads to a 2nd group of 5 points having the same Cl-S and having their own point 5P-s-P4 on their circumconic.

New 5P transformation for EQF

Best regards

Bernard

PS If we find a QL have these 5 points as triple QA-Cu7 points, it has necessary the same Cl-S ...

Message: #3272
Date: 17/11/2018 10:51:04
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I was interested in the question,
... if the 5 triple points give a special 5P.
CABRI-observations show,
... that the last 4 inverses in #3270
... are not equal for an arbitrary 5P.
So the 5 triple points give a special 5P.
What about further properties?

For example:
Symmetric or affin regular 5P have the considered property.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3273
Date: 17/11/2018 10:51:23
From: bernard.keizer
Subject: New point for a 5P

Dear Antreas,
It seems that for 7 points, the CB transformation works only if
6 points are not coconic and the same way for 5 points only if 4
are not concyclic.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3274
Date: 17/11/2018 4:09:48
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm rather surprised by your last message !
I checked your construction carefully with a random 5P and the 4
inverses give the same point.
The property holds when moving the points ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3275
Date: 17/11/2018 5:13:18
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thank you very much indeed, I made a wrong construction!
Please forget my #3272,
... the 5 triple points seem not to give a special 5P,
... so there will be a chance to verify your conjectures.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3276
Date: 19/11/2018 4:46:24
From: chris.vantienhoven
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart, Dear Bernard,

Regarding your construction in QFG-message #3270 in a Pentangle. You noted in message #3272 that the last 4 inverses of the construction weren't equal.

Meanwhile I had noted that they truly are equal.

Meanwhile also Bernard noted in # 3274 that the 4 inverses gave the same point.

Actually your construction in the Pentangle is built from 5 QA-transformations.

This QA-transformation (denote it as QA-Tfx) is as follows:

Let P be a random point.

Let Q1 be the inverse of P wrt (P2, P3, P4),

Q2 be the inverse of P wrt (P3, P4, P1),

Q3 be the inverse of P wrt (P4, P1, P2),

Q4 be the inverse of P wrt (P1, P2, P3).

then QA-Tf (P) =

the inverse of P1 wrt the circle (Q2, Q3, Q4), as well as

the inverse of P2 wrt the circle (Q3, Q4, Q1), as well as

the inverse of P3 wrt the circle (Q4, Q1, Q2), as well as

the inverse of P4 wrt the circle (Q1, Q2, Q3).

I like this transformation because it is exclusively built from inversions.

The calculation of the coordinates of QA-Tfx[(x:y:z)] was extremely difficult.

After many tryouts with many different approaches I found the coordinates of the mapped point.

The first CT-coordinate is:

$$(a^4 (p + q) (p + r) y z - b^4 p (p + q) x z - c^4 p (p + r) x y + b^2 c^2 p x ((p + q + r) x + r y + q z) +$$

$$a^2 b^2 (p + q) (p x + r x - p y) z + a^2 c^2 (p + r) y (p x + q x - p z))$$

$$/(a^2 (p y z (p + q + r) - q r x (x + y + z)) + b^2 p x (z (p + q) - r (x + y)) + c^2 p x (y (p + r) - q (x + z)))$$

At last I tried to find incidences with existing QA-points. There wasn't much to be found, but what I found is very interesting:

- QA-Tfx(QA-P41) lies on the line QA-P3.QA-P41.
- QA-Tfx(QA-P3) as well as QA-Tfx(QA-P41) lie on the cubic QA-Cu1.

(I am sure Eckart will like this)
It also made me think of the newly found 5P-point 5P-s-P4,
because:
· 5P-s-P5(P1,P2,P3,P4,QA-P41) also lies on the line QA-P3.QA-P41
(different point).

Maybe useful in the QA-Cu7-triples-quest?
Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3277
Date: 19/11/2018 10:27:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris,

very interesting QA-transformation,
... first observations:

QA-Tfx maps QA-Cu1 to itself,
... especially the diagonal triangle to the Miquel triangle,
... with generally 3rd intersection QA-P3.

