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Chris van Tienhoven et al.

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Messages #631 - #1294

Contents

1 Introduction

This journal is a compilation of messages from the **Quadri- and Poly-Geometry (QPG)** forum, where mathematicians and geometry enthusiasts exchange ideas on the properties of **quadrilaterals**, **polygons**, and **curves of nth degree**. The discussions cover a wide range of topics, from classical geometric theorems to new discoveries and insights.

The origins of this journal trace back to the Quadri Figures Group (QFG, available at https://groups.io/g/Quadri-Figures-Group), which was active from 2013 until November 2019. In November 2019, the forum transitioned into the Quadri- and Poly-Geometry Group (QPG, available at https://groups.io/g/Quadri-and-Poly-Geometry) forum, which continues to facilitate discussions on quadrilaterals, polygons, and related topics. Over the years, these forums have evolved into valuable resources for exploring both well-established results and novel perspectives in geometry. For both forums, an **annual record of all incoming messages** is compiled in this journal.

This journal is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing users to quickly jump between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting these forum messages, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Bernard Keizer
- Chris van Tienhoven
- César Lozada
- Dao Thanh Oai
- Eckart Schmidt
- Jayendra Jha and Sankalp Savaran
- Michael de Villiers
- Ngo Quang Duong
- Peter Liepa
- Stanley Rabinowitz
- Tran Quang Hung
- Vu Thanh Tung

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

- **Eckart Schmidt**
  email: eckart_schmidt@t-online.de:
  #632 #636 #638 #640 #642 #644 #647 #652 #655 #660 #661 #663 #668 #670 #671 #673 #675 #677 #678 #683 #686 #687 #688 #690 #691 #697 #699 #702 #705 #707 #710 #712 #713 #715 #718 #724 #726 #727 #729 #731 #732 #736 #741 #744 #745 #746 #748 #750 #753 #757 #761 #762 #764 #766 #768 #772 #773 #776 #777 #781 #782 #784 #785 #787 #792 #793 #797 #799 #800 #802 #805 #806 #808 #812 #814 #817 #818 #819 #826 #827 #829 #830 #833 #835 #837 #838 #843 #845 #849 #865 #869 #873 #875 #877 #880 #881 #882 #884 #885 #892 #905 #911 #916 #919 #921 #922 #923 #924 #927 #928 #931 #934 #942 #943 #950 #952 #959 #960 #961 #962 #970 #971 #978 #979 #982 #990 #992 #993 #994 #998 #999 #1000 #1004 #1006 #1009 #1012 #1013 #1014 #1018 #1026 #1027 #1029 #1030 #1031 #1032 #1033 #1037 #1038 #1039 #1043 #1044 #1047 #1049 #1051 #1053 #1054 #1056 #1058 #1061 #1062 #1063 #1064 #1066 #1067 #1068 #1069 #1071 #1074 #1075 #1076 #1077 #1080 #1081 #1083 #1086 #1087 #1089 #1090 #1091 #1096 #1097 #1098 #1099 #1101 #1103 #1104 #1105 #1109 #1110 #1114 #1115 #1118 #1119 #1122 #1124 #1125 #1127 #1128 #1131 #1132 #1133 #1136 #1138 #1140 #1143 #1145 #1147 #1148 #1152 #1153 #1154 #1155 #1156 #1157 #1159 #1163 #1167 #1168 #1170 #1173 #1180 #1182 #1185 #1189 #1193 #1195 #1196 #1201 #1202 #1203 #1205 #1210 #1212 #1213 #1217 #1222 #1224 #1228 #1229 #1242 #1243 #1245 #1246 #1248 #1252 #1253 #1254 #1261 #1271 #1272 #1275 #1276 #1277 #1281 #1283 #1284 #1286 #1290

- **Jayendra Jha and Sankalp Savaran**
  email: archanajha112018@gmail.com:
  #1206 #1207 #1230 #1231 #1233 #1234 #1236 #1238 #1239 #1241 #1263 #1264 #1265 #1266 #1268 #1269 #1279 #1280

- **Michael de Villiers**
  email: profmd1@mweb.co.za:
  #1177 #1179 #1183 #1226 #1232 #1240

- **Ngo Quang Duong**
  email: ngo.quang.duong.1100@gmail.com:
  #1161 #1164 #1165 #1166 #1188 #1198 #1200 #1204 #1209 #1219 #1221 #1256 #1262 #1278 #1293 #1294

- **Peter Liepa**
  email: peter.liepa@gmail.com:
  #1235 #1237 #1267 #1274

- **Stanley Rabinowitz**
  email: Stan.Rabinowitz@comcast.net:
  #1175 #1178 #1184 #1291 #1292

- **Tran Quang Hung**
  email: analgeomatica@gmail.com:
  #631 #820 #824 #888 #891 #894 #900 #901 #904 #907 #915 #917

#918 #929 #930 #932 #933 #937 #954 #955 #958 #974 #983 #984 #1008

- **Vu Thanh Tung**
  email: tungvtt@gmail.com:
  #889 #890 #893 #899 #903 #906 #908 #909 #910 #912 #913 #914 #935 #936 #938 #939 #940 #941 #944 #945 #946 #947 #948 #949 #951 #953 #963 #964 #965 #966 #967 #977 #987 #988 #989 #997 #1003 #1171 #1172 #1176 #1191 #1194 #1197 #1247 #1249 #1257 #1258

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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines.* Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry.* Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Ngo Quang Duong

Location
Living in Hanoi, Vietnam.

Year of Birth / Generation
Born in 1998.
Generation Z (approx. 1997–2012).

Short Biography
Ngo Quang Duong studied at the Vietnam National University, where he initially majored in Mathematics before switching to Software Engineering. He has since been working professionally in the software field, while continuing to pursue his interest in Mathematics in his free time. His background combines formal mathematical training with practical experience in computing and problem solving, giving his work a distinctive blend of theoretical insight and computational intuition.

Themes and Interests

- Geometry, Topology, Analysis, and their interactions
- Classical Geometry, especially triangle geometry and quadri-figure geometry
- Contributions to QFG, including n-angle centers and new uses of coordinate systems
- General mathematical exploration and independent study

Selected Publications

- (with T. T. Vu) *A Generalization of the Droz Farny Line Theorem with Orthologic Triangles*, Forum Geometricorum, Volume 16 (2016), 415–418.
- *Generalization of Musselman's theorem. Some Properties of Isogonal Conjugate Points*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 15–29.
- (with O. T. Dao and P. Yiu) *Golden Sections in an Isosceles Triangle and Its Circumcircle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 5 (2016), 93–97.
- *Generalizations of Lester Circle*, Global Journal of Advanced Research on Classical and Modern Geometry, Volume 10 (2021), 49–61.

Additional Remarks
He has not actively returned to Classical Geometry for some time, but remains mathematically engaged through online communities. He is active on Math StackExchange (MSE), where he contributes under the profile: https://www.math.stackexchange.com/users/821868/duong-ngo

Stanley Rabinowitz

Location
Living in New Hampshire, USA.

Year of Birth / Generation
1947 (Baby Boomer).

Short Biography
Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions
Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, International Journal of Computer Discovered Mathematics 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, International Journal of Computer Discovered Mathematics 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 214–287.

Additional Remarks
Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

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- **subject: Conic transformation**:
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- **subject: Construction of 5P-quartic**:
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- **subject: Convex Hexagon Theorem related to Napoleon Configuration.**:
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- **subject: Cu1 and Cu7**:
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- **subject: Do you known this theorem on tangential quadrilateral before?**:
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- **subject: Focus and 5P-s-Tf6-pivot of 7P-s-Cu1 for a 5P**:
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- **subject: QG-circumquartic and QL-2Px**:
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- **subject: QG-Cubics with CSC-Pivot**:
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- **subject: QG-P16-splitter**:
  #1056 #1057 #1059
- **subject: QG-P2 property**:
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- **subject: Review QA-Cu6**:
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- **subject: Some observations and references concerning Quadrilateral Conics**:
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- **subject: Twin 5P, conics, cubics and quartics and 5P sextic, Newton Line, circles and conic**:
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- **subject: Unknown QG-circle?**:
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- **subject: Unknown QL-circle?**:
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- **subject: Unknown triple on QL-Ci1**:
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- **subject: View angle of QG-diagonals**:
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4 Messages

4.1 Message Index

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4.2 Messages

Message: #631
Date: 2021-01-01
From: analgeomatica@gmail.com
Subject: Re: Some new theorems on Pentagon and Pentagram

Dear Chris, dear Eckart, dear Bernard and geometers,

Happy new year 2021 to all members of QPG.

I still closely follow some of your new discussions on QPG, but I find my knowledge is still limited so I can rarely participate in discussions, but I am always very interested in your discussions.

Returning to this topic, I think Chris's comments on the name "dual of Theorem 4" are very interesting, I discovered that my theorem 5 and my theorem 4 are really different. However the so-called "Dual of Theorem 4" should be the theorem below (please see my figure)

Let Ai , i = 1, 2, . . . , 5, be any five points. Taking subscripts modulo 5, we denote, for i = 1, 2, . . . , 5, the intersection of the lines AiAi+2 and Ai+1Ai+4 by Bi, the second intersection of two circles (BiAi+2Ai+4) and (Bi+3AiAi+2) by Ci+4, the center of circle (Ci+2Bi+1Bi+3) by Li, and the center of circle (Bi+3Ai+1Ai+4) by Ki. Then five lines KiLi , for i = 1, 2, . . . , 5, are concurrent at a point X.

I think that X is a new point? Could you please help me to check it?

So I think that there is a "dual of Theorem 5".

Best Regards
Tran Quang Hung.

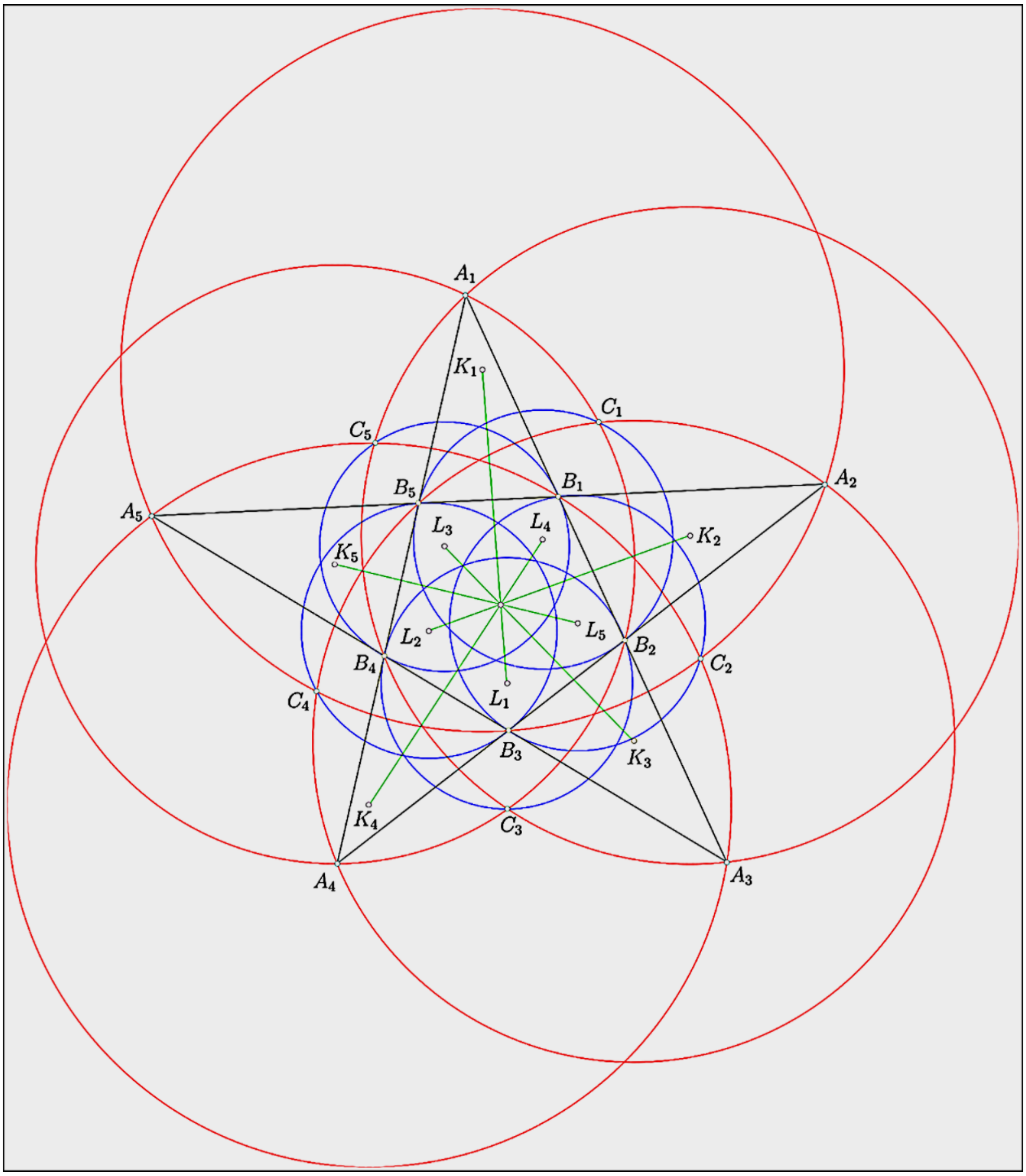

Figure7749.pdf

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Message: #632
Date: 2021-01-03
From: eckart_schmidt@t-online.de
Subject: 7P-s-Cu1

Dear all,

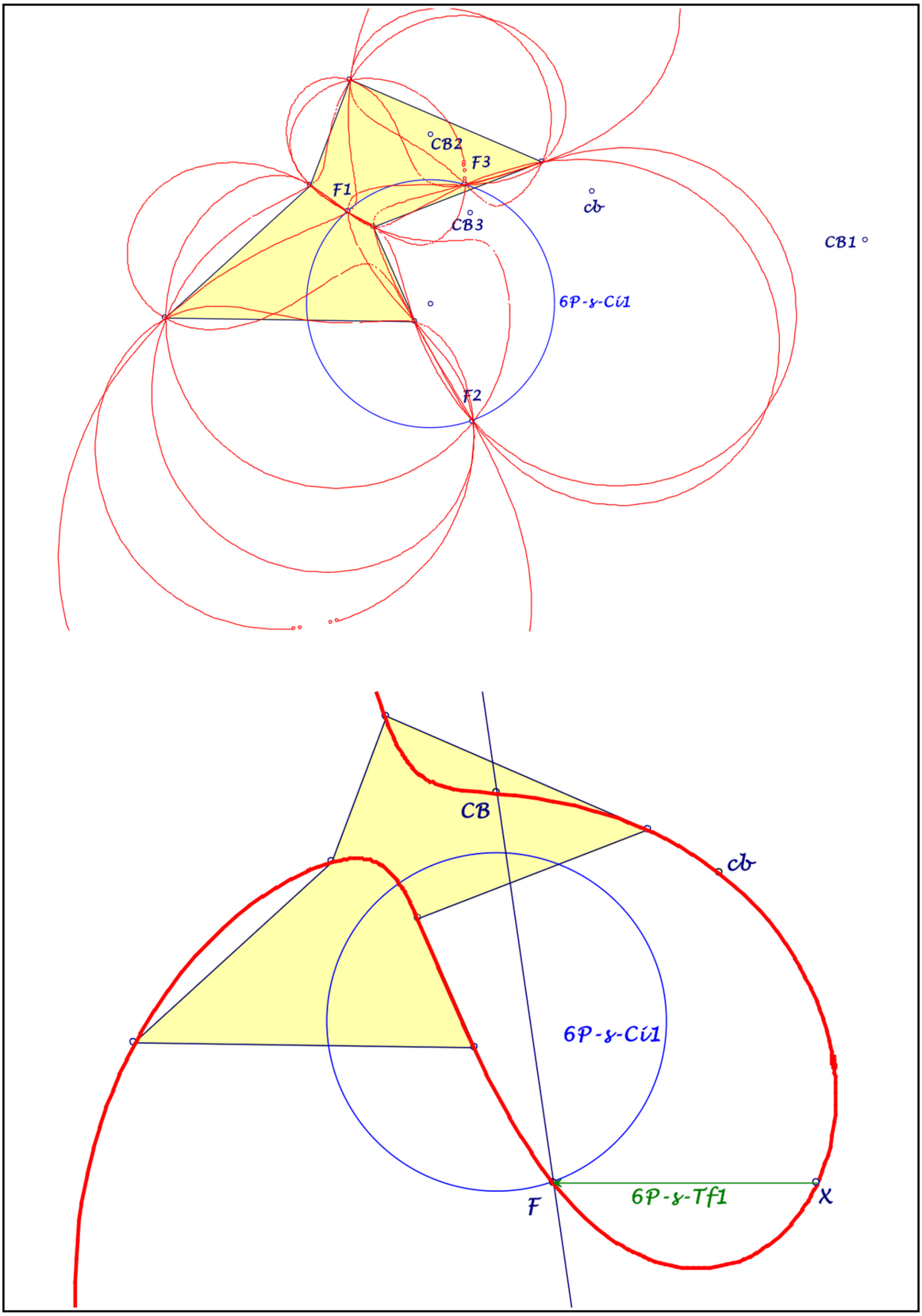
I start the new year with the hope,
... that someone can confirm this investigation:

Let us consider a 6P and its Cayley-Bacharach point cb
... wrt the 6 vertices and the two circular points.
A further point defines a 6P-circumconic as 7P-s-Cu1.

If we take the six 5P-quartics of the 6P
... we get up to three common points F on 6P-s-Ci1,
... which are the fixed points of 6P-s-Tf1,
... one shall be taken for a 7P-s-Cu1,
... which bears the 9th Cayley-Bacharach point CB
... of the 6 vertices, cb and F
... and has an asymptote parallel F.CB.
This 7P-s-Cu1 is the locus of all points X with 6P-s-Tf1(X) = F.

Best regards Eckart

PS. The drawing of the cubic is not exactly constructed!



2021-01-03.pdf

Message: #633
Date: 2021-01-03
From: bernard.keizer@gmail.com
Subject: Re: Some new theorems on Pentagon and Pentagram

```
Dear Chris, dear Eckart, dear Tran
Sorry if I join your debate a little late !
I first tried to reproduce your properties.
It seems to me that it is much easier to describe the figure as
a pentalateral.
5 lines Li, i = 1 to 5 give the 10 vertices Ai and Bi in Tran's
last figure as Aij and the centers of circumcircles of AijAikAjk
as Olm (Ki in the figures of theorems 4 or 5).
The Ci of Tran's last figure are the Miquel points Mi, which are
concyclic on 5L-o-Ci1 (2nd intersections of circles in theorem
1).
I found then easily the 2 different points X of theorems 4 and
5.
But I can't see a difference between theorem 5 in the article
and the socalled dual of theorem 4 in the message 631. Did I
miss something ?
Best regards
Bernard
```

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Message: #634
Date: 2021-01-03
From: van10hoven@gmail.com
Subject: Re: Some new theorems on Pentagon and Pentagram

Dear Tran Quang Hung,

Another description

Here is my description of your most recent point in QPG#631:
* Let L i = Line(A i-1 , A i+1).
* Let C i = Circle(A i-1 , A i+1 , L i-2 ^L i+2) with center P i.
* Let S i = 2 nd intersection point of circles C i-1 ^C i+1.
* Let D i = Circle(S i , L i+1 ^L i+2 , L i-1 ^L i-2) with center Q i.
* Then lines P i.Q i will be concurrent in a point X for i = 1, ... , 5.

Look at the indices and you will see the symmetric construction in the combinatorics in it.
It's an extra level to find the combinatoric logic.
Seeing the combinatoric logic it can bring you new ideas.
For example, what happens when Circle(A i-1 , A i+1 , L i-2 ^L i+2) is replaced by Circle(A i-1 , A i+1 , L i-1 ^L i+1)?
Pentagons spanned in a Pentangle or Pentalateral
Now a bit of theory.
When we consider a Pentagon we have 5 cyclically connected Points.
However when we consider the 5 points without restriction we have a Pentangle.
And we see that the Pentagon was spanned in the Pentangle.
There are 10 ways to span a Pentagon in a Pentangle.
· Type-1: One way is the convex Pentagon we draw by reflex.
· Type-2: There is another one that we usually call the Pentagram.
· Types-3&4: And then we have 8 others subdivided in 2 forms.

See https://www.chrisvantienhoven.nl/ng-items/ng-geninf/ng-1.
Note that a Pentagon also can be spanned in 10 different ways in a Pentalateral.
What I want to clarify is that when you redraw your picture by dragging all 5 points to other ones of the original points, you can simply make a Type-2-Pentagon out of a Type-1-Pentagon, where the original points are preserved. Then your point X has become a point X2 relating to the original points. But it is the same point, it is just one of the ten 5G-versions in a Pentangle.

Maybe not very exciting, but it is good to realize that when you change the description of a 5G-point by systematically interchanging indices (i-2,i-1,i,i+1,i+2) or indices (i,i+1,i+2,i+3,i+4), we actually describe another version of the same Pentagon point. That's what we have to bear in mind considering 5G-points. And it shows the importance of making a combinatoric description of a 5G-point.

Best regards,

Chris

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Message: #635
Date: 2021-01-03
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,

Can you refresh my memory for the construction of the QL-Cu1 in
a 5P.
I know that you need 5P-s-P6. But how is the stepwise
construction.
When I understand it right the 5P-quartic is
5P-s-Tf1(5P-QL-Cu1).

Best regards,

Chris
```

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Message: #636
Date: 2021-01-03
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

I think old #3675 describes a construction of a 5P-quartic.

Best regards Eckart
```

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Message: #637
Date: 2021-01-03
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,
Yes, I read that, but I got stuck in the construction of the
QL-Cu1-like-cubic in a Pentangle.
Therefore I asked you a detailed stepwise construction.
Preferably with the new 5P-names.
Best regards,
Chris
```

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Message: #638
Date: 2021-01-04
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

here a construction of the cubic, whose 5P-s-Tf8 image is the
5P-quartic:

Consider a 5P and its transformation 5P-s-Tf8,
... further the Möbius transformation CSC,
... ... centered in 5P-s-Tf8(5P-s-P5),
        swapping 5P-s-P4 and 5P-s-P6,
... further parallels L to the line Lo,
    connecting the midpoints of
5P-s-Tf8(Pi) and CSC(5P-s-Tf8(Pi)).
The intersections of the circles CSC(L) and the reflection of L
in Lo give the cubic,
... whose 5P-s-Tf8 image is the 5P-quartic.

Best regards Eckart
```

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Message: #639
Date: 2021-01-04
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,

Your answers are still a riddle to me.
1. You talk about 5P-s-Tf8 and the Möbius transformation CSC as
different things.
For me 5P-s-Tf8 is the 5P-CSC-transformation as well as a Möbius
transformation. Maybe you mean with CSC the
QL-Tf1-transformation for 4 remaining points?
2. Then you introduce a line Lo, connecting the midpoints of
5P-s-Tf8(Pi) and CSC(5P-s-Tf8(Pi)). Since i=1-5 you obviously
assume that these midpoints are linear for i=1,2,3,4,5. I cannot
confirm this when CSC = the QL-Tf1 transformation for 4
remaining points.
So please unravel this riddle in a way as simple as possible, so
that even I can understand it (attempt to a joke).

Best regards,
Chris
```

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Message: #640
Date: 2021-01-04
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

give the here used Möbius transformation another name XYZ
... but take the definition of my message:
... ...  centered in 5P-s-Tf8(5P-s-P5),
         swapping 5P-s-P4 and 5P-s-P6,
... then the midpoints of 5P-s-Tf8(Pi) and XYZ(5P-s-Tf8(Pi))
    will be collinear.

Best regards Eckart
```

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Message: #641
Date: 2021-01-04
From: van10hoven@gmail.com
Subject: Re: Locus on 6L and points on 7L with projection

Dear Vu Thanh Tung,

About your four questions in QPG#601:
1. Given a 6L, what is the locus of point X such that 6 projection of X on the component lines lie on a conic?
2. Given a 7L, what is the point(s) X such that 7 projection of X on the component lines lie on a conic? It is (or they are) the intersection of the 7 locus (defined above) of component 6L's.
3. Given a 10L, what is the locus of point X such that 10 projection of X on the component lines lie on a cubic?
4. Given a 11L, what is the point(s) X such that 11 projection of X on the component lines lie on a cubic? It is (or they are) the intersection of the 11 locus (defined above) of component 10L's.

Question1
I found only a clue for your question 1.
The calculation with Mathematica was not difficult, but was running out of time.
However I was able to construct in Cabri 6 different collinear points Pi with the property that all Pi-projection points on the defining lines of the 6-Lines are coconic. Dragging this line the points Pi moved along with the line. Sometimes their number changed, but I couldn't find more than 6 points on this variable line. Therefore the locus must be of degree >= 6.

Questions 2,3,4 are to labor intensive for Mathematica program to solve.
Drawing in Cabri, ... , I hardly dare to start with it.

Best regards,

Chris

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Message: #642
Date: 2021-01-05
From: eckart_schmidt@t-online.de
Subject: Re: Locus on 6L and points on 7L with projection

```
Dear Vu Thanh Tung, dear Chris,

in addition to Chris message #641, question 1:
My CABRI experiences give a degree of >=7 for the locus of point
X.
Is already mentioned, that Clifford's Point 6L-e-P2 lies on this
curve?

Best regards Eckart
```

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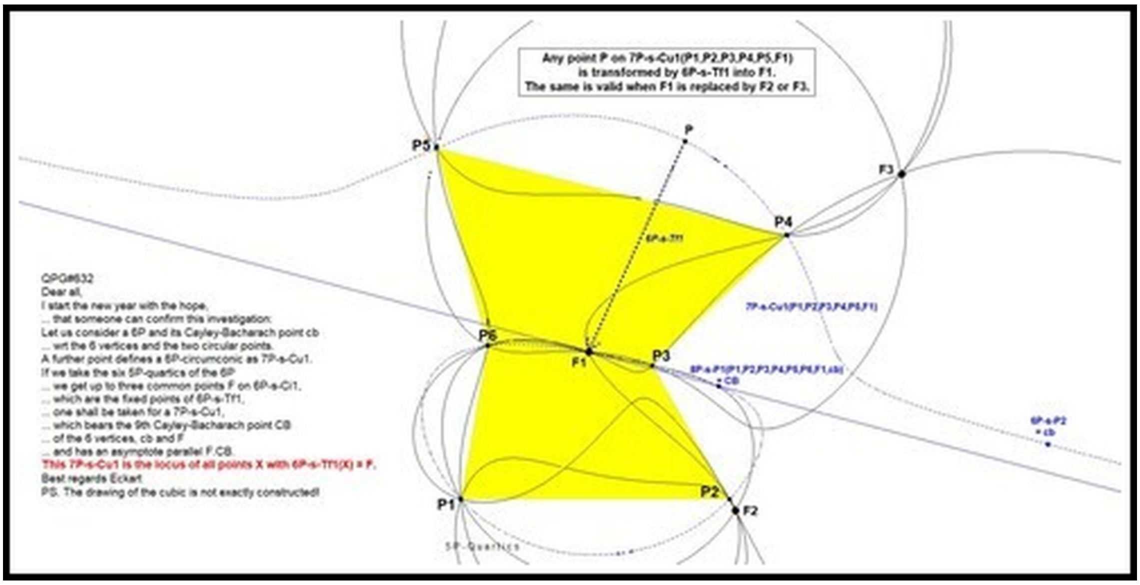
Message: #643
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

Dear Eckart,

Thanks for your last explanation.
Now I understand that there are two Moebius Transformations involved.
Actually everything is at the 5P-level now and can be described in terms of EPG-items, except the 2 nd 5P-Moebius Transformation and the 5P-Quartic.
About your question: “someone can confirm this investigation”.
Yes I can confirm your investigation broadly. The Cabri-drawings confirm it for all points F1, F2 and F3. I made a picture of everything you mentioned using F1 (but checked it works for F2 and F3) incl. EPG-notation. See attachment.
Of course there is a disclaimer. Every particular drawing is limited in scope and sometimes the drawing suggests properties that later appear to be conditional or even untrue.
Given the matter I got some new questions:
1. What is the reason you used the Moebius Transformation centered in 5P-s-Tf8(5P-s-P5), swapping 5P-s-P4 and 5P-s-P6?
2. You mentioned the fixed points of 6P-s-Tf1. Most probably you explained it before. Can you give a reference or explain.

Best regards,
Chris



6P-Configurations 6 versions 5P-s-Qu1.JPG

Message: #644
Date: 2021-01-05
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

thanks for confirmation.
Wrt your question 1:
Bernard has proposed further Möbius transformations
... CSC2 and CSC3, see #3660 and 3679,
... CSC2 is the transformation you ask for.
Wrt your question 2:
The fixed points of 6P-s-Tf1 are not discussed before,
... and I cannot give neither a construction nor properties..

Furthermore: I try in vain to find an exact construction for the
cubic.

Best regards Eckart
```

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Message: #645
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: Locus on 6L and points on 7L with projection

```
On Mon, Jan 4, 2021 at 10:50 PM, Eckart Schmidt wrote:

>
> Is already mentioned, that Clifford's Point 6L-e-P2 lies on
this curve?

Very nice observation Eckart!
It is new to me.

Chris
```

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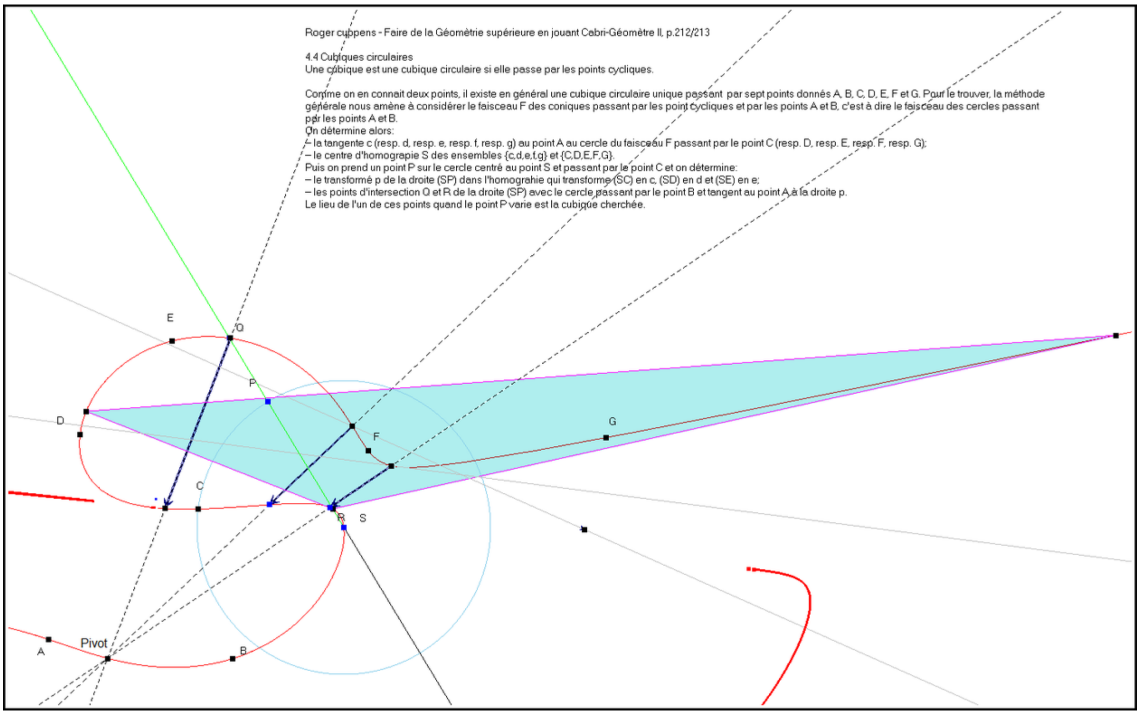
Message: #646
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
On Tue, Jan 5, 2021 at 12:53 AM, Eckart Schmidt wrote:
>
> Furthermore: I try in vain to find an exact construction for
the cubic.

Dear Eckart,

The construction of the circular cubic is written by me in EPG
within the picture at 7P-s-Cu1. ( https://www.chrisvantienhoven.
nl/np-items/np-geninf/np-0/23-mathematics/encyclopedia-of-poly-f
igures/np-objects/artikelen-np/441-7p-s-cu1 )
It's in the French language. Maybe Bernard can translate.
Attached a Cabri-figure and a Cabri-macro of 7P-s-Cu1 that I
once made.

Best regards,
Chris
```



7P-s-Cu1–Circular-Cubic-01.png

Message: #647
Date: 2021-01-05
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

thanks for quick answer, the item 7P-s-Cu1 I had studied,
... but what means
... "centre  d'homographie S des ensembles {c,d,e,f,g} et
{C,D,E,F,G} ..."
Can you give a short explication, thanks in advance?
Your CABRI figure allowed a macro, to confirm my suggestion.

Best regards Eckart
```

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Message: #648
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
On Tue, Jan 5, 2021 at 02:40 AM, Eckart Schmidt wrote:

> the item 7P-s-Cu1 I had studied,
>
> ... but what means
> ... "centre  d'homographie S des ensembles {c,d,e,f,g} et
{C,D,E,F,G} ..."

Dear Eckart,

I had the same problem when I was studying it.
It took me a lot of time to get all information.
Attached the crucial Cabri-figures with all information I hope.
When something is unclear, please let me know.
Best regards,

Chris
```

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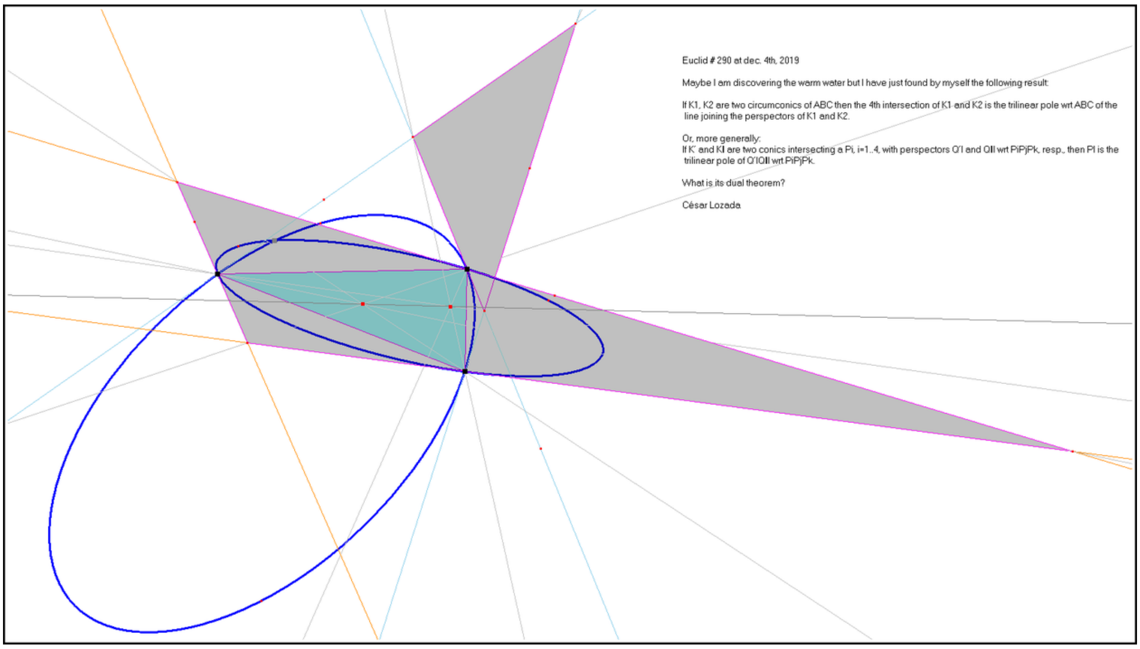
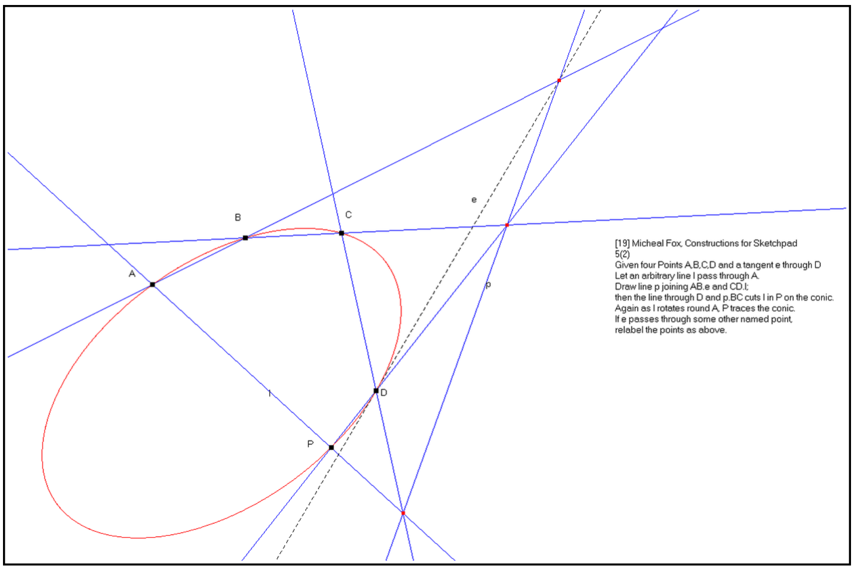
Message: #649
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

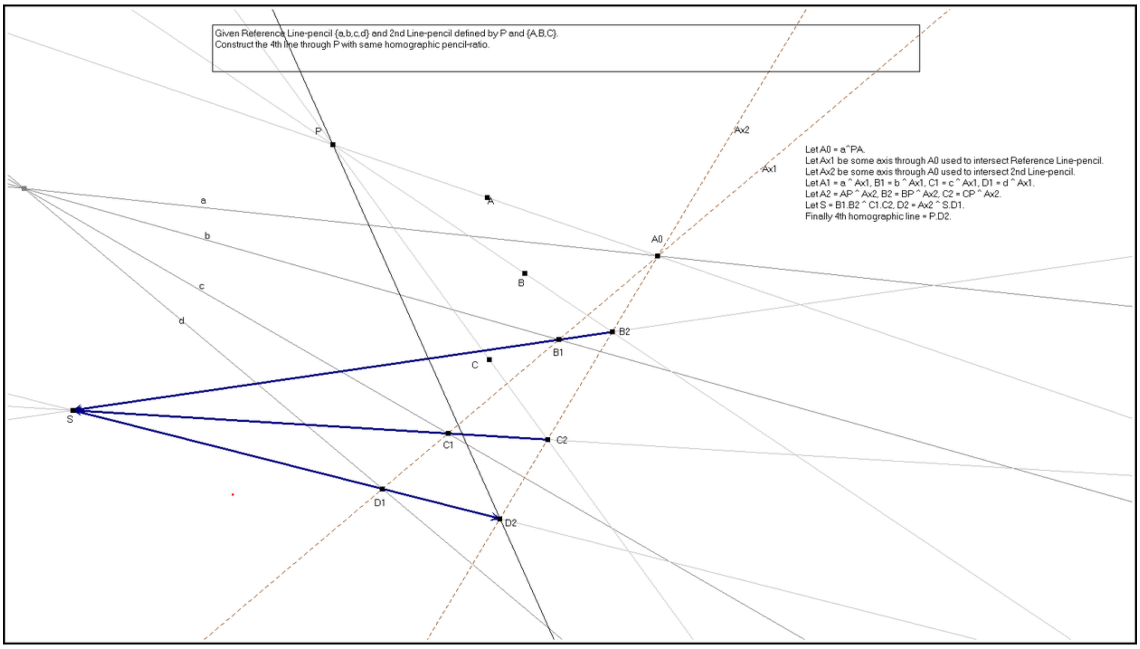
```
Dear Eckart,
Here are the attachments,
Chris
```



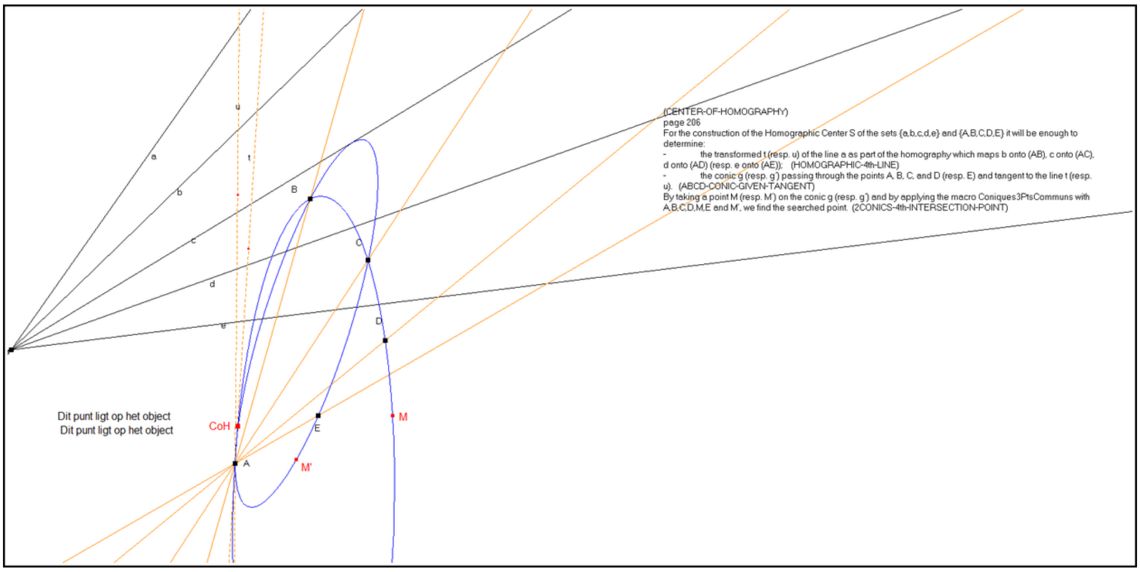
4th-Intersectionpoint-Two-Conics-01.png



Constr–Conic-given-A-B-C-D and D-Tangent-02.png



Homograpic4thLine-01.png



Center-of-Homography-01.png

Message: #650
Date: 2021-01-05
From: bernard.keizer@gmail.com
Subject: Re: Twin 5P, conics, cubics and quartics and 5P sextic, Newton Line,

---

Dear Chris, dear Eckart

I don't try to follow your investigations about 7P-s-Cu1, but I would really be happy if you put the twin cubics and quartics as well as the sextic at 5P level, as you say.
You just need to follow the constructions step by step
1) the 5Pi
2) the 5 Xi = QA-Tf16(Pi) or 5P-s-Tf8(Pi)
3) the 5 isgi = QA-P4(5P without Pi) (isg is for isogonal point)
4) the 5 isg'i = QA-P4(5X without Xi)
5) the 5 5P-s-Tf8(isgi)
6) the 5 5P-s-Tf8(isg'i)
7) the Newton Line L0 contains the middles of 5P-s-P45P-s-P5, 5X-s-P45X-s-P5, Pi5P-s-Tf8(isg'i) and Xi5P-s-Tf8(isgi)
8) the 2 Newton circles one centered in 5P-s-P3 through the middles of Piisgi and the middles of 5P-s-P45P-s-P5 and 5P-s-P55P-s-P6
and the other centered in 5X-s-P3 through the middles of Xiisg'i and the middles of 5X-s-P45X-s-P5 and 5X-s-P55X-s-P6
(please note that 5P-s-P6 = 5X-s-P6 and 5P-s-P5 = 5P-s-Tf8(5X-s-P5)
9) the Newton conic is through the 5 middles of PiXi
10) the 5Pi cubic is a Van Rees circular focal cubic with focus in 5X-s-P5, invariant in the CSC centered in this point and swapping the Pi and the 5P-s-Tf8(isg'i).
It happens that this CSC also swaps 5P-s-P4 and 5P-s-P6. The asymptote of the cubic is the parallel to L0 through 5P-s-P4.
11) the 5Xi cubic is the twin cubic with focus 5P-s-P5, invariant in the CSC centered in this point and swapping the Xi and the 5P-s-Tf8(isgi)
It happens that this CSC also swaps 5X-s-P4 and 5X-s-P6
12) the 5Pi quartic is 5P-s-Tf8(5Xi cubic) It is invariant in a CSC centered in 5P-s-Tf8(5P-s-P4) and swapping the Pi and the isgi
It happens that this CSC also swaps 5P-s-P5 and 5P-s-P6
13) the Xi quartic is the twin quartic or 5P-s-Tf8(5Pi cubic) It is invariant in a CSC centered in 5X-s-P4 and swapping the Xi and the isg'i
It happens that this CSC also swaps 5X-s-P5 and 5X-s-P6
14) Last, there is a 5Pi and 5Xi sextic through the 10 points invariant in 5P-s-Tf8

There are alltogether 5 curves 2 twin cubics, 2 twin quartics and a sextic, 5 CSC, 2 for the cubics, 2 for the quartics and 1 for the sextic and 5 reference Newton curves, 2 lines which are the same, 2 circles and a conic.
Each time, the construction is the same : for a variable point m on the corresponding Newton curve, if the fixed points of the corresponding CSC are F1 anf F2, the intersections between the bisector of F1mF2 and the circle which is it's CSCtransformed give 2 conjugated points on the searched curve.
Thanks to Eckart for checking the properties (I hope I didn't make a mistake).
For Chris, if something is not clear, please let me know.
Best regards
Bernard
PS Eckart, I didn't mention your triangles and rectangular hyperbolas, I just wanted to simplify as much as possible ...

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Message: #651
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: Twin 5P, conics, cubics and quartics and 5P sextic, Newton Line,

---

Dear Bernard,

Thank you very much for your very clear summary!

Best regards,
Chris

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Message: #652
Date: 2021-01-05
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

many, many  thanks for the CABRi attachments in your last
message,
... I shall study them well,
... at the moment I try the obtained CABRI macro.
First observations, perhaps anywhere in EPG:
Any 7 points on 7P-s-Cu1 define the same cubic,
... which is 7P-s-Tf1 invariant with a pivot
... in a common intersection of X.7P-s-Tf1(X) on the curve,
... which will be a new 7P-s-point,
... whose tangential is its 7P-s-Tf1 partner
    (also new 7P-s-point).
7P-s-Cu1 is the locus of 7P-s-Tf1 partner on lines through the
pivot.

Best regards Eckart
```

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Message: #653
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart, dear Bernard,

Eckart, wrt your reaction in QPG#644:
   The fixed points of 6P-s-Tf1 are not discussed before,
   ... and I cannot give neither a construction nor properties..
I found some special properties about 6P-s-Tf1 and the fixed
points.
1. All points in the plane mapped by 6P-s-Tf1 lie on a circle
6P-s-Cix.
2. It is the circle with center 6P-s-P1 passing through
6P-s-Tf1(6P-s-P1).
3. Consequently the 3 fixed points Fi (i=1,2,3) for which
6P-s-Tf1(Fi)=Fi, lie on this circle.
4. I did not find an easy way to construct these fixed points.
Only an nth degree curve touching the circle at F1, F2 and F3.
5. When we consider a circular cubic Cux =
7P-s-Cu1(P1,P2,P3,P4,P5,P6,Px), where Px is some point on
6P-s-Cix, then 6P-s-Tf1(X) = 6P-s-Tf1(Px) for all points X on
Cux. It looks like that the plane field consists of a continuum
of 6P-circumscribed circular cubics, each corresponding with its
own unique point on circle 6P-s-Cix.

Best regards,
Chris
```

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Message: #654
Date: 2021-01-05
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

Dear Eckart, dear Bernard,
I just noticed that the circle of the fixed points is already known in EPG. It is 6P-s-Ci1. Now it has several extra properties. Best regards,
Chris

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Message: #655
Date: 2021-01-07
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

thanks for your observations in #653, point 5 was new for me.
Here two further remarks:
1. The 5P-quartic can be considered as locus
... of fixed points of 6P-s-Tf1 for 6P = 5P plus any point.
2. For 6P = 5P plus any point
... the locus of 6P-s-Tf1(5P-s-P5) is the circle Ci(5P-s-P5),
... wrt Ci(P) see #3575.

Best regards Eckart
```

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Message: #656
Date: 2021-01-10
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,

Regarding your message #652: Wonderful properties.

Here all concepts and notions we built up in EPG start to pay
off.

When we have a 7P-circular cubic we can use a concept for a
9P-cubic to find the pivot of the 7P-cubic. Remarkable.

This pivot you describe is it strictly related to the 7P-s-Tf1
transformation. Is it conceivable that with another
transformation we find another pivot?

Best regards,

Chris
```

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Message: #657
Date: 2021-01-10
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

Dear Eckart, dear Bernard,

I have to relativize my remarks in messages #653 and #654.

1. All points in the plane mapped by 6P-s-Tf1 lie on 6P-s-Ci1. Exceptions are the 6P-vertices and points lying on lines Pi.Pj, their images are indetermined.

2. When we consider a circular cubic Cux = 7P-s-Cu1(P1,P2,P3,P4,P5,P6,Px), where Px is some point on 6P-s-Cix, then 6P-s-Tf1(X) = 6P-s-Tf1(Px) for all points X on Cux.

Hence we have a pencil of circular cubics, all passing through some point on 6P-s-Ci1 and obviously also passing through 6P-s-P2, each corresponding with its own unique point on circle 6P-s-Ci1. Note that this pencil of cubics is just a subset of all circular cubics in the plane. I noticed this in regular drawings the pencil doesn't fill the plane.

Best regards,

Chris

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Message: #658
Date: 2021-01-12
From: bernard.keizer@gmail.com
Subject: Homography

Dear Chris, dear Eckart
I tried to understand Roger Cuppens construction of a cubic through 9 points using the homography.
Many thanks to Chris, as it gave me the opportunity of reading again this publication by Cuppens, which was in my library ...
For Eckart, Cuppens mentions in his bibliography a reference in german Heinrich Schröter Die Theorie der ebenen Kurven dritter Ordnung auf synthetisch-geometrischem Wege.
1) Homography is a projective bijective transformation keeping the cross-ratio.
2) A set of 4 lines define a cross-ratio, which is the same for the 4 points intersection of any line with this set
3) 2 sets of 3 lines through 2 summits define a homography : the intersections of a line of the 1rst set with the corresponding line of the 2nd give 3 points, the se 3 points determine with the 2 summits a conic and the lines of the 2 sets partners in the homography intersect on this conic.
4) A set of 4 curves of degree n through a basis of n(n+3)/2 points (1 for a line, 5 for a conic, 9 for a cubic, 14 for a quartic ...) has the cross-ratio of the 4 tangents in a point of the basis
5) An homography of 2 sets of curves of degrees n and n' determine as intersection of 2 corresponding curves a curve of degree n + n' (n = n' = 1 gives a conic, n = 1 and n' = 2 gives a cubic)
Hence the construction of a cubic through 9 points (for a circular cubic through 7 points, the construction is the same providing you replace the conics through 4 points by circles through 2 points)
1) 9 points A to I
2) set of 5 conics through A,B,C,D and E (resp F,G,H,I)
3) set of 5 tangents e,f,g,h and i through A to these conics
4) The center of homography S between e,f,g,h and i and E,F,G,H and I is the unique point S for which the lines e,f,g,h and i and the lines SE,SF,SG,SH and SI intersect in the same conic through A and S. *This point S is the point named by Cotterill focus of the QA ABCD wrt the cubic, which is a pivotal isocubic wrt the DT of the QA ABCD with pivot S.*
a) Choose the conic defining the homography between g,h,and i and EG,EH and EI (conic though the 3 intersections g',h' and i' of the corresponding lines and the summits A and E.
b) e cuts this conic in A and a 2nd point e' ; the transformed of e in the homography in Ee'

```
c) Draw the conic through E,G,H and I and tangent in E to Ee'
d) Do the same construction by replacing E by F and e by f ; the
4th intersection of the 2 last conics (other than G,H and I) is
S
5) Having S, draw the conic through the 5 intersections of the
lines e,f,g,h and i and the lines SE,SF,SG,SH and SI.
6) For any point P on this conic, AP and SP are transformed in
the homography and SP cuts the conic though A,B,C and D tangent
in A to AP in 2 points Q and R describing the cubic when P
describes the conic. (My construction is here slightly different
from Cuppens, but that doesn't change the result ...)
I suppose the next step is to draw quartics by intersecting a
set of lines and a set of cubics or 2 sets of conics ...
Best regards
Bernard
```

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Message: #659
Date: 2021-01-13
From: bernard.keizer@gmail.com
Subject: Re: Homography

```
Dear Chris, dear Eckart
Sorry, the cubic through the 9 points is not a pivotal isocubic
with pivot S, but it holds that S is the Cotterill focus.
Best regards
Bernard
```

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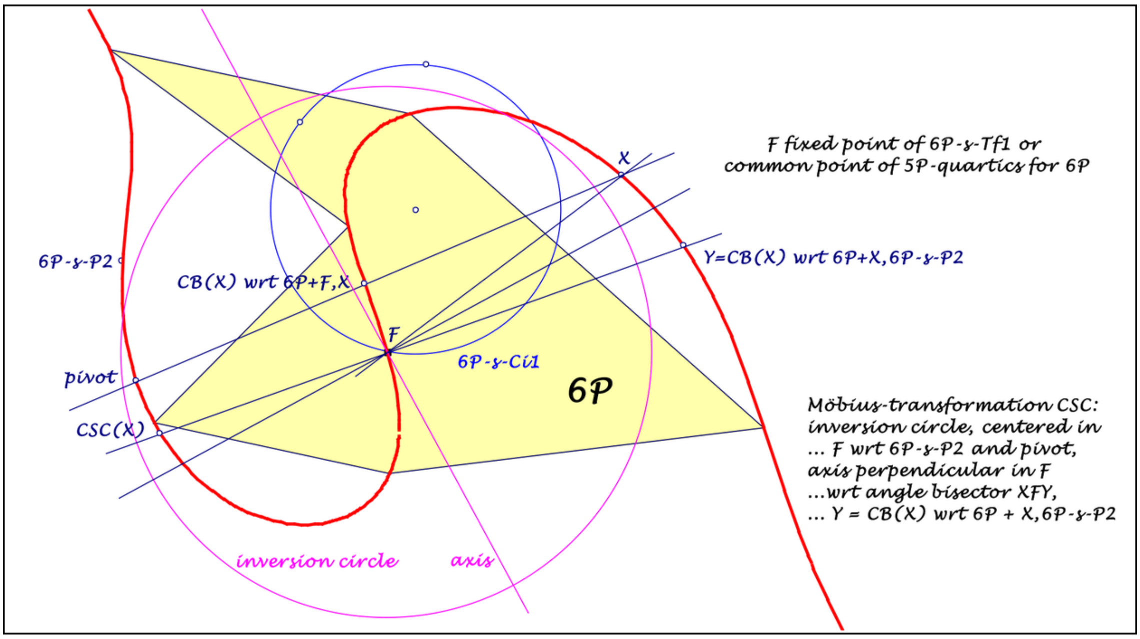
Message: #660
Date: 2021-01-13
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris, dear Bernard,

content of this message is a Möbius transformation for the
6P-cubics in
#632,
... which are 7P-s-Cu1 for a 6P plus a point F on 6P-s-Ci1,
... F fixed point of 6P-s-Tf1 or common point of the
    5P-quartics.
Reason was the observation, that any 6P on QL-Cu1 of a QL
... has a circle 6P-s-Ci1 through the Miquel point QL-P1.
On the other hand: 6P on a fixed 7P-s-Cu1 of a reference 6P
... have a common point for their 6P-s-Ci1,
    not necessary on the cubic,
... but if it is a point on the cubic,
    the point is F in the sense above.
Here a Möbius transformation for the cubics in the preface:
... Inversion circle, centered in F wrt 6P-s-P2 and pivot
    (wrt pivot see #652),
... axis perpendicular in F wrt the angle bisector of  <XFY,
... X any point on the cubic, Y = CB(X) wrt 6P plus X, 6P-s-P2.
 Now it is easy to find QL on the cubic, so that the cubic is
 QL-Cu1.

Best regards Eckart

PS. Please give me time to study your detailed messages,
up to now I use the macro for 7P-s-Cu1 without construction
myself.
```



2021-01-13.pdf

Message: #661
Date: 2021-01-13
From: eckart_schmidt@t-online.de
Subject: Re: Homography

```
Dear Bernard,

thanks a lot for your detailed description
... of the construction for a cubic, defined by 9 points!
I succeeded in following your description and got a macro,
... WONDERFUL!!!

Best regards Eckart
```

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Message: #662
Date: 2021-01-14
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart, dear friends,

I placed the two 7P-points of message #652 in EPG.
They have names;
* 7P-s-P2 CB-pivot of the 7P-Circular Cubic
* 7P-s-P3 7P-s-Cu1 Tangential of 7P-s-P2
I also made some other changes. See "Recently Added" in EPG.
Any comment is welcome.
Best regards,

Chris
```

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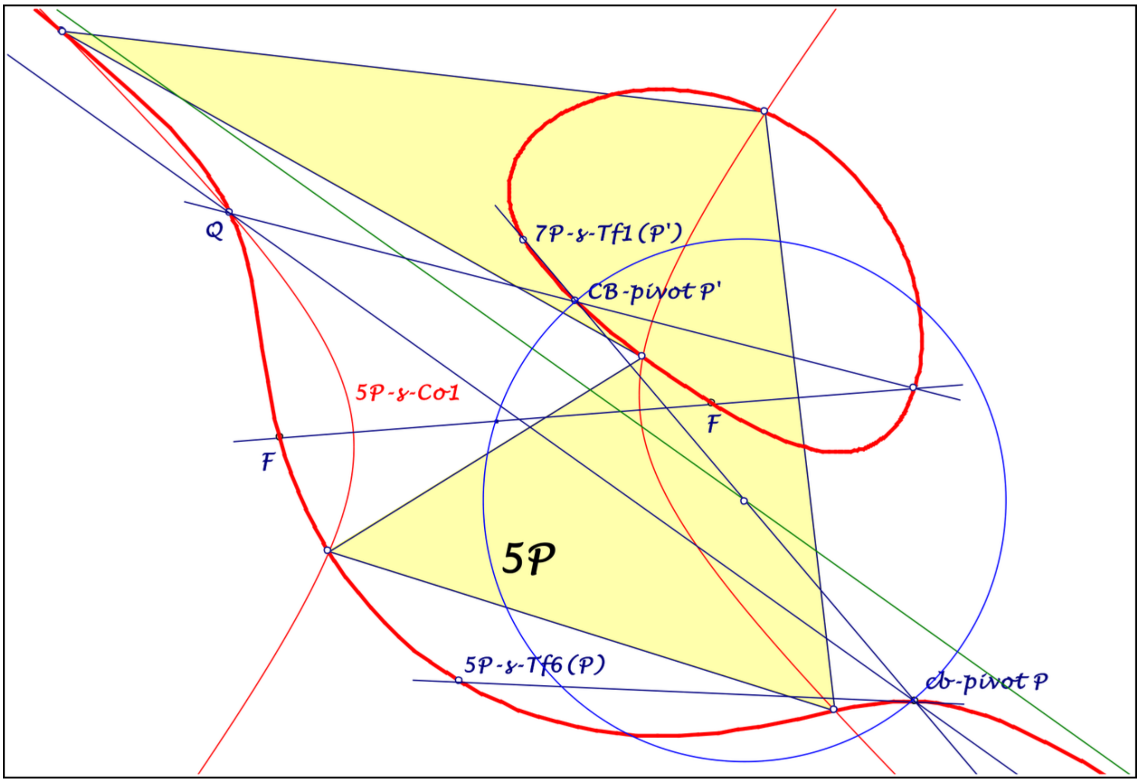
Message: #663
Date: 2021-01-15
From: eckart_schmidt@t-online.de
Subject: 7P-s-Cu1 for a 5P and foci of 5P-s-Co1

```
Dear Bernard, dear Chris,

if we consider for a 5P and the foci of its circumconic the
cubic CU = 7P-s-Cu1,
... this cubic is invariant wrt 5P-s-Tf6 (cb-point)
    with a pivot P,
... collinear with 5P-s-P4 and the 6th intersection Q of CU
    and 5P-s-Co1
... with PQ parallel to the asymptote of CU.

The cubic is also invariant wrt 7P-s-Tf1 (CB-point) of 5P plus
the foci
with a pivot P'
... collinear with Q and the 3rd intersection of CU
    and the main axis of 5P-s-Co1,
... P, P' and 7P-s-Tf1(P') are collinear and 5P-s-Tf6(P)
    is the tangential of P.

The circles 6P-s-Ci1 for 6P = 5P plus a point X on CU have two
common fixed points,
... 5P-s-P5 and R = 6P-s-Tf1(Y) with Y on CU,
... for X = Q we get the radical axis as degenerated circle.

So far an excursion in 5P-/6P-/7P-geometry,
... without Chris' EPG-nominations really impossible to
    describe.

Best regards Eckart
```



2021-01-15.pdf

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Message: #664
Date: 2021-01-15
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1 for a 5P and foci of 5P-s-Co1

Dear Eckart,

Wonderful excursion you describe.
It is a banquet of wonderful ingredients.
These two remarks:
1. I think there was a typo: 7P-s-Tf1(P') should be 7P-s-Tf1(P)
2. 6P-s-Tf1(Y) is a fixed point wrt any 7P-circular cubic for all 6+1 points taken on it.

Therefore 6P-s-Tf1(Y) is a CircuCubical transformation ! !
6P-s-Tf1(Y) produces the same point for any 6+1 points spanned in a circular cubic.
This point is actually 7P-s-P1 and it does not lie on the circular cubic. Nevertheless it is specifically related to the circular cubic. What extra properties does this point have wrt the circular cubic?

Best regards,

Chris

p.s. I can imagine that this coded information for others it is difficult to understand. We grew into this, others did not.
I hope other readers of QPG realize that also less coded and simpler information is available in our group and can be raised and discussed here. In fact these questions are often a source of new insights and discoveries.

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Message: #665
Date: 2021-01-16
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1 for a 5P and foci of 5P-s-Co1

```
Dear Eckart, dear Bernard,

Some more observations:
5P-s-P5 applied to 5P(QA-vertices+QA-P4) = QA-P4.
7P-s-P1 is
* fixed point for all 7P's inscribed in 7P-s-Cu1
* 6P-s-Tf1(X) for 6P inscribed in 7P-s-Cu1 and all X on 7P-s-Cu1
* 6L-s-Tf1(Pi) wrt Perpendicular Bisectors(Pi.Pj), j unequal i
* 6P-s-Ci1 common-circles-center of component 6P's
* QL-P1 for any 7 points on QL-Cu1
* QA-P9 for any 7 points on QA-Cu1
* QA-P41 for any 7 points on QA-Cu7

So far,
Chris
```

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Message: #666
Date: 2021-01-16
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1 for a 5P and foci of 5P-s-Co1

```
Dear Chris, dear Eckart
Beautiful work, indeed ! Very interesting properties ...
For any 7P-s-Cu1, 7P-s-P1 is the focus of the circular cubic !
Any circular cubic has only one real asymptote, which cuts the
curve in a point Q.
Any line through the point Q cuts the curve in 2 points
equidistant from the focus F.
If makes easy to find the focus F as intersection of the
perpendicular bisectors of all segments joining 2 points on the
curve aligned with the point Q.
If F lies on the curve, it is a Van Rees circular focal cubic
(like QL-Cu1 with focus QL-P1 and QA-Cu7 with focus QA-P41).
If not, it is only a circular cubic, like QA-Cu1 with focus
QA-P9.
Best regards
Bernard
```

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Message: #667
Date: 2021-01-16
From: bernard.keizer@gmail.com
Subject: Re: Homography

```
Dear Chris, dear Eckart
I think I understand better the link between Hart, Coppens and
Cotterill !
It deals always with croos-ratio.
Let suppose you have the 9 points A to I and their circumcubic.
You define the focus S of ABCD with Coppens and the same way the
focus T of EFGH.
Then CB(A to H) is the 3rd intersection of ST with the cubic
according to Cotterill.
AS there are 70 points S or T by choosing 2 groups of 4 points
in the 8 points, CB is on 35 segments ST.
CB is also the 6th intersection of the conics ABCDT and EFGHS
with the cubic.
This means that CB has the same cross-ratio as S wrt EFGH and as
T wrt ABCD.
Best regards
Bernard
```

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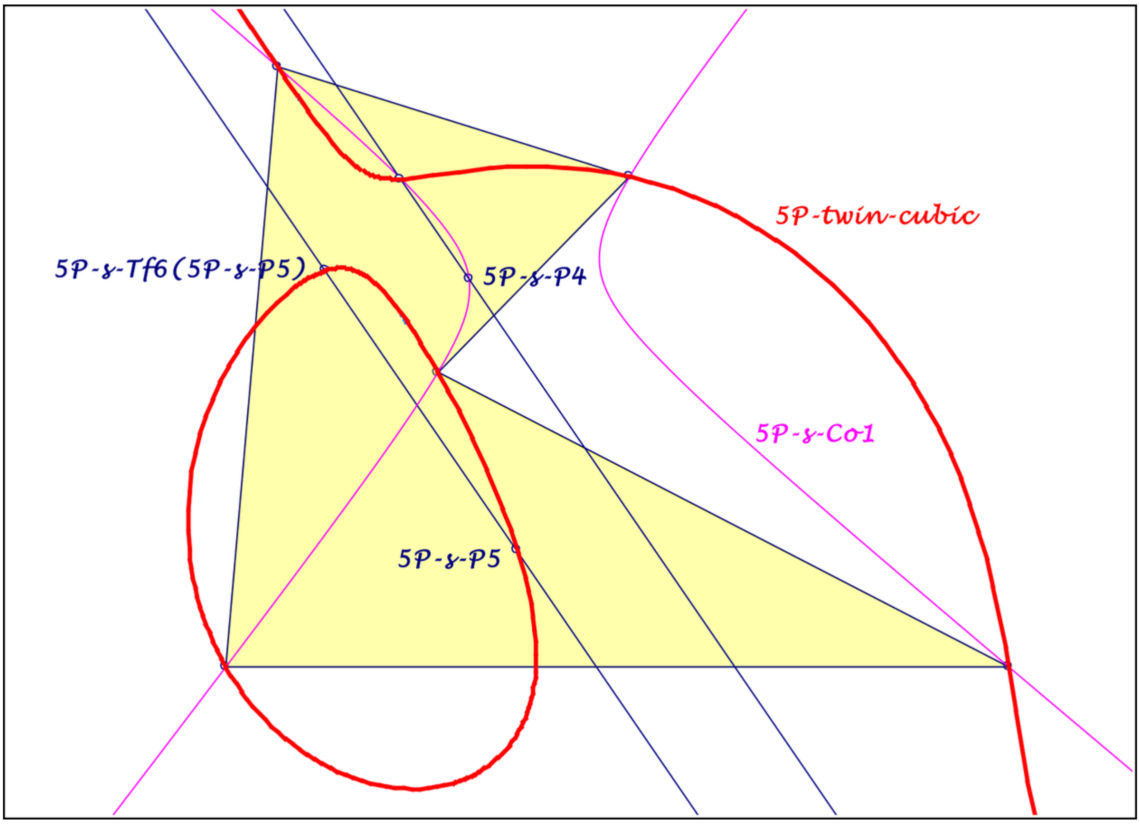
Message: #668
Date: 2021-01-16
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Bernard, dear Chris,

your twin cubic for a 5P (up to now not mentioned in EPG),
... which is 5P-s-Tf8 of the 5P-quartic of 5P-s-Tf8(5P),
... can be described as 7P-s-Cu1 for 5P plus 5P-s-P5
... plus the 2nd intersection of 5P-s-Co1
... ... and a parallel to 5P-s-P5.5P-s-Tf6(5P-s-P5)
        through 5P-s-P4.

Best regards Eckart
```



2021-01-16.pdf

Message: #669
Date: 2021-01-17
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1

Dear Chris, dear Eckart,
Of course, but rather complicate !
Knowing the result, any cubic is a 9P cubic of any 9 of it's points (provided it is not a CB system) and any circular cubic is either a 9P-Cu1 of any 9 of it's real points (same remark) or a 7P-s-Cu1 of any 7 of it's real points (provided it is not a cb system).
The advantage (for me) of the real points is that Geogebra allows the drawing of cubics through 9 points or of quartics through 14 points and allows also the reflexion in a line and the inversion wrt a circle, so that many of the found properties are easy to check !
Unfortunately, it doesn't allow the drawing of circular curves ...
Best regards
Bernard

PS Do we agree (see my message 650) that the cubic and the quartic are the 5P curves and that the twin cubic and quartic are the 5X curves, with 5Xi = 5P-s-Tf8(Pi) and of course quartic and twin quartic are 5P-s-Tf8(twin cubic and cubic).

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Message: #670
Date: 2021-01-17
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Bernard,

excuse that I didn't use the right "twin"-nomination.
With my CABRI I cannot draw circular cubics with 9 given points,
... so I was so excited to get a macro with your message 658.
I used it once more successful wrt "cevian pedal QG" (see
old#1242, new#446)
... as 7P-s-Cu1 for a QG = P1,...,P4
    plus 3 of the 4 intersections Si
... ... of perpendiculars to Pi.Pi+1 in Pi+1
        and to Pi.Pi-1 in Pi-1,
... Si.Si+2 intersect in QL-P1 on the cubic ...

Best regards Eckart
```

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Message: #671
Date: 2021-01-17
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1 for a 5P and foci of 5P-s-Co1

```
Dear Chris,

I cannot confirm your observation:
7P-s-P1 is "6P-s-Ci1 common-circles-center of component 6P’s".
I think it has to be:
7P-s-P1 is "common point of 6P-s-Ci1 for the component 6P’s".

Best regards Eckart
```

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Message: #672
Date: 2021-01-17
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1

Dear Eckart,
I don't get your point !
The construction given by Cuppens of 9P-Cu1 is exactly the same as 7P-s-Cu1 ! (see EQF)
If you know 9 points of a cubic, you are able to draw 9P-s-Cu1.
Taking 7 points of the same cubic (among the 9 or other), you are able to draw 7P-sCu1.
* If the 2 cubics coïncide, it is necessary circular !
* If you get 2 different cubics, the 1rst one is not circular
...
Best regards
Bernard

For example, taking the cubic stelloïd QL-Cu2, you have 27 points on it (centers of the 27 inscribed cardioïds) and some others.
It's easy to check with the given method that it is not circular
...

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Message: #673
Date: 2021-01-18
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

Dear Bernard,

the properties of your remarks in #672 are already known to me
... and I haven't postulated converse positions in #670,
... why do you say "I don't get your point !"?

Best regards Eckart

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Message: #674
Date: 2021-01-18
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,
Sorry if I misunderstood you !
You said in 670 : With my Cabri, I cannot draw circular cubics
with given 9 points.
In fact, you can if you know in advance that the 9 points are on
a circular cubic ...
The essential is that we agree !
Best regards
Bernard
PS Can you confirm my intuition in message 666 that 7P-s-P1 is
the focus of 7P-s-Cu1 ?
```

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Message: #675
Date: 2021-01-18
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Bernard,

yes, I can confirm, that 7P-s-P1 is the focus of 7P-s-Cu1.
I think, this is a relevant observation,
... background for Chris' many properties of 7P-s-P1 in #665.

Best regards Eckart
```

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Message: #676
Date: 2021-01-18
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,

What are the reasons that you think 7P-s-P1 is the focus of
7P-s-Cu1?
According to the papers of Bernard the singular focus of a
circular cubic is the intersection of the tangents to the cubic
at the circular points at infinity.

Best regards,
Chris
```

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Message: #677
Date: 2021-01-18
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

I used the results of our discussion in EPG #152 and followers
... to identify Bernard's observation 7P-s-P1 as focus of
    7P-s-Cu1.

Best regards Eckart
```

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Message: #678
Date: 2021-01-20
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Bernard, dear Chris,

two questions:

(1) Is the following property of a focal circular cubic already
mentioned:
... Tangential of the focus is the intersection of the cubic and
    its asymptote?

(2) What about the 5P-circumcubic through the ten following
points:
... 5 vertices Pi of 5P and 5 QA-Tf2(Pi)
    wrt the remaining vertices Pj (j unequal i)?

Best regards Eckart
```

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Message: #679
Date: 2021-01-20
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,
For the 1rst question, the property is well-known, but not in
EQF !
The explanation is obvious : the focus F and the infinity point
Ω are CSC partners and have the same tangential Q, which is the
intersection between the curve and it's asymptote.
The CSC of Q is q, the 3rd intersection between the line FΩ (the
parallel through F to the asymptote and to the Newton Line) and
the curve.
Best regards
Bernard
```

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Message: #680
Date: 2021-01-20
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-Cu1

```
Dear Eckart,
For the 2nd question, I drew the figure and checked that the 10
points are on the same cubic.
But I have no idea about this cubic.
Best regards
Bernard
```

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Message: #681
Date: 2021-01-21
From: bernard.keizer@gmail.com
Subject: Cayleyan

Dear Chris, dear Eckart
It is well known that any cubic has a hessian, locus of the points for which the polar conics degenerate in 2 lines.
The 2 lines intersect in a point CSC of the initial point ; conversely, the polar conic of the 2nd point is also degenerated in 2 lines intersecting in the 1rst point.
Now the 2 cubics define a 3rd curve, named the cayleyan, which is the envelop of the lines through 2 CSC partners.
For example, QL-Cu1, which is the hessian of QL-Cu2 defines a CSC and a cayleyan. The contact point of a line through 2 CSC partners is the harmonic conjugate of the 3rd intersection with the cubic wrt the 2 CSC partners. (Eckart remembers surely we have discussed this cayleyan and it's properties many years ago). The cayleyan is tangent to the hessian in 3 points ...
QA-Cu1 considered as hessian is invariant in 3 CSC's centered in the Miquel points and defines 3 cayleyans (same definition of the contact point and same property of tangency).
But in fact, any curve invariant in a CSC defines the same way such a cayleyan !
For examples, the twin cubics of a 5P, which is obvious as they are QL-Cu1's, the twin quartics and even the sextix of the 5P (this time, I don't know which is the contact point ...
My own figures are too ugly to be presented, but I'm convinced that Eckart (if he is interested, which I hope) will draw more beautiful curves.
Best regards
Bernard

PS I'm rather disappointed that some of my previous messages remained without any answer (587 about the main pivot triangle of QL-Cu2 or 611 about CB16 and 5L) ...

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Message: #682
Date: 2021-01-21
From: van10hoven@gmail.com
Subject: Re: Cayleyan

Dear Bernard,

Thanks for your concise summary about hessian, caleyan and polar
conics.
I still have little experience with these items. I hope this
will come.
About your message #611.
The subjects were very interesting for me.
However there is few information about it. I think it would help
if we have a discussion once about these subjects. That will
help gathering more information.
There are so many interesting items . . .

Best regards,
Chris

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Message: #683
Date: 2021-01-23
From: eckart_schmidt@t-online.de
Subject: New 5P-s-Cubic

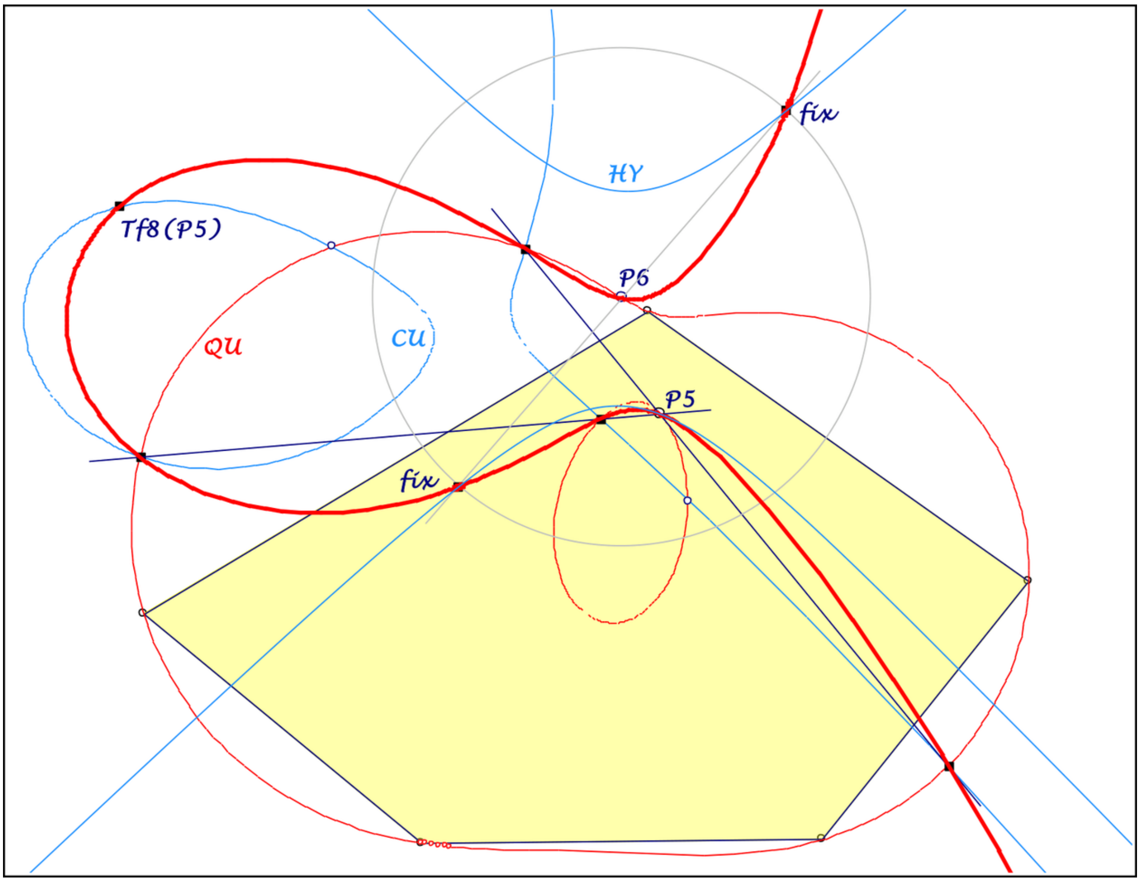
```
Dear Bernard, dear Chris,

for this excursion in 5P-geometry the prefix 5P-s- is omitted.

In July 2019 we discussed for a 5P a circumquartic QU and a
cubic CU as its
Tf8-image (CSC).
These two curves have 6 intersections, 3 pairs of Tf8-partner,
... two pairs collinear with P5 give first 4 points for the
searched cubic.
Adding the two fixed points of Tf8, further Tf8(P5),
... 7P-s-Cu1 gives a focal circular cubic, invariant wrt Tf8,
... with asymptote parallel P5.P6
    through the reflection of Tf8(P5) in P5.P6
... and focus Tf8(P5).
This cubic can be constructed as locus of Tf8-partner on lines
through P5.

Final question to Bernard wrt #681:
The locus of 4th harmonic points of P5 wrt Tf8-partner on the
cubic
... is an orthogonal hyperbola HY, centered in P6,
    through the fixed points of Tf8 and P5.
Is this hyperbola the Cayleyan of the cubic?

Best regards Eckart
```



2021-01-23.pdf

Message: #684
Date: 2021-01-23
From: bernard.keizer@gmail.com
Subject: Re: New 5P-s-Cubic

Dear Eckart,
First I was very intrigued, as you have discovered another beautiful cubic !
Very surprising that only 2 of the 3 copples of CSC partners intersect in P5.
Now this cubic has focus Tf8(P5), Newton Line P5P6 and asymptote parallel to P5P6.
More important, this cubic is invariant in the CSC centered in Tf8(P5) swapping P5 and P6 and swapping also the fixed points of Tf8.
(The QU CSC is centered in Tf8(P4 and swaps also P5 and P6)
The classical cayleyan associated to your cubic as the hessian of another one is the locus of the harmonic conjugate of the 3rd intersection (residual) of the lines through 2 partners in this CSC with the curve (or the envelop of these lines or the envelop of the axes of the inscribed conics in all the QL's inscribed in the curve).
The cayleyan is a sextic of class 3.
Last, you may reverse the points P5 and P6 ; your cubic is also invariant in a CSC centered in P5 with partners aligned with P6.
You will have a 2nd rectangular hyperbola.
I'm not sure if the RH could be cayleyans as the envelop of lines through a fixed point P5 or P6 is clearly the point itself!
Anyhow, your construction gave me the opportunity of revising properties of the Van Rees circular cubic in the monocursal case.
Many thanks for that !
Best regards
Bernard

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Message: #685
Date: 2021-01-23
From: bernard.keizer@gmail.com
Subject: Re: New 5P-s-Cubic

```
PS Of course, there is also a twin of your cubic for the Xi !
Chris has plenty of work before him ...
```

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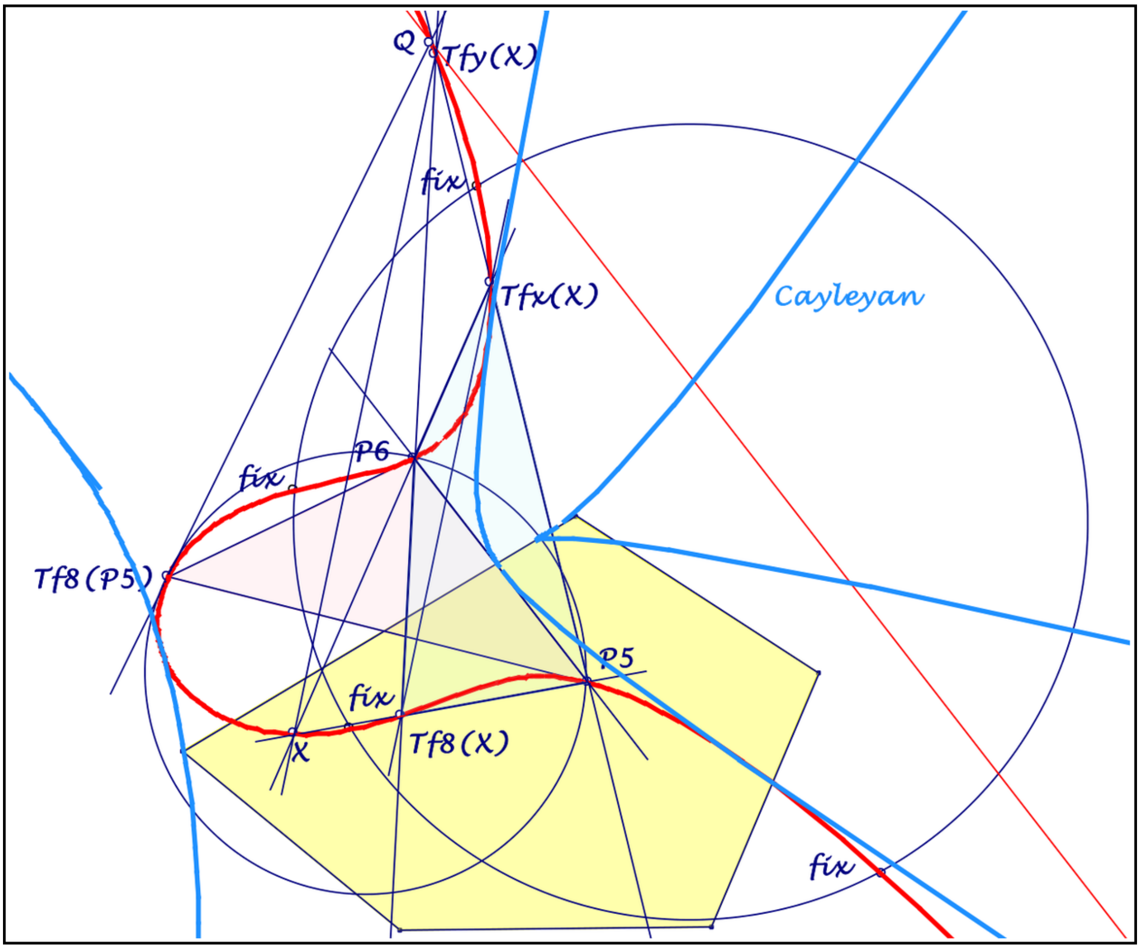
Message: #686
Date: 2021-01-24
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-s-Cubic

```
Dear Bernard,

your message gives a new sight of the cubic in #683,
... bearing the points P5, P6, Tf8(P5) (prefix 5P-s- omitted),
... invariant wrt three Möbius transformations:
... ... Tf8, centered in P6,
        swapping P5 <-->Tf8(P5) or the fixed points of Tfx,
... ... Tfx, centered in P5,
        swapping P6 <--> Tf8(P5) or the fixed points of Tf8,
... ... Tfy, centered in Tf8(P5),
        swapping P5 <--> P6 or the fixed points of Tf8 or Tfx.
The fixed points of Tf8 and Tfx lie on a circle, invariant wrt
Tf8, Tfx, Tfy.
The cubic is 7P-s-Cu1 of the fixed points of Tf8 and Tfx and P5,
P6, Tf8(P5).
For points X on the cubic: X.Tfy(X) parallel Tf8(X).Tfx(X),
... X and Tf8(X) collinear P5,
    X and Tfx(X) collinear P6,
... Tfx(X) and Tfy(X) collinear P5,
    Tf8(X) and Tfy(X) collinear P6.
So we get a special QL on the cubic
... with opposite vertices P5, P6 and Tf8(X),
    Tfx(X) and X,Tfy(X),
... QL-P1 = Tf8(P5),
    QL-L1 = P5.P6 (diagonal of the QL), CSC = Tfy.
The circumcircle of P5, P6, Tf8(X) contacts the cubic in Tf8(X)
... and as focus with tangential
    in the intersection of the cubic and its asymptote.

Best regards Eckart
```



2021-01-24.pdf

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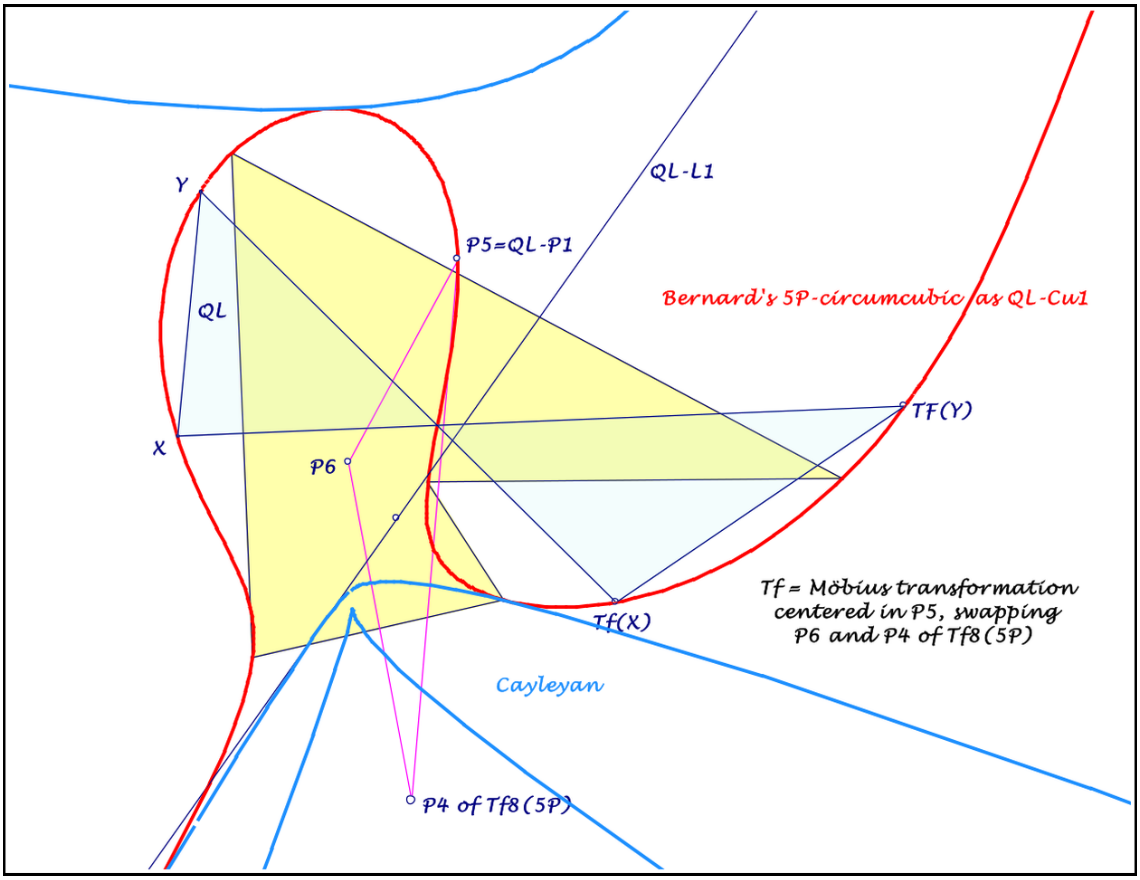
Message: #687
Date: 2021-01-24
From: eckart_schmidt@t-online.de
Subject: Re: Cayleyan

```
Dear Bernard,

attached the cayleyan of your circumscribed 5P-cubic,
but I cannot observe any properties.

Best regards Eckart
```



2021-01-24a.pdf

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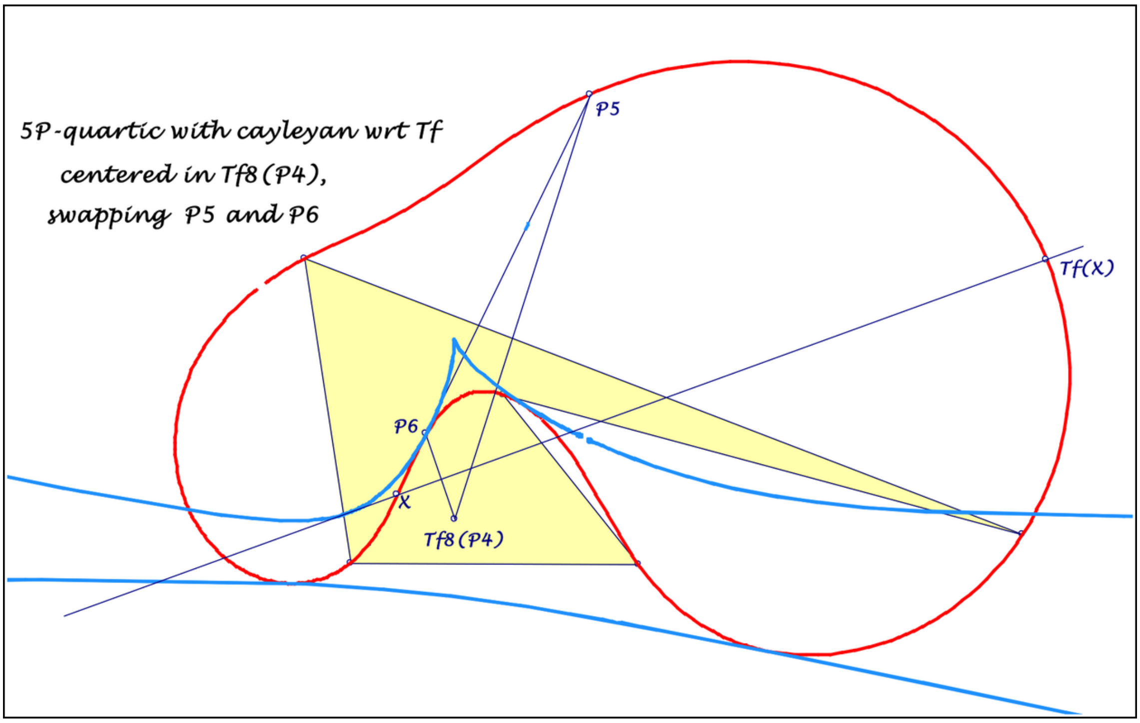
Message: #688
Date: 2021-01-24
From: eckart_schmidt@t-online.de
Subject: Re: Cayleyan

```
Dear Bernard,

attached the cayleyan of the circumquartic of a 5P,
... wrt a Möbius transformation Tf,
... centered in Tf8(P4), swapping P5 and P6,
... but without properties of the contact points of X.Tf(X).

Best regards Eckart
```



2021-01-24b.pdf

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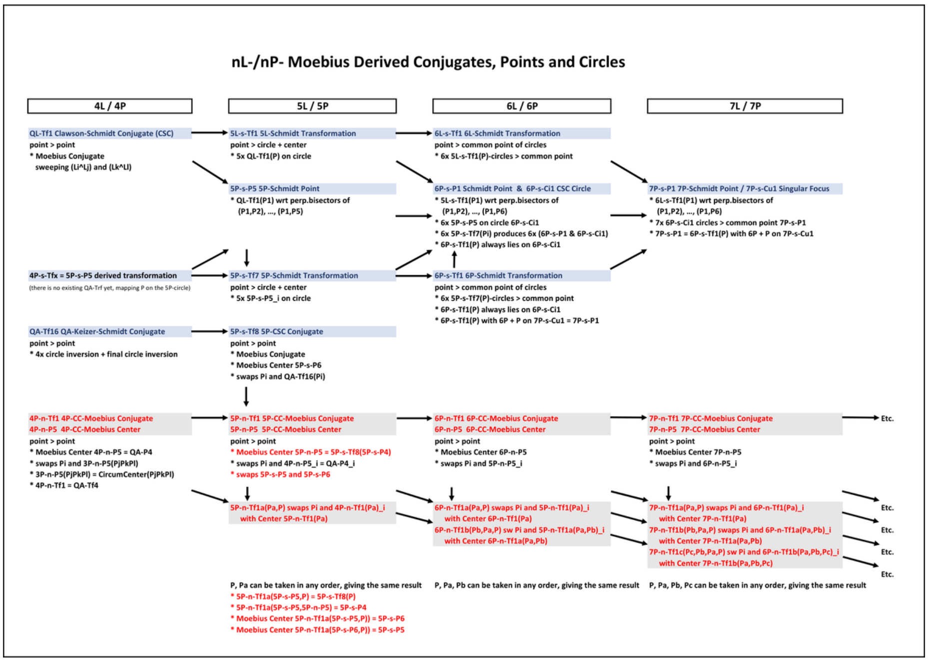
Message: #689
Date: 2021-01-24
From: van10hoven@gmail.com
Subject: Moebius (related) Transformations

Dear Eckart, dear Bernard,

1. I studied all Moebius-related transformations in EQF and EPG.
It helped me understand all kind of relations between items and processes.
See my scheme: nL-nP-Moebius Derived Conjugates, Points and Circles.pdf.
2. I also found a completely new Moebius Transformation for all n-Points. I named it nP-n-Tf1 with Moebius Center nP-n-P5.
For a concise picture see: nP-n-Tf1 & P5 nP-CC Moebius Conjugate-01.png.
For a preliminary description see: nP-n-Tf1 nP-CC-Moebius Conjugate.pdf.
Some results:
· Moebius Center 3P-n-P5 = X(3) in ETC
· Moebius Center 4P-n-P5 = QA-P4 in EQF
· Moebius Center 5P-n-P5 = 5P-s-Tf8(5P-s-P4) in EPG
· 5P-n-Tf1 swaps 5P-s-P5 and 5P-s-P6.
· Moebius Center 5P-n-P5 = 5P-s-Tf8(5P-s-P4).
Hope you like it.

Best regards,
Chris

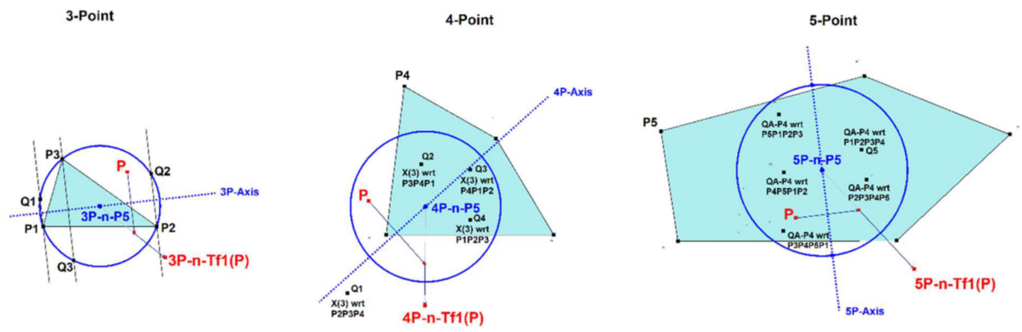


nL-nP-Moebius Derived Conjugates, Points and Circles.pdf

nP-n-P5 nP-CC-Moebius Center

The nP-CC-Moebius Center is the Center of the nP-CC-Moebius Conjugate nP-n-Tf1.

The nP-CC-Moebius Conjugate is a Moebius transformation valid for configurations of n points (n = 3, 4, ….). It maps a point into another point. The transformation is bijective.
See for further information nP-n-Tf1.



The 3P-CC Moebius Pairs (Pi,Qi), where i=1,2,3,
determine the 3P-Moebius-Circle, Center and Axis, where
Pi = Triangle vertices P1, P2, P3
Qi = 2nd intersection point of the line through Pi perpendicular to the Axis and the circumcircle).
The 3P-Moebius Circle is the 3P-circumscribed circle.
The 3P-Moebius Center is 3P-n-P5 = X(3).
The 3P-Moebius Axis can be any line through 3P-n-P5.
The 3P-Moebius Conjugate 3P-n-Tf1 of any point P is:
the reflection of P about the 3P-Moebius Axis followed by the inversion in the 3P-Moebius Circle.

The 4P-CC Moebius Pairs (Pi,Qi), where i=1,2,3,4,
determine the 4P-Moebius-Circle, Center and Axis, where
Pi = Quadrangle vertices P1, P2, P3, P4
Qi = 3P-n-P5 = X(3) wrt Component Triangle PjPkPl, where (i,j,k,l) Î (1,2,3,4) and any 3P-axis in the construction can be chosen.
The 4P-Moebius-Circle, Center and Axis, are constructed from (Pi,Qi) according to the Pi-Qi-standard Moebius construction.
The 4P-Moebius Center 4P-n-P5 = QA-P4.
The 4P-Moebius Conjugate 4P-n-Tf1(P) = QA-Tf4(P) is:
the reflection of P about the 4P-Moebius Axis followed by the inversion in the 4P-Moebius Circle.

The 5P-CC Moebius Pairs (Pi,Qi), i=1,2,3,4,5,
determine the 5P-Moebius-Circle, Center and Axis, where
Pi = Quadrangle vertices P1, P2, P3, P4, P5
Qi = 4P-n-P5 wrt Component Quadrangle PjPkPlPm, where (i,j,k,l,m) Î (1,2,3,4,5).
The 5P-Moebius-Circle, Center and Axis, are constructed from (Pi,Qi) according to the Pi-Qi-standard Moebius construction.
The 5P-Moebius Center is called 5P-n-P5.
The 5P-Moebius Conjugate 5P-n-Tf1(P) is:
the reflection of P about the 5P-Moebius Axis followed by the inversion in the 5P-Moebius Circle.

Properties:

- Moebius Center 3P-n-P5 = X(3) in ETC
- Moebius Center 4P-n-P5 = QA-P4 in EQF
- Moebius Center 5P-n-P5 = 5P-s-Tf8(5P-s-P4) in EPG

nP-n-Tf1 nP-CC-Moebius Conjugate

The nP-CC-Moebius Conjugate is a Moebius transformation valid for configurations of n points (n = 3, 4, ….). It maps a point into another point. The transformation is bijective.

Preamble
The CC-Moebius Conjugate is a Moebius Transformation mapping a point into another point.
When all points on a circle are mapped by a Moebius Transformation the mapped points again form a circle. Therefore it's said that a Moebius transformation maps circles into other circles. When two circles are transformed the angles under which they meet are preserved.
Lines are also mapped into circles. But lines are supposed to be circles with its center in infinity. Therefore also the notion of "generalized circles" is used, meaning that a circle can degenerate into a line and they are of the same kind.
The Moebius transformation is bijective.
The type of Moebius Transformation used here is represented by the function 1/z in the complex plane. It is the sequence of a circle inversion of a point in a unit circle and the reflection about an axis through the center of that unit circle.
The prefix CC- is used to indicate the role of the CircumCenter in this Conjugate. It all starts with the CircumCenter in a triangle.

(drawing) circle axis transform

There are 3 basic elements needed for a CC-Moebius Transformation:

1. A unit-circle, here called the Moebius Circle.
2. The Center of the unit-circle, here called the Moebius Circle Center.
3. An axis through the center of the unit-circle, here called the Moebius Axis.

These three elements here will be called the *Moebius configuration*.
The transformation, mapping P into Q is a sequence of two steps:

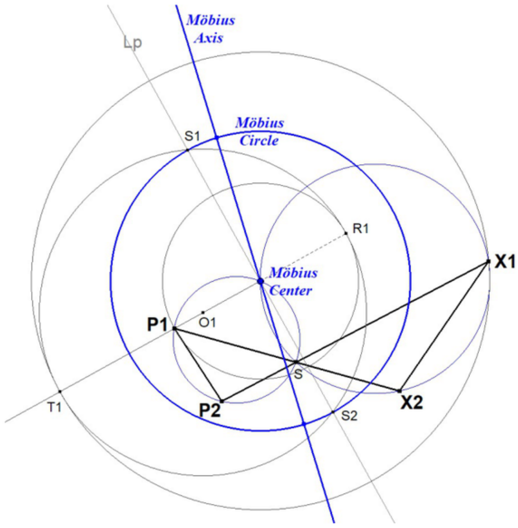
- Let R be the reflection of P about the Moebius Axis.
- Let Q be the inverse of R in the Moebius Circle.

Those two steps can be changed in order. The outcome will be the same.

There can be different points Pi being mapped into Qi.
When two pairs (P1,Q1) and (P2,Q2) are known, then the underlying Moebius Circle with Center and Axis can be constructed.
In many instances more than two pairs (Pi,Qi) are known and turn out to have the same Moebius Circle with Center and Axis. This is the case with the nP-CC-Moebius Conjugate, where always per n-Point n pairs (Pi,Qi) will come forward having the same underlying Moebius Configuration.

The method to construct the Moebius constellation will be called here the *Pi-Qi-standard Moebius construction*. It is described in next picture.

nP-n-Tf1 nP-CC-Moebius Conjugate.pdf



Construction Möbius Center
We know that the Moebius transformation swaps P1 and X1 as well as P2 and X2.
This information is enough to construct the Miquel Point QL-P1 of the Quadrilateral with lines P1.P2, P2.X1, X1.X2, X2.P1, which is the Möbius Center Ce.

Construction Möbius Circle
Now we know Möbius Center Ce.
We know that the Möbius transformation swaps P1 and X1.
1. Draw the halfline Ce.P1
2. Draw circle(Ce,X1) and let it intersect Ce.P1 in T1.
3. Let R1 be the reflection of P1 about Ce.
4. Draw line Lp perpendicular to Ce.P1 at Ce.
5. Let S1, S2 be the intersection points of Lp and Thalescircle (R1,T1) with center O1.
6. The circle with center Ce through both points S1 and S2 is the Möbius Circle.
It has radius SQRT(Ce.P1 * Ce.X1).

Construction Möbius Axis
The Möbius Axis is the bisector of angle P1.Ce.X1, which should coincide with the bisector of angle P2.Ce.X2.

The nP-n-Tf1 Conjugate
After these introductory words we come to the nP-n-Tf1 Conjugate.
Per n-Point there will be a nP-CC Moebius Configuration with Moebius Center nP-n-P5 and an nP-CC Moebius Conjugate nP-n-Tf1. As soon as this information is known the (n+1)P-CC Moebius Configuration can be constructed using the nP-n-P5 Moebius Center of the n-level before.

It starts with *a 3-Point / Triangle*. There the 3P-CC Moebius Configuration is defined by a Moebius Configuration with the 3P-circumcircle as the Moebius Circle and some line through the circumcenter (any line) as the Moebius Axis. Therefore the 3P-Moebius Center 3P-n-P5 will be X(3) and the conjugate 3P-n-Tf1 can be constructed because Moebius Circle, Center and Axis are known.

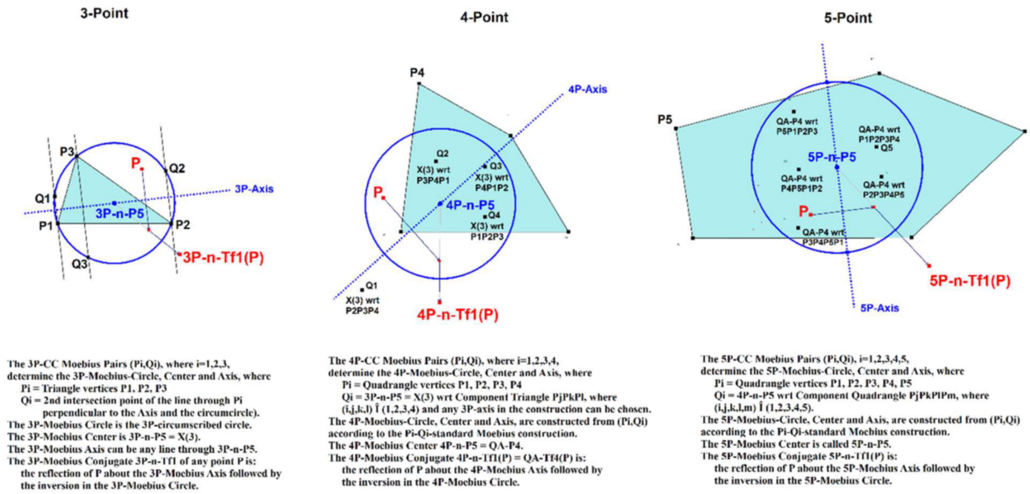
Then *in a 4-Point / Quadrangle* with vertices P1, P2, P3, P4 we have 4 Component Triangles each with its own 3P-Moebius Center Q1=3P-n-P5_1, Q2=3P-n-P5_2, Q3=3P-n-P5_3, Q4=3P-n-P5_4. This leads to 4 pairs (P1,Q1), (P2,Q2), P3,Q4), (P4,Q4) which generate a 4P-Moebius Configuration with a 4P-Moebius Center 4P-n-P5 and a 4P-Moebius Conjugate 4P-n-Tf1.

Then *in a 5-Point / Pentangle* with vertices P1, P2, P3, P4, P5 we have 5 Component Triangles each with its own 4P-Moebius Center Q1=4P-n-P5_1, Q2=4P-n-P5_2, Q3=4P-n-P5_3, Q4=4P-n-P5_4, Q4=4P-n-P5_5. This leads to 4 pairs (P1,Q1), (P2,Q2), P3,Q4), (P4,Q4) which generate a 5P-Moebius Configuration with a 5P-Moebius Center 5P-n-P5 and a 5P-Moebius Conjugate 5P-n-Tf1.

And so forth for a 6-Point, 7-Point, 8-Point, etc..

See next picture for the first three n-levels.

nP-n-Tf1 nP-CC-Moebius Conjugate.pdf



Properties:

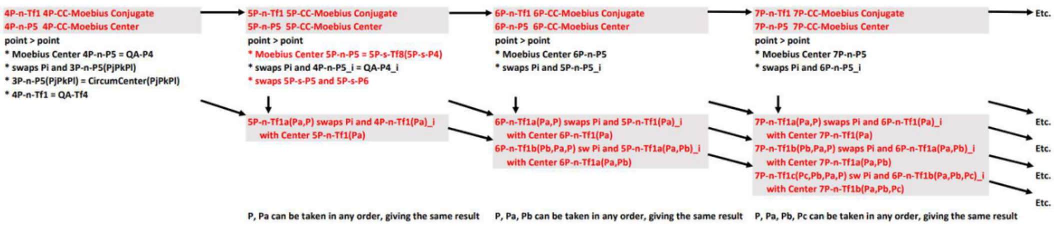
- 5P-n-Tf1 swaps 5P-s-P5 and 5P-s-P6.
- Moebius Center 5P-n-P5 = 5P-s-Tf8(5P-s-P4).

Extension to the nP-n-Tf1 Moebius Conjugate

There are these related Moebius Conjugates:
* nP-n-Tf1a swaps Pi and (n-1)P-n-Tf1(Pa)
  with Center nP-n-Tf1(Pa), Pa being an added random point
* nP-n-Tf1b swaps Pi and (n-1)P-n-Tf1a(Pb)
  with Center nP-n-Tf1a(Pa,Pb), Pa,Pb being added random points
* nP-n-Tf1c swaps Pi and (n-1)P-n-Tf1b(Pc)
  with Center nP-n-Tf1b(Pa,Pb,Pc), Pa,Pb,Pc being added random points

etc. ad infinitum

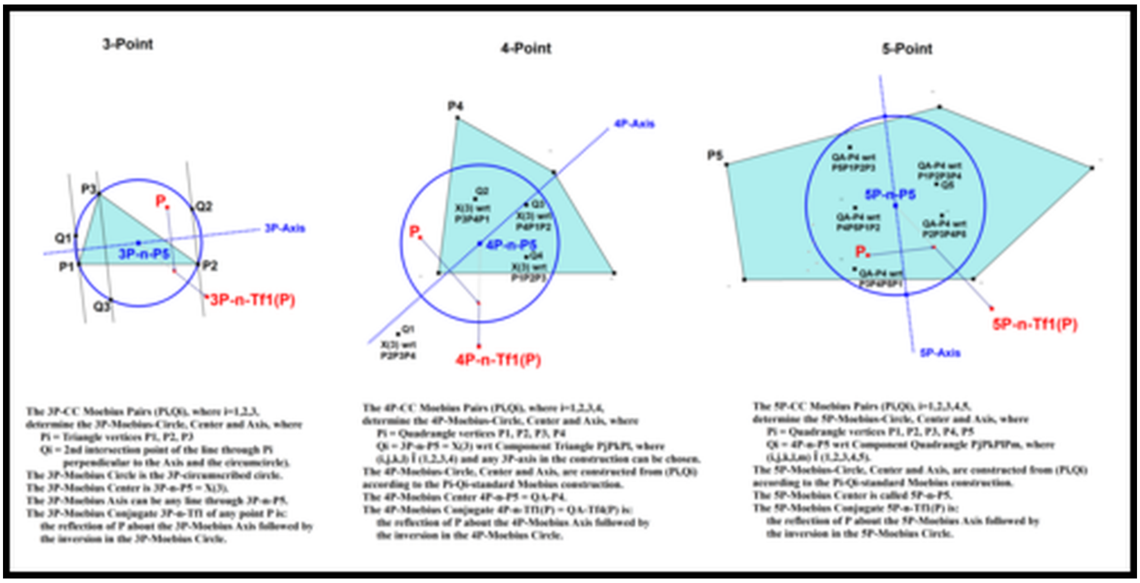
NOTE: the conjugate nP-n-Tf1x(Pi, .. , Pn), Pi,..,Pn can be taken in any order (gives the same result)



Properties:

- 5P-n-Tf1a(5P-s-P5, P) = 5P-s-Tf8(P)
- 5P-n-Tf1a(5P-s-P5, 5P-n-P5) = 5P-s-P4
- Moebius Center 5P-n-Tf1a(5P-s-P5, P) = 5P-s-P6
- Moebius Center 5P-n-Tf1a(5P-s-P6, P) = 5P-s-P5

nP-n-Tf1 nP-CC-Moebius Conjugate.pdf



nP-n-Tf1 _ P5 nP-CC Moebius Conjugate-01.png

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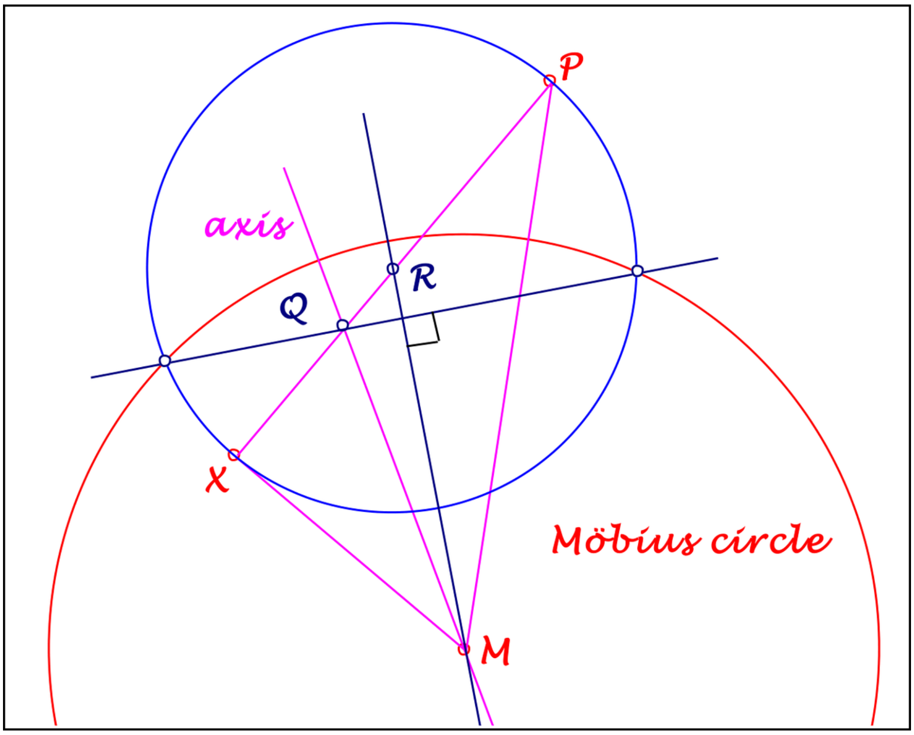
Message: #690
Date: 2021-01-25
From: eckart_schmidt@t-online.de
Subject: Re: Moebius (related) Transformations

```
Dear Chris,

I just started your message wrt the Möbius transformation,
... you prefer a definition with axis and circle,
... but studying geometric relations
... I prefer a definition with center M
    and pair of images X<-->P
... with a simple construction of axis and circle:
The angle bisector of XMP gives the axis, intersecting XP in Q.
The circle with diameter XP, centered in R,
... intersects the perpendicular to MR
    through Q on the Möbius circle.

Best regards Eckart
```



2021-01-25.pdf

Message: #691
Date: 2021-01-25
From: eckart_schmidt@t-online.de
Subject: Re: Moebius (related) Transformations

```
Dear Chris,

very interesting development of the Möbius Conjugate nP-n-Tf1,
... especially the possibility of generalization.
Main results for me:
... for n=4: center 4P-n-P5 = QA-P4 with 4P-n-Tf1 = QA-Tf4,
    swapping QA-P3
and QA-P9
... for n=5: center 5P-n-P5 = 5P-s-Tf8(5P-s-P4)
    with 5P-n-Tf1, swapping
5P-s-P5 and 5P-s-P6,
... already used as 5P-transformation,
    which lets the 5P-quartic invariant,
... first mentioned by Bernard as CSC3, see #3660 and #3679.

Best regards Eckart
```

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Message: #692
Date: 2021-01-25
From: van10hoven@gmail.com
Subject: Re: Moebius (related) Transformations

```
Dear Eckart,

Thanks for your construction. It’s indeed simpler.
About the Möbius transformation it can be defined by either:
* two pairs of points to be swapped by the transformation (the
center is not needed then) or
* a circle and an axis through the center of the circle.

I took the last definition, because often we work with more than
two pairs.
For example two pairs (P1, QA-P4_1) and (P2, QA-P4_2) define a
Moebius Transformation and we know that (P3, QA-P4_3) and (P4,
QA-P4_4) do define the same Moebius Transformation.
Likewise the two pairs (P1, QA-P3_1) and (P2, QA-P3_2) define a
Moebius Transformation BUT we know that (P3, QA-P3_3) and (P4,
QA-P3_4) do NOT define the same Moebius Transformation.
Because of this complication I prefer to define a Moebius
Transformation by the circle and its axis, because one might
think incidentally that a complete set of pairs define a Moebius
Transformation.
But of course it is no problem to use either definition when it
is convenient.

Then another remark about nP-n-Tf1 a :
The advantage of this extended conjugate nP-n-Tf1 a is that for
every n-Point any point P can be referred to as a Moebius Center
of a Moebius Transformation, when Pa is chosen to be
nP-n-Tf1(P).
Note the different use of nP-n-Tf1 and nP-n-Tf1 a.

I noticed that Bernard and you used different Moebius
Transformations centered in different points. Knowing a point in
advance it is easy now to find a corresponding Moebius
Transformation with that center.

Best regards,
Chris
```

Message: #693
Date: 2021-01-25
From: van10hoven@gmail.com
Subject: Quartics

```
Dear Bernard,

In your message #611 you state:
The same way, as 2 quartics intersect in 16 points and a quartic
is defined by 14 points, it is correct that: 14 points define a
quartic (as mentioned by Chris)
13 points have 3 CB16 partners (only add several 14 th points
give different quartics intersecting in 3 other points)
12 points define a transformation associating 3 points to one.

It is not quite clear to me what you mean.
When we have 13 fixed random points P1, …, P13 and 3 other
different points P14a, P14b, P14c.
Then we have 3 different quartics Qu1a(P1, …, P13,P14a),
Qu1b(P1, …, P13,P14b), Qu1c(P1, …, P13,P14c), which will
mutually intersect in 16 points.
Are you saying that the intersection points of Qu1a^Qu1b and
Qu1a^Qu1c and Qu1b^Qu1c are normally not identical, because you
say “add several 14 th points give different quartics
intersecting in 3 other points”?

How can you state then “12 points define a transformation
associating 3 points to one”?

Best regards,

Chris
```

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Message: #694
Date: 2021-01-25
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan

```
Dear Eckart,
Wunderbar !
I don't know how you manage in order to draw such curves as
envelops ...
It appears on your figures that the cayleyan is tangent to the
hessian in 3 points for the cubic and 4 points for the quartic.
For the cubic, the 3 contact points seem to be on a circle
through P5 ; their CSC are aligned (inflexion points of the
hessian ?).
For the quartic ?
I suppose you will be able to draw the same way the cayleyan
associated to the sextic invariant in Tf8 through the 5 Pi and
the 5 Xi with middle of the segments joining CSC partners on the
conic through the middles of the segments PiXi ?
Best regards
Bernard
```

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Message: #695
Date: 2021-01-25
From: bernard.keizer@gmail.com
Subject: Re: Quartics

```
Dear Chris,
It's exactly that !
With 3 different points P14a,b and c, the 3 quartics Qu1a, b and
c intersect in the same 16 points : P1 to P13 + 3 same points.
Hence 13 points have 3 CB16 partners and for a variable P13, P1
to 12 define a transformation associating 3 points P14a,b and c
to P13.
Best regards
Bernard
PS The 2nd transformation I mention is different and a little
bit more complicated, it will be for another time !
```

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Message: #696
Date: 2021-01-26
From: van10hoven@gmail.com
Subject: Re: Quartics

```
Dear Bernard,

1. Ok, then I skip your comment in #611 "(only add several 14 th
points give different quartics intersecting in 3 other points)".
2. What is your reasoning that we always have 3 common
intersection points for three different 13P-quartics?
I know it is like Cayley-Bacharach Theorem for cubics.
Nevertheless why do you think this theorem can be transferred?

Best regards,
Chris
```

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Message: #697
Date: 2021-01-26
From: eckart_schmidt@t-online.de
Subject: Re: Moebius (related) Transformations

```
Dear Chris,

it was a hard work, to understand your nomination 5P-n-Tf1a
... for Bernard's Möbius transformations in #3660 ,#3679,
... good for systematic, but difficult to handle:
... Bernard's CSC1 = 5P-s-Tf8 = 5P-n-Tf1a for Pa =5P-s-P5,
... Bernard's CSC2 = 5P-n-Tf1a(5P-n-Tf1(5P-n-Tf1a))
    for Pa = 5P-s-P5 ,
... Bernard's CSC3 = 5P-n-Tf1.

Best regards Eckart
```

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Message: #698
Date: 2021-01-26
From: van10hoven@gmail.com
Subject: Re: Moebius (related) Transformations

```
Dear Eckart,

I know this nP-n-Tf1a is a difficult transformation.
I considered even skipping it.
But I noticed its potential.
I think later on it can be of use.
Thanks for your perseverance.

Best regards,

Chris
```

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Message: #699
Date: 2021-01-27
From: eckart_schmidt@t-online.de
Subject: Re: Cayleyan

```
Dear Bernard,

wrt #694: It seems, that your observations for your
5P-circumcubic are correct,
... it is very time-consuming,
    to get several CABRI-constructions.
I don't remember your mentioned sextic and cannot reproduce your
description:
"... sextic invariant in Tf8 through the 5 Pi and the 5 Xi with
middle of the segments
... joining CSC partners on the conic
    through the middles of the segments PiXi".

Best regards Eckart
```

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Message: #700
Date: 2021-01-27
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan

```
Dear Eckart,
I only regret that you don't remember your own work !
I described the sextic in old messages 3775 and following and
again more recently in new message 650 point 14.
The construction is always the same : taking a point m on the
conic through the middles of PiXi as middle of 2 conjugate
points X and X', XX' is the bisector of F1mF2, X and X' are the
intersections between the line and it's CSCtransform (circle
through the center of the CSC) and the "cayleyan" is the envelop
of XX'.
You gave me in your old message 3779 a beautiful drawing of the
sextic.
Best regards
Bernard

PS The beauty of the construction is that it works with any CSC
(defined by it's 2 fixed points) and any curve named Newton
curve (line, circle or conic) !
Amazing, isn'it ? For example, your marvellous QL's quartic,
invariant in CSC, CSCdiag and CSC*CSCdiag has 3 "cayleyans" ...
```

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Message: #701
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan

```
Dear Eckart,
Please forget the last sentence in the PS
The "cayleyan" is necessary your strange conic DTinscribed,
tangent to the Steiner axes of CSC and CSCdiag and to the
Steiner line.
As I'm not home, I'm not able to check the locus of the middles
and the property with the tangents to the conic bisectors of
F1mF2 ...
Best regards
Bernard
```

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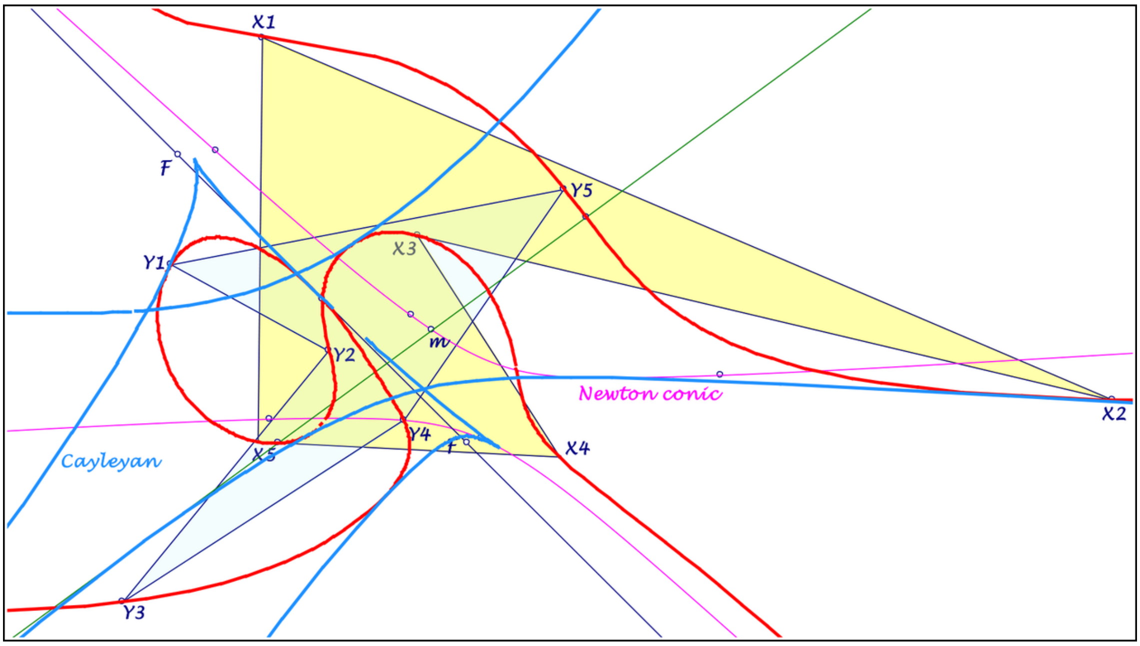
Message: #702
Date: 2021-01-28
From: eckart_schmidt@t-online.de
Subject: Re: Cayleyan

```
Dear Bernard,

thanks for refreshing my memory, attached the Cayleyan of the
sextic,
... using my old drawing in # 3779.

Best regards Eckart
```



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Message: #703
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Moebius (related) Transformations

```
Dear Chris, dear Eckart
I hardly followed your interesting discussion !
Here some remarks :
A Moebius transformation has several possible definitions, but
in the end, we need the 2 fixed points or a circle and one of
it's diameters.
Having 2 copples of partners gives the center as QL-P1.
Having 1 copple of partners and the center gives the bisector as
Steiner axis, then perhaps another construction : the reflection
in the Steiner axis of the line through the partners intersects
the circumcircle of the 2 partners and the center in 2 points
equidistant from the center ; the circle with center the center
of the transformation through the 2 equidistant intersections is
the inversion center.
Now it seems to me that such a transformation is interesting
only if it refers to well-known elements or swaps well-known
elements of the triangle, the QA or the QL or the PA or the PL.
For example with a triangle and the center in X3, you could take
as diameter associated to the circumcircle as inversion circle
either the Euler Line or the Brocard axis ...
For example always with a triangle, the psi transformation in
ETC centered in X2 swaps X13 and X14 and X15 and X16 and the
vertices of the triangle with the vertices of the Brocard
triangle or the Brocard axis and the Parry circle ...
Last example with a triangle, there are 3 CSC centered in a
vertice and swapping the 2 other vertices.
Back to the QL, a monocursal QL-Cu1 is invariant in these 3
CSC's of the triangle QL-P1QL-2P2a and b ...
Hence my interest for the main pivot triangle of QL-Cu2 which
has the same psi transformation as the CSC of the QL's inscribed
in the corresponding QL-Cu1 (see my message 587 remained without
answer).
Best regards
Bernard
```

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Message: #704
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Moebius (related) Transformations

```
Dear Chris, dear Eckart
Perhaps a last example :
in the pentalateral, the circle Ci with center X' mentionned
under 5L-sTf1 in EPG is the locus of the CSC partners of X in
all CSC's centered in a point of the circle 5L-s-Co1 swapping
the 2 foci of the inscribed conic 5L-s-Ci1. (In particular, the
5 CSC's of the 5 QL's of the PL are centered on this circle and
swap these 2 foci).
Best regards
Bernard
```

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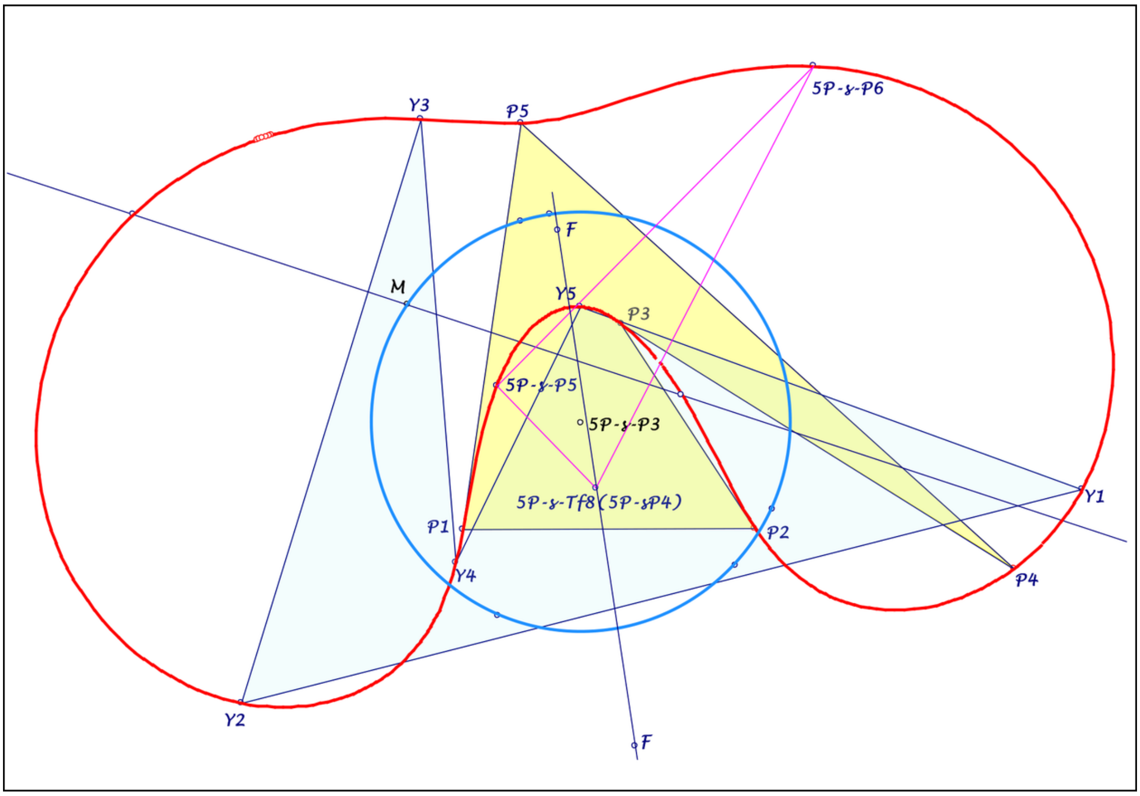
Message: #705
Date: 2021-01-28
From: eckart_schmidt@t-online.de
Subject: Basic Möbius transformation for 5P

```
Dear Bernard, dear Chris,

we got the Möbius transformation 5P-s-Tf8,
... studying the common points of the QA-Cu7-cubics,
... but I think another Möbius transformation is more important:
This will be Chris' 5P-n-Tf1 or Bernard's CSC3,
... centered in 5P-s-Tf8(5P-s-P4), swapping 5P-s-P5 and 5P-s-P6.
If we consider for a 5P = P1...P5 a 2nd 5P = Y1...Y5 with Yi =
5P-n-Tf1(Pi),
... which are the QA-P4 of the 5P,
... the midpoints of PiYi, we get concyclic points on a circle,
... centered in 5P-s-P3, mentioned under this item in EPG.
If we consider for points M on this circle and the fixed points
F of 5P-n-Tf1
... the angle bisectors FMF with 5P-n-Tf1 partner on it,
... we get our well known 5P-quartic,
    evidently invariant under 5P-n-Tf1.
Best regards Eckart
```



2021-01-28b.pdf

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Message: #706
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Basic Möbius transformation for 5P

```
Dear Eckart,
Of course, we agree on this construction, dressed with Chris new
names !
What happens if you consider as starting pentangle the Y
pentangle ?
I suppose we will find the same quartic and the same
transformation with the same circle, but I wonder what the QA-P4
of the 5Y are and the same way the points P3,P4,P5 and P6 for
the 5Y (as these 4 points are needed for the construction) ...
Best regards
Bernard
```

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Message: #707
Date: 2021-01-28
From: eckart_schmidt@t-online.de
Subject: Re: Basic Möbius transformation for 5P

```
Dear Bernard,

for the 2nd 5P = Y1...Y5 we have a different transformation
5P-n-Tf1,
... so we get not the same circle
    and the same quartic as for 5P = P1...P5.

Best regards Eckart
```

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Message: #708
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Basic Möbius transformation for 5P

```
Dear Eckart,
Thanks for your quick answer !
Now if CSC3 = CSC1*CSC2*CSC1, we have also CSC2 = CSC1*CSC3*CSC1
(with CSC1 = Tf8 and CSC3 = 5P-n-Tf2).
You may then describe the 5P cubic through the 5P and the CSC1
of the QA-P4 of the CSC1 of the Pi.
Could CSC2 be 5P-n-Tf2 ?
Best regards
Bernard
PS Please don't forget the 5P sextic invariant in Tf8 !
```

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Message: #709
Date: 2021-01-28
From: bernard.keizer@gmail.com
Subject: Re: Quartics

```
Dear Chris,
I didn't forget your question. I have 2 reasons :
1) algebric calculation
There is an abundant litteratur about the CB transformation for
8 points and it's possible extension.
The equation of a cubic needs 10 coefficients with a factor of
proportionality.
Hence 9 points define generally a unique cubic unless they are
CB9 partners.
If 2 cubics of equations C1 = 0 and C2 = 0 intersect in 9
points, any cubic with equation a*C1 + b*C2 = 0 will pass
through the 9 points.
The same way, the equation of a quartic needs 15 coefficients
with a factor of proportionnality.
Hence 14 points define generally a unique quartic unless they
are CB16 partners.
If 2 quartics of equations Qu1 = 0 and Qu2 = 0 intersect in 16
points, any quartic with equation a*Qu1 + b*Qu2 = 0 will pass
through the 16 points.
That's why 13 points have 3 CB16 partners on both Qu1 and Qu2
and therefore on any other quartic through the 13 points.
2) Figure observation
Geogebra allows the drawing of Implicit curves through 9 (cubic)
or 14 (quartic) points.
I checked the property that several quartics through 13 points
and another intersect in the same 3 other points, which are the
CB16 partners of the 13 points.
Best regards
Bernard
```

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Message: #710
Date: 2021-01-29
From: eckart_schmidt@t-online.de
Subject: Re: Basic Möbius transformation for 5P

Dear Bernard,

wrt your #708: What do you mean with 5P-n-Tf2, I don't find it
in Chris' paper.
Your CSC3 is 5P-n-Tf1, see #705.
If you define CSC3 as 5P-n-Tf2 (line 2), I don't understand your
question
... "Could CSC2 be 5P-n-Tf2 ?" (line 4).
What are my misunderstandings?
Wrt the PS see #702.

Best regards Eckart

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Message: #711
Date: 2021-01-29
From: bernard.keizer@gmail.com
Subject: Re: Basic Möbius transformation for 5P

Dear Eckart,
Reading this message again, it seems we have to create twinCSC2
and twinCSC3.
Then it holds that CSC3 = CSC1*twinCSC2*CSC1 and twinCSC3 =
CSC1*CSC2*CSC1.
The 5P quartic is invariant in CSC3 and the 5X twinquartic in
twinCSC3.
The 5P cubic is invariant in twinCSC2 and the 5X twincubic in
twinCSC2.
The 5P quartic and the twin 5Xcubic as well as the 5X twin
quartic and the 5P cubic are CSC1 partners.
The 5P and 5X sextic is CSC1 invariant.
Best regards
Bernard

PS I know, CSC1 is 5P-s-Tf8 and CSC3 is 5P-n-Tf1, so twinCSC3 is
5X-n-Tf1, with Xi = 5P-s-Tf8(Pi), but what about CSC2 and
twinCSC2 ?

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Message: #712
Date: 2021-01-29
From: eckart_schmidt@t-online.de
Subject: Re: Basic Möbius transformation for 5P

```
Dear Bernard, dear Chris,

excuse, in the attached drawing of #705
... the nomination of point 5P-s-P5 and 5P-s-P6
    has to be changed.

Best regards Eckart
```

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Message: #713
Date: 2021-01-29
From: eckart_schmidt@t-online.de
Subject: Re: Basic Möbius transformation for 5P

```
Dear Bernard,

I think, we should follow Chris' nomination,
... else no other will understand our messages.
Excuse, if I don't follow your twin-geometry,
... for any Möbius transformation leads to another twin-aspect.

Best regards Eckart
```

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Message: #714
Date: 2021-01-29
From: van10hoven@gmail.com
Subject: Re: Quartics

Dear Bernard,

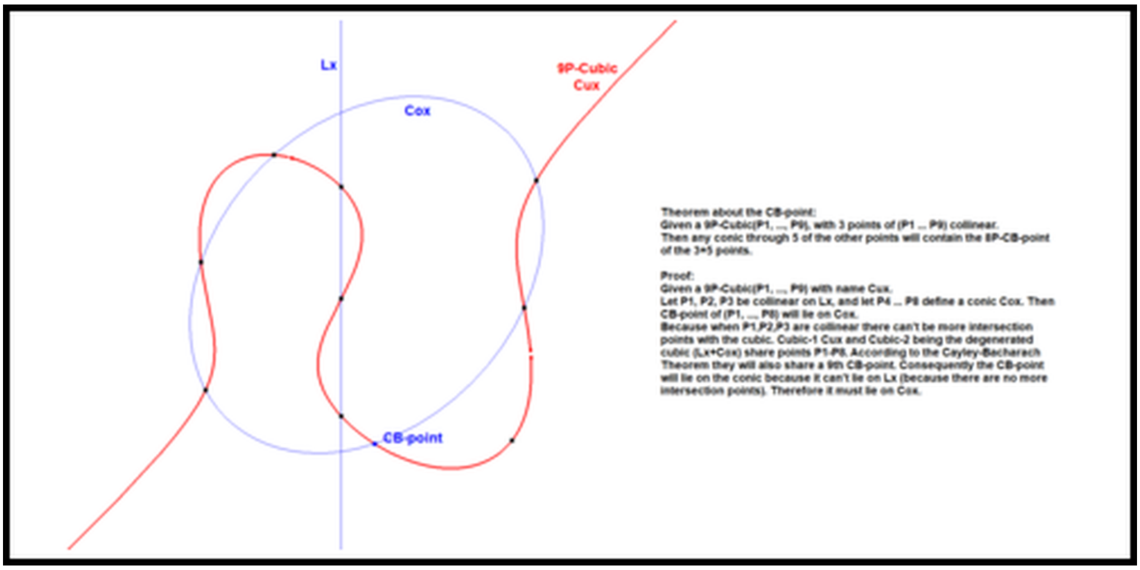
Thanks for the explanation. It helped me in my considerations.
Last days I made just like you a pictures of three Quartics as an example and found out that 13 fixed points produce 3 indeed common CB partners.
See attached file.
I also made a picture of three Quintics as an example and found that 19 fixed point have 6 CB partners. I also found out that some of these points can be imaginary. I think per set of 2 points.

