Journal of the Quadri- and Poly-Geometry Group 2022

Digital Edition

Chris van Tienhoven et al.

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Messages #1295 - #1544

Contents

1 Introduction

This journal is a compilation of messages from the **Quadri- and Poly-Geometry (QPG)** forum, where mathematicians and geometry enthusiasts exchange ideas on the properties of **quadrilaterals**, **polygons**, and **curves of nth degree**. The discussions cover a wide range of topics, from classical geometric theorems to new discoveries and insights.

The origins of this journal trace back to the Quadri Figures Group (QFG, available at https://groups.io/g/Quadri-Figures-Group), which was active from 2013 until November 2019. In November 2019, the forum transitioned into the Quadri- and Poly-Geometry Group (QPG, available at https://groups.io/g/Quadri-and-Poly-Geometry) forum, which continues to facilitate discussions on quadrilaterals, polygons, and related topics. Over the years, these forums have evolved into valuable resources for exploring both well-established results and novel perspectives in geometry. For both forums, an **annual record of all incoming messages** is compiled in this journal.

This journal is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing users to quickly jump between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting these forum messages, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Antreas Hatzipolakis
- Bernard Keizer
- Chris van Tienhoven
- Daniel Hardisky
- Eckart Schmidt
- Francisco Javier García Capitán
- Ivan Pavlov
- James Cooper
- Michael de Villiers
- Stanley Rabinowitz
- Tran Quang Hung
- Trinh Xuan Minh
- Vu Thanh Tung
- Ángel Montesdeoca

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.
Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

- **Ivan Pavlov**
  email: ivan.pavlov@gmail.com:
  #1342 #1350 #1372 #1375 #1385 #1387 #1388 #1390 #1394 #1487

- **James Cooper**
  email: james.cooper@jku.at:
  #1510 #1538

- **Michael de Villiers**
  email: profmd1@mweb.co.za:
  #1457

- **Stanley Rabinowitz**
  email: Stan.Rabinowitz@comcast.net:
  #1380 #1384 #1469

- **Tran Quang Hung**
  email: analgeomatica@gmail.com:
  #1393 #1395 #1521 #1526 #1535

- **Trinh Xuan Minh**
  email: hoingason@gmail.com:
  #1345 #1347

- **Vu Thanh Tung**
  email: tungvtt@gmail.com:
  #1531

- **Ángel Montesdeoca**
  email: amontes@ull.es:
  #1408

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2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Antreas P. Hatzipolakis

Location
Lives in Greece.

Year of Birth / Generation
1952.

Short Biography
Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacinthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: http://www.anthrakitis.blogspot.com/

Chris van Tienhoven

Global Location
Living in the Netherlands.

Year of Birth
1950.

Short Biography
Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. Journal of Geometry and Graphics, Vol. 25, No. 1, 2021.

Other Remarks
Website: www.chrisvantienhoven.nl
Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location
Living in Germany.

Year of Birth / Generation
1939.

Short Biography
Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n-gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke.* B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken.* MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten.* MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks.* PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles.* Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect.* Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen.* MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks.* MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Francisco Javier García Capitán

Location
Priego de Córdoba, Andalucía, Spain.

Year of Birth / Generation
1963.

Short Biography
Francisco Javier García Capitán is a mathematician and long-time secondary school teacher with a strong interest in geometry, elementary mathematics, and computational approaches. He is an active explorer of barycentric coordinates and the author of *Baricentricas.nb*. His work bridges classical geometric insight with modern computational tools.

Themes and Interests

- Elementary mathematics
- Geometry
- Mathematica and hobby programming
- Barycentric coordinates

Selected Publications
International Journal of Geometry

- (with Paul Yiu) *Three mutually tangent congruent circles…*, 5 (2016), 15–18.
- *A structure on the circumcircle*, 10 (2021), 71–83.
- *Infinite points and isogonal conjugate*, 12 (2023), 127–134.
- *Isotomic conjugate and parallelism*, 12 (2023), 89–100.

Forum Geometricorum

- *Means as chords*, 8 (2008), 99–101.
- *Trilinear polars of Brocardians*, 9 (2009), 297–300.
- *Collinearity of the first trisection points…*, 11 (2011), 217–221.
- (with Ehrmann & Myakishev) *Construction of circles…*, 11 (2011), 261–268.
- (with Dergiades & Lim) *On six circumcenters…*, 11 (2011), 269–275.
- *Some simple results on cevian quotients*, 13 (2013), 227–231.
- *A simple construction of an inconic*, 14 (2014), 387–388.
- *Lemniscates and a locus…*, 15 (2015), 123–125.
- *Another construction of the Simson lines…*, 15 (2015), 173–176.

- *Locus of centroids of similar inscribed triangles*, 16 (2016), 257–267.
- *A Family of Triangles...*, 18 (2018), 79–82.

Additional Remarks

- Website: www.garciacapitan.epizy.com
- Blog: www.garciacapitan.blogspot.com

Stanley Rabinowitz

Location
Living in New Hampshire, USA.

Year of Birth / Generation
1947 (Baby Boomer).

Short Biography
Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions
Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, International Journal of Computer Discovered Mathematics 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, International Journal of Computer Discovered Mathematics 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), International Journal of Computer Discovered Mathematics 7 (2022), 214–287.

Additional Remarks
Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location
Born and working in Hanoi, Vietnam.

Year of Birth / Generation
Millennial (approx. 1981–1996).

Short Biography
Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdös–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:
www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

Ángel Montesdeoca Delgado (1949–2024)

Location
Canary Islands, Spain.

Year of Birth / Generation
1949–2024.

Short Biography
Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions
Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at:
www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes
Members of the geometry community remembered Ángel with great affection:

- His "deep knowledge of Projective Geometry" and "great amount of geometrical results" (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: https://amontes.webs.ull.es/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

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4 Messages

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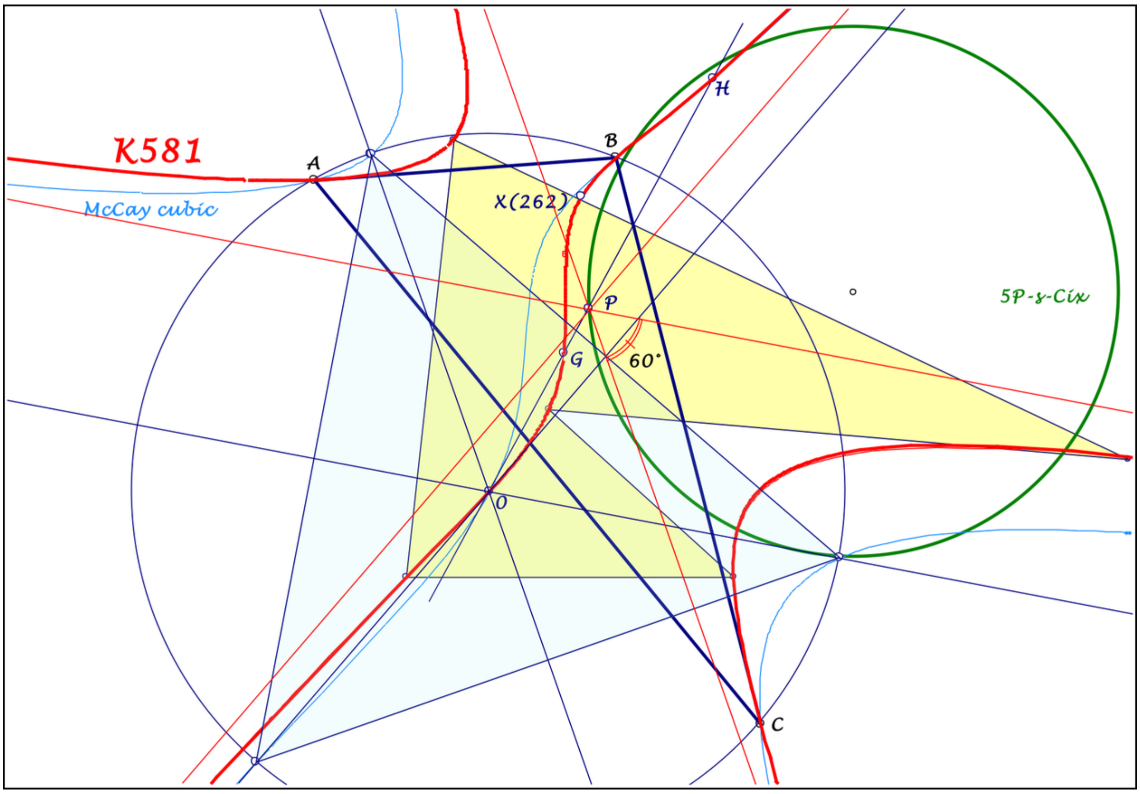
4.2 Messages

Message: #1295
Date: 2022-01-02
From: eckart_schmidt@t-online.de
Subject: Special K60+ 5P-circumcubics

```
Dear all,

in #922 I described new 5P-elements:
    5P-s-Px, 5P-s-Cix, 5P-s-Tfx,
... here a corrected form, for there is a typo:

Consider a 5P = P1...P5
... and for each vertex Pi the quadrangle
    of the remaining points
... with the four centroids of its triangles
... and let Qi be QA-P4 of these four centroids,
... which are concylic on a circle 5P-s-Cix,
    centered in 5P-s-Px.
5P-s-Px divides 5P-n-P1.5P-s-P3 with ratio -2:5,
... the radii of 5P-s-Cix and the 5P-s-P3-circle have ratio 2:3.

Replacing Pi by another point X,
... we get 5 circles with a common point on 5P-s-Cix
... which shall be 5P-s-Tfx(X).
    5P-s-Tfx maps any point to a point on 5P-s-Cix,
... so it is of interest to ask for the locus
... of points with a fixed image P on 5P-s-Cix.

These loci are special 5P-circumconics K581 in Bernard Gibert's
nomination K60+,
... with asymptotes, intersecting in a point under angles
    of 60°,
... but what about the reference triangles ABC???

Attached an approximate drawing,
... starting with a 5P and a fixed point P on its 5P-s-Cix.
The 5 vertices of 5P and by approximation 4 points X with
5P-s-Tf(X) = P
... give the locus for all points X with this property as cubic
    9P-s-Cu1.
This cubic has three asymptotes, intersecting with 60° in P,
... but P is not necessary a point on the cubic.
Several drawings give the assumption,
... that the cubic is K581 with a reference triangle ABC on the
    cubic.
```

```
K581 has a lot of interesting properties (see Bernard Gibert)
... K581 bears X(2), X(3), X(4), X(262) of ABC
... ... and the points of the Thomson cubic on the circumcircle
... ... and the infinity points of the McCay cubic.
... K581 intersects the circumcircle of ABC in 3 further points,
... ... which give an equilateral triangle with altitudes
        parallel to the asymptotes.
... The asymptotes intersect in a point on the Euler line,
... ... dividing OH with ratio 4/5 in the starting point P.

I hope some one can confirm my assumption
... and give a construction of the reference triangle ABC.

Best regards and a Happy New Year
Eckart

PS: Of special interest are fixed points P on 5P-s-Cix wrt
    5P-s-Tfx,
    ... there can be up to 3 such points ...
```



2022-01-02.pdf

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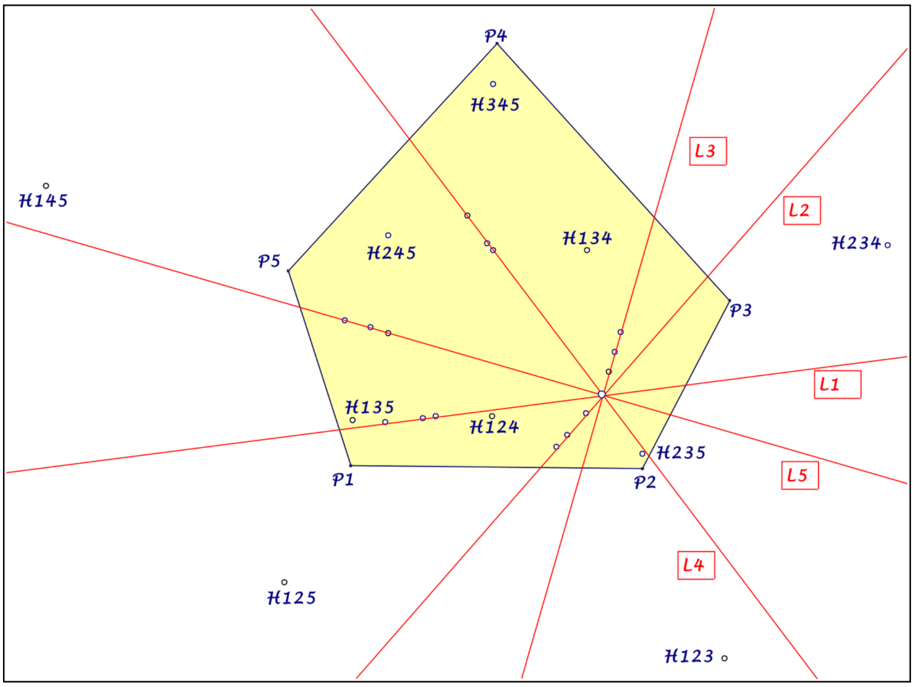
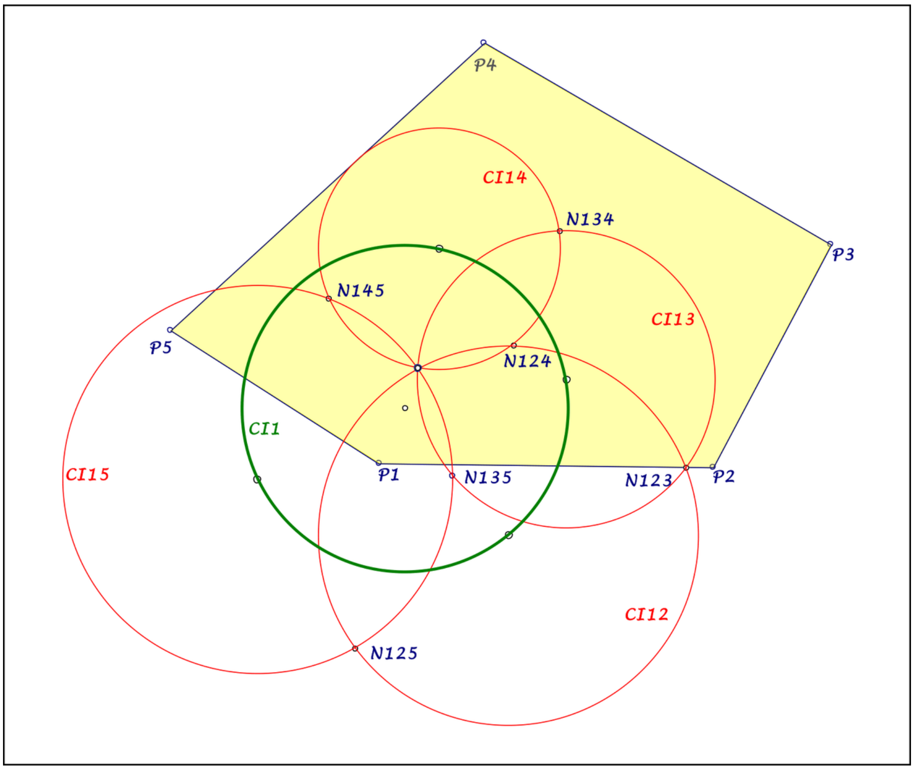
Message: #1296
Date: 2022-01-04
From: eckart_schmidt@t-online.de
Subject: New 5P-points and circle

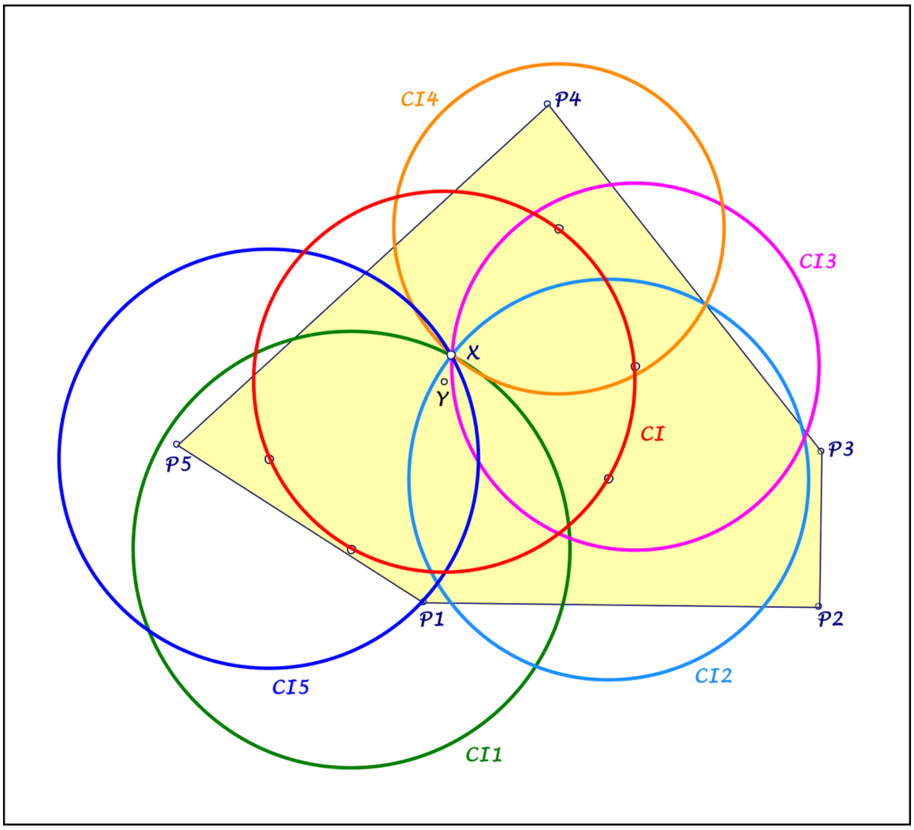
```
Dear all,

consider a 5P = P1...P5 with its 10 triangles PiPjPk:

(1) Take the orthocenters Hijk of these triangles
... and the midpoints of Hijk.Hilm, Hijl.Hikm, Hijm.Hikl,
... which are collinear on a line Li,
... these 5 lines Li have a common point (first attached).

(2) Take the nine-point centers Nijk of the triangles PiPjPk
... and the 10 circles CIij of Nijk, Nijl, Nijm,
... each four circles CIij, CIik, CIil, CIim have a common point
... and concyclic centers on a circle CIi (second attached),
... the five circles CIi have a common point X (third attached)
... and concyclic centers on a circle CI centered in Y.

Best regards Eckart
```



2022-01-04a.pdf



2022-01-04b.pdf



2022-01-04c.pdf

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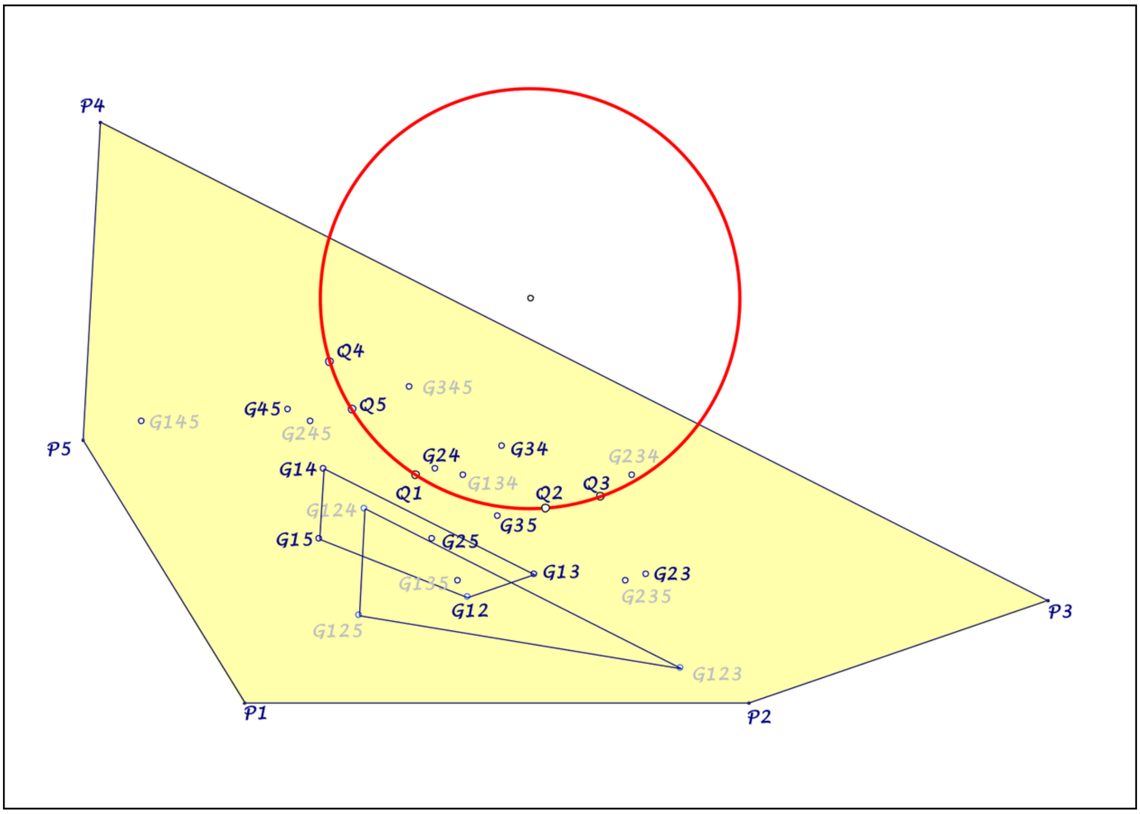
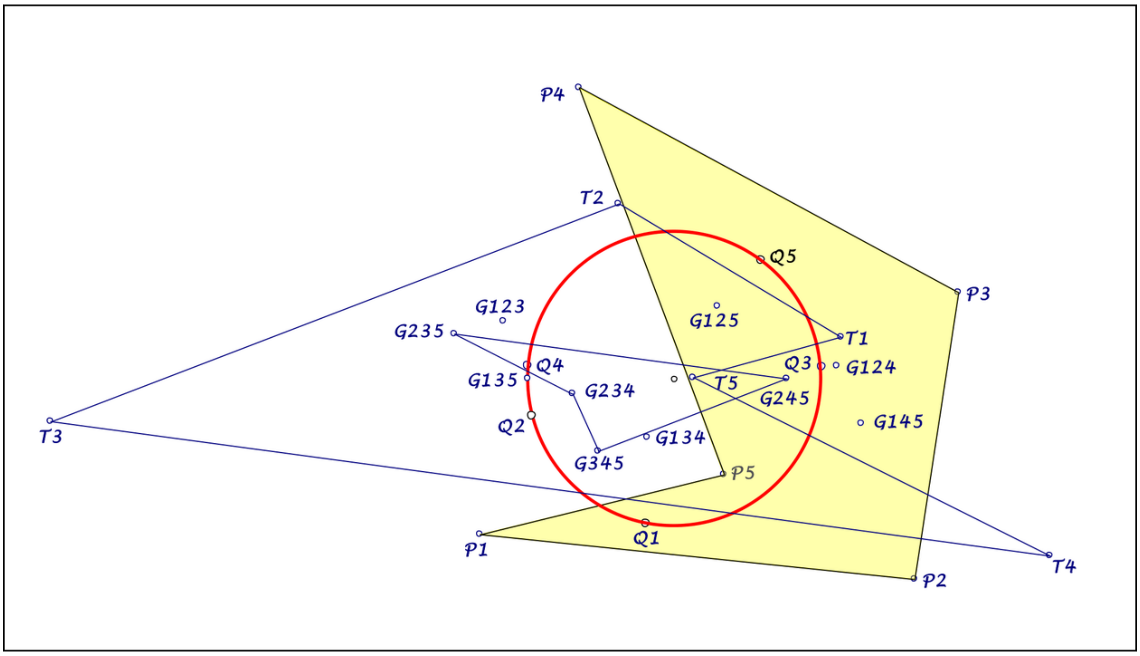
Message: #1297
Date: 2022-01-05
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-points and circles

```
Dear all,

in addition to my last message further 5P-circles

(3) Consider a 5P = P1...P5 with its 10 triangles PiPjPk
... and  take the 10 centroids Gijk of these triangles,
... then the 10 centroids Gij of Gijk, Gijl, Gijk,
... then 5 QA-P4 of Gij, Gik, Gil, Gim as Qi,
... finally the 5 Qi are concyclic,
... (first attached).

(4) Consider a 5P = P1...P5 and take the 5 Ti = QA-P4(PjPkPlPm),
... then the 10 centroids Gijk of Ti, Tj, Tk,
... then the 5 Qi = QA-P4(GjklGjkmGjlmGklm),
... finally the 5 Qi are concyclic,
... (second attached).

Best regards Eckart
```



