

Journal of the Quadri Figures Group 2019

Digital Edition

Chris van Tienhoven et al.

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Messages #3300 - #3906

Contents

1	Introduction	2
2	Authors	3
2.1	Author Index	4
2.2	Author Information	6
3	Subjects	18
4	Messages	25
4.1	Message Index	25
4.2	Messages	30
5	Keyword Index	528
6	Colophon	532

1 Introduction

This journal is a compilation of messages from the

Quadri Figures Group (QFG)

a forum where mathematicians and geometry enthusiasts exchanged ideas on the properties of **quadrilaterals**, **polygons**, and related geometric structures. The discussions covered a wide range of topics, from classical geometric theorems to new discoveries and insights.

The Quadri Figures Group was active from 2013 until November 2019. During these years, the forum developed into a vibrant community and a valuable resource for exploring both well-established results and novel perspectives in geometry. In 2018 and 2019, problems began to arise with Yahoo Groups, the platform that handled the email distribution. Many attachments failed to arrive. In this journal, an effort has been made to recover and include as many of these attachments as possible.

When Yahoo Groups ended its activities in November 2019, the mathematical spirit of QFG did not disappear. Instead, the discussions continued and expanded within the **Quadri- and Poly-Geometry Group (QPG)**, available at <https://groups.io/g/Quadri-and-Poly-Geometry>. QPG took over the baton from QFG, broadening the scope from quadrilaterals to include polygons, poly-figures, and higher-degree curves. Together, the two forums form a continuous line of geometric exploration. An interactive backup of the former Quadri Figures Group is available at <https://groups.io/g/Quadri-Figures-Group>.

This journal was compiled retroactively in 2026 and preserves the annual record of all incoming messages from the Quadri Figures Group. It is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing readers to move quickly between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at <https://www.chrisvantienhoven.nl/>, which aims to systematically classify and analyze geometric structures. By collecting the forum messages of the Quadri Figures Group, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

[↩ Table of Contents](#) [↩ Message Index](#)

2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Benedetto Scimemi
- Bernard Gibert
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- David Fraivert
- Eckart Schmidt
- Stanley Rabinowitz
- Systems Manager
- Tran Quang Hung
- Tsihong Lau
- Vu Thanh Tung
- Wilson Logan
- Ángel Montesdeoca

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.

Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

Antreas Hatzipolakis:

[#3413](#) [#3653](#) [#3691](#) [#3829](#)

Benedetto Scimemi:

[#3512](#) [#3783](#) [#3897](#)

Bernard Gibert:

[#3547](#)

Bernard Keizer:

[#3301](#) [#3303](#) [#3308](#) [#3311](#) [#3312](#) [#3315](#) [#3318](#) [#3319](#) [#3321](#) [#3323](#) [#3324](#)
[#3325](#) [#3326](#) [#3327](#) [#3329](#) [#3334](#) [#3335](#) [#3337](#) [#3340](#) [#3347](#) [#3349](#) [#3350](#)
[#3352](#) [#3354](#) [#3356](#) [#3358](#) [#3361](#) [#3363](#) [#3365](#) [#3367](#) [#3369](#) [#3371](#) [#3373](#)
[#3378](#) [#3385](#) [#3394](#) [#3398](#) [#3402](#) [#3406](#) [#3407](#) [#3408](#) [#3410](#) [#3421](#) [#3422](#)
[#3423](#) [#3427](#) [#3428](#) [#3429](#) [#3431](#) [#3432](#) [#3434](#) [#3437](#) [#3440](#) [#3441](#) [#3446](#)
[#3458](#) [#3485](#) [#3490](#) [#3495](#) [#3506](#) [#3514](#) [#3515](#) [#3517](#) [#3518](#) [#3519](#) [#3524](#)
[#3526](#) [#3527](#) [#3533](#) [#3534](#) [#3535](#) [#3536](#) [#3540](#) [#3545](#) [#3546](#) [#3553](#) [#3554](#)
[#3557](#) [#3559](#) [#3563](#) [#3565](#) [#3566](#) [#3569](#) [#3570](#) [#3572](#) [#3574](#) [#3584](#) [#3585](#)
[#3592](#) [#3593](#) [#3594](#) [#3595](#) [#3597](#) [#3600](#) [#3601](#) [#3602](#) [#3603](#) [#3604](#) [#3605](#)
[#3609](#) [#3610](#) [#3612](#) [#3615](#) [#3617](#) [#3620](#) [#3623](#) [#3625](#) [#3630](#) [#3631](#) [#3632](#)
[#3633](#) [#3638](#) [#3660](#) [#3674](#) [#3676](#) [#3680](#) [#3684](#) [#3692](#) [#3699](#) [#3700](#) [#3704](#)
[#3709](#) [#3710](#) [#3721](#) [#3757](#) [#3758](#) [#3764](#) [#3765](#) [#3769](#) [#3770](#) [#3775](#) [#3780](#)
[#3785](#) [#3788](#) [#3791](#) [#3793](#) [#3797](#) [#3828](#) [#3848](#) [#3873](#) [#3879](#) [#3880](#) [#3884](#)
[#3888](#) [#3901](#) [#3902](#)

Chris van Tienhoven:

[#3415](#) [#3416](#) [#3418](#) [#3425](#) [#3487](#) [#3491](#) [#3523](#) [#3530](#) [#3532](#) [#3538](#) [#3550](#)
[#3567](#) [#3573](#) [#3576](#) [#3578](#) [#3580](#) [#3582](#) [#3586](#) [#3588](#) [#3589](#) [#3619](#) [#3627](#)
[#3628](#) [#3634](#) [#3637](#) [#3639](#) [#3640](#) [#3641](#) [#3644](#) [#3645](#) [#3651](#) [#3655](#) [#3656](#)
[#3662](#) [#3664](#) [#3665](#) [#3670](#) [#3682](#) [#3685](#) [#3687](#) [#3693](#) [#3695](#) [#3701](#) [#3706](#)
[#3714](#) [#3717](#) [#3723](#) [#3724](#) [#3732](#) [#3737](#) [#3738](#) [#3740](#) [#3742](#) [#3746](#) [#3747](#)
[#3750](#) [#3754](#) [#3759](#) [#3767](#) [#3772](#) [#3776](#) [#3800](#) [#3809](#) [#3821](#) [#3861](#) [#3863](#)
[#3877](#) [#3881](#) [#3889](#) [#3890](#) [#3903](#) [#3905](#)

César Lozada:

[#3302](#) [#3305](#) [#3306](#) [#3310](#) [#3681](#) [#3845](#) [#3869](#)

David Fraivert:

[#3801](#)

Eckart Schmidt:

[#3300](#) [#3304](#) [#3307](#) [#3309](#) [#3314](#) [#3316](#) [#3317](#) [#3320](#) [#3322](#) [#3328](#) [#3330](#)
[#3331](#) [#3333](#) [#3336](#) [#3338](#) [#3339](#) [#3341](#) [#3342](#) [#3343](#) [#3344](#) [#3345](#) [#3346](#)
[#3348](#) [#3351](#) [#3353](#) [#3355](#) [#3357](#) [#3359](#) [#3360](#) [#3362](#) [#3364](#) [#3366](#) [#3368](#)
[#3370](#) [#3372](#) [#3374](#) [#3375](#) [#3377](#) [#3380](#) [#3383](#) [#3388](#) [#3389](#) [#3392](#) [#3393](#)

#3395 #3397 #3399 #3400 #3401 #3403 #3404 #3405 #3409 #3411 #3412
#3414 #3420 #3424 #3426 #3430 #3433 #3435 #3436 #3438 #3439 #3442
#3444 #3445 #3450 #3451 #3459 #3460 #3461 #3464 #3466 #3468 #3471
#3476 #3478 #3481 #3482 #3484 #3489 #3492 #3493 #3494 #3498 #3503
#3508 #3509 #3511 #3513 #3516 #3522 #3525 #3528 #3541 #3542 #3544
#3548 #3549 #3551 #3552 #3555 #3558 #3561 #3562 #3564 #3568 #3571
#3575 #3577 #3579 #3581 #3583 #3587 #3590 #3591 #3596 #3598 #3599
#3606 #3607 #3608 #3611 #3613 #3614 #3616 #3618 #3621 #3622 #3624
#3626 #3629 #3635 #3636 #3642 #3643 #3647 #3648 #3649 #3657 #3663
#3668 #3669 #3675 #3679 #3683 #3690 #3694 #3696 #3697 #3698 #3703
#3705 #3712 #3713 #3718 #3719 #3720 #3722 #3726 #3727 #3728 #3729
#3760 #3762 #3763 #3766 #3768 #3771 #3773 #3774 #3777 #3778 #3779
#3781 #3782 #3784 #3786 #3787 #3789 #3790 #3792 #3795 #3798 #3799
#3804 #3805 #3808 #3810 #3817 #3819 #3820 #3822 #3823 #3826 #3834
#3836 #3837 #3842 #3849 #3854 #3858 #3859 #3860 #3862 #3867 #3868
#3871 #3872 #3876 #3883 #3885 #3886 #3887 #3891 #3892 #3893 #3894
#3895 #3896 #3898 #3899 #3900

Stanley Rabinowitz:

#3417 #3419 #3443 #3472 #3474 #3475 #3477 #3479 #3488 #3496 #3497
#3499 #3501 #3504 #3505 #3510

Systems Manager:

#3332 #3543

Tran Quang Hung:

#3376 #3379 #3381 #3382 #3384 #3386 #3387 #3390 #3391 #3396 #3447
#3448 #3452 #3453 #3454 #3455 #3456 #3457 #3462 #3463 #3465 #3467
#3469 #3470 #3473 #3480 #3483 #3486 #3646 #3650 #3652 #3654 #3658
#3659 #3661 #3666 #3667 #3671 #3672 #3673 #3677 #3678 #3686 #3688
#3689 #3702 #3707 #3708 #3711 #3715 #3716 #3725 #3730 #3731 #3733
#3734 #3735 #3736 #3739 #3741 #3743 #3744 #3745 #3748 #3749 #3751
#3752 #3753 #3755 #3756 #3761 #3794 #3796 #3802 #3807 #3813 #3814
#3816 #3818 #3833 #3838 #3840 #3841 #3847

Tsihong Lau:

#3313

Vu Thanh Tung:

#3449 #3500 #3502 #3507 #3520 #3521 #3529 #3531 #3539 #3556 #3560
#3803 #3806 #3811 #3812 #3815 #3824 #3825 #3827 #3830 #3831 #3832
#3835 #3839 #3843 #3844 #3846 #3850 #3851 #3852 #3853 #3855 #3856
#3857 #3864 #3865 #3866 #3870 #3874 #3875 #3878 #3882

Wilson Logan:

#3904 #3906

Ángel Montesdeoca:

#3537

2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location

Lives in Greece.

Year of Birth / Generation

1952.

Short Biography

Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: <http://www.anthrakitis.blogspot.com/>

Benedetto Scimemi (1938–2023)

Location

Italy

Year of Birth / Generation

1938–2023.

Short Biography

Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.es-teri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter’s 60th birthday (the “A5 Meeting”, named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See [CO-Tf3](#) in EPG.
- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location

Living in the Netherlands.

Year of Birth

1950.

Short Biography

Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks

Website: www.chrisvantienhoven.nl

Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location

Living in Germany.

Year of Birth / Generation

1939.

Short Biography

Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n -gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halkes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Stanley Rabinowitz

Location

Living in New Hampshire, USA.

Year of Birth / Generation

1947 (Baby Boomer).

Short Biography

Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions

Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, International Journal of Computer Discovered Mathematics 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, International Journal of Computer Discovered Mathematics 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 214–287.

Additional Remarks

Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location

Born and working in Hanoi, Vietnam.

Year of Birth / Generation

Millennial (approx. 1981–1996).

Short Biography

Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdős–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:

www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

Ángel Montesdeoca Delgado (1949–2024)

Location

Canary Islands, Spain.

Year of Birth / Generation

1949–2024.

Short Biography

Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions

Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes

Members of the geometry community remembered Ángel with great affection:

- His “deep knowledge of Projective Geometry” and “great amount of geometrical results” (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: <https://amontes.webs.ull.es/>

3 Subjects

The list below shows the subjects along with the numbers of related messages.
Click on a number to go to the corresponding page.

subject: "Familiar names for Polygons or n-Gons when $n > 5$ a ":
[#3653](#) [#3655](#)

subject: $(p^4 : q^4 : r^4) :$
[#3867](#)

subject: 5 points on on a cubic:
[#3405](#)

subject: 5P Cubic/Quartic Intersections:
[#3762](#) [#3765](#) [#3766](#) [#3767](#) [#3769](#) [#3770](#) [#3778](#)

subject: 5P orthogonal hyperbola:
[#3579](#) [#3580](#) [#3586](#) [#3587](#) [#3589](#) [#3590](#) [#3592](#)

subject: 5P, CSC and twin conics, cubics and quartics:
[#3757](#) [#3758](#) [#3760](#) [#3764](#) [#3771](#) [#3775](#) [#3779](#) [#3780](#)

subject: 5P-Geometry:
[#3334](#) [#3335](#) [#3336](#) [#3337](#) [#3338](#) [#3339](#) [#3340](#) [#3342](#) [#3346](#) [#3347](#) [#3348](#)
[#3349](#) [#3403](#) [#3412](#) [#3415](#) [#3420](#) [#3421](#) [#3423](#) [#3426](#) [#3429](#) [#3436](#) [#3437](#)
[#3442](#) [#3446](#) [#3522](#) [#3524](#) [#3526](#) [#3544](#) [#3546](#) [#3548](#) [#3549](#) [#3550](#) [#3551](#)
[#3552](#) [#3553](#) [#3554](#) [#3555](#) [#3557](#) [#3559](#) [#3561](#) [#3562](#) [#3563](#) [#3569](#) [#3571](#)
[#3572](#) [#3575](#) [#3576](#) [#3577](#) [#3578](#) [#3584](#) [#3585](#) [#3591](#) [#3593](#) [#3595](#) [#3596](#)
[#3597](#) [#3598](#) [#3600](#) [#3621](#) [#3645](#) [#3647](#) [#3648](#)

subject: 5P-Quartett:
[#3607](#) [#3609](#) [#3610](#)

subject: 5P-Quartic:
[#3599](#) [#3601](#) [#3602](#) [#3603](#) [#3604](#) [#3605](#) [#3606](#) [#3608](#) [#3611](#) [#3612](#) [#3613](#)
[#3614](#) [#3615](#) [#3616](#) [#3617](#) [#3719](#) [#3720](#) [#3721](#) [#3722](#)

subject: 5P-quartics for 6P:
[#3696](#) [#3697](#)

subject: 5P-Quartics for 6P:
[#3763](#)

subject: 5P-s-10P:
[#3564](#) [#3565](#) [#3566](#)

subject: 5P-s-2Px:
[#3768](#) [#3774](#)

subject: 5P-s-2Px giving Guitar-shaped Locus:
[#3772](#)

subject: 5P-s-Cix:
[#3773](#) [#3776](#) [#3783](#) [#3784](#) [#3786](#) [#3892](#) [#3893](#)

subject: 5P-s-Co1:
[#3517](#) [#3518](#) [#3693](#) [#3694](#) [#3699](#) [#3701](#) [#3704](#)

subject: 5P-s-P3,4,5:
[#3581](#) [#3582](#) [#3583](#) [#3588](#) [#3618](#) [#3619](#) [#3620](#) [#3623](#) [#3627](#) [#3628](#) [#3629](#)

[#3630](#) [#3631](#) [#3634](#) [#3636](#) [#3637](#) [#3638](#) [#3639](#) [#3640](#) [#3641](#) [#3642](#) [#3643](#)
[#3644](#)

subject: 5P-s-P4:

[#3333](#) [#3424](#) [#3425](#) [#3594](#)

subject: 5P-s-Tf3 / Co-Tf3:

[#3896](#) [#3897](#) [#3898](#)

subject: 5P=4L ?:

[#3519](#) [#3527](#)

subject: 6P-Geometry:

[#3646](#) [#3650](#) [#3651](#) [#3652](#)

subject: 6P-s-P1 and 7P-s-P1:

[#3860](#) [#3861](#) [#3862](#) [#3863](#) [#3883](#) [#3884](#) [#3886](#)

subject: 7 or 5 P or L:

[#3385](#) [#3389](#) [#3393](#) [#3394](#) [#3404](#) [#3406](#) [#3407](#) [#3408](#) [#3409](#) [#3410](#)

subject: 8th CB-Point on a Cubic:

[#3300](#) [#3301](#) [#3308](#) [#3312](#) [#3313](#)

subject: A conjecture on Newton line and inscribed parabola of quadrilate:

[#3826](#) [#3827](#) [#3828](#)

subject: A conjecture on Newton line and inscribed parabola of quadrilateral:

[#3825](#)

subject: A conjecture on two QA:

[#3753](#) [#3754](#) [#3755](#)

subject: A family of centers in quadrangle:

[#3830](#) [#3831](#) [#3832](#) [#3834](#) [#3835](#) [#3837](#) [#3839](#) [#3840](#) [#3841](#) [#3842](#) [#3843](#)
[#3844](#) [#3846](#) [#3847](#) [#3848](#) [#3849](#) [#3850](#) [#3851](#) [#3852](#)

subject: A new family of lines on quadrangle associated with Kimberling trian:

[#3520](#) [#3521](#)

subject: A new point on quadrangle ?:

[#3529](#) [#3530](#) [#3531](#) [#3537](#) [#3539](#)

subject: A Quartic for Quadrilaterals:

[#3513](#) [#3515](#) [#3516](#)

subject: A tranform on QA with two lines:

[#3671](#) [#3672](#) [#3682](#) [#3686](#)

subject: A transformation on QA from point to line:

[#3794](#) [#3795](#) [#3796](#) [#3800](#) [#3802](#) [#3803](#) [#3804](#) [#3805](#) [#3806](#) [#3807](#) [#3808](#)
[#3809](#) [#3816](#) [#3817](#) [#3818](#) [#3819](#)

subject: A transformation to parallel line and perspective triangle on QA:

[#3677](#) [#3681](#) [#3689](#)

subject: About QA-P4 of centroid quadrangle:

[#3386](#)

subject: Anallagmatic QL/5P-Quartic:
[#3887](#) [#3888](#)

subject: Another family of points on quadrangle:
[#3864](#) [#3878](#) [#3879](#) [#3880](#) [#3882](#)

subject: Another point on the Seebach-Walser Line:
[#3443](#) [#3444](#) [#3447](#) [#3449](#) [#3450](#) [#3456](#) [#3472](#) [#3474](#) [#3475](#) [#3476](#) [#3477](#)
[#3479](#)

subject: Another transformation on QA with two lines:
[#3673](#)

subject: Another transformation with two lines in QA:
[#3716](#) [#3723](#) [#3725](#) [#3726](#)

subject: Apply a transformation to QA:
[#3756](#) [#3759](#) [#3761](#)

subject: Apply P-Hagge circle to QA:
[#3749](#) [#3750](#) [#3752](#)

subject: Apply transformations in triangle to QA:
[#3745](#) [#3747](#) [#3748](#)

subject: Bad news from Yahoo:
[#3877](#) [#3881](#) [#3889](#)

subject: Begonia point of Quadrangle:
[#3391](#) [#3392](#)

subject: CB-Transformations:
[#3435](#)

subject: Center of circle lies on Steiner line:
[#3463](#) [#3466](#)

subject: Coaxial circles in QA:
[#3688](#) [#3691](#) [#3708](#) [#3712](#) [#3714](#) [#3715](#) [#3731](#) [#3732](#) [#3733](#)

subject: Common orthocenter from triangle:
[#3741](#) [#3742](#) [#3743](#)

subject: Common square point for 4 vertices w.r.t. Diagonal Triangle:
[#3853](#) [#3854](#) [#3856](#)

subject: Construction of a point on quadrangle:
[#3556](#) [#3558](#) [#3560](#)

subject: Construction of a point on Quadrangle:
[#3865](#) [#3866](#) [#3868](#) [#3869](#) [#3870](#) [#3871](#) [#3872](#) [#3873](#) [#3874](#) [#3875](#) [#3876](#)

subject: Construction of CO-Tf3(X):
[#3511](#) [#3512](#)

subject: CSC-invariant QA-circles:
[#3782](#)

subject: CTC issue:
[#3547](#)

subject: Deleted Message #3332:
[#3332](#)

subject: Deleted Message #3543:
[#3543](#)

subject: Deltoid for a convex 5P:
[#3649](#)

subject: Dual 4/5 P/L:
[#3536](#) [#3542](#) [#3545](#)

subject: Dual lines in QA:
[#3711](#) [#3713](#) [#3717](#) [#3718](#) [#3724](#)

subject: EPG-question:
[#3820](#) [#3821](#)

subject: epicycloidal curve around the sun:
[#3487](#) [#3488](#)

subject: Euler-Poncelet line in Quadrangle points:
[#3376](#) [#3377](#) [#3379](#)

subject: Feuerbach point lies on OI line:
[#3829](#) [#3833](#) [#3836](#) [#3838](#) [#3845](#)

subject: Fixed points of CB-transformation cb:
[#3622](#) [#3624](#) [#3625](#) [#3626](#) [#3632](#) [#3633](#) [#3635](#)

subject: Four related "Euler Line" analogs in a quadrigon:
[#3496](#) [#3497](#) [#3498](#) [#3525](#)

subject: Generalization of Eckart's observation in number 3842:
[#3855](#) [#3857](#)

subject: Gergonne circle:
[#3454](#) [#3458](#) [#3461](#) [#3462](#)

subject: LAST MESSAGE NEW START WITH A NEW GROUP:
[#3903](#) [#3904](#) [#3905](#) [#3906](#)

subject: last message(s) QFG-Yahoo-group:
[#3890](#)

subject: Line passes through centroid of Quadrangle points:
[#3382](#)

subject: Line passes through Poncelet point in Quadrangle points:
[#3381](#) [#3383](#) [#3384](#)

subject: Loci wrt concentric circles:
[#3885](#)

subject: Miquel circle of 5L:
[#3453](#) [#3459](#)

subject: Miquel circles are concurrent with 5L:
[#3455](#) [#3460](#) [#3491](#)

subject: Nagel Line Analog in a Quadrangle:
[#3499](#) [#3500](#) [#3501](#) [#3502](#) [#3503](#) [#3504](#) [#3505](#) [#3506](#) [#3507](#) [#3509](#) [#3510](#)
[#3514](#) [#3523](#)

subject: Name for line through Gergonne and Poncelet points of a quadrangle:
[#3413](#) [#3416](#) [#3417](#) [#3418](#) [#3419](#)

subject: New 5P transformation:

[#3660](#) [#3663](#) [#3668](#) [#3669](#) [#3670](#) [#3674](#) [#3675](#) [#3676](#) [#3679](#) [#3680](#) [#3683](#)
[#3684](#) [#3685](#) [#3690](#) [#3692](#) [#3695](#) [#3698](#) [#3700](#) [#3703](#) [#3705](#) [#3709](#) [#3710](#)

subject: New aspect of QA-Cu7-triple points:

[#3493](#) [#3494](#) [#3495](#)

subject: New aspects of 5P-geometry:

[#3789](#) [#3791](#) [#3792](#) [#3793](#) [#3797](#) [#3799](#)

subject: New line to line transformation for QL:

[#3810](#) [#3811](#) [#3812](#) [#3813](#) [#3822](#) [#3823](#) [#3824](#)

subject: New Quartic for the 5P of the QA-Cu7-triple Points:

[#3891](#)

subject: Newton line passes through Poncelet point:

[#3387](#) [#3395](#) [#3396](#)

subject: Newton lines are concurrent in QA:

[#3735](#) [#3738](#)

subject: Newton lines are concurrent with 5P:

[#3448](#) [#3451](#) [#3452](#)

subject: No Subject:

[#3331](#)

subject: number 3382, 3386,number 3387:

[#3388](#) [#3390](#)

subject: number 3453, number 3455, number 3459, number 3460:

[#3489](#) [#3490](#) [#3492](#)

subject: On a 5-Gon point:

[#3654](#) [#3656](#) [#3657](#) [#3658](#)

subject: Orthogonal QL-Hyperbolas:

[#3438](#) [#3440](#) [#3441](#)

subject: Perpendicular lines from NPC in triangle:

[#3744](#) [#3746](#) [#3751](#)

subject: Perspective triangles in 6-Gon point:

[#3659](#) [#3664](#) [#3666](#)

subject: QA - A circumconic:

[#3302](#) [#3303](#) [#3304](#) [#3305](#) [#3306](#) [#3309](#) [#3310](#) [#3311](#)

subject: QA-Cu1:

[#3900](#)

subject: QA-Cu7-Geometry:

[#3307](#) [#3314](#) [#3315](#) [#3316](#) [#3320](#) [#3321](#) [#3322](#) [#3323](#) [#3324](#) [#3325](#) [#3326](#)
[#3327](#) [#3328](#) [#3329](#) [#3341](#) [#3345](#) [#3350](#) [#3351](#) [#3352](#) [#3353](#) [#3354](#) [#3355](#)
[#3356](#) [#3357](#) [#3358](#) [#3359](#) [#3360](#) [#3361](#) [#3362](#) [#3363](#) [#3364](#) [#3365](#) [#3366](#)
[#3367](#) [#3368](#) [#3369](#) [#3372](#) [#3373](#) [#3374](#) [#3375](#) [#3378](#) [#3380](#) [#3397](#) [#3398](#)
[#3399](#) [#3401](#) [#3402](#) [#3411](#) [#3427](#) [#3428](#) [#3430](#) [#3431](#) [#3432](#) [#3433](#) [#3434](#)
[#3528](#) [#3532](#) [#3533](#) [#3534](#) [#3535](#) [#3538](#) [#3540](#) [#3541](#) [#3567](#) [#3568](#) [#3570](#)
[#3573](#) [#3574](#)

subject: QA-Geometry:
[#3317](#) [#3318](#) [#3319](#) [#3330](#) [#3370](#) [#3371](#)

subject: QA-P2,4,41 perspective QG-P1,19,18:
[#3414](#) [#3422](#)

subject: QA-P4 of pedal-QA:
[#3781](#) [#3785](#) [#3787](#) [#3788](#) [#3790](#)

subject: QG-2:
[#3344](#)

subject: QL-geometry with point products:
[#3858](#) [#3859](#)

subject: QL-Geometry with QL-Quartet:
[#3484](#) [#3486](#)

subject: QL-Lines wrt QA-Tf2(P):
[#3777](#)

subject: QL-Parabolas:
[#3439](#)

subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33:
[#3727](#) [#3728](#) [#3729](#) [#3730](#)

subject: Quartics for 3 Lines with a CSC-transformation:
[#3894](#) [#3901](#) [#3902](#)

subject: Quasi-Euler Lines for a 5P:
[#3508](#)

subject: Quintupel-Points for a 5P:
[#3445](#) [#3485](#)

subject: Similar figures of seven points:
[#3814](#) [#3815](#)

subject: Six concyclic Poncelet points on QL:
[#3678](#) [#3687](#)

subject: Some new theorems on Pentagon and Pentagram:
[#3702](#) [#3706](#) [#3707](#)

subject: Special QA-Circumcubic:
[#3895](#)

subject: Special QA-Circumquartics:
[#3899](#)

subject: Special QL on QL-Cu1:
[#3343](#)

subject: Steiner line of Gergonne point:
[#3736](#) [#3740](#)

subject: Steiner lines are concurrent in QL:
[#3467](#) [#3468](#) [#3469](#) [#3470](#) [#3471](#) [#3473](#) [#3478](#) [#3480](#) [#3481](#) [#3482](#) [#3483](#)

subject: The theory of a convex quadrilateral and a circle that forms Pascal:
[#3801](#)

subject: Triple points for QG-lines and conics:

[#3400](#)

subject: trisecting the vertex angle of a regular n-Gon:

[#3665](#)

subject: Two cyclologic triangles in QA:

[#3661](#) [#3662](#) [#3667](#)

subject: Two QA have the same Poncelet point:

[#3734](#) [#3737](#) [#3739](#)

subject: Two QL share the same Miquel point and Steiner line:

[#3457](#) [#3464](#) [#3465](#)

subject: Wrt 5P-s-Tf1,2,3,4:

[#3798](#)

[↔ Table of Contents](#) [↔ Message Index](#)

4 Messages

4.1 Message Index

Msg #3300: page 30	Msg #3346: page 73	Msg #3392: page 111
Msg #3301: page 33	Msg #3347: page 75	Msg #3393: page 112
Msg #3302: page 34	Msg #3348: page 76	Msg #3394: page 112
Msg #3303: page 35	Msg #3349: page 78	Msg #3395: page 113
Msg #3304: page 35	Msg #3350: page 79	Msg #3396: page 113
Msg #3305: page 36	Msg #3351: page 80	Msg #3397: page 114
Msg #3306: page 36	Msg #3352: page 81	Msg #3398: page 115
Msg #3307: page 38	Msg #3353: page 82	Msg #3399: page 116
Msg #3308: page 40	Msg #3354: page 82	Msg #3400: page 118
Msg #3309: page 41	Msg #3355: page 83	Msg #3401: page 119
Msg #3310: page 42	Msg #3356: page 83	Msg #3402: page 120
Msg #3311: page 43	Msg #3357: page 84	Msg #3403: page 121
Msg #3312: page 43	Msg #3358: page 84	Msg #3404: page 122
Msg #3313: page 44	Msg #3359: page 85	Msg #3405: page 124
Msg #3314: page 45	Msg #3360: page 86	Msg #3406: page 125
Msg #3315: page 46	Msg #3361: page 87	Msg #3407: page 126
Msg #3316: page 47	Msg #3362: page 88	Msg #3408: page 126
Msg #3317: page 48	Msg #3363: page 89	Msg #3409: page 127
Msg #3318: page 48	Msg #3364: page 90	Msg #3410: page 128
Msg #3319: page 50	Msg #3365: page 92	Msg #3411: page 129
Msg #3320: page 51	Msg #3366: page 93	Msg #3412: page 131
Msg #3321: page 52	Msg #3367: page 94	Msg #3413: page 132
Msg #3322: page 52	Msg #3368: page 95	Msg #3414: page 133
Msg #3323: page 53	Msg #3369: page 97	Msg #3415: page 134
Msg #3324: page 53	Msg #3370: page 98	Msg #3416: page 134
Msg #3325: page 54	Msg #3371: page 99	Msg #3417: page 135
Msg #3326: page 54	Msg #3372: page 99	Msg #3418: page 135
Msg #3327: page 55	Msg #3373: page 100	Msg #3419: page 136
Msg #3328: page 55	Msg #3374: page 100	Msg #3420: page 137
Msg #3329: page 56	Msg #3375: page 101	Msg #3421: page 138
Msg #3330: page 57	Msg #3376: page 102	Msg #3422: page 138
Msg #3331: page 58	Msg #3377: page 102	Msg #3423: page 139
Msg #3332: page 59	Msg #3378: page 103	Msg #3424: page 139
Msg #3333: page 59	Msg #3379: page 104	Msg #3425: page 140
Msg #3334: page 60	Msg #3380: page 104	Msg #3426: page 140
Msg #3335: page 61	Msg #3381: page 105	Msg #3427: page 141
Msg #3336: page 62	Msg #3382: page 105	Msg #3428: page 142
Msg #3337: page 63	Msg #3383: page 106	Msg #3429: page 142
Msg #3338: page 64	Msg #3384: page 106	Msg #3430: page 143
Msg #3339: page 65	Msg #3385: page 107	Msg #3431: page 144
Msg #3340: page 66	Msg #3386: page 108	Msg #3432: page 144
Msg #3341: page 66	Msg #3387: page 109	Msg #3433: page 144
Msg #3342: page 68	Msg #3388: page 109	Msg #3434: page 145
Msg #3343: page 70	Msg #3389: page 110	Msg #3435: page 146
Msg #3344: page 71	Msg #3390: page 110	Msg #3436: page 147
Msg #3345: page 72	Msg #3391: page 111	Msg #3437: page 148

Msg #3438: page 149	Msg #3488: page 190	Msg #3538: page 232
Msg #3439: page 151	Msg #3489: page 190	Msg #3539: page 233
Msg #3440: page 152	Msg #3490: page 191	Msg #3540: page 234
Msg #3441: page 153	Msg #3491: page 192	Msg #3541: page 235
Msg #3442: page 154	Msg #3492: page 193	Msg #3542: page 236
Msg #3443: page 155	Msg #3493: page 194	Msg #3543: page 236
Msg #3444: page 156	Msg #3494: page 195	Msg #3544: page 237
Msg #3445: page 157	Msg #3495: page 195	Msg #3545: page 237
Msg #3446: page 158	Msg #3496: page 196	Msg #3546: page 238
Msg #3447: page 158	Msg #3497: page 197	Msg #3547: page 239
Msg #3448: page 159	Msg #3498: page 198	Msg #3548: page 240
Msg #3449: page 160	Msg #3499: page 199	Msg #3549: page 241
Msg #3450: page 161	Msg #3500: page 200	Msg #3550: page 241
Msg #3451: page 162	Msg #3501: page 201	Msg #3551: page 242
Msg #3452: page 162	Msg #3502: page 202	Msg #3552: page 242
Msg #3453: page 163	Msg #3503: page 203	Msg #3553: page 243
Msg #3454: page 163	Msg #3504: page 203	Msg #3554: page 243
Msg #3455: page 164	Msg #3505: page 204	Msg #3555: page 244
Msg #3456: page 165	Msg #3506: page 205	Msg #3556: page 245
Msg #3457: page 166	Msg #3507: page 206	Msg #3557: page 247
Msg #3458: page 166	Msg #3508: page 207	Msg #3558: page 247
Msg #3459: page 167	Msg #3509: page 208	Msg #3559: page 248
Msg #3460: page 167	Msg #3510: page 208	Msg #3560: page 248
Msg #3461: page 168	Msg #3511: page 209	Msg #3561: page 249
Msg #3462: page 168	Msg #3512: page 209	Msg #3562: page 250
Msg #3463: page 169	Msg #3513: page 210	Msg #3563: page 250
Msg #3464: page 170	Msg #3514: page 211	Msg #3564: page 251
Msg #3465: page 171	Msg #3515: page 211	Msg #3565: page 252
Msg #3466: page 172	Msg #3516: page 212	Msg #3566: page 252
Msg #3467: page 173	Msg #3517: page 213	Msg #3567: page 253
Msg #3468: page 173	Msg #3518: page 214	Msg #3568: page 258
Msg #3469: page 174	Msg #3519: page 214	Msg #3569: page 259
Msg #3470: page 175	Msg #3520: page 215	Msg #3570: page 259
Msg #3471: page 176	Msg #3521: page 215	Msg #3571: page 260
Msg #3472: page 176	Msg #3522: page 216	Msg #3572: page 260
Msg #3473: page 177	Msg #3523: page 217	Msg #3573: page 261
Msg #3474: page 178	Msg #3524: page 218	Msg #3574: page 261
Msg #3475: page 178	Msg #3525: page 219	Msg #3575: page 262
Msg #3476: page 179	Msg #3526: page 220	Msg #3576: page 264
Msg #3477: page 179	Msg #3527: page 220	Msg #3577: page 264
Msg #3478: page 180	Msg #3528: page 221	Msg #3578: page 265
Msg #3479: page 181	Msg #3529: page 223	Msg #3579: page 266
Msg #3480: page 182	Msg #3530: page 224	Msg #3580: page 267
Msg #3481: page 183	Msg #3531: page 225	Msg #3581: page 267
Msg #3482: page 184	Msg #3532: page 226	Msg #3582: page 268
Msg #3483: page 185	Msg #3533: page 229	Msg #3583: page 269
Msg #3484: page 186	Msg #3534: page 230	Msg #3584: page 270
Msg #3485: page 187	Msg #3535: page 230	Msg #3585: page 271
Msg #3486: page 188	Msg #3536: page 231	Msg #3586: page 272
Msg #3487: page 189	Msg #3537: page 232	Msg #3587: page 272

Msg #3588: page 273	Msg #3638: page 311	Msg #3688: page 351
Msg #3589: page 274	Msg #3639: page 311	Msg #3689: page 352
Msg #3590: page 275	Msg #3640: page 312	Msg #3690: page 352
Msg #3591: page 276	Msg #3641: page 314	Msg #3691: page 353
Msg #3592: page 277	Msg #3642: page 315	Msg #3692: page 354
Msg #3593: page 277	Msg #3643: page 315	Msg #3693: page 354
Msg #3594: page 278	Msg #3644: page 316	Msg #3694: page 355
Msg #3595: page 279	Msg #3645: page 316	Msg #3695: page 355
Msg #3596: page 279	Msg #3646: page 317	Msg #3696: page 356
Msg #3597: page 280	Msg #3647: page 319	Msg #3697: page 356
Msg #3598: page 280	Msg #3648: page 319	Msg #3698: page 357
Msg #3599: page 281	Msg #3649: page 320	Msg #3699: page 357
Msg #3600: page 282	Msg #3650: page 321	Msg #3700: page 358
Msg #3601: page 282	Msg #3651: page 322	Msg #3701: page 359
Msg #3602: page 283	Msg #3652: page 323	Msg #3702: page 360
Msg #3603: page 284	Msg #3653: page 324	Msg #3703: page 360
Msg #3604: page 284	Msg #3654: page 325	Msg #3704: page 361
Msg #3605: page 285	Msg #3655: page 326	Msg #3705: page 361
Msg #3606: page 285	Msg #3656: page 327	Msg #3706: page 362
Msg #3607: page 286	Msg #3657: page 327	Msg #3707: page 363
Msg #3608: page 287	Msg #3658: page 328	Msg #3708: page 364
Msg #3609: page 288	Msg #3659: page 329	Msg #3709: page 365
Msg #3610: page 288	Msg #3660: page 330	Msg #3710: page 366
Msg #3611: page 289	Msg #3661: page 331	Msg #3711: page 367
Msg #3612: page 290	Msg #3662: page 331	Msg #3712: page 368
Msg #3613: page 291	Msg #3663: page 332	Msg #3713: page 368
Msg #3614: page 291	Msg #3664: page 332	Msg #3714: page 369
Msg #3615: page 292	Msg #3665: page 333	Msg #3715: page 369
Msg #3616: page 293	Msg #3666: page 334	Msg #3716: page 370
Msg #3617: page 293	Msg #3667: page 335	Msg #3717: page 371
Msg #3618: page 294	Msg #3668: page 335	Msg #3718: page 372
Msg #3619: page 295	Msg #3669: page 336	Msg #3719: page 372
Msg #3620: page 296	Msg #3670: page 336	Msg #3720: page 373
Msg #3621: page 297	Msg #3671: page 337	Msg #3721: page 373
Msg #3622: page 298	Msg #3672: page 338	Msg #3722: page 374
Msg #3623: page 300	Msg #3673: page 339	Msg #3723: page 375
Msg #3624: page 301	Msg #3674: page 340	Msg #3724: page 377
Msg #3625: page 301	Msg #3675: page 340	Msg #3725: page 378
Msg #3626: page 302	Msg #3676: page 341	Msg #3726: page 379
Msg #3627: page 302	Msg #3677: page 341	Msg #3727: page 381
Msg #3628: page 303	Msg #3678: page 342	Msg #3728: page 382
Msg #3629: page 304	Msg #3679: page 343	Msg #3729: page 382
Msg #3630: page 305	Msg #3680: page 343	Msg #3730: page 383
Msg #3631: page 305	Msg #3681: page 345	Msg #3731: page 383
Msg #3632: page 306	Msg #3682: page 346	Msg #3732: page 384
Msg #3633: page 306	Msg #3683: page 347	Msg #3733: page 384
Msg #3634: page 307	Msg #3684: page 348	Msg #3734: page 385
Msg #3635: page 308	Msg #3685: page 348	Msg #3735: page 385
Msg #3636: page 309	Msg #3686: page 349	Msg #3736: page 386
Msg #3637: page 310	Msg #3687: page 350	Msg #3737: page 387

Msg #3738: page 387	Msg #3788: page 423	Msg #3838: page 468
Msg #3739: page 388	Msg #3789: page 424	Msg #3839: page 468
Msg #3740: page 389	Msg #3790: page 426	Msg #3840: page 469
Msg #3741: page 390	Msg #3791: page 427	Msg #3841: page 470
Msg #3742: page 391	Msg #3792: page 428	Msg #3842: page 471
Msg #3743: page 391	Msg #3793: page 429	Msg #3843: page 472
Msg #3744: page 392	Msg #3794: page 429	Msg #3844: page 472
Msg #3745: page 393	Msg #3795: page 430	Msg #3845: page 473
Msg #3746: page 394	Msg #3796: page 430	Msg #3846: page 474
Msg #3747: page 394	Msg #3797: page 431	Msg #3847: page 475
Msg #3748: page 395	Msg #3798: page 431	Msg #3848: page 476
Msg #3749: page 396	Msg #3799: page 432	Msg #3849: page 477
Msg #3750: page 396	Msg #3800: page 433	Msg #3850: page 478
Msg #3751: page 397	Msg #3801: page 435	Msg #3851: page 479
Msg #3752: page 397	Msg #3802: page 436	Msg #3852: page 480
Msg #3753: page 398	Msg #3803: page 436	Msg #3853: page 481
Msg #3754: page 399	Msg #3804: page 437	Msg #3854: page 481
Msg #3755: page 400	Msg #3805: page 437	Msg #3855: page 482
Msg #3756: page 400	Msg #3806: page 438	Msg #3856: page 482
Msg #3757: page 401	Msg #3807: page 439	Msg #3857: page 483
Msg #3758: page 402	Msg #3808: page 439	Msg #3858: page 484
Msg #3759: page 402	Msg #3809: page 440	Msg #3859: page 485
Msg #3760: page 403	Msg #3810: page 442	Msg #3860: page 486
Msg #3761: page 403	Msg #3811: page 443	Msg #3861: page 486
Msg #3762: page 404	Msg #3812: page 444	Msg #3862: page 487
Msg #3763: page 405	Msg #3813: page 445	Msg #3863: page 487
Msg #3764: page 406	Msg #3814: page 445	Msg #3864: page 488
Msg #3765: page 407	Msg #3815: page 446	Msg #3865: page 488
Msg #3766: page 407	Msg #3816: page 447	Msg #3866: page 489
Msg #3767: page 408	Msg #3817: page 448	Msg #3867: page 490
Msg #3768: page 408	Msg #3818: page 448	Msg #3868: page 490
Msg #3769: page 409	Msg #3819: page 449	Msg #3869: page 491
Msg #3770: page 409	Msg #3820: page 449	Msg #3870: page 492
Msg #3771: page 410	Msg #3821: page 450	Msg #3871: page 492
Msg #3772: page 411	Msg #3822: page 451	Msg #3872: page 493
Msg #3773: page 412	Msg #3823: page 452	Msg #3873: page 493
Msg #3774: page 412	Msg #3824: page 453	Msg #3874: page 494
Msg #3775: page 413	Msg #3825: page 454	Msg #3875: page 495
Msg #3776: page 413	Msg #3826: page 455	Msg #3876: page 496
Msg #3777: page 414	Msg #3827: page 455	Msg #3877: page 497
Msg #3778: page 414	Msg #3828: page 456	Msg #3878: page 498
Msg #3779: page 415	Msg #3829: page 457	Msg #3879: page 502
Msg #3780: page 416	Msg #3830: page 458	Msg #3880: page 502
Msg #3781: page 417	Msg #3831: page 459	Msg #3881: page 503
Msg #3782: page 419	Msg #3832: page 461	Msg #3882: page 504
Msg #3783: page 420	Msg #3833: page 462	Msg #3883: page 505
Msg #3784: page 420	Msg #3834: page 464	Msg #3884: page 506
Msg #3785: page 422	Msg #3835: page 465	Msg #3885: page 507
Msg #3786: page 422	Msg #3836: page 467	Msg #3886: page 507
Msg #3787: page 423	Msg #3837: page 467	Msg #3887: page 509

Msg #3888: page [510](#)
Msg #3889: page [511](#)
Msg #3890: page [512](#)
Msg #3891: page [513](#)
Msg #3892: page [515](#)
Msg #3893: page [515](#)
Msg #3894: page [516](#)

Msg #3895: page [518](#)
Msg #3896: page [520](#)
Msg #3897: page [520](#)
Msg #3898: page [521](#)
Msg #3899: page [522](#)
Msg #3900: page [523](#)
Msg #3901: page [524](#)

Msg #3902: page [524](#)
Msg #3903: page [525](#)
Msg #3904: page [526](#)
Msg #3905: page [527](#)
Msg #3906: page [527](#)

[↔ Table of Contents](#)

4.2 Messages

Message: #3300

Date: 01/1/2019 4:30:59

From: eckart_schmidt@t-online.de

Subject: 8th CB-Point on a Cubic

Dear Bernard, dear Chris,

I don't know, whether the following geometry on cubics is already discussed:

Well known is the 9th Cayley-Bacharach point

... as common point of all cubics through 8 points.

Also well known is, that 7 points define a CB-transformation $X \mapsto CB(X)$.

... For 7 points on a cubic the CB-transformation maps the cubic to itself.

... The 3rd intersections of $X.CB(X)$ and the cubic give the same point Q ,

... here named as 8th CB-point of seven points on the cubic.

For the cubic $QL-Cu1$ bearing the 6 QL -points and a point P^*

... the 8th CB-point is $Q = CSC(P)$.

For the cubic $QA-Cu7$ bearing 3 QG -P18, 3 QG -P19 and a point P

... the 8th CB-point is $Q = QA-Tf2(P)$.

In this way we get a transformation $P \mapsto Q$ on a cubic,

... defined by 6 points on the cubic (see attached file):

Take the CB-transformation of the 6 defining points and P

... and consider for points X on the cubic

... the 3rd intersection of $X.CB(X)$ as image Q of P .

By the way:

... $CB(Q)$ is the tangential of Q wrt the cubic.

... The 9th CB-point of $P1,2,3,4,5,6,X$ and $CB(X)$ is P .

Examples (see also #3006 and #3176):

Let E be the infinity point of the asymptote of $QA-Cu7$

... and F be the intersection of $QA-Cu7$ and its asymptote,

... which lies diametral to $QA-P41$ wrt the circumcircle of $QA-P2,4,41$.

(1) For $QA-Cu7$ with the vertices of $QA-Tr1$ and the QG -P19 triangle

... $P \mapsto Q$ maps $QA-P2$ to $QA-P4$, $QA-P41$ to E

... and F to the 3rd intersection of $QA-P2.QA-P4$ and $QA-Cu7$.

For $QA-Cu7$ with the vertices of $QA-Tr1$ and the QG -P18 triangle

... $P \mapsto Q$ maps $QA-P2$ to $QA-P41$, $QA-P4$ to E

... and F to the 3rd intersection of $QA-P2.QA-P41$ and $QA-Cu7$.

For $QA-Cu7$ with the vertices of the QG -P18 and the QG -P19 triangle

... $P \mapsto Q$ is $QA-Tf2$ and maps $QA-P4$ to $QA-P41$, $QA-P2$ to E

... and F to the 3rd intersection of $QA-P4.QA-P41$ and $QA-Cu7$.

(2) For QA-Cu7 with the vertices of QA-Tr1 and QA-P2, QA-P4, QA-P41
 ... the transformation $P \rightarrow Q$ maps F to QA-P2.
 For QA-Cu7 with the vertices of the QG-P18 triangle and QA-P2, QA-P4, QA-P41
 ... the transformation $P \rightarrow Q$ maps F to QA-P41.
 For QA-Cu7 with the vertices of the QG-P19 triangle and QA-P2, QA-P4, QA-P41
 ... the transformation $P \rightarrow Q$ maps F to QA-P4.

(3) For QA-Cu1 with the vertices of the triangles QA-Tr1 and QA-Tr2
 ... the transformation $P \rightarrow Q$ maps QA-P3
 ... to the 3rd intersection of the cubic and the line
 ... from QA-P41 to the intersection of the cubic
 and its asymptote.
 For QA-Cu1 with the vertices of QA-Tr2 and the QG-P16 triangle
 ... the transformation $P \rightarrow Q$ maps QA-P4 to QA-P41.

Finally:

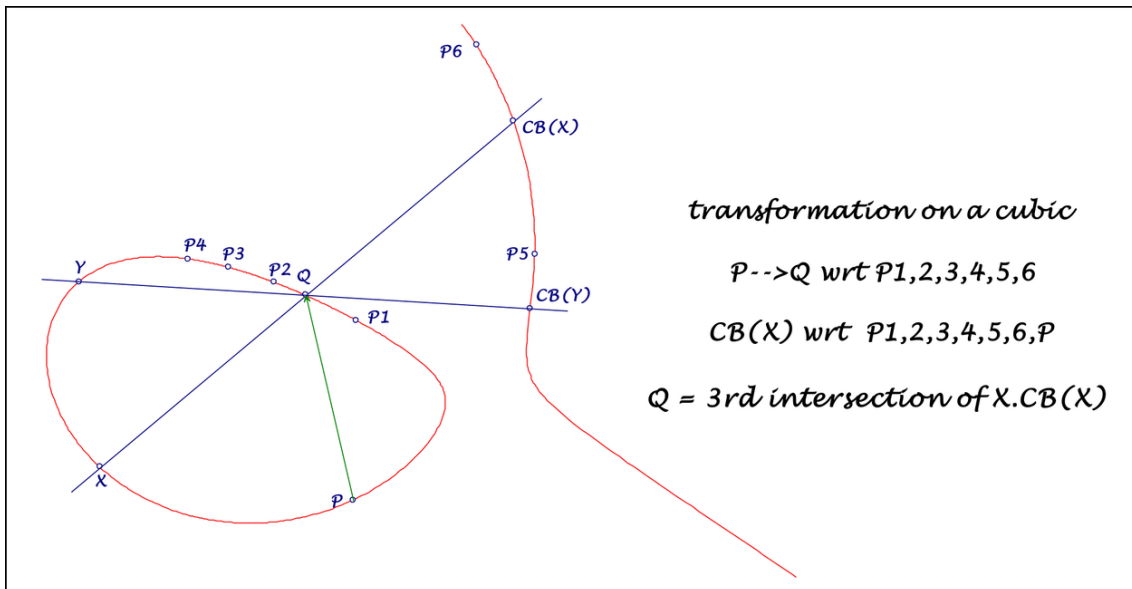
(4) Consider a QL and the 5 triple intersections of its 3 QA-Cu7,
 ... take a QG-version of the QL, interpreted as QA with its QA-Cu7,
 ... and the transformation $P \rightarrow Q$
 ... wrt the 5 triple intersections and a point P6 on QA-Cu7,
 ... then the QG-point S (see #3176) will be mapped to P6.

Remark wrt (2) and (4):

If P is the 6th intersection of the cubic
 ... and a conic through 5 defining points,
 ... then Q is the 6th defining point.
 This leads wrt (2) to three conics:
 ... circumconic of QA-Tr1 through QA-P4 and QA-P41,
 ... circumconic of the QG-P18-triangle through QA-P2 and QA-P4,
 ... circumconic of the QG-P19-triangle through QA-P2 and QA-P41,
 with a common point in the intersection F of QA-Cu7
 and its asymptote.

I hope, there are no grave errors.
 Perhaps the observations can be described simpler.

Best regards Eckart



2019-01-01.pdf

Next → ← Message Index ↑ Subjects

Message: #3301
Date: 03/1/2019 1:10:15
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

Dear Eckart,
I suppose, as long as we are here, the QFG is not dead !
8 points give a 9th CB point
7 points define a CB transformation
On a given cubic, the CB transformation of 7 points is an
isoconjugation and the cubic is a pivotal isocubic with pivot Q,
fixed points the vertices of the tangential QA of Q and
reference triangle the DT of the QA.
All this may be proved with Cotterill's construction.
You make a step further by considering the transformation with 6
fixed points and a variable 7th point P and the pivot Q. Very
clever indeed !
The 9th CB point of P1 to 6, P and X being CB(X), the 9th point
of P1 to 6, X and CB(X) is P ...
For QL-Cu1 (which is a particular QA-Cu1), any point is QA-P4 of
it's tangential triangle and the pivot is QA-P2, intersection of
the 2 perpendicular lines of the QA (which form the degenerated
polar conic of the point) and CSC of QA-P4 ...
For QA-Cu7, which is also a QL-Cu1, many of your properties
derive from the definition of the 3 CSC of the triangle QA-P2,
P4 and P41 ...
Of course, all this is true also for non circular cubics and for
circular cubics, you may have the same properties with 5 points
or 4 fixed points + 1 variable ...
I read your messages always with same pleasure, please go on !
Best regards
Bernard
PS How do you draw your cubics in Cabri ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3302
Date: 07/1/2019 5:30:02
From: César E. Lozada
Subject: QA - A circumconic

Let $qa=\{A_i \ (i=1..4)\}$ be a quadrangle.

Denote:

Kn = the nine-point conic QA-Co1 of qa , with center $On = QAP-1$
(i.e, the conic through all six $M_{i,j}$ midpoints of A_iA_j , $i \neq j$)

Q_i = the perspector of Kn w/r to the triangle $A_jA_kA_l$ ($i=1..4$)

K^* = the circumconic of qa through On

Then

Q_i lies on K^* , for $i=1..4$.

K^* passes also through QA-P16 (QA-Harmonic center)

The diagonal triangle of qa is autopolar w/r to K^*

CT coordinates of center O^* and perspector Q^* of K^* are:

$O^* = p^*(q-r)^2*(2*p+q+r)*(3*(p+q+r)*p-q*r) : :$

$Q^* = p^*(q-r)*(2*p+q+r) : :$

Best regards,
César Lozada

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3303
Date: 07/1/2019 10:55:23
From: bernard.keizer
Subject: QA - A circumconic

Dear César,
Your conic K^* is the dual curve of the parabola inscribed in the dual QL of the QA.
It passes also through the 3 S-points, 3 other intersections of the DT circumscribed conic through QA-P10 and QA-P16, which is the dual curve of the parabola inscribed in DT and tangent to the Newton Line.
The 3 S-points lie on the Dimidium circle QL-Ci6 and form the QL-Tr2, which is also autopolar wrt K^* .
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3304
Date: 07/1/2019 11:05:54
From: eckart_schmidt@t-online.de
Subject: QA-A circumconic

Dear Cesar Lozada,

some remarks wrt #3302:

- (1) What is the perspector of a conic wrt a triangle?
- (2) I think, property 3) holds in general for QA-circumconics.
- (3) In addition to the coordinates of O^* :

O^* is QA-Tf2 of the infinity point
... of the tangents at K_n
... in the intersections with QA-L3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3305
Date: 07/1/2019 11:26:15
From: César E. Lozada
Subject: QA-A circumconic

Dear Eckart,

Thanks a lot for your quick response.

(1) What is the perspector of a conic wrt a triangle?

In the plane of a reference triangle

(<http://mathworld.wolfram.com/ReferenceTriangle.html>), the

perspector of ... and its polar triangle

(<http://mathworld.wolfram.com/PolarTriangle.html>) with respect

to a given conic is called the perspector of that conic. The

perspector is not defined for conics with respect to which is self-polar

(<http://mathworld.wolfram.com/Self-PolarTriangle.html>).

For an inconic (<http://mathworld.wolfram.com/Inconic.html>), the perspector is the Brianchon point

(<http://mathworld.wolfram.com/BrianchonPoint.html>) of the conic.

(<http://mathworld.wolfram.com/Perspector.html>)

(2) I think, property 3) holds in general for QA-circumconics.

It is good to know it.

Thanks again and best regards,

César Lozada

De:

Quadri-Figures-Group@yahoogroups.com

Enviado el: lunes, 7 de enero de 2019 06:06 AM

Para: -Group, Quadri-Figures

Asunto: [Quadri-Figures-Group] QA-A circumconic

Dear Cesar Lozada,

some remarks wrt #3302:

(1) What is the perspector of a conic wrt a triangle?

(2) I think, property 3) holds in general for QA-circumconics.

(3) In addition to the coordinates of O^* :

O^* is QA-Tf2 of the infinity point

... of the tangents at K_n

... in the intersections with QA-L3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3306
Date: 07/1/2019 1:43:10
From: César E. Lozada
Subject: QA - A circumconic

Dear Bernard,

Thank you for your valuable information.
Best regards,
César Lozada

De:
Quadri-Figures-Group@yahoogroups.com
Enviado el: lunes, 7 de enero de 2019 05:55 AM
Para: Quadri-Figures-Group@yahoogroups.com
Asunto: [Quadri-Figures-Group] Re: QA - A circumconic
Dear César,
Your conic K^* is the dual curve of the parabola inscribed in the dual QL of the QA.
It passes also through the 3 S-points, 3 other intersections of the DT circumscribed conic through QA-P10 and QA-P16, which is the dual curve of the parabola inscribed in DT and tangent to the Newton Line.
The 3 S-points lie on the Dimidium circle QL-Ci6 and form the QL-Tr2, which is also autopolar wrt K^* .
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3307
Date: 07/1/2019 2:51:30
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your encouraging message #3301,
... here some further observations,
... perhaps helpful for the 5 QA-Cu7-triple points:

In #3296 I described a transformation TF for a 5P $P_1P_2P_3P_4P_5$,
... which is the CSC of a QL with P_i as QA-Cu7-triple points,
... and defines a new 5P-point M as Miquel point of this QL.

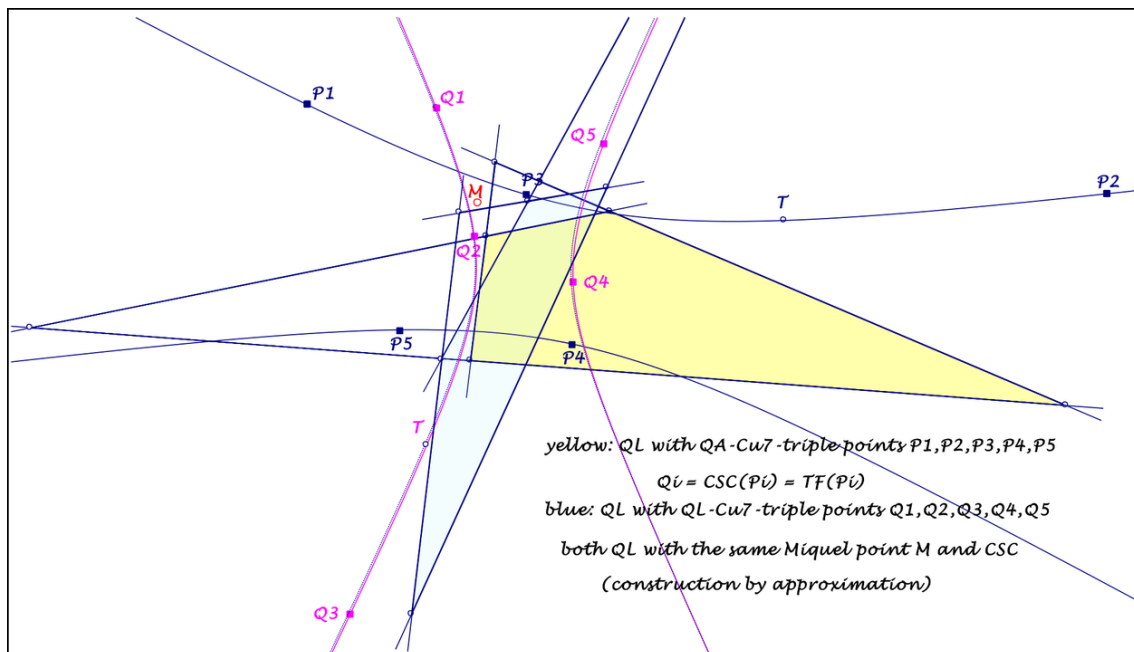
Further properties with $Q_i = TF(P_i)$:
(1) Circumcircles of P_i, P_j, P_k are centered on Q_lQ_m .
(2) $P_iP_j^{\wedge}Q_iQ_j$ is the 2nd intersection
... of the circumcircles of P_i, Q_j, M and P_j, Q_i, M .

In #3288 you asked for the QL with QA-Cu7-triple points Q_i ,
... attached a drawing by approximation as exact as I could.
But I couldn't find no further connections between the two QL.

Best regards Eckart

PS. Wrt your question, how to draw cubics with CABRI:
For pivotal isocubics
... take the line pencil of the pivot
and consider
... the intersections of the lines and their isoconjugate conics.

For nonpivotal cubics
... I studied Gibert's Special Isocubics 1.5.4,
... difficult to understand and to describe!



2019-01-07.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #3308
Date: 07/1/2019 7:11:32
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

Dear Eckart,
Reading again your message, I spent the day in drawings !
Thanks to you, I begin to understand better the QA-Cu7.
Let's start with a QA-Cu7 or bicursal QL-Cu1 (the QA-Cu7 are always bicursal ?) defined with the 3 points QA-P2,P4 and P41 of an unknown QA.
Taking 2 points A and A' on the curve, it's possible to find the corresponding QA by using the properties of the curve.
Let's consider the csc with center QA-P41 and swapping P2 and P4.
Acsc(A') and A'csc(A) intersect in QG-P18
AA' cuts the curve in a 3rd point, which is csc(QG-P18).
The conic through A,A',P4 and 41 and F cuts the cubic in a 6th point QG-P1 and the conic through A,A', P2 and 41 and F cuts the cubic in a 6th point QG-P19. QG-P1 and QG-P19 are csc partners and the 4 points are the vertices of the tangential QA of the same point.
The vertices of the tangential QA of csc(QG-P18) are the points U2,U3,V2 and V3 and the DT of this QA is QG-P1,P18 and P19.
U2V2 and U3V3 intersect in QG-P1 and form 2 of the 3 DT sides of the searched QL ; the 2 other DT vertices are the intersections with AA', which is the 3rd DT side.
Now the CSC of the QL is given by the 2 couples of CSC partners A and A' and QG-P1 and QA-P4.
Having the CSC and the DT, it is possible to find the 4 other vertices of the QL, which are the vertices of the QA having the 1st curve as QA-Cu7.
Finally, it's possible to draw the 2 other QA-Cu7 of the QL and to find the 5 triple points !
3 points for the QA-Cu7 and 2 points A and A' on it give the QL.
I suppose now that the same 3 points for the QA-Cu7 and 2 of the 5 triple points will give also the same QL ...
Last step must be the 5 points alone !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3309
Date: 07/1/2019 9:55:20
From: eckart_schmidt@t-online.de
Subject: QA - A circumconic

Dear Cesar Lozada,

thanks for the explanation of
... the perspector of a conic wrt a triangle.
Is the following property well known?
For any conic, not bearing the quadrangle points A_i ,
... the perspectors of the conic wrt the triangles
 of the quadrangle
... lie on a circumconic of the quadrangle.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3310
Date: 07/1/2019 11:20:20
From: César E. Lozada
Subject: QA - A circumconic

Dear Eckart,

I ignore if your proposed property is well known (I am light-years away from the subject of quadrifigures), But it seems to be true that:

For any conic, not bearing the quadrangle points A_i ,
... the perspectors of the conic wrt the triangles
of the quadrangle
... lie on a circumconic of the quadrangle.

Alternative property:

Given a quadrangle A_i and a conic K , such that Q_i are the
perspectors of K wrt the triangles of the quadrangle,
... the conic K^* circumscribing A_i and passing through any one
of the perspectors Q_i also passes
through the other three perspectors.

Best regards,
César

De:

Quadri-Figures-Group@yahoogroups.com

Enviado el: lunes, 7 de enero de 2019 04:55 PM

Para: -Group, Quadri-Figures

Asunto: [Quadri-Figures-Group] Re: QA - A circumconic

Dear Cesar Lozada,

thanks for the explanation of

... the perspector of a conic wrt a triangle.

Is the following property well known?

For any conic, not bearing the quadrangle points A_i ,

... the perspectors of the conic wrt the triangles of the
quadrangle

... lie on a circumconic of the quadrangle.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3311
Date: 08/1/2019 10:35:35
From: bernard.keizer
Subject: QA - A circumconic

Dear César, dear Eckart
Is it true that both QA's (the Ai and the Qi) have the same DT ?
Any circumconic of the QA bears the vertices of the anticevian triangle of any of it's points ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3312
Date: 08/1/2019 12:02:37
From: bernard.keizer
Subject: 8th CB-Point on a Cubic

Dear Eckart,
Step by step ...
Having the same QA-Cu7 (3 points) and it's csc and the CSC (2 points), it's possible to draw CSC(QA-Cu7), which is a quartic (?) through QL-P1 (center of the CSC).
Both curves intersect in 6 real points, 3 copples of CSC partners QA-P4 and QG-P1, A and A' and 2 points which are at the same time csc and CSC partners.
With A and A', we have the same reconstruction as in my previous message.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3313
Date: 08/1/2019 2:49:28
From: tsihonglau
Subject: 8th CB-Point on a Cubic

Dear all,

>>I suppose, as long as we are here, the QFG is not dead !

I think I have not posted here for a year, so this group became quieter. I started a facebook group "Plane Geometry Research" a year ago.

I posted my research results to it. I have many results last year. I'll post them as soon as possible. I share results to this group.

<https://www.facebook.com/groups/2008519989391030/> Plane Geometry Research

<http://lth.name/> / My website see Plane Geometry Research section

http://lth.name/geometry/involution_three_algebraic_curve.html

<- A recent page, isogonal conjugation=QA-Tf16, this conjugation is mentioned by me in topic #2710

Best regards,
Tsihong Lau

[← Previous](#) [Next →](#) [↩ Message Index](#) [↑ Subjects](#)

Message: #3314
Date: 08/1/2019 2:59:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your detailed message.
You describe QA-Cu7
... as QA-Cu1 of QAs and QL-Cu1 of QLs.

Here my sight:

(1) QA-Cu7 as QA-Cu1 of a new QA:
You get one vertex as reflection of QA-P41
... in the center of the circumcircle of QA-P2,4,41
... and the other vertices Möbius-transformed wrt QA-P2,4,41.

This new quadrangle has
... the triangle QA-P2,4,41 as Miquel triangle,
... its isogonal conjugated as isoconjugation
... and pivot in the infinity point of the bisector of
QA-P2.QA-P4.

(2) QA-Cu7 as QL-Cu1 of a QL:
Consider the new QA in (1),
... which has vertices in pairs on orthogonal lines,
... so it leads to a QL
as described in #1423, last passage,
... wrt two points on QA-Cu7.
This shows once more,
... that two QA-Cu7-triple points give the other three.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3315
Date: 08/1/2019 4:41:43
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm a little bit disappointed.
In 1), you give me another QA of the same curve, formed by the point F (your notation) and the 3 isogonals of the vertices QA-P2,4 and 41.
In 2) you give the corresponding QL with 2 new csc partners on the curve.
Finally I don't understand the remark wrt 2 triple points giving the 3 others (you need the CSC ...)
I'm afraid you have read my messages too quickly !
In my first message, I start with a QA-Cu7 with it's csc.
Then I take 2 random points A and A' on this curve and find
1) the QA having this curve as QA-Cu7
2) the QL having this curve as one of it's QA-Cu7
3) the 5 triple points as intersection of the 3 QA-Cu7
4) the CSC of the QL which is the same as the CSC of the 5 points
The CSC is variable and depends from the 2 points A and A'.
In the 2nd message, I start with the same QA-Cu7 and the CSC and find the 2 points A and A' by intersecting the QA-Cu7 with it's CSC (by the way, it is a sextic, not a quartic)
I try now to find how 2 of the 5 triple points on the same QA-Cu7 could give me the CSC and the 3 other triple points.
Finally, I hope we will find how the 5 triple points and their point T and their CSC give the QL ...
Did I miss something in your message ?
Thanks in advance for your attention
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3316
Date: 09/1/2019 8:26:23
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

please accept my apologies, I am still studying your first message 3308,
... and your message 3312 I have read after my answer.
After a first look in #3308, I wanted only to inform you of my sight
... wrt QA-Cu7 as QA-Cu1 of a QA and QL-Cu1 of a QL,
... for this seems the theme of your message for me.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3317
Date: 09/1/2019 4:06:41
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

Dear Bernard,

WOW

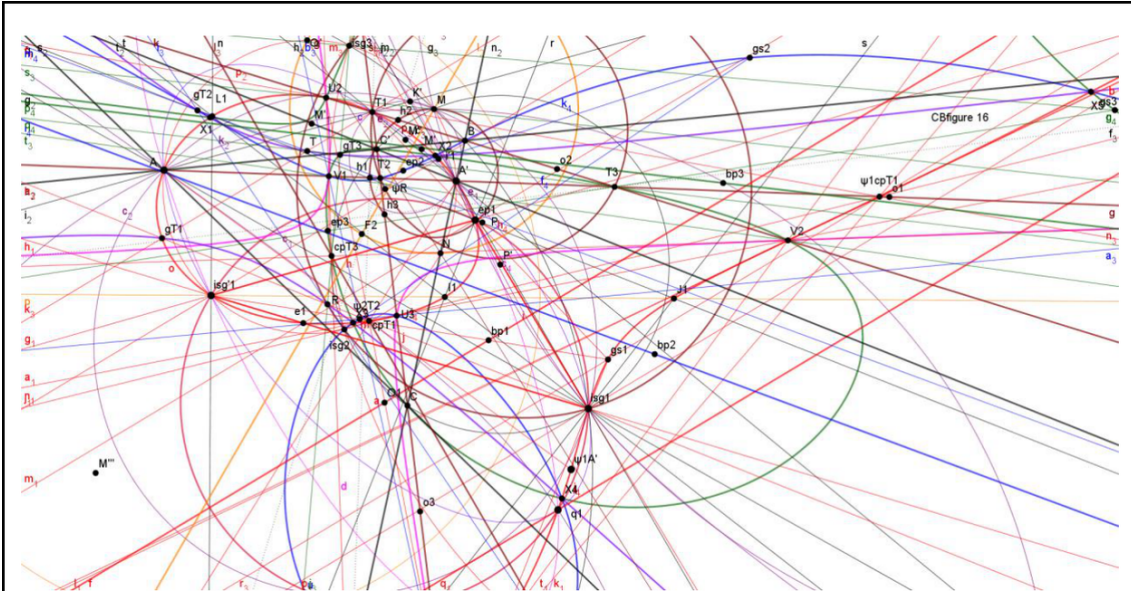
- that was a hard work, to reproduce your #3308!
Using your nomination, your final QL has the lines
... $A.A'$, $\text{csc}(A).\text{csc}(A')$, $A.\text{csc}(A')$, $A'.\text{csc}(A)$.
This is the same QL as in my sight.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3318
Date: 09/1/2019 6:24:17
From: bernard.keizer
Subject: QA-Geometry

Dear Eckart,
This time, I dare to send you a figure and some explanations ...
But I'm afraid it is really difficult to read !
My idea is to find always 5 points in order to find the searched
QL : 3 points for QA-Cu7 and 2 points on it, then the same with
the CSC, defined by 2 points, then the same and 2 of the 5
triple points and last the 5 triple points alone.
I hope we will succeed in finding the mystery this year !
Best regards and thanks for your attention
Bernard



It's hardly possible to read this figure !

The 1rst QA-Cu7 is defined by the 3 points QA-P2 or ep1, QA-P4 or isg1 and QA-P41 or isg'1 (red curve)

2 points A and A' on this curve determine QG-P1 or T1, QG-P18 or cpT1, which is the intersection of Acsc(A') and A'csc(A), QG-P19 or csc(QG-P1) and csc(cpT18), which is the 3rd intersection of AA' with the cubic.

I use $\psi 1$ for csc and ψ for CSC.

The vertices of the tangential QA of csc(QG-P18) give the points U2, V2 and U3,V3.

AA', U2V2 and U3V3 (which intersect in T1 or QG-P1) are the 3 diagonals of the searched QL and give T2 and T3, the 2 other DT vertices.

A and A' are CSC partners as well as T1 or QG-P1 and QA-P4 or isg1 as well as h1 or QG-P17 and cpT1 or QG-P18. This give the CSC with center M or QL-P1.

The CSC and the DT give the vertices B,B',C and C' of the QA having the red curve as QA-Cu7.

The 2 other curves in blue and green are the QA-Cu7 of the QA's A,A',B and B' and A,A',C and C'.

The 3 curves intersect in the 5 triple points X1,2,3,4 and 5 ; the conic is through T and R.

QA-Cu-7 and 2 points.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3319
Date: 10/1/2019 1:05:52
From: bernard.keizer
Subject: QA-Geometry

Dear Eckart,
I think it's a little bit clearer now.
Taking again the initial QA-Cu7 and 5 random points on it.
We consider 3 cubics through the 5 points, the QA-Cu7, the
degeneratated cubic formed by a line through 2 points and the
circle through the 3 others and the degeneratated cubic formed by
the conic through the 5 points and the infinity line.
The line through the 2 points X_1 and X_2 cuts the cubic in U , the
circle through X_3 , X_4 and X_5 cuts the cubic in V and the conic
in W ; the conic intersects the cubic in a 6th point gT
(isogonal of T).
The parallel through gT to the asymptote of the cubic cuts the
conic in T and the cubic in T_1 .
 T lies also on the parallel to X_1X_2 through W .
Last T_1V and TW intersect in P on the circle.
 U and V are CB partners with pivots T_1 and P on the 1st and the
2nd cubic.
 gT and the infinity point of the asymptote are CB partners on
the 1st and 3rd cubic with pivots T_1 and T .
 W and the infinity point of X_1X_2 are CB partners on the 2nd and
3rd cubics with pivots P and T .
This construction is always possible.
For 2 given points, there is only 3 other on the curve such as
the CSC of the 5 points swaps QA-P4 and T_1 (QG-P1 of the
searched QL).
More simply, for 2 given points, T has only one possible
position to allow the construction.
I suppose you will agree with that as it is your conjecture !
Best regards
Bernard
PS For 5 points, there are only 3 possible QA-Cu7 and one
possible QL and, as you already mentionned, all the QL-points
are 5P-points.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3320
Date: 10/1/2019 4:04:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I study your #3319, but:

(1) What are the seven points for these CB-partners?

"U and V are CB partners with pivots T1 and P ..."

(2) What is the QA for QA-P4 in the bold remark?

Thanks in advance!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3321
Date: 10/1/2019 5:57:37
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for your interest !
I took 5 random points on a circular cubic defined by QA-P2, QA-P4 and QA-P41 and the csc with center QA-P41 swapping QA-P2 and QA-P4.

- 1) The 7 points are the 5 points and the 2 circular points
- 2) QA-P4 is defined as above and the pivot T1 of the cubic must be the CSC of QA-P4 with CSC defined by the 5 points

Best regards

Bernard

PS Is it possible, by any chance, that the twin QL I was looking for (my message 3296 and your message 3307) is simply defined by the CSC of the 5 points and the triangle of the QA-P4 taken as DT ?

In this case, the 3 QA-Cu7 of the twin QL are through the 5 CSC of the 5 points and through the 3 copples of CSC partners QA-P4 and QG-P1 ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3322
Date: 11/1/2019 9:58:45
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for your explanations,
... but further questions wrt #3319:
(1) Why are U and V CB-partners
... wrt the 5 points and the circular points?
I only understand CB-partner for infinity points.
(2) You consider three cubics
and later their pivots,
... but what is your understanding of pivot?
(3) What is the relation between CB-partners and pivots?
Excuse my lack of understanding!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3323
Date: 11/1/2019 12:08:38
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

1) U and V are CB partners as there are 2 cubics through the 9 points 5 points + 2 circular points + U and V

2) The pivot for 7 points on a given cubic is your 8th point in message 3300

3) 2 CB partners are on line with the pivot on each cubic

Best regards

Bernard

PS What do you think from my last conjecture about the twin QL ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3324
Date: 11/1/2019 2:59:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

My conjecture about the twin QL doesn't hold, as QL-P1 is not on the Euler circle of the triangle of the QA-P4.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3325
Date: 11/1/2019 4:31:36
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
It could perhaps interest you (or is it obvious ?)
CSC(R) on the DT circumcircle QL-Ci1 is the inverse of QL-P1 in
the inversion of center QL-P10 swapping the DT Euler circle
QL-Ci2 and the DT circumcircle QL-Ci1 (and therefore the QG-P1
and the QG-P17).
Neither R (on the QA-P4 circumcircle) nor this point are in EQF.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3326
Date: 12/1/2019 11:11:16
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
For the twin QL, we have to find a triangle of QG-P17 being the
orthic triangle of a 2nd one of QG-P1.
The CSC of QG-P1 are the QA-P4, the CSC of QG-P17 are the QG-P18
and the QA-P4 and the QG-P18 are perspective with perspector R
on the conic through the 5 CSC of the triple points.
Last, the circumcircle of the QG-P17 must pass through QL-P1.
I've found in the picture only 3 such circles the QG-P17, which
gives the 1st QL with R on the conic of the triple points and
the QA-P41, but the corresponding point R is not on the conic of
the CSC of the triple points.
I give up !
Any other idea ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3327
Date: 12/1/2019 11:24:50
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
The triangle CSC(QA-P2) as DT doesn't work either ...
The fight goes on !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3328
Date: 12/1/2019 3:00:29
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for answering my questions and further information,
... especially # 3319 and # 3325,
... but I can't add new aspects.
I try the following way:
Starting with a 5P and its circumconic,
... the transformation TF (see # 3296),
... which is CSC of the searched QL,
... gives a 2nd 5P and a 2nd circumconic,
... intersecting the 1st circumconic in *4 points* ,
... which may have a connection with the *4 lines* of the QL.
But up to now without result.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3329
Date: 12/1/2019 4:05:47
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
In this case, as it is reversible, the 4 intersection points would have a connection with the 4 lines of both QL's (the 1st one and the one I name the twin) ...
I've just tried the symmetrics of the DT wrt QL-P1, but it doesn't work either.
I think I stop to search blindly.
I have another idea.
As you provided an approximative image of both QL's with the 5 points and their CSC, could you, without too much efforts, put on the figure some elements as Steiner Lines, circle of inversion, both DT's and QA-P4's and both QG-P17 and QG-P18, giving at the end the points S and S' and R and R' (you already put M, T and T').
I can't imagine how difficult it is, but I'm sure we could find some new ideas ...
Thanks in advance
Best regards
Benard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

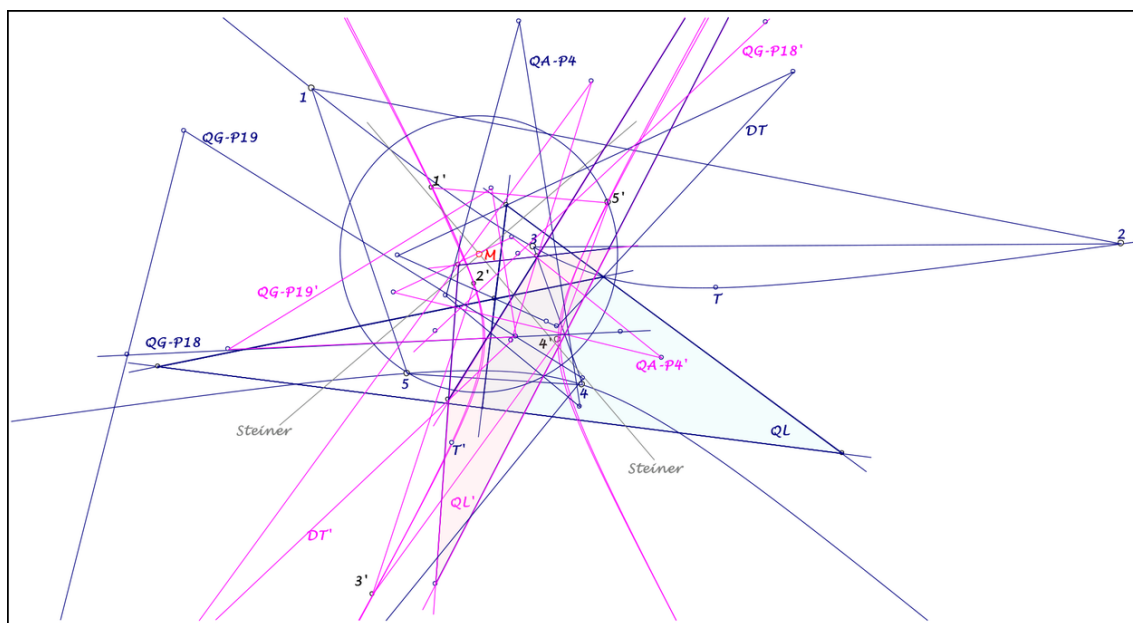
Message: #3330
Date: 13/1/2019 10:14:03
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

Dear Bernard,

I tried a drawing, you demanded for,
... but you see the approximation in the circumconic
of the twin QL.
I am lost in this jungle and hope you find a way!

Best regards Eckart

PS. I couldn't prevent, that the inversion circle bears P5.



2019-01-13.pdf

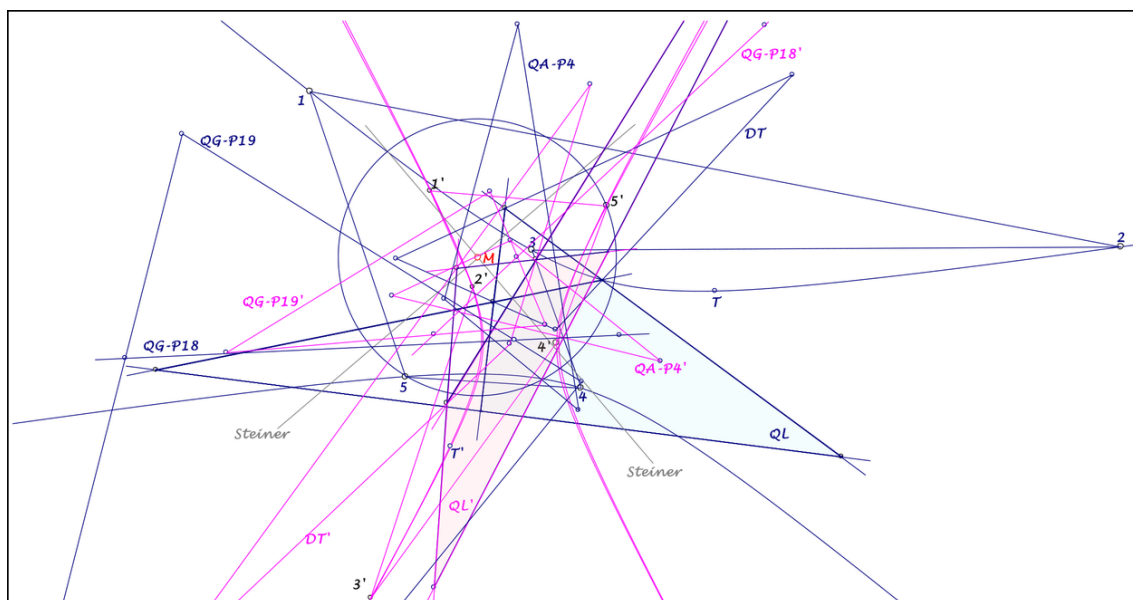
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3331
Date: 13/1/2019 10:33:59
From: eckart_schmidt@t-online.de
Subject: No Subject

Dear Bernard,

excuse, there was a mistake in the drawing
... wrt the triangle of QG-P19'.

Best regards Eckart



[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3332
Date: 2020-03-08
From: System Manager
Subject: Deleted Message
3332

Message number 3332 was a deleted message in Yahoo Groups.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3333
Date: 17/1/2019 12:17:33
From: eckart_schmidt@t-online.de
Subject: 5P-s-P4

Dear Bernard, dear Chris,

I think, the main property of 5P-s-P4
... should be mentioned in EPG:
Let T be 5P-s-P4 of $P_1, 2, 3, 4, 5$:
... T lies on the circumconic 5P-s-Co1.

Further properties:
Let P_i' be 5P-s-P4 of T, P_j, P_k, P_l, P_m ,
... then holds $P_i P_i'$ parallel to the tangent in T .

Let X be a point on 5P-s-C01
... and X_i = 5P-s-P4 of X, P_j, P_k, P_l, P_m ,
... then holds $P_i X_i$ parallel XT .

Perhaps we should study 5P-geometry,
... to get the QL for the 5 QA-Cu7-triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3334
Date: 17/1/2019 4:31:04
From: bernard.keizer
Subject: 5P-Geometry

Dear Chris, dear Eckart,

Eckart invites us to study the 5P-Geometry in order to understand the 5 triple points of the QA-Cu7.

I think I've understood the mystery of these points.

Each 5P defines a conic 5P-s-Co1, a point T on this conic as 5P-s-P4 and a transformation TF, which is the CSC such as 2 points are inverse wrt the circle of the CSC and vice-versa (Eckart gave a construction).

Let's name X_i the 5 points.

For any point X on 5P-s-Co1, the parallel to TX through the X_i intersects the conic in a 2nd point T_i , which is 5P-s-P4 of the 5 points X_j, X_k, X_l, X_m and X (see Eckart 3333).

Let's now define TF_i as the TF of the 5 same 5 points (with X replacing X_i).

There are only 3 positions of X on the conic for which the 6 circles $TF(TX)$ and $TF_i(X_iT_i)$ concur in a point; let's name S_1, S_2 and S_3 these 3 positions and K_1, K_2 and K_3 the corresponding points of concurrence.

For one position, S_1 for example, $CSC_i(K_1)$ defines a point Z_1 on S_1T and 5 points Z_{1i} on X_iT_i .

The curve through the points K_1, S_1 and the 5 X_i, Z_1 and the 5 points Z_{1i} is one of the 3 QA-Cu7 having the 5 points X_i as triple points, Z_1 is QG-P1 and K_1 is QA-P4.

Z_1S_1, Z_2S_2 and Z_3S_3 intersect on the conic in the point T and K_1S_1, K_2S_2 and K_3S_3 intersect on the conic in the point R.

(The reason is that for this curve, if S_1 is one of the 3 points S, each X_i must be one of the 3 points S of the 5 points X, X_j, X_k, X_l and X_m).

Best regards

Bernard

PS The understanding doesn't give me a simple construction ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3335
Date: 18/1/2019 12:01:41
From: bernard.keizer
Subject: 5P-Geometry

Dear Chris, dear Eckart,
Reading my message again, I notice that a few words are missing.
The CSC for 5 points is such as 2 points are inverse wrt the
circle of the CSC of the 3 others
and vice-versa.
On the cubic, the direction of the $XiZi$ or $S1Z1$ is obviously the
direction of the asymptote and Xi and $Z1i$, as well as $S1$ and $Z1$
are isogonal wrt the triangle $QA-P2,4$ and 41 (we know only
 $QA-P4$).
Then, the property is on the curve, which is one of the 3 $QA-Cu7$
of the initial QL, that the isogonal of Xi is $CSCi(QA-P4)$, where
 $CSCi$ is the CSC of the 5 points Xj, Xk, Xl, Xm and the isogonal
of $QG-P17$. The isogonal of Xi is the $QG-P17$ of another QL having
these 5 points as triple $QA-Cu7$ points.
This gives 5 other different QL's, sharing the $QA-Cu7$ with the
initial QL (but of course not the 2 others).
I hope these precisions will help the comprehension of my
message and I'm waiting impatiently for your comments ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3336
Date: 18/1/2019 2:31:49
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

I try to reproduce the constellations of #3334,
... but I stopped at ...
 "There are only 3 positions of X on the conic
 ... for which the 6 circles $TF(TX)$ and $TF_i(X_iT_i)$
 concur in a point".
I observed four positions for X,
... using a 5P with a hyperbola as circumconic,
... for the 5 QA-Cu7-triple points seem always
 to give a hyperbola.
I observed only two positions for X,
... using a 5P with an ellipse as circumconic.

Can you confirm my observations?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3337
Date: 19/1/2019 4:09:20
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks for your interest and your quick answer !
Unfortunately, I won't be able to confirm your observations before a certain time.
My computer is out of order and needs repairing.
If you identify the point of concurrence of the 6 circles, then the CSC and CSCi of this point give 6 points Z and Zi on the parall lines TX and TiXi and the curves through the 12 points X and Xi and Z and Zi must be the searched circular cubics.
If you have 4 points (hyperbola), 3 should be the searched points S1,2,3, isogonals of the QG-P1, the 4th could be the point R ...
If you have only 2 points (ellipse), I don't know what to say.
Beautiful new field, indeed!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3338
Date: 20/1/2019 10:34:36
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

I hope your computer will be quick repaired,
... for my observations don't confirm your results,
... attached a drawing of your constellation in #3334,
... which shows, that your statement in #3337 doesn't hold:
 "If you identify the point of concurrence of the 6 circles,
... then the CSC and CSCi of this point give 6 points Z and Zi
 on the parall lines TX and TiXi
... and the curves through the 12 points X and Xi and Z and Zi
 must be the searched circular cubics."

Best regards Eckart

PS: Your points Si in #3334 are not the points Si in #3337!

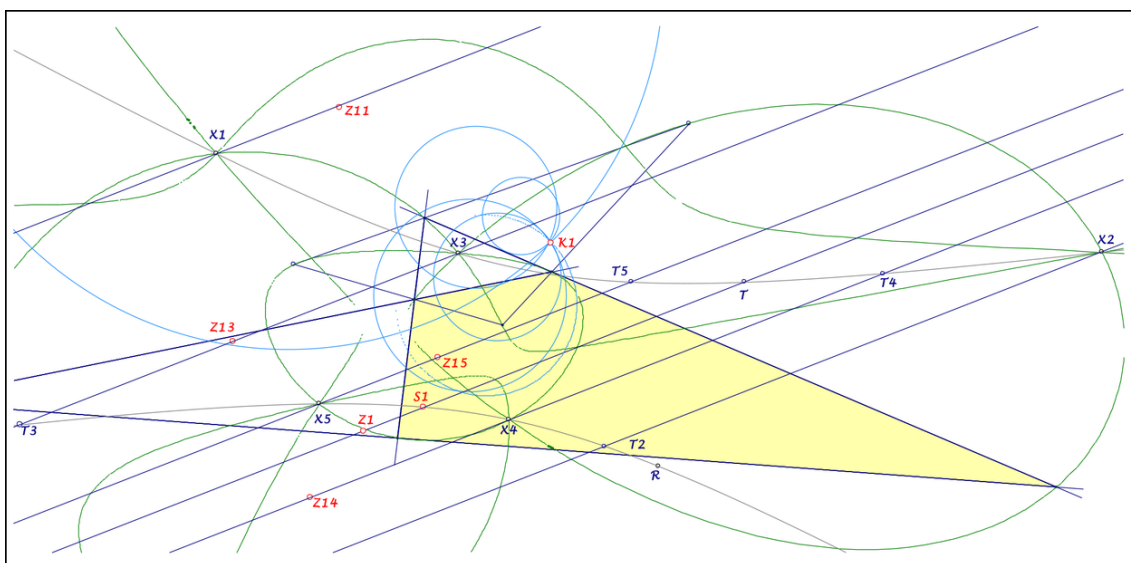
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3339
Date: 20/1/2019 10:40:14
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

I try once more to send the lost attached file of #3338.