Generalizing I came to this preliminary conclusion:

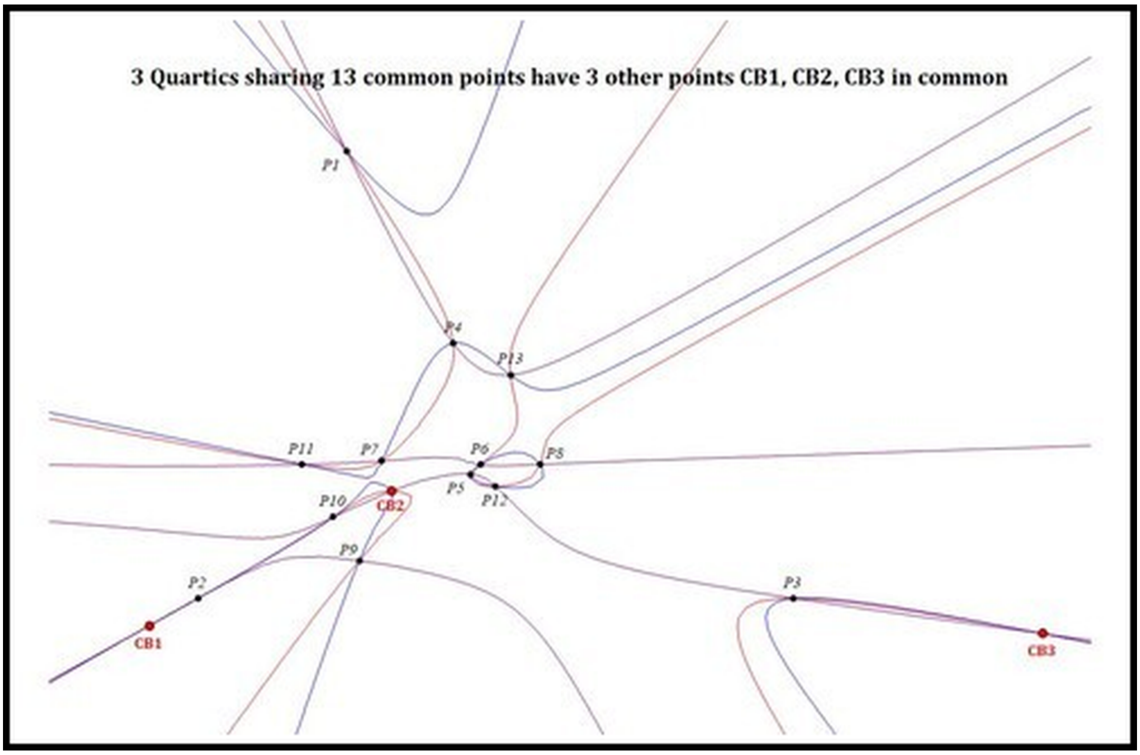
```
n=1 --> 2 points needed for construction,
        1 intersection point of 2 lines, 0 CB-points
n=2 --> 5 points needed for construction,
        4 intersection points of 2 curves, 0 CB-points
n=3 --> 9 points needed for construction,
        9 intersection points of 2 curves, 1 CB-point
n=4 --> 14 points needed for construction,
        16 intersection points of 2 curves, 3 CB-points
n=5 --> 20 points needed for construction,
        25 intersection points of 2 curves, 6 CB-points
n=6 --> 27 points needed for construction,
        36 intersection points of 2 curves, 10 CB-points
n=7 --> 35 points needed for construction,
        49 intersection points of 2 curves, 15 CB-points
etc.
```

In general (conjecture):
   n=n --> n(n+3)/2 points needed for construction,
   n^2 intersection points of 2 curves, (n(n-3)/2) + 1
   CB-points.
I suppose you also had this in mind.
Last but not least playing with these elements I found this theorem.
Given a 9P-Cubic(P1, ..., P9), where 3 points of the set (P1 ... P9) are collinear.
Then any conic through 5 of the remaining points will contain the CB-point wrt the 3+5 designated points.
The CB-point will be the 6 th intersection point of the conic and the cubic. See attached file.

Best regards, Chris



8p-s-tf1 Calyley-Bacharach Point-Theorem Line-Conic-01.png



14P-s-Qu1 14-Point Quartic plus 3 CB-points-01.JPG

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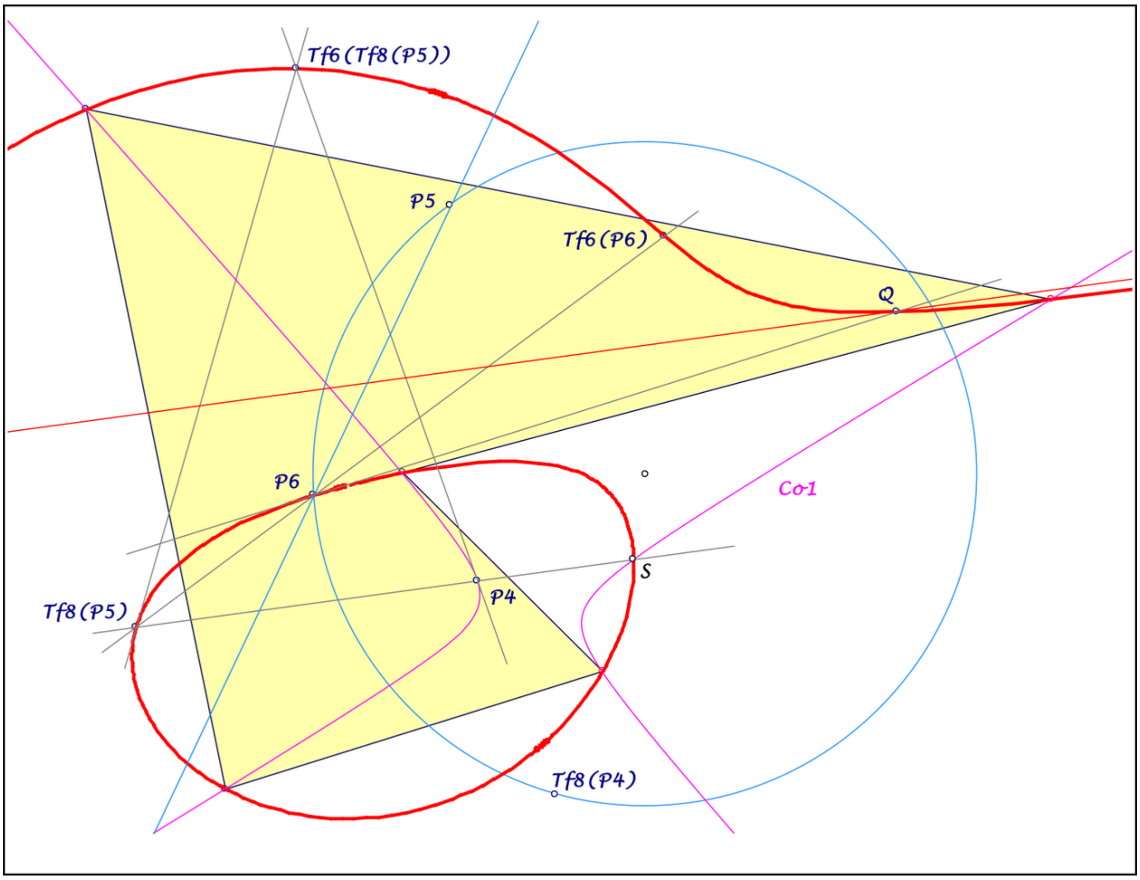
Message: #715
Date: 2021-01-30
From: eckart_schmidt@t-online.de
Subject: Another focal circular 5P-circumcubic

```
Dear Bernard, dear Chris,

may I invite you once more to an excursion in 5P-geometry?
Starting, let us omit the pre-fix 5P-s-.

For a 5P there is a transformation P <--> Ci(P), which maps a
point to a circle,
... bearing the 5 points P5 for pentangles "P plus a QA"
    of the 5P,
... degenerating to a line,
    if P is a point on the circumconic Co1.
The circles/lines Ci(P) have the common point P5 of the 5P.
P and the Cayley-Bacharach point Tf6(P) give the same circle
Ci(P).
Example: Ci(P6) through P5, P6 and Tf8(P4).
Ci(X) is P5.P6 for the 2nd intersection S of Co1 and P4.Tf8(P5).
These properties are already described in #3575.

Now we look for further circles Ci(P) through P5 and P6.
The locus for P is a focal circular circumcubic of the 5P,
... bearing P6, Tf8(P5) and the 2nd intersection of Co1
    and P4.Tf8(P5),
... further the Tf6-images of these points,
... for the cubic is Tf6-invariant with pivot Tf8(P5),
... whose tangential is Tf6(Tf8(P5)).
If we consider 7P-s-Tf1 for "5P plus Tf8(P5) and Tf6(Tf8(P5))",
... the cubic is the locus of 7P-s-Tf1-partner on parallels to
    P4.Tf8(P5).
The asymptote is parallel to P4.Tf8(P5),
... intersecting the cubic on the tangent in P6,
    which is the focus.

Summary:
This focal circular circumcubic of a pentangle is the locus
... of P with Ci(P), centered on the bisector of P5.P6.
... or: of Tf6-partner on lines through Tf8(P5).

Best regards Eckart
```



2021-01-30.pdf

Message: #716
Date: 2021-01-30
From: van10hoven@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Thanks Eckart for all these beautiful properties.
I like your idea of omitting the prefix of QPG-codes when they are all the same and whenever convenient.
By the way, the circle you describe also has a QPG-code:
5P-s-Tf7.
Best regards,
Chris

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Message: #717
Date: 2021-01-30
From: bernard.keizer@gmail.com
Subject: Re: Quartics

Dear Chris,
We perfectly agree !
I'm glad that you reproduce the 3 CB16 points for 3 quartics through the same 13 points.
For your theorem, the demonstration is obvious : if 9 points define a cubic and 3 of the 9 points are aligned, then the line forms with a conic through 5 of the 6 points a degenerated cubic through the 8 points and the intersection between this cubic and the 1rst is the CB point of the 8 points (this CB point cannot be on the line and must lie on the conic).
You may generalise your theorem with the same demonstration : if 14 points define a quartic and 4 of the 14 points are aligned, then the line forms with a cubic through 9 of the 10 other points a degenerated quartic through the 13 points and the intersections between this quartic and the 1rst one are the 3 CB of the 13 points (these points cannot be on the line and must lie on the cubic).
Best regards
Bernard

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Message: #718
Date: 2021-01-30
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard, dear Chris,

this is an addition to #715, using the nomination there.
I described a focal circular circumcubic of a 5P,
... defined by the 5P and P6 and Tf8(P5).
Here are three Möbius transformations, which let the cubic
invariant:
Consider the points P6, Tf6(P6) and S (S 2nd intersection of
P4.Tf8(P5) and Co1),
... a Möbius transformation,
    centered in one of the three points,
... swapping the other two, maps the cubic to itself.

Best regards Eckart
```

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Message: #719
Date: 2021-01-31
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
Very interesting new circular focal circumcubic of the 5P !
Here some remarks and a question :
1) Any 7 points on a cubic define a pivotal CB transformation on this cubic and any 5 points on a circular cubic define a pivotal cb transformation on this cubic.
2) Any circular focal cubic with focus F defines a Moebius transformation and has the property that for any copple of conjugates X and X' the curve is invariant in the 3 Moebius transformations of the triangle FXX'
There are an infinity of cb invariant circular 5P circumcubics (pivot P gives S as 2nd intersection of PP4 and Co1 ...)
I suppose, a lot of them are focal
What is the locus of the foci ? (circle through P5 and P6 ?)
Best regards
Bernard
PS Thanks for your answer about the "cayleyan" of the sextic in 702 and for your explanation about CSC2 in 697 (I find it rather complicate !).
For Chris, thanks to Eckart, I've found a mystake in 650 point 10 centered in P5 and swapping P6 and 5X-P4 (and not P4) and point 11 centered in Tf8(P5) and swapping P6 and 4 (not 5X-P4).

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Message: #720
Date: 2021-01-31
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
I'm less convinced now !
Checking again old figures with the 3 QA-Cu7 and the 5 points, the foci of the 3 QA-Cu7 are the QA-P41 on a circle through P6 of the 5P, which is P1 of the QL, but this circle doesn't pass through P5 ...
Perhaps you will have another idea
Best regards
Bernard

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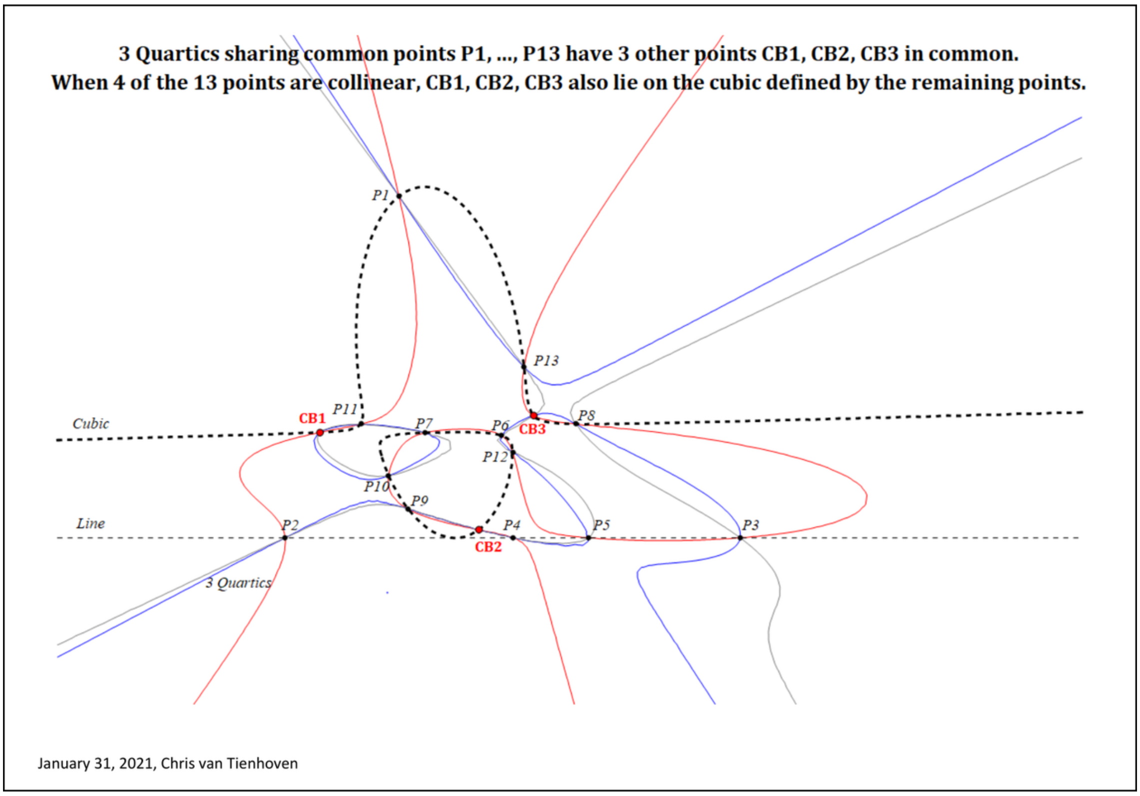
Message: #721
Date: 2021-01-31
From: van10hoven@gmail.com
Subject: Re: Quartics

```
Dear Bernard, dear Eckart,

Bernard, in accordance with your remarks.
Attached a picture of 3 Quartics all through the same 13 points
with 3 CB-points.
When 4 of the 13 points are collinear the 3 CB-points lie on the
cubic defined by the remaining points.

Best regards,
Chris
```



14P-s-Qu1 14P-Quartics where 4 points collinear.pdf

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Message: #722
Date: 2021-02-01
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
I tried in vain to find a trace of such cubics in our discussions about the 5 triple points of QA-Cu7 !
(For Chris, is it possible to put in the forum a direct link to the Archive messages ?)
If my memory is correct, we have now 2 circular focal 5P circumcubics , both cb invariant for the 5P :
1) the 1rst is let say 5P-s-Cu1 (message 687) with focus in P5, Newton Line middles of PiQA-P4(Tf8(4Pj with j not equal to i)) and middles of P5P4 and Tf8(P5)P4 of Tf8(Pi) invriant in the main CSC swapping the Pi and the QA-P4 of Tf8(4Pj with j not equal to i) and P6 and P4 of Tf8(5P).
The pivot of the cb transformation Tf6 is the infinity point of the Newton Line (we have therefore P5cbP5 parallel to the Newton Line).
The asymptote is the parallel in P4 to the Newton Line.
The twin cubic has focus Tf8(P5) and main CSC swapping the Tf8(Pi) and the QA-P4(4Pj with j not equal to i) and P6 and P4.
It is invariant in the cb of the 5Tf8(Pi) with pivot the same point, as the Newton Line is the same (Tf8(P5)cbTf8(P5) is also parallel to the Newton Line).
The asymptote is the parallel to the Newton Line in P4 of the Tf8(5P).
2) the 2nd one we will name 5P-s-Cu2 is described in your message 715 with focus in P6 invariant in the main CSC swapping Tf6(P6) and S.
The pivot of the cb transformation is Tf8(P5).
I suppose we also have a twin cubic with the same focus and pivot of it's cb transformation 5X-s-Tf6 in P5 ...
And what about Tf8 of this 2nd twin cubic ?
3) the 2 cubics Cu1 and Cu2 intersect in 2 points cb partners on the parallel in Tf8(P5) to P5Tf6(P5)
As you always draw such beautiful curves, I hope you will soon put the 2 cubics on the same figure ...
Perhaps it will give new ideas in order to find other such focal circular 5P circumcubics
Best regards
Bernard

Message: #723
Date: 2021-02-01
From: van10hoven@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

Regarding your question in message #722:
For Chris, is it possible to put in the forum a direct link to
the Archive messages?

Yes, there is a link to the old messages of the Quadri-Figures
Group, as well as the messages of the new Quadri- and Poly-
Geometry Group.
See resp. [34] and [66] at
https://www.chrisvantienhoven.nl/eqf-references.

Ah and now I understand you want in our actual group a reference
to the Quadri-Figures Group.
Meanwhile... I made a reference at the first page of the
Quadri-Figures Group
(https://groups.io/g/Quadri-Figures-Group/messages).

Best regards,

Chris
```

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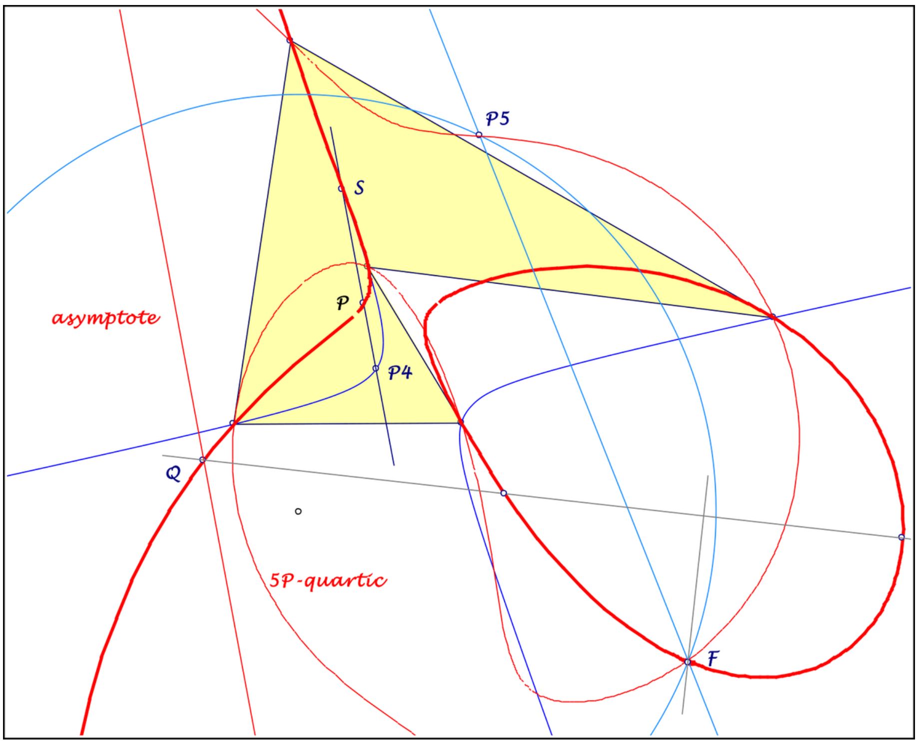
Message: #724
Date: 2021-02-01
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

thanks for your remarks in #719,
... you ask for the locus of the foci of focal circular
    5P-circumcubics.,
Focal circular 5P-circumcubics have their focus on the
5P-quartic.
Circular 5P-circumcubics of a 5P are Tf6 invariant with pivot P
... P.P4 intersects Co1 further in S,
    P.P4 is parallel to the asymptote,
... 5P plus P and S define the cubic as 7P-s-Cu1.
Let us start with a 5P and a given direction of its asymptote
... with a parallel through P4, intersecting Co1 in S,
... and an arbitrary point P on P4.S,
... that defines a circular circumcubic of 5P,
... which has a focus, but must not be focal.
The focus F is the 2nd intersection (beside P5) of the
Tf7(P)-circle and the Tf7(S)-line,
... if F lies on the cubic, F is a point of the 5P-quartic.

Best regards Eckart
```



2021-02-01.pdf

Message: #725
Date: 2021-02-01
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
Wunderbar !
Amazing property, which certainly deserves to be explained in
EPG !!!
(In fact, P5, P6 as well as the QA-P41 are on the quartic of the
5 triple points of the 3 QA-Cu7 of a QC).
I have to study the figure more completely and, if I find time,
to draw my own figure ...
Thanks a lot
Best regards
Bernard
PS Thanks to Chris for the link to the Archive in Quadri-Figures
Group
```

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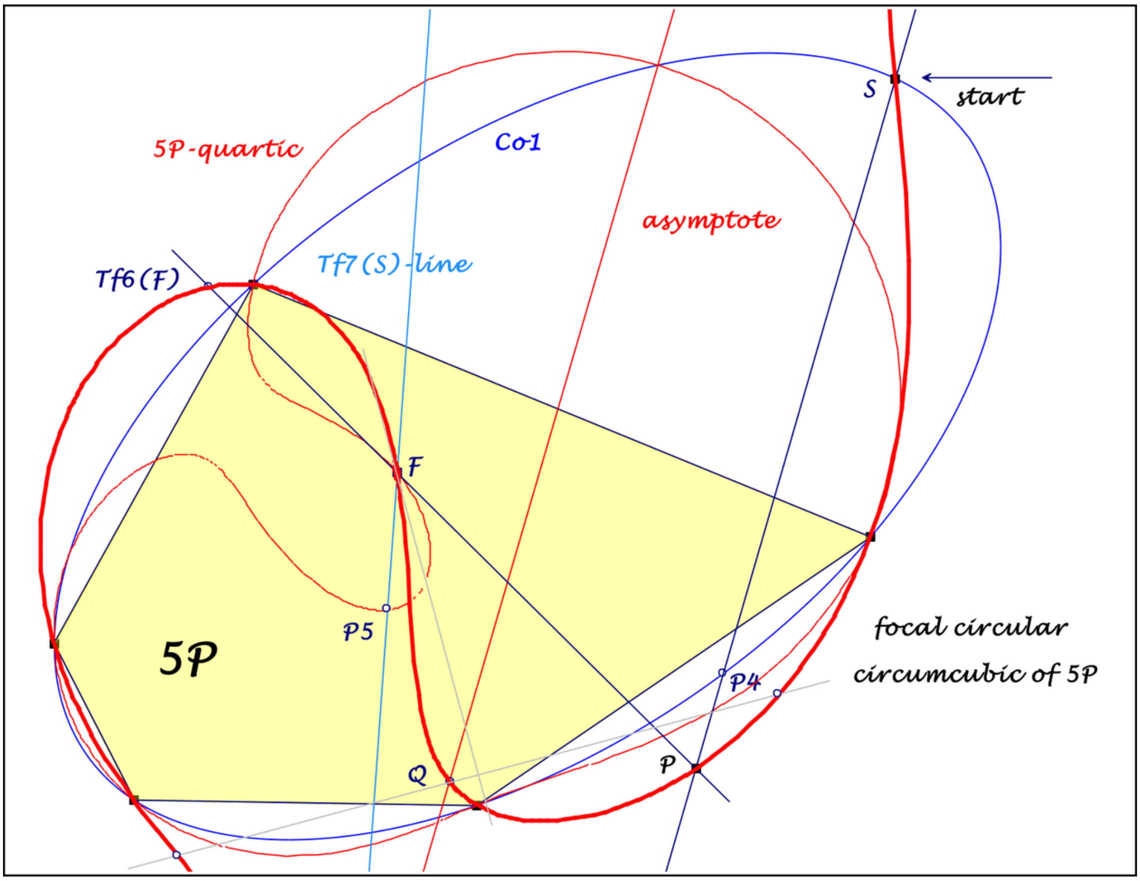
Message: #726
Date: 2021-02-02
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

how to get further focal circular 5P-circumcubics CU:
Consider a 5P and a point S on Co1 with its Tf7(S)-line,
... intersecting the 5P-quartic up to three times in foci F
    for a CU (unequal P5).
The line F.Tf6(F) intersects S.P4 in the corresponding
TF6-pivot P
... and 7P-s-Cu1 of 5P plus S and P gives a cubic CU,
... the tangential of F gives the intersection Q with the
    asymptote.

Best regards Eckart
```



2021-02-02.pdf

Message: #727
Date: 2021-02-02
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

in addition to #724 and 726 in the same nomination:
Focal circular circumcubics of a 5P
... are invariant wrt a Möbius transformation TF,
... centered in the focus F and swapping S and Tf6(F).
Let T be the intersection of the cubic and a parallel to S.P4
through F,
... then TF(T) = Q,
    the intersection of the asymptote and the cubic
... and the cubic is QL-Cu1 of the quadrigon S.T.Tf6(F).Q.

Best regards Eckart

PS: Is remark 2) in 719 true, for it doesn't hold for TF above.
```

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Message: #728
Date: 2021-02-02
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
Thanks for all your precious complements about the construction!
My apologise, as my formulation in 719 is completely wrong.
For a QL-Cu1, monocursal or bicursal, it holds that if M is
QL-P1 and X and X' the points QL-2P2a and b, the curve is
invariant in the 3 Moebius transformations of the triangle MXX';
in the bicursal case, the in- and excenters of the triangle lie
on the curve.
On your figure 715 (bicursal QL-Cu1), S and Tf6(P6) don't seem
to be symmetric wrt the Newton Line ...
Best regards
Bernard
```

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Message: #729
Date: 2021-02-03
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

Dear Bernard,
wrt your remark
"On your figure 715 (bicursal QL-Cu1), S and Tf6(P6) don't seem to be symmetric wrt the Newton Line ..."
I think, the drawing is correct:
The Newton line is a parallel to the asymptote, half the distance to the Miquel point, here P6.
Then the midpoint of S.Tf6(P6) will be on the Newton line.
Best regards Eckart

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Message: #730
Date: 2021-02-03
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
I had no doubt your drawing was correct !
The mentionned property with the 3 CSC holds for the triangle QL-P1QL-2P2a and b.
I'm surprised that it holds also for another copple of CSC partners (S and Tf6(P6)).
Hence my question !
I tried to reproduce your 2nd cubic, but I'm not able to draw Tf6 as I don't use macros and Hart's construction takes too much time.
As I told you, Geogebra allows only the drawing of cubics through 9 real points (not CB partners) and quartics through 14 real points (same condition).
For the 1rst cubic, I have the 5P and the QA-P4 of the QA's of the Tf8, which is more than enough with P5 ...
For this 2nd cubic, I have only the 5P, Tf8(P5), P6 and S, I miss 9nth !
If I dared to ask you for a drawing of the 2 cubics on the same figure (with P4, P5 and P6), it would help me greatly !
Many thanks in advance
Best regards
Bernard

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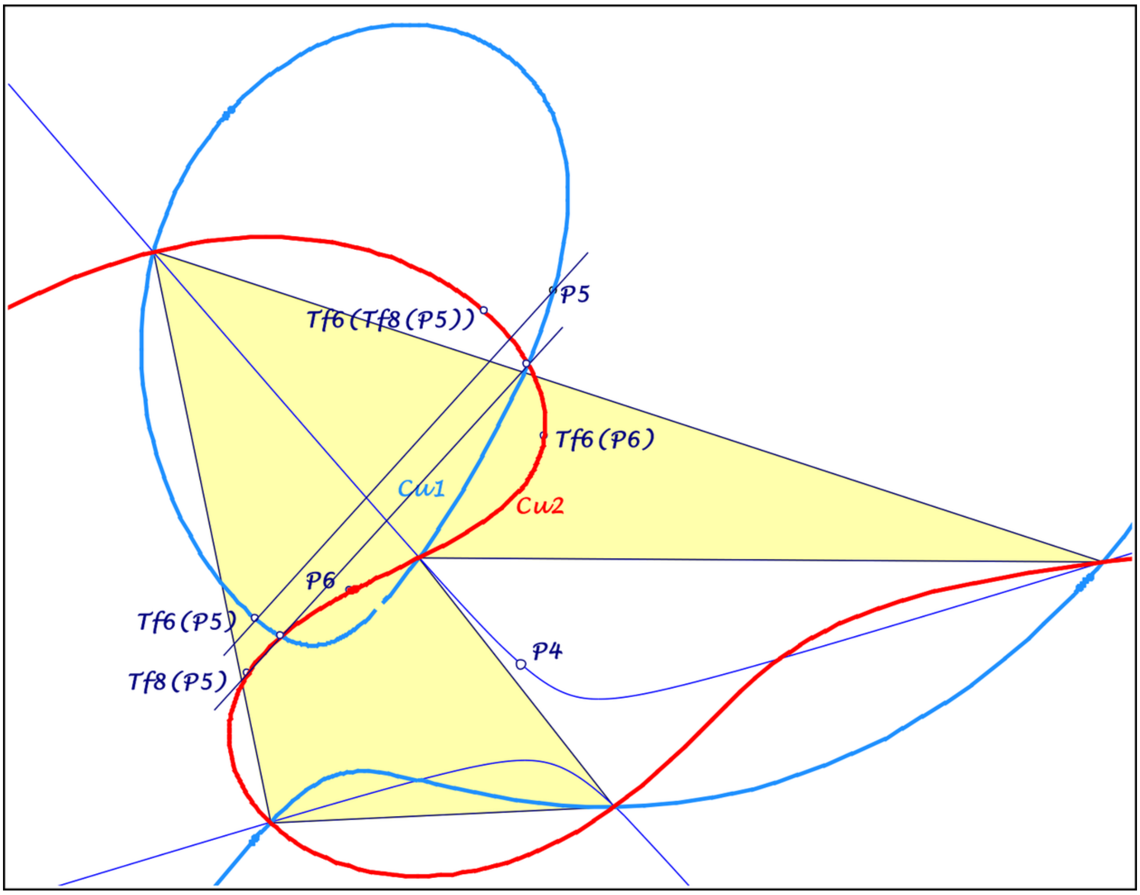
Message: #731
Date: 2021-02-03
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,
attached the two cubics of #722,
...if you need more points, please say.
Best regards Eckart
```



2021-02-03.pdf

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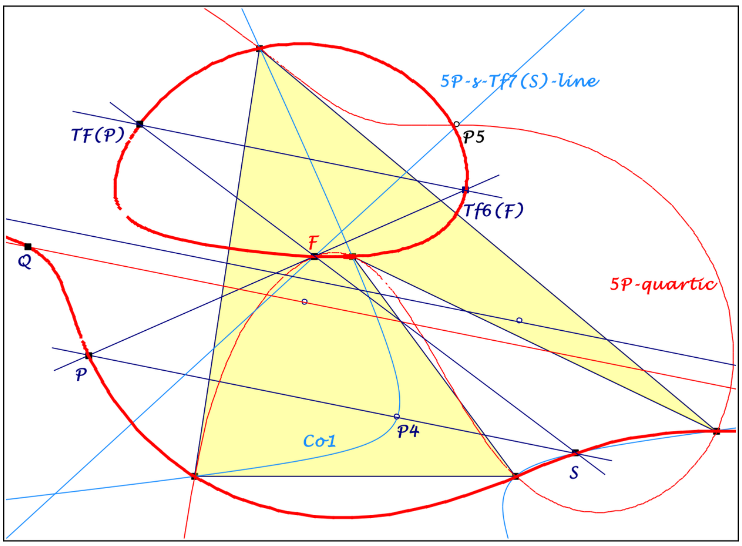
Message: #732
Date: 2021-02-04
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

Dear Chris, dear Bernard,
a short summary of the main points on
... a focal circular circumcubic of a 5P:
Let us start with the focus F of the cubic,
... which has to be a point on the 5P-quartic,
... and defines the cubic.
On the circumconic of the 5P exists a Co1-point S
... whose 5P-s-Tf7(S)-line bears F (construction?).
Now the cubic can be constructed as 7P-s-Cu1 wrt 5P plus F plus S. The line F.Tf6(F) intersects the line S.P4
... in the pivot P for the cb-transformation Tf6, which lets the cubic invariant.
The Möbius transformation TF, which lets the cubic invariant,
... is centered in F, swapping S and Tf6(F).
The Newton line is parallel to S.P.P4 through the midpoint of S.Tf6(F). The asymptote is parallel to S.P.P4 through the reflection of F in the Newton line.
The cubic has the asymptote intersection Q
... on the tangent in F at the circumcircle of F, S, P.
Tf6(F).TF(P) is parallel to S.P.P4, further F, S and TF(P) are collinear.
All conics through these main points F, S, P, Q
... intersect the cubic in two further points X, Y
... and the line XY has a third fixed cubic intersection
... on a parallel to the asymptote through F.
Best regards Eckart



2021-02-04.pdf

Message: #733
Date: 2021-02-04
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
Before studying your new message, I don't resist to give you
immediately a (perhaps) new element.
The cubic Cu2 has a twincubic constructed the same way as Cu2
cbinvariant wrt Tf8(5P) with pivot Tf5 and focus P6.
Then Cu2 and twin Cu2 are Tf8partners ! I hope you will confirm
this property ...
Best regards
Bernard
PS I'm convinced that all these developments of the geometry of
the pentangle belong to the most interesting items we met.
```

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Message: #734
Date: 2021-02-04
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
I remembered an old construction of a pivotal circular circumcubic of 5 points.
5P, pivot P, point P4.
A line Dij through 2 points PiPj, a corresponding circle Cij through the 3 other points.
Dij and Cij form a degenerated circular circumcubic Cuij.
The infinity line and the circumconic form a degenerated circular circumcubic Cu.
The parallel through P4 to Dij intersect the circle in 2 points, one on the circumconic, the other is Tij, the pivot of the degenerated circular circumcubic Cuij.
For a pivot P, the line PTij intersect Dij in Uij and Cij in a 2nd point Vij and PP4 intersect the conic in S and the infinity line in cbS.
Uij and Vij are cbpartners on the circular circumcubic with pivot P.
This property gives beside P and S 10 points Uij and10 points Vij.
This allowed me to draw Cu2 ...
I thank you for your picure in 731, it appears clearly that the 2 cubics Cu1 and Cu2 intersect in a parallel to the Newton Line of Cu1 through Tf8(P5) (id est the line joining the 2 pivots), but these 2 points are not necessary real and I had to use my old construction to get an exact drawing.
Best regards
Bernard
PS The last point you describe in your previous message as Cotterill's focus of F, S, P and Q is q=FP(Q).

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Message: #735
Date: 2021-02-04
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Sorry, in message 733, please read twinCu2 cbinvariant wrt
Tf8(5P) with pivot P5 and focus P6.
Then twinCu2 = Tf8(Cu2).
```

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Message: #736
Date: 2021-02-04
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard, dear Chris,

in my last message #732 there fails a construction
... of the point S on Co1,
    if its 5P-s-Tf7(S)-line F.P5 is given.
Let this line intersect the orthogonal hyperbola HY,
... through P4 and P5, centered in the middle,
    axes parallel to those of Co1,
... in a point U, then U.P4 intersects Co1 in S.
Now a focal circular circumcubic of a 5P can be constructed,
... if the focus F on the 5P-quartic is given.

Best regards Eckart
```

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Message: #737
Date: 2021-02-04
From: van10hoven@gmail.com
Subject: Another Moebius Conjugate with center QA-P4.

```
Dear Bernard and Eckart,

A small thing.
Let Qi (i=1,2,3,4) be the Isogonal conjugates of Pi wrt triangle
PjPkPl in a Quadrangle PiPjPkPl.
There is a Moebius Conjugate swapping Pi and Qi (i=1,2,3,4).
Moebius Center is QA-P4 and Moebius Axis is perpendicular to the
Moebius Axis of QA-Tf4.
QA-P2 and QA-P41 are being swapped by this Moebius Conjugate.

Best regards,
Chris
```

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Message: #738
Date: 2021-02-04
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
Cherry on the cake (french expression), the cubics Cu2 and
twinCu2 are similar in a similitude centered in P6 !
Best regards
Bernard
```

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Message: #739
Date: 2021-02-05
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
I've just finished to read completely your messages 732 and 736!
It gives in fact this time a complete construction of a circular
focal circumcubic of a 5P starting with a focus F on the
circumquartic.
Congratulations
I hope Chris will soon start to introduce these curves in EPG !
Best regards
Bernard
```

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Message: #740
Date: 2021-02-05
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
The 5P formed by the TF of the 5P on Cu2 is similar to the 5P of
the Tf8(5P).
Perhaps obvious, the circumconic of the 5P cuts Cu2 in the point
S and the circumconic of the Tf8(P) cuts Tf8(Cu2) in Tf8(S).
The explanation is following : as CSC3 = CSC1*CSC2*CSC1, with
CSC1,2 and 3 centered in P6, CSC1*CSC2 is the product of 2
Moebius transformations with the same center and is a similitude
(rotation aroud P6 and homothety) and CSC3 is CSC1*similitude.
Best regards
Bernard
```

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Message: #741
Date: 2021-02-06
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard, dear Chris,
circular focal circumcubics for a 5P
... are the loci of Tf6-partner on lines through a pivot point.
What about the locus of these pivots?
Best regards Eckart
```

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Message: #742
Date: 2021-02-06
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
Could it be, by any chance, my twincubic of Cu1, with pivot the
infinity point of the Newton Line and focus Tf8(P5), cb
invariant wrt the Tf8(5P) and Tf8(circumquartic Qu1) ?
Best regards
Bernard
```

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Message: #743
Date: 2021-02-06
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Eckart,
If I'm right, we would be able to associate systematically a
copple of circular focal circumcubics of the 5P having as pivot
and focus the Tf8 of focus and pivot of the other ! That's
exactly the case with Cu1 and Cu2 ...
Best regards
Bernard
```

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Message: #744
Date: 2021-02-06
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

complement, you are right in #742,
The locus for the cb-pivots of focal circular circumconics of a
5P is your
twin cubic!

Best regards Eckart
```

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Message: #745
Date: 2021-02-06
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

```
Dear Bernard,

CABRI-constructions confirm your
... "copple of circular focal circumcubics of the 5P
... having as pivot and focus the Tf8 of focus and pivot of the
other."

Best regards Eckart
```

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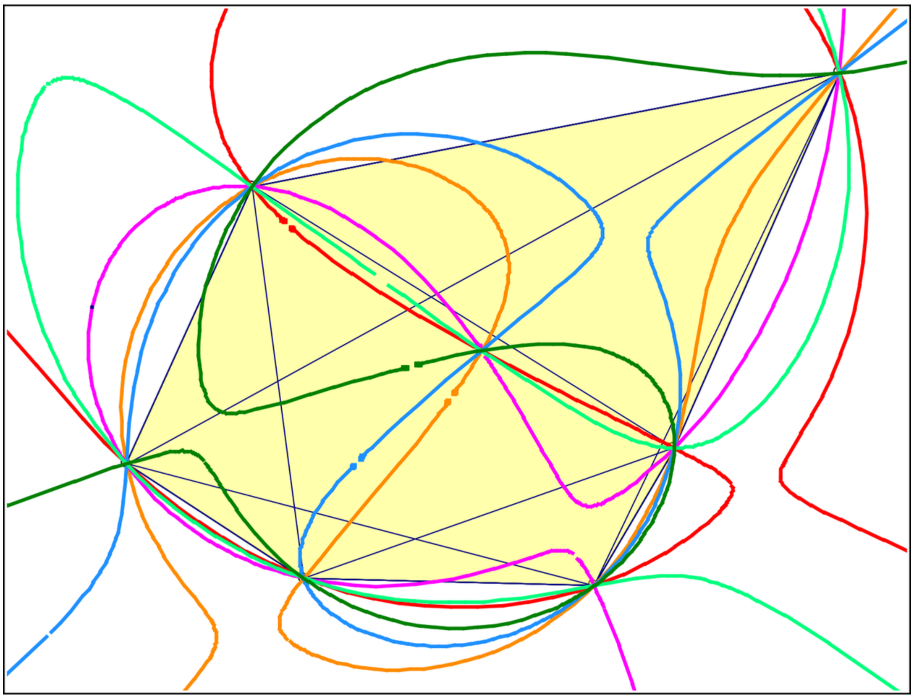
Message: #746
Date: 2021-02-06
From: eckart_schmidt@t-online.de
Subject: New 6P-point?

```
Dear Bernard, dear Chris,

let us omit the focal property and consider
... a Tf6-pivotal circular circumcubic of a 5P as 7P-s-Cu1
... of 5P plus a pivot P
    plus intersection of P.5P-s-P4 and 5P-s-Co1:
These cubics for a 6P (see attached file)
... wrt pivot in a vertex Pi and a 5P in the remaining vertices
... have a common point, perhaps new 6P-s-Px.
... which has the property,
    that this point plus any 5P of the 6P
... have the same 6P-s-P1/6P-s-Ci1 as the reference 6P.

Best regards Eckart
```



2021-02-06.pdf

Message: #747
Date: 2021-02-06
From: bernard.keizer@gmail.com
Subject: Re: New 6P-point?

```
Dear Eckart,
As your 6 cubics are circular circumscribed cubics of the 6P, it
seems you describe the point 6P-s-P2 !
Best regards
Bernard
```

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Message: #748
Date: 2021-02-06
From: eckart_schmidt@t-online.de
Subject: Re: New 6P-point?

```
Dear Bernard,
you are right, it's evident, it was an overhasty message.
Best regards Eckart
```

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Message: #749
Date: 2021-02-07
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

Dear Bernard, dear Eckart,

I redesigned 7P-s-P1 and 7P-s-Cu1 in EPG.
If you have comments or additions please let me know.

I got some questions about 7P-s-Cu1:
1 .Why the notion of a focal circular cubic? Isn't a circular cubic always focal?
2. A circular cubic has ALWAYS an asymptote ?
3. How to construct the asymptote ?
4. How to construct point Q, the intersection point of 7P-s-Cu1 and its asymptote ?
5. Do you agree that the tangential of a Circular Cubic is a 6P-transformation, as well as the tangential of a regular cubic is a 8P-transformation? Is there a construction?

Best regards,
Chris

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Message: #750
Date: 2021-02-07
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

```
Dear Chris,

#724, #732, #736 give answers to some of your questions:
Wrt 1.  7P-s-Cu1 (as in #746) is circular, but not necessary
focal,
... you can test it with the focal property,
... that lines through the asymptote intersection Q
... intersect the cubic in two further points
... with bisector through a fixed point (focus) on the cubic.
For a 5P a focal circular circumscribed cubic
... has its focus on the 5P-quartic (see #724).
Wrt 3. and 4. The construction of the asymptote
... and its intersection Q with the cubic
... you find in #732 with #736.

Best regards Eckart
```

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Message: #751
Date: 2021-02-08
From: van10hoven@gmail.com
Subject: Re: 7P-s-Cu1

Dear Eckart,

Thanks for your explanation of the focal property.
It's only hard to test when I cannot construct asymptote and Q.
Your explanation of this asymptote and Q however are for a
5P-circumcubic and not for a general 7P-circular cubic.
Hope you can help me with a general description.
It will be helpful if you can describe it in such a way so that
I can copy it in the description of 7P-s-Cu1.

Best regards,
Chris

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Message: #752
Date: 2021-02-09
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
With the previous notations,
Tf8(Cu1) is the twin quartic of the Tf8(5P).
Tf8(Cu2) is a cubic of the Tf8(5P), similar to Cu2. Can you
confirm this property ?
Any focal circular circumcubic of the 5P Cu with pivot on the
twin cubic and focus on the 5P quartic has an associate with
pivot as Tf8(focus) and focus = Tf8(pivot).
Both cubics have as Tf8 focal circular circumcubics of the
Tf8(5P).
The only exception seems to be Cu1, which has a quartic of the
Tf8(5P). Can you confirm this property ?
Best regards
Bernard

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Message: #753
Date: 2021-02-10
From: eckart_schmidt@t-online.de
Subject: Re: Another focal circular 5P-circumcubic

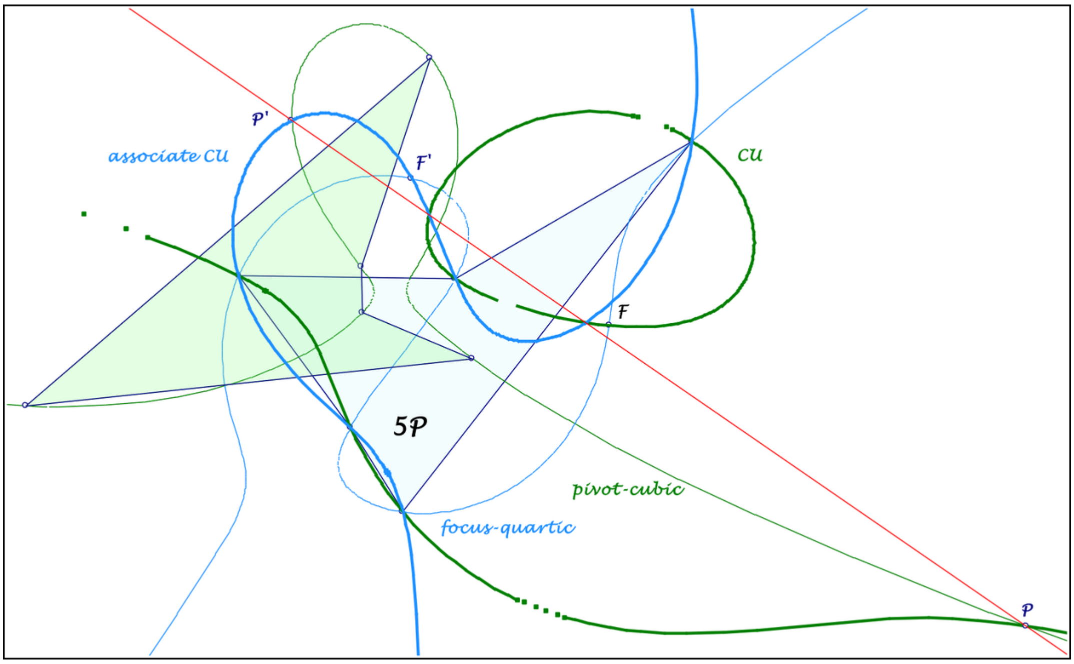
Dear Bernard,

excuse, but I am unable to handle your "twin" nomination,
... please give me an exact definition.
I think, your first property has to be: "Tf8(Cu1) is the quartic of the Tf8(5P)".

Your 2nd property: "Tf8(Cu2) is a cubic of the Tf8(5P), similar to Cu2."
... will be correct, for me very unexpected.

I think, your 3rd property had to be:
... "Any focal circular circumcubic of the 5P ...
... has an associate with pivot as Tf8(focus) and focus = Tf8(pivot).
... Both cubics have as Tf8 circumquartics of the Tf8(5P)."
Your last property then will be no exception.
A further observation (see attached file):
The both cubics have two intersections beside the 5P-vertices (not always real),
... which are Tf6-partner collinear with the two pivots.

Best regards Eckart



2021-02-10.pdf

Message: #754
Date: 2021-02-10
From: bernard.keizer@gmail.com
Subject: Re: Another focal circular 5P-circumcubic

Dear Eckart,
Thanks a lot for answering !
Property 1
By twin, I mean only the same definition wrt the Tf8(5P).
So there are twin 5P, twin conics, twin cubics and twin quartics ...
But you are wright, Tf8(Cu1) is the quartic of the Tf8(5P) (I named it twin quartic)
The same way, the locus of the pivots in the cubic of the Tf8(5P) (I named it the twin cubic)
The locus of the foci is the quartic of the 5P, which is Tf8(twin cubic)
Property 2
Tf8(Cu2) is Cu2 of the 5P (or twin Cu2), similar to Cu2
Very unexpected, indeed, but I tried to explain this in my message 740 (the focus P6 is the same for both cubics and the center of Tf8 ...)
Property 3
You are completely right, both cubics have as Tf8 circumquartics of the Tf8(5P).
In fact, the only exception is precisely Cu2 !
Best regards
Bernard
PS Thanks for your beautiful figures, I've printed the whole collection of these focal circular circumcubics of the 5P ...

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Message: #755
Date: 2021-02-11
From: bernard.keizer@gmail.com
Subject: Re: QL's triangle with the same psi

Dear Chris, dear Eckart
Not a single sign of interest from both of you for this item !
Could please just tell me if it is really not interesting, or if it is perhaps not clear enough ?
Many thanks in advance
Best regards
Bernard

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Message: #756
Date: 2021-02-11
From: van10hoven@gmail.com
Subject: Re: QL's triangle with the same psi

Dear Bernard,

Unfortunately I didn't find time to deepen myself in the Psi-subject.
I am still researching your message #611 so that I can record parts of it in EQF and EPG. And this takes a lot of study and research and implementation time.
Alas I don't have enough spare time to deepen myself in each topic that comes along even how interesting it often is. It certainly is not disinterest for the beautiful subjects you and Eckart often arise. I wish I had more time. Even my own research is disrupted and I don't like that at all.
When I myself ask a question for details, it's because I hope that you or Eckart or someone else will give some text that can be used for EQF/EPG. Unfortunately that is not always the case.
Nevertheless I am very glad with our QPG team, because of the beautiful items and sharing it together. So please forgive me not reacting always. I hope you understand.

Best regards,

Chris

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Message: #757
Date: 2021-02-11
From: eckart_schmidt@t-online.de
Subject: 7P-s-P1

```
Dear Chris, dear Bernard,

there is a very simple construction for the focus of a circular
cubic,
... starting with 7 points,
    interpreted as 5P plus two points X, Y:
The focus 7P-s-P1 is the 2nd intersection beside 5P-s-P5
... of the 5P-s-Tf7-circles of X and Y.

Best regards Eckart
```

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Message: #758
Date: 2021-02-11
From: van10hoven@gmail.com
Subject: Re: 7P-s-P1

```
Dear Eckart,

Very nice construction!
It inspired me to simplify it a little bit.
1. Given 7 random points P1,P2,P3,P4,P5,P6,P7.
2. Denote Xij = 5P-s-P5 wrt 5-Point P1P2P3PiPj.
3. Let O6 be the circumcenter of X45.X46.X56.
4. Let O7 be the circumcenter of X45.X47.X57.
5. Now 7P-s-P1 is the reflection of X45 about O6.O7.

Best regards,
Chris
```

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Message: #759
Date: 2021-02-13
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-P1

```
Dear Eckart,
Very beautiful !
7 points define a unique circular cubic with focus 7P-s-P1.
I suppose it is correct that 7P-s-P1 is 6P-s-P1 of any of the 7
points wrt the 6 others.
Now it is also the 2nd intersection beside 5P-s-P5 of the
5P-s-Tf7 circles of 2 of the 7 points wrt the 5 others ...
The whole construction will give plenty of different circles (42
?) with a common point !
Good job, indeed
Best regards
Bernard
```

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Message: #760
Date: 2021-02-13
From: bernard.keizer@gmail.com
Subject: Re: 7P-s-P1

```
Dear Eckart,
Sorry, 7P-s-P1 is the common point of the 6 versions of
6P-s-Ci1.
It's also interesting to notice that the construction doesn't
work if the 7 points are cb partners ; in this case, each point
is 6P-s-P2 of the 6 others and all the circles are the same.
This circle is then the locus of the foci of the circular
circumcubics ?
Best regards
Bernard
```

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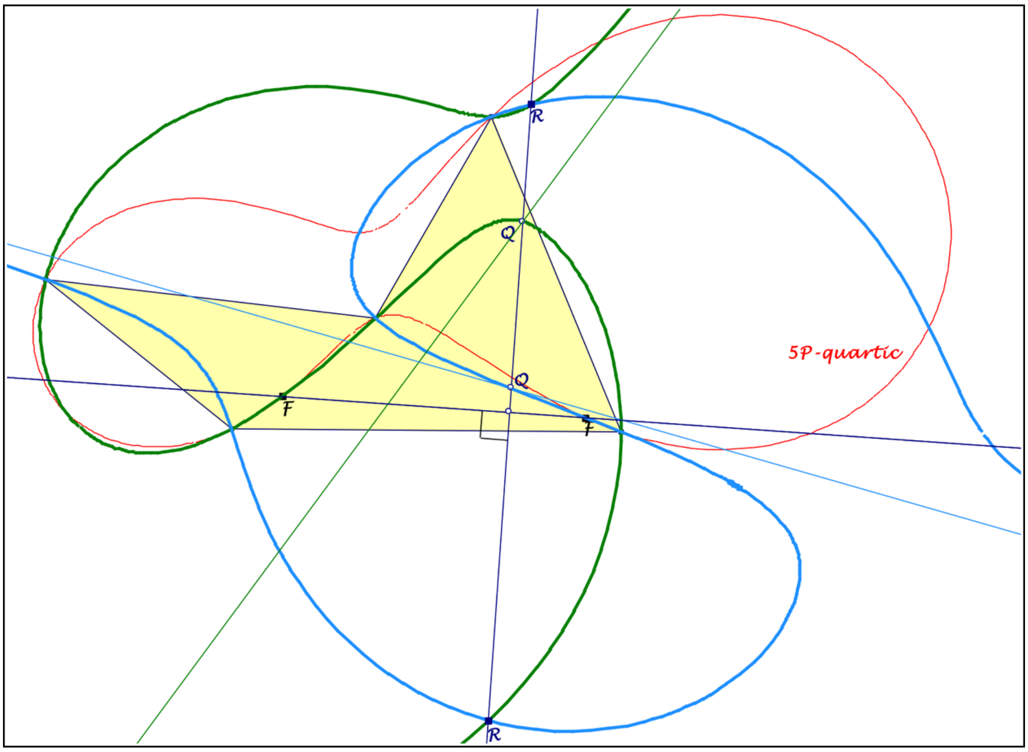
Message: #761
Date: 2021-02-13
From: eckart_schmidt@t-online.de
Subject: Fixed points of 5P-s-Tf6

```
Dear Bernard, dear Chris,

if we ask in general for the fixed points of 5P-s-Tf6,
... the locus will be a curve of higher degree (6?),
... circumscribed the 5P with double points in the vertices.

If we search Tf6-fixed points on the 5P-quartic,
... there are interesting observations wrt their geometry,
... leading to new 5P-points, -lines, -cubics.
I cannot construct these Tf6-fixed points,
... so the results are observations of approximations!

The 5P-quartic can bear two or four fixed points F,
... which can be used as foci of focal circumscribed cubics.
If there are two fixed points,
... we get two cubics CU with two intersections R,
... which are Tf6-partner with bisector through the foci F,
... so the 3rd intersection of the cubics and RR
... will give the asymptote intersections Q of the cubics.
If there are four fixed points F, Fa, Fb, Fc on the 5P-quartic
... three - Fa, Fb, Fc - are collinear on a line L,
... with 4th intersection of L and the 5P-quartic in P5,
... so the cubic CU with focus F is special,
... intersecting the other three in Tf6-partner Ra, Rb, Rc
... with partner-bisectors intersecting in F
... and common partner lines intersection R on CU,
... tangential of F and Tf6-pivot wrt CU.

The asymptote is a perpendicular to L through R.
So it seems, that the special Cu is a focal circular
circumscribed cubic,
... for which the Tf6-pivot and the asymptote intersection
    coincide.

It would be good, if someone could confirm these observations,
... several questions remain open.

Best regards Eckart

PS: On a monopartite focal circular circumscribed cubic of a 5P
... there are two Tf6-fixed points, concyclic with focus and
    Tf6-pivot.
```



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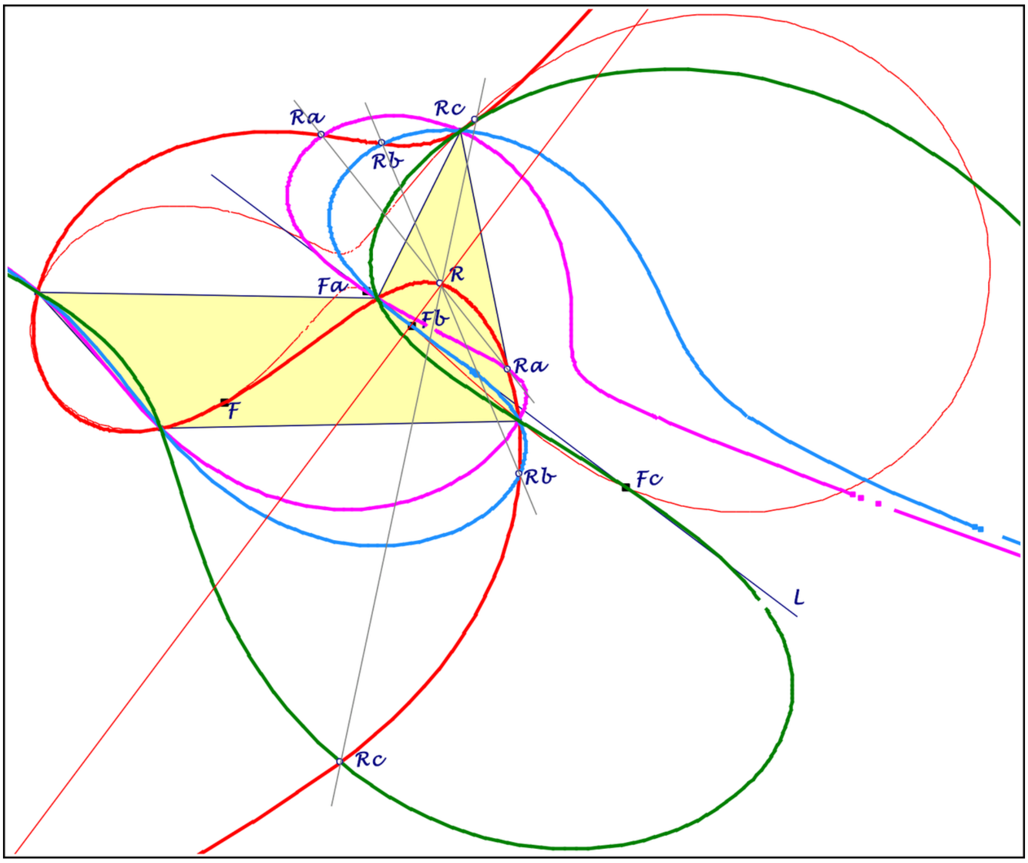

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Message: #762
Date: 2021-02-13
From: eckart_schmidt@t-online.de
Subject: Re: 7P-s-Cu1

Dear Bernard,

CABRI-constructions confirm your interesting remark in #760.

Best regards Eckart

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Message: #763
Date: 2021-02-14
From: van10hoven@gmail.com
Subject: New items in EPG

Dear Eckart and Bernard,

I intent to add these items to EPG, but without specific information I can't describe it. If you have some more information please let me know. Please give simple and specific information that I can copy. I noticed in contact with others that what is simple for us is complicated for someone new in the matter.

```
*  5P-s-Co2 Common Polar Conic of 5P-Pivotal Isocubics
*  6P-s-Tf3 P-Tangential of the Circular Cubic
*  7P-s-L1  Asymptote of the Circular Cubic
*  7P-s-P4  Intersection point of Circular Cubic and Asymptote
*  8P-s-Tf2 P-Tangential of a Cubic
* 11P-s-3P1 CB-points of a Circular Quartic
* 11P-s-Tf1 12th point Tangent at Circular Quartic
* 12P-s-Qu1 12P-Circular Quartic
* 13P-s-3P1 CB-points of a Quartic
* 13P-s-Tf1 14th point Tangent at a Quartic
```

Best regards,
Chris

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Message: #764
Date: 2021-02-14
From: eckart_schmidt@t-online.de
Subject: Re: New items in EPG

```
Dear Chris,

reading your list of new items, I missed mainly:
... the 5P-quartic,
    locus of the foci of focal circular 5P-circumscribed cubics,
... the orthogonal hyperbola through 5P-s-P4, 5P-s-P5,
    centered in the middle,
... ... with axes parallel to those of 5P-s-Co1.
Doing 5P-geometry, these items are relevant,
... first mentioned in July 2019, recently used in #736.
A question wrt 5P-s-Co2: 5P-pivotal isocubics for which
transformation?

Best regards Eckart
```

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Message: #765
Date: 2021-02-14
From: van10hoven@gmail.com
Subject: Re: New items in EPG

```
Dear Eckart,

About 5P-s-Co2 I meant the conic mentioned in QFG#3517.
Maybe my wording isn't right.
Please send me detailed information about all items you
mentioned.
I need:
* a plain general description,
* a picture,
* a construction method,
* if possible coordinates,
* relationship to lower level figures
  (triangles, quadri-figures)
* and enumerated special properties belonging to the item.

Best regards,
Chris
```

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Message: #766
Date: 2021-02-14
From: eckart_schmidt@t-online.de
Subject: Re: New items in EPG

Dear Chris,

if I am not wrong, in #3517 is described the 5P-circumconic 5P-s-Co1.

Best regards Eckart

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Message: #767
Date: 2021-02-14
From: van10hoven@gmail.com
Subject: Re: New items in EPG

Dear Eckart,

5P-s-Co1 is the 5P-Circumscribed Conic.
The conic described in QFG#3517 is the common unique diagonal polar conic for 5 pivotal isocubics having 4 of the 5 points as fixed points of the conjugation and the 5th point as pivot. And this is not the 5P-Circumscribed Conic.

Actually I have not worked much with polar conics, and do not understand QFG#3517 very well. Once Bernard or you asked for attention for this conic. So I am hoping for a better description.

Best regards,

Chris

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Message: #768
Date: 2021-02-14
From: eckart_schmidt@t-online.de
Subject: Re: New items in EPG

```
Dear Chris,

 I have also not worked much with polar conics,
... but I have proved the property,
... that the anticevians of the 5P-vertices wrt the diagonal
    triangle of the remaining QA lie on the conic,
... this can only be right for the circumscribed conic 5P-s-Co1,
    isn' it?

Best regards Eckart
```

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Message: #769
Date: 2021-02-14
From: van10hoven@gmail.com
Subject: Re: New items in EPG

```
Dear Eckart,

QPG#768, your remark “... that the anticevians of the
5P-vertices wrt the diagonal triangle of the remaining QA lie on
the conic, ... this can only be right for the circumscribed
conic 5P-s-Co1, isn' it?”
You are right.
That makes Bernard’s property a property of 5P-s-Co1.
I will include it at the properties at 5P-s-Co1.
Thanks for observation.

Best regards,
Chris
```

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Message: #770
Date: 2021-02-15
From: bernard.keizer@gmail.com
Subject: Re: New items in EPG

```
Dear Chris,
Beautiful work in perspective !
But like Eckart, I miss badly the whole construction explained
in 650 with twin conics, cubics and quartics as well as sextix
...
Best regards
Bernard

PS There was a mistake in points 10 and 11, I've corrected it a
little bit later
```

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Message: #771
Date: 2021-02-15
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of 5P-s-Tf6

Dear Eckart,
1) With 2 points, the 2 asymptotes are the parallels through P4 to the asymptotes of the circumconic (which must be an hyperbola)
2) With 4 points, the foci Fa, Fb and Fc are on a line through P5.
The pivots Pa, Pb and Pc are on a line through P4 and the asymptote of the 3 cubics is the same.
The 2 asymptotes are like in 1) the parallels through P4 to the asymptotes of the circumconic.
Explanation :
the 5P quartic is invariant in a CSC Tfx centered in Tf8(P4) swapping P5 and P6
It's Tf8 is the twin cubic (cubic of the Tf8(5P))
Any focal circular circumcubic of the 5P has pivot P on the twin cubic and focus F on the quartic and *F = Tfx(Tf8(P))
* PP4 cuts the conic in a 2nd point S on the cubic
Tf7(S) is the line P5F
tg(F) is a point Q where the cubic cuts it's asymptote (parallel through Q to PP4)
the cubic is invariant in a CSC centered in F and swapping cbF and S
*Now, if you look for points F such as cbF = F, you must have P = Q and S an infinity point on the circumconic and on PP4
* There are 2 lines Tf7(S), in the case with 4 points, one is P5F, the other in the line L, carrying the 3 foci Fa, Fb and Fc and P5
Tfx(L) is a circle through Tfx(Fa), Tfx(Fb), Tfx(Fc), P6 and Tf8(P4)
*Tf8(Tfx(L)) is a line through Pa, Pb, Pc and P4 !
* It took me time to understand all this, but it gave me the opportunity to revise all properties of the focal circular circumcubics of 5P
Your figures helped me greatly, as usual *
* Best regards
Bernard
PS Could you please make me the pleasure to answer my message 755, I hardly understand your disinterest

---

Message: #772
Date: 2021-02-15
From: eckart_schmidt@t-online.de
Subject: Focus and 5P-s-Tf6-pivot of 7P-s-Cu1 for a 5P

```
Dear Bernard, dear Chris,

7P-s-Cu1 for a 5P plus two further points P6, P7
... gives a circular circumcubic for the 5P,
    which is a Tf6-pivotal curve.

Let us start with any point F,
... constructing the line F.P5,
... the 2nd intersection U of F.P5
    with the orthogonal hyperbola HY
... HY through P4, P5, centered in the middle with axes parallel
    to those of Co1,
... the line U.P4, intersecting Co1 in S,
    intersecting F.Tf6(F) in P.

If we take two of the points F, P, S for P5, P6
... we get the same 7P-s-Co1 with Tf6-pivot P,
... if F on the 5P-quartic,
    the circular cubic is focal with focus F.

Let us now start with any point P,
... constructing the line P.P4,
... the 2nd intersection S of P.P4 with Co1,
... the 2nd intersection F of the Tf7-circle/line for P and S.

Taking P, S for P6, P7, we get 7P-s-Co1 as circular
5P-circumcubic
... with Tf6-pivot P and focus F, F not necessary on the cubic,
... if P on the cubic Tf8 of the 5P-quartic of Tf8(5P)
    (Bernard's twin-cubic),
... ... we get a focal circular cubic with focus F.

Best regards Eckart
```

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Message: #773
Date: 2021-02-15
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 5P-s-Tf6

```
Dear Bernard,

thanks for varifications and explications of my observations in
#761,
... the replacing of my construction by transformations
    was new for me:
... F = Tfx(Tf8(P)) = csc2(csc1(P)).
I think my next message will also be of interest for you.
Wrt your #755:
At the moment I study pentangles,
... it would take days to become familiar with your #587
... and Bernard Gibert's paper about my cubic QL-Cu2,
... excuse my no answer, perhaps later.

Best regards Eckart
```

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Message: #774
Date: 2021-02-15
From: van10hoven@gmail.com
Subject: Re: New items in EPG

```
Dear Bernard,

When you have specific topics to be included please write
documentary to me in the format I described in my last mail to
Eckart.

I think it is better for sending copy for enriching EQF and EPG
directly to my mail and not to the group. It will be less
interesting for the group. When you reply in the Groups.io page,
then use the button "Private" instead of "Reply to the group".
I think it will all take some time to add all intended items.

Best regards,
Chris
```

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Message: #775
Date: 2021-02-16
From: bernard.keizer@gmail.com
Subject: Re: New items in EPG

```
Dear Chris, dear Eckart,
Wrt your discussion about 5P-s-Co1 as the unique diagonal polar
conic of 5 pivotal isocubics having one of the points as the
pivot and the 4 others as fixed points of the isoconjugation,
please see Bernard Gibert Special Isocubics in the Triangle
Plane page 36/37 (with equation).
Any conic has the property that it contains the vertices of the
anticevian triangle of any of it's points wrt the DT of any of
it's inscribed QA's. We just need to consider the 4 vertices of
the QA as fixed points of an isocojugation and the 5th point as
the pivot of the pivotal isocubic wrt the DT of the QA ...
Best regards
Bernard

PS The polar conic of a point wrt a cubic is the conic through
the contact points of the 6 tangents from the point to the
cubic.
```

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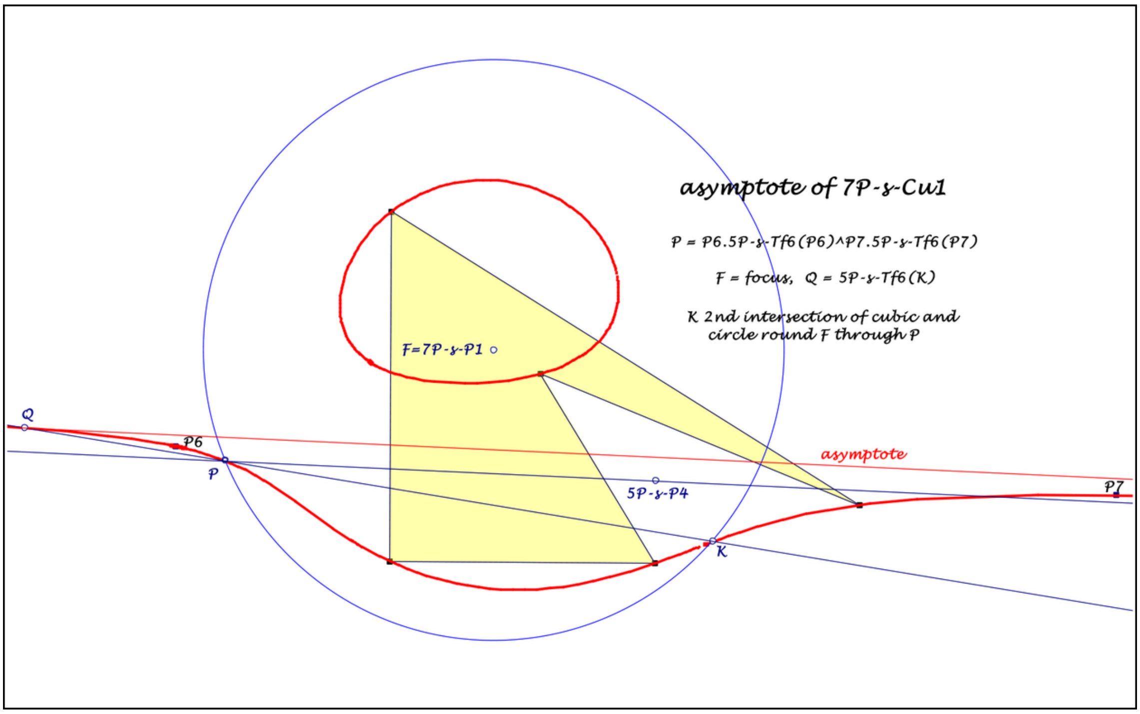
Message: #776
Date: 2021-02-18
From: eckart_schmidt@t-online.de
Subject: Asymptote of 7P-s-Cu1

```
Dear Chris,

in message #763 you asked for
"7P-s-L1 Asymptote of the Circular Cubic
 7P-s-P4 Intersectionpoint of Circular Cubic and  Asymptote",
here a construction:
Starting with 7 points P1, ..., P7
... and its cubic 7P-s-Cu1 with focus F = 7P-s-P1,
... let us take P1, ..., P5 for a 5P
... with 5P-s-Tf6-pivot P = P6.5P-s-Tf6(P6) ^ P7.5P-s-Tf6(P7),
... P.5P-s-P4 is parallel to the asymptote.

Now consider a circle round F through P
... and 2nd intersection K with the cubic,
... then 5P-s-Tf6(K) as 3rd intersection of the cubic and P.K
... is the intersection Q of the cubic and its asymptote.

Best regards Eckart
```



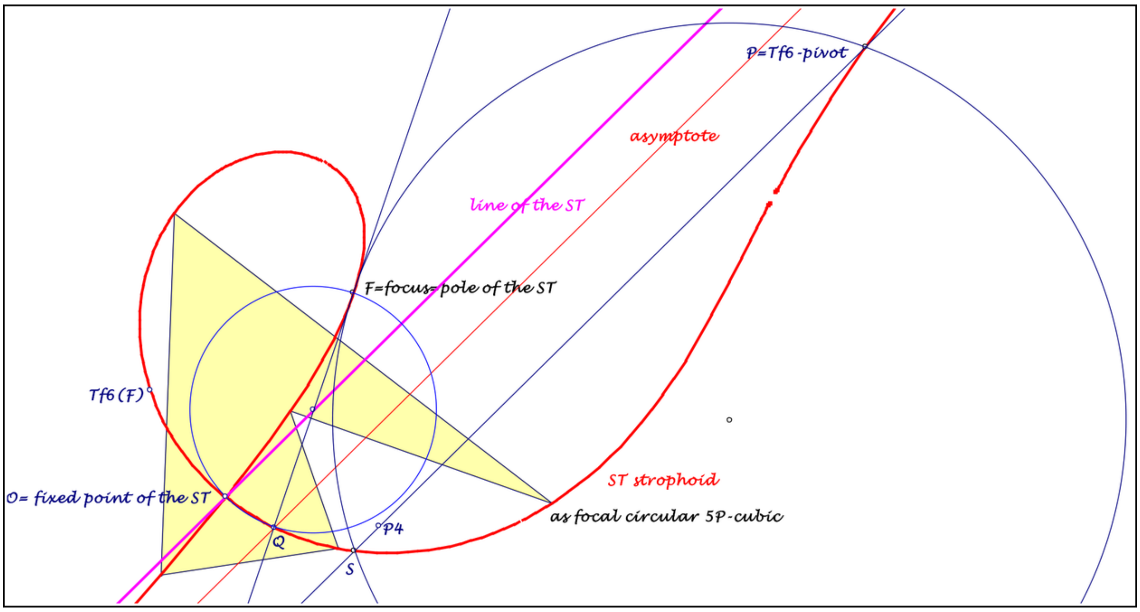
2021-02-17.pdf

Message: #777
Date: 2021-02-19
From: eckart_schmidt@t-online.de
Subject: 5P-strophoid as focal circular 5P-cubic

```
Dear Bernard, dear Chris,

a focal circular 5P-cubic can be mono- or bipartite,
... intermediate it is a strophoid.
There are up to 10 such strophoids for a 5P,
... attached an example,
... approximated with an focus on the 5P-quartic.
What about these 10 foci, not always all real?
To study the attached example,
... further properties are constructed,
... wrt constructions see # 732, #736.

Best regards Eckart
```



2021-02-19.pdf

Message: #778
Date: 2021-02-19
From: bernard.keizer@gmail.com
Subject: Re: 5P-strophoid as focal circular 5P-cubic

```
Dear Eckart,
I reproduced some strophoïds as limit of mono- or bicursal VR's
with pivot on the twin cubic and focus on the quartic, but I
can't find a particular property for the pivots, the foci or the
nodes.
Tf8 of these strophoïds have also a node in Tf8(node of the VR),
but I suppose it is obvious ...
Best regards
Bernard
```

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Message: #779
Date: 2021-02-20
From: van10hoven@gmail.com
Subject: Re: New items in EPG

```
On Mon, Feb 15, 2021 at 10:49 PM, Bernard Keizer wrote:
>
> polar conic of a point wrt a cubic

Dear Bernard,
Is there a construction method of to the polar conic of a point
wrt a cubic?
Best regards,
Chris
```

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Message: #780
Date: 2021-02-20
From: bernard.keizer@gmail.com
Subject: Re: Asymptote of 7P-s-Cu1

```
Dear Chris, dear Eckart
Reading again the list of the new items, it seems we have now
5P-s-Co1 (instead of Co2) and 7P-s-P3 and L1 (thanks to Eckart).
I don't understand 6P-s-P3 and 8P-s-P2 (just draw the cubic and
the tangent and you will get another point on the cubic).
I can't imagine 11P-s-3P1, 11P-s-Tf1 or 12P-s-Qu1, as I don't
know how to draw a circular quartic.
You have already 13P-s3P1.
I don't understand either 13P-s-Tf1 (just draw the quartic and
the tangent and you will get 2 other points on the quartic).
Best regards
Bernard
PS I will start a presentation of QL-Cu2's main pivot triangle
(see message 587), 5P Geometry (see message 650)and 5L Geometry
(see message 230), but it will take time ...
```

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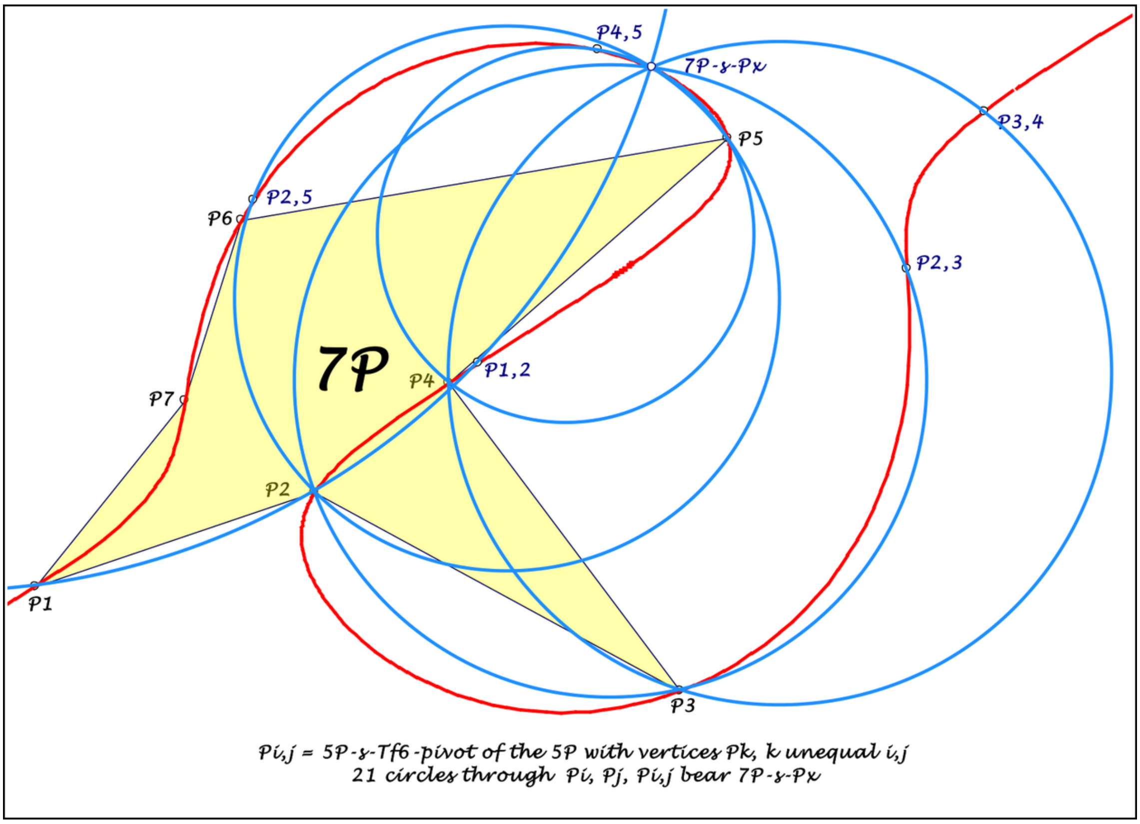
Message: #781
Date: 2021-02-20
From: eckart_schmidt@t-online.de
Subject: New 7P-s-Px?

```
Dear Bernard, dear Chris,

let us consider a 7P and its 7P-s-Cu1 and for any 5P of the 7
points
... the 5P-s-Tf6-pivot Pi,j = Pi.5P-s-Tf6(Pi) ^ Pj.5P-s-Tf6(Pj)
... wrt the remaining two vertices Pi, Pj.
The 21 circles through Pi, Pj, Pi,j have a common point on
7P-s-Cu1.

Best regards Eckart
```



2021-02-20.pdf

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Message: #782
Date: 2021-02-20
From: eckart_schmidt@t-online.de
Subject: New 7P-s-Py

```
Dear Bernard, dear Chris,

perhaps a further new 7P-s-point:
Let us consider a 7P and its 7P-s-Cu1 and for any 5P of the 7
points
... the 5P-s-Tf7-circles for the remaining two vertices,
... which intersect in 5P-s-P5 and another point,
... the same for the 21 possible 5P.

Best regards Eckart
```

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Message: #783
Date: 2021-02-21
From: bernard.keizer@gmail.com
Subject: Re: New 7P-s-Py

```
Dear Eckart,
According to your definition, this point is 7P-s-P1, the focus
of 7P-s-Cu1 (see your message 757).
Best regards
Bernard
```

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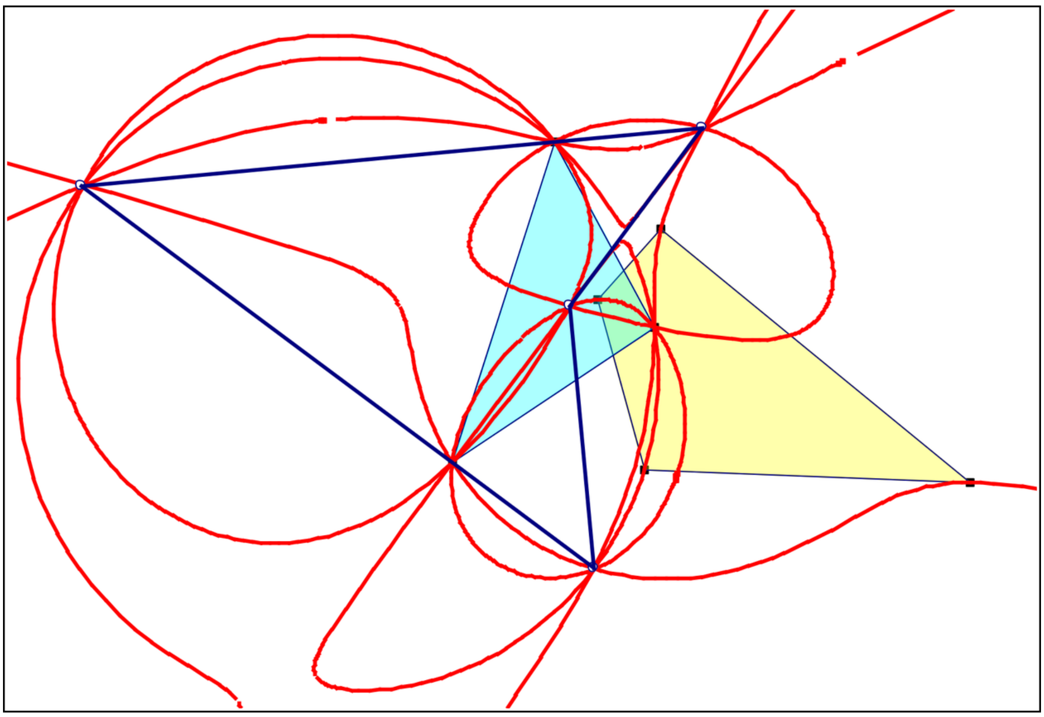
Message: #784
Date: 2021-02-21
From: eckart_schmidt@t-online.de
Subject: ... just for fun ...

```
Dear Bernard, dear Chris,

let us consider a quadrangle P1P2P3P4 and its diagonal triangle
... with the three Möbius transformations,
... ... centered in one vertex of the diagonal triangle,
... ... swapping the other two vertices.
For each vertex Pi of the quadrangle we get three images
Qi1,Qi2,Qi3,
... and for the 7 points P1,P2,P3,P4,Qi1,Qi2,Qi3 a circular
    cubic CUi,
... that means four circular cubics CUi,
... with four triple intersections,
    which give an orthocentric quadrangle,
... whose vertices are the in-/excenters of the diagonal
    triangle above.
For an orthocentric quadrangle the following points are
collinear
... QA-P1,5,10,11,12,13,14,15,18,20,22,24,25,26,32,33,37,38, ...
Excuse my excursus.

Best regards Eckart
```



2021-02-21.pdf

Message: #785
Date: 2021-02-21
From: eckart_schmidt@t-online.de
Subject: Re: New 7P-s-Py

```
Dear Bernard,

you are right, thanks for controlling my observations,
... I confused points in my drawing.

Best regards Eckart
```

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Message: #786
Date: 2021-02-21
From: bernard.keizer@gmail.com
Subject: Re: New 7P-s-Px?

```
Dear Eckart,
I give up !
I spent a few hours to reproduce your property.
It's really interesting.
I hoped at the beginning the point would be the pivot of the
transformation given by the 7 points on 7P-s-Cu1, but I don't
think it is (unless I made a mistake in my construction ...).
So I'm waiting impatiently for another explanation.
Did you try with 9 real points on an ordinary 9P-s-Cu1 ?
That gives 36 pivots for the 7P pivotal transformations. What
can be said about them ?
I've found an old message of yours in olf forum (#3520). Perhaps
you will find it again ...
Best regards
Bernard
```

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Message: #787
Date: 2021-02-21
From: eckart_schmidt@t-online.de
Subject: Re: New 7P-s-Px?

Dear Bernard,

thanks for interest, I made a further interesting observation:
The 3rd intersection of 7P-s-P2.7P-s-Px with the cubic will be
7P-s-Tf1(7P-s-Px) = Q,
... the intersection of the cubic and its asymptote.
You ask "Did you try with 9 real points on an ordinary
9P-s-Cu1 ?"
I couldn't find analog properties.

Best regards Eckart

PS: What about #3520, it's not sent by me?

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Message: #788
Date: 2021-02-21
From: bernard.keizer@gmail.com
Subject: Re: New 7P-s-Px?

Dear Eckart,
Wunderbar !
It's all clear this time.
The focus 7P-s-P1 is on the perpendicular bisector of
7P-s-P2-7P-s-Px ...
I'm rather released, as I feared a mistake in my constructions.
Reading your messages 776 and 781, it gives plenty of
possibilities to draw 7P-s-Cu1 with many real points.
Very beautiful indeed !
Best regards
Bernard
PS I checked again, it was #3420 and 3428, but I didn't reread
the messages ...

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Message: #789
Date: 2021-02-21
From: van10hoven@gmail.com
Subject: Re: New 7P-s-Px?

```
Dear Eckart,

Wonderful constructions in #781 and #787!
Now we (almost) found a wonderful construction of the
intersection point of 7P-s-Cu1 with its asymptote.
The only missing part is now to determine in your construction
of Px in #781 in Cabri which intersection point of the two
Pi-Pj-Pij-circles is Px.
Most of the times the other intersection point is a known point
and then you reflect that point about the connecting line of the
circle centers.
Any more ideas?
Again it shows that working with n-Point items in a systematic
and structured way pays off !

Best regards,
Chris
```

Message: #790
Date: 2021-02-21
From: van10hoven@gmail.com
Subject: Re: Asymptote of 7P-s-Cu1

Dear Bernard,

About your remarks in QPG#780.

[BK] I don't understand 6P-s-P3 and 8P-s-P2 (just draw the cubic and the tangent and you will get another point on the cubic).

Yes I know it is easy. Just like we have the CB-point in a 8P and the CB-Conjugate in a 7P. That’s very easy, but also very useful.

6P-s-P3 and 8P-s-P2 are 6P-/8P-points that exist even without drawing any cubic. They can have other functions and just by naming them we can use them more easily.

[BK] I can't imagine 11P-s-3P1, 11P-s-Tf1 or 12P-s-Qu1, as I don't know how to draw a circular quartic. You have already 13P-s3P1. I don't understand either 13P-s-Tf1 (just draw the quartic and the tangent and you will get 2 other points on the quartic).

I agree. But I love the idea to disclose the realm of 11P-, 12P- and 13P-items. You never know what follows then. Fortunately I found a method to make pictures of them in Mathematica. Only numerical cases.

I really think we are doing groundbreaking and pioneering work in our QPG-Group.

Best regards,
Chris

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Message: #791
Date: 2021-02-21
From: van10hoven@gmail.com
Subject: Re: New 7P-s-Py

```
Dear Eckart,
Beautiful point 7P-s-Py!
In this case we can construct the exact intersection point
7P-s-Py, because the 2 intersection points of the circles are
7P-s-Py and 5P-s-P5 wrt the 5 common reference points used in
the construction.
We can construct 7P-s-Py by reflecting this version of 5P-s-P5
in the connecting line of the circle centers.
Best regards,
Chris
```

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Message: #792
Date: 2021-02-22
From: eckart_schmidt@t-online.de
Subject: Re: New 7P-s-Py

```
Dear Chris,
7P-s-Py is 7P-s-P1, see #783.
Best regards Eckart
```

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Message: #793
Date: 2021-02-22
From: eckart_schmidt@t-online.de
Subject: Re: New 7P-s-Px

```
Dear Chris,
wrt "... which intersection point of the two Pi-Pj-Pij-circles
is Px..."
If we have the circles Pi-Pj-Pij and Pi-Pk-Pik,
... Px is the 2nd intersection beside Pi.
Best regards Eckart
```

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Message: #794
Date: 2021-02-28
From: van10hoven@gmail.com
Subject: Cayley-Bacharach Points using points of tangency

Dear Eckart, dear Bernard, dear friends,

The CB-point of a 9P-cubic is the common point of all cubics with 8 common points.

I wondered what would happen when two of the common points coincide. This means that the cubics are mutually tangent at that point.

Then there is the possibility that cubics are mutually tangent at 4 points. When counting the points of tangency double, we have exactly 8 points, the number needed for constructing a CB-point.

However using the CB theorem, it is not enough to point at it twice, because not only the location of the double point is relevant but also the direction of the tangent at that point. The directions are provided by having 4 reference points (a quadrangle) and a circumscribed reference cubic (like QA-Cu1). With these reference items there is a unique CB-like-point on that cubic. See picture.

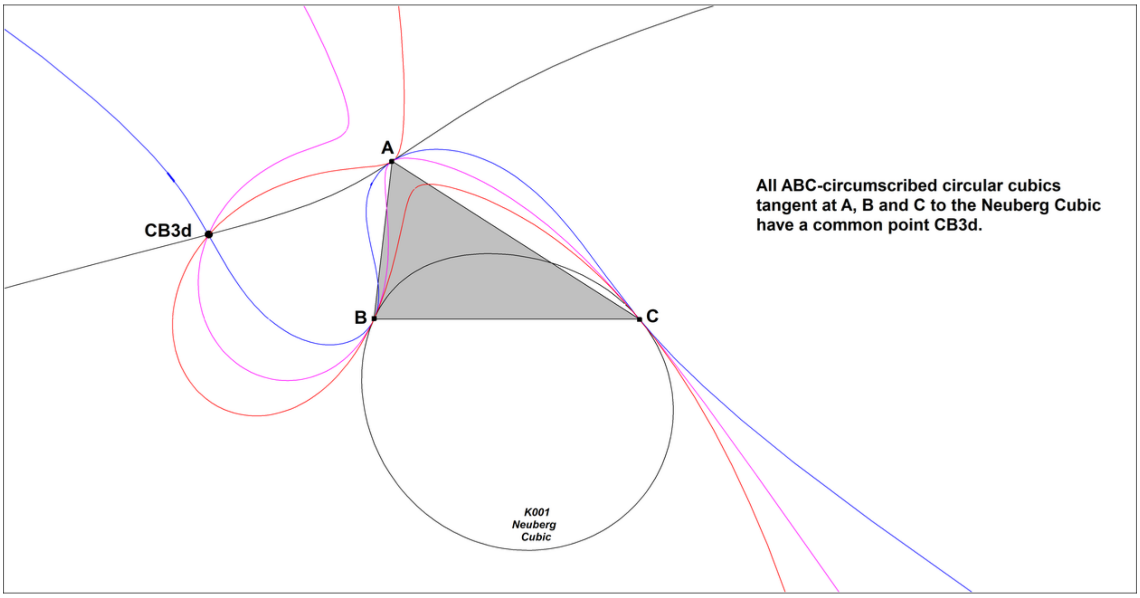
The same is valid for the CB-point of a 7P-cubic.

For example all ABC-circumscribed circular cubics tangent in A, B and C at the Neuberg Cubic (which is a circular cubic) have a common point CB3d. See 2 nd picture.

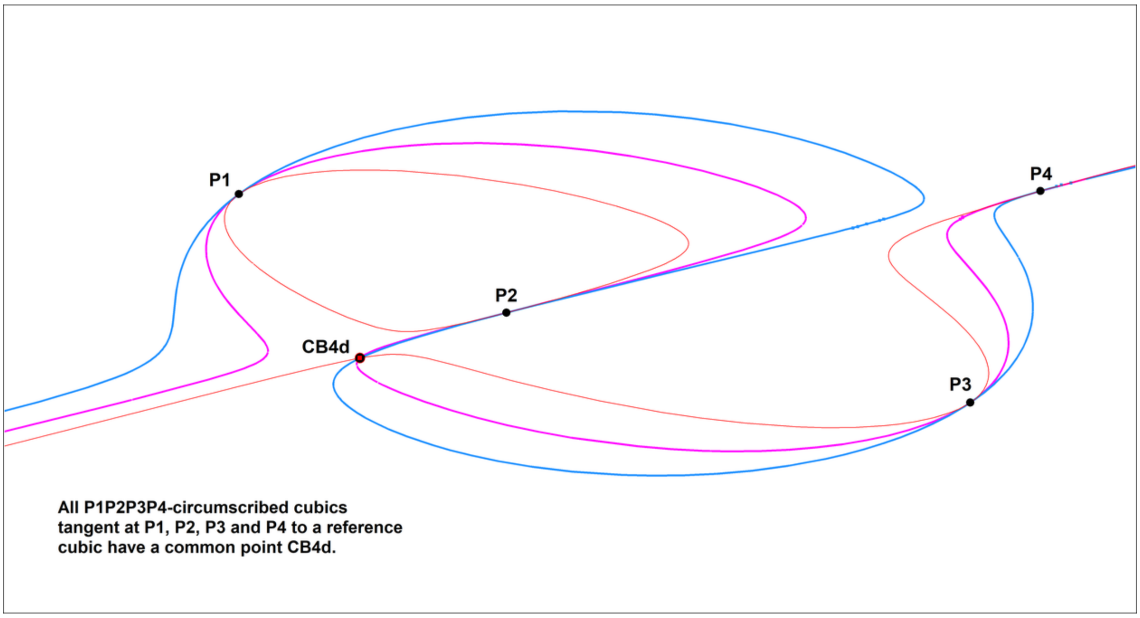
I don’t see an easy way yet to determine these points.

Best regards,

Chris



TR-Cu1-Neuberg Cubic-02.png



8P-s-P1 CB-point of 4 double points.png

Message: #795
Date: 2021-02-28
From: bernard.keizer@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

---

```
Dear Chris,
It seems very interesting ... but I cannot open your files !
They are apparently not recognised by Acrobat Reader.
Are they really pdf files ?
Best regards
Bernard
```

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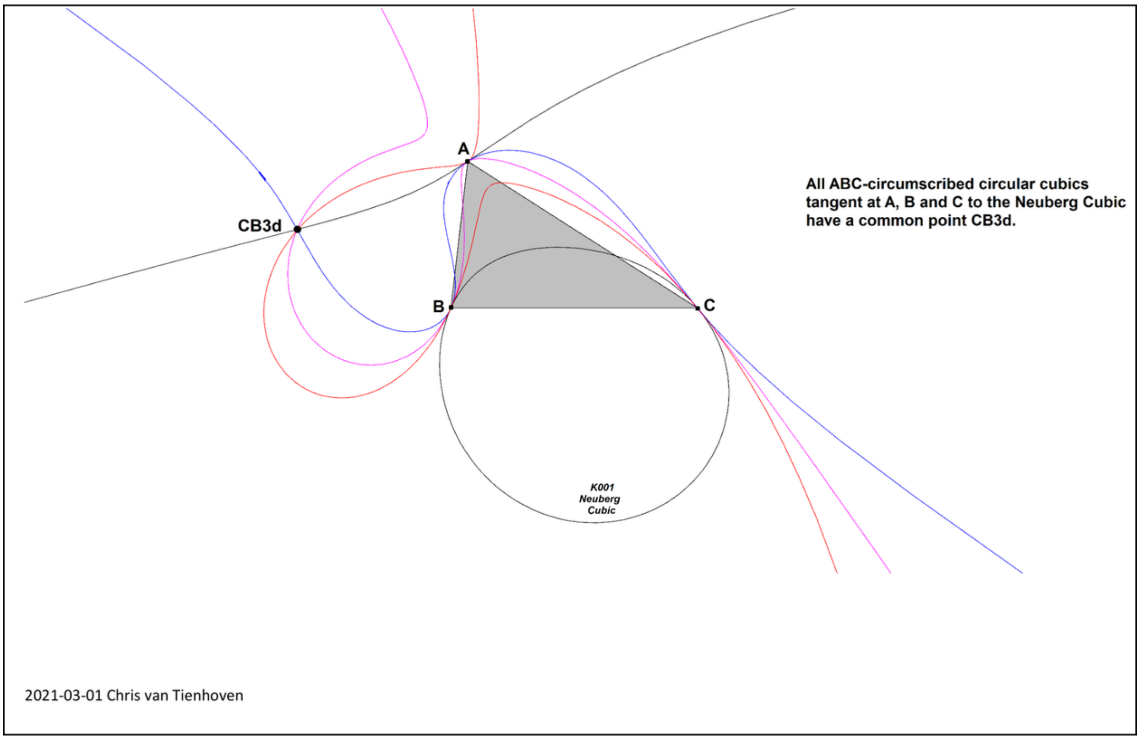
Message: #796
Date: 2021-02-28
From: van10hoven@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

---

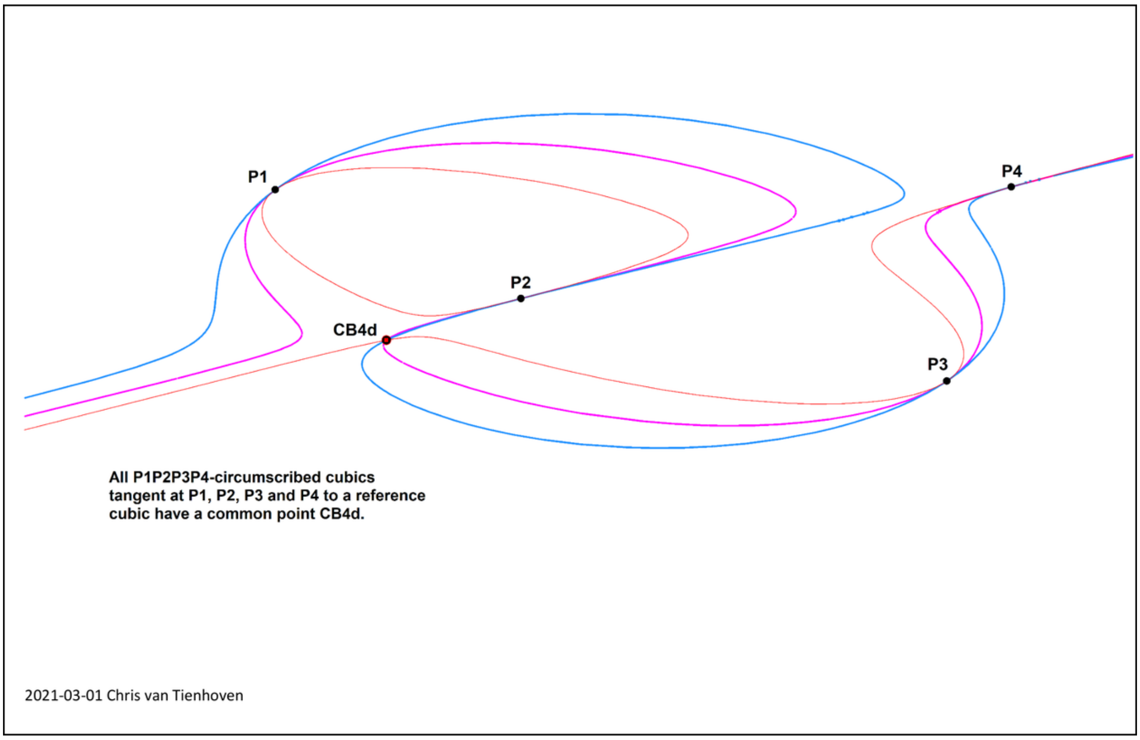
```
Dear Bernard,

Sorry for the inconvenience.
They were Cabri-files.
I think I was tired sending the wrong files.
Attached now one pdf-file with both pictures.

Best regards,
Chris
```



8P-s-P1 CB-point of 4 double points.pdf



8P-s-P1 CB-point of 4 double points.pdf

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Message: #797
Date: 2021-03-01
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

---

```
Dear Chris,

with great interest I have studied your observstions,
especially:
"All ABC-circumscribed circular cubics tangent in A, B and C at
the Neuberg Cubic have a common point CB3d."
The point CB3d on the Neuberg cubic will be
... X2133, isogonal conjugate of X2132,
... with X2132 = X30-ceva conjugate of X74,
... X74 4th intersection of the Neuberg cubic
    and the triangle circumcircle
... as common point of the tangents at the Neuberg cubic
    in the triangle vertices.
Approximately I got the ETC-code for CB3d as +7,854 instead of
+7,855 for X2133.

I hope, you can confirm my result.

Best regards Eckart
```

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Message: #798
Date: 2021-03-01
From: van10hoven@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

---

```
Dear Eckart,
Yes, I can confirm CB3d is X(2133) = Isogonal Conjugate of
X(2132) indeed.
I saw at Bernard Gibert's site that X(74) is the isopivot (or
secondary pivot) and X(2132) could be considered as a tertiary
pivot.
I think it is also interesting to look at the CB4d points of
QA-Cu1, QACu7 and what their special properties will be at these
cubics.
I had no direct results looking at QA-Cu1.
Best regards,
Chris
```

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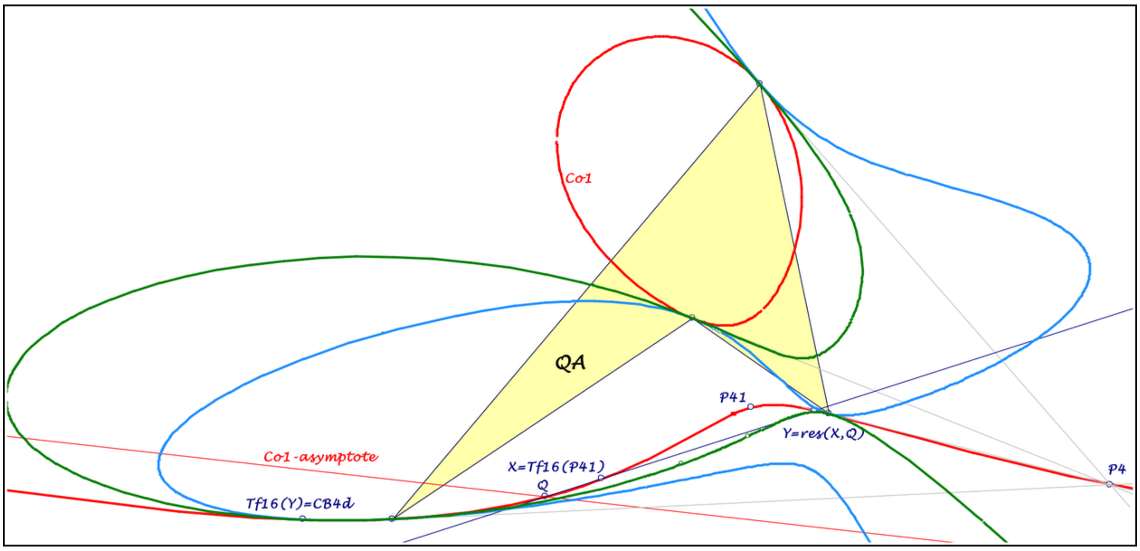
Message: #799
Date: 2021-03-01
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

---

```
Dear Chris,

for a QA and its circumcubic QA-Cu1
... the tangents in the vertices intersect in QA-P4.
You get your point CB4d as follows:
... let X be QA-Tf16(QA-P41),
... let Y be res(X,Q), the 3rd intersection of QA-Cu1 and XQ,
... ... Q intersection of QA-Cu1 and its asymptote,
... then QA-Tf16(Y) = CB4d.

Best regards Eckart
```



2021-03-01.pdf

Message: #800
Date: 2021-03-02
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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Dear Chris,

I just noticed, that CB4d in #799 is the tangential of QA-P41.

Best regards Eckart

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Message: #801
Date: 2021-03-02
From: bernard.keizer@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

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Dear Chris, dear Eckart
I'm bluffed this time, it seems that Cabri has functions which don't exist in Geogebra !
Chris, you ask often for explanations of our constructions, it's your turn !
How do you draw cubics tangent to certain lines ?
How do you calculate the ETC code or coordinates of points on your cubic ?
Last, perhaps a pure geometrical remark :
Using Cotterill's construction, the CB 9th point of 8 points is the residual of the line through the 2 foci of the 2 QA's formed by the 8 points (35 lines).
If the 4 points are the same, there's is only one line, which is the tangent in the focus and the residual is the tangential of the focus on the choosen cubic.
For a QA on QA-Cu1, the focus is QA-P41, tangential of QA-P4 and CB4d would be tgQA-P41 or tgtg(QA-P4).
Is it true ? Is it generalisable that for a pivotal isocubic CB4d is tgtg(pivot) ?
For the Neuberg cubic, the pivot is X30, the infinity point of the Euler Line, the isopivot is X74 = tg(X30) ...
Best regards
Bernard

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Message: #802
Date: 2021-03-02
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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Dear Chris,

if we consider a QA and lines through the vertices,
... intersecting in a point P,
... there is a 9P-s-Cu1 through P,
    tangent to the lines in the vertices,
... and the CB4d of this constellation is the tangential of
QA-Tf2(P).

Best regards Eckart
```

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Message: #803
Date: 2021-03-02
From: bernard.keizer@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

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```
Dear Eckart,
I think you confirm my idea !
QA-Tf2(P) is tg(P) on the cubic and CB4d is tg(tg(P)) ...
I suppose for the Neuberg cubic that we could consider the QA
formed by A,B,C and X30: the pivot is X74 and CB4d is
tg(tg(X74)) on this cubic.
What's the link with CB3d ?
Best regards
Bernard
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Message: #804
Date: 2021-03-02
From: van10hoven@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

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Dear Bernard, dear Eckart,

[BK] How do you draw cubics tangent to certain lines ?
Actually I do not exactly construct the tangent cubic. I cheat a bit. At first I construct a reference cubic Cu_a through 9 points P1a, P2a, …, P9a. Then I choose 4 of the 9 points to be the QA, for example P1a,P2a,P3a,P4a. Then I place 4 other points P1b, P2b, P3b, P4b on the cubic near to the 4 QA-points on the cubic and I chose an external point P9b not on the cubic. I draw the cubic(P1a,P2a,P3a,P4a,P1b,P2b,P3b,P4b,P9b). In the same way I do construct a cubic(P1a,P2a,P3a,P4a,P1b,P2b,P3b,P4b,P9c).
Then I make point P1b glide on the reference cubic to point P1a and make P2b glide on the reference cubic to point P2a and point P3b glide on the reference cubic to point P3a and make P4b glide on the reference cubic to point P4a. Now all of sudden we have 3 cubics (almost) mutually tangent at the points P1a,P2a,P3a,P4a and you will see they coincide in a new point.

[BK] How do you calculate the ETC code or coordinates of points on your cubic ?
Pretty easy. I draw a triangle with sidelengths 6, 9, 13 and do the construction and measure the distance to the 3 rd side. Then I look in the table of Clark Kimberling.
I checked in a QA also for QA-Cu2, QA-Cu3 if CB4b = tgtg(pivot) and it appeared to be true.
Of course no proof yet. But suppose that. . .

What about when we do not know a pivot?
Can we construct it back after constructing CB4b?
Then we first have to have a reference cubic and then chose 4 points P1,P2,P3,P4 on the cubic to represent the QA. Then construct two more cubics tangent at P1,P2,P3,P4 to the reference cubic.
Then draw a tangent from CB4b to the reference cubic. Then draw from the point of tangency another tangent to the cubic. The 2 nd point of tangency should be the pivot.
Same procedure for Circular Cubics around a triangle.

Best regards,

Chris

Message: #805
Date: 2021-03-03
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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Dear Bernard,

CABRI has no special functions wrt this theme.
Wrt your question "How do you draw cubiccs tangent to certain lines ?"
I draw these cubics approximately,
... choosing 2nd points Pi' very very near to the vertices Pi
    on the line through Pi.
Wrt your question "How do you calculate the ETC code ... ?"
I draw a 6-9-13 triangle and construct the constellation keenly,
... then I can measure the necessary distances for the ETC-code,
... often too inexactly to identify precise the point.

Finally a further observation
... wrt a QA and lines through the vertices,
... which are the tangents of a QA-circumscribed conic,
... then the discussed cubics degenerate
... in the QA-circumscribed cubic and lines with a common point.
What about this common point CB4d?

Best regards Eckart

PS. I just see, that Chris has already answered in the same sense.

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Message: #806
Date: 2021-03-03
From: eckart_schmidt@t-online.de
Subject: Re: New items in EPG

```
Dear Chris,

wrt your question for "polar conic" in #779,
... I thought Bernard would answer,
... but here is a reference:
Henry Martyn Cundy and Cyril Frederick Parry:
Some cubic curves associated with a triangle,
Journal of geometry Vol. 53 (1995), 2.15 page 45.

Best regards Eckart
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Message: #807
Date: 2021-03-03
From: van10hoven@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

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```
Dear Eckart,

Intriguing degenerate-point.
I noticed that the point changes when the inscribed QA changes
in a conic, so it is no conical point.
Also when changing the conic around the QA this point also
changes, therefore it is no QA-point.

Best regards,
Chris
```

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Message: #808
Date: 2021-03-03
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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```
Dear Bernard, dear Chris,

let us consider a QA, its QA-Cu1 with tangents in the vertices
through QA-P4,
... then CB3d for cubics through 3 vertices,
    tangent to the tangents above,
... coincide in QA-Tf16(QA-P41).

This property allows the CB3d-construction with CABRI
... for 3 points A, B, C and lines through a fixed point P:
Complete the triangle ABC to a QA by a point D,
... which is the ABC-isogonal conjugate of the inverse of P
    wrt the circumcircle of ABC,
... then P is QA-P4 and CB3d = QA-Tf16(QA-P41).

Best regards Eckart
```

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Message: #809
Date: 2021-03-03
From: van10hoven@gmail.com
Subject: Re: New items in EPG

```
Dear Eckart,

Thank you very much for the reference to the paper of Cundy and
Parry about the Polar Conic.
The mentioned construction using harmonic conjugates seems a
very nice construction for the polar conic to me.
Best regards,

Chris
```

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Message: #810
Date: 2021-03-04
From: bernard.keizer@gmail.com
Subject: Re: New items in EPG

```
Dear Chris, dear Eckart
I didn't know this article, I just notice that Cuppens also uses
some kind of construction with harmonic conjugates ...
Best regards
Bernard
```

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Message: #811
Date: 2021-03-04
From: bernard.keizer@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

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```
Dear Chris, dear Eckart
Many thanks to both of you for your pedagogical answer !
I knew the 6-9-13 triangle, but had never used it until now.
Beautiful construction of CB3d for 3 points and a pencil of
lines through a 4th point.
Now I'm a little confused.
Having 5 points, 4 defining a QA and the 5th the pivot, we get 4
different CB3d with the same pivot and 3/4 points and we have a
CB4d for the 4 points and the same pivot.
But with the same 5 points, we have 5 possible pivots, the 4
others defining a QA, which gives 20 different CB3d and 5
different CB4d.
Is there a link between all these points ?
Best regards
Bernard
```

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Message: #812
Date: 2021-03-04
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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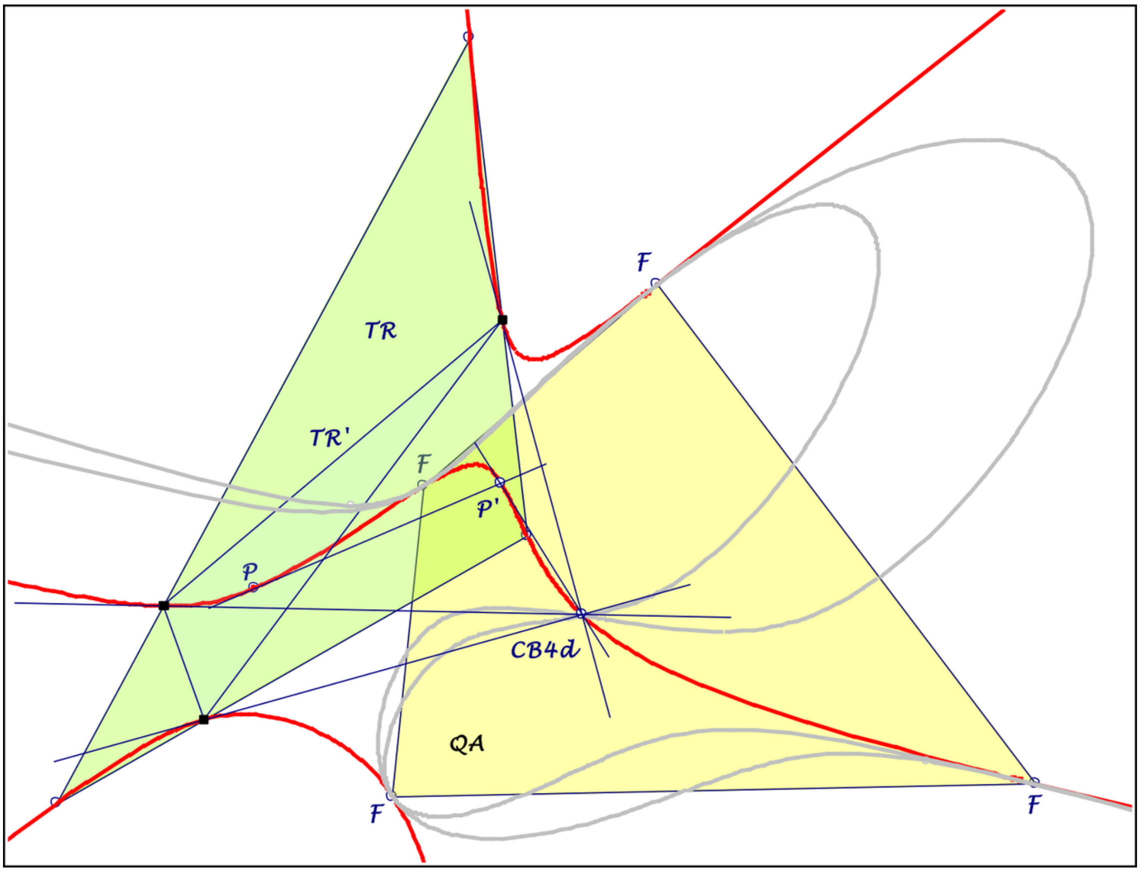
```
Dear Bernard, dear Chris,

just as training wrt this item:
Let us consider a triangle TR and a pivotal isocubic CU
... with pivot P and an isoconjugation with fixed point F
... which can be a vertex of a QA with QA-Tr1 = TR,
... then the tangential of F will be the pivot P
... and CB4d of the QA will be the tangential
    of the tangential of the pivot P,
... this is already discussed, perhaps new:
... CB4d is the tangential of the 3rd intersections
    of CU and TR-sides,
... which give a new triangle TR', Ceva triangle
    of the pivot P wrt TR,
... reference triangle for CU as pivotal isocubic
    with fixed point P
... and pivot P' = QA-Tf2(P), tangential of P,
    contact point of the 4th tangent from CB4d at CU.
Finally CU can be considered as pivotal isocubic  of QA* = TR'
plus P'
... with pivot CB4d and isoconjugation QA*-Tf2 ...

Best regards Eckart
```



2021-03-04.pdf

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Message: #813
Date: 2021-03-05
From: van10hoven@gmail.com
Subject: Re: Cayley-Bacharach Points using points of tangency

---

```
Dear Eckart,

Beautiful analyses Eckart. It helped me to better understand the
pivotal isocubic in a 3P- and 4P-environment.
I think that the rule that CB4d = the tangential of the
tangential of some pivot is only valid for the pivotal isocubic.

Best regards,
Chris
```

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Message: #814
Date: 2021-03-06
From: eckart_schmidt@t-online.de
Subject: Re: Cayley-Bacharach Points using points of tangency

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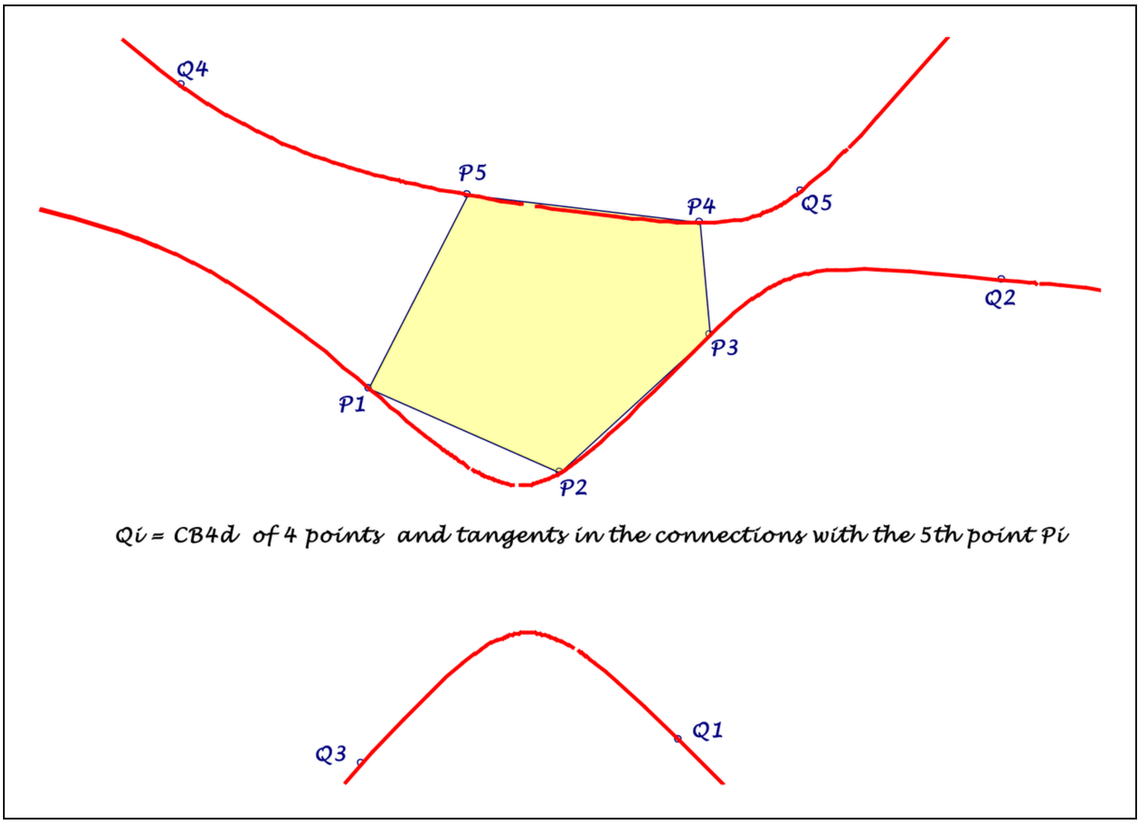
```
Dear Bernard,

wrt your #811:
"... 5 points, we have 5 possible pivots,
     the 4 others defining a QA, which gives
... 5 different CB4d."
The 5 points and the 5 CB4d lie on a cubic.

Best regards Eckart
```



2021-03-06.pdf

Message: #815
Date: 2021-03-07
From: bernard.keizer@gmail.com
Subject: Main pivot triangle of QL-Cu2

```
Dear Chris, dear Eckart, dear Benedetto, dear Bernard G,
This is the 1rst item I promised to Chris.
But, as Max und Moritz would say, Dies war der erste Streich,
doch der nächste kommt sogleich ...
Best regards
Bernard
```

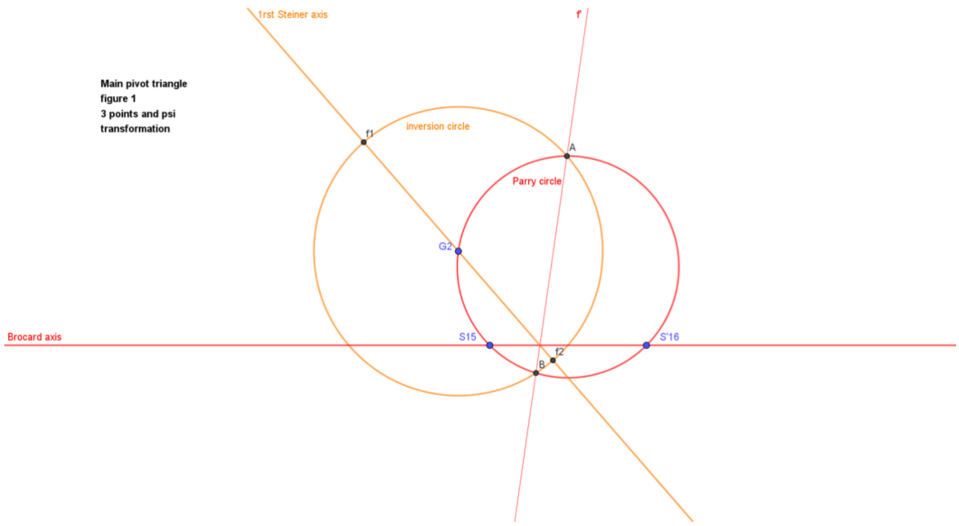
Main pivot triangle of QL-Cu2

A Moebius transformation, named psi by Bernard Gibert (*) (followed by ETC) or mu by Benedetto Scimemi (**) and CSC in EQF associates an axial symmetry and an inversion wrt a circle centered on the axis of symmetry. This transformation can be defined by 3 points, one being the center of the transformation and the 2 others the transformed points. In a triangle, the choosen points will be the centroïd G2 and the isodynamic points S15 and S'16 and the fixed points are the foci of the Steiner inellipse. For a QL, the points will be QL-P1 and the points QL-2P2a and b and the fixed points are QL-2P3a and b. This item shows the links between the triangle and the QL having the same transformation ; it involves the cubics QL-Cu1 and QL-Cu2.

A. 3 points and psi transformation

Having choosen 3 points, one as the center of the transformation G2 and the 2 others as 2 transformed points S15 and S'16, the internal bisector of the angle SGS' is the 1rst Steiner axis, the reflexion of the line SS' in this axis intersects the circle through G, S and S' in 2 points A and B equidistant from G and the inversion circle is the circle with center G through A and B. The 1rst Steiner axis and the inversion circle intersect in 2 points f1 and f2, fixed points of the transformation.

The psi transformation centered in G with fixed points f1 and f2 swaps S and S' and the line through S and S' and the circle through G, S and S'.



(*) Bernard Gibert Orthocorrespondance and Orthopivotal Cubics Forum Geometricorum 2003

(**) Benedetto Scimemi Simple relations regarding the Steiner inellipse of a triangle Forum Geometricorum 2010

B. Triangle and psi transformation

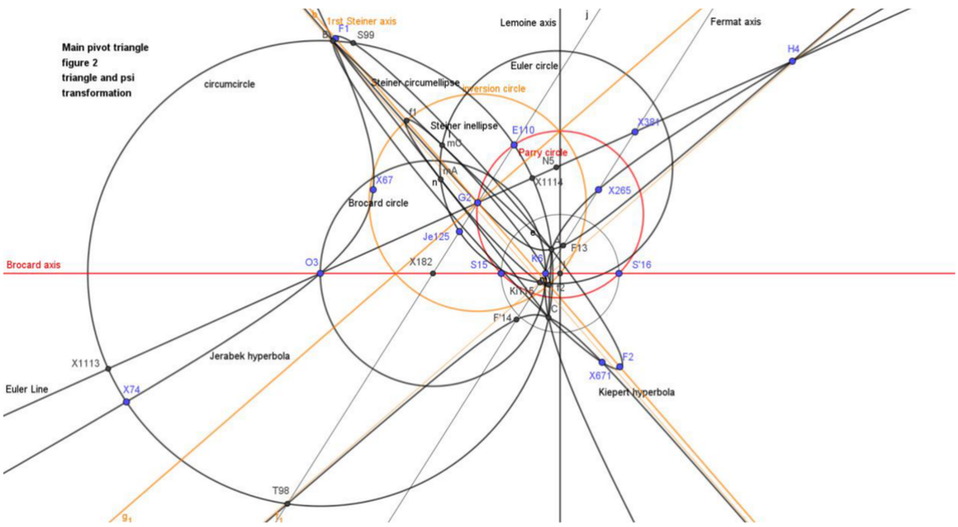
Having the 3 points G2, S15 and S'16, we have immediately the Brocard axis through S and S' and the Parry circle through G, S and S' and we find the foci of the Steiner inellipse f1 and f2 by using the preceding construction ; it follows the foci of the Steiner circumellipse F1 and F2, homothetic of f1 and f2 in the homothety (G2, 2). We have also the Lemoine axis as bisector of SS'.

If you have the QL-Cu2 when QL-Cu1 is monocursal, the Brocard Line cuts the cubic QL-Cu2 in the circumcircle O3. If we have only the 3 points G, S and S', we can use an approximate solution.

For any point O on the Brocard axis, K is the harmonic of O wrt S and S', ψO is the point E110 and ψK the point X111. The points E110 and X111 are the intersections of the circumcircle and the Parry circle. O is in the position of the circumcenter O3 when the psi transform of the circle with diameter OK or Brocard circle is the circumcircle.

It's possible to use other alternative constructions. For example, the Stammler hyperbola of the searched triangle ABC is centered in E110 and passes through the in- and excenters of both triangles ABC and GSS' and through the points O3 and K6. For any point O on the Brocard axis, having the same way K and ψO, O is in the position of O3 when the rectangular hyperbola centered in ψO through the in- and excenters of GSS' passes through O and K.

It follows the Euler Line with the orthocenter H4 and the center of the Euler circle N5. ψH is JE125, the center of the rectangular Jerabek hyperbola through O, K and H and their reflexions in Je125, respectively X265, X67 and X74. The Jerabek hyperbola cuts the circumcircle in X74 and the 3 vertices on the searched triangle ABC.



We may use an alternative construction by drawing the reflexion of the Euler Line in the 1rst Steiner axis ; this line is ψEuler Line through E110, G2, Je125 and X182 (middle of OK and center of the Brocard circle) and cuts the circumcircle in a 2^{nd} point T98, the Tarry point. The diametral point of T98 on the circumcircle is S99, the Steiner point and the ellipse with foci F1 and F2 through the Steiner point is the Steiner circumellipse, which cuts the circumcircle in 3 other points being the vertices of the searched triangle ABC.

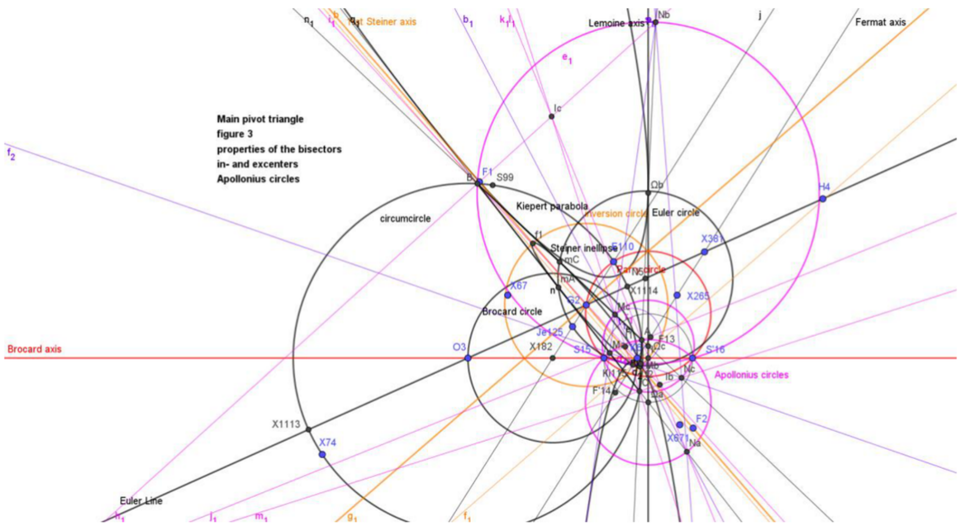
Perhaps a last well known rectangular hyperbola of the triangle ABC : Ki115 is the Kiepert point, middle of H4T98. The Kiepert hyperbola, centered in this point, through the vertices of the triangle ABC, the centroïd G2 and the orthocenter H4, as well as through their reflexions in Ki115, respectively X671 and T98, has it's asymptotes parallel to the Steiner axes and passes through the Fermat points F13 and F'14 and through the Napoleon points Np17 and Np'18, copples of ψpartners.

C. Triangle and properties of the bisectors

Let's draw the 6 bisectors of the triangle ABC :

- they intersect in the in- and excenters of ABC, I, Ia , Ib and Ic
- each of the lines cuts the opposite side of the triangle Ma, Mb and Mc for the internal bisectors, Na, Nb and Nc for the external bisectors ; the middles of MaNa, MbNb and McNc are Ωa, Ωb and Ωc and the circles with diameters MaNa, Mb,Nb and McNc are the Apollonius circles through the opposite vertice of the triangle ABC, which intersect in S15 and S'16. The 6 points Ma, Mb, Mc, Na , Nb and Nc are the vertices of a QL with QL-P1 in Ki115 and QL-P17 in E110, DT is the triangle ABC, the Newton Line is the Lemoine axis, the Steiner Line is the Brocard axis and S15 and S'15 are the Plücker points.

  The CSCdiag centered in E110 swaps the Plücker points S15 and S'16, the Brocard axis and the Parry circle and the circumcircle of the triangle ABC and the Lemoine axis. The parabola with focus Ki 115 and directrix the Brocard axis is QL-Co1, tangent to the 4 lines of this QL and the parabola with focus E110 and directrix the Euler Line is DQL-Co1, tangent to the 3 sides of the triangle ABC, to the Lemoine axis and to the Steiner axes.



The interest of the points I, Ia, Ib and Ic is that they are on the Mac Cay cubic stelloïd.

The interest of the points Ωa, Ωb and ΩC is that they are on the Kjp cubic stelloïd.

D. Properties of the cubic stelloïd, it's hessian and it's cayleyan

1. The stelloïd QL-Cu2

The cubic stelloïd associated to a QL is the curve having as apolar conics the inscribed conics of the QL and as polar conics the rectangular hyperbolas cutting harmonically the sides of the diagonal triangle of the QL and the main axes of all inscribed conics (diagonals of QL's inscribed in the hessian)

It is the locus of the 27 centers of the inscribed cardioïds.

The 3 asymptotes through QL-P1 trisect the angle between the axes of the parabola QL-Co1 and the cardioïd QL-Qu1 ; they cut the curve in 3 points aligned on a parallel to the Newton Line, homothetic in a homothety (QL-P1, 2/3).

This curve has an infinity of pivots triangles, all having QL-2P3a and b as foci of the Steiner inellipse. The main pivot triangle is ABC, which has QL-P1 as centroïd G2, QL-2P3a and b as foci of the Steiner inellipse and QL-2P2a and b as isodynamic points S15 and S'16.

Wrt this main pivot triangle, QL-Cu2 is either a Mac Cay cubic stelloïd, if the Newton Line is the Brocard axis, or a Kjp cubic stelloïd, if the Newton Line is the Lemoine axis.

The Mac Cay cubic stelloïd is K003 in Bernard Gibert's catalogue ; it is pK(K6,O3), an isogonal pK with pivot O3 and isopivot H4, which passes through the in- and excenters of the triangle ABC.

The Kjp cubic stelloïd is K024 in Bernard Gibert's catalogue ; it is nK0+(K6,K6), a non-pivotal isogonal cubic with root in K6, which passes through the centers of the Apollonius circles of the triangle ABC.

2. The hessian QL-Cu1

The hessian of the cubic is the locus of the points for which the rectangular hyperbola is degenerated in 2 orthogonal lines. 2 conjugate points of the hessian are the foci of an inscribed conic and a pair of vertices of an infinity of QL's inscribed in the hessian.

In each point of the hessian pass 2 orthogonal lines, which form the degenerated polar conic of it's conjugate wrt the cubic stelloïd. These 2 lines cut the stelloïd in the contact points of the tangent to the stelloïd from the conjugate point on the hessian.

For example, the Miquel point QL-P1 is the conjugate of the infinity point, the 2 Steiner axes are the degenerated rectangular hyperbola of the infinity point wrt the stelloïd and cut the stelloïd in points where the tangent is parallel to the Newton Line.

The 2 perpendicular lines in each point cut the axis of the parabola (main axis of an inscribed conic) in 2 points , which are harmonic conjugates wrt the Miquel point and the infinity point and therefore symmetric wrt the Miquel point QL-P1, which makes easy to find the second one if we know the first.

Main pivot triangle of QL-Cu2.pdf

The 2 points QL-2P3a and b, intersections of the inversion circle and the 1rst Steiner axis are invariant in the Clawson-Schmidt transformation QL-Tf1, which is the transformation ψ of the main pivot triangle of QL-Cu2.

The hessian of the Mac Cay cubic stelloïd is K048 in Bernard Gibert's catalogue ; it is monocursal and passes through the points X1113 and X1114, on the Euler Line of the triangle ABC.

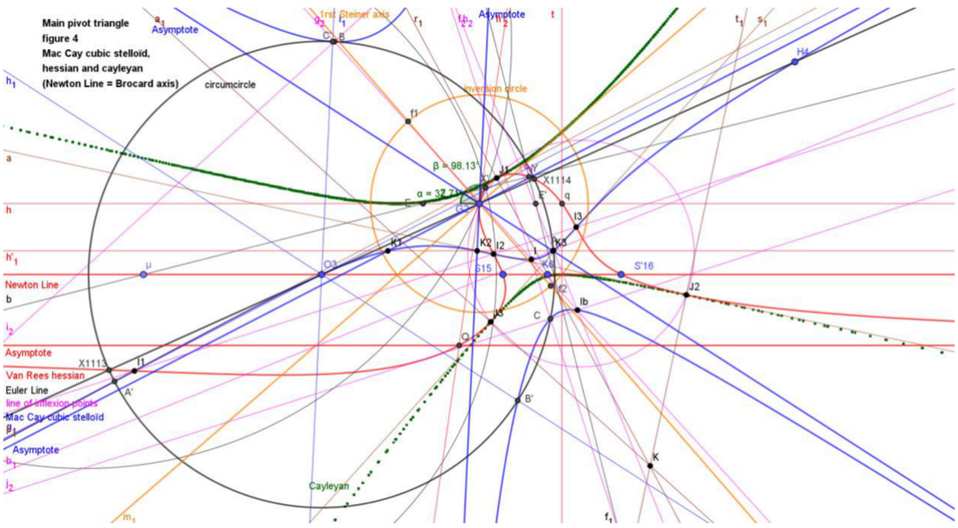
The hessian of the Kjp cubic stelloïd is K193 in Bernard Gibert's catalogue ; it is bicursal.

3. The cayleyan

The cayleyan is the enveloppe of all the lines forming these degenerated rectangular hyperbolas or the enveloppe of the lines through 2 conjugate points (main axis of an inscribed conic). The contact point is the harmonic conjugate of the 3rd intersection point of a line trough 2 conjugates.

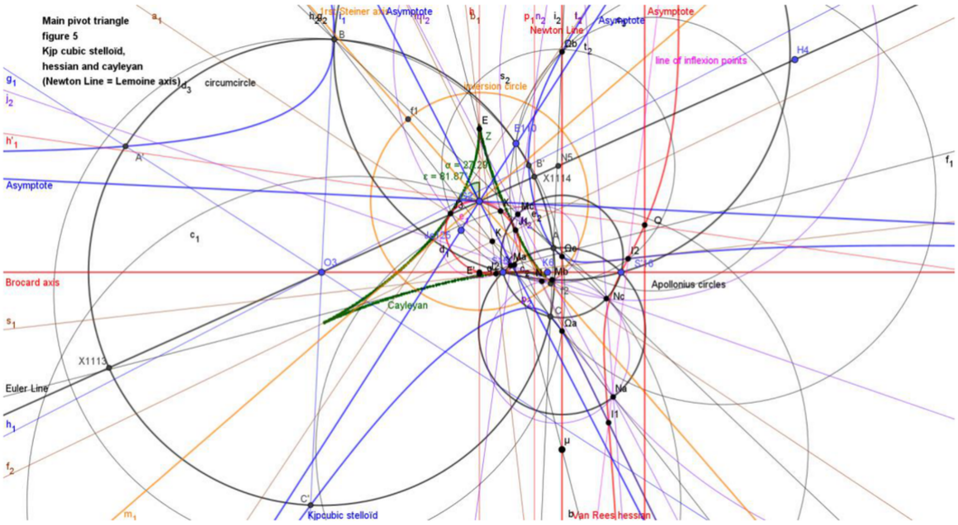
This makes easy to draw the curve : the line joining 2 conjugate points cuts the axis of the parabola in a point ; the perpendicular to this line in the reflection of this point in QL-P1 gives the 3rd point of intersection between the line and the hessian. Both lines form the degenerated hyperbola of the tangential point of the 2 conjugate points, which is the conjugate of the 3rd intersection.

For example, the axis of the parabola is tangent to the cayleyan in a point , which is the harmonic conjugate of the infinity point, the reflection of the 2nd intersection point in QL-P1 and, in the unicursal curve, the Newton Line QL-L1 is tangent to the cayleyan in a point, which is the harmonic conjugate of the infinity point, the middle of QL-2P2a and b.



Last, the cayleyan and the hessian are tangents in 3 points on a circle trough QL-P1 and the conjugates of these 3 points are also their tangentials, the 3 inflexion points and intersection points of the stelloïd and the hessian, which lie on a line conjugate of this circle. The 6 points are therefore the vertices of a QL inscribed in the hessian.

The 3 common tangents to the hessian and the cayleyan pass through the corresponding inflexion points and form the DT of this QL. The normals to the hessian and the cayleyan in the 3 contact points intersect in a point K.



E. Properties of the QL's inscribed in QL-Cu1

The fundamental property of the QL's inscribed in a given curve QL-Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curves QL-Cu1 and Cu2.

If St is the 1rst Steiner axis of the psi transformation, Si are the directions of the sides of the triangle ABC and Di are the directions of the 4 lines, Br and Le the directions of the Brocard and Lemoine axes, we have Σ (St,Si) = (St,Le) = π/2 + (St,Br) and Σ(St,Di) = 2*Σ (St,Di) = 2*(St,Le) = 2*(St,Br).

The sides of the Morley triangles of ABC have as direction wrt St 1/3*Σ(St,Di) m° π/3 and the asymptotes of the Mac Cay cubic are orthogonal to these lines whereas the asymptotes of the Kjp cubic are parallel to these lines ; they are rotated from one to the other in an angle π/6.

The axes of the deltoïds QL-Qu2 of the QL's inscribed in the corresponding hessians are parallel to these asymptotes.

Main pivot triangle of QL-Cu2.pdf

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Message: #816
Date: 2021-03-07
From: van10hoven@gmail.com
Subject: Re: Main pivot triangle of QL-Cu2

```
Dear Bernard,

Thank you very much for your extensive contribution.
I will need some time to work it through, but I will certainly
do so.
I have got quite a backlog with items to include in EPG.