2022-01-05a.pdf



2022-01-05b.pdf

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Message: #1298
Date: 2022-01-09
From: eckart_schmidt@t-online.de
Subject: QA-P4

Dear Chris,

wrt QA-P4 there is the following property in EQF

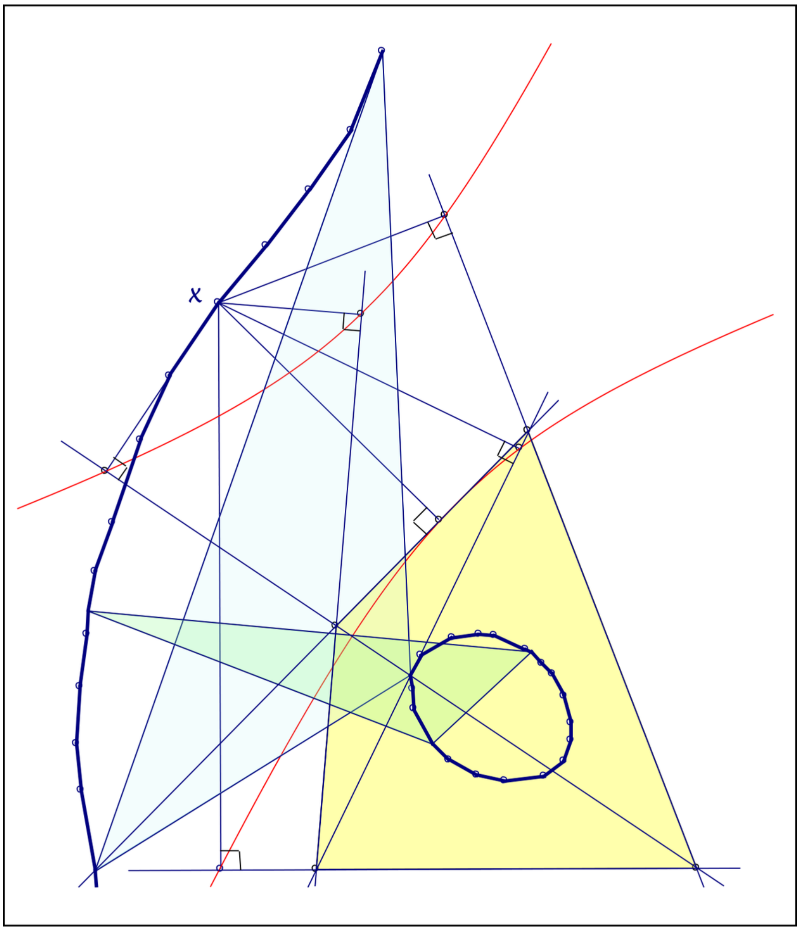
* Let Nij be the feet of the perpendiculars from QA-P4
  <http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4>
  to Pi.Pj..
  Then the 4 versions of circles through Nij, Nik, Nil (for all
  combinations (i,j,k,l) ∈(1,2,3,4)) have one common point,
  which is QA-P4
  <http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p4>
  (Seiichi Kirikami, July 17, 2013).
  See Ref-34, Quadri-Figures-Group, message # 126.

This property holds for any point, not only for QA-P4.

I didn't find the following property:
The 6 pedal points of QA-P4 on PiPj are coconic.
What about the locus of points
... with 6 coconic feet on the QA-lines?
The QA-points are singular points,
... the locus bears beside QA-P4 the QA-Tr1- and
QA-Tr2-vertices
... and the curve is QA-Tf16-invariant, but no cubic
(approximated attached)?

Best regards Eckart

PS: Mathematica gives an equation of degree 7.



2022-01-09.pdf

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Message: #1299
Date: 2022-01-10
From: eckart_schmidt@t-online.de
Subject: Re: QA-P4

```
Dear Chris,

excuse, the locus of points with 6 coconic feet on the QA-lines
... is already described in old#503 and new#1027 ...

Best regards Eckart
```

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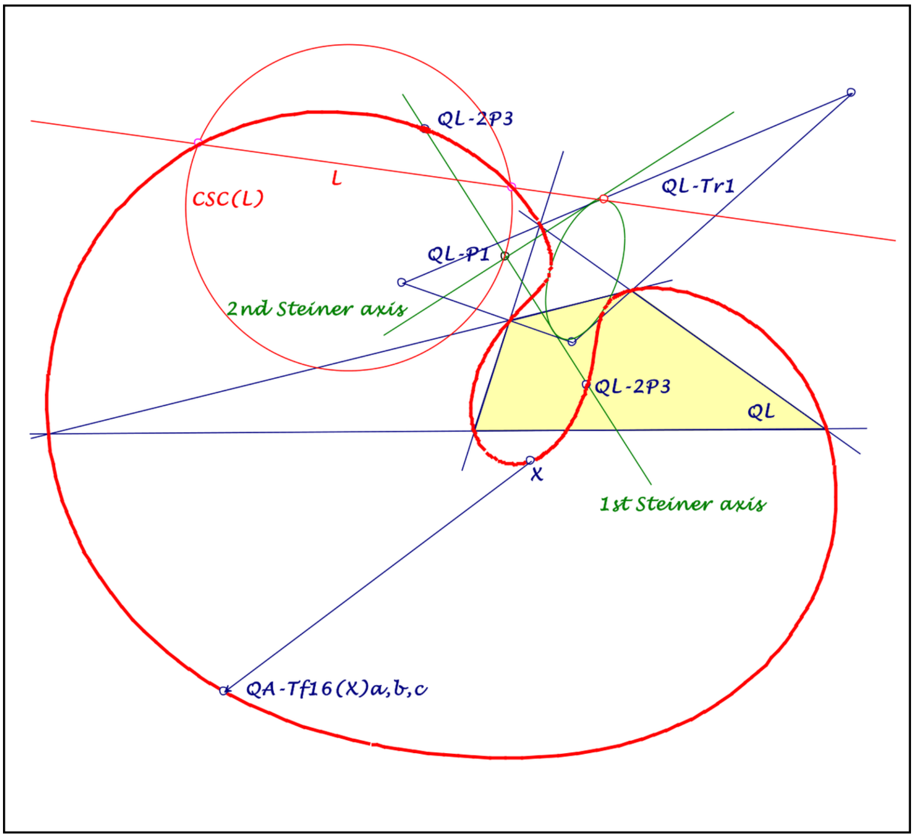
Message: #1300
Date: 2022-01-11
From: eckart_schmidt@t-online.de
Subject: QL-quartic wrt QA-Tf16

```
Dear all,

if we consider the locus of points,
... whose QA-Tf16 images coincide for the 3QG of a QL,
... we get a QL-circumquartic,
... already described with construction in old#364 (attached)
... there defined as locus of points,
... whose circles with opposite QL-points are coaxial.
This QL-circumquartic is invariant wrt CSC and QA-Tf16,
... bearing QL-2P3 and the Miquel points unequal QL-P1
... ... of the Miquel triangles of the three QG.

Best regards Eckart
```



2022-01-11.pdf

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Message: #1301
Date: 2022-01-12
From: eckart_schmidt@t-online.de
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Chris,

the quartic is also mentioned in old#3031,
... where already QA-Tf16 - before definition - is used,
... QA-Tf16 first mentioned in old#3030, not in EQF.

Best regards Eckart
```

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Message: #1302
Date: 2022-01-12
From: van10hoven@gmail.com
Subject: Re: QL-quartic wrt QA-Tf16

```
Thanks Eckart for the details.
Chris
```

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Message: #1303
Date: 2022-01-13
From: bernard.keizer@gmail.com
Subject: Re: New 5P-points and circle

```
Dear Eckart,
I reproduced without difficulty your figures 4b and c.
Amazing property ! Is it a generalisation of QA-P2 for a 5P ?
Best regards
Bernard
```

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Message: #1304
Date: 2022-01-13
From: bernard.keizer@gmail.com
Subject: Re: Special K60+ 5P-circumcubics

```
Dear Eckart,
Very beautiful K60+ 5P circumcubic !
But unfortunately, I'm not able to help you in the construction
of the triangle.
Best regards
Bernard
```

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Message: #1305
Date: 2022-01-13
From: bernard.keizer@gmail.com
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Eckart,
I've already told you several times I found this quartic
fascinating !
I only regret that it hasn't found yet it's place in EQF ...
QA-Tf16, which is described in EQF, is for a QA 3 times the
product of 2 transformations CIC and CSC.
QA-Tf16 = CICi*CSCi for i = 1 to 3.
So I suppose the curve is also the locus of points which have
the same transformed in the 3 CIC.
Another property I like is that the curve is also CSCdiag
invariant.
CSCdiag is the CSC of DQL formed by the 3 diagonals and the
Newton Line ; it is centered in QL-P17 and swaps for example the
2 Plücker points.
For any point P on the curve, the 3 circles through opposite
vertices of the QL intersect in a 2nd point P', which is
CSCdiag(P).
Naturally, this point P' is the reflexion of P in the line of
the centers of the 3 circles.
Any tangent L to your conic cuts the quartic in 4 points (not
necessary all real) ; 2 are CSC partners and the 2 others are
CSCdiag partners.
The construction given for the 2 CSC partners as intersection
between L and CSC(L) is the same for the CSCdiag partners as
intersection between L and CSCdiag(L).
Taking for example the Steiner Line (which is tangent to your
conic) we get the Plücker points as CSCdiag partners and the
intersections with QL-Ci3 as CSC partners.
Having the 6 QL vertices and the 6 Miquel points other than
QL-P1, the 2 fixed points of the CSC and the 2 Plücker points,
it's more than enough to draw the quartic ...
Best regards
Bernard
```

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Message: #1306
Date: 2022-01-13
From: bernard.keizer@gmail.com
Subject: Re: n-angle centers revisited

Dear Ngo Quang Duong,
I hope everything is going well with your thesis !
I'm waiting impatiently for your return to the n-angle centers topic ...
Personnaly, I'm blocked at the same point in the case p = 1, q = 3 and n = 4 (nephroïds).
It's easy to draw for the 2 vertices B and C of a triangle ABC the 4 quadrisectrices (2 times bisectors) and their 16 intersections Pi, i = 1 to 16.
For each Pi, the corresponding n-angle center Ii is the 2nd intersection of APi with the circle through B, C and Pi.
The isogonal conjugate of Ii is gIi.
The locus of the nephroïds tangent to the 3 sides of the triangle is made of 16 lines perpendicular bisectors of the segments IigIi.
Doing the same for the 4 triangles of a QL, we get the 64 centers as intersections of 4 lines (one for each triangle).
I'm convinced that the 64 centers lie on the same quartic, but unfortunately I'm not able to draw it with Geogebra.
Best regards
Bernard

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Message: #1307
Date: 2022-01-14
From: eckart_schmidt@t-online.de
Subject: Re: QL-quartic wrt QA-Tf16

Dear Bernard,
thanks for interest in my last messages and your additional remarks.
I have repeated the properties of your CSCdiag,
... but I did not found the transformation CIC in my weak memory.
Wrt #1303: Excuse, I cannot see any interpretation as QA-Tf2 for 5P.

Best regards Eckart

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Message: #1308
Date: 2022-01-14
From: bernard.keizer@gmail.com
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Eckart,
The QG transformation CIC is mentionned in your notes 2014-04-14
and 2013-12-05.
In 1303, I said X could be a generalisation of QA-P2, not QA-Tf2
(as it involves 9 points circles for a QA and for a 5P).
Best regards
Bernard
```

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Message: #1309
Date: 2022-01-15
From: bernard.keizer@gmail.com
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Eckart,
During the night, I've just realised following property :
Your beautiful quartic is defined as locus of points P such as
the 3 circles through P and 2 opposite QL vertices are coaxial.
In this case, the 3 CICa,b and c coïncide in a point P', which
is CSCdiag(P) and naturally the 3 QA-Tf16a, b and c coïncide in
a point P'', which is CSC*CSCdiag(P).
In fact, the 2nd definition of the quartic as locus of point for
which the 3 QA-Tf16a, b and c coïncide is the same as the 1rst
one !
QA-Tf16 is described in EQF as a QA-transformation.
But nor CIC as a QG-transformation and neither the property that
QA-Tf16 = CICi*CSCi i = 1 to 3 for the 3 QGs of the QA are
described in EQF.
Best regards
Bernard
```

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Message: #1310
Date: 2022-01-15
From: eckart_schmidt@t-online.de
Subject: QA-P4 - QL-P1 - QG-P1-Circumcubic

```
Dear all,

this is an apologia for a QG-cubic,
... already described in old#441,
... but here with new aspects:

This cubic CU is QG-circumscribed
... through the main points of EQF: QG-P1, QL-P1, QA-P4,
... as focal circular cubic with focus QA-P4.

(1) Transformations:
Cu is CSC-invariant with pivot QG-P1,
CU is CIC-invariant (see old#362 or PS),
CU is QA-Tf16-invariant, midpoints on QL-P1.QG-P1,
CU is isogonal-invariant wrt the triangle TR(QG-P1,QL-P1,QA-P4),
... coinciding with QA-Tf16 on CU.
CU is invariant wrt the Möbius transformations of TR,
... one centered in QL-P1, swapping QG-P1 and QA-P4, is CSC.

(2) Points:
QG-vertices and QG-P1, QL-P1, QA-P4,
X0 infinity point of the asymptote,
... parallel QG-P1.QL-P1 through reflection of QA-P4 in
QG-P1.QL-P1,
X1, X1' fixed points of CSC (QL-2P3),
X2, X2' intersections of the Schmidt-circle (see QL-Tf1)
... and a perpendicular to the 1st Steiner axis through QG-P1,
... X2, X2' CSC-partner.
X3, X3' intersections of QL-P1.X2 and QL-P1.X2'
... with the reflection of X2.X2' in the QG-P1-angle-bisector,
... X3, X3' CSC-partner, CIC-partner of X2, X2', isogonal
    conjugates of X2', X2, X4, X4' intersections of a
    perpendicular through QA-P4 wrt the Steiner axis
... and an orthogonal circle to the Schmidt-circle round the
    reflection of QA-P4 in the Steiner axis,
X5, X5' CSC-partner, QA-Tf16-partner, TR-isogonal partner and
TR-Möbius-partner,
... on the TR-angle bisector of QG-P1,
    concyclic with QL-P1 and QA-P4,
... X5, X5' fixed points of CIC,
X6, X6' intersection of a perpendicular
    to the 2nd angle bisector in QA-P4
... and X1.X5 or X1'.X5' and X1X5' or X1'X5,
... X6, X6' isogonal conjugated, QA-Tf16-partner.
```

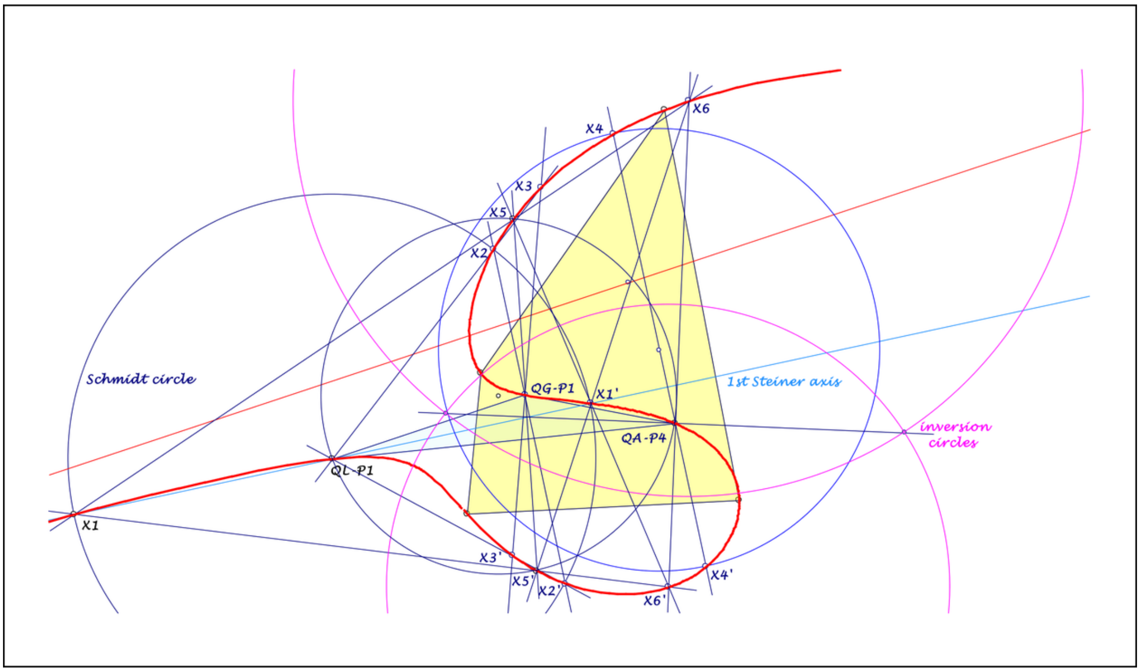
```
The cubic CU is QL-Cu1 of the quadrigon X2X5X3'X5' or
X2'X5'X3X5,
... CU can be constructed
    only with three points QG-P1, QL-P1, QA-P4.

(3) Tangents:
QL-P1-tangential is QA-P4,
QG-P1-tangential is QA-P4,
QA-P4-tangential is the intersection of CU and its asymptote.

(4) Anallagmatic curve:
CU is inverse wrt circles round X6, X6'
... through CSC-partner on the QA-P4-angle bisector.

Best regards Eckart

PS: For a QG CIC(P) is the 2nd intersection of the circles
(P,P1,P3) and (P,P2,P4).
```



2022-01-15.pdf

Message: #1311
Date: 2022-01-15
From: eckart_schmidt@t-online.de
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Bernard,

thanks for information and explications in your last two
messages:
The unexpected property QA-Tf16 = CIC*CSC for a QG is new for me
... and really worth to be mentioned in EQF!
Special thanks for remembering CIC,
... I could use it at once for my next message.