Best regards Eckart



2019-01-20.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3340
Date: 20/1/2019 5:13:27
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for your researches and for your figure!
In fact it seems unfortunately you are right and the points I
named S1 ... are not the points S of the 3 QA-Cu7.
I promise to send you figures as soon as I will be able to do
it!
The mystery of the triple points remains and we have a new
mystery to deal with ...
At this stage, I can only confirm that there are 2 points for an
ellipse.
The points X1,X2,X3,X4,X5,Z1,Z11,Z12,Z13,Z14,Z15 and S1 (but not
K1) are cocubic on a curve which looks like a circular Van Rees
curve QL-Cu1 with TS1 as direction of the asymptote. For one of
the 2 points, the curve is unicursal, for the other, it's
bicursal.
On your own figure, the same points seem also to be on such a
figure ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3341
Date: 26/1/2019 3:51:03
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

a new aspect in QA-Cu7-Geometry:
Remember the construction of QA-Cu7 in EQF:
... Orthogonal lines L1, L2 through QA-P4
... give with the intersections $L1 \wedge QA-Tf2(L2)$ the cubic QA-Cu7.
... Let us consider these QA-cubics CU, replacing QA-P4 by any
point X.
The cubics CU(X) are nonpivotal isocubics with
... reference triangle QA-Tr1,
... isoconjugation QA-Tf2,
... root ??? (see PS)

... bearing X and $QA-Tf2(X)$.
 If we consider $CU(X)$ for a QA ,
 ... bearing a given point Y :
 ... the locus for X is a circle $CI(Y)$
 ... through Y and diametral $QA-Tf2(Y)$.
 A QL has three QG/QA -versions,
 ... which give three circles $CI(Y)$,
 ... which have collinear centers.
 For a $QA-Cu7$ -triple point Y
 ... $CI(Y)$ bears Y , $QA-Tf2(Y)$ and $QA-P4$.

Now we consider $CU(X)$ for the three QG/QA -versions of a QL :
 $CU(X)$ for points X on a QL -line:
 ... each bearing two opposite QL -points
 and a QL -diagonal crosspoint,
 ... with 6 coconic double intersections on the QL -diagonals
 ... and 5 triple points beside X .
 $CU(X)$ for a QL -point X ,
 ... not vertex of the considered QG -version,
 ... degenerates in 3 lines:
 ... two lines are angle bisectors at X
 ... the 3rd is the opposite line of $QG-Tr1$,
 ... bearing three remaining triple intersections
 with the other $CU(X)$.
 $CU(X)$ for infinity points of QL -lines
 ... degenerate in conics (and the line at infinity?)
 ... with three real collinear double intersections
 ... and three remaining real triple intersections.
 $CU(X)$ for points on $QL-Cu1$
 ... have six triple points beside X , *which give a new QL^* .
 $CU(QL-P1)$ lead in this way to a new QL
 ... with the same $QL-P1$, $QL-L2$ and $QL-Co1$.

There will be more properties,
 ... but I found no expected links to the 5 $QA-Cu7$ -triple points.
 I hope, someone can confirm my observations.

Best regards Eckart

PS: The root of $CU(X)$ lies on the line,
 ... connecting the $QA-Tr1$ -tripoles of X and $QA-Tf2(X)$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3342
Date: 28/1/2019 3:15:20
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

consider a 5P, a point P and lines L of its line pencil,
... the prefix "5P-s-" is omitted, so $P1 = 5P-s-P1$ and $Tfx = 5P-s-Tfx$,

... we start with the observation:

$L = Tf2(L \wedge Tf4(L))$ is perpendicular L.

Related properties:

(1) For the line pencil of a point P (see #2792):

... The lines $Tf4(L)$ intersect in $Tf3(P)$,

... The points $X = L \wedge Tf4(L)$ give an orthogonal hyperbola HY,

... through P, $Tf3(P)$ and $P1$,

... centered in the midpoint H of $P.Tf3(P)$

... with asymptotes parallel to the axes of Co1.

(2) For the line pencil of a point P:

... The lines L^* envelope a parabola PB,

... tangent to the axes of Co1 on the polar of $P1$,

... with directrix $P.P1$ and focus F on the polar of $P1$

... in the intersection of $P1.Tf3(P)$ and $Tf2(H)$,

... which intersects the Co1-axes in the contact points.

... The polar of P wrt PB

... is a perpendicular line to PF in F.

(3) For the line pencil of a point P:

... The intersections Y of the orthogonal lines L and L^*

... give a strophoid ST for the line $P.P1$ with fixed point P
and pole F,

... bearing the foci of Co1,

... tangent to PB and intersecting HY on Co1.

(4) For the line pencil of a point P:

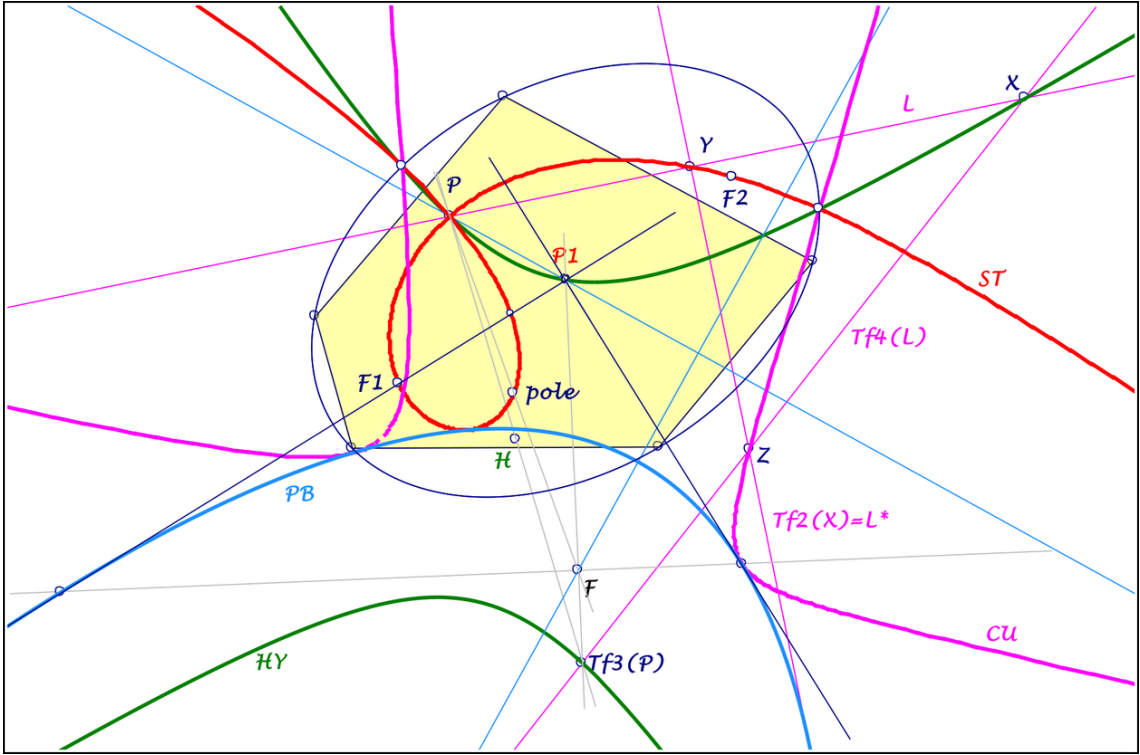
... The intersections Z of $Tf4(L)$ and L^*
give a cubic CU,

... sometimes with double point $Tf3(P)$,

... tangent to PB, intersecting HY and ST on Co1.

I hope, there are no false observations.

Best regards Eckart



2019-01-28.pdf

Message: #3343
Date: 29/1/2019 9:07:23
From: eckart_schmidt@t-online.de
Subject: Special QL on QL-Cu1

Dear Bernard, dear Chris,

well known see #1423:

... Two points X and Y on QL-Cu1 give with the lines
... $X.Y$, $X.CSC(Y)$, $CSC(X).Y$, $CSC(X).CSC(Y)$ a new QL
... with the same QL-P1, QL-L1, QL-Cu1, further QL-Cu2.

Let us now start with two CSC-partners on QL-Cu1:

... The tangents from these points to the parabola QL-Co1
... give a new QL with a lot of common elements:
... QL-Ci3, QL-Co1, QL-Cu1, QL-Cu2, QL-Qu1, QL-Qu2,
... QL-L1,2,3,4,5,6,
... QL-P1,2,4,7,19,22.
There will be more.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3344
Date: 29/1/2019 10:33:23
From: eckart_schmidt@t-online.de
Subject: QG-2

Dear Chris,

cleaning my desk, perhaps of interest ...
In addition to QG-2 some lines QG-P1.QG-Px, or QG-P1.QA-Px,
... whose three versions for a quadrilateral
 have a common QL-point,
... (but not mentioned in QG-2):
... QG-P1.QG-P6.QG-P10 ---> infinity point of QL-L2,
... QG-P1.QG-P17.QA-P12 ---> QL-P10,
... QG-P1.QG-P19 ---> new QL-point,
... QG-P1.QA-P17 ---> new QL-point,
... QG-P1.QG-P18 ---> new QL-point,
... QG-P1.QA-P20 ---> infinity point of QL-L1,
... QG-P1.QA-P23 ---> new QL-point,
... QG-P1.QA-P29 ---> new QL-point,
... QG-P1.QA-P41 ---> new QL-point.

Not only starting with QG-P1, there are further QG-lines,
... leading to new QL-points,
... already mentioned in the discussion "3 QL-versions of QA-Cu7",
... using the QG-point $S = \text{QG-P18.QA-P4} \wedge \text{QG-P19.QA-P41}$ (#2980):
... QG-P1.S ---> new QL-point T (#2981),
... QG-P18.S.QA-P4 ---> new QL-point R (#2974),
... QG-P19.QA-P4 ---> new QL-point U (Chris summary 2983).

There is a curious property for some QG-/QA-points Px
... wrt their three lines QG-P1.Px for a quadrilateral:
... QG-P1a.Pxa, QG-P1b.Pxb and QG-P1c.QA-Tf2(Pxc)
 have a common point.

This property holds
... for QG-Px with $x = 4, 8, 14, 16, 18$,
... for QA-Px with $x = 1, 2, 4, 5, 7, 10$.

Best regards Eckart
PS. No claim for completeness!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3345
Date: 29/1/2019 10:33:48
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

cleaning my desk , perhaps of interest:
If one of the QG-versions for a QL is concyclic,
... the center of the circle lies on the first Steiner axis and
... the Schmidt-circle (see QL-Tf1) is orthogonal
to the circumcircle,
... the corresponding *QA-Cu7* *degenerates to an orthogonal
hyperbola* ,
... which is the conic for the 5 QA-Cu7-triple points,
... QA-DT-circumscribed, centered in the midpoint of QG-P4.QG-P8,
... bearing the midpoint of the circle.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3346
Date: 29/1/2019 10:35:29
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

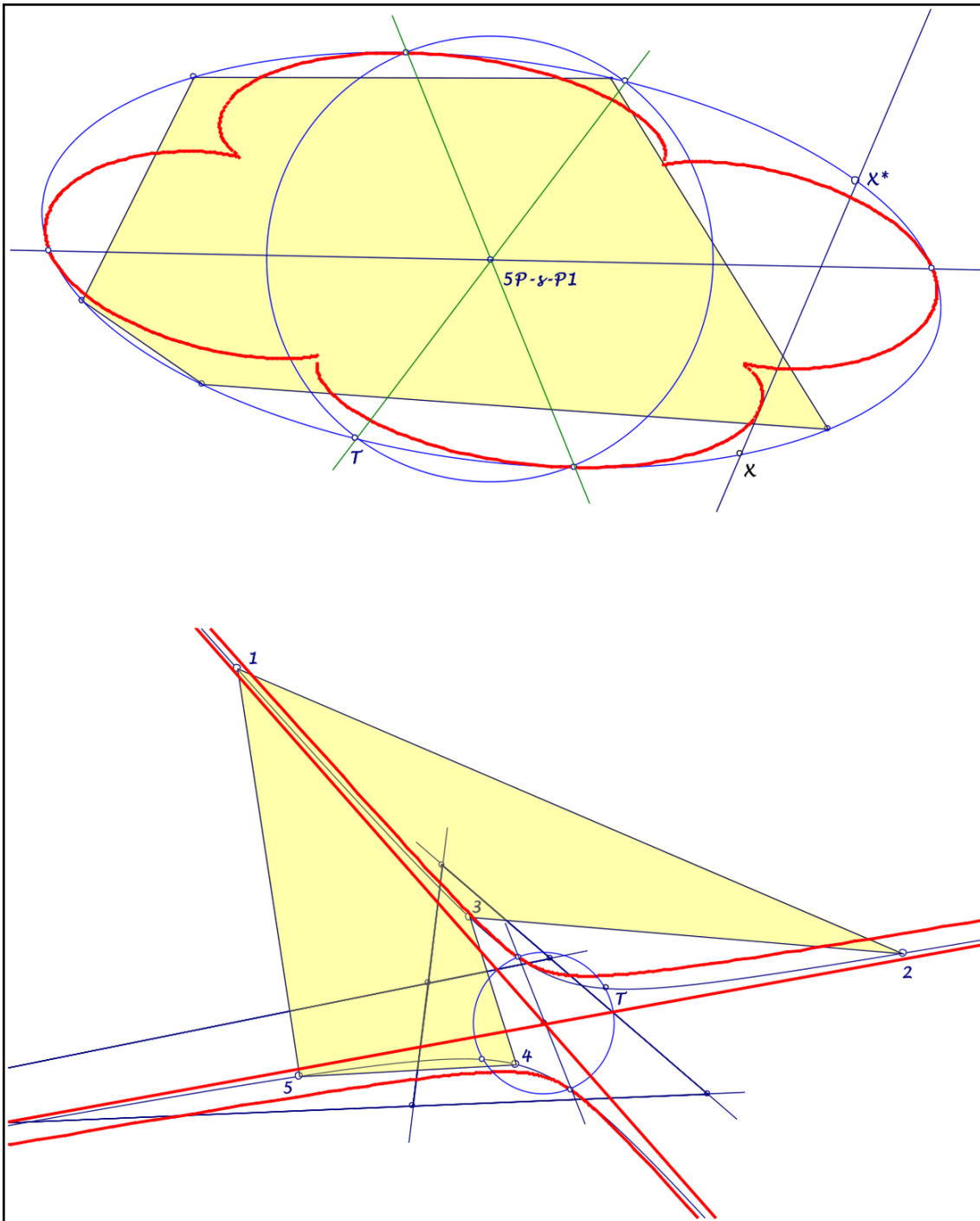
Dear Bernard, dear Chris,

Cleaning my desk, another aspect:
Consider the transformation $*$ for a 5P:
... P^* shall be 5P-s-P4 of $Q_1, 2, 3, 4, 5$
... with $Q_i = 5P-s-P4$ of P, P_j, P_k, P_l, P_m .
 $P \rightarrow P^*$ maps 5P-s-C01 to itself.
Let 5P-s-C01 be an ellipse with points X :
 XX^* envelopes a sextic (see attached file):
... symmetrically 5P-s-P1 inscribed 5P-s-C01
... with 4 contact points and 4 cusps,
... one pair of contact points diametral
... wrt a circle round 5P-s-P1 through $T = 5P-s-P4$.

If 5P-s-C01 is a hyperbola,
... the sextic has no cusps,
... two contact points are the infinity points of 5P-s-C01.

What about this sextic?

Best regards Eckart



2019-01-17.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3347
Date: 30/1/2019 11:21:51
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
My computer was in fact out of order !
I had to buy a new one and to have it reinstalled with all my applications and files, which took time.
During those days, you've started plenty of new items and I don't know where to start ...
Perhaps this simple remembering of old C-B properties.
5 points X_i define a CSC, a point $T = 5P-s-P_4$ and 11 degenerated cubics, one formed by the conic $5P-s-Co_1$ and the infinity line and 10 formed by a line X_iX_j and the circle through the 3 other points.
Consider a point P as pivot of a circular cubic through the 5 points.
The line PT cuts the conic in S .
The circle through 3 points X_k, X_l and X_m cuts the conic in S_{ij} and the parallel through S_{ij} to X_iX_j cuts the conic in T and the circle in T_{ij} , which are the pivots of the 11 degenerated conics.
The lines PT_{ij} cut the lines X_iX_j in U_{ij} and the corresponding circles in V_{ij} .
For each point P , there is one circular cubic through the 5 points X_i , the 10 points U_{ij} , the 10 points V_{ij} and the points P and S , PS being the direction of the asymptote and U_{ij} and V_{ij} being C-B partners on the cubic.
The circular cubics may be monocursal or bicursal in both cases ellipse or hyperbola.
Now adding a supplementary constraint limits the number of solutions.
If we wish $CSC(P)$ on the cubic, there are still several solutions in both cases.
If we wish $CSC(TS)$ and $CSC_i(\text{parallel to } TS \text{ through } X_i)$ having a common point, there are only 2 solutions for S on the ellipse and 4 on the hyperbola and $CSC(\text{common point})$ is the pivot Z_1 of the circular cubic (the common point is not on the cubic this time).
May be this could give you new ideas ...
Best regards
Bernard
PS No idea about your sextic, but they look beautiful

Message: #3348
Date: 30/1/2019 2:57:33
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

I hope, you can reinstall your applications in a new computer,
... to control my observations and give new ideas.
I try to attach a file with a figure,
... so that you can follow my description.

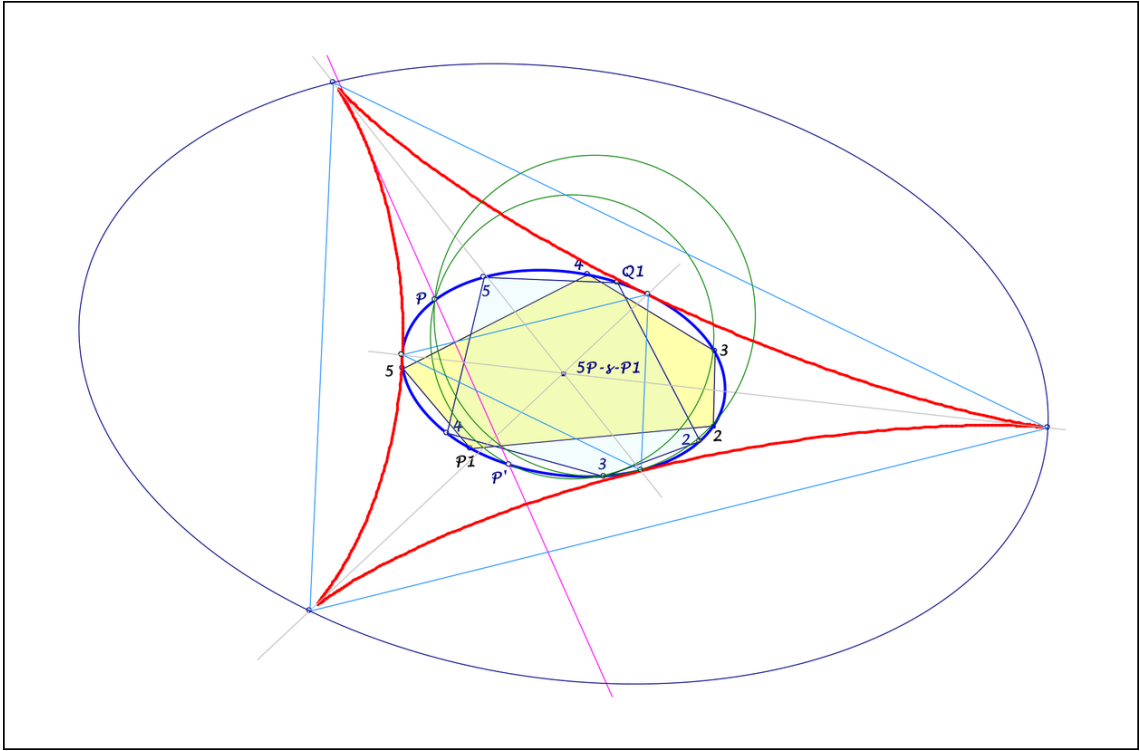
Consider a convex pentagon $P_1, 2, 3, 4, 5$
... and a variable point P on $5P-s-Co1$,
... which leads to a new pentagon $Q_1, 2, 3, 4, 5$ on $5P-s-Co$
... with $Q_i = 5P-s-P_4(P, P_j, P_k, P_l, P_m)$.
... The areas of the two pentagons are equal.

Consider the circles (P, P_i, Q_i)
... with a common 2nd point P' on $5P-s-Co1$,
... the lines PP' envelope a *deltoid* tangent $5P-s-Co1$
... with cusps, if P, P' and $5P-s-P_1$ are collinear.
The triangles of the cusps and the contact points have centroid
 $5P-s-P_1$.
Their Steiner circumconics are homothetic wrt $5P-s-P_1$
and ratio 3.

Best regards Eckart

PS: Are there analogies to the Kantor-Hervey deltoid?

Give me some time, to study your message 3347.



2019-01-29.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3349
Date: 31/1/2019 10:20:33
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,

Your deltoïd is a stretched deltoïd, with the same tangential definition as the KH deltoïd, replacing the inner circle by the inner ellipse.

The definition is the envelop of the lines joining 2 points, which describe the circle/ellipse in opposite directions, one at a speed double than the other. The contact point is the reflection of P' in P .

If you consider the Euler circle of a triangle as the inner circle, you get the Steiner deltoïd tangent to the 3 sides and the 3 altitudes.

With the ellipse, you have worked on this figure with Benedetto Scimemi : if my memory is not wrong, P was the QA-P4 of a QA and P' described the curve through the reflections of QAP4 in the 6 sides of the QA ; the stretched deltoïd was tangent to the 6 sides of the QA.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3350
Date: 01/2/2019 10:34:04
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Very interesting generalisation of QA-Cu7 !
I don't try to reproduce all the mentionned properties, as it takes too much time to draw new figures ...
But I have 2 questions by analogy with QA-Cu7 :
1) I suppose CU(X) is also a pivotal isocubic with pivot one of the DT vertices, fixed points of the transformation the vertices of the tangential QA of the pivot and reference triangle the DT of this QA.
2) What happens for a QL if you take the same QA-point in the 3 QA's (QA-P2 or P41 or any other point) ?
What about the CSC of the 5 triple points in this case ? Is for each of the 3 CU(X) the pivot CSC (X) ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3351
Date: 02/2/2019 9:14:59
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

before studying your #3350
... a new sight for the 5 QA-Cu7-triple points:

Well known are (see #3176):

- ... the QA-Cu7-double points on the QL-Tr1-sides,
- ... the conic CO of the QA-Cu7-triple points
- ... and the new QG-point S.

Consider for an arbitrary point P wrt a QG-version of the QL
... the two circles through P and the double points on a diagonal
... with a second intersection Q.

The QG-transformation $P \rightarrow Q$

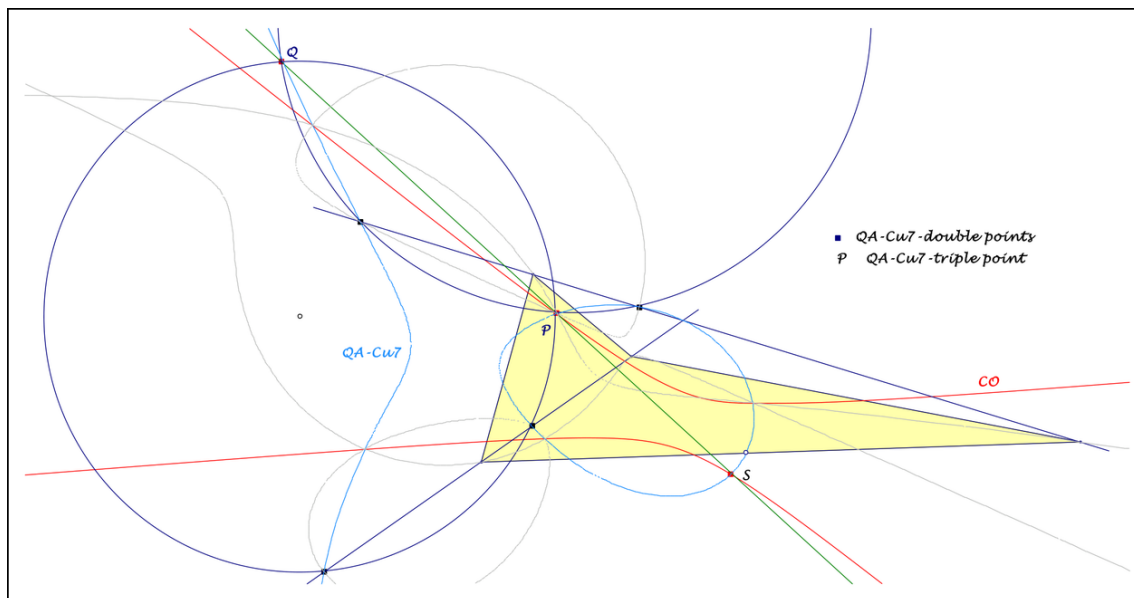
- ... maps QG-P18 to QA-P4,
- ... maps QA-Cu7 to itself, point and image collinear S.

If P is a point on CO and P, Q, S collinear,

... P is a QA-Cu7-triple point.

Best regards Eckart

PS: The QG-transformation $P \rightarrow Q$ should be studied more.



2019-02-02.pdf

Message: #3352
Date: 02/2/2019 11:42:06
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
This time I try to react quickly !
If you consider the csc with center QA-P41 swapping QA-P2 and QA-P4, then QG-P19 is $\text{csc}(\text{QG-P1})$.
Consider now $\text{csc}(\text{QG-P18})$; the 4 points QG-P1, QG-P18, QG-P19 and $\text{csc}(\text{QG-P18})$ have the same tangential.
The double points named U2, U3, V2 and V3 on the QA-Cu7 are the vertices of the tangential QA of $\text{csc}(\text{QG-P18})$; U2V2 and U3V3 intersect in QG-P1 and are 2 diagonals of the QL, U2V3 and U3V2 intersect orthogonally in QG-P18 and U2U3 and V2V3 intersect in QG-P19.
All this is well known.
Now your transformation swaps QG-P18 and QA-P4, but also $\text{csc}(\text{QG-P18})$ and QA-P2, the infinity point of the asymptote and QG-P1 and QG-P19 and QA-P41.
More generally, if the transformation swaps P and Q, it swaps also $\text{csc}(P)$ and $\text{csc}(Q)$.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3353
Date: 02/2/2019 3:09:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt your message #3350 with questions to my message #3341:
wrt 1) CABRI-observations confirm your supposition.
wrt 2) Considering the 3 CU(X) wrt other QG/QA-points as QA-P4,
... I found no observations wrt QA-P2, QA-P41, QG-P19 and S,
... but the cubic CU(QG-P18a) bears
... the double intersections of QA-Cu7a,b and of QA-Cu7a,c.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3354
Date: 03/2/2019 10:33:34
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
When I react quickly, the answer may be not complete !
In fact, your transformation is a special case of a general
property of the Van Rees cubic QL-Cu1 in it's bicursal version.
(If I'm not wrong, this property is not in EQF and wasn't
mentionned before).
Let formulate it in a simple way :
Any circle cuts the circular cubic in 4 real points forming a QA
; the 3rd intersections of 2 opposite sides with the cubic are
isogonal wrt the triangle QA-P2,4 and 41.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3355
Date: 03/2/2019 11:41:53
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt your # 3354:
If I am not wrong,
... QA-P41 is a point at infinity for a cyclic QA,
... so I cannot reproduce your result.
Is there a misunderstanding on my side?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3356
Date: 03/2/2019 1:47:52
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
QA-P2,4 and 41 are the 3 points defining the bicursal Van Rees curve (it could be M, V and V') and have nothing to do with the cyclic QA !
Sorry if I wasn't clear ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3357
Date: 03/2/2019 3:07:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse my lack of comprehension:
What are the 3 points defining QL-Cu1, for example M, V, V'?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3358
Date: 03/2/2019 4:27:18
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
If you deal with a triangle like B. Gibert, you get the
monocursal MacCay hessian K048 or the bicursal Kjp hessian K193
; the defining points are X1 and X15 and X16.
If you deal with the QL, the defining points of QL-Cu1 are QL-P1
and the QL-2P2a and b.
If you deal with the QA, the defining points of QA-Cu7 are
QA-P41 and QA-P2 and QA-P4.
This is always the same focal circular Van Rees cubic.
I hope this time I have been complete and clear ...
Sorry for having been too elliptic
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3359
Date: 03/2/2019 8:22:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt your #3352:

It was a real pleasure,
to reproduce your description
... of the geometry of the QA-Cu7-double points,
... here some further results, perhaps already mentioned:
The bisectors of U2V3 and U3V2
... intersect orthogonal on QA-Cu7 in a point X,
... which is the 3rd intersection of QA-Cu7 and QG-P1.QA-P2,
... concyclic with QG-P18, QA-P2, QA-P4
 and the midpoints of U2,V3 and U3,V2,
... with $\text{csc}(X)$ = 3rd intersection of QA-Cu7 and QG-P1.QA-P4.
The transformation csc swaps U2,V3 and U3,V2.
My new QG-transformation (#3351)
... maps QG-diagonals to itself
... with constant distance product to QG-P1.

Best regards Eckart

PS: There will be a typo in the middle of your message:
"... and U2U3 and V2V3 intersect in QG-P1."

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3360
Date: 03/2/2019 8:40:31
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

many thanks for the explanation
... of the reference triangles for QA-Cu7 and QL-Cu1
... as non-pivotal isogonal circular cubics.
I should study Bernard Gibert's chapter 4 once more,
... to get familiar with Van Rees focals.

Now I could check your interesting property in #3354
... and its application to the QG-point S
... as isogonal conjugate of QG-P1 wrt the triangle QA-P2,4,41.

But: In the bicursal version of QL-Cu1
... the points QL-2P2 are not defined in EQF,
... an old disappointing discussion:
... they are the CSC-partners on a perpendicular to QL-L1
... in the intersection with QL-L6.

Best regards Eckart

PS: In reference [15b] chapter 7
... there are properties of concyclic points on QA-Cu1.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3361
Date: 04/2/2019 10:24:09
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

You may find many interesting properties in <http://bernard.gibert.pagesperso-orange.fr> under Catalogue (MacCay is K03 ...) or in <http://www.mathcurve.com> under corbes 2D, then choose english version and focal circular cubic, also named Van Rees.

For the 2 points, you have 2 solutions :

- 1) keep the same points and change the Newton Line (through the points for monocursal, perpendicular bisector of the segment joining the 2 points for bicursal)
- 2) keep the line and consider 2 couples of CSC points S and S' on the line and V and V' symmetric wrt the line.; the 4 points can't be real at the same time, S and S' are real and V and V' imaginary for monocursal, V and V' are real and S and S' imaginary for bicursal. (That's another way of finding basis points or Poncelet points of a set of circles ...)

Best regards

Bernard

PS Working on your message 3341, I find this question : what are the intersection points of QA-Cu1 and QA-Cu7 (apart of the DT vertices, QA-P4 and P41 and the circular points) ? The same way, what are the intersection points of QA-Cux and QA-Cuy constructed with the same QA-point (the 1st by intersecting a line through the pivot and QA-Tf2 of this line and the 2nd between the perpendicular in the pivot to the line and QA-Tf2 of the 1st line) apart of DT vertices, the pivot and it's QA-Tf2 ? Last, another old question : what is the CSC of QA-P41 (on a diagonal of DT) and what is the CSC line of the circle circumscribed to the 3 QA-P41 of a QL ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3362
Date: 05/2/2019 11:07:02
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for further references wrt Van Rees focals in #3361.

Wrt your PS-questions:

I don't see further intersections of QA-Cu1 and QA-Cu7.

The cubics QA-Cux,y - as you describe - are the same.

CSC(QA-P41) wrt a QG-version of the QL

... is the intersection $X = QG-L1 \wedge QG-P1.QA-P4$.

The line of the collinear CSC(QA-P41),

... considered for a QG-version of the QL,

... is QL-Tf2(Y.QL-P26)

... with Y 4th harmonic point of X wrt QG-2P2.

It needs a bit concentration

... to handle a QG as well as QA as QL!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3363
Date: 07/2/2019 3:00:21
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks a lot for the CSC of QA-P41 ! It was so simple !
By the way, this point is on QA-Cu1 of the QA formed by 2
copples of CSC partners of the QL.
Now, back to QA-Cu1 and QA-Cu7 : both cubics are circular and
should intersect in 7 real points.
Like you, I can't find points other than DT vertices and QA-P4
and P41. Where is the problem ?
The same goes by replacing QA-P4 by any point X : by using the
same construction as for QA-Cu1 and QA-Cu7, this gives 2
different cubics QA-Cux and QA-Cuy ; what are the intersections
other than DT vertices, X and QA-Tf2(X) ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3364
Date: 08/2/2019 12:20:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I try to gather properties of the QA-Cu7-double points,
... first mentioned in QFG-message 2974.

Nomination:

Let us consider a QL

... with its QG-versions QGa,b,c and their cubics QA-Cu7a,b,c
... with double intersections of QA-Cu7b,c in Xa and Ya on QG-L1a.

(1) The QA-Cu7-double intersections Xa, Ya can be constructed
... with QL-geometry (see #2974) as intersections of QG-L1a
... .. and the CSC-circle CIa
of the diameter circle for QG-2P3a,
or:
... with *QA-geometry as QA-Tf2b or QA-Tf2c partner on QG-L1a.*

(2) The 6 QA-Cu7-double points are coconic.

(3) The diameter circle CIa of Xa. Ya bears:
... QA-P4b,c
... and QG-P18b as *orthogonal intersection* of Xa.Yc and Ya.Xc,
... and QG-P18c as *orthogonal intersection* of Xa.Yb and Ya.Xb.

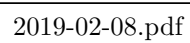
(4) The circles CIb,c intersect in QA-P4a and in QG-P18a.

(5) The radical center of the 3 circles CIa,b,c
... is the point R (see #2974, #3176),
... perspector of the QA-P4- and QG-P18-triangle.

(6) The 3rd intersections of QA-P4.QG-P18 and QA-Cu7
... are the points S (see #2980, #3176)
... on the conic of the QA-Cu7-triple points.

Best regards Eckart

PS: Please use the nomination of X and Y
... so that the properties in (3) are fulfilled.
I think, there will be more properties.



91

Message: #3365
Date: 09/2/2019 12:21:53
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Beautiful and useful figure indeed !

1) I regret that you changed the U_a and V_a , which you used before, in X_a and Y_a .

R , 3 points S and T on the conic of the 5 points, 6 points U_a, b, c and V_a, b, c for the double points, 5 points X_1 to 5 as triple points and Y_1 to 5 for the CSC of the X_i seems more convenient ...

2) If you consider the vertices of the QL A, A', B, B', C and C' and it's CSC, QA-P4 is CSC(QG-P1) and QG-P18 is CSC(QG-P17) ; QG-P1aQA-P4a cuts QG-P1bQG-P1c in CSC(QA-P41a).

3) The csca of QA-Cu7a with center QA-P41a swapping QA-P2a and QA-P4a swaps also QG-P1a and QG-P19a and QA-P18a and the 3rd intersection of AA' and QA-Cu7a, which is csca(QA-P18a) and the 4 double points X_b, X_c, Y_b and Y_c (your notation) are the vertices of the tangential QA of this csca(QA-P18a).

4) The double points X_b and Y_c and X_c and Y_b are also csca conjugates (one of your last messages).

Best regards

Bernard

PS You didn't answer my last message:

1) I'm looking for the intersections of QA-Cu1 and QA-Cu7, then for the same curves QA-Cux and QA-Cuy by replacing QA-P4 by another same point (I took as example QA-P1 and QA-Cux is QA-Cu5 ; both cubics QA-Cu5 and QA-Cuy are through DT vertices, QA-P1 and QA-P20)

2) I'm then looking for the intersections of the 3 same curves for a QL : QA-Cu1 gives a set of cubics through the vertices of DT, QL-P1 and the circular points, QA-Cu7 gives the 5 triple points with the same CSC as the QL.

What is the result for the 3 QA-Cu5 and the 3 QA-Cuy ? Many thanks for your attention ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3366
Date: 10/2/2019 12:02:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

thanks for further observations wrt the QA-Cu7-double intersections.

Please excuse changing the nomination, not projected.

Wrt PS of #3365, CABRI-observation show:

1) The intersections of the generalized QA-Cu1 and QA-Cu7,
... replacing QA-P4 by another point X,
... have only the intersections in the DT vertices,
X and QA-Tf2(X).

2) Wrt the intersections of the 3 QA-Cu5 :
Beside the triple intersections in the QL-points
... there are 5 double intersections.

It seems, that the QA-Cu5-cubics

... of the convex and the "crossed" QG-version of the QL
... give only one of the double intersections.

Wrt the corresponding QA-Cuy cubics

... there are no triple, but a lot of double intersections
... without observations.

With growing interest I am looking for

... the 7th triple point beside the 6 QL-points

... of the three QA-cubics (see #2017, #3015),

... which are similar to QA-Cu7.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3367
Date: 12/2/2019 6:19:20
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for your precise answers wrt my questions in PS of 3365.
I've reproduced your 7th point, but I can't find any particular
property ...
Best regards
Bernard
PS I suppose now that almost everything is clear for the QA-Cu7
5 triple or double points ; only the reverse construction of the
QL starting with the 5 triple points is resisting ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3368
Date: 12/2/2019 6:29:13
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

perhaps of interest: A new aspect of QA-Cu7-geometry,
... looking for the intersections M_i , N_i of QA-Cu7
... and the lines L_i of a reference QG-version of QL.

Consider opposite lines L_1 and L_3 with M_1 , N_1 , M_3 , N_3 :
... M_1M_3 and N_1N_3 intersect orthogonal
(using the right nomination).

$M_1N_1M_3N_3$ as QA:

... Its QA-Tf2 maps $L_1 \wedge L_2 \leftrightarrow L_1 \wedge L_4$ and $L_3 \wedge L_2 \leftrightarrow L_3 \wedge L_4$,
... its QA-P2 is the orthogonal intersection $M_1M_3 \wedge N_1N_3$,
... its QA-P4 is QG-P19 of the reference QG,
... its QA-P9 is QA-P41 of the reference QG,
... its QA-Cu1 is QA-Cu7 of the reference QG.

The QA $M_1N_1M_3N_3$ has three QG-versions:

QG1 = $M_1N_1M_3N_3$:

... Its QL-P1 is QA-P41 of the reference QG,
... *its QL-Cu1 is QA-Cu7 of the reference QG,*
... its QL-Tf1 maps its
QG-P1 to QG-P19 of the reference QG,
... ... as well as QA-P2 $\leftrightarrow$ QA-P4 of the reference QG.

QG2 = $M_1N_1N_3M_3$:

... Its QL-P1 is QA-P4 of the reference QG,
... its QL-Tf1 maps QA-Cu7 of the reference QG to itself.
... its QL-Tf1 maps its QG-P1 to QG-P19 of the reference QG,
... ... as well as QA-P2 $\leftrightarrow$ QA-P41 of the reference QG.

QG3 = $M_1N_3N_1M_3$:

... Its QL-P1 is QA-P2 of the reference QG,
... its QL-Tf1 maps QA-Cu7 of the reference QG to itself,
... its QL-Tf1 maps its QG-P1 to QG-P19 of the reference QG,
... ... as well as QA-P4 $\leftrightarrow$ QA-P41 of the reference QG.

Best regards Eckart

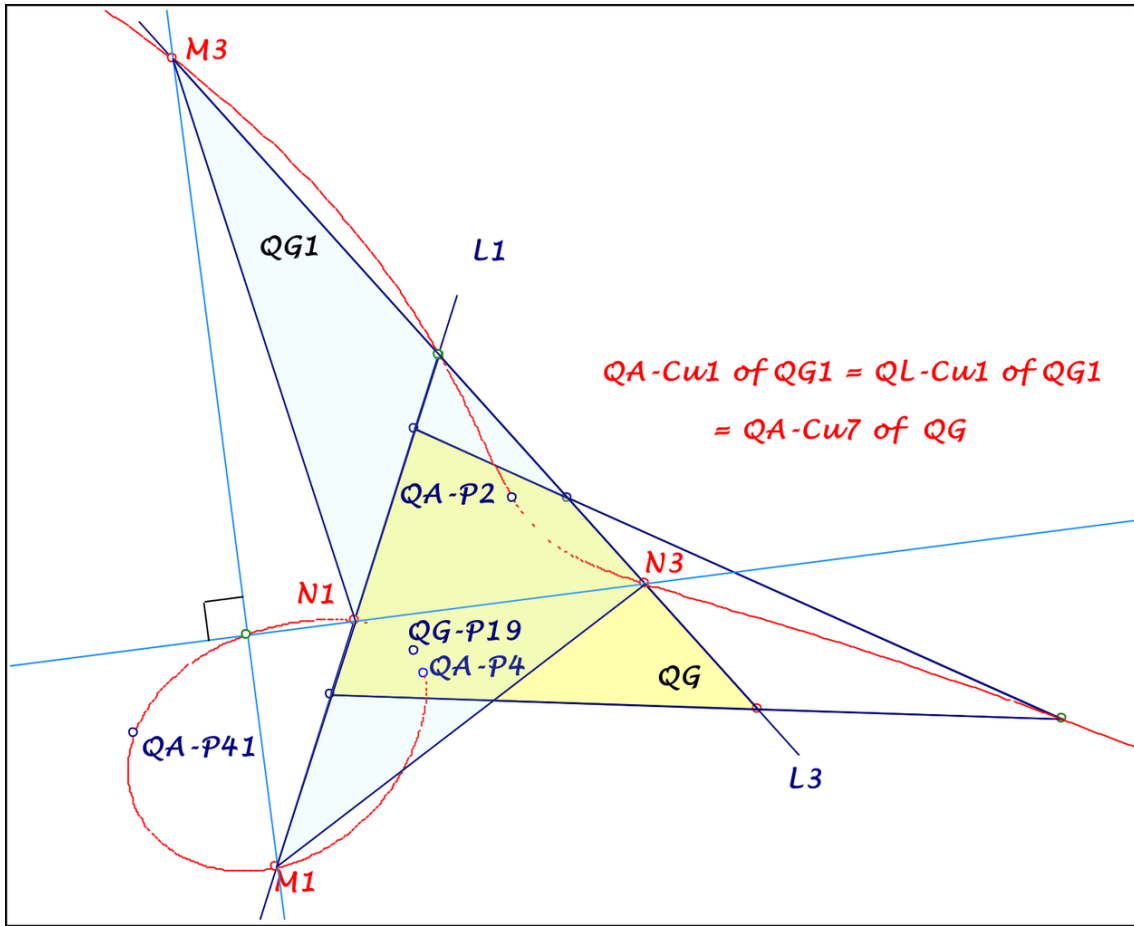
PS. Construction of M_i , N_i on L_i :

Take a point X

... and its harmonic conjugated Y

wrt the QG-points on L_i .

The locus of the centers of the circles $(X, Y, QA-P4)$ is a line,
... intersecting L_i in the center of a circle through QA-P4,
... which intersects L_i in M_i , N_i .



2019-02-12.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3369
Date: 13/2/2019 11:20:03
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
It's always the same story with the (bicursal) Van Rees circular focal cubic !
If the QA is BCB'C', the 2 other QL vertices are A and A' and the intersection of BB' and CC' is QG-P1.
L1 and L3 (your figure) intersect in A.
Then M1, N1, M3 and N3 are the vertices of the tangential QA of A', which is their QA-P4 (and not QA-P19 as you mention).
The orthogonal intersection of M1M3 and N1N3 is QA-P2 of the tangential QA as you mention as well as a DT vertice ; the 2 lines form the degenerated rectangular hyperbola which is the polar conic of the point A' wrt the stelloïd cubic for which the QA-Cu7 is the hessian ...
The csc with center QA-P41 swapping QA-P2 and QA-P4 swaps QA-P1 and QA-P19 and the mentionned intersection and A' as well as M1 and M3 and N1 and N3 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3370
Date: 15/2/2019 9:09:53
From: eckart_schmidt@t-online.de
Subject: QA-Geometry

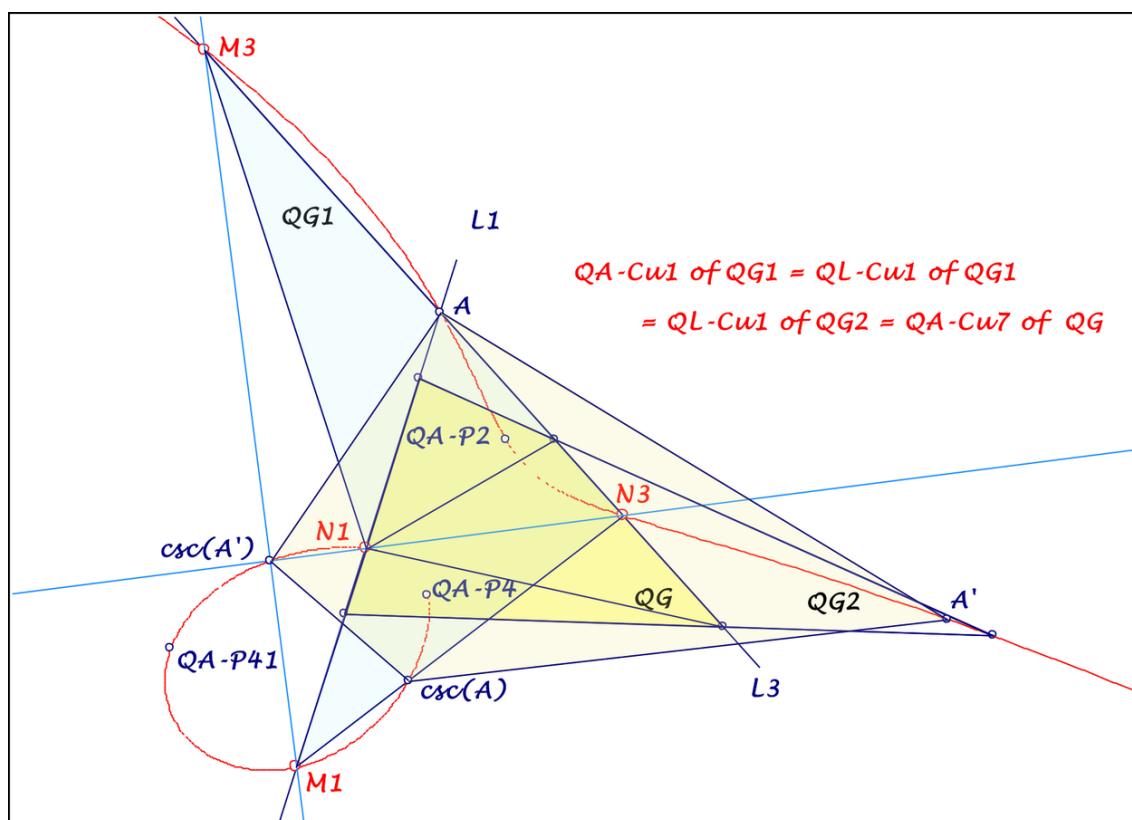
Dear Bernard,

thanks for the interpretation of the configuration in #3368.
 You are right with the correction (except the typo $Q * G^* - P19$):
 " ... of A' , which is their QA-P4 (and not QA-P19 as you mention)."

But I miss the connection with the intersections of QA-Cu7 and $L1, L3$:

... your $A' = \text{QA-P4 of } M1, N1, M3, N3$ is $\text{csc}(\text{QA-Tf2}(\text{csc}(L1 \wedge L3)))$
 ... and your corresponding QL (see #3308) has the lines
 ... AA' , $\text{csc}(A) \cdot \text{csc}(A')$, $A \cdot \text{csc}(A')$, $A' \cdot \text{csc}(A)$
 with $\text{csc}(A) = M1N3 \wedge M3N1$ and $\text{csc}(A') = M1M3 \wedge N1N3$,
 ... which is the QL of another QG2 as QG1 in my description
 ... with lines $M1N1$, $N1M3$, $M3N3$, $N3M1$ (see attached file).

Best regards Eckart



2019-02-15.pdf

Message: #3371
Date: 15/2/2019 12:23:35
From: bernard.keizer
Subject: QA-Geometry

Dear Eckart,
Beautiful and useful figure !
You may add :
QG-P1 (3rd DT vertice of the QA) as CB pivot of the 5 + 2 points
on the cubic and QG-P19 = csc(QG-P1)
QG-P18 as intersection of $Acsc(A')$ and $A'csc(A)$ and csc(QG-P18),
3rd intersection of AA' with the cubic.
The 4 points are the vertices of the tangential QA of another
point, which is it's QA-P4 and it's QA-P2 and the csc of this
QA-P4 is the orthogonal intersection of QG-P1QG-P19 and
QG-P18csc(QGP-18) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3372
Date: 15/2/2019 2:46:53
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I think, your observation
" QG-P18 as intersection of $Acsc(A')$ and $A'csc(A)$
and csc(QG-P18), 3rd intersection of AA' with the cubic."
doesn't hold, for A' isn't $L2^{\wedge}L4$, which has this property.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3373
Date: 15/2/2019 7:12:28
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
But of course A' is the intersection of L_2 and L_4 ! (like A is the intersection of L_1 and L_3)
And the vertices of the tangential QA of A are the same way M_2, M_4, N_2 and N_4 .
 M_2M_4 and N_2N_4 intersect orthogonally in $\text{csc}(A)$ and M_2N_4 and M_4N_2 intersect in $\text{csc}(A')$.
 $A\text{csc}(A)\text{csc}(A')$ is the DT of the QA $M_1N_1M_3N_3$ and $A'\text{csc}(A)\text{csc}(A')$ is the DT of the QA $M_2N_2M_4N_4$
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3374
Date: 15/2/2019 9:04:15
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

in your message #3369 you wrote:
" Then M_1, N_1, M_3 and N_3 are the vertices of the tangential QA of A' , which is their QA-P4 ..."
This A' is not $L_2 \wedge L_4$ (see my #3370).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3375
Date: 17/2/2019 11:27:40
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

in addition to message #3364 wrt the QA-Cu7-double points:

Nomination:

Let us consider a QL

... with its QG-versions QGa,b,c and their cubics $QA-Cu7a,b,c$

... with double intersections of $QA-Cu7b,c$ in Xa and Ya on $QG-L1a$.

Using the right nomination the transformations $*QG-Tf3a,b,c*$ give:

... $QG-Tf3b(Xa) = Yc$, $QG-Tf3c(Xa) = Yb$, $QG-Tf3b(Ya) = Xc$,
 $QG-Tf3c(Ya) = Xb$,

... $XbYc$ and $XcYb$ intersect orthogonal in $QG-P18a$,

... $*QA-Cu1$ and $QL-Cu1$ of the quadrigon $XbXcYcYb$
 are $QA-Cu7a$ of QGa .*

Best regards Eckart

PS: Excuse once more my changed nomination.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3376
Date: 17/2/2019 11:39:34
From: Tran Quang Hung
Subject: Euler-Poncelet line in Quadrangle points

Dear geometers,

I have seen a nice fact which I call by Euler-Ponclet line in Quadrangle points as following

Let A, B, C, D be Quadrangle points

- O_a, O_b, O_c, O_d are circumcenters of triangles BCD, CDA, DAB, ABC .

O is Poncelet point of (O_a, O_b, O_c, O_d) .

- H_a, H_b, H_c, H_d are orthocenters of triangles BCD, CDA, DAB, ABC .

H is Poncelet point of (H_a, H_b, H_c, H_d) .

- N_a, N_b, N_c, N_d are nine-point center of triangles BCD, CDA, DAB, ABC .

N is Poncelet point of (N_a, N_b, N_c, N_d) .

- G_a, G_b, G_c, G_d are circumcenters of triangles BCD, CDA, DAB, ABC .

G is Poncelet point of (G_a, G_b, G_c, G_d) .

Then O, H, N, G lie on a line. I call it by Euler-Ponclet line of $ABCD$.

Note that (H, G, O, N) is also harmonic range.

Is this fact known before?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3377
Date: 17/2/2019 12:45:14
From: eckart_schmidt@t-online.de
Subject: Euler-Poncelet line in Quadrangle points

Dear Tran Quang Hung,

In 2004 I described this line in
<http://eckartschmidt.de/EULER.pdf>.

Sorry in German.

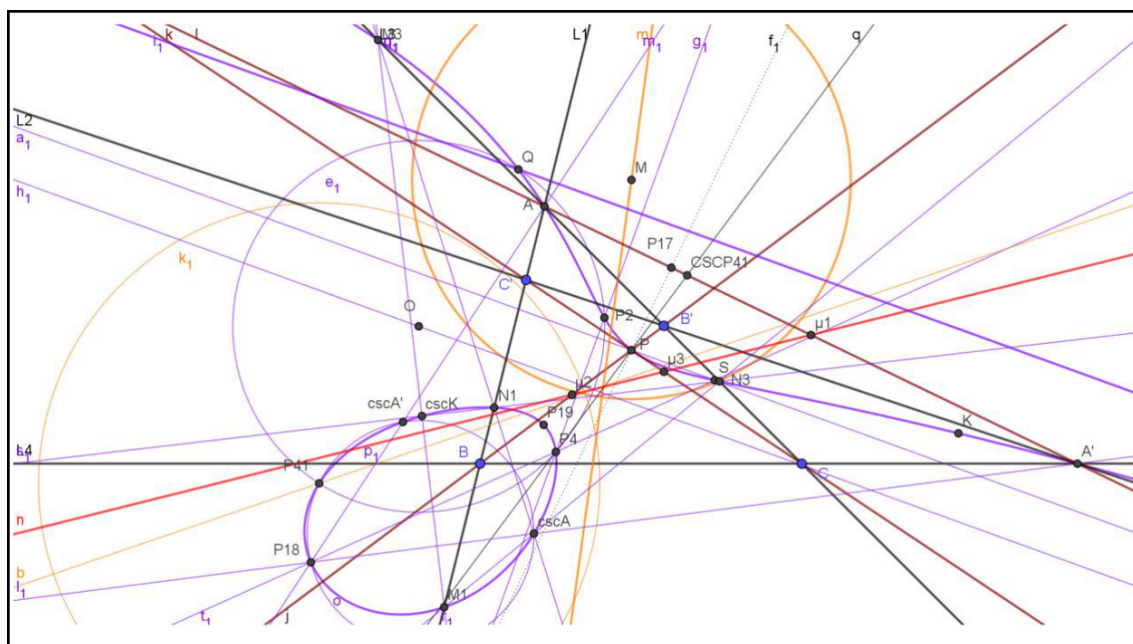
In EQF you find it as QA-L1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3378
Date: 17/2/2019 1:07:21
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
 I made an exact figure myself and I think I understand the misunderstanding between us.
 For me, the vertices of the QL are A, A', B, B', C and C' .
 The QA-Cu7 of the QA $BB'CC'$ has as DT $AA'P$, the rest of the construction is well-known.
 Your 4 points M_1, M_3, N_1 and N_3 are the vertices of the tangential QA of a point K on the curve, hence the points M_1 and M_3 as well as N_1 and N_3 are csc partners and $1M_3$ and N_1N_3 intersect orthogonally in $cscK$.
 And it holds that $A_{cscA'}$ and A'_{cscA} intersect in the crosspoint $QG-P18$ and that $csc(QG-P18)$ is the 3rd intersection of AA' with the curve QA-Cu7.
 I send you my figure (M is QL-P1, P is QG-P1, P17, P18 and P19 are QG points, P2, P4 and P41 are QA points).
 Best regards
 Bernard
 PS Now we have discussed the same properties ten times under 10 different configurations ...
 I wish now to concentrate on the 5P geometry, I'll send you soon new ideas.



QA-Cu7.pdf

Message: #3379
Date: 17/2/2019 1:19:09
From: Tran Quang Hung
Subject: Euler-Poncelet line in Quadrangle points

Thank you very much Mr Eckart.

I have seen it in
<https://chrisvantienhoven.nl/qa-items/qa-lines/qa-l1>

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3380
Date: 17/2/2019 2:16:52
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

excuse, please replace in #3375 QG-Tf3 by QG-Tf2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3381
Date: 17/2/2019 4:45:34
From: Tran Quang Hung
Subject: Line passes through Poncelet point in Quadrangle points

Dear geometers,

Let A, B, C, D be Quadrangle points.
Line da connects A and orthocenter of triangle BCD .
Define similarly lines db, dc, dd .
Then Newton line of (da, db, dc, dd) passes through Poncelet point of (A, B, C, D) .
Is this line known before?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3382
Date: 17/2/2019 4:48:38
From: Tran Quang Hung
Subject: Line passes through centroid of Quadrangle points

Dear geometers,

Let A, B, C, D be Quadrangle points.
Line da connects A and de Longchamps point ($X(20)$) of triangle BCD .
Define similarly lines db, dc, dd .
Then Newton line of (da, db, dc, dd) passes through centroid of (A, B, C, D) .
Is this line known before?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3383
Date: 17/2/2019 8:41:08
From: eckart_schmidt@t-online.de
Subject: Line passes through Poncelet point in Quadrangle points

Dear Tran Quang Hung,

wrt the line in message #3381:
The line is a perpendicular in QA-P2 to QA-L1, not in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3384
Date: 18/2/2019 1:41:02
From: Tran Quang Hung
Subject: Line passes through Poncelet point in Quadrangle points

Dear Mr Eckart,

Thank you very much for pointing out the new line. I shall try to find other properties for this.

Best regards,
Tran Quang Hung.

Vào Th 2, 18 thg 2, 2019 vào lúc 02:44 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt the line in message #3381:
>> The line is a perpendicular in QA-P2 to QA-L1, not in EQF.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3385
Date: 18/2/2019 11:35:21
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Chris, dear Eckart,

For 7 given points X_1 to X_7 and a random point P taken as pivot, X_1X_2 and X_3X_4 intersect in U_1 and the conics through the 5 remaining points intersect in a 4th point (other than X_5, X_6 and X_7) V_1 .

The line U_1V_1 intersect the 2 conics in 2 2nd points (other than V_1) T_{12} and T_{34} , which are the pivots of the CB transformation of the 7 points for the 2 degenerated cubics formed by the 2 copples line/conic.

The line PT_{12} intersect the line X_1X_2 in U_{12} and the conic in V_{12} ; the same goes for PT_{34} .

There is a cubic through P , it's CB conjugate P' and the 21 points U_{ij} and the 21 points V_{ij} .

P' is the tangential of P on the cubic and the cubic is a pivotal isocubic with pivot P , fixed points of the transformation the vertices of the tangential QA of P and reference triangle the DT of the QA.

For 5 points X_1 to X_5 and a pivot P , the construction is exactly the same with copples of lines and circles.

The cubics are circular and isogonal with pivot the infinity point of PT where T is on the conic of the 5 points; the cubics pass through the point S , 2nd intersection of PT with the conic and isogonal of P .

There are 10 points U_{ij} and 10 points V_{ij} ; the isogonal U'_{ij} of the U_{ij} are the intersections of 3 lines X_kV_{lm} .

For 7 lines, we have the dual property: the intersection between 2 lines is a point which forms with the conic tangent to the 5 remaining lines a degenerated curve of 3rd class tangent to the 7 lines ...

For 5 lines, we have the same dual property with intersection of 2 lines and incircle of the triangle formed by the 3 remaining lines ...

This is to start new reflexions !
Best regards
Bernard

Message: #3386
Date: 20/2/2019 12:36:20
From: Tran Quang Hung
Subject: About QA-P4 of centroid quadrangle

Dear geometers,

Given four points A, B, C, D .
 G_a, G_b, G_c, G_d are centroids of triangles BCD, CDA, DAB, ABC .
 G is centroid of $ABCD$.
 $P = QA-P4$ of $ABCD$
Then $QA-P4$ of (G_a, G_b, G_c, G_d) lies on line PG .
Is this point known before?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3387
Date: 20/2/2019 3:48:42
From: Tran Quang Hung
Subject: Newton line passes through Poncelet point

Dear geometers,

Let A, B, C, D be Quadrangle.
Let A_1, A_2, A_3 be the reflections of A in lines BC, CD, DB .
Line da connects point A and Poncelet point of (A, A_1, A_2, A_3) .
Define similarly the lines db, dc, dd .
Then Newton line of (da, db, dc, dd) passes through Poncelet point of A, B, C, D .
Which is this line?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3388
Date: 20/2/2019 9:31:07
From: eckart_schmidt@t-online.de
Subject: number 3382, 3386, number 3387

Dear Tran Quang Hung,
wrt #3382: I found no further properties of this line,
wrt #3386: This point divides $QA-P1.QA-P4$ with ratio $-1:4$,
wrt #3387: If I am not wrong, the line doesn't pass $QA-P2$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3389
Date: 20/2/2019 11:09:54
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

very interesting message #3385, but it needs time to reproduce.
Only some first remarks wrt the 5 QA-Cu7-triple points of a QL:
If we take $P = QG-P1$ of a QG-version of QL,
... we get QA-Cu7 of the QG.
If we take $P = T$
... the cubic degenerates,
... all U_{ij} and V_{ij} lie on the conic of the triple points.
With interest I am looking for the cubic for $P = QL-P1$,
... bearing the 2nd intersection
... of $QL-P1.T$ and the conic of the triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3390
Date: 20/2/2019 11:47:18
From: Tran Quang Hung
Subject: number 3382, 3386,number 3387

Dear Mr Eckart,

Thank you very much for quick reply. I try to check #3387, I still see that the line passes through QA-P2. Could you please check this again?

Best regards,
Tran Quang Hung.

Vào Th 4, 20 thg 2, 2019 vào lúc 15:31 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

>> Dear Tran Quang Hung,
>> wrt #3382: I found no further properties of this line,
>> wrt #3386: This point divides QA-P1.QA-P4 with ratio -1:4,
>> wrt # 3387: If I am not wrong, the line doesn't pass QA-P2.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3391
Date: 21/2/2019 12:31:09
From: Tran Quang Hung
Subject: Begonia point of Quadrangle

Dear geometers,

Follow my post

<https://artofproblemsolving.com/community/c6h1336737p7238120>

Let A, B, C, D be four points.

Lines AB, AC, AD meet CD, DB, BC at Ab, Ac, Ad , reps. Let Ab', Ac', Ad' be the reflections of A in lines $AcAd, AdAb, AbAc$, reps. Then BAb', CAc', DAd' are concurrent at point A' . (A' is called by Begonia point of A wrt BCD follow Darij Grinberg

<http://www.cip.ifi.lmu.de/~grinberg/Begonia.zip>)

Define similarly the points B', C', D' .

Then lines AA', BB', CC', DD' are concurrent at a point. I call this point by Begonia point of A, B, C, D .

Which is this point?

Best regards,
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3392
Date: 21/2/2019 9:28:45
From: eckart_schmidt@t-online.de
Subject: Begonia point of Quadrangle

Dear Tran Quang Hung,

The Begonia point of a quadrangle
... is QA-P23 , Inscribed Square Axes Crosspoint in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3393
Date: 21/2/2019 10:51:20
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

I have to correct my observation:
If we take $P = T$
... the cubic degenerates,
... all V_{ij} lie on the conic of the triple points,
... all U_{ij} are points at infinity.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3394
Date: 21/2/2019 11:01:28
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
I had noticed !
If P is in T , the cubic is the degenerated one formed by the conic and the infinity line (which contains the circular points).
If P is in T_{ij} , the cubic is the degenerated one formed by the circle (which contains the circular points) and the line X_iX_j .
Thanks for your interest
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3395
Date: 22/2/2019 10:45:19
From: eckart_schmidt@t-online.de
Subject: Newton line passes through Poncelet point

Dear Tran Quang Hung,

I checked once more #3387,
... but it doesn't hold.
I don't find my error.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3396
Date: 22/2/2019 1:10:34
From: Tran Quang Hung
Subject: Newton line passes through Poncelet point

Dear Eckart,

You are right, I am very sorry. I looked and checked carelessly on my Geogebra.

Best regards,
Tran Quang Hung.

Vào Th 6, 22 thg 2, 2019 vào lúc 17:03 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahooogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> I checked once more #3387,
>> ... but it doesn't hold.
>> I don't find my error.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

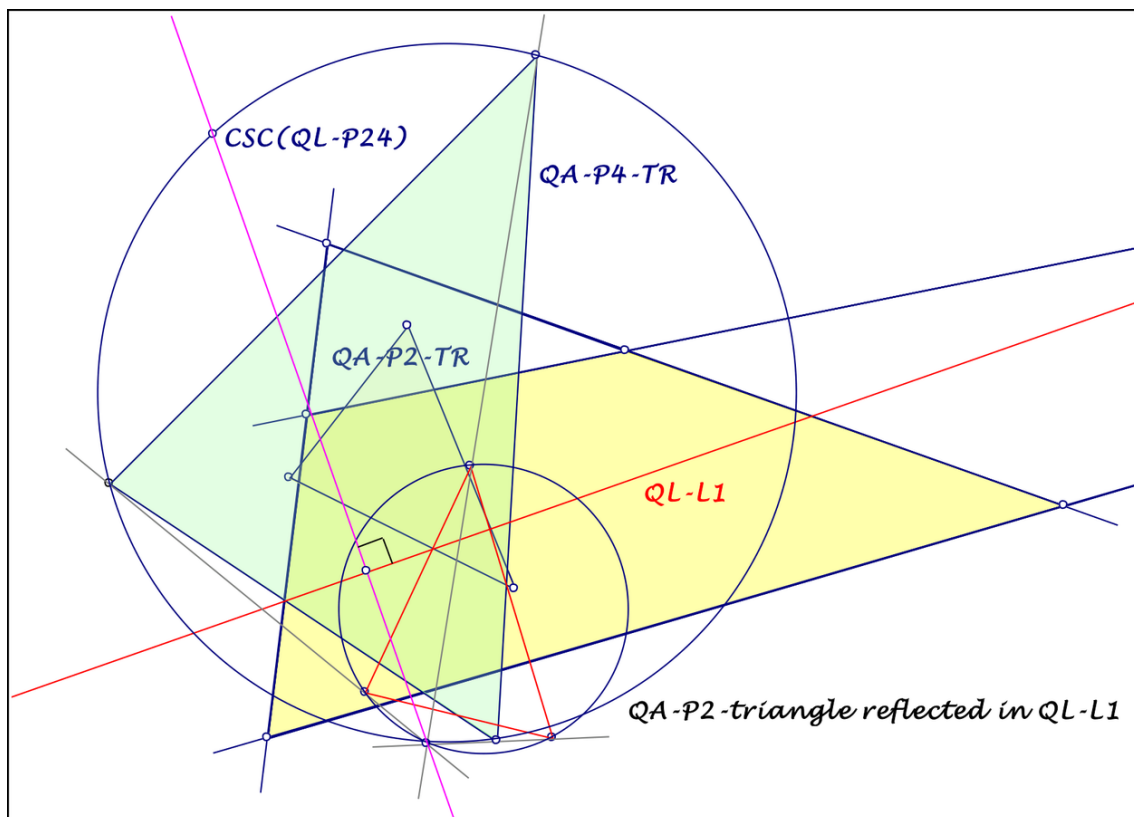
Message: #3397
Date: 22/2/2019 6:49:24
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

a further observation wrt the QA-Cu7-puzzle:
The points QA-P2 and QA-P4
... play a significant role in QA-Cu7-geometry.

Let us consider the QA-P2- and the QA-P4-triangle for a QL,
... they are similar with opposite sense of rotation,
... reflecting one in QL-L1, it is perspective to the other,
... with perspector in the intersection of their circumcircles
... on a perpendicular to QL-L1 through CSC(QL-P24).

Best regards Eckart



2019-02-22.pdf

Message: #3398
Date: 24/2/2019 5:48:39
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Curious and interesting last property !
Best regards
Bernard
PS Did you find something with the circular cubic through the 5
points and pivot QL-P1 ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3399
Date: 25/2/2019 3:34:12
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard, dear Chris,

perhaps of interest:

(1) For a QG = P1P2P3P4 we get

... the QA-Cu7-double points on the diagonal P1P3

... as intersections X13, Y13 of P1P3 and a conic

... through P2, P4, QG-2P2a,b, QG-P18 and QG-P19.

(2) The following 9 points give a CB-System:

... QG-P1, QG-P18, QG-P19, QG-2P2a,b

... and the 4 QA-Cu7-double intersections X13, Y13, X24, Y24.

(3) The 9 points under (2)

are the intersections

... of QA-Cu7 and CU(QG-P18) (see #3341).

(4) The conics through the QG-points QG-P1,18,19 and QG-2P2a,b

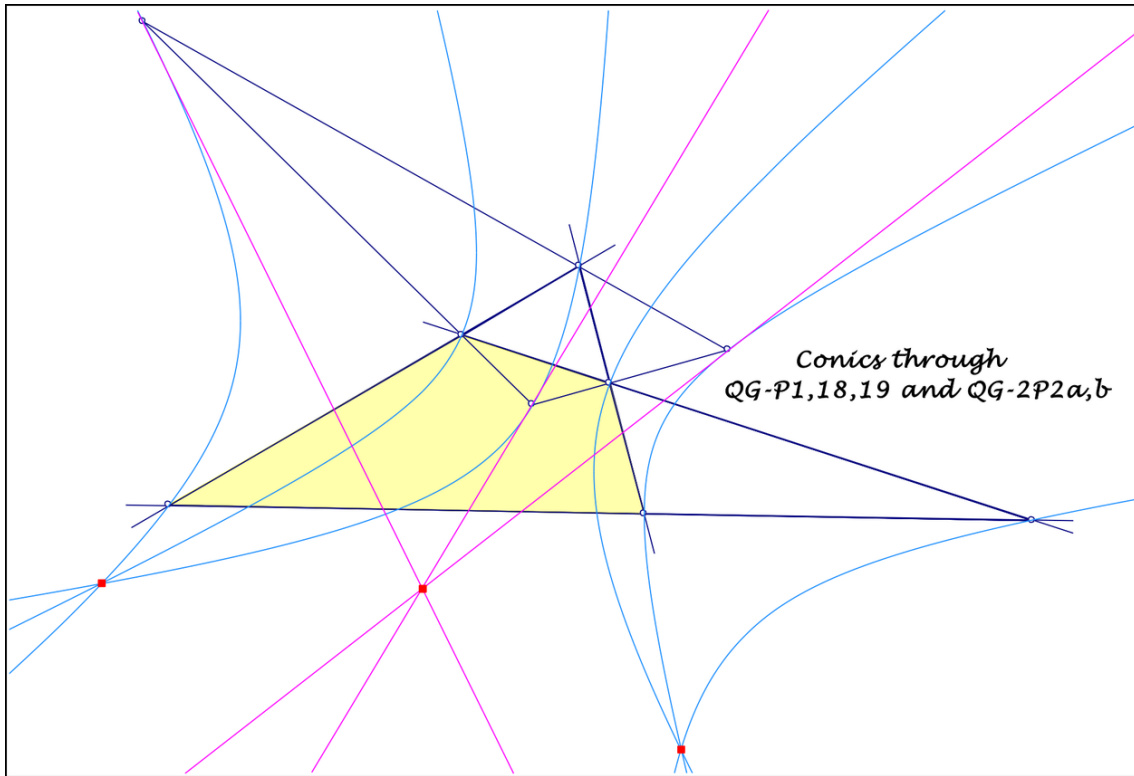
... for the three QG-versions of a QL

... have two triple points QL-2Px.

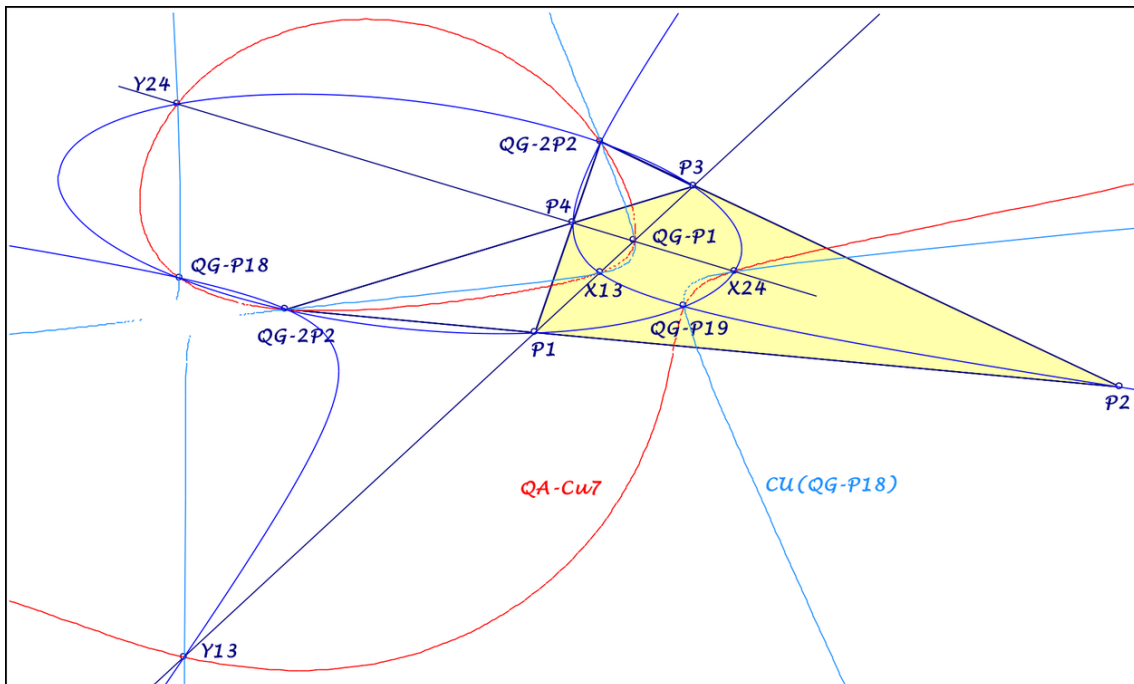
The tangents in QG-P1a,b,c have a common point QL-Px.

What about these QL-points?

Best regards Eckart



2019-02-25a.pdf



2019-02-25.pdf

Message: #3400
Date: 01/3/2019 10:39:33
From: eckart_schmidt@t-online.de
Subject: Triple points for QG-lines and conics

Dear Bernard, dear Chris,

the QA-Cu7-triple points remain mysterious,
... so I looked for simpler examples:
Several QG-lines and their QG-Tf2-conics
... give triple points for a QL wrt its QG-versions.
Triple points of QG-lines can be interpreted
... as perspectors of triangles.

Examples (see also QG-2)

- (1) Lines QG-P1. QG-P3 intersect in QL-P8.
- (2) Lines QG-P1. QG-P5 intersect in QL-P16.
- (3) Lines QG-P1.QG-P12 intersect in QL-P13.
- (4) Lines QG-P1.QG-P19 intersect in V (see #3176).
- (5) Lines QG-P1.QG-P6 are parallel QL-L2.
- (6) Lines QG-P1.QG-P17 intersect in QL-P10.
- (7) Lines QG-P1.S intersect in T (see #3176).
- (1) - (7) The QG-Tf2 conics of the three lines
... have four triple points (two not always real).

Conversely:

For a fixed point P wrt a QL
the QG-Tf2 conics
... of P.QG-P1 have four triple points (two not always real).
(8) If P is a point on QL-Ci1,
... the triple points are concyclic on CSC(QL-Ci1).
(9) If P = QL-P1
... one triple point is the point at infinity of QL-L1.

Further observations:

- (10) Lines QG-P1.QA-P41 intersect in W (see #3176),
... one triple point of their QG-Tf2 conics at infinity.
- (11) If QL-Cu1 is not bipartite,
... the QG-Tf2 conics of QL-L1 intersect in QL-P1 and QL-2P2.
- (12) *QG-Tf2 conics of QL-lines QL-Lx - unequal QL-L1 -
... have triple points
in the intersections - unequal QL-P1 -
... of QL-Cu1 and the circle CSC(QL-Lx).
Who can lighten the background of this observations?
I hope there are no main errors.
Best regards Eckart

Message: #3401
Date: 08/3/2019 8:51:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

perhaps a relevant observation:
Consider a QL and its 3 QG-versions QGa,b,c
... with their cubics QA-Cu7a,b,c
... and their 6 double points and 5 triple points.

The Cayley-Bacharach transformation CBa for
... the 5 triple points and the double points of QA-Cu7b,c
... maps double points of QA-Cu7a,b to QA-Cu7b
... and double points of QA-Cu7a,c to QA-Cu7c.

CBa maps QA-Cu7b,c to itself,
... point and image collinear with the infinity point
... of the asymptote.
CBb and CBc map QA-Cu7a in the same points to itself.
CBa maps points P on QG-L1a
... to the 2nd intersection of PT (for T see #3176)
... and the conic of the 5 triple points.

The CBa,b,c images of the double points
... produce on each QA-Cu7 4 points,
... which give a new QG' with orthogonal diagonals:
... QA-Tr2 of QG' is triangle QA-P2,4,41 of QG,
... *QA-Cu1 of QG' and QL-Cu1 of QG' are QA-Cu7 of QG.*

Likewise, already mentioned:
The double points Xa, Ya of QA-Cu7b,c
... and the double points Xb, Yb of QA-Cu7a,c
... lie on orthogonal lines (see #3364).
The QG-version of these double points,
... which has orthogonal diagonals
... and QG-L1a,b as opposite lines,
... interpreted as QA has QA-Cu7c as QA-Cu1
... interpreted as QL has QA-Cu7c as QL-Cu1 (see #3375).

Best regards Eckart

PS: What do you know about the CB-transformation
... wrt the 5 triple points and the two circular points?
I only remember the images of points at infinity
... in the 2nd intersection of their connection with T
... and the conic of the triple points.

Message: #3402
Date: 09/3/2019 10:32:42
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
I'm not home next week, but I wish to answer now your last question.
The CB transformation wrt the 5 points and the 2 circular points is an isoconjugation, which swaps precisely the points U_{ij} and V_{ij} for a given pivot P (see the construction in my message 3385).
This leads to an infinity of pivotal isocubics ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3403
Date: 14/3/2019 8:33:27
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

if we replace in Hart's construction of the CB-point 8P-s-P1
... the initial conics 12345, 12346, 12356 by circles 345, 346,
356,
... we get the CB-point of 3,4,5,6,7,8 and the two circular
points.

In this way we get a construction for the CB-point
... of six points and the two circular points,
... which can be interpreted as 6P-s-Px,
... or as transformation CB for a 5P,
... especially for the 5 QA-Cu7-triple points.

Properties of this transformation CB (see also #3385):

- (1) CB maps 5P-s-Co1 to the line at infinity,
... X, CB(X) collinear with 5P-s-P4.
- (2) CB for the five QA-Cu7-triple points:
... (a) CB maps QA-Cu7a,b,c to itself,
... ... X, CB(X) collinear QG-P1a,b,c.
... (b) CB changes the double points of QA-Cu7a,b, QA-Cu7b,c,
QA-Cu7a,c.
... (c) CB maps QG-P1 to the tangential wrt QA-Cu7.
... (d) CB maps Bernard's cubics for 5 points in #3385 to itself
... ... X, CB(X) collinear P.
... ... The cubic is the locus of CB-partners on lines through P.
... ... The cubic intersects the conic of the triple points on PT.
- (3) Some remarks wrt Bernard's cubics in #3385:
... (a) If $P = QG-P1$ we get the cubic QA-Cu7.
... (b) For P on a circle (P1P2P3) through 3 of the five points
... ... CB(P) lies on the line P4P5,
... ... and the lines P.CB(P) have a fixed point P0 on (P1P2P3),
... ... the parallel to P4P5 through Po bears 5P-s-P4.
... For $P = P0$ the cubic degenerates in circle (P1P2P3)
and line P4P5.

Best regards Eckart

Message: #3404
Date: 17/3/2019 9:09:30
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

in #3385 you describe a cubic CU
... for 5 points and a point P,
... here my reproduction:

Let C0 be the circumconic of the 5 points,
... T on C0 the point 5P-s-P4 of the 5P
... and P* the 2nd intersection of PT and C0.

Using the CB-transformation CB (see #3403)
... for the 5 points and the two circular points,
... we get the point $CB(P) = P'$.

Your cubic CU is the locus of
... CB-partners on lines through P,
... with asymptote parallel PT
... and P' tangential of P.

If CU is bipartite:
The contact points of tangents parallel to the asymptote
... give a QA1 with diagonal triangle TR.
CU is a pivotal isocubic
... with reference triangle TR,
... isoconjugation: isogonal conjugate ($P \rightarrow P^*$),
... pivot: infinity point of PT.

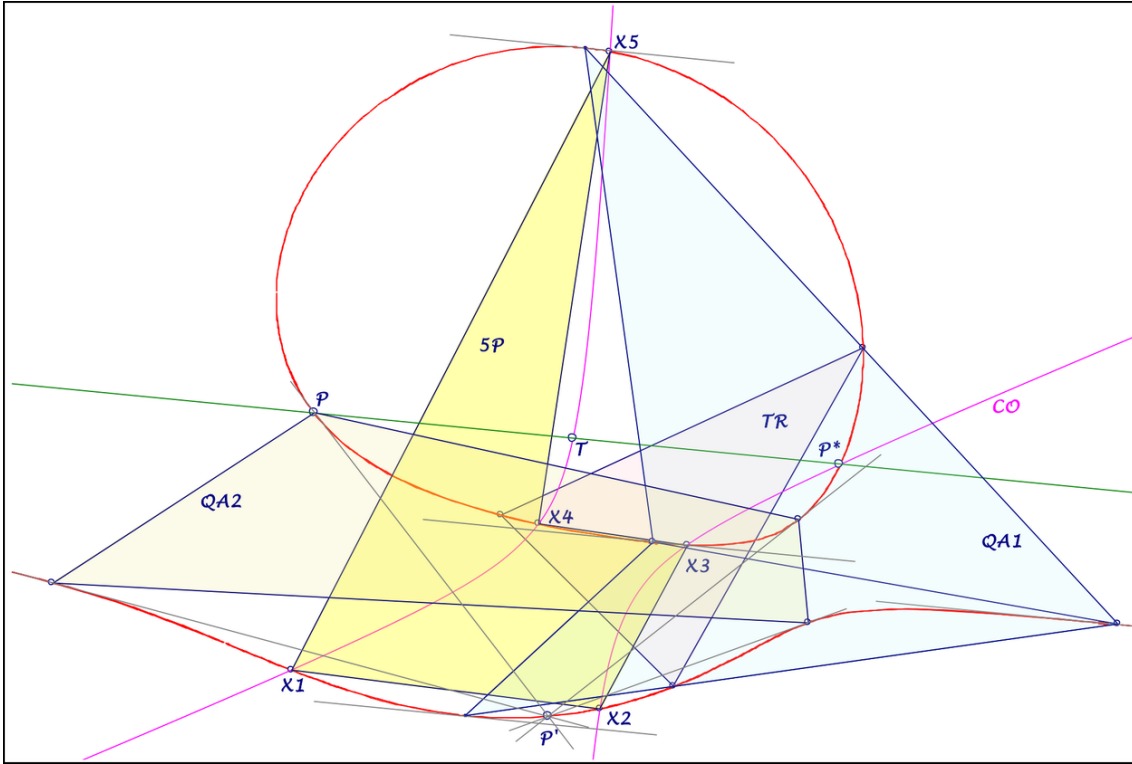
Or:

If we take a QA2 with vertices P
... and its Möbius transformed wrt TR,
... which is the tangential QA of P',
... CU is QA-Cu1 of this QA.

I think, the last observation isn't the same as yours.

Best regards Eckart

PS: The attached file is constructed
... in the reverse order,
... starting with the cubic and a conic.



2019-03-16.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3405
Date: 17/3/2019 10:23:41
From: eckart_schmidt@t-online.de
Subject: 5 points on on a cubic

Dear Bernard, dear Chris,

CABRI leads to the following observations:

Let CU be a cubic,

... which can be described as QA-Cu1 or QL-Cu1,

... take any 5 points on CU

... with its circumconic CO,

... intersecting CU in a 6th point X.

A parallel to the asymptote through X

... intersects CO in $T = 5P-s-P4$ of the 5 points.

Let Y be the 3rd intersection of XT and CU.

The CB-transformation CB (see 3403)

... wrt the 5 points and the 2 circular points

... maps X to the infinity point of XT,

... maps Y to the tangential of Y.

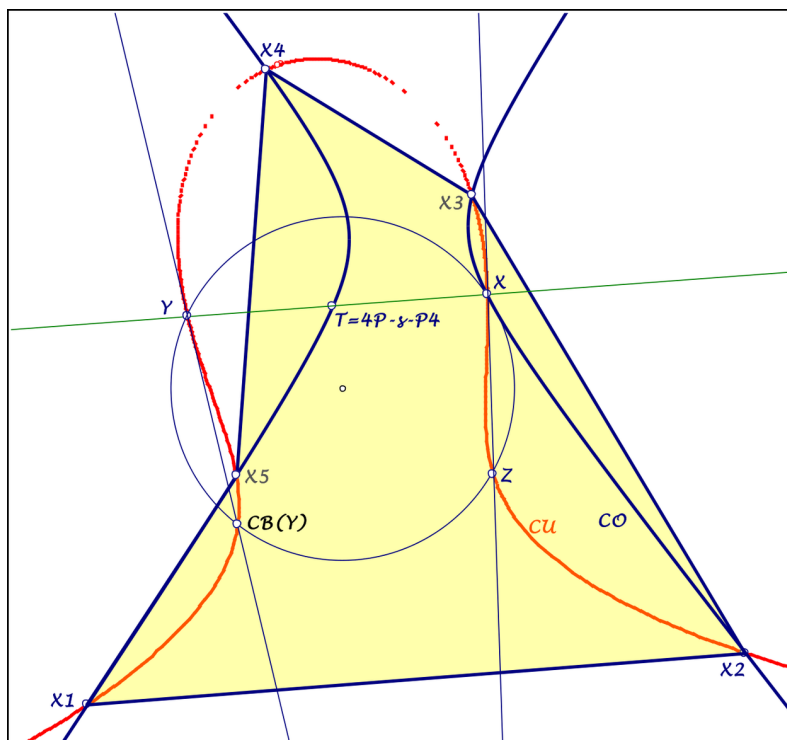
The circle CI = (X,Y,CB(Y))

... with 4th intersection Z on CU

... gives the tangential of X.

Are there other cubics with this properties?

Best regards Eckart



2019-03-17.pdf

Message: #3406
Date: 19/3/2019 12:26:01
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
This time I give up !
I wrote a complete answer, but I couldn't send it ...
In 3403, you use the same construction than in my message 3385
and we naturally agree
In 3405, you described several constructions of a point on the
curve taken as pivot (infinity point of the asymptote or
isopivot) and the observations are perfectly compatible with
mine for the pivot (fixed points are not necessary real)
In 3405 applied to a QA-Cu7 gives 4 concyclic points on the
curve QG-P1 and the point S and their tangentials ; this
property is new for me.
Last obsessing question : for 5 given points, how can we find
the 3 positions of P for which CSC(P) lies on the cubic and the
3 cubics are the QA-Cu7 ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3407
Date: 19/3/2019 7:45:46
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
Reading again your message 3404, I've got an idea.
Let's give the 5 points and their conic, the point T on this conic and the CSC.
For a point P, we have a cubic like described in my message 3385 and your message 3403.
The construction of QA1 as described in your message 3404 leads to a DT which is necessary for a QA-Cu7 QA-P2, 4 and 41 ; CSC(P) is QA-P4 and the perpendicular bisector of QA-P2 and 4 is also parallel to PT ...
What do you think of that ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3408
Date: 21/3/2019 11:57:51
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
Naturally, the vertices of this QA1 are the in- and excenters of the triangle TR or QA-P2, 4 and 41 !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3409
Date: 21/3/2019 3:13:11
From: eckart_schmidt@t-online.de
Subject: 7 or 5 P or L

Dear Bernard,

I think, your description of QA1 in #3407 doesn't hold
... (nomination as in my message 3404):

QA1 is an orthocentric quadrangle,
... QA-P4 is an infinity point and QA-P2 isn't defined.
QA2 has P' as QA-P4 and QA-P41 as QA-Tf2(P').

TR is QA-Tr1 of QA1 and QA-Tr2 of QA2,
... its circumcircle bears
... ... QA-P41 of QA1,
... ... QA-P9 of QA2 and diametral
... ... the intersection of the cubic and its asymptote.