Best regards,
Chris
```

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Message: #817
Date: 2021-03-08
From: eckart_schmidt@t-online.de
Subject: Conic constellations for a pentagon

```
Dear all,

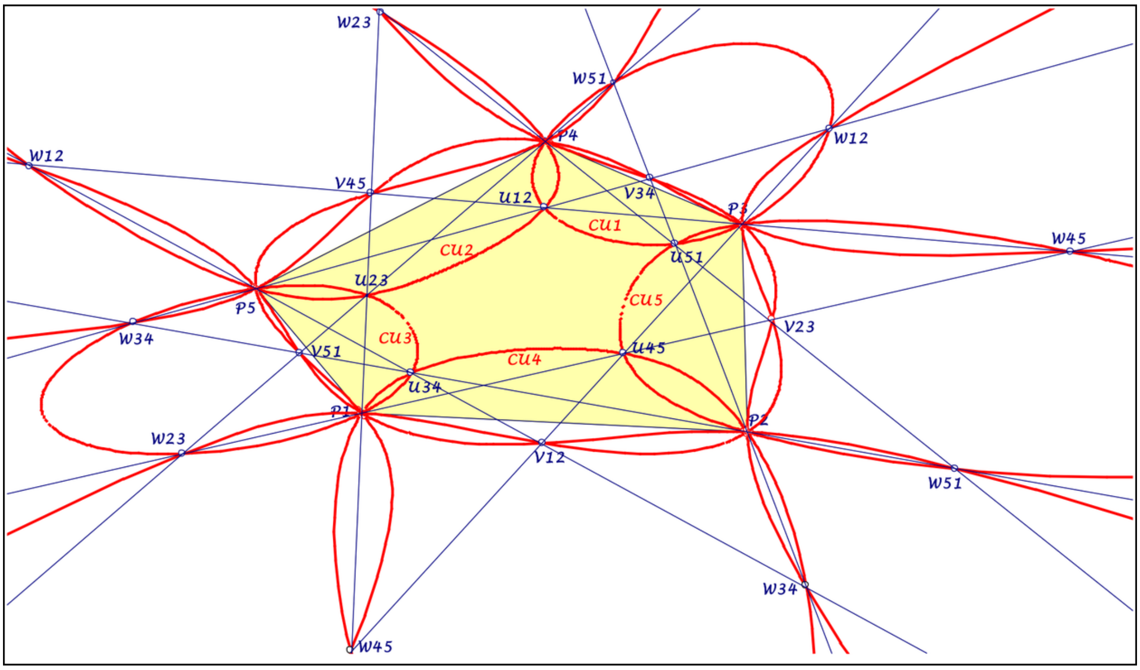
at the moment I study conic constellations for a convex
pentagram,
... similar to those of Tran Quang Hung in
... [1908.00974] Some new theorems on Pentagon and Pentagram
    (arxiv.org) <https://arxiv.org/abs/1908.00974>,
... not with circles, but with conics,
... here only the constellations, perhaps of interest.

Let 5G be a convex pentagon with vertices Pi (subscripts
modulo 5)
... and the 5 circumcubics CUi,
... each bearing 4 vertices Pj, j unequal i,
    tangent in Pj to PiPj,
... which are pivotal isocubics,
... ... pivot Pi, isoconjugation QA-Tf2,
        reference triangle QA-Tr1of the 4 Pj.

These 5 cubics CUi have 20 intersections unequal the vertices,
... 5 intersections Ui,i+1 = CUi^CUi+1 inside 5G,
... 5 intersections Vi,i+1 = CUi^CUi+1 between 5G and its
    circumellipse,
... the outer 10 intersections Wi,i+1 =  CUi^CUi+1 are pairs,
    collinear with Pi-2.

The 20 points and the 5 5G-vertices have a lot of conic
constellations, for example:
(a)  six degenerated 8-point conics as two lines
...  intersecting in a 5G-vertex P,
     each bearing 1 U-, 1 V-, 2 W-points.
(b)  six 8-point conics,
     bearing 4 vertices P and 2 U-, 2 W-points,
(c)  six 8-point conics,
     bearing 4 vertices P and 2 V-, 2W-points,
(d)  six 6-point conics,
     bearing 2 vertices Pi, Pi+1 (or Pi, Pi+2) and 2U-,
     2W-points,
(e)  six 6-point conics,
     bearing 2 vertices Pi, Pi+1 (or Pi, Pi+2) and 2V-,
     2W-points,
(f)  six 6-point conics,
     bearing 2 vertices Pi, Pi+1 (or Pi, Pi+2) and 2U-,
     2V-points.
```

```
Fascinating 5G-s-20P1 constellation, but no relevant
observations!
Best regards Eckart
```



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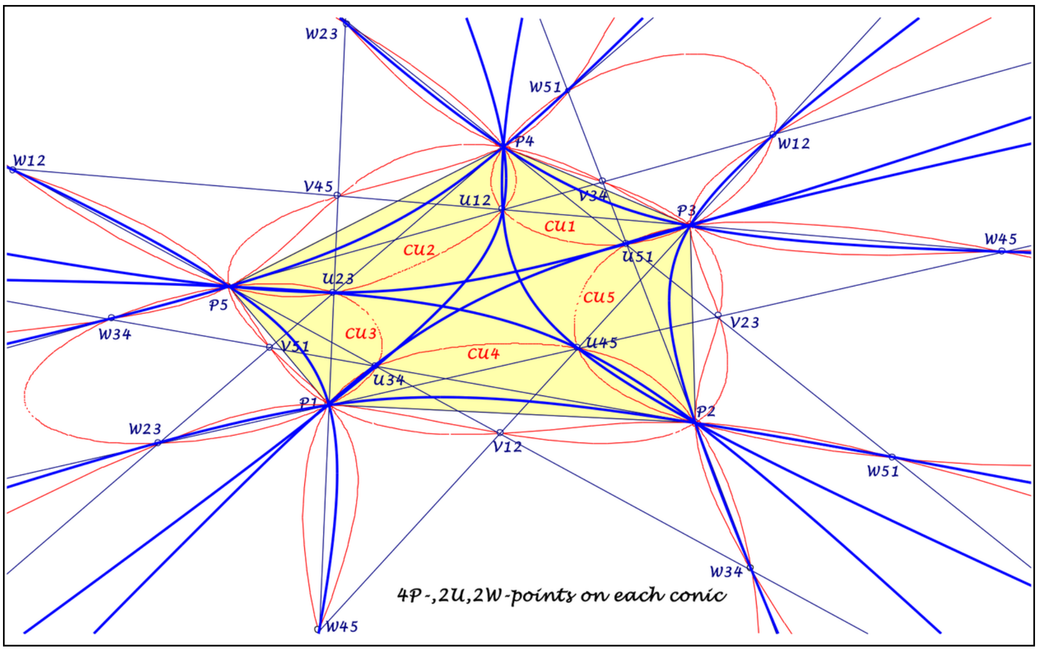

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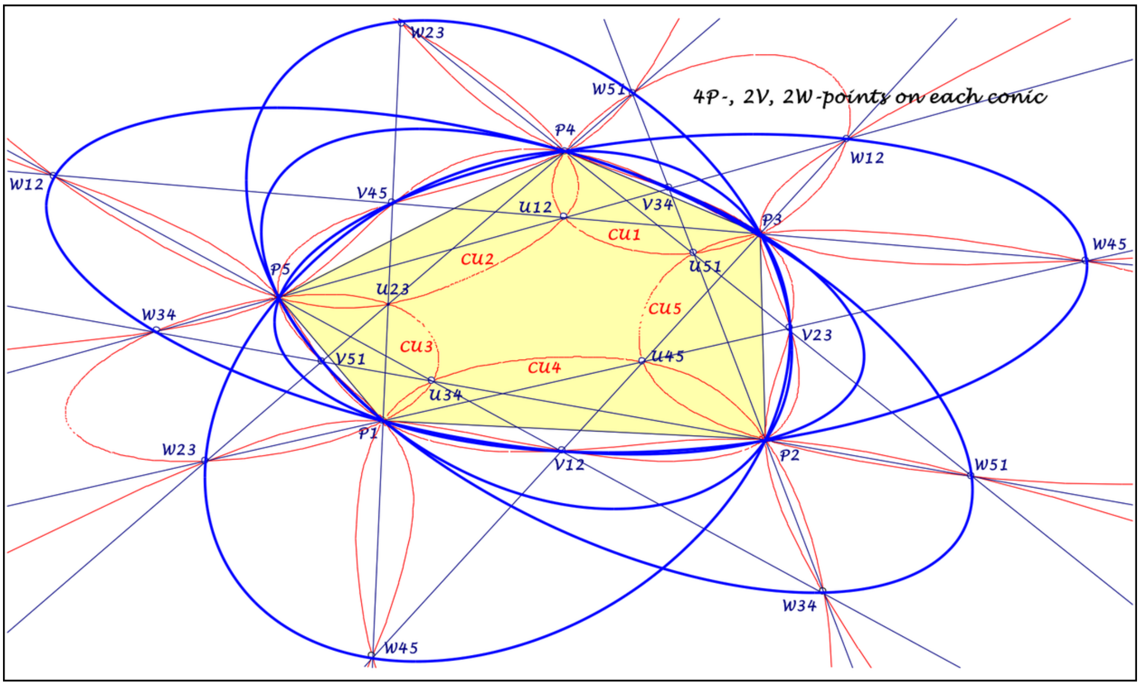

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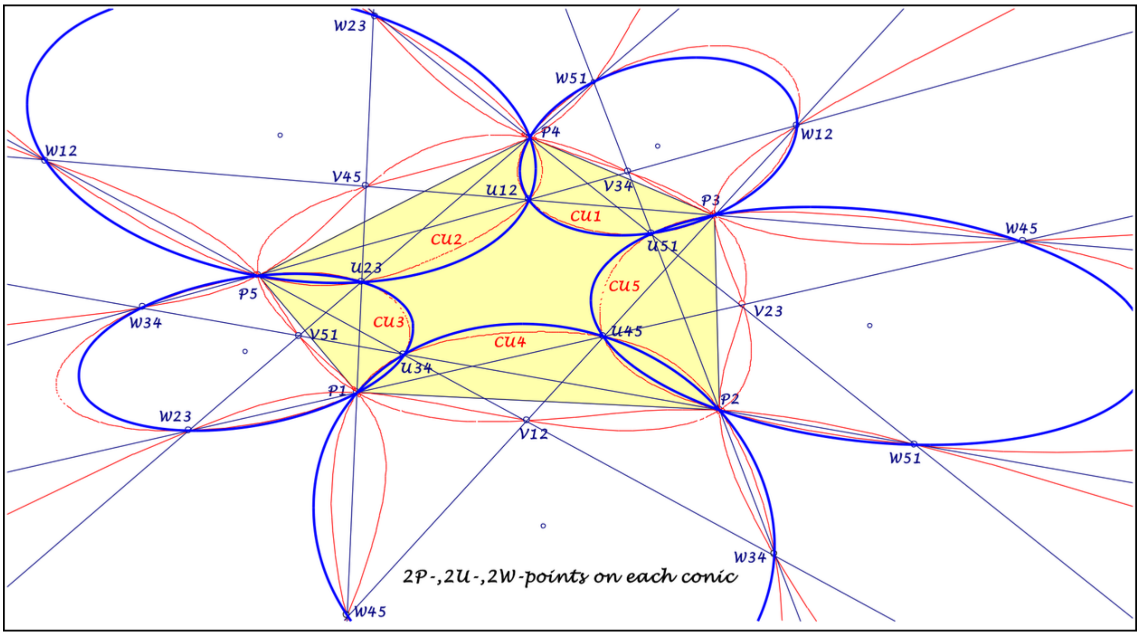

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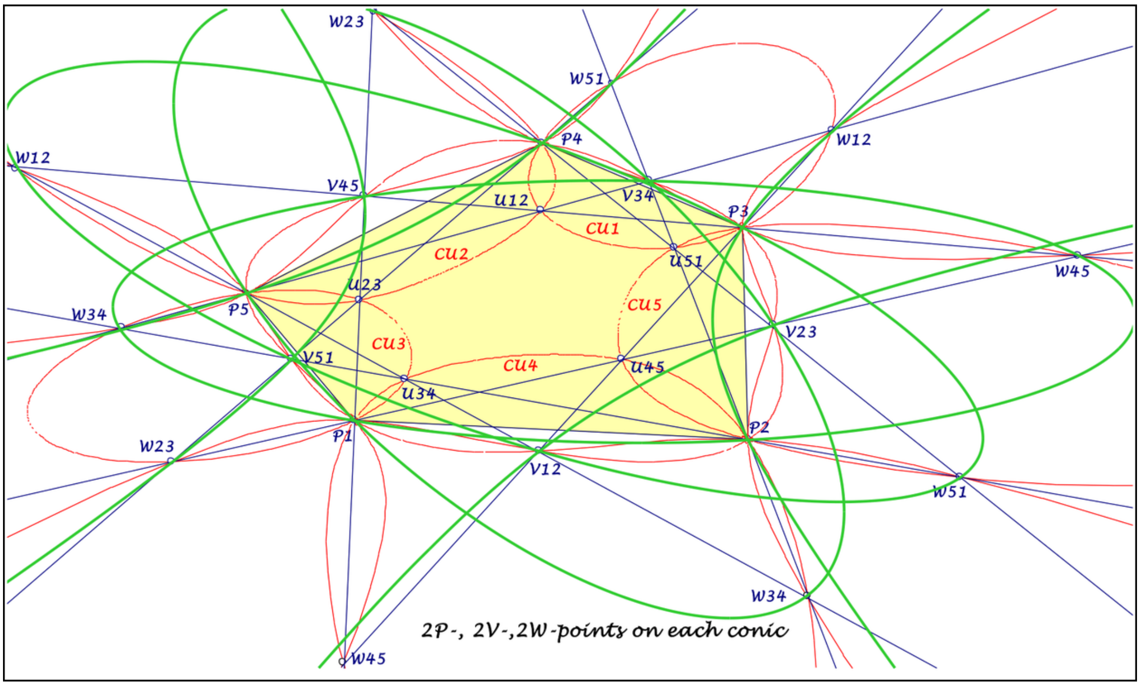

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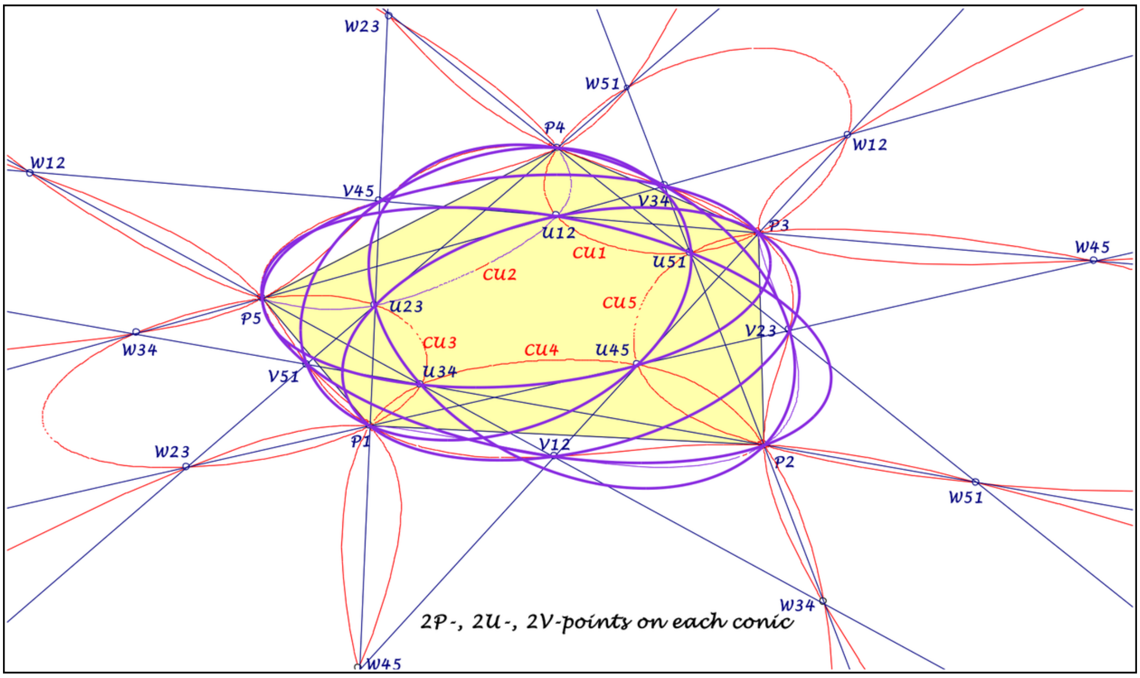

2021-03-08f.pdf

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Message: #818
Date: 2021-03-11
From: eckart_schmidt@t-online.de
Subject: 5 special QG for a 5G

```
Dear all,

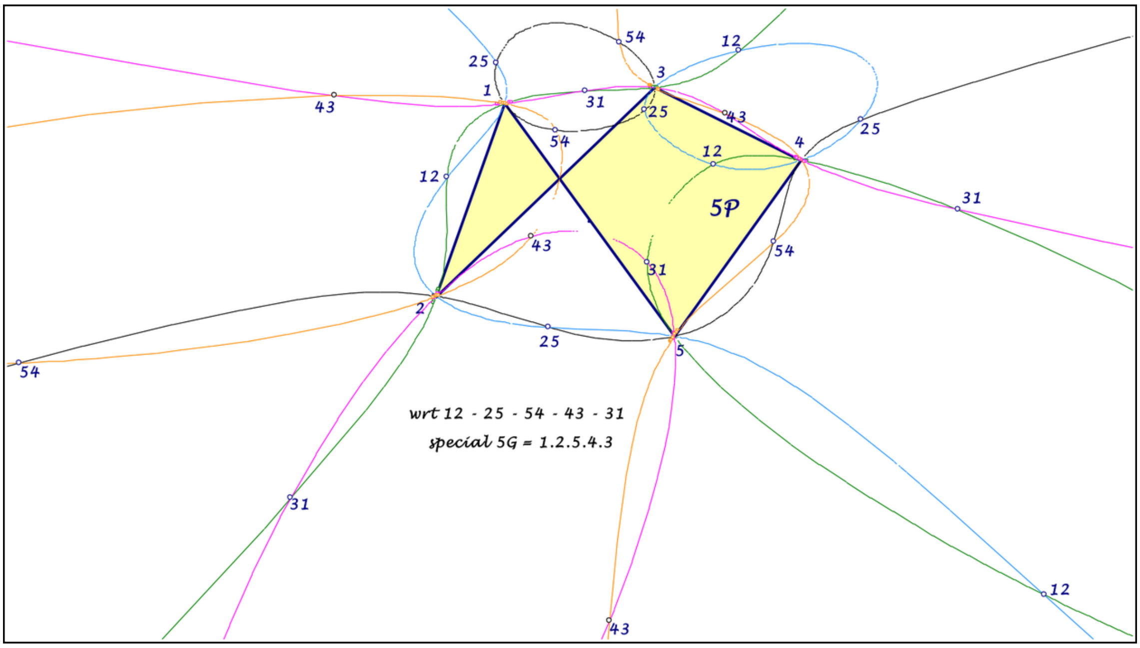
let us consider a 5P and the 5 circumcubics CUi,
... which are pivotal isocubics for the QA of 4 vertices,
... reference triangle QA-Tr1, isoconjugation QA-Tf2,
    pivot 5th vertex Pi.
These cubics have 20 intersections Si,j = CUi^CUj beside the
5P-vertices,
... only for 5 special pairs CUi, CUj there is a
    quadruple of Si,j,
... (not always all real, if 5P is not convex),
... the 5 index pairs can be ordered
        (i,j), (j,k), (k,l), (l,m), (m,i)
... and give a sequence i, j, k, l, m,
        which defines a special 5G = PiPjPkPlPm.
For a convex 5P, this special 5G will be the convex 5G-version.

The constellation can easier be handled for a convex 5G,
... the special quadruples are Si,i+1, cyclic indexed,
... two points inside, two points outside the 5G-circumconic,
... which give a QG with diagonal crosspoint Pi+3.

These 5 QG can be used for studying 5G-points:
Their lines QL-L1 have the common point 5G-s-P2.
Their QL-Cu1 have two common CSC-partner symmetrically 5G-s-P2.
5G-s-P2 is a center of a common inscribed conic of the 5 QG.
If the 5G isn't convex, we have to take the special 5G of the 5
points,
... but then not all intersections have to be real.

Best regards Eckart

PS. Wrt the point constellation see also #817.
```



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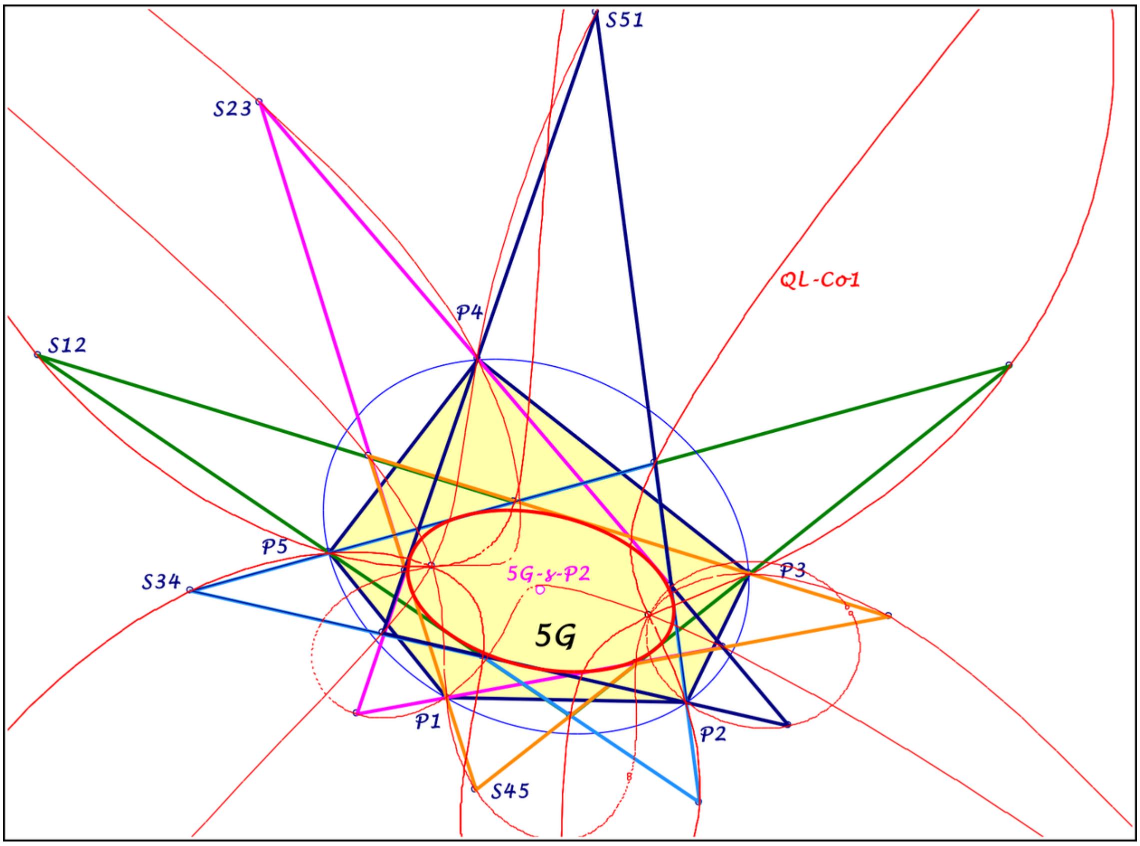


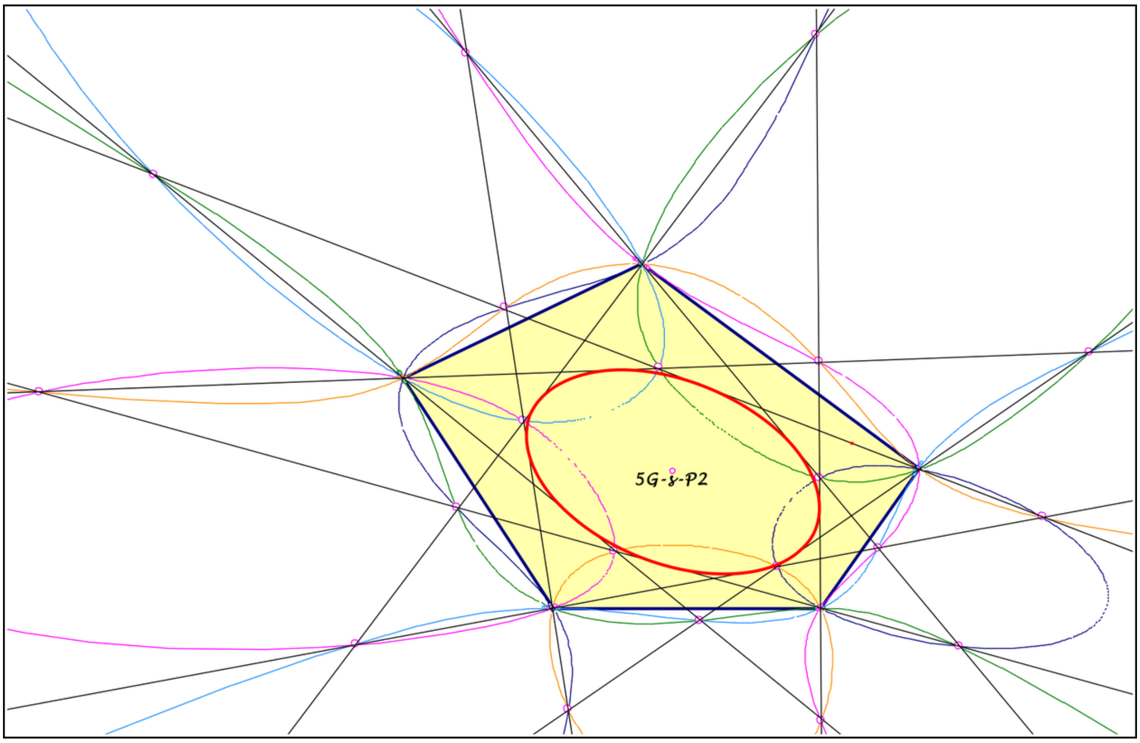
2021-03-11b.pdf

Message: #819
Date: 2021-03-12
From: eckart_schmidt@t-online.de
Subject: 5G-s-Cox

```
Dear Chris,

perhaps worth to be mentioned in EPG:
5G-s-Cox is a conic for convex 5G with center 5G-s-P2,
... whose polars for vertices Pi are the lines Pi-1.Pi+1.
How to get the conic:
The pivotal isocubics for the QA of 4 5G-vertices,
... reference triangle QA-Tr1, isoconjugation QA-Tf2,
    pivot 5th vertex,
... have 20 intersections beside the 5G-vertices,
... in quadruples on 10 lines,
    pairwise intersecting in the 5G-vertices
... and tangent of the conic 5G-s-Cox.

Best regards Eckart
PS: This is a result of #817 and #818.
```



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Message: #820
Date: 2021-03-13
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] 6P- lines connecting orthocenters are

---

```
Dear Chris, dear Eckart, dear Bernard and friends,

I have seen streng concurrent lines with 6P as follows
Let six arbitrary points Ai (i=1,...,6), taking subscripts
modulo 6:
- Lines AiA(i+1) meets lines A(i+2)A(i+3) at Bi.
- Lines AiA(i+1) meets lines A(i+3)A(i+4) at Ci
  (there are only three points C1, C2, C3).
- Line L1 connects orthocenters of triangles C1B1B2, C1B4B5;
  Line L2 connects orthocenters of triangles C2B2B3, C2B5B6;
  Line L3 connects orthocenters of triangles C3B6B1, C3B3B4.
Then lines L1, L2, L3 are concurrent.
Please see also the figure in the attached file.

Sincerely yours,
Tran Quang Hung.
```

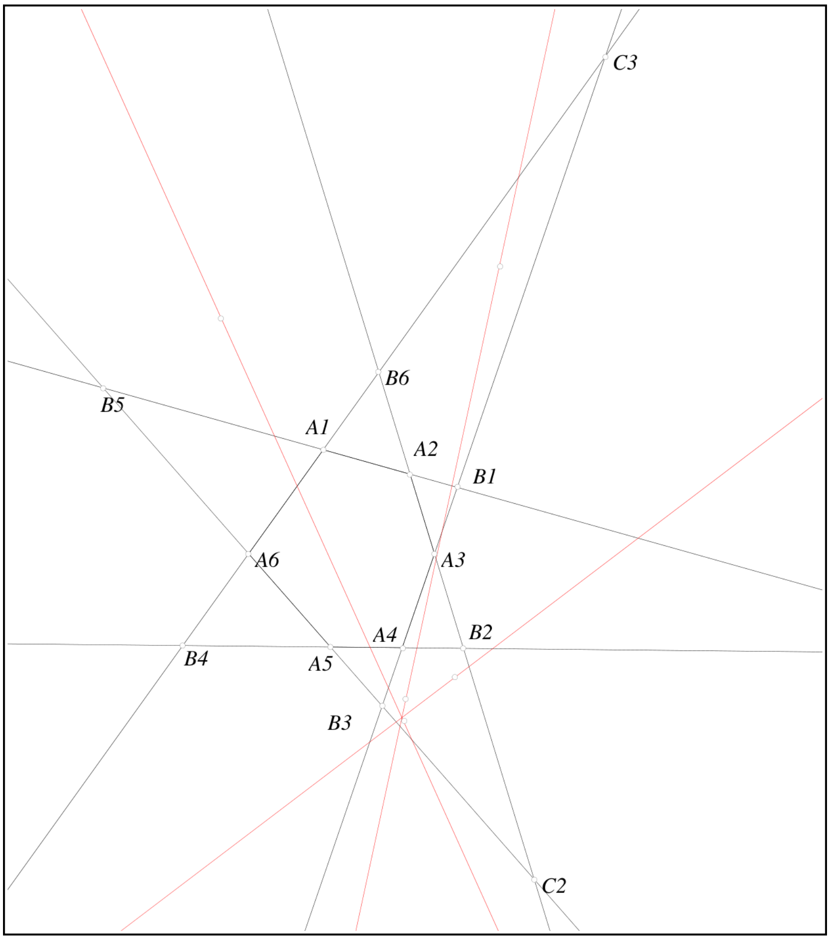

Figure7800.pdf

Message: #821
Date: 2021-03-13
From: cesar_e_lozada@yahoo.es
Subject: 6P- lines connecting orthocenters are concurrent

Dear Mr Hung and others,

Amazing and… algebraically proved!!!!
Since I only handle a little bit of triangles, I used the reference triangle ABC and another triangle with generic vertices.
Congratulations, Mr. Hung. It is a very beautiful result.
The underlined condition "…six arbitrary points no three on a line" seems to be necessary.
Unfortunately, when your construction is applied to ABC and other central triangle, the final point of intersection results not to be a triangle center(*).

Best regards,
César Lozada

(*): For example, for vertices of ABC and A'B'C'=1st circumperp triangle I found:

```
Q = {a*(a^2-(b+c)*a+(b-c)*c)*(a^8-2*a^7*c-3*a^6*b^2+(6*b^2+7*b*c
+3*c^2)*a^5*c+(3*b^4-2*c^4-(8*b+9*c)*b*c^2)*a^4-3*(b^2-c^2)*(b+c
)*(2*b-c)*a^3*c-(b^2-c^2)*(b-c)*(b-2*c)*(b^2+3*b*c+3*c^2)*a^2+2*
(b^2-c^2)*(b-c)*(b^3-2*c^3+(b-c)*b*c)*a*c-(b^2-c^2)^3*c^2),
b*(-(c^2-a^2)*(c-a)*b^7+(c^2+3*c*a+a^2)*(c-a)^2*b^6+(c^3+a^3)*(3
*c^2-7*c*a+3*a^2)*b^5-(3*c^6+3*a^6-(c^4+a^4+(3*c^2-5*c*a+3*a^2)*
c*a)*c*a)*b^4-(c+a)*(3*c^6+3*a^6-(14*c^4+14*a^4-(17*c^2-14*c*a+1
7*a^2)*c*a)*c*a)*b^3+(3*c^8+3*a^8-(5*c^6+5*a^6+(4*c^4+4*a^4-(9*c
^2-c*a+9*a^2)*c*a)*c*a)*c*a)*b^2+(c^2-a^2)*(c-a)*(c^6+a^6-4*(c^4
+a^4-(c^2-c*a+a^2)*c*a)*c*a)*b-(c^2-c*a+a^2)*(c^6+a^6-(c^2-c*a+a
^2)*c^2*a^2)*(c-a)^2),
c*(c^2-(a+b)*c-(a-b)*a)*(c^8-2*c^7*a-3*c^6*b^2+(3*a^2+7*a*b+6*b^
2)*c^5*a-(2*a^4-3*b^4+(9*a+8*b)*a^2*b)*c^4-3*(a^2-b^2)*(a+b)*(a-
2*b)*c^3*a+(a^2-b^2)*(a-b)*(2*a-b)*(3*a^2+3*a*b+b^2)*c^2-2*(a^2-
b^2)*(a-b)*(2*a^3-b^3+(a-b)*a*b)*c*a+(a^2-b^2)^3*a^2)}
```

Sent: Friday, March 12, 2021 10:41 PM
To: Quadri-and-Poly-Geometry@groups.io
Dear Chris, dear Eckart, dear Bernard and friends,
I have seen streng concurrent lines with 6P as follows
Let six arbitrary points Ai (i=1,...,6), taking subscripts modulo 6:
- Lines AiA(i+1) meets lines A(i+2)A(i+3) at Bi.

```
- Lines AiA(i+1) meets lines A(i+3)A(i+4) at Ci (there are only
three points C1=C4, C2=C5, C3=C6).
- Line L1 connects orthocenters of triangles C1B1B2, C1B4B5;
Line L2 connects orthocenters of triangles C2B2B3, C2B5B6;  Line
L3 connects orthocenters of triangles C3B6B1, C3B3B4.
Then lines L1, L2, L3 are concurrent.
Please see also the figure in the attached file.
Sincerely yours,
Tran Quang Hung.
```

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Message: #822
Date: 2021-03-13
From: van10hoven@gmail.com
Subject: Re: 6P- lines connecting orthocenters are concurrent

---

```
Dear Tran Quang Hung and César,

Very nice point indeed.
At first I wondered if it is one of a bicentric pair, but it
isn't.
The reason César did not find symmetric coordinates is because
points in an n-Gon (nG-Point) for n > 3 will never produce
symmetric results, unless it appears to be an nP-point or an
nL-point. This is because a point in a 6-Point or 6-Line will
have symmetric coordinates, but in an n-Point or n-Line 60
different 6-Gons can be spanned and therefore up to 60 different
versions of a 6G-point may exist in a 6-Point or 6-Line.
See nG-1 in EPG.

Best regards,
Chris
```

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Message: #823
Date: 2021-03-13
From: van10hoven@gmail.com
Subject: Re: 6P- lines connecting orthocenters are concurrent

---

```
Dear Tran Quang Hung and César,

A nice example of an nG-point that happened to be an
nP-/nL-point is 5G-s-P4 the Miquel-Catalan point.
It actually is a 5L-point, 5L-o-P2 to be precisely.
That means that, when we have a reference 5-Line with
coefficients for the defining lines L1=(0:0:1), L2=(0:1:0),
L3=(0:0:1), L4=(l1,m1,n1), L5=(l2,m2,n2), then you will find
symmetric coordinates for the 5L-constructed point.

Best regards,
Chris
```

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Message: #824
Date: 2021-03-14
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] 6P- lines connecting orthocenters are

---

```
Dear César and Chris,

Thank you so much for your interest in this problem. I think
that we can
write this problem as
Let ABC be a triangle with three arbitrary points P, Q, R...
which is mentioned by César.
Thanks Chris let me know that this problem is actually for 6G
not 6P.
I hope this problem will contribute to QPG.

Sincerely yours,
Tran Quang Hung.

Vào Th 7, 13 thg 3, 2021 vào lúc 21:27 Chris
<van10hoven@gmail.com> đã
viết:

> Dear Tran Quang Hung and César,
> A nice example of an nG-point that happened to be an
nP-/nL-point is
> 5G-s-P4
> <https://www.chrisvantienhoven.nl/ng-items/ng-geninf/ng-0/27-m
athematics/encyclopedia-of-poly-figures/ng-objects/artikelen-ng/
447-5g-s-p4>
> the Miquel-Catalan point. It actually is a 5L-point, 5L-o-P2
> <https://www.chrisvantienhoven.nl/nl-items/ol-obj/ol-pts/nl-o-
p2> to be
> precisely. That means that, when we have a reference 5-Line
with
> coefficients for the defining lines L1=(0:0:1), L2=(0:1:0),
L3=(0:0:1),
> L4=(l1,m1,n1), L5=(l2,m2,n2), then you will find symmetric
coordinates for
> the 5L-constructed point.
> Best regards,
> Chris
```

Message: #825
Date: 2021-03-15
From: van10hoven@gmail.com
Subject: Newton Point-like point on a circular cubic

Dear Bernard and Eckart,

I found an interesting 7P-point, but it needs some confirmation.
It is very well possible that parts of the constrcution have
been mentioned before.
1. Given a random point P on 7-point-cubic 7P-s-Cu1.
2. Let T1 and T2 be the points of tangency of the tangents from
P to 7P-s-Cu1.
3. Let S3 be the 3rd intersection point of T1.T2 intersecting
7P-s-Cu1.
4. Draw a line through P intersecting 7P-s-Cu1 in Q1 and Q2.
5. Let R1 be the 3rd intersection point of Q1.S3 with 7P-s-Cu1.
Let R2 be the 3rd intersection point of Q2.S3 with 7P-s-Cu1.
R1R2 will be a line through P.
6. Now the Miquel point of the QL with lines Q1Q2, R1R2, Q1R1,
Q2R2 will be a fixed point on 7P-s-Cu1.
7. It appears that when changing point P on the cubic, even then
QL-P1 is a fixed point, static on the cubic.
So it isn't even dependent on P, it is a fixed point on
7P-s-Cu1.

Eckart, can you confirm my construction?
Bernard, I supposed that wrt a circular cubic only 2 tangents
can be drawn from a random point on 7P-s-Cu1. Is this
justifiable?

Best regards,
Chris

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Message: #826
Date: 2021-03-15
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,

a first approximate drawing for your construction confirms your
result,
a second CABRI-construction for a focal circular cubic gave the
focus.
I shall research it more tomorrow.

Best regards Eckart
```

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Message: #827
Date: 2021-03-16
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,

if I am not wrong, a 7P-s-Cu1 can be mono- and bipartite,
... in the monopartite case there will be two tangents
    from a point on the cubic to the cubic,
... in the bipartite case there are up to four tangents
    from a point on the cubic to the cubic.

Best regards Eckart
```

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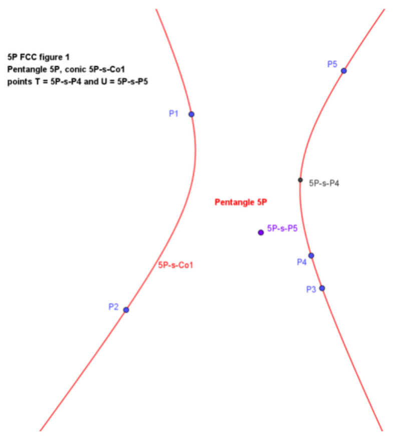
Message: #828
Date: 2021-03-16
From: bernard.keizer@gmail.com
Subject: 5P focal circular circumcubics

```
Dear Chris, dear Eckart
Dieses war der zweite Streich, doch der dritte folgt sogleich !
Thanks in advance to Eckart for reading and correcting if
necessary
Best regards
Bernard
```

Pentangle's focal circular circumcubics

Let's consider a pentangle PA of 5 points Pi with it's circumconic 5P-s-Co1 and it's known points T = 5P-s-P4 and U = 5P-s-P5.



There are 11 degenerated circular circumcubics of the 5 points, 10 formed by a circle through 3 of the 5 points (and the circular points) and the line through the 2 other points and one 11th formed by the circumconic of the 5 points and the infinity line (through the circular points).

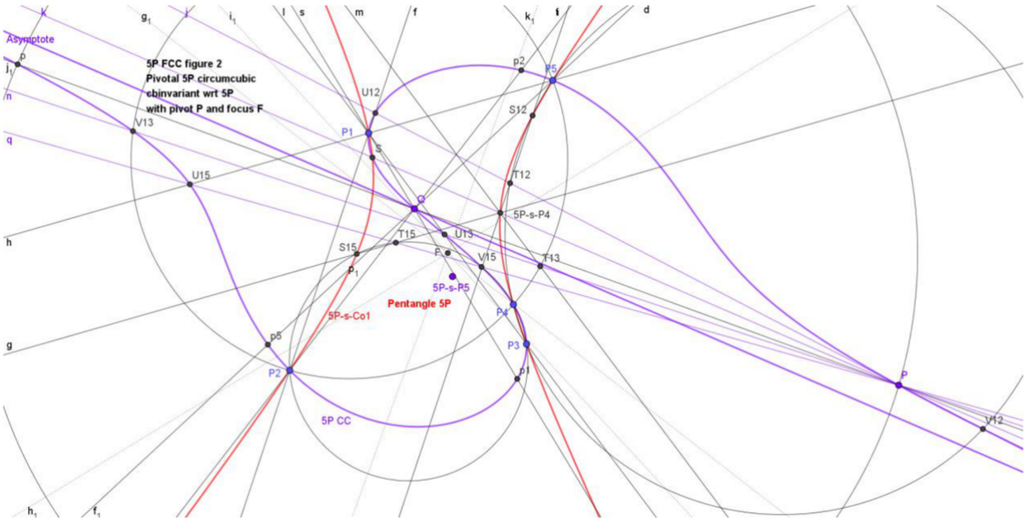
Each of these eleven cubics is a pivotal circumcubic of the 5 points and has a pivot Tij for the 10 1rst and T for the last one. Here is a method to determine the pivots Tij. The parallels through T to the lines PiPj intersect the circles through the 3 other points in 2 points ; one is Sij on the circumcubic, the other is the pivot Tij.

Having the 11 pivots, any pivotal circular circumcubic of the 5 points with pivot P is easy to draw. The line PT cuts the circumconic in a 2nd point (other than T) S and the lines PTij intersect the lines PiPj in a point Uij and the circles through the 3 other points in a 2nd point (other than Tij) Vij. Uij and Vij as well as S and the infinity point of PT are 11 copples of cb partners wrt the 5 points and the 2 circular points. This gives 21 points beside the 5 points and the pivot on the cubic.

For any point of the cubic, the circle Ci(P) pass through the 2 points U = 5P-sP5 and F, the focus of the cubic (for the point S on the circumconic, Ci(P) is the line UF), which allows to find F.

Using Eckart's construction allows to find the point Q where the cubic cuts it's asymptote and the asymptote as parallel to P5P-s-P4 through this point. (Any line through Q cuts the cubic in 2 points equidistant from the focus).

5P FCC.pdf



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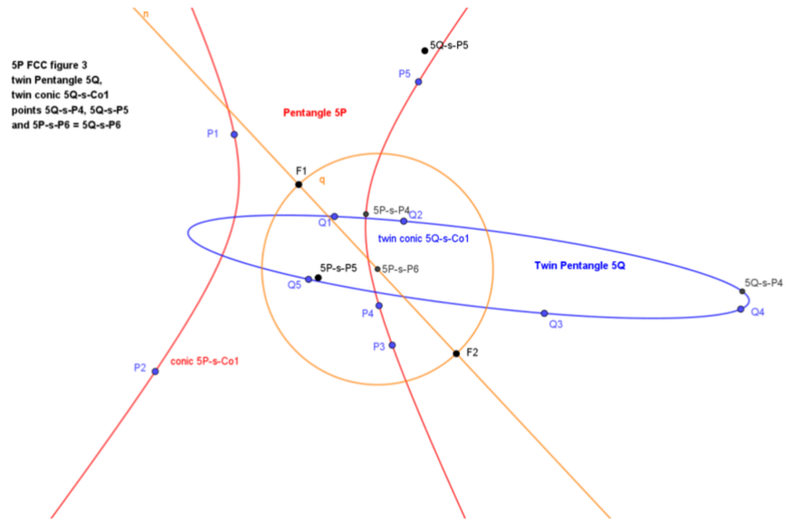
An interesting question is to find under which conditions this circular circumcubic of the 5 points is focal, id est it's focus lies on the cubic. In this case, the 5P CC is a 5P FCC.

This item gives all the points, curves and constructions needed to answer the question.

1. the twin pentangle of 5 points Qi = Tf8(Pi) with the point M = 5P-s-P6, the twin conic 5Q-s-Co1 and the points 5Q-s-P4 and 5Q-s-P5 (note that this point is Tf8(5P-s-P5)).



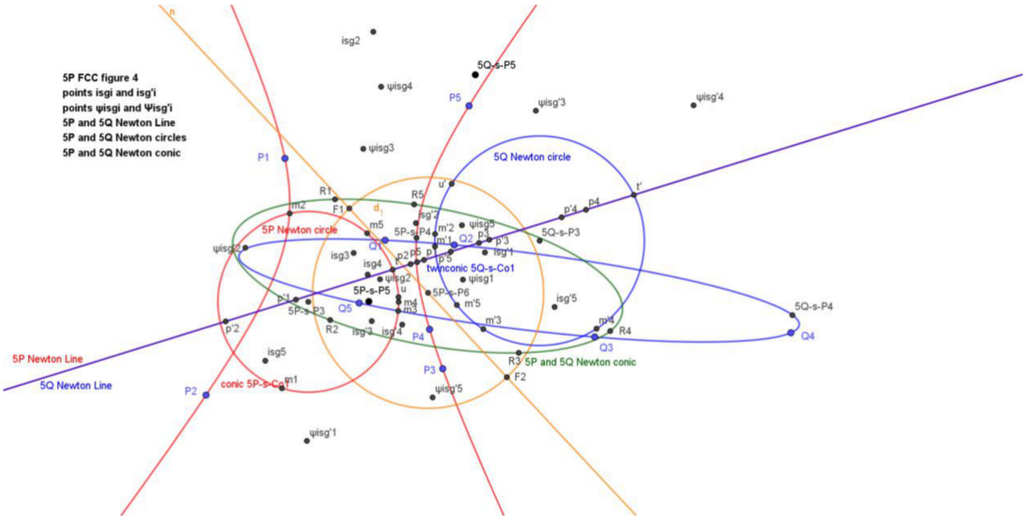
5P FCC.pdf

2. the 20 points isgi = QA-P4(5P less Pi), isg'i = QA-P4(5Q less Qi) and their Tf8 transformed ψisgi and ψisg'i.

3. the middles pi of Piψisg'i and p'i of Qiψisgi are aligned on a Newton Line through the middles t of TU and t' of T'U'.

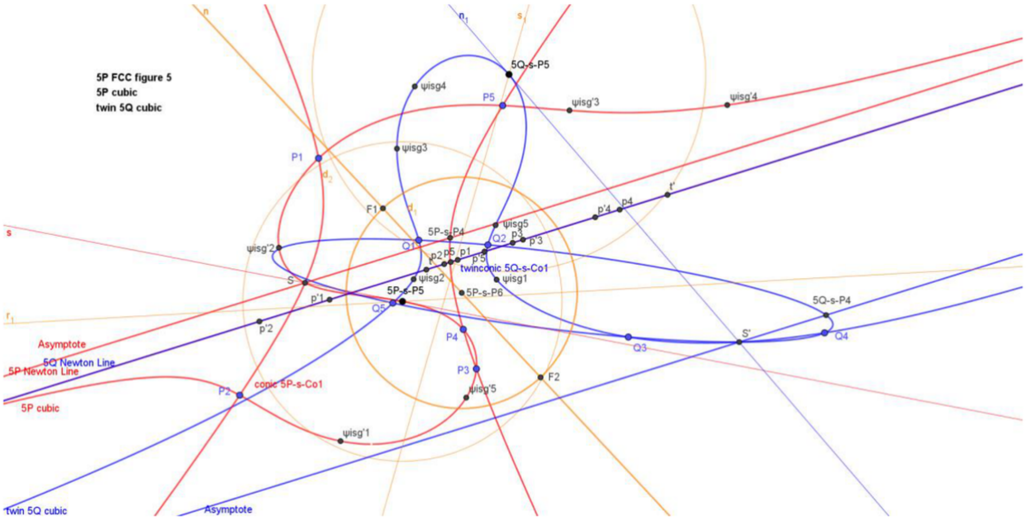
4. the middles mi of Piisgi and m'i of Qiisg'i are on the twin Newton circles, centered in ω = 5P-s-P3 and ω' = 5Q-s-P3, which pass also respectively through the middles m of MU and m' of MU'.

5. last, the middles Ri of PiQi are on the common Newton conic.



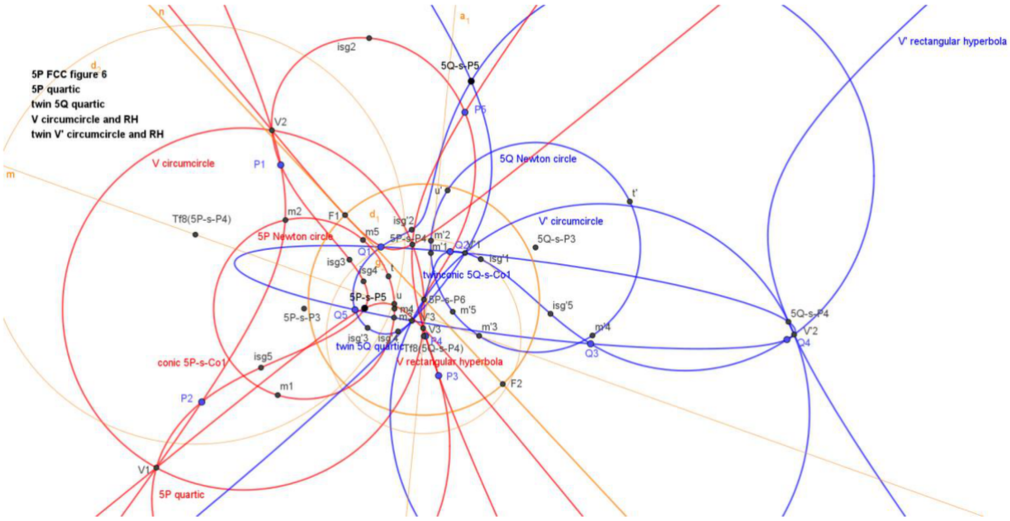
6. the 10 points Pi and ψisg'i are on the 5P cubic, which is a Van Rees focal circular circumcubic of the 5 points cb invariant with pivot in the infinity point of the Newton Line and focus in U = 5P-s-P5 ; it is invariant in the CSC2 centered in 5P-s-P5 and swapping the Pi and the ψisg'i and 5P-s-P6 and 5Q-s-P4.

7. the 10 points Qi and ψisgi are on the twin 5Q cubic, which is a Van Rees focal circular circumcubic of the 5 points cb invariant with pivot in the infinity point of the Newton Line and focus in U' = 5Q-s-P5 ; it is invariant in the twinCSC2 centered in 5Q-s-P5 and swapping the Qi and the ψisgi and 5P-s-P6 and 5P-s-P4.

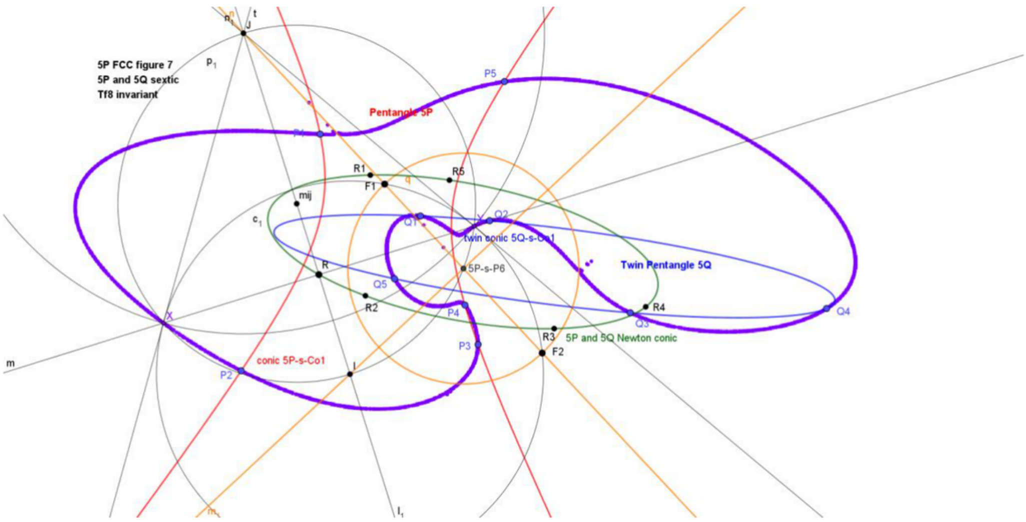


8. the 5P quartic is Tf8(5Q twin cubic), invariant in the CSC3 centered in Tf8(5P-s-P4), which swaps the Pi and the isgi and U = 5P-s-P5 and M = 5P-s-P6. CSC3 is a combination of CSC1 (or Tf8) and twinCSC2 ; CSC3 = CSC1*twinCSC2*CSC1.
   The 5P quartic and the 5P conic intersect beside the 5 points in 3 points V1, V2 and V3 forming the V triangle ; it's circumcircle passes through 5P-s-P4 and 5P-s-P6, it's Euler circle is the 5P Newton circle and it's orthocenter is 5P-s-P5.
   The V rectangular hyperbola passes through the vertices of the V triangle and through the points 5P-s-P5 and 5P-s-P4 and is centered in the middle of these 2 points. It's axes are are parallel to those of the 5P conic.

9. the 5Q quartic is the twin quartic, Tf8(5P cubic), invariant in the twinCSC3 centered in Tf8(5Q-s-P4), which swaps the Qi and the isg'i and U' = 5Q-s-P5 and M = 5P-s-P6. twinCSC3 is a combination of CSC1 (or Tf8) and CSC2 ; twinCSC3 = CSC1*CSC2*CSC1.
   The 5Q quartic and the 5Q conic intersect beside the 5 points in 3 points V'1, V'2 and V'3 forming the V' triangle ; it's circumcircle passes through 5Q-s-P4 and 5P-s-P6, it's Euler circle is the 5Q Newton circle and it's orthocenter is 5Q-s-P5 (Tf8(P-s-P5).
   The V' rectangular hyperbola passes through the vertices of the V' triangle and through the points 5Q-s-P5 and 5Q-s-P4 and is centered in the middle of these 2 points. It's axes are are parallel to those of the 5Q conic.



10. the 5P and 5Q sextic is invariant in Tf8 ; for a variable point R on the 5P and 5Q Newton conic, the 2 intersections between the bisector of the angle F1RF2 and it's Tf8 transformed give 2 Tf8 partners of the sextic.



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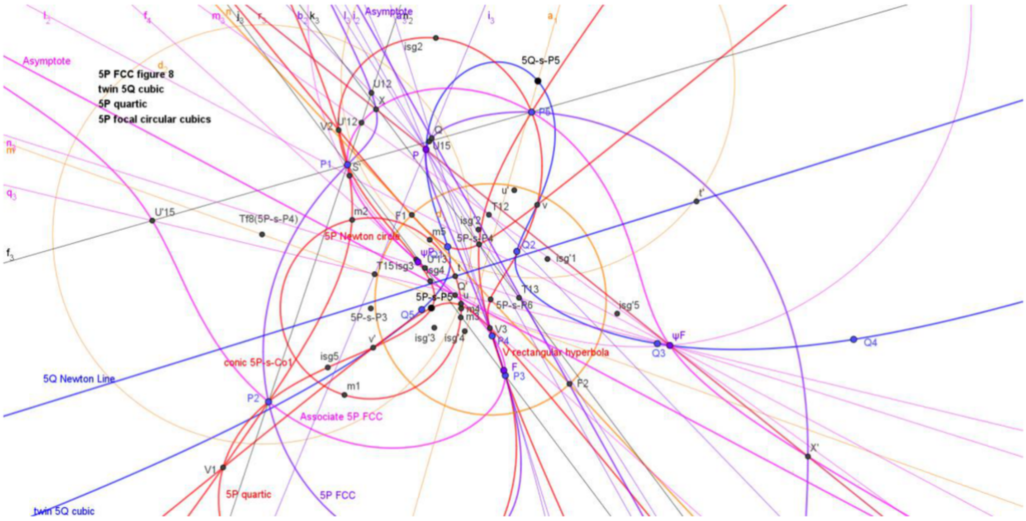
It now possible to answer the question above : a 5P circular circumcubic is focal if and only if it's pivot P lies on the twin 5Q cubic ; it's focus F is then CSC4(P), where CSC4 = twinCSC2*CSC1. As we have CSC3 = CSC1*twinCSC2*CSC1, we have also CSC4 = CSC1*CSC3 (Remember that CSC1 is Tf8).

This 5P FCC has an associate 5P FCC with pivot P' = twinCSC2(P) = Tf8(F) and focus F' = CSC3(F) = Tf8(P).



The 2 associated cubics intersect beside the 5 points Pi and the 2 circular points in 2 points X and X' cb partners wrt the 5 points on the pivot line PTf8(F).

There are many more or less interesting examples of such 5P FCC copples of associated cubics :

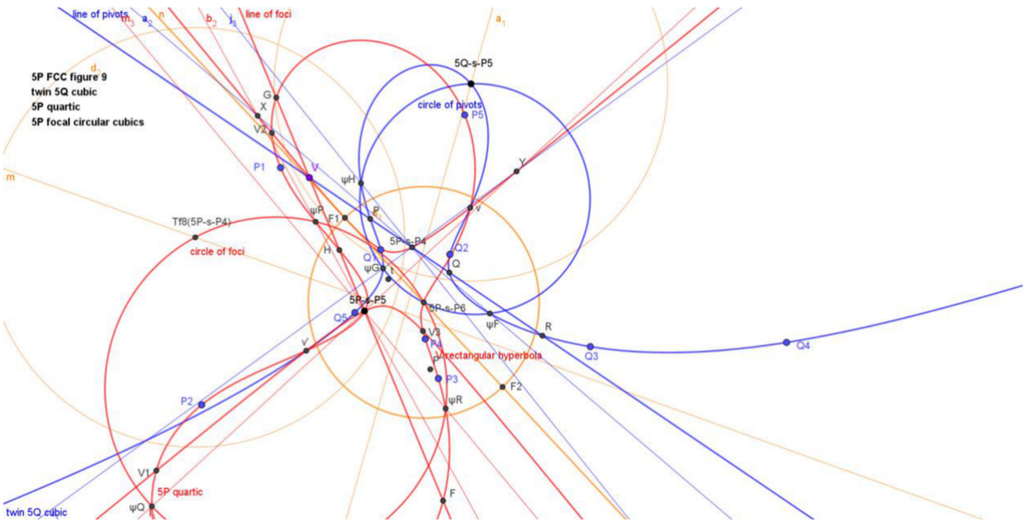
- pivot the infinity point of the Newton Line and focus 5P-s-P5 : this is our 1rst 5P cubic ; it's associate has pivot in 5Q-s-P5 and focus in 5P-s-P6.
- pivot in Qi and focus in isgi and associate with pivot in Tf8(isgi) and focus in Pi ; this gives 10 cubics.
- among the intersections of the twin 5Q cubic and the 5P quartic, 2 points v and v' are particularly interesting as they are at the same time CSC1 (or Tf8) partners and twin CSC2 partners, each point being therefore a fixed point for twin CSC2*CSC1.
  There are 2 associated cubics with pivot = focus in v or v'.

5P FCC.pdf

Perhaps a last interesting property, using the V rectangular hyperbola :

- a line through 5P-s-P4 intersect the 5Q twin cubic in 3 points, possible pivots
- the twinCSC2 transform of this line is a circle through the twinCSC2 transforms of the 3 points, which are 3 other possible pivots on the twin 5Q cubic, and through 5Q-s-P5 and 5P-s-P6 (as twinCSC2 transform of 5P-s-P4)
- the CSC1 (or Tf8) transform of this circle is a line through 5P-s-P5 (as Tf8(5Q-s-P5) and the 3 corresponding foci
- the 2 lines intersect in a point V on the V rectangular hyperbola

- conversely, for a point V on the RH, the 2 lines V5P-s-P4 and V5P-s-P5 are CSC4 partners ; the 1rst one intersects the twin 5Q cubic in 3 pivots P, Q and R and the 2nd intersects the 5P quartic in 3 foci, F, G and H giving 3 5P FCC
- the Tf8 of the 3 foci give in turn 3 other pivots ψF, ψG and ψH on a circle through 5Q-s-P5 on the twin 5Q cubic and the Tf8 of the 3 1rst pivots give 3 other foci ψP, ψQ and ψR on a circle through Tf8(5P-s-P4) on the 5P quartic, giving the 3 associate 5P FCC
- naturally, the lines ψF5P-s-P4 and ψP5P-s-P5 intersect in a point X, the lines ψG5P-s-P4 and ψQ5P-s-P5 intersect in a point Y and the lines ψH5P-s-P4 and ψR5P-s-P5 intersect in a point Z and the 3 points X, Y and Z lie on the RH



5P FCC.pdf

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Message: #829
Date: 2021-03-16
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,

for a bipartite 7P-s-Cu1 with 6 pairs of tangents
... from a fixed point on the cubic to the cubic,
... I got 3 different points using your construction.

Best regards Eckart
```

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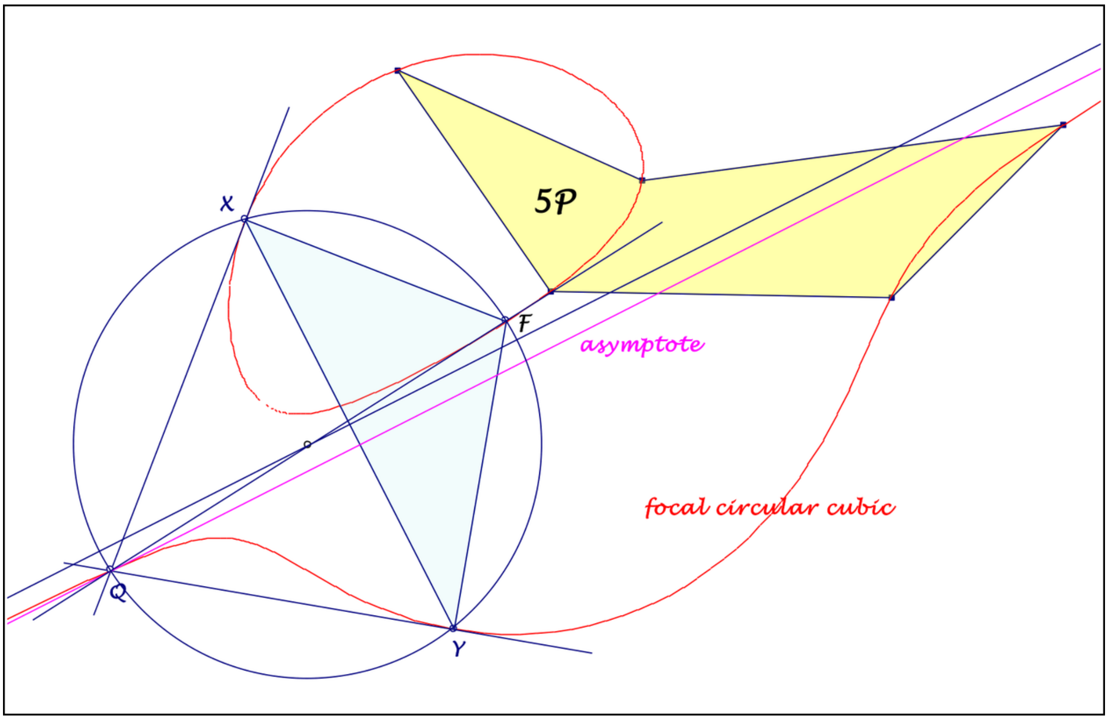
Message: #830
Date: 2021-03-16
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris,

your special points on 7P-s-Cu1 in #825
... become more and more interesting,
... here observations for focal circular cubics:
Your construction gives in the bipartite case
... 3 points on the cubic, one is the focus F,
... the other two X and Y define a Möbius transformation,
... centered in F, mapping the cubic to itself,
... the midpoints of image partners on the cubic give a line,
... parallel to the asymptote, half the distance to the focus.
Tangents to the cubic in X, Y intersect on the cubic
... in the tangential of the focus,
... which is the intersection Q of the cubic and its asymptote,
... F, X, Y, Q are concyclic.

I hope, you can conform these results.

Best regards Eckart



2021-03-16.pdf

Message: #831
Date: 2021-03-16
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Eckart,

Very nice results!
You write:
" Your construction gives in the bipartite case
... 3 points on the cubic, one is the focus F, "

That means that in the bipartite case there always is a focus on the cubic?
Meaning that a bipartite cubic is always a focal cubic?
Or was your starting figure a focal circular cubic?
On my part I also had constructed the three versions of QL-P1 lying on 7P-s-Cu1.
I wasn't sure that all three points were fixed. That's because I don't have a precise construction method for a tangent at the circular cubic coming from a point external to the circular cubic. Maybe you can help me with that.
I wondered if the cubic was self-isogonal wrt the triangle of the three versions of QL-P1 (F,X,Y in your terminology). It looks like it. It also looks like the line X.iso-Tf(X) // asymptote. Maybe you can check this all.
When F,X,Y are fixed points on the cubic we will have a standard and constructible triangle on a random circular cubic with properties of a known isoconjugation. I don't know if this is new.
I go on studying.

Best regards,
Chris

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Message: #832
Date: 2021-03-16
From: van10hoven@gmail.com
Subject: Re: 5P focal circular circumcubics

```
Dear Bernard, dear Eckart,

Thanks for the document about "Pentangle’s focal circular
circumcubics".
It looks very thorough.
For the time being it goes on the pile, until I start updating
EPG.
For me it's now a time of researching.
There are some very interesting properties regarding (circular)
cubics to come.

Best regards,
Chris
```

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Message: #833
Date: 2021-03-16
From: eckart_schmidt@t-online.de
Subject: Tangential for focal circular cubics

Dear all,

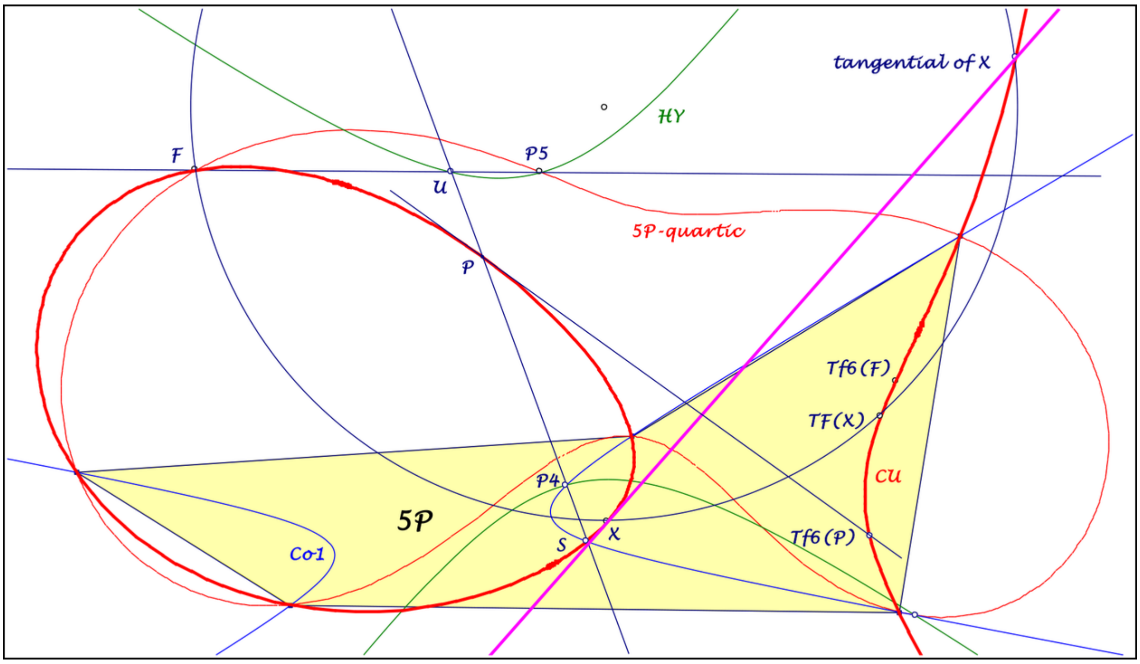
let us consider a focal circular cubic with focus F wrt a 5P,
... then the focus has to be a point on the 5P-quartic
    (not in EPG).
Below the prefix 5P-s- in the nomination will be omitted.

A relevant 5P-point (not in EPG, but in #736)
... is the 6th intersection S of the cubic and the
    5P-circumconic,
... constructed as follows:
The line F.P5 intersects an orthogonal hyperbola HY (not in EPG),
... through P4 and P5, centered in the middle,
    axes parallel to those of Co1,
... in a 2nd point, whose connection with P4
    intersects the circumconic 2nd in S.

To get the tangential for a point X of the cubic
... we use a Möbius transformation TF,
... centered in the focus F, swapping S and Tf6(F),
... which maps the cubic to itself.
The tangential of X is the 4th intersection of the cubic and the circle (F,X, TF(X)).

Some examples:
For the focus F the construction fails, the tangential of F is the TF-image
... of the intersection of the cubic
    and a parallel to the asymptote through F,
... or: the tangential of F is the intersection of the cubic
    and its asymptote (see #732).
The 4th intersection of the circle (F,S,Tf6(F))
... is the common tangential of S and Tf6(F).
The tangential of the Tf6-pivot P is Tf6(P).

Best regards Eckart



2021-03-16a.pdf

Message: #834
Date: 2021-03-17
From: bernard.keizer@gmail.com
Subject: Re: Tangential for focal circular cubics

```
Dear Eckart,
If you read my memo about the 5P FCC, you will find the quartic
as locus of the foci and the twin cubic as locus of the pivots,
as well as the V rectangular hyperbola !
You will find also the CSC1 (or Tf8) and twin CSC2 and the fact
that P and F are partners in the transformation twinCSC2*CSC1.
Then PP4 and FP5 intersect on the RH, as you mention, but you
may have P directly from F and PP4 cuts the circumconic in S ...
Best regards
Bernard
```

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Message: #835
Date: 2021-03-17
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,

I think, your observations are right:
We can interpret a bipartite circular cubic 7P-s-Cu1
... as isogonal pivotal cubic, QA-Cu1 of a special QA:
Your "construction" of fixed points on 7P-s-Cu1 in #825
... gives in the bipartite case three points A, B, C
    for a reference triangle
... and the cubic is isogonal invariant wrt ABC
... with pivot in the infinity point of the asymptote.
The triangle ABC can be taken as Miquel triangle,
... giving three Möbius transformations,
... (centered in one vertex, swapping the other two),
... which give for any point P1 of the cubic three further
    P2, P3, P4,
... which lead to a QA with the cubic as QA-Cu1.

Starting with 4 points of tangency in your "construction",
they give
... a QA,
    whose QA-Cu1 is the reference cubic and whose QA-Tr2 is ABC.

Best regards Eckart

PS: Wrt the question in #831: The starting figure should be a
focal circular cubic.
```

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Message: #836
Date: 2021-03-17
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Eckart,

Thanks for checking it all.
I can confirm your last additions too. When I took the 4 points of tangency as the vertices of the QA and constructed QA-Cu1 it overlapped perfectly. That feels good. Now we are in familiar territory.

So we can conclude that we can span in a bipartite circular cubic different QA's, starting with some point P on the cubic that admits 4 tangents. The points of tangency form the QA and P will have the function of QA-P4 wrt the QA.

There is no such construction in the case of a unipartite circular cubic, assuming that a unipartite circular cubic does not permit 4 real tangents (is there any proof of that?). And probably there will be only real bipartite versions of QA-Cu1.

Best regards,
Chris

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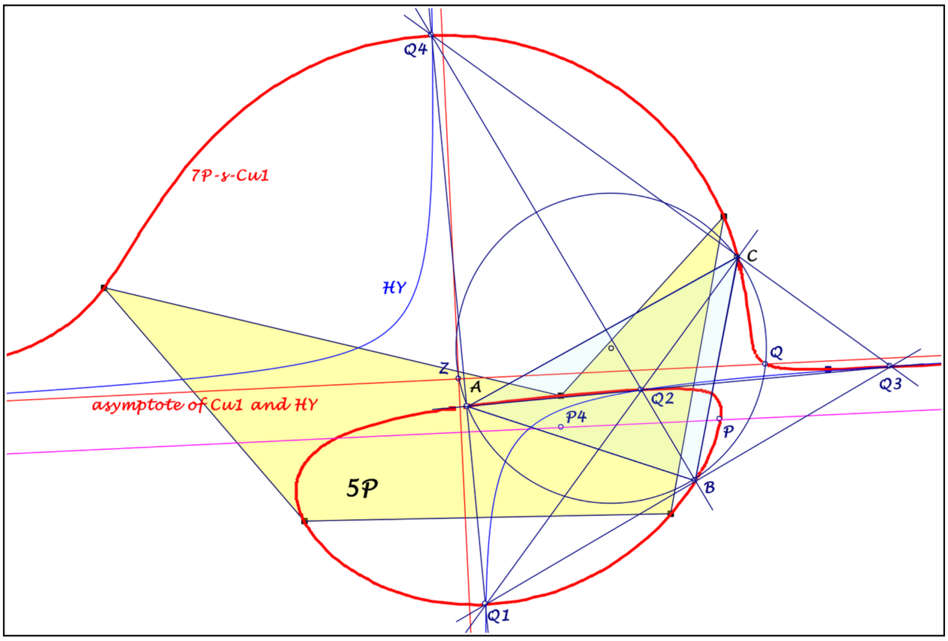
Message: #837
Date: 2021-03-17
From: eckart_schmidt@t-online.de
Subject: New sight of bipartite circular cubics

```
Dear all,

let us start with a bipartite cubic 7P-s-Cu1 with 5 points as
5P,
... the remaining two points give with their Tf6-images
    the Tf6-pivot P,
... the line P.P4 is a parallel to the asymptote,
... so parallels to P.P4 intersect the cubic in two points,
... whose midpoints give an orthogonal hyperbola HY,
... with one asymptote also asymptote of the cubic,
... the hyperbola intersects the cubic in four points,
... vertices of an orthocentric quadrangle,
... whose diagonal triangle ABC gives Chris' points of #825
... and its circumcircle bears the center of HY
... and the intersection Q of the cubic and its asymptote.
For further properties of ABC see former messages.

For bipartite focal circular cubics
... the focus lies on the circumcircle of ABC diametral to Q.

Best regards Eckart
```



2021-03-17.pdf

Message: #838
Date: 2021-03-18
From: eckart_schmidt@t-online.de
Subject: Re: 5P focal circular circumcubics

```
Dear Bernard,

thanks for the 2nd memo, now I shall study it, but give me time,
... for I have difficulties with your nomination
    and your figures,
... already the first pivots Tij were new for me.

Best regards Eckart
```

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Message: #839
Date: 2021-03-18
From: van10hoven@gmail.com
Subject: Cayley-Bacharach related points

Dear friends,

In attached paper I made a description of several constructions using the Cayley-Bacharach point.
The Cayley-Bacharach Theorem states that all cubics through eight distinct points in the plane, no four of them collinear and no seven of them coconic, pass through a unique ninth point.
In drawings and calculations, it often is difficult to calculate or construct intersection points of cubics crossed with other cubics, conics, or lines.
It appears that the Cayley-Bacharach point (CB-point) is a very useful tool in the construction of these intersection points.
This is especially remarkable because the Cayley-Bacharach Point can be constructed by the ruler only (although very complicated). Therefore is some cases the described construction points also can be constructed by the ruler only.
Moreover, the Cayley-Bacharach point can be calculated.
Therefore most described points can be calculated now.

Some remarkable results:
· A construction is shown for the 3 rd intersection point of a line through two cubical points.
· A construction is shown for the 6 th intersection point of a conic through five cubical points.
· A construction of the line through the two unknown intersection points of two 9P-cubics with 7 common points.
· A construction of the line through the two unknown intersection points of two 7P-cubics with 5 common points
· A new pivot point for the 7P-cubic is defined.
· Several series of CB-related points are shown for the 7P-cubic as well as the 9P-cubic.

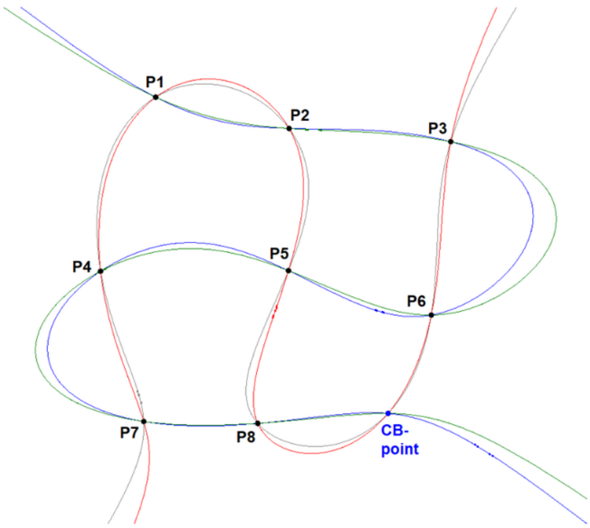
It so very well possible that some results are already known.
But I hope the combination of all items will give a better understanding.

Best regards,
Chris

APPLICATION OF CAYLEY-BACHARACH POINTS

Chris van Tienhoven
March 2021

The Cayley-Bacharach Theorem states that all cubics through eight distinct points in the plane, no four of them collinear and no seven of them coconic, pass through a unique ninth point.
In drawings and calculations, it often is difficult to calculate or construct intersection points of cubics crossed with other cubics, conics, or lines.
It appears that the Cayley-Bacharach point (CB-point) is a very useful tool in the construction of these intersection points.
This is especially remarkable because the Cayley-Bacharach Point can be constructed by the ruler only (although very complicated). See [1]. Therefore is some cases the described construction points also can be constructed by the ruler only.
Moreover, the Cayley-Bacharach can be calculated. See [2]. Therefore most described points can be calculated now.
This provisional paper was written in a flow of four weeks where one property rolled out of the other, and so forth.
Several properties are proven but not all of them.



Some remarkable results:

- A construction is shown for the 3rd intersection point of a line through two cubical points.
- A construction is shown for the 6th intersection point of a conic through five cubical points.
- A construction for the line through the two unknown intersection points of two 9P-cubics with 7 common points.
- A construction for the line through the two unknown intersection points of two 7P-cubics with 5 common points
- A third pivot point for the 7P-cubic is defined.
- Several series of CB-related points are shown for the 7P-cubic as well as the 9P-cubic.

CB-related points-01.pdf

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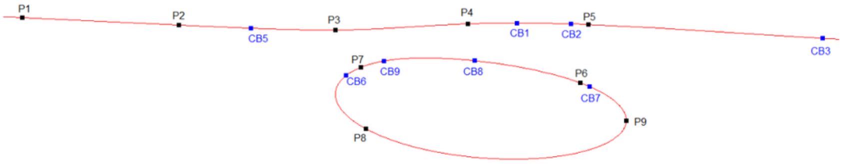
CB-related points-01.pdf

1. CB-related points

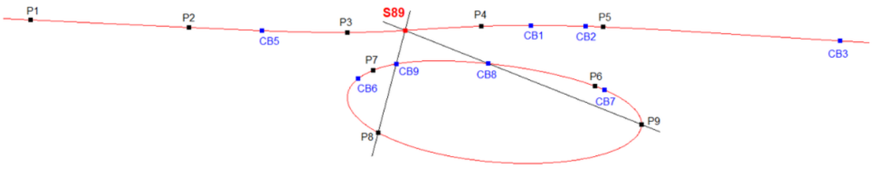
1.1 CB-related points on a 9-Point-Cubic

1. Given 9 points Pi defining a cubic.

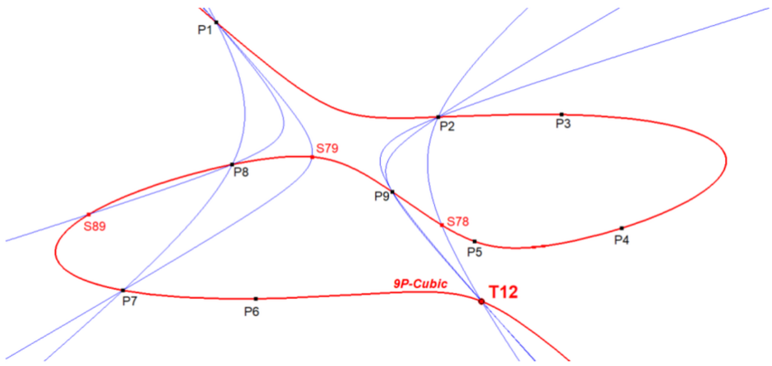
2. Denote CBi = Cayley-Bacharach Point of Pi wrt the rest of the 9 points P1 ... P9.
   There are 9 CBi-points on a 9-point-cubic.



3. Let Sij = Pi.CBj ^ Pj.CBi.
   Sij lies on the cubic. There are 36 different Sij-points.

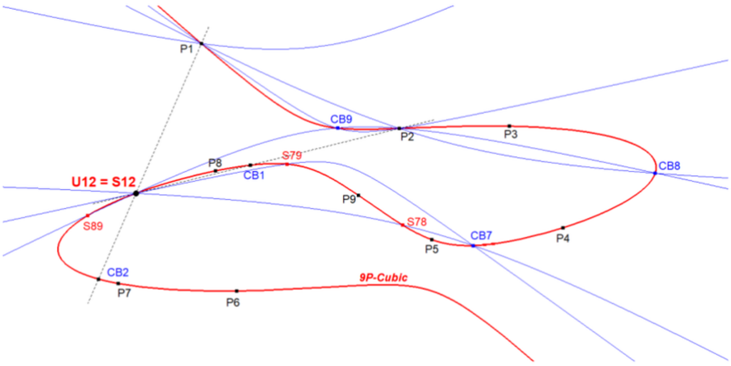


4. All Conics (Pi,Pj,Pk,Pl,Skl) have a common point Tij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,9). Tij is a point on the cubic. There are 36 different Tij-points.

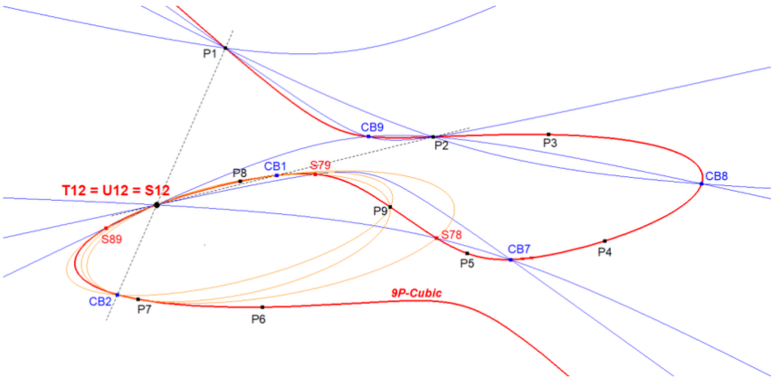


5. All Conics (Pi,Pj,CBk,CBl,Skl) have a common point Uij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,9). Uij is a point on the cubic. It appears that Uij = Sij. The construction method of Sij however is much simpler.

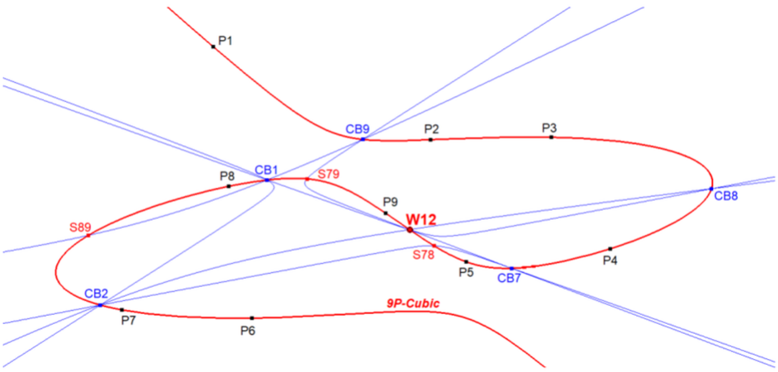
CB-related points-01.pdf



6. All Conics (CBi,CBj,Pk,Pl,Skl) have a common point Vij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,9). Vij is a point on the cubic. It appears that Vij = Sij = Vij. The construction method of Sij however is much simpler.

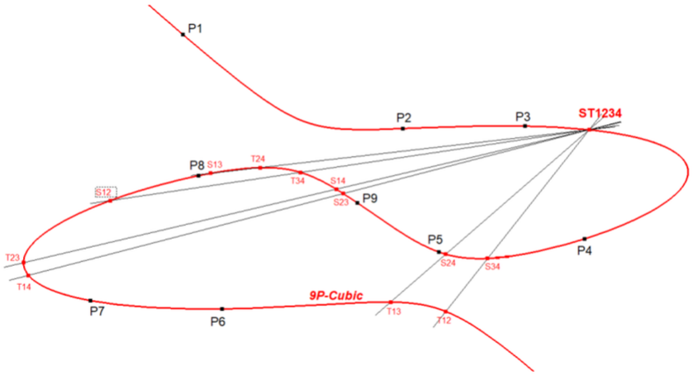


7. All Conics (CBi,CBj,CBk,CBl,Skl) have a common point Wij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,9). Wij is a point on the cubic. This time there is no identity with a known point. It is a new point. There are 36 different Wij-points.

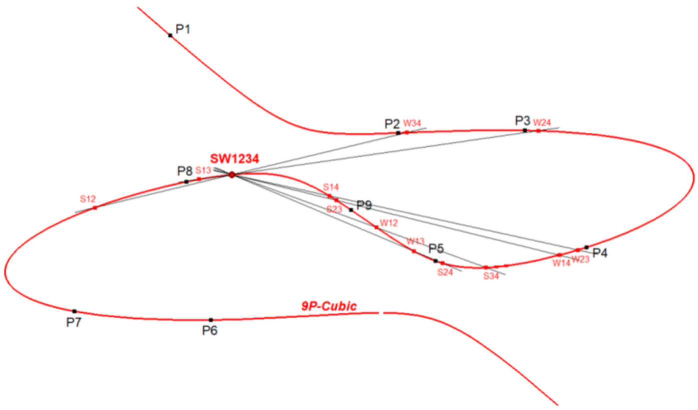


CB-related points-01.pdf

8. Any combination of Sij and Tkl, combining different numbers i,j,k,l cross at the same cubical point, when i,j,k,l are mutually interchanged. For example, S12.T34 ^ T12.S34, S13.T24 ^ T13.S24 and S14.T23 ^ T14.S23 coincide at the same point. We call this pivot-point ST1234. Stated in a general way, every line Sij.Tkl passes through the same cubical pivot point STijkl for all combinations (i,j,k,l), being 4 different numbers from the set (1,2,3,4,5,6,7,8,9). There are 126 versions of STijkl on the 9P-cubic. The order of the numbers i,j,k,l behind the suffix "ST" is not relevant.



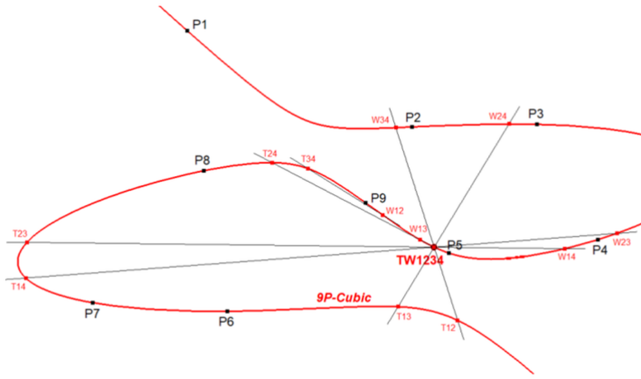
9. Any combination of Sij and Wkl, combining different numbers i,j,k,l cross at the same cubical point, when i,j,k,l are mutually interchanged. For example, S12.W34 ^ W12.S34, S13.W24 ^ W13.S24 and S14.W23 ^ W14.S23 coincide at the same point. We call this pivot-point SW1234.
Stated in a general way, every line Sij.Wkl passes through the same cubical pivot point SWijkl for all combinations (i,j,k,l), being 4 different numbers from the set (1,2,3,4,5,6,7,8,9). There are 126 versions of SWijkl on the 9P-cubic. The order of the numbers i,j,k,l behind suffix “SW” is not relevant.



10. Any combination of Tij and Wkl, combining different numbers i,j,k,l cross at the same cubical point, when i,j,k,l are mutually interchanged. For example, T12.W34 ^ W12.T34, T13.W24 ^

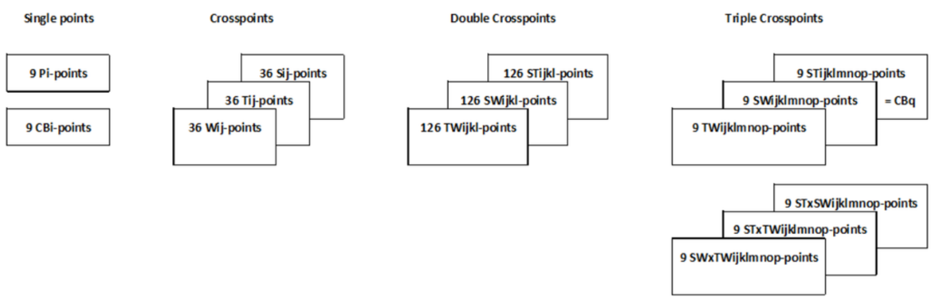
CB-related points-01.pdf

W13.T24 and T14.W23 ^ W14.T23 coincide at the same point. We call this pivot-point TW1234.
Stated in a general way, every line Tij.Wkl passes through the same cubical pivot point TWijkl for all combinations (i,j,k,l), being 4 different numbers from the set (1,2,3,4,5,6,7,8,9).
There are 126 versions of SWijkl on the 9P-cubic. The order of the numbers i,j,k,l behind suffix "TW" is not relevant.



11. Given 9 numbers i1,i2,i3,i4,i5,i6,i7,i8,i9, being different numbers from (1,2,3,4,5,6,7,8,9), it looks like that all 35 versions of:
    - STi1i2i3i4.STi5i6i7i8 passes through a cubical point ST_i9, **which is CBi9!**
    - SWi1i2i3i4.SWi5i6i7i8 passes through another cubical point SW_i9.
    - TWi1i2i3i4.TWi5i6i7i8 passes through another cubical point TW_i9.
    - STi1i2i3i4.SWi5i6i7i8 passes through another cubical point STxSW_i9.
    - STi1i2i3i4.TWi5i6i7i8 passes through another cubical point STxTW_i9.
    - SWi1i2i3i4.TWi5i6i7i8 passes through another cubical point SWxTW_i9.

    These 6 types of points all have 9 versions.
    Making drawings was extremely difficult due to the processing time of the many layers in the drawing.
    Summary of all 9P-(cross-)points:

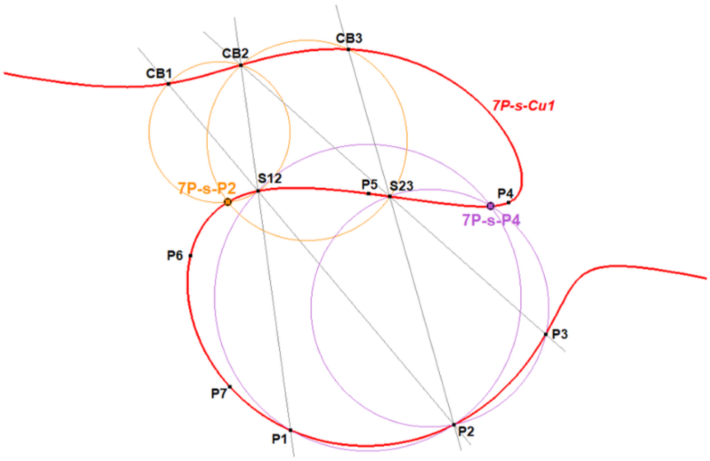


These points have 1, 2, 4, or 8 indices.
For each possible combination of these indices (without order) different versions of these points exist. The indices are taken as different numbers from (1,2,3,4,5,6,7,8,9).

CB-related points-01.pdf

1.2 CB-related points on a 7-Point-Cubic

1. Given 7 points Pi defining a circular cubic. It is a 9-Point-Cubic where two of its points are the circular points at infinity. The same points are lying on each circle. These points are not visible, because they are imaginary points and because they are points at infinity. Let CI1 & CI2 be the circular points at infinity.
2. Denote CBi = Cayley-Bacharach Point of Pi wrt the rest of the 7 points P1 ... P7.
   There are 7 CBi-points on a 7-point-circular cubic.
3. Let Sij = Pi.CBj ^ Pj.CBi. Sij is a point on the circular cubic.
   There are 21 different Sij-points.
   Sij(CI1,CI2) = common point of circles (Sij, CBi, CBj) = 7P-s-P2 = Pz.
4. All Conics(Pi,Pj,Pk,Pl,Skl) have common points Tij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,7). Tij is a point on the circular cubic.
   There are 21 different Tij-points.
   Special case is Tij(CI1,CI2) = common point of conics (CI1, CI2, Sij, Pi, Pj) = common point of circles (Sij, Pi, Pj) = 7P-s-P4 = Px. See [4], Eckart's Px in QPG#781.

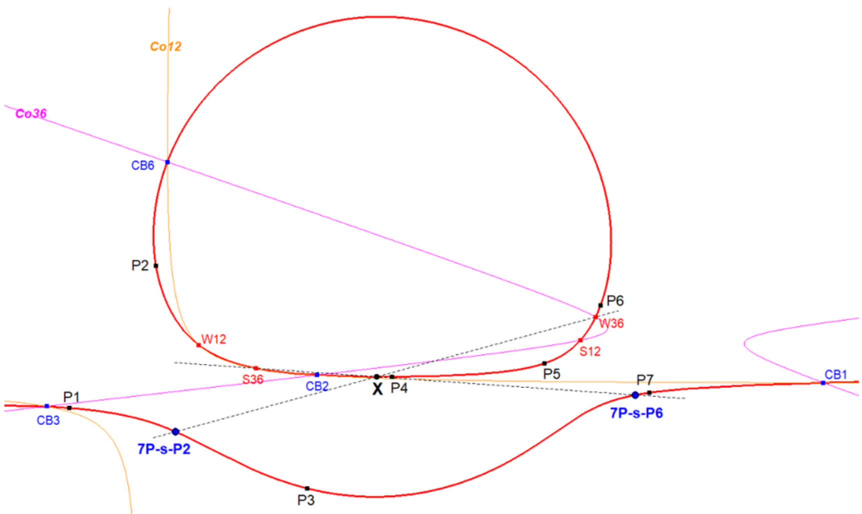


5. All Conics(CBi,CBj,CBk,CBl,Skl) have common points Wij on 9P-s-Cu1, where i,j are fixed numbers and k,l are variable numbers and i,j,k,l are different numbers from the range (1,..,7).
   There are 21 different Wij-points.
   A very special case is Wij(CI1,CI2) = common point of conics (CB(CI1), CB(CI2), CBi, CBj, Sij) = a real point 7P-s-P6 on the cubic. Numerical calculations show that CB(CI1) and CB(CI2) are finite imaginary points. Nevertheless, they have a real common intersection point with conics combined with CBi, CBj, and Sij, which is confirmed by numerical calculations.

   Construction of 7P-s-P6 = 3rd 7P-s-Cu1 Pivot Point.
   It is known that Sij.Wkl ^ Skl.Wij is a point on the cubic,
   - for all (i,j) and (k,l) being different numbers from (1,2,3,4,5,6,7),
   - or (0,0) when representing the circular points at infinity.

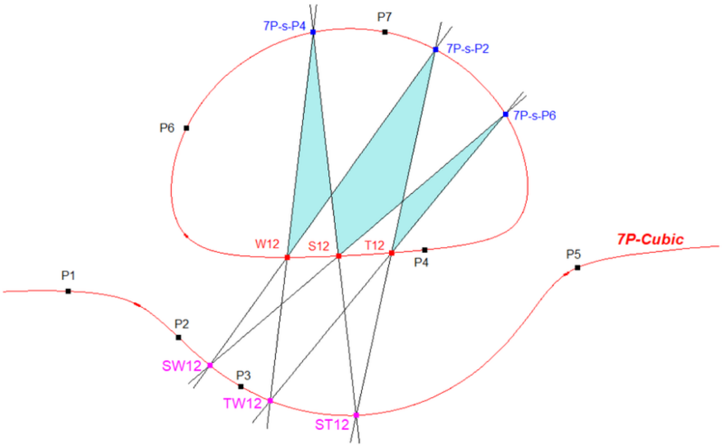
CB-related points-01.pdf

Let's choose ij=36 and let kl=00.
Then S36.W00 ^ S00.W36 will be a point X on the cubic.
The point we want to construct is W00 = 7P-s-P6 and we know that S00 = 7P-s-P2.
To construct W00, we need X = the 3rd intersection point of 7P-s-P2.W36 with the cubic.
This point we can construct according to *).
Now we know X on the cubic and also that the line S36.W00 passes through X, so W00 will be the 3rd intersection point of X.S36, which can be constructed according to *).
*)
We know two conics through 6 points on the cubic because they already are used for the construction of W12 and W36. They are
Co12=Conic (CB1,CB2,CB3,CB6,S36,W12) and
Co36=Conic (CB3,CB6,CB1,CB2,S12,W36).
Therefore the 3rd intersection point X of 7P-s-P2.W36 will be
CB(CB1,CB2,CB3,CB6,S36,W12,7P-s-P2,W36).
Because the degenerated cubic Conic (CB1,CB2,CB3,CB6,S36,W12) + Line(7P-s-P2,W36) passes through 8 points of the cubic the CB point of these 8 points should lie also on the degenerated cubic. It can't lie on the conic, so it will lie on the line. Two common points on the line already are taken, therefore it must be the 3rd common point of the line, which is the 3rd intersection point of the line with the cubic.
In the same way the 3rd intersection point of X.S36 with the cubic can be constructed by using Conic (CB3,CB6,CB1,CB2,S12,W36) and Line(X,S36).

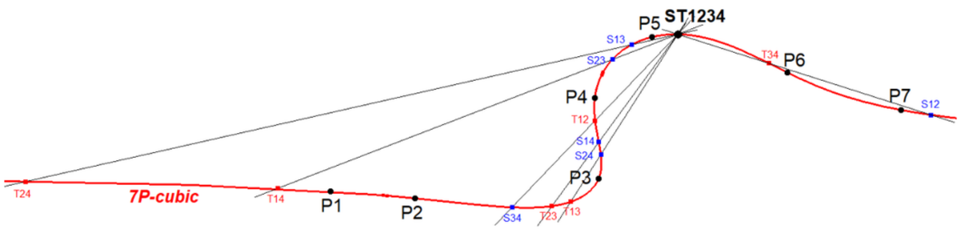


Now we know the location of 7P-s-P6 we can study the relationship with other known points. Right away it can be seen that 7P-s-P6 is the 3rd intersection point of 7P-s-P3.7P-s-P4 with the cubic. The elaborate construction of 7P-s-P6 was necessary to discover its location and this linear property.

6. Knowing 7P-s-P2, 7P-s-P4, 7P-s-P6 we can construct several new crosspoints on the cubic:
   * STij = 7P-s-P2.Tij ^ 7P-s-P4.Sij
   * TWij = 7P-s-P4.Wij ^ 7P-s-P6.Tij
   * SWij = 7P-s-P6.Sij ^ 7P-s-P2.Wij
   for all (i,j) being different numbers from (1,2,3,4,5,6,7).
   The order of (i,j) is not relevant. That gives 21 SWij-points and 21 TWij-points.



7. STij-kl = Sij.Tkl ^ Skl.Tij is another type of point on the circular cubic.
   There are 210 STij-kl-points.
   When i,j represent the circular points at infinity, then Vij-kl will be no longer dependent of ij, since the circular points at infinity are integral. Therefore we have STkl = Pz.Tkl ^ Px.Skl.
   Or STkl = 7P-s-P2.Tkl ^ 7P-s-Tf4.Skl, which is always a point on 7P-s-Cu1.
   There are 21 different STkl-points.
   Any combination of S-T-crosspoints, combining 4 different numbers, for example, 1,2,3,4 cross at the same cubical point:
   ST12-34 = S12.T34 ^ T12.S34
   ST13-24 = S13.T24 ^ T13.S24
   ST14-23 = S14.T23 ^ T14.S23
   It appears that ST12-34 = ST13-24 = ST14-23 = ST23-14 = ST24-13 = ST34-12, therefore order of the numbers is not important and we call this pivot-point ST1234.
   Considered in another way every line Sij.Tkl passes through the same cubical pivot point STijkl for all combinations (i,j,k,l) containing different numbers from (1,2,3,4).
   There are 35 versions of STijkl.



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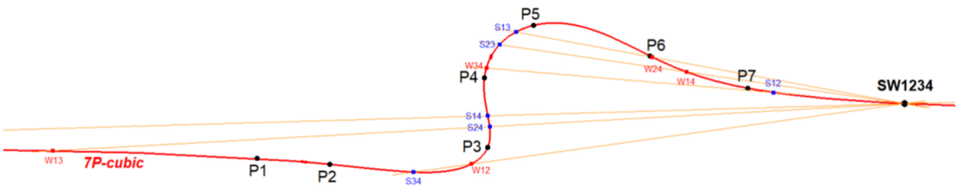
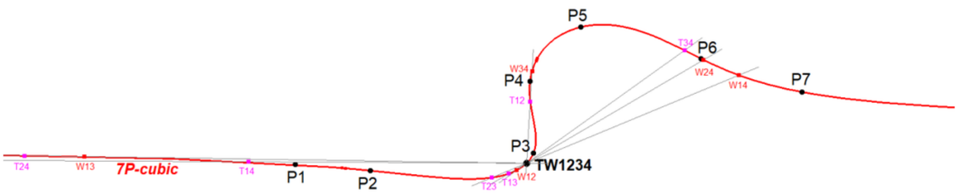
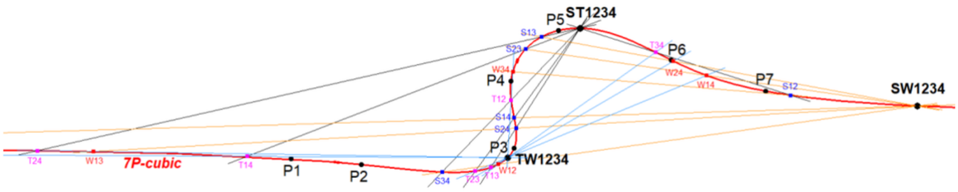
Similar properties for SWijkl.
There are 35 versions of SWijkl.
Similar properties for TWijkl.
There are 35 versions of TWijkl.

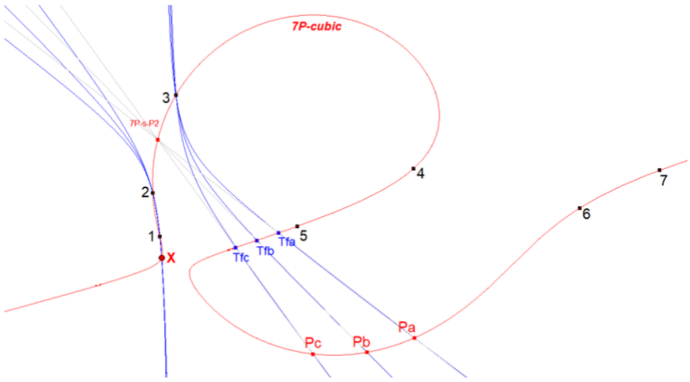




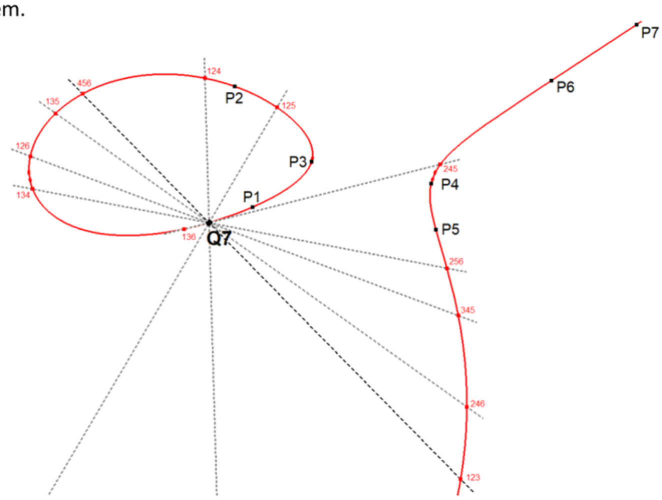
The ST1234, SW1234, TW1234-points in one picture:



12. There is another special series of 7 CB-related points on the 7P-cubic.
Let P1, ..., P7 be the defining points of the 7P-cubic, and let Pa, Pb, Pc be three random points on the cubic.
The conics:
(P1,P2,P3, Pa, CB(P1,...,P7,Pa))
(P1,P2,P3, Pb, CB(P1,...,P7,Pb))
(P1,P2,P3, Pc, CB(P1,...,P7,Pc))
concur in a cubical point X.
X is depending on 3 points on the cubic (P1,P2,P3 in our case). P1, P2, P3 can be variable points on the cubic and/or reference points of the cubic. Pa, Pb, Pc no longer play a role, because the outcome is independent of their locations. Just one random point Pa on the cubic suffices to construct X.



13. Denote P_123 = the point determined by P1,P2,P3 and more abstract P_i1j1k1= the point determined by P_i1,Pj1,Pk1.
There is a crosspoint Q7 = P_i1j1k1.P_l1m1n1 ^ P_i2j2k2.P_l2m2n2, where (i1,j1,k1,l1,m1,n1) as well as (i2,j2,k2,l2,m2,n2) are all different numbers from the set (1,2,3,4,5,6). There are 10 combinations of ijk-lmn and therefore 10 of these lines. They all cross in this point Q7. It's called Q7, because 7 is the missing number in the range (1,...,7) used for the 7P-cubic.
Similarly points Q1, Q2, ..., Q6 can be constructed. Just as the CB-points on the 7P-cubic there are 7 of them.

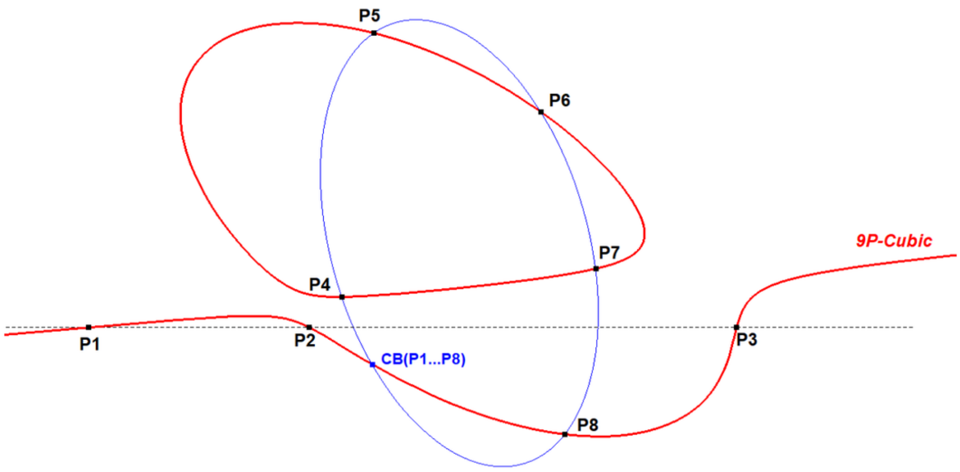


2. Some basic theorems

First, some basic theorems are needed for later results.

2.1 Theorem 1a

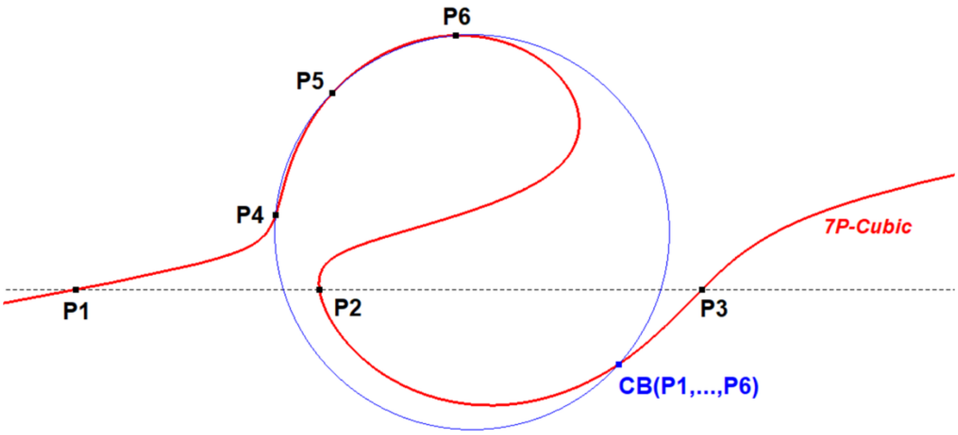
Let P1, …, P9 define a 9-Point Cubic Cu.
When 3 points (Pi,Pj,Pk) of (P1,…,P9) are collinear, then CB (Pi,Pj,Pk, Pl,Pm,Pn,Po,Pp) will be a point coconic with Pl,Pm,Pn,Po,Pp.



Proof
This follows from the definition of a Cayley-Bacharach Point. It is the concurring pivot point of all cubics passing through 8 random points. Since it is valid for all cubics it is also valid for degenerated cubics. We will consider a reference cubic refCu and a cubic degCu degenerated in a conic degCo and a line degLi, having 8 common points P1,…,P8.
We also use the fact that a cubic and a line have 3 intersection points and a cubic and a conic have 6 intersection points. Therefore degCu ^ refCu have 6 intersection points on degCo and 3 intersection points on degLi.
When we have 3 collinear points P1,P2,P3, then automatically the CB point of P1,P2,P3 and 5 other points P4,P5,P6,P7,P8 should be on the conic(P4,P5,P6,P7,P8), because it has to lie on the degenerated conic and there are no more intersection points available on the line, so it must be the 6th intersection point of refCu and conic(P4,P5,P6,P7,P8).

2.2 Theorem 1b

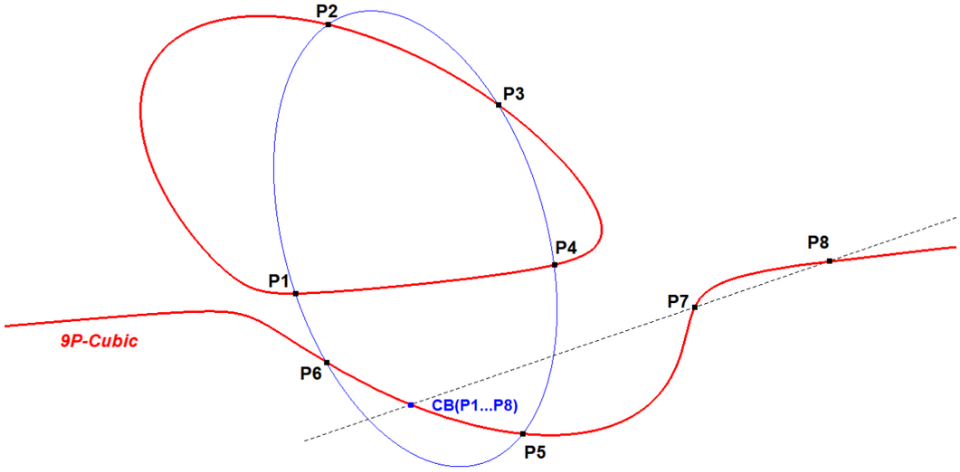
Let P1, …, P7 define a 7-Point Cubic Cu.
A 7-Point Cubic is a cubic passing through the two circular points at infinity and is therefore called a Circular Cubic. Because of the standard circular points at infinity only 7 points are needed to define the cubic instead of 9 points.
A 7-Point Cubic also has a CB-point, except that two of the 8 points are the circular points at infinity. Because they are standard, they won't be mentioned when applied to a Circular Cubic and therefore the CB-point for a Circular Cubic will be applied to 6 points instead of 8 points.
The equivalent of theorem 1a for a circular Cubic is:
When 3 points (Pi,Pj,Pk) of (P1,…,P7) on a 7P-cubic are collinear, then CB (Pi,Pj,Pk, Pl,Pm,Pn) will be a point concyclic with Pl,Pm,Pn.



Proof
The proof is analogous to the proof of theorem 1a.

2.3 Theorem 2a

Let P1, …, P9 define a 9-Point Cubic Cu.
When 6 points (Pi,Pj,Pk,Pl,Pm,Pn) of (P1,…,P9) are coconic, then CB (Pi,Pj,Pk,Pl,Pm,Pn, Po,Pp) will be a point collinear with Po and Pp.

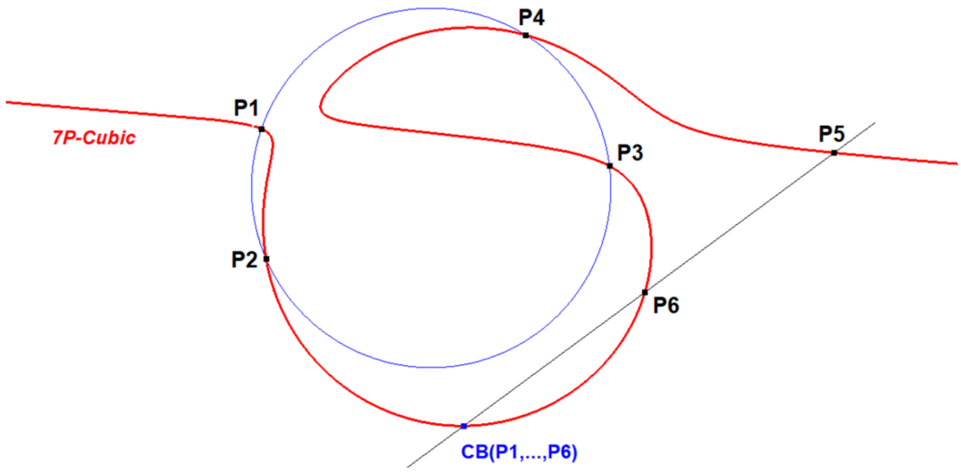


Proof
This follows from the definition of a Cayley-Bacharach Point. It is the concurring pivot point of all cubics passing through 8 random points. Since it is valid for all cubics it is also valid for degenerated cubics. We will consider a reference cubic refCu and a cubic degCu degenerated in a conic degCo and a line degLi, having 8 common points P1,…,P8.
We also use the fact that a cubic and a line have 3 intersection points and a cubic and a conic have 6 intersection points. Therefore degCu^refCu have 6 intersection points on degCo and 3 intersection points on degLi.
When we have 6 points P1,…P6 on a conic, then automatically the CB point of P1,…P6 and two other points P7 and P8 should be on the line P7.P8, because it has to lie on degCo or degLi, but there are no more intersection points available on the conic, so it must be the 3rd intersection point of refCu and P7.P8.

CB-related points-01.pdf

2.4 Theorem 2b

Let P1, …, P7 define a 7-Point Cubic Cu.
A 7-Point Cubic is a cubic passing through the two circular points at infinity and is therefore called a Circular Cubic. Because of the standard circular points at infinity only 7 points are needed to define the cubic instead of 9 points.
A 7-Point Cubic also has a CB-point, except that two of the 8 points are the circular points at infinity. Because they are standard, they won't be mentioned when applied to a Circular Cubic and therefore the CB-point for a Circular Cubic will be applied to 6 points instead of 8 points.
The equivalent of theorem 2a for a circular Cubic is:
When 4 points (Pi,Pj,Pk,Pl) of (P1,…,P9) are coconic, then CB (Pi,Pj,Pk,Pl, Pm,Pn) will be a point collinear with Pm and Pn.



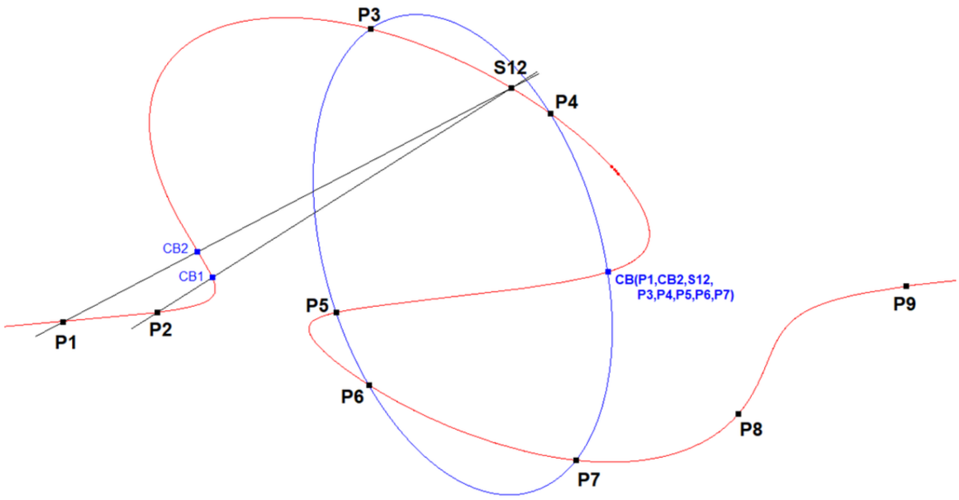
Proof
The proof is analogous to the proof of theorem 2a.

3. Crossing of cubics, conics, and lines

In the following part intersections of a 7P-/9P-cubic with another cubic, conic, circle, or line are discussed.

3.1 9P-Cubic ^ 5P-Conic with 5 common points

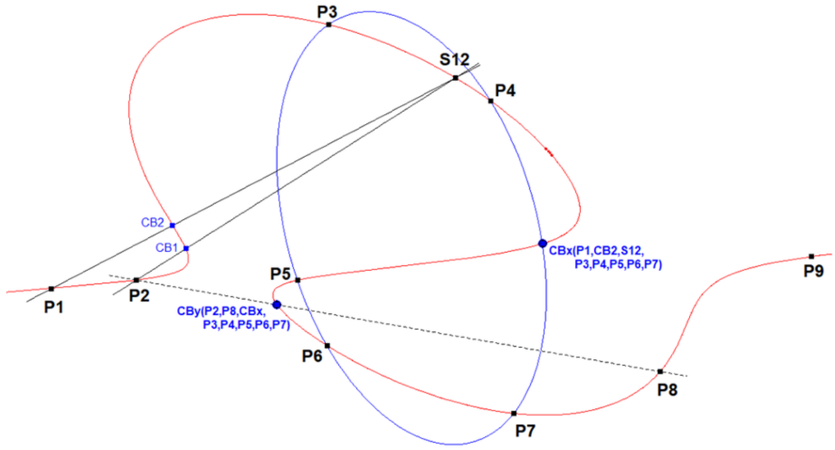
The property that Sij = Pi.CBj^Pj.CBi is a cubical point offers the possibility for constructing the 6th intersection point of a conic passing through 5 points of the 9-Point Cubic (P1, … ,P9).



Proof:
First some point Sij can be constructed from two of the defining points.
Let's assume S12 = P1.CB2 ^ P2.CB1 will be such point.
Now we have three collinear cubical points. Let's take P1, CB2, S12.
Applying theorem 1a it is clear that CB(P1,CB2,S12, P3,P4,P5,P6,P7) is the 6th intersection point of the P3-P4-P5-P6-P7-conic with the 9P-cubic.

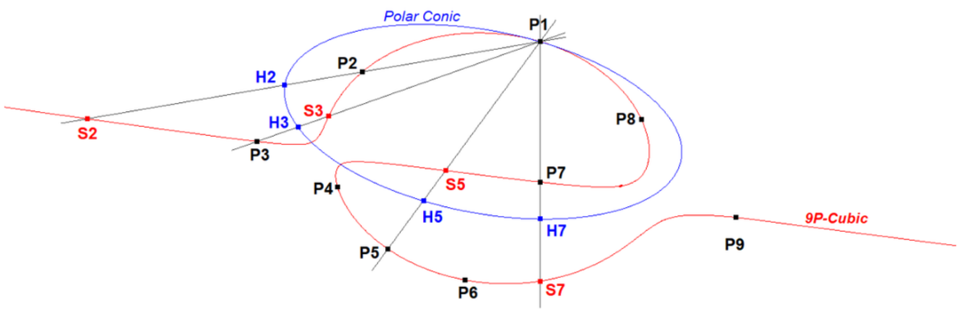
3.2 9P-Cubic ^ 2P-Line with 2 common points

Knowing the 6th intersection point of a 5P-conic intersected with a 9P-cubic it opens the door to finding the 3rd intersection point of a 2P-line intersected with a 9P-cubic.
First find the 6th intersection point of a 5P-conic intersected with a 9P-cubic as described before. Now we have 6 known points on the cubic. When we know two points of a line on the cubic, the 3rd intersection point will be the Cayley-Bacharach point of the 6 points on the conic and the 2 known points on the line.



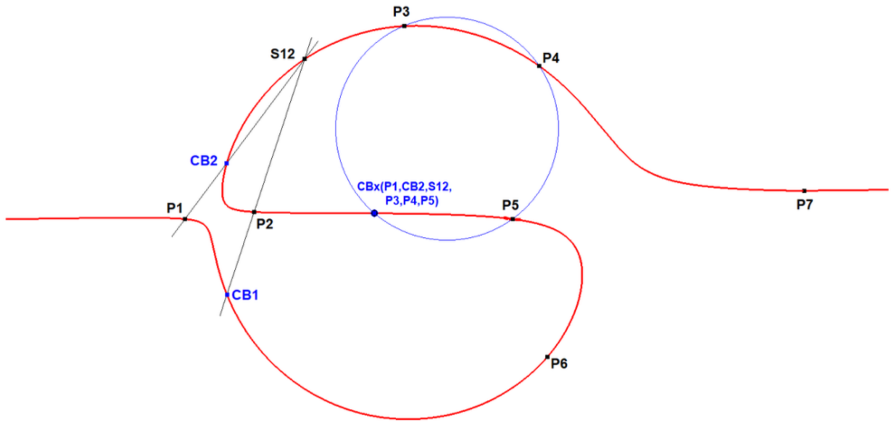
Note: The tangential is defined as the intersection point of the tangent at a point on the cubic with the cubic. Because of the aforementioned the tangential of P1 will be CB(P1,P1,CBx,P3,P4,P5,P6,P7). However, because of the double use of the point P1 calculations and constructions will get stuck. Nevertheless the tangential at least can be approximated by using a second nearby point P1a.

Construction Polar Conic
An application of determining the 3rd intersection point of a line with a 9P-cubic is the construction of the polar conic at some point P on the cubic. Start with constructing the 3rd intersection points Si of P1.Pi with the cubic for 4 points Pi. Then construct the points Hi=harmonic conjugate of P1 wrt Si.Pi. The conic(P1,Hi,Hj,Hk,Hl) will be the P1-polar conic wrt the 9P-cubic. This method can be found at [5].



3.3 7P-Cubic ^ 3P-Circle with 3 common points

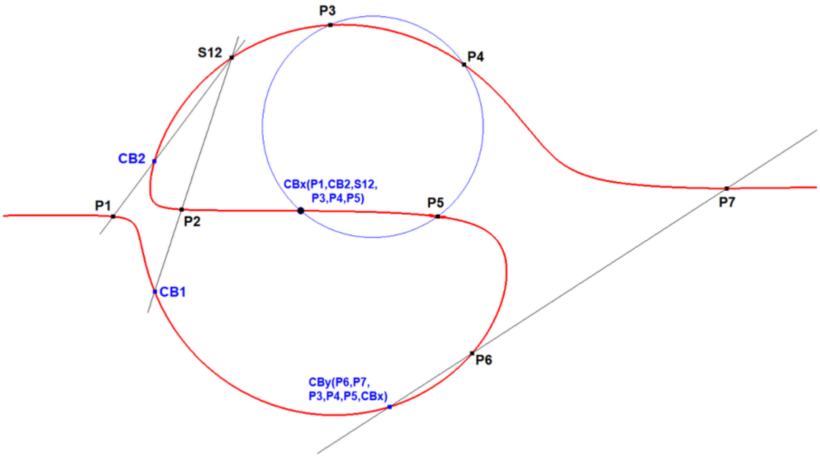
The Sij-property offers the possibility for constructing the 4th intersection point of a circle passing through 3 points of the 7-Point Cubic (P1, … ,P7).
The intersection points between a cubic and a circular cubic are the 2 circular imaginary points at infinity, which they both have in common, and 4 real points.
When we draw a circle through three random points on the cubic, the 4th intersection point can be constructed as shown in the next picture.



Proof:
First some point Sij can be constructed from two of the defining points.
Let's assume S12 = P1.CB2 ^ P2.CB1 will be such point.
Now we have three collinear cubical points. Let's take P1, CB2, S12.
Applying theorem 2a it is clear that CB(P1,CB2,S12, P3,P4,P5) is the 4th intersection point of the P3-P4-P5-circle with the 7P-cubic.

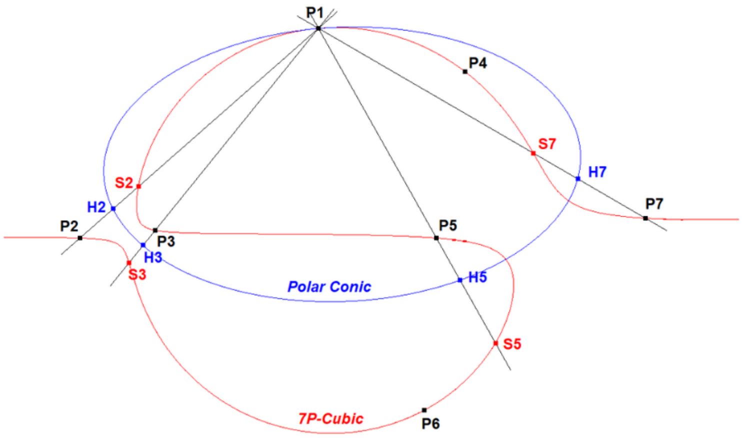
3.4 7P-Cubic ^ 2P-Line with 2 common points

Knowing the 4th intersection point of a 3P-circle intersected with a 7P-cubic it opens the door to finding the 3rd intersection point of a 2P-line intersected with a 7P-cubic.
First find the 4th intersection point of a 3P-circle intersected with the 7P-cubic as described before.
Now we have 4 known points on the cubic. When we know two points of a line on the cubic, the 3rd intersection point will be the Cayley-Bacharach point of the 4 points on the circle and the 2 known points on the line.



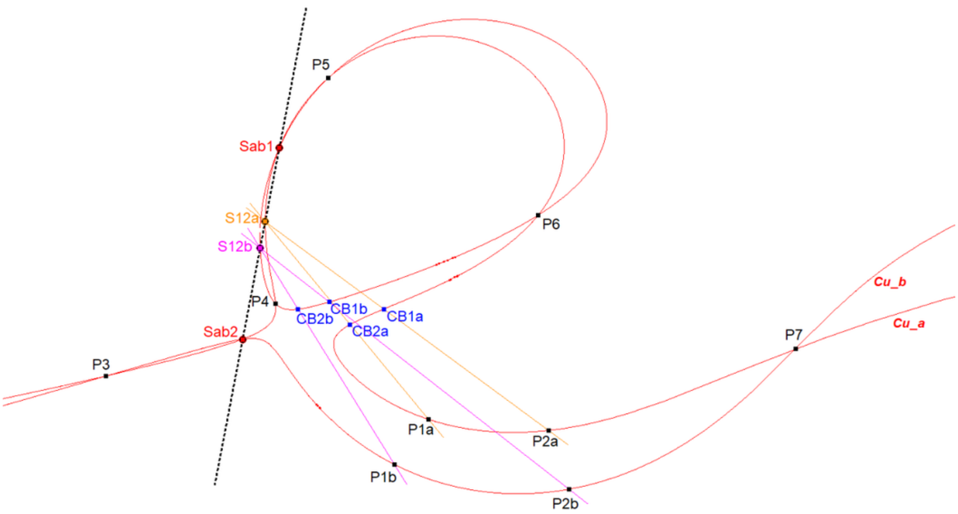
Note: The tangential is defined as the intersection point of the tangent at a point on the cubic with the cubic. Because of the aforementioned the tangential of P1 will be CB(P1,P1,P3,P4,P5,CBx). However, because of the double use of the point P1 calculations and constructions will get stuck. Nevertheless the tangential at least can be approximated by using a second nearby point P1a.

Construction Polar Conic
An application of determining the 3rd intersection point of a line with a 7P-cubic is the construction of the polar conic at some point P1 on the cubic. Start with constructing the 3rd intersection points Si of P1.Pi with the cubic for 4 points Pi. Then construct the points Hi=harmonic conjugate of P1 wrt Si.Pi. The conic(P1,Hi,Hj,Hk,Hl) will be the P1-polar conic wrt the 7P-cubic. This method can be found at [5].



3.5 7P-Cubic ^ 7P-Cubic with 5 common points

Let Cu_a and Cu_b be two circular 7P-cubics with 5 points P3, … , P7 in common and individual points P1a, P2a on Cu_a and P1b, P2c on Cu_b.
Let CBi = Cayley-Bacharach Points of Pi wrt the other 6 points.
Let Sij = Pi.CBj ^ Pj.CBi.
Specifically let S12a = P1aCB2a ^ P2a.CB1a and let S12b = P1bCB2b ^ P2b.CB1b.
Cu_a and Cu_b have 7 points in common, therefore there must be 2 extra intersection points.
Denote them with S1 and S2.
Theorem: S12a ,S12b, S1 and S2 are collinear.

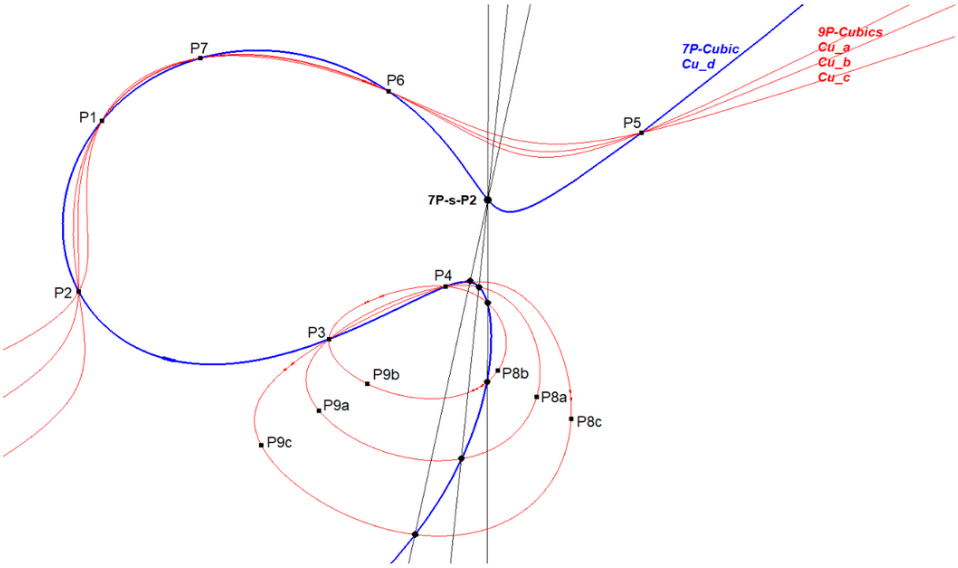


Proof
In general S2 = CB(P1,...,P5,S1) will be a point on **Cu_a** when S1 is on **Cu_a**.
In general S2 = CB(P1,...,P5,S1) will be a point on **Cu_b** when S1 is on **Cu_b**.
Therefore when S1 lies on Cu_a as well as Cu_b, S2 also will lie on Cu_a as well as Cu_b. This is only the case for intersection points, therefore Sab1 and Sab2 will be CB-partners.
And since CB-partners on Cu_a pass through the pivot point S12a and since CB-partners on Cu_b pass through the pivot point S12b, it must be that S12a, S12b, Sab1, and Sab2 are collinear.

CB-related points-01.pdf

3.6 7P-Cubic ^ 9P-Cubic with 7 common points

Theorem: The intersection points of all 9P-cubics intersecting with a 7P-cubic and all together with the same 7 points in common will have intersect-lines through a pivot-point of the 7P-cubic.

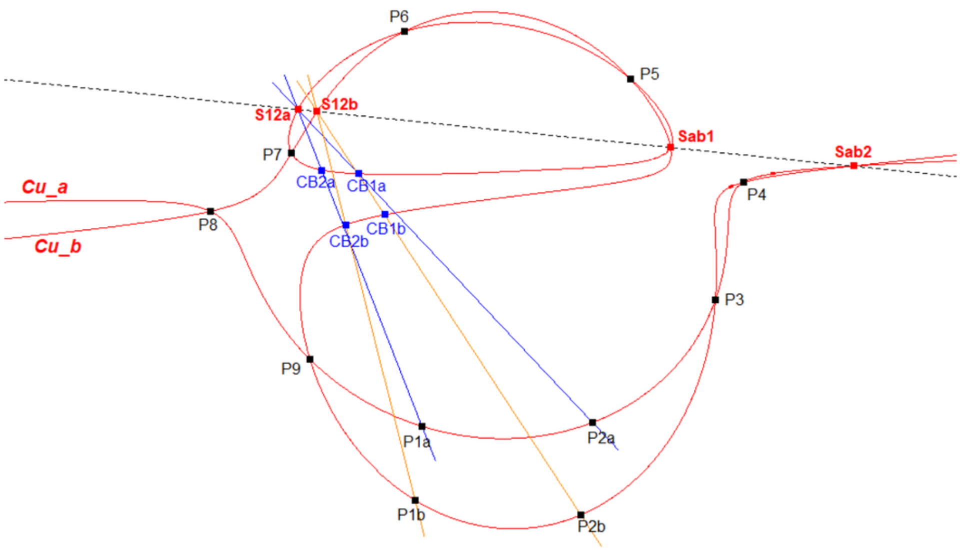


Proof:

1. First, it is good to realize that a 7P-cubic is a 9P-cubic, where the two missing points are the circular points at infinity.
2. There is a pivot point on a circular cubic where all connecting lines between a point on the cubic and its CB-point meet.
3. We discussed earlier that the two intersection points between two 7P-cubics are CB-partners.
4. Therefore all 9P-cubics intersecting with a 7P-cubic and all together with 7 points in common will have intersect-lines through this pivot-point. The name of the pivot point is 7P-s-P2.

3.7 9P-Cubic ^ 9P-Cubic with 7 common points

Let Cu_a and Cu_b be two regular 9P-cubics with 7 points P3, … , P9 in common and individual points P1a, P2a on Cu_a and P1b, P2c on Cu_b.
Let CBi = Cayley-Bacharach Points of Pi wrt the other 8 points.
Let Sij = Pi.CBj ^ Pj.CBi. It is the pivot point 7P-s-P2(P3,P4,P5,P6,P7,P8,P9).
Specifically let S12a = P1aCB2a ^ P2a.CB1a and let S12b = P1bCB2b ^ P2b.CB1b.
Cu_a and Cu_b have 7 points in common, therefore there must be 2 extra intersection points.
Denote them with Sab1 and Sab2.
Theorem: S12a ,S12b, Sab1, and Sab2 are collinear.

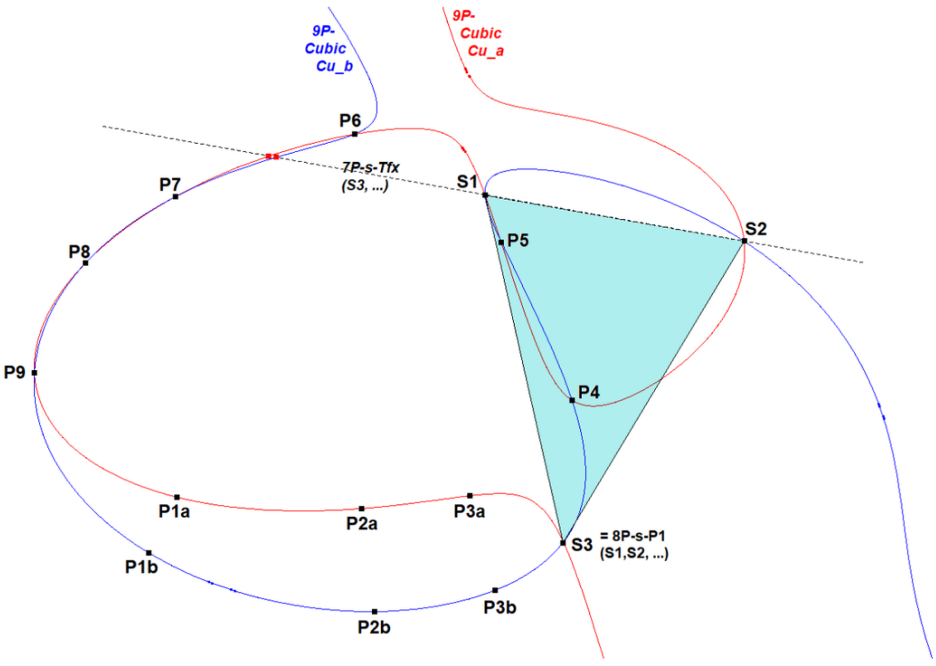


Proof
In general CB(P1,...,P7,S1) will be a point S2 on **Cu_a** when S1 is on **Cu_a**.
In general CB(P1,...,P7,S1) will be a point S2 on **Cu_b** when S1 is on **Cu_b**.
Therefore when S1 lies on Cu_a as well as Cu_b, S2 also will lie on Cu_a as well as Cu_b. This is only the case for intersection points, therefore Sab1 and Sab2 will be CB-partners.
And since CB-partners on Cu_a pass through the pivot point S12a and since CB-partners on Cu_b pass through the pivot point S12b, it must be that S12a, S12b, Sab1, and Sab2 are collinear.

CB-related points-01.pdf

3.8 9P-Cubic ^ 9P-Cubic with 6 common points

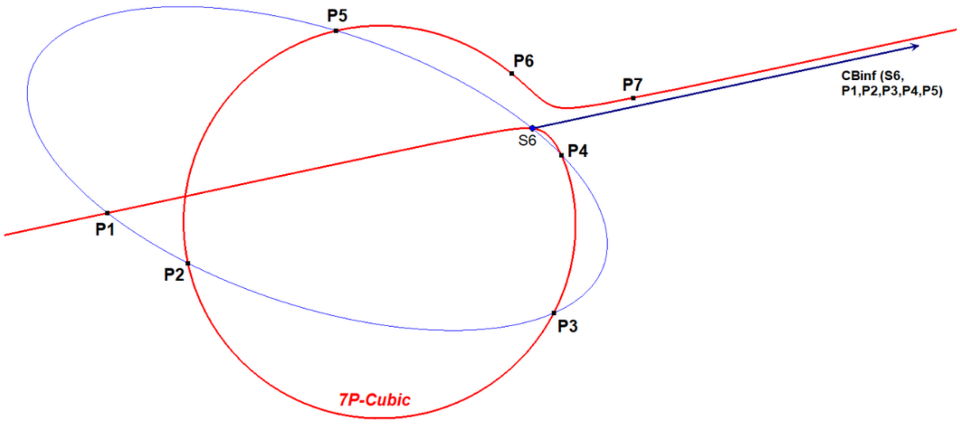
Let Cu_a and Cu_b be two cubics with 6 common points. Since two cubics always have 3x3=9 intersection points, 3 other intersection points are bound to be found. Let S1, S2, S3 be the unknown intersection points of Cu_a and Cu_b.
Note that we now have two cubics each passing through the same 9 points S1, S2, S3, P4, P5, P6, P7, P8, P9, without being identical.
This is possible because, when the CB-point is determined of 8 of these points, then the 9th point will be CB-point. This is because there is a mutual dependency between the points. The points are "inter-dependent". They should be "random". Nine inter-dependent points do not define a cubic.
Nevertheless, we still can find some interesting properties.
Let 7P-s-Tfx be the transformation that transforms 7 common points and separate points P1a,P2a and P1b,P2b of resp. cubics Cu_a, Cu_b into the line that connects the two intersection points.
Let's denote this transformation by 7P-s-Tfx(P3,P4,P5,P6,P7,P8,P9, P1a,P2a, P1b,P2b).
Further denote the Caley-Bacharach Point by 8P-s-P1(P1,P2,P3,P4,P5,P6,P7,P8).
Now line S1.S2 = 7P-s-Tfx(S3,P4,P5,P6,P7,P8,P9, P1a,P2a, P1b,P2b)
line S2.S3 = 7P-s-Tfx(S1,P4,P5,P6,P7,P8,P9, P1a,P2a, P1b,P2b)
line S3.S1 = 7P-s-Tfx(S2,P4,P5,P6,P7,P8,P9, P1a,P2a, P1b,P2b)
and S1 = 8P-s-P1(S2,S3,P1a,P4,P5,P6,P7,P8). (P1a also can be P2a or P3a)
S2 = 8P-s-P1(S3,S1,P1a,P4,P5,P6,P7,P8). (P1a also can be P2a or P3a)
S3 = 8P-s-P1(S1,S2,P1a,P4,P5,P6,P7,P8). (P1a also can be P2a or P3a).
Therefore if one of the 3 intersection points is known, the line through the others can be found (7P-s-Tfx), and if two of the 3 intersection points is known, the last point can be found (8P-s-P1=Cayley-Bacharach Point).
In a way, 7P-s-Tfx can be seen as a cubical polar mapping a point into a line.
Triangle S1S2S3 is self-polar wrt 7P-s-Tfx.
Similarly, 8P-s-P1 can be seen as a cubical pole mapping a line (defined by two points) into a point.
Triangle S1S2S3 is self-polar wrt 8P-s-P1.



CB-related points-01.pdf

3.9 Infinity point of the 7P-Cubic

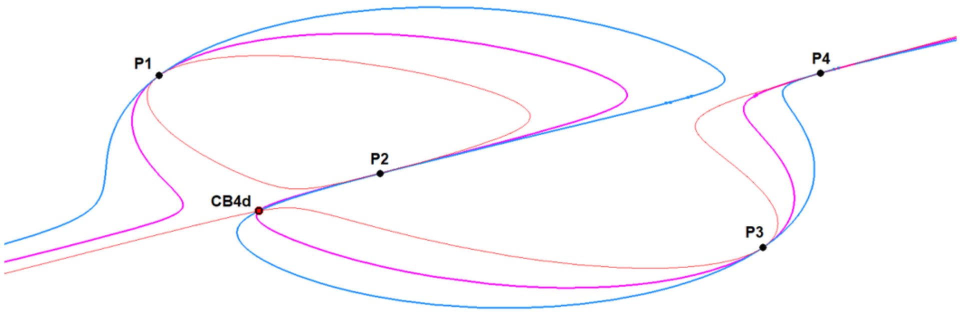
Knowing the 6th intersection point of a 5P-conic it gives the possibility to determine the Infinity Point of a 7P-Cubic again using the Cayley-Bacharach Point.
Let S6 be the 6th intersection point of the 5P-conic(P1,P2,P3,P4,P5). Now CB(S6,P1,P2,P3,P4,P5) will be the Infinity Point of the 7P-Cubic. Especially in calculations, this is very useful.



Proof:
This is easy to deduct:
7P-s-Cu1 is the cubic defined by:
P1,P2,P3,P4,P5,P6,P7 + CI1,CI2 (Circular points at infinity)
When a cubic is intersected with a conic there are 6 intersection points.
A conic plus a line form a degenerated cubic.
There are 9 intersection points with this degenerated cubic, being:
P1,P2,P3,P4,P5,S6,CI1,CI2 leaving one more intersection point that should be on the line CI1.CI2, which is the line at infinity.
This combined with the fact that the CB-point lies on the cubic we will have a point on the cubic at infinity, which only can be the infinity point of the cubic and the asymptote.

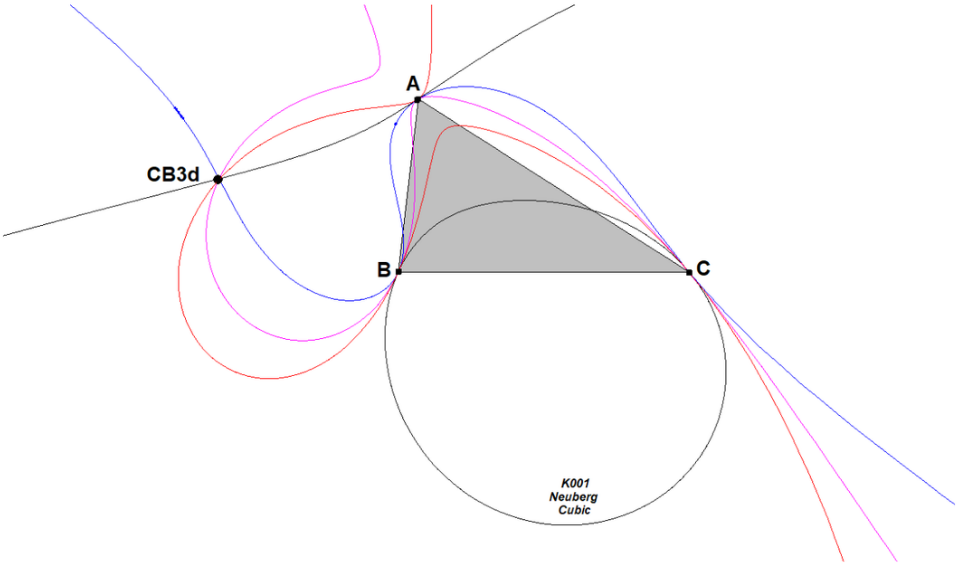
3.10 9P-Cubic ^ 9P-Cubic both mutually tangent at 4 common points

There is a situation that two or more 9P-cubics are mutually tangent at 4 common points.
Important is here the 4 points and the tangents at these points.
When we have one reference 9P-cubic with 4 points on it, then also the tangents are provided.
All P1P2P3P4-circumscribed 9P-cubics tangent at P1, P2, P3 and P4 to the reference cubic have a common point CB4d. It is a special case of the Cayley-Bacharach Theorem for regular cubics circumscribed around a quadrangle.
See [4]. QPG-messages #794-#814 for more information.



3.11 7P-Cubic ^ 7P-Cubic both mutually tangent at 3 common points

There is a situation that two or more 7P-cubics are mutually tangent at 3 common points.
Important are here the 3 points and the tangents at these points.
When we have one reference 7P-cubic with 3 points on it, then also the tangents are provided.
All P1P2P3-circumscribed 7P-cubics tangent at P1, P2, and P3 to the reference cubic have a common point CB3d. It is a special case of the Cayley-Bacharach Theorem for circular cubics circumscribed around a triangle.
In next picture is given an example of the unique CB3d point on the Neuberg Cubic.
This point CB3d appears to be X(2133) in the Encyclopedia of Triangle Centers. See [6].
See [4]. QPG-messages #794-#814 for more information.



4. References

[1] A.S. Hart - Construction by the Ruler alone to determine the ninth Point of Intersection of two Curves of the third Degree. Cambridge and Dublin Mathematical Journal 6 (1851) 181-182.

[2] Qingchun Ren, Jürgen Richter-Gebert & Bernd Sturmfels (2015) Cayley–Bacharach Formulas, The American Mathematical Monthly, 122:9, 845-854, DOI: 10.4169
Available at: https://arxiv.org/pdf/1405.6438v2.pdf

[3] Encyclopedia of Polygon Geometry, n-Point Information
Available at: https://www.chrisvantienhoven.nl/np-items/np-geninf/np-0

[4] Quadri-and-Poly-Geometry (QPG), an Internet forum for discussions on topics related to the Geometry of Quadrilaterals & Polygons.
Available at: https://groups.io/g/Quadri-and-Poly-Geometry

[5] Henry Martyn Cundy and Cyril Frederick Parry, Some cubic curves associated with a triangle, Journal of geometry Vol. 53 (1995), 2.15 page 45.

[6] Encyclopedia of Triangle Centers (ETC)
Available at: https://faculty.evansville.edu/ck6/encyclopedia/etc.html

CB-related points-01.pdf

Message: #840
Date: 2021-03-18
From: bernard.keizer@gmail.com
Subject: Re: New sight of bipartite circular cubics

Dear Eckart,
In your last remark, I suppose you mean for bipartite circular cubics (non necessary focal).
Following your discussion with Chris, I wonder finally what's really new in all these properties !
It is now clear that a circular cubic 7P-s-Cu1 is a QA-Cu1.
Any point of the cubic is then the QA-P4 of it's tangential quadrangle.
Any 7 points on the cubic (including the circular points) define a pivotal CB or cb transformation.
Any line through the point Q where the curve cuts it's asymptote cuts the curve in 2 other points equidistant from the focus F.
Using all these well-known properties, you were able to find the points Q and F (QA-P9).
Now the circle with diameter QF cuts the curve in 3 points (other than Q) which are the vertices of QA-Tr2 and centers of the 3 Moebius transformations of the cubic.
The in- and excenters of QA-Tr2 are the points where the tangents are parallel to the asymptote (centers of anallagmaty of the cubic).
New for me was the rectangular hyperbola through the middles of points on parallels to the asymptote ...
Best regards
Bernard
PS Sorry if I didn't react earlier, I'm busy with the redaction of the last promised item about my 5L's transformation swapping the lines and the VR's

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Message: #841
Date: 2021-03-18
From: van10hoven@gmail.com
Subject: Re: New sight of bipartite circular cubics

Dear Bernard,

Thanks for the concise overview.
I think we came at the same point after walking different ways.
I am glad you got it sorted out so well.

So we can say that:
· Choose on a bipartite 7P-s-Cu1 some point P with 4 real tangents. The 4 points of tangency of P will form a QA.
· Point P will be the QA-P4 point of this QA.
· The cubic 7P-s-Cu1 will be the QA-Cu1 cubic of this QA.

What can we say in a similar way about QA-Cu7 and QL-Cu1?

Best regards,
Chris

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Message: #842
Date: 2021-03-19
From: bernard.keizer@gmail.com
Subject: Re: New sight of bipartite circular cubics