QA-Cu1 is a pivotal isocubic
... with reference triangle QG-P16a,b,c,
... isoconjugation with fixed points with tangential QA-P3,
... pivot QA-P3.

The isoconjugated points on lines through the pivot QA-P3
... are QA-Tfx-partners,
... but there are two other QA-Tfx-partners on these lines!

QA-Tfx maps lines to QA-circumscribed quartics:
... with double points in the QA- vertices,
... through QA-Tfx(QA-P3),
... which is the 3rd intersection of QA-Cu1 and Q.QA-P41,
... with Q intersection of QA-Cu1 and its asymptote.

Best regards Eckart

PS: I hope, this isn't an overhasty observation.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3278
Date: 20/11/2018 9:53:44
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris,

excuse, the last passage in #3277 has to begin:
"QA-Tfx maps lines *through QA-P3* to QA-circumscribed
quartics:"
... and can be more specified as follows:
... with double points in the QA- vertices,
... through QA-P4 and QA-Tfx(QA-P3),
... .. which is the 3rd intersection of QA-Cu1 and Q.QA-P41,
... .. with Q intersection of QA-Cu1 and its asymptote.
Lines through QA-P3 bear two pairs of QA-Tfx-partner,
... one pair on QA-Cu1, collinear with QA-P3,
... one pair äquidistant to QA-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3279
Date: 20/11/2018 11:37:12
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris,

excuse once more, the last passage in #3277 has to begin:
"QA-Tfx maps lines *through QA-P3* to QA-circumscribed *Sextics*
:"

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3280
Date: 20/11/2018 12:14:10
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

As mentioned in message 3270, if a QL has the 5 points as triple points of the 3 QA-Cu7, the QL and the 5P have the same Cl-S. The question was : is it unique ?

There are a double infinity of QL's having this Cl-S.

They are inscribed in a QL-Cu1 having this Cl-S and for one QL-Cu1, there is an infinity.

But a QL-Cu1 having this property is defined by the Cl-S and a Newton Line and there are an infinity of such curves.

Starting with a couple of CSC partners, the line through the 2 points is QG-L1 and carries a infinity of harmonic conjugated points which are 2 possible QG-P1.

The lines through T and these 2 QG-P1 give the isogonal points S on the conic and the points QA-P4 as Cl-S conjugates ; the intersection of the 2 lines SQA-P4 is not generally on the conic.

When it is on the conic, it's the point R.

The CSC of the circle with diameter the segment joining the 2 QG-P1 intersects QG-L1 in the points U1 and V1 and the lines through R in the 2 points QG-P18, which gives their CSC, the 2 points QG-P17.

We have then the 3rd QG-P1, QG-P17 and QG-P18 ...

So I'm convinced now that there are an infinity of QL's having the same triple points (and of course another infinity having the 5 CSC partners of the 5 triple points as triple points).

Do you agree ?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3281
Date: 20/11/2018 1:25:58
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris, dear Eckart
This is indeed a fascinating transformation !
4 fixed points and a variable point P define a transformation
and a variable Cl-S, which swaps P and its transformed.
This has many beautiful new properties on QA-Cu1 (I like the
swapping of DT and Miquel triangle with pivot QA-P3).
If we take now a QL-Cu1 (which is a particular QA-Cu1), what
happens ?
More precisely, if the 4 fixed points are the vertices of a
tangential triangle on QL-Cu1, is it true that for any point P
on QL-Cu1 the Cl-S is centered in P ?
(For any P on QL-Cu1 and any couple of CSC partners X and X',
the bisector of XPX' is the same ...)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3283

Date: 21/11/2018 9:08:01

From: eckart_schmidt@t-online.de

Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris,

wrt the last property in #3276:

For points P on QA-Cu1 holds:

... P , QA-Tfx(P), $5P$ -s- $P4(P1, P2, P3, P4, P)$ and QA-P3 are collinear.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3284
Date: 21/11/2018 10:14:12
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart,

Thanks for your quick answer and your simple and beautiful figure !

