Journal of the Quadri Figures Group 2019

Digital Edition

Chris van Tienhoven et al.

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Messages #3300 - #3906

Contents

1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at https://groups.io/g/Quadri-and-Poly-Geometry. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at https://groups.io/g/Quadri-Figures-Group.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Benedetto Scimemi
- Bernard Gibert
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- David Fraivert
- Eckart Schmidt
- Stanley Rabinowitz
- Systems Manager
- Tran Quang Hung
- Tsihong Lau
- Vu Thanh Tung
- Wilson Logan
- Ángel Montesdeoca

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

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Wilson Logan:
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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Benedetto Scimemi (1938–2023)

Location
Italy

Year of Birth / Generation
1938–2023.

Short Biography
Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.esteri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter's 60th birthday (the "A5 Meeting", named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See CO-Tf3 in EPG.

- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke.* B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken.* MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten.* MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks.* PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles.* Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect.* Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen.* MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks.* MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Stanley Rabinowitz

Location
Living in New Hampshire, USA.

Year of Birth / Generation
1947 (Baby Boomer).

Short Biography
Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions
Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, International Journal of Computer Discovered Mathematics 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, International Journal of Computer Discovered Mathematics 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 214–287.

Additional Remarks
Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

subject: Deleted Message #3543:
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subject: Deltoid for a convex 5P:
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subject: Dual 4/5 P/L:
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subject: Dual lines in QA:
#3711 #3713 #3717 #3718 #3724

subject: EPG-question:
#3820 #3821

subject: epicyloidal curve around the sun:
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subject: Euler-Poncelet line in Quadrangle points:
#3376 #3377 #3379

subject: Feuerbach point lies on OI line:
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subject: Fixed points of CB-transformation cb:
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subject: Four related "Euler Line" analogs in a quadrigon:
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subject: Generalization of Eckart's observation in number 3842:
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subject: Gergonne circle:
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subject: LAST MESSAGE NEW START WITH A NEW GROUP:
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subject: last message(s) QFG-Yahoo-group:
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subject: Line passes through centroid of Quadrangle points:
#3382

subject: Line passes through Poncelet point in Quadrangle points:
#3381 #3383 #3384

subject: Loci wrt concentric circles:
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subject: Miquel circle of 5L:
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subject: Miquel circles are concurrent with 5L:
#3455 #3460 #3491

subject: Nagel Line Analog in a Quadrangle:
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subject: Name for line through Gergonne and Poncelet points of a quadrangle:
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subject: New 5P transformation:
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subject: New aspect of QA-Cu7-triple points:
#3493 #3494 #3495

subject: New aspects of 5P-geometry:
#3789 #3791 #3792 #3793 #3797 #3799

subject: New line to line transformation for QL:
#3810 #3811 #3812 #3813 #3822 #3823 #3824

subject: New Quartic for the 5P of the QA-Cu7-triple Points:
#3891

subject: Newton line passes through Poncelet point:
#3387 #3395 #3396

subject: Newton lines are concurrent in QA:
#3735 #3738

subject: Newton lines are concurrent with 5P:
#3448 #3451 #3452

subject: No Subject:
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subject: number 3382, 3386,number 3387:
#3388 #3390

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subject: On a 5-Gon point:
#3654 #3656 #3657 #3658

subject: Orthogonal QL-Hyperbolas:
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subject: Perpendicular lines from NPC in triangle:
#3744 #3746 #3751

subject: Perspective triangles in 6-Gon point:
#3659 #3664 #3666

subject: QA - A circumconic:
#3302 #3303 #3304 #3305 #3306 #3309 #3310 #3311

subject: QA-Cu1:
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subject: QA-Cu7-Geometry:
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4 Messages

4.1 Message Index

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4.2 Messages

Message: #3300
Date: 01/1/2019 4:30:59
From: eckart_schmidt@t-online.de
Subject: 8th CB-Point on a Cubic

Dear Bernard, dear Chris,

I don't know, whether the following geometry on cubics is
already discussed:
Well known is the 9th Cayley-Bacharach point
… as common point of all cubics through 8 points.
Also well known is, that 7 points define a CB-transformation
X-->CB(X).
… For 7 points on a cubic the CB-transformation maps the cubic
  to itself.
… The 3rd intersections of X.CB(X) and the cubic give the same
point Q,
… here named as 8th CB-point of seven points on the cubic.
For the cubic QL-Cu1 bearing the 6 QL-points and a point P*
… the 8th CB-point is Q = CSC(P).
For the cubic QA-Cu7 bearing 3 QG-P18, 3 QG-P19 and a point P
… the 8th CB-point is Q = QA-Tf2(P).
In this way we get a transformation P-->Q on a cubic,
… defined by 6 points on the cubic (see attached file):
Take the CB-transformation of the 6 defining points and P
… and consider for points X on the cubic
… the 3rd intersection of X.CB(X) as image Q of P.
By the way:
… CB(Q) is the tangential of Q wrt the cubic.
… The 9th CB-point of P1,2,3,4,5,6,X and CB(X) is P.
Examples (s ee also #3006 and #3176):
Let E be the infinity point of the asymptote of QA-Cu7
… and F be the intersection of QA-Cu7 and its asymptote,
… … which lies diametral to QA-P41 wrt the circumcircle
    of QA-P2,4,41.

(1) For QA-Cu7 with the vertices of QA-Tr1 and the QG-P19
triangle
… P-->Q maps QA-P2 to QA-P4, QA-P41 to E
… and F to the 3rd intersection of QA-P2.QA-P4 and QA-Cu7.
For QA-Cu7 with the vertices of QA-Tr1 and the QG-P18 triangle
… P-->Q maps QA-P2 to QA-P41, QA-P4 to E
… and F to the 3rd intersection of QA-P2.QA-P41 and QA-Cu7.
For QA-Cu7 with the vertices of the QG-P18 and the QG-P19
triangle
… P-->Q is QA-Tf2 and maps QA-P4 to QA-P41, QA-P2 to E
… and F to the 3rd intersection of QA-P4.QA-P41 and QA-Gu7.

(2) For QA-Cu7 with the vertices of QA-Tr1 and QA-P2, QA-P4, QA-P41
… the transformation P-->Q maps F to QA-P2.
For QA-Cu7 with the vertices of the QG-P18 triangle and QA-P2, QA-P4, QA-P41
… the transformation P-->Q maps F to QA-P41.
For QA-Cu7 with the vertices of the QG-P19 triangle and QA-P2, QA-P4, QA-P41
… the transformation P-->Q maps F to QA-P4.

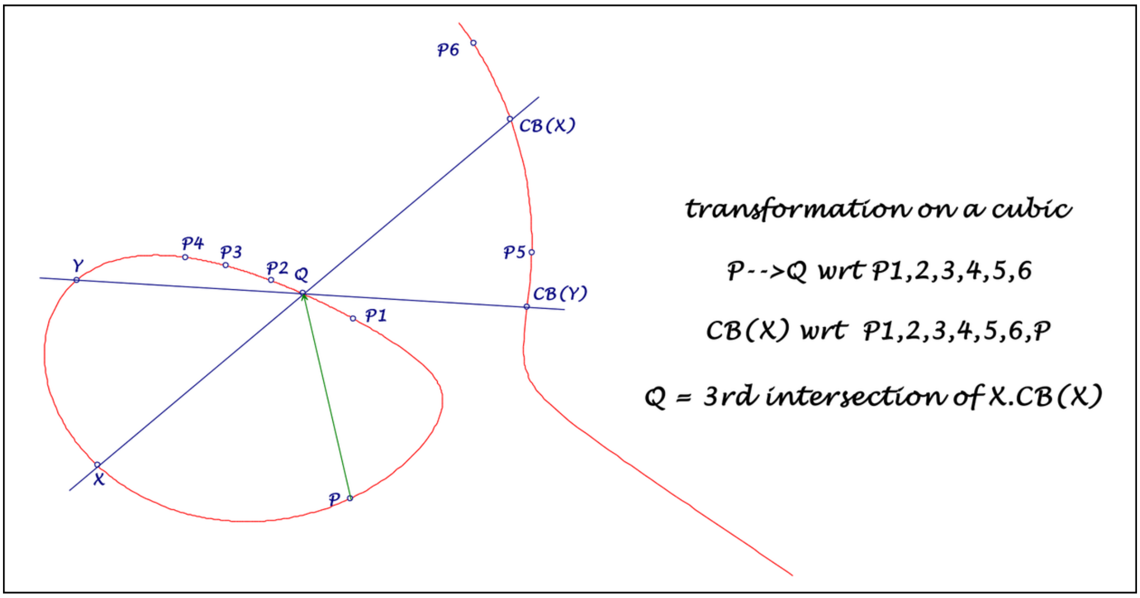
(3) For QA-Cu1 with the vertices of the triangles QA-Tr1 and QA-Tr2
… the transformation P-->Q maps QA-P3
… to the 3rd intersection of the cubic and the line
… … from QA-P41 to the intersection of the cubic
and its asymptote.
For QA-Cu1 with the vertices of QA-Tr2 and the QG-P16 triangle
… the transformation P-->Q maps QA-P4 to QA-P41.

Finally:
(4) Consider a QL and the 5 triple intersections of its 3 QA-Cu7,
… take a QG-version of the QL, interpreted as QA with its QA-Cu7,
… and the transformation P-->Q
… wrt the 5 triple intersections and a point P6 on QA-Cu7,
… … then the QG-point S (see #3176) will be mapped to P6.

Remark wrt (2) and (4):
If P is the 6th intersection of the cubic
… and a conic through 5 defining points,
… then Q is the 6th defining point.
This leads wrt (2) to three conics:
… circumconic of QA-Tr1 through QA-P4 and QA-P41,
… circumconic of the QG-P18-triangle through QA-P2 and QA-P4,
… circumconic of the QG-P19-triangle through QA-P2 and QA-P41,
with a common point in the intersection F of QA-Cu7
and its asymptote.

I hope, there are no grave errors.
Perhaps the observations can be described simpler.

Best regards Eckart



2019-01-01.pdf

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Message: #3301
Date: 03/1/2019 1:10:15
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

Dear Eckart,
I suppose, as long as we are here, the QFG is not dead !
8 points give a 9th CB point
7 points define a CB transformation
On a given cubic, the CB transformation of 7 points is an isoconjugation and the cubic is a pivotal isocubic with pivot Q, fixed points the vertices of the tangential QA of Q and reference triangle the DT of the QA.
All this may be proved with Cotterill's construction.
You make a step further by considering the transformation with 6 fixed points and a variable 7th point P and the pivot Q. Very clever indeed !
The 9th CB point of P1 to 6, P and X being CB(X), the 9th point of P1 to 6, X and CB(X) is P ...
For QL-Cu1 (which is a particular QA-Cu1), any point is QA-P4 of it's tangential triangle and the pivot is QA-P2, intersection of the 2 perpendicular lines of the QA (which form the degenerated polar conic of the point) and CSC of QA-P4 ...
For QA-Cu7, which is also a QL-Cu1, many of your properties derive from the definition of the 3 CSC of the triangle QA-P2, P4 and P41 ...
Of course, all this is true also for non circular cubics and for circular cubics, you may have the same properties with 5 points or 4 fixed points + 1 variable ...
I read your messages always with same pleasure, please go on !
Best regards
Bernard
PS How do you draw your cubics in Cabri ?

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Message: #3302
Date: 07/1/2019 5:30:02
From: César E. Lozada
Subject: QA - A circumconic

```
Let qa={Ai (i=1..4)} be a quadrangle.
Denote:
Kn = the nine-point conic QA-Co1 of qa, with center On = QAP-1
(i.e, the conic through all six Mi,j midpoints of AiAj, i≠j)
Qi = the perspector of Kn w/r to the triangle AjAkAl (i=1..4)
K* = the circumconic of qa through On

Then
Qi lies on K*, for i=1..4.
K* passes also through QA-P16 (QA-Harmonic center)
The diagonal triangle of qa is autopolar w/r to K*

-------------------------------------------------------

CT coordinates of center O* and perspector Q* of K* are:
O* = p*(q-r)^2*(2*p+q+r)*(3*(p+q+r)*p-q*r) : :
Q* = p*(q-r)*(2*p+q+r) : :

Best regards,
César Lozada
```

Message: #3303
Date: 07/1/2019 10:55:23
From: bernard.keizer
Subject: QA - A circumconic

Dear César,
Your conic K* is the dual curve of the parabola inscribed in the dual QL of the QA.
It passes also through the 3 S-points, 3 other intersections of the DT circumscribed conic through QA-P10 and QA-P16, which is the dual curve of the parabola inscribed in DT and tangent to the Newton Line.
The 3 S-points lie on the Dimidium circle QL-Ci6 and form the QL-Tr2, which is also autopolar wrt K*.
Best regards
Bernard

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Message: #3304
Date: 07/1/2019 11:05:54
From: eckart_schmidt@t-online.de
Subject: QA-A circumconic

Dear Cesar Lozada,

some remarks wrt #3302:
(1) What is the perspector of a conic wrt a triangle?
(2) I think, property 3) holds in general for QA-circumconics.
(3) In addition to the coordinates of O*:

O* is QA-Tf2 of the infinity point
… of the tangents at Kn
… in the intersections with QA-L3.

Best regards Eckart

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Message: #3305
Date: 07/1/2019 11:26:15
From: César E. Lozada
Subject: QA-A circumconic

Dear Eckart,

Thanks a lot for your quick response.
(1) What is the perspector of a conic wrt a triangle?
In the plane of a reference triangle
(http://mathworld.wolfram.com/ReferenceTriangle.html), the
perspector of ... and its polar triangle
(http://mathworld.wolfram.com/PolarTriangle.html) with respect
to a given conic is called the perspector of that conic. The
perspector is not defined for conics with respect to which is
self-polar
(http://mathworld.wolfram.com/Self-PolarTriangle.html).

For an inconic (http://mathworld.wolfram.com/Inconic.html), the
perspector is the Brianchon point
(http://mathworld.wolfram.com/BrianchonPoint.html) of the conic.
(http://mathworld.wolfram.com/Perspector.html)

(2) I think, property 3) holds in general for QA-circumconics.
It is good to know it.

Thanks again and best regards,
César Lozada

De:
Quadri-Figures-Group@yahoogroups.com
Enviado el: lunes, 7 de enero de 2019 06:06 AM
Para: -Group, Quadri-Figures
Asunto: [Quadri-Figures-Group] QA-A circumconic
Dear Cesar Lozada,
some remarks wrt #3302:
(1) What is the perspector of a conic wrt a triangle?
(2) I think, property 3) holds in general for QA-circumconics.
(3) In addition to the coordinates of O*:
O* is QA-Tf2 of the infinity point
… of the tangents at Kn
… in the intersections with QA-L3.
Best regards Eckart

Message: #3306
Date: 07/1/2019 1:43:10
From: César E. Lozada
Subject: QA - A circumconic

Dear Bernard,

Thank you for your valuable information.
Best regards,
César Lozada

De:
Quadri-Figures-Group@yahoogroups.com
Enviado el: lunes, 7 de enero de 2019 05:55 AM
Para: Quadri-Figures-Group@yahoogroups.com
Asunto: [Quadri-Figures-Group] Re: QA - A circumconic
Dear César,
Your conic K* is the dual curve of the parabola inscribed in the dual QL of the QA.
It passes also through the 3 S-points, 3 other intersections of the DT circumscribed conic through QA-P10 and QA-P16, which is the dual curve of the parabola inscribed in DT and tangent to the Newton Line.
The 3 S-points lie on the Dimidium circle QL-Ci6 and form the QL-Tr2, which is also autopolar wrt K*.
Best regards
Bernard

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Message: #3307
Date: 07/1/2019 2:51:30
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your encouraging message #3301,
… here some further observations,
… perhaps helpful for the 5 QA-Cu7-triple points:

In #3296 I described a transformation TF for a 5P P1P2P3P4P5 ,
… which is the CSC of a QL with Pi as QA-Cu7-triple points,
… and defines a new 5P-point M as Miquel point of this QL.

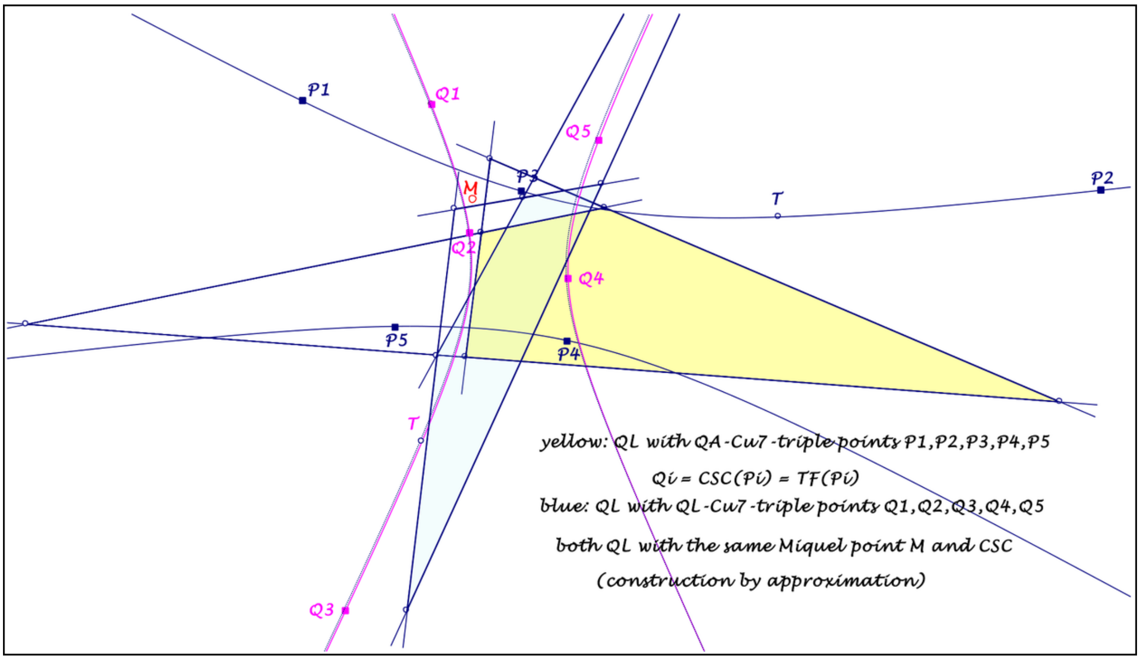
Further properties with Qi = TF(Pi):
(1) Circumcircles of Pi,Pj,Pk are centered on QlQm.
(2) PiPj^QiQj is the 2nd intersection
… of the circumcircles of Pi,Qj,M and Pj,Qi,M.

In #3288 you asked for the QL with QA-Cu7-triple points Qi,
… attached a drawing by approximation as exact as I could.
But I couldn't find no further connections between the two QL.

Best regards Eckart

PS. Wrt your question, how to draw cubics with CABRI:
For pivotal isocubics
… take the line pencil of the pivot
and consider
… the intersections of the lines and their isoconjugate conics.

For nonpivotal cubics
… I studied Gibert's Special Isocubics 1.5.4,
… difficult to understand and to describe!



2019-01-07.pdf

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Message: #3308
Date: 07/1/2019 7:11:32
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

Dear Eckart,
Reading again your message, I spent the day in drawings !
Thanks to you, I begin to understand better the QA-Cu7.
Let's start with a QA-Cu7 or bicursal QL-Cu1 (the QA-Cu7 are always bicursal ?) defined with the 3 points QA-P2,P4 and P41 of an unknown QA.
Taking 2 points A and A' on the curve, it's possible to find the corresponding QA by using the properties of the curve.
Let's consider the csc with center QA-P41 and swapping P2 and P4.
Acsc(A') and A'csc(A) intersect in QG-P18
AA' cuts the curve in a 3rd point, which is csc(QG-P18).
The conic through A,A',P4 and 41 and F cuts the cubic in a 6th point QG-P1 and the conic through A,A', P2 and 41 and F cuts the cubic in a 6th point QG-P19. QG-P1 and QG-P19 are csc partners and the 4 points are the vertices of the tangential QA of the same point.
The vertices of the tangential QA of csc(QG-P18) are the points U2,U3,V2 and V3 and the DT of this QA is QG-P1,P18 and P19.
U2V2 and U3V3 intersect in QG-P1 and form 2 of the 3 DT sides of the searched QL ; the 2 other DT vertices are the intersections with AA', which is the 3rd DT side.
Now the CSC of the QL is given by the 2 copples of CSC partners A and A' and QG-P1 and QA-P4.
Having the CSC and the DT, it is possible to find the 4 other vertices of the QL, which are the vertices of the QA having the 1rst curve as QA-Cu7.
Finally, it's possible to draw the 2 other QA-Cu7 of the QL and to find the 5 triple points !
3 points for the QA-Cu7 and 2 points A and A' on it give the QL.
I suppose now that the same 3 points for the QA-Cu7 and 2 of the 5 triple points will give also the same QL ...
Last step must be the 5 points alone !
Best regards
Bernard

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Message: #3309
Date: 07/1/2019 9:55:20
From: eckart_schmidt@t-online.de
Subject: QA - A circumconic

```
Dear Cesar Lozada,

thanks for the explanation of
… the perspector of a conic wrt a triangle.
Is the following property well known?
F or any conic, not bearing the quadrangle points Ai,
… the perspectors of the conic wrt the triangles
  of the quadrangle
… lie on a circumconic of the quadrangle.

Best regards Eckart
```

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Message: #3310
Date: 07/1/2019 11:20:20
From: César E. Lozada
Subject: QA - A circumconic

Dear Eckart,

I ignore if your proposed property is well known (I am
light-years away from the subject of quadrifigures), But it
seems to be true that:
F or any conic, not bearing the quadrangle points Ai,
… the perspectors of the conic wrt the triangles
  of the quadrangle
… lie on a circumconic of the quadrangle.

Alternative property:
Given a quadrangle Ai and a conic K, such that Qi are the
perspectors of K wrt the triangles of the quadrangle,
… the conic K* circumscribing Ai and passing through any one
  of the perspectors Qi also passes
  through the other three perspectors.

Best regards,
César

De:
Quadri-Figures-Group@yahoogroups.com
Enviado el: lunes, 7 de enero de 2019 04:55 PM
Para: -Group, Quadri-Figures
Asunto: [Quadri-Figures-Group] Re: QA - A circumconic
Dear Cesar Lozada,
thanks for the explanation of
… the perspector of a conic wrt a triangle.
Is the following property well known?
F or any conic, not bearing the quadrangle points Ai,
… the perspectors of the conic wrt the triangles of the
quadrangle
… lie on a circumconic of the quadrangle.
Best regards Eckart

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Message: #3311
Date: 08/1/2019 10:35:35
From: bernard.keizer
Subject: QA - A circumconic

```
Dear César, dear Eckart
Is it true that both QA's (the Ai and the Qi) have the same DT ?
Any circumconic of the QA bears the vertices of the anticevian
triangle of any of it's points ...
Best regards
Bernard
```

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Message: #3312
Date: 08/1/2019 12:02:37
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

```
Dear Eckart,
Step by step ...
Having the same QA-Cu7 (3 points) and it's csc and the CSC (2
points), it's possible to draw CSC(QA-Cu7), which is a quartic
(?) through QL-P1 (center of the CSC).
Both curves intersect in 6 real points, 3 copples of CSC
partners QA-P4 and QG-P1, A and A' and 2 points which are at the
same time csc and CSC partners.
With A and A', we have the same reconstruction as in my previous
message.
Best regards
Bernard
```

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Message: #3313
Date: 08/1/2019 2:49:28
From: tsihonglau
Subject: 8th CB-Point on a Cubic

Dear all,

>>I suppose, as long as we are here, the QFG is not dead !

I think I have not posted here for a year, so this group became
quieter. I started a facebook group "Plane Geometry Research" a
year ago.
I posted my research results to it. I have many results last
year. I'll post them as soon as possilble. I share results to
this group.
https://www.facebook.com/groups/2008519989391030/ Plane Geometry
Research
http://lth.name / My website see Plane Geometry Research section
http://lth.name/geometry/involution_three_algebraic_curve.html
<- A recent page, isogonal conjugation=QA-Tf16, this conjugation
is mentioned by me in topic #2710

Best regards,
Tsihong Lau

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Message: #3314
Date: 08/1/2019 2:59:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your detailed message.
You describe QA-Cu7
… as QA-Cu1 of QAs and QL-Cu1 of QLs.

Here my sight:
(1) QA-Cu7 as QA-Cu1 of a new QA:
You get one vertex as reflection of QA-P41
… in the center of the circumcircle of QA-P2,4,41
… and the other vertices Möbius-transformed wrt QA-P2,4,41.

This new quadrangle has
… the triangle QA-P2,4,41 as Miquel triangle,
… its isogonal conjugated as isoconjugation
… and pivot in the infinity point of the bisector of
QA-P2.QA-P4.

(2) QA-Cu7 as QL-Cu1 of a QL:
Consider the new QA in (1),
… which has vertices in pairs on orthogonal lines,
… so it leads to a QL
as described in #1423, last passage,
… wrt two points on QA-Cu7.
This shows once more,
… that two QA-Cu7-triple points give the other three.

Best regards Eckart

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Message: #3315
Date: 08/1/2019 4:41:43
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm a little bit disappointed.
In 1), you give me another QA of the same curve, formed by the
point F (your notation) and the 3 isogonals of the vertices
QA-P2,4 and 41.
In 2) you give the corresponding QL with 2 new csc partners on
the curve.
Finally I don't understand the remark wrt 2 triple points giving
the 3 others (you need the CSC ...)
I'm afraid you have read my messages too quickly !
In my first message, I start with a QA-Cu7 with it's csc.
Then I take 2 random points A and A' on this curve and find
1) the QA having this curve as QA-Cu7
2) the QL having this curve as one of it's QA-Cu7
3) the 5 triple points as intersection of the 3 QA-Cu7
4) the CSC of the QL which is the same as the CSC of the 5
points
The CSC is variable and depends from the 2 points A and A'.
In the 2nd message, I start with the same QA-Cu7 and the CSC and
find the 2 points A and A' by intersecting the QA-Cu7 with it's
CSC (by the way, it is a sextic, not a quartic)
I try now to find how 2 of the 5 triple points on the same
QA-Cu7 could give me the CSC and the 3 other triple points.
Finally, I hope we will find how the 5 triple points and their
point T and their CSC give the QL ...
Did I miss something in your message ?
Thanks in advance for your attention
Best regards
Bernard

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Message: #3316
Date: 09/1/2019 8:26:23
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

please accept my apologies, I am still studying your first message 3308,
… and your message 3312 I have read after my answer.
After a first look in #3308, I wanted only to inform you of my sight
… wrt QA-Cu7 as QA-Cu1 of a QA and QL-Cu1 of a QL,
… for this seems the theme of your message for me.

Best regards Eckart

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Message: #3317
Date: 09/1/2019 4:06:41
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

```
Dear Bernard,

WOW
- that was a hard work, to reproduce your #3308!
Using your nomination, your final QL has the lines
… A.A', csc(A).csc(A'), A.csc(A'), A'.csc(A).
This is the same QL as in my sight.

Best regards Eckart
```

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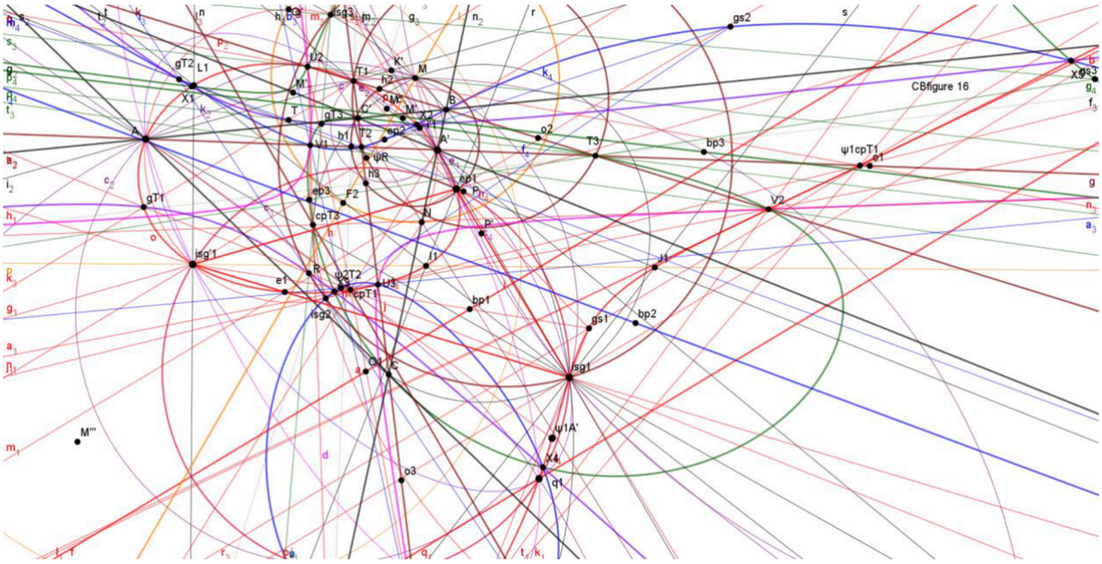
Message: #3318
Date: 09/1/2019 6:24:17
From: bernard.keizer
Subject: QA-Geometry

```
Dear Eckart,
This time, I dare to send you a figure and some explanations ...
But I'm afraid it is really difficult to read !
My idea is to find always 5 points in order to find the searched
QL : 3 points for QA-Cu7 and 2 points on it, then the same with
the CSC, defined by 2 points, then the same and 2 of the 5
triple points and last the 5 triple points alone.
I hope we will succeed in finding the mystery this year !
Best regards and thanks for your attention
Bernard
```



It’s hardly possible to read this figure !

The 1rst QA-Cu7 is defined by the 3 points QA-P2 or ep1, QA-P4 or isg1 and QA-P41 or isg’1 (red curve)

2 points A and A’ on this curve determine QG-P1 or T1, QG-P18 or cpT1, which is the intersection of Acsc(A’) and A’csc(A), QG-P19 or csc(QG-P1) and csc(cpT18), which is the 3rd intersection of AA’ with the cubic.

I use ψ1 for csc and ψ for CSC.

The vertices of the tangential QA of csc(QG-P18) give the points U2, V2 and U3,V3.

AA’, U2V2 and U3V3 (which intersect in T1 or QG-P1) are the 3 diagonals of the searched QL and give T2 and T3, the 2 other DT vertices.

A and A’ are CSC partners as well as T1 or QG-P1 and QA-P4 or isg1 as well as h1 or QG-P17 and cpT1 or QG-P18. This give the CSC with center M or QL-P1.

The CSC and the DT give the vertices B,B’,C and C’ of the QA having the red curve as QA-Cu7.

The 2 other curves in blue and green are the QA-Cu7 of the QA’s A,A’,B and B’ and A,A’,C and C’.

The 3 curves intersect in the 5 triple points X1,2,3,4 and 5 ; the conic is through T and R.

QA-Cu-7 and 2 points.pdf

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Message: #3319
Date: 10/1/2019 1:05:52
From: bernard.keizer
Subject: QA-Geometry

Dear Eckart,
I think it's a little bit clearer now.
Taking again the initial QA-Cu7 and 5 random points on it.
We consider 3 cubics through the 5 points, the QA-Cu7, the degenarated cubic formed by a line through 2 points and the circle through the 3 others and the degenarated cubic formed by the conic through the 5 points and the infinity line.
The line through the 2 points X1 and X2 cuts the cubic in U, the circle through X3, X4 and X5 cuts the cubic in V and the conic in W ; the conic intersects the cubic in a 6th point gT (isogonal of T).
The parallel through gT to the asymptote of the cubic cuts the conic in T and the cubic in T1.
T lies also on the parallel to X1X2 through W.
Last T1V and TW intersect in P on the circle.
U and V are CB partners with pivots T1 and P on the 1rst and the 2nd cubic.
gT and the infinity point of the asymptote are CB partners on the 1rst and 3rd cubic with pivots T1 and T.
W and the infinity point of X1X2 are CB partners on the 2nd and 3rd cubics with pivots P and T.
This construction is always pssible.
For 2 given points, there is only 3 other on the curve such as the CSC of the 5 points swaps QA-P4 and T1 (QG-P1 of the searched QL).
More simply, for 2 given points, T has only one possible position to allow the construction.
I suppose you will agree with that as it is your conjecture !
Best regards
Bernard
PS For 5 points, there are only 3 possible QA-Cu7 and one possible QL and, as you already mentionned, all the QL-points are 5P-points.

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Message: #3320
Date: 10/1/2019 4:04:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

I study your #3319, but:
(1) What are the seven points for these CB-partners?
    "U and V are CB partners with pivots T1 and P …"
(2) What is the QA for QA-P4 in the bold remark?
Thanks in advance!

Best regards Eckart
```

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Message: #3321
Date: 10/1/2019 5:57:37
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for your interest !
I took 5 random points on a circular cubic defined by QA-P2, QA-P4 and QA-P41 and the csc with center QA-P41 swapping QA-P2 and QA-P4.
1) The 7 points are the 5 points and the 2 circular points
2) QA-P4 is defined as above and the pivot T1 of the cubic must be the CSC of QA-P4 with CSC defined by the 5 points
Best regards
Bernard
PS Is it possible, by any chance, that the twin QL I was looking for (my message 3296 and your message 3307) is simply defined by the CSC of the 5 points and the triangle of the QA-P4 taken as DT ?
In this case, the 3 QA-Cu7 of the twin QL are through the 5 CSC of the 5 points and through the 3 copples of CSC partners QA-P4 and QG-P1 ...

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Message: #3322
Date: 11/1/2019 9:58:45
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your explanations,
… but further questions wrt #3319:
(1) Why are U and V CB-partners
… wrt the 5 points and the circular points?
I only understand CB-partner for infinity points.
(2) You consider three cubics
and later their pivots,
… but what is your understanding of pivot?
(3) What is the relation between CB-partners and pivots?
Excuse my lack of understanding!

Best regards Eckart

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Message: #3323
Date: 11/1/2019 12:08:38
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
1) U and V are CB partners as there are 2 cubics through the 9
points 5 points + 2 circular points + U and V
2) The pivot for 7 points on a given cubic is your 8th point in
message 3300
3) 2 CB partners are on line with the pivot on each cubic
Best regards
Bernard
PS What do you think from my last conjecture about the twin QL ?
```

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Message: #3324
Date: 11/1/2019 2:59:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
My conjecture about the twin QL doesn't hold, as QL-P1 is not on
the Euler circle of the triangle of the QA-P4.
Best regards
Bernard
```

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Message: #3325
Date: 11/1/2019 4:31:36
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
It could perhaps interest you (or is it obvious ?)
CSC(R) on the DT circumcircle QL-Ci1 is the inverse of QL-P1 in the inversion of center QL-P10 swapping the DT Euler circle QL-Ci2 and the DT circumcircle QL-Ci1 (and therefore the QG-P1 and the QG-P17).
Neither R (on the QA-P4 circumcircle) nor this point are in EQF.
Best regards
Bernard

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Message: #3326
Date: 12/1/2019 11:11:16
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
For the twin QL, we have to find a triangle of QG-P17 being the orthic triangle of a 2nd one of QG-P1.
The CSC of QG-P1 are the QA-P4, the CSC of QG-P17 are the QG-P18 and the QA-P4 and the QG-P18 are perspective with perspector R on the conic through the 5 CSC of the triple points.
Last, the circumcircle of the QG-P17 must pass through QL-P1.
I've found in the picture only 3 such circles the QG-P17, which gives the 1rst QL with R on the conic of the triple points and the QA-P41, but the corresponding point R is not on the conic of the CSC of the triple points.
I give up !
Any other idea ?
Best regards
Bernard

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Message: #3327
Date: 12/1/2019 11:24:50
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
The triangle CSC(QA-P2) as DT doesn't work either ...
The fight goes on !
Best regards
Bernard
```

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Message: #3328
Date: 12/1/2019 3:00:29
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

thanks for answering my questions and further information,
… especially  # 3319 and # 3325,
… but I can't add new aspects.
I try the following way:
Starting with a 5P and its circumconic,
… the transformation TF (see # 3296),
… which is CSC of the searched QL,
… gives a 2nd 5P and a 2nd circumconic,
… intersecting the 1st circumconic in *4 points* ,
… which may have a connection with the *4 lines* of the QL.
But up to now without result.

Best regards Eckart
```

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Message: #3329
Date: 12/1/2019 4:05:47
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
In this case, as it is reversible, the 4 intersection points would have a connection with the 4 lines of both QL's (the 1rst one and the one I name the twin) ...
I've just tried the symmetrics of the DT wrt QL-P1, but it doen't work either.
I think I stop to search blindly.
I have another idea.
As you provided an approximative image of both QL's with the 5 points and their CSC, could you, without too much efforts, put on the figure some elements as Steiner Lines, circle of inversion, both DT's and QA-P4's and both QG-P17 and QG-P18, giving at the end the points S and S' and R and R' (you already put M, T and T').
I can't imagine how difficult it is, but I'm sure we could find some new ideas ...
Thanks in advance
Best regards
Benard

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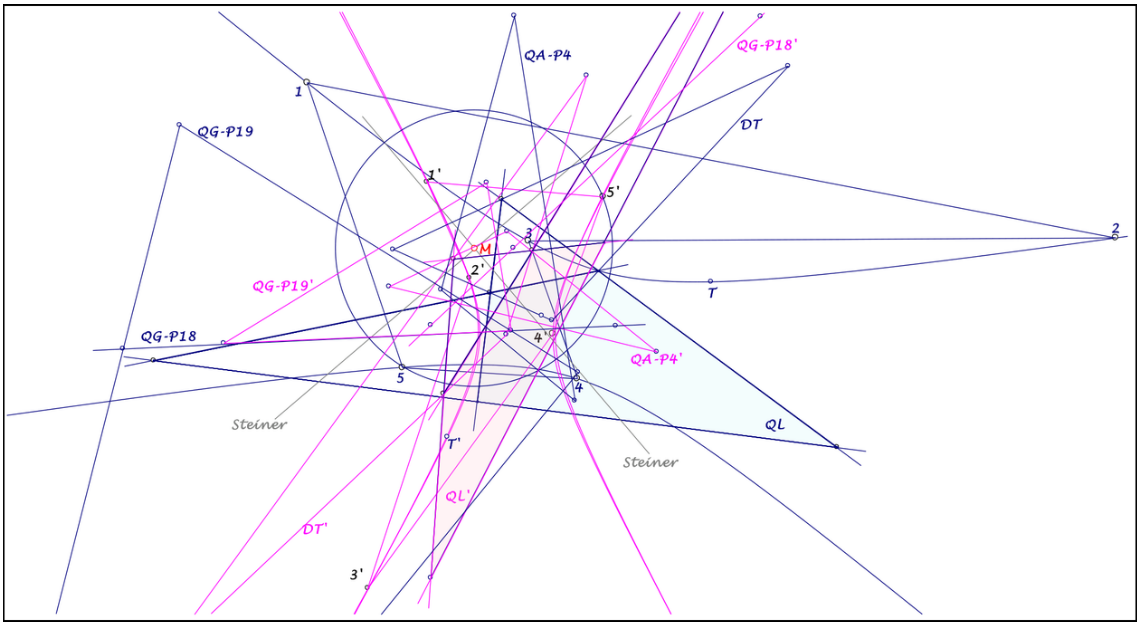
Message: #3330
Date: 13/1/2019 10:14:03
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

```
Dear Bernard,

I tried a drawing, you demanded for,
… but you see the approximation in the circumconic
  of the twin QL.
I am lost in this jungle and hope you find a way!

Best regards Eckart

PS. I couldn't prevent, that the inversion circle bears P5.
```



2019-01-13.pdf

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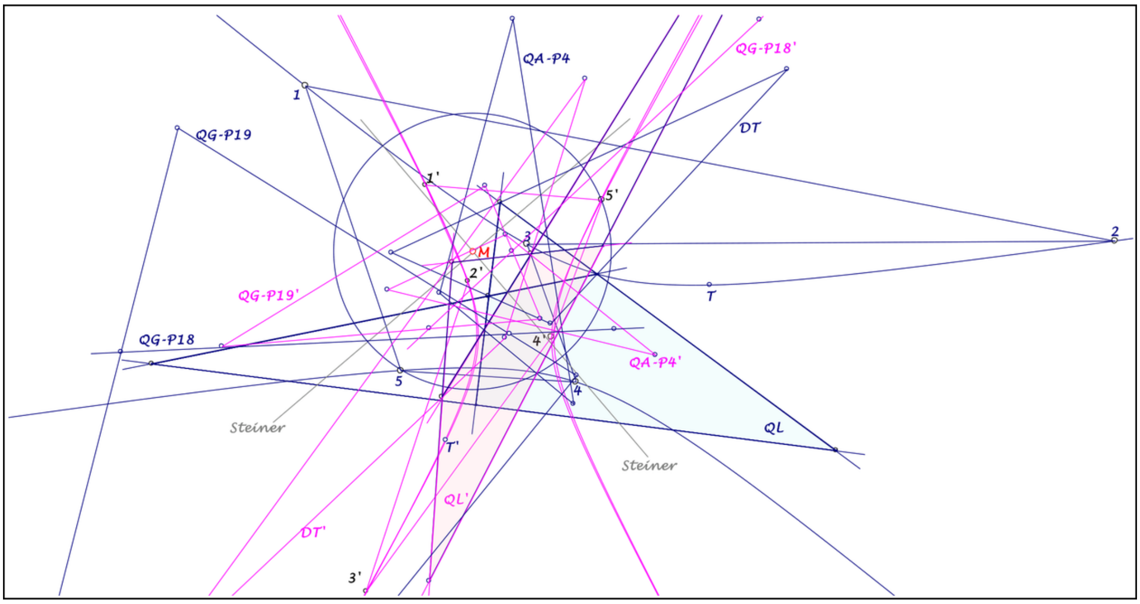
Message: #3331
Date: 13/1/2019 10:33:59
From: eckart_schmidt@t-online.de
Subject: No Subject

```
Dear Bernard,

excuse, there was a mistake in the drawing
… wrt the triangle of QG-P19'.

Best regards Eckart
```



2019-01-13-3331.pdf

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Message: #3332
Date: 2020-03-08
From: System Manager
Subject: Deleted Message
3332

```
Message number 3332 was a deleted message in Yahoo Groups.
```

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Message: #3333
Date: 17/1/2019 12:17:33
From: eckart_schmidt@t-online.de
Subject: 5P-s-P4

```
Dear Bernard, dear Chris,

I think, the main property of 5P-s-P4
… should be mentioned in EPG:
Let T be 5P-s-P4 of P1,2,3,4,5:
… T lies on the circumconic 5P-s-Co1.

Further properties:
Let Pi' be 5P-s-P4 of T,Pj,Pk,Pl,Pm,
… then holds PiPi' parallel to the tangent in T.

Let X be a point on 5P-s-C01
… and Xi = 5P-s-P4 of X,Pj,Pk,Pl,Pm,
… then holds PiXi parallel XT.

Perhaps we should study 5P-geometry,
… to get the QL for the 5 QA-Cu7-triple points.

Best regards Eckart
```

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Message: #3334
Date: 17/1/2019 4:31:04
From: bernard.keizer
Subject: 5P-Geometry

Dear Chris, dear Eckart,

Eckart invites us to study the 5P-Geometry in order to understand the 5 triple points of the QA-Cu7.
I think I've understood the mystery of these points.
Each 5P defines a conic 5P-s-Co1, a point T on this conic as 5P-s-P4 and a transformation TF, which is the CSC such as 2 points are inverse wrt the circle of the CSC and vice-versa (Eckart gave a construction).
Let's name Xi the 5 points.
For any point X on 5p-s-Co1, the parallel to TX through the Xi intersects the conic in a 2nd point Ti, which is 5P-s-P4 of the 5 points Xj,Xk,Xl,Xm and X (see Eckart 3333).
Let's now define TFi as the TF of the 5 same 5 points (with X replacing Xi).
There are only 3 positions of X on the conic for which the 6 circles TF(TX) and TFi(XiTi) concur in a point; let's name S1,S2 and S3 these 3 positions and K1,K2 and K3 the corresponding points of concurrence.
For one position, S1 for example, CSCi(K1) defines a point Z1 on S1T and 5 points Z1i on XiTi.
The curve through the points K1, S1 and the 5 Xi, Z1 and the 5 points Z1i is one of the 3 QA-Cu7 having the 5 points Xi as triple points, Z1 is QG-P1 and K1 is QA-P4.
Z1S1, Z2S2 and Z3S3 intersect on the conic in the point T and K1S1, K2S2 and K3S3 intersect on the conic in the point R.
(The reason is that for this curve, if S1 is one of the 3 points S, each Xi must be one of the 3 points S of the 5 points X, Xj,Xk,Xl and Xm).
Best regards

Bernard

PS The understanding doen't give me a simple construction ...

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Message: #3335
Date: 18/1/2019 12:01:41
From: bernard.keizer
Subject: 5P-Geometry

Dear Chris, dear Eckart,
Reading my message again, I notice that a few words are missing.
The CSC for 5 points is such as 2 points are inverse wrt the circle of the CSC of the 3 others
and vice-versa.
On the cubic, the direction of the XiZi or S1Z1 is obviously the direction of the asymptote and Xi and Z1i, as well as S1 and Z1 are isogonal wrt the triangle QA-P2,4 and 41 (we know only QA-P4).
Then, the property is on the curve, which is one of the 3 QA-Cu7 of the initial QL, that the isogonal of Xi is CSCi(QA-P4), where CSCi is the CSC of the 5 points Xj, Xk, Xl, Xm and the isogonal of QG-P17. The isogonal of Xi is the QG-P17 of another QL having these 5 points as triple QA-Cu7 points.
This gives 5 other different QL's, sharing the QA-Cu7 with the initial QL (but of course not the 2 others).
I hope these precisions will help the comprehension of my message and I'm waiting impatiently for your comments ...
Best regards
Bernard

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Message: #3336
Date: 18/1/2019 2:31:49
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard,

I try to reproduce the constellations of #3334,
… but I stopped at ...
  "There are only 3 positions of X on the conic
   … for which the 6 circles TF(TX) and TFi(XiTi)
     concur in a point".
I observed four positions for X,
… using a 5P with a hyperbola as circumconic,
… for the 5 QA-Cu7-triple points seem always
  to give a hyperbola.
I observed only two positions for X,
… using a 5P with an ellipse as circumconic.

Can you confirm my observations?

Best regards Eckart
```

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Message: #3337
Date: 19/1/2019 4:09:20
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
Thanks for your interest and your quick answer !
Unfortunately, I won't be able to confirm your observations
before a certain time.
My computer is out of order and needs repairing.
If you identify the point of concurrence of the 6 circles, then
the CSC and CSCi of this point give 6 points Z and Zi on the
parall lines TX and TiXi and the curves through the 12 points X
and Xi and Z and Zi must be the searched circular cubics.
If you have 4 points (hyperbola), 3 should be the searched
points S1,2,3, isogonals of the QG-P1, the 4th could be the
point R ...
If you have only 2 points (ellipse), I don't know what to say.
Beautiful new field, indeed!
Best regards
Bernard
```

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Message: #3338
Date: 20/1/2019 10:34:36
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard,

I hope your computer will be quick repaired,
… for my observations don't confirm your results,
… attached a drawing of your constellation in #3334,
… which shows, that your statement in #3337 doesn't hold:
  "If you identify the point of concurrence of the 6 circles,
… then the CSC and CSCi of this point give 6 points Z and Zi
   on the parall lines TX and TiXi
… and the curves through the 12 points X and Xi and Z and Zi
   must be the searched circular cubics."

Best regards Eckart

PS: Your points Si in #3334 are not the points Si in #3337!
```

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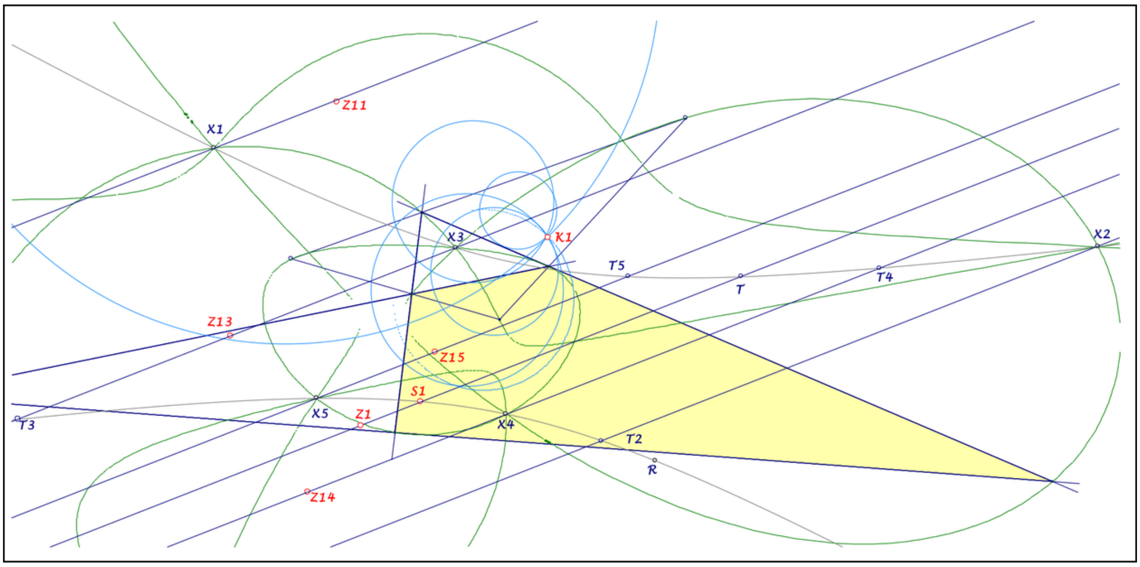
Message: #3339
Date: 20/1/2019 10:40:14
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard,

I try once more to send the lost attached file of #3338.

Best regards Eckart
```



2019-01-20.pdf

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Message: #3340
Date: 20/1/2019 5:13:27
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
Thanks a lot for your researches and for your figure!
In fact it seems unfortunately you are right and the points I
named S1 ... are not the points S of the 3 QA-Cu7.
I promise to send you figures as soon as I will be able to do
it!
The mystery of the triple points remains and we have a new
mystery to deal with ...
At this stage, I can only confirm that there are 2 points for an
ellipse.
The points X1,X2,X3,X4,X5,Z1,Z11,Z12,Z13,Z14,Z15 and S1 (but not
K1) are cocubic on a curve which looks like a circular Van Rees
curve QL-Cu1 with TS1 as direction of the asymptote. For one of
the 2 points, the curve is unicursal, for the other, it's
bicursal.
On your own figure, the same points seem also to be on such a
figure ...
Best regards
Bernard
```

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Message: #3341
Date: 26/1/2019 3:51:03
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

a new aspect in QA-Cu7-Geometry:
Remember the construction of QA-Cu7 in EQF:
… Orthogonal lines L1, L2 through QA-P4
… give with the intersections L1 ^ QA-Tf2(L2) the cubic QA-Cu7.
… Let us consider these QA-cubics CU, replacing QA-P4 by any
  point X.
The cubics CU(X) are nonpivotal isocubics with
… reference triangle QA-Tr1,
… isoconjugation QA-Tf2,
… root ??? (see PS)
```

… bearing X and QA-Tf2(X).
If we consider CU(X) for a QA,
… bearing a given point Y:
… the locus for X is a circle CI(Y)
… through Y and diametral QA-Tf2(Y).
A QL has three QG/QA-versions,
… which give three circles CI(Y),
… which have collinear centers.
For a QA-Cu7-triple point Y
… CI(Y) bears Y, QA-Tf2(Y) and QA-P4.

Now we consider CU(X) for the three QG/QA-versions of a QL:
CU(X) for points X on a QL-line:
… each bearing two opposite QL-points
  and a QL-diagonal crosspoint,
… with 6 coconic double intersections on the QL-diagonals
… and 5 triple points beside X.
CU(X) for a QL-point X,
… not vertex of the considered QG-version,
… degenerates in 3 lines:
… two lines are angle bisectors at X
… the 3rd is the opposite line of QG-Tr1,
… bearing three remaining triple intersections
  with the other CU(X).
CU(X) for infinity points of QL-lines
… degenerate in conics (and the line at infinity?)
… with three real collinear double intersections
… and three remaining real triple intersections.
CU(X) for points on QL-Cu1
… have six triple points beside X, *which give a new QL*.
CU(QL-P1) lead in this way to a new QL
… with the same QL-P1, QL-L2 and QL-Co1.

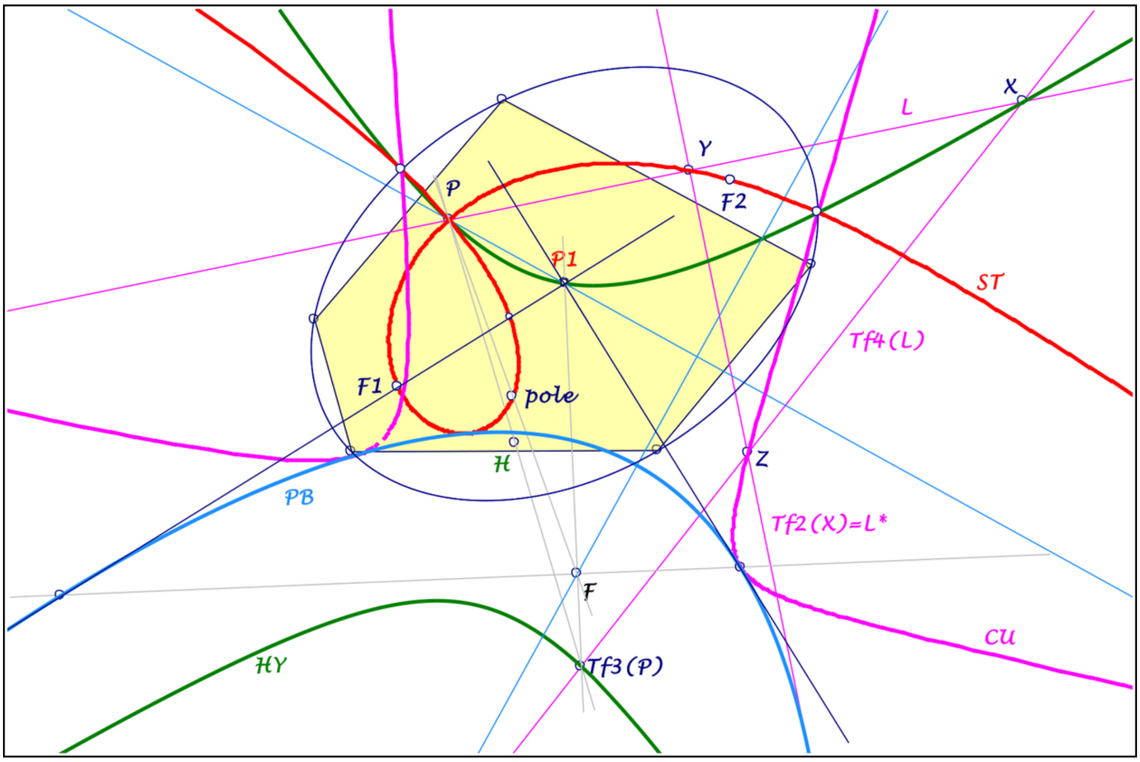
There will be more properties,
… but I found no expected links to the 5 QA-Cu7-triple points.
I hope, someone can confirm my observations.

Best regards Eckart

PS: The root of CU(X) lies on the line,
    … connecting the QA-Tr1-tripoles of X and QA-Tf2(X).

Message: #3342
Date: 28/1/2019 3:15:20
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard, dear Chris,

consider a 5P, a point P and lines L of its line pencil,
… the prefix "5P-s-" is omitted, so P1 = 5P-s-P1 and Tfx =
5P-s-Tfx,
… we start with the observation:
  L = Tf2(L^Tf4(L)) is perpendicular L.

Related properties:
(1) For the line pencil of a point P (see #2792):
… The lines Tf4(L) intersect in Tf3(P),
… The points X = L^Tf4(L) give an orthogonal hyperbola HY,
… … through P, Tf3(P) and P1,
… … centered in the midpoint H of P.Tf3(P)
… … with asymptotes parallel to the axes of Co1.

(2) For the line pencil of a point P:
… The lines L* envelope a parabola PB,
… … tangent to the axes of Co1 on the polar of P1,
… … with directrix P.P1 and focus F on the polar of P1
… … in the intersection of P1.Tf3(P) and Tf2(H),
… … which intersects the Co1-axes in the contact points.
… The polar of P wrt PB
… … is a perpendicular line to PF in F.

(3) For the line pencil of a point P:
… The intersections Y of the orthogonal lines L and L*
… … give a strophoid ST for the line P.P1 with fixed point P
    and pole F,
… … bearing the foci of Co1,
… … tangent to PB and intersecting HY on Co1.

(4) For the line pencil of a point P:
… The intersections Z of Tf4(L) and L*
  give a cubic CU,
… … sometimes with double point Tf3(P),
… … tangent to PB, intersecting HY and ST on Co1.

I hope, there are no false observations.

Best regards Eckart
```



2019-01-28.pdf

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Message: #3343
Date: 29/1/2019 9:07:23
From: eckart_schmidt@t-online.de
Subject: Special QL on QL-Cu1

```
Dear Bernard, dear Chris,

well known see #1423:
… Two points X and Y on QL-Cu1 give with the lines
… X.Y, X.CSC(Y), CSC(X).Y, CSC(X).CSC(Y) a new QL
… with the same QL-P1, QL-L1, QL-Cu1, further QL-Cu2.

Let us now start with two CSC-partners on QL-Cu1:
… The tangents from these points to the parabola QL-Co1
… give a new QL with a lot of common elements:
… QL-Ci3, QL-Co1, QL-Cu1, QL-Cu2, QL-Qu1,QL-Qu2,
… QL-L1,2,3,4,5,6,
… QL-P1,2,4,7,19,22.
There will be more.

Best regards Eckart
```

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Message: #3344
Date: 29/1/2019 10:33:23
From: eckart_schmidt@t-online.de
Subject: QG-2

```
Dear Chris,

cleaning my desk, perhaps of interest ...
In addition to QG-2 some lines QG-P1.QG-Px, or QG-P1.QA-Px,
… whose three versions for a quadrilateral
  have a common QL-point,
… (but not mentioned in QG-2):
… QG-P1.QG-P6.QG-P10 ---> infinity point of QL-L2,
… QG-P1.QG-P17.QA-P12 ---> QL-P10,
… QG-P1.QG-P19 ---> new QL-point,
… QG-P1.QA-P17 ---> new QL-point,
… QG.P1.QG-P18 ---> new QL-point,
… QG-P1.QA-P20 ---> infinity point of QL-L1,
… QG-P1.QA-P23 ---> new QL-point,
… QG-P1.QA-P29 ---> new QL-point,
… QG-P1.QA-P41 ---> new QL-point.

Not only starting with QG-P1, there are further QG-lines,
… leading to new QL-points,
… already mentioned in the discussion "3 QL-versions of QA-Cu7",
… using the QG-point S = QG-P18.QA-P4 ^ QG-P19.QA-P41 (#2980):
… QG-P1.S ---> new QL-point T (#2981),
… QG-P18.S.QA-P4 ---> new QL-point R (#2974),
… QG-P19.QA-P4 ---> new QL-point U (Chris summary 2983).

There is a curious property for some QG-/QA-points Px
… wrt their three lines QG-P1.Px for a quadrilateral:
… QG-P1a.Pxa, QG-P1b.Pxb and QG-P1c.QA-Tf2(Pxc)
  have a common point.

This property holds
… for QG-Px with x = 4, 8, 14, 16, 18,
… for QA-Px with x = 1, 2, 4, 5, 7, 10.

Best regards Eckart
PS. No claim for completeness!
```

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Message: #3345
Date: 29/1/2019 10:33:48
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

cleaning my desk , perhaps of interest:
If one of the QG-versions for a QL is concyclic,
… the center of the circle lies on the first Steiner axis and
… the Schmidt-circle (see QL-Tf1) is orthogonal
  to the circumcircle,
… the corresponding *QA-Cu7* *degenerates to an orthogonal
  hyperbola* ,
… which is the conic for the 5 QA-Cu7-triple points,
… QA-DT-circumscribed, centered in the midpoint of QG-P4.QG-P8,
… bearing the midpoint of the circle.

Best regards Eckart
```

Message: #3346
Date: 29/1/2019 10:35:29
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

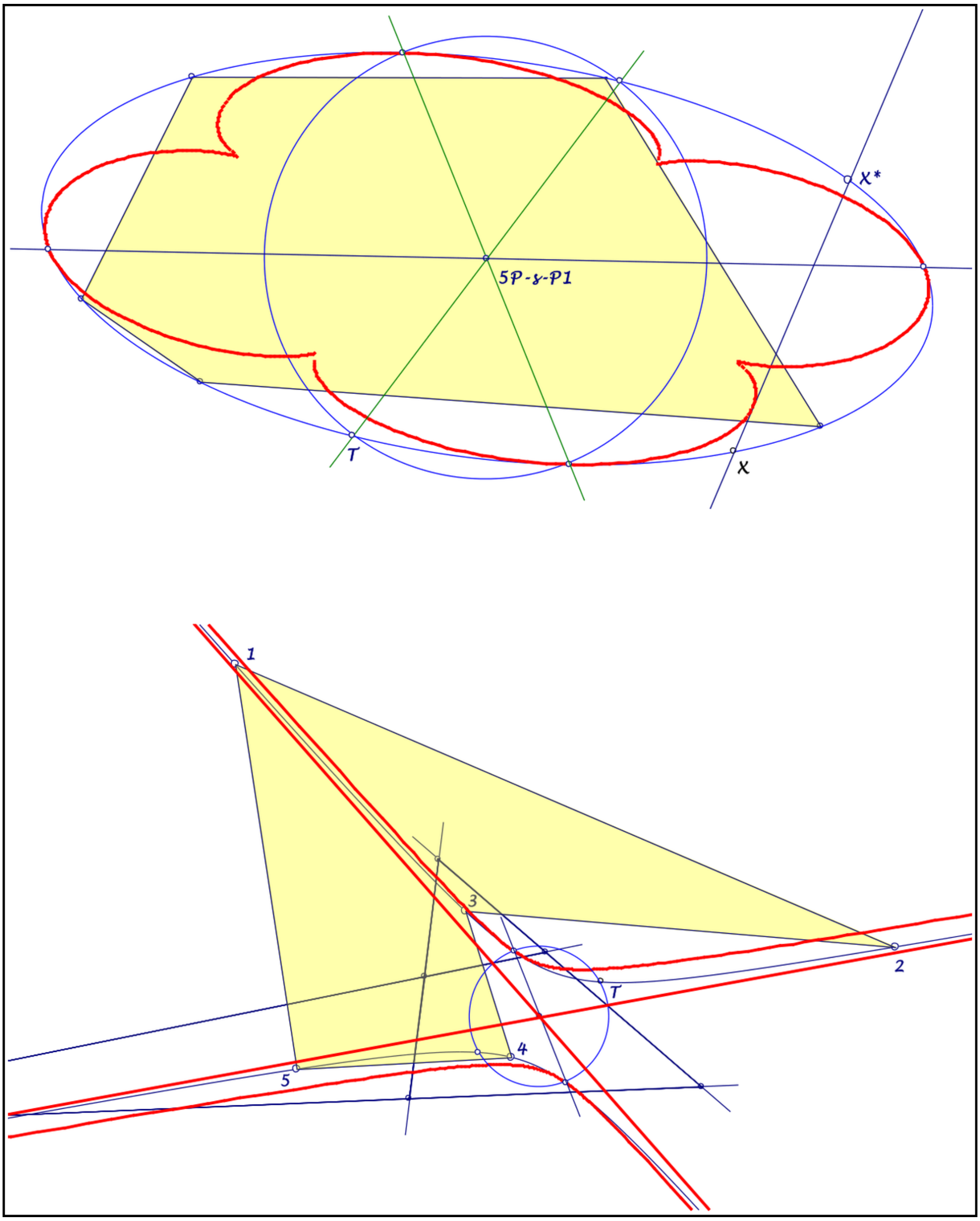
Dear Bernard, dear Chris,

Cleaning my desk, another aspect:
Consider the transformation * for a 5P:
… P* shall be 5P-s-P4 of Q1,2,3,4,5
… with Qi = 5P-s-P4 of P,Pj,Pk,Pl,Pm.
P-->P* maps 5P-s-C01 to itself.
Let 5P-s-C01 be an ellipse with points X:
XX* envelopes a sextic (see attached file):
… symmetrically 5P-s-P1 inscribed 5P-s-C01
… with 4 contact points and 4 cusps,
… one pair of contact points diametral
… … wrt a circle round 5P-s-P1 through T = 5P-s-P4.

If 5P-s-Co1 is a hyperbola,
… the sextic has no cusps,
… two contact points are the infinity points of 5P-s-C01.

What about this sextic?

Best regards Eckart



2019-01-17.pdf

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Message: #3347
Date: 30/1/2019 11:21:51
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
My computer was in fact out of order !
I had to buy a new one and to have it reinstalled with all my applications and files, which took time.
During those days, you've started plenty of new items and I don't know where to start ...
Perhaps this simple remembering of old C-B properties.
5 points Xi define a CSC, a point T = 5P-s-P4 and 11 degenerated cubics, one formed by the conic 5P-s-Co1 and the infinity line and 10 formed by a line XiXj and the circle through the 3 other points.
Consider a point P as pivot of a circular cubic through the 5 points.
The line PT cuts the conic in S.
The circle through 3 points Xk, Xl and Xm cuts the conic in Sij and the parallel through Sij to XiXj cuts the conic in T and the circle in Tij, which are the pivots of the 11 degenerated conics.
The lines PTij cut the lines XiXj in Uij and the corresponding circles in Vij.
For each point P, there is one circular cubic through the 5 points Xi, the 10 points Uij, the 10 points Vij and the points P and S, PS being the direction of the asymptote and Uij and Vij being C-B partners on the cubic.
The circular cubics may be monocursal or bicursal in boyh cases ellipse or hyperbola.
Now adding a supplementary constraint limits the number of solutions.
If we wish CSC(P) on the cubic, there are stil several solutions in both cases.
If we wish CSC(TS) and CSCi(parallel to TS through Xi) having a common point, there are only 2 solutions for S on the ellipse and 4 on the hyperbola and CSC(common point) is the pivot Z1 of the circular cubic (the common point is not on the cubic this time).
May be this could give you new ideas ...
Best regards
Bernard
PS No idea about your sextic, but they look beautiful

Message: #3348
Date: 30/1/2019 2:57:33
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

I hope, you can reinstall your applications in a new computer,
… to control my observations and give new ideas.
I try to attach a file with a figure,
… so that you can follow my description.

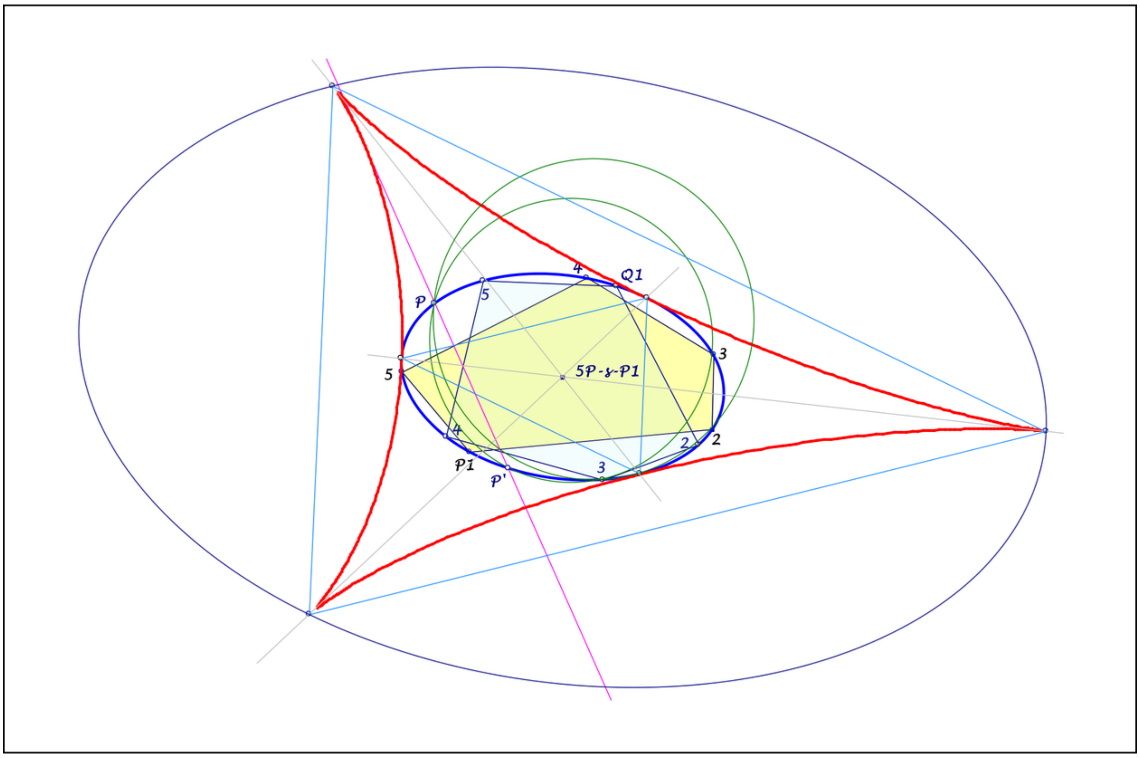
Consider a convex pentagon P1,2,3,4,5
… and a variable point P on 5P-s-Co1,
… which leads to a new pentagon Q1,2,3,4,5 on 5P-s-Co
… with Qi = 5P-s-P4(P,Pj,Pk,Pl,Pm).
… The areas of the two pentagons are equal.

Consider the circles (P,Pi,Qi)
… with a common 2nd point P' on 5P-s-Co1,
… the lines PP' envelope a *deltoid* tangent 5P-s-Co1
… with cusps, if P, P' and 5P-s-P1 are collinear.
The triangles of the cusps and the contact points have centroid
5P-s-P1.
Their Steiner circumconics are homothetic wrt 5P-s-P1
and ratio 3.

Best regards Eckart

PS: Are there analogies to the Kantor-Hervey deltoid?

Give me some time, to study your message 3347.



2019-01-29.pdf

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Message: #3349
Date: 31/1/2019 10:20:33
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Your deltoïd is a stretched deltoïd, with the same tangential definition as the KH deltoïd, replacing the inner circle by the inner ellipse.
The definition is the envelop of the lines joining 2 points, which describe the circle/ellipse in opposite directions, one at a speed dobble than the other. The contact point is the reflection of P' in P.
If you consider the Euler circle of a triangle as the inner circle, you get the Steiner deltoïd tangent to the 3 sides and the 3 altitudes.
With the ellipse, you have worked on this figure with Benedetto Scimemi : if my memory is not wrong, P was the QA-P4 of a QA and P' described the curve through the reflections of QAP4 in the 6 sides of the QA ; the stretched deltoïd was tangent to the 6 sides of the QA.
Best regards
Bernard

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Message: #3350
Date: 01/2/2019 10:34:04
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Very interesting generalisation of QA-Cu7 !
I don't try to reproduce all the mentionned properties, as it takes too much time to draw new figures ...
But I have 2 questions by analogy with QA-Cu7 :
1) I suppose CU(X) is also a pivotal isocubic with pivot one of the DT vertices, fixed points of the transformation the vertices of the tangential QA of the pivot and reference triangle the DT of this QA.
2) What happens for a QL if you take the same QA-point in the 3 QA's (QA-P2 or P41 or any other point) ?
What about the CSC of the 5 triple points in this case ? Is for each of the 3 CU(X) the pivot CSC (X) ?
Best regards
Bernard

Message: #3351
Date: 02/2/2019 9:14:59
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

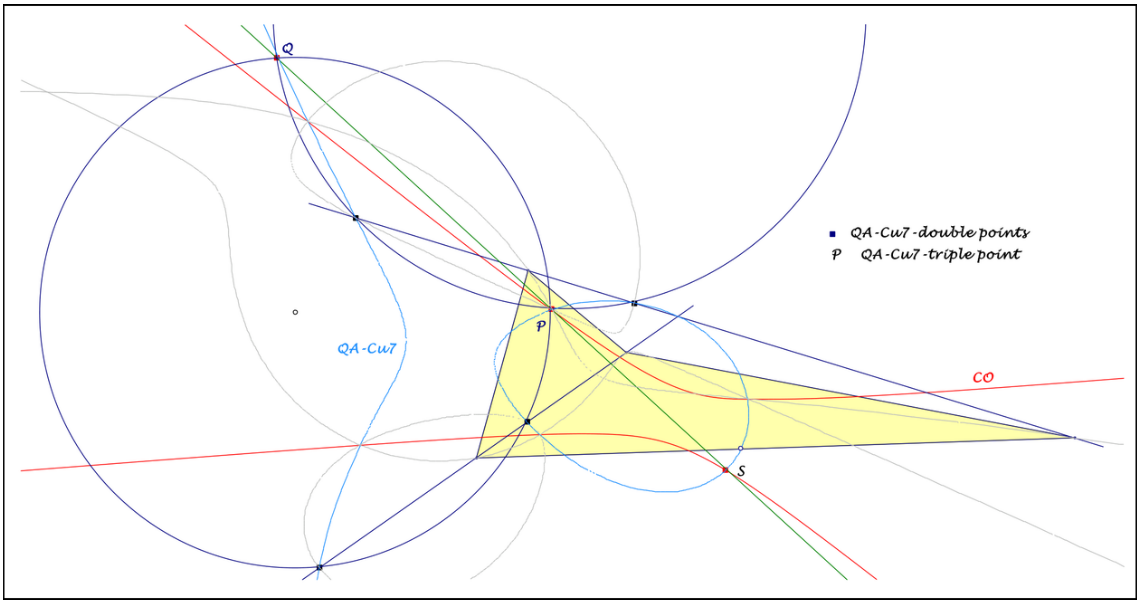
```
Dear Bernard,

before studying your #3350
… a new sight for the 5 QA-Cu7-triple points:

Well known are (see #3176):
… the QA-Cu7-double points on the QL-Tr1-sides,
… the conic CO of the QA-Cu7-triple points
… and the new QG-point S.
Consider for an arbitrary point P wrt a QG-version of the QL
… the two circles through P and the double points on a diagonal
… with a second intersection Q.
The QG-transformation P-->Q
… maps QG-P18 to QA-P4,
… maps QA-Cu7 to itself, point and image collinear S.
If P is a point on CO and P, Q, S collinear,
… P is a QA-Cu7-triple point.

Best regards Eckart

PS: The QG-transformation P-->Q should be studied  more.
```



2019-02-02.pdf

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Message: #3352
Date: 02/2/2019 11:42:06
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
This time I try to react quickly !
If you consider the csc with center QA-P41 swapping QA-P2 and QA-P4, then QG-P19 is csc(QG-P1).
Consider now csc(QG-P18) ; the 4 points QG-P1, QG-P18, QG-P19 and csc(QG-P18) have the same tangential.
The double points named U2, U3, V2 and V3 on the QA-Cu7 are the vertices of the tangential QA of csc(QG-P18) ; U2V2 and U3V3 intersect in QG-P1 and are 2 diagonals of the QL, U2V3 and U3V2 intersect orthogonally in QG-P18 and U2U3 and V2V3 intersect in QG-P19.
All this is well known.
Now your transformation swaps QG-P18 and QA-P4, but also csc(QG-P18) and QA-P2, the infinity point of the asymptote and QG-P1 and QG-P19 and QA-P41.
More generally, if the transformation swaps P and Q, it swaps also csc(P) and csc(Q).
Best regards
Bernard

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Message: #3353
Date: 02/2/2019 3:09:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt your message #3350 with questions to my message #3341:
wrt 1) CABRI-observations confirm your supposition.
wrt 2) Considering the 3 CU(X) wrt other QG/QA-points as QA-P4,
… I found no observations wrt QA-P2, QA-P41, QG-P19 and S,
… but the cubic CU(QG-P18a) bears
… the double intersections of QA-Cu7a,b and of QA-Cu7a,c.

Best regards Eckart

Message: #3354
Date: 03/2/2019 10:33:34
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
When I react quickly, the answer may be not complete !
In fact, your transformation is a special case of a general property of the Van Rees cubic QL-Cu1 in it's bicursal version.
(If I'm not wrong, this property is not in EQF and wasn't mentionned before).
Let formulate it in a simple way :
Any circle cuts the circular cubic in 4 real points forming a QA ; the 3rd intersections of 2 opposite sides with the cubic are isogonal wrt the triangle QA-P2,4 and 41.
Best regards
Bernard

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Message: #3355
Date: 03/2/2019 11:41:53
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

wrt your # 3354:
If I am not wrong,
… QA-P41 is a point at infinity for a cyclic QA,
… so I cannot reproduce your result.
Is there a misunderstanding on my side?

Best regards Eckart
```

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Message: #3356
Date: 03/2/2019 1:47:52
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
QA-P2,4 and 41 are the 3 points defining the bicursal Van Rees
curve (it could be M, V and V') and have nothing to do with the
cyclic QA !
Sorry if I wasn't clear ...
Best regards
Bernard
```

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Message: #3357
Date: 03/2/2019 3:07:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

excuse my lack of comprehension:
What are the 3 points defining QL-Cu1, for example M, V, V'?

Best regards Eckart
```

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Message: #3358
Date: 03/2/2019 4:27:18
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
If you deal with a triangle like B. Gibert, you get the
monocursal MacCay hessian K048 or the bicursal Kjp hessian K193
; the defining points are X1 and X15 and X16.
If you deal with the QL, the defining points of QL-Cu1 are QL-P1
and the QL-2P2a and b.
If you deal with the QA, the defining points of QA-Cu7 are
QA-P41 and QA-P2 and QA-P4.
This is always the same focal circular Van Rees cubic.
I hope this time I have been complete and clear ...
Sorry for having been too elliptic
Best regards
Bernard
```

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Message: #3359
Date: 03/2/2019 8:22:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

wrt your #3352:
It was a real pleasure,
to reproduce your description
… of the geometry of the QA-Cu7-double points,
… here some further results, perhaps already mentioned:
The bisectors of U2V3 and U3V2
… intersect orthogonal on QA-Cu7 in a point X,
… which is the 3rd intersection of QA-Cu7 and QG-P1.QA-P2,
… concyclic with QG-P18, QA-P2, QA-P4
  and the midpoints of U2,V3 and U3,V2,
… with csc(X) = 3rd intersection of QA-Cu7 and QG-P1.QA-P4.
The transformation csc swaps U2,V3 and U3,V2.
My new QG-transformation (#3351)
… maps QG-diagonals to itself
… with constant distance product to QG-P1.

Best regards Eckart

PS: There will be a typo in the middle of your message:
     "... and U2U3 and V2V3 intersect in QG-P1."
```

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Message: #3360
Date: 03/2/2019 8:40:31
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

many thanks for the explanation
… of the reference triangles for QA-Cu7 and QL-Cu1
… as non-pivotal isogonal circular cubics.
I should study Bernard Gibert's chapter 4 once more,
… to get familar with Van Rees focals.

Now I could check your interesting property in #3354
… and its application to the QG-point S
… as isogonal conjugate of QG-P1 wrt the triangle QA-P2,4,41.

But: In the bicursal version of QL-Cu1
… the points QL-2P2 are not defined in EQF,
… an old disappointing discussion:
… they are the CSC-partners on a perpendicular to QL-L1
… in the intersection with QL-L6.

Best regards Eckart

PS: In reference [15b] chapter 7
… there are properties of concyclic points on QA-Cu1.

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Message: #3361
Date: 04/2/2019 10:24:09
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
You may find many interesting properties in
http://bernard.gibert.pagesperso-orange.fr under Catalogue
(MacCay is K03 ...) or in http://wwww.mathcurve.com under corbes
2D, then choose english version and focal circular cubic, also
named Van Rees.
For the 2 points, you have 2 solutions :
1) keep the same points and change the Newton Line (through the
points for monocursal, perpendicular bisector of the segment
joining the 2 points for bicursal)
2) keep the line and consider 2 copples of CSC points S and S'
on the line and V and V' symmetric wrt the line.; the 4 points
can't be real at the same time, S and S' are real and V and V'
imaginary for monocursal, V and V' are real and S and S'
imaginary for bicursal. (That's another way of finding basis
points or Poncelet points of a set of circles ...)
Best regards
Bernard
PS Working on your message 3341, I find this question : what are
the intersection points of QA-Cu1 and QA-Cu7 (apart of the DT
vertices, QA-P4 and P41 and the circular points) ? The same way,
what are the intersection points of QA-Cux and QA-Cuy
constructed with the same QA-point (the 1rst by intersecting a
line through the pivot and QA-Tf2 of this line and the 2nd
between the perpendicular in the pivot to the line and QA-Tf2 of
the 1rst line) apart of DT vertices, the pivot and it's QA-Tf2 ?
Last, another old question : what is the CSC of QA-P41 (on a
diagonal of DT) and what is the CSC line of the circle
circumscribed to the 3 QA-P41 of a QL ?
```

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Message: #3362
Date: 05/2/2019 11:07:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for further references wrt Van Rees focals in #3361.

Wrt your PS-questions:
I don't see further intersections of QA-Cu1 and QA-Cu7.
The cubics QA-Cux,y - as you describe - are the same.
CSC(QA-P41) wrt a QG-version of the QL
… is the intersection X = QG-L1 ^ QG-P1.QA-P4.

The line of the collinear CSC(QA-P41),
… considered for a QG-version of the QL,
… is QL-Tf2(Y.QL-P26)
… with Y 4th harmonic point of X wrt QG-2P2.

It needs a bit concentration
… to handle a QG as well as QA as QL!

Best regards Eckart

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Message: #3363
Date: 07/2/2019 3:00:21
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
Thanks a lot for the CSC of QA-P41 ! It was so simple !
By the way, this point is on QA-Cu1 of the QA formed by 2
copples of CSC partners of the QL.
Now, back to QA-Cu1 and QA-Cu7 : both cubics are circular and
should intersect in 7 real points.
Like you, I can't find points other than DT vertices and QA-P4
and P41. Where is the problem ?
The same goes by replacing QA-P4 by any point X : by using the
same construction as for QA-Cu1 and QA-Cu7, this gives 2
different cubics QA-Cux and QA-Cuy ; what are the intersections
other than DT vertices, X and QA-Tf2(X) ?
Best regards
Bernard
```

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Message: #3364
Date: 08/2/2019 12:20:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I try to gather properties of the QA-Cu7-double points,
… first mentioned in QFG-message 2974.

Nomination:
Let us consider a QL
… with its QG-versions QGa,b,c and their cubics QA-Cu7a,b,c
… with double intersections of QA-Cu7b,c in Xa and Ya on QG-L1a.

(1) The QA-Cu7-double intersections Xa, Ya can be constructed
… with QL-geometry (see #2974) as intersections of QG-L1a
… … and the CSC-circle CIa
of the diameter circle for QG-2P3a,
or:
… with *QA-geometry as QA-Tf2b or QA-Tf2c partner on QG-L1a.*

(2) The 6 QA-Cu7-double points are coconic.

(3) The diameter circle CIa of Xa. Ya bears:
… QA-P4b,c
… and QG-P18b as *orthogonal intersection* of Xa.Yc and Ya.Xc,
… and QG-P18c as *orthogonal intersection* of Xa.Yb and Ya.Xb.