Best regards Eckart

PS: Excuse the topic in #1307
```

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Message: #1312
Date: 2022-01-16
From: bernard.keizer@gmail.com
Subject: Re: QL-quartic wrt QA-Tf16

```
Dear Eckart,
I mentionned this property of QA-Tf16 = CIC*CSC in my messages
1036 and 1088 ...
Best regards
Bernard
```

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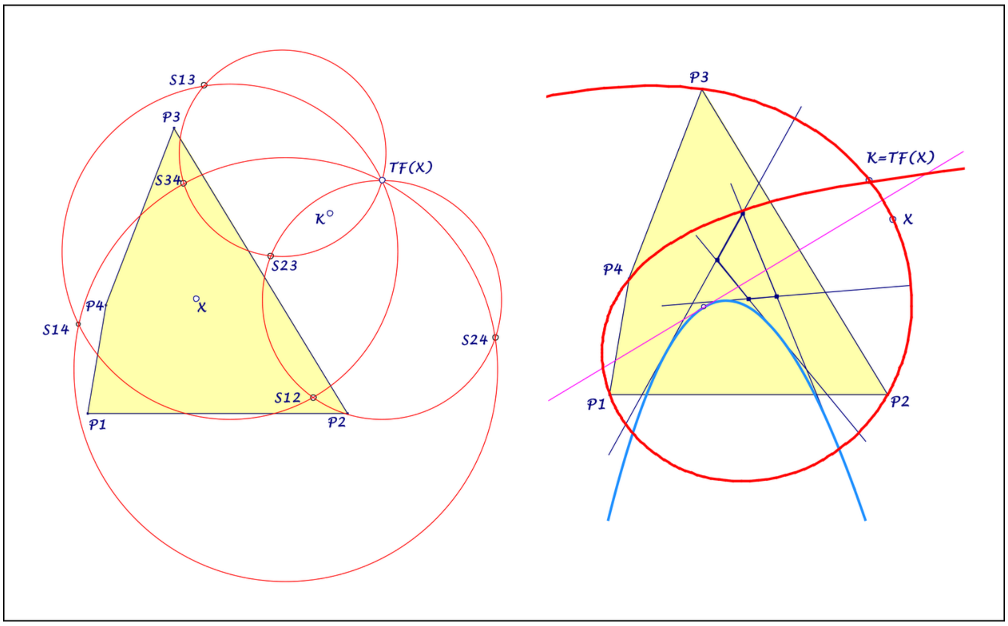
Message: #1313
Date: 2022-01-17
From: eckart_schmidt@t-online.de
Subject: QA-circumcubic with knot

```
Dear all,

may I invite you for a short excursion in QA-geometry?
Let QA = P1P2P3P4, K a fixed point and X a variable point,
... consider the 6 points Sij = QA-P4(Pi,Pj,K,X)
... and get the 4 circles (Sij,Sik,Sil)
... with a common point as image TF(X).
The locus for points X with TF(X) = K
... is a QA-circumscribed cubic CU with knot K.
This cubic is the limit result
... of the 5P-s-Tf6-image wrt 5P = QA plus K
... of circles round K wrt radius --> zero.
A correct construction as follows:
The four bisectors KPi define an inscribed parabola
... and the reflections of K in tangents at the parabola give
the cubic CU.

For the three Miquel points as K
... we get three cubics CU with a common point Z,
... which is the QA-Tr2-isogonal conjugate of QA-P41,
... in the 2nd intersection of QA-Cu1 and a parallel to its
asymptote in QA-P41.

Best regards Eckart
```



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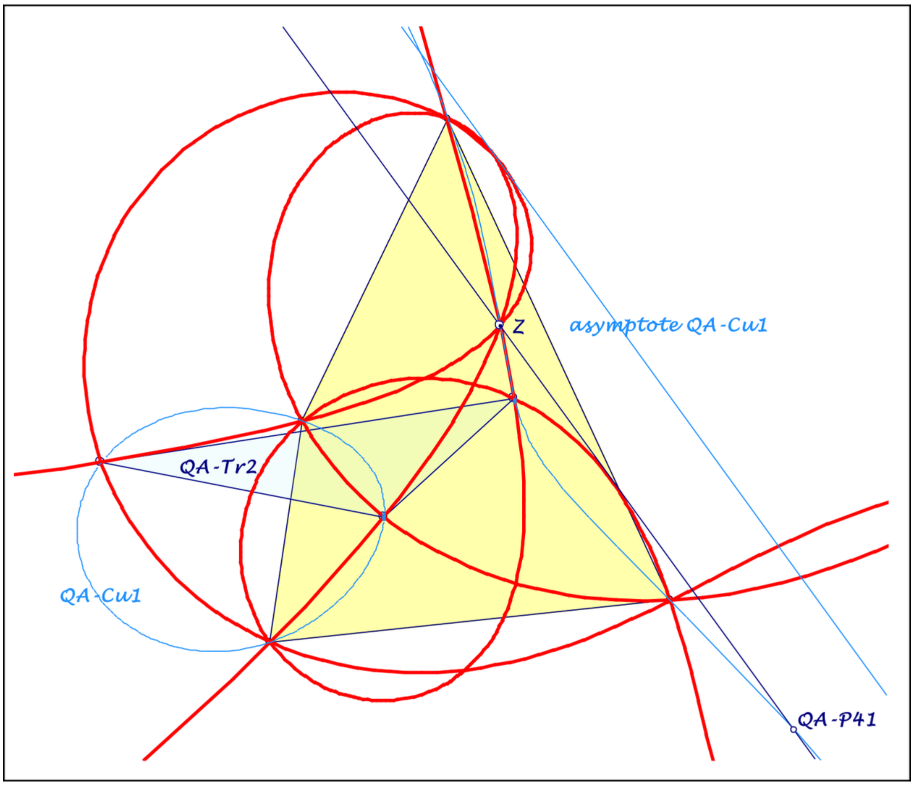

2022-01-17a.pdf

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Message: #1314
Date: 2022-01-17
From: bernard.keizer@gmail.com
Subject: Re: QA-P4 - QL-P1 - QG-P1-Circumcubic

```
Dear Eckart,
Beautiful QG circular focal circumcubic !
For a QA, there are 3 such circular focal circumcubics
intersecting in the QA vertices, in QA-P4 and in the circular
points.
These 7 points are on QA-Cu1 (which is not focal).
They have the same focus QA-P4 and their asymptotes intersect in
the reflexion of QA-P4 in QA-P3 (as the Newton Lines QG-P1QL-P1
intersect in QA-P3).
May be a nice figur of yours with the 3 QG cubics and QA-Cu1
would show other properties (2 last intersections of the 3
cubics 2 by 2, for example) ...
Best regards
Bernard
```

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Message: #1315
Date: 2022-01-17
From: eckart_schmidt@t-online.de
Subject: Re: QA-P4 - QL-P1 - QG-P1-Circumcubic

```
Dear Bernard,

there are no further intersections of these QG-circumcubics:
... for a QA as QA-P4, for a QL as QL-P1 and QL-2P3.

Best regards Eckart
```

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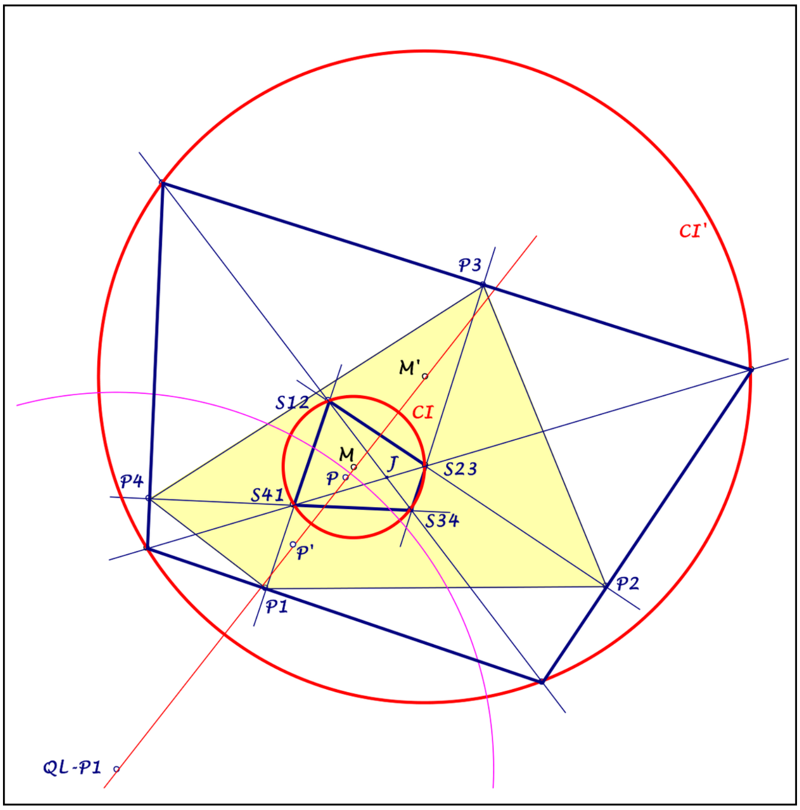
Message: #1316
Date: 2022-01-23
From: eckart_schmidt@t-online.de
Subject: QG-angle bisectors

```
Dear all,

let us consider a QG = P1P2P3P4 (first attached)
... and the inner angle bisectors Wi of <Pi-1.Pi.Pi+1,
... further the intersections Si,i+1 of Wi and Wi+1,
... which are concyclic on a circle CI with center M.
If we take the outer angle bisectors
... we get in this way another circle CI' with center M'.
M and M' are collinear with QL-P1 on the 1st Steiner axis.
The circles CI and CI' are CSC-invariant
... with pivots P and P' on the 1st Steiner axis,
... CSC-images of the circle centers
... and inverse of QL-P1 wrt CI and CI',
... the Schmidt circle intersects CI and CI' orthogonal.
The two cyclic QG are perspective,
... perspector is the incenter J of QG-Tr3.

Wrt the first circles CI for the QG-versions of a QL:
A QL has a convex first QG,
... a non convex and not overturned 2nd QG
... and an overturned 3rd QG,
... the circles CI for the 1st and 2nd QG coincide
... and shall be researched further.

Now let us consider a convex 5L (second attached)
... and the 5 circles CI
... for the convex QG-versions of the QL-components of the 5L,
... which have a common point
... and concyclic centers,
... if no QL-P1 of the QL-components lies inside the 5L.

I hope someone can confirm the last observation
... and lighten the background.

Best regards Eckart
```



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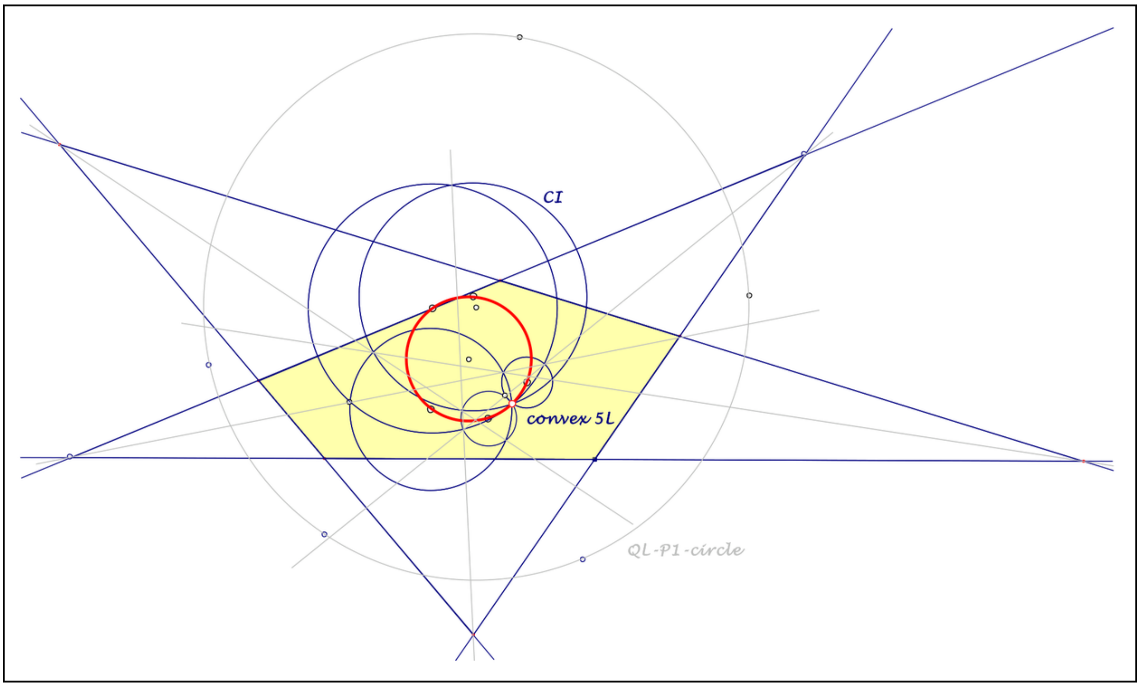

2022-01-23b.pdf

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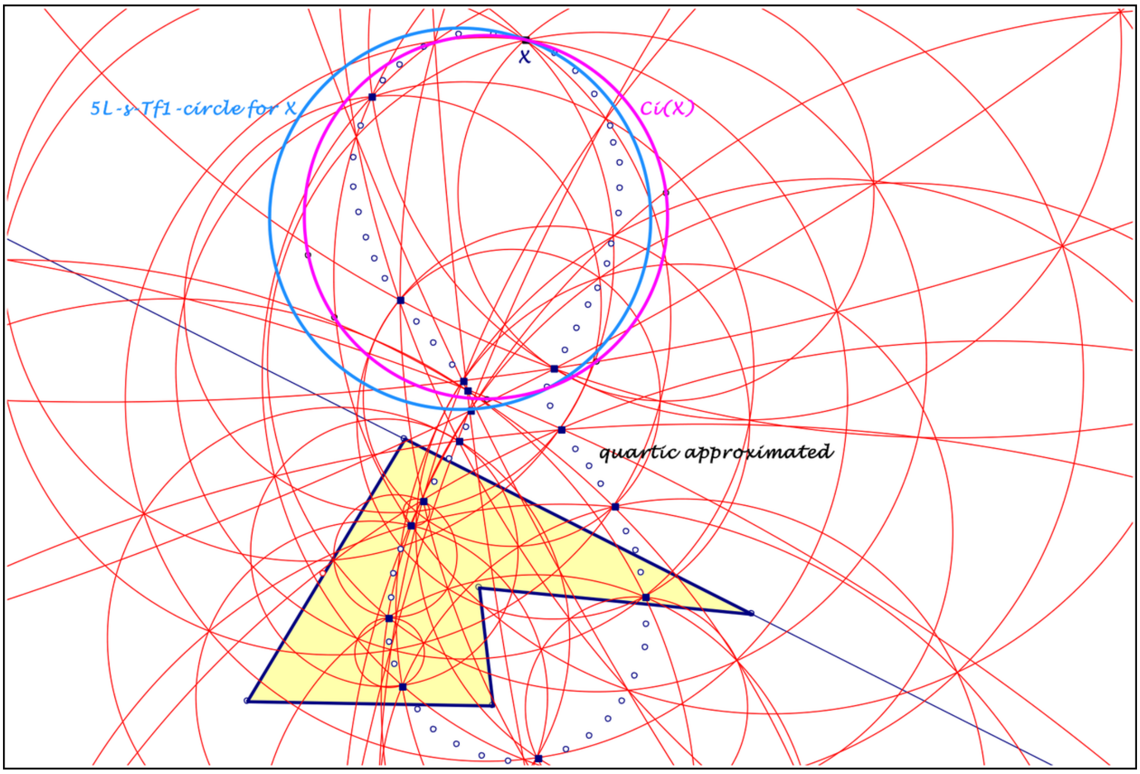
Message: #1317
Date: 2022-02-02
From: eckart_schmidt@t-online.de
Subject: Steiner circles for a 5L

```
Dear all,

are the following observations correct or well known?

A quadrilateral has 8 Steiner circles,
... for the 5 QL of a 5L we get 40 Steiner circles
... with 16 5-times-intersections X,
... in pairs on the Steiner circles.
The centers of the 5 Steiner circles intersecting in X
... are concyclic on a circle through X,
... each X has its circle Ci(X).
The center of each Steiner circle
... is an intersection of the two Ci(X)
... for the two X on the Steiner circle.

The 16 5-times-intersections of the Steiner circles
... are points of a quartic, locus of points Y,
... which lie on their 5L-s-Tf1-circle.
The tangent of Y at the circle
... is the tangent of Y at the quartic.
What about a construction of this quartic?

Each Steiner circle of a 5L bears 4 4-times intersections,
... so we have 40 of these points.
The 5x2 4-times intersections for the 5 QL components,
... which are the fixed points of the 5 CSC,
... lie on the quartic,
... so we have overall 26 points of the quartic.

Best regards Eckart
```



2022-02-02.pdf

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Message: #1318
Date: 2022-02-04
From: bernard.keizer@gmail.com
Subject: Re: Steiner circles for a 5L

```
Dear Eckart,
Very interesting properties, indeed !
I tried to reproduce your quartic locus of the points X lying on
their 5L-s-Tf1(X).
I take the case where the inscribed conic is an ellipse.
I consider the CSCi fixed points Fi and F'i, with the Fi outside
and the F'i inside the Miquel circle.
Then the quartic is made of the 2 ellipses through the 5 Fi and
the 5 F'i.
Curiously, the centers of the 5L-s-Tf1 circles describe also 2
ellipses ...
Are these observations correct ?
Best regards
Bernard
```

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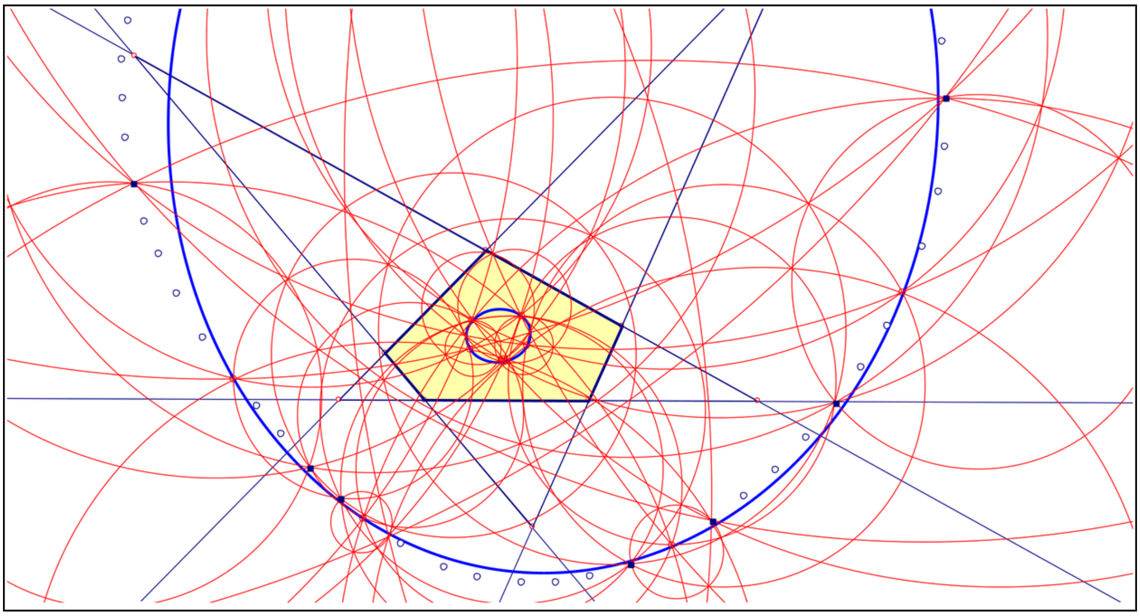
Message: #1319
Date: 2022-02-04
From: eckart_schmidt@t-online.de
Subject: Re: Steiner circles for a 5L

Dear Bernard,

thanks for interest, but the following observation doesn't hold:
"Then the quartic is made of the 2 ellipses through the 5 Fi and the 5 F'i.'"
Your two ellipses bear the 10 4-times-intersections
... but not the 16 5-times-intersections of the Steiner circles,
... see attached file, only partial plotted quartic,
... 10 points don't define a quartic.

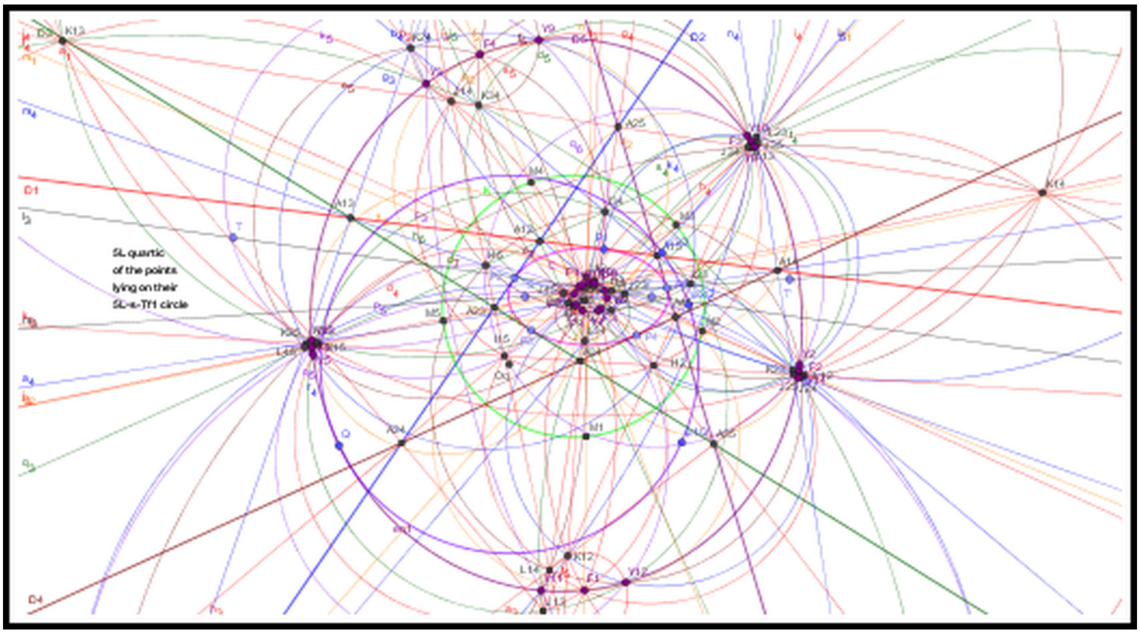
Best regards Eckart



2022-02-04.pdf

Message: #1320
Date: 2022-02-07
From: bernard.keizer@gmail.com
Subject: Re: Steiner circles for a 5L

Dear Eckart,
Of course, 10 points don't define a quartic !
I checked too quickly that points of the 2 ellipses verified
your property, but it is not the case ...
The 2 ellipses are only a proxy.
I reproduced this time carefully your complete figur with the 16
quintuple intersections and the 10 fixed points of the CSCs.
Your quartic is in fact split in 2 parts.
Beautiful curve, but I didn't find a construction.
Best regards
Bernard



5L quartic.ggb

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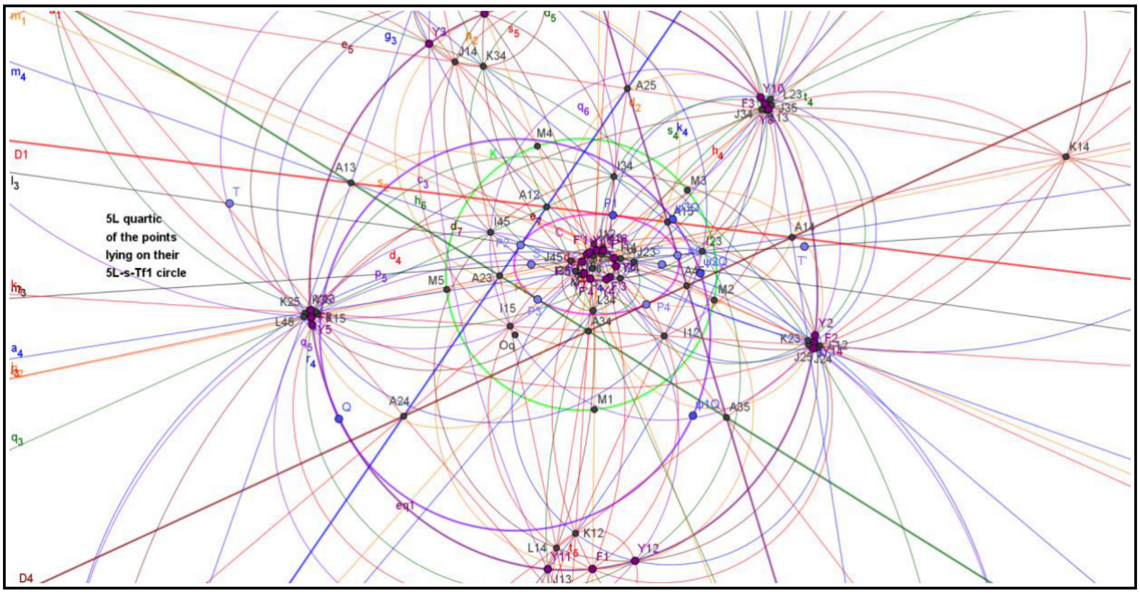
Message: #1321
Date: 2022-02-07
From: bernard.keizer@gmail.com
Subject: Re: Steiner circles for a 5L