1) I use regularly Cl-S as well as CSC ; of course, it's the same for Clawson-Schmidt (Möbius transformation named also ψ by Bernard G. or μ by Benedetto)

For a QL, it is QL-Tf1

2) Thanks for the remark

3) Sorry for the typo, I mean quadrangle

4) Thanks for the property with QA-P3 and the revision of QL-Cu1 as QA-Cu1 (P is QA-P4, CSC(P) is at the same time QA-P2 and QG-P1 ...)

Last, but not least, it wan't my question !

A 5P defines, as we saw, a CSC such as each cople of points are inverse wrt the circle of the CSC of the 3 others and vice-versa.

What is this CSC for the 5 points P,P1,P2,P3,P4,P5 with P on QA-Cu1 and the other pivotal isocubics or QL-Cu1 ? What are the 5 CSC points in this CSC ?

For QL-Cu1, this 2nd CSC is not QL-Tf1 as QA-Tfx(P) is the 3rd intersection of QA-P3P with the curve and not CSC(P).

Thanks in advance for your attention

Best regards

Bernard

PS In the case of the 5 triple points of the QA-Cu7 of a QL, the CSC of the 5 points is the same as QL-Tf1 of the QL (see my previous message)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3285
Date: 21/11/2018 8:20:55
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Bernard,

a further puzzle piece for the five QA-Cu7-triple points wrt a QL:

Let P_i be the 5 triple points

... with their CSC-images P_i^* (see #3270),

... which can be constructed independent of the QL.

Consider the quadrilaterals $P_i.P_j.P_i^*.P_j^*$ and their QL-P1,

... which coincide with QL-P1 of the reference QL.

Now the CSC-image of any point can be constructed,

... as Möbius transformation,

... centered in QL-P1, swapping P_i and P_i^* ,

... independent of the reference QL.

In this way we get a new 5P-transformation.

The quadrilaterals $P_i.P_j.P_i^*.P_j^*$, interpreted as QL,

... have the same QL-P1, but not the same 5 QA-Cu7-triple points.

If we search for QLs with the same triple points,

... they must have CSC-partner as opposite points.

CABRI-observations suggest,

... that there is only one QL with this property.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3286
Date: 21/11/2018 11:32:38
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart,
The new 5P transformation was mentioned in my messages 3271, 3280 and 3284 !
If a QL has the 5 points as triple points of the QA-Cu7, the transformation is the QL-Tf1.
I also mentioned the fact that there a double infinity of QL's having the same QL-Tf1 (all the QL's inscribed in all the QL-Cu1 having the same QL-Tf1).
But I was wondering if there were an infinity of QL's having the 5 points as triple points of the QA-Cu7 and I tried to begin the construction as described in my message 3284.
Apparently, you seem to be convinced that it is unique.
In fact, as I also mentioned somewhere, 5 independent QL-points normally define a unique QL; this remark is an argument in your direction.
But if this is true, the points S as well as R must also be 5P points !
In particular, having already T, if we had also R, it appears possible to find the points S with the QG-Pi on TSi and the CSC of QG-Pi, id est the QA-P4 on RSi ...
Conversely, the 5 points R, T and the 3 Si should give the 5 triple points ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3287
Date: 22/11/2018 9:25:08
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart,
In the end of my message, please read : having T, R and the 3 points S_i , it should be possible to find the points QG-P1i on $T S_i$ and QA-P4i = CSC(QGP1i) on $R S_i$...
I checked if by any chance the 5 points T, R and S_i had the same CSC, but it isn't the case.
I hope you will find something new !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3288
Date: 22/11/2018 6:52:53
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart,
Perhaps this last idea or question.
The CSC of the 5 triple points have by construction the same CSC ; they have their own conic and their own point T or 5P-s-P4.
Always trying to follow your conjecture that there is a unique QL having 5 points as 5 triple points of the QA-Cu7, which QL has the 5 CSC as triple points ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3289
Date: 24/11/2018 6:57:26
From: chris.vantienhoven
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Eckart, Dear Bernard,