Best regards Eckart

PS: It seems, that CSC wrt the 5 points gives here no
observations.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3410
Date: 22/3/2019 10:44:55
From: bernard.keizer
Subject: 7 or 5 P or L

Dear Eckart,
It's always this question of naming !
Once for all, I name QA-P2,4 and 41 the 3 defining points of the QA-Cu7 (id est P2,4 and 41 of one of the 3 QA's of the searched QL).
Back to your figure, the vertices of QA1 (contact points of the tangents parall to the asymptote), which is obviously an orthocentric QA, are the in- and excenters of it's DT.
If we search a point QG-P1 as pivot P in order to define a possible circular QA-Cu7 through 5 points, let's draw the 4 tangents to the cubic parall to the asymptote PT and it's DT ; one of the 3 vertices of this DT is necessary QA-P4=CSC(P) and the asytmote is parallel to the perpendicular bisector of 2 vertices QA-P2 and P4.
I suppose and hope this could give an approximative way of finding the 3 searched QG-P1.
(Drawing a cubic with a random point P, CSC(P) is generally not on the cubic ...)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3411
Date: 23/3/2019 11:12:27
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

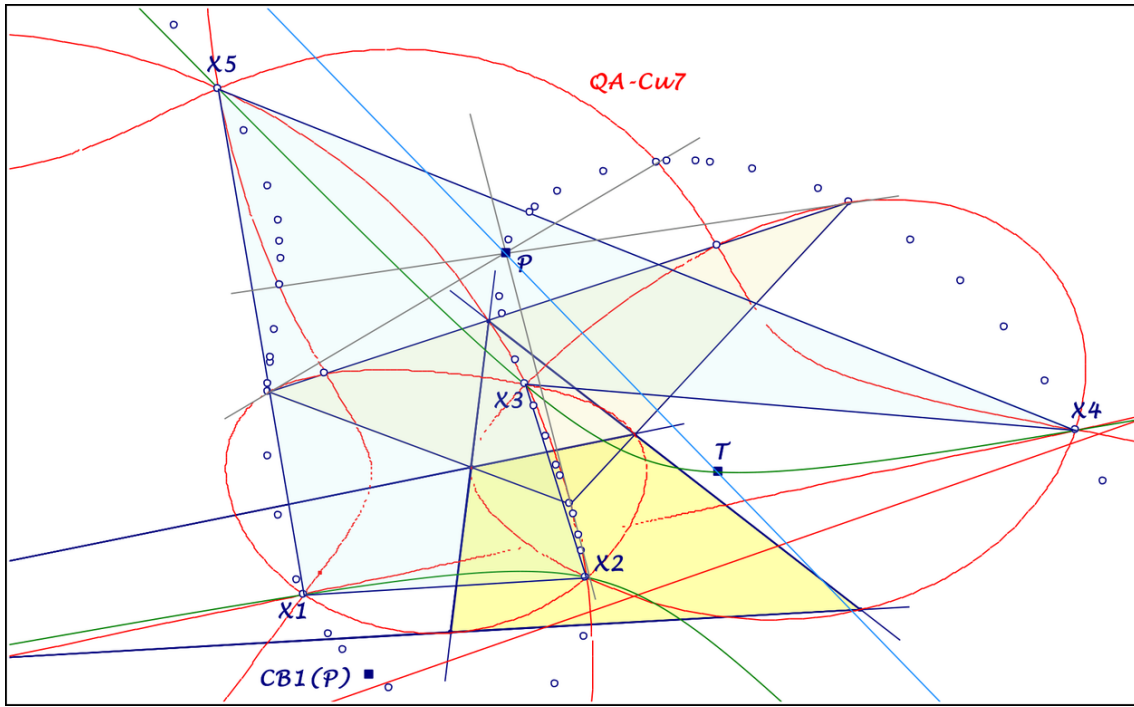
here two observations:

For 5 points (see #3270 and #3285)
... we can construct QL-P1 of the QL
... which has the 5 points as QA-Cu7-triple points
... and consider the CB-transformation CB1
... wrt the 5 points and the 2 circular points (see #3403):

(1) The line
QL-P1.CB1(QL-P1)
bears QL-P8 and QL-P24.
(2) The lines QG-P1.CB1(QG-P1) have a common point P,
... and the CB1-partner on lines through P
... give a cubic CU,
 bearing the 5 points and the 3 QG-P1.
 This cubic is invariant (see attached file)
... wrt the CB-transformations CB1
... and CB2 wrt the 5 points and P, CB1(P).
For points X on the cubic holds:
... X, CB1(X) and P collinear,
... X.CB2(X) parallel PT.

Best regards Eckart

P S. Thanks for clarification in #3410.



2019-03-23.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3412
Date: 24/3/2019 9:29:13
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

I think, the CB-transformation
... wrt 5 points and the two circular points
... is a relevant 5P-transformation.

For a 5P and a pivot P
... the CB-partner on lines through P
... give a circumcubic of the 5P (see #3403, #3404).

The tangential of the pivot P is CB(P).

If we consider a 2nd CB-transformation CB'
... wrt the 5 points, P and CB(P),
... the CB'-partner on parallels to P.5P-s-P4
... give the same cubic.

For a QL with its 5 QA-Cu7-triple points
... and the pivots QG-P1a,b,c we get the cubics QA-Cu7a,b,c.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3413

Date: 24/3/2019 7:23:27

From: Antreas Hatzipolakis

Subject: Name for line through Gergonne and Poncelet points of a quadrangle

----- Forwarded message -----

From: *Stan.Rabinowitz*

In EQF, many notable lines in a quadrilateral have names like the Newton Line, the Steiner line, the Morley line, etc.

But for a quadrangle, lines are designated as QA-L1 or as the QA-P1-P2-P3 line, etc.

I am writing a geometry book and one chapter talks about notable lines in a complete quadrangle. I would like to know if these lines have any well-known names in the literature.

For example, in a quadrangle, QA-L1 is the line through the centroid, the Euler-Poncelet point, and the Gergonne-Steiner point. I am tempted to call this the Gergonne-Poncelet line, but before I do, I thought I would check with this group to see if the line already has a common name other than QA-L1.

Also, I would like to give a reference to the paper in which this collinearity was first proven. Does anyone know who was the first person to prove that these three points colline?

[← Previous](#) [Next →](#) [↩ Message Index](#) [↑ Subjects](#)

Message: #3414
Date: 24/3/2019 7:23:48
From: eckart_schmidt@t-online.de
Subject: QA-P2,4,41 perspective QG-P1,19,18

Dear Bernard, dear Chris,

is the following perspectivity already mentioned?
For a quadrigon
... the triangles QA-P2,4,41 and QG-P1,19,18
... have a perspector on QA-Cu7,
... which is a cyclogonic center of the two triangles.
For a QG-version of a QL this point is
... the CB-partner of QA-P2
for the CB-transformation
... wrt the 5 QA-Cu7-triple points and the two circular points.
For a QL the 3 perspectors give a triangle,
... perspective to the QG-P19- and QA-P4-triangle
... with perspector U (see #3176).
The two perspective triangles define
... in the sense of #2017 the cubic QA-Cu7.

So far a first impression of this perspectivity.

Best regards Eckart

PS:

The cyclogonic center of QG-P1,19,18 and QA-P2,4,41
... is also a point on QA-Cu7,
... its 3 versions for a QL are the CB-partner of QG-P1,
... which give a triangle perspective to the QG-P1-triangle ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3415
Date: 24/3/2019 9:00:21
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,
Very interesting transformations indeed.
Unfortunately, I am still too busy to give it the time it deserves.
Nevertheless, I am following all messages, although not in depth.
I admire the quest you are on with Bernard finding the background of the 5 common points of the 3 versions of QA-Cu7 in a Quadrilaterals.
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3416
Date: 24/3/2019 9:31:51
From: chris.vantienhoven
Subject: Name for line through Gergonne and Poncelet points of a quadrangle

Dear Stan,

In this paper from Jean-Louis Ayme you will find a description of both points QA-P2 and QA-P3:
<http://jl.ayme.pagesperso-orange.fr/Docs/Le%20point%20d'Euler.pdf>

I am not sure if there is a proof of collinearity with QA-P1.

I noticed last years that this collinearity is noticed several times by different persons.

I called the line QA-P1-P2-P3 Line because of the lack of a better name.

I like your name "Gergonne-Poncelet Line" and when not another name is encountered I can include it in EQF.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3417

Date: 24/3/2019 11:43:53

From: staninchelmsford

Subject: Name for line through Gergonne and Poncelet points of a quadrangle

Dear Chris,

Thank you for your quick response.

Ayme's paper does describe all these points and many properties, but he does not show that the centroid, QA-P1, lies on the line through QA-P2 and QA-P3.

I will continue to hunt for references to this fact since there is no point in naming a line that only goes through two points. I will give EQF as my reference until I find additional references. It would also be nice to find out who was the original discoverer of this collinearity.

- stan -

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3418

Date: 25/3/2019 5:24:55

From: chris.vantienhoven

Subject: Name for line through Gergonne and Poncelet points of a quadrangle

Dear Stan,

Ok, thanks.

Please keep me informed.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3419

Date: 25/3/2019 11:48:34

From: staninchelmsford

Subject: Name for line through Gergonne and Poncelet points of a quadrangle

The fact that the centroid (QA-P1) is the midpoint of the line segment joining the Poncelet point (QA-P2) and the Gergonne point (QA-P3) seems to be proven in Theorem 11b of Michal Rolínek and Le Anh Dung, The Miquel Points, Pseudocircumcenter, and Euler-Poncelet Point of a Complete Quadrilateral, Forum Geometricorum, 14(2014)145-153, where O is the Gergonne point, X is the Poncelet point, and G is the centroid.

<http://forumgeom.fau.edu/FG2014volume14/FG201413.pdf>

Was this known before 2014 or is Rolínek the discoverer of this fact?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

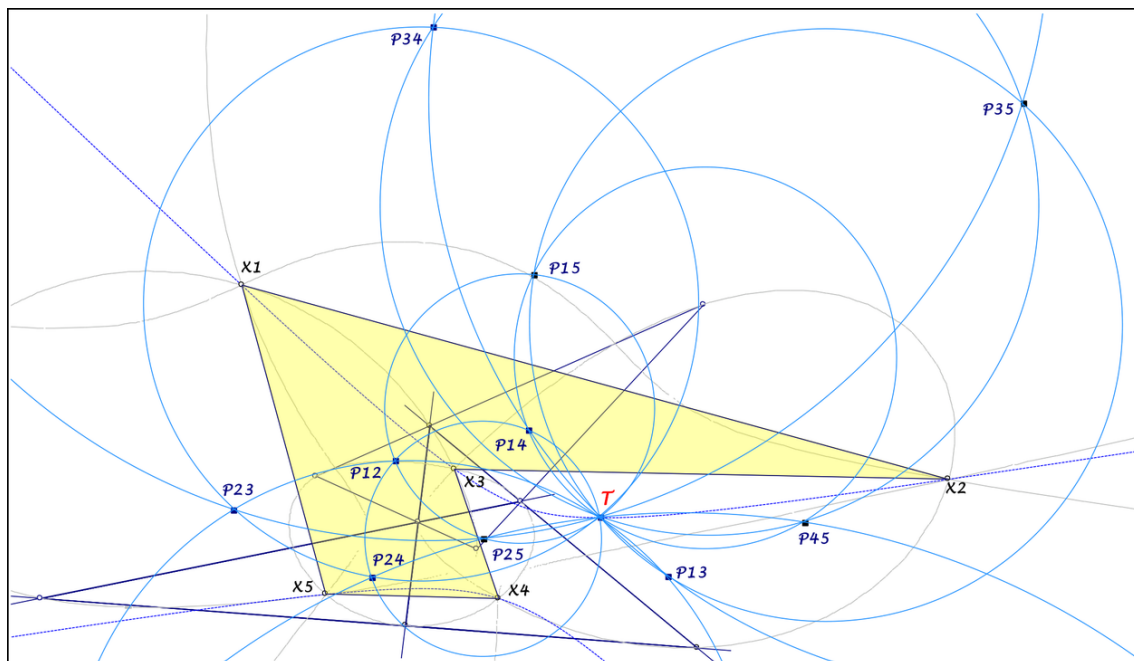
Message: #3420
Date: 27/3/2019 10:06:47
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

perhaps of interest, 10 circles with common point $T = 5P-s-P4$:
Let $X_1X_2X_3X_4X_5$ be a 5P with its CB-transformation
... wrt its 5 vertices and the two circular points.

CB maps lines X_iX_j of the 5P to circles (X_k, X_l, X_m) .
For points P on X_iX_j the lines $P.CB(P)$
... have a common point P_{ij} on the circle (X_k, X_l, X_m) .
... $*T = 5P-s-P4$ is the common point of the 10 circles
 (P_{ij}, P_{jk}, P_{ki}) .
... T and P_{ij} are collinear with the 4th intersection of
 (X_k, X_l, X_m) and $5P-s-Co1$,
... as well as $T.P_{ij}$ parallel $X_i.X_j$.

Best regards Eckart



2019-03-27.pdf

Message: #3421
Date: 28/3/2019 11:31:47
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Each time I spend a few days far from home, you send plenty of interesting messages !
I'll try to answer each one separately in order to avoid too long messages.
I knew already this property, but I can't interpret it.
You should mention that the P_{ij} are the 10 pivots of the degenerated cubics formed by a circle through 3 points and the line through the 2 last points ; of course, T is the pivot of the 11th cubic formed by the conic of the 5 points and the infinity line.
Is there such a property for the 21 pivots in the case of 7 real points, each pivot being on a conic through 5 points (there is no point T in this case) ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3422
Date: 28/3/2019 11:38:01
From: bernard.keizer
Subject: QA-P2,4,41 perspective QG-P1,19,18

Dear Eckart,
Yes we already mentionned this property, at last indirectly.
The QA-Cu7 is the QA-Cu1 of the QA UjVjUkVk, where the U_i, V_j are the double points ; 2 sides intersect orthogonally in QG-P18.
QA-P2,4,41 is the DT of the QA and QG-P1,19,18 is the Miquel triangle of the QA ; they are in perspective with a point, which is the QA-P3 of the QA.
The QA-P2 of this QA is QG-P18 and the QA-P1 is the middle of the middles of UjVj and UkVk (on the diagonals of the QL) ...
It's interesting that the triangle of these 3 QA-P3 is in turn in perspective with the QG-P19 and QA-P4.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3423
Date: 28/3/2019 11:39:18
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart, I regret that Chris is now too busy to take interest in the CB transformations for 7 points or for 5 points and the circular points as well as for the CSC transformation of the 5 points (the dual properties are also interesting for 7 lines).

Only one remark to your message ; considering any copple P and CB(P) in a CB wrt 7 points, they form with the 7 points a CB system and there are 21 other CB transformations where 2 of the 7 points are CB partners wrt the 5 others and P and CB(P).

In particular, if 2 of the 7 points are the circular points, there is a CB transformation for the 5 points, P and CB(P) in which the circular points are CB partners, meaning that the pivot is on the infinity line ...

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3424
Date: 28/3/2019 8:57:40
From: eckart_schmidt@t-online.de
Subject: 5P-s-P4

Dear Chris,

perhaps worth, to be mentioned:
F or the vertices of a QA and QA-P4
... 5P-s-P4 is the reflection of QA-P4 in QA-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3425
Date: 28/3/2019 9:18:31
From: chris.vantienhoven
Subject: 5P-s-P4

Dear Eckart,
A very nice property of 5P-s-P4 in a Quadrangle!
Best regards,
Chris

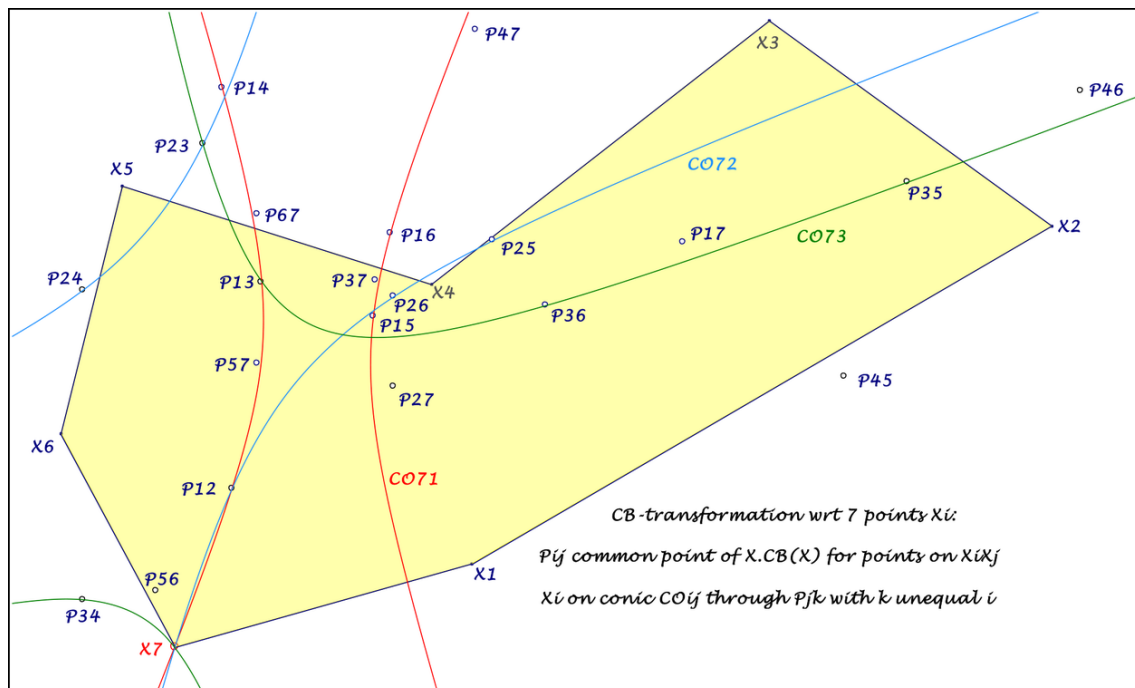
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3426
Date: 29/3/2019 10:56:30
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

thanks for your additional remarks in #3421, #3422, #3423.
Wrt the question in #3421 for the 21 "pivots" in the case of 7
real points
... see attached file, perhaps evident?

Best regards Eckart



2019-03-29.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3427
Date: 29/3/2019 11:44:52
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,

Last, but not least message of yours, it was far the most interesting and promising !

1) Very clever idea of using the CB transformation and applying to the unique point we have apart of the Xi and the point T !
CB1(QL-P1) on the line through QL-P8 and QL-P24 means that
CSC(CB(QL-P1)) is on the line QL-P1U, where U is CSC(QL-P24) and diametral of R on the QA-P4 circle ...

By the way, UR being a diameter of QA-P4 circle and orthogonal to the QG-P18 line means that CSC(UR) is a circle through QL-P1, QL-P24 and CSC(R) and is orthogonal in QL-P24 and CSC(R) to the DT circumcircle QL-Ci1 and in QL-P1 to the DT 9 points circle ; it is therefore centered on the radical axis of both circles, id est the orthic axis of DT.

But I cannot identify this point CB(QL-P1) more precisely ...

2) I reproduced without difficulty your point P, perspector of the QG-P1 and their CB1 partners ; the point T was already identified as the perspector of the QG-P1 and their CB2 partners ...

But I cannot either identify this point more precisely ...

Best regards

Bernard

PS Which method do you use exactly in order to find the CB1 partner ? (you mentionned Hard's method with circles ...).

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3428
Date: 29/3/2019 4:52:30
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
The circle $CSC(UV)$ is also orthogonal to the polar circle of DT : centered in the orthocenter $QL-P10$, this circle is the inversion circle in the inversion swapping the circumcircle and the 9 points circle of DT , swapping in particular $QG-P1$ and $QG-P17$ and $QL-P1$ and $CSC(R)$.
Best regards
Bernard
PS The points R and $CSC(R)$ are simple QL -points and should deserve to be mentionned in EQF ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3429
Date: 30/3/2019 10:37:57
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks for the figure and the answer to my question !
It's not evident at all and interesting anyhow ...
If I understand correctly, each pivot P_{ij} is on 2 conics $C0_{ki}$ and $C0_{kj}$ through X_k with k unequal i and j .
That makes something like 42 different conics, each conic contains 5 P_{ij} and each P_{ij} is on 10 conics.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3430
Date: 30/3/2019 4:29:22
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

wrt #3427:

In 1) you give an interesting excursion in QL-geometry,
... which shows your good overview!

But up to now we only have the line QL-P1.QL-P8.QL-P24 for
CB1(QL-P1),
... similar to the situation for points X on QA-Cu7
... with CB1(X) on the line X.QG-P1.

You asked for the construction of CB1(X)
... wrt 5 points and the two circular points:

I modified my macro of Hart's construction
... as described in #3403,
... sorry, impossible to describe in details.

Best regards Eckart

PS: The last result of #3429 doesn't hold,
... right descibed in the sentence before.
In #3428 I cannot reproduce the orthogonality
... of CSC(UV) and the polar circle of QL-Tr1,
... if U is the perspector of the QG-P19- and QA-P4-triangle
... and V is the perspector of the QG-P19- and QG-P1-triangle.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3431
Date: 30/3/2019 7:03:38
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Sorry for my mistake, it is CSC(UR) as mentionned in 3427
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3432
Date: 31/3/2019 10:46:25
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
What is wrong in 3429 ? I've just reproduced your property !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3433
Date: 31/3/2019 3:40:09
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I very much apologize, your #3429 is correct!
I became in trouble with my own nomination.
In excuse an unexpected splitter:
Consider a QA and a point P,
... take the 4th harmonic point
... of P wrt the QA-Tf2-partner on lines through P
... and you get the QA-circumconic through P.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3434
Date: 01/4/2019 11:42:30
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Eckart,
Thanks for the property !
But it's not totally unexpected.
The 4 points define the QA-Tf2 and the point P defines a pivotal isocubic, locus QA-Tf2 partners on lines through P, which are also CB partners for the 7 points QA vertices and DT vertices. The conic through P and the 4 QA vertices is the polar conic of P wrt the cubic, which is the unique diagonal polar conic : it passes through the 4 points, P and the vertices of the anticevian triangle of P wrt DT and more generally the vertices of the anticevian triangle of any of it's points.
(see Jean-Pierre Ehrmann and Bernard Gibert Special Isocubics ...p30-31)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3435
Date: 01/4/2019 4:41:54
From: eckart_schmidt@t-online.de
Subject: CB-Transformations

Dear Bernard, dear Chris,

let us consider CB-transformations
... CB wrt 7 real points,
... cb wrt 5 real points and the two circular points.

For a "pivot" P we can look for cubics
... as loci of CB-/cb-partner on lines through P.

Well known:
For $P = QA-P4$ and CB-partner
... wrt the vertices of the QA and its QA-Tr1
... we get QA-Cu1 (CB is QA-Tf2).

New perhaps:
For the same $P = QA-P4$ and cb-partner
... wrt the QA-vertices and the 2nd intersection X
... of QA-P3.QA-P41 and the QA-circumconic through $P = QA-P3$
... we get also QA-Cu1.

In addition wrt the 5P of the QA-vertices and X:
... $5P-s-P4 = QA-P3$, $5P-CSC(X) = QA-P41$,
... $5P-s-Tf3(QA-P4) = QA-P2$.

Best regards Eckart

PS: Thanks to Bernard for the background of my splitter in #3433.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

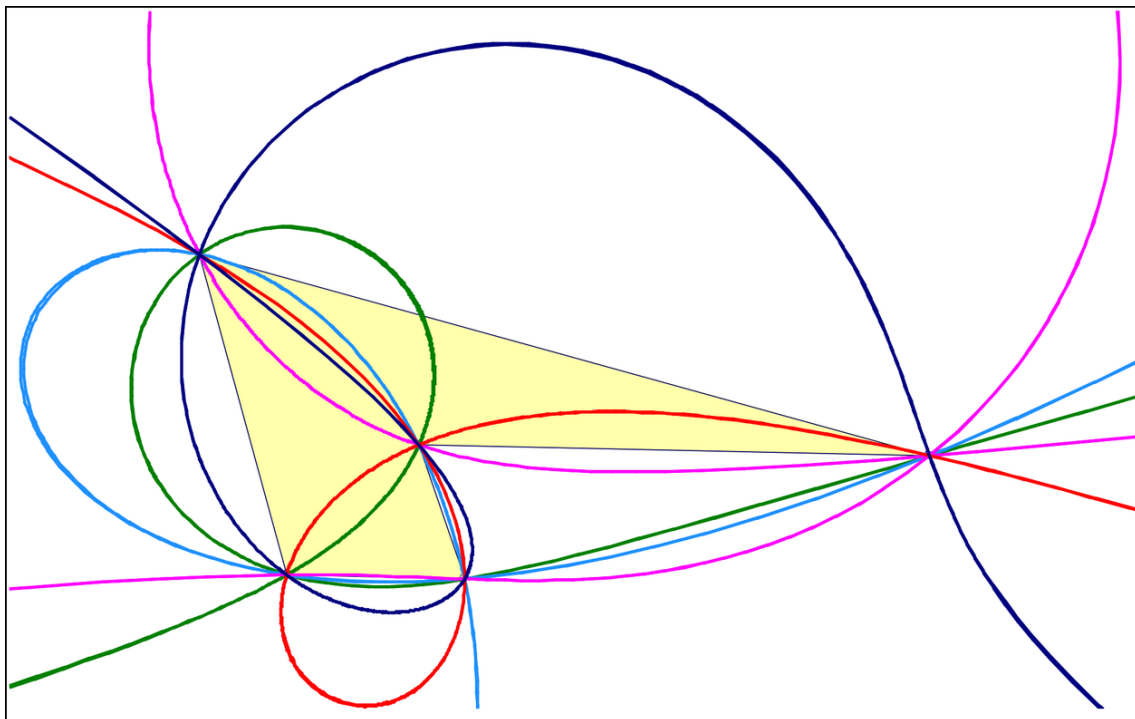
Message: #3436
Date: 04/4/2019 8:50:42
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

attached 5 circumcubics for a 5P
... with curious construction:

Take a circle round one vertex of the 5P
... and its cb-image wrt the 5P and the 2 circular points,
... which is a curve of higher degree.
... Reducing the radius of the circle,
... you will get as limit a cubic,
... locus of the points with cb-image in the chosen vertex.

Best regards Eckart



2019-04-03.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3437
Date: 04/4/2019 11:36:09
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Beautiful curves, indeed (stophoïds ?)
I loose progressively courage and I don't know if we will one day find a simple construction of the QL starting with 5 points taken as triple QA-Cu7 points, which would be an enormous progress in the knowledge of the 5P Geometry identical to the QL-Geometry.
We have the 5 points, the circumconic, the point T, the CSC transformation and the CB transformation wrt the 5 + 2 circulars.
And plenty of ideas, but not enough time to check them ...
For example, the 5 points give 5 QA's and 5 pivotal isocubics with pivot the 5th point. The vertices of the anticevian triangle of the 5th point wrt the DT of the QA of the 4 points give 3 more points having with the 5 th point the same DT as the QA of the 4 points.
What about the DT of the QA's formed by the 3 points S and either T or R ?
The triangles QG-P1 and Si perspective with T give a Reye configuration and another cubic, the same for the QA-P4 and Si perspective with R.
Each morning, I open my computer, hoping desperately that you will have found a solution...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3438
Date: 04/4/2019 11:53:46
From: eckart_schmidt@t-online.de
Subject: Orthogonal QL-Hyperbolas

Dear Bernard,

let us consider a generalization of the splitter in #3433:

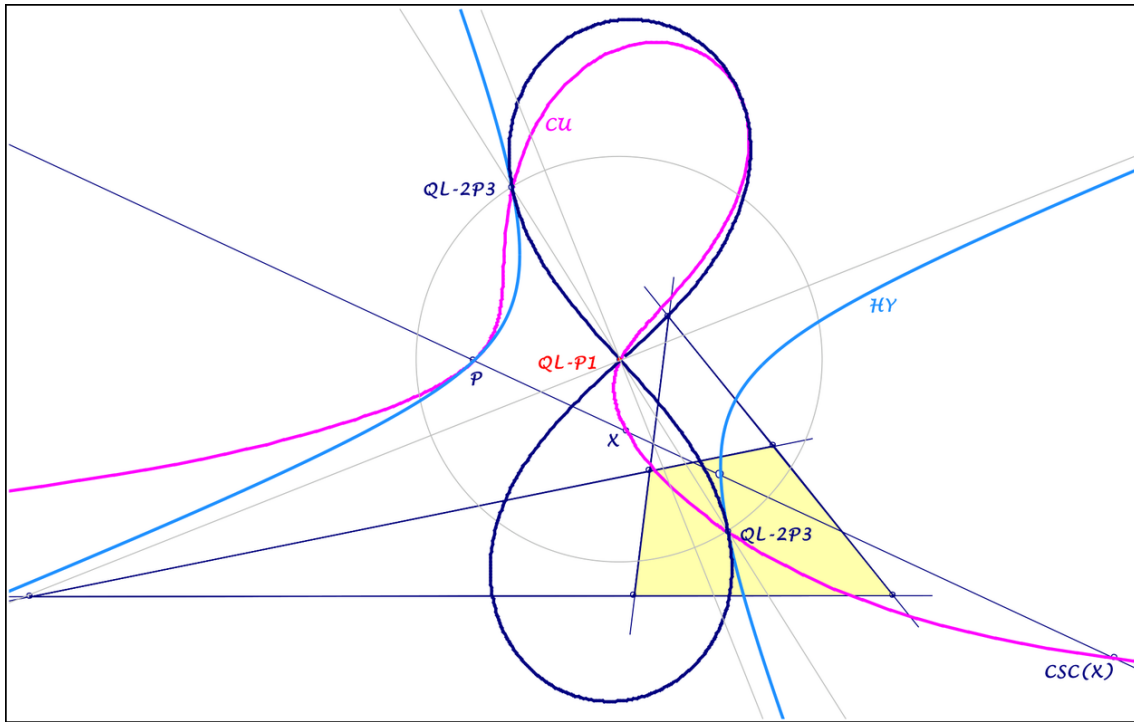
1) For a pivot P
... the CSC-partner on lines through P give a cubic CU
... through P, QL-P1 and QL-2P3.
The 4th harmonic points of P wrt the CSC-partner
... give an orthogonal hyperbola HY,
... centered in QL-P1 through P and QL-2P3
... whose CSC-image is a lemniscate LE,
... centered in QL-P1, tangent to HY in QL-2P3,
... with its foci in the CSC-images of the HY-foci.

2) Now we consider CB-transformations cb,
... defined by 5 real points and the two circular points:
For a pivot P
... the cb-partner on lines through P give a cubic,
... the 4th harmonic points of P wrt the cb-partner
... give a conic with tangent in P through cb(P) ...

What about these conics? Especially:
... cb wrt the 5 QA-Cu7-triple points and pivot QL-P1.

Best regards Eckart

PS-splitter with a well known background:
Let X, Y be two opposite points of the QL
... then $X.cb(X) \wedge Y.cb(Y) = QG-P1$.



2019-04-04.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3439
Date: 05/4/2019 9:21:50
From: eckart_schmidt@t-online.de
Subject: QL-Parabolas

Dear Bernard, dear Chris,

there is an interesting construction
... of QL-parabolas
wrt a defining point P unequal QL-P1:
Consider circles round P with variable radius
... and the radical axes wrt their CSC-image circles,
... which envelop a parabola
... with focus CSC(P) and directrix P.QL-P1,
... tangent to the Steiner axes.
The parabolas degenerate for P on the Steiner axes.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3440
Date: 05/4/2019 10:06:09
From: bernard.keizer
Subject: Orthogonal QL-Hyperbolas

Dear Eckart,

1) Your property in the splitter is very general

For any point, a variable line through the point cuts a conic in 2 points and the harmonic of the point wrt the 2 intersections describe a line, which is the polar of the point wrt the conic. For any point on a cubic, a variable line through the point cuts the cubic in 2 points and the harmonic of the point wrt the 2 intersections describe a conic, which is the polar conic of the point.

2) For any cubic, the tangential QA of a point gives 4 points (not necessary real) which are the fixed points of an isoconjugation wrt the DT of the QA ...

With the CSC partners, you get rectangular hyperbolas through the QL-2P3.

These RH are the polar conics of the infinity points wrt QL-Cu2, the cubic stelloïd associated to the QL.

3) For the point QL-P1, as the CSC partners are on lines symmetric wrt the Steiner axes, I suppose the cubic degenerates in 3 lines 2 Steiner axes and infinity line and the conic is tangent to the Steiner axes (?).

Best regards

Bernard

PS By the way, P is the common tangential of the QL-2P3 and the lemniscate is tangent to the cubic in CSC(P), which is the tangential of P.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3441
Date: 06/4/2019 12:51:38
From: bernard.keizer
Subject: Orthogonal QL-Hyperbolas

Dear Eckart,
In 3), my last remark is not correct.
For P in QL-P1, the RH degenerates in the 2 Steiner axes and is
the polar conic in the cubic stelloid of the infinity point of
the Newton Line QL-L1.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

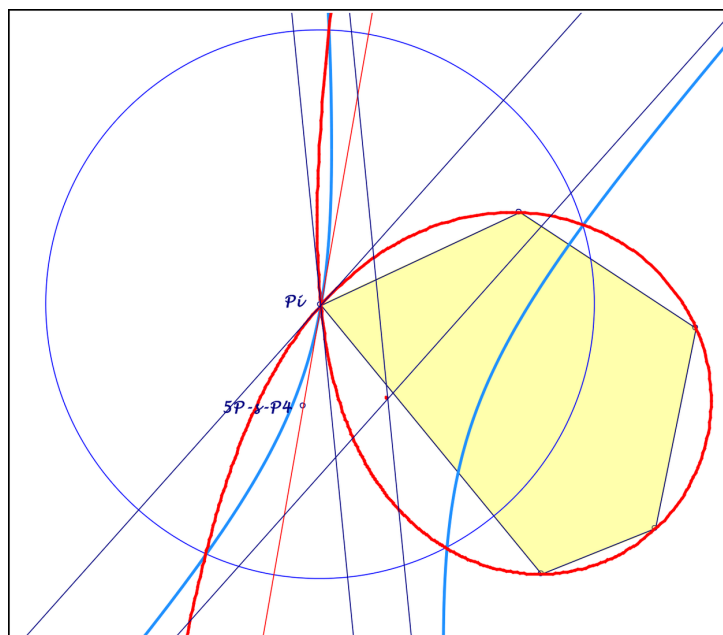
Message: #3442
Date: 10/4/2019 10:30:43
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

in message 3436 I offered 5P-cubics,
... which are the locus of points X
... with $cb(X)$ in one vertex P_i of the 5P,
... $cb = CB$ -transformation, defined by 5P and 2 circular points.
Here is a simple concrete construction:
... Draw a circle round a chosen 5P-vertex P_i
... and the inversions Q_j of the other four 5P-points.
... Consider the conic through P_i, Q_j, Q_k, Q_l, Q_m
... and its inversion wrt the circle,
... which gives the cubic above (see attached file).
The asymptote of the cubic is parallel $P_i.5P-s-P_4$.
If the conic is a hyperbola,
... the cubic has a double point in P_i
... with tangents parallel to the asymptotes of the conic,
... but the cubic must not be a strophoid.
These cubics can also be discussed for QA wrt a point P .

Best regards Eckart

PS: Many thanks for explanations in #3440,
for I am not so familiar with polar conics.



2019-04-10.pdf

Message: #3443

Date: 14/4/2019 9:25:40

From: staninchelmsford

Subject: Another point on the Seebach-Walser Line

EQF lists five points on the QG-Centroids Line, QG-L3 (also known as the Seebach-Walser Line).

I think I've found a new point on this line.

Does anyone know if the following result is already known?

Let ABCD be a quadrigon with diagonal crosspoint E.

Consider the line segment from the midpoint of AB to the centroid of triangle CDE.

Also consider the other 3 similar line segments from the midpoints of the other 3 sides.

Then these 4 line segments meet at a point P.

This point P lies on the Seebach-Walser line.

If $M=QA-P1$ (the vertex centroid of the quadrangle) and $G=QG-P4$ (the area centroid of the quadrigon), then $EP:PM:MG=7:2:3$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3444

Date: 15/4/2019 10:15:52

From: eckart_schmidt@t-online.de

Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford,

if I am not wrong the ratios for your new QG-point P are as follows:

$EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,$

P divides QG-P1.QG-P4 with ratio 3 : 2,

... QG-P1.QG-P8 with ratio -6:1,

... QG-P1. QA-P1 with ratio 4 : 1,

... QG-P1.QG-P15 with ratio 2 : 3,

... QG-P5.QG-P11 with ratio 4 : 1,

... QG-P6.QG-P9 with 4 : 1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3445
Date: 15/4/2019 11:09:32
From: eckart_schmidt@t-online.de
Subject: Quintupel-Points for a 5P

Dear Bernard,

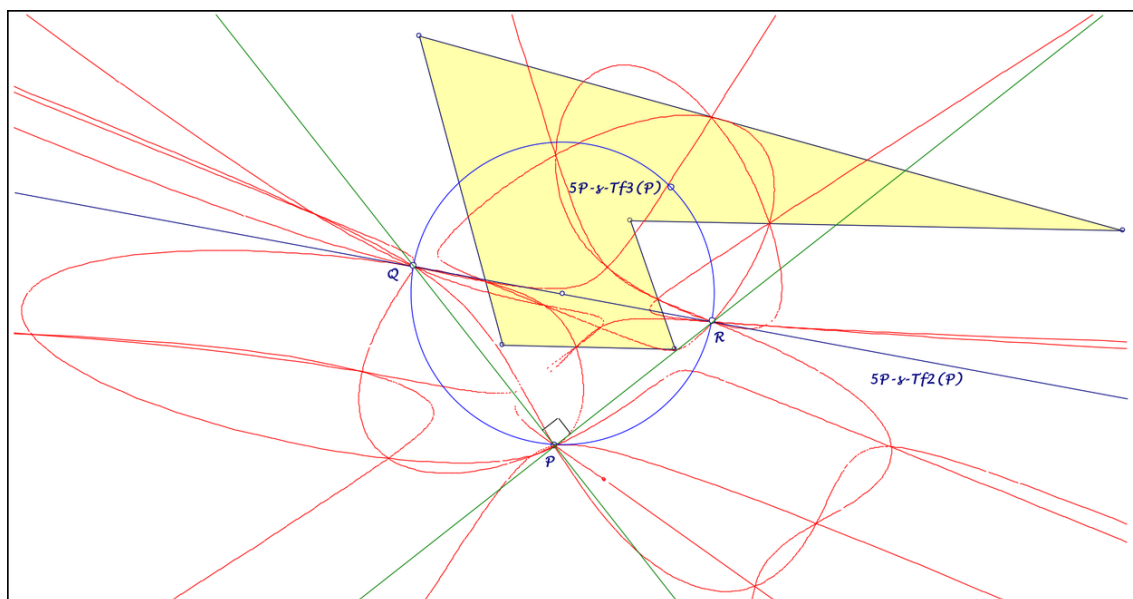
if the mysterious 5 QA-Cu7-triple points of a QL
... become boring, here a new constellation:

Replacing in the construction of QA-Cu7
... QA-P4 by another point P,
... we get cubics CU(P) for a QA (see #3341).

Now consider a 5P and any point P,
... construct the 5 cubics CU(P) wrt 4 points of the 5P,
... and you get 20 triple-points,
... pairwise on the ten 5P-lines, in harmonic position,
... circles with these pairs as diameter bear P,
... further common points: 3 quintupel-points P, Q, R
... with $QR = 5P\text{-s-Tf2}(P)$, PR orthogonal PQ,
... and $5P\text{-s-P3}(P)$ on the circumcircle of PQR.

In this way the 2 quintupel-points Q and R beside P
... are constructible!

Best regards Eckart



2019-04-15.pdf

Message: #3446
Date: 15/4/2019 11:31:16
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Sorry, I wasn't home for a few days.
This is a very beautiful construction indeed !
I checked without difficulty that the cubic obtained by
inverting a conic gives the same result as my construction in
3385 with pivot in one of the 5 points.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3447
Date: 15/4/2019 12:14:40
From: Tran Quang Hung
Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford and Eckart

I see the general propblem

ABCD be a quadrigon with any point P.
Consider the line segment from the midpoint of AB to the
centroid of triangle PCD.
Also consider the other 3 similar line segments from the
midpoints of the other 3 sides.
Then these 4 line segments meet at a point Q.
When P=E diagonal crosspoint, we get above problem?

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 15:15 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Staninchelmsford,
>> if I am not wrong the ratios for your new QG-point P are as
follows:
>> $EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,$

>> P divides
>> QG-P1.QG-P4 with ratio 3 : 2,
>> ... QG-P1.QG-P8 with ratio -6:1,
>> ... QG-P1. QA-P1 with ratio 4 : 1,
>> ... QG-P1.QG-P15 with ratio 2 : 3,
>> ... QG-P5.QG-P11 with ratio 4 : 1,
>> ... QG-P6.QG-P9 with 4 : 1.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3448
Date: 15/4/2019 3:09:55
From: Tran Quang Hung
Subject: Newton lines are concurrent with 5P

Dear geometers,

Let A,B,C,D,E be five points.

Let

d1=Newton line of four lines (AB,BC,CD,DE)

d2=Newton line of four lines (EA,BC,CD,DE)

d3=Newton line of four lines (EA,AB,CD,DE)

d4=Newton line of four lines (EA,AB,BC,DE)

d5=Newton line of four lines (EA,AB,BC,CD)

Then d1,d2,d3,d4,d5 are concurrent. Which is this point?

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3449
Date: 15/4/2019 3:11:10
From: Thanh Tung
Subject: Another point on the Seebach-Walser Line

Dear Tran Quang Hung, Staninchelmsford and Eckart,

For the problem below,
the point of concurrency
Q is centroid of the five points P, A, B, C, D.

Best regards,
Vu Thanh Tung

Le lundi 15 avril 2019 à 17:14:48 UTC+7, Tran Quang Hung
analgeomatrica@gmail.com [Quadri-Figures-Group] a écrit :
Dear Staninchelmsford and Eckart
I see the general problem
ABCD be a quadrigon with any point P.
Consider the line segment from the midpoint of AB to the
centroid of triangle PCD.
Also consider the other 3 similar line segments from the
midpoints of the other 3 sides.
Then these 4 line segments meet at a point Q.
When P=E diagonal crosspoint, we get above problem?
Best regards
Tran Quang Hung.

Và o Th 2, 15 thg 4, 2019 vaì€o luì€c 15:15 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups..com (Quadri-Figures-Group@yahoogroups.com) > Ä'ã viá°¿t:
>> Dear Staninchelmsford,
>> if I am not wrong the ratios for your new QG-point P are as follows:
>> EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5,
>> P divides
>> QG-P1.QG-P4 with ratio 3 : 2,
>> ... QG-P1.QG-P8 with ratio -6:1,
>> ... QG-P1. QA-P1 with ratio 4 : 1,
>> ... QG-P1.QG-P15 with ratio 2 : 3,
>> ... QG-P5.QG-P11 with ratio 4 : 1,
>> ... QG-P6.QG-P9 with 4 : 1.
>>> Best regards Eckart

Message: #3450
Date: 15/4/2019 3:29:52
From: eckart_schmidt@t-online.de
Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford and Tran Quang Hung,

wrt the generalized proplem in #3347:
... P, Q, QA-P4 are collinear
... and Q divides P.QA-P1 with ratio 4:1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3451
Date: 15/4/2019 4:33:19
From: eckart_schmidt@t-online.de
Subject: Newton lines are concurrent with 5P

Dear Tran Quang Hung,

The common point of the Newton lines for the QL-versions of a 5L ... is 5L-s-P1 (see EPG).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3452
Date: 16/4/2019 12:25:49
From: Tran Quang Hung
Subject: Newton lines are concurrent with 5P

Dear Eckart

Now I see, thank you very much.

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 21:33 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> The common point of the Newton lines for the QL-versions of a
5L
>> ... is 5L-s-P1 (see EPG).
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3453
Date: 16/4/2019 1:04:20
From: Tran Quang Hung
Subject: Miquel circle of 5L

Dear geometers,

Let d_1, d_2, d_3, d_4, d_5 be five lines.
 M_1 = Miquel point of (d_2, d_3, d_4, d_5) .
Define similarly, M_2, M_3, M_4, M_5 .
Then M_1, M_2, M_3, M_4, M_5 lie on a circle.
It seems that this property does not like 5L-s-L3?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3454
Date: 16/4/2019 3:02:39
From: Tran Quang Hung
Subject: Gergonne circle

Dear geometers

Let d_1, d_2, d_3, d_4 be four lines.
 A_{12} = d1 meet d2.
Define similarly the points $A_{13}, A_{14}, A_{23}, A_{24}, A_{34}$.

Let
 G_1 = Gergonne point (QA-P3) of $(A_{12}, A_{14}, A_{23}, A_{34})$
 G_2 = Gergonne point (QA-P3) of $(A_{23}, A_{24}, A_{13}, A_{14})$
 G_3 = Gergonne point (QA-P3) of $(A_{12}, A_{13}, A_{24}, A_{34})$

Then circumcircle of triangle $G_1G_2G_3$ passes through Miquel point (QL-P1) of (d_1, d_2, d_3, d_4) .
I call this circle by Gergonne circle of (d_1, d_2, d_3, d_4) . Is this circle and its center known before?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3455
Date: 16/4/2019 3:46:56
From: Tran Quang Hung
Subject: Miquel circles are concurrent with 5L

Dear geometers

Let d_1, d_2, d_3, d_4, d_5 be five lines.
Let circle (M_1) = Miquel circle (QL-Ci3) of (d_2, d_3, d_4, d_5) .
Define similarly circles $(M_2), (M_3), (M_4), (M_5)$.
Then five circles $(M_1), (M_2), (M_3), (M_4), (M_5)$ are concurrent at a point.
Which is this point of 5L?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3456
Date: 16/4/2019 3:47:47
From: Tran Quang Hung
Subject: Another point on the Seebach-Walser Line

Thank You for your interest.

Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 20:29 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Staninchelmsford and Tran Quang Hung,
>> wrt the generalized proplem in #3347:
>> ... P, Q, QA-P4 are collinear
>> ... and Q divides P..QA-P1 with ratio 4:1.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3457
Date: 16/4/2019 5:02:45
From: Tran Quang Hung
Subject: Two QL share the same Miquel point and Steiner line

Dear geometers,

Let d_1, d_2, d_3, d_4 be four lines.
Let A_1 be orthopole of d_1 wrt triangle (d_2, d_3, d_4) .
Let l_1 be orthotransversal of A_1 wrt triangle (d_2, d_3, d_4) .
Define similarly the lines l_2, l_3, l_4 .
Then (d_1, d_2, d_3, d_4) and (l_1, l_2, l_3, l_4) share the same Miquel point (QL-P1) and the same Steiner line (QL-L2).
Miquel circle of (d_1, d_2, d_3, d_4) and (l_1, l_2, l_3, l_4) meet again at X (other than QL-P1). Which is this point?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3458
Date: 16/4/2019 9:39:31
From: bernard.keizer
Subject: Gergonne circle

Dear Tran Quang Hung,
Your circle is the Dimidium circle (see EQF)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3459
Date: 16/4/2019 9:56:15
From: eckart_schmidt@t-online.de
Subject: Miquel circle of 5L

Dear Tran Quang Hung,

it seems, that you are right:
... your Miquel circle for a 5L is not in EPG,
... but it is already discussed years ago
... for example in #710.

Following generalization holds:
... The QL-Tf1-images of a point
... wrt the 5 QL of a 5L are concyclic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3460
Date: 16/4/2019 10:24:08
From: eckart_schmidt@t-online.de
Subject: Miquel circles are concurrent with 5L

Dear Tran Quang Hung,

I am not shure, that this point is in EPG,
... for I am not fit in Chris' nomination,
... but it is also already mentioned 2014 in #710.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3461
Date: 16/4/2019 10:36:58
From: eckart_schmidt@t-online.de
Subject: Gergonne circle

Dear Tran Quang Hung,

this circle is QL-Ci6 in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3462
Date: 16/4/2019 12:07:27
From: Tran Quang Hung
Subject: Gergonne circle

Thank You very much Bernard and Eckart for all quick and clear reply.

Best regards
Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 16:27 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> this circle is QL-Ci6 in EQF.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3463
Date: 16/4/2019 1:17:25
From: Tran Quang Hung
Subject: Center of circle lies on Steiner line

Dear geometers

Let d_1, d_2, d_3, d_4 be four lines.

$A_{12} = d_1 \text{ meets } d_2$.

Define similarly the points $A_{13}, A_{14}, A_{23}, A_{24}, A_{34}$.

M is Miquel point of (d_1, d_2, d_3, d_4) .

Let M_1 be isogonal conjugate of M wrt triangle $A_{23}A_{14}A_{34}$

Let M_2 be isogonal conjugate of M wrt triangle $A_{12}A_{14}A_{23}$

Let M_3 be isogonal conjugate of M wrt triangle $A_{12}A_{24}A_{13}$

Let M_4 be isogonal conjugate of M wrt triangle $A_{24}A_{34}A_{13}$

Then M_1, M_2, M_3, M_4 lie on a circle and center of this circle lies on Steiner line of (d_1, d_2, d_3, d_4) .

Let M_5 be isogonal conjugate of M wrt triangle $A_{12}A_{14}A_{34}$

Let M_6 be isogonal conjugate of M wrt triangle $A_{12}A_{34}A_{23}$

Let M_7 be isogonal conjugate of M wrt triangle $A_{13}A_{14}A_{24}$

Let M_8 be isogonal conjugate of M wrt triangle $A_{23}A_{24}A_{13}$

Then M_5, M_6, M_7, M_8 also lie on a circle and center of this circle lies on Steiner line of (d_1, d_2, d_3, d_4) .

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3464

Date: 16/4/2019 1:51:38

From: eckart_schmidt@t-online.de

Subject: Two QL share the same Miquel point and Steiner line

Dear Tran Quang Hung,

interesting second QL (l1,l2,l3,l4)!

... Not only QL-P1 and QL-L2 are the same,

... but also the *inscribed parabola QL-Co1.*

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3465

Date: 16/4/2019 3:28:58

From: Tran Quang Hung

Subject: Two QL share the same Miquel point and Steiner line

Thank You Eckart for your interest.

Thus $QL(d_1, d_2, d_3, d_4)$ and $QL(l_1, l_2, l_3, l_4)$ share the same Miquel point and the same Steiner line is actually consequence of they share the same inscribed parabola $QL-Co1$. This is very interesting!

Best regards

Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 19:28 '

eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de

[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >

đã viết:

>> Dear Tran Quang Hung,

>> interesting second $QL(l_1, l_2, l_3, l_4)$!

>> ... Not only $QL-P1$ and $QL-L2$ are the same,

>> ... but also the *inscribed parabola $QL-Co1$.*

>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3466
Date: 16/4/2019 3:32:08
From: eckart_schmidt@t-online.de
Subject: Center of circle lies on Steiner line

Dear Tran Quang Hung,

Your two circles (M_1, M_2, M_3, M_4) and (M_5, M_6, M_7, M_8)
... have a radical axis,
... whose QL-Tf1 image circle
... has a tangent in QL-P1 through QL-P4.

Your two circles (M_1, M_2, M_3, M_4) and (M_5, M_6, M_7, M_8)
... have QL-Tf1 image circles,
... which are orthogonal to QL-Ci3
... with radical axis through QL-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3467
Date: 16/4/2019 3:32:26
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Dear geometers,

Let d_1, d_2, d_3, d_4 be four lines.
Let e_1, e_2, e_3 be the diagonal lines of $QL(d_1, d_2, d_3, d_4)$.
Let s_1 be Steiner line of $QL(e_1, e_2, e_3, d_1)$.
Define similarly the lines s_2, s_3, s_4 .
Then s_1, s_2, s_3, s_4 are concurrent. Which is this point?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3468
Date: 16/4/2019 3:46:28
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

the described point is QL-P10.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3469
Date: 16/4/2019 6:38:43
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Thank You Eckart

I also see that

M1 = Miquel point of QL(e1,e2,e3,d1).

M2 = Miquel point of QL(e1,e2,e3,d2).

M3 = Miquel point of QL(e1,e2,e3,d3).

M4 = Miquel point of QL(e1,e2,e3,d4).

Then four points M1, M2, M3, M4 are on a circle and its center lies on Steiner line of QL(d1,d2,d3,d4).

Best regards

Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 22:57 '

eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de

[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

>> Dear Tran Quang Hung,

>> the described point is QL-P10.

>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3470
Date: 16/4/2019 6:42:21
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Dear Eckart and all,

I also see that

n_1 = Newton line of $QL(e_1, e_2, e_3, d_1)$.

n_2 = Newton line of $QL(e_1, e_2, e_3, d_2)$.

n_3 = Newton line of $QL(e_1, e_2, e_3, d_3)$.

n_4 = Newton line of $QL(e_1, e_2, e_3, d_4)$.

n = Newton line of $QL(d_1, d_2, d_3, d_4)$.

Then two Newton lines of $QL(n, e_1, e_2, e_3)$ and $QL(n_1, n_2, n_3, n_4)$ are the same.

Best regards

Tran Quang Hung.

Vào Th 3, 16 thg 4, 2019 vào lúc 23:38 Tran Quang Hung < analgeomatrica@gmail.com > đã viết:

>> Thank You Eckart

>> I also see that

>> M_1 = Miquel point of $QL(e_1, e_2, e_3, d_1)$.

>> M_2 = Miquel point of $QL(e_1, e_2, e_3, d_2)$.

>> M_3 = Miquel point of $QL(e_1, e_2, e_3, d_3)$.

>> M_4 = Miquel point of $QL(e_1, e_2, e_3, d_4)$.

>> Then four points M_1, M_2, M_3, M_4 are on a circle and its center lies on

>> Steiner line of $QL(d_1, d_2, d_3, d_4)$.

>> Best regards

>> Tran Quang Hung.

>> Vào Th 3, 16 thg 4, 2019 vào lúc 22:57 '

eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de

>> [Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com

> đã viết:

>>> Dear Tran Quang Hung,

>>> the described point is QL-P10.

>>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3471
Date: 16/4/2019 9:35:00
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

only two additional remarks:
wrt #3469:
The circle (M1,M2,M3,M4) is
... the circumcircle of the diagonal triangle QL-Tr1.
wrt # 3470:
QL(n1,n2,n3,n4) has the vertices
... midpoints of Aij and Aik.Ajl ^ Ail.Ajk.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3472
Date: 17/4/2019 1:42:48
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Tran Quang Hung,

You wrote:
> When P=E diagonal crosspoint, we get above problem?

Yes. You are correct. Nice generalization.

---In Quadri-Figures-Group@yahoogroups.com, wrote :
Thank You for your interest.
Best regards
Tran Quang Hung.

Vào Th 2, 15 thg 4, 2019 vào lúc 20:29 ' eckart_schmidt@... '
eckart_schmidt@... [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
>>> Dear Staninchelmsford and Tran Quang Hung,
>>> wrt the generalized proplem in #3347:
>>> ... P, Q, QA-P4 are collinear
>>> ... and Q divides P..QA-P1 with ratio 4:1.
>>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3473
Date: 17/4/2019 2:06:34
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Thank You Eckart, I now understand two these messages.

Best regards
Tran Quang Hung.

Vào Th 4, 17 thg 4, 2019 vào lúc 05:43 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> only two additional remarks:
>> wrt #3469:
>> The circle (M1,M2,M3,M4) is
>> ... the circumcircle of the diagonal triangle QL-Tr1.
>> wrt # 3470:
>> QL(n1,n2,n3,n4) has the vertices
>> ... midpoints of Aij and Aik.Ajl ^ Ail.Ajk.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3474
Date: 17/4/2019 2:07:32
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Eckart,

You wrote:

> P, Q, QA-P4 are collinear
> and Q divides P.QA-P1 with ratio 4:1.

I assume this was a typo and you meant "P, Q, QA-P1 are collinear"

Anyhow, this ratio, 4 : 1, appears to be correct. It also agrees with the ratio Tran Quang Hung found.

Earlier, you wrote:

> if I am not wrong the ratios for your new QG-point P are as follows:

> EP : PM : MG = QG-P1.P : P.QA-P4 : QA-P4.QG-P4 = 12 : 3 : 5

Again, I think this was a typo and you meant

EP : PM : MG = QG-P1.P : P.QA-P1 : QA-P1.QG-P4 = ...

I think your ratio 12 : 3 : 5 is wrong though. I think this should be 12 : 3 : 4.5

Also, my original ratio of 7 : 2 : 3 was wrong also.

I will double-check my numbers and write up the results in a separate reply.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3475
Date: 17/4/2019 2:57:02
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

I've rechecked my calculations.

If we call my new point QG-new, then we have 6 points on the Seebach-Walser Line (QG-L3).

The ratios appear to be:

QG-P1 : QG-P8 : QG-new : QA-P1 : QG-P4 : QG-P15 = 10 : 2 : 3 : 5 : 10

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3476
Date: 17/4/2019 9:39:43
From: eckart_schmidt@t-online.de
Subject: Another point on the Seebach-Walser Line

Dear Staninchelmsford,

you are right, correcting my typo, I beg your pardon!
But - excuse - I cannot confirm your ratio 12 : 3 : 4.5
in #3474,
... and I do not understand the notation in #3475.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3477
Date: 17/4/2019 2:54:37
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Eckart,

I appreciate your checking my numbers.
I will go check again.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3478
Date: 17/4/2019 3:37:05
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

wrt $QL' = QL(n1, n2, n3, n4)$ in #3470:
In #3471 I described the vertices of QL' :
... .. midpoints of A_{ij} and $A_{ik}.A_{jl} \wedge A_{il}.A_{jk}$.

Further properties:

... $QL-P1$ of QL' is the midpoint of $QL-P17.QL-P25$.
... $QL-L1$ of QL' is the bisector of $QL-P1$ and $QL-P7$,
... .. is a parallel to $QL-L9$ through $QL-P19$,
... $QL-L2$ of QL' is a parallel to $QL-L6$ through $QL-P11$.
... $QL-L6$ of QL' is $QL-L2$ of the reference QL .
... $QL-L7, QL-L8$ of QL' are the same as for reference QL .
... $QL-L9$ of QL' is $QL-L1$ of the reference QL .
... $QL-Ci1$ of QL' is a circle round $QL-P11$ through $QL-P25$.
... .. If we apply the construction of QL' once more,
... we get a QL'' , homothetic to the reference QL ,
... center $QL-P8$, ratio 4:1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3479
Date: 17/4/2019 4:22:40
From: staninchelmsford
Subject: Another point on the Seebach-Walser Line

Dear Eckart,

You wrote:

> I do not understand the notation in #3475.

Sorry, I had listed the 6 points and then gave the ratios of the consecutive distances between them.

What I should have said was:

QG-P1.QG-P8: QG-P8.

QG-new : QG-new.QA-P1 : QA-P1.QG-P4 : QG-P4.QG-P15 = 10 : 2 : 3
: 5 : 10

But if my number of $4.5 = 9/2$ in message #3474 is wrong, then this would be wrong as well.

I'll check again and report back.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3480
Date: 18/4/2019 2:16:12
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Thank You Eckart for the interesting properties of $QL(n_1, n_2, n_3, n_4)$.

Best regards
Tran Quang Hung.

Vào Th 4, 17 thg 4, 2019 vào lúc 20:57 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> wrt $QL' = QL(n_1, n_2, n_3, n_4)$ in #3470:
>> In #3471 I described the vertices of QL' :
>> ... midpoints of A_{ij} and $A_{ik}.A_{jl} \wedge A_{il}.A_{jk}$.
>> Further properties:
>> ... $QL-P1$ of QL' is the midpoint of $QL-P17.QL-P25$.
>> ... $QL-L1$ of QL' is the bisector of $QL-P1$ and $QL-P7$,
>> ... is a parallel to $QL-L9$ through $QL-P19$,
>> ... $QL-L2$ of QL' is a parallel to $QL-L6$ through $QL-P11$.
>> ... $QL-L6$ of QL' is $QL-L2$ of the reference QL .
>> ... $QL-L7, QL-L8$ of QL' are the same as for reference QL .
>> ... $QL-L9$ of QL' is $QL-L1$ of the reference QL .
>> ... $QL-Ci1$ of QL' is a circle round $QL-P11$ through $QL-P25$.
>> ...
>> If we apply the construction of QL' once more,
>> ... we get a QL'' , homothetic to the reference QL ,
>> ... center $QL-P8$, ratio 4:1.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3481
Date: 18/4/2019 9:25:29
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

If we consider for the $QL(e_1, e_2, e_3, d_i)$
the circles QL-Ci3,
... these circles have a common point in QL-P9.
If we consider for the $QL(e_1, e_2, e_3, d_i)$
the circles QL-Ci4,
... these circles have collinear centers
... on a perpendicular to QL-L7 in QL-P11.
If we consider for the $QL(e_1, e_2, e_3, d_i)$ the circles QL-Ci5,
... the radical axes intersect in QL-P10.
If we consider for the $QL(e_1, e_2, e_3, d_i)$ the circles QL-Ci6,
... the radical axes intersect in QL-P8.
If we consider for the $QL(e_1, e_2, e_3, d_i)$
the circles QL-Ci2,
... we get a common point on QL-Ci1.

What about this point?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3482
Date: 18/4/2019 12:04:18
From: eckart_schmidt@t-online.de
Subject: Steiner lines are concurrent in QL

Dear Tran Quang Hung,

in addition to #3481:

If we consider for the $QL(e_1, e_2, e_3, d_i)$ QL-P1,
... we get points on QL-Ci1 of the reference QL.

If we consider for the $QL(e_1, e_2, e_3, d_i)$ QL-P3,
... we get collinear points
... on a perpendicular to QL-L7 through QL-P11.

If we consider for the $QL(e_1, e_2, e_3, d_i)$ QL-P23,
... we get collinear points.

What about the corresponding line?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3483
Date: 19/4/2019 2:02:02
From: Tran Quang Hung
Subject: Steiner lines are concurrent in QL

Thank You Eckart for your interest in the configuration
QL(e1,e2,e3,di).

Your facts is very interesting. I shall probably need a lot of
time to study the EQF to find more interesting facts like You.

Best regards
Tran Quang Hung.

Vào Th 5, 18 thg 4, 2019 vào lúc 17:04 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahooogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> in addition to #3481:
>> If we consider for the QL(e1,e2,e3,di) QL-P1,
>> ... we get points on QL-Ci1 of the reference QL.
>> If we consider for the QL(e1,e2,e3,di) QL-P3,
>> ... we get collinear points
>> ... on a perpendicular to QL-L7 through QL-P11.
>> If we consider for the QL(e1,e2,e3,di) QL-P23,
... we get collinear points.
>> What about the corresponding line?
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3484
Date: 19/4/2019 10:34:10
From: eckart_schmidt@t-online.de
Subject: QL-Geometry with QL-Quartet

Dear all,

in #3470 Tran Quang Hung used $QL(e_1, e_2, e_3, d_i)$
... with e_1, e_2, e_3 QL-Tr1 sidelines and d_i a QL-line,
... which lead in #3481 and #3482
... to new results and new elements in QL-geometry.

This concept can be generalized,
... replacing QL-Tr1 by QL-Tr2:

Let s_1, s_2, s_3 be QL-Tr2 sidelines
... and consider the four $QL(s_1, s_2, s_3, d_i)$:
... Their QL-L1,4 are parallel to QL-L1,4 of the reference QL.
... Their QL-L2,3 are the same as for the reference QL.
... Their QL-L5 intersect in QL-P1 of the reference QL.
... Their QL-L6 intersect in a *new QL-point*.
... Their QL-Ci1 are centered on QL-L2 of the reference QL.
... Their QL-Ci2 intersect in QL-P1 of the reference QL.
... Their QL-Ci3 intersect in QL-P1 and QL-P6 of the reference QL.
... Their QL-Ci4 have collinear centers on a *new QL-line*.
... Their QL-Ci5 intersect in QL-P1 and a *new QL-point*.
... Their QL-Ci6 intersect in QL-P1 and a *new QL-point*

on QL-P1.QL-P12.
... Their QL-P1 are the same as for the reference QL.
... Their QL-P2,7,19 are collinear on QL-L2 of the reference QL.
... Their QL-P3,4,5,6,12,20,21,22,30 are collinear on *new
QL-lines*.
... Their QL-Co1 are the same as for the reference QL.

What about these new QL-elements?
There will be other generalizations,
... replacing QL-Tr1,2 by QG-Px- or QA-Px-triangles.

Example for QA-P4-triangle with sidelines t_1, t_2, t_3, t_4
... and QL-quartet for $QL(t_1, t_2, t_3, t_4, d_i)$:
... Their QL-L2 intersect in a *new QL-point*.
... Their QL-Ci3 intersect in a *new QL-point*.
... Their QL-Ci4 have collinear center on a *new QL-line*.
... Their QL-Ci5 have common radical center in a *new QL-point*.
... Their QL-Ci6 have common radical center in a *new QL-point*.
... Their QL-P1 lie on the circumcircle of the QA-P4-triangle.
... Their QL-P3 lie collinear on a *new QL-line*.
... ..

Assumption:

The QL-quartet wrt any QG-Px- or QA-Px-triangle
... has its QL-P1 on the circumcircle of the triangle
... (further controlled for QG-P19-, QA-P2-triangle
... and the S-triangle in #3176).
There will be further theorems.

Best regards Eckart

PS: The examples are not complet
... and I hope without mistakes.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3485

Date: 19/4/2019 4:26:56

From: bernard.keizer

Subject: Quintupel-Points for a 5P

Dear Eckart,

Sorry, my preceding message went lost ...

Another intriguing and interesting constellation !

I don't find the 5 triple points boring, on a contrary, I think it's one of the most interesting items we have encountered (with the stelloïd cubic QL-Cu2, the epi- and hypocycloïds tangent to the 4L, the 3 foci of a 4P ...)

Simply, we havent found yet a simple construction of the QL associated to 5 points in general !

The same way, the construction of a triangle knowing 3 points incenter, orthocenter and circumcenter is not so easy ! We know already several constructions of a QL starting with 5 points (5 QL vertices, 2QL-2P3 and DT vertices, 4 circumcenters of the reference triangles and QL-P1 ...)

By the way, is the dual property also correct (5 lines leading to a unique QA) ?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3486
Date: 20/4/2019 1:33:42
From: Tran Quang Hung
Subject: QL-Geometry with QL-Quartet

Dear Eckart

I am happy when my idea is useful to create your new results and new elements in QL-geometry. Thank You so much for these nice results.

Best regards
Tran Quang Hung.