```
Dear Eckart,
For a QA-Cu1, it goes in particular that Q is the QA-P4 of the
QA formed by the vertices of QA-Tr2 and the infinity point of
the asymptote, which is it's tangential QA.
QA-Cu7 is a QL-Cu1
If a QL-Cu1 is bicursal, it has the same properties as a QA-Cu1
with a point and 4 tangents, with QA-P2 as CSC of the point as
QA-P4 ...
If QL-Cu1 is monocursal, a point has only 2 real tangents ; the
contact points are CSC partners, the circle through the point
and the contact points of the 2 tangents intersect the curve in
a 4th point, which is the focus QL-P1. Q is the tangential of
QL-P1 and the tangents from QL-P1 give as contact points the 2
points QL-2P2 on the Newton Line. The asymptote is the parallel
to the Newton Line, it's homothetic in the homothety (QL-P1,2)
...
Best regards
Bernard
```

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Message: #843
Date: 2021-03-20
From: eckart_schmidt@t-online.de
Subject: Re: 5P focal circular circumcubics

```
Dear Bernard,

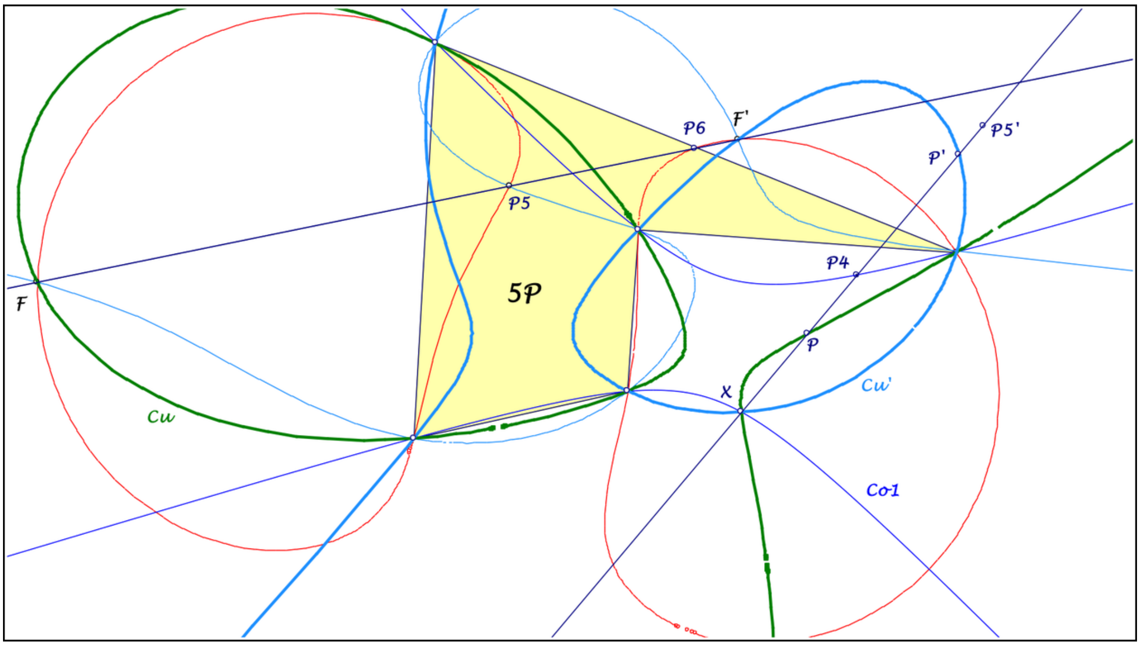
I have studied your 2nd paper "Pentangle's focal circular
circumcubics",
... but not all passages,
    for it's a hard work with your nomination and your figures.
An answer to your leading question "... under which conditions
this circular cubic of 5 points is focal..."
... is first mentioned in #724:
Focal circular 5P-circumcubics have their focus on the
5P-quartic.
You show a way to this result with Möbius transformations,
... is the following summary correct (prefix 5P-s- omitted)?

CSC1 = Tf8
       centered in P6, P5 <--> P5 of Tf8(5P)
       quartic <--> twincubic twinCSC2
       centered in P5 of Tf8(5P), P6 <--> P4
       twincubic <--> twincubic
CSC3 = CSC1*twinCSC2*CSC1 = 5P-n-Tf1
       centered in Tf8(P4), P6 <--> P5
       quartic <--> quartic
CSC4 = CSC1*CSC3 = twinCSC2*CSC1
       Tf6-pivot ---> focus
       twincubic ---> quartic

But the last CSC4 is not a Möbius transformation (see #771).

Here a final remark to your last example on page 6:
P5 is an intersection of the 5P-quartic and the 5P-cubic
(attached),
... there are two further -not always real- intersections on
    the line P5.P6,
... which can be considered as foci F, F' of focal circular
    5P-cubics Cu, Cu',
... with 6th intersection X on Co1,
... whose degenerated Tf7-circle is P5.P6
... and whose connection with P4 bears the pivots P, P'
    and P5' = Tf8(P5)
... and has the direction of the asymptotes of the cubics.


Best regards Eckart
```



2021-03-20.pdf

Message: #844
Date: 2021-03-20
From: bernard.keizer@gmail.com
Subject: Re: 5P focal circular circumcubics

Dear Eckart,
Many thanks for your attentive reading !
Your summary is correct.
CSC1 is Tf8, but why is CSC3 5P-n-Tf1 ? Is twinCSC3 5Q-n-Tf1 ?
What would be a correct name for CSC2 and twin CSC2 ?
You're right, CSC4 is not a Moebius transformation (but it has 2 fixed points on both the quartic and the twin cubic) and I should have avoided to name it CSC !!!
Thanks for your last example with figure : naturally, P4P5' and P5P6 intersect on the rectangular hyperbola and the 5P FCC with pivot P5' and focus P6 (associate of the main 5P cubic with pivot the infinity point of the Newton Line) has the same point S (X on your figure) and the same direction of asymptote.
Best regards
Bernard

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Message: #845
Date: 2021-03-20
From: eckart_schmidt@t-online.de
Subject: Re: 5P focal circular circumcubics

Dear Bernard,

wrt your question "... why is CSC3 5P-n-Tf1? " please have a look in #711 PS and #705.

Best regards Eckart

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Message: #846
Date: 2021-03-21
From: bernard.keizer@gmail.com
Subject: Re: 5P focal circular circumcubics

Dear Eckart,
Thanks to you, I won't die stupid !
I read again Chris message 689 and I understand that CSC3 is 5P-n-Tf1 (and twinCSC3 is 5Q-n-Tf1).
Now, for CSC2 and twin CSC2, it's possible to avoid the naming.
If CSC3 = CSC1*twinCSC2*CSC1, twinCSC2 = CSC1*CSC3*CSC1, which is 5P-s-Tf8*5P-n-Tf1*5P-s-Tf8 (ugly, isn'it ?)
and CSC2 is the same way 5P-s-Tf8*5Q-n-Tf1*5P-s-Tf8.
I regret this necessary 5P-references.
In fact, without a 5P, for a given CSC1 centered in M and swapping 2 pointsS and S' and a twinCSC2 centered in S' and swapping M with a point T, the CSC3 centered in CSC1(T) swaps S and M (S gives S', which gives the infinity point, which gives M) and the transformation CSC1*twinCSC2 (which is not a Moebius transformation) swaps any line through T to a line through S, both lines intersecting on a rectangular hyperbola centered in the middle of TU through T and U, with 2 points symmetric wrt this middle of TU and being at the same time CSC1 and twin CSC2 partners ...
Best regards
Bernard

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Message: #847
Date: 2021-03-21
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard and Eckart,

Given a 7P-cubic 7P-s-Cu1, we now have a construction method to
construct an inscribed QA such that 7P-s-Cu1 = QA-Cu1.
1. Is there a construction method to construct an inscribed QA
such that 7P-s-Cu1 = QA-Cu7 ?
2. Is there a construction method to construct an inscribed QL
such that 7P-s-Cu1 = QL-Cu1 ?

Best regards,
Chris
```

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Message: #848
Date: 2021-03-21
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris, dear Eckart,
I've already answered this question in my message #842 !
Having a bicursal 7P-s-Cu1, it can be a QA-cu1 with 3 QA-Tr2
vertices; if the focus is one of the 3, it is a QL-Cu1. (Again,
QA-Cu7 is a QL-Cu1)
Having a monocursal 7P-s-Cu1, I suppose it is always a QL-Cu1.
Best regards
Bernard
```

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Message: #849
Date: 2021-03-22
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris,

in #847 you wrote:
"Given a 7P-cubic 7P-s-Cu1, we now have a construction method to construct
an inscribed QA such that 7P-s-Cu1 = QA-Cu1.
1. Is there a construction method to construct an inscribed QA such that
7P-s-Cu1 = QA-Cu7 ?
2. Is there a construction method to construct an inscribed QL such that
7P-s-Cu1 = QL-Cu1 ?"

Wrt the introduction:
For a point on a bipartite 7P-s-Cu1 with four tangents at the cubic
... the contact points give a QA, whose QA-Cu1 is 7P-s-Cu1.
Do you have a construction for these contact points?
If 7P-s-Cu1 is monopartite, there will be no QA-construction with 7P-s-Cu1
= QA-Cu1,
... for there exists no point on the cubic with four tangents.

Wrt 2.
If 7P-s-Cu1 is bipartite, there is per introduction a QA with 7P-s-Cu1 =
QA-Cu1.
Further QA' with this QA-Cu1 can be constructed
... with the Möbius transformations of QA-Tr2.
Their QA-Cu1 can only be considered as QL-Cu1,
... if the QA' has vertices pairwise on orthogonal lines (see ref in PS),
... that is the case for the QA' of the in- and excenters of QA-Tr2.
If 7P-s-Cu1 is monopartite, there will be no QA-construction with 7P-s-Cu1
= QL-Cu1,
... for there exists no QA with 7P-s-Cu1 = QA-Cu1.

Wrt 1.
If 7P-s-Cu1 is bipartite, CABRI-observations show,
... that all QA' (see above) have a QA-Cu7 unequal QA-Cu1,
... (see also EQF QA-Cu7, properties, last point).

If 7P-s-Cu1 is monopartite, there will be no QA-construction
with 7P-s-Cu1
= QA-Cu7,
... for QA-Cu7 is always bipartite.

Best regards Eckart

PS:
2016-01-13.pdf (eckartschmidt.de)
<http://eckartschmidt.de/2016-01-13.pdf>
2015-12-04.pdf (eckartschmidt.de)
<http://eckartschmidt.de/2015-12-04.pdf>

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Message: #850
Date: 2021-03-22
From: bernard.keizer@gmail.com
Subject: Pentalateral’s transformation PSI or Triple Moebius

```
Dear Chris, dear Eckart,
Dies war der letzte Streich
I like this 5L's transformation PSI (I had first named it Super
Moebius, but triple Moebius is more appropriate)
I hope you will enjoy it
Thanks to Eckart for our past discussions about this item
Best regards
Bernard
```

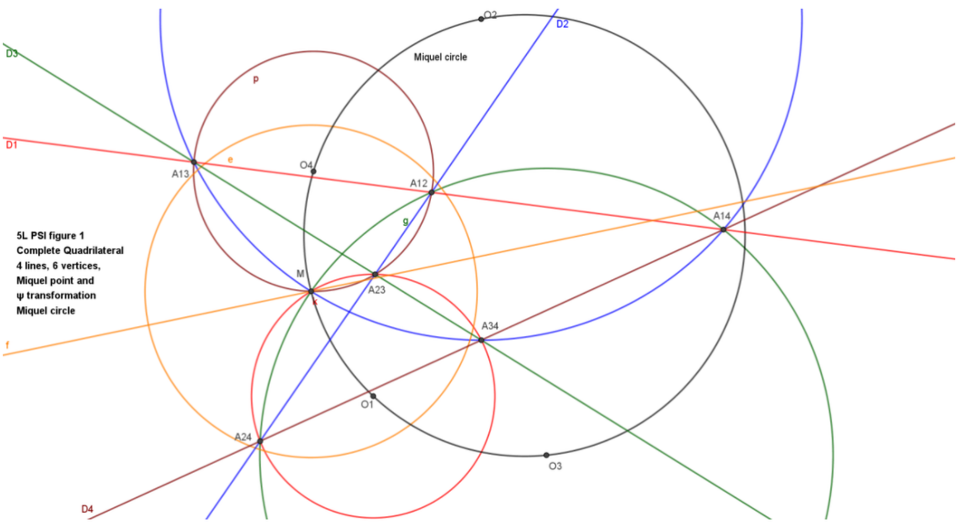
Pentalateral's transformation Ψ or Triple Moebius

A. Complete Quadrilateral

Let's first consider a quadrilateral 4L with it's 6 vertices Aij, it's point QL-P1 and it's Cl-S transformation QL-Tf1, which swaps 2 opposite vertices and the foci of all inscribed conics.

The 4 degenerated cubics formed by a line through 3 vertices and the circumcircle through the 3 other pass through the 6 vertices of the QL, through QL-P1 and through the circular points.

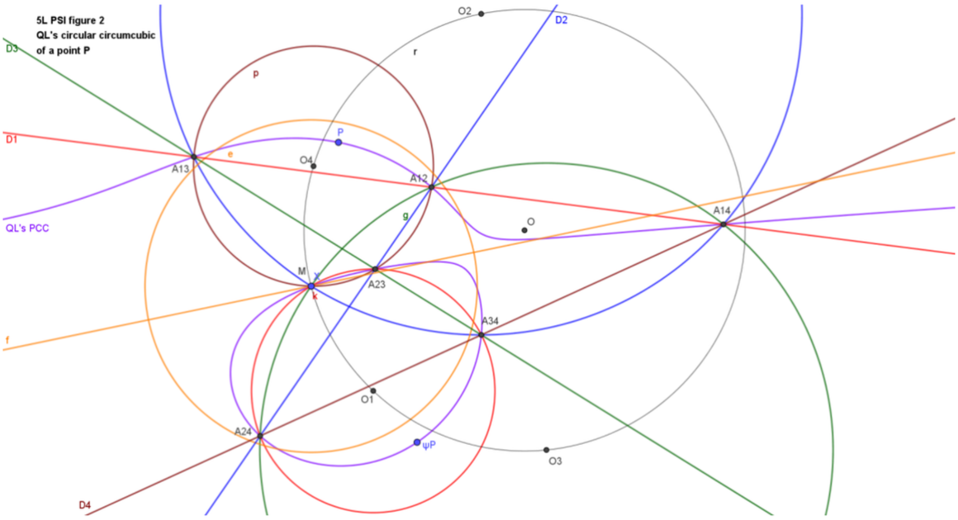
These 9 points form therefore a CB system. The Cl-S swaps each of the 4 lines with the corresponding circle and more generally a point with a point and a line not through QL-P1 with a circle through QL-P1.



There are an infinity of cubics through these 9 points and we need another point P in order to identify a specific circular circumcubic of the 6 vertices of the QL and QL-P1.

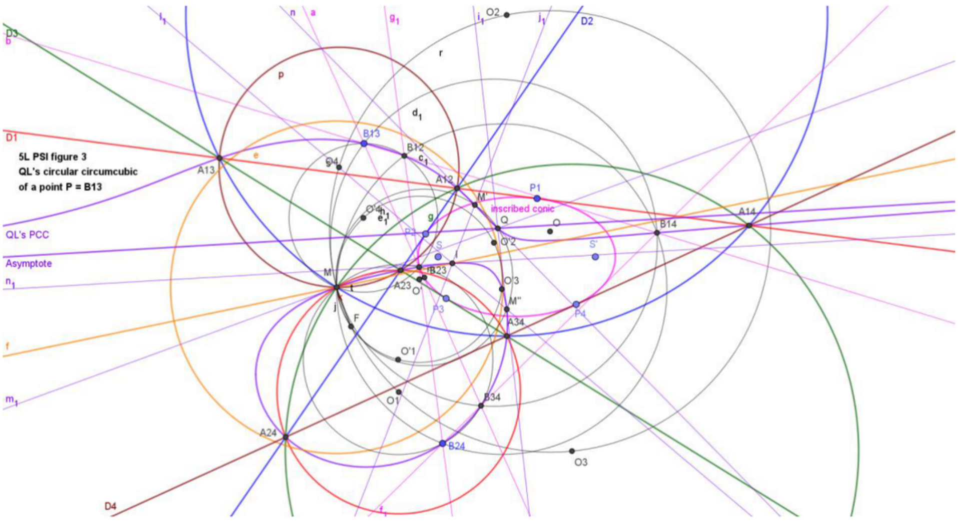
The circular circumcubics of the 6 vertices and a 7th point P pass through QL-P1 and are all invariant in the Cl-S or QL-Tf1 transformation ψ and pass through ψP (CSC partner of P).

5L PSI.pdf

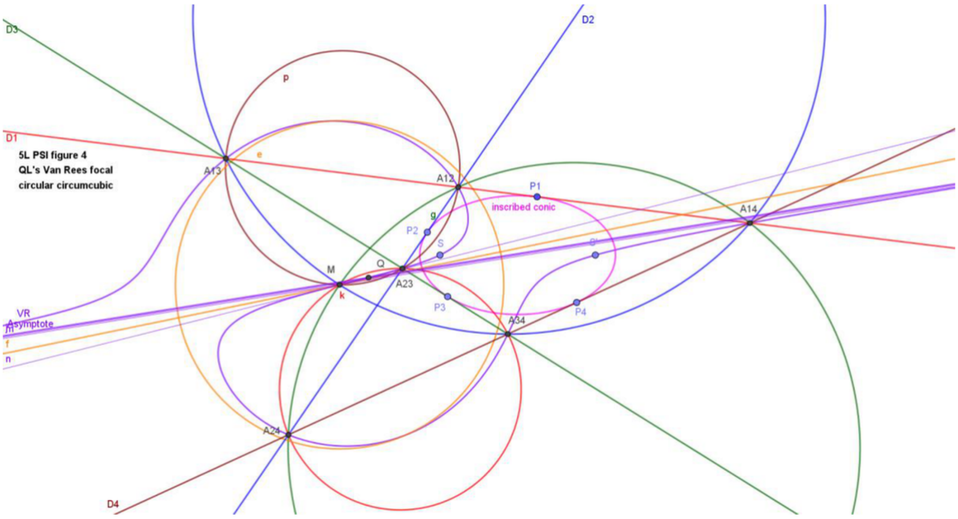


This QL's P circular circumcubic is a 7P-Cu1 and a QA-Cu1 and is generally not focal. It can be seen as the locus of the vertices of the QL's formed by the 2 tangents in P and ψP to all QL's inscribed conics (including the QL's parabola QL-Co1). These cubics are Poncelet-Darboux curves for any inscribed conic, meaning that for any point of the curve the tangents to the conic in this point and it's CSC partner form a 3rd QL inscribed in the curve and circumscribed to the conic.

The focus F of the cubic is the 2nd intersection (apart of QL-P1) of the Miquel circles of the 2 QL's. The perpendicular in M to MF gives the point Q, where the asymptote cuts the curve and the circle with diameter QF gives the 2 other vertices of the triangle QA-Tr2 of the Miquel points. The in- and excenters of this triangle are on the curve, with tangents parallel to the asymptote, which allows to draw it through Q. These 4 points are the centers of anallagmaty of the curve.



There is only one Van Rees focal circular circumcubic of the 6 vertices of a QL, it is QL-Cu1 with focus in QL-P1. This cubic is the locus of the foci of all QL's inscribed conics. For a given inscribed conic with foci S and S', all the QL's inscribed in the cubic and circumscribed to the conic have different VR's intersecting in QL-P1, S and S' and this curve is a particular Poncelet-Darboux curve wrt the conic.

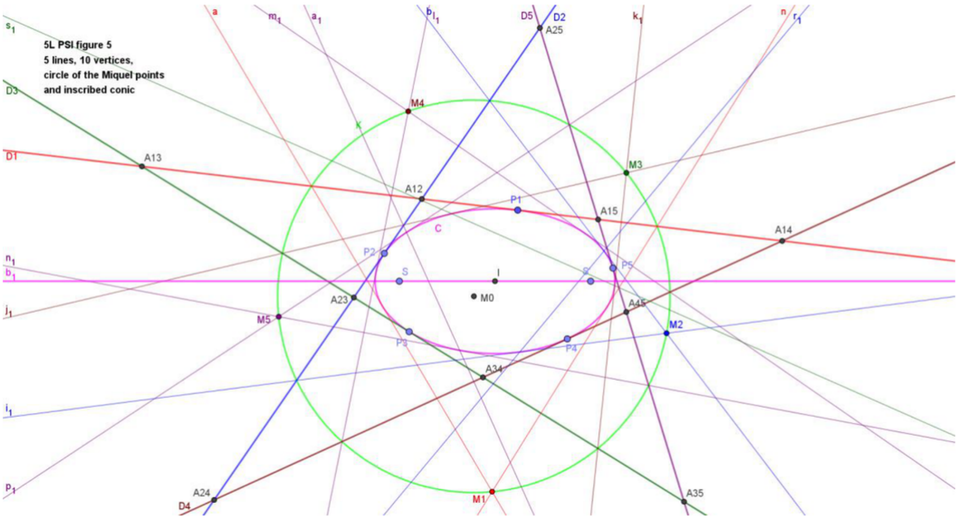


Let's remark in conclusion that the CB partner of the 6 vertices, QL-P1 and P is not ψP. The transformation ψ and the CB transformation of the 6 vertices and QL-P1 are different.

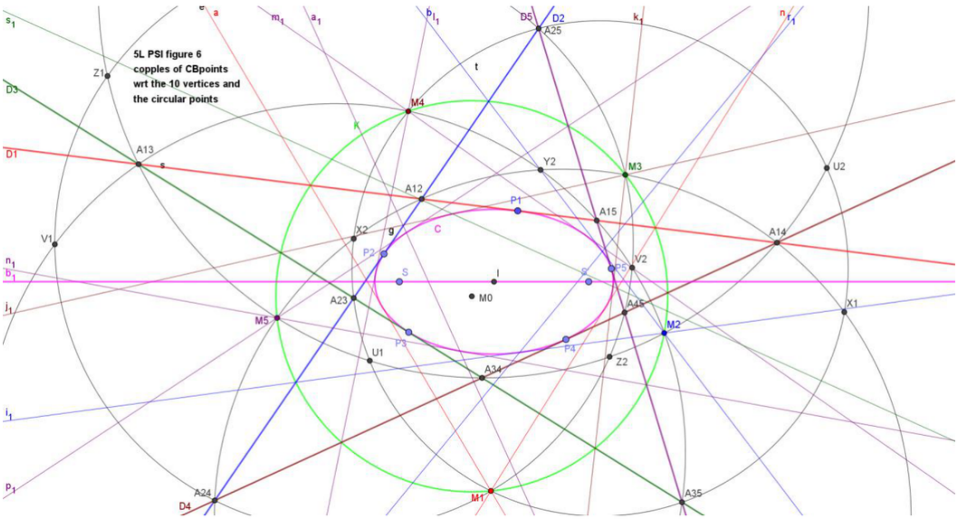
B. Pentalateral

Let's consider now a pentalateral 5L with it's 10 vertices Aij, it's inscribed conic 5L-s-Co1, hereafter inconic with foci S and S' and it's circle of Miquel points 5L-o-Ci1, hereafter Miquel circle.

The Miquel circle is a Poncelet-Darboux curve for the insconic ; the 5 triangles formed by the lines Li and the tangents to the inscribed conic in the corresponding Miquel points Mi of the QL's formed by the 4 other lines are inscribed in the Miquel circle and circumscribed to the inconic.

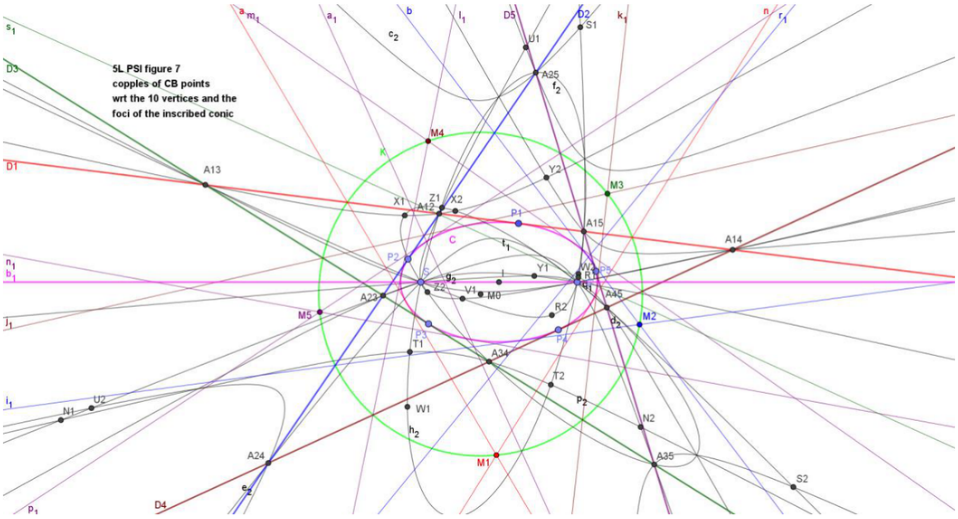


The 10 degenerated quartics formed by 2 lines (through 7 points) and the circles through the 3 last points pass through the 10 vertices and through the circular points (but not through the foci S and S' of the inconic). 2 of these quartics intersect in 2 other points in the intersection of the 2 circles. All the copples of points are CB partners with the 10 vertices and the circular points.



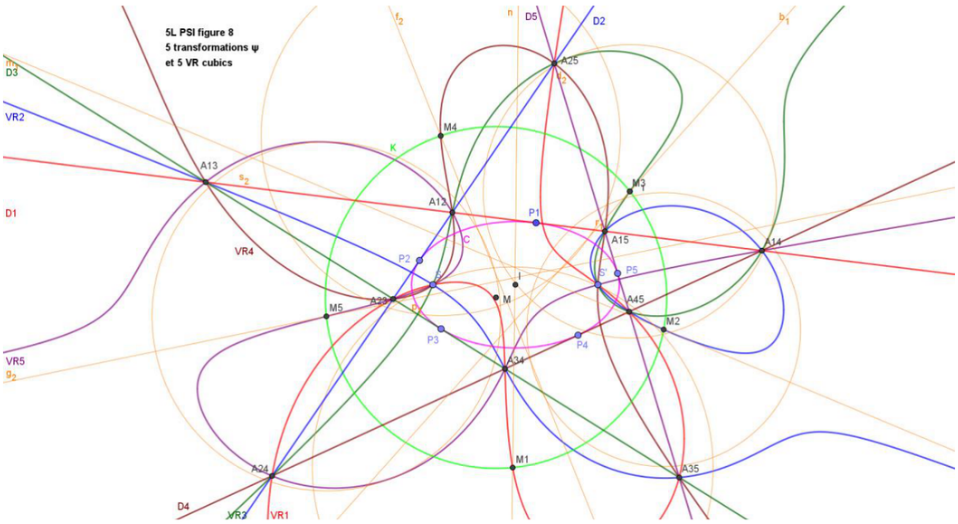
5L PSI.pdf

The same way, 10 degenerated quartics formed by 2 lines (through 7 points) and the conics through the 3 last points and the foci S and S' of the inconic pass through the 10 vertices and these foci S and S', but are not circular. 2 of these quartics intersect in 2 other points in the intersection of the 2 conics. All the copples of points are CB partners with the 10 vertices and the foci S and S'.



Despite all my efforts, I couldn't find interesting properties of these copples of points (anyway not all necessary real).

The 5 degenerated quartics formed by a line and the Van Rees focal circular cubic QL-Cu1 of the QL of the 4 other lines pass through the 10 vertices of the 5L, through the foci S and S' of the inconic and through the circular points. The foci of the VR's are the Miquel points.



These 14 points form therefore a CB system.

There are an infinity of quartics through these 14 points and we need another point P in order to identify a specific circular circumquartic of the 10 vertices of the PL and the foci S and S’.

This time, it is more promising. We are looking for a PL transformation Ψ swapping each of the 5 lines Li with the corresponding VRi and more generally a line tangent to the inconic with a VR and a point with a triple of points.

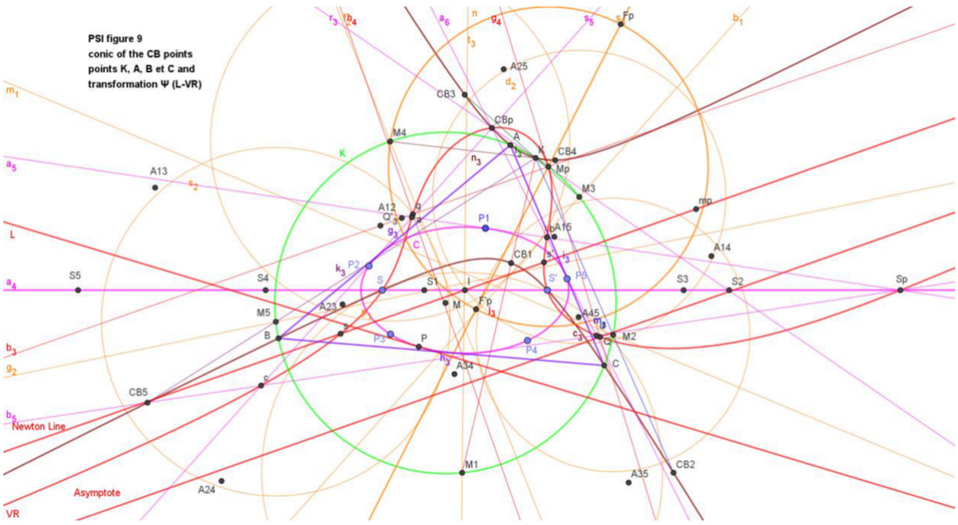
We will see that this transformation Ψ is a triple Moebius transformation, which associates to a vertice Aij of the PL the 3 vertices Akl, Akm and Alm, ψtransformed of Aij in the 3 Moebius transformations ψm, ψl and ψk centered in Mm, Ml and Mk swapping the foci S and S’ of the inconic.

We will use the conic of the CB points for the VR’s and the circle Ci(Q) for a point Q.

Let CBi be the CB of the 6 vertices of the QL of the 4 lines without Li and S and S’ ; it is the 4th intersection with VRi of the circle through Mi, S and S’ and the tangential of S and S’ on this cubic. It is therefore also the CSCi of the 3rd intersection Si of SS’ with VRi. The conic through the 5 CBi intersects the Miquel circle in 4 points K, A, B and C. ABC is circumscribed to the conic and K is aligned with MiCBi.

For any line Lp tangent to the inconic in a point P, we determine the point Mp on the Miquel circle as intersection of the 2nd tangents to the inconic in the points where Lp cuts the Miquel circle and the point CBp as 2nd intersection between the conic of the CB points and KMp. The CSCp centered in Mp swapping S and S’ swaps CBp and a point Sp on SS’. Then VRp is the VR with focus Mp invariant in CSCp with Newton Line the line through I, the middle of SS’ and mp, the middle of SpCBp, which gives the asymptote. This VRp passes through Mp, CBp, Sp, S and S’ and is completely determined.

The transformation Ψ associates to the line Lp the Van Rees focal circular cubic VRp.



For any point Q, the circle Ci(Q) is the locus of the ψtransforms of Q in all ψtransformations centered in a point of the Miquel circle and swapping the foci S and S' of the inconic ; in particular, this circle Ci(Q) passes through the 5 ψtransforms of Q in the 5 ψtransformations CSCi of the 5 VRi.

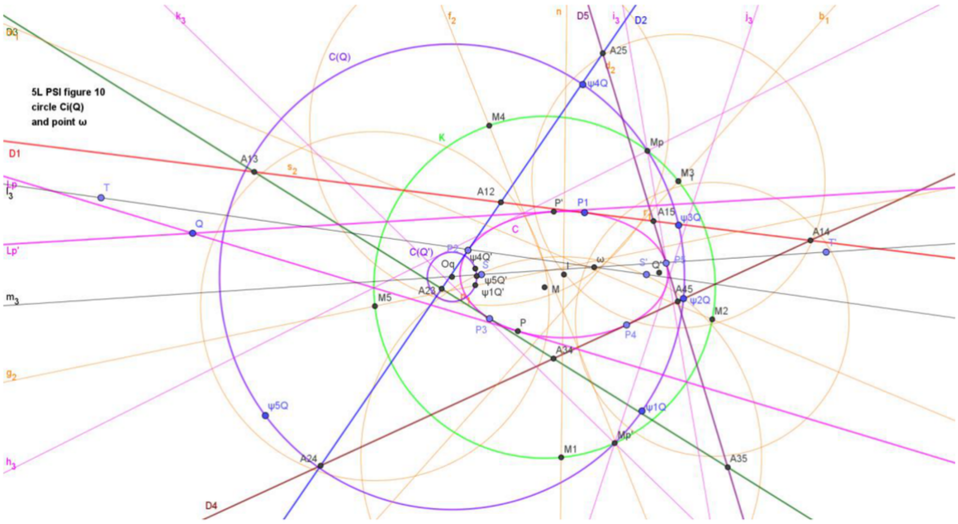
If the point Q is on the Miquel circle, Ci(Q) is a line tangent to the inconic.

If the point Q is on the line of infinity, Ci(Q) is the Miquel circle.

Let T and T' be inverses of S and S' wrt the Miquel circle ; ST' and S'T intersect in a point ω. The inversion with center ω which swaps S and T' and S' and T leaves the Miquel circle globally unchanged. The point Q' is the inverse of Q in this inversion. Then the circles Ci(Q) and Ci(Q') have the same center.

For Q on the Miquel circle, Q' is also on the Miquel circle, QQ' passes through ω and the lines Ci(Q) and Ci(Q') are parallel and symmetric wrt I, the middle of SS'.

Let Lp and L'p be the tangents from Q to the inconic, the circle Ci(Q) intersects the Miquel circle in Mp and M'p, Ci(Q) is ψp(L'p) or ψ'p(Lp), where ψp and ψ'p are the ψtransformations centered in Mp and M'p and swapping S and S'. Ci(Mp) is the line Lp and Ci(M'p) is the line L'p.

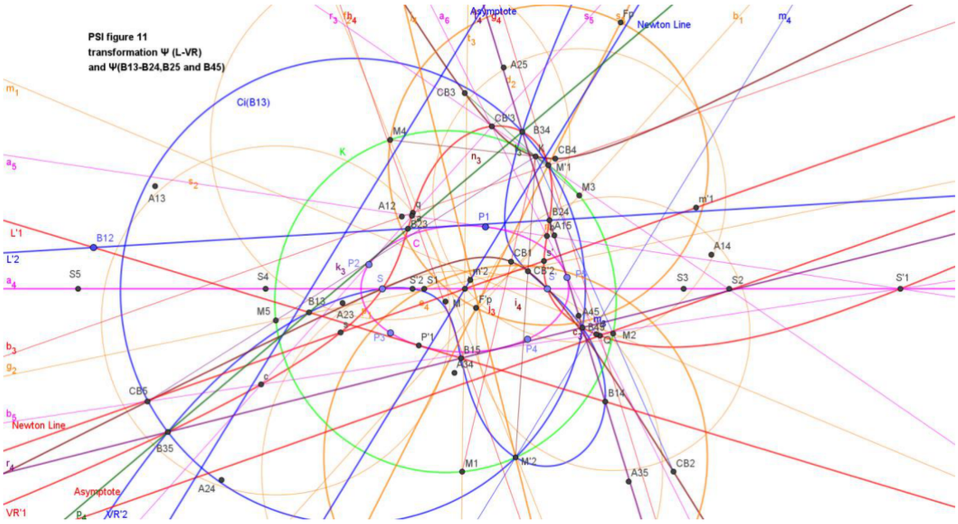


It's now possible to achieve the figur. For a point Q, named this time B12, the 2 tangents to the inscribed conic will be L'1 and L'2. The 2 Vr's associated to L'1 and L'2 are VR'1 and VR'2, which intersect in 3 points B34, B35 and B45 on the circle Ci(B12).

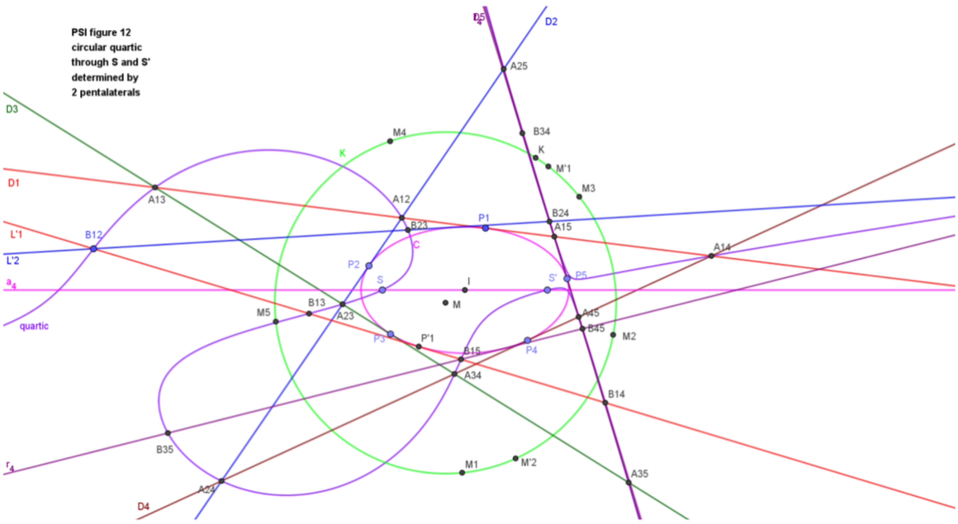
The triangle B34B35B45 is circumscribed to the inconic and it's sides form with the 2 lines L'1 and L'2 a 2nd pentalateral circumscribed to the inconic.

5L PSI.pdf

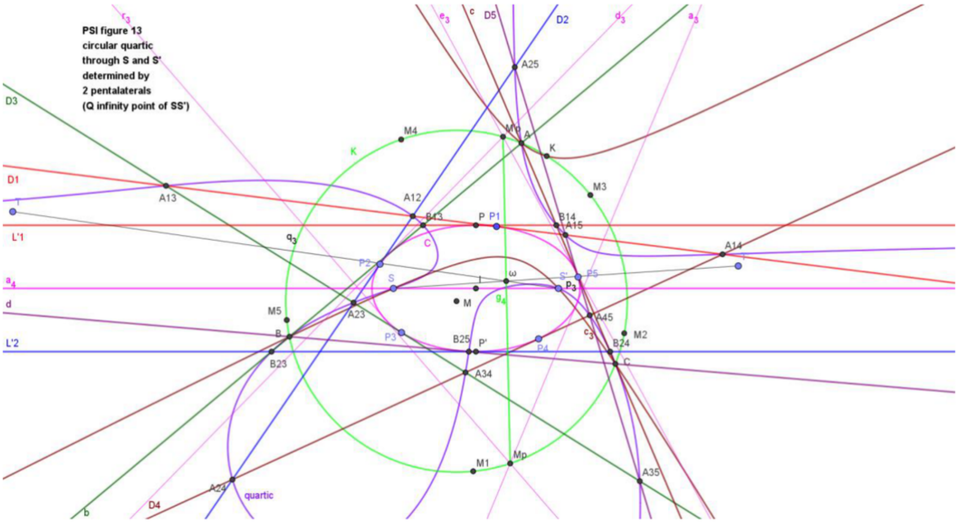
The transformation Ψ associates to the point B12 the triple of points B34, B35 and B45, intersections on the circle Ci(B12) of the 2 VR's associated to the tangents from this point to the inconic. If the VR's are invariant in the ψtransformations ψ1 and ψ2 centered in M'1 and M'2, it's interesting (and useful) to remark that B34, B35 and B45 are on VR'1 the ψ1 transforms of the 3 points where L'2 cuts VR'1 as well as on VR'2 the ψ2 transforms of the 3 points where L'1 cuts VR'2.



The 2 pentalaterals Aij and Bij determine a circular quartic through S and S'. This quartic is a Poncelet-Darboux curve for the inconic, as there are an infinity of petalaterals inscribed in the quartic and circumscribed to the inconic.



If the point B12 is choosen as the infinity point of the line SS’, 2 lines are parallel to SS’ and the 3 others are the sides of the triangle ABC. The points Mp and Mp’ corresponding to the parallel line are aligned with the point ω, the 3 others are A, B and C.



Let’s finally remark that the Ψpartners of a point wrt a pentalateral are not the CB partners of the point wrt the 10 vertices of the pentalateral and the foci of the inconic. The Ψ transformation and the CB transformation are different.

As it involves many Moebius transformations ψ of quadrilaterals, I named this pentalateral’s transformation Ψ or triple Moebius.

5L PSI.pdf

Message: #851
Date: 2021-03-22
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard and Eckart,

Thank you both for your informative answers on my questions about QA-Cu1, QA-Cu7 and QL-Cu1 related to 7P-s-Cu1.
It gave me again a much better insight and helped me a lot.
It appears to me that the distinction between a monopartite and a bipartite cubic for a circular cubic is an essential one.
It looks like the difference between a hyperbola (bipartite) and other conics (monopartite).
I wonder how you can make that distinction algebraically. Is there a kind of eccentricity?
Of course for a general cubic 9P-s-Cu1 we even can have three branches of the cubic, and when I am not wrong even with a single point added.
I think that should also be apparent in the algebraic equation of the curve. But how?
Is there anything known about this as far as you know?

Bernard, could you answer these questions?
1. You used the terminology “monocursal” and “bicursal”. Is that the same as “monopartite” and “bipartite”? What is the background of both terminologies?
2. You describe how to construct in the monocursal case the Miquel Point and the Newton Line. But is there away to construct the defining lines of the QL belonging to these characteristics?

Eckart you asked in between:
Do you have a construction for these contact points? (of the tangents from some point at a bipartite circular cubic to the cubic)
Well, that depends on which methods you admit in a construction.
In my last paper in message #838 I described in paragraph 3.4 a method (by the ruler only) to construct 5 points of the polar conic at some point of a 7P-cubic, using the reference (Cundy and Parry) you offered me so kindly in message #806. This method can be used for a monopartite as well as a bipartite circular cubic.
The polar conic intersects the cubic in the contact points.
Cabri offers a method to pin point the intersection points of a cubic (locus) and a conic (fixed). So there you have your contact points.
But it is not by ruler and compass. I think there should be a better way.

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Algebraically there is the easier way to calculate the equation
of a polar conic by partial differentiation. The solving of the
intersection points of the subsequent equations of the 2 nd and
the 3 rd degree isn’t that easy, but that is where Mathematica
is good at.
I will read your message #849 further at another moment.

Best regards,
Chris
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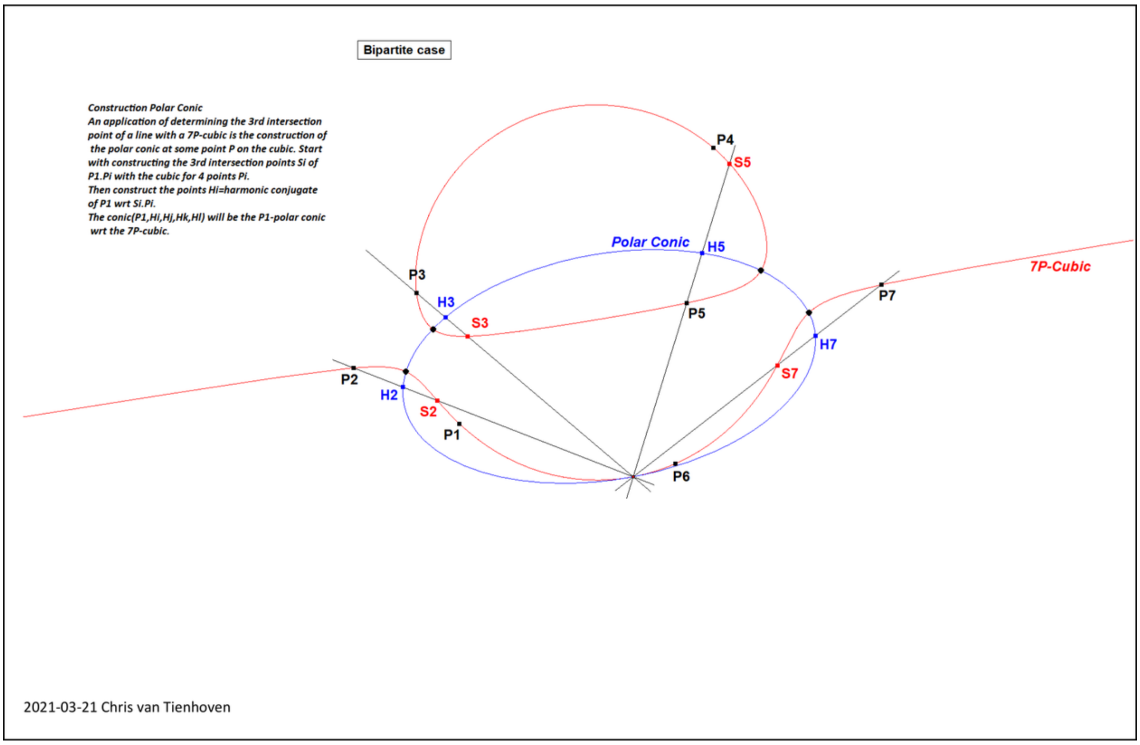

7P-s-Cu1 Pi-Polar Conic-02 - Bipartite-case.pdf

Message: #852
Date: 2021-03-23
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris, dear Eckart
1) I thought unicursal meaned unus cursus, drawn in one time, but it seems it has another definition as rational curve ... I won't use this terminology anymore
2) for a bipartite circular cubic, it is always a QA-Cu1 ; it can be a QL-Cu1 and has an inscribed QA with 2 sides orthogonal ; then it has an infinity.
Once identified QL-P1 among the 3 vertices of QA-Tr2 and the 2 others as QL-2P2a and b, you have the CSC and an infinity of QL's inscribed in the curve (see point 3).
As a QA-Cu7 is always a bipartite QL-Cu1, it can also be the QA-Cu7 of an infinity of QA's not inscribed in the curve
3) for a monopartite circular cubic, it is always a QL-Cu1 : having QL-P1 and the 2 points QL-2P2a and b gives the CSC centered in QL-P1 and swapping the QL-2P2a and b.
Then there is an infinity of QL's inscribed in the cubic : any line cuts the cubic in 3 points, the CSC partners of these points are on a circle through QL-P1 and the 6 points are the vertices of an inscribed QL.
Best regards
Bernard

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Message: #853
Date: 2021-03-23
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris, dear Eckart
The vertices of the QA are not on the QA-Cu7, but the vertices
of it's DT are !
Having a bipartite circular cubic as QA-Cu7, taking 3 points of
the curve as DT vertices, is it correct that any point and the
vertices of it's anticevian triangle wrt this DT have the cubic
as QA-Cu7 ?
Best regards
Bernard
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Message: #854
Date: 2021-03-23
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

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Dear Chris, dear Eckart,
Please forget my last message, it's not that simple !
But knowing QA-P2, QA-P4 and QA-P41 should allow to identify one
( or several ?) QA's having the curve as QA-Cu7 ...
Best regards
Bernard
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Message: #855
Date: 2021-03-24
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

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Dear Bernard and Eckart,

Bernard, thanks for your recent answers.
A last question:
Is there a construction method to construct an inscribed
Triangle in 7P-s-Cu1, such that 7P-s-Cu1 is the Neuberg Cubic?

Best regards,
Chris
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Message: #856
Date: 2021-03-25
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,
For a bipartite circular cubic 7P-s-Cu1, any point on the cubic
and it's 4 tangents define a QA and it's QA-P4.
The triangle QA-Tr2 is the same for all such QA's and defines
the cubic.
The cubic is an isogonal circular pK (X6,P) wrt this triangle, P
being the pivot which lies on the infinity line and is the
infinity point of the asymptote.
The Neuberg cubic is pK(X6,X30), meaning that the pivot is the
infinity point of the Euler Line.
This cubic passes through the circumcenter and the orthocenter
of the reference triangle.
A bipartite circular cubic 7P-s-Cu1 is generally not a Neuberg
cubic.
Best regards
Bernard
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Message: #857
Date: 2021-03-25
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Eckart and Bernard,

Bernard thanks for your remarks about the Neuberg cubic.
I was reviewing all your comments regarding 7P-s-Cu1 in combination with QA-Cu7.
I hoped to find a construction to span a QA in 7P-s-Cu1 such that 7P-s-Cu1=QA-Cu7.
But I could not find such comments, apart from that the cubic should be bipartite.

When studying the configuration I found that there is no general way to span a QA in a 7P-s-Cu1 such that QA-Cu7 will coincide with the circular cubic.
Here is my reasoning.
Let the cubic be bipartite.
Let S be the intersection point of the asymptote and the cubic.
At a bipartite cubic can have up to 4 tangent from a point on the cubic to the cubic.
Taken from S there will be up to 3 tangents to the cubic, because the asymptote will be the 4 th tangent to the cubic at the cubical point at infinity.
These 3 tangents will have up to 3 points of tangency. In the case of QA-Cu7 these 3 points are QA-P2, QA-P4, QA-P41. There is a special property wrt QA-Cu7 that the line QA-P2.QA-P4 is perpendicular to the asymptote of QA-Cu7. However this is not a general feature for a general bipartite circular cubic that two of the three real points of tangency of the tangents taken from S, are perpendicular to the asymptote. Therefore QA-Cu7 is a special type of circular cubic and a general circular cubic cannot be the reference cubic in which a QA can be spanned for which the cubic is QA-Cu7.

Best regards,
Chris

Message: #858
Date: 2021-03-25
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris,
Of course, QA-Cu7 is a special type of circular cubic !
It is a Van Rees focal circular cubic !!!
The Newton Line is the perpendicular bisectot of P2P4, P2 and P4
being the QL-2P2a and b and P41 being QL-P1.
You are looking for a method giving a QA such as QA-P2, QA-P4
and QA-P41 are given.
The DT vertices are on QA-Cu7 such as the circumcircircle of DT
passes through QA-P2 and such as the 3rd intersection between DT
sides and the curve are aligned.
The QA vertices are not on QA-Cu7 but are harmonic wrt the
intersections of QA sides with the curve ...
There are plenty of properties and I'm sure there is a way to
find a QA (or several ?) having this curve as QA-Cu7.
It would perhaps be useful to read Benedetto's article about QA
points and constructions of QA's knowing 4 QA points ...
Best regards
Bernard
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Message: #859
Date: 2021-03-25
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard and Eckart,

A 7-point configuration defines a 7P-circular cubic 7P-s-Cu1.
This cubic can take two forms, one being a continuous curved
line, and the other a curved line being split-up in two parts.
The first is called the monopartite case and the second is
called the bipartite case.
When 7P-s-Cu1 is bipartite then generally from any point on the
cubic 4 real tangents can be drawn to the cubic, giving 4
contact points.
When 7P-s-Cu1 is monopartite then generally from any point on
the cubic only 2 real tangent can be drawn to the cubic, giving
2 contact points. The other two contact points are imaginary.
This can be shown algebraically. The contact points of the
tangents of P lie on the polar conic of P wrt the cubic. The
polar conic is of the 2 nd degree and the cubic is of the 3 rd
degree and consequently both mutually have 6 intersection
points. Because the polar conic always is tangent at the cubic
in P, we have 2 contact points there. This leaves 4 contact
points. In the monopartite case we only have 2 real points. That
leaves 2 imaginary points when the cubic is monopartite.
So we can say that any circular cubic has 4 contact points from
tangents taken from some point P on the curve, two of which will
be imaginary in the case that the cubic is monopartite.
This also has the consequence that we always have for every
point P on the cubic a QA formed by the contact points of the
tangents from P, where 2 of the 4 QA-vertices can be imaginary.
This QA has an invariant Miquel Triangle QA-Tr2 (invariant wrt
the position of P on the cubic), where 2 of its vertices will be
imaginary if the cubic is monopartite.
The thing I want to tell is that we always have a fixed Miquel
Triangle 7P-s-Tr1, related to any circular cubic, related to any
7 points, with the only restriction that 2 of its 3 vertices
will be imaginary when the reference cubic is monopartite.
And like Bernard mentioned in message #856: the cubic is an
isogonal circular pK (X6,P) wrt this triangle, P being the pivot
which lies on the infinity line and is the infinity point of the
asymptote.

Best regards,
Chris

Message: #860
Date: 2021-03-25
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard,

[BK in #858] There are plenty of properties and I'm sure there is a way to find a QA (or several ?) having this curve as QA-Cu7.
I had reasoned that because the cubic was a special curved cubic such that the line through two of the contact points from S are perpendicular to the asymptote it wasn't a general cubic.
But our starting point is the general cubic 7P-s-Cu1 and so we will never find a QA with the property that 7P-s-Cu1 is QA-Cu7.
It is like asking for a general QA being a square.

Best regards,
Chris

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Message: #861
Date: 2021-03-25
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris,
We finally agree !
The question you asked yourself remains : for a given bipartite *focal* circular cubic QL-Cu1 taken as QA-Cu7 (id est QL-P1 is QA-P41 and the QL-2P2a and b are QA-P2 and QA-P4), can we find a QA such as it's QA-Cu7 is this QL-Cu1 ? Then we would have immediately the QA-Cu1 of this QA, through the vertices of the QA, the vertices of the DT of the QA, QA-P4 and QA-P41.
QA-Cu1 and QA-Cu7 intersect in 5 real points, DT vertices QA-P4 and QA-P41 and in the 2 circular points. What about the 2 last points ?
Best regards
Bernard

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Message: #862
Date: 2021-03-25
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard,

Ah, that's where the difference is.
However I never had in mind the starting position of a focal circular cubic QL-Cu1 taken as QA-Cu7.
All the time I wanted to work with the starting position of a general circular cubic, being 7P-s-Cu1.
You asked: “ QA-Cu1 and QA-Cu7 intersect in 5 real points, DT vertices QA-P4 and QA-P41 and in the 2 circular points. What about the 2 last points ? ”.
That’s the situation I described in my document of message #839, paragraph 3.5. A construction is shown for the line through the two unknown intersection points of two 7P-cubics with 5 common points. This line connects two pivot-points, per cubic one pivot point.
In our situation the pivot point of QA-Cu7 is the infinity point of its asymptote, the pivot point of QA-Cu1 is a finite point X on QA-Cu1.
Therefore the 6 th and 7 th intersection points of QA-Cu1 and QA-Cu7 will be on the line through point X on QA-Cu1 parallel to the asymptote of QA-Cu7.
When I look to a picture no real intersection points can be seen. So I suppose they are two imaginary points on this line.

Best regards,
Chris

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Message: #863
Date: 2021-03-26
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard,

I was still thinking about my assumption that the 2 remaining intersection points of QA-Cu1 & QA-Cu7 should be imaginary points lying on a real line.
I made in Mathematica a numerical setup of QA-Cu1 & QA-Cu7 and their points lying on them.
I crossed their equations and found their 9 intersection points.
They were indeed the vertices of the QA-Diagonal Triangle, QA-P4, QA-P41 and 2 imaginary points X1 & X2 indeed !
The line through these imaginary points X1 & X2 was a real line indeed.
I calculated the pivot-points of QA-Cu1 & QA-Cu7, and indeed the line through these pivot points coincided with the line through X1 and X2.

That 2 imaginary points possibly will lie on a real line can easily be seen from this example.
Given P1 = {1, +i, -i}, P2 = {1, -i, +i}. Their connecting line is the cross-product of P1 and P2, being {0, 1, 1}, which is a real line indeed.

Best regards,
Chris

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Message: #864
Date: 2021-03-27
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris,
Thank you very much for your effort !
As I don't use Mathematica, I'm condemned to work on Geogebra figures.
I'm convinced that the 2 last points are not real.
For any QA-Cu1, there are an infinity od QA-Cu7, one for each point of QA-Cu1 taken as QA-P4 and it's tangential QA ...
Conversely, for a QL-Cu1 taken as QA-Cu7, I'm convinced there is only one QA-Cu1.
The problem is to find the DT triangle knowing P2, P4 and P41, which gives the curve QA-Cu7.
The curve is CSC invariant in the CSC centered in P41 and swapping P2 and P4, QA-Tf2 invariant in the Tf2 swaping P4 and P41 and cb invariant in the cb of the 5 points DT vertices, P4 and P41. The cb transformation is pivotal with pivot the infinity point of the asymptote.
Then it holds that any parallel to the asymptote cuts the curve in 2 points being at the same time cb partners wrt the 5 points and isogonal wrt the triangle P2P4P41.
The circle through 3 of the 5 points and the line through the 2 last points intersect the curve in 2 cb partners ; only one line is known P4P41, which intersect the curve in Sic, which is Tf2(S) and cb partner of P2.
Last, the circumconic of the 5 points passes through S, where the curve cut's it's asymptote and 5P-s-P4 of the 5 points lies on this conic and on the asymptote.
After all these observations, I remain convinced that there is only one DT triangle on the curve QA-Cu7, defined by P2, P4 and P41 such as it's circumcircle passes through P2, the 3 vertices are coconic with P4, P41 and S, the 5P-s-P4 of the 3 vertices and P4 and P41 lies on the asymptote and the intersections of the lines through 2 of the 3 DT vertices and the curve are aligned and are cb partners of the 4th intersection of the circle through P4, P41 and the last vertice.
I hope I didn't forget anything ...
Can you give me an indication about the pivot of the cb transformation for QA-Cu1 ; you say it is a finite point ...
Best regards
Bernard
PS Naturally, I'm waiting impatiently for your comments and remarks about my 3 memos ...

Message: #865
Date: 2021-03-27
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

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Dear Bernard,

wrt  "... the pivot of the cb transformation for QA-Cu1 ..."
in #864
... for the 5P = QA-Tr1 plus QA-P4,
    QA-P41 is the isogonal conjugate of
QA-P41 wrt QA-Tr2.
Good luck for your intensive discussion.

Best regards Eckart
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Message: #866
Date: 2021-03-27
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris,
Congratulation for the 2 imaginary points on a real line !
I haven't found yet a simple construction for DT vertices
knowing only QA-Cu7.
But I've found your real line : it is the perpendicular bisector
of P2P4.
The pivots of the cb transformation wrt DT vertices, P4 and P41
are in fact the infinity point of the asymptote for Cu7 and the
point where the perpendicular bisector of P2P4 cuts Cu1 for Cu1.
This 2nd pivot is real as you said.
It is the intersection of the 4 lines, 3 through a vertice of
QA-Tr2 and the 3rd intersection of the corresponding DT side
with Cu1 and the 4th through P4 and the intersection of the DT
circumcircle with Cu1, which lies on the asymptote of Cu7.
Best regards
Bernard

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Message: #867
Date: 2021-03-27
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris, dear Eckart
I can confirm Eckart's property about the pivot of Cu1 in the cb
transformation wrt DT vertices, P4 and P41 as the isogonal wrt
Tr2 of P41.
This confirms also the line of pivots as the perpendicular
bisector of P2P4 with no real intersection with Cu1 and Cu7.
Best regards
Bernard

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Message: #868
Date: 2021-03-27
From: van10hoven@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard and Eckart,

It's a pleasure that we all tried to find the pivot of the cb transformation for QA-Cu1.
I tried to find its coordinates with the algebraic approach but without result yet. The calculations were to intensive.
I was glad that you both found much more information about the pivot.
According to Bernard the line of both pivots is the perpendicular bisector of QA-P2 and QA-P4.
And its intersection with QA-Cu1 is the pivot we were searching for.
Moreover Eckart discovered that the pivot is the isogonal conjugate of QA-P41 wrt QA-Tr2.
It is still a special thing that the 6 th and 7 th intersection point of QA-Cu1 and QA-Cu1 are imaginary points lying on the perpendicular bisector of QA-P2 and QA-P4. Moreover they are CB-partners.
Still there is one thing that bothers me.
We used the property that when we have 5 fixed points (5P) on a circular cubic that point P = X.5P-s-Tf6(X) ^ Y.5P-s-Tf6(Y) is a fixed (pivot) point on the cubic for all points X,Y on the cubic not coinciding with the 5P-vertices.
How can we prove this?

Best regards,
Chris

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Message: #869
Date: 2021-03-28
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

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Dear Bernard, dear Chris,

excuse, that I couldn't follow your discussion,
... perhaps the following assumption is already mentioned:

Starting with QA-Cu7, we can consider this cubic as QA-Cu1,
... taking any point of QA-Cu7 as one QA-vertex
... and the others with the Möbius transformations
    of the triangle P2P4P41.
These QAs are special, their vertices lie in pairs on orthogonal
lines,
... so my assumption:
Starting with a QA-Cu1, we can only consider this cubic as
QA-Cu7
... if the reference QA has vertices in pairs
    on orthogonal lines.
I shall try a construction.

Best regards Eckart

PS: This is the same property to consider QA-Cu1 as QL-Cu1.
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Message: #870
Date: 2021-03-28
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

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Dear Eckart,
Sorry if I'm angry, but this is too much !
We aren't looking for a QA on QA-Cu7, but on QA-Cu1 !!! (Please
read only the message #864)
The problem we deal with with Chris is simple.
Having an ordinary circular cubic, taken as QA-Cu1 leads to an
infinity of QA's and an infinity of QA-Cu7 (but only to one Tr2
defining the cubic).
Conversely, having a *focal* circular cubic QL-Cu1, taken as
QA-Cu7 leads to the points QA-P41 as focus and QA-P2 and QA-P4
as QL-2P2a and b.
We may choose one of the 2 as QA-P4 and the other as QA-P2.
I'm convinced that this QA-Cu7 leads to only one QA having these
3 points as QA-P2, P4 and P41 as well ans the transformation
QA-Tf2 swapping P4 and P41 as well as S and Sic (see EQF).
That's the reason why I'm looking for a simple construction
given the DT of the QA, having only P2, P4 and P41.
Thanks for your help and welcome to the club.
Best regards
Bernard
```

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Message: #871
Date: 2021-03-28
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Chris, dear Eckart
Sorry for Eckart, in fact I was upset against myself that I couldn't fine the solution to our problem.
I think I've found it and I feel much better !
Again, for a given focal circular cubic, defined as QA-Cu7, how can we find the corresponding QA (and it's DT on both cubics).
I was convinced that there was only one solution, but I was wrong ! There are an infinity !!!
Any point Q on the asymptote will be 5P-s-P4 of the 5 points DT vertices, P4 and P41.
The middle of QP41 is a point P on the perpendicular bisector of P2P4, which will be the pivot of Cu1.
The line QP41 will be parallel to the asymptote of Cu1.
The parallel through Q to P4P41 cuts P2Sic in a point T (Sic onCu7).
TP4 cuts the asymptote in a point V (on Cu1).
The circle through P2, T and V is the DT circumcircle.
The parallel through Q to P4P41 cuts the circle in a 2nd point (other than T) U.
P4, P41, S (on Cu7), Q and U define the circumconic of the 5 points DT vertices, P4 and P41.
The circumcircle and the conic intersect in 4 points U and the 3 searched DT vertices, which lie on Cu7.
Alleluiah!
P41 is the QA-P4 of P4 and these 3 DT vertices.
Cu1 is the pivotal cubic cb invariant in the cb transformation wrt the 5 points DT vertices, P4 and P41 with pivot P.
The vertices of the tangential QA of P4 wrt this Cu1 form a QA having the initial cubic as QA-Cu7.
End of this little quest !
Best regards
Bernard

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Message: #872
Date: 2021-03-29
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris, dear Eckart
I'm not sure that the point V (cbpartner of P4 on Cu1) is on the
asymptote of Cu7.
I'm preparing a short memo (the 4th one) about Cu1 and Cu7 or
the way to have Cu7's having one Cu1 (circular cubic) or Cu1's
having one Cu7 (focal circular cubic).
Best regards
Bernard
```

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Message: #873
Date: 2021-03-29
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard, dear Chris,

I regret some remarks out of place by Bernard in #870,
... nevertheless I risk to give the missing construction in #869
... for my assumption:
Starting with a QA-Cu1, we can only consider this cubic as
QA-Cu7
... if the reference QA has vertices in pairs
    on orthogonal lines.

Let us start with a QA with vertices in pairs on two orthogonal
lines
... and its cubic QA-Cu1, intersecting its asymptote in Q
... and its Miquel triangle QA-Tr2,
... which will give QA''-P2, QA''-P4, QA''-P41
    for the searched quadrangle QA''.
The Möbius transformations of the Miquel triangle QA-Tr2
... map Q to three further points,
    which give with Q a 2nd quadrangle QA'.QA'-P4 and QA'-P41
    of this 2nd quadrangle are vertices of QA''-Tr1,
... the diagonal triangle of the searched quadrangle QA''.
The diagonal triangle QA'-Tr1 of the 2nd quadrangle
... has one vertex in QA'-P2,
    the intersection of its orthogonal diagonals,
... the opposite triangle side intersects
    the endless part of QA-Cu1
... in the 3rd vertex of the diagonal triangle QA''-Tr1
    of the searched QA''.
The fixed points of the isoconjugation for QA''-Tr1,
... swapping two QA'-vertices on each part of the cubic
... give the searched quadrangle QA'',
... whose QA''-Cu7 is QA-Cu1 of the reference quadrangle.

Best regards Eckart

PS: The constellation has a lot of further properties.,
... e.g. two lines of QA'' are connections of QA'-vertices
    on one part of the cubic
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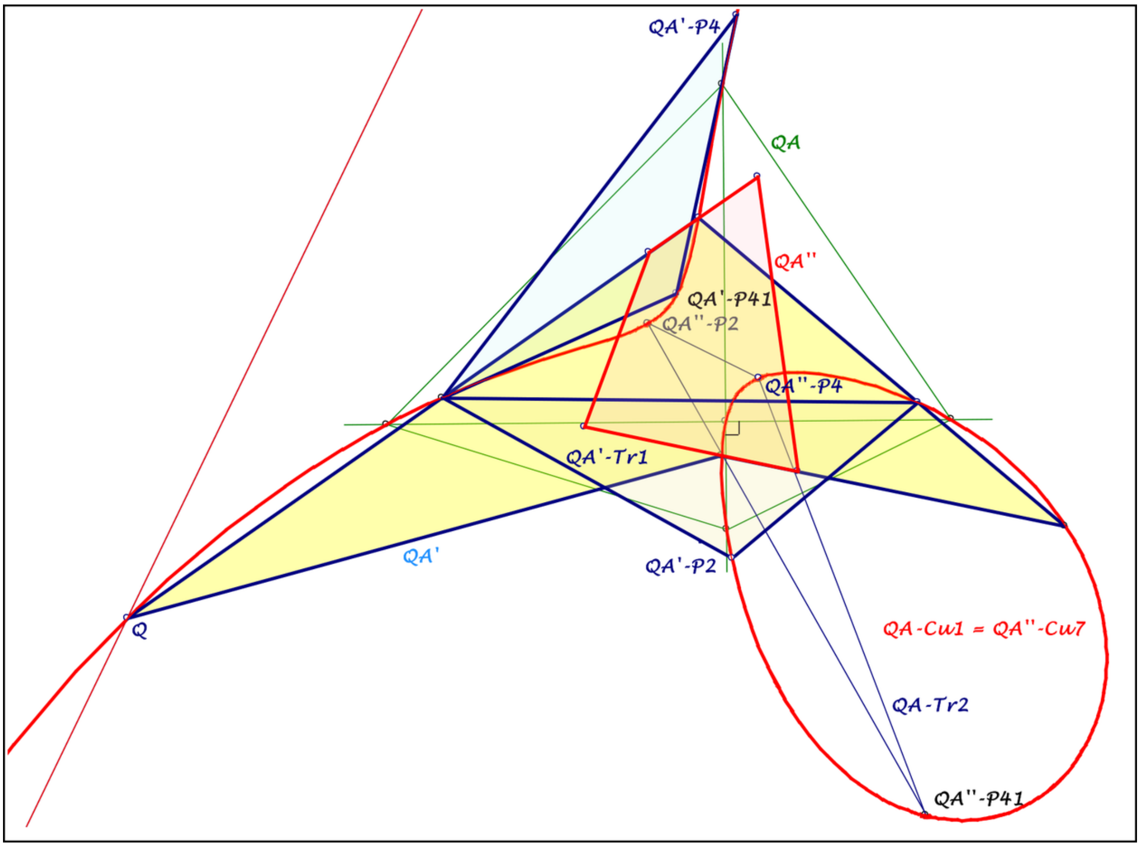

2021-03-29.pdf

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Message: #874
Date: 2021-03-30
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Eckart,
My humble apologise for my unuseful rude remark in #870 !
In fact, we say the same thing with different names or
properties !
Your assumption that QA-Cu7 is necessary the QA-Cu1 of a
reference QA with 2 orthogonal sides is exactly what I'm saying
since the beginning, it is a Van Rees focal circular cubic.
Beautiful construction you give !
I observe that you don't use your reference QA and you find a
solution depending only from the point Q, which is given from
P2, P4 and P41.
My question is : can you do the same with any point of the curve
and it's 3 Moebius transformations ? (for example, why not the
reference QA ?)
I had also noticed that the vertices of the search DT lie on the
endless part of the cubic ...
Best regards
Bernard

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Message: #875
Date: 2021-03-30
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

Dear Bernard,

you are right, we can take any point of QA-Cu1 as vertex of QA',
... even QA as 2nd quadrangle QA'!
Then QA-P4 and QA-P41
... and the QA-Tr1-vertex on the endless part of QA-Cu1 give the
searched DT.
Thanks for this simplification.

Best regards Eckart

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Message: #876
Date: 2021-04-01
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

Dear Eckart,
You're right, it's always better when we understand each other !
I've tried to work on your construction.
First, let's simplify : it's possible to start with the real cubic Cu7, defined with the points P2, P4 and P41 and the point S diametral of P41 on the circumcircle of the 3 points.
(Any isogonal circular cubic wrt the triangle of the 3 points contains it's in- and excenters, only if the cubic contains S, the circular cubic is focal)
Then taking directly any point on the cubic as P'4, we have easily it's Moebius transforms as P'2 and P'4b and P'4c and their tangential P'41.
We have 6 interesting circles through P'41, P'4b, P'4c and P41, P'41, P'2, P'4 and P41. P'41,P'4,P'4b and P2, P'41, P'2,P'4c and P2, P'41, P'4, P'4c and P4, P'41, P'4b, P'2 and P'4.
I have a worrying question and an amusing remark
1) Your construction leads to P'4, P'41 and P'b or P'c (the one on the endless part of the cubic) as DT of the search QA having P2, P4 and P41.
In this case, we will never find a solution like Cu1, as the DT sides intersect Cu7 in a 3rd point (these points are aligned, as mentionned in EQF).
So, we can't have one vertice as tangential of another !
And obviously, if we draw the QA-Cu1 and QA-Cu7 of the founded QA'', it can't be the same curves, but it should be ...
This has been worrying me for 2 days and I prefer ask your opinion on this.
2) More amusing, perhaps obvious, the 3 points P'2, P'4 and P'41 lead to a Cu7 of the cubic.
A focal circular cubic is a circular cubic and can be taken either as Cu1 of an infinity of Cu7 or as Cu7 of an infinity of Cu1 ...
Best regards
Bernard

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Message: #877
Date: 2021-04-01
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard,

excuse, I don't understand your last message #876.
Wrt 1):
You say you work on my construction (I think in #873)
... and you start with a cubic QA-Cu7,
    but I start with a cubic QA-Cu1
... for a QA with vertices in pairs on two orthogonal lines.
Wrt 2):
A focal circular cubic can be mono- or bipartite,
... if it is monopartite, it cannot be taken as QA-Cu1,
    for QA-Cu1 is always bipartite,
... if it is bipartite, it can be taken as QA-Cu1,
... but the corresponding QA must have vertices in pairs
    on two orthogonal lines,
... to be interpreted as QA-Cu7, what seems to be the case
(already mentioned?).
If the conditions are fulfilled there are infinity of solutions.

Best regards Eckart

PS: Is QA-Cu1 the only circular pivotal isocubic wrt QA-Tr1 and
QA-Tf2?
```

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Message: #878
Date: 2021-04-01
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Eckart,
Finally, I no longer understand anything !
Perhaps I wasn't clear enough ...
A QA-Cu1 is always bipartite.
If it is with a QA with 2 orthogonal sides, it is a focal
circular cubic like a bipartite QL-Cu1 or a QL-Cu7.
Do we agree on that ?
On your figure (beautiful as usuel), Q is the diametral point on
the circle through QA''-P2, P4 and P41 and QA''-P41 is the
focus, on the curve.
The best way to have such a curve is in fact to start with a QA
with 2 orthogonal sides and we have a focal bipartite circular
cubic.
Having such a curve, my questions were :
1) can we find one or several QA-Cu1 with a QA such as this
curve is it's QA-Cu7
2) can we find one or several QA-Cu7 of QA's of this curve
The answer is clearly an infinity in both cases.
Last, what is the product of your construction in message #873 ?
I found it very interesting, but I can't interpret the result !
I don't think your cubic can be QA''-Cu7, but may be I'm wrong.
Perhaps you didn't search the same thing as me and you found a
new product ?
Is there something I missed ?
Thanks in advance for your patience and your explanations
Best regards
Bernard
PS All my comments in my message #876 , particularly the 6
circles, were on your own figure !
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Message: #879
Date: 2021-04-02
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Eckart,
I didn't answer your PS.
For any QA, it holds that the unique circular pivotal isocubic
QA-Cu1 has pivot P4 and isopivot P41, defining Tr1 and Tf2.
I hope it answers your question.
I have a last question (found during the night).
I understand that your construction need a focal circular cubic
in order to have P2 as one of the Moebius conjugates of P4 and
P41 as the tangential of both P2 and P4.
But what gives your construction for a simple circular (non
focal) cubic ? For any point on the endless part of QA-Cu1, the
3 Moebius conjugates, the tangential of the 4 points, the
triangle formed by the point, it's Moebius conjugate on the
endless part and the tangential as DT and the Tf2 associating
the tangential to the initial point, last the 4 fixed points of
the isoconjugation.
What about this QA, it's Cu1 and it's Cu7 ?
Best regards
Bernard
```

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Message: #880
Date: 2021-04-02
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard,

I think, we talk at cross purposes,
... let me start once more with a reference QA
... with vertices in pairs on orthogonal lines,
    intersecting in QA-P2,
... and its Miquel triangle: one vertex is QA-P9, later
QA''-P41,
... 2nd vertex on the closed part of the cubic later QA''-P4,
... 3rd vertex on the endless part of the cubic later QA''-P2.
Now I consider the cubic QA-Cu1 and construct a quadrangle QA'',
... whose QA''-Cu7 is QA-Cu1 of the starting QA.
This quadrangle QA'' has its vertices not on QA-Cu1,
... perhaps that is what you expect?

You say that the cubic in #873 cannot be QA''-Cu7,
please try it in the simple form:
Take a QA with the upper properties with its QA-Cu1,
... take A = QA-P4, B = QA-P41 and consider for C
... the diagonal triangle QA-Tr1, the opposite side of QA-P2
... and the intersection C with the endless part of QA-Cu1.
ABC is the diagonal triangle for a QA'' with QA''-Cu7 = QA-Cu1,
... the vertices of QA'' lie on the sides of QA
    through C harmonic conjugated.

So far only wrt the construction in #873.

Best regards Eckart
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Message: #881
Date: 2021-04-02
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard,

question wrt #879:
What about the defined "... Tf2 associating the tangential to
the initial point ..."?
These two points are vertices of the reference triangle.

Best regards Eckart
```

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Message: #882
Date: 2021-04-03
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Chris, dear Bernard,

circular cubics can be unipartite and bipartite, further focal
or nonfocal,
... and you want to consider these cubics as QA-Cu1 or QL-Cu1
    or QA-Cu7.

Is the following summary correct (red means not possible)?
1)  bipartite focal circular cubics can be taken as
    ...  QA-Cu1, QL-Cu1, QA-Cu7.
2)  bipartite nonfocal circular cubics can be taken as
    ...  QA-Cu1, QL-Cu1, QA-Cu7.
3)  monopartite focal circular cubics can be taken as
    ...  QA-Cu1, QL-Cu1, QA-Cu7.
4)  monopartite nonfocal circular cubics can be taken as
    ...  QA-Cu1, QL-Cu1, QA-Cu7.

Background:
Bipartite circular cubics can be considered as QA-Cu1.
... and if they are focal,
... the corresponding QA has vertices on orthogonal lines
... and allows to be taken as QL-Cu1 or QA-Cu7.
Monopartite circular cubics cannot be taken as QA-Cu1 or QA-Cu7,
... which are bipartite.
Focal circular cubics can be taken as QL-Cu1 with focus QL-P1.
Only bipartite focal circular cubics can be taken as QA-Cu7 with
focus QA-P41.

Please correct, if something wrong.

Best regards Eckart
```

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Message: #883
Date: 2021-04-04
From: bernard.keizer@gmail.com
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Eckart,
This seems correct, but I'm not sure that there are monopartite
nonfocal circular cubics ...
Best regards
Bernard
```

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Message: #884
Date: 2021-04-04
From: eckart_schmidt@t-online.de
Subject: Just for fun

```
Dear Chris, dear Bernard,

let us consider a special QL,
... whose QL-P10 is a point of QL-L1,
... then QL-P10 with partner QL-P16
... ... are the fixed points of QL-Tf1.
QL-Cu1 will be a strophoid of QL-L1
... with fixed point QL-P10 and pole QL-P1.
5P of any quadrigon component QG plus QL-P10
... have a 5P-quartic, bearing QL-P1,10,16,
... the focal circular 5P-circumcubic with focus QL-P1 is
QL-Cu1,
... locus of all points X with 5P-s-Tf6(X) = QL-P10.

Happy Easter!
Eckart
```

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Message: #885
Date: 2021-04-05
From: eckart_schmidt@t-online.de
Subject: Re: Newton Point-like point on a circular cubic

```
Dear Bernard,

thanks for confirmation,
... but you can get monopartite nonfocal circular cubics
... with the same construction as for focal cubics (see #727):
Start with a 5P and its quartic and a point F' not on the
quartic,
... let U = F'.P5 ^ HY (2nd intersection),
... ... HY orthogonal hyperbola, bearing P4, P5,
        centered in the middle
... ... with axes parallel to those of Co1,
... let S = U.P4 ^ Co1 (2nd intersection)
... and consider 7P-s-Cu1 of 5P plus F' and S.
Varying F' you get circular cubics monopartite and bipartite,
... control whether the intersections with the quartic are foci,
... if not there will remain an infinity of examples for
... monopartite nonfocal circular cubics.

Best regards Eckart
```

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Message: #886
Date: 2021-04-05
From: bernard.keizer@gmail.com
Subject: Cu1 and Cu7

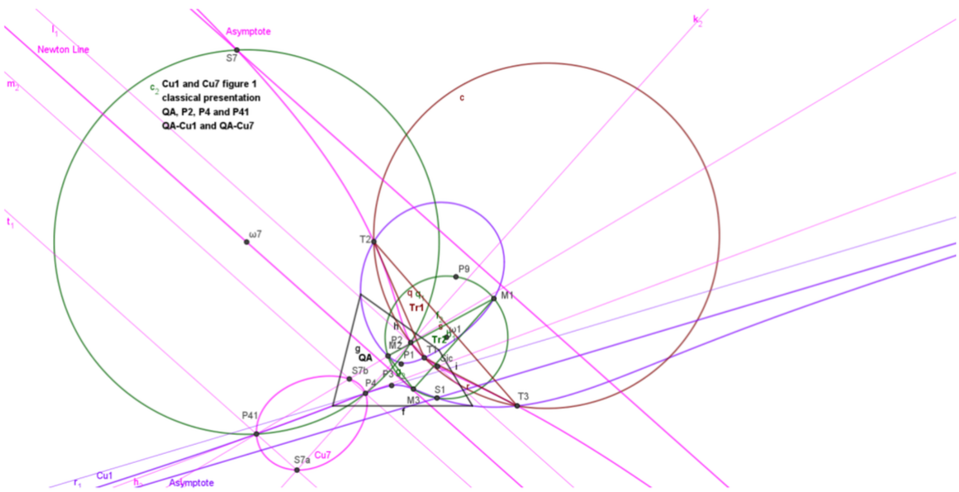
```
Dear Chris, dear Eckart
It's not really a new topic, but the preceding one is full !
It's the 4th time I write completely an item with text, figures
and explanations and I'm still desperately waiting for reactions
...
For this one, I found 2 important things (at last, I hope for
Eckart).
1) I found a general reverse construction for Cu1's of a given
Cu7 (point 3 of the memo)
2) I can confirm to Eckart that his construction leads also to
Cu1's for a given Cu7, but it is a special case, not a general
configuration.
At the end of point 5 of the memo, I suggest we could study the
locus of the points V leadind to Eckart's configuration ...
Best regards
Bernard
```

Cu1 and Cu7

1. Classical presentation

Let's start with the classical presentation of both curves QA-Cu1 and QA-Cu7 of a QA.

All the elements are conform to EQF (P1, P2, P3, P4, P9, DT vertices, triangles of Miquel points for both Cu1 and Cu7, asymptotes of the curves).

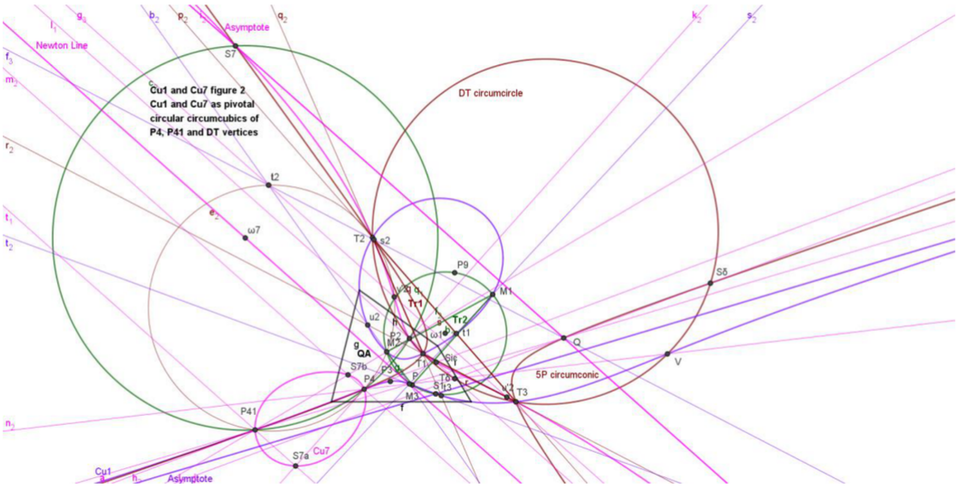


2. Cu1 and Cu7 as 5P circular circumcubics

Both curves are circular and intersect in 5 real points P4, P41 and the DT vertices.

The 2 last points are not real, but lie on a real line, the Newton Line of Cu7, as mentionned by Chris.

It appears that both curves are 5P pivotal circular circumcubics of the 5 points P4, P41 and DT vertices. The pivot of the cb transformation wrt the 5 points is the infinity point of the asymptote for Cu7 and a finite point P, intersection of Cu1 with the Newton Line of Cu7, for Cu1. P is the isogonal transform of P41 wrt the triangle Tr2 of Cu1, as mentionned by Eckart.



Considering the 5 points, the point 5P-s-P4 is a point Q, reflexion of P41 in P. The parallel to P4P41 through Q intersect the circumcircle of DT in 2 points Sδ on the circumconic of the 5 points and Tδ on P2Sic, reflexion of P4 in P. The line P4P intersect the cubic Cu1 in a point V. P is the pivot for Cu1, the infinity point of the asymptote is the pivot for Cu7, Tδ is the pivot for the degenerated cubic formed by the line P4P41 and the circumcircle of DT and Q is the pivot for the degenerated cubic formed by the circumconic of the 5 points and the infinity line. P41 Q is parallel to the asymptote of Cu1. Last, the parallels to the DT sides through Q intersect the circles through P4, P41 and the opposite DT vertice in 2 points si on the circumconic and ti ; the ti are the pivots for the degenerated cubics formed by a circle and the corresponding DT side.

Cu1 and Cu7.pdf

So there are a lot of copples of cb partners on Cu1 P41 and infinity point of the asymptote, P4 and V, vertices of Tr2 and 3rd intersection of Tr1 sides with the curve and on Cu7 P41 and S7a, P4 and S7b, P2 and Sic, S7 and infinity point of the asymptote …

Now, I would like to find a construction of a Cu1, knowing Cu7 and the pivot P (part 3 : reverse construction) and of Cu7's for the same focal circular cubic taken this time as Cu1 (part 4).

3. Reverse construction : Cu1's of a Cu7

It happens that P describes the Newton Line of Cu7, Q it's asymptote and Tδ the line P2Sic.

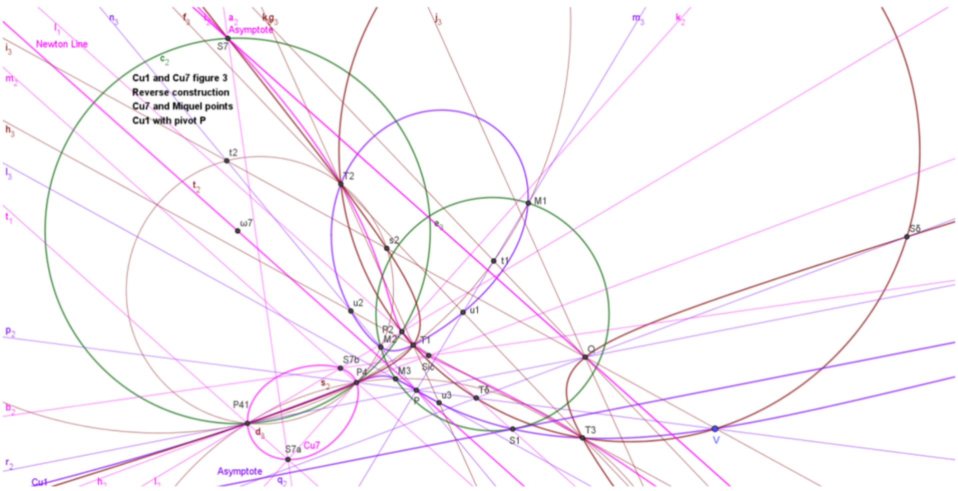
Starting with a point V, the line P4V intersects the Newton Line of Cu7 in P and the line P2Sic in Tδ. Q is the reflexion of P41 in P.

The line TδQ, parallel to P4P41, intersects the circle through P2, Tδ and V in Sδ.

The conic through P4, P41, Q, S7 and Sδ and the circle through P2, Tδ and Sδ intersect in 3 points (other than Sδ) on the curve Cu7 ; these 3 points are the vertices of the searched DT.

It's then possible to draw the 3 lines of DT sides and the 3 circles through P4, P41 and a DT vertice. The parallels in Q to the DT sides intersect the corresponding circles in 2 points si on the circumconic and ti ; the ti will be the pivots for the degenerated cubics formed by a DT side and the circle through P4, P41 and the opposite DT vertice. The lines Pti cut the DT sides in the points ui and the corresponding circles in ti and a 2nd point, which is Mi, one of the Miquel points.

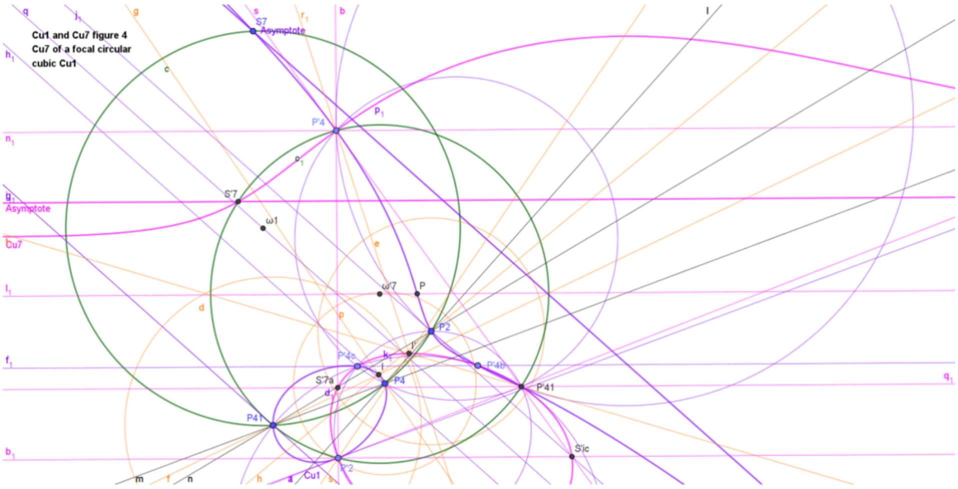
The pivotal circular circumconic of the 5 points P4, P41 and DT vertices with pivot P gives the cubic Cu1 and the tangents from P4 to the cubic give the QA for which Cu1 is QA-Cu1 and Cu7 QA-Cu7.



4. Cu7's of a focal circular cubic Cu1

Naturally, a focal circular cubic can be taken as Cu1 and has an infinity of Cu7's using the same classical construction (a QA on this curve, with 2 orthogonal sides, as mentionned by Eckart, it's P'2, P'4 and P'41, S'7 as diametral of P'41 on the circumcircle of the 3 points, the DT of the QA, with vertices P'2, P'4b and P'4c).

It's interesting to notice that the pivot of the cb transformation of the 5 points P'2, P'4b and P'4c (DT vertices) and P'4 and P'41 is the finite point where the Newton Line of Cu7 cuts Cu1 and the isogonal of P'41 wrt the Miquel triangle P2P4P41 of Cu1.



Cu1 and Cu7.pdf

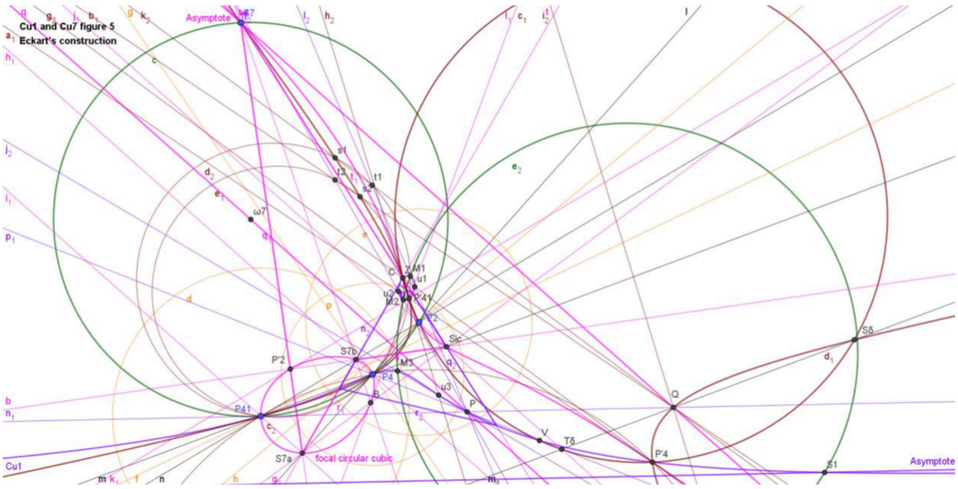
5. Eckart's construction

Eckart has proposed in the message 873 a construction with a point on the endless part of Cu7 and the 2 vertices of it's tangential triangle which lie on the same part as vertices of DT triangle.

It's possible to use the complete construction in point 3. We have immediately the DT circumcircle (through P2) and the circumconic of the DT vertices, P4 and P41. The 4th intersection of the circle and the conic (apart of the DT vertices) is Sδ, the parallel in Sδ to the line P4P41 intersects the conic in Q on the asymptote of Cu7 and the circle in Tδ on the line P2Sic. The middle of P41Q is the pivot P on the Newton Line. P4P intersects the circumcircle in the 2 points Tδ and V.

It's possible to achieve the construction and to draw Cu1 as pivotal circular circumcubic of the 5 points with pivot P and to find the searched QA by drawing the 4 tangents from P4 to the cubic Cu1.

In figure 5, I reproduced Eckart's figure with QA formed by S7, S7a, S7b and Sic, with DT P'2BC and points P'4 and P'41. As mentionned by Eckart, the vertices of the searched QA on Cu1 are on the sides of the initial QA on Cu7 passing through the point C.



It's important to notice that this construction is a special case of the general construction ; the 3 DT vertices are generally not a point and 2 vertices of it's tangential QA on Cu7.

Checking the general configuration, it must be possible to study the locus of the point V for this configuration. But that's another story …

Cu1 and Cu7.pdf

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Message: #887
Date: 2021-04-06
From: bernard.keizer@gmail.com
Subject: Re: Cu1 and Cu7

Dear Eckart,
I was very glad to find that we were both right in our cross researches !
First, as EQF mentions explicitly that the 3rd intersections of DT sides with Cu7 were aligned, I didn't understand your construction, as for the 2 sides from P'41, there isn't apparently a 3rd intersection ! In fact, there is one, it is P'4 and C, as tangentials of P'41 and the 3rd intersection of the last side P'4C with the curve is by definition aligned with P'4 and C.
Second, my general construction works with Sδ (4th intersection of DT circumcircle and 5P conic) as well as with V and it's possible to look at the locus of this point in your configuration.
It seems to be a parabola ?
Best regards
Bernard

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Message: #888
Date: 2021-04-06
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear geometers,

Let A1, A2, A3, A4 be four points.
P is any point.
Bisector of <A1PA2 meets line A1A2 at B1
Bisector of <A2PA3 meets line A2A3 at B2
Bisector of <A3PA4 meets line A3A4 at B3
Bisector of <A4PA1 meets line A4A1 at B4
Bisector of <A1PA3 meets line A1A3 at B5
Bisector of <A2PA4 meets line A2A4 at B6
Then the lines B1B3, B2B4, B5B6 are concurrent at f(P).
Is this transformation new? Which are some f(P)? (for P = Qa-P1, Qa-P2,...)

Best Regards
Tran Quang Hung.

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Message: #889
Date: 2021-04-06
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

```
Dear Tran Quang Hung and all,

After some computation, I can prove that the transformation is
given by:
f(P) = Q   such that    sum vector(QAi) / PAi = vector(0)

For a generalization, let t be a real number.
B1 in  A1A2 such that
       vector(B1A1)/PA1^t + vector(B1A2)/PA2^t = vector(0)
B2 in  A2A3 such that
       vector(B2A2)/PA2^t + vector(B2A3)/PA3^t = vector(0)
B3 in  A3A4 such that
       vector(B3A3)/PA3^t + vector(B3A4)/PA4^t = vector(0)
B4 in  A4A1 such that
       vector(B4A4)/PA4^t + vector(B4A1)/PA1^t = vector(0)
B5 in  A1A3 such that
       vector(B5A1)/PA1^t + vector(B5A3)/PA3^t = vector(0)
B6 in  A2A4 such that
       vector(B6A2)/PA2^t + vector(B6A4)/PA4^t = vector(0)

then B1B3, B2B4, B5B6 are concurrent at Q = f(P) such that:
       sum vector(QAi) / PAi^t = vector(0)
Best regards,
Vu Thanh Tung
```

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Message: #890
Date: 2021-04-06
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

```
Dear Tran Quang Hung and all,

We can rewrite the generalization as follows:
Let X be a triangle center with triangle function X = a^t: b^t :
c^t.
Let X1 be the X-point of triangle PA1A2.
B1 = intersection of PX1 and A1A2.
Define B2,B3,B4,B5,B6 similarly for triangle PA2A3, PA3A4,
PA4A1, PA1A3, PA2A4.
Then B1B3, B2B4, B5B6 are concurrent at Q = f(P) such that:
     sum vector(QAi) / PAi^t = vector(0)
For t = 0, Q = quadrangle centroid.

Best regards,
Vu Thanh Tung
```

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Message: #891
Date: 2021-04-07
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Vu Thanh Tung and all,

Thank you for your contribution. I think we can get more general problem as
follows
Let f(x) be a real function one variable x.
Let X be the triangle center of ABC with barycentric coordinates X(f(BC),f(CA),f(AB)).
With four points A1,A2,A3,A4 and any point P.
Line connecting P and X-center of PA1A2 meets lines A1A2 at B1.
Line connecting P and X-center of PA3A4 meets lines A3A4 at B2.
Line connecting P and X-center of PA2A3 meets lines A2A3 at B3.
Line connecting P and X-center of PA1A4 meets lines A1A4 at B4.
Line connecting P and X-center of PA1A3 meets lines A1A3 at B5.
Line connecting P and X-center of PA2A4 meets lines A2A4 at B6.
Then lines B1B3, B2B4, B5B6 are concurrent at point Q which satisfies
vec(QA1)/f(PA1) + vec(QA2)/f(PA2) + vec(QA3)/f(PA3) + vec(QA4)/f(PA4)=vec(0).

Where f(x)=x, we get my problem.
Where f(x)=x^t, we get the problem of Vu Thanh Tung.

If f(x)=ln(x), f(x) is the trigonometric function, or some real function,
we have some other transformations.

Best Regards
Tran Quang Hung.

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Message: #892
Date: 2021-04-07
From: eckart_schmidt@t-online.de
Subject: Re: Cu1 and Cu7

```
Dear Bernard,

thanks for your 4th paper "Cu1 and Cu7", I have studied it:

Wrt 2): You offer a reference 5P = Tr1 plus QA-P4, QA-P41
... and describe an interesting point constellation
... up to a point V = QA-Tf2(isg(QA-P41)) on QA-Cu1,
    (isg isogonal wrt QA-Tr2) ,
... which you use in part 3 and 5.

Wrt 3) After the construction of the DT I gave up.
If you start with a QA-Cu7, you have a given reference QA,
... and the triangle QA-P2,4,41 gives with its
    Möbius transformations
... for any QA-Cu7-point a QA, whose QA-Cu1
    is the starting QA-Cu7.

Wrt 5): Your interpretation of my construction seems right,
... but starting with QA-Cu1 for a given QA
... with vertices in pairs on two orthogonal lines
... and its Miquel triangle with a point on the endless
    part of QA-Cu1,
... which shall be QA-P2 of the searched QA:
... how to find the tangential triangle of this point
    on the same endless part?

Your final reproduction of my construction I cannot judge
... for I don't know what points are S7a, S7b, B, C.

It is a hard work, trying to understand your paper!

Best regards Eckart
```

Message: #893
Date: 2021-04-07
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

```
Hi Tran Quang Hung,

Unfortunately, the only functions f such that X(f(a),f(b),f(c))
is a triangle center are ones having the form f(x) = x^t.

Best regards,
Vu Thanh Tung
```

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Message: #894
Date: 2021-04-07
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

```
Dear Vu Thanh Tung,

I mean "triangle center" in that it has barycentric coordinates
X(f(BC),f(CA),f(AB)).
I agree that some "triangle centers" in the ETC have the form
f(x)=x^t
but undeniably there are some "triangle centers" that are not
needed in ETC.
I give an example,
I consider point X has barycentric coordinates
X(ln(BC),ln(CA),ln(AB)),
it is (maybe) not in ETC, but it is still a triangle center and
my problem is true with this center.

Sincerely yours,
Trần Quang Hùng
Tổ toán THPT chuyên KHTN, ĐHKHTN, ĐHQGHN.
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Message: #895
Date: 2021-04-08
From: bernard.keizer@gmail.com
Subject: Re: Cu1 and Cu7

```
Dear Eckart,
Thanks for your interest and your attentive reading !
Wrt 2) thanks for your precise definition of V !
Wrt 3) In fact, you don't need which follows the construction of
the DT if you know with your macros how to draw the 5P circular
circumcubic with pivot P!
I just use the intermediate pivots in order to draw this cubic
as a 9P cubic, which is possible with Geogebra.
When having the cubic QA-Cu1, the tangents from P4 give the
search QA for which the 2 cubics are Cu7 and Cu1.
Wrt 5), I was more disappointed, as I deliberatly choosed to
reproduce your construction in your message #873.
S7a and b are defined in my figures 1) and 2) ; they form with
S7 (your point Q) and Sic your initial QA, which is the
tangential QA of your point QA'-P4.
QA'-P2 and B are the Moebius transforms of QA'P4 on the finite
part of Cu7 and C is the last Moebius transform of QA'-P4, on
the infinite part of Cu7 (it is Sic).
QA'-P41 is the tangential of the 4 points QA'-P2, QA'-P4, B and
C and the triangle QA'-P2BC is the DT of your initial QA formed
by S7, Sic, S7a and S7b.
Make it more simple as you did in your message #876 : for any
point QA'-P4 on the endless part of Cu7, it's 3 Moebius
transforms will be the same way QA'-P2, a point B on the finite
part (like QA'-P2) and C on the endless part. Let's QA'-P41 be
the tangential of the 4 points QA'-P2, QA'-P4, B and C ; your
searched DT is formed by the 3 points QA'-P4, C and QA'-P41, the
3 points lie on the endless part ...
Best regards
Bernard
```

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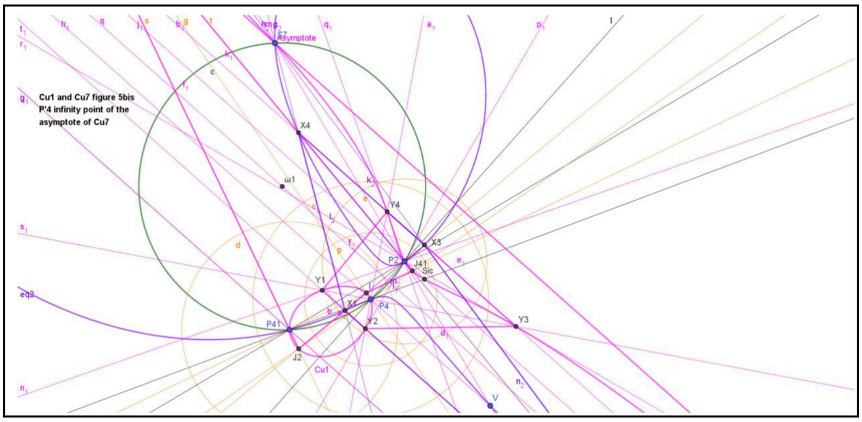
Message: #896
Date: 2021-04-08
From: bernard.keizer@gmail.com
Subject: Re: Cu1 and Cu7

Dear Eckart,
I think I've found a simplification, which might interest you.
Taking any point on the endless part of Cu7 as P'4, the point C is simply the Moebius transform in the transformation with center P2 swapping P4 and P41.
If P41 is the tangential of both P'4 and C, it follows immediately that P'4, C, P'41 and P2 are cocyclic.
It's remarkable that P'4 and C can be swapped in this definition as they are partners in a Moebius transformation and have the same P'41.
Using 2 times your remark that the vertices of your searched QA with DT P'4CP'41 are on the sides of the initial QA (tangential QA of P'4 passing through C), it appears that the 4 vertices of the searched QA are the intersections of the sides through P'4 and C of the tangential QA's of C and P'4 on Cu7.
Cu1 is then the QA-Cu1 of this last QA and Cu7 it's QA-Cu7.
It's possible to find the QA for example when P'4 is the infinity point of the asymptote of Cu7 and it's partner is P2 itself.
One of the Miquel points of this QA is P4 swapping P2 and S7.
I've just added the point V as transform of P41 in the Moebius transformation centered in P4 and swapping P2 and S7.
I've drawn the QA-Cu1 of the QA formed by the Xi as intersections of both tangential QA's of the infinity point and P2.
May be you will find other properties ...
P seems to be the infinity point of the asymtote of Cu7 ?
Best regards
Bernard



Cu1 and Cu7 bis.pdf

Message: #897
Date: 2021-04-08
From: bernard.keizer@gmail.com
Subject: Re: Cu1 and Cu7

```
Dear Chris, dear Eckart,
It's even more beautiful than I thought ! I suppose it could
interest EQF ...
I haven't checked completely, but I think the last QA-Cu1 is
also focal with Miquel triangle P2P4S7 and focus S7.
QA-Cu1 and QA-Cu7 for this QA of the Xi have the same Miquel
circle and the same direction of asymptotes !
Cu1 and Cu7 intersect their asymptotes in the 2 diametral points
of the Miquel circle P41 and S7 !!!
Best regards
Bernard
```

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Message: #898
Date: 2021-04-08
From: van10hoven@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and dear Vu Thanh Tung,

With interest I read your very nice transformation and the comments from Vu Thanh Tung.
I looked if there were relationships with EQF-points.
There was only one thing:
It looks like f(QA-P4) is the point that Vu Thanh Tung introduced in QFG-messages #3529, #3530, #3537, #3539.
See https://groups.io/g/Quadri-Figures-Group/message/20872 or http://www.qfg.epizy.com/message.php?msg=3529.
The only thing is that this came out with algebraic calculations, but I couldn't find affirmation in my drawings. So I am not sure and I don't have enough time to check it completely. Maybe you can check it, one of you or both?

Then I also found an extension:
* Bisector of <A1PA2 meets line A1A2 at B1
* Let C1 be the reflection of B1 in the midpoint of A1A2.
* Similar construction for C2, C3, C4, C5, C6.
* Now the lines C1C3, C2C4, C5C6 are concurrent at g(P).
* And g(QA-P32) is a point on the Centroids Line with points QA-P1, QA-P5, QA-P10, QA-P18, QA-P20, QA-P22, QA-P25, QA-P26, QA-P43.
* No other incidences I found wrt EQF-items.

Best regards,

Chris

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Message: #899
Date: 2021-04-09
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

```
Dear Chris,

Your extension is a special case of my generalization,
with t = -1.
I will write down my proof in general case t, it is pretty
simple.
If t is an even integer number, we can have a nice expression
for barycentric coordinates as it only requires the computation
of PAi^2 which we already have.
Also, I will check your result on f(QA-P4).

Best regards,
Vu Thanh Tung
```

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Message: #900
Date: 2021-04-09
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Chris,

Thank you for your interesting comments. I give here a proof for my generalization.

I wrote
Let f(x) be a real function one variable x.
> Let X be the triangle center of ABC with barycentric coordinates
> X(f(BC),f(CA),f(AB)).
> With four points A1,A2,A3,A4 and any point P.
> Line connecting P and X-center of PA1A2 meets lines A1A2 at B1.
> Line connecting P and X-center of PA3A4 meets lines A3A4 at B2.
> Line connecting P and X-center of PA2A3 meets lines A2A3 at B3.
> Line connecting P and X-center of PA1A4 meets lines A1A4 at B4.
> Line connecting P and X-center of PA1A3 meets lines A1A3 at B5.
> Line connecting P and X-center of PA2A4 meets lines A2A4 at B6.
> Then lines B1B3, B2B4, B5B6 are concurrent at point Q which satisfies
> vec(QA1)/f(PA1) + vec(QA2)/f(PA2) + vec(QA3)/f(PA3) +
> vec(QA4)/f(PA4)=vec(0).

From the construction of B1, we have the calculation of barycentric coordinates
(f(PA1)+f(PA2))B1=f(PA1)A2+f(PA2)A1 or
(1/f(PA1)+1/f(PA2))B1=A1/f(PA1)+A2/f(PA2).

Similarly, we have
(1/f(PA3)+1/f(PA4))B2=A3/f(PA3)+A4/f(PA4).

Define the point
Q=(A1/f(PA1)+A2/f(PA2)+A3/f(PA3)+A4/f(PA4))
     /(f(PA1)+f(PA2)+f(PA3)+A4/f(PA4)).

We easily seen Q= ((1/f(PA1)+1/f(PA2))B1+
     (1/f(PA3)+1/f(PA4))B2)/(f(PA1)+f(PA2)+f(PA3)+A4/f(PA4)).

This means Q lies on line B1B2.

Similarly, since Q has symmetric expression, we have Q lies on
line B3B4,
B5B6. This completes the proof.

Note that from the barycentric expression of Q,

```
Q = (A1/f(PA1)+A2/f(PA2)+A3/f(PA3)+A4/f(PA4))
     / (f(PA1)+f(PA2)+f(PA3)+A4/f(PA4)),
```

we easily see

```
  vec(QA1)/f(PA1) + vec(QA2)/f(PA2)
+ vec(QA3)/f(PA3) + vec(QA4)/f(PA4) = vec(0).
```

Best Regards
Tran Quang Hung.

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Message: #901
Date: 2021-04-09
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] Another transformation on quadrangle

---

Dear geometers,

Let A1, A2, A3, A4 be quadrangle points. P is any point.
Lines A1A2 and A3A4 meet at X1. Isogonal lines of line PX1 in
angles <A1PA2 and <A3PA4 meet lines A1A2 and A3A4 at B1 and B2,
respectively.
Lines A2A3 and A1A4 meet at X2. Isogonal lines of line PX2 in
angles <A2PA3 and <A1PA4 meet lines A2A3 and A1A4 at B3 and B4,
respectively.
Lines A1A3 and A2A4 meet at X3. Isogonal lines of line PX3 in
angles <A1PA3 and <A2PA4 meet lines A1A3 and A2A4 at B5 and B6,
respectively.
Then the lines B1B2, B3B4, B5B6 are concurrent at Q(P).
Is this transformation new? Which are some Q(P)? (for P=Qa-P1,
Qa-P2,...)

Best Regards,
Tran Quang Hung.

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Message: #902
Date: 2021-04-09
From: bernard.keizer@gmail.com
Subject: Re: Cu1 and Cu7

Dear Eckart,
I'm able to confirm my assumptions !
For the QA X1,2,3 and 4 with DT P2, S7 and infinity point of the perpendicular bisector of P2P4, the 2nd cubic is it's QA-Cu1 and the 1rst it's QA-Cu7.
For the QA Y1,2,3 and 4 with DT P4, P41 and same infinity point, the 1rst cubic is it's QA-Cu1 and the 2nd it's QA-Cu7.
Both cubics are circular 5P circumcubics of the 5 points P2, P4,P41,S7 and infinity point.
Can you confirm these assumptions ?
Best regards
Bernard

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Message: #903
Date: 2021-04-09
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Tran Quang Hung and Chris,

Your solution is also similar to mine.
Although, the only function f that makes sense is f(x) = x^t.
Other functions do not as they do not have homogeneous properties.
Take your example f(x) = ln(x).
Consider two triangles with sides (a,b,c) and (2a,2b,2c).
The f-center of the first one is not the same point :
(ln a, ln b, ln c) ≠ (ln 2a, ln 2b, ln 2c):
ln a / ln 2a ≠ n b / ln 2b ≠ ln c/ln 2c
While for f(x) = x^t they are the same point:
(a^t, b^t, c^t) = ((2a)^t, (2b)^t, (2c)^t) :
a^t / (2a)^t = b^t / (2b)^t = c^t / (2c)^t .

Best regards,
Vu Thanh Tung

While for f(x) = x^t, they are the same

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Message: #904
Date: 2021-04-10
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Vu Thanh Tung, dear Chris,

Thank you for your interest.
The truth is I don't understand the meaning of the word "sense"
you used. It is possible that homogeneous properties need to be
given the word "triangle centers" but it is not needed for
barycentric coordinates.
Indeed, I'll leave the term "triangle centers" now away.

I can rewrite my generalization as
Let f(x) be a real function one variable x, f(x) may not need to
be a homogeneous function.
For triangle ABC, we denote point X(f(BC),f(CA),f(AB)) in
barycentric coordinates wrt ABC
(it means f(BC).vec(XA) + f(CA).vec(XB) + f(AB).vec(XC) =
vec(0)).

With four points A1,A2,A3,A4 and any point P.
Line connecting P and point (f(A1A2),f(PA2),f(PA1))
    wrt triangle PA1A2 meets lines A1A2 at B1.
Line connecting P and point (f(A3A4),f(PA4),f(PA3))
    wrt triangle PA3A4 meets lines A3A4 at B2.
Line connecting P and point (f(A2A3),f(PA3),f(PA2))
    wrt triangle PA2A3 meets lines A2A3 at B3.
Line connecting P and point (f(A1A4),f(PA4),f(PA1))
    wrt triangle PA1A4 meets lines A1A4 at B4.
Line connecting P and point (f(A1A3),f(PA3),f(PA1))
    wrt triangle PA1A3 meets lines A1A3 at B5.
Line connecting P and point (f(A2A4),f(PA4),f(PA2))
    wrt triangle PA2A4 meets lines A2A4 at B6.
Then lines B1B3, B2B4, B5B6 are concurrent at point Q which
satisfies
  vec(QA1)/f(PA1) + vec(QA2)/f(PA2)
+ vec(QA3)/f(PA3) + vec(QA4)/f(PA4) = vec(0).

This problem is actually true, as my proof that I present,

>From the construction of B1, we have the calculation of
barycentric
> coordinates
> (f(PA1)+f(PA2))B1=f(PA1)A2+f(PA2)A1 or
> (1/f(PA1)+1/f(PA2))B1=A1/f(PA1)+A2/f(PA2).
> Similarly, we have

```
> (1/f(PA3)+1/f(PA4))B2=A3/f(PA3)+A4/f(PA4).
> Define the point
> Q=(A1/f(PA1)+A2/f(PA2)+A3/f(PA3)+A4/f(PA4))/(f(PA1)+f(PA2)+f(P
A3)+f(PA4)).
> We easily seen Q= ((1/f(PA1)+1/f(PA2))B1+
> (1/f(PA3)+1/f(PA4))B2)/(f(PA1)+f(PA2)+f(PA3)+f(PA4)).
> This means Q lies on line B1B2.
> Similarly, since Q has symmetric expression, we have Q lies on
line B3B4,
> B5B6. This completes the proof.
> Note that from the barycentric expression of Q,
> Q=(A1/f(PA1)+A2/f(PA2)+A3/f(PA3)+A4/f(PA4))/(f(PA1)+f(PA2)+f(P
A3)+f(PA4)),
> we easily seen
> vec(QA1)/f(PA1) + vec(QA2)/f(PA2) + vec(QA3)/f(PA3) +
> vec(QA4)/f(PA4)=vec(0).

I apologize if I don't understand correctly, if your word
"sense" is understood as "beauty", then I would also see that
f(x) = x^t has "sense" with t = 1,2.

Best Regards,
Tran Quang Hung.
```

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Message: #905
Date: 2021-04-11
From: eckart_schmidt@t-online.de
Subject: Re: Cu1 and Cu7

```
Dear Bernard,

thanks for further explanations, hard work once more!
I have reproduced message #895, #896, #897 and #902,
... constructing with CABRI as far as possible,
    not to the last detail,
... I think you are right with your observations,
... but I cannot add new aspects, I am rather unable to cope.