I included the special new transformation QA-Tfx in EQF.
See: QA-Tf16.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3290
Date: 24/11/2018 2:57:14
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris, dear Eckart,
Thanks a lot again ! And many thanks to Eckart ...
It is now possible to describe the 5P Möbius transformation by
applying 5times this QA transformation.
Having 5 points P_i , Q_i is QA-Tf16(P_i) wrt the 4 other points.
There is a unique CSC such as Q_i is CSC(P_i).
2 points P_i and P_j are inverse wrt the circle through the 3
other points Q and vice-versa.
Best regards

Bernard

PS Considering the conic through the Q_i , it's CSC is a quartic
which cuts the conic through the P_i in 8 points : the 5 P_i and 3
other points sharing with the P_i the property that they are on
the same conic and that their CSC are on the conic of the Q_i .
What about these 3 new points ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3291
Date: 30/11/2018 11:59:08
From: bernard.keizer
Subject: QA-Cu7-Geometry / new QA-Transformation

Dear Chris, dear Eckart
This time, I give up !
I've rubbed out all my figures of the 3 QA-Cu7 of a QL, as I couldn't add a single point.
I made a brand new one and replaced all the points, working in particular with the beautiful summary of Eckart in the message 3176 ; by the way, there is a typo about the points R and U, which are diametral on the circle $CSC(QL-Ci1)$ and not $CSC(QL-L1)$.
I made the same construction for the QL formed by the points QG-P17 and QG-P18, which is inscribed in the same QL-Cu1 and found the 3 triple points as well as the 3 points S, and the points T and R ...
I didn't find anything new !
I couldn't find a direct construction of R and the 3 points S from the 5 triple points and I couldn't either find the twin QL having the CSC of the triple points as triple points (which would be a confirmation of Eckart's conjecture that there is only one QL having the 5 points as triple points).
Best regards
Bernard
PS I always hope Eckart will find something new ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

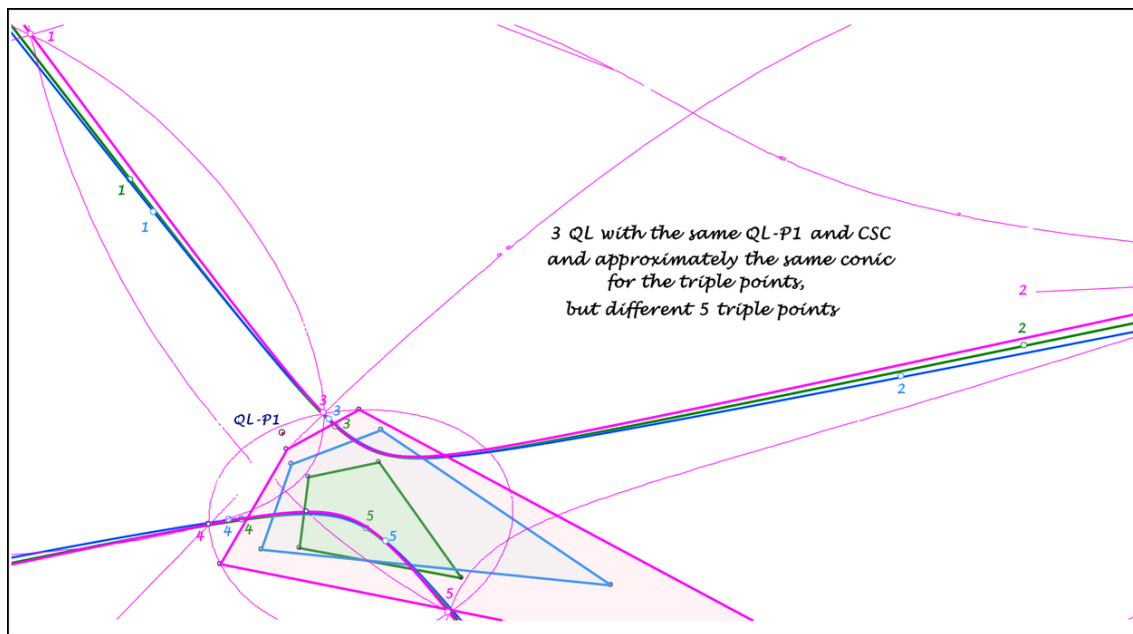
Message: #3292
Date: 01/12/2018 12:05:05
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse, that I haven't reacted to your last messages,
... but I am really confused and disappointed
... wrt the 5 QA-Cu7-triple points,
... for I can't give new aspects.
My situation seems the same as yours.
Attached a drawing of three QL
... with the same QL-P1 and CSC
... and approximately the same conic for the triple points,
... but different triple points.
I don't know, whether the reason is the approximation.