Vào Th 6, 19 thg 4, 2019 vào lúc 15:40 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

```
>> Dear all,
>> in #3470 Tran Quang Hung used QL(e1,e2,e3,di)
>> ... with e1,e2,e3 QL-Tr1 sidelines and di a QL-line,
>> ... which lead in #3481 and #3482
>> ... to new results and new elements in QL-geometry.
>> This concept can be generalized,
>> ... replacing QL-Tr1 by QL-Tr2:
>> Let s1,s2,s3 be QL-Tr2 sidelines
>> ... and consider the four QL(s1,s2,s3,di):
>> ... Their QL-L1,4 are parallel to QL-L1,4 of the reference QL.
>> ... Their QL-L2,3 are the same as for the reference QL.
>> ... Their QL-L5 intersect in QL-P1 of the reference QL.
>> ... Their QL-L6 intersect in a *new QL-point*.
>> ... Their QL-Ci1 are centered on QL-L2 of the reference QL.
>> ... Their QL-Ci2 intersect in QL-P1 of the reference QL.
>> ... Their QL-Ci3 intersect in QL-P1 and QL-P6 of the
reference QL.
>> ... Their QL-Ci4 have collinear centers on a *new QL-line*.
>> ... Their QL-Ci5 intersect in QL-P1 and a *new QL-point*.
>> ... Their QL-Ci6 intersect in QL-P1 and a *new QL-point*
>> on QL-P1.QL-P12.
>> ... Their QL-P1 are the same as for the reference QL.
>> ... Their QL-P2,7,19 are collinear on QL-L2 of the reference
QL.
>> ... Their QL-P3,4,5,6,12,20,21,22,30 are collinear on *new
QL-lines*.
>> ... Their QL-Co1 are the same as for the reference QL.
>> What about these new QL-elements?
>> There will be other generalizations,
>> ... replacing QL-Tr1,2 by QG-Px- or QA-Px-triangles.
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>> Example for QA-P4-triangle with sidelines t1,t2,t3,t4
>> ... and QL-quartet for QL(t1,t2,t3,t4,di) :
>> ... Their QL-L2 intersect in a *new QL-point*.
>> ... Their QL-Ci3 intersect in a *new QL-point*.
>> ... Their QL-Ci4 have collinear center on a *new QL-line*.
>> ... Their QL-Ci5 have common radical center in a *new
QL-point*.
>> ... Their QL-Ci6 have common radical center in a *new
QL-point*.
>> ... Their QL-P1 lie on the circumcircle of the QA-P4-triangle.
>> ... Their QL-P3 lie collinear on a *new QL-line*.
>> ... ...
>> Assumption:
>> The QL-quartet wrt any QG-Px- or QA-Px-triangle
>> ... has its QL-P1 on the circumcircle of the triangle
>> ... (further controlled for QG-P19-, QA-P2-triangle
>> ... ... and the S-triangle in #3176).
>> There will be further theorems.
>> Best regards Eckart
>> PS: The examples are not complet
>> ... and I hope without mistakes.

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[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3487
Date: 20/4/2019 8:55:50
From: chris.vantienhoven
Subject: epicycloidal curve around the sun

Dear friends,

I came across this article "Asteroid's Chance of Earth Impact in 2027 Now 96%".

See:

<https://cneos.jpl.nasa.gov/pd/cs/pdc17/Chodas-Day2-Briefing.pdf?fbclid=IwAR3IPfLw8xpor0cvodUp-FvtXF8gIu95FrNp8m-zbRhXgXDYnjt67dC-TGY>

The interesting part I found to be the curve of the asteroid around the sun.

It is an epicycloidal curve. A curve we discussed before in this group.

Best regards,
 Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3488
Date: 21/4/2019 4:34:01
From: staninchelmsford
Subject: epicyloidal curve around the sun

Do note that each slide is labeled "EXERCISE ONLY" and other web searches yield articles describing this as a "hypothetical asteroid".

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3489
Date: 24/4/2019 10:02:25
From: eckart_schmidt@t-online.de
Subject: number 3453, number 3455, number 3459, number 3460

Dear Chris,

Tran Quang Hung describes in #3453 and #3455 two 5L-elements,
... which I don't find in EPG, see #3459 and #3460.
I think, they are relevant, why not?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3490

Date: 24/4/2019 6:48:59

From: bernard.keizer

Subject: number 3453, number 3455, number 3459, number 3460

Dear Eckart,

The points you mention are not listed in the menu of 5L in EPG, but they are mentionned on the line 5L-s-L1 (5L Newton Line) : see 5L-o-P2, 5L-n-P1 and 5L-n-P3.

Best regards

Bernard

PS If you consider the tangential 5L of the 5 triple points and these points of the 5L, is there a connexion with the CSC of the 5 points ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3491
Date: 24/4/2019 7:56:44
From: chris.vantienhoven
Subject: Miquel circles are concurrent with 5L

Dear Eckart, Bernard and Tran,

Bernard is right.

Regarding Tran's message #3453:

Let d_1, d_2, d_3, d_4, d_5 be five lines. M_1 =Miquel point of (d_2, d_3, d_4, d_5) .

Define similarly, M_2, M_3, M_4, M_5 . Then M_1, M_2, M_3, M_4, M_5 lie on a circle.

This point is $nL-n-P_3$ in a 5-Line. Because it is the version of $nL-n-P_3$ in a 5-Line it is called 5L-n-P3.

There are also versions of this point in a 6-Line, 7-Line, etc..

Regarding Tran's message #3455:

Let d_1, d_2, d_3, d_4, d_5 be five lines. Let circle (M_1) = Miquel circle (QL-Ci3) of (d_2, d_3, d_4, d_5) .

Define similarly circles $(M_2), (M_3), (M_4), (M_5)$. Then five circles $(M_1), (M_2), (M_3), (M_4), (M_5)$ are concurrent at a point.

This point is $nL-n-P_1$ in a 5-Line. Because it is the version of $nL-n-P_3$ in a 5-Line it is called 5L-n-P1. There are also versions of this point in a 6-Line, 7-Line, etc..

Regarding Tran's message #3454:

Note:

* $nL-n-P_i$ is a point valid for 3L, 4L, 5L, etc. Applied in that situations it is named resp. $3L-n-P_i, 4L-n-P_i, 5L-n-P_i$ (i =some number 1,2,3, ...). Infix -n- stand for "natural" number n .

* $5L-s-P_i$ is a point that only occurs in a 5-Line, not in a 6-Line, etc. Infix -s- stand for "special" number.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3492

Date: 25/4/2019 9:07:34

From: eckart_schmidt@t-online.de

Subject: number 3453, number 3455, number 3459, number 3460

Dear Bernard, dear Chris,

thanks for your informations

... and excuse my insecurity in EPG-nomination.

The circle in Tran's message 3453 is 5L-o-Ci1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3493
Date: 28/4/2019 9:37:33
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-Cu7-triple points

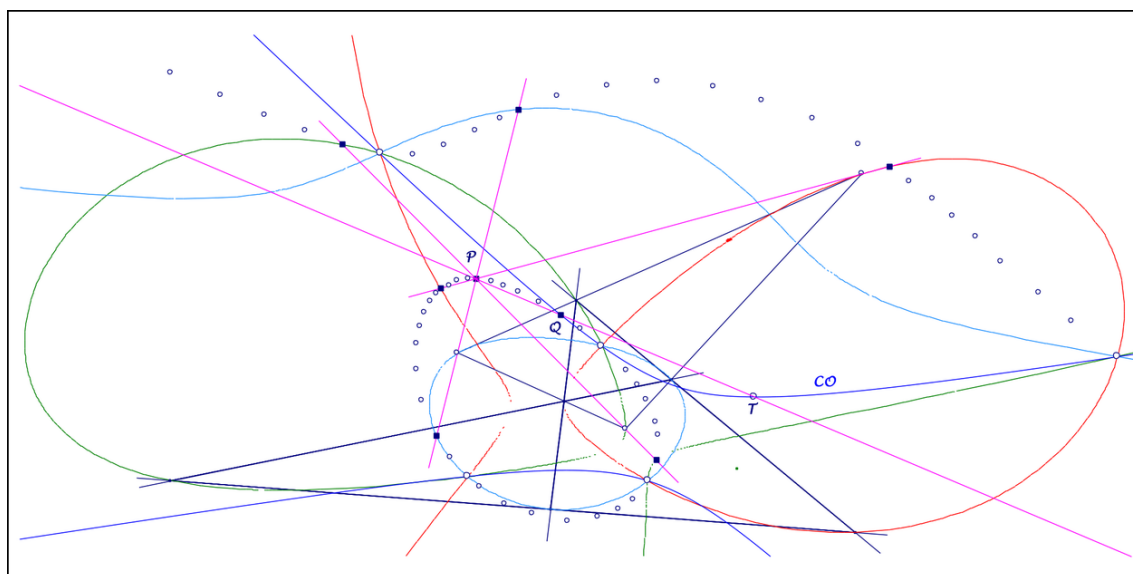
Dear Bernard,

for a QL we can construct
... the 3 QA-Cu7 of its QG-versions,
... the conic CU of their 5 triple points,
... and its 5P-s-P4 as point T on CU.

Here a new aspect (see attached file):
The 5 QA-Cu7-triple points of a QL
... are the common points of all QL-cubics,
... defined by an arbitrary point P
... and the following 8 points:
... infinity point of PT and its 2nd intersection Q with CO
... and 3 pairs of intersections of P.QG-P1 and QA-Cu7
(unequal QG-P1).

Inverted:
Starting with the 5 triple points and arbitrary points P,
... we get these cubics as locus of the cb-partner on lines
through P
... with cb(X) Cayley-Bacharach point of X
... and the 5 triple points and the two circular points.

Best regards Eckart



2019-04-28.pdf

Message: #3494
Date: 28/4/2019 4:36:16
From: eckart_schmidt@t-online.de
Subject: New aspect of QA-Cu7-triple points

Dear Bernard,
excuse, there is a typo in my last message:
Replace CU by C0 in the first passage.
A further remark wrt #3493:
... For the 5P-circumcubics QA-Cu7 the point P is QG-P1.
... For the degenerated line/circle cubics of a 5P
... the point P is the intersection, not on C0,
... of a parallel to the line through T and the circle.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3495
Date: 29/4/2019 4:31:08
From: bernard.keizer
Subject: New aspect of QA-Cu7-triple points

Dear Eckart,
It's interesting, but it's not a new aspect !
Using the construction I gave in my message 3385, you are able
to draw any circular cubic through the 5 points knowing the
pivot P and 4 different cubics.
You may choose 4 degenerated cubics circle + line or 3 cubics
circle + line and cubic conic + infinity line or 4 normal cubics
or 3 normal cubics like the QA-Cu7 and the cubic conic +
infinity line (that's your choice).
You use always the same property that 2 of the cubics intersect
in 2 points aligned with the 2 pivots !
In fact, I think I prefer your other definition of the cubic as
locus of the 2 CB partners on all lines through the choosen
pivot, as you don't need to know any particular non degenerated
cubic in advance.
Again, our problem is precisely how to find the 3 QA-Cu-7 among
all the circular cubics through the 5 points !
You have to use the CSC of the 5 points as the CSC of the pivot
is generally not on the cubic, but unfortunately, there are more
than 3 cubics having this property ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3496

Date: 01/5/2019 12:12:36

From: staninchelmsford

Subject: Four related "Euler Line" analogs in a quadrigon

Let the vertices of a quadrigon be A_1 , A_2 , A_3 , and A_4 .
Let T_i be the component triangle of the quadrigon that does not include vertex A_i .
Let Q_0 be the quadrigon formed by the circumcenters of T_1 , T_2 , T_3 , and T_4 , in order.
Let Q_M be the quadrigon formed by the centroids of T_1 , T_2 , T_3 , and T_4 , in order.
Let Q_N be the quadrigon formed by the nine-point centers of T_1 , T_2 , T_3 , and T_4 , in order.
Let Q_H be the quadrigon formed by the orthocenters of T_1 , T_2 , T_3 , and T_4 , in order.
Let "special-center" be exactly one of "area centroid, diagonal crosspoint, Poncelet point, vertex centroid".
Let O be the special-center of Q_0 .
Let M be the special-center of Q_M .
Let N be the special-center of Q_N .
Let H be the special-center of Q_H .
Then O , M , N , H lie on a straight line (in that order) and $OM:MN:NH=2:1:3$.

Definitions:

area-centroid = QG-P4, also known as the 1st QG-Quasi Centroid

diagonal crosspoint = QG-P1, the intersection of the diagonals

vertex centroid = QA-P1, also known as the quadrangle centroid

Poncelet point = QA-P2, also known as the Euler point or the Euler-Poncelet point

I found these four lines empirically, using Geometer's Sketchpad.

They are analogs of the Euler Line of a Triangle.

When the chosen special-center is the diagonal crosspoint, the line is already known.

It is cataloged in EQF as QG-L4, the 1st QG-Quasi Euler line and was discovered by Myakishev in

<http://forumgeom.fau.edu/FG2006volume6/FG200634.p>

When the chosen special-center is the Poncelet point, the line is already known.

It is mentioned by Eckart Schmidt in

<http://eckartschmidt.de/EULER.pdf>.

My German is a bit rusty, so someone correct me if I am wrong:

Eckart's "Feuerbachpunkt" seems to be QA-P2, the Euler-Poncelet point.

Question 1: Did Eckart discover this line in this paper or is he just reporting previously known work?

Is there an earlier reference?

[Since Eckart is a member of this forum, I hope he can give the definitive answer to this question.]

Question 2: EQF says line QG-L4 is mentioned in Eckart's paper (Ref-15i). I think this is a mistake/typo.

QG-L4 uses the diagonal points of the four quadrilaterals, whereas Eckart is using the Euler-Poncelet Point.

The two lines when the "special-center" chosen is the area centroid or the vertex centroid do not seem to be previously known.

Question 3: Am I overlooking these lines in EQF?

Question 4: Are these lines new? Does anyone have a reference to them in the literature?

I hope I have expressed myself clearly.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3497

Date: 01/5/2019 1:28:17

From: staninchelmsford

Subject: Four related "Euler Line" analogs in a quadrilateral

Also, are these four lines special cases of some more general theorem?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3498
Date: 01/5/2019 3:24:47
From: eckart_schmidt@t-online.de
Subject: Four related "Euler Line" analogs in a quadrigon

Dear Staninchelmsford,

I am glad, that someone is interested in my paper
... "Euler-Gerade eines Vierecks",
... for a design was sent 1999 to the Aulis Verlag,
... but came back 5 years later unpublished.
Sorry, I cannot give further references.

Perhaps of interest:

If we consider for a reference quadrangle and its triangles
... the 4 quadrangles of X_2, X_3, X_4, X_5 (your Q_M, Q_O, Q_H, Q_N),
... their four $QA-P_x$ often are collinear:
... for $QA-P_1$ on the line $QA-P_1.32.33$,
... for $QA-P_2$ on the line $QA-L_1$ (my Euler-line),
... for $QA-P_5$ on the line $QA-L_5$,
... for $QA-P_{10}$ on the parallel to $QA-P_{10}.24$ through $QA-P_{25}$,
... further for $QA-P_{3,16,17,18,19,20,21,22,25,26,27,29,31,35}$
... with no relevant properties at the moment.

If we start with a reference quadrigon,
... the $QG-P_x$ -quadrupel will be collinear
... for $QG-P_1$ on the line $QG-L_4$,
... $QG-P_{2,3,4,12,13,14,15}$.

What is the background for so many collinear quadrupel?

Best regards Eckart

PS: $QG-L_4$ in Ref-15i will be a typo.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3499
Date: 05/5/2019 6:39:18
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

First, recall that the Nagel line of a triangle passes through the following 4 points:

I = incenter (X1)
M = centroid (X2)
S = Spieker center (X10)
N = Nagel point (X8)

and the points form the following ratio:

$$IM : MS : SN = 2 : 1 : 3$$

Now consider a quadrangle with vertices A_1, A_2, A_3 , and A_4 .
Let T_i be the component triangle of the quadrangle that does not include vertex A_i .

Let Q_I be the quadrangle formed by the incenters of T_1, T_2, T_3 , and T_4 .

Let Q_M be the quadrangle formed by the centroids of T_1, T_2, T_3 , and T_4 .

Let Q_S be the quadrangle formed by the Spieker centers of T_1, T_2, T_3 , and T_4 .

Let Q_N be the quadrangle formed by the Nagel points of T_1, T_2, T_3 , and T_4 .

Let I be the vertex centroid of Q_I .

Let M be the vertex centroid of Q_M .

Let S be the vertex centroid of Q_S .

Let N be the vertex centroid of Q_N .

(By vertex centroid, I mean the quadrangle centroid, QA-P1.)

Then I, M, S, N lie on a straight line (in that order)

$$\text{and } IM : MS : SN = 2 : 1 : 3.$$

I discovered this empirically with Geometer's Sketchpad.

Question 1: Is there some simple or obvious reason why this should be so?

Question 2: Is this line already known and/or cataloged in EQF?

Question 3: No corresponding result occurs if "vertex centroid QA-P1" is replaced by QA-P2, QA-P3, QG-P1, QG-P4, or the perimeter centroid. These are the only points I tried.

Does some corresponding line occur using some other quadrangle point instead of QA-P1?

[Yes, I know that QG-P1 and QG-P4 are not quadrangle points. For these, I ordered the quadrigon as A_1, A_2, A_3, A_4 .]

Message: #3500
Date: 06/5/2019 12:54:08
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

Dear all,

Question 1: Is there some simple or obvious reason why this should be so?

Yes, there is. Here is the reason:

For each $i = 1:k$, let U_i, V_i, W_i be three collinear points on the plane such that $U_i V_i : V_i W_i = t = \text{const.}$

Let U, V, W be the k

P - n - P_1 (centroid) of k points $(U_i), (V_i), (W_i)$ respectively then U, V, W are collinear and $UV : VW = t$.

see also

<https://www.chrisvantienhoven.nl/np-items/np-luc/np-n-luc1>

Inspired by the nP - n - l_1 :

nL -MVP Eulerline,

<https://www.chrisvantienhoven.nl/np-items/np-obj/np-lns/np-n-l1>

the line described below for QA can be generalized to nP :

Consider a nP (n points)

Let I be the nP - n - $Luc_1(X_1)$ of this nP .

Let M be the nP - n - $Luc_1(X_2)$ of this nP .

Let S be the nP - n - $Luc_1(X_{10})$ of this nP .

Let N be the nP - n - $Luc_1(X_8)$ of this nP .

Then I, M, S, N lie on a straight line (in that order) and $IM :$

$MS : SN = 2 : 1 : 3$.

May we call it nL -MVP Nagelline of nP ?

Best regards,

Vu Thanh Tung

Le dimanche 5 mai 2019 à 23:39:23 UTC+7,

Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :

First, recall that the Nagel line of a triangle passes through the following 4 points:

I = incenter (X_1)

M = centroid (X_2)

S = Spieker center (X_{10})

N = Nagel point (X_8)

and the points form the following ratio:

$IM:MS:SN=2:1:3$

Now consider a quadrangle with vertices A_1, A_2, A_3 , and A_4 .

Let Q_I be the quadrangle formed by the incenters of T_1, T_2, T_3 , and T_4 .

Let Q_M be the quadrangle formed by the centroids of T_1 , T_2 , T_3 , and T_4 .
 Let Q_S be the quadrangle formed by the Spieker centers of T_1 , T_2 , T_3 , and T_4 .
 Let Q_N be the quadrangle formed by the Nagel points of T_1 , T_2 , T_3 , and T_4 .
 Let I be the vertex centroid of Q_I .
 Let M be the vertex centroid of Q_M .
 Let S be the vertex centroid of Q_S .
 Let N be the vertex centroid of Q_N .
 (By vertex centroid, I mean the quadrangle centroid, QA-P1.)
 Then I, M, S, N lie on a straight line (in that order) and $IM:MS:SN=2:1:3$.
 I discovered this empirically with Geometer's Sketchpad.
 Question 1: Is there some simple or obvious reason why this should be so?
 Question 2: Is this line already known and/or cataloged in EQF?
 Question 3: No corresponding result occurs if "vertex centroid QA-P1" is replaced by QA-P2, QA-P3, QG-P1, QG-P4, or the perimeter centroid. These are the only points I tried.
 Does some corresponding line occur using some other quadrangle point instead of QA-P1?
 [Yes, I know that QG-P1 and QG-P4 are not quadrangle points. For these, I ordered the quadrigon as A_1, A_2, A_3, A_4 .]

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3501
Date: 06/5/2019 3:32:21
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,

Great work!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3502
Date: 06/5/2019 4:43:08
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

Dear all,

It applies not only for nP but also for nL.

Best regards,
Vu Thanh Tung

Le lundi 6 mai 2019 à 08:32:24 UTC+7,
Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :
Dear Thanh Tung,
Great work!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3503
Date: 06/5/2019 11:18:30
From: eckart_schmidt@t-online.de
Subject: Nagel Line Analog in a Quadrangle

Dear S taninchelmsford,

wrt question 2 in #3499:

I have found no corresponding result
... if QA-P1 is replaced by QA-P2 til QA-P41.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3504
Date: 06/5/2019 8:39:14
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

You wrote:

> May we call it
 nL-MVP Nagel line of nP?

I do not think "Nagel line" would be an appropriate name since as you pointed out, the result is true for any set of points/lines with ratio 2:1:3 and is not specific to the Nagel line of a triangle.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3505
Date: 06/5/2019 9:21:16
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

Dear all,

Thanh Tung's response in message 3500 showed that the result has nothing to do with the 4 component triangles of a quadrangle.

To rephrase his result:

Let $\{A_i, B_i, C_i\}$ be any number of collinear triples in the plane with the ratio $A_i B_i : B_i C_i$ being a constant, t . Let A be the centroid of the $\{A_i\}$ and the same for B and C .

Then A, B, C colline and $AB:BC=t$.

I was hoping that a stronger result might hold, namely that instead of $A_i B_i C_i$ being collinear, the result would be that if the $A_i B_i C_i$ were all directly similar triangles, then ABC would also be directly similar to them. I tested this hypothesis out and unfortunately it is false.

I also checked that the common ratio is a necessary condition. I was hoping that merely having collinearity of the given triples would make the centroids colline. This too turns out to be false.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3506
Date: 07/5/2019 9:43:43
From: bernard.keizer
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,
Long time we hadn't seen on this forum such a simple geometric property with a synthetic proof !
Congratulations
There is a well known example for the QL
Let's consider the 4 centroids, circumcenters and orthocenters of the 4 reference triangles.
The 3 centroids of the 3 quadrupels are QL-P8, QL-P6 and QL-P2, which are the centroid, circumcenter and orthocenter of the triangle of the S-points.
Best regards
Bernard
PS I haven't checked if the centroid of the incenters was the incenter of this triangle ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3507
Date: 07/5/2019 6:10:51
From: Thanh Tung
Subject: Nagel Line Analog in a Quadrangle

Hi,

>> I was hoping that a stronger result might hold, namely that instead of $A_iB_iC_i$ being collinear, the result would be that if the $A_iB_iC_i$ were all directly similar triangles, then ABC would also be directly similar to them. I tested this hypothesis out and unfortunately it is false.

It is true, we can prove it by complex number.: we indentify points in the plane as complex number.

Suppose that

$A_iB_iC_i$ were all directly similar triangles and directly similar to a triangle XYZ .

There is an unique complex number t such that $Z = tX + (1-t)Y$ (if t is real then X, Y, Z are collinear and any A_i, B_i, C_i are collinear) and then

$$C_i = tA_i + (1-t)B_i$$

By summing up and divide by the number of triangle $A_iB_iC_i$ we obtain $C = tA + (1-t)B$ wich shows that ABC is directly similar to $A_iB_iC_i$.

Here remark that instead of centroid (equal weighted of points), we can use any weighted scheme.

Best regards,
Vu Thanh Tung

Le mardi 7 mai 2019 à 02:21:22 UTC+7,

Stan.Rabinowitz@comcast.net [Quadri-Figures-Group] a écrit :

>Dear all,

>Thanh Tung's response in message 3500 showed that the result has nothing to do

>with the 4 component triangles of a quadrangle.

>To rephrase his result:

>Let $\{A_i, B_i, C_i\}$ be any number of collinear triples in the plane with the ratio $A_iB_i : B_iC_i$ being a constant, t .

>Let A be the centroid of the $\{A_i\}$ and the same for B and C .

>Then A, B, C colline and $AB : BC = t$.

>I was hoping that a stronger result might hold, namely that instead of $A_iB_iC_i$ being collinear,

>the result would be that if the $A_iB_iC_i$ were all directly similar triangles,

>then ABC would also be directly similar to them.
>I tested this hypothesis out and unfortunately it is false.
>I also checked that the common ratio is a necessary condition.
>I was hoping that merely having collinearity of the given
triples would make the centroids colline.
>This too turns out to be false.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3508
Date: 07/5/2019 9:57:23
From: eckart_schmidt@t-online.de
Subject: Quasi-Euler Lines for a 5P

Dear all,

there are several triple QA-P_{x,y,z} in Euler constellation,
... consider for a 5P and its 5 QA the 3 pentangle of QA-P_{x,y,z}
... and take 5P-n-P1 of these pentangles,
... which are collinear with Euler ratios (see #3500) for:
(1) QA-P1, QA-P32, QA-P33,
(2) QA-P10, QA-P11, QA-P12,
(3) QA-P10, QA-P1, QA-P20,
(4) QA-P14, QA-P1, QA-P24,
(5) QA-P10, QA-P16, QA-P19,
(6) QA-P14, QA-P33, QA-P12.

For (1) the corresponding line is 5P-n-L1,
... the other lines seem to be not in EPG.

The lines for (1), (3), (4)
... have the common point 5P-n-P1,
... which is the 5P-MVP of the triangle point X(2).

The lines for (2), (3), (5)
... have a common point 5P-n-Px,
... which is the 5P-MVP of the triangle point X(?).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3509
Date: 08/5/2019 2:36:26
From: eckart_schmidt@t-online.de
Subject: Nagel Line Analog in a Quadrangle

Dear Bernard,

I think, there is a typo in your #3506:
... The centroid of the S-triangle is QL-P12 (not QL-P8).

Wrt your last remark:
The incenter of the S-triangle
... is not the centroid of the four incenters.
But for example X(20)
... on the Euler line of the S-triangle
... is the centroid of the four X(20).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3510
Date: 08/5/2019 9:35:00
From: staninchelmsford
Subject: Nagel Line Analog in a Quadrangle

Dear Thanh Tung,

Thank you for taking the time to check my calculations.
My test was faulty and I agree with you that the triangles are similar.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3511
Date: 10/5/2019 3:27:43
From: eckart_schmidt@t-online.de
Subject: Construction of CO-Tf3(X)

Dear Benedetto, dear Chris,

is the following construction of CO-Tf3(X) already mentioned?
Consider the four intersections of the conic
... and two orthogonal lines through X:
... QA-P4 of this QA is Co-Tf3(X).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3512
Date: 11/5/2019 7:14:27
From: Benedetto Scimemi
Subject: Construction of CO-Tf3(X)

Dear Eckart, your construction as such is not in the page of Co-Tf3. It could added, if Chris wants, next to the "quick" construction (there reported) for the inverse mapping.
In fact both of them are immediate consequences of the basic CoTf3 property: QA-P2 → QA-P4 for any inscribed quadrangle QA.
When in QA two sides are orthogonal with intersection X, then X=Co-P2, hence $\text{Co-Tf3}^{(-1)} : X \rightarrow \text{QA-P4}$.
When QA is cyclic with circumcenter X, then X=Co-P4, hence $\text{Co-Tf3}^{(-1)} : X \rightarrow \text{QA-P2}$.
A weak point for both constructions is that they do not apply to all points X.
Best regards. Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3513
Date: 12/5/2019 9:42:36
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

Dear Chris, dear Bernard,

in message 364 (2013) I offered a quartic for a QL
... without any feedback.

This quartic QL-Qux is the locus
... for intersections of lines and their CSC-circles,
... taking for lines the tangents at a conic,
... inscribed QL-Tr1, tangent to the Steiner axes and QL-L2.

Main properties:
... QL-Qux bears the six points of a QL,
... is CSC-invariant,
... bearing further QL-2P3 and 3 pairs of QA-Tr2-vertices
... .. unequal QL-P1 for the QG-versions of QL.

There is an interesting new aspect of this quartic:
QL-Qux is invariant wrt the three QA-Tf16
... for the QG-versions of QL,
... which give for QL-Qux-points the same image.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3514
Date: 12/5/2019 7:35:03
From: bernard.keizer
Subject: Nagel Line Analog in a Quadrangle

Dear Eckart,
Indeed !
Thanks for your attention ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3515
Date: 12/5/2019 7:48:24
From: bernard.keizer
Subject: A Quartic for Quadrilaterals

Dear Eckart,
I regret that you mention the absence of feedback !
If my memory is not in default, we discussed this quartic a few time after your 1rst message.
The quartic is not only CSCinvariant, but also CSCdiaginvariant; CSCdiag is the CSC of the DQL, the diagonal QL with the DT sides and the Newton Line as 4 lines (your conic is inscribed in the DQL).
The quartic contains the Plücker points, which are CSCdiagpartners.
I liked this quartic so much that I've described it in the 2nd article on my blog under the name of quartic of generalised Plücker points (with reference to you of course)
<http://bernardkeizer.blogspot.com> complete quadrilateral and cubic stelloïd september 2014 on page 40
Best regards
Bernard
PS This quartic deserves a description in EQF, but that is another story ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3516
Date: 13/5/2019 8:30:36
From: eckart_schmidt@t-online.de
Subject: A Quartic for Quadrilaterals

Dear Bernard,

I very much apologize for my bad general view.
The cases accumulate, that I lost survey,
... thanks for controlling teamwork.
Special thanks for the Plücker points QL-2P1 on the quartic.
But the last property in my message will be new.
You are right, this quartic should be mentioned in EQF.

Finally a remark wrt QA-Tf16 wrt pentangles:
Well known:
QA-Tf16 of a 5P-vertex P_i wrt the remaining vertices
... is CSC(P_i) wrt a QL with the 5P as QA-Cu7-triple points.

Perhaps new:
If we map a 5P $P_1 \dots P_5$ to $Q_1 \dots Q_5$
and $Q_1 \dots Q_5$ to $R_1 \dots R_5$
... with $Q_i = \text{QA-Tf16}(P_i)$ wrt the QA = $P_j P_k P_l P_m$
... and $R_i = \text{QA-Tf16}(Q_i)$ wrt the QA = $Q_j Q_k Q_l Q_m$,
... we get $R_1 \dots R_5 = P_1 \dots P_5$.
This is not the property in EQF: $\text{QA-P16}(\text{QA-P16}(P)) = P$,
... for the reference QAs are different.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3517
Date: 13/5/2019 10:46:55
From: bernard.keizer
Subject: 5P-s-Co1

Dear Eckart,

5P-s-Co1 (curiously not in EQF) is the common unique diagonal polar conic for 5 pivotal isocubics having 4 of the 5 points as fixed points of the conjugation and the 5th point as pivot. This conic contains the vertices of the anticevian triangles of each of the 5 points wrt the DT of the 4 others (15 points). This conic contains also the vertices of the anticevian triangle of any of it's points wrt the DT's of any of 4 points on the conic ! (for example the anticevian triangles of the point T wrt the 5 DT's of 4 of the 5 points (15 points more)). The 5 pivotal cubics are tangent to the conic in the pivot, intersect the conic in 5 points (one being double) and intersect apparently only in the 5 real points (as they are not circular, the other 4 intersections of 2 cubics are not real). CSC of this conic is a quartic (?) through the 5 CSC of the points and the center of the CSC.

Best regards
Bernard

PS Using the unknown points R and S1,S2 and S3, I couldn't find a use for the 5 triple points

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3518
Date: 13/5/2019 11:45:02
From: bernard.keizer
Subject: 5P-s-Co1

Dear Eckart,
I now try always to split my messages , in order to avoid them being lost ...
One use of the points T, R, QG-P1, QA-P4 and S1,S2 and S3 is obviously a desmic configuration as the triangles QG-P1 and S1,2,3 are in perspective with perspector T and the triangles QA-P4 and S1,2,3 are in perspective with perspector R. This gives 2 pivotal isocubics with pivots T and R. Each of the cubics cuts the conic in 2 other points than T or R and S1,2,3, but again, I couldn't find a use for the QL of the 5P.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3519
Date: 13/5/2019 12:03:17
From: bernard.keizer
Subject: 5P=4L ?

Dear Eckart,
Using my construction in message 3385 gives always pivotal circular isocubics through the 5 points ; they generally don't contain the CSC of the pivot.
Pivotal isoubics through the 5 points, a pivot P and the CSC of the pivot are generally not circular.
There are still an infinity of pivotal circular isocubics containing the CSC of the pivot.
What do the 3 QA-Cu7 have as particularity ? They are focal.
Maybe they are the 3 unique focal ...
But I'm not able to find them !
Could you please help me and explain simply how you manage to find the approximate QL ?
Did you find a converging method or something like that ?
I spent now so many hours in drawing curves that I'm a little bit disappointed (it's an understatement)
Many thanks in advance for your help !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3520

Date: 14/5/2019 7:29:07

From: Thanh Tung

Subject: A new family of lines on quadrangle associated with Kimberling trian

Dear all,

I would like to introduce a new family of lines on quadrangle geometry:

Let

- _ n: an integer
- _ ABCD: a Quadrangle
- _ A_n: the Kimberling center $X(n)$ of triangle BCD.
Define B_n, C_n, D_n similarly.
- _ trA: the antipedal triangle of A w.r.t triangle BCD.
Define trB, trC, trD similarly.
- _ A': the isogonal conjugate of A w.r.t triangle trA.
Define B', C', D' similarly.
- _ A*_n: the orthopole of the line A'A_n w.r.t triangle BCD.
Define B*_n, C*_n, D*_n similarly.

Then A*_n, B*_n, C*_n, D*_n are collinear on a line. When n varies this defines a family of lines on quadrangle associated with Kimberling triangle centers. This is a very surprised property. Yet I do not have proof for this.
Is this property known before? And do you have idea how to prove this?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3521

Date: 14/5/2019 8:09:29

From: tulip13091983

Subject: A new family of lines on quadrangle associated with Kimberling t

Dear all,
This is false, please ignore it,
Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3522
Date: 14/5/2019 9:15:03
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

thanks for your numerous stimulations,
... please excuse, if I don't react at once.
... I shall do it, if I see new consequences.

Wrt your question for an approximate QL for a 5P:
... no concept, pure trying up to now.

But perhaps helpful:
If we have one vertex of the QL,
... we can construct the others:

Let P1 be one QL-point,
... then 5P-CSC(P1) gives P3,
... opposite point for a QG-version of the QL.

Using the cb-transformation,
... which gives the CB-point wrt the 5P and the circular points,
... $P1.cb(P1) \wedge P3.cb(P3)$ is a vertex U of QL-Tr1.

The cb-partner on P1P3 give two QA-Cu7-double points X,Y,
... the 5P-CSC-circle of the circle with diameter XY
... gives in its intersections with P1P3
... the further vertices V, W of QL-Tr1.

The 5P-CSC-partner on UV and UW are the other QL-points.

Best regards Eckart

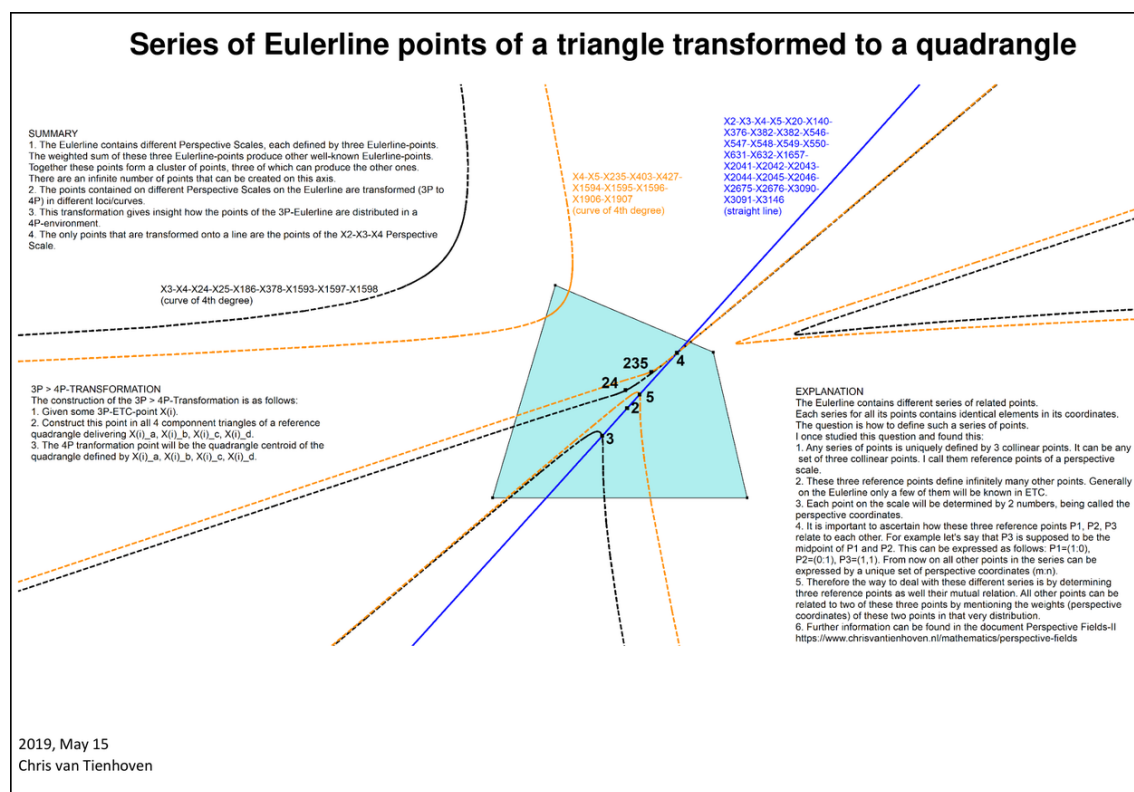
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3523
Date: 15/5/2019 8:46:29
From: chris.vantienhoven
Subject: Nagel Line Analog in a Quadrangle

Dear friends,

I found very nice results about the Nagel-line.
I wondered how points on a triangle line are dispersed in the Quadrangle plane using the centroid of the four $X(i)$ in a Quadrangle-transformation.
Because there are many points on the Eulerline I used the Eulerline as an example to see how different sets of points are dispersed in a Quadrangle.
See enclosed picture.

Best regards,
Chris



QA-X2-X3-X4-Eulerline-points-Centroids-Loci-01.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #3524
Date: 15/5/2019 10:56:15
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for your attention and these new developments ; I have to study them carefully !
Here just a simple discovery found by drawing all kinds of figures.
You showed a certain time ago that 3 points and a CSC allow to find 2 points such as the 5 points have this CSC.(then we have their conic and their point T.
I found that any 2 points on a conic and a CSC have 3 partners on this conic such as the 5 points have this CSC (their point T is apparently a different one). The construction is easy : use the construction with a 3rd variable point and search the position of this 3rd point such as the 2 other points are on the conic ...
Best regards
Bernard
PS I didn't try to apply the construction to the points T and R on the 5 points conic

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3525
Date: 17/5/2019 3:11:15
From: eckart_schmidt@t-online.de
Subject: Four related "Euler Line" analogs ...

Dear all,

this is an addition to #3498:

Consider points X on the Euler line of a triangle
... and their quadrupel for the triangles of a reference QA,
... which give new X -depending QAs.

In #3498 the collinearity of their QA-points is researched,
... here the envelopes of their QA-lines are tested,
... which often give a parabola,
... .. tangent to the corresponding line of the reference QA.

QA-L1 envelope a parabola,
... tangent in QA-P34 to QA-L1 of the reference QA,
... tangent in the midpoint of QA-P32.33 to QA-P1.32.33.

QA-L2 envelope a parabola,
... tangent in QA-P1 to QA-L1 of the reference QA,
... tangent in QA-P4 to QA-P3.4.

QA-L3 envelope a parabola,
... tangent to QA-L3 and QA-P1.32.33 of the reference QA.

QA-L4 envelope a parabola,
... tangent to QA-L4 in a point,
... .. dividing QA-P1.23 with ratio 1:2,
... tangent in QA-P32 to QA-P1.32.33.

QA-L7 envelope a parabola,
... not tangent to QA-L7 of the reference QA ...

The lines QA-P3.4 envelope a parabola,
... not tangent to QA-P3.4 of the reference QA,
... but tangent to QA-L2 in QA-P4
... and tangent to QA-P2.23 in QA-P23.

What is the background of this parabola property?

Best regards Eckart

Message: #3526
Date: 19/5/2019 11:05:21
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for this beautiful construction !
Very clever simultaneous use of CB and CSC transformations of 5 points.
Nothing new in fact in the properties, but new is the use you make of them ...
Only one remark : the points you name X and Y (CB partners on P1P3) as well as V and W (2 other DT vertices) must be harmonic conjugates wrt P1 and P3. For this reason, the 2 circles with diameters X and Y and V and W which are CSC partners are orthogonal to any circle through P1 and P3, in particular the circle through P1, P3 and QLP1 and therefore to it's CSC, id est the line P1P3, which must be a diameter for both circles.
As I cannot draw the CB partner of a point, I had difficulties to use directly your construction.
But I managed a proxy by drawing the 2 circular cubics through the 5 points with pivots in 2 CSC partners P1 and P3, which intersect in X and Y ...
But unfortunately, the property I mention is necessary, but not sufficient !
Anyhow, it's for me already a huge progress.
Many thanks again
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3527
Date: 21/5/2019 9:42:11
From: bernard.keizer
Subject: 5P=4L ?

Dear Eckart,
I checked the centroids of the 5 triple points and of the dual QA of the QL.
They don't coincide (the 4 points cannot be obtained by derivating the complex equation of the 5 points).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3528
Date: 23/5/2019 8:26:35
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

I am glad, that my construction in #3522 was helpful for you.
I made the same observations
... and tried to find a locus for P_1, P_3 ,
but in vain ...

Cleaning my desk before holiday,
... some remarks wrt CB-transformations
... for a $5P = P_1 \dots P_5$ and the searched QL,
... some will be evident or already mentioned:
... Let C_0 be the circumcubic of the $5P$
... and $T = 5P-s-P_4$:

(1) cb , defined by the $5P$ and the circular points:
... cb maps each QA-Cu7 to itself,
... ... point and image collinear $QG-P_1$.
... cb swaps QA-Cu7-double intersections,
... cb maps points X on C_0 to the infinity point of XT .
... cb maps lines $P_i P_j$ to circumcircles of $P_k P_l P_m$.
... $cb(QL-P_1)$ is a point on $QL-P_1, QL-P_8, QL-P_{24}$.

(2) CB , defined by the $5P$ and 2 opposite QL-points
... as $QG-2P_2$ of a QGa version of the QL:
... CB maps QA-Cu7a to itself, point and image collinear QA-P2.
... CB maps $QG-P_{1b.c}$ to the conic C_0 .

(3) CB , defined by the $5P$ and 2 QA-Cu7-double points X_a, Y_a :
... CB maps QA-Cu7b,c to itself,
... ... point and image collinear with the infinity point of the asymptote.
... CB maps $X_a.Y_a$ to the conic C_0 , point and image collinear T ,
... ... especially T as CB of intersection of $X_a.Y_a$ and the tangent in T at C_0 .

(4) CB , defined by the $5P$ and any pair X, Y :
... CB maps the line $X.Y$ to the conic C_0 ,
... ... point and image collinear to a fixed point on C_0 .
In this way the 4 QL-lines give a QA on C_0 ...

(5) CB , defined by the $5P$ and the foci of the circumconic:
... CB maps the the principal axis of C_0 to C_0 .

Finally an application of (1):

For any points X, Y on $P_i.P_j$
... $X.cb(X)^Y.cb(Y)$ gives a point Q_{ij} (Q in #3245).
... The 10 points Q_{ij} give 10 circles (Q_{ij}, Q_{jk}, Q_{ik}) through T ,
... centered in M_{ijk} and $M_{ijk}, M_{ijl}, M_{ijm}$ collinear.
... The 10 points Q_{ij} give 5 conics $(Q_{ij}, Q_{ik}, Q_{il}, Q_{im}, P_i)$
through T .

Best regards Eckart

PS: Next two weeks I make my drawings in the beach sand.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3529

Date: 28/5/2019 4:58:53

From: Thanh Tung

Subject: A new point on quadrangle ?

Dear all,

I describe here a possible new point on quadrangle:

First a transformatin on trianlge:

* a triangle ABC and a point P

* A_0 : second intersection of line PA and circle (PBC) and cyclically defined B_0 , C_0 .

* G_{ab} : centroid of triangle PAB_0 and cyclically defined G_{ac} , G_{bc} , G_{ba} , G_{ca} , G_{cb} .

* A' : intersection of $G_{ab}G_{cb}$ and $G_{bc}G_{ac}$ and cyclically defined B' , C' .

then the lines AA' , BB' , CC' are concurrent at a point $Q = \text{tr}(P, \triangle ABC)$.

Now using the above transformtion we obtain a point on quadrangle:

* a quadrangle $P_1 P_2 P_3 P_4$

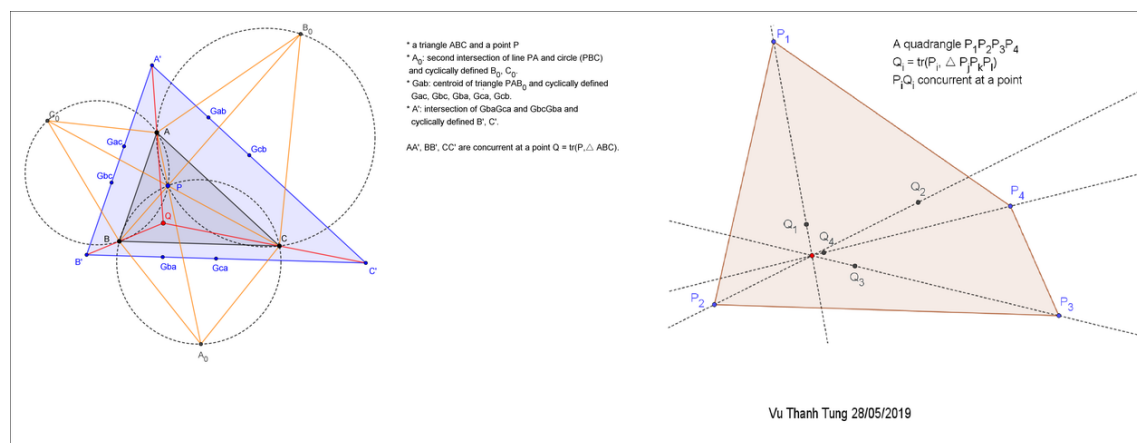
* $Q_i = \text{tr}(P_i, \triangle P_j P_k P_l)$

then the lines $P_i Q_i$ are concurrent at a point.

I am not sure if it describes a new point or it is just a new description of an existing point.

Best regards,

Vu Thanh Tung



QuadranglePoint.png

Message: #3530

Date: 29/5/2019 12:45:20

From: chris.vantienhoven

Subject: A new point on quadrangle ?

Dear Vu Thanh Tung,

I only calculated the coordinates algebraically.

Your triangle transformation of point $P(x:y:z)$ gives a new point with these barycentric coordinates:

$$\begin{aligned} & \{y z (-b^2 x^2 - a^2 x y - b^2 x y + c^2 x y - a^2 y^2) (c^2 x^2 + a^2 x z - b^2 x z + c^2 x z + a^2 z^2) \\ & (c^2 x^2 y^2 - 2 a^2 x^2 y z + b^2 x^2 y z + c^2 x^2 y z - 2 a^2 x y^2 z - a^2 y^3 z + b^2 x^2 z^2 - 2 a^2 x y z^2 - 2 a^2 y^2 z^2 - a^2 y z^3), \\ & x z (-b^2 x^2 - a^2 x y - b^2 x y + c^2 x y - a^2 y^2) (c^2 y^2 - a^2 y z + b^2 y z + c^2 y z + b^2 z^2) \\ & (c^2 x^2 y^2 - b^2 x^3 z - 2 b^2 x^2 y z + a^2 x y^2 z - 2 b^2 x y^2 z + c^2 x y^2 z - 2 b^2 x^2 z^2 - 2 b^2 x y z^2 + a^2 y^2 z^2 - b^2 x z^3), \\ & x y (c^2 x^2 + a^2 x z - b^2 x z + c^2 x z + a^2 z^2) (c^2 y^2 - a^2 y z + b^2 y z + c^2 y z + b^2 z^2) (c^2 x^3 y + 2 c^2 x^2 y^2 + c^2 x y^3 + 2 c^2 x^2 y z + 2 c^2 x y^2 z - b^2 x^2 z^2 - a^2 x y z^2 - b^2 x y z^2 + 2 c^2 x y z^2 - a^2 y^2 z^2)\} \end{aligned}$$

The Quadrangle point you constructed using your transformation has these coordinates:

$$\begin{aligned} & \{a^2 q^2 r^2 (-2 b^2 c^2 p^4 - a^2 c^2 p^3 q - 2 b^2 c^2 p^3 q + c^4 p^3 q - a^2 c^2 p^2 q^2 - a^2 b^2 p^3 r + b^4 p^3 r - 2 b^2 c^2 p^3 r - a^4 p^2 q r + b^4 p^2 q r - 2 b^2 c^2 p^2 q r + c^4 p^2 q r - a^4 p q^2 r + a^2 b^2 p q^2 r - a^2 c^2 p q^2 r - a^2 b^2 p^2 r^2 - a^4 p q r^2 - a^2 b^2 p q r^2 + a^2 c^2 p q r^2 - a^4 q^2 r^2), \\ & b^2 p^2 r^2 (-b^2 c^2 p^2 q^2 - 2 a^2 c^2 p q^3 - b^2 c^2 p q^3 + c^4 p q^3 - 2 a^2 c^2 q^4 + a^2 b^2 p^2 q r - b^4 p^2 q r - b^2 c^2 p^2 q r + a^4 p q^2 r - b^4 p q^2 r - 2 a^2 c^2 p q^2 r + c^4 p q^2 r + a^4 q^3 r - a^2 b^2 q^3 r - 2 a^2 c^2 q^3 r - b^4 p^2 r^2 - a^2 b^2 p q r^2 - b^4 p q r^2 + b^2 c^2 p q r^2 - a^2 b^2 q^2 r^2), \\ & \end{aligned}$$

$$-c^2 p^2 q^2 (c^4 p^2 q^2 - a^2 c^2 p^2 q r + b^2 c^2 p^2 q r + c^4 p^2 q r + a^2 c^2 p q^2 r - b^2 c^2 p q^2 r + c^4 p q^2 r + b^2 c^2 p^2 r^2 - a^4 p q r^2 + 2 a^2 b^2 p q r^2 - b^4 p q r^2 + c^4 p q r^2 + a^2 c^2 q^2 r^2 + 2 a^2 b^2 p r^3 - b^4 p r^3 + b^2 c^2 p r^3 - a^4 q r^3 + 2 a^2 b^2 q r^3 + a^2 c^2 q r^3 + 2 a^2 b^2 r^4)\}$$

It isn't a known QA-point yet.
It doesn't lie on familiar lines either.

The only line I could find to lie on is the line with these 5 points on it:

- QA-P4
- QA-Tf4(QA-P8)
- QA-Tf4(QA-P23)
- QA-Tf4(QA-P32)
- QA-Tf4(QA-P42)

Note:
QA-Tf4 is the QA-Möbius Conjugate.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3531
Date: 29/5/2019 6:34:40
From: Thanh Tung
Subject: A new point on quadrangle?

Hi,

Thank you for your interest in my point construction.

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3532
Date: 01/6/2019 10:32:03
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart, Dear Bernard,

I revisited the constellation of three versions of QA-Cu7 in a Quadrilateral.

I looked at the 6 points that lie on only 2 of the 3 versions of QA-Cu7 in a QL.

Here are some results I found. I don't know what is already mentioned before.

1. They lie per pair on the 3 lines of the QL-Diagonal Triangle.

2. Each line of the QL-DT is also the 3rd diagonal of one of the three Component Quadrigons.

3. The pair of endpoints of this 3rd diagonal is harmonically conjugated with the pair of doublepoints on this line.

4. The CT-coordinates of the 6 doublepoints are:

$\{-a W, -b c n, b c m\}$

$\{a W, -b c n, b c m\}$

$\{a c n, -b W, -a c l\}$

$\{a c n, b W, -a c l\}$

$\{a b m, -a b l, c W\}$

$\{a b m, -a b l, -c W\}$

where $W = \text{Sqrt}[a^2 (1-m)(1-n) - b^2 (m-1)(m-n) + c^2 (n-1)(n-m)]$

5. The 6 doublepoints are coconic (algebraically proven).

6. The equation of the conic they are on:

$+b^2 c^2 (a^2 c^2 l^2 n^2 - b^2 m^2 W^2) (a^2 b^2 l^2 m^2 - c^2 n^2 W^2) x^2$

$+a^2 c^2 (b^2 c^2 m^2 n^2 - a^2 l^2 W^2) (a^2 b^2 l^2 m^2 - c^2 n^2 W^2) y^2$

$+a^2 b^2 (b^2 c^2 m^2 n^2 - a^2 l^2 W^2) (a^2 c^2 l^2 n^2 - b^2 m^2 W^2) z^2$

$-a^2 b^2 c^2 l m (-2 a^2 b^2 c^2 l^2 m^2 n^2 + (a^4 l^4 + b^4 m^4 + c^4 n^4) W^2 - W^6) x y$

$-a^2 b^2 c^2 l n (-2 a^2 b^2 c^2 l^2 m^2 n^2 + (a^4 l^4 + b^4 m^4 + c^4 n^4) W^2 - W^6) x z$

$-a^2 b^2 c^2 m n (-2 a^2 b^2 c^2 l^2 m^2 n^2 + (a^4 l^4 + b^4 m^4 + c^4 n^4) W^2 - W^6) y z = 0$

7. The center of this conic is:

$\{2 a^4 b^4 c^4 l^3 m^3 n^3 (a^4 l^4 m - b^4 m^5 + a^4 l^4 n - 2 a^4 l^3 m n + b^4 m^4 n + c^4 m n^4 - c^4 n^5) - a^2 b^2 c^2 m n (a^8 l^9 m - b^8 l^9 m^9 + a^8 l^9 n - 3 a^8 l^8 m n + 6 a^4 b^4 l^5 m^4 n - 6 a^4 b^4 l^4 m^5 n + b^8 l^8 m^8 n + b^8 m^9 n + 6 a^4 c^4 l^5 m n^4 - 6 a^4 c^4 l^4 m n^5 - 2 b^4 c^4 m^5 n^5 + c^8 l^8 m n^8 - c^8 l^8 n^9 + c^8 m n^9) W^2$
 $+ 2 a^4 l^2 (a^4 b^4 l^5 m^3 - 2 a^4 b^4 l^4 m^4 + b^8 l^8 m^7 + a^4 c^4 l^5 n^3 + 2 b^4 c^4 l^8 m^4 n^3 - 2 a^4 c^4 l^4 n^4 + 2 b^4 c^4 l^8 m^3 n^4 - 6 b^4 c^4 m^4 n^4 + c^8 l^8 n^7) W^4$
 $+ 2 a^2 b^2 c^2 m n (-b^4 l^8 m^5 + 3 a^4 l^4 m n + b^4 m^5 n - c^4 l^8 n^5 + c^4 m n^5) W^6$
 $- 2 a^4 l^3 (b^4 m^3 + c^4 n^3) W^8$
 $+ a^2 b^2 c^2 m n (l^8 m + l^8 n - m n) W^{10},$

$2 a^4 b^4 c^4 l^3 m^3 n^3 (-a^4 l^5 + b^4 l^8 m^4 + a^4 l^4 n - 2 b^4 l^8 m^3 n + b^4 m^4 n + c^4 l^8 n^4 - c^4 n^5) - a^2 b^2 c^2 l^8 n (-a^8 l^9 m + b^8 l^9 m^9 + a^8 l^9 n + a^8 l^8 m n - 6 a^4 b^4 l^5 m^4 n + 6 a^4 b^4 l^4 m^5 n - 3 b^8 l^8 m^8 n + b^8 m^9 n + 6 b^4 c^4 l^8 m^5 n^4 - 2 a^4 c^4 l^5 n^5 - 6 b^4 c^4 l^8 m^4 n^5 + c^8 l^8 m n^8 + c^8 l^8 n^9 - c^8 m n^9) W^2$
 $- 2 b^4 m^2 (-a^8 l^7 m^2 + a^4 b^4 l^4 m^4 - a^4 b^4 l^3 m^5 - 2 a^4 c^4 l^4 m n^3 - b^4 c^4 m^5 n^3 + 6 a^4 c^4 l^4 n^4 - 2 a^4 c^4 l^3 m n^4 + 2 b^4 c^4 m^4 n^4 - c^8 m n^7) W^4$
 $+ 2 a^2 b^2 c^2 l^8 n (-a^4 l^5 m + a^4 l^5 n + 3 b^4 l^8 m^4 n + c^4 l^8 n^5 - c^4 m n^5) W^6$
 $- 2 b^4 m^3 (a^4 l^3 + c^4 n^3) W^8$
 $+ a^2 b^2 c^2 l^8 n (l^8 m - l^8 n + m n) W^{10},$

$- 2 a^4 b^4 c^4 l^3 m^3 n^3 (a^4 l^5 - a^4 l^4 m - b^4 l^8 m^4 + b^4 m^5 + 2 c^4 l^8 m n^3 - c^4 l^8 n^4 - c^4 m n^4) + a^2 b^2 c^2 l^8 m (-a^8 l^9 m^2 + a^4 b^4 l^5 m^5 - b^8 l^9 m^9 + a^8 l^9 n - a^8 l^8 m n - b^8 l^8 m^8 n + b^8 m^9 n + 6 a^4 c^4 l^5 m n^4 + 6 b^4 c^4 l^8 m^5 n^4 - 6 a^4 c^4 l^4 m n^5 - 6 b^4 c^4 l^8 m^4 n^5 + 3 c^8 l^8 m n^8 - c^8 l^8 n^9 - c^8 m n^9) W^2$
 $- 2 c^4 n^2 (6 a^4 b^4 l^4 m^4 - a^8 l^7 n - 2 a^4 b^4 l^4 m^3 n - 2 a^4 b^4 l^3 m^4 n - b^8 m^7 n + 2 a^4 c^4 l^4 n^4 + 2 b^4 c^4 m^4 n^4 - a^4 c^4 l^3 n^5 - b^4 c^4 m^3 n^5) W^4$
 $+ 2 a^2 b^2 c^2 l^8 m (a^4 l^5 m + b^4 l^8 m^5 - a^4 l^5 n - b^4 m^5 n + 3 c^4 l^8 m n^4) W^6$
 $- 2 c^4 (a^4 l^3 + b^4 m^3) n^3 W^8$
 $- a^2 b^2 c^2 l^8 m (l^8 m - l^8 n - m n) W^{10} \}$

I attach a picture of the results.

Best regards,

Chris

Message: #3533
Date: 02/6/2019 3:54:17
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris, dear Eckart,

It's a real pleasure to read news from Chris on the forum, specially about this wonderful item of the 3 QA-Cu7 !

I'm very impressed by the barycentric calculations ...

As Eckart is on the beach, I make a 1rst answer with other properties of these 6 double points ; of corse Eckart will correct or complete ...

1) Let's name U_i and V_i the 2 double points of the QA-Cu7 j and k . U_jV_j and U_kV_k intersect in QG-P1i, U_jV_k and U_kV_j intersect orthogonally in QG-P18i (CSC-QG-P17i) and U_jU_k and V_jV_k in QG-P19.

The CSC of the circle with diameter QG-P1j and k is the circle with diameter U_iV_i and both circles are therefore orthogonal to their diameter and to it's CSC, the circle through QL-P1 and the 2 QL vertices on this diagonal.

2) U_i and V_i are CB partners in the transformation wrt the 5 triple points and the 2 circular points (obvious, as the 2 cubics QA-Cu7j and k pass through the 9 points).

3) The 6 points are coconic, as Chris mentions : as the intersections QG-P18i,j and k are aligned (on the CSC of the Euler circle of DT), this figure forms the socalled mystic hexagam of Pascal.

4) Each QA-Cu7i bears the 4 points U_j, V_j, U_k and V_k ; QA-Cu7i is the QA-Cu1 of the QA $U_jV_jU_kV_k$, which has as DT QG-P1i, QG-P18i and QG-P19i and as Miquel triangle QA-P2,4 and 41 of the corresponding component quadrigon (with QG-P1j and k as one diagonal). As 2 sides are orthogonal, this QA-Cu1 is a QL-Cu1.

5) The 6 double points lie on a cubic with the 3 QG-P18 and the 3 QG-P19 (see Eckart's message 3187).

Best regards

Bernard

PS Would it be possible to calculate the same way the coordinates of the 5 triple points and of their conic ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3534
Date: 03/6/2019 10:48:04
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,

Also perhaps of interest :

- 1) the circle with diameter U_iV_i carries 2 QA-P4 (as CSC of 2 QG-P1) and 2 QG-P18 (as CSC of 2 QG-P17)
- 2) CB(QA-P4) is the intersection of QG-P1QA-P4 with the corresponding diagonal and the CSC of QA-P41.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3535
Date: 04/6/2019 10:59:26
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,

Please cancel in 2) CB(QA-P4) and read simply : the intersection of QG-P1QA-P4 with the corresponding diagonal is the CSC of QA-P41.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3536
Date: 04/6/2019 11:48:09
From: bernard.keizer
Subject: Dual 4/5 P/L

Dear Eckart,
I hope you will come back from holidays with plenty of new ideas and full of strength !
Here a simple idea I had put in my message 3885, but never achieved.
If there is only one QL corresponding to 5 points with the same CSC, the duality QA/QL allows to find a 5L as dual of the 5P by using the same duality.
All points and curves of the 4 figures QA, QL, 5P or 5L belong in fact to the same figure !
For example, the dual of any conic is a conic and the dual of any cubic is a curve of class 3.
The duals of the degenerated cubics formed by a circle and a line are degenerated curves of class 3 formed by the dual point of the line and the dual conic of the circle (which is not necessary a circle) ; the dual of the degenerated cubic formed by the conic of the 5 points and the infinity line is the curve of class 3 formed by the dual of the conic and the point QA-P16 or QL-P13.
There must be correspondances between different points.
For example, I found that QA-P1 lies on the tangent in T to the conic of the 5 points and on the dual of R (tangent to the dual conic) ...
Could you please confirm these observations (my figures are generally less precise than yours).
I would be glad to have your comments ...
This works perhaps also for cubics through 7P linked to the QA/QL : the duals of the degenerated cubics formed by a conic and a line are degenerated curves of class 3 formed by the dual conic and the dual point.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3537
Date: 04/6/2019 12:40:01
From: amontes1949
Subject: A new point on quadrangle ?

Dear Vu Thanh Tung,

If P lies on circumcircle, then $Q = \text{tr}(P, \triangle ABC)$. = centroid of $\{A, B, C, P\}$.

Best regards,
Angel Montesdeoca

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3538
Date: 04/6/2019 2:52:49
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Bernard,

Thanks for the extra properties of the double points. They were new to me. I will study them more extensively.
You asked if it is possible to calculate the coordinates of the 5 triple points and their conic the same way I did for the double points.

I tried to do so, however, trying to find the common points of two known cubics is too complicated in Mathematica. Maybe you have suggestions?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3539
Date: 04/6/2019 5:39:22
From: Thanh Tung
Subject: A new point on quadrangle ?

Dear Angel Montesdeoca and others,

From your message, if P_1, P_2, P_3, P_4 are concyclic then the point Q_i are the same as the centroid $QA-P_1$ of the quadrangle $P_1P_2P_3P_4$.

I have noticed that the triangle transformation that I described in my previous message is still true for any 7 points $P, A, B, C, A_0, B_0, C_0$, i.e we can have a general setting:

General setting:

- * For any point P and two triangles ABC and $A_0B_0C_0$:
- * G_{ab} : centroid of triangle PAB_0 and cyclically defined $G_{ac}, G_{bc}, G_{ba}, G_{ca}, G_{cb}$.
- * A' : intersection of $G_{ab}G_{cb}$ and $G_{bc}G_{ac}$ and cyclically defined B', C' .
then the lines AA', BB', CC' are concurrent at a point Q .

Proof:

The line $B'C'$ is the same as the line $G_{ba}G_{ca}$. This line is parallel to BC as they are both parallel to the line connecting the midpoints of PB, PC .

So $B'C' \parallel BC$, similarly $C'A' \parallel CA, A'B' \parallel AB \Rightarrow$ Two triangles $A'B'C'$ and ABC are homothetic and AA', BB', CC' are concurrent at Q .

Some particular cases:

$A_0B_0C_0$ is inscribed triangle of $\triangle ABC$ (A_0, B_0, C_0 are in the lines BC, CA, AB) then AA', BB', CC concur at $Q = P$.

$A_0B_0C_0$ is the antipedal of P w.r.t ABC then three points: circumcenter of ABC , isogonal conjugate of P w.r.t ABC and Q are collinear.

Consequently if we consider a quadrangle $P_1P_2P_3P_4$, and let $Q_i =$ concurrent point in the general setting of P_i , triangles $P_jP_kP_l$, and the antipedal triangle of P_i w.r.t $P_jP_kP_l$ then P_iQ_i concur at the isogonal center $QA-P_4$ of $P_1P_2P_3P_4$.

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3540
Date: 05/6/2019 10:35:33
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Apparently, my last answer went lost !
The conic of the 5 triple points is the conic through the 3 S points and the points T and R.
The S-points are the isogonals of the QG-P1 wrt the triangles QA-P2,4 and 41 of the reference quadrilaterals), the point T is the perspector of the DT triangle and the S-triangle and the point R is the inverse of QL-P1 in the inversion with center QL-P10 which swaps QL-Ci2 and QL-Ci1.
The intersection between the conic and a QA-Cu7 gives 6 points, the 5 triple points and one S-point.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3541
Date: 06/6/2019 2:24:46
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Bernard,

back from holiday, reading your resumes
... wrt the QA-Cu7-double points:
Thanks for answering Chris' messages.
Are the following observations already mentioned?
Consider lines L of an arbitrary line pencil:
... $L \wedge QL-Tf2(L)$ give a cubic,
... bearing the 6 QL-points (see EQF).

If we can translate QL-Tf2 for a 5P (as we did for QL-Tf1),
... we had a solution of our problem.

Special: Consider lines L through QL-P1
... and the circles QL-Tf1(QL-Tf2(L)) through QL-P1,
... which have centers on an orthogonal hyperbola,
... centered in QL-P6

with asymptotes parallel to the Steiner axes,
... bearing QL-P4, QL-P5 ...

For pencils of points on the QL-lines or QL-Tr1-lines
... the cubic degenerates in the line and a conic.

Finally:
For a line L through a QA-Cu7-double point
... QL-Tf2(L) bears the corresponding other double point.

For a QG-version QGa of the QL with double points U_a, V_a ,
... on the conic Co of the 6 double points,
... the QL-Tf2-partner of lines through U_a and V_a intersect
... on a circumconic Co' of QGa through U_a, V_a and QG-P18b,c.
... The intersections of Co' and a circle with diameter U_aV_a are
QG-P18b,c.
... The intersections of Co' and the conic Co of the 6 double
points
... are the contact points of tangents from QG-P1a to Co .

Best regards Eckart

PS: Thanks for new aspects in #3536, I shall answer later.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3542
Date: 07/6/2019 10:21:28
From: eckart_schmidt@t-online.de
Subject: Dual 4/5 P/L

Dear Bernard,

Reading your message 3536, I cannot reproduce:
"I found that QA-P1 lies on the tangent in T
to the conic of the 5 points and on the dual of R ..."

I only found:
The dual of T is a parallel to QL-P3.4 through QL-P1,
... but this has no connection with the 5P.

Wrt the interesting conic, dual of the conic of the 5P,
... I found no properties.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3543
Date: 2020-03-08
From: System Manager
Subject: Deleted Message
3543

Message number 3543 was a deleted message in Yahoo Groups.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3544
Date: 10/6/2019 10:58:19
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

the point QL-P24 can be constructed
... with the 5 QA-Cu7-triple points:
In my #2799 a new 5P-point was described,
... this point is $CSC(QL-P24) = U$.
To interpret this observation
... your #3427 will be very helpful!

Best regards Eckart

PS: The "CSC-circle of X" in #2799 is defined
... by the CSC-images of X wrt the 5 QL of the 5L.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3545
Date: 10/6/2019 12:02:20
From: bernard.keizer
Subject: Dual 4/5 P/L

Dear Eckart,
I spent some hours in drawing a brandnew figur !
You're completely right, my property with QA-P1 doesn't hold ...
My apologise.
I was convinced until a few days that 5 points were necessary in
order to get a QL (5 QL vertices, DT vertices + pair of QL-2P3,
4 circumcenters of the reference triangles + QL-P1 ...)
But I've realised that 4 points were enough (4 QA vertices give
the dual QL, 2 non CSC vertices of QL and pair of QL-2P3 give
the 2 CSC partners and the QL ...)
Hence my conviction now that the dual 4 P/L and the dual 5 P/L
are in fact the same figure !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3546
Date: 11/6/2019 4:38:44
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Congratulations for this old message for Christmas giving the point U, perspector of QA-P4 and QG-P19.
It seems we come always nearer !
Are the following assumptions correct ?
1) For any point X (not QL-P1, not on the conic), we have CSC(X) and CB(X) and the locus of the foci of the circular cubics through the 5 points is XCB(X)
2) For any point X on the conic, CSC(X) is on a strange curve (quartic ?) CSC(conic) and CB(X) is the infinity point of TX (CB(T) is the infinity point on the tangent in T to the conic).
For QL-P1, CSC is the infinity point on the QL's Newton line and CB is on the line QL-P8CSC(U) and CSC(QL-P8) and CSC(CB(QL-P1)) are on a line through U and QL-P1 (symmetric of the preceding line wrt the 1st Steiner axis)
I think you've put a wedge in the 5P mystery !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3547
Date: 12/6/2019 1:18:41
From: Bernard Gibert
Subject: CTC issue

Dear friends

you might have noticed some issues with CTC recently.
The reason is that Orange decided to switch from http to https without giving me the opportunity to stay in http.
Worse still, they decided that a point in the name of a site would now be prohibited, which was the case with CTC and "bernard.gibert", a default name by Orange long ago...
Before making the change, they announced that an http site would not be migrated unless the dot was replaced by a dash like "bernard-gibert" and that this change was not mandatory.
But they never announced that an unchanged site would no longer be accessible for uploading and that an automatic redirection would be possible after the change.
Moreover, the software I used to transfer my site is not working anymore and I had to change for FileZilla which is far more complicated to me.
So I have been forced to change CTC to <https://bernard-gibert.pagesperso-orange.fr/index.html>, which is a real nuisance for everyone.
I am really sorry for those having links pointing to CTC in their sites and I sincerely apologize.
I hope CTC will survive without too many problems and please feel free to report any bugs you spot. Please also inform those who find some interest in CTC.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3548
Date: 19/6/2019 2:40:04
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

some observations, perhaps of interest:

Consider for a 5P = P1P2P3P4P5,

... the point T = 5P-s-P4 on its circumconic CO = 5P-s Co1,

... the QL with the 5P as QA-Cu7-triple points

... and the pentalateral 5L(T),

whose lines are the bisectors of TPi.

Let the CSC-circle of a point X be the circle (see 5L-s-Tf1)

... through the CSC-images of X wrt the 5 QL-components of 5L(T):

... The CSC-circle of T wrt 5L(T) is a line through U =

CSC(QL-P24);

... all CSC-circles of points on this line bear T.

5L-o-Ci1 is the locus of points,

... whose CSC-circles wrt 5L(T) degenerate collinear.

The transformation 5L-s-Tf1 for 5L(T) maps T at infinity:

... The line T.5L-o-Tf1(T) is orthogonal

... to the collinear degenerated CSC-circle of T (see above).

Let us now consider 5L(X) of bisectors of XPi and X on CO:

... The CSC-circles of X degenerate collinear bearing U.

There are three points X on CO, whose CSC-lines bear X,

... which give a triangle with orthocenter U

... and a circumcircle, which bears QL-P1 and T.

Finally

an additional remark to Bernard's #3427:

A perpendicular to QL-P1.QL-P24 in QL-P24

... bears QL-P16 (see EQF).

A circle with diameter U.QL-P1 bears CSC(QL-P16).

The line QL-P1.CSC(QL-P16) bears R.

How to find QL-P16, starting with the 5P, to get R?

Best regards Eckart

PS for Bernard: T hanks for your encouraging #3546.

Assumption 2) is correct, but I don't understand 1),

... perhaps my knowledge of circular cubics is insufficient.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3549
Date: 19/6/2019 8:22:34
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

the point $U = \text{CSC}(\text{QL-P24})$ for a QL
... is a special point of the 5P of the 5 QA-Cu7-triple points,
... as we discussed in the last messages.
I think, it should be mentioned in EPG as 5P-s-Px,
... for it has a surprising simple construction:
Consider a vertex P_i of the 5P
... and the QL of the bisectors P_iP_j ($j \neq i$),
... then $\text{CSC}(P_i)$ gives the point above
... independent of the chosen vertex P_i .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3550
Date: 20/6/2019 9:50:59
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,

Remarkable point and a remarkable construction!

Questions:

- When I describe it in short as the $5P-4L-\text{CSC}(P_i)$, is there also a $6P-5L-\text{CSC}(P_i)$ and a $7P-6L-\text{CSC}(P_i)$?
- And is there a $6P-5L-\text{Co-Tf2}(P_i)$ and/or $6P-5L-\text{Co-Tf3}(P_i)$?
- Further I lost sight on the connection with QL-P24. For the description in EPG could you please give a concise description of all properties of $5P-4L-\text{CSC}(P_i)$?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3551
Date: 20/6/2019 10:48:44
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

thanks for your interest and stimulation.
Wrt the 1st question in #3550 a first observation:
6P-5L-Tf(Pi) with $Tf = 5L-s-Tf1$ is a special 6P-point,
... independent of the chosen vertex Pi,
... but I cannot give properties at the moment.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3552
Date: 20/6/2019 11:02:27
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

in addition to my last message 3551:

7P-6L-Tf(Pi) with $Tf = 6L-s-Tf1$ is special 7P-point,
... CABRI-controlled.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3553
Date: 20/6/2019 11:27:40
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Very interesting constructions, including the amazing construction of U in your following message !
Wrt your PS, it's only an application of CB properties with 7 points (it works also, as we know, if 2 of the 7 are the circular points) : any pivotal cubic with pivot P wrt 7 points and passing through a point X passes also through CB(X) and P, X and CB(X) are aligned. It follows that the line XCB(X) is the locus of the foci of the cubics (circular or not) passing through the 7 points, X and CB(X).
Wrt your message, QL-P16 is the reflexion of QL-P10 in QL-P1 and lies on QL-Ci1 ; CSC(R) is the 2nd intersection (other than QL-P16) of QL-P1QL-P10 with QL-Ci1.
The 2 intersections of QL-Ci1 (through the QG-P1) and it's CSC (through the QA-P4) are necessary CSC partners. Is it possible that these 2 points are precisely QL-P16 and it's CSC partner ?
If it is not only a coincidence on my figure, it would be surely an interesting property !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3554
Date: 20/6/2019 11:50:27
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Please forget the end of my message, it's in fact only a coincidence !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3555
Date: 20/6/2019 2:35:08
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

wrt your #3550 2nd question:

... There are no points 6P-5L-Co-Tf2(Pi) and 6P-5L-Co-Tf3(Pi).

Wrt QL-P24 in your 3rd question.

For a reference QL the point CSC(QL-P24) is the perspector U
... of the QA-P4 and the QG-P19 triangle (see your #2986).

For the 5P of the corresponding QA-Cu7-triple points

... I found no connection of U with well known 5P-points.

But for the 5L(X), whose lines are the bisectors of X.Pi,

... the CSC-circles wrt 5L(X) for X on the circumconic of 5P

... degenerate collinear with common point U.

Somewhat curious remains the observation,

... that there are 3 points X on the 5P-circumconic,

... which lie on its degenerated CSC-circle

... and give a triangle with orthocenter U

... and circumcircle through QL-P1 and 5P-s-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3556
Date: 20/6/2019 5:50:59
From: Thanh Tung
Subject: Construction of a point on quadrangle

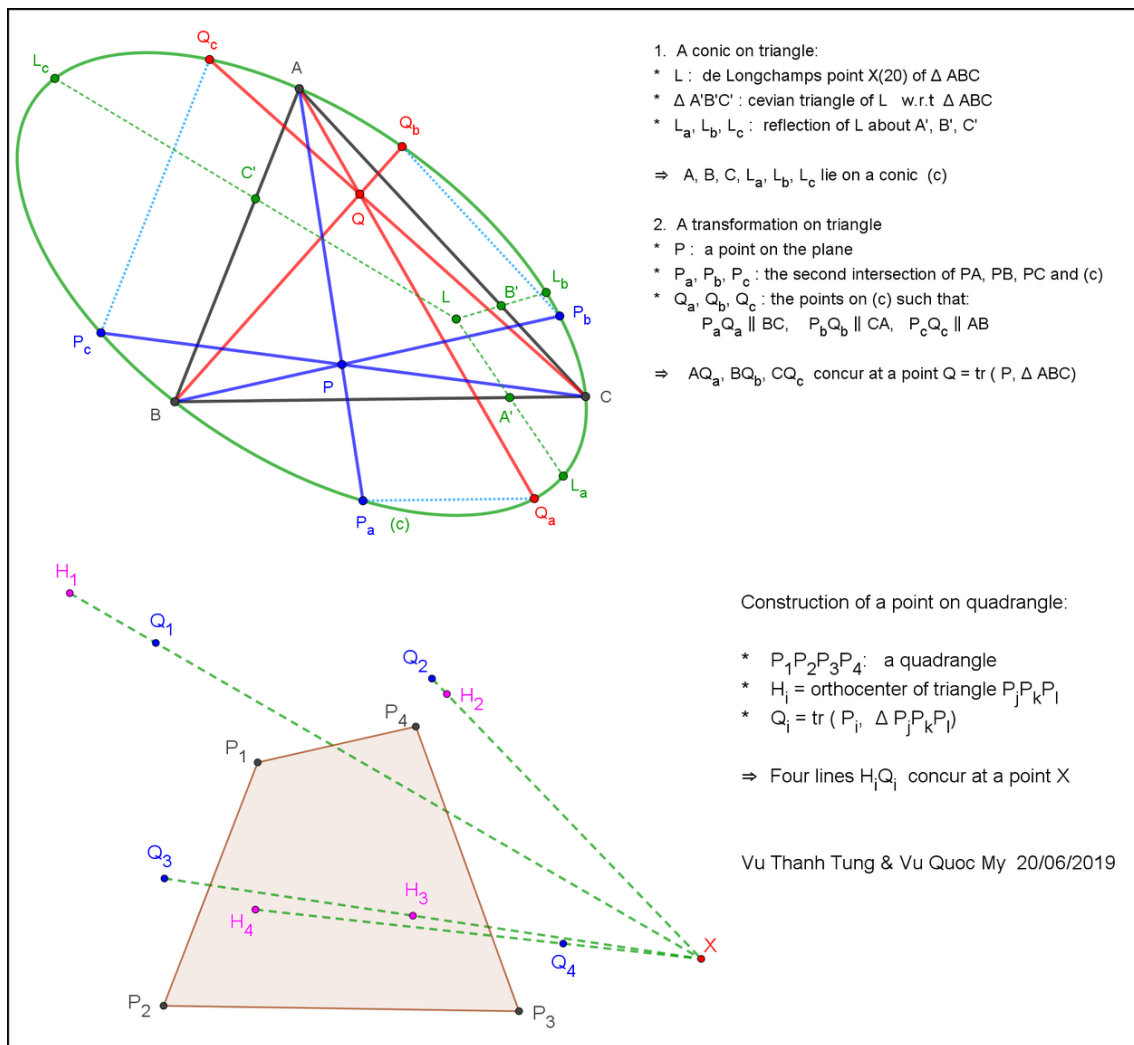
Dear all,

We found a construction of a point on quadrangle as follows:

1. A conic on triangle:
 - * L: de Longchamps point $X(20)$ of ΔABC
 - * $\Delta A'B'C'$: cevian triangle of L w.r.t ΔABC
 - * L_a, L_b, L_c : reflection of L about A', B', C'
 - $\Rightarrow A, B, C, L_a, L_b, L_c$ lie on a conic (c)
2. A transformation on triangle
 - * P: a point on the plane
 - * P_a, P_b, P_c : the second intersection of PA, PB, PC and the conic (c)
 - * Q_a, Q_b, Q_c : the points on (c) such that:
 $P_a Q_a \parallel BC, P_b Q_b \parallel CA, P_c Q_c \parallel AB$
 - $\Rightarrow AQ_a, BQ_b, CQ_c$ concur at a point $Q = \text{tr} (P, \Delta ABC)$
3. Construction of a point on quadrangle:
 - * $P_1 P_2 P_3 P_4$: a quadrangle
 - * H_i = orthocenter of triangle $P_j P_k P_l$
 - * $Q_i = \text{tr} (P_i, \Delta P_j P_k P_l)$
 - $\Rightarrow$ Four lines $H_i Q_i$ concur at a point X

We are not sure if the point constructed as above is new or not.

Best regards,
Vu Thanh Tung & Vu Quoc My



PointOnQuadrangleX20X4TungMy20190620_v4.png

Message: #3557
Date: 20/6/2019 6:17:31
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Of course, it remains that QL-P10 and QL-P16 being symmetric wrt QL-P1, their CSC are also symmetric wrt QL-P1 and the line QL-P10QL-P16 bearing QL-P1 and CSC(R) is symmetric wrt the 1st Steiner axis of the line CSC(QL-P10)CSC(QL-P16) bearing QL-P1 and R.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3558
Date: 20/6/2019 8:50:02
From: eckart_schmidt@t-online.de
Subject: Construction of a point on quadrangle

Dear Vu Thanh Tung & Vu Quoc My,

the described point is QA-P23.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3559
Date: 20/6/2019 9:53:14
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
If my memory is good, you said once that $QL-P1CB(QL-P1)$ bears $CSC(U)$ and $QL-P8$ (it is the line $QL-P8QL-P24$ through $QL-P1$) and you know how to find $CB(QL-P1)$.
Is it possible that $CSC(CB(QL-P1))CB(CSC(CB(QL-P1)))$ bears also $QL-P8$?
In other words, the circular cubic through the 5 triple points with focus $QL-P8$ passes through $QL-P1$ and $CB(QL-P1)$ and through $CSC(CB(QL-P1))$ and $CB(CSC(CB(QL-P1)))$...
If this is true, it gives $QL-P8$ as intersection of the 2 lines $QL-P1CB(QL-P1)$ and $CSC(CB(QL-P1))$ and $CB(CSC(CB(QL-P1)))$.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3560
Date: 21/6/2019 3:11:46
From: Thanh Tung
Subject: Construction of a point on quadrangle

Dear Eckart,
Thank you very much,
Best regards,
Vu Thanh Tung

Sent from Yahoo Mail for iPhone (
<https://overview.mail.yahoo.com/?src=iOS>)
> On Friday, June 21, 2019, 1:49 AM,
'eckart_schmidt@t-online.de'
> eckart_schmidt@t-online.de [Quadri-Figures-Group] wrote:
>> Dear Vu Thanh Tung & Vu Quoc My,
>> the described point is QA-P23.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3562
Date: 21/6/2019 11:03:10
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

for a $nP = (P_1, \dots, P_n)$ we can consider the $(n-1)L(P_i)$,
... whose lines are the bisectors of P_iP_j (j unequal i).
Starting with a 6P
... the CSC-circles of P_i wrt $5L(P_i)$ are the same for all P_i .
For the 6P-components of a 7P
... these circles have a common point.
But I cannot integrate these observations in EPG-geometry.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3563
Date: 21/6/2019 12:26:15
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
There is no attachment ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3564
Date: 24/6/2019 4:02:40
From: eckart_schmidt@t-online.de
Subject: 5P-s-10P

Dear Bernard, dear Chris,

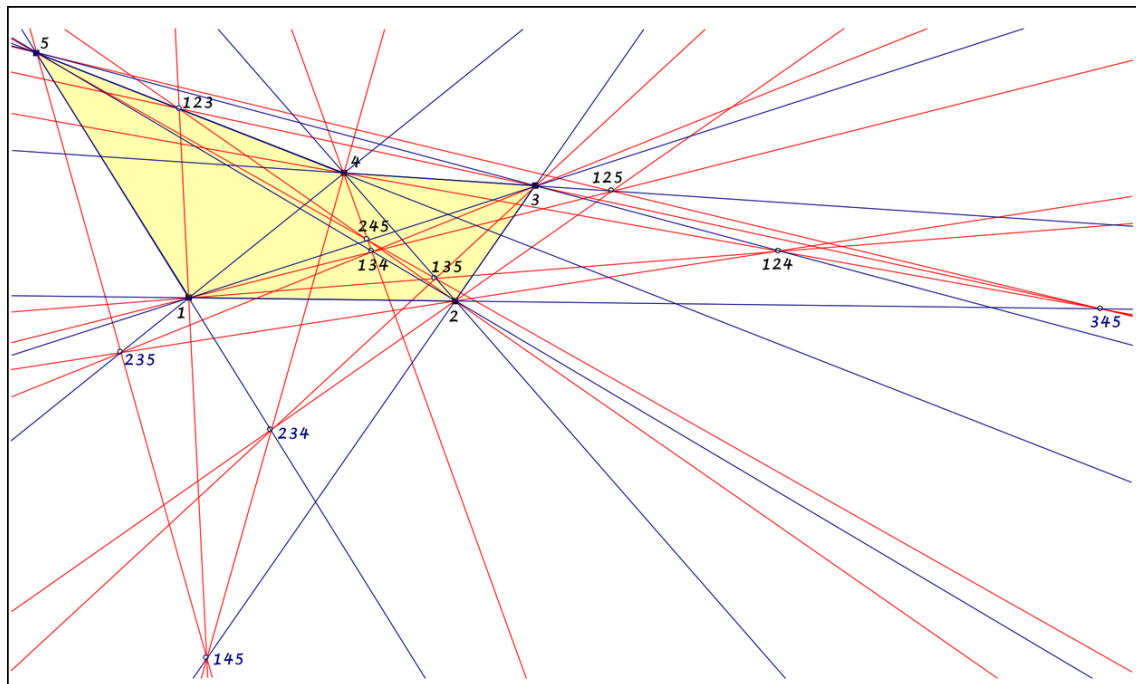
perhaps of interest a
15-lines constellation for a 5P,
... bearing the vertices and 10 triple intersections:

Consider for each vertex P_i of a 5P
... the radical axes of the circles
... (P_i, P_j, P_k) , (P_i, P_l, P_m) and

(P_i, P_j, P_l) , (P_i, P_k, P_m) and (P_i, P_j, P_m) , (P_i, P_l, P_k) ,
... which give 15 lines with 5 triple points in the vertices
... and 10 further triple points.

Each line bears one vertex and 2 further triple points,
... each line $P_i P_j$ bears a further triple point,
... whose 4th harmonic point is collinear
... with three pairs of further triple points.

Best regards Eckart



2019-06-24.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3565
Date: 25/6/2019 10:12:49
From: bernard.keizer
Subject: 5P-s-10P

Dear Eckart,
I reproduced with interest your constellation.
It's possible to draw 5 cubics through the 5 points and each
time 4 of the 10 other triple points (for example 345, 245, 235
and 234 ...), each of the points belonging to 2 cubics.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3566
Date: 25/6/2019 3:54:20
From: bernard.keizer
Subject: 5P-s-10P

Dear Eckart,
Looking for amusing constellations, you may have this one,
already mentionned, but without reaction :
there are 5 pivotal cubics with pivot one of the 5 points and
fixed points of the isoconjugation the 4 others.
The interest is that the conic of the 5 points is the common
unique diagonal polar conic of the 5 cubics ; it is the polar
conic of the pivot wrt the corresponding cubic.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3567
Date: 25/6/2019 4:15:57
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Eckart, Dear Bernard,

I managed to calculate the equation of the conic through the 5 common QA-Cu7 points.

I had to use all the tricks I learned in my years of using Mathematica.

The equation (in QL-CT-coordinates) is not simple and not complicated.

CONIC= $T_{xx} x^2 + T_{yy} y^2 + T_{zz} z^2 + T_{yz} y z + T_{zx} z x + T_{xy} x y$
where:

$T_{xx} = -b^2 c^2 l (b^2 l - c^2 l - b^2 m + c^2 n) (-2 a^2 l + a^2 m + b^2 m - c^2 m + a^2 n - b^2 n + c^2 n) (b^2 m^2 + a^2 m n - b^2 m n - c^2 m n + c^2 n^2)$

$T_{yy} = a^2 c^2 m (-a^2 l + a^2 m - c^2 m + c^2 n) (a^2 l + b^2 l - c^2 l - 2 b^2 m - a^2 n + b^2 n + c^2 n) (a^2 l^2 - a^2 l n + b^2 l n - c^2 l n + c^2 n^2)$

$T_{zz} = a^2 b^2 (a^2 l^2 - a^2 l m - b^2 l m + c^2 l m + b^2 m^2) n (a^2 l - b^2 m - a^2 n + b^2 n) (a^2 l - b^2 l + c^2 l - a^2 m + b^2 m + c^2 m - 2 c^2 n)$

$$\begin{aligned}
\text{Tyz} = & a^2 (a^6 b^2 l^4 m - a^4 b^4 l^4 m + a^4 b^2 c^2 l^4 m - 2 \\
& a^6 b^2 l^3 m^2 - a^4 b^4 l^3 m^2 + 3 a^2 b^6 l^3 m^2 + 2 a^4 b \\
& 2 c^2 l^3 m^2 - 3 a^2 b^4 c^2 l^3 m^2 + a^6 b^2 l^2 m^3 + 5 a^4 \\
& b^4 l^2 m^3 - 4 a^2 b^6 l^2 m^3 - 2 b^8 l^2 m^3 - 3 a^4 b^2 c^2 \\
& l^2 m^3 - 3 a^2 b^4 c^2 l^2 m^3 + 3 b^6 c^2 l^2 m^3 + 3 a^2 b^2 \\
& c^4 l^2 m^3 - b^2 c^6 l^2 m^3 - 3 a^4 b^4 l m^4 - a^2 b^6 l m^4 \\
& + 4 b^8 l m^4 + 6 a^2 b^4 c^2 l m^4 - b^6 c^2 l m^4 - 3 b^4 c^4 \\
& l m^4 + 2 a^2 b^6 m^5 - 2 b^8 m^5 - 2 b^6 c^2 m^5 - a^6 c^2 l^4 \\
& n - a^4 b^2 c^2 l^4 n + a^4 c^4 l^4 n - 2 a^6 b^2 l^3 m n + 4 a \\
& 4 b^4 l^3 m n - 2 a^2 b^6 l^3 m n + 2 a^6 c^2 l^3 m n - 4 a^4 c \\
& 4 l^3 m n + 2 a^2 c^6 l^3 m n + 4 a^6 b^2 l^2 m^2 n - 5 a^4 b^4 \\
& l^2 m^2 n - 2 a^2 b^6 l^2 m^2 n + 3 b^8 l^2 m^2 n - a^6 c^2 l^2 \\
& m^2 n - 7 a^4 b^2 c^2 l^2 m^2 n + 15 a^2 b^4 c^2 l^2 m^2 n - 3 b \\
& 6 c^2 l^2 m^2 n + 3 a^4 c^4 l^2 m^2 n + 2 a^2 b^2 c^4 l^2 m^2 n \\
& - 2 b^4 c^4 l^2 m^2 n - 3 a^2 c^6 l^2 m^2 n + b^2 c^6 l^2 m^2 n \\
& + c^8 l^2 m^2 n - 2 a^6 b^2 l m^3 n - 2 a^4 b^4 l m^3 n + 10 a^2 \\
& b^6 l m^3 n - 6 b^8 l m^3 n + 8 a^4 b^2 c^2 l m^3 n - 9 a^2 b^4 \\
& c^2 l m^3 n - 5 b^6 c^2 l m^3 n - 10 a^2 b^2 c^4 l m^3 n + 7 b^4 \\
& c^4 l m^3 n + 4 b^2 c^6 l m^3 n + 3 a^4 b^4 m^4 n - 6 a^2 b^6 m \\
& 4 n + 3 b^8 m^4 n - 6 a^2 b^4 c^2 m^4 n + 8 b^6 c^2 m^4 n + 3 b \\
& 4 c^4 m^4 n + 2 a^6 c^2 l^3 n^2 - 2 a^4 b^2 c^2 l^3 n^2 + a^4 c \\
& 4 l^3 n^2 + 3 a^2 b^2 c^4 l^3 n^2 - 3 a^2 c^6 l^3 n^2 + a^6 b^2 \\
& l^2 m n^2 - 3 a^4 b^4 l^2 m n^2 + 3 a^2 b^6 l^2 m n^2 - b^8 l^2 \\
& m n^2 - 4 a^6 c^2 l^2 m n^2 + 7 a^4 b^2 c^2 l^2 m n^2 - 2 a^2 b \\
& 4 c^2 l^2 m n^2 - b^6 c^2 l^2 m n^2 + 5 a^4 c^4 l^2 m n^2 - 15 a \\
& 2 b^2 c^4 l^2 m n^2 + 2 b^4 c^4 l^2 m n^2 + 2 a^2 c^6 l^2 m n^2 \\
& + 3 b^2 c^6 l^2 m n^2 - 3 c^8 l^2 m n^2 - 2 a^6 b^2 l m^2 n^2 + \\
& 6 a^4 b^4 l m^2 n^2 - 6 a^2 b^6 l m^2 n^2 + 2 b^8 l m^2 n^2 + 2 \\
& a^6 c^2 l m^2 n^2 - 12 a^2 b^4 c^2 l m^2 n^2 + 10 b^6 c^2 l m^2 \\
& n^2 - 6 a^4 c^4 l m^2 n^2 + 12 a^2 b^2 c^4 l m^2 n^2 + 6 a^2 c^6 \\
& l m^2 n^2 - 10 b^2 c^6 l m^2 n^2 - 2 c^8 l m^2 n^2 + a^6 b^2 m^3 \\
& n^2 - 3 a^4 b^4 m^3 n^2 + 3 a^2 b^6 m^3 n^2 - b^8 m^3 n^2 - 5 a \\
& 4 b^2 c^2 m^3 n^2 + 14 a^2 b^4 c^2 m^3 n^2 - 9 b^6 c^2 m^3 n^2 + \\
& 7 a^2 b^2 c^4 m^3 n^2 - 9 b^4 c^4 m^3 n^2 - 3 b^2 c^6 m^3 n^2 - \\
& a^6 c^2 l^2 n^3 + 3 a^4 b^2 c^2 l^2 n^3 - 3 a^2 b^4 c^2 l^2 n^3 + \\
& b^6 c^2 l^2 n^3 - 5 a^4 c^4 l^2 n^3 + 3 a^2 b^2 c^4 l^2 n^3 + \\
& 4 a^2 c^6 l^2 n^3 - 3 b^2 c^6 l^2 n^3 + 2 c^8 l^2 n^3 + 2 a^6 c \\
& 2 l m n^3 - 8 a^4 b^2 c^2 l m n^3 + 10 a^2 b^4 c^2 l m n^3 - 4 b \\
& 6 c^2 l m n^3 + 2 a^4 c^4 l m n^3 + 9 a^2 b^2 c^4 l m n^3 - 7 b \\
& 4 c^4 l m n^3 - 10 a^2 c^6 l m n^3 + 5 b^2 c^6 l m n^3 + 6 c^8 l \\
& m n^3 - a^6 c^2 m^2 n^3 + 5 a^4 b^2 c^2 m^2 n^3 - 7 a^2 b^4 c^2 \\
& m^2 n^3 + 3 b^6 c^2 m^2 n^3 + 3 a^4 c^4 m^2 n^3 - 14 a^2 b^2 c^4 \\
& m^2 n^3 + 9 b^4 c^4 m^2 n^3 - 3 a^2 c^6 m^2 n^3 + 9 b^2 c^6 m^2 \\
& n^3 + c^8 m^2 n^3 + 3 a^4 c^4 l n^4 - 6 a^2 b^2 c^4 l n^4 + 3 b \\
& 4 c^4 l n^4 + a^2 c^6 l n^4 + b^2 c^6 l n^4 - 4 c^8 l n^4 - 3 a \\
& 4 c^4 m n^4 + 6 a^2 b^2 c^4 m n^4 - 3 b^4 c^4 m n^4 + 6 a^2 c^6 \\
& m n^4 - 8 b^2 c^6 m n^4 - 3 c^8 m n^4 - 2 a^2 c^6 n^5 + 2 b^2 c \\
& 6 n^5 + 2 c^8 n^5)
\end{aligned}$$

$$\begin{aligned}
T_{zx} = & -b^2 (-2 a^8 l^5 + 2 a^6 b^2 l^5 - 2 a^6 c^2 l^5 + 4 a^8 l^4 m - a^6 b^2 l^4 m - 3 a^4 b^4 l^4 m - a^6 c^2 l^4 m + 6 a^4 b^2 c^2 l^4 m - 3 a^4 c^4 l^4 m - 2 a^8 l^3 m^2 - 4 a^6 b^2 l^3 m^2 + 5 a^4 b^4 l^3 m^2 + a^2 b^6 l^3 m^2 + 3 a^6 c^2 l^3 m^2 - 3 a^4 b^2 c^2 l^3 m^2 - 3 a^2 b^4 c^2 l^3 m^2 + 3 a^2 b^2 c^4 l^3 m^2 - a^2 c^6 l^3 m^2 + 3 a^6 b^2 l^2 m^3 - a^4 b^4 l^2 m^3 - 2 a^2 b^6 l^2 m^3 - 3 a^4 b^2 c^2 l^2 m^3 + 2 a^2 b^4 c^2 l^2 m^3 - a^4 b^4 l m^4 + a^2 b^6 l m^4 + a^2 b^4 c^2 l m^4 + 3 a^8 l^4 n - 6 a^6 b^2 l^4 n + 3 a^4 b^4 l^4 n + 8 a^6 c^2 l^4 n - 6 a^4 b^2 c^2 l^4 n + 3 a^4 c^4 l^4 n - 6 a^8 l^3 m n + 10 a^6 b^2 l^3 m n - 2 a^4 b^4 l^3 m n - 2 a^2 b^6 l^3 m n - 5 a^6 c^2 l^3 m n - 9 a^4 b^2 c^2 l^3 m n + 8 a^2 b^4 c^2 l^3 m n + 7 a^4 c^4 l^3 m n - 10 a^2 b^2 c^4 l^3 m n + 4 a^2 c^6 l^3 m n + 3 a^8 l^2 m^2 n - 2 a^6 b^2 l^2 m^2 n - 5 a^4 b^4 l^2 m^2 n + 4 a^2 b^6 l^2 m^2 n - 3 a^6 c^2 l^2 m^2 n + 15 a^4 b^2 c^2 l^2 m^2 n - 7 a^2 b^4 c^2 l^2 m^2 n - b^6 c^2 l^2 m^2 n - 2 a^4 c^4 l^2 m^2 n + 2 a^2 b^2 c^4 l^2 m^2 n + 3 b^4 c^4 l^2 m^2 n + a^2 c^6 l^2 m^2 n - 3 b^2 c^6 l^2 m^2 n + c^8 l^2 m^2 n - 2 a^6 b^2 l m^3 n + 4 a^4 b^4 l m^3 n - 2 a^2 b^6 l m^3 n + 2 b^6 c^2 l m^3 n - 4 b^4 c^4 l m^3 n + 2 b^2 c^6 l m^3 n - a^2 b^4 c^2 m^4 n - b^6 c^2 m^4 n + b^4 c^4 m^4 n - a^8 l^3 n^2 + 3 a^6 b^2 l^3 n^2 - 3 a^4 b^4 l^3 n^2 + a^2 b^6 l^3 n^2 - 9 a^6 c^2 l^3 n^2 + 14 a^4 b^2 c^2 l^3 n^2 - 5 a^2 b^4 c^2 l^3 n^2 - 9 a^4 c^4 l^3 n^2 + 7 a^2 b^2 c^4 l^3 n^2 - 3 a^2 c^6 l^3 n^2 + 2 a^8 l^2 m n^2 - 6 a^6 b^2 l^2 m n^2 + 6 a^4 b^4 l^2 m n^2 - 2 a^2 b^6 l^2 m n^2 + 10 a^6 c^2 l^2 m n^2 - 12 a^4 b^2 c^2 l^2 m n^2 + 2 b^6 c^2 l^2 m n^2 + 12 a^2 b^2 c^4 l^2 m n^2 - 6 b^4 c^4 l^2 m n^2 - 10 a^2 c^6 l^2 m n^2 + 6 b^2 c^6 l^2 m n^2 - 2 c^8 l^2 m n^2 - a^8 l m^2 n^2 + 3 a^6 b^2 l m^2 n^2 - 3 a^4 b^4 l m^2 n^2 + a^2 b^6 l m^2 n^2 - a^6 c^2 l m^2 n^2 - 2 a^4 b^2 c^2 l m^2 n^2 + 7 a^2 b^4 c^2 l m^2 n^2 - 4 b^6 c^2 l m^2 n^2 + 2 a^4 c^4 l m^2 n^2 - 15 a^2 b^2 c^4 l m^2 n^2 + 5 b^4 c^4 l m^2 n^2 + 3 a^2 c^6 l m^2 n^2 + 2 b^2 c^6 l m^2 n^2 - 3 c^8 l m^2 n^2 - 2 a^2 b^4 c^2 m^3 n^2 + 2 b^6 c^2 m^3 n^2 + 3 a^2 b^2 c^4 m^3 n^2 + b^4 c^4 m^3 n^2 - 3 b^2 c^6 m^3 n^2 + 3 a^6 c^2 l^2 n^3 - 7 a^4 b^2 c^2 l^2 n^3 + 5 a^2 b^4 c^2 l^2 n^3 - b^6 c^2 l^2 n^3 + 9 a^4 c^4 l^2 n^3 - 14 a^2 b^2 c^4 l^2 n^3 + 3 b^4 c^4 l^2 n^3 + 9 a^2 c^6 l^2 n^3 - 3 b^2 c^6 l^2 n^3 + c^8 l^2 n^3 - 4 a^6 c^2 l m n^3 + 10 a^4 b^2 c^2 l m n^3 - 8 a^2 b^4 c^2 l m n^3 + 2 b^6 c^2 l m n^3 - 7 a^4 c^4 l m n^3 + 9 a^2 b^2 c^4 l m n^3 + 2 b^4 c^4 l m n^3 + 5 a^2 c^6 l m n^3 - 10 b^2 c^6 l m n^3 + 6 c^8 l m n^3 + a^6 c^2 m^2 n^3 - 3 a^4 b^2 c^2 m^2 n^3 + 3 a^2 b^4 c^2 m^2 n^3 - b^6 c^2 m^2 n^3 + 3 a^2 b^2 c^4 m^2 n^3 - 5 b^4 c^4 m^2 n^3 - 3 a^4 c^4 l n^4 + 6 a^2 b^2 c^4 l n^4 - 3 b^4 c^4 l n^4 - 8 a^2 c^6 l n^4 + 6 b^2 c^6 l n^4 - 3 c^8 l n^4 + 3 a^4 c^4 m n^4 - 6 a^2 b^2 c^4 m n^4 + 3 b^4 c^4 m n^4 + a^2 c^6 m n^4 + b^2 c^6 m n^4 - 4 c^8 m n^4 + 2 a^2 c^6 n^5 - 2 b^2 c^6 n^5 + 2 c^8 n^5)
\end{aligned}$$

$$\begin{aligned}
T_{xy} = & c^2 (-2 a^8 l^5 - 2 a^6 b^2 l^5 + 2 a^6 c^2 l^5 + 3 a^8 l^4 m + 8 a^6 b^2 l^4 m + 3 a^4 b^4 l^4 m - 6 a^6 c^2 l^4 m - 6 a^4 b^2 c^2 l^4 m + 3 a^4 c^4 l^4 m - a^8 l^3 m^2 - 9 a^6 b^2 l^3 m^2 - 9 a^4 b^4 l^3 m^2 - 3 a^2 b^6 l^3 m^2 + 3 a^6 c^2 l^3 m^2 + 14 a^4 b^2 c^2 l^3 m^2 + 7 a^2 b^4 c^2 l^3 m^2 - 3 a^4 c^4 l^3 m^2 - 5 a^2 b^2 c^4 l^3 m^2 + a^2 c^6 l^3 m^2 + 3 a^6 b^2 l^2 m^3 + 9 a^4 b^4 l^2 m^3 + 9 a^2 b^6 l^2 m^3 + b^8 l^2 m^3 - 7 a^4 b^2 c^2 l^2 m^3 - 14 a^2 b^4 c^2 l^2 m^3 - 3 b^6 c^2 l^2 m^3 + 5 a^2 b^2 c^4 l^2 m^3 + 3 b^4 c^4 l^2 m^3 - b^2 c^6 l^2 m^3 - 3 a^4 b^4 l m^4 - 8 a^2 b^6 l m^4 - 3 b^8 l m^4 + 6 a^2 b^4 c^2 l m^4 + 6 b^6 c^2 l m^4 - 3 b^4 c^4 l m^4 + 2 a^2 b^6 m^5 + 2 b^8 m^5 - 2 b^6 c^2 m^5 + 4 a^8 l^4 n - a^6 b^2 l^4 n - 3 a^4 b^4 l^4 n - a^6 c^2 l^4 n + 6 a^4 b^2 c^2 l^4 n - 3 a^4 c^4 l^4 n - 6 a^8 l^3 m n - 5 a^6 b^2 l^3 m n + 7 a^4 b^4 l^3 m n + 4 a^2 b^6 l^3 m n + 10 a^6 c^2 l^3 m n - 9 a^4 b^2 c^2 l^3 m n - 10 a^2 b^4 c^2 l^3 m n - 2 a^4 c^4 l^3 m n + 8 a^2 b^2 c^4 l^3 m n - 2 a^2 c^6 l^3 m n + 2 a^8 l^2 m^2 n + 10 a^6 b^2 l^2 m^2 n - 10 a^2 b^6 l^2 m^2 n - 2 b^8 l^2 m^2 n - 6 a^6 c^2 l^2 m^2 n - 12 a^4 b^2 c^2 l^2 m^2 n + 12 a^2 b^4 c^2 l^2 m^2 n + 6 b^6 c^2 l^2 m^2 n + 6 a^4 c^4 l^2 m^2 n - 6 b^4 c^4 l^2 m^2 n - 2 a^2 c^6 l^2 m^2 n + 2 b^2 c^6 l^2 m^2 n - 4 a^6 b^2 l m^3 n - 7 a^4 b^4 l m^3 n + 5 a^2 b^6 l m^3 n + 6 b^8 l m^3 n + 10 a^4 b^2 c^2 l m^3 n + 9 a^2 b^4 c^2 l m^3 n - 10 b^6 c^2 l m^3 n - 8 a^2 b^2 c^4 l m^3 n + 2 b^4 c^4 l m^3 n + 2 b^2 c^6 l m^3 n + 3 a^4 b^4 m^4 n + a^2 b^6 m^4 n - 4 b^8 m^4 n - 6 a^2 b^4 c^2 m^4 n + b^6 c^2 m^4 n + 3 b^4 c^4 m^4 n - 2 a^8 l^3 n^2 + 3 a^6 b^2 l^3 n^2 - a^2 b^6 l^3 n^2 - 4 a^6 c^2 l^3 n^2 - 3 a^4 b^2 c^2 l^3 n^2 + 3 a^2 b^4 c^2 l^3 n^2 + 5 a^4 c^4 l^3 n^2 - 3 a^2 b^2 c^4 l^3 n^2 + a^2 c^6 l^3 n^2 + 3 a^8 l^2 m n^2 - 3 a^6 b^2 l^2 m n^2 - 2 a^4 b^4 l^2 m n^2 + a^2 b^6 l^2 m n^2 + b^8 l^2 m n^2 - 2 a^6 c^2 l^2 m n^2 + 15 a^4 b^2 c^2 l^2 m n^2 + 2 a^2 b^4 c^2 l^2 m n^2 - 3 b^6 c^2 l^2 m n^2 - 5 a^4 c^4 l^2 m n^2 - 7 a^2 b^2 c^4 l^2 m n^2 + 3 b^4 c^4 l^2 m n^2 + 4 a^2 c^6 l^2 m n^2 - b^2 c^6 l^2 m n^2 - a^8 l m^2 n^2 - a^6 b^2 l m^2 n^2 + 2 a^4 b^4 l m^2 n^2 + 3 a^2 b^6 l m^2 n^2 - 3 b^8 l m^2 n^2 + 3 a^6 c^2 l m^2 n^2 - 2 a^4 b^2 c^2 l m^2 n^2 - 15 a^2 b^4 c^2 l m^2 n^2 + 2 b^6 c^2 l m^2 n^2 - 3 a^4 c^4 l m^2 n^2 + 7 a^2 b^2 c^4 l m^2 n^2 + 5 b^4 c^4 l m^2 n^2 + a^2 c^6 l m^2 n^2 - 4 b^2 c^6 l m^2 n^2 + a^6 b^2 m^3 n^2 - 3 a^2 b^6 m^3 n^2 + 2 b^8 m^3 n^2 - 3 a^4 b^2 c^2 m^3 n^2 + 3 a^2 b^4 c^2 m^3 n^2 + 4 b^6 c^2 m^3 n^2 + 3 a^2 b^2 c^4 m^3 n^2 - 5 b^4 c^4 m^3 n^2 - b^2 c^6 m^3 n^2 + 3 a^6 c^2 l^2 n^3 - 3 a^4 b^2 c^2 l^2 n^3 - a^4 c^4 l^2 n^3 + 2 a^2 b^2 c^4 l^2 n^3 - 2 a^2 c^6 l^2 n^3 - 2 a^6 c^2 l m n^3 + 2 b^6 c^2 l m n^3 + 4 a^4 c^4 l m n^3 - 4 b^4 c^4 l m n^3 - 2 a^2 c^6 l m n^3 + 2 b^2 c^6 l m n^3 + 3 a^2 b^4 c^2 m^2 n^3 - 3 b^6 c^2 m^2 n^3 - 2 a^2 b^2 c^4 m^2 n^3 + b^4 c^4 m^2 n^3 + 2 b^2 c^6 m^2 n^3 - a^4 c^4 l n^4 + a^2 b^2 c^4 l n^4 + a a^2 c^6 l n^4 - a^2 b^2 c^4 m n^4 + b^4 c^4 m n^4 - b^2 c^6 m n^4)
\end{aligned}$$

The Center of the Conic however is pretty long. Too long to mention here.

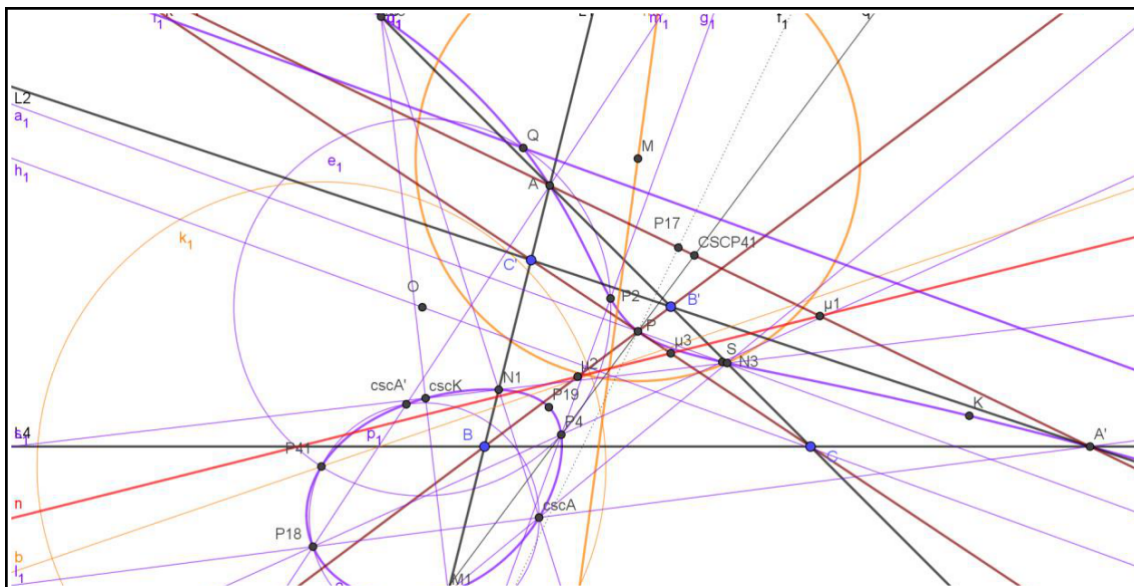
The advantage of having calculated equation and coordinates is that we can look for incidences with other QL-items. However, I could not find any incidence with other QL-items so far.

Also, an advantage is secure plotting. See attached file.

Now for me, there will be the next challenge to calculate the intersection points of QA-Cu7a and this conic. One of these intersection points will be Sa (QFG#2980,#2983,#2984). But I am not very hopeful.

Best regards,

Chris



QA-Cu7.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3568
Date: 25/6/2019 9:00:32
From: eckart_schmidt@t-online.de
Subject: QA-Cu7-Geometry

Dear Chris,

congratulations on the equation of the 5P-conic
... for the QA-Cu7-triple points!
Now our observations will get analytical certification.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3569
Date: 25/6/2019 10:59:26
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Many thanks for the attachment !
I reproduced your construction of U. It's amazing ...
Now, for 5 points, we have the conic, the CSC and the points T and U.
The CSC, T and U are perfectly defined for an ellipse as well as for an hyperbola ...
For any point R, we have the circle with diameter UR, it's CSC circle with center P9, the points P16 and P10, the point P8 as intersection of P9P10 with P1P24 ...
What next ? (We haven't used the CB transformation)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3570
Date: 26/6/2019 10:00:34
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Congratulations, it looks very impressive !
As I don't practice Mathematica myself, I hardly measure the difficulty of these calculations, but I see the possible consequences ...
I hope you will succeed in finding the equation of the 6th degree giving the 6 intersections of the conic and QA-Cu7a and, eliminating the point Sa, the equation of the 5th degree giving the 5 triple points ; it has to be the same with QA-Cu7b and QA-Cu7c ...
Courage, the fight goes on !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3571
Date: 26/6/2019 9:10:20
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

wrt your message 3569:
... "What next ? (We haven't used the CB transformation)".

Here a further observation for the 5P/QL-puzzle wrt QA-Cu7:
Let cb be the CB-transformation wrt 5P and the circular points,
... already mentioned cb(QL-P1) on QL-P1.QL-P24.

If we consider for cb(QL-P1)
... the 5L of the bisectors wrt the 5P-vertices
... and the CSC-circle of cb(QL-P1) wrt this 5L,
... we get a circle through QL-P1, $CSC(QL-P24)=U$,
 $CSC(5P-s-P4)=CSC(T)$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3572
Date: 27/6/2019 8:22:30
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Another proof that all fit's together !
I hope you will find now another point like U.
What about the QA-P3 of the reference QA's on the Dimidium
circle and their CSC, the QG-P16 aligned on the CSC line of this
Dimidium circle ?
What about the point you mentionned as perspector of the QG-P1
and their cb partners ?
Best regards
Bernard
PS I won't be home for about ten days

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3573
Date: 27/6/2019 9:00:45
From: chris.vantienhoven
Subject: QA-Cu7-Geometry

Dear Bernard, Dear Eckart,
[BK] I hope you will succeed in finding the equation of the 6th degree giving the 6 intersections of the conic and QA-Cu7a and, eliminating the point Sa, the equation of the 5th degree giving the 5 triple points
I tried to calculate the intersection points of the conic and QA-Cu7a.
I supposed the intersection points to be of the form (x:y:1)
I calculated the equation of the 6th degree in x. I also eliminated the point Sa and indeed I found an equation of degree 5 of this form:
$$EQx = T_0 + T_1 x + T_2 x^2 + T_3 x^3 + T_4 x^4 + T_5 x^5 = 0.$$

In a similar way, I can calculate an equation of the form
$$EQy = V_0 + V_1 y + V_2 y^2 + V_3 y^3 + V_4 y^4 + V_5 y^5 = 0.$$

However, the coefficients $T_0, T_1, T_2, T_3, T_4, T_5, V_0, V_1, V_2, V_3, V_4, V_5$ are huge. See attachment. (Editorial note: attachment omitted due to excessive length.)
So hidden in both equations are the coordinates (x:y:1) of the 5 common points of the 3 QL-versions of QA-Cu7.
Nice to know, but these equations are algebraically unsolvable. Like Bernard commented: "Courage, the fight goes on."
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3574
Date: 28/6/2019 9:36:51
From: bernard.keizer
Subject: QA-Cu7-Geometry

Dear Chris,
Congratulations again !
I didn't hope you could solve the equation of the 5th degree, but only prove that the equation is the same for the 3 cubics QA-Cu7a,b and c; this would prove that the 5 points are common to the 3 cubics.
Best regards
Bernard
PS The attachment has disappeared ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3575
Date: 02/7/2019 4:30:58
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

here are further observations for the 5P/QL-constellation wrt the 3 QA-Cu7:

For a 5P we can construct the QL-points QL-P1 and QL-P24
... and the QL-transformation $CSC = QL-Tf1$,
... also the CB-transformation $*cb*$ wrt 5P
and the two circular points.

Properties of the transformation $*cb*$

(some already mentioned):

- (a) cb maps each QA-Cu7 to itself with P , $cb(P)$ collinear QG-P1.
- (b) cb maps 5P-s-Co1 to the line at infinity with $P, cb(P)$ collinear $T = 5P-s-P4$.
- (c) cb swaps the double points of two QA-Cu7.
- (d) The lines $S.cb(S)$ (wrt S see #3176) intersect in $T = 5P-s-P4$,
... perspector of the S - and QG-P1-triangle.
- (e) The lines $QG-P19.cb(QG-P19)$ intersect in V ,
... perspector of the QG-P1- and QG-P19-triangle.
- (f) The lines $QA-P41.cb(QA-P41)$ intersect in W ,
... perspector of the QG-P1- and QA-P41-triangle.

Now consider a 5P = $P1...P5$ and an arbitrary point P ,
... further the 5L of the bisectors PP_i
... and the concyclic CSC-images of P wrt the 4L of 5L,
... which give a circle $*Ci(P)*$.

Properties of the $*Ci(P)*$:

- (1) The circles $Ci(P)$ have a common point $U = CSC(QL-P24)$.
- (2) P and $cb(P)$ have the same circle $Ci(P) = Ci(cb(P))$.
- (3) For points P on the conic 5P-s-Co1
... $Ci(P)$ degenerates to a line through U .
- (4) For QL-P1 and its cb -image on QL-P1.P8.P24 we get
... a circle through QL-P1, $U = CSC(QL-P24)$, $CSC(T) = CSC(5P-s-P4)$.
- (5) For the double points of QA-Cu7a,b, which are cb -partner,
... the circles are the same, bearing U , QA-P4c, QA-P41a,b.
- (6) The circles $Ci(QG-P1)$ bear U , QA-P41 and a 2nd common point.
- (7) The circles $Ci(QA-P4)$ bear U , QA-P41 and QA-P4;
... the 2nd intersection of $Ci(QA-P4a,b)$ lies on $Ci(QA-P41c)$.
- (8) The circles $Ci(QG-P18)$ bear U , QA-P41;

- ... the 2nd intersection of $Ci(QG-P18a,b)$ lies on $Ci(QG-P19c)$.
- (9) The circles $Ci(QA-P41)$ bear U , $QA-P41$,
 - ... the 2nd intersection of two $Ci(QA-P4)$
 - ... and a further common point;
 - ... their centers lie on the $QG-P18$ -line of the QL .
- (10) The circles $Ci(QG-P19)$ bear U , $QA-P41$ and a 2nd common point.
- (11) The circles $Ci(QA-P2)$ bear U , $QA-P41$;
 - ... the 2nd intersection of $Ci(QA-P2a,b)$ lies on $U.QA-P41c$.
- (12) The circles $Ci(S)$ degenerate to lines $U.QA-P41$.
- (13) The circles $Ci(P)$ for the six QL -points
 - ... have common point U , 3 double and 4 triple intersections;
 - ... the three double intersections are $QA-P41a,b,c$.
- (14) The locus for points X with $Ci(X)$ through $QL-P1$
 - ... is the locus of cb -partner on lines through $QL-P24$:
 - ... This is a cb -invariant circumcubic of $5P$,
 - ... through $QL-P1$, $cb(QL-P1)$, $QL-P24$, $cb(QL-P24)$
 - ... and the 2nd intersection of $QL-P24.5P-s-P4$ and $5P-s-Co1$,
 - ... whose cb -image is the infinity point of the cubic.
 - ... The tangential of $QL-P24$ wrt this cubic is $cb(QL-P24)$.
 - ... If we consider the CB -transformation wrt $5P$, $QL-P24$, $cb(QL-P24)$,
 - ... this cubic is also the locus of CB -partner on parallels to $QL-P24.5P-s-P4$.

I think the transformations $P \rightarrow Ci(P)$ and $P \rightarrow cb(P)$
 ... will be relevant for the $5P/QL$ -constellation wrt
 $QA-Cu7$ -points,
 ... but the way back from the $5P$ to the QL isn't visible not yet.
 The results above are only $CABRI$ -observations!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3576
Date: 04/7/2019 12:05:38
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,

I included the special new points in a 5P, 6P, 7P as mentioned in messages #3549-3552 in EPG.
See 5P-s-P5, 6P-s-P1, 7P-s-P1.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3577
Date: 04/7/2019 8:47:50
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

there is also a point 4P-3L-Tf:
... With Tf the inversion wrt the circumcircle of the 3L
... you get QA-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3578
Date: 04/7/2019 10:32:12
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart,

4P-3L-Tf = QA-P4 is a nice point fitting in front of the row with 5P-s-P5, 6P-s-P1, 7P-s-P1.

It shows the relationship between 4-Points, 5-Points, 6-Points and 7-Points.

Similar series in an n-Line are:

- QL-Tf1 - 5L-s-Tf1 - 6L-s-Tf1
- X(186) - QL-P28 - 5L-s-P2
- X(256) - QL-P29 - 5L-s-P3
- X (4) - QL-P20 - 5L-s-P7
- 5L-s-P1 - 6L-s-P1 - 7L-s-3P1 - 8L-s-P1

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3579
Date: 06/7/2019 9:46:05
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

Dear Bernard, dear Chris,

in #3575 I described circles $C_i(P)$, here an application:
Consider a 5P, its circumconic 5P-s-Co1 and points P on it:
... the circles $C_i(P)$ degenerate to lines through $U = 5P-s-P_5$
... and the intersections of $P.T = P.5P-s-P_4$ and $C_i(P)$
... give an orthogonal hyperbola
 through $T = 5P-s-P_4$ and $U = 5P-s-P_5$,
... centered in the midpoint of $U.T$
... and intersecting 5P-s-Co1 in three further points X beside T,
... which give a triangle with orthocenter U
... and a circumcircle through $QL-P_1$ and T
... and the property X on $C_i(X)$ for its vertices
 (see also #3548).

Maybe this is a reference triangle to find the lines of the QL
... with the 5P as its QA-Cu7-triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3580
Date: 06/7/2019 12:50:27
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

Deze Eckart,
You're doing a great job finding alle these wonderful properties
of the 3xQA-Cu7-constellation
Right niet In AM on holiday. When I am back I will look further.
Best

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3581
Date: 08/7/2019 8:42:49
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Bernard, dear Chris,

in EPG / 5P-s-P3 there is described a circle with 5P-s-P3 as
center,
... this circle bears the midpoint of $T = 5P-s-P4$ and $U = 5P-s-P5$
... as well as the midpoint of $QL-P1$ (see PS) and $U = 5P-s-P5$.
... This circle is the nine-point circle
 of the triangle in #3579,
... bearing the center of the orthogonal hyperbola in #3579,
... thus 5P-s-P3 is the nine-point center
 of the triangle in #3579.

The orthogonal hyperbola in #3579 can easier be described:
... centered in the middle of TU,

bearing T and U,
... axes parallel to those of 5P-s-Co1.

The intersections -beside T- with 5P-s-Co1 give the triangle in
#3579.

Best regards Eckart

PS: I think, the point $QL-P1$ of the QL with the 5P-vertices as
QA-Cu7-triple points
 ... should be mentioned in EPG as 5P-s-P6.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3582
Date: 08/7/2019 9:56:53
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Deze Eckart,
Could youne exactly describe 5P-s-P6 as a 5P-point?
Probably I misser it.
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3583
Date: 09/7/2019 11:39:59
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

in #3270 (16.11.2018) I described
... a construction for a 5P-CSC-transformation,
... which leads to $QL-P1 = 5P-s-P6$
... for the QL with the vertices of the 5P
... as QA-Cu7-triple points.

CSC for the vertices of a 5P = $P1...P5$:

Let $Q1$ be $P1$

... $Q2$ the inverse of $P1$ wrt the circle $(P3,P4,P5)$,
... $Q3$ the inverse of $P1$ wrt the circle $(P2,P4,P5)$,
... $Q4$ the inverse of $P1$ wrt the circle $(P2,P3,P5)$,
... $Q5$ the inverse of $P1$ wrt the circle $(P2,P3,P4)$,
... then $CSC(P1)$ is
... .. the inverse of $P2$ wrt the circle $(Q3,Q4,Q5)$,
... .. or the inverse of $P3$ wrt the circle $(Q2,Q4,Q5)$,
... .. or the inverse of $P4$ wrt the circle $(Q2,Q3,Q5)$,
... .. or the inverse of $P5$ wrt the circle $(Q2,Q3,Q4)$.

5P-s-P6:

QL-P1 of the QL with lines $Pi.Pj$, $Pi.CSC(Pj)$, $Pj.CSC(Pi)$,
 $CSC(Pi).CSC(Pj)$,
... independent of the chosen 5P-vertices Pi , Pj .

5P-s-P6, a vertex Pi and $CSC(Pi)$ define a 5P-CSC-transformation
for any point.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3584
Date: 09/7/2019 5:58:12
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,

Back from a few days far away from home and computer, I discover your last findings.

It's amazing, I don't know where to start with this point $U = \text{CSC}(\text{QL-P24})$ as extension to the 5P of QA-P4 for the QL (and, if I understand correctly, of the circumcenter X_3 for the triangle), the $\text{Ci}(P)$ and last this rectangular hyperbola fallen from the sky as a cherry on the cake.

I'll try to catch up and to check your properties ...

I'm for the moment totally bluffed ! Congratulations, it's indeed a beautiful and huge work !

Maybe 2 or 3 simple (?) questions :

1) if any 2 cb partners have the same circle $\text{Ci}(P)$ through U , is the converse true ? In this case, which are the 2 cb partners having the circle with diameter UR (CSC of QL-Ci1) as $\text{Ci}(P)$?

2) the same way, if any point on the 5P conic has a line through U as degenerated $\text{Ci}(P)$, which is the line of R ? As you gave so many examples, it's possible that you gave the answer and I missed it ...

3) which is the point on the conic having the CSC of the Dimidium circle as degenerated $\text{Ci}(P)$,

Best regards

Bernard

PS Please be patient, it will take me time to understand and reproduce your beautiful properties

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3585
Date: 09/7/2019 9:26:00
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
As very often, I forgot one last simple (?) question :
at the end of your message 3575, you describe the circular
circumcubic of the 5 points with pivot QL-P24, wich contains
QL-P1 and cb(QL-P1) on the line QL-P1,8 and 24.
More generally, it contains 2 cb partners on any line through
the pivot.
Which are the cb partners on the line QL-P13, 17 and 24 ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3586
Date: 09/7/2019 10:32:42
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

Dear Eckart,

I started to study your messages #3575 and #3579.

First observation:

1. Your circle $C_i(P)$ is actually a 6P-Circle. It is the circle through the six versions of 5P-s-P1. Because the starting point and the vertices can be interchanged producing the same circle it is a 6P-Circle. Let's call it 6P-s-Ci1.
2. The center of 6P-s-Ci1 is 6P-s-P1 !
3. In a 7P the seven versions of 6P-s-Ci1 concur in 7P-s-P1 !

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3587
Date: 10/7/2019 9:34:38
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

Dear Chris,

thanks for a better interpretation of $C_i(P)$ as 6P-s-Ci1,
... but it is the circle through the six 5P-s-P5.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3588
Date: 10/7/2019 3:07:23
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

I read your messages #3270 and #3583 about the construction of a 5P-CSC-transformation.

It's a very peculiar construction and I got some questions.

1. Why do you call $CSC(P1)$ being
... the inverse of $P2$ wrt the circle $(Q3, Q4, Q5)$, etc.?
This deals with inverses at two levels.
The real $CSC=QL-Tf1$ is the sequence of a reflection and an inversion.
2. How did you arrive at taking the MiquelPoint of the QL formed by the 4 lines $Pi.Pj$, $Pi.CSC(Pj)$, $Pj.CSC(Pi)$, $CSC(Pi).CSC(Pj)$?
3. What is according to you the usefulness of this point in relation to the 3 QA-Cu7 configuration?
By the way the first part of your description is identical to the construction of QA-Tf16, but I suppose you know.
Using the functionality of QA-P16 the construction is (when I did interpret it well):
 - Let $Ri = QA-Tf16(Pi)$ wrt QA=5-Point minus Pi .
 - Let $Rj = QA-Tf16(Pj)$ wrt QA=5-Point minus Pj . $5P-s-P6 = QL-P1$ of the QL with lines $Pi.Pj$, $Pi.Rj$, $Pj.Ri$, $Ri.Rj$, which delivers a point independent of the chosen 5-Point-vertices Pi , Pj .

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3589
Date: 10/7/2019 3:28:30
From: chris.vantienhoven
Subject: 5P orthogonal hyperbola

Dear Eckart, Dear Bernard,

I like the double-function of the points T, U, "QL-P1" or 5P-s-P4, 5P-s-P5, 5P-s-P6.
They function in the Reference QL as T, U, "QL-P1", and
they function in the 3xQA-Cu7-5P as 5P-s-P4, 5P-s-P5, 5P-s-P6.

Eckart, you wrote:
"Maybe this is a reference triangle
to find the lines of the QL
... with the 5P as its QA-Cu7-triple points."

It even would be better to find the 5P from these known and related items:
* the conic through the 5P
* T, U
* Sa, Sb, Sc
Don't you think there is a double function for Sa, Sb, Sc?
But, I doubt severely that even when knowing all these items we will be able to find the 5P.
I think there should be an alternative way.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3590
Date: 10/7/2019 8:34:24
From: eckart_schmidt@t-online.de
Subject: 5P orthogonal hyperbola

Dear Chris,

the main problem for Bernard and me is
... to get back from the 5P to the QL,
... whose 5 QA-Cu7-triple points are the vertices of the 5P.
We already can construct the transformation CSC
... and the points QL-P1, QL-P24,
... U = perspector of the QA-P4- and QG-P19-triangle,
... T = perspector of the QG-P1- and the S-triangle.
If we are able to "construct" the QL,
... we can use QL-points for 5P-geometry.
But I think, this is not the background of your message 3589.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3591
Date: 12/7/2019 10:42:37
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

thanks for your interested cooperation,
... but I cannot give answers wrt your questions in #3584.
I tried in vain to get the relevant point R, starting with a 5P,
... but there is a new property of R wrt the QL:
The QL-point R is the CSC-image of
... the 2nd intersection of QL-P1.10.16 and QL-Ci1.
If you study the circles $Ci(P)$ in #3575:
... there are further observations for a QG-version of the QL:
The line $Ci(S)$ and the circles
... $Ci(QA-P2)$, $Ci(QA-P4)$, $Ci(QG-P18)$, $Ci(QG-P19)$, $Ci(QA-P41)$
bear QA-P41
... with a 2nd triple point for $Ci(QG-P19)$ and $Ci(QA-P41)$
... and $Ci(QA-P4)$, $Ci(QA-P41)$ bearing their generating point.
Wrt the perspectors V and W (see #3176):
... $Ci(V)$ bears the 2nd triple point of $Ci(QG-P19)$,
... $Ci(W)$ bears the 2nd triple point of $Ci(QA-P41)$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3592
Date: 12/7/2019 10:44:34
From: bernard.keizer
Subject: 5P orthogonal hyperbola

Dear Eckart,
I started from the end with your rectangular hyperbola.
I understand why X on the conic and the infinity point on TX give the same 2nd intersection of TX with the RH and therefore the same line $Li(X)$ through U and this 2nd intersection (that goes for T with the tangent in T).
I have reproduced your construction and I'm able to answer myself my questions 2) and 3) in message 3584.
The line $Li(R)$ doesn't give anything particular and the point X having $CSC(\text{Dimidium circle})$ as $Li(X)$ doesn't seem to be particular either.
It remains my 1st question in message 3584 and my question in message 3585.
Which are the 2 cb partners X and $cb(X)$ having the circle with diameter UR as $Ci(X) = Ci(cb(X))$?
Which are the 2 cb partners on the line $QL-P13$, 17 and 24 ?
The same question goes for the line $QL-P16$ and 24 ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3593
Date: 12/7/2019 10:54:17
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Your answer crossed mine.
For the point R , I put this property already several times on the forum without reaction :
 $CSC(R)$ is the inverse of $QL-P1$ in the inversion with center $QL-P10$ swapping the $QG-P1$ and the $QG-P17$, id est the inversion with center $QL-P10$ swapping the circumcircle $QL-Ci1$ and the Euler circle $QL-Ci2$.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3594
Date: 12/7/2019 5:09:57
From: bernard.keizer
Subject: 5P-s-P4

Dear Chris, dear Eckart,

Dealing with 7 points P1 to 7, it's well known that they form a CB transformation swapping the 21 conics line/conic. Taking for example the line 67 and the conic 12345 and 45 and 12367, the 2 lines intersect in a point U123 and the 2 conics in a 4th point (other than 123) V123. U123 and V123 are CB partners and the line U123V123 intersects the conics in 2 2nd points (other than V123) T67 and T45. It's possible this way to construct the 21 pivots of the 21 conics for this CB transformation. It is remarkable that for each conic, the pivot is exactly the point T or 5P-s-P4 on the conic depending only from the 5 points on this conic. Perhaps it was obvious after all, but I was surprised for myself ...

Best regards
Bernard

PS Does that mean that the circular points are CB partners for the transformation of any group of 7 points ?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3595
Date: 13/7/2019 10:14:00
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
In fact, this property is easy to prove synthetically !
As R is the perspector of the 3 QA-P4 and the 3 QG-P18, $CSC(R)$ belongs to the 3 CSCcircles of the 3 lines concurring in R . The 3 circles are coaxial and pass through $QL-P1$ and $CSC(R)$; the intersection of the lines $QL-P1CSC(R)$ and $QG-P1QG-P17$ is the same for the 3 couples of $QG-P1$ and $QG-P17$, id est the orthocenter $QL-P10$.
The power of $QL-P10$ wrt the circles is $QL-P10QL-P1 * QL-P10CSC(R)$ = $QL-P10QG-P1 * QL-P10QG-P17$, hence the inversion swapping the circumcircle of DT $QL-Ci1$ and it's Euler circle $QL-Ci2$.
I hope I have convinced you this time ...
Best regards
Bernard
PS $QL-P16$, reflexion of $QL-P10$ in $QL-P1$ is in EQF , but neither this point $CSC(R)$, as you mention 2nd intersection of $QL-P10QL-P1$ with $QL-Ci1$, nor the middle of $QL-P10CSC(R)$ lying on $QL-Ci2$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3596
Date: 13/7/2019 3:36:13
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,
excuse, that I hadn't parat your property of R ,
... thanks for synthetical justification.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3597
Date: 13/7/2019 5:19:33
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
As usual, my figures lack of precision !
Is it possible that under the 2 cbpartners on QL-P24QL-P17, one
is QL-P13 (on the circular circumcubic you described with pivot
QL-P24 and on all circular circumcubics with pivot on this
line)?
(The other is not QL-P17)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3598
Date: 14/7/2019 12:08:46
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard,

sorry, QL-P13 is not a cb-partner on QL-P24.QL-P17,
... but I cannot find any properties,
... as well as for the cb-partner on QL-P24.QL-P16.
But there is following observation (see #3575):
For cb-partner on lines through QL-P24
... the circle $C_i(X)$ bears $U = 5P-s-P5$ and $*QL-P1^*$.
In general:
For cb-partner on lines through a point P
... the circle $C_i(X)$ bear U and a further common point.
What about the point P with further common point R?

Best regards Eckart

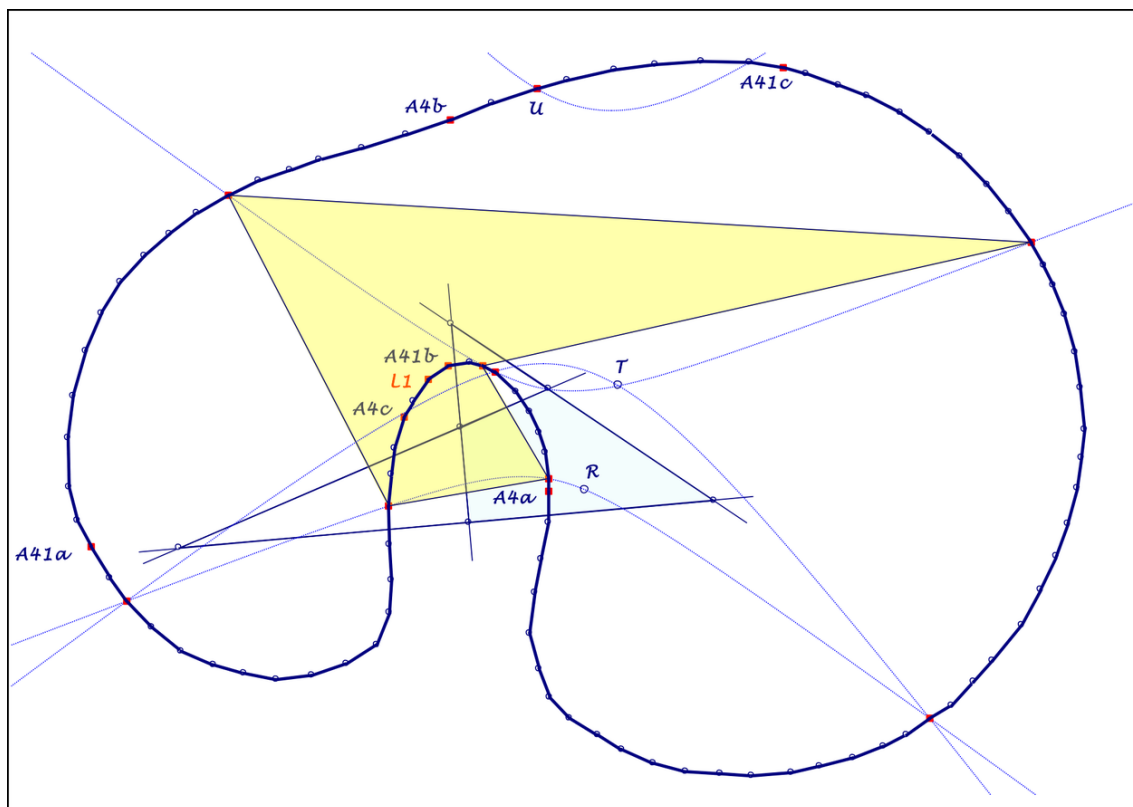
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3599
Date: 14/7/2019 2:32:26
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Chris, dear Bernard,

the circles $C_i(P)$ (see #3575) lead to an interesting quartic
... for the 5P/QL constellation wrt QA-Cu7,
... bearing 16 well known points,
... attached an approximate pointwise drawing.
The quartic is the locus of points X with X on $C_i(x)$,
... bearing the 5 QA-Cu7-triple points,
... the points $U = 5P-s-P5$ and $QL-P1 = 5P-s-P6$,
... the 3 QA-P4 and the 3 QA-P41
... and the 3 intersections beside $T = 5P-s-P4$
... of $5P-s-Co1$
and the orthogonal hyperbola in #3579.

Best regards Eckart



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[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3600
Date: 14/7/2019 4:03:47
From: bernard.keizer
Subject: 5P-Geometry

Dear Eckart,
Thanks a lot for your patience !
As you can see, I follow the ad of Avis "We're second, but we try harder!"
I hoped you would be able to identify the 2 cb partners having the circle with diameter UR as C_i .
But your new property gives more chances to identify the point P such as the C_i 's of all the cb partners on lines through P are centered on the perpendicular bisector of UR ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3601
Date: 14/7/2019 4:18:49
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Beautiful ! Marvellous ! (Thanks for the figur)
According to your definition, I suppose the quartic is cb invariant, which gives 16 more points !
If I'm not wrong, there are 10 points under the 16 which are determined only from the 5 points.
With their cb partners, that makes 20 !
A quartic being determined by 14 points, I think your quartic is entirely constructible from the 5 points !!!
Is this assumption correct ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3602
Date: 14/7/2019 5:22:23
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Sorry, my assumption is not correct !
The summer is very hot in the Dordogne and the brains are suffering too !!!
After a short bath in my swimming pool, I realised that the fact that a point X and it's partner $cb(X)$ have the same $Ci(X)$ doesn't mean that $cb(X)$ is on $Ci(X)$ if X is on $Ci(X)$.
 $cb(QL-P1)$ doesn't lie on the circle through $QL-P1, U$ and $CSC(T)$ and the cb partners of your 3 points on the RH are infinity points on lines through T ...
I very much apologise for my wrong conclusions.
Perhaps, it would be interesting to look for the cb curve of your quartic?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3603
Date: 14/7/2019 10:46:36
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
I think your quartic is the CSC of a cubic through the CSC of the 5 points, the 3 DT vertices QG-P1 (CSC of the QA-P4), the CSC of the QA-P41 (intersections of QG-P1QA-P4 and the DT sides), the CSC of your 3 points intersections(other than T) between the conic of the 5 points and the rectangular hyperbola and QL-P24.
Any circle through QL-P1 cuts the quartic in 3 other points and the CSC of the 3 points are aligned on the cubic
This time, as the cubic may be described from the 5 points, the same goes for the quartic.
I hope this time I didn't make a basic mistake ...
Best regards
Bernard
PS I suppose the cubic may be described as a non pivotal isocubic wrt DT with root the trilinear pole of the line of the CSC(QL-P41)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3604
Date: 14/7/2019 11:44:35
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
The trilinear pole of the line of the CSC(QA-P41) is the point W, perspecter of the QG-P1 and the QA-P41.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3605
Date: 15/7/2019 10:49:48
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
The more I look on the figures, the more I get excited !
In the construction of the rectangular hyperbola, you use the point 5P-s-P3 and the QA-P4 of the 5 QAs.
Your beautiful quartic passes through the 5 QA-P4, which gives you 5 more points; that makes with the 5 P, the points U and QL-P1 and the 3 intersections of the conic with the RH 15 points which depend only from the 5 P.
Your quartic is perfectly dtermined !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3606
Date: 15/7/2019 10:54:53
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

your cubic as CSC of the quartic is a remarkable obvious curve,
... thanks for discovery and detailed description,
... I try to reproduce and get a construction.
What is the isoconjugation for this isocubic?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3607
Date: 15/7/2019 3:24:54
From: eckart_schmidt@t-online.de
Subject: 5P-Quartett

Dear Bernard,

I gather our observations:
The 5 QA-Cu7-triple points wrt a QL lie on a quartic,
... which is the CSC-image of a nonpivotal isocubic:
... .. reference triangle QL-Tr1,
... .. isoconjugation: isogonal conjugate,
... .. point: QL-P24,
... .. root: W = perspector of the QG-P1- and QA-P41-triangle.

Best regards Eckart

PS: Point W is also the isogonal conjugate of T wrt QL-Tr1.

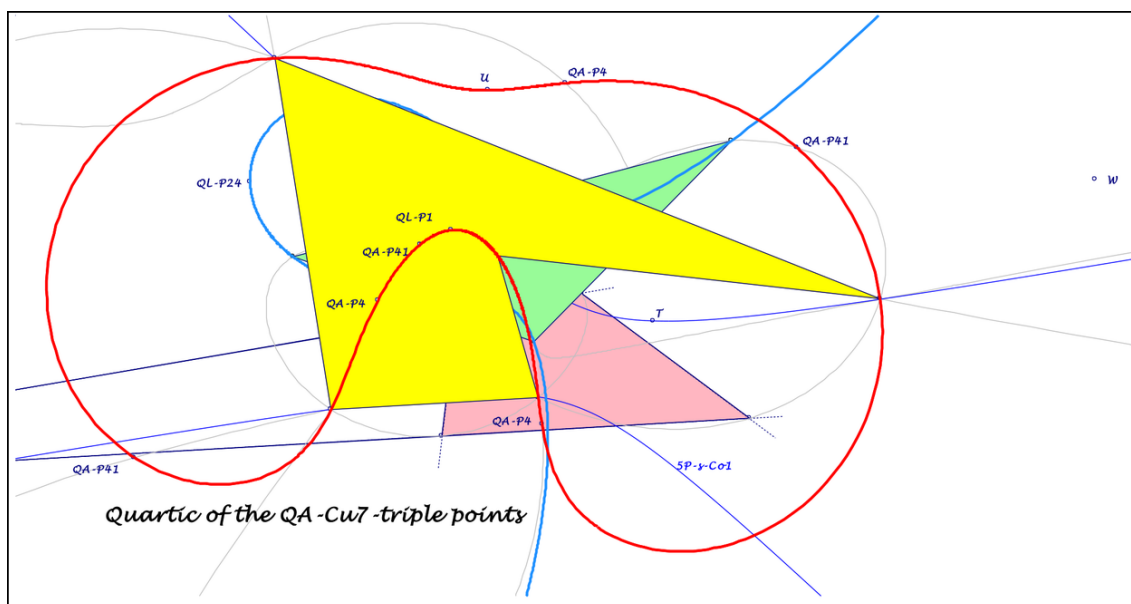
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3608
Date: 15/7/2019 4:50:29
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

I try to transmit a construction of the quartic and its
CSC-cubic (word file).

Best regards Eckart



2019-07-15.docx

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3609
Date: 15/7/2019 5:09:24
From: bernard.keizer
Subject: 5P-Quartett

Dear Eckart,
The cubic is also a pivotal circular circumcubic of the 5 CSC of the triple points.
QL-P24 is the tangential of the middle of UT (on the cubic) and the 6th intersection of the cubic with the conic of the 5 CSC is a point S' tangential of QL-P24.
With T' the point 5P-s-P4 of the 5 CSC, the asymptote of the cubic is T'S' and it seems the pole for the CB transformation wrt the 5 CSC points and the 2 circular points is the infinity point of the asymptote.
Best regards
Bernard
PS Could you please send me your picture in pdf file. Thanks in advance

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3610
Date: 15/7/2019 5:42:16
From: bernard.keizer
Subject: 5P-Quartett

Dear Eckart,
The cubic is also a beautiful Van Rees curve, invariant in the CSC with center QL-P24 swapping the QG-P1 and the QA-P41 ...
But the interest of defining it only from the 5 CSC of the triple points is that it allows the reverse operation with the 5 triple points in order to find what I named the twin QL !
Best regards
Bernard
PS We only need now a definition of R ...

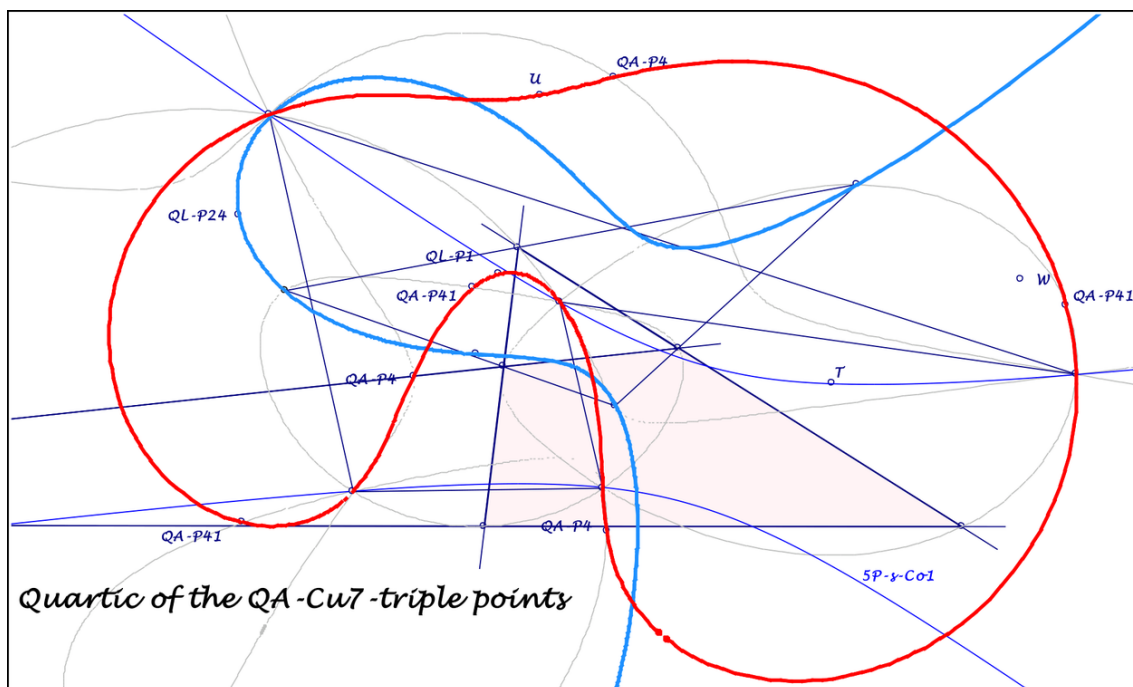
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3611
Date: 16/7/2019 9:01:33
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard, dear Chris,

I try once more, to transmit to QFG a construction
... of the quartic and its CSC-cubic described in #3607.

Best regards Eckart



2019-07-15.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3612
Date: 16/7/2019 10:13:58
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Thank you for the figures ; they look like mine.
I'm waiting impatiently for your comments on my messages 3605,
3609 and 3610.
I tried several ideas wrt the point R (isogonality or trilinear
polar wrt the triangle of the 3 intersections of conic and RH
...), but nothing works !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3613
Date: 16/7/2019 11:57:03
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

wrt your messages #3609, #3610:

- (1) I cannot reproduce,
... that the middle of UT lies on the cubic (see figur).
- (2) The point S' seems interessant with your observed
properties,
... they are new for me, especially S'T' as asymptote
of the cubic,
... the point at infinity seems to be $\text{isog}(\text{QL-P24})$ wrt QL-DT
or $\text{cb}'(\text{S}')$.
- (3) I cannot reproduce,
that the cubic is invariant (see figure)
... "in the CSC with center QL-P24 swapping the QG-P1
and the QA-P41".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3614
Date: 16/7/2019 2:44:26
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

wrt (3) in my last message: I think you mean:
The cubic is invariant
... "in the CSC with center QL-P24 swapping the CSC(QG-P1)
and the CSC(QA-P41)".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3615
Date: 16/7/2019 4:14:54
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,

Thanks for your answer to my messages 3609 and 3610

1) Thanks for having checked the property : it was only a coincidence on my own figur

2) These are classic properties of a circular circumcubic of 5 points applied to the CSC of the 5 triple points.

New is that the pivot can be the infinity point on the line ...

3) Sorry for my mistake : it was obviously the QG-P1 and the CSC of the QA-P41 !

The cubic is the QL-Cu1 of the QL formed by the DT vertices and the points CSC(QA-P41) which lie on the DT sides and are aligned. The asymptote is T'S' which is parallel to the Newton Line of this new QL ...

4) You didn't answer my message 3605 : can you confirm that the QA-P4 of the 5 QA's of the 5P are on your quartic (and therefore their CSC on the cubic)

In this case, having the 5 P (which determine a CSC) and their CSC partners, it's easy to draw the 2 reciprocal cubics through 5 points and the CSC of the QA-P4 of the CSC of the 5 points and vice-versa.

This would lead (so I hope) to the 2 twin QL's ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3616
Date: 16/7/2019 8:46:00
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

excuse, my interpretation of CSC wrt QL-P24 was not right,
... your correction in #3615 is correct.

Of course I can confirm,
... that the five QA-P4 of the 5P lie on the quartic,
... making possible, to define the quartic starting with the 5P.
... Thanks, this was new for me!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3617
Date: 17/7/2019 10:32:19
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Thanks for this confirmation !
Then we have easily the 2 reciprocal cubics, which give 2 new points S and S' as 6th intersection with the 2 conics. What about these 2 new points ?
As these 2 cubics are Van Rees curves, they may be monocursal or bicursal ...
The 2 cubics are defined for an ellipse as well as for an hyperbola (5P may be on an ellipse or an hyperbola and their CSC the same way on an ellipse or an hyperbola ?)
I won't be home again for a couple of days, but as soon as I come back, I'll check new drawings ...
Best regards
Bernard
PS I think we are very near from the goal now

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3618
Date: 18/7/2019 2:54:24
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

searching for an old message I recognized
... that I have not read your #3588,
please accept my apologies!

Wrt 1. Reasons for my observations are only CABRI-constructions.
... CSC(P1) is the real CSC for the QL with QA-Cu7-triple points
in the 5P.

Wrt 2. If we have CSC(Pi), we can construct QL-P1
... and the Steiner axes as well as the inversion circle.

Wrt 3. Perhaps there will be no direct relation
... between QL-P1 and the QA-Cu7-triple points,
... but the transformation CSC is a relevant tool,
... to study the QL/5P constellation wrt QA-Cu7.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3619
Date: 18/7/2019 11:42:07
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Thanks for your answer.

Now I understand that:

* QA-P16(P_i) wrt 4 pts of the 5 TriplePoints $P(i) = QL-Tf1(P_i)$

wrt Ref-QL

* 5P-s-P4 wrt the TriplePoints-5P = T = perspector of 2
Ref-QL-triangles

* 5P-s-P5 wrt the TriplePoints-5P = U = perspector of 2
Ref-QL-triangles

* 5P-s-P6 wrt the TriplePoints-5P = QL-P1 wrt Ref-QL

Which other 5P<>QL-links there are?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3620
Date: 22/7/2019 6:35:02
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris, dear Eckart,
We are looking for links 5P-QL.
As the QL defines the 5 P, every 5P point is a QL point, but we wish a reverse construction from the 5P to the QL !
The points $T = 5P-s-P4$ is the isogonal conjugate of W wrt DT, where W is the perspector of the QG-P1 and the QA-P41 and the trilinear pole of the line through the CSC(QA-P41) wrt DT.
The point $U = 5P-s-P5$ is CSC(QL-P24)
The point $QL-P1 = 5P-s-P6$ is center of the CSC of the initial QL.
Last, but not least, we have this wonderful cubic and it's CSC the quartic.
The cubic is defined by 10 points which are the 5 CSC of the 5P and the 5 CSC of the QA-P4 of the 5 QA's of the 5P ; this cubic is a circular circumcubic of the 5 CSC of the 5P (asymptote passing through the 5P-s-P4 of this 5P and through the 6th intersection between the cubic and the conic of this 5P).
This cubic is also the QL-Cu1 of the QL formed by the QG-P1 and the CSC(QA-P41) with focus QL-P24.
Only the point R which is the CSC of the inverse of QL-P1 in the inversion with center QL-P10 swapping the QG-P1 and the QG-P17 or the circumcircle and the Euler circle of DT, QL-Ci1 and QL-Ci2 remains an enigma for the 5P ...
Best regards
Bernard.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3621
Date: 23/7/2019 10:01:56
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

some new observations wrt a $5P = (P_1, \dots, P_5)$ and a point P :
Let $T = 5P-s-P_4$ and $T_i = 5P-s-P_4$ of P
and four vertices unequal P_i ,
... so we get $5T = (T_1, \dots, T_5)$ with following properties:
(1) $P_i.T_i$ parallel $P.T$ with midpoints on a line through $5P-s-P_1$.
(2) Point P lies on the circumconic of $5T$.
(3) $5P$ and $5T$ have the same area, if they are konvex.
(4) $5T$ degenerates collinear with P , *if $cb(P) = P^*$ wrt $5P$
(wrt cb see #3575),
... on a line orthogonal $Ci(P)$ wrt $5T$ (wrt $Ci(P)$ see #3575).
(5) $Ci(P)$ wrt $5T$ degenerates always collinear.
(6) $cb(P)$ wrt $5T$ is always an infinity point.
(7) If P is a $5P$ -vertex, one vertex of $5T$ is T .
(8) If P is $5P-s-P_4$ of $5T$ on $5P-s-Co_1$,
... $5T$ is the reflection of $5P$ in the 2nd axis of $5P-s-Co_1$.

Best regards Eckart

PS: Excuse, no connection with our 5P-QL-problem.

Thanks to Bernard for the summary of our observations.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3622
Date: 23/7/2019 10:11:42
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard, dear Chris,

several times I used the CB-transformation cb,
... defined by the 5 vertices of a 5P and the circular points.

Here are some doubtful observations
... wrt the 5P of the QA-Cu7-triple points of a QL:

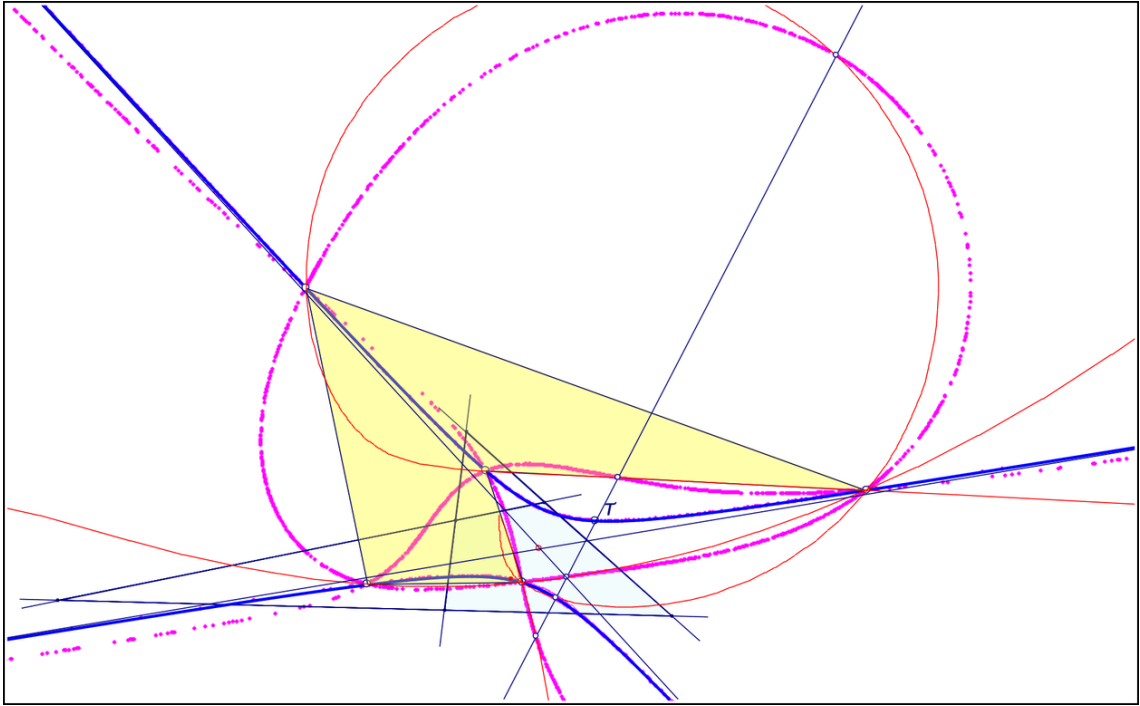
The 5P of the QA-Cu7-triple points seems to be
... a convex QG = P1P2P3P4 and an inner point P5.

Only 5P with this property shall now be considered:
... Looking for points X with $cb(X) = X$,
... we get three curves:
... one circumscribed QG (perhaps a quartic),
... two through one pair of opposite QG-points
... ... and an infinity point of 5P-s-Co1 (perhaps cubics).

Attached a drawing,
... using lines through $T = 5P-s-P4$ and its cb-image,
... intersecting in a 2nd point on 5P-s-Co1
... with cb-image at infinity
... and further intersections, which are cb-fixed.
Perhaps someone can confirm these observations.

Best regards Eckart

PS: What is the geometrical background for the property $X = cb(X)$?



2019-07-23.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3623
Date: 23/7/2019 10:15:19
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Eckart,
Waiting for your comments or new ideas, I try to find new properties.
The cubic described in message 3620 is is circular circumcubic of either the CSC of the 5P or the CSC of the QA-P4 ; both conics cut the cubic in a point Q or q and the asymptote is TQ or tq.
Q and q are CSC in the CSC of the QL of the QG-P1 and the CSC of the QA-P41.
The pivot of the cubic is the infinity point of TQ and tq ...
The point q seems to be the 9th CB point of the 5 CSC of the 5 P and the 3 QG-P1.
I found the same way the 9th CB point of the 5 P and the 3 QG-P1, but I can't identify this point.
By the way, you mentionned some time ago a circular circumcubic of the 5 P and the 3 QG-P1 (and their cb partners), which of course passes through this 9th CB point. Have you identified the pivot ?
Last, you mentionned also that for a point P, the Ci of the 2 cb partners on the lines through P pass through the point U and a 2nd point Q (and are centered on the perpendicular bisector of UQ).
For example, if P is the pivot above, the point Q is the 2nd intersection of the Ci(QG-P1). Have you identified this 2nd intersection ?
Last, can you identify the point P for which the point Q is R ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3624
Date: 23/7/2019 11:02:42
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard, dear Chris,

excuse, a property is missing for the two cubics,
... they bear the 5th inner point P5.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3625
Date: 23/7/2019 3:38:11
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

Dear Eckart,
Sorry, don't understand your definition!
As I already told you, the summer is particularly hot ...
cb-image of what?
intersections of what with what?
Waiting for a pedagogical explanation, many thanks in advance
for your patience !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3626
Date: 24/7/2019 9:39:14
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard,

I try, to describe my observations once more:
Consider special 5P with
... 4 points in the vertices of a convex QG = P1P2P3P4
... and an inner 5th point P5,
... further the cb-transformation, defined by this 5P.
... Looking for points X with $cb(X) = X$,
we get three curves:
... one circumscribed the QG (perhaps a quartic),
... two through P5 and a pair of opposite QG-points
... and an infinity point of 5P-s-Co1 (perhaps cubics).
Wrt the drawing (see #3622):
Take lines L through T = 5P-s-P4,
... their cb-image $cb(L)$ is a curve of higher degree,
... intersecting L in a point on 5P-s-Co1 with cb-image at
infinity,
... further intersections of L and $cb(L)$ are fixed points of cb.

Best regards Eckart

PS: You are right: Hot summer time!
I have the same problem with your message 3623.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3627
Date: 24/7/2019 12:36:22
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard,
You wrote in your message 3620:
"Only the point R which is the CSC of the inverse of QL-P1 in
the inversion with center QL-P10 . . . "
I understand from your message that R is the CSC of an inverse.
But the inverse wrt which circle?
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3628
Date: 24/7/2019 12:38:28
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

I examined the points T,U,W and their background.
They all come forth from points on QA-Cu7 in a QL-environment.
I reconsidered that and found some new points as well.

Here a summary of old and new points:

Point *R* = perspector of QA-P4-triangle and QG-P18-triangle.

Sa = QA-P41a.QG-P19a ^ QA-P4a.QG-P18a

Sb = QA-P41b.QG-P19b ^ QA-P4b.QG-P18b

Sc = QA-P41c.QG-P19c ^ QA-P4c.QG-P18c

Point T = perspector of QL-Tr1 and S-triangle.

Point U = perspector of QA-P4-triangle and QG-P19-triangle. Also
CSC(QL-P24)

Va = QG-P18b.QG-P19b ^ QG-P18c.QG-P19c

Vb = QG-P18c.QG-P19c ^ QG-P18a.QG-P19a

Vc = QG-P18a.QG-P19a ^ QG-P18b.QG-P19b

Point W = perspector of QG-P1-triangle and V-triangle.

New:

Point X = perspector of QG-P1-triangle and QA-P41-triangle.

Point Y = perspector of QG-P1-triangle and QG-P19-triangle.

Point Z = perspector of QG-P19-triangle and V-triangle.

Special properties:

Sa, Sb, Sc, R and T lie on the Triple-Points-Conic (TPC).

QL-Tf1[U] = QL-P24

QL-Tf1[U] and QL-Tf1(R) both lie on QL-Ci1.

QL-Tf1(R) = the 2nd intersection point of QL-Ci1 ^ QL-P1.QL-P10
(1st is QL-P16)

QL-Tf11(T) = the line through QL-P1 parallel to QL-P3.QL-P4

QL-Tf11(X) = a line through QL-P26 (surprise, surprise)

T and X are isogonal conjugated wrt QL-Tr1.

Relationship with the 5 TriplePoints:

- QA-P16(Pi) wrt 4 pts of the 5 TriplePoints P(i) = QL-Tf1(Pi)
wrt Ref-QL
- 5P-s-P4 wrt the TriplePoints-5P = T = perspector
of QL-Tr1 and S-triangle
- 5P-s-P5 wrt the TriplePoints-5P = U = perspector

- of QA-P4-triangle and QG-P19-triangle
- 5P-s-P6 wrt the TriplePoints-5P = QL-P1 wrt Ref-QL

I wonder if the points R, W, X, Y, Z will have some 5P-relationship too.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3629
Date: 24/7/2019 2:27:01
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

thanks for a further summary of our results.
The points X and Y are not new,
... they are W and V in message 3176.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3630
Date: 24/7/2019 3:05:09
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris,
It follows from the given definition that the inversion circle ,
centered in QL-P10, is orthogonal to the 3 circles with diameter
QG-P1QG-P17 and to the circle with diameter QL-P1CSC(R).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3631
Date: 24/7/2019 3:17:50
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris,
Again: T, U and QL-P1 can be defined independently either from
the QL or from the 5P.
R can be defined only from the QL.
The cubic described in my message can also be defined either
from the QL or the 5P.
It is possible to find by approximation the 3 points QG-P1 on
the cubic such as the CSC of the circumcircle of these 3 points
(through CSC(U) and CSC(R)) is the circle with diameter UR.
But it would be nice and more elegant to find a direct
definition of R from the 5P !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3632
Date: 24/7/2019 3:33:51
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

Dear Eckart,
Thanks for the explanation ! I understand now.
Unfortunately, as I'm not able to construct directly a cb partner of a point X, I cannot reproduce your construction of the curve of higher degree.
For $X = cb(X)$, I can only guess that X is it's own tangential on the circular circumcubic of the 5P with pivot X, which means that the cubic is tangent in this point to the curve of higher degree?
Best regards
Bernard
PS My message 3623 had 2 parts, the 1st part concerning the wonderful cubic and the 2nd being in fact composed of precise questions about properties you mentionned before (intersection of the 3 QG-P1cb(QG-P1) or point P such as all cb partners on lines through P have the same 2nd intersection Q with different examples ...). Please just tell me what was unclear

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3633
Date: 24/7/2019 5:23:27
From: bernard.keizer
Subject: Fixed points of CB-transformation cb

Dear Eckart,
For this search, I don't see a difference between 5P on an ellipse or an hyperbola.
Is there any?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3634
Date: 24/7/2019 9:43:09
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

I missed your message #3176 in my busy period.
However, point W was already named in message #3003.
For the sake of unity, I make a new list of these points with
adapted name-giving, following your list.
Point R = perspector of QA-P4-triangle and QG-P18-triangle.

Sa = QA-P41a.QG-P19a ^ QA-P4a.QG-P18a
Sb = QA-P41b.QG-P19b ^ QA-P4b.QG-P18b
Sc = QA-P41c.QG-P19c ^ QA-P4c.QG-P18c

Point T = perspector of QL-Tr1 and S-triangle.
Point U = perspector of QA-P4-triangle and QG-P19-triangle.
Also CSC(QL-P24)
Point V = perspector of QG-P1-triangle and QG-P19-triangle.
Point W = perspector of QG-P1-triangle and QA-P41-triangle.

Xa = QG-P18b.QG-P19b ^ QG-P18c.QG-P19c
Xb = QG-P18c.QG-P19c ^ QG-P18a.QG-P19a
Xc = QG-P18a.QG-P19a ^ QG-P18b.QG-P19b

Point Y = perspector of QG-P1-triangle and X-triangle.
Point Z = perspector of QG-P19-triangle and X-triangle.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

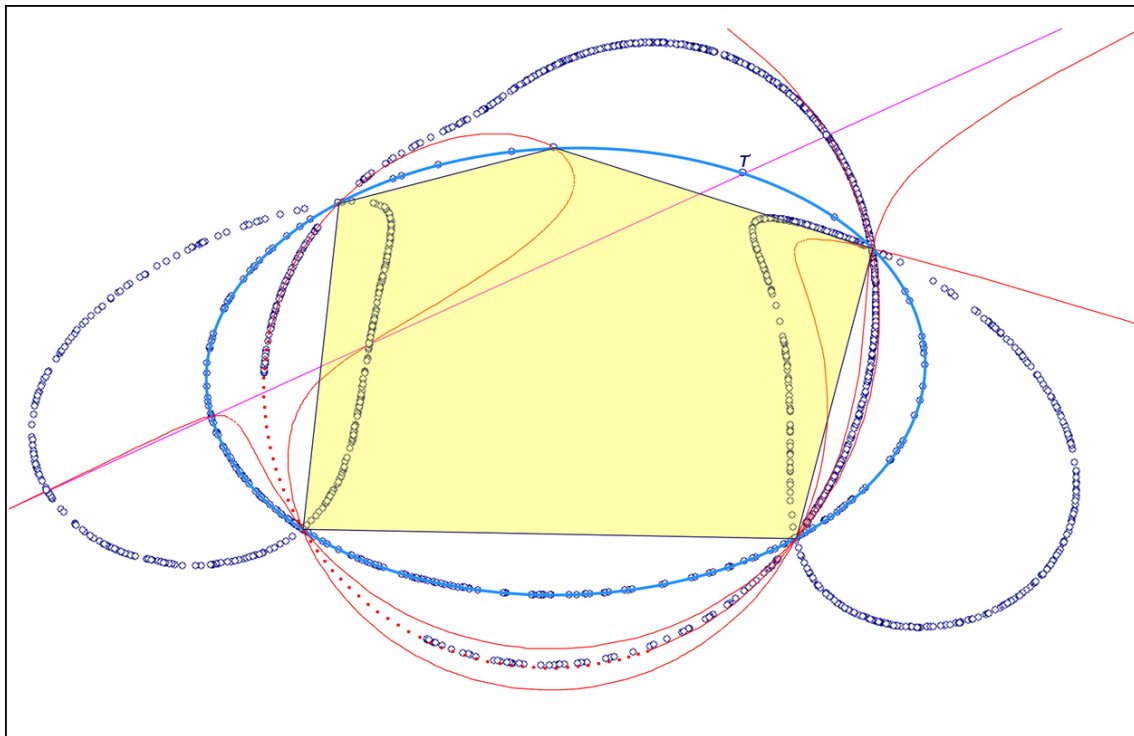
Message: #3635
Date: 25/7/2019 10:20:33
From: eckart_schmidt@t-online.de
Subject: Fixed points of CB-transformation cb

Dear Bernard,

wrt #3633 there are significant other observations
... for the three curves for $X = cb(X)$ wrt a convex 5P,
... have a look on the attached file.

Best regards Eckart

PS: Please have patient with me wrt #3623.



2019-07-24.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3636
Date: 25/7/2019 11:38:16
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

thanks for the last summary.
If we study these numerous perspectivities,
... there will be further related examples
... with a concept, described in #2017:

Two perspective triangles $A_1B_1C_1$, $A_2B_2C_2$
have a third triangle $A_3B_3C_3$
... with $A_3 = B_1C_2 \cap B_2C_1$, $B_3 = A_1C_2 \cap A_2C_1$, $C_3 = B_1A_2 \cap B_2A_1$,
... perspective to $A_1B_1C_1$ in $P_{13} = A_1A_3 \cap B_1B_3 \cap C_1C_3$,
... perspective to $A_2B_2C_2$ in $P_{23} = A_2A_3 \cap B_2B_3 \cap C_2C_3$,
... with collinear perspectors,
... with a pivotal isocubic:
... .. one triangle as reference triangle,
... .. isoconjugation swapping the vertices
 of the other two triangles,
... .. or: isoconjugation swapping the perspectors
 with the other two triangles,
... .. pivot in the perspector of the other two triangles,
... .. bearing the vertices of the three triangles
... .. and their perspectors.

But I have not studied these possibilities.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3637
Date: 25/7/2019 2:08:26
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Wrt your last message: I considered exactly the same. Maybe some other wording.

All these mutually perspective triangles!

Having two perspective triangles it is possible to construct a third triangle like you noted.

This new set of three mutually perspective triangles also forms a desmic configuration with an associated desmic cubic.

See QA-Tr-1.

I found these items:

- * The three triangles P18a.P18b.P18c, P19a.P19b.P19c, Xa,Xb,Xc form a desmic configuration with a degenerated desmic cubic, being the set of 3 lines: P18a.P19a, P18b.P19b, P18c.P19c.

- * Per set of perspective triangles I found no associated desmic cubic passing through the Triple Points.

- * I found no special properties for the desmic cubics, except:

Y lies on QG-P1xQG-P19 desmic cubic and of course also V

V lies on QG-P1xVa.Vb.Vc desmic cubic and of course also Y

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3638
Date: 25/7/2019 7:18:26
From: bernard.keizer
Subject: 5P-s-P3,4,5

Dear Chris, dear Eckart,
I know that repeating is the key of teaching, but I'm now tired of reading always the same properties and points presented with different names !
All these perspective triangles are by definition linked to the QL and won't give what I'm waiting for, a simple definition of R deriving uniquely from the 5P (like T, U, QL-P1 and the cubic merely mentionned).
Of course, it could also be a circular circumcubic of the 5P through the 3 QG-P1 (and their cb partners) with pivot an unidentified point or the 2 cb partners having the circle with diameter UR as $Ci(X)$ or the point such as the cb partners of all lines through it pass through U and R or any other idea giving the QG-P1 directly and uniquely from the 5P ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3639
Date: 25/7/2019 11:25:10
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

Thanks for your essay in #3176. It was very helpful.
Bernard, I appreciate your desire for solving the location of R in a 5P-configuration.
Please have some patience with me. I come from far.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3640
Date: 26/7/2019 9:51:55
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

Searching for a 5P-construction of R, I found another interesting 5P-item.

A transformation mapping a random point P into 2 other points.

Short description:

Let 1.2.3.4.5 be a Pentangle.

Let Cu_1,2,3,4,5 be the QA-Cu7-like cubics with pivot P

Construction (see also QA-Cu-1 and QA-Cu7 in EQF):

Let P1, P2, P3, P4 be the vertices of the Reference Quadrangle.

Let V (u:v:w) be a variable point.

Let Lv be some line through V.

Let IC(Lv) be the Involutary Conjugate (QA-Tf2) of Line Lv.

IC(Lv) is a conic since QA-Tf2 is a transformation of the 2nd degree.

The locus of the intersection of IC (Lv) ^ the perpendicular at V to Lv is a QA-Cubic Type-2.

Properties of this configuration:

- * Besides P there are two other points Q1 & Q2 that lie on all 5 cubics.

- * Q1 & Q2 lie on the polar of P wrt the circumscribed conic of the reference pentangle.

- * When P is on the circumscribed conic then there is only one point Q lying on the P-tangent.

- * The Involutary Conjugate of some point T on the circumscribed conic wrt any component quadrangle lies on the T-tangent.

- * Lines ij and kl intersect in a point on Cu_m (QA-DT-points)

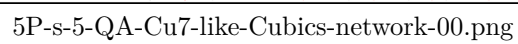
- * Each combination of 3 cubics Cu_i, Cu_j, Cu_k have two intersection points lying on line lm.

See picture.

Maybe it is helpful in some way.

Best regards,

Chris



313

Message: #3641
Date: 27/7/2019 9:23:14
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Bernard, Dear Eckart,

Regarding my last message.

The points Q1 and Q2 generated by a random point P wrt a 5P has extra properties:

- * The 5P-transformation mapping P into Q1 and Q2 is actually a C0-transformation.

For a given point P the points Q1 and Q2 are identical for each spanned Pentangle in a reference conic.

- * The triangle PQ1Q2 is selfpolar wrt the 5P-circumscribed conic.
- * The triangle P0102 is rectangular: PQ1 is perpendicular to PQ2.

Q1 and Q2 can be constructed as follows:

- * Let 5P-Co be the 5P-circumscribed conic.
- * Let Lp be the polar of P wrt 5P-Co.
- * Let S be the inverse Scimemi Transformation (Co-Tf3 -1) of P wrt 5P-Co.
- * Let Lb be the perpendicular bisector of PS.
- * Let M be $PS \wedge Lp$.
- * Let Ci be the circle with center M through P.
- * Now Q1 and Q2 are the intersection points of Ci and Lp.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3642
Date: 27/7/2019 10:30:25
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

wrt #3640, # 3641:
you find your points Q1, Q2 as Q, R in #3445.
I shall study your properties.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3643
Date: 27/7/2019 10:42:26
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5

Dear Chris,

I just noticed a typo in my message 3445:
Replace 5P-s-P3 by 5P-s-Tf3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3644
Date: 27/7/2019 3:32:13
From: chris.vantienhoven
Subject: 5P-s-P3,4,5

Dear Eckart,

It is not the first time that we investigated the same subject and probably it won't be the last. Nice to look at our differences in findings.

Especially I like your property "circles with these pairs as diameter bear P".

$5P-s-Tf3(P)$ in a $5P = \text{Inverse}[C0-Tf3(P)]$ in a $C0$.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3645
Date: 28/7/2019 12:04:12
From: chris.vantienhoven
Subject: 5P-Geometry

Dear Eckart, Dear Bernard,

There is a 5P-CB-Transformation (to be named 5P-s-Tf6) dealing with Circular Cubics and a 7P-CB-Transformation (to be named 7P-s-Tf1), dealing with regular cubics.

* I found a description of 5P-s-Tf6 by Eckart in #3412. For a description in EPG is there more to say about this transformation? Because it is the tangential I suppose that $5P-s-Tf6[5P-s-Tf6[P]] \leftrightarrow P$.

* What can be written in EPG about 7P-s-Tf1 ? I thought that in this case $7P-s-Tf1[7P-s-Tf1[P]] = P$. Is this true?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3646
Date: 28/7/2019 2:01:32
From: Tran Quang Hung
Subject: 6P-Geometry

Dear geometers,

I have found three concurrent lines with six arbitrary points on plane. Please see the attached file.

Let A_i , $i = 1..6$, be six any points on plane. Taking subscripts modulo 6, we denote, for $i = 1..6$, the intersection of the lines $A_iA_{(i+1)}$ and $A_{(i+2)}A_{(i+3)}$ by $B_{(i+3)}$, and the second intersection of the circumcircles of triangles $A_iA_{(i+1)}B_{(i+2)}$ and $A_{(i+1)}A_{(i+2)}B_{(i+3)}$ by $C_{(i+1)}$, and the circumcenter of the triangle $C_iB_{(i+1)}B_{(i+2)}$ by D_i . The lines D_1D_4 , D_2D_5 , and D_3D_6 are concurrent.

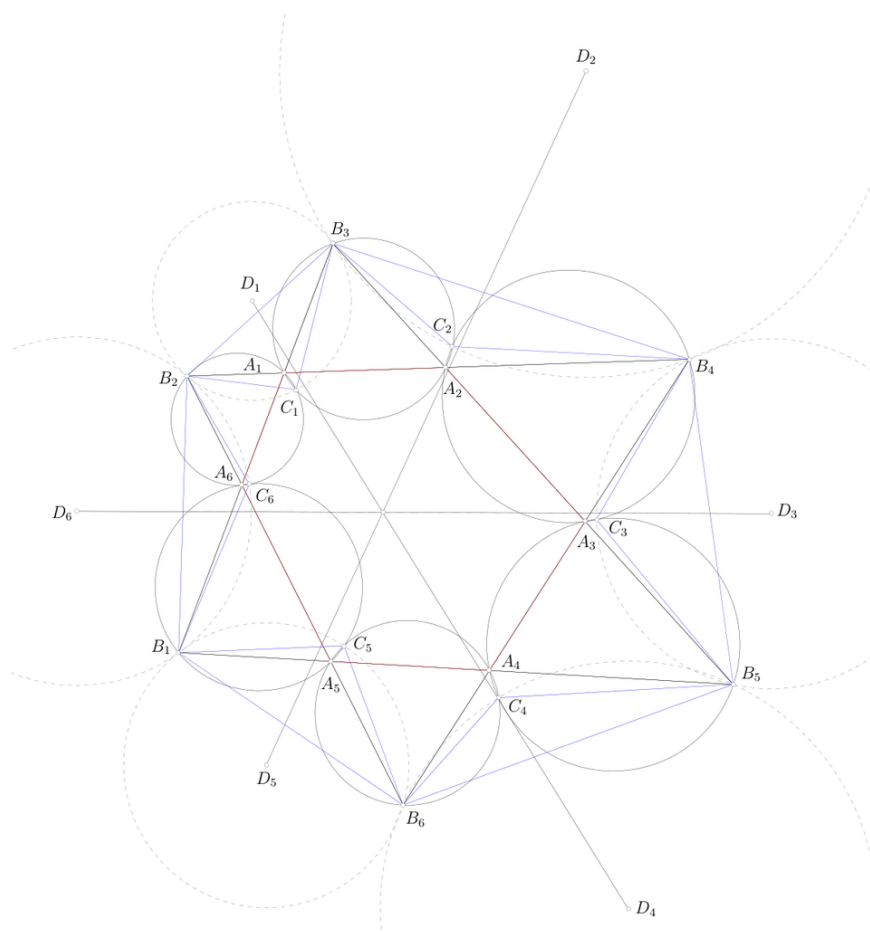
Best regards
Tran Quang Hung.

Concurrent lines in an arbitrary hexagon with twelve circles

Tran Quang Hung

July 12, 2019

Theorem (Tran Quang Hung). Let A_i , $i = 1, 2, \dots, 6$, be six points on plane. Taking subscripts modulo 6, we denote, for $i = 1, 2, \dots, 6$, the intersection of the lines $A_i A_{i+1}$ and $A_{i+2} A_{i+3}$ by B_{i+3} , and the second intersection of the circumcircles of triangles $A_i A_{i+1} B_{i+2}$ and $A_{i+1} A_{i+2} B_{i+3}$ by C_{i+1} , and the circumcenter of the triangle $C_i B_{i+1} B_{i+2}$ by D_i . The lines $D_1 D_4$, $D_2 D_5$, and $D_3 D_6$ are concurrent.



Message: #3647
Date: 28/7/2019 1:11:29
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Chris,

wrt #3645:

Your 5P-s-Tf6 will be my transformation cb:

... For properties have a look in

... #3412, #3420, #3435, #3436, #3438, 3442, #3528 (1), #3575.

I think, it is a relevant transformations for 5P,

... for example: The image of the circle through 3 vertices

... is the line through the 2 remaining vertices.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3648
Date: 28/7/2019 10:27:28
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry

Dear Bernard, dear Chris,

here is a new aspect of the point R,

background are Bernard's messages #3620, #3630.

Take the polar circle of QL-DT,

... then the inverse of QL-P1,

... whose CSC-image is R.

 R is the center of the CSC-circle

 ... of the polarcircle of QL-DT.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3649
Date: 30/7/2019 12:09:06
From: eckart_schmidt@t-online.de
Subject: Deltoid for a convex 5P

Dear Bernard, dear Chris,

just for a change:

Consider a **convex** 5P = P1...P5

... and for points X the 5L of the bisectors of X.Pi:

... For X on the circumconic 5P-s-Co1

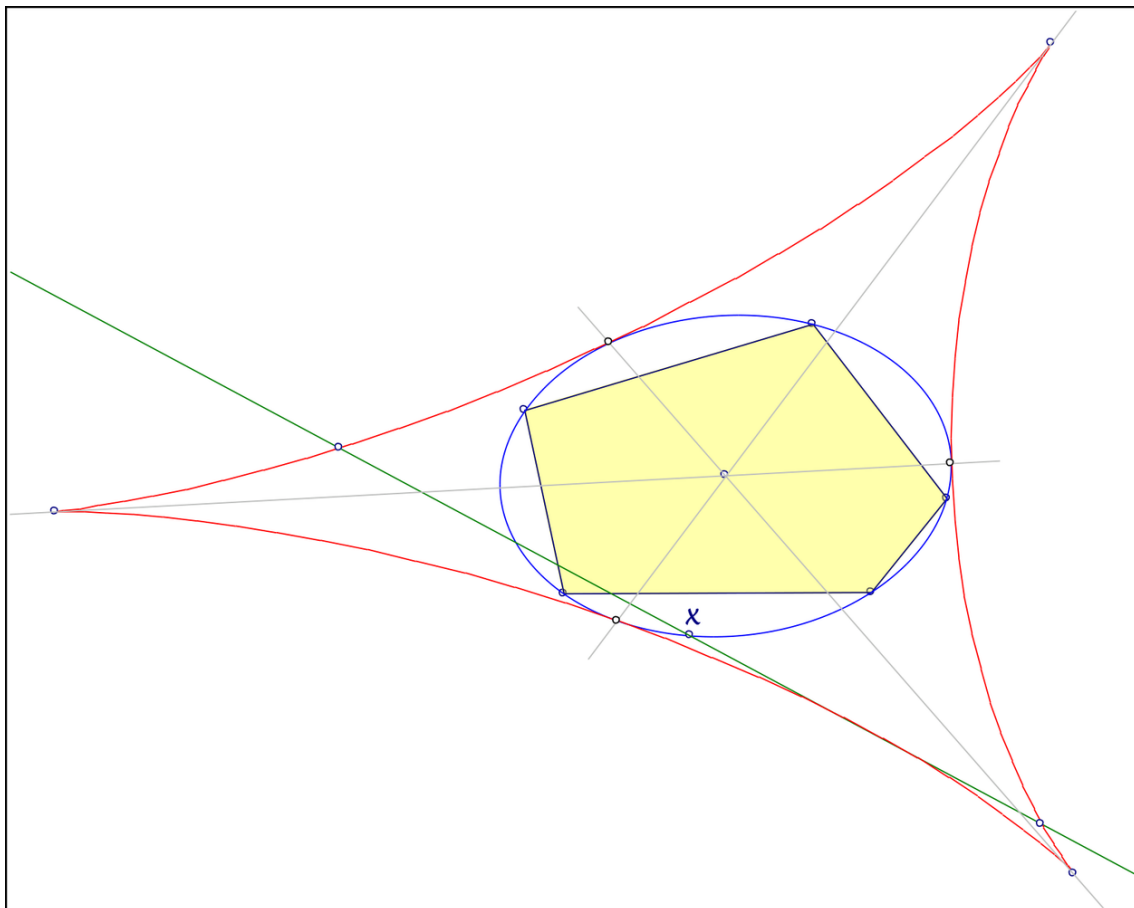
... the lines X.5L-s-Tf1(X) envelop a deltoid,

... circumscribed 5P-s-Co1,

... cusps and contact point collinear 5P-s-P1,

... X midpoint of the intersections of X.5L-s-Tf1(X)
and the deltoid.

Best regards Eckart



2019-07-30.pdf

Message: #3650
Date: 31/7/2019 4:02:48
From: Tran Quang Hung
Subject: 6P-Geometry

Dear geometers,

The concurrent point is generated from six any points, can it be considered as a point of 6-points configuration? I wish You are interested in this?

Best regards
Tran Quang Hung.

Vào CN, 28 thg 7, 2019 vào lúc 07:01 Tran Quang Hung <analgeomatrica@gmail.com> đã viết:
>> Dear geometers,
>> I have found three concurrent lines with six arbitrary points on plane.
>> Please see the attached file.
>> Let A_i , $i = 1..6$, be six any points on plane. Taking subscripts modulo 6,
>> we denote, for $i = 1..6$, the intersection of the lines $A_iA_{(i+1)}$ and $A_{(i+2)}A_{(i+3)}$ by $B_{(i+3)}$, and the second intersection of the circumcircles of triangles $A_iA_{(i+1)}B_{(i+2)}$ and $A_{(i+1)}A_{(i+2)}B_{(i+3)}$ by $C_{(i+1)}$, and the circumcenter of the triangle $C_iB_{(i+1)}B_{(i+2)}$ by D_i . The
>> lines D_1D_4 , D_2D_5 , and D_3D_6 are concurrent.
>> Best regards
>> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3651
Date: 01/8/2019 2:28:48
From: chris.vantienhoven
Subject: 6P-Geometry

Dear Tran Quang Hung,

Nice construction of this point.
The point can be considered as a point of some 6-points configuration.
It is not a point related to 6 random points (a Hexangle) because the points have a certain order.
When you change the points P1 and P2 and do the same construction, then you will find another point.
Therefore it is a Hexagon-point or 6-Gon-point.
See <https://www.chrisvantienhoven.nl/epg-intro>

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3652
Date: 01/8/2019 10:05:46
From: Tran Quang Hung
Subject: 6P-Geometry

Dear Chris,

Thank You very much for your interest. I now understand
6-Gon-point and Hexangle.
I found this theorem when I study Dao's theorem
<http://forumgeom.fau.edu/FG2014volume14/FG201424index.html>

Best regards
Tran Quang Hung.

Vào Th 5, 1 thg 8, 2019 vào lúc 07:28 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Tran Quang Hung,
>> Nice construction of this point.
>> The point
>> can be considered as a point of some 6-points configuration.
>> It is not a point related to 6 random points (a Hexangle)
because the
>> points have a certain order.
>> When you change the points P1 and P2 and do the same
construction, then
>> you will find another point.
>> Therefore it is a Hexagon-point or 6-Gon-point.
>> See <https://www.chrisvantienhoven.nl/epg-intro>
>> Best regards,
>> Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3653

Date: 01/8/2019 10:24:34

From: Antreas Hatzipolakis

Subject: "Familiar names for Polygons or n-Gons when $n > 5$ a "

Dear Chris

You have listed the names of polygons here (
<https://www.chrisvantienhoven.nl/epg-intro>) |

For your information:

John H. Conway and I have named the polygons according to
ancient Greek,

See

<http://mathforum.org/dr.math/faq/faq.polygon.names.html>

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

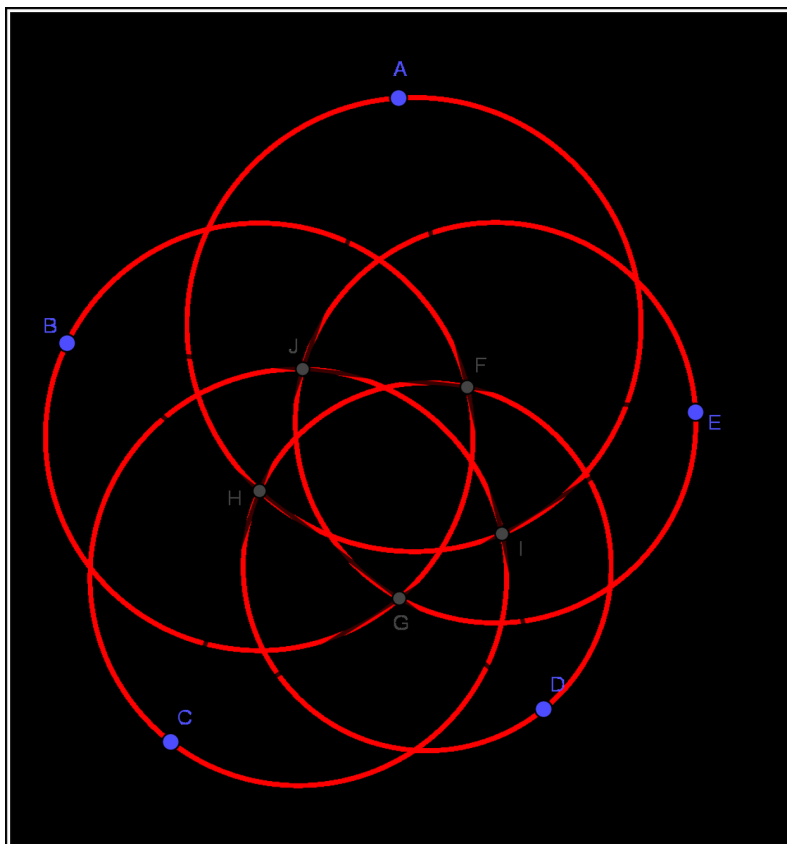
Message: #3654
Date: 01/8/2019 11:43:34
From: Tran Quang Hung
Subject: On a 5-Gon point

Dear geometers,

Base on 5G-s-P2 and problem of Telv
<https://artofproblemsolving.com/community/q2h1813119p12086815>

I proposed a problem as in figure.
Theorem. Let $ABCDE$ be a pentagram, then five red circles (as in figure attached) have the same radical center.
I don't think that my problems and 5G-s-P2 are the same because the circle in 5G-s-P2 have a common vertex of pentagram, but mine is not. Could you please have me to check this?

Best regards
Tran Quang Hung.



Plane13.png

Message: #3655

Date: 01/8/2019 11:56:03

From: chris.vantienhoven

Subject: "Familiar names for Polygons or n-Gons when $n > 5$ a"

Dear Antreas,

Thanks for your extended list.

I made a reference to it in the EPG-intro (
<https://www.chrisvantienhoven.nl/epg-intro>).

I saw that in the extended list

polygons en polyhedra's were named according to ancient Greek.

Actually not only -GONS en -HEDRA'S should use these prefixes,
but also -ANGLES and -LATERALS.

For example:

* A Pentagon = a bounded figure consisting of 5 points connected
with 5 line(-segment)s. (consequently the points have a fixed
order)

* A Pentangle = a figure consisting of 5 random points without
order

* A Pentalateral = a figure consisting of 5 random lines without
order

Again this applies for all numbers, there is a Hexagon, Hexangle
and a Hexalateral, there is a Heptagon, Heptangle and a
Heptalateral, etc.

These differences are also explained in EPG-intro (
<https://www.chrisvantienhoven.nl/epg-intro>).

The distinction between -gons/-angles/-laterals is necessary
because these basic figures / configurations represent
completely different geometries.

This distinction is not new.

For example Coxeter and Greitzner wrote in their book "Geometry
Revisited" (page 52):

Obviously we should discourage the tendency to call a quadrangle
a "quadrilateral". In projective geometry, where the sides are
whole lines instead of mere segments, we need both terms with
distinct meanings.

Best regards,

Chris

Message: #3656
Date: 01/8/2019 4:46:40
From: chris.vantienhoven
Subject: On a 5-Gon point

Dear Tran Quang Hung,

I like your new point, because it is so simple, just one level of construction.

You are right, this new point isn't QG-s-P2.

But special is that it is collinear with QG-s-P2 and QG-s-P3.

Are you able to prove this?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3657
Date: 01/8/2019 4:56:00
From: eckart_schmidt@t-online.de
Subject: On a 5-Gon point

Dear Tran Quang Hung,

the radical center of the 5 red circles isn't 5G-s-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3658
Date: 01/8/2019 5:38:16
From: Tran Quang Hung
Subject: On a 5-Gon point

Dear Chris and Eckart,

Thank You so much for your comments. I had found a related paper since 2014

https://www.jstor.org/stable/10.4169/math.mag.87.1.44?seq=1#page_scan_tab_contents

I also hope this my construction is not direct consequence of 5G-s-P2?

Best regards
Tran Quang Hung.

Vào Th 5, 1 thg 8, 2019 vào lúc 22:00 ' eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Tran Quang Hung,
>> the radical center of the 5 red circles isn't 5G-s-P2.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3659
Date: 02/8/2019 4:23:06
From: Tran Quang Hung
Subject: Perspective triangles in 6-Gon point

Dear geometers,

Let $ABCDEF$ be a hexagon.
The principal diagonals AD , BE , CF meet bound triangle PQR .
The perpendicular bisector of AD , BE , CF meet the side of $ABCDEF$ that created the hexagon $UVXYZW$.
The perpendicular bisectors of principal diagonals UY , VZ , XW meet bound triangle EFG .
Then triangles PQR and EFG are perspective. Is this concurrent point known before?
Please see the attached figure.

Best regards
Tran Quang Hung.

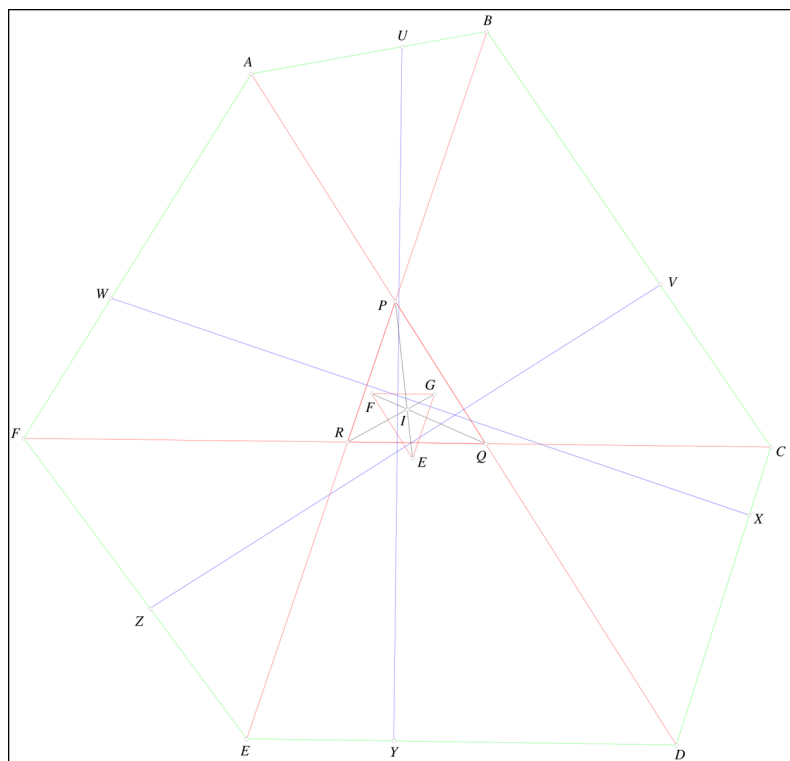


Figure7381.pdf

Message: #3660
Date: 02/8/2019 12:17:28
From: bernard.keizer
Subject: New 5P transformation

Dear Chris, dear Eckart, dear Tran Quang Duong,

The 5P define a transformation $CSC1$, which is the same as the QL's CSC .

The QL-Cu1 of the QL formed by the QG-P1 and the 3 $CSC(QG-P41)$ is a Van Rees cubic with focus QL-P24 ; this cubic contains the CSC of the 5P and the CSC of the QA-P4 of the 5 QA 's of the 5P. The cubic is invariant in a 2nd $CSC2$ with center QL-P24.

The CSC of the cubic is a quartic through the 5P and the 5 QA-P4 of the 5 QA's ; it contains the QA-P4 and QA-P41 of the initial QL as well as the points U and QL-P1.

The quartic is invariant in the transformation $CSC1 * CSC2 * CSC1$, which swaps the QA-P4 and QA-P41 of the QL, the 5P and the QA-P4 of their QA's and the point U and the point QL-P1.

The middle of the segment joining 2 conjugate points on the quartic lies on the circle described by Tran Quang Duong and by Eckart ; it's a generalisation to the quartic of the Newton Line for the QL's cubic.

This transformation seems to be a 3rd $CSC3$ with center $CSC(T)$ and 2 unidentified fixed points (symmetric wrt $CSC(T)$).

Best regards
Bernard

PS I hope Eckart will confirm these observations and find plenty of other properties ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3661
Date: 02/8/2019 2:15:03
From: Tran Quang Hung
Subject: Two cyclologic triangles in QA

Dear geometers,

I see that QA-Tr2 and QA-Tr1 are cyclologic, one of cyclologic center is QA-P3, which is another cyclologic center?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3662
Date: 02/8/2019 7:05:54
From: chris.vantienhoven
Subject: Two cyclologic triangles in QA

Dear

Tran Quang Hung,
The other cyclologic center is QA-P41
QA-Tr1 is formed of the three QA-versions of QG-P1.
QA-Tr2 is formed of the three QA-versions of QL-P1.
These kinds of triangles are called "triple triangles". See
QA-Tr-1.
You can find a list of cyclologic centers at QA-Tr-4 of all kind
of triple triangles.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3663
Date: 02/8/2019 8:26:21
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

very interesting #3660, not only a good summary,
... but also new observations wrt the CSC-transformations,
... which are new for me.

I can confirm all results with CABRI observations,
... adding for CSC3 with center CSC(T)
... the fixed points: necessarily CSC of the fixed points of CSC2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3664
Date: 02/8/2019 11:30:31
From: chris.vantienhoven
Subject: Perspective triangles in 6-Gon point

Dear

Tran Quang Hung,

The point is not known by me.
This point and the point you introduced before are the first
6-Gon-points I encountered.
I am sure there will be more.
There is a lot of uncharted territory here.
Lots of new points to be found relating to n-Gons!
I am waiting for the first 7-Gon-point.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3665

Date: 02/8/2019 11:58:05

From: chris.vantienhoven

Subject: trisecting the vertex angle of a regular n-Gon

Dear friends,

There was a discussion in the Advanced Plane Geometry Group ADGEOM. †)

I think there will be interest in our group too for this problem.

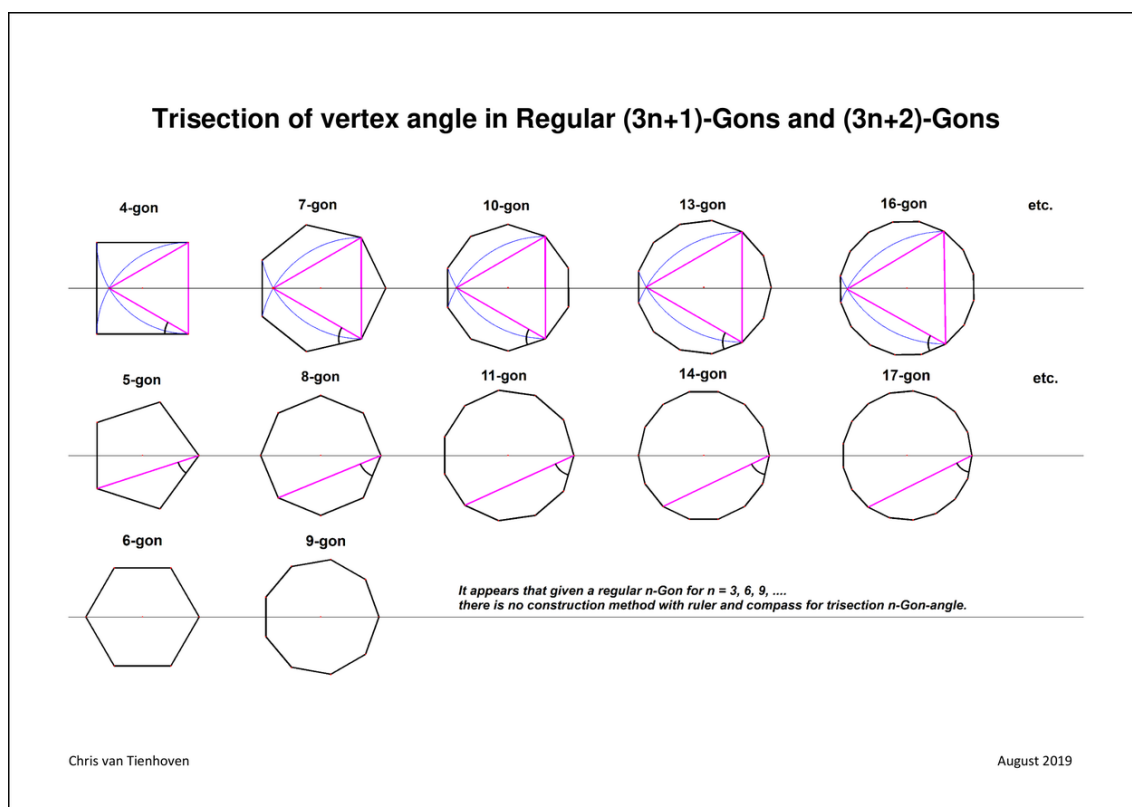
Bui Quang Tuan gave a very nice method for trisecting the vertex angle of a regular heptagon.

I wondered if his method could be generalized. See the attached picture for my generalization.

The method showed for the 7-Gon in the above row is Bui Quang Tuan's initial solution.

Best regards,

Chris



n-Gon-Trisection in Regular n-Gons-03.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[50\]](#).

Message: #3666
Date: 03/8/2019 5:19:25
From: Tran Quang Hung
Subject: Perspective triangles in 6-Gon point

Dear Chris and geometers,

Thank you very much for your encouragement. I am really interested in EQF and EPG. I shall study them much more.

Best regards
Tran Quang Hung.

Vào Th 7, 3 thg 8, 2019 vào lúc 04:30 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahooogroups.com >
đã viết:

>> Dear Tran Quang Hung,
>> The point is not known by me.
>> This point and the point you introduced before are the first
6-Gon-points
>> I encountered.
>> I am sure there will be more.
>> There is a lot of uncharted territory here.
>> Lots of new points to be found relating to n-Gons!
>> I am waiting for the first 7-Gon-point.
>> Best regards,
>> Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3667
Date: 03/8/2019 5:38:49
From: Tran Quang Hung
Subject: Two cyclologic triangles in QA

Dear Chris

Thank you for your interest and new information. I shall study EQF much more.

Best regards
Tran Quang Hung.

Vào Th 7, 3 thg 8, 2019 vào lúc 00:07 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

>> Dear Tran Quang Hung,
>> The other cyclologic center is QA-P41
>> QA-Tr1 is formed of the three QA-versions of QG-P1.
>> QA-Tr2 is formed of the three QA-versions of QL-P1.
>> These kinds of triangles are called "triple triangles". See
QA-Tr-1
>> You can find a list of cyclologic centers at QA-Tr-4
>> of all kind of triple triangles.
>> Best regards,
>> Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3668
Date: 03/8/2019 5:12:07
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,
is the following property already mentioned?
Your CSC2 in #3660 with center QL-P24 swaps QL-P1 and T.
Then your CSC1 and CSC2 can be constructed, starting with a 5P.
Now we can find QA-P4 and QA-P41 on the quartic:
... Let X be a point on the quartic
... and $CSC1(X) = X1$, $CSC2(X1) = X2$, $CSC1(X2) = X3$.
... If X, X1, X2 collinear, then X = QA-P4,
... if X1, X2, X3 collinear, then X = QA-P41.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3669
Date: 03/8/2019 8:40:03
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

excuse, the 2nd passage of my last message 3668 is not correct
... and has to be specified in a next message,
... but the first passage should hold!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3670
Date: 03/8/2019 9:54:31
From: chris.vantienhoven
Subject: New 5P transformation

Dear Bernard,

You write: "The QL-Cu1 of the QL formed by the QG-P1 and the 3 CSC(QG-P41) is a Van Rees cubic with focus QL-P24; this cubic contains the CSC of the 5P and the CSC of the QA-P4 of the 5 QA 's of the 5P."

I suppose you mean the 3 CSC(QA-P41).

I constructed these 3 points and it appeared they are collinear, so I was not able to construct the QL-Cu1 of QG-P1 and the 3 CSC(QA-P41).

Did I misunderstand something?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3671
Date: 04/8/2019 2:22:34
From: Tran Quang Hung
Subject: A tranform on QA with two lines

Dear geometers

Using projective transformation, we shall have a transformation with a line and a point as following

Let A,B,C,D be four points and r a random line.

Line r intersects AB, CD, AC, DB, AD, BC at M_{ab} , M_{cd} , M_{ac} , M_{db} , M_{ad} , M_{bc} .

P is a random point.

PM_{ab} meets CD at P_{cd} .

PM_{cd} meets AB at P_{ab} .

Define similarly the points P_{ac} , P_{db} , P_{ad} , P_{bc} .

Then the lines $P_{ab}P_{cd}$, $P_{ac}P_{db}$, $P_{ad}P_{bc}$ are concurrent.

Best regards

Tran Quang Hung.

Vào CN, 4 thg 8, 2019 vào lúc 06:23 Tran Quang Hung <analgeomatica@gmail.com> đã viết:

>> Dear geometers,

>> When I study the transformation of Quang Duong, I see this concurrent lines.

>> Let A,B,C,D be four points and r a random line.

>> Line r intersects AB, CD, AC, DB, AD, BC at M_{ab} , M_{cd} , M_{ac} , M_{db} , M_{ad} , M_{bc} .

>> d is another random line.

>> P_{ab} , P_{cd} lie on CD, AB such that $M_{ab}P_{ab} \parallel d$, $M_{cd}P_{cd} \parallel d$.

>> Define similarly the points P_{ac} , P_{db} , P_{ad} , P_{bc} .

>> Then the lines $P_{ab}P_{cd}$, $P_{ac}P_{db}$, $P_{ad}P_{bc}$ are concurrent.

>> Best Regards

>> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3672
Date: 04/8/2019 3:38:28
From: Tran Quang Hung
Subject: A tranform on QA with two lines

Dear geometers,

When I study the transformation of Quang Duong, I see this concurrent lines.

Let A, B, C, D be four points and r a random line.

Line r intersects AB, CD, AC, DB, AD, BC at $M_{ab}, M_{cd}, M_{ac}, M_{db}, M_{ad}, M_{bc}$.

d is another random line.

P_{ab}, P_{cd} lie on CD, AB such that $M_{ab}P_{ab} \parallel d, M_{cd}P_{cd} \parallel d$.

Define similarly the points $P_{ac}, P_{db}, P_{ad}, P_{bc}$.

Then the lines $P_{ab}P_{cd}, P_{ac}P_{db}, P_{ad}P_{bc}$ are concurrent.

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3673

Date: 04/8/2019 6:04:56

From: Tran Quang Hung

Subject: Another transformation on QA with two lines

Dear geometers,

When I study the transformation of Quang Duong

<https://chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf6>

, I see the following transformation.

Let A, B, C, D be four points and r a random line.

Line r intersects AB, CD, AC, DB, AD, BC at $M_{ab}, M_{cd}, M_{ac}, M_{db}, M_{ad}, M_{bc}$.

d is another random line.

d meets perpendicular bisectors of segments $M_{ab}M_{cd}, M_{ac}M_{db}, M_{ad}M_{bc}$ at P_1, P_2, P_3 .

The circle (P_1) center at P_1 and passing through M_{ab}, M_{cd} , (P_1) meets AB, CD again at P_{ab}, P_{cd} .

Define similarly the points $P_{ac}, P_{db}, P_{ad}, P_{bc}$.

Then the lines $P_{ab}P_{cd}, P_{ac}P_{db}, P_{ad}P_{bc}$ are concurrent.

When $d=r$, we get the transformation QA-Tf6 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf6>).

Best Regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3674
Date: 04/8/2019 10:42:14
From: bernard.keizer
Subject: New 5P transformation

Dear Chris,
Of course, it was CSC(QA-P41); my apologise for the typo
These 3 points are aligned, which gives the 4th line of the QL,
the 3 others being the DT sides!
Hence the construction of the QL-Cu1 of this QL (the point QL-P1
being QL-P24) ...
Best regards
Bernard
PS The circle circumscribed to the 3 QA-P41 passes through QL-P1
and through W

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3675
Date: 04/8/2019 11:03:50
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Chris,

wrt your #3670:

You can construct the described cubic as QL-Cu1
... starting with a 5P and its CSC:
... using $CSC(5P-s-P5) = CSC(U)$ as QL-P1,
... CSC swapping $5P-s-P6 = QL-P1$ and $5P-s-P4 = T$,
... QL-L1 line through the midpoints of $CSC(P_i)$ and
 $CSC(CSC(P_i))$.
The CSC-image of this cubic is the discussed quartic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3676
Date: 04/8/2019 11:10:57
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Thanks for these precisions!
CSC1 can be constructed from the 5P.
CSC2 can also be constructed from the 5P, as we have 5 couples of CSC2 partners (the CSC(Xi) and the CSC(QA-P4 of the 5 QA's of the Xi).
CSC3 can also be constructed from the 5P, as we know the center CSC(T) and the 2 fixed points, CSC1 of the fixed points of CSC2, as you mentioned.
For your next property, I suppose you try to use the alignment of QG-P1, QA-P4 and CSC(QA-P41) ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3677
Date: 04/8/2019 6:32:19
From: Tran Quang Hung
Subject: A transformation to parallel line and perspective triangle on QA

Dear geometers,

Let A,B,C,D be four points and r a random line.
Line r intersects AB, CD, AC, DB, AD, BC at Rab, Rcd, Rac, Rdb, Rad, Rbc, reps.
Let Mab, Mcd, Mac, Mdb, Mad, Mbc be the midpoints of AB, CD, AC, DB, AD, BC, reps.
Let Pab, Pcd, Pac, Pdb, Pad, Pbc be the reflections of Rab, Rcd, Rac, Rdb, Rad, Rbc in the points Mab, Mcd, Mac, Mdb, Mad, Mbc.

Then

- Midpoints of three segments PabPcd, PacPdb, PadPbc are collinear on line d. Also d is parallel to r.
- Three lines PabPcd, PacPdb, PadPbc form a triangle which is perspective to QA-Diagonal Triangle, which is the perspector?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3678
Date: 04/8/2019 7:03:01
From: Tran Quang Hung
Subject: Six concyclic Poncelet points on QL

Dear geometers,

Let d_1, d_2, d_3, d_4 be a QL.
 $A_{12} = d_1$ meets d_2 . Similarly we have $A_{13}, A_{14}, A_{23}, A_{24}, A_{34}$.

Let
 $P_1 = \text{Poncelet}(A_{13}, A_{14}, A_{23}, A_{24})$
 $P_2 = \text{Poncelet}(A_{12}, A_{14}, A_{23}, A_{34})$
 $P_3 = \text{Poncelet}(A_{12}, A_{13}, A_{24}, A_{34})$

Let $Q_{12} = \text{Poncelet}(A_{12}, P_1, P_2, P_3)$
Define similarly, we have the points $Q_{13}, Q_{14}, Q_{23}, Q_{24}, Q_{34}$.
Then six points $Q_{12}, Q_{13}, Q_{14}, Q_{23}, Q_{24}, Q_{34}$ are concyclic on a circle. Which is this circle of QL?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3679
Date: 05/8/2019 11:56:13
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard, dear Chris,

in #3675 I described a possibility,
... to get the cubic/quartic wrt the QA-Cu7-triple points,
... starting with a 5P and its CSC = CSC1,
... using a construction of a QL-Cu1 with QL-P1,
CSC and QL-L1.

If this construction isn't parat,
... use parallels L of "QL-L1" and take the intersections
... of CSC(L) and the reflection of L in QL-L1.

As Bernard mentioned, we have 3 CSC-transformations:
... CSC1 = 5P-CSC-transformation (see #3583),
... ... mapping the cubic to the quartic,
... CSC2, centered in CSC1(5P-s-P5),
... ... swaps 5P-s-P4 and 5P-s-P6,
... ... maps the cubic to itself,
... CSC3 = CSC1.CSC2.CSC1, centered in CSC1(5P-s-P4),
... ... swaps 5P-s-P6 and 5P-s-P5,
... ... maps the quartic to itself.

Now we can search QA-P4 and QA-P41 on the quartic:
... Let X be a point on the quartic
... ... and $CSC1(X) = X1$, $CSC2(X1) = X2$, $CSC1(X2) = X3$.
... If $x = X2$ and $X1 = X3$, they are two intersections of cubic and
quartic.
... If $X = QA-P4$: X, X1, X2 collinear, bearing $X1 = QG-P1$,
 $X3 = QA-P41$.
... If $X = QA-P41$: X1, X2, X3 collinear, bearing $X2 = QG-P1$,
 $X3 = QA-P4$.

But these properties are not sufficient ...
No clearness, but a question:
... Are the QA-P4 tangentials of 5P-s-P5 wrt the quartic?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3680
Date: 05/8/2019 3:33:45
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Wunderbar ! Weihnachten in August!
All this is perfectly clear, but I think the most important thing in your message is the last conjecture !
I think it holds that the 3 QA-P4 are the tangentials of U on the quartic.
As the CSC1,2 and 3 and the cubic and quartic are defined from the 5P, it gives at last a simple definition of R as the diametral point of U on the circumcircle (through U) of the 3 QA-P4!
Many thanks for this last discovery!
I suppose the game is over now ...
I'm able now to draw the twin QL's having the 5P or their CSC as QA-Cu7 triple points!
Best regards
Bernard
PS It remains a few details, like to understand why it works for an hyperbola and not for an ellipse

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3681

Date: 05/8/2019 5:44:59

From: César E. Lozada

Subject: A transformation to parallel line and perspective triangle on QA

Dear Mr Hung,

Perspector for line (x:y:z):

(CT)

$$Q = ((q+r)*(2*p+q+r)*q*r*x - (q^2 + (p+2*q+2*r)*p + (q+r)^2)*p*r*y - (r^2 + (p+2*q+2*r)*p + (q+r)^2)*p*q*z) * ((2*(p+q+r)*p + (q+r)^2)*x + p*q*y + p*r*z)$$

Best regards,

César Lozada

De:

Quadri-Figures-Group@yahoogroups.com

Enviado el: domingo, 4 de agosto de 2019 12:32 PM

Para: Quadri-Figures-Group@yahoogroups.com

Asunto: [Quadri-Figures-Group] A transformation to parallel line and perspective triangle on QA

> Dear geometers,

> Let A,B,C,D be four points and r a random line.

> Line r intersects AB, CD, AC, DB, AD, BC at Rab, Rcd, Rac, Rdb, Rad, Rbc, reps.

> Let Mab, Mcd, Mac, Mdb, Mad, Mbc be the midpoints of AB, CD, AC, DB, AD, BC, reps.

> Let Pab, Pcd, Pac, Pdb, Pad, Pbc be the reflections of Rab, Rcd, Rac, Rdb, Rad, Rbc in the points Mab, Mcd, Mac, Mdb, Mad, Mbc.

> Then

> - Midpoints of three segments PabPcd, PacPdb, PadPbc are collinear on line d. Also d is parallel to r.

> - Three lines PabPcd, PacPdb, PadPbc form a triangle which is perspective to QA-Diagonal Triangle, which is the perspector?

> Best regards

> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3682
Date: 06/8/2019 1:19:36
From: chris.vantienhoven
Subject: A tranform on QA with two lines

Dear Quang Hung,
Let me first start with your *first Transformation* QA-Tfx(L,P)
described by you in QFG #3671.
You mentioned it before in #2099.

Properties:

- * It is a linear transformation.
- * Regardless of the position of line L, the point QA-Tfx(L,P)
will lie on the 5th Point tangent QA-Tf9(P).
- * For extra properties, see QFG#2101, #2103, #2122

Your 2nd transformation in #3672 is the same as #3671.

Only instead of finite point P you use the infinity point of 2nd
line r.

Your 3rd transformation in #3673 gives a pretty long expression.
It maps 2 lines L1 & L2 into a point X.

Properties/Incidents:

* When L1 *or* L2 are the Line at Infinity, then X = an infinity
point.

* L1=Line at Infinity, L2=QA-L2,
then X = Infinity Point (QA-L9) = QA-Tf2 (QA-P2)

* L1=Line at Infinity, L2=QA-L7,
then X = Infinity Point (QA-P12.QA-P32) = Infinity Point
(QA-P14.QA-P37)

* L1=Line at Infinity, L2=QA-L9,
then X = Infinity Point (QA-L2) = Infinity Point
(QA-P1.QA-P32)

* L2=Line at Infinity, L1=QA-L3,
then X = Infinity Point (QA-L7) = QA-P9041 = InfinityPoint of
the line
through the 3 QL-P18 points in a Quadrangle

* L2=Line at Infinity, L1=QA-L4,
then X = Infinity Point (QA-L9) = QA-Tf2 (QA-P2)

* L2=Line at Infinity, L1=QA-L7,
then X = Infinity Point (QA-L3) = Infinity Point (QA-P1.QA-P5)

* L2=Line at Infinity, L1=QA-L9,
then X = Infinity Point (QA-L4) = QA-Tf2 (QA-P3)

* L1=QA-L2, L2=QA-L2, then X = QA-P2

* L1=QA-L9, L2=QA-L9, then X = QA-P23

* L1=QA-L5, L2=QA-L5, then X = point on QA-Ci1

* L1=QA-L2, L2=QA-L9, then X = some infinity point

* L1=QA-L9, L2=QA-L2, then X = QA-Tf2(QA-P2)

Best regards,
Chris

Message: #3683
Date: 06/8/2019 11:37:45
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

now I can construct the cubic and the quartic,
... starting with a QL or a 5P, CABRI-macros are saved!
You say "game over", but:
... How do you find the QA-P4 on the quartic and get the QL?

My observations:

The point $U = 5P-s-P5$ can have up to six tangents to the quartic.

If all are real:

... two tripl of contact points, each concyclic with U ,
... one tripl gives the QA-P4.

If 5P is convex,

... cubic and quartic can be constructed, but:

... there are no three contact points concyclic with U .

If 5P is not convex,

... there are not always three contact points concyclic with U .

Are the 5P of the triple points special nonconvex 5P?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3684
Date: 06/8/2019 12:16:43
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
I suppose we have to study now the circular quartics !
This point U seems to be a focus like QL-P1 for the circular focal Van Rees cubic.
This focus is the intersection point of the tangents in the QA-P4, in U and in the circular points (hence the circle).
With the little circle playing the role of the Newton Line, the circle of your points V1, 2 and 3 (through QL-P1 and T) plays the role of the asymptote !
But unfortunately, my knowledge in the quartics equals to zero !
Have you any idea of where to start ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3685
Date: 06/8/2019 1:00:59
From: chris.vantienhoven
Subject: New 5P transformation

Dear Benard,
I can't wait to see how you construct the twin QL's having the 5P.
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3686
Date: 06/8/2019 1:13:17
From: Tran Quang Hung
Subject: A transform on QA with two lines

Dear Chris,

Thank You very much for your detail help. It very helpful for me. I really need your help in the upcoming messages.

Best regards
Tran Quang Hung.

Vào Th 3, 6 thg 8, 2019 vào lúc 06:19 van10hoven@gmail.com
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:

>> Dear Quang Hung,
>> Let me first start with your **first Transformation**
QA-Tfx(L,P) described
>> by you in QFG #3671.
>> You mentioned it before in #2099.
>> Properties:
>> It is a linear transformation.
>> Regardless of the position of line L, the point QA-Tfx(L,P)
will lie on
>> the 5th Point tangent QA-Tf9(P).
>> . For extra properties, see QFG#2101, #2103, #2122
>> Your **2 nd transformation** in #3672 is the same as #3671.
Only instead of
>> finite point P you use the infinity point of 2 nd line r.
>> Your **3 rd transformation** in #3673 gives a pretty long
expression.
>> It maps 2 lines L1 & L2 into a point X.
>> Properties/Incidents:
>> . When L1 **or** L2 are the Line at Infinity, then X = an
infinity point.
>> . L1=Line at Infinity, L2=QA-L2, then X = Infinity Point
(QA-L9) = QA-Tf2
>> (QA-P2)
>> . L1=Line at Infinity, L2=QA-L7, then X = Infinity Point
(QA-P12.QA-P32) =
>> Infinity Point (QA-P14.QA-P37)
>> . L1=Line at Infinity, L2=QA-L9, then X = Infinity Point
(QA-L2) = Infinity
>> Point (QA-P1.QA-P32)
>> . L2=Line at Infinity, L1=QA-L3, then X = Infinity Point
(QA-L7) = QA-P9041
>> = InfinityPoint of the line through the 3 QL-P18 points in a
Quadrangle