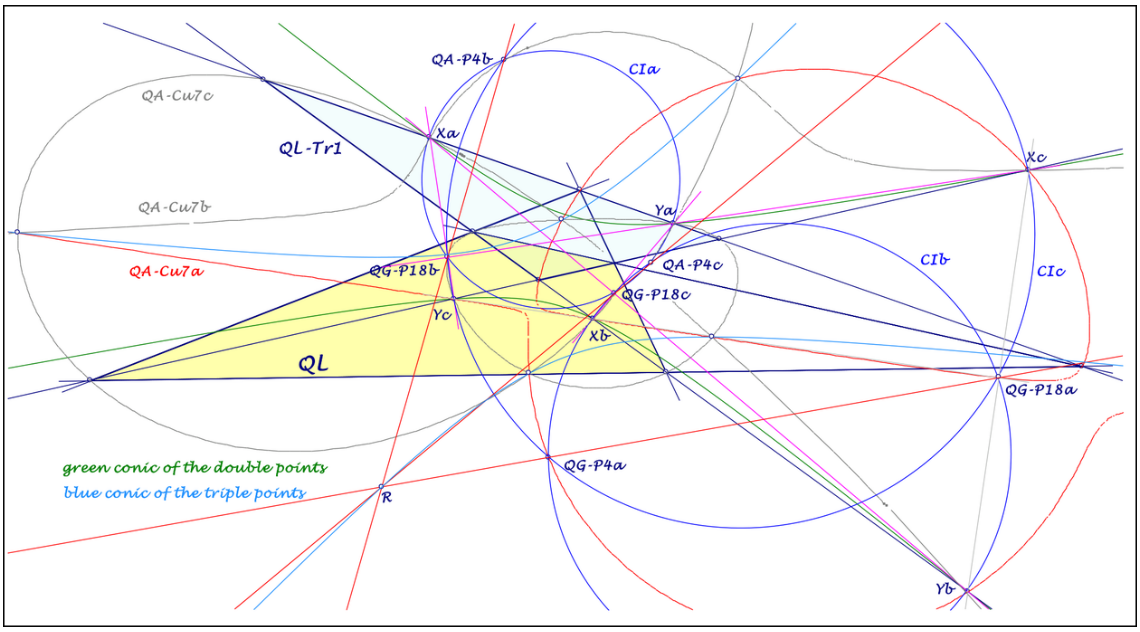
(4) The circles CIb,c intersect in QA-P4a and in QG-P18a.

(5) The radical center of the 3 circles CIa,b,c
… is the point R (see #2974, #3176),
… perspector of the QA-P4- and QG-P18-triangle.

(6) The 3rd intersections of QA-P4.QG-P18 and QA-Cu7
… are the points S (see #2980, #3176)
… on the conic of the QA-Cu7-triple points.

Best regards Eckart

PS: Please use the nomination of X and Y
    … so that the properties in (3) are fulfilled.
    I think, there will be more properties.



2019-02-08.pdf

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Message: #3365
Date: 09/2/2019 12:21:53
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Beautiful and useful figure indeed !
1) I regret that you changed the Ua and Va, which you used before, in Xa and Ya.
R, 3 points S and T on the conic of the 5 points, 6 points Ua,b,c and Va,b,c for the dobble points, 5 points X1 to 5 as triple points and Y1 to 5 for the CSC of the Xi seems more convenient ...
2) If you consider the vertices of the QL A,A',B,B',C and C' and it's CSC, QA-P4 is CSC(QG-P1) and QG-P18 is CSC(QG-P17) ; QG-P1aQA-P4a cuts QG-P1bQG-P1c in CSC(QA-P41a).
3) The csca of QA-Cu7a with center QA-P41a swapping QA-P2a and QA-P4a swaps also QG-P1a and QG-P19a and QA-P18a and the 3rd intersection of AA' and QA-Cu7a, which is csca(QA-P18a) and the 4 dobble points Xb, Xc, Yb and Yc (your notation) are the vertices of the tangential QA of this csca(QA-P18a).
4) The dobble points Xb and Yc and Xc and Yb are also csca conjugates (one of your last messages).
Best regards
Bernard

PS You didn't answer my last message:
1) I'm looking for the intersections of QA-Cu1 and QA-Cu7, then for the same curves QA-Cux and QA-Cuy by replacing QA-P4 by another same point (I took as example QA-P1 and QA-Cux is QA-Cu5 ; both cubics QA-Cu5 and QA-Cuy are through DT vertices, QA-P1 and QA-P20)
2) I'm then looking for the intersections of the 3 same curves for a QL : QA-Cu1 gives a set of cubics through the vertices of DT, QL-P1 and the circular points, QA-Cu7 gives the 5 triple points with the same CSC as the QL.
What is the result for the 3 QA-Cu5 and the 3 QA-Cuy ? Many thaks for your attention ...

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Message: #3366
Date: 10/2/2019 12:02:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

thanks for further observations wrt the QA-Cu7-double
intersections.
Please excuse changing the nomination, not projected.
Wrt PS of #3365, CABRI-observation show:

1) The intersections of the generalized  QA-Cu1 and QA-Cu7,
… replacing QA-P4 by another point X,
… have only the intersections in the DT vertices,
  X and QA-Tf2(X).

2) Wrt the intersections of the 3 QA-Cu5 :
Beside the triple intersections in the QL-points
… there are 5 double intersections.
It seems, that the QA-Cu5-cubics
… of the convex and the "crossed" QG-version of the QL
… give only one of the double intersections.
Wrt the corresponding QA-Cuy cubics
… there are no triple, but a lot of double intersections
… without observations.
With growing interest I am looking for
… the 7th triple point beside the 6 QL-points
… of the three QA-cubics (see #2017, #3015),
… which are similar to QA-Cu7.

Best regards Eckart
```

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Message: #3367
Date: 12/2/2019 6:19:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
Thanks for your precise answers wrt my questions in PS of 3365.
I've reproduced your 7th point, but I can't find any particular
property ...
Best regards
Bernard
PS I suppose now that almost everything is clear for the QA-Cu7
5 triple or dobble points ; only the reverse construction of the
QL starting with the 5 triple points is resisting ...
```

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Message: #3368
Date: 12/2/2019 6:29:13
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

perhaps of interest: A new aspect of QA-Cu7-geometry,
… looking for the intersections Mi, Ni of QA-Cu7
… and the lines Li of a reference QG-version of QL.

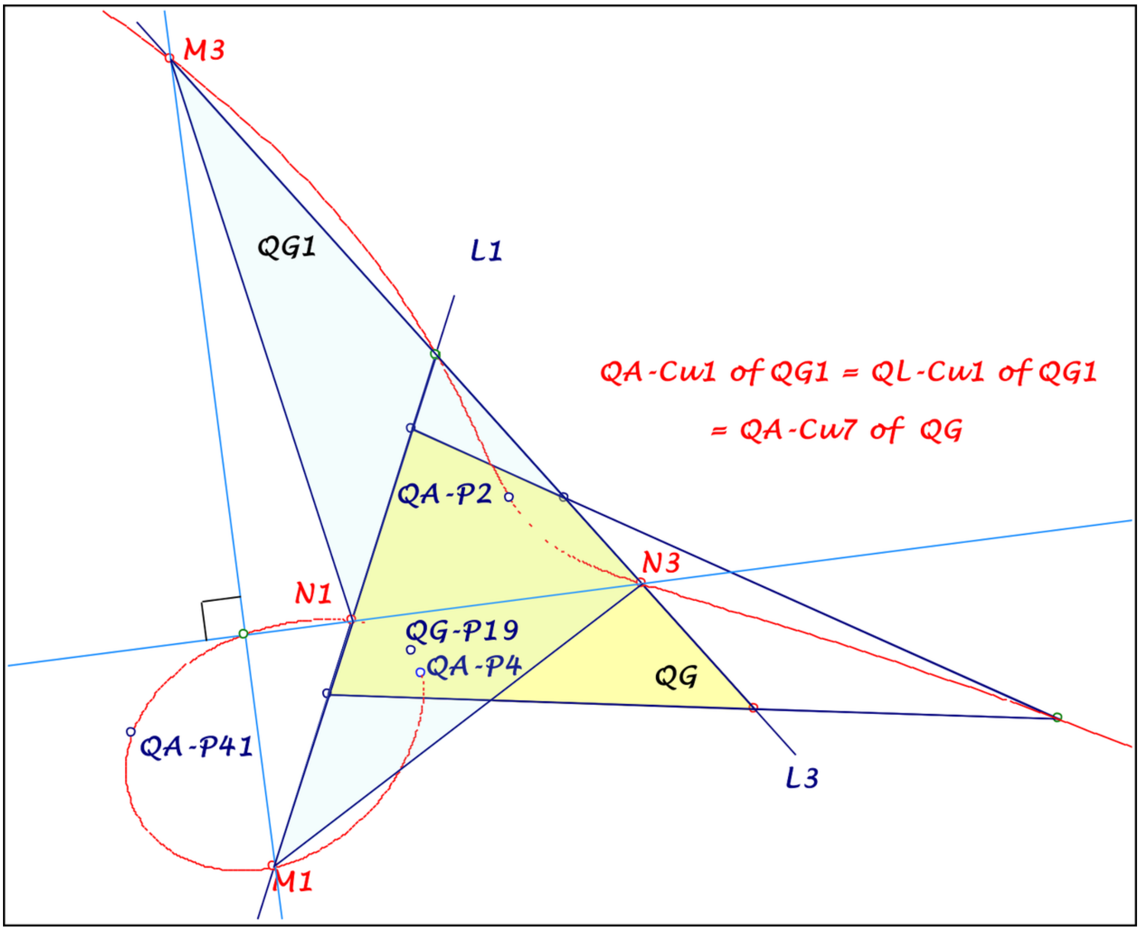
Consider opposite lines L1 and L3 with M1, N1, M3, N3:
… M1M3 and N1N3 intersect orthogonal
(using the right nomination).

M1N1M3N3 as QA:
… Its QA-Tf2 maps L1^L2<-->L1^L4 and L3^L2<-->L3^L4,
… its QA-P2 is the orthogonal intersection M1M3^N1N3,
… its QA-P4 is QG-P19 of the reference QG,
… its QA-P9 is QA-P41 of the reference QG,
… its QA-Cu1 is QA-Cu7 of the reference QG.

The QA M1N1M3N3 has three QG-versions:
QG1 = M1N1M3N3:
… Its QL-P1 is QA-P41 of the reference QG,
… *its QL-Cu1 is QA-Cu7 of the reference QG,*
… its QL-Tf1 maps its
QG-P1 to QG-P19 of the reference QG,
… … as well as QA-P2 <--> QA-P4 of the reference QG.
QG2 = M1N1N3M3:
… Its QL-P1 is QA-P4 of the reference QG,
… its QL-Tf1 maps QA-Cu7 of the reference QG to itself.
… its QL-Tf1 maps its QG-P1 to QG-P19 of the reference QG,
… … as well as QA-P2 <--> QA-P41 of the reference QG.
QG3 = M1N3N1M3:
… Its QL-P1 is QA-P2 of the reference QG,
… its QL-Tf1 maps QA-Cu7 of the reference QG to itself,
… its QL-Tf1 maps its QG-P1 to QG-P19 of the reference QG,
… … as well as QA-P4 <--> QA-P41 of the reference QG.

Best regards Eckart

PS. Construction of Mi, Ni on Li:
Take a point X
… and its harmonic conjugated Y
wrt the QG-points on Li.
The locus of the centers of the circles (X,Y,QA-P4) is a line,
… intersecting Li in the center of a circle through QA-P4,
… which intersects Li in Mi, Ni.



2019-02-12.pdf

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Message: #3369
Date: 13/2/2019 11:20:03
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
It's always the same story with the (bicursal) Van Rees circular
focal cubic !
If the QA is BCB'C', the 2 other QL vertices are A and A' and
the intersection of BB' and CC' is QG-P1.
L1 and L3 (your figure) intersect in A.
Then M1, N1, M3 and N3 are the vertices of the tangential QA of
A', which is their QA-P4 (and not QA-P19 as you mention).
The orthogonal intersection of M1M3 and N1N3 is QA-P2 of the
tangential QA as you mention as well as a DT vertice ; the 2
lines form the degenerated rectangular hyperbola which is the
polar conic of the point A' wrt the stelloïd cubic for which the
QA-Cu7 is the hessian ...
The csc with center QA-P41 swapping QA-P2 and QA-P4 swaps QA-P1
and QA-P19 and the mentionned intersection and A' as well as M1
and M3 and N1 and N3 ...
Best regards
Bernard
```

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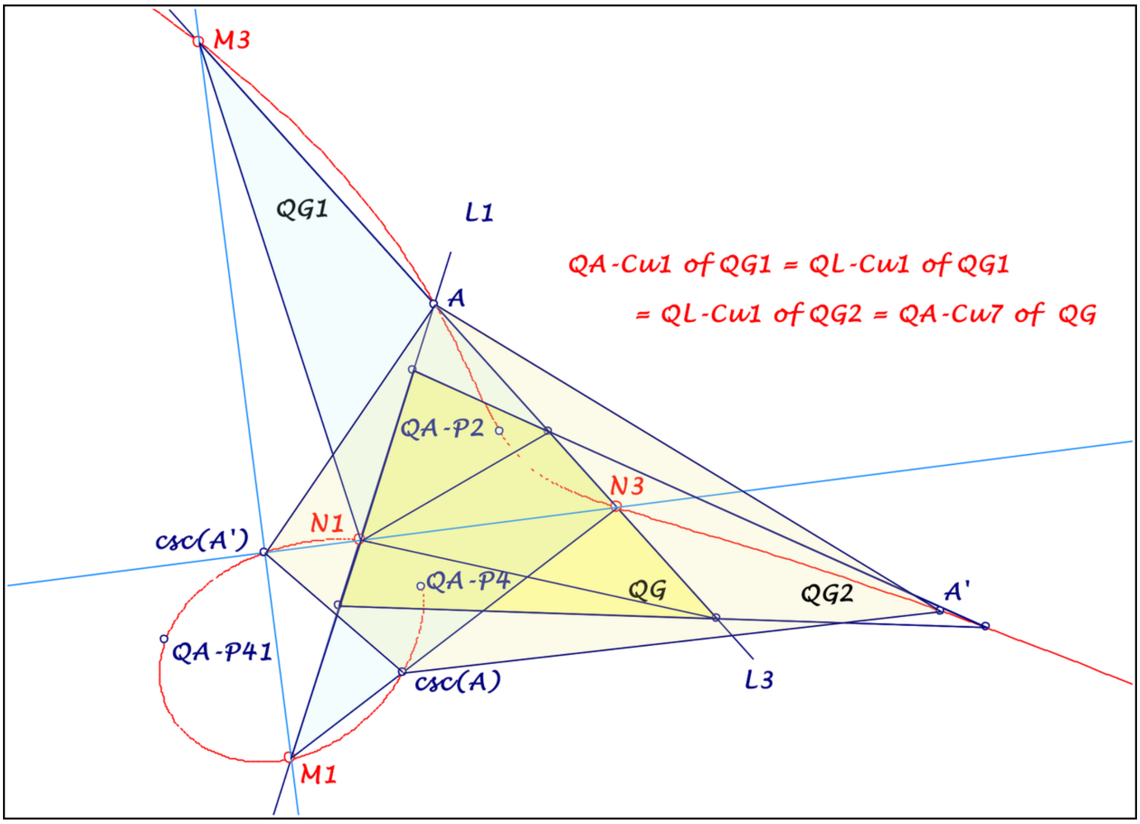
Message: #3370
Date: 15/2/2019 9:09:53
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

```
Dear Bernard,

thanks for the interpretation of the configuration in #3368.
You are right with the correction (except the typo Q *G*-P19):
" … of A', which is their QA-P4 (and not QA-P19 as you
mention)."

But I miss the connection with the intersections of QA-Cu7 and
L1,L3:
… your A' = QA-P4 of M1,N1,M3,N3 is csc(QA-Tf2(csc(L1^L3)))
… and your corresponding QL (see #3308) has the lines
… AA', csc(A).csc(A'), A.csc(A'), A'.csc(A)
… ... with csc(A)  = M1N3^M3N1 and csc(A') = M1M3^N1N3,
… which is the QL of another QG2 as QG1 in my description
… with lines M1N1, N1M3, M3N3, N3M1 (see attached file).

Best regards Eckart
```



2019-02-15.pdf

Message: #3371
Date: 15/2/2019 12:23:35
From: bernard.keizer
Subject: QA-Geometry

Dear Eckart,
Beautiful and useful figure !
You may add :
QG-P1 (3rd DT vertice of the QA) as CB pivot of the 5 + 2 points
on the cubic and QG-P19 = csc(QG-P1)
QG-P18 as intersection of Acsc(A') and A'csc(A) and csc(QG-P18),
3rd intersection of AA' with the cubic.
The 4 points are the vertices of the tangential QA of another
point, which is it's QA-P4 and it's QA-P2 and the csc of this
QA-P4 is the orthogonal intersection of QG-P1QG-P19 and
QG-P18csc(QGP-18) ...
Best regards
Bernard

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Message: #3372
Date: 15/2/2019 2:46:53
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I think, your observation
" QG-P18 as intersection of Acsc(A') and A'csc(A)
and csc(QG-P18), 3rd intersection of AA' with the cubic."
doesn't hold, for A' isn't L2^L4, which has this property.

Best regards Eckart

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Message: #3373
Date: 15/2/2019 7:12:28
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
But of course A' is the intersection of L2 and L4 ! (like A is
the intersection of L1 and L3)
And the vertices of the tangential QA of A are the same way
M2,M4,N2 and N4.
M2M4 and N2N4 intersect orthogonally in csc(A) and M2N4 and M4N2
intersect in csc(A').
Acsc(A)csc(A') is the DT of the QA M1N1M3N3 and A'csc(A)csc(A')
is the DT of the QA M2N2M4N4
Best regards
Bernard
```

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Message: #3374
Date: 15/2/2019 9:04:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

in your message #3369 you wrote:
" Then M1, N1, M3 and N3 are the vertices of the tangential QA
of A', which is their QA-P4 …"
This A' is not L2^L4  (see my #3370).

Best regards Eckart
```

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Message: #3375
Date: 17/2/2019 11:27:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

in addition to message #3364 wrt the QA-Cu7-double points:
Nomination:
Let us consider a QL
… with its QG-versions QGa,b,c and their cubics QA-Cu7a,b,c
… with double intersections of QA-Cu7b,c in Xa and Ya on QG-L1a.
Using the right nomination the transformations *QG-Tf3a,b,c*
give:
… QG-Tf3b(Xa) = Yc, QG-Tf3c(Xa) = Yb, QG-Tf3b(Ya) = Xc,
  QG-Tf3c(Ya) = Xb,
… XbYc and XcYb intersect orthogonal in QG-P18a,
… *QA-Cu1 and QL-Cu1 of the quadrigon XbXcYcYb
   are QA-Cu7a of QGa.*

Best regards Eckart

PS: Excuse once more my changed nomination.
```

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Message: #3376
Date: 17/2/2019 11:39:34
From: Tran Quang Hung
Subject: Euler-Poncelet line in Quadrangle points

```
Dear geometers,

I have seen a nice fact which I call by Euler-Ponclet line
in Quadrangle points as following
Let A, B, C, D be Quadrangle points
- Oa,Ob,Oc,Od are circumcenters of triangles BCD,CDA,DAB,ABC.
  O is Poncelet point of (Oa,Ob,Oc,Od).
- Ha,Hb,Hc,Hd are orthocenters of triangles BCD,CDA,DAB,ABC.
  H is Poncelet point of (Ha,Hb,Hc,Hd).
- Na,Nb,Nc,Nd are nine-point center of triangles
                                                BCD,CDA,DAB,ABC.
  N is Poncelet point of (Na,Nb,Nc,Nd).
- Ga,Gb,Gc,Gd are circumcenters of triangles BCD,CDA,DAB,ABC.
  G is Poncelet point of (Ga,Gb,Gc,Gd).

Then O, H, N, G lie on a line. I call it by Euler-Ponclet line
of ABCD.
Note that  (H,G,O,N) is also harmonic range.
Is this fact known before?

Best regards,
Tran Quang Hung.
```

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Message: #3377
Date: 17/2/2019 12:45:14
From: eckart_schmidt@t-online.de
Subject: Euler-Poncelet line in Quadrangle points

```
Dear Tran Quang Hung,

In 2004 I described this line in
http://eckartschmidt.de/EULER.pdf.
Sorry in German.
In EQF you find it as QA-L1.

Best regards Eckart
```

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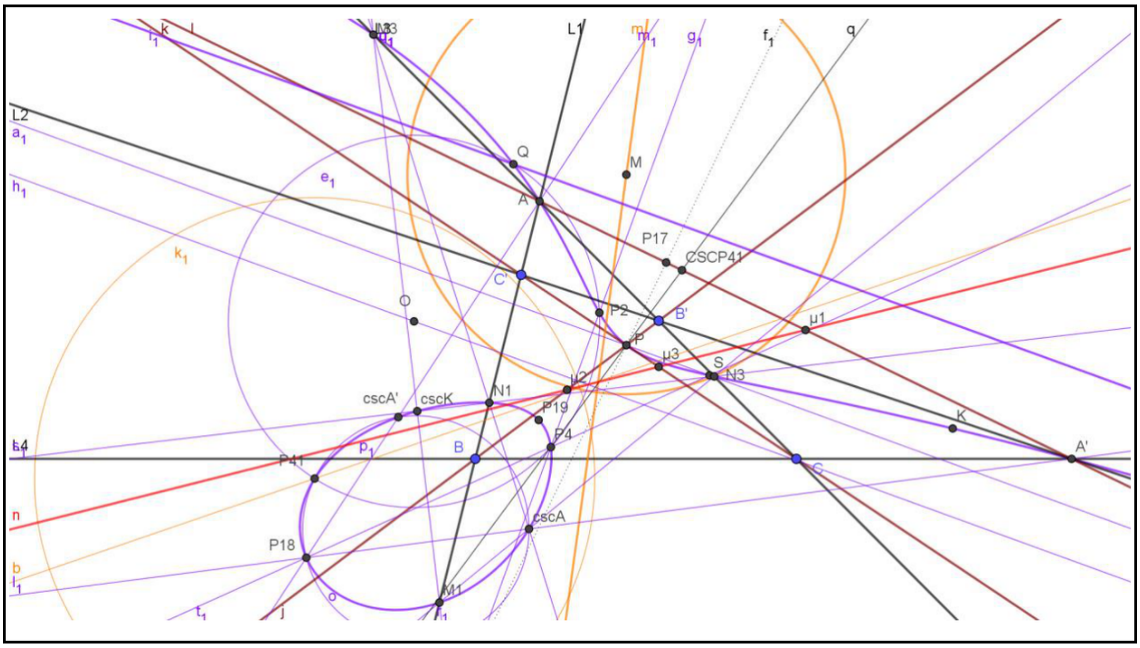
Message: #3378
Date: 17/2/2019 1:07:21
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I made an exact figure myself and I think I understand the misunderstanding between us.
For me, the vertices of the QL are A,A',B,B', C and C'.
The QA-Cu7 of the QA BB'CC' has as DT AA'P, the rest of the construction is weel-known.
Your 4 points M1,M3,N1 and N3 are the vertices of the tangential QA of a point K on the curve, hence the points M1 and M3 as well as N1 and N3 are csc partners and 1M3 and N1N3 intersect orthogonally in cscK.
And it holds that AcscA' and A'cscA intersect in the crosspoint QG-P18 and that csc(QG-P18) is the 3rd intersection of AA' with the curve QA-Cu7.
I send you my figure (M is QL-P1, P is QG-P1, P17, P18 and P19 are QG points, P2,P4 and P41 are QA points).
Best regards
Bernard
PS Now we have discussed the same properties ten times under 10 different configurations ...
I wish now to concentrate on the 5P geometry, I'll send you soon new ideas.



QA-Cu7.pdf

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Message: #3379
Date: 17/2/2019 1:19:09
From: Tran Quang Hung
Subject: Euler-Poncelet line in Quadrangle points

```
Thank you very much Mr Eckart.

I have seen it in
https://chrisvantienhoven.nl/qa-items/qa-lines/qa-l1

Best regards,
Tran Quang Hung.
```

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Message: #3380
Date: 17/2/2019 2:16:52
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

excuse, please replace in #3375 QG-Tf3 by QG-Tf2.

Best regards Eckart
```

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Message: #3381
Date: 17/2/2019 4:45:34
From: Tran Quang Hung
Subject: Line passes through Poncelet point in Quadrangle points

---

```
Dear geometers,

Let A,B,C,D be Quadrangle points.
Line da connects A and orthocenter of triangle BCD.
Define similarly lines db, dc, dd.
Then Newton line of (da,db,dc,dd) passes through Poncelet point
of (A,B,C,D).
Is this line known before?

Best regards,
Tran Quang Hung.
```

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Message: #3382
Date: 17/2/2019 4:48:38
From: Tran Quang Hung
Subject: Line passes through centroid of Quadrangle points

```
Dear geometers,

Let A,B,C,D be Quadrangle points.
Line da connects A and de Longchamps point (X(20)) of triangle
BCD.
Define similarly lines db, dc, dd.
Then Newton line of (da,db,dc,dd) passes through centroid of
(A,B,C,D).
Is this line known before?

Best regards,
Tran Quang Hung.
```

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Message: #3383
Date: 17/2/2019 8:41:08
From: eckart_schmidt@t-online.de
Subject: Line passes through Poncelet point in Quadrangle points

---

Dear Tran Quang Hung,

wrt the line in message #3381:
The line is a perpendicular in QA-P2 to QA-L1, not in EQF.

Best regards Eckart

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Message: #3384
Date: 18/2/2019 1:41:02
From: Tran Quang Hung
Subject: Line passes through Poncelet point in Quadrangle points

---

Dear Mr Eckart,

Thank you very much for pointing out the new line. I shall try
to find other properties for this.

Best regards,
Tran Quang Hung.

Vào Th 2, 18 thg 2, 2019 vào lúc 02:44 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt the line in message #3381:
>> The line is a perpendicular in QA-P2 to QA-L1, not in EQF.
>> Best regards Eckart

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Message: #3385
Date: 18/2/2019 11:35:21
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Chris, dear Eckart,

For 7 given points X1 to X7 and a random point P taken as pivot,
X1X2 and X3X4 intersect in U1 and the conics through the 5
remaining points intersect in a 4th point (other than X5,X6 and
X7) V1.
The line U1V1 intersect the 2 conics in 2 2nd points (other than
V1) T12 and T34, which are the pivots of the CB transformation
of the 7 points for the 2 degenerated cubics formed by the 2
copples line/conic.
The line PT12 intersect the line X1X2 in U12 and the conic in
V12 ; the same goes for PT34.
There is a cubic through P, it's CB conjugate P' and the 21
points Uij and the 21 points Vij.
P' is the tangential of P on the cubic and the cubic is a
pivotal isocubic with pivot P, fixed points of the
transformation the vertices of the tangential QA of P and
reference triangle the DT of the QA.

For 5 points X1 to X5 and a pivot P, the construction is exactly
the same with copples of lines and circles.
The cubics are circular and isogonal with pivot the infinity
point of PT where T is on the conic of the 5 points ; the cubics
pass through the point S, 2nd intersection of PT with the conic
and isogonal of P.
There are 10 points Uij and 10 points Vij ; the isogonal U'ij of
the Uij are the intersections of 3 lines XkVlm.

For 7 lines, we have the dual property : the intersection
between 2 lines is a point which forms with the conic tangent to
the 5 remaining lines a degenerated curve of 3rd class tangent
to the 7 lines ...

For 5 lines, we have the same dual property with intersection of
2 lines and incircle of the triangle formed by the 3 remaining
lines ...

This is to start new reflexions !
Best regards
Bernard

Message: #3386
Date: 20/2/2019 12:36:20
From: Tran Quang Hung
Subject: About QA-P4 of centroid quadrangle

```
Dear geometers,

Given four points A,B,C,D.
Ga, Gb, Gc, Gd are centroids of triangles BCD, CDA, DAB, ABC.
G is centroid of ABCD.
P = QA-P4 of ABCD
Then QA-P4 of (Ga,Gb,Gc,Gd) lies on line PG.
Is this point known before?

Best regards,
Tran Quang Hung.
```

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Message: #3387
Date: 20/2/2019 3:48:42
From: Tran Quang Hung
Subject: Newton line passes through Poncelet point

```
Dear geometers,

Let A,B,C,D be Quadrangle.
Let A1, A2, A3 be the refections of A in lines BC, CD, DB.
Line da connects point A and Poncelet point of (A,A1,A2,A3).
Define similarly the lines db, dc, dd.
Then Newton line of (da,db,dc,dd) passes through Poncelet point
of A,B,C,D.
Which is this line?

Best regards,
Tran Quang Hung.
```

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Message: #3388
Date: 20/2/2019 9:31:07
From: eckart_schmidt@t-online.de
Subject: number 3382, 3386,number 3387

```
Dear Tran Quang Hung,
wrt #3382: I found no further properties of this line,
wrt #3386: This point divides QA-P1.QA-P4 with ratio -1:4,
wrt #3387: If I am not wrong, the line doesn't pass QA-P2.

Best regards Eckart
```

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Message: #3389
Date: 20/2/2019 11:09:54
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

very interesting message #3385, but it needs time to reproduce.
Only some first remarks wrt the 5 QA-Cu7-triple points of a QL:
If we take P = QG-P1 of a QG-version of QL,
… we get QA-Cu7 of the QG.
If we take P = T
… the cubic degenerates,
… all Uij and Vij lie on the conic of the triple points.
With interest I am looking for the cubic for P = QL-P1,
… bearing the 2nd intersection
… of QL-P1.T and the conic of the triple points.

Best regards Eckart

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Message: #3390
Date: 20/2/2019 11:47:18
From: Tran Quang Hung
Subject: number 3382, 3386,number 3387

Dear Mr Eckart,

Thank you very much for quick reply. I try to check #3387, I
still see that the line passes through QA-P2. Could you please
check this again?

Best regards,
Tran Quang Hung.

Vào Th 4, 20 thg 2, 2019 vào lúc 15:31 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt #3382: I found no further properties of this line,
>> wrt #3386: This point divides QA-P1.QA-P4 with ratio -1:4,
>> wrt # 3387: If I am not wrong, the line doesn't pass QA-P2.
>> Best regards Eckart

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Message: #3391
Date: 21/2/2019 12:31:09
From: Tran Quang Hung
Subject: Begonia point of Quadrangle

```
Dear geometers,

Follow my post
https://artofproblemsolving.com/community/c6h1336737p7238120
Let A,B,C,D be four points.
Lines AB, AC, AD meet CD, DB, BC at Ab, Ac, Ad, reps. Let Ab',
Ac', Ad' be the reflections of A in lines AcAd, AdAb, AbAc,
reps. Then BAb', CAc', DAd' are concurrent at point A'. (A' is
called by Begonia point of A wrt BCD follow Darij Grinberg
http://www.cip.ifi.lmu.de/~grinberg/Begonia.zip )
Define simiarly the points B', C', D'.
Then lines AA', BB', CC', DD' are concurrent at a point. I call
this point by Begonia point of A,B,C,D.
Which is this point?

Best regards,
Tran Quang Hung.
```

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Message: #3392
Date: 21/2/2019 9:28:45
From: eckart_schmidt@t-online.de
Subject: Begonia point of Quadrangle

```
Dear Tran Quang Hung,

The Begonia point of a quadrangle
… is QA-P23 , Inscribed Square Axes Crosspoint in EQF.

Best regards Eckart
```

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Message: #3393
Date: 21/2/2019 10:51:20
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

```
Dear Bernard,

I have to correct my observation:
If we take P = T
… the cubic degenerates,
… all Vij lie on the conic of the triple points,
… all Uij are points at infinity.

Best regards Eckart
```

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Message: #3394
Date: 21/2/2019 11:01:28
From: bernard.keizer
Subject: 7 or 5 P or L

```
Dear Eckart,
I had noticed !
If P is in T, the cubic is the degenerated one formed by the
conic and the infinity line (which contains the circular
points).
If P is in Tij, the cubic is the degenerated  one formed by the
circle (which contains the circular points) and the line XiXj.
Thanks for your interest
Best regards
Bernard
```

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Message: #3395
Date: 22/2/2019 10:45:19
From: eckart_schmidt@t-online.de
Subject: Newton line passes through Poncelet point

Dear Tran Quang Hung,

I checked once more #3387,
… but it doesn't hold.
I don't find my error.

Best regards Eckart

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Message: #3396
Date: 22/2/2019 1:10:34
From: Tran Quang Hung
Subject: Newton line passes through Poncelet point

Dear Eckart,

You are right, I am very sorry. I looked and checked carelessly
on my Geogebra.

Best regards,
Tran Quang Hung.

Vào Th 6, 22 thg 2, 2019 vào lúc 17:03 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> I checked once more #3387,
>> … but it doesn't hold.
>> I don't find my error.
>> Best regards Eckart

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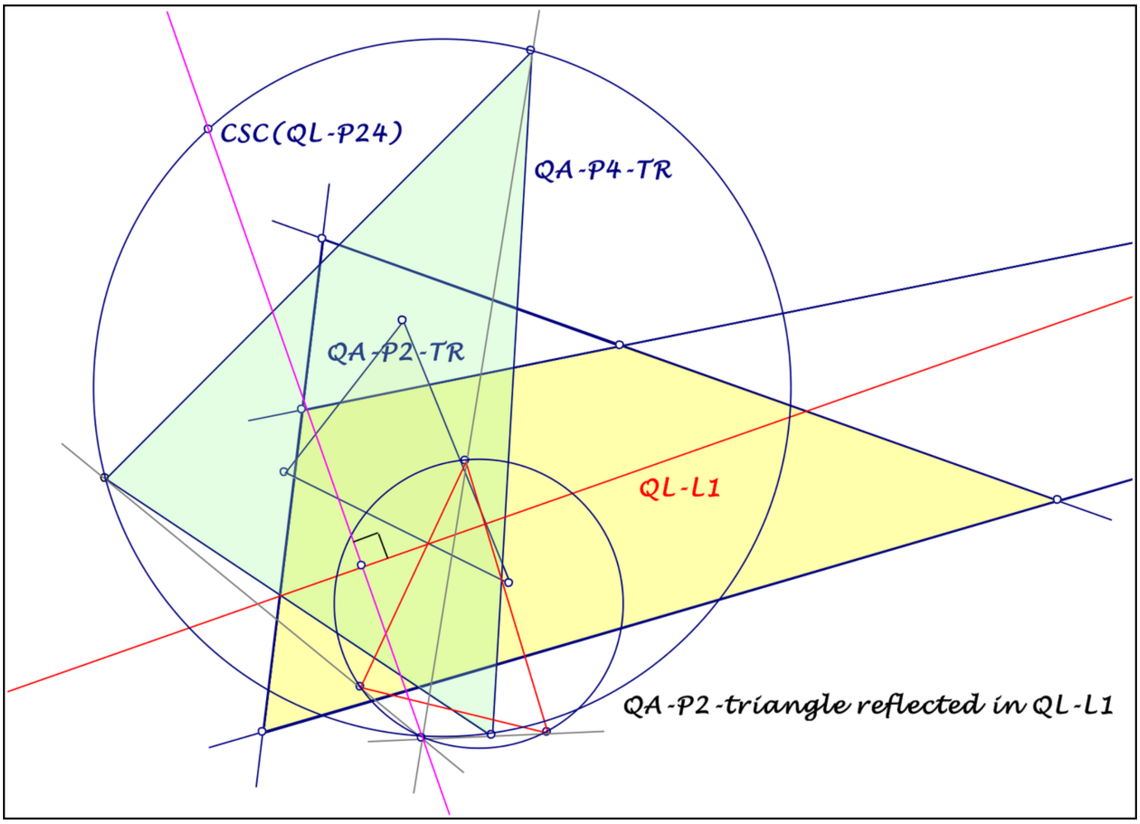
Message: #3397
Date: 22/2/2019 6:49:24
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

a further observation wrt the QA-Cu7-puzzle:
The points QA-P2 and QA-P4
… play a significant role in QA-Cu7-geometry.

Let us consider the QA-P2- and the QA-P4-triangle for a QL,
… they are similar with opposite sense of rotation,
… reflecting one in QL-L1, it is perspective to the other,
… with perspector in the intersection of their circumcircles
… on a perpendicular to QL-L1 through CSC(QL-P24).

Best regards Eckart
```



2019-02-22.pdf

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Message: #3398
Date: 24/2/2019 5:48:39
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
Curious and interesting last property !
Best regards
Bernard
PS Did you find something with the circular cubic through the 5
points and pivot QL-P1 ?
```

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Message: #3399
Date: 25/2/2019 3:34:12
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

perhaps of interest:
(1) For a QG = P1P2P3P4 we get
… the QA-Cu7-double points on the diagonal P1P3
… as intersections X13, Y13 of P1P3 and a conic
… through P2, P4, QG-2P2a,b, QG-P18 and QG-P19.

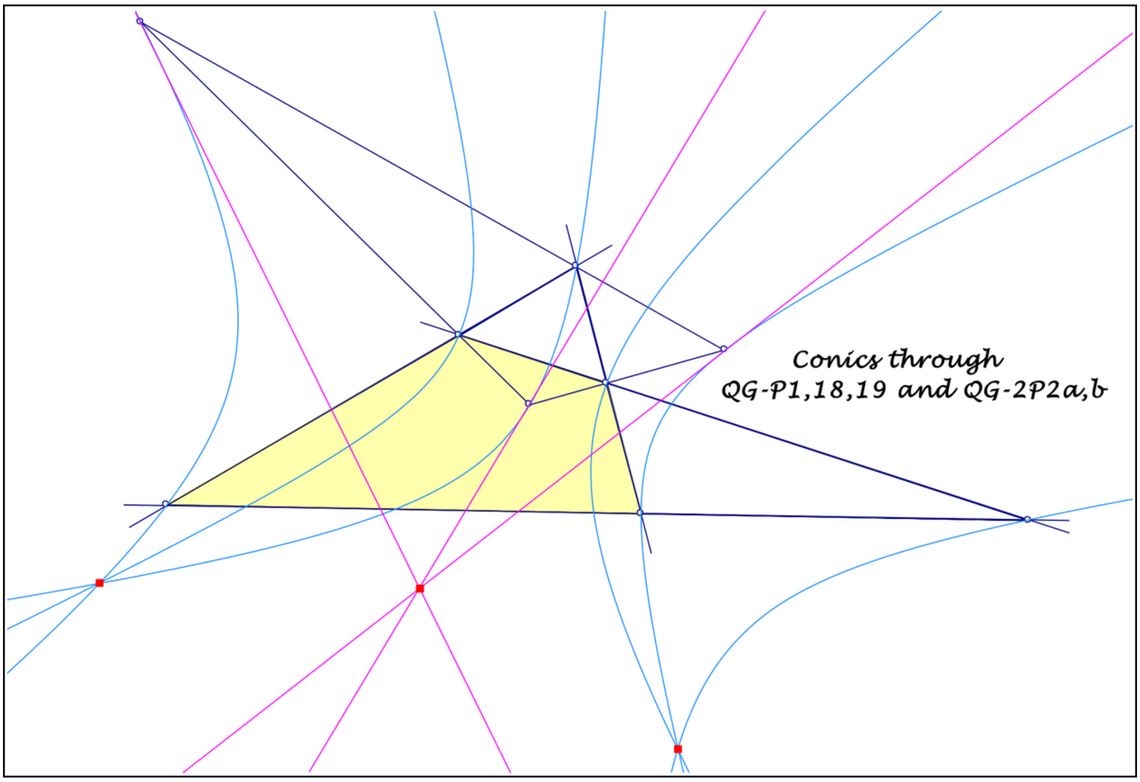
(2) The following 9 points give a CB-System:
… QG-P1, QG-P18, QG-P19, QG-2P2a,b
… and the 4 QA-Cu7-double intersections X13, Y13, X24, Y24.

(3) The 9 points under (2)
are the intersections
… of QA-Cu7 and CU(QG-P18) (see #3341).

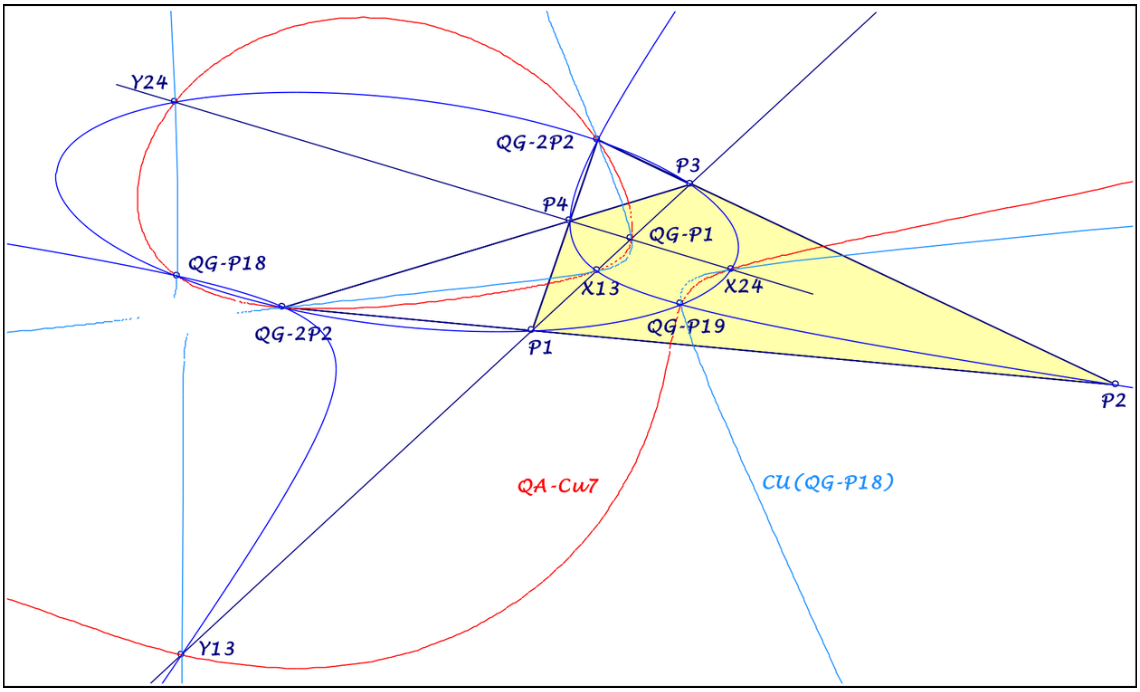
(4) The conics through the QG-points QG-P1,18,19 and QG-2P2a,b
... for the three QG-versions of a QL
... have two triple points QL-2Px.
The tangents in QG-P1a,b,c have a common point QL-Px.

What about these QL-points?

Best regards Eckart



2019-02-25a.pdf



2019-02-25.pdf

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Message: #3400
Date: 01/3/2019 10:39:33
From: eckart_schmidt@t-online.de
Subject: Triple points for QG-lines and conics

Dear Bernard, dear Chris,

the QA-Cu7-triple points remain mysterious,
… so I looked for simpler examples:
Several QG-lines and their QG-Tf2-conics
… give triple points for a QL wrt its QG-versions.
Triple points of QG-lines can be interpreted
… as perspectors of triangles.

Examples (see also QG-2)
(1) Lines QG-P1. QG-P3 intersect in QL-P8.
(2) Lines QG-P1. QG-P5 intersect in QL-P16.
(3) Lines QG-P1.QG-P12 intersect in QL-P13.
(4) Lines QG-P1.QG-P19 intersect in V (see #3176).
(5) Lines QG-P1.QG-P6 are parallel QL-L2.
(6) Lines QG-P1.QG-P17 intersect in QL-P10.
(7) Lines QG-P1.S intersect in T (see #3176).
(1) - (7) The QG-Tf2 conics of the three lines
… have four triple points (two not always real).

Conversely:
For a fixed point P wrt a QL
the QG-Tf2 conics
… of P.QG-P1 have four triple points (two not always real).
(8) If P is a point on QL-Ci1,
… the triple points are concyclic on CSC(QL-Ci1).
(9) If P = QL-P1
… one triple point is the point at infinity of QL-L1.

Further observations:
(10) Lines QG-P1.QA-P41 intersect in W (see #3176),
... one triple point of their QG-Tf2 conics at infinity.
(11) If QL-Cu1 is not bipartite,
… the QG-Tf2 conics of QL-L1 intersect in QL-P1 and QL-2P2.
(12) *QG-Tf2 conics of QL-lines QL-Lx - unequal QL-L1 -
… have triple points
in the intersections - unequal QL-P1 -
… of QL-Cu1 and the circle CSC(QL-Lx).
Who can lighten the background of this observations?
I hope there are no main errors.
Best regards Eckart

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Message: #3401
Date: 08/3/2019 8:51:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

perhaps a relevant observation:
Consider a QL and its 3 QG-versions QGa,b,c
… with their cubics QA-Cu7a,b,c
… and their 6 double points and 5 triple points.

The Cayley-Bacharach transformation CBa for
… the 5 triple points and the double points of QA-Cu7b,c
… maps double points of QA-Cu7a,b to QA-Cu7b
… and double points of QA-Cu7a,c to QA-Cu7c.

CBa maps QA-Cu7b,c to itself,
… point and image collinear with the infinity point
  of the asymptote.
CBb and CBc map QA-Cu7a in the same points to itself.
CBa maps points P on QG-L1a
… to the 2nd intersection of PT (for T see #3176)
  and the conic of the 5 triple points.

The CBa,b,c images of the double points
… produce on each QA-Cu7 4 points,
… which give a new QG' with orthogonal diagonals:
… QA-Tr2 of QG' is triangle QA-P2,4,41 of QG,
… *QA-Cu1 of QG' and QL-Cu1 of QG' are QA-Cu7 of QG.*

Likewise, already mentioned:
The double points Xa, Ya of QA-Cu7b,c
… and the double points Xb, Yb of QA-Cu7a,c
… lie on orthogonal lines (see #3364).
The QG-version of these double points,
… which has orthogonal diagonals
… and QG-L1a,b as opposite lines,
… interpreted as QA has QA-Cu7c as QA-Cu1
… interpreted as QL has QA-Cu7c as QL-Cu1 (see #3375).

Best regards Eckart
PS: What do you know about the CB-transformation
    … wrt the 5 triple points and the two circular points?
    I only remember the images of points at infinity
    … in the 2nd intersection of their connection with T
    … and the conic of the triple points.
```

Message: #3402
Date: 09/3/2019 10:32:42
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm not home next week, but I wish to answer now your last question.
The CB transformation wrt the 5 points and the 2 circular points is an isoconjugation, which swaps precisely the points Uij and Vij for a given pivot P (see the construction in my message 3385).
This leads to an infinity of pivotal isocubics ...
Best regards
Bernard

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Message: #3403
Date: 14/3/2019 8:33:27
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard, dear Chris,

if we replace in Hart's construction of the CB-point 8P-s-P1
… the initial conics 12345, 12346, 12356 by circles 345, 346,
356,
… we get the CB-point of 3,4,5,6,7,8 and the two circular
points.

In this way we get a construction for the CB-point
… of six points and the two circular points,
… which can be interpreted as 6P-s-Px,
… or as transformation CB for a 5P,
… especially for the 5 QA-Cu7-triple points.

Properties of this transformation CB (see also #3385):
(1) CB maps 5P-s-Co1 to the line at infinity,
… X, CB(X) collinear with 5P-s-P4.

(2) CB for the five QA-Cu7-triple points:
… (a) CB maps QA-Cu7a,b,c to itself,
… … X, CB(X) collinear QG-P1a,b,c.
… (b) CB changes the double points of QA-Cu7a,b, QA-Cu7b,c,
      QA-Cu7a,c.
… (c) CB maps QG-P1 to the tangential wrt QA-Cu7.
… (d) CB maps Bernard's cubics for 5 points in #3385 to itself
… … X, CB(X) collinear P.
… … The cubic is the locus of CB-partners on lines through P.
… … The cubic intersects the conic of the triple points on PT.

(3) Some remarks wrt Bernard's cubics in #3385:
… (a) If P = QG-P1 we get the cubic QA-Cu7.
… (b) For P on a circle (P1P2P3) through 3 of the five points
… … CB(P) lies on the line P4P5,
… … and the lines P.CB(P) have a fixed point P0 on (P1P2P3),
… … the parallel to P4P5 through Po bears 5P-s-P4.
… For P = P0 the cubic degenerates in circle (P1P2P3)
  and line P4P5.

Best regards Eckart
```

Message: #3404
Date: 17/3/2019 9:09:30
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

in #3385 you describe a cubic CU
… for 5 points and a point P,
… here my reproduction:

Let CO be the circumconic of the 5 points,
… T on CO the point 5P-s-P4 of the 5P
… and P* the 2nd intersection of PT and CO.

Using the CB-transformation CB (see #3403)
… for the 5 points and the two circular points,
… we get the point CB(P) = P'.

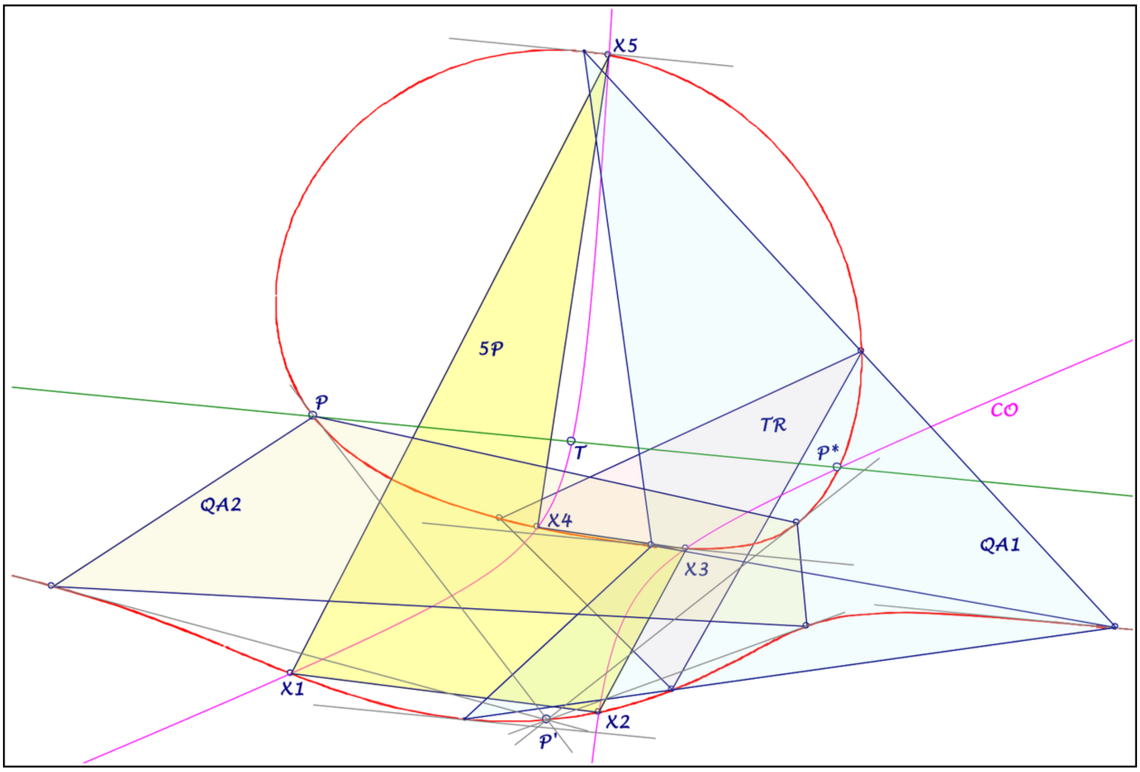
Your cubic CU is the locus of
… CB-partners on lines through P,
… with asymptote parallel PT
… and P' tangential of P.

If CU is bipartite:
The contact points of tangents parallel to the asymptote
… give a QA1 with diagonal triangle TR.
CU is a pivotal isocubic
… with reference triangle TR,
… isoconjugation: isogonal conjugate (P-->P*),
… pivot: infinity point of PT.
Or:
If we take a QA2 with vertices P
… and its Möbius transformed wrt TR,
… which is the tangential QA of P',
… CU is QA-Cu1 of this QA.

I think, the last observation isn't the same as yours.

Best regards Eckart

PS: The attached file is constructed
    … in the reverse order,
    … starting with the cubic and a conic.



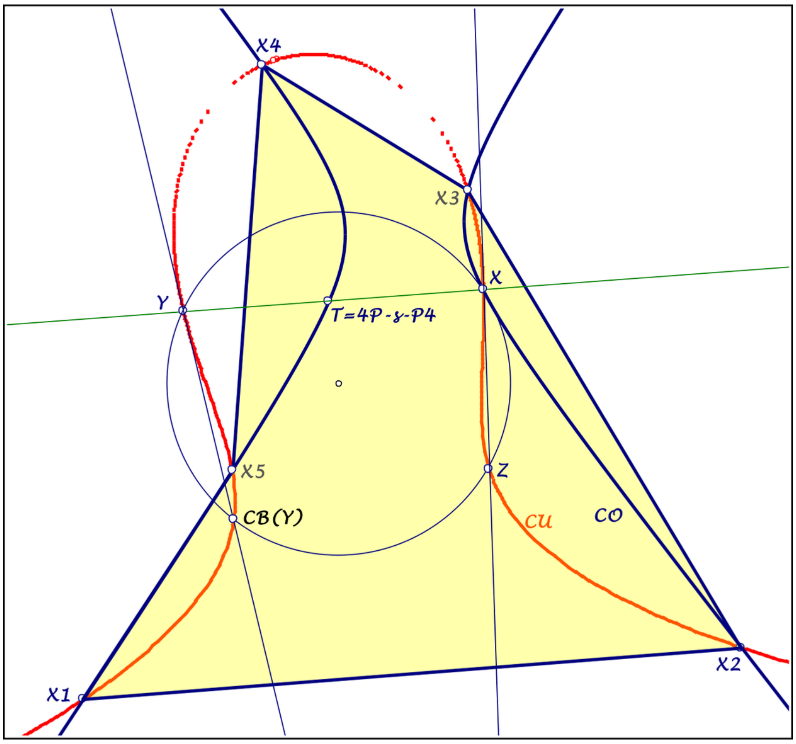
2019-03-16.pdf

Message: #3405
Date: 17/3/2019 10:23:41
From: eckart_schmidt@t-online.de
Subject: 5 points on on a cubic

Dear Bernard, dear Chris,

CABRI leads to the following observations:
Let CU be a cubic,
… which can be described as QA-Cu1 or QL-Cu1,
… take any 5 points on CU
… with its circumconic CO,
… intersecting CU in a 6th point X.
A parallel to the asymptote through X
… intersects CO in T = 5P-s-P4 of the 5 points.
Let Y be the 3rd intersection of XT and CU.
The CB-transformation CB (see 3403)
… wrt the 5 points and the 2 circular points
… maps X to the infinity point of XT,
… maps Y to the tangential of Y.
The circle CI = (X,Y,CB(Y))
… with 4th intersection Z on CU
… gives the tangential of X.
Are there other cubics with this properties?

Best regards Eckart



2019-03-17.pdf

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Message: #3406
Date: 19/3/2019 12:26:01
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
This time I give up !
I wrote a complete answer, but I couldn't send it ...
In 3403, you use the same construction than in my message 3385 and we naturally agree
In 3405, you described several constructions of a point on the curve taken as pivot (infinity point of the asymptote or isopivot) and the observations are perfectly compatible with mine for the pivot (fixed points are not necessary real)
In 3405 applied to a QA-Cu7 gives 4 concyclic points on the curve QG-P1 and the point S and their tangentials ; this property is new for me.
Last obsessing question : for 5 given points, how can we find the 3 positions of P for which CSC(P) lies on the cubic and the 3 cubics are the QA-Cu7 ?
Best regards
Bernard

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Message: #3407
Date: 19/3/2019 7:45:46
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
Reading again your message 3404, I've got an idea.
Let's give the 5 points and their conic, the point T on this conic and the CSC.
For a point P, we have a cubic like described in my message 3385 and your message 3403.
The construction of QA1 as described in your message 3404 leads to a DT which is necessary for a QA-Cu7 QA-P2, 4 and 41 ; CSC(P) is QA-P4 and the perpendicular bisector of QA-P2 and 4 is also parallel to PT ...
What do you think of that ?
Best regards
Bernard

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Message: #3408
Date: 21/3/2019 11:57:51
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
Naturally, the vertices of this QA1 are the in- and excenters of the triangle TR or QA-P2, 4 and 41 !
Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3409
Date: 21/3/2019 3:13:11
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

```
Dear Bernard,

I think, your description of QA1 in #3407 doesn't hold
… (nomination as in my message 3404):
QA1 is an orthocentric quadrangle,
… QA-P4 is an infinity point and QA-P2 isn't defined.
QA2 has P' as QA-P4 and QA-P41 as QA-Tf2(P').

TR is QA-Tr1 of QA1 and QA-Tr2 of QA2,
... its circumcircle bears
... ... QA-P41 of QA1,
... ... QA-P9 of QA2 and diametral
... ... the intersection of the cubic and its asymptote.

Best regards Eckart

PS: It seems, that CSC wrt the 5 points gives here no
observations.
```

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Message: #3410
Date: 22/3/2019 10:44:55
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
It's always this question of naming !
Once for all, I name QA-P2,4 and 41 the 3 defining points of the QA-Cu7 (id est P2,4 and 41 of one of the 3 QA's of the searched QL).
Back to your figure, the vertices of QA1 (contact points of the tangents parall to the asymptote), which is obviously an orthocentric QA, are the in- and excenters of it's DT.
If we search a point QG-P1 as pivot P in order to define a possible circular QA-Cu7 through 5 points, let's draw the 4 tangents to the cubic parall to the asymptote PT and it's DT ; one of the 3 vertices of this DT is necessary QA-P4=CSC(P) and the asymtote is parallel to the perpendicular bisector of 2 vertices QA-P2 and P4.
I suppose and hope this could give an approximative way of finding the 3 searched QG-P1.
(Drawing a cubic with a random point P, CSC(P) is generally not on the cubic ...)
Best regards
Bernard

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Message: #3411
Date: 23/3/2019 11:12:27
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

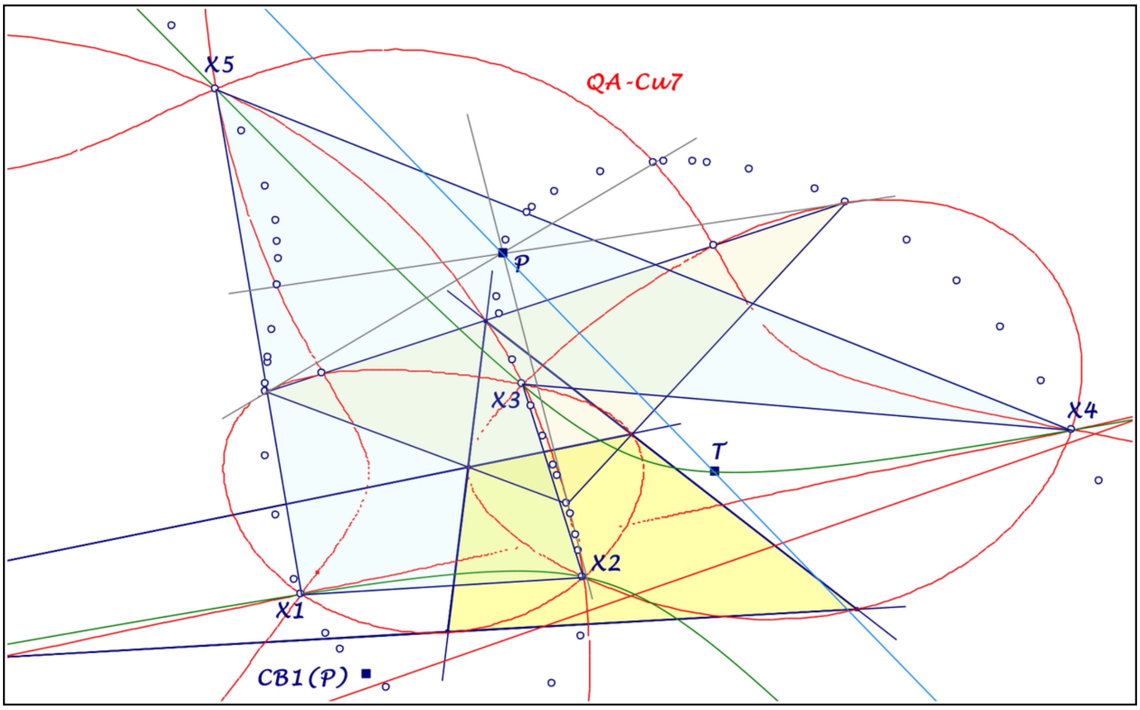
here two observations:

For 5 points (see #3270 and #3285)
… we can construct QL-P1 of the QL
… which has the 5 points as QA-Cu7-triple points
… and consider the CB-transformation CB1
… wrt the 5 points and the 2 circular points (see #3403):

(1) The line
QL-P1.CB1(QL-P1)
bears QL-P8 and QL-P24.
(2) The lines QG-P1.CB1(QG-P1) have a common point P,
… and the CB1-partner on lines through P
… give a cubic CU,
  bearing the 5 points and the 3 QG-P1.
  This cubic is invariant (see attached file)
… wrt the CB-transformations CB1
… and CB2 wrt the 5 points and P, CB1(P).
For points X on the cubic holds:
… X, CB1(X) and P collinear,
… X.CB2(X) parallel PT.

Best regards Eckart

P S. Thanks for clarification in #3410.



2019-03-23.pdf

Message: #3412
Date: 24/3/2019 9:29:13
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

I think, the CB-transformation
… wrt 5 points and the two circular points
… is a relevant 5P-transformation.

For a 5P and a pivot P
… the CB-partner on lines through P
… give a circumcubic of the 5P (see #3403, #3404).

The tangential of the pivot P is CB(P).

If we consider a 2nd CB-transformation CB'
… wrt the 5 points, P and CB(P),
… the CB'-partner on parallels to P.5P-s-P4
… give the same cubic.

For a QL with its 5 QA-Cu7-triple points
… and the pivots QG-P1a,b,c we get the cubics QA-Cu7a,b,c.

Best regards Eckart
```

Message: #3413
Date: 24/3/2019 7:23:27
From: Antreas Hatzipolakis
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

---

```
---------- Forwarded message ---------

From: *Stan.Rabinowitz*

In EQF, many notable lines in a quadrilateral have names like
the Newton Line, the Steiner line, the Morley line, etc.

But for a quadrangle, lines are designated as QA-L1 or as the
QA-P1-P2-P3 line, etc.

I am writing a geometry book and one chapter talks about notable
lines in a complete quadrangle. I would like to know if these
lines have any well-known names in the literature.

For example, in a quadrangle, QA-L1 is the line through the
centroid, the Euler-Poncelet point, and the Gergonne-Steiner
point. I am tempted to call this the Gergonne-Poncelet line, but
before I do, I thought I would check with this group to see if
the line already has a common name other than QA-L1.

Also, I would like to give a reference to the paper in which
this collinearity was first proven. Does anyone know who was the
first person to prove that these three points colline?
```

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Message: #3414
Date: 24/3/2019 7:23:48
From: eckart_schmidt@t-online.de
Subject: QA-P2,4,41 perspective QG-P1,19,18

```
Dear Bernard, dear Chris,

is the following perspectivity already mentioned?
For a quadrigon
… the triangles QA-P2,4,41 and QG-P1,19,18
… have a perspector on QA-Cu7,
… which is a cyclologic center of the two triangles.
For a QG-version of a QL this point is
… the CB-partner of QA-P2
for the CB-transformation
… wrt the 5 QA-Cu7-triple points and the two circular points.
For a QL the 3 perspectors give a triangle,
… perspective to the QG-P19- and QA-P4-triangle
… with perspector U (see #3176).
The two perspective triangles define
… in the sense of #2017 the cubic QA-Cu7.

So far a first impression of this perspectivity.

Best regards Eckart

PS:
The cyclologic center of QG-P1,19,18 and QA-P2,4,41
… is also a point on QA-Cu7,
… its 3 versions for a QL are the CB-partner of QG-P1,
… which give a triangle perspective to the QG-P1-triangle ...
```

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Message: #3415
Date: 24/3/2019 9:00:21
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,
Very interesting transformations indeed.
Unfortunately, I am still to busy to give it the time it deserves.
Nevertheless, I am following all messages, although not in depth.
I admire the quest you are on with Bernard finding the background of the 5 common points of the 3 versions of QA-Cu7 in a Quadrilaterals.
Best regards,
Chris

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Message: #3416
Date: 24/3/2019 9:31:51
From: chris.vantienhoven
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

---

Dear Stan,

In this paper from Jean-Louis Ayme you will find a description of both points QA-P2 and QA-P3:
http://jl.ayme.pagesperso-orange.fr/Docs/Le%20point%20d'Euler.pdf
I am not sure if there is a proof of collinearity with QA-P1.

I noticed last years that this collinearity is noticed several times by different persons.
I called the line QA-P1-P2-P3 Line because of the lack of a better name.
I like your name "Gergonne-Poncelet Line" and when not another name is encountered I can include it in EQF.
Best regards,

Chris

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Message: #3417
Date: 24/3/2019 11:43:53
From: staninchelmsford
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

---

```
Dear Chris,

Thank you for your quick response.
Ayme's paper does describe all these points and many properties,
but he does not show that the centroid, QA-P1, lies on the line
through QA-P2 and QA-P3.

I will continue to hunt for references to this fact since there
is no point in naming a line that only goes through two points.
I will give EQF as my reference until I find additional
references. It would also be nice to find out who was the
original discoverer of this collinearity.

- stan -
```

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Message: #3418
Date: 25/3/2019 5:24:55
From: chris.vantienhoven
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

---

```
Dear Stan,
Ok, thanks.
Please keep me informed.
Best regards,
Chris
```

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Message: #3419
Date: 25/3/2019 11:48:34
From: staninchelmsford
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

---

```
The fact that the centroid (QA-P1) is the midpoint of the line
segment joining the Poncelet point (QA-P2)
and the Gergonne point (QA-P3) seems to be proven in Theorem 11b
of Michal Rolínek and Le Anh Dung, The Miquel Points,
Pseudocircumcenter, and
Euler-Poncelet Point of a Complete Quadrilateral, Forum
Geometricorum, 14(2014)145-153,
where O is the Gergonne point, X is the Poncelet point, and G is
the centroid.

http://forumgeom.fau.edu/FG2014volume14/FG201413.pdf

Was this known before 2014 or is Rolínek the discoverer of this
fact?
```

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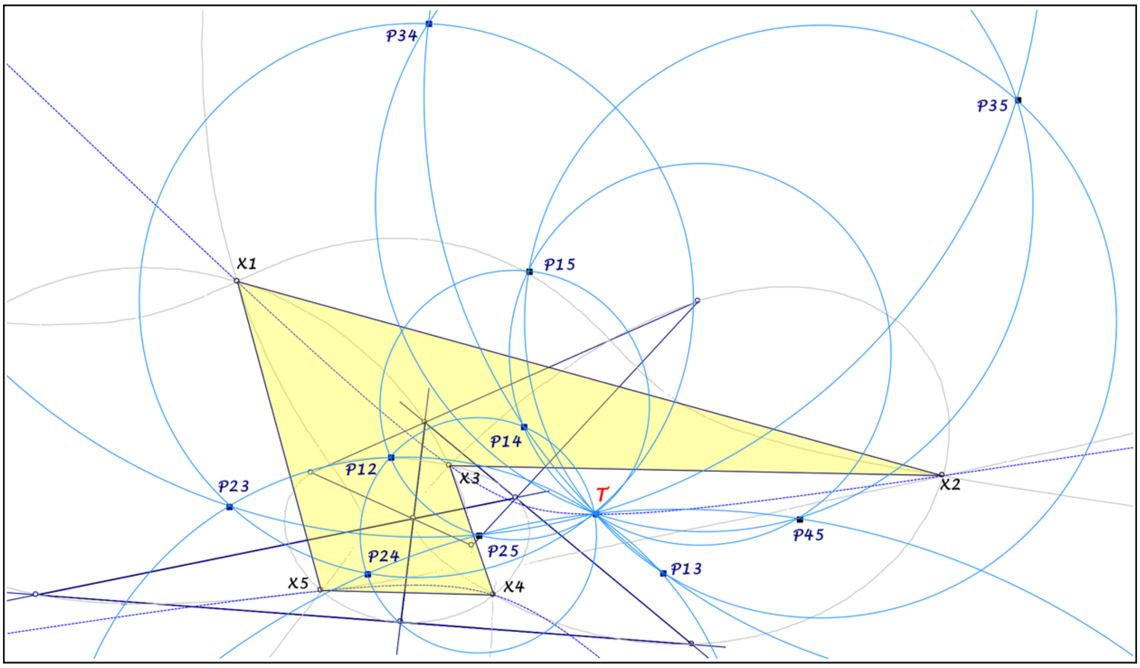
Message: #3420
Date: 27/3/2019 10:06:47
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard, dear Chris,

perhaps of interest, 10 circles with common point T = 5P-s-P4:
Let X1X2X3X4X5 be a 5P with its CB-transformation
… wrt its 5 vertices and the two circular points.

CB maps lines XiXj of the 5P to circles (Xk,Xl,Xm).
For points P on XiXj the lines P.CB(P)
… have a common point Pij on the circle (Xk,Xl,Xm).
… *T = 5P-s-P4 is the common point of the 10 circles
(Pij,Pjk,Pki).*
… T and Pij are collinear with the 4th intersection of
(Xk,Xl,Xm) and 5P-s-Co1,
… … as well as T.Pij parallel Xi.Xj.

Best regards Eckart
```



2019-03-27.pdf

Message: #3421
Date: 28/3/2019 11:31:47
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Each time I spend a few days far from home, you send plenty of interesting messages !
I'll try to answer each one separately in order to avoid too long messages.
I knew already this property, but I can't interpret it.
You should mention that the Pij are the 10 pivots of the degenerated cubics formed by a circle through 3 points and the line through the 2 last points ; of course, T is the pivot of the 11th cubic formed by the conic of the 5 points and the infinity line.
Is there such a property for the 21 pivots in the case of 7 real points, each pivot being on a conic through 5 points (there is no point T in this case) ?
Best regards
Bernard

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Message: #3422
Date: 28/3/2019 11:38:01
From: bernard.keizer
Subject: QA-P2,4,41 perspective QG-P1,19,18

Dear Eckart,
Yes we already mentionned this property, at last indirectly.
The QA-Cu7 is the QA-Cu1 of the QA UjVjUkVk, where the Ui,Vj are the double points ; 2 sides intersect orthogonally in QG-P18.
QA-P2,4,41 is the DT of the QA and QG-P1,19,18 is the Miquel triangle of the QA ; the are in perspective with a point, which is the QA-P3 of the QA.
The QA-P2 of this QA is QG-P18 and the QA-P1 is the middle of the middles of UjVj and UkVk (on the diagonals of the QL) ...
It's interesting that the triangle of these 3 QA-P3 is in turn in perspective with the QG-P19 and QA-P4.
Best regrds
Bernard

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Message: #3423
Date: 28/3/2019 11:39:18
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart, I regret that Chris is now too busy to take
interest in the CB transformations for 7 points or for 5 points
and the circular points as well as for the CSC transformation of
the 5 points (the dual properties are also interesting for 7
lines).
Only one remark to your message ; considering any copple P and
CB(P) in a CB wrt 7 points, they form with the 7 points a CB
system and there are 21 other CB transformations where 2 of the
7 points are CB partners wrt the 5 others and P and CB(P).
In particular, if 2 of the 7 points are the circular points,
there is a CB transformation for the 5 points, P and CB(P) in
which the circular points are CB partners, meaning that the
pivot is on the infinity line ...
Best regards
Bernard

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Message: #3424
Date: 28/3/2019 8:57:40
From: eckart_schmidt@t-online.de
Subject: 5P-s-P4

Dear Chris,

perhaps worth, to be mentioned:
F or the vertices of a QA and QA-P4
… 5P-s-P4 is the reflection of QA-P4 in QA-P3.

Best regards Eckart

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Message: #3425
Date: 28/3/2019 9:18:31
From: chris.vantienhoven
Subject: 5P-s-P4

```
Dear Eckart,
A very nice property of 5P-s-P4 in a Quadrangle!
Best regards,
Chris
```

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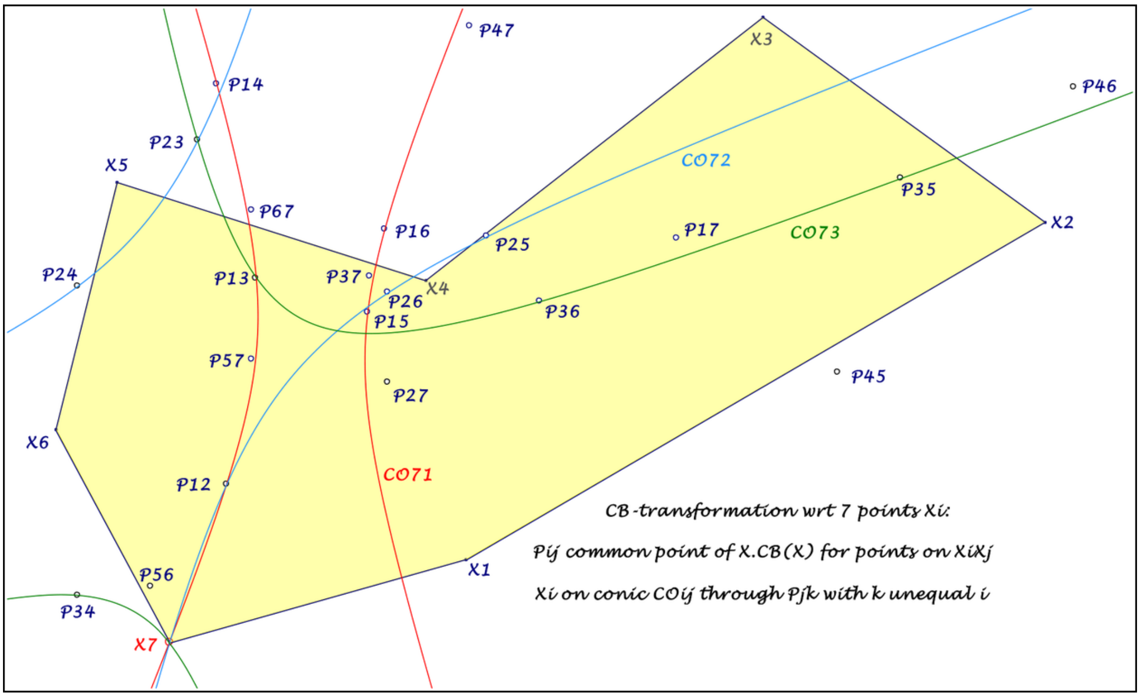
Message: #3426
Date: 29/3/2019 10:56:30
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard,

thanks for your additional remarks in #3421, #3422, #3423.
Wrt the question in #3421 for the 21 "pivots" in the case of 7
real points
… see attached file, perhaps evident?

Best regards Eckart
```



2019-03-29.pdf

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Message: #3427
Date: 29/3/2019 11:44:52
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Last, but not least message of yours, it was far the most interesting and promising !
1) Very clever idea of using the CB transformation and applying to the unique point we have apart of the Xi and the point T !
CB1(QL-P1) on the line through QL-P8 and QL-P24 means that CSC(CB(QL-P1)) is on the line QL-P1U, where U is CSC(QL-P24) and diametral of R on the QA-P4 circle ...
By the way, UR being a diameter of QA-P4 circle and orthogonal to the QG-P18 line means that CSC(UR) is a circle through QL-P1, QL-P24 and CSC(R) and is orthogonal in QL-P24 and CSC(R) to the DT circumcircle QL-Ci1 and in QL-P1 to the DT 9 points circle ; it is therefore centered on the radical axis of both circles, id est the orthic axis of DT.
But I cannot identify this point CB(QL-P1) more precisely ...
2) I reproduced without difficulty your point P, perspector of the QG-P1 and their CB1 partners ; the point T was already identified as the perspector of the QG-P1 and their CB2 partners ...
But I cannot either identify this point more precisely ...
Best regards
Bernard
PS Which method do you use exactly in order to find the CB1 partner ? (you mentionned Hard's method with circles ...).

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Message: #3428
Date: 29/3/2019 4:52:30
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
The circle CSC(UV) is also orthogonal to the polar circle of DT
: centered in the orthocenter QL-P10, this circle is the
inversion circle in the inversion swapping the circumcircle and
the 9 points circle of DT, swapping in particular QG-P1 and
QG-P17 and QL-P1 and CSC(R).
Best regards
Bernard
PS The points R and CSC(R) are simple QL-points and should
deserve to be mentionned in EQF ...
```

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Message: #3429
Date: 30/3/2019 10:37:57
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
Thanks for the figure and the answer to my question !
It's not evident at all and interesting anyhow ...
If I understand correctly, each pivot Pij is on 2 conics COki
and COkj through Xk with k unequal i and j.
That makes something like 42 different conics, each conic
contains 5 Pij and each Pij is on 10 conics.
Best regards
Bernard
```

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Message: #3430
Date: 30/3/2019 4:29:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

wrt #3427:
In 1) you give an interesting excursion in QL-geometry,
… which shows your good overview!

But up to now we only have the line QL-P1.QL-P8.QL-P24 for
CB1(QL-P1),
… similar to the situation for points X on QA-Cu7
… with CB1(X) on the line X.QG-P1.

You asked for the construction of CB1(X)
… wrt 5 points and the two circular points:

I modified my macro of Hart's construction
… as described in #3403,
… sorry, impossible to describe in details.

Best regards Eckart

PS: The last result of #3429 doesn't hold,
    … right descibed in the sentence before.
    In #3428 I cannot reproduce the orthogonality
    … of CSC(UV) and the polar circle of QL-Tr1,
    … if U is the perspector of the QG-P19- and QA-P4-triangle
    … and V is the perspector of the QG-P19- and QG-P1-triangle.
```

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Message: #3431
Date: 30/3/2019 7:03:38
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
Sorry for my mistake, it is CSC(UR) as mentionned in 3427
Best regards
Bernard
```

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Message: #3432
Date: 31/3/2019 10:46:25
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Eckart,
What is wrong in 3429 ? I've just reproduced your property !
Best regards
Bernard
```

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Message: #3433
Date: 31/3/2019 3:40:09
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

```
Dear Bernard,

I very much apologize, your #3429 is correct!
I became in trouble with my own nomination.
In excuse an unexpected splitter:
Consider a QA and a point P,
… take the 4th harmonic point
… of P wrt the QA-Tf2-partner on lines through P
… and you get the QA-circumconic through P.

Best regards Eckart
```

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Message: #3434
Date: 01/4/2019 11:42:30
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for the property !
But it's not totally unexpected.
The 4 points define the QA-Tf2 and the point P defines a pivotal isocubic, locus QA-Tf2 partners on lines through P, which are also CB partners for the 7 points QA vertices and DT vertices.
The conic through P and the 4 QA vertices is the polar conic of P wrt the cubic, which is the unique diagonal polar conic : it passes through the 4 points, P and the vertices of the anticevian triangle of P wrt DT and more generally the vertices of the anticevian triangle of any of it's points.
(see Jean-Pierre Ehrmann and Bernard Gibert Special Isocubics ...p30-31)
Best regards
Bernard

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Message: #3435
Date: 01/4/2019 4:41:54
From: eckart_schmidt@t-online.de
Subject: CB-Transformations

```
Dear Bernard, dear Chris,

let us consider CB-transformations
… CB wrt 7 real points,
… cb wrt 5 real points and the two circular points.

For a "pivot" P we can look for cubics
… as loci of CB-/cb-partner on lines through P.

Well known:
For P = QA-P4 and CB-partner
… wrt the vertices of the QA and its QA-Tr1
… we get QA-Cu1 (CB is QA-Tf2).

New perhaps:
For the same P = QA-P4 and cb-partner
… wrt the QA-vertices and the 2nd intersection X
… of QA-P3.QA-P41 and the QA-circumconic through P = QA-P3
… we get also QA-Cu1.

In addition wrt the 5P of the QA-vertices and X:
… 5P-s-P4 = QA-P3, 5P-CSC(X) = QA-P41,
  5P-s-Tf3(QA-P4) = QA-P2.

Best regards Eckart

PS: Thanks to Bernard for the background of my splitter in
#3433.
```

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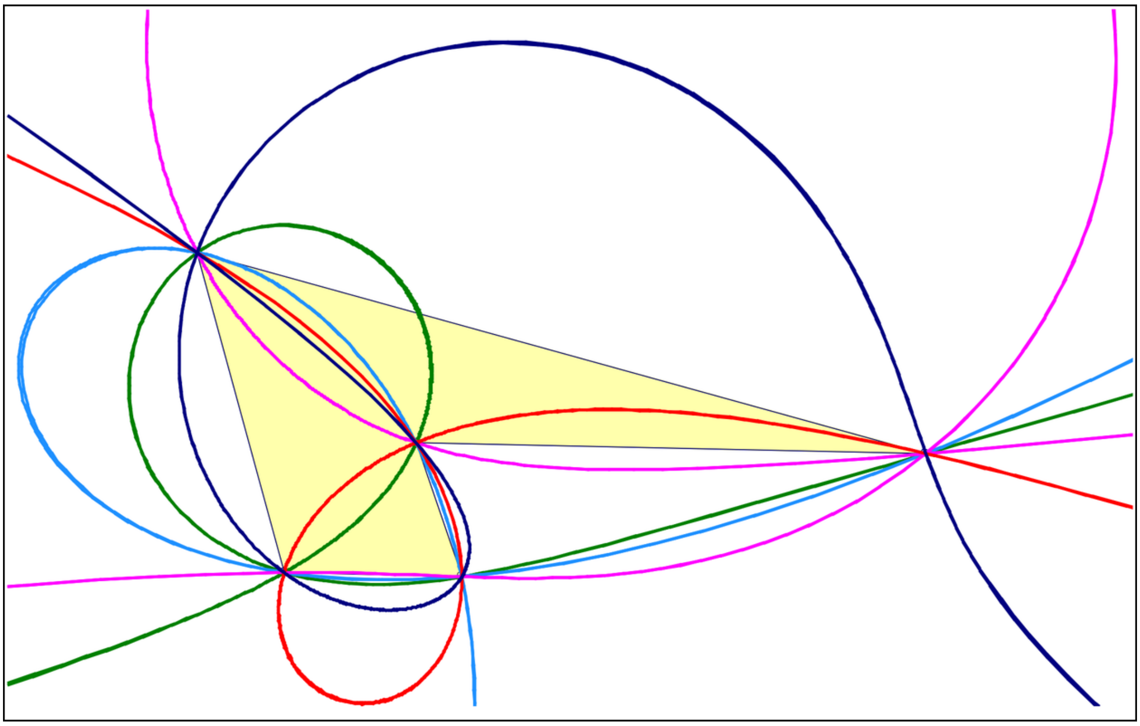
Message: #3436
Date: 04/4/2019 8:50:42
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard,

attached 5 circumcubics for a 5P
… with curious construction:

Take a circle round one vertex of the 5P
… and its cb-image wrt the 5P and the 2 circular points,
… which is a curve of higher degree.
… Reducing the radius of the circle,
… you will get as limit a cubic,
… locus of the points with cb-image in the chosen vertex.

Best regards Eckart
```



2019-04-03.pdf

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Message: #3437
Date: 04/4/2019 11:36:09
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Beautiful curves, indeed (stophoïds ?)
I loose progressively courage and I don't know if we will one day find a simple construction of the QL starting with 5 points taken as triple QA-Cu7 points, which would be an enormous progress in the knowledge of the 5P Geometry identical to the QL-Geometry.
We have the 5 points, the circumconic, the point T, the CSC transformation and the CB transformation wrt the 5 + 2 circulars.
And plenty of ideas, but not enough time to check them ...
For example, the 5 points give 5 QA's and 5 pivotal isocubics with pivot the 5th point. The vertices of the anticevian triangle of the 5th point wrt the DT of the QA of the 4 points give 3 more points having with the 5 th point the same DT as the QA of the 4 points.
What about the DT of the QA's formed by the 3 points S and either T or R ?
The triangles QG-P1 and Si perspective with T give a Reye configuration and another cubic, the same for the QA-P4 and Si perspective with R.
Each morning, I open my computer, hoping desperately that you will have found a solution...
Best regards
Bernard

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Message: #3438
Date: 04/4/2019 11:53:46
From: eckart_schmidt@t-online.de
Subject: Orthogonal QL-Hyperbolas

Dear Bernard,

let us consider a generalization of the splitter in #3433:

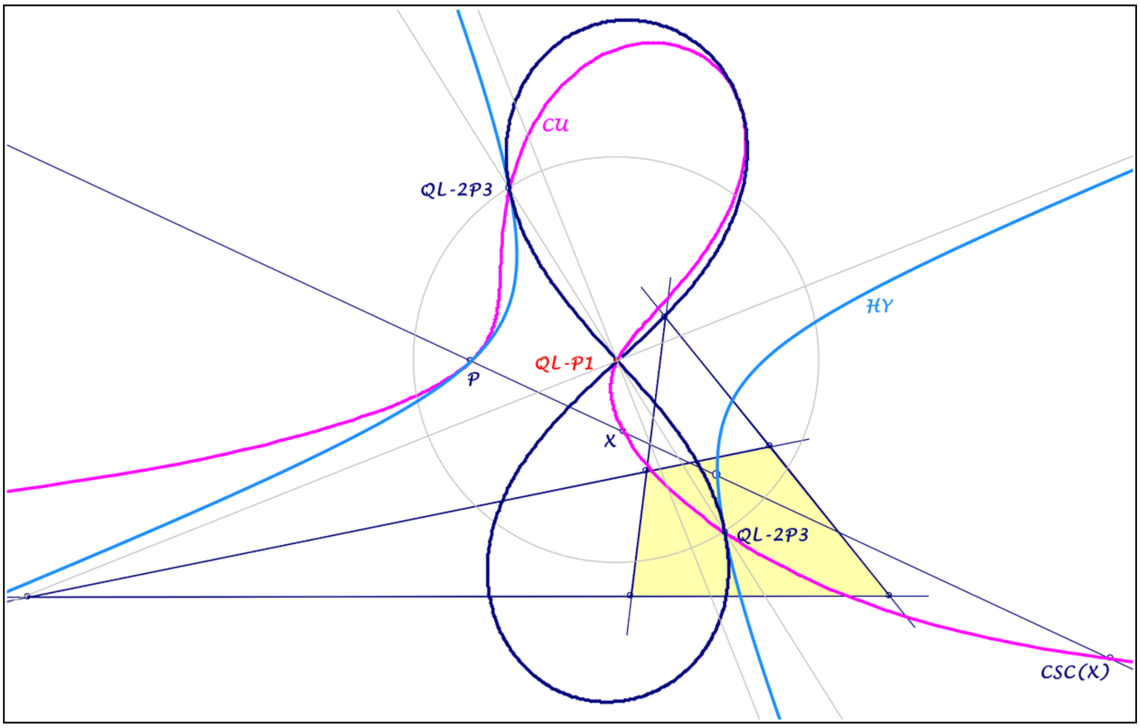
1) For a pivot P
… the CSC-partner on lines through P give a cubic CU
… through P, QL-P1 and QL-2P3.
The 4th harmonic points of P wrt the CSC-partner
… give an orthogonal hyperbola HY,
… centered in QL-P1 through P and QL-2P3
… whose CSC-image is a lemniscate LE,
… centered in QL-P1, tangent to HY in QL-2P3,
… with its foci in the CSC-images of the HY-foci.