```
Dear Eckart,
Sorry, I forgot to convert my Geogebra file in pdf.
It's better this way.
Best regards
Bernard
```



5L quartic.pdf

Message: #1322
Date: 2022-02-07
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-/6P-Elements

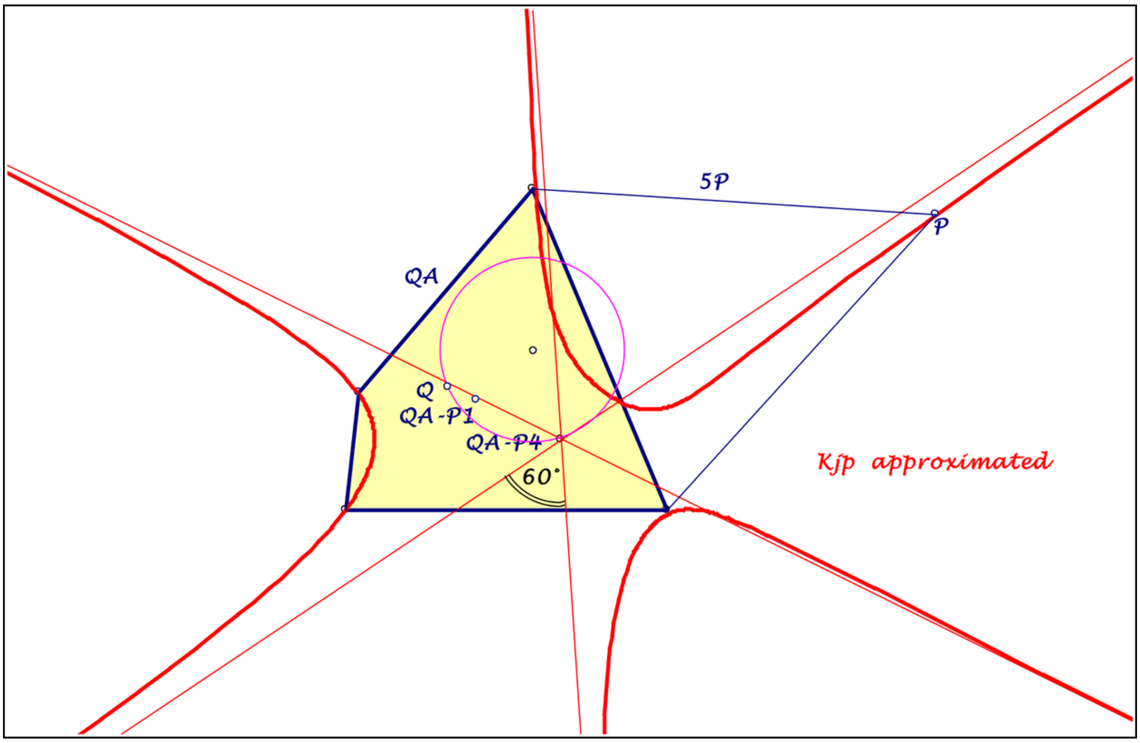
```
Dear all,

in #922 I described a new point 5P-s-Px and circle 5P-s-Cix,
... but I have to correct a typo:
5P-s-Px divides 5P-n-P1.5P-s-P3 with ratio -2:5.

Here four new properties:
(1) The radius of 5P-s-Cix is 2/3 of the radius of the
5P-s-P3-circle.
(2) The radius of 5P-s-Cix is 1/3 of the radius
... of the circumcircle for the triangle TR (first mentioned in
old#3579).
(3) For 5P = QA plus any point
... the circles 5L-s-Cix have a common point Q ,
... QA-P4 of the centroid-quadrangle,
... dividing QA-P1.QA-P4 with ratio -1:4.

Finally:
(4) Let 5P be QA plus a point P
... and consider their circles 5P-s-Cix (see #922):
The locus of P with 5P-s-Cix bearing QA-P4
... is a QA-circumscribed cubic,
... in Bernard Gibert's nomination K024 = Kjp
... with three 60°-angled asymptotes intersecting in QA-P4,
... attached an approximated drawing.
What about a construction of this cubic?

Best regards Eckart
```



2022-02-07.pdf

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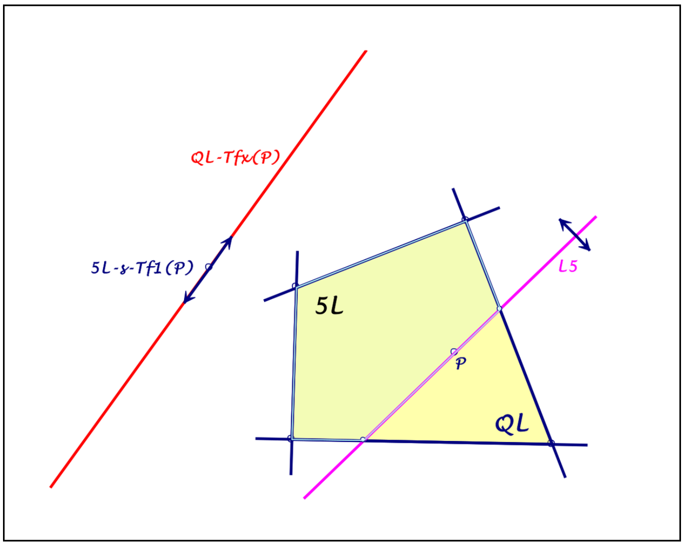
Message: #1323
Date: 2022-02-09
From: eckart_schmidt@t-online.de
Subject: New QL-transformation

```
Dear all,

this new QL-transformation QL-Tfx is a point to line
transformation,
... starting with a QL, a point P and variable lines L5
    through P,
... which lead to 5L = QL plus L5.
The image line QL-Tfx(P) is the locus of points 5L-s-Tf1(P),
... changing L5 (see attached file).
Properties:
(1) Image line of QL-P1 is the line at infinity.
(2) QL-Tfx(QL-P6) is a line through QL-P26,
... and QL-Tfx(QL-P26) is a line through QL-P6.
(3) Points on a line L have image lines through a common point Q
with QL-Tfx(Q) = L,
... e.g. points on QL-L2 have image lines through QL-P4 and
QL-Tfx(QL-P4) = QL-L2.
(4) Points on lines through QL-P1 have parallel image lines,
... e.g. points on QL-P1.4 have image lines parallel QL-L2.
(5) Points on a QL-line have image lines with a common point on
QL-Ci3
... in a circumcenter of a QL-trilateral component.
(6) Image lines of points on QL-Ci3 envelope QL-Co1 and vice
versa.

Best regards Eckart
```



2022-02-09.pdf

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Message: #1324
Date: 2022-02-09
From: bernard.keizer@gmail.com
Subject: Re: New 5P-/6P-Elements

Dear Eckart,
I reproduced without difficulty an approximate drawing of this QA-Kjp cubic.
But I have no idea about a construction.
We would need 2 fixed points of a CSC centered in QA-P4.
The hessian of your cubic will be a focal circular cubic K048 with focus in QA-P4, invariant in this CSC.
The Newton Line, parallel to the asymptote of K048, is the homothetic of the line through the 3 intersections of your Kjp cubic with its 3 asymptotes (Homothety centered in QA-P4 with ratio 3/2) ...
Best regards
Bernard

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Message: #1325
Date: 2022-02-10
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-/6P-Elements

Dear Bernard,

excuse, if I am not so familiar with cubics and their hessians,
... you describe K048 as hessian of my QA-Kjp cubic,
... but the hessian of a Kjp cubic is K193,
... the hessian of the McCay cubic is K048.

Best regards Eckart

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Message: #1326
Date: 2022-02-10
From: bernard.keizer@gmail.com
Subject: Re: New 5P-/6P-Elements

Dear Eckart,
Of course, you are right !
Mac Cay is K003 and its hessian is K048.
Kjp is K024 and its hessian K193.
Is following property correct :
taking any 3 points on a given Kjp with center K, the isogonal
wrt the triangle ABC of the inverse of K wrt the circumcircle of
ABC lies on the cubic ?
Best regards
Bernard

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Message: #1327
Date: 2022-02-10
From: bernard.keizer@gmail.com
Subject: Re: New 5P-/6P-Elements

Dear Eckart,
To precise my question :
Your cubic certainly looks like a Kjp.
But what makes you sure that it is exactly a Kjp ?
If I'm not wrong, taking a triangle ABC and its point X67, then
the QA-P4 of these 4 points is the centroïd X2.
But the Kjp of the triangle ABC, centered in X2 doesn't pass
through X67 ...
Best regards
Bernard

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Message: #1328
Date: 2022-02-10
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-/6P-Elements

```
Dear Bernard,

wrt your last message:
Several very precise drawings with CABRI,
... up to a reference triangle,
... and controlling Kjp-properties
... confirm my assumption.
Your described Kjp-property is evident,
... but why should X67 be a point on Kjp?
X67 is the isogonal conjugate of X23
... and X23 is the inverse of X2 in the circumcircle.
On the other hand: QA-P4 of 4 points is the isogonal conjugate
of one point
... wrt the triangle of the remaining points,
... finally reflected in the circumcircle.

Best regards Eckart
```

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Message: #1329
Date: 2022-02-11
From: bernard.keizer@gmail.com
Subject: Re: New 5P-/6P-Elements

```
Dear Eckart,
According to your definition, your cubic contains an infinity of
QA's having the center K as QA-P4.
According to Bernard Gibert, Kjp of a triangle ABC is centered
in X2.
X2 is the QA-P4 of A,B,C and their X67.
Hence your cubic passes through A,B,C and X67, but Kjp doesn't
pass through X67.
So I think it is not the same cubic.
Best regards
Bernard
```

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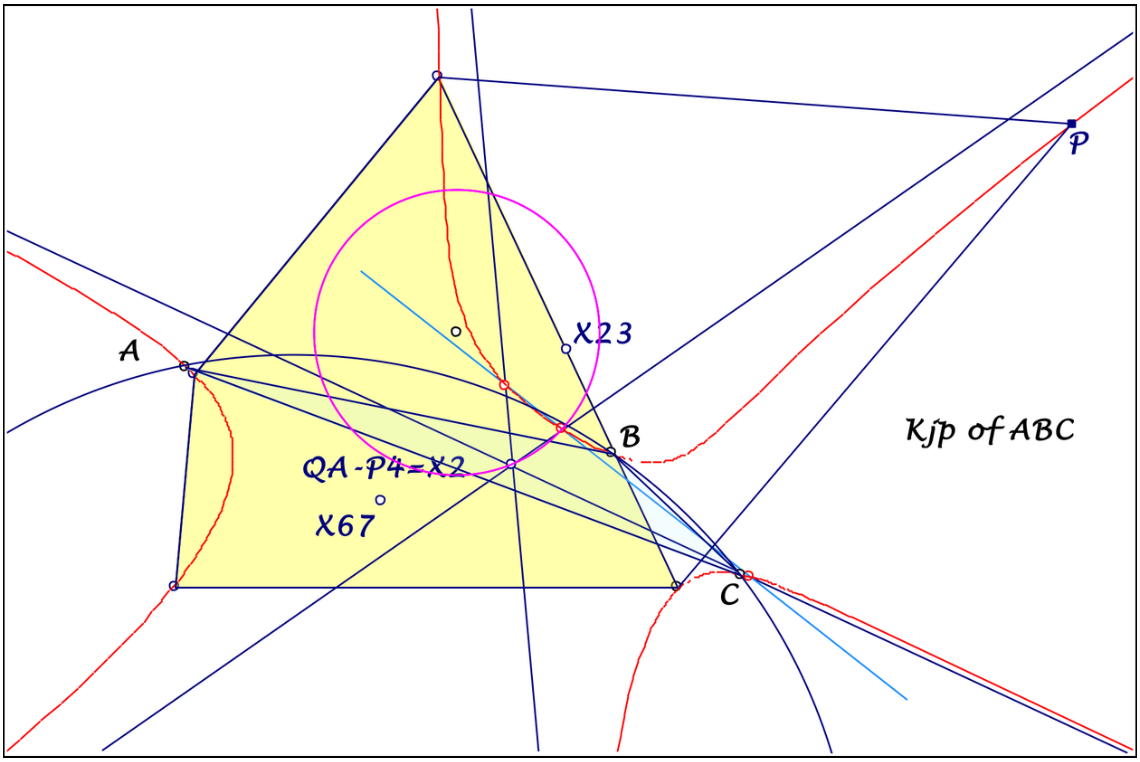
Message: #1330
Date: 2022-02-11
From: eckart_schmidt@t-online.de
Subject: Re: New 5P-/6P-Elements

```
Dear Bernard,

for a reference triangle with X2 = QA-P4 of my cubic
... the point X67 lies not on this cubic (attached).
You use a "center" for a QA and for the Kjp
... what do you mean with this point K,
... is it Bernard's Lemoine point X6,
... which is not mentioned under Kjp?

Best regards Eckart
```



2022-02-11.pdf

Message: #1331
Date: 2022-02-11
From: bernard.keizer@gmail.com
Subject: Re: New 5P-/6P-Elements

Dear Eckart,
This time you convince me !
K, "center of the cubic" is simply QA-P4 of the initial QA and X2 of the main pivot triangle ABC.
It doesn't hold that for any triangle on the cubic the 4th point such as the QA-P4 of the 4 points is this point lies on the cubic.
The fixed points of the CSC I mentionned are the foci of the Steiner inellipse of ABC ...
Then, for any point on the cubic it's possible to find the 2 other vertices of a pivot triangle having the same foci of the Steiner inellipse.
(This construction is mentionned in Bernard Gibert Eckart's cubic).
I had really fun with this item (as in the previous one with the 5L-quartic with the 26 points).
Thanks a lot.
Best regards
Bernard
PS Is it possible that the cubic becomes a Mac Cay cubic with another type of QA ?

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Message: #1332
Date: 2022-02-12
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 / QL-L1

Dear Chris,

is the following QL-property worth to be mentioned in EQF?
QA-P4 of pedal-QA for QL-Cu1-points give QL-L1.

Best regards Eckart

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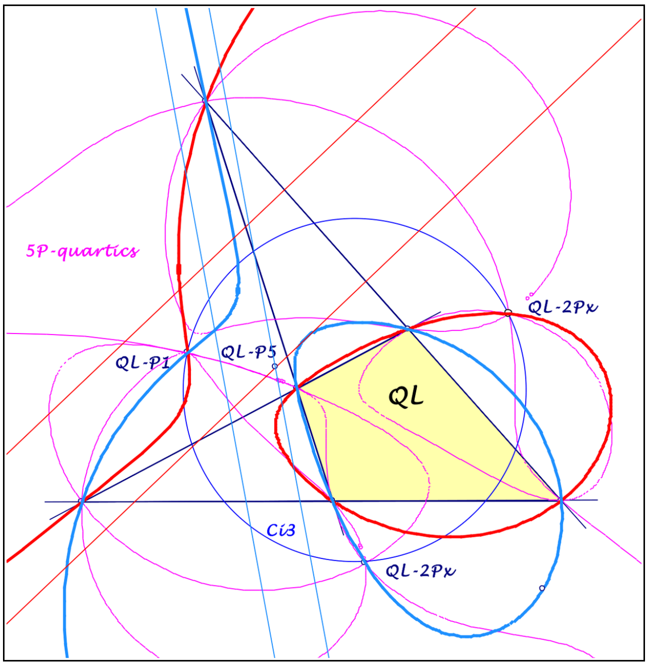
Message: #1333
Date: 2022-02-12
From: eckart_schmidt@t-online.de
Subject: QL-2Px

```
Dear all,

consider for a QL and its three QG the 5P = QG plus QL-P1,
... their 5P-quartics have beside QL-P1 two common points
    QL-2Px,
... which can be constructed as follows:
Take a circle round QL-P7 orthogonal QL-Ci1
... intersecting QL-L2 in two points with CSC-images QL-2Px,
... which are points on QL-Ci3.
Each of QL-2Px can be considered as focus
... of a focal circular 5P-cubic,
... which are the same for the three 5P,
... so we get two such cubics,
... bearing the 6 QL-points and QL-P1
... and invariant wrt CSC.
If we consider the pedal-QA of cubic points
... and the locus of its QA-P4,
... we get for each cubic a line,
... which is the Newton-line of the cubic
... (parallel to the asymptote half the distance to the focus).
These two Newton lines intersect in QL-P5.

Best regards Eckart
```



2022-02-12.pdf

Message: #1334
Date: 2022-02-12
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1 / QL-L1

```
Dear Eckart,
Do you mean QL-P1 ?
Best regards
Bernard
PS Link with the previous item ?
```

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Message: #1335
Date: 2022-02-12
From: bernard.keizer@gmail.com
Subject: Re: QL-Cu1 / QL-L1

```
Dear Eckart,
My apologise, you mean a point on QL-L1.
As the 4 pedals are concyclic, I suppose their QA-P4 is the
center of the circumcircle.
Best regards
Bernard
```

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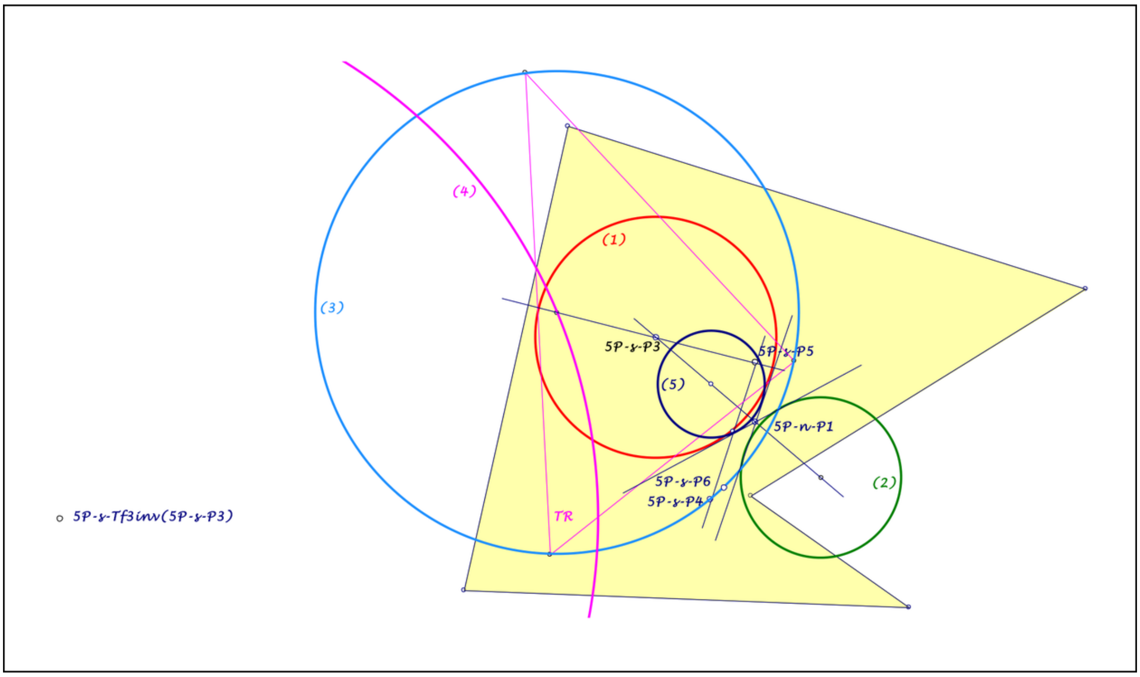
Message: #1336
Date: 2022-02-14
From: eckart_schmidt@t-online.de
Subject: No 5P-circles as item in EPG?

```
Dear Chris,

why is no 5P-circle worth to have an item in EPG?
... Background for this message is old#3789.
5P-circles:
(1) Quang Duong's circle only mentioned at 5P-s-P3,
... centered in 5P-s-P3, through midpoint of 5P-s-P4
    and 5P-s-P5, radius r(1).

(2) For a 5P any point P gives a 6P = 5P plus P with 6P-s-P1 on
this circle,
... center divides 5P-n-P1.5P-s-P3 with ratio -2:5,
    radius r(2) = 2/3 r(1).

