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Quadri- and Poly-Geometry Group
2020

Digital Edition

Chris van Tienhoven et al.

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Messages #61 - #630

Contents

1 Introduction

This journal is a compilation of messages from the **Quadri- and Poly-Geometry (QPG)** forum, where mathematicians and geometry enthusiasts exchange ideas on the properties of **quadrilaterals**, **polygons**, and **curves of nth degree**. The discussions cover a wide range of topics, from classical geometric theorems to new discoveries and insights.

The origins of this journal trace back to the Quadri Figures Group (QFG, available at https://groups.io/g/Quadri-Figures-Group), which was active from 2013 until November 2019. In November 2019, the forum transitioned into the Quadri- and Poly-Geometry Group (QPG, available at https://groups.io/g/Quadri-and-Poly-Geometry) forum, which continues to facilitate discussions on quadrilaterals, polygons, and related topics. Over the years, these forums have evolved into valuable resources for exploring both well-established results and novel perspectives in geometry. For both forums, an **annual record of all incoming messages** is compiled in this journal.

This journal is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing users to quickly jump between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting these forum messages, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- benedetto.scimemi@gmail.com
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- Dao Thanh Oai
- Eckart Schmidt
- Francisco Javier García Capitán
- Michael de Villiers
- Stanley Rabinowitz
- Tran Quang Hung
- Vu Thanh Tung

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

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- **Francisco Javier García Capitán**
  email: garciacapitan@gmail.com:
  #156

- **Michael de Villiers**
  email: profmd1@mweb.co.za:
  #505

- **Stanley Rabinowitz**
  email: Stan.Rabinowitz@comcast.net:
  #297 #301 #459 #564 #619 #621

- **Tran Quang Hung**
  email: analgeomatica@gmail.com:
  #534 #536 #538 #539 #545 #546 #549 #553 #561 #588 #589 #592 #594 #606

- **Vu Thanh Tung**
  email: tungvtt@gmail.com:
  #225 #229 #243 #244 #246 #247 #249 #250 #252 #256 #298 #336 #339 #342 #345 #597 #600 #601

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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Benedetto Scimemi (1938–2023)

Location
Italy

Year of Birth / Generation
1938–2023.

Short Biography
Benedetto Scimemi was born in Padua in 1938. Although he graduated in Physics, he devoted most of his research life to Mathematics. He served as Professor of Algebra and later of Complementary Mathematics at the University of Padua, where he cultivated a deep interest in the foundational and structural aspects of elementary mathematics. He maintained a strong commitment to mathematics education and the training of future secondary-school teachers, serving as President of the Italian Commission for Mathematics Education (CIIM) and as Vice-President of the Italian Mathematical Union (UMI). A brief memorial overview of his life and contributions can be found at the Italian Mathematical Union (UMI): umi.dm.unibo.it/2023/06/13/scomparsa-del-professor-benedetto-scimemi

Themes and Interests

- Classical and modern geometry
- Mathematical exposition and education
- Intersections of mathematics, culture, and the arts
- Music and the mathematics of J. S. Bach

Selected Publications and Academic Work

- A selection of Benedetto Scimemi's publications is available on his Academia.edu profile: independent.academia.edu/BenedettoScimemi
- One of Benedetto Scimemi's notable contributions regarding EPG is the paper *Central Points of the Complete Quadrangle*, in which he investigates the geometry of the complete quadrangle and the special points that arise from its classical configuration. The work provides clear constructions, and insightful commentary on the relationships between central points, diagonal triangles, and perspectivities within the quadrangle: academia.edu/86588221/Central_Points_of_the_Complete_Quadrangle

Additional Remarks

- Benedetto Scimemi had a deep interest in the relationship between mathematics and music, particularly in the works of J. S. Bach. An example of this aspect of his intellectual life can be found here: iicdublino.esteri.it/.../musica-e-matematica-in-j-s-bach-2

- In February 2005, at a special meeting in Bloomington held in honour of Douglas Hofstadter's 60th birthday (the "A5 Meeting", named after the alternating group of order 60), Benedetto presented a pair of his geometric results together with a set of transparencies. Although several well-known triangle geometers such as Clark Kimberling were present, interest in quadrilaterals and pentagons was still minimal at that time. One of the constructions Benedetto presented in Bloomington — originally thought to be a 5P-transformation — was later analysed within the QFG forum and shown to be a rare example of a *conical transformation*. This transformation was subsequently named after him: the *Co-Tf3 Scimemi Transformation*. It was later developed in full detail within the EPG in collaboration with the EPG author. See CO-Tf3 in EPG.

- Benedetto was an active participant in the QFG and QPG forums during the years 2015–2020, contributing insights, discussions, and geometric ideas that influenced several later developments.

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Francisco Javier García Capitán

Location
Priego de Córdoba, Andalucía, Spain.

Year of Birth / Generation
1963.

Short Biography
Francisco Javier García Capitán is a mathematician and long-time secondary school teacher with a strong interest in geometry, elementary mathematics, and computational approaches. He is an active explorer of barycentric coordinates and the author of *Baricentricas.nb*. His work bridges classical geometric insight with modern computational tools.

Themes and Interests

- Elementary mathematics
- Geometry
- Mathematica and hobby programming
- Barycentric coordinates

Selected Publications
International Journal of Geometry

- (with Paul Yiu) *Three mutually tangent congruent circles…*, 5 (2016), 15–18.
- *A structure on the circumcircle*, 10 (2021), 71–83.
- *Infinite points and isogonal conjugate*, 12 (2023), 127–134.
- *Isotomic conjugate and parallelism*, 12 (2023), 89–100.

Forum Geometricorum

- *Means as chords*, 8 (2008), 99–101.
- *Trilinear polars of Brocardians*, 9 (2009), 297–300.
- *Collinearity of the first trisection points…*, 11 (2011), 217–221.
- (with Ehrmann & Myakishev) *Construction of circles…*, 11 (2011), 261–268.
- (with Dergiades & Lim) *On six circumcenters…*, 11 (2011), 269–275.
- *Some simple results on cevian quotients*, 13 (2013), 227–231.
- *A simple construction of an inconic*, 14 (2014), 387–388.
- *Lemniscates and a locus…*, 15 (2015), 123–125.
- *Another construction of the Simson lines…*, 15 (2015), 173–176.

- *Locus of centroids of similar inscribed triangles*, 16 (2016), 257–267.
- *A Family of Triangles...*, 18 (2018), 79–82.

Additional Remarks

- Website: www.garciacapitan.epizy.com
- Blog: www.garciacapitan.blogspot.com

Stanley Rabinowitz

Location
Living in New Hampshire, USA.

Year of Birth / Generation
1947 (Baby Boomer).

Short Biography
Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions
Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, International Journal of Computer Discovered Mathematics 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, International Journal of Computer Discovered Mathematics 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 214–287.

Additional Remarks
Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

3 Subjects

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4 Messages

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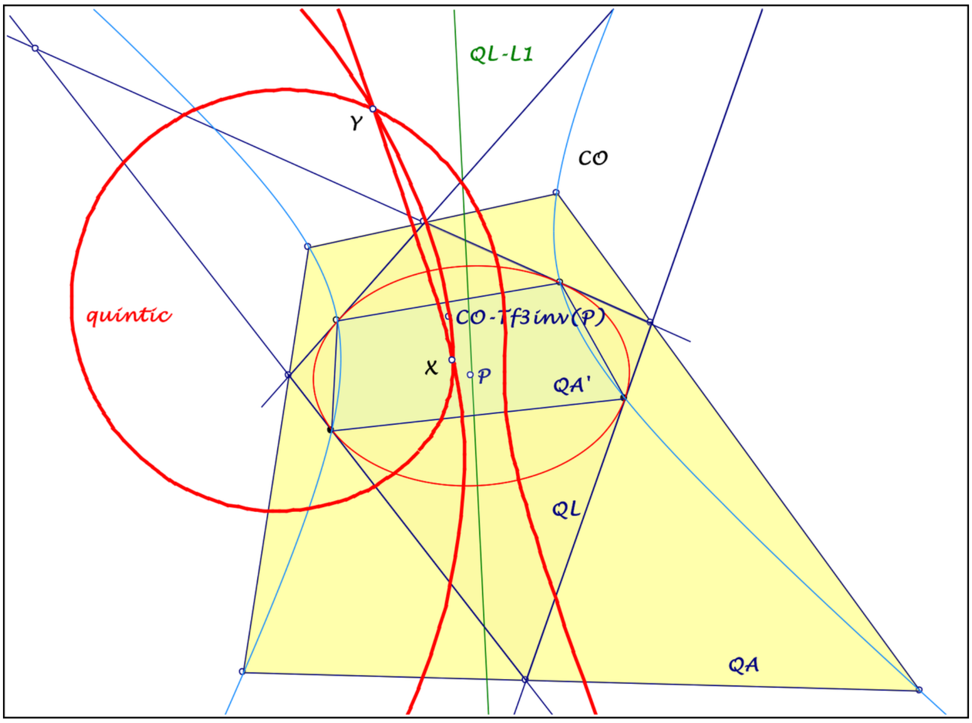
4.2 Messages

Message: #61
Date: 2020-01-01
From: eckart_schmidt@t-online.de
Subject: QA/QL/CO-Excursion

```
Dear Benedetto, dear Bernard, dear Chris,

may I invite you to a curious excursion,
... perhaps someone can lighten the background:
Let us start with a dual QA/QL-constellation (see QA-8 / QL-8),
... consider inscribed conics of QL, centered in P on QL-L1
... and their contact quadrangle QA'.
The vertices of QA and QA' lie on a conic CO.
The locus of CO-Tf3inv(P) wrt CO is a quintic
... with a double point X and a triple point Y
    (Y not always real):
The double point X is QA-P2, center of QA-Co2 on QA-Ci1.
The triple point Y lies on QA-P2.QL-P16.
If we consider a fixed point Q on QL-L1
... and CO-Tf3inv(Q) wrt all CO,
... we get circles through X and Y.
For any fixed point Q we get circles through X,
... for QA-P12 = QL-P10 the circle QL-Ci1 = QA-Ci1.

Best regards Eckart
```



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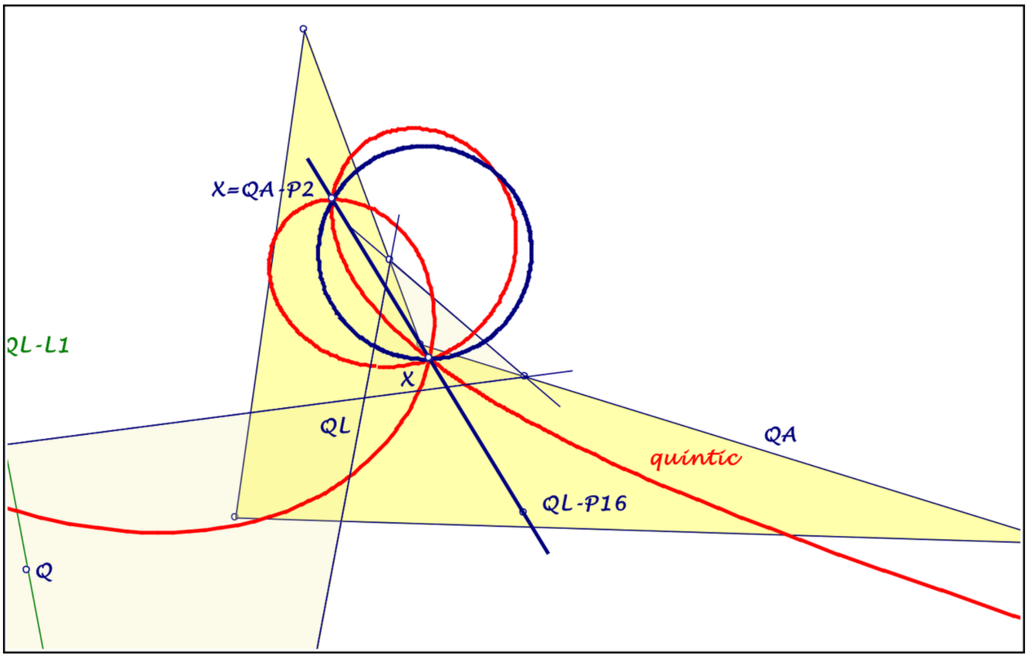

2020-01-01b.pdf

Message: #62
Date: 2020-01-02
From: bernard.keizer@gmail.com
Subject: Re: QA/QL/CO-Excursion

```
Dear Eckart,
The figures are beautiful !
Could you please refresh my memory ?
What is CO-Tf3inv ?
Thanks in advance
Best regards
Bernard
```

Message: #63
Date: 2020-01-02
From: eckart_schmidt@t-online.de
Subject: Re: QA/QL/CO-Excursion

Dear Bernard,

on Chris' homepage under "Information on Conic Sections" you find:
Keizer-Construction for CO-Tf3-1(P):
This construction is based upon Bernard Keizer's construction at Ref-34, QFG#2826.
1. Let P1 be the inverse of P in the Circle CO-Ci1 with the foci as diameter
2. Let P2 be the inverse of P1 in the Orthoptic Circle CO-Ci2
3. CO-Tf3-1(P) = the reflection of P2 in the principal axis
of CO.

Note:
When the principal axis does not connect the foci, the orthoptic circle is not real.
Replace it by the circle with radius (a2 - b2)1/2.

Best regards Eckart

Message: #64
Date: 2020-01-02
From: eckart_schmidt@t-online.de
Subject: CO-Tf3 /CO-Tf3inv

Dear Benedetto, dear Bernard, dear Chris, the following
transformations seem to be new:

(1) QA: point P --> line
... locus of CO-Tf3(P) wrt the QA-circumscribed conics CO
(2) QA: point P --> circle
... locus of CO-Tf3inv(P) wrt the QA-circumscribed conics CO
(3) QL: point P --> conic
... locus of CO-Tf3(P) wrt the QL-inscribed conics CO
(4) QL: point P --> quartic
... locus of CO-Tf3inv(P) wrt the QL-inscribed conics CO

I think, there will be interesting properties, for example:
Wrt (2): The loci of CO-Tf3inv(P) wrt the QA-circumscribed
conics
... are circles through QA-P2, for P = QA-P12 we get QA-Ci1,
... for P = QA-P2 we get a circle round QA-P23 through QA-P2.
Wrt (4): The loci of CO-Tf3inv(P) wrt the QL-inscribed conics
... are quartics with knot on QL-Ci1.

Or:
Modifying in (2) the image of P in the center of the circle:
... QA-P2 --> QA-P23, QA-P3 --> QA-P1, QA-P4 --> QA-P2,
    QA-P12 --> QA-P11,
... QA-L1 --> QA-L4, QA-L2 --> QA-P2.23,
... QA-L4 --> parallel to QA-L1 through midpoint of QA-P2.23,
...

Best regards Eckart

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Message: #65
Date: 2020-01-05
From: eckart_schmidt@t-online.de
Subject: CO-Tf3

```
Dear Benedetto, dear Chris,

perhaps new:
Pedal circles of a point X wrt the triangles of a QA
... have a common point.
This leads to a QA-transformation TF
... with TF(X) = Co-Tf3(X) wrt the QA-circumconic through X.

Best regards Eckart
```

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Message: #66
Date: 2020-01-05
From: eckart_schmidt@t-online.de
Subject: QA-Cu1

```
Dear Chris,

QA-Cu1 is the locus of points X,
... whose common point Y of the pedal circles
    wrt the QA-triangles
... has Simson lines wrt the pedal triangles of X
... with a common point Z.

For X = QA-P4 we get Y = QA-P2 and
... Z intersection of QA-L1 and a parallel to QA-P11.41
    through QA-P6.

Best regards Eckart
```

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Message: #67
Date: 2020-01-06
From: benedetto.scimemi@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] CO-Tf3

Thank you, Eckart,

for discovering this interesting TF. It seems actually new and its definition is so natural that it looks surprising that none of us had met it before. Many consequences come from the connection with CoTf3, for example: TF(P) = QAP2 for all P in QACo2, diagonal points are fixed, for any P on a QA side TF projects P orthogonally on the opposite side ... etc.

>From Geogebra it seems that TF transforms a line into a quintic which passes twice through QAP2 and has another triple point. If I find the .. forces, I plan to look for proofs, by studying TF analytically, a good job to begin the new year.

Best regards. Benedetto

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Message: #68
Date: 2020-01-07
From: bernard.keizer@gmail.com
Subject: Re: CO-Tf3 /CO-Tf3inv

Dear Eckart,
I found these 4 transformations very interesting !
They deserve surely a systematical complete study.
Considering a QA/QL, are there links between the 4 curves ?
Best regards
Bernard

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Message: #69
Date: 2020-01-07
From: bernard.keizer@gmail.com
Subject: Re: QA/QL/CO-Excursion

```
Dear Eckart,
I try to remember the definition of the duality wrt a QA/QL.
The dual of a QA circumscribed conic is a QL inscribed conic.
If a point lies on the dual line of another point, the 2nd point
lies also on the dual line of the 1rst.
You consider a QL inscribed conic with contact points Qi
The dual lines of the Qi pass through the vertices Pi of the
dual QA.
It's possible to consider the QL inscribed conic as
circumscribed to the QA of the Qi and the dual QA circumscribed
conic as a conic inscribed in the QL of the dual lines of the Qi
(with contact points the QA vertices).
Now, the QA of the Qi and the QL of the dual lines have the same
DT as the QA of the Pi and it's dual QL.
*This explains the conic through the 8 points as the
bianticevian conic wrt DT of one of the Pi and one of the Qi.
* But this conic has in turn a dual conic, tangent to the 4 QL
lines and to the 4 dual lines of the Qi.
We have so far 4 conics, 2 pairs of dual conics.
I hope I didn't make any mistake in the reasoning.
Now I'm lost and I hope you will help me to understand ...
Of curse, it doesn't change anything in your beautiful quintic.
I tried to find such curves in Bernard Gibert, but I haven't
succeeded yet !
Best regards
Bernard
```

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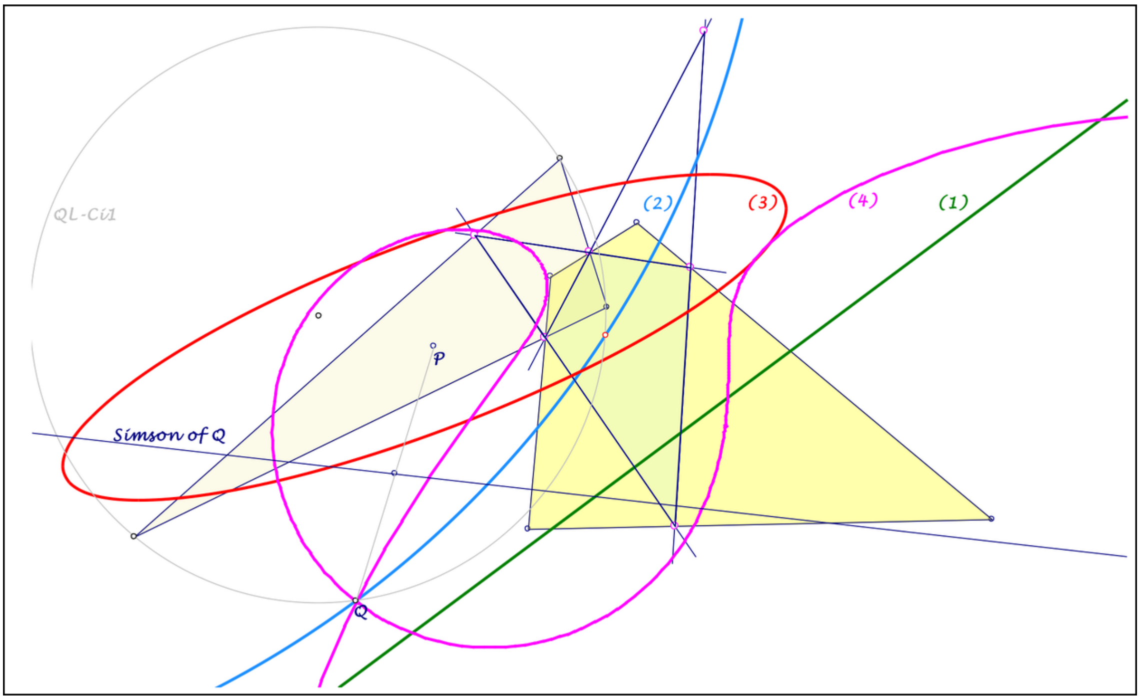
Message: #70
Date: 2020-01-07
From: eckart_schmidt@t-online.de
Subject: Re: CO-Tf3

```
Dear Benedetto,

thanks for your feedback, wrt TF(line):
... Not in general this curve bears QA-P2 as double point
    and has a triple point,
... but I cannot describe conditions and properties.

Studying TF once more, I noticed a typo in #65:
"... with TF(X) = CO-Tf3inv(X) wrt the QA-circumconic through X.

Best regards Eckart
```

Message: #71
Date: 2020-01-07
From: eckart_schmidt@t-online.de
Subject: Re: CO-Tf3 / CO-Tf3inv

```
Dear Bernard,

the curves wrt (2) and (4) for a point P in #68
... have a common point Q on QA-Ci1 = QL-Ci1 with the property,
... that the middle of PQ lies on the Simson line of Q wrt
QA-Tr1 = QL-Tr1,
... which is parallel QL-P1.10.16.

Best regards Eckart
```



2020-01-07.pdf

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Message: #72
Date: 2020-01-08
From: bernard.keizer@gmail.com
Subject: Re: CO-Tf3 / CO-Tf3inv

```
Dear Eckart,
Beautiful curves, indeed !
I'm surprised par your assertion wrt the Simson Line of Q (on
QL-Ci1) wrt DT.
If the Simson Line contains the middle of PQ, then the Steiner
Line contains P and QL-P10.
It is QL-P1QL-P16 only if P lies on this line ?
Do I miss something ?
Best regards
Bernard
```

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Message: #73
Date: 2020-01-08
From: eckart_schmidt@t-online.de
Subject: Re: CO-Tf3 /CO-Tf3inv

```
Dear Bernard,

thanks for review of my #71.
There is a mistake in the last property, it has to be:
... "it is parallel to P.QL-P10".
So your statement:
"If the Simson Line contains the middle of PQ, … ."
... has to be finished in this way.

Best regards Eckart
```

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Message: #74
Date: 2020-01-10
From: eckart_schmidt@t-online.de
Subject: QA-2Px

```
Dear Chris,

perhaps worth to be mentioned in EQF:
QA-Co2 and QA-Cu1 intersect in two points X and Y
... on the line QA-P11.QA-P41,
... they are isoconjugated
... ... wrt the cevian triangle for QA-P4 wrt QA-Tr1 (see PS)
... ... and an isoconjugation with fixed point QA-P4.
... XY is parallel QA-P2.QA-P23 // QA-P3.QA-P32 (not in EQF).

Best regards Eckart

PS. The cevian triangle of QA-P4 wrt QA-Tr1 on QA-Cu1
    ... is perspective QA-Tr2 wrt the QA-Tr2-isogonal of QA-P41.
```

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Message: #75
Date: 2020-01-11
From: eckart_schmidt@t-online.de
Subject: Direction of QL-L1

```
Dear Bernard,

just for fun, a curious method
... to get the direction of the Newton line:

Consider a QL-quadrigon  QG
... and reflect any point P round QG, you get a point Q
... with PQ- always of the same length -  parallel QL-L1.

What about the length of PQ?

Best regards Eckart
```

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Message: #76
Date: 2020-01-11
From: eckart_schmidt@t-online.de
Subject: Re: Direction of QL-L1

```
Dear Bernard,

another aspect of the direction of QL-L1,
here for a 5G = P1 … P5:
The parallels through Pi wrt the Newton line of the remaining QG
... give a circumscribed 5G' with side-midpoints
    in the vertices of 5G.

There is only one 5G with this property,
... easy to construct as follows:
Consider the 4th parallelogram point P of P1,P2,P3,
... then the 4th parallelogram point of P, P4, P5,
... and you get a point Q1,
... which gives, reflected round P1 ... P5,
    the circumscribed 5G' above.

Best regards Eckart
```

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Message: #77
Date: 2020-01-11
From: bernard.keizer@gmail.com
Subject: Re: Direction of QL-L1

```
Dear Eckart,
Your 1rst message seems promising, but I can't understand the
definition !
What is QL-quadrigon QG ?
What is the reflexion of a point round QG ?
For an ordinary QL, the line QL-L1 has as direction the mean
direction of the 4 lines.
This direction is the same for all the QL's inscribed in the
same QL-Cu1.
The direction is given by the cubic stelloïd QL-Cu2, for which
QL-Cu1 is the hessian.
Best regards
Bernard
```

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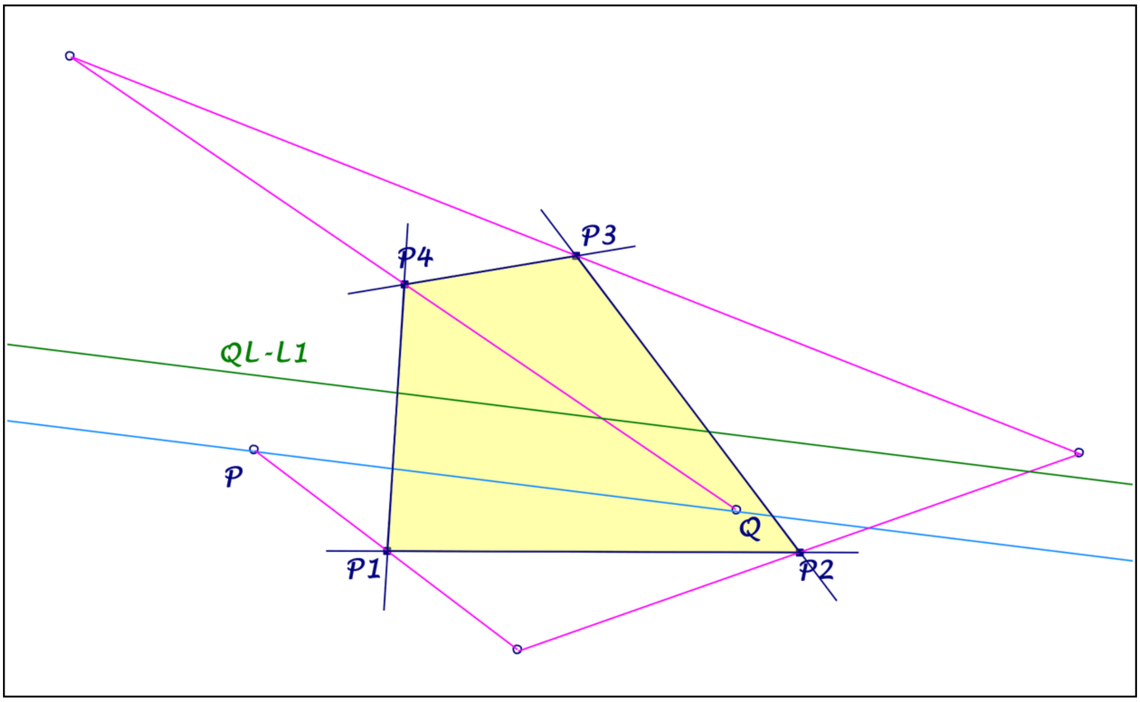
Message: #78
Date: 2020-01-12
From: eckart_schmidt@t-online.de
Subject: Re: Direction of QL-L1

```
Dear Bernard,

wrt your questions in #77:
A QL-quadrigon QG shall be one of the three quadrigon versions
of a QL.
Reflection of a point P round a QG = P1P2P3P4 means:
... Reflect P in P1,
    reflect this image in P2,
    reflect this image in P3, ...
... after reflecting in P4, you get the point Q
    (see attached file).

Best regards Eckart

PS: In #76, second passage, first line, there is a typo:
    … replace 5G by 5G'.
```



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Message: #79
Date: 2020-01-12
From: bernard.keizer@gmail.com
Subject: Re: Direction of QL-L1

Dear Eckart,
Thanks for the explanation !
It's obvious on your figure that PQ = 2*(P1P2 + P3P4) = 2*(P1P4 + P3P2) (in vectors)
I can explain neither the distance PQ nor the direction of the line PQ.
(The direction of the Newton Line is of course the same for the 3 QLquadrigons, but not the distance PQ, which depends of the choosen QG).
Best regards
Bernard
PS It's not true that the direction of the Newton Line is the mean direction of the 4 lines.
But it is true that the angle between the Newton Line and the mean direction of the 4 lines is the same for all QL's inscribed in the same QL-Cu1 and having therefore the same QL-Cu2.
This property is linked to the trisection of the angle between the Newton Line and the 1rst Steiner axis of the CSC ...

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Message: #80
Date: 2020-01-12
From: bernard.keizer@gmail.com
Subject: Re: Direction of QL-L1

```
Dear Eckart,
A figure definitively helps to understand the properties ...
Naming m1 the middle of P1P3 and m2 the middle of P3P4, it's the
same way obvious that
P1P2 = P1m1 + m1m2 + m3P2 (always in vectors)
P3P4 = P3m1 + m1m2 + m3P4
It follows that P1P2 + P3P4 = 2 m1m2 and that PQ = 4 m1m2
This explains the distance PQ, equal to 4*distance m1m2 and the
direction, as m1m2 is the Newton Line per definition ...
Best regards
Bernard
```

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Message: #81
Date: 2020-01-13
From: eckart_schmidt@t-online.de
Subject: Re: Direction of QL-L1

```
Dear Bernard,

thanks for the proof of my observation.
Wrt the constancy of the distance of PQ:
... I stated this for a chosen QG,
... independent of the starting point in the cyclic order,
... but in a vector interpretation there will change the sign.

Best regards Eckart
```

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Message: #82
Date: 2020-01-13
From: eckart_schmidt@t-online.de
Subject: 5L-s-Tfx

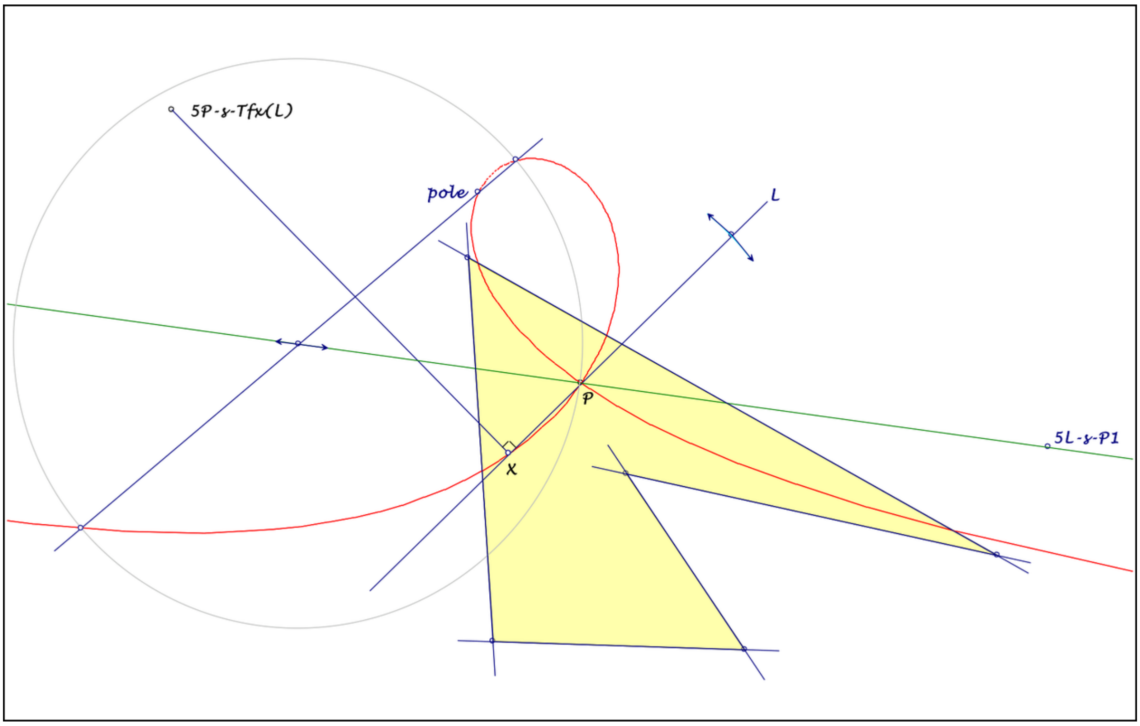
```
Dear Bernard,

let 5L-s-Tfx be a transformation, which maps a line L wrt a 5L
to a point:
The images QL-Tf2(L) wrt the QL-components of a 5L
... have the common point 5L-s-Tfx(L).

Properties:
(1) For the lines Li of a 5L the image 5L-s-Tfx(Li)
... is the contact point of Li and 5L-s-Co1,
... tangents at 5L-s-Co1 give the contact point.
(2) Line pencils of a point give the polar of this point wrt
5L-s-Co1.
(3) Lines through 5L-s-P1 give points at infinity,
... parallel to the conjugated diameter.
(4) For the lines L of a pencil for a point P
... the pedal points X of 5L-s-Tfx(L) on L
... give a strophoid for the line P.5L-s-P1, fixed point P
... and pole constructible as follows:
... ... Consider pairs of orthogonal lines through P,
... ... the lines through their pedal points
        intersect in the pole.
(5) For the lines L of a pencil for a point P
... the parallels L1 through 5L-s-Tfx(L)
    envelope a parabola Pb1,
... the perpendiculars L2 through 5L-s-Tfx(L)
    envelope a parabola Pb2,
... the axes of Pb1 and Pb2 intersect orthogonal
    in the common focus,
... the directrix of Pb2 is P.5L-s-P1,
... the polar of P wrt 5L-s-Co1 is a common tangent
    of the parabolas
... and locus for 5L-s-Tfx(L),
... the contact points of the common tangent at the parabolas
... ... are its intersections with the directrices,
... the two intersections of the parabolas
... ... and the intersection of the directrices are collinear.

Best regards Eckart

PS: I hope there are no fake observations.
```



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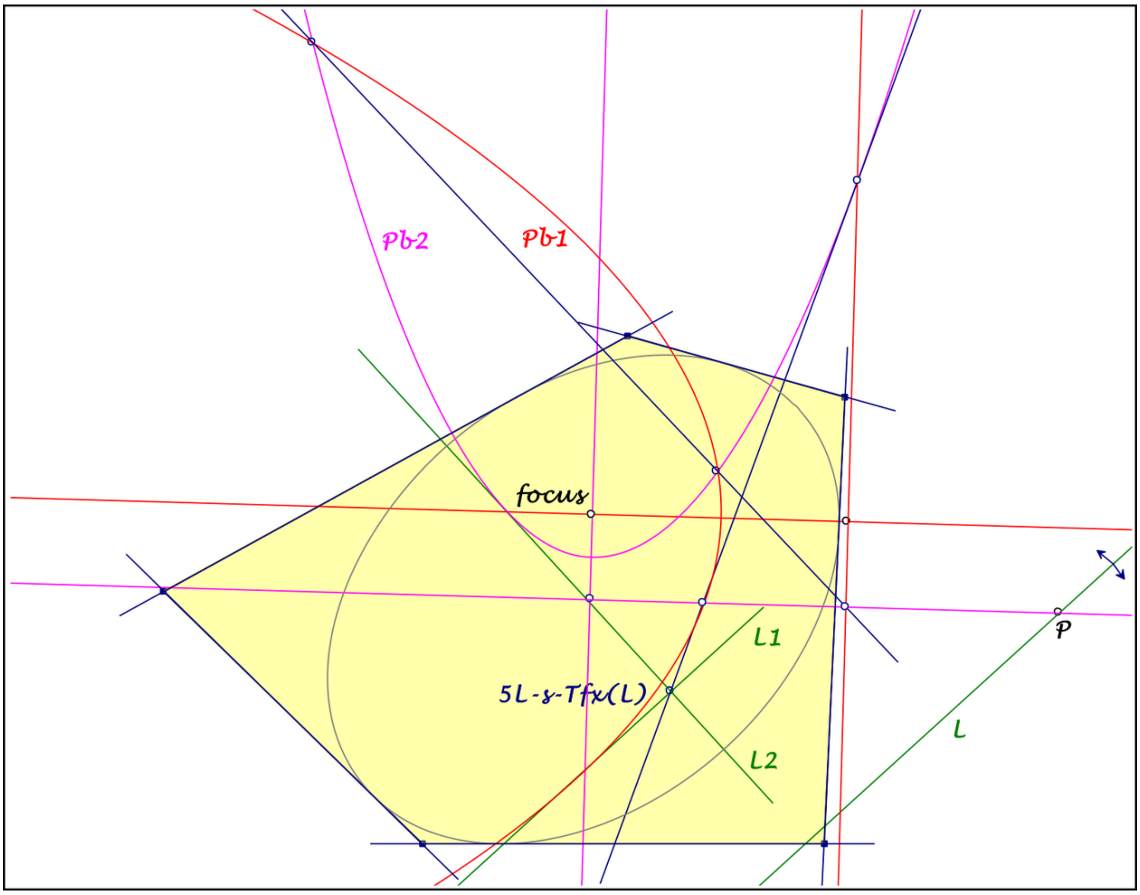

2020-01-13b.pdf

Message: #83
Date: 2020-01-14
From: bernard.keizer@gmail.com
Subject: Re: 5L-s-Tfx

Dear Eckart,
The transformation you describe maps a line wrt a 5L to a point.
Conversely, it maps a point to the line through the transformed
of 2 lines through this point.
May be I'm too quick, but it seems your transformation is the
transformation pole/polar wrt the inscribed conic ...
Best regards
Bernard
PS Did you try already the same with QL-Tf1 ? The transformed of
a line are 5 circles (having a common point ?)
What about the transformed of the 5 lines ?

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Message: #84
Date: 2020-01-14
From: eckart_schmidt@t-online.de
Subject: Re: 5L-s-Tfx

Dear Bernard,

of course 5P-s-Tfx is the polar --> pole transformation wrt
5L-s-Co1!
Excuse my restricted review,
... so only (4) and (5) in #82 will be perhaps of interest.

Best regards Eckart

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Message: #85
Date: 2020-01-15
From: eckart_schmidt@t-online.de
Subject: Re: 5L-s-Tfx

```
Dear Bernard,

wrt your PS-question in #83:
Starting with a line
... we get 5 CSC-circles wrt the 5 QL of a 5L,
... which have radical axes with a common point,
... this can be interpreted as a line
                          --> point 5L-transformation TF.
First observation:
... TF maps lines through a focus of 5L-s-Co1
    to the other focus.
... TF maps the lines of any pencil into a cubic
... ... with a double point, bearing the foci of 5L-s-Co1.

Best regards Eckart
```

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Message: #86
Date: 2020-01-15
From: eckart_schmidt@t-online.de
Subject: Re: 5L-s-Tfx

```
Dear Bernard,

I wasn't sure,
... but we have already discussed a lot of 5L-properties,
... at the moment not to find in QFG-messages and EPG,
... but on my homepage under EQF-Notes:
    http://eckartschmidt.de/EQF-Notes.html
For example #82 (4) in
    http://eckartschmidt.de/2018-01-23.pdf.
For example #85 in
    http://eckartschmidt.de/2017-11-04.pdf
In the EQF-Notes 2017 you will find more notes wrt 5L-geometry,
... especially transformations, not in EPG.
    http://eckartschmidt.de/2017-11-04.pdf

Best regards Eckart
```

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Message: #87
Date: 2020-01-16
From: bernard.keizer@gmail.com
Subject: CB13 and 5P

```
Dear Eckart,

Thanks for your answer and your own reference
Of course, I remember our many exchanges round the Pentalateral,
including the Kantor-Hervey point and the famous Ennacardioïd of
Frank Morley as envelop of the 5 Cardioïds of the 5 QL's.
Here I have a more simple (?) question.
For a QL, we found that the 6 vertices and QL-P1 form with the
circular points a CB system. All the cubics through the 6 points
include naturally 3 points on each line ...
For a 5P, all the quartics include the same way the 10 points
with 4 points on each line.
13 points give 3 complementary points (instead of 1 for 8
points)
12 points make a transformation of 4 points (instead of 2 for 7
points)
But the same way as for the circular cubic, I'd like to find
bicircular quartics through the 10 points (invariant in a CSC to
define with Steiner axes tangent to the inscribed conic ?).
Any idea ?

Best regards
Bernard
```

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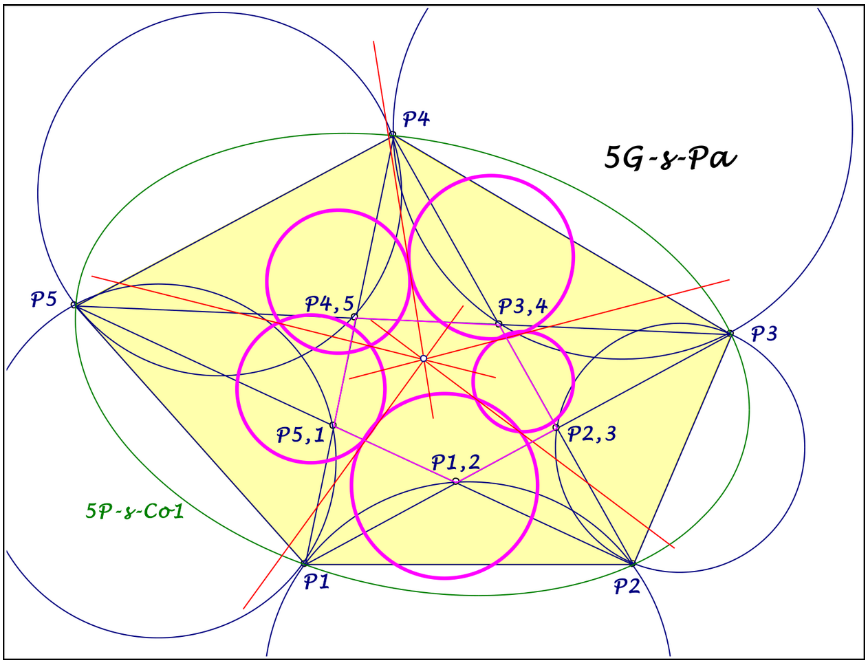
Message: #88
Date: 2020-01-20
From: eckart_schmidt@t-online.de
Subject: New 5G-points

```
Dear Bernard, dear Benedetto, dear Chris,

perhaps of interest:
Consider a 5G = P1P2...P5,
which can be interpreted as 5P and 5L,
... and the points Pi,i+1 = PiPi+2 ^ Pi+1Pi-2
    (see attached file),
... circles (Pi, Pi,i+1, Pi+1)  and their image circles
    wrt CO-Tf3 for 5P-s-Co1,
... which have a common point 5G-s-Pa for their radical axes.
Replacing CO-Tf3 by CO-Tf3inv we get in the same way a point
5G-s-Pb,
... using CO-Tf3 for 5L-s-Co1 we get a point 5G-s-Pc,
... using CO-Tf3inv for 5L-s-Co1 we get a point 5G-s-Pd.
5G-s-Pa, b are collinear with 5P-s-P1, 5G-s-Pc,d are collinear
with 5L-s-P1.

Best regards Eckart

PS: The radical axes of the circles (Pi, Pi,i+1, Pi+1)
... intersect in 5G-s-P2 (see EPG).
The 2nd intersections of the circles (Pi,i+1, Pi, Pi,i-1)
... are concyclic with center 5G-s-P3.
```



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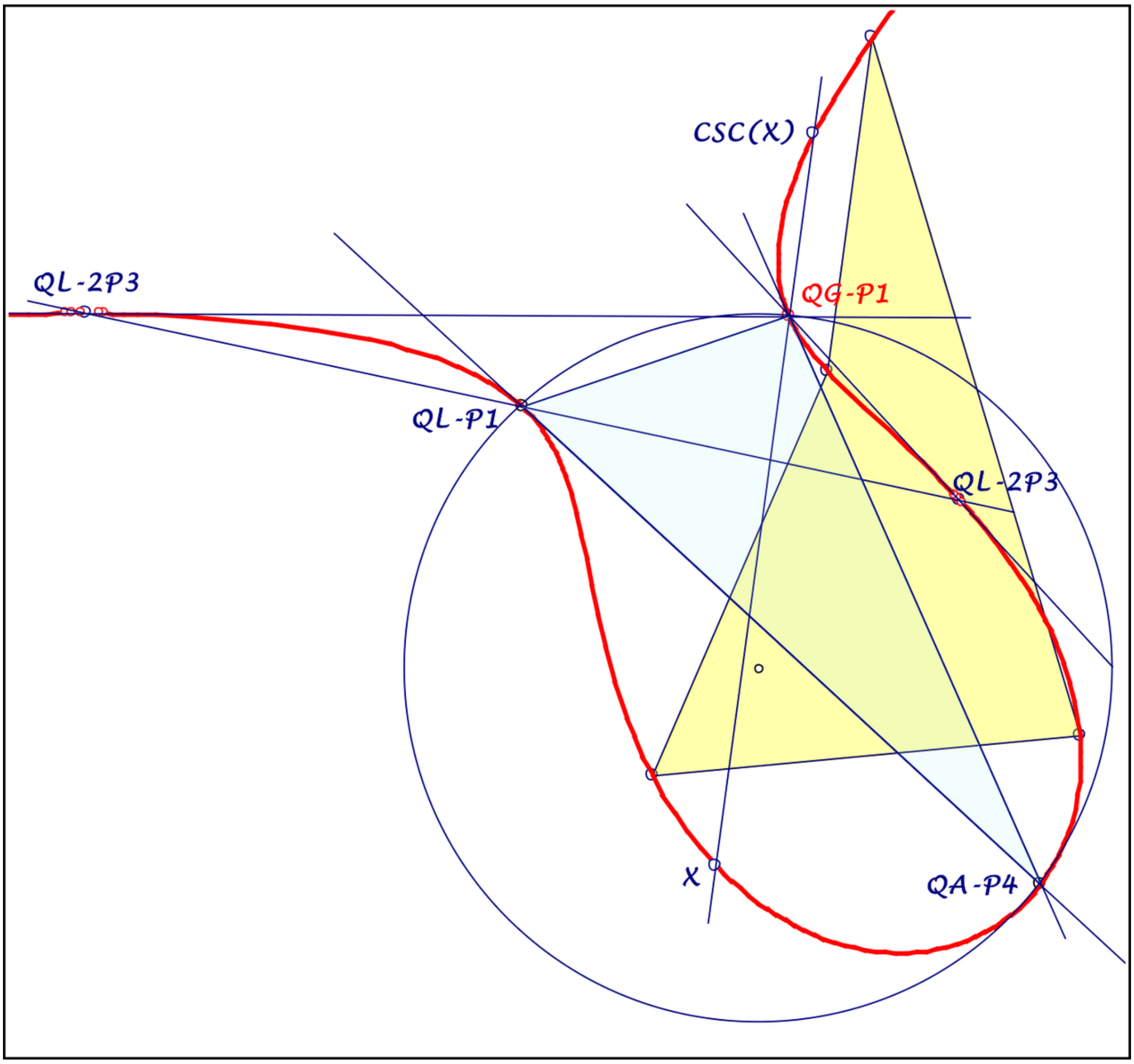
Message: #89
Date: 2020-01-25
From: eckart_schmidt@t-online.de
Subject: QG-Circumcubic

```
Dear Bernard, dear Chris,

perhaps of interest, a QG-circumcubic,
... locus of CSC-partner on lines of the QG-P1-pencil
... through P1, P2, P3, P4, QG-P1, QA-P4, QL-P1, QL-2P3
... and the intersections of the Schmidt circle
... ... with a perpendicular through QG-P1 to the 1st Steiner
axis,
... asymptote parallel QG-P1.QL-P1,
... tangent in QL-P1 through QA-P4 = CSC(QG-P1.QL-P1),
... tangent in QG-P1 through QA-P4 = CSC of the circle
    (QG-P1,QA-P4,QL-P1),
... tangent in QA-P4 (also tangent to the circumcircle
    of TR = (QG-P1,QA-P4,QL-L1))
... ... is the isogonal conjugate wrt TR
        of a parallel to QG-P1.QL-P1 through QA-P4.
... tangents in QL-2P3a,b through QG-P1 = CSC(QG-P1.QL-2P3b,a)
The cubic is not only CSC-invariant,
... but also isogonal invariant wrt the triangle
    TR = (QG-P1,QA-P4,QL-L1),
... swapping QG-P1 and QL-P1 also QL-2P3.
The cubic is a nonpivotal isocubic:
... reference triangle TR = (QG-P1,QA-P4,QL-L1),
... isogonal conjugation,
... root in the midpoint of QG-P1.QL-P1.
The cubic is also invariant wrt the Möbius transformations,
... centered in one vertex of TR, swapping the other two.
A point on the cubic and its images wrt the Möbius
transformations
... give a quadrangle (trapezoid) with Miquel triangle TR.
Finally: The points QG-P1, QA-P4 and QL-2P3a,b are concyclic
... on an TR-isogonal invariant circle,
... centered on the TR-circumcircle in the 2nd intersection
    with the QL-2P3-bisector.

Best regards Eckart

PS: I hope, there are no fake observations.
```



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Message: #90
Date: 2020-01-25
From: bernard.keizer@gmail.com
Subject: Re: QG-Circumcubic

```
Dear Eckart,
Beautiful new curve!
I didn't find fake observations.
There is only a typo, repeted 3 times, in the description of TR:
the 3rd vertice is QL-P1 and not QL-L1.
Your curve is a pivotal focal circular monocursal Van Rees cubic
with focus in QA-P4, Newton Line QL-P1QG-P1 and pivot the
infinity point of the Newton Line.
The asymptote is the reflexion in the Newton Line of the
parallel to this line in QA-P4.
The QA described at the end has 2 perpendicular sides, which
intersect on the curve ...
Best regards
Bernard
```

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...
Message: #91
Date: 2020-01-25
From: bernard.keizer@gmail.com
Subject: Re: QG-Circumcubic

```
Dear Eckart,
Your cubic is also generated by partners on each line through
QL-P1 in the CSC centered in QG-P1 swapping QL-P1 and QA-P4.
In fact, it is more a triangle property and you can easily
generalise your construction following way :
for any triangle P1P2P3, consider the 3 CSC centered in one
vertice and swapping the 2 others, there are 3 Van Rees focal
circular cubics
with foci one of the 3 vertices, Newton Line the opposite side
and pivot the infinity point of the Newton Line.
Each cubic is invariant in the 3 CSC and isogonal invariant wrt
the triangle.
Each cubic with focus in one vertice can be obtained as the
locus of partners on lines through a 2nd vertice in the CSC
centered in the 3rd (and vice-versa).
Best regards
Bernard
```

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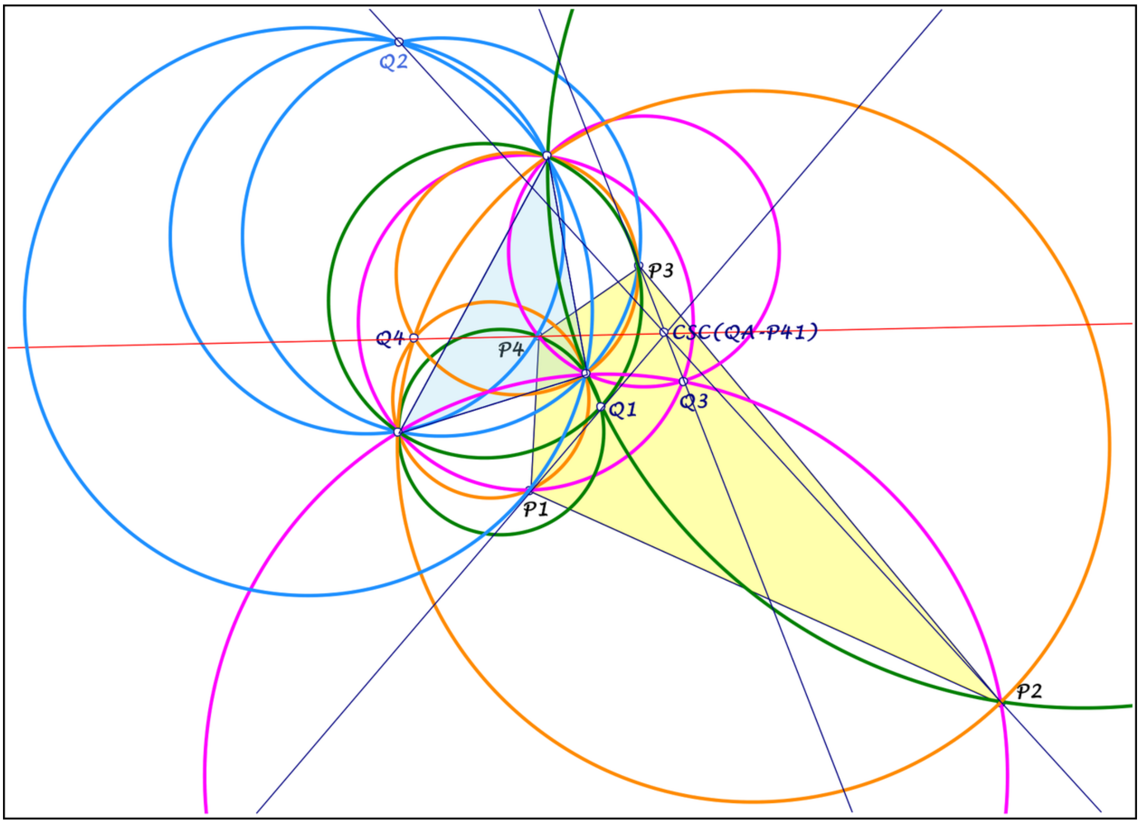
Message: #92
Date: 2020-01-25
From: eckart_schmidt@t-online.de
Subject: CSC(QA-P41)

```
Dear Bernard, dear Chris,

in #2 the point CSC(QA-P41) on QA-Cu1 is mentioned,
… here is a geometric background (see attached file):

Consider a QA = P1P2P3P4
… and for each vector PiPj the circle through Pj
… … and the two diagonal points not on the line PiPj.
The circles for PiPj, PiPk, PiPl give a common point Qi
… and the lines PiQi have the common point CSC(QA-P41).

Best regards Eckart
```



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Message: #93
Date: 2020-01-26
From: eckart_schmidt@t-online.de
Subject: Re: QG-Circumcubic

```
Dear Bernard,

thanks for correction of the typos
... and the interesting interpretations of the cubic,
... so that the asymptote gets an exact description.

But I cannot confirm,
... that the final QA has two perpendicular sides
    intersecting on the cubic.

My sight of the cubic is a generalized monocursal QL-Cu1,
... QL-Cu1 is defined by three points (A,B,C),
... with A = QL-P1 and B, C as QL-2P2 and BC = QL-L1
... and CSC as Möbius transformation TF, centered in A,
    swapping B, C.
Construction:
... Intersections of TF of parallels to BC
     and reflection of the parallels in BC.
In this way every point tripel gives a cubic,
the cubic in #89 by (QA-P4,QG-P1,QL-P1).
Every two pairs of TF-partners X1,X2 and Y1,Y2
give a QG = X1Y1X2Y2,
... which -as QL interpreted- has this cubic as QL-Cu1.
So we can get the cubic in #89 as QL-Cu1
of the QG = QL-2P3a.QG-P1.QL-2P3b.QL-P1.

Best regards Eckart

PS: Wrt #89, I was fascinated
    ... of the reference triangle TR = (QG-P1,QA-P4,QL-P1)
    ... with vertices in the main points of QG-, QA-
        and QL-geometry.
```

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Message: #94
Date: 2020-01-27
From: bernard.keizer@gmail.com
Subject: Re: QG-Circumcubic

```
Dear Eckart,
You're right.
Only for bicursal Van Rees cubics, the QA formed by a point and
the 3 CSC of the point have 2 perpendicular sides.
For monocursal VR, the QA's have 2 parallel sides.
Best regards
Bernard
```

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Message: #95
Date: 2020-01-27
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

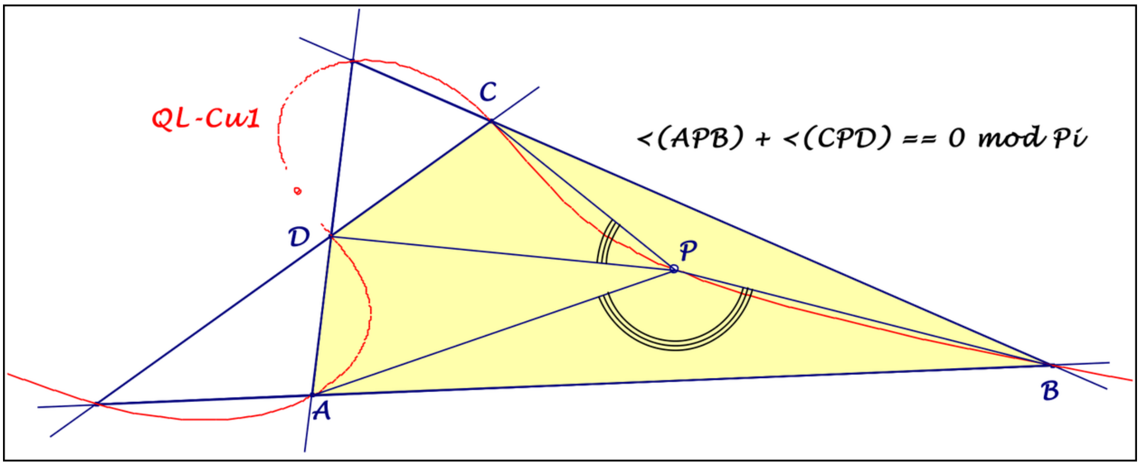
```
Dear Bernard, dear Chris,

is the following property already mentioned?
QL-Cu1 of a QG = ABCD is the locus of points P
… with the property: <(APB) + <(CPD) == 0 mod Pi.

Best regards Eckart

PS: See also EQF Ref 62, last passage,

https://www.mathcurve.com/courbes2d.gb/focaledevanrees/focaledev
anrees.shtml
```



2020-01-27.pdf

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Message: #96
Date: 2020-01-29
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

```
Dear Eckart,
In fact, this simple, beautiful and well-known property is not
in EQF.
(The link you give doesn't work, I suppose if you choose the
english version, you have to name the curve in english too).
I also mentionned several times following property without
reaction :
for any QL inscribed in the same QL-Cu1, the sum of the
directions (or the mean direction) of the 4 lines is the same,
depending of a choosen reference line (for example the Newton
Line).
Best regards
Bernard
```

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Message: #97
Date: 2020-01-29
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

```
Dear Bernard,

please try it once more with this address:
https://www.mathcurve.com/courbes2d.gb/focaledevanrees/focaledev
anrees.shtml

The angle definition of QL-Cu1 wasn't well-known  for me,
... are there references,
... for it leads to a very elementary construction.

Best regards Eckart
```

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Message: #98
Date: 2020-01-30
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

```
Dear Eckart,
I know this property from Mathcurve and from the 3 articles
mentionned in your link by D. Roux et M. Tixier in Quadrature
46, 47 and 65 (in french).
I've put the property in my 1rst article in 2013 on my blog.
Best regards
Bernard
PS You didn't answer my own property of the directions of the 4
lines in 96 ...
Is it not clear or not interesting ?
```

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Message: #99
Date: 2020-01-31
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

```
Dear Bernard,

thanks for your informations,
... can you please give me the side in your blog 2013
    with the angle property?

Your observation wrt the mean direction of the 4 QL-lines is
correct,
... but I tried in vain to find connections with QL-elements!

Best regards Eckart
```

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Message: #100
Date: 2020-02-01
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

Dear Eckart,
The 1rst property is mentionned page 43 in the 1rst article in 2013 on my blog.
The 2nd property is mentionned he same way page 52 in the 2nd article in 2014 on my blog.
I had also mentionned it about the same time in an attached file to a message to the forum (in english).
The sum of the angles between the 4 lines and the Newton Line is twice the angle between the 1rst Steiner axis and the Newton Line, id est the angle between the axis of the cardioïd QL-Qu1 and the parabola QL-Co1.
The asymptotes of the cubic stelloïd QL-Cu2, which are parallel to the axes of the deltoïd QL-Qu2, are the trissectrices of this angle.
I attach this file again to this message.
Best regards
Bernard

Properties of the QL's inscribed in a given curve Cu1

The fundamental property of the QL's inscribed in a given curve Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curve.

All the QL's share :

- the Miquel point P1
- the Schmidt circle, the Steiner axes and the invariant points F1 and F2
- the Newton Line L1, parallel to the asymptote of Cu1 at half a distance (P1 being the railwaywatcher of the 2 lines)
- the direction of the Steiner Line L2
- the points B1 and B2 (bicursal curve) or R1 and R2 (unicursal curve)
- therefore the middle of these 2 points where the line L6 cuts the Newton Line
- the axis of the inscribed parabola (parallel to the Newton Line through the Miquel point)
- the axis of the cardioïd Qu1 (tangent to Cu1 in the Miquel point, through the point where Cu1 cuts it's asymptote, reflection of the axis of the parabola in the 1rst Steiner axis and locus of the point P3)
- the direction of the axes of the inscribed deltoïd Qu2, which is the direction of the asymptotes of the stelloïd Cu2
  the axes of the stelloïd trisect the oriented angle between the 1rst Steiner axis and the axis of the parabola or the oriented angle between the axis of the parabola and the axis of the cardioïd (it's easy to draw them with the Chasles construction directly on the Schmidt circle with a rectangular hyperbola through P1 centered in the middle of P1A, where A is a point where the axis of the parabola cuts the Schmidt circle, with axes of symmetry parallel to the bisectors of the angle between the axis of the parabola and the 1rst Steiner axis)
- the asymptote of the hessian as tangent to the inscribed deltoïd
- last, but not least, as discovered by Eckart and discussed in details by Bernard G., the centers of the 27 inscribed cardioïds lying on the stelloïd.

If we call R the fixed radius of the Schmidt circle, r the variable radius of the Miquel or the Hervey circle of a QL, p the distance from P1 to the Steiner Line and θi the angles between the lines Li and the fixed direction of the Steiner Line, we have for all inscribed QL's :

- Σθi equal to the angle between the axes of the parabola and the cardioïd or twice the angle between the 1rst Steiner axis and the axis of the parabola
- $R^2 = 2pr = 16\, r^2\, \Pi\cos\theta_i$ ($p = 8r\, \Pi\cos\theta_i$)

For a given reference QL, the inscribed conics and their confocals (of which the degenerated ones made of 2 conjugate points) form a tangential set. The QL's inscribed in the hessian are determined by the 4 common tangents to 2 conics of the set of these confocals (for example the 4 lines joining 2 pairs of conjugate points ; the reference QL is obtained by joining 2 pairs of opposite vertices)

Properties of the QL.pdf

Message: #101
Date: 2020-02-02
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

```
Dear Bernard,

thanks for references and properties:

(1) Wrt the angle property/definition of QL-Cu1,
... which give an elementary simple construction:
Take a quadrigon component ABCD of the QL
... and the circles for angles alpha about AB and DC,
... their intersections give QL-Cu1.

(2) Wrt your interesting "mean direction" properties of the 4
QL-lines:
... Your paper already worked through as attachment of QFG #561
... excuse was not parat at the moment on my side.

I hope Chris will read the last messages, to enrich his EQF … …

Best regards Eckart
```

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Message: #102
Date: 2020-02-02
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

Dear Eckart,
I'm glad that you finally take interest in this item !
Perhaps this last curiosity :
I was long time intrigued by the triangle QL-P1QL-2Pa and b, because it seems it doesn't fit with the property of the 4 lines.
In fact, you have to consider as limit inscribed QL the one formed by the 2 lines QL-P1QL-2Pa and QL-P1QL2Pb and 2 lines either through the 2 points QL-2Pa and b for the monocursal QL-Cu1 or parallel to the Newton Line in these 2 points for the bicursal QL-Cu1.
The sum of the directions of the 2 lines QL-P1QL-2Pa and QL-P1QL-2Pb wrt the Newton Line is by definition twice the direction of the 1rst Steiner axis, as these 2 points are CSC partners and the sum of the directions of the 4 lines of these 2 limit QL's wrt the Newton Line has the searched value !
Best regards
Bernard
PS Of course you're right, these beautiful and simple properties deserve to figur in EQF, but sometimes I wonder if there is always a pilot on board ...

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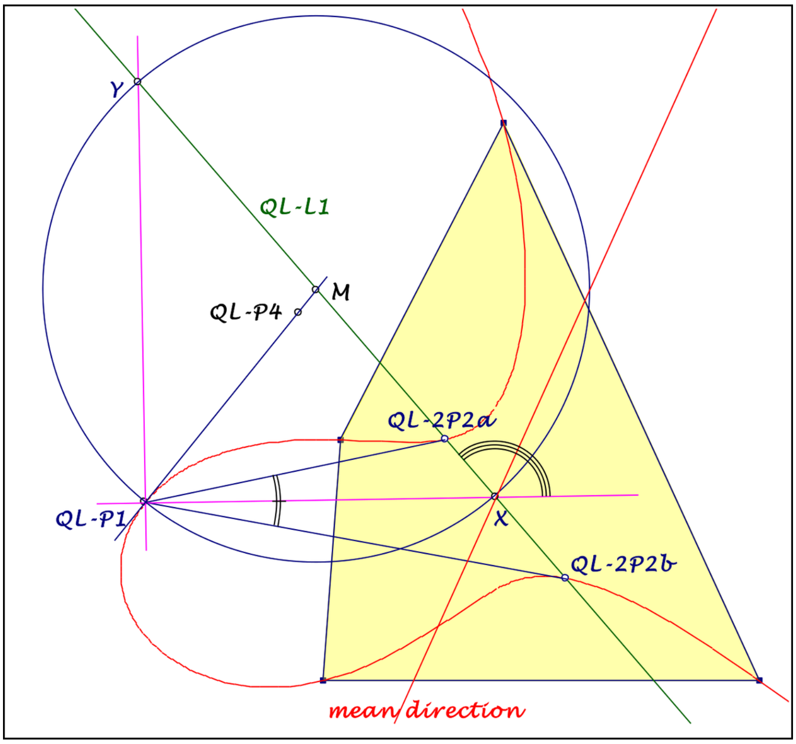
Message: #103
Date: 2020-02-03
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

```
Dear Bernard,

thanks for further properties,
... there will be a typo: replace parallel by perpendicular.
Another sight of your property (see attached file):
Let M be the intersection of QL-L1 and QL-P1.QL-P4,
... and consider the circle Ci centered in M through QL-P1,
... intersecting QL-L1 in two points X and Y
... on the inner and outer angle bisectors of
    QL-2P2a.QL-P1.QL-2P2b (see PS).
The circle Ci is the CSC-image of a line
... orthogonal QL-L1 through QL-L6^QL-L1.
The "mean direction" is given by the outer bisector
of the angle of QL-P1.X.Y.

Best regards Eckart

PS: Other than in EQF, for me QL-2P2a,b are the CSC-partner
... on QL-L1 in the monocursal case of QL-Cu1,
... on a perpendicular to QL-L1 in QL-L1^QL-L6
    in the bicursal case.
```



2020-02-03.pdf

Message: #104
Date: 2020-02-04
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

```
Dear Eckart,
Interesting construction !
QL-P1QL-P4 is the axis of the cardioïd.
The parallel in QL-P1 to the Newton Line is the axis of the
parabola.
As already mentionned, the sum of the 4 oriented directions of
the lines wrt the Newton Line is the oriented angle between the
2 axes (m° 2π).
This angle is equal to the angle YMQL-1, which is twice the
angle MXQL-P1.
The mean direction is 1/4 of this angle and is defined m° π/4
and can be the inner as well as the outer bisector of MXQL-P1.
The 3 asymptotes of the cubic stelloïd (parallel to the axes of
the deltoïd) are the 3 trisectrices of the oriented angle
between the 2 axes of the parabola and the cardioïd.
Best regards
Bernard
PS The line CSC of Ci is the perpendicular bisector of
QL-2PaQL-2Pb and intersects the axis of the parabola on the Van
Rees curve in a point Q' which is the CSC of the intersection Q
between the curve and its asymptote ; Q lies therefore on Ci, it
is the diametral point of QL-P1 .
```

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Message: #105
Date: 2020-02-06
From: eckart_schmidt@t-online.de
Subject: QG-Rose

Dear Bernard,

perhaps of interest,
a curious point --> line transformation TF for QL
... and a curious QG-rose, pedal curve of QG-Co2 wrt TF.

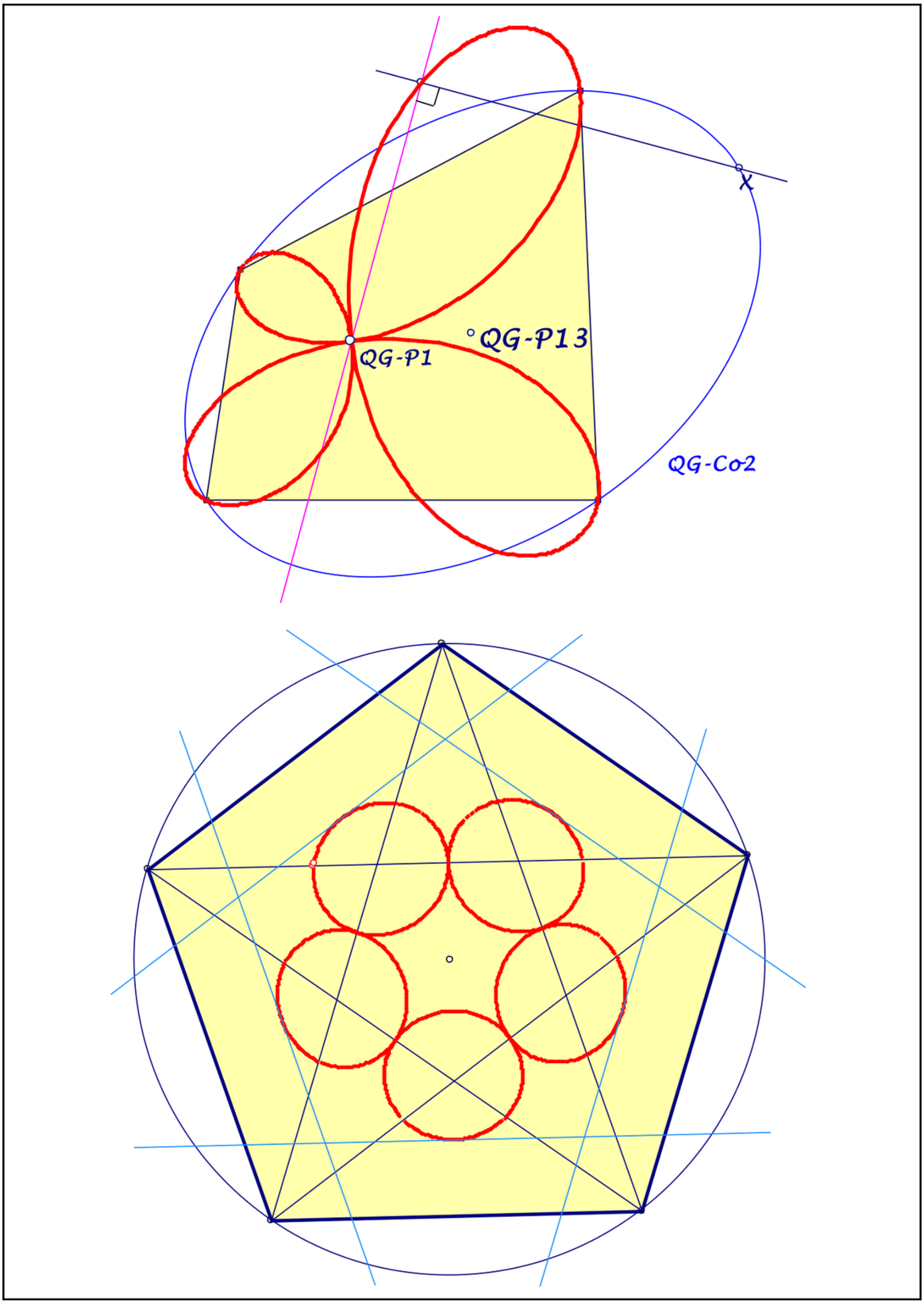
Consider a QL with its 3 QG-versions
... and its cicumconics through a fixed point,
... which have collinear 4th intersections.
This gives a point-line-transformation TF for QL.

TF maps vertices of QL-Tr1 to the opposite QL-Tr1-sideline
... and points on a diagonal to the opposite QL-Tr1-vertex.
TF maps points on QL-lines to this QL-line.
TF(QL-P1) = CSC(QL-Ci6).
TF(P)^QL-Tf2(TF(P)) is a point on the trilinear polar of P wrt QL-Tr1.
TF maps QL-L1 to a quartic tangent to the QL-lines
... and tangent to QL-L9 in QL-P18
... with a cusp ... ...

An application for a QG, interpreted as QL (see attached file):
Consider for points X on QG-Co2
... the locus of pedal points Y on TF(X).
What about this QG-rose?

Finally: As you like interesting figures,
... attached six ellipses for a regular pentagon P1...P5:
    Lines = TF(Pi) wrt Pi+1Pi+2Pi+3Pi+4.
    Ellipses Ei = envelope of TF(P) wrt Pi+1Pi+2Pi+3Pi+4
... and P point on the circumcircle of the pentagon.

Best regards Eckart



2020-02-05.pdf

Message: #106
Date: 2020-02-07
From: bernard.keizer@gmail.com
Subject: Re: QG-Rose

```
Dear Eckart,
Interesting property !
If I understand correctly, I suppose any QG circumscribed conic
has the same way a QG-rose ...
Best regards
Bernard
```

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Message: #107
Date: 2020-02-07
From: eckart_schmidt@t-online.de
Subject: Re: QG-Rose

```
Dear Bernard,

if you take an arbitrary circumscribed conic of the QG,
... you get a sectic with several double points,
... for the TF-images of conic points have no common point,
... but envelope a conic.

Best regards Eckart
```

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Message: #108
Date: 2020-02-08
From: bernard.keizer@gmail.com
Subject: Re: QG-Rose

```
Dear Eckart,
Thanks for the explanation.
Your particulary QG-rose is a sextic with a unique quadruple
point whereas the other sextics have several double points ...
The transformation P to pedal of P on TF(P) is a point to point
transformation and leads apparently to plenty of interesting
properties.
What are the images of the main QL's figures (lines, circles,
conics or even cubics or quartics) ?
I don't begin, as I fear it would take again plenty of time !
Best regards
Bernard
```

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Message: #109
Date: 2020-02-08
From: bernard.keizer@gmail.com
Subject: Re: CB13 and 5P

Dear Eckart,
I regret that you didn't answer this message, but I have to admit it was rather confuse. I've tried to clarify my ideas.
First, it was obviously 5L (and not 5P) with 10 points and 4 on each of the 5 lines leading to quartics like the 6 points on a QL with 3 on each of the 4 lines lead to cubics.
Then I searched an analogy with the CB theorem.
A cubic is generally defined by 9 points, 2 cubics intersect also in 9 points and define a CB system, therefore we have a 9th CB point of 8 points and a CB transformation of 7 points associating 2 points.
The same way, a quartic is defined by 14 points, but 2 quartics intersect in 16 points, therefore we have 3 CB partners of 13 points and a CB transformation of 12 points associating 4 points instead of 2. What about these QA's ?
Last I continued to search an analogy with the QL.
The 6 vertices of the QL, the point QL-P1 and the 2 circular points form a CB system of 9 points and an infinity of circular cubics are circumscribed to the 7 real points. Among them, 4 are degenerated and formed by a line through 3 vertices and the circle through the 3 other vertices, QL-P1 and the circular points.
The same way, the 10 vertices of the 5L, the 2 foci of the inscribed conic and the circular points form a CB system and an infinity of (bi?)circular quartics pass through the 12 real points. Among them, 5 are degenerated and formed by a line through 4 vertices and the circular focal Van Rees curve of the remaining QL, through the 6 other 5L vertices, the 2 foci of the inscribed conic and the 2 circular points.
At this step, I remain blocked ! In particular, I can't find a transformation wrt the inscribed conic swapping each line and the QL-Cu1 of the remaining 4 lines ...
Have you already searched in this direction ?
Have you alreading read something about the CB transformation of 12 points on a quartic ?
Have you any idea ?
Many thanks in advance for your attention and for an answer (even negative).
Best regards
Bernard

Message: #110
Date: 2020-02-10
From: bernard.keizer@gmail.com
Subject: Re: CB13 and 5P

```
Dear Eckart,
3 lines of the QL define also a degenerated cubic through the 6
vertices of the QL, but not through QL-P1 and not circular.
The same way, 4 lines of the 5L define a degenerated quartic
through the 10 vertices, but not through the 2 foci and not
circular.
More interesting perhaps, a pair of conics each through 2 points
on each line define also a degenerated quartic through the 10
vertices, not through the foci and not circular.
Best regards
Bernard
```

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Message: #111
Date: 2020-02-11
From: bernard.keizer@gmail.com
Subject: Re: CB13 and 5P

```
Dear Eckart,
On any quartic, 12 points define a CB transformation associating
4 points (a point has 3 partners).
On any quartic through the 10 vertices of the 5L and the 2 foci
of the inscribed conic, a point has 3 partners.
In particular, on one of the 5 degenerated circular quartics
formed by a line and the Van Rees curve of the 4 others, any
point on the VR has 3 partners also on the VR wrt the CB
transformation of the 12 points.
Best regards
Bernard
PS The 9th CB partner of the 8 points of the VR (6 QL vertices
and the 2 foci) is the CSC of the 3rd intersection of the axis
of the conic through the 2 foci and the VR or the 4th
intersection of the circle through QL-P1 and the 2 foci with the
VR (I hope you will agree at last with this property).
```

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Message: #112
Date: 2020-02-11
From: eckart_schmidt@t-online.de
Subject: Re: CB13 and 5P

```
Dear Bernard,

I try to follow your interesting explanations,
... but I cannot give any stimulations nor judgments,
... for my constructions don't reach your horizon,
... but the last property in #111 holds.

Nevertheless I try to understand the CB-transformation,
... defined by 12 points, associating 4 points.

Best regards Eckart
```

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Message: #113
Date: 2020-02-15
From: eckart_schmidt@t-online.de
Subject: Pivotal Isocurves for a Quadrilateral

```
Dear Bernard, dear Chris,

for a QA there are several pivotal isocubics
... defined by a reference triangle QA-Tr1,
... an isoconjugation QA-Tf2 and a pivot P.
Analog for a QL:
... reference triangle QL-Tr1,
... isoconjugation QL-Tf2 for lines and a pivot line L:
If we consider for points X on the pivot line L
... the two QL-Tf2 partner lines through X
... and their envelope, X running on L, we get a curve:
... ... tangent to the QL-lines in the intersections
        with the pivot line L,
... ... tangent to the sidelines of QL-Tr1 in the intersections
        with QL-Tf2(L),
... ... tangent to the pivot line L in the intersection of L and
        QL-Tf2(L),
... ... tangent to QL-Tf2(L) in ...
You can get the QL-Tf2 partner lines through X
... as tangents from X at  the QL-Tr1 inscribed conic,
... which is the envelope of QL-Tf2 images of lines through X.

Sorry, I cannot offer a drawing,
... CABRI doesn't plot the envelope in the right way,
... for whatever reason ...

What about these curves?

Best regards Eckart
```

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Message: #114
Date: 2020-02-16
From: bernard.keizer@gmail.com
Subject: Re: Pivotal Isocurves for a Quadrilateral

Dear Eckart,
For a QL, your curves are curves of class 3 and are the duals of the pivotal isocubics of the dual QA (with L dual of P).
DT is self dual and the inscribed conics are the duals of the circumscribed conics which are the QA-Tf2 of the lines through the pivot P ...
Best regards
Bernard
PS The contact point on QL-Tf2(L) is the dual of the tangent to the cubic in the pivot, which passes through the isopivot.

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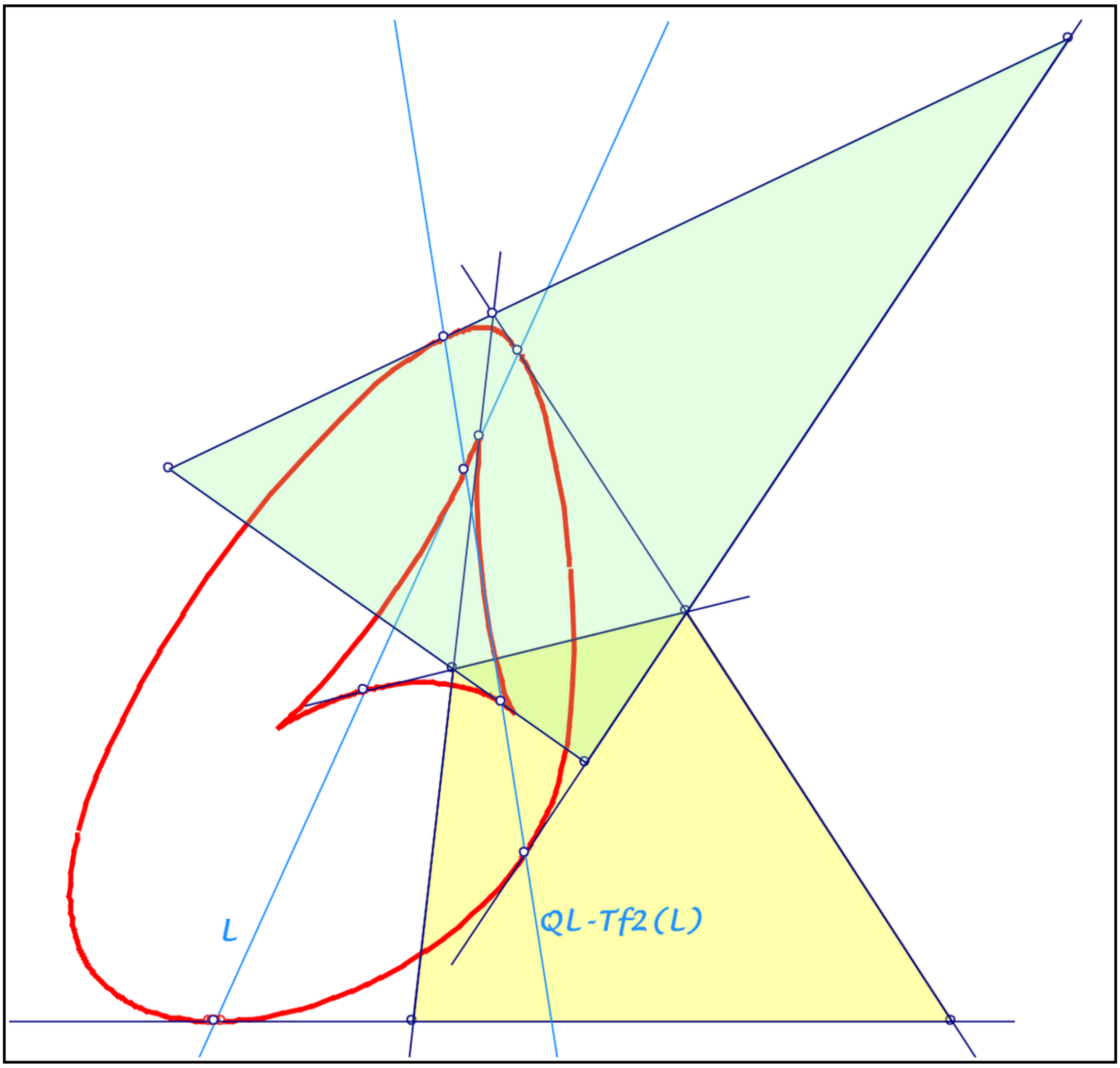
Message: #115
Date: 2020-02-16
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal Isocurves for a Quadrilateral

```
Dear Bernard,

thanks for clearing up, you have a better view!
Attached a drawing, now possible.

Best regards Eckart
```



2020-2-16.pdf

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Message: #116
Date: 2020-02-17
From: bernard.keizer@gmail.com
Subject: Re: Pivotal Isocurves for a Quadrilateral

Dear Eckart,
Thanks for the figure !
I think at last the dual curve of QA-Cu1 should be mentionned in EQF ...
Conversely, duals of circumcubics of the 6 vertices of a QL give curves of class 3 tangent to the 6 sides of the dual QA.
For example, it is the way we succeeded in drawing Siebeck's sextic tangent to the 6 sides in their middles (Siebeck's theorem as generalisation of Marden's theorem).
Again, the dual curve of QL-Cu1 as well as this Siebeck's sextic should be mentionned in EQF...
Best regards
Bernard

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Message: #117
Date: 2020-02-19
From: eckart_schmidt@t-online.de
Subject: 5P with the same Quartic

Dear Bernard,

two observations for 5P with the same quartic:
(1) The 5P have the same point 5P-s-P3
... and the corresponding circle (see 5P-s-P3),
... which is the nine-point circle of the triangle in QFG#3579.
(2) The 5P have the same your transformation CSC3,
... centered in 5PCSC(5P-s-P4), swapping 5P-s-P5,6.