Best regards Eckart
```

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Message: #906
Date: 2021-04-11
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Tran Quang Hung,

I understand that you inspired my general result with function x^t to create your general result with any function f.
Mathematically, it is correct. But it does not make sense to use any function f other than x^t. That's why I did use only the function x^t in the first place.
The reason why that does not make sense was given in my previous email.
It is the same reason why it does not make sense to call a point like (ln a, ln b, ln c) a triangle center while a point like (a^t, b^t, c^t) is a perfect triangle center.

You mentioned the word 'beauty', I should say that it is a beautiful result for all t.

Now I present a general, simple and beautiful result that I found in the first place:

Lemma:
Let x1, x2, x3, x4 be four real numbers.
Let B1, B2, B3, B4, B5, B6 on lines A1A2, A3A4, A2A3, A1A4,A1A3,A2A4 such that:
A1B1/A2B1 = x1/x2
A3B2/A4B2 = x3/x4
A2B3/A3B3 = x2/x3
A1B4/A4B1 = x1/x4
A1B5/A3B5 = x1/x5
A2B6/A4B6 = x2/x4
Here we use the sign distance.
Then B1B2, B3B4, B5B6 are concurrent at a point Q such that:
sum vector(QAi)/xi = vector (0)

Best regards,

Vu Thanh Tung

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Message: #907
Date: 2021-04-11
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

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```
Dear Vu Thanh Tung,

Thank you for your reply and for letting me know your opinion.
Above truth as you say it is my mathematical opinion as far as I
understand it.
Thank you very much for your interest.
Have a nice week.

Best regards
Tran Quang Hung.
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Message: #908
Date: 2021-04-11
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

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```
Dear all,

The lemma should be:
Lemma:
Let x1, x2, x3, x4 be four real numbers. Let B1, B2, B3, B4, B5,
B6 on lines A1A2, A3A4, A2A3, A1A4,A1A3,A2A4 such that:
A1B1/A2B1 = - x1/x2
A3B2/A4B2 = - x3/x4
A2B3/A3B3 = - x2/x3
A1B4/A4B1 = - x1/x4
A1B5/A3B5 = - x1/x3
A2B6/A4B6 = - x2/x4
Here we use the sign distance.
Then  B1B2, B3B4, B5B6 are concurrent at a point Q such that:
sum  vector(QAi)/xi  = vector (0)

Best regards,
Vu Thanh Tung
```

← Previous Next → ↔ Message Index ↑ Subjects

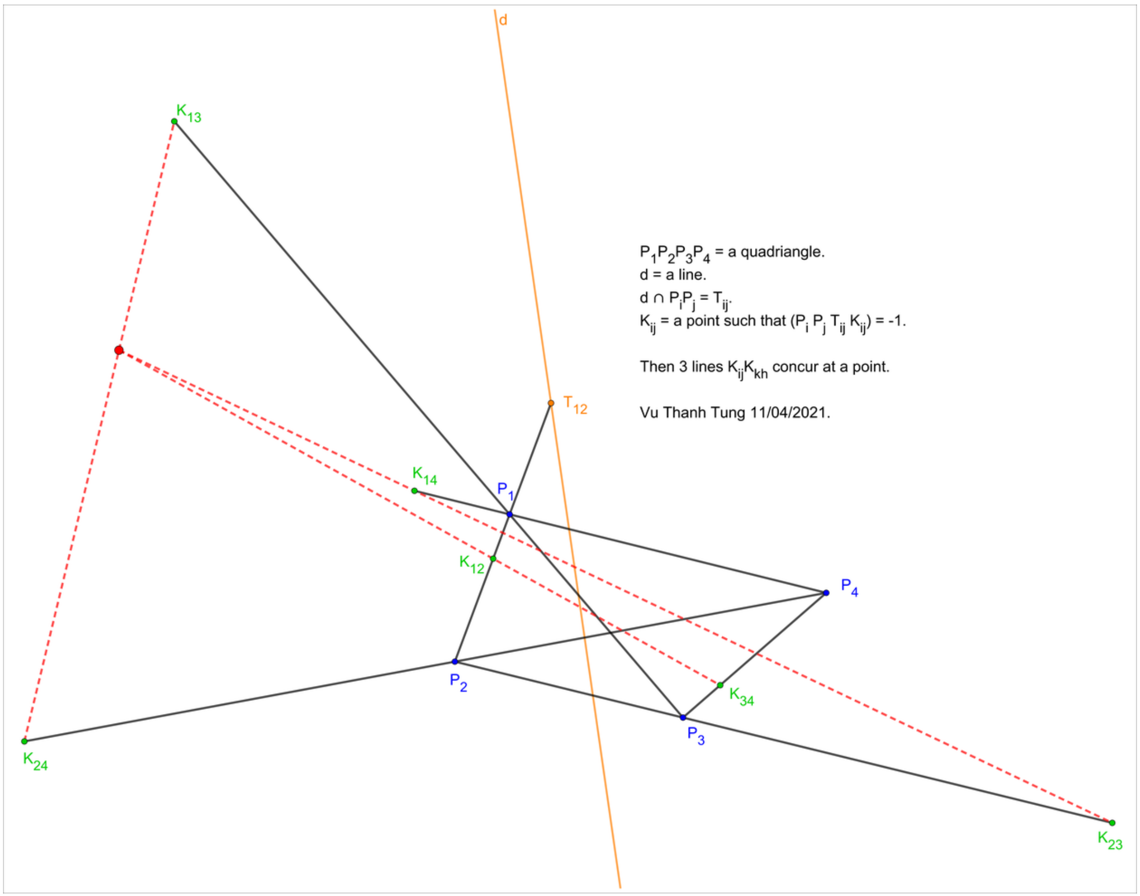
Message: #909
Date: 2021-04-11
From: tungvtt@gmail.com
Subject: A line-to-point transformation in quadriangle geometry

---

Dear all,

I would like to introduce a possibly new line-to-point
transformation in quadriangle geometry.
Let P_1P_2P_3P_4 be a quadriangle.
d is a line in the plane.
d ∩ P_iP_j = T_{ij}.
K_{ij} is a point such that (P_i P_j T_{ij} K_{ij}) = -1.
Then 3 lines K_{ij}K_{lh} concur at a point.
It can be proved by the lemma that I introduced in #908

Best regards,
Vu Thanh Tung



ALine-To-PointQuadriangleTransform.pdf

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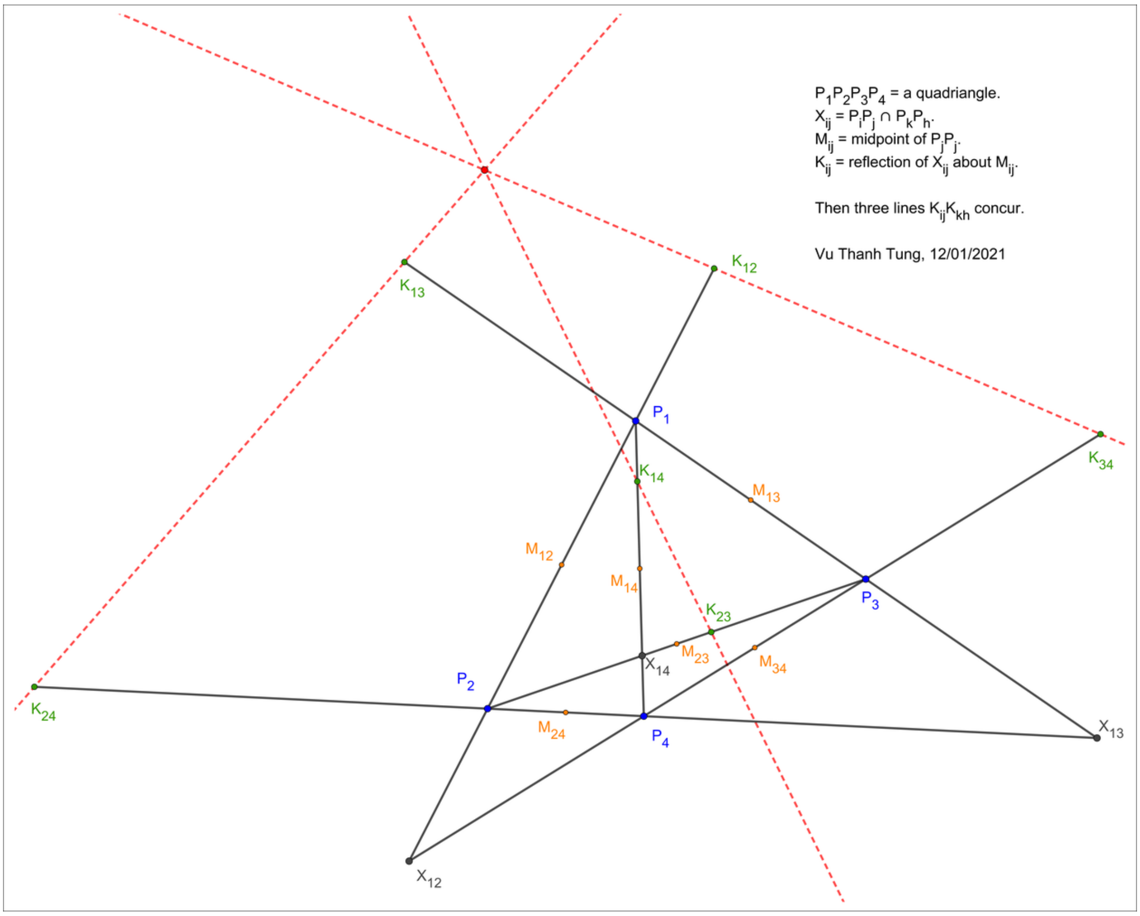
Message: #910
Date: 2021-04-11
From: tungvtt@gmail.com
Subject: A new point on quadriangle

Dear all,

I would like to introduce a possibly new point on quadriangle:
P_1P_2P_3P_4 = a quadriangle.
X_{ij} = P_iP_j ∩ P_kP_h.
M_{ij} = midpoint of P_jP_j.
K_{ij} = reflection of X_{ij} about M_{ij}.
Then three lines K_{ij}K_{kh} concur.

Best regards,
Vu Thanh Tung



AQuadrianglePoint.pdf

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Message: #911
Date: 2021-04-11
From: eckart_schmidt@t-online.de
Subject: Re: A new point on quadriangle

Dear Vu Thanh Tung,

your point in #910 is QA-P5.

Best regards Eckart

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Message: #912
Date: 2021-04-12
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A new point on quadriangle

---

Dear Eckart,

Thank you very much.

Best regards,

Vu Thanh Tung

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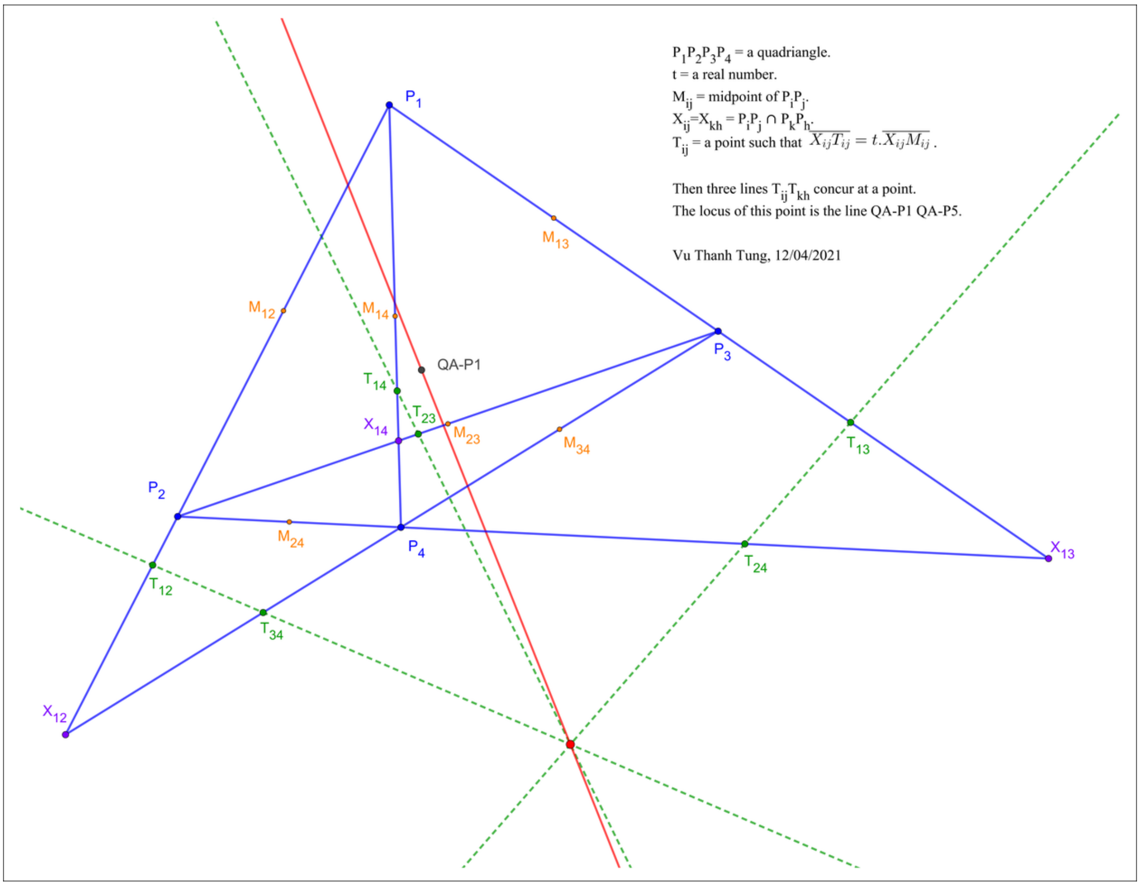
Message: #914
Date: 2021-04-12
From: tungvtt@gmail.com
Subject: Re: A family of points on quadriangle

```
Dear all,

I would like to introduce a family of points as a generalization
of #910:
P_1P_2P_3P_4 = a quadriangle.
t = a real number.
M_{ij} = midpoint of P_iP_j.
X_{ij}=X_{kh} = P_iP_j ∩ P_kP_h.
T_{ij} = a point such that X_{ij}T_{ij} = t. X_{ij}M_{ij}
(signed distance).
Then three lines T_{ij}T_{kh} concur at a point.
The locus of this point is the line QA-P1 QA-P5.

Best regards
```



AFamilyofPointsQuadrianglePointt.pdf

Message: #915
Date: 2021-04-13
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A transformation with point P and

---

Dear geometers,

Let A1, A2, A3, A4 be four points. P is any point. t is a real number.
Let O1 and G1 be the circumcenter and centroid of triangle PA1A2. B1=tG1+(1-t)O1.
Let O2 and G2 be the circumcenter and centroid of triangle PA3A4. B2=tG2+(1-t)O2.
Let O3 and G3 be the circumcenter and centroid of triangle PA2A3. B3=tG3+(1-t)O3.
Let O4 and G4 be the circumcenter and centroid of triangle PA1A4. B4=tG4+(1-t)O4.
Let O5 and G5 be the circumcenter and centroid of triangle PA1A3. B5=tG5+(1-t)O5.
Let O6 and G6 be the circumcenter and centroid of triangle PA2A4. B6=tG6+(1-t)O6.

Then midpoints of segments B1B2, B3B4, B5B6 are collinear, called this line by d(P,t).

Which is line d(P,t) where P is some quadrangle points and t for Bi is orthocenter or NPC center...

Sincerely yours,
Tran Quang Hung.

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Message: #916
Date: 2021-04-13
From: eckart_schmidt@t-online.de
Subject: Re: A transformation with point P and parameter

Dear Tran Quang Hung,

wrt your line d(P,t) in #915:
The direction of the line is independent of the parameter t.
For P = QA-P4 you get lines orthogonal QA-P3.QA-P4.

Best regards Eckart

← Previous Next → ↔ Message Index ↑ Subjects

Message: #917
Date: 2021-04-14
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation with point P and

---

```
Dear Eckart,

Thank you very much for quick answers and interesting property
of QA-P4 with this construction.

Sincerely yours,
Tran Quang Hung.
```

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Message: #918
Date: 2021-04-14
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] Family point lie on QA-L1

```
Dear geometers,

Let A,B,C,D be four points. t is a real number.

Let Ga, Oa be the circumcenter and orthocenter of BCD.
Pa=tGa+(1-t)Oa
Define similarly points Pb, Pc, Pd.
Let Ha, Hb, Hc, Hd be orthocenter of PbPcPd, PcPdPa, PdPaPb,
PaPbPc.

Then QA-P1 of (Ha,Hb,Hc,Hd) lies on QA-L1 of ABCD.

For some t which is QA-P1 of (Ha,Hb,Hc,Hd)?

Sincerely yours,
Tran Quang Hung.
```

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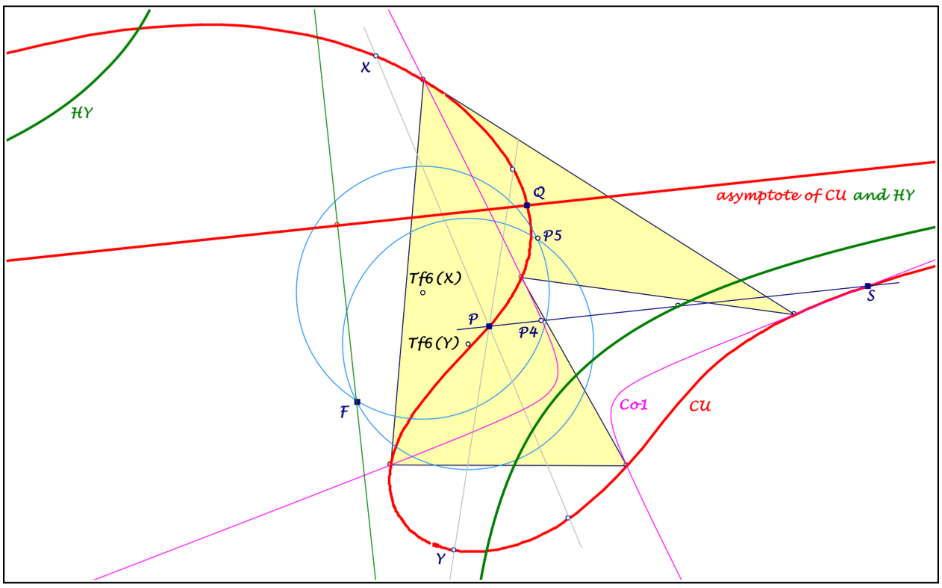
Message: #919
Date: 2021-04-14
From: eckart_schmidt@t-online.de
Subject: Main points for circular 5P-circumcubics

```
Dear all,
if a circular 5P-circumcubic CU is considered as 7P-s-Cu1,
... defined by the 5P-vertices and two further points X, Y,
... we are interested in the following main points:
(1) Focus F,
... which is the 2nd intersection of circles through P5
... centered in Tf7(X) and Tf7(Y).
If the focus lies on the 5P-quartic, the cubic is focal.
(2) Pivot P for Tf6,
... which is the intersection of X.Tf6(X) and Y.Tf6(Y).
(3) 6th intersection S of CU and circumconic Co1,
... which is the 2nd intersection of P.P4 and Co1,
... P.P4 is parallel to the asymptote of CU.
(4) The intersection Q with the asymptote:
Parallels to P.P4 as parallels to the asymptote intersect CU in
two points,
... whose midpoints give an orthogonal hyperbola HY,
... bearing for example the midpoint of P.S,
... one asymptote of this hyperbola bears the focus F,
... the other is the asymptote of CU, intersecting CU in Q.
If the cubic is focal, Q is a point on the tangent in F at the
circle (F,S,P).
Best regards Eckart

PS. I think, perhaps this summary can be helpful.
```



2021-04-14.pdf

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Message: #920
Date: 2021-04-15
From: bernard.keizer@gmail.com
Subject: Re: Main points for circular 5P-circumcubics

```
Dear Eckart,
On your figure, the centers of the 2 circles are Tf7(X) and
Tf7(Y), not Tf6 ...
On a 7P-s-Cu1, there are 21 possibilities of choosing the 5P, id
est 21 pivots Pij, 21 points P4ij and 21 points Sij.
The points F and Q as well as the rectangular hyperbola are
defined by the cubic and are the same for the 21 5P.
The intersections of the RH and the cubic are the points of
anallagmaty, where the tangents to the cubic are parallel to the
asymptote (2 for unipartite curve, 4 for bipartite).
Is it correct that the 2nd asymptote of the RH passes through F?
Is it correct that the center of the RH lies on the circle with
diameter FQ?
Best regards
Bernard
```

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Message: #921
Date: 2021-04-15
From: eckart_schmidt@t-online.de
Subject: Re: Main points for circular 5P-circumcubic

```
Dear Bernard,

thanks for correction,
... but the description in the text is correct, the figure
contains a typo.
Wrt your last two questions,
... several constructions with CABRI confirm my observations.

Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #922
Date: 2021-04-15
From: eckart_schmidt@t-online.de
Subject: New 5P-/6P-elements?

```
Dear Chris,

consider a 5P = P1...P5
... and for each vertex Pi the quadrangle of the remaining
    points
... with the four centroids of its triangles
... and let Qi be QA-P4 of these four centroids,
... which are concylic on a circle 5P-s-Cix,
    centered in 5P-s-Px.
5P-s-Px divides 5P-n-P1.5P-s-P3 with ratio -4:1.

Replacing Pi by another point P,
... we get 5 circles with a common point on 5P-s-Cix
... which shall be 5P-s-Tfx(P),

If we consider for a 6P = P1...P6
... the circles 5P-s-Ci(Pi) wrt the 5P of the remaining
    vertices,
... we get six circles with a common point 6P-s-Px.

What about the properties of
... 5Ps-Px, 5P-s-Ci1, 5P-s-Tfx, 6P-s-Px?

Best regards Eckart
```

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Message: #923
Date: 2021-04-16
From: eckart_schmidt@t-online.de
Subject: Construction of 5P-quartic

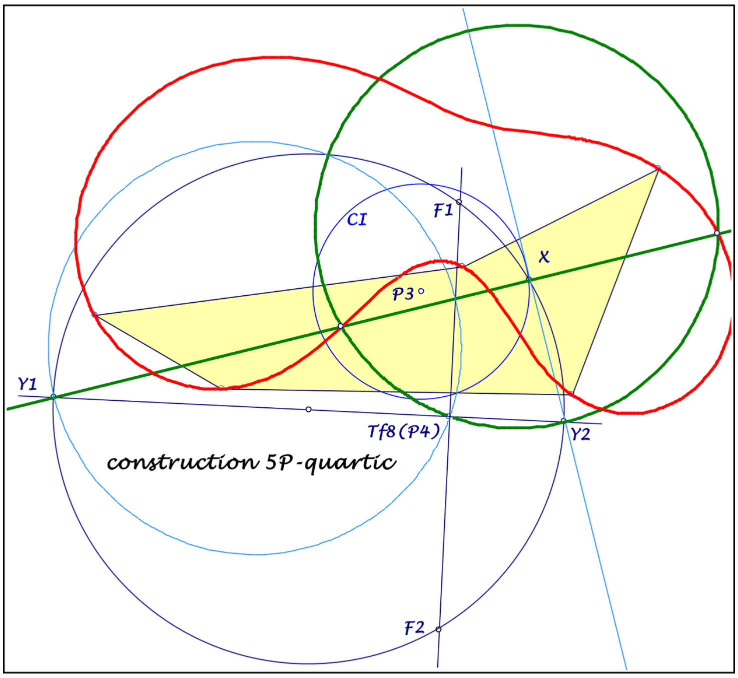
```
Dear Bernard, dear Chris,

if we mention the 5P-quartic,
... we consider the quartic as 5P-s-Tf8 of Bernard's twincubic,
... but there is a direct construction already in old#3710
    and old#3719,
... here a detailed version:

Let CI be the circle, centered in 5P-s-P3, mentioned there in
EPG,
... let X be a point on CI and F1,F2 the fixed points
    of 5P-n-Tf1 = CSC3,
... let Y1,Y2 be the intersections of circle (X,F1,F2)
... ... and the 2nd Steiner axis of 5P-n-Tf1,
... then XYi intersect their 5P-n-Tf1 image circle on the
    quartic.

Best regards Eckart

PS: CSC3 is a Möbius transformation,
... centered in 5P-s-Tf8(5P-s-P4), swapping 5P-s-P5, 5P-s-P6,
... named by Chris as 5P-n-Tf1, but not to find in EPG.
```



2021-03-21.pdf

Message: #924
Date: 2021-04-17
From: eckart_schmidt@t-online.de
Subject: Re: 5P-s-2Px

```
Dear Bernard, dear Chris,

this is a plea for two points X, Y on the 5P-quartic,
... already mentioned in old#3720,
... which are the common points of the 5P-quartic QU
... and the orthogonal hyperbola HY (see below)
... and Bernard's twincubic CU (5P-s-Tf8 of QU).
X, Y are Tf8-partner on the quartic.
Midpoint of X.Y is the midpoint of P4.P5.
X, Y are the only foci of focal circular circumscribed
5P-cubics,
... which are at the same time the cb-pivot of the cubic.

Best regards Eckart

PS. HY orthogonal hyperbola through P4, P5,
... centered in the middle,
... with axes parallel to those of 5P-s-Co1.
```

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Message: #925
Date: 2021-04-17
From: bernard.keizer@gmail.com
Subject: Re: Construction of 5P-quartic

```
Dear Eckart,
Why don't you consider directly the bisectors of the angle F1XF2
and their CSC3 transforms ?
Best regards
Bernard
```

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Message: #926
Date: 2021-04-17
From: bernard.keizer@gmail.com
Subject: Re: 5P-s-2Px

Dear Eckart,
These 2 points intersection of the quartic, the twin cubic and the rectangular hyperbola are mentionned in my 2nd memo (see message #828) as fixed points of the transformation P(ivot)-F(ocus).
Best regards
Bernard

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Message: #927
Date: 2021-04-17
From: eckart_schmidt@t-online.de
Subject: Re: Construction of 5P-quartic

Dear Bernard,

thanks for simplifying the construction.

Best regards Eckart

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Message: #928
Date: 2021-04-17
From: eckart_schmidt@t-online.de
Subject: Re: 5P-s-2Px

Dear Bernard,

excuse the passage in your paper is marked in my exemplar,
... but I didn't remember writing the message #924.

Best regards Eckart

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Message: #929
Date: 2021-04-18
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A transformation from point to line on

---

```
Dear geometers,

Given four points A1, A2, A3, A4. P is any point

B1 lies on A1A2 such that PB1 is perpendicular to A3A4.
B2 lies on A3A4 such that PB2 is perpendicular to A1A2.
B3 lies on A2A3 such that PB3 is perpendicular to A1A4.
B4 lies on A1A4 such that PB4 is perpendicular to A2A3.
B5 lies on A1A3 such that PB5 is perpendicular to A2A4.
B6 lies on A2A4 such that PB6 is perpendicular to A1A3.

Then midpoints of B1B2, B3B4, B5B6 are collinear on line d(P).

Is this transformation known before?

Sincerely yours,
Tran Quang Hung.
```

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Message: #930
Date: 2021-04-18
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point to line on

---

```
Dear geometers,

I have found more transformations on this configuration as
follows
Given four points A1, A2, A3, A4. P is any point
B1 lies on A1A2 such that PB1 is parallel to A3A4.
B2 lies on A3A4 such that PB2 is parallel to A1A2.
B3 lies on A2A3 such that PB3 is parallel to A1A4.
B4 lies on A1A4 such that PB4 is parallel to A2A3.
B5 lies on A1A3 such that PB5 is parallel to A2A4.
B6 lies on A2A4 such that PB6 is parallel to A1A3.
Then, the lines B1B2, B3B4, B5B6 are concurrent at a point Q(P).
```

More generally,
Given four points A1, A2, A3, A4. P is any point.
A line d meets lines A1A2, A3A4, A2A3, A1A4, A1A3, A2A4 at C1, C2, C3, C4, C5, C6, respectively.
B1 is the intersection on lines PC2 and A1A2.
B2 is the intersection on lines PC1 and A3A4.
B3 is the intersection on lines PC4 and A2A3.
B4 is the intersection on lines PC3 and A1A4.
B5 is the intersection on lines PC6 and A1A3.
B6 is the intersection on lines PC5 and A2A4.
Then, the lines B1B2, B3B4, B5B6 are concurrent at a point Q(d,P).
Where d=infinity line, we get above transformation with parallel lines.

Sincerely yours,
Tran Quang Hung.

Vào CN, 18 thg 4, 2021 vào lúc 01:22 Tran Quang Hung via groups.io
<analgeomatica=gmail.com@groups.io> đã viết:

> Dear geometers,
>
> Given four points A1, A2, A3, A4. P is any point
>
> B1 lies on A1A2 such that PB1 is perpendicular to A3A4.
> B2 lies on A3A4 such that PB2 is perpendicular to A1A2.
> B3 lies on A2A3 such that PB3 is perpendicular to A1A4.
> B4 lies on A1A4 such that PB4 is perpendicular to A2A3.
> B5 lies on A1A3 such that PB5 is perpendicular to A2A4.
> B6 lies on A2A4 such that PB6 is perpendicular to A1A3.
>
> Then midpoints of B1B2, B3B4, B5B6 are collinear on line d(P).
>
> Is this transformation known before?
>
> Sincerely yours,
> Tran Quang Hung.
>

Message: #931
Date: 2021-04-18
From: eckart_schmidt@t-online.de
Subject: Re: A transformation from point to line on quadrangle

---

```
Dear  Tran Quang Hung,

your simple transformation P --> Q(P) in #930 seems very
interesting:
    * QA-P1 --> QA-P22,
    * QA-P2 --> infinity point of QA-L2
    * QA-P3 --> infinity point of QA-P3.QA-P4
    * Points on the circumconic of QA-Tr1,
      centered in QA-P1 are mapped at infinity.
    * QA-Tf2-partner are mapped at their midpoint, examples:
    * QA-P1,20, QA-P4,41,QA-P5,17, QA-P6,30, QA-P10,16,
      QA-P12,23, QA-P18,19, QA-P21,27
    * If P --> Q, then reflection of P in Q
      will also be mapped at Q.
    * For P on QA-Cu1: P and Q(P) are collinear with QA-P4.
There will be more properties!

Best regards Eckart
```

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Message: #932
Date: 2021-04-18
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point to line on

---

Dear Eckart,

Thank you very much for your interest with nice properties.
For my second transformation
More generally,
Given four points A1, A2, A3, A4. P is any point.
> A line d meets lines A1A2, A3A4, A2A3, A1A4, A1A3, A2A4 at
> C1, C2, C3, C4, C5, C6, respectively.
> B1 is the intersection on lines PC2 and A1A2.
> B2 is the intersection on lines PC1 and A3A4.
> B3 is the intersection on lines PC4 and A2A3.
> B4 is the intersection on lines PC3 and A1A4.
> B5 is the intersection on lines PC6 and A1A3.
> B6 is the intersection on lines PC5 and A2A4.
> Then, the lines B1B2, B3B4, B5B6 are concurrent at a point
Q(d,P).

It is "pure" projective geometry, so it has dual as a
transformation on quadrilateral.
Given four lines a1, a2, a3, a4.
p is any line.
D is any point.
The lines connecting D with the intersections of pairs
(a1,a2), (a3,a4), (a2,a3), (a1,a4), (a1,a3), (a2,a4)
denote by c1, c2, c3, c4, c5, c6, respectively.
b1 is the line connecting intersections of pairs lines (p,c2)
and (a1,a2)
b2 is the line connecting intersections of pairs lines (p,c1)
and (a3,a4)
b3 is the line connecting intersections of pairs lines (p,c4)
and (a2,a3)
b4 is the line connecting intersections of pairs lines (p,c3)
and (a1,a4)
b5 is the line connecting intersections of pairs lines (p,c6)
and (a1,a3)
b6 is the line connecting intersections of pairs lines (p,c5)
and (a2,a4)

Then, the intersections of pairs lines (b1,b2), (b3,b4), (b5,b6)
are collinear.

Sincerely yours,
Tran Quang Hung.

Vào CN, 18 thg 4, 2021 vào lúc 15:57 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:

> Dear Tran Quang Hung,
>
>
>
> your simple transformation P --> Q(P) in #930 seems very
interesting:
>
>   - QA-P1 --> QA-P22,
>   - QA-P2 --> infinity point of QA-L2
>   - QA-P3 --> infinity point of QA-P3.QA-P4
>   - Points on the circumconic of QA-Tr1, centered in QA-P1
>   are mapped at infinity.
>   - QA-Tf2-partner are mapped at their midpoint, examples:
>   - QA-P1,20, QA-P4,41,QA-P5,17, QA-P6,30, QA-P10,16,
>   QA-P12,23, QA-P18,19, QA-P21,27
>   - If P --> Q, then reflection of P in Q will also be mapped
at Q.
>   - For P on QA-Cu1: P and Q(P) are collinear with QA-P4.
>
> There will be more properties!
>
> Best regards Eckart

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Message: #933
Date: 2021-04-19
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point to line on

---

```
Dear Eckart and friends,

Is the transformation in #929 known before? I see it is quite
simple?

Sincerely yours,
Tran Quang Hung.
```

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Message: #934
Date: 2021-04-19
From: eckart_schmidt@t-online.de
Subject: Re: A transformation from point to line on quadrangle

---

```
Dear Tran Quang Hung,

your transformation P --> d(P) in #929 is not listed in EQF,
...   here some properties:
      d(QA-P2) is a line through QA-P6, orthogonal QA-P3.QA-P4 =
      QA-L4,
      d(QA-P3) is a line through QA-P3, orthogonal QA-P3.QA-P4 =
      QA-L4,
      points on QA-L1 give lines orthogonal QA-P3.QA-P4 = QA-L4,
      points P on a line through QA-P2 give parallel lines d(P),
      points P on a line, not bearing QA-P2, give lines d(P),
      which envelope a parabola.

Best regards Eckart
```

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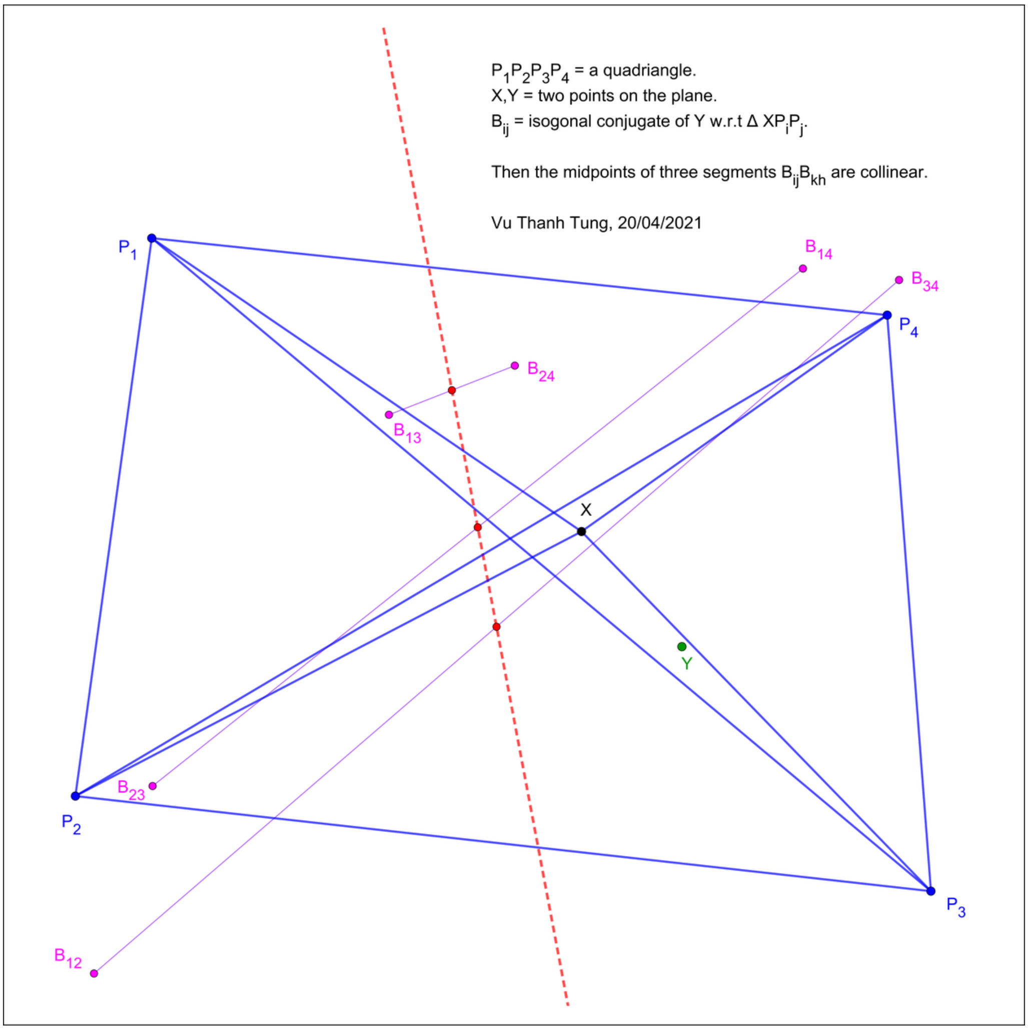
Message: #935
Date: 2021-04-20
From: tungvtt@gmail.com
Subject: A transformation on quadriangle with midponts of isogonal conju-ate

---

Dear all,

I found an interesting transformation on quadriangle :
P_1P_2P_3P_4 = a quadriangle.
X,Y = two points on the plane.
B_{ij} = isogonal conjugate of Y w.r.t Δ XP_iP_j.
Then the midpoints of three segments B_{ij}B_{kh} are collinear
on a line d(Y,X, P_1P_2P_3P_4).
Is this transformation is new ?

Best regards,
Vu Thanh Tung



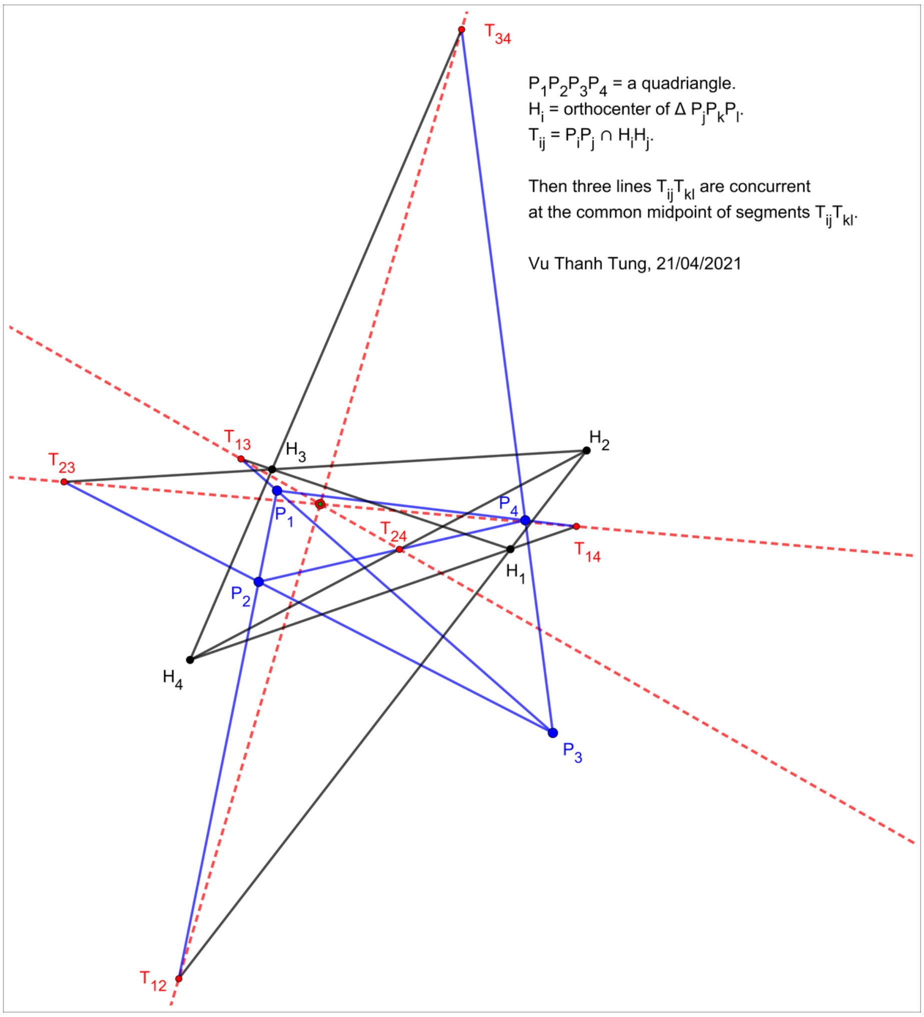
QuadriangleIsogonalMidpointColinear.pdf

Message: #936
Date: 2021-04-20
From: tungvtt@gmail.com
Subject: A point on quadriangle

```
Dear all,

I would like to propose a point on quadriangle.
The construction is as follows:
Let P1 P2 P3 P4 be a quadriangle.
Let H_i be the  orthocenter of triangle Pj Pk Pl.
Let Tij be the intersection of  PiPj and HiHj.
Then three lines T12T34, T13T24, T14T23 are concurrent and the
concurrent point is  the common midpoint of segments T12T34,
T13T24, T14T23.
Is this a new point on quadriangle?

Best regards,
Vu Thanh Tung
```



OrthocenterPointQuadriangle.pdf

Message: #937
Date: 2021-04-21
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point to line on

---

```
Dear Eckart,

Thank you very much for your confirmation with nice and new
properties of
this transformation.

Best regards
Tran Quang Hung.
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #938
Date: 2021-04-20
From: tungvtt@gmail.com
Subject: A point on quadriangle with incenters

```
Dear all,

I would like to propose a point on quadriangle similar to #936.
The construction is as follows:
Let P1 P2 P3 P4 be a quadriangle.
Let X_i be the  incenter of triangle Pj Pk Pl.
Let Tij be the intersection of  PiPj and XkXl.
Then three lines T12T34, T13T24, T14T23 are concurrent at a
point.
Is this a new point on quadriangle?

Best regards,
Vu Thanh Tung
```

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Message: #939
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A point on quadriangle with incenters

---

Dear all,

I found that if we replace the incenter with any triangle center of
barycentric coordinates (a^t,b^t,c^t), like the symedian point (t=2)
then the conclusion still holds.

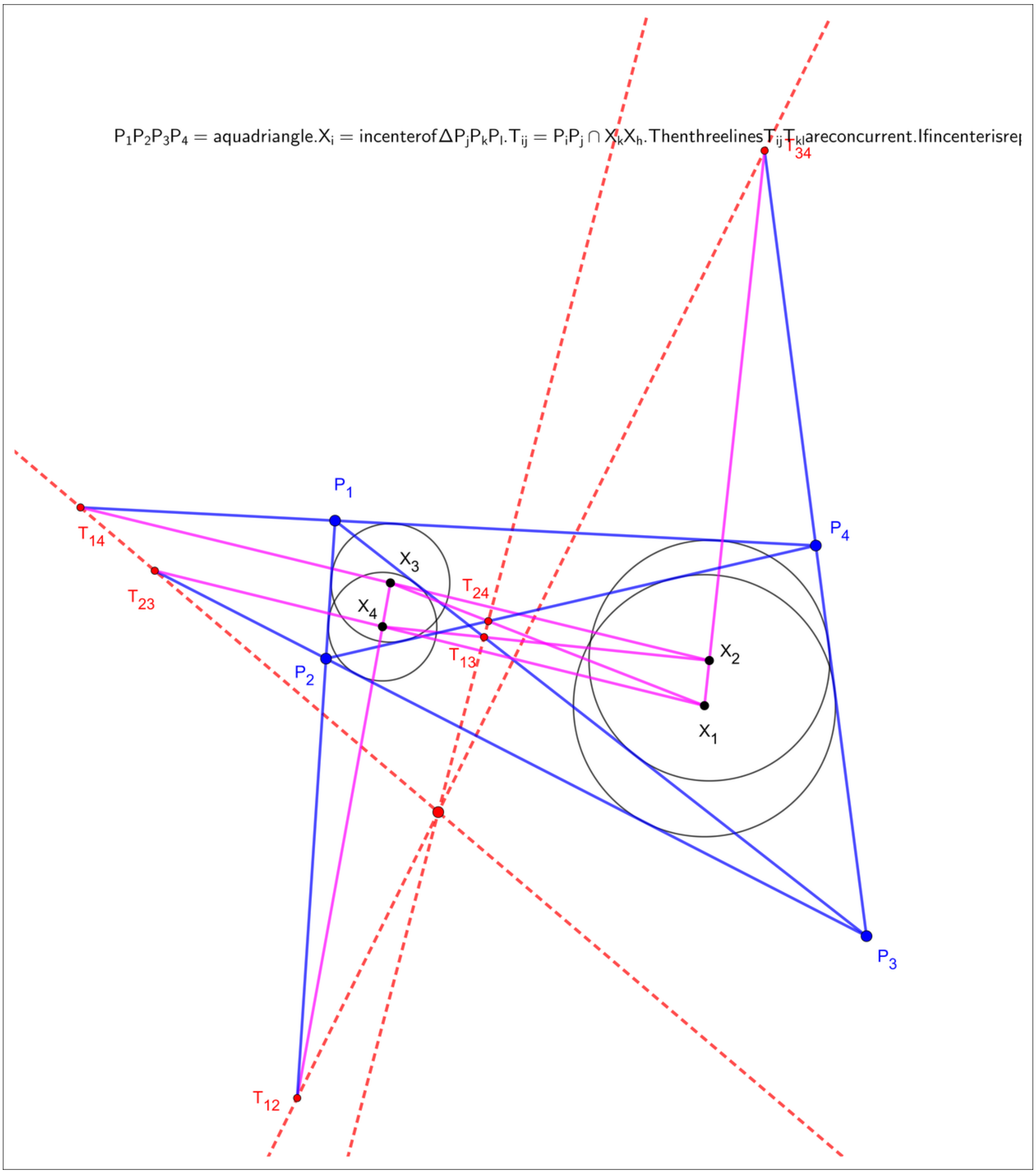
Best regards,
Vu Thanh Tung

On Wed, Apr 21, 2021 at 12:12 PM Vu Thanh Tung via groups.io
<tungvttgmail.com@groups.io> wrote:

> Dear all,
>
> I would like to propose a point on quadriangle similar to #936.
> The construction is as follows:
>
> Let P1 P2 P3 P4 be a quadriangle.
> Let X_i be the incenter of triangle Pj Pk Pl.
> Let Tij be the intersection of PiPj and XkXl.
>
> Then three lines T12T34, T13T24, T14T23 are concurrent at a point.
>
> Is this a new point on quadriangle?
>
> Best regards,
>
> Vu Thanh Tung
>
>
>

Message: #940
Date: 2021-04-20
From: tungvtt@gmail.com
Subject: Re: A point on quadriangle with incenters

```
Dear all,
Please see the attached file for a illustration.
Best regards,
Vu Thanh Tung
```



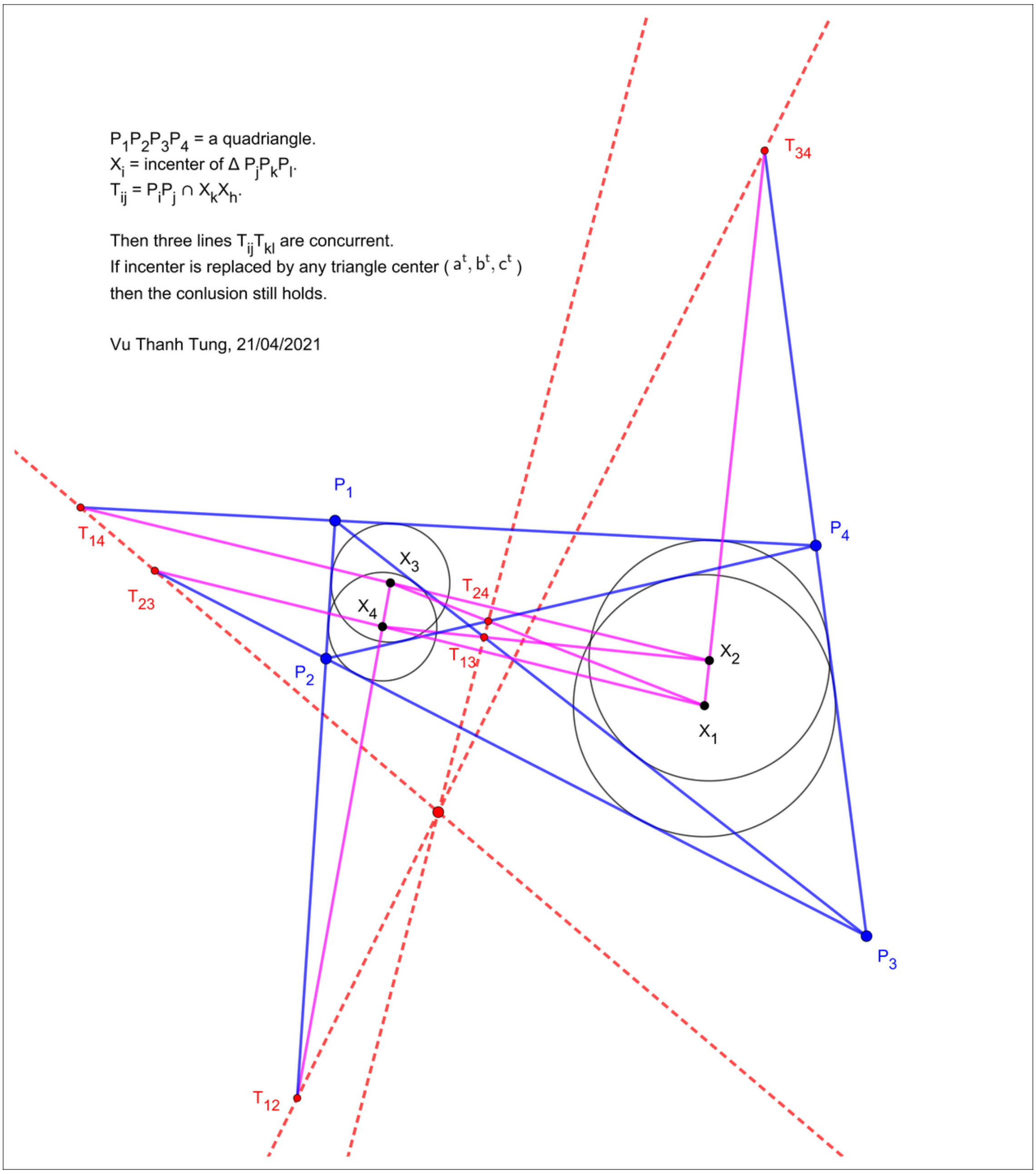
IncenterPointQuadriangle.pdf

Message: #941
Date: 2021-04-20
From: tungvtt@gmail.com
Subject: Re: A point on quadriangle with incenters

```
Dear all,

Please find the correct file in the attachment.

Best regards,
Vu Thanh Tung
```



IncenterPointQuadriangle.pdf

Message: #942
Date: 2021-04-21
From: eckart_schmidt@t-online.de
Subject: Re: A point on quadriangle

```
Dear Vu Thanh Tung,
your point in #936 is QA-P23.
Best regards Eckart
```

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Message: #943
Date: 2021-04-21
From: eckart_schmidt@t-online.de
Subject: Re: A point on quadriangle with incenters

```
Dear  Vu Thanh Tung,
your point in #939 for the symedian point is QA-P16.
Best regards Eckart

PS. Your point for the incenter will be new.
```

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Message: #944
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A point on quadriangle with incenters

---

```
Dear Eckart,

Thank you for your reply and interest.

Best regards,
Vu Thanh Tung
```

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Message: #945
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Another point-to-point transformation in quadriangle with isogonal

---

```
Dear all,

I found a point-to-point transformation  in quadriangle with
isogonal conjugate as follows:
Let P1 P2 P3 P4 be a quadriangle.
Q is any point on the plane.
Let X_i be the  isogonal conjugate of Q w.r.t. triangle Pj Pk
Pl.
Let Tij be the intersection of  PiPj and XiXj.
Then three lines T12T34, T13T24, T14T23 are concurrent at a
point.
Is this a new transformation in quadriangle ?

Best regards,
Vu Thanh Tung
```

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Message: #946
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Dear all,

Interestingly, we can obtain a point-to-line transformation by
chosing other intersection points as follows:
Let P1 P2 P3 P4 be a quadriangle.
Q is any point on the plane.
Let X_i be the  isogonal conjugate of Q w.r.t. triangle Pj Pk
Pl.
Let Uij be the intersection of  PiPj and XkXl.
Then six points T12,T34, T13,T24, T14,T23 are collinear on a
line d(Q, P1 P2 P3 P4).
Is this a new transformation in quadriangle ?

Best regards,
Vu Thanh Tung
```

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Message: #947
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Dear all,

I correct the typos as  follows:
Then six points *U12,U34, U13,U24, U14,U23* are collinear on a
line d(Q, P1 P2 P3 P4).

Best regards,
Vu Thanh Tung

On Wed, Apr 21, 2021 at 03:36 AM, Vu Thanh Tung wrote:
> Dear all,
> Interestingly, we can obtain a point-to-line transformation by
chosing
> other intersection points as follows:
> Let P1 P2 P3 P4 be a quadriangle.
> Q is any point on the plane.
> Let X_i be the  isogonal conjugate of Q w.r.t. triangle Pj Pk
Pl.
> Let Uij be the intersection of  PiPj and XkXl.
> Then six points T12,T34, T13,T24, T14,T23 are collinear on a
line d(Q, P1
> P2 P3 P4).
> Is this a new transformation in quadriangle ?
> Best regards,
> Vu Thanh Tung
```

Message: #948
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Dear all,

Two transformation that I described in #945 and #946 are true
for isotomic conjugate.
So I conjecture that it is true for any conjugate of the type
(x,y,z) -> (a^t/x,b^t/y,c^t/z).

Best regards,
Vu Thanh Tung
```

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Message: #949
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Two other lemma for concurrency and collinearity in Quadriangle

---

```
Dear all,

I introduced a lemma at #908 to explain some concurency on
Quadriangle.
I would like to introduce two more lemmas to explain some more
concurency and collinearity.

Let P1 P2 P3 P4 be a quadriangle.
Let D = (dij)   (1<=i,j<=4) be a 4*4 real symmetric  matrix
(dij = dji ).

Lemma 1:
Let X1 be the point having barrycentric ( d34, d42, d23) w.r.t
triangle P2 P3 P4 and define X2,X3,X4 cyclically.
Let Tij be the intersection of XiXj and PkPl.
Then three lines T12T34, T13T24, T14T23 are concurrent at a
point.

Lemma 2:
Let Y1 be the point having barrycentric ( d12, d13, d14) w.r.t
triangle P2 P3 P4 and define Y2,Y3,Y4 cyclically.
Let Nji be the intersection of YiYj and PiPj.
Let Uij be the intersection of YiYj and PkPl.
Then:
1. three lines N12N34, N13N24, N14N23 are concurrent at a point.
2. six points U12, U34, U13, U24, U14, U23 are collinear on a
line.

Application:
_  #938 and #939 are consequence of Lemma 1 with dij = d(Pi,Pj)
(incenter),  or dij = d(Pi,Pj)^t (general case).
_  #945, #946 and #948 are consequence of Lemma 1 with dij =
d(Pi,Pj)^2/s(QPiPj) (isogonal conjugate), dij =
1/s(QPiPj) (isotomicconjugate) or dij = s(Pi,Pj)^t/ S(QPiPj)
(general case).
Here d(X,Y) and S(XYZ) is the distance and signed area function.

Best regards,

Vu Thanh Tung
```

Message: #950
Date: 2021-04-21
From: eckart_schmidt@t-online.de
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Dear Vu Thanh Tung,

your tansformation in #945 is a Möbius transformation,
... centered in QA-P4, swapping QA-P2 and QA-P41.

Best regards Eckart
```

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Message: #951
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Another point-to-point transformation

---

```
Dear Eckart,

It is interesting. Thank you for your interest.

Best regards,
Vu Thanh Tung


On Wed, Apr 21, 2021 at 8:45 PM Eckart Schmidt
<eckart_schmidt@t-online.de>
wrote:
> Dear Vu Thanh Tung,
> your tansformation in #945 is a Möbius transformation,
> ... centered in QA-P4, swapping QA-P2 and QA-P41.
> Best regards Eckart
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #952
Date: 2021-04-21
From: eckart_schmidt@t-online.de
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Dear  Vu Thanh Tung,

your point --> line transformation in #946/947
... has some interesting properties, for example:
(1) The Miquel points are mapped to lines through QA-P2
... and a vertex of the diagonal triangle QA-Tr1.
(2) QA-P2 is mapped to a line through QA-P41
... which is the image of the circle through QA-P2,4,9
... wrt your transformation in #945.

There will be more properties!

Best regards Eckart
```

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Message: #953
Date: 2021-04-21
From: tungvtt@gmail.com
Subject: Re: Another point-to-point transformation in quadriangle with

---

```
Thanks Eckart.
Can't wait to see more properties.
```

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Message: #954
Date: 2021-04-22
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A transformation from point divide Euler

---

```
Dear geometers,

Let A,B,C,D be four points. t is a real number.
Let Oa, Ha be the circumcenter and orthocenter of triangle BCD,
respectively.
Xa=tOa+(1-t)Ha.
Define similarly points Xb,Xc,Xd. of triangles CDA, DAB, ABC,
respectively.

Line XaXb meets line AB at Pab.
Line XcXd meets line CD at Pcd.
M1 is the midpoint of PabPcd.

Line XaXd meets line AD at Pad.
Line XcXb meets line CB at Pcb.
M2 is the midpoint of PadPcb.

Line XaXc meets line AC at Pac.
Line XbXd meets line BD at Pbd.
M3 is the midpoint of PabPcd.

Then, three points M1, M2, M3 lie on a line d(t).
Which is d(t) for some parameter t, i.e. Xa is the circumcenter,
orthocenter, NPC center?

Sincerely yours,
Tran Quang Hung.
```

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Message: #955
Date: 2021-04-22
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point divide

---

Dear geometers,

I am sorry, I corrected the message in red text.
Let A,B,C,D be four points. t is a real number.
> Let Oa, Ha be the circumcenter and orthocenter of triangle
> BCD, respectively.
> Xa=tOa+(1-t)Ha.
> Define similarly points Xb,Xc,Xd. of triangles CDA, DAB, ABC,
> respectively.
> Line XaXb meets line AB at Pab.
> Line XcXd meets line CD at Pcd.
> M1 is the midpoint of PabPcd.
> Line XaXd meets line AD at Pad.
> Line XcXb meets line CB at Pcb.
> M2 is the midpoint of PadPcb.
> Line XaXc meets line AC at Pac.
> Line XbXd meets line BD at Pbd.
> M3 is the midpoint of PacPbd.
> Then, three points M1, M2, M3 lie on a line d(t).
> Which is d(t) for some parameter t, i.e. Xa is the
> circumcenter, orthocenter, NPC center?

Sincerely yours,
Trần Quang Hùng
Tổ toán THPT chuyên KHTN, ĐHKHTN, ĐHQGHN.

Vào Th 5, 22 thg 4, 2021 vào lúc 00:16 Tran Quang Hung via
groups.io
<analgeomatica=gmail.com@groups.io> đã viết:

> Dear geometers,
>
> Let A,B,C,D be four points. t is a real number.
>
> Let Oa, Ha be the circumcenter and orthocenter of triangle
BCD,
> respectively.
> Xa=tOa+(1-t)Ha.
> Define similarly points Xb,Xc,Xd. of triangles CDA, DAB, ABC,
respectively.
>
> Line XaXb meets line AB at Pab.

```
> Line XcXd meets line CD at Pcd.
> M1 is the midpoint of PabPcd.
>
> Line XaXd meets line AD at Pad.
> Line XcXb meets line CB at Pcb.
> M2 is the midpoint of PadPcb.
>
> Line XaXc meets line AC at Pac.
> Line XbXd meets line BD at Pbd.
> M3 is the midpoint of PabPcd.
>
> Then, three points M1, M2, M3 lie on a line d(t).
>
> Which is d(t) for some parameter t, i.e. Xa is the
> circumcenter, orthocenter, NPC center?
>
> Sincerely yours,
> Tran Quang Hung.
```

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Message: #956
Date: 2021-04-22
From: bernard.keizer@gmail.com
Subject: Re: Two other lemma for concurrency and collinearity in Quadrangle

---

```
Dear Vu Thanh Tung
Very interesting properties indeed !
Did you try to prove them analytically (in DT coordinates, it
seems possible ...)
Best regards
Bernard
```

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Message: #957
Date: 2021-04-22
From: bernard.keizer@gmail.com
Subject: Re: A transformation from point divide Euler line

```
Dear Tran Quang Hung
I reproduced your properties for the circumcenter (a line), for
the orthocenter (a point).
For the centroïds, the line is the infinity line.
Question 1: did you try to prove the property analytically (in
DT coordinates ...) ?
Question 2: what about other triangle lines ? (Brocard axis ?)
Best regards
Bernard
```

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Message: #958
Date: 2021-04-23
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point divide

---

Dear Bernard,

Thank you for your interest.
I have tried to prove this property. I think I can prove it by
Cartesian coordinates.
I see this property is true with the Euler line only.

Sincerely yours,
Tran Quang Hung.

Vào Th 5, 22 thg 4, 2021 vào lúc 15:23 Bernard Keizer <
bernard.keizer@gmail.com> đã viết:

> Dear Tran Quang Hung
> I reproduced your properties for the circumcenter (a line),
for the
> orthocenter (a point).
> For the centroïds, the line is the infinity line.
> Question 1 : did you try to prove the property analytically
> (in DT coordinates ...) ?
> Question 2 : what about other triangle lines ?
> (Brocard axis ?)
> Best regards
> Bernard

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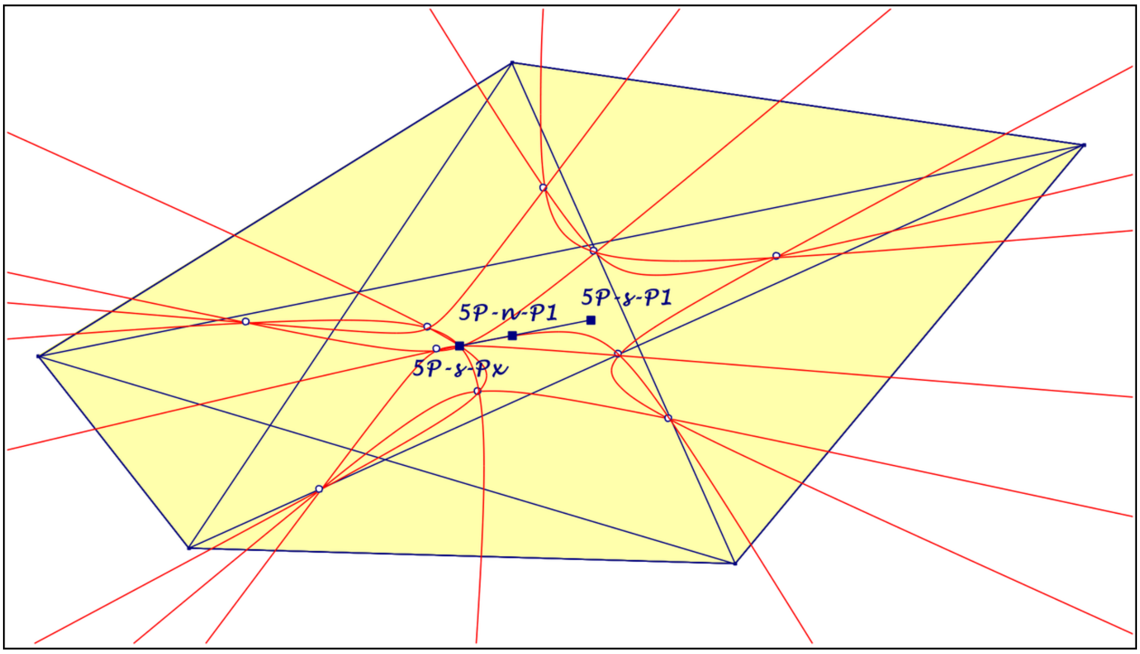
Message: #959
Date: 2021-04-23
From: eckart_schmidt@t-online.de
Subject: New 5P-point ?

```
Dear Chris, dear Bernard,

a vertex Pi of a 5P = P1...P5 is vertex of 6 triangles PiPjPk,
... their centroids define a conic COi,
... the 5 conics have a common point 5P-s-Px,
... collinear with 5P-n-P1, 5P-s-P1, dividing with ratio -2:5.

Another observation:
A vertex Pi of a 5P = P1...P5 is vertex of 6 triangles PiPjPk,
... their circumcenters define a quadrilateral QLi,
... the 5 CSC-images of Pi wrt QLi coincide in 5P-s-P5.

Best regards Eckart
```



2021-04-23.pdf

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Message: #960
Date: 2021-04-24
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-point ?

```
Dear Chris, dear Bernard,

in #922 I proposed a new 5P-point, but with wrong ratio:
... corrected ratio is -4:10 = -2:5,
... let this point be X.
In #959 I proposed a further new 5P-point,
... let this point be Y.
X divides 5P-n-P1.5P-s-P3 with ratio -2:5,
Y divides 5P-n-P1.5P-s-P1 with ratio -2:5.
Perhaps there is an interesting Möbius transformation,
... centered in 5P-n-P1, swapping X, 5P-s-P3 and Y, 5P-s-P1.

Further observation:
Let r be the radius of the circle round X, mentioned in #922,
... let R be the radius of the Quang Duong's circle,
    centered in 5P-s-P3,
... then holds r : R = 2 : 3.

Best regards Eckart
```

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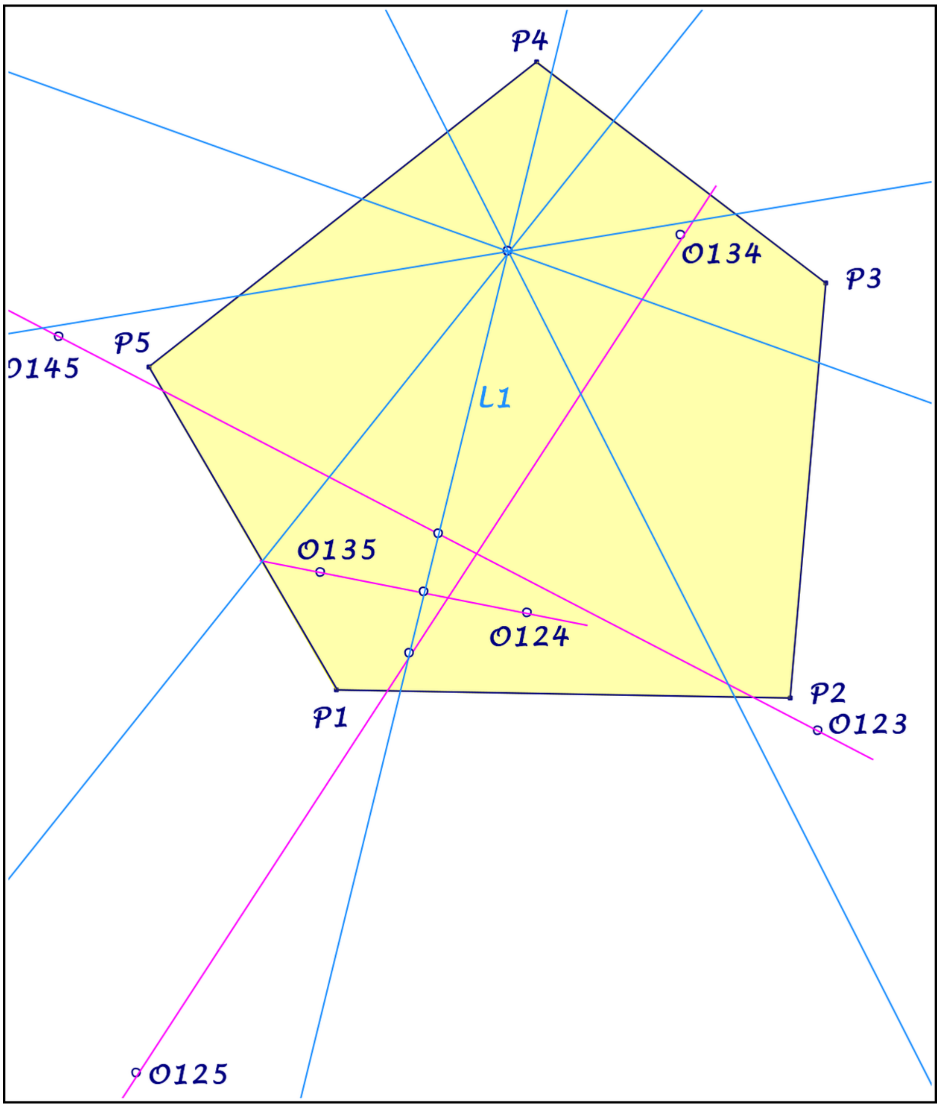
Message: #961
Date: 2021-04-25
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-point ?

Dear Chris, dear Bernard,

in #922 and #959 I proposed new 5P-points, here a third one:
Consider for every vertex Pi of a 5P = P1...P5
... the 6 orthocenters Oijk of the triangles PiPjPk
    with vertex Pi,
... the 3 midpoints of Oijk.Oilm, Oijl.Oikm, Oijm.Oikl
    are collinear
... and the 5 lines of these midpoints have a common point.

Best regards Eckart



2021-04-25.pdf

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Message: #962
Date: 2021-04-25
From: eckart_schmidt@t-online.de
Subject: New QA-transformations ?

```
Dear  Vu Thanh Tung,

are the following transformations well known?

(1) Point to point:
Consider for a quadrangle P1...P4 and a point P
... the six nine-point centers of PPiPj = Qij
... with 4 circles CIi = (Qij,Qik,Qil),
... which have a common point.
This holds also for the six centroids.

(2) Point to line:
Consider for a quadrangle P1...P4 and a point P
... the six orthocenters of PPiPj = Qij
... and the three midpoints of Qij.Qkl,
... which are collinear.
This holds for all ETC-points on the Euler line.
If we vary this point and hold P fix, the lines are parallel.

Best regards Eckart
```

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Message: #963
Date: 2021-04-25
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?

```
Dear Eckart,

I do not know (1).
For (2), Tran Quang Hung did mention it recently at #915, and I
do believe he also mentioned it one year ago.

Best regards,
Vu Thanh Tung
```

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Message: #964
Date: 2021-04-25
From: tungvtt@gmail.com
Subject: Re: Two other lemma for concurrency and collinearity in Quadriangle

---

```
Hi Bernard,

I do not have proof for them yet. I will try to prove it in DT
coordinates.

Best regards,
Vu Thanh Tung
```

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Message: #965
Date: 2021-04-25
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?

```
Dear Eckart,

Did you try
(3)  4 circles CI'i = (Qjk,Qkl,Qlj)  have common.
I also believe that they also holds for quadrilateral.

Best regards,
Vu Thanh Tung
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #966
Date: 2021-04-25
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?

```
Dear Eckart,

Did you try
(3)  4 circles CI'i = (Qjk,Qkl,Qlj)  have common *point*.
I also believe that they also holds for quadrilateral.

Best regards,
Vu Thanh Tung

On Sun, Apr 25, 2021 at 05:01 PM, Vu Thanh Tung wrote:
> Dear Eckart,
> Did you try
> (3)  4 circles CI'i = (Qjk,Qkl,Qlj)  have common.
> I also believe that they also holds for quadrilateral.
> Best regards,
> Vu Thanh Tung
```

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Message: #967
Date: 2021-04-25
From: tungvtt@gmail.com
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

```
Dear all,
With inspiration from #962, I found two more properties in this
configuration.

P1 P2 P3 P4 = a quadriangle.
X,Y = two points on the plane.
Bij = isogonal conjugate of Y w.r.t Δ X Pi Pj.

Then
(1) the midpoints of three segments Bij Bkl are collinear on a
line d(Y,X, P1 P2 P3 P4).  (#935)
(2) for each i, define circle Cli = (Bij Bik Bil) then 4 circles
Cl1, Cl2, Cl3, Cl4 have a common point.
(3) for each i, define circle Cl'i = (Bjk Bkl Blj) then 4
circles Cl'1, Cl'2, Cl'3, Cl'4 also have a common point.

Best regards,

Vu Thanh Tung
```

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Message: #968
Date: 2021-04-26
From: bernard.keizer@gmail.com
Subject: Re: A transformation from point divide Euler line

Dear Tran Quang Hung
I continued searching properties of your points.
At the beginning, I thought that all points from the Euler Line verify your property and only these points.
I could in fact reproduce the property for ETC points X2, X3, X4, X5 and X20, but if I'm not wrong, the property doesn't hold for X22 and X23 !
Then I found that some other points also verify the property.
Defining the barycenter of the 4 points QA-P1 as K, it's obvious that K = 1/4(P1,P2,P3,P4) = 1/4(P1, 3G1) = 1/4(P1, X, 2Y) = 1/2(middle of P1X, Y), where X is the complement of Y.
In other words, if the property holds for a point Y on the Euler Line, the middle of PiYi is the reflexion in K of the complement X of Y !
For example, naming G the centroïd, O the circumcenter, H the orthocenter, N the center of NPC and L the de Longchamps point and g, o, h, n and l the middles of the segments joining the points Gi, Oi, Hi, Ni and Li to each vertice Pi, it appears that
the middle gi of PiGi is the reflexion of Gi in K
the middle hi of PiHi is the relexion in K of Oi
the middle oi of PiOi is the reflexion in K of Ni
the middle li of PiLi is the reflexion in K of Hi
All these points gi, hi, oi, ni and li verify your property (for hi, the 3 middles are in K)
Best regards
Bernard

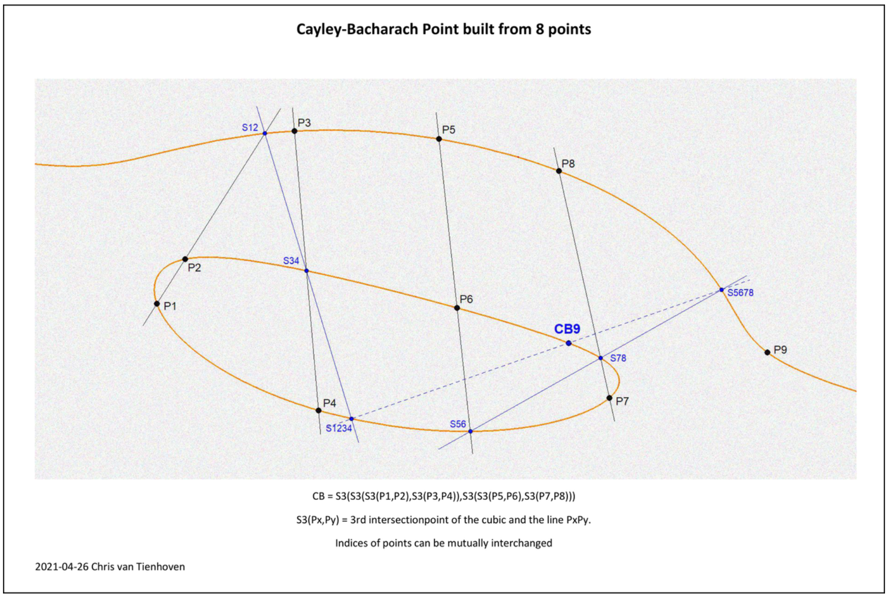
← Previous Next → ↔ Message Index ↑ Subjects

Message: #969
Date: 2021-04-26
From: van10hoven@gmail.com
Subject: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Bernard, Eckart, dear friends,

I found an amazing “construction” of the Cayley-Bacharach Point.
Let P1,P2,P3,P4,P5,P6,P7,P8,P9 be the 9 defining points of a
cubic CU.
* Let CB9 = Cayley-Bacharach Point of P1, ..., P8.
* Let S12 = 3rd intersection point of CU with P1P2.
* Let S34 = 3rd intersection point of CU with P3P4.
* Let S56 = 3rd intersection point of CU with P5P6.
* Let S78 = 3rd intersection point of CU with P7P8.
* Let S1234 = 3rd intersection point of CU with S12S34.
* Let S5678 = 3rd intersection point of CU with S56S78
* Now CB9 = 3rd intersection point of CU with S1234S5678.
The indices 1,2,3,4,5,6,7,8,9 can be mutually interchanged.
In one formula:
* CB = S3(S3(S3(P1,P2),S3(P3,P4)),S3(S3(P5,P6),S3(P7,P8)))
where S3(Px,Py) = 3rd intersectionpoint of the cubic and the
line PxPy.
See attached picture.
You can see very well how the position of the CB-point changes
when one of the defining point changes by following the
connecting lines.
Best regards,
Chris
```



8P-s-P1-Cayley-Bacharach Point-10-Alternative-Construction.pdf

---

Message: #970
Date: 2021-04-26
From: eckart_schmidt@t-online.de
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Chris,

really an amazing observation, gratulation!
Is there an analogon for 6 points plus circular points?

Best regards Eckart
```

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Message: #971
Date: 2021-04-26
From: eckart_schmidt@t-online.de
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Chris,

first drawings with CABRI confirm an analogon for
... the Cayley Bacharach point for 6 points
    plus the circular points,
... using 7P-s-Cu1 and for S78 the infinity point
    of the asymptote.

Best regards Eckart
```

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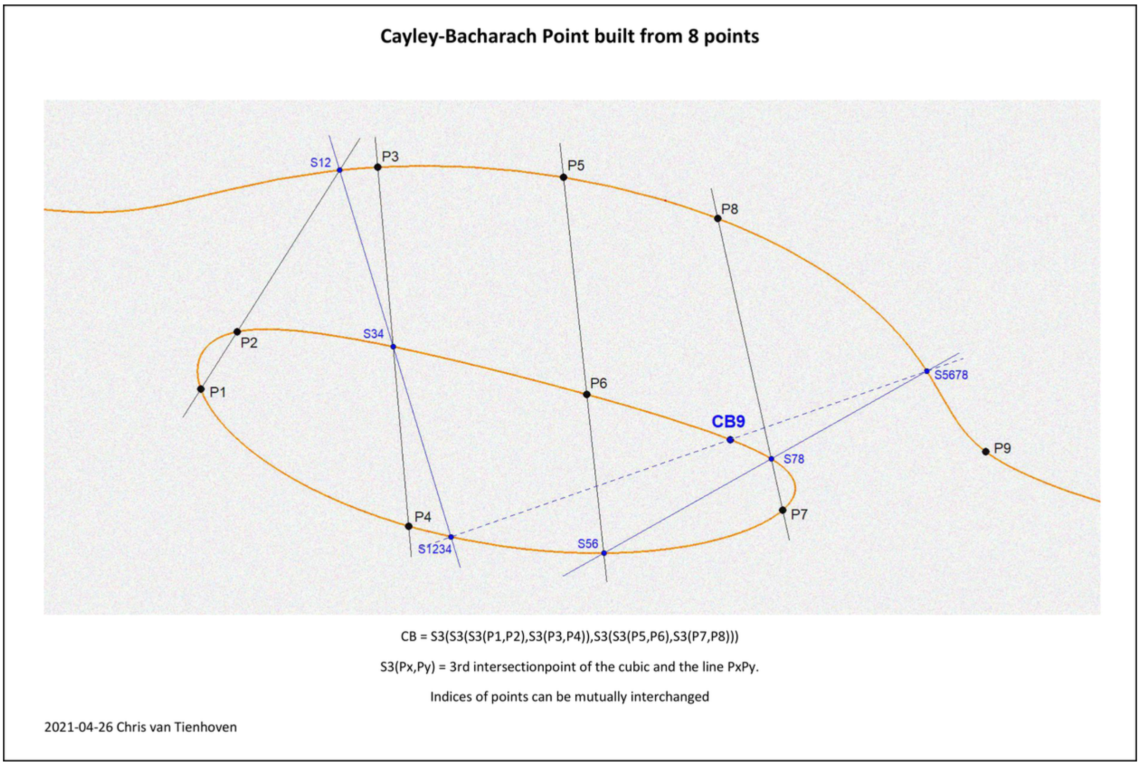
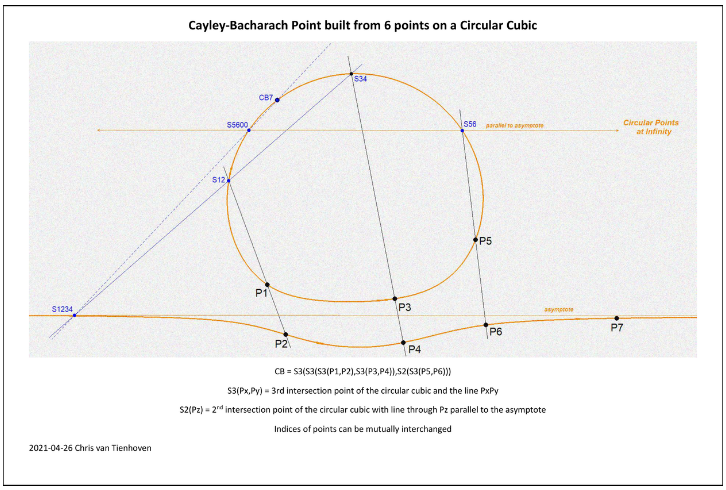
Message: #972
Date: 2021-04-26
From: van10hoven@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Eckart,
Right, see my new picture I was sending you.
Best regards,
Chris
```



8P-s-P1-Cayley-Bacharach Point-10-Alternative-Construction.pdf



8P-s-P1-Cayley-Bacharach Point-10-Alternative-Construction.pdf

---

Message: #973
Date: 2021-04-27
From: bernard.keizer@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

Dear Chris,
You use without saying Cotterill's property that any conic through 4 points of a cubic intersect the cubic in 2 other points having as S3 (your notation) a fixed point depending only of the 4 points and named by Cotterill the focus of the QA.
For 8 points on a cubic, there are 35 lines through the CB joining the foci of groups of 4 points.
In your 1rst example with an ordinary cubic, S1234 is the focus of P1,2,3,4 and S5678 the focus of P5,6,7,8.
In your 2nd example with a circular cubic, S5600 is the focus of P5,6 and the circular points.
The interest of your construction is that you use degenerated conics formed by 2 lines (in the 2nd case, S00 is the infinity point of the asymptote).
If the cubic is a pivotal isocubic of 4 points P1 to 4 with pivot P, S12 and S34 coïncide on the cubic (DT vertice) and the focus is the tangential of the 3 DT vertices, which is also tgP.
If the cubic is a pivotal circular cubic with pivot QA-P4, the focus is tgQA-P4 or QA-P41.
Best regards
Bernard
PS I'm always waiting for your comments on my 4 memos, in particular the 3rd one about the triple Moebius, which was completely new ...

Message: #974
Date: 2021-04-27
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point divide

---

```
Dear Bernard,

My problem is true for the point divide Euler in constant ratio
t.
Let Oa, Ha be the circumcenter and orthocenter of triangle BCD,
> respectively.
> Xa=tOa+(1-t)Ha.
> Define similarly points Xb,Xc,Xd. of triangles CDA, DAB, ABC,
respectively.

Constant t does not depend on the elements of the triangle.

Sincerely yours,
Tran Quang Hung.
```

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Message: #975
Date: 2021-04-27
From: van10hoven@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Bernard,
Thanks for your analyses about the alternative construction of
the Cayley-Bacharach Point.
It's a beautiful property of Cotterill that you mentioned.
Do you have one or more reference(s) for it?
Best regards,
Chris
```

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Message: #976
Date: 2021-04-28
From: bernard.keizer@gmail.com
Subject: Re: A transformation from point divide Euler line

```
Dear Tran Quan Hung
I think I understand what you mean.
t is a real number
In fact, I checked a certain number of Euler Line points.
The property holds for X2,3,4,5,20,140 and 382 (t=
2/3,1,0,1/2,2,3/4,-1).
But it doesn't hold for X21,22,23,24,25 ...
But I think you have to complete your definition, as the
property holds also for the relexions in K of all points defined
with t real number dividing OH.
(As mentionned in my previous message, for a point Xi of your
list, the middle of PiXi is the reflexion in K of the complement
Yi of Xi, which also belongs to your list)
Best regards
Bernard
```

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Message: #977
Date: 2021-04-28
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?
: Re: A transformation from point divide Euler line

```
Dear Eckart,

Here what I found for quadriangle:

1) Point to point:
Consider for a quadrangle P1...P4 and a point P
... the six circumcenter of PPiPj = Qij
... with 4 circles CIi = (Qjk,Qkl,Qlj),
... which have a common point.
This holds also for the six centroids.

Best regards,
Vu Thanh Tung

On Sun, Apr 25, 2021 at 05:02 PM, Vu Thanh Tung wrote:
> Dear Eckart,
> Did you try
> (3)  4 circles CI'i = (Qjk,Qkl,Qlj)  have common *point*.
> I also believe that they also holds for quadrilateral.
> Best regards,
> Vu Thanh Tung
```

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Message: #978
Date: 2021-04-28
From: eckart_schmidt@t-online.de
Subject: Re: A transformation from point divide Euler line

```
Dear Tran Quang Hung, dear Bernard,

I think, that Tran Quang Hung uses for Xa triangle points
... but Bernard's points gi, hi, oi,
... depend also on the 4th point,
... that is quite another problem.

Best regards Eckart
```

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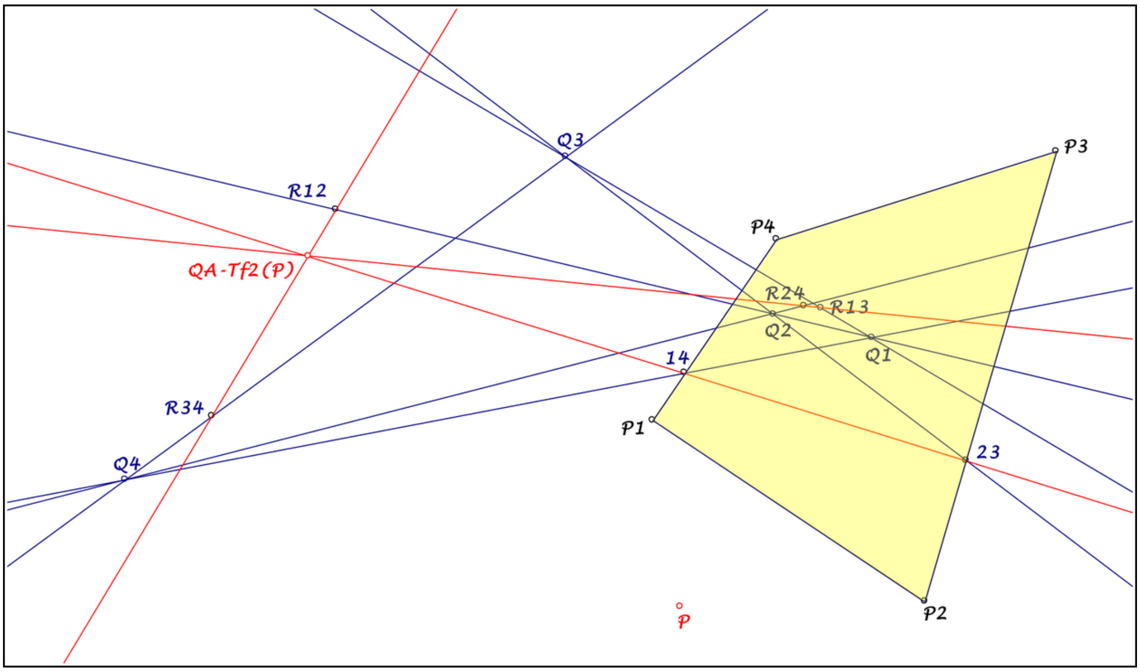
Message: #979
Date: 2021-04-28
From: eckart_schmidt@t-online.de
Subject: New interpretation of QA-Tf2

```
Dear Vu Thanh Tung, dear Chris,

consider a QA = P1...P4 and a point P
... the trilinear poles Qi of P,Pi wrt the triangle PjPkPl,
... further the intersections Rij = Qi.Qj ^ Pi.Pj  ,
... then the lines Rij.Rkl have a common point,
... which is the QA-Tf2-image of P.

Analog there is an interpretation of QL-Tf2.

Best regards Eckart
```



2021-04-28.pdf

Message: #980
Date: 2021-04-28
From: bernard.keizer@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Chris,
I've referred several times in the Forum to this beautiful short
article, which deserves certainly to be in the references of EQF
(like Marden and Siebeck):
Thomas Cotterill A Geometrical property of curves of the third
order.
Best regards
Bernard
PS You didn't answer my PS
```

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Message: #981
Date: 2021-04-28
From: van10hoven@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Bernard,

Thanks for the reference.
Unfortunately I could not find any match at Google.
Do you have a link of a pdf?
Also of Marden and Siebeck?
About your PS, I wish I had enough time to read all your
beautiful documents or even one.
I am very very busy in many ways. That’s why I can’t make any
promises about that.
I am really sorry.

Chris
```

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Message: #982
Date: 2021-04-28
From: eckart_schmidt@t-online.de
Subject: Re: New QA-transformations ?

```
Dear Vu Thanh Tung,

excuse my late answer, but I searched for properties
... for your transformations in #966 and #977,
... which are right observations.
Only for the centroids I found a property:
The image Y of a point X divides X.QA-P2 with ratio 2:1.

Best regards Eckart
```

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Message: #983
Date: 2021-04-29
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation from point divide

---

```
Dear Bernard, dear Eckart,

I mean as Eckart understand, I consider
Xa=tOa+(1-t)Ha.
Xb=tOb+(1-t)Hb.
Xc=tOc+(1-t)Hc.
Xd=tOd+(1-t)Hd.
Real number t is common for all triangles ABC,BCD,CDA,DAB.
This property seems true with Euler line only.

Sincerely yours,
Tran Quang Hung.

Vào Th 4, 28 thg 4, 2021 vào lúc 20:59 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:
>  Dear Tran Quang Hung, dear Bernard,
> I think, that Tran Quang Hung uses for Xa triangle points
> ... but Bernard's points gi, hi, oi , ... depend also on the
4th point,
> ... that is quite another problem.
> Best regards Eckart
```

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Message: #984
Date: 2021-04-29
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A property of four circles

---

```
Dear geometers,

We consider four random circles
(A,Ra), (B,Rb), (C,Rc), (D,Rd).
Let A1 be the radical center of (B,Rb), (C,Rc), (D,Rd). Define
similarly, B1, C1, D1.

Consider circle (A,A1) (Circle center A and passes through A1),
and similarly (B,B1), (C,C1), (D,D1).

Let A2 be the radical center of circles (B,B1), (C,C1), (D,D1).
Define similarly, B2, C2, D2.

Then we have two quadrangle points (A1,B1,C1,D1) and
(A2,B2,C2,D2) are homothetic.

Which is the homothetic center in terms of Ra, Rb, Rc, Rd?

If we consider Ra=R(BCD) (circumradius of triangle BCD),
Rb=R(CDA), Rc=R(DAB), Rd=R(ABC), is the homothetic center in
EQF?

Sincerely yours,
Tran Quang Hung.
```

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Message: #985
Date: 2021-04-29
From: bernard.keizer@gmail.com
Subject: Re: A transformation from point divide Euler line

Dear Tran Quang Hung,
If you consider the 2 points O'a and H'a, reflexions of Oa and Ha in K (QA-P1 of the 4 points A,B,C and D, the property with the aligned middles is true
1) for the point Xa,Xb,Xc and Xd such as Xa = tOa + (1-t)Ha ...
2) for the points X'a,X'b,X'c and X'd such as X'a = tO'a + (1-t)H'd ... t being the same real number
I can't understand why the existence of the points X'a sharing the same property as the Xa could damage this beautiful property!
Best regards
Bernard

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Message: #986
Date: 2021-04-29
From: bernard.keizer@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

Dear Chris,
I cannot either find this article on Google and I don't remember where I founded it !
Exact reference is A Geometrical Property of Curves of the third order by Thomas Cotterill, M.A., Late Fellow of St. John's College, Cambridge
At the end of the article is mentionned London, March 12, 1851
Marden and Siebeck are everywhere (for example Wikipedia)
Best regards
Bernard

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Message: #987
Date: 2021-04-29
From: tungvtt@gmail.com
Subject: Re: Two other lemma for concurrency and collinearity in Quadrangle

---

```
Dear Bernard and all,

I  would like to correct the second lemma as follows:

Let D = (dij)   (1<=i,j<=4) be a 4*4 real *antisymmetric*
*matrix* (dij = - dji).
Lemma 2:
Let Y1 be the point having barrycentric ( d12, d13, d14) w.r.t
triangle P2 P3 P4 and define Y2,Y3,Y4 cyclically.
Let Nji be the intersection of YiYj and PiPj.
Let Uij be the intersection of YiYj and PkPl.
Then:
1. three lines N12N34, N13N24, N14N23 are concurrent at a point.
2. six points U12, U34, U13, U24, U14, U23 are collinear on a
line.

So Lemma 1 is true for *symmetric matrix* and *Lemma 2 i* s true
for antisymmetric matrix .
And application:
_  #945, #946 and #948 are consequence of Lemma 1 with
   dij = d(Pi,Pj)^2/s(QPiPj) (isogonal conjugate),
   dij = 1/s(QPiPj) (isotomicconjugate) or
   dij = s(Pi,Pj)^t/ S(QPiPj) (general case).
Here d(X,Y) and S(XYZ) is the distance and signed area function.
   dij = - dji as s(QPiPj) = - s(QPjPi)

Best regards,
Vu Thanh Tung
```

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Message: #988
Date: 2021-04-29
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?

```
Hi Eckart,

Yes, because the 6 centroids are image of the six midpoints of
PiPj under the homothety of center P and factor 2/3.
Similarly, For your transformation (1) in #962 with six
centroids,  the image Y of a point X divides X.QA-P3 with ratio
2:1.

Best regards,
Vu Thanh Tung

On Wed, Apr 28, 2021 at 10:18 AM, Eckart Schmidt wrote:
> Dear Vu Thanh Tung,
> excuse my late answer, but I searched for properties
> ... for your transformations in #966 and #977,
> ... which are right observations.
> Only for the centroids I found a property:
> The image Y of a point X divides X.QA-P2 with ratio 2:1.
> Best regards Eckart
```

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Message: #989
Date: 2021-04-29
From: tungvtt@gmail.com
Subject: Re: New interpretation of QA-Tf2

```
Dear Eckart,

We have a pont-to-line transformation from this configuration:
consider a QA = P1...P4 and a point P
... the trilinear poles Qi of P,Pi wrt the triangle PjPkPl,
... further the intersections Rij = Qi.Qj ^ Pk.Pl ,
... then six points Rij are collinear on a line,
I think this can be explained by Lemma 2 in #949 and #987.

Best regards,
Vu Thanh Tung
```

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Message: #990
Date: 2021-04-29
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

```
Dear  Vu Thanh Tung, dear Bernard, dear Chris,

this are very unexpected observations
... wrt the transformation in #935 and #967(1),
... mapping two points X,Y wrt a QA to a line d(X,Y),
... here used for a 5P = P1...P5:
Take X = Pi and Y a point on the 5P-quartic
... and the QA of the remaining 5P-vertices,
... then the five lines d(Pi,Y) are parallel.

What about the direction of these 5 parallels?
Consider the line Y.5P-s-P5
... as degenerated 5P-s-Tf7-circle of a point Z,
... then the line Z.5P-s-P4 gives the direction of the
    parallels.
If Y1 nd Y2 on the quartic are collinear with 5P-s-P5,
... the sets of parallels have the same direction.

Best regards Eckart
```

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Message: #991
Date: 2021-04-30
From: bernard.keizer@gmail.com
Subject: Re: Alternative construction of the Cayley-Bacharach Point

---

```
Dear Chris,
I finally remembered where I Found this reference.
In fact, it was given by Eckart in the message #2481 (old forum)
!
Best regards
Bernard
```

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Message: #992
Date: 2021-04-30
From: eckart_schmidt@t-online.de
Subject: Re: New interpretation of QA-Tf2

```
Dear Vu Thanh Tung,

thanks for the new aspect for a transformation in #989.
The line of the six points bears QA-Tf2(P),
... but I cannot identify the direction.

Best regards Eckart
```

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Message: #993
Date: 2021-05-03
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

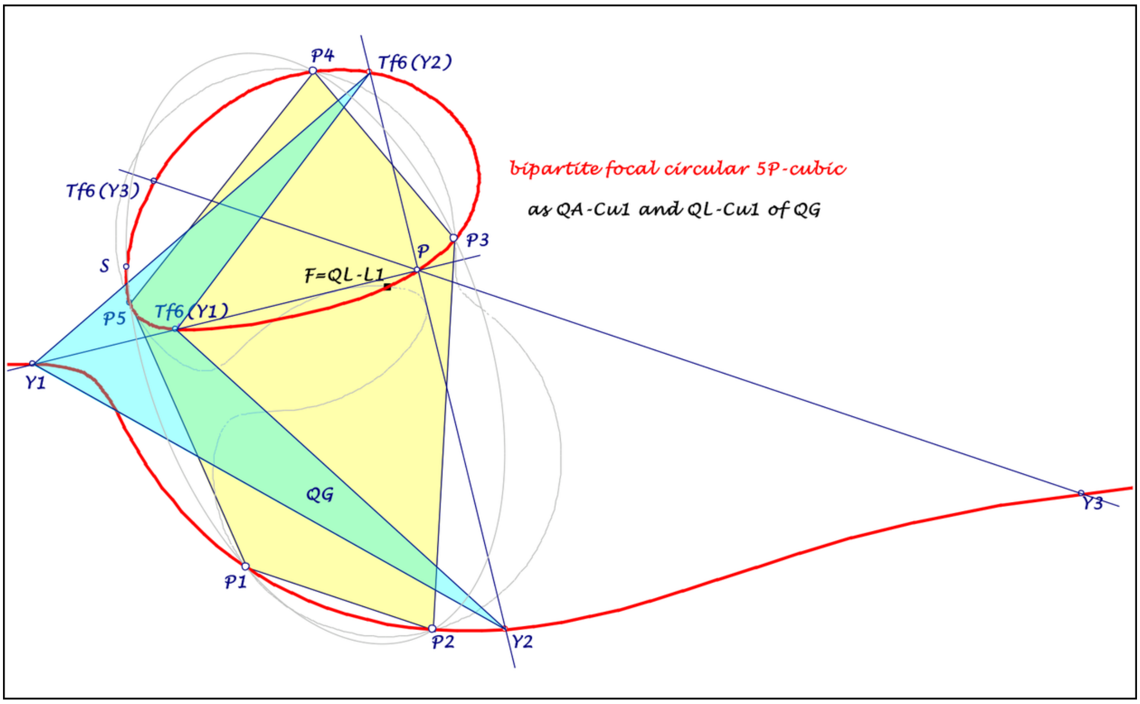
```
Dear  Vu Thanh Tung, dear Bernard, dear Chris,

here are further unexpected observations
... wrt the transformation in  #967(2),
... mapping two points X,Y wrt a QA to a single point Z,
... here once more used for a 5P = P1...P5,
... leading for a bipartite circular focal 5P-cubic to a
    quadrigon,
... whose QA-Cu1 and QL-Cu1 coincide with this cubic.

Take X = Pi, Y any point and the QA of the remaining
5P-vertices,
... then the five image points Zi of Y can coincide,
... the locus for these Y is a higher 5P-circumcurve (septic?),
... which is Tf6-invariant:
... if for Y the five images Zi coincide, then also for Tf6(Y).

Let us finally consider a bipartite focal circular 5P-cubic,
... you will find on it three pairs of Tf6-partner
    with coinciding 5 images Zi,
... two pairs are also partner wrt the Möbius transformation
    of the cubic,
... centered in the focus F, swapping S and Tf6(F),
... S 6th intersection of the cubic and the 5P-circumconic,
... these two pairs give a quadrigon QG with orthogonal
    diagonals,
... whose QA-Cu1 and QL-Cu1 are the reference cubic,
... QG-P1 is the cb-pivot, QL-P1 is the focus of the cubic
... and QA-P4 is the 3rd intersection of the cubic and FS.

Best regards Eckart
```



2021-05-03.pdf

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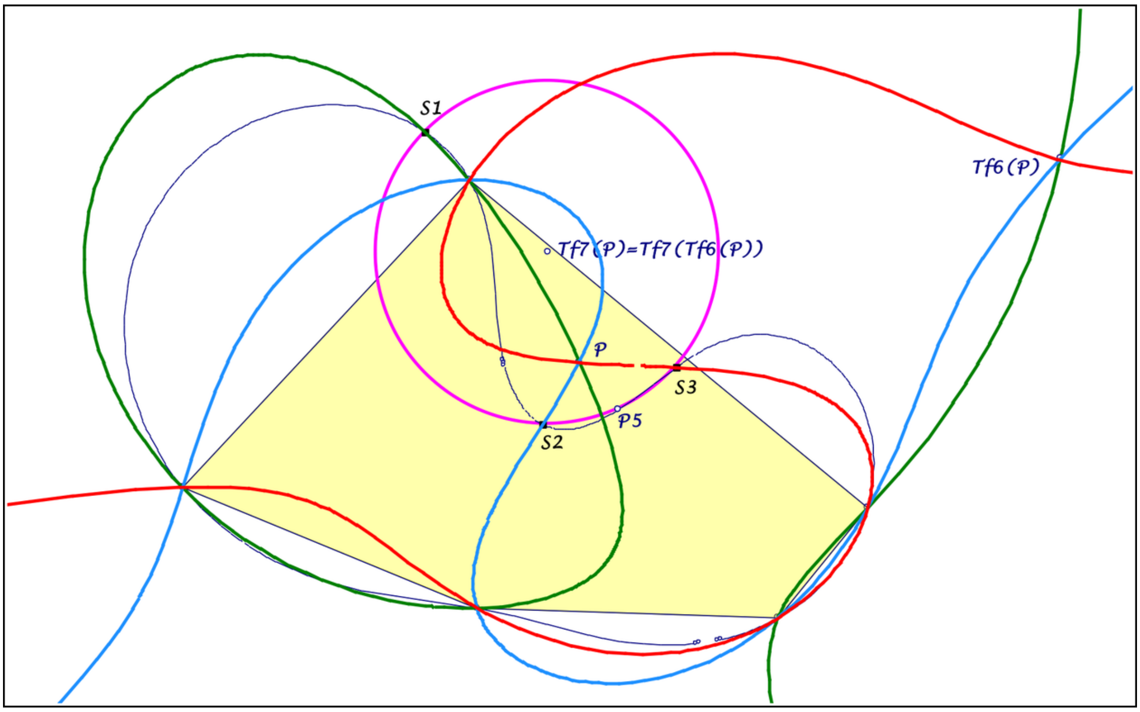
Message: #994
Date: 2021-05-04
From: eckart_schmidt@t-online.de
Subject: Preimages of 5P-s-Tf7

```
Dear Chris,

a point P and its Tf6-image have the same Tf7-image (see EPG),
... now let us start with Tf7(P) and its circle CI through P5
    centered in Tf7(P),
... let CI intersect the 5P-quartic in S1, S2, S3
    (not always real),
... then the circular 5P-circumcubics with focus Si
... intersect in two common points P and Tf6(P).

Best regards Eckart
```



2021-05-04.pdf

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Message: #995
Date: 2021-05-05
From: bernard.keizer@gmail.com
Subject: Re: New interpretation of QA-Tf2

```
Dear Eckart, dear Vu Thang Tung
I reproduced with great interest your figur and it's properties.
Beautiful work, indeed, congratulations !
I think we deal only with harmonic conjugations.
Perhaps this remark :
For any point X on PiPj, the harmonic of X wrt Pi and Pj is
QA-Tf2(X).
For any point P, not on the 6 sides PiPj, any line through
QA-Tf2(P) cuts the 6 sides PiPj in points Rij and the QA-Tf2 of
this line is a DT circumconic through P, which cuts the 6 sides
PiPj in the harmonic of Rij wrt Pi and Pj.
There are 3 lines RijRkl by Eckart through QA-Tf2(P) and one
line by Vu Thang Tung through the Rij (different from those of
Eckart).
We could add the lines PPi, the QA-Tf2 of these lines are
circumDT conics through Pi and QA-Tf2(P) with the same property.
We may even consider the line PQA-Tf2(P) and its QA-Tf2, which
is the circumDT conic through P and QA-Tf2(P).
We find always the same property that lines and conics intersect
the 6 QA sides PiPjin harmonic conjugates wrt Pi and Pj.
Best regards
Bernard
```

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Message: #996
Date: 2021-05-05
From: bernard.keizer@gmail.com
Subject: Re: Preimages of 5P-s-Tf7

```
Dear Eckart,
Very nice property !
Perhaps one question :
Taking the 5 points as the 5 triple points of the QA-Cu7 of the
3 QA's of a QL and the CI circle as the circle through the 3
QA-P4 and 5P-s-P5 (this point being CSC(QL-P24), what are the
points Tf6 partners having this circle as CI ?
Best regards
Bernard
```

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Message: #997
Date: 2021-05-05
From: tungvtt@gmail.com
Subject: Re: New QA-transformations ?

Dear Eckart and all,

Here what I found for Quadrilateral:

Consider a Quadrilateral (L1 L2 L3 L4) and a random line d.
Tij is the X(n)-center of the triangle bounded by threr lines d
, Li, Lj
Then
(1) Three midpoints of the segments Tij Tkl are collinear for
X(n) = centroid, orthocenter, circumcenter, nine point center,
or any triangle center such that OX = t OH with a real number t.
(2) Let Cli = circle (Tij Tik Til) then four circles Cl1, Cl2,
Cl3, Cl4 are concurrent at a single point if X(n)=circumcenter
or orthocenter.
(3) Let Cl'i = circle (Tjk Tkl Tlj) then four circles Cl'1,
Cl'2, Cl'3, Cl'4 are concurrent at a single point if
X(n)=circumcenter.

I think (1) has been mentioned before in this forum, but I did
not find the reference number yet.
(2) and (3) seems new for me.

Best regards,

Vu Thanh Tung

On Thu, Apr 29, 2021 at 03:41 AM, Vu Thanh Tung wrote:

>
> Hi Eckart,
>
> Yes, because the 6 centroids are image of the six midpoints of
PiPj under
> the homothety of center P and factor 2/3.
> Similarly, For your transformation (1) in #962 with six
centroids, t he
> image Y of a point X divides X.QA-P3 with ratio 2:1.
>
> Best regards,
>
> Vu Thanh Tung

Message: #998
Date: 2021-05-05
From: eckart_schmidt@t-online.de
Subject: Re: Preimages of 5P-s-Tf7

```
Dear Bernard,

wrt the 5P of the QA-Cu7-intersections of the 3 QA-versions of a
QL:
... The 3 QA-P4a,b,c  lie on the 5P-quartic
... and have a circumcircle through 5P-s-P5,
... which can be considered as 5P-s-Tf7-circle
... of two different 5P-s-Tf6-partner on a line XaXbXc
... through Xa = QG-P1a.QA-P4a ^ QG-L1a, Xb, Xc analog.

Best regards Eckart
```

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Message: #999
Date: 2021-05-06
From: eckart_schmidt@t-online.de
Subject: Re: A property of four circles

```
Dear Tran Quang Hung,

wrt the last question in #984: The homothetic center is not in
EQF,
... I found no properties of this point.

But if we take the same radius for the circles,
... we get a fixed QA-point, independent of the chosen radius,
... this point lies on the line QA-P7.QA-P8, but where?

Best regards Eckart
```

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Message: #1000
Date: 2021-05-06
From: eckart_schmidt@t-online.de
Subject: Re: New QA-transformations ?

```
Dear Vu Thanh Tung,

with interest I have studied your #997, here some remarks:

Wrt (2): The 6 points Tij  are the vertices of a quadrilateral
... and its Miquel point QL-P1 is your single point of the four
    circles.

Wrt (3): The 6 circumcircles (d,Li,Lj) with center Tij
... have 4 triple points, not on the line d,
... but lying on a circle CI,
... bearing the Miquel point QL-P1 of the reference QL
... and your common point of the circles CI'i.
Example: Let d be the Newton line QL-L1,
... then the circle CI = QL-Tf1(QL-L1) and your common point is
    QL-Tf1(QL-P7).
Final application for pentalaterals 5L:
... Let d be a line Li of the 5L and QL the remaining 4 lines,
... then the 5 circles CI coincide with the circle 5L-o-Ci1
... and your 5 final points coincide with 5L-n-P1.

Best regards Eckart
```

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Message: #1001
Date: 2021-05-07
From: bernard.keizer@gmail.com
Subject: Re: Preimages of 5P-s-Tf7

Dear Eckart,
Thanks for your quick answer !
In fact, if the 3 foci of the cubics are the QA-P4, the pivots of the cb transformation are Tf8(QA-P41) which are aligned, as the circle through the QA-P41 passes through QL-P1 = 5P-s-P6.
The 3 associated cubics have their foci in QA-P41 and their pivots in Tf8(QA-P4) = QG-P1 and are the QA-Cu7.
All this is really interesting and leads to other questions for which you have perhaps already the answer :
1) which is the CSC of these 3 new cubics (centered in QA-P4 and swapping QA-P41 and ?)
2) there are 2 more associated cubics which can be interesting, with focus = pivot in 2 intersections of the quartic and the twincubic
3) there are 3 pairs of associociared cubics with foci in your points Vi and in the 5P-n-Tf1 of the Vi
(these 14 cubics through the 5 triple points are QL cubics ...)
In the general case of 5 points, the 5P-n-Tf1 of a circle through 5P-s-P5 is a circle through 5P-s-P6, this explains why the 3 cubics with foci in the 3 points Si have aligned pivots and therefore 2 Tf6 partners as common points ; what can be said about the 3 associated cubics, I suppose they lead to 3 pairs of Tf6 partners (6 coconic points) like the QA-Cu7 ...
As you can see, this was a very stimulating message ...
Best regards
Bernard

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Message: #1002
Date: 2021-05-07
From: bernard.keizer@gmail.com
Subject: Re: Preimages of 5P-s-Tf7

Dear Eckart,
In my enthousiasm, I almost forgot the mother of all these cubics, with focus in P5 and pivot in the infinity point of the asymptote and it's associate, with focus in P6 = QL-P1 and pivot in Tf8(P5) = QL-P24. These are also QL cubics ...
Best regards
Bernard

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Message: #1003
Date: 2021-05-07
From: tungvtt@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] New QA-transformations ?

```
Dear Eckart,
It is interesting.
Thank you for your interest.
Best regards,
Vu Thanh Tung

On Fri, May 7, 2021 at 2:44 AM Eckart Schmidt
<eckart_schmidt@t-online.de>
wrote:
>  Dear Vu Thanh Tung,
> with interest I have studied your #997, here some remarks:
> Wrt (2): The 6 points Tij  are the vertices of a quadrilateral
> ... and its Miquel point QL-P1 is your single point of the
four circles.
> Wrt (3): The 6 circumcircles (d,Li,Lj) with center Tij
> ... have 4 triple points, not on the line d,
> ... but lying on a circle CI,
> ... bearing the Miquel point QL-P1 of the reference QL
> ... and your common point of the circles CI'i.
> Example: Let d be the Newton line QL-L1,
> ... then the circle CI = QL-Tf1(QL-L1) and your common point
is
> QL-Tf1(QL-P7).
> Final application for pentalaterals 5L:
> ... Let d be a line Li of the 5L and QL the remaining 4 lines,
> ... then the 5 circles CI coincide with the circle 5L-o-Ci1
> ... and your 5 final points coincide with 5L-n-P1.
> Best regards Eckart
```

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Message: #1004
Date: 2021-05-08
From: eckart_schmidt@t-online.de
Subject: Re: Preimages of 5P-s-Tf7

```
Dear Bernard,

thanks for your remarks and stimulations in #1001.
You ask for the "CSC of these three new cubics (centered in
QA-P4 and swapping QA-P41 and ?)"
The Möbius transformation for these focal cubics is described in
#727,
... centered in the focus F, here QA-P4, swapping S and Tf6(F),
... S 6th intersection of Co1 and the quartic.
Why do you ask for the image of QA-P41,
... which is not a point of the cubic?
The image of QA-P41 seems to be a point on S.P (P pivot),
the image of P is the 3rd intersection of the cubic and S.QA-P4.

I have not the overview as you,
I don't remember, what are associated cubics?

Best regards Eckart
```

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Message: #1005
Date: 2021-05-08
From: bernard.keizer@gmail.com
Subject: Re: Preimages of 5P-s-Tf7

```
Dear Eckart,
Sorry, I meant Tf8(QA-P41), which is the pivot !
But thanks for the answer anyhow ...
If a cubic has focus F and pivot P, the associated cubic has
focus Tf8(P) and pivot Tf8(F).
Best regards
Bernard
```

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Message: #1006
Date: 2021-05-09
From: eckart_schmidt@t-online.de
Subject: Typo in 5P-s-P5 and 5P-s-P6

```
Dear Chris,

I think there is a relevant typo in item 5Ps-P5 and 5P-s-P6,
... in the last passage QA-Cu1 has to be replaced by QA-Cu7.

Best regards Eckart
```

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Message: #1007
Date: 2021-05-09
From: van10hoven@gmail.com
Subject: Re: Typo in 5P-s-P5 and 5P-s-P6

```
Dear Eckart,

Thanks for mentioning.
I adjusted the typos.

Best regards,
Chris
```

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Message: #1008
Date: 2021-05-09
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A property of four circles

---

Dear Eckart,

Thank you very much for your interest.
Actually, we can choose some others radius for these circles to create some new EQF point i.e.
Ra= inradius of triangle BCD.
Ra= f(BCD) where f(BCD) is a function on sides of BCD with degree of f is 1.
....

But for which radius, we will meet some EQF centers?
For the equal radius, the new point is interesting.
I notice that, if we take circles (A,kRa), (B,kRb), (C,kRc), (D,kRd) then the radical center of these circles lies on a fixed line where k changes.

Sincerely yours,
Tran Quang Hung.

Vào Th 6, 7 thg 5, 2021 vào lúc 01:23 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:
> Dear Tran Quang Hung,
> wrt the last question in #984: The homothetic center is not in EQF,
> ... I found no properties of this point.
> But if we take the same radius for the circles,
> ... we get a fixed QA-point, independent of the chosen radius,
> ... this point lies on the line QA-P7.QA-P8, but where?
> Best regards Eckart

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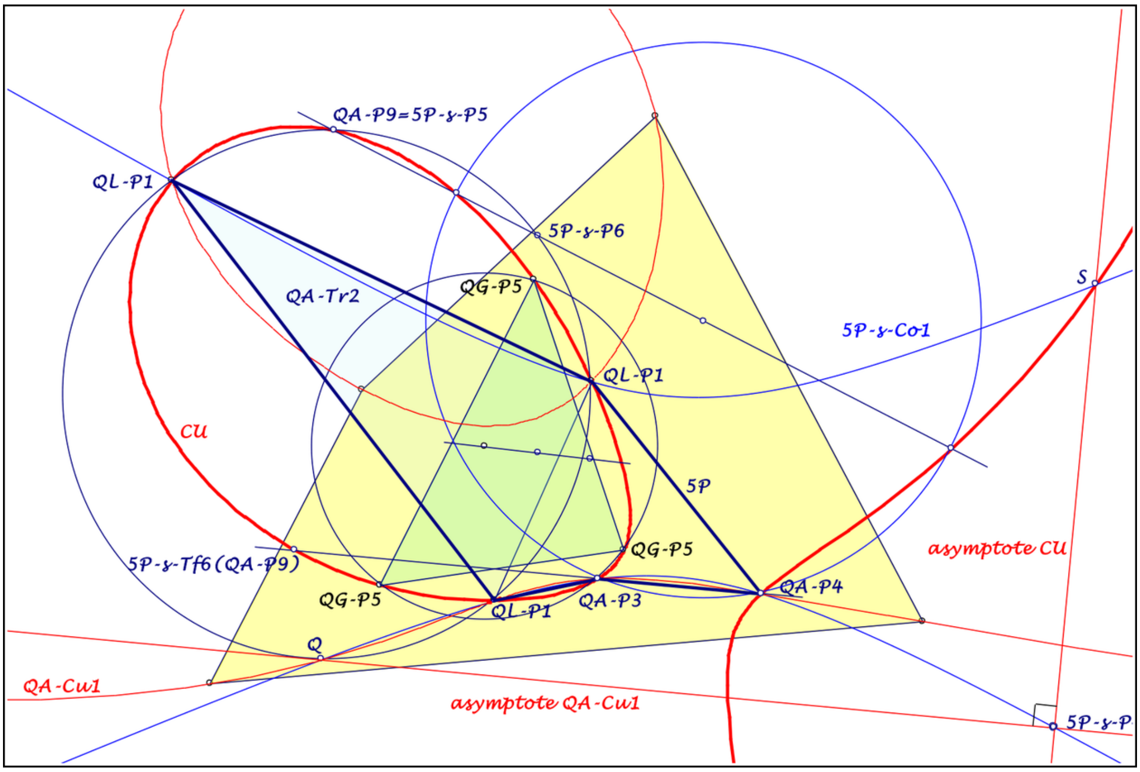
Message: #1009
Date: 2021-05-11
From: eckart_schmidt@t-online.de
Subject: QA-geometry with 5P-cubic

Dear all,

may I invite you to an excursion in QA-geometry,
... using methods of the QA-Cu7-discussion?
Bernard will recognize his "mother of the 5P-cubics".

Let us start with a line pencil for the point QA-P3
... and consider pairs of orthogonal lines L and L'
through QA-P3
... with intersections L' ^ QA-Tf4(L),
... which give a focal circular cubic CU
... with following properties:
(1) CU bears QA-P3, QA-P4, QA-P9, the 3 Miquel points QL-P1
... and the three QG-P5 of the quadrigon versions.
(2) CU is invariant wrt QA-Tf4 and the isogonal conjugate
of QA-Tr2.
(3) QA-Tf4 of the QA-Tr2-circumcircle is a circle,
... centered in the reflection of QA-P15 in QA-P1,
... bearing QA-P3 and the three QG-P5 (QA-Tf4-images of the
Miquel points).
(4) CU is a focal circular circumcubic of the 5P = QA-Tr2 plus
QA-P3 and QA-P4
... with focus QA-P9, which is 5P-s-P5 of the 5P,
... invariant wrt 5P-s-Tf6 with cb-pivot in the infinity point
of the asymptote,
... which is orthogonal to the asymptote of QA-Cu1.
(5) The 5P-quartic degenerates in two circles,
... the first is the circumcircle of QA-Tr2,
... the second is orthogonal to the first through QA-P3, QA-P4,
... centered on 5P-s-P5.5P-s-P6.
(6) The circumconic 5P-s-Co1 is the QA-Tr2- isogonal conjugate
of QA-P3.QA-P4
... and bears the intersection Q of QA-Co1 and its asymptote,
... which intersects the conic once more in 5P-s-P4.
(7) The asymptote of CU is orthogonal to the asymptote of
QA-Cu1,
... the intersection of CU and its asymptote is the 6th
intersection S of CU and 5P-s-Co1.
(8) The cubic is also invariant wrt a Möbius transformation,
... centered in the focus QA-P9 = 5P-s-P5,
swapping S and 5P-s-Tf6(focus),
... which is the 3rd intersection of CU and QA-P3.QA-P4.

Best regards Eckart



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Message: #1010
Date: 2021-05-13
From: bernard.keizer@gmail.com
Subject: Re: QA-geometry with 5P-cubic

Dear Eckart,
Wunderbar !
A brandnew QA-cubic !!!
Let's name QA-Cu8 this cubic.
Cu7 was a pivotal isogonal cubic wrt the triangle P2P4P41,
invariant in the 3 CSC centered in one vertice and swapping the
2 others.
It was the QA-Cu1 of the orthocentric QA formed by the in- and
excenters of this triangle.
It was a focal circular cubic QL-Cu1 with QL-P1 in P41 and the
QL-2P2a and b in P2 and P4, pivot in the infinity point of the
perpendicular bisector of P2P4.
It was a 5P focal circumcubic of the 5 points Tr1 vertices, P4
and P41, like Cu1.
Cu8 is a pivotal isogonal cubic wrt the triangle P3P4P9,
invariant in the 3 CSC centered in one vertice and swapping the
2 others.
It is the QA-Cu1 of the orthocentric QA formed by the in- and
excenters of this triangle.
(The last Moebius transformation in your point 8 centered in P9
swaps P3 and P4).
It is a focal circular cubic QL-Cu1 with QL-P1 in P9 and the
QL-2P2a and b in P3 and P4.
It is a 5P focal circumcubic of the 5 points Tr2 vertices, P3
and P4, like Cu1.
It seems that the 2 last intersections of Cu1 and Cu7 and of Cu1
and Cu8, which are cb partners wrt the 2 groups of 5 points are
not real.
Beautiful work, indeed !
This time I hope Chris will put this new QA cubic in EQF !
Best regards
Bernard
PS There is a typo in point 6, read QA-Cu1 and not QA-Co1

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Message: #1011
Date: 2021-05-13
From: bernard.keizer@gmail.com
Subject: Re: QA-geometry with 5P-cubic

Dear Eckart,
You found me perhaps particulary enthousiastic about your new cubic QA-Cu8.
In fact, I was precisely working on following idea about QA-Cu7.
For a QL, we have 3 QA's and the 5 triple points of the 3 QA-Cu7 ...
Each Cu7 is a pivotal circular focal circumcubic of the 5 triple points invariant in the cb transformation wrt these points.
Focus is P41 and pivot is a DT vertice.
Now the QA belongs to 3 QL's and there are 3 groups of 5 triple points on Cu7.
Let's consider the 3 associated cubics of Cu7 wrt the 3 groups of 5 points ; they are also pivotal circular focal cubics wrt the 3 groups of points with focus in P4 and pivots in CSC(P41).
What can be said about these 3 cubics, which are also QA cubics?
For a QL and it's 3 QA's, the 3 QA-Cu8 intersect necessary in the same vertice of Tr2, but I didn't find particular properties for the other intersections ...
I reach the limits of my ability for using macros with Geogebra!
Thanks in advance for your attention
Best regards
Bernard

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Message: #1012
Date: 2021-05-15
From: eckart_schmidt@t-online.de
Subject: Re: QA-geometry with 5P-cubic

```
Dear Bernard,

thanks for your remarks and detailed descriptions wrt "QA-Cu8",
... thanks also for correcting the typo and further
    stimulations,
... but I also found no properties for the 3 "QA-Cu8" of a QL,
... reason will be, that "QA-Cu8" is not QA-circumscribed.

Best regards Eckart
```

Message: #1013
Date: 2021-05-15
From: eckart_schmidt@t-online.de
Subject: Möbius transformations for 5P-geometry

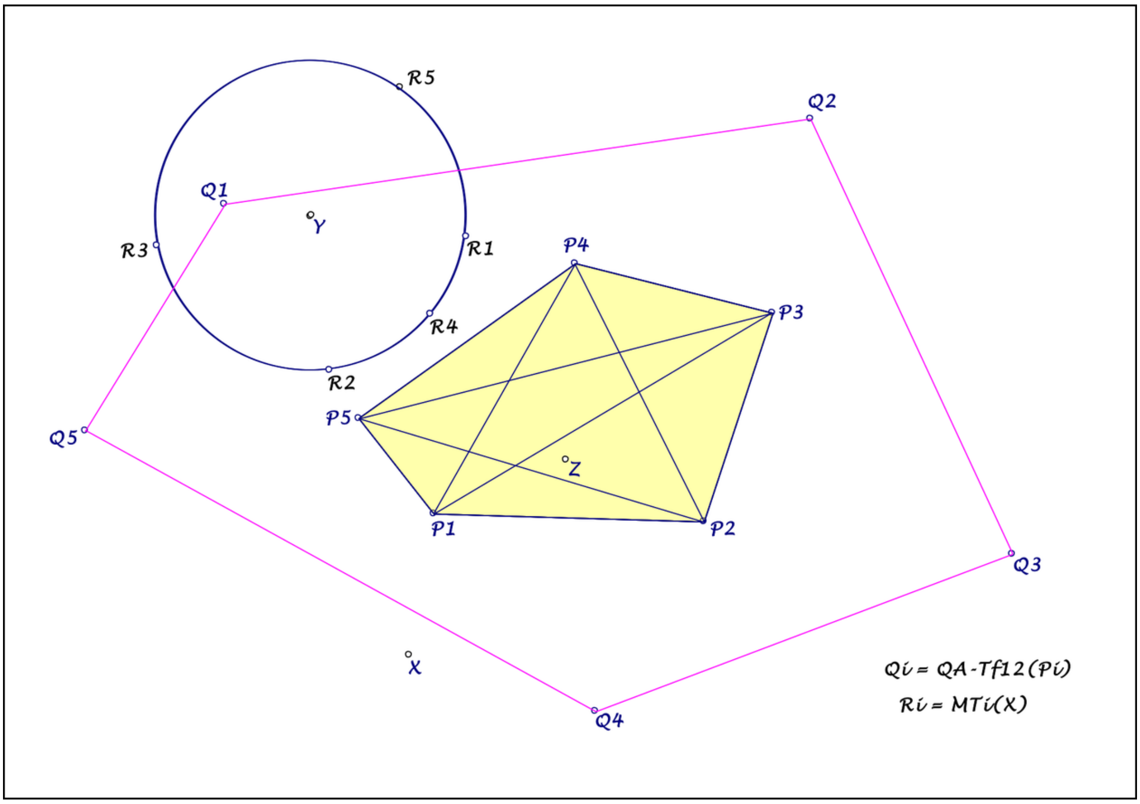
```
Dear Bernard, dear Chris,

perhaps of interest, some new aspects of 5P-geometry.
Preface:
Consider a 5P = P1...P5 and the QA-P4 of its quadrangles,
... which give a new 5P = Q1...Q5 with Qi = QA-P4 of 5P minus Pi
... leading to five Möbius transformations MTi,
... centered in a fixed point Z, swapping Pi and Qi,
... which give for a point X five concyclic images MTi(X)
... on a circle CI, centered in Y.
If X = 5P-s-P5 / 5P-s-P6, the circle CI bears 5P-s-P6 / 5P-s-P5,
... if Z = 5P-s-P5 / 5P-s-P6, the circle CI bears
    5P-sP5 / 5P-s-P6.
For any Z the transformation X --> Y is a Möbius transformation,
... mapping eg. for Z = 5P-s-P5 / 5P-s-P6 the circle CI to the
    bisector of X.Z.

New aspects:
If we take otherwise a second 5P = Q1...Q5,
... for example Qi = QA-Tfx(Pi) wrt 5P minus Pi,
... there exists sometimes a single point Z with the property,
... that  the five MTi(X) are concyclic and lead to a Möbius
    transformation X --> Y,
... this is the case, approximate CABRI observations,
    if we get the 2nd 5P with
... QA-Tf2, Tf3, Tf4, Tf12, Tf14, Tf16 and the isogonality
    wrt QA-Tr2.
But I cannot construct these Z-points,
... which will be interesting 5P-points with their Möbius
    transformations.

Final remark:
For a 6P you get an analog example,
using 5P-s-Tf8 to get a 2nd 6P.

Best regards Eckart
```



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Message: #1014
Date: 2021-05-17
From: eckart_schmidt@t-online.de
Subject: Re: Möbius transformations for 5P-geometry

```
Dear  Bernard, dear Chris,

excuse, please read #1013 critically, there are mistakes,
... the new aspects doesn't hold in general for the
    transformations
... QA-Tf2, QA-Tf16, isogonality wrt QA-Tr2.
Perhaps someone can confirm the property for the other
transformations.
There can be more than one center Z for the transformation.
If we take for the 2nd 5P the points QA-P1 for 5P minus Pi,
... I found 3 centers Z for a Möbius transformation X --> Y.

Best regards Eckart
```

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Message: #1015
Date: 2021-05-17
From: van10hoven@gmail.com
Subject: new QL-Line through Least Squares Point

```
Dear friends,

I came across a QL-Line through QL-P26 with very simple
coefficients.
The three QL-QG-Circumscribed Conics (see QL-3QG1) through the
Miquel Point (QL-P1) mutually have per pair a 4th intersection
point. Their three intersection points are collinear on a line
QL-Lx.

1st coefficient in CT-coefficients:
* (b^2) (c^2) l (m-n) (2 a^2 (l - m) (n - l) + b^2 (l - m) (m -
n) + c^2 (n - l) (m - n))

This line also passes through:
* QL-P26 (Least Squares Point)
* QL-Tf1(QL-P17)
* QL-Tf1(QL-P24)
* QL-Tf3(QL-P2.QL-P6)
* QL-Tf3(QL-P3.QL-P4).

Best regards,
Chris
```

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Message: #1016
Date: 2021-05-17
From: bernard.keizer@gmail.com
Subject: Re: new QL-Line through Least Squares Point

```
Dear Chris,
This line is not very new !
It is the perpendicular bisector of QL-P1QL-P6 and the CSC of
the Dimidium circle (see EQF).
Best regards
Bernard
```

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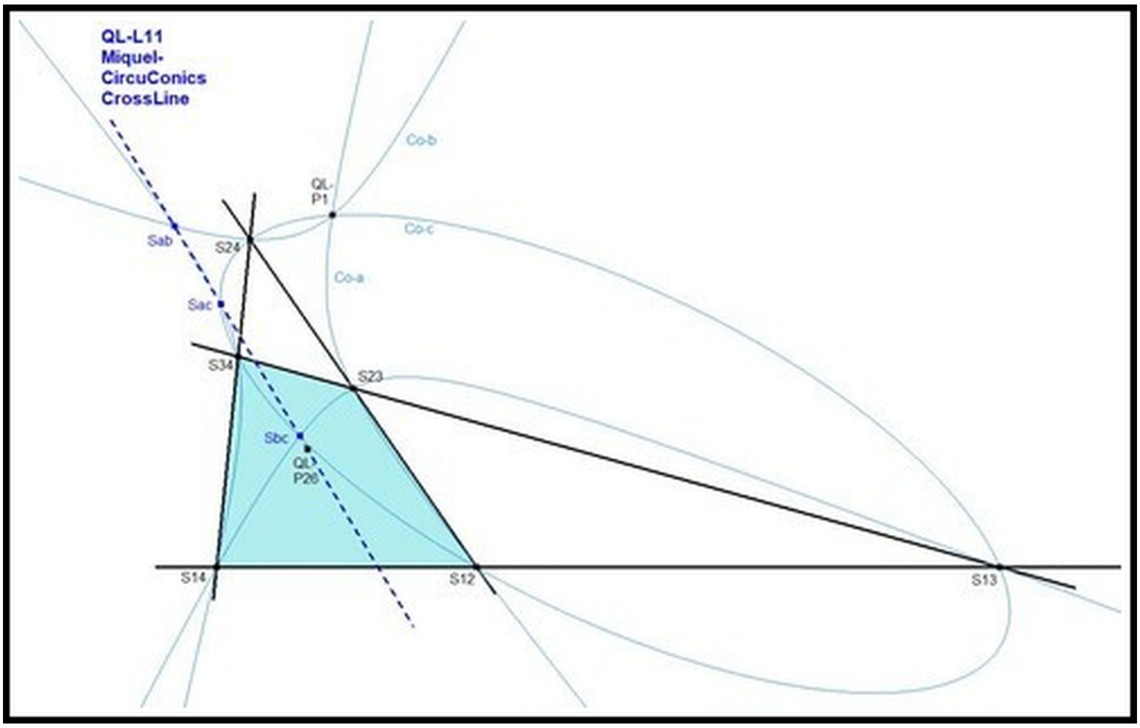
Message: #1017
Date: 2021-05-17
From: van10hoven@gmail.com
Subject: Re: new QL-Line through Least Squares Point

Dear Bernard,

In my construction and calculation Lx is not the perpendicular bisector of QL-P1 and QL-P6.
I will attach a picture of the construction.
By the way I did find that QL-Tf3(Lx) = QL-P6.

Best regards,
Chris



QL-L11 Miquel-CircuConics CrossLine-03.JPG

Message: #1018
Date: 2021-05-17
From: eckart_schmidt@t-online.de
Subject: Re: new QL-Line through Least Squares Point

```
Dear Chris, dear Bernard,

Chris' new line bears the three QG-P16 points (see EQF),
... but I cannot confirm Bernard's first property.

Best regards Eckart
```

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Message: #1019
Date: 2021-05-17
From: van10hoven@gmail.com
Subject: Re: new QL-Line through Least Squares Point

Dear Eckart, dear Bernard,

It's not only that the new line bears the 3 QL-versions of QG-P16 . . .

The 3 intersection points of the 3 conics coincide with the 3 QL-versions of QG-P16.
And as mentioned in EQF at item QL-P26:

* QL-P26 lies on the line CSC(QL-P17).CSC(QL-P24), which is the perpendicular bisector of QL-P1.CSC(QL-P6).
* The three QL-versions of QG-P16 (Schmidt Point) are collinear on former line, which is also the QL-Tf1 image of the Dimidium circle QL-Ci6 . . .

Which also gives a bridge to Bernard's remark.

Best regards,

Chris

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Message: #1020
Date: 2021-05-17
From: bernard.keizer@gmail.com
Subject: Re: new QL-Line through Least Squares Point

Dear Chris, dear Eckart
I think we all agree !
If QL-Tf3(Lx) = QL-P6, then the perpendicular bisector of QL-P1QL-P6 is CSC(Dimidium circle).
There is a typo in EQF at P26, read P6 and not CSC(P6) ...
By the way, dear Eckart, the Dimidium circle bears the QA-P3 of the 3 component QA's and their CSC are the QG-P16, in fact you confirm my remark.
Best regards
Bernard

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Message: #1021
Date: 2021-05-17
From: van10hoven@gmail.com
Subject: Re: new QL-Line through Least Squares Point

```
Dear Bernard,

There is no typo. Lx = the perpendicular bisector of
QL-P1.CSC(QL-P6).
Therefore we didn't agree in the beginning.
Another nice feature is that Lx = QL-Tf6(QL-P1).
Best regards,

Chris
```

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Message: #1022
Date: 2021-05-17
From: bernard.keizer@gmail.com
Subject: Re: new QL-Line through Least Squares Point

```
Dear Chris,
My apologise, in fact there is no typo and Lx is the
perpendicular bisector of QL-P1CSC(QL-P6).
But it holds obviously that Lx is CSC(Dimidium circle) !
Best regards
Bernard
```

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Message: #1023
Date: 2021-05-17
From: van10hoven@gmail.com
Subject: Re: new QL-Line through Least Squares Point

```
On Mon, May 17, 2021 at 08:32 AM, Bernard Keizer wrote:
> But it holds obviously that Lx is CSC(Dimidium circle) !

True !

Chris
```

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Message: #1024
Date: 2021-05-19
From: van10hoven@gmail.com
Subject: EQF and EPG more and more known

Dear friends,

I notice that our efforts assembled in the Encyclopedia of Quadri-Figures and Encyclopedia of Polygon Geometry become more and more accessible.

For example, when you Google on keyword "QL-P12" then the first result will guide you to the right page in EQF. When you do the same in Bing, then the result is even outlined and giving the right definition found at EQF.

And this is the case for (almost) all items of EQF and EPG!

Moreover César Lozada made a beautiful inquiry program to quickly find items from the encyclopedias ETC, CTC, TTW, EQF, EPG and from the groups Hyacinthos, Adgeom, Anopolis, Quadri-Figures Group (QFG), Euclid and our Quadri- and Poly-Geometry Group (QPG).

Here is the site: www.geolinker.epizy.com (http://www.geolinker.epizy.com).

Best regards,

Chris

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Message: #1025
Date: 2021-05-19
From: van10hoven@gmail.com
Subject: Re: new QL-Line through Least Squares Point

```
Dear Eckart and Bernard,
I added the new line Lx with name "QL-L11" in EQF.
Best regards,
Chris
```

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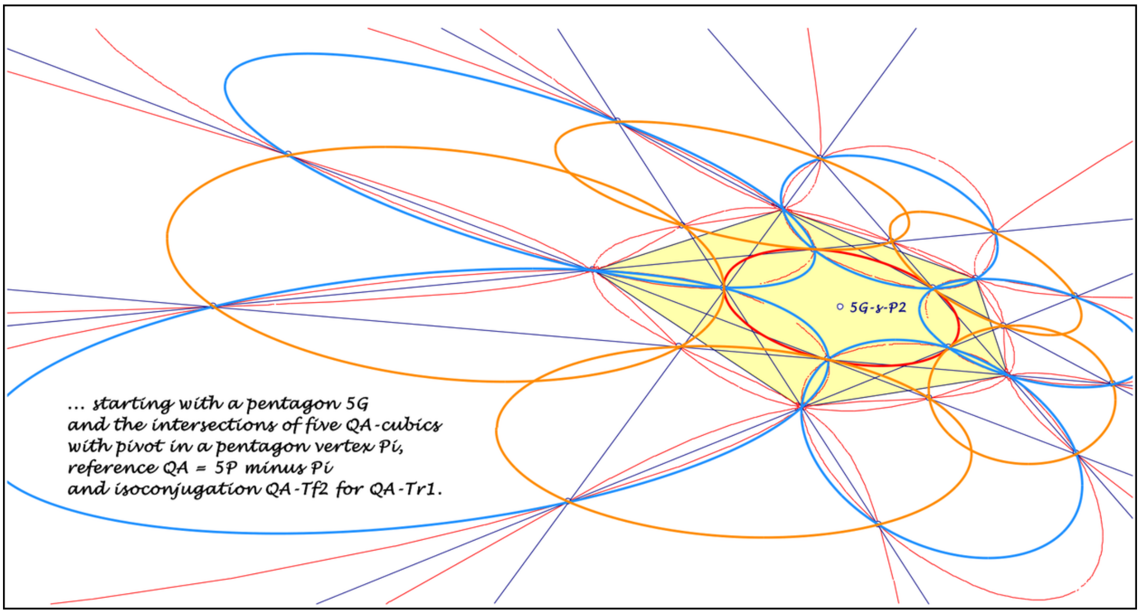
Message: #1026
Date: 2021-05-23
From: eckart_schmidt@t-online.de
Subject: Just for fun

```
... attached a conic cloud for a pentagon ...

Season's Greetings
Eckart
```



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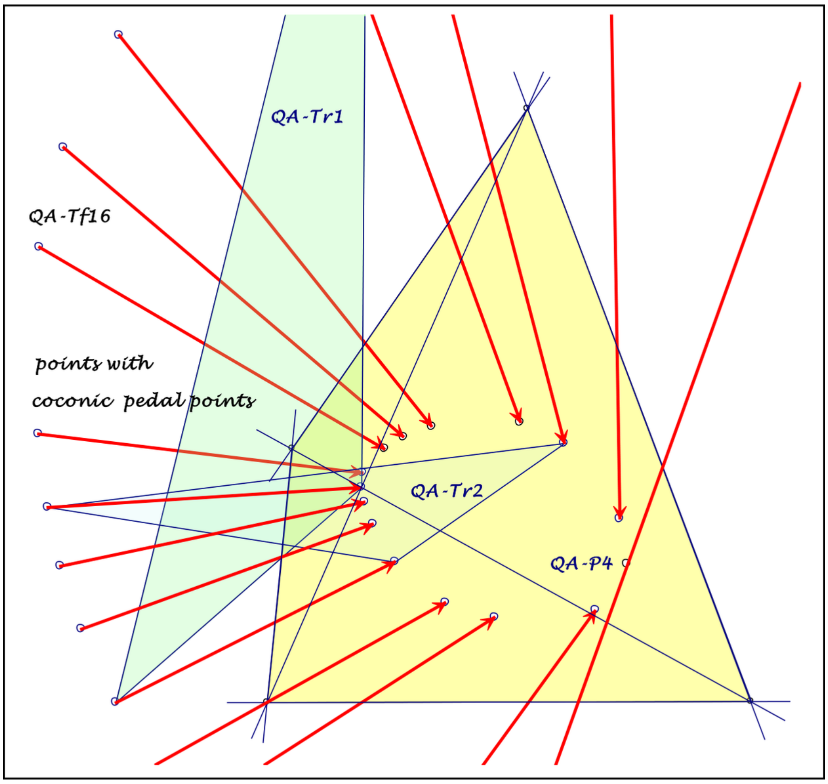
Message: #1027
Date: 2021-05-24
From: eckart_schmidt@t-online.de
Subject: Coconic QA-pedal points

Dear Bernard, dear Chris,

for QA-P4 we find a property in EQF wrt pedal points (5th
passage from the
end),
... which is equivalent to "QA-P4 has 6 coconic pedal points on
PiPj",
... I didn't find this in EQF.
If we look for further points with this property,
... we get a septic with Mathematica,
    but what about a construction?
... The septic will be circumscribed QA-Tr1 and QA-Tr2.
Remarkable: The septic is QA-Tf16-invariant!
But CABRI drawings show unexpected a cubic-like curve.