2) Now we consider CB-transformations cb,
… defined by 5 real points and the two circular points:
For a pivot P
… the cb-partner on lines through P give a cubic,
… the 4th harmonic points of P wrt the cb-partner
… give a conic with tangent in P through cb(P) ...

What about these conics? Especially:
… cb wrt the 5 QA-Cu7-triple points and pivot QL-P1.

Best regards Eckart

PS-splitter with a well known background:
Let X, Y be two opposite points of the QL
… then X.cb(X) ^ Y.cb(Y) = QG-P1.



2019-04-04.pdf

Message: #3439
Date: 05/4/2019 9:21:50
From: eckart_schmidt@t-online.de
Subject: QL-Parabolas

```
Dear Bernard, dear Chris,

there is an interesting construction
… of QL-parabolas
wrt a defining point P unequal QL-P1:
Consider circles round P with variable radius
… and the radical axes wrt their CSC-image circles,
… which envelop a parabola
… with focus CSC(P) and directrix P.QL-P1,
… tangent to the Steiner axes.
The parabolas degenerate for P on the Steiner axes.

Best regards Eckart
```

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Message: #3440
Date: 05/4/2019 10:06:09
From: bernard.keizer
Subject: Orthogonal QL-Hyperbolas

Dear Eckart,
1) Your property in the splitter is very general
For any point, a variable line through the point cuts a conic in 2 points and the harmonic of the point wrt the 2 intersections describe a line, which is the polar of the point wrt the conic.
For any point on a cubic, a variable line through the point cuts the cubic in 2 points and the harmonic of the point wrt the 2 intersections describe a conic, which is the polar conic of the point.
2) For any cubic, the tangential QA of a point gives 4 points (not necessary real) which are the fixed points of an isoconjugation wrt the DT of the QA ...
With the CSC partners, you get rectangular hyperbolas through the QL-2P3.
These RH are the polar conics of the infinity points wrt QL-Cu2, the cubic stelloïd associated to the QL.
3) For the point QL-P1, as the CSC partners are on lines symmetric wrt the Steiner axes, I suppose the cubic degenerates in 3 lines 2 Steiner axes and infinity line and the conic is tangent to the Steiner axes (?).
Best regards
Bernard
PS By the way, P is the common tangential of the QL-2P3 and the lemniscate is tangent to the cubic in CSC(P), which is the tangential of P.

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Message: #3441
Date: 06/4/2019 12:51:38
From: bernard.keizer
Subject: Orthogonal QL-Hyperbolas

```
Dear Eckart,
In 3), my last remark is not correct.
For P in QL-P1, the RH degenerates in the 2 Steiner axes and is
the polar conic in the cubic stelloïd of the infinity point of
the Newton Line QL-L1.
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

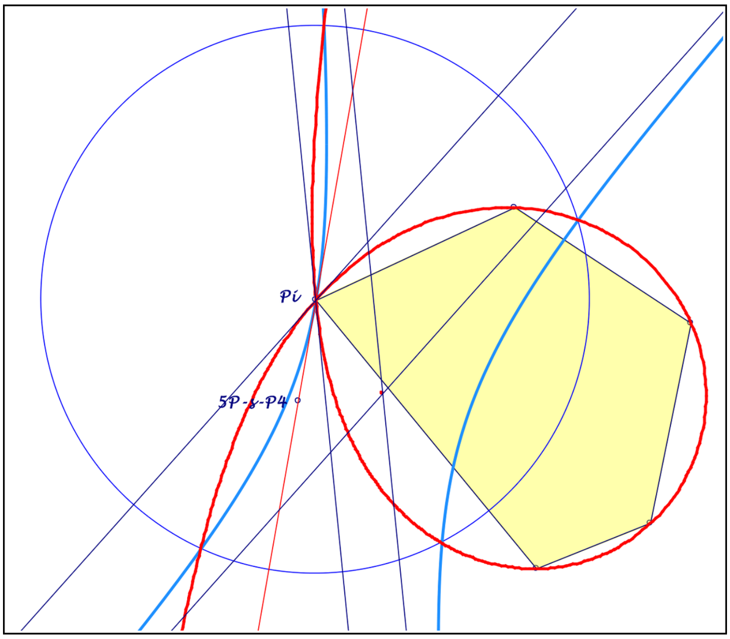
Message: #3442
Date: 10/4/2019 10:30:43
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

in message 3436 I offered 5P-cubics,
… which are the locus of points X
… with cb(X) in one vertex Pi of the 5P,
… cb = CB-transformation, defined by 5P and 2 circular points.
Here is a simple concrete construction:
… Draw a circle round a chosen 5P-vertex Pi
… and the inversions Qj of the other four 5P-points.
… Consider the conic through Pi,Qj,Qk,Ql,Qm
… and its inversion wrt the circle,
… which gives the cubic above (see attached file).
The asymptote of the cubic is parallel Pi.5P-s-P4.
If the conic is a hyperbola,
… the cubic has a double point in Pi
… with tangents parallel to the asymptotes of the conic,
… but t he cubic must not be a strophoid.
These cubics can also be discussed for QA wrt a point P.

Best regards Eckart

PS: Many thanks for explanations in #3440,
    for I am not so familar with polar conics.



2019-04-10.pdf

Message: #3443
Date: 14/4/2019 9:25:40
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

```
EQF lists five points on the QG-Centroids Line, QG-L3 (also
known as the Seebach-Walser Line).
I think I've found a new point on this line.

Does anyone know if the following result is already known?

Let ABCD be a quadrigon with diagonal crosspoint E.
Consider the line segment from the midpoint of AB to the
centroid of triangle CDE.
Also consider the other 3 similar line segments from the
midpoints of the other 3 sides.
Then these 4 line segments meet at a point P.

This point P lies on the Seebach-Walser line.
If M=QA-P1 (the vertex centroid of the quadrangle) and G=QG-P4
(the area centroid of the quadrigon),
then EP:PM:MG=7:2:3.
```

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Message: #3444
Date: 15/4/2019 10:15:52
From: eckart_schmidt@t-online.de
Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford,

if I am not wrong the ratios for your new QG-point P are as follows:
EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,

P divides QG-P1.QG-P4 with ratio 3 : 2,
… QG-P1.QG-P8 with ratio -6:1,
… QG-P1. QA-P1 with ratio 4 : 1,
… QG-P1.QG-P15 with ratio 2 : 3,
… QG-P5.QG-P11 with ratio 4 : 1,
… QG-P6.QG-P9 with 4 : 1.

Best regards Eckart

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Message: #3445
Date: 15/4/2019 11:09:32
From: eckart_schmidt@t-online.de
Subject: Quintupel-Points for a 5P

Dear Bernard,

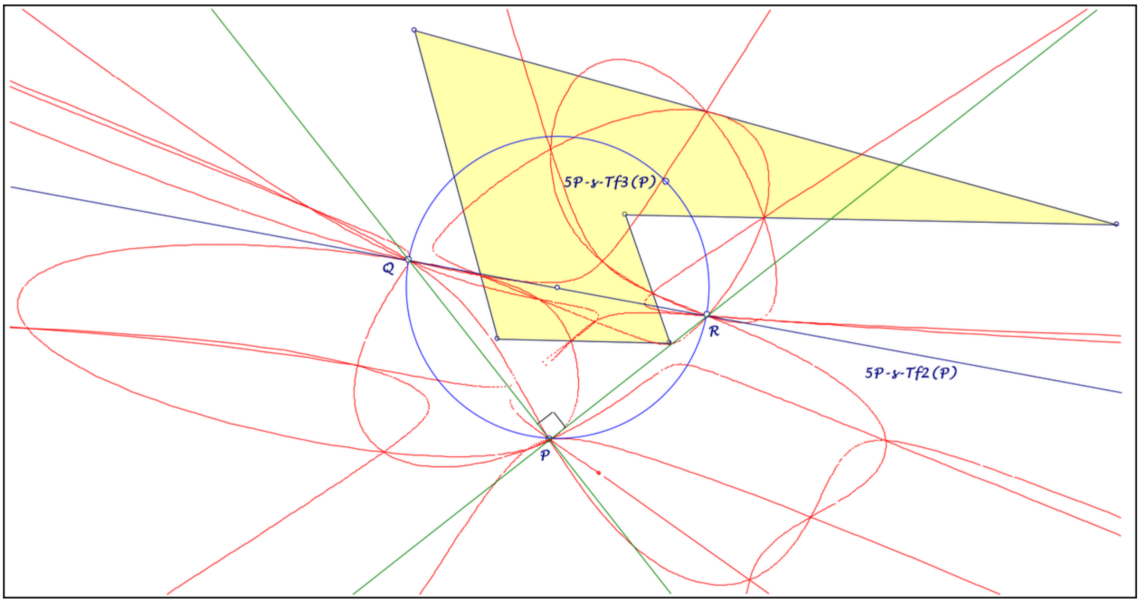
if the mysterious 5 QA-Cu7-triple points of a QL
… become boring, here a new constellation:

Replacing in the construction of QA-Cu7
… QA-P4 by another point P,
… we get cubics CU(P) for a QA (see #3341).

Now consider a 5P and any point P,
… construct the 5 cubics CU(P) wrt 4 points of the 5P,
… and you get 20 triple-points,
… … pairwise on the ten 5P-lines, in harmonic position,
… … circles with these pairs as diameter bear P,
… further common points: 3 quintupel-points P, Q, R
… … with QR = 5P-s-Tf2(P), PR orthogonal PQ,
… … and 5P-s-P3(P) on the circumcircle of PQR.

In this way the 2 quintupel-points Q and R beside P
… are constructible!

Best regards Eckart



2019-04-15.pdf

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Message: #3446
Date: 15/4/2019 11:31:16
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Sorry, I wasn't home for a few days.
This is a very beautiful construction indeed !
I checked without difficulty that the cubic obtained by
inverting a conic gives the same result as my construction in
3385 with pivot in one of the 5 points.
Best regards
Bernard

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Message: #3447
Date: 15/4/2019 12:14:40
From: Tran Quang Hung
Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford and Eckart

I see the general propblem

ABCD be a quadrigon with any point P.
Consider the line segment from the midpoint of AB to the
centroid of triangle PCD.
Also consider the other 3 similar line segments from the
midpoints of the other 3 sides.
Then these 4 line segments meet at a point Q.
When P=E diagonal crosspoint, we get above problem?

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 15:15 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Staninchelmsford,
>> if I am not wrong the ratios for your new QG-point P are as
follows:
>> EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,

```
>> P divides
>> QG-P1.QG-P4 with ratio 3 : 2,
>> … QG-P1.QG-P8 with ratio -6:1,
>> … QG-P1. QA-P1 with ratio 4 : 1,
>> … QG-P1.QG-P15 with ratio 2 : 3,
>> … QG-P5.QG-P11 with ratio 4 : 1,
>> … QG-P6.QG-P9 with 4 : 1.
>> Best regards Eckart
```

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Message: #3448
Date: 15/4/2019 3:09:55
From: Tran Quang Hung
Subject: Newton lines are concurrent with 5P

```
Dear geometers,

Let A,B,C,D,E be five points.
Let
d1=Newton line of four lines (AB,BC,CD,DE)
d2=Newton line of four lines (EA,BC,CD,DE)
d3=Newton line of four lines (EA,AB,CD,DE)
d4=Newton line of four lines (EA,AB,BC,DE)
d5=Newton line of four lines (EA,AB,BC,CD)
Then d1,d2,d3,d4,d5 are concurrent. Which is this point?

Best regards
Tran Quang Hung.
```

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Message: #3449
Date: 15/4/2019 3:11:10
From: Thanh Tung
Subject: Another point on the Seebach-Walser Line

Dear Tran Quang Hung, Staninchelmsford and Eckart,

For the problem below,
the point of concurrency
Q is centroid of the five points P, A, B ,C, D.

Best regards,
Vu Thanh Tung

Le lundi 15 avril 2019 à 17:14:48 UTC+7, Tran Quang Hung
analgeomatica@gmail.com [Quadri-Figures-Group] a écrit :
Dear Staninchelmsford and Eckart
I see the general propblem
ABCD be a quadrigon with any point P.
Consider the line segment from the midpoint of AB to the
centroid of triangle PCD.
Also consider the other 3 similar line segments from the
midpoints of the other 3 sides.
Then these 4 line segments meet at a point Q.
When P=E diagonal crosspoint, we get above problem?
Best regards
Tran Quang Hung.

Và o Th 2, 15 thg 4, 2019 vaÌ€o luÌc 15:15 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups..com (
Quadri-Figures-Group@yahoogroups.com) > Ä‘ã viá°¿t:
>> Dear Staninchelmsford,
>> if I am not wrong the ratios for your new QG-point P are as
follows:
>> EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,
>> P divides
>> QG-P1.QG-P4 with ratio 3 : 2,
>> … QG-P1.QG-P8 with ratio -6:1,
>> … QG-P1. QA-P1 with ratio 4 : 1,
>> … QG-P1.QG-P15 with ratio 2 : 3,
>> … QG-P5.QG-P11 with ratio 4 : 1,
>> … QG-P6.QG-P9 with 4 : 1.
>>> Best regards Eckart

Message: #3450
Date: 15/4/2019 3:29:52
From: eckart_schmidt@t-online.de
Subject: Another point on the Seebach-Walser Line

```
Dear Staninchelmsford and Tran Quang Hung,

wrt the generalized proplem in #3347:
… P, Q, QA-P4 are collinear
… and Q divides P.QA-P1 with ratio 4:1.

Best regards Eckart
```

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Message: #3451
Date: 15/4/2019 4:33:19
From: eckart_schmidt@t-online.de
Subject: Newton lines are concurrent with 5P

```
Dear Tran Quang Hung,

The common point of the Newton lines for the QL-versions of a 5L
… is 5L-s-P1 (see EPG).

Best regards Eckart
```

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Message: #3452
Date: 16/4/2019 12:25:49
From: Tran Quang Hung
Subject: Newton lines are concurrent with 5P

```
Dear Eckart

Now I see, thank you very much.

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 21:33 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> The common point of the Newton lines for the QL-versions of a
5L
>> … is 5L-s-P1 (see EPG).
>> Best regards Eckart
```

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Message: #3453
Date: 16/4/2019 1:04:20
From: Tran Quang Hung
Subject: Miquel circle of 5L

```
Dear geometers,

Let d1, d2, d3, d4, d5 be five lines.
M1 = Miquel point of (d2, d3, d4, d5).
Define similarly, M2, M3, M4, M5.
Then M1, M2, M3, M4, M5 lie on a circle.
It seems that this property does not like 5L-s-L3?

Best regards
Tran Quang Hung.
```

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Message: #3454
Date: 16/4/2019 3:02:39
From: Tran Quang Hung
Subject: Gergonne circle

```
Dear geometers

Let d1, d2, d3, d4 be four lines.
A12 = d1 meest d2.
Define similarly the points A13, A14, A23, A24, A34.

Let
G1 = Gergonne point (QA-P3) of (A12, A14, A23, A34)
G2 = Gergonne point (QA-P3) of (A23, A24, A13, A14)
G3 = Gergonne point (QA-P3) of (A12, A13, A24, A34)

Then circumcircle of triangle G1G2G3 passes through Miquel point
(QL-P1) of (d1,d2,d3,d4).
I call this circle by Gergonne circle of (d1, d2, d3, d4). Is
this circle and its center known before?

Best regards
Tran Quang Hung.
```

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Message: #3455
Date: 16/4/2019 3:46:56
From: Tran Quang Hung
Subject: Miquel circles are concurrent with 5L

```
Dear geometers

Let d1, d2, d3, d4, d5 be five lines.
Let circle (M1) = Miquel circle (QL-Ci3) of (d2,d3,d4,d5).
Define similarly circles (M2), (M3), (M4), (M5).
Then five circles (M1), (M2), (M3), (M4), (M5) are concurrent at
a point.
Which is this point of 5L?

Best regards
Tran Quang Hung.
```

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Message: #3456
Date: 16/4/2019 3:47:47
From: Tran Quang Hung
Subject: Another point on the Seebach-Walser Line

```
Thank You for your interest.

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 20:29 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Staninchelmsford and Tran Quang Hung,
>> wrt the generalized proplem in #3347:
>> … P, Q, QA-P4 are collinear
>> … and Q divides P..QA-P1 with ratio 4:1.
>> Best regards Eckart
```

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Message: #3457
Date: 16/4/2019 5:02:45
From: Tran Quang Hung
Subject: Two QL share the same Miquel point and Steiner line

Dear geometers,

Let d1, d2, d3, d4 be four lines.
Let A1 be orthopole of d1 wrt triangle (d2,d3,d4).
Let l1 be orthotranversal of A1 wrt triangle (d2,d3,d4).
Define similarly the lines l2, l3, l4.
Then (d1,d2,d3,d4) and (l1,l2,l3,l4) share the same Miquel point (QL-P1) and the same Steiner line (QL-L2).
Miquel circle of (d1,d2,d3,d4) and (l1,l2,l3,l4) meet again at X (other than QL-P1). Which is this point?

Best regards
Tran Quang Hung.

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Message: #3458
Date: 16/4/2019 9:39:31
From: bernard.keizer
Subject: Gergonne circle

Dear Tran Quang Hung,
Your circle is the Dimidium circle (see EQF)
Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3459
Date: 16/4/2019 9:56:15
From: eckart_schmidt@t-online.de
Subject: Miquel circle of 5L

```
Dear Tran Quang Hung,

it seems, that you are right:
… your Miquel circle for a 5L is not in EPG,
… but it is already discussed years ago
… for example in #710.

Following generalization holds:
… The QL-Tf1-images of a point
… wrt the 5 QL of a 5L are concyclic.

Best regards Eckart
```

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Message: #3460
Date: 16/4/2019 10:24:08
From: eckart_schmidt@t-online.de
Subject: Miquel circles are concurrent with 5L

```
Dear Tran Quang Hung,

I am not shure, that this point is in EPG,
… for I am not fit in Chris' nomination,
… but it is also already mentioned 2014 in #710.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3461
Date: 16/4/2019 10:36:58
From: eckart_schmidt@t-online.de
Subject: Gergonne circle

```
Dear Tran Quang Hung,

this circle is QL-Ci6 in EQF.

Best regards Eckart
```

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Message: #3462
Date: 16/4/2019 12:07:27
From: Tran Quang Hung
Subject: Gergonne circle

```
Thank You very much Bernard and Eckart for all quick and clear
reply.

Best regards
Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 16:27 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> this circle is QL-Ci6 in EQF.
>> Best regards Eckart
```

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Message: #3463
Date: 16/4/2019 1:17:25
From: Tran Quang Hung
Subject: Center of circle lies on Steiner line

```
Dear geometers

Let d1, d2, d3, d4 be four lines.
A12 = d1 mees d2.
Define similarly the points A13, A14, A23, A24, A34.
M is Miquel point of (d1,d2,d3,d4).
Let M1 be isogonal conjugate of M wrt triangle A23A14A34
Let M2 be isogonal conjugate of M wrt triangle A12A14A23
Let M3 be isogonal conjugate of M wrt triangle A12A24A13
Let M4 be isogonal conjugate of M wrt triangle A24A34A13

Then M1, M2, M3, M4 lie on a circle and center of this circle
lies on Steiner line of (d1,d2,d3,d4).
Let M5 be isogonal conjugate of M wrt triangle A12A14A34
Let M6 be isogonal conjugate of M wrt triangle A12A34A23
Let M7 be isogonal conjugate of M wrt triangle A13A14A24
Let M8 be isogonal conjugate of M wrt triangle A23A24A13
Then M5, M6, M7, M8 also lie on a circle and center of this
circle lies on Steiner line of (d1,d2,d3,d4).

Best regards
Tran Quang Hung.
```

Message: #3464
Date: 16/4/2019 1:51:38
From: eckart_schmidt@t-online.de
Subject: Two QL share the same Miquel point and Steiner line

Dear Tran Quang Hung,

interesting second QL (l1,l2,l3,l4)!
… Not only QL-P1 and QL-L2 are the same,
… but also the *inscribed parabola QL-Co1.*

Best regards Eckart

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Message: #3465
Date: 16/4/2019 3:28:58
From: Tran Quang Hung
Subject: Two QL share the same Miquel point and Steiner line

Thank You Eckart for your interest.

Thus QL(d1,d2,d3,d4) and QL(l1,l2,l3,l4) share the same Miquel point and the same Steiner line is actually consequence of they share the same inscribed parabola QL-Co1. This is very interesting!

Best regards
Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 19:28 ' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Tran Quang Hung,
>> interesting second QL (l1,l2,l3,l4)!
>> … Not only QL-P1 and QL-L2 are the same,
>> … but also the *inscribed parabola QL-Co1.*
>> Best regards Eckart

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Message: #3466
Date: 16/4/2019 3:32:08
From: eckart_schmidt@t-online.de
Subject: Center of circle lies on Steiner line

```
Dear Tran Quang Hung,

Your two circles (M1,M2,M3,M4) and (M5,M6,M7,M8)
… have a radical axis,
… whose QL-Tf1 image circle
… has a tangent in QL-P1 through QL-P4.

Your two circles (M1,M2,M3,M4) and (M5,M6,M7,M8)
… have QL-Tf1 image circles,
… which are orthogonal to QL-Ci3
… with radical axis through QL-P4.

Best regards Eckart
```

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Message: #3467
Date: 16/4/2019 3:32:26
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

```
Dear geometers,

Let d1, d2, d3, d4 be four lines.
Let e1, e2, e3 be the diagonal lines of QL(d1,d2,d3,d4).
Let s1 be Steiner line of QL(e1,e2,e3,d1).
Define similarly the lines s2,s3,s4.
Then s1,s2,s3,s4 are concurrent. Which is this point?

Best regards
Tran Quang Hung.
```

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Message: #3468
Date: 16/4/2019 3:46:28
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

```
Dear Tran Quang Hung,

the described point is QL-P10.

Best regards Eckart
```

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Message: #3469
Date: 16/4/2019 6:38:43
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

```
Thank You Eckart

I also see that
M1 = Miquel point of QL(e1,e2,e3,d1).
M2 = Miquel point of QL(e1,e2,e3,d2).
M3 = Miquel point of QL(e1,e2,e3,d3).
M4 = Miquel point of QL(e1,e2,e3,d4).
Then four points M1, M2, M3, M4 are on a circle and its center
lies on Steiner line of QL(d1,d2,d3,d4).

Best regards
Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 22:57 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> the described point is QL-P10.
>> Best regards Eckart
```

Message: #3470
Date: 16/4/2019 6:42:21
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Dear Eckart and all,

I also see that
n1 = Newton line of QL(e1,e2,e3,d1).
n2 = Newton line of QL(e1,e2,e3,d2).
n3 = Newton line of QL(e1,e2,e3,d3).
n4 = Newton line of QL(e1,e2,e3,d4).
n = Newton line of QL(d1,d2,d3,d4).
Then two Newton lines of QL(n,e1,e2,e3) and QL(n1,n2,n3,n4) are the same.

Best regards
Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 23:38 Tran Quang Hung < analgeomatica@gmail.com > đã viết:
>> Thank You Eckart
>> I also see that
>> M1 = Miquel point of QL(e1,e2,e3,d1).
>> M2 = Miquel point of QL(e1,e2,e3,d2).
>> M3 = Miquel point of QL(e1,e2,e3,d3).
>> M4 = Miquel point of QL(e1,e2,e3,d4).
>> Then four points M1, M2, M3, M4 are on a circle and its center lies on
>> Steiner line of QL(d1,d2,d3,d4).
>> Best regards
>> Tran Quang Hung.

>> Vào Th 3, 16 thg 4, 2019 vào lúc 22:57 ' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
>> [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>>> Dear Tran Quang Hung,
>>> the described point is QL-P10.
>>> Best regards Eckart

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Message: #3471
Date: 16/4/2019 9:35:00
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

only two additional remarks:
wrt #3469:
The circle (M1,M2,M3,M4) is
… the circumcircle of the diagonal triangle QL-Tr1.
wrt # 3470:
QL(n1,n2,n3,n4) has the vertices
… midpoints of Aij and Aik.Ajl ^ Ail.Ajk.

Best regards Eckart

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Message: #3472
Date: 17/4/2019 1:42:48
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Tran Quang Hung,

You wrote:
> When P=E diagonal crosspoint, we get above problem?

Yes. You are correct. Nice generalization.

---In Quadri-Figures-Group@yahoogroups.com, wrote :
Thank You for your interest.
Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 20:29 ' eckart_schmidt@... '
eckart_schmidt@... [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
>>> Dear Staninchelmsford and Tran Quang Hung,
>>> wrt the generalized proplem in #3347:
>>> … P, Q, QA-P4 are collinear
>>> … and Q divides P..QA-P1 with ratio 4:1.
>>> Best regards Eckart

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Message: #3473
Date: 17/4/2019 2:06:34
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Thank You Eckart, I now understand two these messages.

Best regards
Tran Quang Hung.

Vào Th 4, 17 thg 4, 2019 vào lúc 05:43 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> only two additional remarks:
>> wrt #3469:
>> The circle (M1,M2,M3,M4) is
>> … the circumcircle of the diagonal triangle QL-Tr1.
>> wrt # 3470:
>> QL(n1,n2,n3,n4) has the vertices
>> … midpoints of Aij and Aik.Ajl ^ Ail.Ajk.
>> Best regards Eckart

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Message: #3474
Date: 17/4/2019 2:07:32
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

```
Dear Eckart,

You wrote:
> P, Q, QA-P4 are collinear
> and Q divides P.QA-P1 with ratio 4:1.

I assume this was a typo and you meant "P, Q, QA-P1 are
collinear"
Anyhow, this ratio, 4 : 1, appears to be correct. It also agrees
with the ratio Tran Quang Hung found.
Earlier, you wrote:
>if I am not wrong the ratios for your new QG-point P are as
follows:
> EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5

Again, I think this was a typo and you meant
EP : PM : MG = QG-P1.P : P.QA- P1 : QA- P1.QG-P4 = ...
I think your ratio 12 : 3 : 5 is wrong though. I think this
should be 12 : 3 : 4.5
Also, my original ratio of 7 : 2 : 3 was wrong also.
I will double-check my numbers and write up the results in a
separate reply.
```

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Message: #3475
Date: 17/4/2019 2:57:02
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

```
I've rechecked my calculations.
If we call my new point QG-new, then we have 6 points on the
Seebach-Walser Line (QG-L3).

The ratios appear to be:

QG-P1 : QG-P8 : QG-new : QA-P1 : QG-P4 : QG-P15 = 10 : 2 : 3 : 5
: 10
```

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Message: #3476
Date: 17/4/2019 9:39:43
From: eckart_schmidt@t-online.de
Subject: Another point on the Seebach-Walser Line

```
Dear Staninchelmsford,

you are right, correcting my typo, I beg your pardon!
But - excuse - I cannot confirm your ratio 12 : 3 : 4.5
in #3474,
… and I do not understand the notation in #3475.

Best regards Eckart
```

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Message: #3477
Date: 17/4/2019 2:54:37
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

```
Dear Eckart,

I appreciate your checking my numbers.
I will go check again.
```

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Message: #3478
Date: 17/4/2019 3:37:05
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

```
Dear Tran Quang Hung,

wrt QL' = QL(n1,n2,n3,n4) in #3470:
In #3471 I described the vertices of QL':
… … midpoints of Aij and Aik.Ajl ^ Ail.Ajk.

Further properties:
… QL-P1 of QL' is the midpoint of QL-P17.QL-P25.
… QL-L1 of QL' is the bisector of QL-P1 and QL-P7,
… … is a parallel to QL-L9 through QL-P19,
… QL-L2 of QL' is a parallel to QL-L6 through QL-P11.
… QL-L6 of QL' is QL-L2 of the reference QL.
… QL-L7, QL-L8 of QL' are the same as for reference QL.
… QL-L9 of QL' is QL-L1 of the reference QL.
… QL-Ci1 of QL' is a circle round QL-P11 through QL-P25.
… … … If we apply the construction of QL' once more,
… we get a QL'', homothetic to the reference QL,
… center QL-P8, ratio 4:1.

Best regards Eckart
```

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Message: #3479
Date: 17/4/2019 4:22:40
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Eckart,

You wrote:
> I do not understand the notation in #3475.
Sorry, I had listed the 6 points and then gave the ratios of the consecutive distances between them.
What I should have said was:

QG-P1.QG-P8: QG-P8.
QG-new : QG-new.QA-P1 : QA-P1.QG-P4 : QG-P4.QG-P15 = 10 : 2 : 3 : 5 : 10

But if my number of 4.5 = 9/2 in message #3474 is wrong, then this would be wrong as well.
I'll check again and report back.

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Message: #3480
Date: 18/4/2019 2:16:12
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

```
Thank You Eckart for the interesting properties of
QL(n1,n2,n3,n4).

Best regards
Tran Quang Hung.

Vào Th 4, 17 thg 4, 2019 vào lúc 20:57 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt QL' = QL(n1,n2,n3,n4) in #3470:
>> In #3471 I described the vertices of QL':
>> … … midpoints of Aij and Aik.Ajl ^ Ail.Ajk.
>> Further properties:
>> … QL-P1 of QL' is the midpoint of QL-P17.QL-P25.
>> … QL-L1 of QL' is the bisector of QL-P1 and QL-P7,
>> … … is a parallel to QL-L9 through QL-P19,
>> … QL-L2 of QL' is a parallel to QL-L6 through QL-P11.
>> … QL-L6 of QL' is QL-L2 of the reference QL.
>> … QL-L7, QL-L8 of QL' are the same as for reference QL.
>> … QL-L9 of QL' is QL-L1 of the reference QL.
>> … QL-Ci1 of QL' is a circle round QL-P11 through QL-P25.
>> … … …
>> If we apply the construction of QL' once more,
>> … we get a QL'', homothetic to the reference QL,
>> … center QL-P8, ratio 4:1.
>> Best regards Eckart
```

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Message: #3481
Date: 18/4/2019 9:25:29
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

```
Dear Tran Quang Hung,

If we consider for the QL(e1,e2,e3,di)
the circles QL-Ci3,
… these circles have a common point in QL-P9.
If we consider for the QL(e1,e2,e3,di)
the circles QL-Ci4,
… these circles have collinear centers
… on a perpendicular to QL-L7 in QL-P11.
If we consider for the QL(e1,e2,e3,di) the circles QL-Ci5,
… the radical axes intersect in QL-P10.
If we consider for the QL(e1,e2,e3,di) the circles QL-Ci6,
… the radical axes intersect in QL-P8.
If we consider for the QL(e1,e2,e3,di)
the circles QL-Ci2,
… we get a common point on QL-Ci1.

What about this point?

Best regards Eckart
```

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Message: #3482
Date: 18/4/2019 12:04:18
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

in addition to #3481:

If we consider for the QL(e1,e2,e3,di) QL-P1,
… we get points on QL-Ci1 of the reference QL.

If we consider for the QL(e1,e2,e3,di)QL-P3,
… we get collinear points
… on a perpendicular to QL-L7 through QL-P11.

If we consider for the QL(e1,e2,e3,di) QL-P23,
… we get collinear points.

What about the corresponding line?

Best regards Eckart

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Message: #3483
Date: 19/4/2019 2:02:02
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

```
Thank You Eckart for your interest in the configuration
QL(e1,e2,e3,di).

Your facts is very interesting. I shall probably need a lot of
time to study the EQF to find more interesting facts like You.

Best regards
Tran Quang Hung.

Vào Th 5, 18 thg 4, 2019 vào lúc 17:04 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> in addition to #3481:
>> If we consider for the QL(e1,e2,e3,di) QL-P1,
>> … we get points on QL-Ci1 of the reference QL.
>> If we consider for the QL(e1,e2,e3,di) QL-P3,
>> … we get collinear points
>> … on a perpendicular to QL-L7 through QL-P11.
>> If we consider for the QL(e1,e2,e3,di) QL-P23,
… we get collinear points.
>> What about the corresponding line?
>> Best regards Eckart
```

Message: #3484
Date: 19/4/2019 10:34:10
From: eckart_schmidt@t-online.de
Subject: QL-Geometry with QL-Quartet

Dear all,

in #3470 Tran Quang Hung used QL(e1,e2,e3,di)
… with e1,e2,e3 QL-Tr1 sidelines and di a QL-line,
… which lead in #3481 and #3482
… to new results and new elements in QL-geometry.

This concept can be generalized,
… replacing QL-Tr1 by QL-Tr2:

Let s1,s2,s3 be QL-Tr2 sidelines
… and consider the four QL(s1,s2,s3,di):
… Their QL-L1,4 are parallel to QL-L1,4 of the reference QL.
… Their QL-L2,3 are the same as for the reference QL.
… Their QL-L5 intersect in QL-P1 of the reference QL.
… Their QL-L6 intersect in a *new QL-point*.
… Their QL-Ci1 are centered on QL-L2 of the reference QL.
… Their QL-Ci2 intersect in QL-P1 of the reference QL.
… Their QL-Ci3 intersect in QL-P1 and QL-P6 of the reference QL.
… Their QL-Ci4 have collinear centers on a *new QL-line*.
… Their QL-Ci5 intersect in QL-P1 and a *new QL-point*.
… Their QL-Ci6 intersect in QL-P1 and a *new QL-point*

on QL-P1.QL-P12.
… Their QL-P1 are the same as for the reference QL.
… Their QL-P2,7,19 are collinear on QL-L2 of the reference QL.
… Their QL-P3,4,5,6,12,20,21,22,30 are collinear on *new QL-lines*.
… Their QL-Co1 are the same as for the reference QL.

What about these new QL-elements?
There will be other generalizations,
… replacing QL-Tr1,2 by QG-Px- or QA-Px-triangles.

Example for QA-P4-triangle with sidelines t1,t2,t3,t4
… and QL-quartet for QL(t1,t2,t3,t4,di) :
… Their QL-L2 intersect in a *new QL-point*.
… Their QL-Ci3 intersect in a *new QL-point*.
… Their QL-Ci4 have collinear center on a *new QL-line*.
… Their QL-Ci5 have common radical center in a *new QL-point*.
… Their QL-Ci6 have common radical center in a *new QL-point*.
… Their QL-P1 lie on the circumcircle of the QA-P4-triangle.
… Their QL-P3 lie collinear on a *new QL-line*.
… … …

```
Assumption:
The QL-quartet wrt any QG-Px- or QA-Px-triangle
… has its QL-P1 on the circumcircle of the triangle
… (further controlled for QG-P19-, QA-P2-triangle
… … and the S-triangle in #3176).
There will be further theorems.

Best regards Eckart

PS: The examples are not complet
    … and I hope without mistakes.
```

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Message: #3485
Date: 19/4/2019 4:26:56
From: bernard.keizer
Subject: Quintupel-Points for a 5P

```
Dear Eckart,
Sorry, my preceding message went lost ...
Another intriguing and interesting constellation !
I don't find the 5 triple points boring, on a contrary, I think
it's one of the most interesting items we have encountered (with
the stelloïd cubic QL-Cu2, the epi- and hypocycloïds tangent to
the 4L, the 3 foci of a 4P ...)
Simply, we havent found yet a simple construction of the QL
associated to 5 points in general !
The same way, the construction of a triangle knowing 3 points
incenter, orthocenter and circumcenter is not so easy ! We know
already  several constructions of a QL starting with 5 points (5
QL vertices, 2QL-2P3 and DT vertices, 4 circumcenters of the
reference triangles and QL-P1 ...)
By the way, is the dual property also correct (5 lines leading
to a unique QA) ?
Best regards
Bernard
```

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Message: #3486
Date: 20/4/2019 1:33:42
From: Tran Quang Hung
Subject: QL-Geometry with QL-Quartet

Dear Eckart

I am happy when my idea is useful to create your new results and new elements in QL-geometry. Thank You so much for these nice results.

Best regards
Tran Quang Hung.

Vào Th 6, 19 thg 4, 2019 vào lúc 15:40 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear all,
>> in #3470 Tran Quang Hung used QL(e1,e2,e3,di)
>> … with e1,e2,e3 QL-Tr1 sidelines and di a QL-line,
>> … which lead in #3481 and #3482
>> … to new results and new elements in QL-geometry.
>> This concept can be generalized,
>> … replacing QL-Tr1 by QL-Tr2:
>> Let s1,s2,s3 be QL-Tr2 sidelines
>> … and consider the four QL(s1,s2,s3,di):
>> … Their QL-L1,4 are parallel to QL-L1,4 of the reference QL.
>> … Their QL-L2,3 are the same as for the reference QL.
>> … Their QL-L5 intersect in QL-P1 of the reference QL.
>> … Their QL-L6 intersect in a *new QL-point*.
>> … Their QL-Ci1 are centered on QL-L2 of the reference QL.
>> … Their QL-Ci2 intersect in QL-P1 of the reference QL.
>> ... Their QL-Ci3 intersect in QL-P1 and QL-P6 of the reference QL.
>> ... Their QL-Ci4 have collinear centers on a *new QL-line*.
>> ... Their QL-Ci5 intersect in QL-P1 and a *new QL-point*.
>> ... Their QL-Ci6 intersect in QL-P1 and a *new QL-point*
>> on QL-P1.QL-P12.
>> … Their QL-P1 are the same as for the reference QL.
>> … Their QL-P2,7,19 are collinear on QL-L2 of the reference QL.
>> … Their QL-P3,4,5,6,12,20,21,22,30 are collinear on *new QL-lines*.
>> … Their QL-Co1 are the same as for the reference QL.
>> What about these new QL-elements?
>> There will be other generalizations,
>> … replacing QL-Tr1,2 by QG-Px- or QA-Px-triangles.

```
>> Example for QA-P4-triangle with sidelines t1,t2,t3,t4
>> … and QL-quartet for QL(t1,t2,t3,t4,di) :
>> … Their QL-L2 intersect in a *new QL-point*.
>> … Their QL-Ci3 intersect in a *new QL-point*.
>> … Their QL-Ci4 have collinear center on a *new QL-line*.
>> … Their QL-Ci5 have common radical center in a *new
QL-point*.
>> … Their QL-Ci6 have common radical center in a *new
QL-point*.
>> … Their QL-P1 lie on the circumcircle of the QA-P4-triangle.
>> … Their QL-P3 lie collinear on a *new QL-line*.
>> … ... ...
>> Assumption:
>> The QL-quartet wrt any QG-Px- or QA-Px-triangle
>> … has its QL-P1 on the circumcircle of the triangle
>> … (further controlled for QG-P19-, QA-P2-triangle
>> … ... and the S-triangle in #3176).
>> There will be further theorems.
>> Best regards Eckart
>> PS: The examples are not complet
>> ... and I hope without mistakes.
```

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Message: #3487
Date: 20/4/2019 8:55:50
From: chris.vantienhoven
Subject: epicyloidal curve around the sun

```
Dear friends,

I came across this article “Asteroid’s Chance of Earth Impact in
2027 Now 96%”.
See:
https://cneos.jpl.nasa.gov/pd/cs/pdc17/Chodas-Day2-Briefing.pdf?
fbclid=IwAR3IPfLw8xporOcvodUp-FvtXF8gIu95FrNp8m-zbRhXgXDYnjt67dC
-TGY
The interesting part I found to be the curve of the asteroid
around the sun.
It is an epicycloidal curve. A curve we discussed before in this
group.

Best regards,
Chris
```

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Message: #3488
Date: 21/4/2019 4:34:01
From: staninchelmsford
Subject: epicyloidal curve around the sun

```
Do note that each slide is labeled "EXERCISE ONLY" and other web
searches yield articles describing this as a "hypothetical
asteroid".
```

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Message: #3489
Date: 24/4/2019 10:02:25
From: eckart_schmidt@t-online.de
Subject: number 3453, number 3455, number 3459, number 3460

```
Dear Chris,

Tran Quang Hung describes in #3453 and #3455 two 5L-elements,
… which I don't find in EPG, see #3459 and #3460.
I think, they are relevant, why not?

Best regards Eckart
```

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Message: #3490
Date: 24/4/2019 6:48:59
From: bernard.keizer
Subject: number 3453, number 3455, number 3459, number 3460

```
Dear Eckart,
The points you mention are not listed in the menu of 5L in EPG,
but they are mentionned on the line 5L-s-L1 (5L Newton Line) :
see 5L-o-P2, 5L-n-P1 and 5L-n-P3.
Best regards
Bernard
PS If you consider the tangential 5L of the 5 triple points and
these points of the 5L, is there a connexion with the CSC of the
5 points ?
```

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Message: #3491
Date: 24/4/2019 7:56:44
From: chris.vantienhoven
Subject: Miquel circles are concurrent with 5L

Dear Eckart, Bernard and Tran,

Bernard is right.

Regarding Tran's message #3453:
Let d1, d2, d3, d4, d5 be five lines. M1=Miquel point of (d2, d3, d4, d5).
Define similarly, M2, M3, M4, M5.Then M1, M2, M3, M4, M5 lie on a circle.
This point is nL-n-P3 in a 5-Line. Because it is the version of nL-n-P3 in a 5-Line it is called 5L-n-P3.
There are also versions of this point in a 6-Line, 7-Line, etc..
Regarding Tran's message #3455:
Let d1, d2, d3, d4, d5 be five lines. Let circle (M1) = Miquel circle (QL-Ci3) of (d2,d3,d4,d5).
Define similarly circles (M2), (M3), (M4), (M5). Then five circles (M1), (M2), (M3), (M4), (M5) are concurrent at a point.
This point is nL-n-P1 in a 5-Line. Because it is the version of nL-n-P3 in a 5-Line it is called 5L-n-P1. There are also versions of this point in a 6-Line, 7-Line, etc..
Regarding Tran's message #3454:
Note:
* nL-n-Pi is a point valid for 3L, 4L, 5L, etc. Applied in that situations it is named resp. 3L-n-Pi, 4L-n-Pi, 5L-n-Pi (i=some number 1,2,3, …). Infix -n- stand for "natural" number n.
* 5L-s-Pi is a point that only occurs in a 5-Line, not in a 6-Line, etc. Infix -s- stand for "special" number.

Best regards,
Chris

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Message: #3492
Date: 25/4/2019 9:07:34
From: eckart_schmidt@t-online.de
Subject: number 3453, number 3455, number 3459, number 3460

```
Dear Bernard, dear Chris,

thanks for your informations
… and excuse my insecurity in EPG-nomination.
The circle in Tran's message 3453 is 5L-o-Ci1.

Best regards Eckart
```

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Message: #3493
Date: 28/4/2019 9:37:33
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-Cu7-triple points

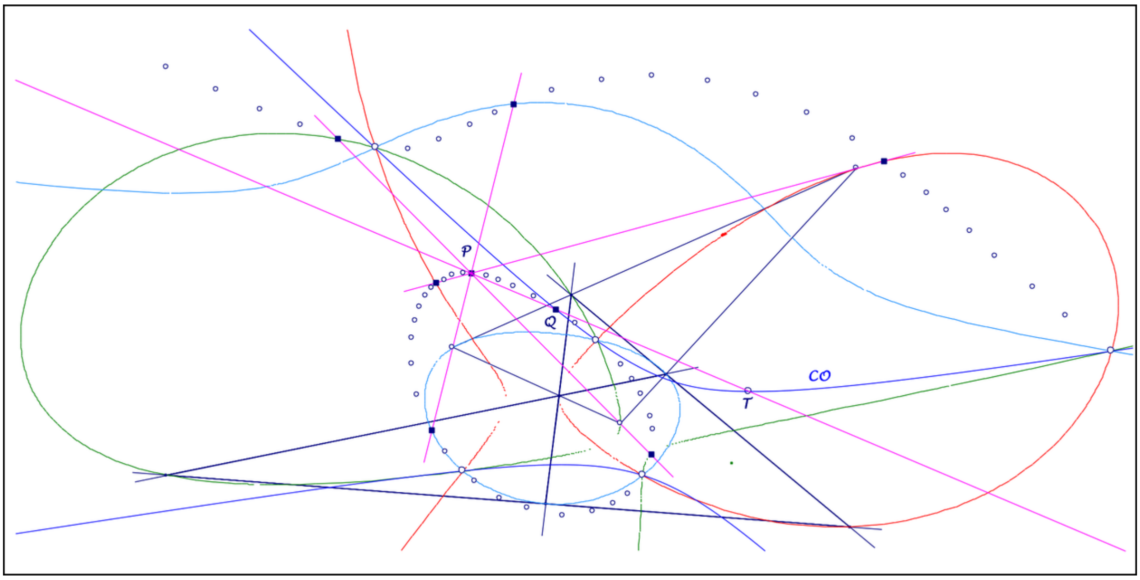
Dear Bernard,

for a QL we can construct
… the 3 QA-Cu7 of its QG-versions,
… the conic CU of their 5 triple points,
… and its 5P-s-P4 as point T on CU.

Here a new aspect (see attached file):
The 5 QA-Cu7-triple points of a QL
… are the common points of all QL-cubics,
… defined by an arbitrary point P
… and the following 8 points:
… infinity point of PT and its 2nd intersection Q with CO
… and 3 pairs of intersections of P.QG-P1 and QA-Cu7
(unequal QG-P1).

Inverted:
Starting with the 5 triple points and arbitrary points P,
… we get these cubics as locus of the cb-partner on lines
through P
… with cb(X) Cayley-Bacharach point of X
… and the 5 triple points and the two circular points.

Best regards Eckart



2019-04-28.pdf

Message: #3494
Date: 28/4/2019 4:36:16
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-Cu7-triple points

```
Dear Bernard,
excuse, there is a typo in my last message:
Replace CU by CO in the first passage.
A further remark wrt #3493:
… For the 5P-circumcubics QA-Cu7 the point P is QG-P1.
… For the degenerated line/circle cubics of a 5P
… … the point P is the intersection, not on CO,
… … of a parallel to the line through T and the circle.
Best regards Eckart
```

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Message: #3495
Date: 29/4/2019 4:31:08
From: bernard.keizer
Subject: New aspect of QA-Cu7-triple points

```
Dear Eckart,
It's interesting, but it's not a new aspect !
Using the construction I gave in my message 3385, you are able
to draw any circular cubic through the 5 points knowing the
pivot P and 4 different cubics.
You may choose 4 degenerated  cubics circle + line or 3 cubics
circle + line and cubic conic + infinity line or 4 normal cubics
or 3 normal cubics like the QA-Cu7 and the cubic conic +
infinity line (that's your choice).
You use always the same property that 2 of the cubics intersect
in 2 points aligned with the 2 pivots !
In fact, I think I prefer your other definition of the cubic as
locus of the 2 CB partners on all lines through the choosen
pivot, as you don't need to kow any particular non degenerated
cubic in advance.
Again, our problem is precisely how to find the 3 QA-Cu-7 among
all the circular cubics through the 5 points !
You have to use the CSC of the 5 points as the CSC of the pivot
is generally not on the cubic, but unfortunately, there are more
than 3 cubics having this property ...
Best regards
Bernard
```

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Message: #3496
Date: 01/5/2019 12:12:36
From: staninchelmsford
Subject: Four related "Euler Line" analogs in a quadrigon

Let the vertices of a quadrigon be A_1, A_2, A_3, and A_4.
Let T_i be the component triangle of the quadrigon that does not include vertex A_i.
Let Q_O be the quadrigon formed by the circumcenters of T_1, T_2, T_3, and T_4, in order.
Let Q_M be the quadrigon formed by the centroids of T_1, T_2, T_3, and T_4, in order.
Let Q_N be the quadrigon formed by the nine-point centers of T_1, T_2, T_3, and T_4, in order.
Let Q_H be the quadrigon formed by the orthocenters of T_1, T_2, T_3, and T_4, in order.
Let "special-center" be exactly one of "area centroid, diagonal crosspoint, Poncelet point, vertex centroid".
Let O be the special-center of Q_O.
Let M be the special-center of Q_M.
Let N be the special-center of Q_N.
Let H be the special-center of Q_H.
Then O, M, N, H lie on a straight line (in that order) and OM:MN:NH=2:1:3.

Definitions:
area-centroid = QG-P4, also known as the 1st QG-Quasi Centroid
diagonal crosspoint = QG-P1, the intersection of the diagonals
vertex centroid = QA-P1, also known as the quadrangle centroi
Poncelet point = QA-P2, also known as the Euler point or the Euler-Poncelet point
I found these four lines empirically, using Geometer's Sketchpad.
They are analogs of the Euler Line of a Triangle.
When the chosen special-center is the diagonal crosspoint, the line is already known.
It is cataloged in EQF as QG-L4, the 1st QG-Quasi Euler line and was discovered by Myakishev in
http://forumgeom.fau.edu/FG2006volume6/FG200634.p
When the chosen special-center is the Poncelet point, the line is already known.
It is mentioned by Eckart Schmidt in
http://eckartschmidt.de/EULER.pdf.
My German is a bit rusty, so someone correct me if I am wrong:
Eckart's "Feuerbachpunkt" seems to be QA-P2, the Euler-Poncelet point.
Question 1: Did Eckart discover this line in this paper or is he just reporting previously known work?

```
Is there an earlier reference?
[Since Eckart is a member of this forum, I hope he can give the
definitive answer to this question.]
Question 2: EQF says line QG-L4 is mentioned in Eckart's paper
(Ref-15i). I think this is a mistake/typo.
QG-L4 uses the diagonal points of the four quadrigons, whereas
Eckart is using the Euler-Poncelet Point.
The two lines when the "special-center" chosen is the area
centroid or the vertex centroid
do not seem to be previously known.

Question 3: Am I overlooking these lines in EQF?
Question 4: Are these lines new? Does anyone have a reference to
them in the literature?

I hope I have expressed myself clearly.
```

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Message: #3497
Date: 01/5/2019 1:28:17
From: staninchelmsford
Subject: Four related ”Euler Line” analogs in a quadrigon

```
Also, are these four lines special cases of some more general
theorem?
```

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Message: #3498
Date: 01/5/2019 3:24:47
From: eckart_schmidt@t-online.de
Subject: Four related "Euler Line" analogs in a quadrigon

Dear Staninchelmsford,

I am glad, that someone is interested in my paper
… "Euler-Gerade eines Vierecks",
… for a design was sent 1999 to the Aulis Verlag,
… but came back 5 years later unpublished.
Sorry, I cannot give further references.

Perhaps of interest:
If we consider for a reference quadrangle and its triangles
… the 4 quadrangles of X2, X3, X4, X5 (your Q_M,Q_O,Q_H,Q_N),
… their four QA-Px often are collinear:
… for QA-P1 on the line QA-P1.32.33,
… for QA-P2 on the line QA-L1 (my Euler-line),
… for QA-P5 on the line QA-L5,
… for QA-P10 on the parallel to QA-P10.24 through QA-P25,
… further for QA-P3,16,17,18,19,20,21,22,25,26,27,29,31,35
… with no relevant properties at the moment.

If we start with a reference quadrigon,
… the QG-Px-quadrupel will be collinear
… for QG-P1 on the line QG-L4,
… QG-P2,3,4,12,13,14,15.

What is the background for so many collinear quadrupel?

Best regards Eckart

PS: QG-L4 in Ref-15i will be a typo.

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Message: #3499
Date: 05/5/2019 6:39:18
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

First, recall that the Nagel line of a triangle passes through the following 4 points:

I = incenter (X1)
M = centroid (X2)
S = Spieker center (X10)
N = Nagel point (X8)
and the points form the following ratio:
IM : MS : SN = 2 : 1 : 3

Now consider a quadrangle with vertices A_1, A_2, A_3, and A_4.
Let T_i be the component triangle of the quadrangle that does not include vertex A_i.
Let Q_I be the quadrangle formed by the incenters of T_1, T_2, T_3, and T_4.
Let Q_M be the quadrangle formed by the centroids of T_1, T_2, T_3, and T_4.
Let Q_S be the quadrangle formed by the Spieker centers of T_1, T_2, T_3, and T_4.
Let Q_N be the quadrangle formed by the Nagel points of T_1, T_2, T_3, and T_4.
Let I be the vertex centroid of Q_I.
Let M be the vertex centroid of Q_M.
Let S be the vertex centroid of Q_S.
Let N be the vertex centroid of Q_N.
(By vertex centroid, I mean the quadrangle centroid, QA-P1.)
Then I, M, S, N lie on a straight line (in that order)
and IM : MS : SN = 2 : 1 : 3.
I discovered this empirically with Geometer’s Sketchpad.
Question 1: Is there some simple or obvious reason why this should be so?
Question 2: Is this line already known and/or cataloged in EQF?
Question 3: No corresponding result occurs if “vertex centroid QA-P1” is replaced by QA-P2, QA-P3, QG-P1, QG-P4, or the perimeter centroid. These are the only points I tried.

Does some corresponding line occur using some other quadrangle point instead of QA-P1?

[Yes, I know that QG-P1 and QG-P4 are not quadrangle points. For these, I ordered the quadrigon as A_1, A_2, A_3, A_4.]

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Message: #3500
Date: 06/5/2019 12:54:08
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

Dear all,

Question 1: Is there some simple or obvious reason why this should be so?
Yes, there is. Here is the reason:
For each i =1:k, let Ui, Vi, Wi be three collinear points on the plane such that UiVi : ViWi = t = const.
Let U,V,W be the k
P-n-P1 (centroid) of k points (Ui), (Vi), (Wi) respectively then U, V, W are collinear and UV : VW = t.

see also
https://www.chrisvantienhoven.nl/np-items/np-luc/np-n-luc1

Inspired by the nP-n-l1:
nL-MVP Eulerline,
https://www.chrisvantienhoven.nl/np-items/np-obj/np-lns/np-n-l1
the line described below for QA can generalized to nP:
Consider a nP (n points)
Let I be the nP-n-Luc1(X(1)) of this nP.
Let M be the nP-n-Luc1(X(2)) of this nP.
Let S be the nP-n-Luc1(X(10)) of this nP.
Let N be the nP-n-Luc1(X(8)) of this nP.
Then I, M, S, N lie on a straight line (in that order) and IM : MS : SN = 2 : 1 : 3.

May we call it nL-MVP Nagelline of nP?

Best regards,
Vu Thanh Tung

Le dimanche 5 mai 2019 à 23:39:23 UTC+7, Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :
First, recall that the Nagel line of a triangle passes through the following 4 points:
I = incenter (X1)
M = centroid (X2)
S = Spieker center (X10)
N = Nagel point (X8)
and the points form the following ratio:
IM:MS:SN=2:1:3
Now consider a quadrangle with vertices A_1, A_2, A_3, and A_4.
Let Q_I be the quadrangle formed by the incenters of T_1, T_2, T_3, and T_4.

Let Q_M be the quadrangle formed by the centroids of T_1, T_2,
T_3, and T_4.
Let Q_S be the quadrangle formed by the Spieker centers of T_1,
T_2, T_3, and T_4.
Let Q_N be the quadrangle formed by the Nagel points of T_1,
T_2, T_3, and T_4.
Let I be the vertex centroid of Q_I.
Let M be the vertex centroid of Q_M.
Let S be the vertex centroid of Q_S.
Let N be the vertex centroid of Q_N.
(By vertex centroid, I mean the quadrangle centroid, QA-P1.)
Then I, M, S, N lie on a straight line (in that order) and
IM:MS:SN=2:1:3.
I discovered this empirically with Geometer's Sketchpad.
Question 1: Is there some simple or obvious reason why this
should be so?
Question 2: Is this line already known and/or cataloged in EQF?
Question 3: No corresponding result occurs if “vertex centroid
QA-P1”□ is replaced by
QA-P2, QA-P3, QG-P1, QG-P4, or the perimeter centroid. These are
the only points I tried.
Does some corresponding line occur using some other quadrangle
point instead of QA-P1?
[Yes, I know that QG-P1 and QG-P4 are not quadrangle points.
For these, I ordered the quadrigon as A_1, A_2, A_3, A_4.]

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Message: #3501
Date: 06/5/2019 3:32:21
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,

Great work!

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Message: #3502
Date: 06/5/2019 4:43:08
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

```
Dear all,

It applies not only for nP but also for nL.

Best regards,
Vu Thanh Tung

Le lundi 6 mai 2019 à 08:32:24 UTC+7,
Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :
Dear Thanh Tung,
Great work!
```

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Message: #3503
Date: 06/5/2019 11:18:30
From: eckart_schmidt@t-online.de
Subject: Nagel Line Analog in a Quadrangle

```
Dear S taninchelmsford,

wrt question 2 in #3499:

I have found no corresponding result
... if QA-P1 is replaced by QA-P2 til QA-P41.

Best regards Eckart
```

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Message: #3504
Date: 06/5/2019 8:39:14
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

```
You wrote:
> May we call it
  nL-MVP Nagel line of nP?

I do not think "Nagel line" would be an appropriate name since
as you pointed out, the result is true for any set of
points/lines with ratio 2:1:3 and is not specific to the Nagel
line of a triangle.
```

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Message: #3505
Date: 06/5/2019 9:21:16
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

```
Dear all,

Thanh Tung's response in message 3500 showed that the result has
nothing to do with the 4 component triangles of a quadrangle.
To rephrase his result:
Let {Ai,Bi,Ci} be any number of collinear triples in the plane
with the ratio AiBi:BiCi being a constant, t. Let A be the
centroid of the {Ai} and the same for B and C.
Then A,B,C colline and AB:BC=t.

I was hoping that a stronger result might hold, namely that
instead of AiBiCi being collinear, the result would be that if
the AiBiCi were all directly similar triangles, then ABC would
also be directly similar to them. I tested this hypothesis out
and unfortunately it is false.

I also checked that the common ratio is a necessary condition. I
was hoping that merely having collinearity of the given triples
would make the centroids colline. This too turns out to be
false.
```

Message: #3506
Date: 07/5/2019 9:43:43
From: bernard.keizer
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,
Long time we hadn't seen on this forum such a simple geometric property with a synthetic proof !
Congratulations
There is a well known example for the QL
Let's consider the 4 centroïds, circumcenters and orthocenters of the 4 reference triangles.
The 3 centroïds of the 3 quadrupels are QL-P8, QL-P6 and QL-P2, which are the centroïd, circumcenter and orthocenter of the triangle of the S-points.
Best regards
Bernard
PS I haven't checked if the centroïd of the incenters was the incenter of this triangle ...

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Message: #3507
Date: 07/5/2019 6:10:51
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

Hi,

>> I was hoping that a stronger result might hold, namely that instead of AiBiCi being collinear, the result would be that if the AiBiCi were all directly similar triangles, then ABC would also be directly similar to them. I tested this hypothesis out and unfortunately it is false.

It is true, we can prove it by complex number.: we indentify points in the plane as complex number.

Suppose that
AiBiCi were all directly similar triangles and
directly similar to a triangle XYZ.
There is an unique complex number t such that Z = t*X+(1-t)*Y
(if t is real then X,Y,Z are collinear and any Ai,Bi,Ci are collinear) and then
Ci = t*Ai + (1-t)*Bi
By summing up and divide by the number of triangle AiBiCi we obtain C=t*A+(1-t)*B wichi shows that ABC is directly similar to AiBiCi.

Here remark that instead of centroid (equal weighted of points), we can use any weighted scheme.

Best regards,
Vu Thanh Tung

Le mardi 7 mai 2019 à 02:21:22 UTC+7, Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :
>Dear all,
>Thanh Tung's response in message 3500 showed that the result has nothing to do
>with the 4 component triangles of a quadrangle.
>To rephrase his result:
>Let {Ai,Bi,Ci} be any number of collinear triples in the plane with the ratio AiBi:BiCi being a constant, t.
>Let A be the centroid of the {Ai} and the same for B and C.
>Then A,B,C colline and AB:BC=t.
>I was hoping that a stronger result might hold, namely that instead of AiBiCi being collinear,
>the result would be that if the AiBiCi were all directly similar triangles,

>then ABC would also be directly similar to them.
>I tested this hypothesis out and unfortunately it is false.
>I also checked that the common ratio is a necessary condition.
>I was hoping that merely having collinearity of the given triples would make the centroids colline.
>This too turns out to be false.

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Message: #3508
Date: 07/5/2019 9:57:23
From: eckart_schmidt@t-online.de
Subject: Quasi-Euler Lines for a 5P

Dear all,

there are several triple QA-Px,y,z in Euler constellation,
… consider for a 5P and its 5 QA the 3 pentangle of QA-Px,y,z
… and take 5P-n-P1 of these pentangels,
… which are collinear with Euler ratios (see #3500) for:
(1) QA-P1, QA-P32, QA-P33,
(2) QA-P10, QA-P11, QA-P12,
(3) QA-P10, QA-P1, QA-P20,
(4) QA-P14, QA-P1, QA-P24,
(5) QA-P10,QA-P16, QA-P19,
(6) QA-P14, QA-P33, QA-P12.

For (1) the corresponding line is 5P-n-L1,
… the other lines seem to be not in EPG.

The lines for (1), (3), (4)
… have the common point 5P-n-P1,
… which is the 5P-MVP of the triangle point X(2).

The lines for (2), (3), (5)
… have a common point 5P-n-Px,
… which is the 5P-MVP of the triangle point X(?).

Best regards Eckart

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Message: #3509
Date: 08/5/2019 2:36:26
From: eckart_schmidt@t-online.de
Subject: Nagel Line Analog in a Quadrangle

Dear Bernard,

I think, there is a typo in your #3506:
… The centroid of the S-triangle is QL-P12 (not QL-P8).

Wrt your last remark:
The incenter of the S-triangle
… is not the centroid of the four incenters.
But for example X(20)
… on the Euler line of the S-triangle
… is the centroid of the four X(20).

Best regards Eckart

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Message: #3510
Date: 08/5/2019 9:35:00
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,

Thank you for taking the time to check my calculations.
My test was faulty and I agree with you that the triangles are similar.

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Message: #3511
Date: 10/5/2019 3:27:43
From: eckart_schmidt@t-online.de
Subject: Construction of CO-Tf3(X)

```
Dear Benedetto, dear Chris,

is the following construction of CO-Tf3(X) already mentioned?
Consider the four intersections of the conic
… and two orthogonal lines through X:
… QA-P4 of this QA is Co-Tf3(X).

Best regards Eckart
```

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Message: #3512
Date: 11/5/2019 7:14:27
From: Benedetto Scimemi
Subject: Construction of CO-Tf3(X)

```
Dear Eckart, your construction as such is not in the page of
Co-Tf3. It could added, if Chris wants, next to the "quick"
construction (there reported) for the inverse mapping.
In fact both of them are immediate consequences of the basic
CoTf3 property: QA-P2  -> QA-P4 for any inscribed quadrangle QA.
When in QA two sides are orthogonal with intersection X, then
X=Co-P2, hence  Co-Tf3^(-1) : X --> QA-P4.
When QA is cyclic with circumcenter X,  then X=Co-P4,
hence Co-Tf3^(-1) : X --> QAP2.
A weak point for both constructions is that they do not apply to
all points  X.
Best regards. Benedetto
```

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Message: #3513
Date: 12/5/2019 9:42:36
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

Dear Chris, dear Bernard,

in message 364 (2013) I offered a quartic for a QL
… without any feedback.

This quartic QL-Qux is the locus
… for intersections of lines and their CSC-circles,
… taking for lines the tangents at a conic,
… inscribed QL-Tr1, tangent to the Steiner axes and QL-L2.

Main properties:
… QL-Qux bears the six points of a QL,
… is CSC-invariant,
… bearing further QL-2P3 and 3 pairs of QA-Tr2-vertices
… … unequal QL-P1 for the QG-versions of QL.

There is an interesting new aspect of this quartic:
QL-Qux is invariant wrt the three QA-Tf16
… for the QG-versions of QL,
… which give for QL-Qux-points the same image.

Best regards Eckart

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Message: #3514
Date: 12/5/2019 7:35:03
From: bernard.keizer
Subject: Nagel Line Analog in a Quadrangle

Dear Eckart,
Indeed !
Thanks for your attention ...
Best regards
Bernard

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Message: #3515
Date: 12/5/2019 7:48:24
From: bernard.keizer
Subject: A Quartic for Quadrilaterals

Dear Eckart,
I regret that you mention the absence of feedback !
If my memory is not in default, we discussed this quartic a few time after your 1rst message.
The quartic is not only CSCinvariant, but also CSCdiaginvariant;
CSCdiag is the CSC of the DQL, the diagonal QL with the DT sides and the Newton Line as 4 lines (your conic is inscribed in the DQL).
The quartic contains the Plücker points, which are CSCdiagpartners.
I liked this quartic so much that I've described it in the 2nd article on my blog under the name of quartic of generalised Plücker points (with reference to you of course)
http://bernardkeizer.blogspot.com complete quadrilateral and cubic stelloïd september 2014 on page 40
Best regards
Bernard
PS This quartic deserves a description in EQF, but that is another story ...

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Message: #3516
Date: 13/5/2019 8:30:36
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

```
Dear Bernard,

I very much apologize for my bad general view.
The cases accumulate, that I lost survey,
... thanks for controlling teamwork.
Special thanks for the Plücker points QL-2P1 on the quartic.
But the last property in my message will be new.
You are right, this quartic should be mentioned in EQF.

Finally a remark wrt QA-Tf16 wrt pentangles:
Well known:
QA-Tf16 of a 5P-vertex Pi wrt the remaining vertices
… is CSC(Pi) wrt a QL with the 5P as QA-Cu7-triple points.

Perhaps new:
If we map a 5P P1...P5 to Q1...Q5
and Q1...Q5 to R1...R5
… with Qi = QA-Tf16(Pi) wrt the QA = PjPkPlPm
… and Ri = QA-Tf16(Qi) wrt the QA = QjQkQlQm,
… we get R1...R5 = P1...P5.
This is not the property in EQF: QA-P16(QA-P16(P)) = P,
… for the reference QAs are different.

Best regards Eckart
```

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Message: #3517
Date: 13/5/2019 10:46:55
From: bernard.keizer
Subject: 5P-s-Co1

Dear Eckart,

5P-s-Co1 (curiously not in EQF) is the common unique diagonal polar conic for 5 pivotal isocubics having 4 of the 5 points as fixed points of the conjugation and the 5th point as pivot.
This conic contains the vertices of the anticevian triangles of each of the 5 points wrt the DT of the 4 others (15 points).
This conic contains also the vertices of the anticevian triangle of any of it's points wrt the DT's of any of 4 points on the conic ! (for example the anticevian triangles of the point T wrt the 5 DT's of 4 of the 5 points (15 points more).
The 5 pivotal cubics are tangent to the conic in the pivot, intersect the conic in 5 points (one being dobble) and intersect apparently only in the 5 real points (as they are not circular, the other 4 intersections of 2 cubics are not real).
CSC of this conic is a quartic (?) through the 5 CSC of the points and the center of the CSC.

Best regards
Bernard

PS Using the unknown points R and S1,S2 and S3, I couldn't find a use for the 5 triple points

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Message: #3518
Date: 13/5/2019 11:45:02
From: bernard.keizer
Subject: 5P-s-Co1

Dear Eckart,
I now try always to split my messages , in order to avoid them being lost ...
One use of the points T, R, QG-P1, QA-P4 and S1,S2 and S3 is obviuously a desmic configuration as the triangles QG-P1 and S1,2,3 are in perspective with perspector T and the triangles QA-P4 and S1,2,3 are in perspective with perspector R.
This gives 2 pivotal isocubics with pivots T and R. Each of the cubics cuts the conic in 2 other points than T or R and S1,2,3, but again, I couldn't find a use for the QL of the 5P.
Best regards
Bernard

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Message: #3519
Date: 13/5/2019 12:03:17
From: bernard.keizer
Subject: 5P=4L ?

Dear Eckart,
Using my construction in message 3385 gives always pivotal circular isocubics through the 5 points ; they generally don't contain the CSC of the pivot.
Pivotal isoubics through the 5 points, a pivot P and the CSC of the pivot are generally not circular.
There are still an infinity of pivotal circular isocubics containing the CSC of the pivot.
What do the 3 QA-Cu7 have as particularity ? They are focal.
Maybe they are the 3 unique focal ...
But I'm not able to find them !
Could you please help me and explain simply how you manage to find the approximate QL ?
Did you find a converging method or something like that ?
I spent now so many hours in drawing curves that I'm a little bit disappointed (it's an understatement)
Many thanks in advance for your help !
Best regards
Bernard

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Message: #3520
Date: 14/5/2019 7:29:07
From: Thanh Tung
Subject: A new family of lines on quadrangle assiciated with Kimberling trian

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```
Dear all,

I would like to introduce a new family of lines on
quadrangle geometry:

Let
_ n:    an integer
_ ABCD: a Quadrangle
_ A_n:  the Kimberling center X(n) of triangle BCD.
        Define B_n, C_n, D_n similarly.
_ trA:  the antipedal triangle of A w.r.t triangle BCD.
        Define trB, trC, trD similarly.
_ A':   the isogonal conjugate of A w.r.t triangle trA.
        Define B', C', D' similarly.
_ A*_n: the orthopole of the line A'A_n w.r.t triangle BCD.
        Define B*_n, C*_n, D*_n similarly.

Then A*_n, B*_n, C*_n ,D*_n are collinear on a line. When n
varis this define a family of lines on
quadrangle assiciated with Kimberling triangle centers.
This is a very surprised property. Yet I do not have proof for
this.
Is this property known before? And do you have idea how to prove
this?

Best regards,
Vu Thanh Tung
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Message: #3521
Date: 14/5/2019 8:09:29
From: tulip13091983
Subject: A new family of lines on quadrangle assiciated with Kimberling t

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Dear all,
This is false, please ignore it,
Best regards,
Vu Thanh Tung
```

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Message: #3522
Date: 14/5/2019 9:15:03
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

thanks for your numerous stimulations,
… please excuse, if I don't react at once.
… I shall do it, if I see new consequences.

Wrt your question for an approximate QL for a 5P:
… no concept, pure trying up to now.

But perhaps helpful:
If we have one vertex of the QL,
… we can construct the others:

Let P1 be one QL-point,
… then 5P-CSC(P1) gives P3,
… opposite point for a QG-version of the QL.

Using the cb-transformation,
… which gives the CB-point wrt the 5P and the circular points,
… P1.cb(P1) ^ P3.cb(P3) is a vertex U of QL-Tr1.

The cb-partner on P1P3 give two QA-Cu7-double points X,Y,
… the 5P-CSC-circle of the circle with diameter XY
… gives in its intersections with P1P3
… … the further vertices V, W of QL-Tr1.

The 5P-CSC-partner on UV and UW are the other QL-points.

Best regards Eckart

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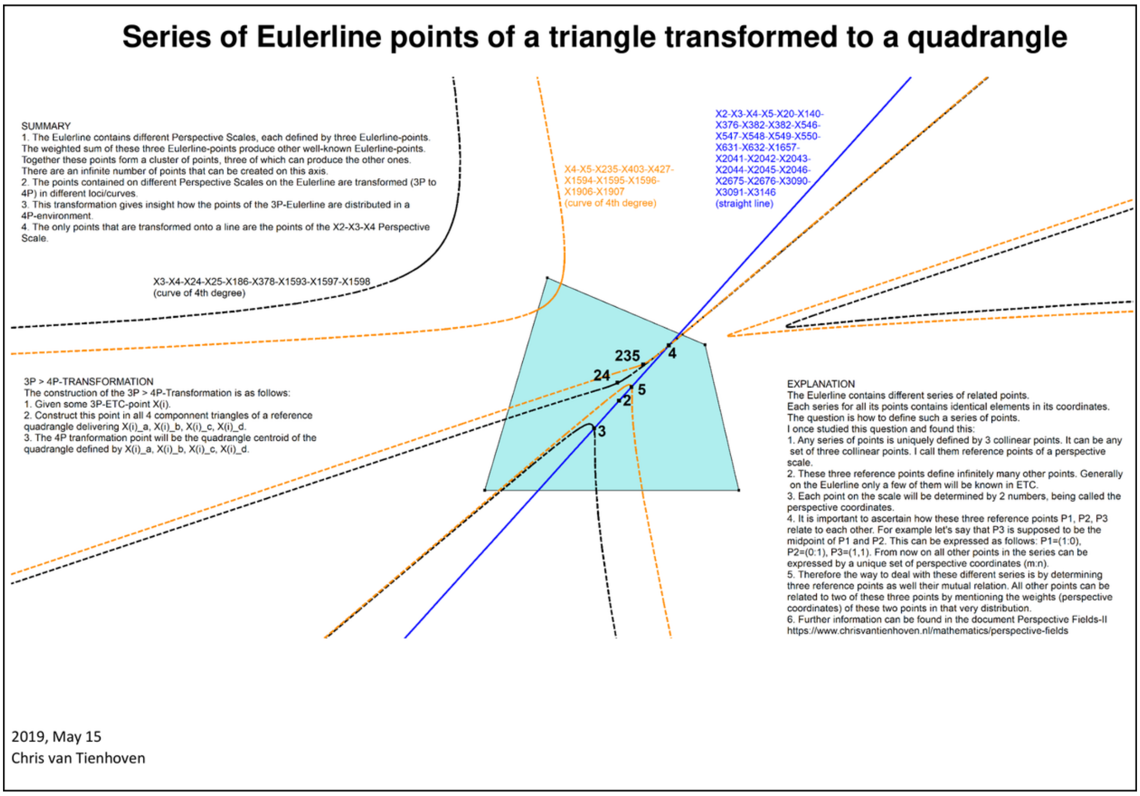
Message: #3523
Date: 15/5/2019 8:46:29
From: chris.vantienhoven
Subject: Nagel Line Analog in a Quadrangle

```
Dear friends,

I found very nice results about the Nagel-line.
I wondered how points on a triangle line are dispersed in the
Quadrangle plane using the centroid of the four X(i) in a
Quadrangle-transformation.
Because there are many points on the Eulerline I used the
Eulerline as an example to see how different sets of points are
dispersed in a Quadrangle.
See enclosed picture.

Best regards,
Chris
```



QA-X2-X3-X4-Eulerline-points-Centroids-Loci-01.pdf

Message: #3524
Date: 15/5/2019 10:56:15
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for your attention and these new developments ; I have to study them carefully !
Here just a simple discovery found by drawing all kinds of figurs.
You showed a certain time ago that 3 points and a CSC allow to find 2 points such as the 5 points have this CSC.(then we have their conic and their point T.
I found that any 2 points on a conic and a CSC have 3 partners on this conic such as the 5 points have this CSC (their point T is apparently a different one). The construction is easy : use the construction with a 3rd variable point and search the position of this 3rd point such as the 2 other points are on the conic ...
Best regards
Bernard
PS I didn't try to apply the construction to the points T and R on the 5 points conic

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Message: #3525
Date: 17/5/2019 3:11:15
From: eckart_schmidt@t-online.de
Subject: Four related "Euler Line" analogs ...

```
Dear all,

this is an addition to #3498:
Consider points X on the Euler line of a triangle
… and their quadrupel for the triangles of a reference QA,
… which give new X-depending QAs.

In #3498 the collinearity of their QA-points is researched,
… here the envelopes of their QA-lines are tested,
… which often give a parabola,
… … tangent to the corresponding line of the reference QA.

QA-L1 envelope a parabola,
… tangent in QA-P34 to QA-L1 of the reference QA,
… tangent in the midpoint of QA-P32.33 to QA-P1.32.33.

QA-L2 envelope a parabola,
… tangent in QA-P1 to QA-L1 of the reference QA,
… tangent in QA-P4 to QA-P3.4.

QA-L3 envelope a parabola,
… tangent to QA-L3 and QA-P1.32.33 of the reference QA.

QA-L4 envelope a parabola,
… tangent to QA-L4 in a point,
… … dividing QA-P1.23 with ratio 1:2,
… tangent in QA-P32 to QA-P1.32.33.

QA-L7 envelope a parabola,
… not tangent to QA-L7 of the reference QA ...

The lines QA-P3.4 envelope a parabola,
… not tangent to QA-P3.4 of the reference QA,
… but tangent to QA-L2 in QA-P4
… and tangent to QA-P2.23 in QA-P23.

What is the background of this parabola property?

Best regards Eckart
```

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Message: #3526
Date: 19/5/2019 11:05:21
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for this beautiful construction !
Very clever simultaneous use of CB and CSC transformations of 5 points.
Nothing new in fact in the properties, but new is the use you make of them ...
Only one remark : the points you name X and Y (CB partners on P1P3) as well as V and W (2 other DT vertices) must be harmonic conjugates wrt P1 and P3. For this reason, the 2 circles with diameters X and Y and V and W which are CSC partners are orthogonal to any circle through P1 and P3, in particular the circle through P1, P3 and QLP1 and therefore to it's CSC,id est the line P1P3, which must be a diameter for both circles.
As I cannot draw the CB partner of a point, I had difficulties to use directly your construction.
But I managed a proxy by drawing the 2 circular cubics through the 5 points with pivots in 2 CSC partners P1 and P3, which intersect in X and Y ...
But unfortunately, the property I mention is necessary, but not sufficient !
Anyhow, it's for me already a huge progress.
Many thanks again
Best regards
Bernard

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Message: #3527
Date: 21/5/2019 9:42:11
From: bernard.keizer
Subject: 5P=4L ?

Dear Eckart,
I checked the centroÏds of the 5 triple points and of the dual QA of the QL.
They don't coincide (the 4 points cannot be obtained by derivating the complex equation of the 5 points).
Best regards
Bernard

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Message: #3528
Date: 23/5/2019 8:26:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I am glad, that my construction in #3522 was helpful for you.
I made the same observations
… and tried to find a locus for P1,P3,
but in vain ...

Cleaning my desk before holiday,
… some remarks wrt CB-transformations
… for a 5P = P1...P5 and the searched QL,
… some will be evident or already mentioned:
… Let CO be the circumcubic of the 5P
… and T = 5P-s-P4:

(1) cb, defined by the 5P and the circular points:
… cb maps each QA-Cu7 to itself,
… … point and image collinear QG-P1.
… cb swaps QA-Cu7-double intersections,
… cb maps points X on CO to the infinity point of XT.
… cb maps lines PiPj to circumcircles of PkPlPm.
… cb(QL-P1) is a point on QL-P1.QL-P8.QL-P24.

(2) CB, defined by the 5P and 2 opposite QL-points
… as QG-2P2 of a QGa version of the QL:
… CB maps QA-Cu7a to itself, point and image collinear QA-P2.
… CB maps QG-P1b.c to the conic CO.

(3) CB, defined by the 5P and 2 QA-Cu7-double points Xa, Ya:
… CB maps QA-Cu7b,c to itself,
… … point and image collinear with the infinity point of the asymptote.
… CB maps Xa.Ya to the conic CO, point and image collinear T,
… … especially T as CB of intersection of Xa.Ya and the tangent in T at CO.

(4) CB, defined by the 5P and any pair X, Y:
… CB maps the line X.Y to the conic CO,
… … point and image collinear to a fixed point on CO.
In this way the 4 QL-lines give a QA on CO ...

(5) CB, defined by the 5P and the foci of the circumconic:
… CB maps the the principal axis of CO to CO.

Finally an application of (1):

For any points X, Y on Pi.Pj
… X.cb(X)^Y.cb(Y) gives a point Qij (Q in #3245).
… The 10 points Qij give 10 circles (Qij,Qjk,Qik) through T,
… centered in Mijk and Mijk, Mijl, Mijm collinear.
… The 10 points Qij give 5 conics (Qij,Qik,Qil,Qim,Pi)
  through T.

Best regards Eckart

PS: Next two weeks I make my drawings in the beach sand.

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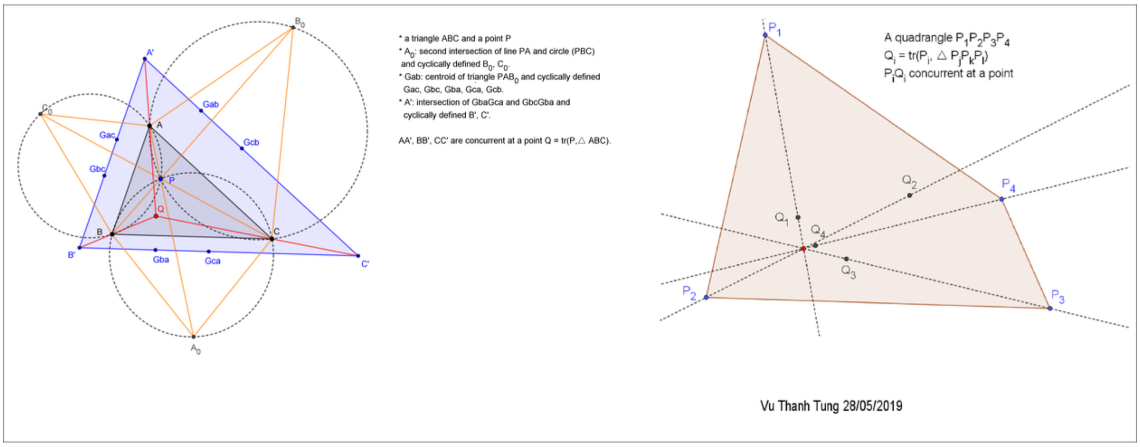
Message: #3529
Date: 28/5/2019 4:58:53
From: Thanh Tung
Subject: A new point on quadrangle ?

Dear all,

I describe here a possible new point on quadrangle:
First a transformatin on trianlge:
* a triangle ABC and a point P
* A_0: second intersection of line PA and circle (PBC)
and cyclically defined B_0, C_0.
* Gab: centroid of triangle PAB_0 and cyclically defined
Gac, Gbc, Gba, Gca, Gcb.
* A': intersection of GabGcb and GbcGac and
cyclically defined B', C'.
then the lines AA', BB', CC' are concurrent at a point Q =
tr(P, △ABC).
Now using the above transformtion we obtain a point on
quadrangle:
* a quadrangle P1 P2 P3 P4
* Qi = tr(Pi, △PjPkPl)
then the lines PiQi are concurrent at a point.
I am not sure if it describes a new point or it is just a new
description of an existing point.

Best regards,
Vu Thanh Tung



QuandranglePoint.png

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Message: #3530
Date: 29/5/2019 12:45:20
From: chris.vantienhoven
Subject: A new point on quadrangle ?