Best regards Eckart

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Message: #118
Date: 2020-02-19
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

```
Dear Chris,

perhaps worth to be mentioned:
QL-Cu1 is the locus of points P with
... <XPY == <Y'PX' mod 180° for opposite points X, X' and Y, Y'
... (private letter from R. Stärk 08.08.2010).

Best regards Eckart
```

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Message: #119
Date: 2020-02-20
From: bernard.keizer@gmail.com
Subject: Re: 5P with the same Quartic

```
Dear Eckart,
These intriguing properties are consequences of the properties
of the 5P and it's quartic,
as 5P-s-P1,2,3,4,5 and 6 depend only of the 5P, but 5P-s-P3 is
necessary the same as given from the quartic.
The circumconic and the quartic of each 5P intersect in 8
points, the 5P and the vertices of the mentionned triangle.
The circumcircle of this triangle intersect the circumconic in 4
points, the 3 vertices and 5P-s-P4.
The CSC and the other points of the 2 5P's are different, but
CSC(5P-s-P4) give the same point ...
I just checked that the quartic is not the quartic of the 5P of
the QA-P4 of 4 of the 5P.
Do you see a possible application of these properties ?
Best regards
Bernard
```

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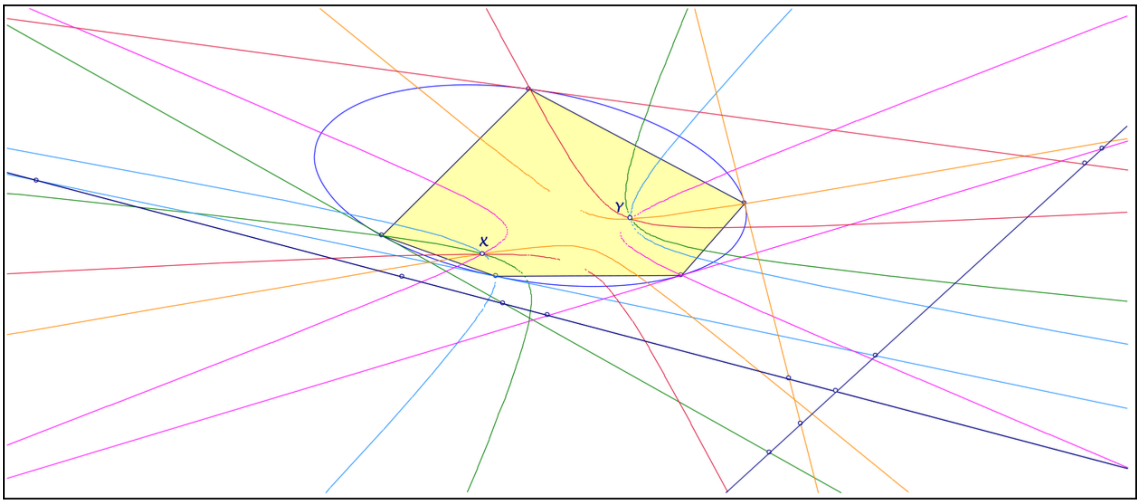
Message: #120
Date: 2020-02-21
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Pa,b

```
Dear Chris,

perhaps of interest:

The five image conics of the tangents in Pi at 5P-s-Co1
... wrt QA-Tf2 of the remaining QA
... have two common points X and Y,
... whose preimages lie five by five on two lines,
... which are the polares of X and Y wrt 5P-s-Co1.

Best regards Eckart
```



2020-02-21.pdf

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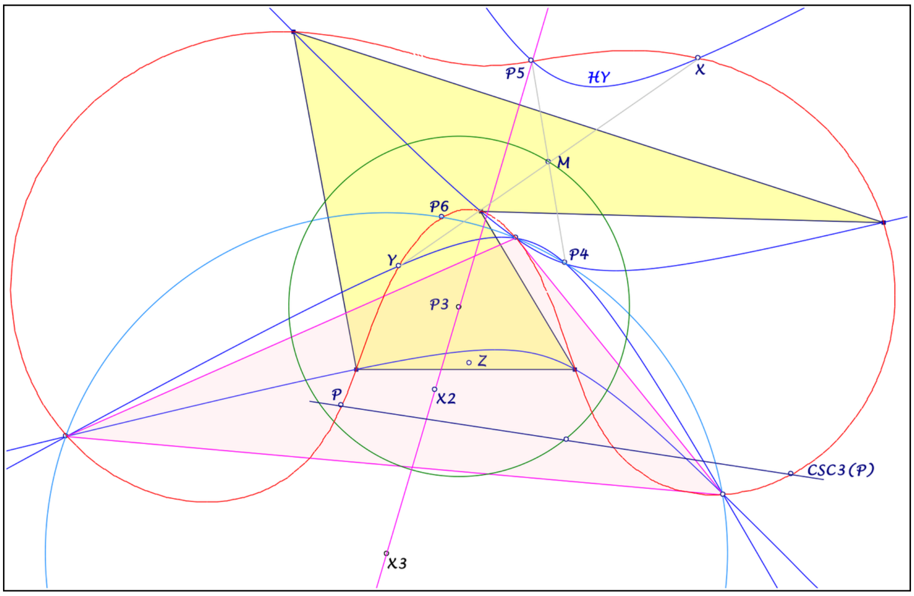
Message: #121
Date: 2020-02-23
From: eckart_schmidt@t-online.de
Subject: 5P-s-P3,4,5,6 Geometry

```
Dear Bernard, dear Chris,

here some well known, some new properties of the points
5P-s-P3,4,5,6,
... further the midpoint M of 5P-s-P4, 5P-s-P5
... and the point Z = CSC(5P-s-P4),
... which is the center of Bernard's CSC3,
    swapping 5P-s-P5, 5P-s-P6.

(1) Reference triangle TR, relevant for 5P-geometry,
... has as vertices the intersections -unequal 5P-s-P -
    of 5P-s-Co1
... and an orthogonal hyperbola HY through 5P-s-P4, 5P-s-P5,
    centered in the middle,
... ... with axes parallel to those of 5P-s-Co1.
The circumcircle of TR bears 5P-s-P4 and 5P-s-P6
... and is centered in the reflection of 5P-s-P5 in 5P-s-P3.
The nine-point circle of TR is centered in 5P-s-P3.
The Euler line of TR is 5P-s-P3.5P-s-P5,
... X4 = 5P-s-P5, X3 = reflection of 5P-s-P5 in 5P-s-P3,
... ... X2 divides 5P-s-P3.5P-s-P5 with ratio -1:4.
The Simson line of 5P-s-P4 wrt TR bears the midpoint of
5P-s-P4.5P-s-P5
... with direction orthogonal to the connection
    with isogonal conjugate of 5P-s-P4.
The Simson line of 5P-s-P6 wrt TR
... is parallel to Z.5P-s-P5 through the midpoint of 5P-s-P6
... ... and the 2nd intersection of 5P-s-P4.5P-s-P6 and HY.
The line Z.5P-s-P5 is the CSC-image of the TR-circumcircle.

(2) The 5P-quartic bears the 5P-vertices and 5P-s-P5, 5P-s-P6
... as well as the TR-vertices and the QA-P4 of four 5P-vertices
... and  two further intersections X, Y with the hyperbola HY,
... ... which are CSC3-partner symmetric M.
The 5P-quartic is invariant wrt CSC3,
... CSC3-partner on the quartic have concyclic midpoints
... ... on the nine-point circle of TR (see above).

(3) The CSC-image of HY is a generalized limacon:
... the midpoints for intersections of lines through 5P-s-P6
    and the quartic
... are concyclic on a circle through M and 5P-s-P6
... and the middle of CSC(5P-s-P5) and the CSC-image
... ... of the 2nd intersection of 5P-s-P5.5P-s-P6 and HY,
```

```
... which are collinear with 5P-s-P6.
The limacon bears the double point 5P-s-P6 and X, Y, Z.

(4) The CSC3-image of HY gives also a limacon
... bearing 5P-s-P6, X, Y and the double point Z.

Best regards Eckart

PS. What about the point 5P-s-P2?
```



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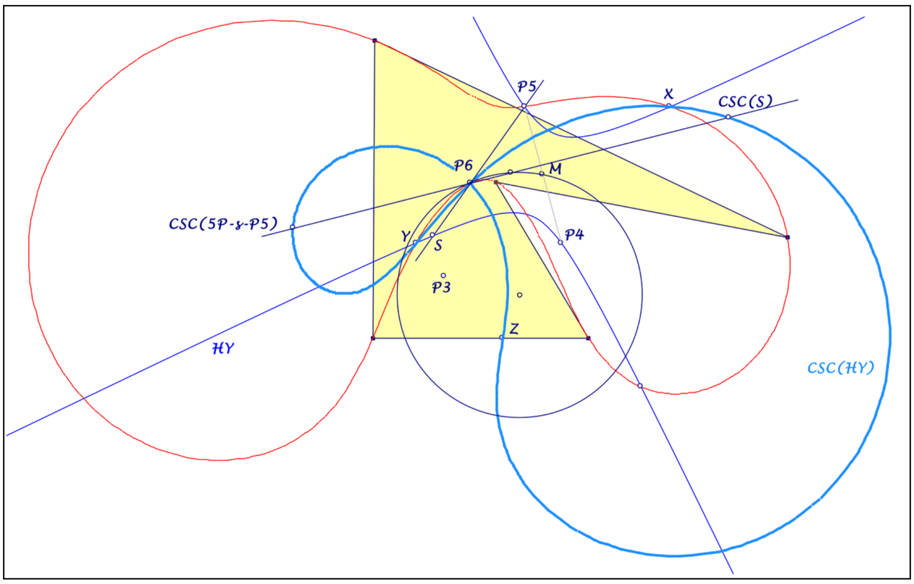

2020-02-23b.pdf

Message: #122
Date: 2020-02-23
From: bernard.keizer@gmail.com
Subject: Re: 5P-s-2Pa,b

```
Dear Eckart,
Very interesting points, mixing 5P and 5L properties !
I had tried in the same way to draw the CSCi(Li), which give 5
circles tangent in Mi to the Miquel circle or the CSCi(Pi), but
without any interesting result ...
Best regards
Bernard
```

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Message: #123
Date: 2020-02-23
From: van10hoven@gmail.com
Subject: Re: 5P-s-2Pa,b

```
Dear Eckart,
Thanks for the nice observation.
I remember we discussed the common points X and Y before.
I wish I had more time to discuss with you and Bernard and the
group like in old times.
But duty calls for me.
Maybe later.
Best regards,
Chris
```

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Message: #124
Date: 2020-02-24
From: bernard.keizer@gmail.com
Subject: Re: 5P-s-P3,4,5,6 Geometry

```
Dear Eckart,
Many interesting properties, some well known as you say and some
new !
Here only some remarks
The Newton circle contains also the middle of P5P6 (CSC3
partners).
Naming F and F' the invariant points of CSC3 anf mp the middle
of PCSC3(P), FF' is the Steiner axis, bisector of PZCSC(P) and
of P5ZP6, and PCSC(P) is the bisector of FmpF' (it is the same
property as for the CSC with the Newton circle instead of the
Newton Line).
Perhaps more interesting, the line PCSC(P) cuts the quartic in 2
other points Q and CSC3(Q), the middle mq is on the Newton
circle and the line is also the bisector of FmqF'. (On your
figure, P5P6 or XY have the same property ...).
Z is CSC(P4) and P5 is CSC3(P6) ; the TR circumcircle is through
P4 and P6 and I don't see how the line ZP5 can be either CSC or
CSC3 of this circle ?
Limaçons de Pascal are normally inverses of a conic wrt one of
it's foci, I suppose that's why you say generalized limaçons.
Anyhow, it gives beautiful curves ...
Best regards
Bernard
```

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Message: #125
Date: 2020-02-24
From: eckart_schmidt@t-online.de
Subject: Re: 5P-s-P3,4,5,6 Geometry

```
Dear Bernard,

thanks for your remarks, you are right,
... Z.5P-s-P5 isn't a CSC of the TR-circumcircle, excuse!

I cannot confirm your observation, that for P and CSC3(P) on the
quartic
... the two other intersections of P.CSC3(P) and the quartic are
also
CSC3-partner,
... but their midpoint lies on the "Newton" circle.

Some further observations;
The quartic intersects the TR-sidelines beside the vertices in
CSC3-partner.
CSC3-circles of the TR-sidelines intersect in Z
... and three points on the quartic with circumcircle through
5P-s-P5.
X and Y are the only points, which are CSC- and CSC3-partner.

Best regards Eckart
```

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Message: #126
Date: 2020-02-25
From: bernard.keizer@gmail.com
Subject: Re: 5P-s-P3,4,5,6 Geometry

Dear Eckart,
The 3 copples of CSC3 partners on the TR sides are on 4 pairs of circles, which are CSC3 partners (like the 4 lines and 4 circles of the QL).
It's a kind of complete quadricircular, TR is the DT, the Euler circle is the Newton circle and the bicircular quartic is a kind of Van Rees curve ...
Maybe there are plenty of other properties !
In fact, it seems you have found the reference DT of the 5P.
Congratulations !!!
Your 3 points apart of Z are the CSC3 of the TR vertices and their circumcircle is the CSC3 of the TR circumcircle, logically through P5, CSC3 of P6.
Best regards
Bernard

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Message: #127
Date: 2020-02-25
From: eckart_schmidt@t-online.de
Subject: Re: Points on QA-Cu1

```
Dear Bernard, dear Chris,

some observations wrt QA-Tf16,
... which maps QA-Cu1 to itself,
... perhaps not all mentioned in EQF:

(1) Fixed points of QA-Tf16 lie on QA-Cu1,
... they have tangential QA-P3
... and give a QA' with diagonal triangle of QG-P16 points.
(2) For P on QA-Cu1 QA-Tf16 is QA-Tf2 of QA'.
(3) For P on QA-Cu1 QA-Tf16(P) = res(P,QA-P3).
(4) For P on QA-Cu1 QA-Tf16(P) = cb(P) wrt QA-P4,P1,P2,P3,P4.
(5) QA-Tf16(P) = 5PCSC(P) wrt P,P1,P2,P3,P4
(6) QA-Tf16(P), cb(P) wrt QA-P4,P1,2,3,4 are collinear with
    QA-P4.

QA-Cu1 is the locus of points
... with 5PCSC(P) wrt P,P1,2,3,4 = cb(P) wrt QA-P4,P1,2,3,4.

Best regards Eckart
```

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Message: #128
Date: 2020-02-26
From: bernard.keizer@gmail.com
Subject: Re: Points on QA-Cu1

```
Dear Eckart,
You have already mentionned (1) to (4) before.
If I'm not wrong, (5) is new and leads to the beautiful
definition of QA-Cu1 in red.
I don't understand (6), as the 2 points are the same after (4) ;
perhaps a typo ?
Best regards
Bernard
```

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Message: #129
Date: 2020-02-26
From: eckart_schmidt@t-online.de
Subject: Re: Points on QA-Cu1

```
Dear Bernard,

observation (6) holds for any point,
... (4) only for points on QA-Cu1.

Best regards Eckart
```

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Message: #130
Date: 2020-02-27
From: bernard.keizer@gmail.com
Subject: Re: Points on QA-Cu1

```
Dear Eckart,
Thanks for the explanation !
If I understand correctly, it holds for a 5 P with vertices Pi
and QA-P4 isgi and CSC partners Qi and QA-P4 isg'i :
cb(Pi) wrt (Pj, j not equal to i, isgi) aligned with Qi and isgi
cb(Qi) wrt (Qj,j not equal to i, isg'i aligned with Pi and isg'i
Best regards
Bernard
PS As you know, I don't know how to draw the cb of a point
without drawing circular cubics and I hardly can check this
beautiful property ...
```

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Message: #131
Date: 2020-02-28
From: bernard.keizer@gmail.com
Subject: Re: Points on QA-Cu1

```
Dear Eckart,
As announced, I tried to reproduce your property by drawing a
pivotal circular circumcubic of P1,2,3 and 4 and isg5 with pivot
in P5.
It takes time, but I've succeeded !
cb(P5) is the tangential of P5 on this cubic and lies on the
line Q5isg5.
This point isn't on any of the twin cubics or quartics of the
5P.
Best regards
Bernard
```

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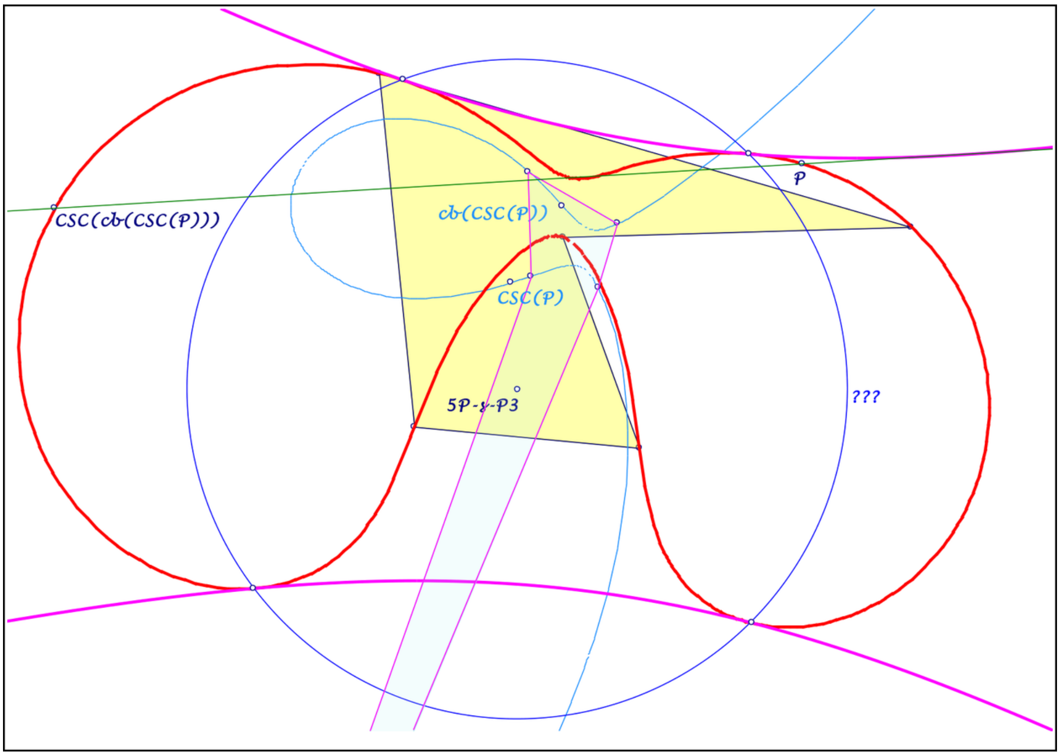
Message: #132
Date: 2020-02-29
From: eckart_schmidt@t-online.de
Subject: Transformation for 5P-Quartic

```
Dear Bernard,

your CSC3 for 5P,
... centered in CSC(5P-s-P4) swaps 5P-s-P5 and 5P-s-P6
... and maps the 5P-quartic to itself.
Here is another transformation with this property:
... CSC maps a point P of the quartic
... to a point CSC(P) on the cubic,
... which is cb-invariant wrt CSC(5P),
... so we get cb(CSC(P)) on the cubic
... and CSC(cb(CSC(P))) = TF(P) on the quartic again.
The lines P.TF(P) with P and TF(P) on the quartic
... envelope a conic (see attached file) ...

I made several constructions with following observations:
... If there are common points of the quartic and the conic
    (up to four),
... they are contact points on a circle round 5P-s-P3.

Best regards Eckart
```



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Message: #133
Date: 2020-02-29
From: eckart_schmidt@t-online.de
Subject: cb-approximation

```
Dear Bernard,

perhaps helpful an approximation for cb:

In QFG-message 3575 I defined for a 5P circles Ci(P)
... with the property that P and cb(P) have the same circle
Ci(P).
If you construct for six points Pi and an arbitrary point X
... some circles Ci(X) wrt five of the six points
... and variegate X so that the circles coincide,
... you have an approximation of cb for the six Pi.

Best regards Eckart
```

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Message: #134
Date: 2020-03-02
From: bernard.keizer@gmail.com
Subject: Re: cb-approximation

```
Dear Eckart,

Thanks a lot for this simple construction !
In this case, cb(6 points) = CB(6 points + 2 circular points)
and cb(6 points) is cb of each of the 6 points wrt the 5 others
...
Best regards
Bernard
```

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Message: #135
Date: 2020-03-02
From: bernard.keizer@gmail.com
Subject: Re: Transformation for 5P-Quartic

Dear Eckart,
Wunderbar !
Great, this idea of using the property that the cubic of the 5 CSC and of the 5 CSC of the QA-P4 is cb invariant wrt the 5 CSC!
In fact and more precisely, the cubic is a Van Rees focal circular cubic with focus in CSC(5P-s-P5) and is cb invariant wrt the 5 CSC, the cb transformation on the cubic being pivotal with pivot the infinity point of the Newton Line (locus of the middles of the CSC and the CSC of the QA-P4).
This means in particular that the asymptote is the parallel to the Newton Line through the point 5P-s-P4 of the 5 CSC and is the reflexion of the parallel to the Newton Line in CSC(P5) in this Newton Line. It means also that the cb partners are on parallel to the Newton Line.
In particular, the Newton Line cuts the cubic in 2 points S and S', which are at the same time CSC2 and cb partners.
The cb partners are isogonal conjugates wrt the triangle CSC(5P-s-P5)SS'.
The asymptote cuts the cubic in a point Q, which is cb partner of the infinity point of the Newton Line.
The parallel to the Newton Line through CSC cuts the cubic in a 2nd point q, which is cb(5P-s-P5) and CSC2(Q) and lies, as well known, on the perpendicular bisector of SS'.
Last, the 2 points where the 2nd Steiner axis of CSC2 cuts the cubic are 2 points Z and Z', centers of anallagmaty of the cubic; these 2 points are their own cb partners, as the tangents in these points are parallel to the Newton Line.
It follows from these properties that the middles of CSC(S)CSC(S') and of CSC(Q) and CSC(q) are on the circle with center 5P-s-P3 through the middles of CSC3 partners.
Much more interesting, I can confirm your property of the conic as envelop of the lines through the CSC partners on the quartic of 2 cb partners on the cubic.
You have as particulars tangents to the cubic the lines CSC(S)CSC(S'), CSC(Q)5P-s-P6, CSC(q)CSC(5P-s-P5) and the tangents to the quartic in CSC(Z) and in CSC(Z'), these 5 lines defining exactly your conic.
I can also confirm your observation that the conic is tangent to the quartic in 4 points on a circle centered in 5P-s-P3, but I can't find more.

That's all for today, but I'm sure there are other properties
(for example, the cubic is also invariant in the 2 other CSC
centered in a point S or S' and swapping CSC(5P-s-P5) and S' or
S ...).
Best regards
Bernard
PS The bicircular quartic is also an anallagmatic curve, but I
don't know the centers ...

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Message: #136
Date: 2020-03-02
From: bernard.keizer@gmail.com
Subject: Re: Transformation for 5P-Quartic

Dear Eckart,
It isn't true that the cb partners are isogonal conjugates wrt
the triangle CSC(5P-s-P5)SS'.
I hope the rest is correct!
Best regards
Bernard

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Message: #137
Date: 2020-03-02
From: eckart_schmidt@t-online.de
Subject: Reference Triangle for 5P-Quartic

```
Dear Bernard,

here another sight of the reference triangle TR for a
5P-quartic:
Consider the circles Ci(P) (see QFG-3575)
… for points P on 5P-s-Co1, which degenerate to lines,
… and their intersections with PPi  (Pi fixed 5P-vertex).
… The loci are 5 orthogonal hyperbolas HYi ,
… with common points 5P-s-P5 and the TR-vertices.
… The centers of HYi lie on the nine-point circle of TR,
… centered in 5P-s-P3 through the midpoint of 5P-s-P4,5.

Best regards Eckart
```

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Message: #138
Date: 2020-03-02
From: eckart_schmidt@t-online.de
Subject: Re: Reference Triangle for 5P-Quartic

```
Dear Bernard,

what about the observation,
… that there are 5P with no real reference triangle?

Best regards Eckart
```

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Message: #139
Date: 2020-03-03
From: eckart_schmidt@t-online.de
Subject: Re: Transformation for 5P-Quartic

```
Dear Bernard,

WHOW, that was a hard work to reproduce
… your many interesting and most new properties
… which also confirm and specify my observations!
Admirable how you have kept overview in the relations of cubic
and quartic!

Beside your correction in #136
… and the addition CSC(5P-s-P5) in line18
… and the tangent CSC(q)5P-s-P5 instead of CSC(q)CSC(5P-s-P5),
I think all results are correct, my compliments!

Wrt the anallagmaty of the quartic
… your anallagmatic centers Z and Z' for the cubic can be used:
… For the quartic the anallagmatic centers are
… CSC(Z).5P-s-P5 ^ CSC(Z').5P-s-P6 and CSC(Z).5P-s-P6 ^
CSC(Z').5P-s-P5.

Best regards Eckart
```

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Message: #140
Date: 2020-03-03
From: bernard.keizer@gmail.com
Subject: Re: Transformation for 5P-Quartic

```
Dear Eckart,
Thanks for your compliments !
Using the 10 points CSC(Pi) and CSC(QA-P4i) on the cubic and the
cb partners, you get 10 more tangents to your conic through the
points Pi and QA-P4i of the quartic.
Have you identified the center of the conic ?
Thanks also a lot for the centers of anallagmaty of the quartic
(they are both on the 2nd Steiner axis of the CSC3 and both
circles of inversion pass through he fixed points of this CSC3).
Best regards
Bernard
```

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Message: #141
Date: 2020-03-03
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as Isocubic

Dear Bernard, dear Chris,

QL-Cu1 can be considered as isocubic:
... reference triangle with vertices QL-P1, QL-2P2a,b (see PS),
... isoconjugation: isogonal conjugacy *,
… bipartite: pivotal with pivot in the infinity point of QL-P1,
… … for P on QL-Cu1 are P*, CSC(P) and QL-P1 collinear,
… non bipartite: nonpivotal with root in QL-L1^QL-L6
… … for P on QL-Cu1 holds P* = CSC(P).

Bernard, I became observant in your messages 135, 136
… wrt the isogonal conjugate of CSC(5P-s-P5)SS'.
Your 5P-cubic can also be considered as pivotal isocubic.
The corresponding isogonality
… swaps the intersections of 5P-cubic and 5P-quartic,
… swaps your Q and q as well as your Z and Z'.

Best regards Eckart

PS: If QL-Cu1 is bipartite, QL-2P2a,b are the CSC-partner
... on a QL-L1-perpendicular line through QL-L1^QL-L6.

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Message: #142
Date: 2020-03-03
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1 as Isocubic

Dear Eckart,
For a monocursal QLCu1, the CSC partners are isogonal wrt any reference triangle of any inscribed QL.
Here, as I mentionned already, you may consider the QL formed by the lines CSC(5P-s-P5)S, CSC(5P-s-P5)S' and 2 times the Newton Line.
Then CSC(5P-s-P5)SS' is a dobble reference triangle and the CSC partners are isogonals wrt this triangle.
The 5P cubic is a monocursal QL-Cu1. The CSC2 partners are isogonal wrt this tiangle.
New for me is the fact that the intersections between the cubic and the quartic were CSC2 partners.
Best regards
Bernard
PS We have a really comfortable forum and so many new properties that it would be highly regrettable that they don't figur one day in EQF ...

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Message: #143
Date: 2020-03-04
From: bernard.keizer@gmail.com
Subject: Re: Reference Triangle for 5P-Quartic

Dear Eckart,
I tried to find a possible answer to your observation.
The vertices of the TR triangle are the 3 intersections (other than the 5P) between the circumconic and the circumquartic of the 5P.
The CSC of the circumconic is another quartic, a kind of limaçon with node in 5P-s-P6 through the CSC of the 5P and the CSC of the 3 TR vertices.
The CSC of the circumquartic is a cubic passing through the same 8 points.
The TR circumcircle passes through 5P-s-P4 and 6 and it's CSC is a line through CSC(5P-s-P4) and the CSC of the 3 TR vertices.
If the circle has no intersections with the quartic, the CSC line has no intersections with the cubic, which could mean that this line is parallel to the Newton Line and intersect the line ZZ' outside of the segment ZZ' ? The line has no other intersection than CSC(5P-s-P4) with the limaçon.
The same way, looking for a reverse construction for a given QL-Cu1, it appears that there are many 5Ps on the QL-Cu1 for which the cubic is cbinvariant ; the circumconic of the 5P must pass through the point Q, in this case the point 5P-s-P4 is on the asymptote of the cubic.
There are then as many quartics CSCs of the cubic than 5Ps defining different CSCs ...
Best regards
Bernard

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Message: #144
Date: 2020-03-07
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1 as Isocubic

```
Dear Bernard,

the main result of my #141 should be the general case,
... for it is not mentioned in EQF.
Wrt your cubic:
... Your cubic must not be monocursal
... and the points S, S' must not be real.
If I am not wrong, the isogonality wrt CSC(5P-s-P5)SS'
... gives only CSC-partner for the intersections of cubic
    and quartic.

Best regards Eckart

PS. What do you mean with
    "... two times the Newton line ..." for a QL?
```

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Message: #145
Date: 2020-03-07
From: eckart_schmidt@t-online.de
Subject: Circum-Conic, -Cubic and -Quartic for a 5P

```
Dear Bernard,

I gather some aspects of my current sight of our 5P-geometry,
... using your cubic for the pentangle CSC(5P),
... based on the points 5P-s-P3,4,5,6
    and the transformations CSC,
... of course well known for you.

For a 5P we get your cubic as "QL-Cu1" (see QFG 3675 and PS)
... with "QL-P1" = CSC(5P-s-P5),
... "CSC" swapping 5P-s-P6 and 5P-s-P4,
... "QL-L1" line through the midpoints of CSC(Pi) and
    "CSC"(CSC(Pi)).
This is not a circumcubic of 5P,
... but the CSC-image of this cubic is a circumquartic QU of 5P.

If we construct your cubic for the CSC-image of 5P,
... we get a circumcubic CU of 5P,
... as "QL-Cu1" with "QL-P1" = 5P-s-P5,
... "CSC" swapping 5P-s-P6 and 5P-s-P4 of CSC(5P),
... "QL-L1" = Newton line through the midpoints of Pi and
    "CSC"(Pi).

Finally the circumconic CO = 5P-s-Co1 of 5P.

Transformations beside CSC:
... CSC3, centered in CSC(5P-s-P4), swapping 5P-s-P5, 5P-s-P6,
... ... maps the quartic QU to itself,
... CSCx, centered in 5P-s-P5, swapping 5P-s-P6, 5P-s-P4
    of CSC(5P),
... ... maps the cubic CU to itself (see above).

Intersections (beside the 5P-vertices):
... CO^CU: 2nd intersection of CO
... ... and a parallel to the Newton-line of CU through 5P-s-P4.
... CO^QU: Vertices of the reference triangle for 5P,
    not always real
... ... (intersections unequal 5P-s-P4 of CO
... ... and a circle round the reflection of 5P-s-P5 in 5P-s-P3
... ... through 5P-s-P4 and 5P-s-P6).
... CU^QU: 5P-s-P5 and two intersections on 5P-s-P5.5P-s-P6,
    not always real.

Final remarks:
```

```
cb(P) = Cayley-Bacharach point wrt 5P, P and circular points,
... maps the cubic CU to itself
... with image in the 3rd intersection of CU and 5P-s-P5.CSCx(P)
... ... on a parallel to the Newton line through P,
... special: cb(5P-s-P5) = CSCx(CO^CU).
There are several pairs of points with middle on the Newton line
of CU:
... 5P-s-P4, 5P-s-P5 and 5P-s-P4, cb(5P-s-P5)
    and CSC(5P-s-P5), 5P-s-P4 of CSC(5P).
... and the final intersections of CU ^ QU on 5P-s-P5.5P-s-P6.

The properties of your cubic in #135 can directly be translated,
... replacing 5P-s-P4 by 5P-s-P4 of CSC(5P),
... ... 5P-s-P5 by CSC(5P-s-P5) = 5P-s-P5 of CSC(5P),
... ... 5P-s-P6 by 5P-s-P6 and CSC2 by CSCx.

Best regards Eckart

PS: The cubic QL-Cu1 can easily be constructed with QL-P1,
    CSC and QL-L1,
    ... taking the intersections of CSC-circles of parallels
        to QL-L1
    ... with the reflection of the parallels in QL-L1.
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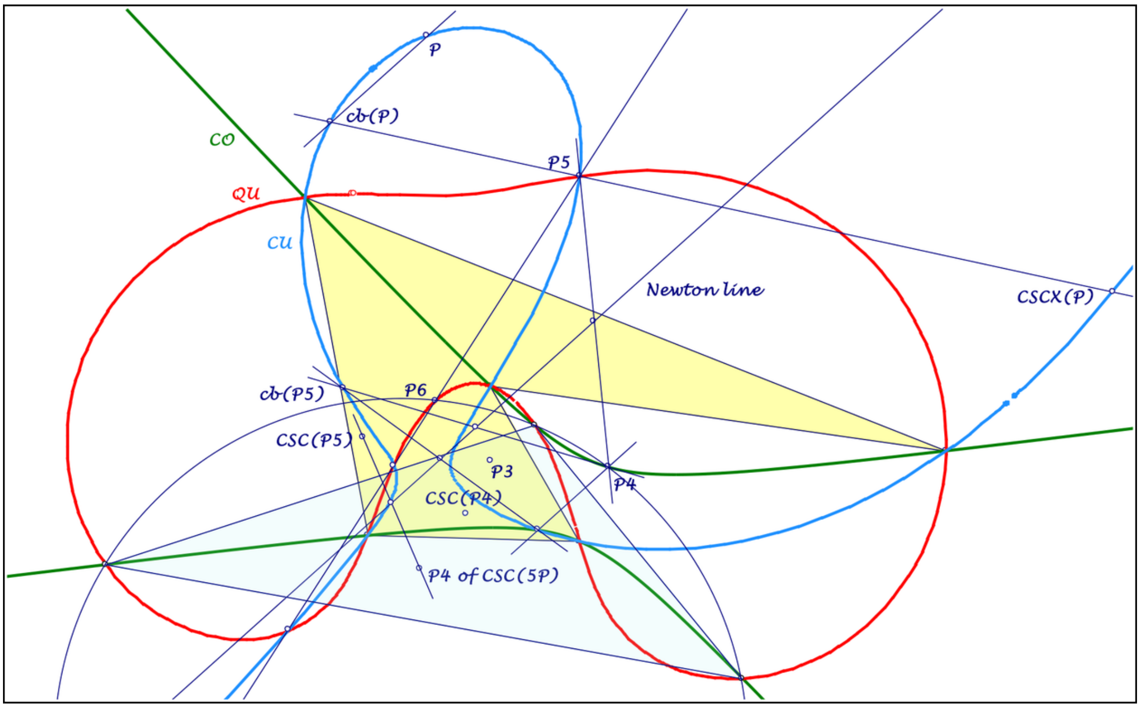

2020-03-07.pdf

Message: #146
Date: 2020-03-07
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1 as Isocubic

Dear Eckart,
I answer your 2 questions in the case of the monocursal Van Rees.
1) The intersections between the quartic of the 5P and the cubic of the CSC of the 5P are obviously CSC partners, as the cubic and the quartic are CSC partners ; but they are also CSC2 partners on the cubic and CSC3 partners on the quartic.
2) For a VR with it's CSC centered in the focus M, any parallel to the Newton Line cuts the curve in 2 real points P and cb(P) ; it holds that CSC(P) is aligned with M and cb(P) and that cb(CSC(P)) is aligned with M and P (the transformation cb(P) to CSC(P) is pivotal with pivot M).
If you consider now the QL formed by the 2 lines MP and Mcb(P) and the 2 parallels to the newton Line, CSC(M) is the infinity point on the Newton Line.
The the CSC partners on the curve are isogonal wrt the 2 triangles MPcb(P) and MCSC(P)cb(CSC(P)) (which are reference triangles of this QL).
(The it holds that the sum of the directions wrt a given line of the 4 lines of this QL is 2*Newton Line + 2*1rst Steiner axis)
In the limit position, with S and S' cb as well as CSC partners on the Newton Line, you get 2 times the Newton Line and the lines MS and MS' (tangents from M to the cubic).
The sum of the directions is the same and MSS' is a dobble reference triangle (hence the property that CSC partners are isogonal wrt this triangle).
Best regards
Bernard
PS For the bicursal curve, the parallel to the Newton Line give CSC partners isogonal wrt a triangle MVV' with V and V' symmetric wrt the Newton Line.
What are then the cb partners ?

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Message: #147
Date: 2020-03-07
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1 as Isocubic

```
Dear Eckart,
Please correct my PS !
For the bicursal curve, the perpendiculars to the Newton Line
give CSC partners and isogonal partners wrt MVV'.
The parallels to the Newton Line give cb partners like in the
monocursal case.
With my apologise
Best regards
Bernard
```

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Message: #148
Date: 2020-03-08
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as Isocubic

```
Dear Bernard,

thanks for explanations in #146,
... reason for my misunderstanding:
You considered CSC of the cubic,
... I considered CSC of the 5P.

Best regards Eckart
```

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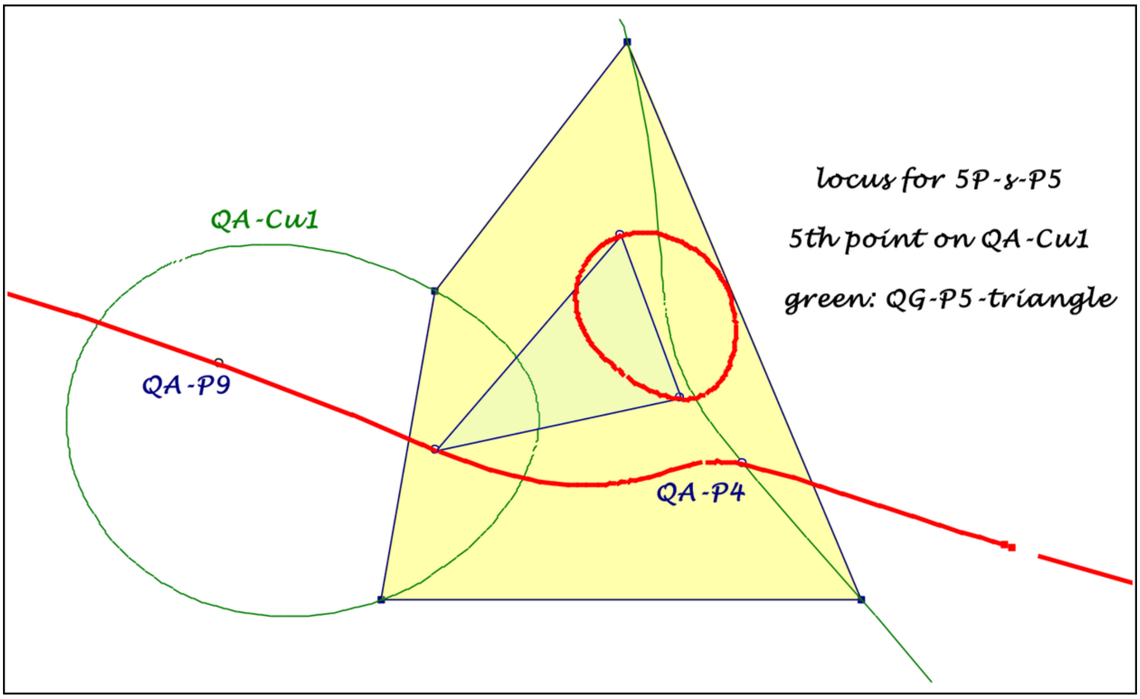
Message: #149
Date: 2020-03-08
From: eckart_schmidt@t-online.de
Subject: 5P-Geometry for QA

```
Dear Bernard, dear Chris,

if we take for a QA = P1P2P3P4 a variable point P5 on QA-Cu1
... and consider the loci of 5P-points 5P-s-Pi,
... there are three interesting observation:

For 5P-s-P1 we get the cubic QA-Cu1.
For 5P-s-P4 we get a pivotal isocubic:
... reference triangle QA-Tr1,
... isoconjugation QA-Tf2,
... pivot: reflection of QA-P4 in QA-P3,
... bearing QA-P3 and the infinity point of QA-Cu1.
For 5P-s-P5 we get a pivotal isocubic (see attached file):
... reference triangle QG-P5-triangle,
... isoconjugation swaps QA-P4 and QA-P9,
... pivot QA-P4,
... bearing QA-P4 and QA-P9.

Best regards Eckart
```



2020-03-08.pdf

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Message: #150
Date: 2020-03-08
From: bernard.keizer@gmail.com
Subject: Re: Circum-Conic, -Cubic and -Quartic for a 5P

Dear Eckart,
I always like your summaries and figurs !
It is remarkable that the 5P and their P3,4,5,6 and the CSC, giving the CSC(P) of the 5P and their P3,4,5,6 define the twin CO,CU and QU as well as the twin CI, circumcircles of the TR triangles.
The 5P and the CSC(P) define the 2 twin conics.
P6 is the same and P5 of the CSC is CSC(P5).
The Newton Line is the same through the middles of P4P5 and P4P5 of the CSC(P).
The CSC2 is centered in CSC(P5) and swaps P6 and P4 of the CSC(P), the CSCx is centered in P5 and swaps P6 and P4 of the CSC(P).
Then we have the 2 twin cubics and their CSC, the 2 twin quartics.
The twin cubics are invariant in the CSC 2 or x descibed above and in the cb transformations of the 5P or of the CSC(P).
The 2 quartics are invariant in 2 transformations CSC3 or y (CSC3 = CSC*CSC2*CSC) and CSC*cb*CSC.
The twin CI are centered in the reflexions of P5 in P3 and pass through P4 and P6.
Last, as mentionned in my message 143 (to which you didn't answer), you could add the CSC of the 2 CI, which are 2 lines through CSC(P4) and through the CSC of the TR vertices and the same for the CSC(P) and the CSC of the 2 conics, which are quartics (limaçons or strophoïds ?) through the 5P or the CSC of the 5P and through the CSC of the TR vertices.
The interest is that the line CSC(CI) intersect the corresponding cubic and limaçon in these CSC of the TR vertices.
Thanks for your initiative of beginning to summarise properties of the pentangle, as you see it was very stimulating !
Best regards
Bernard

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Message: #151
Date: 2020-03-08
From: eckart_schmidt@t-online.de
Subject: 5P/6P-Geometry for QA

```
Dear Bernard, dear Chris,

if we take for a QA = P1P2P3P4 two points P5, P6,
... which are QA-Tf2-partner on QA-Cu1,
... and consider the locus for 6P-s-P1, we get a line.

For a 6P the six 5P-s-P5 are concyclic (not in EPG)
... on a circle 6P-s-Ci1, centered in 6P-s-P1.
For the 6P, described above,
... these circles have two common points in QA-P9 and
    QA-Tf4(QA-P41),
... and their bisector is the locus for the 6P-s-P1.

Best regards Eckart
```

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Message: #152
Date: 2020-03-08
From: bernard.keizer@gmail.com
Subject: circular cubic, focus and asymptote

```
Dear Chris, dear Eckart,
You often asked for simple old properties not in EQF.
A circular cubic has a focus F where the tangents in the
circular points intersect.
It has also a real asymptote and cuts it's asymptote
in a point Q.
Any line through Q cuts the circular cubic in 2 points
equidistant from the focus F.
For QA-Cu1, the focus is QA-P9, for QL-Cu1, the focus is QL-P1.
Q is not in EQF fo the 2 curves, but the property holds and is
not in EQF.
Best regards
Bernard
```

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Message: #153
Date: 2020-03-08
From: van10hoven@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Bernard,
You are right, these are beautiful and relatively simple properties for a circular cubic.
I didn't check them.
Do you have any references or proof for them?
Best regards,
Chris

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Message: #154
Date: 2020-03-09
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Chris,
It's a real pleasure to have an answer from you.
It's a long time that it hadn't happened !
1) My reference is a serie of 3 aticles by Roux and Tixier in the french revue Quadrature
They are mentionned in my 1rst article on my blog (reference 43 in EQF) in the bibliography with number 29.
They are also mentionned in Mathcurve under circular focal cubic.
2) The proof uses the Punktechnung auf der Zirkularkurve in Geometrie auf der Zirkularkurve Eckart Schmidt reference 15b in EQF
If we name F the focus, Q the intersection of the asymptote with the cubic, K1 and K2 the circular points, Ω the infinity point of the asymptote and X and Y the 2 intersections of a line through Q with the cubic, we have
on the asymptote Q + 2Ω = 0
on the line through Q Q + X + Y = 0
on the infinity line K1 + K2 + Ω = 0
and therefore X + Y + 2K1 + 2K2 = 0, which means that X and Y are on a circle bitangent in the circular points to the cubic, id est a circle centered in F.
Best regards
Bernard

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Message: #155
Date: 2020-03-09
From: van10hoven@gmail.com
Subject: Archive backups of Yahoo Quadri-Figures Group,

Dear friends,

Archive Quadri-Figures Group

It is a pleasure telling you that the old Quadri-Figures Group at Yahoo is fully migrated to a separate group at Groups.io. No messages can be sent to it. It’s just meant to be an archive. Not only the messages can be inspected, but also the corresponding attachments. It was quite a job to recollect them!
When you are looking for a QFG-message with a specific number (e.g. #255), then use the Search option with keyword “Message: 255” and you will be directed to the right message.
Thanks to Wilson Logan for his invaluable support.
See: https://groups.io/g/Quadri-Figures-Group

Also César Lozada made another and beautiful archive backup of QFG.
See: www.qfg.99on.com (http://www.qfg.99on.com).
You can go to a specific nnnn number message by typing:
www.qfg.99on.com/message.php?msg=nnnn (http://www.qfg.99on.com/message.php?msg=nnnn).
You can go to a specific tttt topic by typing:
www.qfg.99on.com/message.php?topic=tttt (http://www.qfg.99on.com/message.php?topic=tttt).

Archive Hyacinthos
César also made an accessible archive for Hyacinthos:
www.hyacinthos.99on.com (http://www.hyacinthos.99on.com)
You can go to a specific nnnn number message by typing:
www.hyacinthos.99on.com/message.php?msg=nnnn (http://www.hyacinthos.99on.com/message.php?msg=nnnn).
You can go to a specific tttt topic by typing:
www.hyacinthos.99on.com/message.php?topic=tttt (http://www.hyacinthos.99on.com/message.php?topic=tttt).

Archive Advanced Plane Geometry
César also made an accessible archive for Advanced Plane Geometry: www.adgeom.99on.com (http://www.adgeom.99on.com).
You can go to a specific nnnn number message by typing:
www.adgeom.99on.com/message.php?msg=nnnn (http://www.adgeom.99on.com/message.php?msg=nnnn).

```
You can go to a specific tttt topic by typing:
www.adgeom.99on.com/message.php?topic=tttt (
http://www.adgeom.99on.com/message.php?topic=tttt ).

Now for the foreseeable future, we will be able to find our way
back to our groups as well as many new interested geometrists in
a more distant future.

Best regards,
Chris
```

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Message: #156
Date: 2020-03-09
From: garciacapitan@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Archive backups of Yahoo

---

Excellent!

El lun., 9 mar. 2020 a las 21:28, Chris (<van10hoven@gmail.com>)
escribió:

> Dear friends,
> *Archive Quadri-Figures Group*
>
> It is a pleasure telling you that the old Quadri-Figures Group
at Yahoo is
> fully migrated to a separate group at Groups.io. No messages
can be sent to
> it. It’s just meant to be an archive. Not only the messages
can be
> inspected, but also the corresponding attachments. It was
quite a job to
> recollect them!
> When you are looking for a QFG-message with a specific number
(e.g. #255),
> then use the Search option with keyword “Message: 255” and you
will be
> directed to the right message.
> Thanks to Wilson Logan for his invaluable support.
> See: https://groups.io/g/Quadri-Figures-Group
> Also César Lozada made another and beautiful archive backup of
QFG.
> See: www.qfg.99on.com.
> You can go to a specific nnnn number message by typing:
> www.qfg.99on.com/message.php?msg=nnnn.
> You can go to a specific tttt topic by typing:
> www.qfg.99on.com/message.php?topic=tttt.

> *Archive Hyacinthos*
>
> César also made an accessible archive for Hyacinthos:
> www.hyacinthos.99on.com
> You can go to a specific nnnn number message by typing:
> www.hyacinthos.99on.com/message.php?msg=nnnn.
> You can go to a specific tttt topic by typing:
> www.hyacinthos.99on.com/message.php?topic=tttt.

> *Archive Advanced Plane Geometry*
> César also made an accessible archive for Advanced Plane
Geometry:

```
> www.adgeom.99on.com.
> You can go to a specific nnnn number message by typing:
> www.adgeom.99on.com/message.php?msg=nnnn.
> You can go to a specific tttt topic by typing:
> www.adgeom.99on.com/message.php?topic=tttt.
> Now for the foreseeable future, we will be able to find our
way back to
> our groups as well as many new interested geometrists in a
more distant
> future.
> Best regards,
> Chris
---
Francisco Javier García Capitán
http://garciacapitan.000webhostapp.com/
```

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Message: #157
Date: 2020-03-10
From: eckart_schmidt@t-online.de
Subject: Re: Archive backups of Yahoo Quadri-Figures Group,

```
Dear Chris,

thank you very much for this archive,
... now we can make concrete references!

Best regards Eckart
```

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Message: #158
Date: 2020-03-10
From: eckart_schmidt@t-online.de
Subject: Re: Circum-Conic, -Cubic and -Quartic for a 5P

```
Dear Bernard,

your message 150 is a good summary
... of observations wrt the twin aspect,
... thanks a lot!
I cannot add new observations,
... there are a lot of transformations
... but less points and lines for applications.
For example: What about the point 5P-s-P2?

Best regards Eckart

PS. Wrt CSC2 there will be a typo.
```

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Message: #159
Date: 2020-03-10
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

that is an amazing property in #152! But I suppose,
... that my "Punktrechnung auf der Zirkularkurve"
    cannot prove it,
... for the focus as common tangential of the circular points
... must not be a point of the circular cubic.
You use my "Punktrechnung" in a different interpretation,
... for me the infinity point of the asymptote is the
    "Nullpunkt'".
But nevertheless I cannot follow your last conclusion,
... can you give a short explanation?
Can you also precise your references? I cannot find them.
Is the focus of QA-Cu7 the point QA-P41
and for your 5P-cubic the point CSC(5P-s-P5)?

Best regards Eckart

PS: Perhaps I am no longer familiar with my own calculation (15
years ago).
```

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Message: #160
Date: 2020-03-11
From: eckart_schmidt@t-online.de
Subject: 6P-Geometry for 5P

```
Dear Bernard, dear Chris,

if you consider a 5P and a variable point P6
... on the 5P-circumcubic CU (see QPG #145)
... and take 6P-s-P1 of these 6P,
... you get a line through 5P-s-P5,
... parallel to 5P-s-Tf4 of the Newton line of CU.
This line is Ci(P) (see QFG #3575) for a point P,
... which is the 2nd intersection of 5P-s-Co1
... ... and a perpendicular to the Newton line through 5P-s-P4.

Best regards Eckart
```

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Message: #161
Date: 2020-03-11
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Eckart,
After spending many time for finding the articles on the web, I only found this : Quadrature Roux et Tixier give the references (already given in the biblio of my first article on my blog).
In particular, the last of the 3 articles is available at mathcurve.com/focaledevanrees/cubiques tixier.
The Punktrechnung auf der Zirkularkurve holds for any circular curve, provided it is not nodal.
As I mentionned in 152, the focus can be on the curve (QL-P1 for QL-Cu1) or not (QA-P9 for QA-Cu1) ; QA-Cu7 and the twin cubics are QL-Cu1 with QL-P1 in QA-P41 or in 5P-s-P5 and CSC(5P-s-P5).
In all these examples, the property holds !
I used the same calculation as yours, the point Q is the Hauptpunkt (H in your article) with the properties (3) and(7) on page 2 of your article.
Property (3) is used on 3 lines (asymptote, line through Q and infinity line).
The key is perhaps the fact that the infinity line cuts any circle in the circular points and the center of the circle is the pole of the infinity line.
Property (7) is used for the circle centered in the focus through the points X, Y and the 2 circular points.
(X + Y + K1 + K2) + K1 + K2 = 0 means that the 4 points X, Y , K1 and K2 are concyclic on a circle tangent to the cubic in the circular points.
The pole of the infinity line is the focus of the curve (as it is by definition the intersection point of the tangents in the circular points) and the center of the circle.
I must admit it is not really intuitive and it took me time to understand this proof ...
Best regards
Bernard

Message: #162
Date: 2020-03-12
From: bernard.keizer@gmail.com
Subject: Inflexion points of QL-Cu1 and QL-Cu2

Dear Chris, dear Eckart,
For any (non nodal) circular cubic, it is well known that the 3 inflexion points are aligned.
This property can be proved by using Eckart's Punktrechnung.
In particular, the cubic stelloïd QL-Cu2 and it's hessian QL-Cu1 intersect in 3 real points (the 6 others are imaginary) which are their common inflexion points and are aligned.
Best regards
Bernard

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Message: #163
Date: 2020-03-12
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

Dear Bernard,

if you use my "Punktrechnung" with Q = H
… and "Nullpunkt" = O in the infinity point of the asymptote,
… your interpretations of (3) must be corrected:
Wrt collinear on the asymptote:
… Q + O + O = H (evident, see above).
Wrt collinear on the line through Q:
… X + Y + Q = H, that means X + Y = O.
Wrt collinear on the line at infinity:
… K1 + K2 + O = H, that means K1 + K2 = H (see (8)).
This doesn't give your conclusion.

Best regards Eckart

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Message: #164
Date: 2020-03-12
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

my "Punktrechnung" allows the following conclusion:
Any circle through two points X and Y bears the circular points
K1 and K2.
If these points lie on a circular cubic,
    holds (see 7.) X + Y + K1 + K2 = H.
With K1 + K2 = H (see (8)) we get X + Y + H = H,
... that means X, Y, H are collinear (see (3)).

Best regards Eckart
```

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Message: #165
Date: 2020-03-12
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Eckart,
Trying to understand what is worrying you, I read again the article in french by Roux and Tixier and your Geometrie auf der Zirkularkurve.
1) They use a kind of Punktrechnung, but not exactly the same, which explains some differences.
X, Y and Z are aligned if X + Y + Z = O, where O is an inflexion point and not H in your approach
The same way, if 6 points are coconic, the sum is also O instead of 2H.
2) With your Punktrechnung, you prove that any 2 points on the curve are aligned with H, which is remarkable, but obviously wrong !
You have to consider a circle with center the focus of the cubic, which intersects the curve in X and Y and the circular points K1 and K2.
As the focus is by definition the pole of the infinity line, this circle is tangent in K1 and K2 to the lines FK1 and FK2 and therefore to the cubic.
(F is the tangential of K1 and of K2 on the cubic).
That's why we have X + Y + 2K1 + 2K2 = 2H in your Punktrechnung.
It follows X + Y = 0 and X + Y + H = H.
I hope you will be convinced this time !
Otherwise, I don't know what to do ...
Best regards
Bernard

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Message: #166
Date: 2020-03-13
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

Dear Bernard,
please forget my #164, you are right,
… it was an overhasty unchecked wrong result!
I have to revise passage 7. of my "Punktrechnung".
Thanks for your explanations, I try to understand.
Best regards Eckart

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Message: #167
Date: 2020-03-13
From: bernard.keizer@gmail.com
Subject: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

Dear Eckart,
Maybe this could be interesting ...
1) Let's consider the 6 vertices of a QL A, A', B, B', C and C' and the CSC center M = QL-P1.
It is well known that these 7 points form with the 2 circular points a CB system ; these 7 points form a cb sysyem.
In particular, A and A' are cb partners wrt the cb transformation of the 5 points B, B', C, C' and M.
2) Any cubic cb invariant in this transformation with pivot P on AA' will pass through A and A'.
All these cubics are CSC invariant, but the locus of the middles of CSC partners, which contains the middles of AA', BB' and CC' is generally a cubic (line only for QL-Cu1).
The tangentials of the point M on these cubics are the intersection Q of the cubic and it's asymptote.
3) The conic of the 5 points B, B', C, C' and M intersect the cubics in a 6th point S.
The 5P-s-P4 of the 5 points is a point T on the conic and T, S and P are aligned.
The degenerated cubic formed by the conic and the infinity line is also cb invariant with pivot T (it doen't pass through A and A', as T is not on AA').
The asymptotes of the cb invariant cubics with pivot on AA' have their asymptotes parallel to the lines TS.
4) Let's in particular define 4 cubics :
Cu0 formed by the circle through A, B, C and M and the line through A', B' and C' ; the pivot is the 2nd intersection of AA' and the circle. (There are 4 cubics like this).
Cu1 is QA-Cu1 of the QA BB'CC' ; the pivot is CSC(QA-P41) and QG-P1 and QA-P4 are cb as well as CSC partners.
Cu2 is the QL-Cu1 of the QL ; the pivot is the pedal QG-P17 of QG-P1 on AA'.
Cu3 is the cb invariant cubic with pivot in the infinity point of AA'.
5) Some observations :
Q3 and S3 are the same point.
The lines TS0, TS1, TS2 and TS3, parallel in T to the 4 asymptotes, make 2 by 2 angles equal to the angles of the corresponding tangents to the 4 cubics in M, id est the lines MQ0, MQ1, MQ2 and MQ3, but in reverse rotation.
More precisely, let's start with the parallel TS3 to AA' in T.
Any line through T making a clockwise angle θ with TS3 cuts the conic of the 5 points in S and the line AA' in a point T.

```
The cubic with pivot P is cb invariant wrt the 5 points, pass
through A and A' and is also CSC invariant. The tangent in M to
the cubic makes with MQ3 the same angle θ, but counterclockwise.
It is remarkable that this generation of cubics contains the
well-known QA-Cu1 and QL-Cu1 !
Best regards
Bernard
```

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Message: #168
Date: 2020-03-14
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

```
Dear Eckart,
The property holds, as mentionned, for QA-Cu1 and focus QA-P9
(not on the cubic) and for QL-Cu1 with focus QL-P1 (on the
cubic).
It is interesting to notice that this property gives for any
circular (non nodal) cubic an easy way to find the focus.
Lines through the point Q where the cubic cuts it's asymptote
intersect the cubic in pairs of points and their perpendicular
bisectors pass through a fixed point (not necessary on the
cubic), which is the searched focus.
Useful property ...
Best regards
Bernard
```

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Message: #169
Date: 2020-03-14
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
For an ordinary QL, there are 4 degenerated cubics like Cu0 ;
their foci are obviously the circumcenters of the reference
triangle, which lie concyclic on the Miquel circle.
There are 3 Cu1, which are the QA-Cu1 of the 3 QA's of the QL
whith foci the QA-P9.
There is one Cu2 which is the QL-Cu1 with focus in QL-P1.
There are again 3 Cu3 and it is easy to find their foci by using
the property mentionned in 168.
It's amazing to notice that all these foci lie on the Miquel
circle of the QL.
Best regards
Bernard
PS Of course, for an ordinary QA, there are 3 different QL's and
QA-P9 belongs to 3 different Miquel circles, as mentionned in
EQF ...
```

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Message: #170
Date: 2020-03-14
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

wrt #168: I used this destination of the focus already for the
foci in #159.
Here two other examples:
… focus for the cubic in QFG-message 414: QL-P1,
… focus for the cubic in QFG-message 441: QA-P4.

Best regards Eckart
```

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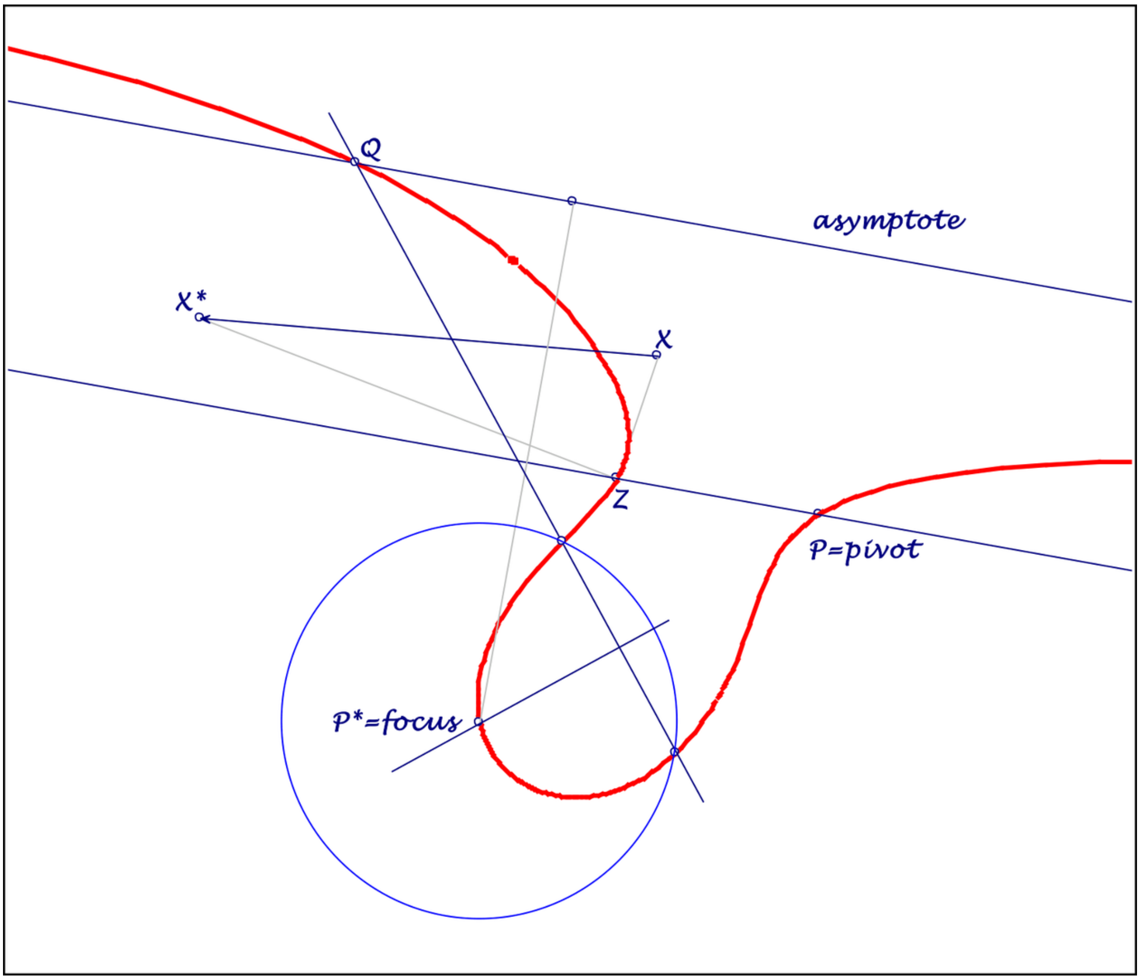
Message: #171
Date: 2020-03-15
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

many circular cubics are of the following structure (see
attached file):
Consider a "CSC" X-->X*, centered in Z, and a pivot P.
The "CSC"-partner on lines through P give a cubic,
... with asymptote parallel PZ through the reflection
    of P* in PZ,
... and focus P*.
You find examples
... in QFG-message 1237 for a QL and QL-Tf1,
... in QFG-message 1272 for a QA and QA-Tf4.

Best regards Eckart
```



2020-03-15.pdf

Message: #172
Date: 2020-03-15
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

```
Dear Eckart,
I had completely forgotten these beautiful curves !
They are are all Van Rees focal circular cubics with focus on
the curve ...
I've printed again your 4 notes, I'll read them  and I try to
answer you in the coming days.
But unfortunately I fear we have all other matters to worry
about ...
Best regards
Bernard
```

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Message: #173
Date: 2020-03-16
From: eckart_schmidt@t-online.de
Subject: QG/QL-Circumquartic

```
Dear Bernard, dear Chris,

I think this is a remarkable circumquartic of a QG,
... their 3 versions for a QL give the same CSC-invariant
    QL-circumquartic,
... already described in QFG#364.
Construction (see attached file):
Consider a QL and a conic, tangent to the diagonals and the
Steiner axes,
... the CSC-partner on the tangents at this conic
... give an unipartite circumquartic for the QL,
... intersecting the conic on QL-Ci6, bearing QL-2P3.
If we consider this quartic for a QG, it bears the following 10
points:
... QG-vertices, QG-2P2, QL-2P3 and two Miquel points
    unequal QL-P1.
The quartic is CSC-invariant (evident)
... and "CSC"-invariant, centered in QL-P1,
... ... swapping the two Miquel points unequal QL-P1,
... and invariant wrt a QG-transformation X-->X^
... ... with X^ 2nd intersection of circles through X
        and opposite QG-vertices.
This quartic for a QL is the locus of points X,
... whose circles (X,Y,Z) with Y, Z opposite QL-points
... have a common 2nd intersection.

The QG-transformation X-->X^, described in QFG#362 (not in EQF),
... has wrt this quartic following property:
The bisectors of XX^ envelope a conic,
... centered in QL-P6, tangent to QL-L1 and the Steiner axes,
... with axes parallel to those of the conic above.

Best regards Eckart

PS: If you are interested in such quartics,
... some examples on my homepage,
... also to find in QFG-messages.
    http://eckartschmidt.de/2013-12-05.pdf
    http://eckartschmidt.de/2013-12-06.pdf
    http://eckartschmidt.de/2013-12-08.pdf
    http://eckartschmidt.de/2015-11-13.pdf
    http://eckartschmidt.de/2015-11-21.pdf
    http://eckartschmidt.de/2015-12-03.pdf
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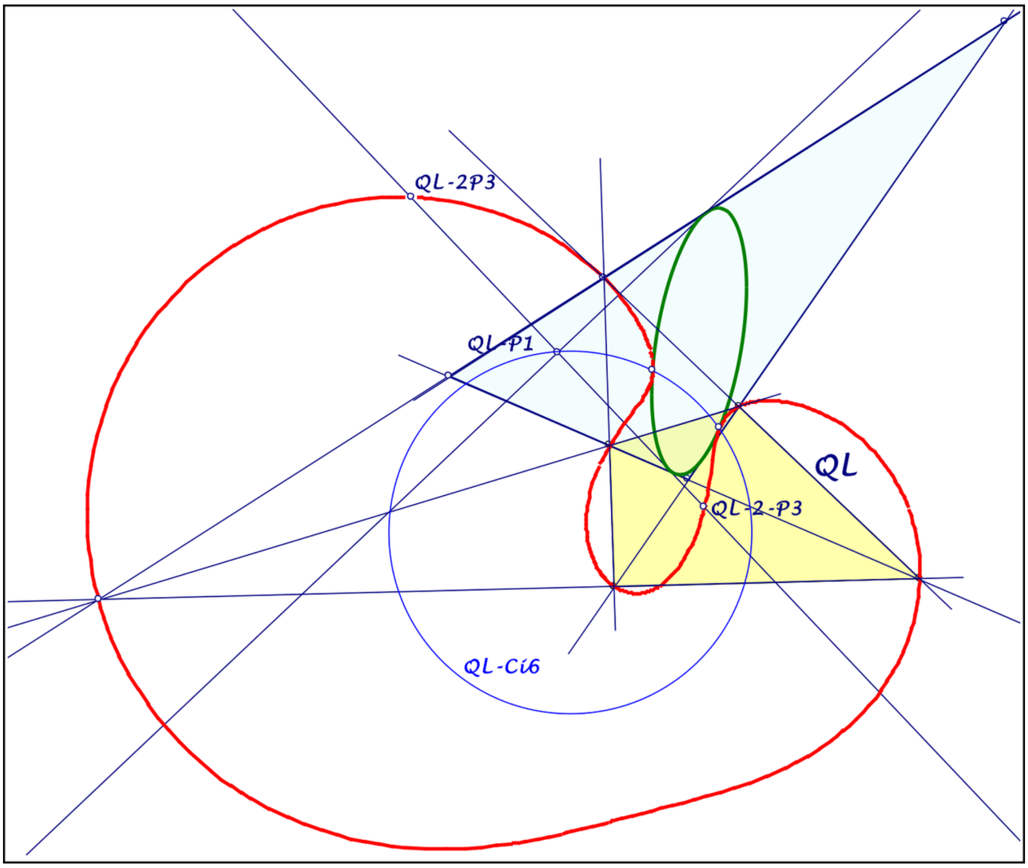
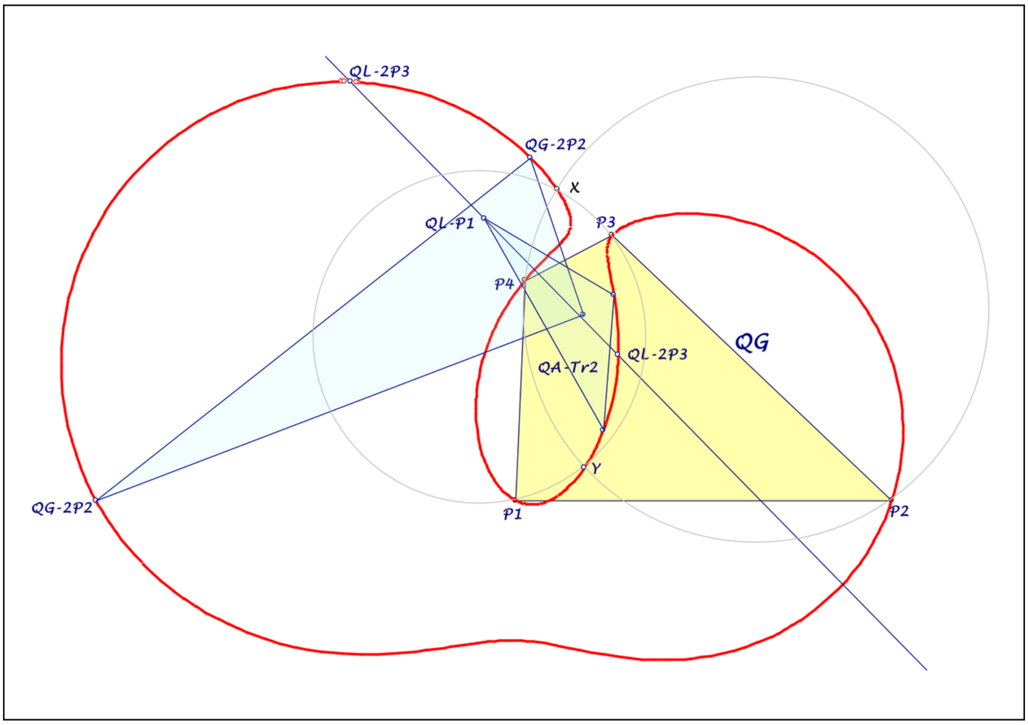


2020-03-16a.pdf



2020-03-16b.pdf

Message: #174
Date: 2020-03-16
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Eckart,
Thanks again for reminding me all these beautiful constructions!
As mentionned, all your curves are Van Rees focal circular cubics.
They are invariant in a CSC centered in the focus and swapping either the 2 intersections with the Newton Line (monocursal) or the 2 points symmetric wrt this line (bicursal).
In 414, the CSC is the the same as given from the QG (P1 and P3 are CSC, as well as P2 and P4).
The Newton Line is the perpendicular bisector of the segment joining the middles of P1P3 and P2P4 and therefore orthogonal to the Newton Line of the QL of this QA (both lins intersect in QA-P1).
In 441, the 2nd CSC is centered in the focus QA-P4 swapping QG-P1 and QL-P1 ; the Newton Line is QG-P1QL-P1.
441 is an example of 1237 (old forum) or 171 (new forum) ; the 2nd CSC is centered in the CSC of the pivot P swapping P and the center Z of the 1rst CSC and the Newton Line is the line PZ.
In 1272, the 2nd CSC is centered in QA-P9 and swaps QA-P3 and QA-P4 ; the Newton Line is QA-P3QA-P4 (the asymptote of this cubic is parallel to the asymptote of QA-Cu1).
Best regards
Bernard

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Message: #175
Date: 2020-03-16
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
I think we now know and understand many things better than
before!
While waiting impatiently for your comments, I notice among your
last curves the 441.
As mentionned, QG-P1 and QA-P4 are CSC artners, but also cb
partners in the cb(B,B',C,C',M).
Any circular circumcubic of these 5 points with pivot on the
line QG-P1QA-P4 will pass through these 2 points.
This is the case for QACu1 with pivot in CSC(QA-P41) and the 441
(through QA-P4,QG-P1 and QL-P1), with pivot in QA-P4.
Best regards
Bernard
```

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Message: #176
Date: 2020-03-16
From: bernard.keizer@gmail.com
Subject: Re: QG/QL-Circumquartic

```
Dear Eckart,
If my memory is not wrong, we already discussed your 1rst
quartic!
It is not only CSC, but also CSCdiag invariant (CSCdiag is the
CSC of DQL formed by the sides of DT and the Newton Line, is
centered in QL-P17 and swapping for example the Plücker points).
The quartic pass through QL-P17 and the 2 fixed points of CSC
diag as well as through the Plücker points.
Any tangent to your cubic cuts the quartic in 4 points, 2 CSC
partners and 2 CSCdiag partners.
Best regards
Bernard
```

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Message: #177
Date: 2020-03-16
From: bernard.keizer@gmail.com
Subject: Re: QG/QL-Circumquartic

```
Sorry, please read conic and not cubic
```

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Message: #178
Date: 2020-03-17
From: bernard.keizer@gmail.com
Subject: Re: QG/QL-Circumquartic

```
Dear Eckart,
Reading again your message 364 (old forum), QL-L2 is in fact
also tangent to your conic.
Hence the Plücker points as CSCdiag partners and the
intersections of QL-L2 and Miquel circle as CSC partners.
As I told you already, I named this quartic on my blog the
generalised Plücker points quartic ...
Best regards
Bernard
```

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Message: #179
Date: 2020-03-17
From: eckart_schmidt@t-online.de
Subject: Re: QG/QL-Circumquartic

```
Dear Bernard,

as mentioned, the quartic is already treated in QFG#364,
... but I think #173 gives some new aspects.
Your remarks wrt CSCdiag are correct,
... but the quartic doesn't pass through QL-P17.
Your last statement isn't correct,
... a tangent to the conic must not have intersections
    with the quartic,
... there can be two intersections,
    which are CSC- or CSCdiag-partners.

Here another QG-quartic:
... Consider the CSC-partner on circles round QG-P5
... and you get a monopartite circumquartic of the QG,
... CSC-invariant and bearing QL-2P3,
... also invariant wrt the transformation X-->X^ in #173.
Midpoints of X.X^ for quartic-points are collinear on
QL-P1.CSC(QG-P5),
... bisectors of X.X^ intersect in QG-P5,
... lines X.X^ envelope a parabola,
... ... tangent to the diagonals
... ... tangent to the quartic on a circle with diameter
QL-P1.QG-P5
... ... focus CSC(QG-P5), directrix QL-P1.QG-P5.
... CSC-partner on tangents to this parabola give the quartic,
... partner of X-->X^ on tangents to the parabola give also the
    quartic.

Best regards Eckart
```

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Message: #180
Date: 2020-03-17
From: bernard.keizer@gmail.com
Subject: Re: QG/QL-Circumquartic

Dear Eckart,
My apologise, the quartic pass neither through QL-P1 nor through QL-P17.
But it passes through the fixed points of CSC (which are CSCdiag partners) and those of CSCdiag (which are CSC partners).
For any line tangent to your conic, the intersections wih the CSC circle and the CSCdiag circle of the line give 4 points (not necessary real) on the quartic.
2 points are CSC partners, 2 other are CSCdiag partners.
Best regards
Bernard

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Message: #181
Date: 2020-03-17
From: bernard.keizer@gmail.com
Subject: Re: QG/QL-Circumquartic

Dear Eckart,
Your conic is also tangent to the Steiner axes of CSCdiag.
If my memory is not wrong, you mentionned somewhere that the circles CSC of the tangents describe a rectangular hyperbola through the circumcenters of the reference triangles of the QL (?).
Best regards
Bernard

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Message: #182
Date: 2020-03-17
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

is my following interpretation of #174 right:
... If we have a circular cubic, pivotal wrt a CSC (see #171),
... you describe a 2nd CSC for the cubic, centered in the focus,
... ... swapping the pivot and the center of the 1st CSC.

Example:
The circular cubic in QFG#441
... has pivot QG-P1 wrt the regular CSC and focus QA-P4.
Your 2nd CSC is centered in QA-P4,
... swapping  QG-P1 and QL-P1.
This aspect was new for me.

Best regards Eckart
```

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Message: #183
Date: 2020-03-18
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Bernard,

wrt #167: you describe
... cb-invariant pivotal circumscribed cubics of special 5P,
... which are a QG and its Miquel point with pivot on QG-L1,
... the cubics  are also CSC-invariant wrt QG.
This is very special, but quite interesting,
... for it contains QA-Cu1 and QL-Cu1.
Please give me time, to study it.

Best regards Eckart
```

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Message: #184
Date: 2020-03-18
From: bernard.keizer@gmail.com
Subject: Re: circular cubic, focus and asymptote

Dear Eckart,
Alleluiah !
It's exactly that !
Your construction uses in fact a well-known property of the monocursal VR.
Any monocursal VR has in fact 3 real CSC, each centered in one of the vertices of the triangle MSS'.
The main CSC is centered in the focus and swaps S and S' ; SS' is the Newton Line.
Q is the tangential of M, M is the tangential of S and S'.
S is the tangential of f'1 and f'2, symmetric wrt S' and S' the tangential of f1 and f2 symmetric wrt S.
The points f1 and f2 and f'1 and f'2 (2 copples of CSC partners) are concyclic and they are the fixed points of the CSCs centered in S and S'.
The partners on VR in both CSCs are aligned with the other point (hence your construction).
f1f'1 and f2f'2 and f1f'2 and f2f'1 intersect in the points Z and Z', where the tangents are parallel to the Newton Line (centers of anallagmaty).
I named the QL formed by the 6 points the main QL of the monocursal VR.
Best regards
Bernard
PS For bicursal VR, the points S and S' are not real. Using the same triangle, renamed MVV', the Newton Line is the perpendicular bisector of VV' and the centers of anallagmaty are the in- and excenters of the triangle ...

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Message: #185
Date: 2020-03-18
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
It is even more interesting than I thought !
For a QL, the 4 curves QL-Cu1 and the 3 QA-Cu1 of the 3 QAs of
the QL are cbinvariant wrt the 3 cb : cb(B,B',C,C',M),
cb(A,A',B,B',M) and cb(A,A',C,C',M).
The corresponding pivots are on the 3rd diagonal QG-P17,
CSC(QA-P41) and the 2 DT vertices other than QG-P1.
Best regards
Bernard
```

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Message: #186
Date: 2020-03-18
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
Of course, the same way, for a QA, there are 3 QL-Cu1 and 1
QA-Cu1 and the 4 curves are cbinvariant wrt the 3 cb :
cb(B,B',C,C' and one of the Miquel vertices).
The corresponding pivots are the CSC(QA-P41) and QG-P17
Best regards
Bernard
```

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Message: #187
Date: 2020-03-18
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Bernard,

only a short addition to #167:
Beside QA-Cu1 and QL-Cu1 you consider the cubic Cu3,
... cb-invariant with pivot in the infinity point of AA'.
The focus of this cubic is 5P-s-P5 of B,B',C,C',M, not a point
on Cu3.

Best regards Eckart
```

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Message: #188
Date: 2020-03-18
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Bernard,

another description of the focus for Cu3:
2nd intersection of QL-Ci3 and a perpendicular in M to MS.

Best regards Eckart
```

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Message: #189
Date: 2020-03-19
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
Interesting that F3 is 5P-s-P5 of (B,B',C,C',M).
In this case, it is the focus of your VR focal circular cubic of
the 5 points in message 145 ...
Best regards
Bernard
```

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Message: #190
Date: 2020-03-19
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Bernard,
I risk the following conjecture:
Your considered cubics in #167 have foci on QL-Ci3
... in the 2nd intersection of QL-Ci3
    and a perpendicular to MQ in M.
Best regards Eckart
```

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Message: #191
Date: 2020-03-20
From: eckart_schmidt@t-online.de
Subject: Re: circular cubic, focus and asymptote

```
Dear Bernard,

after decoding your nomination in #184
... I have with great interest reproduced your message,
... background of my #171.
Thanks a lot, I try to apply it.

Best regards Eckart
```

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Message: #192
Date: 2020-03-23
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

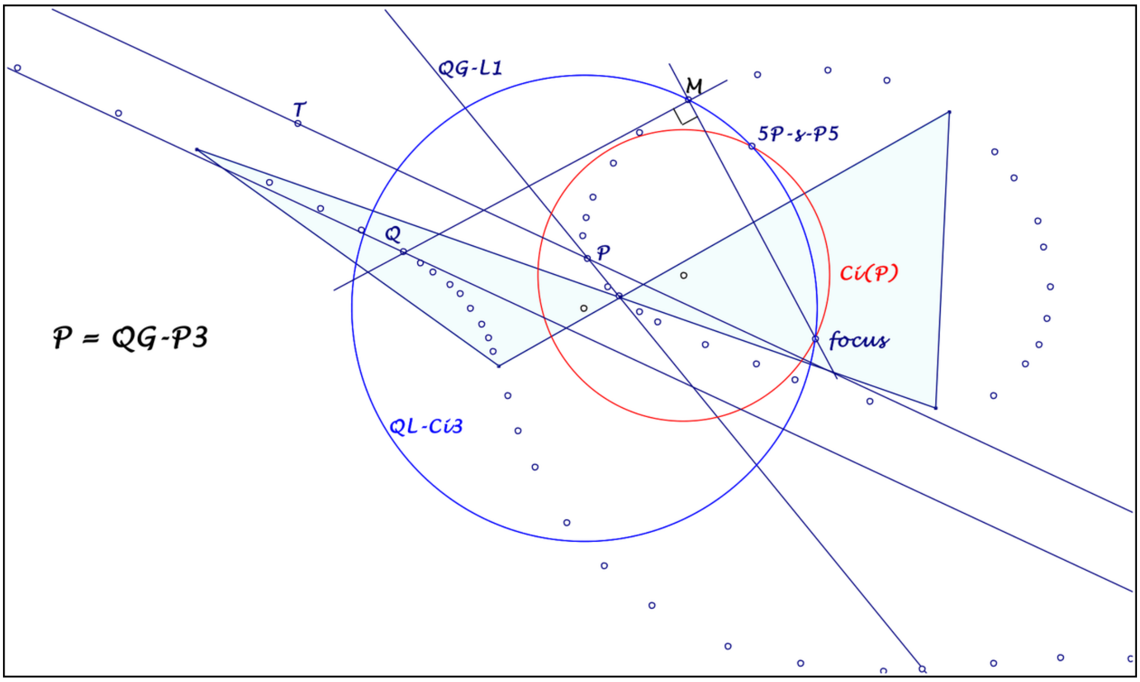
---

```
Dear Bernard,

there is a very simple construction of the foci for your cubics
in #167,
... using the mapping P --> Ci(P) in QFG-message 3575:
The focus lies on QL-Ci3 in the intersection with Ci(pivot)
unequal 5P-s-P5.

In the general case of cb-invariant pivotal 5P-circumcubics
holds:
... the Ci(X) for the pivot and any point of the cubic
    give 5P-s-P5 and the focus.

Best regards Eckart
```



2020-03-23.pdf

Message: #193
Date: 2020-03-24
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
Very interesting !
Finally, your circle Ci(X) is a key for almost everything ...
Best regards
Bernard
```

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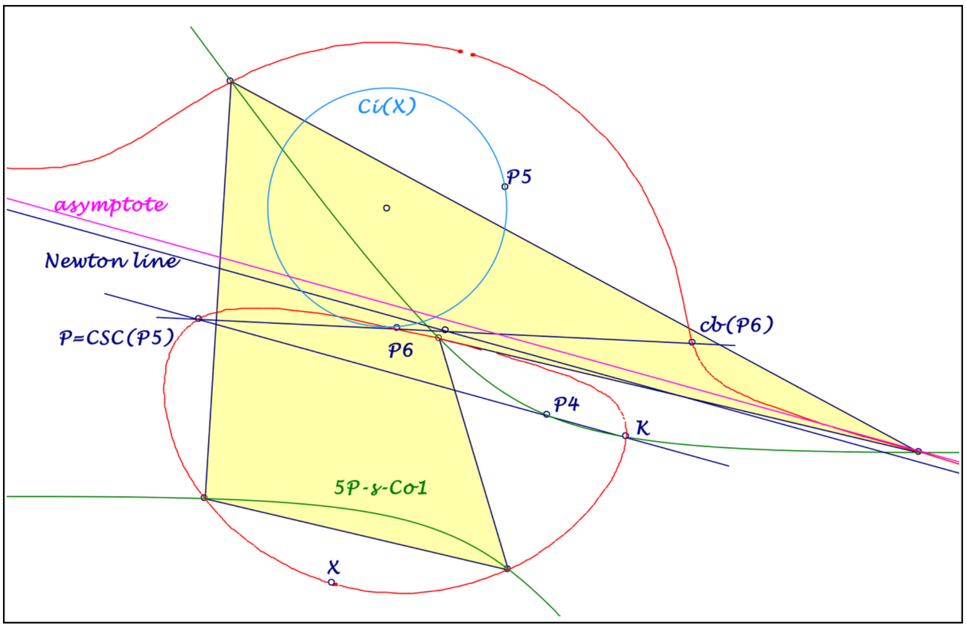
Message: #194
Date: 2020-03-26
From: eckart_schmidt@t-online.de
Subject: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard, dear Chris,
I think, this theme will be relevant for 5P-geometry,
... here a first example, already indicated in QFG-message 3575.
Consider a 5P and the points P4 = 5P-s-P4, P5 = 5P-s-P5,
P6 = 5P-s-P6,
... with the cb- and the CSC-transformation for 5P,
... further the transformation X-->Ci(x) (see QFG-message 3575)
... and the special pivot P = CSC(P5):
We consider the locus of cb-partner on lines through P,
... which gives a cb-invariant 5P-circumcubic
... through the pivot P, P6
    and the 2nd intersection K of P.P4 and 5P-s-Co1.
The cubic is the locus of points X with Ci(X) bearing P5 and P6,
... P6 is the focus of the cubic (see #192).
Now we define a new CSC-transformation,
... centered in P6, swapping cb(P6) and K.
If we take as Newton line a parallel to P.P4
... through the midpoint of P and cb(P6),
... we can construct the cubic:
... New CSC-partner on parallels to P.P4,
... reflected in the Newton line, give the cubic.
The asymptote is parallel to P.P4.
... through the reflection of P6 in the Newton line.
Best regards Eckart
```



2020-03-26.pdf

Message: #195
Date: 2020-03-26
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

Dear Eckart,

Before answering your message 194, I'd like to end properly this item.
Any circular circumcubic of the 7 points QL-P1 + vertices of a QL are
* CSC invariant
* cb invariant wrt (B,B',C,C',M), (A,A',B,B',M) and (A,A',B,B',M) : the 3 pivots Pa, Pb and Pc are the 3rd intersections with the diagonals AA', CC' and BB' and the circumcircle of Pa, Pb and Pc pass through M = QL-P1
* any parallel to the asymptote give 2 points and the CSC of the 2 points are aligned with M and the other points ; therefore, the 2 CSC are also on a line parallel to the asymptote (this gives in particular the lines through the 5P-s-P4 and the points Pa and Sa, Pb and Sb and Pc and Sc and the line through M and q and the asymptote through the infinity point and Q = CSC(q))
* the locus of the foci is QL-Ci3, the focus of each cubic being the 2nd intersection of the perpendicular in M to MQ (your conjecture in 190)
This includes naturally in particular QL-Cu1 and the 3 QA-Cu1 of the 3 QA's of the QL.

Best regards
Bernard

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Message: #196
Date: 2020-03-27
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and QL-Cu1 as cb and CSC invariant cubics

---

```
Dear Eckart,
The beauty of all this is that any of these curves is entirely
determined by the 6 vertices of the QL, the point QL-P1 and any
point X.
The cubic is defined with 9 real points : the 7 points, X and
CSC(X).
Having Q as tangential of M gives q = CSC(Q) and the asymptote
is the parallel in Q to Mq.
The rest follows ...
Best regards
Bernard
```

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Message: #197
Date: 2020-03-28
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

if we consider the cb-partner X, Y wrt a 5P
... on lines through a pivot P on 5P-s-Co1,
... we have: PX*PY = const.
What about the corresponding circle round P?

Best regards Eckart
```

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Message: #198
Date: 2020-03-29
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

in addition to #197, pivot on 5P-s-Co1, the corresponding circle
... intersects the pivotal cb-invariant 5P-circumcubic
... in four points, which give a rectangle.

Best regards Eckart
```

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Message: #199
Date: 2020-03-30
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

on the cubic QL-Cu1 of a quadrigon P1P2P3P4
... there are CSC-partner P5 and P5',
... so that the cb-invariant pivotal cubic (see #194)
... of P1P2P3P4P5 and P1P2P3P4P5' wrt their 5PCSC(5P-s-P5)
    as pivot
... are QL-Cu1 of the initial quadrigon.
What about these QG-points P5, P5'?
First observation: 5P-s-P6 of the two 5P is QL-P1.

Best regards Eckart
```

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Message: #200
Date: 2020-03-30
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

QA-Cu1 for a QA = P1P2P3P4
... is a pivotal cb-invarant circumcubic
... of P1P2P3P4+QA-P4 with pivot QA-P3,
in general for a 5th point P5 on QA-Cu1:
... of P1P2P3P4P5 with pivot QA-Tf2(QA-Tf16(P5)).

Best regards Eckart
```

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Message: #201
Date: 2020-03-30
From: bernard.keizer@gmail.com
Subject: Re: Pivotal cb-invariant 5P-circumcubics

Dear Eckart,
I regret that you didn't answer my messages 195 and 196 ...
Apart of that, I admire your energy and your curiousity for digging always new properties in this rather trubbled period !
(Of course, I also regret that we are apparently now only the both of us on this forum).
First, let me try under your control a short summary of what we know about circular circumcubics of 5 points :

* the cubics are cbinvariant wrt the 5 points
* they have 2 caracteristic points the pivot P(on the cubic) and the focus F (not necessary on the cubic ; if F is on the cubic, it is a Van Rees focal circular cubic)
* for X on the cubic, P, X and cb(X) are aligned (in particular, cb(P) is the tangential of P)
* X and cb(X) have the same Ci(X) through U = 5P-s-P5 and F
* the cubics have a real asymptote, parallel to PT, where T is 5P-s-P4 (T is on the circumconic of the 5 points, not on the circumcubic)
* the cubic cuts it's asymptote in a point Q (tangential of the focus for the Van Rees curves) and the line PT in S, 6th intersection of the cubic and the circumconic (cb(S) is the infinity point of the asymptote).

You give in messages 145 and 194 2 beautiful examples of Van Rees circular circumcubics of 5 points :
In 145, your cubic CU has focus U = 5P-s-P5 and the CSC swaps 5P-s-P6 and the 5P-s-P4 of the CSC of the 5 points.
The pivot of the cb is the infinity point of the asymptote and cb(pivot) is the point Q and S tangential of U ...
In 194, the cubic has focus in 5P-s-P6 and pivot is CSC(U).
Best regards
Bernard

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Message: #202
Date: 2020-03-30
From: bernard.keizer@gmail.com
Subject: Re: Pivotal cb-invariant 5P-circumcubics

Dear Eckart,
I make here a separate answer to your message 197.
As in summary in the preceding message, the asymptote is parallel to PT.
It is PT if P is on the degenerated cubic formed by the circumconic and the infinity line.
If P is on the infinity line, like in 145, Q = S = cb(P) = tangential of P
If P is on the circumconic, Q = S = P and cb(P) is the tangential of P and the infinity point of the asymptote, which means that the asymptote is tangent in P to the cubic.
But any circular cubic is anallagmatic, the centers of anallagmaty being the points of the cubic where the tangents are parallel to the asymptote.
Your property means that the pivot on the circumconic is a center of anallagmaty of the circular circumcubic.
Best regards
Bernard
PS I didn't find your points P5 and CSC(5) on QL-Cu1, but I suppose you can have them approximately, starting with the QL and one of it's QAs and a 5th point on the curve such as the parallel through T passes through CSC(U) on QL-Cu1 ?

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Message: #203
Date: 2020-04-01
From: bernard.keizer@gmail.com
Subject: 5L-n-P1 centric focus

Dear Eckart,
I always try to find circular quartics through the 10 vertices and the 2 foci of the inscribed conic of a 5L.
Remember: there are 5 degenerated quartics formed by the QL-Cu1 of 4 lines and the 5th line which pass by definition through the 12 points.
The 12 points form therefore a cb12 system for circular quartics exactly as the 7 points 6 vertices of a QL and the focus QL-P1 form a cb7 system for circular cubics.
Another point is necessary to have a determined circumcubic of the 7 points or a circumquartic of the 12 points.
I looked again the point X, 5L-s-P1 or centric focus of the 5L.
I found properties, which are not in EQF and, if I'm not wrong, had never been mentionned before.
The reflexions Xi of X in the 5 lines are on a rectangular hyperbola passing through X.
The 5 CSCi of X are aligned (it is the degenerated Ci(X) wrt the Xi, which passes through 5P-s-P5 of the Xi).
This line is the tangent to the inscribed conic in the point 5P-s-P4 of the 5 contact points.
I hope you will be able to confirm these observations.
I'm sure there are other properties to find.
Best regards
Bernard

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Message: #204
Date: 2020-04-01
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

your #201 is a good summary of our observations
... wrt pivotal cb-invariant circumcubics of 5P, up to now ...
... perhaps you can confirm:  focus = CSC3(CSC(pivot)).

I have reproduced the properties for QA-Cu7 of a QG
... as pivotal cb-invariant circumcubic of the 5 common
QA-Cu7-points
... with 5P-s-P4 = T, 5P-s-P5 = CSC(QL-P24), 5P-s-P6 = QL-P1:
... ... pivot P = QG-P1, focus F = QA-P41.

Two observations for this cubic:
... focus on the 5P-quartic,
... S.CSC(P) orthogonal 5P-s-P5.CSC(P).

Finally general properties of 5P-points:
... 5P-s-P6, cb(5P-s-P6), CSC(5P-s-P5) are collinear,
... CSC(5P-s-P4), 5P-s-P5, 5P-s-P6 on Ci(5P-s-P6).

Best regards Eckart
```

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Message: #205
Date: 2020-04-02
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

wrt #197, #198 and your answer in #202:
Thanks for describing the pivot on the circumconic
... as center of anallagmaty with tangent to the cubic parallel
to the asymptote,
... but not necessary the asymptote, as you wrote.
Is it also anallagmaty, if we get cb(X) with X on the cubic
... by an inversion, centered in P, and reflection in P?

Wrt your PS in #202 and my #199:
I cannot give a construction of P5,
... but an approximation by changing P5 on QL-Cu1,
... so that the pivot 5PCSC(5P-s-P5) for P1P2P3P4P5
    lies on QL-Cu1.

Best regards Eckart
```

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Message: #206
Date: 2020-04-02
From: bernard.keizer@gmail.com
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Eckart,
Very interesting your formula focus = CSC3(CSC(pivot)) !
I've checked, like you, that it works for your 2 cubics in 145
and 194 as well as for the 3 QA-Cu7.
As I don't practice the macros, I hardly can confirm this
general property without much efforts.
Perhaps, you could also verify easily that CSC3(CSC(diagonal
AA')) is the Miquel circle QL-Ci6 wrt the 5P (B,B',C,C',M).
Best regards
Bernard
```

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Message: #207
Date: 2020-04-02
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P1

```
Dear Bernard,

your message #203 is an interesting new start in 5L-geometry,
... but I cannot confirm your last property.
I shall have a look on further results.

Best regards Eckart

PS. There is a typo in the middle of the message:
... replace 5L-s-P1 by 5L-n-P1.
```

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Message: #208
Date: 2020-04-02
From: bernard.keizer@gmail.com
Subject: Re: 5L-n-P1 centric focus

```
Dear Eckart,
In fact the line of the CSCi of X is not tangent to the
inscribed conic.
But I hope the rectangular hyperbola through X and it's
reflexions in the 5 lines holds !
Best regards
Bernard
```

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Message: #209
Date: 2020-04-03
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P1 centric focus

```
Dear Bernard,

it is correct, that the line of the CSCi of 5L-n-P1 is tangent
to 5L-s-Co1,
... but the contact point is not 5P-s-P4 of the contact points
of 5L-s-Co1.

5L-n-P1 is a point on 5L-o-Ci1
... and all points on this circle lead to a tangent at 5L-s-Co1.

Best regards Eckart
```

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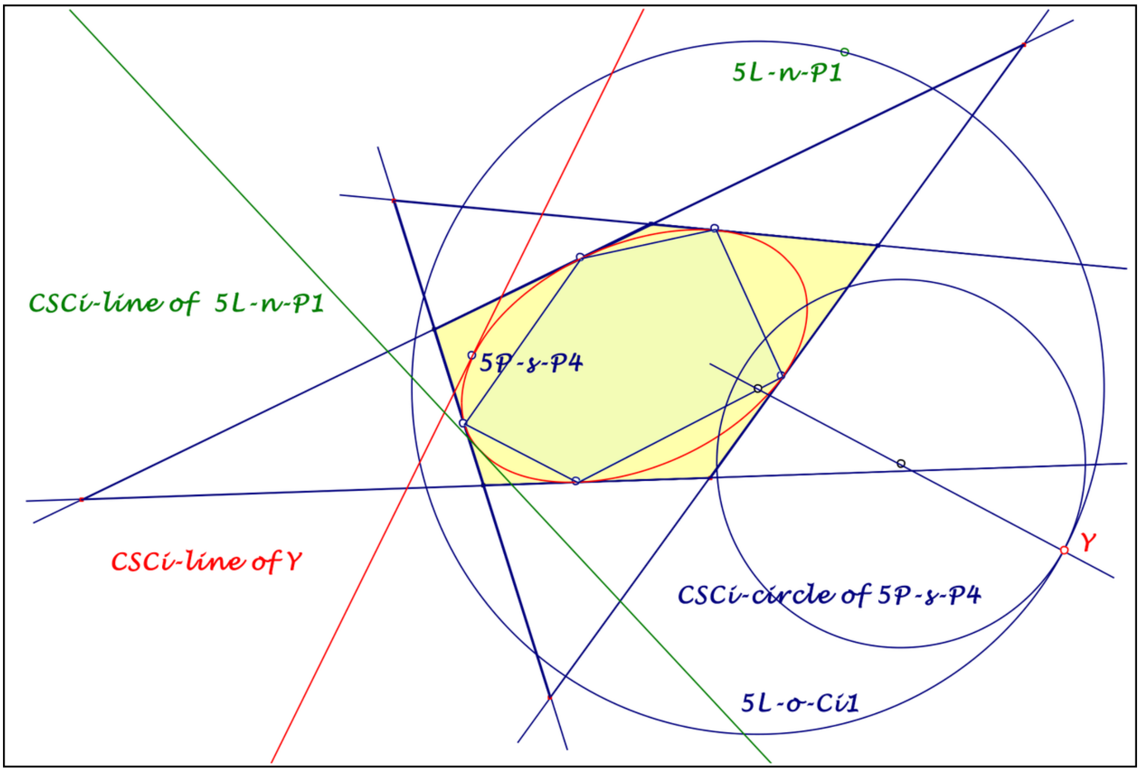
Message: #210
Date: 2020-04-03
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P1

```
Dear Bernard,

in addition to my last message:
The CSCi-circle for 5P-s-P4 of the contact-5P of 5L-s-Co1
... contacts 5L-o-Ci1 in a point,
    whose CSCi-line is the tangent in 5P-s-P4 at 5L-s-Co1.

Best regards Eckart
```



2020-04-03.pdf

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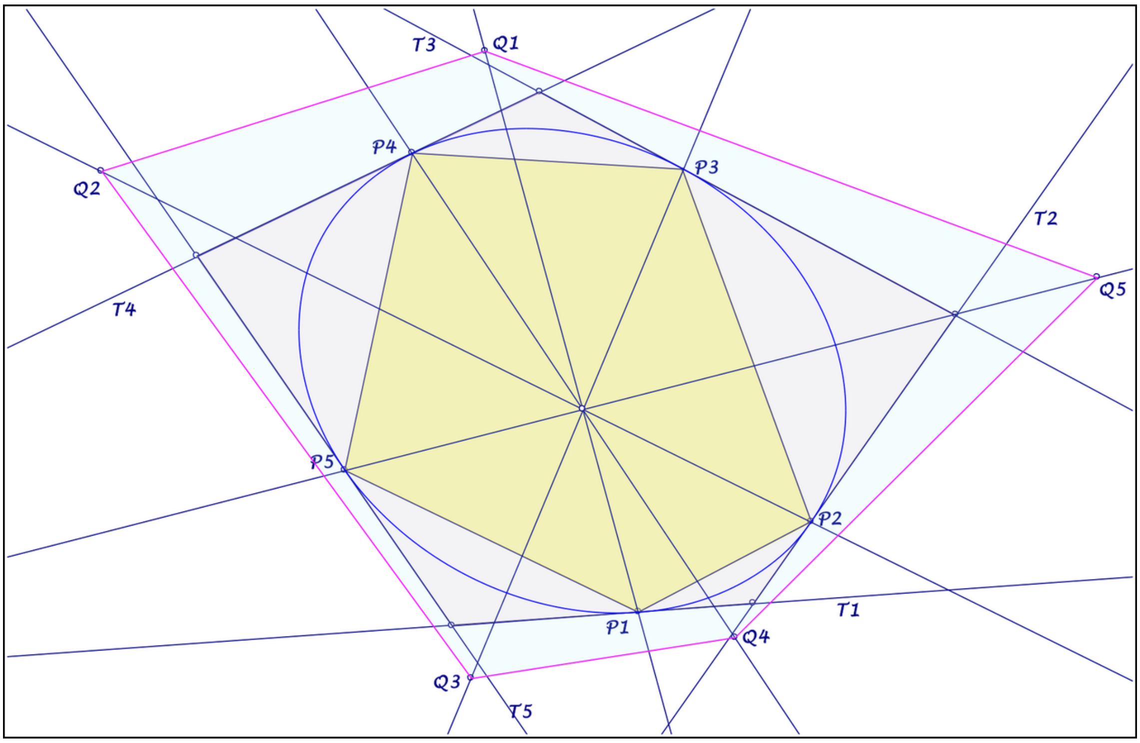
Message: #211
Date: 2020-04-04
From: eckart_schmidt@t-online.de
Subject: New 5P-point, 5L-line?

```
Dear Chris, dear Bernard,

Consider a 5P = P1...P5 and its circumconic
... with tangents T1, ... ,T5, which give a 5L,
... and for each tangent Ti the dual point Qi
    (see QA-8/QL-8 )
... wrt the QL of the other tangents.
The new 5P = Q1...Q5 is perspective to P1...P5
(see attached file).
Analog:
Consider a 5L=L1...L5 and its inscribed conic
... with contact points C1, … ,C5, which give a 5P,
... and for each point Ci the dual line Mi (see QA-8/QL-8)
... wrt the QA of the other contact points.
The new 5L = M1 ... M5 is line perspective to L1 ... L5.

Unfortunately I cannot give properties.

Best regards Eckart
```



2020-04-04.pdf

Message: #212
Date: 2020-04-05
From: bernard.keizer@gmail.com
Subject: Re: 5L-n-P1 centric focus

```
Dear Eckart,
This give a remarkable transformation point M of circle of the 5
Miquel points Mi (through X = 5L-n-P1) to tangent in P to the
inscribed conic of the 5 lines.
Further obvious (?) observations :
CSCi(Li) is a circle tangent in Mi to the Mi circle and passing
through CSCi(Pi)
CSCi(Mi circle) is a parallel to Li through CSCi(X)
The quartic through the 12 points (10 vertices of the 5L + 2
foci of the inscribed conic) and the points M and P is defined
only for the 5 positions of M in Mi and of P in Pi and give then
the 5 degenerated circular quartics formed by a circular focal
VR of 4 of the 5 lines (through Mi) and the 5th line Li (through
Pi).
Best regards
Bernard
```

Message: #213
Date: 2020-04-06
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

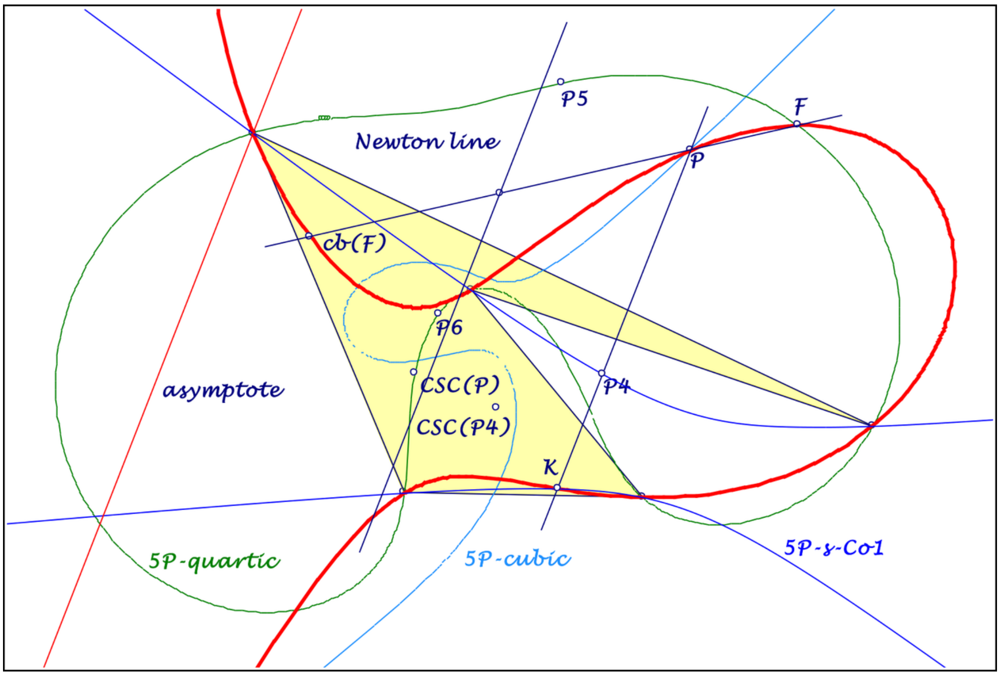
```
Dear Bernard,

this is a generalization of the special example in #194.
Here a construction is described for pivotal cb-invariant
5P-circumcubics
... with pivot P on your 5P-cubic.
These cubics bear their focus F = CSC3(CSC(P)) on the
5P-quartic.