(3) Circumcircle of the triangle TR (see old#3579, old#3581,
old#3784 and EPG 5P-s-P3).,
... centered in the reflection of 5P-s-P5 in 5P-s-P3,
    radius r(3) = 2r(1),
... bearing 5P-s-P4 and 5P-s-P6,
... circle (1) is the nine-point circle of TR.
Radii of these circles: r(2) : r(1) : r(3) = 2 : 3 : 4.

(4) "New" circle (see old#3773),
... centered in 5P-s-Tf3inv(5P-s-P3),
... through the reflection of 5P-s-P5 in 5P-s-P3,
    center of circle (3).
5P-s-Tf3 maps the circle (4) to the Quang Duong circle (1).

(5) Wrt circle (5) see #1297 (3),
... center divides 5P-n-P1.5P-s-P3 with ratio 4:5,
    radius r(5) = 4/9 r(1).
The circles (1), (2), (5) have common tangents, intersecting in
5P-n-P1
... and their centers are collinear.

Best regards Eckart

PS: Only main properties are here mentioned,
... further observations in the references.
```



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Message: #1337
Date: 2022-02-14
From: van10hoven@gmail.com
Subject: Re: No 5P-circles as item in EPG?

Dear Eckart,

About your question " why is no 5P-circle worth to have an item in EPG? ".
I do not quite understand your question.
Did I ever made any remark that made you think so?
Nevertheless I appreciate your eagerness very much.
Additionally, there are personal reasons and circumstances and that is because of lack of time.
I hope you can respect that.

Best regards,

Chris

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Message: #1338
Date: 2022-02-15
From: bernard.keizer@gmail.com
Subject: Re: No 5P-circles as item in EPG?

Dear Eckart,
I also like very much your 5L-circle through the 10 Plücker points (message #1131).
Best regards
Bernard

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Message: #1339
Date: 2022-02-19
From: eckart_schmidt@t-online.de
Subject: New concept for nP-transformations

```
Dear all,

these are generalizations of the concept for QA-Tf16,
... replacing the inverse in the circumcircle
... by transformations for the QA-triangle components:
Starting with a QA = P1P2P3P4, a point P
... and a triangle transformation TF,
... consider Qi = TF(P) wrt PjPkPl
... and the 4 images Ri = TF(Pi) wrt QjQkQl.
For TF = inverse wrt the circumcircle we get QA-Tf16,
... the 4 last images Ri coincide.
(1) For TF = isogonal conjugate the 4 Ri coincide too
... and define a point to point transformation
... with fixed points in the QA-Tr1-vertices,
... mapping QA-P4 to the reflection of QA-P4 in QA-P2,
... mapping QA-circumconics to conics with the same foci.
(2) For TF = isotomic conjugate the 4 Ri are collinear
... and define a point to line transformation,
... but I found no properties.

There is an analog generalization for 5P
... using a QA-transformation TF
... with Qi = TF(P) wrt PjPkPlPm and Ri = TF(Pi) wrt PjPkPlPm.
(3) For TF = QA-Tf16 the 5 Ri coincide
... and we get a 5P-transformation, mapping a point to a point.
(4) Analog for a 6P and TF = 5P-s-Tf6 for the 5P
... we get a point to point 6P-transformation.

Perhaps someone can enrich these exemplary observations
... and lighten the background.

Best regards Eckart
```

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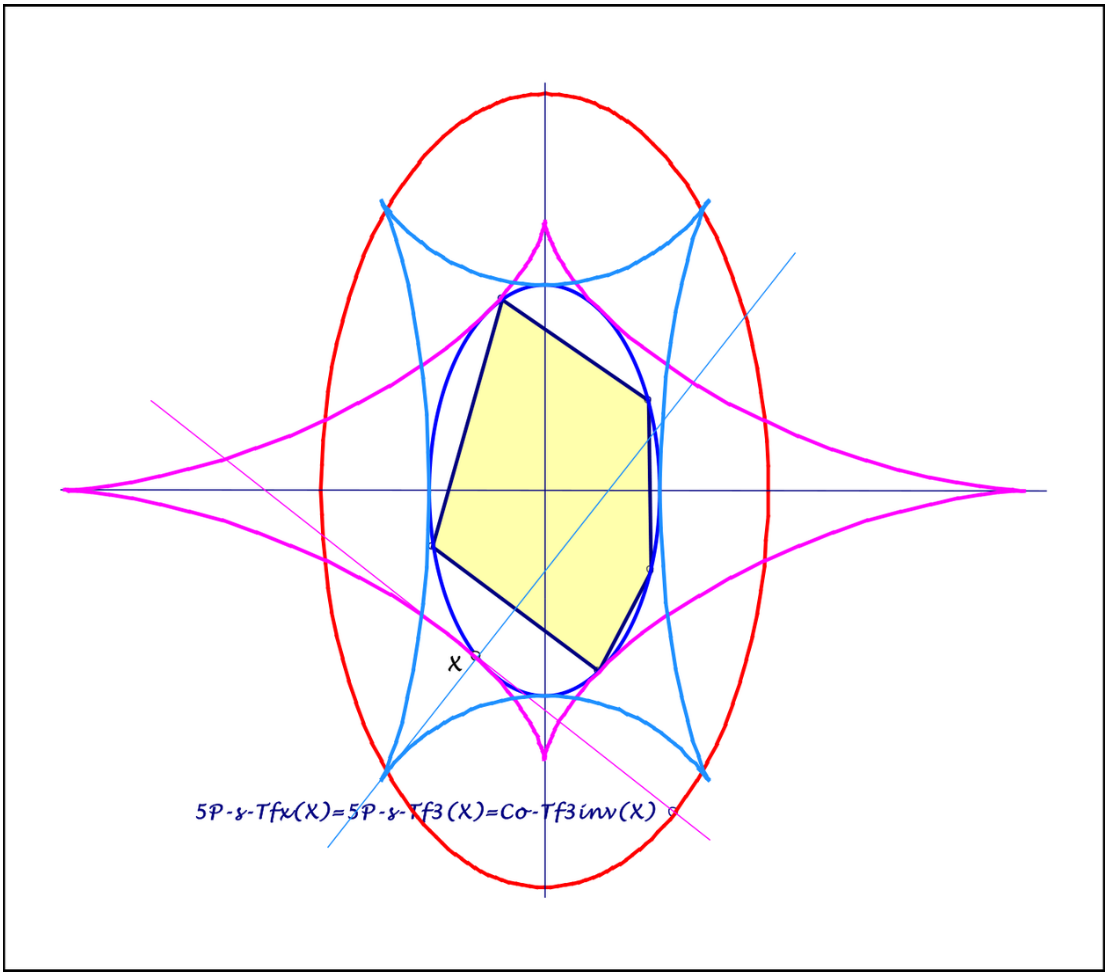
Message: #1340
Date: 2022-02-21
From: eckart_schmidt@t-online.de
Subject: Re: New concept for nP-transformations

Dear all,

here is an application for the new transformation (3) in #1339,
here 5P-s-Tfx:
For points X on 5P-s-Co1 the image 5P-s-Tfx(X) is the same wrt
all inscribed 5P of the conic,
... 5P-s-Tfx maps the circumconic 5P-s-Co1 to a new conic with
the same axes,
... with 5P-s-Tfx(X) = 5P-s-Tf3(X) = Co-Tf3inv(X),
... so 5P-s-Co1 is the locus of the Frégier points of the new
conic (see Co-Tf3).
By the way, there are two nice quartics, tangent to 5P-s-Co1,
... as envelopes of X.5P-s-Tfx(X) and their perpendiculars in X.

Best regards Eckart



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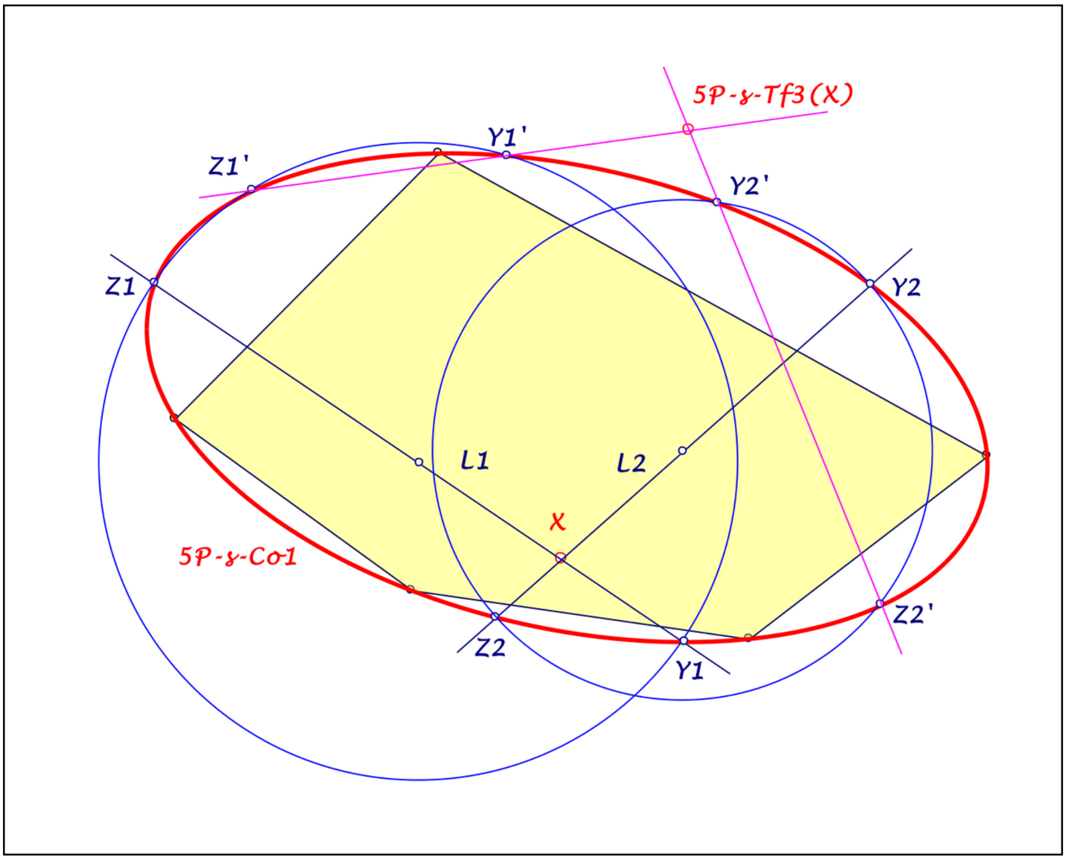
Message: #1341
Date: 2022-02-22
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 = Co-Tf3inv

```
Dear Chris,

the relevant property that 5P-s-Tf3 is a conical transformation
... is only indirect mentioned in the last property of
    Co-Tf3 = 5P-s-Tf3inv.
Co-Tf3 and 5P-s-Tf3 are conical transformations wrt 5P-s-Co1,
... independent of the 5P inscribed the conic
... and independent of stretching the 5P from 5P-s-P1.
A very simple construction of Co-Tf3(X) is described in
old#3511,
... here attached a simple construction for 5P-s-Tf3(X):
Take two lines through X, each intersecting the conic in two
points Y and Z,
... let the circles with diameter YZ intersect the conic further
    in Y' and Z',
... then the two lines Y'Z' intersect in 5P-s-Tf3(X).

Best regards Eckart
```



2022-02-22.pdf

Message: #1342
Date: 2022-03-05
From: ivan.pavlov@gmail.com
Subject: On a property of the Isogonal center

```
Let ABCD be a quadrangle. Let (ca) be the circumcircle of BCD,
and similarly define (cb), (cc) and (cd). Let Iab be the
internal similitude center of (ca) and (cb) and similarly define
Iac, Iad, Ibc, Ibd and Icd.

Then the circles defined by the following triples of points
(Iab, Iac, Iad), (Iab, Ibc, Ibd), (Iac, Ibc, Icd) and (Iad, Ibd,
Icd) are concurrent and their common point is the Isogonal
Center.

n.b.
This probably follows from the fact that the Isogonal Center
lies on all 6 circles of similitude of the circumscribed circles
of the component triangles.
```

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Message: #1343
Date: 2022-03-06
From: eckart_schmidt@t-online.de
Subject: Re: On a property of the Isogonal center

```
Dear  Ivan Pavlov,

in addition to your observation:
Replacing "internal similitude center" in your last message
... by "intersection of common tangents"
... you get finally QA-P9.

Best regards Eckart
```

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Message: #1344
Date: 2022-03-07
From: eckart_schmidt@t-online.de
Subject: Curious QA-Px

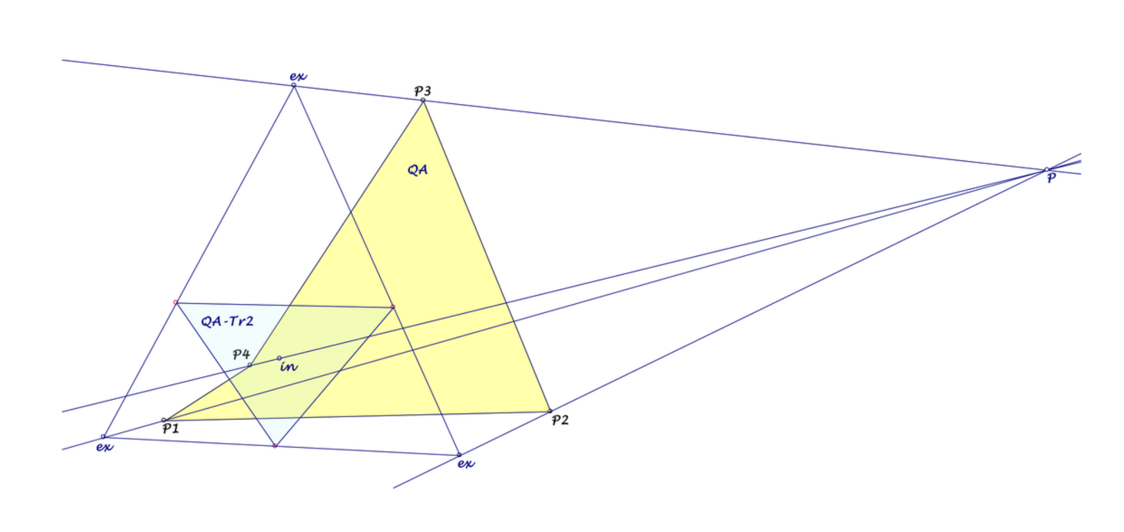
```
Dear all,

consider for the vertices Pi of a QA
... the 3 circles through Pi and two other vertices,
... take pairwise the intersection of common tangents,
... which are collinear on a line Li,
... and take finally the perpendicular through Pi wrt Li,
... which have a common point P = QA-Px.

For a convex QA
... the point P is the excenter of QA-Tr2
... wrt the Miquel point of the convex QG-component of QA.

For a nonconvex QA with "inner" point P4 (attached)
... the point P is the perspector of the triangle P1P2P3
... and the excenter triangle of QA-Tr2
... on the line connecting P4 and the incenter of QA-Tr2.

Best regards Eckart
```



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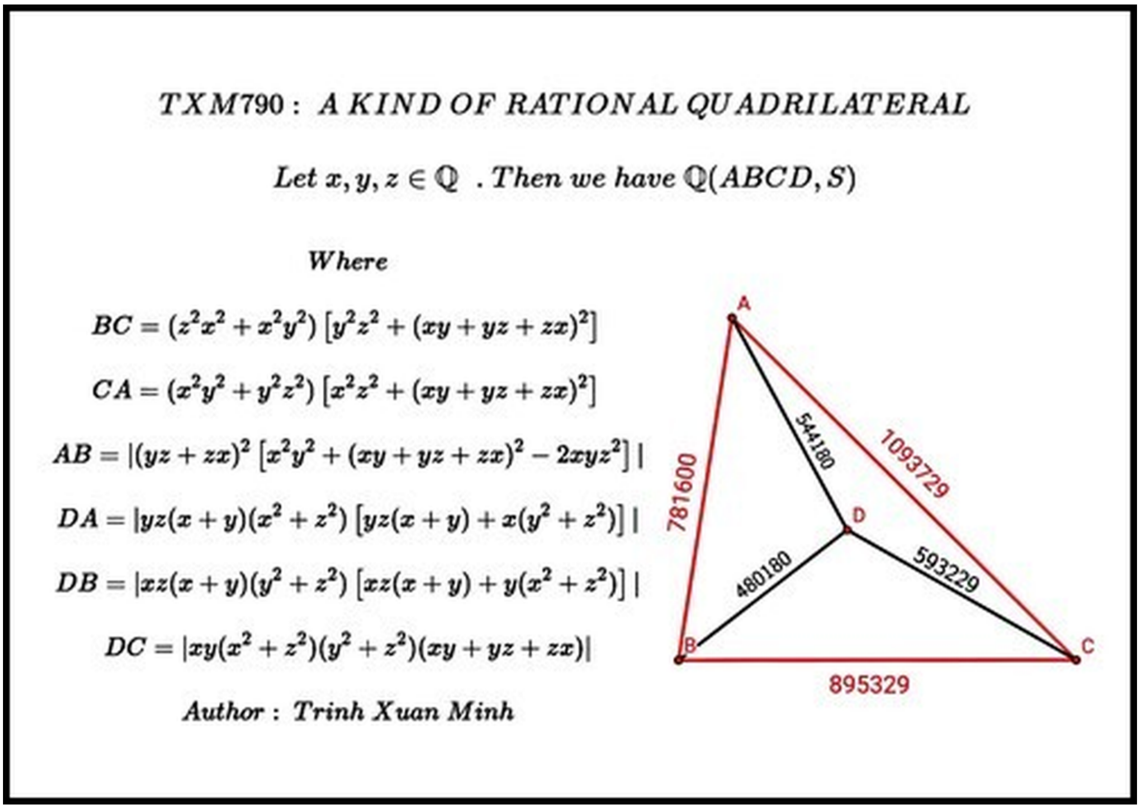
Message: #1345
Date: 2022-03-08
From: hoingason@gmail.com
Subject: A KIND OF RATIONAL QUADRILATERAL

```
Hello everyone
I am a new member, I came to know about this group through the
introduction of Mr. Seiichi Kirikami, pleased to introduce to
everyone one of my formulas for rational quadrilaterals.

This is my group on facebook where I post my formulas. Welcome
everyone! https://www.facebook.com/groups/453397409364044
Best regards
```



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Message: #1346
Date: 2022-03-08
From: van10hoven@gmail.com
Subject: Re: A KIND OF RATIONAL QUADRILATERAL

Dear Trịnh Xuân Minh,
Welcome to our group and thank you for your contribution.
Maybe you can explain a bit more about how a rational
quadrilateral is defined.
Best regards,
Chris van Tienhoven

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Message: #1347
Date: 2022-03-09
From: hoingason@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A KIND OF RATIONAL QUADRILATERAL

Thank you for your interest in this and for raising an
interesting question. I have developed a theory (RP = Rational
Point) with the aim of studying rational points in a new way.
Where I define a rational polygon
corresponding to each set of rational points. For example, S_3
is used to represent a set of 3 rational points (in other words,
a rational triangle).
The same definition for S_n is a set of n rational points.
The rational point here means that the distance between any two
points of S_n is a rational number.
In the absence of further explanation, S_n is said to be a
perfect rational set if any triangle formed by any of its three
points has a rational area.

Vào 4:50, Th 4, 9 thg 3, 2022 Chris <van10hoven@gmail.com> đã
viết:
> Dear Trịnh Xuân Minh,
> Welcome to our group and thank you for your contribution.
> Maybe you can explain a bit more about how a rational
quadrilateral is defined.
> Best regards,
> Chris van Tienhoven

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Message: #1348
Date: 2022-03-09
From: bernard.keizer@gmail.com
Subject: Re: A KIND OF RATIONAL QUADRILATERAL

```
Dear Gi Cung Co,
It looks very interesting !
Are there other examples of rational QA's (3 points is a
triangle and 4 points a quadrangle) ?
In your example, what are x, y and z ? (I would like to
reproduce your figur with Geogebra) ?
What are the properties of a rational QA ?
Thanks in advance
Best regards
Bernard
```

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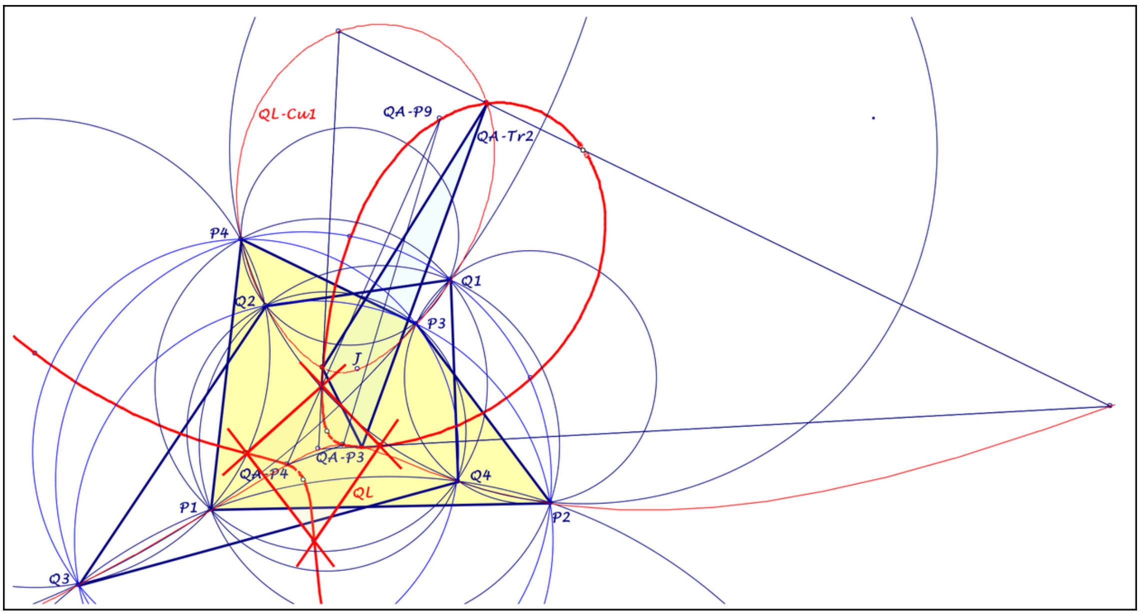
Message: #1349
Date: 2022-04-02
From: eckart_schmidt@t-online.de
Subject: Interesting QA-cubic

```
Dear all,

although discussion seems dead,
... perhaps someone will read this message.