```
Dear Vu Thanh Tung,

I only calculated the coordinates algebraically.
Your triangle transformation of point P(x:y:z) gives a new point
with these barycentric coordinates:

{y z (-b^2 x^2 - a^2 x y - b^2 x y + c^2 x y - a^2 y^2) (c^2 x^2
+ a^2 x z - b^2 x z + c^2 x z + a^2 z^2)
(c^2 x^2 y^2 - 2 a^2 x^2 y z + b^2 x^2 y z + c^2 x^2 y z - 2 a^2
x y^2 z - a^2 y^3 z + b^2 x^2 z^2 - 2 a^2 x y z^2 - 2 a^2 y^2
z^2 - a^2 y z^3),

x z (-b^2 x^2 - a^2 x y - b^2 x y + c^2 x y - a^2 y^2) (c^2 y^2
- a^2 y z + b^2 y z + c^2 y z + b^2 z^2)
(c^2 x^2 y^2 - b^2 x^3 z - 2 b^2 x^2 y z + a^2 x y^2 z - 2 b^2 x
y^2 z + c^2 x y^2 z - 2 b^2 x^2 z^2 - 2 b^2 x y z^2 + a^2 y^2
z^2 - b^2 x z^3),

x y (c^2 x^2 + a^2 x z - b^2 x z + c^2 x z + a^2 z^2) (c^2 y^2 -
a^2 y z + b^2 y z + c^2 y z + b^2 z^2) (c^2 x^3 y + 2 c^2 x^2
y^2 + c^2 x y^3 + 2 c^2 x^2 y z + 2 c^2 x y^2 z - b^2 x^2 z^2 -
a^2 x y z^2 - b^2 x y z^2 + 2 c^2 x y z^2 - a^2 y^2 z^2)}

The Quadrangle point you constructed using your transformation
has these coordinates:

{a^2 q^2 r^2 (-2 b^2 c^2 p^4 - a^2 c^2 p^3 q - 2 b^2 c^2 p^3 q +
c^4 p^3 q - a^2 c^2 p^2 q^2 - a^2 b^2 p^3 r + b^4 p^3 r - 2 b^2
c^2 p^3 r - a^4 p^2 q r + b^4 p^2 q r - 2 b^2 c^2 p^2 q r + c^4
p^2 q r - a^4 p q^2 r + a^2 b^2 p q^2 r - a^2 c^2 p q^2 r - a^2
b^2 p^2 r^2 - a^4 p q r^2 - a^2 b^2 p q r^2 +     a^2 c^2 p q
r^2 - a^4 q^2 r^2),

b^2 p^2 r^2 (-b^2 c^2 p^2 q^2 - 2 a^2 c^2 p q^3 - b^2 c^2 p q^3
+ c^4 p q^3 - 2 a^2 c^2 q^4 + a^2 b^2 p^2 q r - b^4 p^2 q r -
b^2 c^2 p^2 q r + a^4 p q^2 r - b^4 p q^2 r - 2 a^2 c^2 p q^2 r
+ c^4 p q^2 r + a^4 q^3 r - a^2 b^2 q^3 r - 2 a^2 c^2 q^3 r -
b^4 p^2 r^2 - a^2 b^2 p q r^2 - b^4 p q r^2 + b^2 c^2 p q r^2 -
a^2 b^2 q^2 r^2),
```

```
-c^2 p^2 q^2 (c^4 p^2 q^2 - a^2 c^2 p^2 q r + b^2 c^2 p^2 q r +
c^4 p^2 q r + a^2 c^2 p q^2 r - b^2 c^2 p q^2 r + c^4 p q^2 r +
b^2 c^2 p^2 r^2 - a^4 p q r^2 + 2 a^2 b^2 p q r^2 - b^4 p q r^2
+ c^4 p q r^2 + a^2 c^2 q^2 r^2 + 2 a^2 b^2 p r^3 - b^4 p r^3 +
b^2 c^2 p r^3 - a^4 q r^3 + 2 a^2 b^2 q r^3 + a^2 c^2 q r^3 + 2
a^2 b^2 r^4)}
```

It isn't a known QA-point yet.
It doesn’t lie on familiar lines either.

The only line I could find to lie on is the line with these 5 points on it:

- QA-P4
- QA-Tf4(QA-P8)
- QA-Tf4(QA-P23)
- QA-Tf4(QA-P32)
- QA-Tf4(QA-P42)

Note:
QA-Tf4 is the QA-Möbius Conjugate.

Best regards,
Chris

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Message: #3531
Date: 29/5/2019 6:34:40
From: Thanh Tung
Subject: A new point on quadrangle ?

Hi,

Thank you for your interest in my point construction.

Best regards,
Vu Thanh Tung

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Message: #3532
Date: 01/6/2019 10:32:03
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart, Dear Bernard,

I revisited the constellation of three versions of QA-Cu7 in a Quadrilateral.
I looked at the 6 points that lie on only 2 of the 3 versions of QA-Cu7 in a QL.
Here are some results I found. I don’t know what is already mentioned before.
1. They lie per pair on the 3 lines of the QL-Diagonal Triangle.

2. Each line of the QL-DT is also the 3 rd diagonal of one of the
   three Component Quadrigons.

3. The pair of endpoints of this 3 rd diagonal is harmonically
   conjugated with the pair of doublepoints on this line.

4. The CT-coordinates of the 6 doublepoints are:
   {-a W,   -b c n,   b c m}
   { a W,   -b c n,   b c m}
   {a c n, -b W,   -a c l}
   {a c n,   b W,   -a c l}
   {a b m, -a b l,   c W}
   {a b m, -a b l, -c W}
   where W = Sqrt[a 2 (l-m)(l-n)-b 2 (m-l)(m-n)+c 2 (n-l)(n-m)]

5. The 6 doublepoints are coconic (algebraically proven).

6. The equation of the conic they are on:
+b 2 c 2 (a 2 c 2 l 2 n 2 -b 2 m 2 W 2) (a 2 b 2 l 2 m 2 -c 2 n 2 W 2) x 2
+a 2 c 2 (b 2 c 2 m 2 n 2 -a 2 l 2 W 2) (a 2 b 2 l 2 m 2 -c 2 n 2 W 2) y 2
+a 2 b 2 (b 2 c 2 m 2 n 2 -a 2 l 2 W 2) (a 2 c 2 l 2 n 2 -b 2 m 2 W 2) z 2
-a 2 b 2 c 2 l m (-2 a 2 b 2 c 2 l 2 m 2 n 2 +(a 4 l 4 +b 4 m 4 +c 4 n 4) W 2 -W 6) x y
-a 2 b 2 c 2 l n (-2 a 2 b 2 c 2 l 2 m 2 n 2 +(a 4 l 4 +b 4 m 4 +c 4 n 4) W 2 -W 6) x z
-a 2 b 2 c 2 m n (-2 a 2 b 2 c 2 l 2 m 2 n 2 +(a 4 l 4 +b 4 m 4 +c 4 n 4) W 2 -W 6) y z = 0

7. The center of this conic is:

{2 a 4 b 4 c 4 l 3 m 3 n 3 (a 4 l 4 m-b 4 m 5 +a 4 l 4 n-2 a 4 l
3 m n+b 4 m 4 n+c 4 m n 4 -c 4 n 5)-a 2 b 2 c 2 m n (a 8 l 9
m-b 8 l m 9 +a 8 l 9 n-3 a 8 l 8 m n+6 a 4 b 4 l 5 m 4 n-6 a 4 b
4 l 4 m 5 n+b 8 l m 8 n+b 8 m 9 n+6 a 4 c 4 l 5 m n 4 -6 a 4 c 4
l 4 m n 5 -2 b 4 c 4 m 5 n 5 +c 8 l m n 8 -c 8 l n 9 +c 8 m n 9
) W 2
+2 a 4 l 2 (a 4 b 4 l 5 m 3 -2 a 4 b 4 l 4 m 4 +b 8 l m 7 +a 4 c
4 l 5 n 3 +2 b 4 c 4 l m 4 n 3 -2 a 4 c 4 l 4 n 4 +2 b 4 c 4 l m
3 n 4 -6 b 4 c 4 m 4 n 4 +c 8 l n 7) W 4
+2 a 2 b 2 c 2 m n (-b 4 l m 5 +3 a 4 l 4 m n+b 4 m 5 n-c 4 l n
5 +c 4 m n 5) W 6
-2 a 4 l 3 (b 4 m 3 +c 4 n 3) W 8
+a 2 b 2 c 2 m n (l m+l n-m n) W 10 ,

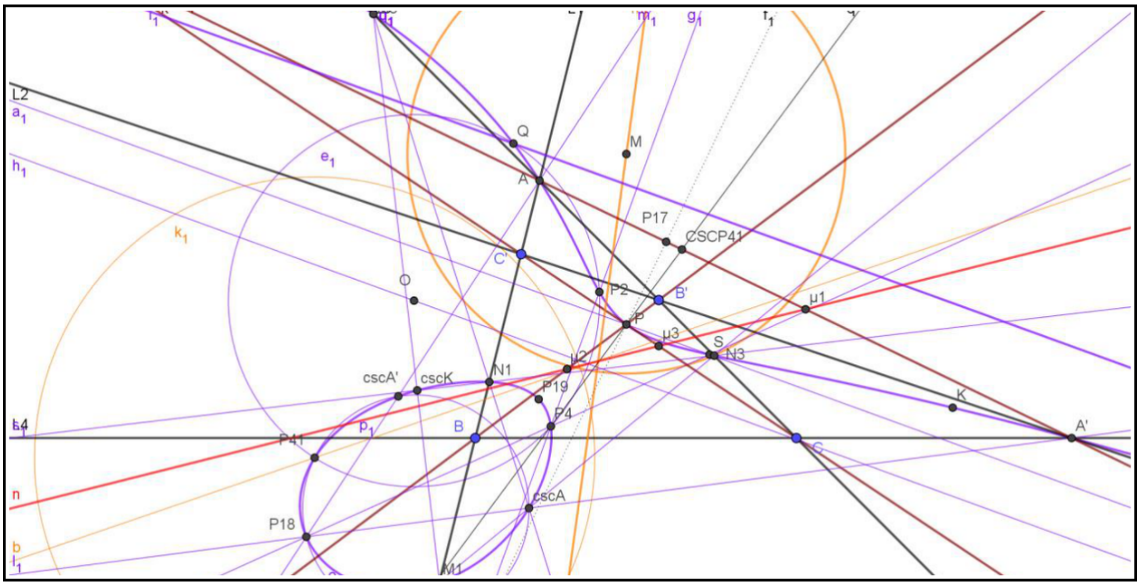
2 a 4 b 4 c 4 l 3 m 3 n 3 (-a 4 l 5 +b 4 l m 4 +a 4 l 4 n-2 b 4
l m 3 n+b 4 m 4 n+c 4 l n 4 -c 4 n 5)-a 2 b 2 c 2 l n (-a 8 l 9
m+b 8 l m 9 +a 8 l 9 n+a 8 l 8 m n-6 a 4 b 4 l 5 m 4 n+6 a 4 b 4
l 4 m 5 n-3 b 8 l m 8 n+b 8 m 9 n+6 b 4 c 4 l m 5 n 4 -2 a 4 c 4
l 5 n 5 -6 b 4 c 4 l m 4 n 5 +c 8 l m n 8 +c 8 l n 9 -c 8 m n 9
) W 2
-2 b 4 m 2 (-a 8 l 7 m+2 a 4 b 4 l 4 m 4 -a 4 b 4 l 3 m 5 -2 a 4
c 4 l 4 m n 3 -b 4 c 4 m 5 n 3 +6 a 4 c 4 l 4 n 4 -2 a 4 c 4 l 3
m n 4 +2 b 4 c 4 m 4 n 4 -c 8 m n 7) W 4
+2 a 2 b 2 c 2 l n (-a 4 l 5 m+a 4 l 5 n+3 b 4 l m 4 n+c 4 l n 5
-c 4 m n 5) W 6
-2 b 4 m 3 (a 4 l 3 +c 4 n 3) W 8
+a 2 b 2 c 2 l n (l m-l n+m n) W 10 ,

-2 a 4 b 4 c 4 l 3 m 3 n 3 (a 4 l 5 -a 4 l 4 m-b 4 l m 4 +b 4 m
5 +2 c 4 l m n 3 -c 4 l n 4 -c 4 m n 4)+a 2 b 2 c 2 l m (-a 8 l
9 m+2 a 4 b 4 l 5 m 5 -b 8 l m 9 +a 8 l 9 n-a 8 l 8 m n-b 8 l m
8 n+b 8 m 9 n+6 a 4 c 4 l 5 m n 4 +6 b 4 c 4 l m 5 n 4 -6 a 4 c
4 l 4 m n 5 -6 b 4 c 4 l m 4 n 5 +3 c 8 l m n 8 -c 8 l n 9 -c 8
m n 9) W 2
-2 c 4 n 2 (6 a 4 b 4 l 4 m 4 -a 8 l 7 n-2 a 4 b 4 l 4 m 3 n-2 a
4 b 4 l 3 m 4 n-b 8 m 7 n+2 a 4 c 4 l 4 n 4 +2 b 4 c 4 m 4 n 4
-a 4 c 4 l 3 n 5 -b 4 c 4 m 3 n 5) W 4
+2 a 2 b 2 c 2 l m (a 4 l 5 m+b 4 l m 5 -a 4 l 5 n-b 4 m 5 n+3 c
4 l m n 4) W 6
-2 c 4 (a 4 l 3 +b 4 m 3) n 3 W 8
-a 2 b 2 c 2 l m (l m-l n-m n) W 10 }

I attach a picture of the results.

Best regards,

Chris



QA-Cu7.pdf

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Message: #3533
Date: 02/6/2019 3:54:17
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris, dear Eckart,
It's a real pleasure to read news from Chris on the forum, specially about this wunderful item of the 3 QA-Cu7 !
I'm very impressed by the barycentric calculations ...
As Eckart is on the beach, I make a 1rst answer with other properties of these 6 double points ; of corse Eckart will correct or complete ...
1) Let's name Ui and Vi the 2 double points of the QA-Cu7 j and k. UjVj and UkVk intersect in QG-P1i, UjVk and UkVj intersect orthogonally in QG-P18i (CSC-QG-P17i) and UjUk and VjVk in QG-P19.
The CSC of the circle with diameter QG-P1j and k is the circle with diameter UiVi and both circles are therefore orthogonal to their diameter and to it's CSC, the circle through QL-P1 and the 2 QL vertices on this diagonal.
2) Ui and Vi are CB partners in the transformation wrt the 5 triple points and the 2 circular points (obvious, as the 2 cubics QA-Cu7j and k pass through the 9 points).
3) The 6 points are coconic, as Chris mentions : as the intersections QG-P18i,j and k are aligned (on the CSC of the Euler circle of DT), this figure forms the socalled mystic hexagam of Pascal.
4) Each QA-Cu7i bears the 4 points Uj,Vj, Uk and Vk ; QA-Cu7i is the QA-Cu1 of the QA UjVjUkVk, which has as DT QG-P1i, QG-P18i and QG-P19i and as Miquel triangle QA-P2,4 and 41 of the corresponding component quadrigon (with QG-P1j and k as one diagonal). As 2 sides are orthogonal, this QA-Cu1 is a QL-Cu1.
5) The 6 double points lie on a cubic with the 3 QG-P18 and the 3 QG-P19 (see Eckart's message 3187).
Best regards
Bernard
PS Would it be possible to calculate the same way the coordinates of the 5 triple points and of their conic ?

Message: #3534
Date: 03/6/2019 10:48:04
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Chris,

Also perhaps of interest :
1) the circle with diameter UiVi carries 2 QA-P4 (as CSC of  2
QG-P1) and 2 QG-P18 (as CSC of 2 QG-P17)
2) CB(QA-P4) is the intersection of QG-P1QA-P4 with the
corresponding diagonal and the CSC of QA-P41.

Best regards
Bernard
```

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Message: #3535
Date: 04/6/2019 10:59:26
From: bernard.keizer
Subject: QA-Cu7-Geometry

```
Dear Chris,

Please cancel in 2) CB(QA-P4) and read simply : the intersection
of QG-P1QA-P4 with the corresponding diagonal is the CSC of
QA-P41.

Best regards
Bernard
```

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Message: #3536
Date: 04/6/2019 11:48:09
From: bernard.keizer
Subject: Dual 4/5 P/L

Dear Eckart,
I hope you will come back from holidays with plenty of new ideas and full of strengh !
Here a simple idea I had put in my message 3885, but never achieved.
If there is only one QL corresponding to 5 points with the same CSC, the duality QA/QL allows to find a 5L as dual of the 5P by using the same duality.
All points and curves of the 4 figures QA, QL, 5P or 5L belong in fact to the same figure !
For example, the dual of any conic is a conic and the dual of any cubic is a curve of class 3.
The duals of the degenerated cubics formed by a circle and a line are degenerated curves of class 3 formed by the dual point of the line and the dual conic of the circle (which is not necessary a circle) ; the dual of the degenerated cubic formed by the conic of the 5 points and the infinity line is the curve of class 3 formed by the dual of the conic and the point QA-P16 or QL-P13.
There must be correspondances between different points.
For example, I found that QA-P1 lies on the tangent in T to the conic of the 5 points and on the dual of R (tangent to the dual conic) ...
Could you please confirm these observations (my figures are generally less precise than yours).
I would be glad to have your comments ...
This works perhaps also for cubics through 7P linked to the QA/QL : the duals of the degenerated cubics formed by a conic and a line are degenerated curves of class 3 formed by the dual conic and the dual point.
Best regards
Bernard

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Message: #3537
Date: 04/6/2019 12:40:01
From: amontes1949
Subject: A new point on quadrangle ?

Dear Vu Thanh Tung,

If P lies on circumcircle, then Q = tr(P, △ ABC). = centroid of {A,B,C,P}.

Best regards,
Angel Montesdeoca

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Message: #3538
Date: 04/6/2019 2:52:49
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Bernard,

Thanks for the extra properties of the double points. They were new to me. I will study them more extensively.
You asked if it is possible to calculate the coordinates of the 5 triple points and their conic the same way I did for the double points.
I tried to do so, however, trying to find the common points of two known cubics is too complicated in Mathematica. Maybe you have suggestions?

Best regards,
Chris

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Message: #3539
Date: 04/6/2019 5:39:22
From: Thanh Tung
Subject: A new point on quadrangle ?

```
Dear Angel Montesdeoca and others,

From your message, if P1, P2, P3, P4 are concyclic then the
point Qi are the same as the centroid QA-P1 of the quadrangle
P1P2P3P4.
I have noticed that the triangle transformation that I described
in my previous message is still true for any 7 points P, A, B,
C, A_, B_0, C_0, i.e we can have a general setting:

General setting:
* For any point P and two triangles ABC and  A_0B_0C_0 :
* Gab: centroid of triangle PAB_0 and cyclically defined
  Gac, Gbc, Gba, Gca, Gcb.
* A': intersection of GabGcb and GbcGac
  and cyclically defined B', C'.
  then the lines AA', BB', CC' are concurrent at a point Q.

Proof:
The line B'C' is the same as the line GbaGca. This line is
parallel to BC as they are both paralelel to the line connecting
the midpoints of PB,PC.
So B'C' // BC, similarly C'A'//CA, A'B'//AB   =>  Two triangles
A'B'C' and ABC are homothetic and AA', BB', CC' are concurrent
at Q.

Some particular cases:
A_0B_0C_0 is inscribed triangle of △ ABC (A_0, B_0, C_0 are in
the lines BC, CA, AB) then AA', BB', CC concur at Q = P.
A_0B_0C_0 is the antipedal of P w.r.t ABC then three points:
circumcenter of ABC, isogognal conjugate of P w.r.t ABC and Q
are collinear.
Consequently  if we consider a quadrangle P1P2P3P4, and let Qi=
concurrent point in the general setting of Pi,
triangles PjPkPl, and the antipedal triangle of Pi w.r.t PjPkPl
then PiQi concur at the isogognal center QA-P4 of P1P2P3P4.

Best regards,
Vu Thanh Tung
```

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Message: #3540
Date: 05/6/2019 10:35:33
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Apparently, my last answer went lost !
The conic of the 5 triple points is the conic through the 3 S points and the points T and R.
The S-points are the isogonals of the QG-P1 wrt the triangles QA-P2,4 and 41 of the reference quadrigons), the point T is the perspector of the DT triangle and the S-triangle and the point R is the inverse of QL-P1 in the inversion with center QL-P10 which swaps QL-Ci2 and QL-Ci1.
The intersection between the conic and a QA-Cu7 gives 6 points, the 5 triple points and one S-point.
Best regards
Bernard

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Message: #3541
Date: 06/6/2019 2:24:46
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

back from holiday, reading your resumes
… wrt the QA-Cu7-double points:
Thanks for answering Chris' messages.
Are the following observations already mentioned?
Consider lines L of an arbitrary line pencil:
… L ^ QL-Tf2(L) give a cubic,
… bearing the 6 QL-p0ints (see EQF).

If we can translate QL-Tf2 for a 5P (as we did for QL-Tf1),
… we had a solution of our problem.

Special: Consider lines L through QL-P1
… and the circles QL-Tf1(QL-Tf2(L)) through QL-P1,
… which have centers on an orthogonal hyperbola,
… … centered in QL-P6

with asymptotes parallel to the Steiner axes,
… … bearing QL-P4, QL-P5 ...

For pencils of points on the QL-lines or QL-Tr1-lines
… the cubic degenerates in the line and a conic.

Finally:
For a line L through a QA-Cu7-double point
… QL-Tf2(L) bears the corresponding other double point.

For a QG-version QGa of the QL with double points Ua, Va,
… on the conic Co of the 6 double points,
… the QL-Tf2-partner of lines through Ua and Va intersect
… on a circumconic Co' of QGa through Ua, Va and QG-P18b,c.
… The intersections of Co' and a circle with diameter UaVa are
QG-P18b,c.
… The intersections of Co' and the conic Co of the 6 double
points
… are the contact points of tangents from QG-P1a to Co.

Best regards Eckart

PS: Thanks for new aspects in #3536, I shall answer later.

Message: #3542
Date: 07/6/2019 10:21:28
From: eckart_schmidt@t-online.de
Subject: Dual 4/5 P/L

```
Dear Bernard,

Reading your message 3536, I cannot reproduce:
"I found that QA-P1 lies on the tangent in T
to the conic of the 5 points and on the dual of R …"

I only found:
The dual of T is a parallel to QL-P3.4 through QL-P1,
… but this has no connection with the 5P.

Wrt the interesting conic, dual of the conic of the 5P,
… I found no properties.

Best regards Eckart
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Message: #3543
Date: 2020-03-08
From: System Manager
Subject: Deleted Message
3543

```
Message number 3543 was a deleted message in Yahoo Groups.
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Message: #3544
Date: 10/6/2019 10:58:19
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

the point QL-P24 can be constructed
… with the 5 QA-Cu7-triple points:
In my #2799 a new 5P-point was described,
… this point is CSC(QL-P24) = U.
To interpret this observation
… your #3427 will be very helpful!

Best regards Eckart

PS: The "CSC-circle of X" in #2799 is defined
… by the CSC-images of X wrt the 5 QL of the 5L.

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Message: #3545
Date: 10/6/2019 12:02:20
From: bernard.keizer
Subject: Dual 4/5 P/L

Dear Eckart,
I spent some hours in drawing a brandnew figur !
You're completely right, my property with QA-P1 doesn't hold ...
My apologise.
I was convinced until a few days that 5 points were necessary in
order to get a QL (5 QL vertices, DT vertices + pair of QL-2P3,
4 circumcenters of the reference triangles + QL-P1 ...)
But I've realised that 4 points were enough (4 QA vertices give
the dual QL, 2 non CSC vertices of QL and pair of QL-2P3 give
the 2 CSC partners and the QL ...)
Hence my conviction now that the dual 4 P/L and the dual 5 P/L
are in fact the same figure !
Best regards
Bernard

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Message: #3546
Date: 11/6/2019 4:38:44
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Congratulations for this old message for Christmas giving the
point U, perspector of QA-P4 and QG-P19.
It seems we come always nearer !
Are the following assumptions correct ?
1) For any point X (not QL-P1, not on the conic), we have CSC(X)
and CB(X) and the locus of the foci of the circular cubics
through the 5 points is XCB(X)
2) For any point X on the conic, CSC(X) is on a strange curve
(quartic ?) CSC(conic) and CB(X) is the infinity point of TX
(CB(T) is the infinity point on the tangent in T to the conic).
For QL-P1, CSC is the infinity point on the QL's Newton line and
CB is on the line QL-P8CSC(U) and CSC(QL-P8) and CSC(CB(QL-P1))
are on a line through U and QL-P1 (symmetric of the preceding
line wrt the 1rst Steiner axis)
I think you've put a wedge in the 5P mystery !
Best regards
Bernard

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Message: #3547
Date: 12/6/2019 1:18:41
From: Bernard Gibert
Subject: CTC issue

```
Dear friends

you might have noticed some issues with CTC recently.
The reason is that Orange decided to switch from http to https
without giving me the opportunity to stay in http.
Worse still, they decided that a point in the name of a site
would now be prohibited, which was the case with CTC and
“bernard.gibert”, a default name by Orange long ago…
Before making the change, they announced that an http site would
not be migrated unless the dot was replaced by a dash like
“bernard-gibert” and that this change was not mandatory.
But they never announced that an unchanged site would no longer
be accessible for uploading and that an automatic redirection
would be possible after the change.
Moreover, the software I used to transfer my site is not working
anymore and I had to change for FileZilla which is far more
complicated to me.
So I have been forced  to change CTC to
https://bernard-gibert.pagesperso-orange.fr/index.html , which
is a real nuisance for everyone.
I am really sorry for those having links pointing to CTC in
their sites and I sincerely apologize.
I hope CTC will survive without too many problems and please
feel free to report any bugs you spot. Please also inform those
who find some interest in CTC.

Best regards
Bernard
```

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Message: #3548
Date: 19/6/2019 2:40:04
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard, dear Chris,

some observations, perhaps of interest:
Consider for a 5P = P1P2P3P4P5,
… the point T = 5P-s-P4 on its circumconic CO = 5P-s Co1,
… the QL with the 5P as QA-Cu7-triple points
… and the pentalateral 5L(T),
  whose lines are the bisectors of TPi.
Let the CSC-circle of a point X be the circle (see 5L-s-Tf1)
… through the CSC-images of X wrt the 5 QL-components of 5L(T):
… The CSC-circle of T wrt 5L(T) is a line through U =
CSC(QL-P24);
… all CSC-circles of points on this line bear T.
  5L-o-Ci1 is the locus of points,
… … whose CSC-circles wrt 5L(T) degenerate collinear.
The transformation 5L-s-Tf1 for 5L(T) maps T at infinity:
… The line T.5L-o-Tf1(T) is orthogonal
… … to the collinear degenerated CSC-circle of T (see above).
Let us now consider 5L(X) of bisectors of XPi and X on CO:
… The CSC-circles of X degenerate collinear bearing U.
There are three points X on CO, whose CSC-lines bear X,
… which give a triangle with orthocenter U
… and a circumcircle, which bears QL-P1 and T.

Finally
an additional remark to Bernard's #3427:
A perpendicular to QL-P1.QL-P24 in QL-P24
… bears QL-P16 (see EQF).
A circle with diameter U.QL-P1 bears CSC(QL-P16).
The line QL-P1.CSC(QL-P16) bears R.
How to find QL-P16, starting with the 5P, to get R?

Best regards Eckart

PS for Bernard: T hanks for your encouraging #3546.
Assumption 2) is correct, but I don't understand 1),
… perhaps my knowledge 0f circular cubics is insufficient.
```

Message: #3549
Date: 19/6/2019 8:22:34
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

the point U = CSC(QL-P24) for a QL
… is a special point of the 5P of the 5 QA-Cu7-triple points,
… as we discussed in the last messages.
I think, it should be mentioned in EPG as 5P-s-Px,
… for it has a surprising simple construction:
Consider a vertex Pi of the 5P
… and the QL of the bisectors PiPj (j unequal i),
… then CSC(Pi) gives the point above
… independent of the chosen vertex Pi.

Best regards Eckart

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Message: #3550
Date: 20/6/2019 9:50:59
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,

Remarkable point and a remarkable construction!

Questions:
· When I describe it in short as the 5P-4L-CSC(Pi), is there also a 6P-5L-CSC(Pi) and a 7P-6L-CSC(Pi)?
· And is there a 6P-5L-Co-Tf2(Pi) and/or 6P-5L-Co-Tf3(Pi)?
· Further I lost sight on the connection with QL-P24. For the description in EPG could you please give a concise description of all properties of 5P-4L-CSC(Pi)?

Best regards,

Chris

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Message: #3551
Date: 20/6/2019 10:48:44
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

thanks for your interest and stimulation.
Wrt the 1st question in #3550 a first observation:
6P-5L-Tf(Pi) with Tf = 5L-s-Tf1 is a special 6P-point,
… independent of the chosen vertex Pi,
… but I cannot give properties at the moment.

Best regards Eckart
```

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Message: #3552
Date: 20/6/2019 11:02:27
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

in addition to my last message 3551:

7P-6L-Tf(Pi) with Tf = 6L-s-Tf1 is special 7P-point,
… CABRI-controlled.

Best regards Eckart
```

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Message: #3553
Date: 20/6/2019 11:27:40
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Very interesting constructions, including the amazing construction of U in your following message !
Wrt your PS, it's only an application of CB properties with 7 points (it works also, as we know, if 2 of the 7 are the circular points) : any pivotal cubic with pivot P wrt 7 points and passing through a point X passes also through CB(X) and P, X and CB(X) are aligned. It follows that the line XCB(X) is the locus of the foci of the cubics (circular or not) passing through the 7 points, X and CB(X).
Wrt your message, QL-P16 is the reflexion of QL-P10 in QL-P1 and lies on QL-Ci1 ; CSC(R) is the 2nd intersection (other than QL-P16) of QL-P1QL-P10 with QL-Ci1.
The 2 intersections of QL-Ci1 (through the QG-P1) and it's CSC (through the QA-P4) are necessary CSC partners. Is it possible that these 2 points are precisely QL-P16 and it's CSC partner ?
If it is not only a coïncidence on my figure, it would be surely an interesting property !
Best regards
Bernard

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Message: #3554
Date: 20/6/2019 11:50:27
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Please forget the end of my message, it's in fact only a coïncidence !
Best regards
Bernard

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Message: #3555
Date: 20/6/2019 2:35:08
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

wrt your #3550 2nd question:
… There are no points 6P-5L-Co-Tf2(Pi) and 6P-5L-Co-Tf3(Pi).

Wrt QL-P24 in your 3rd question.
For a reference QL the point CSC(QL-P24) is the perspector U
… of the QA-P4 and the QG-P19 triangle (see your #2986).

For the 5P of the corresponding QA-Cu7-triple points
… I found no connection of U with well known 5P-points.

But for the 5L(X), whose lines are the bisectors of X.Pi,
… the CSC-circles wrt 5L(X) for X on the circumconic of 5P
… degenerate collinear with common point U.

Somewhat curious remains the observation,
… that there are 3 points X on the 5P-circumconic,
… which lie on its degenerated CSC-circle
… and give a triangle with orthocenter U
… and circumcircle through QL-P1 and 5P-s-P4.

Best regards Eckart

Message: #3556
Date: 20/6/2019 5:50:59
From: Thanh Tung
Subject: Construction of a point on quadrangle

Dear all,

We found a construction of a point on quadrangle as follows:

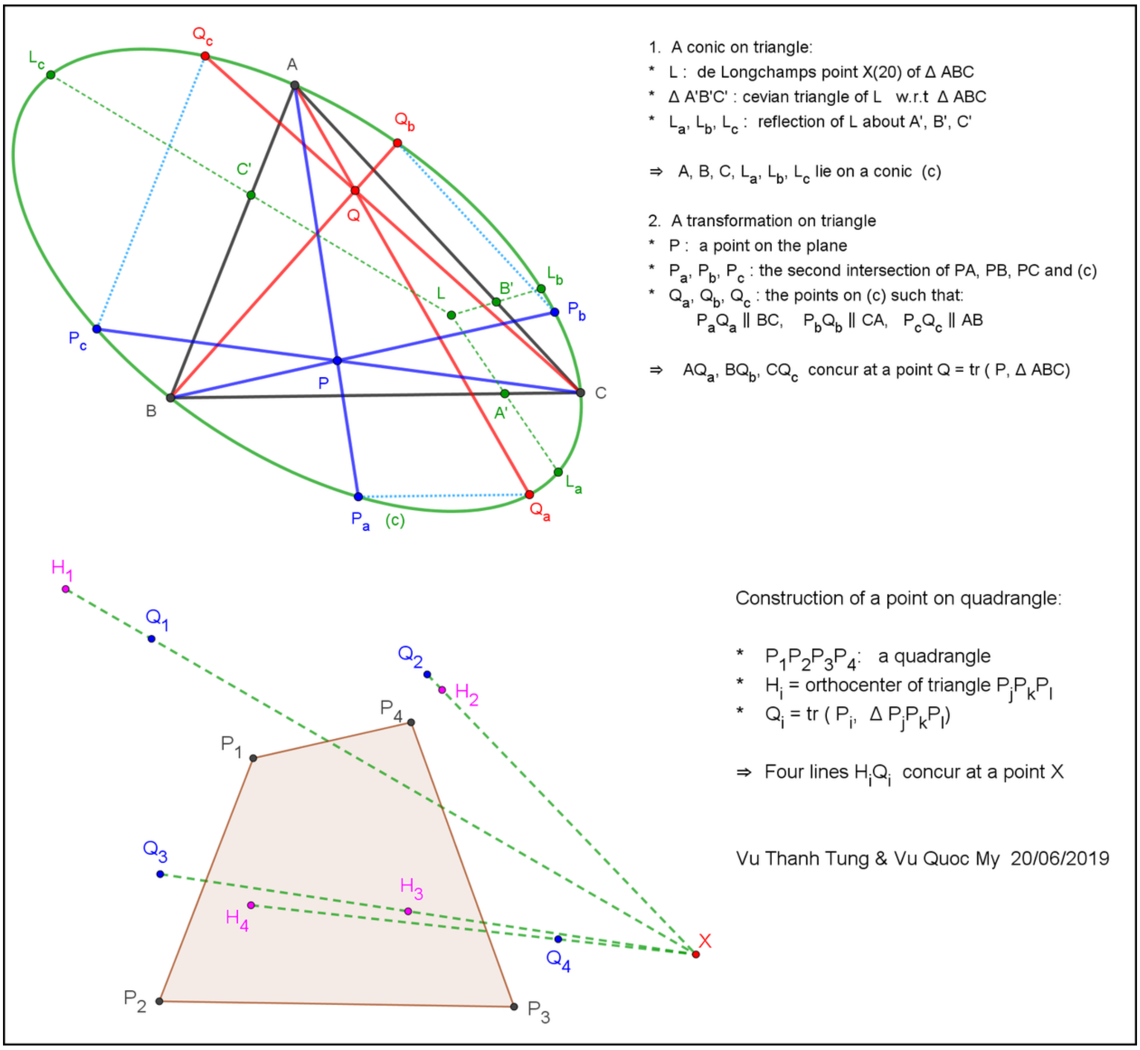
1. A conic on triangle:
   * L: de Longchamps point X(20) of Δ ABC
   * Δ A'B'C': cevian triangle of L w.r.t Δ ABC
   * L_a, L_b, L_c: reflection of L about A', B', C'
   ⇒ A, B, C, L_a, L_b, L_c lie on a conic (c)

2. A transformation on triangle
   * P: a point on the plane
   * P_a, P_b, P_c : the second intersection of PA, PB, PC and the conic (c)
   * Q_a, Q_b, Q_c : the points on (c) such that: P_a Q_a // BC, P_b Q_b // CA, P_c Q_c // AB
   ⇒ AQ_a, BQ_b, CQ_c concur at a point Q = tr (P, Δ ABC)

3. Construction of a point on quadrangle:
   * P_1 P_2 P_3 P_4: a quadrangle
   * H_i = orthocenter of triangle P_j P_k P_l
   * Q_i = tr (P_i, Δ P_j P_k P_l)
   ⇒ Four lines H_i Q_i concur at a point X

We are not sure if the point constructed as above is new or not.

Best regards,
Vu Thanh Tung & Vu Quoc My



PointOnQuadrangleX20X4TungMy20190620_v4.png

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Message: #3557
Date: 20/6/2019 6:17:31
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
Of course, it remains that QL-P10 and QL-P16 being symmetric wrt
QL-P1, their CSC are also symmetric wrt QL-P1 and the line
QL-P10QL-P16 bearing QL-P1 and CSC(R) is symmetric wrt the 1rst
Steiner axis of the line CSC(QL-P10)CSC(QL-P16) bearing QL-P1
and R.
Best regards
Bernard
```

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Message: #3558
Date: 20/6/2019 8:50:02
From: eckart_schmidt@t-online.de
Subject: Construction of a point on quadrangle

```
Dear Vu Thanh Tung & Vu Quoc My,

the described point is QA-P23.

Best regards Eckart
```

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Message: #3559
Date: 20/6/2019 9:53:14
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
If my memory is good, you said once that QL-P1CB(QL-P1) bears
CSC(U) and QL-P8 (it is the line QL-P8QL-P24 through QL-P1) and
you know how to find CB(QL-P1).
Is it possible that CSC(CB(QL-P1))CB(CSC(CB(QL-P1))) bears also
QL-P8?
In other words, the circular cubic through the 5 triple points
with focus QL-P8 passes through QL-P1 and CB(QL-P1) and through
CSC(CB(QL-P1) and CB(CSC(CB(QL-P1))) ...
If this is true, it gives QL-P8 as intersection of the 2 lines
QL-P1CB(QL-P1) and CSC(CB(QL-P1)) and CB(CSC(CB(QL-P1))).
Best regards
Bernard
```

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Message: #3560
Date: 21/6/2019 3:11:46
From: Thanh Tung
Subject: Construction of a point on quadrangle

```
Dear Eckart,
Thank you very much,
Best regards,
Vu Thanh Tung

Sent from Yahoo Mail for iPhone (
https://overview.mail.yahoo.com/?.src=iOS )
> On Friday, June 21, 2019, 1:49 AM,
'eckart_schmidt@t-online.de'
> eckart_schmidt@t-online.de [Quadri-Figures-Group] wrote:
>> Dear Vu Thanh Tung & Vu Quoc My,
>> the described point is QA-P23.
>> Best regards Eckart
```

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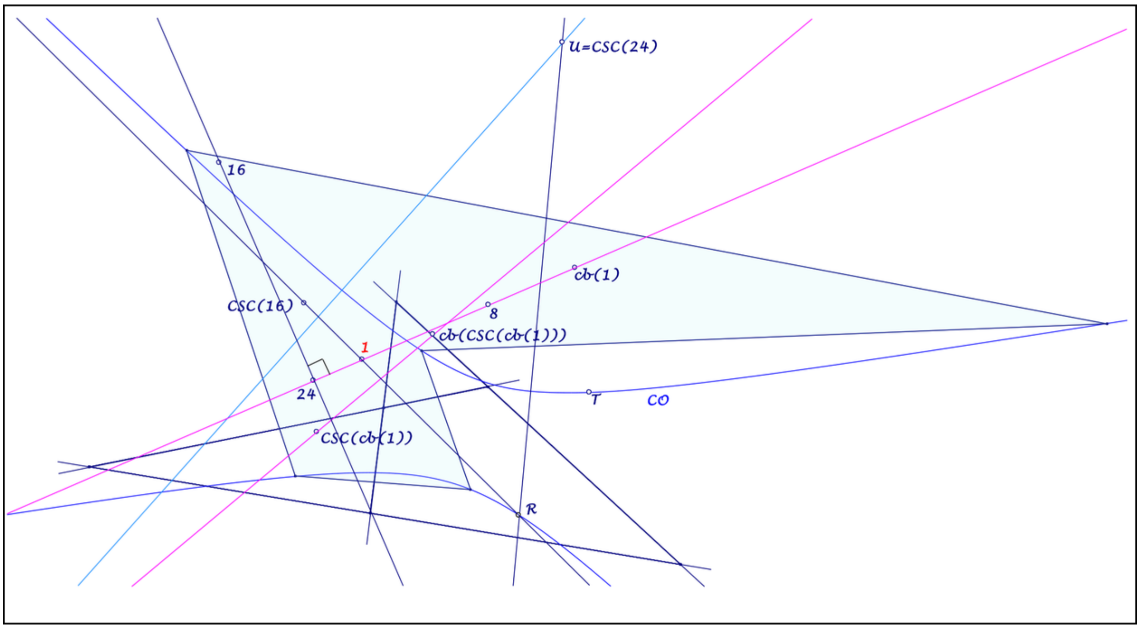
Message: #3561
Date: 21/6/2019 9:14:03
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

sorry, your assumption doesn't hold,
… perhaps the attached construction is helpful.

Best regards Eckart



2019-06-21.pdf

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Message: #3562
Date: 21/6/2019 11:03:10
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

for a nP = (P1,...,Pn) we can consider the (n-1)L(Pi),
… whose lines are the bisectors of PiPj (j unequal i).
Starting with a 6P
… the CSC-circles of Pi wrt 5L(Pi) are the same for all Pi.
For the 6P-components of a 7P
… these circles have a common point.
But I cannot integrate these observations in EPG-geometry.

Best regards Eckart
```

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Message: #3563
Date: 21/6/2019 12:26:15
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
There is no attachment ...
Best regards
Bernard
```

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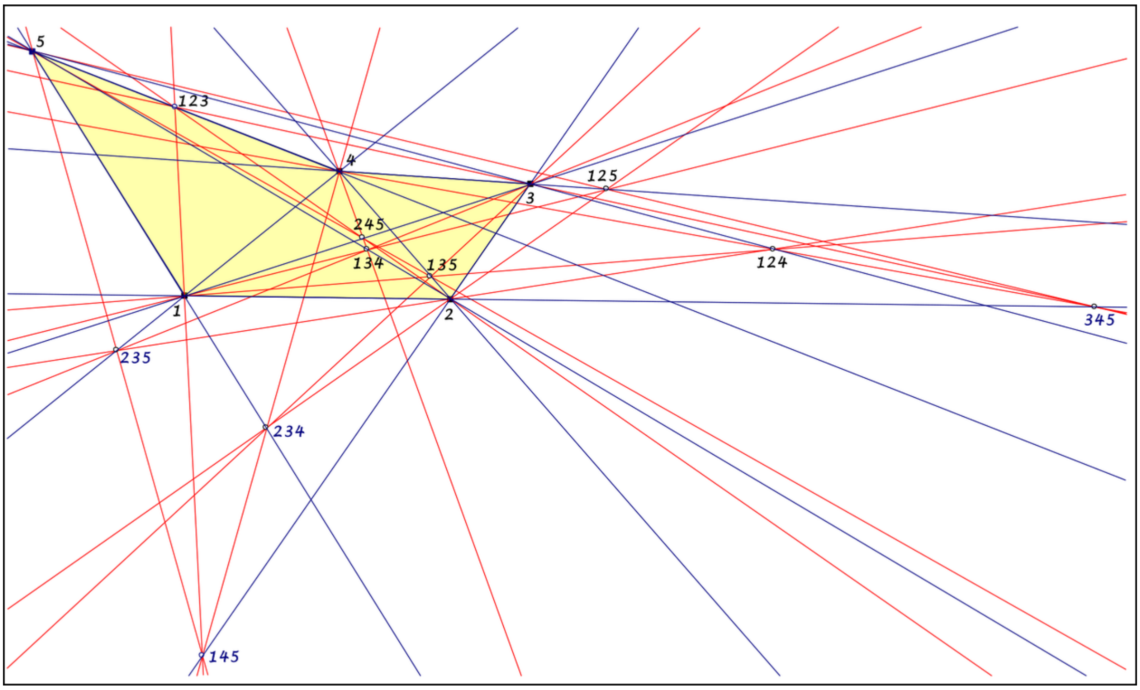
Message: #3564
Date: 24/6/2019 4:02:40
From: eckart_schmidt@t-online.de
Subject: 5P-s-10P

```
Dear Bernard, dear Chris,

perhaps of interest a
15-lines constellation for a 5P,
… bearing the vertices and 10 triple intersections:

Consider for each vertex Pi of a 5P
… the radical axes of the circles
… (Pi,Pj,Pk), (Pi,Pl,Pm) and

(Pi,Pj,Pl), (Pi,Pk,Pm)and (Pi,Pj,Pm), (Pi,Pl,Pk),
… which give 15 lines with 5 tripel points in the vertices
… and 10 further tripel points.

Each line bears one vertex and 2 further triple points,
… each line PiPj bears a further triple point,
… whose 4th harmonic point is collinear
… … with three pairs of further triple points.

Best regards Eckart
```



2019-06-24.pdf

Message: #3565
Date: 25/6/2019 10:12:49
From: bernard.keizer
Subject: 5P-s-10P

```
Dear Eckart,
I reproduced with interest your constellation.
It's possible to draw 5 cubics through the 5 points and each
time 4 of the 10 other triple points (for example 345, 245, 235
and 234 ...), each of the points belonging to 2 cubics.
Best regards
Bernard
```

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Message: #3566
Date: 25/6/2019 3:54:20
From: bernard.keizer
Subject: 5P-s-10P

```
Dear Eckart,
Looking for amusing constellations, you may have this one,
already mentionned, but without reaction :
there are 5 pivotal cubics with pivot one of the 5 points and
fixed points of the isoconjugation the 4 others.
The interest is that the conic of the 5 points is the common
unique diagonal polar conic of the 5 cubics ; it is the polar
conic of the pivot wrt the corresponding cubic.
Best regards
Bernard
```

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Message: #3567
Date: 25/6/2019 4:15:57
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart, Dear Bernard,

I managed to calculate the equation of the conic through the 5 common QA-Cu7 points.
I had to use all the tricks I learned in my years of using Mathematica.
The equation (in QL-CT-coordinates) is not simple and not complicated.

CONIC= $T_{xx} x^2 + T_{yy} y^2 + T_{zz} z^2 + T_{yz} y z + T_{zx} z x + T_{xy} x y$
where:
$T_{xx} = -b^2 c^2 l (b^2 l - c^2 l - b^2 m + c^2 n) (-2 a^2 l + a^2 m + b^2 m - c^2 m + a^2 n - b^2 n + c^2 n) (b^2 m^2 + a^2 m n - b^2 m n - c^2 m n + c^2 n^2)$

$T_{yy} = a^2 c^2 m (-a^2 l + a^2 m - c^2 m + c^2 n) (a^2 l + b^2 l - c^2 l - 2 b^2 m - a^2 n + b^2 n + c^2 n) (a^2 l^2 - a^2 l n + b^2 l n - c^2 l n + c^2 n^2)$

$T_{zz} = a^2 b^2 (a^2 l^2 - a^2 l m - b^2 l m + c^2 l m + b^2 m^2) n (a^2 l - b^2 m - a^2 n + b^2 n) (a^2 l - b^2 l + c^2 l - a^2 m + b^2 m + c^2 m - 2 c^2 n)$

Tyz = a 2 (a 6 b 2 l 4 m - a 4 b 4 l 4 m + a 4 b 2 c 2 l 4 m - 2
a 6 b 2 l 3 m 2 - a 4 b 4 l 3 m 2 + 3 a 2 b 6 l 3 m 2 + 2 a 4 b
2 c 2 l 3 m 2 - 3 a 2 b 4 c 2 l 3 m 2 + a 6 b 2 l 2 m 3 + 5 a 4
b 4 l 2 m 3 - 4 a 2 b 6 l 2 m 3 - 2 b 8 l 2 m 3 - 3 a 4 b 2 c 2
l 2 m 3 - 3 a 2 b 4 c 2 l 2 m 3 + 3 b 6 c 2 l 2 m 3 + 3 a 2 b 2
c 4 l 2 m 3 - b 2 c 6 l 2 m 3 - 3 a 4 b 4 l m 4 - a 2 b 6 l m 4
+ 4 b 8 l m 4 + 6 a 2 b 4 c 2 l m 4 - b 6 c 2 l m 4 - 3 b 4 c 4
l m 4 + 2 a 2 b 6 m 5 - 2 b 8 m 5 - 2 b 6 c 2 m 5 - a 6 c 2 l 4
n - a 4 b 2 c 2 l 4 n + a 4 c 4 l 4 n - 2 a 6 b 2 l 3 m n + 4 a
4 b 4 l 3 m n - 2 a 2 b 6 l 3 m n + 2 a 6 c 2 l 3 m n - 4 a 4 c
4 l 3 m n + 2 a 2 c 6 l 3 m n + 4 a 6 b 2 l 2 m 2 n - 5 a 4 b 4
l 2 m 2 n - 2 a 2 b 6 l 2 m 2 n + 3 b 8 l 2 m 2 n - a 6 c 2 l 2
m 2 n - 7 a 4 b 2 c 2 l 2 m 2 n + 15 a 2 b 4 c 2 l 2 m 2 n - 3 b
6 c 2 l 2 m 2 n + 3 a 4 c 4 l 2 m 2 n + 2 a 2 b 2 c 4 l 2 m 2 n
- 2 b 4 c 4 l 2 m 2 n - 3 a 2 c 6 l 2 m 2 n + b 2 c 6 l 2 m 2 n
+ c 8 l 2 m 2 n - 2 a 6 b 2 l m 3 n - 2 a 4 b 4 l m 3 n + 10 a 2
b 6 l m 3 n - 6 b 8 l m 3 n + 8 a 4 b 2 c 2 l m 3 n - 9 a 2 b 4
c 2 l m 3 n - 5 b 6 c 2 l m 3 n - 10 a 2 b 2 c 4 l m 3 n + 7 b 4
c 4 l m 3 n + 4 b 2 c 6 l m 3 n + 3 a 4 b 4 m 4 n - 6 a 2 b 6 m
4 n + 3 b 8 m 4 n - 6 a 2 b 4 c 2 m 4 n + 8 b 6 c 2 m 4 n + 3 b
4 c 4 m 4 n + 2 a 6 c 2 l 3 n 2 - 2 a 4 b 2 c 2 l 3 n 2 + a 4 c
4 l 3 n 2 + 3 a 2 b 2 c 4 l 3 n 2 - 3 a 2 c 6 l 3 n 2 + a 6 b 2
l 2 m n 2 - 3 a 4 b 4 l 2 m n 2 + 3 a 2 b 6 l 2 m n 2 - b 8 l 2
m n 2 - 4 a 6 c 2 l 2 m n 2 + 7 a 4 b 2 c 2 l 2 m n 2 - 2 a 2 b
4 c 2 l 2 m n 2 - b 6 c 2 l 2 m n 2 + 5 a 4 c 4 l 2 m n 2 - 15 a
2 b 2 c 4 l 2 m n 2 + 2 b 4 c 4 l 2 m n 2 + 2 a 2 c 6 l 2 m n 2
+ 3 b 2 c 6 l 2 m n 2 - 3 c 8 l 2 m n 2 - 2 a 6 b 2 l m 2 n 2 +
6 a 4 b 4 l m 2 n 2 - 6 a 2 b 6 l m 2 n 2 + 2 b 8 l m 2 n 2 + 2
a 6 c 2 l m 2 n 2 - 12 a 2 b 4 c 2 l m 2 n 2 + 10 b 6 c 2 l m 2
n 2 - 6 a 4 c 4 l m 2 n 2 + 12 a 2 b 2 c 4 l m 2 n 2 + 6 a 2 c 6
l m 2 n 2 - 10 b 2 c 6 l m 2 n 2 - 2 c 8 l m 2 n 2 + a 6 b 2 m 3
n 2 - 3 a 4 b 4 m 3 n 2 + 3 a 2 b 6 m 3 n 2 - b 8 m 3 n 2 - 5 a
4 b 2 c 2 m 3 n 2 + 14 a 2 b 4 c 2 m 3 n 2 - 9 b 6 c 2 m 3 n 2 +
7 a 2 b 2 c 4 m 3 n 2 - 9 b 4 c 4 m 3 n 2 - 3 b 2 c 6 m 3 n 2 -
a 6 c 2 l 2 n 3 + 3 a 4 b 2 c 2 l 2 n 3 - 3 a 2 b 4 c 2 l 2 n 3
+ b 6 c 2 l 2 n 3 - 5 a 4 c 4 l 2 n 3 + 3 a 2 b 2 c 4 l 2 n 3 +
4 a 2 c 6 l 2 n 3 - 3 b 2 c 6 l 2 n 3 + 2 c 8 l 2 n 3 + 2 a 6 c
2 l m n 3 - 8 a 4 b 2 c 2 l m n 3 + 10 a 2 b 4 c 2 l m n 3 - 4 b
6 c 2 l m n 3 + 2 a 4 c 4 l m n 3 + 9 a 2 b 2 c 4 l m n 3 - 7 b
4 c 4 l m n 3 - 10 a 2 c 6 l m n 3 + 5 b 2 c 6 l m n 3 + 6 c 8 l
m n 3 - a 6 c 2 m 2 n 3 + 5 a 4 b 2 c 2 m 2 n 3 - 7 a 2 b 4 c 2
m 2 n 3 + 3 b 6 c 2 m 2 n 3 + 3 a 4 c 4 m 2 n 3 - 14 a 2 b 2 c 4
m 2 n 3 + 9 b 4 c 4 m 2 n 3 - 3 a 2 c 6 m 2 n 3 + 9 b 2 c 6 m 2
n 3 + c 8 m 2 n 3 + 3 a 4 c 4 l n 4 - 6 a 2 b 2 c 4 l n 4 + 3 b
4 c 4 l n 4 + a 2 c 6 l n 4 + b 2 c 6 l n 4 - 4 c 8 l n 4 - 3 a
4 c 4 m n 4 + 6 a 2 b 2 c 4 m n 4 - 3 b 4 c 4 m n 4 + 6 a 2 c 6
m n 4 - 8 b 2 c 6 m n 4 - 3 c 8 m n 4 - 2 a 2 c 6 n 5 + 2 b 2 c
6 n 5 + 2 c 8 n 5)

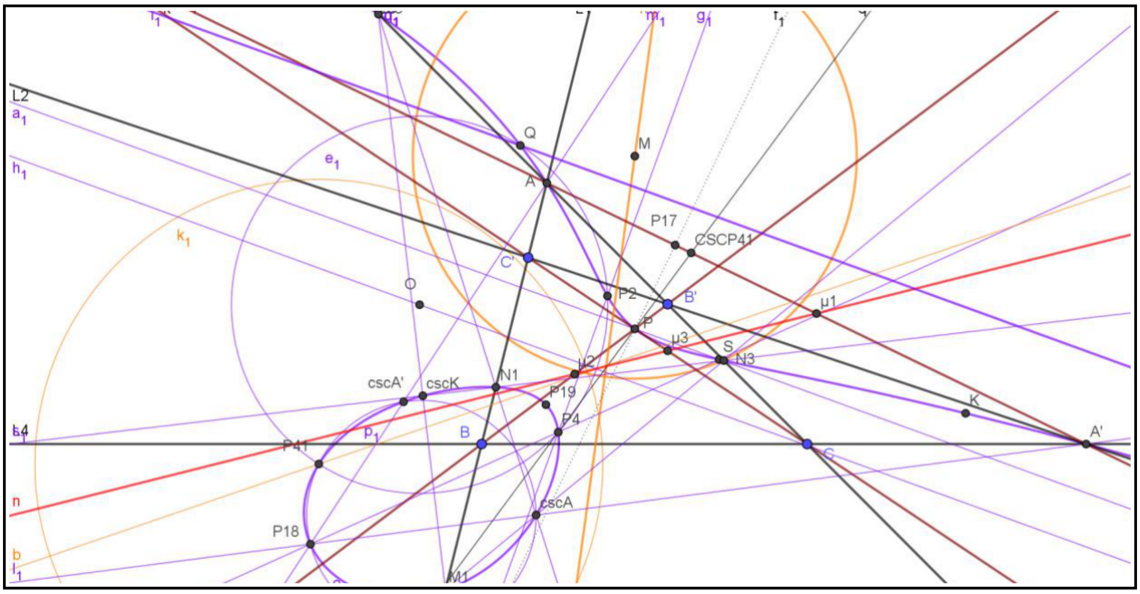
Tzx = -b 2 (-2 a 8 l 5 + 2 a 6 b 2 l 5 - 2 a 6 c 2 l 5 + 4 a 8 l
4 m - a 6 b 2 l 4 m - 3 a 4 b 4 l 4 m - a 6 c 2 l 4 m + 6 a 4 b
2 c 2 l 4 m - 3 a 4 c 4 l 4 m - 2 a 8 l 3 m 2 - 4 a 6 b 2 l 3 m
2 + 5 a 4 b 4 l 3 m 2 + a 2 b 6 l 3 m 2 + 3 a 6 c 2 l 3 m 2 - 3
a 4 b 2 c 2 l 3 m 2 - 3 a 2 b 4 c 2 l 3 m 2 + 3 a 2 b 2 c 4 l 3
m 2 - a 2 c 6 l 3 m 2 + 3 a 6 b 2 l 2 m 3 - a 4 b 4 l 2 m 3 - 2
a 2 b 6 l 2 m 3 - 3 a 4 b 2 c 2 l 2 m 3 + 2 a 2 b 4 c 2 l 2 m 3
- a 4 b 4 l m 4 + a 2 b 6 l m 4 + a 2 b 4 c 2 l m 4 + 3 a 8 l 4
n - 6 a 6 b 2 l 4 n + 3 a 4 b 4 l 4 n + 8 a 6 c 2 l 4 n - 6 a 4
b 2 c 2 l 4 n + 3 a 4 c 4 l 4 n - 6 a 8 l 3 m n + 10 a 6 b 2 l 3
m n - 2 a 4 b 4 l 3 m n - 2 a 2 b 6 l 3 m n - 5 a 6 c 2 l 3 m n
- 9 a 4 b 2 c 2 l 3 m n + 8 a 2 b 4 c 2 l 3 m n + 7 a 4 c 4 l 3
m n - 10 a 2 b 2 c 4 l 3 m n + 4 a 2 c 6 l 3 m n + 3 a 8 l 2 m 2
n - 2 a 6 b 2 l 2 m 2 n - 5 a 4 b 4 l 2 m 2 n + 4 a 2 b 6 l 2 m
2 n - 3 a 6 c 2 l 2 m 2 n + 15 a 4 b 2 c 2 l 2 m 2 n - 7 a 2 b 4
c 2 l 2 m 2 n - b 6 c 2 l 2 m 2 n - 2 a 4 c 4 l 2 m 2 n + 2 a 2
b 2 c 4 l 2 m 2 n + 3 b 4 c 4 l 2 m 2 n + a 2 c 6 l 2 m 2 n - 3
b 2 c 6 l 2 m 2 n + c 8 l 2 m 2 n - 2 a 6 b 2 l m 3 n + 4 a 4 b
4 l m 3 n - 2 a 2 b 6 l m 3 n + 2 b 6 c 2 l m 3 n - 4 b 4 c 4 l
m 3 n + 2 b 2 c 6 l m 3 n - a 2 b 4 c 2 m 4 n - b 6 c 2 m 4 n +
b 4 c 4 m 4 n - a 8 l 3 n 2 + 3 a 6 b 2 l 3 n 2 - 3 a 4 b 4 l 3
n 2 + a 2 b 6 l 3 n 2 - 9 a 6 c 2 l 3 n 2 + 14 a 4 b 2 c 2 l 3 n
2 - 5 a 2 b 4 c 2 l 3 n 2 - 9 a 4 c 4 l 3 n 2 + 7 a 2 b 2 c 4 l
3 n 2 - 3 a 2 c 6 l 3 n 2 + 2 a 8 l 2 m n 2 - 6 a 6 b 2 l 2 m n
2 + 6 a 4 b 4 l 2 m n 2 - 2 a 2 b 6 l 2 m n 2 + 10 a 6 c 2 l 2 m
n 2 - 12 a 4 b 2 c 2 l 2 m n 2 + 2 b 6 c 2 l 2 m n 2 + 12 a 2 b
2 c 4 l 2 m n 2 - 6 b 4 c 4 l 2 m n 2 - 10 a 2 c 6 l 2 m n 2 + 6
b 2 c 6 l 2 m n 2 - 2 c 8 l 2 m n 2 - a 8 l m 2 n 2 + 3 a 6 b 2
l m 2 n 2 - 3 a 4 b 4 l m 2 n 2 + a 2 b 6 l m 2 n 2 - a 6 c 2 l
m 2 n 2 - 2 a 4 b 2 c 2 l m 2 n 2 + 7 a 2 b 4 c 2 l m 2 n 2 - 4
b 6 c 2 l m 2 n 2 + 2 a 4 c 4 l m 2 n 2 - 15 a 2 b 2 c 4 l m 2 n
2 + 5 b 4 c 4 l m 2 n 2 + 3 a 2 c 6 l m 2 n 2 + 2 b 2 c 6 l m 2
n 2 - 3 c 8 l m 2 n 2 - 2 a 2 b 4 c 2 m 3 n 2 + 2 b 6 c 2 m 3 n
2 + 3 a 2 b 2 c 4 m 3 n 2 + b 4 c 4 m 3 n 2 - 3 b 2 c 6 m 3 n 2
+ 3 a 6 c 2 l 2 n 3 - 7 a 4 b 2 c 2 l 2 n 3 + 5 a 2 b 4 c 2 l 2
n 3 - b 6 c 2 l 2 n 3 + 9 a 4 c 4 l 2 n 3 - 14 a 2 b 2 c 4 l 2 n
3 + 3 b 4 c 4 l 2 n 3 + 9 a 2 c 6 l 2 n 3 - 3 b 2 c 6 l 2 n 3 +
c 8 l 2 n 3 - 4 a 6 c 2 l m n 3 + 10 a 4 b 2 c 2 l m n 3 - 8 a 2
b 4 c 2 l m n 3 + 2 b 6 c 2 l m n 3 - 7 a 4 c 4 l m n 3 + 9 a 2
b 2 c 4 l m n 3 + 2 b 4 c 4 l m n 3 + 5 a 2 c 6 l m n 3 - 10 b 2
c 6 l m n 3 + 6 c 8 l m n 3 + a 6 c 2 m 2 n 3 - 3 a 4 b 2 c 2 m
2 n 3 + 3 a 2 b 4 c 2 m 2 n 3 - b 6 c 2 m 2 n 3 + 3 a 2 b 2 c 4
m 2 n 3 - 5 b 4 c 4 m 2 n 3 - 3 a 2 c 6 m 2 n 3 + 4 b 2 c 6 m 2
n 3 + 2 c 8 m 2 n 3 - 3 a 4 c 4 l n 4 + 6 a 2 b 2 c 4 l n 4 - 3
b 4 c 4 l n 4 - 8 a 2 c 6 l n 4 + 6 b 2 c 6 l n 4 - 3 c 8 l n 4
+ 3 a 4 c 4 m n 4 - 6 a 2 b 2 c 4 m n 4 + 3 b 4 c 4 m n 4 + a 2
c 6 m n 4 + b 2 c 6 m n 4 - 4 c 8 m n 4 + 2 a 2 c 6 n 5 - 2 b 2
c 6 n 5 + 2 c 8 n 5)

Txy = c 2 (-2 a 8 l 5 - 2 a 6 b 2 l 5 + 2 a 6 c 2 l 5 + 3 a 8 l 4 m + 8 a 6 b 2 l 4 m + 3 a 4 b 4 l 4 m - 6 a 6 c 2 l 4 m - 6 a 4 b 2 c 2 l 4 m + 3 a 4 c 4 l 4 m - a 8 l 3 m 2 - 9 a 6 b 2 l 3 m 2 - 9 a 4 b 4 l 3 m 2 - 3 a 2 b 6 l 3 m 2 + 3 a 6 c 2 l 3 m 2 + 14 a 4 b 2 c 2 l 3 m 2 + 7 a 2 b 4 c 2 l 3 m 2 - 3 a 4 c 4 l 3 m 2 - 5 a 2 b 2 c 4 l 3 m 2 + a 2 c 6 l 3 m 2 + 3 a 6 b 2 l 2 m 3 + 9 a 4 b 4 l 2 m 3 + 9 a 2 b 6 l 2 m 3 + b 8 l 2 m 3 - 7 a 4 b 2 c 2 l 2 m 3 - 14 a 2 b 4 c 2 l 2 m 3 - 3 b 6 c 2 l 2 m 3 + 5 a 2 b 2 c 4 l 2 m 3 + 3 b 4 c 4 l 2 m 3 - b 2 c 6 l 2 m 3 - 3 a 4 b 4 l m 4 - 8 a 2 b 6 l m 4 - 3 b 8 l m 4 + 6 a 2 b 4 c 2 l m 4 + 6 b 6 c 2 l m 4 - 3 b 4 c 4 l m 4 + 2 a 2 b 6 m 5 + 2 b 8 m 5 - 2 b 6 c 2 m 5 + 4 a 8 l 4 n - a 6 b 2 l 4 n - 3 a 4 b 4 l 4 n - a 6 c 2 l 4 n + 6 a 4 b 2 c 2 l 4 n - 3 a 4 c 4 l 4 n - 6 a 8 l 3 m n - 5 a 6 b 2 l 3 m n + 7 a 4 b 4 l 3 m n + 4 a 2 b 6 l 3 m n + 10 a 6 c 2 l 3 m n - 9 a 4 b 2 c 2 l 3 m n - 10 a 2 b 4 c 2 l 3 m n - 2 a 4 c 4 l 3 m n + 8 a 2 b 2 c 4 l 3 m n - 2 a 2 c 6 l 3 m n + 2 a 8 l 2 m 2 n + 10 a 6 b 2 l 2 m 2 n - 10 a 2 b 6 l 2 m 2 n - 2 b 8 l 2 m 2 n - 6 a 6 c 2 l 2 m 2 n - 12 a 4 b 2 c 2 l 2 m 2 n + 12 a 2 b 4 c 2 l 2 m 2 n + 6 b 6 c 2 l 2 m 2 n + 6 a 4 c 4 l 2 m 2 n - 6 b 4 c 4 l 2 m 2 n - 2 a 2 c 6 l 2 m 2 n + 2 b 2 c 6 l 2 m 2 n - 4 a 6 b 2 l m 3 n - 7 a 4 b 4 l m 3 n + 5 a 2 b 6 l m 3 n + 6 b 8 l m 3 n + 10 a 4 b 2 c 2 l m 3 n + 9 a 2 b 4 c 2 l m 3 n - 10 b 6 c 2 l m 3 n - 8 a 2 b 2 c 4 l m 3 n + 2 b 4 c 4 l m 3 n + 2 b 2 c 6 l m 3 n + 3 a 4 b 4 m 4 n + a 2 b 6 m 4 n - 4 b 8 m 4 n - 6 a 2 b 4 c 2 m 4 n + b 6 c 2 m 4 n + 3 b 4 c 4 m 4 n - 2 a 8 l 3 n 2 + 3 a 6 b 2 l 3 n 2 - a 2 b 6 l 3 n 2 - 4 a 6 c 2 l 3 n 2 - 3 a 4 b 2 c 2 l 3 n 2 + 3 a 2 b 4 c 2 l 3 n 2 + 5 a 4 c 4 l 3 n 2 - 3 a 2 b 2 c 4 l 3 n 2 + a 2 c 6 l 3 n 2 + 3 a 8 l 2 m n 2 - 3 a 6 b 2 l 2 m n 2 - 2 a 4 b 4 l 2 m n 2 + a 2 b 6 l 2 m n 2 + b 8 l 2 m n 2 - 2 a 6 c 2 l 2 m n 2 + 15 a 4 b 2 c 2 l 2 m n 2 + 2 a 2 b 4 c 2 l 2 m n 2 - 3 b 6 c 2 l 2 m n 2 - 5 a 4 c 4 l 2 m n 2 - 7 a 2 b 2 c 4 l 2 m n 2 + 3 b 4 c 4 l 2 m n 2 + 4 a 2 c 6 l 2 m n 2 - b 2 c 6 l 2 m n 2 - a 8 l m 2 n 2 - a 6 b 2 l m 2 n 2 + 2 a 4 b 4 l m 2 n 2 + 3 a 2 b 6 l m 2 n 2 - 3 b 8 l m 2 n 2 + 3 a 6 c 2 l m 2 n 2 - 2 a 4 b 2 c 2 l m 2 n 2 - 15 a 2 b 4 c 2 l m 2 n 2 + 2 b 6 c 2 l m 2 n 2 - 3 a 4 c 4 l m 2 n 2 + 7 a 2 b 2 c 4 l m 2 n 2 + 5 b 4 c 4 l m 2 n 2 + a 2 c 6 l m 2 n 2 - 4 b 2 c 6 l m 2 n 2 + a 6 b 2 m 3 n 2 - 3 a 2 b 6 m 3 n 2 + 2 b 8 m 3 n 2 - 3 a 4 b 2 c 2 m 3 n 2 + 3 a 2 b 4 c 2 m 3 n 2 + 4 b 6 c 2 m 3 n 2 + 3 a 2 b 2 c 4 m 3 n 2 - 5 b 4 c 4 m 3 n 2 - b 2 c 6 m 3 n 2 + 3 a 6 c 2 l 2 n 3 - 3 a 4 b 2 c 2 l 2 n 3 - a 4 c 4 l 2 n 3 + 2 a 2 b 2 c 4 l 2 n 3 - 2 a 2 c 6 l 2 n 3 - 2 a 6 c 2 l m n 3 + 2 b 6 c 2 l m n 3 + 4 a 4 c 4 l m n 3 - 4 b 4 c 4 l m n 3 - 2 a 2 c 6 l m n 3 + 2 b 2 c 6 l m n 3 + 3 a 2 b 4 c 2 m 2 n 3 - 3 b 6 c 2 m 2 n 3 - 2 a 2 b 2 c 4 m 2 n 3 + b 4 c 4 m 2 n 3 + 2 b 2 c 6 m 2 n 3 - a 4 c 4 l n 4 + a 2 b 2 c 4 l n 4 + aa 2 c 6 l n 4 - a 2 b 2 c 4 m n 4 + b 4 c 4 m n 4 - b 2 c 6 m n 4)

The Center of the Conic however is pretty long. Too long to mention here.
The advantage of having calculated equation and coordinates is that we can look for incidences with other QL-items. However, I could not find any incidence with other QL-items so far.
Also, an advantage is secure plotting. See attached file.
Now for me, there will be the next challenge to calculate the intersection points of QA-Cu7a and this conic. One of these intersection points will be Sa (QFG#2980,#2983,#2984). But I am not very hopeful.

Best regards,

Chris



QA-Cu7.pdf

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Message: #3568
Date: 25/6/2019 9:00:32
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Chris,

congratulations on the equation of the 5P-conic
… for the QA-Cu7-triple points!
Now our observations will get analytical certification.

Best regards Eckart

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Message: #3569
Date: 25/6/2019 10:59:26
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Many thanks for the attachment !
I reproduced your construction of U. It's amazing ...
Now, for 5 points, we have the conic, the CSC and the points T and U.
The CSC, T and U are perfectly defined for an ellipse as well as for an hyperbola ...
For any point R, we have the circle with diameter UR, it's CSC circle with center P9, the points P16 and P10, the point P8 as intersection of P9P10 with P1P24 ...
What next ? (We haven't used the CB transformation)
Best regards
Bernard

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Message: #3570
Date: 26/6/2019 10:00:34
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Congratulations, it looks very impressive !
As I don't practice Mathematica myself, I hardly measure the difficulty of these calculations, but I see the possible consequences ...
I hope you will succeed in finding the equation of the 6th degree giving the 6 intersections of the conic and QA-Cu7a and, eliminating the point Sa, the equation of the 5th degree giving the 5 triple points ; it has to be the same with QA-Cu7b and QA-Cu7c ...
Courage, the fight goes on !
Best regards
Bernard

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Message: #3571
Date: 26/6/2019 9:10:20
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

wrt your message 3569:
… "What next ? (We haven't used the CB transformation)".

Here a further observation for the 5P/QL-puzzle wrt QA-Cu7:
Let cb be the CB-transformation wrt 5P and the circular points,
… already mentioned cb(QL-P1) on QL-P1.QL-P24.

If we consider for cb(QL-P1)
… the 5L of the bisectors wrt the 5P-vertices
… and the CSC-circle of cb(QL-P1) wrt this 5L,
… we get a circle through QL-P1, CSC(QL-P24)=U,
CSC(5P-s-P4)=CSC(T).

Best regards Eckart

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Message: #3572
Date: 27/6/2019 8:22:30
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Another proof that all fit's together !
I hope you will find now another point like U.
What about the QA-P3 of the reference QA's on the Dimidium
circle and their CSC, the QG-P16 aligned on the CSC line of this
Dimidium circle ?
What about the point you mentionned as perspector of the QG-P1
and their cb partners ?
Best regards
Bernard
PS I won't be home for about ten days

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Message: #3573
Date: 27/6/2019 9:00:45
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Bernard, Dear Eckart,
[BK] I hope you will succeed in finding the equation of the 6th degree giving the 6 intersections of the conic and QA-Cu7a and, eliminating the point Sa, the equation of the 5th degree giving the 5 triple points
I tried to calculate the intersection points of the conic and QA-Cu7a.
I supposed the intersection points to be of the form (x:y:1)
I calculated the equation of the 6th degree in x. I also eliminated the point Sa and indeed I found an equation of degree 5 of this form:
EQx = T0 + T1 x + T2 x^2 + T3 x^3 + T4 x^4 + T5 x^5 = 0.
In a similar way, I can calculate an equation of the form
EQy = V 0 + V1 y + V2 y^2 + V3 y^3 + V4 y^4 + V5 y^5 = 0.
However, the coefficients T0, T1, T2, T3, T4, T5, V0, V1, V2, V3, V4, V5 are huge. See attachment. (Editorial note: attachment omitted due to excessive length.)
So hidden in both equations are the coordinates (x:y:1) of the 5 common points of the 3 QL-versions of QA-Cu7.
Nice to know, but these equations are algebraically unsolvable.
Like Bernard commented: "Courage, the fight goes on.".
Best regards,
Chris

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Message: #3574
Date: 28/6/2019 9:36:51
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Congratulations again !
I didn't hope you could solve the equation of the 5th degree, but only prove that the equation is the same for the 3 cubics QA-Cu7a,b and c; this would prove that the 5 points are common to the 3 cubics.
Best regards
Bernard
PS The attachment has disappeared ...

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Message: #3575
Date: 02/7/2019 4:30:58
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

here are further observations for the 5P/QL-constellation wrt
the 3 QA-Cu7:

For a 5P we can construct the QL-points QL-P1 and QL-P24
… and the QL-transformation CSC = QL-Tf1,
… also the CB-transformation *cb* wrt 5P
  and the two circular points.

Properties of the transformation *cb*
(some already mentioned):
(a) cb maps each QA-Cu7 to itself with P, cb(P) collinear QG-P1.
(b) cb maps 5P-s-Co1 to the line at infinity with P,cb(P)
collinear T = 5P-s-P4.
(c) cb swaps the double points of two QA-Cu7.
(d) The lines S.cb(S) (wrt S see #3176)
    intersect in T = 5P-s-P4,
    … perspector of the S- and QG-P1-triangle.
(e) The lines QG-P19.cb(QG-P19) intersect in V,
    … perspector of the QG-P1- and QG-P19-triangle.
(f) The lines QA-P41.cb(QA-P41) intersect in W,
    … perspector of the QG-P1- and QA-P41-triangle.

Now consider a 5P = P1...P5 and an arbitrary point P,
… further the 5L of the bisectors PPi
… and the concyclic CSC-images of P wrt the 4L of 5L,
… which give a circle *Ci(P)*.

Properties of the *Ci(P)*:
(1) The circles Ci(P) have a common point U = CSC(QL-P24).
(2) P and cb(P) have the same circle Ci(P) = Ci(cb(P)).
(3) For points P on the conic 5P-s-Co1
    … Ci(P) degenerates to a line through U.
(4) For QL-P1 and its cb-image on QL-P1.P8.P24 we get
    … a circle through QL-P1, U = CSC(QL-P24), CSC(T) =
    CSC(5P-s-P4).
(5) For the double points of QA-Cu7a,b , which are cb-partner,
    … the circles are the same, bearing U, QA-P4c, QA-P41a,b.
(6) The circles Ci(QG-P1) bear U, QA-P41 and a 2nd common point.
(7) The circles Ci(QA-P4) bear U, QA-P41 and QA-P4;
    … the 2nd intersection of Ci(QA-P4a,b) lies on Ci(QA-P41c).
(8) The circles Ci(QG-P18) bear U, QA-P41;

… the 2nd intersection of Ci(QG-P18a,b) lies on Ci(QG-P19c).

(9) The circles Ci(QA-P41) bear U, QA-P41,
… the 2nd intersection of two Ci(QA-P4)
… and a further common point;
… their centers lie on the QG-P18-line of the QL.

(10) The circles Ci(QG-P19) bear U, QA-P41 and a 2nd common point.

(11) The circles Ci(QA-P2) bear U, QA-P41;
… the 2nd intersection of Ci(QA-P2a,b) lies on U.QA-P41c.

(12) The circles Ci(S) degenerate to lines U.QA-P41.

(13) The circles Ci(P) for the six QL-points
… have common point U, 3 double and 4 triple intersections;
… the three double intersections are QA-P41a,b,c.

(14) The locus for points X with Ci(X) through QL-P1
… is the locus of cb-partner on lines through QL-P24:
… This is a cb-invariant circumcubic of 5P,
… through QL-P1, cb(QL-P1), QL-P24, cb(QL-P24)
… and the 2nd intersection of QL-P24.5P-s-P4 and 5P-s-Co1,
… whose cb-image is the infinity point of the cubic.
… The tangential of QL-P24 wrt this cubic is cb(QL-P24).
… If we consider the CB-transformation wrt 5P, QL-P24, cb(QL-P24),
… this cubic is also the locus of CB-partner on parallels to QL-P24.5P-s-P4.

I think the transformations P --> Ci(P) and P --> cb(P)
… will be relevant for the 5P/QL-constellation wrt QA-Cu7-points,
… but the way back from the 5P to the QL isn't visible not yet.
The results above are only CABRI-observations!

Best regards Eckart

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Message: #3576
Date: 04/7/2019 12:05:38
From: chris.vantienhoven
Subject: 5P-Geometry

```
Dear Eckart,

I included the special new points in a 5P, 6P, 7P as mentioned
in messages #3549-3552 in EPG.
See 5P-s-P5, 6P-s-P1, 7P-s-P1.

Best regards,
Chris
```

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Message: #3577
Date: 04/7/2019 8:47:50
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

there is also a point 4P-3L-Tf:
… With Tf  the inversion wrt the circumcircle of the 3L
… you get QA-P4.

Best regards Eckart
```

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Message: #3578
Date: 04/7/2019 10:32:12
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,

4P-3L-Tf = QA-P4 is a nice point fitting in front of the row
with 5P-s-P5, 6P-s-P1, 7P-s-P1.
It shows the relationship between 4-Points, 5-Points, 6-Points
and 7-Points.
Similar series in an n-Line are:
· QL-Tf1 – 5L-s-Tf1 – 6L-s-Tf1
· X(186) – QL-P28 – 5L-s-P2
· X(256) – QL-P29 – 5L-s-P3
· X (4) – QL-P20 – 5L-s-P7
· 5L-s-P1 – 6L-s-P1 – 7L-s-3P1 – 8L-s-P1

Best regards,

Chris

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Message: #3579
Date: 06/7/2019 9:46:05
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

```
Dear Bernard, dear Chris,

in #3575 I described circles Ci(P), here an application:
Consider a 5P, its circumconic 5P-s-Co1 and points P on it:
…  the circles Ci(P) degenerate to lines through U = 5P-s-P5
…  and the intersections of P.T = P.5P-s-P4 and Ci(P)
…  give an orthogonal hyperbola
   through T = 5P-s-P4 and U = 5P-s-P5,
…  centered in the midpoint of U.T
…  and intersecting 5P-s-Co1 in three further points X beside T,
…  which give a triangle with orthocenter U
…  and a circumcircle through QL-P1 and T
…  and the property X on Ci(X) for its vertices
   (see also #3548).

Maybe this is a reference triangle to find the lines of the QL
…  with the 5P as its QA-Cu7-triple points.

Best regards Eckart
```

Message: #3580
Date: 06/7/2019 12:50:27
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

```
Deze Eckart,
You're doing a great job finding alle these wonderful properties
of the 3xQA-Cu7-constellation
Right niet In AM on holiday. When I am back I will look further.
Best
```

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Message: #3581
Date: 08/7/2019 8:42:49
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

```
Dear Bernard, dear Chris,

in EPG / 5P-s-P3 there is described a circle with 5P-s-P3 as
center,
… this circle bears the midpoint of T = 5P-s-P4 and U = 5P-s-P5
… as well as the midpoint of QL-P1 (see PS) and U = 5P-s-P5.
… This circle is the nine-point circle
  of the triangle in #3579,
… bearing the center of the orthogonal hyperbola in #3579,
… thus 5P-s-P3 is the nine-point center
  of the triangle in #3579.

The orthogonal hyperbola in #3579 can easier be described:
… centered in the middle of TU,

bearing T and U,
… axes parallel to those of 5P-s-Co1.

The intersections -beside T- with 5P-s-Co1 give the triangle in
#3579.

Best regards Eckart

PS: I think, the point QL-P1 of the QL with the 5P-vertices as
QA-Cu7-triple points
     … should be mentioned in EPG as 5P-s-P6.
```

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Message: #3582
Date: 08/7/2019 9:56:53
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

```
Deze Eckart,
Could youne exactly describe 5P-s-P6 as a 5P-point?
Probably I misser it.
Best regards,
Chris
```

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Message: #3583
Date: 09/7/2019 11:39:59
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

in #3270 (16.11.2018) I described
… a construction for a 5P-CSC-transformation,
… which leads to QL-P1 = 5P-s-P6
… for the QL with the vertices of the 5P
as QA-Cu7-triple points.