If you take a parallel to P.P4 through the midpoint of P.cb(F)
as Newton line
... and a new CSC, centered in F, which swaps K and cb(F),
... ... K = 2nd intersection of P.P4 and the 5P-circumconic,
... you get the announced 5P-circumcubic with asymptote
... in a parallel to P.P4 through the reflection
    of F in the Newton line.

Best regards Eckart

PS: Can it be, that all Van Rees circular 5P-circumcubics
    ... have their pivot on the 5P-cubic
    ... and their focus on the 5P-quartic?
```



2020-04-06.pdf

Message: #214
Date: 2020-04-08
From: bernard.keizer@gmail.com
Subject: Re: Pivotal cb-invariant 5P-circumcubics

Dear Eckart,
Beautiful, simple construction !
Your PS is simply amazing and this property has plenty of consequences.
Any line contains 2 cb partners and any point P on the line as pivot gives a circular circumcubic of the 5 points with focus CSC3(CSC1(P)).
The locus of the foci is CSC3(CSC1(line)), which is a circle through P5, as CSC1(line) is a circle through P6.
This line cuts the 5P cubic in 3 points which are the 3 pivots of the 3 VR of the 5P ; their 3 foci are the 3 intersections (other than P5) of the circle CSC3(CSC1(line)) with the 5P quartic.
(As CSC3 is CSC1*CSC2*CSC1, CSC3*CSC1 = CSC1*CSC2).
For the 5 points M,B,B',C and C' of a QL, it is easy to check that CSC3*CSC1(diagonal AA') is the Miquel circle.
AA' cuts the 5P cubic in 3 points : one is the pedal of QG-P1 on AA' (QG-P17 if I'm not wrong).
This gives the Van Rees QL-Cu1 ; CSC1(h1) is QA-P4 of BB'CC' and CSC3(QA-P4) is M or QL-P1.
As well-known, QG-P1 and QA-P4 are also cb partners ; AA' and QG-P1QA-P4 intersect in CSC(QA-P41) which is the pivot of QA-Cu1 of BB'CC'.
As this point is not on the 5P cubic, QA-Cu1 is not a VR and the focus QA-P9 is CSC3*CSC1(CSC(QA-P41)) on the Miquel circle and not on QACu1.
The line QG-P1QA-P4 cuts the 5P cubic in 3 points : one is QG-P1 and leads to a VR circumcubic of the 5 points with pivot QA-P4.
For 5 points in general, the line P6cb(P6) cuts the 5P cubic in 3 finite points : one is CSC(P5) and leads to your 5P circumcubic with pivot in P6 described in your message 194.
This construction is generalised precisely in your message 213.
The line P5cb(P5) is parallel to the Newton Line and cuts the 5P cubic in 3 points : one is the infinity point of the Newton Line and leads to the circumcubic with focus P5 described in your message 145.
For the 5 triple points of the QA-Cu7, we had already noticed that the pivots were the QG-P1 on the 5P cubic and the foci the QA-P41 on the 5P quartic (on a circle through P6 = QL-P1) ; the QA-P4 are the CSC1 of the QG-P1 and the QA-P41 are the CSC3 of the QA-P4. (CSC3*CSC1 of the DT circumcircle is the circle of the QA-P41 and CSC3*CSC1 of QL-P24 is QL-P1).
Best regards

```
Bernard
PS We never found a simple property of the 5 triple points when
they lead to a QL ...
```

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Message: #215
Date: 2020-04-08
From: eckart_schmidt@t-online.de
Subject: 5P-/5L-Transformations

```
Dear Chris, dear Benedetto,

in EPG we have the transformation 5P-s-Tf2,
... which maps a point to a line,
... using the collinear 5 QA-Tf2-images of the point.
The corresponding transformation 5L-s-Tfx for a 5L is not in
EPG:
... which maps a line to a point,
... using the common point of the 5 QL-Tf2 images of the line.
The inverse of this transformation 5L-s-Tfx,
... which maps a point to a line,
... using the collinear 5L-s-Tfx-images of the line-pencil,
... has the property, that its image is the polar wrt 5L-s-Co1.
This is the conical transformation CO-Tf1 wrt 5L-s-Co1 in EPG.

Best regards Eckart
```

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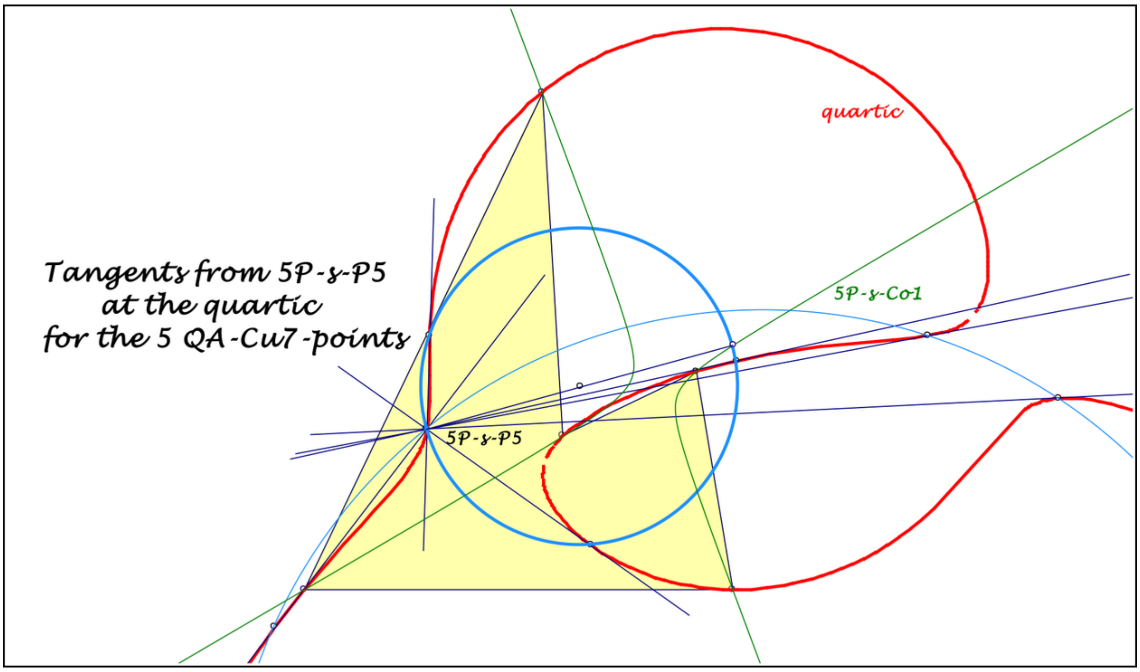
Message: #216
Date: 2020-04-10
From: eckart_schmidt@t-online.de
Subject: Re: Pivotal cb-invariant 5P-circumcubics

```
Dear Bernard,

wrt your #214 thanks for the extensive elaboration,
... back to the 5 QA-Cu7-triple-points,
... interesting background for your PS.
I searched once more for a simpler property,
... but I always come back to my first observation
    (see QFG #3690):

For the 5P of the QA-Cu7-triple-points (see attached file)
... there are always three tangents from 5P-s-P5 at the quartic
... with contact points concyclic with 5P-s-P5
... and a diametral point of 5P-s-P5 on 5P-s-Co1.
These contact points are the QA-P4 for the 3 QG of the QL,
... which allow  a construction of the QL
    (see last passage of #214).
If there are further three tangents from 5P-s-P5 at the quartic,
... their contact points are also concyclic with 5P-s-P5,
... but without the diametral point of 5P-s-P5 on 5P-s-Co1.

Best regards Eckart
```



2020-04-10.pdf

Message: #217
Date: 2020-04-11
From: bernard.keizer@gmail.com
Subject: Re: Pivotal cb-invariant 5P-circumcubics

Dear Eckart,
Always using your amazing property in message 213, there are 10 more possible VR circumcubics of a 5P :
the pivots are either the CSC of the 5P or the CSC of the QA-P4 of 4 of the 5P (these points are CSC2 partners)
the corresponding foci are the CSC3(P) = QA-P4 or the CSC3(QA-P4) = P.
For the 5 triple points of the QA-Cu7, there are 6 more possible VR circumcubics of the 5 triple points :
the pivots are either the QG-P1 (which gives the QA-Cu7) or the CSC2 of the QG-P1, which are the CSC1 of the QA-P41
the corresponding foci are the QA-P41 (QA-Cu7) or the QA-P4
Best regards
Bernard

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Message: #218
Date: 2020-04-12
From: bernard.keizer@gmail.com
Subject: Re: 5L-n-P1 centric focus

Dear Eckart,
For 5 fixed points Pi, we define Ci(P) as the circle of the CSCi(P) wrt the 5 QLs of the 5L formed by the perpendicular bisectors of PPi.
Here some more properties in order to introduce plenty of questions in a next message.
Let's consider the inscribed conic of the 5 perpendicular bisectors with foci S and S' and the circle of the centers Mi of the 5 CSCi.
More generally, let's consider the CSC centered in a point M of this circle and swapping the 2 foci.
For any point X, Ci(X) is the locus of CSC(X) with M describing the Mcircle (in particular through the 5 CSCi(X)).
For example, for Aij intersection of 2 lines Li and Lj of the 5L, Ci(Aij) is the circle centered in Oij through Mi, Mj, Akl, Akm and Alm.
For M on the Mcircle, Ci(M) is a line L tangent in P to the inscribed conic.
Ci(P) is CSC(L) and is a circle tangent in M to the Mcircle.
(This gives M knowing P on the conic).
CSC(Micircle) is a line parallel to L and tangent to the conic in the reflection P' of P in the center of the conic. (It is therefore Ci(M') where M' is another point of the Mcircle).
Best regards
Bernard

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Message: #219
Date: 2020-04-12
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P1 centric focus

```
Dear Bernard,

it took some time, to reproduce your message 218, some remarks:
... Why do you start with 5 fixed points and not with a 5L?
... Your Mcircle is 5L-o-Ci1, the center of Ci(X)
    is 5L-s-Tf1(X).
... Some of your properties are already mentioned
    in 5L-s-Tf1 in EPG.
... The CSC, centered in a point of 5L-o-Ci1, swapping the foci,
    seems promising.

Best regards and Happy Easter
Eckart
```

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Message: #220
Date: 2020-04-12
From: van10hoven@gmail.com
Subject: Re: 5P-/5L-Transformations

```
Dear Eckart,

You are right.

5L-s-Tfx(L) = the intersection point of the 5 versions of
QL-Tf2(L) and 5L-s-Tfx -1 (P) = the line connecting the 5
versions of QL-Tf2 -1 (P).
I will include the features later in EPG.

Best regards,

Chris
```

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Message: #221
Date: 2020-04-13
From: bernard.keizer@gmail.com
Subject: Re: 5L-n-P1 centric focus

Dear Eckart,
Many thanks for your answer and your references in EPG 5L-o-Ci1 and 5L-s-Tf1.
My idea is always to find circular quartics through my 12 points (vertices of the 5L + 2 foci of inscribed conic), defined by another point and a transformed of this point. This is exactly like the circular cubics through the 7 points (6 vertices of the QL + QL-P1) defined by any point and it's CSC (see my previous messages).
First, I started with the 5P with X and cb(X) defining 2 5Ls, hoping they would have some interesting unknown properties (same 5L-o-Ci1, same inscribed conic, 10 vertices on the same quartic ...), but I didn't find any property. But of course, it remains that the properties which work separately for a QL must work for the other (for example, if X is on the conic of the 5P, Ci(X) is a line and X is on 5L-o-Ci1 and cb(X) must also be on the circumconic and on 5L'-o-Ci1). Here I need you badly, as I don't have your knowledge in this matter.
Then, I came back to the 5L, which is always my ultimate goal.
With the definition of Ci(X) as locus of the CSC of X in the CSC centered on 5L-o-Ci1, I don't think it is possible to find another point having the same Ci. I hope you will confirm this.
So I tried to define cb(X) as the cb partner of X wrt the 5 reflections of X in the 5 lines. I think that with the same definition cb(cb(X)) is not X, as there is no reason that the reflections of cb(X) in the 5 lines are also the reflections of X in the 5 lines perpendicular bisectors of the segments joining cb(X) to the reflections of X in the 5 lines. I hope you will also confirm this.
My last hope is that X, cb(X), cb(cb(X)) and so on form a chain of points on a quartic through my 12 points.
But here I reach the limits of my ability to draw figures ...
Starting with a conic of foci S and S' and 5 lines tangent to the conic, it's easy to determine 10 vertices of the 5L and the 5L-o-Ci1 (like your figur in message 210).
Starting with a point X and using macros as you do, I suppose you could better than me determine the mentionned chain of X, cb(X), cb(cb(X)) and so on ...
If it doesn't work, I give up !
Thanks in advance for your interest
Best regards and Happy Easter to you too.   Bernard

Message: #222
Date: 2020-04-13
From: eckart_schmidt@t-online.de
Subject: 5L-o-Ci1

Dear Bernard, dear Chris,

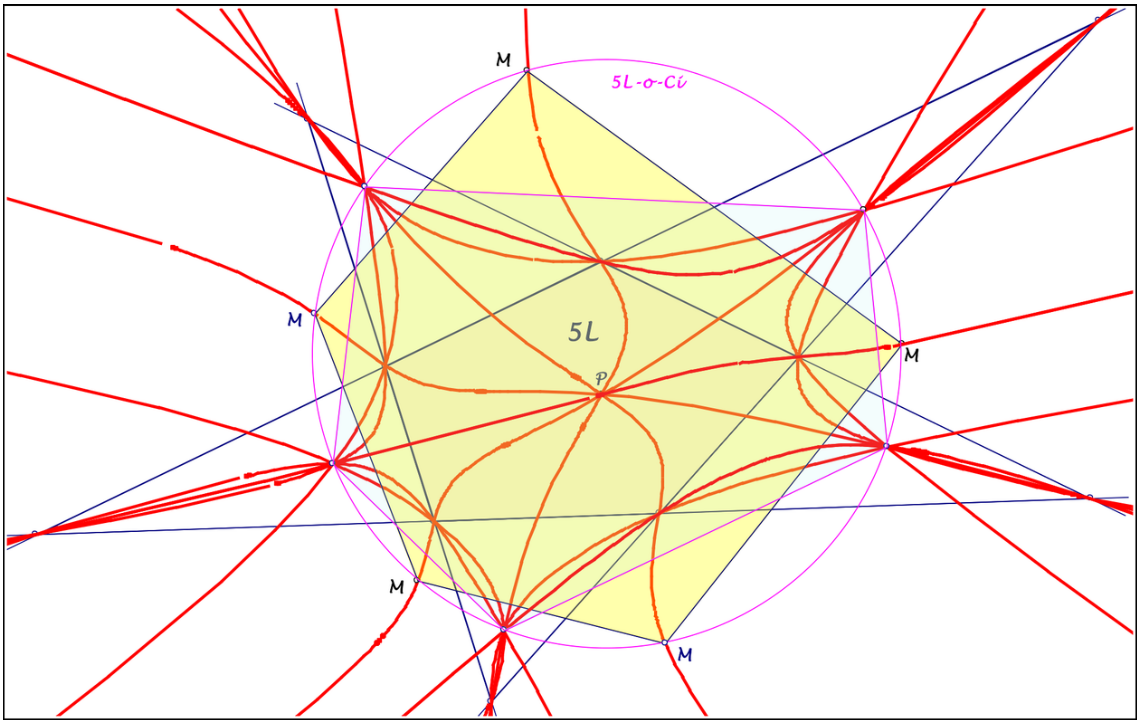
Clifford's circle 5L-o-Ci1 is the circle of the concyclic QL-P1
... for the quadrilaterals of a 5L (see EPG).

But wrt a pivot 5L-o-Ci1 bears further points:
Consider a QL and lines L through a pivot P,
... that leads to 5L and their Clifford circles 5L-o-Ci1.
The intersections of L and the corresponding 5L-o-Ci1
... give a QL-circumscribed cubic through
... QL-P1 and the pivot P and CB(P) wrt QL+QL-P1.

If we now consider these cubics for the 5 QL of a 5L,
... its 5L-o-Ci1 bears the 5 QL-P1
... and up to 5 common points of the 5 cubics.

What about these five points, depending of the pivot P?

Best regards Eckart



2020-04-13.pdf

Message: #223
Date: 2020-04-14
From: bernard.keizer@gmail.com
Subject: Re: 5L-n-P1 centric focus

```
Dear Eckart,
I've just rediscovered your notes 754 and 762 in the former
Forum.
Perhaps there will be other ideas ...
But I count on you !
Best regards
Bernard
```

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Message: #224
Date: 2020-04-16
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P1 centric focus

```
Dear Bernard,

wrt your #221, I have difficulties in understanding.
In the second passage I don't see the connections.
Then you consider a 5L and define a transformation X-->Ci(X),
... this transformation is bijective:
... Take points Y on Ci(X) and their circles Ci(Y),
... the common point of these circles is X.
Then you define a new cb-transformation
... and consider the chain X, cb(X), cb(cb(X)), ...
... I constructed first points, but found no sign,
... ... that the chain will be a quartic through your 12 points.

Excuse my lack of comprehension.
Perhaps you find helpful information
... in the QFG-messages 769, 780, 790.

Best regards Eckart
```

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Message: #225
Date: 2020-04-16
From: tungvtt@gmail.com
Subject: Minimum sum of distance square

```
Dear all,

Just some curious questions:

Define the distance of a point P and a curve (c) as the minimum
distance between P and a point on (c).

Question 1: Given a nP = P1P2...Pn (n>=4), how to construct a
circle (c) such that sum of d(Pi,(c))^2 is minimum ?

Question 1': Given a nP = P1P2...Pn and one or two other points
Qi, how to construct a circle (c) passing through all points Qi
such that sum of d(Pi,(c))^2 is minimum ?

Question 2: Given a nP = P1P2...Pn (n>=6), how to construct a
conic (c) such that sum of d(Pi,(c))^2 is minimum ?

Question 2': Given a nP = P1P2...Pn and one to four other points
Qi, how to construct a conic (c) passing all points Qi such that
sum of d(Pi,(c))^2 is minimum ?

Best regards,

Vu Thanh Tung
```

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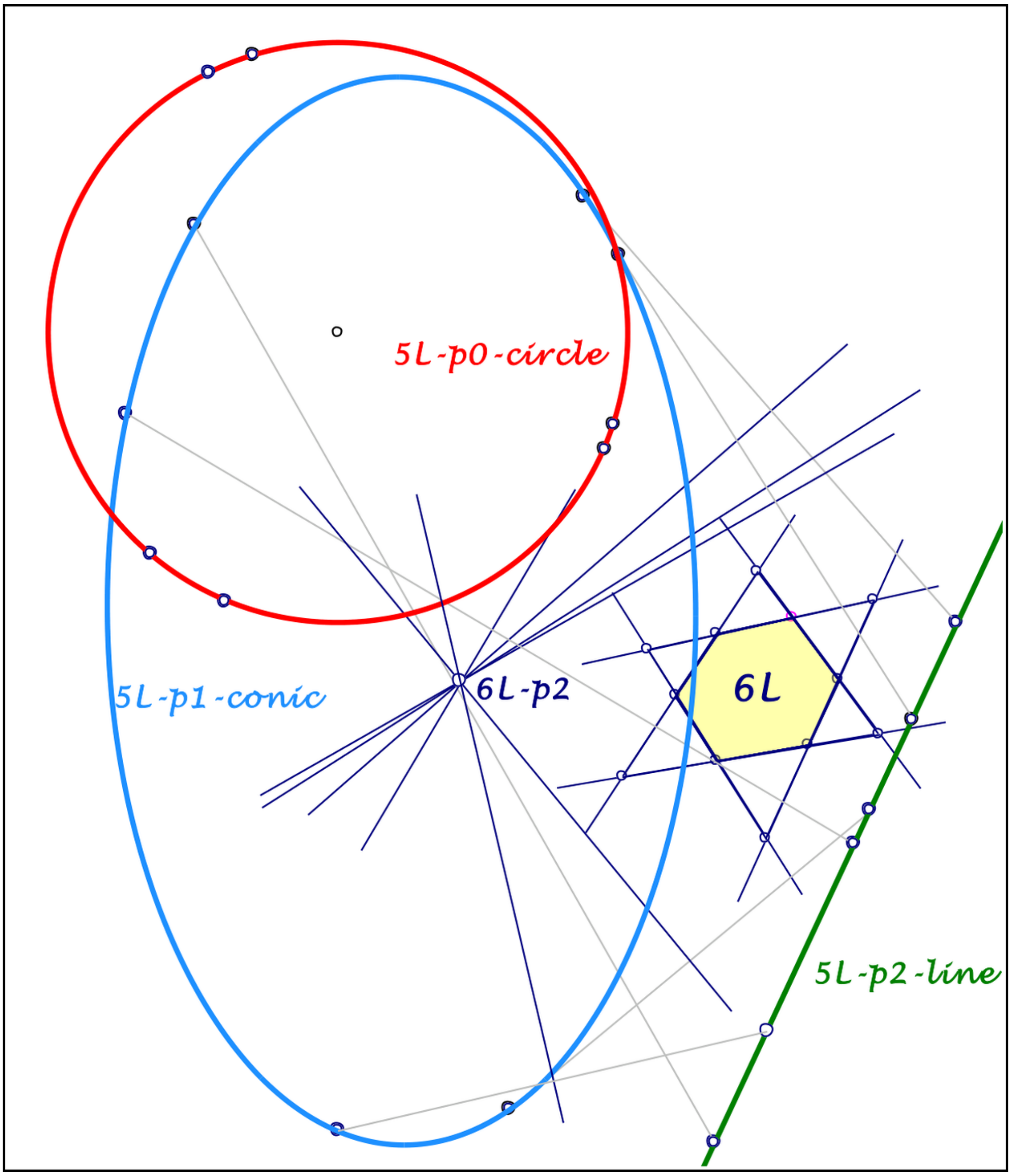
Message: #226
Date: 2020-04-17
From: eckart_schmidt@t-online.de
Subject: 5L Morley Points for 6L

```
Dear Chris, dear Bernard,

in 2014 we discussed Morley points
... (see for example nL-n-pi in EPG or #897 in QFG).

For a QL = 4L:
... 4L-p0 = QL-P4,
... 4L-p1 = QL-P3,
... 4L-p2 = QL-P29 (reflection of QL-P3 in QL-P2).

For a 5L:
... 5L-p0 = 5L-n-P3 (center of the QL-P4-circle),
... 5L-p1 = reflection of 5L-n-P3 in 5L-s-P4,
... 5L-p2 = common point of the perpendiculars
... ... through QL-P3 wrt the omitting line.

With the Morley points of a 5L we can make 6L-geometry:
The six 5L-p0 for a 6L are concyclic.
The six 5L-p1 for a 6L lie on a conic.
The six 5L-p2 for a 6L are collinear.
The six bisectors of 5L-p1.5L-p2 intersect in 6L-p2.
I think, not all properties are mentioned in EPG.

Best regards Eckart
```



2020-04-17.pdf

Message: #227
Date: 2020-04-18
From: van10hoven@gmail.com
Subject: Re: 5L Morley Points for 6L

Dear Eckart, dear Bernard,

That’s a long time ago we researched the Morley points.
I had to pick up an old scheme to find entrance again to Morley’s Miracles.
I will attach this scheme to this message. I hope it will help you.
It helps me to convert Morley’s codes to EPG-codes using the scheme and then search them in nL-Objects (https://www.chrisvantienhoven.nl/nl-items/nl-geninf/nl-0).

Eckart, about your comments, all properties can be found in EPG.
I copy here your last remarks and add my own comment in *bold* and italic.
The six 5L-p0 *= 5L-n-P3* for a 6L are concyclic. *(definition of nL-n-P3)*
The six 5L-p1 *= 5L-n-P7* for a 6L lie on a conic. *(see last property nL-n-P7)*
The six 5L-p2 *= 5L-o-P1* for a 6L are collinear. *(see 2 nd property nL-n-P7)*
The six bisectors of 5L-p1.5L-p2 *= 5L-n-P7.5L-o-P1* intersect in 6L-p2 *= 6L-e-P1. (see 1 st property nL-e-P1)*
I agree it is worthwhile developing a 6L-geometry. Morley can help us.

Best regards,

Chris

Recursive points/lines/circles in n-Lines

	old 5L-n-otation	Morley-notation	General nL-Points	n = 3	n = 4	n=5 HC-point	Ref	Construction / Properties
nL-n-P1	5L-P3		nL-Miquel Point / Centric Focus	--	QL-P1	no	[1,4]	Common point n mL-Center circles (nL-n-Ci1)
							[3]	Node Mono EnnaCardioid circumscribing n mL-EnnaCardioids
nL-n-P2			nL-Morley Centroid	X(2)	QL-P22	no	#880	Ratiopoint nL-n-P3.nL-n-P4 ((n-2) : 2)
								Homothetic Center Ref. n-Line & n-Line [Parallels through mL-n-P5]
nL-n-P3	5L-P2	a1/p0	nL-Morley's Centercircle Center / Cent	X(3)	QL-P4	1st/2nd yes	[2]	Center of nL-n-Ci1. Also called Kantor's point.
nL-n-P4		h	nL-Morley's 2nd Orthocenter	X(4)	QL-P2	no	[2]	POOL [mL-n-P5]
							[2]	External Homothetic Center of nL-n-Ci1 & nL-n-Ci2
nL-n-P5			nL-Morley's Second Circle Center	X(5)	QL-P30	0/1/2nd yes	[2]	Ratiopoint nL-n-P3.nL-n-P4 ((n-2) : 1)
nL-n-P6	5L-P5		nL-Least Squared Distances Points	X(6)	QL-P26	no	[5]	Point with Least Sum of Squared Distances to reference lines
nL-n-P7		p1	nL-Hervey Point / SVP Point	X(4)	QL-P3	1st/2nd yes	[2]	SVP [nL-n-P3.mL-n-P3]
							[2,4]	Fixed Ratiopoint [mL-n-P3(i), mL-n-P7(i)], where i=1,...,n.
nL-n-P8	5L-P6		nL-MVP Centroid	X(2)	QL-P12	0/1/2nd yes	#869	Point where Sum of vectors to all nL-vertices is zero vector
							#878	SVP [nL-n-P3.(Li^Lj)] / (n(n-1)/2)
							#873	nP-Centroid of all nL-vertices (Li^Lj)
							#881	MVP [mL-MVP Centroids]
nL-n-P9			nL-MVP Circumcenter	X(3)	QL-P6	no	#881	MVP [mL-MVP Circumcenters]
nL-n-P10			nL-MVP Orthocenter	X(4)	QL-P2	no	#881	MVP [mL-MVP Orthocenters]
nL-n-P11			nL-MVP Nine-pointcenter	X(5)	QL-Px	no	#881	MVP [mL-MVP Nine-pointcenters]
			nL-POOL[QL-P20]					
			General nL-recursive pi-/gi-points					
nL-n-pi			nL-Morley's intermediate recursive pi points				[2]	nL-pi = Ratiopoint(nL-p(i-1),nL-g(i-1)) (n : (i-n))
								Startingpoints(i=0,1) are always nL-n-p0=nL-n-P3, nL-n-p1=nL-n-P7.
nL-n-gi			nL-Morley's intermediate recursive gi points				[2]	nL-gi = Centroid(mL-pi(j)), where j=1,...,n
nL-n-Tf1			nL-Orthopole					
nL-n-Tf2			nL-Orthopolar					
			General nL-Circles/Lines/Curves					
nL-n-L1			nL-Morley Eulerline	EulerLine	QL-P2.P4		#880	Line through nL-n-P2, nL-n-P3, nL-n-P4, nL-n-P5.
nL-n-L2			nL-MVP Eulerline	EulerLine	QL-P2.P6		#880	Line through nL-n-P8, nL-n-P9, nL-n-P10, nL-n-P11.
nL-n-iL1			nL-Morley's Axes					
nL-n-Ci1			nL-Centercircle (center nL-n-P3)	CircumCircle	QL-Ci3		[2]	Circle of concyclic points mL-n-P3. Also called Kantor's Circle.
nL-n-Ci2			nL-Second Circle (center nL-n-P5)	EulerCircle	QL-Cix		[2]	Radius nL-n-Ci2 = nL-R / (n-1), where nL-R = radius nL-n-Ci1.
nL-n-Cv1			nL-Mono EnnaCardioid		QL-Qu1		[3],	nL-Curve circumscribing lower-level mL-EnnaCardioids
			n=ODD Points/Lines/Curves					
nL-o-P1	5L-P9	p(n-1)/2	nL-Morley's 1st Orthocenter	X(4)	--		[2]	nL-o-P1 will be 3L-n-p1, 5L-n-p2, 7L-n-p3, 9L-n-p4, 11L-n-p5, etc.
								POOL(mL-e-P1)
nL-o-P2	5L-P1		nL-Clifford's Circle Center	X(3)	--			Center of n concyclic mL-Clifford's Points (nL-e-P2)
nL-o-L1			nL-Line of Inscribed EnnaDeltoid Cente	X(3).X(4)			[2]	Perpendicular Bisector of nL-n-p((n-3)/2).nL-n-p((n-1)/2)
nL-o-L2			nL-Morley's Line of Orthocenters	X(4).X(4)	--		[2]	Line connecting nL-o-P1 and nL-n-P4
								POOL(nL-e-L2) preserving ratio's (checked locus 4-Line to 5-Line)
nL-o-2L1			nL-Orthogonal Reflective Axes					Lines through nL-o-P1
nL-o-Ci1			nL-Clifford's Circle (center=nL-o-P2)	CircumCircle	--			Circle of n concyclic mL-Clifford's Points (nL-e-P2)
								... the ratiopoint of (2n+1)L-p(n-1) and the centroid of 2nL-p(n-1) with
								is the same point as
			n=EVEN Points/Lines/Curves					... the common point for perpendiculars of 2nL-p(n-1) wrt the line left
nL-e-P1		p(n/2-1)	nL-Morley's EnnaDeltoid Center	--	QL-P3		[2]	nL-e-P1 will be 4L-n-p1, 6L-n-p2, 8L-n-p3, 10L-n-p4, etc.
								CPPB [mL-o-P1.mL-n-p((n-4)/2))]
nL-e-P2			nL-Clifford's Point	--	QL-P1			Common point n mL-Clifford's circles (nL-o-Ci1)
nL-e-L1			nL-Morley's Ortho Directrix	--	QL-L2		[2]	Line of n collinear versions of mL-o-P1, when n = even
								Perpendicular Bisector of nL-n-p((n-2)/2).nL-n-p(n/2)
nL-e-L2			nL-Morley's Alternate Line of Orthocenters		QL-P3.P30		[2]	Line connecting nL-e-P1 and nL-n-P5
nL-e-Cv1			nL-Morley's Inscribed EnnaDeltoid	--	QL-Qu2		[2]	Center is nL-e-P1

infix:

n	=	point/figure occurring for n = **n**atural number > 2
e	=	point/figure occurring only for n=**e**ven
o	=	point/figure occurring only for n=**o**dd
s	=	point/figure occurring only for some **s**pecial n

References:

[1] Morley's paper: On the metric Geometry of the Plane n-line
[2] Morley's paper: Orthocentric properties of the Plane n-line
[3] Morley's paper: Extensions of Clifford's Chain Theorem
[4] R. Goormaghtigh: The Hervey Point of the general n-LINE
[5] J.L. Coolidge: Two geometrical applications of the method of least squares
#nnn Refers to Yahoo Quadri-Figures Group message # nnn

Abbreviations/Terminoloy:

n-Line	A system of n random coplanar lines
nL-	indicates item that is referring to an n-Line
mL-	means (n-1)L: referring to the nLine of the first level below n
omitted line	an n-Line has n m-Lines
	When referring to an m-Line there is always an omitted line.
SVP	Sum Vector Point = Endpoint of n Summarized radial Vectors
MVP	Mean Vector Point = Endpoint SumVector/n
Ratiopoint	Let X=Ratiopoint Q1.Q2 (m:n)
	Let d1=distance(X,Q1), d2=distance(X,Q2)
	d1 is positive when X lies on the same side of Q1 as Q2
	d1 is negative when X and Q2 lie at different sides of Q1
	Corresponding properties for d2. Now m:n = d1:d2.
CPPB	Common Point Perpendicular Bisectors of mL-points
POOL	Perpendicular On Omitted Line through mL-points

MVP properties:

#888	The MVP of QL-P3 in a 5-line divides 5L-p1.5L-p2 (Morley points) in the ratio 2:3 (ES)
#882	The 5L-MVP-Orthocenter divides Morley's 5L-1st Orthocenter and 5L-2nd Orthocenter in the ratio 9:1 (ES)
#887	Not the same property in a 7-Line (ES)
#890	MVP of (n-1)L-pi-1 for n-lines divide nL-pi-1.nL-pi with ratio i/(n-i). (ES)

Compiled by Chris van Tienhoven. The content of this diagram was found in cooperation with Bernard Keizer and Eckart Schmidt.

nL-points Morley vs EPG.pdf

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Message: #228
Date: 2020-04-18
From: van10hoven@gmail.com
Subject: Re: Minimum sum of distance square

```
Dear Vu Thanh Tung,

What is your definition of "sum of d(Pi,(c))" ?
since (c) has infinite many points.

Best regards,

Chris
```

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Message: #229
Date: 2020-04-19
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Minimum sum of distance square

---

```
Dear Chris,

Here I mean d(Pi,(c))  the minimum distance of P to all points
in (c).
d(Pi,(c)) = inf{ d(P,Q) | Q in (c) }.
Anyway I think that the questions that I posed do not have
treatable answer.

Best regards,
Vu Thanh Tung

On Sun, Apr 19, 2020 at 1:33 AM Chris <van10hoven@gmail.com>
wrote:
> Dear Vu Thanh Tung,
> What is your definition of "sum of d(Pi,(c))" ?
> since (c) has infinite many points.
> Best regards,
> Chris
```

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Message: #230
Date: 2020-04-19
From: bernard.keizer@gmail.com
Subject: New 5L transformation

Dear Eckart,
Thanks for your patience and your interest (as well as for the last references). I've finally found what I had been desperatly searching for so long time.
I consider always the 12 points 10 vertices of the 5L and the 2 foci of the inscribed conic.
They form with the 2 circular points a CB14 system as the 5 copples VR of 4 lines and 5th line are degenerated circular quartics through these 12 points.
So I was convinced that there must be a transformation swapping each VR and each 5th line and more generally each tangent to the conic to a new VR ; the same transformation must give as imges of a point a triple of points forming a triangle in which the conic is inscribed.
First remark : for any point P, the circle CiP cuts the circle 5L-o-Ci in 2 points M and M' ; if you consider the CSC and CSC' centered in M and M' and swapping S and S' and the tangents L and L' from P to the conic, then it holds that CiP is either CSC(L') or CSC'(L).
I consider now the intersections Ti and T'i of L and L' with Li.
CiTi is CSC(Li) or CSCi(L) through M, Mi and CSCi(P) and cuts VRi in 3 points other than Mi. (These 3 points are the CSCi of the 3 intersections of L with VRi).
This gives 15 points (not necessary all real) on a VR with focus M, invariant in CSC and passing through S and S'.
The same way, we get 15 points (same remark) on a VR' with focus M', invariant in CSC' and also passing through S and S'.
VR and VR' intersect in S and S' and in 3 other points Q1, Q2 and Q3 on CiP.
Q1Q2, Q1Q3 and Q2Q3 are tangent to the conic and the 3 lines form with the 2 lines L and L' (also tangent to the conic) a 2nd 5L having the same inscribed conic and the same 5L-o-Ci as the initial 5L.
The image of P is the triple Q1Q2Q3, the images of L and L' are VR and VR'.
The quartic through the 12 points and P and Q1 contains Q2 and Q3 as well as the other vertices of the 2nd 5L.
This quartic is invariant in the described transformation. I suppose it is circular (or bicircular ?).
Best regards Bernard
I don't dare to send you a figure, it's too ugly !

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Message: #231
Date: 2020-04-20
From: eckart_schmidt@t-online.de
Subject: Re: 5L Morley Points for 6L

```
Dear Chris,

excuse my comment, but it is difficult for me,
... to find references for observations
... wrt different nominations of Morley, EPG and old
QFG-messages.
So your scheme is helpful for me, thanks.

My message should be a short summary of the first Morley points
... without studying the recursive definitions.

Best regards Eckart
```

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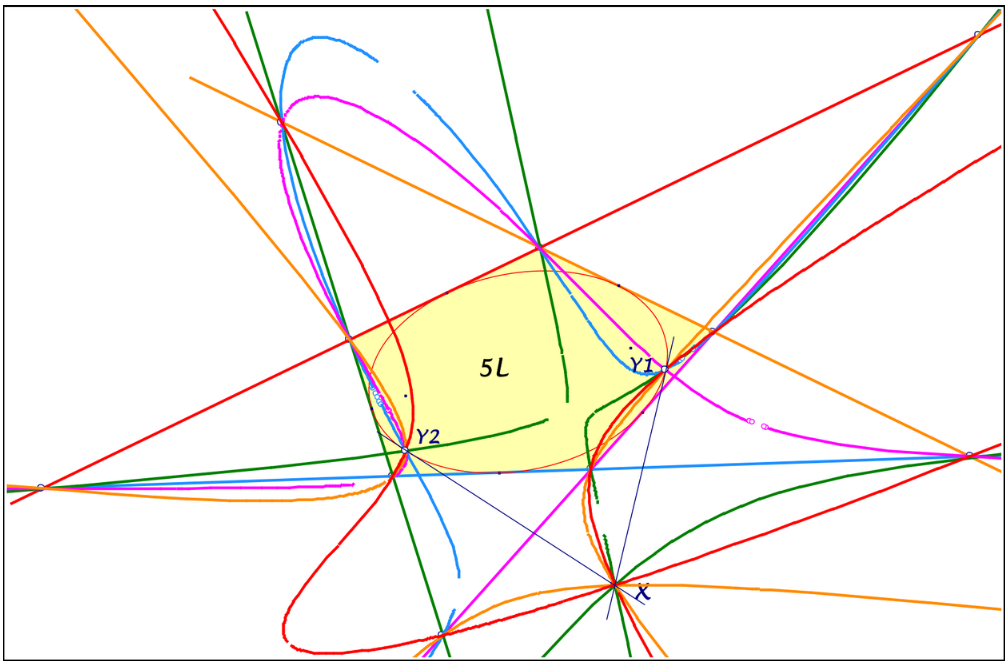
Message: #232
Date: 2020-04-20
From: eckart_schmidt@t-online.de
Subject: Re: New 5L transformation

```
Dear Bernard,

studying your interesting message 230
… I remember my QFG-message 2610
… with a QL-circumscribed cubic,
… defined for a QL by a point X,
… which leads to a 5L-configuration,
… perhaps of interest for you (see attached file):
You searched for degenerated quartics
… for the 12 points of ten 5L-points
  and the two foci of 5L-s-Co1.
Replacing the two foci by two points Y1, Y2 on 5L-s-Co1,
… there are 5 degenerated quartics through these 12 points,
… each a line of the 5L and the cubic above
  for the remaining QL,
… taking the defining point X with 5L-s-Co1-polar Y1Y2.

Best regards Eckart

PS. In #2610 there is a typo in the last line of the definition,
replace X by Z.
Wrt a simple alternative construction of the CB-point see
QFG-message 2611.
```



2020-04-20.pdf

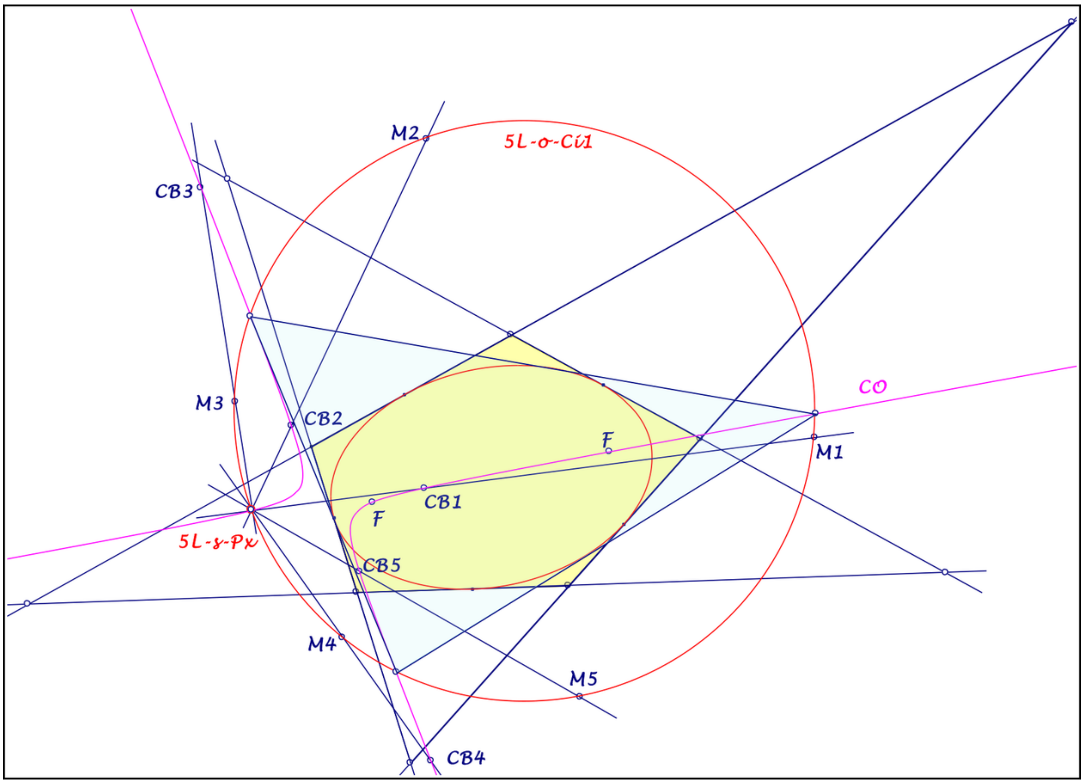
Message: #233
Date: 2020-04-22
From: eckart_schmidt@t-online.de
Subject: New Point for 5L-Geometry

```
Dear Bernard, dear Chris,

let us consider a 5L with the foci of 5L-s-Co1
… and the 5 Cayley-Bacharach points CB
… for 6 vertices of QL-components and the two foci,
… which define a conic CO, bearing the two foci.
The lines, connecting CB and the corresponding Miquel-point,
… give a common new 5L-point 5L-s-Px,
… which is also a point on the conic CO and on the circle
  5L-o-Ci1.
The conic CO intersects 5L-o-Ci in three further points,
… which give a triangle TR = ABC tangent to 5L-s-Co1
… with Ci(A) = BC, Ci(B) = AC, Ci(C) = AB
  (Ci Bernard's degenerated circle in #218).
The TR-isogonal conjugate of CO is the main axis of 5L-s-Co1.

There will be more properties,
... perhaps TR as reference triangle of 5L?

Best regards Eckart
```



2020-04-22.pdf

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Message: #234
Date: 2020-04-23
From: bernard.keizer@gmail.com
Subject: Re: New 5L transformation

```
Dear Eckart,
I've printed and read your messages 2610 and 2611.
Does this give also a transformation giving for a point P 3
points on CiP and for a line tangent to the conic a new cubic ?
My complete construction gave for a point P 3 points on CiP and
for a line tangent to the conic a Van Rees circular focal cubic.
It gave also for a point a 2nd 5L and a non degenerated circular
(?) quartic through the vertices of both 5L and S and S'.
Best regards
Bernard
```

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Message: #235
Date: 2020-04-23
From: bernard.keizer@gmail.com
Subject: Re: New Point for 5L-Geometry

```
Dear Eckart,
Beautiful !
A new point, new properties ...
As mentionned in an old message, the CBi are the CSCi of the 3rd
intersections Si of SS' with the corresponding VRi and are
therefore cocyclic with Si and Mi.
But Si and CBi are not isogonals wrt TR and isog(Si) is on CO
and isog(CBi) on SS' !
I'm waiting impatiently for new developments ...
Best regards
Bernard
```

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Message: #236
Date: 2020-04-24
From: eckart_schmidt@t-online.de
Subject: Re: New 5L-transformation

```
Dear Bernard,

I try in vain to follow your #230,
... with line 20 it comes out of my horizon ...

With the triangle in #233 you get a possibility
... for a transformation, mapping P to 3 points on CiP,
... taking the triangle vertices as center for CSC,
    swapping S and S'.
But its not your searched transformation.

Best regards Eckart
```

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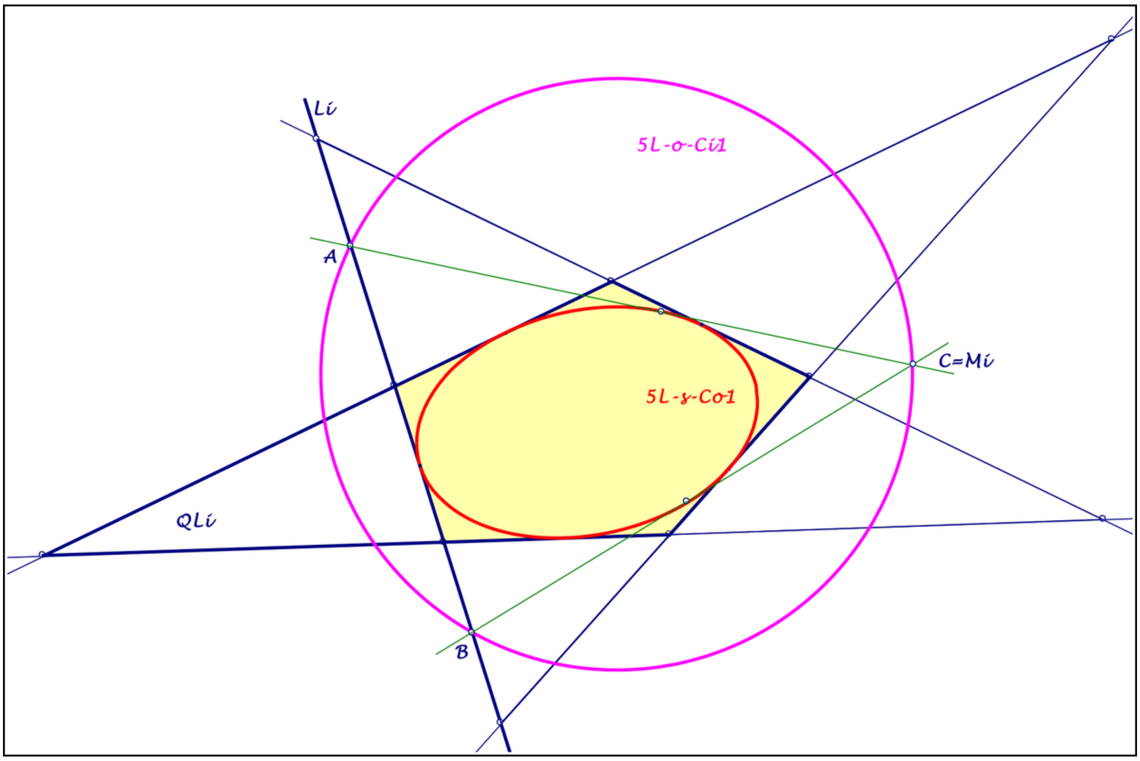
Message: #237
Date: 2020-04-24
From: eckart_schmidt@t-online.de
Subject: 5L-o-Ci1

```
Dear Bernard,

is the following property evident?

Let a 5L-s-Co1-tangent intersect 5L-o-Ci1 in A and B,
... then the 2nd tangents from A and B at 5L-s-Co1
... intersect in a point C on 5L-o-Ci1.
For a 5L-line this point C is the Miquel point of the other four
5L-lines.

Best regards Eckart
```



2020-04-24.pdf

Message: #238
Date: 2020-04-25
From: bernard.keizer@gmail.com
Subject: Re: 5L-o-Ci1

Dear Eckart,
I don't understand why you add always new items, it is always the same !
(I suggest respectually that you use the command reply to the group ...)
You gave me some references, which I read carefully.
The property you mentionned here is in your message 790.
Please read again my message 230 : for any point P, the 2 tangents L and L' to the conic, CiP cuts 5L-o-Ci1 in M and M'.
Defining the CSC and CSC' centered in M and M' and swapping S and S', it holds that CiP = CSC(L') = CSC'(L).
The mentionned property is the special case for a point on 5L-o-Ci1.
For a point on 5L-s-Co1, L = L' and M = M' and it holds that CiP = CSC(L), which is a circle tangent in M to 5L-o-Ci1.
CSC(5L-o-Ci1) is a line parallel to L and tangent to 5L-s-Co1 (in the opposite point of the contact point of L).
Best regards
Bernard

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Message: #239
Date: 2020-04-25
From: bernard.keizer@gmail.com
Subject: Re: New 5L-transformation

Dear Eckart,
I've perhaps found a new construction, which should suit you.
For any line L tangent to the conic, you associate the corresponding point M on the circle.
You define the CSC centered in M and swapping S and S'.
Then you need the Newton Line and you look for CSC conjugates and you have 3 possibilities :
1) S and S' are CSC partners by construction
2) define the point CB as the 2nd intersection of the line 5L-s-PxM with the conic CO in your message 233 and determinate CSC(CB)
3) draw the parallel to L tangent to the conic (it is CSC(5L-o-Ci1) see message 238) : then you have 2 other CSC partners reflexion of M in the parallel and center of 5L-o-Ci1.
The Van Rees circular focal cubic VR with focus M, invariant in the CSC and with Newton Line the line through the 3 middles of CSC partners is perfectly determined.
My transformation associates to the line L the VR here described and to any point X of the line L the 3 intersections (other than M) of CiX with VR.
I hope this time I haven't made a basic mystake and I will convince you.
Best regards
Bernard
PS It doesn't change anything to the construction in message 230, but it is considerably more simple !

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Message: #240
Date: 2020-04-25
From: bernard.keizer@gmail.com
Subject: Re: New 5L-transformation

Dear Eckart,
I've already found a basic mystake in the 3rd possibility, as the center of 5L-o-Ci1 is not on the cubic VR !
This new construction needs your conic CO in order to have the point CB, which is a good example of our collaboration!
Many thanks   Bernard

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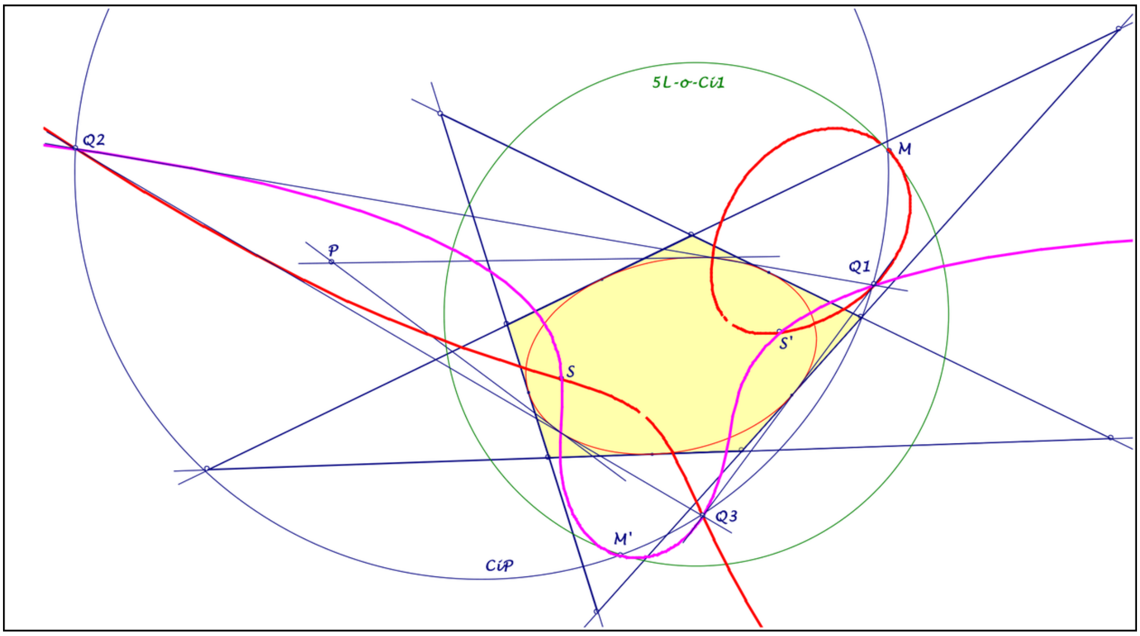
Message: #241
Date: 2020-04-26
From: eckart_schmidt@t-online.de
Subject: Re: New 5L transformation

```
Dear Bernard,

with #239 2) I could finally made a construction for your
message #230.

Best regards Eckart
```



2020-04-26.pdf

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Message: #242
Date: 2020-04-27
From: bernard.keizer@gmail.com
Subject: Re: New 5L transformation

Dear Eckart,
Alleluiah !
I'm so glad that we finally understand each other.
Looking again at your figures 233 and 241, it appears that this 5L transformation is rather a triangle transformation ...
If you start with a triangle ABC, an inscribed conic with foci S and S', you find the point K as 4th intersection of the conic CO = Isog(SS') wrt ABC (through A, B, C, S and S').
Any other point A' on the circumcircle defines with it's tangents to the conic and the 2nd intersections B' and C' of these tangents with the circumcircle another triangle circumscribed to the conic.
We may define Ci(P) as the locus of CSC(P) with CSC centered in a point of the circumcircle and swapping S and S'.
The 5L leads to the inscribed conic 5L-s-Co1 and to the circumcircle of the Mi 5L-o-Co1 and the 5 CB points give the conic CO and the triangle ABC and the point K = 5L-s-Px.
Of course, for a point P in one vertice of the 5L, the construction gives back the 5L.
Now for another point P, it gives a 2nd 5L and, if I'm not wrong, the 10 vertices of the 2 5L's are on a quartic through S and S'.
To say it another way, a triangle, it's circumcircle and an inscribed conic and 2 independant points lead to the construction 2 5L's and of a quartic through to their vertices and S and S'.
At the beginning, I was looking for circular quartics, at the end I find quartics, but I'm not sure they are circular or bicircular ...
Best regards
Bernard

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Message: #243
Date: 2020-04-27
From: tungvtt@gmail.com
Subject: A new point on quadrangle?

Dear all,

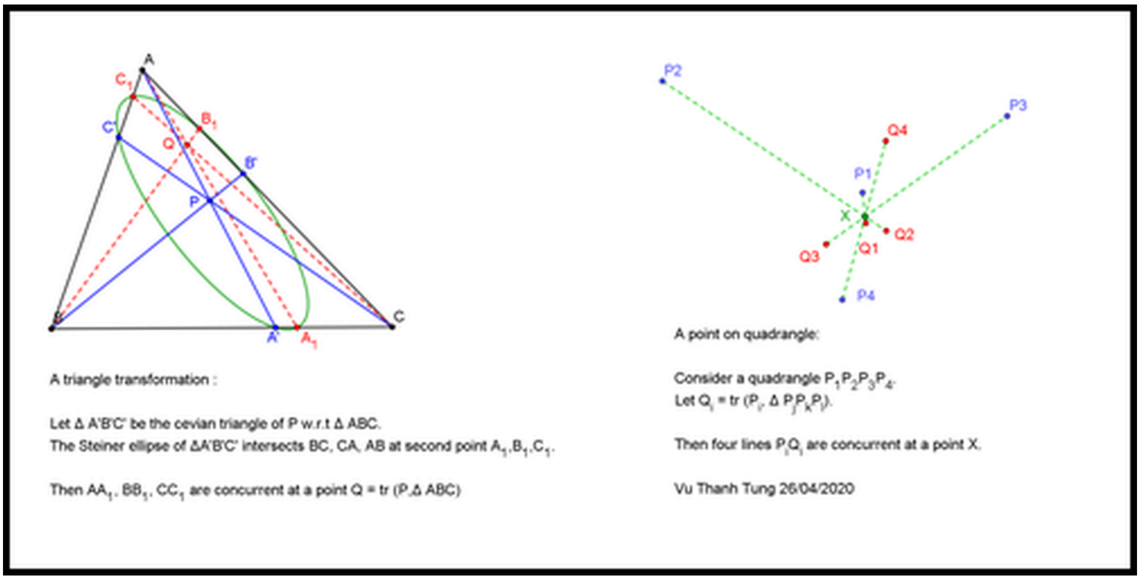
I found a point in quadrangle constructed as following:

1. A triangle transformation :
Let Δ A'B'C' be the cevian triangle of P w.r.t Δ ABC.
The Steiner ellipse of ΔA'B'C' intersects BC, CA, AB at second point A1,B1,C1.
Then AA1, BB1, CC1 are concurrent at a point Q = tr (P,Δ ABC)

2. A point on quadrangle:
Consider a quadrangle P1 P2 P3 P4.
Let Qi = tr (Pi, Δ PjPkPl).
Then four lines PiQi are concurrent at a point X.
Is X a new point in quadrangle geometry ?

Best regards,

Vu Thanh Tung



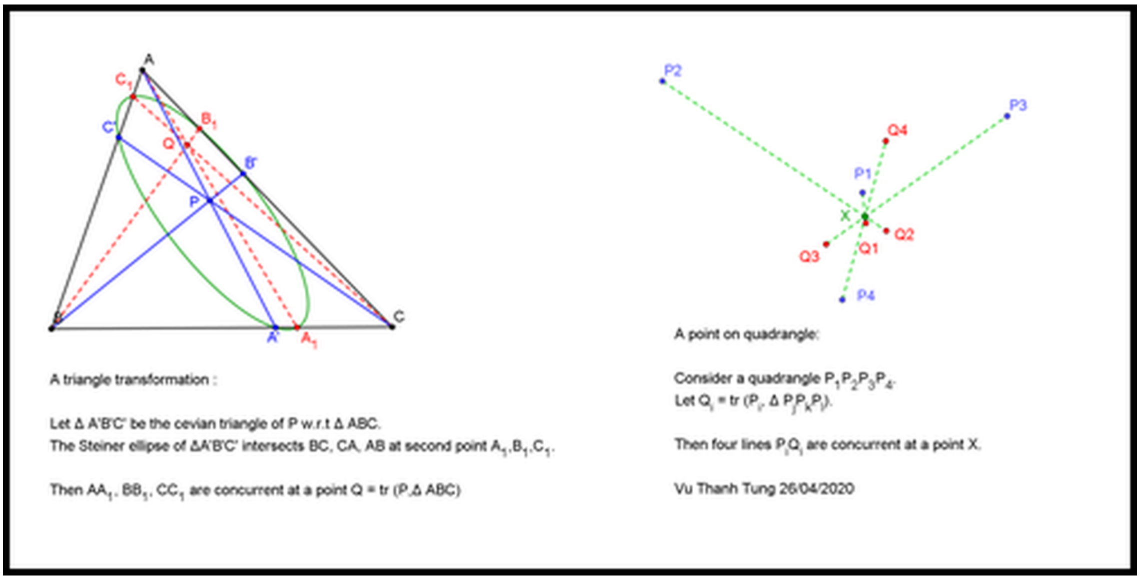
NewPointSteinerEllipse.png

Message: #244
Date: 2020-04-27
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A new point on quadrangle?

```
Please see attached figure

On Mon, Apr 27, 2020 at 10:45 PM Vu Thanh Tung via groups.io
<tungvtt=
gmail.com@groups.io> wrote:

> Dear all,
> I found a point in quadrangle constructed as following:
> *1. A triangle transformation* :
> Let Δ A'B'C' be the cevian triangle of P w.r.t Δ ABC.
> The Steiner ellipse of ΔA'B'C' intersects BC, CA, AB at second
point
> A1,B1,C1.
> Then AA1, BB1, CC1 are concurrent at a point Q = tr (P,Δ ABC)
> *2. A point on quadrangle:*
> Consider a quadrangle P1 P2 P3 P4.
> Let Qi = tr (Pi, Δ PjPkPl).
> Then four lines PiQi are concurrent at a point X.
> Is X a new point in quadrangle geometry ?
> Best regards,
> Vu Thanh Tung
```



NewPointSteinerEllipse.png

Message: #245
Date: 2020-04-27
From: eckart_schmidt@t-online.de
Subject: Re: New point on quadrangle

Dear Vu Thanh Tung,
your new point X isn't in EQF, but can be described as follows:
… X is a point on a parallel L to QA-P1.QA-P16 through QA-P5,
… the 4th harmonic point of QA-P5
… wrt the intersections of the parallel L and its QA-Tf2-conic.
Best regards Eckart

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Message: #246
Date: 2020-04-27
From: tungvtt@gmail.com
Subject: Re: New point on quadrangle

Dear Eckart,
Thank you very much,
Best regards,
Vu Thanh Tung

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Message: #247
Date: 2020-04-27
From: tungvtt@gmail.com
Subject: Re: New point on quadrangle

Dear Eckart and all,

I realized that if instead of Steiner ellipse, using any other
of circumconic like Feuerbach hyperbola, Kiepert hyperbola,
Jerabeck hyperbola, ...
The result of concurrency still hold true.

Best regards,

Vu Thanh Tung

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Message: #248
Date: 2020-04-28
From: cesar_e_lozada@yahoo.es
Subject: Re: [Quadri-and-Poly-Geometry] New point on quadrangle

---

```
Dear Mr Tung and all,

You are right:. any circumconic around a cevian triangle leads
to the explained concurrence.
See the preamble before X(36598) in ETC.

Best regards,
César Lozada

-----------------------------------------------------------
From: Quadri-and-Poly-Geometry@groups.io
[mailto:Quadri-and-Poly-Geometry@groups.io] On Behalf Of Vu
Thanh Tung
Sent: Tuesday, April 28, 2020 2:41 AM
To: Quadri-and-Poly-Geometry@groups.io
Subject: Re: [Quadri-and-Poly-Geometry] New point on quadrangle

Dear Eckart and all,
I realized that if instead of Steiner ellipse, using any other
of circumconic like Feuerbach hyperbola, Kiepert hyperbola,
Jerabeck hyperbola, ...
The result of concurrency still hold true
Best regards,
Vu Thanh Tung
```

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Message: #249
Date: 2020-04-28
From: tungvtt@gmail.com
Subject: Re: New point on quadrangle

Dear César Lozada and all,

I read the preamble before X(36598) in ETC.
If I understand correctly,
this explains the triangle transformation
(part 1. in my message #243)
but does not explain the quadrangle point
(part 2. in my message #243).
Maybe part 2. can be used to construct new points on ETC too.

Best regards,

Vu Thanh Tung

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Message: #250
Date: 2020-04-28
From: tungvtt@gmail.com
Subject: New transformation in quadrangle

Dear all,

Inspired from #243, I propose a new transformation in quadrangle:

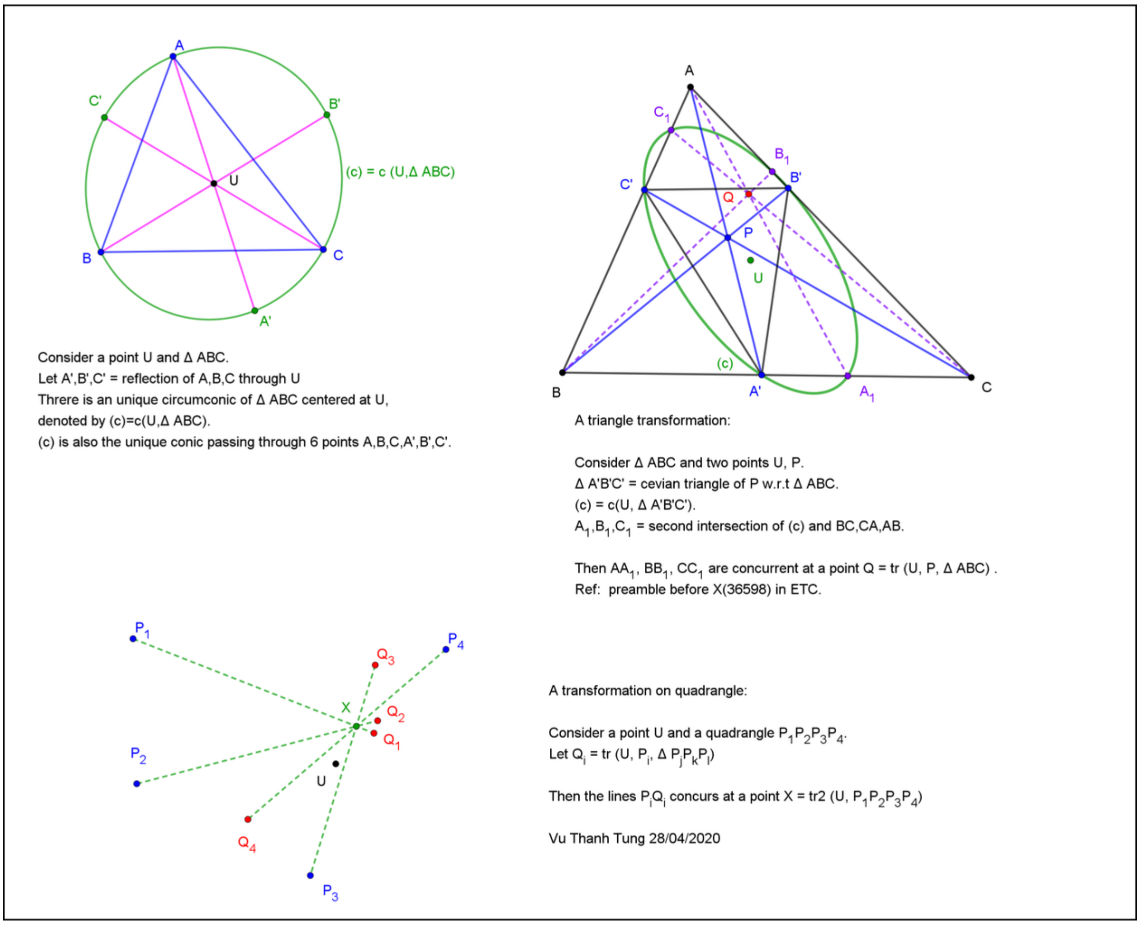
1. Conic construction:
Consider a point U and Δ ABC.
Let A',B',C' = reflection of A,B,C through U.
A,B,C,A',B',C' lie on the unique circumconic (c) = c (U, Δ ABC) centered at U.

2. A triangle transformation:
Consider Δ ABC and two points U, P.
Δ A'B'C' = cevian triangle of P w.r.t Δ ABC.
(c) = c(U, Δ A'B'C').
A1,B1,C1 = second intersection of (c) and BC,CA,AB.
Then AA1, BB1, CC1 are concurrent at a point Q = tr (U, P, Δ ABC) .

Ref: preamble before X(36598) in ETC.

3. A quadrangle transformation:
Consider a point U and a quadrangle P1 P2 P3 P4.
Let Qi = tr (U, Pi, Δ Pj Pk Pl)
Then the lines Pi Qi concurs at a point X = tr2 (U, P1 P2 P3 P4)

Best regards,
Vu Thanh Tung



QuadrangleTransformation1.pdf

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Message: #251
Date: 2020-04-29
From: eckart_schmidt@t-online.de
Subject: Re: New transformation in quadrangle

```
Dear Vu Thanh Tung,

in #243 you describe generalized quadrangle points,
... defined by triangle centers X(n),
... which are the center of circumconics for the triangles
    of the construction,
... for example in #243 the centroid X(2) as center
    of the Steiner ellipse.

Your quadrangle transformation in #250
... maps X(n) of the diagonal triangle QA-Tr1
... to the quadrangle point, described above for X(n).

Best regards Eckart
```

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Message: #252
Date: 2020-04-29
From: tungvtt@gmail.com
Subject: Re: New transformation in quadrangle

```
Dear Eckart,

Thank you, it is interesting.

Best regards,
Vu Thanh Tung
```

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Message: #253
Date: 2020-04-29
From: bernard.keizer@gmail.com
Subject: Re: New 5L transformation

Dear Eckart,
In fact, it seems we are rediscovering properties of Poncelet's porism theorem in the special case of triangles inscribed in a circle and circumscribed to a conic ...
Did you already know these properties before ?
In this case, do you have any reference ?
I've found on the web Chapter XIV Properties of two triangles J.W. Russell 1893
Best regards
Bernard

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Message: #254
Date: 2020-04-29
From: van10hoven@gmail.com
Subject: Re: New transformation in quadrangle

Dear Vu Thanh Tung and Eckart,

Point U(x:y:z) is transformed into:
{x (-x + y + z) (-q^2 r^2 x^2 (-x + y + z)^2 + p^2 r^2 y^2 (x - y + z)^2 + p^2 q^2 z^2 (x + y - z)^2),

y (+x - y + z) (+q^2 r^2 x^2 (-x + y + z)^2 - p^2 r^2 y^2 (x - y + z)^2 + p^2 q^2 z^2 (x + y - z)^2),

z (+x + y - z) (+q^2 r^2 x^2 (-x + y + z)^2 + p^2 r^2 y^2 (x - y + z)^2 - p^2 q^2 z^2 (x + y - z)^2)},

using DT-coordinates.
Combining both your comments this transformation maps any ETC-point into a QA-point with this formula.
Question: are there more Triangle centers or are there more Quadrangle centers?

Best regards,
Chris

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Message: #255
Date: 2020-04-29
From: eckart_schmidt@t-online.de
Subject: Re: New transformation in quadrangle

```
Dear Vu Thanh Tung, dear Chris,

some remarks wrt the new QA-transformation:
\vspace{1.0cm}
fixed points: QA-P1 and vertices on the sidelines of the
diagonal triangle,
... remarkable: QA-P11 mapped to QA-P38,
... lines through QA-P16 bear a triple point of its image curve
... ... on the QA-circumconic through QA-P1 and QA-P16,
... ... for the line QA-P10.QA-P16 the triple point is QA-P16.

Best regards Eckart
```

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Message: #256
Date: 2020-04-30
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] New transformation in quadrangle

---

Dear Eckart and Chris,

Thank you very much,
For Chris' question: *are there more Triangle centers or are there more Quadrangle centers*
I think that they are equal, and I have two different arguments.
Argument 1: No matter Triangle or Quadrangle, since we always label a center by an integer, the set of centers is infinite and countable, thus number of centers is roughly equal to the that of integers.
Argument 2: No matter Triangle or Quadrangle, for a fixed triangle or quadrangle, the set of centers is the entire plane, as we always find a fitting process to force a point to be a center.
For example of Triangle, a point (x:y:z) we always find a triangle center function f(a,b,c) such that x=f(a,b,c), y = f(b,a,c), z=f(c,a,b).
Thus, the number of number of centers is roughly equal to the number of the points on the entire plane.

Best regards,
Vu Thanh Tung

On Thu, Apr 30, 2020 at 1:35 AM eckart_schmidt@t-online.de <
eckart_schmidt@t-online.de> wrote:
> Dear Vu Thanh Tung, dear Chris,
> some remarks wrt the new QA-transformation:
> … fixed points: QA-P1 and vertices on the sidelines of the
> diagonal triangle,
> … remarkable: QA-P11 mapped to QA-P38,
> … lines through QA-P16 bear a triple point of its image curve
> … … on the QA-circumconic through QA-P1 and QA-P16,
> … … for the line QA-P10.QA-P16 the triple point is QA-P16.
> Best regards Eckart

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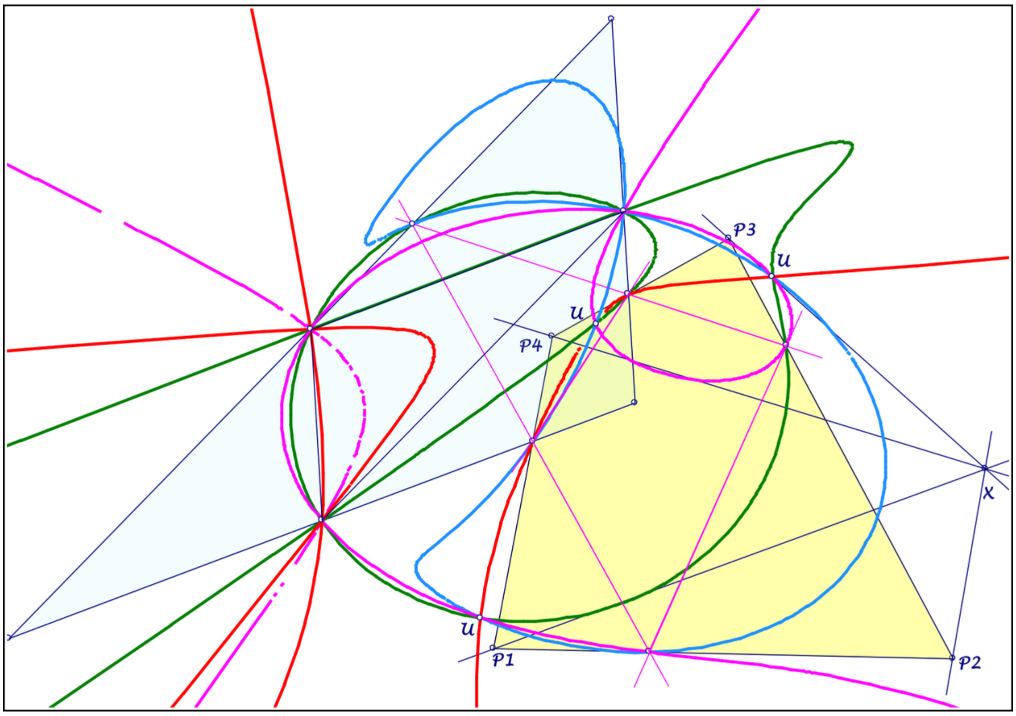
Message: #257
Date: 2020-04-30
From: eckart_schmidt@t-online.de
Subject: Re: New transformation in quadrangle

```
Dear Vu Thanh Tung, dear Chris,

in #250 for a QA = P1...P4 a new quadrangle transformation U -->
X is given,
... what about the preimages U of a given point X?
Consider for a triangle and two points
... the center Z of the common circumconic
    of their cevian triangles.
For the QA and the given point X
... take for the triangles PjPkPl one point X
    and the other point on line XPi,
... then the loci for Z give four quartics,
... circumscribed the medial triangle
    of the diagonal triangle QA-Tr1,
... double intersections in the vertices of the dual
quadrilateral
... and triple intersections U with image X
    wrt the new QA-transformation.
There can be up to three real preimages.

Best regards Eckart
```



2020-04-30.pdf

Message: #258
Date: 2020-05-01
From: eckart_schmidt@t-online.de
Subject: Re: New 5L-transformation

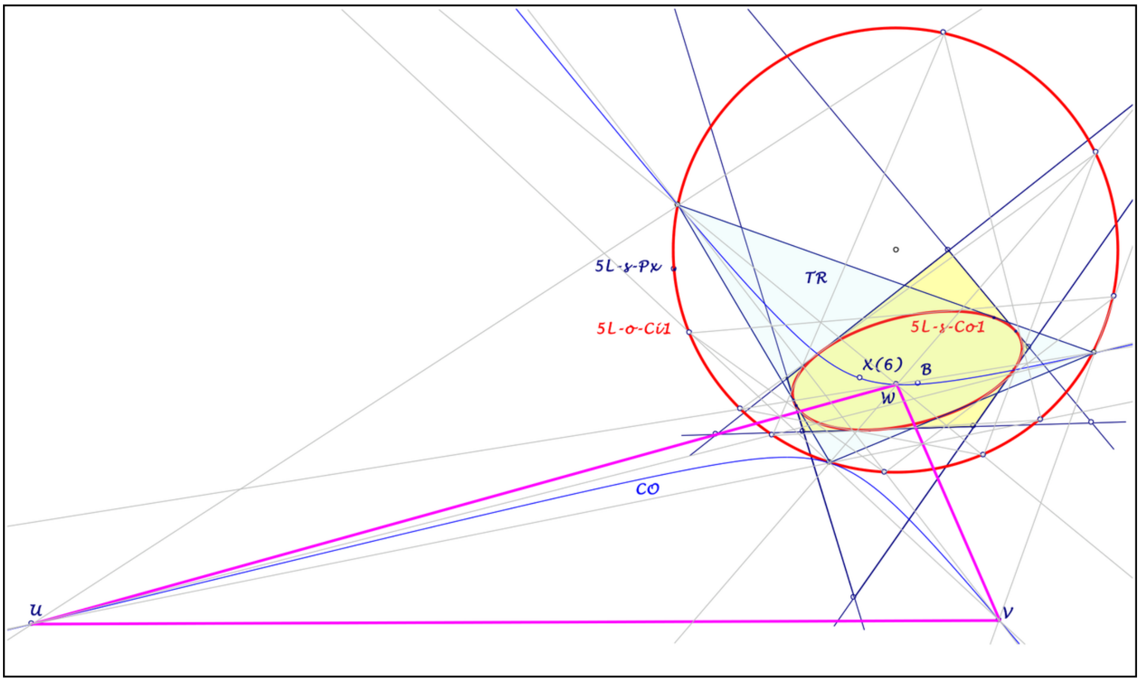
```
Dear Bernard,

without knowledge of your reference in #253
... I considered in 2008 triangles between conics,
... see on my homepage: http://eckartschmidt.de/Persp.pdf ,
... here an application for 5L-geometry:

For a 5L-constellation with 5L-s-Co1 and 5L-o-Ci1
... let us consider "inter-triangles"
... circumscribed 5L-s-Co1 with circumcircle 5L-o-Ci1.

There is an example TR in #233.
Wrt this triangle TR there are three other inter-triangles,
... perspective TR with perspectors U, V, W,
... which have the same polar wrt 5L-s-Co1 and 5L-o-Ci1,
... points on the TR-circumconic CO through
... ... the Lemoine point X(6) and the Brianchon point B.
The triangle UVW is self-polar wrt 5L-s-Co1 and 5L-o-Ci1
... and has the polar circle 5L-o-Ci1,

Best regards Eckart
```



2020-05-01.pdf

Message: #259
Date: 2020-05-01
From: bernard.keizer@gmail.com
Subject: Poncelet, triangle, QL and 5L

Dear Chris, dear Eckart, dear geometers
Chris, I remember you always liked the properties generalising for nL's triangle properties.
Eckart, I try a summary of many of our last discoveries with the help of Poncelet.

1) Poncelet porism or closure theorem
* if 2 triangles are inscribed in the same conic, they are circumscribed to a 2nd conic and conversely
* the property holds when replacing the 2 triangles by 2 n-gons ; if the vertices lie on a conic, the sides touch a 2nd conic and conversely
* the property holds when replacing the 1rst conic by a cubic, a quartic or a curve of higher degree.

2) Triangle
* any triangle circumscribed to a conic is inscribed in any conic through it's 3 vertices
* any triangle inscribed in the same conic is circumscribed to the 1rst one
* it holds if the 2nd conic is circular, id est it is the circumcircle of the 1rst triangle (for example, any triangle inscribed in 5L-o-Ci1 is circumscribed to 5L-Co1)

3) Quadrilateral
* any QL circumscribed to a conic is inscribed in any cubic through the 6 vertices
* any QL inscribed in this cubic is circumscribed to the conic
* the tangents to the conic in 2 CSC partners define a 2nd QL
* the 12 vertices are on the same cubic, which passes through QL-P1 and is CSC invariant
* if the cubic is circular, it is QL-Cu1 of both QL's (for Eckart, it gives a simple construction of the VR knowing QL-P1, CB and it's CSC partner on SS')

4) 5L
* any 5L is circumscribed to a conic and inscribed in any quartic through the 10 vertices
* any 5L inscribed in the quartic is circumscribed to the same conic
* the tangents to the conic frm a point and it's 3 5L-Tfx partners (see new 5L transformation) define a 2nd 5L

```
* the 20 vertices are on the same quartic, which passes through
the 2 foci of the conic
* which of these quartics is circular ? (passing perhaps through
the vertices of Eckart's triangle ABC and the point 5L-Px)

Best regards
Bernard
PS Many thanks in advance to all members of the forum for the
answer to my last question or any reference to Poncelet's porism
```

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Message: #260
Date: 2020-05-02
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Bernard,

reading your #259, I stopped at the second point.
Where is my misunderstanding?
Consider two quadrigons in an ellipse,
… then the lines of the first and one line of the second
… define the inscribed conic,
… which must not contact the other lines of the second
quadrigon.

Best regards Eckart
```

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Message: #261
Date: 2020-05-02
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Eckart,
Thanks for your interest, I hope it is a new start !
You may read Poncelet closure theorem on Wikipedia or in
Mathword ...
You cannot consider 2 random quadrigons in a conic, which in
fact generally don't touch another conic.
You have to consider a quadrigon in a conic circumscribing
another conic.
Then take either a point on the 1rst conic or a line tangent to
the 2nd.
Draw one tangent from the point to the 2nd conic and again the
tangent from the 2nd intersection of the 1rst tangent and so on.
The 4th operation close the quadrigon in the starting point.
Or take one of the 2 intersections of the tangent to the 2nd
conic with the 1rst and do the same as above.
The result is the same.
Best regards
Bernard
```

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Message: #262
Date: 2020-05-02
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Bernard,
I think, in #259 your formulation in point 2 of 1) isn't right,
... as well as in point 3 of 2) wrt the 5L-example,
... as well as in point 2 of 3) …
In #261 you describe poncelet's porism in my used understanding:
For a n-gon with circumscribed and inscribed conic
every point on the circumconic
... can be the vertex of another n-gon
    with these circum- and inscribed conic.
Perhaps of interest:
For a 5G with 5P-s-Co1 and 5L-s-Co1
... the 5G between the conics have the same 5G-s-P1,2.
Best regards Eckart
```

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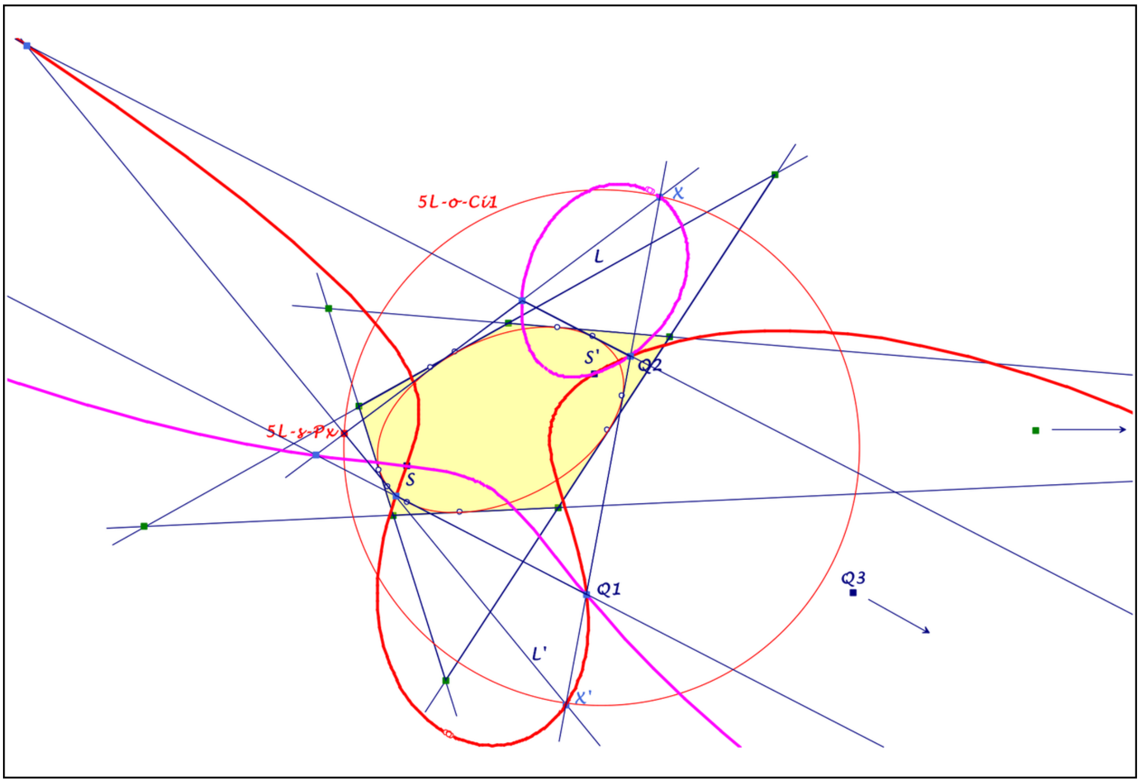
Message: #263
Date: 2020-05-02
From: eckart_schmidt@t-online.de
Subject: Re: New 5L transformation

Dear Bernard,

attached a drawing of the 20 +2 points
… for the final quartic in #230 for P = 5L-s-Px in #233.
Special:
L, L' intersect VR, VR' on 5L-s-Ci1 in X, X',
... XX' bears two intersections Q1, Q2 of VR and VR',
... Q3 is a point at infinity,
… L1, L2, Q1Q2, Q1Q3, Q2Q3 give the 2nd 5L.

Best regards Eckart



2020-05-02.pdf

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Message: #264
Date: 2020-05-03
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Eckart,
I tried in vain to understand and interpret your message !
What is wrong in 1.2, 2.3 and 3.2 ?
You give no precision and no explanation ...
I don't want to die stupid
Best regards
Bernard
```

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Message: #265
Date: 2020-05-03
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Bernard,

wrt #259: 1.2 applied to quadrigons:
"... if 2 quadrigons are inscribed in the same conic, they are
circumscribed to a 2nd conic."
This doesn't hold (see #260).

Best regards Eckart
```

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Message: #266
Date: 2020-05-05
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Eckart,
Many thanks for your attention and corrections.
Finally, my formulation in 1.2 was wrong and I understand your
message 260, for which I was responsible !
I have to check it carefully and rewrite totally.
3.2 and 4.2 are also wrong for the same reason, but I think 2.3
is correct ?
1.3 is also wrong : it holds only for socalled Poncelet curves,
curves of degree n-1 through the 1/2 n(n-1) vertices of a n-gon
circumscribed to a conic.
(If there is one, there are an infinity ...)
All conics and cubics are Poncelet curves but only the Lüroth
quartics.
Best regards
Bernard.
```

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Message: #267
Date: 2020-05-05
From: eckart_schmidt@t-online.de
Subject: Re: New 5L transformation

```
Dear Bernard,

in addition to #263:
The points Q1 and Q2 for the special quartic wrt 5L-s-Px
… lie on the axes of 5L-s-Co1.

Best regards Eckart
```

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Message: #268
Date: 2020-05-05
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Eckart,
Sorry, you're right, 2.3 is also wrong !
Best regards
Bernard
```

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Message: #269
Date: 2020-05-05
From: bernard.keizer@gmail.com
Subject: Re: New 5L transformation

```
Dear Eckart,
Sorry that I didn't answer immediately, as I'm involved in
Lüroth quartics and it takes me me time and efforts !
With your new property, I checked immediately that the quartic
through the 12 points and Q1 and Q2 passes also through X and X'
and that the 2nd tangents in Q1 and Q2 to the conic are
parallel.
Best regards
Bernard
```

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Message: #270
Date: 2020-05-07
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Eckart,
This time I checked carefully all the formulations !
I hope there is no longer basic mistakes ...
For Lüroth, see Wikipedia, which gives 2 references of articles
in german
Best regards
Bernard
```

Poncelet, Darboux, Lüroth, triangle, QL and 5L

1. Poncelet porism or closure theorem
   - If a n-gon is inscribed in a 1rst conic and circumscribed to a 2^{nd} conic, there are an infinity of n-gons sharing the same circumconic and inconic

2. Poncelet curves and Darboux theorem
   - Poncelet curves are curves of degree n-1 passing through the ½ n(n-1) vertices of n-gons (conics for triangles, cubics for QL's, Lüroth quartics for 5L's …)
   - If a n-gon is inscribed in a Poncelet curve and circumscribed to a conic, there are an infinity of n-gons sharing the circumscribed curve and the inscribed conic

3. Triangle
   - If 2 triangles are inscribed in the same 1rst conic, they are circumscribed to a 2^{nd} conic and conversely (and they are self-polar wrt a 3rd conic and conversely)
   - In this case, there are an infinity of triangles sharing the same circumconic and inconic
   - For example, if the conics are the circumcircle and incircle of a triangle, there are an infinity of triangles sharing the same circumcircle and incircle : they are poristic triangles.
   - Another example is given in the 5L ; there are an infinity of triangles inscribed in 5L-o-Ci1 and circumscribed to 5L-s-Co1

4. Quadrilateral
   - If a quadrigon is inscribed in a cubic and circumscribed to a conic, there are an infinity of quadrigons sharing the circumscribed cubic and the inscribed conic
   - The tangents to the conic in 2 CSC partners define a 2^{nd} quadrigon
   - The 12 vertices of the 2 QL's lie on the same cubic, which passes through QL-P1 and is CSC invariant (if the cubic is circular, it is QL-Cu1 of both QL's)

5. Pentalateral and Lüroth quartics
   - If a pentagon is inscribed in a quartic and circumscribed to a conic, there are an infinity of pentagons sharing the circumscribed quartic and the inscribed conic
   - The quartic through the 10 vertices is a Lüroth quartic (for example : 4 of the 5 lines, 2 conics each through 2 points of each line, 2 lines and the circumcircle of the 3 remaining points, 2 lines and the conic through the 3 remaining points and the 2 foci of the conic, a line and the Van Rees focal circular cubic of the QL of the 4 other lines or simply any quartic through the 10 vertices of the 5L and 4 random points …)
   - The tangents to the conic from a point and it's 3 5L-Tfx partners define a 2^{nd} 5L. 5L-Tfx swaps each line and the VR of the QL of the 4 other lines.
   - The 20 vertices are on the same quartic, which passes through the 2 foci of the inscribed conic and is 5L-Tfx invariant (which of these quartics is circular ?)

Poncelet.pdf

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Message: #271
Date: 2020-05-08
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Bernard,

excuse my new doubts:

Are there typos wrt n-gon and nL?
... Perhaps in 2.1: "... 1/2 n(n-1) vertices of n-gons…)

In 4.1 you describe 2.2 for quadrigons, but I cannot confirm:
... "If a quadrigon is inscribed in a cubic and circumscribed
    to a conic,
... there are an infinity of quadrigons …"

Perhaps the same in 5.1.

Best regards Eckart
```

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Message: #272
Date: 2020-05-09
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

Dear Eckart,
No need to apologise !
On a contrary, I'm glad that you continue to take interest in this fasinating item.
I've just discovered that there are links to the Marden/Siebeck theorem ...
I'm afraid it is far beyond my geometrical knowledge !
As I am confined home in the Dordogne, I can't access to the library of the Institut Henri Poincaré in Paris, as I did when I discovered Frank Morley for example.
For 2 conics, Poncelet used n-gon ; it is sometimes reproduced by closed n-gon.
For Poncelet curves, I found complete n-gon, which is rather nL, as you mention !
In 4.1 and 5.1, I used in fact the description and construction of 2.2
Take any cubic through the 6 vertices of a QL with 4 lines tangent to conic or any quartic through the 10 vertices of a 5L with 5 lines tangent to the conic 5L-s-Co1.
Any other line tangent to the conics cuts the cubic in 3 points or the quartic in 4 points.
The 2nd tangents in the 3 or the 4 intersection points give with the line a new QL of 4 lines or a new 5L of 5 lines.
Best regards
Bernard
PS The 2 main references on Google I found are Lüroth quartics with Lüroth and Morley and Poncelet curves ...

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Message: #273
Date: 2020-05-09
From: van10hoven@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Bernard, dear Eckart,

Are you familiar with this reference?
Frank Morley, On the Lüroth Quartic Curve
* Lüroth, Math. Annalen, Vol. I
American Journal of Mathematics, Vol. XXXVI.

Best regards,
Chris
```

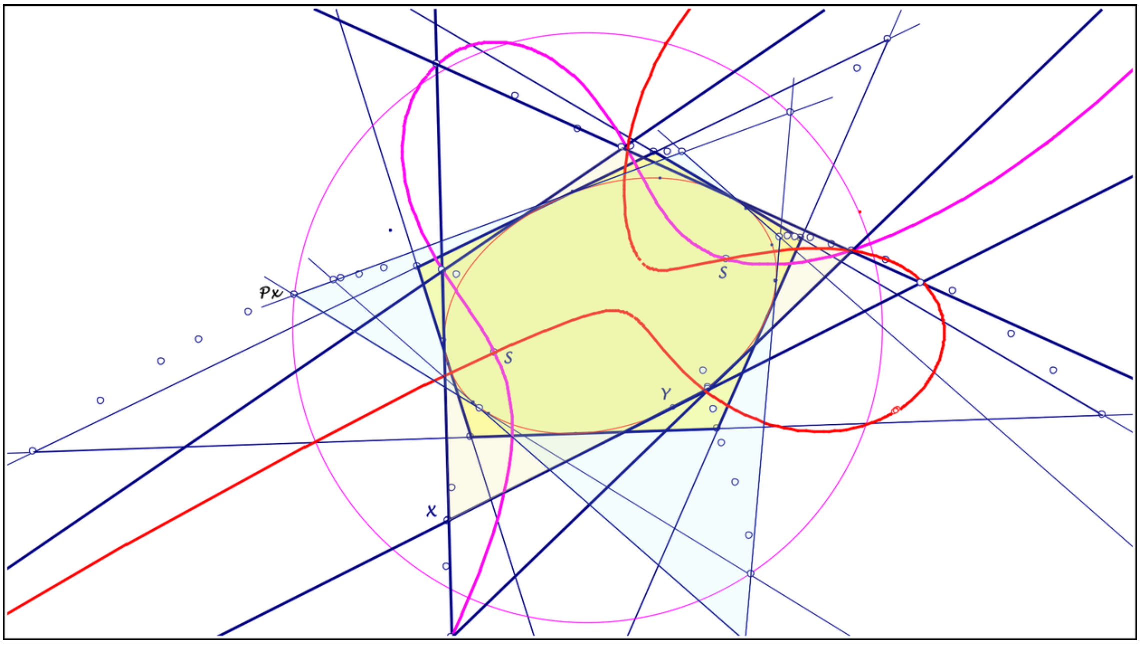
Message: #274
Date: 2020-05-10
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

```
Dear Bernard,

attached a Lüroth labyrinth in approximation
... as sign, that I am interested in this item:
Following your #230, I started with a 1st 5L (yellow)
... and a 2nd 5L (blue) wrt the point Px in #233 on 5L-s-Ci1.
With your new 5L-transformation
... I draw points of the quartic in approximation,
... so you can make a handmade curve.
Every point X can be a vertex of an inscribed 5L (brown),
... tangent to the common conic 5L-s-Co1:
Let Y be the contact point of one tangent from X at 5L-s-Co1,
... The cubic VR=VR' for X in the sense of #230
... intersects the other tangent in 3 points,
... their tangents at 5L-s-Co1 give the last 3 lines
    of the 3rd 5L.