```

>> . L2=Line at Infinity, L1=QA-L4, then X = Infinity Point
(QA-L9) = QA-Tf2
>> (QA-P2)
>> . L2=Line at Infinity, L1=QA-L7, then X = Infinity Point
(QA-L3) = Infinity
>> Point (QA-P1.QA-P5)
>> . L2=Line at Infinity, L1=QA-L9, then X = Infinity Point
(QA-L4) = QA-Tf2
>> (QA-P3)
>> . L1=QA-L2, L2=QA-L2, then X = QA-P2
>> . L1=QA-L9, L2=QA-L9, then X = QA-P23
>> . L1=QA-L5, L2=QA-L5, then X = point on QA-Ci1
>> . L1=QA-L2, L2=QA-L9, then X = some infinity poin
>>> . L1=QA-L9, L2=QA-L2, then X = QA-Tf2(QA-P2)
>> Best regards,
>> Chris

```

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3687
Date: 06/8/2019 1:46:11
From: chris.vantienhoven
Subject: Six concyclic Poncelet points on QL

Dear Quang Hung,

Regarding your message QFG#3678.

The Triangle P1.P2.P3 in your construction is a QL-Triple Triangle for QA-P2, because the Quadrangles you choose are the three Component Quadrilaterals of a Quadrilateral (see QL-3QG1). For every QA-point and every QG-point such Triple Triangle can be constructed.

otes:

1. The circle is the nine-point circle of triangle P1.P2.P3
2. The Poncelet point of P1.P2.P3.P,
where P can be any point in the plane, lies on this circle.
3. The equation of the circle is extreme long.
4. There are no known QL-points lying on this circle.
5. Its center has no connections with other QL-points either.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3688
Date: 06/8/2019 2:43:46
From: Tran Quang Hung
Subject: Coaxial circles in QA

Dear geometers,

P_1, P_2, P_3, P_4 be the defining Quadrangle Points.
Let $S_1 = P_1P_2$ cuts P_3P_4 , $S_2 = P_1P_3$ cuts P_2P_4 and $S_3 = P_1P_4$ cuts P_2P_3 .
Line S_2S_3 meets lines P_1P_2 and P_3P_4 at S_{12} and S_{34} .
Line S_1S_2 meets lines P_1P_4 and P_2P_3 at S_{14} and S_{23} .
Line S_3S_1 meets lines P_1P_3 and P_2P_4 at S_{13} and S_{24} .
 r is a random line.
 r meets lines $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$ at the points $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}$.
Let R_{34}' be the point such that $(S_1, S_{34}; R_{34}, R_{34}') = -1$ (Harmonic conjugate).
Let R_{12}' be the point such that $(S_1, S_{12}; R_{12}, R_{12}') = -1$.
We define similarly the points $R_{13}', R_{14}', R_{23}', R_{24}'$.
Now the circles with diameters $(R_{12}'R_{34}'), (R_{13}'R_{24}'), (R_{14}', R_{23}')$ are coaxial.
This means its centers are collinear and they have a common radical axis. Thus we have the transformation of a line r to two lines? Which are these lines?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3689
Date: 06/8/2019 4:45:37
From: Tran Quang Hung
Subject: A transformation to parallel line and perspective triangle on QA

Thank You very much Cesar for your calculation.

Best Regards
Tran Quang Hung.

Vào Th 2, 5 thg 8, 2019 vào lúc 22:47 César E. Lozada
cesar_e_lozada@yahoo.es [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
>> Dear Mr Hung,
>> Perspector for line (x:y:z):
>> (CT)
>>
$$Q = ((q+r)*(2*p+q+r)*q*r*x - (q^2+(p+2*q+2*r)*p+(q+r)^2)*p*r*y -$$

>>
$$(r^2+(p+2*q+2*r)*p+(q+r)^2)*p*q*z)*((2*(p+q+r)*p+(q+r)^2)*x+p*q*$$

>>
$$y+p*r*z)$$

>> Best regards,
>> César Lozada

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3690
Date: 06/8/2019 5:06:43
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Chris, dear Bernard,

I try to describe interim the way back
... from a 5P to the QL with these QA-Cu7-triple points:

Consider a 5P, its CSC and its quartic:
... If there are three tangents from 5P-s-P5 at the quartic
... with contact points concyclic with 5P-s-P5
... .. and the diametral point of 5P-s-P5 on 5P-s-Co1,
... they are QA-P4,
... take the CSC-images of QA-P4, they are QG-P1,
... and consider the CSC-partner on the QG-P1-triangle sides,
... which are the opposite points of the searched QL.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3691
Date: 06/8/2019 5:46:23
From: Antreas Hatzipolakis
Subject: Coaxial circles in QA

[Tran Quang Hung]:

Dear geometers,

P_1, P_2, P_3, P_4 be the defining Quadrangle Points.
Let $S_1 = P_1P_2$ cuts P_3P_4 , $S_2 = P_1P_3$ cuts P_2P_4 and $S_3 = P_1P_4$ cuts P_2P_3 .
Line S_2S_3 meets lines P_1P_2 and P_3P_4 at S_{12} and S_{34} .
Line S_1S_2 meets lines P_1P_4 and P_2P_3 at S_{14} and S_{23} .
Line S_3S_1 meets lines P_1P_3 and P_2P_4 at S_{13} and S_{24} .
 r is a random line.
 r meets lines $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$ at the points $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}$.
Let R_{34}' be the point such that $(S_1, S_{34}; R_{34}, R_{34}') = -1$ (Harmonic conjugate).
Let R_{12}' be the point such that $(S_1, S_{12}; R_{12}, R_{12}') = -1$.
We define similarly the points $R_{13}', R_{14}', R_{23}', R_{24}'$.
Now the circles with diameters $(R_{12}'R_{34}'), (R_{13}'R_{24}'), (R_{14}', R_{23}')$ are coaxial.
This means its centers are collinear and they have a common radical axis. Thus we have the transformation of a line r to two lines? Which are these lines?

.... and to one point: the radical trace of the circles = intersection of the radical axis and the diacentric line.

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3692
Date: 06/8/2019 10:30:25
From: bernard.keizer
Subject: New 5P transformation

Dear Chris, dear Eckart,
As already mentionned, repetition is the key of teaching !
But the occasion is too beautiful !
Doing exactly the same with the CSC of the 5 points (which have obviously the same CSC) gives the twin QL !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3693
Date: 06/8/2019 11:45:21
From: chris.vantienhoven
Subject: 5P-s-Co1

Dear Bernard, Dear Eckart,

Regarding your common unique diagonal polar conic for 5 pivotal isocubics having 4 of the 5 points as fixed points of the conjugation and the 5th point as pivot (QFG#3517).
How do you construct a pivotal isocubic given 4 points and a pivot?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3694
Date: 07/8/2019 12:11:53
From: eckart_schmidt@t-online.de
Subject: 5P-s-Co1

Dear Chris,

wrt: " How do you construct a pivotal isocubic given 4 points and a pivot?"

If the 4 points are fixed points of the isoconjugation:

... Take the 4 fixed points as QA with diagonal triangle Tr

... and a line pencil of the 5th point.

... The isoconjugated points on the lines

... ... for the Tr-isoconjugation with the fixed 4 points

... give the searched cubic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3695
Date: 07/8/2019 12:20:54
From: chris.vantienhoven
Subject: New 5P transformation

Dear Bernard and Eckart,

Congratulations !

After a year of hard work you succeeded finding the way back from 5P to 4L.

A major achievement.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3696
Date: 07/8/2019 12:23:36
From: eckart_schmidt@t-online.de
Subject: 5P-quartics for 6P

Dear Chris,

if we construct six 5P-quartics for a 6P,
... we get three common points.
What about these 6P-s-3Px?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3697
Date: 07/8/2019 12:25:34
From: eckart_schmidt@t-online.de
Subject: 5P-quartics for 6P

Dear Chris,

I was too overhasty with my last message:
If we construct the six 5P-quartics for a 6P,
... there are three points, *where five quartics intersect.*

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3698
Date: 07/8/2019 12:27:39
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard, dear Chris,

if you are interested,
... there is a construction for tangents at the 5P-quartic:
Let P be a point on the quartic and $Q = \text{CSC}(P)$ on the cubic.
The cubic can be considered as $QL\text{-}Cu1$ (see #3675),
... so we can construct a tangent in Q at the cubic (see EQF),
... whose CSC -image is a circle through P ,
... and the tangent in P at the circle is also the tangent at the quartic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3699
Date: 07/8/2019 1:01:38
From: bernard.keizer
Subject: 5P-s-Co1

Dear Chris,
Very surprising question coming from you !
The construction is given in EQF under $QA\text{-}Cu\text{-}1$, the 5 pivotal isocubics are $QA\text{-}Cubics$ of type 1.
(each cubic contains the DT vertices of it's QA and the vertices of the cevian triangle of the pivot wrt this DT)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3700
Date: 07/8/2019 4:23:42
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Beautiful and useful construction, indeed.
But we need not only the tangent in the point to the quartic,
but also the tangents from the point to the quartic !
Any idea ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3701
Date: 07/8/2019 10:40:51
From: chris.vantienhoven
Subject: 5P-s-Co1

Dear Bernard,

[BK] Very surprising question coming from you !
I think that's because my brain is wired in another way than your brain. That's why we work so good together!
But actually, I wish my memory was better.
And therefore some unavoidable questions.
Isn't it so that a polar conic is always:

a) wrt a cubic
b) wrt a reference point (on or outside the cubic)
That are two variables.
When we have a 5P and we construct a pivotal isocubic using one point as a pivot and the other four as fixed points of the conjugation, then we have the cubic (condition b).
To produce a polar conic we still need some more input:
a) which reference point ?

Further you mention "diagonal polar conic". What do you mean in this case with "diagonal"?
You wrote: "This conic contains the vertices of the anticevian triangles of each of the 5 points wrt the DT of the 4 others (15 points).".
I tried to construct the conic using this property, but did not succeed.
Could you please explain. Or is my brain is playing games with me?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3702

Date: 08/8/2019 3:16:50

From: Tran Quang Hung

Subject: Some new theorems on Pentagon and Pentagram

Dear geometers,

I had found "Some new theorems on Pentagon and Pentagram" in the paper

<https://arxiv.org/abs/1908.00974>

I look forward the consideration of geometers on Quadri-Figures-Group.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3703

Date: 08/8/2019 9:06:42

From: eckart_schmidt@t-online.de

Subject: New 5P transformation

Dear Bernard,

have you really a drawing of the "twin" QL for a 5P?

My observation:

... The 5P of the QA-Cu7-triple points is a special 5P,

... not all 5P have a QL, whose QA-Cu7-triple points give the 5P.

Up to now I have not found an example,

... that the CSC-5P of the QA-Cu7-triple points

... shows the properties of #3690 , to go the way back to a QL.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3704
Date: 08/8/2019 3:53:04
From: bernard.keizer
Subject: 5P-s-Co1

Dear Chris,
Yes, indeed !
The diagonal polar conic is the polar conic of the pivot wrt the pivotal isocubic (diagonal because it contains the 4 fixed points see Bernard Gibert Special Isocubics page 30).
The circumscribed polar conic is the polar conic of the isopivot (isoconjugate of the pivot) wrt the same cubic (circumscribed because it contains the vertices of the reference triangle DT).
The 1st mentionned conic contains the vertices of the anticevian triangle of any of it's points wrt DT (that means that the point and the 3 vertices of it's anticevian triangle form an inscribed QA having the same DT).
This was only a curiosity, but I'm glad you took interest in it!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3705
Date: 10/8/2019 9:30:07
From: eckart_schmidt@t-online.de
Subject: New 5P transformation

Dear Bernard,

wrt messages #3680, #3685, 3692:
After a lot of CABRI-constructions I am convinced,
... that the 5P of the QA-Cu7-triple points are special 5P!
Only if there are for a 5P three with 5P-s-P5 concyclic
... contact points
of tangents from 5P-s-P5 at the 5P-quartic
... with diametral point of 5P-s-P5 on 5P-s-Co1,
... there will be a QL with this 5P for its QA-Cu7-triple points.
I found no example for a 5P of triple points,
... whose CSC-5P has this property.

Best regards Eckart

PS: Of course there will be a simpler description of these 5P.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3706

Date: 10/8/2019 1:28:50

From: chris.vantienhoven

Subject: Some new theorems on Pentagon and Pentagram

Dear Quang Hung,

I am reading your document at
<https://arxiv.org/pdf/1908.00974.pdf>.

I need some time to read and elaborate it in fully.
Here is my first reaction about Theorem I, Miquel's Pentagram Theorem.
I think this is the most well-known theorem about Pentagons and it is good to mention it.
This theorem is sometimes referred to as Jiang Zemin's problem, as this former president of China talked about the theorem the end of 1999 as he visited Macau.
(see Mathworld).

>From the Article of Jean-Louis Ayme "'LA CHAÎNE INACHEVÉE DE WILLIAM KINGDON CLIFFORD"', page 4, available at:
<http://jl.ayme.pagesperso-orange.fr/Docs/Etude%204.pdf>, comes this phrase (in the French language): "En 1838, Auguste Miquel découvre à partir d'une idée d'Eugène Catalan, "'Le pentagramme'" formé par cinq droites en position générale."

Meaning that Miquel was brought to the idea of a Pentagram by Eugène Catalan. He used it to find a general theorem about 5 circles, the "Miquel's Five Circles Theorem".
I myself am astonished about another property:

- * The points is not only a 5G-point but also a 5L-point: it is 5L-o-P2, which is the 5L-version of nL-o-P2: nL-Clifford's Circle Center.
- * Moreover the circle through the 2nd-circle-points is 5L-o-Ci1, which is the 5L-version of nL-o-Ci1: nL-Clifford's Circle. See also Jean-Louis Ayme's document.

Also I found these properties:

- * When the 5-Gon is considered as a 5-Point, then it appears that the Miquel Point QL-P1 of each component 4-Point (Quadrangle) lies on a corresponding Pentagram circle.

* When the 5-Gon is considered as a 5-Line, then it appears that the Miquel Point QL-P1 of each Component 4-Line (Quadrilateral) coincides with the corresponding 2nd intersection point of a pair of construction circles. Accidentally is the circle through these 2nd intersection points the circle that defines 5G-s-P4, which explains why $5G-s-P4 = 5L-0-P2$.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3707

Date: 10/8/2019 1:40:36

From: Tran Quang Hung

Subject: Some new theorems on Pentagon and Pentagonagram

Dear van Tienhoven,

Thank You so much for your new information. I focus in the Theorem 4 and Theorem 5 in my paper, it was for arbitrary pentagram, maybe 5-gon? I wish You and Mr Eckart will consider them soon.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3708
Date: 10/8/2019 1:51:31
From: Tran Quang Hung
Subject: Coaxial circles in QA

Thank You very much Mr Antreas for your interest. I hope Mr van Tienhoven and Mr Eckart will help me to check this result.

Thank you very much and best regards
Tran Quang Hung.

Vào Th 3, 6 thg 8, 2019 vào lúc 22:50 Antreas Hatzipolakis anopolis72@gmail.com [Quadri-Figures-Group] <Quadri-Figures-Group@yahoogroups.com> đã viết:

```
>> [Tran Quang Hung]:
>> Dear geometers,
>> P1, P2, P3, P4 be the defining Quadrangle Points.
>> Let
>> S1 = P1P2 cuts P3P4, S2 = P1P3 cuts P2P4 and S3 = P1P4
>> cuts P2P3.
>> Line S2S3 meets lines P1P2 and P3P4 at S12 and S34.
>> Line S1S2 meets lines P1P4 and P2P3 at S14 and S23.
>> Line S3S1 meets lines P1P3 and P2P4 at S13 and S24.
>> r is a random line.
>> r meets lines P1P2, P1P3, P1P4, P2P3, P2P4, P3P4 at the
>> points R12, R13,
>> R14, R23, R24, R34.
>> Let R34' be the point such that (S1, S34; R34, R34') = -1
>> (Harmonic
>> conjugate).
>> Let R12' be the point such that (S1, S12; R12, R12') = -1.
>> We define similarly the points R13', R14', R23', R24'.
>> Now the circles with diameters (R12'R34'), (R13'R24'),
>> (R14',R23') are
>> coaxial.
>> This means its centers are collinear and they have a common
>> radical axis.
>> Thus we have the transformation of a line r to two lines?
>> Which are these
>> lines?
>> .... and to one point: the radical trace of the circles =
>> intersection of
>> the radical axis and the diacentric line.
>> APH
```

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3709
Date: 10/8/2019 4:08:31
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
Of course, I completely agree with you !
It took me time and drawings to be definitively convinced that the twin QL doesn't exist ...
In a certain way, I'm rather released, as I could hardly accept the idea that a 5P is always equivalent to a 4L (and to its dual 4P).
It's well known that the points T, U , QL-P1 can be defined for any 5P.
But R can now also be defined for an ordinary 5P with the help of the cubic/quartic as diametral of U on the circle through the 3 real tangentials of U wrt the quartic (which contains U) or in another way as the intersection of the 3 normals to the quartic in these points.
The quartic is circular and U is its focus, meaning that the circular points are also tangentials of U.
This leads to 2 triangles of "QA-P4" and "QA-P41" on the quartic and of "QG-P1" on the cubic with CSC("QA-P41") aligned and being CSC2("QG-P1"). (QL-P1 doesn't lie on the Euler circle of the "QG-P1" triangle).
The 5P are the triple points of the 3 QA-Cu7 of a QL with DT the QG-P1 only if R lies on the 5P conic.
Best regards
Bernard
PS We should now study this strange quartic which functions like a Van Rees cubic with the CSC3 as CSC and a circle as "Newton Line" ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3710
Date: 10/8/2019 4:20:44
From: bernard.keizer
Subject: New 5P transformation

Dear Eckart,
For example, I forgot one interesting property of the quartic :
2 CSC3 conjugate points on the quartic not only have the middle
of the segment joining them on the circle acting as Newton Line,
but are concyclic with the 2 fixed points (and harmonic wrt
these 2 points on the circle), which is a wellknown property of
the Van Rees cubic and gives an easy way of drawing the quartic)
...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3711
Date: 10/8/2019 6:08:37
From: Tran Quang Hung
Subject: Dual lines in QA

Dear geometers,

Dear geometers,

P_1, P_2, P_3, P_4 be the defining Quadrangle Points.
Let $S_1 = P_1P_2$ cuts P_3P_4 , $S_2 = P_1P_3$ cuts P_2P_4 and $S_3 = P_1P_4$ cuts P_2P_3 .
Line S_2S_3 meets lines P_1P_2 and P_3P_4 at S_{12} and S_{34} .
Line S_1S_2 meets lines P_1P_4 and P_2P_3 at S_{14} and S_{23} .
Line S_3S_1 meets lines P_1P_3 and P_2P_4 at S_{13} and S_{24} .
 r is a random line.
 r meets lines S_1S_3, S_3S_2, S_2S_1 at T_2, T_1, T_3 .

Construct T_1', T_2', T_3' such that
 $(S_{12}, S_{34}; T_1, T_1') = -1$
 $(S_{14}, S_{23}; T_3, T_3') = -1$
 $(S_{13}, S_{24}; T_2, T_2') = -1$
Then T_1', T_2', T_3' are collinear on a line. I call it by d .
Thus we have $d = f(P_1, P_2, P_3, P_4, r)$, and also $r = f(P_1, P_2, P_3, P_4, d)$. So I call r and d are dual wrt $QA(P_1, P_2, P_3, P_4)$.
Which are the others properties of r and d ?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3712
Date: 10/8/2019 8:59:39
From: eckart_schmidt@t-online.de
Subject: Coaxial circles in QA

Dear Tran Quang Hung,

wrt your message #3688 some observations:
Let T_f be the transformation, that maps a line
... to the line of the centers of the last three circles.

- (1) $T_f(QA-L3) = QA-P16.QA-P20$,
... parallel to the line $QA-P5.17.19.21$ (see PS).
- (2) For lines L through $QA-P10$:
 $T_f(L)$ bears $QA-P20$,
... $L \wedge T_f(L)$ give a circumconic of $QA-Tr1$ through $QA-P10$ and $QA-P20$,
... which is $QA-Tf2(QA-P1.QA-P16)$.
- (3) T_f of a QA-line bears the midpoint of the opposite line.
- (4) Fixed lines of T_f are the lines of $QA-Tr1$.

Best regards Eckart

PS for Chris: I didn't find in EQF
... the parallelism of $QA-P16.20$ and $QA-P5.17.19.21$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3713
Date: 10/8/2019 10:05:35
From: eckart_schmidt@t-online.de
Subject: Dual lines in QA

Dear Tran Quang Hung,

wrt your #3711 with an interesting property:
Let T_f be the line transformation, that maps $r \leftrightarrow d$.
For lines L of a line pencil wrt a point P
... $T_f(L)$ envelope a conic tangent $QA-Tr1$
... in the cevian points of $QA-Tf2(P)$.
Example: Inscribed Steiner ellipse for $QA-P16$.
There will be more properties!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3714
Date: 10/8/2019 10:14:00
From: chris.vantienhoven
Subject: Coaxial circles in QA

Dear Quang Hung, dear Eckart,

I have nothing to add to the observations of Eckart.
The calculations aren't easy. The DT-calculations are easier,
but still the results are long.

Eckart, the parallelism of QA-P16.20 and QA-P5.17.19.21 can be
found at QA-3, 10th line of examples.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3715
Date: 11/8/2019 9:46:38
From: Tran Quang Hung
Subject: Coaxial circles in QA

Dear Chris, dear Eckart,

Thank You very much for your interest. I was very impressed with
Quang Duong's transformation QA-Tf6 (<http://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf6>).

I found some these transformations around its configuration.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3716

Date: 11/8/2019 10:16:34

From: Tran Quang Hung

Subject: Another transformation with two lines in QA

Dear geometers,

P_1, P_2, P_3, P_4 be the defining Quadrangle Points.

r is a random line.

r meets lines $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$ at the points $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}$.

Let T_1, T_2, T_3 be the midpoints of segments $R_{12}R_{34}, R_{13}R_{24}, R_{14}R_{23}$.

d is another random line.

Line d_1, d_2, d_3 pass through T_1, T_2, T_3 and are parallel to d .

d_1 meets lines P_1P_2, P_3P_4 at points M_{12}, M_{34} .

Let R_{12}' be the reflection of R_{12} in M_{12} .

Let R_{34}' be the reflection of R_{34} in M_{34} .

Line d_1' connects points R_{12}' and R_{34}' .

Define similarly the lines d_2' and d_3' .

Then d_1', d_2' and d_3' are concurrent at a point P .

This points is a transformation of lines r, d to point P in QA.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3717
Date: 11/8/2019 10:17:55
From: chris.vantienhoven
Subject: Dual lines in QA

Dear Quang Hung, Dear Eckart.

This transformation QA-Tfx in QFG#3711 is much more "harmonic" than the one in QFG#3688.
Calculations are much simpler.
Some results:

1. $QA-Tfx(QA-Tfx(r)) = r$
2. QA-Tfx doesn't produce interesting results for QA-L1 till QA-L9
3. However $QA-Tfx(\text{Infinity Line}) = QA-Lx = \text{line through the midpoints of } (S12, S34), (S13, S24), (S14, S23)$

As a bycatch the infinity point of QA-Lx (denote as QA-Px) has some interesting properties:

Let $QA-Py = QA-Tf2(QA-Px)$, then:

QA-Py = a point on the Nine-point Conic QA-Co1.

QA-Py = the intersection point of these 4 lines:

- $QA-P1.QA-Tf2(\text{InfinityPoint}(QA-P1.QA-P16))$
- $QA-P5.QA-Tf2(\text{InfinityPoint}(QA-P5.QA-P17))$
- $QA-P10.QA-Tf2(\text{InfinityPoint}(QA-P10.QA-P16))$
- $QA-P19.QA-Tf2(\text{InfinityPoint}(QA-P1.QA-P5))$

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3718
Date: 11/8/2019 8:35:04
From: eckart_schmidt@t-online.de
Subject: Dual lines in QA

Dear Chris, dear Tran Quang Hung,

the line-line transformation in #3711
... is QL-Tf2 of the dual QL in the sense of QA-8/QL-8 in EQF.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3719
Date: 12/8/2019 9:56:50
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

wrt the property in #3710, which "... gives an easy way of drawing the quartic."

I tried it in the following way:

Let CI be the circle, centered in 5P-s-P3, mentioned there in EPG,

Let F1,2 be the fixed points of CSC3

... and M the center of a circle through F1,2,
intersecting CI in X

The lines X.CSC3(M) intersect the circle round M on the quartic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3720
Date: 12/8/2019 10:01:32
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

have we already mentioned, that
... the QA-P4 for the five QA of the 5P lie on the quartic?
Corresponding points have midpoints on the circle
... centered in 5P-s-P3 and mentioned there in EPG.
There are two points on the quartic,
... which are CSC-partner wrt the two 5P,
... points on the orthogonal hyperbola in # 3579, #3581,
... with midpoint in the middle of 5P-s-P4.5.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3721
Date: 12/8/2019 11:36:19
From: bernard.keizer
Subject: 5P-Quartic

Dear Eckart,
Of course, we have already mentionned this property ! (see messages 3615,3616 and 3617)
That's precisely why the CSC of the 5P and of the QA-P4 of the 5 QA's of the 5P define the cubic !
The middles of the segments joining the CSC1 partners on the cubic lie on the Newton Line and the middles of the segments joining the CSC3 partners on the quartic lie on the circle acting as a "Newton Line".
By the way, it is possible to generalise these quartics : taking any cevian with the DT vertices gives a new QL with a Van Reeves cubic with "QL-Cu1" on the DT circumcircle and a CSC2.
The CSC of the cubic gives a quartic invariant in a CSC3 with a circle instead of the Newton Line.
(With the Newton Line of the QL as cevian, we get DQL and a Van Reeves cubic with "QL-Cu1" in QL-P17).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3722
Date: 12/8/2019 9:10:57
From: eckart_schmidt@t-online.de
Subject: 5P-Quartic

Dear Bernard,

I apologize for my careless reading your messages,
... I confused the QA-P4 for the 4P of a 5P with the QA-P4
for the QG of a QL.

Perhaps of interest, a curious observation:
Consider a nonconvex 5P, so that the circumconic is a hyperbola
... with an obtused angled triangle TR, describe in #3579
... and the pedal point X of U=5P-sP5 on the longest side of TR.
The circle Ci(X) (see #3575) intersects the quartic in U and
points Y,
... which are contact points of tangents from U at the quartic.
For a 5P of QA-Cu7-triple points
... these contact points are not the QA-P4,
... which are the remaining three contact points,
concylic with U.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3723

Date: 12/8/2019 9:44:18

From: chris.vantienhoven

Subject: Another transformation with two lines in QA

Dear Quang Hung,

Your transformation as described in QFG#3716 is very interesting and has a very special application.

Let's denote the transformation as QA-Tfx.

There are two variable lines:

- r, that intersects the edges of the Quadrangle,
- d, that defines an infinity point that is used to construct parallel lines in the construction.

I substituted for both lines the QA-Lines QA-L1 up to and including QA-L9 as well as the line at infinity, here called QA-L0.

Denote $P_{ij} = \text{QA-Tfx}(\text{QA-Li}, \text{QA-Lj})$.

Finite and Infinite Points

- all P_{0i} -points and P_{ii} -points are Infinity Points
- all P_{i0} -points are not defined
- all other P_{ij} -points are Finite Points

General collinearities

$P_{0j}, P_{1j}, P_{2j}, P_{3j}, P_{4j}, P_{5j}, P_{6j}, P_{7j}, P_{8j}, P_{9j}$ are collinear for $j=1,2,\dots,9$

P_{i1}, P_{i4}, P_{i9} are collinear for $i=0,1,2,\dots,9$

Extra properties

$P_{ij} = \text{QA-Tfx}(\text{QA-Li}, \text{QA-Lj})$.

"Infinity Point" is abbreviated as "IP".

$P_{0i} = P_{ii}$, they are the infinity points of QA - Li, therefore:

$P_{01} = P_{11} = \text{IP}(\text{QA-P1.QA-P2})$

$P_{02} = P_{22} = \text{IP}(\text{QA-P1.QA-P32})$

$P_{03} = P_{33} = \text{IP}(\text{QA-P1.QA-P5})$

$P_{04} = P_{44} = P_{17} = \text{QA-Tf2}(\text{QA-P3})$

$P_{05} = P_{55} = \text{IP}(\text{QA-P10.QA-P11})$

$P_{06} = P_{66} = \text{IP}(\text{QA-P1.QA-P15})$

$P_{07} = P_{77} = \text{InfinityPoint of QA-L7,}$

the line through the 3 QL-P18 points in a Quadrangle

$P_{08} = P_{88} = \text{InfinityPoint of QA-L8}$

$P_{09} = P_{99} = \text{QA-Tf2}(\text{QA-P2}) = \text{Infinity Point}$

$P_{14} = \text{QA-P3}$

$P_{17} = P_{04} = P_{44} = \text{QA-Tf2}(\text{QA-P3}) = \text{InfinityPoint}$

$P_{19} = P_{29} = \text{QA-P2}$

$P_{24} = 5P-s-P_4 (P_1, P_2, P_3, P_4, QA-P_4)$
 P_{12} lies on $QA-P_2.QA-P_{33}$
 P_{41} lies on $QA-P_1.QA-Tf_2(QA-P_2)$
 P_{42} lies on $QA-P_{23}.QA-P_{33}$
 P_{69} lies on $QA-P_{14}.QA-P_{24}$
 P_{92} lies on $QA-P_1.QA-P_6.QA-P_{23}$

Special application

I proved algebraically that for all points $P_{ij} = QA-Tfx(QA-L_i, QA-fixed-line-L_j)$ are collinear on the 5th-point-tangent($InfinityPoint(QA-fixed-line-L_j)$).
 Note that the 5th-point-tangent ($QA-Tf_9$) of an infinity point is not necessarily the line at infinity.
 In fact, in general, it isn't.
 The advantage of this exercise with transformation $QA-Tfx$ is that it helps us to construct the 5th-point-tangent at an infinity point!
 Let L_1 and L_2 be two random lines and let P_0 be the Infinity Point of some line named L_0 .
 Then 5 th -point-tangent line $QA-Tf_9(P_0)$ simply can be constructed by applying $QA-Tfx(L_1, L_0)$ and $QA-Tfx(L_2, L_0)$ and by connecting both obtained $QA-Tfx$ -points.
 In general, a hyperbola can be constructed knowing 3 points on the conic and an asymptote.
 So I tried it in Cabri and the 4 th QA -point also lies very neatly on the constructed hyperbola.

Quartic

The envelope of all 5 th -point-tangents at an infinity point is a quartic touching all six sides of the Quadrangle. It is the same quartic as mentioned by Eckart in QFG#141.

This is as far as I got.

Best regards,
 Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3724
Date: 13/8/2019 6:58:58
From: chris.vantienhoven
Subject: Dual lines in QA

Dear Quang Hung, Dear Eckart.

Very nice property that the transformation QA-Tfx in QFG#3711 = QL-Tf2 of the dual QL.

It shows, even more, the harmonic properties of the transformation.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3725

Date: 13/8/2019 7:12:33

From: Tran Quang Hung

Subject: Another transformation with two lines in QA

Dear Chris,

Thank You so much for very detail explanation and the beautiful properties from this transformation. I learned a lot from your comments.

I see that you used infinity line of projective geometry in many times. If we take the infinity line back to plane, we shall have projective version as following:

Let P_1, P_2, P_3, P_4 be the defining Quadrangle Points.

r is a random line.

r meets lines $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$ at the points $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}$.

d is another random line.

d meets r at point X .

Let T_1, T_2, T_3 be the harmonic conjugate points: $(R_{12}, R_{34}; X, T_1) = (R_{13}, R_{24}; X, T_2) = (R_{14}, R_{23}; X, T_3) = -1$.

P is a point on d .

Line PT_1 meets the lines P_1P_2, P_3P_4 at points M_{12}, M_{34} .

Line d meets the lines P_1P_2, P_3P_4 at points M_{12}', M_{34}' .

Let R_{12}' and R_{34}' be the harmonic conjugate points:

$(M_{12}, M_{12}'; R_{12}, R_{12}') = (M_{34}, M_{34}'; R_{34}, R_{34}') = -1$

Line d_{12}' connects the points R_{12}' and R_{34}' .

Define similarly the lines d_{23}' and d_{34}' .

Then d_{12}', d_{23}' and d_{34}' are concurrent at a point Q .

Thus this is a transformation of lines r, d and P to point Q in QA.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3726

Date: 14/8/2019 9:20:30

From: eckart_schmidt@t-online.de

Subject: Another transformation with two lines in QA

Dear Quang Hung, dear Chris,

thanks to Quang Hung for the transformation in #3716
... and to Chris for the interesting observations in #3723.

Here another aspect:

If we consider a line r of a line pencil wrt a point P
... and lines d with any direction,
... QA-Tfx(r, d) give -varying d - a cubic $Cu(r)$,
... intersecting r on QA-Co1 and a common infinity point.

The cubic $Cu(r)$ has three asymptotes,
... two parallels and a parallel to r .
The cubics $Cu(r)$ can have up to three common points Q, R, S
... on the circumconic of QA through P ,
... on tangents from P at the quartic, mentioned in #3723,
first in #141

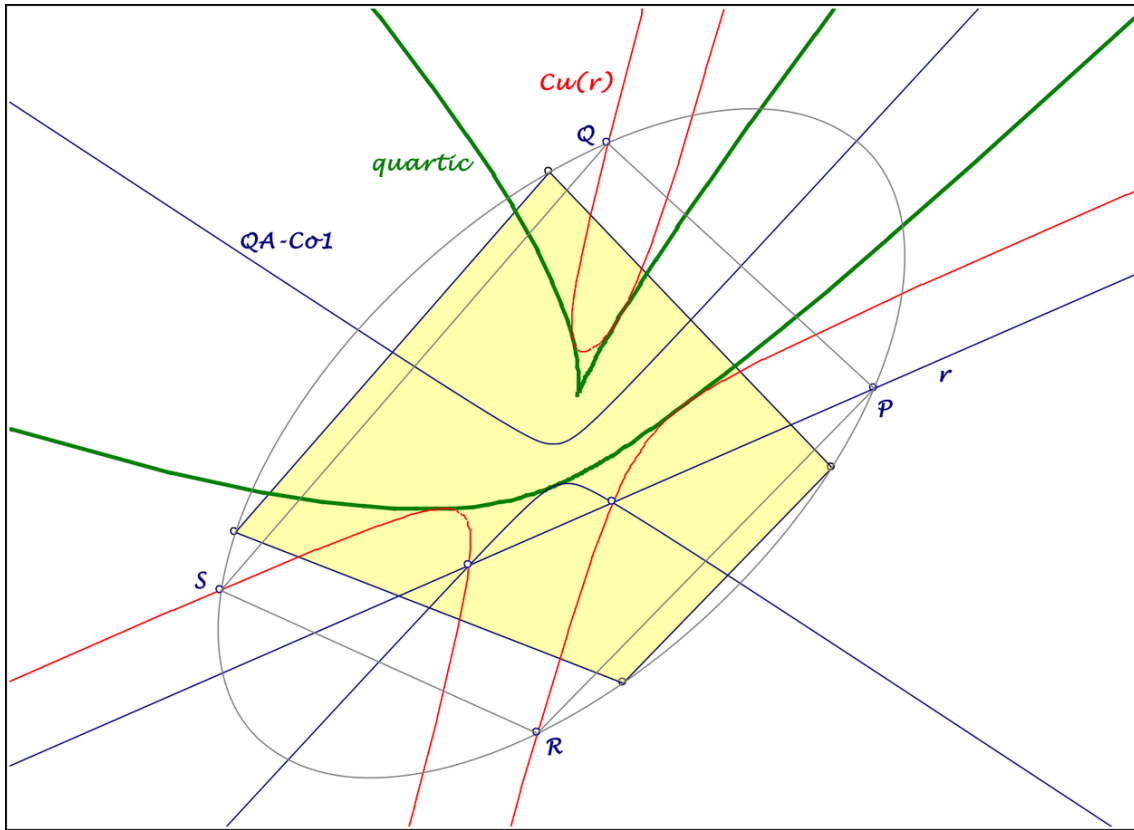
If there are three common points Q, R, S of the $Cu(r)$,
... PQRS is a QA with the same quartic.
If r is tangent to this quartic, the cubic $Cu(r)$ degenerates
... in three lines,
two of them coincide, the 3rd is r .

I hope someone can confirm these observations
... and lighten the background (see attached file).

What about the contact points of $Cu(r)$ and the quartic?
If a construction of the quartic isn't parat:
Consider a circle $C1$, centered in QA-P3 through QA-P2,
... intersecting QA-P3.QA-P4 in points X ,
... and a circle $Ci2$, centered in M on QA-Co1 through QA-P2,
... intersecting $X.QA-P2$ in N :
... The lines NM envelope the quartic.

Best regards Eckart

PS: I will be ready for two weeks of holiday ...



2019-08-15.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3727

Date: 15/8/2019 11:31:37

From: eckart_schmidt@t-online.de

Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

Dear Quang Hung, dear Chris,

This is an application of Quang Hung's transformation

... QA-Tfx(r,d)

in #3716, see also #3723, #3726:

Consider a reference QA = P1P2P3P4 and a point P,

... which will be the first point of a 2nd QA,

... further the cubics Cu(r) for lines r through P.

If these cubics have three common points Q, R, S,

... they are points on the circumconic of P1P2P3P4 through P

... and give the 2nd QA = PQRS

... with the same points QA-P1,2,3,4,6,7,8,23,32,33

as the reference QA,

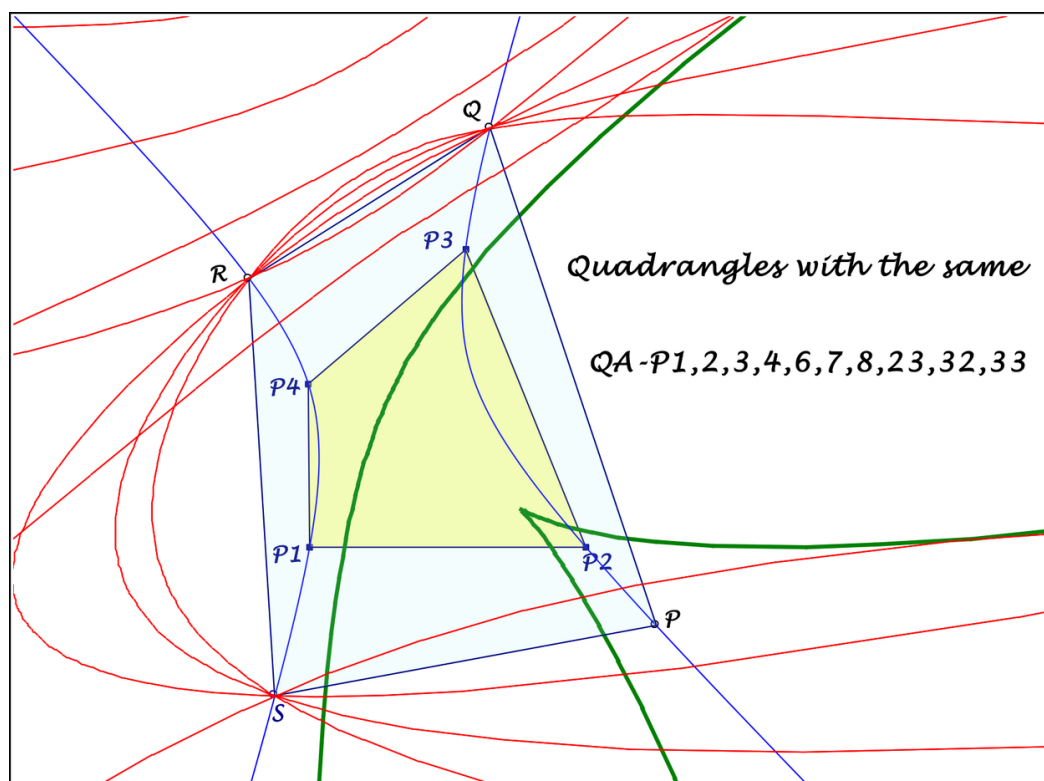
... in consequence with the same lines QA-L1,2,4 and

QA-P4.8.23.32, QA-P1.7.17, QA-P1.32.33.

The two QA have the same quartic (see #3726).

Best regards Eckart

PS. I hope, someone can confirm these CABRI-observations.



2019-08-15.pdf

Message: #3728
Date: 15/8/2019 3:25:03
From: eckart_schmidt@t-online.de
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

Dear Quang Hung, dear Chris,

in addition to #3727:

The two quadrangles have also the same QA-Co1,
... so it easy to construct the 2nd QA:

Reflect the point P in points of QA-Co1 and you get a conic,
... intersecting the QA-circumconic Co through P
... in four points Q, R, S and the diametral point of P wrt Co:
... PQRS is the 2nd QA in the sense of #3727.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3729
Date: 15/8/2019 9:41:33
From: eckart_schmidt@t-online.de
Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

Dear Quang Hung, dear Chris, dear Benedetto,

in #3727 and #3728 I described two QA with 10 common points,
... there is a further unexpected property:
... the two QA have the same parameter SIGMA.

This parameter is described in #1683 (first in #1471):
QA-P4 is the homothetic center
... of the 1st circumcenter quadrangle
... and the 1st isogonal conjugate quadrangle:
... The corresponding ratio is the parameter Sigma.

The parameter SIGMA is also Benedetto's "lambda" in EQF-ref [36].

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3730

Date: 16/8/2019 2:08:43

From: Tran Quang Hung

Subject: Quadrangles with the same QA-P1,2,3,4,6,7,8,23,32,33

Dear Eckart, dear Chris, dear Benedetto,
Thank You very much for your enthusiastic contribution. I can check with Cabri the simple properties on line and conic but I can't work with cubic. I am still studying your comments hard. They are very interesting and new with me.
Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3731

Date: 17/8/2019 7:54:08

From: Tran Quang Hung

Subject: Coaxial circles in QA

Dear Chris, dear Eckart,

I wonder that we can change the harmonic conjugate with any ratio k . As following
 P_1, P_2, P_3, P_4 be the defining Quadrangle Points.
Let $S_1 = P_1P_2$ cuts P_3P_4 , $S_2 = P_1P_3$ cuts P_2P_4 and $S_3 = P_1P_4$ cuts P_2P_3 .
Line S_2S_3 meets lines P_1P_2 and P_3P_4 at S_{12} and S_{34} .
Line S_1S_2 meets lines P_1P_4 and P_2P_3 at S_{14} and S_{23} .
Line S_3S_1 meets lines P_1P_3 and P_2P_4 at S_{13} and S_{24} .
 r is a random line.
 r meets lines $P_1P_2, P_1P_3, P_1P_4, P_2P_3, P_2P_4, P_3P_4$ at the points $R_{12}, R_{13}, R_{14}, R_{23}, R_{24}, R_{34}$.
Let k be a real number.
Let R_{34}' be the point such that $(S_1, S_{34}; R_{34}, R_{34}') = k$.
Let R_{12}' be the point such that $(S_1, S_{12}; R_{12}, R_{12}') = k$.
We define similarly the points $R_{13}', R_{14}', R_{23}', R_{24}'$.
Now the circles with diameters $(R_{12}'R_{34}'), (R_{13}'R_{24}'), (R_{14}', R_{23}')$ are coaxial.
Could You please help me to check it?

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3732
Date: 17/8/2019 8:20:29
From: chris.vantienhoven
Subject: Coaxial circles in QA

Dear Quang Hung,

I like your method of not taking a harmonic conjugate of three points and instead of that an anharmonic point with fixed cross-ratio k .

However, in my drawings, I cannot confirm that the circles with diameters $(R_{12}'R_{34}')$, $(R_{13}'R_{24}')$, (R_{14}',R_{23}') are coaxial.

In my drawings these three circles have different radical axes coinciding in a Radical Center X .

The locus of X for variable k is a Quartic passing through QA-P11, the circumcenter of the QA-Diagonal Triangle.

Best regards,
Chris

p.s. More about anharmonic ratios can be found at Perspective Fields (<https://www.chrisvantienhoven.nl/mathematics/perspective-fields>) part II.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3733
Date: 18/8/2019 1:05:45
From: Tran Quang Hung
Subject: Coaxial circles in QA

Dear Chris,

Thank You very much for your confirmation. Now I know about Perspective Fields (<https://www.chrisvantienhoven.nl/mathematics/perspective-fields>),

It looks very interesting for me. I shall study it hard.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3734
Date: 18/8/2019 8:10:55
From: Tran Quang Hung
Subject: Two QA have the same Poncelet point

Let A, B, C, D be a QA with Gergone point P (QA-P3).
Let Pa, Pb, Pc, Pd be the isogonal conjugate of P wrt triangles BCD, CDA, DAB, ABC , resp.
 $QA-P2(A,B,C,D) = QA-P2(Pa,Pb,Pc,Pd)$.

Moreover

Let Ma, Mb, Mc, Md be the midpoints of segments APa, BPb, CPc, DPd .

Then we have the "area" identity $[ABCD] + [PaPbPcPd] = 2[MaMbMcMd]$
 $[ABCD] = 1/2 \text{cross product}(\text{vec}(AC), \text{vec}(BD))$, if $ABCD$ is convex
then $|[ABCD]| = \text{Area}(ABCD)$.

We can compare with the properties of Orthocenter Quadrangle of (A,B,C,D) . If Ha, Hb, Hc, Hd be the Orthocenter Quadrangle of (A,B,C,D) . Then
 $QA-P2(A,B,C,D) = QA-P2(Ha,Hb,Hc,Hd)$ and $[ABCD] = [Ha,Hb,Hc,Hd]$.

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3735
Date: 19/8/2019 4:02:07
From: Tran Quang Hung
Subject: Newton lines are concurrent in QA

Dear geometers,

Let (A,B,C,D) be a QA.
 $Lab, Lbc, Lcd, Lda, Lac, Lbd$ are the perpendicular bisectors of segments AB, BC, CD, DA, AC, BD respectively.
Then Newton lines of the QL
 $QL-L-1(Lab, Lcd, Lad, Lbc)$
 $QL-L-1(Lac, Lbd, Lcd, Lda)$
 $QL-L-1(Lab, Lbc, Lac, Lbd)$
are concurrent. Which is this point of QA?

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3736
Date: 19/8/2019 5:54:46
From: Tran Quang Hung
Subject: Steiner line of Gergonne point

Dear geometers

Let (A, B, C, D) be a QA.
 $L_{ab}, L_{bc}, L_{cd}, L_{da}, L_{ac}, L_{bd}$ are the perpendicular bisectors of segments AB, BC, CD, DA, AC, BD respectively.
 L_{ab} meets L_{cd} at P_1 .
 L_{bc} meets L_{da} at P_2 .
 L_{ac} meets L_{bd} at P_3 .
Then Gergonne point $P = QA-P3(A, B, C, D)$ lies on circle $(P_1P_2P_3)$.
Let St be the Steiner line of P wrt triangle $P_1P_2P_3$. (Steiner line is dilation center P of Simson line with ratio 2).
Let St' be the orthopolar line of St wrt $QA(A, B, C, D)$ i.e $St' = QA-Tf8(A, B, C, D, St)$.
Then St' passes through Poncelet point of $QA(A, B, C, D)$.
Which are the other points of $QA(A, B, C, D)$ on this line?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3737
Date: 19/8/2019 10:46:03
From: chris.vantienhoven
Subject: Two QA have the same Poncelet point

Dear Quang Hung,

Nice properties regarding QA's built from points relating to the
4 Component Triangles QA-4Tr1

One thing I cannot confirm.

In my drawings and calculations Ma, Mb, Mc, Md are collinear.

Can you affirm?

The line they define is a line through QA-P1 with long
coordinates.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3738
Date: 19/8/2019 12:07:48
From: chris.vantienhoven
Subject: Newton lines are concurrent in QA

Dear Quang Hung,

Your point in #3735 is QA-P32: Centroid of the Circumcenter
Quadrangle.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3739
Date: 19/8/2019 12:27:29
From: Tran Quang Hung
Subject: Two QA have the same Poncelet point

Dear Chris,

Thank You very much for your comments. Your confirmation is very nice and right. I affirm that Ma, Mb, Mc, Md are collinear. Actually I was confused when I define Ma, Mb, Mc, Md , but by chance, we see them collinear. Thank You.

I now rewrite my message,

Let A, B, C, D be a QA with Gergone point P (QA-P3).

Let Pa, Pb, Pc, Pd be the isogonal conjugate of P wrt triangles BCD, CDA, DAB, ABC , resp.

$QA-P2(A,B,C,D) = QA-P2(Pa,Pb,Pc,Pd)$.

Moreover

Let Na, Nb, Nc, Nd be the midpoints of segments CPa, DPb, APc, BPd .

Then we have the "area" identity $[ABCD] + [PaPbPcPd] = 2[NaNbNcNd]$.

$[ABCD] = 1/2 \text{cross product}(\text{vec}(AC), \text{vec}(BD))$, if $ABCD$ is convex then $|[ABCD]| = \text{Area}(ABCD)$.

We can compare with the properties of Orthocenter Quadrangle of (A,B,C,D) .

If Ha, Hb, Hc, Hd be the Orthocenter Quadrangle of (A,B,C,D) .

Then $QA-P2(A,B,C,D) = QA-P2(Ha,Hb,Hc,Hd)$ and $[ABCD] = [HaHbHcHd]$.

Best Regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3740
Date: 19/8/2019 7:17:12
From: chris.vantienhoven
Subject: Steiner line of Gergonne point

Dear Quang Hung,

Regarding your message #3736.

I checked everything algebraically and it all stands.

QA-P3 on P1.P2.P3-circle

The points P1, P2, P3 are also the three QA-versions of Quadrignon Point QG-P5 (= 1st QG-Quasi Circumcenter). It is known from the circle through these 3 QA-versions that QA-P3 lies on it. See QG-P5.

The center of the circle through these 3 QA-versions of QG-P5 is the Reflection of QA-P15 in QA-P1. It lies on QA-L6 = QA-Newton-Morley Line.

Well actually it is also the common point of the Eulerlines of triangles QG-P5a.QG-P5b.QL-P4c, QG-P5b.QG-P5c.QL-P4a, QG-P5c.QG-P5a.QL-P4b (a/b/c denoting the QA-versions of the involved points).

QA-P2 of QA-Tf8(Tr-SteinerLine)

The Steiner line St for triangle P1.P2.P3 happens to be the line QA-P4.QA-P5.

Apart from QA-P2, there are no other known QA-points on St' = QA-Tf8(St).

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3741
Date: 21/8/2019 3:27:37
From: Tran Quang Hung
Subject: Common orthocenter from triangle

Dear geometers

I wrote on ADGEOM †)

>> Let ABC be a triangle and any point P.
>> Ha, Hb, Hc are the orthocenters of triangles PBC, PCA, PAB
reps.
>> Then the triangles AHbHc, BHcHa, CHaHb have the same
orthocenter.
>> Is this theorem known before and which is this common
orthocenter in term
>> of P?
> I think this problem is interesting with QA.
> We let the above common center by the transformation
 $Xp = Tf(ABC, P)$.
> Now let $QA(A, B, C, D)$.
> $Xd = Tf(ABC, D)$

$Xa = Tf(BCD, A)$
 $Xb = Tf(CDA, B)$
 $Xc = Tf(DAB, C)$

We have some properties of $QA(Xa, Xb, Xc, Xd)$
- $QA(Xa, Xb, Xc, Xd)$ and $QA(A, B, C, D)$ have the same Poncelet point
i.e $QA-P2(Xa, Xb, Xc, Xd) = QA-P2(A, B, C, D)$.
- Newton line of $QL(AXa, BXb, CXc, DXc)$ passes through common
Poncelet point of $QA(A, B, C, D)$ and $QA(Xa, Xb, Xc, Xd)$.
- The line passes through Centroid of $QA(A, B, C, D)$ and
 $QA(Xa, Xb, Xc, Xd)$ is perpendicular to Newton line of $QL(AXa, BXb,$
 $CXc, DXc)$.
- Equal area $[XaXbXcXd] = [ABCD]$.
Because of the above properties, can we call $QA(A, B, C, D)$ and
 $QA(Xa, Xb, Xc, Xd)$ by dual QA?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[50\]](#).

Message: #3742
Date: 21/8/2019 8:17:50
From: chris.vantienhoven
Subject: Common orthocenter from triangle

Dear Quang Hung,

I can confirm all your properties except that the line passing through Centroid of $QA(A,B,C,D)$ and $QA(Xa,Xb,Xc,Xd)$ is perpendicular to the Newton line of $QL(AXa, BXb, CXc, DXc)$. I wouldn't prefer the name dual, because this name mostly is used for a transformation mapping points into lines and a corresponding transformation mapping lines into points, which isn't the case here.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3743
Date: 21/8/2019 9:09:24
From: Tran Quang Hung
Subject: Common orthocenter from triangle

Dear Chris,

Thank You very much for your reply. You are right, the property "the line passing through Centroid of $QA(A,B,C,D)$ and $QA(Xa,Xb,Xc,Xd)$ is perpendicular to the Newton line of $QL(AXa, BXb, CXc, DXc)$." is wrong. I was false.

I agree with you, "dual" name is better for projective properties. But two $QA(A, B, C, D)$ and $QA(Xa, Xb, Xc, Xd)$ are very similar, I try to find some other properties.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3744

Date: 21/8/2019 3:52:52

From: Tran Quang Hung

Subject: Perpendicular lines from NPC in triangle

Dear geometers,

Let ABC be a triangle and P is any point.

Let N_a, N_b, N_c be NPC centers of triangles PBC, PCA, PAB , reps.

Perpendicular lines from N_a, N_b, N_c to PA, PB, PC are concurrent at point N_p .

We denote by the transformation $N_p = Tf(ABC, P)$.

Let (A, B, C, D) be QA.

$N_a = Tf(DBC, A)$

$N_b = Tf(CDA, B)$

$N_c = Tf(DAB, C)$

$N_d = Tf(ABC, D)$

We have property

$QA-P1(A, B, C, D) = \text{Reflection of } QA-P2(N_a, N_b, N_c, N_d) \text{ in}$

$QA-P3(N_a, N_b, N_c, N_d)$.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3745
Date: 21/8/2019 5:17:44
From: Tran Quang Hung
Subject: Apply transformations in triangle to QA

Dear geometers,

Let ABC be a triangle and any point P . We had have many transformations $Tf(ABC,P)$, these transformations can return points, lines or circles.

So what happen when we apply them for QA? We shall have

Points $Pa = Tf(BCD,A)...$

or lines $La = Tf(BCD,A)...$

or circles $Ca = Tf(BCD,A)...$

We shall have new QA, QL and set of four circles.

I think we shall have many new points and properties?

I look forward to hearing from You.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3746

Date: 21/8/2019 10:43:09

From: chris.vantienhoven

Subject: Perpendicular lines from NPC in triangle

Dear Quang Hung,

QA-P1(Na,Nb,Nc,Nd) = point on QA-P1. QA-P32. QA-P33

QA-P2(Na,Nb,Nc,Nd) = QA-P33

QA-P3(Na,Nb,Nc,Nd) = P1.P32.P33 & QA-P2.QA-P23
& QA-P14.QA-P37

QA-P4(Na,Nb,Nc,Nd) = point on QA-P2.QA-P23

QA-P5(Na,Nb,Nc,Nd) = point on QA-P1.QA-P40 & QA-P5.QA-P24
& QA-P14.QA-P20

Special for is that QA-P3(Na,Nb,Nc,Nd) also is:

common point of the 3 QA-versions (see QA-3QG1)

of the lines QG-P7.QL-P2

(QG-P7 = 2nd Quasi Nine-point Center,

QL-P2 = point made from Component Nine-point Centers)

It's all connected!

An alternative:

Let: Perpendicular lines from Na, Nb, Nc to BC, CA, AB are
concurrent at point Mp.

We denote by the transformation $M_p = \text{Tf2}(ABC, P)$.

Now:

$M_a = \text{Tf2}(DBC, A)$

$M_b = \text{Tf2}(CDA, B)$

$M_c = \text{Tf2}(DAB, C)$

$M_d = \text{Tf2}(ABC, D)$

and

QA-P1(M_a, M_b, M_c, M_d) = point on QA-P1. QA-P11 & QA-P5. QA-P12 &
QA-P13. QA-P20 & QA-P30. QA-P34

QA-P2(M_a, M_b, M_c, M_d) = QA-P3

QA-P3(M_a, M_b, M_c, M_d) = QA-P32

QA-P4(M_a, M_b, M_c, M_d) = QA-P1

QA-P5(M_a, M_b, M_c, M_d) = point on P1.P32.P33 & P2.P8

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3747
Date: 21/8/2019 10:50:27
From: chris.vantienhoven
Subject: Apply transformations in triangle to QA

Dear Quang Hung,

Different geometers have been exploring several triangle points and transformations wrt the Components Triangles of a Quadrangle.

But I am sure there is more to explore that has not been discovered yet.

Of course we also have the domain of exploring triangle points and transformations wrt the Components Triangles of a Quadrilateral. Often forgotten because people tend to think in "points" and not in "lines".

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3748
Date: 22/8/2019 1:55:14
From: Tran Quang Hung
Subject: Apply transformations in triangle to QA

Dear Chris,

Thank You for your reply. I agree with you that we are often more familiar with points than with lines.

I also felt that I was working with QA more familiar than QL but the fact that the idea of QL was great and I really found myself lucky in exploring some new properties.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3749
Date: 22/8/2019 3:56:23
From: Tran Quang Hung
Subject: Apply P-Hagge circle to QA

Dear geometers,

Let ABC be a triangle and any point P.
A'B'C' is circumcevian triangle of P.
A'', B'', C'' are reflection on A', B', C' in lines BC, CA, AB
reps.
Then circle (A''B''C'') passes through orthocenter H of ABC and it
was called by P-Haage circle of P wrt ABC.
Let (Op) = Tf(ABC,P).
Let QA(A,B,C,D).
(Oa)=Tf(BCD,A), (Ob)=Tf(CDA,B), (Oc)=Tf(DAB,C), (Od)=Tf(ABC,D).
We have the parallel lines P2(A,B,C,D)P3(Oa,Ob,Oc,Od) //
P3(A,B,C,D)P2(Oa,Ob,Oc,Od).

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3750
Date: 22/8/2019 10:40:09
From: chris.vantienhoven
Subject: Apply P-Hagge circle to QA

Dear Quang Hung,

TQH: We have the parallel lines P2(A,B,C,D)P3(Oa,Ob,Oc,Od) //
P3(A,B,C,D)P2(Oa,Ob,Oc,Od).
The line QA-P2(A,B,C,D).QA-P3(Oa,Ob,Oc,Od) = QA-P2.QA-P23, and
the line QA-P3(A,B,C,D).QA-P2(Oa,Ob,Oc,Od) = QA-P3.QA-P32
which are known parallel lines.

See: QA-3: List of parallel QA-Lines

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3751
Date: 22/8/2019 10:54:25
From: Tran Quang Hung
Subject: Perpendicular lines from NPC in triangle

Thank You dear Chris for nice and interesting observation.
The case "perpendicular lines to BC, CA, AB" is very new with me.
I shall study it more.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3752
Date: 22/8/2019 11:10:09
From: Tran Quang Hung
Subject: Apply P-Hagge circle to QA

Dear Chris, thank You very much for quick response. I study some more properties of four circles (Oa), (Ob), (Oc), (Od) but I do not see any thing.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3753
Date: 22/8/2019 11:40:35
From: Tran Quang Hung
Subject: A conjecture on two QA

Dear geometers,

I have a conjecture on two QA as following

Let $QA(A,B,C,D)$ and $QA(A',B',C',D')$ be two QA such that

- They have common Poncelet point QA-P2.
- They have equal area $[ABCD] = [A'B'C'D']$.

Then Newton line of $QL(AA',BB',CC',DD')$ passes through the common Poncelet point?

Could You please confirm this conjecture? I can't check it by my computer.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3754
Date: 22/8/2019 4:57:00
From: chris.vantienhoven
Subject: A conjecture on two QA

Dear Quang Hung,

I did a tryout with a QA and its QA-P2.
I rotated the QA through an angle of 15 degrees around QA-P2.
Obviously, the rotated QA will have the same area and the same QA-P2.
Then it was easy to connect corresponding vertices giving the 4 lines of a quadrilateral QL.
However, the Newton Line (QL-L1) of the QL did not pass through the common QA-P2.
My conclusion: when there is one example that doesn't hold, the conjecture will not hold.

Some extra remarks to play with:

1. QA-P2 is the center of the unique rectangular conic passing through the vertices of the QA. Let's say QA1.
2. Therefore a second QA (QA2) will have another unique rectangular conic passing through the vertices of the QA2.
3. When we have the only condition that the QA's have equal areas, then there are no corresponding vertices and there will be different sets of 4 lines that can be drawn.
4. The Newton line is the locus of all conics inscribed in the 4 lines of the QL.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3755
Date: 23/8/2019 1:51:46
From: Tran Quang Hung
Subject: A conjecture on two QA

Dear Chris

Thank You so much for counterexample. I met at least 2 such QA and I quickly concluded, I was not right. Thank you for very detailed instructions.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3756
Date: 23/8/2019 2:04:42
From: Tran Quang Hung
Subject: Apply a transformation to QA

Dear geometers

Let ABC be a triangle and P is any point.
 PA meets circle (PBC) again at A' .
 A'' is projection of A' on BC .
Define similarly the points B'' , C'' . Then AA'' , BB'' , CC'' are concurrent at point P^* .
Let us call this transformation by $X_p = Tf(ABC, P)$.
Let (A, B, C, D) be a QA.
 $X_a = Tf(BCD, A)$, $X_b = Tf(CDA, B)$, $X_c = Tf(DBA, C)$, $X_d = Tf(BAC, D)$.

We properties of $QA(X_a, X_b, X_c, X_d)$

- They have the same Poncelet point $QA-P2(A, B, C, D) = QA-P2(X_a, X_b, X_c, X_d)$.
- Equal area $[ABCD] = [X_a X_b X_c X_d]$
- Newton line of $QL(AX_a, BX_b, CX_c, DX_d)$ passes through common Poncelet point of these two QA.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3757

Date: 23/8/2019 10:23:57

From: bernard.keizer

Subject: 5P, CSC and twin conics, cubics and quartics

Dear Chris, dear Eckart,

Before leaving definitively the item of the 5 QA-Cu7 triple points, perhaps these simple observations, which deserve perhaps a place in the properties or curves of the 5P.

Giving 5 points X_i and the CSC, let's name $Y_i = \text{CSC}(X_i)$, $\text{isgi} = \text{QA-P4}(X_j \text{ with } j \text{ not equal to } i)$ and the same way $\text{isg}'_i = \text{QA-P4}(Y_j \text{ with } j \text{ not equal to } i)$.

The twin conics are the conics of the X_i and of the Y_i with the points T and T' .

The middles of $X_i\text{isgi}$ and of $Y_i\text{isg}'_i$ lie on 2 circles, which we could name Newton circles.

The middles of $X_i\text{CSC}(\text{isg}'_i)$ and of $Y_i\text{CSC}(\text{isgi})$ lie on the same line, which we name Newton Line.

The points X_i and $\text{CSC}(\text{isg}'_i)$ and Y_i and $\text{CSC}(\text{isgi})$ define the twin cubics which are Van Rees circular focal cubics with foci U and $\text{CSC}(U)$; the cubics are invariant under CSC transformations centered in U and $\text{CSC}(U)$.

The cubics are circular circumcubics of the $5X_i$ or the $5Y_i$ and the asymptotes of the cubics are the parallel to the Newton Line through T and T' , the pivot being the same for both cubics, id est the infinity point of the Newton Line; the asymptote cuts it's cubic in a point Q or Q' which is the 6th intersection between the conic and the cubic (or the 2nd intersection of the asymptote and the conic).

The middles of UT and $\text{CSC}(U)T'$ lie on the Newton Line and on the Newton circles; the middles of $UQL\text{-}P1$ and $\text{CSC}(U)QL\text{-}P1$ lie also on the Newton circles.

The CSC of the 2 twin cubics define the twin quartics through the X_i and the isgi and through the Y_i and the isg'_i ; they pass through $QL\text{-}P1$ and through U and $\text{CSC}(U)$; the quartics are invariant under CSC transformations centered in $\text{CSC}(T)$ and $\text{CSC}(T')$.

There are surely other observations to be made, but I stop here for today ...

I hope Eckart will confirm these 1st observations.

Best regards

Bernard

PS As mentionned by Eckart, if the 5P are 5 QA-Cu7 triple points, there isn't a twin QL !

Message: #3758
Date: 23/8/2019 1:45:18
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Chris, dear Eckart,
Of course, the points U and CSC(U) can be obtained directly with Eckart's construction from the Xi or the Yi.
But they are also the the tangentials of Q and Q' on the twin cubics.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3759
Date: 24/8/2019 8:01:46
From: chris.vantienhoven
Subject: Apply a transformation to QA

Dear Quang Hung,

Nice example again of quadrangles with equal areas and equal QA-P2.
The line QA-P2.QA-P33 is perpendicular to the Newton Line of QL (AXa, BXb, CXc, DXd) .

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3760
Date: 25/8/2019 4:49:03
From: eckart_schmidt@t-online.de
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Bernard,

Uff, hard work after holiday,
... to reproduce all the properties of your twin-geometry
... for a constellation, which was one and a half year our
terrain.
... Some properties were new for me, but I found no errors!

Have you researched the twin-aspect
... for the orthogonal hyperbolas and the triangles,
... mentioned in #3579 and 3581?

The 5P of QA-Cu7-triple points are special,
... is there perhaps a twin-property,
... which doesn't hold for an arbitrary 5P?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3761
Date: 26/8/2019 5:18:54
From: Tran Quang Hung
Subject: Apply a transformation to QA

Dear Chris

Thank You very much for your interest. I try to find some more
configurations with equal are and equal P2.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3762
Date: 26/8/2019 8:28:49
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard, dear Chris,

in #3599 a quartic and in #3603 a cubic are discussed,
... which are 5P-CSC partner.

Here some observations wrt their intersections:
There are up to 3 pairs of intersections, which are CSC-partner:
... One pair lies on the orthogonal hyperbola, described in #3579,
... with the same midpoint as 5P-s-P4 and 5P-s-P5.
... The other two pairs - not always real - are collinear with
5P-s-P5.

If we consider the first pair X, Y,
... the QG = (X, 5P-s-P4, Y, 5P-s-P5) is a parallelogram.
... the QG = (X, 5P-s-P5, Y, CSC(5P-s-P5)) has a cubic QL-Cu1,
... bearing 5P-s-P6, its CSC is the 5P-CSC,
... its QL-L1 bears the midpoints of 5P-s-P4.5P-s-P5 and
5P-s-P5.CSC(5P-s-P5).

Perhaps there is a connection for the 5P of QA-Cu7-triple points
... of the last QG, interpreted as QL, and the QL for the QA-Cu7.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3763
Date: 27/8/2019 12:15:05
From: eckart_schmidt@t-online.de
Subject: 5P-Quartics for 6P

Dear Bernard, dear Chris,

if we consider the six 5P-quartics for a 6P,
... we have one, two or three common points.

What about these points?

Best regards Eckart



2019-08-27.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3764
Date: 27/8/2019 1:01:46
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Eckart,

Thanks for your answer !

The 5 X_i define a conic, a rectangular hyperbola, a CSC with center M , a point T and a point U and the 3 points V_1, V_2 and V_3 , a cubic and a quartic.

The $Y_i = \text{CSC}(X_i)$ define the same elements.

The circumcircle of the V_i , the circumconic and the RH intersect in the same 4 points V_i and T .

The quartics are the CSC of the cubics.

The conics and the cubics intersect in 6 points, the X_i and a point Q .

The conics and the quartics intersect in 8 points, the X_i and the V_i .

The beauty of all these curves and points is that U' is $\text{CSC}(U)$
...

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3765
Date: 28/8/2019 10:18:57
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
I checked your properties with the 3 pairs of CSC partners.
You have obviously the same results with the twin cubic and quartic replacing U and T by CSC(U) and T'.
For the QL-Cu1, it's not surprising, as you start with 2 copples of CSC partners (X and Y and U and CSC(U)).
For your point 5P-s-P6, you and me understand what it means, but it unfortunately, it isn't yet in EPG ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3766
Date: 29/8/2019 10:22:31
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,

perhaps of interest for your twin-geometry:
Let Cu and Qu be 5P-cubic and 5P-quartic
... and Cu' and Qu' the corresponding twin-objects:

There are two tripl of collinear intersections of $Cu \wedge Qu'$ and $Cu' \wedge Qu$
... on two lines through 5P-s-P6, which are CSC-partner.
The quartics Qu and Qu' have a common tangent in 5P-s-P6.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3767
Date: 29/8/2019 11:40:13
From: chris.vantienhoven
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,
You commented that 5P-s-P6 isn't in EQF and you are right.
It is in the pipeline with many other items to be included in EQF.
It takes a lot of effort to realize that and I will do it when all things work together.
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3768
Date: 29/8/2019 4:38:09
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px

Dear Chris,

I think, the following pair of points isn't mentioned till now:
Consider for tangents T_g at 5P-s-Co1 in 5P-vertices
... the conics QA-Tf2(T_g) wrt the remaining QA:
These five conics have two common points.
What about these points?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3769
Date: 29/8/2019 6:00:32
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
Thanks a lot for this new property !
Now perhaps another one : the middles of the segments X_iY_i (with Y_i and X_i CSC partners) lie on a rectangular hyperbola and the 10 points lie on a curve obviously with 5P-s-P6 as double point which contains all the points CSC partners with the middle of the segment joining them on this curve.
This curve is by definition CSC invariant.
Any idea about this curve ? Does it contain the isg and their CSC and the isg' and their CSC ?
I reach the limits of my ability and perhaps willing to draw new curves ...
Perhaps you will be able to help me !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3770
Date: 29/8/2019 7:43:15
From: bernard.keizer
Subject: 5P Cubic/Quartic Intersections

Dear Eckart,
Sorry for the typo, the middle is of course on the RH !
Best regards
Bernard

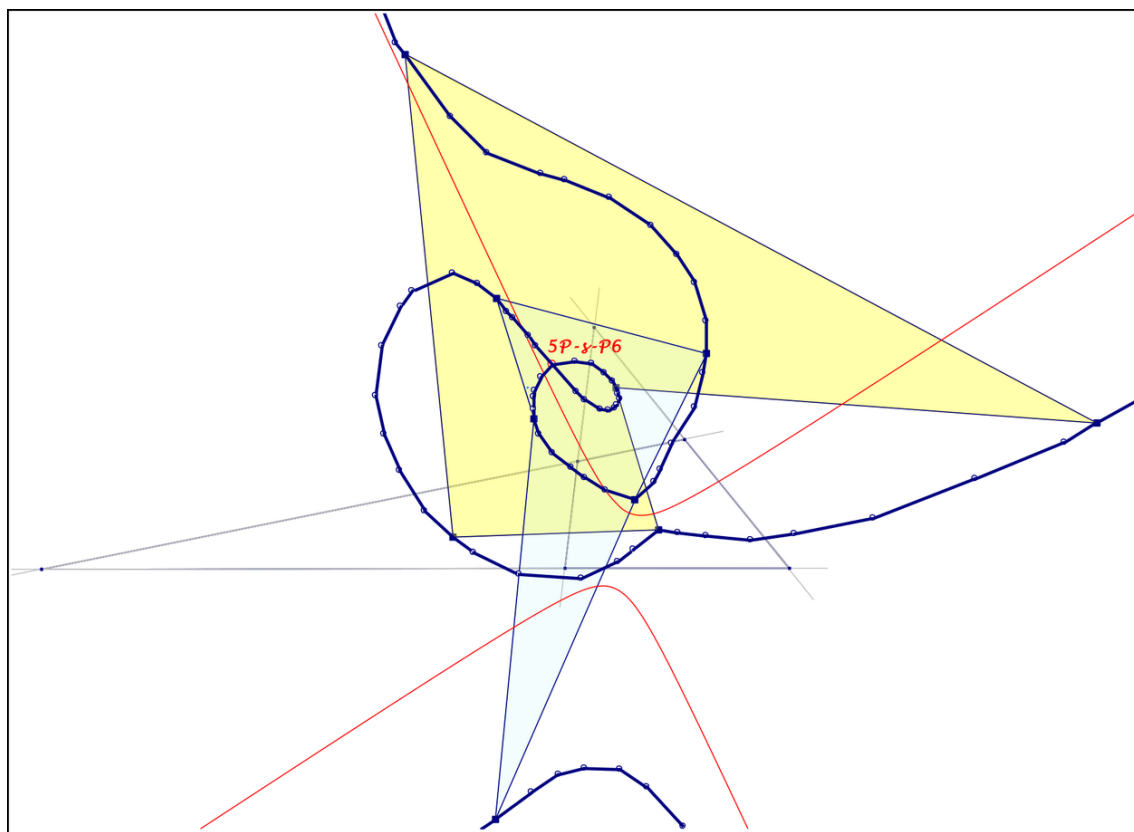
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3771
Date: 30/8/2019 11:14:44
From: eckart_schmidt@t-online.de
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Bernard,

wrt your #3769, attached an approximately drawing of the curve,
... it will be a sextic, CSC-invariant,
... but not bearing the isg and their CSC.

Best regards Eckart



2019-08-30.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3772

Date: 30/8/2019 12:37:51

From: chris.vantienhoven

Subject: 5P-s-2Px giving Guitar-shaped Locus

Dear Eckart,

That are two remarkable 5P-points indeed.

Let's call these points X1 and X2.

I found these properties:

1. QA-Tf2(Xi) wrt any QA(PiPjPkPl) = (Xi-polar $\wedge$ Pm-tangent) wrt 5P-circumscribed-conic.

2. When the 5P = QA + QA-DT-point Si, then X1 & X2 are the intersection points of the two QA-sides through Si and the QA-DT-side opposite Si.

3. Spanning a QA P1.P2.P3.P4 in a reference Conic C0 and some extra point P5 on this conic, the locus of X1 and X2 with fixed QA and variable P5 gives *a very special guitar-shaped-locus* (see attached picture) with these properties:

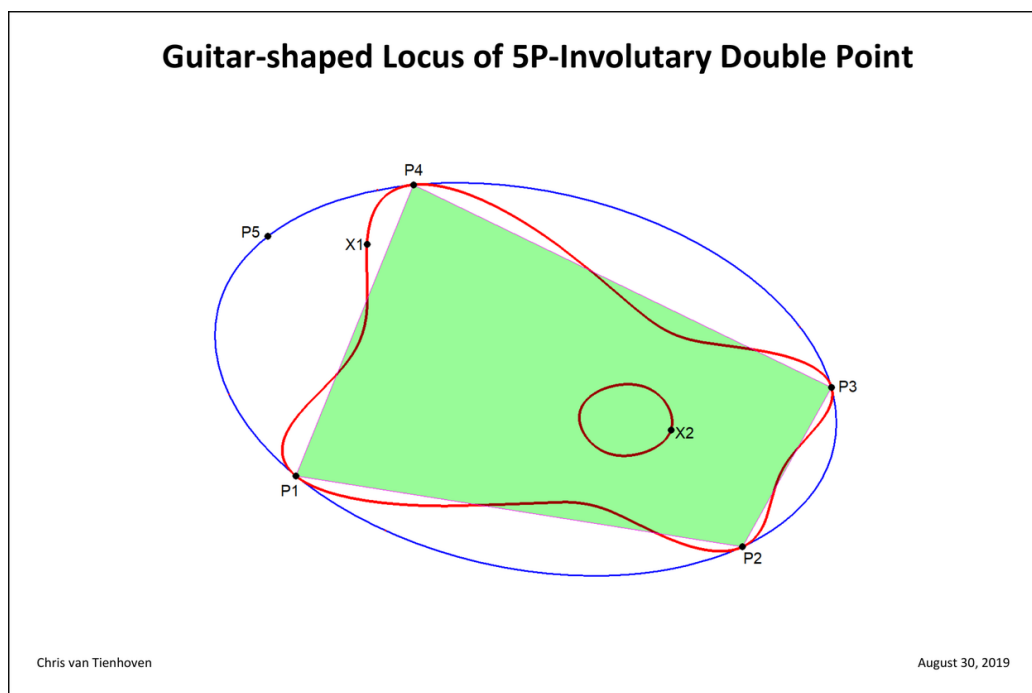
a. It is tangent to C0 at the QA-vertices.

b. Each of the 6 QA-sides is intersected at the two vertices and two other points, each intersected by one of the defining QA-Tf2(tangent)-conics. Therefore these points can be easily constructed and calculated.

Regardless of these special properties somehow I think there should be some other simple property related to these points.

Best regards,

Chris



5p-s-2P99 Guitar-shaped Locus-01.pdf

Message: #3773
Date: 01/9/2019 2:51:03
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

Dear Chris, dear Benedetto,

background for this circle is the message 2910:
Consider a 5P and for each vertex P_i
... the four midpoints of $P_i.P_j$ and the quadrangle
... of their images wrt $5P-s-Tf3_{inv} = Co-Tf3$ wrt $5P-s-Co1$.
The five QA-P4 of these quadrangles are concyclic.

Best regards Eckart

PS: Thanks to Chris for further observations for 5P-s-2Px.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3774
Date: 01/9/2019 8:59:13
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px

Dear Chris,

just studying my old messages,
... I found the 5P-s-2Px already in #2912.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3775
Date: 02/9/2019 12:13:13
From: bernard.keizer
Subject: 5P, CSC and twin conics, cubics and quartics

Dear Eckart,
Thanks for your answer, specially for the drawing !
Is it possible to generalise ?
Having a CSC, any Newton Line gives a Van Rees focal circular cubic, which is CSC invariant.
We found 2 copples cubic/quartic CSC partners with 2 Newton circles.
Now, this sextic is CSC invariant with a rectangular hyperbola as Newton conic ...
The construction could be the same as the Van Rees cubic : for a point m taken as middle, the line through 2 CSC partners is the bisector of the angle F_1mF_2 with F_1 and F_2 fixed points of the CSC and the 2 CSC partners are on a circle through the 2 fixed points, harmonic conjugates of these 2 fixed points on the circle.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3776
Date: 02/9/2019 2:28:21
From: chris.vantienhoven
Subject: 5P-s-Cix

Dear Eckart,

Peculiar 5P-Circle you found.
Do you see special properties for this circle?
I couldn't find any.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3777
Date: 03/9/2019 1:17:46
From: eckart_schmidt@t-online.de
Subject: QL-Lines wrt QA-Tf2(P)

Dear Chris,

in earlier messages already mentioned:

For a QL the three QA-Tf2 images of a point are collinear.

... Where can I find this in EQF?

This gives a QL-transformation TF, which maps points to lines.

Some examples, perhaps new:

$TF(QL-P1) = QL-P26.CSC(QL-P24) = CSC(QL-Ci6)$

$TF(QL-P8) =$ line through QL-P23, parallel to its QL-Tf2-image.

$TF(QL-P13) =$ line through QL-P19, parallel QL-L9.

$TF(\text{infinity point of } QL-L1) = QL-L9.$

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3778
Date: 04/9/2019 9:22:04
From: eckart_schmidt@t-online.de
Subject: 5P Cubic/Quartic Intersections

Dear Bernard,

if I am not wrong, the following doesn't hold (#3769, #3775)

"... the middles of the segments X_iY_i (with Y_i and X_i CSC partners) lie on a rectangular hyperbola".

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3779

Date: 04/9/2019 9:53:04

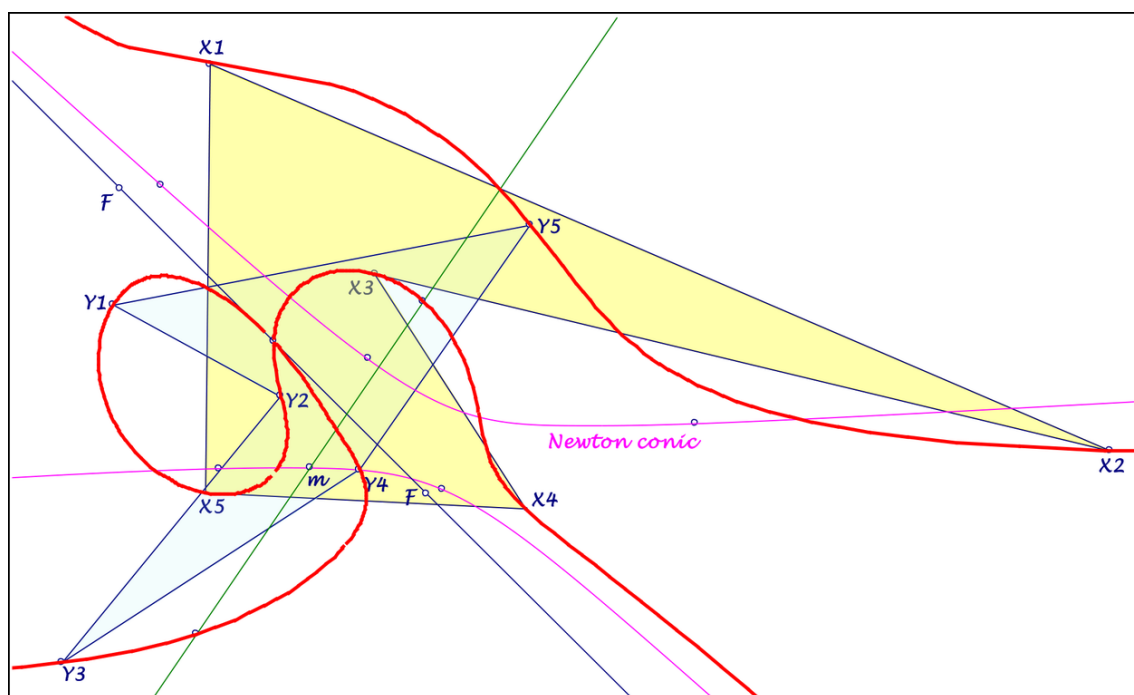
From: eckart_schmidt@t-online.de

Subject: 5P, CSC and twin conics, cubics and quartics

Dear Bernard,

for the twin 5P $X_1 \dots X_5$ and $Y_1 \dots Y_5$ with $Y_i = \text{CSC}(X_i)$
... and a Newton conic through the midpoints of $X_i Y_i$
... we can construct the sextic as you described in #3775,
... see attached file.

Best regards Eckart



2019-09-04.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3780

Date: 04/9/2019 11:19:22

From: bernard.keizer

Subject: 5P, CSC and twin conics, cubics and quartics

Dear Eckart,

Thanks for your correction about the RH and your confirmation about the construction of the sextic.

Both cubics may be constructed the same way with the common Newton Line and the CSC2 with centers U or $CSC(U)$ and both quartics with the 2 Newton circles and the CSC3 with centers $CSC(T)$ or $CSC(T')$.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3781
Date: 07/9/2019 11:58:33
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

Dear Bernard, dear Chris,

this is a very curious excursion in QL-geometry,
... orientated at a QL-transformation TF,
... which maps a point to QA-P4 of its pedal QA.

Well known is, that $TF(QL-Cu1)$ gives $QL-L1$ (see EQF),
... TF of CSC-partner on $QL-Cu1$ is their midpoint.
But there are further points X with $TF(X)$ on $QL-L1$,
... their locus is a QL circumscribed quartic (sextic?) through $QL-P1$.

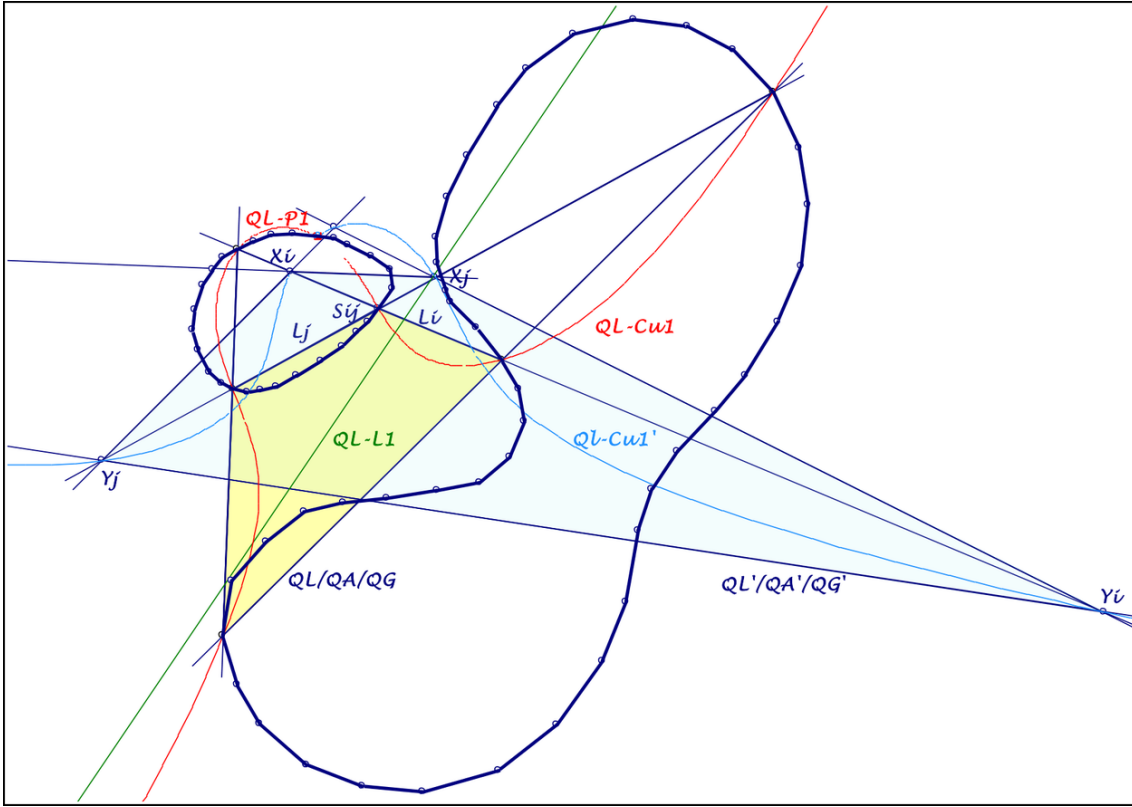
Fixed points of TF are $QL-2P3$
... and four other points X_i, Y_i and X_j, Y_j
... on two special lines L_i, L_j of the QL.
... The intersection S_{ij} of L_i and L_j
... is the nearest QL-point to the line of $QG-2P3$
... for the convex QG-version of the QL.
The points X_i, Y_i and X_j, Y_j are CSC-partner
... and easy to construct.

$X_i X_j Y_i Y_j$ can be considered as
(1) quadrigon QG'
with L_i, L_j as diagonals and $QG'-P1 = S_{ij}$.
(2) quadrangle QA' with $QA'-P4$
as opposite QL-point of S_{ij} ,
(3) *quadrilateral QL' with the same CSC as the reference QL.*

What about the quartic (sextic?), mentioned above?

Best regards Eckart

PS: It's the first time, that the convex QG-version of a QL
plays a role.



2019-09-06.pdf

Message: #3782
Date: 08/9/2019 2:08:37
From: eckart_schmidt@t-online.de
Subject: CSC-invariant QA-circles

Dear Bernard, dear Chris,

if you are interested, here a short excursion in QA-geometry:
Consider a QA, its QA-Tr2=ABC with incenter I and excenters
Ia,Ib,Ic,
... further the 3 QG-versions, interpreted as QL with QL-Tf1=CSC:
... The locis for points X, which are concyclic with the 3 CSC(X)
... are three circles Cia,b,c,
... .. centered in the excenters of QA-Tr2 on QA-Cu1
... .. with radius $ra = \sqrt{A \cdot Ia \cdot I \cdot Ia}$, analog rb, rc,
... .. Cia,b intersect orthogonal in QLc-2P3, analog Cib,c and
... .. Cic,a,
... .. with radical center I.

I think, these three QA-circles will be the only,
... which are invariant wrt the three CSC.

Consider for example Cia:
... CSCa-partner on Cia are collinear with I,
... CSCb-partner on Cia are collinear with Ic,
... CSCc-partner on Cia are collinear with Ib.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3783
Date: 08/9/2019 7:47:17
From: Benedetto Scimemi
Subject: 5P-s-Cix

Dear Eckart, dear Chris, after a long geometric ... vacation, I happened to read my name in the address of #3773, where Eckart describes an interesting 5P-circle that he discovered in 5P geometry. Thanks to Eckart who thus re-awoke my curiosity. It seems to me, however, that Co-Tf3 does not play any essential role in his definition. Its action (a similarity) can be actually ignored by giving the following simpler statement:

Given a pentagon $A_1...A_5$, for each index $i = 1...5$ let O_i denote the isoptic point (QA-P4) of the A_i -complementary quadrangle (e.g. O_1 is QA-P4 of $A_2A_3A_4A_5$ etc.) . Then the 5 midpoints $M_i = (A_i + O_i)/2$ lie on a circle 5P-Ci-?

I tried - without success - to prove this unexpected result by angular properties. I was also disappointed, like you probably were, not finding any other property neither for this circle or the original one of Eckart. I will try more.

Incidentally, if a somewhat similar situation is considered within quadrangles, one can state:
In a quadrangle $A=A_1...A_4$ for each index $i=1...4$ let O_i denote the circumcenter of the A_i -complementary triangle (e.g. O_1 is the circumcenter of $A_2A_3A_4$). Then the 4 midpoints $(A_i + O_i)/2$ are the vertices of a quadrangle QA-Q? which is negatively similar to QA.
I wonder: are this similarity and its fixed-point already present in EQF ?

P.S. I take this opportunity to suggest to Chris, as I perhaps already did years ago, that - if he ever finds the time - he could add some pages of EQF, regarding *QA-quadrangles*. For example, the above 01020304 could be denoted by QA-Qx (and similarly QA-Qy for nine-points-, QA-Qz for orthocenters- etc). There exist in fact interesting relations among them and a nice sort of algebra can be introduced in their set (map composition and midpoints).

Best regards Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3784
Date: 08/9/2019 9:58:06
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

Dear Benedetto, dear Chris,

thanks to Benedetto for his interest
... and his simpler definition in #3783
for the 5P-circle 5P-s-Cix in # 3773.

But now I noticed, that this circle plays a role
... in our discussion wrt the QA-Cu7-triple points:

Consider the orthogonal hyperbola (see #3579, #3581)
... through 5P-s-P5 and centered in the middle of 5P-s-P4.5
... with axes parallel to those of 5P-s-Co1,
... intersecting 5P-s-Co1 in three points unequal 5P-s-P4,
... which give a triangle with nine-point circle 5P-s-Cix.

Best regards Eckart

[← Previous](#) [Next →](#) [↩ Message Index](#) [↑ Subjects](#)

Message: #3785
Date: 08/9/2019 11:23:24
From: bernard.keizer
Subject: QA-P4 of pedal-QA

Dear Eckart,
Very interesting new quartic !
I try to reproduce your property.
But I could not find TF in EQF.
Could you please give me the exact reference where it is
mentionned ...
Many thaks in advance
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3786
Date: 09/9/2019 9:25:03
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

Dear Benedetto,

I think, my message 3784 isn't correct:
your described circle in #3783 isn't my circle in #3773,
... but in #3784
I described your circle,
... which is Quang Duong's circle, see 5P-s-P3.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3787
Date: 09/9/2019 10:36:03
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

Dear Bernard,
the transformation TF isn't in EQF, but this property under
QL-Cu1, 6, QL-Cu1 is the locus of all points P
... for which the feet of the perpendiculars from P to the 4
quadrilateral lines
... are concyclic (all lie on a circle). "
QA-P4 of a cyclic QA is the center of the circumcircle
I thought, this was the reason for $TF(QL-Cu1) = QL-L1$,
... but you are right, this isn't mentioned in EQF, but it holds!
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3788
Date: 10/9/2019 4:59:50
From: bernard.keizer
Subject: QA-P4 of pedal-QA

In fact, your formulation was rather elliptic !
Sure, the QA-P4 of 4 concyclic points is the circumcenter (the
isogonal of one of the 4 points wrt the triangle of the 3 others
is at the infinity and it's inverse is the circumcenter), but I
don't see why it follows that this circumcenter is precisely the
middle of the segment joining 2 CSC partners on QL-Cu1 ...
Anyhow, as you say, the property holds !
For the points X_i and j and Y_i and j , they are the intersections
of a line of the QL (through 3 QL vertices) and the reference
circumcircle through the 3 other QL vertices.
For 2 lines of the QL, the points are real, for the 2 others,
they are imaginary.
I checked that the 6 mentioned points are fixed points of TF.
But that doesn't help to understand the quartic !
I haven't found anything until now, but I'm still searching ...
Did you try to draw the CSC curve of the quartic ? It could be a
cubic through the 6 QL vertices with asymptote parallel to the
Newton Line ...
Best regards
Bernard
PS The fact that QL' has the same CSC as QL is obvious, as you
start with 2 couples of CSC partners ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

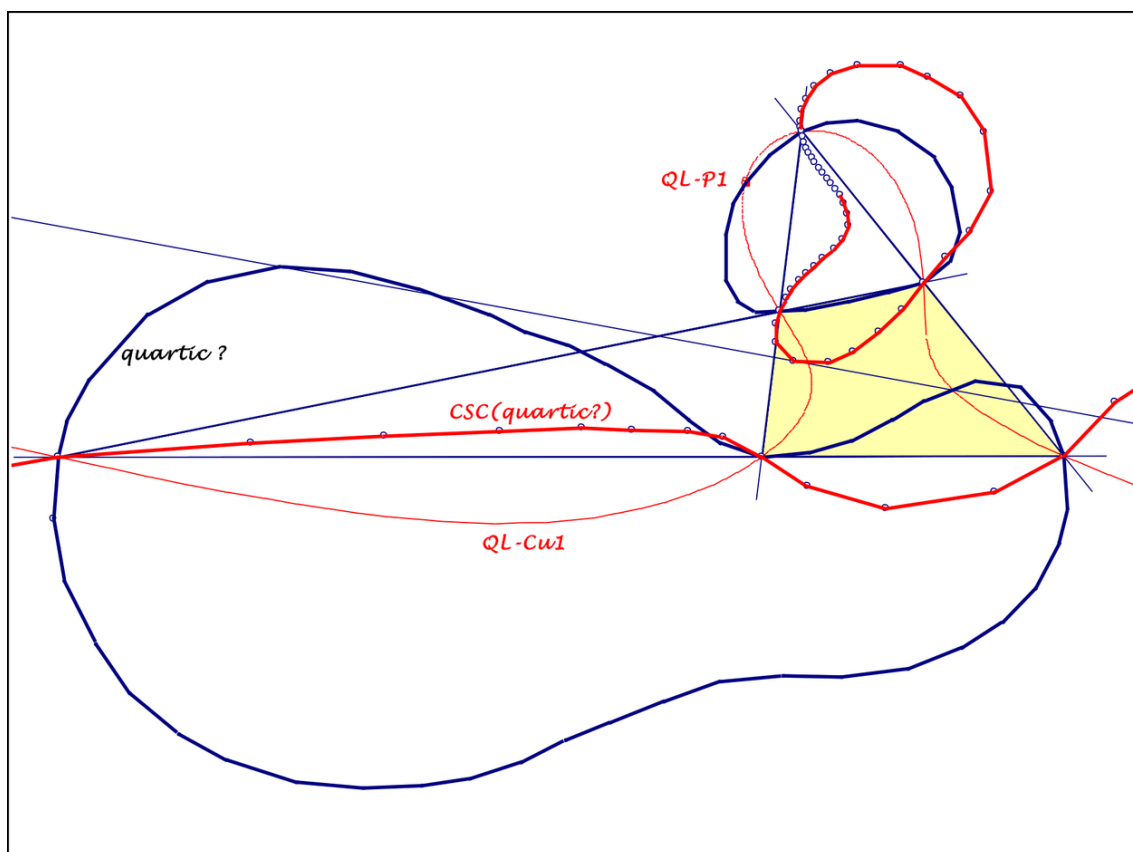
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3790
Date: 11/9/2019 11:19:56
From: eckart_schmidt@t-online.de
Subject: QA-P4 of pedal-QA

Dear Bernard,

attached a drawing for the CSC of the supposed quartic,
... perhaps a quintic?
I found no properties.

Best regards Eckart



2019-09-11.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3791
Date: 15/9/2019 10:18:16
From: bernard.keizer
Subject: New aspects of 5P-geometry

Dear Eckart, my previous answer went lost, I don't know why ...
I'm glad that you begin to gather properties of the 5P geometry.
I'm sad that you didn't mention either the cubic CSC(quartic) or
the twin cubic (circular, also through the 5P and the point U,
like the quartic.

Last, I asked for precisions wrt Tf3 : Quang Duong circle
(centered in P3 and not P2) carries the middles of the segments
Xiisg'i, UT and UM ; you mentionned that the Px is Tf3(midpoint
of UT, what are the Tf3 of the 6 other points ?

Thanks in advance

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3792
Date: 18/9/2019 11:45:37
From: eckart_schmidt@t-online.de
Subject: New aspects of 5P-geometry

Dear Bernard,

excuse, that I haven't mentioned your cubic and the twin-aspect,
... which is orientated at the 5P-CSC transformation.
The properties in #3789 are dependent on the transformation
5P-s-Tf3,
... which is only defined by the conic 5P-s-Co1,
... that is a weak point of this view,
... but 5P-s-Tf3 maps lines to lines,
... circles to circles, conics to conics inclusive center.

Background for my research was the aspect,
... that the mentioned triangle TR may be a reference triangle
for 5P-geometry:
... 5P-s-P5 is the orthocenter X4 of TR,
... 5P-s-P3 is the nine-point center X5 of TR,
... the Quang Duong circle is the nine-point circle of TR,
... bearing the midpoints of 5P-s-P4,5 and 5P-s-P5,6,
... 5P-s-P4 is the TR-isogonal conjugate of the infinity point
... of the tangent in the TR circumcenter at the new circle,
... and for TR = ABC with obtuse angle at A:
... The pedal point of 5P-s-P4 on BC
... connected with the middle of 5P-s-P4,5
... is parallel to the line through the points 5P-s-Px,y.

Wrt the transformation 5P-CSC:
... The circles CSC(AB), CSC(AC), CSC(BC) through 5P-s-P6
... intersect further pairwise and collinear
... on your cubic in CSC(A), CSC(B), CSC(C),
... with centers on C.5P-s-P4, B.5P-s-P4, A.5P-s-P4
... concyclic with 5P-s-P4,6 and CSC(5P-s-P5).
... CSC of this circle is a line through 5P-s-P5,
... which is the degenerated circle Ci(P) (see #3575)
... with P intersection of 5P-s-P4.5P-s-P6 and 5P-s-Co1.

But all these properties without systematic and concept.
I searched in vain for a relevant TR-isoconjugation.

Best regards Eckart

PS: Thanks for correcting the typo:
... Center of the Quang Duong circle is 5P-s-P3.

[← Previous](#) [Next →](#) [↩ Message Index](#) [↑ Subjects](#)

Message: #3793
Date: 18/9/2019 5:25:13
From: bernard.keizer
Subject: New aspects of 5P-geometry

Dear Eckart,
Thanks for this long message, I understand better what you are looking for !
I have to investigate more in this transformation Tf3.
Is it possible (if it swaps points to points, lines to lines, circles to circles ...) that it is a similitude ?
Hence my question about other examples , Tf3(P4,5 or 6) or Tf3(circumcircle of TR) ?
The line P_xP_y is parallel to the Simson or Steiner Line of P₄ wrt this circle.
The Steiner Lines of P₄ and P₆ intersect obviously in P₅ (orthocenter of TR), but more interesting, the Steiner Line of P₆ passes through CSC(P₄).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3794
Date: 18/9/2019 6:11:55
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear geometers,

Let A,B,C,D be a QA.
P is any point.
H₁ is midpoint of the segment connecting two orthocenters of triangles PAB, PCD.
H₂ is midpoint of the segment connecting two orthocenters of triangles PBC, PAD.
H₃ is midpoint of the segment connecting two orthocenters of triangles PAC, PBD.
Then H₁, H₂, H₃ are collinear on line d.
Thus we have a transformation on QA from point P to line d. Is this transformation known before?

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3795
Date: 18/9/2019 10:00:39
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Quang Hung,

at the moment I cannot answer
... whether the transformation is in EQF,
... but there is a nice property for points on the cubic QA-Cu1:
For P on QA-Cu1 the corresponding line
... is a perpendicular to P.QA-P3 through QA-P2.
Thus QA-Cu1 is the locus for points,
... whose corresponding line bears QA-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3796
Date: 19/9/2019 9:38:56
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear Eckart,

Thank You very much for interesting property of QA-cu1 and this transformation.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3797
Date: 19/9/2019 11:44:14
From: bernard.keizer
Subject: New aspects of 5P-geometry

Dear Eckart,
The fixed point of the transformation is 5P-s-P1, the center of the circumconic (property in EQF).
On your beautiful figure with message 3789, it holds that the distances between 5P-s-P1 and X and 5P-s-P1 and Tf3(X) are in the same ratio for $X = P_y$ and $X = P_x$.
It seems also that $P_yP_1P_3$ and $P_xP_1\text{mid}(P_4P_5)$ have the same bisector.
Could the transformation Tf3 be the product of an axial symmetry and an homothety with center 5p-s-P1 ?
I don't know if it was obvious, please just tell me if it is correct ...
Thanks in advance
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3798
Date: 19/9/2019 2:30:24
From: eckart_schmidt@t-online.de
Subject: Wrt 5P-s-Tf1,2,3,4

Dear Chris, dear Bernard,

the transformation 5P-s-Tf3 is conical, it is Co-Tf3inv.
5P-s-Tf3 maps lines to lines,
... circles to circles, conics to conics, inclusive center,
... this should be mentioned in EPG under 5P-s-Tf3.
I think, that the constructions of 5P-s-Tf1,2,3,4inv in #2758
... are also worth to be mentioned in EPG.

Best regards Eckart

PS for Bernard wrt #3797:
5P-s-Tf3(P) is the reflection of P in the main axis of 5P-s-Co1,
... stretched from 5P-s-P1 with factor $(a^2+b^2)/(a^2-b^2)$ (see Co-Tf3).

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3799
Date: 19/9/2019 8:48:29
From: eckart_schmidt@t-online.de
Subject: New aspects of 5P-geometry

Dear Bernard,

thanks for your remarks wrt #3792:
The Simson line of P_4 is a better description of my parallel to $P_x.P_y$.

New for me: "... Steiner line of P_6 ...",
... this is a better description of my degenerated circle $C_i(P)$.
Thus $CSC(P_4)$ is the intersection of the Steiner line of P_6
... and the line through $CSC(A)$, $CSC(B)$, $CSC(C)$.

New: $Tf_3(P_4)$ is the intersection of the Simson line of P_4
... and a perpendicular to the Simson line of P_6 through $Tf_3(P_6)$.

Best regards Eckart

PS. Wrt 5P-s- Tf_3 see #3798.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3800

Date: 19/9/2019 10:15:52

From: chris.vantienhoven

Subject: A transformation on QA from point to line

Dear Quang Hung and dear Eckart,

Very nice construction of the transformation of Tran Quang Hung in message #3794.

Some remarks of mine regarding this transformation,

X(4)-Transformation

Let QA-Tfx4a(P) = Tran Quang Hung's Ortholine.

Let QA-Tfx4b(P) = line through P perpendicular to QA-Tfx4a(P).

The coordinates of QA-Tfx4b(P) are way simpler than the coordinates of QA-Tfx3b(P),

Properties:

1. Let (i,j,k,l) be the four indices of reference points (P1,P2,P3,P4).

Let L_{ij} = Line (Pi,Pj) and C_i = Circle (Pj,Pk,Pl).

Let P_{ij} = 2nd intersection point of $C_i \wedge L_{ij}$.

Then QA-Tfx4b (P_{ij}) = L_{ij} .

2. When P is a point on C_i ,

then QA-Tfx4b(P) is a line through P_i .

3. The locus of QA-Tfx4a(P) with variable P on C_i is a hyperbola tangent to the nine-point circle of triangle(Pj,Pk,Pl) and with center X(5) of triangle(Pj,Pk,Pl).

X(2)-Transformation

When doing the same construction with the centroid instead of the orthocenter,

it appears that the 3 midpoints coincide in one point

QA-Tfx2a(P) with these coordinates:

$\{4px+3qx+3rx+2py+qy+ry+2pz+qz+rz,$
 $px+2qx+rx+3py+4qy+3ry+pz+2qz+rz,$
 $px+qx+2rx+py+qy+2ry+3pz+3qz+4rz\}$

where $P=\{x,y,z\}$ and $P4=\{p,q,r\}$.

X(3)-Transformation

When doing the same construction with the circumcenter instead of the orthocenter,

it again appears that the midpoints are collinear and

QA-Tfx3a(P) // QA-Tfx4a(P) & QA-Tfx3b(P) coincides with QA-Tfx4b(P).

X(5)-Transformation

When doing the same construction with the nine-point center instead of the orthocenter,

it again appears that the midpoints are collinear and
 QA-Tfx5a(P) // QA-Tfx4a(P) & QA-Tfx5b(P) coincides with
 QA-Tfx4b(P).
 For me the main results of this exercise are the bycatches:
 QA-Tfx2a(P) with these CT-coordinates:

$$\{4px+3qx+3rx+2py+qy+ry+2pz+qz+rz, \\ px+2qx+rx+3py+4qy+3ry+pz+2qz+rz, \\ px+qx+2rx+py+qy+2ry+3pz+3qz+4rz\}$$

QA-Tfx3b(P) = QA-Tfx4b(P) = QA-Tfx5b(P)
 with these CT-coefficients:

$$\{-y z (-r y + q z) (-c^2 p q x^2 - b^2 p r x^2 - a^2 q r x^2 + \\ c^2 p^2 x y - b^2 p r x y + c^2 p r x y - a^2 q r x y + b^2 p^2 \\ x z + b^2 p q x z - c^2 p q x z - a^2 q r x z + a^2 p^2 y z + \\ a^2 p q y z + a^2 p r y z),$$

$$x z (-r x + p z) (c^2 q^2 x y - b^2 p r x y - a^2 q r x y + c^2 \\ q r x y - c^2 p q y^2 - b^2 p r y^2 - a^2 q r y^2 + b^2 p q x z \\ + b^2 q^2 x z + b^2 q r x z + a^2 p q y z - c^2 p q y z + a^2 \\ q^2 y z - b^2 p r y z),$$

$$-x y (-q x + p y) (c^2 p r x y + c^2 q r x y + c^2 r^2 x y - c^2 \\ p q x z - a^2 q r x z + b^2 q r x z + b^2 r^2 x z - c^2 p q y z \\ + a^2 p r y z - b^2 p r y z + a^2 r^2 y z - c^2 p q z^2 - b^2 p \\ r z^2 - a^2 q r z^2)\}$$

Remarkable is that the coefficients of QA-Tfx4b(P) are composed
 of corresponding coefficients of lines and the equations of
 corresponding circles.

Best regards,
 Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3801

Date: 19/9/2019 10:24:04

From: davidfraivert

Subject: The theory of a convex quadrilateral and a circle that forms Pascal

Dear Geometers,

The following six articles present the main definitions and properties of "the theory of a convex quadrilateral and a circle that forms Pascal points on its sides".

The general case of a convex quadrilateral:

1)

The theory of a convex quadrilateral and a circle that forms "Pascal points" - the properties of "Pascal points" on the sides of a convex quadrilateral.

http://dx.doi.org/10.18642/jmsaa_7100121666

2)

Properties of the tangents to a circle that forms Pascal points on the sides of a convex quadrilateral.

<http://forumgeom.fau.edu/FG2017volume17/FG201726.pdf>

The case of a cyclic quadrilateral:

3)

The Theory of an Inscriptible Quadrilateral and a Circle that Forms Pascal Points.

http://dx.doi.org/10.18642/jmsaa_7100121742

4)

Pascal-points quadrilaterals inscribed in a cyclic quadrilateral.

<https://doi.org/10.1017/mag.2019.54>

The case of an orthodiagonal quadrilateral:

5)

Properties of a Pascal points circle in a quadrilateral with perpendicular diagonals.

<http://forumgeom.fau.edu/FG2017volume17/FG201748.pdf>

6)

A Set of Rectangles Inscribed in an Orthodiagonal Quadrilateral and Defined by Pascal-Points Circles.

<http://www.heldermann.de/JGG/JGG23/JGG231/jgg23002.htm>

Regards,
David Fraivert

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3802
Date: 20/9/2019 7:27:10
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear Eckart, dear Chris,

Thank You very much for your interest. Thank you for pointing out more interesting transformations besides that. Thus these transformations will be true for all point on Euler line and divide OH in the same ratio.

Best regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3803
Date: 20/9/2019 9:18:37
From: Thanh Tung
Subject: A transformation on QA from point to line

Dear all,

I found another property of this transformation:
Let given 5 points P_1, P_2, P_3, P_4, P_5 .
Let h_1 = Tran Quang Hung's Ortholine of P_1 w.r.t quadrangle $P_2P_3P_4P_5$, define h_2, h_3, h_4, h_5 similarly,
Then h_1, h_2, h_3, h_4, h_5 concur at a point.
What is this point w.r.t 5P: $P_1P_2P_3P_4P_5$?

Best regards,
Vu Thanh Tung

Le vendredi 20 septembre 2019 à 12:27:15 UTC+7, Tran Quang Hung analgeomatrica@gmail.com [Quadri-Figures-Group] a écrit :
> Dear Eckart, dear Chris,
> Thank You very much for your interest. Thank you for pointing out more interesting transformations besides that.
> Thus these transformations will be true for all point on Euler line and divide OH in the same ratio.
> Best regards
> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3804
Date: 20/9/2019 9:31:07
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Chris, dear Quang Huong, dear Thanh Tung

wrt #3803: The point is 5P-s-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3805
Date: 20/9/2019 10:10:42
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Chris, dear Quang Hung,

with interest I have read Chris' #3800
wrt the $X(4)$ -transformation,
... here a special property:
Points X on QA-Cu1 are mapped by QA-Tfx4b to $X.QA-P3$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3806

Date: 20/9/2019 11:13:43

From: Thanh Tung

Subject: A transformation on QA from point to line

Dear all,

We can rewrite #3794, #3800 and #3802 as the following form:

Definition: Given a constant t . For a triangle ABC with circumcenter O and orthocenter H , the t -Eulerline of triangle ABC is the point T such that $OT = t OH$.

$ABCD$ is a quadrangle, P is any point, t is constant

T_1 is midpoint of the segment

connecting two t -Eulerline of triangles PAB , PCD .

T_2 is midpoint of the segment

connecting two t -Eulerline of triangles PBC , PAD .

T_3 is midpoint of the segment

connecting two t -Eulerline of triangles PAC , PBD .

Then :

_ T_1, T_2, T_3 are collinear on line $d(t)$.

_ All line $d(t)$ are parallel

_ the distance between $d(t_1)$ and $d(t_2)$

is linearly of proportional to $|t_1 - t_2|$

Best regards,

Vu Thanh Tung

Le vendredi 20 septembre 2019 à 15:10:49 UTC+7,

'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de

[Quadri-Figures-Group] a écrit :

> Dear Chris, dear Quang Hung,

> with interest I have read Chris' #3800 wrt the

$X(4)$ -transformation,

> ... here a special property:

> Points X on $QA-Cu1$ are mapped by $QA-Tfx4b$ to $X.QA-P3$.

> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3807

Date: 20/9/2019 3:30:19

From: Tran Quang Hung

Subject: A transformation on QA from point to line

Dear Vu Thanh Tung, dear Chris, dear Eckart,

I mentioned the general problem $d(t)$ on the above message
>> Thus these transformations will be true for all point on Euler line and
>> divide OH in the same ratio.
> I guess that if we have five points A1, A2, A3, A4, A5
> $d1 = d(t)(A1, A2, A3, A4)$
> Define similarly $d2, d3, d4, d5$.
> Then $d1, d2, d3, d4, d5$ are concurrent for all t .
> Best regards
> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3808

Date: 20/9/2019 3:32:27

From: eckart_schmidt@t-online.de

Subject: A transformation on QA from point to line

Dear Chris, dear Quang Hung, dear Thanh Tung,

If we consider with Chris' nomination in #3800
... QA-Tfx4a for the vertices of a 5P wrt the remaining QA,
... we get as common point 5P-s-Tf3(5P-s-P5).

If we consider QA-Tfx4b
... we get as common point 5P-s-P4 (see #3804).

If we consider points P,
... whose QA-Tfx4a-lines wrt the 4P of the 5P are parallel
... or whose QA-Tfx4b-lines coincide,
... we get the fixed points of the Cayley-Bacharach transformation
... wrt the 5P-vertices and the circular points.
Very unexpected properties!

Best regards Eckart

PS: Sorry, I used in #3804 already QA-Tfx4b!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3809

Date: 21/9/2019 1:16:13

From: chris.vantienhoven

Subject: A transformation on QA from point to line

Dear friends,

I mentioned in message #3800 the transformation QA-Tfx2a(P) with these simple coordinates:

$$\begin{aligned} &\{4px+3qx+3rx+2py+qy+ry+2pz+qz+rz, \\ &\quad px+2qx+rx+3py+4qy+3ry+pz+2qz+rz, \\ &\quad px+qx+2rx+py+qy+2ry+3pz+3qz+4rz\} \end{aligned}$$

where $P=\{x,y,z\}$ and $P4=\{p,q,r\}$.

It is remarkable that the variables $\{a,b,c\}$ do not occur in the coordinates of the mapped point.

It can be constructed as the midpoint of Centroid(P,P1,P2) and Centroid(P,P3,P4).

It is the same point as the midpoint of Centroid(P,P1,P3) and Centroid(P,P2,P4) and the midpoint of Centroid(P,P1,P4) and Centroid(P,P2,P3).

I investigated the transformation a bit further with these results.

- QA-Tfx2a(QA-Px) lies on QA-P1.QA-Px (1 : 2)
- when QA-Px, QA-Py, QA-Pz are collinear then QA-Tfx2a(QA-Px), QA-Tfx2a(QA-Py), QA-Tfx2a(QA-Pz) are collinear

Let QA-Tfx2a(QA-Px) be denoted as Tfx, then:

- Tf1 = QA-P1
- Tf3 = QA-P34
- Tf10 = QA-P26
- Tf20 = QA-P10
- Tf24 = QA-P14
- Tf2 & Tf34 lie on QA-L1
- Tf5 & Tf18 & Tf25 & Tf26 & Tf43 lie on QA-L3
- Tf6 & Tf23 lie on QA-L4
- Tf15 lies on QA-L6
- Tf2 is collinear with QA-P36, QA-P37
- Tf4 is collinear with QA-P3, QA-P6
- Tf4 is collinear with QA-P20, QA-P28
- Tf7 is collinear with QA-P1, QA-P4
- Tf12 is collinear with QA-P3, QA-P36
- Tf12 is collinear with QA-P5, QA-P39
- Tf12 is collinear with QA-P13, QA-P20
- Tf16 is collinear with QG-P3, QG-P12
- Tf16 is collinear with QA-P10, QA-P27
- Tf16 is collinear with QA-P19, QA-P43
- Tf19 is collinear with QA-P20, QA-P21, QA-P31

- Tf21 is collinear with QA-P5, QA-P31
- Tf23 is collinear with QA-P1, QA-P6
- Tf29 is collinear with QA-P2, QA-P43
- Tf29 is collinear with QA-P5, QA-P35
- Tf29 is collinear with QA-P10, QA-P34
- Tf30 is collinear with QA-P3, QA-P11
- Tf30 is collinear with QA-P29, QA-P37
- Tf30 is collinear with QA-P35, QA-P39
- Tf31 is collinear with QA-P16, QA-P43
- Tf35 is collinear with QA-P3, QA-P43
- Tf36 is collinear with QA-P30, QA-P43
- Tf37 is collinear with QA-P5, QA-P11
- Tf37 is collinear with QA-P13, QA-P43
- Tf39 is collinear with QA-P37, QA-P43

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3810
Date: 21/9/2019 9:56:52
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

Dear Chris, dear Quang Hung, dear Thanh Tung,

this is an analogon to Quang Hung's transformation in #3794
... for a QL and a line, with application to a 5L.

Consider a QL and a line L
... and the 3 midpoints of orthocenters
... for two trilaterals (L, L_i, L_j) and (L, L_k, L_l) ,
... which are collinear on a line L' .
Example: L' of QL-L2 is QL-L1.

More interesting for a 5L:
The 5 lines L' of the 5L-lines wrt the remaining QL
... have a common point,
... which divides $5L-n-P5.5L-n-Px$ with ratio $-1:3$,
... $5L-n-Px$ is Morley's "first orthocenter of the 5-line" in
... F. Morley: Orthocentric properties of the plane n-line
(theorem 3), 1903.

Replacing the orthocenter X_4 by X_2, X_3, X_5 ,
... we get corresponding line to line transformations for a QL.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3811
Date: 22/9/2019 12:06:27
From: Thanh Tung
Subject: New line to line transformation for QL

Dear all,

Again this is very nice transformation. And again, as Quang Hung pointed out in the 5P case and Eckart in the 5L case, this transformation can be generalized to any point in the Euler line.

In other words:

Definition: Given a constant t . For a triangle ABC with circumcenter O and orthocenter H , the t -Eulerline of triangle ABC is the point T such that $OT = t OH$.

$t=0$: $T=X(3)$; $t=1$: $T=X(4)$; $t=1/3$: $T=X(2)$,
 $t=1/2$: $T=X(5)$, $t=-1$: $T=X(20)$

Given a quadrilateral $L_1 L_2 L_3 L_4$, a line L and a constant t
 T_2 is midpoint of the segment connecting two t -Eulerline of triangles formed by two trilaterals (L, L_1, L_2) and (L, L_3, L_4)
 T_3 is midpoint of the segment connecting two t -Eulerline of triangles formed by two trilaterals (L, L_1, L_3) and (L, L_2, L_4)
 T_4 is midpoint of the segment connecting two t -Eulerline of triangles formed by two trilaterals (L, L_1, L_4) and (L, L_2, L_3)

Then T_2, T_3, T_4 are collinear on a line $d(L, t)$.

When t varies it appears that the line $d(L, t)$ tangent to a fixed curve (a parabola ?) What is this curve ?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3812
Date: 22/9/2019 4:31:15
From: Thanh Tung
Subject: New line to line transformation for QL

Dear all,

In the case of X_2 ($t=1/3$), consider a 5L (L_1, L_2, L_3, L_4, L_5)
let d_1 = transforming line of L_1 w.r.t. QL (L_2, L_3, L_4, L_5) and
define d_2, d_3, d_4, d_5 similarly.

Then the 5 lines d_i are concurrent at a point.

This point is different to the concurrent point mentioned
in the case of X_4 ($t=1$) by Eckard. What is this point?

In the case of X_3 ($t=0$), the transforming lines of one line L
w.r.t any QL (L_1, L_2, L_3, L_4) perpendicular to the line L .

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3813
Date: 22/9/2019 6:01:19
From: Tran Quang Hung
Subject: New line to line transformation for QL

Dear Chris, dear Eckart, dear Thanh Tung,

It is very interesting transformation with 4L and 5L.
Thank You so much for new idea and contributing.
I also think about the similar things on Quadrigon Object?

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3814
Date: 22/9/2019 6:03:06
From: Tran Quang Hung
Subject: Similar figures of seven points

Dear geometers,

I see the following problems
Let $A_1, A_2, A_3, A_4, A_5, A_6$ be six points and any point P .
Then midpoints of the segments connecting $QA-P_1(P, A_i, A_j, A_k)$ and $QA-P_1(P, A_m, A_n, A_p)$ are the same point for all different sets of numbers (i, j, k, m, n, p) .
Note that this point is not Centroid of $(A_1, A_2, A_3, A_4, A_5, A_6)$ or Centroid of $(P, A_1, A_2, A_3, A_4, A_5, A_6)$?
So we have the transformation of Seven points $X = T_{fx}(P, [A_1, A_2, A_3, A_4, A_5, A_6])$.
Now we have seven points $A_1, A_2, A_3, A_4, A_5, A_6, A_7$.
Let $X_i = T_{fx}(A_i, [A_1, \dots, A_{(i-1)} A_{(i+1)}, \dots, A_7])$, we have seven points $X_1, X_2, X_3, X_4, X_5, X_6, X_7$.
And then two figures $(A_1, A_2, A_3, A_4, A_5, A_6, A_7)$ and $(X_1, X_2, X_3, X_4, X_5, X_6, X_7)$ are similar with ratio 8.

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3815
Date: 22/9/2019 6:25:55
From: Thanh Tung
Subject: Similar figures of seven points

Dear all,

For the first part, one can prove that $X = \text{Tfx}(P, [A1, A2, A3, A4, A5, A6])$ is the point such that $GX = \frac{1}{4} GP$ (in vector) where G is the centroid of the $6P[A1, A2, A3, A4, A5, A6]$.

For the second part, the $7P(X1, X2, X3, X4, X5, X6, X7)$ is image of the $7P(A1, A2, A3, A4, A5, A6, A7)$ under the Homothetic transformation of center G and ratio $\frac{1}{8}$, here G is common centroid of the two $7Ps$.

The result then can be extended to $2n-P$, the ratio of the first part will be $\frac{1}{(n+1)}$ and the second part will be $\frac{1}{2(n+1)}$.

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3816

Date: 22/9/2019 9:35:19

From: Tran Quang Hung

Subject: A transformation on QA from point to line

Dear Chris, dear Eckart, dear Vu Thanh Tung,

Follow X(3)-Transformation of Chris, I also see that
Let A, B, C, D be a QA. P is any point.

- (01) is the circle with diameter is the segment connecting circumcenters of triangles PAB , PCD .
- (02) is the circle with diameter is the segment connecting circumcenters of triangles PBC , PAD .
- (03) is the circle with diameter is the segment connecting circumcenters of triangles PAC , PBD .

Then three circles (01), (02), and (03) are coaxial. Thus we shall have a transformation from P to this radical axis?

But the same thing is not right for orthocenter, and if we change

- (01) is the circle with diameter is the segment connecting two points that divide OH segment of triangles PAB , PCD in ratio t .
 - (02) is the circle with diameter is the segment connecting two points that divide OH segment of triangles PBC , PAD in ratio t .
 - (03) is the circle with diameter is the segment connecting two points that divide OH segment of triangles PAC , PBD in ratio t .
- For which number t then (01), (02), (03) are coaxial?

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3817
Date: 22/9/2019 1:54:02
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Quang Hung,

CABRI-observations wrt #3816:

"For which number t then (01), (02), (03) are coaxial?"

This holds only for $t=0$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3818
Date: 22/9/2019 2:13:50
From: Tran Quang Hung
Subject: A transformation on QA from point to line

Dear Eckart,

Thank You very much. I hope the Transformation from P to this common radical axis is new.

I guess this result (coaxial circles) is also true for X3 of 4L?

Best Regards
Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3819
Date: 22/9/2019 4:20:57
From: eckart_schmidt@t-online.de
Subject: A transformation on QA from point to line

Dear Quang Hung,

wrt #3816, #3818:
If we take this transformation
... for a point and the 5 QA of a 5P,
... and consider the case, that the 5 lines coincide,
... we get the fixed points of the Cayley-Bacharach transformation
... wrt the 5P-vertices and the circular points (as in #3808).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3820
Date: 22/9/2019 10:14:24
From: eckart_schmidt@t-online.de
Subject: EPG-question

Dear Chris,

what is the nomination in EPG for
... Morley's first orthocenter
... and point 5L-P6 in #710.

Excuse my uncertainty in using EPG.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3821
Date: 22/9/2019 10:46:54
From: chris.vantienhoven
Subject: EPG-question

Dear Eckart,

Here is a conversion list for the codes mentioned in QFG#710.

5L-P1 now is the 5L-version of nL-o-P2, being 5L-o-P2.

5L-P2 now is the 5L-version of nL-n-P3, being 5L-n-P3.

5L-P3 now is the 5L-version of nL-n-P1, being 5L-n-P1.

5L-P4 now is 5L-s-P1

5L-P5 now is the 5L-version of nL-n-P6, being 5L-n-P6.

5L-P6 now is the 5L-version of nL-n-P8, being 5L-n-P8.

5L-P7 now is 5L-s-P2

5L-P8 now is 5L-s-P3

5L-P9 (Morley's 1st Orthocenter) now is the 5L-version of nL-o-P1, being 5L-o-P1.

Points with suffix "nL-n-" are points with versions for each n.

Points with suffix "nL-o-" are points with versions for odd n.

Points with suffix "5L-s-" are specific points only occurring for 5-Lines.

Like you can see the points we discerned at the beginning of our research happened to be a set of points occurring for all n, or all odd n, or only specific for n=5.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3822
Date: 23/9/2019 11:10:48
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

Dear Chris, dear Quang Hung, dear Thanh Tung,

in #3810 I described a QL-transformation,
... orientated to the orthocenter X_4
... mapping lines to lines $L \rightarrow L'$,
... with an application for 5L,
 leading to a point,
... which divides $5L-n-P5.5L-o-P1$ with ratio $-1:3$.

Taking the transformation, orientated to the centroid X_2 ,
... and applicated to a 5L, we get a point,
... which divides $5L-n-P8.5L-s-P1$ with ratio $-2:5$.

Only for X_2 and X_4 on the Euler line
... the transformation leads to a 5L-point!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3823
Date: 23/9/2019 9:32:49
From: eckart_schmidt@t-online.de
Subject: New line to line transformation for QL

Dear Thanh Tung,

wrt your #3811:

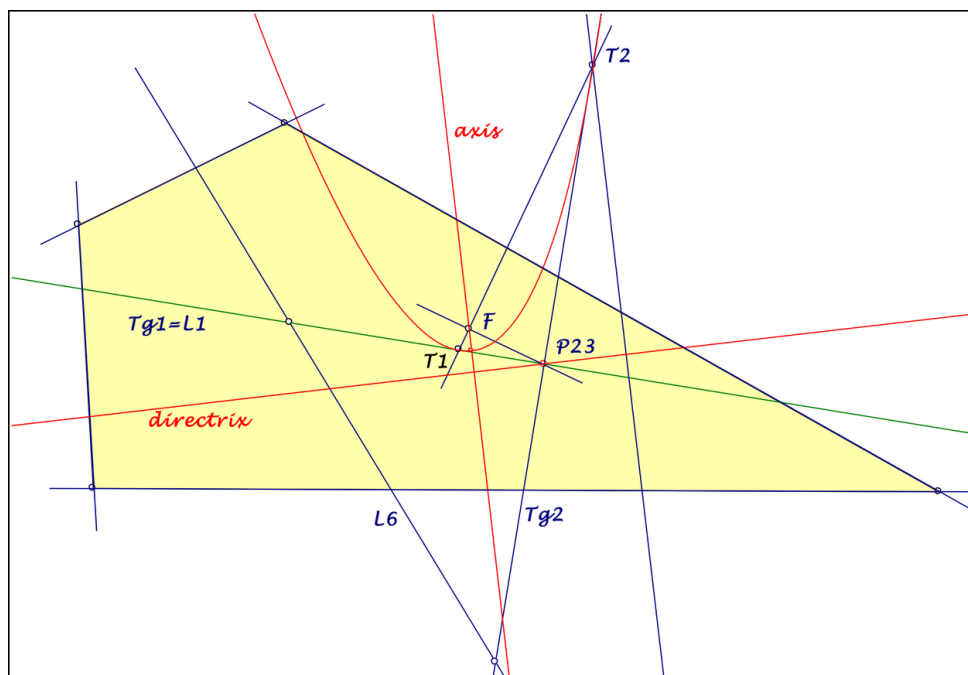
"When t varies it appears that the line $d(L,t)$ tangent to a fixed curve (a parabola ?) What is this curve ?"

You are right, the lines $d(L,t)$ envelop a parabola.

Example for the Newton line QL-L1 (see attached file),
... here an independent construction of this parabola:

QL-P23 is a point of the directrix
... and intersection of two orthogonal tangents:
... Tg1 = QL-L1 with contact point T1,
... which divides QL-P23 and QL-L1^QL-L6 with ratio 1:2,
... Tg2 is perpendicular to QL-L1 in QL-P23 with contact point T2,
... which is the reflection of Tg2^QL-L6 in QL-P23.
... Focus F is the pedal point of QL-P23 on T1T2.
... Direction of the axis is given with the line T1T2
... reflected in Tg2.
... Directrix is a perpendicular to the axis through QL-P23.

Best regards Eckart



2019-09-23.pdf

Message: #3824
Date: 24/9/2019 4:03:53
From: Thanh Tung
Subject: New line to line transformation for QL

Dear Eckart,

I think that the parabola is the QL-Co1 (Inscribed Parabola) of the QL formed by the X_2, X_3, X_4, X_5 - transformations of line L , or in general by any t_1, t_2, t_3, t_4 - Eulerlinepoint transformation lines.

Best regards,
Vu Thanh Tung

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3825

Date: 24/9/2019 4:55:18

From: Thanh Tung

Subject: A conjecture on Newton line and inscribed parabola of quadrilateral

Dear all,

I have a conjecture
on Newton line and inscribed parabola of quadrilateral as
follows:

- _ Consider a $QL(L_1, L_2, L_3, L_4)$ with newton line d
and inscribed parabola c .
- _ Let $U_{ij} = L_i \wedge L_j$ with $U'_{13} U_{23} = t \cdot U_{13} U_{23}$;
 $U'_{14} U_{24} = t \cdot U_{14} U_{24}$ (in vector) where t is a real number
- _ Let $L'1$ = line passing through U'_{13} and U'_{14} .
- _ Let $d(t)$ be the Newton line of the $QL(L'1, L_2, L_3, L_4)$.

Then:

- _ The line $L'1$ envelops the parabola c .
In other words,
the inscribed parabola of the $QL(L'1, L_2, L_3, L_4)$
remains the same when t varies.
- _ The line Newton line $d(t)$ is always parallel to the line d .
- _ The distance between $d(t_1)$ and $d(t_2)$ is proportional to
 $|t_1 - t_2|$.

Are these observations true?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3826

Date: 25/9/2019 10:45:45

From: eckart_schmidt@t-online.de

Subject: A conjecture on Newton line and inscribed parabola of quadrilate

Dear Thanh Tung,

wrt #3825:

CABRI constructions confirm your observations.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3827

Date: 25/9/2019 2:44:56

From: Thanh Tung

Subject: A conjecture on Newton line and inscribed parabola of quadrilate

Dear Eckart,

Thank you for your confirmation.

Best regards,

Vu Thanh Tung

Le mercredi 25 septembre 2019 à 15:45:50 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :

> Dear Thanh Tung,
> wrt #3825:
> CABRI constructions confirm your observations.
> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3828

Date: 26/9/2019 11:53:22

From: bernard.keizer

Subject: A conjecture on Newton line and inscribed parabola of quadrilate

Dear Thanh Tung, dear Eckart,

This interesting property is easy to prove analytically.

Writing the equation of the parabola with focus M on the y -axis and directrix parallel to the x -axis as $y = x^2$, the equation of the tangent in a point with x -coordinate x_0 is $y = 2xx_0 - x_0^2$. The 4 lines of the QL are tangent to the parabola in points with x -coordinate x_1, x_2, x_3 and x_4 .

The vertices of the QL have x -coordinates $x_{ij} = 1/2 (x_i + x_j)$ and y -coordinates $y_{ij} = x_i x_j$

Then, with your notations, the line $L'1$ through $U'13$ and $U'14$ is the tangent to the parabola in the point with x -coordinate $(1 - t)x_2 + tx_1$.

If $L'1$ is tangent to the parabola, it forms with 3 other tangents a new QL with the same Miquel point (focus of the parabola) and the same Steiner Line (directrix of the parabola) and the Newton Lines are all parallel, as perpendicular to the directrix ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3829
Date: 30/9/2019 5:17:31
From: Antreas Hatzipolakis
Subject: Feuerbach point lies on OI line

[Tran Quang Hung]:

- > Let ABC be a triangle.
- > OI line meets BC, CA, AB at A', B', C', resp.
- > Let Oa, Ob, Oc be the circumcenters of the triangles
- > AB'C', BC'A', CA'B' respectively.
- > Then Feuerbach point of triangles OaObOc
- > lies on the OI line of ABC.
- > Which is this point?

[APH]

Dear Tran

The same is true for the Brocard axis OK instead of the OI line:
The point X(115) of OaObOc lies on the Brocard axis
I think this generalization is true:
Let ABC be a triangle and P a point.

Denote:

D = the Poncelet point of ABCP

L = the isogonal conjugate of the rectangular circumhyperbola
centered at D (it is a line)

A', B', C' = the intersections of L and BC, CA, AB, resp.

Oa, Ob, Oc = the circumcenters of AB'C', BC'A', CA'B', resp.

Then the D point of OaObOc lies on the line L

For P = I: L = OI line, D = X(11)

For P = G: L = OK line, D = X(115)

Now, call the point D of OaObOc as (P, ABC) and let P1P2P3P4 be
a quadrangle.

Which are the properties of the quadrangle (P1, P2P3P4) (P2,
P3P4P1) (P3, P4P1P2) (P4, P1P2P3)?

Is it cyclic?

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3830
Date: 30/9/2019 6:14:50
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

I found an interesting family of centers in quadrangle geometry.
The construction is as the following:

1. A triangle transformation:
 - _ Given an integer n , The barycentric coordinates of a point P w.r.t $\triangle ABC$ is $(x : y : z)$.
 - _ The transformation $Q = \text{Tr}(P, \triangle ABC, n)$ maps P to the point Q with barycentric coordinates $(x^n : y^n : z^n)$ w.r.t. $\triangle ABC$
2. A point on quadrangle geometry.
 - _ Consider a quadrangle $P_1 P_2 P_3 P_4$ and an integer n .
 - _ Let $Q_i = \text{Tr}(P_i, \triangle P_j P_k P_l, n)$.

Then 4 lines $P_i Q_i$ concur at a point $X(n)$.

Some existing points:

- _ $n = 0$: $X(0) = \text{QA-P1}$: Quadrangle centroid (the triangle transformation is fixed to the centroid)
- _ $n = -1$: $X(-1) = \text{QA-P5}$: Isotomic center (the triangle transformation is the isotomic transformation)

Is this family of points known before ?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3831
Date: 30/9/2019 7:50:08
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

Please find attached a proof.
 Best regards,

Vu Thanh Tung

A family of points in Quadrangle

Vu Thanh Tung

October 1, 2019

Abstract

A family of points in Quadrangle is given.

In this short note, I define a family of points in Quadrangle.

Firstly a family of transformation of a point with respect to a triangle is defined as the following.

Definition 0.1. For each integer n , the transformation $tr_n(P, \triangle ABC)$ maps the point P of barycentric coordinates $P = (x : y : z)$ with respect to $\triangle ABC$ to the point $Q = (x^n : y^n : z^n)$.

Now we are able to construct a family of points in quadrangle.

Theorem 1 (Vu Thanh Tung, [1]). Consider a quadrangle $P_1P_2P_3P_4$ and an integer n . Define $Q_1 = tr_n(P_1, \triangle P_2P_3P_4)$, $Q_2 = tr_n(P_2, \triangle P_3P_4P_1)$, $Q_3 = tr_n(P_3, \triangle P_4P_1P_2)$ and $Q_4 = tr_n(P_4, \triangle P_1P_2P_3)$.

Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point $X(n)$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4, \triangle P_3P_4P_1, \triangle P_4P_1P_2, \triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and we can write:

$$P_4 = \left(-\frac{s_1}{s_4}\right).P_1 + \left(-\frac{s_2}{s_4}\right).P_2 + \left(-\frac{s_3}{s_4}\right).P_3 \quad (1)$$

The point Q_4 has barycentric coordinates (s_1^n, s_2^n, s_3^n) with respect to $\triangle P_1P_2P_3$ and we can write:

$$Q_4 = \frac{1}{s_1^n + s_2^n + s_3^n} (s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3) \quad (2)$$

Define the point $X(n)$ as the following weighted center of $P_1P_2P_3P_4$:

$$X(n) = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n} (s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3 + s_4^n.P_4) \quad (3)$$

$X(n)$ clearly lies on the line P_4Q_4 as we can write:

$$X(n) = \frac{s_1^n + s_2^n + s_3^n}{s_1^n + s_2^n + s_3^n + s_4^n}.Q_4 + \frac{s_4^n}{s_1^n + s_2^n + s_3^n + s_4^n}.P_4 \quad (4)$$

$X(n)$ divides segment P_4Q_4 as $\frac{X(n)Q_4}{X(n)P_4} = \frac{s_4^n}{s_1^n + s_2^n + s_3^n}$.

Similarly. $X(n)$ lies on the lines P_1Q_1, P_2Q_2, P_3Q_3 . $\square$

Corollary 1. • For $n = 0$ we get $X(0) = QA-P1(P_1P_2P_3P_4)$ (quadrangle centroid).

• For $n = -1$ we get $X(-1) = QA-P5(P_1P_2P_3P_4)$ (quadrangle isotomic center).

Proof. For $n = 0$ the triangle transformation becomes $tr_0(P, \triangle ABC) =$ centroid of $\triangle ABC$.

For $n = -1$, the triangle transformation becomes

$tr_{-1}(P, \triangle ABC) =$ isotomic conjugate of P with respect to $\triangle ABC$. $\square$

The two points $QA-P1(P_1P_2P_3P_4)$ and $QA-P5(P_1P_2P_3P_4)$ are described in [2] and [3] respectively.

References

- [1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Sep 30, 2019, <https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3830>
- [2] ENCYCLOPEDIA OF QUADRI-FIGURES, *QA-P1: QA-Centroid or Quadrangle Centroid*, <https://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p1>
- [3] ENCYCLOPEDIA OF QUADRI-FIGURES, *QA-P5: Isotomic Center*, <https://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p5>

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: tungvtt@gmail.com

Message: #3832
Date: 01/10/2019 3:35:29
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear all,

I will reword the result as following:

Result:

_ Given a quadrangle $P_1 P_2 P_3 P_4$ and an integer n .
_ Let Q_i be the n -power of P_i w.r.t. $\triangle P_j P_k P_l$.
Then 4 lines $P_i Q_i$ concur at a point $X(n)$.

ex: $X(0)$ = quadrangle centroid,
 $X(-1)$ = quadrangle isotomic center.

May we call the point X_n the n -power point of the quadrangle $P_1 P_2 P_3 P_4$?

Here, the n -power point of $P = (x : y : z)$ w.r.t. a $\triangle ABC$ is the point $P^n = (x^n : y^n : z^n)$.

0-power of point P = centroid of $\triangle ABC$, (-1) -power of point P = isotomic conjugate of P w.r.t. $\triangle ABC$.

The n -power point P^n can be recursively constructed by isotomic conjugate and the Paul Yiu's construction of multiplication of points given in this paper:

The uses of homogeneous barycentric coordinates in plane Euclidean geometry

(<http://math.fau.edu/yiu/PSRM2015/yiu/LAPTOPbackup030317/Geometry98/Barycentric/IJMEST/barycentric.pdf>)

Also I think that playing with multiplication of points can give new points in quadrangle.

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3833
Date: 01/10/2019 8:05:02
From: Tran Quang Hung
Subject: Feuerbach point lies on OI line

Dear Mr Antreas and César,

It is interesting with L = the isogonal conjugate of the rectangular circumhyperbola centered at D (it is a line).

Let ABC be a triangle and P a point.

Denote:

D = the Poncelet point of $ABCP$

L = the isogonal conjugate of the rectangular circumhyperbola centered at D (it is a line)

$L = Tf(ABC, P)$

Let $P_1P_2P_3P_4$ be a quadrangle.

$L_1 = Tf(P_2P_3P_4, P_1)$

$L_2 = Tf(P_3P_4P_1, P_2)$

$L_3 = Tf(P_2P_4P_1, P_1)$

$L_4 = Tf(P_1P_2P_3, P_4)$

Then L_1, L_2, L_3, L_4 are concurrent at a point. Which is this point of $QA(P_1, P_2, P_3, P_4)$?

Best regards

Tran Quang Hung.

Vào Th 3, 1 thg 10, 2019 vào lúc 06:30 Tran Quang Hung < analgeomatrica@gmail.com > đã viết:

>> Dear Mr Antreas and César,

>> Thank you very much for your interest and nice generalization.

>> Apply this generalization to a quadrangle which is interesting,

>> but I have not seen a special property on this configuration?

>> Do you see anything?

>> Best regards

>> Tran Quang Hung.

>> Vào Th 2, 30 thg 9, 2019 vào lúc 23:50 César E. Lozada cesar_e_lozada@yahoo.es

>> [Anopolis] < Anopolis@yahoogroups.com > đã viết:

>>> Dear geometers,

>>> Also $X(56)$ of triangle $0a0b0c$ lies on OI line of ABC . Which is this point?

>>> Best Regards

>>> Tran Quang Hung.