Best regards Eckart

PS: Excuse the limits of the drawing, but I couldn't do it better.



2018-12-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3293
Date: 03/12/2018 11:50:52
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm rather glad that your 3 QL's with the same CSC have different triple points and different conics !
That goes in direction of your conjecture that 5 triple points lead to a unique QL.
As I put in a recent post-scriptum, the CSC of a conic is a quartic.
The 5 triple points X_i define a conic and a CSC as well as 5 other points $Y_i = CSC(X_i)$ with their own conic and by construction the same CSC.
The CSC of this 2nd conic is a quartic and intersects the 1st conic in 8 points, the 5 points X_i and 3 other points sharing with the X_i the property that they are coconic and that their CSC are also coconic.
Shortly, there cannot be 10 points or more on the same conic.
I checked several drawings and these 3 additionnal points are not the 3 points S or the point T .
If we could identify the twin QL with the same CSC having the points Y_i as triple points, we would identify the 2nd conic and get a construction of the 5 points among the 8 intersections of the 1st conic and the quartic CSC of the 2nd.
In fact, I think we have discovered most of the properties of these 5 triple points.
We only need a confirmation of your conjecture and a construction of a QL having 5 random points as QA-Cu7 triple points (we have already the construction of the CSC and of the point T).
Best regards
Bernard
PS
Always counting 5 points in order to define a QL, the 5 points should give a unique QL.
The 5 points define a CSC and a conic and 3 points and the CSC define the 2 other points and the conic.
I suppose the CSC and the conic should define the 5 points ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3294
Date: 04/12/2018 12:18:56
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Please forget my last sentence in the post-scriptum.
It appears that there are several groups of 5 points on the same conic and having the same CSC.
(Of course, the point T is different and the 2nd conics of the CSC are also different).
I don't know how interpret this property ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3295
Date: 04/12/2018 11:45:07
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
In fact, you could as well forget my whole message ! My apologise ...
If there are several groups of 5 points on the same conic and having the same CSC, there are also several QL's referring to these groups of 5 points and having the same CSC.
Each QL has a twin QL referring to the CSC of it's 5 points, but the 10 CSC don't need to be coconic ...
This marks perhaps the end of this long quest and I feel like you a little bit disappointed.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3296
Date: 23/12/2018 8:43:38
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

QFG seems dead,
... but the mystery of the 5 QA-Cu7-triple points remains.
Nevertheless
some remarks wrt the 5P-transformation TF,
... mentioned in #3270 and #3285, but not in EPG.
Definition of TF wrt a 5P with vertices P_i :
Let Q_1 be one P_i
... and Q_j be the inverse of P_i wrt the circumcircle (P_k, P_l, P_m) :
... $TF(P_i) =$ inverse of P_j wrt the circumcircle (Q_k, Q_l, Q_m) ,
... $TF(X) =$ CSC wrt the $QG = P_i.P_j.TF(P_i).TF(P_j)$, interpreted as
QL.
TF is CSC of a QL with QA-Cu7-triple points $P_1, 2, 3, 4, 5$.
If there is a unique QL with this property (my conviction),
... all QL-points can be translated to 5P-points,
... for example QL-P1 see #3285.

Properties:

- (1) $TF(X)$ wrt $P_1, P_2, P_3, P_4, P_5 = TF(X)$ wrt $TF(P_1), TF(P_2), TF(P_3), TF(P_4), TF(P_5)$.
- (2) If four P_i are concyclic (but not five),
TF of any point is the fifth P_i .
- (3) The TF-image of the circumcircle of P_i, P_j, P_k is a circle,
... centered on $P_l P_m$ with P_l, P_m inverse.
- (4) The TF-image of a line $P_i P_j$ is a circle
... orthogonal intersecting the circumcircle of P_k, P_l, P_m .
- (5) $TF(P_i)$ and $TF(P_j)$ are inverse
... wrt the circumcircle of P_k, P_l, P_m .