Perhaps helpful for consideration.

Best regards Eckart
```



2020-05-10.pdf

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Message: #275
Date: 2020-05-11
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

```
Dear Bernard,

excuse, there are typos in my last message:
... line 4:  replace 5L-s-Ci by 5L-o-Ci,
... line 11: replace X by Y.

Best regards Eckart
```

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Message: #276
Date: 2020-05-11
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, triangle, QL and 5L

```
Dear Chris,
Thanks for your interest !
This is in the 1rst of of the 2 references I gave Eckart in my
message 272.
I'm seeking desperatly for a reference to a *circular* Lüroth
quartic ...
Best regards
Bernard
```

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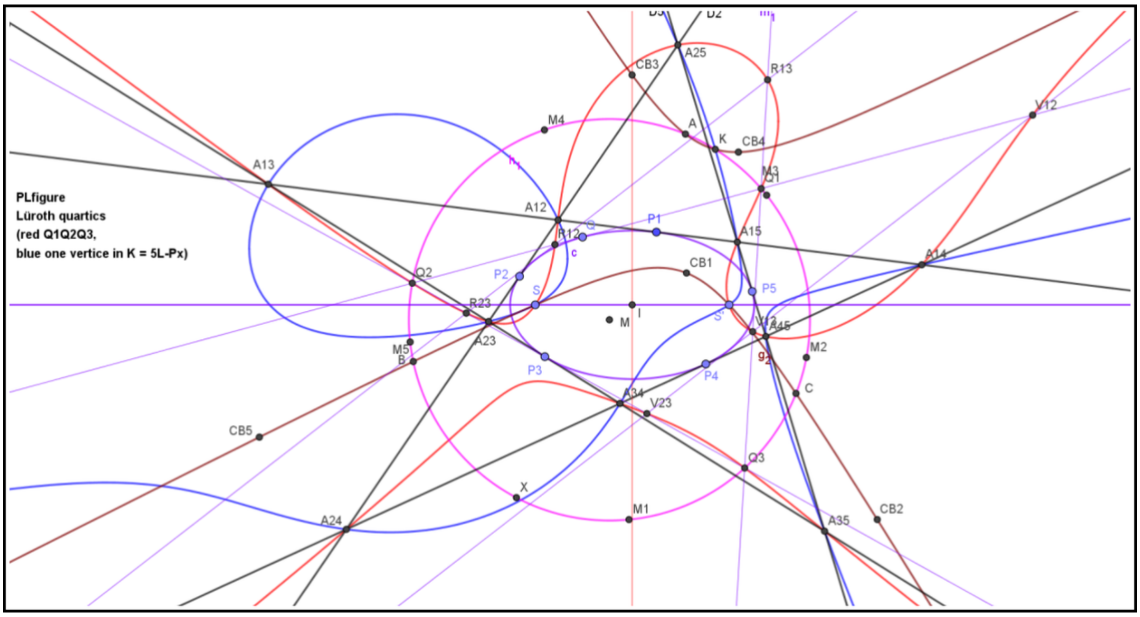
Message: #277
Date: 2020-05-12
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

Dear Eckart,
Thanks for this renewed sign of interest
I think I've understood at last !
Your quartic with the point K = 5L-Px is a special case of a more general construction.
For any triangle Q1Q2Q3 inscribed in 5L-o-Ci1 and circumscribed to 5L-s-Co1, it is possible to draw a Lüroth quartic through the 10 vertices Aij of the initial 5L, the 2 foci S and S' of the conic and 2 vertices of the triangle Q1Q2Q3 (the last vertice is on the quartic).
Q1Q2Q3 are the 3 5L-Tfx of an infinity point P and CiP is the circle 5L-o-Ci1.
The quartic cuts Qij in 2 other points Rij and Vij and the 2 lines through the Rij and the Vij are parallel and tangent to the conic. The 2 lines and the 3 sides of Q1Q2Q3 form the 2nd 5L inscribed in the same quartic with vertices the infinity point P in the direction given by the 2 parallel lines, the vertices Q1,Q2 and Q3 and the 6 points Rij and Vij.
This holds in particular if Q1Q2Q3 is your triangle ABC or if one of the vertices is in K = 5L-s-Px or in X = 5L-n-P1.
This time, I send you a figur !
Best regards
Bernard
PS Unfortunately, I don't think these quartics are circular ...



Lroth quartics.pdf

Message: #278
Date: 2020-05-13
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

```
Dear Bernard,

thanks for the figure, I am fascinated of your drawn quartics,
... what is your construction?

Can you confirm my following observations?
... The construction of the 3rd 5L in #274
... for any point X with two tangents at 5L-s-Co1
... give a 5L with the same 5L-s-Px and conic C0,
... further the same 5L-s-Co1 and 5L-s-Tf1 as for the
    reference 5L.
This holds especially for all 5L wrt reference 5L
... inscribed a quartic and circumscribed a conic.

Best regards Eckart

PS. Is there a possibility to judge the circularity without
equation?
```

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Message: #279
Date: 2020-05-14
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

Dear Eckart,
I use no particular construction.
Geogebra allows to draw conics through 5 points, cubics through 9 points (provided they are not in a CB configuration), quartics through 14 points (same remark), quintics through 20 points ...
There is a special command CourbeImplicite(points).
Back to the problem
All quartics through the 10 vertices of a 5L are Lüroth quartics and verify Poncelet porism.
Now, some are circular, some are through S and S', some are through a point and it's 5L-Tfx partners ...
The 5 degenerated line + VR of the 4 other lines are circular through S and S'. The 10 vertices , S and S' and the circular points are therefore in a CB14 configuration.
But all circular don't contain necessary the points S and S' (for exaple the 12 degenerated formed by 2 lines (which contain 7 vertices) and the circle through the 3 remaining points are circular and not through S and S'.
I checked following ideas :
1) For a quartic through the 10 vertices and 4 random points, any 6th line tangent to the conic 5L-s-Co1 cuts the quartic in 4 points and the 2nd tangents to the conic in each of these points form with the 6th line a 2nd 5L, which has not the same 5L-o-Ci1.
2) For a quartic through the 10 vertices, the 2 points S and S' and 2 random points, it is the same as above.
3) For a quartic through the 10 vertices, any random point X and it's 3 5L-Tfx partners, it's different !
The quartic is 5L-Tfx invariant and passes through the points S and S', the 2nd 5L defined by the 2 tangents from X to the conic and the 3 sides of the triangle of it's 5L-Tfx partners (inscribed in Ci(P) and circumscribed to the conic 5L-s-Co1) has the same circle 5L-o-Ci1 and the same conic 5L-s-Co1 by definition, it has also the same conic CO through the CB and therefore the same triangle ABC and the same point K = 5L-Px.
I suppose this will confirm your conjectures and I hope it will give you plenty of new ideas (what about the parabola inscribed in ABC with focus in K ?).
Best regards
Bernard

PS I think your last remark is correct, but I hoped to find something with special points X, like we know the circular cubic through the 6 vertices of a QL passes through QL-P1 if it is CSC invariant and if it passes through the 3 vertices of the orthic triangle of DT (and their CSC partners).

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Message: #280
Date: 2020-05-14
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

Dear Eckart,
Sorry, the formulation of the PS is not correct : the cubic through the 6 vertices of a QL, a point X and it's CSC partner passes through QL-P1 and is CSC invariant ; it is QL-Cu1 if X is one vertice of the orthic triangle of DT.
The same way, the quartic through the 10 vertices of a 5L, a point X and it's 3 5L-Tfx partners passes through S and S' and is 5L-Tfx invariant ; for which points X is it circular ?
Best regards
Bernard
PS By the way, there are 10 and not 12 degenerated circular quartics formed by 2 lines and the circle through the 3 remaining points and there are 10 degenerated not circular quartics formed by the 2 lines and the conic through the 3 remaining points and the points S and S'.

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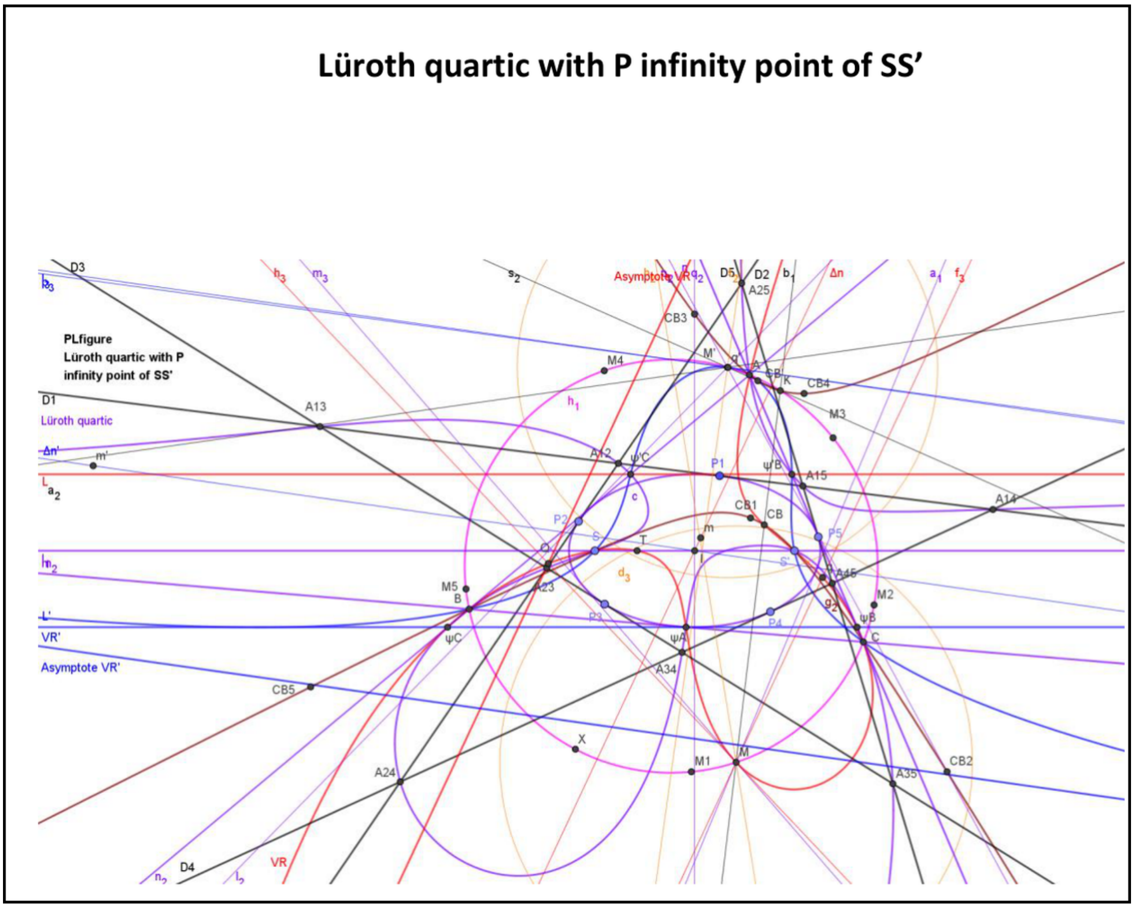
Message: #281
Date: 2020-05-16
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

```
Dear Eckart,
Perhaps of interest for you.
The 3 5L-Tfx partners of the infinity point of SS' are your
points A, B and C.
The 2 tangents to the conic parallel to SS' define with the
sides of ABC the 2nd 5L.
I send you a figur with L and L' tangent to the conic and
parallel to SS', their associated points M and M' and their
5L-Tfx transforms which are the VR of the 3 sides of ABC and L'
or L.
The 10 vertices of the 2nd 5L are the infinity point of SS', the
points A, B and C and their CSC in the 2 CSCs centered in M and
M' and swapping S and S'. (My notation for CSC is ψ).
Last, the quartic through the 22 points.
Best regards
Bernard
```



Lroth quartic with P infinity point of SS_.pdf

Message: #282
Date: 2020-05-18
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle,, QL and 5L

```
Dear Bernard,

thanks for your last two messages
... with detailed interesting properties,
... especially wrt the background for the new 5L-point,
... but with my handicap of no construction for the quartic
... I can not give new aspects.

Best regards Eckart
```

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Message: #283
Date: 2020-05-18
From: eckart_schmidt@t-online.de
Subject: Octic for 5L

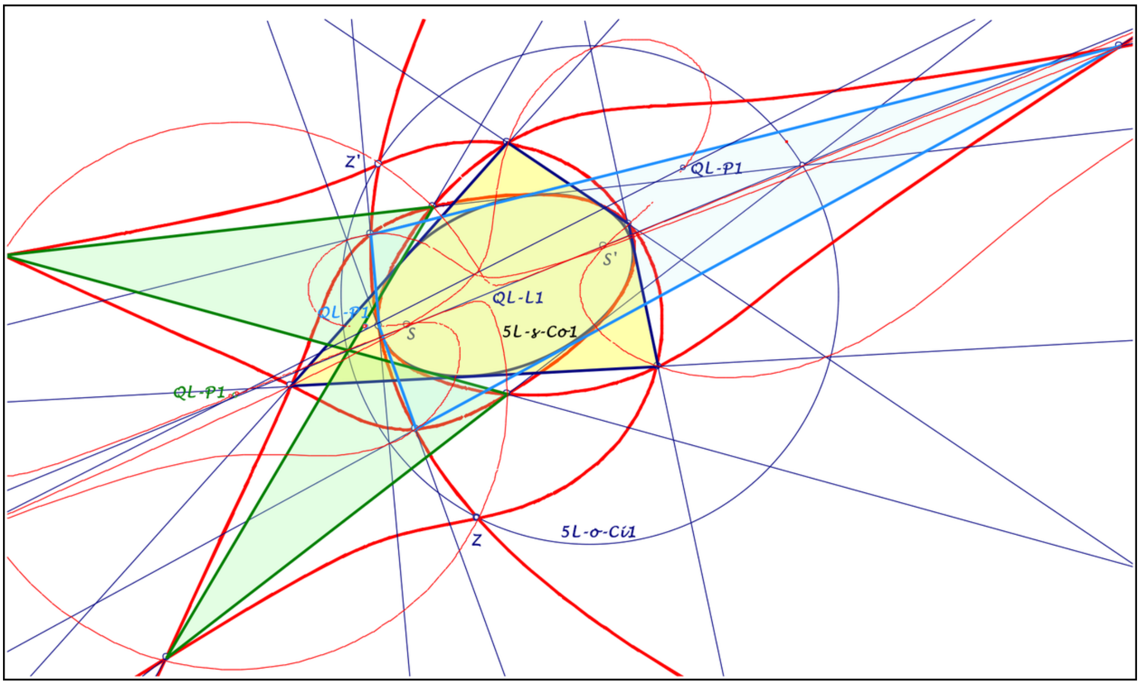
```
Dear Bernard,

let us start with a 5L, its 5L-o-Ci1 and its 5L-s-Co1 and the
foci S, S'
... and consider a point X on Ci1 with its tangents to Co1,
... further the circumconic of X, S, S'
... ... and the 2nd intersections of the tangents and Ci1;
let Y be the 4th intersection of the circumconic and Ci1
... with its tangents to Co1,
... there are two points Z, Z' on Ci1 where X = Y.

The locus of the 4 intersections of the tangents (unequal X,Y)
is an octic:
... tangent in four points to Co1
... with 14 double points, two of them Z, Z'.
Each double point unequal Z, Z'
... bears on a tangent to Co1 another double point,
... defining 12 tangents to Co1,
... which give three QG with vertices in the double points
... and QG-2P2 on Ci1.
The QG have the same Newton line QL-L1 = SS'
... and their Miquel points are collinear,
... their cubics QL-Cu1 bear S, S' and have a common point (?).

Remark wrt the figure:
The double points are approximated, so there are inaccuracies.
... The reference 5L is not shown,
... the octic is the same for all 5L with the same Ci1 and Co1.
Final question:
Define the foci S, S' and the 12 considered double points
(unequal X,Y)
... a special Lüroth quartic for the reference 5L?

Best regards Eckart
```



2020-05-18.pdf

Message: #284
Date: 2020-05-18
From: eckart_schmidt@t-online.de
Subject: Re: Octic for 5L

```
Dear Bernard,

there is a typo in my last message #283,
... replace X,Y by Z,Z' in line next to the last.

Best regards Eckart
```

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Message: #285
Date: 2020-05-20
From: eckart_schmidt@t-online.de
Subject: Re: Octic for 5L

```
Dear Bernard,

in addition to #283:
... The three QG are cyclic, their circumcircles bear Z, Z'.
... The line of the collinear QL-P1
    is the common 1st Steiner axis.
... The three QL-Cu1 have common points in the foci S, S'
... ... and a common point on the line of the collinear QL-P1.

Best regards Eckart
```

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Message: #286
Date: 2020-05-22
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

Dear Bernard,

is the following observation true?
... A Lüroth quartic of a 5L intersects 5L-o-Ci1 in 6 points.

If this holds: Is there a special Lüroth quartic
... for the following constructed 6 points on 5L-o-Ci1?

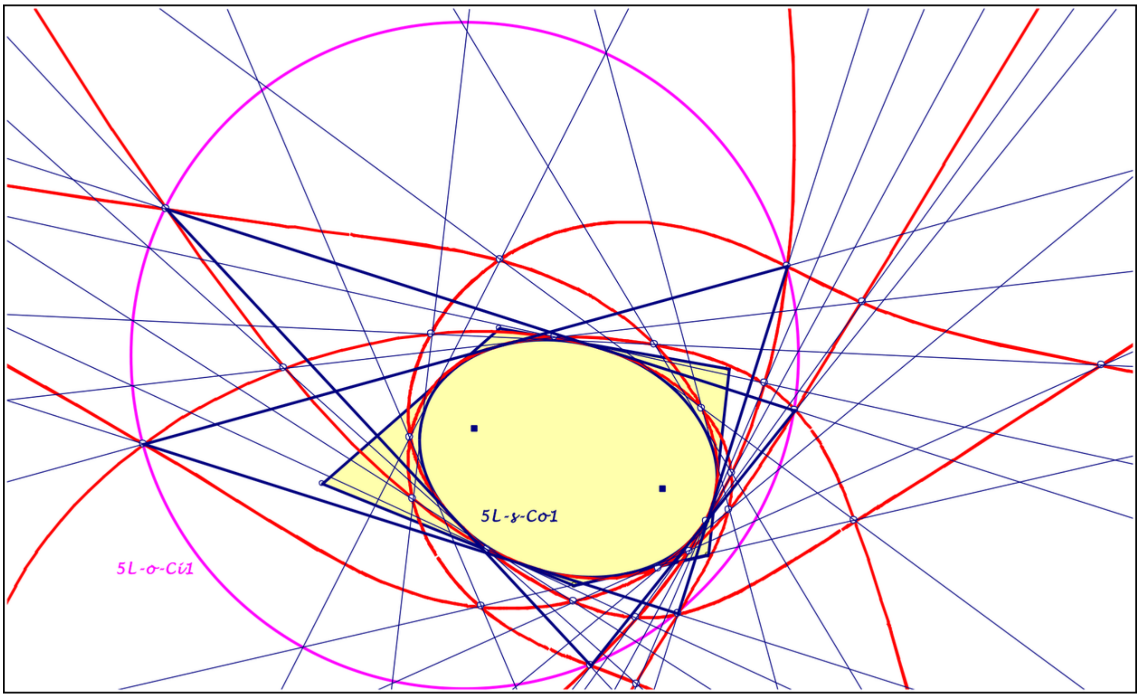
Consider a point X on Ci1, its two tangents at Co1
... and a 3rd tangent for a Co1 circumscribed triangle TR(X).
Let analog TR(Y) be a 2nd triangle wrt Y,
... 4th intersection of Ci1 and the circumconic of TR(X)
    through the foci of Co1.

The locus for the 9 intersections of the sidelines pairwise of TR(X) and TR(Y)
... is a curve with 24 double points,
    in pairs on 24 tangents at Co1,
... (or: on 6 curves, contacting Co1, bearing 8 of them).
Six of these double points lie on Ci1,
... vertices of two Co1 circumscribed triangles.

Perhaps the Lüroth quartic through these six points
... is your searched circular case?

Best regards Eckart

PS. All my observations are made for the case,
... that Co1 is an ellipse inside Ci1.



2020-05-22.pdf

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Message: #287
Date: 2020-05-23
From: eckart_schmidt@t-online.de
Subject: ”New” 5L-Point

```
Dear Chris,

studying 5L wrt Lüroth quartics, I rediscovered a 5L-point, not
in EPG,
... but mentioned in QFG-messages #769, #780, #790, #2665,
    #2669:

Let F1, F2 be the foci of 5L-s-Co1,
... F1°, F2° their inverses wrt 5L-o-Ci1,
... then 5L-s-Px = F1F2° ^ F1°F2.
Properties can be found in the cited messages.

Here another aspect:
Consider any two triangles,
... circumscribed 5L-s-Co1 and inscribed 5L-o-Ci1,
    which give a 6L,
... its 6L-s-Tf1 maps 5L-o-C1 to itself,
... and for any point X on 5L-o-Ci1 holds X.6L-s-Tf1(X)
    bears 5L-s-Px.

Best regards Eckart
```

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Message: #288
Date: 2020-05-23
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

Dear Eckart,
You have to be patient, as I have now for a few weeks my 2 last grandchildren with me and have less time to spend for geometry.
If I understand correctly, any infinity point give as 5L-Tfx partners the 3 vertices of a triangle circumscribed to the conic and inscribed in the circle.
The isogonal of this infinity point wrt the triangle is a point on the circle, which gives in turn a 2nd triangle corresponding to a 2nd infinity point and a 3rd point on the circle ...
For example, the triangle ABC gives the point K. (K is the isogonal wrt ABC of the infinity point of the line SS', but K doesn't belong to the Lüroth quartic through A, B and C).
Your conjecture is that it ends with 2 triangles circumscribed to the conic and inscribed in the circle and the vertices of the 2 triangles on a same Lüroth quartic through the 10 vertices of the 5L and the 2 foci of the conic.
I'm not able now to reproduce your property, but I will do my best ...
Best regards
Bernard

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Message: #289
Date: 2020-05-24
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

```
Dear Eckart,
Sorry, I wasn't correct !
Any infinity point gives in fact as 5L-Tfx partners the 3
vertices of a triangle TR, but Y is the isogonal wrt TR of the
infinity point of SS' !
And it is correct that for ABC, Y is K = 5L-Px, but for the next
steps, you use always the same infinity point on SS' ...
Best regards
Bernard
```

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Message: #290
Date: 2020-05-25
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

```
Dear Eckart,
I finally succeeded to reproduce your construction and the
stange curves as locus of the 9 intersections of the 2 triangles
with vertice in X and in Y.
A well-known property of the ellipse is that the bisector of 2
tangents in a point X is also the bisector of the angle SXS'.
A given tangent L to the  conic 5L-s-Co1 cuts the circle
5L-o-Ci1 in 2 points X2 and X3 : the corresponding point X1 is
the intersection (on the circle) of the reflexions of L in the
bisectors of SX2S' and SX3S' and the point Y1 is the
intersection (also on the circle) of the parallels in X2 and X3
to SS' in the same bisectors.
Therefore X1 = Y1 and X4 = Y4 in the 2 positions of L = X2X3 or
X5X6 parallel to SS'.
This leads to 2 triangles X1X2X3 and X4X5X6 with X2X3 and X5X6
parallel to SS'.
In fact there is a Lüroth quartic through the 10 vertices of the
5L and the 6 points X1, X2, X3, X4, X5 and X6 (but not through S
and S').
But I don't understand your construction and I'm not able to say
if the quartic is circular or not !
Best regards
Bernard
```

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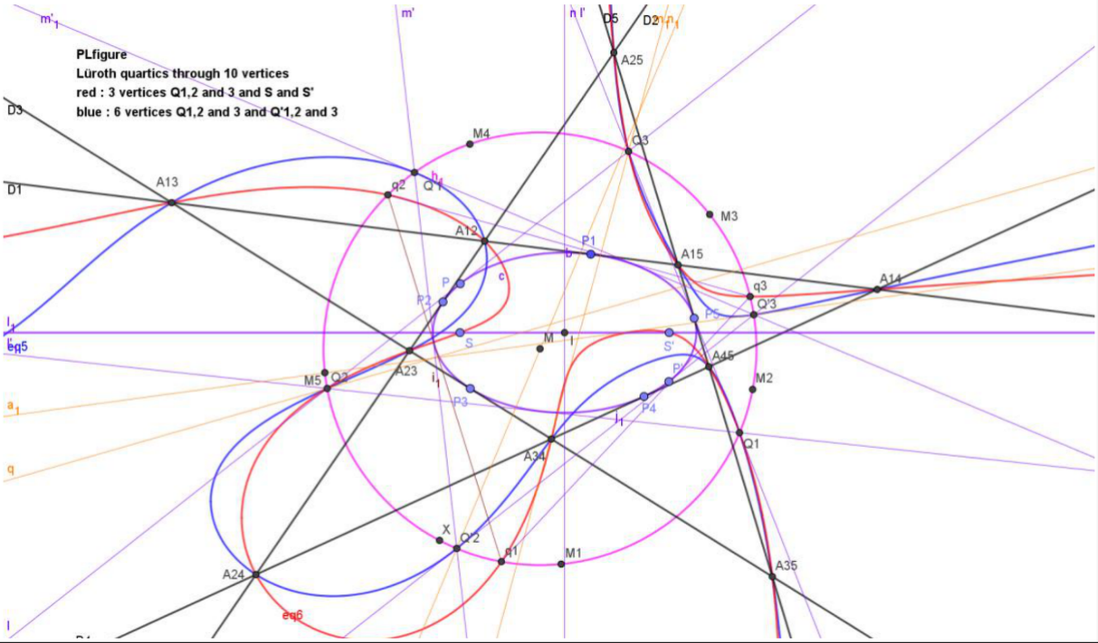
Message: #291
Date: 2020-05-26
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

```
Dear Bernard,

you are right, in #286 and #290
... we discuss the same two triangles
    between 5L-o-Ci1 and 5L-s-Co1,
... defined by two tangents Tg at Co1, parallel SS'.
These triangles TR define a Lüroth quartic for the 5L,
... but my approximate drawings show,
... that the quartic bears the foci S, S'.
If we consider your cubics VR, VR' (see #230)
... for the Co1-contact point of one tangent Tg,
... the cubic VR=VR' intersects
... the other two triangle sides on the quartic.
The cubics VR=VR' for the two contact points of Tg
... have 3 double points - unequal S, S' - on Ci1
... (what about these 3 points?).
For each triangle TR there are two diametral points on Co1
... whose cubics VR=VR' are circumscribed TR
... (what about these 4 points?).
It seems, that there is a point on Co1,
... whose VR=VR' intersects Ci1 on the quartic
... (what about this point?).

I regret, that I cannot draw the quartic!

Best regards Eckart
```

Message: #292
Date: 2020-05-27
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

Dear Eckart,
I suppose this will answer some of your questions.
Any infinity point defines a direction.
This point has 3 5L-Tfx partners vertices of a triangle inscribed in the circle and circumscribed to the conic ; the 3 sides of the triangle define with the 2 tangent to the conic parallel to the direction a 2nd 5L and a quartic through the vertices of the 2 5L's and through S and S'. This gives a 1rst Lüroth quartic.
But the 2 parallel lines define also 2 triangles inscribed in the circle and circumscribed to the conic and the 6 vertices of these 2 triangles define with the 10 vertices of the 5L a 2nd Lüroth quartic, which generally doesn't not pass through S and S'.
The 2 quartics coïncide only for a particular direction ...
On the attached figur, a triangle Q1Q2Q3 is inscribed in the circle and circumscribed to the conic.
The 10 vertices of the 5L define with Q1, Q2 and Q3 and S and S' a 1rst Lüroth quartic, which cuts the circle in the points q1, q2 and q3 which is also circumscribed to the conic.
The parallel to Q2Q3 tangent to the conic defines a 2nd triangle Q'1Q'2Q'3 also circumscribed to the conic and inscribed in the circle and the quartic through the 10 vertices of the 5L and the 6 vertices of the 2 triangles define a 2nd Lüroth quartic not passing through S and S'.
Best regards
Bernard

2 Lüroth quartics



2 Lroth quartics.pdf

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Message: #293
Date: 2020-05-28
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

```
Dear Bernard,

thanks once more for explanations and drawing,
... but they don't answer my questions in #291.
There I consider a special Lüroth quartic,
... defined by two tangents at Co1, parallel SS',
... this quartic will bear S and S'.
I can destinate 30 points of this quartic,
... 10 5L-vertices, S and S',
... 6 points of the triangles, generated by the two parallels,
... and on each triangle side two further points (see #291),
... but I have no general construction.
Can you send me a drawing of this quartic?
Thanks in advance.

Best regards Eckart
```

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Message: #294
Date: 2020-05-29
From: bernard.keizer@gmail.com
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

Dear Eckart,
Many thanks for your interest
Your 30 points are not on the same Lüroth quartic.
Like in the general case, the infinity point of SS' determines 3 5L-Tfx partners, which are your points A, B and C on the CB conic (your 1rst question).
Then you have 2 different Lüroth quartics through the 10 vertices of the 5L :
one is through A, B and C and S and S' and determines a 2nd 5L with the 2 tangents Q2Q3 and Q'2Q'3 parallel to SS' ; it determines a 2nd triangle abc circumscribed to the conic and inscribed in the circle, which in turn determines another infinity point and a 3rd 5L inscribed in the same quartic.
(Please, notice that the 2 VR's through A, B and C have thrir foci in Q1 and Q'1).
the second is through Q1, 2 and 3 and Q'1, 2 and 3, *but not through S and S'* , and determines also 2 other 5L's inscribed in this quartic (your 2nd question).
Here an attached file
Best regards
Bernard
PS I didn't understand the last question

2 Lüroth quartics with the infinity point of SS’

Q2Q3 and Q’2Q’3 parallel to SS’

blue : 5L vertices, Q1, 2 and 3 and Q’1, Q’2 and Q’3

red : 5L vertices, S and S’, A, B and C and a, b and c

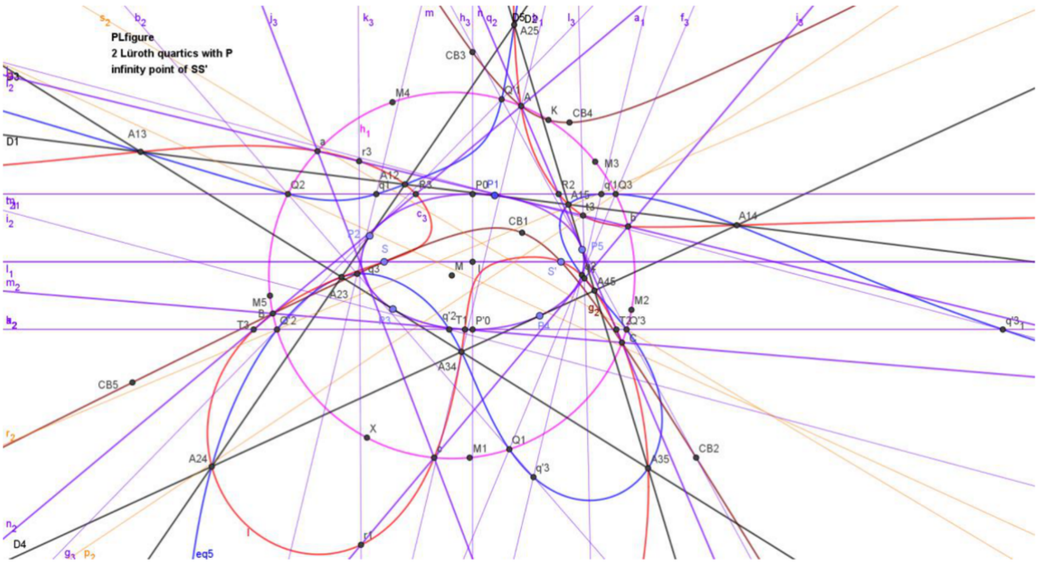
Each point defines 2 tangents to the conic and has 3 5L-Tfx partners, determining a triangle circumscribed to the conic ; the 2 tangents and the 3 sides of the triangle determine a 2nd 5L and the 2 5l’s determine a Lüroth quartic.

The Tfx transformation associates to a point it’s 3 partners and to a tangent to the conic the VR of the QL formed by the triangle and the other tangent from the point to the conic.

Each infinity point defines a triangle of Tfx partners on the circle and the 2 tangents are parallel.

The 3 partners of the infinity point of SS’ are A, B and C.

The red quartic through A, B and C and S and S’ determines a 2nd triangle abc.



2 Lroth quartics with the infinity point of SS_.pdf

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Message: #295
Date: 2020-05-29
From: eckart_schmidt@t-online.de
Subject: New 5L-s-Px and 5P-s-Lx

```
Dear Bernard, dear Chris,

Consider a 5L
... with contact points Xi wrt 5L-s-Co1
... and dual point Yi of Li wrt the other 5L-lines.
The lines XiYi have a common point.

Consider a 5P
... with tangents Mi wrt 5P-s-Co1,
... and dual lines Ni of Pi wrt the other 5P-points.
The intersections of MiNi are collinear.

Best regards Eckart
```

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Message: #296
Date: 2020-05-30
From: eckart_schmidt@t-online.de
Subject: Re: Poncelet, Darboux, Lüroth, triangle, QL and 5L

```
Dear Bernard,

you will be right,
... that my Lüroth quartic in #291 and 293 does not bear S
    and S'.
I very much apologize for my lack of understanding,
... but my approximate drawings led to false observations.
Further apologies wrt my 1st question,
... that I haven't recognized my triangle ABC.
Finally forget my 3rd question,
... the suspected observation will not hold,
... my figures are too inexact.

Thanks for your appreciative claryfications.

Best regards Eckart
```

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Message: #297
Date: 2020-05-30
From: Stan.Rabinowitz@comcast.net
Subject: Another 4 points on QA-Co2

```
It is known that the four orthocenters of all QA-component
triangles of a quadrangle lie on a hyperbola, QA-Co2.

Call these four orthocenters H1, H2, H3, and H4.
If we form the orthocenter of all triples of points selected
from {H1, H2, H3, H4},
then these four new orthocenters also lie on QA-Co2.

Is this either well-known or obvious?
```

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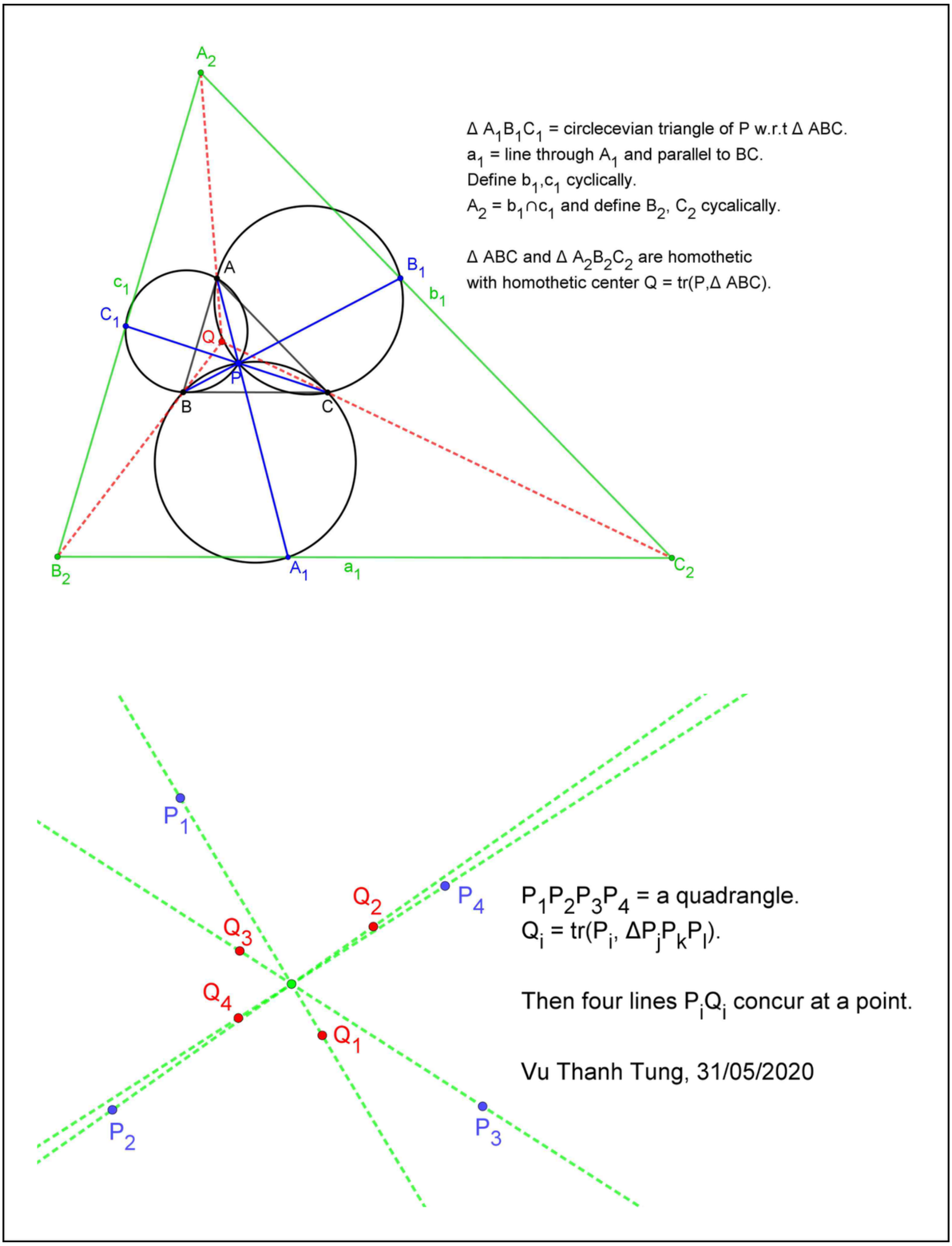
Message: #298
Date: 2020-05-30
From: tungvtt@gmail.com
Subject: A new point on quadrangle?

```
Dear all,

*Definition* :
Given a point P and a triangle ABC,
A1 = the point, other than P, that (PBC) intersects PA.
Define B1,C1 cyclically.
Triangle A1B1C1 is called circlecevian triangle of P w.r,t
triangle ABC

*A triangle transformation:*
Let A1B1C1 be the circlecevian triangle of a point P with
respect to triangle ABC.
The lines through A1,B1,C1 parallel to BC,CA,AB form a triangle
tA2B2C2 hat is homothetic  to triangle ABC.
Let tr(P, ABC) be the homothetic centers.

*A (new ?) point on Quadrangle* (seems have not been listed):
Given a quadrangle P1P2P3P4. Let Q_i = tr(Pi, PjPkPl).
Then four lines PiQi concur at a (new ?) point on Quadrangle.

Barycentric coordinates:
CT-coordinates:
{a^2 q^2 r^2 (2 b^2 c^2 p^4 + a^2 c^2 p^3 q + 2 b^2 c^2 p^3 q -
c^4 p^3 q + a^2 c^2 p^2 q^2 + a^2 b^2 p^3 r - b^4 p^3 r +
2 b^2 c^2 p^3 r + a^4 p^2 q r - b^4 p^2 q r + 2 b^2 c^2 p^2 q r
-
c^4 p^2 q r + a^4 p q^2 r - a^2 b^2 p q^2 r + a^2 c^2 p q^2 r +
a^2 b^2 p^2 r^2 + a^4 p q r^2 + a^2 b^2 p q r^2 -
a^2 c^2 p q r^2 + a^4 q^2 r^2),
b^2 p^2 r^2 (b^2 c^2 p^2 q^2 + 2 a^2 c^2 p q^3 + b^2 c^2 p q^3 -
c^4 p q^3 + 2 a^2 c^2 q^4 - a^2 b^2 p^2 q r + b^4 p^2 q r +
b^2 c^2 p^2 q r - a^4 p q^2 r + b^4 p q^2 r + 2 a^2 c^2 p q^2 r
-
c^4 p q^2 r - a^4 q^3 r + a^2 b^2 q^3 r + 2 a^2 c^2 q^3 r +
b^4 p^2 r^2 + a^2 b^2 p q r^2 + b^4 p q r^2 - b^2 c^2 p q r^2 +
a^2 b^2 q^2 r^2),
c^2 p^2 q^2 (c^4 p^2 q^2 - a^2 c^2 p^2 q r + b^2 c^2 p^2 q r +
c^4 p^2 q r + a^2 c^2 p q^2 r - b^2 c^2 p q^2 r + c^4 p q^2 r +
b^2 c^2 p^2 r^2 - a^4 p q r^2 + 2 a^2 b^2 p q r^2 - b^4 p q r^2
+
c^4 p q r^2 + a^2 c^2 q^2 r^2 + 2 a^2 b^2 p r^3 - b^4 p r^3 +
b^2 c^2 p r^3 - a^4 q r^3 + 2 a^2 b^2 q r^3 + a^2 c^2 q r^3 +
2 a^2 b^2 r^4)}
```

```
DT-coordinates:
{-p^2 (b^2 c^4 p^4 q^2 - b^2 c^4 p^2 q^4 + c^6 p^2 q^4 +
b^4 c^2 p^4 r^2 + a^6 p^2 q^2 r^2 - 2 a^4 b^2 p^2 q^2 r^2 +
a^2 b^4 p^2 q^2 r^2 - 2 a^4 c^2 p^2 q^2 r^2 +
2 a^2 b^2 c^2 p^2 q^2 r^2 + a^2 c^4 p^2 q^2 r^2 - a^6 q^4 r^2 +
2 a^4 b^2 q^4 r^2 - a^2 b^4 q^4 r^2 - a^4 c^2 q^4 r^2 +
a^2 c^4 q^4 r^2 + b^6 p^2 r^4 - b^4 c^2 p^2 r^4 - a^6 q^2 r^4 -
a^4 b^2 q^2 r^4 + a^2 b^4 q^2 r^4 + 2 a^4 c^2 q^2 r^4 -
a^2 c^4 q^2 r^4), -q^2 (-a^2 c^4 p^4 q^2 + c^6 p^4 q^2 +
a^2 c^4 p^2 q^4 - a^4 b^2 p^4 r^2 + 2 a^2 b^4 p^4 r^2 -
b^6 p^4 r^2 - b^4 c^2 p^4 r^2 + b^2 c^4 p^4 r^2 +
a^4 b^2 p^2 q^2 r^2 - 2 a^2 b^4 p^2 q^2 r^2 + b^6 p^2 q^2 r^2 +
2 a^2 b^2 c^2 p^2 q^2 r^2 - 2 b^4 c^2 p^2 q^2 r^2 +
b^2 c^4 p^2 q^2 r^2 + a^4 c^2 q^4 r^2 + a^4 b^2 p^2 r^4 -
a^2 b^4 p^2 r^4 - b^6 p^2 r^4 + 2 b^4 c^2 p^2 r^4 -
b^2 c^4 p^2 r^4 + a^6 q^2 r^4 -
a^4 c^2 q^2 r^4), -r^2 (-a^4 c^2 p^4 q^2 + b^4 c^2 p^4 q^2 +
2 a^2 c^4 p^4 q^2 - b^2 c^4 p^4 q^2 - c^6 p^4 q^2 +
a^4 c^2 p^2 q^4 - b^4 c^2 p^2 q^4 - a^2 c^4 p^2 q^4 +
2 b^2 c^4 p^2 q^4 - c^6 p^2 q^4 - a^2 b^4 p^4 r^2 + b^6 p^4 r^2
+
a^4 c^2 p^2 q^2 r^2 + 2 a^2 b^2 c^2 p^2 q^2 r^2 +
b^4 c^2 p^2 q^2 r^2 - 2 a^2 c^4 p^2 q^2 r^2 -
2 b^2 c^4 p^2 q^2 r^2 + c^6 p^2 q^2 r^2 + a^6 q^4 r^2 -
a^4 b^2 q^4 r^2 + a^2 b^4 p^2 r^4 + a^4 b^2 q^2 r^4)
```

Best regards,

Vu Thanh Tung



QA_Homothetic_Circlecevian_PointQA.pdf

Message: #299
Date: 2020-05-31
From: bernard.keizer@gmail.com
Subject: Re: Another 4 points on QA-Co2

```
Dear Stanley,
A well-known property of a rectangular hyperbola is that the
orthocenter of any inscribed triangle lies on the RH.
Hence your property ...
Best regards
Bernard
```

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Message: #300
Date: 2020-05-31
From: bernard.keizer@gmail.com
Subject: Re: Another 4 points on QA-Co2

```
Dear Stanley,
Surprisingly, this property is not in EQF :
the orthocenter Hi of the triangle PjPkPl is the symmetric of Pi
wrt QA-P2.
QA-P2 is also the QA-P2 of the Hi and the orthocenter of HjHkHl
is Pi.
Best regards
Bernard
```

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Message: #301
Date: 2020-06-01
From: Stan.Rabinowitz@comcast.net
Subject: conic associated with a cyclic quadrangle and its symmedian points

---

First, I'm not sure if this group is appropriate for posts about specific types of quadrilaterals, such as cyclic quadrilaterals or tangential quadrilaterals or if the topics of discussion should be limited to general position quadrangles, quadrilaterals, and quadrigons. Also, are figures allowed in postings or should posts be kept to text only?

In any case, I have discovered an interesting property of cyclic quadrigons and would like to know if the result is already known.

Let ABCD be a cyclic quadrigon.
It has 3 diagonal points, X, Y, and Z.
Let K_A be the symmedian point of △BCD, with corresponding definitions for K_B, K_C, and K_D.
THEN
(a) The 4 points and the 4 symmedian points lie on a conic.
(b)The line joining any two symmedian points passes through a diagonal point.

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Message: #302
Date: 2020-06-01
From: van10hoven@gmail.com
Subject: Re: conic associated with a cyclic quadrangle and its symmedian

---

Dear Stanley,

Our group certainly is interested in specific types of quadrilaterals, such as cyclic quadrilaterals or tangential quadrilaterals. Thanks for sharing!
Because we are a limited group figures are allowed in postings. When the data storage for figures exceeds a certain limit groups from Groups.io have to be paid. But because we are a limited group the chance we exceed this limit is very small. So please feel free to send your pictures included.

Your exercise with the Symmedian point in a cyclic QA is very interesting.
Somehow the Symmedian point stayed underexposed in EQF. So far it only occured in relationship with QL-P26 (https://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p26) (Least Squares Point).

Admit me to restate your discovery.
In a cyclic QA:
1. The Symmedian-QA will be coconic with the reference points of the QA.
2. The Reference-QA and the Symmedian-QA will share the same QA-Diagonal Triangle.

Best regards,
Chris

Message: #303
Date: 2020-06-02
From: eckart_schmidt@t-online.de
Subject: Re: conic associated with a cyclic quadrangle and its symmedian

---

```
Dear Stanley, dear Chris,

perhaps of interest wrt EQF-elements
... for special QA and QL (written 2013 in German language):
<http://eckartschmidt.de/KVKrv.pdf>
<http://eckartschmidt.de/KVKrv.pdf>
<http://eckartschmidt.de/STKrv.pdf>

Best regards Eckart
```

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Message: #304
Date: 2020-06-04
From: eckart_schmidt@t-online.de
Subject: Re: A new point on quadrangle?

```
Dear Vu Thanh Tung ,

I thought, Chris would answer, but ...
Your new quadrangle point is not in EQF
... and I found no connections with other QA-elements,
... background will be your unexpected interesting construction.
I tried to find an analogon for line geometry,
... but I found only a line-->point transformation for
triangles,
... following your way:
Given a line L and a trilateral abc with vertices A, B, C,
... let a1 be the line, other than L, through a^L,
... that is tangent to the incircle of abc,
... define b1, c1 cyclically and A1, B1, C1 vertices of a1b1c1.
ABC and A1B1C1 are point perspective.
Also if we consider lines a2, b2, c2 parallel a, b, c  through
A1, B1, C1
... we get a triangle A2B2C2 homothetic ABC.

But I found no properties, replacing L by the 4th line of a
quadrilateral.

Best regards Eckart
```

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Message: #305
Date: 2020-06-05
From: bernard.keizer@gmail.com
Subject: 3rd conic of a 5

```
Dear Eckart,
Any triangle inscribed in 5L-o-Ci1 and circumscribed to 5L-s-Co1
is also self-polar wrt a 3rd conic.
The director circle of the conic is orthogonal to 5L-o-Ci1
(Gaskin's theorem).
I can't identify this 3rd conic.
Can you help me ?
Best regards
Bernard
```

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Message: #306
Date: 2020-06-05
From: bernard.keizer@gmail.com
Subject: Chain of circular Poncelet curves and transformations

---

Dear Eckart,
Before leaving (provisory) the item of circular Poncelet curves, here a few ideas :
As well known, the QL's CSC transformation swaps a line and a circular conic (id est a circle) and the CSC of a point is the 2nd intersection (other than QL-P1) of all circles CSC of the pencil of lines through the point. There is only one circular circumcubic of the 6 vertices of a QL (through QL-P1).
The 5L transformation swaps a line and a circular cubic and the partners of a point are the 3 intersections (other than the foci S and S' of the conic) of the 2 circular cubics 5L-Tf of the 2 tangents from the point to the conic (on the circle CiP). I conjectured, but couldn't identify, that there is only one circular quartic through the 10 vertices of the 5L (and the 2 foci of the conic).
I suppose this goes on and there is a 6L transformation swapping a line and a circular quartic and the partners of a point will be the 6 intersections of 2 circular quartics (partners of which lines ?) on a circular cubic. I suppose the same way that the is only one circular quintic through the 15 vertices and which other points ?
For a 6L, the are 15 vertices, 15 points QL-P1 of the remaining 4 lines and your transformation 6L-Tf1 swapping these 15 copple of points as well as the copples of foci of the 6 conics.
Best regards
Bernard
PS As I already told you, I intend to read books by Poncelet and Morley in the library of the Institut Henri Poincaré in Paris, but not before automn or winter ...

Message: #307
Date: 2020-06-06
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
First step : the center of this 3rd conic is the pole of the
infinity line wrt this conic.
For 2 parallel tangents to the conic 5L-s-Co1 Q2Q3 and Q'2Q'3,
the 3rd vertices of the 2 triangles inscribed in 5L-o-Ci1 are Q1
and Q'1.
The lines Q1Q'1 pass through a fixed point of the 5L, which is
the center of this 3rd conic.
(Q1 being the pole of Q2Q3 and Q'1 the pole of Q'2Q'3, the
intersection of the 2 lines, id est the infinity point, is the
pole of Q1Q'1 and Q1Q'1 contains the pole of the infinity line).
Best regards
Bernard
```

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Message: #308
Date: 2020-06-06
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

here only a short remark:
The center of your 3rd conic of a 5L is the "new" point in #287.

Best regards Eckart
```

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Message: #309
Date: 2020-06-06
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

excuse my question:
Is your 3rd conic for a 5L the same for all triangles
... tangent 5L-s-Co1 and inscribed 5L-o-Ci1?

Best regards Eckart
```

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Message: #310
Date: 2020-06-06
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
Thanks for your quick answer.
It's remarquable that the center of the conic, which I found
only by reasoning and determining the point with 2 particular
directions of parallel tangent to 5L-s-Co1 coincides with your
new point in 287 !
Of course, I immediately cheched this property ...
What about the conic, now ?
Best regards
Bernard
```

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Message: #311
Date: 2020-06-06
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
The answer is yes !
(See for example J. W. Russell 1893 Properties of 2 triangles)
Thanks for your interest ...
Best regards
Bernard
```

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Message: #312
Date: 2020-06-07
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
You instilled with your question some kind of a doubt and I'm no
longer so sure of my answer !
I suppose my brain continued to function during the night ...
In fact, there are 2 different considerations :
1) if 2 triangles are inscribed in a same conic, they are
circumscribed to a 2nd conic and conversely
if 2 triangles are either inscribed in a same conic or
circumscribed to a 2nd, they are self polar wrt a 3rd conic and
conversely
2) if there is a triangle inscribed in a conic and circumscribed
to a 2nd conic, there are an infinity *with the same conics*
(Poncelet's porism).
But Poncelet's porism doesn't apply to the 3rd conic !
There are different 3rd conics for each copple of conics
inscribed in 5L-o-Ci1 and circumscribed to 5L-s-Co1 !
For copples of triangles with one side parallel, the 3rd conics
have the same center, which is your new point in 287.
Name Li the lines, Aij the 10 vertices of the 5L and Mi the
QL-P1 of the 4 lines without Li, there are 10 3rd conics Coij
associated to the 10 vertices.
Mi is the pole of Li and Mj the pole of Lj wrt Coij and Aij is
the pole of MiMj ...
My apologies for my wrong answer !
Best regards
Bernard
```

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Message: #313
Date: 2020-06-07
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

Dear Eckart,
Reading again the article by J.W. Russell, I find in chapter XIV properties of two triangles on page 153 EX 21 following statement :
If two conics β and γ be situated that one triangle can be circumscribed to β so as to be inscribed in γ, then an infinite number of such triangles can be drawn, and all of these will be self-conjugate for a third conic α ; also the two conics β and γ are reciprocal for α.
After all, my 1rst answer was perhaps correct !
I'm now in trubble, can you help me ?
Best regards
Bernard

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Message: #314
Date: 2020-06-07
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

yesterday I tried hopeless and in vain, to find the common 3rd
conic
... for which the Poncelet triangles of 5L-o-Ci1 and 5L-s-Co1
    are selfpolar,
... using as center the "new" point of  #287 and others.

Today my CABRI has an unexpected breakdown,
so I cannot research ...
But I found the following passage in
... Encyklopädie der Mathematischen Wissenschaften III C 1,
... Kegelschnitte und Kegelschnittsysteme 28
    von Friedrich Dingeldey:
... "G. Kohn zeigte,
    dass jedes Poncelet'sche Polygon von ungerader Seitenzahl
... sich selbst polar ist
    in Bezug auf einen gewissen Kegelschnitt k',
... und zwar ist jede Seite die Polare ihrer Gegenecke.
... Die Kurve k' ist dieselbe für alle Poncelet'schen Polygone,
... die denselben umschriebenen und denselben eingeschriebenen
    Kegelschnitt besitzen,
... und hat mit diesen beiden ein gemeinsames Poldreieck."

Searching goes on, if my CABRI is repaired.

Best regards Eckart
```

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Message: #315
Date: 2020-06-07
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
Wunderbar ! Many thanks for this confirmation.
Then, my first answer was correct.
There is a conic, centered in the new point, wrt which all
Poncelet triangles of the circle and the conic are self polar
!!!
Mi is the pole of Li and Aij is the pole of MiMj ...
I hope you will identify it soon.
Best regards
Bernard
```

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Message: #316
Date: 2020-06-09
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

I am rather handicapt, only with 15 minutes demo-version of
CABRI.
Have you a drawing of the 3rd conic?
Can you send me an example?
Thanks in advance.

Best regards Eckart
```

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Message: #317
Date: 2020-06-09
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
If I had one, I would have sent it to you immediately !
Unfortunately, I'm not able to draw this 3rd conic ...
I hoped you would succeed.
Best regards
Bernard
PS As you said, searching goes on !
```

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Message: #318
Date: 2020-06-11
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

at the moment I suppose,
... that this 3rd conic doesn't exist,
... contrary to two references.
I experienced in many cases under different concepts,
... to get an approximated example for the 3rd conic,
... but without success !!!

What about the following argumentation:
(1) The circumcircle of a self-polar triangle wrt a conic
... intersects always this conic.
(2) Consider now a conic CO inside a circle CI
... with Poncelet triangles tangent to CO with circumcircle CI.
If there is an intersection X of the 3rd conic and CI,
... its polar wrt the 3rd conic must be the tangent
... but it is also the opposite side of the Poncelet triangle,
... that can't be!

Where is my wrong conclusion?

Best regards Eckart
```

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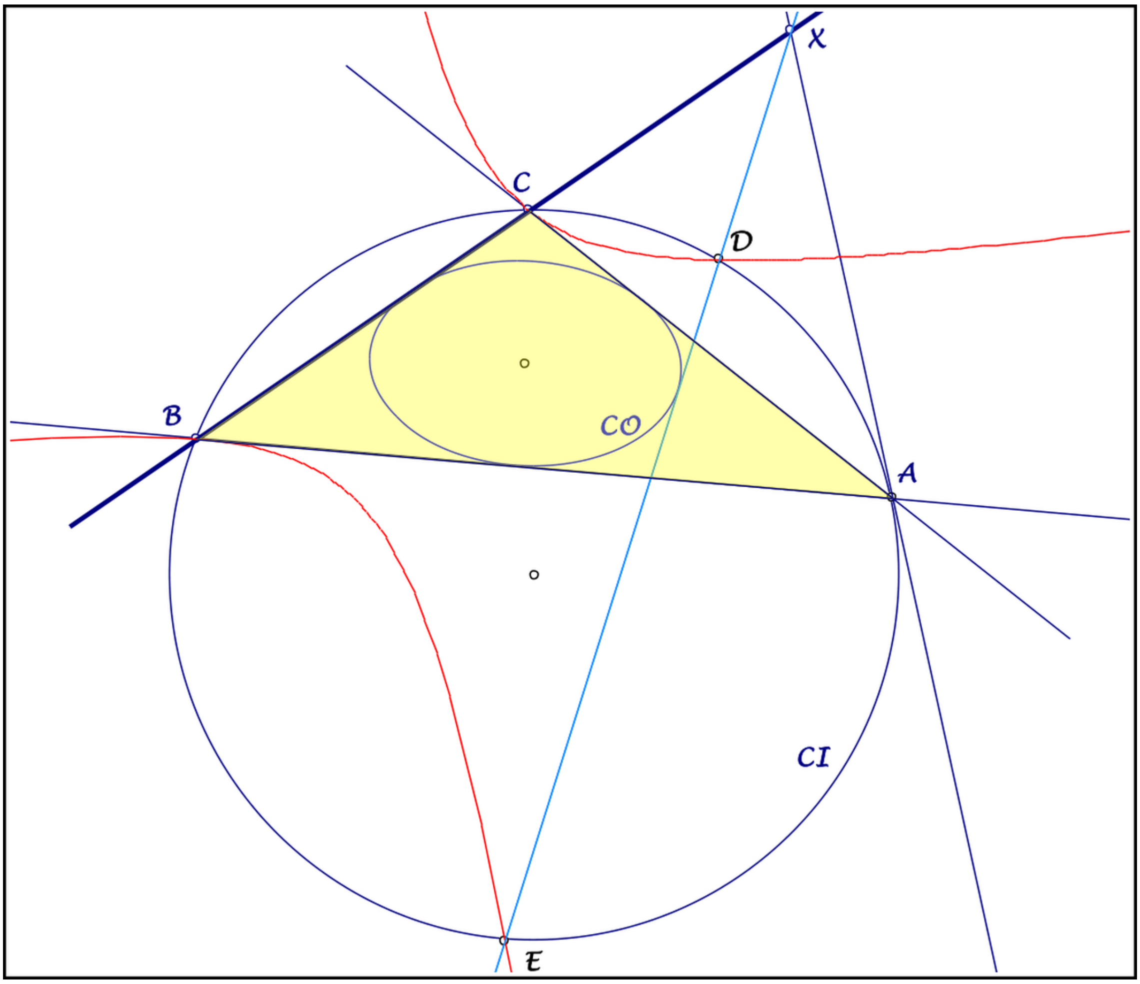
Message: #319
Date: 2020-06-11
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic of a 5L

```
Dear Bernard,

perhaps of interest the following aspect:
Consider an ellipse CO inside a circle CI
... with a Poncelet triangle ABC,
... let X be the intersection of BC and the tangent at CI in A,
... let D, E be the intersections of CI and the 2nd tangent
    from X at CO.
For the conic through B,C,D,E with tangents AB and AC
... the line BC is polar of A.

Best regards Eckart
```



2020-06-11.pdf

Message: #320
Date: 2020-06-11
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic of a 5L

```
Dear Eckart,
I'm convinced that the 3rd conic exists, but it has not your new
point in 287 as center !
We are looking for a conic which is a polar conic for all
Poncelet's triangles.
In particular, each of the 5 lines Li cuts the circle in Ai and
Bi and MiAiBi is a Poncelet triangle.
Li cuts the 3rd conic in 2 points Vi and Wi harmonic conjugates
wrt Ai and Bi.
(Any tangent to the conic cuts the same way the circle and the
3rd conic in 2 pairs of harmonic conjugates).
I tried (in vain until now) to find 5 points Vi on Li such as
the conic through the 5 Vi coincides exactly with the conic of
the 5 Wi ...
I'm not totally desperate now !
Best regards
Bernard
```

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Message: #321
Date: 2020-06-11
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic for a 5L

```
Dear Bernard,

I think, the following statement in #320 can not hold:
... "Li cuts the 3rd conic in 2 points Vi and Wi
    harmonic conjugates wrt Ai and Bi."
Ai and Bi are points on the circle,
... Vi, Wi are unequal,
    one inside and the other outside the circle,
... so there are two parts for the 3rd conic,
... but that cannot be, even if it is a hyperbola.

Best regards Eckart

PS. Can you find a mistake in my argumentation in message 318?
```

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Message: #322
Date: 2020-06-12
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic for a 5L

Dear Eckart,
Unfortunately, I don't understand your argumentation.
I just summarise where we are.
We deal with 3 conics C1 (here a circle), C2 and C3 such as there are an infinity of triangles inscribed in C1, circumscribed to C2 and selfpolar wrt C3.
If C1 and C3 have real intersections, the polar of these intersections are tangent to C2, but there is nothing to say about intersections between C2 and C3 !
Any tangent to C2 cuts C1 and C3 in 2 pairs of harmonic conjugates. That is, if I'm not wrong, a general property of polar conics of a triangle ...
>From any point of C1, the tangents to C2 and C3 form a harmonic range.
The director circle of a conic is orthogonal to all circumcircles of the selfpolar triangles wrt this conic (Gaskin's theorem).
The 3 conics C1, C2 and C3 have a common selfpolar triangle (end of your your article in 314).
Theoratically, it must be possible to find this triangle, knowing C1 and C2, which gives another circumcircle of a selfpolar triangle, different from C1.
Best regards
Bernard
PS C2 can be an ellipse as well as an hyperbola

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Message: #323
Date: 2020-06-12
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic for a 5L

```
Dear Bernard,

I have found my mistake,
... please forget my messages #318 and #321.

Now I can offer a first construction
... of the 3rd conic for Poncelet triangles
... inscribed a circle C1 and circumscribed a hyperbola C2,
... C2 intersecting C1 in X and Y:

Consider the tangents to C2 in X and Y, intersecting C1 in X'
and Y'.
The 3rd conic is defined and can be constructed
... through the points X', Y' with tangents XX' and YY',
... centered in the "new" point of #287.

Best regards Eckart
```

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Message: #324
Date: 2020-06-12
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic for a 5L

```
Dear Eckart,
Wunderbar! Beautiful idea to consider XX' and YY' as flat
Poncelet triangles!
All the announced properties hold (Mi pole of Li, Aij pole of
MiMj, harmonic divisions on tangents to C2, director circle of
C3 orthogonal to C1, common self polar triangle for the 3
conics).
For the self polar triangle common to 2 conics, I found this
construction : to any point Z, associate by a point to point
transformation the point Z', intersection of the polars of Z wrt
the 2 conics.
For 2 lines through Z, the transforms are 2 conics, which
intersect in 4 points, Z' and the 3 vertices of the searched
self polar triangle.
Now, the mystery remains entire for the 3rd conic C3 if C2 is an
ellipse and the point 287 inside C2, which is inside C1 ! (There
is no circle centered in this point and orthogonal to C1.
Best regards
Bernard
```

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Message: #325
Date: 2020-06-13
From: eckart_schmidt@t-online.de
Subject: Re: 3rd conic for a 5L

Dear Bernard,

if C2 is a parabola, the construction in #323 also can be used
... using as center of the 3rd conic the focus of the parabola,
... which is a point on C1.

If C2 is an ellipse, there are no degenerated Poncelet
triangles,
... which I didn't respect in my argumentation in #318.
So I think, the argumentation holds in this case.

Best regards Eckart

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Message: #326
Date: 2020-06-14
From: bernard.keizer@gmail.com
Subject: Re: 3rd conic for a 5L

Dear Eckart,
Thanks for these ultime precisions.
I suppose this puts an end to our long quest !
I'm glad that we (re)discovered together all these beautiful
properties I never heard about before ...
Best regards
Bernard

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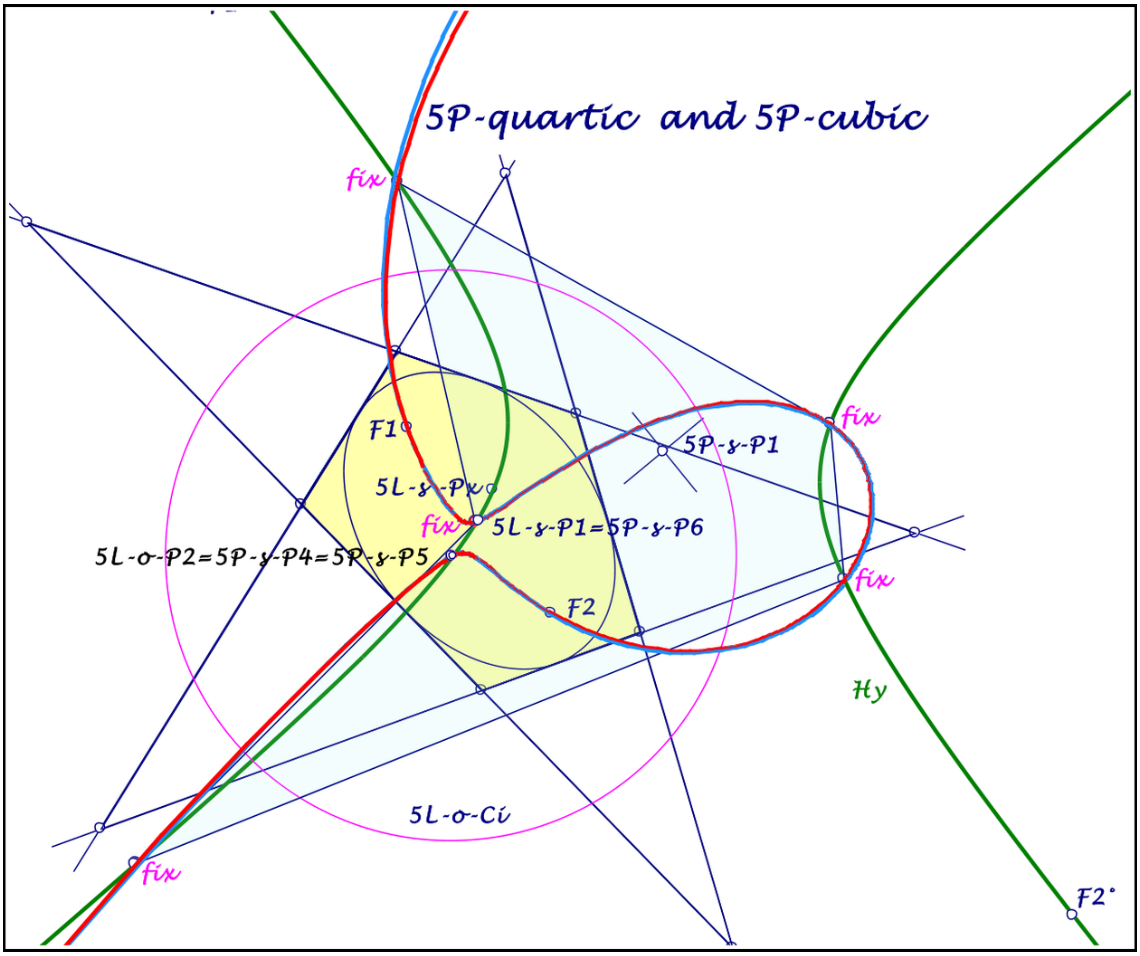
Message: #327
Date: 2020-06-15
From: eckart_schmidt@t-online.de
Subject: Fixed points of 5L-s-Tf1

```
Dear Bernard,

in QFG-message #769 I described the 5 fixed points of 5L-s-Tf1,
... which must not all be real (!!!)
... on an orthogonal hyperbola HY through 5L-o-P2,
... the inverses F1° and F2° of the foci of 5L-s-Co1
    wrt 5L-o-Ci1
... and through the new 5L-point 5L-s-Px of EPG-message #287,
... centered in the middle of F1° and F2°.

The intersections of HY and its 5L-s-Tf1 image
... give the fixed points of 5L-s-Tf1 (unequal 5L-o-P2).

The quartic for the 5P of the 5 fixed points of 5L-s-Tf1
... bears F1, F2 and 5L-o-P2 = 5P-s-P4 = 5P-s-P5,
... further 5L-s-P1 = 5P-s-P6.

The 5P-quartic is invariant wrt the 5P-CSC-transformation
... with fixed points F1 and F2,
... so the 5P-quartic coincides with the 5P-cubic (?).

I would be obliged if you could confirm these curious
properties.
The attached file is not my first figure with these properties,
... but it is hard, to start a 5L with 5 fixed points of 5L-Tf1.

Best regards Eckart
```



2020-06-15.pdf

Message: #328
Date: 2020-06-16
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Eckart,
I studied some time ago your message 769, which I found very
interesting !
It's not difficult to find the RH, but I never succeeded in
drawing it's 5L-CSC (I only found that it contains necessary F1
and F2 as well as 5L-P1 and the reflexion of P1 in P4) and so
far I'm unable to confirm or infirm your properties ...
Could you help me ? I suppose it is a cubic, as there are 6
intersections with the conic.
I would be glad to join you in this new item.
Best regards
Bernard
```

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Message: #329
Date: 2020-06-17
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Eckart,
I succeeded in drawing the 5L CSC of the RH, but it lacks of
precision !
My apologise, it is of course not a cubic, perhaps a quartic,
but why only 6 intersections with the conic ?
Anyhow, I'll try to check your other properties, just be patient
...
Best regards
Bernard
```

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Message: #330
Date: 2020-06-17
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Bernard,

thanks for your interest in my unexpected observations.
Perhaps you can find the fixed points of 5L-s-Tf1 on the
hyperbola
... using the intersections with the quartic in #769, 33
... (5L-P1old is 5L-o-P2new, 5L-CSCold is 5L-s-Tf1new).
What about the coincidence of the 5P-quartic and 5P-cubic?

Best regards Eckart
```

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Message: #331
Date: 2020-06-18
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5L-s-Tf1

Dear Eckart,
I achieved the construction mit Müh und Not.
I'm able to confirm all your properties and to add some comments.
1) The center M of the circle is 5P-s-P4 as well as P5
2) The center I of the conic is 5P-s-P6 and CSC1 has the foci F1 and F2 as fixed points
3) The cubic is a monocursal Van Rees circular focal cubic with focus CSC1(M), invariant in the CSC2 centered in this point and swapping I and M.
The cubic is also invariant in another CSC4 centered in M and swapping I and CSC1(M) and contains the fixed points of this new CSC4.
The Newton Line is IM and the asymptote is the parallel yo IM, homothetic in a homothety centered in CSC1(M) with ratio 2.
The 5P cubic coïncides with it's twin cubic and contains the 5 fixed points Zi and their CSC1 patners as well as the QA-P4 of 4 of the 5 points of each group.
The 5P quartic is invariant in a CSC3 = CSC1*CSC2*CSC1 centered in CSC1(M), but CSC3 = CSC4*CSC1 = CSC2.
The 5P quartic also coïncides with it's twin quartic and I think it is degenerated and formed by the cubic and the infinity line.
Hence the coïncidence ...
Best regards
Bernard
PS These are with some others classical properties of the monocursal VR, but it is amazing to have found it where nobody expected it ! Congratulations

---

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Message: #332
Date: 2020-06-18
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Eckart,
I forgot this obvious, but I suppose interesting remark.
The cubic can be constructed by knowing only the conic with
center I and foci S and S', giving the CSC1 and the center M of
the circle, giving it's CSC1, which is the center of the CSC2
swapping I and M as well as S and S'.
The RH was constructed from S and S' and their inverses s and s'
wrt the circle, centered in the middle J of ss' and passing
through s, s', M and K, intersection of Ss' and S's.
Your 5 fixed points can be easily and precisely obtained as the
5 intersections (other than M) between the cubic and the RH.
Best regards
Bernard
```

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Message: #333
Date: 2020-06-19
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Bernard,

thanks for your detailed confirmation of my observations
... and your description of further properties.
The 5P of the fixed points are special 5P
... on an orthogonal hyperbola
... with 5P-s-P4 = 5P-s-P5 and CSC2 = CSC3
... and 5P-s-P3 infinity point of perpendiculars to
    5P-s-P5.5P-s-P6,
... its cubic and the infinity line give its quartic.
The inversion circle centered in 5P-s-P4 = 5P-s-P5
... for the foci of the orthogonal hyperbola bears 5P-s-P1.

Are there more specific 5P-properties?

It seems, that the condition 5P-s-P4 = 5P-s-P5
... for a 5P on an orthogonal hyperbola
... is sufficient, that the properties above hold.

Best regards Eckart
```

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Message: #334
Date: 2020-06-20
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Bernard,

there are further properties for the 5P of the fixed points of
5L-s-Tf1:
   The Cayley-Bacharach transformation wrt the 5 points and
   the two circular points maps the 5P-cubic/quartic to itself
   with pivot 5P-s-P6, 5P-s-P6<-->CSC(5P-s-P5), 5P-s-P5 <-->
   infinity point of 5P-s-P5.6
   The Cayley-Bacharach transformation wrt the 5 points and
   the fixed points of 5P-CSC maps the 5P-cubic/quartic to
   itself with pivot in the infinity point of 5P-s-P5.6,
   5P-s-P5<-->5P-s-P6.

Best regards Eckart
```

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Message: #335
Date: 2020-06-20
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 5L-s-Tf1

```
Dear Bernard,

the Cayley-Bacharach transformation for 5P
and two circular points
... is for the 5P of the fixed 5P-s-Tf1 points and points of its
5P-cubic/quartic
... your CSC4 ...
It seems, I lost control!

Best regards Eckart
```

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Message: #336
Date: 2020-06-20
From: tungvtt@gmail.com
Subject: A point on quadrangle

Dear all,

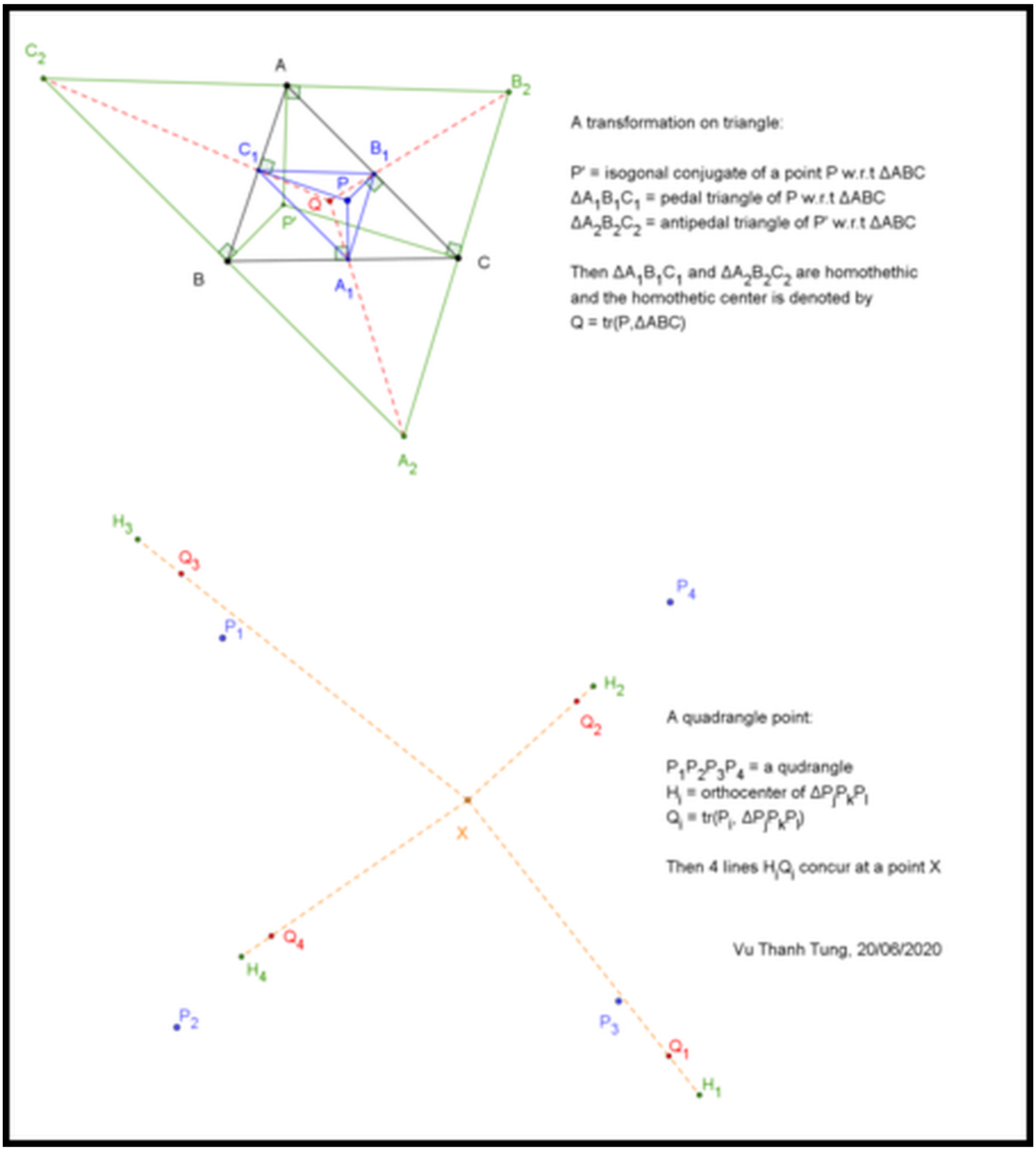
I found a point on QT constructed as follows:

A transformation on triangle:
P' = isogonal conjugate of a point P w.r.t ΔABC
ΔA_1B_1C_1 = pedal triangle of P w.r.t ΔABC
ΔA_2B_2C_2 = antipedal triangle of P' w.r.t ΔABC
Then ΔA_1B_1C_1 and ΔA_2B_2C_2 are homothethic
and the homothetic center is denoted by
Q = tr(P,ΔABC)

A quadrangle point:
P_1P_2P_3P_4 = a qudrangle
H_i = orthocenter of ΔP_jP_kP_l
Q_i = tr(P_i, ΔP_jP_kP_l)
Then 4 lines H_iQ_i concur at a point X

Unfortunately, I could not compute the coordinates for this point.
Is this a new point on QA ?

Best regards,
Vu Thanh Tung



QA-HomotheticPoint22.png

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Message: #337
Date: 2020-06-20
From: eckart_schmidt@t-online.de
Subject: Re: A point on quadrangle

Dear Vu Thanh Tung,

can it be, that the Qi in the 2nd figure
are not constructed as in the 1st figure?

Best regards Eckart

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Message: #338
Date: 2020-06-20
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5L-s-Tf1

Dear Eckart,
We discussed already the same kind of properties in messages 194 and following ...
The fact that the 9 points 5 fixed points + fixed points of CSC1 + 5P-s-P5 and P6 form a CB system is obvious as we have 2 different cubics through the 9 points : the 5P cubic and the degenerated cubic formed by the RH and the line F1F2 ...
The points P4 and P5 of the 5 CSC of the 5 fixed points is CSC1(P4 or P5), their P6 is the same by definition and the cubic is also cb invariant wrt the 5 CSC1 with the same pivot ...
Best regards
Bernard

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Message: #339
Date: 2020-06-20
From: tungvtt@gmail.com
Subject: Re: A point on quadrangle

Hi Eckart,

Sorry, I had typos. Here is the correction.

A transformation on triangle:
P' = isogonal conjugate of a point P w.r.t ΔABC
ΔA_1B_1C_1 = pedal triangle of *P'* w.r.t ΔABC
ΔA_2B_2C_2 = antipedal triangle of *P* w.r.t ΔABC
Then ΔA_1B_1C_1 and ΔA_2B_2C_2 are homothethic
and the homothetic center is denoted by
Q = tr(P,ΔABC)

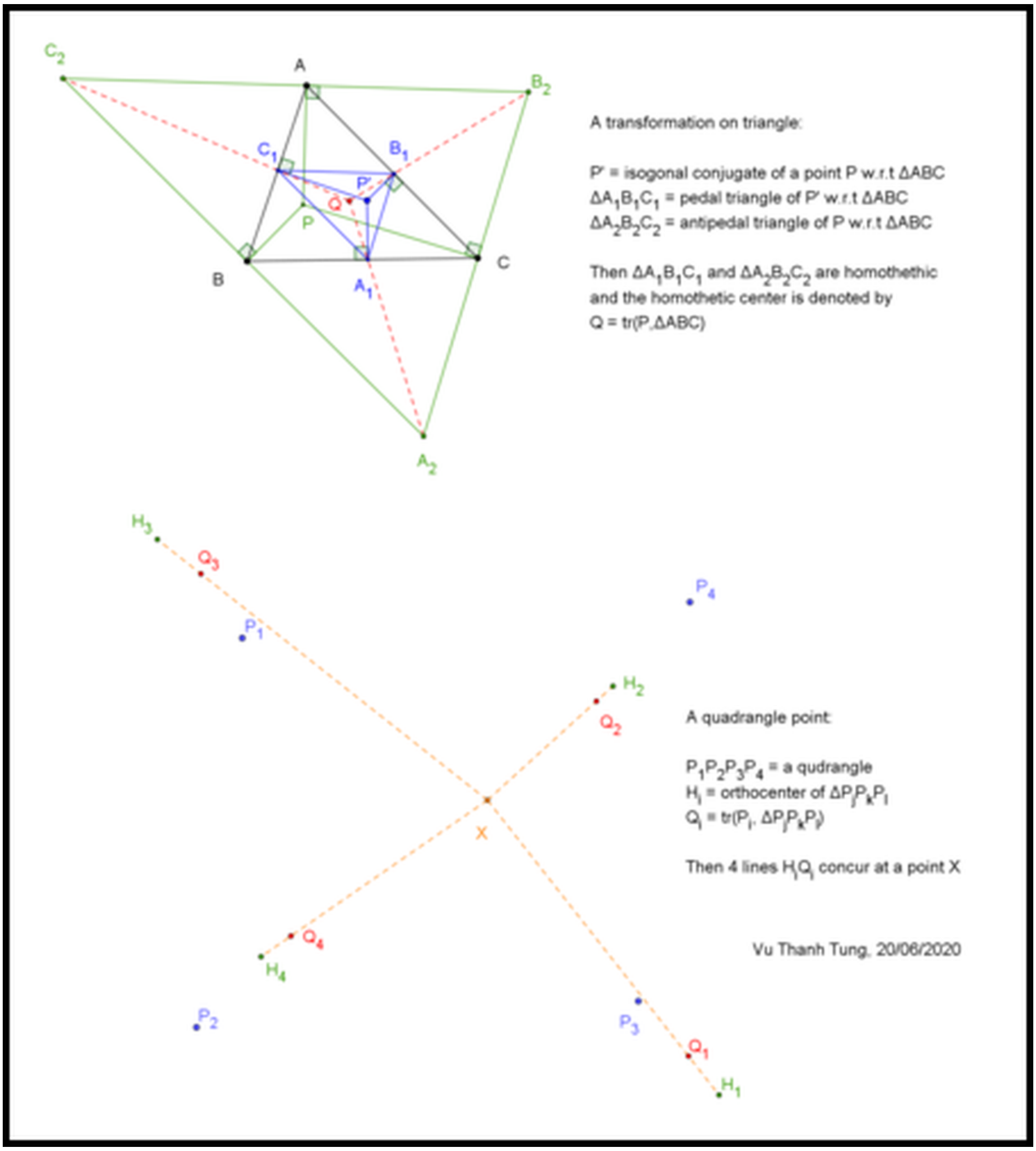
A quadrangle point:
P_1P_2P_3P_4 = a qudrangle
H_i = orthocenter of ΔP_jP_kP_l
Q_i = tr(P_i, ΔP_jP_kP_l)
Then 4 lines H_iQ_i concur at a point X

I also got the coordinates for X:
1st CT-coordinates for X is:
q r (-2 a^2 (b^2 - c^2) p (q - r) - (b^2 - c^2)^2 p (2 p + q +
r) +
a^4 (2 p^2 + 4 q r + 3 p (q + r)))

1st DT-coordinates for X is :
2 a^2 (b^2 + c^2) p^2 (p^2 - q^2 - r^2) +
a^4 (-p^4 + 4 q^2 r^2 + p^2 (q^2 + r^2)) +
p^2 (b^4 (-p^2 + q^2 - 3 r^2) + c^4 (-p^2 - 3 q^2 + r^2) +
2 b^2 c^2 (-p^2 + q^2 + r^2))

Best regards,

Vu Thanh Tung



QA-HomotheticPoint2.png

Message: #340
Date: 2020-06-21
From: eckart_schmidt@t-online.de
Subject: Re: A point on quadrangle

```
Dear Vu Thanh Tung,

your new point is QA-P1.QA-P4 ^ QA-P2.QA-P33.

Best regards Eckart
```

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Message: #341
Date: 2020-06-21
From: eckart_schmidt@t-online.de
Subject: 5L-s-Tf1

```
Dear Chris,

in EPG under 5P-s-Tf1 you cite further properties in my
messages.
Here should also be listed #769
... and changed old nomination:
... 5L-P1 by 5L-o-P2, 5L-P4 by 5L-s-P1, QL-P1-circle by 5L-o-Ci1
... and the mistake of "three" fixed points by "five" fixed
points.

Best regards Eckart
```

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Message: #342
Date: 2020-06-21
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A point on quadrangle

---

```
Dear Eckart,

Thank you very much.
Best regards,
Vu Thanh Tung

On Sun, Jun 21, 2020 at 3:03 PM eckart_schmidt@t-online.de <
eckart_schmidt@t-online.de> wrote:

> Dear Vu Thanh Tung,
> your new point is QA-P1.QA-P4 ^ QA-P2.QA-P33.
> Best regards Eckart
```

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Message: #343
Date: 2020-06-21
From: van10hoven@gmail.com
Subject: Re: 5L-s-Tf1

```
Dear Eckart,

Very secure of yours and necessary for full understanding.
Changes are done.

Best regards,
Chris
```

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Message: #344
Date: 2020-06-22
From: bernard.keizer@gmail.com
Subject: Monocursal VR

Dear Eckart,
Having 3 points M, S and S', there is one monocursal VR such as M is the focus, the CSC centered in M swaps S and S' and SS' is the Newton Line ...
This is well known, M is QL-P1 and S and S' the QL-2P2a and b.
Apparently less well known (or just forgotten), this VR is also invariant in the 2 CSC's centered in S and S' and swapping M and S' and M and S.
The fixed points of these 2 CSC's are on the VR, are cocyclic and are pairwise CSC partners in the 1rst CSC.
These 2 CSC's are pivotal transformations for the cubic, the pivot being the other point S' or S.
You found in the 5 fixed points of 5LTf1 5 points of the VR having S as 5P-s-P4 and 5 and S' as 5P-s-P6 and the pivotal transformation is a cb transformation wrt the 5 points.
(Any group of 5 points of the VR coconic with S has the same property ?)
I have 2 questions :
1) How can these points be constructed starting with the VR ?
How can we find then the conic with foci as the fixed points of the CSC centered in S' and the circle centered in S ?
2) I suppose there are 5 other points having S' as 5P-s-P4 and 5 and S as 5p-s-P6. How can we find the same way the conic with foci as the fixed points of the CSC centered in S and the circle centered in S' ?
Best regards
Bernard

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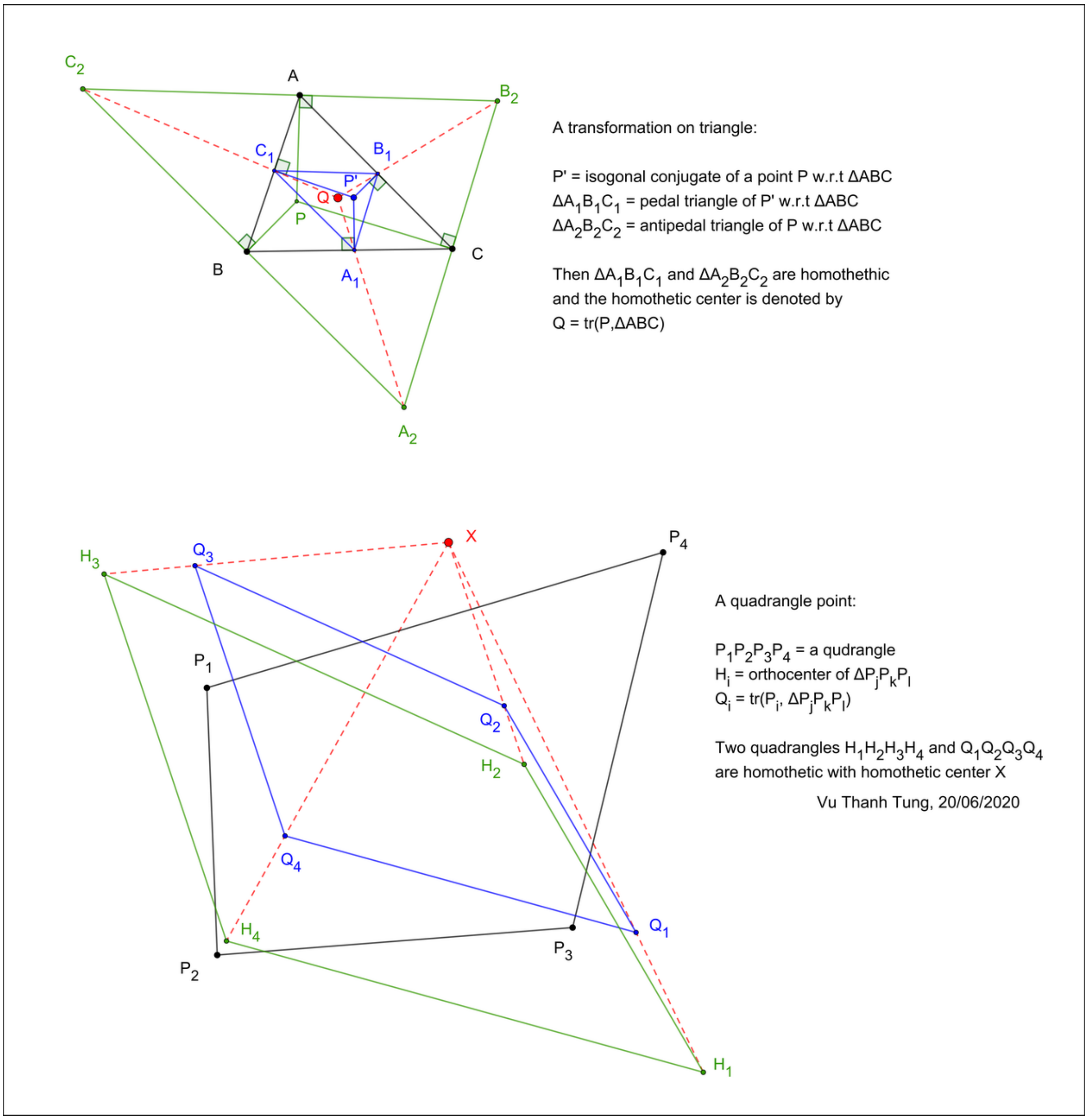
Message: #345
Date: 2020-06-23
From: tungvtt@gmail.com
Subject: Re: A point on quadrangle

Hi all,

There is an additional interesting property in this configuration:
Two quadrangles H_1H_2H_3H_4 and Q_1Q_2Q_3Q_4 are homothetic with homothetic center X.