Thanks in advance for clearance.

Best regards Eckart



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Message: #1028
Date: 2021-05-25
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

```
Dear Eckart,
I suppose calculation and drawing are both right !
The 4 vertices of the QA belong to your locus.
So your septic is perhaps a cubic and 4 points.
I checked with many lines that there are always 3 points with
this property !
But I'm not able to draw simply the cubic ...
Is it correct that QA-Tf16(QA-P4) is the infinity point of the
asymptote of QA-Cu1 ?
Is it also correct that the vertices of Tr1 and Tr2 form with
QA-P4 and the circular points a CB system ?
Could your cubic be circular ?
Very interesting item ...
Best regards
Bernard
```

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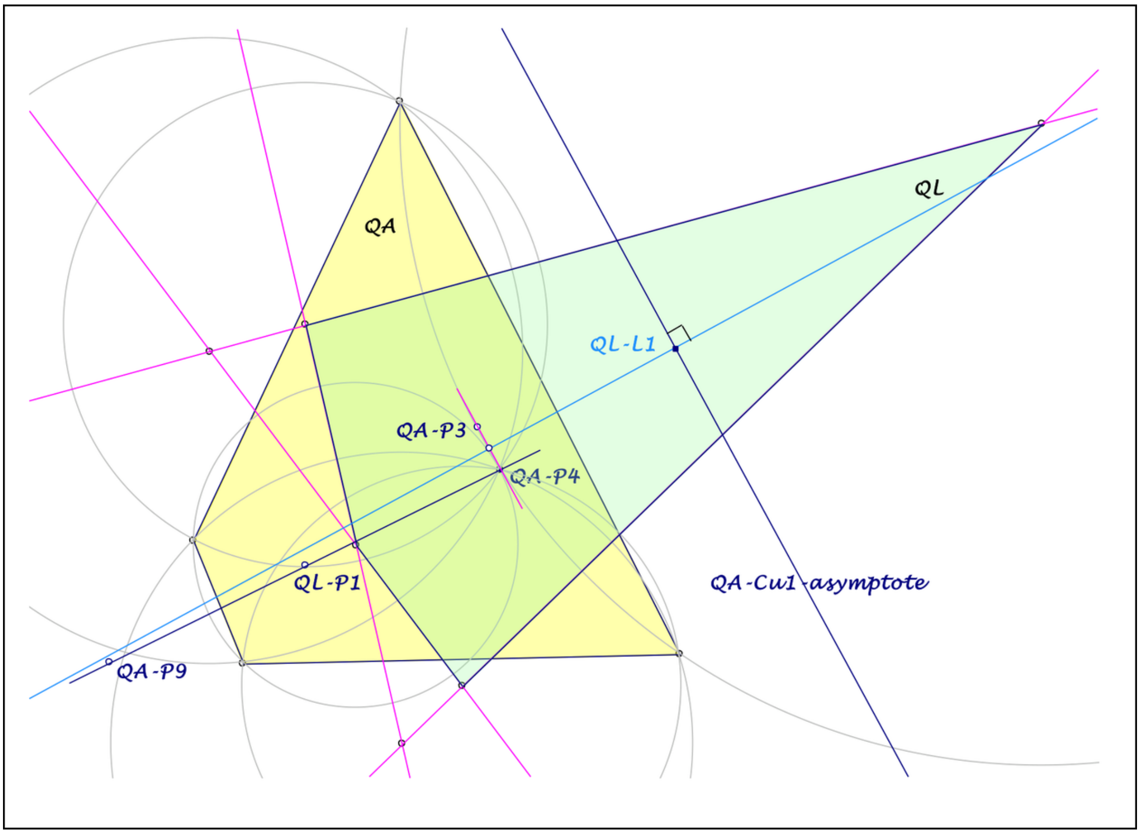
Message: #1029
Date: 2021-05-25
From: eckart_schmidt@t-online.de
Subject: QL for QA

```
Dear Bernard, dear Chris,

let us start with a reference QA = P1...P4
... and the images of PiPj wrt QA-Tf16,
... which are circles through Pi, Pj, QA-P4,
... and whose 6 centers give an interesting QL:
QL-P1 is the midpoint of QA-P4 and QA-P9,
... which are the fixed points of its CSC,
... the QL-L1 is the bisector of QA-P3.QA-P4.
There will be more properties.

Best regards Eckart
.
```



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Message: #1030
Date: 2021-05-25
From: eckart_schmidt@t-online.de
Subject: Re: Coconic QA-pedal points

```
Dear Bernard,

thanks for interest,
... but a new drawing shows, that the curve can never be a
cubic,
... it will be the septic of calculation with Mathematica,
... but the QA-vertices have not to be points of the curve,
... for the conic for the pedal points is not defined,
... correct will be, that QA-Tf16(QA-P4) is the infinity point
of the
asymptote.

Best regards Eckart
```

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Message: #1031
Date: 2021-05-27
From: eckart_schmidt@t-online.de
Subject: Point to circle transformation for QA

```
Dear Bernard, dear Chris,

let us start with a QA = P1...P4 and a vertex Pi,
... let a line Li through Pi intersect the 3 sides PjPk in Ql
    (j, k unequal i),
... the 3 perpendiculars of PjPk in Ql intersect in 3 points,
... whose loci - varying Li through Pi - give conics with a
    common point Ri.
Now consider the Möbius transformations
... centered in QA-P4, swapping Pi and Ri,
... which give for a point X four images on a circle CI(X)

The transformation X --> CI(X) maps points to circles
... centered in QA-Tf4(X), bearing QA-Tf16(X).

The transformation maps Pi to the circumcircle of the other 3
vertices.
QA-P3  --> circle through QA-P41 round QA-P9.
QA-P9  --> circle through QA-P2 round QA-P3.
QA-P41 --> circle through QA-P3, centered in QA-Tf4(QA-P41)
           (see EQF).
QA-P2  --> circle through QA-P9, centered in QA-Tf4(QA-P2)
           (see EQF).
QL-P1  --> circle through QG-P1, centered in QG-P5.
QG-P5  --> circle through QG-P18, centered in QL-P1.
QG-P18 --> circle through QG-P5, centered in QA-Tf4(QG-P18).
If CI(X) bears Y, then CI(Y) bears X.

Best regards Eckart
```

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Message: #1032
Date: 2021-06-02
From: eckart_schmidt@t-online.de
Subject: 5P-s-2Px not in EPG

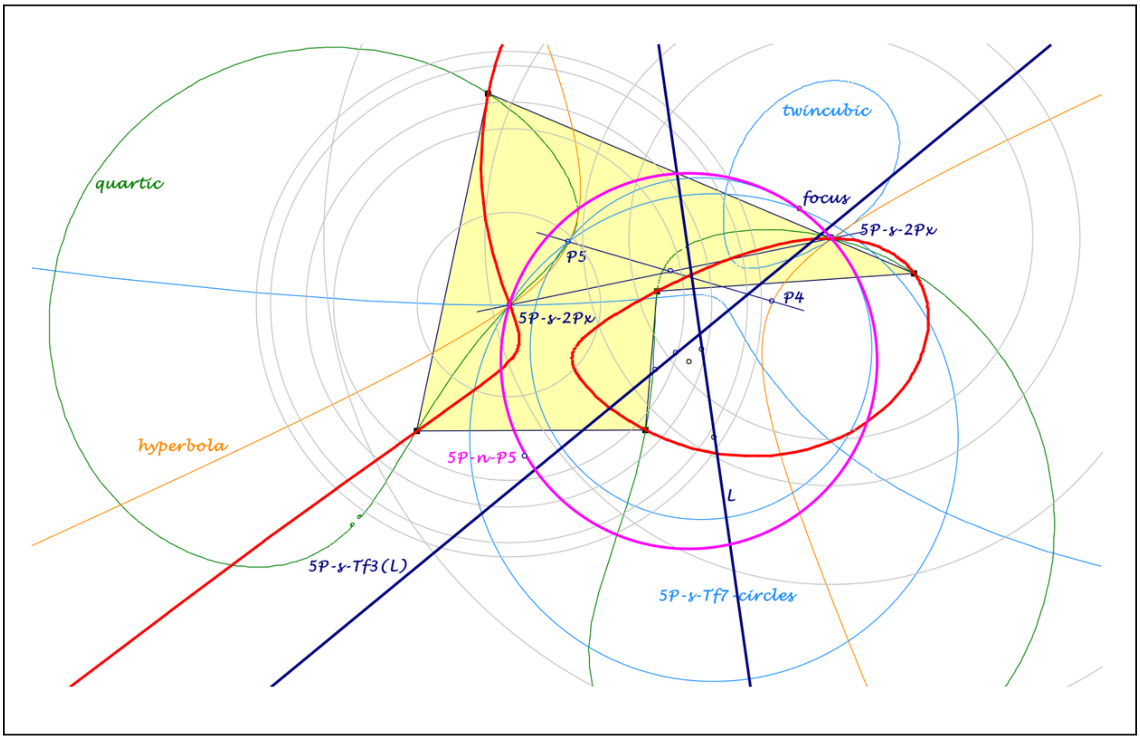
```
Dear Chris, dear Bernard,

there are several 5P-elements not in EPG,
... for example 5P-quartic, Bernard's twin cubic and an
    orthogonal hyperbola,
... discussed since July 2019 ...
How shall someone else understand our actual discussion,
... if these elements are not in EPG?
There are two common points of these three curves above,
... not in EPG, but described in Aug. 2019.
These 5P-s-2Px are 5P-s-Tf8 as well as 5P-n-Tf1 partner
... symmetric wrt midpoint 5P-s-P4.5P-s-P5
... and elements of their 5P-s-Tf7-circles.

In  #1031 I described the following QA-transformation:
... which maps a point to a circle
... centered in QA-Tf4(X), bearing QA-Tf16(X).
For a 5P there are for a point X five circles wrt the
QA-components of the 5P,
... for a 5P-s-2Px point these 5 circles have the same center
    in the partner point.

If we consider the circular 7P-s-Cu1 cubic for 5P plus 5P-s-2Px
... the transformation 5P-s-Tf7 gives a line L,
... which is the bisector of 5P-s-P5 and the focus of the cubic,
... its 5P-s-Tf3 image is a parallel to the asymptote
    of the cubic.
By the way: Replacing 5P-s-2Px by any two points this also
holds.
But special for 5P-s-2Px will be,
... that they are concyclic with the focus and 5P-n-P5,
... the corresponding circle is the 5P-n-Tf1 image of the line
    through 5P-s-2Px.

Best regards Eckart
```



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Message: #1033
Date: 2021-06-03
From: eckart_schmidt@t-online.de
Subject: Re: Coconic QA-pedal points

```
Dear Bernard,

the septic wrt coconic pedal points for a QA
... is already researched in my note attached in QFG old#503,
... also there a construction is described,
... my memory seems to be damaged.

Best regards Eckart
```

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Message: #1034
Date: 2021-06-04
From: bernard.keizer@gmail.com
Subject: Re: 5P-s-2Px not in EPG

```
Dear Eckart,
I remember I described some time ago the Quadriforum as the
flying dutchman without crew or pilot !
(das Schrecken aller Frommen, der fliegender Holländer selbst
...)
In fact, I can't understand Chris.
After asking for different memos, he no longer seems to be
willing to put these new items in EQF or EPG ...
You're right, nonone can follow our discussions, we are
repeating often the same items without progress.
It's a little bit discouraging !
Best regards
Bernard
```

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Message: #1035
Date: 2021-06-04
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

Dear Eckart,
Many thanks to you, I was very interested by this curve !
The curve in invariant in the 3 CSC and in the 3 CIC as well as in QA-Tf16.
Your construction is a little bit complicate, but it gives the searched result, which was the purpose.
(There are again 3 such hyperbolas and 3 such CCC)
As I didn't have enough points, I took T = CCC(S1 or S2 or M1 or M2 with your notation) on the hyperbola, then CCCC(T) on the curve and again CCCC(CCC(T)) on the hyperbola several times.
If you had another idea, thanks in advance (for example, the center of the hyperbola ?)
Anyhow, a very interesting item, thanks again
Best regards
Bernard

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Message: #1036
Date: 2021-06-06
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

```
Dear Eckart,
Please forget my elucubration about CCC(CCC(T)) ...
I hadn't understood the meaning of reverse construction !
I think this beautiful curve needs a complete renewed and
detailed description in EQF (with definition, description,
equation and construction.
I've found with your own definitions in message 503 that
QA-Tf16(P) = CIC(CSC(P)) =CSC(CIC(P)).
Taking one example, if we name U13, V13, U24 and V24 the 4
points on the lines P1P3 and P2P4,
U13 and V13 as well as U24 and V24 are CIC partners,
U13 and V24 as well as U24 and V13 are CSC partners
U13 and U24 as well as V13 and V24 are TF16 partners
This doen't help me, as it doesn't give new points ...
Can you help me ?
Best regards
Bernard
PS I tried the 2 other CSC's centered in M1 or M2 (your
notation) and swapping QL-P1 and M2 or M1, but it gives for the
4 points U and V 13 and 24 4 new points having the same CCC
point
```

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Message: #1037
Date: 2021-06-09
From: eckart_schmidt@t-online.de
Subject: Re: Coconic QA-pedal points

```
Dear Bernard,

it needed some time to reproduce my own construction of the
septic,
... "the inverse transformation of CCC" is the difficulty,
... not correct formulated, for CCC(X) has 8 preimages,
... here a construction starting with a point Y on the
hyperbola:
Let L13 be a parallel through Y of the bisector m13 of P1P3,
... let CI be the CSC-circle of L13, intersecting m13 in Z1, Z2,
... let CI1 be the circle round Z1 through P1 and P3,
,,, let Ci2 be the circle round Z2 through P1 and P3.
Analog you get two circles Ci3 and Ci4,
... starting with L24,
    parallel through Y of the bisector m24 of P2P4
... The 8 intersections C1^C3, C1^C4, C2^C3, C2^C4
    are points of the septic.

Best regards Eckart
```

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Message: #1038
Date: 2021-06-10
From: eckart_schmidt@t-online.de
Subject: Review QA-Cu6

```
Dear all,

why always looking for new elements
... and not making geometry with them?
Here another way to QA-Cu6:

Starting with a line L and a QA with its three QG-versions,
... we get three image lines with QL-Tf2,
... which have a common point M.
Let us consider the transformation L-->M
... for a line pencil of a point P,
... then we get a cubic CU
... with a knot K on the circumconic of QA plus P
... in the 2nd intersection with the polar of QA-Tf2(P).
If P on QA-P1.QA-P16, the cubic bears QA-P22.
If P = QA-P16, the cubic is QA-Cu6.
This cubic bears several points (see EQF):
... vertices of QA, QA-P22, QA-P1 (knot),
    vertices of the medial triangle TR of QA-Tr1,
New property:
    The tangents in the vertices of TR intersect in QA-P31.
Finally: QA-Cu6 bears the vertices of a cevian triangle of TR
... for a point with the 1st DT coordinate
    p^2(p^2-q^2-r^2)(p^2q^2+p^2r^2-2q^2r^2-q^4-r^4),
... what about this point?

Best regards Eckart
```

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Message: #1039
Date: 2021-06-10
From: eckart_schmidt@t-online.de
Subject: QL-Ci5-splitter

```
Dear Chris,

for a quadrangle the three circle center of QL-Ci5 are collinear
... on a line parallel QA-L4 through the center of the QA-Tr2
circumcircle.

Best regards Eckart
```

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Message: #1040
Date: 2021-06-11
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

```
Dear Eckart,
Help, please !
I perfectly understand  the construction of the 8 points for any
point on the conic.
More generally, using symmetric notations for CIC 1 to 3 and CSC
1 to 3 and QA-Tf16 = CICi*CSCi, any point X has 7 partners being
CICi(X), CSCi(X) and TF16(X).
X, CIC1(X), CSC1(X) and TF16(X) have the same CCC1(X), together
with 4 other points, which are for example CSC2 of your 4 points
T13 and T24.
CIC2(X), CIC3(X), CSC2(X) and CSC3(X) have also the same CCC1,
together with 4 other points (but this 2nd CCC1 is different).
My problem is the construction of the hyperbola.
I have only 2 points M1 (QL-P1 in your notation), CCC1 of T2,
T3, M2, M3 (S1, S2, M1 and M2 in your notation) and the 4 real
points T13 and T24.
The other points T12 and T34 and T114 and T23 are not real.
Can you help me ?
Many thanks in advance
Best regards
Bernard
PS I spent a lot of time on this figure and I stil find it
fascinating ...
```

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Message: #1041
Date: 2021-06-11
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

Dear Eckart,
Sorry, a part of the sentence is missing !
M1 (QL-P1) is the CCC1 of the points M1 (QL-P1), T1 (QG-P1),
QA-P4, the infinity point and the 4 real points T13 and T24.
T is the CCC1 of the points T2, T3, M2, M3 (S1, S2, M1 and M2 in
your notation) and 4 other points, for example CSC2 of the
points T13 and T24.
Best regards
Bernard

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Message: #1042
Date: 2021-06-13
From: bernard.keizer@gmail.com
Subject: Re: Coconic QA-pedal points

Dear Eckart,
If I'm not wrong, QA-Cu1 is also invariant in the 3 CIC, in the
3 CSC as well in QA-Tf16.
Any point on QA-Cu1 has a CCC point. What is the locus of these
CCC points ?
It seems it is a curve also through QL-P1 and T, but the points
T13 and T24 as well as their CSC2 are not on QA-Cu1.
Where is the the problem ?
Best regards
Bernard

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Message: #1043
Date: 2021-06-13
From: eckart_schmidt@t-online.de
Subject: Re: Coconic QA-pedal points

```
Dear Bernard,

excuse, if I haven't studied your #1040/1041 in detail,
... the nomination was too confusing and my concentration gave
    up,
... but I understand your problem, constructing the hyperbola.
Using my nomination in old#503 for a quadrigon,
... we have the following points surely:
... infinity points of m13 and m24, further QL-P1 and T,
... perhaps new, that T is a point of the hyperbola.
We get further points as CCC(Tij) beside CCC(T13) and CCC(T24),
... but there are special quadrangles,
    for which these Tij are not real.
This is not mentioned in my paper, thanks for your doubts.
At the moment I cannot solve this problem,
... but an unfounded observation:
If the quadrangle is not convex, the points Tij unequal T13 and
T24 exists,
... but for the convex case I cannot find the conditions
    for no Tij unequal T13 and T24.

Best regards Eckart

PS: The hyperbolas of different quadrigons of the quadrangle are
different,
... my constructions are based on one fixed quadrigon.
```

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Message: #1044
Date: 2021-06-13
From: eckart_schmidt@t-online.de
Subject: QG-circumquartic and QL-2Px

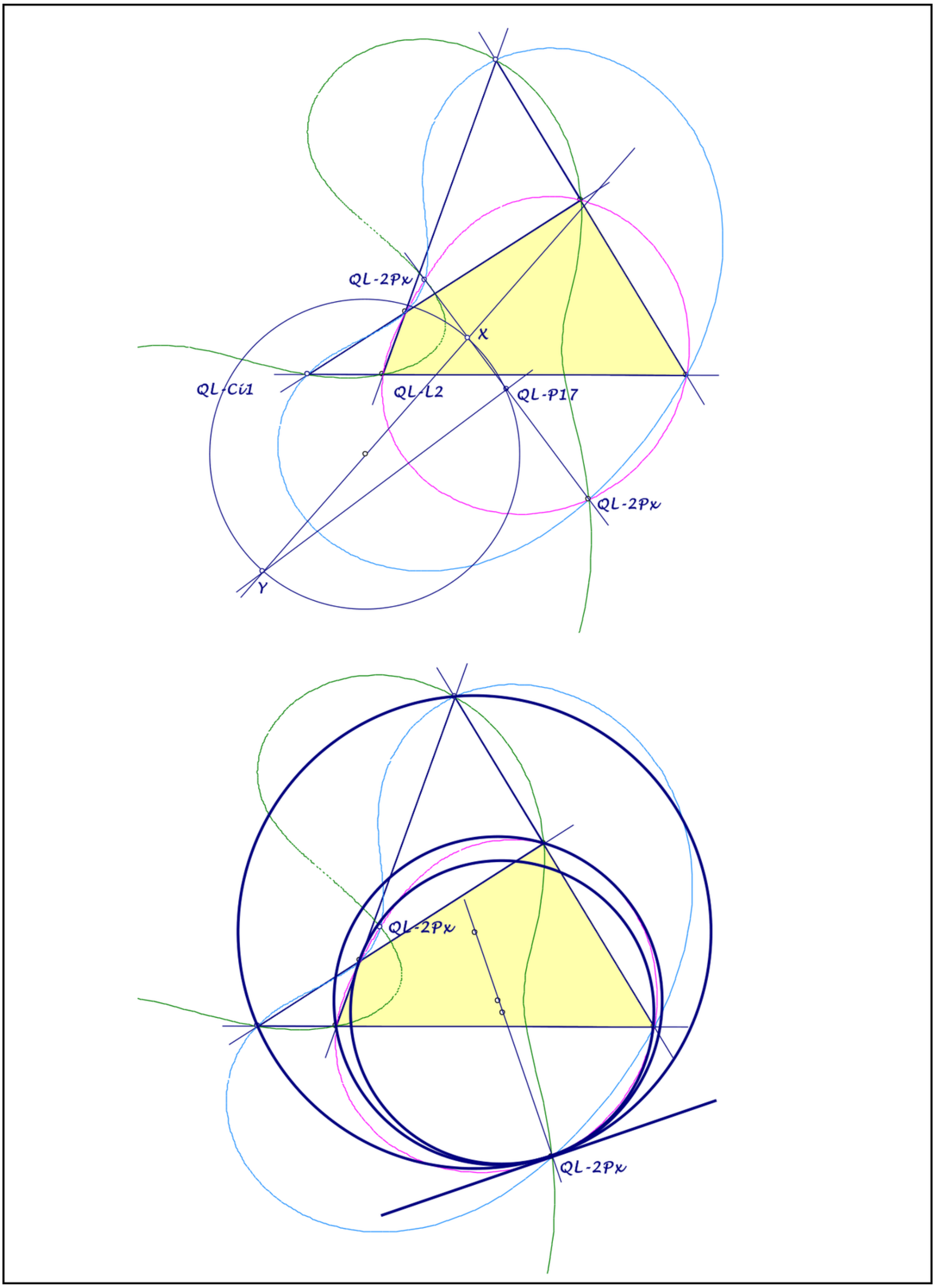
Dear all,

in message old#362 a quartic for a QG is described,
... here only the construction, properties in the message:
We need the vertices M1 and M2 (unequal QL-P1) of the Miquel triangle
... and the fixed points F1 and F2 of CSC=QL-Tf1.
1. Circle through QG-P1, M1, M2 with a variable point X.
2. Lx angle bisector of <F1XF2.
3. Intersections of Lx and its CSC-circle are points of the quartic.

The quartic is the locus of points,
... where circles through opposite QG-vertices are tangent.

The following will be new:
Consider the three quartics for the QG-versions of a QL,
... we get two triple intersections QL-2Px,
... which are CSC-partner with midpoint QL-P17
... and can be constructed as follows:
Let the line QL-L2 intersect QL-Ci1 in X and Y,
... one of the lines X.QL-P17 or Y.QL-P17 bears CSC-partner,
... which are QL-2Px with the property:
QL-2Px are the points,
... where circles through opposite QL-points have a common tangent.

Best regards Eckart



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Message: #1045
Date: 2021-06-14
From: bernard.keizer@gmail.com
Subject: QA-Cu1 and associated quartic

Dear Chris, dear Eckart
Using symmetric notations, let's consider Eckart's transformations CIC1,2,3 and CSC1,2,3 as well as CIC1*CSC1= CIC2*CSC2 = CIC3*CSC3 = QA-Tf16.
The DT vertices are T1, T2 and T3 and the Miquel points M1, M2 and M3.
Let's consider also Eckart's construction CCC1,2,3 (see message #503).
For any point X on QA-Cu1, CIC1(X) is the 2nd intersection of the circles XP1P3 and XP2P4.
Let's consider now the 4 points intersections of the circles XP1P3 and CSC1(X)P2P4 and the circles CSC1(X)P1P3 and XP2P4.
These 4 points have the same CCC1 as X, CIC1(X), CSC1(X) and QA-Tf16(X).
For X describing QA-Cu1, these 4 new points describe a quartic associated to the cubic QA-Cu1.
This quartic passes through the 4 QA-vertices and through the 4 points named T13 and T24 by Eckart (these points have the same CCC1 - M1 - as M1,T1, QA-P4 and the infinity point).
The quartic passes also through T2, T3, M2 and M3 and through CSC2(T13 and T24) (these 8 points have the same CCC1, the point T).
The locus of CCC1(X) is an hyperbola through M1 and T with asymptotes parallel to the bisectors m13 and m24 of P1P3 and P2P4.
This hyperbola is easy to draw with the CCC1 of certain points of QA-Cu1 (QA-P3 or intersection Q of QA-Cu1 with it's asymptote ...).
The beauty of these 3 curves is that for any point on the hyperbola, we get 4 points on the cubic QA-Cu1 and 4 points on the associated quartic.
QA-Cu1 and it's associated quartic form a septic and both parts of the septic are invariant in CIC1,2,3, in CSC1,2,3 and in QA-Tf16.
Best regards
Bernard

Message: #1046
Date: 2021-06-16
From: bernard.keizer@gmail.com
Subject: Bipartite QL-Cu1 and associted quartics

```
Dear Chris, dear Eckart,
The construction given in the previous topic works obviously for
a bipartite QL-Cu1, as this curve is the QA-Cu1 of many QA's
with 2 perpendicular sides.
M1 is QL-P1, M2 and M3 ere the QL-2P2a and b, T1 is QA-P2, QA-P4
is CSC(QA-P2) and the QA is the tangential QA of QA-P4.
It gives many rectangular hyperbolas and many associated
quartics, passing through the 4 vertices of the QA's and the
points named T13 and T24 by Eckart.
Best regards
Bernard
```

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Message: #1047
Date: 2021-06-16
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and associated quartic

```
Dear Bernard,

thanks for your great interest in my constructions,
... but I have two problems in understanding:

(1) Wrt "These 4 points have the same CCC1 as X, CIC1(X),
CSC1(X) and QA-Tf16(X)."
CCC is a QG-transformation,
so the order of the points is relevant
... and I found no order to confirm your observation.

(2) Sorry, I cannot reproduce "The quartic passes also through
T2, T3, M2 and M3..."
Is there a geometric background for the quartic?
Perhaps we should consider other QG-hyperbolas,
... centered in a special point, through QL-P1
... with asymptotes parallel to the bisectors of P1P3 and P2P4,
... using the "invers" of CCC in the sense of #1037.

Best regards Eckart
```

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Message: #1048
Date: 2021-06-16
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and associated quartic

Dear Eckart,
Thanks for your answer !
First, you are completely right, the quartic doesn't pass through M2, M3,T2 and T3 which are points of the cubic.
But the quartic passes through the 4 points having the same CCC1 as these 4 points, id est the CSC2 of your 4 points T13 and T24.
For the 1rst question, using your construction with the centers Z1, Z2, Z3 and Z4 of the 4 circles XP1P3, XP2P4, CSC1(X)P1P3 and CSC1(X)P2P4 and CCC1(X) as the intersection between the 2 lines CSC1(Z1)CSC1(Z3) and CSC1(Z2)CSC1(Z4), I don't see an order in the 4 points. (Or is it implicit with P1 and P3 as CSC1 partners as well as P2 and P4 ?).
Of course, there are 3 such hyperbolas ...
Perhaps, you will find the explanation.
Best regards
Bernard

Message: #1049
Date: 2021-06-17
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and associated quartic

Dear Bernard,

once more controlled, I can only repeat:
(1) Wrt "These 4 points have the same CCC1 as X, CIC1(X), CSC1(X) and QA-Tf16(X)."
CCC is a QG-transformation,
so the order of the points is relevant
... and I found no order to confirm your observation.
By the way wrt
"... id est the CSC2 of your 4 points T13 and T24...":
... Can CCC be a Möbius transformation CSC?
I cannot find my misunderstandings.

Best regards Eckart

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Message: #1050
Date: 2021-06-17
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and associated quartic

Dear Eckart,
I checked also my figures.
This could be perhaps an explanation.
I started with a convex QA with vertices P1, P2, P3 and P4 in this order conterclockwise.
Then using macros, it turns that :
CCC1(X) = (P1,P2,P4,P3,X) = (P1,P4,P2,P3,X)
CCC2(X) = (P1,P3,P4,P2,X) = (P1,P4,P3,P2,X)
CCC3(X) = (P1,P2,P3,P4,X) = (P1,P3,P2,P4,X)
CCC1 describes an hyperbola through M1, CCC2 a 2nd hyperbola through M2 and CCC3 a 3rd hyperbola through M3.
So I can only confirm your property that the order of the points is relevant for this QG transformation !
Best regards
Bernard

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Message: #1051
Date: 2021-06-17
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and associated quartic

```
Dear Bernard,

sorry, I don't understand the nomination in #1050,
... but I think now, the reason for our discussion is,
... that you use another CCC, as I defined in old#503:
There Z3 is CSC(Ci(X,P1,P3) = Ci(CSC(X),P2,P4) = your Z4 in
#1048.

Best regards Eckart
```

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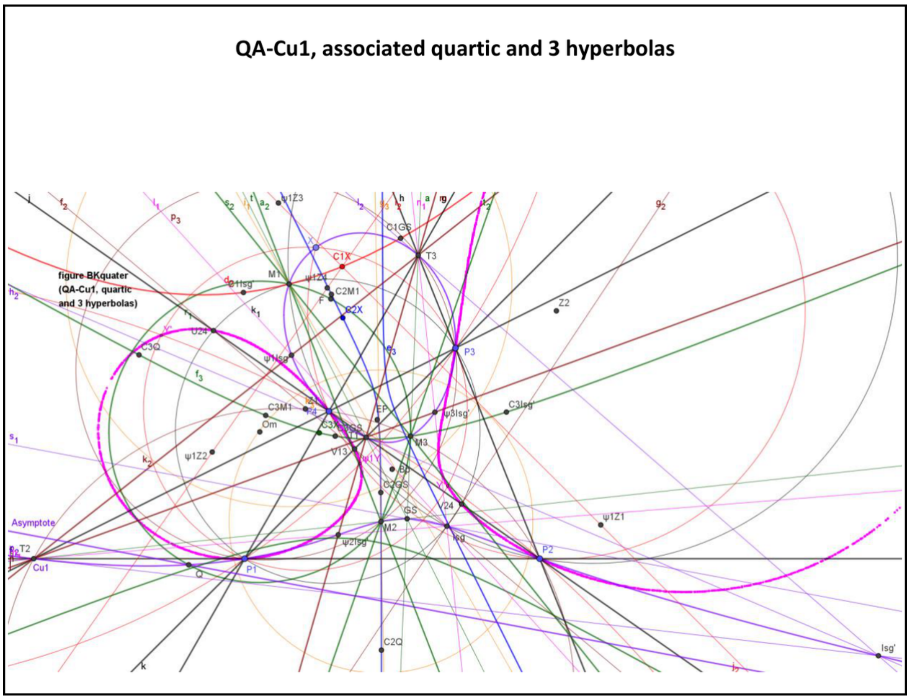
Message: #1052
Date: 2021-06-17
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and associated quartic

Dear Eckart,
What a strange idea !
Of course, I use the same construction as yours in the message #503.
If P1 and P3 are CSC1 partners, as well as P2 and P4, M1 is your QL-P1 and P1P3 and P2P4 intersect in T1.
Z1 is the center of Ci(X,P1,P3) and Z3 is the center of CSC(Ci(X,P1,P3)), which is Ci(CSC(X),P1,P3 and not Ci(CSC(X),P2,P4) !
It's exactly your figure on page 3 of the mentionned message.
I send you this time a figure with the cubic QA-Cu1, the associated quartic and the 3 hyperbolas.
(I use my own denomination, but I suppose it will be obvious (EP is QA-P2, GS is QA-P3, Isg is QA-P4 and Isg' is QA-P41).
I suppose you will recognize the Z1,Z2,Z3 znd Z4 as well as CCC(X), named C1X, C2X and C3X on the 3 hyperbolas.
By definition of your reverse construction, each point on an hyperbola has 8 prefigures, 4 on the cubic and 4 on the quartic.
The quartic is bipartite ...
I hope there will be no longer misunderstanding this time.
Best regards
Bernard



QA-Cu1 and quartic.pdf

Message: #1053
Date: 2021-06-18
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and associated quartic

```
Dear Bernard,

Please accept my apologies and forget my last message #1051,
...  it was a false hasty interpretation of CSC.
You describe in #1048 the same CCC as in old#503,
... so we consider the same quartic.

But my misunderstanding wrt (1) in #1047 and #1049 was quite
another
... as I realized in the meantime:
... I compared CCC(X) wrt the quadrigon of the first 4 points
    and of the second four points,
... which are of course different,
    excuse my wrong interpretation.

Wrt #1050:
If CCC(X) = (P1,P2,P3,P4,X) means CCC(P) wrt the QG P1P2P3P4,
... I cannot confirm (P1,P2,P3,P4,X) = (P1,P3,P2,P4,X),
... but (P1,P2,P3,P4,X) = (P1,P4,P3,P2,X).

Best regards Eckart
```

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Message: #1054
Date: 2021-06-18
From: eckart_schmidt@t-online.de
Subject: Re: QA-Cu1 and associated quartic

```
Dear Bernard,

excuse the typo in my last message, last passage:
... replace P by X in  CCC(P).

Best regards Eckart
```

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Message: #1055
Date: 2021-06-18
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 and associated quartic

```
Dear Eckart,
As I told you, using macros, when I put C1X as final point,
Geogebra returns as initial points (P1,P2,P4,P3,X) !
This has no other meaning ...
Obviously, for you and for me C1X is CCC(X) wrt the QG P1P2P3P4
with QG-P1 in T1.
The same way, C2X is CCC(X) wrt the QG P1P3P2P4 with QG-P1 in T2
and C3X is CCC(X) wrt the QG P1P2P4P3.
Best regards
Bernard
```

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Message: #1056
Date: 2021-06-19
From: eckart_schmidt@t-online.de
Subject: QG-P16-splitter

```
Dear Chris,

let QG1, QG2, QG3 be the QG-versions of a quadrangle QA,
... then QG1-P16 and QG2-P16 are CSC-partner wrt QG3,
... a nice property connecting QA-, QL- and QG-geometry.

Best regards Eckart
```

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Message: #1057
Date: 2021-06-19
From: bernard.keizer@gmail.com
Subject: Re: QG-P16-splitter

Dear Eckart,
The QG-P1 are the CSC1,2,3 of QA-P4.
The QG-P16 are the CSC1,2,3 of QA-P3.
For any point X, CSC2(X) and CSC3(X) are CSC1 partners.
Let's name T1, T2 and T3 the 3 QG-P1 and V1, V2 and V3 the 3 QG-P16.
V2 and V3 are CSC1 partners, as well as T2 and T3.
QA-P3 is on line with M1T1, M2T2 and M3T3.
The CSC of these 3 lines are lines M1V1, M2V2 and M3V3 through QA-P4.
Best regards
Bernard

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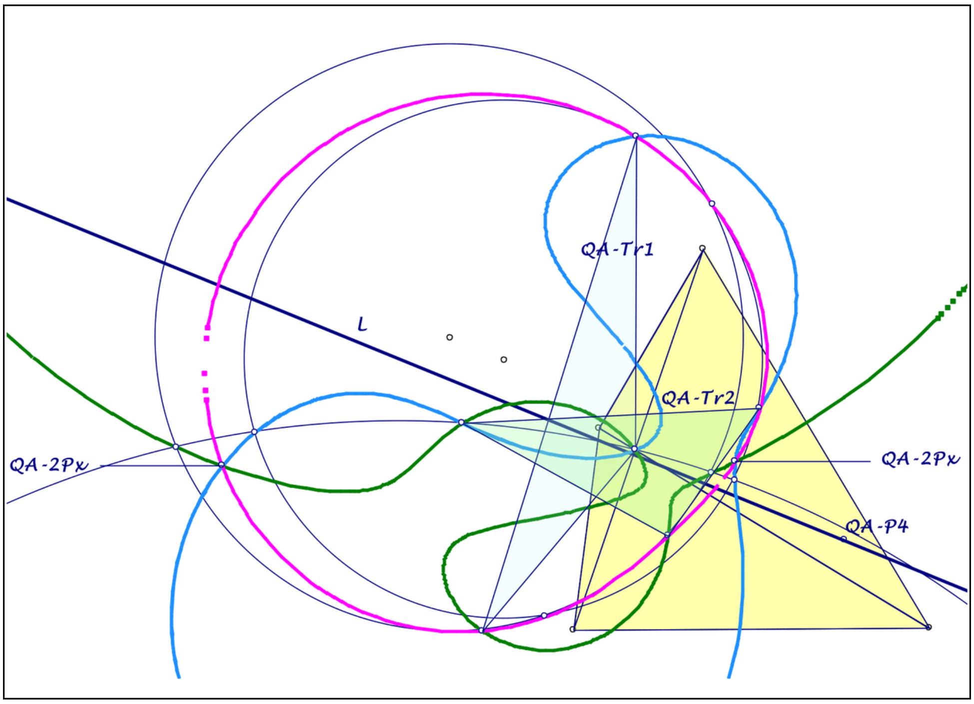
Message: #1058
Date: 2021-06-19
From: eckart_schmidt@t-online.de
Subject: New QA-2Px

```
Dear all,

let us consider a QA and the line pencil of QA-P4 with lines L,
... which have 3 image circles wrt the CSC of the QG-versions of
QA,
... with intersections on 3 quartics,
... each bearing two QA-Tr1-vertices and the Miquel points
    of two QG versions
... and the fixed CSC-points of the 3rd QG-version,
... further two common points QA-2Px.
The two Miquel points and the two QA-Tr1-vertices give a new QG
... with the same CSC and Miquel point as the 3rd QG,
... intersections of opposite sides are QA-P3 and QG-P16
    of the 3rd QG.
Each quartic is invariant wrt the CSC-transformation of a
QG-version.
The two common points QA-2Px are the only points,
... whose CSC-images wrt the the 3 QG-versions are collinear
    on a line through QA-P4.
What about a construction of QA-2Px?

Best regards Eckart
```



2021-06-20.pdf

Message: #1059
Date: 2021-06-20
From: van10hoven@gmail.com
Subject: Re: QG-P16-splitter

```
Dear Eckart,

Very nice property indeed.
Even then QG-P16 is eminently an example of a point connecting
QA-, QL- and QG-geometry.
Another example of connecting QA-, QL- and QG-geometry is QG-P5.
QG-P5 in a QA-Quadrigon = the 2nd intersectionpoint of the
MiquelCircles (QL-Ci3) of the other 2 QA-Quadrigons (the 1st
intersectionpoint is QA-P9).

Best regards,
Chris
```

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Message: #1060
Date: 2021-06-20
From: bernard.keizer@gmail.com
Subject: Re: New QA-2Px

```
Dear Eckart,
I checked your property !
I suppose we have the same construction if we take another
pencil of lines (for example with QA-P3 instead of QA-P4).
Naturally, 2 circles are CSC partners of the line and are CSC
partners wrt the 3rd CSC, the 2 intersections being also CSC
partners in this 3rd CSC.
Each  quartic is CSC invariant in the corresponding CSC, but the
QA-2Px are not CSC partners ...
I have no idea of a construction of these points.
Best regards
Bernard
```

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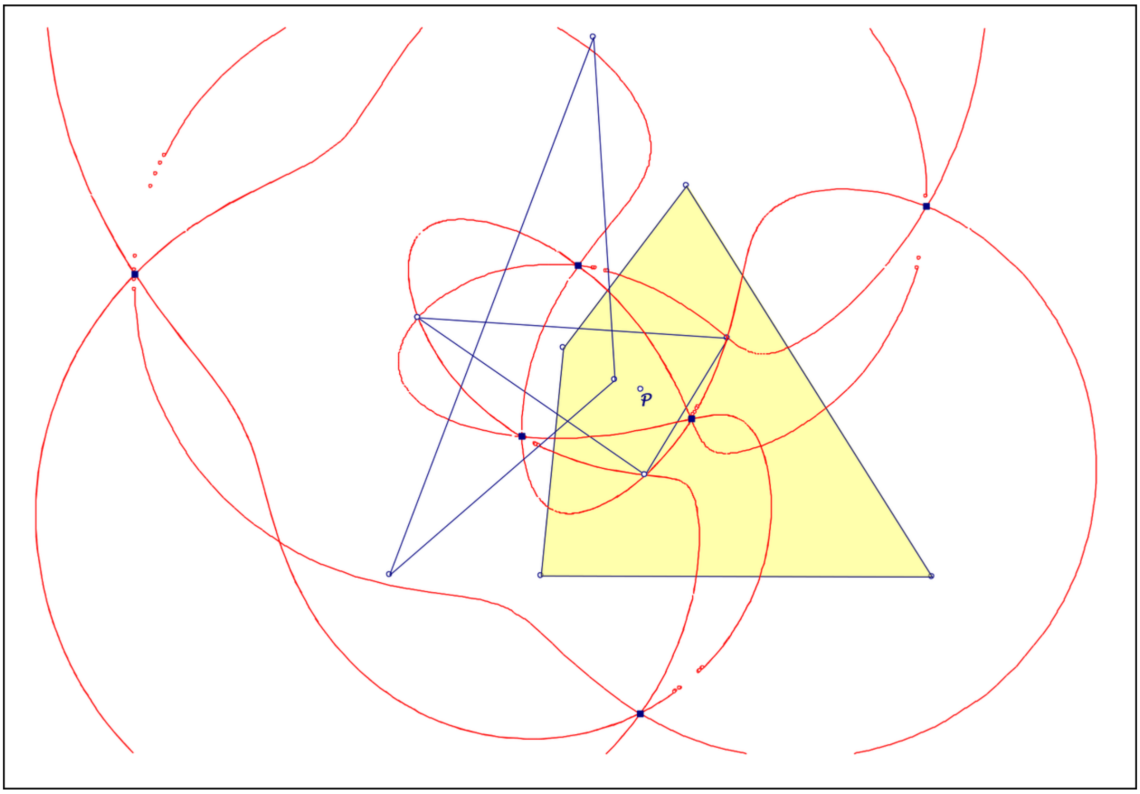
Message: #1061
Date: 2021-06-22
From: eckart_schmidt@t-online.de
Subject: Re: New QA-2Px

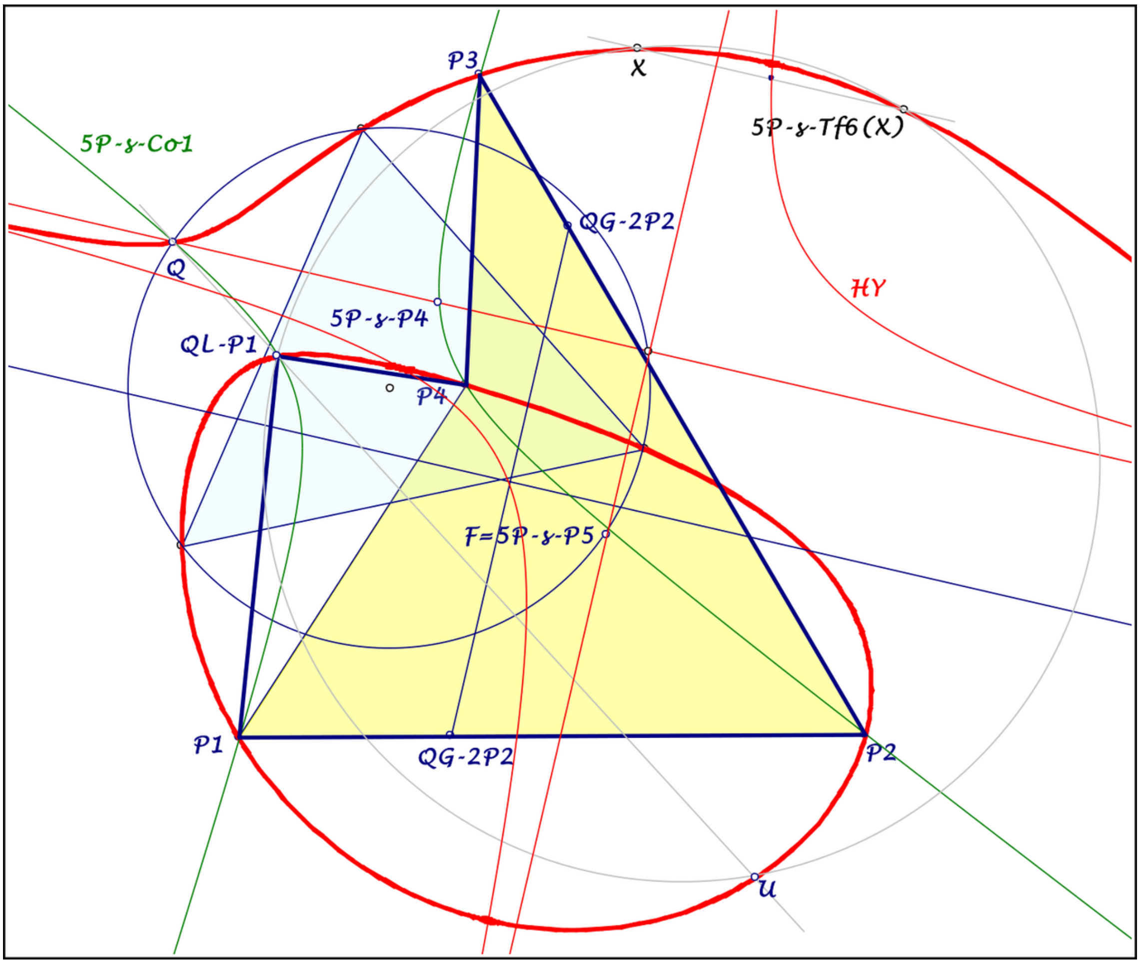
```
Dear Bernard,

if we replace the line pencil of QA-P4 by another line pencil,
... there can be up to six triple intersections,
    see attached file.
The quartics have not to be CSC-invariant and not bearing the
QA-Tr1-vertices.
But the last property holds,
... the 3 CSC-images of each point are collinear
    on a line through P.

Best regards Eckart
```



2021-06-22a.pdf

Message: #1062
Date: 2021-06-24
From: eckart_schmidt@t-online.de
Subject: Re: Pedal Cevian Quadrigons

```
Dear all,

this is a new sight of the circular QG-circumcubic,
... which is the locus of points, whose pedal QG is a cevian QG,
... first mentioned 2012 on my homepage Fucev.pdf
(eckartschmidt.de),
<http://eckartschmidt.de/Fucev.pdf>
... discussed in old#155, old#1242, #446, #447, #450, #452,
#453.

Consider a QG, expanded to a 5P = QG plus Miquel point QL-P1,
... and take the circular 5P-circumcubic with focus 5P-s-P5.

The asymptote is parallel to the bisector of QG-2P2,
intersecting the cubic in
... Q = 2nd intersection of 5P-s-Co1 and a parallel
    to the asymptote through 5P-s-P4.
The pivot wrt 5P-s-Tf6 is the point at infinity of the
asymptote.
The midpoints of 5P-s-Tf6-partner on the cubic give an
orthogonal hyperbola HY,
... intersecting the cubic in in- and excenters
    of the reference triangle,
... used in the cited papers above for this cubic
    as pivotal isogonal circular cubic,
... on the cubic are the isogonal partner the 5P-s-Tf6-partner.
One asymptote of the orthogonal hyperbola is the asymptote of
the cubic,
... the other bears the focus F,
... their intersection lies on the circumcircle
    of the reference triangle,
... bearing diametral 5P-s-P5 and Q,
... with Simson lines parallel to the asymptotes.
Let U be the 3rd intersection of Q.QL-P1 and the cubic,
... then circles through two 5P-s-Tf6-partner and QL-P1 bear U.
If a QG-circumconic intersects the cubic twice,
... then the 3rd intersection of the connecting line
    is also point U,

Further properties can be found in the cited papers,
... I am fascinated of this item!

Best regards Eckart
```



2021-06-24.pdf

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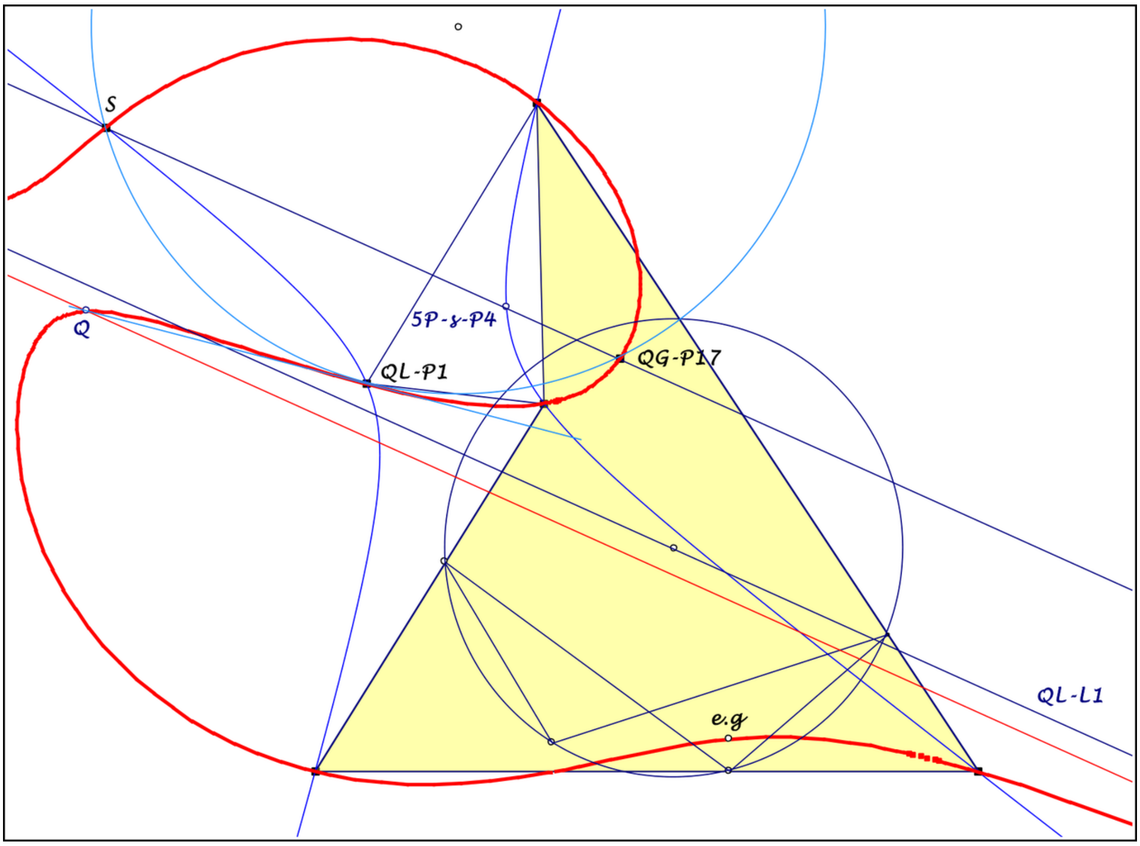
Message: #1063
Date: 2021-06-26
From: eckart_schmidt@t-online.de
Subject: Cyclic Pedal Quadrigons

Dear all,

I am not sure, whether this cubic is already mentioned:
The locus of points with cyclic pedal quadrigons
... is a QG-circumscribed focal circular cubic.

Properties:
(1) The cubic bears the points QL-P1 and QG-P17.
(2) The asymptote is parallel to the Newton line QL-L1.,
... through the reflection of QL-P1 in QL-L1
(3) The focus is the Miquel point QL-P1.
(4) The cubic is invariant wrt QL-Tf1 = CSC.
(5) The circle centers lie on the Newton line QL-L1.
Let us now consider the cubic as circumcubic of a 5P = QG plus QL-P1:
(6) The 5P-s-Tf6-pivot is QG-P17.
(7) The cubic bears the point S, 6th intersection of the cubic and 5P-s-Co1,
... which is the 2nd intersection of 5P-s-Co1 and QG-P17.5P-s-P4.
(8) The intersection Q of the cubic and its asymptote
... is the intersection with the tangent in QL-P1 at the circle through QL-P1, QG-P17,S.
(9) The cubic is a focal circular cubic 7P-s-Cu1
... wrt the QG plus QL-P1, QG-P17 and S.

Best regards Eckart



2021-06-26.pdf

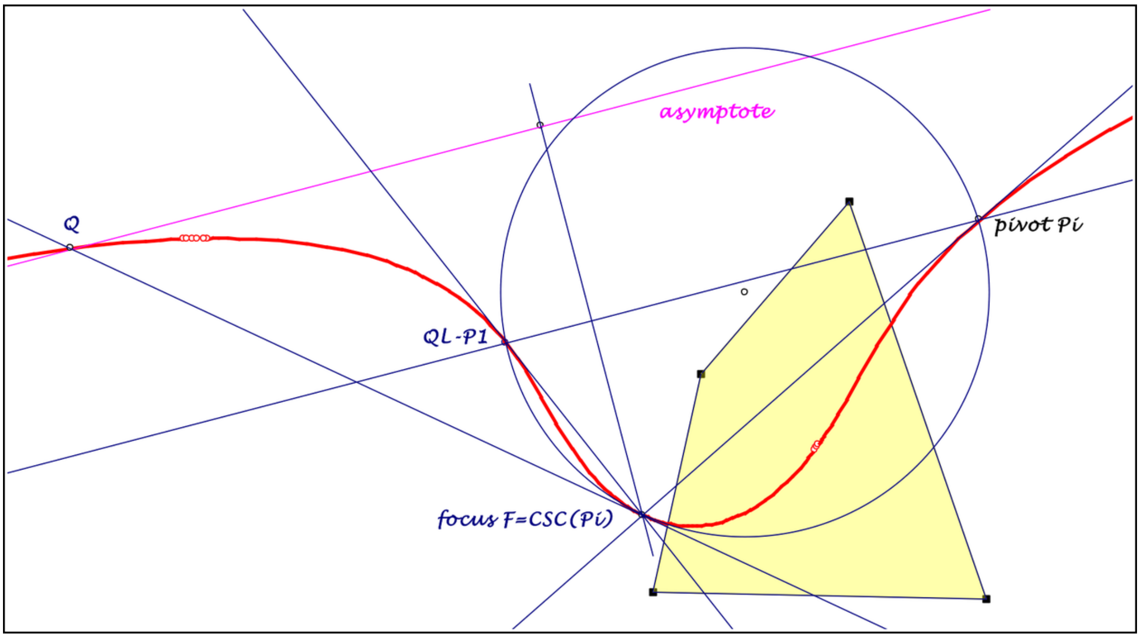
Message: #1064
Date: 2021-06-27
From: eckart_schmidt@t-online.de
Subject: QG-Cubics with CSC-Pivot

```
Dear all,

let us consider a QG and lines through a pivot Pi,
... which intersect their CSC-circles on a focal cubic,
... not necessary QG-circumscribed,
... but bearing Pi and QL-P1 and the focus F = CSC(Pi),
... which is the common tangential of Pi and QL-P1,
... the asymptote is parallel Pi.QL-P1
... through the reflection of F in Pi.QL-P1,
... intersecting the cubic on the tangent in F
    at the circle (Pi,QL-P1,F).

If we take as pivot QG-P1, we get the cubic in old#2382,
... QG-circumscribed, focal and circular,
... which is 7P-s-Cu1 of 7P = QG plus QG-P1,QL-P1,QA-P4,
... focus is QA-P4 and the intersection with the asymptote is
... the tangential of QA-P4.
Further properties in the cited message.

Best regards Eckart
```



2021-06-27.pdf

Message: #1065
Date: 2021-06-28
From: bernard.keizer@gmail.com
Subject: Re: QG-Cubics with CSC-Pivot

```
Dear Eckart,
You have already mentionned this construction several times
before.
In fact, you don't need the QG at all !
All you need is a CSC1 and 2 CSC1 partners.
You get always a pivotal monopartite focal circular cubic, each
of the 2 points being the pivot of CSC2 or 3 partners in the
CSC2 or 3 swapping the other point (center of this CSC2 or 3)
and the center of CSC1.
Best regards
Bernard
```

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Message: #1066
Date: 2021-06-28
From: eckart_schmidt@t-online.de
Subject: Re: QG-Cubics with CSC-Pivot

```
Dear Bernard,

with my message I wanted to point out
... the possibilities of so constructed cubics for QG-geometry
... as the cited example show.

Best regards Eckart
```

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Message: #1067
Date: 2021-06-29
From: eckart_schmidt@t-online.de
Subject: Re: Cyclic Pedal Quadrigons

```
Dear all,

the cubic for points with a cyclic pedal quadrigon bears QG-P18
(see EQF).

Best regards Eckart
```

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Message: #1068
Date: 2021-06-29
From: eckart_schmidt@t-online.de
Subject: Re: Cyclic Pedal Quadrigons

```
Dear all,

in #1063 and #1067 I describe only QL-Cu1 for a quadrigon,
(see EQF),
... please apologize profusely that I didn't realize this
    well known property.

Best regards Eckart
```

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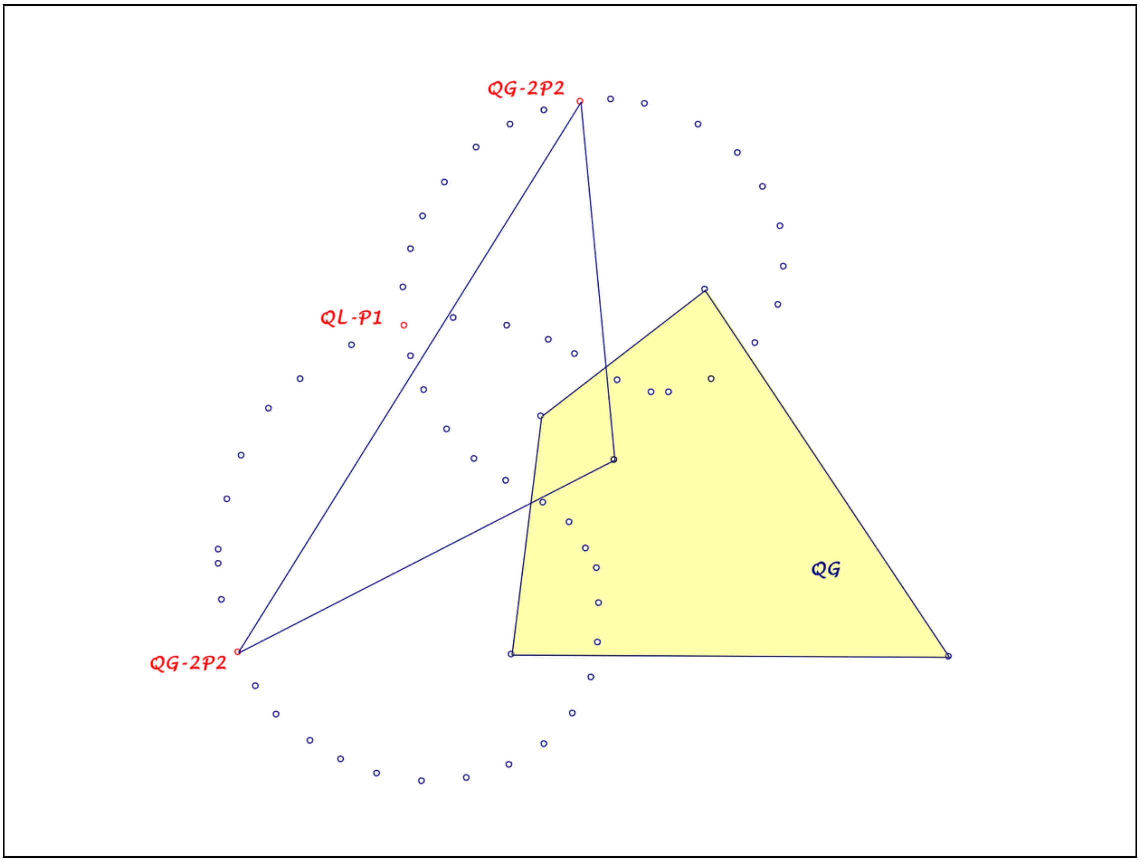
Message: #1069
Date: 2021-07-03
From: eckart_schmidt@t-online.de
Subject: Pedal QG with incircle

```
Dear all,

what about the attached approximately drawn QG-quartic,
... locus of points for pedal QG with incircle?
Researched by the property P1P2+P3P4 = P2P3+P4P1.

Best regards Eckart
```



2021-07-03.pdf

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Message: #1070
Date: 2021-07-04
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
Very interesting item !
I think your quartic is circular and therefore anallagmatic.
The 4 inversion centers are points on the 4 lines of the QG.
If, for example, we take the side below, it cuts the quartic in 2 points, one is QG-2P2, the other between the QG vertices.
Then draw the circle through these 2 points and QL-P1 ; the tangent in QL-P1 cuts the side in one center of inversion.
It's possible to do the same with the side on the right, which gives also only 2 points, one being the other QG-2P2 and the other between the QG vertices.
The same construction gives a 2nd inversion center on this side.
For the 2 other sides, we have 4 intersections with the curve, one being always a point QG-2P2.
It's possible to use these inversion centers to get a certain number of points and draw the quartic.
Best regards
Bernard

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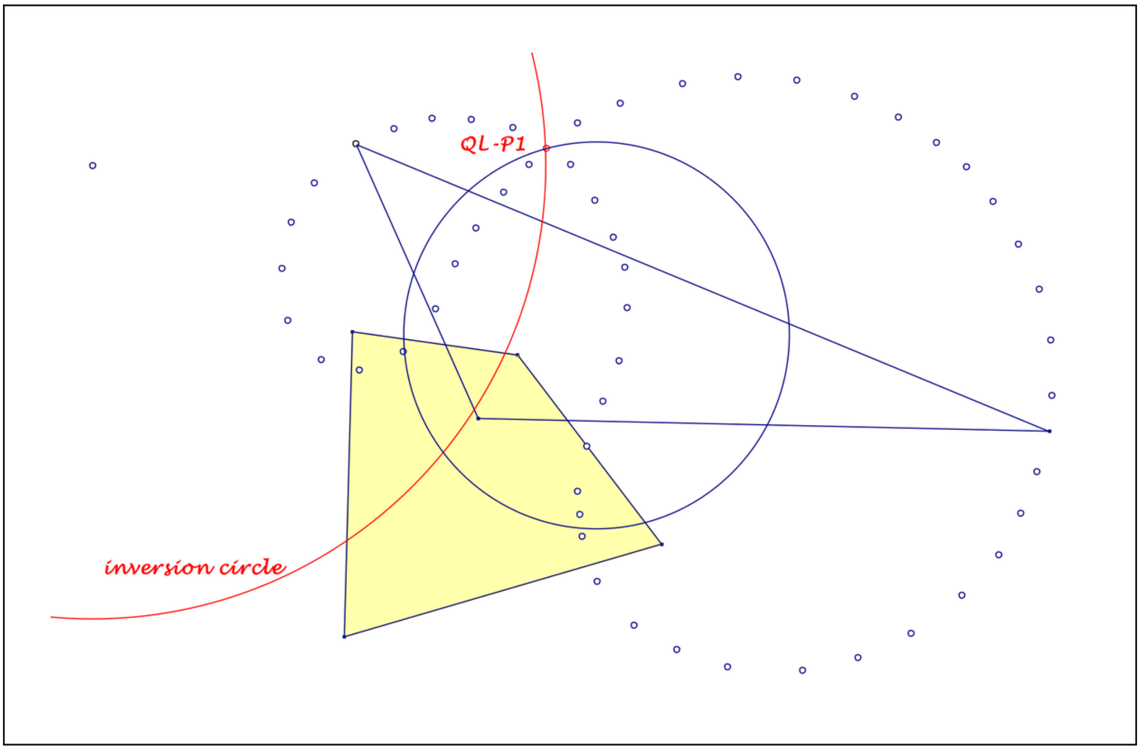
Message: #1071
Date: 2021-07-05
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

Dear Bernard,

thanks for your interest, but some questions:
How do you justify, that the quartic is circular?
Wrt anallagmatic: I cannot reproduce your inversion centers,
... I only see the attached drawn inversion circle, but it is not yours.

But I have another very doubtful property (problems wrt precision):
Lines through QL-P1 intersect the quartic in two points,
... whose midpoints may be concyclic.

Best regards Eckart



2021-07-05.pdf

Message: #1072
Date: 2021-07-05
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
Your quartic looks like a smashed or stretched limaçon and I notice that any circle cuts the curve in only 4 points (instead of 8 as expected).
That's why I supposed it is circular or bicircular.
Sorry for my wrong circles of inversion, how did you find yours ?
It looks your property with the concyclic middles of points aligned with QL-P1 holds.
But I think I'll soon give up, it takes too much time for finding points of the curve ...
Best regards
Bernard

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Message: #1073
Date: 2021-07-07
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
I'm not an expert in these circular quartics, but I think these properties are general.
The curves are invariant in an inversion wrt a certain circle.
The middles of points aligned with a certain point (here QL-P1) are concyclic on a circle through this point.
The curves are envelops of circles bitangent in these 2 points ; these circles have a special relation with the inversion circle.
The centers of these circles described a certain center conic named deferent.
See Mathcurve at bicircular quartic.
Best regards
Bernard
PS Here I reach my top of competence ...

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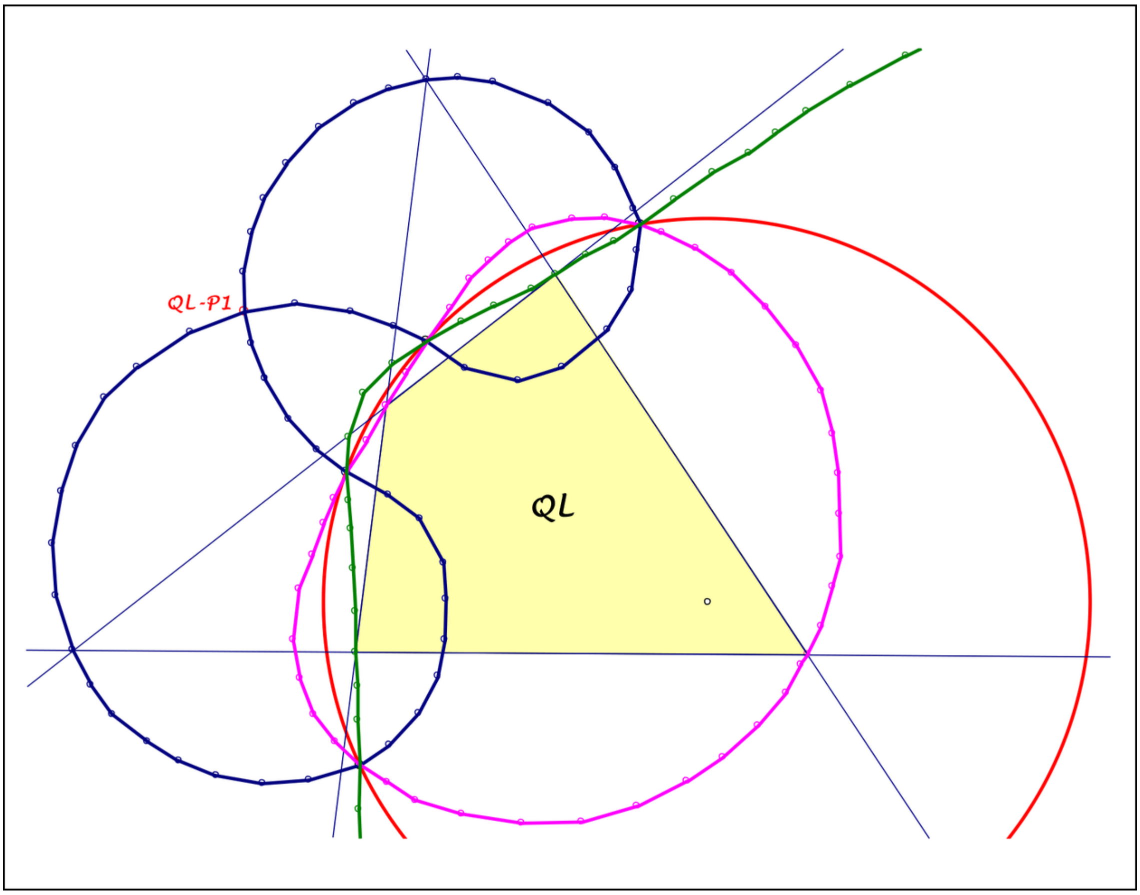
Message: #1074
Date: 2021-07-07
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

Dear Bernard,

a further observation wrt the quartic for pedal QG with incircle:
If we start with a QL, we get wrt the QG-versions of the QL
... three quartics with four concyclic common points.
The pedal QA of these points wrt the QL
... have QG-versions, each with an incircle.
Is there a name for such quadrangles?

Best regards Eckart
PS. Wrt anallagmatic, there is a 2nd inversion circle.



2021-07-07.pdf

Message: #1075
Date: 2021-07-09
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

```
Dear Bernard,

many, many thanks for your helpful remarks and references in
#1073.
First I thought, that "Mathcurve,  Rational Bicircular Quartic"
... would identify the quartic,
... but I tried in vain to confirm the definitions 1) and 3)
... by CABRI-constructions.
I am working on, give me time.

Best regards Eckart
```

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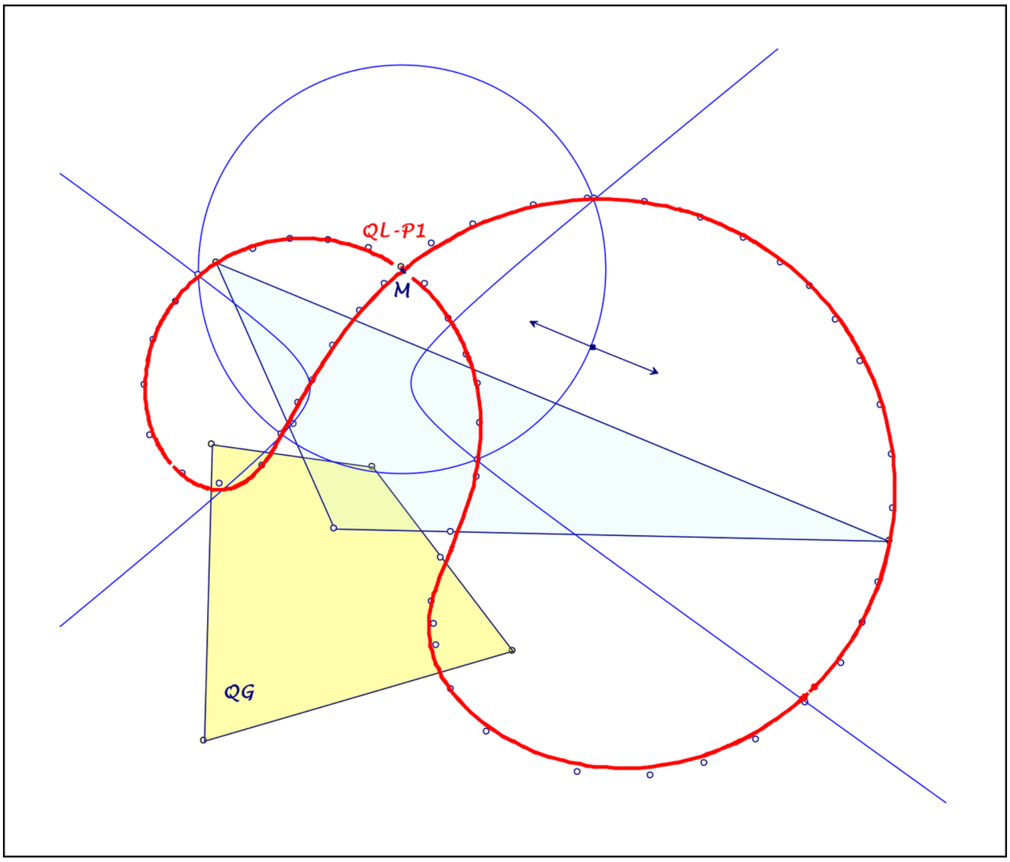
Message: #1076
Date: 2021-07-12
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

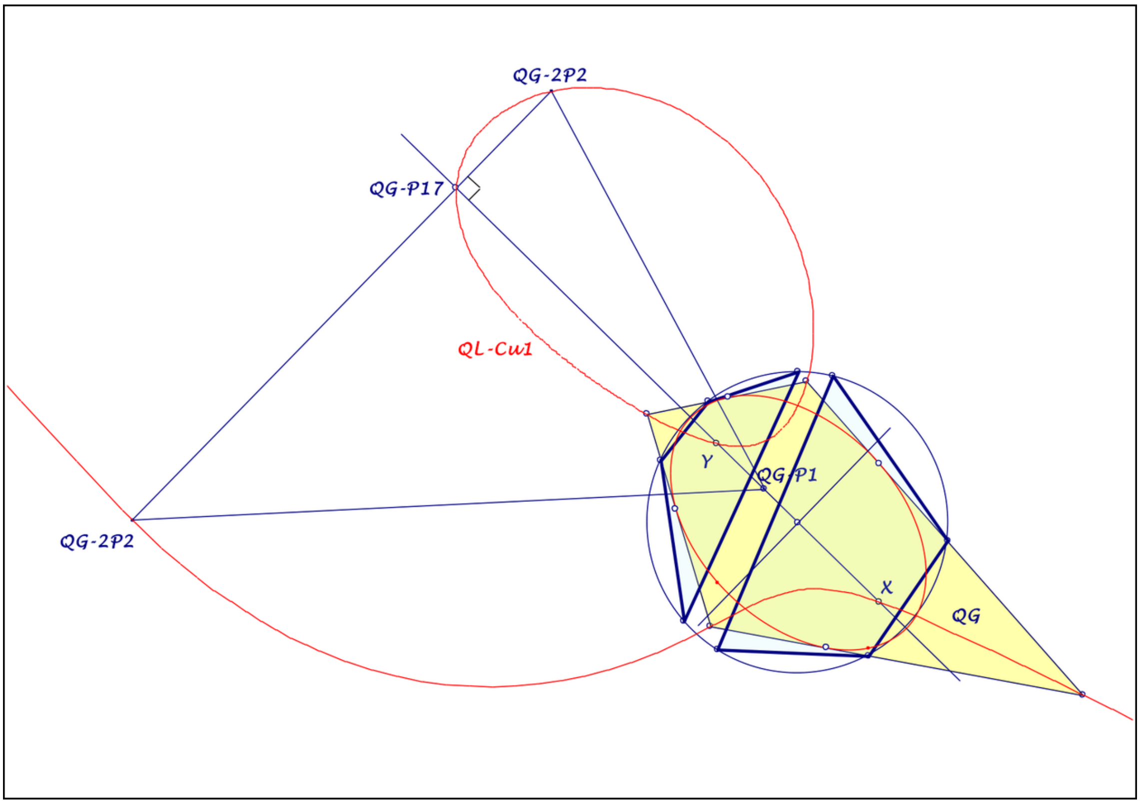
```
Dear Bernhard,

with  "Mathcurve,  Rational Bicircular Quartic" 3) I can make
... a better drawing of the pointwise approximation in #1071,
... but I have problems with the accuracy ...
How to get the attached drawing:
... Start with a QG and 5 points of the quartic
... two will be the intersections QG-2P2 of opposite sides,
... take a circle, centered in a point M with variable radius,
... consider the conic through the inverses of the 5 points,
... the locus of the intersections of the conic and the circle,
... varying the radius, give a quartic,
... changing the center M, you can adapt the quartic.
I am not sure, whether the adapted center M of the circle
... must be the Miquel point, it would be good,
... if we had 5 well known points of the quartic unequal QL-P1.

Best regards Eckart
```



2021-07-12.pdf

Message: #1077
Date: 2021-07-18
From: eckart_schmidt@t-online.de
Subject: Inscribed Quadrigons

```
Dear all,

are you interested in inscribed quadrigons, here some remarks:

Inscribed quadrigons of a reference quadrigon QG can be
... pedal: vertices in pedal points of a point wrt QG,
... cyclic: vertices concyclic,
... cevian: vertices divide the QG-sides
    in ratios with product 1(see #446, #1062),
... contacting: vertices are the contact points
    of an inscibed conic.

Let us start with pedal inscribed quadrigons,
... the locus of points, whose pedal quadrigon is cyclic,
    is QL-Cu1 (see EQF),
... bearing two points X, Y, whose pedal quadrigons
    are cyclic and cevian,
... only real, if QL-Cu1 is bipartite,
... X, Y are CSC-partner on a perpendicular to QG-L1
    through QL-P1,
... both pedal quadrigons are perspective wrt QG-P1
... and have the same circumcircle,
    centered in the midpoint of XY.

If we finally consider an inscribed conic with foci X, Y,
... which has a circumconic through QG-2P2,
... we get a contacting quadrigon, which is cevian,
... but that holds in general independent of X,Y.

Best regards Eckart
```



2021-07-18.pdf

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Message: #1078
Date: 2021-07-18
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
Sorry that I couldn't answer earlier your message 1074, but I wasn't home in the last ten days !
For the pedal QA's of your 4 points, if P1P2 + P3P4 = P1P4 + P2P3 and P1P2 + P3P4 = P1P3 + P2P4, we have also P1P3 + P2P4 = P1P4 + P2P3.
If a point belongs to 2 quartics, it belongs necessary also to the 3rd.
The 4 such pedal QA's have the property that any of the 4 points is the same wrt the triangle of the 4 others.
(We have the same property with orthocentric QA's, any of the 4 points being the orthocenter of the 4 others)
I suppose this point is also a well known ETC point, but I couldn't find which one.
Perhaps you will find it ...
Best regards
Bernard

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Message: #1079
Date: 2021-07-19
From: bernard.keizer@gmail.com
Subject: Re: Inscribed Quadrigons

Dear Eckart,
Do I understand correctly that there are the same way n points intersections between the quartic and QL-Cu1 for which the pedal QA's are at the same time cyclic and having an incircle ?
Best regards
Bernard

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Message: #1080
Date: 2021-07-19
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

```
Dear Bernard,

if we consider a QA with incircles for every QG-version,
... the 4th point is always X(176) of the other three,
... but the four pedal QA in #1074 are not of this type,
... my observation doesn't hold, as new constructions show,
... please excuse ....

Best regards Eckart
```

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Message: #1081
Date: 2021-07-20
From: eckart_schmidt@t-online.de
Subject: Re: Inscribed Quadrigons

```
Dear Bernard,

if you consider the intersections of the quartic and QL-Cu1,
... there are beside QL-P1 and QG-2P2 two further common points,
... whose pedal QG are cyclic with incircle,
... but I cannot find any properties.

Best regards Eckart
```

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Message: #1082
Date: 2021-07-21
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

```
Dear Eckart,
Thanks a lot for this discovering of the point X176 on the line
IGe (I incenter, Ge gergonne point), center of the inner Soddy
circle.
But it seems I was wrong : the pedal QA is necessary convex and
only one of the 4 vertices is X176 of the triangle of the 3
others (the point is inside the triangle of the 3 others).
The property doesn't hold for the 3 other points !
I think the property holds for your 4 pedal QA's in 1074 ?
Best regards
Bernard
PS According to Mathcurve, the inverse of your quartic wrt a
certain circle centered in QL-P1 should be a rectangular
hyperbola ...
```

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Message: #1083
Date: 2021-07-23
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

```
Dear Bernard,

I think, we make triangle geometry:
If a quadrangle has the property,
... that every of its quadrigon versions has an incircle,
... then every QA-vertex is X(176) of the three other
... and the lines X(1).X(7) intersect in X(482).

But the four common points of the three quartics
... for the QG-versions of a quadrilateral
... have not this property, sorry.

So I cannot confirm your properties in #1082.

Best regards Eckart
```

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Message: #1084
Date: 2021-07-23
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
Drawing again carefully point per point your quartic, I find this property :
* for any circle centered in QL-P1, the inverse of any 5 points wrt the circle give an hyperbola and the inverse of this hyperbola is a quartic similar to yours with node in QL-P1, but it is not exactly yours !
* for any other circle centered in a point M, we find the same way an hyperbola and the inverse of the hyperbola gives different types of curves, certain also similar to yours, but all have M as node !
So I give up.
Thanks anyhow for this interesting item, I had a lot of fun ...
Best regards
Bernard

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Message: #1085
Date: 2021-07-24
From: bernard.keizer@gmail.com
Subject: Re: Pedal QG with incircle

Dear Eckart,
I made a lot of other drawings. I added points on lines through QL-P1 or through circles through QL-P1 (only 2 points as QL-P1 is always a dopple point).
I'm convinced that we have to choose any inversion circle centered in QL-P1 (for exemple the inversion circle of the CSC swapping the QG-P2).
The rest depends of the precision of the 3 choosen points (other than the QG-P2).
Beautiful item, indeed ...
Best regards
Bernard

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Message: #1086
Date: 2021-07-24
From: eckart_schmidt@t-online.de
Subject: Re: Pedal QG with incircle

```
Dear Bernard,

thanks for further interest, some remarks:
... our constructions give the same quartic,
... if you change the inversion circle round QL-P1,
... you get always the same quartic,
... perhaps really only the precision is our problem.

Best regards Eckart
```

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Message: #1087
Date: 2021-07-26
From: eckart_schmidt@t-online.de
Subject: Coconic pedal 6P for a QA

```
Dear Bernard, dear Chris,

have we already discussed coconic pedal 6P for a QA?
The locus for points with this property
is a curve SE of degree 7,
... bearing the vertices of QA-Tr1 and QA-Tr2,
... further QA-P4, whose pedal conic bears QA-P2
    and is centered in QA-P6,
... the pedal conics of QA-Tr2-vertices
    degenerate in two lines.
The curve SE is QA-Tf16-invariant!

Best regards Eckart
```

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Message: #1088
Date: 2021-07-26
From: bernard.keizer@gmail.com
Subject: Re: Coconic pedal 6P for a QA

```
My dear Eckart,
We already discussed this septic in detail in messages 1027 to
1043 !
As mentionned in my message 1036,
QA-Tf16 = CICi*CSCi = CSCi*CICi (i = 1 to 3),
with CSC and CIC defined in your old message 503.
SE is invariant in CSC, in CIC and in QA-Tf16.
Best regards
Bernard
PS As Chris never answers, it's not surprising that we sometimes
dig old items ...
```

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Message: #1089
Date: 2021-07-26
From: eckart_schmidt@t-online.de
Subject: Re: Coconic pedal 6P for a QA

```
Dear Bernard,

thanks for clearance, my memory evolves more and more into a
problem.

Best regards Eckart
```

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Message: #1090
Date: 2021-07-28
From: eckart_schmidt@t-online.de
Subject: Just for fun

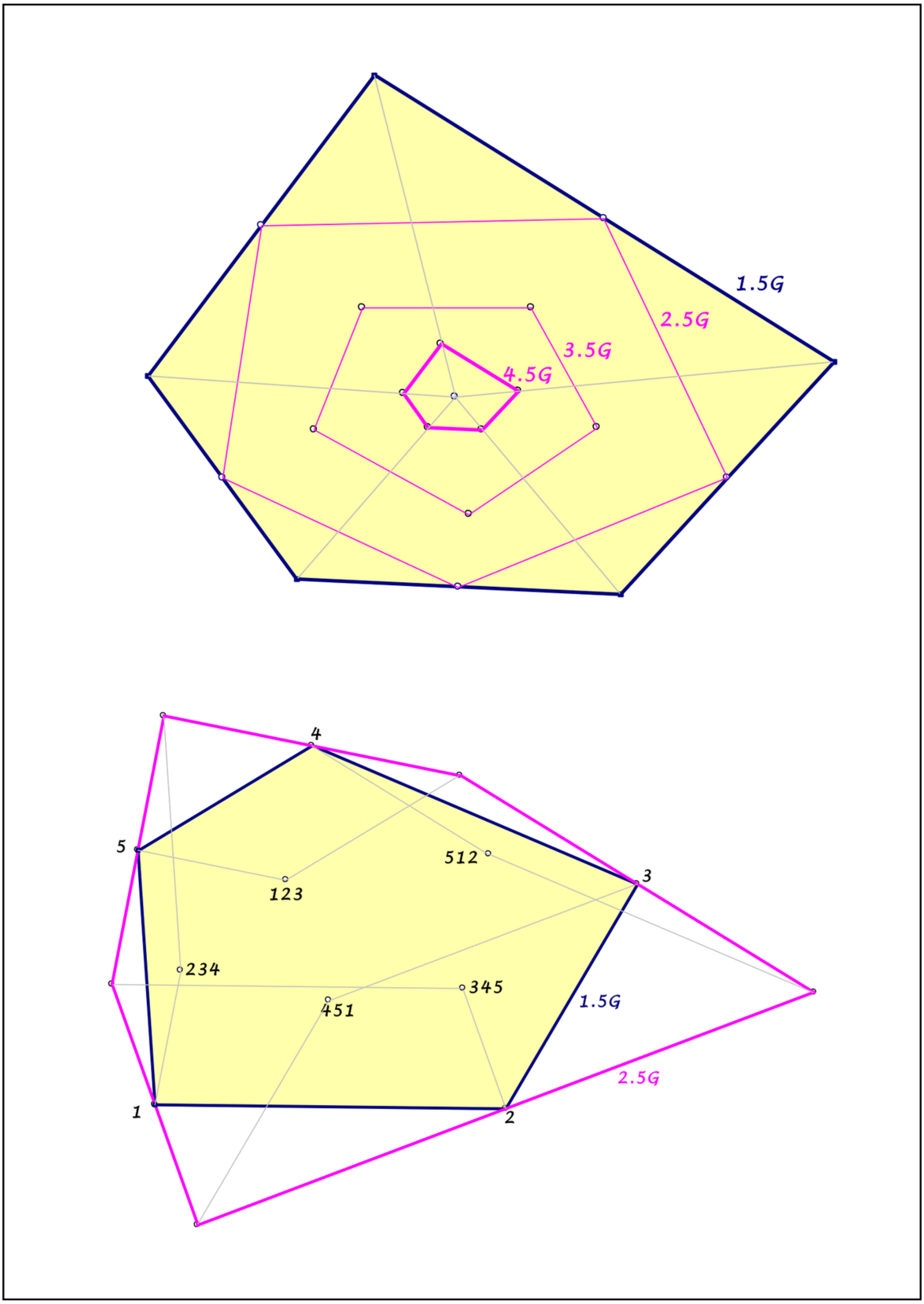
```
Dear all,

here two of a lot of properties out of my Staatsexamensarbeit
... "Baryzentrische  Abbildungen von Polygonen" (Kiel 1966):

Start with a pentagon as reference 5G
... 2nd 5G: midpoints of 2 consecutive points of the 1st 5G,
... 3rd 5G: centroids of 3 consecutive points of the 2nd 5G,
... 4th 5G: parallelogram points of 3 consecutive points
            of the 3rd 5G:
The last 5G is homothetic to the reference 5G, ratio 1/6,
center 5P-s-P1.

Start with a pentagon as reference 5G,
... 2nd 5G: parallelogram points of two consecutive points
... ... and the parallelogram point of the next three points,
... 3rd 5G: midpoints of 2 consecutive points of the 2nd 5G:
The 3nd 5G is the reference 5G.

Best regards Eckart
```



2021-07-28.pdf

Message: #1091
Date: 2021-07-30
From: eckart_schmidt@t-online.de
Subject: QL-Ci2

```
Dear Chris,

if we consider for the QL-inscribed conics the contact QA,
... their QA-P36 lie on QL-Ci2.

Best regards Eckart
```

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Message: #1092
Date: 2021-07-31
From: bernard.keizer@gmail.com
Subject: Re: Just for fun

```
Dear Eckart,
These properties are easy to prove by using the barycentric
definition of your different points (middle, centroïd or
parallelogram points).
Amusing, indeed !
Best regards
Bernard
PS The centroïd of the 5 points is 5P-n-P1 and not 5P-s-P1,
which is the center of the circumconic
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1093
Date: 2021-07-31
From: van10hoven@gmail.com
Subject: Re: QL-Ci2

```
Dear Eckart,

Nice property indeed.

I noticed that the Diagonal Triangle QA-DT of the contact-QA of
all QL-inscribed conics is fixed and coincides with the Diagonal
Triangle QL-DT of the reference-QL. Therefore QA-P36 of the
contact-QA (which lies on the medial circle of QA-DT) will lie
on the medial circle of QL-DT of the reference-QL.

Best regards,

Chris
```

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Message: #1094
Date: 2021-08-01
From: bernard.keizer@gmail.com
Subject: Re: QL-Ci2

```
Dear Chris, dear Eckart,
For the same reason, QA-P2 of all contact-QAs lies on QL-Ci1 !
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #1095
Date: 2021-08-01
From: bernard.keizer@gmail.com
Subject: Re: QL-Ci2

```
This goes naturally also for QA-P30 !
```

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Message: #1096
Date: 2021-08-01
From: eckart_schmidt@t-online.de
Subject: Re: QL-Ci2

```
Dear Bernard,

wrt "QA-P2 of all contact-QAs lies on QL-Ci1":
... this is already in EQF, item QL-Ci1.

Best regards Eckart
```

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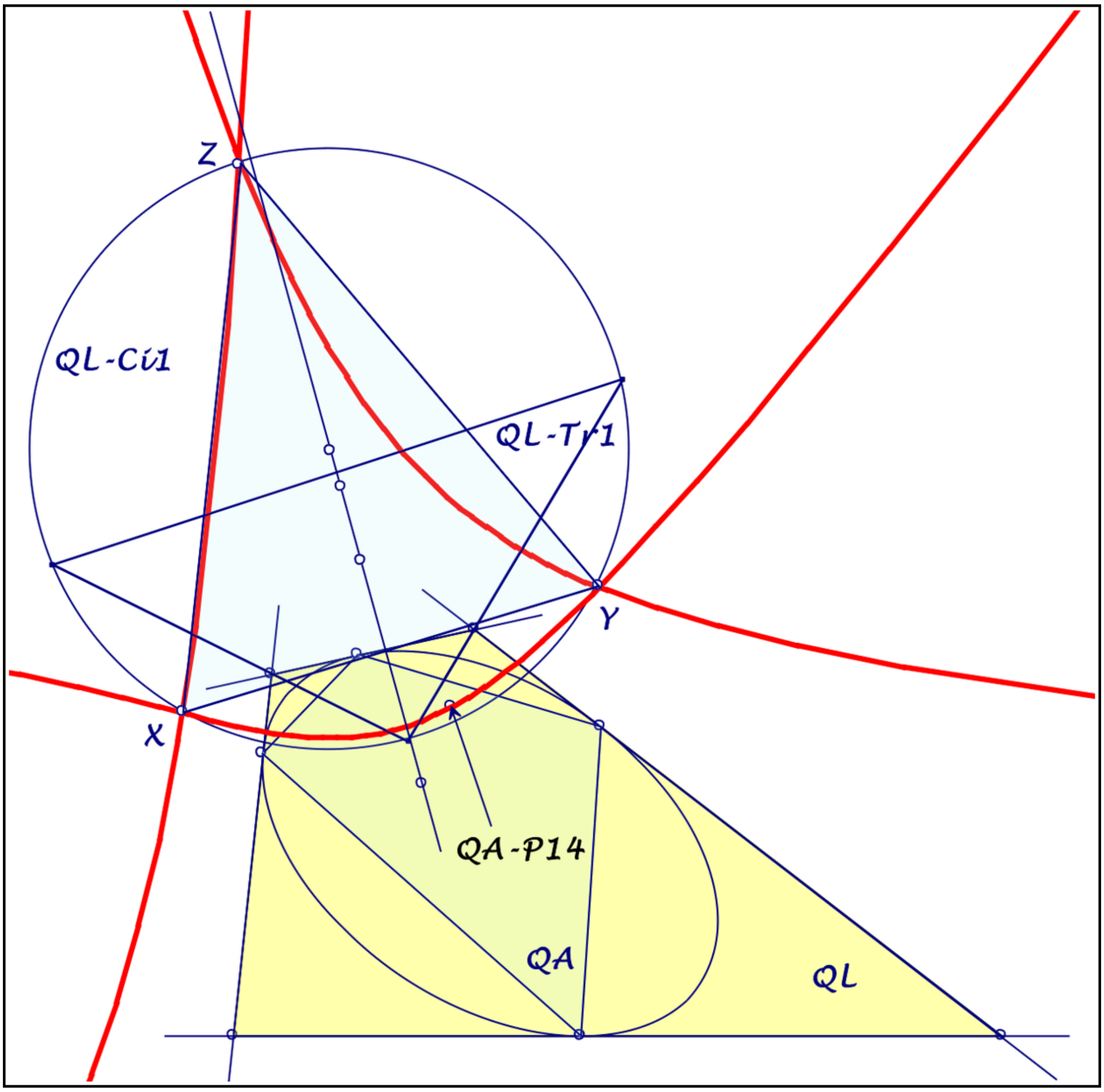
Message: #1097
Date: 2021-08-02
From: eckart_schmidt@t-online.de
Subject: Unknown triple on QL-Ci1

```
Dear all,
what about the three double points X, Y, Z on QL-Ci1
... for the curve of QA-P14 for contact QA of a QL?
The triangle XYZ has the same Euler line as QL-Tr1:
...  same circumcenter,
     centroid of QL-Tr1 becomes orthocenter of XYZ.
Such triangles XYZ can easy be constructed inscribed QL-Ci1,
... but what is the orientation for XYZ?
Best regards Eckart
```



2021-08-02.pdf

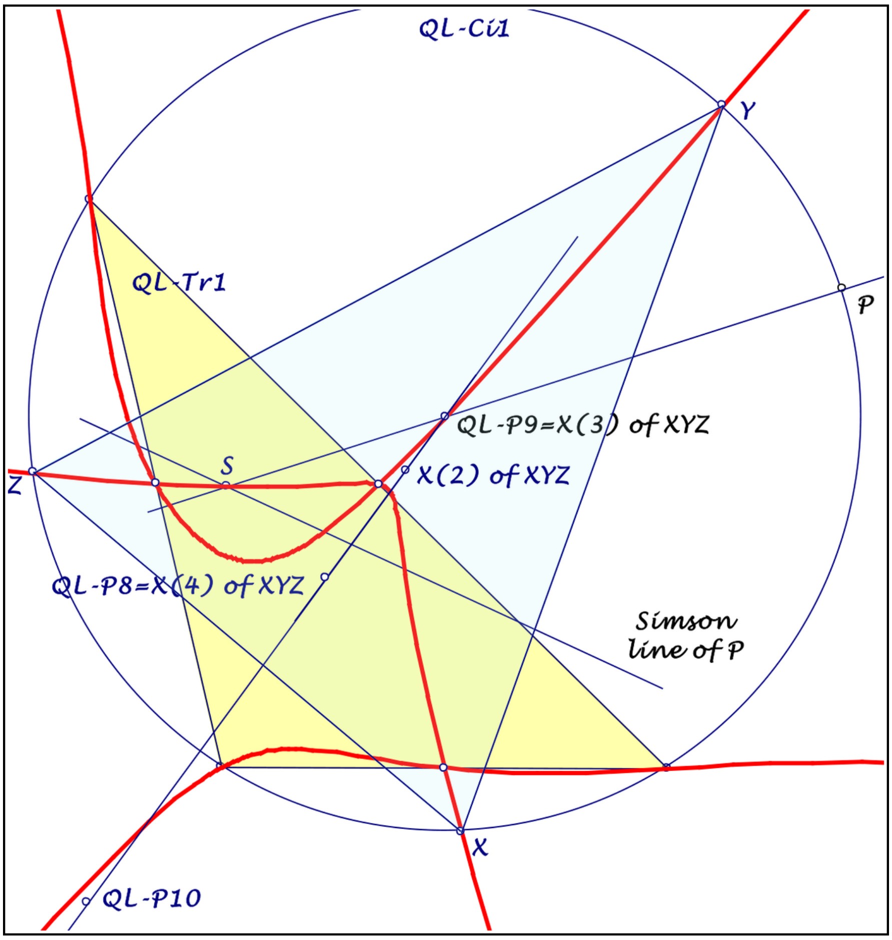
Message: #1098
Date: 2021-08-03
From: eckart_schmidt@t-online.de
Subject: Re: Unknown triple on QL-Ci1

```
Dear all,

in addition to my last message a construction of
... the three double points X, Y, Z on QL-Ci1
... for the curve of QA-P14 for contact QA of a QL.
Consider QL-Ci1 with points P and their Simson lines wrt QL-Tr1,
... intersecting P.QL-P9 (QL-P9 circumcenter of QL-Tr1)
    in points S
... whose locus is a higher curve, circumscribed QL-Tr1,
... bearing the midpoints of the QL-Tr1-sides and QL-P9,
... finally X, Y, Z in further intersections with QL-Ci1.
I hope, someone can confirm this.

Best regards Eckart

PS. What about this curve in triangle geometry?
```



2021-08-04.pdf

Message: #1099
Date: 2021-08-08
From: eckart_schmidt@t-online.de
Subject: QA-P4

Dear Chris,

if I am not wrong the following property of QA-P4
... holds for every point, as well as [1] in old#126:
* Let Nij be the feet of the perpendiculars from QA-P4
  <http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4> to
  Pi.Pj.. Then the 4 versions of circles through Nij, Nik, Nil
  (for all
  combinations (i,j,k,l) ∈(1,2,3,4)) have one common point,
  which is
  QA-P4
  <http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4>
  (Seiichi Kirikami, July 17, 2013). See Ref-
  <http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html>
  34, Quadri-Figures-Group, message # 126.
Best regards Eckart

Message: #1100
Date: 2021-08-09
From: bernard.keizer@gmail.com
Subject: Complex coordinates

Dear Chris, dear Eckart
A friend of mines, with whom I had many exchanges about the QL, has just published in the french mathematical revue Quadrature the 1rst part of an article dedicated to the use of complex coordinates for demonstrating some properties of the QL.
Complex coordinates are particulary adapted in 3 cases :

* general descrition of circles with the origin in QL-P4
* description of the Cl-S and of the Van Rees curve QL-Cu1 with origin in QL-P1
* descrition of the epi- or hypocycloïds with origin in the center of the inner circle

In particular, he took interest in the points QL-P28 and QL-P29 (without knowing them, as he doesn't use a computer ...) and found separately almost all metric properties.
Reading again the description of QL-P28 and QL-P29 in EQF, I suggest that we name the X186 circle the 2nd Miquel circle and the X265 circle the 2nd Steiner circle (the 1rst being in fact the Steiner Line).
More important, I would be pleased that the main property of these points of 2nd generation figur in the description of QL-P28 and QL-P29 : the perpendicular bisectors of the segments X186X265 concur in a point, which is the 2nd Kantor-Hervey point and the center of the 2nd Hervey circle. The property is a generalisation of the Kantor-Hervey theorem.
This goes also for the next Hofstadter points X5961 and 5962 and we would have the 3rd Miquel, Steiner and Hervey circles.
The Hervey circles are the inner circles of hypocycloïds with 3, 5 and 7 cusps, as shown in the figur in the attached file.
Best regards
Bernard

PS Of couse, the article of my friend makes a good publicity for the QL, EQF, the Quadriforum and my own blog ...

QL and hypocycloïds with 2n + 1 cusps

or

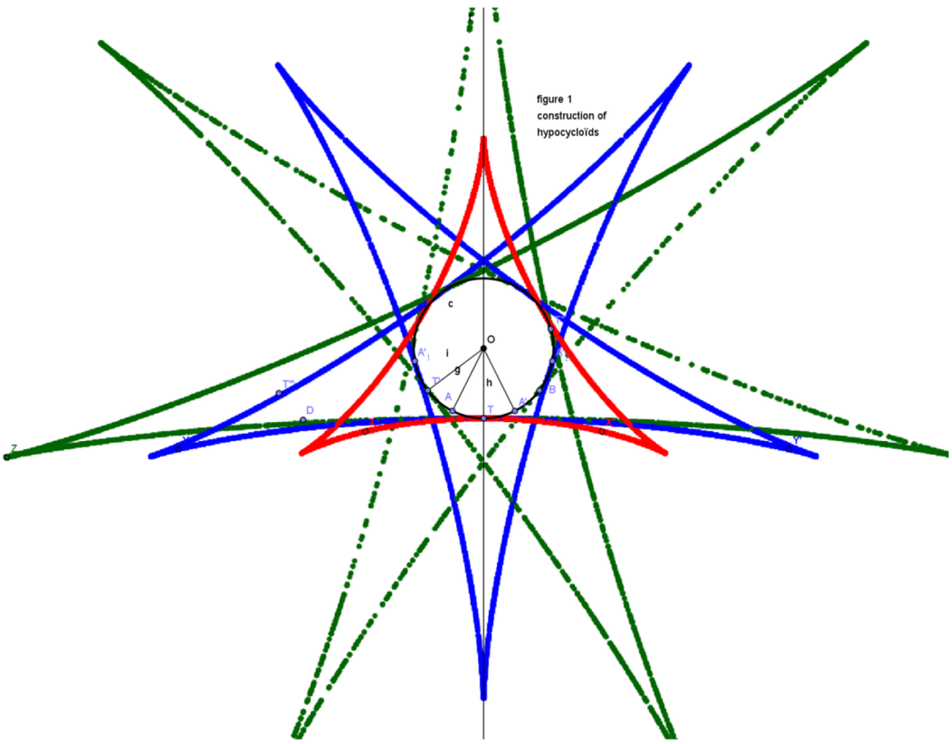
generalization of the Kantor-Hervey theorem

1) Hypocycloïds with 2n + 1 cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio n+1/n (precisely the first point A being the primary point and the other the secondary point B at a speed n+1/n the speed of the first).

The lines through the 2 points envelop a hypocycloid with 2n + 1 cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio n+1/n.(we have TB/n+1 = TA/n = AB)

For example, if n=1, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For n=2 or 3, it's a hypocycloid with 5 or 7 cusps …



QL and hypocyclods.pdf

2) Hypocycloïds with 2n + 1 cusps tangent to 4 lines

Let’s now consider the Hofstadter points for r integer (ETC X3, X4, X186, X265, X5961, X5962 …) ; these points for the reference triangles of the QL derive from the same points for the triangle OjOkOl in the similitude with center QL-P1 which transforms Oi in QL-P4 and the line Li in the Steiner Line QL-L2.
(This remark allows perhaps the calculation of the barycentric coordinates of the points and the demonstration of the following properties)

Following observations can be made :

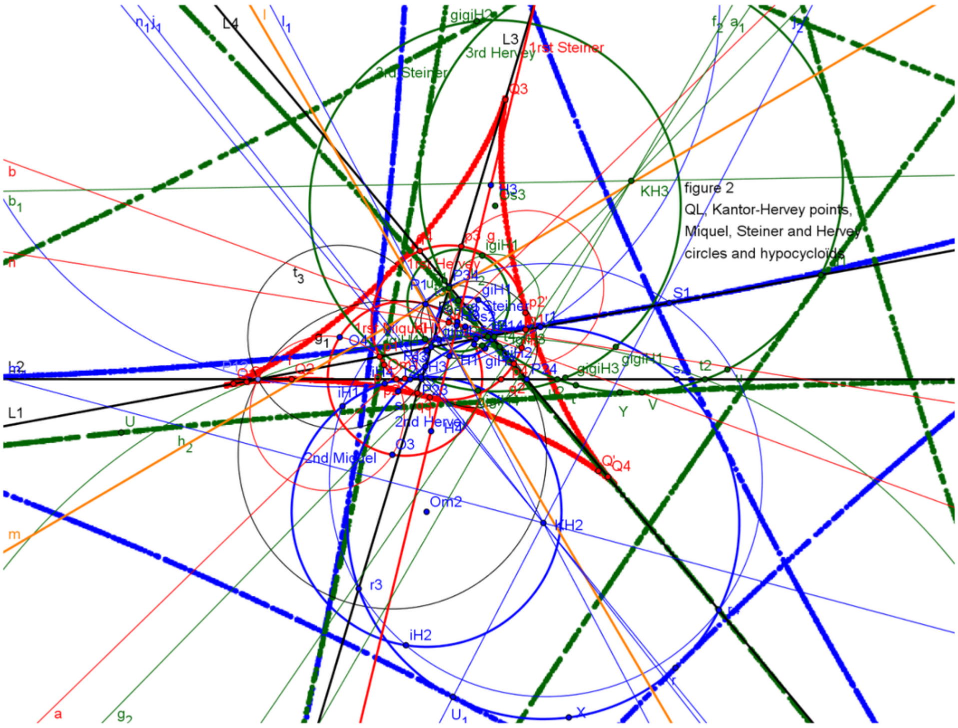
- the same points for the 4 reference triangles are on circles (X3 on the 1rst Miquel circle, X4 on the 1rst Steiner ‘’circle’’, which is the Steiner Line, X186 on the 2^{nd} Miquel circle, X265 on the 2^{nd} Steiner circle …)
- the circles for isogonal points or Miquel and Steiner circles of the same rang are Cl-S partners (property mentioned by Tsihong Lau)
- the perpendicular bisectors of the segments joining 2 isogonal points of rang n in the list of Hofstadter points concur in a point named nth Kantor-Hervey point (QL-P3 for n=1)(property mentioned separately by Eckart and me the same day)
- the circles through each vertice Pkl of the QL (intersection of the lines Lk and Ll) and 2 same Hofstadter points of the reference triangles having Pkl as vertice cut the 2 other lines Li and Lj in 2 points pi and pj, qi and qj, ri and rj, si and sj, ti and tj, ui and uj …(p for X3, q for X4, r for X186, s for X265, t for X5961, u for X5962 …) (construction mentioned by Eckart)
- the 8 points pi and qi are on the 1rst Hervey circle with center QL-P3, the 8 points ri and si are on the 2^{nd} Hervey circle with center the 2^{nd} Kantor-Hervey point, the 8 points ti and ui are on the 3^{rd} Hervey circle with center the 3^{rd} Kantor-Hervey point
- there are on each socalled Hervey circle 2n + 1 points T which divide the 4 arcs of circle in the same ratio n/n+1 :
    - for n = 1 3 points forming an equilateral triangle
    - for n = 2 5 points forming a pentagon
    - for n = 3 7 points forming a heptagon
    - …

QL and hypocyclods.pdf

- The construction in part 1 gives from any of these points a hypocycloïd with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloïds with 2n + 1 cusps and the inner circles of these hypocycloïds are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocyclods.pdf

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Message: #1101
Date: 2021-08-10
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

wrt "2nd Hervey circle":
The bisectors of X3.X4 of QL-triangles intersect in QL-P3 =
Kantor-Hervey point,
... center of the circle QL-Ci4 = Hervey circle,
... which bears X(4) of the triangles for the cyclic QA of the
X3
    of the QL-triangles.
The bisectors of X186.X265 of the QL-triangles intersect in a
new QL-point
... your 2nd Kantor-Hervey point.
The X186 of the triangles for the cyclic QA of the X265 of the
QL-triangles
... and the X265 of the triangles for the cyclic QA of the X186
    of the QL-triangles
... are concyclic on new QL-circles,
    but not with center in your 2nd Kantor-Hervey point.
What is your 2nd Hervey circle?

Is there a misunderstanding on my side?

Best regards Eckart
```

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Message: #1102
Date: 2021-08-10
From: van10hoven@gmail.com
Subject: Re: QA-P4

```
Dear Eckart,
Your transformation is QA-Tf12.
Best regards,
Chris
```

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Message: #1103
Date: 2021-08-10
From: eckart_schmidt@t-online.de
Subject: Re: QA-P4

```
Dear Chris,

for QA-Tf12 you take circles through pedal points Nij,Nik,Njk,
... but Seiichi takes circles through pedal points Nij,Nik,Nil.

Best regards Eckart
```

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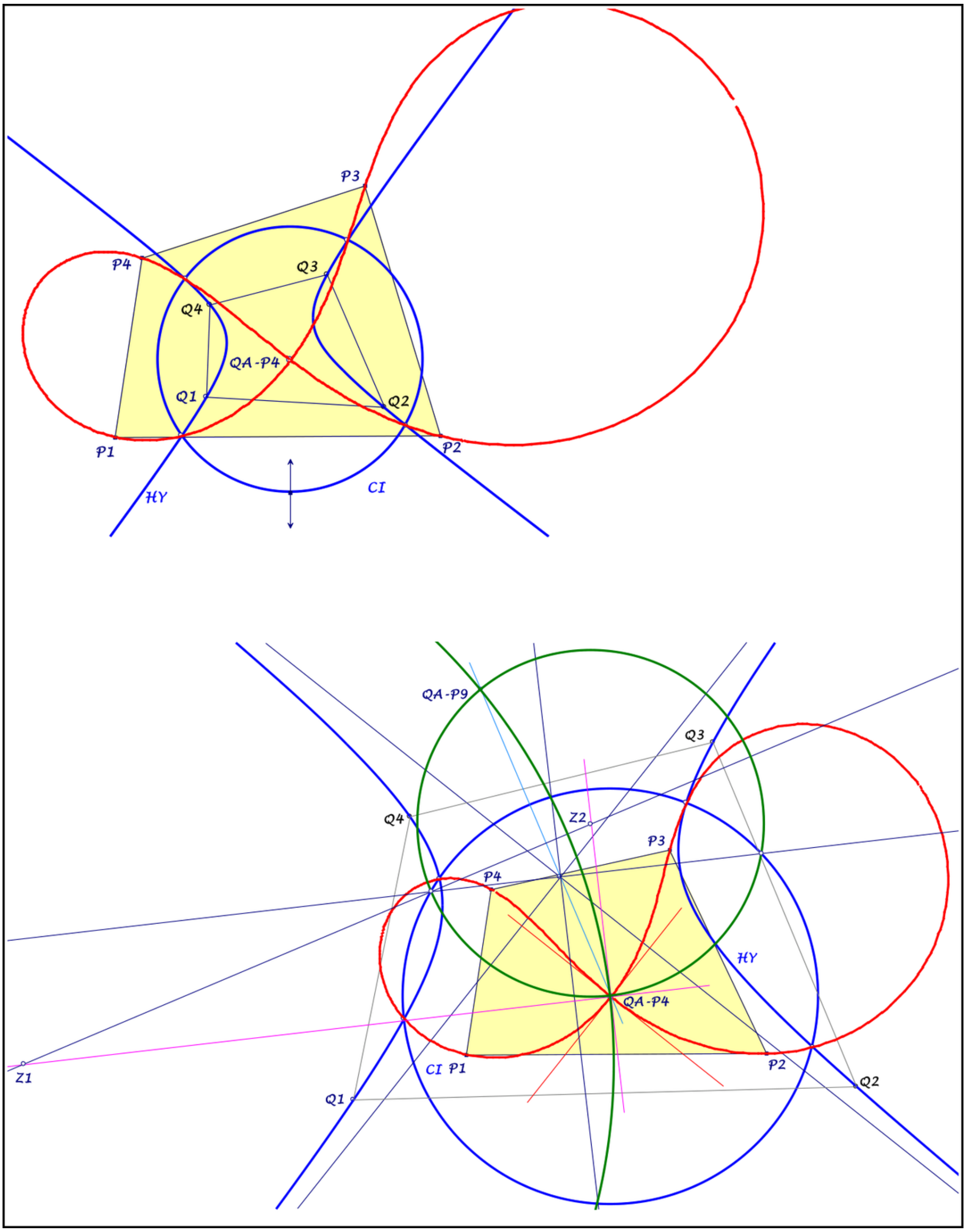
Message: #1104
Date: 2021-08-10
From: eckart_schmidt@t-online.de
Subject: Special QA-quartic

```
Dear all,

may I invite you for a geometric excursion,
... studying a special bicircular quartic,
... see mathcurve.com "Rational Bicircular Quartics" [3].

Let us start with a special point locus:
Consider a quadrangle QA and a point P
... the 5P = QA plus P and the image 5P-s-Tf8(P),
... the locus for P with 5P-s-Tf8(P) on QA-Co2
... is a QA circumscribed quartic, to construct as follows:

Take for the QA  circles CI round QA-P4 with variable radius
... and for the QA-vertices Pi the inverses Qi wrt CI,
... which define an orthogonal hyperbola HY,
... intersecting Ci in points, which give the quartic,
    varying the radius.

Properties:
(1) The quartic has a double point in QA-P4,
... with tangents parallel to the asymptotes of HY.
(2) The inverse of QA-P9 wrt CI is the center of HY,
...  which is QA-P2 of the Q-quadrangle,
... independent of the CI-radius (and vice versa).
(3) The inverse of QA-P3 wrt CI is QA-P41 of the Q-quadrangle,
... independent of the CI-radius (and vice versa).
(4) The quartic is anallagmatic,
... the inversion circles intersect in QA-P4 and QA-P9,
... their centers Z1,Z2 are the intersections of the bisector
    of QA-P4.QA-P9
... and parallels to the axes of Hy through QA-P4,
... or: The axes of HY intersect Ci on the inversion circles.
(5) Lines through QA-P4 intersect the quartic in two other
points,
... whose midpoints lie on a circle,
... which intersects QA-Cu1 in QA-P3, QA-P4 and two further
    points,
... which are the intersections of QA-Cu1 and the quartic
    unequal QA-vertices.

Best regards Eckart
```



2021-08-10.pdf

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Message: #1105
Date: 2021-08-11
From: eckart_schmidt@t-online.de
Subject: QA-Tf16

```
Dear all,

QA-Tf16(P) is 5P-s-Tf8('P) wrt 5P = QA plus P,
... so the quartic in #1104 is the locus of points P
... with QA-Tf16(P) on QA-Co2.

Best regards Eckart
```

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Message: #1106
Date: 2021-08-11
From: bernard.keizer@gmail.com
Subject: Re: Special QA-quartic

```
Dear Eckart,
Very interesting circumQA bicircular quartic !
I checked all your properties without difficulty.
The 2 last points you mention are obviously the contact points
with the 2 other tangents from QA-P4 to the quartic.
One remark : the quartic is independant from the choosen CI and
HY, take any CI, the corresponding Q QA and HY, then the inverse
of HY wrt CI, you get always the same quartic !
You may easely have with the same construction 3 other circumQA
bicircular quartics : the 3 Cl-S transforms of QA-Co2 with nodes
in the 3 Miquel points.
Best regards
Bernard
```

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Message: #1107
Date: 2021-08-11
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
The construction of the 2nd and the other Hervey circles is explained in the points 4 and 5 on the 2nd page of my attached file.
Best regards
Bernard

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Message: #1108
Date: 2021-08-11
From: bernard.keizer@gmail.com
Subject: Re: Special QA-quartic

Dear Eckart,
In fact, for a QA of 4 points Pi, for any point P, draw any circle CI centered in P and take the inverses Qi of the 4 Pi in CI.
Then the inverse of the QA-Co2 of the QA of the Qi wrt CI is a bicircular circumquartic of the Pi with node in P (independant of the choosen CI centered in P).
For 5 points, we have 5 bicircular circumquartics of 4/5 points with node in the 5th.
Best regards
Bernard

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Message: #1109
Date: 2021-08-11
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

following the construction of the Hervey circles
... on page 2 point 4 of your paper, I stopped at
"... the circles through each vertex Pkl of the QL (...)
... and 2 same Hofstadter points of the reference triangles
    having Pkl as vertex
... cut the 2 other lines Li and Lj in 2 points pi and pj ...".
There are 6 of these circles,
... some have no intersections with the other lines Li and Lj,
... and if there are intersections, there are two pi and two pj.
So I cannot realize point 5. Where is my misunderstanding?
Thanks in advance for clearance.

Best regards Eckart
```

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Message: #1110
Date: 2021-08-11
From: eckart_schmidt@t-online.de
Subject: Re: Special QA-quartic

```
Dear Bernard,

just for fun:
Consider a QG and any circle CI round QL-P1,
... further a 2nd QG as CI-inverse of the 1st QG,
... the CI-inverse of any EQF-point X of the 2nd QG
... is CSC(X) wrt X and CSC of the 1st QG.

Best regards Eckart
```

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Message: #1111
Date: 2021-08-11
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
Thanks a lot for your attention and your perseverance !
My humble apologise for this typo !
I hadn't checked this formulation for years ...
The correct sentence would be : circles through Pij and the 2
Hofstadter points Xk and Xl cut the lines Li and Lj in second
points (other than Pij) pi and pj.
the same goes for the isogonal Hofstadter points and gives 2
other points qi and qj on Li and Lj.
Alltogether, we have 8 points (2 on each line) lying on the same
Hervey circle and point 5 is correct and the figure below too.
Best regards
Bernard

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Message: #1112
Date: 2021-08-11
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
There are, as you say, 6 circles for each Hofstadter point.
Each circle carries 2 Hofstadter points X and 2 points p.
There are 4 Hofstadter points and 4 points p, each of these 8
points belonging to 3 circles.
Best regards
Bernard

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Message: #1113
Date: 2021-08-12
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

```
Dear Chris, dear Eckart,
Thanks to Eckart, I realise that my formulation for the
construction of the Hervey circles was wrong !
I've corrected and send you a new attached file ...
My apologise for the inconvenient and my thanks to Eckart.
Best regards
Bernard
```

QL and hypocycloïds with 2n + 1 cusps

or

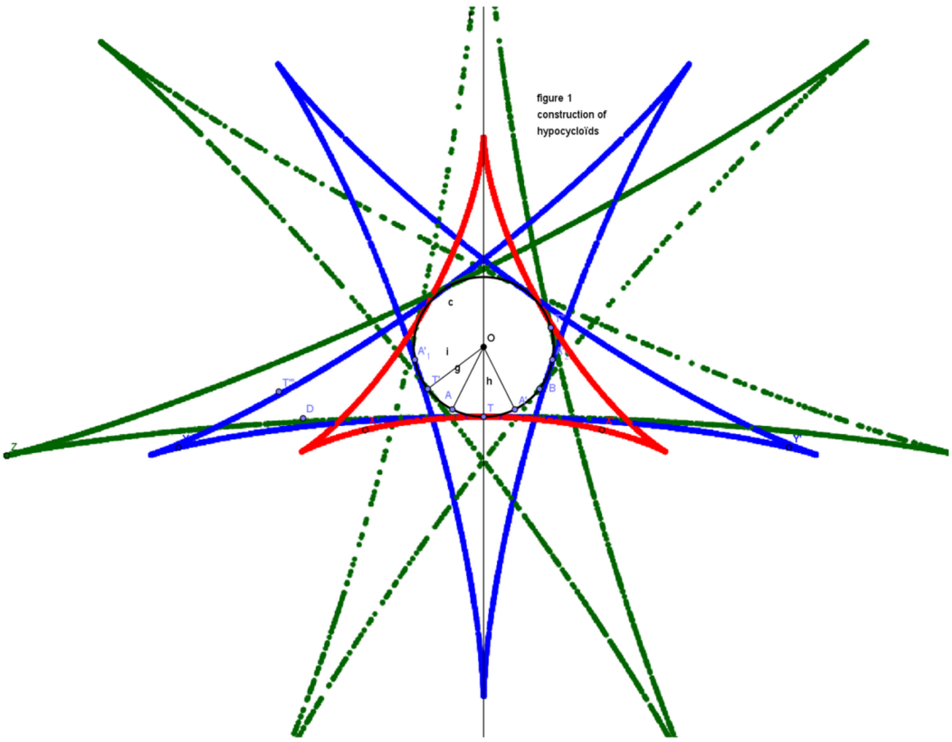
generalization of the Kantor-Hervey theorem

1) Hypocycloïds with 2n + 1 cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio n+1/n (precisely the first point A being the primary point and the other the secondary point B at a speed n+1/n the speed of the first).

The lines through the 2 points envelop a hypocycloid with 2n + 1 cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio n+1/n.(we have TB/n+1 = TA/n = AB)

For example, if n=1, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For n=2 or 3, it's a hypocycloid with 5 or 7 cusps …



QL and hypocyclods.pdf

2) Hypocycloïds with 2n + 1 cusps tangent to 4 lines

Let's now consider the Hofstadter points for r integer (ETC X3, X4, X186, X265, X5961, X5962 …) ; these points for the reference triangles of the QL derive from the same points for the triangle OjOkOl in the similitude with center QL-P1 which transforms Oi in QL-P4 and the line Li in the Steiner Line QL-L2.
(This remark allows perhaps the calculation of the barycentric coordinates of the points and the demonstration of the following properties)

Following observations can be made :

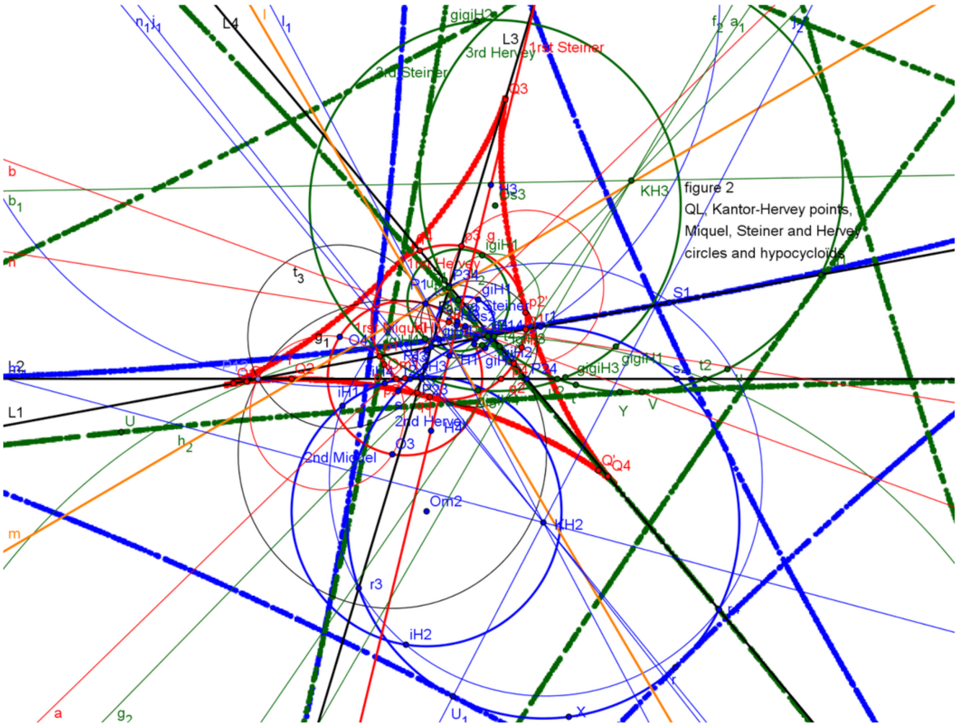
- the same points for the 4 reference triangles are on circles (X3 on the 1rst Miquel circle, X4 on the 1rst Steiner ''circle'', which is the Steiner Line, X186 on the 2nd Miquel circle, X265 on the 2nd Steiner circle …)
- the circles for isogonal points or Miquel and Steiner circles of the same rang are Cl-S partners (property mentioned by Tsihong Lau)
- the perpendicular bisectors of the segments joining 2 isogonal points of rang n in the list of Hofstadter points concur in a point named nth Kantor-Hervey point (QL-P3 for n=1)(property mentioned separately by Eckart and me the same day)
- the circles through each vertice Pij of the QL and 2 same Hofstadter points Xk and Xl of the reference triangles having Pij as vertice cut the 2 lines Li and Lj in 2 points pi and pj, qi and qj, ri and rj, si and sj, ti and tj, ui and uj …(p for X3, q for X4, r for X186, s for X265, t for X5961, u for X5962 …) (construction mentioned by Eckart)
- there are 6 such circles, each circle carrying 2 points Xk and Xl and 2 intersections pi and pj (or qi and qj …). Each of the 4 points X and 4 points p (or q …) belongs to 3 circles. 2 of the 6 circles intersect in one point X and one point p (or q…).
- the 8 points pi and qi are on the 1rst Hervey circle with center QL-P3, the 8 points ri and si are on the 2nd Hervey circle with center the 2nd Kantor-Hervey point, the 8 points ti and ui are on the 3rd Hervey circle with center the 3rd Kantor-Hervey point
- there are on each socalled Hervey circle 2n + 1 points T which divide the 4 arcs of circle in the same ratio n/n+1 :
    - for n = 1 3 points forming an equilateral triangle

- for n = 2 5 points forming a pentagon
- for n = 3 7 points forming a heptagon
- …

- The construction in part 1 gives from any of these points a hypocycloïd with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloïds with 2n + 1 cusps and the inner circles of these hypocycloïds are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocyclods.pdf

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Message: #1114
Date: 2021-08-12
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

thanks for clearance and correction,
really a good generalization!

Best regards Eckart
```

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Message: #1115
Date: 2021-08-12
From: eckart_schmidt@t-online.de
Subject: Fixed points of QA-Tf16

```
Dear Chris, dear Bernard,

QA-Tf16 for a reference QA has four fixed points,
... defining a new quadrangle QA' with vertices on QA-Cu1,
... QA' is four times perspective to QA,
... perspectors are the in- and excenters of QA-Tr2.
QA and QA' have the same QA-Tr2 and QA-Cu1, QA'' = QA.
The CB-point 8P-s-P1 of QA plus QA'
... is the intersection of QA-Cu1 and its asymptote.
There will be more properties.

Best regards Eckart
```

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Message: #1116
Date: 2021-08-12
From: van10hoven@gmail.com
Subject: Re: QA-P4

```
Dear Eckart,

When you take the circles through pedal points Nij,Nik,Nil,
then you have QA-Tfx(P) = P, which is no interesting option.

Chris
```

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Message: #1117
Date: 2021-08-13
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of QA-Tf16

```
Dear Eckart,
The new thing in your message is that the vertices of QA' are
fixed points for QA-Tf16 (and vice-versa).
We discussed already several time this QA' without knowing this
new property.
The vertices of QA' are the isogonals of the vertices of QA wrt
Tr2.
QA, QA' and the QA of the in- and excenters are in a Reye
configuration on QA-Cu1 (the vertices of one of the 3 QAs are
the perspectors of the 2 others).
QA-P3 is the QA-P4 of QA' and the infinity point of the
asymptote is the QA-P4 of the QA of the in- and excenters, the 3
points being therefore aligned.
QA is the tangential QA of QA-P4, QA' is the tangential QA of
QA-P3 and the QA of the in- and excenters is the tangential QA
of the infinity point of the asymptote.
The same way, the 3 QAs formed by Tr1 + QA-P4, DT of QA' + QA-P3
and Tr2 + infinity point are also in a Reye cofiguration !
They are the tangential QAs of QA-P41, QA'-P41 and Q (P41 of the
QA of the in- and excenters), which are therefore aligned) ...
Best regards
Bernard
```

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Message: #1118
Date: 2021-08-13
From: eckart_schmidt@t-online.de
Subject: Re: QA-P4

```
Dear Chris,

your last message describes my first message #1099,
... I think, you should eliminate this property in EQF under
QA-P4.

Best regards Eckart
```

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Message: #1119
Date: 2021-08-13
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of QA-Tf16

```
Dear Bernard,

thanks for recognizing the QA-Tf16-fixed points and further
properties,
... but they are not the QA-Tr2-isogonals of the QA-vertices!

In addition: QA-quartics, defined by a point P:
... Consider a QA and circles CI round P
... with their inverse image QA':
... The intersections of CI and QA'-Cu1
... give a QA-circumscribed quartic,
... bearing the fixed points of QA-Tf16,
... further the defining point P and its QA-Tf16-image,
... for the quartic is QA-Tf16-invariant.

Best regards Eckart
```

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Message: #1120
Date: 2021-08-13
From: bernard.keizer@gmail.com
Subject: Re: Fixed points of QA-Tf16

```
Dear Eckart,
Sorry for the mystake, I reacted only with my memory !
Not QA and QA', but their DT vertices and their QA-P4 are
isogonal wrt Tr2 (as one of the perspectors is the infinity
point of the asymptote).
Best regards
Bernard
```

Message: #1121
Date: 2021-08-14
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

```
Dear Chris, dear Eckart
This time, I checked completely the given construction for an
astroïd tangent to the 4 lines.
Start with the in- and excenters on 8 circles (being at the same
time Miquel and Steiner circles, invariant under Cl-S).
Then, keeping the same 4 points on each circle, consider the
inverses of these points in the circumcircles and their
isogonals, you get 4*4 lines, whic hintersect in 8 points.
These 8 points are on a rectangular hyperbola, each point
belongs to 4 lines and each line carries 2 points.
Each of these points is the center of an astroïd tangent to the
4 lines.
For each point, you get a Miquel circle, a Steiner circle and a
Hervey circle, constructed with the points pi and qi on each
line, exactly as explained in my memo.
Then you have to find to 4 points p0 on the Hervey circle which
divide the arcs of circle piqi in the ratio (-1,+3).
Last, let's a point p describe the Hervey circle and q being the
point of the circle for which  arc(p0q) = -3arc(p0p).
pq is the current tangent to the astroïd and the contact point
is the point Q reflexion in p of the middle m of pq.
The astroïd is the envelop of pq or the locus of Q.
Here an attached file for the astroïd centered in X4.
Best regards
Bernard
```

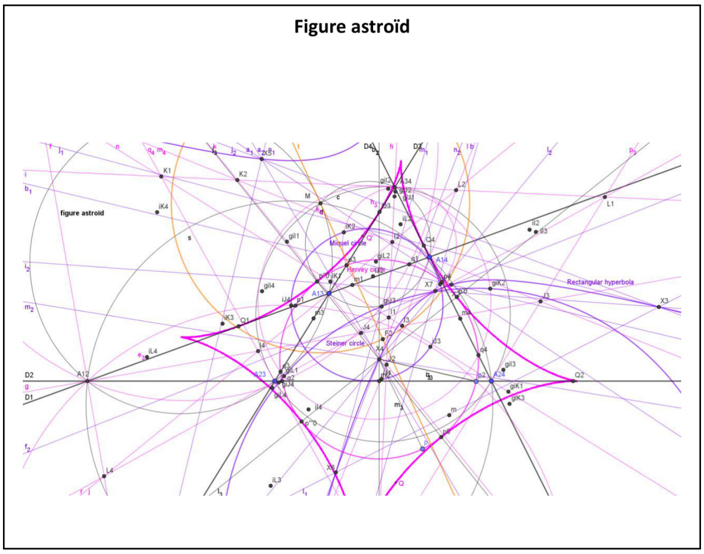

figure astrod.pdf

Message: #1122
Date: 2021-08-15
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

sorry, in vain I try to reproduce your construction of an
astroid.
Can you please explain:
"Then, keeping the same 4 points on each circle,
... consider the inverses of these points in the circumcircles
    and their isogonals,
... you get 4*4 lines, which intersect in 8 points."
What points have to be connected for the 4*4 lines?
Your figur is no help, for I don't understand your nomination.
Is this construction already earlier discussed?

Best regards Eckart
```

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Message: #1123
Date: 2021-08-16
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
Many thanks for your interest !
In fact, you need only the 4 circumcircles (which intersect in QL-P1 = M) and the 16 in- and excenters of the 4 reference triangles (here named I, J, K and L 1 to 4).
The 2 invariant points of the Cl-S QL-2P3a and b = F1 and F2 can help, but are not necessary.
The are 2 ways for finding the 8 Miquel, Steiner and Hervey circles.
1) you draw blind the 16 inverses of the in- and excenters in the respective circumcircles and their isogonals wrt the respective triangles (iX for inverse, giX for isogonal of the inverse).
You get 16 pairs of points and 16 perpendicular bisectors, which intersect in 8 points (2 on each line and each point on 4 lines).
Identify one of the 8 points and it's 4 lines (X4 in my figure).
That gives you the 4 copples of points iJ3, iJ4, iK2 (concyclic on a Miquel circle) and iL1 and giJ3, giJ4, giK2 and giL1 (concyclic on the Steiner circle).
Then draw the circle through A12 and iJ3 and iJ4, you get the primary points p1 and p2 ; the same way, the circle through A34 and iK2 and iL1 gives the primary points p3 and p4.
The circle through A12, giJ3 and giJ4 gives the secondary points q1 and q2 and the circle through A34, giK2 and giL1 gives the secondary points q3 and q4.
The rest is clear, I hope, the 8 points p and q 1 to 4 are on the Hervey circle centered in X4.
2) If you don't want to do it completely blind, first indentify the 8 circles through the in- and excenters : 4 pass through F1 and F2 and 4 are centered on F1F2.
Ech circle carries 4 points and each point belongs to 2 circles (the 8 circles are the Steiner circles).
In my exemple, the points J3, J3, IK2 and iL1 are on the same circle through F1 and F2.
Taking the inverses and their isogonals gives the Miquel and Steiner circle described in point 1) and the 4 perpendicular bisectors through X4.
The difference between the current Hervey circle is that we deal with 4 points instead of one (in- and excenters instead of circumcenter).
My naming is perhaps not the best, if you have a better idea, I would be grateful ...

We discussed these 8 centers of astroïds (on a rectangular hyperbola) many years ago, but not the construction with Hervey circles, as I hadn't found these properties ...
We have the same construction with the 27 centers of cardioïds on 4*9 lines (sides of Morley triangles of the reference triangles), each center on 4 lines and 3 centers on each line.
The 27 centers are on the cubic stelloïd QL-Cu2. The construction of the cardioïds leads to the same kind of Miquel, Steiner and Hervey circles for each center, provided we are ale to identify the corresponding Hofstadter points Hr (r = 3/2 and -1/2 for the 8 astroïds, 1/3 and 2/3 for the 27 cardioïds, 1/4 and 3/4 for the 64 nephroïds ...)
I hope it will be a little clearer now, but don't hesitate if it i not, I find this item fascinating !
Best regards
Bernard

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Message: #1124
Date: 2021-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

Dear Bernard,

thanks a lot for your detailed explications,
... now I can construct the 8 centers of the astroids,
... in your short version the construction of the bisectors
    failed.
Give me time for the rest.

Best regards Eckart

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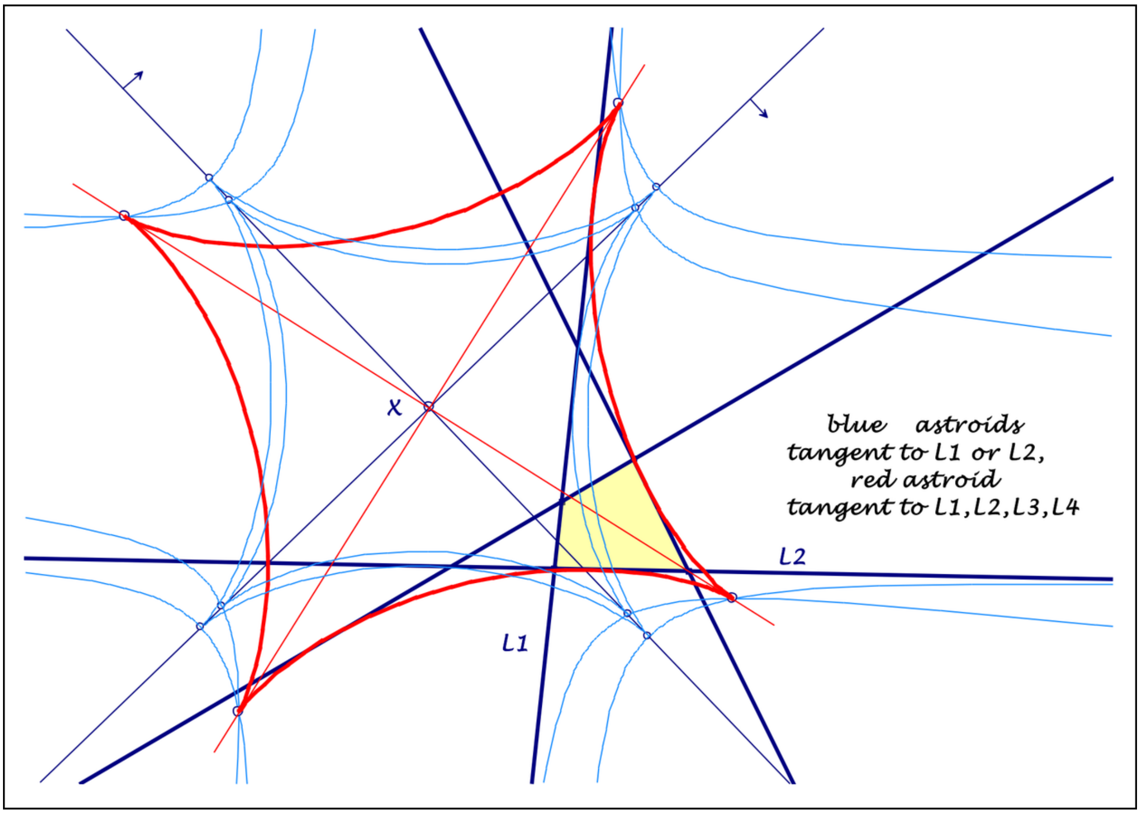
Message: #1125
Date: 2021-08-17
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

once more I lost control in your description, but I found
another
construction
... of the astroids tangent to the QL-lines,
    possible with CABRI,
... starting with your construction of the centers:

Consider for a QL the four trilaterals
... and for each in- and excenter the bisector
... of its isogonal conjugate and its inverse
    wrt the circumcircle,
... you get 4*4 lines, which have 8 intersections of four lines,
... which will be the centers of the astroids tangent
    to the four QL-lines.