(1) Related quadrangle
Let us start with a QA = P1P2P3P4 with its cubic QA-Cu1
... and its Miquel triangle QA-Tr2 with incenter J.
Let Qi be the 3rd intersection of QA-Cu1 and PiJ,
... you get these points as further 4-times-intersections
    of the 16 lines
... connecting Pi with the in- and excenters of QA-Tr2,
... or as inverses of Pi wrt the anallagmatic circles of QA-Cu1
    (see EQF).
The Q-quadrangle has the same QA-Tr2 and QA-Cu1,
... its QA-Tf2 swaps QA-Tr1 and QA-Tr2 of the reference QA.
The Cayley-Bacharach point
... of the vertices of the reference and the related quadrangle
... is the intersection of QA-Cu1 and its asymptote.

(2) Related quadrilateral
```

The following 12 quadruples are concyclic: (Pi,Pj,Qi,Qj) and
(Pi,Pj,Qk,Ql),
... the 3 circle pairs (Pi,Pj,Qi,Qj),(Pk,Pl,Qk,Ql) are inverse
    wrt an anallagmatic circle,
... their centers lie harmonical on the sides of the triangle
    of the excenters
... and are the opposite vertices of a quadrilateral QL.
QL-P1 of this QL is QA-P9, CSC swaps QA-P3 and QA-P4,
... QL-L1 is orthogonal to the asymptote of QA-Cu1
... ... and is the QA-Tf4-image
        of the circle round QA-P9 through QA-P4.
QL-Ci2 is the circumcircle of QA-Tr2.

(3) The cubic
The 12 circle centers lie on a cubic, which is QL-Cu1 of QL,
... bearing further the QA-Tr2-vertices and QA-P3, QA-P4, QA-P9.
The cubic is invariant wrt the 3 Möbius transformations of the
triangle QA-P3,4,9 (e.g. QA-Tf4).
The cubic is QA-Cu1 of quadrangles
... with one chosen vertex on the cubic
    and its Möbius transformed images.
The cubic is the locus of points, whose isogonal conjugate wrt
QA-Tr2 is the image
... of the Möbius transformation, centered in QA-P9,
    swapping QA-P3 and QA-P4.

Best regards Eckart



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Message: #1350
Date: 2022-04-04
From: ivan.pavlov@gmail.com
Subject: Re: Interesting QA-cubic

```
I am reading the messages. It's difficult for me to follow and
contribute because I use Geogebra, and still don't have enough
tools to work in quadrangle geometry.
This will come, in time...

Regards,
Ivan
```

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Message: #1351
Date: 2022-04-04
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

```
Dear Eckart,
discussion is always possible, but without participation of
Chris, it is a little bit discouraging !
You already mentionned this new cubic QA-Cux, which is a QL-Cu1
with focus QL-P9 and CSC swapping QA-P3 and QA-P4, but here with
new interesting aspects.
Newton Line of the QL is the perpendicular bisecyor of
QA-P3QA-P4.
On your figure, the curve named QL-Cu1 is in fact QA-Cu1 and the
other is QL-Cu1.
Best regards
Bernard
PS for Ivan Geogebra allows the drawing of cubics through 9
points with the function Implicit curve(P1 to P9) ...
```

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Message: #1352
Date: 2022-04-05
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

Dear Bernard,
you are right, the nomination of the cubic in the figure must be QA-Cu1,
... thanks for correction and additional remarks.
By the way: The related Q-quadrangle has as vertices
... the contact points of tangents from QA-P3 at QA-Cu1.
Best regards Eckart

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Message: #1353
Date: 2022-04-05
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

Dear Eckart,
According to your construction, the P-quadrangle and the Q-quadrangle are in perspective with the quadrangle of the in- and excenters of Tr2.
The 12 points are in a Reye configuration and define QA-Cu1.
In particular, the 3 tangentials of the vertices of the 3 quadrangles are aligned : QA-P4 is the tangential of the P-quadrangle, QA-P3 is the tangential (or QA-P4) of the Q-quadrangle and the infinity point of the asymptote is the tangential of the in-and excenters quadrangle.
Conversely, for any point on QA-Cu1 taken as QA-P4, you may define it's tangential QA and it's points QA-P3 (isogonal of QA-P4 wrt Tr2), QA-P2 and QA-P41.
Then you have 2 focal circular cubics :
1) QA-Cu7, with focus QA-P41 and CSC swapping QA-P2 and QA-P4 (this cubic contains the vertices of QA-Tr1)
2)QA-Cux , with focus QA-P9 (also focus of QA-Cu1) and CSC swapping QA-P3 and QA-P4 (which is your cubic).
The foci of the QA-Cu7 are all different, but the foci of your QA-Cux are allways QA-P9 and all the QA-Cux cubics pass through the vertices of Tr2).
Best regards
Bernard
PS Of course, no need to repeat that your cubic QA-Cux also deserves a place in EQF ...

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Message: #1354
Date: 2022-04-06
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

Dear Eckart,
Perhaps of interest for you (may be these properties are already well known).
Taking a reference triangle and the QA of it's in and excenters, the 7 vertices form with the 2 circular points a CB system.
Hence there are an infinity of QA-Cu1 having this triangle as Tr2.
You only need one point more : taking a point on the circumcircle of the triangle as the focus F (QA-P9), it's diametral wrt the circumcircle is the point S, where the cubic cuts it's asymptote.
The asymptote is parallel to the simson Line of F, as you mention in EQF.
S belongs to the cubic and it's 3 CSC partners give 3 more points (3rd intersections of the cubic with the 3 sides of Tr2).
You have then 11 points, which is more than needed to draw a cubic ...
3 of these QA-Cu1 are QL-Cu1, when F is in a vertice of the triangle : the Newton Line is then the perpendicular bisector of the opposite side of the triangle.
Best regards
Bernard
PS Again for Ivan, it's then easy to draw a QA-Cu1 or a QL-Cu1 by using the function Implicit curve of Geogebra ...

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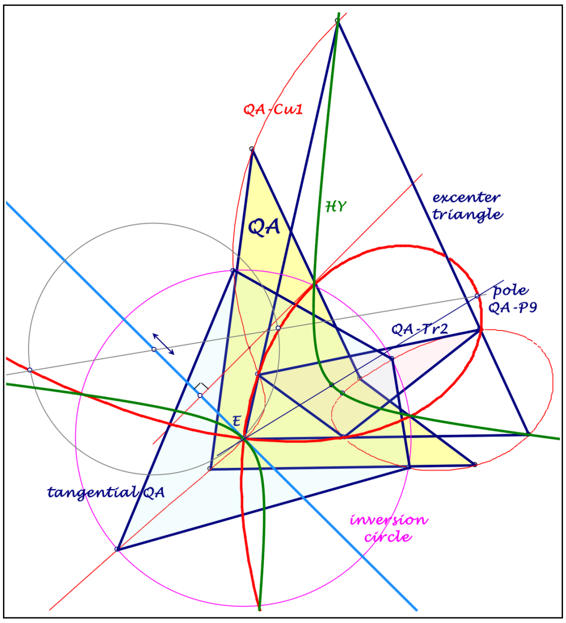
Message: #1355
Date: 2022-04-07
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

Dear Bernard,

thanks for #1353 with the comparison of QA-Cu7 and QA-Cux
... for tangential QAs on QA-Cu1 of a reference QA, that was new for me.
Some further observations for the case,
... that your tangential QA wrt QA-Cu1 is cyclic,
... then the circumcircle is an anallagmatic inversion circle,
... centered in one QA-Tr2-excenter E, corresponding QA-Tr2-vertex and incenter are inverse.
QA-Cu7 of this cyclic tangential QA degenerates to its QA-Co1,
... which is an orthogonal HY, also bearing the QA-Tr2-in/excenters
... and the invers of QA-P9 in the inversion circle.
QA-Cux of this cyclic tangential QA is a QA-Tr2-circumscribed strophoid,
... which is the inverse of HY wrt the inversion circle,
... the strophoid has the fixed point in the excenter E of QA-Tr2,
... ... its line through the excenter E, orthogonal to the QA-Cu1 asymptote
... ... and the pole QA-P9.

Best regards Eckart



2022-04-07.pdf

Message: #1356
Date: 2022-04-07
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

Dear Eckart,
For any point P on a cubic QA-Cu1
1) the 3 CSC partners form with P a tangentiel QA : the 4 points have the same tangential, which is their QA-P4
2) the 4 inverses in the 4 inversion circles centered in the in- and excenters of Tr2 (centers of anallagmaty) form a 2nd tangential QA, *which is the Q-QA of the P-QA.
* the QA-P4 of the Q-QA is the QA-P3 of the P-QA (and vice-versa).
Simple beautiful geometry, isn't it ?
Best regards
Bernard
PS Of course useful for my construction of QA-Cu1 or (bicursal) QL-Cu1, as we have 4 more points, namely the inverses of S in the 4 inversion circles ...

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Message: #1357
Date: 2022-04-07
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

Dear Bernard,

questions to #1356:
wrt 1): Do you mean with CSC the Möbius transformations of QA-Tr2,
... centered in one vertex, swapping the other two vertices?
wrt 2) If I am not wrong, QA-Cu1 has only 3 inversion circles,
... what about an inversion circle, centered in the QA-Tr2-incenter?

Best regards Eckart

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Message: #1358
Date: 2022-04-08
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

```
Dear Eckart,
wrt 1) yes, that's what I mean
wrt 2) QA-Cu1 has 4 inversion circles, centered in the in- and
excenters (vertices of the triangle are inverses of the
excenters in the circle centered in the incenter).
Best regards
Bernard
```

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Message: #1359
Date: 2022-04-08
From: bernard.keizer@gmail.com
Subject: Re: Interesting QA-cubic

```
Dear Eckart,
Of course, the construction given for QA-Cu1 works, as
mentionned, for bipartite QL-Cu1.
For monopartite QL-Cu1, which is also invariant in the 3 CSC of
the triangle, let M1 be the focus and M2M3 be the Newton Line.
This time, the asymptote is the parallel to the Newton Line,
homothetic in the homothety (M1,2) and S is the intersection of
this asymptote and the tangent in M1 to the circumcircle of the
triangle.
Let B and B' and C and C' be the invariant points of the 2 CSC
centered in M2 and M3. Complete the QL with A and A'.
A and A' are the centers of anallagmaty, centers of the
inversion circles. A and A' lie on the 2nd Steiner Line.
The 6 points form the main QL of QL-Cu1.
They form with M1, M2 and M3 a CB system and there are an
infinity of cubics through the 9 points.
You need one point more. Taking the point S gives the searched
QL-Cu1.
Best regards
Bernard
```

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Message: #1360
Date: 2022-04-08
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

Dear Bernard,

wrt the anallagmatic of QA-Cu1:
I don't understand your 4th inversion circle, centered in the incenter of QA-Tr2.
There are only 3 inversion circles of QA-Cu1 (see EQF),
... which are centered in the QA-Tr2-excenters,
... incenter and corresponding QA-Tr2-vertex are invers.

Best regards Eckart

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Message: #1361
Date: 2022-04-08
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

Dear Bernard,

am I right, this is the background for your last messages:
You look for cubics QA-Cu1 and QL-Cu1,
... circumscribed a triangle TR,
... with one vertex F as focus
... and invariant wrt the Möbius transformations TF of TR.
For QA-Cu1:
Take a point P on the TR-circumcircle diametral F,
... then P and its three TF-images give a QA for QA-Cu1.
For QL-Cu1:
Newton-line QL-L1 is
... for a bipartite QL-Cu1 the bisector of the TR-vertices unequal F,
... for a monopartite QL-Cu1 the opposite sideline of F wrt TR,
... CSC is the TF, centered in F,
... consider QL-L1-parallels L and their reflection L' in QL-L1,
... the intersections of L' and CSC(L) give the cubic QL-Cu1.

Best regards Eckart

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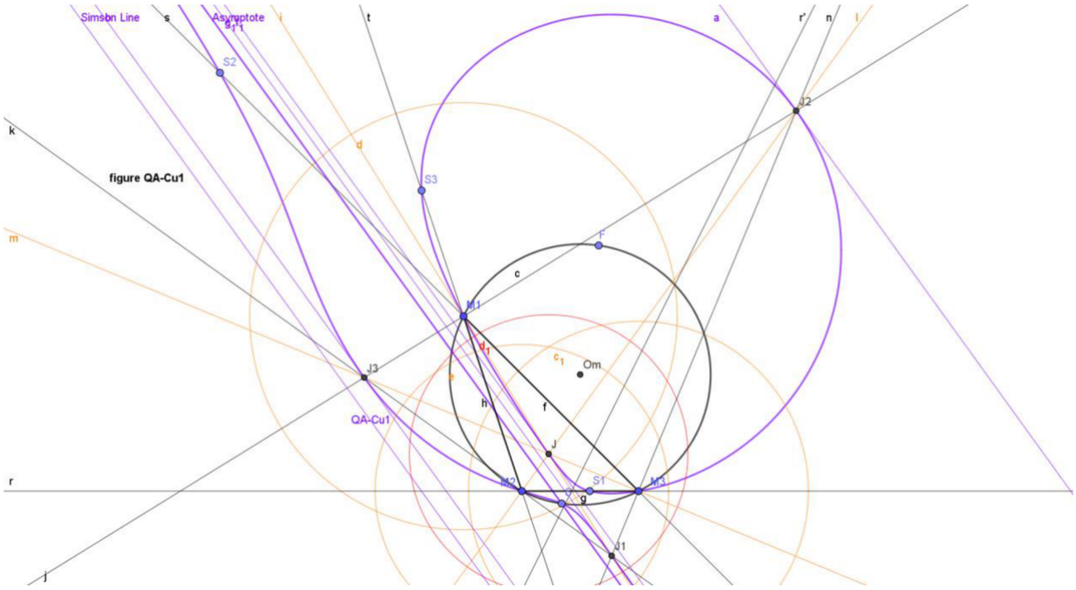
Message: #1362
Date: 2022-04-08
From: bernard.keizer@gmail.com
Subject: QA-Cu1 and QL-Cu1

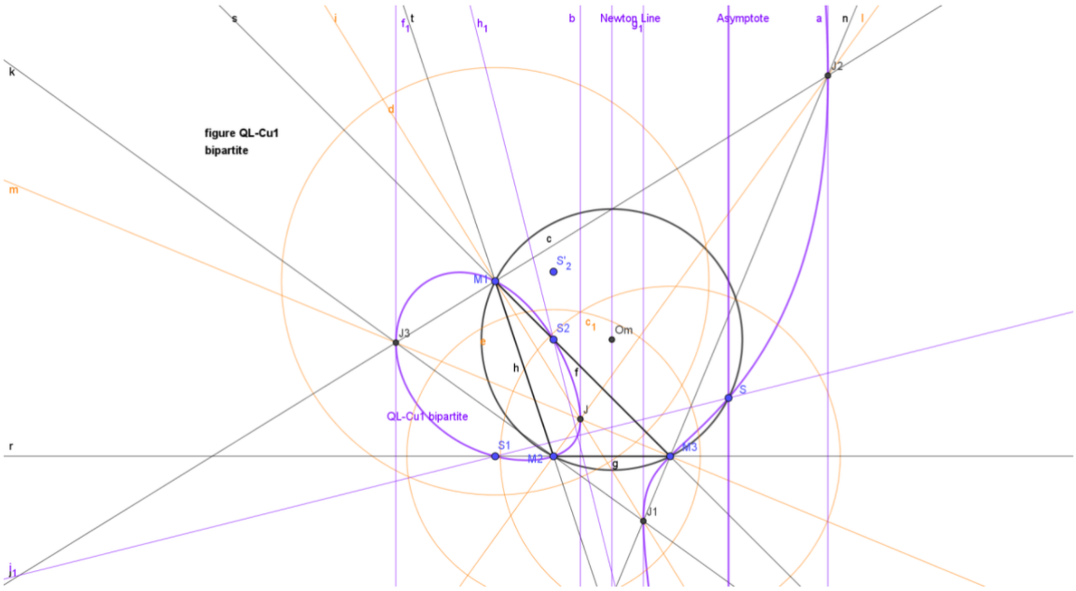
Dear Eckart,
Here the figures of the 3 curves QA-Cu1, QL-Cu1 (bipartite and monopartite) with the same reference triangle.
I only wanted to construct in the 3 cases the curves with 9 real points (not in a CB system), in order to use the command Implicit curve of Geogebra.
Naturally, you're right, QA-Cu1 is the QA-Cu1 of the QA of S and it's 3 CSC with a focus F.
The same goes for QL-Cu1 bipartite (in this case, the QA of S and it's CSC partners has 2 orthogonal sides) and the focus is one vertice (M1 on the figure) and the Newton Line is the perpendicular bisector of the opposite side (M2M3 on the figure).
QL-Cu1 monopartite has one vertice as focus (M1 on the figure) and the opposite side (M2M3 on the figure) as Newton Line.
There is a mistake in EQF : QA-Cu1 and the bipartite QL-Cu1 have 4 centers of anallagmaty, which are the in- and excenters. The circle of inversion centered in the incenter is not real, but you may obtain the inversion in combining a reflexion in the incenter and an inversion wrt a real circle (in red on the 1rst figure) centered in the incenter (the inversion swaps each vertice of the triangle and the corresponding excenter).
The tangents to the curve in the in- and excenters are parallel to the asymptote.
QL-Cu1 monopartite has only 2 real centers of anallagmaty (A and A' on the figue), where the tangents to the curve are parallel to the asymptote.
Naturally, QL-Cu1 is the QL-Cu1 of the QL with vertices in the 6 points A, A', B, B', C and C' (which I name the main QL of the curve).
I hope these explanations and the figure will convince you.
Best regards
Bernard

QA-Cu1 and QL-Cu1 (bipartite and monopartite)

1) QA-Cu1

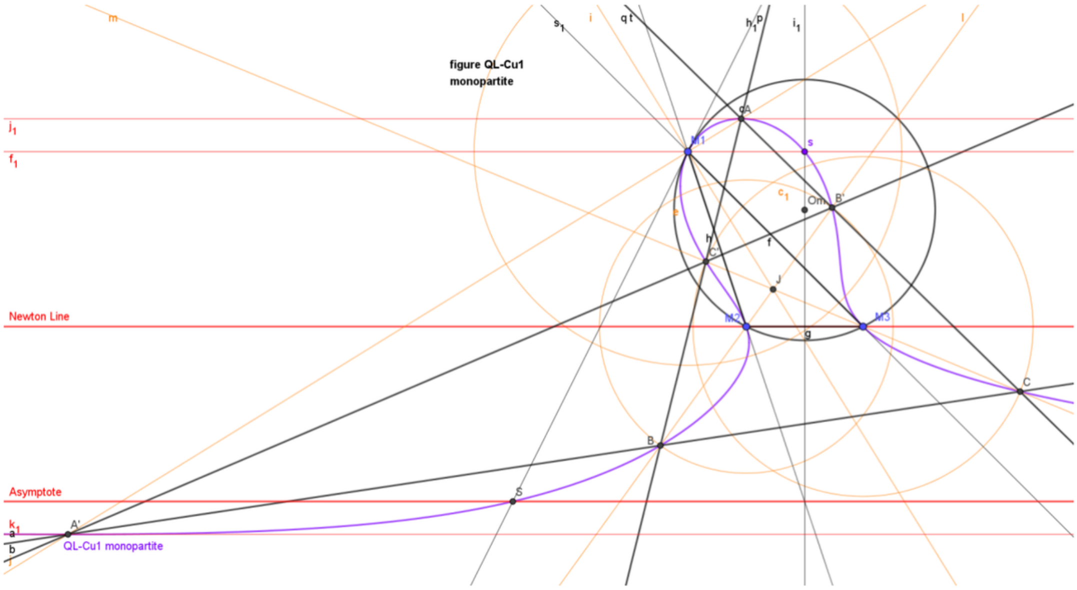


2) Bipartite QL-Cu1



QA-Cu1 and QL-Cu1.pdf

3) Monopartite QL-Cu1



QA-Cu1 and QL-Cu1.pdf

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Message: #1363
Date: 2022-04-09
From: eckart_schmidt@t-online.de
Subject: Re: Interesting QA-cubic

```
Dear Bernard,

Thanks for the detailed explanations,
... the 4th not real inversion circle for QA-Cu1 was new for me.