CSC for the vertices of a 5P = P1...P5:
Let Q1 be P1
… Q2 the inverse of P1 wrt the circle (P3,P4,P5),
… Q3 the inverse of P1 wrt the circle (P2,P4,P5),
… Q4 the inverse of P1 wrt the circle (P2,P3,P5),
… Q5 the inverse of P1 wrt the circle (P2,P3,P4),
… then CSC(P1) is
… … the inverse of P2 wrt the circle (Q3,Q4,Q5),
… … or the inverse of P3 wrt the circle (Q2,Q4,Q5),
… … or the inverse of P4 wrt the circle (Q2,Q3,Q5),
… … or the inverse of P5 wrt the circle (Q2,Q3,Q4).

5P-s-P6:
QL-P1 of the QL with lines Pi.Pj, Pi.CSC(Pj), Pj.CSC(Pi),
CSC(Pi).CSC(Pj),
… independent of the chosen 5P-vertices Pi, Pj.

5P-s-P6, a vertex Pi and CSC(Pi) define a 5P-CSC-transformation
for any point.

Best regards Eckart

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Message: #3584
Date: 09/7/2019 5:58:12
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Back from a few days far away from home and computer, I discover your last findings.
It's amazing, I don't know where to start with this point U = CSC(QL-P24) as extension to the 5P of QA-P4 for the QL (and, if I understand correctly, of the circumcenter X3 for the triangle), the Ci(P) and last this rectangular hyperbola fallen from the sky as a cherry on the cake.
I'll try to catch up and to check your properties ...
I'm for the moment totally bluffed ! Congratulations, it's indeed a beautiful and huge work !
Maybe 2 or 3 simple (?) questions :
1) if any 2 cb partners have the same circle Ci(P) through U, is the converse true ? In this case, which are the 2 cb partners having the circle with diameter UR (CSC of QL-Ci1) as Ci(P) ?
2) the same way, if any point on the 5P conic has a line through U as degenerated Ci(P), which is the line of R ? As you gave so many examples, it's possible that you gave the answer and I missed it ...
3) which is the point on the conic having the CSC of the Dimidium circle as degenerated Ci(P) ,
Best regards
Bernard
PS Please be patient, it will take me time to understand and reproduce your beautiful properties

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Message: #3585
Date: 09/7/2019 9:26:00
From: bernard.keizer
Subject: 5P-Geometry

```
Dear Eckart,
As very often, I forgot one last simple (?) question :
at the end of your message 3575, you describe the circular
circumcubic of the 5 points with pivot QL-P24, wich contains
QL-P1 and cb(QL-P1) on the line QL-P1,8 and 24.
More generally, it contains 2 cb partners on any line through
the pivot.
Which are the cb partners on the line QL-P13, 17 and 24 ?
Best regards
Bernard
```

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Message: #3586
Date: 09/7/2019 10:32:42
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

```
Dear Eckart,

I started to study your messages #3575 and #3579.
First observation:
1. Your circle Ci(P) is actually a 6P-Circle. It is the circle
through the six versions of 5P-s-P1. Because the starting point
and the vertices can be interchanged producing the same circle
it is a 6P-Circle. Let’s call it 6P-s-Ci1.
2. The center of 6P-s-Ci1 is 6P-s-P1 !
3. In a 7P the seven versions of 6P-s-Ci1 concur in 7P-s-P1 !

Best regards,
Chris
```

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Message: #3587
Date: 10/7/2019 9:34:38
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

```
Dear Chris,

thanks for a better interpretation of Ci(P) as 6P-s-Ci1,
... but it is the circle through the six 5P-s-P5.

Best regards Eckart
```

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Message: #3588
Date: 10/7/2019 3:07:23
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

I read your messages #3270 and #3583 about the construction of a 5P-CSC-transformation.

It’s a very peculiar construction and I got some questions.

1. Why do you call CSC(P1) being
… … the inverse of P2 wrt the circle (Q3,Q4,Q5), etc.?
This deals with inverses at two levels.
The real CSC=QL-Tf1 is the sequence of a reflection and an inversion.

2. How did you arrive at taking the MiquelPoint of the QL formed by the 4 lines Pi.Pj, Pi.CSC(Pj), Pj.CSC(Pi), CSC(Pi).CSC(Pj) ?

3. What is according to you the usefulness of this point in relation to the 3 QA-Cu7 configuration?
By the way the first part of your description is identical to the construction of QA-Tf16, but I suppose you know.
Using the functionality of QA-P16 the construction is (when I did interpret it well):
· Let Ri = QA-Tf16(Pi) wrt QA=5-Point minus Pi.
· Let Rj = QA-Tf16(Pj) wrt QA=5-Point minus Pj.
5P-s-P6 = QL-P1 of the QL with lines Pi.Pj, Pi.Rj, Pj.Ri, Ri.Rj, which delivers a point independent of the chosen 5-Point-vertices Pi, Pj.

Best regards,
Chris

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Message: #3589
Date: 10/7/2019 3:28:30
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

Dear Eckart, Dear Bernard,

I like the double-function of the points T, U, "QL-P1" or
5P-s-P4, 5P-s-P5, 5P-s-P6.
They function in the Reference QL as T, U, "QL-P1", and
they function in the 3xQA-Cu7-5P as 5P-s-P4, 5P-s-P5, 5P-s-P6.

Eckart, you wrote:
"Maybe this is a reference triangle
to find the lines of the QL
… with the 5P as its QA-Cu7-triple points."

It even would be better to find the 5P from these known and
related items:
* the conic through the 5P
* T, U
* Sa, Sb, Sc
Don't you think there is a double function for Sa, Sb, Sc?
But, I doubt severely that even when knowing all these items we
will be able to find the 5P.
I think there should be an alternative way.

Best regards,
Chris

Message: #3590
Date: 10/7/2019 8:34:24
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

```
Dear Chris,

the main problem for Bernard and me is
… to get back from the 5P to the QL,
… whose 5 QA-Cu7-triple points are the vertices of the 5P.
We already can construct the transformation CSC
… and the points QL-P1, QL-P24,
… U = perspector of the QA-P4- and QG-P19-triangle,
… T = perspector of the QG-P1- and the S-triangle.
If we are able to "construct" the QL,
… we can use QL-points for 5P-geometry.
But I think, this is not the background of your message 3589.

Best regards Eckart
```

Message: #3591
Date: 12/7/2019 10:42:37
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

thanks for your interested cooperation,
… but I cannot give answers wrt your questions in#3584.
I tried in vain to get the relevant point R, starting with a 5P,
… but there is a new property of R wrt the QL:
The QL-point R is the CSC-image of
… the 2nd intersection of QL-P1.10.16 and QL-Ci1.
If you study the circles Ci(P) in #3575:
… there are further observations for a QG-version of the QL:
The line Ci(S) and the circles
… Ci(QA-P2), Ci(QA-P4), Ci(QG-P18), Ci(QG-P19), Ci(QA-P41)
  bear QA-P41
… with a 2nd triple point for Ci(QG-P19) and Ci(QA-P41)
… and Ci(QA-P4), Ci(QA-P41) bearing their generating point.
Wrt the perspectors V and W (see #3176):
… Ci(V) bears the 2nd triple point of Ci(QG-P19),
… Ci(W) bears the 2nd triple point of Ci(QA-P41).

Best regards Eckart

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Message: #3592
Date: 12/7/2019 10:44:34
From: bernard.keizer
Subject: 5P orthogonal hyperbola

Dear Eckart,
I started from the end with your rectangular hyperbola.
I understand why X on the conic and the infinity point on TX give the same 2nd intersection of TX with the RH and therefore the same line Li(X) through U and this 2nd intersection (that goes for T with the tangent in T).
I have reproduced your construction and I'm able to answer myself my questions 2) and 3) in message 3584.
The line Li(R) doesn't give anything particular and the point X having CSC(Dimidium circle) as Li(X) doesn't seem to be particular either.
It remains my 1rst question in message 3584 and my question in message 3585.
Which are the 2 cb partners X and cb(X) having the circle with diameter UR as Ci(X) = Ci(cb(X)) ?
Which are the 2 cb partners on the line QL-P13, 17 and 24 ?
The same question goes for the line QL-P16 and 24 ?
Best regards
Bernard

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Message: #3593
Date: 12/7/2019 10:54:17
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Your answer crossed mine.
For the point R, I put this property already several times on the forum without reaction :
CSC(R) is the inverse of QL-P1 in the inversion with center QL-P10 swapping the QG-P1 and the QG-P17, id est the inversion with center QL-P10 swapping the circumcircle QL-Ci1 and the Euler circle QL-Ci2.
Best regards
Bernard

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Message: #3594
Date: 12/7/2019 5:09:57
From: bernard.keizer
Subject: 5P-s-P4

Dear Chris, dear Eckart,

Dealing with 7 points P1 to 7, it's well known that they form a CB transformation swapping the 21 copples line/conic.
Taking for example the line 67 and the conic 12345 and 45 and 12367, the 2 lines intersect in a point U123 and the 2 conics in a 4th point (other than 123) V123.
U123 and V123 are CB partners and the line U123V123 intersects the conics in 2 2nd points (other than V123) T67 and T45.
It's possible this way to construct the 21 pivots of the 21 conics for this CB transformation.
It is remarkable that for each conic, the pivot is exactly the point T or 5P-s-P4 on the conic depending only from the 5 points on this conic.
Perhaps it was obvious after all, but I was surprised for myself ...

Best regards
Bernard

PS Does that mean that the circular points are CB partners for the transformation of any group of 7 points ?

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Message: #3595
Date: 13/7/2019 10:14:00
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
In fact, this property is easy to prove synthetically !
As R is the perspector of the 3 QA-P4 and the 3 QG-P18, CSC(R) belongs to the 3 CSCcircles of the 3 lines concurring in R. The 3 circles are coaxial and pass through QL-P1 and CSC(R) ; the intersection of the lines QL-P1CSC(R) and QG-P1QG-P17 is the same for the 3 copples of QG-P1 and QG-P17, id est the orthocenter QL-P10.
The power of QL-P10 wrt the circles is QL-P10QL-P1*QL-P10CSC(R) = QL-P10QG-P1*QL-P10QG-P17, hence the inversion swapping the circumcircle of DT QL-Ci1 and it's Euler circle QL-Ci2.
I hope I have convinced you this time ...
Best regards
Bernard
PS QL-P16, reflexion of QL-P10 in QL-P1 is in EQF, but neither this point CSC(R), as you mention 2nd intersection of QL-P10QL-P1 with QL-Ci1, nor the middle of QL-P10CSC(R) lying on QL-Ci2.

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Message: #3596
Date: 13/7/2019 3:36:13
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,
excuse, that I hadn't parat your property of R,
… thanks for synthetical justification.
Best regards Eckart

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Message: #3597
Date: 13/7/2019 5:19:33
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
As usual, my figures lack of precision !
Is it possible that under the 2 cbpartners on QL-P24QL-P17, one is QL-P13 (on the circular circumcubic you described with pivot QL-P24 and on all circular circumcubics with pivot on this line)?
(The other is not QL-P17)
Best regards
Bernard

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Message: #3598
Date: 14/7/2019 12:08:46
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

sorry, QL-P13 is not a cb-partner on QL-P24.QL-P17,
… but I cannot find any properties,
… as well as for the cb-partner on QL-P24.QL-P16.
But there is following observation (see #3575):
For cb-partner on lines through QL-P24
… the circle Ci(X) bears U =5P-s-P5 and *QL-P1*.
In general:
For cb-partner on lines through a point P
… the circle Ci(X) bear U and a further common point.
What about the point P with further common point R?

Best regards Eckart

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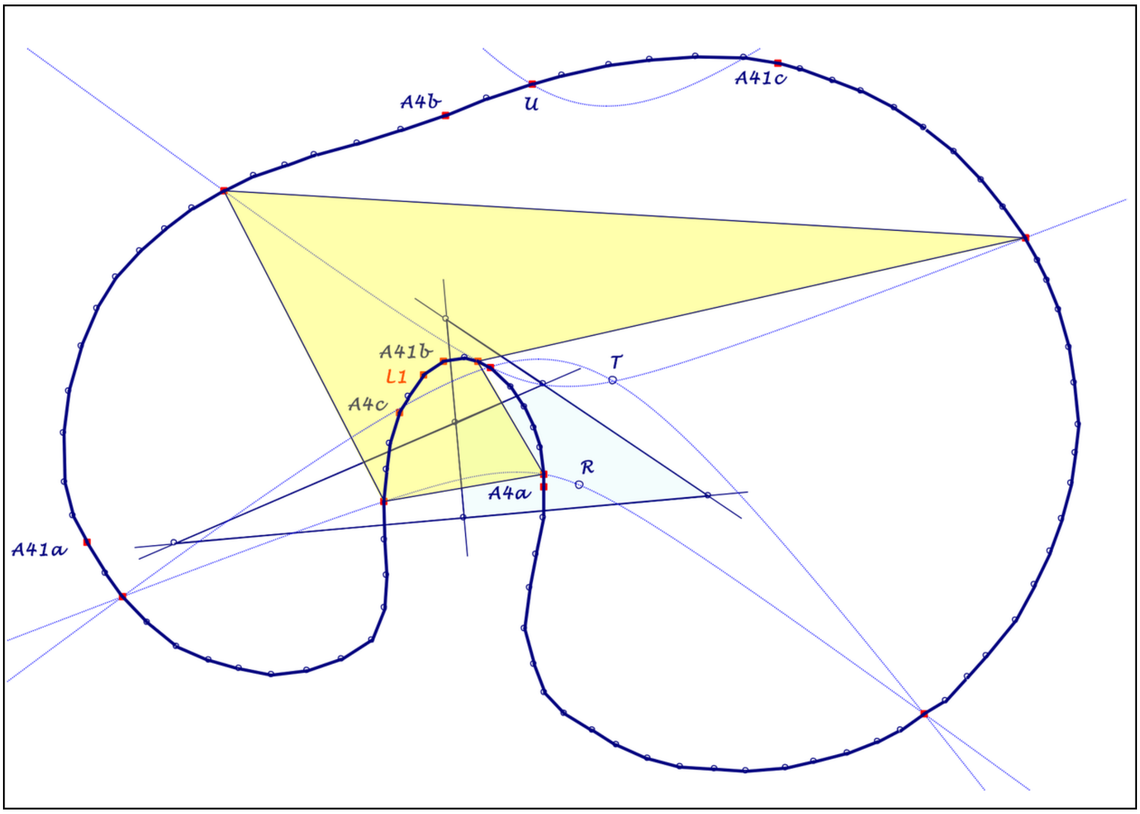
Message: #3599
Date: 14/7/2019 2:32:26
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Chris, dear Bernard,

the circles Ci(P) (see #3575) lead to an interesting quartic
… for the 5P/QL constellation wrt QA-Cu7,
… bearing 16 well known points,
… attached an approximate pointwise drawing.
The quartic is the locus of points X with X on Ci(x),
… bearing the 5 QA-Cu7-triple points,
… the points U = 5P-s-P5 and QL-P1 = 5P-s-P6,
… the 3 QA-P4 and the 3 QA-P41
… and the 3 intersections beside T = 5P-s-P4
… … of 5P-s-Co1
and the orthogonal hyperbola in #3579.

Best regards Eckart



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Message: #3600
Date: 14/7/2019 4:03:47
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for your patience !
As you can see, I follow the ad of Avis "We're second, but we try harder!"
I hoped you would be able to identify the 2 cb partners having the circle with diameter UR as Ci.
But your new property gives more chances to identify the point P such as the Ci's of all the cb partners on lines through P are centered on the perpendicular bisector of UR ...
Best regards
Bernard

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Message: #3601
Date: 14/7/2019 4:18:49
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Beautiful ! Marvellous ! (Thanks for the figur)
According to your definition, I suppose the quartic is cb invariant, which gives 16 more points !
If I'm not wrong, there are 10 points under the 16 which are determined only from the 5 points.
With their cb partners, that makes 20 !
A quartic being determined by 14 points, I think your quartic is entirely constructible from the 5 points !!!
Is this assumption correct ?
Best regards
Bernard

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Message: #3602
Date: 14/7/2019 5:22:23
From: bernard.keizer
Subject: 5P-Quartic

```
Dear Eckart,
Sorry, my assumption is not correct !
The summer is very hot in the Dordogne and the brains are
suffering too !!!
After a short bath in my swimming pool, I realised that the fact
that a point X and it's partner cb(X) have the same Ci(X) doen't
mean that cb(X) is on Ci(X) if X is on Ci(X).
cb(QL-P1) doesn't lie on the circle through QL-P1,U and CSC(T)
and the cb partners of your 3 points on the RH are infinity
points on lines through T ...
I very much apologise for my wrong conclusions.
Perhaps, it would be interesting to look for the cbcurve of your
quartic?
Best regards
Bernard
```

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Message: #3603
Date: 14/7/2019 10:46:36
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
I think your quartic is the CSC of a cubic through the CSC of the 5 points, the 3 DT vertices QG-P1 (CSC of the QA-P4), the CSC of the QA-P41 (intersections of QG-P1QA-P4 and the DT sides), the CSC of your 3 points intersections(other than T) between the conic of the 5 points and the rectangular hyperbola and QL-P24.
Any circle through QL-P1 cuts the quartic in 3 other points and the CSC of the 3 points are aligned on the cubic
This time, as the cubic may be described from the 5 points, the same goes for the quartic.
I hope this time I didn't make a basic mistake ...
Best regards
Bernard
PS I suppose the cubic may be described as a non pivotal isocubic wrt DT with root the trilinear pole of the line of the CSC(QL-P41)

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Message: #3604
Date: 14/7/2019 11:44:35
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
The trilinear pole of the line of the CSC(QA-P41) is the point W, perspector of the QG-P1 and the QA-P41.
Best regards
Bernard

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Message: #3605
Date: 15/7/2019 10:49:48
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
The more I look on the figures, the more I get excited !
In the construction of the rectangular hyperbola, you use the point 5P-s-P3 and the QA-P4 of the 5 QAs.
Your beautiful quartic passes through the 5 QA-P4, which gives you 5 more points; that makes with the 5 P, the points U and QL-P1 and the 3 intersections of the conic with the RH 15 points which depend only from the 5 P.
Your quartic is perfectly dtermined !
Best regards
Bernard

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Message: #3606
Date: 15/7/2019 10:54:53
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

your cubic as CSC of the quartic is a remarkable obvious curve,
… thanks for discovery and detailed description,
… I try to reproduce and get a construction.
What is the isoconjugation for this isocubic?

Best regards Eckart

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Message: #3607
Date: 15/7/2019 3:24:54
From: eckart_schmidt@t-online.de
Subject: 5P-Quartett

Dear Bernard,

I gather our observations:
The 5 QA-Cu7-triple points wrt a QL lie on a quartic,
… which is the CSC-image of a nonpivotal isocubic:
… … reference triangle QL-Tr1,
… … isoconjugation: isogonal conjugate,
… … point: QL-P24,
… … root: W = perspector of the QG-P1- and QA-P41-triangle.

Best regards Eckart

PS: Point W is also the isogonal conjugate of T wrt QL-Tr1.

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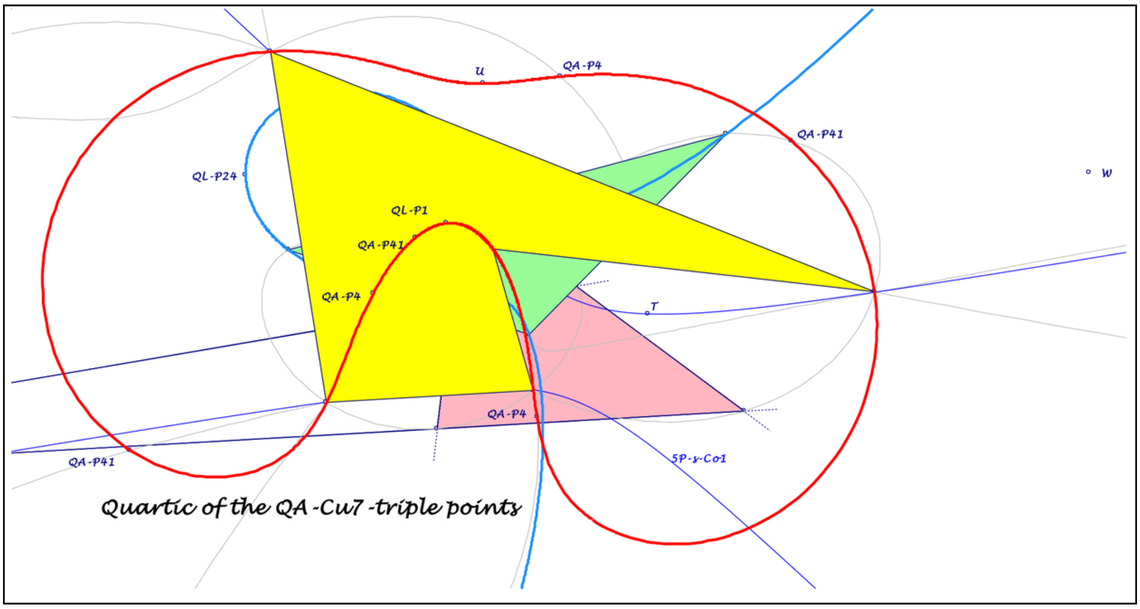
Message: #3608
Date: 15/7/2019 4:50:29
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

I try to transmit a construction of the quartic and its CSC-cubic (word file).

Best regards Eckart



2019-07-15.docx

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Message: #3609
Date: 15/7/2019 5:09:24
From: bernard.keizer
Subject: 5P-Quartett

Dear Eckart,
The cubic is also a pivotal circular circumcubic of the 5 CSC of the triple points.
QL-P24 is the tangential of the middle of UT (on the cubic) and the 6th intersection of the cubic with the conic of the 5 CSC is a point S' tangential of QL-P24.
With T' the point 5P-s-P4 of the 5 CSC, the asymptote of the cubic is T'S' and it seems the pole for the CB transformation wrt the 5 CSC points and the 2 circular points is the infinity point of the asymtote.
Best regards
Bernard
PS Could you please send me your picture in pdf file. Thanks in advance

---

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Message: #3610
Date: 15/7/2019 5:42:16
From: bernard.keizer
Subject: 5P-Quartett

Dear Eckart,
The cubic is also a beautiful Van Rees curve, invariant in the CSC with center QL-P24 swapping the QG-P1 and the QA-P41 ...
But the interest of defining it only from the 5 CSC of the triple points is that it allows the reverse operation with the 5 triple points in order to find what I named the twin QL !
Best regards
Bernard
PS We only need now a definition of R ...

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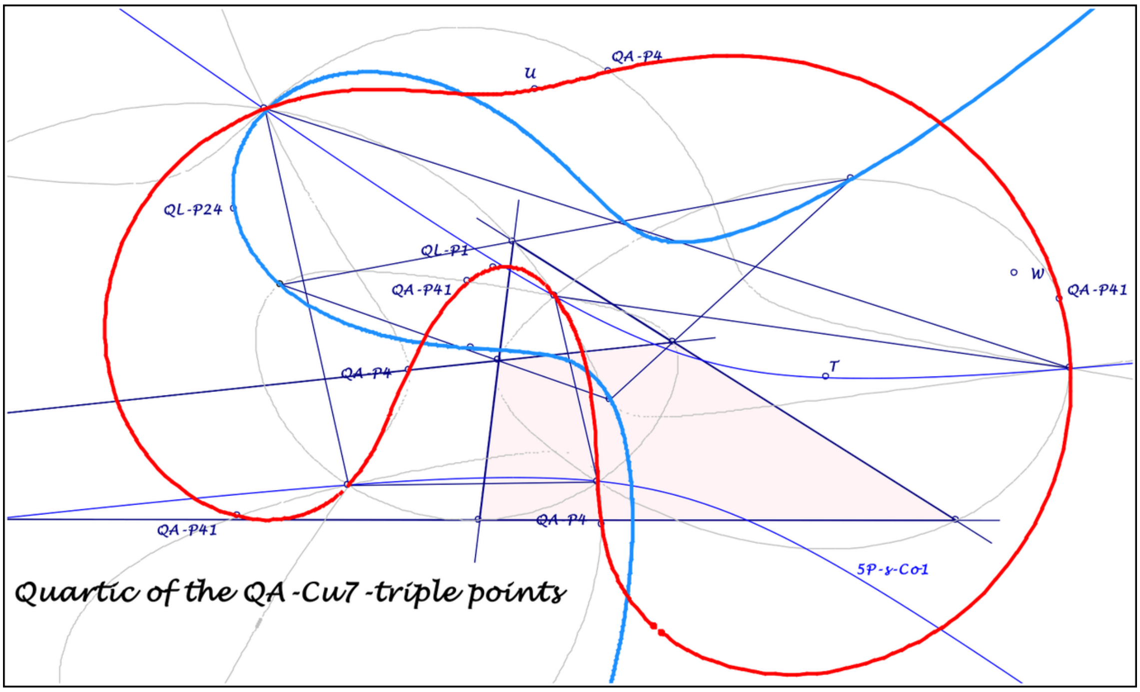
Message: #3611
Date: 16/7/2019 9:01:33
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard, dear Chris,

I try once more, to transmit to QFG a construction
… of the quartic and its CSC-cubic described in #3607.

Best regards Eckart
```



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Message: #3612
Date: 16/7/2019 10:13:58
From: bernard.keizer
Subject: 5P-Quartic

```
Dear Eckart,
Thank you for the figures ; they look like mine.
I'm waiting impatiently for your comments on my messages 3605,
3609 and 3610.
I tried several ideas wrt the point R (isogonality or trilinear
polar wrt the triangle of the 3 intersections of conic and RH
...), but nothing works !
Best regards
Bernard
```

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Message: #3613
Date: 16/7/2019 11:57:03
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard,

wrt your messages #3609, #3610:
(1) I cannot reproduce,
... that the middle of UT lies on the cubic (see figur).
(2) The point S' seems interessant with your observed
properties,
… they are new for me, especially S'T' as asymptote
  of the cubic,
… the point at infinity seems to be isog(QL-P24) wrt QL-DT
  or cb'(S').
(3) I cannot reproduce,
that the cubic is invariant (see figure)
… "in the CSC with center QL-P24 swapping the QG-P1
   and the QA-P41".

Best regards Eckart
```

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Message: #3614
Date: 16/7/2019 2:44:26
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard,

wrt (3) in my last message: I think you mean:
The cubic is invariant
… "in the CSC with center QL-P24 swapping the CSC(QG-P1)
    and the CSC(QA-P41)".

Best regards Eckart
```

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Message: #3615
Date: 16/7/2019 4:14:54
From: bernard.keizer
Subject: 5P-Quartic

```
Dear Eckart,
Thanks for your answer to my messages 3609 and 3610
1) Thanks for having checked the property : it was only a
coïncidence on my own figur
2) These are classic properties of a circular circumcubic of 5
points applied to the CSC of the 5 triple points.
New is that the pivot can be the infinity point on the line ...
3) Sorry for my mistake : it was obviously the QG-P1 and the CSC
of the QA-P41 !
The cubic is the QL-Cu1 of the QL formed by the DT vertices and
the points CSC(QA-P41) which lie on the DT sides and are
aligned. The asymptote is T'S' which is parallel to the Newton
Line of this new QL ...
4) You didn't answer my message 3605 : can you confirm that the
QA-P4 of the 5 QA's of the 5P are on your quartic (and therefore
their CSC on the cubic)
In this case, having the 5 P (which determine a CSC) and their
CSC partners, it's easy to draw the 2 reciprocical cubics
through 5 points and the CSC of the QA-P4 of the CSC of the 5
points and vice-versa.
This would lead (so I hope) to the 2 twin QL's ...
Best regards
Bernard
```

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Message: #3616
Date: 16/7/2019 8:46:00
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

excuse, my interpretation of CSC wrt QL-P24 was not right,
… your correction in #3615 is correct.

Of course I can confirm,
… that the five QA-P4 of the 5P lie on the quartic,
… making possible, to define the quartic starting with the 5P.
… Thanks, this was new for me!

Best regards Eckart

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Message: #3617
Date: 17/7/2019 10:32:19
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Thanks for this confirmation !
Then we have easily the 2 reciprocical cubics, which give 2 new points S and S' as 6th intersection with the 2 conics. What about these 2 new points ?
As these 2 cubics are Van Rees curves, they may be monocursal or bicursal ...
The 2 cubics are defined for an ellipse as well as for an hyperbola (5P may be on an ellipse or an hyperbola and their CSC the same way on an ellipse or an hyperbola ?)
I won't be home again for a copple of days, but as soon as I come back, I'll check new drawings ...
Best regards
Bernard
PS I think we are very near from the goal now

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Message: #3618
Date: 18/7/2019 2:54:24
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

searching for an old message I recognized
… that I have not read your #3588,
please accept my apologies!

Wrt 1. Reasons for my observations are only CABRI-constructions.
… CSC(P1) is the real CSC for the QL with QA-Cu7-triple points
in the 5P.

Wrt 2. If we have CSC(Pi), we can construct QL-P1
… and the Steiner axes as well as the inversion circle.

Wrt 3. Perhaps there will be no direct relation
… between QL-P1 and the QA-Cu7-triple points,
… but the transformation CSC is a relevant tool,
… to study the QL/5P constellation wrt QA-Cu7.

Best regards Eckart

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Message: #3619
Date: 18/7/2019 11:42:07
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Thanks for your answer.
Now I understand that:
* QA-P16(Pi) wrt 4 pts of the 5 TriplePoints P(i) = QL-Tf1(Pi) wrt Ref-QL
* 5P-s-P4 wrt the TriplePoints-5P = T = perspector of 2 Ref-QL-triangles
* 5P-s-P5 wrt the TriplePoints-5P = U = perspector of 2 Ref-QL-triangles
* 5P-s-P6 wrt the TriplePoints-5P = QL-P1 wrt Ref-QL

Which other 5P<>QL-links there are?

Best regards,
Chris

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Message: #3620
Date: 22/7/2019 6:35:02
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris, dear Eckart,
We are looking for links 5P-QL.
As the QL defines the 5 P, every 5P point is a QL point, but we wish a reverse construction from the 5P to the QL !
The points T = 5P-s-P4 is the isogonal conjugate of W wrt DT, where W is the perspector of the QG-P1 and the QA-P41 and the trilinear pole of the line through the CSC(QA-P41) wrt DT.
The point U = 5P-s-P5 is CSC(QL-P24)
The point QL-P1 = 5P-s-P6 is center of the CSC of the initial QL.
Last,but not least, we have this wunderful cubic and it's CSC the quartic.
The cubic is defined by 10 points which are the 5 CSC of the 5P and the 5 CSC of the QA-P4 of the 5 QA's of the 5P ; this cubic is a circular circumcubic of the 5 CSC of the 5P (asymptote passing through the 5P-s-P4 of this 5P and through the 6th intersection between the cubic and the conic of this 5P).
This cubic is also the QL-Cu1 of the QL formed by the QG-P1 and the CSC(QA-P41) with focus QL-P24.
Only the point R which is the CSC of the inverse of QL-P1 in the inversion with center QL-P10 swapping the QG-P1 and the QG-P17 or the circumcircle and the Euler circle of DT, QL-Ci1 and QL-Ci2 remains an enigma for the 5P ...
Best regards
Bernard.

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Message: #3621
Date: 23/7/2019 10:01:56
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

some new observations wrt a 5P = (P1,...,P5) and a point P:
Let T = 5P-s-P4 and Ti = 5P-s-P4 of P
and four vertices unequal Pi,
… so we get 5T = (T1,...,T5) with following properties:
(1) Pi.Ti parallel P.T with midpoints on a line through 5P-s-P1.
(2) Point P lies on the circumconic of 5T.
(3) 5P and 5T have the same area, if they are konvex.
(4) 5T degenerates collinear with P, *if cb(P) = P* wrt 5P
(wrt cb see #3575),
… on a line orthogonal Ci(P) wrt 5T (wrt Ci(P) see #3575).
(5) Ci(P) wrt 5T degenerates always collinear.
(6) cb(P) wrt 5T is always an infinity point.
(7) If P is a 5P-vertex, one vertex of 5T is T.
(8) If P is 5P-s-P4 of 5T on 5P-s-Co1,
… 5T is the reflection of 5P in the 2nd axis of 5P-s-Co1.

Best regards Eckart

PS: Excuse, no connection with our 5P-QL-problem.

Thanks to Bernard for the summary of our observations.

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Message: #3622
Date: 23/7/2019 10:11:42
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard, dear Chris,

several times I used the CB-transformation cb,
… defined by the 5 vertices of a 5P and the circular points.

Here are some doubtful observations
… wrt the 5P of the QA-Cu7-triple points of a QL:

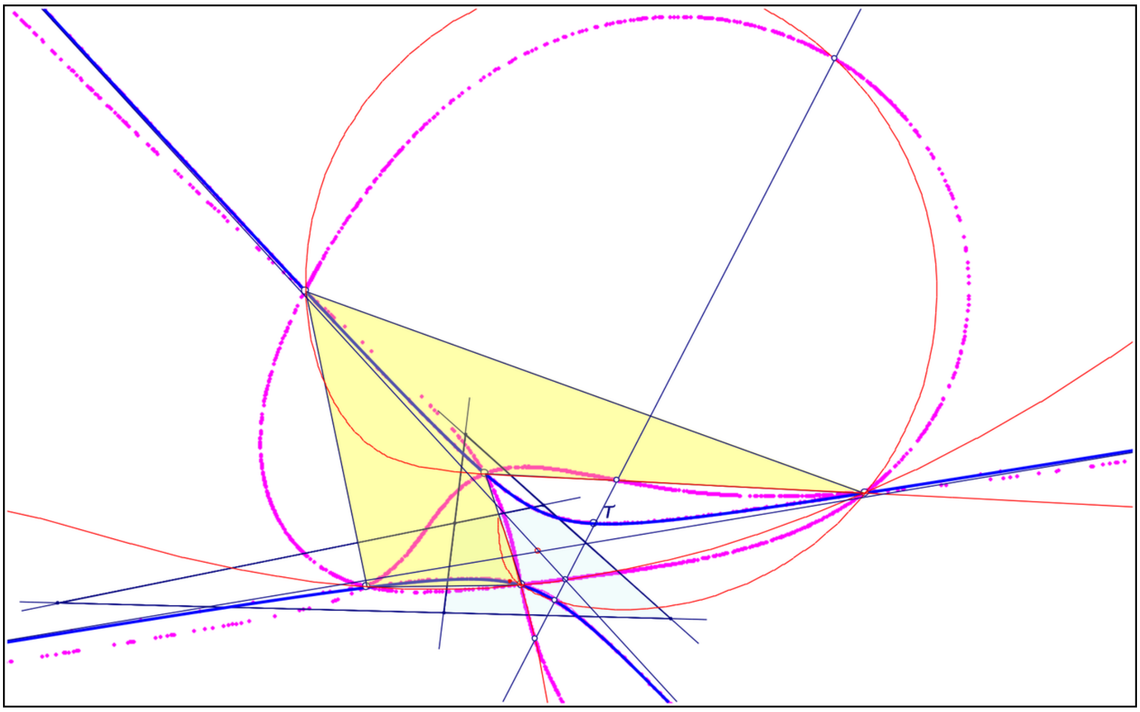
The 5P of the QA-Cu7-triple points seems to be
… a convex QG = P1P2P3P4 and an inner point P5.

Only 5P with this property shall now be considered:
… Looking for points X with cb(X) = X,
… we get three curves:
… one circumscribed QG (perhaps a quartic),
… two through one pair of oppsite QG-points
… … and an infinity point of 5P-s-Co1 (perhaps cubics).

Attached a drawing,
… using lines through T = 5P-s-P4 and its cb-image,
… intersecting in a 2nd point on 5P-s-Co1
  with cb-image at infinity
… and further intersections, which are cb-fixed.
Perhaps someone can confirm these observations.

Best regards Eckart

PS: What is the geometrical background for the property X =
cb(X)?



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Message: #3623
Date: 23/7/2019 10:15:19
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Eckart,
Waiting for your comments or new ideas, I try to find new properties.
The cubic described in message 3620 is is circular circumcubic of either the CSC of the 5P or the CSC of the QA-P4 ; both conics cut the cubic in a point Q or q and the asymptote is TQ or tq.
Q and q are CSC in the CSC of the QL of the QG-P1 and the CSC of the QA-P41.
The pivot of the cubic is the infinity point of TQ and tq ...
The point q seems to be the 9th CB point of the 5 CSC of the 5 P and the 3 QG-P1.
I found the same way the 9th CB point of the 5 P and the 3 QG-P1, but I can't identify this point.
By the way, you mentioned some time ago a circular circumcubic of the 5 P and the 3 QG-P1 (and their cb partners), which of course passes through this 9th CB point. Have you identified the pivot ?
Last, you mentionned also that for a point P, the Ci of the 2 cb partners on the lines through P pass through the point U and a 2nd point Q (and are centered on the perpendicular bisector of UQ).
For example, if P is the pivot above, the point Q is the 2nd intersection of the Ci(QG-P1). Have you identified this 2nd intersection ?
Last, can you identify the point P for which the point Q is R ?
Best regards
Bernard

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Message: #3624
Date: 23/7/2019 11:02:42
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

```
Dear Bernard, dear Chris,

excuse, a property is missing  for the two cubics,
… they bear the 5th inner point P5.

Best regards Eckart
```

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Message: #3625
Date: 23/7/2019 3:38:11
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

```
Dear Eckart,
Sorry, don't understand your definition!
As I already told you, the summer is particulary hot ...
cb-image of what?
intersections of what with what?
Waiting for a pedagogical explanation, many thanks in advance
for your patience !
Best regards
Bernard
```

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Message: #3626
Date: 24/7/2019 9:39:14
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

```
Dear Bernard,

I try, to describe my observations once more:
Consider special 5P with
… 4 points in the vertices of a convex QG = P1P2P3P4
… and an inner 5th point P5,
… further the cb-transformation, defined by this 5P.
… Looking for points X with cb(X) = X,
we get three curves:
… one circumscribed the QG (perhaps a quartic),
… two through P5 and a pair of opposite QG-points
… … and an infinity point of 5P-s-Co1 (perhaps cubics).
Wrt the drawing (see #3622):
Take lines L through T = 5P-s-P4,
… their cb-image cb(L) is a curve of higher degree,
… intersecting L in a point on 5P-s-Co1 with cb-image at
infinity,
… further intersections of L and cb(L) are fixed points of cb.

Best regards Eckart

PS: You are right: Hot summer time!
    I have the same problem with your message 3623.
```

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Message: #3627
Date: 24/7/2019 12:36:22
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

```
Dear Bernard,
You wrote in your message 3620:
"Only the point R which is the CSC of the inverse of QL-P1 in
the inversion with center QL-P10  . . . "
I understand from your message that R is the CSC of an inverse.
But the inverse wrt which circle?
Best regards,
Chris
```

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Message: #3628
Date: 24/7/2019 12:38:28
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

I examined the points T,U,W and their background.
They all come forth from points on QA-Cu7 in a QL-environment.
I reconsidered that and found some new points as well.

Here a summary of old and new points:
Point *R* = perspector of QA-P4-triangle and QG-P18-triangle.

Sa = QA-P41a.QG-P19a ^ QA-P4a.QG-P18a
Sb = QA-P41b.QG-P19b ^ QA-P4b.QG-P18b
Sc = QA-P41c.QG-P19c ^ QA-P4c.QG-P18c

Point T = perspector of QL-Tr1 and S-triangle.
Point U = perspector of QA-P4-triangle and QG-P19-triangle. Also CSC(QL-P24)

Va = QG-P18b.QG-P19b ^ QG-P18c.QG-P19c
Vb = QG-P18c.QG-P19c ^ QG-P18a.QG-P19a
Vc = QG-P18a.QG-P19a ^ QG-P18b.QG-P19b

Point W = perspector of QG-P1-triangle and V-triangle.

New:
Point X = perspector of QG-P1-triangle and QA-P41-triangle.
Point Y = perspector of QG-P1-triangle and QG-P19-triangle.
Point Z = perspector of QG-P19-triangle and V-triangle.

Special properties:
Sa, Sb, Sc, R and T lie on the Triple-Points-Conic (TPC).
QL-Tf1[U] = QL-P24
QL-Tf1[U] and QL-Tf1(R) both lie on QL-Ci1.
QL-Tf1(R) = the 2nd intersection point of QL-Ci1 ^ QL-P1.QL-P10 (1st is QL-P16)
QL-Tf11(T) = the line through QL-P1 parallel to QL-P3.QL-P4
QL-Tf11(X) = a line through QL-P26 (surprise, surprise)

T and X are isogonal conjugated wrt QL-Tr1.
Relationship with the 5 TriplePoints:
- QA-P16(Pi) wrt 4 pts of the 5 TriplePoints P(i) = QL-Tf1(Pi) wrt Ref-QL
- 5P-s-P4 wrt the TriplePoints-5P = T = perspector of QL-Tr1 and S-triangle
- 5P-s-P5 wrt the TriplePoints-5P = U = perspector

```
  of QA-P4-triangle and QG-P19-triangle
• 5P-s-P6 wrt the TriplePoints-5P = QL-P1 wrt Ref-QL

I wonder if the points R, W, X, Y, Z will have some
5P-relationship too.

Best regards,

Chris
```

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Message: #3629
Date: 24/7/2019 2:27:01
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

```
Dear Chris,

thanks for a further summary of our results.
The points X and Y are not new,
… they are W and V in message 3176.

Best regards Eckart
```

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Message: #3630
Date: 24/7/2019 3:05:09
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris,
It follows from the given definition that the inversion circle , centered in QL-P10, is orthogonal to the 3 circles with diameter QG-P1QG-P17 and to the circle with diameter QL-P1CSC(R).
Best regards
Bernard

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Message: #3631
Date: 24/7/2019 3:17:50
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris,
Again: T, U and QL-P1 can be defined independently either from the QL or from the 5P.
R can be defined only from the QL.
The cubic described in my message can also be defined either from the QL or the 5P.
It is possible to find by approximation the 3 points QG-P1 on the cubic such as the CSC of the circumcircle of these 3 points (through CSC(U) and CSC(R)) is the circle with diameter UR.
But it would be nice and more elegant to find a direct definition of R from the 5P !
Best regards
Bernard

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Message: #3632
Date: 24/7/2019 3:33:51
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

```
Dear Eckart,
Thanks for the explanation ! I understand now.
Unfortunately, as I'm not able to construct directly a cb
partner of a point X, I cannot reproduce your construction of
the curve of higher degree.
For X = cb(X), I can only guess that X is it's own tangential on
the circular circumcubic of the 5P with pivot X, which means
that the cubic is tangent in this point to the curve of higher
degree?
Best regards
Bernard
PS My message 3623 had 2 parts, the 1rst part concerning the
wunderful cubic and the 2nd being in fact composed of precise
questions about properties you mentionned before (intersection
of the 3 QG-P1cb(QG-P1) or point P such as all cb partners on
lines through P have the same 2nd intersection Q with different
examples ...). Please just tell me what was unclear
```

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Message: #3633
Date: 24/7/2019 5:23:27
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

```
Dear Eckart,
For this search, I don't see a difference between 5P on an
ellipse or an hyperbola.
Is there any?
Best regards
Bernard
```

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Message: #3634
Date: 24/7/2019 9:43:09
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

```
Dear Eckart,

I missed your message #3176 in my busy period.
However, point W was already named in message #3003.
For the sake of unity, I make a new list of these points with
adapted name-giving, following your list.
Point R = perspector of QA-P4-triangle and QG-P18-triangle.

Sa = QA-P41a.QG-P19a ^ QA-P4a.QG-P18a
Sb = QA-P41b.QG-P19b ^ QA-P4b.QG-P18b
Sc = QA-P41c.QG-P19c  ^ QA-P4c.QG-P18c

Point T = perspector of QL-Tr1 and S-triangle.
Point U = perspector of QA-P4-triangle and QG-P19-triangle.
          Also CSC(QL-P24)
Point V = perspector of QG-P1-triangle and QG-P19-triangle.
Point W = perspector of QG-P1-triangle and QA-P41-triangle.

Xa = QG-P18b.QG-P19b ^ QG-P18c.QG-P19c
Xb = QG-P18c.QG-P19c ^ QG-P18a.QG-P19a
Xc = QG-P18a.QG-P19a ^ QG-P18b.QG-P19b

Point Y = perspector of QG-P1-triangle and X-triangle.
Point Z = perspector of QG-P19-triangle and X-triangle.

Best regards,
Chris
```

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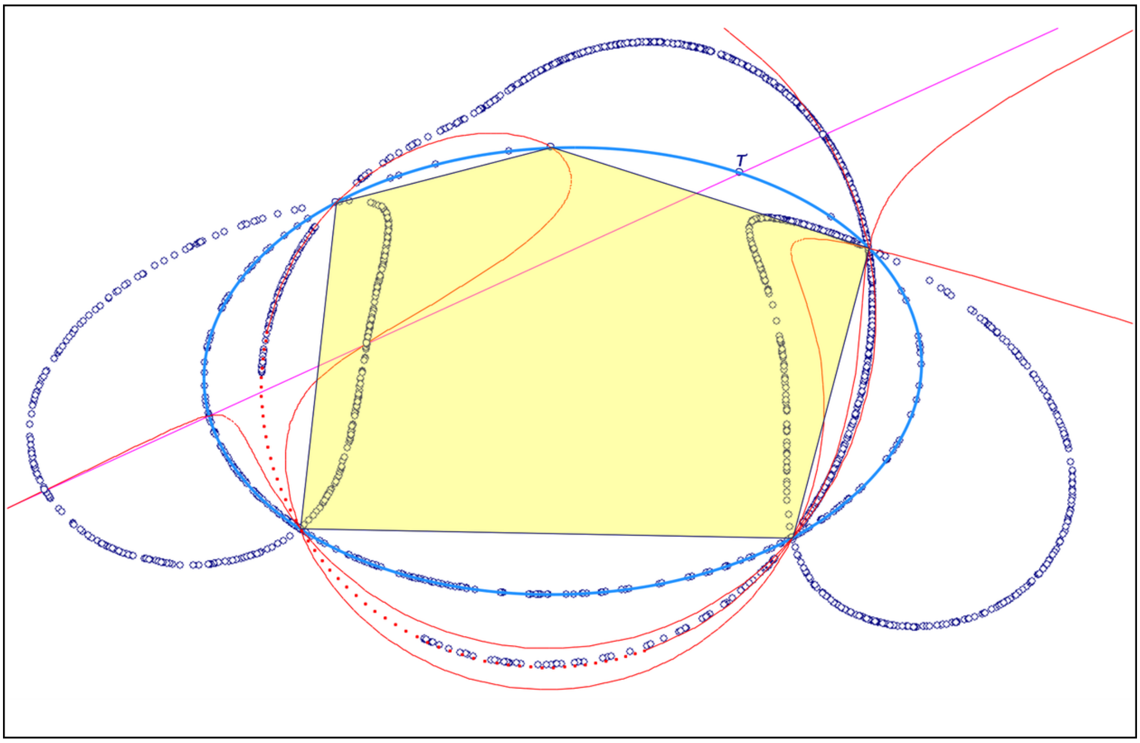
Message: #3635
Date: 25/7/2019 10:20:33
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard,

wrt #3633 there are significant other observations
… for the three curves for X = cb(X) wrt a convex 5P,
… have a look on the attached file.

Best regards Eckart

PS: Please have patient with me wrt #3623.



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Message: #3636
Date: 25/7/2019 11:38:16
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

```
Dear Chris,

thanks for the last summary.
If we study these numerous perspectivities,
… there will be further related examples
… with a concept, described in #2017:

Two perspective triangles A1B1C1, A2B2C2
have a third triangle A3B3C3
… with A3 = B1C2∩B2C1, B3 = A1C2∩A2C1, C3 = B1A2∩B2A1,
… perspective to A1B1C1 in P13 = A1A3∩B1B3∩C1C3,
… perspective to A2B2C2 in P23 = A2A3∩B2B3∩C2C3,
… with collinear perspectors,
… with a pivotal isocubic:
… … one triangle as reference triangle,
… … isoconjugation swapping the vertices
     of the other two triangles,
… … or: isoconjugation swapping the perspectors
     with the other two triangles,
… … pivot in the perspector of the other two triangles,
… … bearing the vertices of the three triangles
… … and their perspectors.

But I have not studied these possibilities.

Best regards Eckart
```

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Message: #3637
Date: 25/7/2019 2:08:26
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Wrt your last message: I considered exactly the same. Maybe some other wording.
All these mutually perspective triangles!
Having two perspective triangles it is possible to construct a third triangle like you noted.
This new set of three mutually perspective triangles also forms a desmic configuration with an associated desmic cubic.
See QA-Tr-1.

I found these items:
* The three triangles P18a.P18b.P18c, P19a.P19b.P19c, Xa,Xb,Xc form a desmic configuration with a degenerated desmic cubic, being the set of 3 lines: P18a.P19a, P18b.P19b, P18c.P19c.
* Per set of perspective triangles I found no associated desmic cubic passing through the Triple Points.
* I found no special properties for the desmic cubics, except:

Y lies on QG-P1xQG-P19 desmic cubic and of course also V
V lies on QG-P1xVa.Vb.Vc desmic cubic and of course also Y

Best regards,
Chris

Message: #3638
Date: 25/7/2019 7:18:26
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris, dear Eckart,
I know that repeating is the key of teaching, but I'm now tired of reading always the same properties and points presented with different names !
All these perspective triangles are by definition linked to the QL and won't give what I'm waiting for, a simple definition of R deriving uniquely from the 5P (like T, U, QL-P1 and the cubic merely mentionned).
Of course, it couls also be a circular circumcubic of the 5P through the 3 QG-P1 (and their cb partners) with pivot an unidentified point or the 2 cb partners having the circle with diameter UR as Ci(X) or the point such as the cb partners of all lines through it pass through U and R or any other idea giving the QG-P1 directly and uniquely from the 5P ...
Best regards
Bernard

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Message: #3639
Date: 25/7/2019 11:25:10
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Thanks for your essay in #3176. It was very helpful.
Bernard, I appreciate your desire for solving the location of R in a 5P-configuration.
Please have some patience with me. I come from far.

Best regards,
Chris

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Message: #3640
Date: 26/7/2019 9:51:55
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

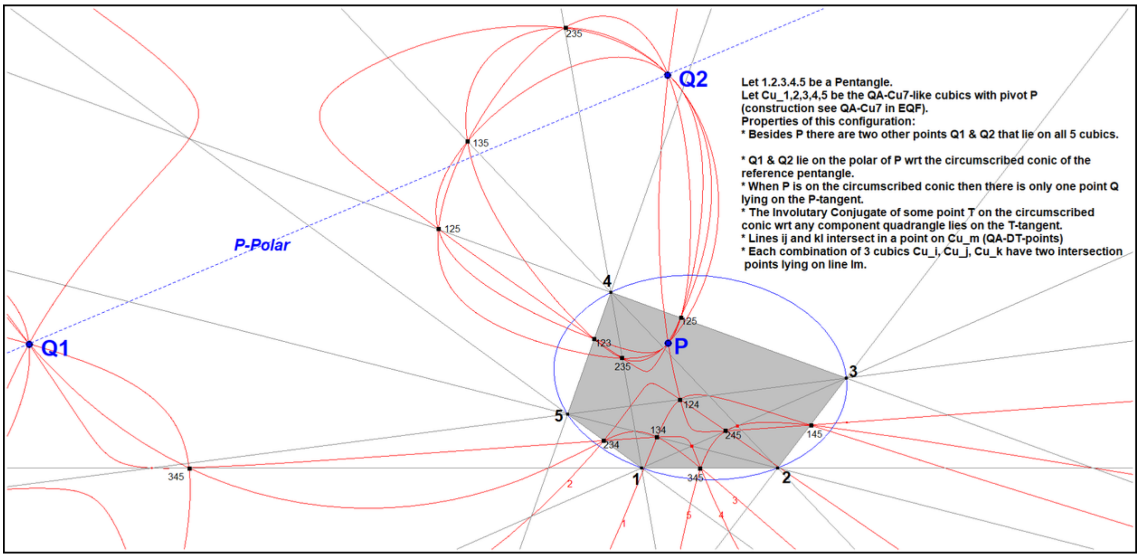
Searching for a 5P-construction of R, I found another interesting 5P-item.
A transformation mapping a random point P into 2 other points.
Short description:
Let 1.2.3.4.5 be a Pentangle.
Let Cu_1,2,3,4,5 be the QA-Cu7-like cubics with pivot P
Construction (see also QA-Cu-1 and QA-Cu7 in EQF):
Let P1, P2, P3, P4 be the vertices of the Reference Quadrangle.
Let V (u:v:w) be a variable point.
Let Lv be some line through V.
Let IC(Lv) be the Involutary Conjugate (QA-Tf2) of Line Lv.
IC(Lv) is a conic since QA-Tf2 is a transformation of the 2nd degree.
The locus of the intersection of IC (Lv) ^ the perpendicular at V to Lv is a QA-Cubic Type-2.

Properties of this configuration:
* Besides P there are two other points Q1 & Q2 that lie on all 5 cubics.
* Q1 & Q2 lie on the polar of P wrt the circumscribed conic of the reference pentangle.
* When P is on the circumscribed conic then there is only one point Q lying on the P-tangent.
* The Involutary Conjugate of some point T on the circumscribed conic wrt any component quadrangle lies on the T-tangent.
* Lines ij and kl intersect in a point on Cu_m (QA-DT-points)
* Each combination of 3 cubics Cu_i, Cu_j, Cu_k have two intersection points lying on line lm.

See picture.
Maybe it is helpful in some way.

Best regards,

Chris



5P-s-5-QA-Cu7-like-Cubics-network-00.png

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Message: #3641
Date: 27/7/2019 9:23:14
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

Regarding my last message.

The points Q1 and Q2 generated by a random point P wrt a 5P has
extra properties:
* The 5P-transformation mapping P into Q1 and Q2 is actually a
C0-transformation.

For a given point P the points Q1 and Q2 are identical for each
spanned Pentangle in a reference conic.
* The triangle PQ1Q2 is selfpolar
  wrt the 5P-circumscribed conic.
* The triangle PO1O2 is rectangular:
  PQ1 is perpendicular to PQ2.

Q1 and Q2 can be constructed as follows:
* Let 5P-Co be the 5P-circumscribed conic.
* Let Lp be the polar of P wrt 5P-Co.
* Let S be the inverse Scimemi Transformation (Co-Tf3 -1)
  of P wrt 5P-Co.
* Let Lb be the perpendicular bisector of PS.
* Let M be PS ^ Lp.
* Let Ci be the circle with center M through P.
* Now Q1 and Q2 are the intersection points of Ci and Lp.

Best regards,
Chris

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Message: #3642
Date: 27/7/2019 10:30:25
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

```
Dear Chris,

wrt #3640, # 3641:
you find your points Q1, Q2 as Q, R in #3445.
I shall study your properties.

Best regards Eckart
```

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Message: #3643
Date: 27/7/2019 10:42:26
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

```
Dear Chris,

I just noticed a typo in my message 3445:
Replace 5P-s-P3 by 5P-s-Tf3.

Best regards Eckart
```

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Message: #3644
Date: 27/7/2019 3:32:13
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

It is not the first time that we investigated the same subject and probably it won't be the last. Nice to look at our differences in findings.

Especially I like your property "circles with these pairs as diameter bear P".

5P-s-Tf3(P) in a 5P = Inverse[CO-Tf3(P)] in a CO.

Best regards,
Chris

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Message: #3645
Date: 28/7/2019 12:04:12
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart, Dear Bernard,

There is a 5P-CB-Transformation (to be named 5P-s-Tf6) dealing with Circular Cubics and a 7P-CB-Transformation (to be named 7P-s-Tf1), dealing with regular cubics.
* I found a description of 5P-s-Tf6 by Eckart in #3412. For a description in EPG is there more to say about this transformation? Because it is the tangential I suppose that 5P-s-Tf6[5P-s-Tf6[P]] <> P.
* What can be written in EPG about 7P-s-Tf1 ? I thought that in this case 7P-s-Tf1[7P-s-Tf1[P]] = P. Is this true?

Best regards,
Chris

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Message: #3646
Date: 28/7/2019 2:01:32
From: Tran Quang Hung
Subject: 6P-Geometry

Dear geometers,

I have found three concurrent lines with six arbitrary points on plane. Please see the attached file.

Let Ai, i = 1..6, be six any points on plane. Taking subscripts modulo 6, we denote, for i = 1.. 6, the intersection of the lines AiA(i+1) and A_(i+2)A_(i+3) by B_(i+3), and the second intersection of the circumcircles of triangles AiA(i+1)B(i+2) and A(i+1)A(i+2)B(i+3) by C(i+1), and the circumcenter of the triangle CiB(i+1)B(i+2) by Di. The lines D1D4, D2D5, and D3D6 are concurrent.

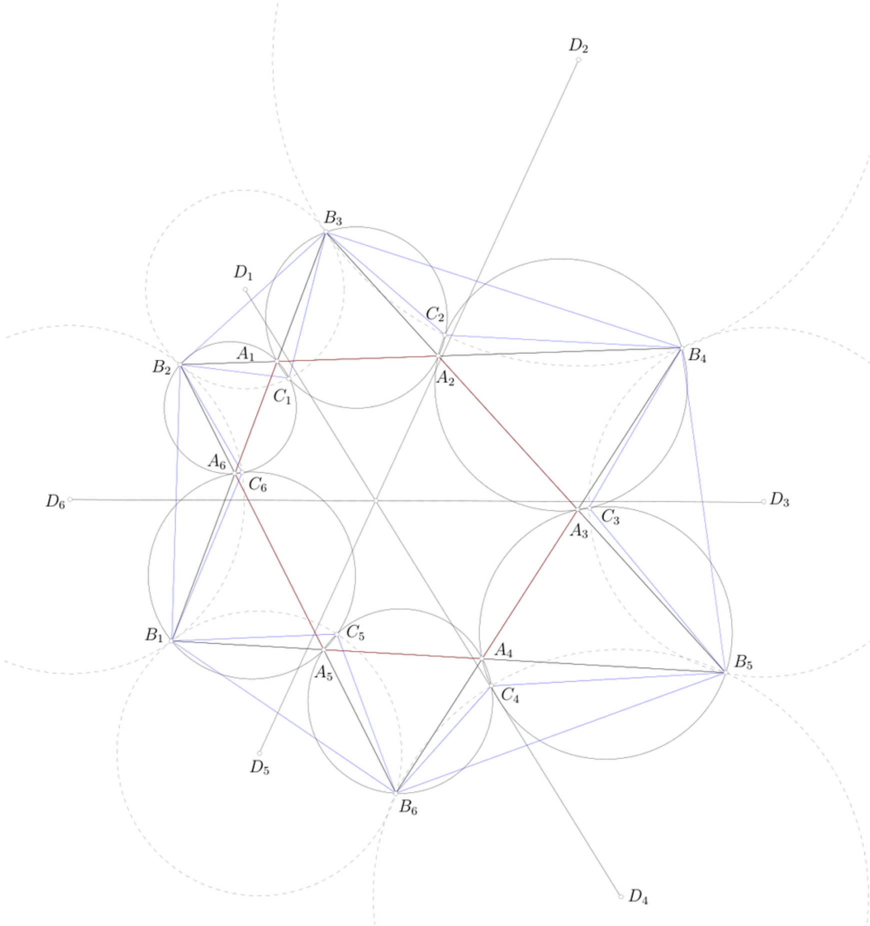
Best regards
Tran Quang Hung.

Concurrent lines in an arbitrary hexagon with twelve circles

Tran Quang Hung

July 12, 2019

Theorem (Tran Quang Hung)**.** Let A_i, $i = 1, 2, \ldots, 6$, be six points on plane. Taking subscripts modulo 6, we denote, for $i = 1, 2, \ldots, 6$, the intersection of the lines A_iA_{i+1} and $A_{i+2}A_{i+3}$ by B_{i+3}, and the second intersection of the circumcircles of triangles $A_iA_{i+1}B_{i+2}$ and $A_{i+1}A_{i+2}B_{i+3}$ by C_{i+1}, and the circumcenter of the triangle $C_iB_{i+1}B_{i+2}$ by D_i. The lines D_1D_4, D_2D_5, and D_3D_6 are concurrent.



derakynay1994.pdf

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Message: #3647
Date: 28/7/2019 1:11:29
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Chris,

wrt #3645:
Your 5P-s-Tf6 will be my transformation cb:
… For properties have a look in
… #3412, #3420, #3435, #3436, #3438, 3442, #3528 (1), #3575.

I think, it is a relevant transformations for 5P,
… for example: The image of the circle through 3 vertices
… is the line through the 2 remaining vertices.

Best regards Eckart
```

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Message: #3648
Date: 28/7/2019 10:27:28
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

```
Dear Bernard, dear Chris,

here is a new aspect of the point R,
background are Bernard's messages #3620, #3630.
Take the polar circle of QL-DT,
… then the inverse of QL-P1,
… whose CSC-image is R.
  R is the center of the CSC-circle
  ... of the polarcircle of QL-DT.

Best regards Eckart
```

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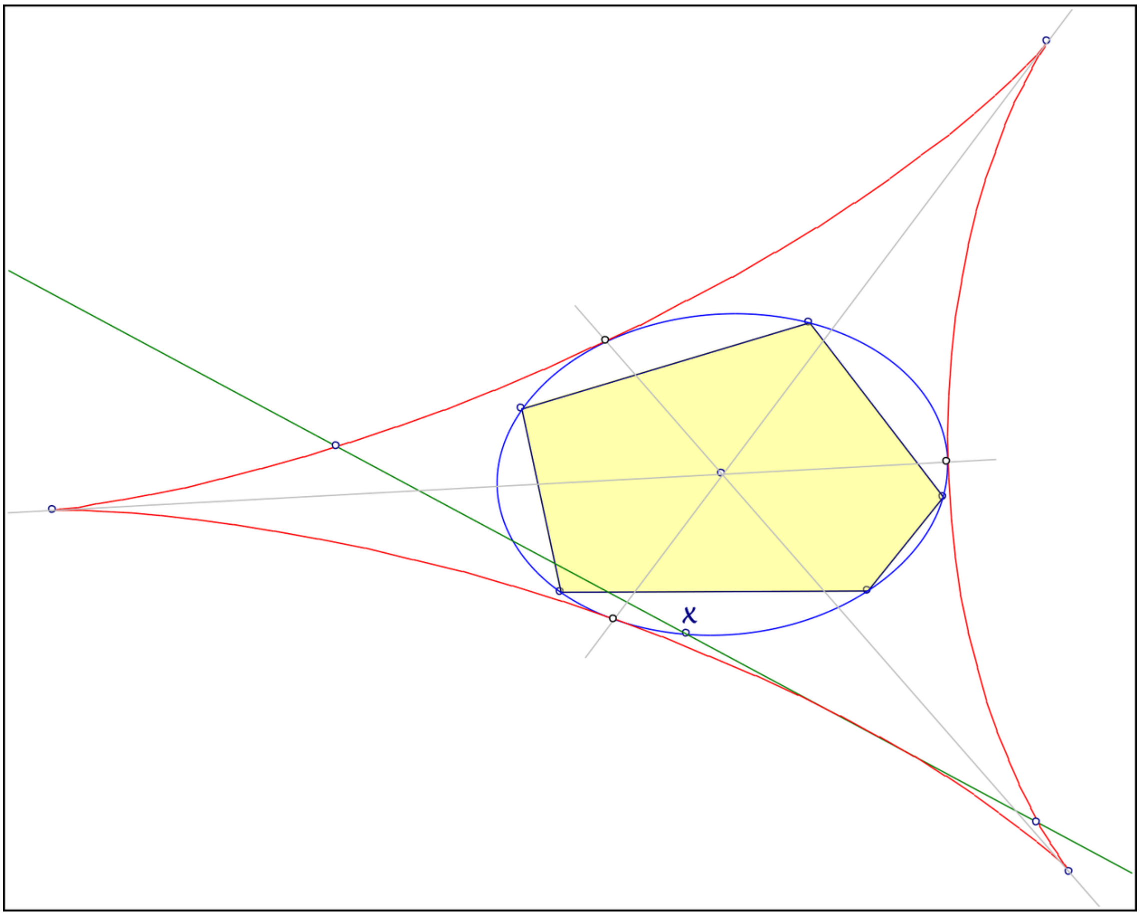
Message: #3649
Date: 30/7/2019 12:09:06
From: eckart_schmidt@t-online.de
Subject: Deltoid for a convex 5P

```
Dear Bernard, dear Chris,

just for a change:
Consider a *convex* 5P = P1...P5
… and for points X the 5L of the bisectors of X.Pi:
… For X on the circumconic 5P-s-Co1
… the lines X.5L-s-Tf1(X) envelop a deltoid,
… circumscribed 5P-s-Co1,
… cusps and contact point collinear 5P-s-P1,
… X midpoint of the intersections of X.5L-s-Tf1(X)
  and the deltoid.

Best regards Eckart
```



2019-07-30.pdf

Message: #3650
Date: 31/7/2019 4:02:48
From: Tran Quang Hung
Subject: 6P-Geometry

Dear geometers,

The concurrent point is generated from six any points, can it be
considered as a point of 6-points configuration? I wish You are
interested in this?

Best regards
Tran Quang Hung.

Vào CN, 28 thg 7, 2019 vào lúc 07:01 Tran Quang Hung <
analgeomatica@gmail.com > đã viết:
>> Dear geometers,
>> I have found three concurrent lines with six arbitrary points
on plane.
>> Please see the attached file.
>> Let Ai, i = 1..6, be six any points on plane. Taking
subscripts modulo 6,
>> we denote, for i = 1.. 6, the intersection of the lines
AiA(i+1) and
>> A_(i+2)A_(i+3) by B_(i+3), and the second intersection of the
>> circumcircles of triangles AiA(i+1)B(i+2) and
A(i+1)A(i+2)B(i+3) by
>> C(i+1), and the circumcenter of the triangle CiB(i+1)B(i+2)
by Di. The
>> lines D1D4, D2D5, and D3D6 are concurrent.
>> Best regards
>> Tran Quang Hung.

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Message: #3651
Date: 01/8/2019 2:28:48
From: chris.vantienhoven
Subject: 6P-Geometry

```
Dear Tran Quang Hung,

Nice construction of this point.
The point can be considered as a point of some 6-points
configuration.
It is not a point related to 6 random points (a Hexangle)
because the points have a certain order.
When you change the points P1 and P2 and do the same
construction, then you will find another point.
Therefore it is a Hexagon-point or 6-Gon-point.
See https://www.chrisvantienhoven.nl/epg-intro

Best regards,
Chris
```

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Message: #3652
Date: 01/8/2019 10:05:46
From: Tran Quang Hung
Subject: 6P-Geometry

Dear Chris,

Thank You very much for your interest. I now understand
6-Gon-point and Hexangle.
I found this theorem when I study Dao's theorem
http://forumgeom.fau.edu/FG2014volume14/FG201424index.html

Best regards
Tran Quang Hung.

Vào Th 5, 1 thg 8, 2019 vào lúc 07:28 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> Nice construction of this point.
>> The point
>> can be considered as a point of some 6-points configuration.
>> It is not a point related to 6 random points (a Hexangle)
because the
>> points have a certain order.
>> When you change the points P1 and P2 and do the same
construction, then
>> you will find another point.
>> Therefore it is a Hexagon-point or 6-Gon-point.
>> See https://www.chrisvantienhoven.nl/epg-intro
>> Best regards,
>> Chris

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Message: #3653
Date: 01/8/2019 10:24:34
From: Antreas Hatzipolakis
Subject: ”Familiar names for Polygons or n-Gons when n>5 a ”

Dear Chris

You have listed the names of polygons here (
https://www.chrisvantienhoven.nl/epg-intro) |

For your information:
John H. Conway and I have named the polygons according to
ancient Greek,

See
http://mathforum.org/dr.math/faq/faq.polygon.names.html

APH

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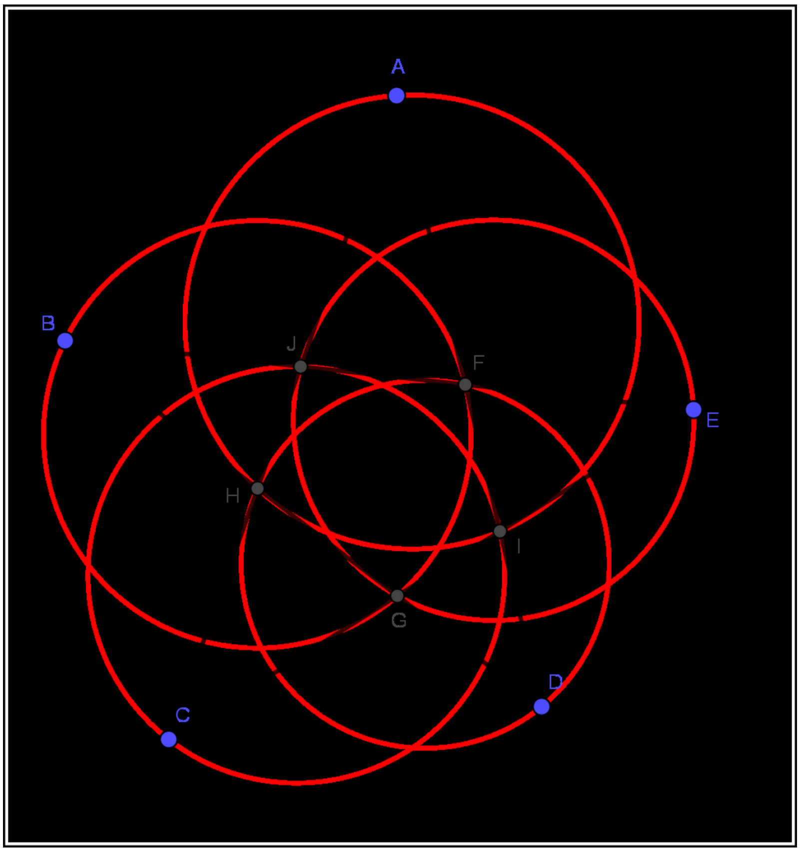
Message: #3654
Date: 01/8/2019 11:43:34
From: Tran Quang Hung
Subject: On a 5-Gon point

Dear geometers,

Base on 5G-s-P2 and problem of Telv
https://artofproblemsolving.com/community/q2h1813119p12086815

I proposed a problem as in figure.
Theorem. Let ABCDE be a pentagram, then five red circles (as in figure attached) have the same radical center.
I don't think that my problems and 5G-sP2 are the same because the circle in 5G-s-P2 have a common vertex of pentagram, but mine is not. Could you please have me to check this?

Best regards
Tran Quang Hung.



Plane13.png

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Message: #3655
Date: 01/8/2019 11:56:03
From: chris.vantienhoven
Subject: ”Familiar names for Polygons or n-Gons when n>5 a ”

Dear Antreas,

Thanks for your extended list.
I made a reference to it in the EPG-intro (
https://www.chrisvantienhoven.nl/epg-intro).
I saw that in the extended list
polygons en polyhedra’s were named according to ancient Greek.
Actually not only -GONS en -HEDRA’S should use these prefixes,
but also -ANGLES and -LATERALS.
For example:
* A Pentagon = a bounded figure consisting of 5 points connected
with 5 line(-segment)s. (consequently the points have a fixed
order)
* A Pentangle = a figure consisting of 5 random points without
order
* A Pentalateral = a figure consisting of 5 random lines without
order
Again this applies for all numbers, there is a Hexagon, Hexangle
and a Hexalateral, there is a Heptagon, Heptangle and a
Heptalateral, etc.

These differences are also explained in EPG-intro (
https://www.chrisvantienhoven.nl/epg-intro).
The distinction between -gons/-angles/-laterals is necessary
because these basic figures / configurations represent
completely different geometries.
This distinction is not new.

For example Coxeter and Greitzner wrote in their book “Geometry
Revisited” (page 52):
Obviously we should discourage the tendency to call a quadrangle
a "quadrilateral". In projective geometry, where the sides are
whole lines instead of mere segments, we need both terms with
distinct meanings.

Best regards,

Chris

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Message: #3656
Date: 01/8/2019 4:46:40
From: chris.vantienhoven
Subject: On a 5-Gon point

```
Dear Tran Quang Hung,

I like your new point, because it is so simple, just one level
of construction.
You are right, this new point isn't QG-s-P2.
But special is that it is collinear with QG-s-P2 and QG-s-P3.
Are you able to prove this?

Best regards,

Chris
```

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Message: #3657
Date: 01/8/2019 4:56:00
From: eckart_schmidt@t-online.de
Subject: On a 5-Gon point

```
Dear Tran Quang Hung,

the radical center of the 5 red circles isn't 5G-s-P2.

Best regards Eckart
```

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Message: #3658
Date: 01/8/2019 5:38:16
From: Tran Quang Hung
Subject: On a 5-Gon point

```
Dear Chris and Eckart,

Thank You so much for your comments. I had found a related paper
since 2014

https://www.jstor.org/stable/10.4169/math.mag.87.1.44?seq=1#page
_scan_tab_contents

I also hope this my construction is not direct consequence of
5G-s-P2?

Best regards
Tran Quang Hung.

Vào Th 5, 1 thg 8, 2019 vào lúc 22:00 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> the radical center of the 5 red circles isn't 5G-s-P2.
>> Best regards Eckart
```

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Message: #3659
Date: 02/8/2019 4:23:06
From: Tran Quang Hung
Subject: Perspective triangles in 6-Gon point

```
Dear geometers,

Let ABCDEF be a hexagon.
The principal diagonals AD, BE, CF meet bound triangle PQR.
The perpendicular bisector of AD, BE, CF meet the side of ABCDEF
that created the hexagon UVXYZW.
The perpendicular bisectors of principal diagonals UY, VZ, XW
meet bound triangle EFG.
Then triangles PQR and EFG are perspective. Is this concurrent
point known before?
Please see the attached figure.

Best regards
Tran Quang Hung.
```

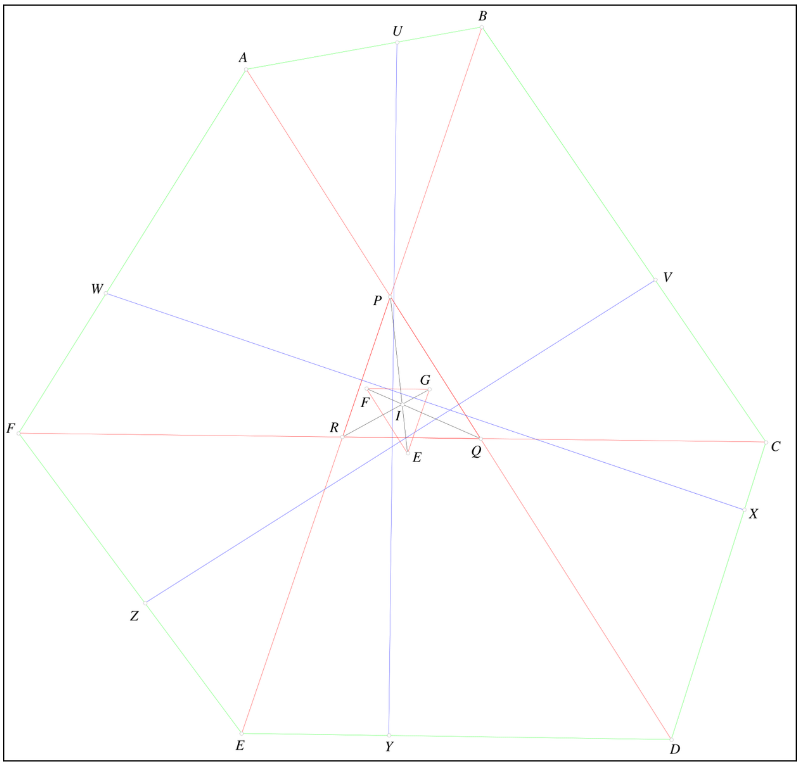

Figure7381.pdf

Message: #3660
Date: 02/8/2019 12:17:28
From: bernard.keizer
Subject: New 5P transformation

Dear Chris, dear Eckart, dear Tran Quang Duong,

The 5P define a transformation CSC1, which is the same as the QL's CSC.
The QL-Cu1 of the QL formed by the QG-P1 and the 3 CSC(QG-P41) is a Van Rees cubic with focus QL-P24 ; this cubic contains the CSC of the 5P and the CSC of the QA-P4 of the 5 QA 's of the 5P.
The cubic is invariant in a 2nd CSC2 with center QL-P24.
The CSC of the cubic is a quartic through the 5P and the 5 QA-P4 of the 5 QA's ; it contains the QA-P4 and QA-P41 of the initial QL as well as the points U and QL-P1.
The quartic is invariant in the transformation CSC1*CSC2*CSC1, which swaps the QA-P4 and QA-P41 of the QL, the 5P and the QA-P4 of their QA's and the point U and the point QL-P1.
The middle of the segment joining 2 conjugate points on the quartic lies on the circle described by Tran Quang Duong and by Eckart ; it's a generalisation to the quartic of the Newton Line for the QL's cubic.
This transformation seems to be a 3rd CSC3 with center CSC(T) and 2 unidentified fixed points (symmetric wrt CSC(T)).

Best regards
Bernard

PS I hope Eckart will confirm these observations and find plenty of other properties ...

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Message: #3661
Date: 02/8/2019 2:15:03
From: Tran Quang Hung
Subject: Two cyclologic triangles in QA

Dear geometers,

I see that QA-Tr2 and QA-Tr1 are cyclologic, one of cyclologic center is QA-P3, which is another cyclologic center?

Best regards
Tran Quang Hung.

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Message: #3662
Date: 02/8/2019 7:05:54
From: chris.vantienhoven
Subject: Two cyclologic triangles in QA

Dear

Tran Quang Hung,
The other cyclologic center is QA-P41
QA-Tr1 is formed of the three QA-versions of QG-P1.
QA-Tr2 is formed of the three QA-versions of QL-P1.
These kinds of triangles are called “triple triangles”. See QA-Tr-1.
You can find a list of cyclologic centers at QA-Tr-4 of all kind of triple triangles.

Best regards,
Chris

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Message: #3663
Date: 02/8/2019 8:26:21
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

very interesting #3660, not only a good summary,
… but also new observations wrt the CSC-transformations,
… which are new for me.

I can confirm all results with CABRI observations,
… adding for CSC3 with center CSC(T)
… the fixed points: necessarily CSC of the fixed points of CSC2.

Best regards Eckart

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Message: #3664
Date: 02/8/2019 11:30:31
From: chris.vantienhoven
Subject: Perspective triangles in 6-Gon point

Dear

Tran Quang Hung,

The point is not known by me.
This point and the point you introduced before are the first
6-Gon-points I encountered.
I am sure there will be more.
There is a lot of uncharted territory here.
Lots of new points to be found relating to n-Gons!
I am waiting for the first 7-Gon-point.

Best regards,
Chris

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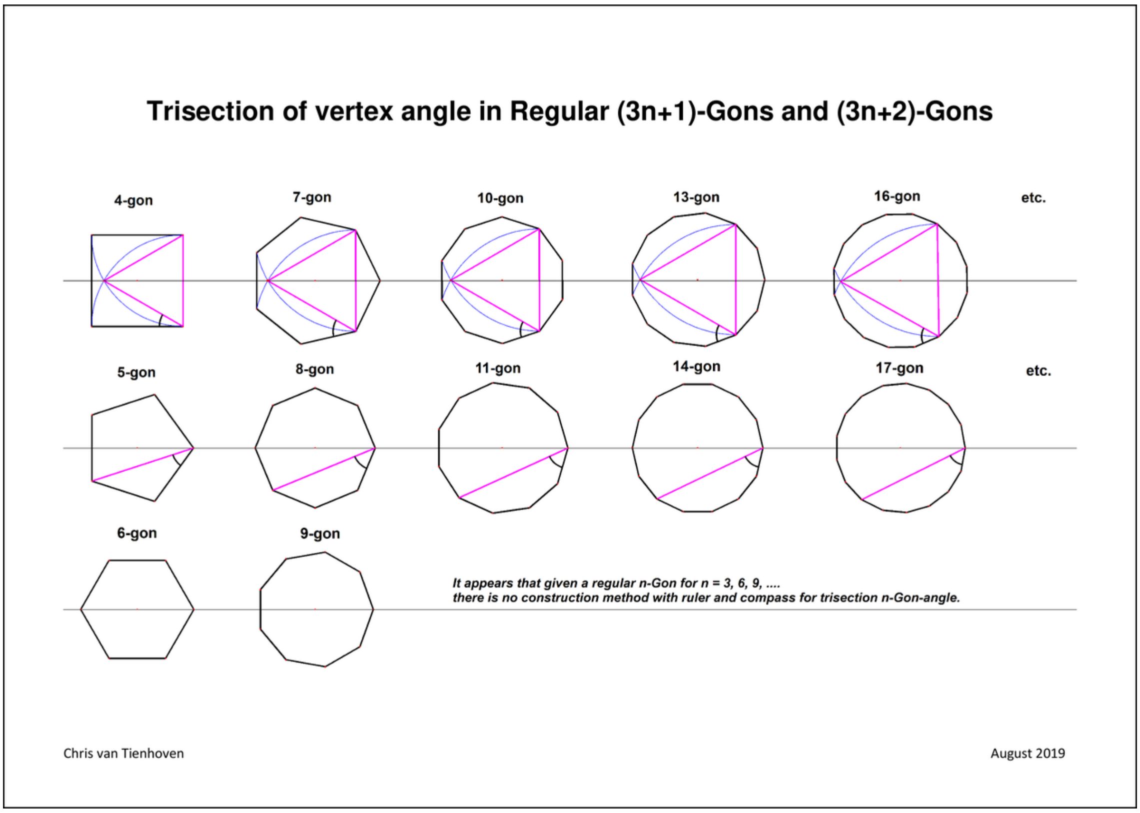
Message: #3665
Date: 02/8/2019 11:58:05
From: chris.vantienhoven
Subject: trisecting the vertex angle of a regular n-Gon

```
Dear friends,

There was a discussion in the Advanced Plane Geometry Group
ADGEOM.  †)
I think there will be interest in our group too for this
problem.

Bui Quang Tuan gave a very nice method for trisecting the vertex
angle of a regular heptagon.
I wondered if his method could be generalized. See the attached
picture for my generalization.
The method showed for the 7-Gon in the above row is Bui Quang
Tuan's initial solution.

Best regards,
Chris
```



n-Gon–Trisection in Regular n-Gons-03.pdf

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #3666
Date: 03/8/2019 5:19:25
From: Tran Quang Hung
Subject: Perspective triangles in 6-Gon point

Dear Chris and geometers,

Thank you very much for your encouragement. I am really
interested in EQF and EPG. I shall study them much more.

Best regards
Tran Quang Hung.

Vào Th 7, 3 thg 8, 2019 vào lúc 04:30 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> The point is not known by me.
>> This point and the point you introduced before are the first
6-Gon-points
>> I encountered.
>> I am sure there will be more.
>> There is a lot of uncharted territory here.
>> Lots of new points to be found relating to n-Gons!
>> I am waiting for the first 7-Gon-point.
>> Best regards,
>> Chris

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Message: #3667
Date: 03/8/2019 5:38:49
From: Tran Quang Hung
Subject: Two cyclologic triangles in QA

Dear Chris

Thank you for your interest and new information. I shall study EQF much more.

Best regards
Tran Quang Hung.

Vào Th 7, 3 thg 8, 2019 vào lúc 00:07 van10hoven@gmail.com [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Tran Quang Hung,
>> The other cyclologic center is QA-P41
>> QA-Tr1 is formed of the three QA-versions of QG-P1.
>> QA-Tr2 is formed of the three QA-versions of QL-P1.
>> These kinds of triangles are called "triple triangles". See QA-Tr-1
>> You can find a list of cyclologic centers at QA-Tr-4
>> of all kind of triple triangles.
>> Best regards,
>> Chris

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Message: #3668
Date: 03/8/2019 5:12:07
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,
is the following property already mentioned?
Your CSC2 in #3660 with center QL-P24 swaps QL-P1 and T.
Then your CSC1 and CSC2 can be constructed, starting with a 5P.
Now we can find QA-P4 and QA-P41 on the quartic:
… Let X be a point on the quartic
… and CSC1(X) = X1, CSC2(X1) = X2, CSC1(X2) = X3.
… If X, X1, X2 collinear, then X = QA-P4,
… if X1, X2, X3 collinear, then X = QA-P41.