Best regards,
Vu Thanh Tung



QA-HomotheticPoint2.pdf

Message: #346
Date: 2020-06-23
From: eckart_schmidt@t-online.de
Subject: Re: Monocursal VR

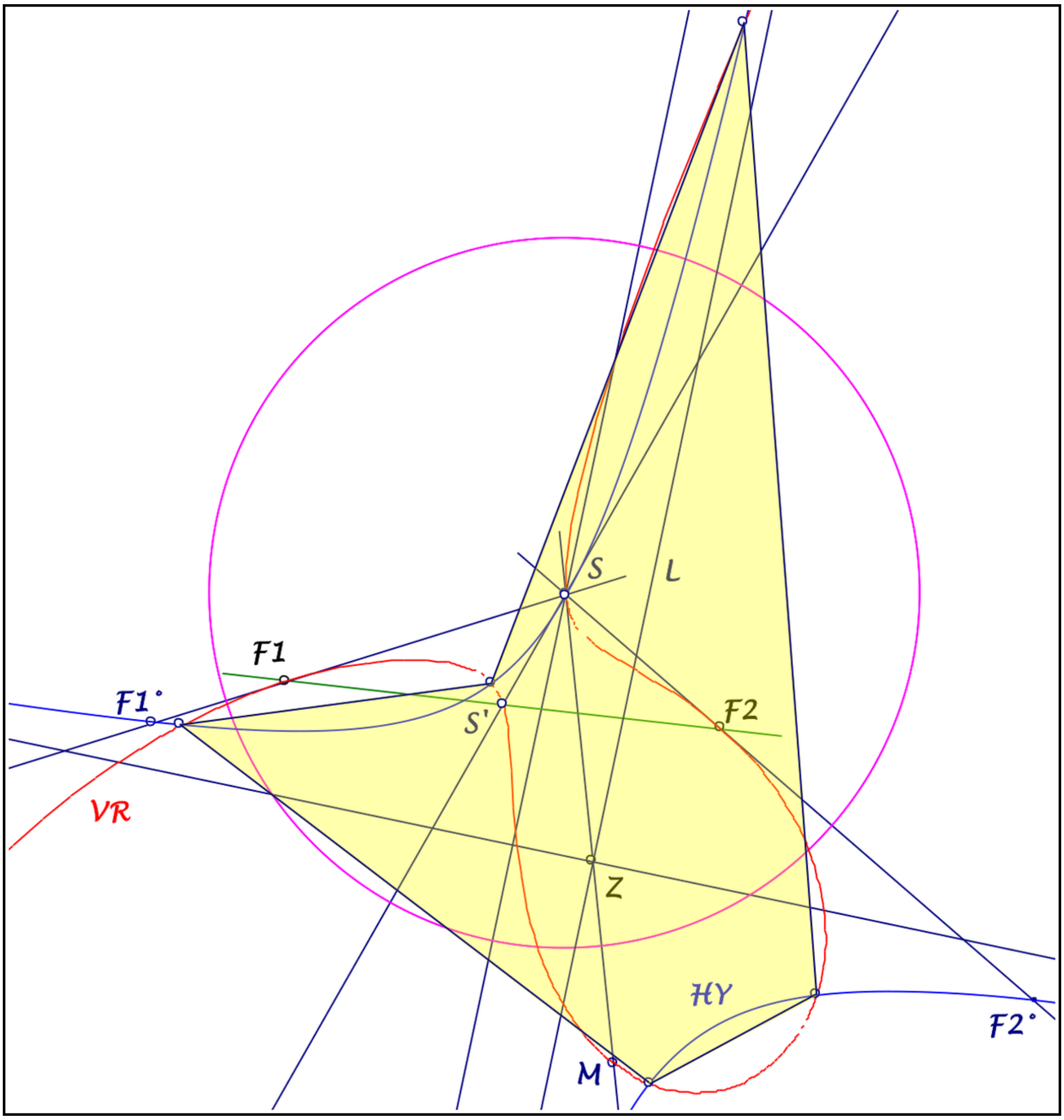
```
Dear Bernard,

wrt your first question in #344:

Let us start with three points M, S, S',
... which define a VR as you describe
... and which shall give S = 5P-s-P4 = 5P-s-P5, S' = 5P-s-P6,
... and M = 5P-CSC(5P-s-P5) of a 5P on VR
... its degenerated quartic.
Let F1, F2 be the fixed points of the CSC,
... centered in S', swapping S and M.
Let Z be any point on SM,
... and L a parallel to the angle bisector of F1,S,F2 through Z.
Let HY be an orthogonal hyperbola,
... through S, centered in Z with asymptote L.
The intersections of HY and VR - unequal S -
... give - if real - the 5P with the properties above.
Varying Z on SM we get in infinity of these 5P.

Wrt the background,
... that this 5P is the 5P of fixed points of 5L-s-Tf1:
... S = 5L-o-P2, S' = 5L-s-P1, F1,F2 are the foci of 5L-s-Co1,
... 5L-o-Ci1 is the inversion circle of Fi and HY ^ S.Fi.

Best regards Eckart
```



2020-06-23.pdf

Message: #347
Date: 2020-06-23
From: bernard.keizer@gmail.com
Subject: Re: Monocursal VR

```
Dear Eckart,
That's beautiful !
There are an infinity of these 5P and of circles 5L-o-Ci1 as
well as conics 5L-s-Co1. This answers my 1rst question.
But for the same reason, there are also with the same
construction an infinity of circles 5L-o-Ci1 centered in S' with
corresponding conics with foci G1 and G2 (on the VR) fixed
points of the CSC centered in S and swapping M and S'. This
answers my 2nd question.
Thanks a lot for these new developments !
Best regards
Bernard
```

Message: #348
Date: 2020-06-26
From: eckart_schmidt@t-online.de
Subject: Re: QA-2Px

```
Dear Chris, dear Bernard,

may I invite you to an excursion in QA-geometry?
Let us consider the two intersections X,Y (see also #74)
... of QA-Cu1 and QA-Co2 unequal QA-vertices,
... they lie on QA-P11.QA-P41, polar of QA-P4 wrt QA-Co2.
For any QA-triangle the isoconjugation, swapping X and Y
... maps XY to QA-Co2.
The line XY is parallel QA-P2.23 and QA-P3,32.

QA-Cu1 is a pivotal isocubic (see also EQF)
... reference triangle: cevian triangle QA-Trx of QA-P4
    wrt QA-Tr1
... isoconjugation, swapping X and Y (fixed point QA-P4),
    pivot QA-P41.
The three triangles QA-Tr1, QA-Tr2 and QA-Trx on QA-Cu1 are
pairwise perspective,
... QA-Tr1,2 wrt QA-P3, QA-Tr1,x wrt QA-P4, QA-Pr2,x
    wrt QA-P41*,
... ... QA-P41* is the isogonal conjugate of QA-P41 wrt QA-Tr2.
Circles through corresponding vertices of the 3 triangles
... touch QA-Cu1 in the QA-Tr1-vertex.
The vertices of QA-Trx,
... which are the 3rd intersections of QA-Co1 and
    the QA-Tr1 sides
... have a common tangential Z = QA-Tf16(QA-Tf2(QA-P41*))
    on QA-Cu1.

QA-Cu1 is a pivotal cubic for the cb-transformation
... wrt the vertices of QA-Tr1, X, Y and the circular points
... and pivot P1 in the QA-Tf4-image of the 3rd intersection
    of QA-Cu1 and Z.QA-P41*.
QA-Cu1 is a pivotal cubic for the cb-transformation
... wrt the vertices of QA-Tr2, X, Y and the circular points
... and pivot P2 in the tangential of QA-P3.
QA-Cu1 is a pivotal cubic for the cb-transformation
... wrt the vertices of QA-Trx, X, Y and the circular points
... and pivot Px in the 3rd intersection of QA-Cu1
    and P2.QA-P41*.

I hope, there are no mistakes in my observations!
Best regards Eckart
```

Message: #349
Date: 2020-06-28
From: bernard.keizer@gmail.com
Subject: Re: QA-2Px

```
Dear Eckart,
If I'm not wrong, the vertices of QA-Trx are the 3 CSC partners
of P41.
Tr1, 2 and x are in a classical Reye configuration on QA-Cu1,
the 3 perspectors P3, P4 and P41* are aligned, as well a the 3
tangentials.
For the rest, I didn't check your properties, but any group of 5
points on a circular cubic gives a pivotal cb transformation ...
Best regards
Bernard
```

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Message: #350
Date: 2020-06-28
From: eckart_schmidt@t-online.de
Subject: Re: QA-2Px

```
Dear Bernard,

if I am not wrong,
neither P3, P4, P41* are aligned nor their tangentials.

Best regards Eckart
```

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Message: #351
Date: 2020-06-28
From: bernard.keizer@gmail.com
Subject: Re: QA-2Px

Dear Eckart,
My apologise !
Of course, the 3 triangles are not in a Reye configuration.
We have in fact 3 Reye configurations with Tr1 and 2 and perspector P3, Tr1 and x and perspector P4 and Tr2 and x with perspector P41*.
It holds that the tangentials of the vertices of Tr1 and Tr2 (id est P41 and the point Q where the asymptote cuts the cubic QA-Cu1) are aligned with tgP3 and the same way that the tangential of the vertices of Trx is tgP41, also aligned with Q and tg(P41*).
I hope this time that I didn't make another mistake ...
Sorry that I was too quick to answer
Best regards
Bernard

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Message: #352
Date: 2020-06-29
From: bernard.keizer@gmail.com
Subject: Re: QA-2Px

Dear Eckart,
This time I checked your main properties wrt X and Y on P11P41 (polar of P4 wrt Co2) as well as Z, which is tgP41.
Can you confirm that the vertices of Trx are the 3 CSC partners of P41 ?
Best regards
Bernard

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Message: #353
Date: 2020-06-30
From: eckart_schmidt@t-online.de
Subject: Re: QA-2Px

Dear Bernard,

you are right,
… the CSC of QA-P41 for the QA-quadrigons are the vertices of QA-Trx.
The CSC of QA-P41 for the QL-quadrigons are collinear.

Best regards Eckart

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Message: #354
Date: 2020-06-30
From: eckart_schmidt@t-online.de
Subject: 5P-2Px

Dear Chris, dear Bernard,

in QFG-message #293 I described a QA-transformation:
Consider for the line pencil of a point X the QA-Tf2-conics,
... the polares of X wrt these conics have a common point.
Have a look in passage 8:
For a point Q on a circumconic Co of a quadrangle
... there are three points X with QA-Tfx(X) = Q.
... All four points lie on a circumconic of QA-Tr1,
... which is the QA-Tf2 image of the tangent in Q at Co.
Now consider a 5P: 4 points for a QA and the 5th point for Q:
The corresponding 5 conics have two common points.
What about the 5P-line through these two points?

There is an analogon for a 5L, which gives two lines.
What about the 5L-point, intersection of these two lines?

Best regards Eckart

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Message: #355
Date: 2020-07-01
From: eckart_schmidt@t-online.de
Subject: Re: 5P-2Px

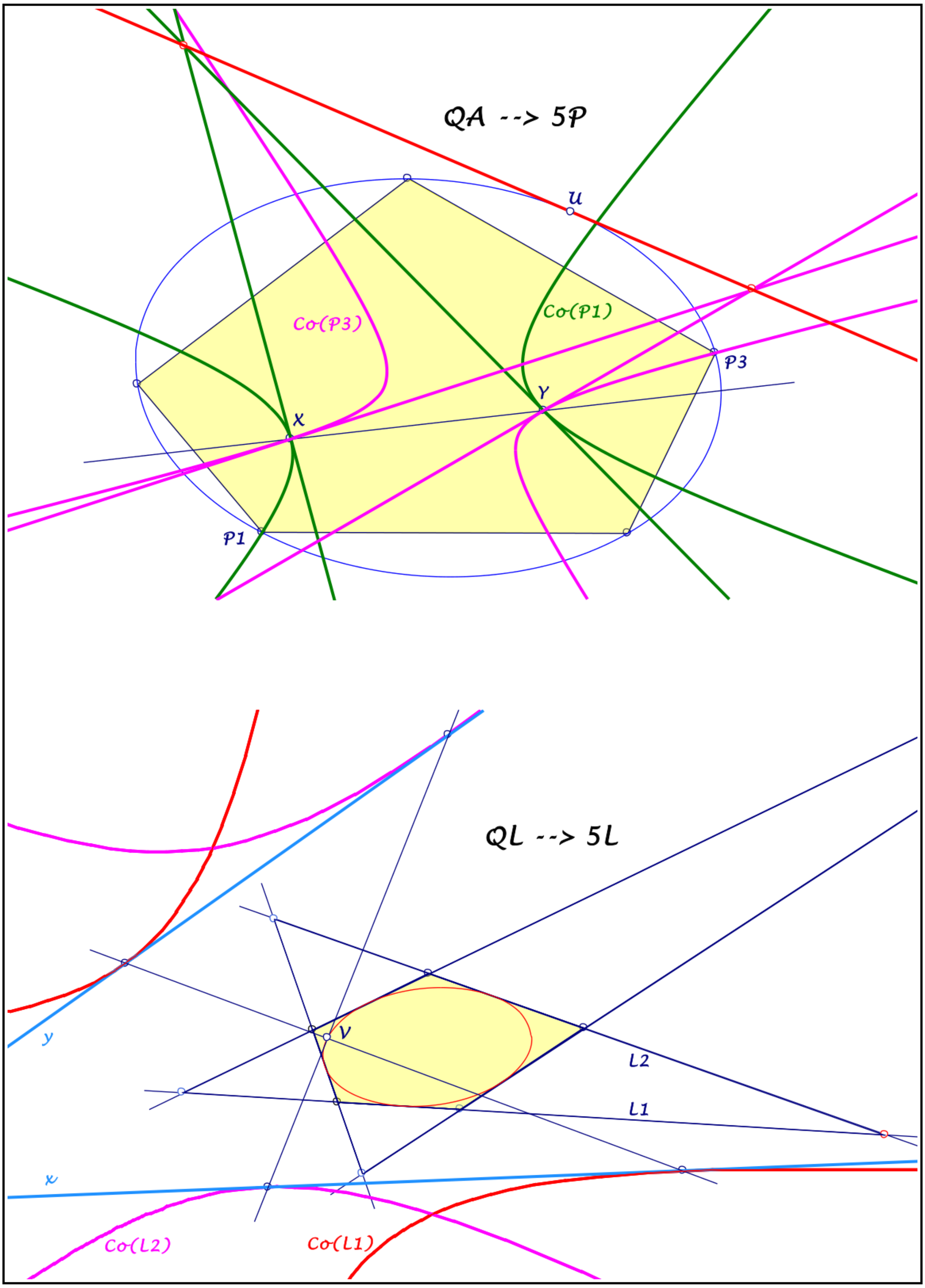
```
Dear Chris, dear Bernard,

in #293old and #354new I described conics for points wrt a QA,
... which give for a 5P five conics with two common points,
... and mentioned an analogon for 5L,
... here further aspects, perhaps of interest:

QA --> 5P
Consider for a QA  and a point Q the conic Co(Q),
... which is the QA-Tf2 image of the tangent in Q
    at the circumconic of QA through Q.
This gives for a 5P = P1...P5 five conics Co(Pi) with two common
points X, Y.
The poles of XY wrt Co(Pi) are collinear on a tangent at
5P-s-Co1
... with contact point U, a new special 5P-s-Px on 5P-s-Co1.

QL--> 5L
Consider for a QL and a line L the inscribed conic with contact
point T of L,
... the QL-Tf2 images of lines through T envelope a conic Co(L).
This gives for a 5L = L1...L5 five conics Co(Li) with two common
tangents x, y.
The polares of x ^ y wrt Co(Li) have a common point V, a new
special 5L-s-Px on 5L-s-Co1.

Best regards Eckart
```



2020-07-01.pdf

Message: #356
Date: 2020-07-03
From: bernard.keizer@gmail.com
Subject: Re: 5P-2Px

Dear Eckart,
I reproduced without difficulty your points X, Y and U.
I even tried to draw the QA-Tf2 of U on the tangent in U, but I could not find any property of U.
It is a new point of 5P on the circumconic.
Congratulations
Best regards
Bernard
PS I hadn't the courage of doing the same for V, but I suppose it's a kind of dual property ...

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Message: #357
Date: 2020-07-03
From: eckart_schmidt@t-online.de
Subject: Re: 5P-s-2Px

Dear Chris, dear Bernard,

these points 5P-s-2Px are already mentioned in #2912old
... but with another background.

Best regards Eckart

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Message: #358
Date: 2020-07-05
From: eckart_schmidt@t-online.de
Subject: 5P-s-P2 / 5L-s-Lx

```
Dear Chris, dear Bernard,

perhaps new a property of 5P-s-P2:
Consider the dual 5L of a 5P,
... with lines dual of the 5P-vertices wrt the QA
    of the other 5P-vertices.
... 5P-s-P2 lies on 5L-s-Co1 of this 5L.

Certainly new an analogon of 5P-s-P2 for a 5L=L1...L5:
Let Mi be the line QL-Tf2(Li) wrt the QL of the other L-lines.
Let Ni be the line QL-Tf2(Mi) wrt the QL of the other M-lines.
The 5L L1...L5 and N1...N5 are line perspective wrt a new
5L-line,
... which is tangent to 5P-s-Co1 of the dual 5P
    of the reference 5L.

Best regards Eckart
```

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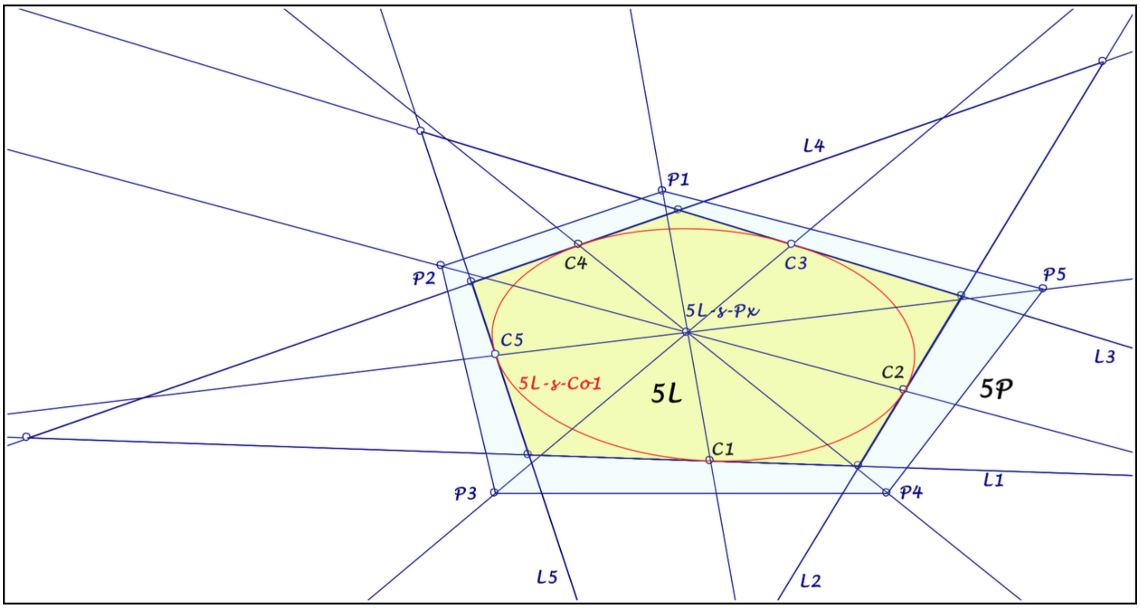
Message: #359
Date: 2020-07-05
From: eckart_schmidt@t-online.de
Subject: New 5L-Point, 5P-Line

```
Dear Chris, dear Bernard,

if you consider a 5L=L1...L5 and its dual 5P=P1...P5
... with vertices Pi dual of Li wrt the other L-lines,
... and the contact points Ci of Li wrt 5L-s-Co1,
... the lines PiCi have a common new point 5L-s-Px.

If you consider a 5P=P1...P5 and its dual 5L=L1...L5
... with lines Li dual of Pi wrt the other P-points,
... and the tangents Ti in Pi at 5P-s-Co1,
... the intersections LiTi are collinear on a new line 5P-s-Lx.

Best regards Eckart
```



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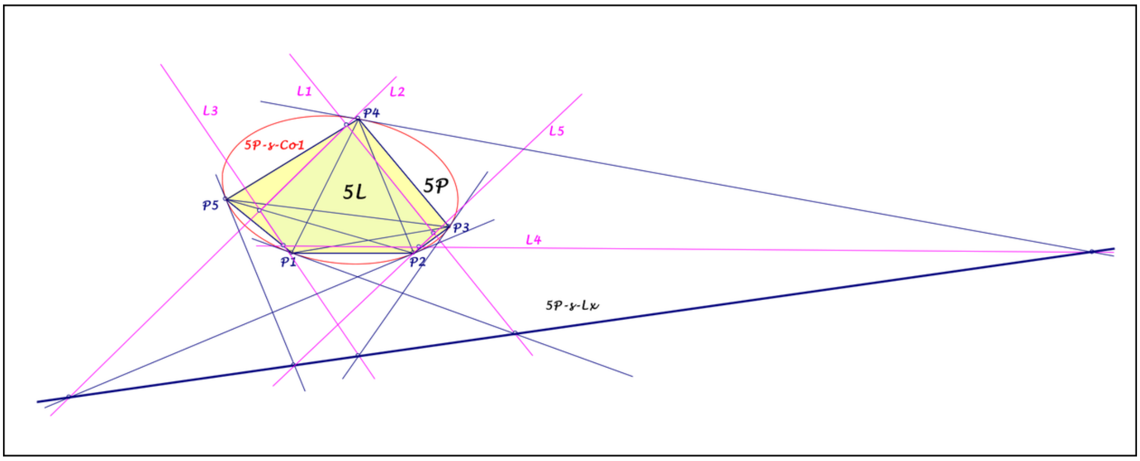

2020-07-05b.pdf

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Message: #360
Date: 2020-07-06
From: bernard.keizer@gmail.com
Subject: Re: New 5L-Point, 5P-Line

Dear Eckart,
Beautiful and amazing ! This is indeed a new very promising item.
First you define the duality for a 5P/5L. It's the 1rst time you mention this duality ?
Then you find this dual property the CiPi through a fixed point and the intersections of TiLi aligned.
I suppose the point and the line are dual as the intersection of TiLi is the dual of CiPi.
But there must be plenty of other properties !
For example, 5P-s-Co1 and L-s-Co1 are also dual and the dual of points of one conic are tangents to the other and conversely.
If 5P-s-P2 lies on 5L-s-Co1, it's dual is a tangent to 5P-s-Co1 ...
Last questions for today : is the 5P of the Ci (contact point of Li with 5L-s-Co1) the dual of the 5L of the Ti (tangent in Pi to 5P-s-Co1) ?
PiPj with Pi vertice of the 5P is the dual of Aij (vertice of the 5L) and conversely CiCj is the dual of the intersection of TiTj ...
In this case, 5L-s-Co1 of the Li is 5P-s-Co1 of the Ci and 5P-s-Co1 of the Pi is the 5L-s-Co1 of the Ti and there must be another point and line 5P-s-Px and 5P-s-Lx.
Last, could we define the dual of a point in the plane as the line through the dual points (on 5P-s-Co1) of the tangents to the conic 5L-s-Co1 ?
Best regards
Bernard
PS Why don't you draw the 5P, the 5L and the 2 conics on the same figure ? It would help to see the converse properties ...

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Message: #361
Date: 2020-07-07
From: eckart_schmidt@t-online.de
Subject: Re: New 5L-Point, 5P-Line

```
Dear Bernard,

more and more it happens,
... that my observations are already mentioned in an earlier
    message,
... wrt these elements in #211 without reaction.

But I have difficulties to understand your remarks:
In the first case of #359 I described the dual 5P for a 5L,
... in the second case I described the dual 5L for a 5P.
Please respect, that the dual 5L for a dual 5P for a 5L
isn't the starting 5L
... and that the dual 5P for a dual 5L for a 5P
    isn't the starting 5P!
So I cannot draw a 5P and a 5L in one figur
... with 5P dual of 5L and 5L dual of 5P.

Wrt your "last question for today":
... For the 5P of the Ci and the 5L of the Ti
... no one is the dual of the other
... neither in the first nor in the second case in #359.

Please excuse, that I cannot judge your further remarks,
... for I don't know, whether you use the first or second case
of #359,
... both contain a 5P of Ci and a 5L of Ti.

Best regards Eckart
```

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Message: #362
Date: 2020-07-08
From: bernard.keizer@gmail.com
Subject: Re: New 5L-Point, 5P-Line

Dear Eckart,
Sorry if I missed your message 211, I suppose I was too busy with the search of the super CSC transformation associating lines tangent to the conic and VRs with focus on the circle ...
But I have more and more the impression that we are alone as passengers of a ghost ship without a pilot and a crew ! That gives perhaps less enthousiasm for digging new properties ...
Anyhow, I had understood that the 5L of the dual 5P of a 5L is not the initial 5L and the same with the dual 5P of the dual 5 of a 5P ...
But the fact that the 5P of the Ci and the 5L of the Ti are not dual figures in both configurations is not a problem !
The same way, a QA/QL defines a duality, but 4 copples of dual points/lines are dual figures in this duality, but the resulting QA and QL are not dual ...
How do you define the dual line of a point or the dual point of a line in both configurations 5L to 5P or 5P to 5L ?
I suggested in the 1rst configuration to draw the tangents from the point to the 5L conic and to define the dual line through their dual points.
(This leads to the tangent in Pi to the circumconic as dual of Ci contact point of Li with the inscribed conic).
Thanks for your attention
Best regards
Bernard

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Message: #363
Date: 2020-07-09
From: eckart_schmidt@t-online.de
Subject: Re: New 5L-Point, 5L-Line

```
Dear Bernard,

wrt "How do you define the dual line of a point or the dual
point of a line
... in both configurations 5L to 5P or 5P to 5L ?"
I use only the duality QA-8/QL-8 of EQF,
... in #359 I define for example a "dual" 5P for a 5L with
... "vertices Pi dual of Li wrt the other L-lines".
Perhaps my nomination "dual" for 5P/5L isn't wise,
... for it is not reciprocal.
Sorry, I see no point/line duality wrt a 5P or 5L
... beside the pole/polar duality wrt 5P-s-Co1/5L-s-Co1.

Best regards Eckart
```

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Message: #364
Date: 2020-07-11
From: bernard.keizer@gmail.com
Subject: 3 circular quartics of 5L

Dear Eckart,
I consider the vertices Aij of a 5L, Oij the centers of their Ci circle and Bij the other preimage of Oij.
I use your points T and T', inverses of S and S' wrt 5L-o-Ci1 and ω the intersection of TS' and T'S.
Bij is the inverse of Aij in an inversion centered in ω which swaps T and S' and T' and T and leaves the circle Ci1 globally invariant.
The inverse of line Li through the Aij is a circle Ci through the Bij and ω. The inverse of a line L tangent to the conic 5L-Co1 in P is a circle C.
The inverse of the conic 5L-s-Co1 is a circular quartic Qu1 which envelops the circles C, the contact point Q being the inverse of P.
The center of the circle C is a point O' which describes a 2nd conic Co2. The perpendicular bisector of ωQ is tangent in O' to the conic Co2.
The inverse of Co2 is a 2nd circular quartic Qu2 which envelops the circles inverses of these perpendicular bisectors, the contact point being the point R, inverse of O' and reflection of ω in the tangent in P to the 1rst conic.
Last, for any point M on the circle 5L-o-Ci1, it's inverse is a point M' on the circle and the inverse of the tangent in M to the circle Ci1 is a circle through ω tangent in M' to the same circle Ci1.
The center O'm of this circle describes a conic Co3 and the perpendicular bisector of ωM' is tangent in O'm to Co3.
The inverse of Co3 is 3rd circular quartic Qu3 which envelops the circles inverses of these perpendicular bisectors, the contact point being Rm inverse of O'm and reflection of ω in the tangent in M to the circle Ci1.
Best regards
Bernard

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Message: #365
Date: 2020-07-12
From: eckart_schmidt@t-online.de
Subject: Re: 3 circular quartics of 5L

Dear Bernard,

in vain I try to complete your interesting excursion in
5L-geometry.
But is your following passage correct?
"Last, for any point M on the circle 5L-o-Ci1, it's inverse is a
point M'
on the circle
and the inverse of the tangent in M to the circle Ci1 is a
circle through ω
tangent in M' to the same circle Ci1."
Thanks in advance for explanation.

Best regards Eckart

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Message: #366
Date: 2020-07-12
From: van10hoven@gmail.com
Subject: Re: New 5L-Point, 5P-Line

```
Dear Eckart, dear Bernard,

Beautiful new 5L-Point, 5P-Line.
I like their simplicity!
Let me rephrase them in my own words.

*Pentalateral case*
· Given Reference Pentalateral 5L (L1,L2,L3,L4,L5).
· Let Pi be QL-Tf10(Li) wrt ref-QL (LjLkLlLm).
· Let Qi be the points of tangency of inscribed conic 5L-s-Co1
  with resp. L1,L2,L3,L4,L5.
· then lines PiCi are concurrent in a new center 5L-s-P11.
Integers i,j,k,l,m each being different numbers from
(1,2,3,4,5).
Let Cox be the Circumscribed Conic of (P1,P2,P3,P4,P5).
Now line 5L-s-L1 will be tangent to Cox. (like mentioned by
Bernard)

*Pentangle case*
· Given Reference Pentangle 5P (P1,P2,P3,P4,P5).
· Let Li be QA-Tf11(Pi) wrt ref-QA (PjPkPlPm).
· Let Ti be the tangents of circumscribed conic 5P-s-Co1
  with resp. P1,P2,P3,P4,P5.
· then points Li ^ Ti are collinear on a new line 5L-s-L1.
Integers i,j,k,l,m each being different numbers from
(1,2,3,4,5).
Let Cox be the Inscribed Conic of (L1,L2,L3,L4,L5).
Now point 5P-s-P2 will be lying on Cox. (like mentioned by
Bernard)
The new 5L-point and 5P-line are items rooted in harmonic
properties.
That explains the incidences with resp. 5L-s-L1 and 5P-s-P2.

I didn’t find any other incidences with known EPG-items.

Best regards,
Chris
```

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Message: #367
Date: 2020-07-12
From: eckart_schmidt@t-online.de
Subject: Re: New 5L-Point, 5P-Line

```
Dear Chris,

thanks for interest, but if I am not wrong,
... there are two typos:
wrt the pentalateral case: 5L-s-L2 will be tangent to Cox,
wrt the pentangle case: the new line isn't 5L-s-L1 but 5P-s-L1.

Best regards Eckart
```

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Message: #368
Date: 2020-07-12
From: van10hoven@gmail.com
Subject: Re: New 5L-Point, 5P-Line

```
Dear Eckart,

You are quite right.
Thanks for correcting.

Best regards,
Chris
```

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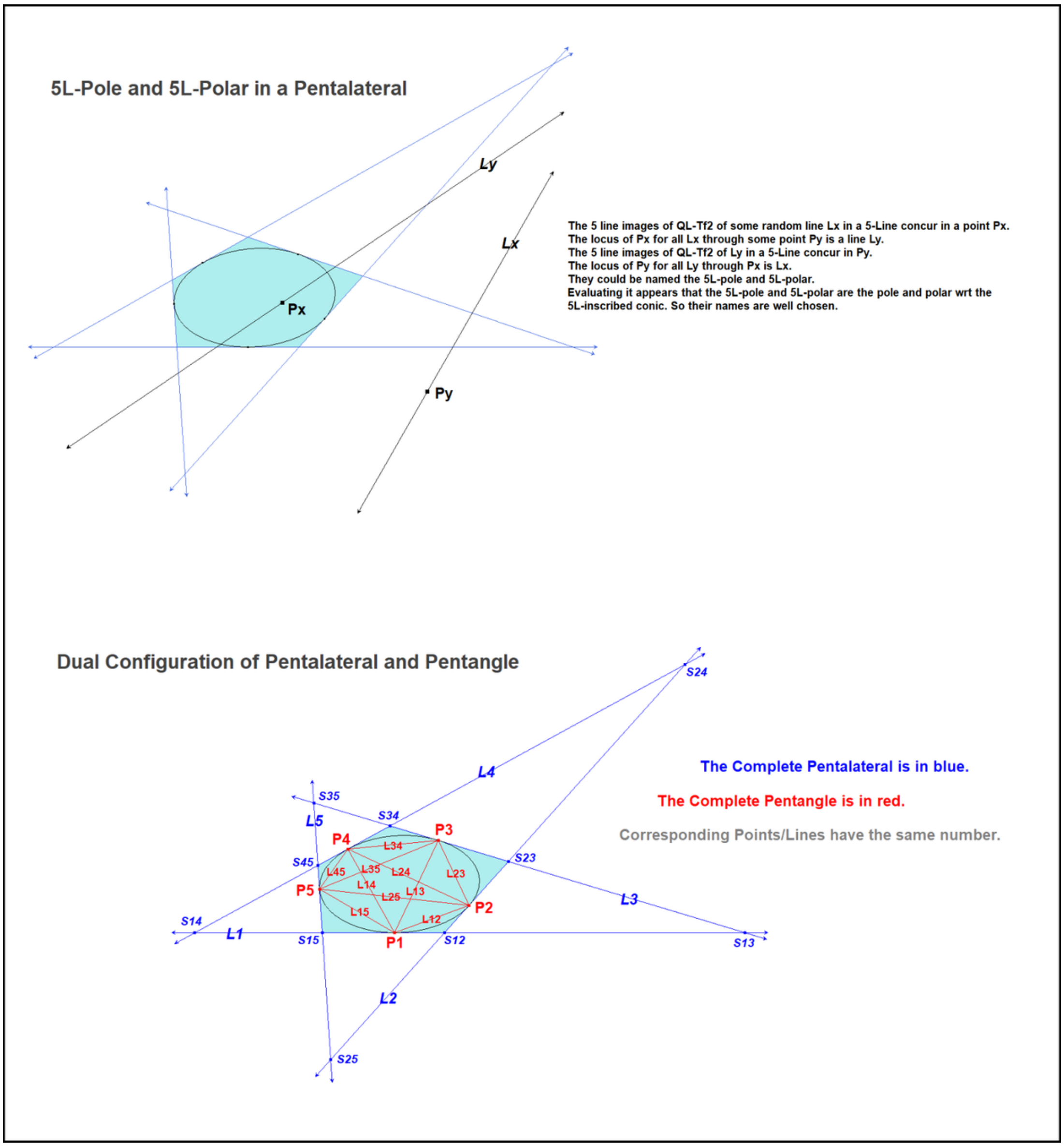
Message: #369
Date: 2020-07-12
From: van10hoven@gmail.com
Subject: 5P-/5L-Duals

```
Dear Eckart and Bernard,

Because of the discussing of the topic about duals in Pentangles
and Pentalaterals I attach an old file of mine about the
subject.
It was attached before in QFG-message #2731.

Best regards,
Chris
```



5L-s-Tf2-Dual Penta Configuration-01.pdf

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Message: #370
Date: 2020-07-12
From: eckart_schmidt@t-online.de
Subject: Re: 3circular quartics of 5L

```
Dear Bernard,

please forget my last message,
... for I succeeded reproducing your message 364.
There was a misunderstanding of the inversion in the 3rd set,
... centered in w which swaps T and S' and T' and S.
This inversion was new for me.

Best regards Eckart.
```

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Message: #371
Date: 2020-07-12
From: eckart_schmidt@t-online.de
Subject: Re: 5P-/5L-Duals

```
Dear Chris,

your cited duality for 5P/5L is the polarity wrt
5L-s-Co1/5L-s-Co1,
I considered quite another duality (see #211):
The dual 5L of a 5P=P1...P5 has lines,
... which are the dual lines of Pi wrt the QA
    of the other vertices.
The dual 5P of a 5L=L1...L5 has vertices,
... which are the dual points of Li wrt the QL
    of the other lines.
This duality is not reciprocal!

Best regards Eckart
```

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Message: #372
Date: 2020-07-13
From: eckart_schmidt@t-online.de
Subject: Re: 3 circular quartics of 5L

```
Dear Bernard,

in #364 you use an inversion,
... consisting of a reflection in the point w and an inversion
... wrt a circle round w with radius sqr(wSxwT')=sqr(wS'xwT).
If you use only the inversion wrt the circle,
... you get nearly the same observations.
This was my first way, to follow your interesting
5L-relationships.

Best regards Eckart
```

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Message: #373
Date: 2020-07-13
From: eckart_schmidt@t-online.de
Subject: Re: 3 circular quartics of 5L

```
Dear Bernard,

I think, background for your observations in #364
... will be the following general concept:
Consider any circle, centered in Z, with its inversion
... and any conic with points P:
The inverse of tangents in P at the conic
... will be a circles C through Z,
... which envelope a quartic Qu
... with contact points Q inverse of P.
The bisectors of ZQ envelope a 2nd conic
... bearing the centers of the circles C.
This 2nd conic allows a new start ...

Therefore part of your observations
... are not special for 5L-o-Ci1 and 5L-s-Co1.

Best regards Eckart
```

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Message: #374
Date: 2020-07-13
From: van10hoven@gmail.com
Subject: Re: 5P-/5L-Duals

```
Dear Eckart,

I noticed about your other duality.

Because it is not reciprocal it is not a real dual in the common
sense of the word.

Therefore I think it is a beautiful set of complementing
transformations in a class of its own.

Best regards,

Chris
```

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Message: #375
Date: 2020-07-17
From: eckart_schmidt@t-online.de
Subject: Clawson's Splitter (33)

```
Dear Bernard, dear Chris,

in reference [22] of EQF Clawson defines
... the Apollonian point of A and B wrt a point O:
Consider for A and B the inversions A' and B' wrt any circle
round O
... and take the inverse of the middle of A'B'.

Clawson's result (33) is part of the following property:
For A, B CSC-partner on QL-Cu1 and arbitrary point O
... the loci for the Apollonian points are circles,
... especially for O = QL-P1 the circle CSC(QL-L1),
... another example for O = QL-P10 the circle QL-Ci1.
For O on QL-L1 the circles degenerate to lines,
... which envelope a hyperbola,
... ... centered in QL-P1, axis 1st Steiner axis,
... ... tangent to QL-L1 with contact point QL-L1 ^ QL-L6,
... ... asymptotes tangent to QL-Cu1, foci in QL-2P3.
For O on QL-Cu1 the circle bears O.

There is also an application in QA-geometry:
For a pivotal isocubic wrt QA-Tr1, QA-Tf2 and pivot P = O,
... the locus of Apollonian points for QA-Tf2-partner A, B
    on the cubic
... is part of the circumconic of the QA through P = O.
This can be generalized.

Perhaps there will be other applications.

Best regards Eckart
```

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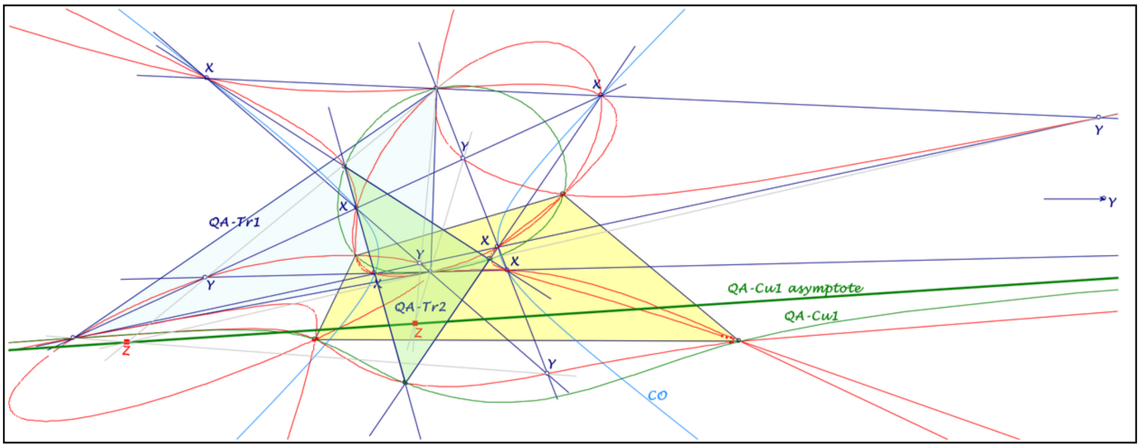
Message: #376
Date: 2020-07-17
From: eckart_schmidt@t-online.de
Subject: Re: Clawson's Splitter (33)

```
Dear Bernard, dear Chris,

Clawson's Apollonian point (see #375) leads to
... a transformation, defined by two points A,B,
... which maps a point O to the Apollonius point of O wrt A,B.
Here an application in QL-geometry for the Plücker points
QL-2P1:
The circle QL-Ci1 will be mapped to itself,
... chords of point and image have a common point,
... ... which is the 4th harmonic point of QL-P9 wrt QL-2P1a,b,
... poles of these chords lie on QL-L1.
The circle QL-Ci5 will also be mapped to itself,
... chords of point and image have a common point on QL-L1,
... ... which is the inverse of QL-P7 wrt QL-Ci5,
... poles of these chords lie on QL-L2.

Best regards Eckart
```

Message: #377
Date: 2020-07-19
From: eckart_schmidt@t-online.de
Subject: QA-Tf2-partner on asymptote of QA-Cu1

```
Dear Bernard, dear Chris,

most of the QA-cubics are pivotal isocubics
... wrt QA-Tr1, QA-Tf2 and different pivots,
... which here shall be the Miquel points, vertices of QA-Tr2.
The 3 corresponding isocubics have
... triple intersections in the QA-points and QA-Tr1-vertices,
... double intersections in 6 points X on a conic CO,
... in pairs QA-Tf2 isoconjugated on the sides of QA-Tr2,
... in pairs collinear with a QA-Tr1-vertex,
... two lines through each QA-Tr1-vertex,
... ... bearing two double intersections X.
These six lines give 6 further intersections Y,
... in pairs QA-Tf2-isoconjugated and collinear
    with a QA-Tr2-vertex.
If we connect each Y with the 3rd QA-Tr1-vertex,
... we get two QA-Tf2-isoconjugated
... triple intersections Z on the asymptote of QA-Cu1.

Best regards Eckart
```



2020-07-19.pdf

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Message: #378
Date: 2020-07-20
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu1

```
Dear Eckart,
Taking any triangle of 3 random points as pivots, the 3 QA-Cu
intersect in 7 triple points QA vertices and DT vertices and in
6 dobble points being 3 pairs of QA-Tf2 conjugates on the sides
of the triangle.
I didn't check the other properties in this general case (6
points coconic, pairwise aligned with DT vertices ...)
Best regards
Bernard
```

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Message: #379
Date: 2020-07-20
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu1

```
Dear Eckart,
Your 2 last QA-Tf2 partners on the asymptote of QA-Cu1 can be
obtained as intersections of 2 QA-Cu pivotal isocubics with
pivots the infinity point of the asymptote and the point where
the asymptote cuts QA-Cu1 ...
Best regards
Bernard
```

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Message: #380
Date: 2020-07-21
From: eckart_schmidt@t-online.de
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu1

```
 Dear Bernard,

thanks for background for the first steps of my excursion
... to the QA-Tf2-partner on the asymptote of QA-Cu1,
... but the further properties of the double intersections
... "6 points coconic, pairwise aligned with DT vertices..."
... don't hold in general.

There will be more examples for tripel of QL- or QG-points,
... for the triangle of QG-P18 there exist
    the final QA-Tf2-partner,
... but for QG-P13 not ...
I shall look for other examples.

Best regards Eckart
```

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Message: #381
Date: 2020-07-22
From: eckart_schmidt@t-online.de
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu

```
Dear Bernard,

replacing the Miquel points in the construction of #377 by other
QL- or QG-points,
... we get several examples for a final line,
    defined by two QA-Tf2-partner:
Some results up to now:
... no final QA-Tf2-partner for
    QL-P2, QL-P8, QL-P13, QG-P8, QG-P13 ...
... with final QA-Tf2-partner:
    QG-P15, QG-P16, QG-P18, QG-P19, ...

Starting with the QG-P15-triple, we get the line QL-P16.QL-P20.

Best regards Eckart
```

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Message: #382
Date: 2020-07-24
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu

```
Dear Eckart,
In fact very interesting construction !
I tried with the famous S-points, but up to now, I wasn't able
to find 6 real points X ...
I give up for the moment.
Best regards
Bernard
```

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Message: #383
Date: 2020-07-25
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA-Cu

Dear Eckart,
Perhaps you will find an explanation ?
I made several different figures with the S-points, I come always to the same result.
QATf2 of the S-points are aligned on QA-P10QA-P16, which is normal, as the conic through the DT vertices, P10, P16 and the S-points is QA-Tf2 of the line P10P16.
Then I find only 2 real of the 6 points X (as intersections of SiSj and the DTcircumconic through QA-Tf2 of Si and Sj).
Can you confirm (and explain ?) this strange property ?
Best regards
Bernard

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Message: #384
Date: 2020-07-25
From: eckart_schmidt@t-online.de
Subject: Re: QA-Tf2-partner on asymptote of QA

```
Dear Bernard,

it wasn't difficult, to find a QA with 6 X on the sides
of the S-triangle (see #377),
... but they are not coconic,
    that is a condition for the final QA-Tf2-partner.
I researched all QG-points, but found no further examples as in
#381 mentioned,
... perhaps there will be QL-points beside QL-P1 ...

Here the results up to now:
... QL-P1-triangle  ---> QA-Tf2-partner on the asymptote
                         of QA-Cu1,
... QG-P15-triangle ---> QA-Tf2-partner on QA-P16.QA-P20,
... QG-P16-triangle ---> QA-Tf2-partner on the tangent in QA-P3
                         to QA-Cu1,
... QG-P18-triangle ---> QA-Tf2-partner on a line,
                         not identified,
... QG-P19-triangle ---> QA-Tf2-partner on a line,
                         not identified.

Remarkable:
QL-P1 and QG-P16 as well as QG-P18 and QG-P19 are
QA-Tf2-partner,
... but QA-Tf2 of QG-P15 is the point at infinity of QL-L1 ...

Best regards Eckart
```

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Message: #385
Date: 2020-07-26
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA

Dear Eckart,
I was intrigued and very interested by your construction.
I noticed that QL-P1 and QG-P16 are QA-Tf2 partners on the same pivotal isocubic QA-Cu1 with pivot QA-P4.
In both cases, the 6 points X are coconic.
The lines Z1Z2 are tangents to QA-Cu1 in 2 QA-Tf2 partners QA-P3 and infinity point of the asmptote (aligned with the pivot QA-P4).
I made the construction with QG-P18 and QG-P19, which are QA-Tf2 partners on a pivotal isocubic QA-Cux with pivot P (?).
In both cases, the 6 points X are coconic.
The lines Z1Z2 are tangents to QA-Cux in 2 QA-Tf2 partners (aligned with the pivot P).
I wonder if this could be a general property of your construction.
For any pivot P and pivotal isocubic QA-Cux with pivot P and isoconjugation QA-Tf2, any 3 points on the cubic would have the same property that the points X are coconic and lead to a line tangent to the cubic, the same for the QA-Tf2 partners of the 3 points ...
Best regards
Bernard

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Message: #386
Date: 2020-07-26
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf2-partner on asymptote of QA

Dear Eckart,
In fact, I suppose it can not be any 3 points on the same pivotal isocubic.
The 3 points must have the same tangential and Z1Z2 is the tangent to the cubic in the 4th point having the same tangential (passing through this tangential).
Best regards
Bernard

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Message: #387
Date: 2020-07-26
From: eckart_schmidt@t-online.de
Subject: Re: QA-Tf2-partner on asymptote of QA

```
Dear Bernard,

you have a better overview, my compliment!
I can confirm your observation:
For a pivotal isocubic of a QA wrt QA-Tr1, QA-Tf2 and pivot P
... and 3 points on the cubic with tangential T
... my construction in #377 leads to two QA-P2-partner,
... defining a line,
    tangent to the cubic in the 4th point with tangential T.

Best regards Eckart
```

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Message: #388
Date: 2020-07-26
From: eckart_schmidt@t-online.de
Subject: Another approach to duality

```
Dear Chris, dear Bernard,

somewhat curious, but perhaps of interest:
Consider for a QL three conics, bearing a given point P
... and the vertices of a QG-version of the QL,
... which have three collinear double intersections unequal
    QL-points.
The poles of the corresponding line wrt the three conics
... give a triangle, perspective QL-Tr1.
Mapping the starting point to the center of perspectivity
... we get QA-Tf2 for the dual QA of the starting QL.

Best regards Eckart
```

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Message: #389
Date: 2020-07-28
From: eckart_schmidt@t-online.de
Subject: Simple QL-geometry?

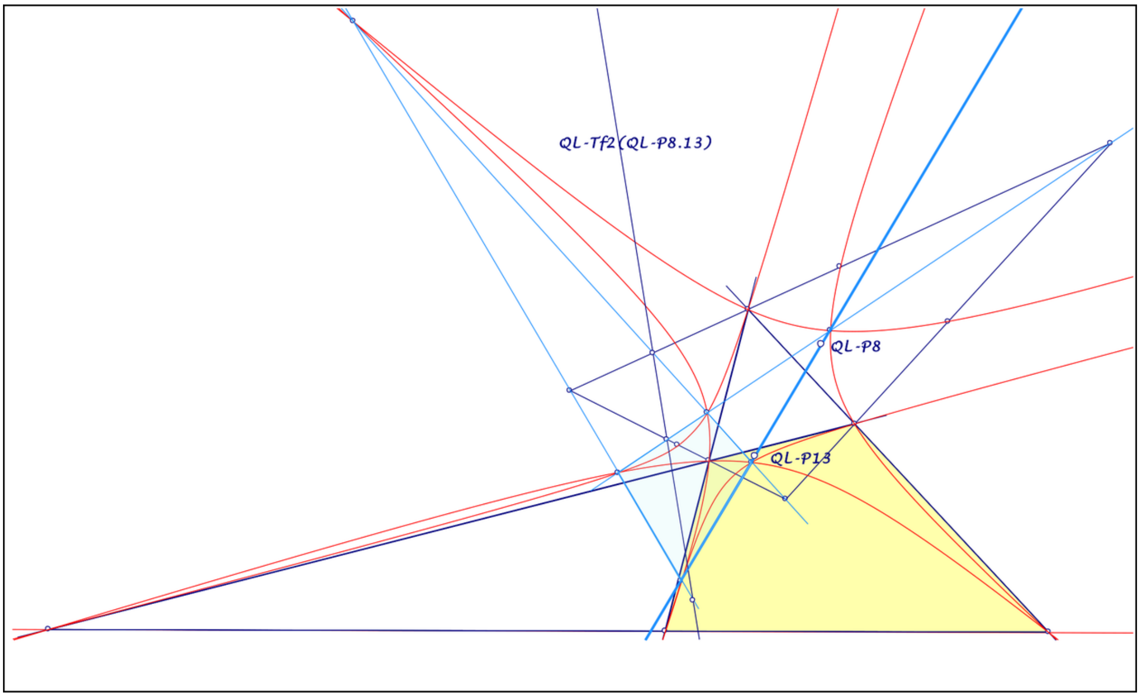
```
Dear Bernard, dear Chris,

leading in a short excursion in QL-geometry:
Consider for a QL the diagonal midpoints on QL-L1
... and 3 conics through a midpoint and the other 4 QL-points,
... which have 3 collinear double intersections in QG-P15
    on QL-L9 (see EQF QL-P18).
Three circles through two midpoints and the corresponding double
intersection
... have a common point in QL-L1 ^ QL-L6.
Three circles through two double intersections and the remaining
midpoint
... have a common point on QL-L1, dividing QL-P7.QL-P12
    with ratio -3/2.

Finally another aspect:
Replacing in the construction above the diagonal midpoints
... by the side midpoints of the diagonal triangle QL-Tr1,
... we get 3 conics with six double intersections,
    unequal QL-points,
... which are the points of a 2nd QL,
... three lines bear a QL-Tr1 vertex,
    the 4th line is QL-P8.QL-P13,
... each of the three lines intersects
    the opposite QL-Tr1 side on QL-Tf2(QL-P8.QL-P13).

What about this unexpected constellation?

Best regards Eckart
```



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Message: #390
Date: 2020-07-29
From: eckart_schmidt@t-online.de
Subject: Re: Simple QL-geometry?

```
Dear Bernard, dear Chris,
wrt the last question in #389:
The figur is a special case of a general constellation:
Consider a QL and a point P with its cevian triangle TR wrt
QL-Tr1,
... further the three circumconics of the QG-versions
    through a TR-vertex,
... with six double intersections unequal QL-points,
    which give a 2nd QL,
... three lines through a QL-Tr1 vertex and a 4th line L
    through P,
... each of the three lines intersects the opposite QL-Tr1 side
    on QL-Tf2(L)
The line L through P can be constructed as follows:
... Take two vertices of TR with the corresponding conics,
... which have two intersections unequal QL points,
... one intersection lies on the TR-side, the other on L.
Best regards Eckart
```

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Message: #391
Date: 2020-07-29
From: bernard.keizer@gmail.com
Subject: Re: Simple QL-geometry?

```
Dear Eckart,
I reproduced without difficulty your figure in the general case,
but I didn't find any new property ...
For example, the Miquel point of the 2nd QL is not on the TR
circumcircle.
I only noticed that the line joining P to a DT vertex is the
tangent to the corresponding conic in the corresponding TR
vertex (on the opposite side of DT).
Best regards
Bernard
```

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Message: #392
Date: 2020-07-30
From: bernard.keizer@gmail.com
Subject: 5P, CSC and twin conics

```
Dear Eckart,
Considering 5Pi, the CSC with center m and the CSC Qi, the
conics through the 2 groups of 5 points are the twin conics.
The 2 conics define in turn another CSC, with the 2 pairs of
foci or with the 4 common tangents to the 2 conics.
The 2 CSC are different and I can't find any interesting
property ...
The 4 tangents being tangents to the conic 5L-s-Co1 lead to
points on the circle 5L-o-Ci1 for both 5L ...
Did you already investigate in this direction ?
Best regards
Bernard
```

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Message: #393
Date: 2020-07-31
From: eckart_schmidt@t-online.de
Subject: New QL-line

```
Dear Bernard, dear Chris,

consider a reference QL and the following four QL
... defined by the three sidelines of QL-Tr1 and a QL-line.
  * Their 4 QL-P1 lie on QL-Ci1 of the ref-QL.
  * Their 4 QL-L2 have the common point QL-P10 of the ref-QL.
  * Their 4 QL-Ci1 have triple intersections in the vertices
    of the dual QA.
  * Their 4 QL-Ci2 have a common point, 4th intersection
    of QL-Ci1 and QA-Co1 of the dual QA of the ref-QL.
  * Their 4 QL-Ci3 have the common point QL-P9 of the ref-QL.
  * Their 4 QL-Ci4 have collinear centers on a line
    through QL-P11, orthogonal QL-L7.
  * Their 4 QL-P23 are collinear on a new QL-line.
  * Their 4 QL-Tr1 give triple of vertices of the dual QA
    of the ref-QL.
Finally special for Bernard:
Their 4 cubics wrt their 5 QA-Cu7-triple points
... have triple intersections in the vertices of the dual QA
    of the ref-QL.

Best regards Eckart
```

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Message: #394
Date: 2020-07-31
From: eckart_schmidt@t-online.de
Subject: New QA-elements

```
Dear Bernard, dear Chris,

this is an analogon to #393, but for a quadrangle QA
Consider a ref-QA and the following four QA,
... defined by the vertices of QA-Tr1 and a QA-point.
  * Their 4 QA-P2 are concyclic on QA-Ci2 of the ref-QA,
    which is QL-Ci2 of the dual QL of the ref-QA.
  * Their 4 QA-P11 are concyclic on QL-Ci3 of the dual QL
    of the ref-QA.
  * Their 4 QA-P12 are collinear on a line through QA-P11,
    which is QL-L2 of the dual QL of the ref-QA.
  * Their 4 QA-Ci1 have a common point on QA-Ci2,
    which is QL-P1 of the dual QL of the ref-QA.
  * Their 4 QA-Ci1 have double intersections in the points
    of the dual QL of the ref-QA.
There will be more properties.

Best regards Eckart
```

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Message: #395
Date: 2020-08-01
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

Dear Eckart,
Very interesting properties ! (not totally new ...)
They have 2 keys :
1) DT is one of the 4 reference triangles for your 4 ql
This explains immediately the properties 1, 2 and 5 and less immediately 6.
2) Perhaps less obvious, the vertices of the dual QA are the perspectors between the reference triangles of a QL and it's DT.
3 vertices of a QA form the anticevian triangle of the 4th wrt DT, or to say the reverse way, DT is the cevian triangle of a vertex of the QA wrt the triangle of the 3 other vertices.
Shortly, which matters is that the dt of your 4 ql are the reference triangles of the dual QA.
This explains the property 8 and immediately 3 and 4 (the common point is QA-P2 of the dual QA).
It explains also the QA-Cu7 of the 4 ql.
May be there are other properties ? What about the Newton Lines or the Ci6 circles or the s-points ?
Best regards
Bernard

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Message: #396
Date: 2020-08-01
From: bernard.keizer@gmail.com
Subject: Re: New QA-elements

Dear Eckart,
This is exactly the dual figure of the preceding one.
As DT is self dual and each vertex of the QA is the dual of a line of the QL, your 4 ql and your 4 qa are dual figurs in the duality QA/QL.
We have the same 2 keys :
1) DT is one of the 4 reference triangles of the 4 qa
2) For the same perspective properties, the 4 reference triangles of the dual ql are the dt of your 4 qa.
This explains easily your properties and perhaps other to come ?
Best regards
Bernard

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Message: #397
Date: 2020-08-03
From: eckart_schmidt@t-online.de
Subject: Inscribed QL-quartic

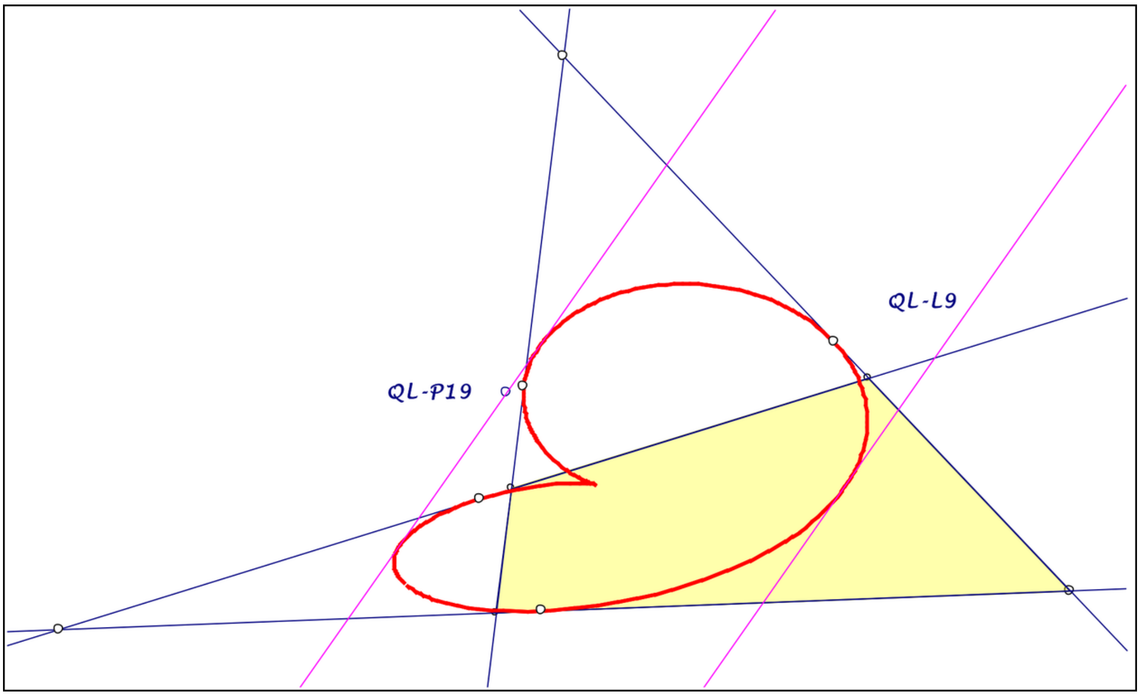
```
Dear Bernard, dear Chris,

this seems to be an interesting quartic, inscribed a QL:

Consider parallel lines and on every line the centroid
... of the 4 intersections with the QL-lines.
... For example: QL-P23 on QL-L1, QL-P19 on QL-L3,
    QL-L4^QL-L9 on QL-L4.
The locus for these centroids on parallel lines is a line again.
This gives a line to line transformation L --> L',
... parallel lines have the same image (QL-L1, QL-L4 --> QL-L9),
... if L and L' are parallel, L' is a QL-line Li.
For lines L of a pencil the image lines L' envelope a quartic,
... independent of the chosen pencil.
The intersections for images of orthogonal lines give a conic,
tangent to the quartic.

Properties of the quartic:
    * The quartic contacts the QL- lines in the centroids
      of the 3 QL-points of each QL-line.
    * The contact-QA has centroid QL-P12.
    * Further tangents: QL-L9 and a parallel to QL-L9
      through QL-P19 as double tangent.
The tangents L' to the quartic extend the reference QL to a 5L:
    * The locus of 5L-s-P1 is QL-L1.
    * The locus of 5L-s-P2 is a circle,
      bearing the X(186) of the QL-trilaterals.
    * The locus of 5L-s-P3 is a circle,
      bearing the X(265) of the QL-trilaterals.
    * The locus of 5L-n-P1 is the circle QL-Ci3.
What about the cusp of the quartic?

Best regards Eckart
```



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Message: #398
Date: 2020-08-03
From: eckart_schmidt@t-online.de
Subject: Re: Inscribed QL-quartic

```
Dear Bernard, dear Chris,
excuse for the last four points in #397,
... they hold in general,
    not special for tangents L' of the quartic.
Best regards Eckart
```

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Message: #399
Date: 2020-08-04
From: bernard.keizer@gmail.com
Subject: Re: Inscribed QL-quartic

```
Dear Eckart,
Beautiful and amazing construction, totally new for me !
But the result is no less, as you will see ...
Your quartic is a stretched cardioïd and is a quartic of class
3.
It's dual should be a cubic of class 4 through the vertices of
the dual QA.
Bingo ! In fact, your quartic is the dual of QA-Cu6, which
contains the vertices of the QA, the middles of DT sides and the
points QA-P1 and QA-P22, complement of QA-P1 wrt DT.
The double tangent to the quartic is the dual line of QA-P1,
which is the Newton Line of DDT (DT of the QL formed by DT and
the Newton Line).
The medians of DT are tangent to the quartic and duals of the
middles of DT sides.
The cusp of your quartic is the dual point of the unique tangent
to the cubic which crosses it.
I haven't identified the dual line of QA-P22 and the dual point
of QL-L9, perhaps you will be able to do it ...
I had really pleasure to discover this quartic and rediscover
the cubic QA-Cu6 and it's property.
Thanks a lot for these moments, rather rare and precious in
those troubled times ...
Best regards
Bernard
```

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Message: #400
Date: 2020-08-04
From: bernard.keizer@gmail.com
Subject: Re: Inscribed QL-quartic

```
Dear Eckart,
I found a mistake in my message : the duals of the middles of DT
sides, tangent to your quartic, are not the medians, but the
lines joining the DT vertices and the middles of the diagonal
segments (on the Newton Line).
Best regards
Bernard
```

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Message: #401
Date: 2020-08-04
From: eckart_schmidt@t-online.de
Subject: Re: Inscribed QL-quartic

```
Dear Bernard,

fantastic identification of the quartic, my compliment!
Duality is often a fascinating instrument.
The cusp will be the dual of the inflexion tangent of QA-Cu6.
The dual of QL-L9 is the 2nd  intersection of QA-Cu6 and
QA-P1.QA-P16.
But I cannot give an interpretation of the dual of QA-P22.

Best regards Eckart
```

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Message: #402
Date: 2020-08-04
From: eckart_schmidt@t-online.de
Subject: Re: New QL-line

```
Dear Bernard,

thanks for your messages 395 and 396,
... which explain my observations.

In addition wrt your last question in #395:
The 4 QL-L1 give a QL
... with diagonal triangle in the medial triangle of QL-Tr1,
... with Newton line through QL-P19 orthogonal to QL-L5,
... with Miquel point in the middle of QL-P17.QL-P25,
... with Morley point in the middle of QL-P2.QL-P9,
... with QL-P12 in the middle of QL-P8.QL-P12,
... with QL-P15 in the middle of QL-P8.QL-P15,
... with QL-P14 in QL-P15, QL-P18 in QL-P12 of the ref-QL,
... with QL-P23 in the reflection of QL-P7 in QL-P5
    of the ref-QL,
... with same QL-P8, QL-P11 as the ref-QL,
... with QL-P16 = QL-P17 ...

There can be more properties,
... but I don't see the reason for so many relations,
... perhaps the first property?
I'm sure, you will lighten the background.

Best regards Eckart
```

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Message: #403
Date: 2020-08-04
From: bernard.keizer@gmail.com
Subject: Re: Inscribed QL-quartic

Dear Eckart,
If you consider the point where the Newton Line of the QL interects the QL's parabola with focus QL-P1 and directory QL-L2, this point is also the intersection between QL's and DQL's Newton Lines. This point has as dual line the line QA-P1QA-P22 through QA-P8. (I remember we have already encountered this point on a rectangular hyperbola through the S-points).
DQL's Newton Line is tangent in this point to QL's parabola.
>From this point you have one double tangent to your quartic as dual of QA-P1 (this is precisely DQL's Newton Line) and a single tangent, which is the dual of QA-P22.
Best regards
Bernard

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Message: #404
Date: 2020-08-05
From: eckart_schmidt@t-online.de
Subject: Re: Inscribed QL-quartic

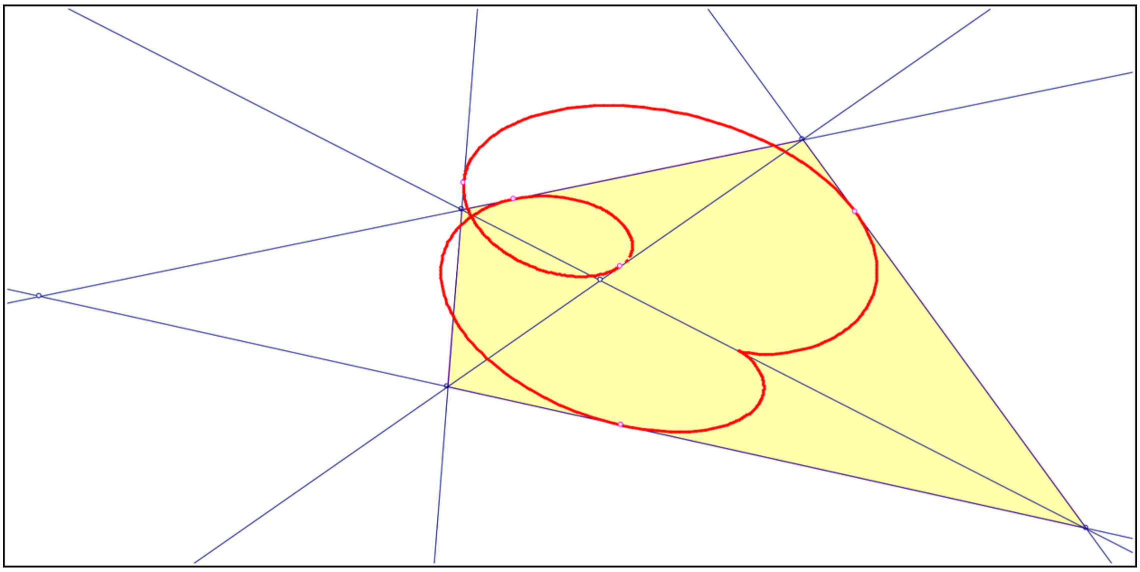
```
Dear Bernard,

there is a corresponding higher curve for a QA:
Consider parallel lines and on every line the centroid
... of the 6 intersections with the QA-lines.
The locus for these centroids on parallel lines is a line again.
This gives a line to line transformation L --> L',
... parallel lines have the same image,
... if L and L' are parallel, L' is a QA-line.
For lines L of a pencil the image lines L'
envelope a higher curve (not a quartic),
... independent of the chosen pencil.

I can only identify the contact points:
A QA-line bears two QA-vertices Pi, Pj and a QA-Tr1-vertex Q,
... let X divide QPi with ratio 2:1 and Y divide XPj
    with ratio 2:3,
... then Y is the contact point of the curve.

Can you find further properties of this curve?

Best regards Eckart
```



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Message: #405
Date: 2020-08-05
From: bernard.keizer@gmail.com
Subject: Re: Inscribed QL-quartic

```
Dear Eckart
Please, give me time !
I hardly follow the rythmus of your inventions ...
Tis seems more complicate and you don't give many clues this
time ! (What are the L' for some well-known QA-lines ?)
A first sight on your beautiful figure, the curve is of class 5,
with a cusp, a node and 2 double tangents (it looks like a
quartic, some kind of a smashed limçon).
The dual figure should be a quintic through the 6 QL vertices
with an inflexion tangent for the cusp, a double tangent for the
node and 2 nodes for the 2 double tangents.
I wonder how you manage to draw your own curve.
If you are able to generate the tangents, you might have their
dual points which describe the dual curve ?
Best regards
Bernard
```

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Message: #406
Date: 2020-08-05
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

Dear Eckart,
I haven't got enough time to check all your properties.
But I think I understand the beginning !
In your construction, the 6 vertices of the 2nd QL formed by the 4 Newton Lines of your 4 previous QL's are the middles of the segments joining the vertices of the originel QL to the corresponding DT vertices. It follows that this 2nd QL has the medial triangle of DT as own DT. It explains proposition 1 and 9.
Even better, it explains that the middles of the diagonal degments of this second QL are the middles of the diagonal segments of DQL and that the Newton Line of this 2nd QL is precisely the famous dual of QA-P1 or DQL's Newton Line (which passes through QL-P19 ! (THis is the double tangent to your 1rst quartic ...) proposition 2
Naturally, that explains also why P12 becomes the middle of P8P12 and P18 in P12 (propositions 5 and 7 final) and I suppose also proposition 6 and 7 beginning.
I continue with the Miquel point of this 2nd QL, on the medial of the medial of DT ...
The fight goes on ...
Best regards
Bernard

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Message: #407
Date: 2020-08-05
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

Dear Eckart
Reading again your message and my partial answer, your
proposition 9 in fact is not correct !
Both QL's have the same P8, as DT and it's medial have the same
centroïd, but P9 and P10 of the second are P11 and P9.
Best regards
Bernard

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Message: #408
Date: 2020-08-06
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

Dear Eckart,
I checked the rest of your properties and a few more !
newP11 is the middle of old P9P11
newP16 is the reflexion of oldP9 in new P1 (and not old P17 as
mentioned in your 10th proposition)
newP24 is on P8newP1 as 2nd intersection between new Dimidium
circle and new DT circumcircle or old midDT circumcircle
The best is coming :
new P17 is old P1 as 1rst intersection between the same 2
circles.
old and new Dimidium circles carries as wellknown old and new
S-points
Among many others, the old S-points are on conics through DT
vertices, P8, P13 and P24, through midDT vertices, P8 and QA-P1
of the dual QA, DDT vertices, the 4 vertices of dual QA and P13.
The beauty is that the 2nd conic passes also through new P24 and
is therefore the conic carrying the new S-points as 3 other
intersections between new Dimidium circle and new DT
circumcircle or old midDT circumcircle. The 2 groups of S-points
are therefore coconic on this conic playing a double role.
Fascinating geometry of the dual QA/QL
Best regards
Bernard

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Message: #409
Date: 2020-08-06
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

```
Dear Eckart,
I was intrigued myself by my resul about newP17 being oldP1.
In fact, it is even more beautiful than I thought !
Considering your 2nd QL, it appears that it's DT is old midDT of
the 1rst and that it's Newton Line is the same as that of old
DDT.
One step more, the DDT of this 2nd QL has a Newton Line parallel
to the Newton Line of the 1rst and that the circumcenter of this
DDT is on the Steiner Line of the 1rst QL.
This new DDT has the same focus and the same directrix as the
1rst QL !
Bingo, the inscribed parabola is the same, tangent, as well
known, to the 3 sides of midDT and to the Newton Line of DDT,
precisely in the point where the Newton Line of DQL meets the
Newton Line of QL.
Best regards
Bernard
PS It is then more easily to understand that old P2 and new P2
could be on the Steiner Line through old P9 ...
```

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Message: #410
Date: 2020-08-06
From: eckart_schmidt@t-online.de
Subject: Re: New QL line

```
Dear Bernard,

Thanks for your numerous, detailed remarks and explications!
The 2nd QL of the Newton lines for the four QL of QL-Tr1 plus Li
... seems a very interesting element in QL-geometry.

Your correction in #407 is correct, thanks.

The background for the double tangent in #406
can also be described
... as the line of the collinear intersections
... of corresponding Newton lines and QL-lines.

Wrt #408 10th property: I wanted to say newP16 = newP17,
... but that doesn't hold, newP16 = middle of P17.P10, excuse.
That the two S-point triple are coconic,
is really a fascinating property.

Best regards Eckart

PS. I wrote this message without studying your #409.
```

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Message: #411
Date: 2020-08-07
From: bernard.keizer@gmail.com
Subject: Re: Inscribed QL-quartic

```
Dear Eckart,
I reproduced carefully your construction of the lines L'.
Then, as mentionned, I drew the dual points of these lines,
which describe a curve through the 6 QL vertices with several
nodes.
I didn't find this time any particular property.
Best regards
Bernard
```

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Message: #412
Date: 2020-08-09
From: eckart_schmidt@t-online.de
Subject: QA-circumisocubics with QG-Px

Dear Bernard, dear Chris,

several QG-Px triple for a QA are perspective QA-Tr1 (see
QA-Tr-2):
... that are QG-P2,4,8,12,13,15,16,17,18,19.
Without QG-P4,8,12,13 they lead to the following cubics:

Consider for a QA = P1P2P3P4 and a triangle ABC of QG-Px
... with A = QG-Px of P1P2P3P4, B = QG-Px of P1P2P4P3,
    C = QG-Px of P1P4P2P3
... the 12 lines APi, BPi, CPi (i = 1,2,3,4)
    with 4 triple intersections
... Q1 = AP3^BP4^CP2, Q2 = AP4^BP3^CP1, Q3 = AP1^BP2^CP4,
    Q4 = AP2^BP1^CP3,
... which give a 2nd QA and a common point P for the lines PiQi.
That defines a pivotal isocubic with
... reference triangle ABC,
... an isoconjugation, which swaps Pi and Qi
... and pivot in the common point P of the lines PiQi,
... which bears the QA-vertices and the vertices of QA-Tr1
... and can also be considered as a pivotal isocubic
    wrt QA-Tr1 and QA-Tf2.

For QG-P2 we get QA-Cu3, for QG-P15 we get QA-Cu2, for QG-P16 we
get QA-Cu1,
... for QG-P17 we get a cubic with pivot QA-P12,
... for the reflection of QG-P1 in QG-P12 we get QA-Cu4,
    for QG-L1^QG-L3 we get QA-Cu5.
For QG-P18 and QG-P19 we get the same cubic,
... already described in QFG message 2017, example 2,
... bearing more than 25 known points:
Reference triangle QA-Tr1,
... isoconjugation QA-Tf2,
... pivot perspector of the QG-P18 and QG-P19 triangle.
Eight of the vertices of the three triangles for QG-P18, QG-P19
and QA-Tr1
... have the remaining vertex as Cayley-Bacharach point.

Best regards Eckart

PS: For ABC = Miquel triangle we get QA-Cu1,
... but there will be no more QL-Px for these cubics.

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Message: #413
Date: 2020-08-12
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

```
Dear Bernard, dear Chris,

in EQF QA-Cu7 is described as pivotal isocubic:
... reference triangle: QA-P2.QA-P4.QA-P41,
... isoconjugation: isogonal conjugate,
... pivot: infinity point of QA-L2.

QA-Cu7 can also be described  as nonpivotal isocubic
... wrt QA-Tr1 and the QG-P18- and QG-P19-triangle of a QA.
These three triangles are pairwise line perspective
wrt a line L.

QA-Cu7 is a nonpivotal isocubic wrt
... reference triangle QA-Tr1,
... isoconjugation QA-Tf2,
... root tripol of line L wrt QA-Tr1.

QA-Cu7 is a nonpivotal isocubic wrt
... reference triangle QG-P18-triangle,
... isoconjugation swapping QA-P2 and QA-P4,
... root tripol of line L wrt QG-P18-triangle.

QA-Cu7 is a nonpivotal isocubic wrt
... reference triangle QG-P19-triangle,
... isoconjugation swapping QA-P2 and QA-P41,
... root tripol of line L wrt QG-P19-triangle.

Best regards Eckart

PS: Perhaps some properties are already mentioned
... in our discussion of the QA-Cu7-triple points.
```

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Message: #414
Date: 2020-08-13
From: eckart_schmidt@t-online.de
Subject: Re: New QL-line

```
Dear Bernard,

please excuse, that I didn't react to your #409,
... but I tried in vain to find further properties
    of the 2nd QL.
I reproduced your 2nd surprising result,
... the common parabola of the first QL and the DDT
    of the 2nd QL,
... but I am not so familiar with DDT,
    so I cannot judge consequences.

Best regards Eckart
```

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Message: #415
Date: 2020-08-14
From: bernard.keizer@gmail.com
Subject: Re: New QL-line

```
Dear Eckart,
You don't need to be familiar with DDT ! (DQL and DDT were
already mentionned by J.W. Clawson, of course not with these
names, which I invented ...)
Just read the last remark on QL-P17 in EQF !
Best regards
Bernard
```

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Message: #416
Date: 2020-08-14
From: eckart_schmidt@t-online.de
Subject: Fixed points of QA-Tf16

```
Dear Bernard, dear Chris,

in #127 the fixed points of QA-Tf16 are mentioned as
... vertices of a newQA on oldQA-Cu1 with tangential oldQA-P3
... and diagonal triangle of oldQG-P16 points.
Here some further properties:

NewQA and oldQA have the same QA-Tr2.

NewQA is four times perspective to oldQA
... with perspectors in the in/excenters of oldQA-Tr2.

NewQA-Tr1 and oldQA-Tr1 are perspective
... wrt the infinity point of the asymptote of QA-Cu1.
Not corresponding vertices of newQA-Tr1 and oldQA-Tr1
... are collinear with a Miquel point.

The Cayley-Bacharach point of oldQA and newQA
... is Q the intersection of QA-Cu1 and its asymptote.

The Cayley-Bacharach point of oldQA-Tr1, newQA-Tr1 and the
circular points
... is the tangential of Q wrt QA-Cu1.

Best regards Eckart

PS: Perhaps the first properties are already mentioned.
```

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Message: #417
Date: 2020-08-15
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of QA-Tf16

```
Dear Eckart,
Yes, you have already mentionned almost all these properties
several times before !
The 3 QA's (old and new and in-and excenters) are in a Reye
configuration on QA-Cu1.
The 3 tangentials are aligned and form themselves the beginning
of a Reye configuration with the 3 other points having the same
tangential (DT vertices of both QA's and vertices of the Miquel
triangle) ...
For the last property of CB and cb points, is it a generalisable
property ?
The CB point of the vertices of 2 QA's is the tangential of the
3rd QA in the Reye configuration ?
The cb point of the DT vertices of these 2 QA's is the
tangential of this tangential ?
Best regards
Bernard
```

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Message: #418
Date: 2020-08-16
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of QA-Tf16

```
Dear Eckart,
At last, the 1rst CB property is easy to understand.
The 2 QA's being in a Reye configuration with the in- and
excenters of the Miquel triangle, the 3 tangentials are aligned
and the 3 tangentials of the tangentials are also aligned.
The CB of the 8 vertices of the 2 QA's is the 3rd intersection
with QA-Cu1 of the line joining the foci (in the sense of
Cotterill) of the 2 QA's.
The foci of the 2 QA's are their QA-P41 or the tangentials of
their tangentials.
The CB point is therefore the tangential of the tangential of
the QA of the in- and excenters, id est the tangential of the
infinity point of the asymptote, which is the point Q.
Best regards
Bernard
PS For the cb point of the 6 vertices of the 2 DT's, you have to
consider the focus of the QA formed by the 3 vertices of one DT
and one of the vertices of the other anf the focus of the QA
formed by the 2 other vertices of the 2nd DT and the circular
points (circles through these 2 points). I haven't checked this
construction yet.
```

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Message: #419
Date: 2020-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of QA-Tf16

```
Dear Bernard,

I think, you have to correct your question
"The CB point of the vertices of 2 QA's
... is the tangential of the tangential of the 3rd QA
    in the Reye configuration ?"
Now I can confirm this property with CABRI
... and additional: CB for the 2 QA is QA-P41 of the 3rd QA,
... or: CB of the 2 QA is the tangential
    of the 3rd diagonal triangle.

Best regards Eckart

PS: This should be an answer to your message 417,
... but I just found your message 418
... with the same results and explicit argumentation, thanks.
```

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Message: #420
Date: 2020-08-17
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of QA-Tf16

```
Dear Eckart,
Thanks for the confirmation !
What about the cb of the 2 diagonal triangles ?
Is it always the tangential of the tangential of the 3rd
diagonal triangle ?
This time, I don't see an obvious exlanation ...
Best regards
Bernard
PS At last, is it correct that the cb of 3 points and their
isogonals wrt the Miquel triangle is always tgQ on QA-Cu1 ?
```

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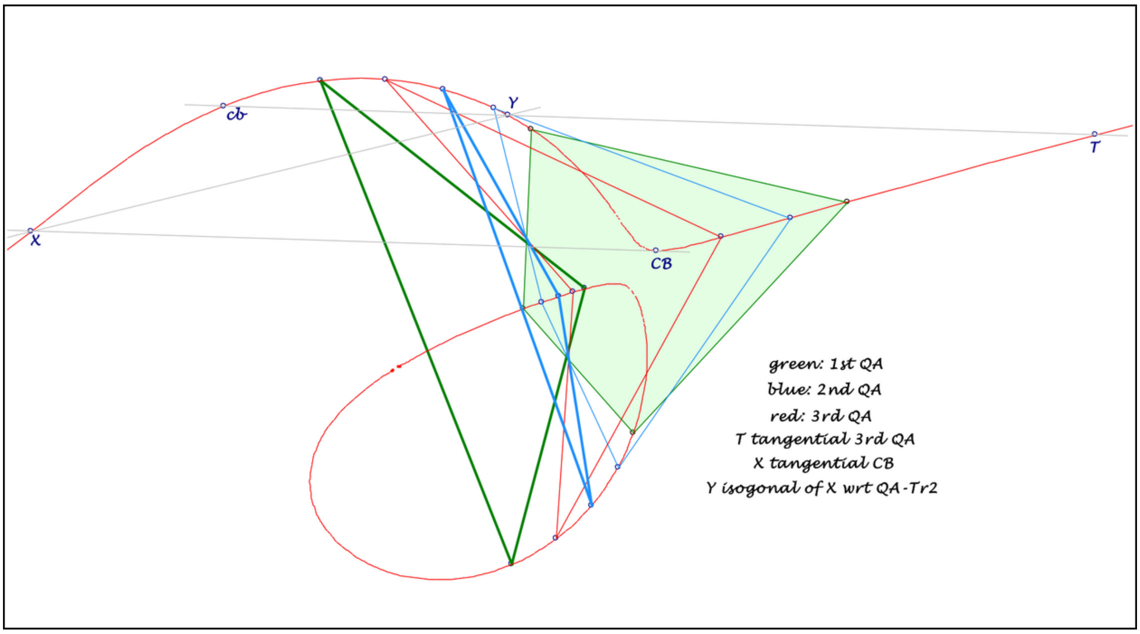
Message: #421
Date: 2020-08-17
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of QA-Tf16

```
Dear Bernard,

wrt the cb of the 2 diagonal triangles:
... it isn't always the tangential of the tangential
    of the 3rd diagonal triangle,
... but I think, the following holds in general:

Let T be the tangential of the 3rd QA
... with CB as tangential of T
... and X tangential of CB
... and Y isogonal conjugate of X wrt QA-Tr2,
... then cb is the 3rd intersection of YT and QA-Cu1.

Best regards Eckart
```



2020-08-17.pdf

Message: #422
Date: 2020-08-17
From: eckart_schmidt@t-online.de
Subject: QL-Triple Triangles

```
Dear Chris,

in EQF there is no item "QL-Triple Triangles"
(see also old#3139)
... with vertices QG-Px for the QG-versions of a QL,
... most of the properties are to be found singly in EQF,
... here a summary under two aspects:

Collinear degenerated QL-triple triangles:
... for QG-P2 on QL-L1,
... for QG-P10 on QL-L6,
... for QG-P12 on QL-L1,
... for QG-P15 on QL-L9,
... for QG-P16 on a parallel to QL-L2 through QL-P26,
... for QG-P18 on CSC(QL-Ci2) .

QL-triple triangles perspective QL-Tr1:
... for QG-P3 perspector QL-P8,
... ... perspectrix line at infinity,
... for QG-P5 perspector QL-P16,
... ... perspectrix QL-L1,
... for QG-P6 perspector infinity point of QL-L2,
... ... perspectrix QL-L6,
... for QG-P9 perspector  QL-P16,
... ... perspectrix parallel QL-P17.23
... ... ... through intersection of QL-L1 and bisector
            of QL-P17.23,
... for QG-P10 perspector infinity point of QL-L2,
... ... perspectrix QL-L6,
... for QG-P12 perspector QL-P13,
... ... perspectrix QL-L1,
... for QG-P13 perspector  QL-P13,
... ... perspectrix QL-L1,
... for QG-P17 perspector QL-P10,
... ... perspectrix radical axis of QL-Ci1 and polarcircle
        of Ql-Tr1,
... for QG-P19 perspector ..., perspectrix ...,
    see also #3176 (4).

Best regards Eckart
```

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Message: #423
Date: 2020-08-17
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of QA-Tf16

```
Dear Bernard,

wrt your last question in #420, you are right:
"...that the cb of 3 points and their isogonals wrt the Miquel
triangle is always tgQ on QA-Cu1."
There is an analogon for QA-Tf2:
The cb of 3 points on QA-Cu1 and their QA-Tf2 images
... is the 3rd intersection of QA-Cu1 and a line, connecting Q
... and the 4th intersection of QA-Cu1 and the circumcircle
    of QA-Tr1.

Best regards Eckart
```

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Message: #424
Date: 2020-08-17
From: van10hoven@gmail.com
Subject: Re: QL-Triple Triangles

```
Dear Eckart,

Thanks for exploring QL-Triple Triangles wrt the QG-points.
I already hoped for a long time someone would do this exercise.
I will include them in EQF.
It would be interesting to find similar results for the
QA-points.

Best regards,
Chris
```

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Message: #425
Date: 2020-08-18
From: eckart_schmidt@t-online.de
Subject: Cayley Bacharach points on QA-Cu1

```
Dear Bernard,

if we consider any triangle on QA-Cu1
... and its QA-Tr2-isogonal or QA-Tf2 image,
... the cb for the two triangles is a fixed point on QA-Cu1
    (see #423).

The same holds for quadrangles wrt CB.

Wrt the QA-Tr2-isogonal conjugate the fixed point is
... the 6th intersection of QA-Cu1 and the conic
... through the in-/ex-centers of QA-Tr2 and Q.

Wrt QA-Tf2 the fixed point is
... the 6th intersection of QA-Cu1
... and the QA-circumconic through QA-P41.

Best regards Eckart
```

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Message: #426
Date: 2020-08-18
From: eckart_schmidt@t-online.de
Subject: QA-Tr-2

```
Dear Chris,

in message old2017 (example 2) I found a notice, that you found
the perspectors
... of the pairwise perspective QA-triple triangles of QG-P1,
    QG-P18, QG-P19,
... but these results are not mentioned in QA-Tr-2.

Best regards Eckart
```

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Message: #427
Date: 2020-08-18
From: bernard.keizer@gmail.com
Subject: Re: Cayley Bacharach points on QA-Cu1

Dear Eckart,
Q is QA-P41 for the QA of the in-/excenters of QA-Tr2
I suppose this CB property holds also for QA-Tf16 (conic through the fixed points and their QA-P41, which is tgQA-P3).
I suppose that the property holds in fact for any transformation (conic through the fixed points and their QA-P41) ?
Best regards
Bernard

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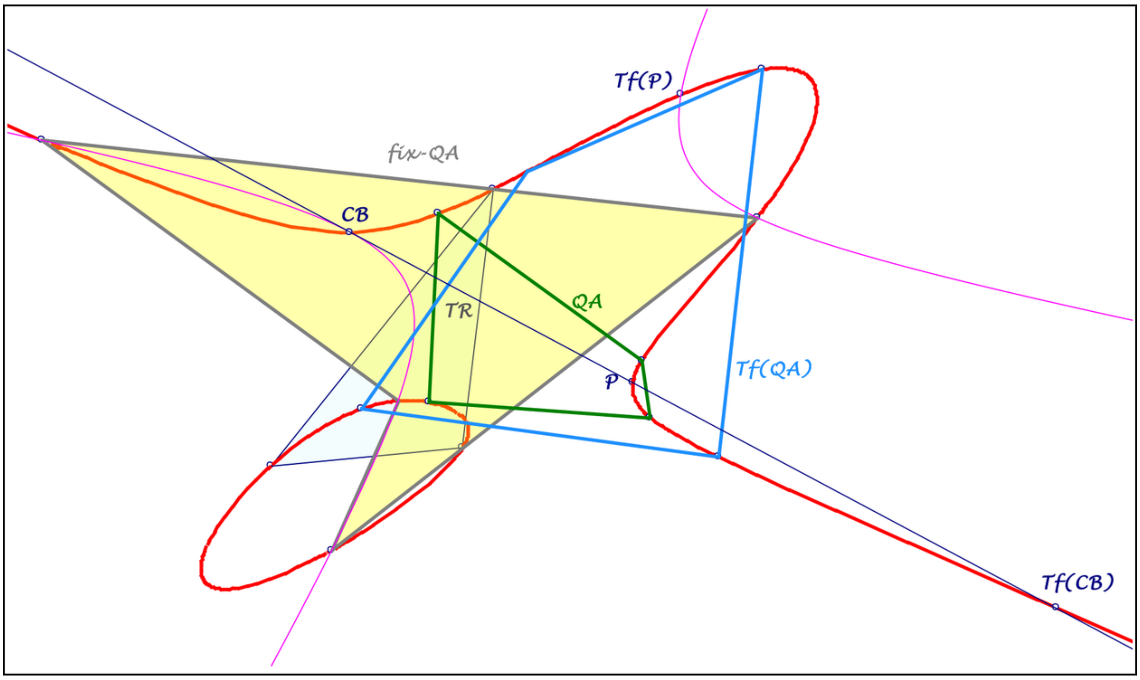
Message: #428
Date: 2020-08-19
From: eckart_schmidt@t-online.de
Subject: Re: Cayley Bacharach points on QA-Cu1

```
Dear Bernard,

wrt your question in #427:
Let us consider a pivotal isocubic Cu,
... reference triangle TR, pivot P and isoconjugation Tf
... with four fixed points, which give the quadrangle fixQA.
Any quadrangle QA and its Tf-image on Cu,
... give a Cayley Bacharach point CB on Cu,
... which is the 6th intersection of Cu and a circumconic
    of fixQA through Tf(P).
P, CB, Tf(CB) are collinear on a tangent at the circumconic
in CB.

Best regards Eckart
```



2020-08-19.pdf

Message: #429
Date: 2020-08-20
From: bernard.keizer@gmail.com
Subject: Re: Cayley Bacharach points on QA-Cu1

```
Dear Eckart,
Many thanks for this clear formulation and for the beautiful
figure as illustration !
The 2 QA's (QA and it's Tf image) are in a Reye configuration
with the diagonal QA (pivot P and vertices of the DT of fixQA).
CB is therefore tgtgP (with tgP = TF(P)).
I don't know why tgP and tgtgP are coconic with the vertices of
fixQA, you kow better the QA and you will probably find an
explanation ...
It was a time where you would have made barycentric calculations
in order to prove the property and Chris would have put the
result in EQF !
Regrets, regrets ...
Best regards
Bernard
```

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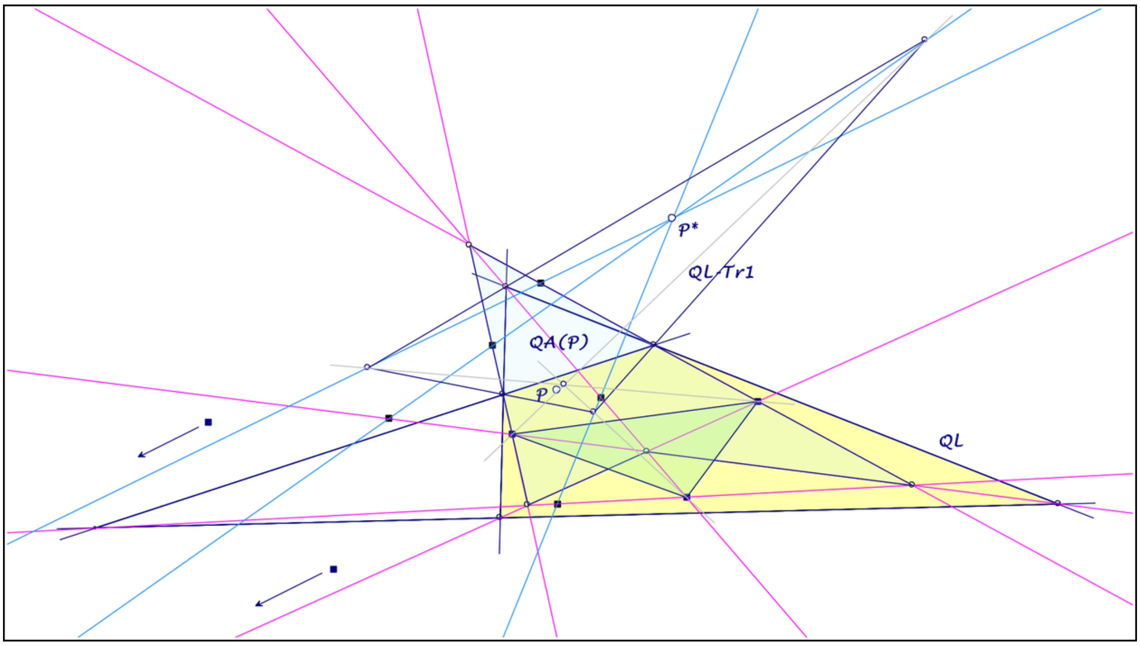
Message: #430
Date: 2020-08-21
From: eckart_schmidt@t-online.de
Subject: New QL-aspect

```
Dear Bernard, dear Chris,

I think, this is really a new aspect:
Consider a QL, its three QA-versions, each with three
QG-versions.
So we get nine QG and for a point P nine QG-Tf2 images,
... which give an interesting point constellation:

Nine points in pairs on six lines through a QL-point,
... with four triple intersections, defining a QA(P),
... with three double intersections in three of the nine points
... which give the diagonal triangle of QA(P)
... and are QG-Tf2(P) wrt the QG-versions of QL.
The remaining six of the nine points
... are in pairs collinear with a QL-Tr1-vertex,
... the corresponding lines have a common point,
... which is the isogonal conjugate of P wrt QL-Tr1.

If the point P is on QL-Cu1,
... QA(P) degenerates in the point CSC(P).
If P is a point on QL-L1,
... the loci for the vertices of QA(P) are conics
... circumscribed the QL-triangles through QL-2P2.
If P is a QL-Tr1-vertex,
... the diagonal triangle of QA(P) has two QG-P18
    and a QG-P19 as vertices.
If P is a point on QL-L1,
... the loci of the vertices of the diagonal triangle of QA(P)
... ... are conics isogonal conjugates of the QL-Tr1-lines
... ... wrt the triangle of QL-P1 and QL-2P2.

Last, not least:
The diagonal triangles of QL and QA(P) are perspective ...

Best regards Eckart
```



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Message: #431
Date: 2020-08-23
From: eckart_schmidt@t-online.de
Subject: Point/Line Transformation for QL

```
Dear Bernard, dear Chris,

this transformation can be defined alternative:
 1. For a QL the three QA-Tf2-images of a point are collinear.
... For QL-P1 we get a line, which is CSC(QL-Ci6).
... For the points of QL-L1 we get an inscribed quartic
    through QL-P18.
 2. For a QL the three circumconics of the QG-versions
    through a fixed point

... have three collinear double intersections.
Both properties lead to the same point/line transformation for a
QL.

This transformation gives for the points at infinity
... the interesting QL-inscribed quartic in #397.

Best regards Eckart
```

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Message: #432
Date: 2020-08-24
From: bernard.keizer@gmail.com
Subject: Re: New QL-aspect

```
Dear Eckart,
I'm not generally fan of QG constellation,
I prefer the dual QA/QL.
But this time, I must admit that you have found an interesting
item with the 9 QG-Tf2 of a point !
Congratulations. What about the same construction starting with
a QA and it's 3 QL's giving also 9 QG's ?
Best regards
Bernard
```

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Message: #433
Date: 2020-08-24
From: bernard.keizer@gmail.com
Subject: Re: Point/Line Transformation for QL

```
Dear Eckart,
I suppose it is in 2. the 3 circumconics of the QA-versions ...
Very beautiful that you find this way again your quartic in 397!
Best regards
Bernard
```

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Message: #434
Date: 2020-08-27
From: eckart_schmidt@t-online.de
Subject: QA-Cu7

Dear Bernard,

this is a detailed research wrt property 7 under QA-Cu7 in EQF:
Let us consider a QA = P1P2P3P4,
... each line PiPj intersects QA-Cu7 in a diagonal point
... and two points, dividing PiPj harmonical, Qij inside and and Rij outside.