```

>>> Vào Th 2, 30 thg 9, 2019 vào lúc 19:24 Tran Quang Hung
>>> <analgeomatrica@gmail.com> đã viết:
>>> Let ABC be a triangle.
>>> OI line meets BC, CA, AB at A', B', C'.
>>> Let Oa, Ob, Oc be circumcenters of triangles AB'C', BC'A',
>>> CA'B' respectively.
>>> Then Feuerbach point of triangles OaObOc lies on OI line
>>> of ABC. Which is this point?
>>> Best Regards
>>> Tran Quang Hung.
>>> *Q = midpoint OF X(56) and X(23981)*
>>> =  $a^3(a+b-c)(a-b+c)(a^5-(b+c)a^4-2(b-c)^2a^3+2(b^3+c^3)a^2+(b^4+c^4-(4b^2-bc+4c^2)bc)a-(b+c)(b^4+c^4-(2b^2-bc+2c^2)bc))((b+c)a^2-2bca-(b^2-c^2)(b-c)) : : (barys)$ 
>>> = lies on these lines: {1, 3}, {1403, 6769}, {6767, 8069}
>>> = midpoint of X(56) and X(23981)
>>> = [ 0.0183414481164834, 0.3097215998853760,
3.4177765520868950 ]
>>> César Lozada

```

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3834
Date: 01/10/2019 1:57:41
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

Dear Thanh Tung,

very interesting messages!
Wrt #3831, 2. first constructions show,
... that the lines from the QA-vertices
... to the n-power point for $n=2$
... wrt the remaining triangle
... have the common point QA-P16.
The n-power point for $n=3$ gives QA-P10.
I hope, someone can confirm these results.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3835
Date: 01/10/2019 6:21:52
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Eckart,
 I am able to confirm your observations.
 My calculation gives the 1st CT-Coordinate of the n-power point is: $p (p^{(n-1)} - (-p -q -r) ^{(n-1)})$
 Please see the proof in attachement.
 Applying for $n=2,3$ we obtain the coordinates of QA-P16 and QA-P10. Also, the formula (3) in attachement gives an easy generalization to Polygon Geometry.
 Best regards, Vu Thanh Tung

n^{th} -power point of Quadrangle

Vu Thanh Tung

October 1, 2019

Abstract

The notation of n^{th} -power point Quadrangle is given.

In this short note, I define a family of points in Quadrangle. For each integer n , the n^{th} -power of a point with respect to a triangle is defined as follows:

Definition 0.1. For each integer n , the n^{th} -power of a point P with barycentric coordinates $(x : y : z)$ with respect to $\triangle ABC$ to the point P^n with barycentric coordinates $(x^n : y^n : z^n)$ with respect to $\triangle ABC$.

Now we are able to construct a family of points in quadrangle.

Theorem 1 (Vu Thanh Tung, [1]). Consider a quadrangle $P_1P_2P_3P_4$ and an integer n . Let Q_1 be the n^{th} -power of P_1 with respect to $\triangle P_2P_3P_4$, Q_2 be the n^{th} -power of P_2 with respect to $\triangle P_3P_4P_1$, Q_3 be the n^{th} -power of P_3 with respect to $\triangle P_4P_1P_2$, Q_4 be the n^{th} -power of P_4 with respect to $\triangle P_1P_2P_3$.

Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point X_n called the n^{th} -power point of Quadrangle $P_1P_2P_3P_4$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4, \triangle P_3P_4P_1, \triangle P_4P_1P_2, \triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and we can write:

$$P_4 = \left(-\frac{s_1}{s_4}\right).P_1 + \left(-\frac{s_2}{s_4}\right).P_2 + \left(-\frac{s_3}{s_4}\right).P_3 \quad (1)$$

The point Q_4 has barycentric coordinates $(s_1^n : s_2^n : s_3^n)$ with respect to $\triangle P_1P_2P_3$ and we can write:

$$Q_4 = \frac{1}{s_1^n + s_2^n + s_3^n} (s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3) \quad (2)$$

Define the point X_n as the following weighted center of $P_1P_2P_3P_4$:

$$X_n = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n} (s_1^n.P_1 + s_2^n.P_2 + s_3^n.P_3 + s_4^n.P_4) \quad (3)$$

X_n clearly lies on the line P_4Q_4 as we can write:

$$X_n = \frac{s_1^n + s_2^n + s_3^n}{s_1^n + s_2^n + s_3^n + s_4^n}.Q_4 + \frac{s_4^n}{s_1^n + s_2^n + s_3^n + s_4^n}.P_4 \quad (4)$$

X_n divides segment P_1Q_4 as $\frac{\overline{X_nQ_4}}{\overline{X_nP_4}} = \frac{s_4^n}{s_1^n + s_2^n + s_3^n}$.
Similarly. X_n lies on the lines P_1Q_1, P_2Q_2, P_3Q_3 . □

Corollary 1 (Coordinates). *The 1st CT-Coordinate of X_n is :*

$$p(p^{n-1} - (-p - q - r)^{n-1}) \quad (5)$$

Proof. We note that $p : q : r = s_1 : s_2 : s_3$ and $s_4 = -s_1 - s_2 - s_3$. From (1) and (4) we have:

$$X_n = \frac{1}{s_1^n + s_2^n + s_3^n + s_4^n} \left(\sum_{i=1}^3 s_i (s_i^{n-1} - s_4^{n-1}) \cdot P_i \right) \quad (6)$$

The conclusion then follows. □

Corollary 2 (Some known points).

- $X_{-1} = QA-P5(P_1P_2P_3P_4)$ (Quadrangle isotomic center).
- $X_0 = QA-P1(P_1P_2P_3P_4)$ (Quadrangle centroid).
- $X_2 = QA-P16(P_1P_2P_3P_4)$ (QA-Harmonic Center).
- $X_3 = QA-P10(P_1P_2P_3P_4)$ (Centroid of the QA-Diagonal Triangle).

The point X_2, X_3 were observed by Eckart Schmidt.

References

- [1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Sep 30, 2019, <https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3830>
- [2] ENCYCLOPEDIA OF QUADRI-FIGURES, , <https://www.chrisvantienhoven.nl/mathematics/encyclopedia>

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: tungvtt@gmail.com

Message: #3836
Date: 01/10/2019 6:36:20
From: eckart_schmidt@t-online.de
Subject: Feuerbach point lies on OI line

Dear Quang Hung,

the point asked for in # 3833 is QA-P4.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3837
Date: 01/10/2019 8:27:20
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

Dear Thanh Tung,

thanks for message 3835. In addition:
For $n = 4$ we get the 2nd intersection (beside QA-P21)
... of the line QA-P1.P16.P21 and the QA-circumconic
through QA-P21.

For $n = 5$ we get the 2nd intersection (beside QA-P19)
... of the line QA-P10.P16.P19.P31 and the QA-circumconic
through QA-P19.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3838
Date: 02/10/2019 1:01:55
From: Tran Quang Hung
Subject: Feuerbach point lies on OI line

Dear Eckart,
Thank You very much for your affirmation.
Best Regards
Tran Quang Hung.

Vào Th 3, 1 thg 10, 2019 vào lúc 23:43 '
eckart_schmidt@t-online.de ' eckart_schmidt@t-online.de
[Quadri-Figures-Group] < Quadri-Figures-Group@yahoogroups.com >
đã viết:
>> Dear Quang Hung,
>> the point asked for in # 3833 is QA-P4.
>> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3839
Date: 02/10/2019 2:43:05
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Eckart,
Thank you for the observations.
Best regards,
Vu Thanh Tung

Le mercredi 2 octobre 2019 à 01:29:13 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
Dear Thanh Tung,
thanks for message 3835. In addition:
For $n = 4$ we get the 2nd intersection (beside QA-P21)
... of the line QA-P1.P16.P21 and the QA-circumconic
through QA-P21.
For $n = 5$ we get the 2nd intersection (beside QA-P19)
... of the line QA-P10.P16.P19.P31 and the QA-circumconic
through QA-P19.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3840
Date: 02/10/2019 11:02:25
From: Tran Quang Hung
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Eckart and geometers,

We can see a general problem.

Let $f(x)$ be a real function of x .

If P has barycentric coordinate (x,y,z) in triangle ABC . We consider $f(P)$ has barycentric coordinate $(f(x),f(y),f(z))$ wrt ABC i.e denote by $f(P,ABC)$.

Now with quadrangle $P_1P_2P_3P_4$.

$Q_1 = f(P_1,P_2P_3P_4)$

$Q_2 = f(P_2,P_3P_4P_1)$

$Q_3 = f(P_3,P_4P_1P_2)$

$Q_4 = f(P_4,P_1P_2P_3)$

Then the line $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ are concurrent.

Proof.

Let s_1, s_2, s_3 , and s_4 be signed area of triangles $P_2P_3P_4, P_3P_4P_1, P_1P_2P_3$, and $P_2P_3P_4$ respectively.

P_1 has coordinate (s_2,s_3,s_4) wrt triangle $P_2P_3P_4$.

So that Q_1 has coordinate $(f(s_2),f(s_3),f(s_4))$ wrt $P_2P_3P_4$. i.e $[f(s_2)+f(s_3)+f(s_4)]Q_1 = f(s_2)P_2 + f(s_3)P_3 + f(s_4)P_4$.

We let $X = [f(s_1)P_1 + f(s_2)P_2 + f(s_3)P_3 + f(s_4)P_4] / (f(s_1)+f(s_2)+f(s_3)+f(s_4))$

Thus $[(f(s_1)+f(s_2)+f(s_3)+f(s_4))]X = f(s_1)P_1 + [f(s_2)+f(s_3)+f(s_4)]Q_1$.

Therefore P_1Q_1 goes through X . Similarly, P_2Q_2, P_3Q_3, P_4Q_4 go through X .

With $f(x)=x^n$ we have problem of Vu Thanh Tung.

For which other function f ?

Question: Can we use a homogeneous function $f(x,y,z)$ from $R^3 \rightarrow R$ instead of one variable function $f(x)$? Or what is the form of three variables function $f(x,y,z)$ can be used for this concurrent property instead of one variable function $f(x)$?

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3841
Date: 02/10/2019 11:15:06
From: Tran Quang Hung
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Eckart,

We also easily seen problem for n-point.

Let $f(x)$ be a real function of x .

If P has barycentric coordinate (x,y,z) in triangle ABC . We consider $f(P)$ has barycentric coordinate $(f(x),f(y),f(z))$ wrt ABC i.e denote by $f(P,ABC)$.

Now with quadrangle $P_1P_2P_3P_4$.

$Q_1 = f(P_1, P_2P_3P_4)$

$Q_2 = f(P_2, P_3P_4P_1)$

$Q_3 = f(P_3, P_4P_1P_2)$

$Q_4 = f(P_4, P_1P_2P_3)$

Then the line $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ are concurrent at $f(P_1P_2P_3P_4)$.

Thus with 5-point $P_1P_2P_3P_4P_5$. Then lines

$d_1 = P_1f(P_2P_3P_4P_5)$

$d_2 = P_2f(P_3P_4P_5P_1)$

$d_3 = P_3f(P_4P_5P_1P_2)$

$d_4 = P_4f(P_5P_1P_2P_3)$

$d_5 = P_5f(P_1P_2P_3P_4)$

are concurrent at $f(P_1P_2P_3P_4P_5)$.

.....

Similarly by we can define inductive for n-points.

Best regards

Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3842
Date: 02/10/2019 12:19:35
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Chris,

there are interesting properties
... of Paul Yiu's multiplication of points, cited by Thanh Tung
in #3832.

Here some splitter, perhaps evident wrt DT-coordinates:
The product of two isoconjugated points wrt a triangle
... is the the isoconjugate of the centroid.
The product of two QA-Tf2-partner wrt QA-Tr1 is QA-P16.
The products of a point (unequal QA-P1) and a QA-vertex wrt the
remaining QA-triangle
... are collinear, bearing QA-P16.

Examples:
... For points on QA-L3 we get collinearity on QA-P1.QA-P16.
... If L is the line for a point P, intersecting P.QA-P10 in Q,
... then Q gives the line P.QA-P16.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3843
Date: 02/10/2019 7:11:29
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Tran Quang Hung,

I think your generalization is not true in general.
Firstly if the function f is not homogeneous one cannot apply it to the signed area
because if we scale up the quadrangle, the transformation $P \rightarrow f(P)$ will give different barycentric coordinates.
Rather we can apply f to the normalized barycentric coordinates.
But even then there is counterexamples.
Take for example P_1, P_2, P_3, P_4 such that $P_4 = 5P_1 + 4P_2 - 8P_3$
and $f(x) = 1/(1+x)$.
I do think that only x^n is the only function that works.
But I agree with you that for some forms of three variable f that is homogeneous and bisymmetric (some kind of triangle center function) that work.

Example1: $f(x,y,z) = x(y+z)$ works,
The construction is as follows:
Let A_1, B_1, C_1 be the cevian triangle of ABC .
 A_2, B_2, C_2 are the midpoint of A_1B_1, B_1C_1, C_1A_1
Then AA_2, BB_2, CC_2 concur at a point Q .
That transforms
 $P = (x,y,z) \rightarrow Q = (x(y+z), y(z+x), z(x+y)) = \text{tr}(P, ABC)$
For $QA-P_1, P_2, P_3, P_4$, let $Q_i = \text{tr}(P_i, P_j, P_k, P_l)$ then 4 lines P_i, Q_i are concurrent.

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3844
Date: 02/10/2019 7:40:41
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Eckart,
They are very nice properties.
Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3845
Date: 02/10/2019 9:51:02
From: César E. Lozada
Subject: Feuerbach point lies on OI line

Then L_1, L_2, L_3, L_4 are concurrent at a point. Which is this point of $QA(P_1, P_2, P_3, P_4)$?

Answer: $Q = QAP_4(A, B, C, D)$

With respect to ABC , given D ,
 $Q = \text{IsogonalConjugate}(\text{AntigonalConjugate}(D))$

I conjecture $Q = \text{IsogonalConjugate}(\text{AntigonalConjugate}(P_i))$
w/r to $P_j P_k P_l$.

Best regards,
César Lozada

De: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
Enviado el: martes, 1 de octubre de 2019 02:05 am
Para: Anopolis@yahoogroups.com;
Quadri-Figures-Group@yahoogroups.com
Asunto: [Quadri-Figures-Group] Re: [EGML] Feuerbach point lies on OI line

> Dear Mr Antreas and César,
> It is interesting with L = the isogonal conjugate
> of the rectangular circumhyperbola centered at D (it is a
line).
> Let ABC be a triangle and P a point.
> Denote:
> D = the Poncelet point of $ABCP$
> L = the isogonal conjugate of the rectangular circumhyperbola
> centered at D (it is a line)
> $L = Tf(ABC, P)$
> Let $P_1 P_2 P_3 P_4$ be a quadrangle.
> $L_1 = Tf(P_2 P_3 P_4, P_1)$
> $L_2 = Tf(P_3 P_4 P_1, P_2)$
> $L_3 = Tf(P_2 P_4 P_1, P_1)$
> $L_4 = Tf(P_1 P_2 P_3, P_4)$
> Then L_1, L_2, L_3, L_4 are concurrent at a point.
> Which is this point of $QA(P_1, P_2, P_3, P_4)$?
> Best regards
> Tran Quang Hung.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3846
Date: 03/10/2019 2:59:11
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Tran Quang Hung,

If $f(x)$ works then $x + t \cdot f(x)$ works, and in this case the concurrent points obtained from two functions are the same. For now I can find only functions of the form x^n or $x + t \cdot x^n$ that work. The point obtained in my example is the same as the point obtained by function $x - x^2$ or function x^2 and is the point QA-P16.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à 00:11:34 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Tran Quang Hung,
>

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3847
Date: 03/10/2019 3:03:52
From: Tran Quang Hung
Subject: A family of centers in quadrangle

Dear Thanh Tung, dear Eckart,

Thank You for your interest in my idea. I give a similar proof of Thanh Tung in his file and the concurrent point in quadrangle $P_1P_2P_3P_4$ is

$$X = \frac{[f(s_1)P_1 + f(s_2)P_2 + f(s_3)P_3 + f(s_4)P_4]}{(f(s_1)+f(s_2)+f(s_3)+f(s_4))}$$

Could You please check my proof?

>> Let $f(x)$ be a real function of x .
>> If P has barycentric coordinate (x,y,z) in triangle ABC .
>> We consider $f(P)$ has barycentric coordinate $(f(x),f(y),f(z))$
>> wrt ABC i.e denote by $f(P,ABC)$.
>> Now with quadrangle $P_1P_2P_3P_4$.
>> $Q_1 = f(P_1,P_2P_3P_4)$
>> $Q_2 = f(P_2,P_3P_4P_1)$
>> $Q_3 = f(P_3,P_4P_1P_2)$
>> $Q_4 = f(P_4,P_1P_2P_3)$
>> Then the line $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ are concurrent.
>> Proof. Let s_1, s_2, s_3 , and s_4 be signed area of triangles
>> $P_2P_3P_4, P_3P_4P_1, P_1P_2P_3$, and $P_2P_3P_4$ respectively.
>> P_1 has coordinate (s_2,s_3,s_4) wrt triangle $P_2P_3P_4$.
>> So that Q_1 has coordinate $(f(s_2),f(s_3),f(s_4))$ wrt $P_2P_3P_4$.
>> i.e. $[f(s_2)+f(s_3)+f(s_4)]Q_1 = f(s_2)P_2+f(s_3)P_3+f(s_4)P_4$.
>> We let $X = [f(s_1)P_1 + f(s_2)P_2 + f(s_3)P_3 +$
>> $f(s_4)P_4]/(f(s_1)+f(s_2)+f(s_3)+f(s_4))$
>> Thus $[(f(s_1)+f(s_2)+f(s_3)+f(s_4)]X =$
>> $f(s_1)P_1 + [f(s_2)+f(s_3)+f(s_4)]Q_1$.
>> Therefore P_1Q_1 goes through X . Similarly, P_2Q_2, P_3Q_3, P_4Q_4
>> go through X .

Best Regards

Tran Quang Hung.

Vào Th 5, 3 thg 10, 2019 vào lúc 07:59 Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] <
Quadri-Figures-Group@yahoogroups.com > đã viết:
> Dear Tran Quang Hung,
>

[← Previous](#) [Next →](#) [↩ Message Index](#) [↑ Subjects](#)

Message: #3848
Date: 03/10/2019 11:52:16
From: bernard.keizer
Subject: A family of centers in quadrangle

Dear Eckart,
Bernard Gibert also uses the product of 2 points (see SITP).
If I'm not wrong, your 1st property is exactly the definition
of the isoconjugation !
With 4 fixed points in the QA vertices, the product of 2 QA-Tf2
partners is QA-P16, which is itself QA-Tf2 of the centroid of
DT. Hence the definition of the pivotal isocubics as $pK(QA-P16,$
pivot) ...
If I understand correctly, you extend the property to any
triangle.
Anyhow, this construction of a new family of centers in a
quadrangle is really fascinating !
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3849
Date: 03/10/2019 4:16:29
From: eckart_schmidt@t-online.de
Subject: A family of centers in quadrangle

Dear Bernard, dear Thanh Tung,

many thanks to Bernard:

... Studied in SITP years ago, with macros on my PC,
... but no longer parat for me "multiplication of points"!
 $X*Y$ = isoconjugate of the centroid
... wrt an isoconjugation, swapping X and Y .

Perhaps of interest wrt "square roots of a point",
... only real for points inside the reference triangle:
Consider a quadrangle $P_1P_2P_3P_4$
... and for each vertex P_i the anticevian Q_i
 inside the remaining triangle,
... (if P_i inside the remaining triangle: $P_i = Q_i$).
... Let R_i be the square root of Q_i
 inside the remaining triangle of P_i .
... The lines P_iR_i have a common point.

What about this new QA-point?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3850
Date: 03/10/2019 5:30:57
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear
Bernard and
Eckart,

I think that there is a generalization:

For a point $P = (x : y : z)$,
w.r.t. triangle ABC and a real number n ,
let $R = (|x|^n, |y|^n, |z|^n)$.
Call $R = f(P, ABC)$.

Consider a QA $(P_1 P_2 P_3 P_4)$,
Let $R_i = f(P_i, P_j P_k P_l)$ then the four lines $P_i R_i$ concur at a
point.

Then for $n=1/2$ we get the Eckart's problem.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à 21:16:32 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Bernard, dear Thanh Tung,
>

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3851
Date: 03/10/2019 6:02:55
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Quang Hung ,

The problem here is dealing with normalized and unnormalized barycentric coordinates.

Formally one should use only normalized barycentric coordinates, i.e. (x, y, z) with $x + y + z = 1$. For homogeneous functions like $f(x) = x^n$ (n integer), applying them to normalized and unnormalized barycentric coordinates give the same points. However in general applying a function to normalized and unnormalized barycentric coordinates would give different points.

For example: choose the non-homogeneous function $f(x) = 1/(1+x)$. Applying to the same point $(3, 2, -4) = (6, 4, -8)$ gives different coordinates.

So the question should be asked is:

Let $f(x)$ be a real function of x .

If P has barycentric coordinate (x, y, z) in triangle ABC . We consider $f(P)$ has *normalized* barycentric coordinate $(f(x), f(y), f(z))$ wrt ABC i.e denote by $tr(P, ABC)$.

Now with quadrangle $P_1P_2P_3P_4$.

$$Q_1 = tr(P_1, P_2P_3P_4)$$

$$Q_2 = tr(P_2, P_3P_4P_1)$$

$$Q_3 = tr(P_3, P_4P_1P_2)$$

$$Q_4 = tr(P_4, P_1P_2P_3)$$

Are the lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concurrent?

In this case

$$f(-s_2/s_1) + f(-s_3/s_1) + f(-s_4/s_1)$$

$$Q_1 = (f(-s_2/s_1) \cdot P_2 + f(-s_3/s_1) \cdot P_3 + f(-s_4/s_1) \cdot P_4)$$

In general it cannot reduce to form

$$[f(s_2) + f(s_3) + f(s_4)]Q_1 = f(s_2)P_2 + f(s_3)P_3 + f(s_4)P_4.$$

Best regards,
Vu Thanh Tung

On Thursday, October 3, 2019, 8:03 AM, Tran Quang Hung
analgeomatrica@gmail.com [Quadri-Figures-Group] wrote:

> Dear Thanh Tung, dear Eckart,

> Thank You for your interest in my idea. I give a similar proof
of Thanh

> Tung in his file and the concurrent point in quadrangle
P1P2P3P4 is
> $X = [f(s_1)P_1 + f(s_2)P_2 + f(s_3)P_3 +$
 $f(s_4)P_4] / (f(s_1) + f(s_2) + f(s_3) + f(s_4))?$
> Could You please check my proof?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3852
Date: 03/10/2019 6:19:24
From: Thanh Tung
Subject: A family of centers in quadrangle

Dear Bernard and Eckart,

The concurrent point described in message #3850 has the
1st CT-Coordinate:
 $(p + q + r) |p|^n + p |p+q+r|^n.$
with $|t|$ = absolute value of t .
I think they are new center as there is no existing center with
coordinates involving absolute value for now.

Best regards,
Vu Thanh Tung

Le jeudi 3 octobre 2019 à 22:31:02 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Bernard and Eckart,
> I think that there is a generalization:
>

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3853
Date: 03/10/2019 6:39:29
From: Thanh Tung
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

Dear all,

Let consider a quadrangle $P_1 P_2 P_3 P_4$ with Diagonal Triangle Δ . Then the square point of $P_1 P_2 P_3 P_4$ with respect to Δ are the same.

Proof:

If the barycentric coordinates of P_1 w.r.t. Diagonal Triangle Δ is (p, q, r) then the barycentric coordinates of 3 other points are: $(p, -q, r)$, $(p, q, -r)$, $(p, -q, -r)$. The common square point is (p^2, q^2, r^2) w.r.t. Δ and so it has a simple first DT-Coordinate: p^2 .

Also for an even integer n 4 points $P_1 P_2 P_3 P_4$ have the same n -power point w.r.t. Diagonal Triangle Δ this point has first DT-Coordinate: p^n .

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3854
Date: 03/10/2019 7:19:36
From: eckart_schmidt@t-online.de
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

Dear Thanh Tung,

the common square point for the QA-vertices wrt QA-Tr1 is QA-P16.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3855
Date: 03/10/2019 7:21:05
From: Thanh Tung
Subject: Generalization of Eckart's observation in number 3842

Dear all,

Eckart's observation in #3842 is intersecting:
The products of a point (unequal QA-P1) and a QA-vertex wrt the remaining QA-triangle are collinear, bearing QA-P16.

I found a generalization of this as follows:
Consider a quadrangle $P_1 P_2 P_3 P_4$, a point P and an integer n .
 $Q_i = n$ -power of P_i w.r.t. triangle $P_j P_k P_l$.
 $R_i =$ multiplication of P and Q_i
w.r.t. triangle $P_j P_k P_l$.
Then four points R_1, R_2, R_3, R_4 are collinear in a line $d(P)$.
All line $d(P)$ passes through a fixed point.
What is this fixed point w.r.t. quadrangle $P_1 P_2 P_3 P_4$?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3856
Date: 03/10/2019 7:23:20
From: Thanh Tung
Subject: Common square point for 4 vertices w.r.t. Diagonal Triangle

Dear Eckart,

Thank you very much,

Best regards,
Vu Thanh Tung

Le vendredi 4 octobre 2019 à 00:19:42 UTC+7,
'eckart_schmidt@t-online.de' eckart_schmidt@t-online.de
[Quadri-Figures-Group] a écrit :
> Dear Thanh Tung,
> the common square point for the QA-vertices wrt QA-Tr1 is
QA-P16.
> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3857

Date: 03/10/2019 7:52:36

From: Thanh Tung

Subject: Generalization of Eckart's observation in number 3842

Dear all,

I observed that the fixed point is the $(n+1)$ -power point $X(n+1)$ that was introduced in message #3832.

Best regards,
Vu Thanh Tung

Le vendredi 4 octobre 2019 à 00:22:07 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :

> Dear all,
> Eckart's observation in #3842 is intersecting:
> The products of a point (unequal QA-P1) and a QA-vertex
> wrt the remaining QA-triangle are collinear, bearing QA-P16.
> I found a generalization of this as follows:
> Consider a quadrangle $P_1 P_2 P_3 P_4$, a point P and an integer n .
> $Q_i = n$ -power of P_i w.r.t. triangle $P_j P_k P_l$.
> $R_i =$ multiplication of P and Q_i w.r.t. triangle $P_j P_k P_l$.
> Then four points R_1, R_2, R_3, R_4 are collinear in a line $d(P)$.
> All line $d(P)$ passes through a fixed point.
> What is this fixed point w.r.t. quadrangle $P_1 P_2 P_3 P_4$?
> Best regards,
> Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3858
Date: 04/10/2019 8:56:59
From: eckart_schmidt@t-online.de
Subject: QL-geometry with point products

Dear Thanh Tung, dear Bernard,

there are also interesting aspects
... for multiplication of points in QL-geometry.

Here first observations:

- (1) Reference triangle QL-Tr1:
 - (a) Squares of opposite QL-points coincide
... in the cevian points of QL-P13.
 - (b) Products of opposite QL-points
... are their midpoints.
 - (c) Products of opposite QL-points and QL-P1
... divide harmonic a QL-Tr1 side.
 - (d) Products of two QL-Tr2-vertices
... are the intersection of QL-L1 and the QL-Tr2-side.
 - (e) Product of all three QL-Tr2-vertices is QL-P23.
- (2) Reference triangle QL-Tr2:
 - (a) Products of two QL-Tr1-vertices are collinear.
 - (b) Products of two CSC-partner on QL-Cu1 are collinear.
 - (c) Products of opposite QL-points
... coincide in the intersection of line a) and b).

So far, there will be more!

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3859
Date: 04/10/2019 12:46:18
From: eckart_schmidt@t-online.de
Subject: QL-geometry with point products

Dear Bernard, dear Thanh Tung,

in addition to #3858 some nice observations:

- (1) Wrt QL-Tr1:
 - Squares of points on the QL-lines
 - ... give a common QL-Tr1 inscribed conic,
 - ... tangent in the cevians of QL-P13.
- (2) Wrt QL-Tr2: Squares of QL-Tr1-vertices
 - ... are collinear with the common product of opposite QL-points.
- (3) Wrt QL-Tr2: The product of the three QL-Tr1-vertices
 - ... is the product of QL-P8 and two opposite QL-points
 - ... on QL-P8.QL-P13.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3860
Date: 05/10/2019 12:44:03
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Chris,

are the following properties hidden in EPG?

- (1) For a 6P the six 5P-s-P5 are concyclic on 6P-s-Cix
... centered in 6P-s-P1.
- (2) for a 7P the seven 6P-s-Cix
... have the common point 7P-s-P1.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3861
Date: 05/10/2019 3:03:51
From: chris.vantienhoven
Subject: 6P-s-P1 and 7P-s-P1

Dear Eckart,

Both properties are in the pipeline of being added to EPG.
6P-s-Cix will become 6P-s-Ci1.
Thanks for reminding me.
There are lots of other items that are ready to be released too.
However, I am very busy in this period. It will come later.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3862
Date: 06/10/2019 3:56:49
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Chris,

in addition to #3861:
For a 5P and any point P
... we can consider 5P-s-P5 wrt P and the 4P of the 5P,
... which give 5 concyclic points on $C_i(P)$ (wrt $C_i(P)$ see #3575).

If we consider for a 6P and any point P
... the six $C_i(P)$ wrt the 5P of the 6P
... we get six circles with a common point
... on the circle 6P-s- C_{ix} round 6P-s-P1 (see # 3861).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3863
Date: 08/10/2019 9:16:35
From: chris.vantienhoven
Subject: 6P-s-P1 and 7P-s-P1

Dear Eckart,

Again, thanks for reminding. I will include these beautiful transformations too.
When 5P-s-Tf6 = 5P-CB-Conjugate and 5P-s-Tf7 = your 5P-Schmidt Transformation (circle + center), what is their relationship? It looks like there is a relationship in your message #3575, but I lost oversight.
When 6P-s-Tf1 = your 6P- Schmidt Transformation (common circle-point), can you describe all properties of 5P-s-Tf7 and 6P-s-Tf1?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3864
Date: 08/10/2019 5:34:23
From: Thanh Tung
Subject: Another family of points on quadrangle

Dear all,

I have found a new two-parameters family of points on Quadrangle.

This new family of points is described as follows:

Consider a quadrangle $P_1 P_2 P_3 P_4$ and two integers m, n .
Let

- _ Q_i = m -th power of P_i w.r.t. triangle $P_j P_k P_l$
- _ R_i = n -th power of the Cyclocevian Conjugate of P_i w.r.t. triangle $P_j P_k P_l$
- _ S_i = product of Q_i and R_i w.r.t triangle $P_j P_k P_l$

Then four lines $P_i S_i$ concur at a point $T(m, n)$.

Some particular cases:

- _ $n=0$:
 $T(0, m)$ is the n -th power point that I described in the message #3830, or #3832
- _ $T(0, 1)$ = QA-P38: Montesdeoca-Hutson Point

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3865
Date: 08/10/2019 7:11:52
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear all,

- _ Let $P_1 P_2 P_3 P_4$ be a Quadrangle.
 - _ Q_i = orthocenter of the triangle formed by the midpoints of $P_i P_j, P_i P_k, P_i P_l$.
 - _ R_i = orthocenter of the triangle formed by the midpoints of $Q_i Q_j, Q_i Q_k, Q_i Q_l$.
- Then four lines $P_i Q_i$ concur at a point.
Is this a new point on Quadrangle or it is a new construction of an existing point?

Best regards,
Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3866
Date: 08/10/2019 7:30:41
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear all,

Also, two Quadrangles $P_1P_2P_3P_4$ and $R_1R_2R_3R_4$ are homothetic and the concurrent point is the homothetic center of these two Quadrangles.

This is very similar to the construction of the point QA-P8 (Midray Homothetic Center).

Best regards,
Vu Thanh Tung

Le mercredi 9 octobre 2019 à 00:12:23 UTC+7, Thanh Tung tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear all,
> _ Let $P_1 P_2 P_3 P_4$ be a Quadrangle.
> _ Q_i = orthocenter of the triangle
> formed by the midpoints of P_iP_j , P_iP_k , P_iP_l .
> _ R_i = orthocenter of the triangle
> formed by the midpoints of Q_iQ_j , Q_iQ_k , Q_iQ_l .
> Then four lines P_iQ_i concur at a point.
> Is this a new point on Quadrangle or it is a new construction
> of an existing point?
> Best regards,
> Vu Thanh Tung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3867
Date: 08/10/2019 8:21:18
From: eckart_schmidt@t-online.de
Subject: $(p^4 : q^4 : r^4)$

Dear Chris,

well known for a QA with vertex $(p:q:r)$ (DT-coordinates):
... $(p^2:q^2:r^2) = \text{QA-P16}$.

Perhaps new:

... $(p^4:q^4:r^4) = \text{2nd intersection of QA-P1.16}$
... .. and QA-Tr1-circumconic through QA-P1 and QA-P18.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3868
Date: 08/10/2019 9:31:04
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung,

there will be a typo in # 3865:
"Then four lines $PiRi$ concur at a point."
This point is QA-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3869
Date: 08/10/2019 11:27:22
From: César E. Lozada
Subject: Construction of a point on Quadrangle

Dear Thanh and Eckart,

If "orthocenter" is replaced with "circumcenter" then the four lines $PiRi$ concur at QA-P8.

Best regards,
César Lozada

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: Tuesday, October 8, 2019 3:31 PM
> To: Quadri-Figures-Group,
> Subject: [Quadri-Figures-Group]
> Re: Construction of a point on Quadrangle
> Dear Thanh Tung,
> there will be a typo in # 3865:
> "Then four lines $PiRi$ concur at a point."
> This point is QA-P2.
> Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3870
Date: 09/10/2019 2:01:47
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear Eckart & César Lozada,

Thank you very much,

Best regards,
Vu Thanh Tung

Le mercredi 9 octobre 2019 à 04:27:24 UTC+7, César E. Lozada
cesar_e_lozada@yahoo.es [Quadri-Figures-Group] a écrit:
> Dear Thanh and Eckart,
> If "orthocenter" is replaced with "circumcenter" then the four
lines PiRi concur at QA-P8.
> Best regards,
> César Lozada

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3871
Date: 09/10/2019 8:59:36
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung, dear César,

the construction in #3865
... gives for orthocenter, circumcenter and centroid
... the QA-points QA-P2, QA-P8, QA-P1.

CABRI observations show,
... that there are three other points on the Euler line,
... dividing X3.X4 in special ratios,
... depending on the reference quadrangle,
... which lead to three further QA-points.
What about these three QA-points?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3872
Date: 10/10/2019 10:37:07
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung, dear César,

the construction in #3865 can be modified:
Let $E(t)$ be a point on the Euler line,
... dividing $X(3).X(4)$ with ratio t .
_ Let $P_1 P_2 P_3 P_4$ be a Quadrangle.
_ $Q_i = E(t)$ of the triangle formed by P_j, P_k, P_l .
_ $R_i = E(t)$ of the triangle formed by Q_j, Q_k, Q_l .
Then the four lines $P_i R_i$ concur at a point
... for $t = 0$ (orthocenter) in QA-P4,
... for $t = 1$ (nine-point center) in QA-P7,
... for $t = 1:2$ (centroid) in QA-P1.
CABRI observations show,
... that there are three other points on the Euler line,
... dividing $X_3.X_4$ in special ratios,
... depending on the reference quadrangle,
... which lead to three further QA-points.
What about these three QA-points?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3873
Date: 10/10/2019 11:57:21
From: bernard.keizer
Subject: Construction of a point on Quadrangle

Dear Eckart, dear Thanh Tung, dear César
Beautiful idea to start directly with the 4 reference triangles
instead of the midtriangles !
I reproduced without difficulty the property for centroid and
circumcenter.
I only observe that the ratio of similitude between the QA of
the R_i and the initial QA is $-1/3$ for centroid (synthetically
obvious with QA-P1 as perspector) and -0.117 for circumcenter.
So far, I have no idea of a synthetic or analytical general
proof ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3874
Date: 10/10/2019 6:41:59
From: Thanh Tung
Subject: Construction of a point on Quadrangle

Dear Bernard, Eckart, César,

Since for centroid, nine-pointcircle, orthocenter of 4 reference triangle we obtain 3 different quadrangle homothetic to the reference quadrangle, they are also homothetic in pair.

What can say about the homothetic center of each pair ? Are they new centers ?

The same question to ask with centroid, circumcenter, orthocenter quadrangle of 4 midray triangle. They are homothetic in pair and what about the homothetic center of each pair?

Best regards,
Vu Thanh Tung

Le jeudi 10 octobre 2019 à 17:01:27 UTC+7,
bernard.keizer@yahoo.com [Quadri-Figures-Group] a écrit :
> Beautiful idea to start directly with the 4 reference triangles instead of the midtriangles !
> I reproduced without difficulty the property for centroid and circumcenter.
> I only observe that the ratio of similitude between the QA of the Ri and the initial QA is $-1/3$
> for centroid (synthetically obvious with QA-P1 as perspector) and -0.117 for circumcenter.
> So far, I have no idea of a synthetic or analytical general proof ...
> Best regards
> Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3875

Date: 11/10/2019 3:55:04

From: Thanh Tung

Subject: Construction of a point on Quadrangle

Dear all,

I would like to clarify my last statement.

Definition:

Let $P_1 P_2 P_3 P_4$ be a Quadrangle and a natural number n .

$Q_i = X_n$ of the triangle formed by the midpoints of $P_j P_k$, $P_k P_l$, $P_l P_j$.

$R_i = X_n$ of the triangle formed by the midpoints of $P_i P_j$, $P_i P_k$, $P_i P_l$.

The Quadrangle $Q_1 Q_2 Q_3 Q_4$ can be called the X_n -midpoint (first generation) of the Quadrangle $P_1 P_2 P_3 P_4$,

The Quadrangle $R_1 R_2 R_3 R_4$ can be called the X_n -midray (first generation) of the Quadrangle $P_1 P_2 P_3 P_4$.

The X_n -midpoint (of X_n -midray) p -th generation of Quadrangle $P_1 P_2 P_3 P_4$ is defined recursively.

(say the n -th generation is the first generation of the $(n-1)$ -th generation),

denoted by n -th- X_n -midpoint or p -th- X_n -midray of Quadrangle $P_1 P_2 P_3 P_4$ for short.

The Quadrangle can be considered as the 0 -th- X_n -midpoint or 0 -th- X_n -midray of itself.

***Result*:**

For any natural integer n , the n -th- X_2 -midpoint or midray is homothetic to the

Reference Quadrangle and the homothetic center is their common centroid.

For even n natural integer n , the n -th- X_3, X_4 -midpoint (or midray) are homothetic to the Reference Quadrangle.

The above Quadrangles are then homothetic in pair.

The homothetic centers of each pair form a new family of Quadrangle centers.

We can also mix X_n -midpoint midray in the process of creating new centers.

For example: The second- X_3 -midpoint of the fourth- X_4 -midray is homothetic to the

second- X_4 -midpoint of the second- X_3 -midray is homothetic (both homothetic to the Reference Quadrangle).

This make this new family of Quadrangle centers even larger.

Best regards,
Vu Thanh Tung

Le jeudi 10 octobre 2019 à 23:42:59 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Bernard, Eckart, César,
> Since for centroid, nine-pointcircle, orthocenter of

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3876
Date: 13/10/2019 8:35:14
From: eckart_schmidt@t-online.de
Subject: Construction of a point on Quadrangle

Dear Thanh Tung, dear César, dear Bernard,

I have to correct my #3872:
Studying a nonconvex QA,
... I observed 8 points $E(t)$ on the Euler line,
... which lead to 8 QA-points in the sense of #3872:
... QA-P1, QA-P4, QA-P7 (collinear),
... *four points on QA-Co2*
... and another point.
What about the last five points?

Best regards Eckart

PS: For a convex QA I observed only 6 points
... there seem to be only two points on QA-Co2.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3877
Date: 16/10/2019 8:26:33
From: chris.vantienhoven
Subject: Bad news from Yahoo

Yahoo has made the decision to no longer allow users to upload content to the Yahoo Groups site. Beginning October 21, you won't be able to upload any more content to the site, and as of December 14 all previously posted content on the site will be permanently removed. You'll have until that date to save anything you've uploaded.

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3878

Date: 16/10/2019 6:42:36

From: Thanh Tung

Subject: Another family of points on quadrangle

Dear all,

As one of the last messages before this Group is gone, please find in attachment my proof of #3864.

Best regards,
Vu Thanh Tung

Le mardi 8 octobre 2019 à 22:36:09 UTC+7, Thanh Tung
tulip13091983@yahoo.com [Quadri-Figures-Group] a écrit :

> Dear all,
> I have found a new two-parameters family of points on
Quadrangle.
> This new family of points is described as follows:
> Consider a quadrangle $P_1 P_2 P_3 P_4$ and two integers m, n .
> Let
> _ Q_i = m -th power of P_i w.r.t. triangle $P_j P_k P_l$
> _ R_i = n -th power of the
> Cyclocevian Conjugate of P_i w.r.t. triangle $P_j P_k P_l$
> _ S_i = product of Q_i and R_i w.r.t triangle $P_j P_k P_l$
> Then four lines $P_i S_i$ concur at a point $T(m, n)$.
> Some particular cases:
> _ $n=0$: $T(0, m)$ is the n -th power point that I described in
the message #3830, or #3832
> _ $T(0, 1)$ = QA-P38 : Montesdeoca-Hutson Point
> Best regards,
> Vu Thanh Tung

Power Cyclocevian Family of Points of Quadrangle

Vu Thanh Tung

October 16, 2019

In this short note, I define a family of points in Quadrangle. For each integer n , the n^{th} -power of a point with respect to a triangle is defined as follows:

Definition 0.1 (Power of a point). *For each integer n , the n^{th} -power of a point P with barycentric coordinates $(x : y : z)$ with respect to $\triangle ABC$ to the point P^n with barycentric coordinates $(x^n : y^n : z^n)$ with respect to $\triangle ABC$.*

Definition 0.2 (Cyclocevian Conjugate). *Let P be a point not on a sideline of a reference triangle $\triangle ABC$. Let $\triangle A'B'C'$ be the cevian triangle of P with respect to $\triangle ABC$. The circumcircle of $\triangle A'B'C'$ meets each sideline in two (not necessarily distinct) points A', A'' ; B', B'' ; and C', C'' . Then the lines AA'' , BB'' , and CC'' concur in a point known as the cyclocevian conjugate of P with respect to $\triangle ABC$.*

Now we are able to construct a family of points in quadrangle.

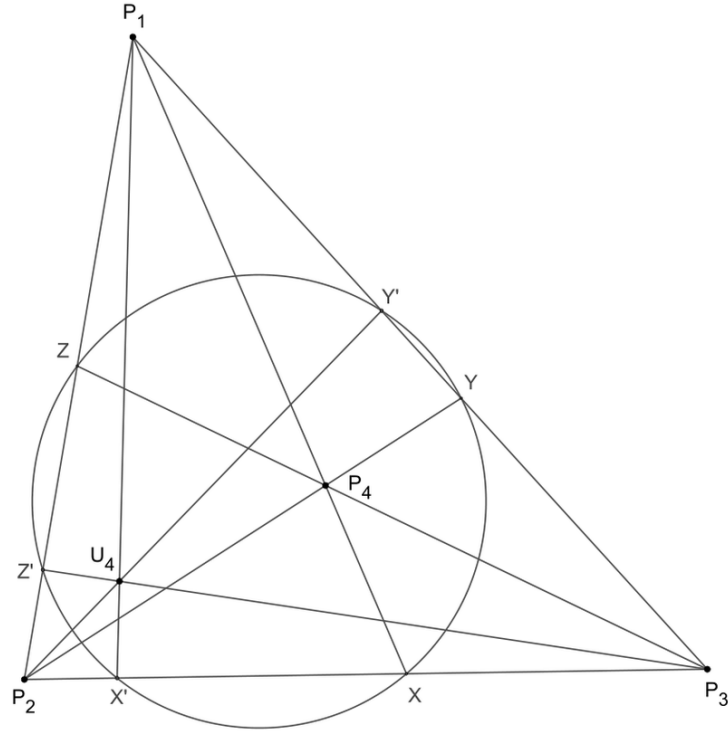
Theorem 1 (Vu Thanh Tung [1]). *Consider a quadrangle $P_1P_2P_3P_4$ and two integers m, n . For $\{i, j, k, l\} = \{1, 2, 3, 4\}$, let*

- Q_i be the m^{th} -power of P_i ,
- U_i be the cyclocevian conjugate of P_i ,
- R_i be the n^{th} -power of U_i ,
- S_i be the product of Q_i and R_i ,

all with respect to $\triangle P_jP_kP_l$. Then the four lines $P_1Q_1, P_2Q_2, P_3Q_3, P_4Q_4$ concur at a point $T_{m,n}$.

Proof. Let s_1, s_2, s_3, s_4 be the signed area of $\triangle P_2P_3P_4, \triangle P_3P_4P_1, \triangle P_4P_1P_2, \triangle P_1P_2P_3$ respectively. We note that $s_1 + s_2 + s_3 + s_4 = 0$. The barycentric coordinates of P_4 with respect to $\triangle P_1P_2P_3$ is $(s_1 : s_2 : s_3)$ and then the point Q_4 has barycentric coordinates $(s_1^m : s_2^m : s_3^m)$ with respect to $\triangle P_1P_2P_3$.

Let $X = P_4P_1 \cap P_2P_3$, $Y = P_4P_2 \cap P_3P_1$, $Z = P_4P_3 \cap P_1P_2$. Let (c) be the circumcircle of $\triangle XYZ$. $(c) \cap P_2P_3 = \{X, X'\}$, $(c) \cap P_3P_1 = \{Y, Y'\}$, $(c) \cap P_1P_2 = \{Z, Z'\}$. Then P_1X' , P_2Y' , P_3Z' concur at the point U_4 .



For $i = 1, 2, 3, 4$, let ρ_i be the power of P_i with respect to (c) . Then we have $\overline{P_2X} \cdot \overline{P_2X'} = \rho_2$, $\overline{P_3X} \cdot \overline{P_3X'} = \rho_3$, so:

$$\frac{\overline{P_2X'}}{\overline{P_3X'}} = \frac{\rho_2}{\rho_3} \cdot \frac{\overline{P_3X}}{\overline{P_2X}} = -\frac{\rho_2 \cdot s_2}{\rho_3 \cdot s_3}. \quad (1)$$

Similarly,

$$\frac{\overline{P_3Y'}}{\overline{P_1Y'}} = -\frac{\rho_3 \cdot s_3}{\rho_1 \cdot s_1}, \quad \frac{\overline{P_1Z'}}{\overline{P_2Z'}} = -\frac{\rho_1 \cdot s_1}{\rho_2 \cdot s_2}. \quad (2)$$

It means that the point U_4 has the barycentric coordinates $(\frac{1}{\rho_1 \cdot s_1} : \frac{1}{\rho_2 \cdot s_2} : \frac{1}{\rho_3 \cdot s_3})$ with respect to $\triangle P_2P_3P_4$. Thus, the point R_4 has the barycentric coordinates $(\rho_1^{-n} \cdot s_1^{-n} : \rho_2^{-n} \cdot s_2^{-n} : \rho_3^{-n} \cdot s_3^{-n})$ and the point S_4 has the barycentric coordinates $(\rho_1^{-n} \cdot s_1^{m-n} : \rho_2^{-n} \cdot s_2^{m-n} : \rho_3^{-n} \cdot s_3^{m-n})$ with respect to $\triangle P_2P_3P_4$. For $i = 1, 2, 3, 4$ let $f_i = \rho_i^{-n} \cdot s_i^{m-n}$. We can write:

$$S_4 = \frac{1}{f_1 + f_2 + f_3} (f_1 \cdot P_1 + f_2 \cdot P_2 + f_3 \cdot P_3) \quad (3)$$

Let the point $T_{m,n}$ be defined as:

$$T_{m,n} = \frac{1}{f_1 + f_2 + f_3 + f_4} (f_1 \cdot P_1 + f_2 \cdot P_2 + f_3 \cdot P_3 + f_4 \cdot P_4) \quad (4)$$

$T_{m,n}$ is on the line P_4S_4 as:

$$T_{m,n} = \frac{f_1 + f_2 + f_3}{f_1 + f_2 + f_3 + f_4} \cdot S_4 + \frac{f_4}{f_1 + f_2 + f_3 + f_4} \cdot P_4. \quad (5)$$

Similarly, $T_{m,n}$ is on the line P_1S_1 , P_2S_2 , P_3S_3 . □

References

- [1] Vu Thanh Tung, *message 3830*, Quadrilateral Geometry & Polygon Geometry, Oct 8, 2019, <https://groups.yahoo.com/neo/groups/Quadri-Figures-Group/conversations/messages/3864>
- [2] ENCYCLOPEDIA OF QUADRI-FIGURES, , <https://www.chrisvantienhoven.nl/mathematics/encyclopedia>

Thanh Tung Vu, Namdinh, Vietnam
E-mail address: tungvtt@gmail.com

Message: #3879

Date: 17/10/2019 10:34:02

From: bernard.keizer

Subject: Another family of points on quadrangle

Dear Than Tung,

I would be very pleased to read your analytical proof, but your attached files don't appear on the forum !

Could you please send it to me as word- or pdf-file at my private e-mail address bernard.keizer@wanadoo.fr

Many thanks in advance

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3880

Date: 17/10/2019 11:24:07

From: bernard.keizer

Subject: Another family of points on quadrangle

Dear Than Tung,

Please forget my last message !

Your attachment doesn't appear on the forum (as it should), but I found it in the mailbox.

I will read it with a great interest

Thanks a lot

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3881
Date: 17/10/2019 2:51:42
From: chris.vantienhoven
Subject: Bad news from Yahoo

Dear friends,

Well, here we are. Yahoo Groups will close on 14th December 2019 & will delete all user content. It will continue as an email list. Only until 28 th October you will be able to send messages in the usual way. After that period, messages only can be send as emails to quadri-figures-group@yahoogroups.com. These messages should reach all members.

But of course, this is not what we want. The Groups Hyacinthos, Anopolis and Advanced Plane Geometry (ADGEOM) have the same problem. We are looking together for a solution. We already archived all content related to our groups externally.

I think we found a new and better Groups-provider. I will do my best to migrate all content to this new provider and keep you informed.

You can also make a backup yourself with the software of PG Offline at <http://www.personalgroupware.com/downloads.htm>. I think it is important when several of us have a backup

* It also has the advantage of looking up older QFG-messages very quickly.

After 14th December 2019 this no longer will be possible via Yahoo Groups.

Could you let me know when you do?

I trust the new situation will be better than before, so it is all for the best.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3882
Date: 17/10/2019 4:01:47
From: Thanh Tung
Subject: Another family of points on quadrangle

Dear Bernard,

Thank you for your interest,

Best regards,
Vu Thanh Tung

Le jeudi 17 octobre 2019 à 16:24:12 UTC+7,
bernard.keizer@yahoo.com [Quadri-Figures-Group] a écrit :
> Dear Than Tung,
> Please forget my last message !
> Your attachment doesn't appear on the forum (as it should),
but I found it in the mailbox.
> I will read it with a great interest
> Thanks a lot
> Best regards
> Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3883
Date: 18/10/2019 3:35:57
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Chris,

In #3863 you ask for a relationship between the transformations
..."cb(P)" = 5P-s-Tf6 and "Ci(P)" = 5P-s-Tf7 in your nomination.

In #3575 these transformations are described with QA-Cu7
background.

Here only the observations wrt 5P-geometry:

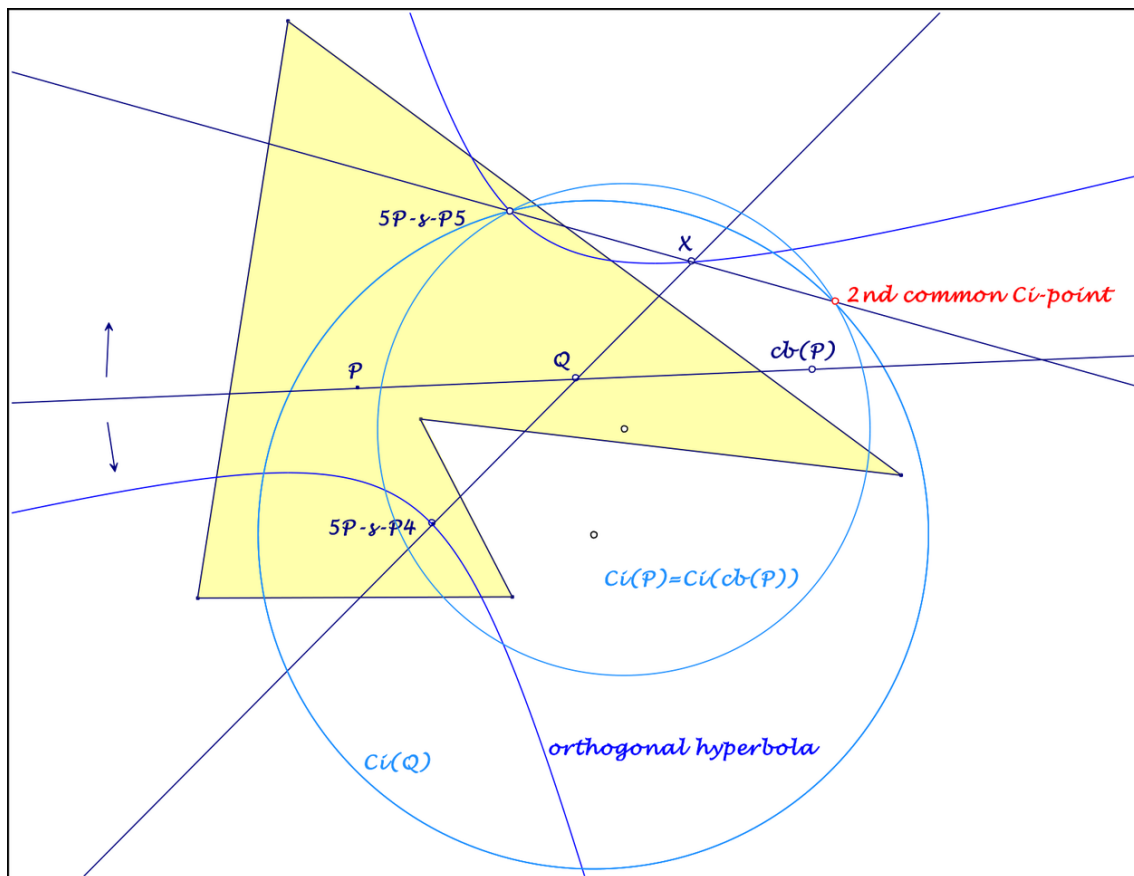
"cb(P)" is the Cayley-Bacharach point of
... the 5P-vertices, the two circular points and P.
... cb(P) maps 5P-s-Co1 to the line at infinity
... .. with P, cb(P) and 5P-s-P4 collinear.
"Ci(P)" uses the 5L of the bisectors P*P*_i,
... the CSC-images of P wrt the 4L of the 5L give a circle Ci(P).

Properties:

(1) P and cb(P) have the same Ci-circle.
(2) For points P on 5P-s-Co1
... Ci(P) degenerates to a line through 5P-s-P5
... and the 2nd intersection of P.5P-s-P4 and an orthogonal
hyperbola
... through 5P-s-P4,5, centered in the middle with axes parallel
to the 5P-s-Co1-axes.
(3) For 5P-s-P6 we get a circle through 5P-s-P5,6 and
5P-CSC(5P-s-P4).
(4) *For cb-partner on lines through a point Q*
... their Ci-circles have a common 2nd point beside 5P-s-P5:
... Example: For Q = 5P-CSC(5P-s-P5) this 2nd Ci-point is 5P-s-P6.
... In general: 2nd intersection of Ci(Q) and X.5P-s-P5
... .. with X = 2nd intersection of Q.5P-s-P4 and the orthogonal
hyperbola in (2).

Best regards Eckart

PS: Is there already a nomination for the 5P-CSC?



2019-10-18.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3884
Date: 18/10/2019 7:24:28
From: bernard.keizer
Subject: 6P-s-P1 and 7P-s-P1

Dear Eckart,
 If I'm not wrong, you never gave the general construction of the 2nd point (apart of U) of the circles $Ci(P)$ for P and $cb(P)$ on lines through a point Q .
 Very interesting indeed
 Thanks a lot
 Best regards
 Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3885
Date: 20/10/2019 11:26:49
From: eckart_schmidt@t-online.de
Subject: Loci wrt concentric circles

Dear Chris, dear Bernard,

concentric circles can be mapped to other circles
... and the loci of their intersections considered:

(1) For 5P wrt 5P-s-Tf3:

... Let C_i be circles, centered in Z ,
and C_i' their 5P-s-Tf3-image circles.
... The intersections of C_i and C_i'
... give a new circle $C_i(Z)$, bearing 5P-s-P1,
... depending only of the circumconic 5P-s-Co1.
... The axes of 5P-s-Co1 intersect $C_i(Z)$ beside 5P-s-P1 diametral.

(2) For 5P wrt 5P-s-Tf3inv = Co-Tf3:
... analog ...

(3) For QL wrt CSC:

... Let C_i be circles, centered in Z ,
and C_i' their CSC-image circles.
... The intersections of C_i and C_i'
... give a quartic, CSC-invariant, through QL-2P3,
... intersecting the 1st Steiner axis further
in two harmonic points wrt QL-2P3
... and the 2nd Steiner axis in two CSC-partner.
... The six quartics for the QL-points
have common points in QL-2P3
... and triple points in two pairs of CSC-partner.
... The radical axes of C_i and C_i' envelope a parabola,
... which is also the envelope of $X.CSC(X)$
for points X on the quartic:
... Focus $CSC(Z)$ and directrix $Z.QL-P1$,
... orthogonal tangents from Z to the parabola.

(4) For 5P wrt 5P-CSC:

... analog ...
... but the five quartics have no triple points.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3886
Date: 21/10/2019 11:36:21
From: eckart_schmidt@t-online.de
Subject: 6P-s-P1 and 7P-s-P1

Dear Bernard,

thanks for interest wrt #3883.
If we take for Q a vertex of a 6P
... and for 5P the remaining 5P,
... the 2nd Ci-point is concyclic
... with the 5P-s-P5 for the 5P of the 6P.

The corresponding circle bears 12 points:
... the six 2nd Ci-points and the six 5P-s-P5,
... is centered in 6P-s-P1,
... and will be relevant for 6P-geometry.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3887
Date: 25/10/2019 9:18:57
From: eckart_schmidt@t-online.de
Subject: Anallagmatic QL/5P-Quartic

Dear Bernard, dear Chris,

for I am not familiar with quartics,
... I hope you can judge the following observations
... wrt the QL/5P-quartic for a 5P
 (see #3599, #3603, #3607, #3675).

The QL/5P-quartic is anallagmatic,
... that means,

there is an inversion wrt a circle,
... which maps the quartic to itself.

Here I only consider the case,
... that the quartic is a one-line curve.

This quartic has two double tangents
... with contact points S_1, T_1 and S_2, T_2 ,
... which are concyclic on a circle, centered in $5P-s-P_3$.

The intersection Z of the double tangents is center of an
inversion
... which maps $S_1 \rightarrow T_1$, $S_2 \rightarrow T_2$ and the quartic to itself.

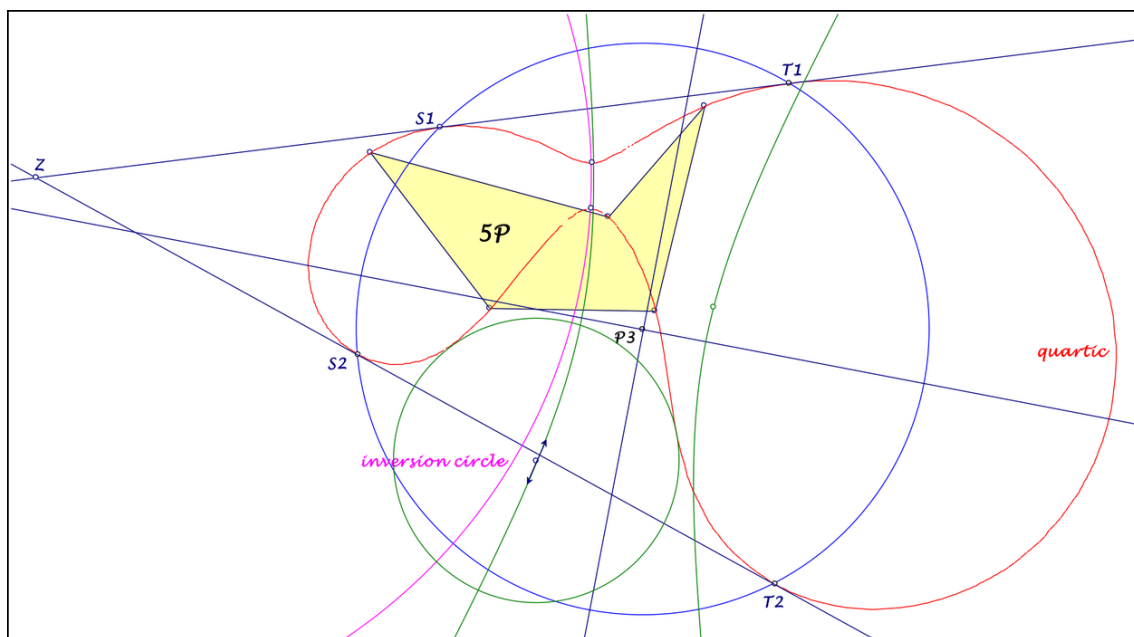
All anallagmatic curves have the property,
... that they are the envelope of a set of circles,
... which intersect the inversion circle orthogonal
... and contact the curve in inverse points.

For our quartic the enveloping circles
... are centered on a conic with center $5P-s-P_3$
... and axes parallel to those of QA-Co2 of $S_1T_1T_2S_2$,
... bearing the two intersections of the quartic
 and the inversion circle.

I hope, someone can confirm these observations.
What about the circle of the contact points S_1, T_1, S_2, T_2 ?

Best regards Eckart

PS: It will be interesting,
 ... to study the anallagmatic property
 of the quartic wrt a QL.



2019-10-24.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3888

Date: 29/10/2019 8:00:06

From: bernard.keizer

Subject: Anallagmatic QL/5P-Quartic

Dear Eckart,

I found this property very interesting !

I'm not a specialist of cubics or quartics either ...

I only found in <http://www.mathcurve.com>

under anallagmatic curves some commentars which may interest you (you may read them in french or in english).

Circular cubics (in particular the Van Rees curve) and bicircular quartics are examples of anallagmatic curves.

This is particularly interesting as the bicircular quartic of the 5P and the Van Rees cubic of the CSC of the 5P are CSC partners ...

In german, they use Anallagmatische Kurve, Kreiskubik and Doppelkreisquartik ...

Best regards

PS I'm really admirative that you continue until the last moment of existence of the forum to dig new properties

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3889
Date: 29/10/2019 8:02:11
From: chris.vantienhoven
Subject: Bad news from Yahoo

Dear friends,

As you know, Yahoo will delete the contents of all Yahoo-groups on 14/12.

Only for a very short period of time, it will be possible to send messages in the usual way to the Yahoo-Group.

I already made a new group in Groups.io, but it will take some time before our old messages will be migrated. I will keep you informed.

It can happen that there will a period we will not be able to send messages to our group until the migration has been done. I am dependent on Groups.io in this.

I will put the archives in Public Domain. This means that I give permission to anyone (person or institution) to host the archives in a web site.

You can download the archives by:

1. <http://www.httrack.com/>

or

2. <http://www.personalgroupware.com/downloads.htm>

(Read the details before starting to download)

Best regards,
Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3890
Date: 29/10/2019 11:41:03
From: Fam. van Tienhoven
Subject: last message(s) QFG-Yahoo-group

Dear friends,

As you know, Yahoo will delete the contents of Yahoo-groups in 14/12.

Only for a very short time it will be possible to send messages in the usual way to the Yahoo-Group.

I already made a new group in Groups.io, but it will take some time before our old messages will be migrated. I will keep you informed.

It can happen that there will a period we will not be able to send messages to our group until the migration has been done. I am dependent on Groups.io in this.

I will put the archives in Public Domain. This means that I give the permission to anyone (person or institution) to host the archives in a web site.

You can download the archives by:

1. <http://www.httrack.com/>

or

2. <http://www.personalgroupware.com/downloads.htm>

(Read the details before starting to download)

Best regards,
Chris van Tienhoven

p.s. this message came to you by regular email to
quadri-figures-group@yahoogroups.com

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3891

Date: 01/11/2019 10:04:01

From: eckart_schmidt@t-online.de

Subject: New Quartic for the 5P of the QA-Cu7-triple Points

Dear Bernard, dear Tsihong Lau,

I try once more a QFG-message,
... for I have an interesting assumption
wrt Bernard's twin 5P
... for the QA-Cu7-triple points and their 5P-CSC-images:

Consider first an arbitrary $5P = P_1P_2P_3P_4P_5$
... and its 5P-CSC- image $P_1'P_2'P_3'P_4'P_5'$,
... further the 5L with lines P_iP_i'
and its inscribed conic 5L-s-Co1,
... then the 5P-CSC-partner on the tangents of 5L-s-Co1
... will give a curious sextic:
... 5P-CSC-invariant,
... bearing the 10 vertices of the twin 5P
... and 5P-s-P6 and the fixed points of 5P-CSC.

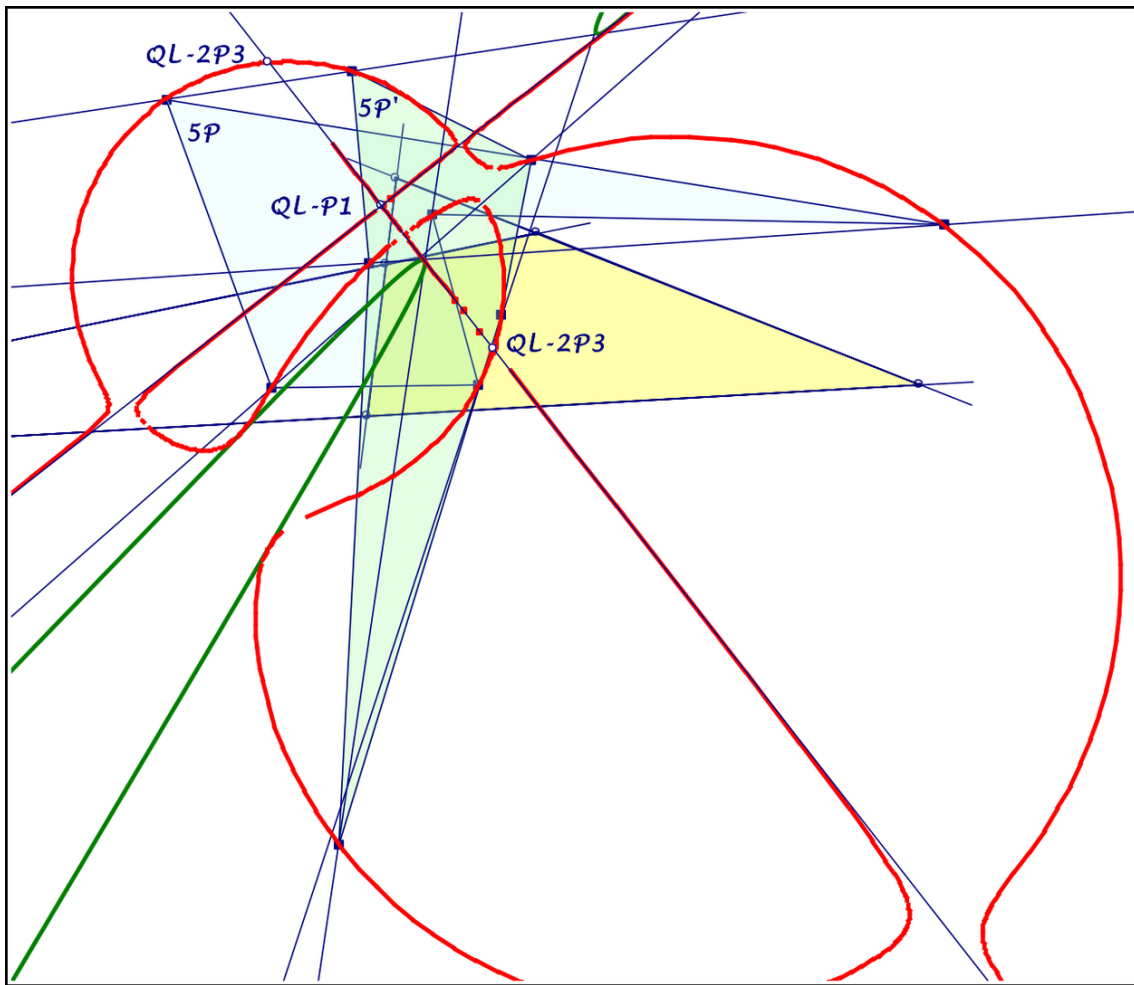
For the 5P of the QA-Cu7-triple points of a QL
... this sextic degenerates to the Steiner axes
... and a quartic:
5P-CSC-invariant,
... bearing the 10 vertices of the twin 5P and QL-2P3.

This shows once more,
... that the QA-Cu7-triple points give a special 5P.

A construction has the handycap
... of approximate QA-Cu7-triple points for a QL,
... so the attached drawing is imprecised,
... but several times repeated.

Best regards Eckart

PS. Tsihong Lau mentioned in a private mail
... such a quartic in general,
... but I don't know his construction.
If this mail doesn't appear in QFG,
... I shall send it to your private address.



2019-11-01.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3892
Date: 03/11/2019 11:26:31
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

Dear Chris,

the circle 5P-s-Cix in #3773 bears 5P-n-P2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3893
Date: 03/11/2019 12:23:01
From: eckart_schmidt@t-online.de
Subject: 5P-s-Cix

Dear Chris,

excuse, please forget my last message,
... it doesn't hold.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3894

Date: 04/11/2019 9:04:13

From: eckart_schmidt@t-online.de

Subject: Quartics for 3 Lines with a CSC-transformation

Dear Bernard, dear Chris,

there is a construction concept for CSC-invariant quartics,
... starting with 3 lines and a CSC-transformation,
... already used in #1354, #1365, #1376.

Consider a CSC-transformation
... with its Steiner axes and three other lines,
... which give a 5L with inconic 5L-s-Co1:
... CSC-partner on tangents at this conic give the quartic,
... evidently CSC-invariant.

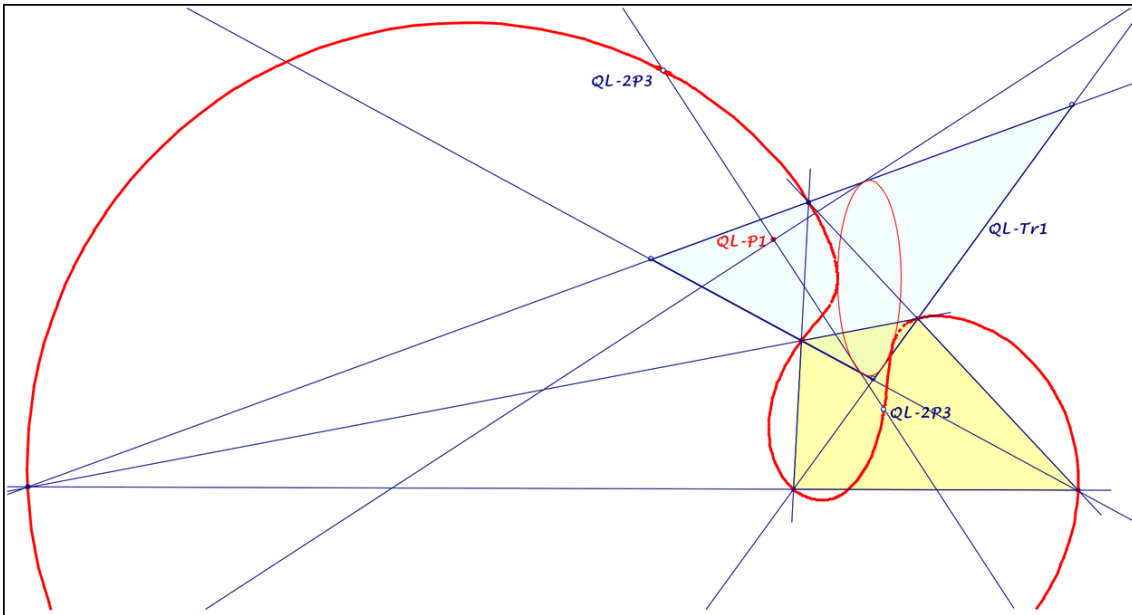
Examples:

For a QL and its CSC (see attached file)
... wrt the Steiner axes and the three diagonals
... we get a CSC-invariant circumquartic of the QL.

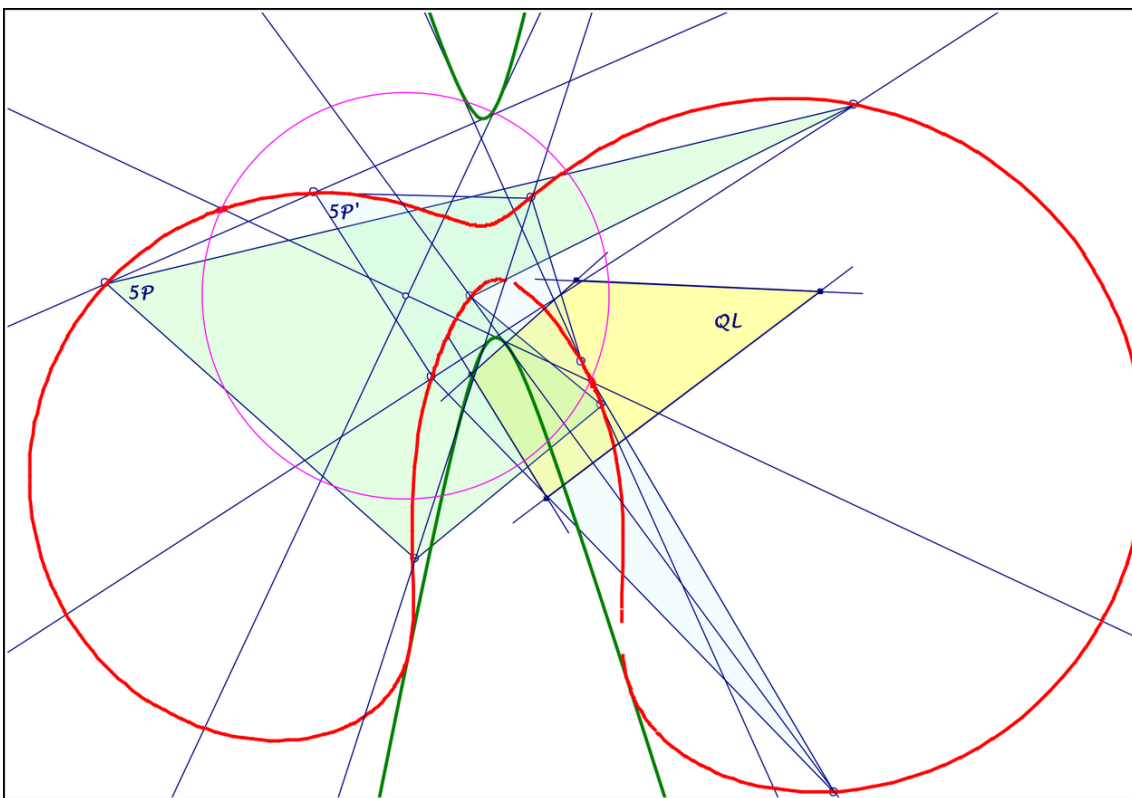
For the 5P = P1P2P3P4P5 of the QA-Cu7-triple points,
... its 5P-CSC with Steiner axes
... and any three of the 5 lines $P_i.CSC(P_i)$
... we get a 5P-circumquartic, evidently bearing $CSC(P_i)$.

The 2nd example allows a better construction
... for the quartic in #3891 (see attached file).

Best regards Eckart



2019-11-03.pdf



2019-11-01a.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #3895
Date: 07/11/2019 9:34:53
From: eckart_schmidt@t-online.de
Subject: Special QA-Circumcubic

Dear Bernard, dear Chris,

in QL-geometry midpoints of QL-Tf1-partner on QL-Cu1 give QL-L1,
... here an analog question in QA-geometry:
... What about the midpoints of QA-Tf2-partner on QA-Cu1?
QA-Cu1 is QA-Tf2-invariant,
... QA-Tf2-partner on QA-Cu1 are collinear with QA-P4.

(1) The midpoints of QA-Tf2-partner on lines through QA-P4
... give parts of a QA-circumcubic QU.
(2) QU is the locus of points X with $cb(X) = QA-P4$,
... cb Cayley-Bacharach transformation
... wrt the QA-vertices, QA-P4 and the circular points.
(3) QU is the QA-Tf16-image
... of a QA-circumconic through QA-P4.
(4) QU is the QA-Tf4-image of a conic
... through QA-Tf4 of the QA-vertices
... ... and QA-Tf4 of the midpoint of QA-P4.41.
(5) Finally a very curious way, to get QU:
... Consider circles round QA-P4
... and their cb -image, a curve of higher degree:
... The limit of the curve for radius zero is QU.

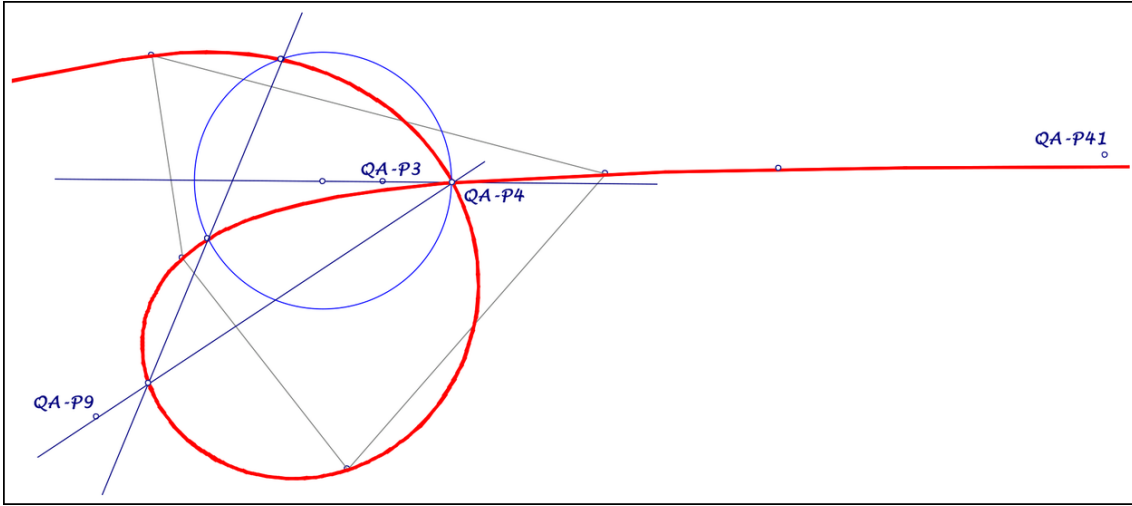
Properties:

QU bears the QA-vertices,
... QA-P4 as double point and the midpoint of QA-P4.41 ...
QU has as asymptote QA-P3.4.
QU has tangents in QA-P4,
... which are parallel to the asymptotes of the QA-circumconic
through QA-P4.

A special point (see attached file):

... Circles through QA-P4, centered on QA-P3.4, intersect the
cubic
... collinear with the midpoint of QA-Tf2-partner on QA-P4.9.

Best regards Eckart



2019-11-07.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3896
Date: 08/11/2019 11:13:52
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 / Co-Tf3

Dear Benedetto, dear Chris,

an unexpected property, perhaps already mentioned:
For all QA-circumconics holds:
 $\text{Co-Tf3}(\text{QA-P2}) = \text{QA-P4}$.
Or:
5P-s-Tf3 maps QA-P4 of a 5P-QA-version
... to QA-P2 of this 5P-QA-version.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3897
Date: 09/11/2019 5:04:13
From: Benedetto Scimemi
Subject: 5P-s-Tf3 / Co-Tf3

Dear Eckart, thanks for your last message. In fact, the
"unexpected" property you mention is reported in Chris's EQF,
section INFORMATIONS ON CONICS , page C0-Tf3.

.....

4. For *any* quadrangle Q inscribed in *any* conic C0, the
C0-Tf3 image of QA-P2 is QA-P4

Also, in the final table:

Any Conic circumscribing Reference Quadrangle

QA-P2 (<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p2>)

QA-P4 (<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4>)

.....

Warning: In a Pentangle the Scimemi Transformation C0-Tf3 wrt
the circumscribed conic is identical to the inverse of the
5P-s-Tf3 Transformation.

This confusion of names should have been avoided, but now it is
difficult to change.

Best regards. Benedetto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3898
Date: 10/11/2019 2:43:18
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 / Co-Tf3

Dear Benedetto,

thanks for your references and excuse,
... that I read careless your Co-Tf3 article.
It is unusual to make geometry only with a reference conic
... but interesting:
Main property of Co-Tf3 / Co-Tf3inv is,
... that they map lines to lines, circles to circles,
... .. conics to conics, inclusive their centers.

A short excursion:
Consider a reference conic, centered in Z
... and circles CI, centered in P:
... the images of CI wrt Co-Tf3 and Co-Tf3inv
... are circles CI1, CI2,
... with centers on the reflection of PZ in the main axis.
Varying the radius of CI,
... The radical center of CI, CI1, CI2 give a line through Z
... and the intersections of CI1 and CI2 give a circle through Z.
What about this circle, if P is a focus of the reference conic
Co?

Best regards Eckart

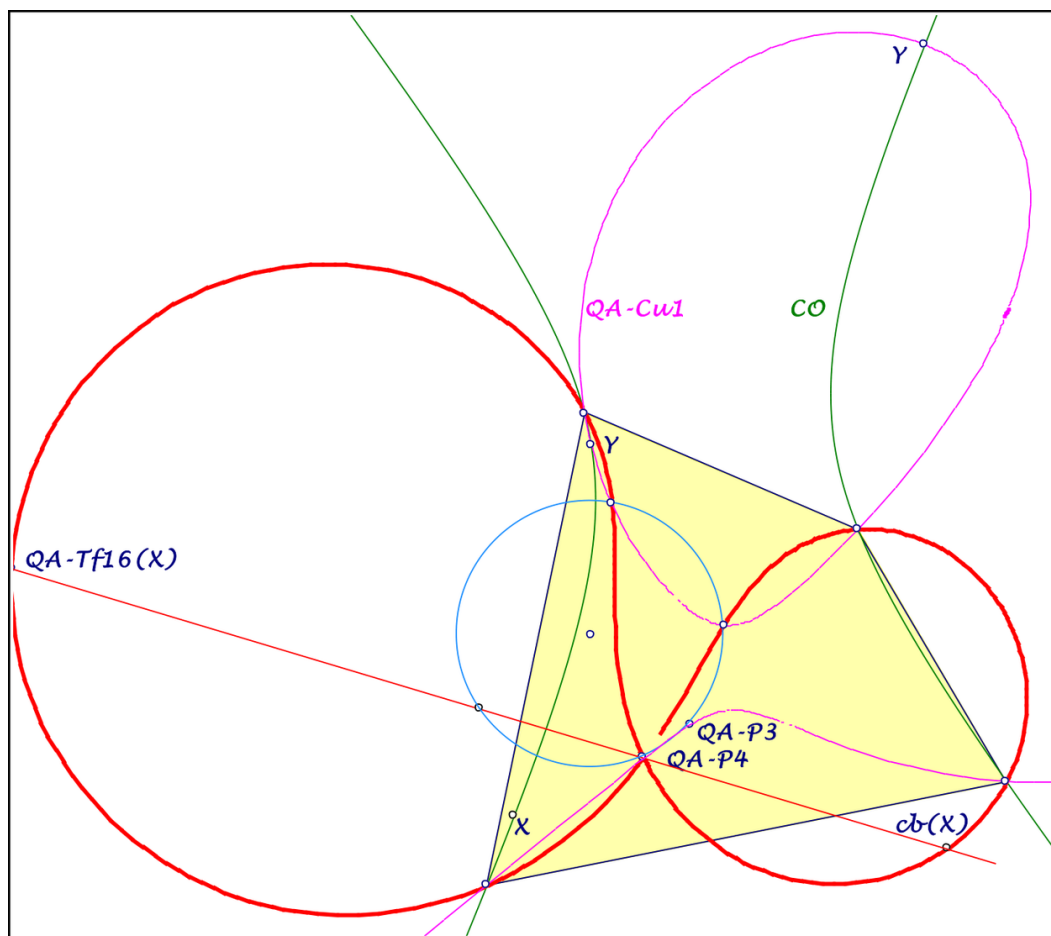
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3899
Date: 11/11/2019 12:10:19
From: eckart_schmidt@t-online.de
Subject: Special QA-Circumquartics

Dear Bernard, dear Chris,

this is a generalization of #3895:
Consider a QA and a circumconic CO ,
... QA-Tf16 of CO gives a QA-circumquartic
... with double point in QA-P4.
The same quartic we get as cb-image of CO ,
... cb Cayley-Bacharach transformation
... wrt QA-vertices, QA-P4 and the circular points.
The QA-Tf16- and the cb-image of a CO -point X
... are collinear with QA-P4,
... their midpoints give a circle through QA-P3, QA-P4
... and QA-Tf16 of further intersections Y of CO and QA-Cu1
... with Y , QA-Tf16(Y), QA-P3 collinear (see attached file).

Best regards Eckart



2019-11-11.pdf

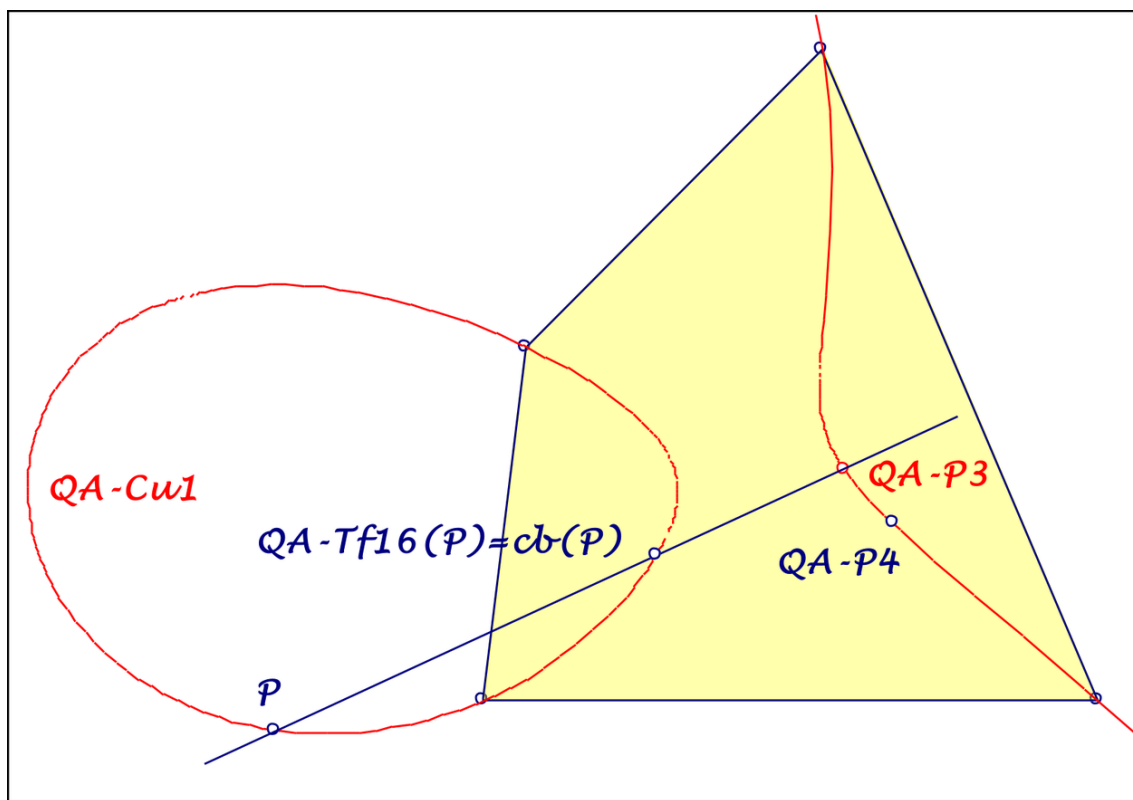
Message: #3900
Date: 11/11/2019 12:56:18
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

Dear Bernard, dear Chris,

for points P on QA-Cu1 holds:
... QA-Tf16(P) = $cb(P)$ on QA-Cu1,
... collinear with P and QA-P3.

Best regards Eckart

PS: cb = Cayley-Bacharach transformation
... wrt QA-vertices, QA-P4 and circular points.



2019-11-11a.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3901
Date: 14/11/2019 11:32:16
From: bernard.keizer
Subject: Quartics for 3 Lines with a CSC-transformation

Dear Eckart,
I wasn't home for a few weeks.
I was (and stil I'm) very interested by your CSC invariant quartics.
We can find easily a few other in the QL environment : for example with the 2 axes of the CSC and the 3 lines QA-P1QA-P4 or the 2 axes of the CSC centered in CSC(T) and the 3 lines QA-P4QA-P41.
The main property is following : is it true that 5 points X_i are the 5 triple points of the 3 QA-Cu7 of a QL if and only if the 5 lines X_iY_i are tangent to a conic also tangent to the 2 axes of the CSC of the 5 points ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3902
Date: 17/11/2019 10:42:46
From: bernard.keizer
Subject: Quartics for 3 Lines with a CSC-transformation

Dear Eckart,
Conversely, a CSC and a conic give with your construction a CSC invariant quartic.
Any 3 points on the quartic define with the CSC the 2 other points (one of your old constructions).
The 2 last points are generally not on the quartic.
If they are one the quartic, the 5 points are 5 QL triple points.
5 points on the quartic are QL triple points if and only if their CSC is the quartic's CSC ; the CSC of the 5 points lie of course on the quartic and the 5 lines joining the 5 points and their CSC are tangent to the conic.
In this case, what can be said about the 5 contact points of the 5 tangents ?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3903

Date: 17/11/2019 9:32:55

From: chris.vantienhoven

Subject: LAST MESSAGE NEW START WITH A NEW GROUP

Dear friends,

The time of our Yahoo Quadri-Figures-Group has come to an end.

Fortunately, there is a good alternative.

I opened a new group at Groups.io with the name

Quadri-and-Poly-Geometry.

This provider gives a lot of new features as you will discover soon.

Alas it was not possible to migrate the Yahoo-messages to the new Group preserving all message-numbers. Since we use these numbers a lot, I decided not to migrate, because all our references will become a mess.

In order to still be able to read old QFG-messages I will place an archive of all our QFG-messages at my site soon.

The address of our new group is

<https://groups.io/g/Quadri-and-Poly-Geometry>.

I will send all members a personal invitation to join the new group.

I am sure we will have a wonderful time together with the new group.

Best regards,

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3904

Date: 17/11/2019 9:38:55

From: Wilson Logan

Subject: LAST MESSAGE NEW START WITH A NEW GROUP

Maybe this can help:

I have developed a utility to load Yahoo Groups messages to groups.io
Currently it loads messages and attachments.
While at the moment it doesn't do Files & Photos uploading, that will be added as a feature.
Here is a demo video of the uploader as it is now.
<https://youtu.be/zZCFdEYiRK4> (<https://youtu.be/zZCFdEYiRK4>)

Cheers,
Wilson Logan.

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: 17 November 2019 20:33
> To: Quadri-Figures-Group@yahoogroups.com
> Subject: [Quadri-Figures-Group] LAST MESSAGE & NEW START WITH
A NEW GROUP
> The time of our Yahoo Quadri-Figures-Group has come to an end.
> Fortunately, there is a good alternative.
> I opened a new group at Groups.io with the name
Quadri-and-Poly-Geometry.
> This provider gives a lot of new features as you will discover
soon.
> Alas it was not possible to migrate the Yahoo-messages to the
new Group preserving all message-numbers.
> Since we use these numbers a lot, I decided not to migrate,
because all our references will become a mess.
> In order to still be able to read old QFG-messages
> I will place an archive of all our QFG-messages at my site
soon.
> The address of our new group is
<https://groups.io/g/Quadri-and-Poly-Geometry>.
> I will send all members a personal invitation to join the new
group.
> I am sure we will have a wonderful time together with the new
group.
> Best regards,
> Chris van Tienhoven

Message: #3905
Date: 17/11/2019 10:11:06
From: chris.vantienhoven
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

Dear Wilson,

I will have a look at it.
When your utility preserves message-numbers we maybe can migrate the Yahoo-messages to a separate group at Groups.io so that we can look up older messages easily.
Either way, the new group can start.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #3906
Date: 17/11/2019 10:35:08
From: Wilson Logan
Subject: LAST MESSAGE NEW START WITH A NEW GROUP

Yes of course.
I would recommend starting a sub-group called "Archive" or similar and load the messages there.

Cheers,
Wilson.

> From: Quadri-Figures-Group@yahoogroups.com
[mailto:Quadri-Figures-Group@yahoogroups.com]
> Sent: 17 November 2019 21:11
> To: Quadri-Figures-Group@yahoogroups.com
> Subject: RE: [Quadri-Figures-Group] LAST MESSAGE & NEW START
WITH A NEW GROUP
> Dear Wilson,
> I will have a look at it.
> When your utility preserves message-numbers we maybe can
migrate the Yahoo-messages
> to a separate group at Groups.io so that we can look up older
messages easily.
> Either way, the new group can start.
> Best regards,
> Chris

[← Previous](#) [↔ Message Index](#) [↑ Subjects](#)

5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

5P-s-P1:

[#3342](#) [#3346](#) [#3348](#) [#3586](#) [#3621](#) [#3649](#) [#3789](#) [#3797](#) [#3798](#) [#3885](#)

5P-s-P4:

[#3333](#) [#3334](#) [#3346](#) [#3347](#) [#3348](#) [#3403](#) [#3404](#) [#3405](#) [#3412](#) [#3420](#) [#3424](#)
[#3425](#) [#3435](#) [#3442](#) [#3493](#) [#3528](#) [#3548](#) [#3555](#) [#3571](#) [#3575](#) [#3579](#) [#3581](#)
[#3589](#) [#3594](#) [#3599](#) [#3609](#) [#3619](#) [#3620](#) [#3621](#) [#3622](#) [#3626](#) [#3628](#) [#3675](#)
[#3679](#) [#3720](#) [#3723](#) [#3725](#) [#3762](#) [#3784](#) [#3789](#) [#3792](#) [#3804](#) [#3808](#) [#3883](#)

5P-s-P5:

[#3576](#) [#3578](#) [#3579](#) [#3581](#) [#3589](#) [#3598](#) [#3599](#) [#3619](#) [#3620](#) [#3628](#) [#3675](#)
[#3679](#) [#3683](#) [#3690](#) [#3705](#) [#3762](#) [#3784](#) [#3789](#) [#3792](#) [#3808](#) [#3860](#) [#3862](#)
[#3883](#) [#3886](#)

6P-s-P1:

[#3576](#) [#3578](#) [#3586](#) [#3860](#) [#3862](#) [#3886](#)

CSC:

[#3300](#) [#3301](#) [#3307](#) [#3308](#) [#3312](#) [#3315](#) [#3317](#) [#3318](#) [#3319](#) [#3321](#) [#3325](#)
[#3326](#) [#3327](#) [#3328](#) [#3329](#) [#3334](#) [#3335](#) [#3337](#) [#3338](#) [#3343](#) [#3347](#) [#3350](#)
[#3352](#) [#3359](#) [#3360](#) [#3361](#) [#3362](#) [#3363](#) [#3364](#) [#3365](#) [#3369](#) [#3370](#) [#3371](#)
[#3372](#) [#3373](#) [#3378](#) [#3397](#) [#3400](#) [#3406](#) [#3407](#) [#3409](#) [#3410](#) [#3423](#) [#3427](#)
[#3428](#) [#3430](#) [#3431](#) [#3435](#) [#3437](#) [#3438](#) [#3439](#) [#3440](#) [#3490](#) [#3495](#) [#3513](#)
[#3515](#) [#3516](#) [#3517](#) [#3519](#) [#3522](#) [#3524](#) [#3526](#) [#3533](#) [#3534](#) [#3535](#) [#3536](#)
[#3544](#) [#3545](#) [#3546](#) [#3548](#) [#3549](#) [#3550](#) [#3553](#) [#3555](#) [#3557](#) [#3559](#) [#3562](#)
[#3569](#) [#3571](#) [#3572](#) [#3575](#) [#3583](#) [#3584](#) [#3588](#) [#3590](#) [#3591](#) [#3592](#) [#3593](#)
[#3595](#) [#3602](#) [#3603](#) [#3604](#) [#3606](#) [#3607](#) [#3608](#) [#3609](#) [#3610](#) [#3611](#) [#3613](#)
[#3614](#) [#3615](#) [#3616](#) [#3617](#) [#3618](#) [#3620](#) [#3623](#) [#3627](#) [#3628](#) [#3631](#) [#3634](#)
[#3648](#) [#3660](#) [#3663](#) [#3670](#) [#3674](#) [#3675](#) [#3676](#) [#3679](#) [#3680](#) [#3690](#) [#3692](#)
[#3698](#) [#3703](#) [#3705](#) [#3709](#) [#3720](#) [#3721](#) [#3757](#) [#3758](#) [#3762](#) [#3764](#) [#3765](#)
[#3766](#) [#3769](#) [#3771](#) [#3775](#) [#3777](#) [#3778](#) [#3779](#) [#3780](#) [#3781](#) [#3782](#) [#3788](#)
[#3790](#) [#3791](#) [#3792](#) [#3793](#) [#3799](#) [#3858](#) [#3883](#) [#3885](#) [#3888](#) [#3891](#) [#3894](#)
[#3901](#) [#3902](#)

Cayley-Bacharach, CB:

[#3300](#) [#3301](#) [#3319](#) [#3320](#) [#3322](#) [#3323](#) [#3371](#) [#3385](#) [#3399](#) [#3401](#) [#3402](#)
[#3403](#) [#3404](#) [#3405](#) [#3411](#) [#3412](#) [#3414](#) [#3420](#) [#3423](#) [#3427](#) [#3434](#) [#3435](#)
[#3436](#) [#3437](#) [#3438](#) [#3442](#) [#3493](#) [#3495](#) [#3522](#) [#3526](#) [#3528](#) [#3533](#) [#3534](#)
[#3535](#) [#3546](#) [#3553](#) [#3559](#) [#3569](#) [#3571](#) [#3572](#) [#3575](#) [#3584](#) [#3585](#) [#3592](#)
[#3594](#) [#3598](#) [#3600](#) [#3601](#) [#3602](#) [#3609](#) [#3613](#) [#3621](#) [#3622](#) [#3623](#) [#3625](#)
[#3626](#) [#3632](#) [#3635](#) [#3638](#) [#3645](#) [#3647](#) [#3808](#) [#3819](#) [#3863](#) [#3883](#) [#3884](#)
[#3895](#) [#3899](#) [#3900](#)

Cotterill:

[#3301](#)

Isoconjugation, isoconjugate:

[#3301](#) [#3307](#) [#3314](#) [#3341](#) [#3402](#) [#3404](#) [#3440](#) [#3566](#) [#3606](#) [#3607](#) [#3636](#)
[#3694](#) [#3704](#) [#3792](#) [#3842](#) [#3844](#) [#3848](#) [#3849](#) [#3850](#) [#3852](#)

Isocubics:

#3307 #3341 #3402 #3434 #3437 #3517 #3518 #3519 #3693 #3699 #3704
#3848

Miquel-point, Miquel Point:

#3307 #3419 #3453 #3454 #3457 #3463 #3465 #3469 #3470 #3491 #3706
#3707 #3828

Miquel-triangle, Miquel Triangle:

#3314 #3422 #3533

Newton Line:

#3303 #3306 #3361 #3381 #3382 #3387 #3413 #3441 #3448 #3451 #3452
#3470 #3490 #3515 #3546 #3615 #3660 #3684 #3709 #3710 #3721 #3735
#3741 #3742 #3743 #3753 #3754 #3755 #3756 #3757 #3759 #3761 #3775
#3780 #3788 #3823 #3824 #3825 #3828

Pivotal Isocubic:

#3301 #3307 #3341 #3350 #3385 #3402 #3404 #3434 #3437 #3517 #3518
#3603 #3607 #3636 #3693 #3694 #3699 #3701 #3704 #3848

QA-Cu1:

#3300 #3301 #3314 #3316 #3360 #3361 #3362 #3363 #3365 #3366 #3368
#3375 #3401 #3404 #3405 #3422 #3435 #3533 #3782 #3795 #3796 #3805
#3806 #3807 #3895 #3899 #3900

QA-Cu7:

#3300 #3301 #3307 #3308 #3312 #3314 #3315 #3316 #3318 #3319 #3321
#3333 #3334 #3335 #3336 #3340 #3341 #3344 #3345 #3350 #3351 #3352
#3358 #3359 #3360 #3361 #3362 #3363 #3364 #3365 #3366 #3367 #3368
#3369 #3370 #3375 #3378 #3389 #3397 #3399 #3400 #3401 #3403 #3406
#3407 #3410 #3411 #3412 #3414 #3415 #3422 #3430 #3437 #3438 #3445
#3493 #3494 #3495 #3516 #3519 #3522 #3528 #3532 #3533 #3540 #3541
#3544 #3548 #3549 #3555 #3567 #3568 #3571 #3573 #3575 #3579 #3581
#3583 #3588 #3589 #3590 #3599 #3607 #3618 #3622 #3628 #3640 #3679
#3680 #3690 #3703 #3705 #3709 #3722 #3757 #3760 #3762 #3784 #3789
#3883 #3891 #3894 #3901

QA-P3:

#3300 #3416 #3417 #3419 #3422 #3424 #3435 #3454 #3498 #3499 #3500
#3525 #3572 #3661 #3682 #3686 #3723 #3725 #3726 #3734 #3736 #3739
#3740 #3744 #3746 #3750 #3751 #3752 #3795 #3796 #3805 #3806 #3807
#3809 #3816 #3895 #3899 #3900

QA-P4:

#3300 #3301 #3308 #3312 #3314 #3319 #3320 #3321 #3324 #3325 #3326
#3329 #3334 #3335 #3341 #3344 #3349 #3351 #3352 #3353 #3358 #3359
#3361 #3362 #3363 #3364 #3365 #3366 #3368 #3369 #3370 #3371 #3374
#3386 #3388 #3390 #3397 #3407 #3409 #3410 #3414 #3422 #3424 #3427
#3430 #3435 #3437 #3444 #3445 #3447 #3449 #3450 #3456 #3472 #3474
#3484 #3486 #3511 #3512 #3518 #3525 #3530 #3531 #3534 #3535 #3539
#3546 #3553 #3555 #3575 #3577 #3578 #3584 #3590 #3591 #3595 #3599
#3603 #3605 #3615 #3616 #3620 #3623 #3628 #3634 #3660 #3668 #3670

#3676 #3679 #3680 #3683 #3684 #3690 #3709 #3720 #3721 #3722 #3723
#3725 #3726 #3727 #3729 #3730 #3740 #3746 #3751 #3757 #3773 #3781
#3783 #3787 #3788 #3809 #3816 #3836 #3838 #3872 #3876 #3895 #3896
#3897 #3899 #3900

QA-P41:

#3300 #3308 #3314 #3321 #3326 #3344 #3352 #3353 #3355 #3358 #3361
#3362 #3363 #3368 #3369 #3400 #3409 #3435 #3503 #3534 #3535 #3575
#3591 #3599 #3603 #3604 #3607 #3610 #3613 #3614 #3615 #3620 #3623
#3628 #3634 #3660 #3662 #3667 #3668 #3670 #3674 #3676 #3679 #3709

QA-Tf16:

#3313 #3513 #3516 #3588 #3895 #3899 #3900

QL-Cu1:

#3300 #3301 #3308 #3314 #3316 #3340 #3341 #3343 #3354 #3357 #3358
#3360 #3368 #3375 #3400 #3401 #3405 #3533 #3615 #3620 #3660 #3670
#3674 #3675 #3679 #3698 #3721 #3762 #3765 #3781 #3787 #3788 #3858
#3895

Quartic:

#3312 #3315 #3513 #3515 #3516 #3517 #3546 #3599 #3601 #3602 #3603
#3605 #3606 #3607 #3608 #3611 #3615 #3616 #3620 #3622 #3626 #3660
#3668 #3675 #3679 #3680 #3683 #3690 #3698 #3700 #3705 #3709 #3710
#3719 #3720 #3721 #3722 #3723 #3725 #3726 #3727 #3732 #3733 #3762
#3764 #3765 #3766 #3775 #3781 #3785 #3788 #3789 #3790 #3791 #3885
#3887 #3888 #3891 #3894 #3899 #3902

Reye configuration:

#3437

Simson line:

#3736 #3799

Steiner Axes:

#3439 #3440 #3441 #3513 #3541 #3618 #3891 #3894

Steiner Line:

#3329 #3413 #3457 #3463 #3465 #3467 #3469 #3470 #3736 #3740 #3793
#3799 #3828

Tangential:

#3300 #3301 #3308 #3349 #3350 #3352 #3365 #3369 #3371 #3373 #3374
#3378 #3385 #3403 #3404 #3405 #3412 #3440 #3490 #3575 #3609 #3632
#3645

Van Rees:

#3340 #3354 #3356 #3358 #3360 #3361 #3362 #3369 #3610 #3617 #3660
#3670 #3684 #3709 #3710 #3757 #3775 #3888

circular cubic:

#3301 #3321 #3337 #3338 #3347 #3354 #3360 #3361 #3398 #3495 #3526
#3546 #3548 #3559 #3645 #3775 #3888

circular points:

[#3321](#) [#3322](#) [#3323](#) [#3361](#) [#3365](#) [#3394](#) [#3401](#) [#3402](#) [#3403](#) [#3404](#) [#3405](#)
[#3411](#) [#3412](#) [#3414](#) [#3420](#) [#3423](#) [#3430](#) [#3435](#) [#3436](#) [#3438](#) [#3442](#) [#3493](#)
[#3522](#) [#3528](#) [#3533](#) [#3553](#) [#3571](#) [#3575](#) [#3594](#) [#3609](#) [#3622](#) [#3684](#) [#3709](#)
[#3808](#) [#3819](#) [#3883](#) [#3895](#) [#3899](#) [#3900](#)

[↔ Table of Contents](#) [↔ Message Index](#)

6 Colophon

Sources & Contact

Web address (QPG Forum): <https://groups.io/g/Quadri-and-Poly-Geometry>

EPG Encyclopedia (content reference): <https://www.chrisvantienhoven.nl>

Editorial correspondence: van10hoven@gmail.com

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- Volume 3 (2015), messages #917–#1402
- Volume 4 (2016), messages #1403–#2169
- Volume 5 (2017), messages #2170–#2799
- Volume 6 (2018), messages #2780–#3299
- Volume 7 (Jan. 2019–Oct. 2019), messages #3300–#3906

Successor Journal:

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Volumes of the successor journal:

- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60
- Volume 2 (2020), messages #61–#630
- Volume 3 (2021), messages #631–#1294
- Volume 4 (2022), messages #1295–#1544
- Volume 5 (2023), messages #1545–#2051
- Volume 6 (2024), messages #2052–#2559
- Volume 7 (2025), messages #2560–#2897

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