Finally for Chris a property of 5P-s-P4:

Let T be 5P-s-P4 wrt $P_1, 2, 3, 4, 5$,
... let P_i' be 5P-s-P4 wrt T, P_j, k, l, m ,
... then $P_i P_i'$ // tangent in T at the conic through $P_1, 2, 3, 4, 5$.

Perhaps relevant for the 5 QA-Cu7-triple points.
What about the 5P-point $TF(5P-s-P4)$?

Merry Christmas and a Happy New Year!
Best regards Eckart

Message: #3297
Date: 24/12/2018 11:08:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris, dear Eckart,
Merry Christmas and Happy New Year to both of you !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3298
Date: 24/12/2018 11:54:18
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart, Bernard and other QFG-friends,

Thank you all for your wonderful contributions that I love so much.

Last year I didn't have the right circumstances for many contributions of myself.

Maybe next year better.

I wish you all the best and hope you have a beautiful Christmas time and a prosperous New Year.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3299
Date: 30/12/2018 3:19:24
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

in message #2965 I gave some descriptions for QA-Cu7,
... I think, the following is new:

QA-Cu7 can be described as nonpivotal isocubic:
... reference triangle QG-Tr3 of a quadrigon version,
... isoconjugation QG-Tf2 of this quadrigon version;
... root QG-Tr3-tripol of a line,
... .. connecting the QG-P19 of the other two quadrigon versions.

Best regards
Eckart

[← Previous](#) [↔ Message Index](#) [↑ Subjects](#)

5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

5P-s-P1:

[#2830](#) [#2852](#) [#2907](#) [#2910](#) [#2914](#)

5P-s-P4:

[#3263](#) [#3266](#) [#3267](#) [#3271](#) [#3276](#) [#3283](#) [#3288](#) [#3296](#)

5P-s-P5:

[#3276](#)

CSC:

[#2846](#) [#2884](#) [#2886](#) [#2889](#) [#2890](#) [#2893](#) [#2900](#) [#2901](#) [#2902](#) [#2918](#) [#2920](#)
[#2937](#) [#2944](#) [#2945](#) [#2951](#) [#2955](#) [#2956](#) [#2960](#) [#2969](#) [#2974](#) [#2980](#) [#2984](#)
[#2985](#) [#2986](#) [#2988](#) [#2995](#) [#3004](#) [#3008](#) [#3014](#) [#3016](#) [#3017](#) [#3031](#) [#3061](#)
[#3068](#) [#3131](#) [#3134](#) [#3135](#) [#3137](#) [#3139](#) [#3142](#) [#3148](#) [#3159](#) [#3161](#) [#3162](#)
[#3163](#) [#3176](#) [#3180](#) [#3189](#) [#3190](#) [#3232](#) [#3234](#) [#3235](#) [#3236](#) [#3237](#) [#3238](#)
[#3240](#) [#3245](#) [#3249](#) [#3260](#) [#3261](#) [#3262](#) [#3265](#) [#3266](#) [#3268](#) [#3270](#) [#3271](#)
[#3280](#) [#3281](#) [#3282](#) [#3284](#) [#3285](#) [#3286](#) [#3287](#) [#3288](#) [#3290](#) [#3291](#) [#3292](#)
[#3293](#) [#3294](#) [#3295](#) [#3296](#)

Cardioid:

[#3152](#)

Cayley-Bacharach, CB:

[#2800](#) [#2838](#) [#2868](#) [#2869](#) [#2884](#) [#2887](#) [#2892](#) [#2893](#) [#2897](#) [#2899](#) [#2964](#)
[#3006](#) [#3052](#) [#3054](#) [#3058](#) [#3086](#) [#3116](#) [#3176](#) [#3178](#) [#3179](#) [#3180](#) [#3181](#)
[#3185](#) [#3189](#) [#3190](#) [#3191](#) [#3192](#) [#3194](#) [#3195](#) [#3196](#) [#3200](#) [#3201](#) [#3202](#)
[#3203](#) [#3206](#) [#3207](#) [#3208](#) [#3209](#) [#3210](#) [#3212](#) [#3213](#) [#3216](#) [#3217](#) [#3218](#)
[#3219](#) [#3220](#) [#3221](#) [#3222](#) [#3223](#) [#3224](#) [#3232](#) [#3242](#) [#3244](#) [#3247](#) [#3248](#)
[#3249](#) [#3253](#) [#3266](#) [#3273](#)

Central conic:

[#2946](#)

Cotterill:

[#3220](#)

Hypocycloid:

[#3126](#) [#3128](#)

Isoconjugation, isoconjugate:

[#2830](#) [#2838](#) [#2843](#) [#2845](#) [#2846](#) [#2856](#) [#2858](#) [#2867](#) [#2869](#) [#2875](#) [#2882](#)
[#2883](#) [#2884](#) [#2886](#) [#2889](#) [#2890](#) [#2891](#) [#2965](#) [#3015](#) [#3074](#) [#3079](#) [#3081](#)
[#3086](#) [#3087](#) [#3092](#) [#3095](#) [#3098](#) [#3115](#) [#3117](#) [#3160](#) [#3183](#) [#3186](#) [#3187](#)
[#3189](#) [#3191](#) [#3192](#) [#3200](#) [#3201](#) [#3217](#) [#3222](#) [#3223](#) [#3248](#) [#3266](#) [#3277](#)
[#3299](#)

Isocubics:

[#2856](#) [#2875](#) [#3079](#) [#3098](#) [#3284](#)

Miquel-point, Miquel Point:

#2832 #2851 #2868 #2883 #2885 #2989 #2990 #3027 #3031 #3045 #3110
#3131 #3132 #3151 #3249

Miquel-triangle, Miquel Triangle:

#2847 #2868 #2869 #2885 #2893 #2901 #3040 #3110 #3115 #3116 #3132
#3189 #3208 #3277 #3281

Newton Line:

#2827 #2831 #2835 #2837 #2839 #2840 #2844 #2870 #2887 #2900 #2918
#2920 #2970 #3004 #3014 #3041 #3045 #3061 #3070 #3071 #3077 #3119
#3131 #3180 #3225 #3226 #3239 #3280

Pivotal Isocubic:

#2838 #2846 #2867 #2869 #2882 #2884 #2957 #2965 #3018 #3074 #3079
#3081 #3086 #3087 #3092 #3095 #3098 #3115 #3117 #3118 #3160 #3200
#3201 #3217 #3222 #3224 #3266 #3277 #3284 #3299

QA-Cu1:

#2850 #2867 #2868 #2869 #2882 #2883 #2884 #2886 #2889 #2890 #2894
#2899 #2901 #2918 #2919 #3015 #3030 #3031 #3033 #3065 #3073 #3095
#3110 #3116 #3132 #3134 #3135 #3149 #3150 #3189 #3192 #3195 #3198
#3207 #3208 #3214 #3232 #3237 #3276 #3277 #3278 #3281 #3282 #3283
#3284

QA-Cu7:

#2960 #2962 #2963 #2965 #2966 #2968 #2970 #2972 #2973 #2974 #2976
#2980 #2981 #2984 #2986 #2993 #3001 #3003 #3004 #3005 #3006 #3014
#3015 #3017 #3081 #3117 #3118 #3132 #3134 #3135 #3139 #3142 #3148
#3158 #3160 #3161 #3163 #3176 #3181 #3183 #3186 #3187 #3188 #3189
#3190 #3191 #3198 #3202 #3207 #3208 #3212 #3213 #3216 #3217 #3218
#3220 #3221 #3223 #3224 #3232 #3236 #3237 #3238 #3247 #3248 #3251
#3254 #3256 #3261 #3265 #3266 #3268 #3270 #3271 #3276 #3280 #3284
#3285 #3286 #3288 #3291 #3292 #3293 #3296 #3299