For each QG-version of the QA the product of the 4 ratios for Qij or Rij is 1.
The three lines QijQkl have a common point Q.

If QA convex:
The lines RijRkl are orthogonal QijQkl, intersecting on QA-Cu7 in a QG-P18 point.
The lines QijRkl and RijQkl intersect on QA-Cu7 in a QG-P19 point.

If QA not convex:
The lines QijRkl are orthogonal RijQkl, intersecting on QA-Cu7 in a QG-P18 point.
The lines QijQkl and RijRkl intersect on QA-Cu7 in a QG-P19 point.

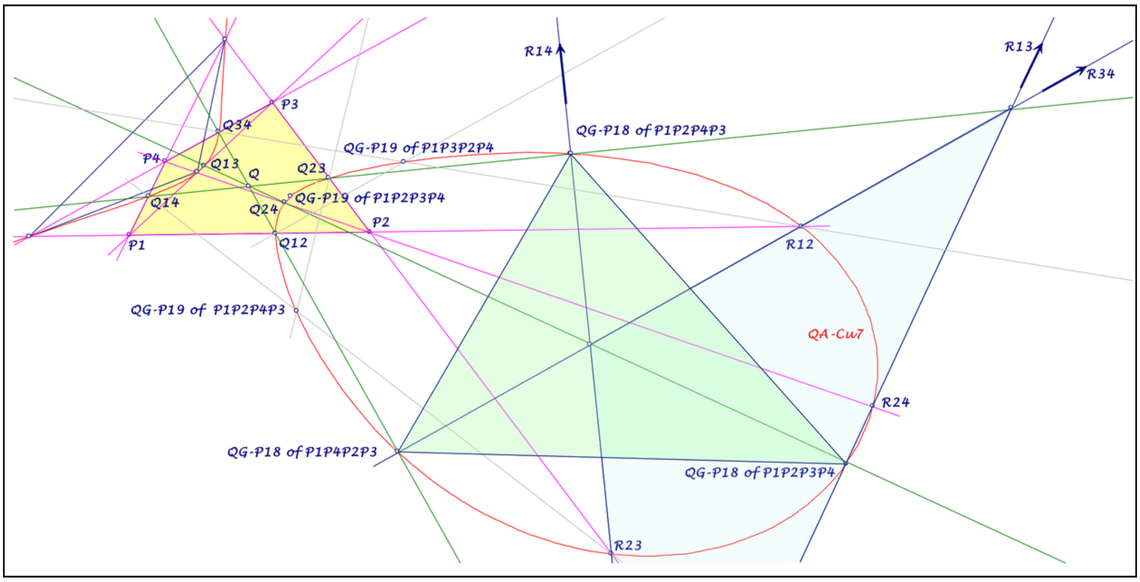
Perhaps new:
QA-Cu7 is the locus of points
... whose three QG-Tf2-images degenerate in a point.

This gives the possibility, to construct Qij and Rij:
... For points P on PiPj two QG-Tf2 images lie on PkPl,
... the 3rd on a conic through Pi and Pj,
... which intersects PkPl in the points Qkl and Rkl.

Wrt the new QA-point Q (4th harmonical point of QG-P19 on QijQkl):
For a convex QA Q is the excenter of the QG-P18-triangle
... ... on the angle bisector through QG-P18
        of the convex QG of QA.
For a non convex QA Q is the perspector
... of QA and the in-/ex-center quadrangle
    of the QG-P18 triangle.

Best regards Eckart

```
PS: I hope someone can describe this point constellation
    in a simpler way,
    ... especially without differentiate between convex
        and non convex QA.
```



2020-08-27.pdf

Message: #435
Date: 2020-08-29
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Chris,

perhaps worth to be mentioned in EQF:

Tangential of a QL-point wrt QL-Cu1
... is QG-P18 of a QG version of QL.

Best regards Eckart
```

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Message: #436
Date: 2020-08-29
From: van10hoven@gmail.com
Subject: Re: QL-Cu1

```
Dear Eckart,

I do not quite understand this.
Is what you are saying this:
Taking the tangent at some point P on QL-Cu1, this tangent will
recut QL-Cu1 in one of the three QL-versions of QG-P18?
If so I cannot confirm this in Cabri-drawing.

Best regards,
Chris
```

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Message: #437
Date: 2020-08-29
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

Dear Chris,

with QL-point I mean one of the 6 intersections of the 4 QL-lines.

Best regards Eckart

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Message: #438
Date: 2020-08-29
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1

Dear Chris, dear Eckart
Each diagonal of the QL intersects QL-Cu1 in a 3rd point QG-P17, vertice of the orthic triangle of DT.
The common tangential of 2 conjugate QL vertices is QG-P18, which is CSC(QG-P17).
This property belongs to the oldest known properties of VR !
What's new, Pussycat ?
Best regards
Bernard

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Message: #439
Date: 2020-08-30
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1

Dear Bernard,

I haven't said, that the property is new, but it was not in EQF.

Best regards Eckart

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Message: #440
Date: 2020-09-01
From: eckart_schmidt@t-online.de
Subject: Re: New QL-aspect

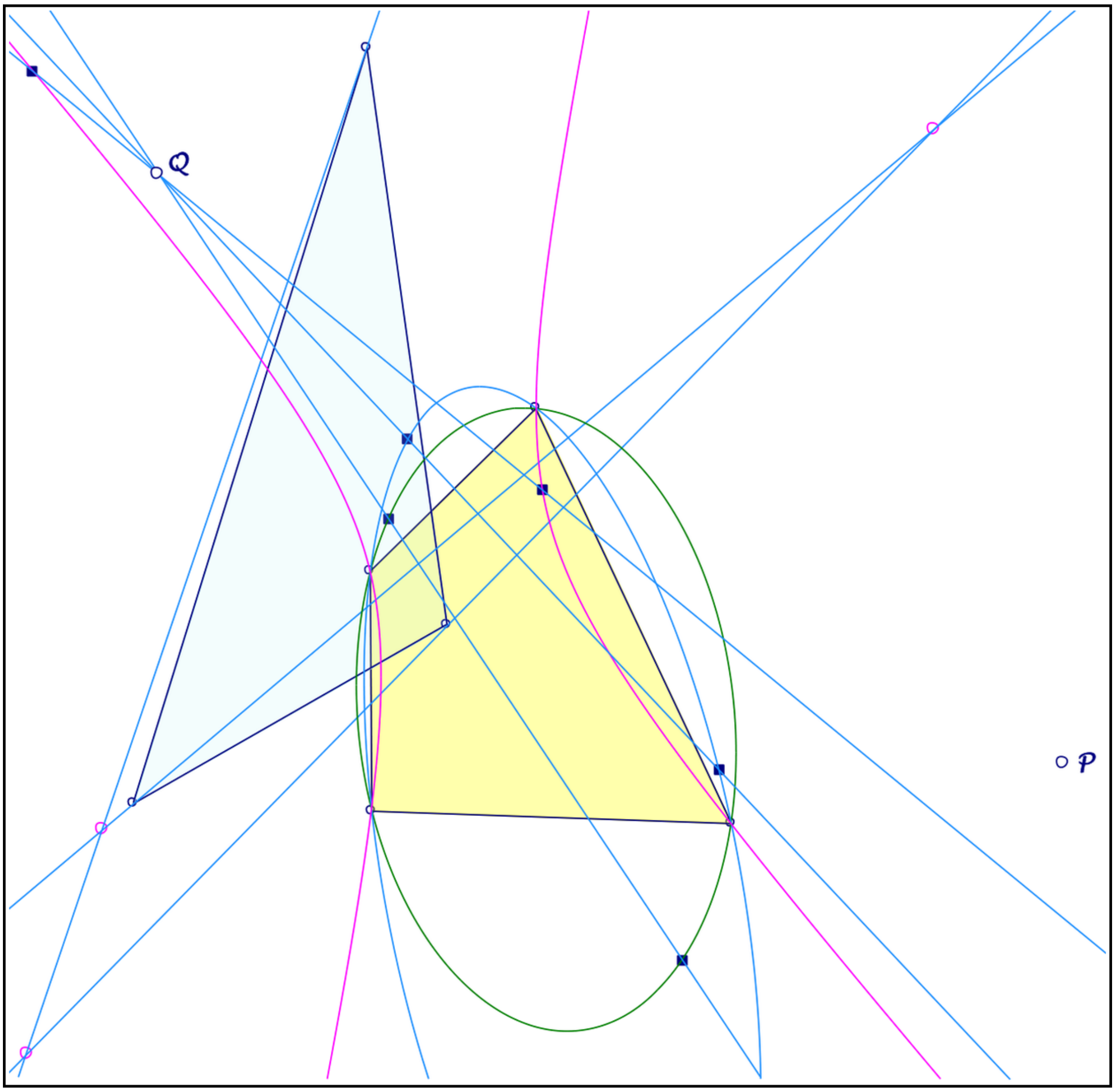
```
Dear Bernard,

here another observation:

Consider a QL = L1L2L3L4 with vertices Pij = Li^Lj and points Pi
on Li
... with QG-Tf2-images Qij, Qik, Qil of Pi,
    which lie on Lj, Lk, Ll.
The lines QijPkl for a fixed i intersect in Ri
... and the locus for Ri, varying Pi on Li is a conic COi,
... circumscribed Pjk, Pkl, Pjl, tangent to QL-Cu1,
... the four conics COi contact in the vertices of QL,
... detailed described in QFG-message old1415.

Wrt your question in message 432 (see attached figure):
QG-Tf2 gives for the 9 QG of a QA 9 images for a point P:
... 3 points in pairs collinear with a vertex of QA-Tr1,
... 6 points in pairs on 3 QA-circumconics
... with polars of P through QA-Tf2(P)
... and a common point  Q for the 3 lines of the pairs
    of the 6 points.
The QA-transformation P --> Q is a CSC
... centered in QA-P4, swapping QA-P2 and QA-P41,
... mapping QA-Cu7 to itself.
For P on a QA-line PiPj,
... two of the 3 points lie on PkPl,
... the 3rd on a conic through Pi, Pj,
    two QA-Tr1-vertices and one of the 6 points,
... this conic is QA-Tf2-invariant.

Best regards Eckart
```



2020-09-01.pdf

Message: #441
Date: 2020-09-01
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu7

```
Dear Eckart,
Your point Q is a new QA-point.
2 of the 3 lines RijRkl intersect on the 3rd QijQkl and give
perhaps 3 other interesting points ?
There are 6 points Qij and 6 points Rij, the 12 points on
QA-Cu7.
QA-Cu7 is QA-Cu1 of the 6 QAs QijQklRijRkl and is a QL-Cu1 as
each time 2 sides of the corresponding QA are orthogonal.
4 of the 12 points belong to the double points of the 3 QA-Cu7
of the same QL.
I didn't find a better description of Q.
Best regards
Bernard
```

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Message: #442
Date: 2020-09-02
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu7

```
Dear Eckart,
Wrt my 1rst remark, the 3 other interesting points are on your
figure obviously with Q the in- and excenters of the QG-P18
triangle.
Best regards
Bernard
```

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Message: #443
Date: 2020-09-03
From: bernard.keizer@gmail.com
Subject: Re: New QL-aspect

```
Dear Eckart,
Beautiful !
It's always a pleasure to ask you an innocent question and to
receive a complete answer and an illustrating figure ...
These 9 QG-Tf2 of a point wrt a QL or a QA are really
interesting.
Best regards
Bernard
```

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Message: #444
Date: 2020-09-03
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu7

```
Dear Bernard,

thanks for your additional remarks in #441 and 442.
If I am not wrong, we already discussed this partly
... in QFG-messages old3188 and old3189,
... but I haven't noticed this, excuse.

Best regards Eckart
```

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Message: #445
Date: 2020-09-03
From: eckart_schmidt@t-online.de
Subject: New QA-points and conics

```
Dear Bernard, dear Chris,

let us consider a quadrangle QA and its three QG-versions
... and triangles TR of QG-points as the QG-P18-TR.
The three pivotal isocubics
... with reference triangle QA-Tr1, isoconjugation QA-Tf2
... and pivot one of the TR-vertices
... often have six double intersections,
... which are the QA-Tf2-partner on the TR-sidelines,
... not always real, but on a conic CO,
... for example for the QG-P19-TR on an orthogonal hyperbola.
The polars of the TR-vertices wrt CO
... give a triangle, perspective TR,
... and the perspector is a new QA-point:
QG-P16 leads to a point on QA-Cu1,
... which is QA-Tf16 of the QA-Tr2-isogonal conjugate
    of QA-Tf16 of QA-P41.
QL-P1 leads also to a point on QA-Cu1,
... which is QA-Tf16 of the tangential of Q
    (Q intersection of QA-Cu1 and its asymptote).

There are also examples for QG-P15, QG-P18, ...

Best regards Eckart
```

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Message: #446
Date: 2020-09-07
From: eckart_schmidt@t-online.de
Subject: Pedal Cevian Quadrigons

```
Dear Bernard, dear Chris,

this theme is already described in EQF message old155,
... but incomplete and I cannot remember a discussion,
... so I try to offer it once more in a revised version,
... for I think it should be mentioned in EQF.

Best regards Eckart
```

EQF-Note 2020-09-07

Background for these notes is
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Pedal Cevian Quadrigons

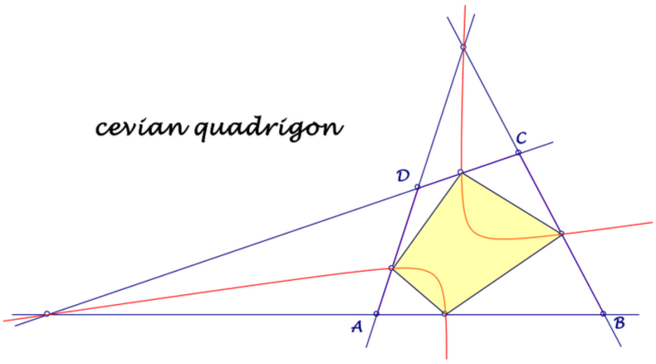
For a triangle the locus of points, whose pedal triangles are cevian triangles is the Darboux cubic. Defining cevian quadrigons in a corresponding way there is a pivotal isogonal circular cubic for quadrigons. This is a revision of QFG-message old155 or http://eckartschmidt.de/Fucev.pdf.

For a quadrigon ABCD with sides a = AB, b = BC, c = CD, d = DA cevian quadrigons shall be defined as follows:

Definition: A cevian quadrigon is an inscribed quadrigon, dividing the sides in ratios with product 1.

For example inscribed conics of a quadrigon give cevian quadrigons.

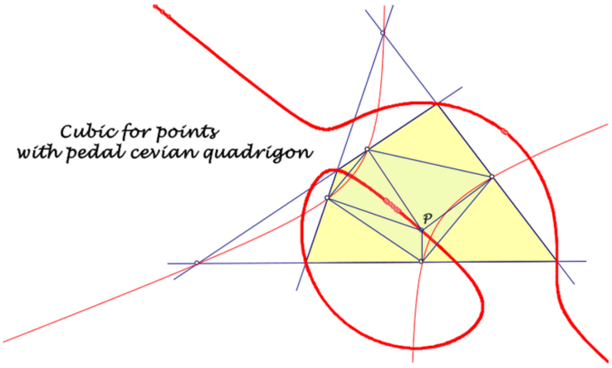
Property: The vertices of a cevian quadrigon and the intersections of opposite sides of the reference quadrigon lie on a conic.



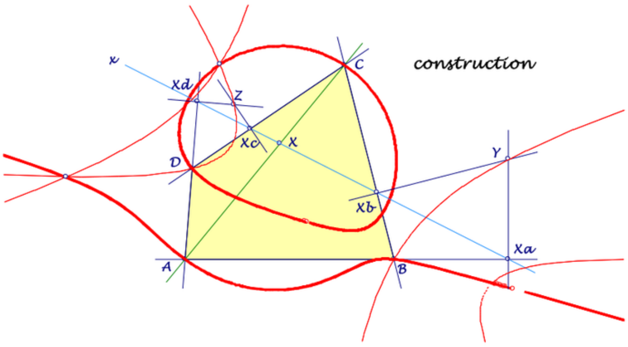
Theorem: The locus of points, whose pedal quadrigon is a cevian quadrigon, can be described as pivotal isogonal circular cubic, containing the Miquel point with an asymptote parallel to the bisector of the intersections of opposite sides.

Let us start with a construction of the cubic, independent of its properties pivotal, isogonal, circular.

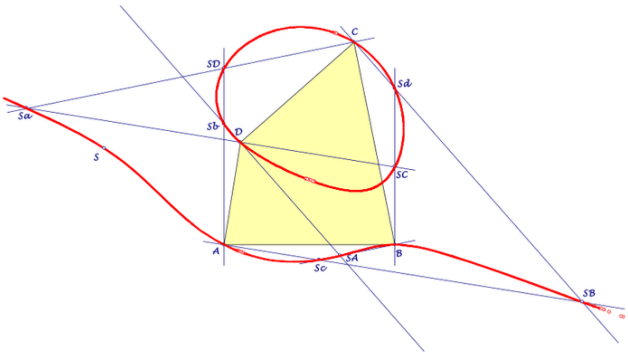
pedal cevian QG.pdf



Construction: Consider points X on a diagonal and lines x through X, which cut the sidelines in Xa, Xb, Xc, Xd. Let Y be the intersection of perpendiculars in Xa and Xb to the corresponding sidelines, let Z be the intersection of perpendiculars in Xc and Xd to the corresponding sidelines. The loci of Y and Z changing the lines x through X are conics, the intersections of these conics are points of the cubic.



Special points: If the pedal points are vertices of the quadrigon, we get special points of the cubic. Let a_B be a perpendicular to a in B, …
Intersections of perpendiculars in close-by points wrt opposite sides:
$a_A \wedge c_D = S_b$, $b_B \wedge d_A = S_c$, $c_C \wedge a_B = S_d$, $d_D \wedge b_C = S_a$.
(Intersections of perpendiculars wrt close-by sides in opposite points:
$a_A \wedge b_C = S_D$, $b_B \wedge c_D = S_A$, $c_C \wedge d_A = S_B$, $d_D \wedge a_B = S_C$.)

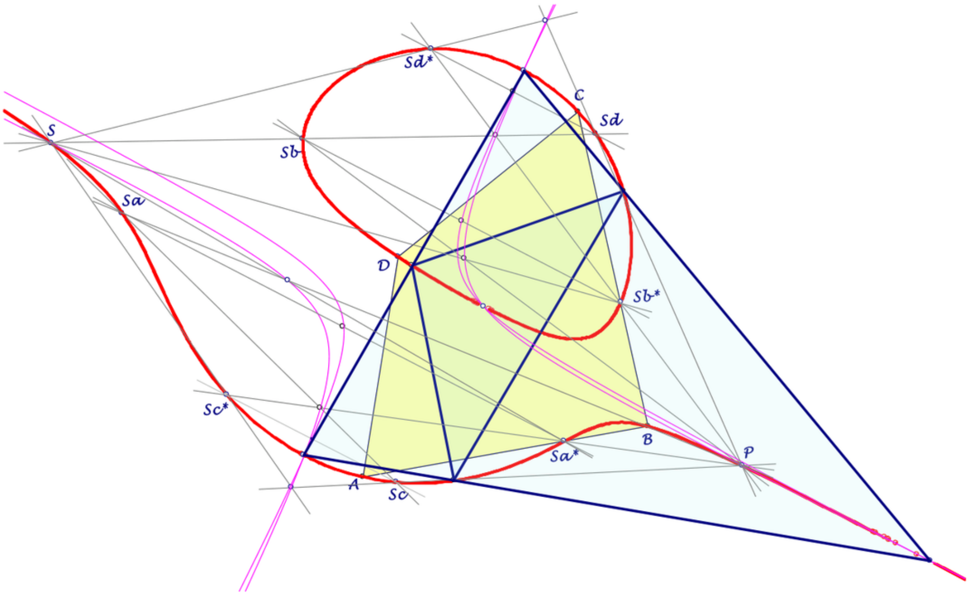


pedal cevian QG.pdf

To describe the cubic as pivotal isogonal cubic, take for pivot the infinity point of the asymptote, which is parallel to the bisector L of the intersections of opposite sides of the quadrigon. Isogonal points X, X* on the cubic lie therefore on a parallel to L.

Isogonal points:
Sa* = intersection of a parallel to L through Sa and the quadrigon side a, analog for Sb*, Sc*, Sd*. The midpoints of Sa.Sa*, Sb.Sb*, Sc.Sc*, Sd.Sd* define an orthogonal hyperbola Hy_1 as polar conic of the infinity point of the cubic.

Reference triangle: Sa.Sc and Sb.Sd intersect in the Miquel point S, Sa*.Sc* and Sb*.Sd* intersect in the conic pole P of the quadrigon. The intersections S.Sa ^ P.Sa*, S.Sa* ^ P.Sa analog for Sb, Sc, Sd define a second orthogonal hyperbola Hy_2.
The intersections of Hy_1 and Hy_2 give the in-/ex-centers of the reference triangle, which is the orthic triangle of the excenters.



Eckart Schmidt
eckart_schmidt@t-online.de
http://eckartschmidt.de

pedal cevian QG.pdf

Message: #447
Date: 2020-09-08
From: eckart_schmidt@t-online.de
Subject: Re: Pedal Cevian Quadrigons

```
Dear Bernard, dear Chris,

please excuse, there is already a message wrt this theme,
... five years ago #1242,
    I forgot this version, painful impression ...

Best regards Eckart
```

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Message: #448
Date: 2020-09-09
From: eckart_schmidt@t-online.de
Subject: QA-P23 property

```
Dear Chris,

the vertices of QA-Tr1 and a vertex Pi of the QA give a new QA,
... whose QA-Co2 has in Pi a tangent through QA.-P23.

By the way:
The centers of the four QA-Co2 are concyclic
... on a circle through QA-P29,36, centered in QA-P13.

Best regards Eckart
```

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Message: #449
Date: 2020-09-10
From: eckart_schmidt@t-online.de
Subject: Special CB-Point

```
Dear Bernard, dear Chris,

the 9th CB-point of the QG-vertices P1,P2,P3,P4
... and the four 4th parallelogram points of
P1P2P3,P2P3P4,P3P4P1,P4P1P2
... give the reflection of QG-P15 in QG-P1.

Best regards Eckart
```

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Message: #450
Date: 2020-09-14
From: bernard.keizer@gmail.com
Subject: Re: Pedal Cevian Quadrigons

```
Dear Eckart,
Very beautiful, this generalisation of the Darboux cubic for 4
lines !
It's easy to draw the cubic with your 8 special points, as it
gives with the 4 vertices of the QA 12 points ...
On the top of page 3, I suppose some words are missing : the
asymptote is parallel to the bisector of the bisectors of 2
opposite sides (it is the mean direction of the 4 lines of the
QL).
I suppose there are 3 such cubics for a QL, each based on one of
the 3 QA's.
(There are for the same reason 3 different cubics for the 3 QG's
of a QA, which are QA's of 3 different QL's).
The 3 cubics have parallel asymptotes. If you consider their
intersections, the QL vertices are double points, QL-P1 is a
triple point, what about the other ?
The locus of the points having their projections on the 4 lines
as vertices of a cevian quadrigon should be the union of these 3
cubics ...
Best regards
Bernard
Of course, these cubics should be mentionned in EQF ...
```

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Message: #451
Date: 2020-09-15
From: eckart_schmidt@t-online.de
Subject: QG-Strophoids

Dear Bernard, dear Chris,

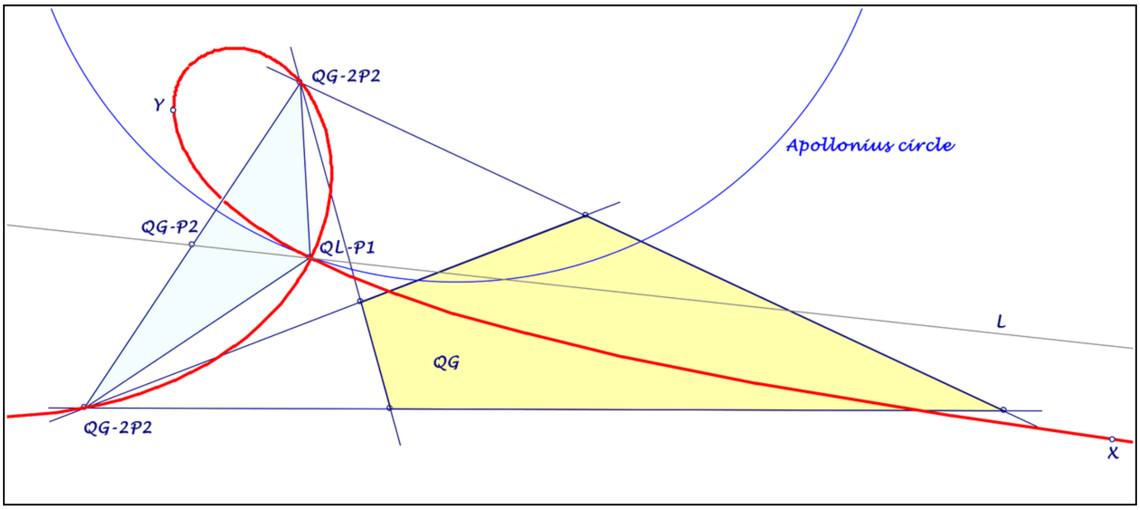
let us start with a reference triangle TR with vertices
... in the Miquel point QL-P1 and the intersections QG-2P2
    of opposite QG-sides,
... consider the isogonal conjugate of the Apollonius circle
    through QL-P1,
... which gives a strophoid.
The three strophoids of the QG-versions of a QL have a common
point X.
X is the intersection of a parallel to QL-L1 through QL-P1
... and a perpendicular to QL-L1 in the intersection with QL-L6,
... or X is CSC of the intersection of QL-Cu1 and its asymptote
... and therefore a point on QL-Cu1.

The defining line L of the strophoid is QL-P1.QG-P2,
... the fixed point is QL-P1,
... the pole Y is CSC of the intersection of L
    and the asymptote of QL-Cu1,
... or Y is the isogonal conjugate of the 2nd intersection of L
    and the Apollonius circle.
The strophoid is the CSC-image of an TR circumscribed orthogonal
hyperbola,
... centered in QG-P2, bearing the intersections
    of the asymptote of QL-Cu1
... with QL-P1.QL-P4 and with the line L.
The strophoid is the pedal curve of QL-P1 wrt the tangents
of a parabola
... with focus in the reflection of QL-P1 in the pole Y
    and directrix L.
The strophoid is the locus of contact points
... of tangents from QL-P1 to conics with foci in QG-2P2.

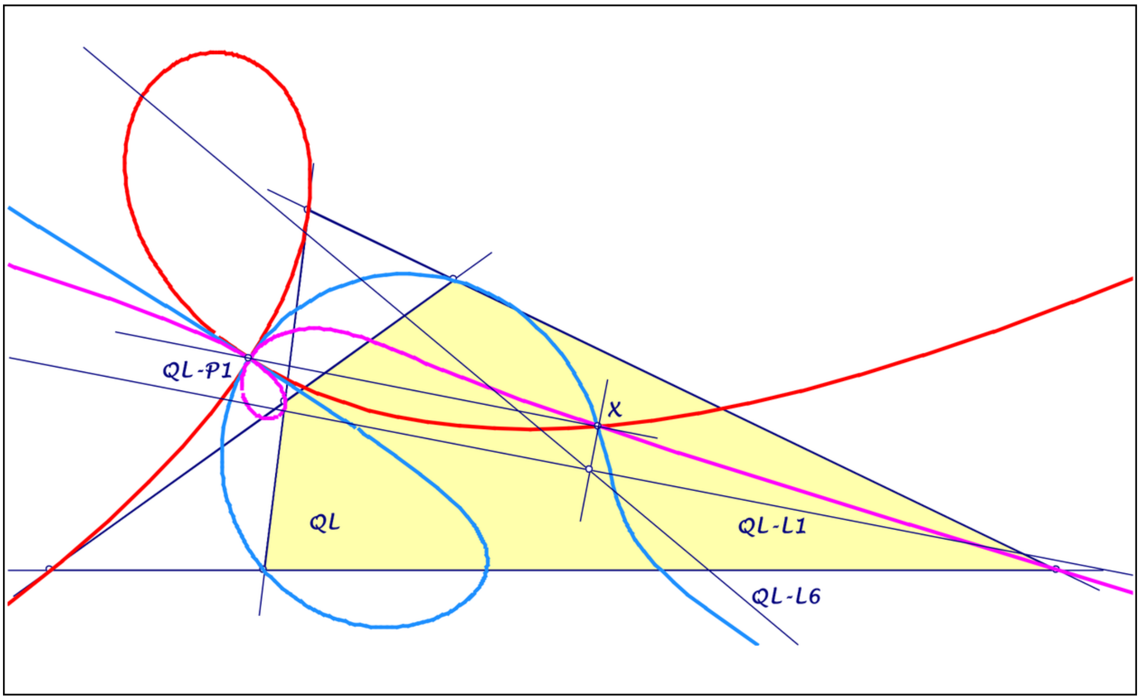
Further properties:
The pedal point of QL-P1 on QG-L1 lies on the strophoid.
Tangents in the knot QL-P1 are the Steiner axes or angle
bisectors of TR.
Tangents in QG-2P2 intersect on the strophoid
... in the isogonal image of the reflection of QL-P1 in QG-L1,
... which is the intersection with the tangent in QL-P1
    to the Apollonius circle.

Best regards Eckart

```
 PS: For a QL the CSC-lines for the Apollonius circles
... give a trilateral with sides orthogonal to those of QL-Tr1,
... perspective QL-Tr1 with perspector QL-P16.
```



2020-09-15a.pdf



2020-09-15b.pdf

Message: #452
Date: 2020-09-15
From: eckart_schmidt@t-online.de
Subject: Re: Pedal Cevian Quadrigons

```
Dear Bernard,

thanks for interest for this several times worked out theme.
Some remarks to your message:

The asymptote is parallel to the bisector of QG-2P2 of the QG,
... so I cannot understand
    "... mean direction of the 4 lines of the QL".
For a QL there are three such cubics,
... not with parallel asymptotes,
... but with triple point in QL-P1,
    whose collinear degenerated pedal QA
... gives cevian QG for the QG-versions of the QL,
... with ratios for opposite sides already with product 1.
Beside the QL-vertices there are 8 double intersections,
... which give each two cevian QG for two QG-versions of the QL.

Best regards Eckart
```

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Message: #453
Date: 2020-09-16
From: bernard.keizer@gmail.com
Subject: Re: Pedal Cevian Quadrigons

```
Dear Eckart,
My apologise, I'm always lost with the QG-properties !
For each QA of the QL, the asymptote of the cubic is the
perpendicular bisector of the segment joining the 2 last
opposite QL vertices.
Of course, the 3 asymptotes are not parallel !
The pedal QA of QL-P1 is the tangent to the parabola
perpendicular to the Newton Line and parallel to the Steiner
Line.
Thanks for your patience and for correcting me !
Best regards
Bernard
```

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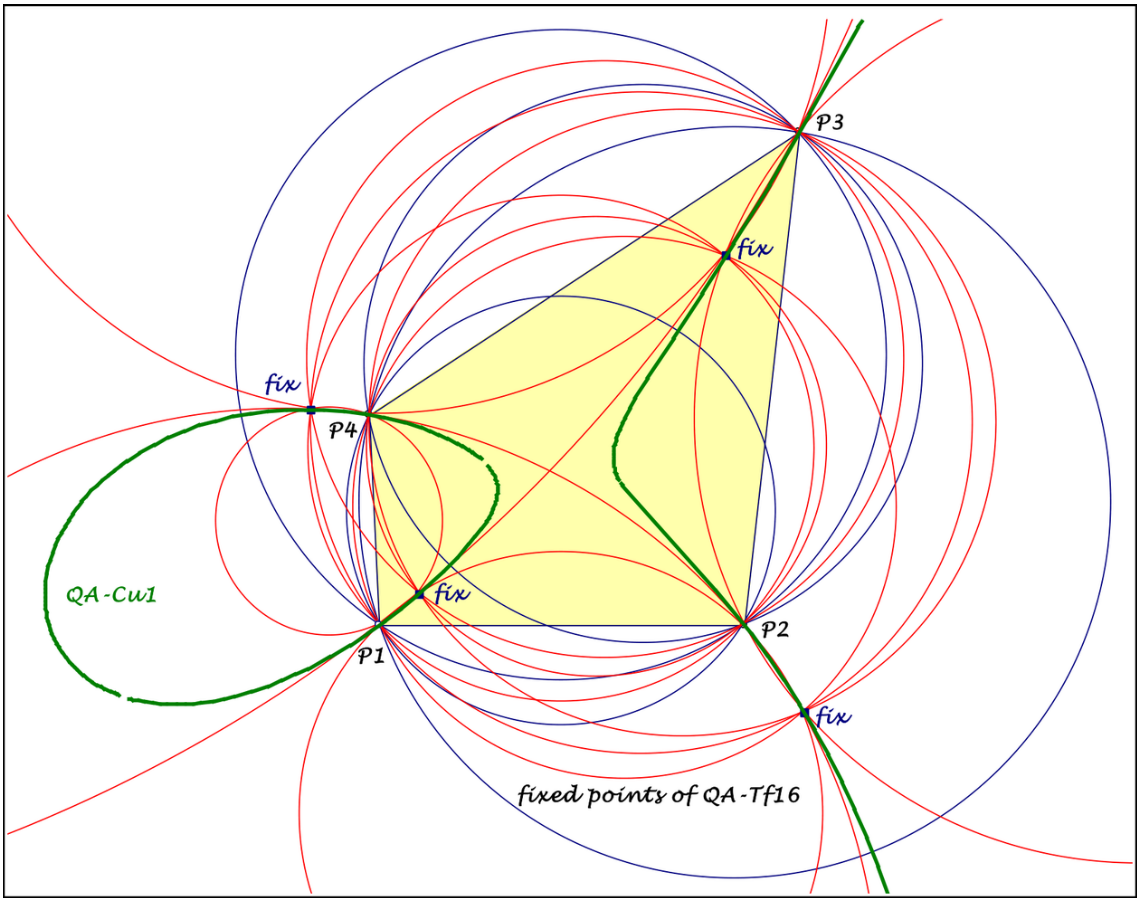
Message: #454
Date: 2020-09-19
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of QA-Tf16

```
Dear Bernard,

unexpected, perhaps evident:
Consider the six pairs of circumcircles of QA-triangles,
... each pair has two inversion circles,
... and the 12 inversion circles
    have four six-time intersections,
... which are the fixed points of QA-Tf16 on QA-Cu1.

Best regards Eckart
```



2020-09-19.pdf

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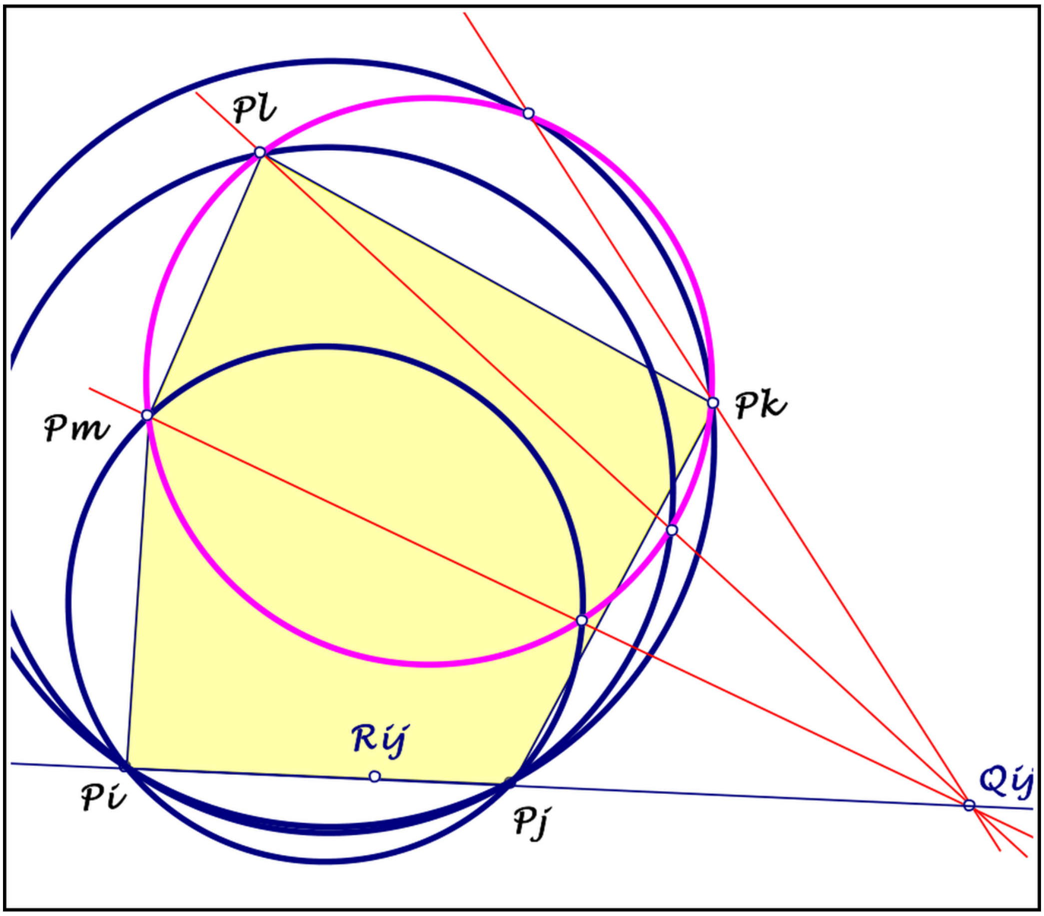
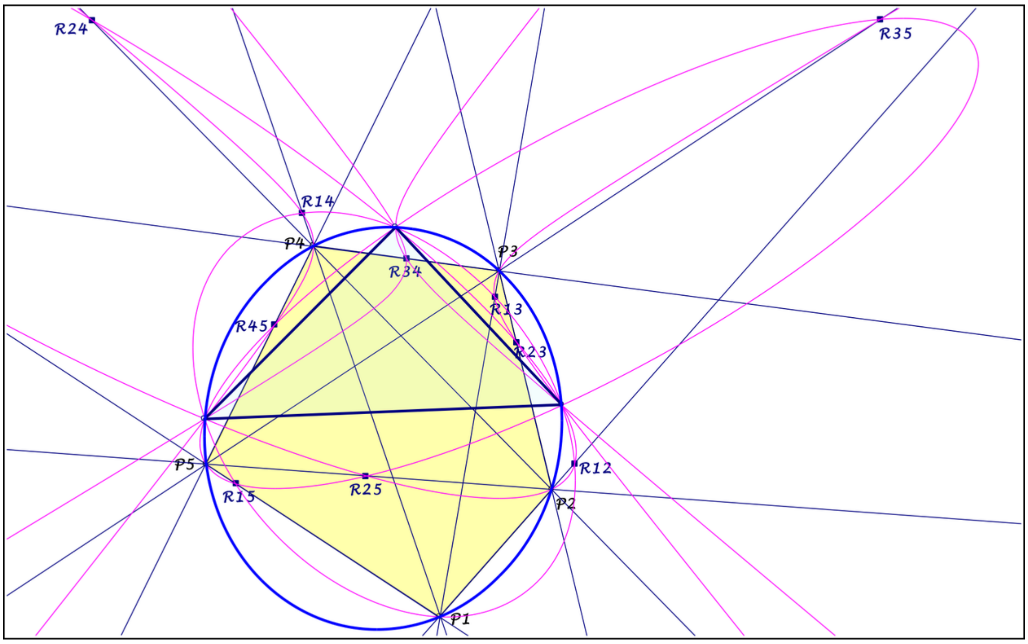
Message: #455
Date: 2020-09-20
From: eckart_schmidt@t-online.de
Subject: Special Triangle on 5P-Co1

```
Dear Bernard, dear Chris,

curious construction for a 5P, unexpected triangle on 5P-Co1:
Consider for a 5P = P1...P5 the 10 pairs Pi,Pj of vertices
... and for each pair the 3 circles (PiPjPk), (PiPjPl), (PiPjPm)
... and their radical axes with the circle (PkPlPm),
... which have a common point Qij on PiPj
... with 4th harmonic point Rij wrt Pi,Pj (see attached file).
The 5 conics through Pi, Rij, Rik, Ril, Rim
... have 3 common points on the circumconic of 5P.

What about this triangle for a 5P?

Best regards Eckart
```



2020-09-20a.pdf



2020-09-20b.pdf

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Message: #456
Date: 2020-09-22
From: bernard.keizer@gmail.com
Subject: Re: Special Triangle on 5P-Co1

```
Dear Eckart,
I reproduced without difficulty your beautiful figure.
There are plenty of interesting aligments.
Each vertice Pi is on 3 lines through QjkQlm (the 10 points Qij
are on 15 lines with 2 points on each line and 3 lines through
each point).
Rij,Rik and Rjk are aligned (the 10 points are on 10 lines with
3 points on each line and 3 lines through each point).
But I didn't find any explanation for your triangle.
I suppose there are dual properties with the duality wrt the
conic ...
Best regards
Bernard
```

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Message: #457
Date: 2020-09-22
From: eckart_schmidt@t-online.de
Subject: Re: Special Triangle on 5P-Co1

```
Dear Bernard,

in addition to #455 a new 5P-point on 5P-Co1:

Let us consider the 10 midpoints Mij of Qij and Rij on PiPj,
... on 10 lines as triple points,
... and the 5 conics through Pi, Mij, Mik, Mil, Mim,
... which have a common point on 5P-Co1.

What about this new 5P-point?

Best regards Eckart
```

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Message: #458
Date: 2020-09-23
From: bernard.keizer@gmail.com
Subject: Re: Special Triangle on 5P-Co1

```
Dear Eckart,
No idea !
Best regards
Bernard
```

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Message: #459
Date: 2020-09-25
From: Stan.Rabinowitz@comcast.net
Subject: results for tetrahedra

```
I am very proud to announce that my paper, "Arrangement of
Central Points on the Faces of a Tetrahedron", has been
published in the International Journal of Computer Discovered
Mathematics.

http://www.journal-1.eu/2020/TetrahedronFaces.pdf

You may be wondering what this has to do with Quadri-Geometry.
Well, as you move one vertex of a tetrahedron down to the
opposite face, the result is a quadrangle. It is possible that
new results might be found for quadrangles as limiting cases of
related results for tetrahedra. For example, if a result in my
paper involves drawing lines from each vertex of a tetrahedron
to the Gergonne point, say, of the opposite face, one can ask if
there is a similar result in the plane when you start with a
quadrangle and draw lines from each vertex to the Gergonne point
of the opposite component triangle.

I hope you will find this paper interesting.
```

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Message: #460
Date: 2020-09-26
From: bernard.keizer@gmail.com
Subject: Re: results for tetrahedra

```
Dear Stanley,
Not only interesting, but fascinating ! Congratulations. I will
certainly read it completely.
The fact that some properties of the tetrahedron may be used for
the QA  goes for example for the Reye configuration and the
resulting pivotal isocubic ...
Best regards
Bernard
```

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Message: #461
Date: 2020-09-26
From: van10hoven@gmail.com
Subject: Re: Special Triangle on 5P-Co1

```
Dear Eckart,

This inscribed triangle in 5P-s-Co1 is most special.
I investigated if there were some incidences,
but I couldn't find any.

Best regards,
Chris
```

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Message: #462
Date: 2020-09-26
From: van10hoven@gmail.com
Subject: Re: results for tetrahedra

```
Dear Stanley,

I have always been interested in tetrahedra, but never found
time to explore them.
You helped me a lot by making this beautiful exposé.
A flat tetrahedron is a quadrangle and I wonder how to imply
tetrahedra properties into quadri geometry.
I am going to study it further.

Best regards,
Chris
```

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Message: #463
Date: 2020-10-06
From: bernard.keizer@gmail.com
Subject: P-D and M-S curves

```
Dear Eckart,
I hope you will find this new item interesting !
Thanks in advance for your comments.
Best regards
Bernard
```

Poncelet'porism

Poncelet-Darboux and Marden-Siebeck curves

Examples from the triangle, the QL and the PL

1) Poncelet's porism

If there is a polygon inscribed in a conic and circumscribed to another, there are an infinity.

(I've respected the wording of Poncelet, but in fact it is a nL, as you already mentionned).

2) Poncelet-Darboux and Marden-Siebeck curves

A Poncelet-Darboux curve is a curve passing through the ½ n(n-1) vertices of a nL tangent to a conic.

A Marden-Siebeck curve is a curve tangent to the ½ n(n-1) sides of a nP inscribed in a conic.

Both curves are dual wrt the conic, each one is

- the locus of the poles wrt the conic of the tangents to the other
- the envelop of the polars wrt the conic of the points of the other

There are an infinity of nL's inscribed in the P-D curve and circumscribed to the conic.

There are an infinity of nP's inscribed in the conic and circumscribed to the M-S curve.

3) Triangle

A good example in the PL figure are the 5 triangles formed by a Miquel point of 4 of the 5 lines and the 5th line, which are inscribed in the circle of the Miquel points and circonscribed to the inscribed conic. The 5 triangles formed by the contact point of a line with the inscribed conic and the polar wrt the inscribed conic of the corresponding Miquel point are the same way inscribed in the inscribed conic and circumscribed to a 2^{nd} conic, which is the dual of the circle of the Miquel points wrt the inscribed conic. (A conic is a curve of 2^{nd} degree as well as a curve of class 2).

Any circle Ci(P) is also a P-D curve wrt the inscribed conic …

It's easy to draw the dual conics of these circles wrt the inscribed conic.

P-D and M-S curves.pdf

4) QL

Any cubic through the 6 vertices of a QL is a P-D curve wrt any inscribed conic.

The dual curve wrt the conic is a M-S curve of class 3 wrt the conic.

The cubic may be degenerated, for example :

- one of the 4 reference circles through the intersections of 3/4 lines and the 4th line
- a conic through the vertices of a QA of the QL and the diagonal through the 2 remaining points (the 2 other diagonals intersect in a fixed point, which is the pole of the fixed diagonal)

In these cases, the dual M-S curve is also degenerated in a point and a conic.

If the cubic is not degenerated, if it passes through 2 partners in the CSC transformation swapping the 4 circles and the 4 lines, it also passes through the Miquel point and is circular.

The best known is the Van Rees circular pivotal cubic QL-Cu1.

It's relatively easy to draw it's dual M-S curve of class 3.

5) PL

Any quartic through the 10 vertices of a PL (socalled Lüroth quartic) is a P-D curve wrt the inscribed conic.

The dual curve wrt the conic is a M-S curve of class 4 wrt the conic.

The quartic may be degenerated, for example :

- 2 conics through the 10 vertices with 2 points of each line on each conic (not circular)
- One of the 5 VR's of the 5 QL's of 4/5 lines and the 5th line

The dual M-S curves are also degenerated, in the 1rst case in 2 conics and in the 2nd case in a point and a curve of class 3.

If the quartic is not degenerated, if it passes through a point and it's 3 partners in the transformation swapping the VR's and the lines, it also passes through the foci of the conic and is circular. One we have studied is obtained for the infinity point of the axis of the conic through it's foci and it's 3 partners, which are 3 of the 4 intersections between the circle of the Miquel points and the conic of the CB points, where CB is on each VR the 9th CB point of the 6 vertices of the QL, the Miquel point and the foci of the inscribed conic.

It's relatively easy to draw it's dual M-S curve of class 4.

P-D and M-S curves.pdf

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Message: #464
Date: 2020-10-16
From: eckart_schmidt@t-online.de
Subject: Re: P-D and M-S curves

```
Dear Bernard,

back from holiday, I shall study your message #463,
... give me some time.

Best regards Eckart
```

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Message: #465
Date: 2020-10-16
From: eckart_schmidt@t-online.de
Subject: QG-2P6

```
Dear Chris,

is there a reason,
... that the item QG-2P6 is not listed in "Quadrigon Objects"
... and that there is no item "Circumscribed Square Centers",
... which are as the inscribed square cenrters van Aubel points
... on the circle with diameter in the diagonal midpoints,
... inversion circle round QA-P1 for QG-P2 and QG-P12.

Best regards Eckart

PS. These remarks arised reading the message of Dario
Pellegrinetti.
```

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Message: #466
Date: 2020-10-16
From: oaidt.evnpsc@gmail.com
Subject: A new theorem with 4 point in the plane

Dear Geometer,

I posed a new theorem with 4 points in the plane. Please see the File attachment. I hope that You like the theorem.

Best regards
Sincerely
Dao Thanh Oai

A New Theorem with 4 Points in Plane

Proposed by DAO THANH OAI, Thai Binh, Viet Nam

October 17, 2020

Abstract

Some days ago, I discover a new property of 4 points in the plane. The egg configration in [1] since 2013.

1 Problem

Problem 1. Let four points A, B, C, D in the plain, the perpendicular of AB meets the perpendicular of CD at P, then always exist point S such that $(\overrightarrow{\text{PD}}, \overrightarrow{\text{PC}}) \equiv 2(AD, AS) \equiv 2(BS, BC)$ and $(\overrightarrow{\text{PB}}, \overrightarrow{\text{PA}}) \equiv 2(CB, CS) \equiv 2(DS, DA)$.

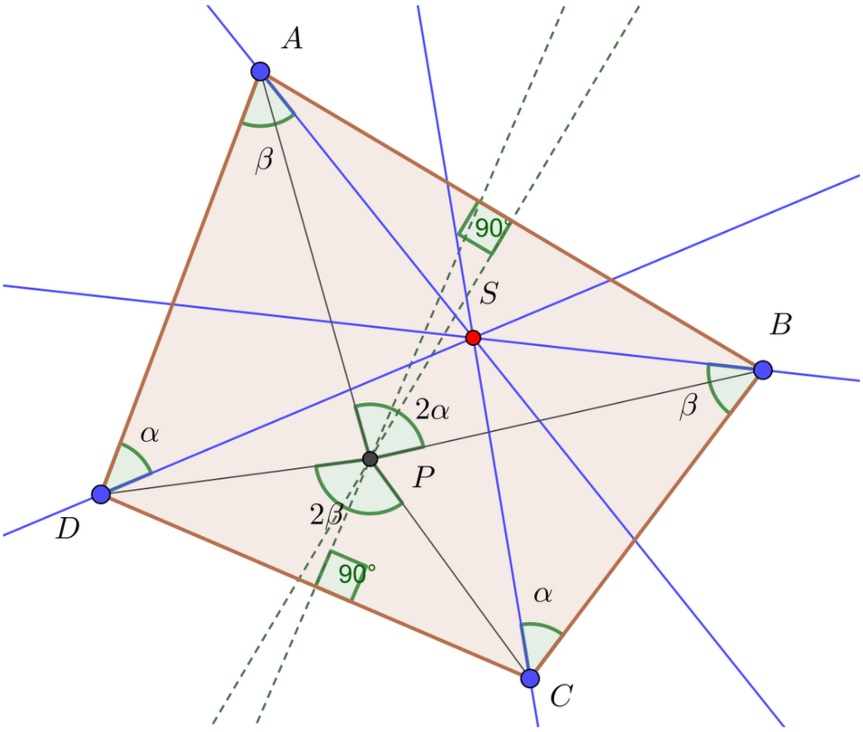

Figure 1

A_new_theorem_with_4_point_in_the_plane.pdf

2 Applycation

If the problem was proved. Then we can apply the theorem to proof two facemous theorem:

1. Proof Napoleon theorem: Apply the theorem with $\beta = 120^0$ and $\alpha = 30^0$ (See Figure 1).

2. Proof Bottema theorem: Apply the theorem with $\beta = 45^0$ and $\alpha = 90^0$ (See Figure 1).

References

[1] Posed by Dao Thanh Oai, solution by Luis Gonzalez, *with case N is fixed point* https://artofproblemsolving.com/community/q3h557966p3244119, 2013

[2] https://en.wikipedia.org/wiki/Bottema%27s_theorem

[3] https://en.wikipedia.org/wiki/Napoleon%27s_theorem

E-mail address: daothanhoai@hotmail.com

A_new_theorem_with_4_point_in_the_plane.pdf

Message: #467
Date: 2020-10-17
From: eckart_schmidt@t-online.de
Subject: Re: [Quadri-and-Poly-Geometry] A new theorem with 4 point in the

---

```
Dear Dao Than Oai,

the point S = QL-Tf1(P).
There are two points P and P' for a quadrigon with
... PP' // SS' and P, S' as well as P', S collinear QL-P1.

Best regards Eckart
```

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Message: #468
Date: 2020-10-17
From: oaidt.evnpsc@gmail.com
Subject: Re: A new theorem with 4 point in the plane

```
Dear Mister Eckart!

Thanks you very much for your reply.
--
Dao Thanh Oai
```

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Message: #469
Date: 2020-10-18
From: van10hoven@gmail.com
Subject: Re: QG-2P6

Dear Eckart

The item QG-2P6 is not listed in "Quadrigon Objects" because I forgot to do it. Thanks for mentioning. I will do it soon.

The centers of the Circumscribed Squares are not mentioned because they belong to the many items that certainly should deserve a place but there has not been an appropriate time yet.

Best regards,

Chris

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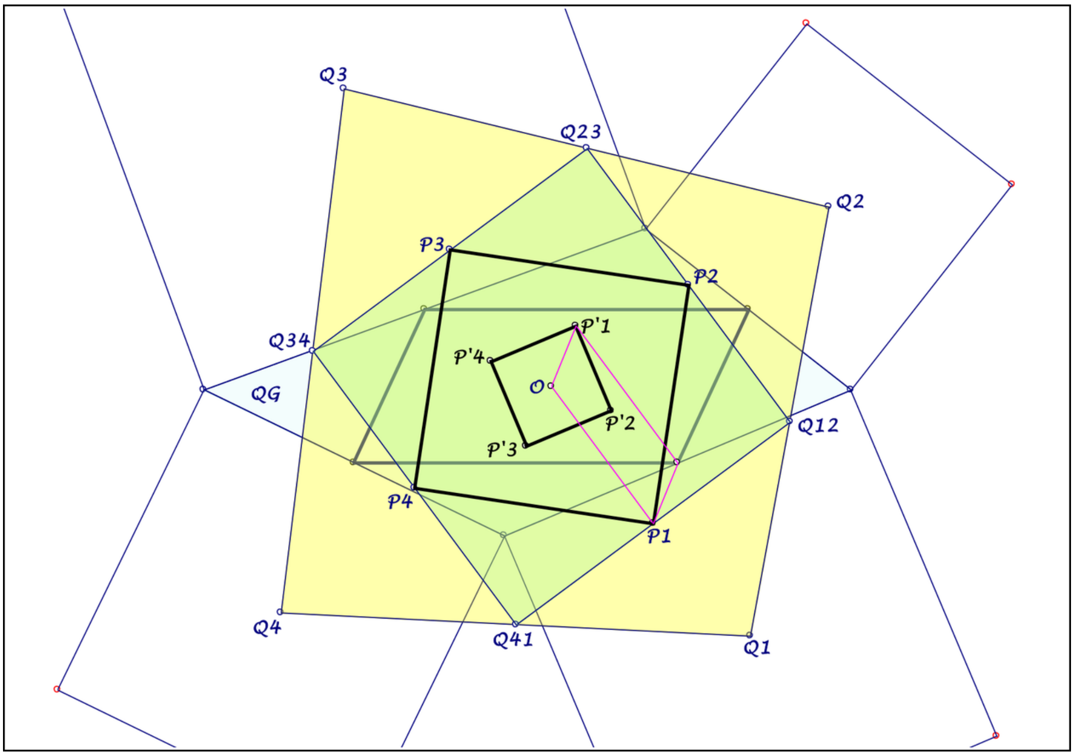
Message: #470
Date: 2020-10-21
From: eckart_schmidt@t-online.de
Subject: Varignon parallelograms and Van Aubel configuration

```
Dear all,

any parallelogram is presentable as vector sum
... of two squares with reverse circulation
... (see F.Bachmann/E.Schmidt: n-gons, page 147/148).
Here a new construction of these squares:

Let us start with a parallelogram as
... Varignon parallelogram of a quadrigon QG.
Consider the centers Qi of outer squares for the sides of QG,
... and the midpoints Qi,i+1 of QiQi+1, which give a square,
... and once more the midpoints Pi of Qi-1,iQi,i+1,
    which give a 2nd square.
Analog we get the vertices P'i,
... starting with the inner squares for the sides of the QG.
Let O be the common center of the squares,
... then the 4th parallelogram points of  Pi, O, P'i
... give the vertices of the starting parallelogram.

Best regards Eckart
```



2020-10-21.pdf

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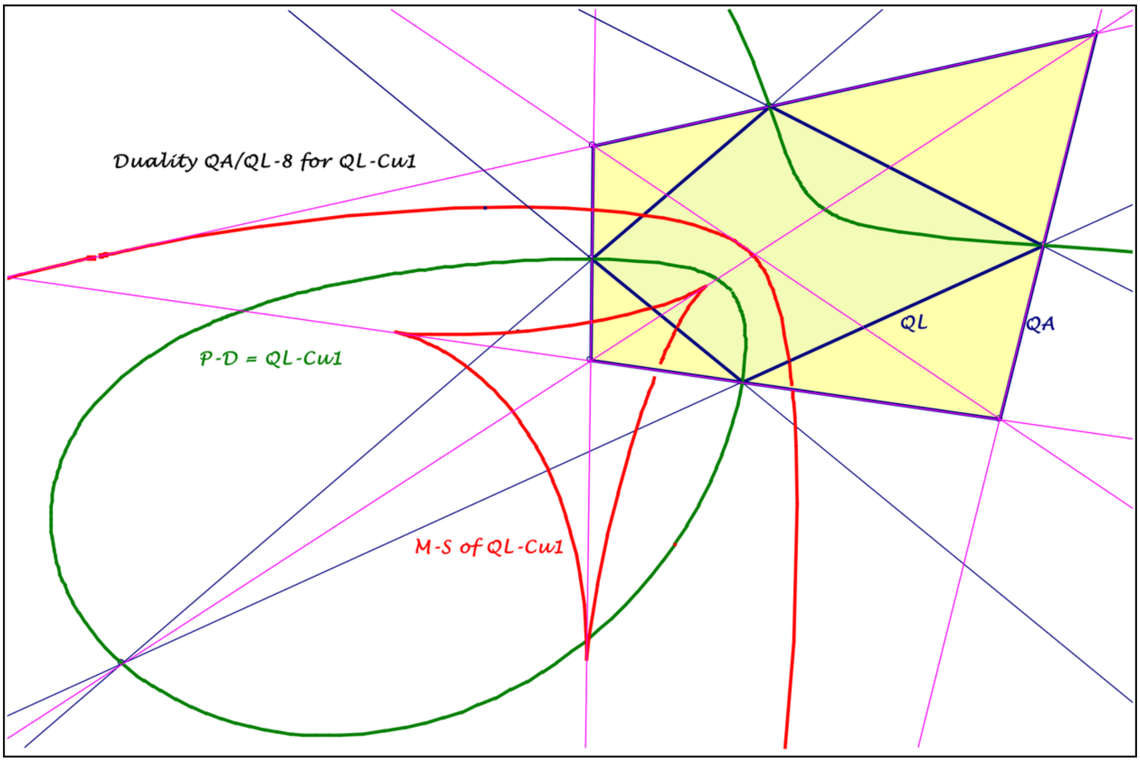
Message: #471
Date: 2020-10-21
From: eckart_schmidt@t-online.de
Subject: Re: P-D and M-S curves

```
Dear Bernard,

I studied your message #463, not detailed,
... for I cannot judge your generalizations,
... but for n = 4 wrt QL-Cu1.
You describe the conic duality for P-D and M-S curves,
... QL-Cu1 is a P-D curve (see 4)),
... passing through the 6 vertices of a QL,
    tangent to any inscribed conic Co,
... the poles of the tangents at QL-Cu1 wrt the conic Co
... give a M-S curve of class 3 tangent to the six sides of a QA
... with vertices in the poles of the QL-lines wrt Co.
You can replace the conic duality above for QL-Cu1
... by the duality in the sense of QA/QL-8,
    independent of a conic,
... and you get also a M-S curve (attached) in a unique mapping.
Have we already discussed this M-S curve?

Best regards Eckart
```



2020-10-21a.pdf

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Message: #472
Date: 2020-10-21
From: bernard.keizer@gmail.com
Subject: Re: P-D and M-S curves

Dear Eckart,
Thanks for your interesting answer !
(M-S curve is not correct, this is my fault, as Siebeck came before Marden, like Poncelet before Darboux. I use now S-M instead of M-S curve).
You give the duality QL/QA as alternative for the use of any inscribed conic.
QL-Cu1 is a P-D curve for any inscribed conic in the QL means that there are an infinity of QL's inscribed in this cubic and circumscribed to the same conic.
The same way, the dual of QL-Cu1 is a S-M curve for any circumscribed conic of the QA means that there are an infinity of QA's inscribed in the same conic and circumscribed to the S-M curve.
The advantage of this duality is, as you mention, that it is independant of the conic ; the disadvantage is that the inscribed conic in the QL is not the same as the circumscribed conic of the QA ...
The starting point all my reflexion was the discovery on Google of 3 "Poncelet curves" by Isabelle Chalendar ; the 3 curves are in fact partial S-M curves, duals of a conic, a cubic and a quartic wrt a circle.
We never discussed this S-M curve dual of QL-Cu1 before, but we discussed a special Siebeck curve tangent to the 6 sides of a QA in their middles as dual (in your sense) of a special isotomic cubic of the dual QL.
Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

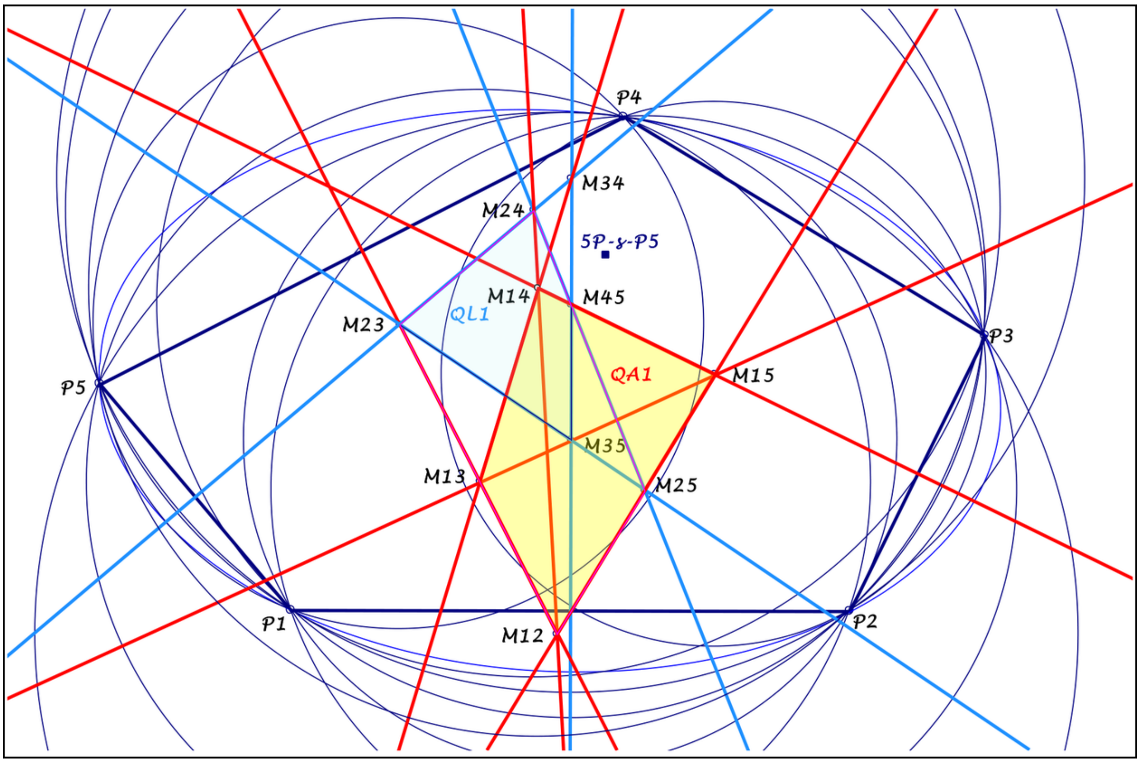
Message: #473
Date: 2020-10-22
From: eckart_schmidt@t-online.de
Subject: 5 QA and 5 QL for a 5P

```
Dear all,

consider for a 5P with vertices Pi
... the 10 circles (PkPlPm) with centers Mij,
... which lie on 10 bisectors, Mij,Mik,Mjk collinear,
    as triple points,
... interpretable as
... a quadrangle QAi with vertices Mij,Mik,Mil,Mim
... and a quadrilateral QLi with vertices Mkl,
    k and l unequal i.

Observation: CSC wrt QLi maps Pi to 5P-s-P5.
Perhaps we can get with these QA and QL new 5P-points.

Best regards Eckart
```



2020-10-22.pdf

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Message: #474
Date: 2020-10-22
From: van10hoven@gmail.com
Subject: Re: QG-2P6

Dear Eckart,

About your question about the item "Circumscribed Square Centers".
I intent to add the information in EQF.
Therefore I gathered all information about the subject in the discussions we had before so that we haven an orderly overview.

History
* In September 2012 we discussed in private mails all types of squares in relationship to a Quadrigon and shared several results with him. You then remarked that the Van Aubel Points lie on the Thales Circle with diameter the midpoints of a Quadrigon.
* December 2013 we had a public discussion about circumscribed squares at the Yahoo Quadri-Figures Group (QFG). Unfortunately Yahoo stopped supporting these groups, but all messages were migrated to Groups.io at https://groups.io/g/Quadri-Figures-Group/messages?expanded=1 (https://groups.io/g/Quadri-Figures-Group/messages?expanded=1) , as well as to this site http://www.qfg.99on.com/message.php?msg=370 (http://www.qfg.99on.com/message.php?msg=370). I made a recap of all messages about Circumscribed Squares even pasting the attachments that were exchanged for a good overview.
* March 2018 I raised the question again in my “Introduction to Quadrilateral Geometry” in QFG as well as two other discussion groups (ADGEOM as well as Anopolis).

All relevant messages can be found in attached recap.
Now I am going to work on their presence in EQF.

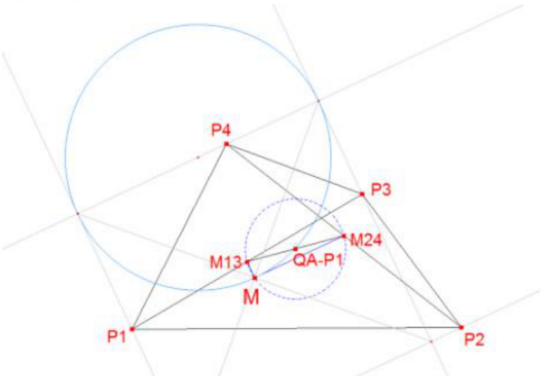
Best regards,

Chris

Posts from the Quadri-Figures Group about QG-Circumscribed Squares

QFG#366 08/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16803

Let P1, P2, P3, P4 be the vertices of a Quadrigon.
Let M13 be midpoint P1.P3.
Let M24 be midpoint P2.P4.
Let M be some point on the circle with diameter M13.M24.
Draw lines through P1 and P3 parallel to M.M13.
Draw lines through P2 and P4 parallel to M.M24.
These 2+2 parallel lines form a rectangle.
Question:
How to construct and possibly calculate point M such that this rectangle is a square ?
Best regards,
Chris
See attachment:



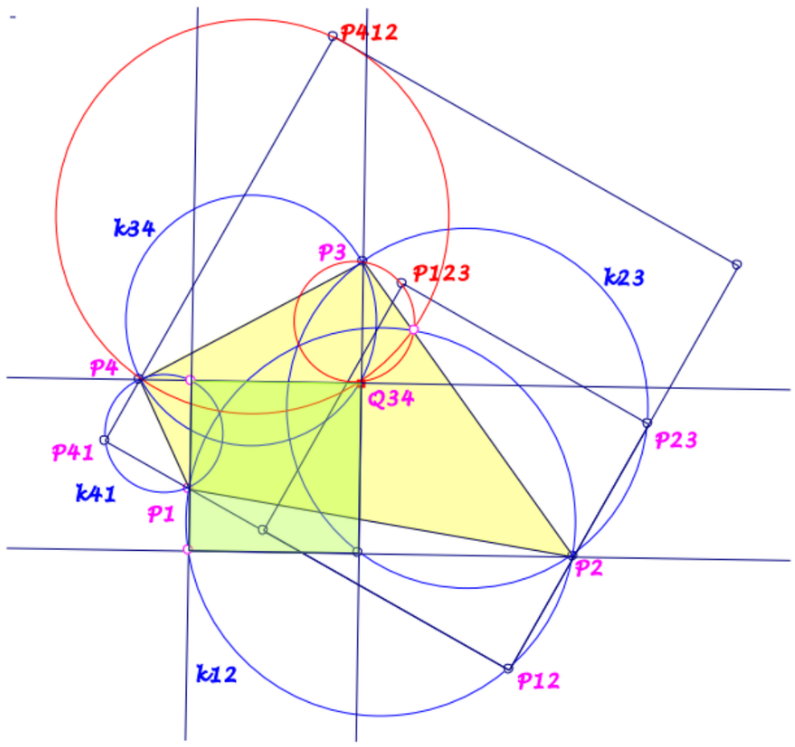
Let P1.P2.P3.P4 be random Quadrigon.
Let M13 be midpoint P1.P3.
Let M24 be midpoint P2.P4.
Let M be some point on the circle with diameter M13.M24.
Draw lines through P1 and P3 parallel to M.M13.
Draw lines through P2 and P4 parallel to M.M24.
These 2+2 parallel lines form a rectangle.
Question:
For which M this rectangle is a square ?

QG-2P6 Circumscribed Squares-01.pdf

QFG#367 09/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16804

Dear Chris,
here is a construction of the described square (see attachment):
Let P1P2P3P4 be a quadrigon and Cij the Thales circles about PiPj with
... P12 point on C12,
P41=P1P12^C41 and P23=P12P2^C23 (second intersections).
Construct a square about P41P12 with P412 as opposite point of P12
... and a square about P12P23 with P123 as opposite point of P12.
The loci of these opposite points are circles C412 and C123.
One of the intersections of these circles lies on C34.
This point is a vertex Q34 of the searched quadrigon ...
Best regards Eckart

Attachment:



QG-2P6 Circumscribed Squares-01.pdf

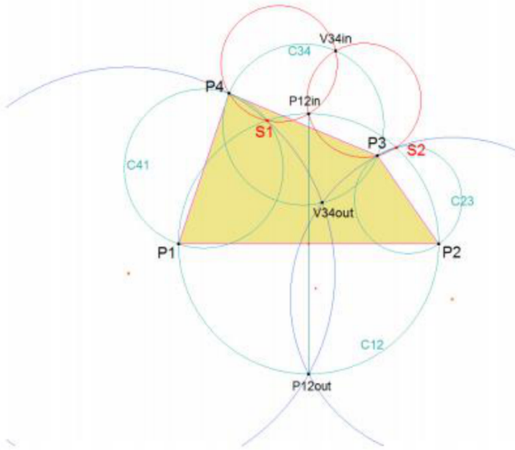
QFG#370 09/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16807

Dear Eckart,
Very nice construction of a circumscribed square around a "quadrilateral" !
I took the liberty to alter your construction a bit. See also attached files.
Construction two circumscribed squares around a quadrigon:
Let P1P2P3P4 be a quadrigon and Cij the Thales circles about PiPj .
C12 is intersected by the perpendicular bisector of P1.P2 in 2 points.
Let P12in be the inward point (direction QA-Centroid).
Let P12out be the outward point (other direction).
Let S1 be 2nd intersection point of circles C12 and C14.
Let S2 be 2nd intersection point of circles C12 and C23.
Let C14in be the circle through S1, P4 and P12in.
Let C23in be the circle through S2, P3 and P12in.
Let V34in be the 2nd intersection point of C14in and C23in.
V34in = the Vertex of the 1st circumscribed square connecting P3 and P4.
Let C14out be the circle through S1, P4 and P12out.
Let C23out be the circle through S2, P3 and P12out.
Let V34out be the 2nd intersection point of C14out and C23out.
V34out = the Vertex of the 2nd circumscribed square connecting P3 and P4.
V34in and V34out are the constructed vertices of the circumscribed squares.
By connecting these points with P3 and P4 the first sidelines of the squares are constructed.
By drawing parallel lines through P1 and P2 the other sidelines of the squares are constructed.
Now we have 2 circumscribed squares.
The centers of the squares lie indeed on the Thales Circle of M13.M24, where M13 and M24 are the midpoints of the two diagonals of the Quadrigon.
Some other interesting properties:
* The diagonals of the two circumscribed squares produce 2 other intersection points, also on the Thales Circle of M13.M24.
* The connecting line of the two centers of the circumscribed squares passes through this point: QG-P1.QG-P6 ^ QG-P5.QG-P8 ^ QA-P1.QG-P11, which is a point on the Nine-point Conic QA-Co1.
For QA-Co1 see: http://www.chrisvantienhoven.nl/other-quadrangle-objects/15-mathematics/quadrangle-objects/artikelen-qa/76-qa-co1.html
See also QG/5: http://www.chrisvantienhoven.nl/quadrigon-objects/16-mathematics/encyclopedia-of-quadri-figures/quadrigon-objects/artikelen-qg/165-qg-5.html
I think there will be more of these intersection points on this circumscribed-square-center line.
* A first check didn't produce concurrencies of the QG-lines of the 3 QA-Quadrigons or the 3 QL-Quadrigons.
Best regards,
Chris

Attachments:

QG-2P6 Circumscribed Squares-01.pdf

construction of two circumscribed squares around a quadrigon



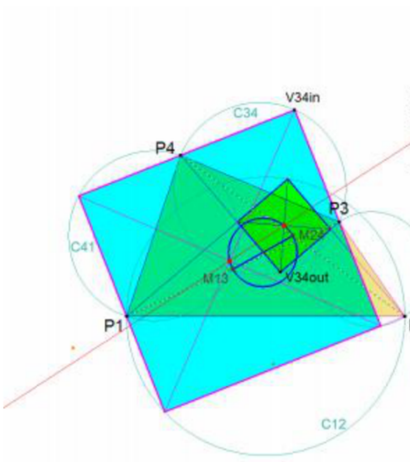
Construction two circumscribed squares around a quadrigon:

Let P1P2P3P4 be a quadrigon and Cij the Thales circles about PiPj .
C12 is intersected by the perpendicular bisector of P1.P2 in 2 points.
Let P12in be the inward point (direction QA-Centroid).
Let P12out be the outward point (other direction).
Let S1 be 2nd intersection point of circles C12 and C14.
Let S2 be 2nd intersection point of circles C12 and C23.

Let C14in be the circle through S1, P4 and P12in.
Let C23in be the circle through S2, P3 and P12in.
Let V34in be the 2nd intersection point of C14in and C23in.
V34in = the Vertex of the 1st circumscribed square connecting P3 and P4.

Let C14out be the circle through S1, P4 and P12out.
Let C23out be the circle through S2, P3 and P12out.
Let V34out be the 2nd intersection point of C14out and C23out.
V34out = the Vertex of the 2nd circumscribed square connecting P3 and P4.

two centers of circumscribed squares of a Quadrigon lying on the Thales circle of M13.M24



V34in and V34out are the constructed vertices of the circumscribed squares.
By connecting these points with P3 and P4 the first sidelines of the squares are constructed.
By drawing parallel lines through P1 and P2 the other sidelines of the squares are constructed.

The centers of the two circumscribed squares lie on theThales cricle of M13.M24,
where M13 and M24 are the resp. midpoints of P1.P3 and P2.P4.

QG-2P6 Circumscribed Squares-01.pdf

QFG#371 10/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16808

Dear Chris,
thank you for the description of the construction for the second circumscribed square. I had supposed it, but wasn't able to find! Reason: The calculation for point M gave two solutions. Now I can give you the coefficients of the circumscribed-square-center line (see attached Mathematica file).
Best regards Eckart

Circumscribed-square-center line (message 370)

Reference triangle for barycentric coordinates:

P1P2P3 with P4:={p,q,r}.

$$
\begin{aligned}
\{&(q+r)\left(2p^2+3pq+2q^2+3pr+4qr+2r^2\right) SA^2 \\
&-(p+q)\, r\,(p+q+2r)\, SC^2 \\
&+\left(p^3+2p^2q+4p^2r+4pqr+pr^2-2qr^2-2r^3\right) SA\, SB \\
&+r\left(p^2-2pr-4qr-3r^2\right) SB\, SC \\
&-\left(p^3+4p^2q+5pq^2+2q^3+4pqr+3q^2r+2pr^2+2qr^2+r^3\right) SA\, SC,\ -p\,(q+r)\,(2p+q+r)\, SA^2 \\
&-2\,(p+r)^2\,(p+2q+r)\, SB^2 \\
&-(p+q)\, r\,(p+q+2r)\, SC^2 \\
&+\left(p^3+4p^2q+10pq^2+4q^3+2p^2r+12pqr+10q^2r+3pr^2+8qr^2+2r^3\right) SA\, SB \\
&+\left(2p^3+8p^2q+10pq^2+4q^3+3p^2r+12pqr+10q^2r+2pr^2+4qr^2+r^3\right) SB\, SC \\
&+(3p+4q+3r)\left(p^2+2pq+q^2+pr+2qr+r^2\right) SA\, SC, \\
&-p\,(q+r)\,(2p+q+r)\, SA^2 \\
&+(p+q)\left(2p^2+4pq+2q^2+3pr+3qr+2r^2\right) SC^2 \\
&-p\left(3p^2+4pq+2pr-r^2\right) SA\, SB \\
&-\left(2p^3+2p^2q-p^2r-4pqr-4pr^2-2qr^2-r^3\right) SB\, SC \\
&-\left(p^3+2p^2q+3pq^2+2q^3+2p^2r+4pqr+5q^2r+4qr^2+r^3\right) SA\, SC\}
\end{aligned}
$$

Eckart Schmidt
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QG-2P6 Circumscribed Squares-01.pdf

QFG#372 12/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16809
Dear Chris,
there is a very simple construction of the circumscribed squares of a quadrigon, using the van Aubel constellation:
Given a quadrigon, place a square outward on each side, and connect the centers of opposite squares. The second intersections of these lines with the Thales circles over the sides are the vertices of an circumscribed square. Placing the squares inside the quadrigon, we get the second circumscribed square.
Best regards Eckart

QFG#373 12/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16810

Dear Eckart,
Remarkably simple indeed!
I already had noticed long before the relation with the Van Aubel configuration. But this simple construction is new to me.
Best regards,
Chris

QG-2P6 Circumscribed Squares-01.pdf

QFG#374 13/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16811

Dear Chris,
there is a further aspect wrt the circumscribed squares of a quadrigon:
Reflecting a random point round the Varignon parallelogram of a quadrigon we get a quadrigon with the same midpoints of the sides as the reference quadrigon. All these quadrigons have circumscribed squares of the same size and parallel sides (see attachments).
Best regards Eckart

Attachments:

Circumscribed-squares of a Quadrigon

Reference triangle for barycentric coordinates:

P1P2P3 with P4:={p,q,r}.

Centroids of the circumscribed squares:

(For the second centroid change the sign of S)

$$\{-2p^2\,S\,SA - p\,q\,S\,SA - p\,r\,S\,SA - 2p^2\,S\,SB - 2p\,r\,S\,SB + 3p^2\,SA\,SB + 2p\,q\,SA\,SB + 3p\,r\,SA\,SB - 2p^2\,S\,SC - 4p\,q\,S\,SC - 2q^2\,S\,SC - 4p\,r\,S\,SC - 3q\,r\,S\,SC - r^2\,S\,SC + 3p^2\,SA\,SC + 3p\,q\,SA\,SC + 4p\,r\,SA\,SC + 4p^2\,SB\,SC + 4p\,q\,SB\,SC + 5p\,r\,SB\,SC + 2q\,r\,SB\,SC + r^2\,SB\,SC + p\,r\,SC^2 + q\,r\,SC^2,$$

$$(-p\,S + r\,S - q\,SA - r\,SA + p\,SC + q\,SC)\,(p\,SA - r\,SC),$$

$$-p^2\,S\,SA - 3p\,q\,S\,SA - 2q^2\,S\,SA - 4p\,r\,S\,SA - 4q\,r\,S\,SA - 2r^2\,S\,SA + p\,q\,SA^2 + p\,r\,SA^2 - 2p\,r\,S\,SB - 2r^2\,S\,SB + p^2\,SA\,SB + 2p\,q\,SA\,SB + 5p\,r\,SA\,SB + 4q\,r\,SA\,SB + 4r^2\,SA\,SB - p\,r\,S\,SC - q\,r\,S\,SC - 2r^2\,S\,SC + 4p\,r\,SA\,SC + 3q\,r\,SA\,SC + 3r^2\,SA\,SC + 3p\,r\,SB\,SC + 2q\,r\,SB\,SC + 3r^2\,SB\,SC\}$$

Coefficients of the circumscribed-square-center line

$$\{(q+r)(2p^2+3pq+2q^2+3pr+4qr+2r^2)\,SA^2$$
$$-(p+q)\,r\,(p+q+2r)\,SC^2$$
$$+(p^3+2p^2q+4p^2r+4pqr+pr^2-2qr^2-2r^3)\,SA\,SB$$
$$+r\,(p^2-2pr-4qr-3r^2)\,SB\,SC$$
$$-(p^3+4p^2q+5pq^2+2q^3+4pqr+3q^2r+2pr^2+2qr^2+r^3)\,SA\,SC,\ -p\,(q+r)(2p+q+r)\,SA^2$$
$$-2(p+r)^2(p+2q+r)\,SB^2$$
$$-(p+q)\,r\,(p+q+2r)\,SC^2$$
$$+(p^3+4p^2q+10pq^2+4q^3+2p^2r+12pqr+10q^2r+3pr^2+8qr^2+2r^3)\,SA\,SB$$
$$+(2p^3+8p^2q+10pq^2+4q^3+3p^2r+12pqr+10q^2r+2pr^2+4qr^2+r^3)\,SB\,SC$$
$$+(3p+4q+3r)(p^2+2pq+q^2+pr+2qr+r^2)\,SA\,SC,$$
$$-p\,(q+r)(2p+q+r)\,SA^2$$
$$+(p+q)(2p^2+4pq+2q^2+3pr+3qr+2r^2)\,SC^2$$
$$-p\,(3p^2+4pq+2pr-r^2)\,SA\,SB$$
$$-(2p^3+2p^2q-p^2r-4pqr-4pr^2-2qr^2-r^3)\,SB\,SC$$
$$-(p^3+2p^2q+3pq^2+2q^3+2p^2r+4pqr+5q^2r+4qr^2+r^3)\,SA\,SC\}$$

Side-length of a circumscribed square (For the second square change sign of S)

$$(p\,SA - r\,SC)\,/\,\sqrt{(-2(p+q+r)(p\,S + r\,S - p\,SA - r\,SC) + (q+r)^2\,SA + (p+r)^2\,SB + (p+q)^2\,SC)}$$

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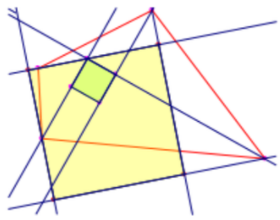
QG-2P6 Circumscribed Squares-01.pdf

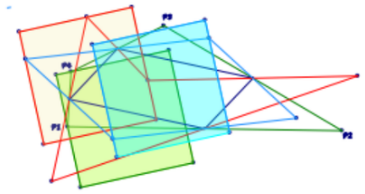
Some remarks wrt circumscribed squares of a quadrigon

For a quadrigon there are two circumscribed squares, see message 366, 370. Possible constructions see message 367, 370, 372. Properties of the circumscribed-square-center line see message 370. Results in calculation see attached Mathematica file.



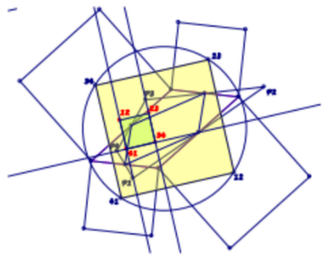
New aspect:

Consider the Varignon parallelogram of the quadrigon. Reflecting a random point *P* round this parallelogram we get a quadrigon with the same midpoints of the sides as the reference quadrigon. All these quadrigons have circumscribed squares of the same size and parallel sides. (The drawing shows only one square component).

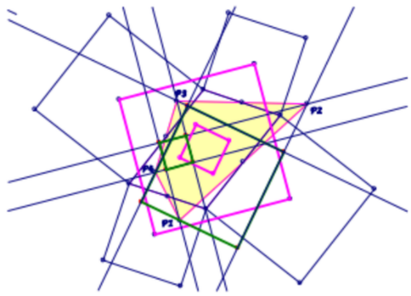


(If the point *P* runs along a circle round a vertex of the reference quadrigon, the centers of the circumscribed squares run along circles with the same radius round the centers of the circumscribed squares of the reference quadrigon.)

Now we consider the parallelogram with the same Varignon parallelogram as the reference quadrigon and draw squares over / under the sides. The centers of these squares give again a square (Napoleon square). The sides of such a Napoleon square are parallel to those of that circumscribed square of the reference quadrigon with opposite orientation.



The ratio of side-lengths of circumscribed squares of a quadrigon equals the ratio of side-lengths of the Napoleon squares described in the last paragraph. This ratio depends only on the Varignon parallelogram of the reference triangle.



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QG-2P6 Circumscribed Squares-01.pdf

QFG#375 13/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16812

Dear Chris,
the Varignon parallelogram was the background for my examination paper at university. So I have described a further aspect wrt the circumscribed squares of a quadrigon (see attachment).
Best regards Eckart

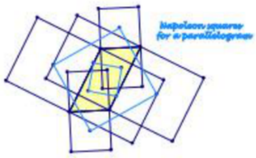
Attachment:

Circumscribed squares of a quadrigon
– an additional aspect –

This is a reference to my professor
Prof. Dr. Friedrich Bachmann

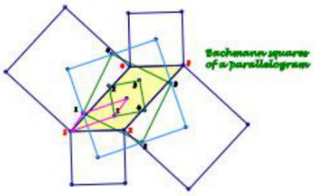
Napoleon squares of a parallelogram

Given a parallelogram, place a square outward / inside of each side, the centers of these squares give two new squares, here called the Napoleon squares (see [1], [2]).



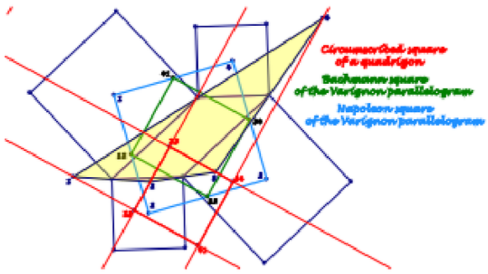
Bachmann squares of a parallelogram

The midpoints of the sides of the Napoleon squares of a parallelogram give two further squares, here called Bachmann squares. The parallelogram is the vectorial sum of these Bachmann squares (see [1], [3]).

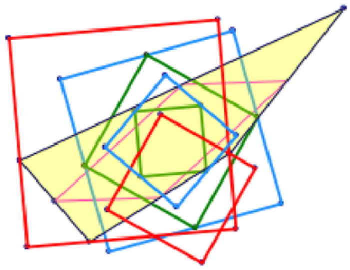


Circumscribed squares of a quadrigon

For a quadrigon there are two circumscribed squares (see QFG message 370).
For all quadrigons with the same Varignon parallelogram these circumscribed squares have the same size. Their sides are parallel to the sides of the Bachmann squares for the Varignon parallelogram.



Finally: There are three pairs of squares, which have the same ratio of size.



References

[1] E. Schmidt: Affin-reguläre n-Ecke und ihre regulären Komponenten. – MNU XXXIX,4.
[2] My homepage 05-5.
[3] F. Bachmann / E. Schmidt: n-Ecke.
– B.I - Hochschultaschenbücher 471/471a,
Bibliographisches Institut Mannheim 1970

Eckart Schmidt
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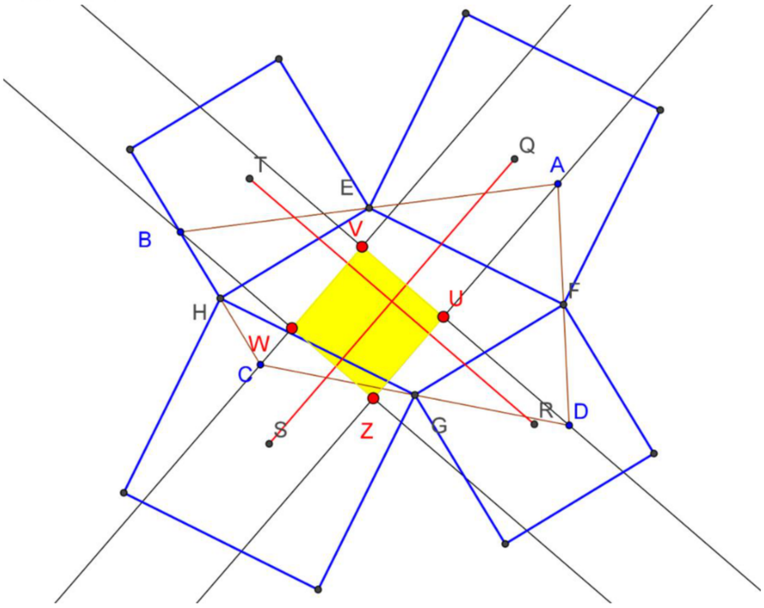
QG-2P6 Circumscribed Squares-01.pdf

QFG#376 13/12/2013 Dao Thanh Oai
https://groups.io/g/Quadri-Figures-Group/message/16813

Dear Mr Chris and Mr Eckart,
ABCD are a quadrilateral
EFGH are a parallelogram
TQRS is a square
then VWZU are squar
Best regards
Sincerely
Dao Thanh Oai

Attachment:



QFG#377 14/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16814

Dear Dao, Dear Eckart,
Very nice properties of the circumscribed square of a quadrilateral.
Thank you !
Best regards,
Chris

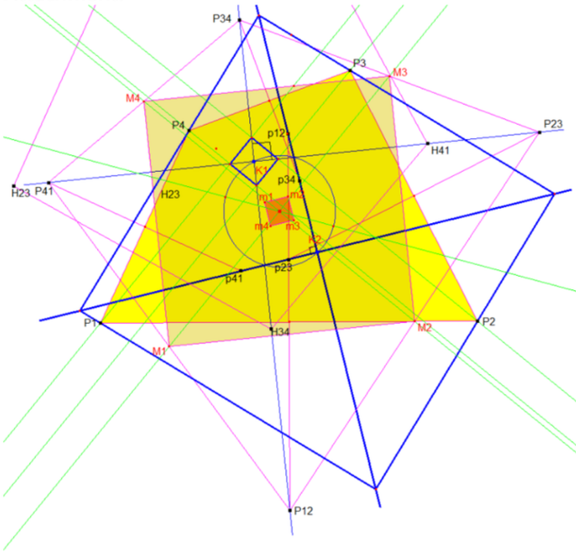
QG-2P6 Circumscribed Squares-01.pdf

QFG#378 14/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16815

Dear Friends,
I found some old notes of mine about squares related to a Quadrigon and the Van Aubel Configuration.
Here are the notes. See also attached figure.
Notes:
Pij and pij are erected at Pi.Pj such that P3.P34.P4.p34 is a square.
Pij is erected outwardly and pij is erected inwardly.
Now P12.P34 _|_ P23.P41 and P12.P34 = P23.P41
and p12.p34 _|_ p23.p41 and p12.p34 = p23.p41.
Let Mj=midpoint(Pij,Pjk) and mj=midpoint(pij,pjk).
M1.M2.M3.M4 and m1.m2.m3m4 are squares for obvious reasons.
Area(P1.P2.P3.P4) = Area(M1.M2.M3.M4) - Area(m1.m2.m3.m4)
P1.P2.P3.P4, M1.M2.M3.M4 and m1.m2.m3.m4 share the same centroid.
Let H41 be OrthoCenter(P12,P23,P34),
and H12 be OrthoCenter(P23,P34,P41),
and H23 be OrthoCenter(P34,P41,P12),
and H34 be OrthoCenter(P41,P12,P23).
Now Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4).
Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) is general property of a quadrangle.
Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4) because M1.M2.M3.M4 is Varignon-parallelogram.
The blue squares in the figure are formed by the lines parallel to the diagonals of the brown squares and passing through the vertices of P1.P2.P3.P4.
K1 and K2 happen to be their centers.
These squares are circumscribed squares of P1.P2.P3.P4.
Special property:
Area[M1.M2.M3.M4] / Area[m1.m2.m3.m4] = Area[N1.N2.N3.N4] / Area[n1.n2.n3.n4]
N1.N2.N3.N4 and n1.n2.n3.n4 are the blue squares.
K1 and K2 have the same distance d to the Centroid of the Reference Quadrigon.
In a quadrilateral there are 3 distances per QL-Quadrigon: da, db, dc.
Then da + db + dc = 0. (distances are signed).
1. the midpoint of the midpoints of the diagonals of the Reference Quadrigon (which is QA-P1) is the center of the Varignon Squares of the Internal and External Van Aubel Quadrigons.
2. the intersection points of the diagonals of the circumscribed squares also lie on the Thales Circle with diameter the midpoints of the diagonals of the Reference Quadrigon.
Best regards,
Chris van Tienhoven

QG-2P6 Circumscribed Squares-01.pdf

Attachment:



K1 and K2 have the same distance d to the Centroid of the Reference Quadrigon.
In a quadrilateral there are 3 distances per QL-Quadrigon: da, db, dc.
Then da + db + dc = 0. (distances are signed).

The blue squares are the formed by the lines parallel to the diagonals of the brown squares and passing through the vertices of P1.P2.P3.P4.
K1 and K2 happen to be their centers.
These squares are circumscribed squares of P1.P2.P3.P4.
Special property:
Area[M1.M2.M3.M4] / Area[m1.m2.m3.m4] = Area[N1.N2.N3.N4] / Area[n1.n2.n3.n4]
N1.N2.N3.N4 and n1.n2.n3.n4 are the blue squares.

Pij and pij are erected at Pi.Pj such that P3.P34.P4.p34 is a square.
Pij is erected outwardly and pij is erected inwardly.

Now P12.P34 _|_ P23.P41 and P12.P34 = P23.P41
and p12.p34 _|_ p23.p41 and p12.p34 = p23.p41.

Let Mj=midpoint(Pij,Pjk) and mj=midpoint(pij,pjk).
M1.M2.M3.M4 and m1.m2.m3m4 are squares for obvious reasons.

Area(P1.P2.P3.P4) = Area(M1.M2.M3.M4) - Area(m1.m2.m3.m4)
P1.P2.P3.P4, M1.M2.M3.M4 and m1.m2.m3.m4 share the same centroid.

Let H41 be OrthoCenter(P12,P23,P34),
and H12 be OrthoCenter(P23,P34,P41),
and H23 be OrthoCenter(P34,P41,P12),
and H34 be OrthoCenter(P41,P12,P23).
Now Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4).
Area(H12,H23,H34,H41) = Area(P12,P23,P34,P41) is general property of a quadrangle.
Area(P12,P23,P34,P41) = 2 * Area(M1.M2.M3.M4) because M1.M2.M3.M4 is Varignon-parallelogram.

Eckart Schmidt, september 18, 2012
die Quadratmitten liegen auf dem Thales-Kreis ueber den Diagonalenmitten.

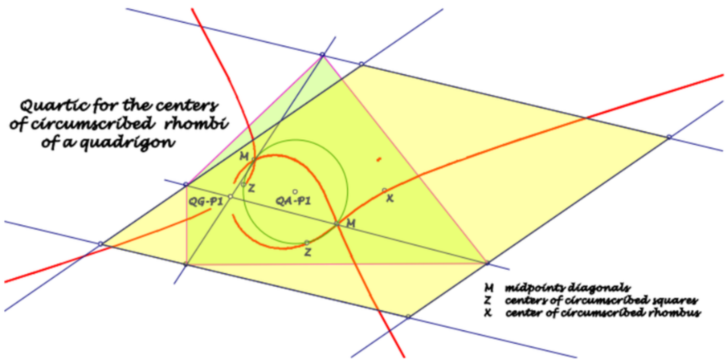
own observation:
1. the midpoint of the midpoints of the diagonals of the Reference Quadrigon (which is QA-P1) is the center of the Varignon Squares of the Internal and External Van Aubel Quadrigons.
2. the intersection points of the diagonals of the circumscribed squares also lie on the Thales Circle with diameter the midpoints of the diagonals of the Reference Quadrigon.

QG-2P6 Circumscribed Squares-01.pdf

QFG#384 15/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16821

Dear Chris,
in addition to circumscribed squares of a quadrigon:
The centers of circumscribed rectangles lie on the Thales circle about the midpoints of the diagonals.
The centers of circumscribed rhombi lie on a quartic (see attachment). The quartic contains the diagonal crosspoint QG-P1, the midpoints M of the diagonals and the centers Z of the circumscribed squares. Construction and equation can be given.
Best regards Eckart

Attachment:



QFG#385 15/12/2013 Chris
https://groups.io/g/Quadri-Figures-Group/message/16822

Dear Eckart,
Nice quartic with these familiar points on it.
I tried to construct the circumscribed rhombi.
In my picture I found 4 circumscribed rhombi.
Do you have a construction method?
I am interested in the equation too.
Best regards,
Chris

QG-2P6 Circumscribed Squares-01.pdf

QFG#386 16/12/2013 Eckart
https://groups.io/g/Quadri-Figures-Group/message/16823

Dear Chris,
attached a description of the construction for the circumscribed rhombi of a quadrigon (I'm sure, there will be a simpler way). The equation of the quartic is easy to calculate, see attachment.
Best regards Eckart

Attachments:

Equation of the quartic for the centers of circumscribed rhombi of a quadrigon (reference triangle P1P2P3 with P4(p:q:r))

$$\begin{aligned}\mathrm{quad}[\{x\_, y\_, z\_\}] := & -b^2 (r^2 x^4 + p^2 z^4) + (c^2 p^2 + a^2 p r - b^2 p r + c^2 p r + a^2 r^2) y^4 \\ & +2 b^2 (p + r) x z (r x^2 + p z^2) - 2 (a^2 - c^2) y (r^2 x^3 - p^2 z^3) - b^2 (p^2 + 4 p r + r^2) x^2 z^2 \\ & + (p + 2 q + 3 r) (c^2 p + 2 c^2 q - a^2 r + b^2 r) x^2 y^2 \\ & + (3 p + 2 q + r) (b^2 p - c^2 p + 2 a^2 q + a^2 r) y^2 z^2 \\ & -2 (c^2 p^2 + 2 c^2 p q + 2 c^2 p r + a^2 q r - b^2 q r + c^2 q r - b^2 r^2 + c^2 r^2) x y^3 \\ & -2 (a^2 p^2 - b^2 p^2 + a^2 p q - b^2 p q + c^2 p q + 2 a^2 p r + 2 a^2 q r + a^2 r^2) y^3 z \\ & +2 (a^2 - c^2) x y z (2 p r x + r^2 x - p^2 z - 2 p r z) + \\ & 2 (a^2 p^2 - b^2 p^2 + 3 a^2 p q - 3 b^2 p q + c^2 p q + 2 a^2 q^2 - 2 b^2 q^2 + 2 c^2 q^2 + 3 a^2 p r - \\ & 4 b^2 p r + 3 c^2 p r + a^2 q r - 3 b^2 q r + 3 c^2 q r - b^2 r^2 + c^2 r^2) x y^2 z\end{aligned}$$

Eckart Schmidt
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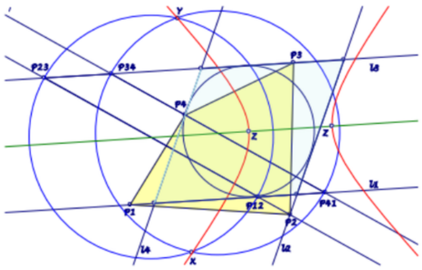
http : // eckartschmidt.de

Construction of circumscribed rhombi for a quadrigon



Let $P_1P_2P_3P_4$ be a quadrigon.

- l_1 variable line through P_1 with a variable point P_{12},
- l_3 parallel to l_1 through P_3,
- $l_2 = P_{12}P_2$,
- l_4 parallel to l_2 through P_4,
- $P_{12}P_{23}P_{34}P_{41}$ circumscribed parallelogram.
- Thales circles over $P_{12}P_{23}$ and $P_{34}P_{41}$ with intersections X, Y.
- Changing P_{12} on l_1 the locus of X, Y is a conic.
- $Z_{1,2}$ intersections of the conic and the mid-parallel of l_1 and l_3, centers of circumscribed rhombi.
- Circle round Z_1 or Z_2 tangent to l_1, l_3.
- A tangent through P_2 at this circle and a parallel through P_4 give a circumscribed rhombus.
- Changing the line l_1 through P_1 the centers Z_i give the quartic for the centers of circumscribed rhombi.

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QG-2P6 Circumscribed Squares-01.pdf

QFG#2923 09/03/2018 Chris
https://groups.io/g/Quadri-Figures-Group/message/19366

Subject: Introduction to Quadrilateral Geometry
11. Quadrigons and Squares
For readers who missed earlier paragraphs of this introduction:
a quadrigon is in ordinary speech a quadrilateral, a figure bounded by 4 lines.
There are several puzzlers about Quadrigons and Squares.
1. Construct a Square inscribed in a random Quadrigon.
2. Construct a Square circumscribing a random Quadrigon.
3. Construct a Square perspective with a random Quadrigon.
There are solutions for all these questions.
I won't give them right away. I just want to start a discussion here.
Who has a solution for one of these questions?
Chris van Tienhoven
www.chrisvantienhoven.nl

This is the last paragraph in the series of this introduction on Quadrilateral Geometry. Later on I will make another series about Polygon Geometry.

QFG#2925 10/03/2018 Bernard
https://groups.io/g/Quadri-Figures-Group/message/19368

Dear Chris, dear Eckart,
For the 2 1rst questions, see C.M. Hebbert The inscribed and circumscribed squares of a QL ... 1914
Best regards
Bernard

QFG#2926 10/03/2018 Eckart
https://groups.io/g/Quadri-Figures-Group/message/19369

Dear Chris,
wrt:
2. Construct a Square
circumscribing a random Quadrigon.
Let Pi be the vertices of the quadrigon
and Li the lines of the square with Pi on Li.
Any line L1* through P1 leads to a circumscribed rectangle
with lines Li*.
Consider the square L1*, L2*, L3*, L.
The 2nd intersection of
... a circle with diameter P3P4
... and a circle through P3, L3*^L
... ... and the pedal point of P2 on P1P3
... is L3^L4 of the searched square.
Replacing P2 by P4, the construction gives the 2nd square
... with contrary circulation.
Best regards Eckart

QG-2P6 Circumscribed Squares-01.pdf

Message: #475
Date: 2020-10-22
From: van10hoven@gmail.com
Subject: Re: 5 QA and 5 QL for a 5P

Dear Eckart,

I think this opens a myriad of new possibilities to find new points and other items for a Pentangle.

Wonderful!

In a Pentangle(P1,P2,P3,P4,P5) we used to work with Component Quadrangles (Pj,Pk,Pl,Pm), but now we can work with other Component Quadrangles QAi(Mij,Mik,Mil,Mim) and even with Component Quadrilaterals QLi(Ljkl,Ljkm,Ljlm,Llmn),
where Llmn = the collinear line (Mlm,Mmn,Mnl), etc.

About your example where the Clawson-Schmidt Conjugate wrt QLi maps Pi to 5P-s-P5.
We can reverse the process.
Given a quadrilateral QL1(L2,L3,L4,L5) and a random point R.
Let P1 be the Clawson-Schmidt Conjugate QL-Tf1(R) wrt QL1.
Let P2 be the reflection of P1 over L2.
Let P3 be the reflection of P1 over L3.
Let P4 be the reflection of P1 over L4.
Let P5 be the reflection of P1 over L5.
Now we have a Pentangle (P1,P2,P3,P4,P5) for which R = 5P-s-P5.

Best regards,

Chris

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Message: #476
Date: 2020-10-22
From: eckart_schmidt@t-online.de
Subject: Re: [Quadri-and-Poly-Geometry] QG-2P6

```
Dear Chris,

many thanks for the detailed overview wrt "Circumscribed Square
Centers",
... it was a good time of cooperation, but years ago,
... with interesting results, many not yet parat for me,
... especially message 384.