Best regards Eckart

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Message: #3669
Date: 03/8/2019 8:40:03
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

excuse, the 2nd passage of my last message 3668 is not correct
… and has tobe specified in a next message,
… but the first passage should hold!

Best regards Eckart

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Message: #3670
Date: 03/8/2019 9:54:31
From: chris.vantienhoven
Subject: New 5P transformation

Dear Bernard,

You write: “The QL-Cu1 of the QL formed by the QG-P1 and the 3 CSC(QG-P41) is a Van Rees cubic with focus QL-P24; this cubic contains the CSC of the 5P and the CSC of the QA-P4 of the 5 QA 's of the 5P.”
I suppose you mean the 3 CSC(QA-P41).
I constructed these 3 points and it appeared they are collinear, so I was not able to construct the QL-Cu1 of QG-P1 and the 3 CSC(QA-P41).
Did I misunderstand something?

Best regards,

Chris

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Message: #3671
Date: 04/8/2019 2:22:34
From: Tran Quang Hung
Subject: A tranform on QA with two lines

Dear geometers

Using projective transformation, we shall have a transformation with a line and a point as following
Let A,B,C,D be four points and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Mab, Mcd, Mac, Mdb, Mad, Mbc.
P is a random point.
PMab meets CD at Pcd.
PMcd meets AB at Pab.
Define similarly the points Pac, Pdb, Pad, Pbc.
Then the lines PabPcd, PacPdb, PadPbc are concurrent.

Best regards
Tran Quang Hung.

Vào CN, 4 thg 8, 2019 vào lúc 06:23 Tran Quang Hung <
analgeomatica@gmail.com > đã viết:
>> Dear geometers,
>> When I study the transformation of Quang Duong, I see this concurrent lines.
>> Let A,B,C,D be four points and r a random line.
>> Line r intersects AB, CD, AC, DB, AD, BC at Mab, Mcd, Mac, Mdb, Mad, Mbc.
>> d is another random line.
>> Pab, Pcd lie on CD, AB such that MabPab // d, McdPcd // d.
>> Define similarly the points Pac, Pdb, Pad, Pbc.
>> Then the lines PabPcd, PacPdb, PadPbc are concurrent.
>> Best Regards
>> Tran Quang Hung.

Message: #3672
Date: 04/8/2019 3:38:28
From: Tran Quang Hung
Subject: A tranform on QA with two lines

```
Dear geometers,

When I study the transformation of Quang Duong, I see this
concurrent lines.
Let A,B,C,D be four points and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Mab, Mcd, Mac, Mdb,
Mad, Mbc.
d is another random line.
Pab, Pcd lie on CD, AB such that MabPab // d, McdPcd // d.
Define similarly the points Pac, Pdb, Pad, Pbc.
Then the lines PabPcd, PacPdb, PadPbc are concurrent.

Best Regards
Tran Quang Hung.
```

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Message: #3673
Date: 04/8/2019 6:04:56
From: Tran Quang Hung
Subject: Another transformation on QA with two lines

```
Dear geometers,

When I study the transformation of Quang Duong
https://chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf6
, I see the following transformation.
Let A,B,C,D be four points and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Mab, Mcd, Mac, Mdb,
Mad, Mbc.
d is another random line.
d meets perpendicular bisectors of segments MabMcd, MacMdb,
MadMbc at P1, P2, P3.
The circle (P1) center at P1 and passing through Mab, Mcd, (P1)
meets AB, CD again at Pab, Pcd.
Define similarly the points Pac, Pdb, Pad, Pbc.
Then the lines PabPcd, PacPdb, PadPbc are concurrent.
When d=r, we get the transformation QA-Tf6 ( http://www.chrisvan
tienhoven.nl/qa-items/qa-transformations/qa-tf6 ).

Best Regards
Tran Quang Hung.
```

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Message: #3674
Date: 04/8/2019 10:42:14
From: bernard.keizer
Subject: New 5P transformation

```
Dear Chris,
Of course, it was CSC(QA-P41); my apologise for the typo
These 3 points are aligned, which gives the 4th line of the QL,
the 3 others being the DT sides!
Hence the construction of the QL-Cu1 of this QL (the point QL-P1
being QL-P24) ...
Best regards
Bernard
PS The circle circumscribed to the 3 QA-P41 passes through QL-P1
and through W
```

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Message: #3675
Date: 04/8/2019 11:03:50
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

```
Dear Chris,

wrt your #3670:

You can construct the described cubic as QL-Cu1
… starting with a 5P and its CSC:
… using CSC(5P-s-P5) = CSC(U) as QL-P1,
… CSC  swapping 5P-s-P6 = QL-P1 and 5P-s-P4 = T,
… QL-L1 line through the midpoints of CSC(Pi) and
  CSC(CSC(Pi)).
The CSC-image of this cubic is the discussed quartic.

Best regards Eckart
```

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Message: #3676
Date: 04/8/2019 11:10:57
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Thanks for these precisions!
CSC1 can be constructed from the 5P.
CSC2 can also be constructed from the 5P, as we have 5 copples of CSC2 partners (the CSC(Xi) and the CSC(QA-P4 of the 5 QA's of the Xi).
CSC3 can also be constructed from the 5P, as we know the center CSC(T) and the 2 fixed points, CSC1 of the fixed points of CSC2, as you mentionned.
For your next property, I suppose you try to use the alignment of QG-P1, QA-P4 and CSC(QA-P41) ...
Best regards
Bernard

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Message: #3677
Date: 04/8/2019 6:32:19
From: Tran Quang Hung
Subject: A transformation to parallel line and perspective triangle on QA

Dear geometers,

Let A,B,C,D be four points and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Rab, Rcd, Rac, Rdb, Rad, Rbc, reps.
Let Mab, Mcd, Mac, Mdb, Mad, Mbc be the midpoints of AB, CD, AC, DB, AD, BC, reps.
Let Pab, Pcd, Pac, Pdb, Pad, Pbc be the reflections of Rab, Rcd, Rac, Rdb, Rad, Rbc in the points Mab, Mcd, Mac, Mdb, Mad, Mbc.

Then
- Midpoints of three segments PabPcd, PacPdb, PadPbc are collinear on line d. Also d is parallel to r.
- Three lines PabPcd, PacPdb, PadPbc form a triangle which is perspective to QA-Diagonal Triangle, which is the perspector?

Best regards
Tran Quang Hung.

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Message: #3678
Date: 04/8/2019 7:03:01
From: Tran Quang Hung
Subject: Six concyclic Poncelet points on QL

Dear geometers,

Let d1, d2, d3, d4 be a QL.
A12 = d1 meets d2. Similarly we have A13, A14, A23, A24, A34.

Let
P1 = Poncelet(A13,A14,A23,A24)
P2 = Poncelet(A12,A14,A23,A34)
P3 = Poncelet(A12,A13,A24,A34)

Let Q12 = Poncelet (A12,P1,P2,P3)
Define similarly, we have the points Q13, Q14, Q23, Q24, Q34.
Then six points Q12, Q13, Q14, Q23, Q24, Q34 are concyclic on a circle. Which is this circle of QL?

Best regards
Tran Quang Hung.

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Message: #3679
Date: 05/8/2019 11:56:13
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

```
Dear Bernard, dear Chris,

in #3675 I described a possibility,
… to get the cubic/quartic wrt the QA-Cu7-triple points,
… starting with a 5P and its CSC = CSC1,
… using a construction of a QL-Cu1 with QL-P1,
  CSC and QL-L1.

If this construction isn't parat,
… use parallels L of "QL-L1" and take the intersections
… of CSC(L) and the reflection of L in QL-L1.

As Bernard mentioned, we have 3 CSC-transformations:
… CSC1 = 5P-CSC-transformation (see #3583),
… … mapping the cubic to the quartic,
… CSC2, centered in CSC1(5P-s-P5),
… … swaps 5P-s-P4 and 5P-s-P6,
… … maps the cubic to itself,
… CSC3 = CSC1.CSC2.CSC1, centered in CSC1(5P-s-P4),
… … swaps 5P-s-P6 and 5P-s-P5,
… … maps the quartic to itself.

Now we can search QA-P4 and QA-P41 on the quartic:
… Let X be a point on the quartic
… … and CSC1(X) = X1, CSC2(X1) = X2, CSC1(X2) = X3.
… If x = X2 and X1 = X3, they are two intersections of cubic and
quartic.
… If X = QA-P4: X, X1, X2 collinear, bearing X1 = QG-P1,
  X3 = QA-P41.
… If X = QA-P41: X1, X2, X3 collinear, bearing X2 = QG-P1,
  X3 = QA-P4.

But these properties are not sufficient ...
No clearness, but a question:
… Are the QA-P4 tangentials of 5P-s-P5 wrt the quartic?

Best regards Eckart
```

Message: #3680
Date: 05/8/2019 3:33:45
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Wunderbar ! Weihnachten in August!
All this is perfectly clear, but I think the most important thing in your message is the last conjecture !
I think it holds that the 3 QA-P4 are the tangentials of U on the quartic.
As the CSC1,2 and 3 and the cubic and quartic are defined from the 5P, it gives at last a simple definition of R as the diametral point of U on the circumcircle (through U) of the 3 QA-P4!
Many thanks for this last discovery!
I suppose the game is over now ...
I'm able now to draw the twin QL's having the 5P or their CSC as QA-Cu7 triple points!
Best regards
Bernard
PS It remains a few details, like to understand why it works for an hyperbola and not for an ellipse

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Message: #3681
Date: 05/8/2019 5:44:59
From: César E. Lozada
Subject: A transformation to parallel line and perspective triangle on QA

Dear Mr Hung,

Perspector for line (x:y:z):
(CT)
Q = ((q+r)*(2*p+q+r)*q*r*x-(q^2+(p+2*q+2*r)*p+(q+r)^2)*p*r*y-(r^
2+(p+2*q+2*r)*p+(q+r)^2)*p*q*z)*((2*(p+q+r)*p+(q+r)^2)*x+p*q*y+p
*r*z)

Best regards,
César Lozada

De:
Quadri-Figures-Group@yahoogroups.com
Enviado el: domingo, 4 de agosto de 2019 12:32 PM
Para: Quadri-Figures-Group@yahoogroups.com
Asunto: [Quadri-Figures-Group] A transformation to parallel line and perspective triangle on QA
> Dear geometers,
> Let A,B,C,D be four points and r a random line.
> Line r intersects AB, CD, AC, DB, AD, BC at Rab, Rcd, Rac, Rdb, Rad, Rbc, reps.
> Let Mab, Mcd, Mac, Mdb, Mad, Mbc be the midpoints of AB, CD, AC, DB, AD, BC, reps.
> Let Pab, Pcd, Pac, Pdb, Pad, Pbc be the reflections of Rab, Rcd, Rac, Rdb, Rad, Rbc in the points Mab, Mcd, Mac, Mdb, Mad, Mbc.
> Then
> - Midpoints of three segments PabPcd, PacPdb, PadPbc are collinear on line d. Also d is parallel to r.
> - Three lines PabPcd, PacPdb, PadPbc form a triangle which is perspective to QA-Diagonal Triangle, which is the perspector?
> Best regards
> Tran Quang Hung.

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Message: #3682
Date: 06/8/2019 1:19:36
From: chris.vantienhoven
Subject: A tranform on QA with two lines

Dear Quang Hung,
Let me first start with your *first Transformation* QA-Tfx(L,P)
described by you in QFG #3671.
You mentioned it before in #2099.
Properties:
* It is a linear transformation.
* Regardless of the position of line L, the point QA-Tfx(L,P)
  will lie on the 5th Point tangent QA-Tf9(P).
* For extra properties, see QFG#2101, #2103, #2122
Your 2nd transformation in #3672 is the same as #3671.
Only instead of finite point P you use the infinity point of 2nd
line r.
Your 3rd transformation in #3673 gives a pretty long expression.
It maps 2 lines L1 & L2 into a point X.
Properties/Incidents:
* When L1 *or* L2 are the Line at Infinity, then X = an infinity
point.
* L1=Line at Infinity, L2=QA-L2,
  then X = Infinity Point (QA-L9) = QA-Tf2 (QA-P2)
* L1=Line at Infinity, L2=QA-L7,
  then X = Infinity Point (QA-P12.QA-P32) = Infinity Point
  (QA-P14.QA-P37)
* L1=Line at Infinity, L2=QA-L9,
  then X = Infinity Point (QA-L2) = Infinity Point
  (QA-P1.QA-P32)
* L2=Line at Infinity, L1=QA-L3,
  then X = Infinity Point (QA-L7) = QA-P9041 = InfinityPoint of
  the line
  through the 3 QL-P18 points in a Quadrangle
* L2=Line at Infinity, L1=QA-L4,
  then X = Infinity Point (QA-L9) = QA-Tf2 (QA-P2)
* L2=Line at Infinity, L1=QA-L7,
  then X = Infinity Point (QA-L3) = Infinity Point (QA-P1.QA-P5)
* L2=Line at Infinity, L1=QA-L9,
  then X = Infinity Point (QA-L4) = QA-Tf2 (QA-P3)
* L1=QA-L2, L2=QA-L2, then X = QA-P2
* L1=QA-L9, L2=QA-L9, then X = QA-P23
* L1=QA-L5, L2=QA-L5, then X = point on QA-Ci1
* L1=QA-L2, L2=QA-L9, then X = some infinity point
* L1=QA-L9, L2=QA-L2, then X = QA-Tf2(QA-P2)
Best regards,
Chris

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Message: #3683
Date: 06/8/2019 11:37:45
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

```
Dear Bernard,

now I can construct the cubic and the quartic,
… starting with a QL or a 5P, CABRI-macros are saved!
You say "game over", but:
… How do you find the QA-P4 on the quartic and get the QL?

My observations:
The point U = 5P-s-P5 can have up to six tangents to the
quartic.
If all are real:
… two tripel of contact points, each concyclic with U,
… one tripel gives the QA-P4.

If 5P is convex,
… cubic and quartic can be constructed, but:
… there are no three contact points concyclic with U.
If 5P is not convex,
... there are not always three contact points concyclic with U.
Are the 5P of the triple points special nonconvex 5P?

Best regards Eckart
```

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Message: #3684
Date: 06/8/2019 12:16:43
From: bernard.keizer
Subject: New 5P transformation

```
Dear Eckart,
I suppose we have to study now the circular quartics !
This point U seems to be a focus like QL-P1 for the circular
focal Van Rees cubic.
This focus is the intersection point of the tangents in the
QA-P4, in U and in the circular points (hence the circle).
With the little circle playing the role of the Newton Line, the
circle of your points V1, 2 and 3 (through QL-P1 and T) plays
the role of the asymptote !
But unfortunately, my knowledge in the quartics equals to zero !
Have you any idea of where to start ?
Best regards
Bernard
```

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Message: #3685
Date: 06/8/2019 1:00:59
From: chris.vantienhoven
Subject: New 5P transformation

```
Dear Benard,
I can't wait to see how you construct the
twin QL's having the 5P.
Best regards,
Chris
```

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Message: #3686
Date: 06/8/2019 1:13:17
From: Tran Quang Hung
Subject: A tranform on QA with two lines

Dear Chris,

Thank You very much for your detail help. It very helpful for me. I really need your help in the upcoming messages.

Best regards
Tran Quang Hung.

Vào Th 3, 6 thg 8, 2019 vào lúc 06:19 van10hoven@gmail.com [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Quang Hung,
>> Let me first start with your *first Transformation* QA-Tfx(L,P) described
>> by you in QFG #3671.
>> You mentioned it before in #2099.
>> Properties:
>> It is a linear transformation.
>> Regardless of the position of line L, the point QA-Tfx(L,P) will lie on
>> the 5th Point tangent QA-Tf9(P).
>> · For extra properties, see QFG#2101, #2103, #2122
>> Your *2 nd transformation* in #3672 is the same as #3671. Only instead of
>> finite point P you use the infinity point of 2 nd line r.
>> Your *3 rd transformation* in #3673 gives a pretty long expression.
>> It maps 2 lines L1 & L2 into a point X.
>> Properties/Incidents:
>> . When L1 *or* L2 are the Line at Infinity, then X = an infinity point.
>> . L1=Line at Infinity, L2=QA-L2, then X = Infinity Point (QA-L9) = QA-Tf2
>> (QA-P2)
>> . L1=Line at Infinity, L2=QA-L7, then X = Infinity Point (QA-P12.QA-P32) =
>> Infinity Point (QA-P14.QA-P37)
>> . L1=Line at Infinity, L2=QA-L9, then X = Infinity Point (QA-L2) = Infinity
>> Point (QA-P1.QA-P32)
>> . L2=Line at Infinity, L1=QA-L3, then X = Infinity Point (QA-L7) = QA-P9041
>> = InfinityPoint of the line through the 3 QL-P18 points in a Quadrangle

```
>> . L2=Line at Infinity, L1=QA-L4, then X = Infinity Point
(QA-L9) = QA-Tf2
>> (QA-P2)
>> . L2=Line at Infinity, L1=QA-L7, then X = Infinity Point
(QA-L3) = Infinity
>> Point (QA-P1.QA-P5)
>> . L2=Line at Infinity, L1=QA-L9, then X = Infinity Point
(QA-L4) = QA-Tf2
>> (QA-P3)
>> . L1=QA-L2, L2=QA-L2, then X = QA-P2
>> . L1=QA-L9, L2=QA-L9, then X = QA-P23
>> . L1=QA-L5, L2=QA-L5, then X = point on QA-Ci1
>> . L1=QA-L2, L2=QA-L9, then X = some infinity poin
>>> . L1=QA-L9, L2=QA-L2, then X = QA-Tf2(QA-P2)
>> Best regards,
>> Chris
```

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Message: #3687
Date: 06/8/2019 1:46:11
From: chris.vantienhoven
Subject: Six concyclic Poncelet points on QL

```
Dear Quang Hung,

Regarding your message QFG#3678.
The Triangle P1.P2.P3 in your construction is a QL-Triple
Triangle for QA-P2, because the Quadrangles you choose are the
three Component Quadrigons of a Quadrilateral (see QL-3QG1). For
every QA-point and every QG-point such Triple Triangle can be
constructed.
otes:
1. The circle is the nine-point circle of triangle P1.P2.P3
2. The Poncelet point of P1.P2.P3.P,
   where P can be any point in the plane, lies on this circle.
3. The equation of the circle is extreme long.
4. There are no known QL-points lying on this circle.
5. Its center has no connections with other QL-points either.

Best regards,
Chris
```

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Message: #3688
Date: 06/8/2019 2:43:46
From: Tran Quang Hung
Subject: Coaxial circles in QA

Dear geometers,

P1, P2, P3, P4 be the defining Quadrangle Points.
Let S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4 cuts P2P3.
Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
r is a random line.
r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points R12, R13, R14, R23, R24, R34.
Let R34' be the point such that (S1, S34; R34, R34') = -1 (Harmonic conjugate).
Let R12' be the point such that (S1, S12; R12, R12') = -1.
We define similarly the points R13', R14', R23', R24'.
Now the circles with diameters (R12'R34'), (R13'R24'), (R14',R23') are coaxial.
This means its centers are collinear and they have a common radical axis. Thus we have the transformation of a line r to two lines? Which are these lines?

Best regards
Tran Quang Hung.

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Message: #3689
Date: 06/8/2019 4:45:37
From: Tran Quang Hung
Subject: A transformation to parallel line and perspective triangle on QA

```
Thank You very much Cesar for your calculation.

Best Regards
Tran Quang Hung.

Vào Th 2, 5 thg 8, 2019 vào lúc 22:47 César E. Lozada
cesar_e_lozada@yahoo.es [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Mr Hung,
>> Perspector for line (x:y:z):
>> (CT)
>> Q = ((q+r)*(2*p+q+r)*q*r*x-(q^2+(p+2*q+2*r)*p+(q+r)^2)*p*r*y-
(r^2+(p+2*q+2*r)*p+(q+r)^2)*p*q*z)*((2*(p+q+r)*p+(q+r)^2)*x+p*q*
y+p*r*z)
>> Best regards,
>> César Lozada
```

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Message: #3690
Date: 06/8/2019 5:06:43
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

```
Dear Chris, dear Bernard,

I try to describe interim the way back
… from a 5P to the QL with these QA-Cu7-triple points:

Consider a 5P, its CSC and its quartic:
… If there are three tangents from 5P-s-P5 at the quartic
… with contact points concyclic with 5P-s-P5
… … and the diametral point of 5P-s-P5 on 5P-s-Co1,
    they are QA-P4,
… take the CSC-images of QA-P4, they are QG-P1,
… and consider the CSC-partner on the QG-P1-triangle sides,
… which are the opposite points of the searched QL.

Best regards Eckart
```

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Message: #3691
Date: 06/8/2019 5:46:23
From: Antreas Hatzipolakis
Subject: Coaxial circles in QA

[Tran Quang Hung]:

Dear geometers,

P1, P2, P3, P4 be the defining Quadrangle Points.
Let S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4 cuts P2P3.
Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
r is a random line.
r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points R12, R13, R14, R23, R24, R34.
Let R34' be the point such that (S1, S34; R34, R34') = -1 (Harmonic conjugate).
Let R12' be the point such that (S1, S12; R12, R12') = -1.
We define similarly the points R13', R14', R23', R24'.
Now the circles with diameters (R12'R34'), (R13'R24'), (R14',R23') are coaxial.
This means its centers are collinear and they have a common radical axis. Thus we have the transformation of a line r to two lines? Which are these lines?

****

.... and to one point: the radical trace of the circles = intersection of the radical axis and the diacentric line.

APH

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Message: #3692
Date: 06/8/2019 10:30:25
From: bernard.keizer
Subject: New 5P transformation

Dear Chris, dear Eckart,
As already mentionned, repetition is the key of teaching !
But the occasion is too beautiful !
Doing exactly the same with the CSC of the 5 points (which have obviously the same CSC) gives the twin QL !
Best regards
Bernard

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Message: #3693
Date: 06/8/2019 11:45:21
From: chris.vantienhoven
Subject: 5P-s-Co1

Dear Bernard, Dear Eckart,

Regarding your common unique diagonal polar conic for 5 pivotal isocubics having 4 of the 5 points as fixed points of the conjugation and the 5th point as pivot (QFG#3517).
How do you construct a pivotal isocubic given 4 points and a pivot?

Best regards,
Chris

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Message: #3694
Date: 07/8/2019 12:11:53
From: eckart_schmidt@t-online.de
Subject: 5P-s-Co1

```
Dear Chris,

wrt: " How do you construct a pivotal isocubic given 4 points
and a pivot?"
If the 4 points are fixed points of the isoconjugation:
… Take the 4 fixed points as QA with diagonal triangle Tr
… and a line pencil of the 5th point.
… The isoconjugated points on the lines
… … for the Tr-isoconjugation with the fixed 4 points
… give the searched cubic.

Best regards Eckart
```

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Message: #3695
Date: 07/8/2019 12:20:54
From: chris.vantienhoven
Subject: New 5P transformation

```
Dear Bernard and Eckart,

Congratulations !
After a year of hard work you succeeded finding the way back
from 5P to 4L.
A major achievement.

Best regards,
Chris
```

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Message: #3696
Date: 07/8/2019 12:23:36
From: eckart_schmidt@t-online.de
Subject: 5P-quartics for 6P

```
Dear Chris,

if we construct six 5P-quartics for a 6P,
… we get three common points.
What about these 6P-s-3Px?

Best regards Eckart
```

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Message: #3697
Date: 07/8/2019 12:25:34
From: eckart_schmidt@t-online.de
Subject: 5P-quartics for 6P

```
Dear Chris,

I was too overhasty with my last message:
If we construct the six 5P-quartics for a 6P,
… there are three points, *where five quartics intersect.*

Best regards Eckart
```

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Message: #3698
Date: 07/8/2019 12:27:39
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard, dear Chris,

if you are interested,
… there is a construction for tangents at the 5P-quartic:
Let P be a point on the quartic and Q = CSC(P) on the cubic.
T he cubic can be considered as QL-Cu1 (see #3675),
… so we can construct a tangent in Q at the cubic (see EQF),
… whose CSC-image is a circle through P,
… and the tangent in P at the circle is also the tangent at the quartic.

Best regards Eckart

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Message: #3699
Date: 07/8/2019 1:01:38
From: bernard.keizer
Subject: 5P-s-Co1

Dear Chris,
Very surprising question coming from you !
The construction is given in EQF under QA-Cu-1, the 5 pivotal isocubics are QA-Cubics of type 1.
(each cubic contains the DT vertices of it's QA and the vertices of the cevian triangle of the pivot wrt this DT)
Best regards
Bernard

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Message: #3700
Date: 07/8/2019 4:23:42
From: bernard.keizer
Subject: New 5P transformation

```
Dear Eckart,
Beautiful and useful construction, indeed.
But we need not only the tangent in the point to the quartic,
but also the tangents from the point to the quartic !
Any idea ?
Best regards
Bernard
```

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Message: #3701
Date: 07/8/2019 10:40:51
From: chris.vantienhoven
Subject: 5P-s-Co1

Dear Bernard,

[BK] Very surprising question coming from you !
I think that's because my brain is wired in another way than your brain. That's why we work so good together!
But actually, I wish my memory was better.
And therefore some unavoidable questions.
Isn't it so that a polar conic is always:

a) wrt a cubic
b) wrt a reference point (on or outside the cubic)
That are two variables.
When we have a 5P and we construct a pivotal isocubic using one point as a pivot and the other four as fixed points of the conjugation, then we have the cubic (condition b).
To produce a polar conic we still need some more input:
a) which reference point ?

Further you mention "diagonal polar conic". What do you mean in this case with "diagonal"?
You wrote: "This conic contains the vertices of the anticevian triangles of each of the 5 points wrt the DT of the 4 others (15 points).".
I tried to construct the conic using this property, but did not succeed.
Could you please explain. Or is my brain is playing games with me?

Best regards,
Chris

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Message: #3702
Date: 08/8/2019 3:16:50
From: Tran Quang Hung
Subject: Some new theorems on Pentagon and Pentagram

```
Dear geometers,

I had found "Some new theorems on Pentagon and Pentagram" in the
paper

https://arxiv.org/abs/1908.00974

I look forward the consideration of geometers
on Quadri-Figures-Group.

Best regards
Tran Quang Hung.
```

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Message: #3703
Date: 08/8/2019 9:06:42
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

```
Dear Bernard,

have you really a drawing of the "twin" QL for a 5P?

My observation:
… The 5P of the QA-Cu7-triple points is a special 5P,
… not all 5P have a QL, whose QA-Cu7-triple points give the 5P.

Up to now I have not found an example,
… that the CSC-5P of the QA-Cu7-triple points
… shows the properties of #3690 , to go the way back to a QL.

Best regards Eckart
```

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Message: #3704
Date: 08/8/2019 3:53:04
From: bernard.keizer
Subject: 5P-s-Co1

Dear Chris,
Yes, indeed !
The diagonal polar conic is the polar conic of the pivot wrt the pivotal isocubic (diagonal because it contains the 4 fixed points see Bernard Gibert Special Isocubics page 30).
The circumscribed polar conic is the polar conic of the isopivot (isoconjugate of the pivot) wrt the same cubic (circumscribed beacause it contains the vertices of the reference triangle DT).
The 1rst mentionned conic contains the vertices of the anticevian triangle of any of it's points wrt DT (that means that the point and the 3 vertices of it's anticevian triangle form an inscribed QA having the same DT).
This was only a curiosity, but I'm glad you took interest in it!
Best regards
Bernard

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Message: #3705
Date: 10/8/2019 9:30:07
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

wrt messages #3680, #3685, 3692:
After a lot of CABRI-constructions I am convinced,
… that the 5P of the QA-Cu7-triple points are special 5P!
Only if there are for a 5P three with 5P-s-P5 concyclic
… contact points
of tangents from 5P-s-P5 at the 5P-quartic
… with diametral point of 5P-s-P5 on 5P-s-Co1,
… there will be a QL with this 5P for its QA-Cu7-triple points.
I found no example for a 5P of triple points,
… whose CSC-5P has this property.

Best regards Eckart

PS: Of course there will be a simpler description of these 5P.

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Message: #3706
Date: 10/8/2019 1:28:50
From: chris.vantienhoven
Subject: Some new theorems on Pentagon and Pentagram

Dear Quang Hung,

I am reading your document at
https://arxiv.org/pdf/1908.00974.pdf.

I need some time to read and elaborate it in fully.
Here is my first reaction about Theorem I, Miquel’s Pentagram
Theorem.
I think this is the most well-known theorem about Pentagons and
it is good to mention it.
This theorem is sometimes referred to as Jiang Zemin's problem,
as this former president of China talked about the theorem the
end of 1999 as he visited Macau.
(see Mathworld).

>From the Article of Jean-Louis Ayme ''LA CHAÎNE INACHEVÉE DE
WILLIAM KINGDON CLIFFORD”, page 4, available at:
http://jl.ayme.pagesperso-orange.fr/Docs/Etude%204.pdf, comes
this phrase (in the French language): “En 1838, Auguste Miquel
découvre à partir d'une idée d’Eugène Catalan, ''Le
pentagramme'' formé par cinq droites en position générale.“

Meaning that Miquel was brought to the idea of a Pentagram by
Eugène Catalan. He used it to find a general theorem about 5
circles, the “Miquel’s Five Circles Theorem”.
I myself am astonished about another property:

* The points is not only a 5G-point but also a 5L-point: it is
5L-o-P2, which is the 5L-version of nL-o-P2: nL-Clifford's
Circle Center.
* Moreover the circle through the 2nd-circle-points is 5L-o-Ci1,
which is the 5L-version of nL-o-Ci1: nL-Clifford’s Circle. See
also Jean-Louis Ayme’s document.

Also I found these properties:

* When the 5-Gon is considered as a 5-Point, then it appears
that the Miquel Point QL-P1 of each component 4-Point
(Quadrangle) lies on a corresponding Pentagram circle.

```
* When the 5-Gon is considered as a 5-Line, then it appears that
the Miquel Point QL-P1 of each Component 4-Line (Quadrilateral)
coincides with the corresponding 2nd intersection point of a
pair of construction circles. Accidentally is the circle through
these 2nd intersection points the circle that defines 5G-s-P4,
which explains why 5G-s-P4 = 5L-0-P2.

Best regards,
Chris
```

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Message: #3707
Date: 10/8/2019 1:40:36
From: Tran Quang Hung
Subject: Some new theorems on Pentagon and Pentagram

```
Dear van Tienhoven,

Thank You so much for your new information. I focus in the
Theorem 4 and Theorem 5 in my paper, it was for arbitrary
pentagram, maybe 5-gon? I wish You and Mr Eckart will consider
them soon.

Best regards
Tran Quang Hung.
```

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Message: #3708
Date: 10/8/2019 1:51:31
From: Tran Quang Hung
Subject: Coaxial circles in QA

Thank You very much Mr Antreas for your interest. I hope Mr van Tienhoven and Mr Eckart will help me to check this result.

Thank you very much and best regards
Tran Quang Hung.

Vào Th 3, 6 thg 8, 2019 vào lúc 22:50 Antreas Hatzipolakis anopolis72@gmail.com [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> [Tran Quang Hung]:
>> Dear geometers,
>> P1, P2, P3, P4 be the defining Quadrangle Points.
>> Let
>> S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4 cuts P2P3.
>> Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
>> Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
>> Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
>> r is a random line.
>> r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points R12, R13,
>> R14, R23, R24, R34.
>> Let R34' be the point such that (S1, S34; R34, R34') = -1 (Harmonic
>> conjugate).
>> Let R12' be the point such that (S1, S12; R12, R12') = -1.
>> We define similarly the points R13', R14', R23', R24'.
>> Now the circles with diameters (R12'R34'), (R13'R24'), (R14',R23') are
>> coaxial.
>> This means its centers are collinear and they have a common radical axis.
>> Thus we have the transformation of a line r to two lines? Which are these
>> lines?
>> .... and to one point: the radical trace of the circles = intersection of
>> the radical axis and the diacentric line.
>> APH

Message: #3709
Date: 10/8/2019 4:08:31
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Of course, I completely agree with you !
It took me time and drawings to be definitively convinced that the twin QL doesn't exist ...
In a certain way, I'm rather released, as I could hardly accept the idea that a 5P is always equivalent to a 4L (and to it's dual 4P).
It's well known that the points T, U , QL-P1 can be defined for any 5P.
But R can now also be defined for an ordinary 5P with the help of the cubic/quartic as diametral of U on the circle through the 3 real tangentials of U wrt the quartic (which contains U) or in another way as the intersection of the 3 normals to the quartic in these points.
The quartic is circular and U is it's focus, meaning that the circular points are also tangentials of U.
This leads to 2 triangles of "QA-P4" and "QA-P41" on the quartic and of "QG-P1" on the cubic with CSC("QA-P41") aligned and being CSC2("QG-P1"). (QL-P1 doesn't lie on the Euler circle of the "QG-P1" triangle).
The 5P are the triple points of the 3 QA-Cu7 of a QL with DT the QG-P1 only if R lies on the 5P conic.
Best regards
Bernard
PS We should now study this strange quartic which functions like a Van Rees cubic with the CSC3 as CSC and a circle as "Newton Line" ...

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Message: #3710
Date: 10/8/2019 4:20:44
From: bernard.keizer
Subject: New 5P transformation

```
Dear Eckart,
For example, I forgot one interesting property of the quartic :
2 CSC3 conjugate points on the quartic not only have the middle
of the segment joining them on the circle acting as Newton Line,
but are concyclic with the 2 fixed points (and harmonic wrt
these 2 points on the circle), which is a wellknown property of
the Van Rees cubic and gives an easy way of drawing the quartic)
...
Best regards
Bernard
```

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Message: #3711
Date: 10/8/2019 6:08:37
From: Tran Quang Hung
Subject: Dual lines in QA

```
Dear geometers,

Dear geometers,

P1, P2, P3, P4 be the defining Quadrangle Points.
Let S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4
cuts P2P3.
Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
r is a random line.
r meets lines S1S3, S3S2, S2S1 at T2, T1, T3.

Construct T1', T2', T3' such that
(S12,S34;T1,T1') = -1
(S14,S23;T3,T3') = -1
(S13,S24;T2,T2') = -1
Then T1', T2', T3' are collinear on a line. I call it by d.
Thus we have d = f(P1,P2,P3,P4,r), and also r =
f(P1,P2,P3,P4,d). So I call r and d are dual wrt
QA(P1,P2,P3,P4).
Which are the others properties of r and d?

Best regards
Tran Quang Hung.
```

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Message: #3712
Date: 10/8/2019 8:59:39
From: eckart_schmidt@t-online.de
Subject: Coaxial circles in QA

```
Dear Tran Quang Hung,

wrt your message #3688 some observations:
Let Tf be the transformation, that maps a line
… to the line of the centers of the last three circles.

(1) Tf(QA-L3) = QA-P16.QA-P20,
    … parallel to the line QA-P5.17.19.21 (see PS).
(2) For lines L through QA-P10:
    Tf(L) bears QA-P20,
    … L ^ Tf(L) give a circumconic of QA-Tr1 through QA-P10 and
    QA-P20,
    … which is QA-Tf2(QA-P1.QA-P16).
(3) Tf of a QA-line bears the midpoint of the opposite line.
(4) Fixed lines of Tf are the lines of QA-Tr1.

Best regards Eckart

PS for Chris: I didn't find in EQF
… the parallelism of QA-P16.20 and QA-P5.17.19.21.
```

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Message: #3713
Date: 10/8/2019 10:05:35
From: eckart_schmidt@t-online.de
Subject: Dual lines in QA

```
Dear Tran Quang Hung,

wrt your #3711 with an interesting property:
Let Tf be the line transformation, that maps r <--> d.
For lines L of a line pencil wrt a point P
… Tf(L) envelope a conic tangent QA-Tr1
… in the cevian points of QA-Tf2(P).
Example: Inscribed Steiner ellipse for QA-P16.
There will be more properties!

Best regards Eckart
```

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Message: #3714
Date: 10/8/2019 10:14:00
From: chris.vantienhoven
Subject: Coaxial circles in QA

```
Dear Quang Hung, dear Eckart,

I have nothing to add to the observations of Eckart.
The calculations aren’t easy. The DT-calculations are easier,
but still the results are long.

Eckart, the parallelism of QA-P16.20 and QA-P5.17.19.21 can be
found at QA-3, 10th line of examples.

Best regards,
Chris
```

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Message: #3715
Date: 11/8/2019 9:46:38
From: Tran Quang Hung
Subject: Coaxial circles in QA

```
Dear Chris, dear Eckart,

Thank You very much for your interest. I was very impressed with
Quang Duong's transformation QA-Tf6 ( http://www.chrisvantienhov
en.nl/qa-items/qa-transformations/qa-tf6 ).
I found some these transformations around its configuration.

Best regards
Tran Quang Hung.
```

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Message: #3716
Date: 11/8/2019 10:16:34
From: Tran Quang Hung
Subject: Another transformation with two lines in QA

Dear geometers,

P1, P2, P3, P4 be the defining Quadrangle Points.
r is a random line.
r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points
R12, R13, R14, R23, R24, R34.

Let T1, T2, T3 be the midpoints of segments R12R34, R13R24,
R14R23.
d is another random line.
Line d1, d2,d3 pass through T1, T2, T3 and are parallel to d.
d1 meets lines P1P2, P3P4 at points M12, M34.
Let R12' be the reflection of R12 in M12.
Let R34' be the reflection of R34 in M34.
Line d1' connects points R12' and R34'.
Define similarly the lines d2' and d3'.
Then d1', d2' and d3' are concurrent at a point P.
This points is a transformation of lines r,d to point P in QA.

Best regards
Tran Quang Hung.

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Message: #3717
Date: 11/8/2019 10:17:55
From: chris.vantienhoven
Subject: Dual lines in QA

```
Dear Quang Hung, Dear Eckart.

This transformation QA-Tfx in QFG#3711 is much more “harmonic”
then the one in QFG#3688.
Calculations are much simpler.
Some results:

1. QA-Tfx(QA-Tfx( r )) = r
2. QA-Tfx doesn’t produce interesting results
   for QA-L1 till QA-L9
3. However QA-Tfx(Infinity Line) = QA-Lx = line through the
   midpoints of (S12,S34), (S13,S24), (S14,S23)

As a bycatch the infinity point of QA-Lx (denote as QA-Px) has
some interesting properties:
Let QA-Py = QA-Tf2(QA-Px), then:
QA-Py = a point on the Nine-point Conic QA-Co1.
QA-Py = the intersection point of these 4 lines:
· QA-P1.QA-Tf2(InfinityPoint (QA-P1.QA-P16))
· QA-P5.QA-Tf2(InfinityPoint (QA-P5.QA-P17))
· QA-P10.QA-Tf2(InfinityPoint (QA-P10.QA-P16))
· QA-P19.QA-Tf2(InfinityPoint (QA-P1.QA-P5))

Best regards,
Chris
```

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Message: #3718
Date: 11/8/2019 8:35:04
From: eckart_schmidt@t-online.de
Subject: Dual lines in QA

```
Dear Chris, dear Tran Quang Hung,

the line-line transformation in #3711
… is QL-Tf2 of the dual QL in the sense of QA-8/QL-8 in EQF.

Best regards Eckart
```

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Message: #3719
Date: 12/8/2019 9:56:50
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard,

wrt the property in #3710, which "... gives an easy way of
drawing the quartic."

I tried it in the following way:
Let CI be the circle, centered in 5P-s-P3, mentioned there in
EPG,
Let F1,2 be the fixed points of CSC3
... and M the center of a circle through F1,2,
    intersecting CI in X
The lines X.CSC3(M) intersect the circle round M on the quartic.

Best regards Eckart
```

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Message: #3720
Date: 12/8/2019 10:01:32
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard,

have we already mentioned, that
… the QA-P4 for the five QA of the 5P lie on the quartic?
  Corresponding points have midpoints on the circle
… centered in 5P-s-P3 and mentioned there in EPG.
There are two points on the quartic,
… which are CSC-partner wrt the two 5P,
… points on the orthogonal hyperbola in # 3579, #3581,
… with midpoint in the middle of 5P-s-P4.5.

Best regards Eckart
```

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Message: #3721
Date: 12/8/2019 11:36:19
From: bernard.keizer
Subject: 5P-Quartic

```
Dear Eckart,
Of course, we have already mentionned this property ! (see
messages 3615,3616 and 3617)
That's precisely why the CSC of the 5P and of the QA-P4 of the 5
QA's of the 5P define the cubic !
The middles of the segments joining the CSC1 partners on the
cubic lie on the Newton Line and the middles of the segments
joining the CSC3 partners on the quartic lie on the circle
acting as a "Newton Line".
By the way, it is possible to generalise these quartics : taking
any cevian with the DT vertices gives a new QL with a Van Reeves
cubic with "QL-Cu1" on the DT circumcircle and a CSC2.
The CSC of the cubic gives a quartic invariant in a CSC3 with a
circle instead of the Newton Line.
(With the Newton Line of the QL as cevian, we get DQL and a Van
Reeves cubic with "QL-Cu1" in QL-P17).
Best regards
Bernard
```

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Message: #3722
Date: 12/8/2019 9:10:57
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

```
Dear Bernard,

I apologize for my careless reading your messages,
… I confused the QA-P4 for the 4P of a 5P with the QA-P4
  for the QG of a QL.

Perhaps of interest, a curious observation:
Consider a nonconvex 5P, so that the circumconic is a hyperbola
… with an obtused angled triangle TR, describe in #3579
… and the pedal point X of U=5P-sP5 on the longest side of TR.
The circle Ci(X) (see #3575) intersects the quartic in U and
points Y,
… which are contact points of tangents from U at the quartic.
For a 5P of QA-Cu7-triple points
… these contact points are not the QA-P4,
… which are the remaining  three contact points,
  concyclic with U.

Best regards Eckart
```

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Message: #3723
Date: 12/8/2019 9:44:18
From: chris.vantienhoven
Subject: Another transformation with two lines in QA

Dear Quang Hung,

Your transformation as described in QFG#3716 is very interesting and has a very special application.
Let's denote the transformation as QA-Tfx.
There are two variable lines:
· r, that intersects the edges of the Quadrangle,
· d, that defines an infinity point that is used to construct parallel lines in the construction.

I substituted for both lines the QA-Lines QA-L1 up to and including QA-L9 as well as the line at infinity, here called QA-L0.

Denote Pij = QA-Tfx(QA-Li,QA-Lj).

Finite and Infinite Points
· all P0i-points and Pii-points are Infinity Points
· all Pi0-points are not defined
· all other Pij-points are Finite Points

General collinearities
P0j, P1j, P2j, P3j, P4j, P5j, P6j, P7j, P8j, P9j are collinear for j=1,2,...,9
Pi1, Pi4, Pi9 are collinear for i=0,1,2,...,9

Extra properties
Pij = QA-Tfx(QA-Li,QA-Lj).
"Infinity Point" is abbreviated as "IP".
P0i = Pii, they are the infinity points of QA - Li, therefore:
P01 = P11 = IP(QA-P1.QA-P2)
P02 = P22 = IP(QA-P1.QA-P32)
P03 = P33 = IP(QA-P1.QA-P5)
P04 = P44 = P17 = QA-Tf2(QA-P3)
P05 = P55 = IP(QA-P10.QA-P11)
P06 = P66 = IP(QA-P1.QA-P15)
P07 = P77 = InfinityPoint of QA-L7,
            the line through the 3 QL-P18 points in a Quadrangle
P08 = P88 = InfinityPoint of QA-L8
P09 = P99 = QA-Tf2(QA-P2) = Infinity Point
P14 = QA-P3
P17 = P04 = P44 = QA-Tf2(QA-P3) = InfinityPoint
P19 = P29 = QA-P2

P24 = 5P-s-P4 (P1,P2,P3,P4,QA-P4)
P12 lies on QA-P2.QA-P33
P41 lies on QA-P1.QA-Tf2(QA-P2)
P42 lies on QA-P23.QA-P33
P69 lies on QA-P14.QA-P24
P92 lies on QA-P1.QA-P6.QA-P23

Special application
I proved algebraically that for all points Pij =
QA-Tfx(QA-Li,QA-fixed-line-Lj) are collinear on the
5th-point-tangent(InfinityPoint(QA-fixed-line-Lj)).
Note that the 5th-point-tangent (QA-Tf9) of an infinity point is
not necessarily the line at infinity.
In fact, in general, it isn't.
The advantage of this exercise with transformation QA-Tfx is
that it helps us to construct the 5th-point-tangent at an
infinity point!
Let L1 and L2 be two random lines and let P0 be the Infinity
Point of some line named L0.
Then 5 th -point-tangent line QA-Tf9(P0) simply can be
constructed by applying QA-Tfx(L1,L0) and QA-Tfx(L2,L0) and by
connecting both obtained QA-Tfx-points.
In general, a hyperbola can be constructed knowing 3 points on
the conic and an asymptote.
So I tried it in Cabri and the 4 th QA-point also lies very
neatly on the constructed hyperbola.

Quartic
The envelope of all 5 th -point-tangents at an infinity point is
a quartic touching all six sides of the Quadrangle. It is the
same quartic as mentioned by Eckart in QFG#141.

This is as far as I got.

Best regards,
Chris

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Message: #3724
Date: 13/8/2019 6:58:58
From: chris.vantienhoven
Subject: Dual lines in QA

```
Dear Quang Hung, Dear Eckart.

Very nice property that the transformation QA-Tfx in QFG#3711 =
QL-Tf2 of the dual QL.
It shows, even more, the harmonic properties of the
transformation.

Best regards,
Chris
```

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Message: #3725
Date: 13/8/2019 7:12:33
From: Tran Quang Hung
Subject: Another transformation with two lines in QA

Dear Chris,

Thank You so much for very detail explanation and the beautiful properties from this transformation. I learned a lot from your comments.

I see that you used infinity line of projective geometry in many times. If we take the infinity line back to plane, we shall have projective version as following:
Let P1, P2, P3, P4 be the defining Quadrangle Points.
r is a random line.
r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points R12, R13, R14, R23, R24, R34.
d is another random line.
d meets r at point X.
Let T1, T2, T3 be the harmonic conjugate points: (R12,R34;X,T1) = (R13,R24;X,T2) = (R14,R23;X,T3) = -1.
P is a point on d.
Line PT1 meets the lines P1P2, P3P4 at points M12, M34.
Line d meets the lines P1P2, P3P4 at points M12', M34'.
Let R12' and R34' be the be the harmonic conjugate points:
(M12,M12';R12,R12') = (M34,M34';R34,R34') = -1
Line d1' connects the points R12' and R34'.
Define similarly the lines d2' and d3'.
Then d1', d2' and d3' are concurrent at a point Q.
Thus this is a transformation of lines r, d and P
to point Q in QA.

Best regards
Tran Quang Hung.

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Message: #3726
Date: 14/8/2019 9:20:30
From: eckart_schmidt@t-online.de
Subject: Another transformation with two lines in QA

Dear Quang Hung, dear Chris,

thanks to Quang Hung for the transformation in #3716
… and to Chris for the interesting observations in #3723.

Here another aspect:
If we consider a line r of a line pencil wrt a point P
… and lines d with any direction,
… QA-Tfx(r,d) give -varying d- a cubic Cu(r),
… intersecting r on QA-Co1 and a common infinity point.

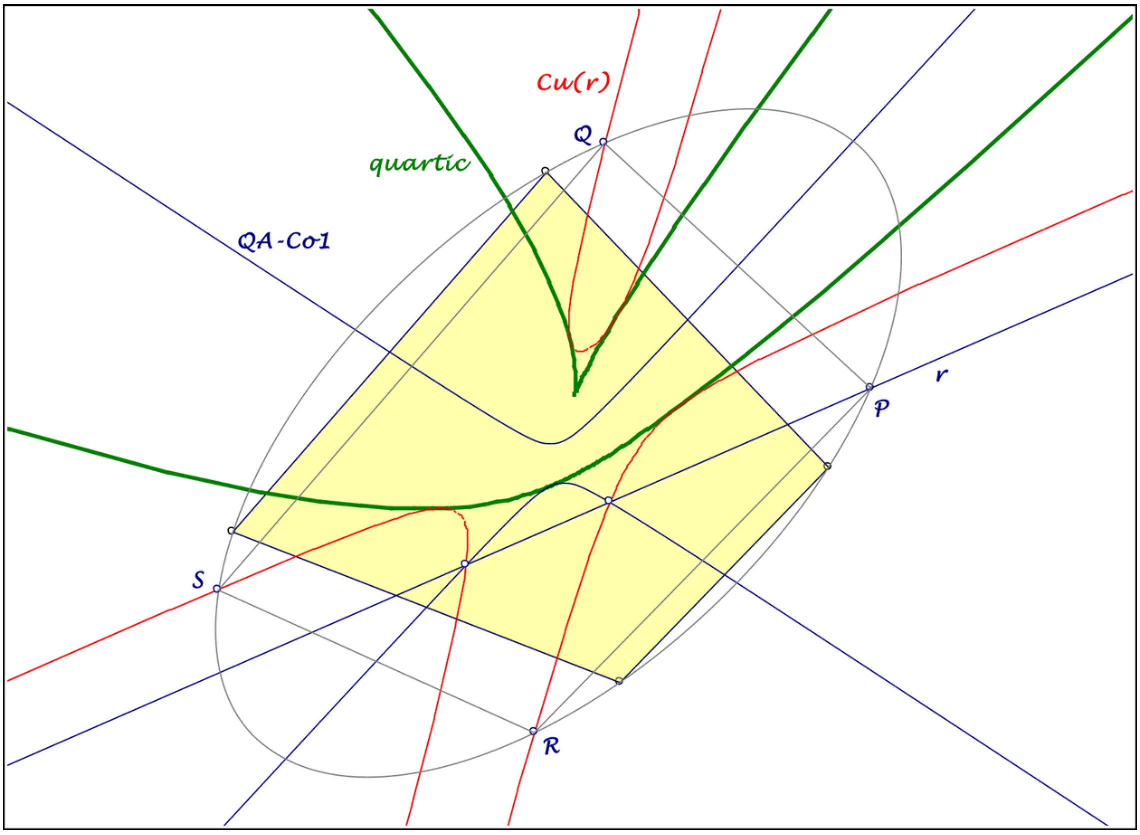
The cubic Cu(r) has three asymptotes,
… two parallels and a parallel to r.
The cubics Cu(r) can have up to three common points Q,R,S
… on the circumconic of QA through P,
… on tangents from P at the quartic, mentioned in #3723,
  first in #141
If there are three common points Q,R,S of the Cu(r),
… PQRS is a QA with the same quartic.
If r is tangent to this quartic, the cubic Cu(r) degenerates
… in three lines,
  two of them coincide, the 3rd is r.

I hope someone can confirm these observations
… and lighten the background (see attached file).

What about the contact points of Cu(r) and the quartic?
If a construction of the quartic isn't parat:
Consider a circle C1, centered in QA-P3 through QA-P2,
… intersecting QA-P3.QA-P4 in points X,
… and a circle Ci2, centered in M on QA-Co1 through QA-P2,
… intersecting X.QA-P2 in N:
… The lines NM envelope the quartic.

Best regards Eckart

PS: I will be ready for two weeks of holiday ...



2019-08-15.pdf

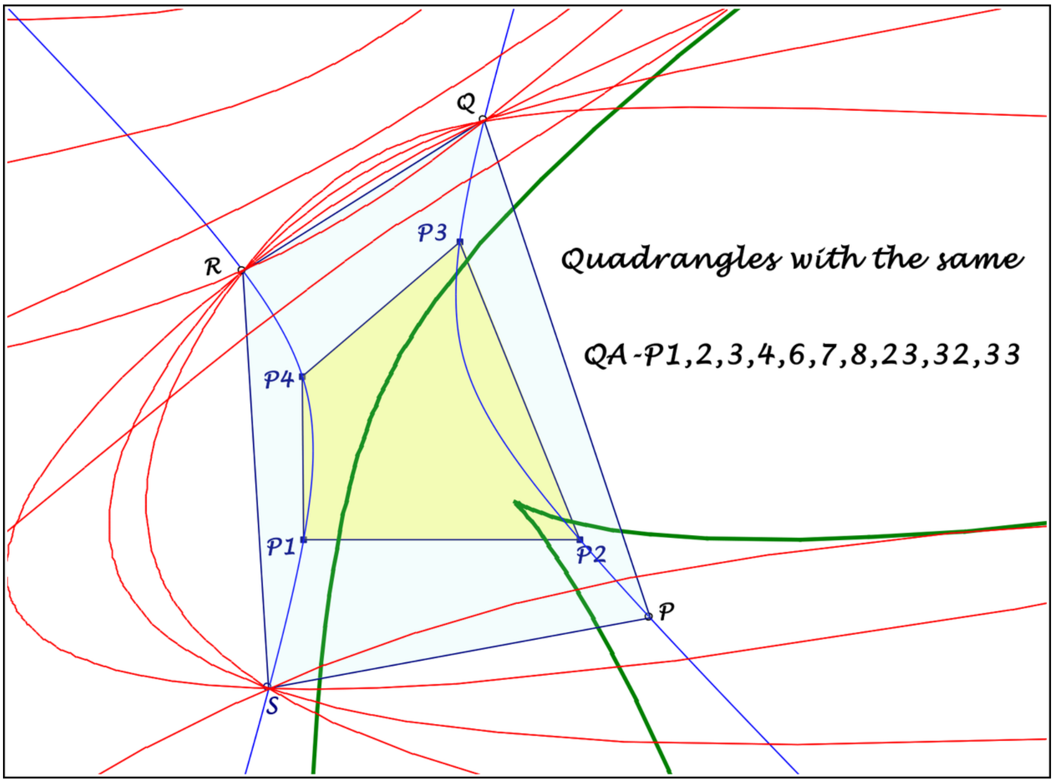
Message: #3727
Date: 15/8/2019 11:31:37
From: eckart_schmidt@t-online.de
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

```
Dear Quang Hung, dear Chris,

This is an application of Quang Hung's transformation
… QA-Tfx(r,d)
in #3716, see also #3723, #3726:
Consider a reference QA = P1P2P3P4 and a point P,
… which will be the first point of a 2nd QA,
… further the cubics Cu(r) for lines r through P.
If these cubics have three common points Q, R, S,
… they are points on the circumconic of P1P2P3P4 through P
… and give the 2nd QA = PQRS
… with the same points QA-P1,2,3,4,6,7,8,23,32,33
  as the reference QA,
… in consequence with the same lines QA-L1,2,4 and
QA-P4.8.23.32, QA-P1.7.17, QA-P1.32.33.
The two QA have the same quartic (see #3726).

Best regards Eckart

PS. I hope, someone can confirm these CABRI-observations.
```



2019-08-15.pdf

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Message: #3728
Date: 15/8/2019 3:25:03
From: eckart_schmidt@t-online.de
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

```
Dear Quang Hung, dear Chris,

in addition to #3727:
The two quadrangles have also the same QA-Co1,
… so it easy to construct the 2nd QA:

Reflect the point P in points of QA-Co1 and you get a conic,
… intersecting the QA-circumconic Co through P
… in four points Q, R, S and the diametral point of P wrt Co:
… PQRS is the 2nd QA in the sense of #3727.

Best regards Eckart
```

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Message: #3729
Date: 15/8/2019 9:41:33
From: eckart_schmidt@t-online.de
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

```
Dear Quang Hung, dear Chris, dear Benedetto,

in #3727 and #3728 I described two QA with 10 common points,
… there is a further unexpected property:
… the two QA have the same parameter SIGMA.

This parameter is described in #1683 (first in #1471):
QA-P4 is the homothetic center
… of the 1st circumcenter quadrangle
… and the 1st isogonal conjugate quadrangle:
… The corresponding ratio is the parameter Sigma.

The parameter SIGMA is also Benedetto's "lambda" in EQF-ref
[36].

Best regards Eckart
```

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Message: #3730
Date: 16/8/2019 2:08:43
From: Tran Quang Hung
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

```
Dear Eckart, dear Chris, dear Benedetto,
Thank You very much for your enthusiastic contribution. I can
check with Cabri the simple properties on line and conic but I
can't work with cubic. I am still studying your comments hard.
They are very interesting and new with me.
Best regards
Tran Quang Hung.
```

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Message: #3731
Date: 17/8/2019 7:54:08
From: Tran Quang Hung
Subject: Coaxial circles in QA

```
Dear Chris, dear Eckart,

I wonder that we can change the harmonic conjugate with any
ratio k. As following
P1, P2, P3, P4 be the defining Quadrangle Points.
Let S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4 cuts
P2P3.
Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
r is a random line.
r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the points
R12, R13, R14, R23, R24, R34.
Let k be a real number.
Let R34' be the point such that (S1, S34; R34, R34') = k.
Let R12' be the point such that (S1, S12; R12, R12') = k.
We define similarly the points R13', R14', R23', R24'.
Now the circles with diameters (R12'R34'), (R13'R24'),
(R14',R23') are coaxial.
Could You please help me to check it?

Best Regards
Tran Quang Hung.
```

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Message: #3732
Date: 17/8/2019 8:20:29
From: chris.vantienhoven
Subject: Coaxial circles in QA

Dear Quang Hung,

I like your method of not taking a harmonic conjugate of three points and instead of that an anharmonic point with fixed cross-ratio k.
However, in my drawings, I cannot confirm that the circles with diameters (R12'R34'), (R13'R24'), (R14',R23') are coaxial.
In my drawings these three circles have different radical axes coinciding in a Radical Center X.
The locus of X for variable k is a Quartic passing through QA-P11, the circumcenter of the QA-Diagonal Triangle.

Best regards,
Chris

p.s. More about anharmonic ratios can be found at Perspective Fields (https://www.chrisvantienhoven.nl/mathematics/perspective-fields) part II.

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Message: #3733
Date: 18/8/2019 1:05:45
From: Tran Quang Hung
Subject: Coaxial circles in QA

Dear Chris,

Thank You very much for your confirmation. Now I know about Perspective Fields (https://www.chrisvantienhoven.nl/mathematics/perspective-fields),
It looks very interesting for me. I shall study it hard.

Best regards
Tran Quang Hung.

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Message: #3734
Date: 18/8/2019 8:10:55
From: Tran Quang Hung
Subject: Two QA have the same Poncelet point

```
Let A, B, C, D be a QA with Gergone point P (QA-P3).
Let Pa, Pb, Pc, Pd be the isogonal conjugate of P wrt triangles
BCD, CDA, DAB, ABC, resp.
QA-P2(A,B,C,D) = QA-P2(Pa,Pb,Pc,Pd).

Moreover
Let Ma, Mb, Mc, Md be the midpoints of segments APa, BPb, CPc,
DPd.
Then we have the "area" indentity [ABCD]+[PaPbPcPd]=2[MaMbMcMd]
[ABCD] = 1/2cross product(vec(AC),vec(BD)), if ABCD is convex
then |[ABCD]| = Area(ABCD).

We can compare with the properties of Orthocenter Quadrangle of
(A,B,C,D). If Ha, Hb, Hc, Hd be the Orthocenter Quadrangle of
(A,B,C,D). Then
QA-P2(A,B,C,D) = QA-P2(Ha,Hb,Hc,Hd) and [ABCD] = [Ha,Hb,Hc,Hd].

Best Regards
Tran Quang Hung.
```

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Message: #3735
Date: 19/8/2019 4:02:07
From: Tran Quang Hung
Subject: Newton lines are concurrent in QA

```
Dear geometers,

Let (A,B,C,D) be a QA.
Lab, Lbc, Lcd, Lda, Lac, Lbd are the perpendicuar bisectors of
segments AB, BC, CD, DA, AC, BD respectively.
Then Newton lines of the QL
QL-L-1(Lab, Lcd, Lad, Lbc)
QL-L-1(Lac, Lbd, Lcd, Lda)
QL-L-1(Lab, Lbc, Lac, Lbd)
are concurrent. Which is this point of QA?

Best Regards
Tran Quang Hung.
```

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Message: #3736
Date: 19/8/2019 5:54:46
From: Tran Quang Hung
Subject: Steiner line of Gergonne point

Dear geometers

Let (A,B,C,D) be a QA.
Lab, Lbc, Lcd, Lda, Lac, Lbd are the perpendicuar bisectors of segments AB, BC, CD, DA, AC, BD respectively.
Lab meets Lcd at P1.
Lbc meets Lad at P2.
Lac meets Lbd at P3.
Then Gergonne point P=QA-P3(A,B,C,D) lies on circle (P1P2P3).
Let St be the Steiner line of P wrt triangle P1P2P3. (Steiner line is delation center P of Simson line with ratio 2).
Let St' be the orthopolar line of St wrt QA(A,B,C,D) i.e St'=QA-Tf8(A,B,C,D,St).
Then St' passes through Poncelet point of QA(A,B,C,D).
Which are the other points of QA(A,B,C,D) on this line?

Best regards
Tran Quang Hung.

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Message: #3737
Date: 19/8/2019 10:46:03
From: chris.vantienhoven
Subject: Two QA have the same Poncelet point

```
Dear Quang Hung,

Nice properties regarding QA's built from points relating to the
4 Component Triangles QA-4Tr1
One thing I cannot confirm.
In my drawings and calculations Ma, Mb, Mc, Md are collinear.
Can you affirm?
The line they define is a line through QA-P1 with long
coordinates.

Best regards,

Chris
```

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Message: #3738
Date: 19/8/2019 12:07:48
From: chris.vantienhoven
Subject: Newton lines are concurrent in QA

```
Dear Quang Hung,

Your point in #3735 is QA-P32: Centroid of the Circumcenter
Quadrangle.
Best regards,

Chris
```

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Message: #3739
Date: 19/8/2019 12:27:29
From: Tran Quang Hung
Subject: Two QA have the same Poncelet point

Dear Chris,

Thank You very much for your comments. Your confirmation is very nice and right. I affirm that Ma, Mb, Mc, Md are collinear. Actually I was confused when I define Ma, Mb, Mc Md, but by chance, we see them collinear. Thank You.
I now rewrite my message,
Let A, B, C, D be a QA with Gergone point P (QA-P3).
Let Pa, Pb, Pc, Pd be the isogonal conjugate of P wrt triangles BCD, CDA, DAB, ABC, resp.
QA-P2(A,B,C,D) = QA-P2(Pa,Pb,Pc,Pd).

Moreover
Let Na, Nb, Nc, Nd be the midpoints of segments CPa, DPb, APc, BPd.
Then we have the "area" indentity [ABCD]+[PaPbPcPd]=2[NaNbNcNd].
[ABCD] = 1/2cross product(vec(AC),vec(BD)), if ABCD is convex then |[ABCD]| = Area(ABCD).
We can compare with the properties of Orthocenter Quadrangle of (A,B,C,D).
If Ha, Hb, Hc, Hd be the Orthocenter Quadrangle of (A,B,C,D).
Then QA-P2(A,B,C,D) = QA-P2(Ha,Hb,Hc,Hd) and [ABCD] = [HaHbHcHd].

Best Regards
Tran Quang Hung.

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Message: #3740
Date: 19/8/2019 7:17:12
From: chris.vantienhoven
Subject: Steiner line of Gergonne point

Dear Quang Hung,

Regarding your message #3736.

I checked everything algebraically and it all stands.

QA-P3 on P1.P2.P3-circle
The points P1, P2, P3 are also the three QA-versions of Quadrigon Point QG-P5 (= 1st QG-Quasi Circumcenter). It is known from the circle through these 3 QA-versions that QA-P3 lies on it. See QG-P5.

The center of the circle through these 3 QA-versions of QG-P5 is the Reflection of QA-P15 in QA-P1. It lies on QA-L6 = QA-Newton-Morley Line.
Well actually it is also the common point of the Eulerlines of triangles QG-P5a.QG-P5b.QL-P4c, QG-P5b.QG-P5c.QL-P4a, QG-P5c.QG-P5a.QL-P4b (a/b/c denoting the QA-versions of the involved points).

QA-P2 of QA-Tf8(Tr-SteinerLine)
The Steiner line St for triangle P1.P2.P3 happens to be the line QA-P4.QA-P5.
Apart from QA-P2, there are no other known QA-points on St’ = QA-Tf8(St).

Best regards,
Chris

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Message: #3741
Date: 21/8/2019 3:27:37
From: Tran Quang Hung
Subject: Common orthocenter from triangle

Dear geometers

I wrote on ADGEOM †)
>> Let ABC be a triangle and any point P.
>> Ha, Hb, Hc are the orthocenters of triangles PBC, PCA, PAB reps.
>> Then the triangles AHbHc, BHcHa, CHaHb have the same orthocenter.
>> Is this theorem known before and which is this common orthocenter in term
>> of P?
> I think this problem is interesting with QA.
> We let the above common center by the transformation Xp=Tf(ABC,P).
> Now let QA(A,B,C,D).
> Xd = Tf(ABC,D)

Xa = Tf(BCD,A)
Xb = Tf(CDA,B)
Xc = Tf(DAB,C)

We have some properties of QA(Xa,Xb,XcXd)
- QA(Xa,Xb,Xc,Xd) and QA(A,B,C,D) have the same Poncelet point i.e QA-P2(Xa,Xb,Xc,Xd)= QA-P2(A,B,C,D).
- Newton line of QL(AXa, BXb, CXc, DXc) passes through common Poncelet point of QA(A,B,C,D) and QA(Xa,Xb,Xc,Xd).
- The line passes through Centroid of QA(A,B,C,D) and QA(Xa,Xb,Xc,Xd) is perpendicular to Newton line of QL(AXa, BXb, CXc, DXc).
- Equal area [XaXbXcXd]=[ABCD].
Because of the above properties, can we call QA(A,B,C,D) and QA(Xa,Xb,Xc,Xd) by dual QA?

Best regards
Tran Quang Hung.

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [50].

Message: #3742
Date: 21/8/2019 8:17:50
From: chris.vantienhoven
Subject: Common orthocenter from triangle

Dear Quang Hung,

I can confirm all your properties except that the line passing through Centroid of QA(A,B,C,D) and QA(Xa,Xb,Xc,Xd) is perpendicular to the Newton line of QL(AXa, BXb, CXc, DXc).
I wouldn't prefer the name dual, because this name mostly is used for a transformation mapping points into lines and a corresponding transformation mapping lines into points, which isn't the case here.

Best regards,

Chris

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Message: #3743
Date: 21/8/2019 9:09:24
From: Tran Quang Hung
Subject: Common orthocenter from triangle

Dear Chris,

Thank You very much for your reply. You are right, the property "the line passing through Centroid of QA(A,B,C,D) and QA(Xa,Xb,Xc,Xd) is perpendicular to the Newton line of QL(AXa, BXb, CXc, DXc)." is wrong. I was false.

I agree with you, "dual" name is better for projective properties. But two QA (A, B, C, D) and QA (Xa, Xb, Xc, Xd) are very similar, I try to find some other properties.

Best regards
Tran Quang Hung.

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Message: #3744
Date: 21/8/2019 3:52:52
From: Tran Quang Hung
Subject: Perpendicular lines from NPC in triangle

```
Dear geometers,

Let ABC be a triangle and P is any point.
Let Na, Nb, Nc be NPC centers of triangles PBC, PCA, PAB, reps.
Perpendicular lines from Na, Nb, Nc to PA, PB, PC are concurrent
at point Np.
We denote by the transformation Np=Tf(ABC,P).
Let (A,B,C,D) be QA.
Na = Tf(DBC,A)
Nb = Tf(CDA,B)
Nc = Tf(DAB,C)
Nd = Tf(ABC,D)

We have property
QA-P1(A,B,C,D) = Reflection of QA-P2(Na,Nb,Nc,Nd) in
QA-P3(Na,Nb,Nc,Nd).

Best regards
Tran Quang Hung.
```

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Message: #3745
Date: 21/8/2019 5:17:44
From: Tran Quang Hung
Subject: Apply transfomations in triangle to QA

```
Dear geometers,

Let ABC be a triangle and any point P. We had have many
transformations Tf(ABC,P), these transformations can return
points, lines or circles.
So what happen when we apply them for QA? We shall have
Points Pa = Tf(BCD,A)...
or lines La = Tf(BCD,A)...
or circles Ca = Tf(BCD,A)...
We shall have new QA, QL and set of four circles.
I think we shall have many new points and properties?
I look forward to hearing from You.

Best regards
Tran Quang Hung.
```

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Message: #3746
Date: 21/8/2019 10:43:09
From: chris.vantienhoven
Subject: Perpendicular lines from NPC in triangle

```
Dear Quang Hung,

QA-P1(Na,Nb,Nc,Nd) = point on QA-P1. QA-P32. QA-P33
QA-P2(Na,Nb,Nc,Nd) = QA-P33
QA-P3(Na,Nb,Nc,Nd) = P1.P32.P33 & QA-P2.QA-P23
                     & QA-P14.QA-P37
QA-P4(Na,Nb,Nc,Nd) = point on QA-P2.QA-P23
QA-P5(Na,Nb,Nc,Nd) = point on QA-P1.QA-P40 & QA-P5.QA-P24
                     & QA-P14.QA-P20

Special for is that QA-P3(Na,Nb,Nc,Nd) also is:
common point of the 3 QA-versions (see QA-3QG1)
of the lines QG-P7.QL-P2
(QG-P7 = 2nd Quasi Nine-point Center,
 QL-P2 = point made from Component Nine-point Centers)
It's all connected!

An alternative:
Let: Perpendicular lines from Na, Nb, Nc to BC, CA, AB are
concurrent at point Mp.
We denote by the transformation Mp = Tf2 (ABC, P).

Now:
Ma = Tf2(DBC,A)
Mb = Tf2(CDA,B)
Mc = Tf2(DAB,C)
Md = Tf2(ABC,D)
  and
QA-P1(Ma,Mb,Mc,Md) = point on QA-P1. QA-P11 &  QA-P5. QA-P12  &
QA-P13. QA-P20  &  QA-P30. QA-P34
QA-P2(Ma,Mb,Mc,Md) = QA-P3
QA-P3(Ma,Mb,Mc,Md) = QA-P32
QA-P4(Ma,Mb,Mc,Md) = QA-P1
QA-P5(Ma,Mb,Mc,Md) = point on P1.P32.P33 & P2.P8

Best regards,
Chris
```

Message: #3747
Date: 21/8/2019 10:50:27
From: chris.vantienhoven
Subject: Apply transfomations in triangle to QA

Dear Quang Hung,

Different geometers have been exploring several triangle points
and transformations wrt the Components Triangles of a
Quadrangle.
But I am sure there is more to explore that has not been
discovered yet.
Of course we also have the domain of exploring triangle points
and transformations wrt the Components Triangles of a
Quadrilateral. Often forgotten because people tend to think in
"points" and not in "lines".

Best regards,
Chris

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Message: #3748
Date: 22/8/2019 1:55:14
From: Tran Quang Hung
Subject: Apply transfomations in triangle to QA

Dear Chris,

Thank You for your reply. I agree with you that we are often
more familiar with points than with lines.
I also felt that I was working with QA more familiar than QL but
the fact that the idea of QL was great and I really found myself
lucky in exploring some new properties.

Best regards
Tran Quang Hung.

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Message: #3749
Date: 22/8/2019 3:56:23
From: Tran Quang Hung
Subject: Apply P-Hagge circle to QA

```
Dear geometers,

Let ABC be a triangle and any point P.
A'B'C' is circumcevian triangle of P.
A", B", C" are reflection on A', B', C' in lines BC, CA, AB
reps.
Then circle (A"B"C") passes through orthocenter H of ABC and it
was called by P-Haage circle of P wrt ABC.
Let (Op) = Tf(ABC,P).
Let QA(A,B,C,D).
(Oa)=Tf(BCD,A), (Ob)=Tf(CDA,B), (Oc)=Tf(DAB,C), (Od)=Tf(ABC,D).
We have the parallel lines P2(A,B,C,D)P3(Oa,Ob,Oc,Od) //
P3(A,B,C,D)P2(Oa,Ob,Oc,Od).

Best Regards
Tran Quang Hung.
```

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Message: #3750
Date: 22/8/2019 10:40:09
From: chris.vantienhoven
Subject: Apply P-Hagge circle to QA

```
Dear Quang Hung,

TQH: We have the parallel lines P2(A,B,C,D)P3(Oa,Ob,Oc,Od) //
P3(A,B,C,D)P2(Oa,Ob,Oc,Od).
The line QA-P2(A,B,C,D).QA-P3(Oa,Ob,Oc,Od) = QA-P2.QA-P23, and
the line QA-P3(A,B,C,D).QA-P2(Oa,Ob,Oc,Od) = QA-P3.QA-P32
which are known parallel lines.

See: QA-3: List of parallel QA-Lines

Best regards,
Chris
```

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Message: #3751
Date: 22/8/2019 10:54:25
From: Tran Quang Hung
Subject: Perpendicular lines from NPC in triangle

```
Thank You dear Chris for nice and interesting observation.
The case "perpendicular lines to BC, CA, AB" is very new with
me.
I shall study it more.

Best regards
Tran Quang Hung.
```

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Message: #3752
Date: 22/8/2019 11:10:09
From: Tran Quang Hung
Subject: Apply P-Hagge circle to QA

```
Dear Chris, thank You very much for quick response. I study some
more properties of four circles (Oa), (Ob), (Oc), (Od) but I do
not see any thing.

Best regards
Tran Quang Hung.
```

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Message: #3753
Date: 22/8/2019 11:40:35
From: Tran Quang Hung
Subject: A conjecture on two QA

```
Dear geometers,

I have a conjecture on two QA as following
Let QA(A,B,C,D) and QA(A',B',C',D') be two QA such that
- They have common Poncelet point QA-P2.
- They have equal area [ABCD] = [A'B'C'D'].

Then Newton line of QL(AA',BB',CC',DD') passes through the
common Poncelet point?
Could You please confirm this conjecture? I can't check it by my
computer.

Best regards
Tran Quang Hung.
```

Message: #3754
Date: 22/8/2019 4:57:00
From: chris.vantienhoven
Subject: A conjecture on two QA

Dear Quang Hung,

I did a tryout with a QA and its QA-P2.
I rotated the QA through an angle of 15 degrees around QA-P2.
Obviously, the rotated QA will have the same area and the same QA-P2.
Then it was easy to connect corresponding vertices giving the 4 lines of a quadrilateral QL.
However, the Newton Line (QL-L1) of the QL did not pass through the common QA-P2.
My conclusion: when there is one example that doesn't hold, the conjecture will not hold.

Some extra remarks to play with:
1. QA-P2 is the center of the unique rectangular conic passing through the vertices of the QA. Let's say QA1.
2. Therefore a second QA (QA2) will have another unique rectangular conic passing through the vertices of the QA2.
3. When we have the only condition that the QA's have equal areas, then there are no corresponding vertices and there will be different sets of 4 lines that can be drawn.
4. The Newton line is the locus of all conics inscribed in the 4 lines of the QL.

Best regards,

Chris

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Message: #3755
Date: 23/8/2019 1:51:46
From: Tran Quang Hung
Subject: A conjecture on two QA

```
Dear Chris

Thank You so much for counterexample. I met at least 2 such QA
and I quickly concluded, I was not right. Thank you for very
detailed instructions.

Best regards
Tran Quang Hung.
```

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Message: #3756
Date: 23/8/2019 2:04:42
From: Tran Quang Hung
Subject: Apply a transformation to QA

```
Dear geometers

Let ABC be a triangle and P is any point.
PA meets circle (PBC) again at A'.
A" is projection of A' on BC.
Define similarly the points B", C". Then AA", BB", CC" are
concurrent at point P*.
Let us call this transformation by Xp = Tf(ABC,P).
Let (A,B,C,D) be a QA.
Xa = Tf(BCD,A), Xb = Tf(CDA,B), Xc = Tf(DBA,C), Xd = Tf(BAC,D).

We properties of QA(Xa,Xb,Xc,Xd)
- They have the same Poncelet point QA-P2(A,B,C,D) =
QA-P2(Xa,Xb,Xc,Xd).
- Equal area [ABCD] = [XaXbXcXd]
- Newton line of QL(AXa,BXb,CXc,DXd) passes through common
Poncelet point of these two QA.

Best regards
Tran Quang Hung.
```

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Message: #3757
Date: 23/8/2019 10:23:57
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Chris, dear Eckart,

Before leaving definitively the item of the 5 QA-Cu7 triple points, perhaps these simple observations, which deserve perhaps a place in the properties or curves of the 5P.
Giving 5 points Xi and the CSC, let's name Yi = CSC(Xi), isgi = QA-P4(Xj with j not equal to i) and the same way isg'i = QA-P4(Yj with j not equal to i).
The twin conics are the conics of the Xi and of the Yi with the points T and T'.
The middles of Xiisgi and of Yiisg'i lie on 2 circles, which we could name Newton circles.
The middles of XiCSC(isg'i) and of YiCSC(isgi) lie on the same line, which we name Newton Line.
The points Xi and CSC(isg'i) and Yi and CSC(isgi) define the twin cubics which are Van Rees circular focal cubics with foci U and CSC'U) ; the cubics are invariant under CSC transformations centered in U and CSC(U).
The cubics are circular circumcubics of the 5Xi or the 5 Yi and the asymptotes of the cubics are the parallel to the Newton Line through T and T', the pivot being the same for both cubics, id est the infinity point of the Newton Line ; the asymptote cuts it's cubic in a point Q or Q' which is the 6th intersection between the conic and the cubic (or the 2nd intersection of the asymptote and the conic).
The middles of UT and CSC(U)T' lie on the Newton Line and on the Newton circles ; the middles of UQL-P1 and CSC(U)QL-P1 lie also on the Newton circles.
The CSC of the 2 twin cubics define the twin quartics through the Xi and the isgi and through the Yi and the isg'i ; they pass through QL-P1 and through U and CSC(U) ; the quartics are invariant under CSC transformations centered in CSC(T) and CSC(T').
There are surely other observations to be made, but I stop here for today ...
I hope Eckart will confirm these 1rst observations.
Best regards
Bernard

PS As mentionned by Eckart, if the 5P are 5 QA-Cu7 triple points, there isn't a twin QL !

Message: #3758
Date: 23/8/2019 1:45:18
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

```
Dear Chris, dear Eckart,
Of course, the points U and CSC(U) can be obtained directly with
Eckart's construction from the Xi or the Yi.
But they are also the the tangentials of Q and Q' on the twin
cubics.
Best regards
Bernard
```

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Message: #3759
Date: 24/8/2019 8:01:46
From: chris.vantienhoven
Subject: Apply a transformation to QA

```
Dear Quang Hung,

Nice example again of quadrangles with equal areas and equal
QA-P2.
The line QA-P2.QA-P33 is perpendicular to the Newton Line of
QL(AXa,BXb,CXc,DXd).

Best regards,
Chris
```

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Message: #3760
Date: 25/8/2019 4:49:03
From: eckart_schmidt@t-online.de
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Bernard,

Uff, hard work after holiday,
… to reproduce all the properties of your twin-geometry
… for a constellation, which was one and a half year our terrain.
… Some properties were new for me, but I found no errors!

Have you researched the twin-aspect
… for the orthogonal hyperbolas and the triangles,
… mentioned in #3579 and 3581?

The 5P of QA-Cu7-triple points are special,
… is there perhaps a twin-property,
… which doesn't hold for an arbitrary 5P?

Best regards Eckart

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Message: #3761
Date: 26/8/2019 5:18:54
From: Tran Quang Hung
Subject: Apply a transformation to QA

Dear Chris

Thank You very much for your interest. I try to find some more configurations with equal are and equal P2.

Best regards
Tran Quang Hung.

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Message: #3762
Date: 26/8/2019 8:28:49
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard, dear Chris,

in #3599 a quartic and in #3603 a cubic are discussed,
… which are 5P-CSC partner.

Here some observations wrt their intersections:
There are up to 3 pairs of intersections, which are CSC-partner:
… One pair lies on the orthogonal hyperbola, described in #3579,
… with the same midpoint as 5P-s-P4 and 5P-s-P5.
… The other two pairs - not always real - are collinear with 5P-s-P5.

If we consider the first pair X, Y,
… the QG = (X,5P-s-P4,Y,5P-s-P5) is a parallelogram.
… the QG = (X,5P-s-P5,Y,CSC(5P-s-P5)) has a cubic QL-Cu1,
… bearing 5P-s-P6, its CSC is the 5P-CSC,
… its QL-L1 bears the midpoints of 5P-s-P4.5P-s-P5 and 5P-s-P5.CSC(5P-s-P5).

Perhaps there is a connection for the 5P of QA-Cu7-triple points
… of the last QG, interpreted as QL, and the QL for the QA-Cu7.

Best regards Eckart

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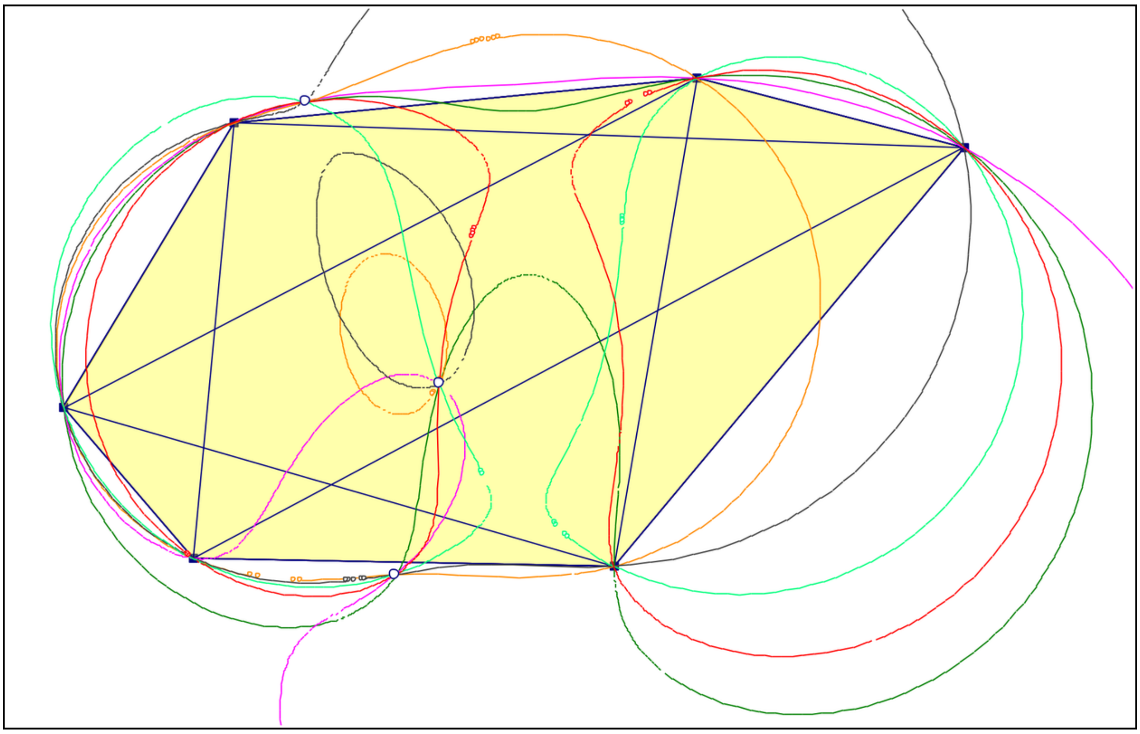
Message: #3763
Date: 27/8/2019 12:15:05
From: eckart_schmidt@t-online.de
Subject: 5P-Quartics for 6P

Dear Bernard, dear Chris,

if we consider the six 5P-quartics for a 6P,
… we have one, two or three common points.