Best regards Eckart
```

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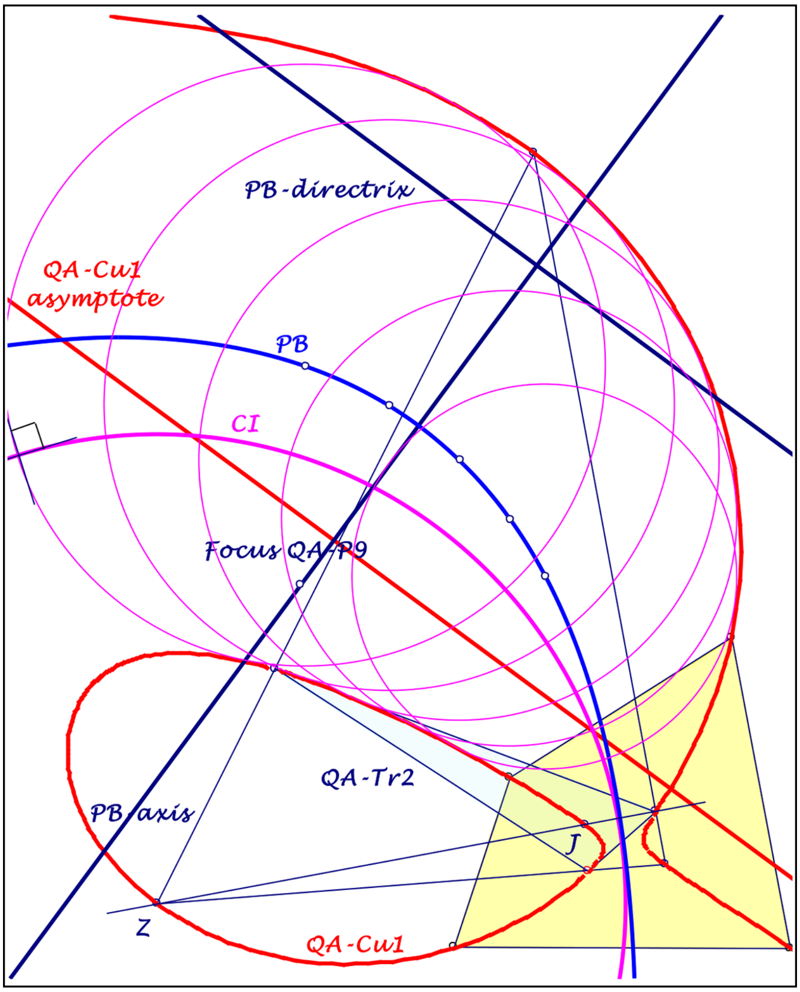
Message: #1364
Date: 2022-04-14
From: eckart_schmidt@t-online.de
Subject: QA-Cu1 as envelope of circles

```
Dear all,

cissoid, strophoid and trisectrix are envelopes of circles,
... which intersect a circle orthogonal
... and are centered on a parabola.
What about QA-Cu1?
QA-Cu1 is bipartite and anallagmatic,
... the real inversion circles are centered in the excenters of
QA-Tr2,
... swapping the incenter J and the corresponding QA-Tr2-vertex.
Consider the inversion circle CI, centered in Z on the closed
part of QA-Cu1,
... which has no intersection with QA-Cu1.
Lines through Z intersect QA-Cu1 in two further points,
... whose bisectors envelope a parabola PB.
Circles, centered on the parabola
... and orthogonal intersecting the inversion circle
... touch two times QA-Cu1 and envelope this cubic.
The parabola has the focus QA-P9
... and an axis orthogonal to the asymptote of QA-Cu1.

Best regards Eckart
```



2022-04-14.pdf

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Message: #1365
Date: 2022-04-15
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 as envelope of circles

Dear all,
QL-Cu1 as anallagmatic curve and envelope of circles
... is already described on my homepage (2011):
Focus.pdf (eckartschmidt.de) <http://eckartschmidt.de/Focus.pdf>
I shall describe it once more in a next message.
Best regards Eckart

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Message: #1366
Date: 2022-04-15
From: eckart_schmidt@t-online.de
Subject: Re: QL-Cu1 as envelope of circles

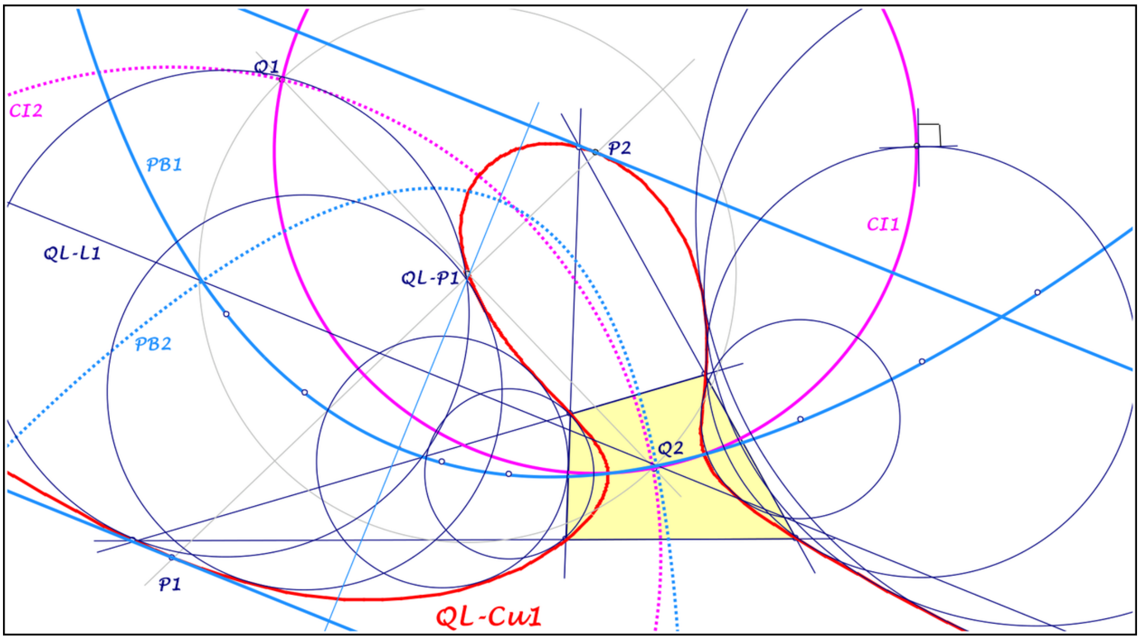
```
Dear all,

QL-Cu1 is anallagmatic:

Let Q1 and Q2 be the fixed points of CSC = QL-Tf1
... and P1 and P2 the CSC-partner on the 2nd Steiner axis,
... then the inversion circles CI of QL-Cu1
... are centered in P1 and P2, bearing Q1 and Q2.

QL-Cu1 as envelope of circles:
The circles are centered on parabolas PB1 and PB2
... with focus QL-P1 and directrix parallel QL-L1
    through P1 or P2,
... intersecting orthogonal the corresponding inversion circle.

Best regards Eckart
```



2022-04-15.pdf

Message: #1367
Date: 2022-04-16
From: eckart_schmidt@t-online.de
Subject: QA-Cu7 as envelope of circles

Dear all,

background for this observation is old#2965 (1):
QA-Cu7 is bipartite and anallagmatic,
... the real inversion circles
... are centered in the excenters of triangle TR = (QA-P2, QA-P4, QA-P41),
... swapping the incenter J and the corresponding TR-vertex.
Consider the inversion circle CI, centered in Z on the closed part of QA-Cu7,
... which has no intersection with QA-Cu7.
Lines through Z intersect QA-Cu7 in two further points,
... whose bisectors envelope a parabola PB.
Circles, centered on the parabola
... and orthogonal intersecting the inversion circle
... touch two times QA-Cu7 and envelope this cubic.
The parabola has the focus QA-P41
... and an axis parallel QA-L2.
Another parallel to the axis through Z
... intersects the cubic in two points,
... whose bisector is the tangent in the PB-vertex

Best regards Eckart

PS: The last property holds also for the parabola in #1364.

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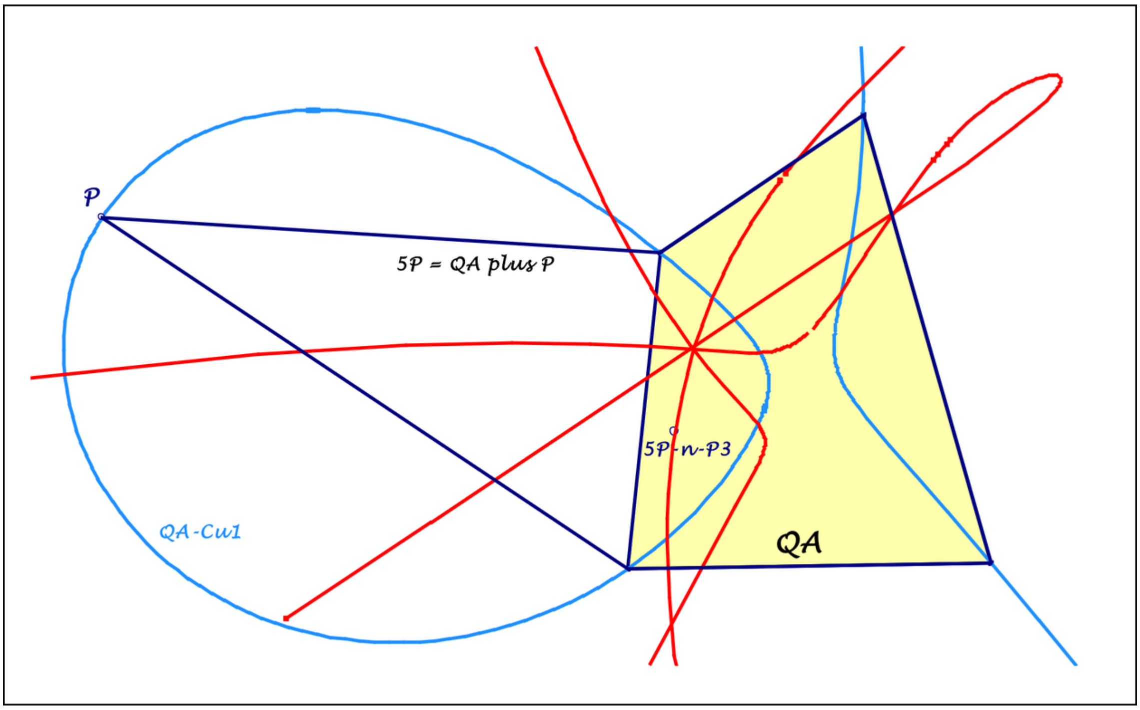
Message: #1368
Date: 2022-04-19
From: eckart_schmidt@t-online.de
Subject: New QA-point?

```
Dear all,

the locus (sextic?) of 5P-n-P3 for 5P = QA plus P on QA-Cu1
... has a 4-times intersection (attached).
What about this point?

Best regards Eckart
```



2022-04-19.pdf

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Message: #1369
Date: 2022-04-22
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 as envelope of circles

Dear Eckart,
All these properties are well known (see Math curve anallagmatic curves), already mentionned several times in particular by yourself and partly in EQF (see QA-Cu1).
The interest of your messages is to refresh these items and to precise the circles of inversion, the deferent curve and the definition of the curve as envelop of circles ...
Naturally, these properties hold for QA-Cu1, for bipartite QL-Cu1, like QA-Cu7 of your beautiful QA-Cux (why not name this curve definitely QA-Cu8 ?) as well as for monopartite QL-Cu1.
But they hold also for other curves like simple lines or circles as well as for curves of higher degree, like Cassini ovals or bicircular quartics ...
Best regards
Bernard

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Message: #1370
Date: 2022-04-22
From: bernard.keizer@gmail.com
Subject: Re: QA-Cu1 as envelope of circles

Dear Eckart,
I've forgotten : Also well-known, QL-Qu1 is a bicircular quartic, envelop of circles centered on a circle (QL-Ci3) through a point (QL-P1) ...
Best regards
Bernard

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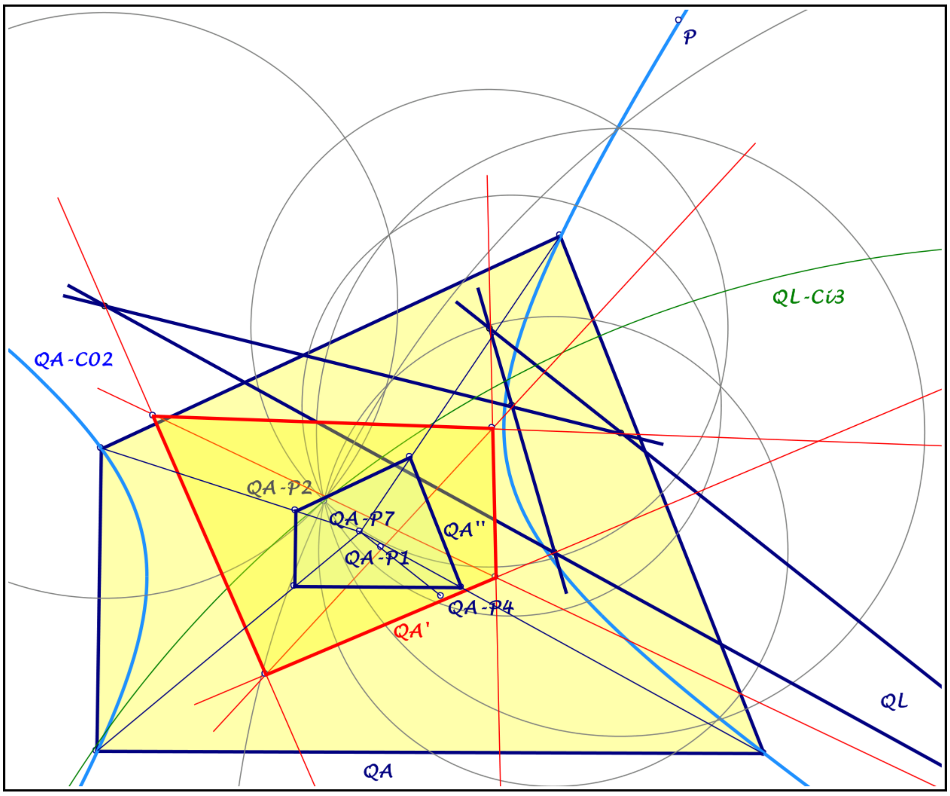
Message: #1371
Date: 2022-04-23
From: eckart_schmidt@t-online.de
Subject: Just for fun

```
Dear all,

... perhaps of interest, a short excursion in QA/QL-geometry:
Let us start with a quadrangle QA  = P1P2P3P4
... and a point P on the orthogonal hyperbola QA-Co2,
... consider the 6 nine-point circles for P, Pi, Pj,
... which have a common point in QA-P2.
... The centers of the 6 circles
... are the points of a related QL,
... whose circle QL-Ci3 bears QA-P2.
Varying P on QA-Co2, the loci for the circle centers
... give 6 lines of a related quadrangle QA',
... which is the QA-nine-point center quadrangle,
... whose QA-P4 is QA-P2 of the reference QA.
The 2nd QA-nine-point center quadrangle QA''
... will be homothetic to the reference QA
... wrt QA-P7 and ratio of QA-P1 dividing QA-P7.QA-P4.

Best regards Eckart
```



2022-04-23.pdf

Message: #1372
Date: 2022-04-29
From: ivan.pavlov@gmail.com
Subject: A cyclic polygon property

Dear All,

Here is something probably obvious, but very nice nonetheless.

Let ABCDE be a cyclic pentagon. Then:
- the Mathot points E', A', B', C', and D' resp. of ABCD, BCDE, CDEA, DEAB and EABC lie on a circle homothetic to the circumcircle of the pentagon
- the lines AA', BB', CC', DD' and EE' are concurrent at the center of homothety and the ratio is 2

Let's call the center of homothety from above, Mathot point of the cyclic pentagon ABCDE.

Let ABCDEF be a cyclic hexagon. Then:
- the Mathot points F', A', B', C', D', E' of the pentagons ABCDE, BCDEF, CDEAB, etc. lie on a circle homothetic to the circumcircle of the hexagon
- the six lines AA', BB', CC', etc. are concurrent at the center of homothety and the ratio is 3

et sic in infinitum...

Ivan Pavlov

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Message: #1373
Date: 2022-04-30
From: eckart_schmidt@t-online.de
Subject: Re: A cyclic polygon property

```
Dear  Ivan Pavlov,

Mathot points for cyclic n-gons were new for me,
... I have read your message with great interest.
In EQF/EPG no properties of cyclic n-gons are considered,
... but sometimes related points lie on a circle
... and Mathot points may be of interest:

Examples:
(1) Consider for the trilaterals of a QL
... the 4 circumcenters on QL-Ci3,
... whose Mathot point is QL-P5.
(2) The same Mathot point QL-P5 for
...  the orthocenters of the 4 triangles of the QA
... formed by the circumcenters of the 4 QL-trilaterals,
... which lie concyclic on QL-Ci4.
(3) Consider QL-P4 for the 5 QL of a 5L,
... which are concyclic on 5L-n-Ci1,
... their Mathot point is 5L-s-P5.

There will be more, I shall have a look.

Best regards Eckart
```

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Message: #1374
Date: 2022-05-01
From: eckart_schmidt@t-online.de
Subject: Re: A cyclic polygon property

```
Dear Ivan Pavlov,

excuse I haven't read your message with care:
... I have taken the common point of  AA', BB', CC',
... as new Mathot point,
... but it is not the center of homothety for the circles.
With your definition example (3) will give the Mathot point,
... which is the reflection of 5L-n-P3 in 5L-s-P4
... or dividing 5L-s-P4.5L-s-P5 with ratio -3:4.

Best regards Eckart
```

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Message: #1375
Date: 2022-05-01
From: ivan.pavlov@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A cyclic polygon property

---

```
Dear Eckart,

My wording was bad. The common point of AA', BB', etc. is the
internal center of similitude of the two circles, and this is
what I called Mathot point for n-gons. Homothety center is
ambiguous, because there are two of them.

Best regards,
Ivan
```

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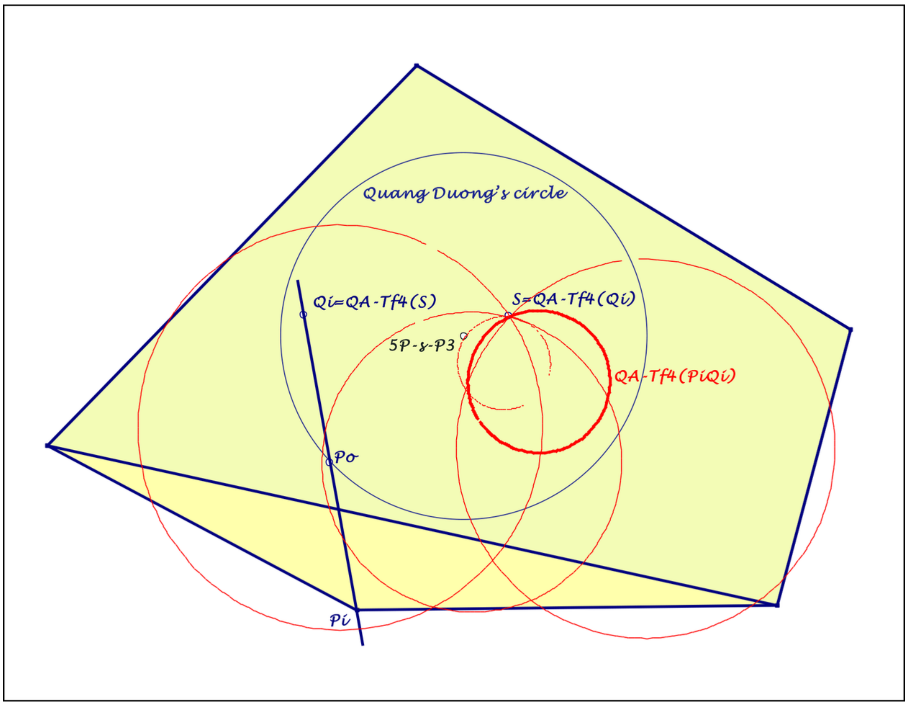
Message: #1376
Date: 2022-05-07
From: eckart_schmidt@t-online.de
Subject: New 5P-points

Dear all,

consider a 5P and an arbitrary point P,
... further the QA-Tf4-image circles
... of the 5 lines PiP wrt the quadrangle PjPkPlPm.
There exists a point P = Po,
... whose 5 circles have a common point S.
The points Qi on PiPo with QA-Tf4(Qi) = S wrt PjPkPlPm
... are the reflections of Pi in Po.
Construction of S:
Po lies on the Quang Duong's circle round 5P-s-P3.
Take a point P on this circle,
... reflect Pi in P and take the QA-Tf4-image wrt PjPkPlPm,
... whose locus is a line through S,
... two of these lines give S.
Construction of Po:
Consider Qi = QA-Tf4(S) wrt PjPkPlPm
... and the midpoint of QiPi.

What about further properties of Po and S?

Best regards Eckart



2022-05-07.pdf

Message: #1377
Date: 2022-05-08
From: eckart_schmidt@t-online.de
Subject: 6P-n-P5

```
Dear Chris,

please help, I lost control:
... I have a macro 6P-n-P5 in CABRI,
... but I found no description of 6P-n-P5 in EPG.
What about this point?
Thanks in advance.

Best regards Eckart
```

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Message: #1378
Date: 2022-05-08
From: van10hoven@gmail.com
Subject: Re: 6P-n-P5

```
Dear Eckart,
6P-n-P5 is the 6P-version of nP-n-P5, which is the nP-CC-Moebius
Center.
Right now I'm on a walking trip through France, so I can't track
down now any more details.
Best regards,
Chris
```

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Message: #1379
Date: 2022-05-09
From: eckart_schmidt@t-online.de
Subject: nP-n-P5 and (n-1)P-n-Tf1

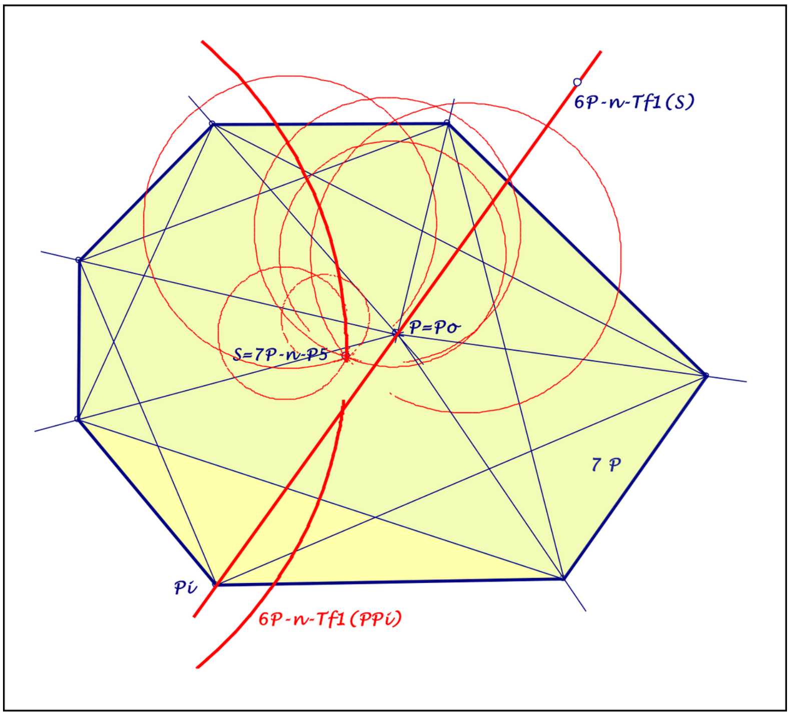
```
Dear all,

this is a generalization of my message #1376,
... background for nomination and properties is Chris' message
#689.
I think the following is a relevant general nP-theorem:

Consider a nP = P1...Pn and an arbitrary point P,
... further the image circles wrt (n-1)P-n-Tf1
... of the lines PPi for the (n-1)P-components of nP
... with vertices unequal Pi.
There exists a point P = Po,
... whose n circles (n-1)P-n-Tf1(PPi) have a common point S.
The common point S is nP-n-P5
... and the point P = Po is the midpoint
... of Pi and (n-1)P-n-Tf1(S).