Let X be the center of an astroid
... with any two orthogonal axes through X,
... each QL-line defines as tangent an astroid,
... whose cusps describe quartics, varying the orthogonal axes,
... the common points of these 4 quartics are the cusps
    of the X-astroid.

I shall try furthermore to reproduce your construction.

Best regards Eckart
```



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Message: #1126
Date: 2021-08-17
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
For the time I feel a little bit confused !
Reading again old messages in my personal papers, I found that we had already discussed many of these beautiful curves in the messages 2280 to 2300.
It seems after all that my memory is not in a better form than yours !
You already drawn at that time astroïds , cardioïds, nephhroïds and diverse epi- and hypocycloïds ...
You found that the rectangular hyperbola of the centers was a polar conic of the cubic stelloïd.
Best regards
Bernard

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Message: #1127
Date: 2021-08-18
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

Dear Bernard,

we didn't realize a reproduction of the old QFG-messages 2280 - 2300 in 2017
... under the item
    "QL, epi and hypocycloïds and n-angle centers"
... and Chris doesn't react ...
But my construction in #1125 seems new.
Have you a complete old message #2301,
... I only get a damaged form?

Best regards Eckart

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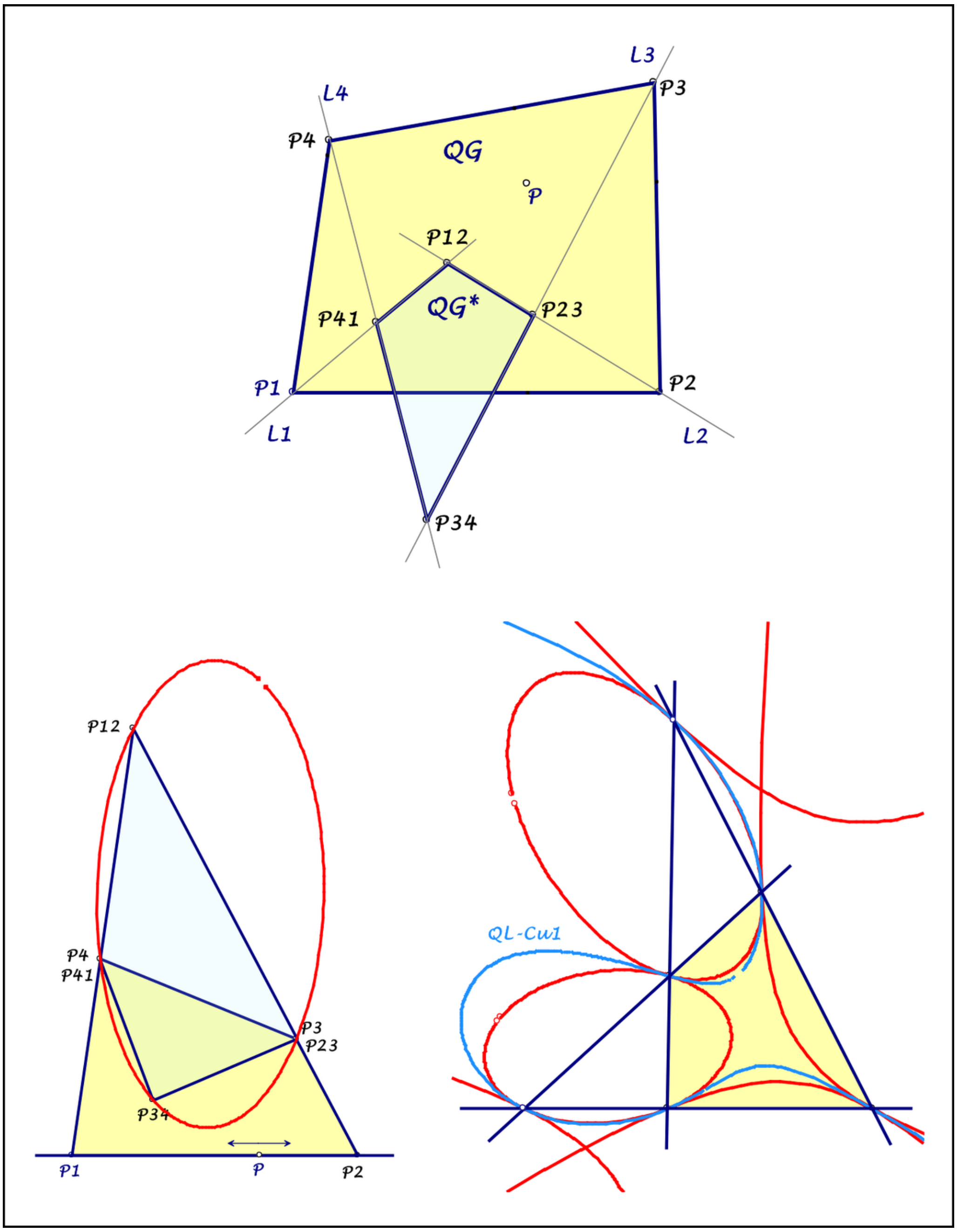
Message: #1128
Date: 2021-08-18
From: eckart_schmidt@t-online.de
Subject: Isogonal conjugate QG of a point

```
Dear all,

this is a resume of the next to last passage 4.3 of
...  Forum Geometricorum, Volume 12 (2012) 161-168,
     (see EQF Ref [27]),
... where an isogonal conjugate QG for a point P is defined as
follows:
Let QG = P1P2P3P4 be a quadrigon and P any point,
... let Li be the reflection of the line PPi in the
    angle bisector at Pi,
... let Pij be the intersection of Li and Lj (see attached file)
... then QG* = P12P23P34P41 is defined as isogonal conjugate
    of P wrt P1P2P3P4.
The authors use the points W = QA-P4 and S = QL-P1,
... but not the right way of quadrigon and quadrilateral.

Properties:
(1) The sides of QG* and the sides of the pedal quadrigon of P
... are perpendicular to each other.
(2) QG* of QL-P1 degenerates collinear
...  in the intersections of QL-L3 and the QG-sides.
(3) QG* of QA-P4 is a parallelogram
... centered in QA-P2 with diagonals through QG-2P2.
(4) QG* for a point P on QL-Cu1 degenerates in CSC(P).
(5) CB-point 8P-s-P1 of QG plus QG* is QG-P1.
(6) QG* of P for the three QG of a QL
... give always the same four points.
(7) Let P be a point on Pi.Pi+1,
... the locus of Pi+2,i+3 is a conic through Pi+2, Pi+3
    and one QG-2P2,
... these four conics contact in the QG-vertices and QG-2P2
... with common tangents at QL-Cu1 (see attached file).

Best regards Eckart
```



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Message: #1129
Date: 2021-08-20
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
I think the whole construction can now be resumed in 3 steps (only the last is new) :
1) For a triangle, the n-sectrices in 2 vertices intersect in n*n points
2) For any point P in the plane, you may associate a point I, 2nd intersection of the circle through 2 vertices and P and the line through the 3rd vertice and P.
gI being the isogonal of I wrt the triangle, you may associate directly the perpendicular bisector of IgI to P. (This construction is fortunately independant of the choice of the 2 vertices).
For the n*n points, you get n*n lines, which intersect in n*n*n points, a point being on 4 lines (one of each triangle) and a line carrying n points.
(That' why you have 8 centers for the astroïds, 27 centers for the cardioïds and 64 points for the nephroïds).
It's easy to find for the centers of the astroïds a rectangular hyperbola, polar conic of the cubic stelloïd, and for the centers of the cardioïds the cubic stelloïd QL-Cu1.
But unfortunately, despite all my efforts, I wasn't able to find a quartic through the 64 points ...
3) Having the center X of an epi- or hypocycloïd tangent to the 4 lines, you have to identify the 4 lines intersecting in this point and the 4 n-angle centers and their 4 isogonals (one for each triangle). Naming these 4 points Ii and gIi, i = 1 to 4, the circles through Aij and Ik and Il cut the lines Li and Lj in a 2nd point (other than Aij), which are the primary points pi and pj. The same way, the circles through Aij and gIk and gIl cut the lines Li and Lj in a 2nd point (other than Aij), which are the secundary points qi and qj.
The 4 points Ii, i = 1 to 4, are on the Miquel circle, the 4 points gIi, i = 1 to 4, are on the Steiner circle, both circles being Cl-S partners.
The 8 points pi and qi, i = 1 to 4, are on the Hervey circle, centered in the point X and the axes of the epi- or hypocycloïd cut the 4 angles piXqi in the same ratio, given by the curve.
Please just tell me if we agree on this basis !
(Your construction in 1125 is clever, but why don't you take directly my general construction ?)
Best regards
Bernard

```
PS The message 2301 is one of yours, I find it undamaged in the
old forum by searching message: 2301
```

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Message: #1130
Date: 2021-08-20
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

```
Dear Eckart,
We deal often with triangle properties !
The transformation I mention associates I and gI (and their
bisector) to P.
Consider the Moebius transformation centered in a vertice and
swapping the 2 other.
Then gI is the transform of P in this Moebius transformation and
I the transform of gP.
Best regards
Bernard
```

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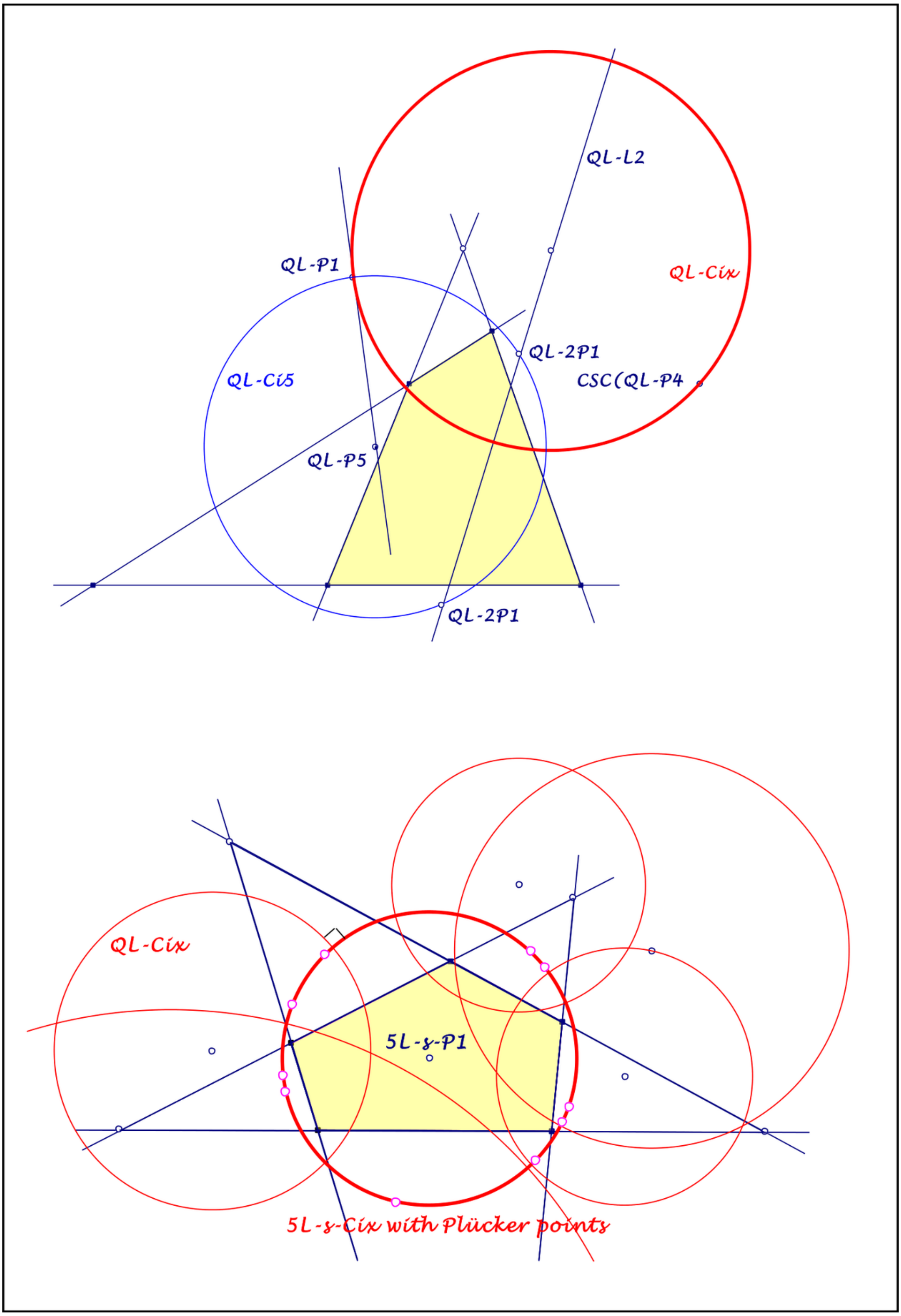
Message: #1131
Date: 2021-08-20
From: eckart_schmidt@t-online.de
Subject: 5L-circle with 10 Plücker points

```
Dear all,

a new QL-circle QL-Cix leads to a 5L-s-Cix, 6L-s-Cox, 7L-s-3Px:

Let us start with a new QL-circle QL-Cix,
... centered on QL-L2,
... bearing QL-P1 and CSC(QL-P4),
... tangent to QL-P1.QL-P5.
Some Properties:
(1) QL-Cix is orthogonal QL-Ci5,.
(2) QL-Cix is orthogonal to circles with a QL-diagonal
    as diameter.
(3) QL-Cix is orthogonal to all circles through the Plücker
points QL-2P1
... with a common point for the radical axes in the inverse of
    QL-P7 wrt QL-Cix.
(4) The Plücker points QL-2P1 are invers wrt QL-Cix.
(5) Radical axis of QL-Cix and QL-Ci1 is QL-L1.

A 5L has five QL-components with their QL-Cix,
... whose radical axes have a common point in 5L-s-P1,
... which is the center of a circle 5L-s-Cix orthogonal
    to the four QL-Cix,
... intersecting the axes of the conic 5L-s-Co1 in points,
... whose connections are tangent to 5L-s-Co1,
... 5L-s-Cix bears the 10 Plücker points QL-2P1
    of the five QL-components,
... which must not all be real.

For a 6L we get six 5L-s-Cix with centers on a conic 6L-s-Cox,
... centered in 6L-s-P1.
For a 7L we get seven conics 6L-s-Cox with three common points
7P-s-3Px
... with a circumcircle centered in 7P-n-P5.

Best regards Eckart

PS. DT-equation of QL-Cix can be given.
```



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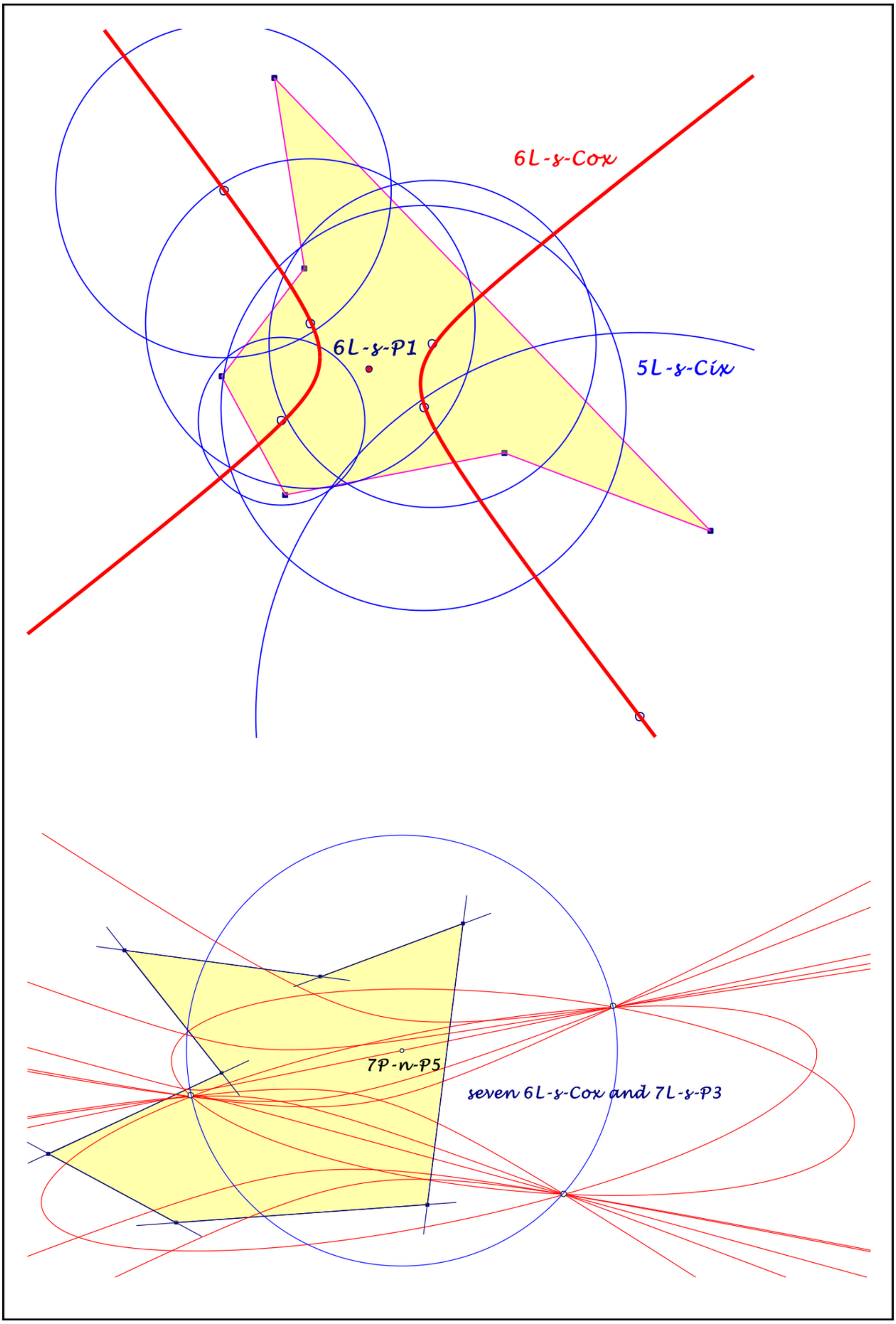

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Message: #1132
Date: 2021-08-20
From: eckart_schmidt@t-online.de
Subject: Re: 5L-circle with 10 Plücker points

```
Dear all,

please excuse,
... I just notice that there is a typo in the last attachment:
    replace 7P by 7L.
So I proved not the right items in EPG,
... the curve 6L-s-Cox is 6L-s-Co1 and 7L-s-3Px is 7P-s-3P1,
... already in EPG!

Best regards Eckart
```

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Message: #1133
Date: 2021-08-21
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

thanks for your numerous detailed explications,
... please excuse my lack of understanding, I give up!
I tried in vain to understand your construction  in #1129:
... wrt 1) : What are n-sectrices in two vertices?
... What are the n*n points?
... wrt 2) I cannot confirm, that
... "the construction is fortunately independent of the choice
    of the two vertices".
... In 2014 old#2305 I had also problems with this matter.

Best regards Eckart
```

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Message: #1134
Date: 2021-08-21
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
This time I'm very disappointed, as it took me time and energy
in order to simplify the presentation !
I'm ready to continue the discussion, as I do not completely
abandon the idea of convincing you.
But I must understand where you block.
1) point 1 (see earlier messages about n-angle centers !)
Take a triangle ABC.
The n-sectrices in a vertice B are the lines dividing the
oriented angle CBA in n parts m° pi/n.
Do the same for C with angle BCA.
You may have bisectrices, trisectrices, quadrisectrices,
n-sectrices.
The n*n points Pi are the intersections of the n lines in B with
the n lines in C.
2) You're right, the points I and gI are not the same for the 3
combinations of 2 vertices, but globally, it gives the same
group of points in a different order and that's which counts.
Again, for any point P, gI and I are the transforms of P and gP
in the Moebius transformations centered in a vertice and
swapping the 2 others.
The bisectrices give the 4 in- and excenters (which are at the
same time their isogonals), the next step in the chain
inverse/isogonal being the 2*2*2 = 8 centers of the astroïds.
The trisectrices give the 3*3*3 = 27 centers of the cardioïds.
The quadrisectrices give the 4*4*4 = 64 centers of the
nephroïds.
So far for today !
Best regards
Bernard
My apologise that I'm not as good in pedagogy as you are
yourself, but I wasn't a teacher in my activity ...

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Message: #1135
Date: 2021-08-21
From: bernard.keizer@gmail.com
Subject: Re: 5L-circle with 10 Plücker points

```
Dear Eckart,
At last a simple, beautiful and totally new property !
Congratulations.
First, I checked the property and I began, as usual, to draw
figures with the QL-P1,4,5,6 and even 17.
(The Cl-Sdiag which swaps the DT circumcircle and the Newton
Line swaps also the Plücker points).
Then I received your last message 1133 and I feel a little bit
discouraged.
So I give up for the moment.
Best regards
Bernard
```

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Message: #1136
Date: 2021-08-21
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

```
Dear Bernard,

your patient seems endless, but my questions remain:
I try to understand the construction for astroids in #1129,
... no problems with the centers X of the astroids,
... so starting with 3):
Why identifying the 4 lines? They give X, needed no more.
... n-angle centers are X3, their isogonals X4,
... leading to the Hervey circle, but not centered in X.
Or are the points I in 3) the points I in 2), replacing P by X?
Then they are not unique.
What do you mean with
    "...but globally, it gives the same group of points ..."?
Excuse once more my inability.

Best regards Eckart
```

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Message: #1137
Date: 2021-08-23
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
Background of all this is that the n-angle centers are the n*n*n points from which you see the 3 sides of a triangle under angles 1/n * the angles in the vertices.
That explains the number of solutions, as the angles are given m° 2π/n.
The n*n*n lines constructed as bisectors of such a n-angle and i's isogonal conjugate form the locus of the centers of the epi- or hypocycloïds tangent to the 3 sides of a triangle.
Hence for a QL the intersections of 4 lines, one in each triangle.
My construction in 3 steps again :
1) n*n*n intersections P of the n-sectrices in 2 vertices of each triangle.
2) 2nd intersections I of the line through the 3rd vertice and P with the circle through the 2 vertices and P.
(If you choose 2 other vertices, you get n*n*n points Q and *the same points I in a different order*).
Take the simple example of the bisectors : starting with 2 vertices give the incenter and an excenter, taking 2 other vertices gives also the incenter and another excenter and globally you have always the 4 same points incenter and excenters. The same goes for all n-centers, which are globally the same group of points !
Then gI (isogonal of I wrt the triangle) and the 4 n*n perpendicular bisectors of the segments IgI, intersecting in n*n*n points, each point on 4 lines and each line carrying n points.
Hence the n*n*n centers of the epi- or hypocycloïds.
3) The points I in 3) are the points I from 2), each point I corresponds to a point P.
They are not X3 or X3 from ETC and not derived from the n*n*n points X (what a strange idea ?)
My construction for example for the deltoïds neds the 4 circumcenters Oi and the 4 orthocenters Hi.
The 4 Oi are cocyclic on the Miquel circle, the 4 Hi are ''cocyclic'' on the Steiner Line (which is a circle), both circles are CSC partners.
We have only one perpendicular bisector of OiHi in each triangle, intersecting in the Kantor-Hervey point, center of the deltoïd tangent to the 4 lines.

The consider the 6 circles AijOkOl, the intersect 2 by 2 in the
Oi and in the points pi, 2nd intersections (other than Aij) with
the lines Li.
The same way, the 6 circles AijHjHk intersect 2 by 2 in the
points Hi and in the points qi, 2nd intersections (other than
Aij) with he lines Li.
The pi are the primary points, the qi the secundary points and
the 8 points pi and qi lie on the same cicle centered in the
Kantor-Hervey point, the Hervey circle.
*This construction is absolutely general, provided you are able
to recognise the 4 n-angle centers and their isogonals
corresponding to the 4 lines (one in each triangle) intersecting
in one of the n*n*n centers of the epi- or hypocycloïds.*
I hope we are now not too far of understanding eachother ...
Best regards
Bernard
I wonder how you were able a few years ago do draw many of these
curves, does that mean that you had another construction of the
n-angle centers ?

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Message: #1138
Date: 2021-08-23
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

Dear Bernard,

what is the "... same ratio, given by the curve ..."
... in the last part of #1129?

Best regards Eckart

PS. I just see your last message, not yet studied, thanks.

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Message: #1139
Date: 2021-08-23
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
My answer went lost, I repeat it shortly !
The ratio p/q defines precisely all these the curves in the tangential definition.
p anq are integers mutually prime (epicycloïds if of the same sign, hypocycloïds if opposite sign).
Examples : deltoïd (-1,2) , cardioïd (1, 2) , astroïd (-1,3) , nephroïd (1, 3) ...
Best regards
Bernard

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Message: #1140
Date: 2021-08-24
From: eckart_schmidt@t-online.de
Subject: Re: Complex coordinates

Dear Bernard,

can it be, that you use another definition of the n-angle center ... as Chris in old#1872?

Best regards Eckart

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Message: #1141
Date: 2021-08-24
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
Chris mentionned in 1872 n-angle centers for n integer! (In this case, it is the same as the Hofstadter points).
But we have also r-angle centers for r rational or even for r real ! (In this case, as I put already, Hofstadter points are misleading).
Just read the messages after 1872 (part are your own messages!)
(p,q) epi- or hypocycloïds need the p/p+q angle center and it's isogonal, which is q/p+q angle center.
All this stuff has been discussed several times on the forum ...
Best regards
Bernard
PS You didn't answer my question in 1137 : how did you draw astroïds or nephroïds without using n-angle centers ?

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Message: #1142
Date: 2021-08-25
From: bernard.keizer@gmail.com
Subject: Re: Complex coordinates

Dear Eckart,
I've found an old attached file I had put on the forum years ago.
I should rewrite it completely with the whole construction of the n-angle centers and of the Miquel, Steiner and Hervey circles for each center of an E or an H.
I should also add many examples of drawings ...
But all this takes much time and as I am with my 2 young grandchildren (6 and 4 years old), I haven't got enough time !
Are you now totally convinced or do you have always questions ?
I would be so happy to read soon a positive message from you ...
In the mean time, I send you this old file unchanged.
Best regards
Bernard

QL, epi- and hypocycloïds and n-angle centers

1) Epi- and hypocycloïds E and H

Let's define an epicycloïd E or hypocycloïd H as the envelop of lines MN, the 2 points M and N describing a circle with center O at different speeds p and q. Let's choose p and q integers and mutually prime, same sign for epicycloïds, opposite sign for hypocycloïds with $p > 0$ and $p > |q|$.

2) Epi- or hypocycloïds tangent to the 4 lines of a QL

- There are $(p+q)^3$ such curves
- Their centers (centers of the inner circle) are at the intersection of 4 sets of $(p+q)^2$ lines for each reference triangle (p + q centers on each line and each center on 4 lines, one of each set)
- These lines are the perpendicular bisectors of copples of isogonal conjugate points wrt the reference triangles from which the sides of the triangle are seen under angles proportional to the opposite angles, the ratio being $p/(p + q)$ and $q/(p + q)$
- These points are the socalled n-angle centers (*) and can be obtained in chains by alterning inversion and isogonality wrt the reference triangle (like Hofstadter, for a point Xr, the inverse is X2-r and the isogonal X1-r)
- There are $(p + q)^2$ such points for each reference triangle and for the QL the $4*(p + q)^2$ points are on $(p+q)^3$ circles of 4 points (one for each triangle) with (p + q) circles through each point
- The circles for isogonal centers are Cl-S conjugates
- The $(p+q)^3$ centers of the epi-or hypocycloïds are on curves (stelloïds ?) with degree p + q

(*) These points coincide with the Hofstadter points only if $p + q = 1$

QL, E, H and n-angle.pdf

3) Examples

- For p + q = 1, there are only 4 lines and we find the generalized Kantor-Hervey theorem (the deltoïd is p = 2, q = -1, the other are the H2n+1)
- For p + q = 2, there are 4 points, the beginning of the chain is for p = q = 1 and give the 4 in- and excenters of the reference triangle ; the points are all selfisogonal. We remember the 16 points are on 8 circles of 4 points, each point being on 2 circles. These circles (socalled Steiner circles) are Cl-S invariant.
- The next step in the same chain (inverse and isogonal of the inverse) is for p = 3 and q = -1 and give the 8 pairs of Cl-S conjugate circles ; the centers are the centers of the 8 astroïds tangent to the QL and are on a rectangular hyperbola, which is a conic stelloïd (see figure below)
- For p + q = 3, there are 9 points, the beginning of the chain is with p = 2 and q = 1 and gives the 27 centers of the cardioïds tangent to the QL, which lie on the cubic stelloïd.
- For p + q = 4, there are 16 points, the beginning of the chain is with p = 3 and q = 1 and gives the 64 centers of the astroïds tangent to the QL, which lie on a quartic (stelloïd ?)

QL, E, H and n-angle.pdf

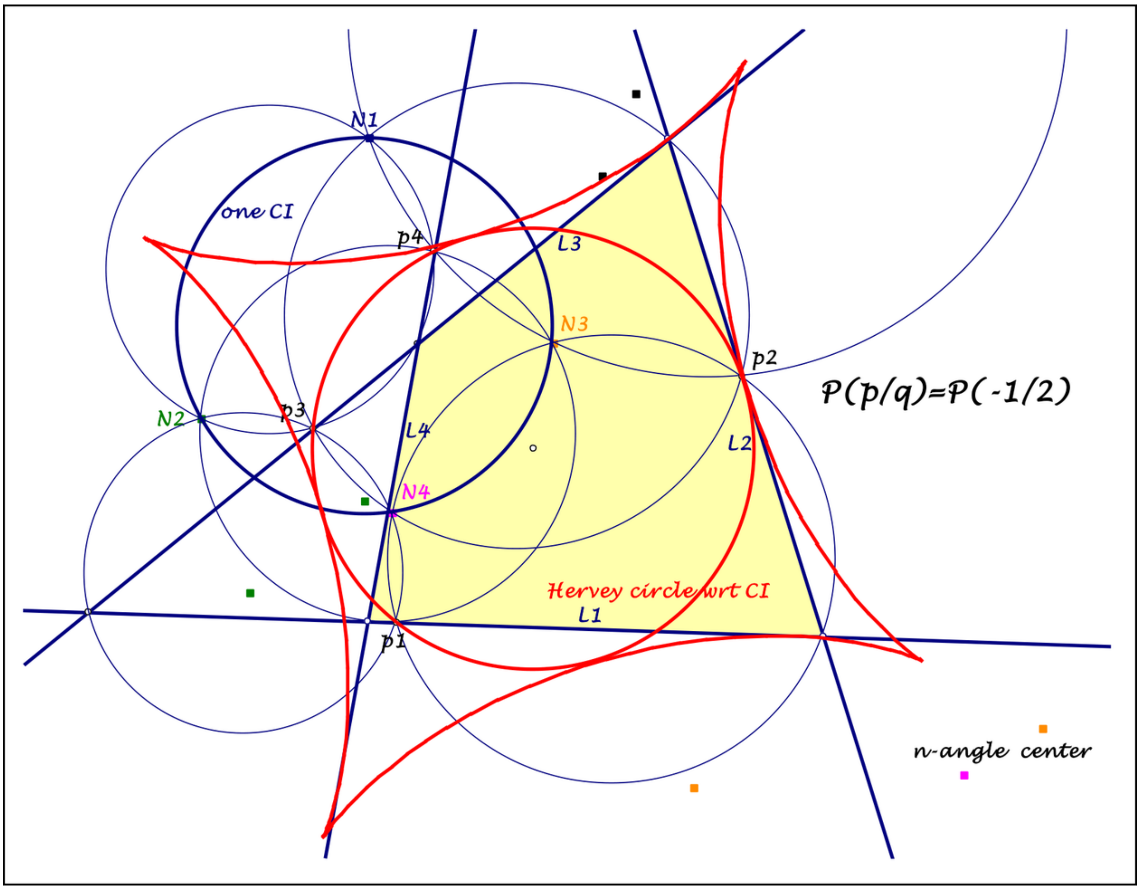
Message: #1143
Date: 2021-08-26
From: eckart_schmidt@t-online.de
Subject: Hervey circles for n-angle points wrt a QL

```
Dear Bernard,

excuse , if this is already well known to you,
... I couldn't reproduce your construction
    due to the nomination,
... so I have tried to study the background,
... you will recognize your concept.
Let us consider a QL = L1...L4 with vertices Aij = Li^Lj,
... let Ni be the n-angle centers P(n) (n integer)
... for the four triangles of the QL (see old#1872),
... the Ni are concyclic on a circle CI.
The six circles AijNkNl give a triple intersection pi on each
line,
... concyclic on the so called Hervey circle for P(n).
The Hervey circle for gP(n) is the same.
For P(2)=X(3) or P(-1)=X(4) the Hervey circle
... is the incircle of a deltoid, tangent to the lines
    of the QL.
For P(3)=X(186) or P(-2)=X(265) the Hervey circle
... is the incircle of what curve tangent to the lines
    of the QL?

If n isn't an integer, here only the case P(-1/2) shall be
considered
attached:
... for a triangle there are four of these points,
... the isogonal conjugates of the inverses
    of the in- and excenters,
... that gives for a QL 16 points on 8 circles,
    bearing from each triangle a point (see old#1873).
Taking these 8 circles as circles CI (see above), we get 8
Hervey circles,
... which are the incircles of 8 astroids,
    tangent to the lines of the QL.
In the case P(1/2) there exists no such Hervey circle.
I hope, you can confirm my sight of your concept.

Best regards Eckart
```



2021-08-26.pdf

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Message: #1144
Date: 2021-08-27
From: bernard.keizer@gmail.com
Subject: Re: Hervey circles for n-angle points wrt a QL

Dear Eckart,
I'm very glad that you apparently continue to look at this wunderful item !
But I'm sad that you only take partsof a whole complete construction ...
I can't see why and where you block.
You choose 2 examples where you don't have to construct the n-angle centers, as you have already the beginning of the chain inverse/isogonal !
It's regrettable, as you could check that my construction is correct ! (3 steps in 1137).
Then your questions :
The CI circle is a Miquel circle for P(2), P(3) or P(4) and a Steiner circle for P(-1), P(-2) or P(-3).
The Miquel circle gives the primary points pi and the Steiner circle gives with the same construction the secundary points qi, the 8 points pi and qi being on the Hervey circle.
The curve for X186 and X265 is a hypocycloïd with 5 cusps as mentionned and drawn in the attached file to the message 1113.
The CI circle on your figure is a Steiner circle for P(-1/2), which gives not the pi points as you mention, but the qi points ; the same construction with P(3/2) gives the corresponding Miquel circle (both circles being Cl-S partners) and the real pi points. The contact points with the curve are the reflexions of the middles mi of piqi in pi. (see message 1121)
Last, again, the background of all this is in the attached file of the message 1142.
Best regards
Bernard

Message: #1145
Date: 2021-08-28
From: eckart_schmidt@t-online.de
Subject: Re: Hervey circles for n-angle points wrt a QL

```
Dear Bernard,

I try to give another simple description
... of the incircles of the 8 astroids tangent to the QL-lines:
Consider a QL with lines Li, intersections Aij and triangles Tri
= LjLkLl =
AjkAklAlj:
Start with the 4 in-/excenters of the 4 Tri, that are 16 point
... four by four on the well known 8 Steiner circles CI,
... take one circle Ci with its 4 points Xi,
... let Yi be the QA-P4 of Tri plus Xi, also concyclic,
... the 6 circles AijYkYl give on each line Li a triple
    intersection Zi,
... concyclic on the incircle of one of the 8 astroids,
... 2nd intersections of this circle with the line Li
    let be Zi',
... Zi and Zi' lead to the contact points and cusps
    of the astroid
... as discussed earlier, but not as you describe in #1144.

Best regards Eckart

PS: Aware I have done it without terms as
... Hofstadter points, n-angle points,
    Miquel and Hervey circles...,
... so I hope, that others can do it also ad once (as in #1125).
```

Message: #1146
Date: 2021-08-29
From: bernard.keizer@gmail.com
Subject: Über den Steg

```
Lieber Eckart,
'' Wir sind uns einmal im Leben so nahe gewesen, dass nichts
unsere Freund- und Bruderschaft mehr zu hemmen schien und nur
noch ein kleiner Steg zwischen uns war. Indem Du ihn eben
betreten wolltest, fragte Ich Dich : " Willst Du zu mir über den
Steg ? '' - aber da wolltest Du nicht mehr und als Ich nochmals
bat, schwiegst Du. Seitdem sind Berge und reissende Ströme, und
was nur trennt und fremd macht, zwischen uns geworfen, und wenn
wir auch zueinander wollten, wir könnten es nicht mehr !
Gedenkst Du aber jetzt jenes kleinen Steges, so hast Du nicht
Worte mehr - nuch noch Schlurzen und Verwunderung. ''
Friedrich Nietzche Die fröhliche Wissenchaft Über den Steg
mit besten geometrischen Grüssen
Bernard
```

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Message: #1147
Date: 2021-08-31
From: eckart_schmidt@t-online.de
Subject: Re: Über den Steg

```
Dear Bernard,

meanwhile I regret my efforts trying to follow your last
messages,
... my understanding and geometric horizon is limited,
... but thanks for numerous detailed explications.

To react on your Nietzsche citation
... is not the place here in the group.

Best regards Eckart
```

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Message: #1148
Date: 2021-09-18
From: eckart_schmidt@t-online.de
Subject: View angle of QG-diagonals

Dear all,

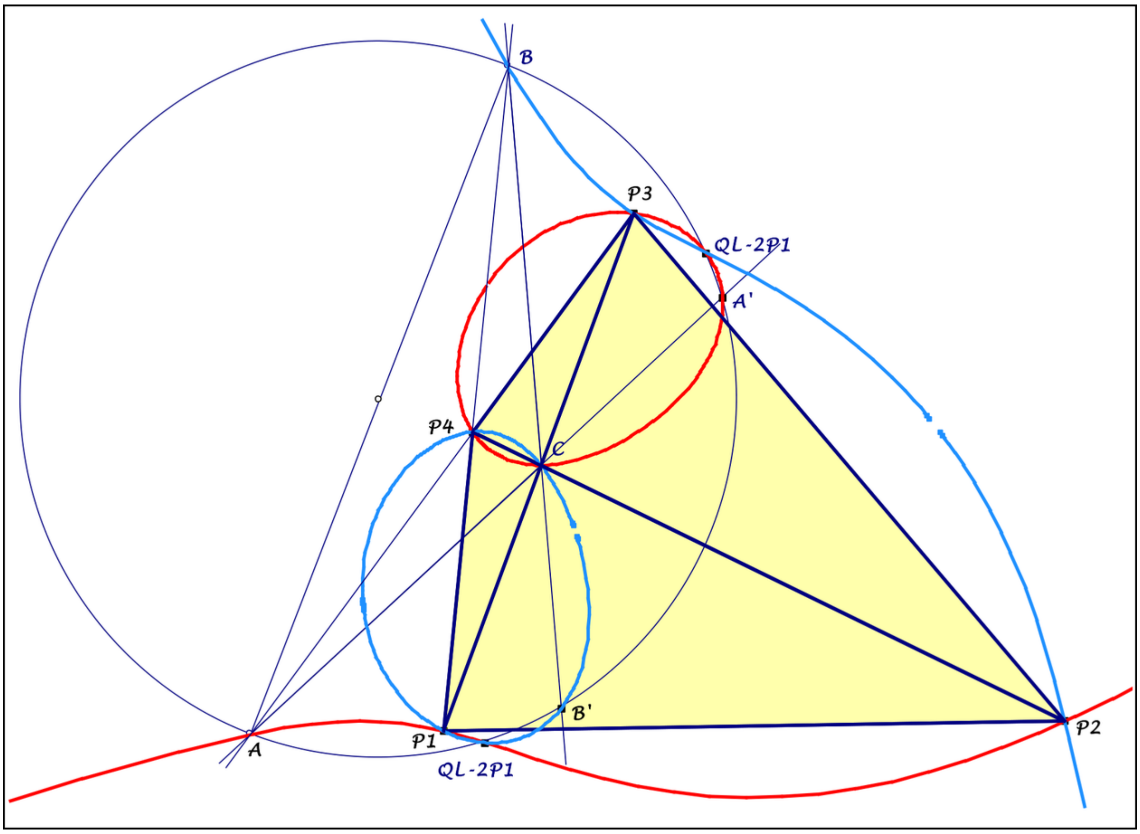
here the locus of points is described,
... which have the same view angle for the diagonals of a quadrigon.

Let us consider a quadrigon QG = P1P2P3P4 (see attached figure),
... its intersections of opposite sides QG-2P2a,b = A,B:
... A = P1P2 ^ P3P4, B = P2P3 ^ P4P1,
... and the diagonal crosspoint QG-P1 = C,
... further a positive orientation for angles A-->C--->B
... and the helpful points A' and B',
... which are the 2nd intersections of AC and BC
... with the circle through A, B and the Plücker points QL-2P1.

The locus of points X with <P1XP3 == <P2XP4 mod Pi
... is a focal circular circumcubic of the quadrigon
... through the Plücker points QL-2P1 and A',
... which is the cubic QL-Cu1 of the quadrigon P1P2P4P3.

The locus of points X with <P1XP3 == - <P2XP4 mod Pi
... is a focal circular circumcubic of the quadrigon
... through the Plücker points QL-2P1 and B',
... which is the cubic QL-Cu1 of the quadrigon P1P4P2P3.

Best regards Eckart



2021-09-18.pdf

Message: #1149
Date: 2021-09-20
From: anopolis72@gmail.com
Subject: Concurrent Aubert lines

[Ivan Pavlov] (*)

The Aubert line of complete quadrilateral ABCD-EF passes through
orthocenters of ADE, BCE, ABF, CDF. Let's denote it with
a(ABCD).
Let XYZ be a triangle with excenters Ex, Ey, Ez, respectively.
Let a(X, Y, Z, Ey) and a(X, Z, Y, Ez) intersect in point Px.
Let a(Z, X, Y, Ex) and a(X, Z, Y, Ez) intersect in point Py.
Let a(Z, X, Y, Ex) and a(X, Y, Z, Ey) intersect in point Pz.
Then the lines XPx, YPy, ZPz are concurrent and their common
point is X(43672).
True of false?
[Ercole Suppa]:
The property: "the lines XPx, YPy, ZPz are concurrent and their
common point is X(43672)" is true.
I verified it using barycentric coordinates.

(*)
Romantics of Geometry #8910
https://www.facebook.com/groups/parmenides52/permalink/436214012
3899660/

Note
The Aubert line is also known as Steiner Line of quadrilateral
Is this property already known ?

APH

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Message: #1150
Date: 2021-09-21
From: van10hoven@gmail.com
Subject: Re: Concurrent Aubert lines

```
Dear Antreas,

Thanks for the message.
I was not familiar with this alternative name of the Steiner
line.
I wonder who Aubert is and the origin of the name, apart from
the reference you mentioned.
The X(43672)-property is new to me. Very nice indeed.

Best regards,
Chris
```

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Message: #1151
Date: 2021-09-21
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Concurrent Aubert lines

---

Chris wrote:
> Dear Antreas,
> Thanks for the message.
> I was not familiar with this alternative name of the Steiner
line.
> I wonder who Aubert is and the origin of the name, apart from
the
> reference you mentioned.
> The X(43672)-property is new to me. Very nice indeed.
> Best regards,
> Chris

Dear Chris

I have no idea who Aubert was. I guess some "minor" geometer of
the golden era of Steiner, Neuberg, Lemoine etc
As for the line
Some write the Steiner line as Aubert line and some mention that
its alternative name is Steiner line
See for example the attacked excerpt from
  Evan Chen, Euclidean Geometry in Mathematical Olympiads

Regards
APH

Theorem 10.5 (Gauss-Bodenmiller Theorem). *The circles with diameters $\overline{AC}$, $\overline{BD}$, $\overline{PQ}$ are coaxial. Their radical axis is a line passing through each of the four orthocenters of the triangles PAB, PCD, QAD, QBC.*

The radical axis is sometimes called the **Steiner line** (or sometimes **Aubert line**). The figure is shown in Figure 10.3B.

01 001.jpg

Message: #1152
Date: 2021-09-22
From: eckart_schmidt@t-online.de
Subject: 5L-s-P1 and 5G-s-P1,2,5 collinear

```
Dear Chris,

5L-s-P1 is the 4th harmonic point
... of 5G-s-P1 wrt 5G-s-P2 and 5G-s-P5
... for every 5G-version of the 5L.

Best regards Eckart
```

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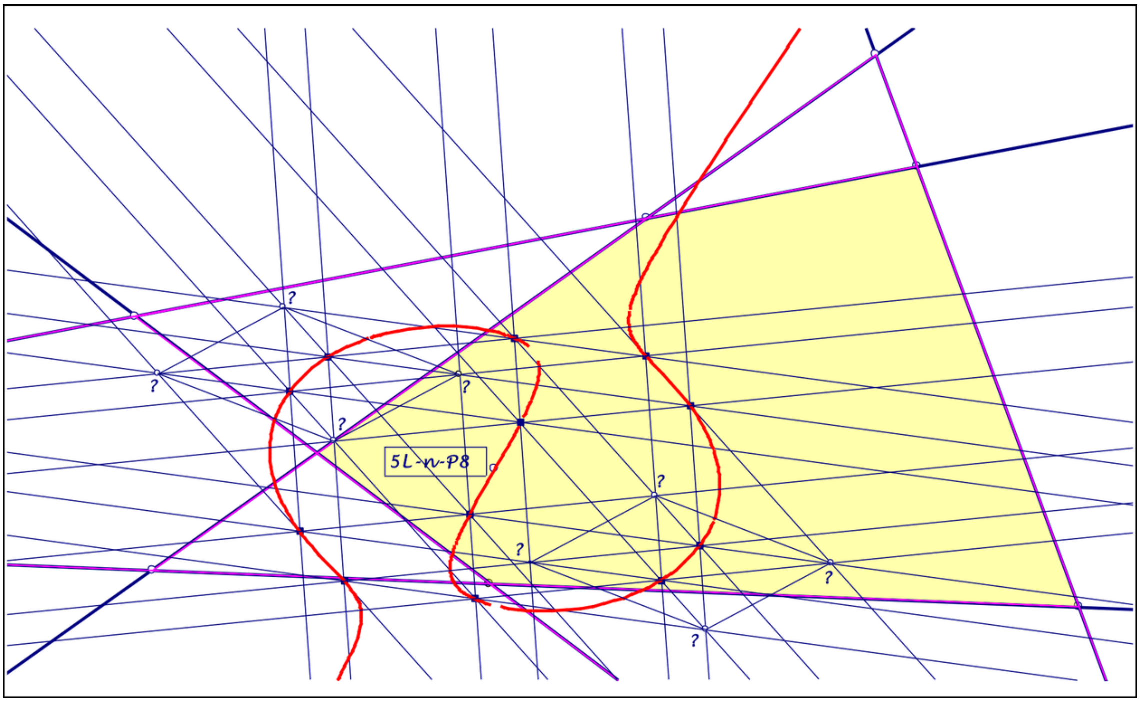
Message: #1153
Date: 2021-09-22
From: eckart_schmidt@t-online.de
Subject: 5G-geometry for a 5L

Dear all,

a pentalateral 5L has 12 pentagon-versions 5G,
... whose centroids 5P-n-P1 shall be researched.
Let Li be the lines of the 5L and Pij the intersections of Li
and Lj,
... then P12P23P34P45P51 and P12P34P51P23P45, etc ...
... have 5P-n-P1 with midpoint 5L-n-P8.
These 6 pairs of 5P-n-P1 with common midpoint
... give pairwise a net of four sets of parallel lines
... symmetric 5L-n-P8 (see attached figure),
... every 5P-n-P1 is the intersection of 4 lines.
This net bears 8 triple points of lines,
... which give two parallelograms, symmetric 5L-n-P8.
The 12 5P-n-P1 lie on a cubic, symmetric 5L-n-P8.
What about this cubic?

Best regards Eckart



2021-09-22.pdf

Message: #1154
Date: 2021-09-24
From: eckart_schmidt@t-online.de
Subject: New QG-transformation

```
Dear all,

this is an interesting combination of a QA- and a
QL-transformation,
... giving a QG-transformation QG-Tfx:
... X --> QA-Tf16(QL-Tf1(X)) = QL-Tf1(QA-Tf16(X)),
... with a very simple construction,
    already described in old#362:
... QG-Tfx(X) = 2nd intersection
    of the circles (X,Pi,Pi+2) and (X,Pi+1,Pi+3).
Some properties:
(1) QG-Tfx maps QA-P4 to QL-P1 and vice versa.
(2) QG-Tfx maps QG-2P2a,b to the Miquel points unequal QL-P1.
(3) QG-Tfx maps lines to sextics
... with double points in the vertices of the QG,
... degenerating for the QG-sidelines and diagonals.
(4) QG-Tfx maps the QG-diagonals to itself.
(5) QG-Tfx maps a QG-sideline PiPi+1 to a circle
... through Pi+2, Pi+3, QG-P1 and a Miquel point unequal QL-P1,
(6) The centers of these four circles give a parallelogram
... with center QA-P9 and sides perpendicular to the diagonals.
(7) QG-Tfx maps lines through a QG-vertex Pi
... to a QG-circumquartic through QG-P1 with double point
    in Pi+2.
(8) The three sextics as QG-Tfx-images of QL-L1 wrt the
    QG-versions of QL
... have four common points on QL-Ci1.
(9) QA-Cu1 is invariant wrt QG-Tfx for the three QG-versions
    of QA.
(10) The three QG-Tfx-images of QA-Tf16(Pi) give Pj, Pk, Pl.
(11) The fixed points of QL-Tf1 are as well QG-Tfx-partner as
     QA-Tf16-partner.
(12) The fixed points of QG-Tfx give a quartic,
     detailed described in old#362, see also #1044.

Best regards Eckart
```

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Message: #1155
Date: 2021-09-24
From: eckart_schmidt@t-online.de
Subject: Re: QL-Ci2

```
Dear Chris,

if we consider for the QL-inscribed conics the contact QA,
... their QA-P29 lie on QL-Ci2.

Best regards Eckart
```

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Message: #1156
Date: 2021-09-25
From: eckart_schmidt@t-online.de
Subject: QA-Tf14

```
Dear Chris,

well known:
    QA-Tf2 maps QA-Co1 at infinity,

perhaps new:
    QA-Tf14 maps QA-Co2 at infinity,
    ... direction orthogonal to the tangent.

Best regards Eckart
```

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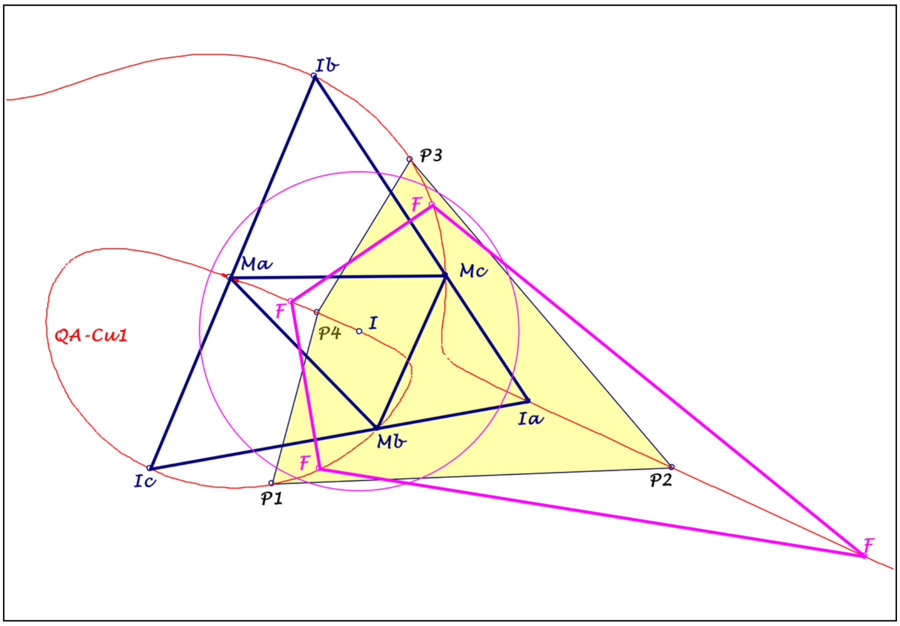
Message: #1157
Date: 2021-09-26
From: eckart_schmidt@t-online.de
Subject: Fixed points of QA-Tf16

Dear all,

consider a quadrangle P1P2P3P4,
... its Miquel triangle MaMbMc with in-/excenters J,Ja,Jb.Jc
... and the Möbius transformation TFa (analog TFb, TFc),
... centered in J, swapping Ja and Ma
... with the same inversion circle as TFb, TFc.
Each of these Möbius transformations TFa, TFb, Tfc
... maps the vertices of QA to the fixed points F of QA-Tf16.
Some properties: The fixed points of QA-Tf16
... are points on the cubic QA-Cu1, 3rd intersections of JPi,
... ... with common tangential QA-P3,
... are 4times perspective to the reference QA wrt J,Ja,Jb,Jc,
... have the CB-point Q with P1,P2,P3,P4,
... ... Q intersection of QA-Cu1 and its asymptote.
Final remark :
The 4 circumcircles of the QA-triangles
... have pairwise two inversion circles,
... which give 12 circles with four new 6times intersections,
... which are the fixed points of QA-Tf16.

Best regards Eckart
PS: See also: eckartschmidt.de/Invzk.pdf, written 2009 in German.



2021-09-20.pdf

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Message: #1158
Date: 2021-09-29
From: van10hoven@gmail.com
Subject: QL-Ci2 5L-s-P1

```
Dear Eckart,

I added your properties in messages #1152, #1155 in EQF and EPG.
See Recently Added (
https://www.chrisvantienhoven.nl/recently-added ).
Thanks for your continuing efforts.

Best regards,
Chris
```

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Message: #1159
Date: 2021-10-05
From: eckart_schmidt@t-online.de
Subject: QL-Cu1

```
Dear Chris,

as you publish in "Journal of Geometry and Graphics"
... perhaps of interest:
In Volume 24 (2020), No. 1, 9-28 Daniel Hu
... researched the cubic QL-Cu1
... as locus of points, whose pairs of lines to opposite
    QL-points
... share the same angle bisectors.

Best regards Eckart
```

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Message: #1160
Date: 2021-10-06
From: van10hoven@gmail.com
Subject: Re: QL-Cu1

```
Dear Eckart,

Thank you for the reference.
Best regards,

Chris
```

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Message: #1161
Date: 2021-10-07
From: ngo.quang.duong.1100@gmail.com
Subject: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Chris, Eckart,

It is so good to see the Quadri-Figures and Polygon Geometry are
still very active. I hope you all (including other members of
the group) to be safe in this pandemic.

I haven't appeared for a while, since I was very busy studying.
Recently, I came back to geometry. In this post, I would like to
share my point of view about Steiner's 8th theorem: [Steinter's
8th theorem](https://drive.google.com/file/d/1TPIH85VhC79E6Ok2XK
JAidh58uNH1j8n/view?usp=sharing)

Best regards,

Ngo Quang Duong
```

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Message: #1162
Date: 2021-10-07
From: anopolis72@gmail.com
Subject: Re: Concurrent Aubert lines

```
1st and 2nd Aubert points of various triangles are listed now in
ETC by C. Lozada

X(45010) - X(45124)

APH

On Mon, Sep 20, 2021 at 6:09 PM Antreas Hatzipolakis
<anopolis72@gmail.com>
wrote:
> [Ivan Pavlov] (*)
>
> The Aubert line of complete quadrilateral ABCD-EF passes
through
> orthocenters of ADE, BCE, ABF, CDF. Let's denote it with
a(ABCD).
> Let XYZ be a triangle with excenters Ex, Ey, Ez, respectively.
> Let a(X, Y, Z, Ey) and a(X, Z, Y, Ez) intersect in point Px.
> Let a(Z, X, Y, Ex) and a(X, Z, Y, Ez) intersect in point Py.
> Let a(Z, X, Y, Ex) and a(X, Y, Z, Ey) intersect in point Pz.
> Then the lines XPx, YPy, ZPz are concurrent and their common
> point is X(43672).
> True of false?
> [Ercole Suppa]:
> The property: "the lines XPx, YPy, ZPz are concurrent and
> their common point is X(43672)" is true.
> I verified it using barycentric coordinates.
>
> (*)
> Romantics of Geometry #8910
> https://www.facebook.com/groups/parmenides52/permalink/4362140
123899660/
>
> Note
> The  Aubert line is also known as Steiner Line of
quadrilateral
> Is this property already known ?
>
> APH
```

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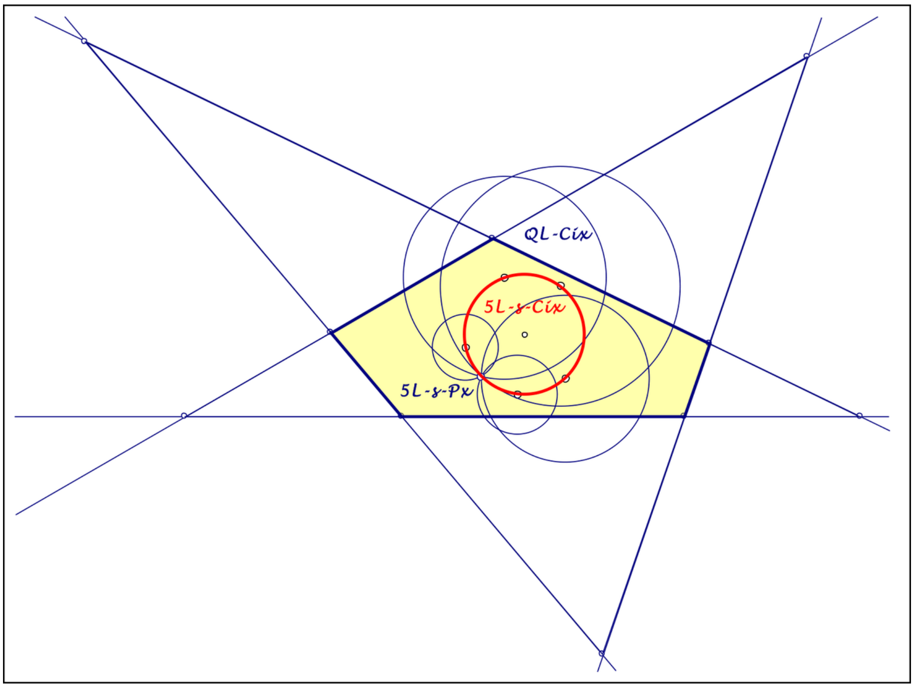
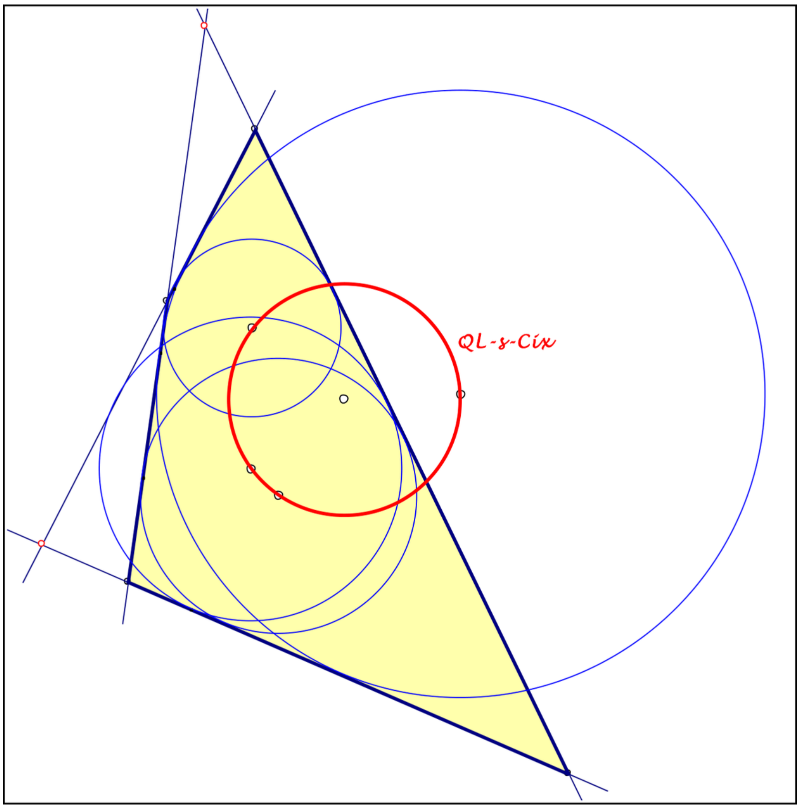
Message: #1163
Date: 2021-10-08
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

with interest I have reproduced your construction:
Taking for the vectors ei = Li-1^Li.Li^Li+1
... you get a special circle QL-Cix for a QL.
Using these circles for the 5 QL of a 5L,
... you get a common point 5L-s-Px.
Moreover the centers of these 5 circles
... lie on a further circle 5L-s-Cix (see attached file).
Final remark:
You can get the special circle QL-s-Cix for a QL
... also in the following way:
Every QL has a convex QG (see attached file),
... consider the incenters
    of the two "QG-including" QL-triangles
... and consider the two "QG-outside" QL-triangles
... ... and their excenters wrt the intersections
        of opposite QG-sides,
... then the two in- and the two excenters
    determine the circle QL-Cix.

I hope, you can confirm my observations,
... I try to find properties of these new QL-/5L-elements.

Best regards Eckart
```



2021-10-08a.pdf



2021-10-08b.pdf

Message: #1164
Date: 2021-10-08
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart,

Thank you so much for your interest and quick reply.
About your remark

> Every QL has a convex QG (see attached file),
> ... consider the incenters of the two "QG-including "
QL-triangles
> ... and consider the two "QG-outside" QL-triangles
> ... ... and their excenters wrt the intersections of opposite
QG-sides,
> ... then the two in- and the two excenters determine the
circle QL-Cix.

Yes, this is true.

In your figure (2021-10-08b.pdf), let the convex QG be ABCD, E is the intersection of AD and BC, F is the intersection of AB and CD, such that: EAD, EBC, FAB, FDC are collinear, in those orders.

According to the idea in my note, four vectors e1, e2, e3, e4 are vector(AB), vector(BC), vector(CD), vector(DA).

The construction in my note is independent of the figure - in other words, we don't have to specify which QG is convex in a given QL.

The aim of the idea in my note is to emphasize the **symmetry** between the four lines. Also, the construction allows us to state the 8th theorem in a more precise and formal way.

Ngo Quang Duong

Message: #1165
Date: 2021-10-08
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart,

> Taking for the vectors ei = Li-1^Li.Li^Li+1
> ... you get a special circle QL-Cix for a QL.
> Using these circles for the 5 QL of a 5L,
> ... you get a common point 5L-s-Px.
> Moreover the centers of these 5 circles
> ... lie on a further circle 5L-s-Cix (see attached file).

This is a nice and unexpected property for me. I believe your idea can be generalized to nL as well.

The Steiner circles, or QL-Cix circles, have been considered as (1/2)-angle center circles (I refer to the topic "Four n-angle centers", which starts with the [message 1843](https://groups.io/g/Quadri-Figures-Group/topic/71470170#18334))

The 5L-s-Cix is kind of similar to the circle passing through 5 Miquel centers of 5 QL in a 5L (this circle also contains the common points of the 5 Miquel circles). This might also be generalized to nL.

I have a more general construction than n angle center. I will write it down as clearly as possible and share it with the group later.

Ngo Quang Duong

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Message: #1166
Date: 2021-10-08
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart,

> The 5L-s-Cix is kind of similar to the circle passing through 5 Miquel centers of 5 QL in a 5L (**this circle also contains the common points of the 5 Miquel circles**). This might also be generalized to nL.

My mistake, the circle does not contain the common points of the 5 Miquel circles.

When I mention the "Miquel center", I refer to QL-P4.

Ngo Quang Duong

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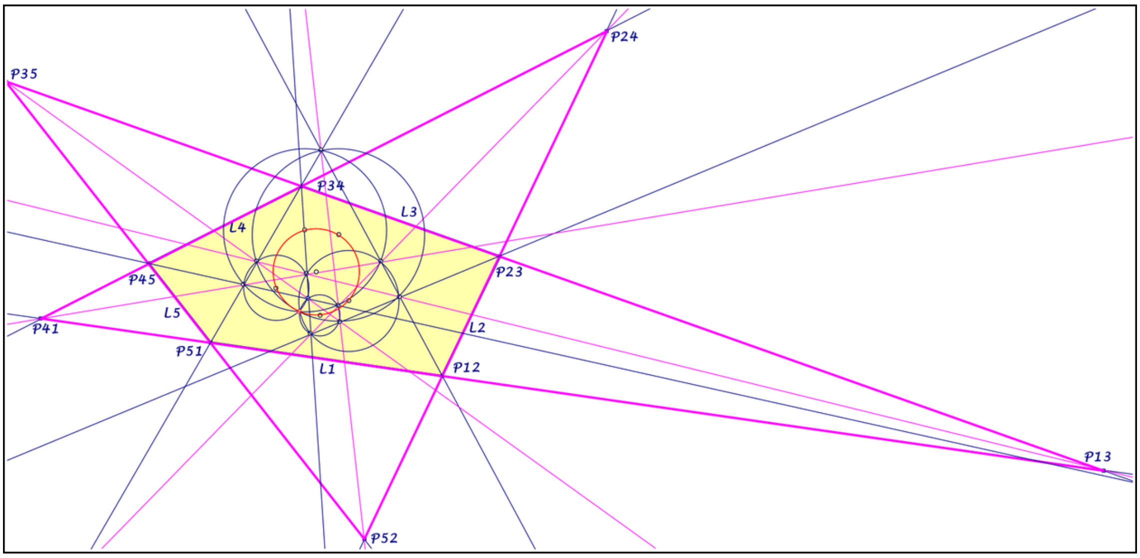
Message: #1167
Date: 2021-10-10
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

if a 5L has a convex 5G, then there is also the following
construction
... for the new elements 5L-s-Cix and 5L-s-Px in message #1163:
Consider a 5L with lines Li and intersections Sij = Li^Lj,
... let 5G = S12.S23.S34.S45.S51 be the convex 5G version,
... consider a second 5G = S13.S35.S52.S24.S41,
... take the angle bisectors for the first and second 5G,
... these 10 bisectors have 10 triple intersections on 5 circles
... with common point 5L-s-Px and concyclic centers on 5L-s-Cix.

Best regards Eckart.
```



2021-10-10.pdf

Message: #1168
Date: 2021-10-11
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

first properties of QL-Cix in #1163:
QL-Cix is a CSC-invariant circle,
... orthogonal to the inversion circle of CSC,
... centered on the 1st Steiner axis.

Best regards Eckart
```

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Message: #1169
Date: 2021-10-11
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart, dear Ngo Quang Duong,
The 8 circles are CSC invariant ! (see EQF)
4 are centered on the 1rst Steiner axis and 4 on the 2nd.
Each circle of the 1rst set is orthogonal to each circle of the 2nd.
The 8 circles are orthogonal to the inversion circle of the CSC.
Now 2 remarks :
1) If we use I as in- or excenter, iI as inverse of I in the corresponding circumcenter and giI the isogonal of this inverse, the points I are 1/2 angle centers, the points iI are 3/2 angle centers and the points giI are -1/2 angle centers. Ngo Quang Duong's method allows easily to identify the 8 circles of iI (which I persist to name Miquel centers) and the 8 circles of giI (which I persist to name Steiner circles). Taking the 4 copples of iI and giI on each circle and the 4 perpendicular bisectors gives as intersections the 8 centers of the 8 astroïds tangent to the 4 lines. The 8 points are on a rectangular hyperbola and are the centers of the inner circles of the astroïds (which I persist to name Hervey circles). (For a complete description and construction, see my message 1121).
2) Consider the 2nd intersection (apart of the in- and excenters) of a circle from the 1rst set and a circle of the 2nd set. You get another 16 points, by construction on the same 8 circles(4 on each circle and 2 circles through each point). But these 16 new points are also on 12 other circles (4 on each circle and 3 circles through each point). The 12 circles are through QL-P1 and form 6 pairs of orthogonal circles, the 2nd intersections being the 6 vertices of the QL.
Best regards
Bernard

Message: #1170
Date: 2021-10-12
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Bernard,

sorry, the properties 5L-s-Cix and 5L-s-Px (#1163)
... seem not holding in general for 5L without a convex 5G.
I try to clear my doubts.

Best regards Eckart
```

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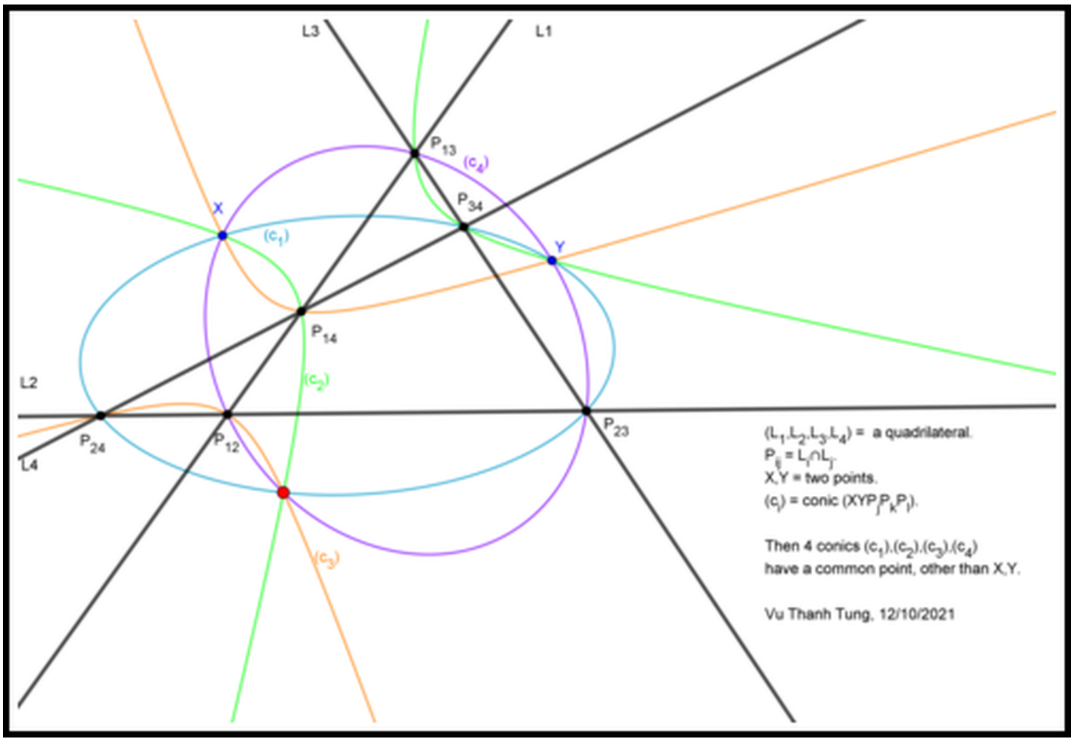
Message: #1171
Date: 2021-10-12
From: tungvtt@gmail.com
Subject: Concurrent conics as transformation for Quadrilateral,

---

```
Dear all,

I found an interesting transformation as follows:
(L1,L2,L3,L4) =  a Quadrilateral.
Pij = Li∩Lj.
X,Y = two points.
(ci) = conic (XYPjPkPl).
Then 4 conics (c1),(c2),(c3),(c4) have a common point, other
than X,Y.
See the figure attached.
That transforms a Pair of Points to a point:  (X, Y) -> Z.
By construction (Y, Z) -> X and (Z, X) -> Y. So the role of
X,Y,Z are symetrical.
This means that Triangle XYZ has a special property to the
Quadrilateral (L1,L2,L3,L4).
Which can be called the relation of Triangle XYZ to
(L1,L2,L3,L4), and which special Triangle has this relation
to  (L1,L2,L3,L4)?

Best regards,
Vu Thanh Tung
```



CommonPointConicQuadrilateral.png

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Message: #1172
Date: 2021-10-12
From: tungvtt@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

```
Dear all,

Please see
https://www.dropbox.com/s/zs45wzymprb795l/CommonPointConicQuadri
lateral.png
for a better figure.

Best regards,
Vu Thanh Tung

On Tue, Oct 12, 2021 at 07:05 AM, Vu Thanh Tung wrote:
> Dear all,
>
> I found an interesting transformation as follows:
>
> (L1,L2,L3,L4) =  a Quadrilateral.
> Pij = Li∩Lj.
> X,Y = two points.
> (ci) = conic (XYPjPkPl).
>
> Then 4 conics (c1),(c2),(c3),(c4) have a common point, other
than X,Y.
> See the figure attached.
>
> That transforms a Pair of Points to a point:  (X, Y) -> Z.
> By construction (Y, Z) -> X and (Z, X) -> Y. So the role of
X,Y,Z are
> symetrical.
> This means that Triangle XYZ has a special property to the
Quadrilateral
> (L1,L2,L3,L4).
>
> Which can be called the relation of Triangle XYZ to
(L1,L2,L3,L4), and
> which special Triangle has this relation to  (L1,L2,L3,L4)?
>
> Best regards,
> Vu Thanh Tung
```

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Message: #1173
Date: 2021-10-12
From: eckart_schmidt@t-online.de
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

```
Dear Vu Thanh Tung,

only some first observations wrt your transformation (X,Y)-->Z
in #1171:
If you consider X and Y on QL-Cu1, Z is also a point on QL-Cu1,
... Z = X.QL-Tf1(Y) ^ Y.QL-Tf1(X),
... QL-Tf1(Z) is the 3rd intersection of the line XY
    and the cubic QL-Cu1.
If Y = QL-Tf1(X), Z is the common tangential of X and Y.
If we take Y = QL-P1 as fixed point we get a transformation X
--> Z,
... which maps a line to a quartic through the six QL-points,
... which can have a triple point in QL-P1.
If we take Y = QL-P1 and a line through Y,
... the image of the line is the reflection
    in the first Steiner axis.
So far, there will be more ...

Best regards Eckart
```

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Message: #1174
Date: 2021-10-12
From: bernard.keizer@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

```
Dear Vu Thanh Tong,
Z is the 9th CB point of the 6 QL vertices and X and Y.
There are an infinity of cubics through the 6 QL vertices, X, Y
and Z.
4 degenerated cubics are formed by a conic through 3 non aligned
QL vertices and X, Y and Z and the line through the 3 other
aligned QL vertices.
In particular, if X and Y are the circular points, Z is QL-P1
and the 4 conics are the circumcircles.
Best regards
Bernard
```

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Message: #1175
Date: 2021-10-13
From: Stan.Rabinowitz@comcast.net
Subject: What is this circle called?

```
A convex quadrilateral has four escribed circles.
(An escribed circle is outside the quadrilateral and is tangent
to three consecutive sides of the quadrilateral.)

The centers of the four escribed circles lie on a circle.
Is this a known circle?
What is this circle called? Does it have a name? Does it have
any interesting properties?
```

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Message: #1176
Date: 2021-10-13
From: tungvtt@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

```
Dear Bernard,

Thanks you very much for your anwser.

Best regards,

Vu Thanh Tung
```

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Message: #1177
Date: 2021-10-13
From: ”Stanley Rabinowitz” <Stan.Rabinowitz@comcast.net>
Subject: [Quadri-and-Poly-Geometry] What is this circle called?

```
Hi Stan & others

I'm not aware that this circle has a specific name, but perhaps
others here would know? Also I do recall some time (years?) ago
seeing it on Antonio Gutierrez's site as Problem 564:
[ http://www.gogeometry.com/problem/p564_complete_quadrilateral_incenter_excenter_concyclic.htm |
http://www.gogeometry.com/problem/p564_complete_quadrilateral_incenter_excenter_concyclic.htm ]

Also see problem 569 which produces a circumscribed (tangential)
quadrilateral at: [ http://www.gogeometry.com/problem/p569_quadrilateral_excircles_tangency_point_congruence.htm |
http://www.gogeometry.com/problem/p569_quadrilateral_excircles_tangency_point_congruence.htm ]

Regards
Michael


To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Wednesday, October 13, 2021 4:02:42 PM

A convex quadrilateral has four escribed circles.
(An escribed circle is outside the quadrilateral and is tangent
to three consecutive sides of the quadrilateral.)

The centers of the four escribed circles lie on a circle.
Is this a known circle?
What is this circle called? Does it have a name? Does it have
any interesting properties?
```

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Message: #1178
Date: 2021-10-13
From: Stan.Rabinowitz@comcast.net
Subject: Re: What is this circle called?

```
Thanks. That was very helpful.

If this circle doesn't already have an established name, I
propose calling it the Bevan Circle of the quadrilateral.
I would call the center of this circle the Bevan Point of the
quadrilateral.

If in quadrilateral ABCD, you make D move to C, then you get a
triangle with its three excircles.
Their centers determine a circle whose center is known as the
Bevan Point of the triangle.
The Bevan Point of a triangle lies on the Bevan Line which
passes through O and I, the circumcenter and incenter of the
triangle.

In the quadrilateral case, if the quadrilateral is bicentric (so
that it has both a circumcenter O and incenter I),
then the Bevan Point of the quadrilateral, V, is collinear with
O and I.
The Bevan Line, OIV, also passes through the Miquel point of the
quadrilateral.

The Bevan Point and Circle exist for all quadrilaterals (not
just bicentric ones). Is the Bevan Point one of the points
listed in EQF?
```

Message: #1179
Date: 2021-10-13
From: ”Stan Rabinowitz” <Stan.Rabinowitz@comcast.net>
Subject: Re: [Quadri-and-Poly-Geometry] What is this circle called?

Dear Stan

Yes, I agree calling it the Bevan circle for the quadrilateral would be a good choice for a name.

As you probably already noted, the general result for a quadrilateral, is quite easy to prove with a bit of angle chasing.

Your reply stirred my memory, which reminded me that Hans Humenberger from the University of Vienna has a paper accepted for the Mathematics Magazine which relates to a specific generalization of the Bevan point of a triangle to a quadrilateral & several related results. The title, which may have changed now, is " Bicentric quadrilaterals: The Bevan point, the Bevan circle, six collinear points, and special homotheties".

I'm not sure when his article will appear, but I'm guessing it should be no later than 2022.

Regards
Michael

" Mathematics - this may surprise or shock some - is never deductive in creation ." - Paul Halmos

" Every human activity, good or bad, except mathematics, must come to an end ." - Paul Erdos
---
Prof Dr Michael de Villiers, retired University of KwaZulu-Natal
Professor Extraordinare, University of Stellenbosch
Homepage: [http://dynamicmathematicslearning.com/homepage4.html | http://dynamicmathematicslearning.com/homepage4.html]
Dynamic Geometry Sketches: [http://dynamicmathematicslearning.com/JavaGSPLinks.htm | http://dynamicmathematicslearning.com/JavaGSPLinks.htm]
[http://www.keypress.com/]

Visit the SA Mathematics Olympiad at [http://www.samf.ac.za/Default2.aspx | http://www.samf.ac.za/Default2.aspx]

To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Wednesday, October 13, 2021 7:49:13 PM

Thanks. That was very helpful.

If this circle doesn't already have an established name, I propose calling it the Bevan Circle of the quadrilateral.
I would call the center of this circle the Bevan Point of the quadrilateral.

If in quadrilateral ABCD, you make D move to C, then you get a triangle with its three excircles.
Their centers determine a circle whose center is known as the Bevan Point of the triangle.
The Bevan Point of a triangle lies on the Bevan Line which passes through O and I, the circumcenter and incenter of the triangle.

In the quadrilateral case, if the quadrilateral is bicentric (so that it has both a circumcenter O and incenter I),
then the Bevan Point of the quadrilateral, V, is collinear with O and I.
The Bevan Line, OIV, also passes through the Miquel point of the quadrilateral.

The Bevan Point and Circle exist for all quadrilaterals (not just bicentric ones). Is the Bevan Point one of the points listed in EQF?

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Message: #1180
Date: 2021-10-13
From: eckart_schmidt@t-online.de
Subject: Re: What is this circle called?

```
Dear all,

not a name for the circle, but some properties:
(1) The circle is QL-Tf1-invariant.
(2) Lines through QL-Tf1-partner on the circle
... have a common point X on the 1st Steiner axis,
... X is the inverse of the Miquel point QL-P1,
... QL-Tf1(X) is the center of the circle.
(3) The circle intersects orthogonal
... the inversion circle of QL-Tf1.

Best regards Eckart
```

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Message: #1181
Date: 2021-10-13
From: van10hoven@gmail.com
Subject: Re: What is this circle called?

Dear Stanley and friends,

Your question:
A convex quadrilateral has four escribed circles.
(An escribed circle is outside the quadrilateral and is tangent to three consecutive sides of the quadrilateral.)
The centers of the four escribed circles lie on a circle. Is this a known circle?
What is this circle called? Does it have a name? Does it have any interesting properties?
The mentioned circle is 1 of 8 circles mentioned by Steiner.
In 1828 Jakob Steiner published in Gergonne's Annales 10 rules on the complete quadrilateral.
For a complete description see Jean Pierre Ehrmann's paper - Steiner's Theorems on the Complete Quadrilateral, Forum Geometricorum 4 (2004) 35-52.

Rules 8, 9, 10 are:
(8) Each of the four possible triangles has an incircle and three excircles.
The centers of these 16 circles lie, four by four, on eight new circles.

(9) These eight new circles form two sets of four, each circle of one set being
orthogonal to each circle of the other set. The centers of the circles of each
set lie on a same line. These two lines are perpendicular.

(10) Finally, these last two lines intersect at the point F (Miquel Point).
For a picture see **QL-8P1* (http://www.chrisvantienhoven.nl/ql-items/ql-mult-pts-lns/ql-8p1) : Steiner Angle Bisector Center Octet.*

Best regards,
Chris

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Message: #1182
Date: 2021-10-14
From: eckart_schmidt@t-online.de
Subject: Re: What is this circle called?

```
Dear all,

let ABCD be the convex quadrigon QG of a quadrilateral
... with AB^CD = E and AD^BC = F.
The Bevan circle is defined
... by the incenters of BEC and CFD
... and the excenters of AFB wrt F and AED wrt E.
There is an analogon circle QL-s-Cix in #1163, defined
... by the incenters of AFB and AED
... and the excenters of BEC wrt E and CFD wrt F.
This analogon has the interesting property in 1167.

Best regards Eckart
```

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Message: #1183
Date: 2021-10-14
From: ”Chris” <van10hoven@gmail.com>
Subject: Re: [Quadri-and-Poly-Geometry] What is this circle called?

```
Ah, thank you very much for that clarification.
I suspected this result was known for some time, and Steiner
sure was so prolific!
```

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Message: #1184
Date: 2021-10-14
From: Stan.Rabinowitz@comcast.net
Subject: Re: What is this circle called?

```
Thanks Chris!
```

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Message: #1185
Date: 2021-10-14
From: eckart_schmidt@t-online.de
Subject: Re: Steiner’s 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Bernard,

sorry once more:
... please forget the applications of QL-Cix
    for a 5L in #1163 and #1167,
... I have to rewrite these messages,
    for the properties don't hold in general.

Best regards Eckart
```

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Message: #1186
Date: 2021-10-15
From: bernard.keizer@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

Dear Eckart, dear Vu Thanh Tung
Just for fun.
On QL-Cu1, we may use a Rechnung auf der Zirkularkurve
(calculation on the circular curve).
Let's name M = QL-P1, Ω = the infinity point, I and J are the
circular points and Q the point where the curve cut's it's
asymptote.
On a line X + Y + Z = 0 (on the infinity line, Ω + I + J = 0)
On a conic U + V + W + X + Y + Z = 0 (on a circle through M, X +
Y + Z + M + I +J =0)
CSC(X) = Ω - M + X
Let's name now A, B and C 3 non aligned QL vertices and CSC(A),
CSC(B) and CSC(C) the 3 aligned QL vertices.
If X, Y and Z form a CB group with the 6 QL vertices, they are
on a conic through A, B and C.
A, B and C are on a circle through M.
X + Y + Z + A + B + C = 0
A + B + C + M + I + J = 0
It follows X + Y + Z = M + I + J = M - Ω or X + Y + Z + Ω - M =
0
2 of the 3 points are aligned with the CSC of the *3rd*
But Ω - M = M- Ω (as 2M = 2Ω = -Q)
We have also X + Y + Z + M + I + J = 0
*The 3 points X, Y and Z are on a circle through M
* X, Y and Z and their CSC form a 2nd Q inscribed in the same
QL-Cu1
*In other words, any circle through QL-P1 cuts the curve QL-Cu1
in 3 points X, Y and Z, which form with the vertices of any
inscribed QL a CB* *system*
Best regards
Bernard
PS The 3 S-points form also a CB system with the 6 QL vertices ;
they are also on a circle through QL-P1, but they are not on QL-
Cu1 ...

Message: #1187
Date: 2021-10-15
From: van10hoven@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

Dear Bernard,

Thanks for your exercise on QL-Cu1 and CSC.
You use equations like
* for a conic U + V + W + X + Y + Z = 0 (on a circle through M,
X + Y + Z + M + I +J = 0
* and for a transformation CSC(X) = Ω - M + X

What is the theoretical background of these equations?
Do you have references ?

Best regards,
Chris

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Message: #1188
Date: 2021-10-15
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart, dear Bernard

To Eckart: I advise you to consider my idea instead of using a convex QG since it depends on the figure - as you said, it does not hold in general.

To Bernard: I believe that you have understood my idea/my method of constructing 8 Steiner circles. Your remarks seem interesting, can you add some figures to illustrating them?

Currently, I am working on the properties of n-angle centers.
I have a generalization for them, but my explanation is not good enough yet. I will come back later.

Ngo Quang Duong

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Message: #1189
Date: 2021-10-15
From: eckart_schmidt@t-online.de
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

```
Dear Bernard, dear Chris,

16 years ago I studied these calculations on cubics:
"Geometrie auf der Zirkularkurve (eckartschmidt.de)
<http://eckartschmidt.de/Zirkul..pdf> ",
... at the moment I am not able to follow at once Bernard's
    calculations,
... but they show the possibilities to prove observations
    on cubics,
thanks.

By the way, is the following property evident?
... CSC on QL-Cu1 is the isogonal conjugate
... wrt any XYZ (in the sense of #1172) on QL-Cu1,
... e.g. X,Y = QL-2P2 and Z = QL-P1 for one-piece QL-Cu1.

Best regards Eckart
```

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Message: #1190
Date: 2021-10-16
From: bernard.keizer@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

Dear Chris, dear Eckart
It's so nice to meet both of you on a common item like in the good old times !
1) For Chris : I have 2 references, the 1rst is Eckart's article Geometrie auf der Zirkularcurve (my apologise to Eckart, Geometrie and not Rechnung), which is an application to QA-Cu1. This article is referred in EQF under 15b. Eckart refers himself to Fred Lang Geometry and Group Structures of Some Cubics. The 2nd reference is a serie of articles in the french revue Quadature(n°s 9, 10, 11, 35, 46, 47 and 65. In particular the n° 46 is dedicated to QL-Cu1. I used this calculation, which differs slightly from Eckart's.
2) For Eckart : yes, the property is obvious that CSC partners are isogonal wrt any triangle XYZ on QL-Cu1 such as it's circumcircle passes through QL-P1.
A QL defines a unique QL-Cu1, but a QL-Cu1 has an infinity of inscribed QL's, sharing the CSC and the Newton Line. Any triangle XYZ on QL-Cu1 with the given property is one of the reference triangles of a QL inscribed in the same curve and has the property that the locus of the foci of the inscribed conics of this 2nd QL is the same curve !
Best regards
Bernard
PS This group structure applies to any non singular cubic (no cusp and no node) and not only to circular cubics ...

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Message: #1191
Date: 2021-10-16
From: tungvtt@gmail.com
Subject: (n-2)-degree curve passing through midpoints in nP

Dear all,

It is known that there is an unique (n-2)-degree curve passing through n(n-1)/2 - 1 points on the plane (in a general positions).
For example, 5 points determine a conic, 9 points determine a cubic.

Now for a nP, there is n(n-1)/2 segments which vertex are 2 of these n points. Consider n(n-1)/2 midpoints of these segments.
Is this true that these n(n-1)/2 points lie on a (n-2)-degree curve ?

It is true for n=4, the conic QA-Co1.

Best regards,

Vu Thanh Tung

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Message: #1192
Date: 2021-10-16
From: bernard.keizer@gmail.com
Subject: Re: Concurrent conics as transformation for Quadrilateral,

---

Dear Chris, dear Eckart
I 've found this memo, which I had put on the Forum many years ago !
May be it could be useful ...
Best regards
Bernard

Properties of the QL's inscribed in a given curve Cu1

The fundamental property of the QL's inscribed in a given curve Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curve.

All the QL's share :

- the Miquel point P1
- the Schmidt circle, the Steiner axes and the invariant points F1 and F2
- the Newton Line L1, parallel to the asymptote of Cu1 at half a distance (P1 being the railwaywatcher of the 2 lines)
- the direction of the Steiner Line L2
- the points B1 and B2 (bicursal curve) or R1 and R2 (unicursal curve)
- therefore the middle of these 2 points where the line L6 cuts the Newton Line
- the axis of the inscribed parabola (parallel to the Newton Line through the Miquel point)
- the axis of the cardioïd Qu1 (tangent to Cu1 in the Miquel point, through the point where Cu1 cuts it's asymptote, reflection of the axis of the parabola in the 1rst Steiner axis and locus of the point P3)
- the direction of the axes of the inscribed deltoïd Qu2, which is the direction of the asymptotes of the stelloïd Cu2
  the axes of the stelloïd trisect the oriented angle between the 1rst Steiner axis and the axis of the parabola or the oriented angle between the axis of the parabola and the axis of the cardioïd (it's easy to draw them with the Chasles construction directly on the Schmidt circle with a rectangular hyperbola through P1 centered in the middle of P1A, where A is a point where the axis of the parabola cuts the Schmidt circle, with axes of symmetry parallel to the bisectors of the angle between the axis of the parabola and the 1rst Steiner axis)
- the asymptote of the hessian as tangent to the inscribed deltoïd
- last, but not least, as discovered by Eckart and discussed in details by Bernard G., the centers of the 27 inscribed cardioïds lying on the stelloïd.

If we call R the fixed radius of the Schmidt circle, r the variable radius of the Miquel or the Hervey circle of a QL, p the distance from P1 to the Steiner Line and θi the angles between the lines Li and the fixed direction of the Steiner Line, we have for all inscribed QL's :

- Σθi equal to the angle between the axes of the parabola and the cardioïd or twice the angle between the 1rst Steiner axis and the axis of the parabola
- $R^2 = 2pr = 16\, r^2\, \Pi\cos\theta_i$ ($p = 8r\, \Pi\cos\theta_i$)

For a given reference QL, the inscribed conics and their confocals (of which the degenerated ones made of 2 conjugate points) form a tangential set. The QL's inscribed in the hessian are determined by the 4 common tangents to 2 conics of the set of these confocals (for example the 4 lines joining 2 pairs of conjugate points ; the reference QL is obtained by joining 2 pairs of opposite vertices)

Properties of the QL.pdf

Message: #1193
Date: 2021-10-16
From: eckart_schmidt@t-online.de
Subject: QL-L1

```
Dear all,

a QL has two obtuse angled trilateral components,
... their polar circles have QL-L1 as radical axis.

Best regards Eckart
```

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Message: #1194
Date: 2021-10-17
From: tungvtt@gmail.com
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

Dear Eckart and others,

Today I draw the configuration again and find some more properties.

Let us remind the configuration and previous found properties :
P1 P2 P3 P4 = a quadriangle.
X,Y = two points on the plane.
Bij = isogonal conjugate of Y w.r.t Δ X Pi Pj.

Then
(1) the midpoints of three segments Bij Bkl are collinear on a line d(Y,X, P1 P2 P3 P4). (#935)
(2) for each i, define circle Cli = (Bij Bik Bil) then 4 circles Cl1, Cl2, Cl3, Cl4 have a common point.
(3) for each i, define circle Cl'i = (Bjk Bkl Blj) then 4 circles Cl'1, Cl'2, Cl'3, Cl'4 also have a common point.

Here is the new ones:
(4) Let Oi be the circumcenter of Δ Bij Bik Bil (the center of Cli) then 4 points O1,O2,O3,O4 lie on a circle.
(5) Let O'i be the circumcenter of Δ Bjk Bkl Blj (the center of Cl'i) then 4 points O'1,O'2,O'3,O'4 lie on a circle.
(6) Let Hi be the orthocenter of Δ Bij Bik Bil) then 4 points H1,H2,H3,H4 lie on a line.
(7) Let H'i be the orthocenter of Δ Bjk Bkl Blj then 4 points H'1,H'2,H'3,H'4 lie on a line.
(8) The lines defined in (6) and (7) are parallel.

I tested the hypotheses if 6 points Bij are midpoints of XjXi, or intersection of Lj, Lj for a QA X or a QL L ,
so the above properties can be explained by known QA ones, or QL ones.
But these hypotheses seem false.

Best regards,
Vu Thanh Tung

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Message: #1195
Date: 2021-10-17
From: eckart_schmidt@t-online.de
Subject: New QG-point

```
Dear all,

let QG = ABCD be a quadrigon with AB^CD = E and AD^BC = F:
The Apollonius circles of
... ABF through F, BCE through E, CDF through F, DAE through E
... have a common point of their radical axes
... and a common orthogonal circle round this point,
... if it is outside the Apollonius circles.

Best regards Eckart
```

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Message: #1196
Date: 2021-10-18
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

```
Dear Vu Thanh Tung,

if you consider the transformation
... Y --> common point of (3) in #1194 for X = QA-P4,
... you get QA-Tf16.

Best regards Eckart
```

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Message: #1197
Date: 2021-10-18
From: tungvtt@gmail.com
Subject: Re: A transformation on quadriangle with midponts of isogonal

---

```
Dear Eckart,

It is very unexpected observation.

Best regards,
Vu Thanh Tung

On Mon, Oct 18, 2021 at 12:46 AM, Eckart Schmidt wrote:
> Dear Vu Thanh Tung,
> if you consider the transformation
> ... Y --> common point of (3) in #1194 for X = QA-P4,
> ... you get QA-Tf16.
> Best regards Eckart
```

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Message: #1198
Date: 2021-10-19
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Eckart, Bernard, Chris,

As already mentioned in the previous message, I introduce a note
on the r-angle center.

[Generalization of QL-Ci3](https://drive.google.com/file/d/1TPIH
85VhC79E6Ok2XKJAidh58uNH1j8n/view?usp=sharing)

On this note, I suggest that the term r-angle center should not
be used. Moreover, I give an alternative statement for the
generalization of the Miquel circle - Steiner line,
Kantor-Hervey point.

Ngo Quang Duong
```

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Message: #1199
Date: 2021-10-20
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Eckart
Thanks for your interest
I'll try to gather some figures, just be patient !
But my construction is explained in the messages 1113, 1121,
1129 (with figures) and 1142 (without figure)
I don't find the generalisation of the Miquel circle in your
attached file ?
I'm impatiently waiting, as it was precisely the object of my
exchanges with Eckart to find a generalisation of the Miquel,
Steiner and Hervey circles...
Best regards
Bernard
```

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Message: #1200
Date: 2021-10-20
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Keizer,

My apologies. I copied the wrong link - it is the link in #1161

Here is the link [Google Drive: generalization of
QL-Ci3](https://drive.google.com/file/d/10nKVZyV20X7kBShQEHdsO90
0wGjWP2oR/view?usp=sharing)

Ngo Quang Duong
```

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Message: #1201
Date: 2021-10-21
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

if I am not wrong, there is a typo:
... in 2.3(6), O4 (?) has to be replaced by another point.
But replacing O4 by O2 I also didn't succeed
... in concyclic O1,O2,O3,O4.

Best regards Eckart
```

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Message: #1202
Date: 2021-10-21
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral-Ngo

---

```
Dear Ngo Quang Duong,

excuse, the construction 2.3. holds , if you replace O4 in (6)
by O2.

Best regards Eckart
```

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Message: #1203
Date: 2021-10-21
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

excuse once more my uncertainness,
... replacing in 2.3 (6) only O4 by O2 doesn't lead to
    concyclic Oi.

Best regards Eckart
```

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Message: #1204
Date: 2021-10-21
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Eckart,

I am so sorry for that. Those were my typos.

I have updated the file on [google drive: generalization of
QL-Ci3](https://drive.google.com/file/d/10nKVZyV20X7kBShQEHds090
0wGjWP2oR/view). This is still the old link, but you can see my
update. The construction steps should be true now.

However, for sure, I also share my [GeoGebra
file](https://drive.google.com/file/d/10JqaBLuYki2JJyir7zJ-LNcIm
wiNget5/view?usp=sharing).

Ngo Quang Duong
```

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Message: #1205
Date: 2021-10-22
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong,

now I succeeded in constructing the four Oi.
Here two additions to 3.1:

The circumcircles O1P23P34, O2P13P34, O4P13P23
... have a common point C3 on the circle O1O2O3O4
... with QL-Tf1(C3) isogonal conjugate of O3 wrt l1, l2, l4.

The circumcircles O2P13P14, O3P12P14, O4P12P13
... are cocurrent in a point on the circumcircle O1O2O3O4,
... the 2nd intersection with the line O1.O2'.O3'.O4',
... independent of the choice of O2.

Best regards Eckart
```

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Message: #1206
Date: 2021-10-22
From: archanajha112018@gmail.com
Subject: Regular polygon with respect to two other polygon

Hello Friends , my name is jayendra Jha and my friend name is Sankalp savaran and we both are new on this Quadrilateral group and we don't know that there is a separate group for polygon . Thanks for making such group. Here is the link of Figure :
____
https://1drv.ms/w/s!AkhDFBaJJuungRYx-92SuXTTAVG1

____
Description

____
See Figure 5.0:
There is two Square OA1A2A3 & OB1B2B3 &Let m(A3,B3) = m1 and Let m(A1,B1) =m2 & reflect point O through m2&m1 at O1&O2 then we get O1O2A2B2 as Square.

____
See Figure 6.0
Let B1OB2 & A1OA2 be two equilaterial triangle and Let m(A2,B2) =X and Reflect point O through X at O1 then A1B1O will be new Equilaterial triangle.

____
My question: We have problems in generalising it for Pentagon, Hexagon etc . We just able to find them in Square and Equilaterial triangle

____
My next question: Is this is finded before?

____
Best regards
Jayendra Jha and sankalp savaran.

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Message: #1207
Date: 2021-10-22
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Regular polygon with respect to two

---

Hello friends , I am again sending the same link here :
https://1drv.ms/w/s!AkhDFBaJJuungRYx-92SuXTTAVG1

On Fri, 22 Oct 2021, 7:49 pm ARCHANA JHA via groups.io,
<archanajha112018gmail.com@groups.io> wrote:
> *Hello Friends , my name is jayendra Jha and my friend name is Sankalp
> savaran and we both are new on this Quadrilateral group and we don't know
> that there is a separate group for polygon . Thanks for making such group.
> Here is the link of Figure
> :________https://1drv.ms/w/s!AkhDFBaJJuungRYx-92SuXTTAVG1
> <https://1drv.ms/w/s!AkhDFBaJJuungRYx-92SuXTTAVG1>*__________
> Description
> ___________
> See Figure 5.0:
> There is two Square OA1A2A3 & OB1B2B3 &Let m(A3,B3) = m1 and Let m(A1,B1)
> =m2 & reflect point O through m2&m1 at O1&O2 then we get O1O2A2B2 as Square.
> _____________
> See Figure 6.0
> Let B1OB2 & A1OA2 be two equilaterial triangle and Let m(A2,B2) =X and
> Reflect point O through X at O1 then A1B1O will be new Equilaterial
> triangle.
> __________
> My question: We have problems in generalising it for Pentagon, Hexagon etc
> . We just able to find them in Square and Equilaterial triangle
> _____________
> My next question: Is this is finded before?
> ________
> Best regards
> Jayendra Jha and sankalp savaran.

Message: #1208
Date: 2021-10-22
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Eckart
If I understand correctly, you also generalise the Steiner Line
(which is a circle) as QL-Tf1(Miquel circle) = Steiner circle.
D4 is is the isogonal of O4 wrt l1,l2,l3 and C4 (on the Miquel
circle) is QL-Tf1(D4) ...
Beautiful work, indeed !
Best regards
Bernard
PS What about the Hervey circle centered in the Kantor-Hervey
point ?
```

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Message: #1209
Date: 2021-10-22
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Eckart, dear Bernard

To Eckart: I haven't noticed that property. Indeed, the common
points of the circle O1.O2.O3.O4 and the line O1.O'2.O'3.O'4 are
O1 and the common point of the circumcircles of triangles
O2.P14.P13, O3.P12.P14, O4.P12.P13.

To Bernard: Yes, this generalizes both the Miquel circle and
Steiner line. However, I have no idea to generalize the Hervey
circle yet. Moreover, I think this generalization of the Miquel
circle and Steiner line is too verbose.

Ngo Quang Duong
```

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Message: #1210
Date: 2021-10-23
From: eckart_schmidt@t-online.de
Subject: QL-Ci3

```
Dear Chris,

can you please prove the following property of QL-Ci3 in EQF:
    The 2nd intersection point of QL-Ci3
    <http://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3>
    with the
    circumcircle of a Component Triangle Li.Lj.Lk
    is Incenter or Excenter of the Triangle formed by the
    Reflections of
    Li, Lj, Lk in Ll, where (i,j,k,l) ∈ (1,2,3,4).
    See Ref-
    <http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-
    mathematics/quadrangle-objects/188-references.html>
     33 Anopolis message # 431.  †)
I think, it doesn't hold.

Best regards Eckart
```

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†) Editorial note: The referenced message(s) can be found as listed in EPG-References [33].

Message: #1211
Date: 2021-10-23
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] QL-Ci3

On Sat, Oct 23, 2021 at 10:34 AM Eckart Schmidt
<eckart_schmidt@t-online.de>
wrote:

> Dear Chris,
> can you please prove the following property of QL-Ci3 in EQF:
> *The 2nd intersection point of QL-Ci3
> <http://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3>
with the
> circumcircle of a Component Triangle Li.Lj.Lk
<http://Li.Lj.Lk>*
> * is Incenter or Excenter of the Triangle formed by the
Reflections of Li,
> Lj, Lk in Ll, where (i,j,k,l) ∈ (1,2,3,4). *
> *See Ref-
> <http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html>33
> Anopolis message # 431.*
> I think, it doesn't hold.
> Best regards Eckart

Dear Eckart

I do not know what this says, but the reflection theorem for a
triangle is this
Let ABC be a triangle and L a line.
The reflections of L in BC, CA, AB bound a triangle A*B*C*
The incenter or an excenter of A*B*C* lies on the circumcircle.
It is the perspector of ABC, A*B*C*.

Note
If L passes through the orthocenter H, the reflections concur
(on the circumcircle)

APH

Message: #1212
Date: 2021-10-23
From: eckart_schmidt@t-online.de
Subject: Re: QL-Ci3

```
Dear Andreas,

thanks for your clearance,
... the right version will be:
    The 2nd intersection point of QL-Ci3
    <http://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3>
    with the
    circumcircle of a Component Triangle Li.Lj.Lk
    <http://li.lj.lk/>
    is Incenter or Excenter of the Triangle formed by
    the Reflections of Ll in the sidelines of Li, Lj, Lk , where
    (i,j,k,l)
    ∈ (1,2,3,4).
    See Ref-
    <http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-
    mathematics/quadrangle-objects/188-references.html>
     33 Anopolis message # 431.

Best regards Eckart
```

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Message: #1213
Date: 2021-10-23
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Bernard,

here is  a simple construction of the generalized Miquel circle:

Starting with O1 and its isogonal conjugate O1* wrt l2,l3,l4,
P23,P34,P24,
... then X = QL-Tf1(O1*) is a point of the generalized Miquel
    circle.
Choose O2 on the circle through X,P13,P14
... and take the circle through X,O1,O2.

The 2nd intersection of the circles (X,O1,O2) and (X,P12,P14)
is O3.
The 2nd intersection of the circles (X,O1,O2) and (X,P12,P13)
is O4.

Best regards Eckart
```

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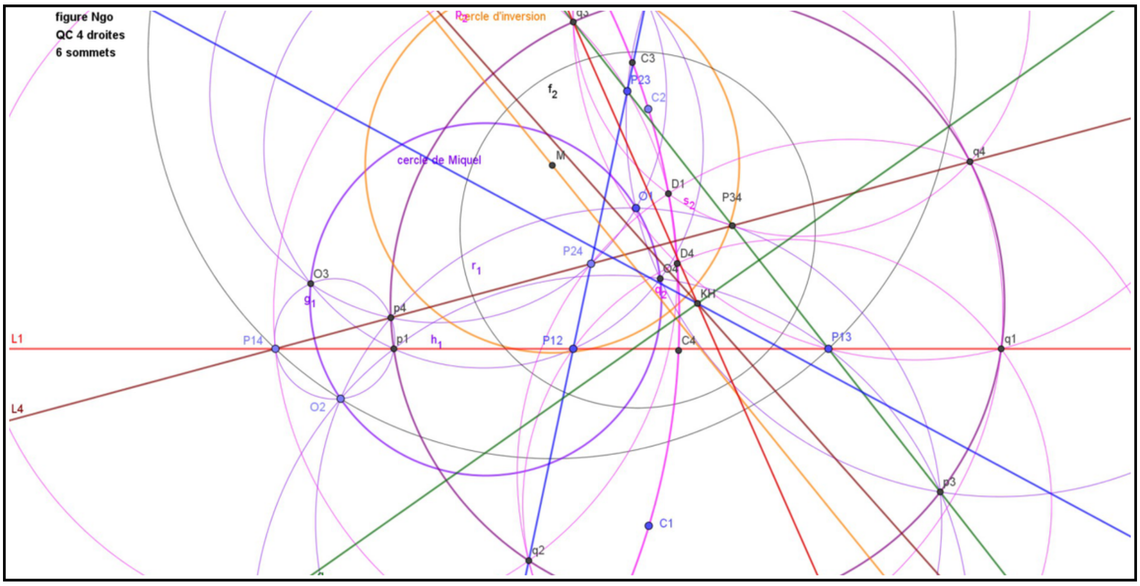
Message: #1214
Date: 2021-10-23
From: bernard.keizer@gmail.com
Subject: Re: Steiner’s 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Ngo Quang Duong, dear Eckart
I reproduced your construction of the Miquel circle and it's CSC
the Steiner circle.
I added the Hervey circle centered in the Kantor-Hervey point
where the perpendicular bisectors of OiDi concur.
I used my construction given in my previous messages already
mentionned 1113, 1121, 1129 and 1142.
The 6 circles through AijOkOl give the points pi and the 6
circles through AijDkDl give the points Qi.
pi and qi are the intersections of the line Li with the Hervey
circle.
I suppose this Hervey circle is the inner circle of an epi- or
hypocycloïd tangent to the 4 lines.
The ratio will be given by comparing the angles, for example,
p1KHp4 and q1KHq4 ...
Best regards
Bernard
```



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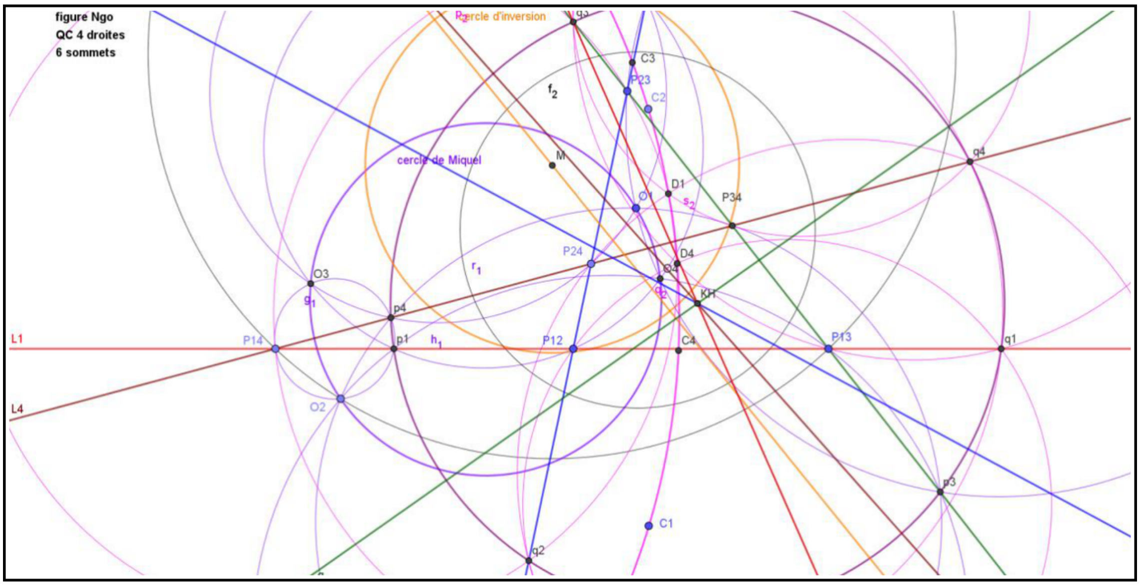
Message: #1215
Date: 2021-10-23
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Sorry, I forgot to convert the file from Word to pdf
Perhaps better this way
Best regards
Bernard
```



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Message: #1216
Date: 2021-10-24
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Ngo quang Duong, dear Eckart
For n integer, you have only one n-angle center in each triangle, only one point KH and one Hervey circle and one curve tangent to the 4 lines.
For n rational, n = p/q with p and q integers, there are a finite number of n-angle centers in each triangle, of points KH, of Hervey circles and of curves tangent to the 4 lines.
(8 astroïds, 27 cardioïds, 64 nephroïds ...)
For n not rational, there are an infinity of n-angle centers in each triangle and an infinity of points KH and of Hervey circles and the curves have an infinity of axes of symmetry and of cusps (on a circle with finite radius), which makes them not really interesting ...
I think you have found a way to generate these kind of n-angle centers, Miquel, Steiner and Hervey circles and the related curve.
Congrtulations
Best regards
Bernard
PS The points Ci on my figure are wrong, they are the CSC of the Di on the Miquel circle (and not the CSC of the Oi on the Steiner circle

---

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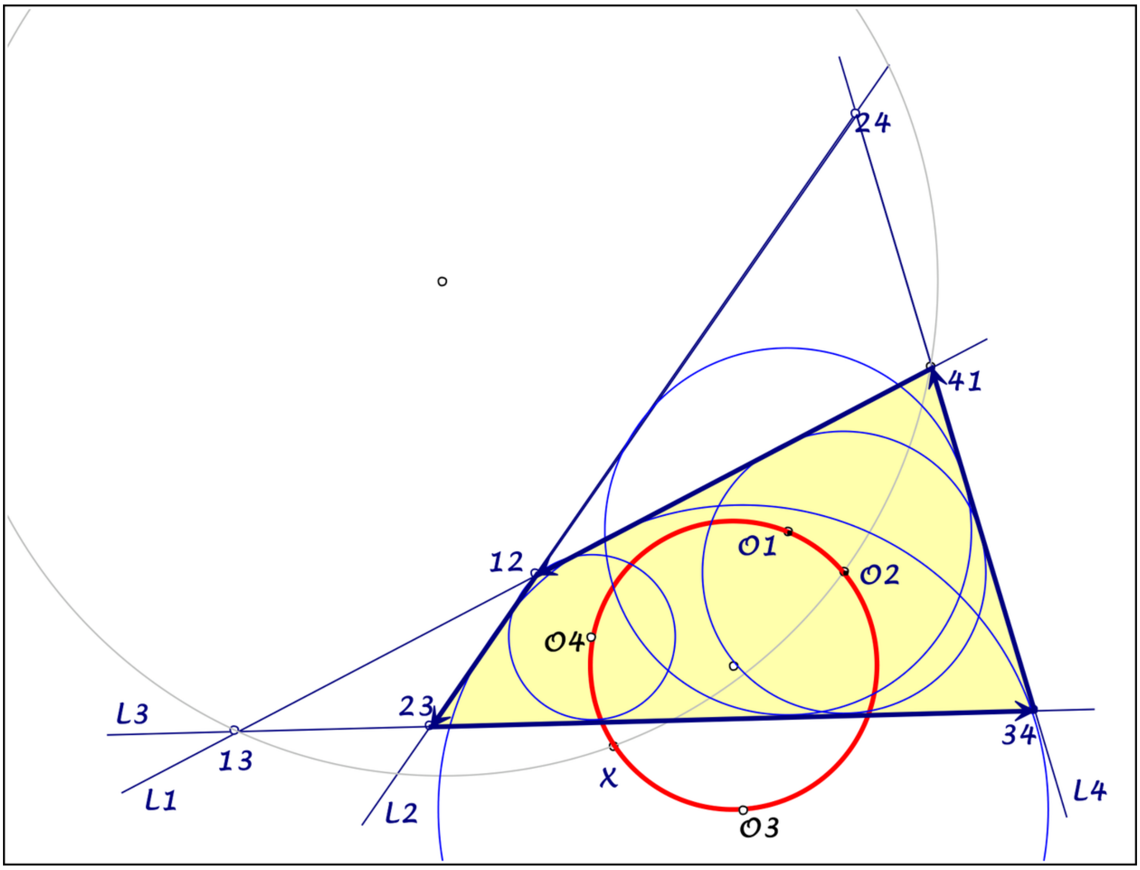
Message: #1217
Date: 2021-10-24
From: eckart_schmidt@t-online.de
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Ngo Quang Duong,

here is a connection of your "Steiner-paper" and your "Miquel-paper":
Consider a QL with its convex QG = P12P23P34P41,
... so that the QG is inside the triangles P12P14P24 and P12P13P23.
Take in the sense of your "Miquel-paper"
... the incenter of the 1st triangle as O1
... and the incenter of the 2nd triangle as O2,
... then your generalized Miquel circle is a Steiner circle,
... in the sense of your "Steiner-paper"
... for lines with directions round the QG.

Best regards Eckart



2021-10-24.pdf

Message: #1218
Date: 2021-10-24
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

```
Dear Eckart, dear Ngo Quang Duong
As alreaady mentionned separately by Chris and by me, the 8
Steiner circles described in EQF are at the same time Miquel and
Steiner circles in the sense of Ngo Quang Duong and me, as they
are CSC invariant.
At the next step (inverse in circumcircle of in- and excenters
and their isogonals), you find the 8 Miquel circles and the 8
Steiner circles, which are 8 copples of CSC partners and the 8
Kantor-Hervey points (on a rectangular hyperbola) and the 8
Hervey circles, which are the inner circles of the 8 astroïds
tangent to the 4 lines.
Best regards
Bernard
```

← Previous Next → ↔ Message Index ↑ Subjects

Message: #1219
Date: 2021-10-25
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart, dear Bernard,

Indeed, the generalization of the Miquel circle in my note also generalizes 8 Steiner circles. I did not notice that fact.

Currently, I am studying the messages in the topic [Complex coordinates](https://groups.io/g/Quadri-and-Poly-Geometry/topic/complex_coordinates/84769387) and I feel really overwhelmed. This is going to take quite of a time.

Perhaps, I have to re-produce those coordinates calculations. If I am not mistaken, this is how Bernard's friend chose complex coordinates:

- The Miquel circle is the unit circle.
- 4 circumcenters of 4 component triangles lie on the unit circle, namely O1(a1), O2(a2), O3(a3), O4(a4). (a1, a2, a3, a4 are complex numbers, whose magnitude equal 1)
- The Miquel point QL-P1 has affix M(1) (M lies on the unit circle as well).
- Pij is the reflection of M in OiOj.

The hypocycloids, which are mentioned in the topic "Complex coordinates" have (2n+1) cusps. The shape of a hypocycloid should be determined by the ratio of two rolling circles, instead of the number of cusps. I guess, is it (2n + 1)/(2n - 1) Have you determined that ratio?

The problem, as I see so far, is that **the Hervey circle is defined as the inner circle of the hypocycloid that tangent to 4 lines of the QL**. There should be an alternative construction and definition, which is independent of the hypocycloid, for the Hervey circle.

Best regards,

Ngo Quang Duong

Message: #1220
Date: 2021-10-26
From: bernard.keizer@gmail.com
Subject: Re: Steiner's 8th theorem on the angle bisectors in Quadrilateral

---

Dear Eckart, dear Ngo Quang Duong
1) The use of complex coordinates was more a curiosity and doesn't bring new properties. My friend found only all the properties already mentionned in EQF for the points QL-P28 and P29 and proved with barycentric coordinates ...
2) I use the tangential definition of epi- and hypocycloïds (2 points describing the same circle at different speeds p and q ; the number of cusps is p + q)
3) I was curious and reproduced the simple construction mentionned by Eckart in the message 1213.
I've put O1 in the real circumcenter and D1 in the real orthocenter of the triangle P23P24P34. Then O2 is not necessary the real circumcenter and the same for O3 and O4.
What is then the meaning of this QA O1O2O3O4 ? The perpendicular bisector of O1H1 is the locus of the deltoïds inscribed in the triangle ...
Best regards
Bernard

← Previous Next → ↔ Message Index ↑ Subjects

Message: #1221
Date: 2021-10-26
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Steiner’s 8th theorem on the angle bisectors in Quadrilateral

---

Dear Bernard, dear Eckart,

> What is then the meaning of this QA O1O2O3O4

This question of Bernard is indeed the key.
I have managed to "connect" the QA O1O2O3O4 (in the generalization) and the QA formed by 4 circumcenters of 4 component triangles. I wrote it as an equivalent statement of the generalization.

Best regards,
Ngo Quang Duong

Generalization of QL-Ci3 (Miquel circle)

Ngo Quang Duong

October 27, 2021

1 QL-Ci3

Given quadrilateral $\ell_1\ell_2\ell_3\ell_4$.

O_1 is the circumcenter of $\ell_2\ell_3\ell_4$.

O_2 is the circumcenter of $\ell_1\ell_3\ell_4$.

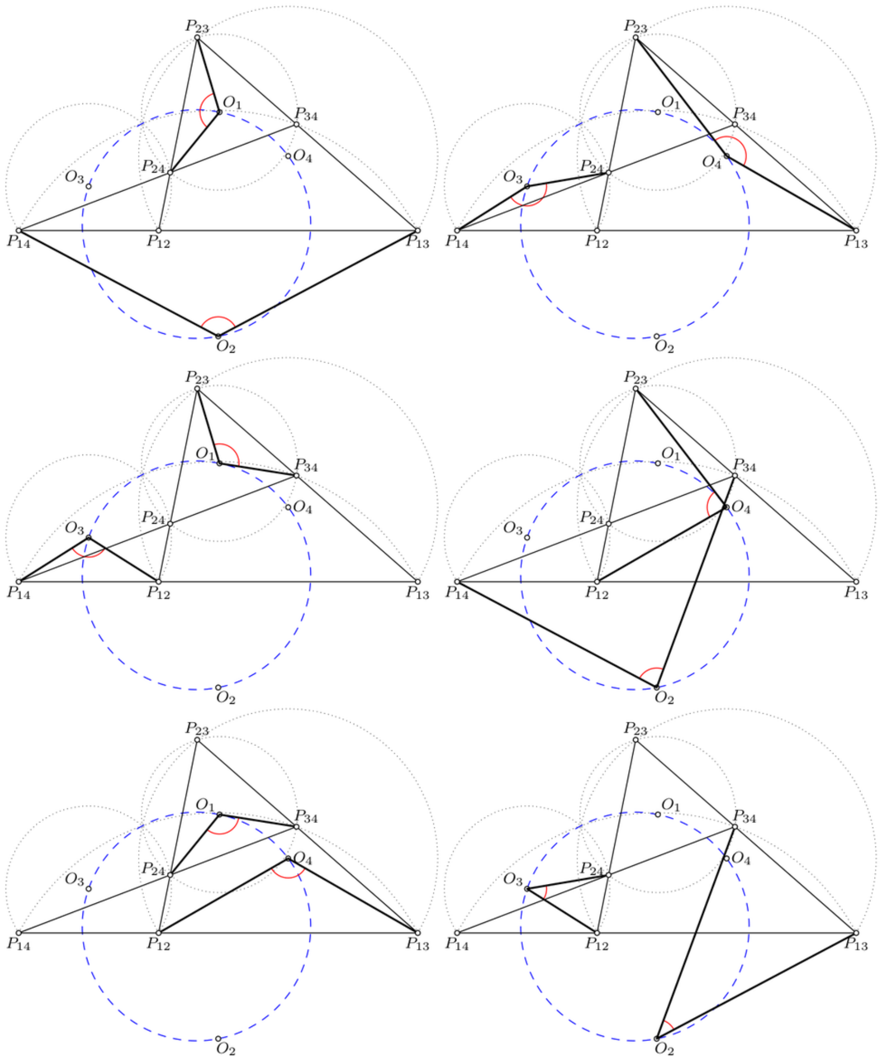
O_3 is the circumcenter of $\ell_1\ell_2\ell_4$.

O_4 is the circumcenter of $\ell_1\ell_2\ell_3$.

Then six following relations (using oriented(directed) angles) hold true:

$$(O_1P_{23}, O_1P_{24}) \equiv -(O_2P_{14}, O_2P_{13}) \pmod{\pi} \quad (1)$$
$$(O_3P_{24}, O_3P_{14}) \equiv -(O_4P_{13}, O_4P_{23}) \pmod{\pi} \quad (2)$$
$$(O_1P_{23}, O_1P_{34}) \equiv -(O_3P_{14}, O_3P_{12}) \pmod{\pi} \quad (3)$$
$$(O_2P_{14}, O_2P_{34}) \equiv -(O_4P_{23}, O_4P_{12}) \pmod{\pi} \quad (4)$$
$$(O_1P_{24}, O_1P_{34}) \equiv -(O_4P_{13}, O_4P_{12}) \pmod{\pi} \quad (5)$$
$$(O_2P_{13}, O_2P_{34}) \equiv -(O_3P_{24}, O_3P_{12}) \pmod{\pi}. \quad (6)$$

generalization-of-ql-ci3.pdf



2

generalization-of-ql-ci3.pdf

2 Generalization

2.1 Comments

There have been a generalization by using r-angle center.

Circumcenter is 2-angle center, since $(OB, OC) \equiv 2(AB, AC) \pmod{\pi}$, where O is the circumcenter of triangle ABC.

Orthocenter is (-1)-angle center.

Point at infinity (in extended complex plane) is 0-angle center.

The former generalization for Miquel circle is:

> "Four r-angle centers of four component triangles of a quadrilateral are concyclic or collinear."

r-angle centers work really well for integer r, since for each integer r, there exists exactly one r-angle center.

However, if r is a non-integer, let's say, rational, then everything starts becoming chaotic.

For example, let $r = \frac{1}{2}$, we obtain four r-angle centers, which are the incenter and three excenters. Then, we get 8 Steiner circle[1], and there is no consistent (does not depend on the figure), and simple method to determine which quadruples of in/excenters are concyclic, in general.

I don't know what would happen if r is irrational. It could be a disaster, according to the rational (and non-integer) case. The term r-angle center, for real number r, is NOT a safe and rigorous way to generalize the Miquel circle.

2.2 Alternative statements

Statement 1

I suggest a more verbose statement for the generalization of the Miquel circle.

Given quadrilateral $\ell_1\ell_2\ell_3\ell_4$.

O_1, O_2, O_3, O_4 are four points such that:

$$(O_1P_{23}, O_1P_{24}) \equiv -(O_2P_{14}, O_2P_{13}) \pmod{\pi} \tag{7}$$
$$(O_3P_{24}, O_3P_{14}) \equiv -(O_4P_{13}, O_4P_{23}) \pmod{\pi} \tag{8}$$
$$(O_1P_{23}, O_1P_{34}) \equiv -(O_3P_{14}, O_3P_{12}) \pmod{\pi} \tag{9}$$
$$(O_2P_{14}, O_2P_{34}) \equiv -(O_4P_{23}, O_4P_{12}) \pmod{\pi} \tag{10}$$
$$(O_1P_{24}, O_1P_{34}) \equiv -(O_4P_{13}, O_4P_{12}) \pmod{\pi} \tag{11}$$
$$(O_2P_{13}, O_2P_{34}) \equiv -(O_3P_{24}, O_3P_{12}) \pmod{\pi}. \tag{12}$$

Then O_1, O_2, O_3, O_4 are concyclic or collinear.

Statement 2

Given quadrilateral $\ell_1\ell_2\ell_3\ell_4$.

J_1, J_2, J_3, J_4 are the circumcenters of the component triangles, then $\triangle J_1J_2J_3 \sim \triangle\ell_1\ell_2\ell_3$, $\triangle J_1J_2J_4 \sim \triangle\ell_1\ell_2\ell_4$, $\triangle J_1J_3J_4 \sim \triangle\ell_1\ell_3\ell_4$, $\triangle J_2J_3J_4 \sim \triangle\ell_2\ell_3\ell_4$.

K_1, K_2, K_3, K_4 are four points such that:

$$(K_1J_3, K_1J_4) \equiv (K_2J_3, K_2J_4) \pmod{\pi}, \tag{13}$$
$$(K_1J_4, K_1J_2) \equiv (K_3J_4, K_3J_2) \pmod{\pi}, \tag{14}$$
$$(K_1J_2, K_1J_3) \equiv (K_4J_2, K_4J_3) \pmod{\pi}, \tag{15}$$
$$(K_2J_3, K_2J_1) \equiv (K_4J_3, K_4J_1) \pmod{\pi}, \tag{16}$$
$$(K_3J_1, K_3J_2) \equiv (K_4J_1, K_4J_2) \pmod{\pi}, \tag{17}$$
$$(K_2J_1, K_2J_4) \equiv (K_3J_1, K_3J_4) \pmod{\pi}. \tag{18}$$

O_1, O_2, O_3, O_4 are four points such that:

$$J_1J_2J_3.K_4 \sim \ell_1\ell_2\ell_3.O_4,$$
$$J_1J_2J_4.K_3 \sim \ell_1\ell_2\ell_4.O_3,$$
$$J_1J_3J_4.K_2 \sim \ell_1\ell_3\ell_4.O_2,$$
$$J_2J_3J_4.K_1 \sim \ell_2\ell_3\ell_4.O_1.$$

[1] `https://chrisvantienhoven.nl/ql-items/ql-mult-pts-lns/ql-8p1`

Then O_1, O_2, O_3, O_4 are concyclic.
Besides, K_1, K_2, K_3, K_4 are concyclic (this follows Miquel's six circles theorem).

2.3 How to construct four such points?

The main idea of this construction is constructing parallel lines to create equal oriented angles.
I suggest a quite simple construction as follow:

(1) Pick a random point O_1, which does not coincide with any vertices of the quadrilateral.

(2) Construct O'_2 such that $O'_2P_{14} \parallel O_1P_{24}$, $O'_2P_{13} \parallel O_1P_{23}$.

(3) Construct O'_3 such that $O'_3P_{12} \parallel O_1P_{23}$, $O'_3P_{14} \parallel O_1P_{34}$.

(4) Construct O'_4 such that $O'_4P_{12} \parallel O_1P_{24}$, $O'_4P_{13} \parallel O_1P_{34}$.

(5) Pick a random point O_2 on the circumcircle of triangle $O'_2P_{14}P_{13}$.

(6) Construct O''_3 such that $O''_3P_{12} \parallel O_2P_{13}$, $O''_3P_{24} \parallel O_2P_{34}$.

(7) Construct O''_4 such that $O''_4P_{23} \parallel O_2P_{34}$, $O''_4P_{12} \parallel O_2P_{14}$.

(8) O_3 is the common point other than P_{12} of the circumcircles of triangles $O''_3P_{12}P_{24}$ and $O'_3P_{12}P_{14}$.

(9) O_4 is the common point other than P_{12} of the circumcircle of triangles $O'_4P_{12}P_{13}$ and $O''_4P_{12}P_{23}$.

3 Other properties

3.1 Relation with QL-Tf1

- The circumcircles of triangles $O_1P_{24}P_{34}$, $O_3P_{14}P_{24}$, $O_2P_{14}P_{34}$ are concurrent at a point on the circle that passes through O_1, O_2, O_3, O_4. Let the point of concurrence be C_4.
- QL-Tf1(C_4) is the isogonal conjugate of O_4 with respect to triangle $\ell_1\ell_2\ell_3$.

3.2 Generalization of Kantor-Hervey point

D_1 is the isogonal conjugate of O_1 with respect to triangle $\ell_2\ell_3\ell_4$.
D_2 is the isogonal conjugate of O_2 with respect to triangle $\ell_1\ell_3\ell_4$.
D_3 is the isogonal conjugate of O_3 with respect to triangle $\ell_1\ell_2\ell_4$.
D_4 is the isogonal conjugate of O_4 with respect to triangle $\ell_1\ell_2\ell_3$.
If $D_1 \not\equiv O_1$, $D_2 \not\equiv O_2$, $D_3 \not\equiv O_3$, $D_4 \not\equiv O_4$, then the perpendicular bisectors of O_1D_1, O_2D_2, O_3D_3, O_4D_4 are concurrent.

3.3 Further ideas

I haven't try this yet, but I believe that, this generalization might lead to a generalization of the nL-Center Circle (nL-n-Ci1) as well.

generalization-of-ql-ci3.pdf

Message: #1222
Date: 2021-10-27
From: eckart_schmidt@t-online.de
Subject: Re: Steiner’s 8th theorem on the angle bisectors in Quadrilateral

---

Dear Ngo Quang Duong,

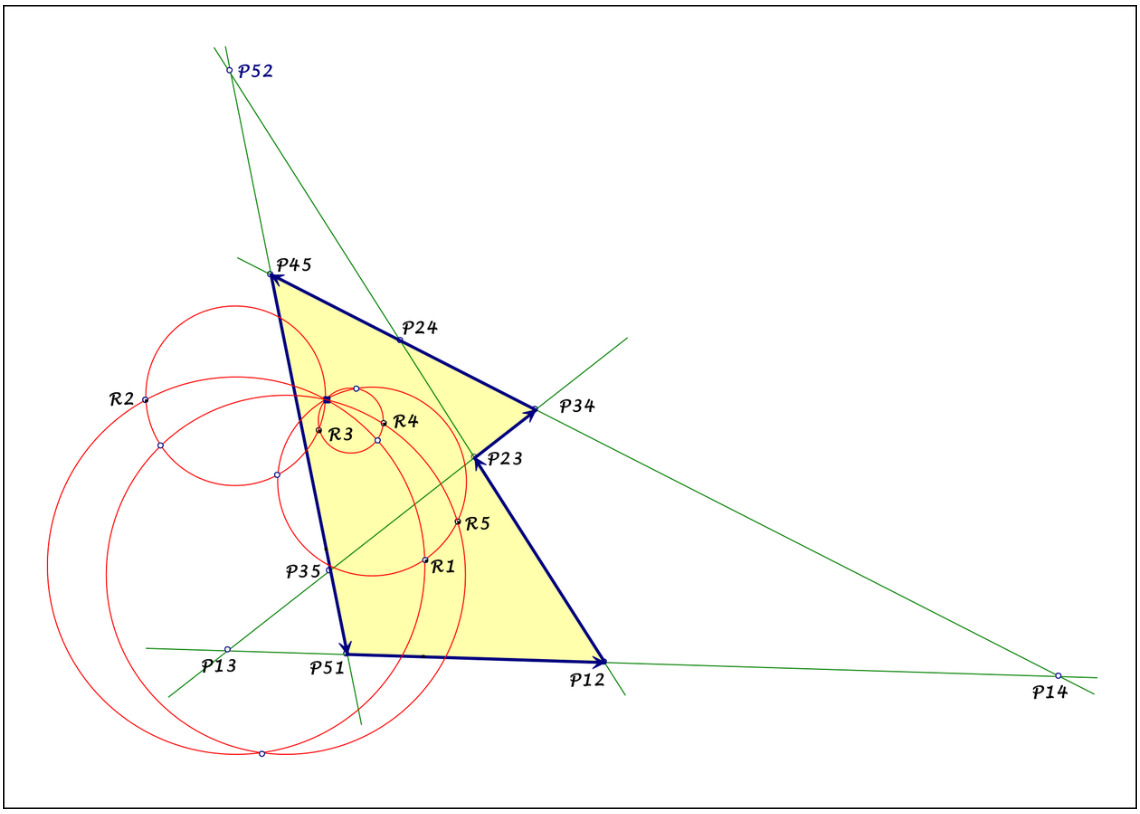
in #1161 you consider a nL with directions for its lines,
... changing all directions will be the same case.
... For a trilateral we get 4 possibilities,
... for a QL we get 8, for a nL we get 2^(n-1) possibilities.
In theorem 3 you describe wrt a trilateral
... a "generalized incenter" for each possibility of the
    directions,
... which give the in- and excenters of the trilateral.
... If the directions lead round the trilateral,
    we get the incenter,
... else the excenter wrt the false directed line.
In theorem 5 you describe wrt a quadrilateral
... a circle for each possibility of the directions,
... which give the 8 Steiner circles.
Using the directions of a 5L for its 5 QL,
... we get 5 circles with a common point
    (and concyclic centers).
Using the directions of a 6L for its 6 5L,
... we get 6 concyclic points for a circle.