What about these points?

Best regards Eckart



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Message: #3764
Date: 27/8/2019 1:01:46
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Eckart,

Thanks for your answer !
The 5 Xi define a conic, a rectangular hyperbola, a CSC with center M, a point T and a point U and the 3 points V1,V2 and V3, a cubic and a quartic.
The Yi = CSC(Xi) define the same elements.
The circumcircle of the Vi, the circumconic and the RH intersect in the same 4 points Vi and T.
The quartics are the CSC of the cubics.
The conics and the cubics intersect in 6 points, the Xi and a point Q.
The conics and the quartics intersect in 8 points, the Xi and the Vi.
The beauty of all these curves and points is that U' is CSC(U) ...

Best regards
Bernard

Message: #3765
Date: 28/8/2019 10:18:57
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
I checked your properties with the 3 pairs of CSC partners.
You have obviously the same results with the twin cubic and
quartic replacing U and T by CSC(U) and T'.
For the QL-Cu1, it's not surprising, as you start with 2 copples
of CSC partners (X and Y and U and CSC(U)).
For your point 5P-s-P6, you and me understand what it means, but
it unfortunately, it isn't yet in EPG ...
Best regards
Bernard

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Message: #3766
Date: 29/8/2019 10:22:31
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,

perhaps of interest for your twin-geometry:
Let Cu and Qu be 5P-cubic and 5P-quartic
… and Cu' and Qu' the corresponding twin-objects:

There are two tripel of collinear intersections of Cu^Qu' and
Cu'^Qu
… on two lines through 5P-s-P6, which are CSC-partner.
The quartics Qu and Qu' have a common tangent in 5P-s-P6.

Best regards Eckart

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Message: #3767
Date: 29/8/2019 11:40:13
From: chris.vantienhoven
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,
You commented that 5P-s-P6 isn't in EQF and you are right.
It is in the pipeline with many other items to be included in EQF.
It takes a lot of effort to realize that and I will do it when all things work together.
Best regards,
Chris

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Message: #3768
Date: 29/8/2019 4:38:09
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px

Dear Chris,

I think, the following pair of points isn't mentioned till now:
Consider for tangents Tg at 5P-s-Co1 in 5P-vertices
… the conics QA-Tf2(Tg) wrt the remaining QA:
These five conics have two common points.
What about these points?

Best regards Eckart

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Message: #3769
Date: 29/8/2019 6:00:32
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
Thanks a lot for this new property !
Now perhaps another one : the middles of the segments XiYi (with Yi and Xi CSC partners) lie on a rectangular hyperbola and the 10 points lie on a curve obviously with 5P-s-P6 as dobble point which contains all the points CSC partners with the middle of the segment joining them on this curve.
This curve is by definition CSC invariant.
Any idea about this curve ? Does it contain the isg and their CSC and the isg' and their CSC ?
I reach the limits of my ability and perhaps willing to draw new curves ...
Perhaps you will be able to help me !
Best regards
Bernard

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Message: #3770
Date: 29/8/2019 7:43:15
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
Sorry for the typo, the middle is of course on the RH !
Best regards
Bernard

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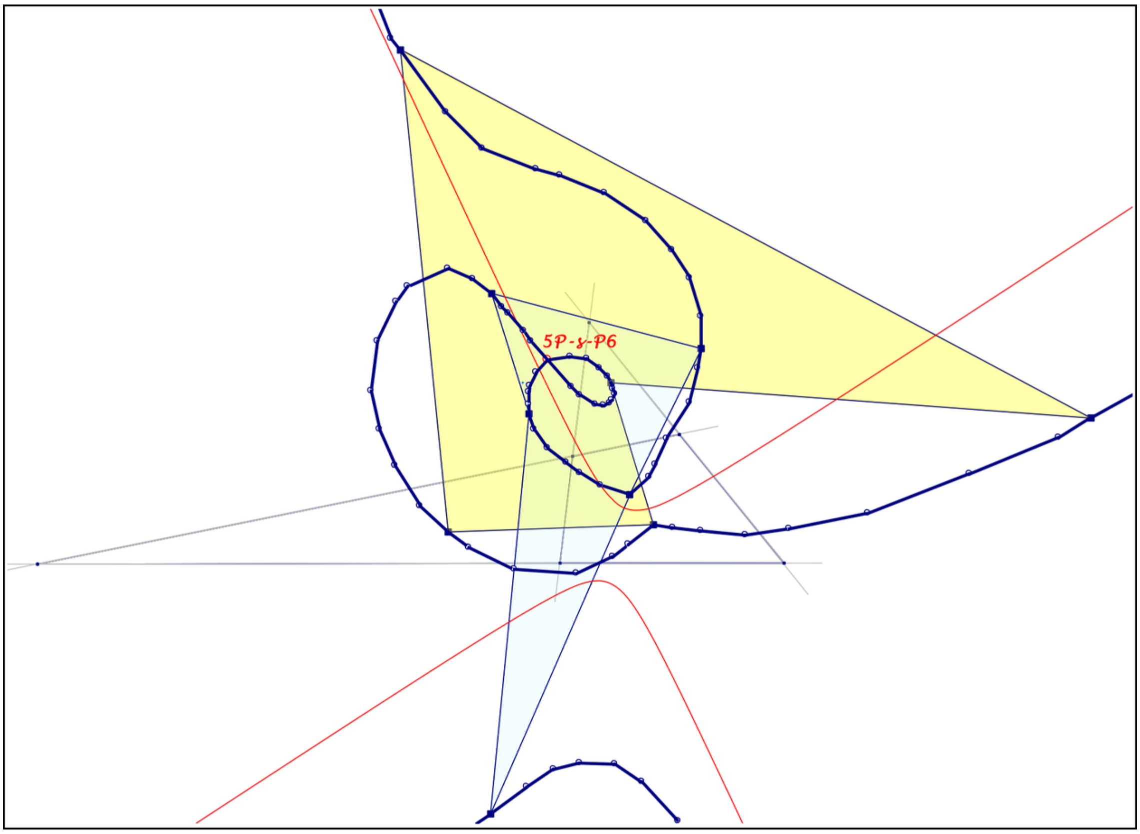
Message: #3771
Date: 30/8/2019 11:14:44
From: eckart_schmidt@t-online.de
Subject: 5P, CSC and twin conics, cubics and quartics

```
Dear Bernard,

wrt your #3769, attached an approximately drawing of the curve,
… it will be a sextic, CSC-invariant,
… but not bearing the isg and their CSC.

Best regards Eckart
```



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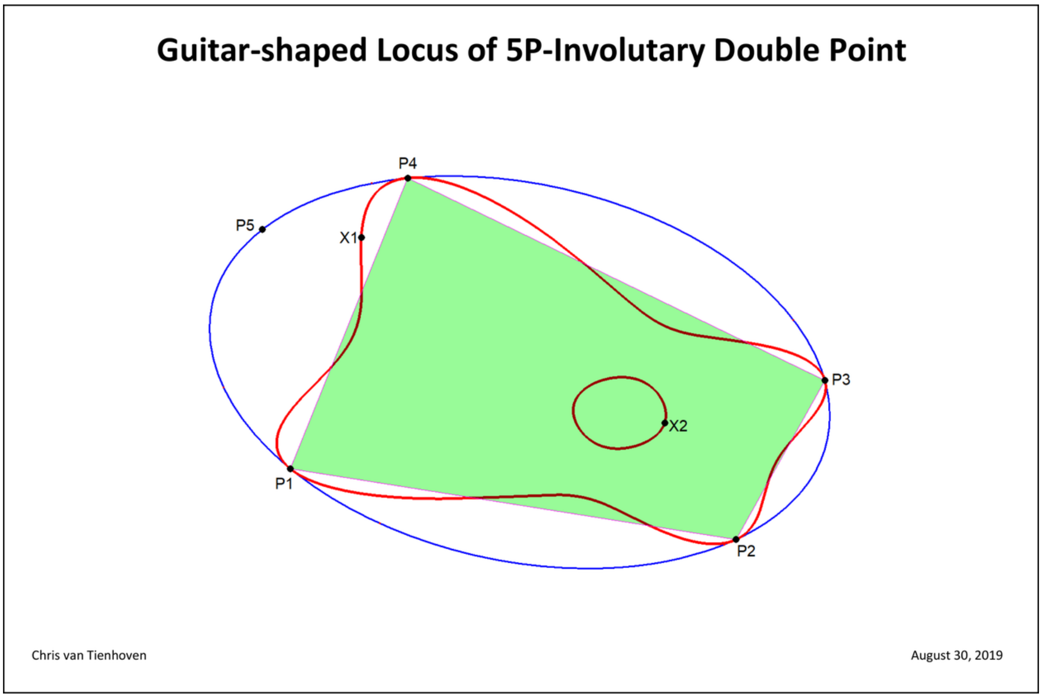
Message: #3772
Date: 30/8/2019 12:37:51
From: chris.vantienhoven
Subject: 5P-s-2Px giving Guitar-shaped Locus

Dear Eckart,
That are two remarkable 5P-points indeed.
Let's call these points X1 and X2.
I found these properties:
1. QA-Tf2(Xi) wrt any QA(PiPjPkPl) = (Xi-polar ^ Pm-tangent) wrt 5P-circumscribed-conic.
2. When the 5P = QA + QA-DT-point Si, then X1 & X2 are the intersection points of the two QA-sides through Si and the QA-DT-side opposite Si.
3. Spanning a QA P1.P2.P3.P4 in a reference Conic C0 and some extra point P5 on this conic, the locus of X1 and X2 with fixed QA and variable P5 gives *a very special guitar-shaped-locus* (see attached picture) with these properties:
a. It is tangent to C0 at the QA-vertices.
b. Each of the 6 QA-sides is intersected at the two vertices and two other points, each intersected by one of the defining QA-Tf2(tangent)-conics. Therefore these points can be easily constructed and calculated.
Regardless of these special properties somehow I think there should be some other simple property related to these points.
Best regards,
Chris



5p-s-2P99 Guitar-shaped Locus-01.pdf

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Message: #3773
Date: 01/9/2019 2:51:03
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

```
Dear Chris, dear Benedetto,

background for this circle is the message 2910:
Consider a 5P and for each vertex Pi
… the four midpoints of Pi.Pj and the quadrangle
… of their images wrt 5P-s-Tf3inv = Co-Tf3 wrt 5P-s-Co1.
The five QA-P4 of these quadrangles are concyclic.

Best regards Eckart

PS: Thanks to Chris for further observations for 5P-s-2Px.
```

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Message: #3774
Date: 01/9/2019 8:59:13
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px

```
Dear Chris,

just studying my old messages,
… I found the 5P-s-2Px already in #2912.

Best regards Eckart
```

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Message: #3775
Date: 02/9/2019 12:13:13
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

```
Dear Eckart,
Thanks for your answer, specially for the drawing !
Is it possible to generalise ?
Having a CSC, any Newton Line gives a Van Rees focal circular
cubic, which is CSC invariant.
We found 2 copples cubic/quartic CSC partners with 2 Newton
circles.
Now, this sextic is CSC invariant with a rectangular hyperbola
as Newton conic ...
The construction could be the same as the Van Rees cubic : for a
point m taken as middle, the line through 2 CSC partners is the
bisector of the angle F1mF2 with F1 and F2 fixed points of the
CSC and the 2 CSC partners are on a circle through the 2 fixed
points, harmonic conjugates of these 2 fixed points on the
circle.
Best regards
Bernard
```

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Message: #3776
Date: 02/9/2019 2:28:21
From: chris.vantienhoven
Subject: 5P-s-Cix

```
Dear Eckart,

Peculiar 5P-Circle you found.
Do you see special properties for this cirlce?
I couldn't find any.

Best regards,
Chris
```

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Message: #3777
Date: 03/9/2019 1:17:46
From: eckart_schmidt@t-online.de
Subject: QL-Lines wrt QA-Tf2(P)

Dear Chris,

in earlier messages already mentioned:
For a QL the three QA-Tf2 images of a point are collinear.
… Where can I find this in EQF?
This gives a QL-transformation TF, which maps points to lines.

Some examples, perhaps new:
TF(QL-P1) = QL-P26.CSC(QL-P24) = CSC(QL-Ci6)
TF(QL-P8) = line through QL-P23, parallel to its QL-Tf2-image.
TF(QL-P13 = line through QL-P19, parallel QL-L9.
TF(infinity point of QL-L1) = QL-L9.

Best regards Eckart

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Message: #3778
Date: 04/9/2019 9:22:04
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,

if I am not wrong, the following doesn't hold (#3769, #3775)
"... the middles of the segments XiYi (with Yi and Xi CSC
partners) lie on a rectangular hyperbola".

Best regards Eckart

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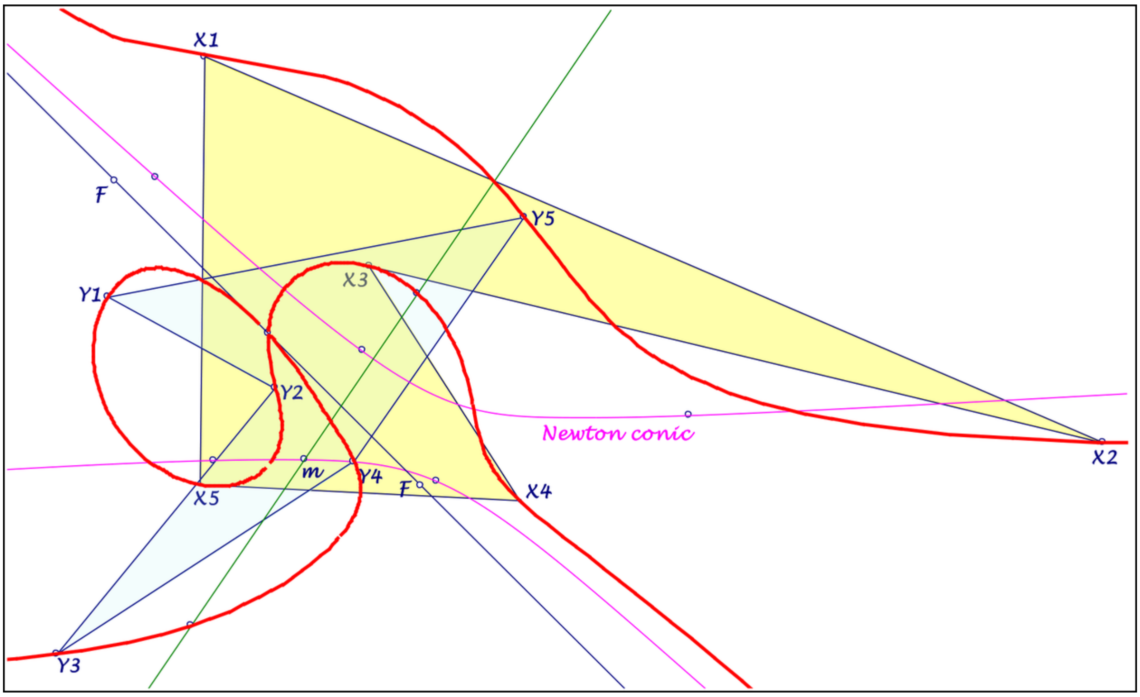
Message: #3779
Date: 04/9/2019 9:53:04
From: eckart_schmidt@t-online.de
Subject: 5P, CSC and twin conics, cubics and quartics

```
Dear Bernard,

for the twin 5P X1...X5 and Y1...Y5 with Yi = CSC(Xi)
… and a Newton conic through the midpoints of XiYi
… we can construct the sextic as you described in #3775,
… see attached file.

Best regards Eckart
```



2019-09-04.pdf

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Message: #3780
Date: 04/9/2019 11:19:22
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

```
Dear Eckart,
Thanks for your correction about the RH and your confirmation
about the construction of the sextic.
Both cubics may be constructed the same way with the common
Newton Line and the CSC2 with centers U or  CSC(U) and both
quartics with the 2 Newton circles and the CSC3 with centers
CSC(T) or CSC(T').
Best regards
Bernard
```

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Message: #3781
Date: 07/9/2019 11:58:33
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

Dear Bernard, dear Chris,

this is a very curious excursion in QL-geometry,
… orientated at a QL-transformation TF,
… which maps a point to QA-P4 of its pedal QA.

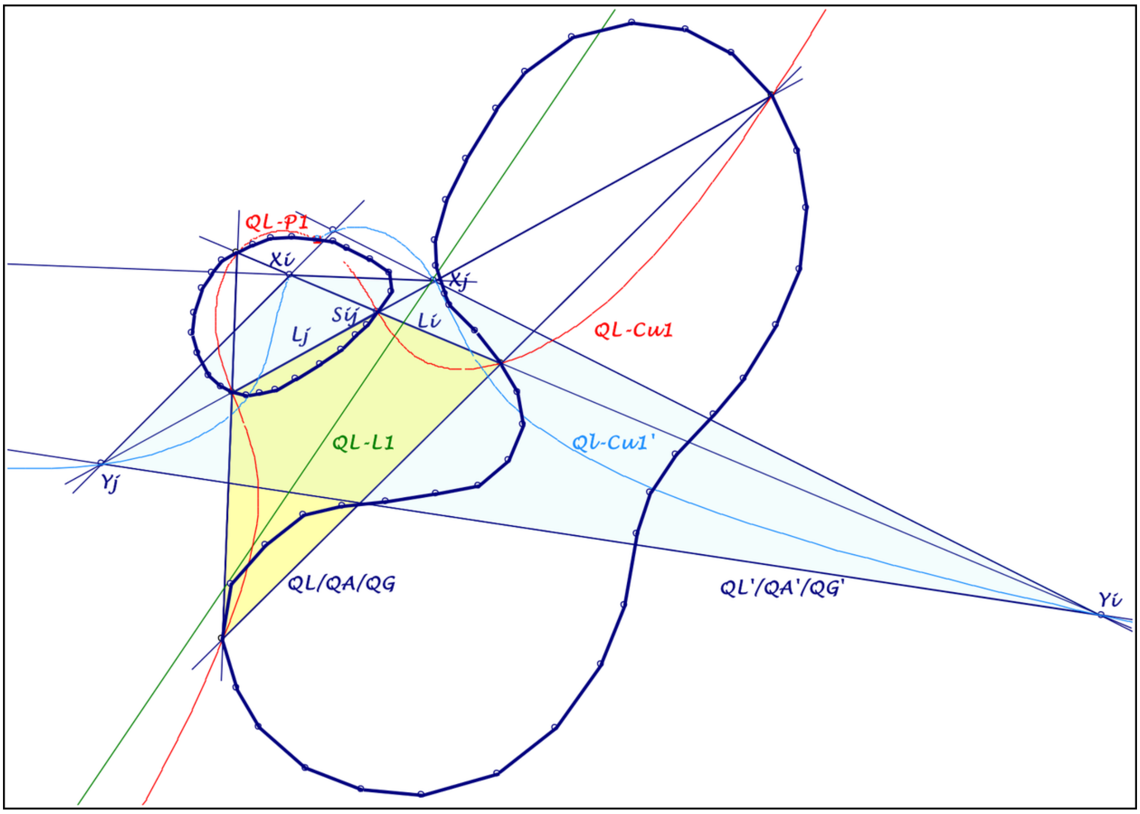
Well known is, that TF(QL-Cu1) gives QL-L1 (see EQF),
… TF of CSC-partner on QL-Cu1 is their midpoint.
But there are further points X with TF(X) on QL-L1,
… their locus is a QL circumscribed quartic (sextic?) through QL-P1.
Fixed points of TF are QL-2P3
… and four other points Xi,Yi and Xj,Yj
… on two special lines Li, Lj of the QL.
… The intersection Sij of Li and Lj
… is the nearest QL-point to the line of QG-2P3
… for the convex QG-version of the QL.
The points Xi,Yi and Xj,Yj are CSC-partner
… and easy to construct.

XiXjYiYj can be considered as
(1) quadrigon QG'
with Li, Lj as diagonals and QG'-P1 = Sij.
(2) quadrangle QA' with QA'-P4
as opposite QL-point of Sij,
(3) *quadrilateral QL' with the same CSC as the reference QL.*

What about the quartic (sextic?), mentioned above?

Best regards Eckart

PS: It's the first time, that the convex QG-version of a QL plays a role.



2019-09-06.pdf

Message: #3782
Date: 08/9/2019 2:08:37
From: eckart_schmidt@t-online.de
Subject: CSC-invariant QA-circles

Dear Bernard, dear Chris,

if you are interested, here a short excursion in QA-geometry:
Consider a QA, its QA-Tr2=ABC with incenter I and excenters Ia,Ib,Ic,
… further the 3 QG-versions, interpreted as QL with QL-Tf1=CSC:
… The locis for points X, which are concyclic with the 3 CSC(X)
… are three circles Cia,b,c,
… … centered in the excenters of QA-Tr2 on QA-Cu1
… … with radius ra=sqr(A.Ia*I.Ia), analog rb, rc,
… … Cia,b intersect orthogonal in QLc-2P3, analog Cib,c and Cic,a,
… … with radical center I.

I think, these three QA-circles will be the only,
… which are invariant wrt the three CSC.

Consider for example Cia:
… CSCa-partner on Cia are collinear with I,
… CSCb-partner on Cia are collinear with Ic,
… CSCc-partner on Cia are collinear with Ib.

Best regards Eckart

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Message: #3783
Date: 08/9/2019 7:47:17
From: Benedetto Scimemi
Subject: 5P-s-Cix

Dear Eckart, dear Chris, after a long geometric ... vacation, I happened to read my name in the address of #3773, where Eckart describes an interesting 5P-circle that he discovered in 5P geometry. Thanks to Eckart who thus re-awoke my curiosity. It seems to me, however, that Co-Tf3 does not play any essential role in his definition. Its action (a similarity) can be actually ignored by giving the following simpler statement:

Given a pentagon A1...A5 , for each index i = 1...5 let Oi denote the isoptic point (QA-P4) of the Ai-complementary quadrangle (e.g. O1 is QA-P4 of A2A3A4A5 etc.) .
Then the 5 midpoints Mi = (Ai +Oi)/2 lie on a circle 5P-Ci-?

I tried - without success - to prove this unexpected result by angular properties. I was also disappointed, like you probably were, not finding any other property neither for this circle or the original one of Eckart. I will try more.

Incidentally, if a somewhat similar situation is considered within quadrangles, one can state:
In a quadrangle A=A1...A4 for each index i=1...4 let Oi denote the circumcenter of the Ai-complementary triangle (e.g. O1 is the circumcenter of A2A3A4). Then the 4 midpoints (Ai +Oi)/2 are the vertices of a quadrangle QA-Q? which is negatively similar to QA.
I wonder: are this similarity and its fixed-point already present in EQF ?

P.S. I take this opportunity to suggest to Chris, as I perhaps already did years ago, that - if he ever finds the time - he could add some pages of EQF, regarding *QA-quadrangles*. For example, the above O1O2O3O4 could be denoted by QA-Qx (and similarly QA-Qy for nine-points-, QA-Qz for orthocenters- etc). There exist in fact interesting relations among them and a nice sort of algebra can be introduced in their set (map composition and midpoints).

Best regards Benedetto

Message: #3784
Date: 08/9/2019 9:58:06
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

```
Dear Benedetto, dear Chris,

thanks to Benedetto for his interest
… and his simpler definition in #3783
for the 5P-circle 5P-s-Cix in # 3773.

But now I noticed, that this circle plays a role
… in our discussion wrt the QA-Cu7-triple points:

Consider the orthogonal hyperbola (see #3579, #3581)
… through 5P-s-P5 and centered in the middle of 5P-s-P4.5
… with axes parallel to those of 5P-s-Co1,
… intersecting 5P-s-Co1 in three points unequal 5P-s-P4,
… which give a triangle with nine-point circle 5P-s-Cix.

Best regards Eckart
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Message: #3785
Date: 08/9/2019 11:23:24
From: bernard.keizer
Subject: QA-P4 of pedal-QA

```
Dear Eckart,
Very interesting new quartic !
I try to reproduce your property.
But I could not find TF in EQF.
Could you please give me the exact reference where it is
mentionned ...
Many thaks in advance
Best regards
Bernard
```

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Message: #3786
Date: 09/9/2019 9:25:03
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

```
Dear Benedetto,

I think, my message 3784 isn't correct:
your described circle in #3783 isn't my circle in #3773,
… but in #3784
I described your circle,
… which is Quang Duong's circle, see 5P-s-P3.

Best regards Eckart
```

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Message: #3787
Date: 09/9/2019 10:36:03
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

Dear Bernard,
the transformation TF isn't in EQF, but this property under
QL-Cu1, 6, QL-Cu1 is the locus of all points P
… for which the feet of the perpendiculars from P to the 4
quadrilateral lines
… are concyclic (all lie on a circle). "
QA-P4 of a cyclic QA is the center of the circumcircle ... ...
I thought, this was the reason for TF(QL-Cu1) = QL-L1,
… but you are right, this isn't mentioned in EQF, but it holds!
Best regards Eckart

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Message: #3788
Date: 10/9/2019 4:59:50
From: bernard.keizer
Subject: QA-P4 of pedal-QA

In fact, your formulation was rather elliptic !
Sure, the QA-P4 of 4 concyclic points is the circumcenter (the
isogonal of one of the 4 points wrt the triangle of the 3 others
is at the infinity and it's inverse is the circumcenter), but I
don't see why it follows that this circumcenter is precisely the
middle of the segment joining 2 CSC partners on QL-Cu1 ...
Anyhow, as you say, the property holds !
For the points Xi and j and Yi and j, they are the intersections
of a line of the QL (through 3 QL vertices) and the reference
circumcircle through the 3 other QL vertices.
For 2 lines of the QL, the points are real, for the 2 others,
they are imaginary.
I checked that the 6 mentionned points are fixed points of TF.
But that doesn't help to understand the quartic !
I haven't found anything until now, but I'm stil searching ...
Did you try to draw the CSC curve of the quartic ? It could be a
cubic through the 6 QL vertices with asymptote parallel to the
Newton Line ...
Best regards
Bernard
PS The fact that QL' has the same CSC as QL is obvious, as you
start with 2 copples of CSC partners ...

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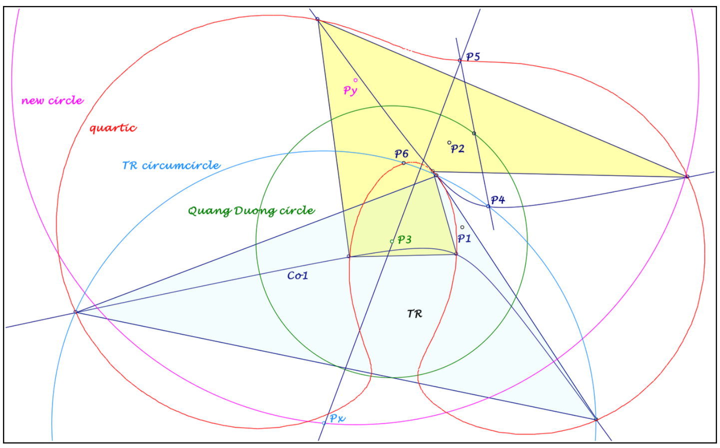
Message: #3789
Date: 10/9/2019 8:51:15
From: eckart_schmidt@t-online.de
Subject: New aspects of 5P-geometry

```
Dear Bernard, dear Chris,

in our discussion wrt the QA-Cu7-triple points
… we found new 5P-elements and their properties
… wrt the points 5P-s-P1,2,3,4,5,6,
… a reference triangle TR ( see #3579, #3581, #3784)
… and circles:
… … Quang Duong circle, centered in 5P-s-P2,
… … circumcircle of TR, centered in 5P-s-Px
… … and a new circle, centered in 5P-s-Py (see #3773),
… further the circumconic 5P-s-Co1, centered in 5P-s-P1,
… a circumquartic (see #3599),
… finally the transformation 5P-s-Tf3.

Properties:
(1)   5P-s-P5 is the orthocenter of TR (see #3579).
(2)   5P-s-P4 and 5P-s-P6 lie on the circumcircle of TR
      (see #3579).
(3)   5P-s-P5 und 5P-s-P6 lie on the quartic (see #3599).
(4)   5P-s-Px is the reflection of 5P-s-P5 in 5P-s-P3.
(5)   5P-s-Px is a point of the new circle.
(6)   5P-s-Tf3(5P-s-Py) = 5P-s-P3.
(7)   5P-s-Tf3(5P-s-Px) is the midpoint of 5P-s-P4,5.
(8)   5P-s-Tf3 maps the new circle to the Quang Duong circle.

Best regards Eckart
```



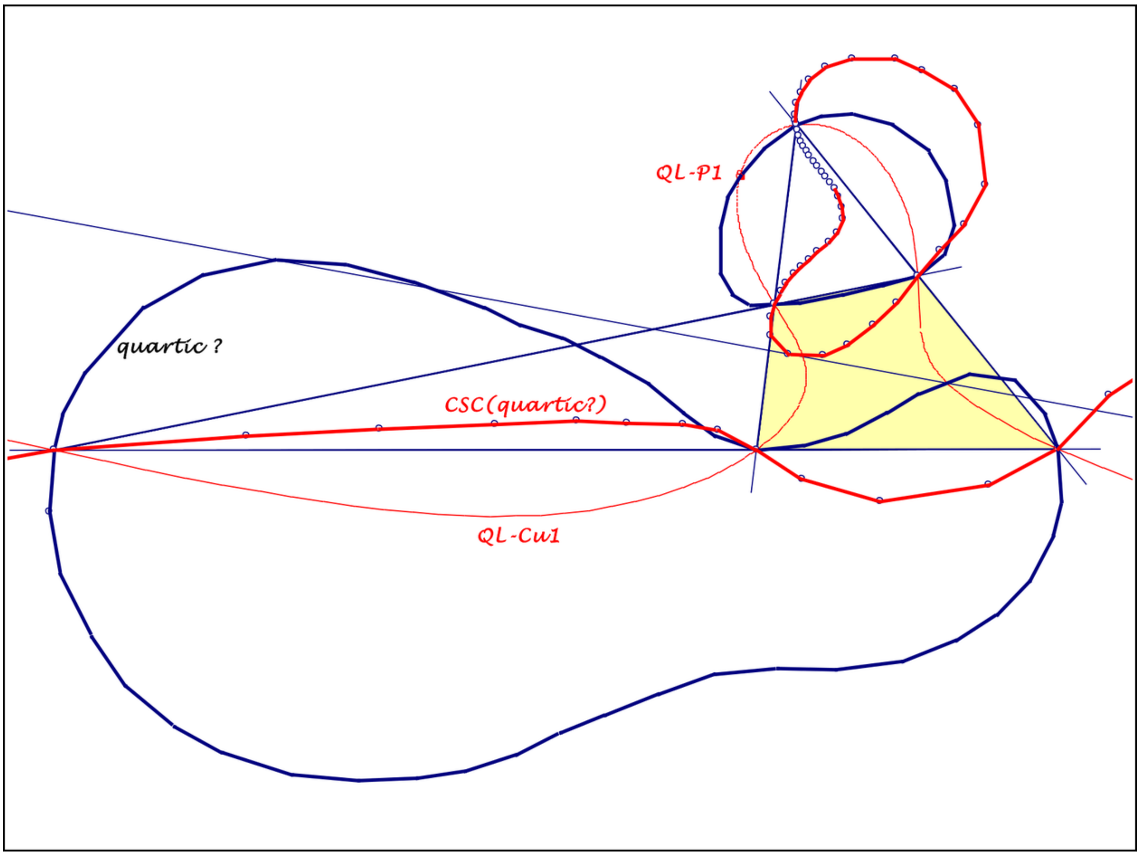
2019-09-10.pdf

Message: #3790
Date: 11/9/2019 11:19:56
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

```
Dear Bernard,

attached a drawing for the CSC of the supposed quartic,
… perhaps a quintic?
I found no properties.

Best regards Eckart
```



2019-09-11.pdf

Message: #3791
Date: 15/9/2019 10:18:16
From: bernard.keizer
Subject: New aspects of 5P-geometry

```
Dear Eckart, my previous answer went lost, I don't know why ...
I'm glad that you begin to gather properties of the 5P geometry.
I'm sad that you didn't mention either the cubic CSC(quartic) or
the twin cubic (circular, also through the 5P and the point U,
like the quartic.
Last, I asked for precisions wrt Tf3 : Quang Duong circle
(centered in P3 and not P2) carries the middles of the segments
Xiisg'i, UT and UM ; you mentionned that the Px is Tf3(midpoint
of UT, what are the Tf3 of the 6 other points ?
Thanks in advance
Best regards
Bernard
```

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Message: #3792
Date: 18/9/2019 11:45:37
From: eckart_schmidt@t-online.de
Subject: New aspects of 5P-geometry

```
Dear Bernard,

excuse, that I haven't mentioned your cubic and the twin-aspect,
… which is orientated at the 5P-CSC transformation.
The properties in #3789 are dependent on the transformation
5P-s-Tf3,
… which is only defined by the conic 5P-s-Co1,
… that is a weak point of this view,
… but 5P-s-Tf3 maps lines to lines,
… circles to circles, conics to conics inclusive center.

Background for my research was the aspect,
… that the mentioned triangle TR may be a reference triangle
  for 5P-geometry:
… 5P-s-P5 is the orthocenter X4 of TR,
… 5P-s-P3 is the nine-point center X5 of TR,
… the Quang Duong circle is the nine-point circle of TR,
… … bearing the midpoints of 5P-s-P4,5 and 5P-s-P5,6,
… 5P-s-P4 is the TR-isogonal conjugate of the infinity point
… … of the tangent in the TR circumcenter at the new circle,
… and for TR = ABC with optuse angle at  A:
… … The pedal point of 5P-s-P4 on BC
… … connected with the middle of 5P-s-P4,5
… … is parallel to the line through the points 5P-s-Px,y.

Wrt the transformation 5P-CSC:
… The circles CSC(AB), CSC(AC), CSC(BC) through 5P-s-P6
… … intersect further pairwise and collinear
… … on your cubic in CSC(A), CSC(B), CSC(C),
… … with centers on C.5P-s-P4, B.5P-s-P4, A.5P-s-P4
… … concyclic with 5P-s-P4,6 and CSC(5P-s-P5).
… CSC of this circle is a line through 5P-s-P5,
… … which is the degenerated circle Ci(P) (see #3575)
… … with P intersection of 5P-s-P4.5P-s-P6 and 5P-s-Co1.

But all these properties without systematic and concept.
I searched in vain for a relevant TR-isoconjugation.

Best regards Eckart

PS: Thanks for correcting the typo:
… Center of the Quang Duong circle is 5P-s-P3.
```

Message: #3793
Date: 18/9/2019 5:25:13
From: bernard.keizer
Subject: New aspects of 5P-geometry

Dear Eckart,
Thanks for this long message, I understand better what you are looking for !
I have to investigate more in this transformation Tf3.
Is it possible (if it swaps points to points, lines to lines, circles to circles ...) that it is a similitude ?
Hence my question about other examples , Tf3(P4,5 or 6) or Tf3(circumcircle of TR) ?
The line PxPy is parallel to the Simson or Steiner Line of P4 wrt this circle.
The Steiner Lines of P4 and P6 intersect obviously in P5 (orthocenter of TR), but more interesting, the Steiner Line of P6 passes trough CSC(P4).
Best regards
Bernard

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Message: #3794
Date: 18/9/2019 6:11:55
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear geometers,

Let A,B,C,D be a QA.
P is any point.
H1 is midpoint of the segment connecting two orthocenters of triangles PAB, PCD.
H2 is midpoint of the segment connecting two orthocenters of triangles PBC, PAD.
H3 is midpoint of the segment connecting two orthocenters of triangles PAC, PBD.
Then H1, H2, H3 are collinear on line d.
Thus we have a transformation on QA from point P to line d. Is this transformation known before?

Best regards
Tran Quang Hung.

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Message: #3795
Date: 18/9/2019 10:00:39
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

```
Dear Quang Hung,

at the moment I cannot answer
… whether the transformation is in EQF,
… but there is a nice property for points on the cubic QA-Cu1:
For P on QA-Cu1 the corresponding line
… is a perpendicular to P.QA-P3 through QA-P2.
Thus QA-Cu1 is the locus for points,
… whose corresponding line bears QA-P2.

Best regards Eckart
```

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Message: #3796
Date: 19/9/2019 9:38:56
From: Tran Quang Hung
Subject: A transformation on QA from point to line

```
Dear Eckart,

Thank You very much for interesting property of QA-cu1 and this
transformation.

Best regards
Tran Quang Hung.
```

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Message: #3797
Date: 19/9/2019 11:44:14
From: bernard.keizer
Subject: New aspects of 5P-geometry

Dear Eckart,
The fixed point of the transformation is 5P-s-P1, the center of the circumconic (property in EQF).
On your beautiful figure with message 3789, it holds that the distances between 5P-s-P1 and X and 5P-s-P1 and Tf3(X) are in the same ratio for X = Py and X = Px.
It seems also that PyP1P3 and PxP1mid(P4P5) have the same bisector.
Could the transformation Tf3 be the product of an axial symmetry and an homothety with center 5p-s-P1 ?
I don't know if it was obvious, please just tell me if it is correct ...
Thanks in advance
Best regards
Bernard

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Message: #3798
Date: 19/9/2019 2:30:24
From: eckart_schmidt@t-online.de
Subject: Wrt 5P-s-Tf1,2,3,4

Dear Chris, dear Bernard,

the transformation 5P-s-Tf3 is conical, it is Co-Tf3inv.
5P-s-Tf3 maps lines to lines,
… circles to circles, conics to conics, inclusive center,
… this should be mentioned in EPG under 5P-s-Tf3.
I think, that the constructions of 5P-s-Tf1,2,3,4inv in #2758
… are also worth to be mentioned in EPG.

Best regards Eckart

PS for Bernard wrt #3797:
5P-s-Tf3(P) is the reflection of P in the main axis of 5P-s-Co1,
… stretched from 5P-s-P1 with factor (a^2+b^2)/(a^2-b^2) (see Co-Tf3).

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Message: #3799
Date: 19/9/2019 8:48:29
From: eckart_schmidt@t-online.de
Subject: New aspects of 5P-geometry

Dear Bernard,

thanks for your remarks wrt #3792:
The Simson line of P4 is a better description of my parallel to Px.Py.

New for me: "... Steiner line of P6 …",
… this is a better description of my degenerated circle Ci(P).
Thus CSC(P4) is the intersection of the Steiner line of P6
… and the line through CSC(A), CSC(B), CSC(C).

New: Tf3(P4) is the intersection of the Simson line of P4
… and a perpendicular to the Simson line of P6 through Tf3(P6).

Best regards Eckart

PS. Wrt 5P-s-Tf3 see #3798.

Message: #3800
Date: 19/9/2019 10:15:52
From: chris.vantienhoven
Subject: A transformation on QA from point to line

```
Dear Quang Hung and dear Eckart,

Very nice construction of the transformation of Tran Quang Hung
in message #3794.
Some remarks of mine regarding this transformation,

X(4)-Transformation
Let QA-Tfx4a(P) = Tran Quang Hung's Ortholine.
Let QA-Tfx4b(P) = line through P perpendicular to QA-Tfx4a(P).
The coordinates of QA-Tfx4b(P) are way simpler than the
coordinates of QA-Tfx3b(P),

Properties:
1. Let (i,j,k,l) be the four indices of reference points
(P1,P2,P3,P4).
   Let Lij = Line (Pi,Pj) and Ci = Circle (Pj,Pk,Pl).
   Let Pij = 2nd intersection point of Ci ^ Lij.
   Then QA-Tfx4b (Pij) = Lij.
2. When P is a point on Ci,
   then QA-Tfx4b(P) is a line through Pi.
3. The locus of QA-Tfx4a(P) with variable P on Ci is a hyperbola
   tangent to the nine-point circle of triangle(Pj,Pk.Pl)
   and with center X(5) of triangle(Pj,Pk.Pl).

X(2)-Transformation
When doing the same construction with the centroid instead of
the orthocenter,
it appears that the 3 midpoints coincide in one point
QA-Tfx2a(P)  with these coordinates:
{4px+3qx+3rx+2py+qy+ry+2pz+qz+rz,
  px+2qx+rx+3py+4qy+3ry+pz+2qz+rz,
  px+qx+2rx+py+qy+2ry+3pz+3qz+4rz}
where P={x,y,z} and P4={p,q,r}.

X(3)-Transformation
When doing the same construction with the circumcenter instead
of the orthocenter,
it again appears that the midpoints are collinear and
QA-Tfx3a(P) // QA-Tfx4a(P)   &  QA-Tfx3b(P) coincides with
QA-Tfx4b(P).

X(5)-Transformation
When doing the same construction with the nine-point center
instead of the orthocenter,
```

```
it again appears that the midpoints are collinear and
QA-Tfx5a(P) // QA-Tfx4a(P)   &  QA-Tfx5b(P) coincides with
QA-Tfx4b(P).
For me the main results of this exercise are the bycatches:
QA-Tfx2a(P)  with these CT-coordinates:

{4px+3qx+3rx+2py+qy+ry+2pz+qz+rz,
  px+2qx+rx+3py+4qy+3ry+pz+2qz+rz,
  px+qx+2rx+py+qy+2ry+3pz+3qz+4rz}

QA-Tfx3b(P) = QA-Tfx4b(P) = QA-Tfx5b(P)
with these CT-coefficients:
{-y z (-r y + q z) (-c^2 p q x^2 - b^2 p r x^2 - a^2 q r x^2 +
c^2 p^2 x y - b^2 p r x y + c^2 p r x y - a^2 q r x y + b^2 p^2
x z + b^2 p q x z - c^2 p q x z - a^2 q r x z + a^2 p^2 y z +
a^2 p q y z + a^2 p r y z),

x z (-r x + p z) (c^2 q^2 x y - b^2 p r x y - a^2 q r x y + c^2
q r x y - c^2 p q y^2 - b^2 p r y^2 - a^2 q r y^2 + b^2 p q x z
+ b^2 q^2 x z + b^2 q r x z + a^2 p q y z - c^2 p q y z + a^2
q^2 y z - b^2 p r y z),

-x y (-q x + p y) (c^2 p r x y + c^2 q r x y + c^2 r^2 x y - c^2
p q x z - a^2 q r x z + b^2 q r x z + b^2 r^2 x z - c^2 p q y z
+ a^2 p r y z - b^2 p r y z + a^2 r^2 y z - c^2 p q z^2 - b^2 p
r z^2 - a^2 q r z^2)}

Remarkable is that the coefficients of QA-Tfx4b(P) are composed
of corresponding coefficients of lines and the equations of
corresponding circles.

Best regards,
Chris
```

Message: #3801
Date: 19/9/2019 10:24:04
From: davidfraivert
Subject: The theory of a convex quadrilateral and a circle that forms Pascal

---

Dear Geometers,

The following six articles present the main definitions and properties of "the theory of a convex quadrilateral and a circle that forms Pascal points on its sides".
The general case of a convex quadrilateral:
1)
The theory of a convex quadrilateral and a circle that forms "Pascal points" - the properties of "Pascal points" on the sides of a con- vex quadrilateral.
http://dx.doi.org/10.18642/jmsaa_7100121666
2)
Properties of the tangents to a circle that forms Pascal points on the sides of a convex quadrilateral.
http://forumgeom.fau.edu/FG2017volume17/FG201726.pdf
The case of a cyclic quadrilateral:
3)
The Theory of an Inscribable Quadrilateral and a Circle that Forms Pascal Points.
http://dx.doi.org/10.18642/jmsaa_7100121742
4)
Pascal-points quadrilaterals inscribed in a cyclic quadrilateral.
https://doi.org/10.1017/mag.2019.54
The case of an orthodiagonal quadrilateral:
5)
Properties of a Pascal points circle in a quadrilateral with perpendicular diagonals.
http://forumgeom.fau.edu/FG2017volume17/FG201748.pdf
6)
A Set of Rectangles Inscribed in an Orthodiagonal Quadrilateral and Defined by Pascal-Points Circles.
http://www.heldermann.de/JGG/JGG23/JGG231/jgg23002.htm

Regards,
David Fraivert

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Message: #3802
Date: 20/9/2019 7:27:10
From: Tran Quang Hung
Subject: A transformation on QA from point to line

```
Dear Eckart, dear Chris,

Thank You very much for your interest. Thank you for pointing
out more interesting transformations besides that. Thus these
transformations will be true for all point on Euler line and
divide OH in the same ratio.

Best regards
Tran Quang Hung.
```

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Message: #3803
Date: 20/9/2019 9:18:37
From: Thanh Tung
Subject: A transformation on QA from point to line

```
Dear all,

I found another property  of this transformation:
Let given 5 points P1, P2, P3, P4, P5.
Let h1 = Tran Quang Hung's Ortholine of P1 w.r.t quadrangle
P2P3P4P5,  define h2, h3, h4, h5 similarly,
Then h1, h2, h3, h4 ,h5 concur at a point.
What is this point w.r.t 5P:   P1P2P3P4P5 ?

Best regards,
Vu Thanh Tung

Le vendredi 20 septembre 2019 à 12:27:15 UTC+7, Tran Quang Hung
analgeomatica@gmail.com [Quadri-Figures-Group] a écrit :
> Dear Eckart, dear Chris,
> Thank You very much for your interest. Thank you for pointing
out more interesting transformations besides that.
> Thus these transformations will be true for all point on Euler
line and divide OH in the same ratio.
> Best regards
> Tran Quang Hung.
```

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Message: #3804
Date: 20/9/2019 9:31:07
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

```
Dear Chris, dear Quang Huong, dear Thanh Tung

wrt #3803: The point is 5P-s-P4.

Best regards Eckart
```

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Message: #3805
Date: 20/9/2019 10:10:42
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

```
Dear Chris, dear Quang Hung,

with interest I have read Chris' #3800
wrt the X(4)-transformation,
… here a special property:
Points X on QA-Cu1 are mapped by QA-Tfx4b to X.QA-P3.

Best regards Eckart
```

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Message: #3806
Date: 20/9/2019 11:13:43
From: Thanh Tung
Subject: A transformation on QA from point to line

```
Dear all,

We can rewrite #3794, #3800 and #3802 as the following form:
Definition:  Given a constant t.  For a triangle ABC with
circumcenter O and orthocenter H, the  t-Eulerline of triangle
ABC is the point T such that  OT = t OH.
ABCD is a quadrangle, P is any point, t is constant
T1 is midpoint of the segment
   connecting two t-Eulerline of triangles PAB, PCD.
T2 is midpoint of the segment
   connecting two t-Eulerline of triangles PBC, PAD.
T3 is midpoint of the segment
   connecting two t-Eulerline of triangles PAC, PBD.

Then :
_ T1, T2, T3 are collinear on line d(t).
_ All line d(t) are parallel
_ the distance between d(t1) and d(t_2)
  is linearly of proportional to | t1 - t2 |

Best regards,
Vu Thanh Tung

Le vendredi 20 septembre 2019 à  15:10:49 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Chris, dear Quang Hung,
> with interest I have read Chris' #3800 wrt the
X(4)-transformation,
> … here a special property:
> Points X on QA-Cu1 are mapped by QA-Tfx4b to X.QA-P3.
> Best regards Eckart
```

Message: #3807
Date: 20/9/2019 3:30:19
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear Vu Thanh Tung, dear Chris, dear Eckart,

I mentioned the general problem d(t) on the above message
>> Thus these transformations will be true for all point on
Euler line and
>> divide OH in the same ratio.
> I guess that if we have five points A1, A2, A3, A4, A5
> d1= d(t)(A1, A2, A3, A4)
> Define similarly d2, d3, d4, d5.
> Then d1, d2, d3, d4, d5 are concurrent for all t.
> Best regards
> Tran Quang Hung.

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Message: #3808
Date: 20/9/2019 3:32:27
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Chris, dear Quang Hung, dear Thanh Tung,

If we consider with Chris' nomination in #3800
… QA-Tfx4a for the vertices of a 5P wrt the remaining QA,
… we get as common point 5P-s-Tf3(5P-s-P5).

If we consider QA-Tfx4b
… we get as common point 5P-s-P4 (see #3804).

If we consider points P,
… whose QA-Tfx4a-lines wrt the 4P of the 5P are parallel
… or whose QA-Tfx4b-lines coincide,
… we get the fixed points of the Cayley-Bacharach transformation
… wrt the 5P-vertices and the circular points.
Very unexpected properties!

Best regards Eckart

PS: Sorry, I used in #3804 already QA-Tfx4b!

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Message: #3809
Date: 21/9/2019 1:16:13
From: chris.vantienhoven
Subject: A transformation on QA from point to line

```
Dear friends,

I mentioned in message #3800 the transformation QA-Tfx2a(P) with
these simple coordinates:
    {4px+3qx+3rx+2py+qy+ry+2pz+qz+rz,
      px+2qx+rx+3py+4qy+3ry+pz+2qz+rz,
      px+qx+2rx+py+qy+2ry+3pz+3qz+4rz}
where P={x,y,z} and P4={p,q,r}.
It is remarkable that the variables {a,b,c} do not occur in the
coordinates of the mapped point.
It can be constructed as the midpoint of Centroid(P,P1,P2) and
Centroid(P,P3,P4).
It is the same point as the midpoint of Centroid(P,P1,P3) and
Centroid(P,P2,P4) and
the midpoint of Centroid(P,P1,P4) and Centroid(P,P2,P3).

I investigated the transformation a bit further with these
results.
· QA-Tfx2a(QA-Px) lies on QA-P1.QA-Px (1 : 2)
· when QA-Px, QA-Py, QA-Pz are collinear then QA-Tfx2a(QA-Px),
  QA-Tfx2a(QA-Py), QA-Tfx2a(QA-Pz) are collinear

Let QA-Tfx2a(QA-Px) be denoted as Tfx, then:
· Tf1  = QA-P1
· Tf3  = QA-P34
· Tf10 = QA-P26
· Tf20 = QA-P10
· Tf24 = QA-P14
· Tf2 & Tf34 lie on QA-L1
· Tf5 & Tf18 & Tf25 & Tf26 & Tf43 lie on QA-L3
· Tf6 & Tf23 lie on QA-L4
· Tf15 lies on QA-L6
· Tf2  is collinear with QA-P36, QA-P37
· Tf4  is collinear with QA-P3,  QA-P6
· Tf4  is collinear with QA-P20, QA-P28
· Tf7  is collinear with QA-P1,  QA-P4
· Tf12 is collinear with QA-P3,  QA-P36
· Tf12 is collinear with QA-P5,  QA-P39
· Tf12 is collinear with QA-P13, QA-P20
· Tf16 is collinear with QG-P3,  QG-P12
· Tf16 is collinear with QA-P10, QA-P27
· Tf16 is collinear with QA-P19, QA-P43
· Tf19 is collinear with QA-P20, QA-P21, QA-P31
```

- Tf21 is collinear with QA-P5, QA-P31
- Tf23 is collinear with QA-P1, QA-P6
- Tf29 is collinear with QA-P2, QA-P43
- Tf29 is collinear with QA-P5, QA-P35
- Tf29 is collinear with QA-P10, QA-P34
- Tf30 is collinear with QA-P3, QA-P11
- Tf30 is collinear with QA-P29, QA-P37
- Tf30 is collinear with QA-P35, QA-P39
- Tf31 is collinear with QA-P16, QA-P43
- Tf35 is collinear with QA-P3, QA-P43
- Tf36 is collinear with QA-P30, QA-P43
- Tf37 is collinear with QA-P5, QA-P11
- Tf37 is collinear with QA-P13, QA-P43
- Tf39 is collinear with QA-P37, QA-P43

Best regards,

Chris

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Message: #3810
Date: 21/9/2019 9:56:52
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

```
Dear Chris, dear Quang Hung, dear Thanh Tung,

this is an analogon to Quang Hung's transformation in #3794
… for a QL and a line, with application to a 5L.

Consider a QL and a line L
… and the 3 midpoints of orthocenters
… for two trilaterals (L,Li,Lj) and (L,Lk,Ll),
… which are collinear on a line L'.
  Example: L' of QL-L2 is QL-L1.

More interesting for a 5L:
The 5 lines L' of the 5L-lines wrt the remaining QL
… have a common point,
… which devides 5L-n-P5.5L-n-Px with ratio -1:3,
… 5L-n-Px is Morley's "first orthocenter of the 5-line" in
… F. Morley: Orthocentric properties of the plane n-line
  (theorem 3), 1903.

Replacing the orthocenter X4 by X2, X3, X5,
… we get corresponding line to line transformations for a QL.

Best regards Eckart
```

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Message: #3811
Date: 22/9/2019 12:06:27
From: Thanh Tung
Subject: New line to line transformation for QL

```
Dear all,

Again this is very nice transformation. And again, as Quang Hung
pointed out in the 5P case and Eckart in the 5L case, this
transformation can be generalized to any point in the Euler
line.

In other words:
Definition:  Given a constant t.  For a triangle ABC with
circumcenter O and orthocenter H, the  t-Eulerline of trianlge
ABC is the point T such that  OT = t OH.
t=0  : T=X(3);    t=1:  T=X(4);     t=1/3: T= X(2),
t=1/2: T=X(5),    t=-1: T=X(20)

Given a quadrilateral L1 L2 L3 L4 , a line L and a constant t
T2 is midpoint of the segment connecting two t-Eulerline of
triangles formed by two trilaterals (L,L1, L2) and (L,L3,L4)
T3 is midpoint of the segment connecting two t-Eulerline of
triangles formed by two trilaterals (L,L1, L3) and (L,L2,L4)
T4 is midpoint of the segment connecting two t-Eulerline of
triangles formed by two trilaterals (L,L1, L4) and (L,L2,L3)

Then T2, T3, T4 are collinear on a line d(L,t).
When t varies it appears that the line d(L,t) tangent to a fixed
curve (a parabola ?)  What is this curve ?

Best regards,
Vu Thanh Tung
```

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Message: #3812
Date: 22/9/2019 4:31:15
From: Thanh Tung
Subject: New line to line transformation for QL

```
Dear all,

In the case of X2 (t=1/3), consider a 5L (L1, L2, L3, L4, L5)
let d1 = transforming line of L1 w.r.t. QL (L2, L3, L4, L5) and
define d2, d3, d4, d5 similarly.
Then the 5 lines di are concurrent at a point.
This point is different to the concurrent point mentioned
in the case of X4 (t=1) by Eckard. What is this point?

In the case of X3 (t=0), the transforming lines of one line L
w.r.t any QL (L1, L2,L3, L4) perpendicular to the line L.

Best regards,
Vu Thanh Tung
```

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Message: #3813
Date: 22/9/2019 6:01:19
From: Tran Quang Hung
Subject: New line to line transformation for QL

Dear Chris, dear Eckart, dear Thanh Tung,

It is very interesting transformation with 4L and 5L.
Thank You so much for new idea and contributing.
I also think about the similar things on Quadrigon Object?

Best Regards
Tran Quang Hung.

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Message: #3814
Date: 22/9/2019 6:03:06
From: Tran Quang Hung
Subject: Similar figures of seven points

Dear geometers,

I see the following problems
Let A1,A2,A3,A4,A5,A6 be six points and any point P.
Then midpoints of the segments connecting QA-P1(P,Ai,Aj,Ak) and
QA-P1(P,Am,An,Ap) are the same point for all different sets of
numbers (i,j,k,m,n,p).
Note that this point is not Centroid of (A1,A2,A3,A4,A5,A6) or
Centroid of (P,A1,A2,A3,A4,A5,A6)?
So we have the transformation of Seven points X =
Tfx(P,[A1,A2,A3,A4,A5,A6]).
Now we have seven points A1,A2,A3,A4,A5,A6,A7.
Let Xi = Tfx(Ai,[A1,...,A(i-1)A(i+1),...,A7]), we have seven
points X1,X2,X3,X4,X5,X6,X7.
And then two figures (A1,A2,A3,A4,A5,A6,A7) and
(X1,X2,X3,X4,X5,X6,X7) are similar with ratio 8.

Best Regards
Tran Quang Hung.

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Message: #3815
Date: 22/9/2019 6:25:55
From: Thanh Tung
Subject: Similar figures of seven points

Dear all,

For the first part, one can prove that X=
Tfx(P,[A1,A2,A3,A4,A5,A6]) is the point such that GX= 1/4 GP (in vector) where G is the centroid of the 6P[A1,A2,A3,A4,A5,A6].

For the second part, the 7P(X1,X2,X3,X4,X5,X6,X7) is image of the 7P(A1,A2,A3,A4,A5,A6,A7) under the Homothetic transformation of center G and ratio 1/8, here G is common centroid of the two 7Ps.

The result then can be extended to 2n-P, the ratio of the first part will be 1/(n+1) and the second part will be 1/2(n+1).

Best regards,
Vu Thanh Tung

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Message: #3816
Date: 22/9/2019 9:35:19
From: Tran Quang Hung
Subject: A transformation on QA from point to line

```
Dear Chris, dear Eckart, dear Vu Thanh Tung,

Follow X(3)-Transformation of Chris, I also see that
Let A,B,C,D be a QA.  P is any point.
(O1) is the circle with diameter is the segment
     connecting circumcenters of triangles PAB, PCD.
(O2) is the circle with diameter is the segment
     connecting circumcenters of triangles PBC, PAD.
(O3) is the circle with diameter is the segment
     connecting circumcenters of triangles PAC, PBD.

Then three circles (O1), (O2), and (O3) are coaxial. Thus we
shall have a transformation from P to this radical axis?
But the same thing is not right for orthocenter, and if we
change
(O1) is the circle with diameter is the segment connecting two
points that devide OH segment of triangles PAB, PCD in ratio t.
(O2) is the circle with diameter is the segment connecting two
points that devide OH segment of triangles PBC, PAD in ratio t.
(O3) is the circle with diameter is the segment connecting two
points that devide OH segment of triangles PAC, PBD in ratio t.
For which number t then (O1), (O2), (O3) are coaxial?

Best regards
Tran Quang Hung.
```

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Message: #3817
Date: 22/9/2019 1:54:02
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

```
Dear Quang Hung,

CABRI-observations wrt #3816:
    "For which number t then (O1), (O2), (O3) are coaxial?"

This holds only for t=0.

Best regards Eckart
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Message: #3818
Date: 22/9/2019 2:13:50
From: Tran Quang Hung
Subject: A transformation on QA from point to line

```
Dear Eckart,

Thank You very much. I hope the Transformation from P to this
common radical axis is new.

I guess this result (coaxial circles) is also true for X3 of 4L?

Best Regards
Tran Quang Hung.
```

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Message: #3819
Date: 22/9/2019 4:20:57
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Quang Hung,

wrt #3816, #3818:
If we take this transformation
… for a point and the 5 QA of a 5P,
… and consider the case, that the 5 lines coincide,
… we get the fixed points of the Cayley-Bacharach transformation
… wrt the 5P-vertices and the circular points (as in #3808).

Best regards Eckart

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Message: #3820
Date: 22/9/2019 10:14:24
From: eckart_schmidt@t-online.de
Subject: EPG-question

Dear Chris,

what is the nomination in EPG for
… Morley's first orthocenter
… and point 5L-P6 in #710.

Excuse my uncertainty in using EPG.

Best regards Eckart

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Message: #3821
Date: 22/9/2019 10:46:54
From: chris.vantienhoven
Subject: EPG-question

Dear Eckart,

Here is a conversion list for the codes mentioned in QFG#710.
5L-P1 now is the 5L-version of nL-o-P2, being 5L-o-P2.
5L-P2 now is the 5L-version of nL-n-P3, being 5L-n-P3.
5L-P3 now is the 5L-version of nL-n-P1, being 5L-n-P1.
5L-P4 now is 5L-s-P1
5L-P5 now is the 5L-version of nL-n-P6, being 5L-n-P6.
5L-P6 now is the 5L-version of nL-n-P8, being 5L-n-P8.
5L-P7 now is 5L-s-P2
5L-P8 now is 5L-s-P3
5L-P9 (Morley's 1st Orthocenter) now is the 5L-version of nL-o-P1, being 5L-o-P1.
Points with suffix "nL-n-" are points with versions for each n.
Points with suffix "nL-o-" are points with versions for odd n.
Points with suffix "5L-s-" are specific points only occurring for 5-Lines.
Like you can see the points we discerned at the beginning of our research happened to be a set of points occurring for all n, or all odd n, or only specific for n=5.

Best regards,
Chris

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Message: #3822
Date: 23/9/2019 11:10:48
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

Dear Chris, dear Quang Hung, dear Thanh Tung,

in #3810 I described a QL-transformation,
… orientated to the orthocenter X4
… mapping lines to lines L --> L',
… with an application for 5L,
  leading to a point,
… which divides 5L-n-P5.5L-o-P1 with ratio -1:3.

Taking the transformation, orientated to the centroid X2,
… and applicated to a 5L, we get a point,
… which divides 5L-n-P8.5L-s-P1 with ratio -2:5.

Only for X2 and X4 on the Euler line
… the transformation leads to a 5L-point!

Best regards Eckart

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Message: #3823
Date: 23/9/2019 9:32:49
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

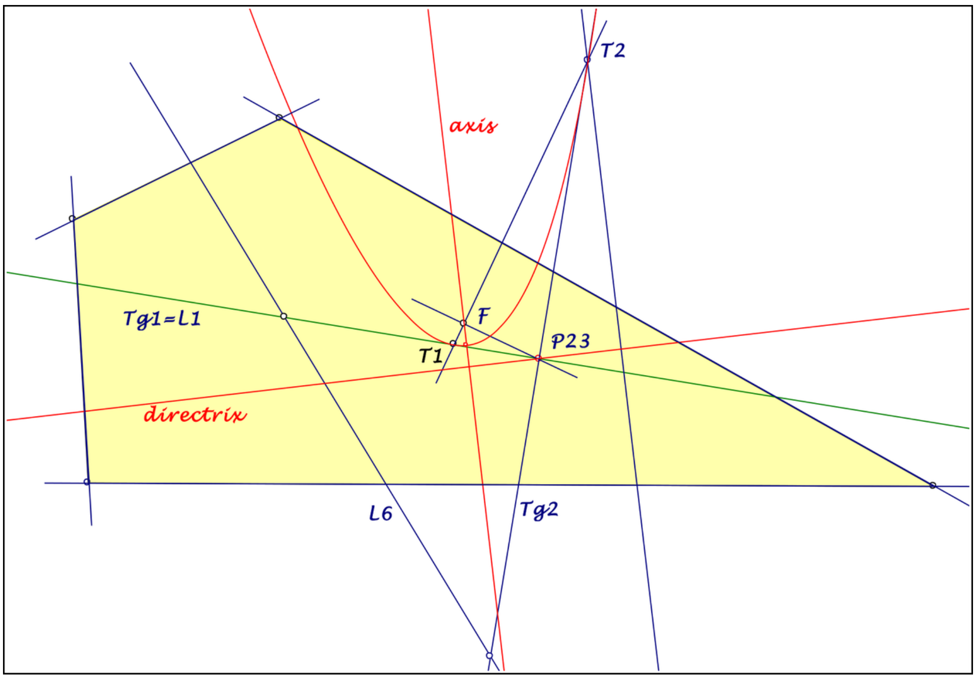
```
Dear Thanh Tung,

wrt your #3811:
"When t varies it appears that the line d(L,t) tangent to a
 fixed curve (a parabola ?)  What is this curve ?"
You are right, the lines d(L,t) envelop a parabola.

Example for the Newton line QL-L1 (see attached file),
… here an independent construction of this parabola:

QL-P23 is a point of the directrix
… and intersection of two orthogonal tangents:
… Tg1 = QL-L1 with contact point T1,
… … which divides QL-P23 and QL-L1^QL-L6 with ratio 1:2,
… Tg2 is perpendicular to QL-L1 in QL-P23 with contact point T2,
… … which is the reflection of Tg2^QL-L6 in QL-P23.
… Focus F is the pedal point of QL-P23 on T1T2.
… Direction of the axis is given with the line T1T2
  reflected in Tg2.
… Directrix is a perpendicular to the axis through QL-P23.

Best regards Eckart
```



2019-09-23.pdf

Message: #3824
Date: 24/9/2019 4:03:53
From: Thanh Tung
Subject: New line to line transformation for QL

```
Dear Eckart,

I think that the parabola is the QL-Co1 (Inscribed Parabola)
of the QL formed by the X2, X3, X4, X5 - transformations of line
L, or in general by any t1, t2, t3, t4 - Eulerlinepoint
transformation lines.

Best regards,
Vu Thanh Tung

Best regards Eckart
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Message: #3825
Date: 24/9/2019 4:55:18
From: Thanh Tung
Subject: A conjecture on Newton line and inscribed parabola of quadrilateral

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Dear all,

I have a conjecture
on Newton line and inscribed parabola of quadrilateral as
follows:

_  Consider a QL(L1, L2, L3, L4) with newton line d
   and insribed parabola c.
_  Let Uij  = Li ^ Lj with U'13 U23 = t . U13 U23;
   U'14 U24 = t . U14 U24 (in vector ) where t is a real number
_  Let L'1 = line passing through U'13 and U'14.
_  Let d(t) be the Newton line of the QL(L'1, L2, L3, L4) .

Then:
_  The line L'1 envelops the parabola c.
   In other words,
   the inscribed parabola of the QL(L'1, L2, L3, L4)
   remains the same when t varies.
_  The line Newton line d(t) is always parallel to the line d.
_  The distance between d(t1) and d(t2) is proportional to
   | t1 - t2 |.

Are these observations true?

Best regards,
Vu Thanh Tung
```

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Message: #3826
Date: 25/9/2019 10:45:45
From: eckart_schmidt@t-online.de
Subject: A conjecture on Newton line and inscribed parabola of quadrilate

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Dear Thanh Tung,

wrt #3825:
CABRI constructions confirm your observations.

Best regards Eckart

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Message: #3827
Date: 25/9/2019 2:44:56
From: Thanh Tung
Subject: A conjecture on Newton line and inscribed parabola of quadrilate

---

Dear Eckart,

Thank you for your confirmation.

Best regards,
Vu Thanh Tung

Le mercredi 25 septembre 2019 à 15:45:50 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Thanh Tung,
> wrt #3825:
> CABRI constructions confirm your observations.
> Best regards Eckart

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Message: #3828
Date: 26/9/2019 11:53:22
From: bernard.keizer
Subject: A conjecture on Newton line and inscribed parabola of quadrilate

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```
Dear Thanh Tung, dear Eckart,
This interesting property is easy to prove analytically.
Writing the equation of the parabola with focus M on the y-axis
and directrix parallel to the x-axis as y = x 2, the equation of
the tangent in a point with x-coordinate x0 is y = 2xx0 - x0 2.
The 4 lines of the QL are tangent to the parabola in points with
x-coordinate x1, x2, x3 and x4.
The vertices of the QL have x-coordinates xij = 1/2 (xi + xj)
and y-cordinates yij = xi*xj
Then, with your notations, the line L'1 through U'13 and U'14 is
the tangent to the parabola in the point with x-coordinate
(1 - t)x2 + tx1.
If L'1 is tangent to the parabola, it forms with 3 other
tangents a new QL with the same Miquel point (focus of the
parabola) and the same Steiner Line (directrix of the parabola)
and the Newton Lines are all parallel, as perpendicular to the
directrix ...
Best regards
Bernard
```

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Message: #3829
Date: 30/9/2019 5:17:31
From: Antreas Hatzipolakis
Subject: Feuerbach point lies on OI line

[Tran Quang Hung]:

> Let ABC be a triangle.
> OI line meets BC, CA, AB at A', B', C', resp.
> Let Oa, Ob, Oc be the circumcenters of the triangles
> AB'C', BC'A', CA'B' respectively.
> Then Feuerbach point of triangles OaObOc
> lies on the OI line of ABC.
> Which is this point?

[APH]

Dear Tran

The same is true for the Brocard axis OK instead of the OI line:
The point X(115) of OaObOc lies on the Brocard axis
I think this generalization is true:
Let ABC be a triangle and P a point.

Denote:
D = the Poncelet point of ABCP
L = the isogonal conjugate of the rectangular circumhyperbola
centered at D (it is a line)
A', B', C' = the intersections of L and BC, CA, AB, resp.
Oa, Ob, Oc = the circumcenters of AB'C', BC'A', CA'B', resp.
Then the D point of OaObOc lies on the line L

For P = I: L = OI line, D = X(11)
For P = G: L = OK line, D = X(115)

Now, call the point D of OaObOc as (P, ABC) and let P1P2P3P4 be
a quadrangle.
Which are the properties of the quadrangle (P1, P2P3P4) (P2,
P3P4P1) (P3, P4P1P2) (P4, PQP2P3)?
Is it cyclic?

APH

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Message: #3830
Date: 30/9/2019 6:14:50
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

I found an interesting family of centers in quadrangle geometry.
The construction is as the following:

1. A triangle transformation:
_ Given an integer n, The barycentric coordinates of a point P w.r.t △ABC is (x : y : z).
_ The transformation Q = Tr (P, △ABC, n) maps P to the point Q with barycentric coordinates (x^n : y^n : z^n) w.r.t. △ABC

2. A point on quadrangle geometry.
_ Consider a quadrangle P1 P2 P3 P4 and an integer n.
_ Let Qi = Tr (Pi, △ Pj Pk Pl, n).

Then 4 lines Pi Qi concur at a point X(n).

Some existing points:
_ n = 0: X(0) = QA-P1: Quadrangle centroid (the triangle transformation is fixed to the centroid)
_ n =-1: X(-1)= QA-P5: Isotomic center (the triangle transformation is the isotomic transformation)

Is this family of points known before ?

Best regards,
Vu Thanh Tung

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Message: #3831
Date: 30/9/2019 7:50:08
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

Please find attached a proof.
Best regards,

Vu Thanh Tung

A family of points in Quadrangle

Vu Thanh Tung

October 1, 2019

Abstract

A family of points in Quadrangle is given.

In this short note, I define a family of points in Quadrangle.

Firstly a family of transformation of a point with respect to a triangle is defined as the following.

Definition 0.1. *For each integer n, the transformation $tr_n(P, \triangle ABC)$ maps the point P of barycentric coordinates $P = (x : y : z)$ with respect to $\triangle ABC$ to the point $Q = (x^n : y^n : z^n)$.*

Now we are able to construct a family of points in quadrangle.

Theorem 1 (Vu Thanh Tung, [1])**.** *Consider a quadrangle $P_1P_2P_3P_4$ and an integer n. Define $Q_1 = tr_n(P_1, \triangle P_2P_3P_4)$, $Q_2 = tr_n(P_2, \triangle P_3P_4P_1)$ $Q_3 = tr_n(P_3, \triangle P_4P_1P_2)$ and $Q_4 = tr_n(P_4, \triangle P_1P_2P_3)$.*

Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point $X(n)$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4$, $\triangle P_3P_4P_1$, $\triangle P_4P_1P_2$, $\triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and we can write:

$$P_4 = (-\frac{s_1}{s_4}).P_1 + (-\frac{s_2}{s_4}).P_2 + (-\frac{s_3}{s_4}).P_3 \tag{1}$$

The point Q_4 has barycentric coordinates (s_1^n, s_2^n, s_3^n) with respect to $\triangle P_1P_2P_3$ and we can write:

$$Q_4 = \frac{1}{s_1^n + s_2^n + s_3^n}\left(s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3\right) \tag{2}$$

Define the point $X(n)$ as the following weighted center of $P_1P_2P_3P_4$:

$$X(n) = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n}\left(s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3 + s_4^n.P_4\right) \tag{3}$$

$X(n)$ clearly lies on the line P_4Q_4 as we can write:

$$X(n) = \frac{s_1^n + s_2^n + s_3^n}{s_1^n + s_2^n + s_3^n + s_4^n}.Q_4 + \frac{s_4^n}{s_1^n + s_2^n + s_3^n + s_4^n}.P_4 \tag{4}$$

1

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$X(n)$ divides segment P_4Q_4 as $\frac{\overline{X(n)Q_4}}{\overline{X(n)P_4}} = \frac{s_4^n}{s_1^n+s_2^n+s_3^n}$.

Similarly. $X(n)$ lies on the lines P_1Q_1, P_2Q_2, P_3Q_3. □

Corollary 1.

- *For $n = 0$ we get $X(0) = QA\text{-}P1(P_1P_2P_3P_4)$ (quadrangle centroid).*
- *For $n = -1$ we get $X(-1) = QA\text{-}P5(P_1P_2P_3P_4)$ (qudrangle isotomic center).*

Proof. For $n = 0$ the triangle transformation becomes $\text{tr}_0(P, \triangle ABC) =$ centroid of $\triangle ABC$.

For $n = -1$, the triangle transformation becomes
$\text{tr}_{-1}(P, \triangle ABC) =$ isotomic conjugate of P with respect to $\triangle ABC$. □

The two points QA-P1($P_1P_2P_3P_4$) and QA-P5($P_1P_2P_3P_4$) are described in [2] and [3] respectively.

References

[1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Sep 30, 2019, https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3830

[2] ENCYCLOPEDIA OF QUADRI-FIGURES, *QA-P1: QA-Centroid or Quadrangle Centroid* , https://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p1

[3] ENCYCLOPEDIA OF QUADRI-FIGURES, *QA-P5: Isotomic Center* , https://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p5

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: tungvtt@gmail.com

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Message: #3832
Date: 01/10/2019 3:35:29
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

I will reword the result as following:

Result:
_ Given a quadrangle P1 P2 P3 P4 and an integer n.
_ Let Qi be the n-power of Pi w.r.t. △Pj Pk Pl.
Then 4 lines Pi Qi concur at a point X(n).

ex: X(0) = quadrangle centroid,
X(-1) = quadrangle isotomic center.

May we call the point Xn the n-power point of the quadrangle P1 P2 P3 P4?
Here, the n-power point of P =(x : y :z) w.r.t. a △ABC is the point P^n= (x^n : y^n :z^n).
0-power of point P = centroid of △ABC, (-1)-power of point P = isotomic conjugate of P w.r.t. △ABC.

The n-power point P^n can be recursively constructed by isotomic conjugate and the Paul Yiu's construction of multiplication of points given in this paper:
The uses of homogeneous barycentric coordinates in plane Euclidean geometry
(http://math.fau.edu/yiu/PSRM2015/yiu/LAPTOPbackup030317/Geometry98/Barycentric/IJMEST/barycentric.pdf)

Also I think that playing with multiplication of points can give new points in quadrangle.

Best regards,
Vu Thanh Tung

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Message: #3833
Date: 01/10/2019 8:05:02
From: Tran Quang Hung
Subject: Feuerbach point lies on OI line

Dear Mr Antreas and César,

It is interesting with L = the isogonal conjugate of the rectangular circumhyperbola centered at D (it is a line).

Let ABC be a triangle and P a point.
Denote:
D = the Poncelet point of ABCP
L = the isogonal conjugate of the rectangular circumhyperbola
    centered at D (it is a line)
L = Tf(ABC,P)

Let P1P2P3P4 be a quadrangle.
L1 = Tf(P2P3P4,P1)
L2 = Tf(P3P4P1,P2)
L3 = Tf(P2P4P1,P1)
L4 = Tf(P1P2P3,P4)
Then L1, L2, L3, L4 are concurrent at a point. Which is this point of QA(P1,P2,P3,P4)?

Best regards
Tran Quang Hung.

Vào Th 3, 1 thg 10, 2019 vào lúc 06:30 Tran Quang Hung < analgeomatica@gmail.com > đã viết:

>> Dear Mr Antreas and César,
>> Thank you very much for your interest and nice generalization.
>> Apply this generalization to a quadrangle which is interesting,
>> but I have not seen aspecial property on this configuration?
>> Do you see anything?
>> Best regards
>> Tran Quang Hung.
>> Vào Th 2, 30 thg 9, 2019 vào lúc 23:50 César E. Lozada cesar_e_lozada@yahoo.es
>> [Anopolis] < Anopolis@yahoogroups.com > đã viết:
>>> Dear geometers,
>>> Also X(56) of triangle OaObOc lies on OI line of ABC. Which is this point?
>>> Best Regards
>>> Tran Quang Hung.

```
>>> Vào Th 2, 30 thg 9, 2019 vào lúc 19:24 Tran Quang Hung
>>> <analgeomatica@gmail.com> đã viết:
>>> Let ABC be a triangle.
>>> OI line meets BC, CA, AB at A', B', C'.
>>> Let Oa, Ob, Oc be circumcenters of triangles AB'C', BC'A',
>>> CA'B' respectively.
>>> Then Feuerbach point of triangles OaObOc lies on OI line
>>> of ABC. Which is this point?
>>> Best Regards
>>> Tran Quang Hung.
>>> *Q = midpoint OF X(56) and X(23981)*
>>> = a^3*(a+b-c)*(a-b+c)*(a^5-(b+c)*a^4-2*(b-c)^2*a^3+2*(b^3+c^
3)*a^2+(b^4+c^4-(4*b^2-b*c+4*c^2)*b*c)*a-(b+c)*(b^4+c^4-(2*b^2-b
*c+2*c^2)*b*c))*((b+c)*a^2-2*b*c*a-(b^2-c^2)*(b-c)) : : (barys)
>>> = lies on these lines: {1, 3}, {1403, 6769}, {6767, 8069}
>>> = midpoint of X(56) and X(23981)
>>> = [ 0.0183414481164834, 0.3097215998853760,
3.4177765520868950 ]
>>> César Lozada
```

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Message: #3834
Date: 01/10/2019 1:57:41
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

```
Dear Thanh Tung,

very interesting messages!
Wrt #3831, 2. first constructions show,
… that the lines from the QA-vertices
… to the n-power point for n=2
… wrt the remaining triangle
… have the common point QA-P16.
The n-power point for n=3 gives QA-P10.
I hope, someone can confirm these results.

Best regards Eckart
```

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Message: #3835
Date: 01/10/2019 6:21:52
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Eckart,
I am able to confirm your observations.
My calculation gives the 1st CT-Coordinate of the n-power point is: p (p^(n-1) - (-p -q -r) ^(n-1))
Please see the proof in attachement.
Applying for n=2,3 we obtain the coordinates of QA-P16 and QA-P10. Also, the formula (3) in attachement gives an easy generalization to Polygon Geometry.
Best regards,   Vu Thanh Tung

n^{th}-power point of Quadrangle

Vu Thanh Tung

October 1, 2019

Abstract

The notation of n^{th}-power point Quadrangle is given.

In this short note, I define a family of points in Quadrangle. For each integer n, the n^{th}-power of a point with respect to a triangle is defined as follows:

Definition 0.1. *For each integer n, the n^{th}-power of a point P with barycentric coordinates $(x : y : z)$ with respect to $\triangle ABC$ to the point P^n with barycentric coordinates $(x^n : y^n : z^n)$ with respect to $\triangle ABC$.*

Now we are able to construct a family of points in quadrangle.

Theorem 1 (Vu Thanh Tung, [1])**.** *Consider a quadrangle $P_1P_2P_3P_4$ and an integer n. Let Q_1 be the n^{th}-power of P_1 with respect to $\triangle P_2P_3P_4$, Q_2 be the n^{th}-power of P_2 with respect to $\triangle P_3P_4P_1$, Q_3 be the n^{th}-power of P_3 with respect to $\triangle P_4P_1P_2$, Q_4 be the n^{th}-power of P_4 with respect to $\triangle P_1P_2P_3$.*

Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point X_n called the n^{th}-power point of Quadrangle $P_1P_2P_3P_4$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4$, $\triangle P_3P_4P_1$, $\triangle P_4P_1P_2$, $\triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and we can write:

$$P_4 = (-\frac{s_1}{s_4}).P_1 + (-\frac{s_2}{s_4}).P_2 + (-\frac{s_3}{s_4}).P_3 \tag{1}$$

The point Q_4 has barycentric coordinates $(s_1^n : s_2^n : s_3^n)$ with respect to $\triangle P_1P_2P_3$ and we can write:

$$Q_4 = \frac{1}{s_1^n + s_2^n + s_3^n}\left(s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3\right) \tag{2}$$

Define the point X_n as the following weighted center of $P_1P_2P_3P_4$:

$$X_n = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n}\left(s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3 + s_4^n.P_4\right) \tag{3}$$

X_n clearly lies on the line P_4Q_4 as we can write:

$$X_n = \frac{s_1^n + s_2^n + s_3^n}{s_1^n + s_2^n + s_3^n + s_4^n}.Q_4 + \frac{s_4^n}{s_1^n + s_2^n + s_3^n + s_4^n}.P_4 \tag{4}$$

1

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X_n divides segment P_4Q_4 as $\frac{\overline{X_nQ_4}}{\overline{X_nP_4}} = \frac{s_4^n}{s_1^n+s_2^n+s_3^n}$.

Similarly. X_n lies on the lines P_1Q_1, P_2Q_2, P_3Q_3. □

Corollary 1 (Coordinates)**.** *The 1st CT-Coordinate of* X_n *is :*

$$p\big(p^{n-1} - (-p-q-r)^{n-1}\big) \tag{5}$$

Proof. We note that $p : q : r = s_1 : s_2 : s_3$ and $s_4 = -s_1 - s_2 - s_3$. From (1) and (4) we have:

$$X_n = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n}\Big(\sum_{i=1}^{3} s_i\big(s_i^{n-1} - s_4^{n-1}\big).P_i\Big) \tag{6}$$

The conclusion then follows. □

Corollary 2 (Some known points)**.**

- $X_{-1} = QA\text{-}P5(P_1P_2P_3P_4)$ *(Qudrangle isotomic center).*
- $X_0 = QA\text{-}P1(P_1P_2P_3P_4)$ *(Quadrangle centroid).*
- $X_2 = QA\text{-}P16(P_1P_2P_3P_4)$ *(QA-Harmonic Center).*
- $X_3 = QA\text{-}P10(P_1P_2P_3P_4)$ *(Centroid of the QA-Diagonal Triangle).*

The point X_2, X_3 were observed by Eckart Schmidt.

References

[1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Sep 30, 2019, `https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3830`

[2] ENCYCLOPEDIA OF QUADRI-FIGURES, , `https://www.chrisvantienhoven.nl/mathematics/encyclopedia`

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: `tungvtt@gmail.com`

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Message: #3836
Date: 01/10/2019 6:36:20
From: eckart_schmidt@t-online.de
Subject: Feuerbach point lies on OI line

```
Dear Quang Hung,

the point asked for in # 3833 is QA-P4.

Best regards Eckart
```

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Message: #3837
Date: 01/10/2019 8:27:20
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

```
Dear Thanh Tung,

thanks for message 3835. In addition:
F or  n = 4 we get the 2nd intersection (beside QA-P21)
… of the line QA-P1.P16.P21 and the QA-circumconic
  through QA-P21.

For n = 5 we get the 2nd intersection (beside QA-P19)
… of the line QA-P10.P16.P19.P31 and the QA-circumconic
  through QA-P19.

Best regards Eckart
```

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Message: #3838
Date: 02/10/2019 1:01:55
From: Tran Quang Hung
Subject: Feuerbach point lies on OI line

```
Dear Eckart,
Thank You very much for your affirmation.
Best Regards
Tran Quang Hung.

Vào Th 3, 1 thg 10, 2019 vào lúc 23:43 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Quang Hung,
>> the point asked for in # 3833 is QA-P4.
>> Best regards Eckart
```

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Message: #3839
Date: 02/10/2019 2:43:05
From: Thanh Tung
Subject: A family of centers in quadrangle

```
Dear Eckart,
Thank you for the observations.
Best regards,
Vu Thanh Tung

Le mercredi 2 octobre 2019 à  01:29:13 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
Dear Thanh Tung,
thanks for message 3835. In addition:
For n = 4 we get the 2nd intersection (beside QA-P21)
… of the line QA-P1.P16.P21 and the QA-circumconic
  through QA-P21.
For n = 5 we get the 2nd intersection (beside QA-P19)
… of the line QA-P10.P16.P19.P31 and the QA-circumconic
  through QA-P19.