Best regards Eckart
```

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Message: #477
Date: 2020-10-23
From: eckart_schmidt@t-online.de
Subject: QL-Quartet Geometry

```
Dear all,

QL-quartet means four QL wrt a reference QL,
... taking the trilateral QL-Tr1 and a QL-line Li.

Here some observations wrt this QL-quartet:
  * Their QL-P1 lie on QL-Ci1.
  * Their QL-L2 intersect in QL-P10.
  * Their QL-Ci1 have 4 triple intersections in the dual points
  of the QL-lines.
  * Their QL-Ci3 have a common point in QL-P9.
  * Their QL-Ci4 have collinear centers
    on a line through QL-P11, orthogonal QL-L7.
  * Their QL-Ci5 have a common radical center in QL-P10.
  * Their QL-Ci6 have a common radical center in QL-P8.

Special case for QL-Ci2:
  * The 4 QL-Ci2 have double intersections
    in the midpoints of the sides of the dual QA.
  * The 4 QL-Ci2 have a common point on QL-Ci1,
    which is QA-P2 of the dual QA
  * and focus of an inscribed parabola of QL-Tr1
    with directrix QA-P4.QL-P10 (QA-P4 of the dual QA).
  * This parabola is the envelope of the QL-Tf2-images
    for lines of a pencil of a point on QL-L1.

What about this point and the parabola?

Best regards Eckart
```

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Message: #478
Date: 2020-10-24
From: eckart_schmidt@t-online.de
Subject: New 5P elements

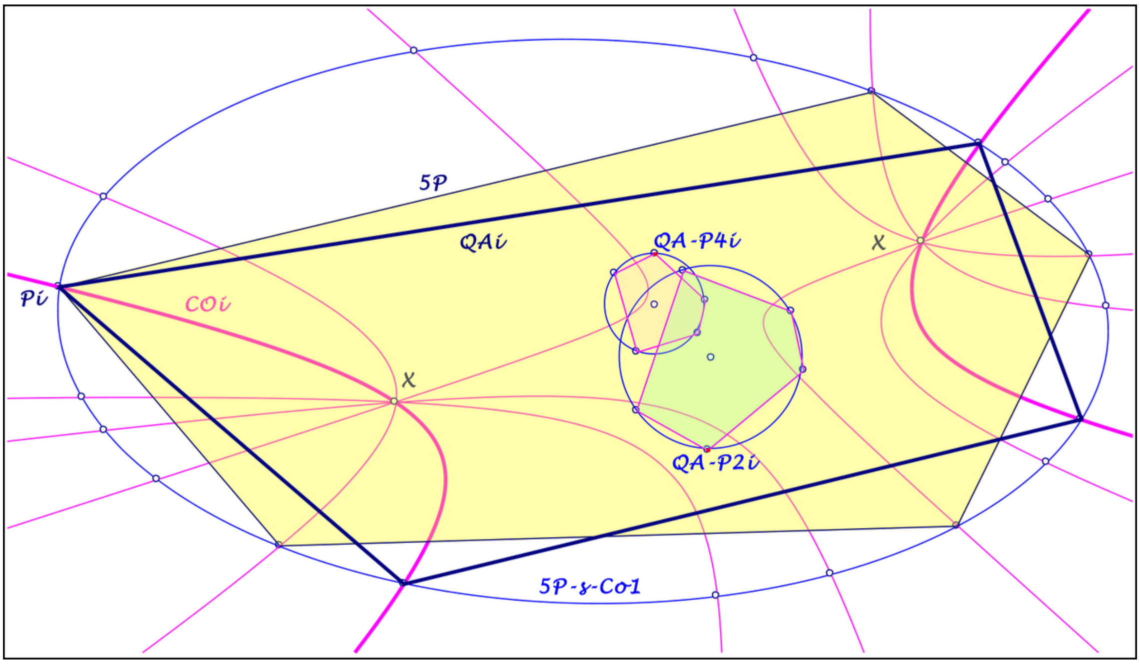
```
Dear all,

let us consider a 5P, its 5P-s-Co1 with tangents Ti in the
vertices Pi:
... Their QA-Tf2-conics COi wrt the QA
    of the remaining 5P-vertices
... have two common points X,
... their QA-Tf2-images wrt wrt the QA-components of 5P
... lie on the polar of X wrt 5P-s-Co1, which is 5P-s-Tf2(X).
What about these 5P-s-2Px?

A conic COi bears Pi
... and gives with three further intersections
    with 5P-s-Co1 a quadrigon QAi,
... whose QA-P2 as well as QA-P4 are concyclic on two circles
... with QA-P2i = 5P-s-Tf3(QA-P4i).

What about the two circles and their centers?

Best regards Eckart
```



2020-10-24.pdf

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Message: #479
Date: 2020-10-25
From: bernard.keizer@gmail.com
Subject: Re: QL-Quartet Geometry

```
Dear Eckart,
You have already mentionned part of these properties, but I
don't remember when and I can't find the corresponding message.
I remember perfectly that we discussed also the QL with the 4
Newton Lines of your 4 quartets, which had the same inscribed
parabola as the original QL ...
Can you help me ?
Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #480
Date: 2020-10-25
From: bernard.keizer@gmail.com
Subject: Re: New 5P elements

```
Dear Eckart,
Beautiful new 5P (and 5L) points !
I reproduced your points X, but I don't find any particular
property.
I hoped they would lie on the main axis of the conic 5P-s-Co1,
but it isn't the case ...
Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #481
Date: 2020-10-26
From: eckart_schmidt@t-online.de
Subject: Re: [Quadri-and-Poly-Geometry] QL-Quartet Geometry

```
Dear Bernard,

excuse I cannot remember, that I mentioned these properties
before,
... but an eighty years old memory may have gaps.
Wrt the second remark, I cannot confirm,
... that the four Newton lines of a QL-quartet
... have the same inscribed parabola as the reference QL.
QL-Tr1 plus QL-L1 give a QL with Newton line tangent to QL-Co1.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #482
Date: 2020-10-26
From: bernard.keizer@gmail.com
Subject: Re: QL-Quartet Geometry

```
Dear Eckart,
Finally, I've found the reference.
The messages are from 393 to 415.
By the way, not the QL of the 4 Newton Lines, but it's DQL,
formed by the sides of midDT and the Newton Line of the DQL of
the original QL have the same parabola as this original QL.
The parabola of QL is inscribed in midDT and tangent to the
Newton Line of DQL.
You wrote new P17 is old P1 ...
Best regards
Bernard
```

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Message: #483
Date: 2020-10-29
From: eckart_schmidt@t-online.de
Subject: "Isogonal conjugation wrt a QL"?

```
Dear all,

I think this nomination - ref [27], page 187 - is not in the
sense of EPQ:
The "isogonal conjugate" of a point P wrt a quadrigon QG =
P1P2P3P4 is there
... another quadrigon QG* with sides, which are the reflections
    of PPi in the angle bisector at Pi.
The "isogonal conjugate" of QA-P4 is a parallelogram,
... centered in QA-P2 with diagonals through QG-2P2.
The "isogonal conjugate" of QL-P1
... degenerates in the infinity point of QL-L1.

Further properties:
The "isogonal conjugates" of points P on QL-Cu1
... degenerate in CSC(P) on QL-Cu1.
The "isogonal conjugates" wrt the three QG-components of a QA
... give three QG,
    each pair with a common diagonal through a QA-Tr1-vertex.
The "isogonal conjugates" wrt the three QG-components of a QL
... give the three QG-components of a quadrangle
... with six lines through the six QL-points.
The last property can be interpreted as:
... The "isogonal conjugate" of a point wrt a QL gives a QA.

But finally:
The diagonal crosspoint of the "isogonal conjugate" of a point P
wrt a quadrigon QG
... is QG-Tf2(P),
    which is the regular isogonal conjugate of P wrt QG-Tr3.

Best regards Eckart
```

← Previous Next → ↩ Message Index ↑ Subjects

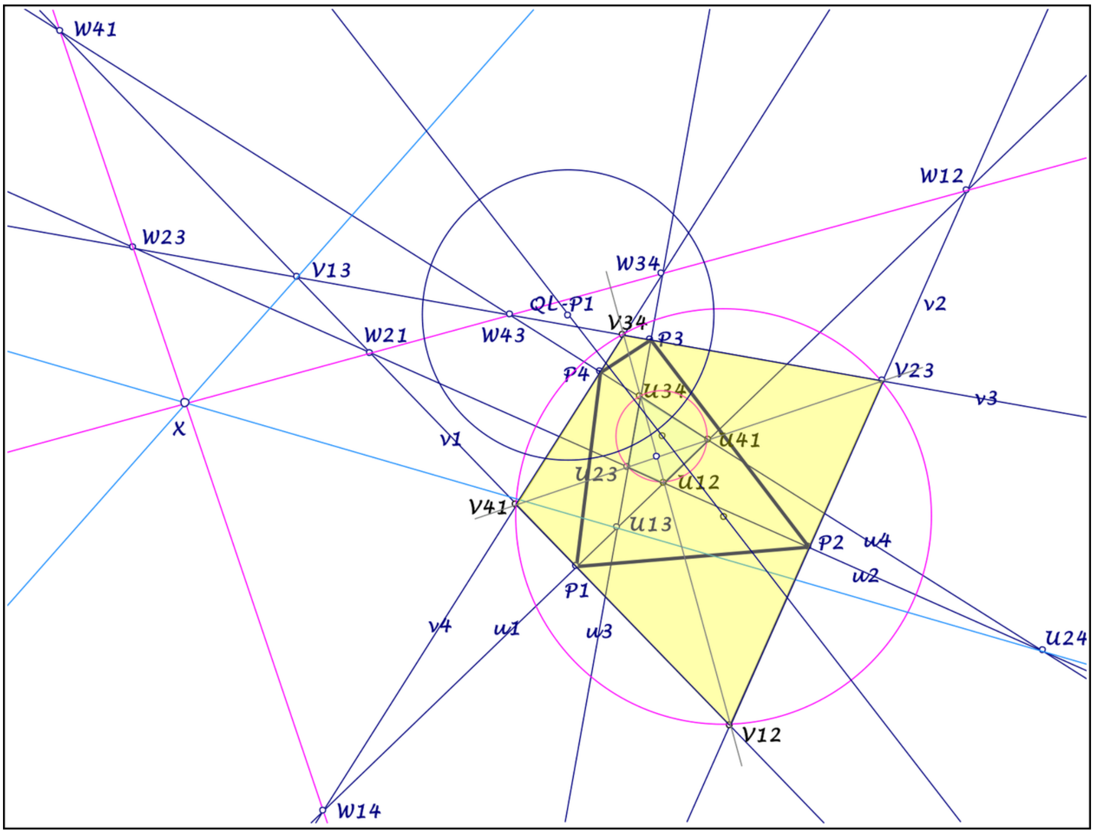
Message: #484
Date: 2020-10-30
From: eckart_schmidt@t-online.de
Subject: QG-angle bisectors

```
Dear all,

consider for a QG = P1...P4 the inner/outer angle bisectors
ui/vi at Pi
... with intersections Uij = ui ^ uj, Vij = vi ^ vj:
... U12,U23,U34,U41 and V12,V23,V34,V41 give two perspective QG
... on two circles CI, centered on the Steiner axis,
... orthogonal to the inversion circle of CSC,
... CSC-invariant with partners collinear
    with the CSC-image of the center of CI.
Let ui ^ vj = Wij unequal Wji = uj ^ vi
... with W12,W21,W34,W43 and W23,W32,W41,W14
    collinear on L1 and L2,
... then L1, L2, U13U24, V13V24 have a common point X.

What about this common point?

Best regards Eckart
```



2020-10-30.pdf

Message: #485
Date: 2020-10-30
From: eckart_schmidt@t-online.de
Subject: QG-P18

Dear Chris,

perhaps worth to be mentioned in EQF:
QG-P18 is the common tangential of QG-2P2a,b wrt QL-Cu1.

Best regards Eckart

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Message: #486
Date: 2020-10-30
From: bernard.keizer@gmail.com
Subject: Re: QG-angle bisectors

Dear Eckart,
Your QG is a QL !
2 vertices are missing P5 and P6.
There are in fact 8 points on 8 circles (and not 2), centered 4 on each Steiner axis.
With your notation, X = V56
See EQF QL-8P1
Best regards
Bernard

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Message: #487
Date: 2020-10-30
From: van10hoven@gmail.com
Subject: Re: QG-P18

Thanks,

Chris

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Message: #488
Date: 2020-10-31
From: eckart_schmidt@t-online.de
Subject: Re: [Quadri-and-Poly-Geometry] QG-angle bisectors

```
Dear Bernard,

excuse my bounded horizon,
you are right, thanks for clearance.
Steiner's theorem on the complete quadrilateral
is well known to me,
... but I haven't interpreted the quadrigon as quadrilateral.

Best regards Eckart
```

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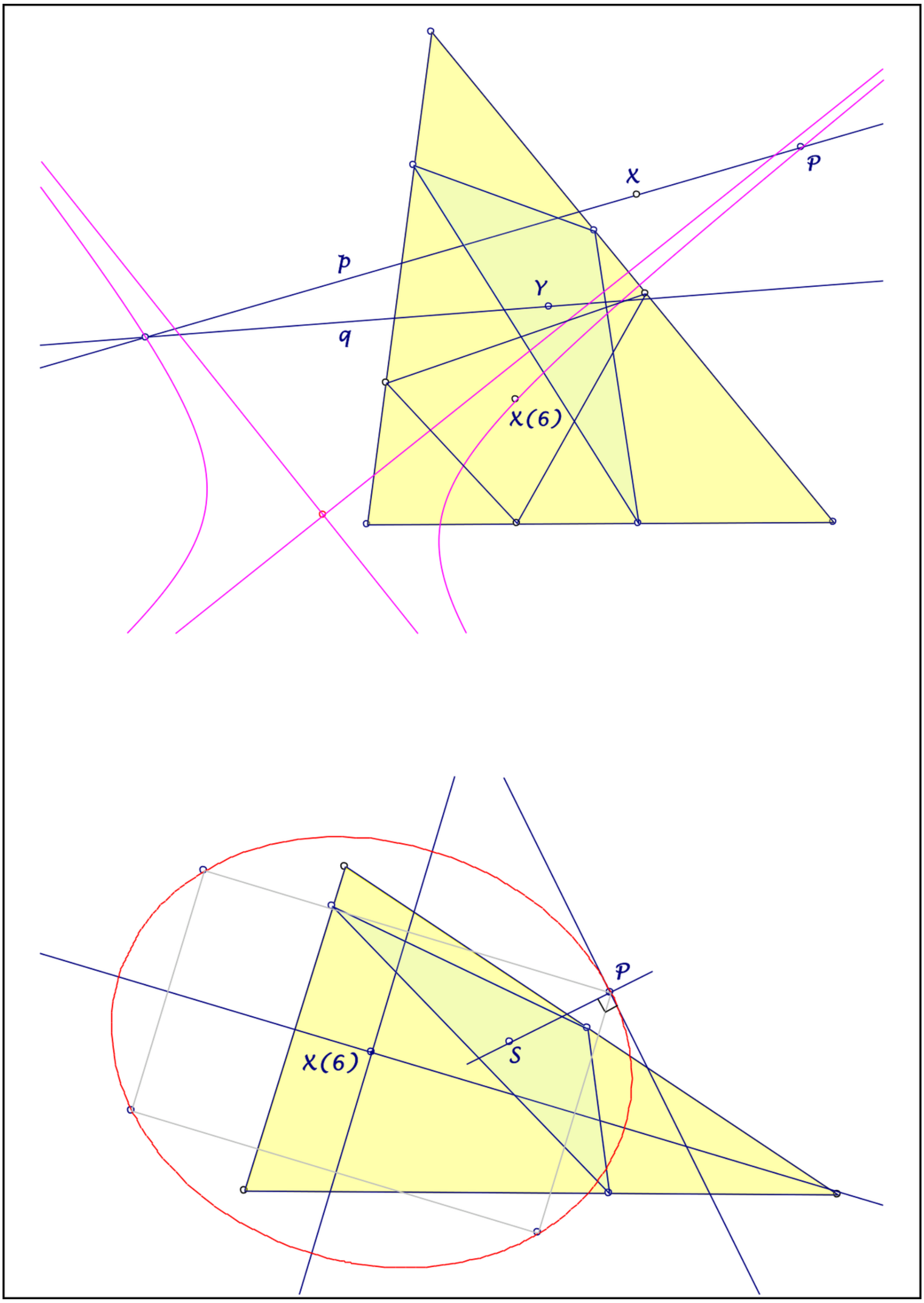
Message: #489
Date: 2020-11-01
From: eckart_schmidt@t-online.de
Subject: Generalization of QL-P26

```
Dear all,

the Lemoine point X(6) of a triangle is the point,
... such that the sum of the squares of the distances
    to the sides is minimal.
The Lemoine point can be generalized for n-lines,
for a QL it is QL-P26.

The following reference gives two remarks (Gerono 1828):
"G. Berkhan und W.FR. Meyer: Neuere Dreiecksgeometrie", page
1272/1273
<https://www.deutsche-digitale-bibliothek.de/item/KCQIVEWFBTAV2B
LY6GNPJBRXFTRZJSKG>
  * The Lemoine point of a n-line is the centroid of its pedal
  n-point.
  * The loci for points with constant square sum are certain
  ellipses.
With the results of QFG-message old583
... I found a general construction for the Lemoine point
    of a n-line
... and a construction of the certain ellipses
    for constant square sum.

Consider a n-line L1,...Ln and any point P with varying lines p
through P,
... let Y be the centroid of the pedal n-point of points X on p.
Varying X on p, the point Y gives a line q
... and the intersections of p and q
    give an orthogonal hyperbola,
... which have for all points P a common point,
    the Lemoine point.
Attached the construction for a triangle.

The certain ellipses with constant square sum through a chosen
point P
... are homothetic centered in the Lemoine point,
... with axes parallel to the asymptotes
    of the orthogonal hyperbola wrt P.
The point P and its reflections in the axes and center
give 4 points of the ellipse,
... a 5th element for a construction is the tangent in P,
... perpendicular to the line,
    connecting P with the centroid S of its pedal n-point.
```

Attached such an ellipse for a triangle.

Best regards Eckart

PS: I have constructed the Lemoine point for a 5L,
... but found no connections with known 5L-points.



2020-11-01.pdf

Message: #490
Date: 2020-11-01
From: van10hoven@gmail.com
Subject: Re: Generalization of QL-P26

Dear Eckart,

Did you include the information of nL-n-P6: nL-Least Squared Distances Point into your considerations?

Best regards,

Chris

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Message: #491
Date: 2020-11-02
From: bernard.keizer@gmail.com
Subject: Re: Generalization of QL-P26

Dear Chris, dear Eckart,
For a QL, consider for each reference triangle the circumconic through it's centroïd and through the corresponding vertice of the dual QA (perspector of the triangle and DT).
This conic passes also through the 2nd intersection of the circumcircle of the reference triangle with the Dimidium circle (the other being QL-P1) and through the S-points.
Consider now the isogonal line of the conic wrt the triangle (through the Lemoine point X6 of the triangle).
P26 is the common point of these 4 lines.
Best regards
Bernard
PS Léon Ripert (see EQF) gives a construction of P26 with the symmedians, which is generalisable to n lines

Dimidium circle and Lemoine point : P6, P13, P17, P24 and P26 revisited

A. Dimidium circle (cf figure1 Dimidium circle)

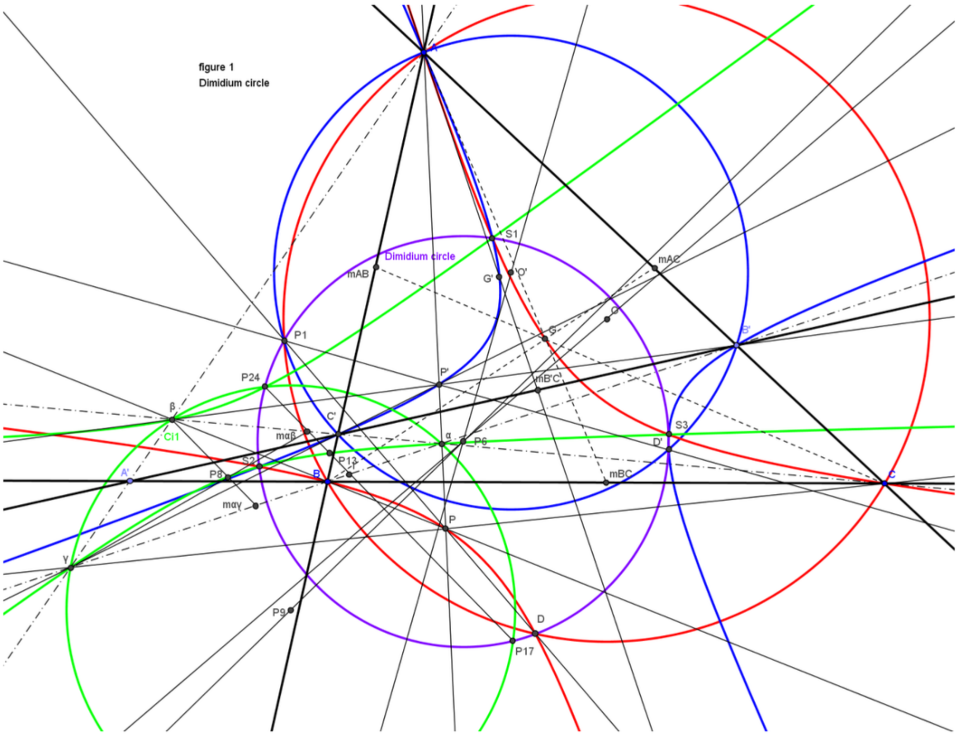
For the reference triangle ABC with centroïd G and circumcenter O, let P be the pole of the 4th line of equation lx + my + nz = 0 ; P has coordinates 1/l , 1/m , 1/n and is the perspector of the triangle ABC and the diagonal triangle αβγ with circumcenter P9 and centroïd P8.

The isotomic conjugate of P, IsoP, has coordinates l, m, n and it's polar is the line with equation x/l + y/m + z/n = 0, which is parallel to the Newton Line L1.

Let D be the second intersection of the line P1P with the circumcircle of ABC ; D has coordinates [a^2 (l – n) + b^2 (m – n)] [a^2 (l – m) + c^2 (n – m)].

D belongs to the line P1P, the circumcircle of ABC, the Dimidium circle and the conic through A, B, C, G and P, with equation (m – n) yz + (n – l) xz + (l – m) xy = 0 ; D is the reflexion of P1 in P6O and is the 4th intersection between the Dimidium circle and the conic.

There are 4 conics like this and 4 points D like this : they are all on the Dimidium circle as 4th intersection with the 4 conics, the 3 others being the 3 well-known S-points and P6 is on the 4 perpendicular bisectors of the lines P1P.



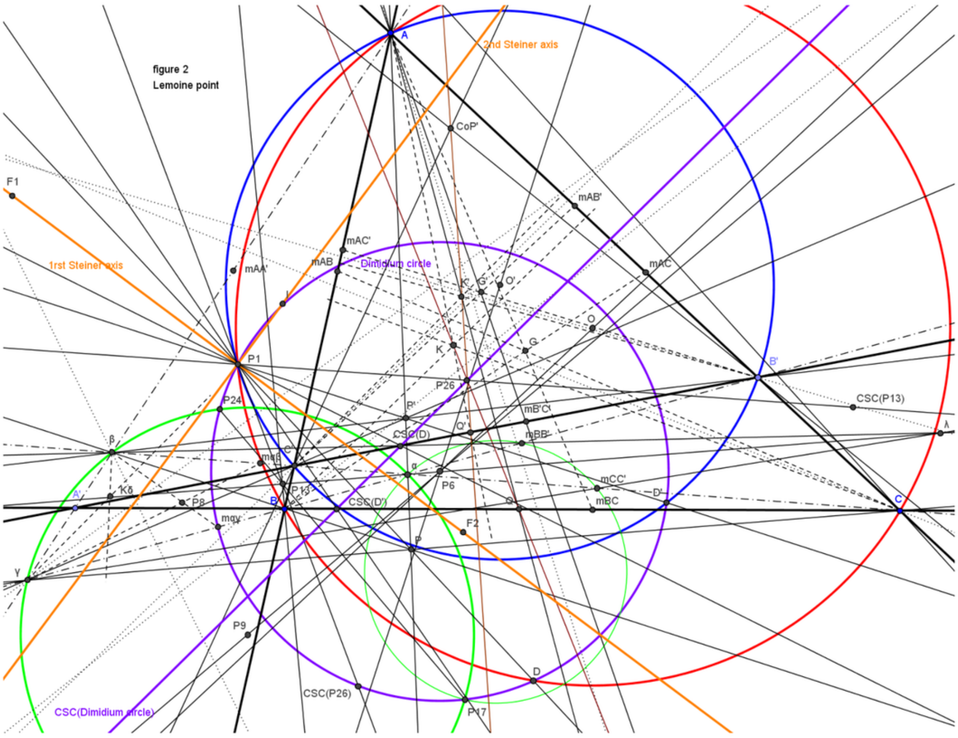
Dimidium and Lemoine.pdf

It's particularly interesting that the same property goes for the diagonal triangle : P17 is the Miquel point of the diagonal QL, formed by the 3 diagonals and the Newton Line, P13 is the pole of the Newton Line and P24 is the second intersection of P17P13 with the circumcircle of the diagonal triangle αβγ Ci1.
P24 belongs to the line P17P13, the circumcircle Ci1, the Dimidium circle and the conic through α, β, γ, P8 and P13.
P24 is on the Dimidium circle as 4th intersection with the conic, the 3 others being the 3 well-known S-points and P6 is on the perpendicular bisector of the line P17P24.
On this way, P6 is stronger than P3, as the Kantor-Hervey points belongs to 4 lines (the perpendicular bisectors of the Euler segments) whereas the point P6 belongs to 5 lines (the perpendicular bisectors of the lines P1P and P17P13).
As we already know (cf my message 164 on diagonal quadrilaterals), the sides of the DT-QL are the diagonals of the QL and it's 4th line is the Newton Line of the QL. The equation of the Newton Line of the QL is the same as the equation of the 4th line in CT-cordinates with l^2, m^2 and n^2 instead of l, m and n ; the DT-coordinates of P17, P13 and P24 are the same as the CT-coordinates of P1, P and D and the equation of the conic associated to the DT-QL is the same as the CT-coordinates of the conic associated to the QL with the same modification.

B. Lemoine point (cf figure2 Lemoine point)

The centroïd G and the Lemoine point K of ABC have respectively coordinates (1,1,1) and (a^2,b^2,c^2) ; the centroïd G' and the Lemoine point K' of AB'C' have respectively coordinates [-- , l(n-l) ,l(m – l)] and [-- , $b^2/l(n-l)$, $c^2/l(m-l)$].
The equation of the symmedian of AB'C' in A is $(n – l)\, y / b^2 + (l – m)\, z / c^2 = 0$. The symmedian of AB'C' in A, of A'BC' in B and of A'B'C in C cuts respectively BC, AC and AB in 3 points Q, R and S ; these 3 points are on a line of equation $(m – n)\, x / a^2 + (n – l)\, y / b^2 + (l – m)\, z / c^2 = 0$, which contains the point K, the point CoP, isogonal conjugate of P, of coordinates a^2l, b^2m,c^2n and the point CoD, isogonal conjugate of D, as infinity point.
The point P26 or Lemoine point of the QL is the intersection of the 4 lines KCoP (if the 4th line is through K, the point P26 is in K, as the distance to the 4th line is 0 and the point which minimise the sum of the squares of the distances to the 3 sides of ABC is precisely the Lemoine point K of the triangle).
The lines KCoP are the isogonal conjugates of the 4 conics we met in the 1rst part.

Dimidium and Lemoine.pdf



It's interesting to consider now the isotomic conjugates of the conics ; they are lines through the centroïds and through the isotomic conjugates of the points P, IsoP and of the point CoP26, IsoCoP26, which is the isotomic conjugate of the isogonal conjugate of P26 (the operation IsoCo, product of an isogonal and an isotomic transformation, is a collineation) ; this point has the same CT-coordinates as P26, divided respectively by a^2, b^2 and c^2.

Dimidium and Lemoine.pdf

Message: #492
Date: 2020-11-02
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of QL-P26

```
Dear Chris,

thanks for information wrt nL-n-P6,
... it shows, that I am not familiar with your nomination,
... I found no lead from QL-P26 to this item,
... but I think, there are some simplifying aspects
    in my message.

Best regards Eckart
```

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Message: #493
Date: 2020-11-02
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of QL-P26

```
Dear Bernard,

thanks for a new construction of QL-P26,
... but I think, it cannot be generalized.

Best regards Eckart
```

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Message: #494
Date: 2020-11-03
From: bernard.keizer@gmail.com
Subject: Re: Generalization of QL-P26

```
Dear Eckart,
It's not really a new construction!
I've put this attached file many years ago and the article by
Léon Ripert (ref 45 in EQF) was wriitten more than 100 years
ago!
Léon Ripert mentions in his article that the construction is
generalisable to a n lateral anfd gives a reference (NA 1902),
but I couldn't find it ...
Best regards
Bernard
```

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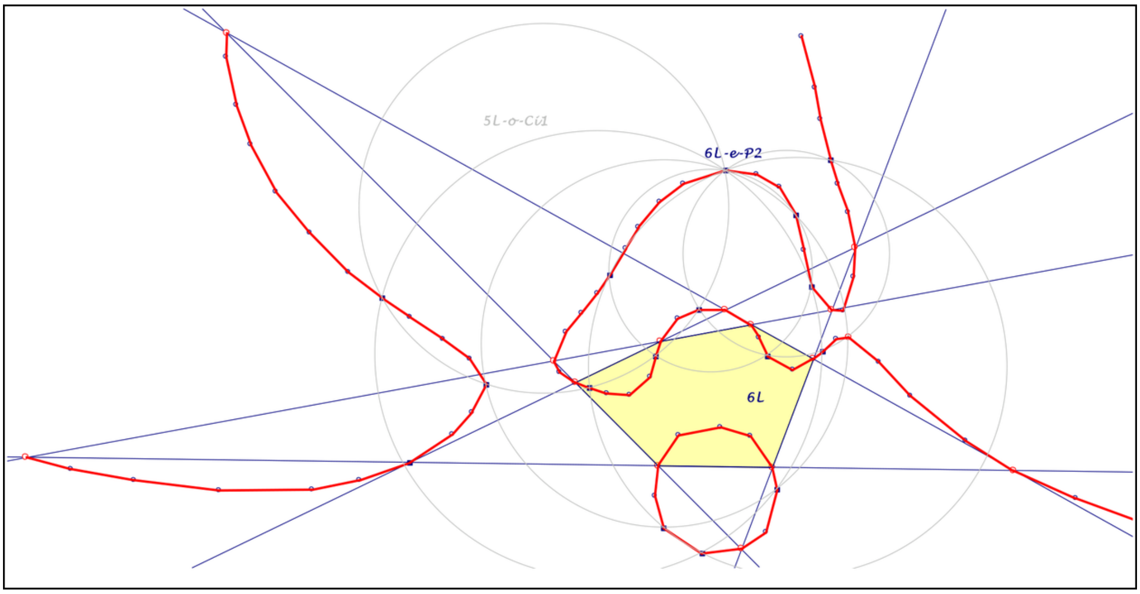
Message: #495
Date: 2020-11-05
From: eckart_schmidt@t-online.de
Subject: Conical pedal 6-points of a 6-line

```
Dear all,

for a quadrilateral points with a cyclic pedal quadrangle lie on
QL-Cu1,
... for a pentalateral only the foci of the inscribed conic
5L-s-Co1 have cyclic pedal 5P,
... for a 6-line there is a higher curve Cv for points
    with a conical pedal 6P:
    Cv is circumscribed the 6L (15 points),
    Cv bears the Miquel points QL-P1 of QL-components
    of the 6-line (15 points),
    ... which are the 6L-Tf1-images of the 6L-vertices
        with degenerated conics,
    Cv bears 6L-e-P2 ...
Attached an approximated drawing of Cv. What about this curve?

Best regards Eckart
```



2020-11-04.pdf

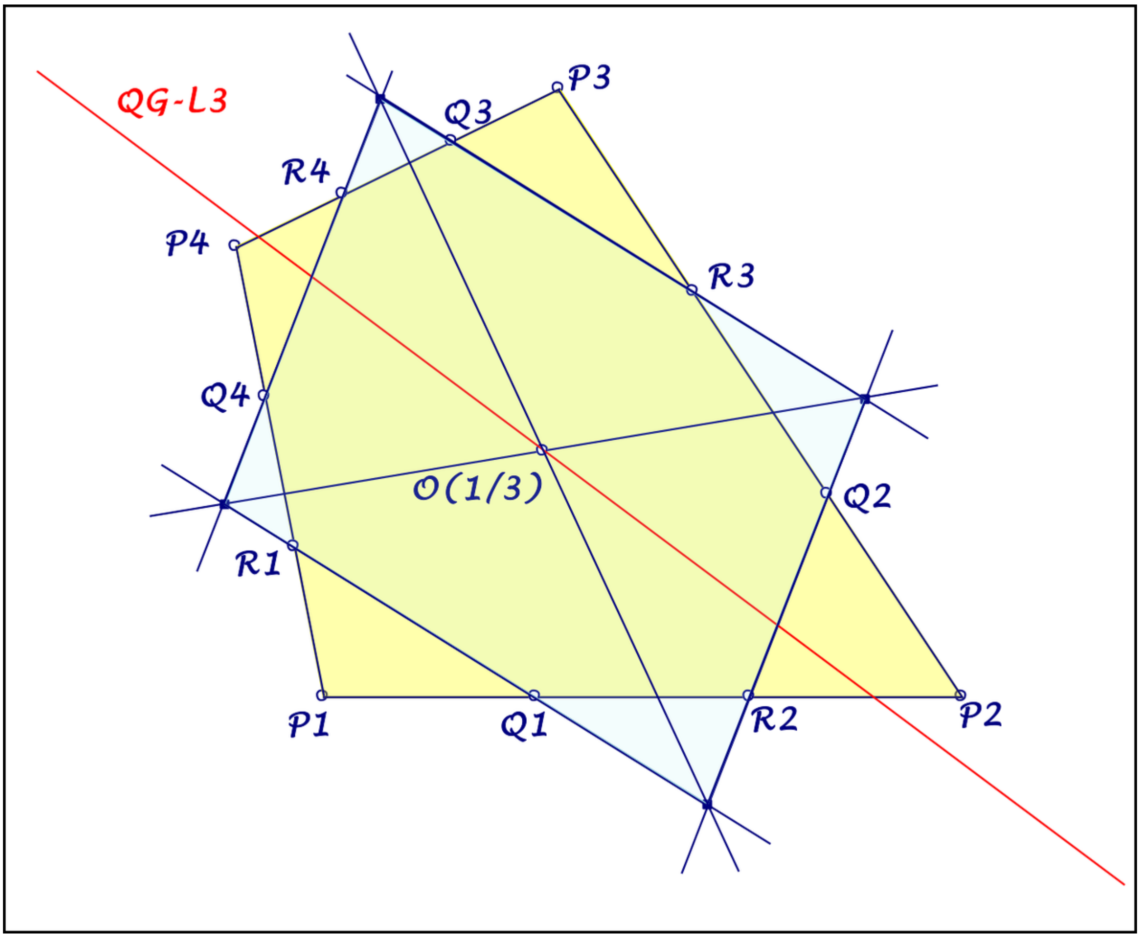
Message: #496
Date: 2020-11-07
From: eckart_schmidt@t-online.de
Subject: QG-L3 and Wittenbauer Parallelograms

```
Dear all,

this is a result wrt the following reference:
<https://ijgeometry.com/wp-content/uploads/2015/03/4.pdf>

Consider a quadrigon P1...P4 and following points on the sides:
... Qi divides PiPi+1 with ratio r,
    Ri divides Pi Pi-1 with ratio r,
... which give the sides QiRi for a Wittenbacher parallelogram
... with diagonal crosspoint O(r) on QG-L3:
QG-L3 is the locus of diagonal crosspoints of Wittenbacher
parallelograms.

Best regards Eckart
```



2020-11-07.pdf

Message: #497
Date: 2020-11-08
From: eckart_schmidt@t-online.de
Subject: Generalization of QG-L3 to nG-n-Lx

```
Dear all,

in the sense of #496 there is a generalization of QG-L3 to
nG-n-Lx:
Consider a n-gon P1...Pn and following points on the sides:
... Qi divides PiPi+1 with ratio r, Ri divides Pi Pi-1
    with ratio r,
... which give the sidelines QiRi for a second n-gon,
... the locus of nP-n-P1 gives a line
    through nP-n-P1 of the reference n-gon.

Best regards Eckart
```

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Message: #498
Date: 2020-11-08
From: bernard.keizer@gmail.com
Subject: Re: Conical pedal 6-points of a 6-line

```
Dear Eckart,
Very interesting item !
I spent a lot of time in drawing figures.
I can't identify the curve, but I found new points !
For the vertices, obviously it remains only 5 of the 6 pedals.
For the Miquel points, the conic is made of 2 lines 4 on a
Simson Line and 2 others.
The vertices and the Miquel points are on 20 circles.
There are 10 copples of circles Aij, Aik, Ajk, Mlm, Mln and Mmn
on a circle and vice-versa on the other.
The intersections of 2 corresponding circles give also (when
they are real) 2 points on your curve with the conic degenerated
in 2 Simson Lines.
Best regards
Bernard
```

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Message: #499
Date: 2020-11-08
From: van10hoven@gmail.com
Subject: Re: Generalization of QG-L3 to nG-n-Lx

Dear Eckart,

Great achievement. This Centroids Line will be the first general item in an n-Gon.

Because of the construction you also prescribed a method for finding items in an n-Gon. Quite different from the level-up constructions in an n-Point (see nP-n-Luc1) or an n-Line (see nL-n-Luc1 till nL-n-Luc5).
I will include the Centroids Line in EPG in due time as nG-n-L1 Centroids Line.

About the line in 4-Gon/Quadrigon I noticed that:
QG-P1, QG-P4, QG-P8, QG-P15, QA-P1 lie on QG-L3 = 4G-n-L1.
The ratio r involved to construct these points are:
* QG-P15: r = 0
* QG-P4: r = 1/3
* QA-P1: r = 1/2
* QG-P8: r = 2/3
* QG-P1: r = 1

Best regards,
Chris

Message: #500
Date: 2020-11-09
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of QG-L3 to nG-n-Lx

```
Dear Chris,

thanks for interest, here a remark wrt QG-L3 independent of the
generalization:
Every line intersects the sides of a QG
... in points, which divide the sides with product 1,
... for QG-L3 their 4th harmonic points on the sides
    are collinear
... on a line through QG-P1, parallel to QL-L1,
    orthogonal to QG-P5.QG-P15;
... the Thales circles about the harmonic points on the sides
... have radical center in a new QG-point,
... which is the intersection of QG-P1.QG-P17 and QG-P5.QG-P15.

Best regards Eckart
```

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Message: #501
Date: 2020-11-09
From: van10hoven@gmail.com
Subject: Re: Generalization of QG-L3 to nG-n-Lx

```
Dear Eckart,

I checked your interesting new results in a 4-Gon.
I looked to your first property that QG-L3 divides the sides
with product 1.
I checked it in a Quadrigon/4-Gon and can confirm your results.
Then I checked it for a 5-Gon and again the product of ratios
was 1.
Then I tried the algebraical approach.

Surprisingly I found the property is valid for any random line
wrt any random n-Gon!
I checked it algebraically for a random line wrt a 3-Gon, 4-Gon,
5-Gon and 6-Gon and it holds.
So I feel confident enough for this conjecture:
* Given an n-Gon P1.P2. … .Pn and a random line L.
* Each side of the n-Gon can be presented as PiPj,
  where i and j are cyclic consecutive numbers
  from the set (1,2, …, n).
* Let line L intersect the n sides PiPj of the n-Gon in Sij.
* Let dij = signed distance (Pi,Sij)
  and eij = signed distance (Sij,Pj).
* Now Product(dij) = Product(dij) or P dij = P eij.
It's so simple that I think it should have been known for a long
time.
I proved it algebraically for n=3, n=4, n=5 and n=6.
About your harmonic properties I could not generalize it for n=4
and further.
It looks like we are finally going to disclose the realm of the
n-Gon.

Best regards,
Chris
```

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Message: #502
Date: 2020-11-09
From: bernard.keizer@gmail.com
Subject: Re: Generalization of QG-L3 to nG-n-Lx

```
Dear Chris, dear Eckart,
It seems you are rediscovering Menelaüs theorem for a n-gon !
(see Mathworld)
Best regards
Bernard
```

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Message: #503
Date: 2020-11-10
From: eckart_schmidt@t-online.de
Subject: 5L-n-P3 on 5L-s-P4.5

```
Dear Chris,

is the following observation correct and in EPG?
5L-n-P3 ratiopoint 5L-s-P4.5(-3:2)

Best regards Eckart
```

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Message: #504
Date: 2020-11-10
From: bernard.keizer@gmail.com
Subject: Re: Generalization of QG-L3 to nG-n-Lx

Dear Chris, dear Eckart,
There is also a generalisation of the Ceva theorem for a n-gon
... (see Mathworld again)
Best regards
Bernard

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Message: #505
Date: 2020-11-10
From: profmd1@mweb.co.za
Subject: Re: [Quadri-and-Poly-Geometry] Generalization of QG-L3 to nG-n-Lx

---

Hi everyone

Yes, I also recall a 1995 paper on the Ceva generalization, as
well as the Menelaus generalization, to n-gons by Branko
Grunbaum & Shepard in the Mathematics Magazine.
Here's a download link to the paper:
http://www-irm.mathematik.hu-berlin.de/~agricola/elemgeo_dateien
/grunbaum-shepard.pdf |
http://www-irm.mathematik.hu-berlin.de/~agricola/elemgeo_dateien
/grunbaum-shepard.pdf

Regards
Michael

From: "Bernard Keizer" <bernard.keizer@gmail.com>
To: Quadri-and-Poly-Geometry@groups.io
Sent: Tuesday, 10 November, 2020 15:08:02
Subject: Generalization of QG-L3 to nG-n-Lx

Dear Chris, dear Eckart,
There is also a generalisation of the Ceva theorem for a n-gon
... (see Mathworld again)
Best regards
Bernard

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Message: #506
Date: 2020-11-10
From: van10hoven@gmail.com
Subject: Re: 5L-n-P3 on 5L-s-P4.5

```
Dear Eckart,

This ratiopoint property can be found in the drawings of 5L-s-P4
and 5L-s-P5.
I just added it at 5L-n-P4.

Best regards,

Chris
```

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Message: #507
Date: 2020-11-10
From: van10hoven@gmail.com
Subject: Re: Generalization of QG-L3 to nG-n-Lx

```
Dear Bernard and Michael,

Thanks for all information.
Sometimes it is strange that somehow some well-known information
did not get through.

Best regards,

Chris
```

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Message: #508
Date: 2020-11-11
From: eckart_schmidt@t-online.de
Subject: Re: 5L-n-P3 on 5L-s-P4.5

```
Dear Chris,

thanks for information,
... but there will be a typo in your message:
... Replace 5L-n-P4 by 5L-n-P3.

Best regards Eckart
```

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Message: #509
Date: 2020-11-11
From: oaidt.evnpsc@gmail.com
Subject: Brother of Japanese theorem for cyclic quadrilaterals

---

```
Dear all,

I proposed to You a problem: Brother of Japanese theorem for
cyclic quadrilaterals

In https://mathoverflow.net/questions/368925

Best regards
Dao Thanh Oai
```

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Message: #510
Date: 2020-11-11
From: eckart_schmidt@t-online.de
Subject: Re: Conical pedal 6-points of a 6-line

```
Dear Bernard,

excuse my late answer,
I tried to add some further observations, but in vain.
Thanks for the unexpected further 20 points -not always real- of
the curve.
It would be good, to find a transformation, which maps the curve
to itself.

Best  regards Eckart
```

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Message: #511
Date: 2020-11-11
From: bernard.keizer@gmail.com
Subject: Re: Conical pedal 6-points of a 6-line

```
Dear Eckart,
Really fascinating curve !
I've found further points, intersections (when real) between
each line and the corresponding circle of Miquel points.
This time, the conic is not degenerated !
So we have possibly 7 points of intersection between a line and
the curve.
The curve could be a septic ... (But in this case, it should
intersect a circle in 14 points and I find possibly 8, 10 if it
is circular ?)
Is it correct that the 2 corresponding circles in 498 are 6L-Tf1
partners ? (also the 2 intersections)
Is it correct that the line and the corresponding Miquel circle
are the same way 6L-Tf1 partners ? (also the 2 intersections)
Last, is it possible that your curve is 6L-Tf1 invariant ?
Best regards
Bernard
```

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Message: #512
Date: 2020-11-11
From: bernard.keizer@gmail.com
Subject: Re: Conical pedal 6-points of a 6-line

```
Dear Eckart,
Last idea, but I suppose you had it already, does your curve
pass through the foci of the 6 inscribed conics (which are also
6L-Tf1 partners, as you mention in EPF) ?
I'm not able to check this idea myself ...
Best regards
Bernard
```

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Message: #513
Date: 2020-11-12
From: van10hoven@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Eckart, dear friends,

I looked to the harmonic conjugates of the intersection points
of a line with the edges of a 4-gon. I found algebraically that:
When in a 4-Gon we have any line through the diagonal crosspoint
QG-P1, then the harmonic conjugates of the intersection points
with the 4 edges are collinear on another line through QG-P1.
Let's call this conditional transformation QG-Tfx.

Properties:
* QG-Tfx maps a line through QG-P1 into a line through QG-P1
* QG-Tfx(QG-Tfx(L)) = L
* QG-Tfx(QG-diagonal) = same QG-diagonal
* Let S12 = P1.P2^P3.P4 and S14 = P1.P4^P2.P3, then
  QG-Tfx(QG-P1.S12) = QG-P1.S13 and vice versa.
* Example: QG-Tfx(QG-P1.QG-P2.QA-P10) =
QG-P1.QG-P12.QG-P13.QA-P14

Best regards,

Chris
```

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Message: #514
Date: 2020-11-12
From: van10hoven@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Eckart, dear friends,

I just noticed that (L, QG-Tfx(L)) and (QG-diagonal-1,
QG-diagonal-2) are a harmonic set of lines.
In this way the property much better can be described.

Best regards,

Chris
```

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Message: #515
Date: 2020-11-12
From: eckart_schmidt@t-online.de
Subject: Re: Conical pedal 6-points of a 6-line

```
Dear Bernard,

it is really a fascinating curve, thanks for further unexpected
points!
Here a summary of 63 points of the curve (not all real) up to
now:
15 vertices of the 6L,
15 6L-s-Tf1 images of the vertices,
... which are the double points of the 5L-o-Ci1,
6L-e-P2 as common point of the six 5L-o-Ci1.
20 intersections of the circumcircles of opposite trilaterals,
12 intersections of a 6L-line and the 5L-o-Ci1 of the remaining
lines.

But sorry, there is a NO for all your questions in #511 and 512,
judged with CABRI.

Best regards Eckart
```

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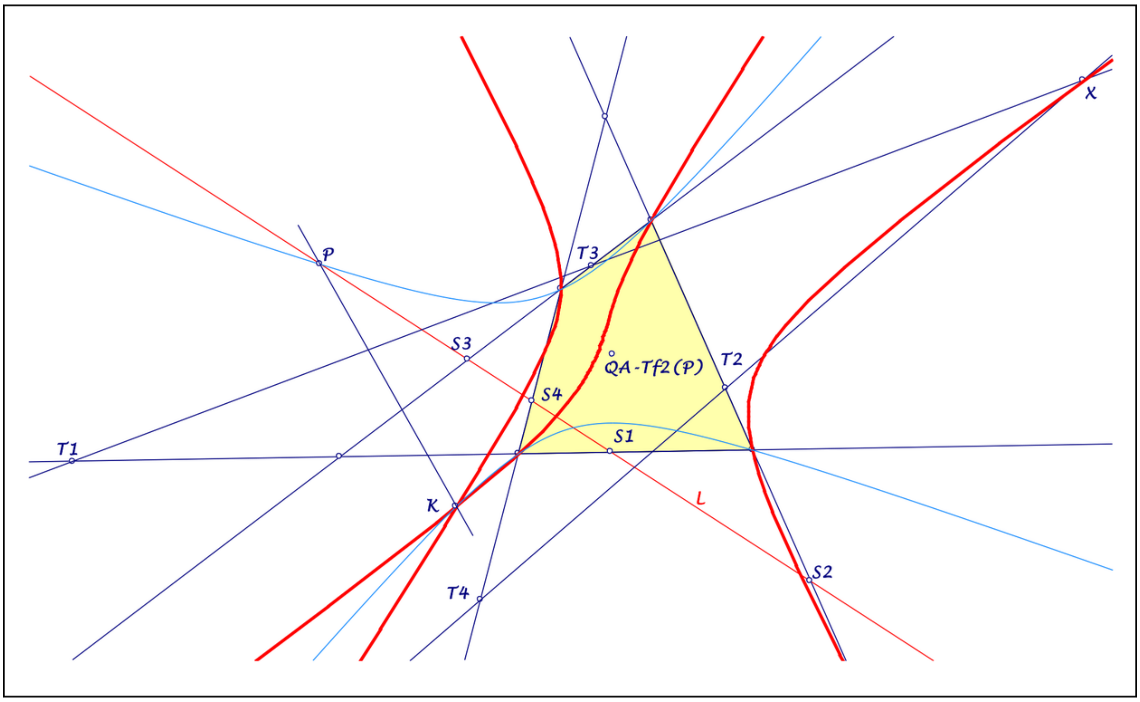
Message: #516
Date: 2020-11-15
From: eckart_schmidt@t-online.de
Subject: New QG-cubics and a QG-quartic

```
Dear all,

let L be a line, intersecting the QG-sides in Si
... with 4th harmonic point Ti  on the sides,
... Si and Ti are QA-Tf2-partner on the sides,
... with intersection X of Ti.Ti+2 and Ti+1.Ti+3.
For lines L of a pencil wrt a point P (unequal QG-P1, see #513)
... we get for X a QG- circumcubic with a double point K,
... which is the 2nd intersection of a QG-circumconic
    through QL-P1
... and its polar for QA-Tf2(P).
These QG-circumcubics depend on the chosen pencil point P.

Let P = QL-P1 with QA-Tf2(P) = QG-P16
... and consider the intersections Y = L ^ X.QG-P16;
... the locus of Y is a QG-circumquartic,
    bearing 11 wellknown points:
... the vertices of QG and the vertices of QG-Tr1,
... further the points QL-P1 (triple point) and QG-P16,
... K for QL-P1 and Q
    (intersection of QA-Cu1 and its asymptote).

Best regards Eckart
```



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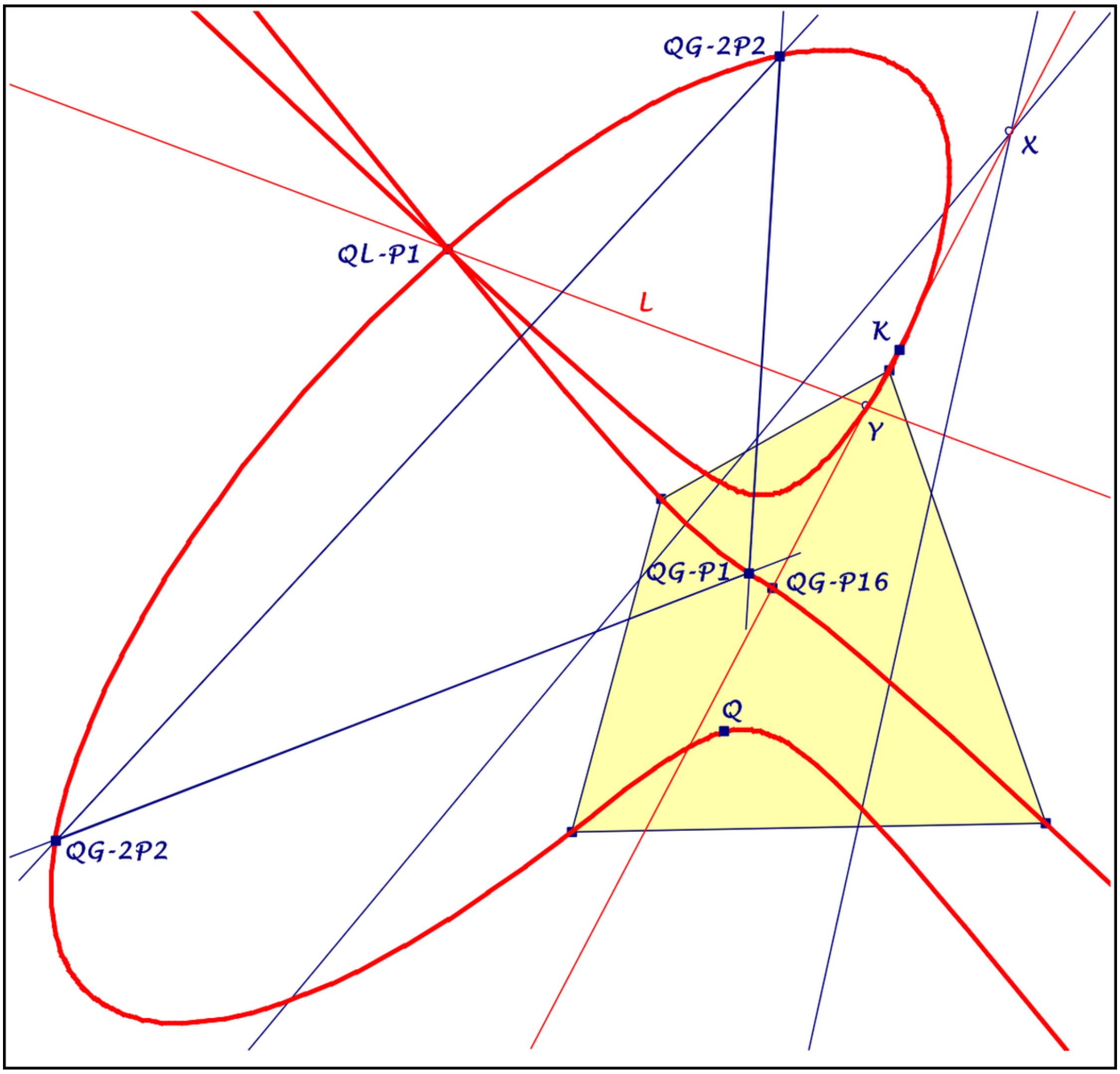
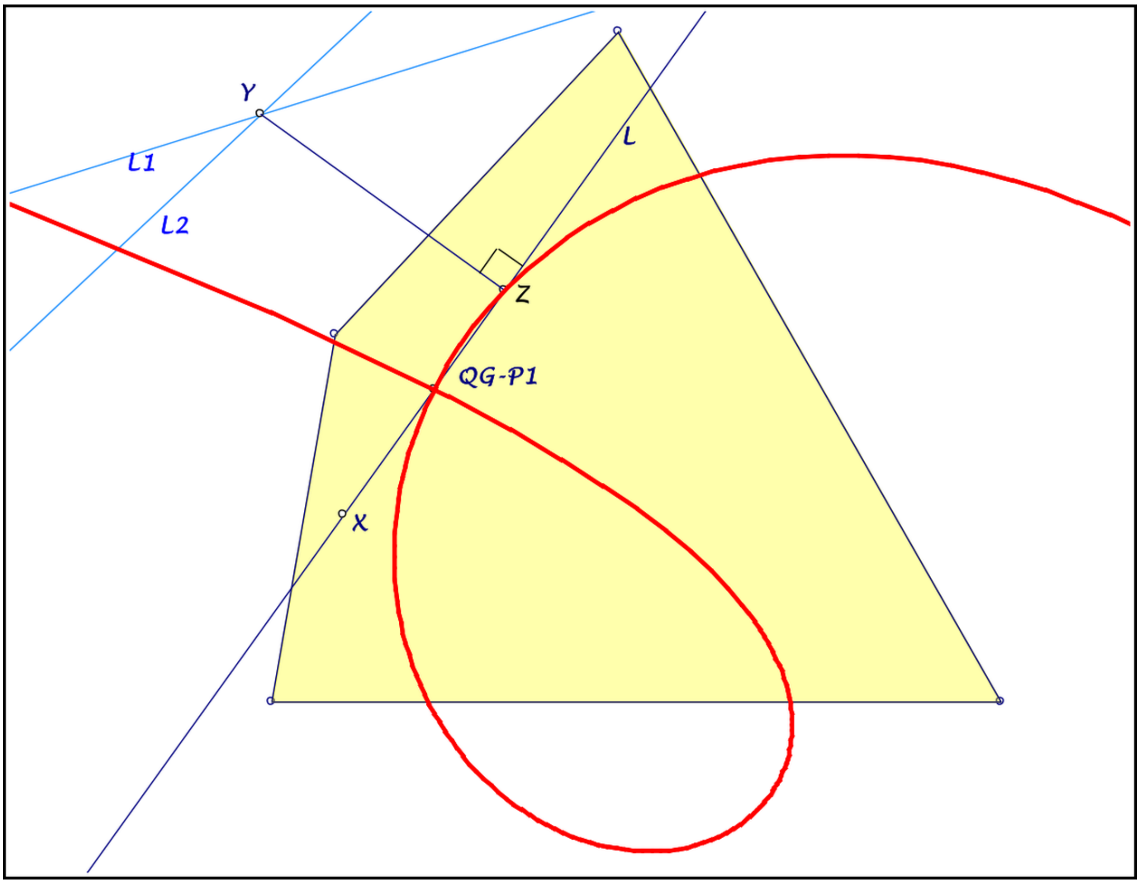


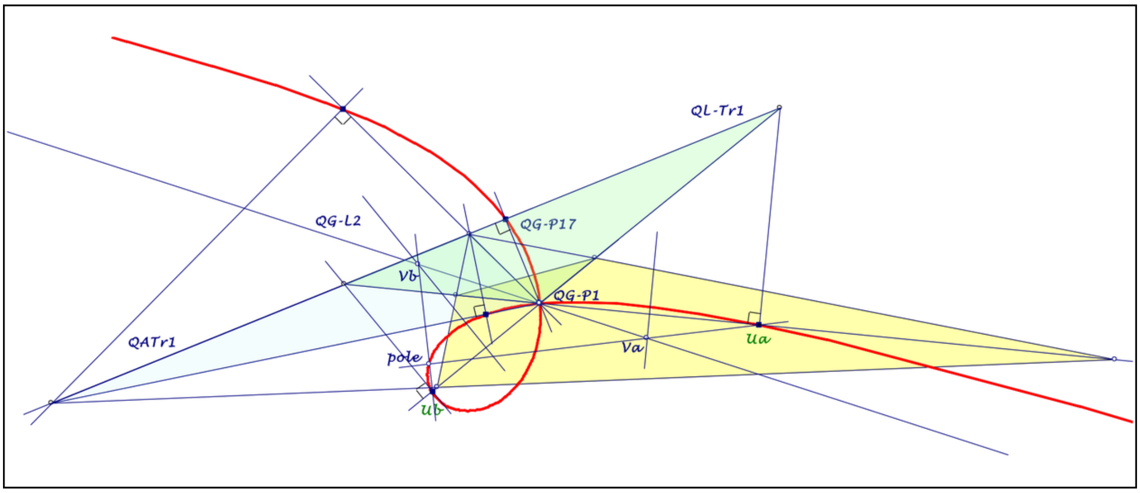
2020-11-15b.pdf

Message: #517
Date: 2020-11-17
From: eckart_schmidt@t-online.de
Subject: QG-Strophoid

```
Dear all,

a curious construction leads to a QG-strophoid,
... bearing the vertices of the orthic triangles
    of QA-Tr1 and QL-Tr1.

Consider lines L through QG-P1 and points X on it
... with dual lines L1, L2 of X wrt the QA- and
QL-interpretation
    of quadrigon QG,
... intersecting in Y on QG-L1 for all X on L
    and pedal point Z of Y on L
The locus of Z is a strophoid varying L through QG-P1.

The strophoid can be described
... with the defining line QG-L2,
    knot QG-P1 and a pole,
    constructed as follows:
Let Ua,b be the pedal points of QL-Tr1-vertices
unequal QG-P1 on the opposite QL-Tr1-side
... and Va,b the intersection of QG-L2
    and the bisectors of Ua,bQG-P1,
... then the intersection UaVa ^ UbVb
    gives the pole of the strophoid.

Best regards Eckart
```



2020-11-17a.pdf



2020-11-17b.pdf

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Message: #518
Date: 2020-11-19
From: bernard.keizer@gmail.com
Subject: Re: QG-Strophoid

```
Dear Eckart,
There are now too many such cubics and quartics with esoteric
constructions for the QA, the QL and the QG !
Some are vey interesting, other are a little bit complicate ...
Why don't you try to make a short list or catalog of all these
curves ? (the same as Bernard Gibert for a triangle, of course
with less details, properties and without equation).
But it could help to remember all this precious material.
Best regards
Bernard
```

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Message: #519
Date: 2020-11-19
From: eckart_schmidt@t-online.de
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Chris,

if I am not wrong, for a line L through QG-P1 holds
QG-Tfx(L) = QA-Tf2(L).

Best regards Eckart
```

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Message: #520
Date: 2020-11-20
From: eckart_schmidt@t-online.de
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

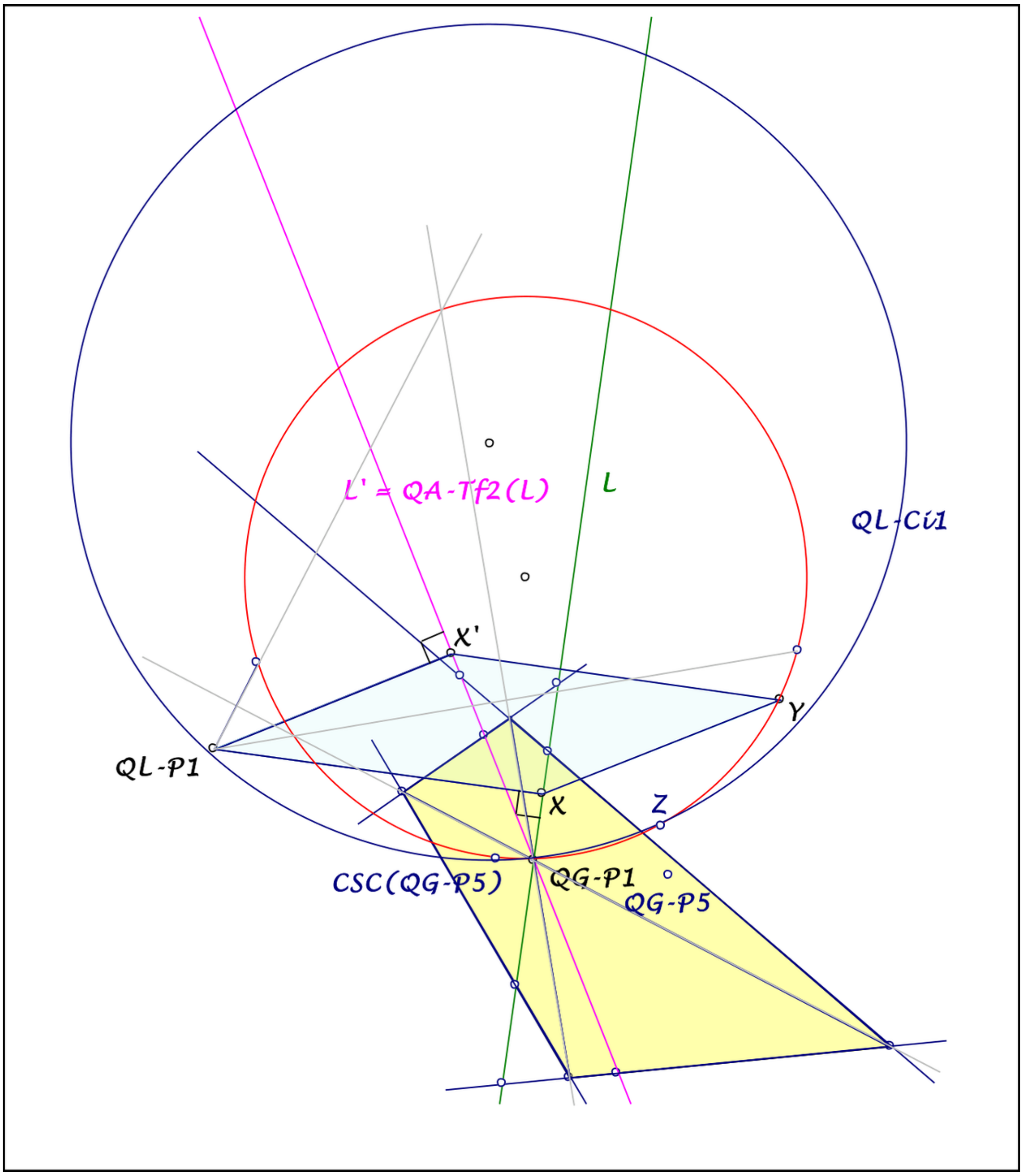
---

```
Dear Chris,

your #513 leads to a new QG-circle:

Consider a QG and lines L, L' = QG-Tfx(L) = QA-Tf2(L) through
QG-P1,
... further the pedal points X, X' of QL-P1 on L, L'
... and the 4th parallelogram point Y of X, QL-P1, X':
The locus of Y
... is a circle CI through QG-P1
    and the reflections of QL-P1 in the QG-diagonals,
... bearing CSC(QG-P5) ...
For a quadrilateral the three circles CI have a common point Z
on QL-Ci1,
... which is the reflection in QL-L2
    of the 2nd intersection of QL-Ci1 and QL-P1.10.16,
... this point is the focus of an QL-Tr1 inscribed parabola
... ... tangent to QL-L2 with directrix QL-P1.10.16.

Best regards Eckart
```



2020-11-20.pdf

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Message: #521
Date: 2020-11-20
From: van10hoven@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Eckart,

Very interesting!
I didn't find any more incidences with QA-circle Cl
and QL-point Z.
Best regards,

Chris
```

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Message: #522
Date: 2020-11-23
From: bernard.keizer@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Chris, dear Eckart
The most interesting result of your construction is this QL
parabola DT inscribed and tangent to the Steiner Line, with
focus Z and directrix P1P10P16.
Z is the transform of P1 in the following triangle
transformation :
Consider a triangle (here DT), any point X (here P1) and the 3
reflexions X1, X2 and X3 of X in the 3 sides (here the diagonals
of DT).
The 3 circles through 2 of the 3 reflexions and the
corresponding vertice have a common point Z on the circumcircle.
What is this transformation ?
Best regards
Bernard
```

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Message: #523
Date: 2020-11-23
From: bernard.keizer@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Chris, dear Eckart
I forgot to mention that the Steiner Line of Z wrt the triangle
pass through X and the parabola with focus Z and directrix this
Steiner Line is inscribed in the triangle.
Best regards
Bernard
```

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Message: #524
Date: 2020-11-23
From: eckart_schmidt@t-online.de
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Bernard,

your triangle transformation
... maps a point X to a point Z on  the circumcircle
... with a Simson line parallel to X.X(4),
... half the distance to Z.
All points on X.X(4) have the same image Z.

Best regards Eckart
```

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Message: #525
Date: 2020-11-23
From: bernard.keizer@gmail.com
Subject: Harmonic Conjugate of a line through the Diagonal Crosspoint in a 4-Gon

---

```
Dear Eckart,
Thanks a lot for your quick answer !
XX4 is the Steiner Line of Z and Z is the same for any X on the
same line.
Beautiful, I didn't know this construction. Is it well-known ?
For the QL, the construction with DT and P1 gives your point Z.
The same construction with DT and P2 (Morley point) gives P17
and the DQL parabola with focus P17 and directrix P2P10.
Best regards
Bernard
```

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Message: #526
Date: 2020-11-24
From: van10hoven@gmail.com
Subject: new items in EQF

```
Dear friends,

I added several new items in EQF:
* QA-Tf17: 1st QA-Hung's Transformation
* QA-Tf18: 2nd QA-Hung's Transformation
* QL-Qu3:  Schmidt Quartic
* QL-Tr-1: QL-Triple Triangles
* QG-Ci5:  QG-Six-point Circle

Also I adapted several items. See Recently added (
https://www.chrisvantienhoven.nl/recently-added ).

Best regards,
Chris
```

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Message: #527
Date: 2020-11-24
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF

Dear Chris,
I'm very glad that you put Eckart's beautiful quartic in EQF !
I only regret that you don't mention other interesting properties.
This quartic pass also through the Plücker points QL-2P1.
Let's consider the Cl-Sdiag which is the Cl-S of the DQL (3 diagonals and Newton Line).
This Cl-Sdiag is centered in QL-P17 and swaps precisely the Plücker points.
The conic mentionned by Eckart is also tangent to the Steiner axes of this Cl-Sdiag (in other words, QL-P1 and QL-P17 belong to the ortoptic circle of the conic).
The quartic passes also through the fixed points of Cl-Sdiag.
Any tangent to the conic cuts the quartic in 4 points, 2 are Cl-S partners and the 2 other Cl-Sdiag partners.
The quartic is invariant in both Cl-S and Cl-Sdiag (and of course in a 3rd Cl-S, which is the product of the 2 Cl-S and Cl-Sdiag).
The construction you mention works for both Cl-S, intersections of a tangent to the conic with it's Cl-S partner or with it's Cl-Sdiag partner are points of the quartic.
Best regards
Bernard

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Message: #528
Date: 2020-11-25
From: van10hoven@gmail.com
Subject: Re: new items in EQF

Dear Bernard,

Thanks for your remarks and I am glad you appreciate it!
It was an earlier remark of yours that made me register it.
* QL-2P1 was already mentioned in the picture, but I will make an explicit note in the properties.
* About the properties regarding Cl-Sdiag, is there a reference to one or more messages in QFG? Then I can refer to that.

Best regards,

Chris

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Message: #529
Date: 2020-11-25
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF

Dear Chris,
Thanks for your answer !
I discussed this item with Eckart several times on the Forum.
The last one was in the messages 173, then 176 to 171.
In fact, there are other properties to mention (under Eckart's control).
Let Px be CSC(P17) or CSCdiag(P1) and CSCx the CSC centered in Px and swapping P1 and P17.
The Steiner axes of the 3 CSCs are the bisectors of the triangle P1P17Px.
Then the quartic is invariant in the 3 CSC, CSCdiag and CSCx.
The fixed points of CSC are CSCdiag as well as CSCx partners and the fixed points of CSCdiag are CSC as well as CSCx partners.
The conic is centered in the pole of P1P17 wrt the Dimidium circle and the orthoptic circle of the conic is centered in the same point, through P1 and P17 and orthogonal to the Dimidium circle.
The quartic is tangent to the conic in 2 points of the Dimidium circle.
The centers of the circles CSC(tangents to the conic) described a rectangular hyperbola centered in the Dimidium point (center of the Dimidium circle) through the 4 circumcenters of the reference triangles of the QL ...
Best regards
Bernard

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Message: #530
Date: 2020-11-25
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF

Sorry please read 176 to 181

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Message: #531
Date: 2020-11-25
From: van10hoven@gmail.com
Subject: Re: new items in EQF

```
Dear Bernard,

Thanks for the information.
I made a note of it in the description of QL-Qu3.

Best regards,
Chris
```

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Message: #532
Date: 2020-11-27
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF

```
Dear Chris,

thanks for the new items in EQF and the numerous "recently
added",
now the properties can be used and can be cited as references
without searching for the message, where they are mentioned.

Best regards Eckart
```

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Message: #533
Date: 2020-11-27
From: van10hoven@gmail.com
Subject: Re: new items in EQF

```
Dear Eckart,

You're welcome, it's my pleasure.
And this is just the beginning.
I have got another set of items in the back pocket that were
waiting too long for being registered in the Encyclopedia for
Polygon Geometry.
I just need some more time to present it all in an appropriate
way. It is coming soon.

Best regards,
Chris
```

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Message: #534
Date: 2020-11-29
From: analgeomatica@gmail.com
Subject: Some new theorems on Pentagon and Pentagram

```
Dear Mr Chris, Mr Eckart and geometers,

I am delighted to be back in the discussions about Quadri- and
Poly-Geometry Group (QPG). I apologize to Mr Chris for missing a
few messages
and I have not been able to rejoin the discussion until today.
The first conversation on QPG, I would like to send You some new
results I found on Pentagon and Pentagram at
https://arxiv.org/abs/1908.00974
I hope they are new and I look forward to your discussion of
these results.

Thank you very much and best regards
Tran Quang Hung.
```

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Message: #535
Date: 2020-11-30
From: eckart_schmidt@t-online.de
Subject: Re: Some new theorems on Pentagon and Pentagram

```
Dear  Tran Quang Hung,

with great interest I have studied your theorem 4,
... which leads to a new 5G-point,
... worth to be mentioned in EPG, congratulations!

Best regards Eckart
```

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Message: #536
Date: 2020-11-30
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Some new theorems on Pentagon and

---

```
Dear Mr Eckart, thank you very much for your deep interest.
I also hope you will study Theorem 5.

Best wishes
Tran Quang Hung.

Vào Th 2, 30 thg 11, 2020 vào lúc 15:16 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:

> Dear  Tran Quang Hung,
> with great interest I have studied your theorem 4,
> ... which leads to a new 5G-point,
> ... worth to be mentioned in EPG, congratulations!
> Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #537
Date: 2020-11-30
From: eckart_schmidt@t-online.de
Subject: Re: Some new theorems on Pentagon and Pentagram

```
Dear  Tran Quang Hung,

the point X in Theorem 5 is not the point X in Theorem 4,
... but a new 5G-point for EPG, congratulations once more!
In vain I searched for relations with other 5G-/5P-/5L-points.

Best regards Eckart

PS. I hope Chris can confirm these remarks.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #538
Date: 2020-12-01
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Some new theorems on Pentagon and

---

```
Dear Mr Eckart,

Thank you so much for your interest.

Best regards
Tran Quang Hung.

Vào Th 2, 30 thg 11, 2020 vào lúc 20:34 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:

> Dear  Tran Quang Hung,
> the point X in Theorem 5 is not the point X in Theorem 4,
> ... but a new 5G-point for EPG, congratulations once more!
> In vain I searched for relations with other 5G-/5P-/5L-points.
> Best regards Eckart
> PS. I hope Chris can confirm these remarks.
```

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Message: #539
Date: 2020-12-01
From: analgeomatica@gmail.com
Subject: Generalization of Vittas' theorem for Quadrigon

Dear geometers,

Vitas' theorem for the cyclic quadrilateral (cyclic quadrigon) is really
beautiful, I like it very much. You can see details here

https://jcgeometry.org/Articles/Volume1/JCG2012V1pp32-39.pdf

I have tried to find a similar version for any quadrilateral and I got the
following result. Using QPG language I should call this a Quadrigon property

Let ABCD be a convex quadrigon.
Let P be Euler-Poncelet of (A,B,C,D).
AC meets BD at I.
Let A', B', C', D' be midpoints of IA, IB, IC, ID, reps.
Let M, N be midpoints of AC, BD, resp.
Parallel line from A' to PN meets parallel line from B' to PM at O1.
Let G1 be the centroid of triangle IAB.
We have line d1 = O1G1.
Define similarly the lines d2, d3, d4.
Then four lines d1, d2, d3, d4 are concurrent.

Note that where ABCD is cyclic. Then PN_|_AC, PM_|_AB. Thus O1 is the
circumcenter of IAB or d1 is the Euler line of IAB. We get Vittas' theorem.

I give my thanks and best wishes to architect Kostas Vittas, I
consider you a great friend who inspired me to geometry.

Best Regards
Tran Quang Hung.

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Message: #540
Date: 2020-12-01
From: van10hoven@gmail.com
Subject: Re: Some new theorems on Pentagon and Pentagram

Dear Tran Quang Hung,

I have read the most of your paper SOME NEW THEOREMS ON PENTAGON AND PENTAGRAM.
Excellent work.

But I got some remarks/questions for good understanding.

1. You note in theorem 5 (also theorem 4):
“the intersection of the lines AiAi+1 and Ai+2Ai+3 by Bi+3”
I think you better use:
“the intersection of the lines AiAi+1 and Ai+2Ai+3 *by Bi+4* ”
(the omitted number)

The same with:
“… the center of circle (AiCi+1Ai+2) by Li”
I think you better use:
“… the center of circle (AiCi+1Ai+2) *by Li+1* ”
(the in-between number).

The same with:
“Follow Miquel’s theorem then five points Bi+1, Ci, Ai+1, Ci+2, and Bi+4 lies on a circle (Ki) with taking subscripts modulo 5.”
I think you better use now:
“Follow Miquel’s theorem then five points Bi-1, Ci-1, Ai, Ci+1, and Bi+1 lies on a circle (Ki) with taking subscripts modulo 5.”
(right away the use of indices make a lot more sense)
When you work it out this way you will see the logic mirrored in the picture and the combinations.
QPG deals for a large part with combinatorics.

2. Your remark about the Miquel’s theorem I do not quite understand.
I can see the five described points surprisingly being concyclic, however do you use
* Miquel’s Pentagram Theorem (theorem 1),
  if so, which pentagram do you use?
or
* Miquel Five Circles Theorem (theorem 2),
  if so which 5 circles do you use?

3. Why is theorem 5 the dual of theorem 4?

```
4. For a good understanding. Am I right in saying:
a. Theorem 6 is for a special 5-Gon,
   where the points Bi lie on circle (O).
   It's not applicable for a random 5-Gon.
b. Theorem 7 is for a special 5-Gon,
   where the points Ai lie on circle (O).
   It's not applicable for a random 5-Gon.
c. Theorem 8 is for a special 5-Gon,
   where the points Ki lie on circle (O).
It's not applicable for a random 5-Gon.

Best regards,
Chris
```

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Message: #541
Date: 2020-12-01
From: van10hoven@gmail.com
Subject: Re: Some new theorems on Pentagon and Pentagram

```
Dear Tran Quang Hung, dear Eckart,

I constructed the QG-points of Theorem 4 and 5 as well.
I only find one relationship with other 5G-/5P-/5L-points.
I intent to register these QG-points in EPG soon:
  * 5G-s-P1 5G-Common Newton Lines Point
  * 5G-s-P2 5G-Bicircles Crosspoint
  * 5G-s-P3 5G-Inner Miquel Points Center
  * 5G-s-P4 5G-Miquel-Catalan Point (theorem 1)
  * 5G-s-P5 1st 5G-Hung's Point (QFG#3654)
  * 5G-s-P6 2nd 5G-Hung's Point (theorem 4)
  * 5G-s-P7 3rd 5G-Hung's Point (theorem 5)
These 4 points are collinear: 5L-s-P1, 5G-s-P1, 5G-s-P2,
5G-s-P5.
Further 5G-s-P4 = 5L-o-P2 (the 5L- version of nL-o-P2).

Best regards,
Chris
```

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Message: #542
Date: 2020-12-02
From: eckart_schmidt@t-online.de
Subject: Re: Some new theorems on Pentagon and Pentagram

```
Dear Chris, dear Tran Quang Hung,

the constellations in theorem 4 and 5 can also be described as
follows:
Let Ai and Ci as in your nomination and Bi in Chris' nomination:

Theorem 4:
... Take Ki
    as intersection of the bisectors for Ai+2.Ci+2 and
    Ai-2.Ci-2,
... take Li
    as intersection of the bisectors for Ci.Bi+2 and Ci.Bi-2,
... then the common point of Ki.Li is the result of theorem 4.

Theorem 5:
... Take Ki
    as intersection of the bisectors of Ci-1.Ai and Ai.Ci+1,
... take Li
    as intersection of the bisectors of Ai-1.Ci and Ci.Ai+1,
... then the common point of Ki.Li is the result of theorem 5.