If we take special directions for the lines of a nL,
... we get new points for odd n and new circles for even n.
Let these special directions for Li be defined by the vectors
Pi-1,i.Pi,i+1.
For a triangle we get the incenter.
For a QL we get a circle, CSC-invariant,
... centered on the 1st Steiner axis,
... orthogonal to the inversion circle of CSC ...
... For a QL with convex QG = P12P23P34P41 this circle bears
... ... the excenters of the two QG-outside triangles
... ... and the incenters of the other two triangles.
Using the special directions of a 5L also for its 5 QL,
... we get 5 circles with a common point
    (and concyclic centers).
     ... Construction (see attached file):
     ... Let Ri be the generalized incenters of the trilaterals
         Li-1LiLi+1,
     ... then consider the circles
     ... Ci12 = (R1,R2,R1P52^R2R3),
     ... Ci23 = (R2,R3,R2P13^R3R4),
     ... Ci34 = (R3,R4,R3P24^R4R5),
     ... Ci45 = (R4,R5,R4P35^R5R1),

```
     ... Ci51 = (R5,R1,R5P41^R1R2),
     ... which have a common point.
 Using the special directions of a 6L also for the 6 5L,
... we can analog consider circles (see attached file)
... Ci12 = (R1,R2,R1P62^R2R3), ... ,
    Ci61 = (R6,R1,R6P51^R1R2),
... the 2nd intersections of neighbored circles are concyclic.

These constructions for n=5 and n=6 don't hold for n > 6.

Best regards Eckart
```



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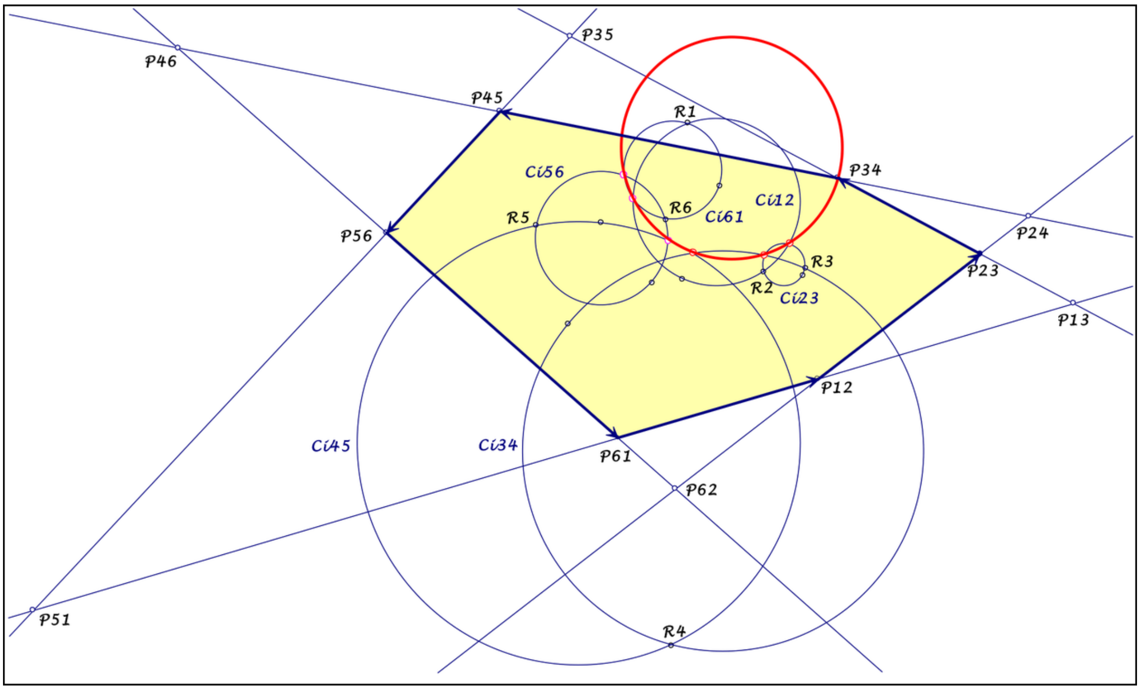

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Message: #1223
Date: 2021-10-29
From: oaidt.evnpsc@gmail.com
Subject: Do you known this theorem on tangential quadrilateral before?

```
Dear Geometers,

Let A B C D be a tangential quadrilateral, P be arbitrary point
in the plane then two red lengths are equal. See the Figure in:

https://mathoverflow.net/questions/407395/does-this-theorem-on-t
angential-quadrilateral-have-a-name

Best regards
Dao Thanh Oai
```

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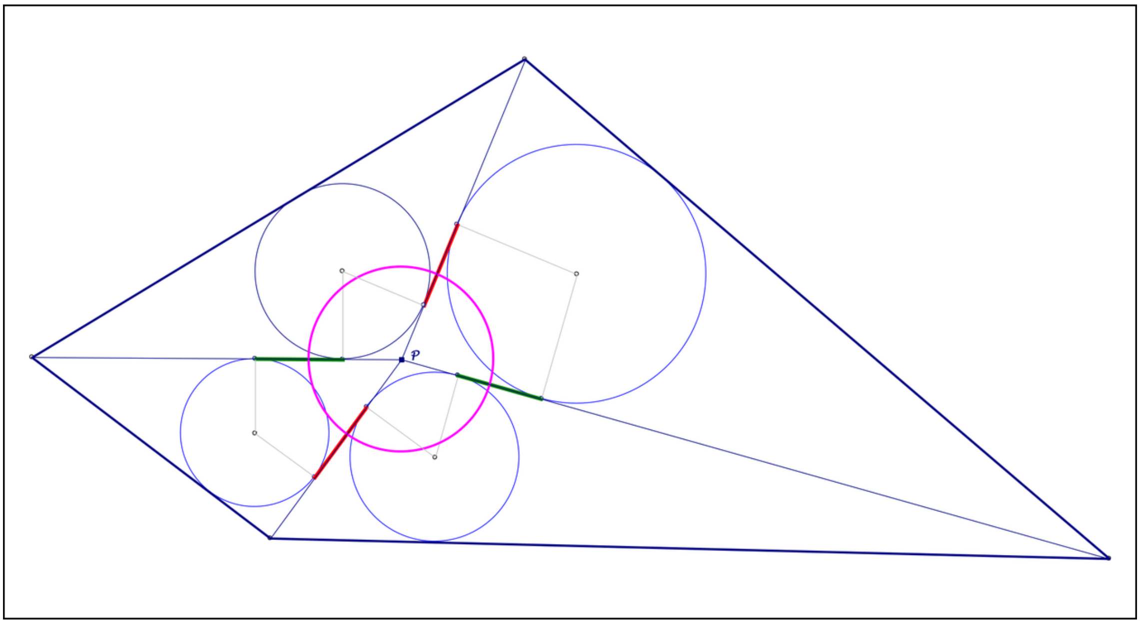
Message: #1224
Date: 2021-10-30
From: eckart_schmidt@t-online.de
Subject: Re: Do you known this theorem on tangential quadrilateral before?

Dear Dao Thanh Oai,

here is another aspect wrt your constellartion
... of 4 incircles for a quadrangle wrt a point P,
... perhaps interesting for Chris,
... for it leads to a new QA-point for EQF:
Consider an arbitrary QA as attached:
There is a point P = QA-Px,
... so that the red and green lines have the same length
... with the same inversion circle wrt P.
What about this point?

Best regards Eckart
<https://groups.io/g/Quadri-and-Poly-Geometry/message/1223>



2021-10-30.pdf

Message: #1225
Date: 2021-10-30
From: bernard.keizer@gmail.com
Subject: n-angle centers revisited

Dear Ngo Quang Duong, dear Eckart
Thanks to Ngo Quang Duong construction, I think I now understand better these n-angle centers and the Miquel, Steiner and Hervey circles.
In particular, I'm able to answer my own question in message 1220.
A QL has 4 triangles and 12 angles, which are in 6 pairs either equal or supplementary.
For a triangle, a n-angle center is a point from which the 3 sides are seen under angles equal to n times the angles opposed to the sides.
For n integer, there is only one point.
If we take the circumcenter, we must have on Ngo Quand Duong's figure P12O3P24 = P13O2P34, which a necessary, but not sufficient condition !
We also must have P12O3P24 = 2*P12P14P24 = 2*P13P14P34 = P132P34, which is not the case if we take O2 randomly on the circle through X, P13, P14 in Eckart's construction. In this case, O1 is the real circumcenter, but O2, O3 and O4 are only pseudocircumcenters and we can't have the QL's deltoïd.
For n = p/q, p and q being relative integers, the value of n times the triangle angles can take only a finite number m° 2π/n and the 4 n-angle centers of a Miquel circle must have the same value wrt the corresponding sides, which makes their identification relatively easy.
For example, if n = 1/2, the point associated to an incenter can be either the incenter of the other triangle having the same angle or the excenter in the same angle and vice-versa ; the 3rd and 4th points are then determined.
If n = 1/3, there are 3 points associated to a point of a triangle, the 3 points being on the same circle through the 2 other vertices of the triangle having the same 3rd vertice as the initial triangle ; the 3rd and 4th triangle are then determined.
If n = 1/4, there are 4 points associated to a point of a triangle, the 4 points being on a circle ...
I was able using this property to find easily the 8 centers of the astroïds tangent to the 4 lines and the 27 centers of the cardioïds tangent to the 4 lines. I try now to find the 64 centers of the nephroïds tangent to the 4 lines.
Best regards
Bernard

```
PS I'm not home next week, but I will continue as soon as I
return
```

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Message: #1226
Date: 2021-10-31
From: ”Dao Thanh Oai” <oaidt.evnpsc@gmail.com>
Subject: [Quadri-and-Poly-Geometry] Do you known this theorem on tangential quadrilateral before?

```
Dear Dao

Sorry for the slow reply, but was away for the weekend.
The result is a straightforward generalization of the following
nice, elementary theorem I've taken the liberty of calling the '
Tangential Quadrilateral Theorem of Gusić & Mladinić - see: [
http://dynamicmathematicslearning.com/tangent-incircles-investig⌋
ate.html |
http://dynamicmathematicslearning.com/tangent-incircles-investig⌋
ate.html ]
with some references.

This result also generalizes to a tangential hexagon as shown
here: [
http://dynamicmathematicslearning.com/tangent-hex-apply.html |
http://dynamicmathematicslearning.com/tangent-hex-apply.html ]
and other variations are possible.
Hope that helps.
Regards
Michael


To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Saturday, October 30, 2021 7:00:40 AM

Dear Geometers,

Let A B C D be a tangential quadrilateral, P be arbitrary point
in the plane then two red lengths are equal. See the Figure in:

[ https://mathoverflow.net/questions/407395/does-this-theorem-on⌋
-tangential-quadrilateral-have-a-name |
https://mathoverflow.net/questions/407395/does-this-theorem-on-t⌋
angential-quadrilateral-have-a-name ]

Best regards
Dao Thanh Oai
```

Message: #1227
Date: 2021-10-31
From: oaidt.evnpsc@gmail.com
Subject: Re: Do you known this theorem on tangential quadrilateral before?

```
Dear Professor Eckart (
https://groups.io/g/Quadri-and-Poly-Geometry/message/1223 ) and
Michael,

Thanks you very much for your answer.

Best regards
Dao Thanh Oai
```

← Previous Next → ↩ Message Index ↑ Subjects

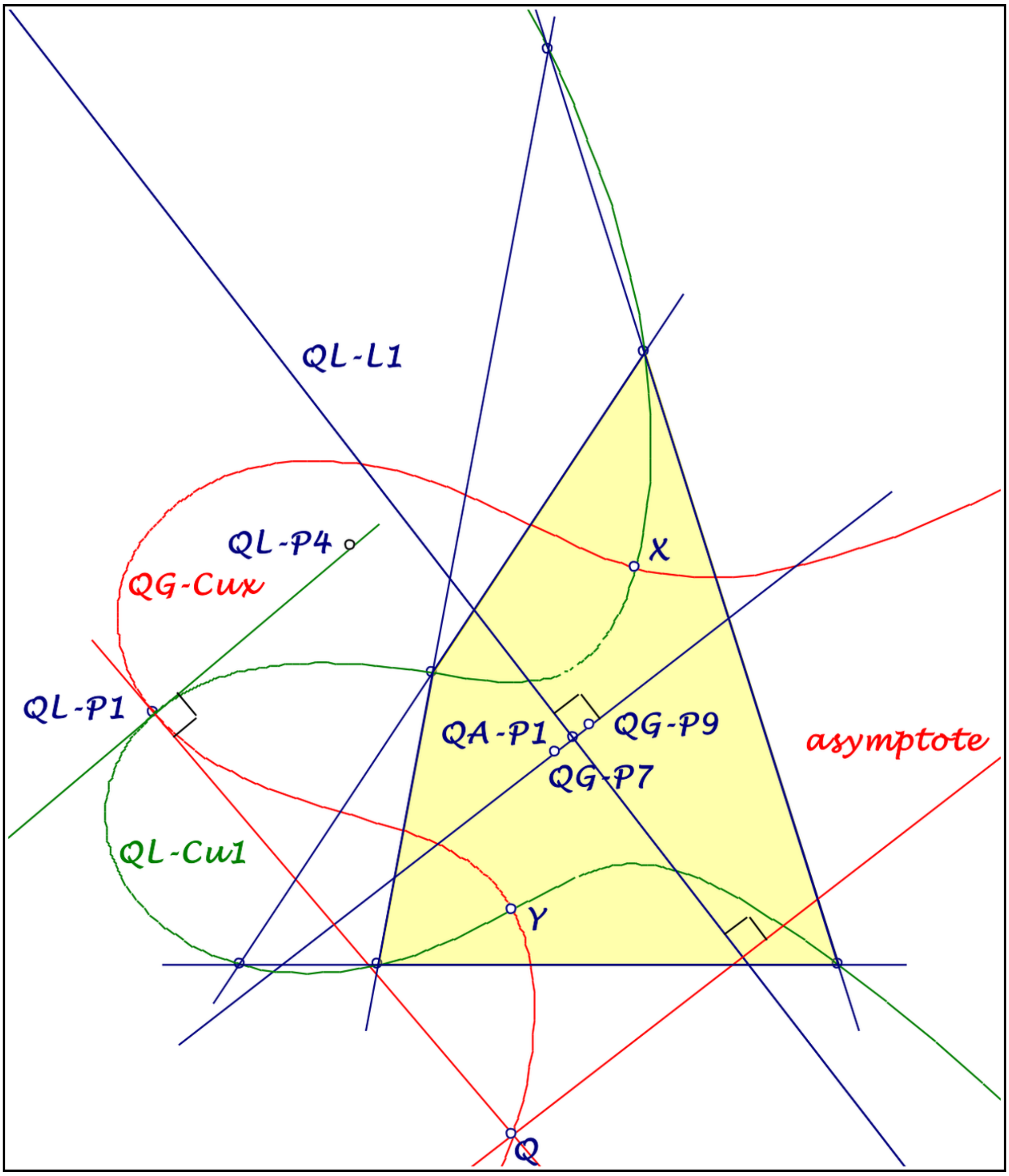
Message: #1228
Date: 2021-11-02
From: eckart_schmidt@t-online.de
Subject: New QG-cubic

```
Dear all,

QL-P1 is the center of the same inversion circle
... for the 3 pairs of opposite QL-points.
The locus for centers with the same inversion circle
... for the 2 pairs of opposite QG-vertices
... is a cubic QG-Cux, bearing QL-P1.
QG-Cux is CSC-invariant,
... midpoints of CSC-partner lie on QG-P7.QG-P9,
... intersecting orthogonal QL-L1 in QA-P1.
QG-Cux bears the two common points X, Y of all cubics,
... CSC-invariant with Newton line through QA-P1,
...  X, Y CSC-partner, äquidistant wrt QA-P1.
QG-Cux intersects orthogonal QL-Cu1,
... tangent in QL-P1 orthogonal QL-P1.QL-P4.
QG-Cux is a focal circular cubic,
... focus QL-P1, asymptote orthogonal QL-L1,
... intersecting QG-Cux in Q on the tangent in QL-P1.

If we consider the three QG-Cux for a quadrangle,
... we get three intersections
... in the QA-Tf4 images of the Miquel points,
... which are the three QG-P5 (see EQF),
... a triangle, perspective to the Miquel triangle,
... the perspector not in EQF.

Best regards Eckart
```



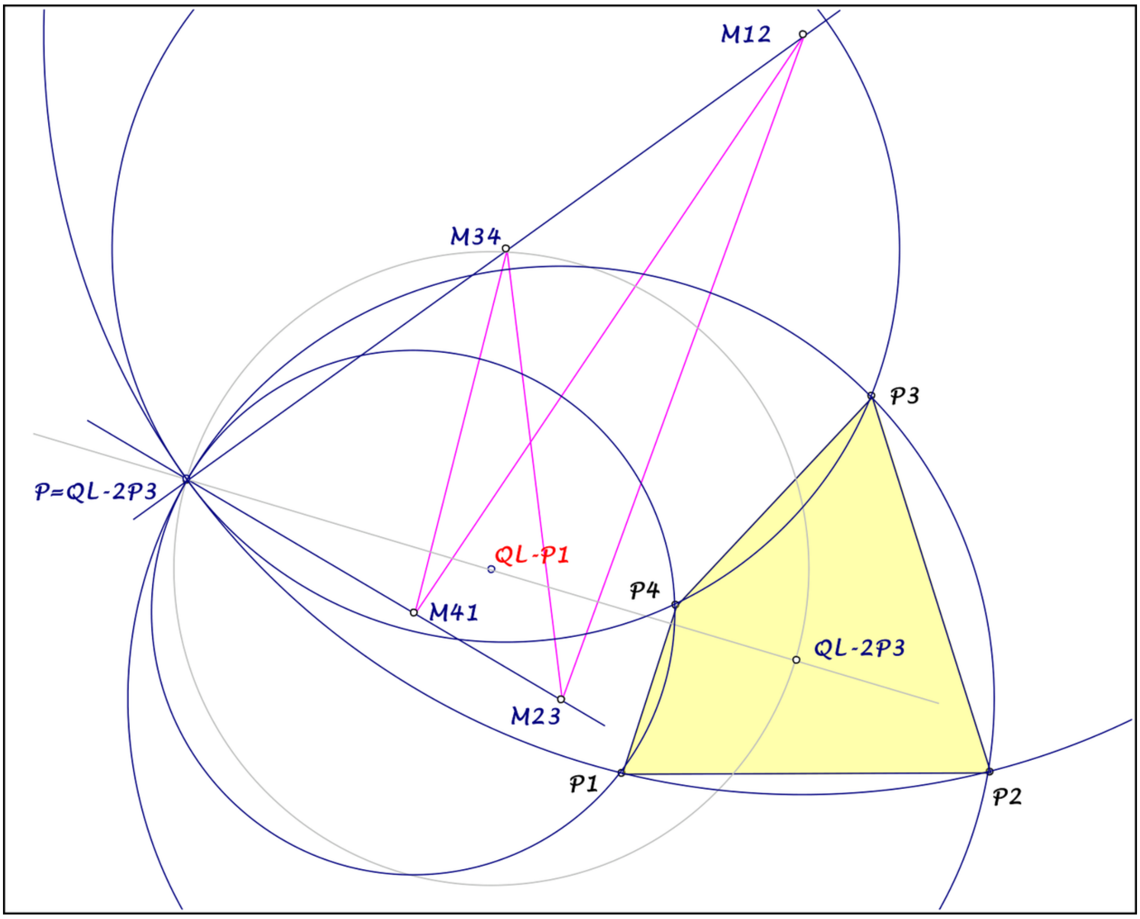
2021-10-29.pdf

Message: #1229
Date: 2021-11-06
From: eckart_schmidt@t-online.de
Subject: QL-2P3 fixed points of CSC

```
Dear all,

consider a quadrigon QG = P1P2P3P4
... and an arbitrary point P for triangles PPiPi+1,
... further the circumcenters of these triangles,
... which give a new quadrigon QG'.
If we ask for the points P,
... which are the diagonal crosspoint of its QG',
... we get the fixed points of CSC QL-2P3,
... now the 4 circles are pairwise CSC-partner.

Best regards Eckart
PS: Perhaps well known:
... QG' concyclic for points on QL-Cu1,
... for QL-P1 the circumcircle of QG' is QL-Ci3.
```



2021-11-05.pdf

Message: #1230
Date: 2021-11-07
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Extension of Van abuel's theorem.

Figure :
https://www.dropbox.com/s/eip2h3kaprx56mj/1636294759791.png?dl=0

______
Let ABCD be quadrilateral and Let Define 4 Square ABIJ , BCLK , CDEF , ADGH
and Let N,M,O,P be midpoint of (G,J);(H,E);(I,L);(F,K) then NP is perpendicular to MO.

_____
Is this known ?

____
Best Regards
Jayendra Jha and sankalp savaran.

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Message: #1231
Date: 2021-11-07
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Extension of Van abuel's theorem.

Figure :
https://www.dropbox.com/s/0hxg2zqtw4y3ayo/Screenshot_2021-11-07-20-32-52-379_org.geogebra.android.calculator.suite.png?dl=0

______
Let NP and MO intersect at U.
Let Q,R be m(M,O);m(N,P)
Let X,Y be m(A,D);m(B,C)
Let m(X,Y) be A1 .

___
Reflect U at A1 to get U'.

____
Then QRUU' will becomes a Rectangle .

____
Best regards
JJ and SS.

On Sun, 7 Nov 2021, 7:58 pm ARCHANA JHA via groups.io,
<archanajha112018gmail.com@groups.io> wrote:

> Figure :
>
https://www.dropbox.com/s/eip2h3kaprx56mj/1636294759791.png?dl=0
> _______
> Let ABCD be quadrilateral and Let Define 4 Square ABIJ , BCLK
, CDEF ,
> ADGH and Let N,M,O,P be midpoint of (G,J);(H,E);(I,L);(F,K)
then NP is
> perpendicular to MO.
> _____
> Is this known ?
> ____
> Best Regards
> Jayendra Jha and sankalp savaran.
>
>

Message: #1232
Date: 2021-11-08
From: ”ARCHANA JHA” <archanajha112018@gmail.com>
Subject: [Quadri-and-Poly-Geometry] Extension of Van abuel’s theorem.

```
Yes, this is very well known, and generalizes further to similar
parallelograms on the sides of the original quadrilateral - see
this URL: [ http://dynamicmathematicslearning.com/aubelparm.html
| http://dynamicmathematicslearning.com/aubelparm.html ]
Click on the links in the dynamic page as well as have a look at
several relevant papers at the bottom.

regards
Michael


To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Sunday, November 7, 2021 4:28:23 PM

Figure :
[
https://www.dropbox.com/s/eip2h3kaprx56mj/1636294759791.png?dl=0
|
https://www.dropbox.com/s/eip2h3kaprx56mj/1636294759791.png?dl=0
]
______
Let ABCD be quadrilateral and Let Define 4 Square ABIJ , BCLK ,
CDEF , ADGH and Let N,M,O,P be midpoint of
(G,J);(H,E);(I,L);(F,K) then NP is perpendicular to MO.
_____
Is this known ?
____
Best Regards
Jayendra Jha and sankalp savaran.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1233
Date: 2021-11-08
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Extension of Van abuel's theorem.

```
Dear Michael, Thanks you very much for link .
```

```
we don't have  much knowledge about Quadrilateral and polygon
but we are
interested in some known theorem regarding it. Thanks for your
review.
```

```
Best regards
JJ and SS.
```

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Message: #1234
Date: 2021-11-08
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Interesting Results on General

---

```
Dear Friends , please verify whether it is known or not !

Figure :
https://www.dropbox.com/s/de96awka2xtzu3h/1636375713471.png?dl=0
________
Let ABCD be General Quadrilateral. Let E,F,G,H be apex of
equilaterial
triangle made on base AD,AB,BC,CD.
Then m(E,F);m(F,G);m(C,D) gives you an equilateral triangle.
_______
Similarly we can find other 3 equilaterial triangle Cyclically
in General
Quadrilateral ABCD .
_______
Best regards
JJ and SS.
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1235
Date: 2021-11-09
From: peter.liepa@gmail.com
Subject: Re: Interesting Results on General Quadrilateral .

```
Dear JJ and SS

This looks like Problem 5, Figure 12 of Abel, Mean Geometry (
http://zacharyabel.com/papers/Mean-Geo_A07.pdf )

Best regards,
Peter
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1236
Date: 2021-11-10
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Interesting Results on General

---

Dear Peter and friends, thank you very much. The Reference is nice .
I checked that problem 04 and 06 is Assymetrical propeller Theorem which we
know but that Problem 05 was not known for us .

____

During time we also Find this can you please check it.

____

Link:
https://www.dropbox.com/s/z47odnvgy310fot/1636510807888.png?dl=0

____

Let ABCD be quadrilateral , Let Apex of equilaterial triangle made on base
AB,BC,CD,DA as {G,H,E,F} . Then G(ABG) ;G(GFE) ;G(GHE) makes equilaterial
triangle and Cyclically we get other 3 equilaterial triangle.(see link)

______

We have not yet geberalised this for isosceles triangle , and N polygon as
we first want to verify then we will try to work on its Generalisation.

_____

So kindly confirm it whether it is correct or not.

_____

Best Regards
JJ and SS

Thanks for the review.

On Wed, 10 Nov 2021, 5:09 am Peter Liepa,
<peter.liepa@gmail.com> wrote:
> [Edited Message Follows]
> Dear JJ and SS
> This looks like Problem 5, Figure 12 of Abel, Mean Geometry
> <http://zacharyabel.com/papers/Mean-Geo_A07.pdf>
> Best regards,
> Peter

Message: #1237
Date: 2021-11-09
From: peter.liepa@gmail.com
Subject: Re: Interesting Results on General Quadrilateral .

Dear JJ and SS

I don't know much about this topic, but I thought the reference
might be useful for you.
At this point you know more than I do. Good luck with your
investigation!

Best regards,
Peter

← Previous   Next →   ↩ Message Index   ↑ Subjects

Message: #1238
Date: 2021-11-10
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Interesting Results on General

---

Dear Peter, Thanks for giving the time.

_____

Yes the Reference was helpful for us.

_____

Best REGARDS
JJ and SS.

On Wed, 10 Nov 2021, 10:15 am Peter Liepa,
<peter.liepa@gmail.com> wrote:
> Dear JJ and SS
> I don't know much about this topic, but I thought the
reference might be
> useful for you.
> At this point you know more than I do. Good luck with your
investigation!
> Best regards,
> Peter

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Message: #1239
Date: 2021-11-10
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Probably a new Generalisation of Van

---

Respected Michael and all my dear friends , we checked many references but
as you know we are not much experience People . So kindly check whether
this is new or old.

_______,
See figure :
https://www.dropbox.com/s/cqpqgh9ze2qkcqx/1636532202027.png?dl=0

____
Conjecture : Let ABFE , BCGH, CDIJ, DAKL be Square on the sides of General
Quadrilateral ABCD.
Now makes Square FGTS, HIMN, JKOP, LERQ .
Here , JP intersect IM at U and similarly define {V,W,Z} then
:UV is perpendicular to WZ .

______
Note: We have not work on its Generalisation by replacing the Square by
parallelogram And other generalisation regarding N polygon .

____
When you confirm then we will try to work on its Generalisation. We try to
match the above mentioned Figure with different Figure available on Google.
May be in case we discovered the old one then please let us know.

______

Best Regards
JJ and SS.

Message: #1240
Date: 2021-11-10
From: ”archanajha112018” <archanajha112018@gmail.com>
Subject: [Quadri-and-Poly-Geometry] Probably a new Generalisation of Van Abuels (Is it known?)!

```
Responding very quickly: the arrangement does look different &
possibly new.

It probably follows from this generalization of Van Aubel: [
https://www.cut-the-knot.org/m/Geometry/DaosVanAubel.shtml |
https://www.cut-the-knot.org/m/Geometry/DaosVanAubel.shtml ]

To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Wednesday, November 10, 2021 10:40:32 AM
Respected Michael and all my dear friends , we checked many
references but as you know we are not much experience People .
So kindly check whether this is new or old.
_______,
See figure :
[
https://www.dropbox.com/s/cqpqgh9ze2qkcqx/1636532202027.png?dl=0
|
https://www.dropbox.com/s/cqpqgh9ze2qkcqx/1636532202027.png?dl=0
]
________
Conjecture : Let ABFE , BCGH, CDIJ, DAKL be Square on the sides
of General Quadrilateral ABCD.
Now makes Square FGTS, HIMN, JKOP, LERQ .
Here , JP intersect IM at U and similarly define {V,W,Z} then
:UV is perpendicular to WZ .
______
Note: We have not work on its Generalisation by replacing the
Square by parallelogram And other generalisation regarding N
polygon .
____
When you confirm then we will try to work on its Generalisation.
We try to match the above mentioned Figure with different Figure
available on Google. May be in case we discovered the old one
then please let us know.
______

Best Regards
JJ and SS.
```

Message: #1241
Date: 2021-11-10
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Probably a new Generalisation of Van

---

Respected sir , you can reply us at any manageable time as we will not be in rush. During the time, we find a interesting pattern in this above mentioned Configuration .

____
We will try to explain the pattern in very easy manner .

____
See Figure 01:
https://www.dropbox.com/s/cqpqgh9ze2qkcqx/1636532202027.png?dl=0

____
Here we will give some name so that you can understand it in easy manner.

____
Let called the Squares
"ABFE, BCGH, CDIJ, DAKL" as "First layer of Square ".

_____
As you know by joining the centroid of Square oppositely we get 90° Angle know as "Van Abuels theorem".

____
Now , Let called Square "FGTS, HINM, JKOP, LERQ" as "Second layer of Square".

____
Where UV & WZ are perpendicular to each other.
We called this theorem as "Magical Theorem ".

____
See Figure 02 :
https://www.dropbox.com/s/mftf3fzwonbw14u/1636555327601.png?dl=0

____
See the 4 Red Square where O1, O2, O3, O4 are G(R,S,A1,B1) ;G(TMC1D1);G(N,O,U,V);G(P, Q,W,Z) then O1O3 are perpendicular to O2O4.
We called this 4 Red Square as "Third Layer of Square " which follows the
property similar to Van Abuels theorem ".

____
Similarly we can find
4th,5th,6th and .....n th layer of Square.

____
Where
1st Layer of Square : Van Abuels theorem
2nd Layer of Square: Magical Theorem
3rd layer of Square : Van Abuels theorem
.

.
.
2n Layer of Square : Magical Theorem.
2n+1 Layer of Square : Van Abuels Theorem.

The pattern Repeat itself continuously. Hope you understand and if you find
difficulties in understanding it then please let me know.

And Thanks for confirming that the given theorem is probably new to you .

Thanks for the link . And we will try further to generalised it for
parallelogram and N POLYGON. Like 4n polygon.

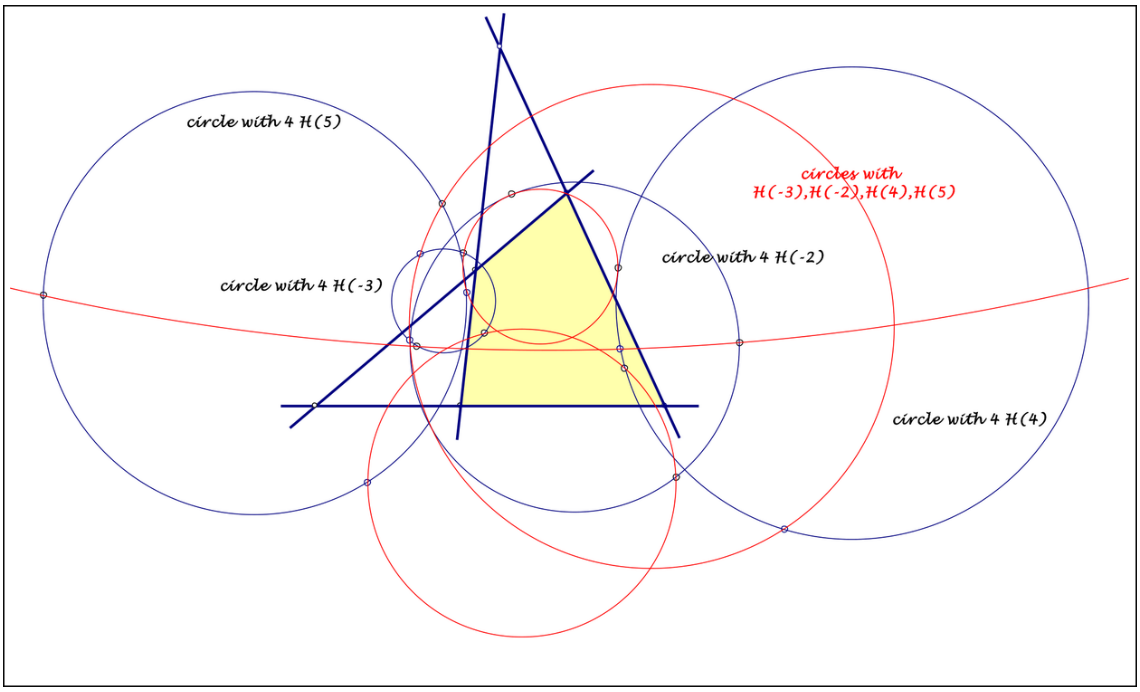
Can you provide us the Reference of Van Abuels theorem where people.
Generalised it for N polygon? So that we get some ideas from there .

Best regards
JJ and SS.

Thanks for the review.

Message: #1242
Date: 2021-11-12
From: eckart_schmidt@t-online.de
Subject: Concyclic Hofstadter points

```
Dear all,

this is an addition to 5L-s-P2,
... where Chris has described the Hofstadter points H(n),
... defined for n integer unequal 0 and 1.
Let "inv" be the inversion wrt the triangle circumcircle
... and "isg" the isogonal conjugate wrt the triangle.

Well known will be the following properties:
(1) H(n+1) = inv(isg(H(n)) starting with H(2) = X(3),
    H(n-1)  = isg(inv(H(n)) starting with H(-1) = X(4),
    isg(H(n)) = H(1-n), inv(H(n)) = H(2-n).

Perhaps new will be:
(2) Inversion partners of Hofstadter points are collinear
    with X(3)
    and two pairs of inversion partners are concyclic:
    for example H(4), H(-2) and H(5), H(-3),
    in general H(1+n), H(1-n) and H(1+m), H(1-m),
    or special H(n), H(1+n), H(1-n), H(2-n),
    the circles are orthogonal to the circumcircle.

Finally wrt the 4 same H(n) for the 4 triangles of a
quadrilateral,
    which are concyclic (see EPG, 5L-s-P2):
(3) For isogonal conjugate H(n) and H(1-n)
    the circles for a QL are CSC-partner,
    for example for n=3:
    the X(186)-circle is the CSC-image of the X(265)-circle.

Best regards Eckart
```

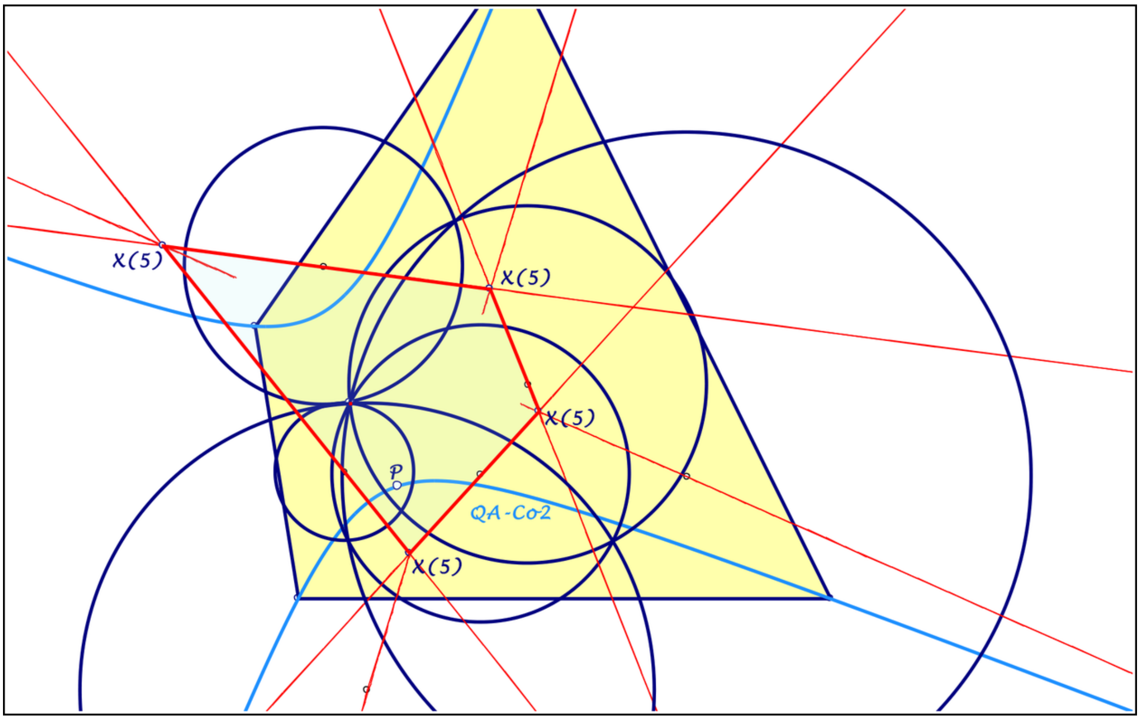


2021-11-12.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #1243
Date: 2021-11-15
From: eckart_schmidt@t-online.de
Subject: Just for fun

```
Dear all,

may I invite you for a short excursion in QA-geometry?
Let QA = P1P2P3P4 be a quadrangle and P any point,
... consider the six triangles PPiPj and their
    nine-point circles.
The locus for points P with a common point for these six circles
... is the orthogonal hyperbola QA-Co2,
... the common point is always the center QA-P2 of QA-Co2.
The loci for the centers of these six circles (varying P on
QA-Co2)
... are six lines with four triple intersections,
... which are the nine-point centers X(5) of the QA-triangles.

Best regards Eckart
```



2021-11-14.pdf

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Message: #1244
Date: 2021-11-15
From: bernard.keizer@gmail.com
Subject: Re: Concyclic Hofstadter points

Dear Eckart,
All these properties are well known (the properties in 2) follow from the inversion wrt the circumcenter).
Perhaps less known : the isogonals of 2 inverse points wrt the circumcircle have their middle on the Euler circle (9 points circle).
Hence H(1-n) and H(n-1) being the isog of H(n) and it's inverse H(2-n) have their middle on the Euler circle.
Of course, all these properties are true for the n-angle centers, which generalise the Hofstadter points for n rational ...
Best regards
Bernard

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Message: #1245
Date: 2021-11-17
From: eckart_schmidt@t-online.de
Subject: QG-P5

Dear Chris,

inversion circles round QG-P5 coincide
... for neighbored vertices of a QG.

Best regards Eckart

PS: "inversion circle" round Z for X, Y means
... a circle with radius sqr[ZX*ZY].

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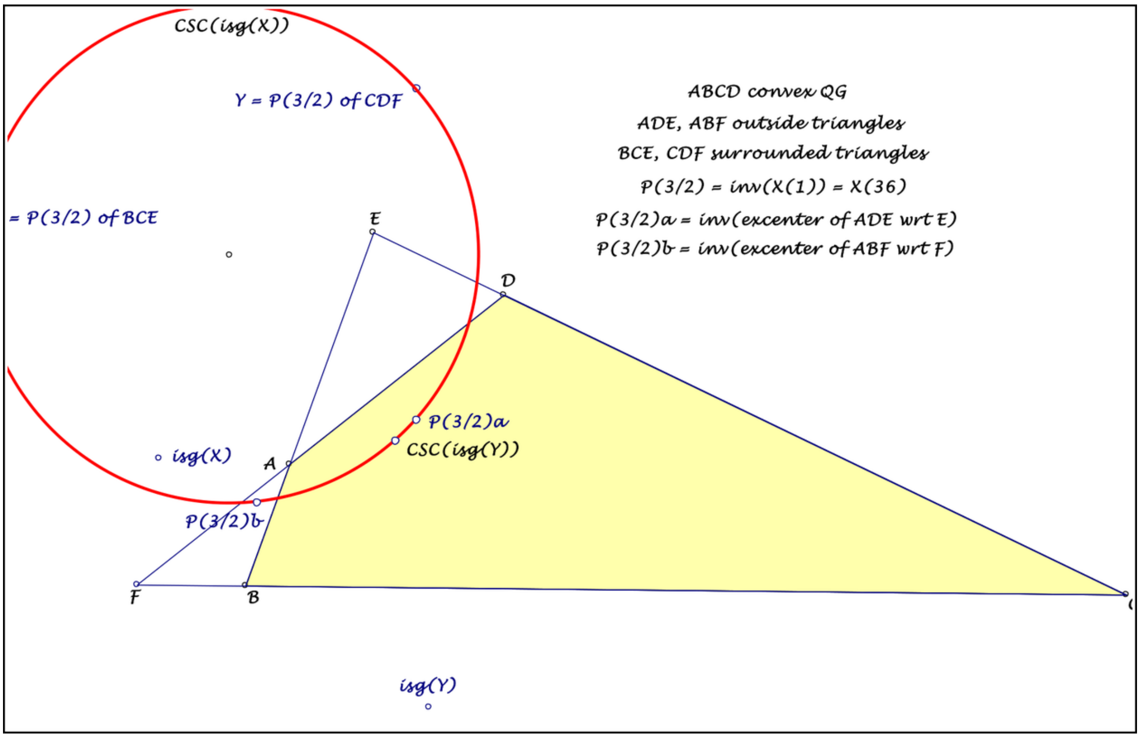
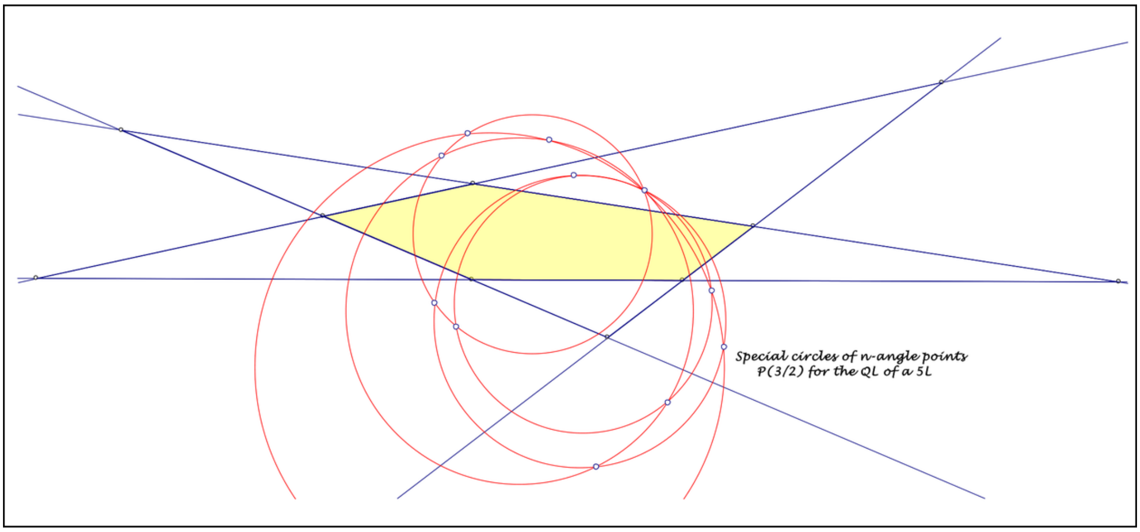
Message: #1246
Date: 2021-11-19
From: eckart_schmidt@t-online.de
Subject: Concyclic generalized n-angle centers

```
Dear all,

in QFG #1873 I generalized n-angle centers P(p/q)
... as points P with angle (PA,PB) == p/q C mod pi/q, etc,
... there are q^2 such centers,
... with the recursions P(1+p/q) = inv(isg(P(p/q))), P(-1+p/q) =
isg(inv((P(p/q))).
Let us only consider q = 2, starting with in- and excenters for
P(1/2),
... second restriction: consider only the incenter X(1) for
    P(1/2),
... then P(3/2) = X(36) and P(5/2) = inv(isg(X(36))) not in ETC.
For a QL with its four triangles
... these four P(p/2) must not be concyclic,
... but let us consider the following circle:
Each QL has a convex QG-version (see attached)
... with a pair of outside triangles
... and a pair of surrounding triangles.
... Take P(p/2) of the surrounding triangles
... and their partners CSC(isg(P(p/2))),
... which give four concyclic points,
... whose circle Ci bears two further P(p/2),
... but related from excenters (see attached for p = 3),
... Ci is one of the 8 circles for the 16 points P(p/2)
    in general.
Finally:
These five circles CI for a 5L have a common point.

Best regards Eckart
```



2021-11-19a.pdf



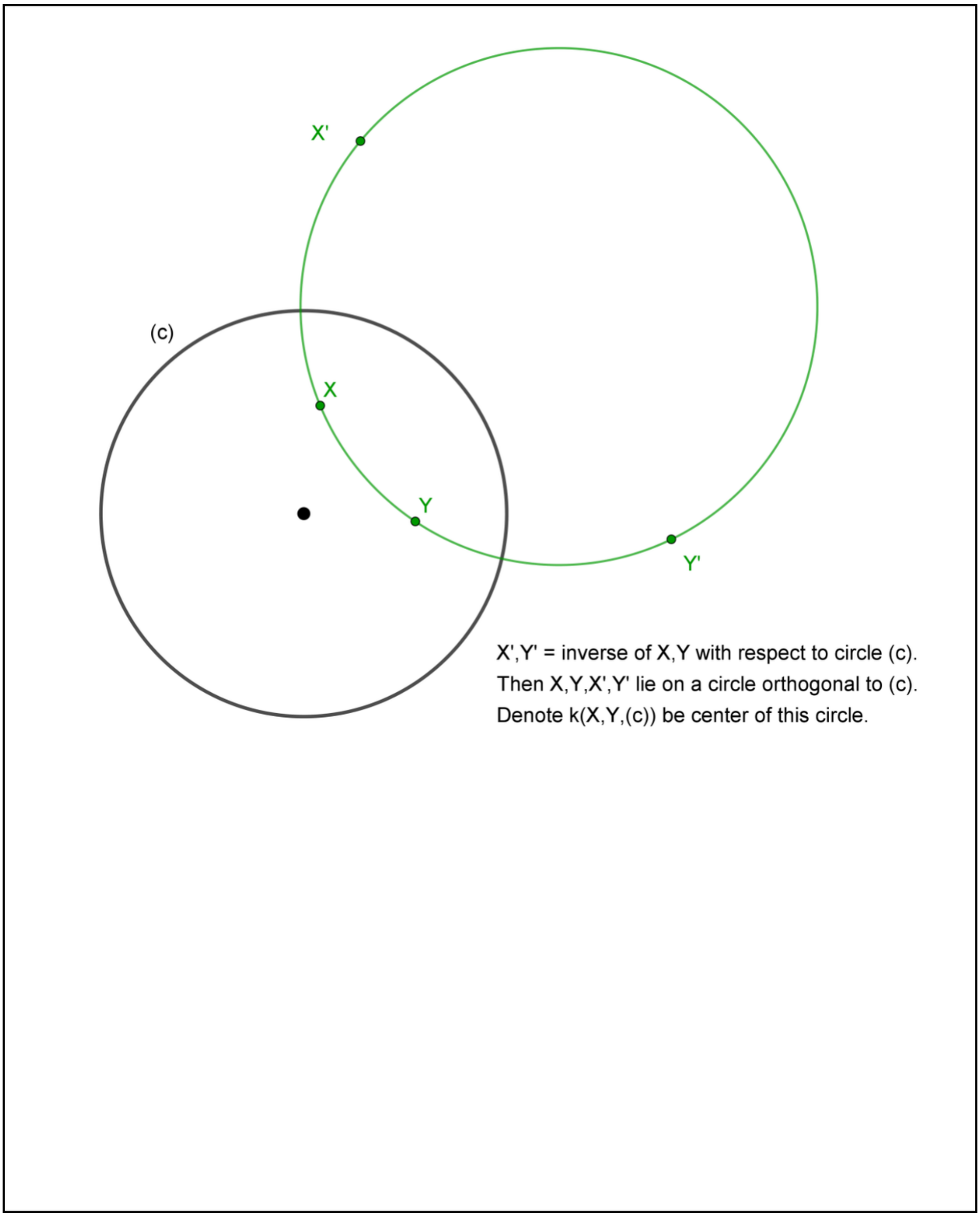
2021-11-19b.pdf

← Previous Next → ↔ Message Index ↑ Subjects

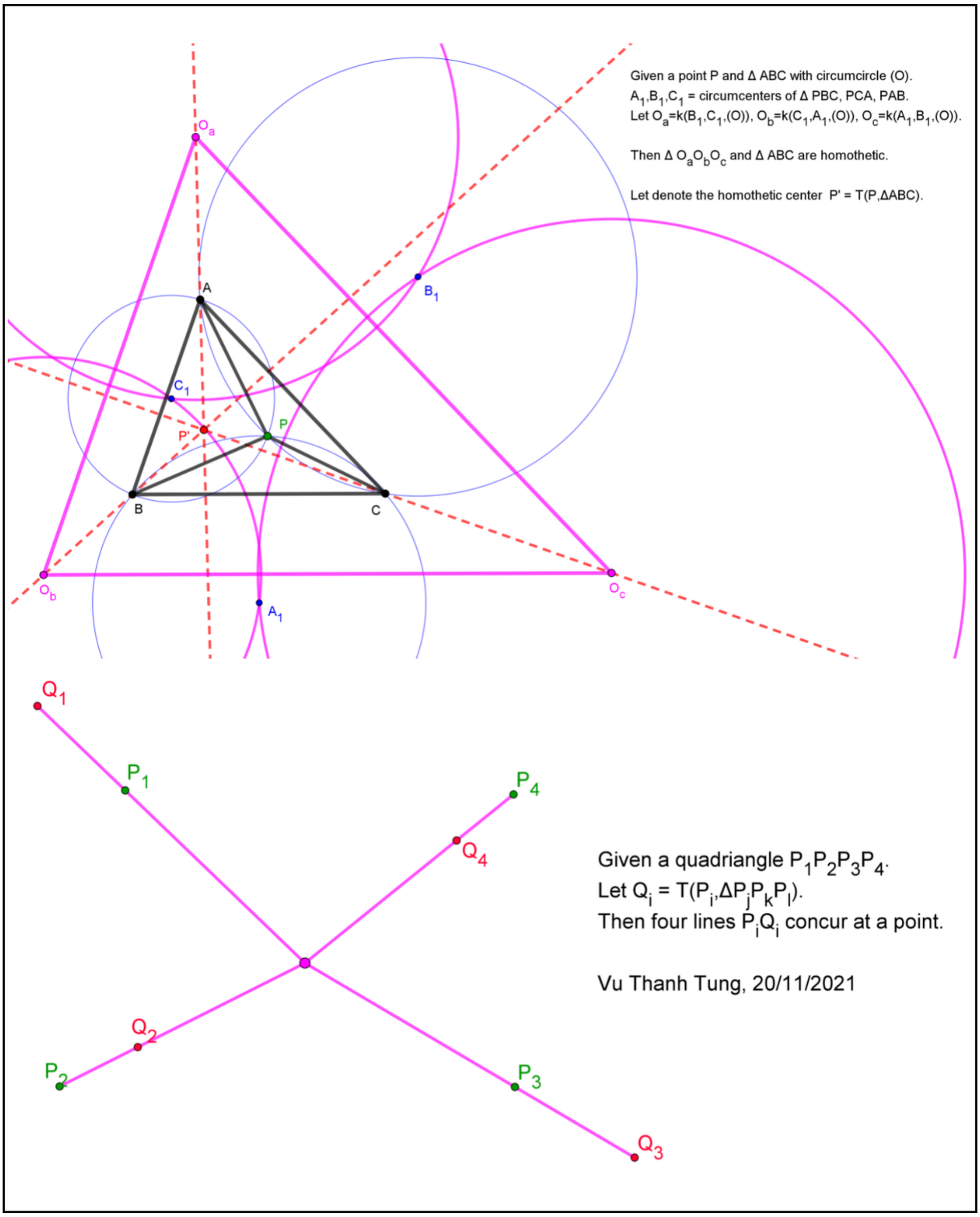
Message: #1247
Date: 2021-11-20
From: tungvtt@gmail.com
Subject: A quadriangle point

```
Dear all,

I found a point in quadriangle as follows:

*Definition* :
X',Y' = inverse of X,Y wrt cirlce (c),
then X,Y,X',Y' lie on a circle  orthogonal to (c).
Denote k(X,Y,(c)) be the center of this circle.

*A transformation on triangle:*
Given a point P and Δ ABC with circumcircle (O).
A1,B1,C1 = circumcenters of Δ PBC, PCA, PAB.
Let Oa=k(B1,C1,(O)), Ob=k(C1,A1,(O)), Oc=k(A1,B1,(O)).

Then Δ OaObOc and Δ ABC are homothetic.
Let denote homothetic center P'=T(P,Δ ABC).
Ref: Euclid 3259 https://groups.io/g/euclid/message/3259  †)

*A point on quadriangle:*
Let P1P2P3P4 be a quadriangle.
Let Qi = T(Pi,Δ PjPkPl).
Then four lines PiQi concur at a point.
What is this point?

Best regards,
Vu Thanh Tung
```

†) Editorial note: The referenced message(s) can be found as listed in EPG-References [72].



QuadranglePoint.pdf



QuadranglePoint.pdf

Message: #1248
Date: 2021-11-20
From: eckart_schmidt@t-online.de
Subject: Re: A quadriangle point

```
Dear Vu Thanh Tung,

the point is QA-P4.

Best regards Eckart
```

← Previous   Next →   ↔ Message Index   ↑ Subjects

Message: #1249
Date: 2021-11-20
From: tungvtt@gmail.com
Subject: Re: A quadriangle point

```
Dear Eckart,

Thank you very much,

Best regards,
Vu Thanh Tung

On Sat, Nov 20, 2021 at 12:04 PM, Eckart Schmidt wrote:
> Dear Vu Thanh Tung,
> the point is QA-P4.
> Best regards Eckart
```

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Message: #1250
Date: 2021-11-21
From: bernard.keizer@gmail.com
Subject: Re: Concyclic generalized n-angle centers

```
Dear Eckart,
If I'm not wrong, isg(P(3/2) is X80 in ETC and inv(isg(P(3/2))
is X10260.
The triangle of the P(3/2) is in perspective with ABC with
perspector X35 = H(1/2).
The same way, we have H(-1/2) = X79, H(1/3) = X357, H(2/3) =
X358, H(1/4) = X1127 and H(3/4) = X1129.
Best regards
Bernard
```

← Previous Next → ↩ Message Index ↑ Subjects

Message: #1252
Date: 2021-11-22
From: eckart_schmidt@t-online.de
Subject: Re: Concyclic generalized n-angle centers

```
Dear Bernard,

I checked the first results,
... but the Hofstadter points are always listened in old#1872.

Best regards Eckart
```

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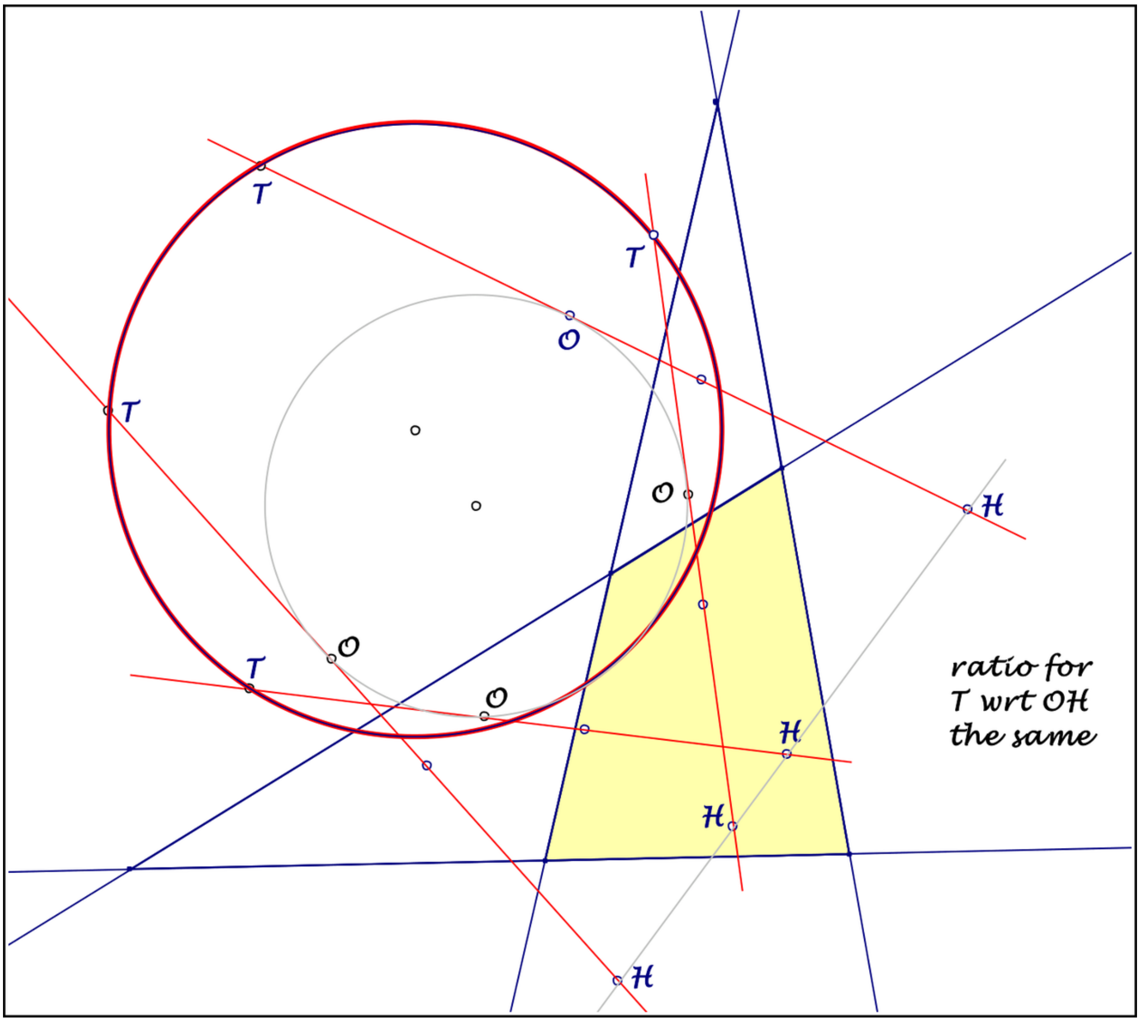
Message: #1253
Date: 2021-11-22
From: eckart_schmidt@t-online.de
Subject: Unknown QL-circle?

```
Dear all,

there is a circle for a quadrilateral (unequal QL-Ci1, QL-L2),
... dividing X(3).X(4) on the Euler lines of the QL-triangles
... in the same ratio, depending on the QL.

What about this circle, how to construct?

Best regards Eckart
```



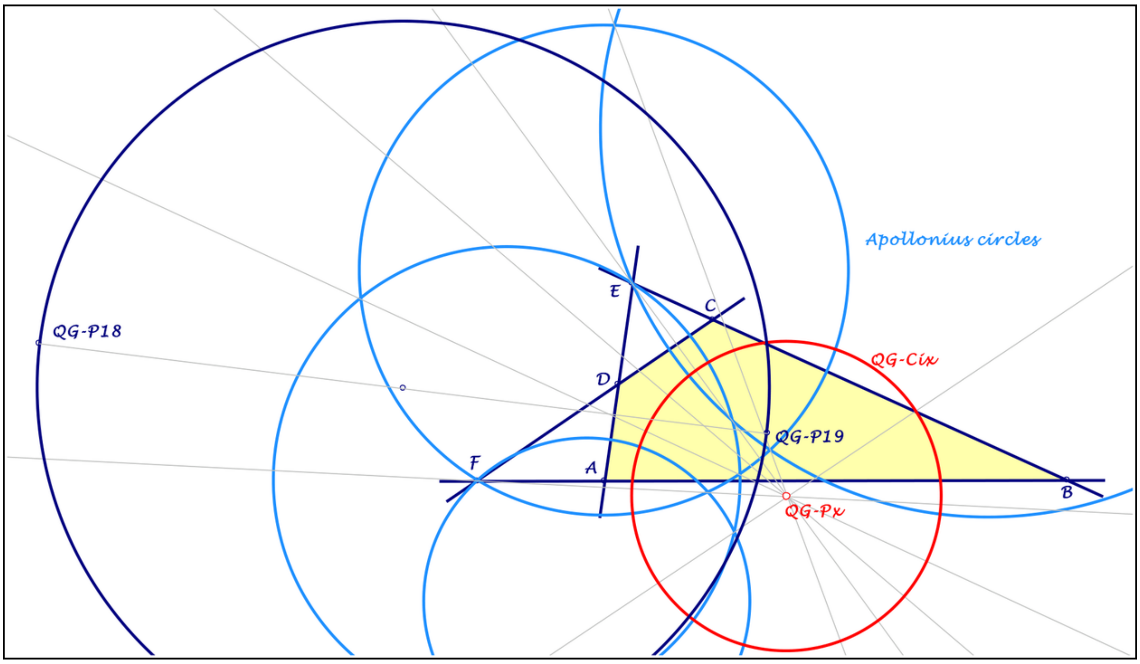
2021-11-22.pdf

Message: #1254
Date: 2021-11-23
From: eckart_schmidt@t-online.de
Subject: Unknown QG-circle?

```
Dear all,

consider a quadrigon ABCD and the intersections E, F of opposite
sides,
... which give four triangles EAB, EDC, FAD, FBC,
... take their Apollonius circles through E and F,
... which have radical axes with a common point QG-Px,
... center of a circle QG-Cix, not always real,
... which is orthogonal to the four Apollonius circles
... and orthogonal to the circle with diameter QG-P18.QG-P19.

Best regards Eckart
```



2021-11-23.pdf

Message: #1256
Date: 2021-11-23
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Unknown QL-circle?

Dear Eckart,

It is a strange circle, indeed.

I did some calculations in the complex plane, where the Miquel circle is the unit circle.

The ratio is simple. However, I have not come up with any construction yet.

Perhaps, my calculations might give you some idea.

Best regards,

Ngo Quang Duong

1 Eckart's problem

Problem 1. *$\ell_1\ell_2\ell_3\ell_4$ is a quadrilateral.*
M is the Miquel point of $\ell_1\ell_2\ell_3\ell_4$.
O_1, H_1 are the circumcenter, orthocenter of $\ell_2\ell_3\ell_4$.
O_2, H_2 are the circumcenter, orthocenter of $\ell_1\ell_3\ell_4$.
O_3, H_3 are the circumcenter, orthocenter of $\ell_1\ell_2\ell_4$.
O_4, H_4 are the circumcenter, orthocenter of $\ell_1\ell_2\ell_3$.
There exists $t \in \mathbb{R} \cup \{\infty\}$ other than 0, 1, and ∞ such that

$$\frac{\overline{T_1H_1}}{\overline{T_1O_1}} = \frac{\overline{T_2H_2}}{\overline{T_2O_2}} = \frac{\overline{T_3H_3}}{\overline{T_3O_3}} = \frac{\overline{T_4H_4}}{\overline{T_4O_4}} = t$$

and T_1, T_2, T_3, T_4 are concyclic.
How to construct these four points?

2 Finding the common ratio

I suggest an approach which uses complex numbers.

In the complex plane, let the Miquel circle be the unit circle, and the affixes of M, O_1, O_2, O_3, O_4 be

$$M(1) \qquad O_1(p_1) \qquad O_2(p_2) \qquad O_3(p_3) \qquad O_4(p_4). \tag{1}$$

P_{12} – the intersection of ℓ_1 and ℓ_2 (which is also the reflection of M in O_3O_4), has affix $p_3 + p_4 - p_3p_4$.

Analogously

$$\begin{aligned}
&P_{12}(p_3 + p_4 - p_3p_4)\\
&P_{13}(p_2 + p_4 - p_2p_4)\\
&P_{14}(p_2 + p_3 - p_2p_3)\\
&P_{23}(p_1 + p_4 - p_1p_4)\\
&P_{24}(p_1 + p_3 - p_1p_3)\\
&P_{34}(p_1 + p_2 - p_1p_2)
\end{aligned} \tag{2}$$

The orthocenter K_1 of triangle $O_2O_3O_4$ has affix $p_2 + p_3 + p_4$.

Since $M.O_2O_3O_4.K_1$ and $M.P_{34}P_{24}P_{23}.H_1$ are directly similar, then the orthocenter H_1 of triangle $\ell_2\ell_3\ell_4$ has affix $p_1 + p_2 + p_3 + p_4 - p_1(p_2 + p_3 + p_4)$.

$$\begin{aligned}
&H_1\left((p_1 + p_2 + p_3 + p_4)(1 - p_1) + p_1^2\right)\\
&H_2\left((p_1 + p_2 + p_3 + p_4)(1 - p_2) + p_2^2\right)\\
&H_3\left((p_1 + p_2 + p_3 + p_4)(1 - p_3) + p_3^2\right)\\
&H_4\left((p_1 + p_2 + p_3 + p_4)(1 - p_4) + p_4^2\right)
\end{aligned} \tag{3}$$

T_1, T_2, T_3, T_4 has affixes

$$\begin{aligned}
&T_1\left(\frac{(p_1+p_2+p_3+p_4)(1-p_1)+p_1^2-t\cdot p_1}{1-t}\right)\\
&T_2\left(\frac{(p_1+p_2+p_3+p_4)(1-p_2)+p_2^2-t\cdot p_2}{1-t}\right)\\
&T_3\left(\frac{(p_1+p_2+p_3+p_4)(1-p_3)+p_3^2-t\cdot p_3}{1-t}\right)\\
&T_4\left(\frac{(p_1+p_2+p_3+p_4)(1-p_4)+p_4^2-t\cdot p_4}{1-t}\right)
\end{aligned}\tag{4}$$

T_1, T_2, T_3, T_4 are concyclic if and only if

$$\frac{T_1-T_2}{T_1-T_3}\div\frac{T_4-T_2}{T_4-T_3}\in\mathbb{R}.$$

where

$$\frac{T_1-T_2}{T_1-T_3}=\frac{p_3+p_4+t}{p_2+p_4+t}\cdot\frac{p_2-p_1}{p_3-p_1}$$
$$\frac{T_4-T_2}{T_4-T_3}=\frac{p_3+p_1+t}{p_2+p_1+t}\cdot\frac{p_2-p_4}{p_3-p_4}$$

Besides, since O_1, O_2, O_3, O_4 are concyclic, then

$$\frac{p_2-p_1}{p_3-p_1}\div\frac{p_2-p_4}{p_3-p_4}\in\mathbb{R}$$

Therefore

$$\frac{T_1-T_2}{T_1-T_3}\div\frac{T_4-T_2}{T_4-T_3}\in\mathbb{R}\Longleftrightarrow\frac{(p_3+p_4+t)(p_1+p_2+t)}{(p_2+p_4+t)(p_1+p_3+t)}\in\mathbb{R}.$$

To find t such that T_1, T_2, T_3, T_4, I will solve the equation

$$\frac{(p_3+p_4+t)(p_1+p_2+t)}{(p_2+p_4+t)(p_1+p_3+t)}=\frac{\left(\frac{1}{p_3}+\frac{1}{p_4}+t\right)\left(\frac{1}{p_1}+\frac{1}{p_2}+t\right)}{\left(\frac{1}{p_2}+\frac{1}{p_4}+t\right)\left(\frac{1}{p_1}+\frac{1}{p_3}+t\right)}.$$

From above, I obtain a quadratic equation of t. If I was not mistaken

$$t=\frac{\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}+\frac{1}{p_4}-\frac{p_1+p_2+p_3+p_4}{p_1p_2p_3p_4}}{\frac{1}{p_1p_2p_3p_4}-1}\qquad\text{or}\qquad t=0.$$

ql-circle-of-eckart.pdf

Message: #1257
Date: 2021-11-24
From: tungvtt@gmail.com
Subject: Re: Unknown QL-circle?

Dear Ngo Quang Duong,

>From your last expression for t, I think you have already found the construction:

First note that t = |sum(pi) - sum(pi*pj*pk)| / |p1p2p3p4 -1| = 4*G1G3/XM,
where G1,G3 are the centroids of the quadriangle P1P2P3P4 and Q1Q2Q3Q4, such that
angle<OM,OQi> = sum angle<OM,Qj> (j=1..4,j#i); and Q1,Q2,Q3,Q4 on the Miquel circle.
The point X is the point on the Miquel circle such that angle<OM,OX> = sum angle<OM,Qi> (i=1..4).

Please check, I am maybe mistaken.

Best regards,

Vu Thanh Tung

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Message: #1258
Date: 2021-11-24
From: tungvtt@gmail.com
Subject: Re: Unknown QL-circle?

```
Dear Ngo Quang Duong,

There is some typos, the angle equation should be:
angle<OM,OQi> = sum angle<OM,OPj> (j=1..4,j#i);  and Q1,Q2,Q3,Q4
on the Miquel circle.
angle<OM,OX> = sum angle<OM,OPi>  (i=1..4).

Best regards,
Vu Thanh Tung

On Wed, Nov 24, 2021 at 12:24 AM, Vu Thanh Tung wrote:

> Dear Ngo Quang Duong,
>
> From your last expression for t, I think you have already
found the
> construction:
> First note that t = |sum(pi) - sum(pi*pj*pk)| / |p1p2p3p4 -1|
=
> 4*G1G3/XM,
> where G1,G3 are the centroids of the quadriangle P1P2P3P4
and  Q1Q2Q3Q4,
> such that
> angle<OM,OQi> = sum angle<OM,Qj> (j=1..4,j#i);  and
Q1,Q2,Q3,Q4 on the
> Miquel circle.
> The point X is the point on the Miquel circle such that
> angle<OM,OX> = sum angle<OM,Qi>  (i=1..4).
> Please check, I am maybe mistaken.
> Best regards,
> Vu Thanh Tung
```

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Message: #1259
Date: 2021-11-24
From: bernard.keizer@gmail.com
Subject: Re: Unknown QL-circle?

```
Dear Eckart, dear Ngo Quang Duong, dear Vu Thanh Tung
Beautiful property and nice calculations with complex
coordinates !
Congratulations to the three of you
Best regards
Bernard
```

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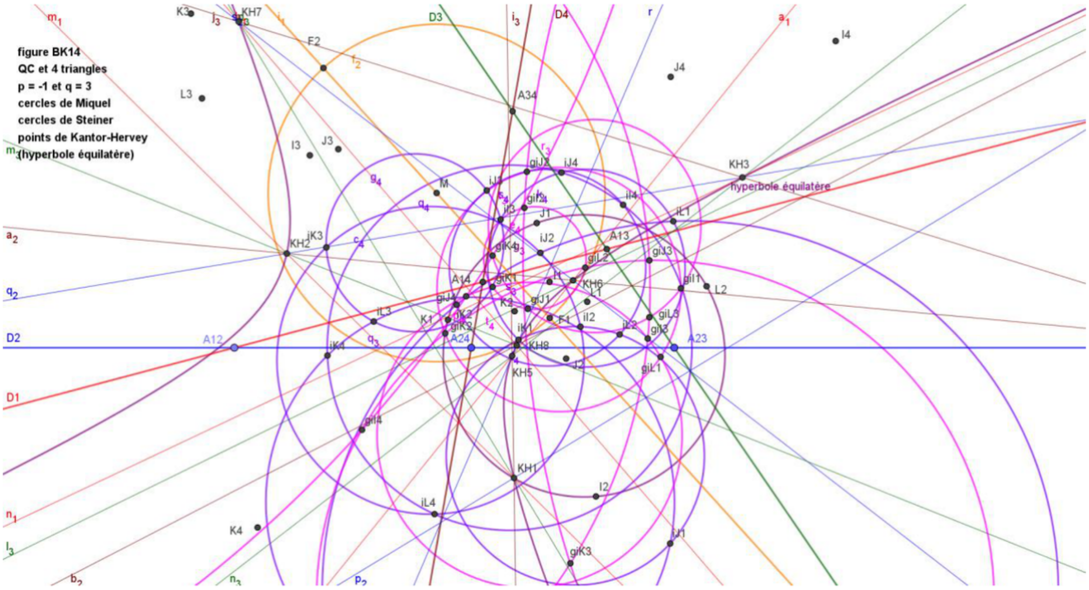
Message: #1260
Date: 2021-11-24
From: bernard.keizer@gmail.com
Subject: Re: n-angle centers revisited

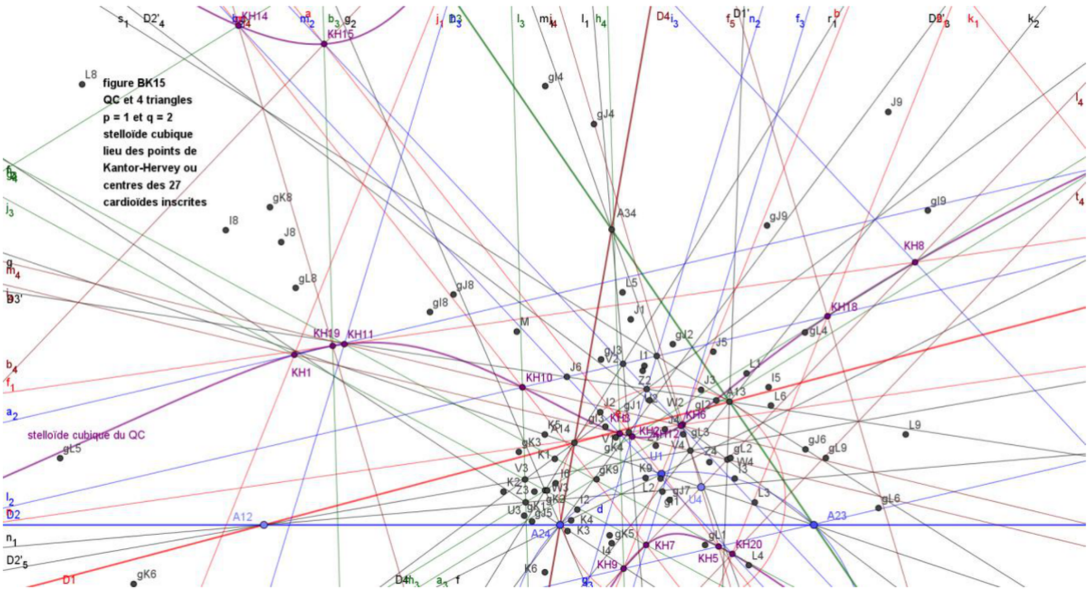
Dear Ngo Quang Duong, dear Eckart,
You will find in the attached file 4 figurs, the 1rst for the 8 centers of astroïds tangent to 4 lines (p = 3, q = -1, n = 2) and the 2nd for the 27 centers of cardioïds tangent to 4 lines (p = 1, q = 2, n = 3).
The 3rd shows the method : there are for a triangle n^2 n-angle centers on n circles through each copple of vertices, each point is the intersection of 3 circles (one for each angle and the opposite side) for a QL, there are 4*n^2 n-angle centers. the key for finding the Miquel circle is that any n-angle center of a triangle may be associated to the n n-angle centers on the same circle of the 3 other triangles (depending on the choosen vertice among the 6 , the 2 sides intersect in the opposite vertice).
To be more precise, any point In of Tr1 (A23A24A34) on a circle through A23 and A24 may be associated to the n points J of the same circle of Tr2 (A13A14A34) through A13 and A14 (the common angle being A34). Each n-angle center is on n Miquel circles. In turn, any point Jn in Tr2 (A13A14A34) on a circle through A13 and A34 may be associated to the n points K of the same circle of Tr3 (A12A14A24) through A12 and A24 (the common angle being AA14).
The rest is well known : the isogonal of a n-angle center in the it's triangle has the same property and may be associated with the isogonal of the 3 associated points of the 3 other triangles, giving the Steiner circles, CSC of the Miquel circles. The perpendicular bisectors of the 4 segments joining the 4 associated points and their isogonals in the 4 triangles give as intersection a Kantor-Hervey point, center of a Hervey circle. There are n^3 such points and circles.
This works for the 8 centers of astroïds and the cardioïds, but not for the nephroïds.
I've identified the 64 points, but they are not on a quartic as I expected !
I've spent much too much time on this construction and I give up temporaly, I hope you will be able to help me.
Many thanks in advance for your attention.
Best regards
Bernard

(rectangular hyperbola)



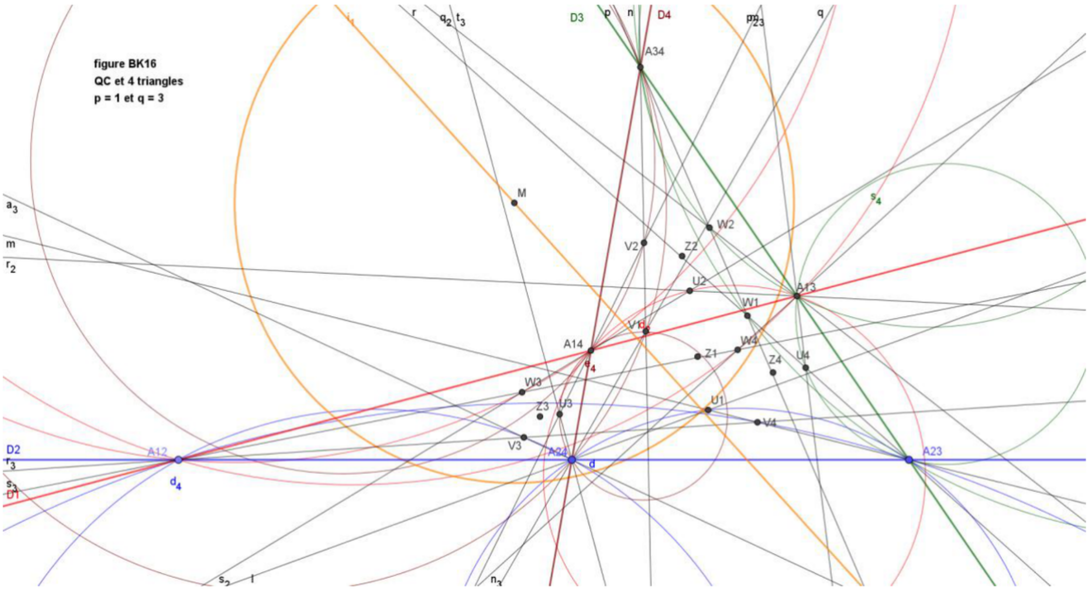
27 centers of cardioïds tangent to 4 lines

(cubic stelloïd)

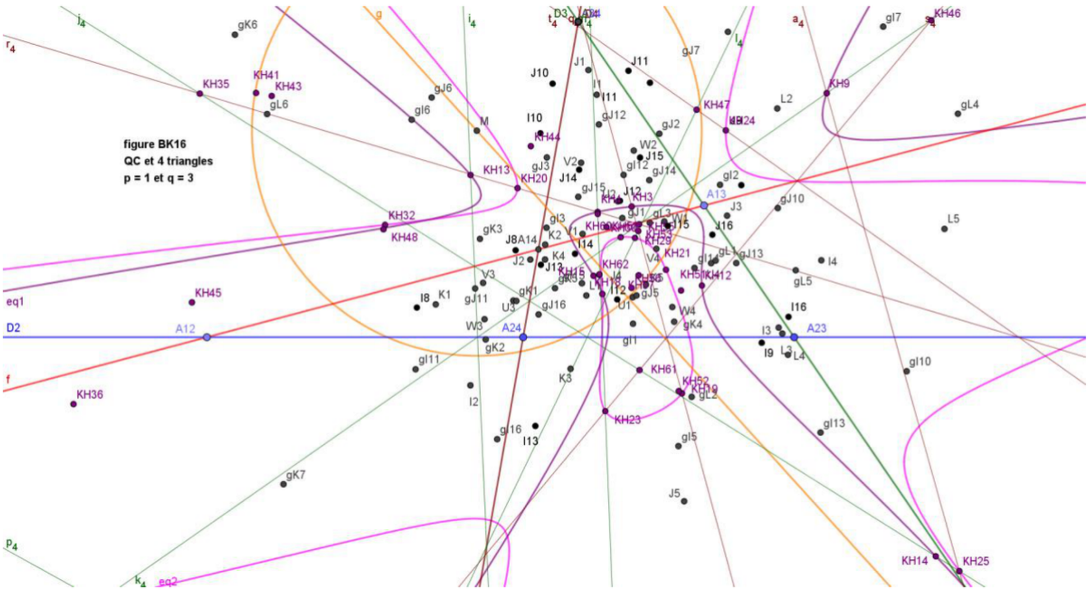


n-angle centers.pdf

(quadrisectrices)



64 centers of nephroïds tangent to 4 lines



n-angle centers.pdf

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Message: #1261
Date: 2021-11-25
From: eckart_schmidt@t-online.de
Subject: Re: Unknown QL-circle?

```
Dear Ngo Quang Duong, dear Vu Thanh Tung,

thanks for your interest and calculations,
... meanwhile I have calculated t
... in DT-coordinates used in EQF,
... but the term is very extensive
... without interpretations wrt a construction.

Best regards Eckart
```

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Message: #1262
Date: 2021-11-25
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Unknown QL-circle?

Dear Eckart, dear Bernard, dear Vu Thanh Tung

I have never used CT-coordinate nor DT-coordinate before. In my opinion, CT and DT-coordinate are not symmetric with respect to 4 lines.

An old topic in Quadri-Figures-Group - [Reconstruction of Quadrilateral when O1, O2, O3, O4 are known](https://groups.io/g/Quadri-Figures-Group/topic/71470394), gave me the idea of an alternative coordinate system (using the complex plane, where the Miquel circle is the unit circle).

On the other hand, the complex plane would be appropriate, especially for the Miquel circle and related objects (since the Miquel point and Miquel circle expose many directly similar figures). I am also using it to work with **n angle centers**.

About the construction of the new circle. It is straightforward to interpret a construction from complex numbers directly. However, I aim to find a more elegant construction.

Ngo Quang Duong

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Message: #1263
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Circle Related to Irregular Hexagon

---

Let P1P2P3P4P5P6 be irregular Hexagon such that
P1P2 Parallel P3P6 Parallel to P4P5 &
P2P3 Parallel to P1P4 Parallel to P5P6 &
P1P6 Parallel to P2P5 Parallel to P3P4 .
AND LET G(P1P2P3P4P5P6)=O.

___

Let Q1= O.t+(1-t).P1
Q2=O.t+(1-t).P6 similarly define Q3,Q4,Q5,Q6 Cyclically .

_____

Let line L1 pass through Q1 and Perpendicular to line OP1 and
Let Line L2
pass through Q6 and Perpendicular to line OP6 .

_____

Let Line L1 and L2 intersect at R1 and define {R2,R3,R4,R5,R6}
Cyclically
then point {R1,R2,R3,R4,R5,R6} lies on same Circle .

______

Note : For t=1/2 ,{R1,....,R6} becomes Circumcentre .

____

Let Line P1P6 intersect Line P2P3 at X1 and
Line P1P2 intersect Line P3P4 =X2.

____

define {X3,X4,X5,X6} Cyclically

______

Then 6 Orthocenters of
(ΔP1P2X1) ; (ΔP2P3X2);.........;(ΔP6P1X6) lies on the same
Circle.

____

*[Note] * = We will check it on other irregular polygon but
before that we
want to varify whether it is new or old?
Best regards
JJ and SS.

Message: #1264
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Circle Related to Irregular Hexagon

---

```
Correction in this line:

____
Let Q1= O.t+(1-t).P1
       Q2=O.t+(1-t).P6 similarly define Q3,Q4,Q5,Q6 Cyclically

_____
Hello in the above line there will be Correction .
There will be Q6 in place of Q2 and it will be written as :

________
Let Q1= O.t+(1-t).P1
       Q6=O.t+(1-t).P6 similarly define Q3,Q4,Q5,Q6 Cyclically

______
For t=1/2 , R1,R2,...,R6 becomes Circumcentre of
(ΔOP6P1).....(ΔOP1P2).

_______
It will holds true for all the values of t where t is Real
number.
```

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Message: #1265
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Irregular polygon Theory . Is it known?

Hello friends , Just read it from link then you don't have to come here
for reading.

____
First Note : We show results from Quadrilateral to Octagon and Similarly
we can Generalise it for any N irregular polygon by making the line
Parallel.

______
Second Note: just varify whether it is new or old so that we can work
further on it and find some more related research.

____
Third Note : After Reading tell us whether such conditions had been applied
to irregular polygon before or not ?

_____
See link below , just read it from link only :
https://1drv.ms/w/s!AkhDFBaJJuungSQUu7lUcaghoDDc

_____
Description :
“Generalisation Of Napoleon Configuration on Irregular N POLYGON”
Hello friends , Here is the Generalisation of Napoleon Configuration on N
irregular polygon.
What is Napoleon configuration ?
Answer: Centroid of equilaterial triangle made on three sides of Triangle
P1P2P3 in same sence gives
Equilaterial triangle and such structures is called Napoleon Configuration .
Here we try to generalised it with some condition .
Conjecture 01 :
Let P1P2P3P4 be Parallelogram then Centroid of Square made on base
(P1P2);(P2P3);(P3P4);(P4,P1) in
same sence makes Square . See Figure 01:
See Figure 01:
Conjecture 02:
Let P1P2P3P4P5 be a irregular Pentagon such that :
P1P2 is parallel to P3P5;

P2P3 is parallel to P1P4;
P3P4 is parallel to P2P5;
P4P5 is parallel to P3P1;
P5P1 is parallel to P2P4 then Centroid of Regular Pentagon made on base
(P1P2), (P2P3),
(P3P4),(P4P5),(P5P1) makes Regular Pentagon.
See Figure 02:
Conjecture 03:
Let P1P2P3P4P5P6 be irregular Hexagon such that
P1P2 Parallel to P3P6 Parallel to P4P5;
P2P3 Parallel to P1P4 Parallel to P5P6 ;
P3P4 Parallel to P2P5 Parallel to P1P6;
Then Centroid of Regular hexagon made on base (P1P2); (P2P3) ;(P3P4);(P4P5) ;(P5,P6);(P6P1) in
same sence makes Regular hexagon.(see figure 03).
Figure 03:
Conjecture 04:
Let P1P2P3P4P5P6P7 be Irregular Heptagon (7 sided polygon) such that
P1P2 Parallel to P3P7 Parallel to P4P6 ;
P2P3 Parallel to P1P4 Parallel to P7P5;
P3P4 Parallel to P2P5 Parallel to P1P6;
P4P5 Parallel to P3P6 Parallel to P2P7;
P5P6 Parallel to P4P7 Parallel to P3P1;
P6P7 Parallel to P1P5 Parallel to P2P4;
P7P1 Parallel to P2P6 Parallel to P3P5 then Centroid of Regular Heptagon
made on base
(P1P2),.....,(P7P1) makes Regular Heptagon . See Figure 04.
Figure 04:
Conjecture 05:
Let P1P2.....P8 be regular Octagon such that
P1P2 Parallel to P3P8 Parallel to P4P7 Parallel to P5P6;
P2P3 Parallel to P1P4 Parallel to P5P8 Parallel to P6P7;
P3P4 Parallel to P2P5 Parallel to P1P7 Parallel to P7P8;
P4P5 Parallel to P3P6 Parallel to P2P7 Parallel to P1P8. Then Centroid of
octagon made on base
P1P2,...,P8P2 makes Regular octagon.
See Figure 05:
Note : similarly we can Generalised it For N polygon by making sides
Parallel .

Best regards
JJ and SS

Message: #1266
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Irregular polygon Theory . Is it

---

```
Hello friends please don't read above things . As I have to make
one correction.
After making that correction you can read. I miss something
which is important to mention above.
```

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Message: #1267
Date: 2021-11-26
From: peter.liepa@gmail.com
Subject: Re: Some observations and references concerning Quadrilateral Conics

---

Dear all,

[This is a restoration of the original post, which I accidently deleted.]

I want to bring to your attention some topics that are relevant to quadrilaterals (and, by duality, to quadrangles), but are not found in the current version of the EQF. They generalize and expand upon material that is already in the EQF. The concepts are mostly projective.

1) The conics tangent to the lines of a quadrilateral form a range that is referred to as the Inscribed Quadrilateral Conics (QL-Co-1).

The director circles of these conics all have a common radical axis, namely the Steiner Line (QL-L2).

The centers of the director circles (and of the conics) lie on the Newton line (QL-L1).

A corollary of this is that the three circles whose diameters are the diagonals of the quadrilateral are coaxal (the Gauss-Bodenmiller Theorem, which is already mentioned in EQF), and that the diagonal midpoints are collinear.

There are many references to this in the classical literature, and some in the modern literature.

For example, Salmon's Conic Sections, pg 277, Ex 3. (https://archive.org/details/ atreatiseonconi09salmgoog/ page/n296/mode/2up?q=%22The+ director+circles%22&view= theater (https://archive.org/details/atreatiseonconi09salmgoog/page/n296/mode/2up?q=%22The+director+circles%22&view=theater))

Also, Russell, Geometry, pgs 218-220. (https://archive.org/details/ cu31924059551501/page/n241/ mode/2up?view=theater (https://archive.org/details/cu31924059551501/page/n241/mode/2up?view=theater))

There are several more available upon request. The Salmon reference is the first one I noticed, and probably not the best. But I didn't want to overwhelm with several references.

A couple of references mentioned the duals of these concepts (e.g. dual of director circles), but I did not get around to understanding what they were saying.

2) The eleven point conic for a quadrangle is a generalization of the nine point conic. The latter is mentioned in EQF.

3) The eleven tangent conic for a quadrilateral is a generalization of the nine tangent conic. Neither are mentioned in EQF.

A modern reference for 2) and 3) is Pamfilos, A Gallery of Conic by Five Elements. (https://forumgeom.fau.edu/FG2014volume14/FG201431.pdf) (https://forumgeom.fau.edu/FG2014volume14/FG201431.pdf)

A classical reference is Hatton, Projective Geometry, pg 250. (https://archive.org/details/ cu31924060184045/page/n265/mode/2up?view=theater) (https://archive.org/details/cu31924060184045/page/n265/mode/2up?view=theater)

Best Regards,
Peter Liepa

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Message: #1268
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Irregular polygon Theory . Is it

---

```
Hello friends ,_______________,

We have also put the same theorem on Euclid group but put here
as it belongs to polygon so there is a Need of little bit
modification in all theorem like in  Case of Hexagonal
Configuration holds true when P1P4, P2P5,P3P6  are concurrent at
Centroid of Hexagon in Conjecture 03 and also in Octagon and
Similar modification needs to be done everywhere

So we here take time to modify it and after those modification
we will write Above all thing in Single General Form.

So you can ignore the above mentioned things for some days.
```

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Message: #1269
Date: 2021-11-26
From: archanajha112018@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Circle Related to Irregular Hexagon

---

```
Correction in this line:
> ____
> Let Q1= O.t+(1-t).P1
>       Q2=O.t+(1-t).P6 similarly define Q3,Q4,Q5,Q6 Cyclically
> _____
> Hello in the above line there will be Correction .
> There will be Q6 in place of Q2 and it will be written as :
> ________
> Let Q1= O.t+(1-t).P1
>       Q6=O.t+(1-t).P6 similarly define Q3,Q4,Q5,Q6 Cyclically
> ______
> For t=1/2 , R1,R2,...,R6 becomes Circumcentre of
(ΔOP6P1).....(ΔOP1P2).
> _______
> It will holds true for all the values of t where t is Real
number.
> ____________________________
```

>

Additional Property :
Case of Orthocentre was always holds true .

____
Case of Circumcentre holds true if and only if
P1P4, P2P5,P3P6 are concurrent at Centroid of Hexagon .

____
Best REGARDS
Jayendra Jha and sankalp savaran.

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Message: #1270
Date: 2021-11-26
From: van10hoven@gmail.com
Subject: Re: Some observations and references concerning Quadrilateral Conics

---

Dear Peter,

Very interesting subject and thanks for the references.
I like especialy the book of Hatton – The principles of Projective Geometry applied tot he straight line and conic.
It shows how many items regarding quadrangles and quadrilaterals were known for a long time.
According to Pamfilos (your 3rd reference) this is the definition of *eleven point conic in a quadrangle* :
A particular property of pencils, together with its dual for ranges, is of interest for our subject. For instance, in the case of a pencil D of conics through points A, B, C, D, it is known that the polars of a fixed point X with respect to all members of the pencil pass through a point Y . This defines a quadratic transformation Y = f(X), ...

The image of a line e under this transformation is a conic ke circumscribing triangle EFG and passing through eight additional points, therefore called an eleven point conic ([1, p. 97], [9, p. 66]). Six of the points are the harmonic conjugates W = V (X, Y) of the intersection point V = (XY,e), where X, Y are taken from {A, B, C, D}. The two remaining points, if real, are the intersection points of ke with line e and simultaneously the contact points of two members of the pencil D, which are tangent to e (a case handled in §8.1).

Remark:
The quadratic transformation Y=f(X) is QA-Tf2 in EQF.
According to Pamfilos (your 3rd reference) this is the definition of *eleven lines conic in a quadrilateral* :
The dual to the previous property relates to the range D □ of conics k tangent to four lines a, b, c, d (see Figure

7). According to this, the poles of a line h with respect to the members of D □ lie on a line h and the transformation h = F □ (h) is a quadratic one of the same nature as the previous one, differing only in that it operates on the dual projective plane P □. Line h can be found by a simple criterion, resulting by considering the triangle of diagonals (efg in Figure 7). Lines h, h intersect each side s of this triangle at points X, Y , which are harmonic conjugate with respect to (U, V), where U, V are the vertices of the quadrilateral lying on s. The images h under F □ of all lines h passing through a fixed point Q are the tangents of a conic kQ inscribed in the triangle efg and tangent to eight additional lines, therefore called an eleven tangents conic. Six of these lines are the harmonic conjugates Q(s, s) of Q with respect to all pairs (s, s) taken from {a, b, c, d}. The two remaining tangents, if real, are the tangents through Q of the members of D □ passing through Q (a case handled in §6.1).

Remark:
The quadratic transformation h = F □ (h) is QL-Tf2 in EQF.
The names of Eleven-point-conic and Eleven-lines-conic were new to me. Also the use of director circles.

Best regards,
Chris

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Message: #1271
Date: 2021-11-28
From: eckart_schmidt@t-online.de
Subject: QG-P2 property

```
Dear Chris,

wrt the following property of QG-P2:
"The 3 QL-Versions of QG-P2
 are 3 points on the Newton Line ..."
we can add:
" ... with centroid QL-P12."

Best regards Eckart
```

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Message: #1272
Date: 2021-11-28
From: eckart_schmidt@t-online.de
Subject: Circles through QL-points

```
Dear all,

most of these properties will be well known,
... especially the four circumcircles of the QL-triangles,
... centered on the Miquel circle QL-Ci3.
But the six QL-points give 12 further circles through 3
QL-points,
... the degenerated QL-lines not counted,
... par four points centered on the bisectors of opposite
QL-points,
... which define the QG-P5 triangle of the QL,
... which has interesting properties, see EQF.

Perhaps new:
The circumcircle of the QG-P5 triangle,
... cuts QL-Ci1 orthogonal in QL-P16 and QL-P17
... and its center is the pole of QL-P16.QL-P17 wrt QL-Ci1,
... Simson line of QL-P16 is parallel QL-L1 through ???,
... isogonal conjugate of QL-P16 is infinity point
    of perpendiculars to QL-L1,
... Simson line of QL-P17 is parallel QL-L9 through QL-P18,
... isogonal conjugate of QL-P17 is infinity point
    of perpendiculars to QL-L9.

Best regards Eckart
```

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Message: #1273
Date: 2021-11-28
From: van10hoven@gmail.com
Subject: Re: QG-P2 property

```
Dear Eckart,

Thanks!
I noted it in EQF at QG-P2 and QL-L1.

Best regards,
Chris
```

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Message: #1274
Date: 2021-11-28
From: peter.liepa@gmail.com
Subject: Re: Some observations and references concerning Quadrilateral Conics

---

Dear Chris,

Thank you for adding some details on eleven points/lines conics, and the relations to Q*-Tf2.

Although I did not know about eleven-point/line conics and the theorem about director circles until very recently, once I knew about them they seemed to pop up whenever I was browsing books or papers for unrelated reasons.

So this post will list additional references, for anybody who is interested in learning more. Many of the books are century old or more. It is not accurate to call this "forgotten geometry", but modern texts tend to overlook this sort of thing in favor of more modern and general techniques and concepts.

Eleven point/line conics are discussed in:
Baker, Vol II, pgs 41-42 - https://archive.org/details/dli.ernet.524768/page/41/mode/2up?view=theater&q=eleven),
(https://archive.org/details/dli.ernet.524768/page/41/mode/2up?view=theater&q=eleven),) which is referenced in Vigara, Non-euclidean shadows of classical projective theorems - https://arxiv.org/pdf/1412.7589.pdf

As for the director circles of inscribed conics ..

Baker, Principles of Geometry Vol II, pg 80 - https://archive.org/details/dli.ernet.524768/page/79/mode/2up?view=theater

I've wondered whether there is a dual concept for conics through 4 points. The above reference has a section called The dual of a director circle of a conic - https://archive.org/details/dli.ernet.524768/page/93/mode/2up?view=theater&q=director - but I have not been able to make much sense of it. Maybe somebody else here will have the time and interest.

Russell Pure Geometry , pgs 218-220 -
https://archive.org/details/cu31924059551501/page/n241/mode/2up

Berger, Geometry II, pgs 239-241. (sorry, no URL) Director circles are called orthoptic circles in this book.

Taylor, *Ancient and Modern Geometry of Conics*, pg 280 -
https://archive.org/details/anintroductiont02taylgoog/page/n375/mode/2up?view=theater&q=director

There is a reference to a dual concept in Article 69 - https://archive.org/details/anintroductiont02taylgoog/page/n269/mode/2up - but again, I haven't had the time or motivation to understand it.

The above list is not meant to be comprehensive. Rather, once I noticed the concept in Salmon, it started jumping out of various other books and papers, even though I was not looking for it.

Best regards,
Peter

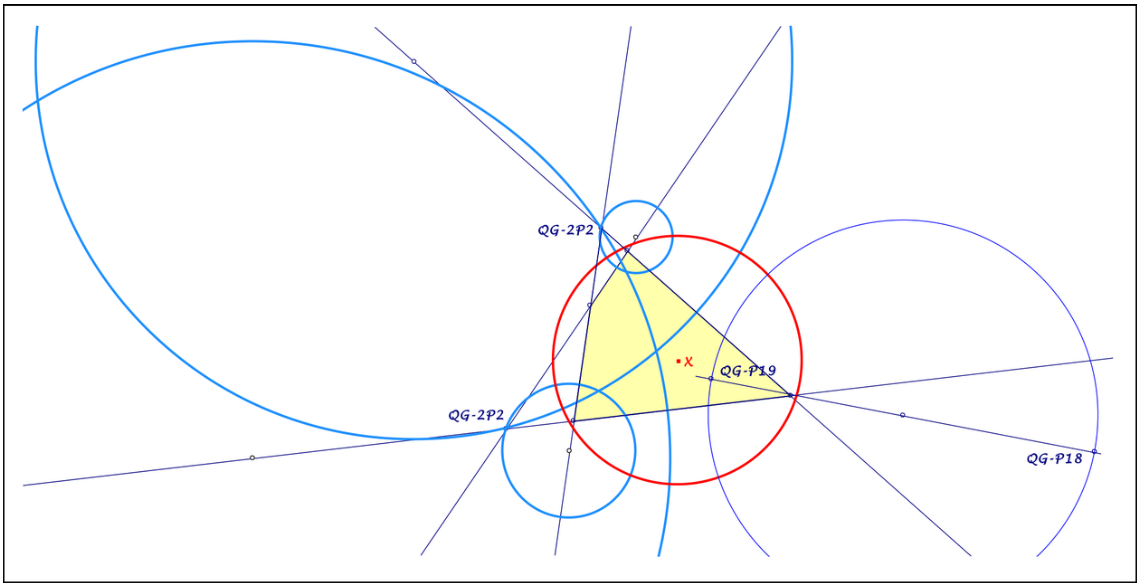
Message: #1275
Date: 2021-11-30
From: eckart_schmidt@t-online.de
Subject: New QG-point and circle

Dear all,

consider a QG and its intersections QG-2P2 of opposite sides,
... if we interpret QG as QL, these two points are vertices of the four
QL-triangles,
... their Apollonius circles have radical axes through a common point X,
... which is center of a circle orthogonal to the four Apollonius circles,
... also orthogonal to the circle with diameter QG-P18.QG-P19.

What about this circle and its center, not always real?

Best regards Eckart



2021-11-29.pdf

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Message: #1276
Date: 2021-11-30
From: eckart_schmidt@t-online.de
Subject: Re: Some observations and references concerning Quadrilateral Conics

---

Dear Peter Liepa, dear Chris,

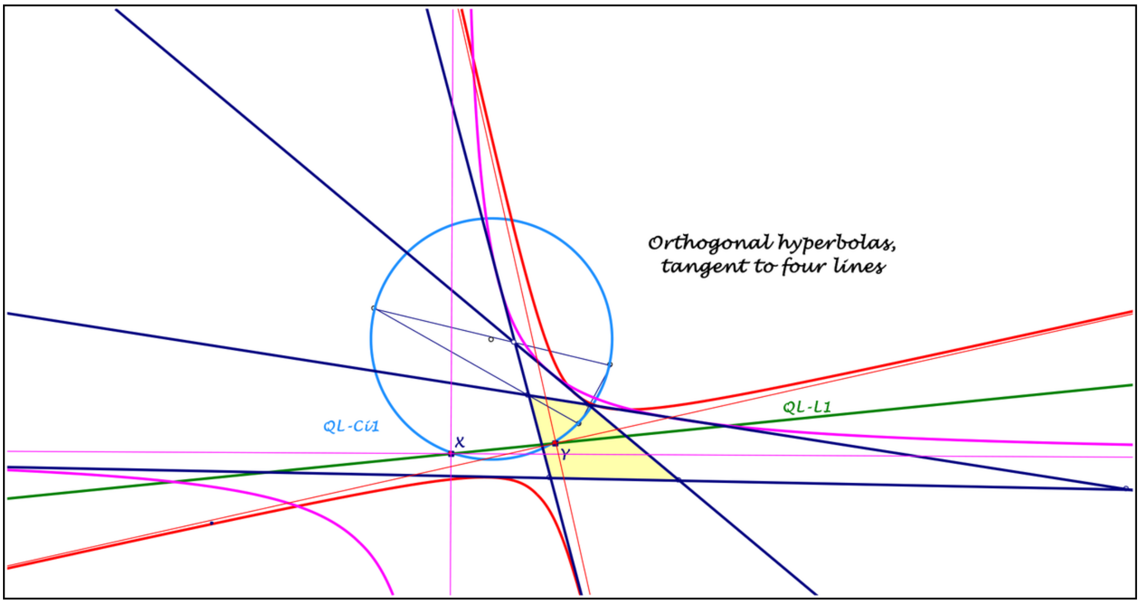
I had a first look in the given references,
... which gather many properties of quadrilaterals and
    quadrangles,
... for example (Russell Pure Geometry, pg.221):

Ex 14: Show that two, and only two, rectangular hyperbolas
... can be drawn to touch four given lines:

The centers of these orthogonal hyperbolas
... are the intersections X, Y of QL-Ci1 and QL-L1,
... but X, Y are not always real!

Assumption: These centers will be real,
... if the convex QG of the QL has two acute
     and two obtuse opposite angles.

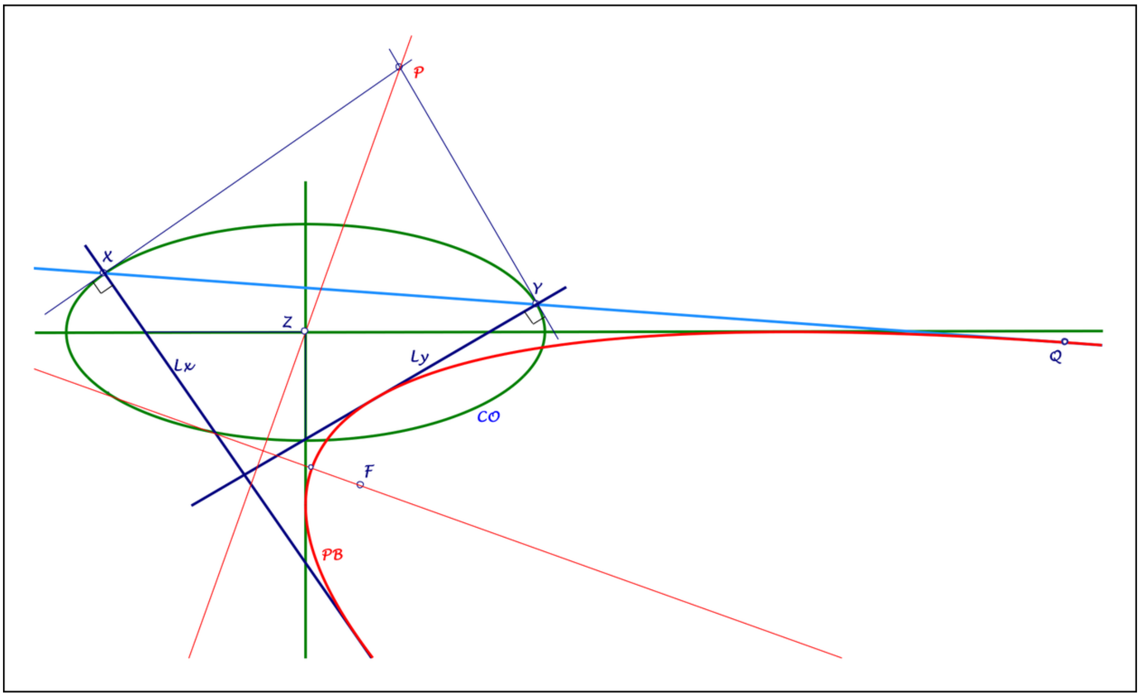
I think this is not in EQF.
Best regards Eckart



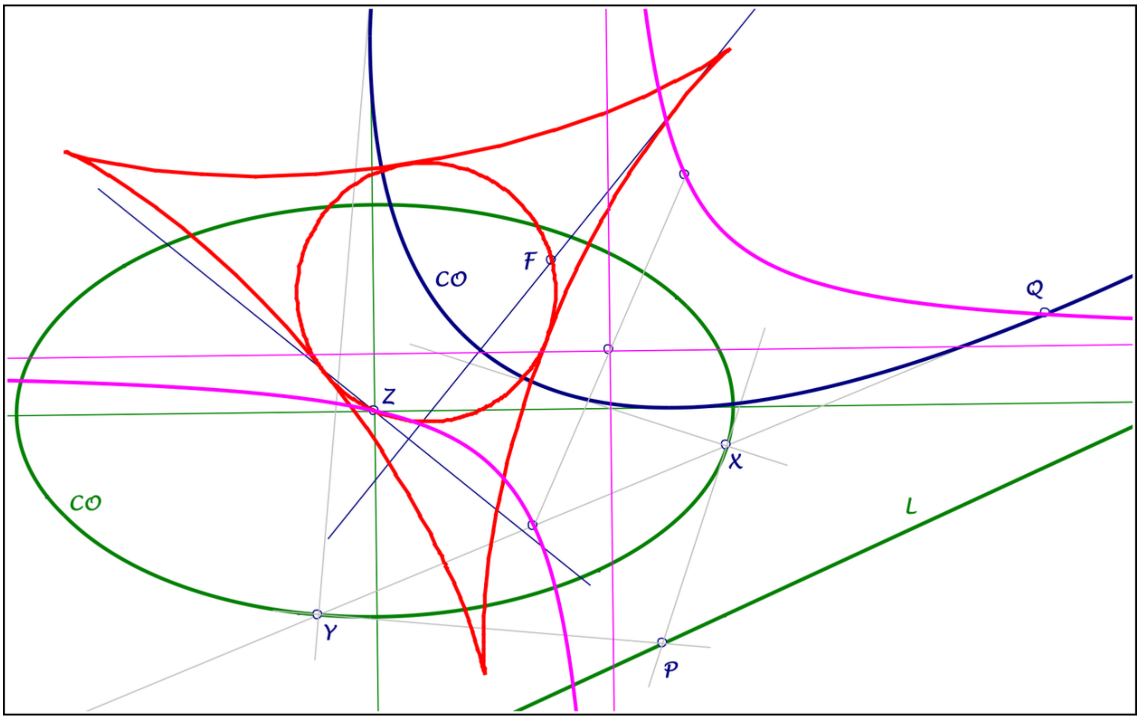
2021-11-30.pdf

Message: #1277
Date: 2021-12-01
From: eckart_schmidt@t-online.de
Subject: Conic transformation

```
Dear all,

if someone is interested in conic transformations,
... here are some aspects, starting with a transformation,
... which maps a point wrt a conic to a parabola,
... background is a reference in #1274: Russell Pure Geometry,
pg.143, Ex9:

Let CO be a conic centered in Z and P an arbitrary point
... with intersections X, Y of the polar of P and CO
... and perpendiculars Lx, Ly to the tangents in X, Y.
The quadrilateral of Lx, Ly and the axes of CO
... has a parabola PB = QL-Co1, tangent to XY in Q,
... with directrix connecting P and the CO-center Z.
If P is a running point on a line L,
... we get for the focus F of PB a circle,
... we get for the axis of PB a deltoid round the circle
... and for the contact point Q of XY and PB an orthogonal
hyperbola,
... ... with asymptotes parallel to the axes of CO,
... ... through the center Z of CO and the pole R of L wrt CO,
... ... centered in the midpoint of R and its inverse
         wrt CO-Tf3.

Best regards Eckart
```



2021-12-01a.pdf



2021-12-01b.pdf

Message: #1278
Date: 2021-12-02
From: ngo.quang.duong.1100@gmail.com
Subject: Re: n-angle centers revisited

```
Dear Bernard,

Pardon me for not replying sooner. So far, I cannot catch up
with your ideas and approaches. Though I do interest in this
topic of n-angle center.

Due to the lack of my knowledge, I'm not acquainted with
algebraic curves. Most of the time, my approaches are elementary
or some appropriate coordinate systems.

I need more time to study your messages (including this topic
and [Complex coordinates](https://groups.io/g/Quadri-and-Poly-Ge
ometry/topic/complex_coordinates/84769387), I assume) and
represent them in my understanding.

Best regards,

Ngo Quang Duong
```

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Message: #1279
Date: 2021-12-02
From: archanajha112018@gmail.com
Subject: [Quadri-and-Poly-Geometry] Convex Hexagon Theorem related to Napoleon

---

```
Hello friends , here is our theory Regarding convex Hexagon here
:
https://math.stackexchange.com/questions/4321214/is-this-origina
l-generalisation-of-napoleon-configuration

Vairfy whether it is new or old?

Best regards
JJ and SS.
```

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Message: #1280
Date: 2021-12-02
From: archanajha112018@gmail.com
Subject: Re: Convex Hexagon Theorem related to Napoleon Configuration.

```
*We are Writing the description of the link here .
But you are requested to read it from link only as there is
FIGURE also .
```

```
We are writing the description here because if in case the link
will dead the description will be available:
```

```
"Description of theorem given in link"
*
Here is our speculate:

Conjecture 01 (generalization of the Napoleon configuration):
Let A1A2A3A4A5A6 be an irregular hexagon and let B1, B2, B3, B4,
B5, B6 be the zenith of an equilateral triangle primarily based
on {A1A2, A2A3, A3A4, A4A5, A5A6, A6A1} both inwards and
outwards, then G (ΔB1B2B3); G (ΔB3B4B5); G (ΔB5B6B1) ends in an
equilateral triangle.

Generalization of Assumption 01: Let C1 = B1.t + (1-t) B2 in
Assumption 01 and outline analogously {C2, C3, C4, C5, C6}
Cyclic then G (ΔC1C2C3); G (ΔC3C4C5); G (ΔC5C6C1) kinds an
equilateral triangle for each actual worth of t.

Note: right here the attribute G (ΔABC) means the focus of
gravity of ΔABC.

Note: Points {A1, A2, …, A6} are capricious factors, they'll
coincide and be completely different, then there is no such
thing as a consequence.

Important point to: Since the factors {A1, …., A6} are
capricious, i.e. if factors A1 and A2 coincide and equally
factors (B1, B2) and (C1, C2) coincide, we get Napoleon
configurations.

We first give our speculate right here:
https://teams.io/g/euclid/message/2249   †)
( https://groups.io/g/euclid/message/2249 )
```

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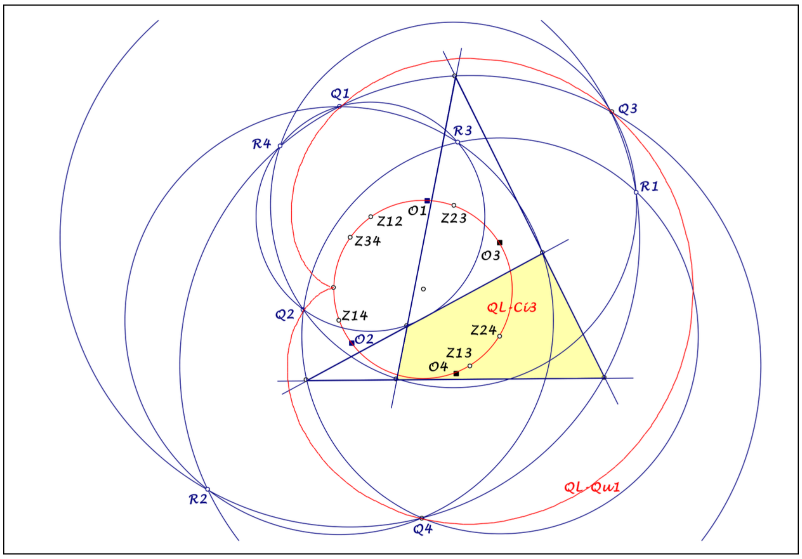
†) Editorial note: The referenced message(s) can be found as listed in EPG-References [72].

Message: #1281
Date: 2021-12-03
From: eckart_schmidt@t-online.de
Subject: Circles, centered on QL-Ci3

Dear all,

the Miquel circle QL-Ci3 is named by Clawson
... as "circumcentric circle" (see Ref. [22] Part 1, 2, (6)),
bearing
... for example QL-P1 and the centers Oi of the circumcircles
    of the QL-triangles.
Here six further circles are described, centered on QL-Ci3:
Consider a QL with lines L1,L2,L3,L4
... and the isogonal conjugate Qi of the infinity points of Li
... wrt the triangles LjLkLl on their circumcircles,
Qi are the CSC images of the contact points of the parabola
QL-Co1,
... so they are points of the cardioid QL-Qu1.
Circles Ci,j through Qi, Qj, Lk^Ll have their center Zi,j also
on QL-Ci3,
... these six circles have four further triple intersections
    Ri = Cj,k^Ck,l^Cj,l,
... so the five points Qi,Qj,Rk,Rl,Lk^Ll are concyclic
    on a circle with center on QL-Ci3,
... bisectors of Qi.Rj (i#j) bear the centers Zi,k and Zi,l,
... perpendiculars in QL-P1 to QL-P1.Li^Lj bear Zk,l,
... circles through Qi, Qj, Zi,j bear QL-P1.

Best regards Eckart



2021-11-29.pdf

Message: #1282
Date: 2021-12-05
From: bernard.keizer@gmail.com
Subject: Re: n-angle centers revisited

```
Dear Ngo Quang Duong,
Thanks anyway for your interest !
I no longer expected an answer ...
How can I help you ? Do you read french ?
Do you need more explanations or new references ?
Best regards
Bernard
```

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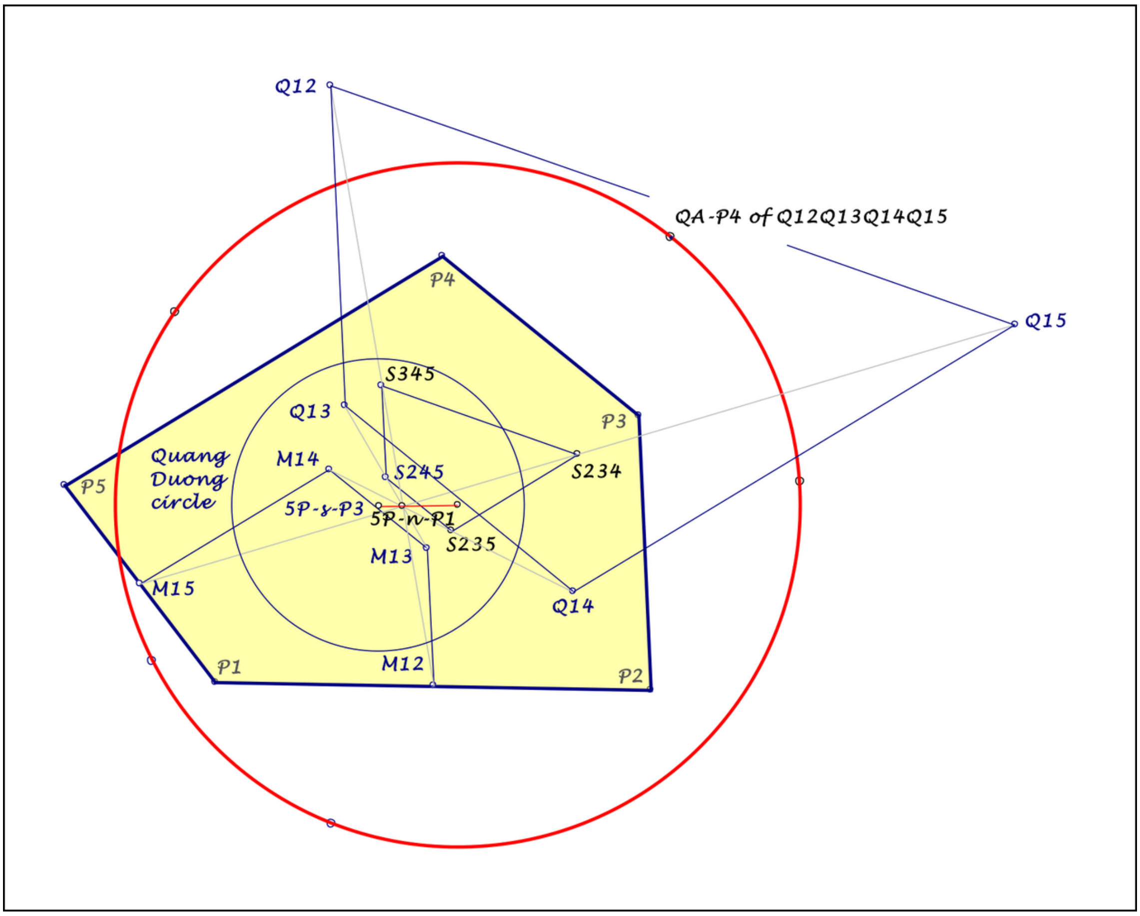
Message: #1283
Date: 2021-12-07
From: eckart_schmidt@t-online.de
Subject: Related Quang Duong circles

```
Dear all,

consider a 5P = P1...P5 and take for each vertex Pi (see
attached drawing)
... the reflected midpoints Mij of PiPj in the centroid Sklm
    of PkPlPm as Qij,
... the 5 points QA-P4 of Qij, i fix and j unequal i,
    are concyclic
... on a circle with a center, dividing 5P-n-P1.5P-s-P3
    with ratio -7/10
... and radius 7/3 of the radius of the Quang Duong circle
    (see 5P-s-P3),
... finally this circle is the expanded Quang Duong circle
    with ratio -7/3 from 5P-n-P1.

If we change the reflection:
consider a 5P = P1...P5 and take for each vertex Pi
... the reflected centroids Sklm of PkPlPm in the midpoint Mij
    of PiPj as Qij,
... the 5 points QA-P4 of Qij, i fix and j unequal i,
    are concyclic
... on a circle with a center, dividing 5P-n-P1.5P-s-P3
    with ratio -8/5
... and radius 8/3 of the radius of the Quang Duong circle
    (see 5P-s-P3),
... finally this circle is the expanded Quang Duong circle
    with ratio 8/3 from 5P-n-P1.

Best regards Eckart
```



2021-12-07.pdf

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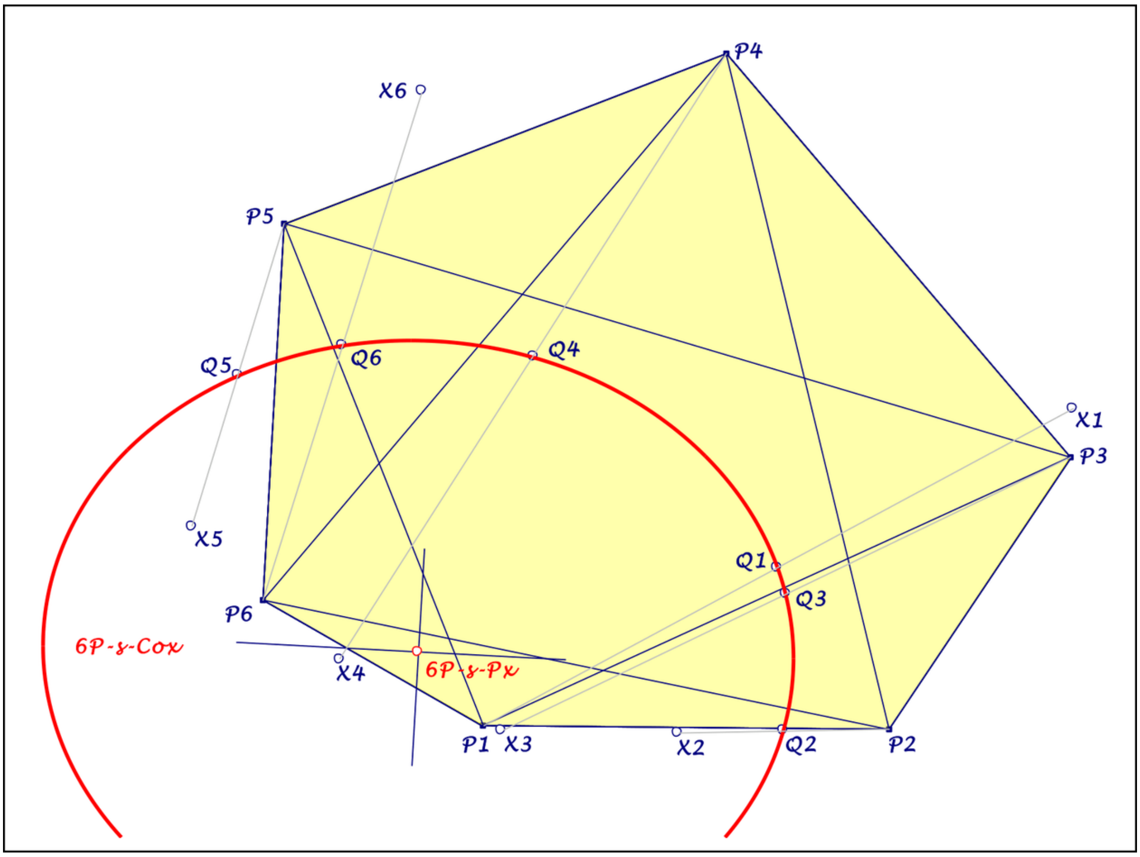
Message: #1284
Date: 2021-12-09
From: eckart_schmidt@t-online.de
Subject: New 6P-s-Conic and 7P-s-Line

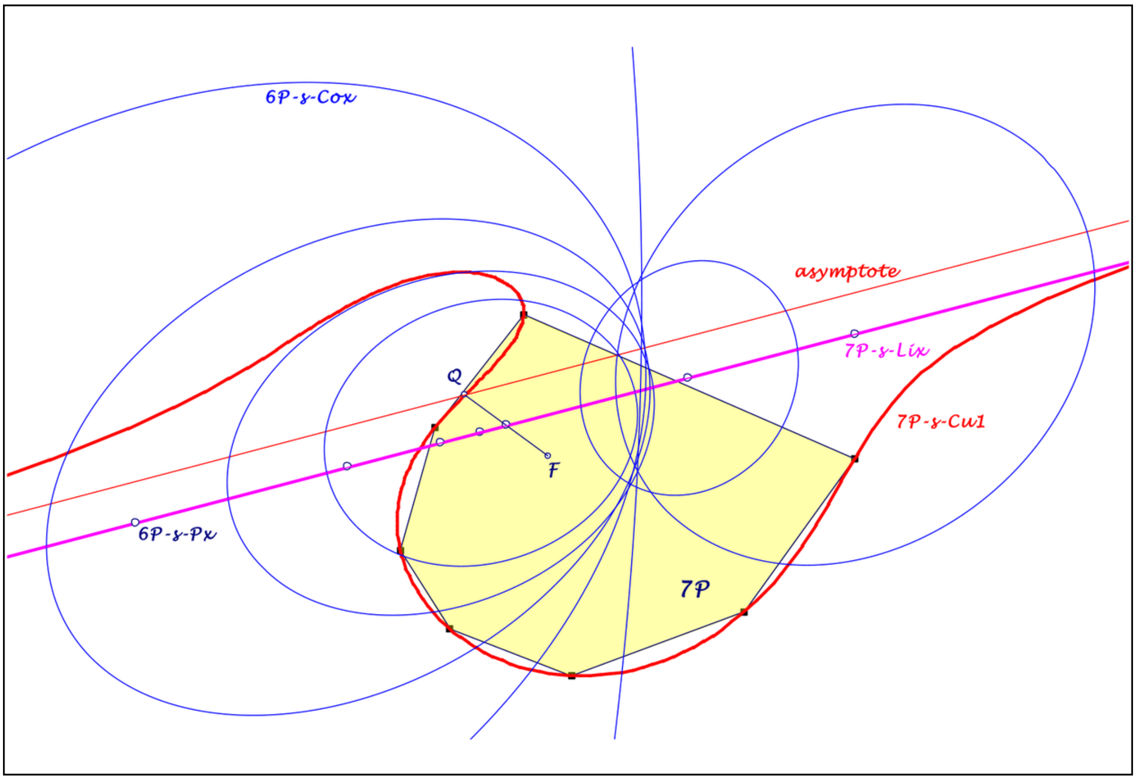
```
Dear all,

it seems, that there is a generalization of Quang Duong's circle
(see 5P-s-P3):
... starting with a 6P = P1...P6,
... let Qi be the midpoint of Pi and Xi = 5P-s-P4
    of the other vertices,
... these 6 Qi lie on a conic 6P-s-Cox,
... for a 7P the centers 6P-s-Px of the 7 conics 6P-s-Cox
    are collinear
... on a line parallel to the asymptote of 7P-s-Cu1
... through the midpoint of F.Q,
... F = focus, Q = intersection of the cubic and its asymptote.

Best regards Eckart
```



2021-12-08.pdf



2021-12-08a.pdf

Message: #1285
Date: 2021-12-13
From: bernard.keizer@gmail.com
Subject: Re: New 6P-s-Conic and 7P-s-Line

```
Dear Eckart,
I've found these properties very interesting !
It is remarkable that any 7 real points on a circular cubic have
the same 7P-s-Line.
For example, for QA-Cu1, it is the parallel to the asymptote
through the middle of QA-P9Q2 (circumcenter of the triangle
QA-Tr2).
For QL-Cu1, it is the Newton Line.
I suppose it is possible to find different combinations of 7
points on these cubics (4 QA vertices + any triple of QA points
or QL vertices + another QL- point ...).
Beautiful, indeed ! Congratulations
Best regards
Bernard
```

Message: #1286
Date: 2021-12-13
From: eckart_schmidt@t-online.de
Subject: 6P-s-P3

```
Dear Chris,

sorry, I lost control:
Where is the point 6P-s-P3 defined,
... I only found it mentioned in #780 and #790, not in EPG.
Thanks in advance.

Best regards Eckart
```

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Message: #1287
Date: 2021-12-13
From: van10hoven@gmail.com
Subject: Re: 6P-s-P3

```
Dear Eckart,

6P-s-P3 was named by Bernard referring to 6P-s-Tf3 which I
proposed in #763.
All items in #763 were proposals and are not realized yet.

Best regards,
Chris
```

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Message: #1288
Date: 2021-12-20
From: bernard.keizer@gmail.com
Subject: Merry Christmas and Happy New Year

```
Dear Chris, dear Eckart, dear Ngo Quang Duong, dear all,
I wish you all a merry Christmas and a happy Newyear 2022
without Covid.
I wish in particular that many of the beautiful properties
recently discovered find their place in EQF.
I hope also that Ngo Quang Duong will continue to dig the item
n-angle centers ...
Best regards
Bernard
```

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Message: #1289
Date: 2021-12-20
From: van10hoven@gmail.com
Subject: Re: Merry Christmas and Happy New Year

```
Dear QPG-friends,

Wishing you a merry Christmas and a happy New Year.
May it be all that you hope it will be!
Best regards,

Chris
```

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Message: #1290
Date: 2021-12-23
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as focal circular cubic

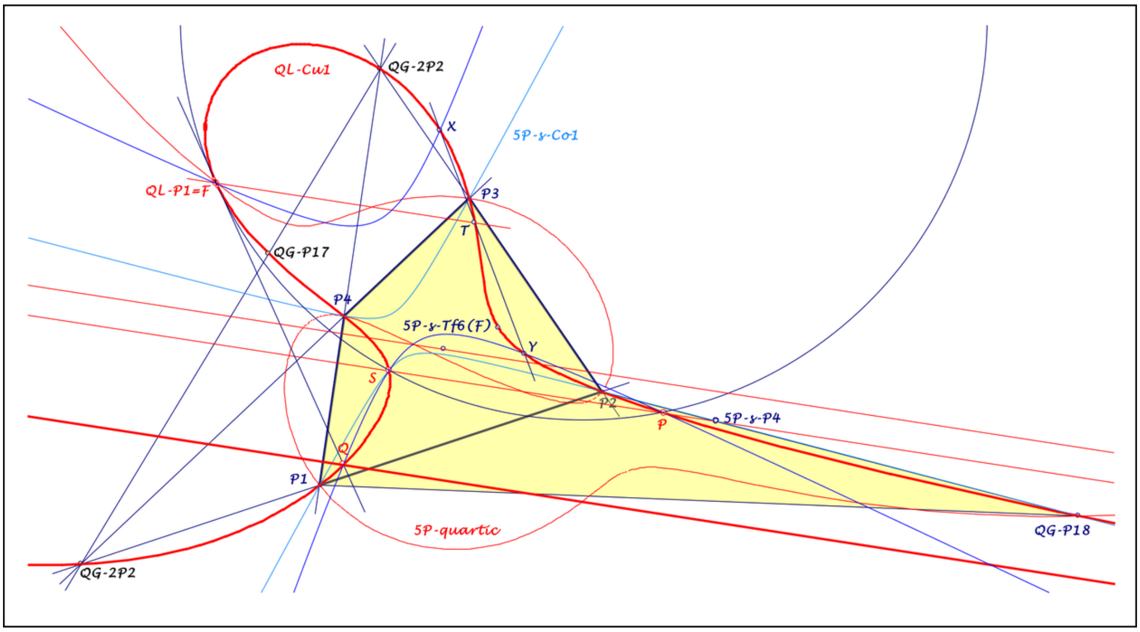
```
Dear all,

for a quadrigon QG = P1P2P3P4 the cubic QL-Cu1
... can be considered as focal circular circumcubic
... of the 5P = QG plus QG-P18 with focus F = QL-P1,
... for QL-P1 lies on the quartic of this 5P.
This aspect here shall be researched
... with the observations in #727, #732, #736.
The results may be already mentioned,
... but perhaps nevertheless interesting:
... QL-Cu1 for a QG  bears also QG-P17 and QG-2P2,
... their 5P-s-Tf7 circles intersect beside 5P-s-P5
    in the focus F = QL-P1.
... QL-Cu1 is CSC-invariant, for example CSC(QG-P17) = QG-P18,
... QL-Cu1 is 5P-s-Tf6-invariant with a pivot P on QL-Cu1.
... The 2nd intersection of P.5P-s-P4 and 5P-s-Co1 gives
    S = CSC(5P-s-Tf6(F)),
... as well as the 3rd intersection of F.CSC(P) and QL-Cu1,
... S is the 6th intersection of QL-Cu1 and the circumconic of
5P.
... S.P.5P-s-P4 is parallel to the asymptote of QL-Cu1,
... a parallel through the midpoint of S.5P-s-Tf6(F)
    is the Newton line
... and a parallel through the reflection of F
    in the Newton line is the asymptote,
... intersecting QL-Cu1 in Q on the tangent in F
    at the circle (F,S,P).
... For conics through the main points F, P, S, Q
... and intersections X, Y with the cubic
... the line XY has a 3rd intersection T = CSC(Q)
... on a parallel to the asymptote through F.

Finally:
QL-Cu1 is also QL-Cu1 of the quadrigon S,T,5P-s-Tf6(F),Q.

Merry Christmas and a healthy start in the New Year.

Best regards Eckart
```



2021-12-21.pdf

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Message: #1291
Date: 2021-12-26
From: Stan.Rabinowitz@comcast.net
Subject: another type of quasicircumcenter

```
Let ABCD be a quadrigon.
Let P be the intersection of the perpendicular bisectors of AC
and BD.
P is almost a circumcenter, since PA=PC and PB=PD.
Dao has shown that the incenters of triangles PAB, PBC, PCD, PDA
lie on a circle.

My question is: Is this point a known center? What is it called?
Does it have any other properties?
```

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Message: #1292
Date: 2021-12-26
From: Stan.Rabinowitz@comcast.net
Subject: Re: another type of quasicircumcenter

```
Correction: (the quadrigon must have an incircle)

Let ABCD be a quadrigon.
Let P be the intersection of the perpendicular bisectors of AC
and BD.
P is almost a circumcenter, since PA=PC and PB=PD.
Dao has shown that the incenters of triangles PAB, PBC, PCD, PDA
lie on a circle *if the quadrigon is tangential*.

My question is: Is this point a known center? What is it called?
Does it have any other properties?
```

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Message: #1293
Date: 2021-12-31
From: ngo.quang.duong.1100@gmail.com
Subject: Re: Merry Christmas and Happy New Year

```
Dear Bernard, dear Chris, dear Eckart, and all,

Thank you so much for your wishes.
I wish you all a Happy New Year.
I hope that you all will be safe and sound in this pandemic.

To Bernard: Pardon me for not being available a few weeks
recently. I have been working on my thesis. I will come back to
the topic "n-angle center" after my thesis, in the next three
weeks, at most.

Best regards,

Ngo Quang Duong
```

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Message: #1294
Date: 2021-12-31
From: ngo.quang.duong.1100@gmail.com
Subject: Re: n-angle centers revisited

```
Dear Bernard,

As mentioned earlier in [message 1293](https://groups.io/g/Quadr
i-and-Poly-Geometry/message/1293), I am busy with my thesis. So
I will be back later.

About the topic "n-angle center", I would re-read [Complex
coordinates topic](https://groups.io/g/Quadri-and-Poly-Geometry/
topic/complex_coordinates/84769387) and organize the ideas
first.

Unfortunately, I don't know French. I will ask you after reading
the topic carefully and compiling the ideas.

Best regards,

Ngo Quang Duong
```

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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

- **QA-P4**:
#650 #665 #689 #691 #705 #706 #708 #722 #730 #737 #799 #801 #808 #836 #840 #841 #842 #854 #856 #857 #858 #861 #862 #863 #864 #865 #868 #870 #871 #875 #880 #892 #898 #899 #900 #903 #904 #906 #907 #908 #916 #917 #922 #931 #932 #933 #934 #937 #950 #951 #973 #993 #996 #1001 #1004 #1009 #1013 #1027 #1028 #1029 #1030 #1031 #1041 #1045 #1046 #1052 #1057 #1058 #1060 #1061 #1064 #1087 #1099 #1104 #1106 #1117 #1118 #1120 #1128 #1145 #1154 #1196 #1197 #1248 #1249 #1283

- **QA-P41**:
#665 #666 #720 #725 #737 #799 #800 #801 #808 #854 #857 #858 #861 #862 #863 #865 #868 #870 #875 #880 #882 #892 #950 #951 #952 #973 #1001 #1004 #1005 #1031 #1052 #1104 #1117

- **QA-Tf16**:
#650 #799 #808 #1014 #1027 #1028 #1029 #1030 #1031 #1032 #1035 #1036 #1040 #1042 #1045 #1047 #1049 #1087 #1088 #1105 #1115 #1117 #1119 #1154 #1157 #1196 #1197

- **QL-Cu1**:
#635 #637 #660 #665 #666 #681 #703 #727 #728 #729 #818 #841 #842 #847 #848 #849 #851 #852 #861 #862 #864 #869 #870 #878 #882 #884 #993 #1010 #1046 #1068 #1077 #1079 #1081 #1100 #1128 #1129 #1148 #1159 #1173 #1186 #1187 #1189 #1190 #1228 #1229 #1285 #1290

- **Quartic**:
#635 #636 #638 #643 #650 #655 #658 #668 #669 #691 #693 #694 #700 #705 #706 #707 #709 #711 #717 #724 #725 #726 #732 #736 #750 #752 #753 #754 #761 #763 #764 #771 #772 #777 #778 #780 #790 #833 #834 #843 #844 #884 #885 #919 #923 #924 #926 #990 #994 #998 #1001 #1004 #1009 #1032 #1044 #1045 #1047 #1048 #1052 #1053 #1058 #1060 #1069 #1070 #1071 #1072 #1073 #1074 #1075 #1076 #1079 #1081 #1082 #1084 #1086 #1104 #1105 #1106 #1119 #1129 #1154 #1173 #1260 #1290

- **Reye configuration**:
#1117

- **Sextic**:
#650 #684 #694 #699 #700 #702 #708 #711 #719

- **Simson line**:
#1062 #1098 #1272

- **Steiner Axes**:
#701

- **Steiner Line**:
#701 #1100 #1137 #1149 #1150 #1151 #1162 #1198 #1208 #1209 #1267

- **Stelloid, stelloïd**:
#672 #1123 #1126 #1129

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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
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Journal of the Quadri- and Poly-Geometry Group

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Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

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Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393