QA-P3:

#2805 #2809 #2841 #2850 #2867 #2868 #2882 #2883 #2885 #2918 #2919
#2920 #2946 #2948 #2989 #2990 #2992 #3029 #3030 #3031 #3033 #3036
#3065 #3066 #3067 #3070 #3071 #3078 #3080 #3082 #3083 #3085 #3088
#3110 #3116 #3198 #3256 #3263 #3276 #3277 #3278 #3279 #3281 #3282
#3283 #3284

QA-P4:

#2801 #2803 #2805 #2809 #2811 #2817 #2863 #2865 #2866 #2868 #2882
#2883 #2885 #2886 #2889 #2890 #2891 #2907 #2913 #2914 #2915 #2916
#2917 #2918 #2919 #2946 #2947 #2955 #2957 #2958 #2960 #2965 #2969
#2971 #2974 #2980 #2983 #2984 #2986 #2991 #2998 #3004 #3006 #3008
#3010 #3012 #3013 #3014 #3017 #3018 #3024 #3028 #3036 #3038 #3048
#3049 #3053 #3057 #3063 #3064 #3065 #3066 #3067 #3070 #3071 #3072
#3073 #3077 #3078 #3080 #3081 #3082 #3083 #3085 #3088 #3089 #3094
#3095 #3110 #3116 #3117 #3119 #3131 #3132 #3135 #3137 #3139 #3142
#3159 #3160 #3162 #3168 #3171 #3176 #3184 #3189 #3208 #3217 #3220

#3224 #3228 #3229 #3232 #3234 #3235 #3236 #3237 #3254 #3256 #3263
#3278 #3280 #3284 #3286

QA-P41:

#2811 #2868 #2869 #2882 #2891 #2955 #2957 #2958 #2960 #2965 #2970
#2971 #2983 #2984 #2991 #2998 #3004 #3006 #3015 #3018 #3019 #3073
#3081 #3117 #3132 #3139 #3142 #3163 #3176 #3189 #3208 #3217 #3232
#3234 #3235 #3236 #3237 #3254 #3256 #3263 #3276 #3277 #3278

QA-Tf16:

#3289 #3290

QL-Cu1:

#2804 #2827 #2828 #2831 #2846 #2884 #2899 #2900 #2901 #2918 #2919
#2931 #2954 #2955 #2956 #2958 #2960 #2965 #2970 #2971 #2973 #3004
#3008 #3014 #3016 #3041 #3045 #3061 #3087 #3097 #3100 #3131 #3134
#3135 #3163 #3178 #3180 #3189 #3198 #3208 #3237 #3280 #3281 #3282
#3284 #3286 #3291

Quartic:

#3031 #3036 #3048 #3049 #3053 #3057 #3061 #3065 #3066 #3068 #3070
#3071 #3073 #3074 #3080 #3083 #3086 #3087 #3088 #3092 #3095 #3097
#3130 #3193 #3194 #3290 #3293

Simson line:

#3035 #3076 #3250

Steiner Axes:

#2887 #2899 #2901 #2902 #3031 #3068 #3131 #3178 #3180 #3205

Steiner Line:

#3123

Tangential:

#2868 #2869 #2891 #2894 #2900 #3110 #3219 #3220 #3223 #3224 #3232
#3234 #3235 #3236 #3248 #3249 #3254 #3266 #3277 #3281 #3282

Van Rees:

#2828 #3008 #3014 #3041 #3097 #3154

circular cubic:

#3154 #3192 #3249 #3266

circular points:

#2835 #2967 #2976 #3004 #3134 #3154 #3179 #3181 #3183 #3185 #3189
#3191 #3192 #3198 #3212 #3220 #3222 #3223 #3242 #3244 #3247 #3248
#3266

[↔ Table of Contents](#) [↔ Message Index](#)

6 Colophon

Sources & Contact

Web address (QPG Forum): <https://groups.io/g/Quadri-and-Poly-Geometry>

EPG Encyclopedia (content reference): <https://www.chrisvantienhoven.nl>

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- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
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- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393