Best regards Eckart
```

Message: #543
Date: 2020-12-02
From: bernard.keizer@gmail.com
Subject: QL-Qu3 as triangle quartic

Dear Chris, dear Eckart
Looking again at Eckart's beautiful quartic recently added by Chris in EQF as QL-Qu3, I've found following property.
Let's consider a triangle, the 3 vertices Pi, the 3 pairs of bisectors (internal and external), intersecting in the 4 points I (incenter) and Ji (excenters) and the 3 CSCs centered in a vertice and swapping the 2 others, having fixed points on the internal bisectors.
The 3 vertices and the 4 in/excenters form with the 2 circular points a CB system.
2 pairs of bisectors in 2 of the 3 vertices of the triangle form a QL with 6 vertices in the 4 in/excenters and in these 2 vertices ; the Newton Line is the perpendicular bisector of the segment joining these 2 vertices and the Miquel point is the 3rd vertice of the triangle.
It's easy to draw the Van Rees focal circular cubic of each of the 3 QLs obtained this way. (The 3 circular VRs pass through the 3 vertices of the triangle and their 4 in/excenters).
Each VR is the locus of the foci of the conics inscribed in the corresponding QL.
Let's now use your construction of the tangents to one of these conics and their intersections (when real) with the 2 circles CSC partners of these tangents in the 2 CSCs other than the CSC of the VR.
You get a quartic of type QL-Qu3 invariant in the 3 CSCs, passing through the 2 pairs of fixed points of these 2 other CSCs and tangent to the conic in 2 points on the circle centered on the perpendicular bisector of the segment joining the 2 vertices of the triangle other than the Miquel point of the VR and orthogonal to the orthoptic circle of the conic.
Best regards
Bernard

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Message: #544
Date: 2020-12-02
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

Dear Tran Quang Hung,

Very interesting construction you found as a generalzation of Vittas' theorem.
It starts with a point P=QA-P2, leading to a point X where d1,d2,d3,d4 concur.

Here a summary of my remarks:
1. I found a subsequent construction for P=QA-P2 yielding your point X.
2. There is another similar point Y with a similar construction starting with P=QA-P3.
3. It can be generalized to a transformation when point P is not fixed on QA-P2 or QA-P3, but is situated on a certain line, or a certain conic or another certain conic, yielding loci of points Z.

Let me explain:

1. IA'O1B' is a parallelogram. IA'O1B' together with the 3 corresponding parallelograms compose a super parallelogram O1O2O3O4, which also can be constructed by drawing lines through A' and C' parallel to PN and lines through B' and D' parallel to PM. The vertices of this composed parallelogram are O1, O2, O3 and O4. It's an easier way to construct O1, O2, O3 and O4.
A second even simpler construction is by drawing lines through A,B,C,D instead of A',B',C',D'. Draw lines through A and C parallel to PM and through B and D parallel to PN (note M,N are interchanged).
Now again we find similar points O1, O2, O3 and O4 and we follow further the same procedure with G1, G2, G3 and G4 like you described. It delivers the same point X.
The coordinates of X are pretty complicated.
Point X is collinear with QG-P4 and QA-P3.

2. Do the same construction as 1., only let P = QA-P3. Note that the directions of M.QA-P2 and N.QA-P2 are the same as resp. N.QA-P3 and M.QA-P3, only M,N are interchanged.
This delivers a point Y with coordinates pretty much complicated as those of X.
Point Y is collinear with QG-P4 and QA-P2.

3. When we are looking for points P for which d1, d2, d3, d4 concur in a point Z, then algebraically it appears that this is the case when P lies on:

  1. QA-Co1 = QA-Nine-point Conic, then locus of Z consists of points from conic (I,O1,O2,O3,O4)
  2. QL-L1 = QL-Newton Line, then locus of Z = Infinity Point of QL-Newton Line
  3. QG-Unknown Conic through the midpoints of the QG-Diagonals M and N. The asymptotes are parallel to the QG-Diagonals and its center is Ratiopoint QG-P1.QG-P4 (-1,3). Then locus of Z consists of points at the line at infinity.

Note that curves 1, 2, 3 all pass through the midpoints of the QG-Diagonals M and N.
Last but not least, note that Z always is collinear with QG-P4 and the reflection of P in QA-P1.

Best regards,
Chris

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Message: #545
Date: 2020-12-03
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Some new theorems on Pentagon and

---

Dear Mr Chris, dear Mr Eckart and geometers,

Thank you so much for your interest and for the profound
analysis of my
paper. It took me 2 days to carefully monitor the analysis and
would like
to reply to you as follows

Mr Chris wrote

1.
> You note in theorem 5 (also theorem 4):
> “the intersection of the lines AiAi+1 and Ai+2Ai+3 by Bi+3”
> I think you better use:
> “the intersection of the lines AiAi+1 and Ai+2Ai+3 *by Bi+4*”
> * (the omitted number)*The same with:
> “… the center of circle (AiCi+1Ai+2) by Li”
> I think you better use:
> “… the center of circle (AiCi+1Ai+2) *by Li+1*”
> * (the in-between number).*The same with:
> “Follow Miquel’s theorem then five points Bi+1, Ci, Ai+1,
Ci+2, and Bi+4
> lies on a circle (Ki) with taking subscripts modulo 5.”
> I think you better use now:
> “Follow Miquel’s theorem then five points *Bi-1, Ci-1, Ai,
Ci+1, and Bi+1* lies
> on a circle (Ki) with taking subscripts modulo 5.” (right away
the use of
> indices make a lot more sense)
> When you work it out this way you will see the logic mirrored
in the
> picture and the combinations.

I understand the new notation. Actually at first when writing
this article
I considered the symbols "modulo subscripts" to be equal, but as
you said,
it needs more logic, I will fix it soon (and figure drawing) for
version 2
of this article.

Mr Chris wrote
QPG deals for a large part with combinatorics.

I realized this and what it means, point notation is a form of
combinatorial use, as is polygons. That tells you the importance
of
combinatorial methods in Euclidean geometry,

Mr Chris wrote

Your remark about the Miquel's theorem I do not quite
understand.
> I can see the five described points surprisingly being
concyclic, however
> do you use
> Miquel's Pentagram Theorem (theorem 1), if so, which pentagram
do you use?
> or
> Miquel Five Circles Theorem (theorem 2), if so which 5 circles
do you use?

In Theorem 1, I used the reference
https://mathworld.wolfram.com/MiquelsPentagramTheorem.html

In Theorem 2, I used the reference
http://mathworld.wolfram.com/MiquelFiveCirclesTheorem.html

As I think, in Theorem 1 (figure 1), we have a red star called
pentagram,
in Theorem 2 (figure 2), we have 5 circles (except for the
circle of
conclusion).

In Theorem 3, I used reference,
https://www.nakanihon.co.jp/gijyutsu/Shimada/
Computationalgeometry/chapter040901.html by the help of Hiroshi
Okumura
from Japan. I was surprised that I used to think Theorem 1 and
Theorem 3
were the same, but actually after careful analysis, I found
these two
theorems independent of each other and they are both really
beautiful.
Apparently the pentagram and the pentagon are very different.

Mr Chris wrote
3.
> Why is theorem 5 the dual of theorem 4?

Actually "dual" is my own word, I find a rather large correspondence of
these two theorems, although I have not been able to explicitly point out
this, and therefore I call it "dual".

Mr Chris wrote
4.
> For a good understanding. Am I right in saying:
> a. Theorem 6 is for a special 5-Gon, where the points Bi lie on circle
> (O). It’s not applicable for a random 5-Gon.
> b. Theorem 7 is for a special 5-Gon, where the points Ai lie on circle
> (O). It’s not applicable for a random 5-Gon.
> c. Theorem 8 is for a special 5-Gon, where the points Ki lie on circle
> (O). It’s not applicable for a random 5-Gon.

Yes, for the Theorem 6,7,8. I am trying to apply the "main" theorems of the
theorem 4 and the theorem 5 to special pentagrams and pentagons and I get
the theorems 6, 7, 8 which add collinearity. I haven't had time to delve
further what happens in Theorem 4 and Theorem 5 if the pentagrams and the
pentagon are circumscribed? I hope I can update this in the next version of
the article.

Mr Chris wrote
I constructed the QG-points of Theorem 4 and 5 as well.
> I only find one relationship with other 5G-/5P-/5L-points.
> I intent to register these QG-points in EPG soon:
> · 5G-s-P1 5G-Common Newton Lines Point
> · 5G-s-P2 5G-Bicircles Crosspoint
> · 5G-s-P3 5G-Inner Miquel Points Center
> · 5G-s-P4 5G-Miquel-Catalan Point (theorem 1)
> · 5G-s-P5 1st 5G-Hung’s Point (QFG#3654)
> · 5G-s-P6 2nd 5G-Hung’s Point (theorem 4)
> · 5G-s-P7 3rd 5G-Hung’s Point (theorem 5)
>
> These 4 points are collinear: 5L-s-P1, 5G-s-P1, 5G-s-P2, 5G-s-P5.
> Further 5G-s-P4 = 5L-o-P2 (the 5L- version of nL-o-P2).

the constellations in theorem 4 and 5 can also be described as follows:
> Let Ai and Ci as in your nomination and Bi in Chris' nomination:

Mr Eckart wrote
Theorem 4:
> ... Take Ki as intersection of the bisectors for Ai+2.Ci+2 and Ai-2.Ci-2,
> ... take Li as intersection of the bisectors for Ci.Bi+2 and Ci.Bi-2,
> ... then the common point of Ki.Li is the result of theorem 4.
> Theorem 5:
> ... Take Ki as intersection of the bisectors of Ci-1.Ai and Ai.Ci+1,
> ... take Li as intersection of the bisectors of Ai-1.Ci and Ci.Ai+1,
> ... then the common point of Ki.Li is the result of theorem 5.

Thank you very much, there are actually quite a bit of things I haven't
discovered on these configurations yet. I'm only actually working on 5G, I
really want to find their links with the 5P and 5L.

Finally, I would like to thank you for your deep interest in my post and for sending important comments, and for updating https://chrisvantienhoven.nl/mathematics/encyclopedia-of-poly-geometry.
Actually, these studies are still very primitive, I would like to update your comments and use the QPG references for the next version.

Best Regards
Tran Quang Hung.

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Message: #546
Date: 2020-12-03
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Generalization of Vittas' theorem for

---

Dear Mr Chris and geometers,

You wrote

Let me explain:
>
> 1. IA'O1B' is a parallelogram. IA'O1B' together with the 3 corresponding
> parallelograms compose a super parallelogram O1O2O3O4, which also can be
> constructed by drawing lines through A' and C' parallel to PN and lines
> through B' and D' parallel to PM. The vertices of this composed
> parallelogram are O1, O2, O3 and O4. It's an easier way to construct O1,
> O2, O3 and O4.
> A second even simpler construction is by drawing lines through A,B,C,D
> instead of A',B',C',D'. Draw lines through A and C parallel to PM and
> through B and D parallel to PN (note M,N are interchanged).
> Now again we find similar points O1, O2, O3 and O4 and we follow further
> the same procedure with G1, G2, G3 and G4 like you described. It delivers
> the same point X.
> The coordinates of X are pretty complicated.
> Point X is collinear with QG-P4 and QA-P3.

As I think (but maybe I'm not sure, can you help me verify?), we can change
A', B', C', D' into the points which divide segment IA, IB, IC, ID, resp,
in the same ratio. However, the meaning of the midpoint is that it extends
directly to Vittas' theorem.

You wrote

2. Do the same construction as 1., only let P = QA-P3. Note that the

> directions of M.QA-P2 and N.QA-P2 are the same as resp.
N.QA-P3 and
> M.QA-P3, only M,N are interchanged.
> This delivers a point Y with coordinates pretty much
complicated as those
> of X.
> Point Y is collinear with QG-P4 and QA-P2.
>
> 3. When we are looking for points P for which d1, d2, d3, d4
concur in a
> point Z, then algebraically it appears that this is the case
when P lies on:
> 1. QA-Co1 = QA-Nine-point Conic, then locus of Z consists of
points from
> conic (I,O1,O2,O3,O4)
> 2. QL-L1 = QL-Newton Line, then locus of Z = Infinity Point of
QL-Newton
> Line
> 3. QG-Unknown Conic through the midpoints of the QG-Diagonals
M and N. The
> asymptotes are parallel to the QG-Diagonals and its center is
Ratiopoint
> QG-P1.QG-P4 (-1,3). Then locus of Z consists of points at the
line at
> infinity.
> Note that curves 1, 2, 3 all pass through the midpoints of the
> QG-Diagonals M and N.
> Last but not least, note that Z always is collinear with QG-P4
and the
> reflection of P in QA-P1.

Since QA-P3 and QA-P2 are reflections in the midpoint of MN, I
think these
two points are equivalent in this Theorem. But for the locus,
It's really
new to me, I think it's an interesting finding, I'll try to
think more
about these locus.

Thank you very much and best regards
Tran Quang Hung.

Message: #547
Date: 2020-12-03
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris, dear Tran Quang Hung,

with great interest I have studied Chris' message 544,
... here two remarks to 3.1. and 3.3.

Wrt 3.1. I think, if P on QA-Co1, the locus for Z must be the
conic (I,G1,G2,G3,G4) not (I,O1,O2,O3,O4),
... centered in QG-P8 through QG-P1,
    QG-P4 and the centroids of Pi,Pi+1,QG-P1
... with asymptotes parallel to the asymptotes of QA-Co1
... and tangent in QG-P1 through QL-P8.

Wrt 3.3. The unknown QG-conic for P with Z at infinity of
P.QG-P8 bears also QG-P8
... and gives with its 3 versions for a QA two triple points.
What about these triple points?

Best regards Eckart
```

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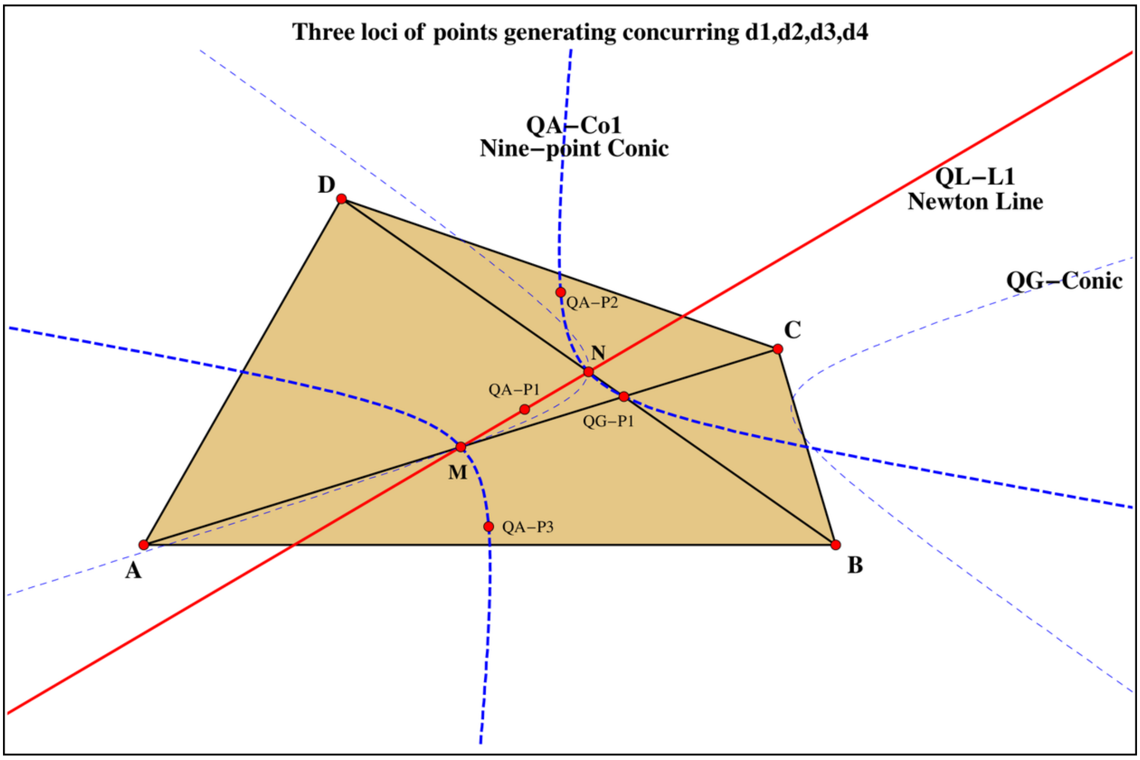
Message: #548
Date: 2020-12-03
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

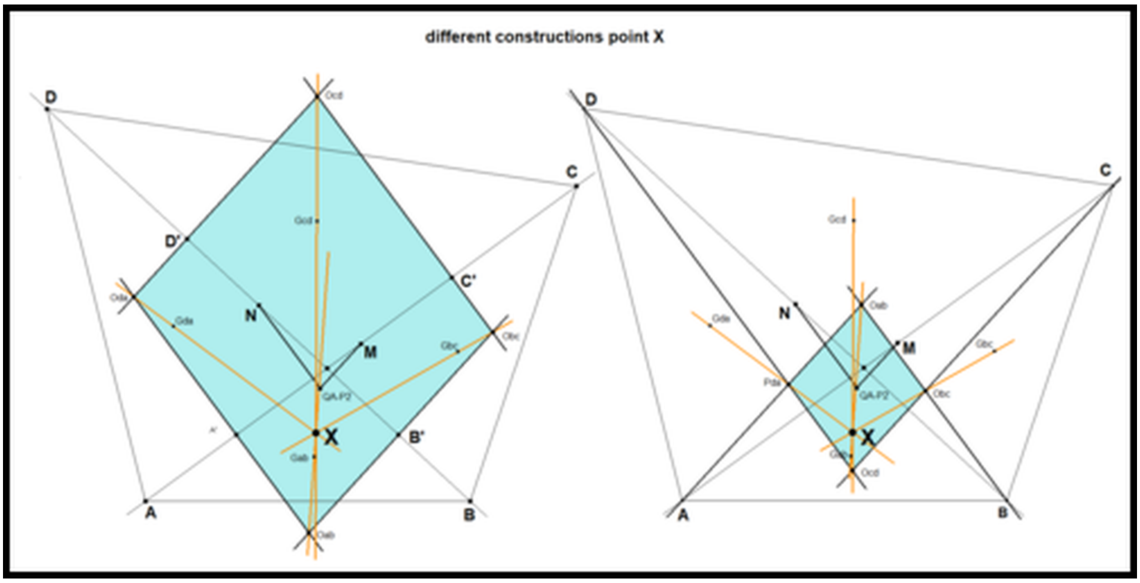
```
Dear Tran Quang Hung,

Find attached these pictures:
1. a picture for two possible constructions of your
   generalized Vittas-point
2. a picture with the three loci of points
   generating concurring d1,d2,d3,d4
I hope it will explain a lot.
A picture often tells more than a thousand of words.

Best regards,
Chris
```



QG-Px Generalized Hung_s QG-point-01.pdf



QG-Px Hung_s Point-21.png

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Message: #549
Date: 2020-12-04
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Generalization of Vittas' theorem for

---

Thank you very much Mr Chris and Mr Eckart for your comments and the figures.

I would like to write back my remark (we can change A', B', C', D' by thpoints which divide IA, IB, IC, ID in the same ratio), this claim is correct.

Let ABCD be a quadrigon and M, N are midpoints of AC, BD.
P is a point on QA-Co1 (QA-Nine-point Conic) or QL-L1 (QL-Newton Line) or QG-Unknown Conic (mentioned by Chris above).
AC meets BD at I.
A', B', C', D' lie on line IA, IB, IC, ID such that IA'/IA = IB'/IB = IC'/ IC= ID'/ID = t (t is a real number).
Let O1 be the point such that A'O1 // PN and B'O1 // PM. Define similarly, O2, O3, O4.
Let G1, G2, G3, G4 be the centroids of IAB, IBC, ICD, IDA.
Then lines O1G1, O2G2, O3G3, O4G4 are concurrent.

Best Regards
Tran Quang Hung.

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Message: #550
Date: 2020-12-03
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

Dear Tran Quang Hung, dear Eckart,

Eckart, about your remarks
* "Wrt 3.1. I think, if P on QA-Co1, the locus for Z
  must be the conic (I,G1,G2,G3,G4) not (I,O1,O2,O3,O4),"
* "Wrt 3.3. The unknown QG-conic
  for P with Z at infinity of P.QG-P8 bears also QG-P8"

Thanks for correcting (your 1 st remark) and I forgot to mention
it in the multitude of new items (your 2 nd remark).

Tran Quang Hung, about your remark
* (we can change A', B', C', D' by the points
  which divide IA, IB, IC, ID in the same ratio)

Sorry that I overlooked your remark in first instance.
You are completely right.
I noticed that the locus of the concurring-d1-d2-d3-d4-point
again is the conic (I,G1,G2,G3,G4).

Best regards,

Chris

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Message: #551
Date: 2020-12-04
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

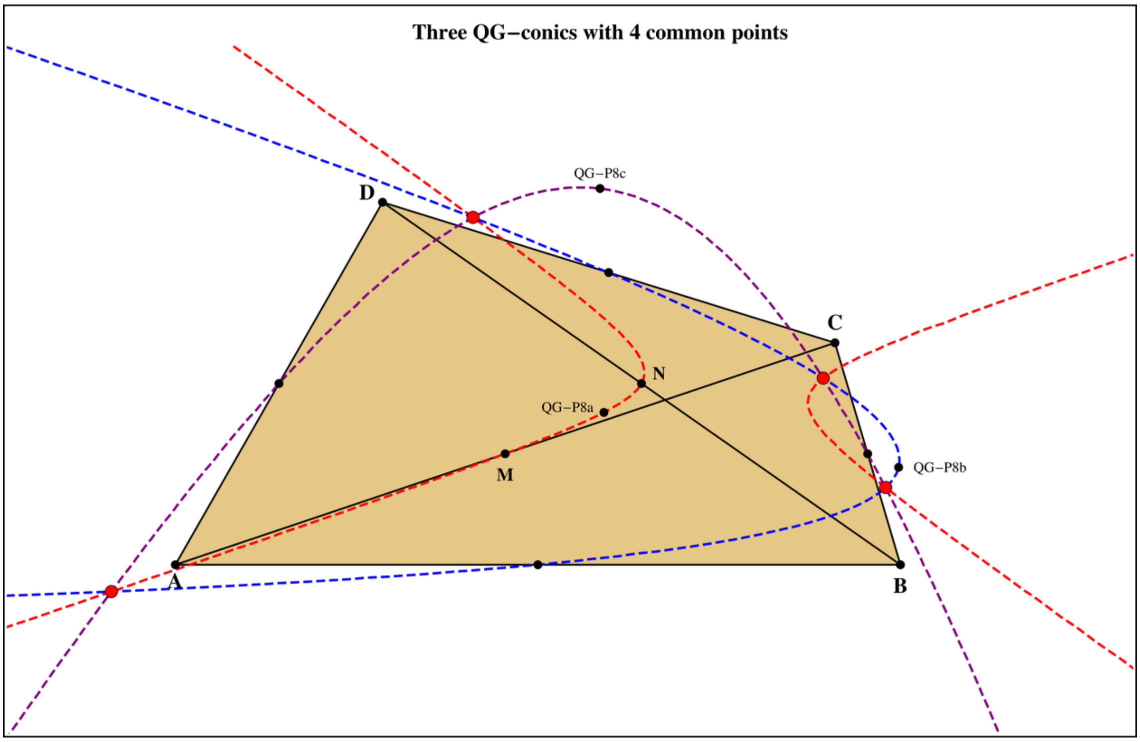
Dear Eckart, dear Tran Quang Hung,

Eckart, regarding your message #547
"The unknown QG-conic for P with Z at infinity of P.QG-P8 bears also QG-P8
... and gives with its 3 versions for a QA two triple points.".

I made pictures in Cabri with varying results. Somehow I couldn't trust my pictures in Cabri.
Sometimes I found 1 common point for the 3 versions of the unknown conic, sometimes 2, sometimes zero.
Therefore I choose for the elaborate way, calculating and plotting in Mathematica.
It wasn't difficult to calculate the equations of the three conics. But pretty elaborate to find their intersection points (if any).
Printing the conics showed that there are even 4 intersection points.
I even succeeded in calculating the coordinates, but they were very very long and complicated too.
However that is no objection for printing in specific settings.
See attached picture. I feel satisfied with the picture.
Questions arise. Did it all go well? What are these points?

Tran Quang Hung, in case you do not know, the 3 versions of a QG-item in a QA can be constructed as follows.
A QA (Quadrangle) has no order in its vertices, but a QG (Quadrigon) has a cyclic order in its vertices (order matters).
Since we can distinguish 3 cyclic orders within 4 random points (1,2,3,4 and 1,2,4,3 and 1,3,2,4) you can construct a QG-items three times in a QA.
Another example of combinatorics in Quadri Geometry.
See QA-3QG1. This property we use to draw 3 QG-conics in a QA.

Best regards,
Chris



QG-Px Hung_s Point-30-Three QG-Conics in QA.pdf

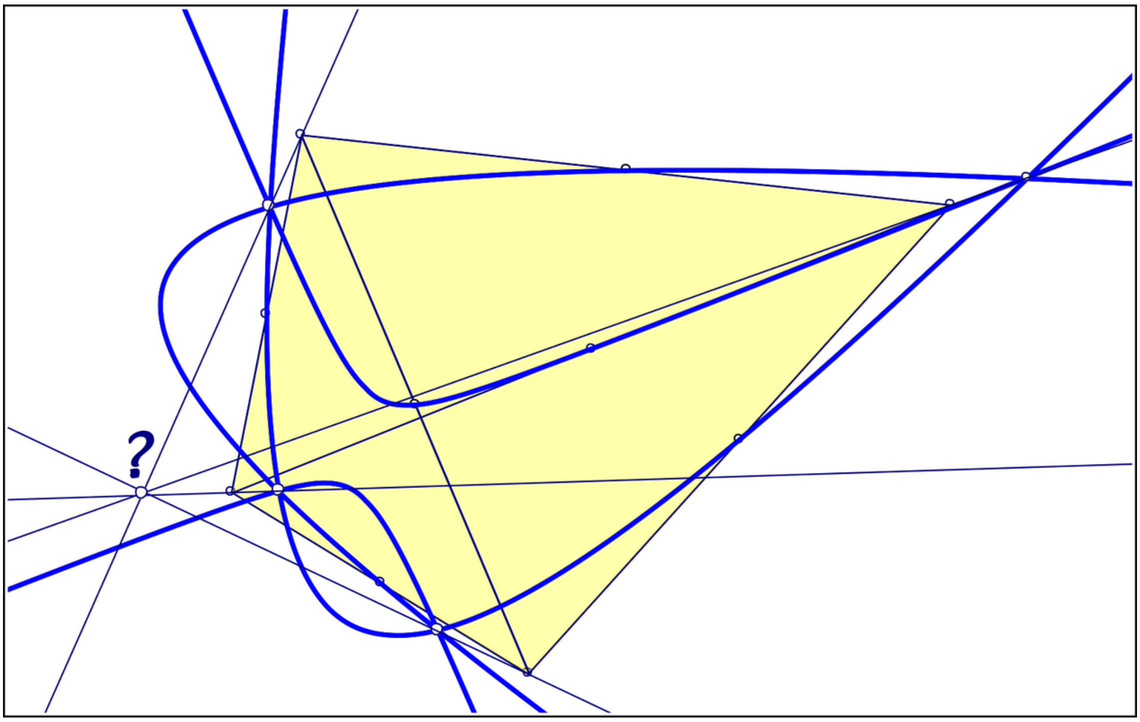
Message: #552
Date: 2020-12-04
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

you are right,
... there will be 4 common points for a QA
    wrt its 3 QG-conics of 3.3. in #544.
These 4 points give a 2nd QA, perspective to the reference QA.

What about this new QA-point?
... on QA-L3 ???

Best regards Eckart
```



2020-12-04.pdf

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Message: #553
Date: 2020-12-05
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Generalization of Vittas' theorem for

---

Dear Mr Chris, dear Mr Eckart,

You wrote

> Tran Quang Hung, in case you do not know,
> the 3 versions of a QG-item in a
> QA can be constructed as follows.
> A QA (Quadrangle) has no order in its vertices,
> but a QG (Quadrigon) has a
> cyclic order in its vertices (order matters).
> Since we can distinguish 3 cyclic orders
> within 4 random points (1,2,3,4
> and 1,2,4,3 and 1,3,2,4) you can construct a QG-items
> three times in a QA.
> Another example of combinatorics in Quadri Geometry.
> See QA-3QG1.
> This property we use to draw 3 QG-conics in a QA.

I now understand more. Thus with one property of QG we will have two similar properties of QA or QL.

Thank you very much for your interest and best regards
Tran Quang Hung.

Message: #554
Date: 2020-12-05
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

I am not quite sure, whether the quadrangles in #552 are
perspective,
... CABRI has its limits, I hope you can confirm it.

But there are curious properties, comparing the two quadrangles,
... for example:
(1) Their QA-P1 is the same, the CB-point of their 8 vertices.
(2) They have the same line QA-L3, corresponding points
    with ratio 2:3 wrt QA-P1,
    ... further the same lines QA-L1,2,4, QA-P1.4, QA-P1.32,
        corresponding points with ratio 5:2 wrt QA-P1.
(3) They have the same Sigma (2016-04-17.pdf (eckartschmidt.de).
<http://eckartschmidt.de/2016-04-17.pdf>

There will be more properties, but up to now, I am rather
confused.

Best regards Eckart
```

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Message: #555
Date: 2020-12-05
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

Dear Eckart,

Very nice extra properties you did find. It shows there is more to explore.

The question about perspectivity of both quadrangles also occurred to me, but I couldn't find enough evidence. I am not sure either. It can be true or maybe it is not true or it is true under circumstances.
I am still walking around the problem. I made pictures in Mathematica that seemed to confirm but also pictures that seemed to reject (but because of the huge coordinates that can be precision differences). I also noticed that in many cases there aren't always four real common intersection points.
Calculating for evidence I run into limits for calculation time and lack of memory on my computer.

Also I noticed that the coordinates are identical, only several plus- and minus-signs are different, hence the 4 coordinates. Example of terms within coordinates of the 4 common points are a+b+c, a+b-c, a-b+c, a-b-c. I wouldn't expect that when each common points has to relate to a specific vertex of the reference quadrangle. How to determine which common point relates to which reference point?
Earlier I would expect logical sequences, like (1,1,1), (1,a+b,a+c), (b+a,1,b+c) and (c+a, b+a,1), that clearly point to specific vertices.

We'll see.

Best regards,
Chris

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Message: #556
Date: 2020-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

wrt the perspectivity of the two quadrangles in #552:
Drawn figures lead to the speculation of a perspectivity,
... the construction below seems to give an approximation,
... but the precision of CABRI doesn't confirm a perspectivity
    center.

Approximation on QA-L3:
Consider the inversion circle round QA-P22,
swapping QA-P5 and QA-P26,
... which intersects QA-L3 in Q between QA-P5 and QA-P26,
... consider the inversion circle round QA-P22,
    swapping Q and QA-P10
... and take the reflection of QA-P1 in the last circle.

Best regards Eckart

PS: The other results in #554 can be CABRI-confirmed a million
times more exactly.
```

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Message: #557
Date: 2020-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

I think, we should forget the perspectivity of the two
quadrangles of #552
... and replace by the following property:
There is a common point
... of tangents in vertices S of the 2nd QA
... at circumconics of the reference QA through S.
This new QA-point is
... a point on QA-L3, dividing QA-P1.10 with ratio 9:4.

Best regards Eckart
```

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Message: #558
Date: 2020-12-06
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Eckart,

Regarding your last message:
"There is a common point
 ... of tangents in vertices S of the 2nd QA
 ... at circumconics of the reference QA through S.".
I studied yesterday also the tangents to the conics at Si
(i=1,2,3,4), but couldn't find any common point.
Do you have a picture?

Best regards,
Chris
```

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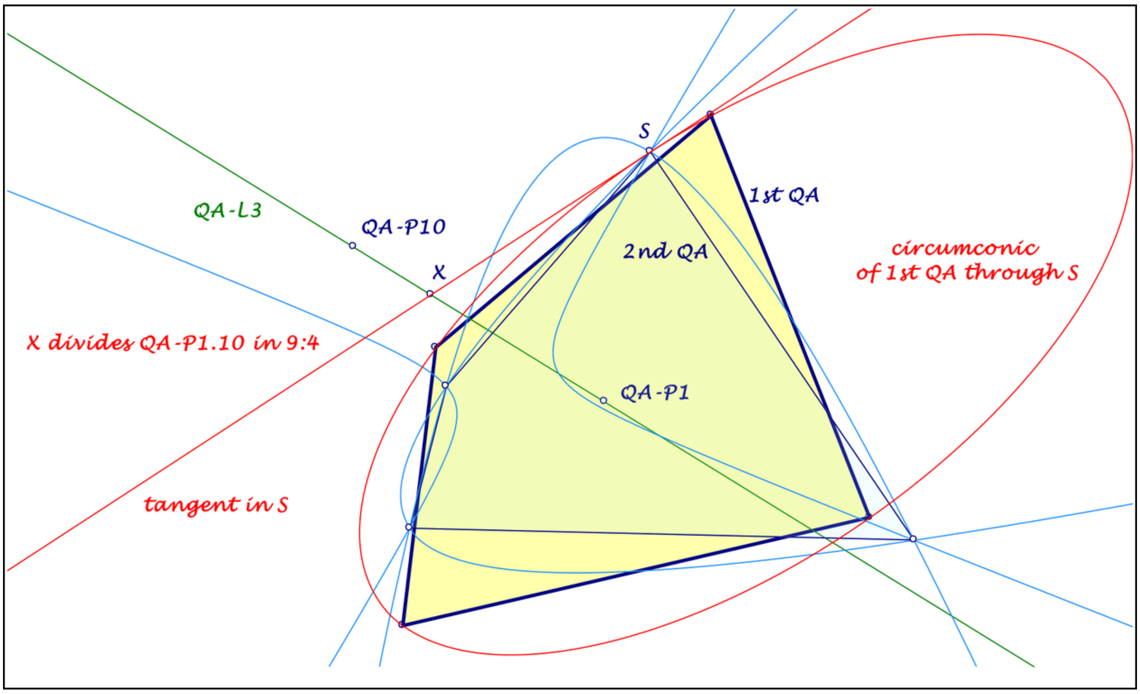
Message: #559
Date: 2020-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

attached the wished drawing,
... I have constructed only one tangent,
... the intersection with QA-L3 is independent of the chosen
    vertex S.

If we take tangents in Pi
... at circumconics through Pi,S1,S2,S3,S4,
... we get also a common point
... on QA-L3, dividing QA-P1.10 in 9:2.

Best regards Eckart
```



2020-12-06.pdf

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Message: #560
Date: 2020-12-06
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

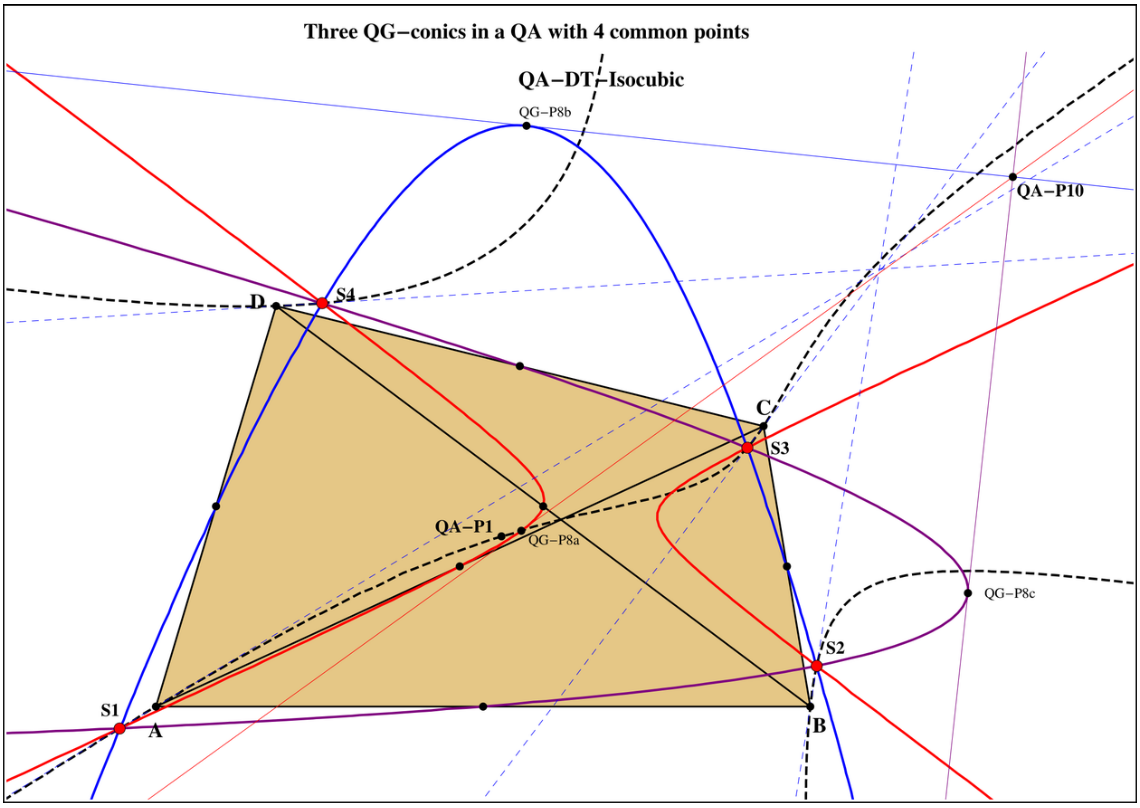
Dear Eckart,

Now I see what you mean. It is an ABCD-circumscribed conic through one of the 4 points Si.
It is very curious that for S1, S2, S3 and S4 the tangents all pass through the same point X.
It made me curious for the locus of points of tangency we have when we construct tangents from X to all circumscribed possible conics about ABCD. When I constructed the locus it appeared that the locus is the QA-Type 2 Cubic with pivot X as described in QA-Cu-1 (
https://www.chrisvantienhoven.nl/qa-items/qa-cubics/qa-cu-1).
These cubics are QA- and QA-DT-circumscribed IsoCubics.
Consequently all four Si-points lie on this cubic.

The equation of the cubic is:
q (p + q) r (p + q + 2 r) (p q + 4 p r + 4 q r + 4 r^2) x^2 y -
p (p + q) r (p + q + 2 r) (p q + 4 p r + 4 q r + 4 r^2) x y^2 -
q r (p + r) (p + 2 q + r) (4 p q + 4 q^2 + p r + 4 q r) x^2 z +
p r (q + r) (2 p + q + r) (4 p^2 + 4 p q + 4 p r + q r) y^2 z +
p q (p + r) (p + 2 q + r) (4 p q + 4 q^2 + p r + 4 q r) x z^2 -
p q (q + r) (2 p + q + r) (4 p^2 + 4 p q + 4 p r + q r) y z^2

I made a picture. See attachment.
Note that the tangents taken from the 3 QA-versions of QG-P8 to the corresponding QA-version of the QG-unknown conic, concur in QA-P10. Proven algebraically.

Best regards,
Chris



QG-Px Cux-Isocubic-01.pdf

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Message: #561
Date: 2020-12-07
From: analgeomatica@gmail.com
Subject: Generalization of Vittas' theorem for Quadrigon

```
Dear Mr Chris, dear Mr Eckart,

Thank you so much for your interest and your interesting
discussion. I think I have found another version of
Generalization of Vittas' theorem as follows.
Let ABCD be a quadrigon and M, N are midpoints of AC, BD.
P is Euler Poncelet point of (A,B,C,D)
AC meets BD at I.
A', B', C', D' are midpoint of IA, IB, IC, ID, resp.
Let O1 be the point such that A'O1 // PN and B'O1 // PM. Define
similarly, O2, O3, O4.
Let H1 be the point such that AH1 // PM and BH1 // PN. Define
similarly, H2, H3, H4.
Then the lines O1H1, O2H2, O3H3, O4H4 are concurrent.
Note that where ABCD is cyclic. Then PN_|_AC, PM_|_AB. Thus O1
is the circumcenter of IAB, H1 is orthocenter of IAB. We get
Vittas' theorem.

---------------------------------------------------------------

For more general theorem by idea of Mr Chris I obtain
Let ABCD be a quadrigon and M, N are midpoints of AC, BD.
P is a point on QA-Co1 (QA-Nine-point Conic) or QL-L1 (QL-Newton
Line) or
QG-Unknown Conic (I am not sure about this conic, is this the
same with conic of the first generalization?).
AC meets BD at I.
A1, B1, C1, D1 lie on line IA, IB, IC, ID such that IA1/IA =
IB1/IB = IC1/IC= ID1/ID = t1 (t1 is a real number).
Let O1 be the point such that A1O1 // PN and B1O1 // PM. Define
similarly, O2, O3, O4.
A2, B2, C2, D2 lie on line IA, IB, IC, ID such that IA2/IA =
IB2/IB = IC2/IC= ID2/ID = t2 (t2 is a real number).
Let H1 be the point such that A2H1 // PM and B2H1 // PN. Define
similarly, H2, H3, H4.
Then the lines O1H1, O2H2, O3H3, O4H4 are concurrent.

Best Regards
Tran Quang Hung.
```

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Message: #562
Date: 2020-12-08
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Chris,

if I am not wrong, there is a typo in your #560:
The cubic of the contact points is of type 1 in QA-Cu-1.
It is an pivotal isocubic
... with reference triangle QA-Tr1,
... isoconjugation QA-Tf2
... and pivot X.
Another example:
QA-Cu1 is the locus for contact points of tangents
... from QA-P4 at QA-circumconics.

Best regards Eckart
```

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Message: #563
Date: 2020-12-08
From: bernard.keizer@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Tran, dear Chris, dear Eckart
I found your last exchanges about the generalisation of Vitas
theorem very interesting.
Sorry if I join the debate a little late, but it took me time to
reproduce and understand the properties.
Here some remarks about the 3 QG conics
The pairs of asymptotes are parallel to the pairs of opposite
sides of the QA
The centers are the reflexions of the points QG-P8 in QG-P1
The conics pass through the middles of opposite sides and
through the points QG-P8
The lines QA-P10QGP8 are tangent to the 3 conics and *parallel
to the the lines through the middles of opposite sides
* The 3 conics intersect in 4 points S1 to 4
The pivotal isocubic through the vertices of the QA and the
vertices of DT with pivot X dividing QA-P1QA-P10 in the ratio
(9,-4) pass through the points S 1 to 4, is QA-Tf2 invariant and
pass through QA-P1 and QA-P20.
The QA of the S 1 to 4 has the same QA-P1 as the original QA and
this QA-P1 is the 9th CB point of the vertices of the 2 QA's.
*Last, the QA of the S 1 to 4 is not in perspective with the
original QA
* Best regards
Bernard
```

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Message: #564
Date: 2020-12-08
From: Stan.Rabinowitz@comcast.net
Subject: sum of reciprocals

```
A cyclic quadrigon has successive sides of lengths a, b, c, d
such that 1/a+1/c=1/b+1/d.

Does such a figure have a special name in the literature?

Has this type of figure been studied? Anyone have any
references?
```

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Message: #565
Date: 2020-12-08
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Bernard,

thanks for gathering properties and adding new aspects wrt the 3
QG-conics,
... I am still studying this theme,
... excuse, that I haven't answered your #543,
... give me some time.

Best regards Eckart
```

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Message: #566
Date: 2020-12-08
From: van10hoven@gmail.com
Subject: Re: sum of reciprocals

```
Hi Stanley,

I am not familiar with these kind of cyclic quadrigons.
Don't know either a name of it.

Best regards,
Chris
```

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Message: #567
Date: 2020-12-09
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Tran Quang Hung, dear Chris, dear Bernard,

let us generalize the first construction in #539
... and take for P any point on the conic QA-Co1,
... further let A',B',C',D' divide IA,IB,IC,ID in a chosen
    ratio,
... then a common final point exists
... and lies always on the conic through I,G1,G2,G3,G4,
... whose 3 versions for a quadrangle we discussed
    in the last messages.

Best regards Eckart
```

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Message: #568
Date: 2020-12-09
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

Dear Eckart, Bernard and friends,

Eckart about your remark in message #547
"The unknown QG-conic ... with its 3 versions for a QA two triple points. What about these triple points?".
I pondered about the 4 common points of these 3 QG-conics and came to some conclusions. Maybe or not maybe well-known.

Give a certain set of points/lines forming a reference figure.
From this reference figure we will be able to construct all kind of other points/lines/curves.
When we constrict ourselves to invertible transformations/construction methods, all these points/lines/curves form a set of elements that are interdependent in such a way that can be constructed from each other in all circumstances. However we have to delimit ourselves in the types of construction methods we use. When we delimit ourselves to ruler and compass method we get a set of elements that are "interdependent".
Now to our problem with the 4 common points of the three QG-conics.
We know that the intersection points of two conics CANNOT be constructed by ruler and compass (see Projective conic sections–constructions at http://mathafou.free.fr/themes_en/conique7.html). Therefore the four common points we are discussing do not belong to the inter dependent set of QA-QG-elements that are to our disposal. They either cannot be constructed from our set of QA-QG-elements by ruler and compass.
I think we have to classify these points as elements from another class, e.g. the conical class.
We might say that with our reference system of 4 points we created a figure that became a reference system in itself and cannot be rolled back to our first reference system (non-invertible).
It means that the 4 common points cannot "easily" be constructed with ruler and compass from the QA-/QG-points we know. Attempts to do so will be in vain.

My loosely made comments undoubtedly will be described elsewhere in some solid theory. Might someone know, I would be interested to know where.

Moreover I noticed this theoretical consideration is backed by the algebra.
In the calculation of the 4 common points I found algebraic Cardano-elements, indicating that they cannot be derived from "crisscrossing" with other simpler elements.
Forgive my unscientific approach, but it makes the special status of the 4 common point plausible.

Last but not least, when this is true for four common points of three conics it also will be true for five common points of three cubics type QA-Cu7, the problem we discussed one year ago extensively.

Best regards,

Chris

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Message: #569
Date: 2020-12-09
From: eckart_schmidt@t-online.de
Subject: Re: QL-Qu3 as triangle quartic

Dear Bernard,

I have followed your construction in #543 and got quartics
... for special QL of a triangle
    (two pairs of orthogonal angle bisectors)
... and any inscribed conic of this QL,
... but what are applications of this set of quartics?
The quartics QL-Qu3 don't exist for these special QL.

Best regards Eckart

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Message: #570
Date: 2020-12-09
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Eckart,

About your remark in QPG#562

"if I am not wrong, there is a typo in your #560:
 The cubic of the contact points is of type 1 in QA-Cu-1."
You are quite right. It should be a cubic of type 1 in QA-Cu-1.
Thanks for correcting.

Best regards,
Chris
```

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Message: #571
Date: 2020-12-09
From: van10hoven@gmail.com
Subject: Re: Generalization of Vittas' theorem for Quadrigon

```
Dear Eckart,

About your remark in message #567:
... and lies always on the conic through I,G1,G2,G3,G4,
... whose 3 versions for a quadrangle we discussed
    in the last messages.

That's not quite correct.
The conic through I,G1,G2,G3,G4 is NOT the conic whose 3
versions for a quadrangle we discussed in the last messages.
The conic whose 3 versions for a quadrangle we discussed in the
last messages was the conic through the midpoint of the
QG-diagonals and QG-P8 and with its center in the Reflection of
QG-P8 in QG-P1.
For the rest I agree with your remarks in message #567.

Best regards,
Chris
```

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Message: #572
Date: 2020-12-09
From: bernard.keizer@gmail.com
Subject: Re: QL-Qu3 as triangle quartic

Dear Eckart,
Many thanks for your interest !
I don't know particular applications for this set of triangle quartics, I only wanted to show a possible reverse construction ...
For this special QL, 2 DT sides are also the 2 Steiner axes and the 3rd DT side is the Steiner Line.
Any DT inscribed conic suits and give a QL-Qu3, the quartics exist, but are undetermined.
For example, taking as 4 lines the 2 pairs of Steiner axes in QL-P1 and QL-P17, the QL has for QL-P1 the point QL-Px = CSC(QL-P17) = CSCdiag(QL(P1) and for QL-P17 the 2nd intersection between the DT circumcircle and the orthoptic circle of the conic ...
Best regards
Bernard

Message: #573
Date: 2020-12-10
From: van10hoven@gmail.com
Subject: which name suits for a geometry of triangles, quadrilaterals,

---

```
Dear friends,

Surveying what we are doing the last decades I noticed we
enlarged the domain of triangle geometry by far. Thanks to the
internet, drawing software (Sketchpad, Cabri, Geogebra), and
calculation software (Mathematica).
Not only triangle geometry ( ETC (
https://faculty.evansville.edu/ck6/encyclopedia/etc.html ) )
reinvented itself, but also the geometry of quadrilaterals ( EQF
), pentalaterals, polygons, etc. ( EPG ).
What they have in common is that all described items are
dependent on n points and/or n lines. The reference system is a
configuration consisting of n points and/or n-lines
(n=2,3,4,5,6, …)
This is a special point of view in geometry.

Other points of view in geometry are Euclidean geometry, affine
geometry, projective geometry, topology, algebraic geometry,
complex geometry, etc.
Since I am describing specifically the geometry that deals with
triangles, quadrilaterals, pentalaterals, polygons, etc. I
wonder what is a good generic name for this type of geometry.
I think this type of geometry is pretty new and for large parts
hardly disclosed.
Anyone who can help me with suggestions for naming this type of
geometry?

Best regards,
Chris
```

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Message: #574
Date: 2020-12-10
From: eckart_schmidt@t-online.de
Subject: Re: Generalization of Vittas' theorem for Quadrigon

Dear Chris,

you are right, my last remark in #567 is wrong,
... lacking in concentration I confused the conics in #547,
... please excuse.

Best regards Eckart

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Message: #575
Date: 2020-12-11
From: bernard.keizer@gmail.com
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

Dear Chris,
For me, we are studying properties of figures with tools.
For example, using complex coordinates, we may obtain the 2 foci of the Steiner inellipse of the triangle (which are not ETC centers) or the 3 foci of Siebeck's curve of the QA (tangent to the 6 sides in their middles).
Another example is the use of the inverse geometry by Frank Morley in studying the plane n-line and many other figures like hypo- and epicycloïds.
ETC is the Encyclopedia of Triangle Centers with barycentric or tripolar coordinates), triangle geometry is the study of the properties of the triangle (see Lalesco for example).
EQF is the Encyclopedia of Quadrifigures (including quadrigons, quadrangles and quadrilaterals) and quadrigeometry is the study of the properties of these figures.
EPF could be the Encyclopedia of Polyfigures (including polygons, polyangles and polylaterals) and polygeometry the study of the properties of these figures.
I find the name of the group perfect (Quadri- and Poly-Geometry).
Best regards
Bernard

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Message: #576
Date: 2020-12-11
From: eckart_schmidt@t-online.de
Subject: Special QA-elements

```
Dear all,

take a QA and a circumconic CO through an arbitrary fixed point
P,
... consider a variable point Q on CO
    and its anticevian triangle wrt QA-Tr1,
... which give a 2nd quadrangle QA' on CO.
Varying Q we can study the loci for points QA-Px of QA'.
 1. Fixed points for the loci of QA-P10,11,12,13 (evident).
 2. Lines through P for the loci of QA-P16,19,31.
 3. Circles through P for the loci of QA-P2,4,28,29,30,35,36.
 4. Conics through P for the loci of
    QA-P5,6,7,8,14,20,22,24,25,26,32,33,37,38.

More interesting are the loci of QA-Px, if we also take P =
QA-Px of QA .
wrt 2. The QA-P16-line is QA-P1.QA-P16.QA-P21,
       The QA-P19-line is ???
       The QA-P31-line is ???
       Further points on QA-P10.QA-P16.QA-P19.QA-P31
       will lead in the same way to lines.
wrt 3. The QA-P2- and QA-P30-circle are QA-Ci1
       (independent of P).
       The QA-P29- and QA-P36-circle are QA-Ci2
       (independent of P).
       The QA-P4-circle bears QA-P12 and ???
       The QA-P28-circle bears QA-P13 and ???
       The QA-P35-circle bears QA-P35
       and is centered on QA-P11.QA-P12 with ratio 2:3.
wrt 4. The QA-P5-conic is a circumconic of QA through QA-P5.
       The QA-P20-conic is a circumconic of QA-Tr1 through
       QA-P5, QA-P20.
       The QA-P38-conic is a circumconic of QA through QA-P38.

Best regards Eckart
```

Message: #577
Date: 2020-12-11
From: bernard.keizer@gmail.com
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

```
Dear Chris,
I realise my message was perhaps not clear enough : you should
replace "Quadrilaterals and Polygons" by "Quadri-Figures and
Poly-Figures" , "we discuss topics on Quadrilateral Geometry and
Polygon Geometry" by " we discuss topics on Quadri-Geometry and
Poly-Geometry " and "Encyclopedia of Polygon -Geometry" by
"Encyclopedia of Poly-Figures".
Best regards
Bernard
```

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Message: #578
Date: 2020-12-11
From: eckart_schmidt@t-online.de
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

```
Dear Bernard,
by the way,
you find the foci of the Steiner inellipse of a triangle
... under 39163 and 39162 in ETC.
Best regards Eckart
```

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Message: #579
Date: 2020-12-11
From: bernard.keizer@gmail.com
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

```
Dear Eckart,
Thanks for the information !
In fact, I never looked at more than 1000 points ...
Best regards
Bernard
```

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Message: #580
Date: 2020-12-13
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

Dear friends,

There has been an update of many new items in EPG.

These items have been added.

5L-1 Notes on the Pentalateral
5L-s-P11 5L-Schmidt Point
5L-s-Tf2 5L-Inscribed Conical Pole
5L-s-Tf3 5L- Inscribed Conical Polar

5P-1 Notes on the Pentangle
5P-s-P6 5P-Miquel Point
5P-s-2P1 5P-Involutary Double Point
5P-s-L1 5P-Schmidt Line
5P-s-Tf5 5P-Conical Pole
5P-s-Tf6 5P-cb Conjugate
5P-s-Tf7 5P-Schmidt Transformation

6P-s-P2 6P-Cayley Bacharach Point for a Circular Cubic
6P-s-Ci1 6P-CSC-Circle
6P-s-Tf1 6P-Schmidt Transformation
6P-s-Tf2 7th point Tangent

7P-s-Cu1 7P-Circular Cubic
7P-s-Tf1 7P-CB Conjugate

8P-s-Tf1 9th point Tangent
9P-s-Cu1 9P-Cubic
14P-s-Qu1 14P-Quartic

nG-n-L1 nG-Centroids Line

5G-s-P4 5G-Miquel-Catalan Point
5G-s-P5 1st 5G-Hung’s Point
5G-s-P6 2nd 5G-Hung’s Point
5G-s-P7 3rd 5G-Hung’s Point

6G-s-P1 1st 6G-Hung’s Point
6G-s-P2 2nd 6G-Hung’s Point

It’s been a tremendous job to gather all information.
Thanks to all QFG-/QPG-contributors, especially Eckart and Bernard.

```
Some information was gathered by myself.
I think Eckart and Bernard will like especially the
cubics-related items and Tran Quang Hung the 5G- and 6G-points.
When you have any additions or comments please let me know.

Best regards,

Chris
```

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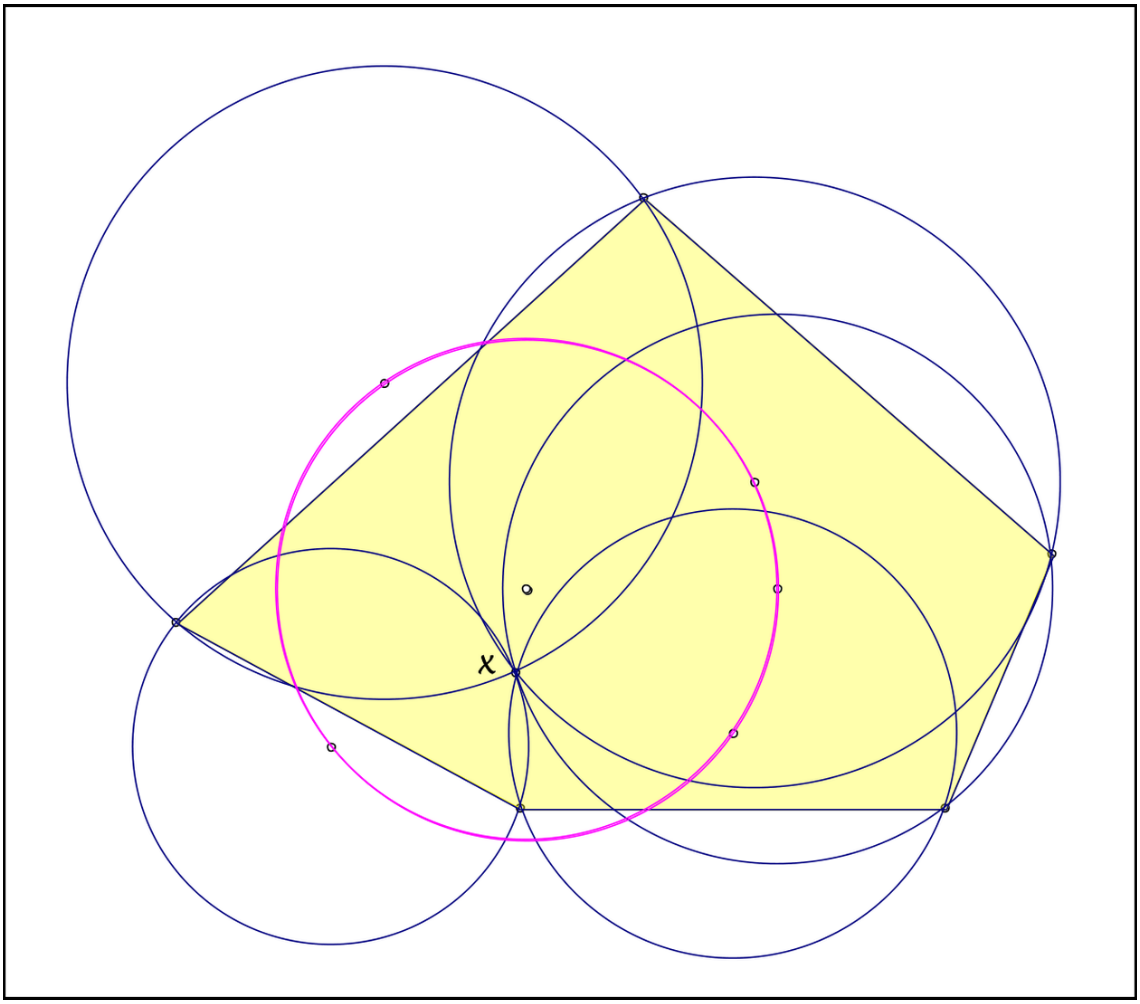
Message: #581
Date: 2020-12-13
From: eckart_schmidt@t-online.de
Subject: New 5G-point/circle

```
Dear all,

what about this 5G-point X,
... whose circles (X,Pi,Pi+1) are concyclic centered?
I found no construction, no properties.
For a QG the locus for these points is QL-Cu1,
... for a 5G  there is only one point X,
... for a 6G there will be no such point in general.

Best regards Eckart
```



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Message: #582
Date: 2020-12-14
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF/EPG

```
Dear Chris,

your update of EPG with new items
... is indeed a surprising Christmas present,
... thanks a lot for this extensive working out!
I shall study it carefully,
... perhaps with new stimulations.

Best regards Eckart
```

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Message: #583
Date: 2020-12-15
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF/EPG

Dear Chris,
Beautiful work, indeed !
Thanks a lot.
It will encourage to search new properties ...
Thanks also for your kind reference to the contributors.
I had a quick glance and have a 1rst reaction : CB14 works as well for a quartic as CB9 for a cubic
There is a 10 points bicircular quartic or a 12 points circular quartic, 13 points have 3 CB14 partners (as 2 quartics intersect in 16 points) and 12 points define a CB transformation associating 3 points to a 13th random point ...
Best regards
Bernard
PS May be I read too quickly, but I don't find the transformation for 5 points centered in 5P-s-P6 such as the transforms of 2 points are inverse wrt the circle of the 3 others ?

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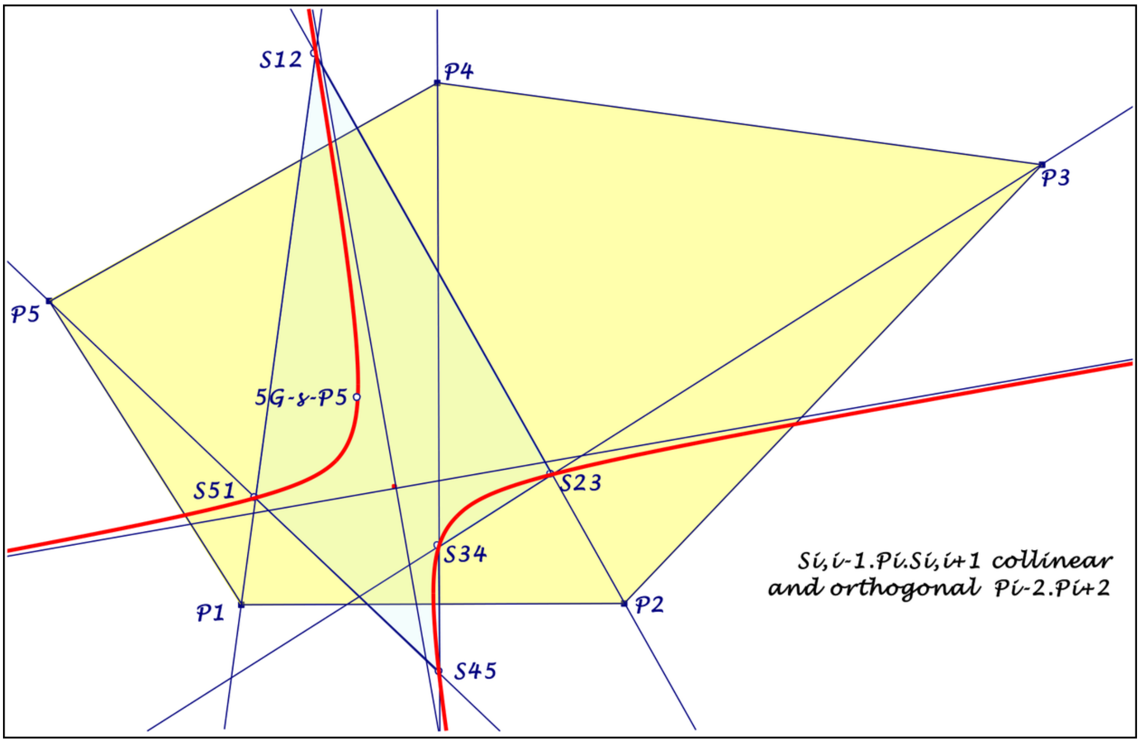
Message: #584
Date: 2020-12-15
From: eckart_schmidt@t-online.de
Subject: Orthogonal 5G-hyperbola

```
Dear all,

is the following orthogonal 5G-hyperbola well known?

Consider for a 5G = P1...P5 the perpendiculars Li from Pi wrt
Pi-2Pi+2
... and the intersections Li ^ Li+1, which define a new 5G,
... with an orthogonal circumhyperbola, bearing 5G-s-P5.

Best regards Eckart
```



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Message: #585
Date: 2020-12-15
From: bernard.keizer@gmail.com
Subject: Stelloïds

Dear Chris, dear Eckart
As Chris seems decided to generalise many items to the polyfigures, I think the stelloïds should have their place.
For n points Pi, a stelloïd is the locus of the points P for which the mean direction of the lines PPi is the same (see Mathcurve).
For n = 2, the curve is the rectangular hyperbola
For n = 3, the curve is the cubic stelloïd QL-Cu1, either Mac Cay or Kjp for the triangle
For n = 4, the curve is a quartic stelloïd ?
All the polar curves of a stelloïd are also stelloïds (the 1rst polars of the cubic stelloïds are rectangular hyperbolas ...)
Best regards
Bernard

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Message: #586
Date: 2020-12-15
From: van10hoven@gmail.com
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

Dear Bernard,

Thanks for your suggestions in naming the geometry of triangles, quadrilaterals, pentalaterals, etc.
People from outside often tend to talk about quadrilaterals and polygons, but they also include the varieties (quadri-figures and poly-figures) we distinguish.
We know how essential the differences in quadri-/poly-figures are (-angles, -laterals, -gons).
It is often a struggle for me to find the right tone, the right words for the right public.
Also in the naming of our encyclopedias.
For us as insiders the name can be the only right one. For outsiders the name can be unclear or even inexplicable.
I haven't made up fully my mind yet.

Best regards,
Chris

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Message: #587
Date: 2020-12-15
From: bernard.keizer@gmail.com
Subject: QL's triangle with the same psi

Dear Chris, dear Eckart
A QL has only but one QL-Cu1 and QL-Cu2
QL-Cu2 is a cubic stelloïd with many pivots triangles, but only one is the main pivots triangle.
We may associate this main pivot triangle of the cubic stelloïd to the QL.
The construction is given by Bernard Gibert in the article Inscribed cardioïds and Eckart's Cubics on page 18-19 with figures 18 for Mac Cay and 19 for Kjp
Mac Cay is K003 in Bernard Gibert catalogue, with monocursal hessian K048 and Kjp is K024 with bicursal hessian K193.
The main interest is that the QL and this triangle have many common properties.
As well known, QL-P1 is the centroïd X2 of the triangle and the points QL-2P2 are the isodynamic points X15 and X16 of the triangle.
If the cubic is Mac Cay, the Newton Line is the Brocard Line through the 2 points, which cuts QL-Cu2 in the circumcenter X3 of the triangle. This gives in turn the Euler Lineof the triangle, which cuts QL-Cu1 in 2 diametral points of the circumcircle.
If the cubic is Kjp, the Newton Line is the Lemoine axis, perpendicular bisector of the 2 points, which cuts QL-Cu2 in 3 points ; these 3 points are the centers of the 3 Apollonius circles, through the 2 points X15 and X16,each circle through a vertice of the triangle ...
The triangle and the QL have the same psi transformation (CSC for the QL), centered in X2 = QL-P1 with fixed points the foci of the Steiner inellipse , which are X39162 and X39163 of the triangle (many thanks again to Eckart) and swapping X15 and X16 = QL-2P2a and b.
The 40000 ETC points of this triangle are QL points ...
Best regards
Bernard

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Message: #588
Date: 2020-12-16
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] new items in EQF/EPG

```
Dear Mr Chris and friends,

Thank you so much for the update of some my findings on your EQF
and EPG, I am happy to see that.
I believe that geometry of quadri-figure and poly-figure has
turned into a major study of plane Euclidean geometry.
I am very pleased to see You are interested in some my new
contributions.
Thank You Mr Chris, Mr Eckart, Mr Bernard, for your help in
contributions more properties around my discovering.
This is really a meaningful Christmas gift for me.
Once again, thank You very much.

Best regards
Tran Quang Hung.
```

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Message: #589
Date: 2020-12-16
From: analgeomatica@gmail.com
Subject: which name suits for a geometry of triangles, quadri laterals, pentalaterals, . . .

Dear Chris, dear Eckart, dear Benard and friends,

Thank you for your messages.
I'm really interested in geometry, so I'm very interested in this discussion.

First of all I have some thoughts on the development of both Mathematics and Computing. It seems that with the help of Cabri, Geogebra, Mathematica even more artificial intelligence technology, we seem to have broken the barriers in geometry. Now some of the visual properties we have are easy to test with a computer. Obviously finding "proofs" (in the classical sense) may seem difficult and unnecessary, the test of Cabri and Geogebra might consider it a solution.

The idea of n-point and n-line geometry is really interesting, points and lines are the basic objects of Euclidean plane geometry. I also have another idea why don't we check n-circle, with 3 random circles, we have radical center? ... I am also very thankful to EQF and EQG, since I heard about EQF and QEG group and the page and web by Chris, I learned a lot more about geometry, I used to be familiar with triangle and quadrilateral. Obviously the work you're doing is a pretty interesting geometric classification. Regarding classified geometry , I also have the idea that we always consider n-points or n-lines in Euclidean plane with Euclidean distance, so embding them in other geometries with other distances gives us have other types of properties, so giving the idea of n-points or n-lines is very general, we can apply that view to existing geometries like affine or projective, but still Euclidean picture is the most abundant.

I also did a few studies on a configuration of point-and-line, circles, and I find them closely related to "Incidence Geometry", which is a fairly new discipline in geometry and classification geometry.

https://en.m.wikipedia.org/wiki/Incidence_geometry

Clearly, the above studies on EQF, PQF in the web of Chris are the properties that are deeper than the n-points and n-lines in the Euclidean plane, the Euclidean geometry is still the richest. But what about other geometries?

Here are some of my comments, thank you very much.

Best regards
Tran Quang Hung.

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Message: #590
Date: 2020-12-16
From: eckart_schmidt@t-online.de
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

Dear Tran Quang Hung,

thanks to formulate the basis of my hobby geometry in your #589:

"First of all I have some thoughts on the development of both Mathematics and Computing.
It seems that with the help of Cabri, Geogebra, Mathematica even more artificial intelligence technology,
we seem to have broken the barriers in geometry.
Now some of the visual properties we have are easy to test with a computer.
Obviously finding "proofs" (in the classical sense) may seem difficult and unnecessary, the test of Cabri and Geogebra might consider it a solution."

Best regards Eckart

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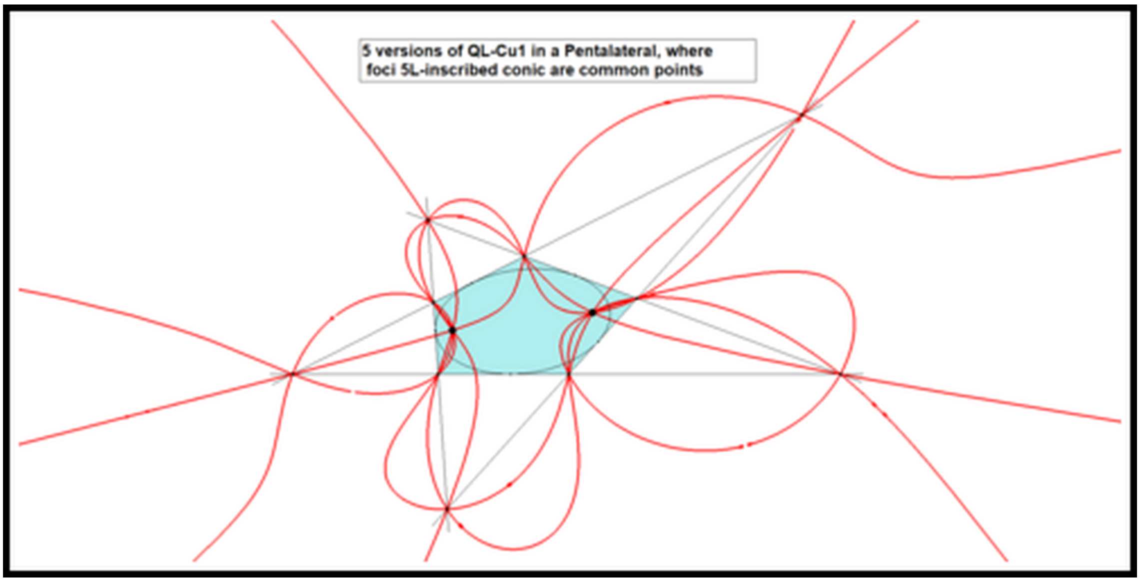
Message: #591
Date: 2020-12-16
From: van10hoven@gmail.com
Subject: Re: New 5G-point/circle

```
Dear Eckart,

Looks like a promising point X for a 5G.
I had a quick view on it, but didn't find further clues.
One thing I found that the 5 versions of QL-Cu1 in a 5-Line have
2 common points, being the foci of the 5L-inscribed conic.
See attached picture.

Best regards,
Chris
```



5L-s-2P1 Common QL-Cu1 points-01.png

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Message: #592
Date: 2020-12-17
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] New 5G-point/circle

Dear Eckart,

Did you construct the exact X point? How to make it with geogebra?

Best Regards
Tran Quang Hung.

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Message: #593
Date: 2020-12-17
From: eckart_schmidt@t-online.de
Subject: Re: New 5G-point/circle

Dear Tran Quang Hung,

sorry I have no construction for this point,
... I search in vain for any further property.

Best regards Eckart
<https://groups.io/g/Quadri-and-Poly-Geometry/message/592>

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Message: #594
Date: 2020-12-17
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] New 5G-point/circle

Dear Eckart

Thank you. So the drawings in your pdf file are approximate? But
why do you
claim there is only one point X?

Best Regards
Tran Quang Hung.

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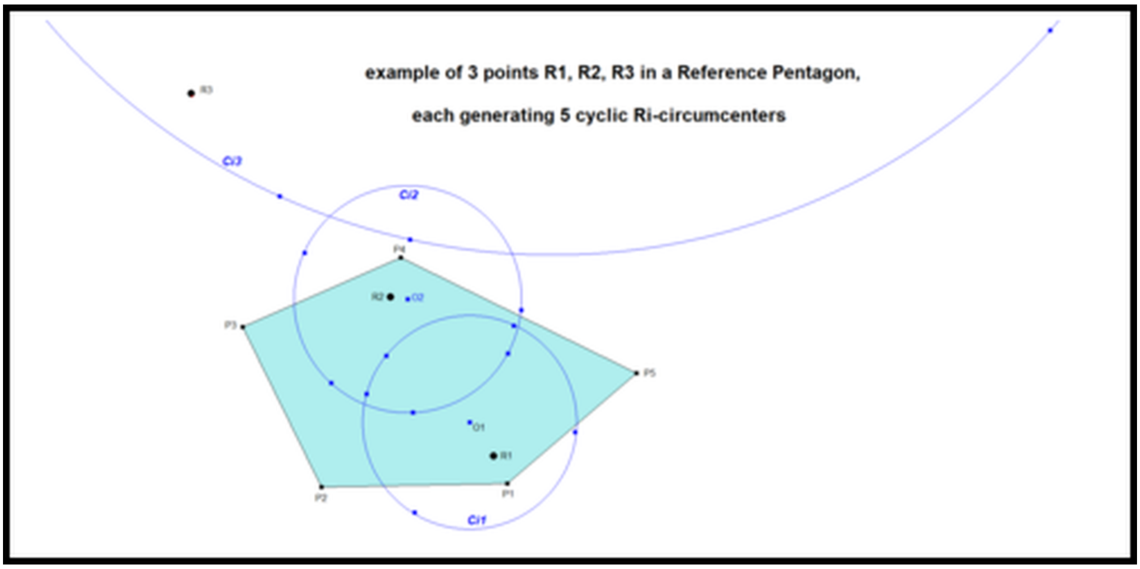
Message: #595
Date: 2020-12-17
From: van10hoven@gmail.com
Subject: Re: New 5G-point/circle

```
Dear Eckart and Tran Quang Hung,

I looked again at the 5G-point.
However I found in at least one example of a 5-Gon 3 points that
generate P-cyclic circumcenters.
See attached picture.
Therefore it is fair to say that there is not one single point
with that property.
For me it is uncertain yet how many points have this property.

Best regards,

Chris
```



5G-1st-P-circumPentangle is cyclic-13.png

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Message: #596
Date: 2020-12-17
From: eckart_schmidt@t-online.de
Subject: QG-P5 and new QG-elements

```
Dear all,

for a quadrigon P1...P4 holds the following property
... Pi.QG-P5 * Pi+1.QG-P5 = const, independent of i,
... this leads to a circle round QG-P5 with radius sqr(const).

If we consider the four Möbius transformations,
... centered in QG-P5, swapping Pi and Pi+1,
... we get four concyclic images of a point,
... this leads to a transformation, mapping a point to a circle.

If we replace QG-P5 by another point T,
... then the four transformations give concyclic images,
... if T is a point on a quartic (?),
... QG-circumscribed, CSC-invariant,
... bearing QG-P1, QG-P5 and two Miquel points (unequal QL-P1),
... which are the vertices of a rectangle.

What about this QG-quartic?

Best regards Eckart
```

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Message: #597
Date: 2020-12-18
From: tungvtt@gmail.com
Subject: Poins on 5L with concyclic projection

Dear all,

Given a 5L, what is the point(s) X such that the 5 projection of X on each line of this 5L are concyclic ?
It is (they are) the intersection of 5 QL-Cu1 of the component QAs.

Best regards,
Vu Thanh Tung

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Message: #598
Date: 2020-12-18
From: eckart_schmidt@t-online.de
Subject: Re: New 5G-point/circle

Dear Chris, dear Tran Quang Hung,

only a short reaction:
Chris is right, there will be more points X in #581.

If you take the 2nd intersection X of circles
... through Pi, Pi+1, Pi+2 and Pi, Pi-1, Pi-2,
... two pairs of the five circles (X,Pi,Pi+1) coincide
... and the centers of the five circles are evidently concyclic.
Further points X can (often) be found on the circles
... through three vertices Pi, Pi+1, Pi+2 of the 5G,
... then two of the five circles coincide.
But not all points X lie on a circle through Pi, Pi+1, Pi+2.
So far, thanks for interest!

Best regards Eckart

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Message: #599
Date: 2020-12-18
From: eckart_schmidt@t-online.de
Subject: Re: Points on 5L with concyclic projection

```
Dear  Vu Thanh Tung,

for a 5L  the points with concyclic projections
... will be the foci of the inscribed conic 5L-s-Co1,
... as common points of the QL-Cu1 of the QL-components.

Best regards Eckart
```

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Message: #600
Date: 2020-12-18
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Points on 5L with concyclic projection

---

```
Dear Eckart,

It is interesting.

Thank you very much,
Vu Thanh Tung
```

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Message: #601
Date: 2020-12-18
From: tungvtt@gmail.com
Subject: Locus on 6L and points on 7L with projection

Dear all,

Given a 6L, what is the locus of point X such that 6 projection of X on the component lines lie on a conic ?
Given a 7L, what is the point(s) X such that 7 projection of X on the component lines lie on a conic ? It is (or they are) the intersection of the 7 locus (defined above) of component 6Ls.
Similarly, we can ask:
Given a 10L, , what is the locus of point X such that 10 projection of X on the component lines lie on a cubic?
Given a 11L, what is the point(s) X such that 11 projection of X on the component lines lie on a cubic? It is (or they are) the intersection of the 11 locus (defined above) of component 10Ls.
and et cetera.

Best regards,
Vu Thanh Tung

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Message: #602
Date: 2020-12-18
From: van10hoven@gmail.com
Subject: Re: Orthogonal 5G-hyperbola

```
Dear Eckart,

Very nice and surprising construction of this 5G-Orthogonal
Hyperbola !
It will become 5G-s-Co1 in EPG.

Best regards,
Chris
```

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Message: #603
Date: 2020-12-20
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear friends,

I added some very interesting new items in EPG, especially
related to n-Gons.
* 2P-s-iRg1 Regular LH-n-Gons (Lighthouse n-Gons)
* nG-n-iRg1 Regular PDN-n-Gons (Petr-Douglas-Neumann)
* 5G-s-Co1  5G-Schmidt Orthogonal Hyperbola
Any suggestions/comments are welcome.

Best regards,

Chris
```

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Message: #604
Date: 2020-12-20
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

Dear Bernard,

Thank you very much for your interesting messages #583, #585, #587 and your encouraging words.
I am sorry I didn't react earlier.

About #583
* CB14 works as well for a quartic as CB9 for a cubic
* There is a 10 points bicircular quartic or a 12 points circular quartic,
* 13 points have 3 CB14 partners (as 2 quartics intersect in 16 points) and
* 12 points define a CB transformation associating 3 points to a 13th random point ...

I agree, they deserve a place in EPG. They are new to me. Do you have more information about these items? Could you precook it a bit? Preferably I would like to have a general description, some picture, a construction method, extra properties and references.
* the transformation for 5 points centered in 5P-s-P6 such as the transforms of 2 points are inverse wrt the circle of the 3 others ?
Could you explain this? Which specific transformation?

About #585, #587
This stelloid is indeed very interesting for EPG. I never payed much attention to it.
It is interesting to see that a whole bunch of similar curves can be constructed from just 2 (or more) points. I will study them.

Best regards,
Chris

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Message: #605
Date: 2020-12-21
From: van10hoven@gmail.com
Subject: Re: which name suits for a geometry of triangles, quadrilaterals,

---

```
Dear Tran Quang Hung,

Thank you for your comments in QPG-message #589.
I find it always interesting to hear another ones insights.

They are very interesting to me.
I agree that incidence geometry is very appealing.
It does not only deal with the incidence of points occurring on
the same line, but also with lines sharing the same common
point. Even in my view 2 coinciding points or 2 coinciding lines
are incidences.
The reason why so many points are lined up was for me the reason
to develop my theory of Perspective fields (https://www.chrisvan
tienhoven.nl/mathematics/perspective-fields). It's fascinating.

Best regards,

Chris
```

Message: #606
Date: 2020-12-22
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] which name suits for a geometry of

---

```
Dear Chris, dear Eckart and friends,

Thank You for the comments, I know about your Perspective fields
<https://www.chrisvantienhoven.nl/mathematics/perspective-fields
>, that's an interesting part of research, I'll spend some time
delving into it.

Best regards
Tran Quang Hung.
```

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Message: #607
Date: 2020-12-22
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF/EPG

Dear Chris,

wrt your question to Bernard
* the transformation for 5 points centered in 5P-s-P6 such as
the transforms of 2 points are inverse wrt the circle of the 3
others ?

This relevant 5P-CSC transformation
... is first mentioned at the end of Bernard's #3266,
... with a construction in #3270 and #3285.

I am glad, that this transformation will finally appear in EPG.

Best regards Eckart

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Message: #608
Date: 2020-12-22
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF/EPG

Dear Chris, dear Eckart
In the description of 5P-s-P6, R1 and R2 are the CSC of P1 and
P2 and R1 and R2 are inverse wrt the circumcircle P3P4P5.
The same way, R3, R4 and R5 are the CSC of P3, P4 and P5 and P1
and P2 are inverse wrt the circumcircle R3R4R5.
It is remarkable that the Pi and the Ri have the same 5P-s-P6
and the same CSC.
Best regards
Bernard

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Message: #609
Date: 2020-12-22
From: bernard.keizer@gmail.com
Subject: Re: Stelloïds

```
Dear Chris,
I'm glad that you take interest in stelloïds for n points.
All I know is taken from Mathcurve or Bernard Gibert for 3
points. (K003 and K024 are good examples).
Best regards
Bernard
```

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Message: #610
Date: 2020-12-22
From: bernard.keizer@gmail.com
Subject: Re: QL's triangle with the same psi

```
Dear Chris, dear Eckart
I hoped I would find more interest in this this main pivot
triangle of QL-Cu2.
Any pivot triangle on QL-Cu2 has the same foci of the Steiner
inellipse, but only this main pivot triangle has the described
properties.
I think it could interest EQF and I gave the precise reference
of the construction by Bernard Gibert.
Best regards
Bernard
```

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Message: #611
Date: 2020-12-22
From: bernard.keizer@gmail.com
Subject: CB16 and 5L

Dear Chris, dear Eckart
Some time ago, I had put a wrong message CB13 and 5P on the forum.
It was in fact CB16 and 5L, but the idea was correct !
I have no reference, only personal reflexion.
As 2 cubics intersect in 9 points and a cubic is also defined by 9 points, it is well-known that:
* 9 points define a cubic
* 8 points define a 9th CBpoint
* 7 points define a transformation

For a QL, we may choose as 7 points the 6 vertices and QL-P1 and we get a QL transformation
These 7 points are on the 4 degenerated cubics formes by a line and a circle
The transformation swapping the lines and the circles is a 2nd transformation, the CSC transformation associating a point and it's CSC partner
The 2 transformations are different

The same way, as 2 quartics intersect in 16 points and a quartic is defined by 14 points, it is correct that :
* 14 points define a quartic (as mentionned by Chris)
* 13 points have 3 CB16 partners (only add several 14th points give different quartics intersecting in 3 other points)
* 12 points define a transformation associating 3 points to one

For the 5L, we may choose as 12 points the 10 vertices and the 2 foci of the inscribed conic, which gives a 5L transformation.
These 12 points are on 5 degenerated circular quartics formed by a line and the QL-Cu1 of the 4 others and form necessary a CBsystem with the circular points (but 2 points are missing ?)
There is a 2nd 5L transformation swapping the lines and the QL-Cu1 and associating also 3 points to one (see many exchanges between Eckart and me).
The 2 transformations are different.
Merry Christmas and a better Newyear to all of you
Best regards
Bernard

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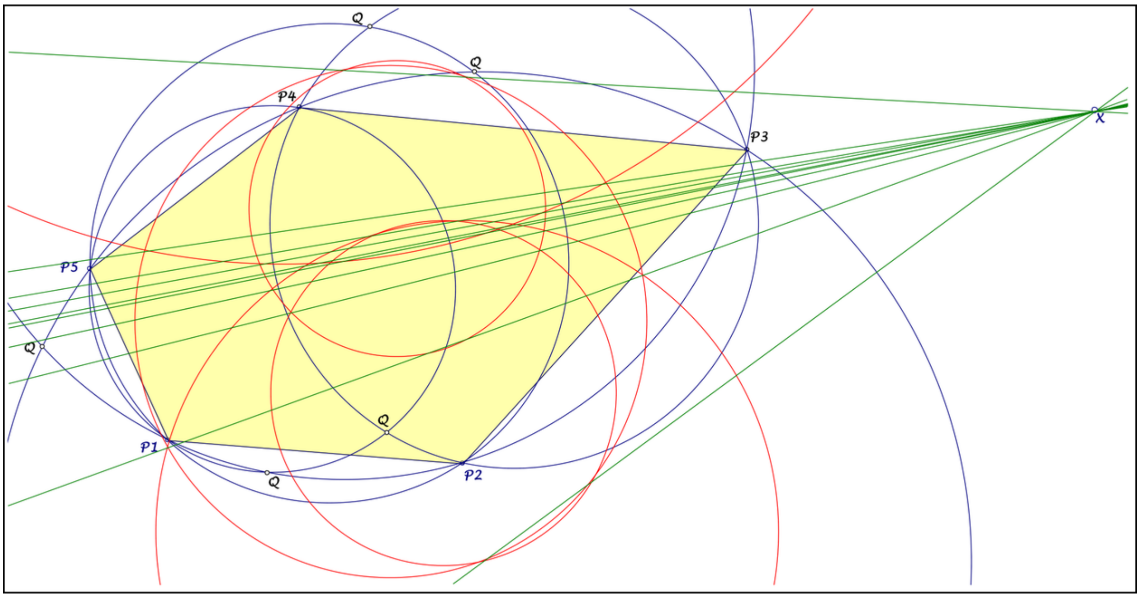
Message: #612
Date: 2020-12-22
From: eckart_schmidt@t-online.de
Subject: Another new 5G-point

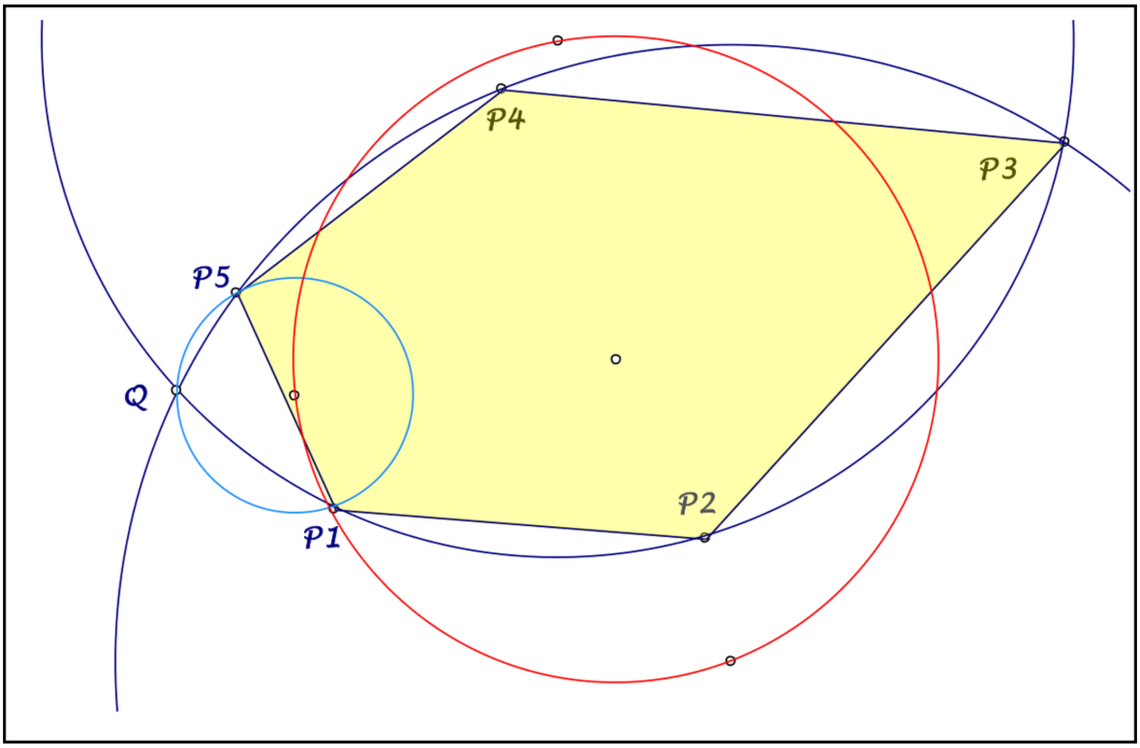
```
Dear all,

for a 5G = P1...5 the 5 circles through 3 consecutive vertices
... have 5 further intersections Q,
... whose 5 circles (Q,Pi,Pi+1) (i=1,...5)
    are centered on a new circle,
... two pairs of centers coincide (see 2nd figure).
The radical axes of these 10 new circles have a common point X.

Best regards and Merry Christmas
Eckart
```



2020-12-22.pdf



2020-12-22a.pdf

Message: #613
Date: 2020-12-23
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

Dear Bernard, Dear Eckart,

Again, regarding the transformation for 5 points centered in 5P-s-P6 such as the transforms of 2 points are inverse wrt the circle of the 3 others
I read the QFG-messages, but it still puzzles me.
Can you be more specific and give me a stepwise constrcution of the 5P-transformation?

Best regrds,
Chri

Message: #614
Date: 2020-12-23
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear Chris,
You gave yourself the construction in the description of QA-Tf16
and 5P-s-P6 !
R1 = QA-Tf16(P2,P3,P4,P5) and so on
R1 and R2 are inverse wrt the circumcircle of P3P4P5 and so on
P1 and P2 are inverse wrt the circumcircle of R3R4R5 and so on
R1 and P1 as well as R2 and P2 are CSC partners in the CSC
centered in 5P-s-P6 with the QL given in your description.
The other R3, R4 and R5 are CSC partners of P3, P4 and P5 in the
same CSC.
By definition, the 5 Ri and the  5 Pi have the same 5P-s-P6 and
the same CSC.
Best regards
Bernard
```

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Message: #615
Date: 2020-12-23
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear Bernard,

Yes, I understand. You describe the different QA's with their
transformations.
But what is the 5P-transformation you were talking about?
Which point is transformed into which other point, by which
5P-transformation in which way?
Best regards,

Chris
```

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Message: #616
Date: 2020-12-23
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF/EPG

```
Dear Chris,

wrt the transformation 5P-CSC (see #3270):
5P-CSC is a Möbius transformation,
... centered in 5P-s-P6, swapping P1 and X1,
... P1 = vertex of the 5P and X1 constructed as follows:

Let Q1 be P1
... Q2 the inverse of P1 wrt (P3,P4,P5),
    Q3 the inverse of P1 wrt (P2,P4,P5),
... Q4 the inverse of P1 wrt (P2,P3,P5),
    Q5 the inverse of P1 wrt (P2,P3,P4),
... then X1 = CSC(P1) is
... ... the inverse of P2 wrt the circle (Q3,Q4,Q5),
... ... the inverse of P3 wrt the circle (Q2,Q4,Q5),
... ... the inverse of P4 wrt the circle (Q2,Q3,Q5),
... ... the inverse of P5 wrt the circle (Q2,Q3,Q4).

Best regards Eckart
```

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Message: #617
Date: 2020-12-23
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

Dear Eckart, dear Bernard

Eckart, what you actually describe in your last message is that
P1 is swapped with X1=QA-Tf16(P1) wrt P2P3P4P5.
My conclusion is that you are describing the QA-P16
Transformation, which is a 4P-Transformation and not a
5P-Transformation.
But you are experts, so I suppose you mean something else than
what you describe.
When you talk about a Möbius transformation centered in 5P-s-P6,
then please describe the radius of the fixed circle involved and
the axis involved.
After all the Möbius transformation is a sequence of an
inversion wrt a fixed circle and a reflection about a fixed
line. Like described in QA-Tf4 (https://www.chrisvantienhoven.nl/qa-items/qa-transformations/qa-tf4) QA-Möbius Conjugate.
Knowing that I will be able to estimate the kind of
transformation you both mean.

Best regards,

Chris

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Message: #618
Date: 2020-12-23
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF/EPG

Dear Chris,

if I describe a Möbius transformation,
... centered in M, swapping A and B,
... the axis is the angle bisector of MA and MB
... and the radius of the inversion circle is sqr(MA*MB).

Best regards Eckart

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Message: #619
Date: 2020-12-23
From: Stan.Rabinowitz@comcast.net
Subject: Name for point in cyclic quadrangle?

```
Let ABCD be a cyclic quadrangle.
It is known [1] that the lines from each vertex to the
nine-point center of the opposite triangle concur at a point K.
The point K is also the center of the circle through the
centroids of the four component triangles.

Does the point K have a name in the literature?
What are some references to books or papers that discuss this
point?

[1] Yaglom, Complex Numbers in Geometry, Academic Press 1968,
pp. 54-59
```

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Message: #620
Date: 2020-12-23
From: eckart_schmidt@t-online.de
Subject: Re: Name for point in cyclic quadrangle?

```
Dear Stanley,

your point K for a cyclic quadrangle is QA-P7
... on a circle round QA-P10 with 2/3 of the radius of QA-Tr1.

Best regards Eckart
```

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Message: #621
Date: 2020-12-23
From: Stan.Rabinowitz@comcast.net
Subject: Re: Name for point in cyclic quadrangle?

Thanks Eckart.

I am wondering if this point could be considered the "nine-point center" of the cyclic quadrangle
or if some other point deserves that name. Maybe call it the "cyclic nine-point center" to distinguish it from
the various "quasi nine-point centers".

EQF doesn't seem to catalog special points associated only with cyclic quadrangles.

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Message: #622
Date: 2020-12-24
From: van10hoven@gmail.com
Subject: Best wishes

```
Dear Friends,

I wish everybody happy days and a good and fruitful 2021.
Thanks for being such great friends!

Best regards,
Chris
```

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Message: #623
Date: 2020-12-26
From: van10hoven@gmail.com
Subject: Re: Another new 5G-point

```
Dear Eckart,

I couldn't get your results.
Could you be a bit more specific.
I constructed:
Q1=Ci(P4,P5,P1)^Ci(P1,P2,P3)
Q2=Ci(P5,P1,P2)^Ci(P2,P3,P4)
Q3=Ci(P1,P2,P3)^Ci(P3,P4,P5)
Q4=Ci(P2,P3,P4)^Ci(P4,P5,P1)
Q5=Ci(P3,P4,P5)^Ci(P5,P1,P2)

And new circles:
Ci(Q3,P5,P1)
Ci(Q4,P1,P2)
Ci(Q5,P2,P3)
Ci(Q1,P3,P4)
Ci(Q2,P4,P5)
In my picture their centers weren't concyclic.
Did I use the right circles?

Then you write:
"... two pairs of centers coincide (see 2nd figure)." Two pairs
of which centers?
"The radical axes of these 10 new circles have a common point
X." I suppose you mean next circles?
Ci(P1,P2,P3) … Ci(P5,P1,P2)
Ci(Q3,P5,P1) … Ci(Q2,P4,P5)
Ten circles have 45 radical axis. Is this what you mean?

So far,
Chris
```

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Message: #624
Date: 2020-12-26
From: eckart_schmidt@t-online.de
Subject: Re: Another new 5G-point

```
Dear Chris,

the 5 circles (Q,Pi,Pi+1) (i=1,...,5) are for fixed Q,
... each Q gives 5 circles, centered on a new circle,
... two pairs of centers of these 5 circles for a fixed Q
    coincide (see 2nd drawing).
If you take pairs of the 5 new circles (red in the drawings)
... the radical axes have a common point.
Excuse the typo in my remark:
... the 10 radical axes of these 5 new circles...

Best regards Eckart
```

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Message: #625
Date: 2020-12-27
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear Eckart, dear Bernard,

I placed 5P-s-Tf8 5P-CSC Conjugate in EPG.
Any comments are welcome.

Best regards,
Chris
```

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Message: #626
Date: 2020-12-28
From: bernard.keizer@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear Chris,
Only one comment : why didn't you put the property given in my
message 614 that the points X1 and X2 are inverse wrt the
circumcircle P3P4P5 ?
Having X1 = QA-Tf16(P1) wrt P2P3P4P5, you have immediately X2,
X3, X4 and X5 as inverses of X1 wrt the circumcircles P3P4P5,
P2P4P5, P2P3P5 and P2P3P4.
Conversely, the points P1 and P2 are inverse wrt the
circumcircle X3X4X5 ?
(It is a well-known and easy to prove geometrically property of
the Moebius transformation : the transformed of 2 points inverse
wrt a circle are themselves inverse wrt the transformed of the
circle)
Best regards
Bernard
```

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Message: #627
Date: 2020-12-28
From: bernard.keizer@gmail.com
Subject: Twin 5P, conics, cubics and quartics and 5P sextic, Newton Line,

---

Dear Chris, dear Eckart
We had with Eckart many exchanges about these items in the old messages 3775 to 3780.
Please read in particular my message 3775.
Chris, you have now started, as you have the 5P, the CSC
5P-s-Tf8 = 5X-s-Tf8 centered in 5P-s-P6 = 5X-s-P6 and the 5X.
5P and 5X are the twin pentangles with the same CSC.
Their circumconics are the twin conics 5P-s-Co1 and 5X-s-Co1.
Now you may have the twin circumcubics and the twin circumquartics (the circumquartic of a pentangle is the CSC of the circumcubic of the other and vice-versa).
The 2 circumcubics are Van Rees circular cubics, invariant in 2 other CSC's and having the same Newton Line, the 2 quartics are also invariant in 2 other CSC's and have the middles of their CSCpartners on 2 Newton circles through the middles of PiQA-P4(other Pj) or XiQA-P4(other Xj) and centered in 5P-s-P3 or 5X-s-P3.
The points T and T' are 5P-s-P4 or 5X-s-P4, U and U' are 5P-s-P5 or 5X-s-P5 and are CSC partners in 5P-s-Tf8.
Last, the vertices of the 2 twin pentangles are on a sextic, invariant in the 5P-s-Tf8 and having the middles of their CSCpartners on the conic through the middles of the segments PiXi.
This is my Christmas present as a contribution the Pentangle Geometry.
Happy Newyear
Best regards
Bernard
PS In old times, Eckart had validated all these properties, I hope I didn't introduce any mystake ...

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Message: #628
Date: 2020-12-28
From: eckart_schmidt@t-online.de
Subject: Re: new items in EQF/EPG

```
Dear Chris,

thanks for the new item 5P-s-Tf8.
Our preliminary nomination 5P-CSC had the reason,
... that for the 5P of the QA-Cu7-triple points of a QL
... 5P-s-Tf8 is the CSC of this QL.

Wrt Bernard's relevant property in #626:
... Bernard has mentioned this property already in old #3266.

Best regards Eckart
```

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Message: #629
Date: 2020-12-29
From: van10hoven@gmail.com
Subject: Re: new items in EQF/EPG

```
Dear Bernard, dear Eckart,

I have processed your comments about 5P-s-Tf8 in EPG.

Best regards,
Chris
```

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Message: #630
Date: 2020-12-30
From: bernard.keizer@gmail.com
Subject: Re: Twin 5P, conics, cubics and quartics and 5P sextic, Newton Line,

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```
Dear Chris, dear Eckart,
The twin pentalaterals follow of course the twin pentangles and
5P-s-Co1 is also 5L-s-Co1 with 2 twin Miquel circles ...
I haven't found any relevant property between the vertices or
the Miquel points of these twin pentalerals, but I don't give up
competely.
For example, the 4 common tangents to the 2 conics define a QL
in which the 2 conics are inscribed and the QL-Cu1 of this QL
passes through the 2 pairs of foci of the conics.
Eckart, have you already searched in this direction ?
Best regards
Bernard
```

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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393