Best regards Eckart
```

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Message: #3840
Date: 02/10/2019 11:02:25
From: Tran Quang Hung
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Eckart and geometers,

We can see a general problem.
Let f(x) be a real function of x.
If P has barycentric coordinate (x,y,z) in triangle ABC. We consider f(P) has barycentric coordinate (f(x),f(y),f(z)) wrt ABC i.e denote by f(P,ABC).
Now with quadrangle P1P2P3P4.
Q1 = f(P1,P2P3P4)
Q2 = f(P2,P3P4P1)
Q3 = f(P3,P4P1P2)
Q4 = f(P4,P1P2P3)
Then the line P1Q1, P2Q2, P3Q3, P4Q4 are concurrent.

Proof.
Let s1, s2, s3, and s4 be signed area of triangles P2P3P4, P3P4P1, P1P2P3, and P2P3P4 respectively.
P1 has coordinate (s2,s3,s4) wrt triangle P2P3P4.
So that Q1 has coordinate (f(s2),f(s3),f(s4)) wrt P2P3P4. i.e [f(s2)+f(s3)+f(s4)]Q1= f(s2)P2+f(s3)P3+f(s4)P4.
We let X = [f(s1)P1 + f(s2)P2 + f(s3)P3 + f(s4)P4] / (f(s1)+f(s2)+f(s3)+f(s4))
Thus [(f(s1)+f(s2)+f(s3)+f(s4)]X = f(s1)P1 + [f(s2)+f(s3)+f(s4)]Q1.
Therefore P1Q1 goes through X. Similarly, P2Q2, P3Q3, P4Q4 go through X.
With f(x)=x^n we have problem of Vu Thanh Tung.
For which other function f?

Question: Can we use a homogeneous function f(x,y,z) from R^3->R instead of one variable function f(x)? Or what is the form of three variables function f(x,y,z) can be used for this concurrent property instead of one variable function f(x)?

Best regards
Tran Quang Hung.

Message: #3841
Date: 02/10/2019 11:15:06
From: Tran Quang Hung
Subject: A family of centers in quadrangle

```
Dear Thanh Tung, dear Eckart,

We also easily seen problem for n-point.
Let f(x) be a real function of x.
If P has barycentric coordinate (x,y,z) in triangle ABC. We
consider f(P) has barycentric coordinate (f(x),f(y),f(z)) wrt
ABC i.e denote by f(P,ABC).

Now with quadrangle P1P2P3P4.
Q1 = f(P1,P2P3P4)
Q2 = f(P2,P3P4P1)
Q3 = f(P3,P4P1P2)
Q4 = f(P4,P1P2P3)
Then the line P1Q1, P2Q2, P3Q3, P4Q4 are concurrent at
f(P1P2P3P4).
Thus with 5-point P1P2P3P4P5. Then lines
d1 = P1f(P2P3P4P5)
d2 = P2f(P3P4P5P1)
d3 = P3f(P2P1P4P5)
d4 = P4f(P2P3P1P5)
d5 = P5f(P2P3P4P1)
are concurrent at f(P1P2P3P4P5).
.....

Similarly by we can define inductive for n-points.

Best regards
Tran Quang Hung.
```

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Message: #3842
Date: 02/10/2019 12:19:35
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

```
Dear Thanh Tung, dear Chris,

there are interesting properties
… of Paul Yiu's multiplication of points, cited by Thanh Tung
  in #3832.

Here some splitter, perhaps evident wrt DT-coordinates:
The product of two isoconjugated points wrt a triangle
… is the the isoconjugate of the centroid.
The product of two QA-Tf2-partner wrt QA-Tr1 is QA-P16.
The products of a point (unequal QA-P1) and a QA-vertex wrt the
remaining QA-triangle
… are collinear, bearing QA-P16.

Examples:
… For points on QA-L3 we get collinearity on QA-P1.QA-P16.
… If L is the line for a point P, intersecting P.QA-P10 in Q,
… … then Q gives the line P.QA-P16.

Best regards Eckart
```

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Message: #3843
Date: 02/10/2019 7:11:29
From: Thanh Tung
Subject: A family of centers in quadrangle

```
Dear Tran Quang Hung,

I think your generalization is inot true in general.
Firstly if the function f is not homogeneous one cannot apply it
to the signed area
because if we scale up the quadrangle , the transformation P ->
f(P) will give different barycentric coordinates.
Rather we can apply f to the normalized barycentric coordinates.
But even then there is  counterexamples.
Take for example P1 P2 P3 P4 such that P4 = 5 P1 + 4 P2 - 8 P3
and f(x) = 1/(1+x).
I do think that only x^n  is the only function that works.
But I agree with you that for some forms of three variable f
that is homogeneous and bisymmetric (some kind of triangle
center function) that work.

Example1: f(x,y,z) = x ( y + z ) works,
The construction is as follows:
Let A1 B1 C1 be the cevian triangle of ABC.
A2, B2, C2 are the midpoint of A1B1 ,B1C1, C1A1
Then AA2, BB2 ,CC2 concur at a point Q.
That transforms
    P = (x,y,z) -> Q = (x(y+z), y(z+x), z(x+y)) = tr(P, ABC)
For QA-P1 P2 P3 P4, let Qi = tr(Pi, Pj Pk Pl) then 4 lines Pi Qi
are concurrent.

Best regards,
Vu Thanh Tung
```

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Message: #3844
Date: 02/10/2019 7:40:41
From: Thanh Tung
Subject: A family of centers in quadrangle

```
Dear Eckart,
They are very nice properties.
Best regards,
Vu Thanh Tung
```

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Message: #3845
Date: 02/10/2019 9:51:02
From: César E. Lozada
Subject: Feuerbach point lies on OI line

Then L1, L2, L3, L4 are concurrent at a point. Which is this point of QA(P1,P2,P3,P4)?

Answer: Q=QAP4(A,B,C,D)

With respect to ABC, given D,
Q = IsogonalConjugate(AntigonalConjugate(D))

I conjecture Q = IsogonalConjugate(AntigonalConjugate(Pi)) w/r to PjPkPl.

Best regards,
César Lozada

De: Quadri-Figures-Group@yahoogroups.com [mailto:Quadri-Figures-Group@yahoogroups.com]
Enviado el: martes, 1 de octubre de 2019 02:05 am
Para: Anopolis@yahoogroups.com; Quadri-Figures-Group@yahoogroups.com
Asunto: [Quadri-Figures-Group] Re: [EGML] Feuerbach point lies on OI line

> Dear Mr Antreas and César,
> It is interesting with L = the isogonal conjugate
> of the rectangular circumhyperbola centered at D (it is a line).
> Let ABC be a triangle and P a point.
> Denote:
> D = the Poncelet point of ABCP
> L = the isogonal conjugate of the rectangular circumhyperbola
> centered at D (it is a line)
> L=Tf(ABC,P)
> Let P1P2P3P4 be a quadrangle.
> L1 = Tf(P2P3P4,P1)
> L2 = Tf(P3P4P1,P2)
> L3 = Tf(P2P4P1,P1)
> L4 = Tf(P1P2P3,P4)
> Then L1, L2, L3, L4 are concurrent at a point.
> Which is this point of QA(P1,P2,P3,P4)?
> Best regards
> Tran Quang Hung.

Message: #3846
Date: 03/10/2019 2:59:11
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Tran Quang Hung,

If f(x) works then x + t . f(x) works, and in this case the concurrent points obtained from two functions are the same.
For now I can find only functions of the form x^n or x + t.x^n that work.
The point obtained in my example is the same as the point obtained by function x - x^2 or fuonction x^2 and is the point QA-P16.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à 00:11:34 UTC+7, Thanh Tung tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Tran Quang Hung,
> ....

Message: #3847
Date: 03/10/2019 3:03:52
From: Tran Quang Hung
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Eckart,

Thank You for your interest in my idea. I give a similar proof
of Thanh Tung in his file and the concurrent point in quadrangle
P1P2P3P4 is
X = [f(s1)P1 + f(s2)P2 + f(s3)P3 + f(s4)P4] /
(f(s1)+f(s2)+f(s3)+f(s4))?
Could You please check my proof?

>> Let f(x) be a real function of x.
>> If P has barycentric coordinate (x,y,z) in triangle ABC.
>> We consider f(P) has barycentric coordinate (f(x),f(y),f(z))
>> wrt ABC i.e denote by f(P,ABC).
>> Now with quadrangle P1P2P3P4.
>> Q1 = f(P1,P2P3P4)
>> Q2 = f(P2,P3P4P1)
>> Q3 = f(P3,P4P1P2)
>> Q4 = f(P4,P1P2P3)
>> Then the line P1Q1, P2Q2, P3Q3, P4Q4 are concurrent.
>> Proof. Let s1, s2, s3, and s4 be signed area of triangles
>> P2P3P4, P3P4P1, P1P2P3, and P2P3P4 respectively.
>> P1 has coordinate (s2,s3,s4) wrt triangle P2P3P4.
>> So that Q1 has coordinate (f(s2),f(s3),f(s4)) wrt P2P3P4.
>> i.e. [f(s2)+f(s3)+f(s4)]Q1= f(s2)P2+f(s3)P3+f(s4)P4.
>> We let X = [f(s1)P1 + f(s2)P2 + f(s3)P3 +
>> f(s4)P4]/(f(s1)+f(s2)+f(s3)+f(s4))
>> Thus [(f(s1)+f(s2)+f(s3)+f(s4)]X =
>> f(s1)P1 + [f(s2)+f(s3)+f(s4)]Q1.
>> Therefore P1Q1 goes through X. Similarly, P2Q2, P3Q3, P4Q4
>> go through X.
Best Regards
Tran Quang Hung.

Vào Th 5, 3 thg 10, 2019 vào lúc 07:59 Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
> Dear Tran Quang Hung,
> ....

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Message: #3848
Date: 03/10/2019 11:52:16
From: bernard.keizer
Subject: A family of centers in quadrangle

```
Dear Eckart,
Bernard Gibert also uses the product of 2 points (see SITP).
If I'm not wrong, your 1rst property is exactly the definition
of the isoconjugation !
With 4 fixed points in the QA vertices, the product of 2 QA-Tf2
partners is QA-P16, which is itself QA-Tf2 of the centroïd of
DT. Hence the definition of the pivotal isocubics as pK(QA-P16,
pivot) ...
If I understand correctly, you extend the property to any
triangle.
Anyhow, this construction of a new family of centers in a
quadrangle is really fascinating !
Best regards
Bernard
```

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Message: #3849
Date: 03/10/2019 4:16:29
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

Dear Bernard, dear Thanh Tung,

many thanks to Bernard:
… Studied in SITP years ago, with macros on my PC,
… but no longer parat for me "multiplication of points"!
  X*Y = isoconjugate of the centroid
… wrt an isoconjugation, swapping X and Y.

Perhaps of interest wrt "square roots of a point",
… only real for points inside the reference triangle:
Consider a quadrangle P1P2P3P4
… and for each vertex Pi the anticevian Qi
  inside the remaining triangle,
… (if Pi inside the remaining triangle: Pi = Qi).
… Let Ri be the square root of Qi
  inside the remaining triangle of Pi.
… The lines PiRi have a common point.

What about this new QA-point?

Best regards Eckart

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Message: #3850
Date: 03/10/2019 5:30:57
From: Thanh Tung
Subject: A family of centers in quadrangle

```
Dear
Bernard and
Eckart,

I think that there is a generalization:

For a point P = ( x : y :z),
  w.r.t. triangle ABC and a real number n,
  let  R = ( |x|^n , |y|^n,  |z|^n).
  Call R = f(P, ABC).

Consider a QA (P1 P2 P3 P4),
Let Ri = f(Pi, Pj Pk Pl)  then the four lines Pi Ri concur at a
point.

Then for n=1/2 we get the Eckart's problem.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à  21:16:32 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Bernard, dear Thanh Tung,
> ..............................
```

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Message: #3851
Date: 03/10/2019 6:02:55
From: Thanh Tung
Subject: A family of centers in quadrangle

```
Dear Quang Hung ,

The problem here is dealing with normalized and unnormalized
barycentric coordinates.

Formally one should use only normalized barycentric coordinates,
i.e. (x, y ,z) with x + y + z = 1.
For homogeneous functions like f(x) = x^n (n integer), applying
them to normalized and unnormalized barycentric coordinates give
the same points. However in general applying a function to
normalized and unnormalized barycentric coordinates would give
different points.

For example: choose the non-homogeneous function f(x) = 1/(1+x).
Applying to to same point (3,2-4) = (6,4,-8) gives different
coordinates.

So the question should be asked is:
Let f(x) be a real function of x.
If P has barycentric coordinate (x,y,z) in triangle ABC. We
consider f(P) has *normalized* barycentric coordinate
(f(x),f(y),f(z)) wrt ABC i.e denote by tr (P,ABC).
Now with quadrangle P1P2P3P4.
  Q1 = tr (P1,P2P3P4)
  Q2 = tr (P2,P3P4P1)
  Q3 = tr (P3,P4P1P2)
  Q4 = tr (P4,P1P2P3)
Are the lines P1Q1, P2Q2, P3Q3, P4Q4  concurrent?

In this case
f(-s2/s1)  + f(-s3/s1). + f(-s4/s1))
Q1 = ( f(-s2/s1) .P2 + f(-s3/s1).P3 + f(-s4/s1)).P4)
In general it cannot reduce to form
[f(s2)+f(s3)+f(s4)]Q1= f(s2)P2+f(s3)P3+f(s4)P4.

Best regards,
Vu Thanh Tung

On Thursday, October 3, 2019, 8:03 AM, Tran Quang Hung
analgeomatica@gmail.com [Quadri-Figures-Group] wrote:
> Dear Thanh Tung, dear Eckart,
> Thank You for your interest in my idea. I give a similar proof
of Thanh
```

> Tung in his file and the concurrent point in quadrangle
P1P2P3P4 is
> X = [f(s1)P1 + f(s2)P2 + f(s3)P3 +
f(s4)P4]/(f(s1)+f(s2)+f(s3)+f(s4))?
> Could You please check my proof?

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Message: #3852
Date: 03/10/2019 6:19:24
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Bernard and Eckart,

The concurrent point described in message #3850 has the
1st CT-Coordinate:
(p + q +r) |p|^n + p |p+q+r|^n.
with |t| = absolute value of t.
I think they are new center as there is no existing center with
coordinates involving absolute value for now.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à 22:31:02 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Bernard and Eckart,
> I think that there is a generalization:
> ....

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Message: #3853
Date: 03/10/2019 6:39:29
From: Thanh Tung
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

---

```
Dear all,

Let consider a quadrangle P1 P2 P3 P4 with Diagonal Triangle Δ.
Then the square point of P1 P2 P3 P4 with respect to Δ are the
same.

Proof:
If the barycentric coordinates of P1 w.r.t. Diagonal
Triangle Δ is (p, q, r) then the barycentric coordinates of 3
other points are: (p, -q ,r), (p, q, -r), (p, -q, -r).
The common square point is (p^2, q^2, r^2) w.r.t. Δ and so it
has a simple first DT-Coordinate: p^2.
Also  for an even integer n 4 points P1 P2 P3 P4 have the
same n-power point w.r.t. Diagonal Triangle Δ this point has
first DT-Coordinate: p^n..

Best regards,
Vu Thanh Tung
```

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Message: #3854
Date: 03/10/2019 7:19:36
From: eckart_schmidt@t-online.de
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

---

```
Dear Thanh Tung,

the common square point for the QA-vertices wrt QA-Tr1 is
QA-P16.

Best regards Eckart
```

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Message: #3855
Date: 03/10/2019 7:21:05
From: Thanh Tung
Subject: Generalization of Eckart's pbservation in number 3842

```
Dear all,

Eckart's observation in #3842 is intersecting:
The products of a point (unequal QA-P1) and a QA-vertex wrt the
remaining QA-triangle are collinear, bearing QA-P16.

I found a generalization of this as follows:
Consider a quadrangle P1 P2 P3 P4 , a point P and an integer n.
Qi = n-power of Pi w.r.t. triangle Pj Pk Pl.
Ri = multiplication of P and Qi
w.r.t. triangle Pj Pk Pl..
Then four points R1, R2, R3, R4 are collinear in a line d(P).
All line d(P) passes through a fixed point.
What is this fixed point w.r.t.  quadramgle P1 P2 P3 P4 ?

Best regards,
Vu Thanh Tung
```

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Message: #3856
Date: 03/10/2019 7:23:20
From: Thanh Tung
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

---

```
Dear Eckart,

Thank you very much,

Best regards,
Vu Thanh Tung

Le vendredi 4 octobre 2019 à  00:19:42 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Thanh Tung,
> the common square point for the QA-vertices wrt QA-Tr1 is
QA-P16.
> Best regards Eckart
```

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Message: #3857
Date: 03/10/2019 7:52:36
From: Thanh Tung
Subject: Generalization of Eckart's pbservation in number 3842

Dear all,

I observed that the fixed point is the (n+1)-power point X(n+1)
that was introduced in message #3832.

Best regards,
Vu Thanh Tung

Le vendredi 4 octobre 2019 à 00:22:07 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear all,
> Eckart's observation in #3842 is intersecting:
> The products of a point (unequal QA-P1) and a QA-vertex
> wrt the remaining QA-triangle are collinear, bearing QA-P16.
> I found a generalization of this as follows:
> Consider a quadrangle P1 P2 P3 P4, a point P and an integer n.
> Qi = n-power of Pi w.r.t. triangle Pj Pk Pl.
> Ri = multiplication of P and Qi w.r.t. triangle Pj Pk Pl..
> Then four points R1, R2, R3, R4 are collinear in a line d(P).
> All line d(P) passes through a fixed point.
> What is this fixed point w.r.t. quadramgle P1 P2 P3 P4 ?
> Best regards,
> Vu Thanh Tung

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Message: #3858
Date: 04/10/2019 8:56:59
From: eckart_schmidt@t-online.de
Subject: QL-geometry with point products

```
Dear Thanh Tung, dear Bernard,

there are also interesting aspects
… for multiplication of points in QL-geometry.

Here first observations:
(1) Reference triangle QL-Tr1:
(a) Squares of opposite QL-points coincide
    … in the cevian points of QL-P13.
(b) Products of opposite QL-points
    … are their midpoints.
(c) Products of opposite QL-points and QL-P1
    … divide harmonic a QL-Tr1 side.
(d) Products of two QL-Tr2-vertices
    … are the intersection of QL-L1 and the QL-Tr2-side.
(e) Product of all three QL-Tr2-vertices is QL-P23.

(2) Reference triangle QL-Tr2:
(a) Products of two QL-Tr1-vertices are collinear.
(b) Products of two CSC-partner on QL-Cu1 are collinear.
(c) Products of opposite QL-points
    … coincide in the intersection of line a) and b).

So far, there will be more!

Best regards Eckart
```

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Message: #3859
Date: 04/10/2019 12:46:18
From: eckart_schmidt@t-online.de
Subject: QL-geometry with point products

```
Dear Bernard, dear Thanh Tung,

in addition to #3858 some nice observations:
(1) Wrt QL-Tr1:
    Squares of points on the QL-lines
    … give a common QL-Tr1 inscribed conic,
    … tangent in the cevians of QL-P13.

(2) Wrt QL-Tr2: Squares of QL-Tr1-vertices
    … are collinear with the common product of opposite
    QL-points.

(3) Wrt QL-Tr2: The product of the three QL-Tr1-vertices
    … is the product of QL-P8 and two opposite QL-points
    … on QL-P8.QL-P13.

Best regards Eckart
```

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Message: #3860
Date: 05/10/2019 12:44:03
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

```
Dear Chris,

are the following properties hidden in EPG?

(1) For a 6P the six 5P-s-P5 are concyclic on 6P-s-Cix
    … centered in 6P-s-P1.

(2) for a 7P the seven 6P-s-Cix
    … have the common point 7P-s-P1.

Best regards Eckart
```

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Message: #3861
Date: 05/10/2019 3:03:51
From: chris.vantienhoven
Subject: 6P-s-P1 and 7P-s-P1

```
Dear Eckart,

Both properties are in the pipeline of being added to EPG.
6P-s-Cix will become 6P-s-Ci1.
Thanks for reminding me.
There are lots of other items that are ready to be released too.
However, I am very busy in this period. It will come later.

Best regards,
Chris
```

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Message: #3862
Date: 06/10/2019 3:56:49
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

```
Dear Chris,

in addition to #3861:
For a 5P and any point P
… we can consider 5P-s-P5 wrt P and the 4P of the 5P,
… which give 5 concyclic points on Ci(P) (wrt Ci(P) see #3575).

If we consider for a 6P and any point P
… the six Ci(P) wrt the 5P of the 6P
… we get six circles with a common point
… on the circle 6P-s-Cix round 6P-s-P1 (see # 3861).

Best regards Eckart
```

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Message: #3863
Date: 08/10/2019 9:16:35
From: chris.vantienhoven
Subject: 6P-s-P1 and 7P-s-P1

```
Dear Eckart,

Again, thanks for reminding. I will include these beautiful
transformations too.
When 5P-s-Tf6 = 5P-CB-Conjugate and 5P-s-Tf7 = your 5P-Schmidt
Transformation (circle + center), what is their relationship? It
looks like there is a relationship in your message #3575, but I
lost oversight.
When 6P-s-Tf1 = your 6P- Schmidt Transformation (common
circle-point), can you describe all properties of 5P-s-Tf7 and
6P-s-Tf1?

Best regards,

Chris
```

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Message: #3864
Date: 08/10/2019 5:34:23
From: Thanh Tung
Subject: Another family of points on quadrangle

```
Dear all,

I have found a new two-parameters family of points on
Quadrangle.
This new family of points is described as follows:
Consider a quadrangle P1 P2 P3 P4 and two integers m, n.
Let
_  Qi =  m-th power of Pi w.r.t. triangle Pj Pk Pl
_  Ri =  n-th power of the Cyclocevian Conjugate of Pi
   w.r.t. triangle Pj Pk Pl
_  Si =  product of Qi and Ri w.r.t triangle Pj Pk Pl
Then four lines Pi Si concur at a point T(m, n).
Some particular cases:
_  n=0:
   T(0, m) is the n-th power point that I described
   in the message #3830, or #3832
_  T(0, 1) = QA-P38: Montesdeoca-Hutson Point

Best regards,
Vu Thanh Tung
```

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Message: #3865
Date: 08/10/2019 7:11:52
From: Thanh Tung
Subject: Construction of a point on Quadrangle

```
Dear all,

_  Let P1 P2 P3 P4 be a Quadrangle.
_  Qi = orthocenter of the triangle
   formed by the midpoints of PiPj, PiPk, PiPl.
_  Ri = orthocenter of
the triangle formed by the midpoints of  QiQj, QiQk, QiQl.
Then four lines PiQi  concur at a point.
Is this a new point on Quadrangle or it is a new construction of
an existing point?

Best regards,
Vu Thanh Tung
```

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Message: #3866
Date: 08/10/2019 7:30:41
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear all,

Also, two Quadrangles P1P2P3P4 and R1R2R3R4 are homothetic and the concurrent point is the homothetic center of these two Quadrangles.

This is very similar to the construction of the point QA-P8 (Midray Homothetic Center).

Best regards,
Vu Thanh Tung

Le mercredi 9 octobre 2019 à 00:12:23 UTC+7, Thanh Tung tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear all,
> _ Let P1 P2 P3 P4 be a Quadrangle.
> _ Qi = orthocenter of the triangle
> formed by the midpoints of PiPj, PiPk, PiPl.
> _ Ri = orthocenter of the triangle
> formed by the midpoints of QiQj, QiQk, QiQl.
> Then four lines PiQi concur at a point.
> Is this a new point on Quadrangle or it is a new construction
> of an existing point?
> Best regards,
> Vu Thanh Tung

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Message: #3867
Date: 08/10/2019 8:21:18
From: eckart_schmidt@t-online.de
Subject: (p$^4 : q^4 : r^4$)

Dear Chris,

well known for a QA with vertex (p:q:r) (DT-coordinates):
… (p^2:q^2:r^2) = QA-P16.

Perhaps new:
… (p^4:q^4:r^4) = 2nd intersection of QA-P1.16
… … and QA-Tr1-circumconic through QA-P1 and QA-P18.

Best regards Eckart

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Message: #3868
Date: 08/10/2019 9:31:04
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung,

there will be a typo in # 3865:
"Then four lines PiRi concur at a point."
This point is QA-P2.

Best regards Eckart

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Message: #3869
Date: 08/10/2019 11:27:22
From: César E. Lozada
Subject: Construction of a point on Quadrangle

Dear Thanh and Eckart,

If “orthocenter” is replaced with “circumcenter” then the four lines PiRi concur at QA-P8.

Best regards,
César Lozada

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: Tuesday, October 8, 2019 3:31 PM
> To: Quadri-Figures-Group,
> Subject: [Quadri-Figures-Group]
> Re: Construction of a point on Quadrangle
> Dear Thanh Tung,
> there will be a typo in # 3865:
> "Then four lines PiRi concur at a point."
> This point is QA-P2.
> Best regards Eckart

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Message: #3870
Date: 09/10/2019 2:01:47
From: Thanh Tung
Subject: Construction of a point on Quadrangle

```
Dear Eckart & César Lozada,

Thank you very much,

Best regards,
Vu Thanh Tung

Le mercredi 9 octobre 2019 à 04:27:24 UTC+7, César E. Lozada
cesar_e_lozada@yahoo.es [Quadri-Figures-Group] a écrit:
> Dear Thanh and Eckart,
> If “orthocenter” is replaced with “circumcenter” then the four
lines PiRi concur at QA-P8.
> Best regards,
> César Lozada
```

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Message: #3871
Date: 09/10/2019 8:59:36
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

```
Dear Thanh Tung, dear César,

the construction in #3865
… gives for orthocenter, circumcenter and centroid
… the QA-points QA-P2, QA-P8, QA-P1.

CABRI observations show,
… that there are three other points on the Euler line,
… dividing X3.X4 in special ratios,
… depending on the reference quadrangle,
… which lead to three further QA-points.
What about these three QA-points?

Best regards Eckart
```

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Message: #3872
Date: 10/10/2019 10:37:07
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung, dear César,

the construction in #3865 can be modified:
Let E(t) be a point on the Euler line,
… dividing X(3).X(4) with ratio t.
_ Let P1 P2 P3 P4 be a Quadrangle.
_ Qi = E(t) of the triangle formed by Pj, Pk, Pl.
_ Ri = E(t) of the triangle formed by Qj, Qk, Ql.
Then the four lines PiRi concur at a point
… for t = 0 (orthocenter) in QA-P4,
… for t = 1 (nine-point center) in QA-P7,
… f or t =1:2 (centroid) in QA-P1.
CABRI observations show,
… that there are three other points on the Euler line,
… dividing X3.X4 in special ratios,
… depending on the reference quadrangle,
… which lead to three further QA-points.
What about these three QA-points?

Best regards Eckart

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Message: #3873
Date: 10/10/2019 11:57:21
From: bernard.keizer
Subject: Construction of a point on Quadrangle

Dear Eckart, dear Than Tung, dear César
Beautiful idea to start directly with the 4 reference triangles
instead of the midtriangles !
I reproduced without difficulty the property for centroïd and
circumcenter.
I only observe that the ratio of similitude between the QA of
the Ri and the initial QA is -1/3 for centroïd (synthetically
obvious with QA-P1 as perspector) and -0.117 for circumcenter.
So far, I have no idea of a synthetic or analytical general
proof ...
Best regards
Bernard

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Message: #3874
Date: 10/10/2019 6:41:59
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear Bernard, Eckart, César,

Since for centroid, nine-pointcircle, orthocenter of 4 reference triangle we obtain 3 different quadrangle homothetic to the reference quadrangle, they are also homothetic in pair.
What can say about the homothetic center of each pair ? Are they new centers ?
The same question to ask with centroid, circumcenter, orthocenter quadrangle   of 4 midray triangle. They are homothetic in pair and what about the homothetic center of each pair?

Best regards,
Vu Thanh Tung

Le jeudi 10 octobre 2019 à 17:01:27 UTC+7, bernard.keizer@yahoo.com [Quadri-Figures-Group] a écrit :
> Beautiful idea to start directly with the 4 reference triangles instead of the midtriangles !
> I reproduced without difficulty the property for centroïd and circumcenter.
> I only observe that the ratio of similitude between the QA of the Ri and the initial QA is -1/3
> for centroïd (synthetically obvious with QA-P1 as perspector) and -0.117 for circumcenter.
> So far, I have no idea of a synthetic or analytical general proof ...
> Best regards
> Bernard

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Message: #3875
Date: 11/10/2019 3:55:04
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear all,

I would like to clarify my last statement.

Definition:
Let P1 P2 P3 P4 be a Quadrangle and a natural number n.
Qi = Xn of the triangle formed by the midpoints of PjPk, PkPl, PlPj .
Ri = Xn of the triangle formed by the midpoints of PiPj, PiPk, PiPl.

The Quadrangle Q1 Q2 Q3 Q4 can be called the Xn-midpoint (first generation) of the Quadrangle P1 P2 P3 P4,
The Quadrangle R1 R2 R3 R4 can be called the Xn-midray (first generation) of the Quadrangle P1 P2 P3 P4.

The Xn-midpoint (of Xn-midray) p-th generation of Quadrangle P1 P2 P3 P4 .is defined recursively.
(say the n-th generation is the first generation of the (n-1)-th generation),
denoted by n-th-Xn-midppint or p-th-Xn-midray of Quadrangle P1 P2 P3 P4 for short.

The Quadrangle can be consider as the 0-th-Xn-midpoint or 0-th-Xn-midray of itself.

Result:
For any natural integer n, the n-th-X2-midpoint or midray is homothetic to the
Reference Quadrangle and the homothetic center is their common centroid.
For even n natural integer n, the n-th-X3,X4-midpoint (or midray) are homothethic to the Reference Quadrangle.
The above Quadrangles are then homothetic in pair.
The homothetic centers of each pair form a new family of Quadrangle centers.
We can also mixing Xn-midpoint midray in the process of creating new centers.
For example: The second-X3-midpoint of the fourth-X4-midray is homothetic to the
second-X4-midpoint of the second-X3-midray is homothetic (both homothetic to the Reference Quadrangle).

This make this new family of Quadrangle centers even larger.

Best regards,
Vu Thanh Tung

Le jeudi 10 octobre 2019 à 23:42:59 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Bernard, Eckart, César,
> Since for centroid, nine-pointcircle, orthocenter of ......

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Message: #3876
Date: 13/10/2019 8:35:14
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung, dear César, dear Bernard,

I have to correct my #3872:
Studying a nonconvex QA,
… I observed 8 points E(t) on the Euler line,
… which lead to 8 QA-points in the sense of #3872:
… QA-P1, QA-P4, QA-P7 (collinear),
… *four points on QA-Co2*
… and another point.
What about the last five points?

Best regards Eckart

PS: For a convex QA I observed only 6 points
    … there seem to be only two points on QA-Co2.

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Message: #3877
Date: 16/10/2019 8:26:33
From: chris.vantienhoven
Subject: Bad news from Yahoo

```
Yahoo has made the decision to no longer allow users to upload
content to the Yahoo Groups site. Beginning October 21, you
won't be able to upload any more content to the site, and as of
December 14 all previously posted content on the site will be
permanently removed. You'll have until that date to save
anything you've uploaded.

Chris
```

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Message: #3878
Date: 16/10/2019 6:42:36
From: Thanh Tung
Subject: Another family of points on quadrangle

Dear all,

As one of the last messages before this Group is gone, please find in attachement my proof of #3864.

Best regards,
Vu Thanh Tung

Le mardi 8 octobre 2019 à 22:36:09 UTC+7, Thanh Tung tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear all,
> I have found a new two-parameters family of points on Quadrangle.
> This new family of points is described as follows:
> Consider a quadrangle P1 P2 P3 P4 and two integers m, n.
> Let
> _ Qi = m-th power of Pi w.r.t. triangle Pj Pk Pl
> _ Ri = n-th power of the
> Cyclocevian Conjugate of Pi w.r.t. triangle Pj Pk Pl
> _ Si = product of Qi and Ri w.r.t triangle Pj Pk Pl
> Then four lines Pi Si concur at a point T(m, n).
> Some particular cases:
> _ n=0: T(0, m) is the n-th power point that I described in the message #3830, or #3832
> _ T(0,1) = QA-P38 : Montesdeoca-Hutson Point
> Best regards,
> Vu Thanh Tung

Power Cyclocevian Family of Points of Quadrangle

Vu Thanh Tung

October 16, 2019

In this short note, I define a family of points in Quadrangle. For each integer n, the n^{th}-power of a point with respect to a triangle is defined as follows:

Definition 0.1 (Power of a point). *For each integer n, the n^{th}-power of a point P with barycentric coordinates $(x : y : z)$ with respect to $\triangle ABC$ to the point P^n with barycentric coordinates $(x^n : y^n : z^n)$ with respect to $\triangle ABC$.*

Definition 0.2 (Cyclocevian Conjugate). *Let P be a point not on a sideline of a reference triangle $\triangle ABC$. Let $\triangle A'B'C'$ be the cevian triangle of P with respect to $\triangle ABC$. The circumcircle of $\triangle A'B'C'$ meets each sideline in two (not necessarily distinct) points $A', A''; B', B''$; and C', C''. Then the lines AA'', BB'', and CC'' concur in a point known as the cyclocevian conjugate of P with respect to $\triangle ABC$.*

Now we are able to construct a family of points in quadrangle.

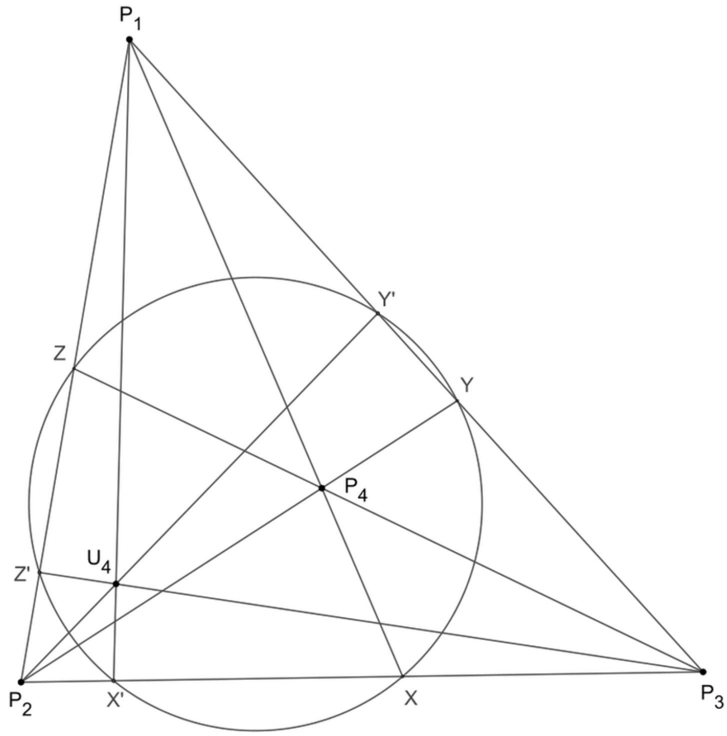
Theorem 1 (Vu Thanh Tung [1]). *Consider a quadrangle $P_1P_2P_3P_4$ and two integers m, n. For $\{i, j, k, l\} = \{1, 2, 3, 4\}$, let*

- *Q_i be the m^{th}-power of P_i,*
- *U_i be the cyclocevian conjugate of P_i,*
- *R_i be the n^{th}-power of U_i,*
- *S_i be the product of Q_i and R_i,*

all with respect to $\triangle P_jP_kP_l$. Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point $T_{m,n}$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4$, $\triangle P_3P_4P_1$, $\triangle P_4P_1P_2$, $\triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and then the point Q_4 has barycentric coordinates $(s_1^m : s_2^m : s_3^m)$ with respect to $\triangle P_1P_2P_3$.

Let $X = P_4P_1 \cap P_2P_3$, $Y = P_4P_2 \cap P_3P_1$, $X = P_4P_3 \cap P_1P_2$. Let (c) be the circumcircle of $\triangle XYZ$. $(c) \cap P_2P_3 = \{X, X'\}$, $(c) \cap P_3P_1 = \{Y, Y'\}$, $(c) \cap P_1P_2 = \{Z, Z'\}$. Then P_1X', P_2Y', P_3Z' concur at the point U_4.



For $i = 1, 2, 3, 4$, let ρ_i be the power of P_i with respect to (c). Then we have $\overline{P_2X}.\overline{P_2X'} = \rho_2$, $\overline{P_3X}.\overline{P_3X'} = \rho_3$, so:

$$\frac{\overline{P_2X'}}{\overline{P_3X'}} = \frac{\rho_2}{\rho_3}.\frac{\overline{P_3X}}{\overline{P_2X}} = -\frac{\rho_2.s_2}{\rho_3.s_3}. \tag{1}$$

Similarly,

$$\frac{\overline{P_3Y'}}{\overline{P_1Y'}} = -\frac{\rho_3.s_3}{\rho_1.s_1}, \quad \frac{\overline{P_1Z'}}{\overline{P_2Z'}} = -\frac{\rho_1.s_1}{\rho_2.s_2}. \tag{2}$$

It means that the point U_4 has the barycentric coordinates $(\frac{1}{\rho_1.s_1} : \frac{1}{\rho_2.s_2} : \frac{1}{\rho_3.s_3})$ with respect to $\triangle P_2P_3P_4$. Thus, the point R_4 has the barycentric coordinates $(\rho_1^{-n}.s_1^{-n} : \rho_2^{-n}.s_2^{-n} : \rho_3^{-n}.s_3^{-n})$ and the point S_4 has the barycentric coordinates $(\rho_1^{-n}.s_1^{m-n} : \rho_2^{-n}.s_2^{m-n} : \rho_3^{-n}.s_3^{m-n})$ with respect to $\triangle P_2P_3P_4$. For $i = 1, 2, 3, 4$ let $f_i = \rho_i^{-n}.s_i^{m-n}$. We can write:

$$S_4 = \frac{1}{f_1 + f_2 + f_3}\big(f_1.P_1 + f_2.P_2 + f_3.P_3\big) \tag{3}$$

Let the point $T_{m,n}$ be defined as:

$$T_{m,n} = \frac{1}{f_1 + f_2 + f_3 + f_4}\big(f_1.P_1 + f_2.P_2 + f_3.P_3 + f_4.P_4\big) \tag{4}$$

$T_{m,n}$ is on the line P_4S_4 as:

$$T_{m,n} = \frac{f_1 + f_2 + f_3}{f_1 + f_2 + f_3 + f_4}.S_4 + \frac{f_4}{f_1 + f_2 + f_3 + f_4}.P_4. \qquad (5)$$

Similarly, $T_{m,n}$ is on the line P_1S_1, P_2S_2, P_3S_3. □

References

[1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Oct 8, 2019, https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3864

[2] ENCYCLOPEDIA OF QUADRI-FIGURES, , https://www.chrisvantienhoven.nl/mathematics/encyclopedia

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: tungvtt@gmail.com

Message: #3879
Date: 17/10/2019 10:34:02
From: bernard.keizer
Subject: Another family of points on quadrangle

Dear Than Tung,
I would be very pleased to read your analytical proof, but your attached files don't appear on the forum !
Could you please send it to me as word- or pdf-file at my private e-mail address bernard.keizer@wanadoo.fr
Many thanks in advance
Best regards
Bernard

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Message: #3880
Date: 17/10/2019 11:24:07
From: bernard.keizer
Subject: Another family of points on quadrangle

Dear Than Tung,
Please forget my last message !
Your attachment doesn't appear on the forum (as it should), but I found it in the mailbox.
I will read it with a great interest
Thanks a lot
Best regards
Bernard

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Message: #3881
Date: 17/10/2019 2:51:42
From: chris.vantienhoven
Subject: Bad news from Yahoo

Dear friends,

Well, here we are. Yahoo Groups will close on 14th December 2019 & will delete all user content. It will continue as an email list. Only until 28 th October you will be able to send messages in the usual way. After that period, messages only can be send as emails to quadri-figures-group@yahoogroups.com.
These messages should reach all members.

But of course, this is not what we want. The Groups Hyacinthos, Anopolis and Advanced Plane Geometry (ADGEOM) have the same problem. We are looking together for a solution. We already archived all content related to our groups externally.

I think we found a new and better Groups-provider. I will do my best to migrate all content to this new provider and keep you informed.

You can also make a backup yourself with the software of PG Offline at http://www.personalgroupware.com/downloads.htm.
I think it is important when several of us have a backup

* It also has the advantage of looking up older QFG-messages very quickly.
After 14th December 2019 this no longer will be possible via Yahoo Groups.
Could you let me know when you do?
I trust the new situation will be better than before, so it is all for the best.

Best regards,
Chris

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Message: #3882
Date: 17/10/2019 4:01:47
From: Thanh Tung
Subject: Another family of points on quadrangle

```
Dear Bernard,

Thank you for your interest,

Best regards,
Vu Thanh Tung

Le jeudi 17 octobre 2019 à  16:24:12 UTC+7,
bernard.keizer@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Than Tung,
> Please forget my last message !
> Your attachment doesn't appear on the forum (as it should),
but I found it in the mailbox.
> I will read it with a great interest
> Thanks a lot
> Best regards
> Bernard
```

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Message: #3883
Date: 18/10/2019 3:35:57
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Chris,

In #3863 you ask for a relationship between the transformations
…"cb(P)" = 5P-s-Tf6 and "Ci(P)" = 5P-s-Tf7 in your nomination.

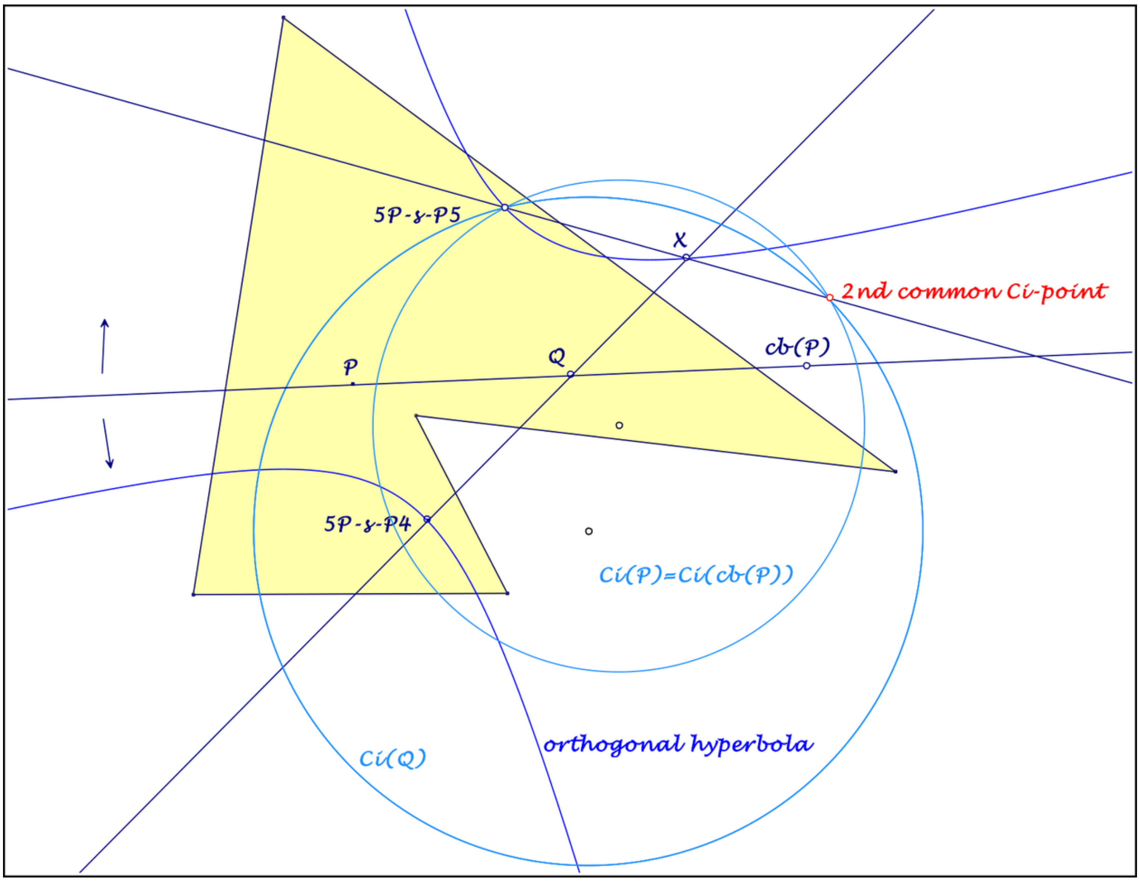
In #3575 these transformations are described with QA-Cu7 background.

Here only the observations wrt 5P-geometry:
"cb(P)" is the Cayley-Bacharach point of
… the 5P-vertices, the two circular points and P.
… cb(P) maps 5P-s-Co1 to the line at infinity
… … with P, cb(P) and 5P-s-P4 collinear.
"Ci(P)" uses the 5L of the bisectors PPi,
… the CSC-images of P wrt the 4L of the 5L give a circle Ci(P).

Properties:
(1) P and cb(P) have the same Ci-circle.
(2) For points P on 5P-s-Co1
… Ci(P) degenerates to a line through 5P-s-P5
… and the 2nd intersection of P.5P-s-P4 and an orthogonal hyperbola
… through 5P-s-P4,5, centered in the middle with axes parallel to the 5P-s-Co1-axes.
(3) For 5P-s-P6 we get a circle through 5P-s-P5,6 and 5P-CSC(5P-s-P4).
(4) *For cb-partner on lines through a point Q*
… their Ci-circles have a common 2nd point beside 5P-s-P5:
… Example: For Q = 5P-CSC(5P-s-P5) this 2nd Ci-point is 5P-s-P6.
… In general: 2nd intersection of Ci(Q) and X.5P-s-P5
… … with X = 2nd intersection of Q.5P-s-P4 and the orthogonal hyperbola in (2).

Best regards Eckart

PS: Is there already a nomination for the 5P-CSC?



2019-10-18.pdf

Message: #3884
Date: 18/10/2019 7:24:28
From: bernard.keizer
Subject: 6P-s-P1 and 7P-s-P1

```
Dear Eckart,
If I'm not wrong, you never gave the general construction of the
2nd point (apart of U) of the circles Ci(P) for P and cb(P) on
lines through a point Q.
Very interesting indeed
Thanks a lot
Best regards
Bernard
```

Message: #3885
Date: 20/10/2019 11:26:49
From: eckart_schmidt@t-online.de
Subject: Loci wrt concentric circles

```
Dear Chris, dear Bernard,

concentric circles can be mapped to other circles
… and the loci of their intersections considered:
(1) For 5P wrt 5P-s-Tf3:
… Let Ci be circles, centered in Z,
  and Ci' their 5P-s-Tf3-image circles.
… The intersections of Ci and Ci'
… give a new circle Ci(Z), bearing 5P-s-P1,
… depending only of the circumconic 5P-s-Co1.
… The axes of 5P-s-Co1 intersect Ci(Z) beside 5P-s-P1 diametral.

(2) For 5P wrt 5P-s-Tf3inv = Co-Tf3:
… analog ...

(3) For QL wrt CSC:
… Let Ci be circles, centered in Z,
  and Ci' their CSC-image circles.
… The intersections of Ci and Ci'
… give a quartic, CSC-invariant, through QL-2P3,
… intersecting the 1st Steiner axis further
  in two harmonic points wrt QL-2P3
… and the 2nd Steiner axis in two CSC-partner.
… The six quartics for the QL-points
  have common points in QL-2P3
… and triple points in two pairs of CSC-partner.
… The radical axes of Ci and Ci' envelope a parabola,
… which is also the envelope of X.CSC(X)
  for points X on the quartic:
… Focus CSC(Z) and directrix Z.QL-P1,
… orthogonal tangents from Z to the parabola.

(4) For 5P wrt 5P-CSC:
… analog ...
… but the five quartics have no triple points.

Best regards Eckart
```

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Message: #3886
Date: 21/10/2019 11:36:21
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Bernard,

thanks for interest wrt #3883.
If we take for Q a vertex of a 6P
… and for 5P the remaining 5P,
… the 2nd Ci-point is concyclic
… with the 5P-s-P5 for the 5P of the 6P.

The corresponding circle bears 12 points:
… the six 2nd Ci-points and the six 5P-s-P5,
… is centered in 6P-s-P1,
… and will be relevant for 6P-geometry.

Best regards Eckart

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Message: #3887
Date: 25/10/2019 9:18:57
From: eckart_schmidt@t-online.de
Subject: Anallagmatic QL/5P-Quartic

Dear Bernard, dear Chris,

for I am not familiar with quartics,
… I hope you can judge the following observations
… wrt the QL/5P-quartic for a 5P
(see #3599, #3603, #3607, #3675).

The QL/5P-quartic is anallagmatic,
… that means,

there is an inversion wrt a circle,
… which maps the quartic to itself.

Here I only consider the case,
… that the quartic is a one-line curve.

This quartic has two double tangents
… with contact points S1,T1 and S2,T2,
… which are concyclic on a circle, centered in 5P-s-P3.

The intersection Z of the double tangents is center of an inversion
… which maps S1-->T1, S2-->T2 and the quartic to itself.

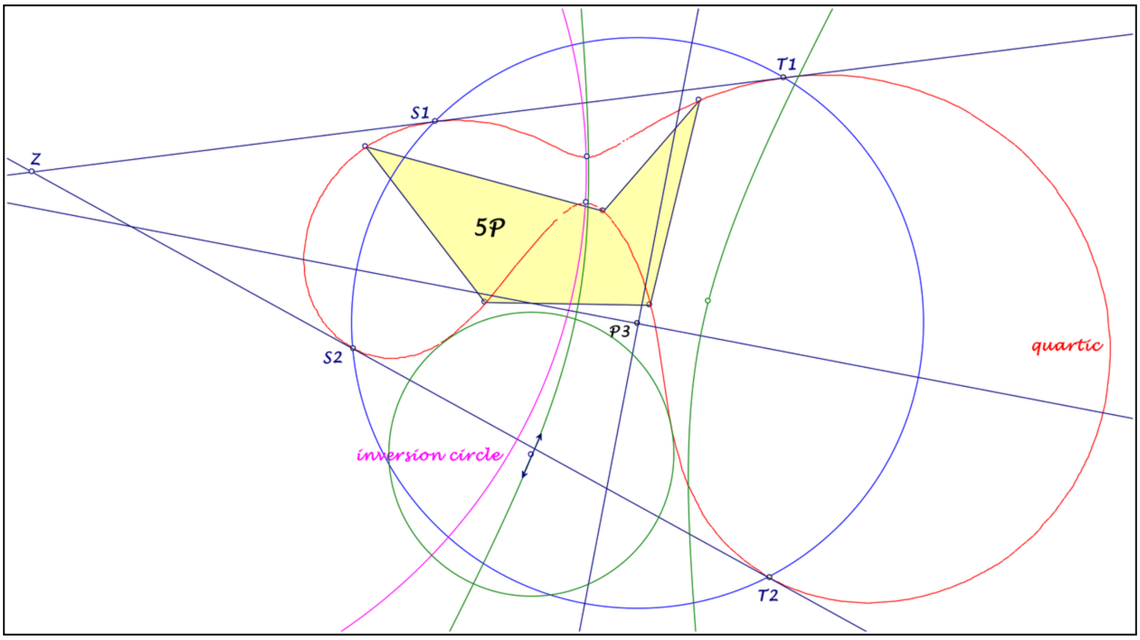
All anallagmatic curves have the property,
… that they are the envelope of a set of circles,
… which intersect the inversion circle orthogonal
… and contact the curve in inverse points.

For our quartic the enveloping circles
… are centered on a conic with center 5P-s-P3
… and axes parallel to those of QA-Co2 of S1T1T2S2,
… bearing the two intersections of the quartic
and the inversion circle.

I hope, someone can confirm these observations.
What about the circle of the contact points S1,T1,S2,T2?

Best regards Eckart

PS: It will be interesting,
… to study the anallagmatic property
of the quartic wrt a QL.



2019-10-24.pdf

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Message: #3888
Date: 29/10/2019 8:00:06
From: bernard.keizer
Subject: Anallagmatic QL/5P-Quartic

```
Dear Eckart,
I found this property very interesting !
I'm not a specialist of cubics or quartics either ...
I only found in http://www.mathcurve.com
under anallagmatic curves some commentars which may interest you
(you may read them in french or in english).
Circular cubics (in particular the Van Rees curve) and
bicircular quartics are examples of anallagmatic curves.
This is particularly interesting as the bicircular quartic of
the 5P and the Van Rees cubic of the CSC of the 5P are CSC
partners ...
In german, they use Anallagmatische Kurve, Kreiskubik and
Doppelkreisquartik ...
Best regards
PS I'm really admirative that you continue until the last moment
of existence of the forum to dig new properties
```

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Message: #3889
Date: 29/10/2019 8:02:11
From: chris.vantienhoven
Subject: Bad news from Yahoo

```
Dear friends,

As you know, Yahoo will delete the contents of all Yahoo-groups
on 14/12.
Only for a very short period of time, it will be possible to
send messages in the usual way to the Yahoo-Group.
I already made a new group in Groups.io, but it will take some
time before our old messages will be migrated. I will keep you
informed.
It can happen that there will a period we will not be able to
send messages to our group until the migration has been done. I
am dependent on Groups.io in this.
I will put the archives in Public Domain. This means that I give
permission to anyone (person or institution) to host the
archives in a web site.
You can download the archives by:
1. http://www.httrack.com/
   or
2. http://www.personalgroupware.com/downloads.htm
(Read the details before starting to download)

Best regards,
Chris van Tienhoven
```

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Message: #3890
Date: 29/10/2019 11:41:03
From: Fam. van Tienhoven
Subject: last message(s) QFG-Yahoo-group

```
Dear friends,

As you know, Yahoo will delete the contents of Yahoo-groups in
14/12.
Only for a very short time it will be possible to send messages
in the usual way to the Yahoo-Group.
I already made a new group in Groups.io, but it will take some
time before our old messages will be migrated. I will keep you
informed.
It can happen that there will a period we will not be able to
send messages to our group until the migration has been done. I
am dependent on Groups.io in this.
I will put the archives in Public Domain. This means that I give
the permission to anyone (person or institution) to host the
archives in a web site.
You can download the archives by:
1. http://www.httrack.com/
   or
2. http://www.personalgroupware.com/downloads.htm
(Read the details before starting to download)

Best regards,
Chris van Tienhoven

p.s. this message came to you by regular email to
quadri-figures-group@yahoogroups.com
```

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Message: #3891
Date: 01/11/2019 10:04:01
From: eckart_schmidt@t-online.de
Subject: New Quartic for the 5P of the QA-Cu7-triple Points

Dear Bernard, dear Tsihong Lau,

I try once more a QFG-message,
… for I have an interesting assumption
wrt Bernard's twin 5P
… for the QA-Cu7-triple points and their 5P-CSC-images:

Consider first an arbitrary 5P=P1P2P3P4P5
… and its 5P-CSC- image P1'P2'P3'P4'P5',
… further the 5L with lines PiPi'
and its inscribed conic 5L-s-Co1,
… then the 5P-CSC-partner on the tangents of 5L-s-Co1
… will give a curious sextic:
… 5P-CSC-invariant,
… bearing the 10 vertices of the twin 5P
… and 5P-s-P6 and the fixed points of 5P-CSC.

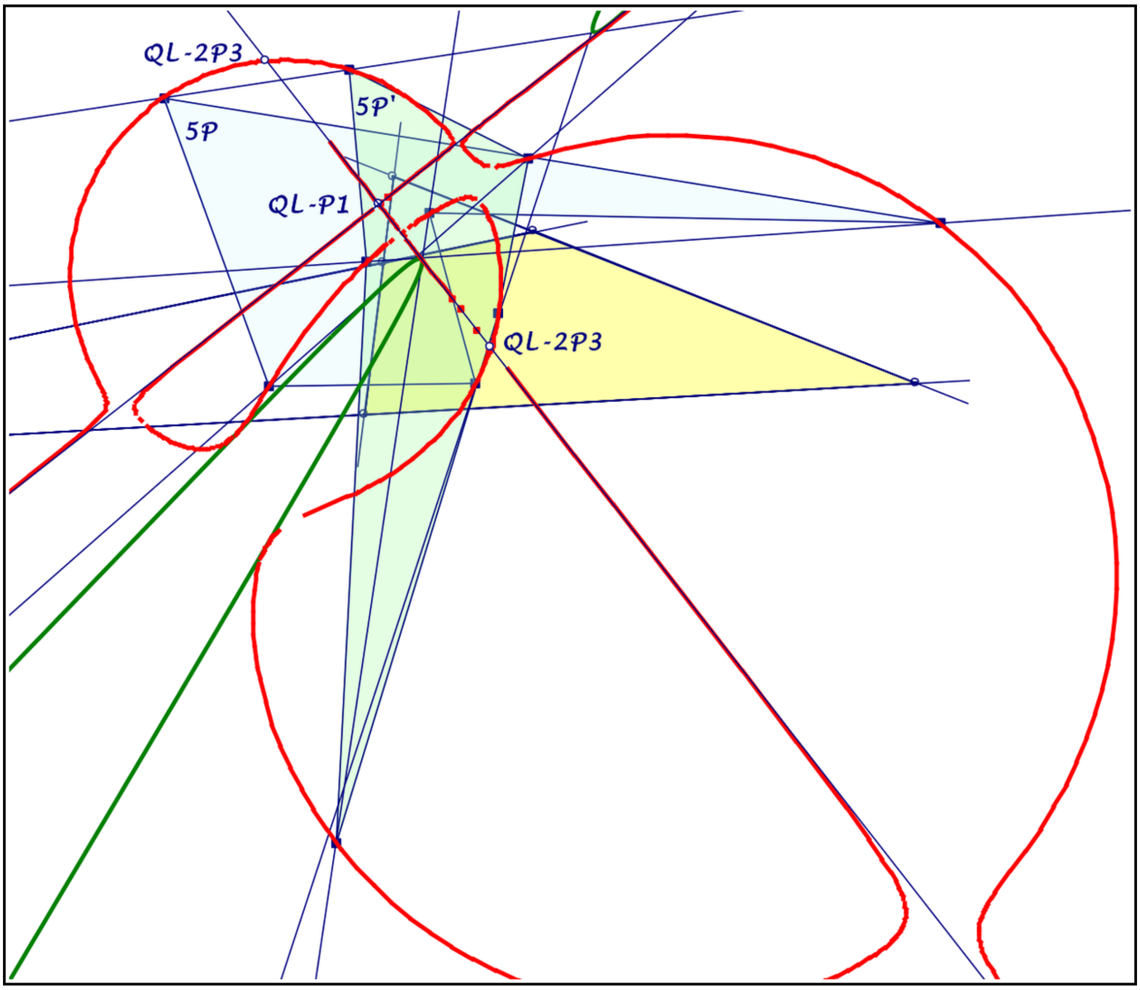
For the 5P of the QA-Cu7-triple points of a QL
… this sextic degenerates to the Steiner axes
… and a quartic:
5P-CSC-invariant,
… bearing the 10 vertices of the twin 5P and QL-2P3.

This shows once more,
… that the QA-Cu7-triple points give a special 5P.

A construction has the handycap
… of approximate QA-Cu7-triple points for a QL,
… so the attached drawing is impricised,
… but several times repeated.

Best regards Eckart

PS. Tsihong Lau mentioned in a private mail
... such a quartic in general,
... but I don't know his construction.
If this mail doesn't appear in QFG,
... I shall send it to your private address.



2019-11-01.pdf

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Message: #3892
Date: 03/11/2019 11:26:31
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

```
Dear Chris,

the circle 5P-s-Cix in #3773 bears 5P-n-P2.

Best regards Eckart
```

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Message: #3893
Date: 03/11/2019 12:23:01
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

```
Dear Chris,

excuse, please forget my last message,
… it doesn't hold.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3894
Date: 04/11/2019 9:04:13
From: eckart_schmidt@t-online.de
Subject: Quartics for 3 Lines with a CSC-transformation

Dear Bernard, dear Chris,

there is a construction concept for CSC-invariant quartics,
… starting with 3 lines and a CSC-transformation,
… already used in #1354, #1365, #1376.

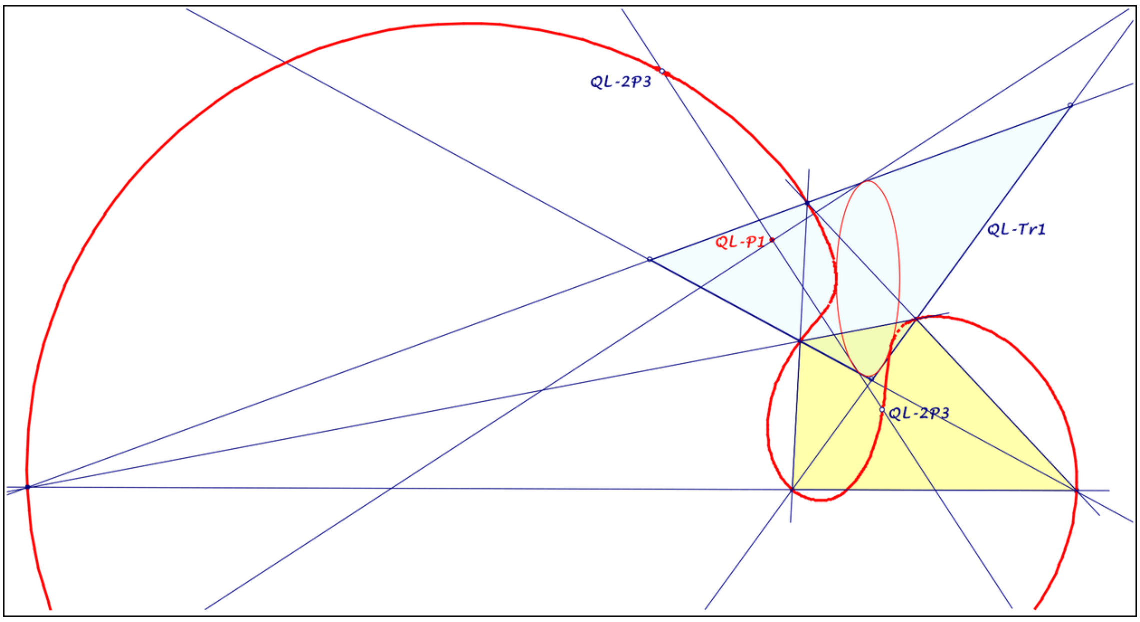
Consider a CSC-transformation
… with its Steiner axes and three other lines,
… which give a 5L with inconic 5L-s-Co1:
… CSC-partner on tangents at this conic give the quartic,
… evidently CSC-invariant.

Examples:
For a QL and its CSC (see attached file)
… wrt the Steiner axes and the three diagonals
… we get a CSC-invariant circumquartic of the QL.

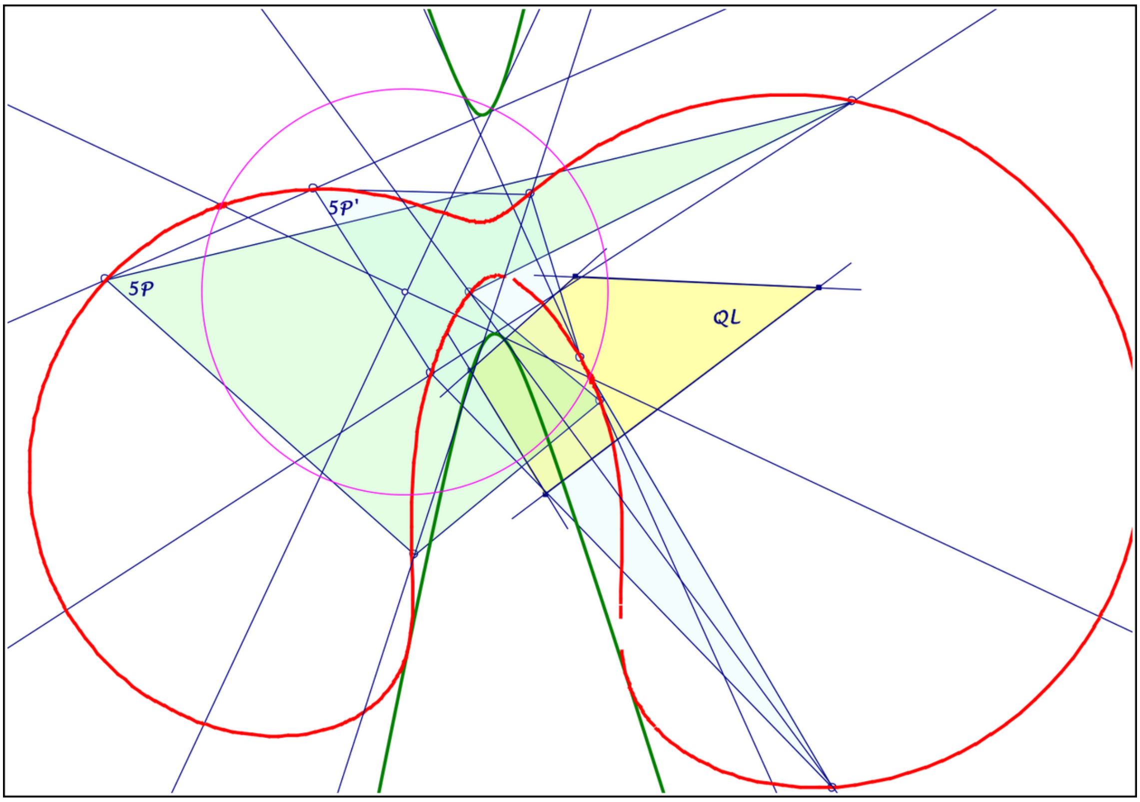
For the 5P = P1P2P3P4P5 of the QA-Cu7-triple points,
… its 5P-CSC with Steiner axes
… and any three of the 5 lines Pi.CSC(Pi)
… we get a 5P-circumquartic, evidently bearing CSC(Pi).

The 2nd example allows a better construction
… for the quartic in #3891 (see attached file).

Best regards Eckart



2019-11-03.pdf



2019-11-01a.pdf

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Message: #3895
Date: 07/11/2019 9:34:53
From: eckart_schmidt@t-online.de
Subject: Special QA-Circumcubic

Dear Bernard, dear Chris,

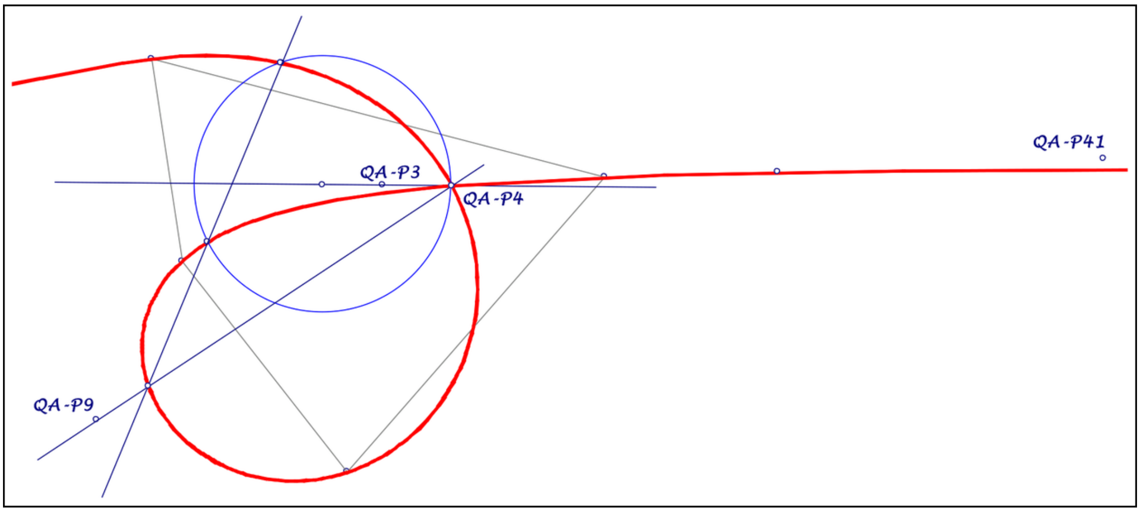
in QL-geometry midpoints of QL-Tf1-partner on QL-Cu1 give QL-L1,
… here an analog question in QA-geometry:
… What about the midpoints of QA-Tf2-partner on QA-Cu1?
QA-Cu1 is QA-Tf2-invariant,
… QA-Tf2-partner on QA-Cu1 are collinear with QA-P4.

(1) The midpoints of QA-Tf2-partner on lines through QA-P4
… give parts of a QA-circumcubic QU.
(2) QU is the locus of points X with cb(X) = QA-P4,
… cb Cayley-Bacharach transformation
… wrt the QA-vertices, QA-P4 and the circular points.
(3) QU is the QA-Tf16-image
… of a QA-circumconic through QA-P4.
(4) QU is the QA-Tf4-image of a conic
… through QA-Tf4 of the QA-vertices
… … and QA-Tf4 of the midpoint of QA-P4.41.
(5) Finally a very curious way, to get QU:
… Consider circles round QA-P4
… and their cb-image, a curve of higher degree:
… The limit of the curve for radius zero is QU.

Properties:
QU bears the QA-vertices,
… QA-P4 as double point and the midpoint of QA-P4.41 ...
  QU has as asymptote QA-P3.4.
  QU has tangents in QA-P4,
… which are parallel to the asymptotes of the QA-circumconic
through QA-P4.

A special point (see attached file):
… Circles through QA-P4, centered on QA-P3.4, intersect the
cubic
… collinear with the midpoint of QA-Tf2-partner on QA-P4.9.

Best regards Eckart



2019-11-07.pdf

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Message: #3896
Date: 08/11/2019 11:13:52
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 / Co-Tf3

```
Dear Benedetto, dear Chris,

an unexpected property, perhaps already mentioned:
For all QA-circumconics holds:
Co-Tf3(QA-P2) = QA-P4.
Or:
5P-s-Tf3 maps QA-P4 of a 5P-QA-version
… to QA-P2 of this 5P-QA-version.

Best regards Eckart
```

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Message: #3897
Date: 09/11/2019 5:04:13
From: Benedetto Scimemi
Subject: 5P-s-Tf3 / Co-Tf3

```
Dear Eckart,  thanks for your last message. In fact, the
"unexpected" property you mention is reported in Chris's EQF,
section INFORMATIONS ON CONICS , page CO-Tf3.
.......

4. For *any* quadrangle Q inscribed in *any* conic CO, the
CO-Tf3 image of QA-P2 is QA-P4 .......

Also, in the final table:
Any Conic circumscribing Reference Quadrangle
QA-P2 (http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p2)
QA-P4 (http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4)
........

Warning: In a Pentangle the Scimemi Transformation CO-Tf3 wrt
the circumscribed conic is identical to the inverse of the
5P-s-Tf3 Transformation.

This confusion of names should have been avoided, but now it is
difficult to change.

Best regards.  Benedetto
```

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Message: #3898
Date: 10/11/2019 2:43:18
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 / Co-Tf3

```
Dear Benedetto,

thanks for your references and excuse,
… that I read careless your Co-Tf3 article.
It is unusual to make geometry only with a reference conic
… but interesting:
Main property of Co-Tf3 / Co-Tf3inv is,
… that they map lines to lines, circles to circles,
… … conics to conics, inclusive their centers.

A short excursion:
Consider a reference conic, centered in Z
… and circles CI, centered in P:
… the images of CI wrt Co-Tf3 and Co-Tf3inv
  are circles Ci1, CI2,
… with centers on the reflection of PZ in the main axis.
Varying the radius of CI,
… The radical center of CI, CI1, CI2 give a line through Z
… and the intersections of CI1 and CI2 give a circle through Z.
What about this circle, if P is a focus of the reference conic
Co?

Best regards Eckart
```

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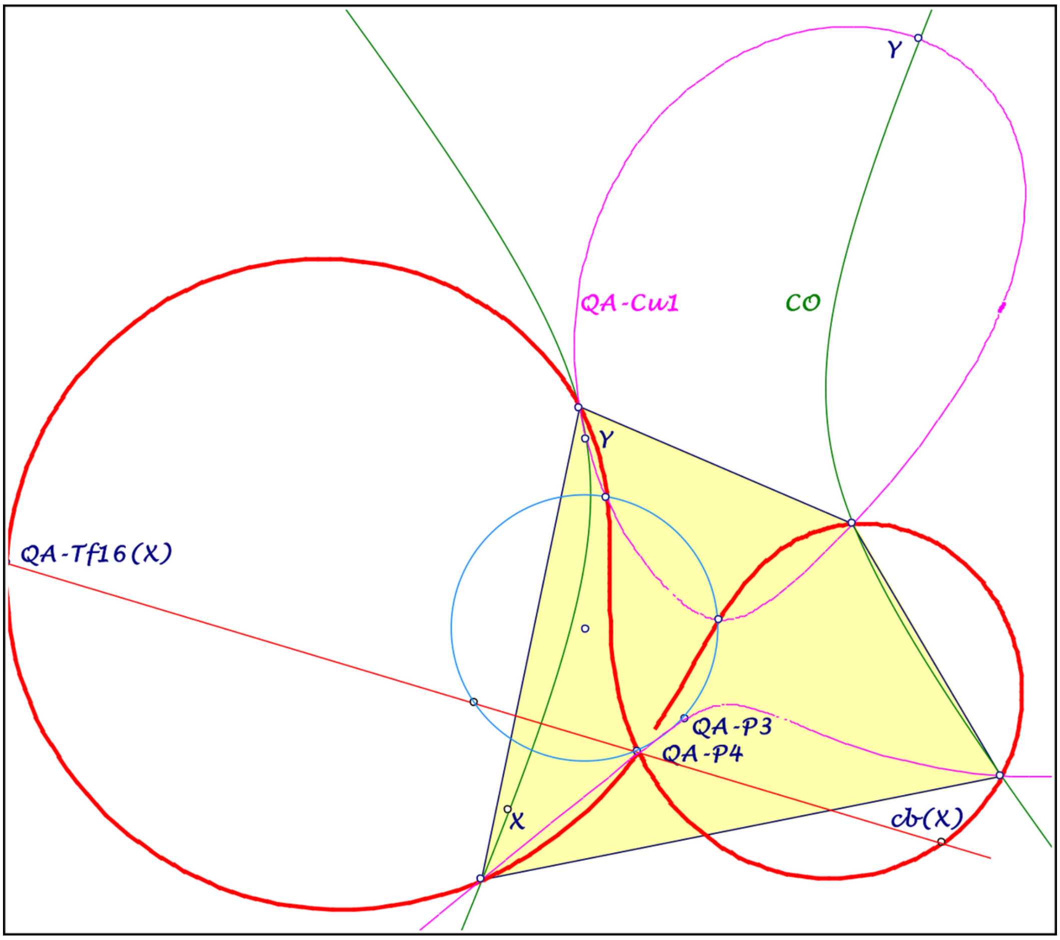
Message: #3899
Date: 11/11/2019 12:10:19
From: eckart_schmidt@t-online.de
Subject: Special QA-Circumquartics

Dear Bernard, dear Chris,

this is a generalization of #3895:
Consider a QA and a circumconic CO,
… QA-Tf16 of CO gives a QA-circumquartic
… with double point in QA-P4.
The same quartic we get as cb-image of CO,
… cb Cayley-Bacharach transformation
… wrt QA-vertices, QA-P4 and the circular points.
The QA-Tf16- and the cb-image of a CO-point X
… are collinear with QA-P4,
… their midpoints give a circle through QA-P3, QA-P4
… and QA-Tf16 of further intersections Y of CO and QA-Cu1
… with Y, QA-Tf16(Y), QA-P3 collinear (see attached file).

Best regards Eckart



2019-11-11.pdf

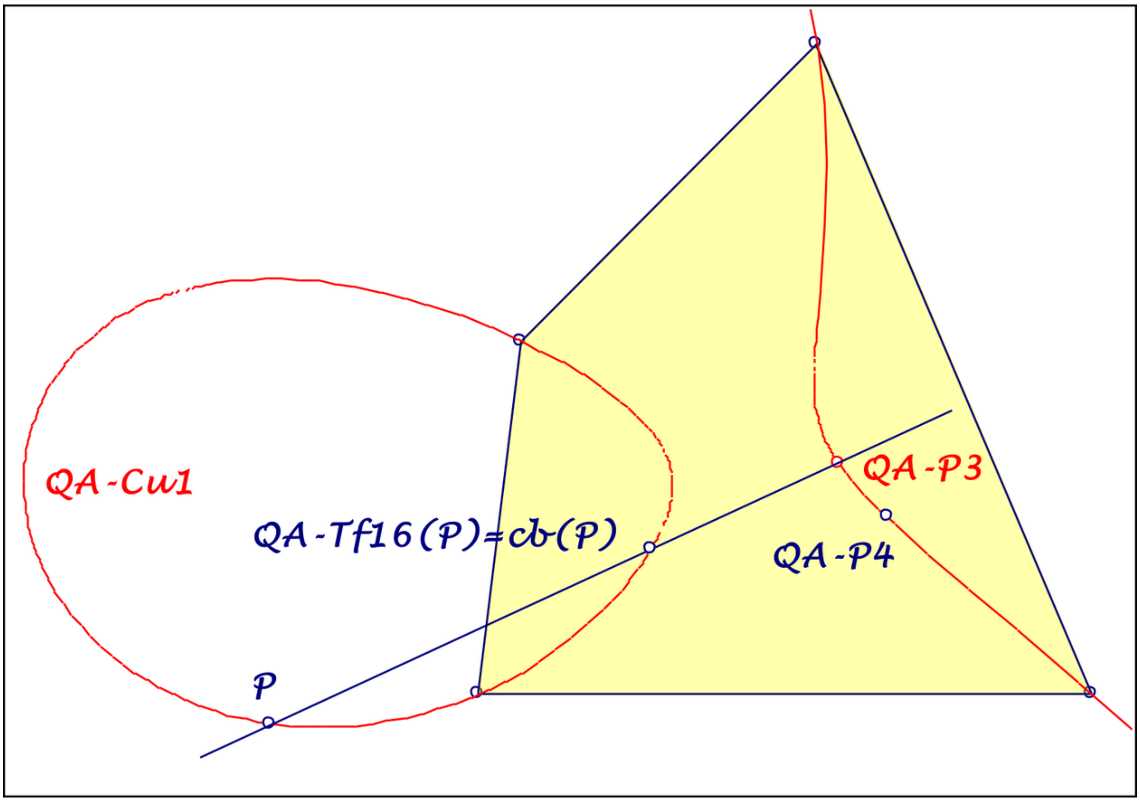
Message: #3900
Date: 11/11/2019 12:56:18
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

```
Dear Bernard, dear Chris,

for points P on QA-Cu1 holds:
… QA-Tf16(P) = cb(P) on QA-Cu1,
… collinear with P and QA-P3.

Best regards Eckart

PS: cb = Cayley-Bacharach transformation
    … wrt QA-vertices, QA-P4 and circular points.
```



2019-11-11a.pdf

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Message: #3901
Date: 14/11/2019 11:32:16
From: bernard.keizer
Subject: Quartics for 3 Lines with a CSC-transformation

```
Dear Eckart,
I wasn't home for a few weeks.
I was (and stil I'm) very interested by your CSC invariant
quartics.
We can find easily a few other in the QL environment : for
example with the 2 axes of the CSC and the 3 lines QA-P1QA-P4 or
the 2 axes of the CSC centered in CSC(T) and the 3 lines
QA-P4QA-P41.
The main property is following : is it true that 5 points Xi are
the 5 triple points of the 3 QA-Cu7 of a QL if and only if the 5
lines XiYi are tangent to a conic also tangent to the 2 axes of
the CSC of the 5 points ?
Best regards
Bernard
```

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Message: #3902
Date: 17/11/2019 10:42:46
From: bernard.keizer
Subject: Quartics for 3 Lines with a CSC-transformation

```
Dear Eckart,
Conversely, a CSC and a conic give with your construction a CSC
invariant quartic.
Any 3 points on the quartic define with the CSC the 2 other
points (one of your old constructions).
The 2 last points are generally not on the quartic.
If they are one the quartic, the 5 points are 5 QL triple
points.
5 points on the quartic are QL triple points if and only if
their CSC is the quartic's CSC ; the CSC of the 5 points lie of
course on the quartic and the 5 lines joining the 5 points and
their CSC are tangent to the conic.
In this case, what can be said about the 5 contact points of the
5 tangents ?
Best regards
Bernard
```

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Message: #3903
Date: 17/11/2019 9:32:55
From: chris.vantienhoven
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

```
Dear friends,

The time of our Yahoo Quadri-Figures-Group has come to an end.

Fortunately, there is a good alternative.
I opened a new group at Groups.io with the name
Quadri-and-Poly-Geometry.
This provider gives a lot of new features as you will discover
soon.
Alas it was not possible to migrate the Yahoo-messages to the
new Group preserving all message-numbers. Since we use these
numbers a lot, I decided not to migrate, because all our
references will become a mess.
In order to still be able to read old QFG-messages I will place
an archive of all our QFG-messages at my site soon.
The address of our new group is
https://groups.io/g/Quadri-and-Poly-Geometry.
I will send all members a personal invitation to join the new
group.
I am sure we will have a wonderful time together with the new
group.

Best regards,

Chris van Tienhoven
```

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Message: #3904
Date: 17/11/2019 9:38:55
From: Wilson Logan
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

```
Maybe this can help:

I have developed a utility to load Yahoo Groups messages to
groups.io
Currently it loads messages and attachments.
While at the moment it doesn't do Files & Photos uploading, that
will be added as a feature.
Here is a demo video of the uploader as it is now.
https://youtu.be/zZCFdEYiRK4 ( https://youtu.be/zZCFdEYiRK4 )

Cheers,
Wilson Logan.

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: 17 November 2019 20:33
> To: Quadri-Figures-Group@yahoogroups.com
> Subject: [Quadri-Figures-Group] LAST MESSAGE & NEW START WITH
A NEW GROUP
> The time of our Yahoo Quadri-Figures-Group has come to an end.
> Fortunately, there is a good alternative.
> I opened a new group at Groups.io with the name
Quadri-and-Poly-Geometry.
> This provider gives a lot of new features as you will discover
soon.
> Alas it was not possible to migrate the Yahoo-messages to the
new Group preserving all message-numbers.
> Since we use these numbers a lot, I decided not to migrate,
because all our references will become a mess.
> In order to still be able to read old QFG-messages
> I will place an archive of all our QFG-messages at my site
soon.
> The address of our new group is
https://groups.io/g/Quadri-and-Poly-Geometry.
> I will send all members a personal invitation to join the new
group.
> I am sure we will have a wonderful time together with the new
group.
> Best regards,
> Chris van Tienhoven
```

Message: #3905
Date: 17/11/2019 10:11:06
From: chris.vantienhoven
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

```
Dear Wilson,

I will have a look at it.
When your utility preserves message-numbers we maybe can migrate
the Yahoo-messages to a separate group at Groups.io so that we
can look up older messages easily.
Either way, the new group can start.

Best regards,
Chris
```

Message: #3906
Date: 17/11/2019 10:35:08
From: Wilson Logan
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

```
Yes of course.
I would recommend starting a sub-group called “Archive” or
similar and load the messages there.

Cheers,
Wilson.

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: 17 November 2019 21:11
> To: Quadri-Figures-Group@yahoogroups.com
> Subject: RE: [Quadri-Figures-Group] LAST MESSAGE & NEW START
WITH A NEW GROUP
> Dear Wilson,
> I will have a look at it.
> When your utility preserves message-numbers we maybe can
migrate the Yahoo-messages
> to a separate group at Groups.io so that we can look up older
messages easily.
> Either way, the new group can start.
> Best regards,
> Chris
```

---

5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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Miquel-triangle, Miquel Triangle:
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Newton Line:
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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal: Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393