Wrt my example in #1376:
n =5; S = 5P-n-P5 = 5P-s-Tf8(5P-s-P4), 4P-n-Tf1 = QA-Tf4.

Best regards Eckart
```



2022-05-09.pdf

Message: #1380
Date: 2022-05-10
From: Stan.Rabinowitz@comcast.net
Subject: central quadrilaterals

Ercole Suppa and I have been investigating quadrilaterals formed from a given quadrilateral by placing triangle centers in each of four triangles associated with the given quadrilateral. Our paper, *The Shape of Central Quadrilaterals* , has just been published by the International Journal of Computer Discovered Mathematics.

Link: http://www.journal-1.eu/2022/6.%20Stanley%20Rabinowitz,%20Ercole%20Suppa.%20The%20Shape%20of%20Central%20Quadrilaterals,%20pp.%20131-180..pdf

We also have another paper, *Relationships between a Central Quadrilateral and its Reference Quadrilateral* , that we have submitted (but it has not been published yet).

Link to preprint: http://www.stanleyrabinowitz.com/download/centralquadrilateralpairs.pdf

Here is a typical result that we have found:

Let E be the Euler-Poncelet point (QA-P2) of a Hjelmslev quadrilateral ABCD. (A Hjelmslev quadrilateral is a quadrilateral with right angles at two opposite vertices.) Let F, G, H, and I be the de Longchamps points (X20 points) of △ EAB, △ EBC, △ ECD, and △ EDA, respectively. Then [FGHI]=2[ABCD], where [PQRS] denotes the area of quadrilateral PQRS.

We would be happy to receive any comments that you may have about these papers. Let us know if you find any errors as there is still time for us to make corrections to the second paper before publication.

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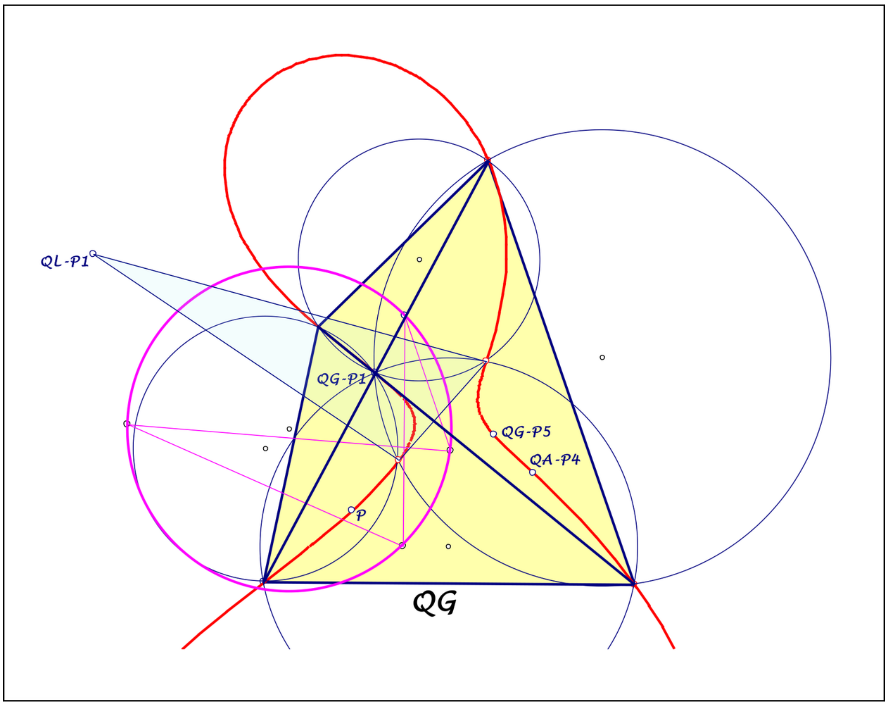
Message: #1381
Date: 2022-05-12
From: eckart_schmidt@t-online.de
Subject: Re: central quadriloaterals

Dear Stanley Rabinowitz,

only a first look in your extensive work:
Your idea wrt "quarter triangles"
... opens a new field of quadrigon geometry,
... for it is only used in EQF for QG-P8,9,10,11.
New aspect for example:
Take the circumcircles of the quarter triangles
... and the four inverses of a point P,
... which are concyclic for P=QA-P4,
... but not only,
... the locus for points with concyclic inverses
... is a circumquartic of the quadrigon,
... CSC=QL-Tf1-invariant, bearing the following 10 points:
... ... vertices of the reference quadrigon,,
... ... QG-P1 and QA-P4,
... ... QG-P5 and its CSC-image,
... ... the two vertices unequal QL-P1 of the Miquel-triangle.
The quartic is the 5P-quartic (see #923)
... of 5P = reference quadrigon plus QG-P1 (attached).

Best regards Eckart



2022-05-12.pdf

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Message: #1382
Date: 2022-05-13
From: bernard.keizer@gmail.com
Subject: Re: central quadrilaterals

Dear Stanley,
Huge work, very interesting indeed !
I personnally prefer the 4 triangles formed by 3/4 points of the quadrangle.
You define then QAs formed by the ETC points of the 4 triangles.
Only some reflexions :
1) it is also possible to define QLs formed by ETC lines of the 4 triangles.
2) the same way, if you consider the 4 triangles formed by 3/4 lines of a QL, you may define QAs formed by ETC points of the 4 triangles or QLs formed by ETC lines of the 4 triangles.
3) it's even possible to generalise to 5Ps or 5Ls and, considering the 5 QAs formed by 4/5 points of the pentangle or the 5 QLs formed by 4/5 lines of the pentalateral, to define new 5Ps or 5Ls formed by the 5 QA-points or the 5 QL-lines of the 5 QAs or QLs.
(Only a well-known property as illustration : the 5 Newton Lines of the 5 QLs formed by 4/5 lines of a 5L concur in the center of the inscribed conic of the 5 lines)
Very stimulating in fact
Best regards
Bernard

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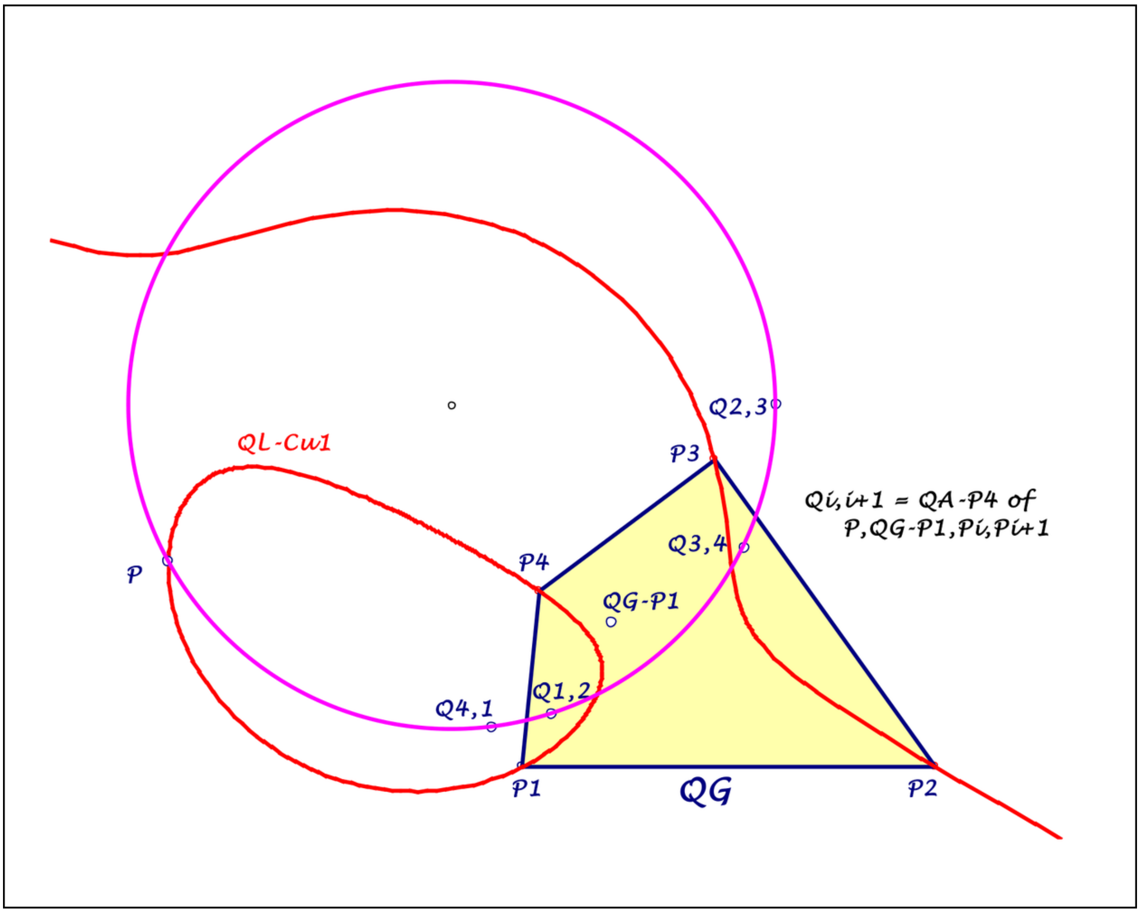
Message: #1383
Date: 2022-05-13
From: eckart_schmidt@t-online.de
Subject: QL-Cu1 for a quadrigon

```
Dear all,

is the following property already mentioned?
consider a QG, its QL-Cu1 and any point P on QL-Cu1,
... then the following 5 points are concyclic:
... P and Qi,i+1 = QA-P4 of P, QG-P1, Pi, Pi+1.
For P = QL-P1 we get a special QG-circle
... through QL-P1, QA-P4, QA-P9.

Best regards Eckart
```



2022-05-13.pdf

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Message: #1384
Date: 2022-05-13
From: Stan.Rabinowitz@comcast.net
Subject: Re: central quadrilaterals

Bernard wrote:

> I personally prefer the 4 triangles formed by 3/4 points of the quadrangle.

We studied these as well. The first paper, The Shape of Central Quadrilaterals , had a whole section on them (section 6).
We called them half triangles in that paper.

The second paper, Relationships between a Central Quadrilateral and its Reference Quadrilateral , was getting too long,
so we omitted half triangles. It also made the paper consistent with all results involving triangles formed by lines to the vertices
from a given point, called the radiator.

We hope to publish our results about relationships involving half triangles (component triangles) in a future paper.
Watch for it!

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Message: #1385
Date: 2022-05-15
From: ivan.pavlov@gmail.com
Subject: Parallel QA-L2 lines

Dear All,

This is more of a triangle topic but anyway... Let Ha, Hb and Hc be the feet of the altitudes of ABC.
Then the QA-L2 lines of AHaHbHC, BHaHbHC, and CHaHbHC are parallel.

Kind Regards,
Ivan

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Message: #1386
Date: 2022-05-16
From: eckart_schmidt@t-online.de
Subject: QA-circles, defined by a point

Dear all,

let us consider a QA = P1P2P3P4
... and the Moebius transformations,
... centered in QA-P4, swapping Pi, Pj,
... which give for any point 6 concyclic images:
Examples:
QA-P3 defines a circle with center QA-P9 bearing QA-P41,
QA-P9 defines a circle with center QA-P3 bearing QA-P2,
QA-P2 defines a circle with center QA-Tf4(QA-P2) bearing QA-P9.
QA-P41 defines a circle with center QA-Tf4(QA-P41) bearing
QA-P3.

The center is always QA-Tf4 of the defining point
... and the circle bears QA-Tf16 of the defining point.

Taking as definition point one of the fixed points of QA-Tf16,
... which are contact points of tangents from QA-P3 to QA-Cu1,
... the circle bears also the defining point.
The locus of defining points, which lie on its circle,
... seems to be a quartic, symmetric wrt QA-P4.
The fixed points of QA-Tf16 and their reflections in QA-P4
... have a common circumconic, centered in QA-P4.

Best regards Eckart

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Message: #1387
Date: 2022-05-21
From: ivan.pavlov@gmail.com
Subject: Quadrangle Centers as Limit Points

Dear all,

Let ABCD be a quadrangle and n a natural number.

Define the recurrent series A(n+1) = X(n) of B(n)C(n)D(n), where X(n) is the n-th triangle center from ETC.
Similarly define B(n+1), C(n+1) and D(n+1).

If the limits exist and are equal: lim A(n) = lim B(n) = lim C(n) = lim D(n) denote the limit point with QA-Xn of ABCD.

It is clear, for example, that QA-X2 of ABCD is the centroid.

Numerically, I determined that QA-X3 and QA-X5 exist.

What is remarkable, is that QA-X2, QA-X3 and QA-X5 are collinear.

Has someone examined such limits before?

Often the limit points are two, e.g. for X1 .
Sometimes the limit points are four, e.g. for X8 .

Kind regards,
Ivan

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Message: #1388
Date: 2022-05-27
From: ivan.pavlov@gmail.com
Subject: Which is this Quadrangle point?

```
Dear all,

Given four points W, X, Y and Z define by □ (W, XYZ) the line
between W and the G-Ceva conjugate of W wrt triangle XYZ, where
G is the centroid of XYZ.

It turns out that given a quadrangle P1P2P3P4, the four lines □
(P1, P2P3P4), □ (P2, P1P3P4), □ (P3, P1P2P4), and □ (P4, P1P2P3)
are concurrent.
Which is their common point wrt the quadrangle?

Kind regards,
Ivan

n.b. It is known that the G-Ceva conjugate is the center of the
circumconic of XYZ with perspector W.
```

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Message: #1389
Date: 2022-05-28
From: eckart_schmidt@t-online.de
Subject: Re: Which is this Quadrangle point?

```
Dear Ivan Pavlov,

sorry, I don't understand  "the G-Ceva conjugate of W wrt
triangle XYZ".
If I take "the isotomic conjugate of W wrt the cevian triangle
of G in XYZ"
... your common point of the lines will be QA-P16.

Best regards Eckart

PS:
If I take "the isotomic conjugate of W wrt the anticevian
triangle of G in XYZ"
... the lines will be parallel to QA-P1.QA-P16.
```

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Message: #1390
Date: 2022-05-29
From: ivan.pavlov@gmail.com
Subject: Re: Which is this Quadrangle point?

```
Dear Eckart,

G means the centroid, and the general definition of Ceva
conjugate is for example here -
https://mathworld.wolfram.com/CevaConjugate.html

Another characterization would be as follows. The G-Ceva
conjugate of a point P wrt triangle ABC is the center of the
circumconic of ABC which has P as its perspector.

Kind Regards,
Ivan
```

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Message: #1391
Date: 2022-05-29
From: eckart_schmidt@t-online.de
Subject: Re: Quadrangle Centers as Limit Points

```
Dear Ivan Pavlov,

back from holiday, I studied your limit points of QA-points:
... QA-X3 = QA-P4, same point for all QA = A(n)B(n)C(n)D(n),
... QA-X5 = QA-P7,
and starting with n=4 all points A(n),B(n),C(n),D(n) lie on
QA-Co2.

Best regards Eckart

PS: Thanks for explanations wrt "G-ceva conjugate of W",
But I think my interpretation was right.
```

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Message: #1392
Date: 2022-06-10
From: eckart_schmidt@t-online.de
Subject: Circumcubics K003/K024 for 4P/5P/6P

```
Dear all,

(1) Preface: 5P/6P-elements, which are not in EPG:
In #922, #1295, #1322 new 5P/6P-elements are described,
... here once more the definitions, properties in the messages:
5P-s-Px, 5P-s-Cix:
     Consider for each vertex Pi the QA of the remaining points
     ... with the four centroids of its triangle components
     ... and Qi = QA-P4 of these centroids,
     ... the Qi are concyclic on a circle 5P-s-Cix,
         centered in 5P-s-Px.
5P-s-Tfx:
     Replace in a 5P each vertex Pi by a point X
     ... and get 5 circles 5P-s-Cix with a common point
     ... on 5P-s-Cix of the reference 5P,
         which shall be 5P-s-Tfx(X).
6P-s-Px:
     Consider for a 6P the six circles 5P-s-Cix for the
     5P-components
     ... and you get a common point 6P-s-Px.

(2) QA-circumcubics, defined by a point P (approximated
attached)
Circles 5P-s-Cix for 5P = QA plus any point X
... have a common point Q, dividing QA-P1.QA-P4 with ratio -1:4.
The locus of points X, whose circles 5P-s-Cix for 5P = QA plus X
... are bearing  further a defining point P unequal Q,
... gives a QA-circumcubic (properties below).

(3) 5P-circumcubics, defined by a point P (approximated
attached)
5P-s-Tfx maps any point X  to the circle 5P-s-Cix
The locus of points X
... with 5P-s-Tfx(X) = 5P-s-Tfx(P) on 5P-s-Cix
... gives a 5P-circumcubic (properties below).

(4) 6P-circumcubic (approximated attached)
The locus of points X
... with 5P-s-Tfx(X) = 6P-s-Px for all six 5P = PiPjPkPlPm
... gives a 6P- circumcubic (properties below).

(5) Common properties for the circumcubics (2), (3), (4)
(a) The circumcubics are tripartite.
```

(b) The circumcubics have 60°-asymptotes with a common point.
(c) The circumcubics are K003 or K024
    (Bernard Gibert's nomination),
    ... K003, if one part has only one asymptote.
    ... K024, if each part has two asymptotes.
(d) The circumcubics intersect their asymptotes collinear.

(6) Final remarks

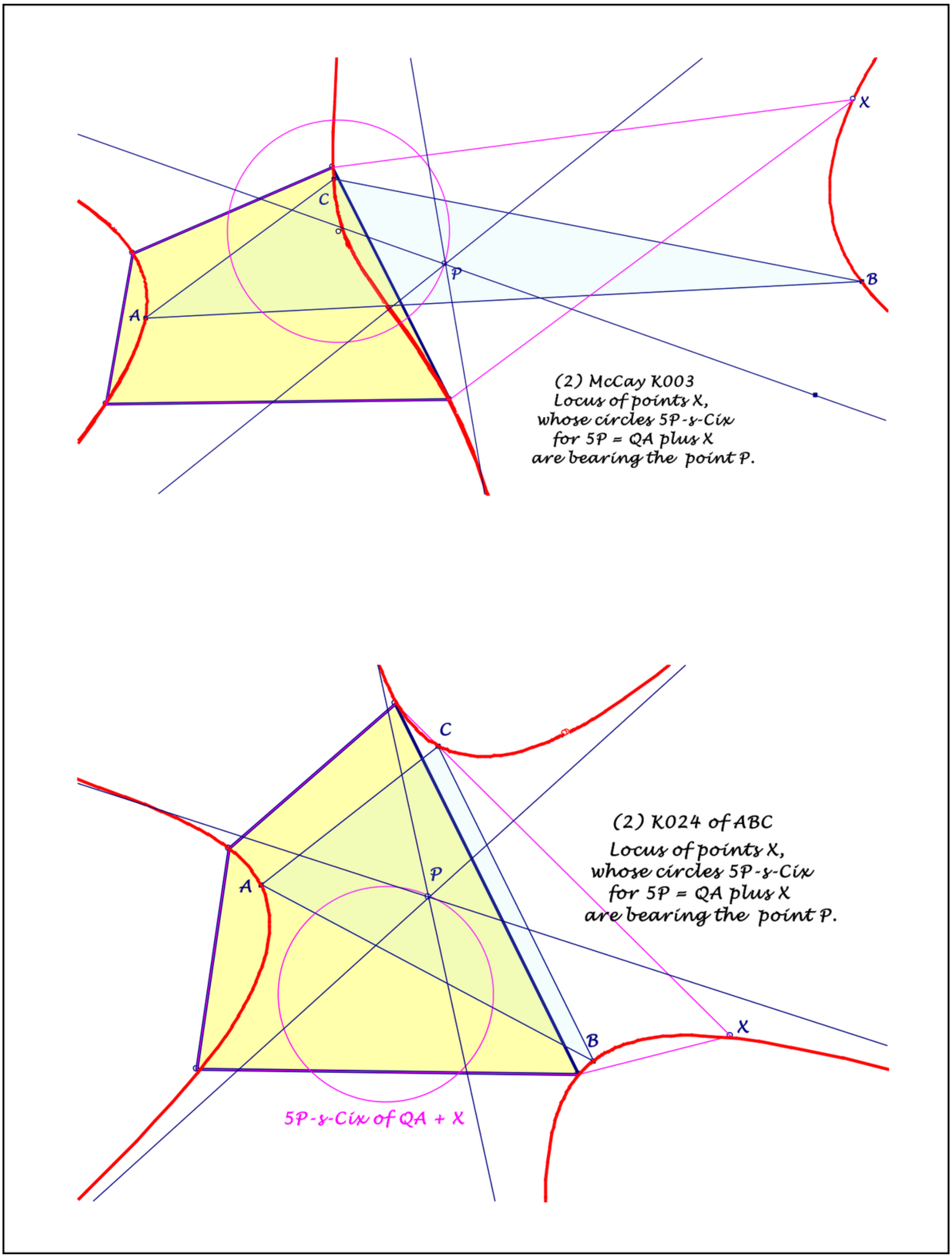
Wrt (2):
The reference triangle ABC for the QA-circumcubics
... has P as centroid, which is the common point of the
    asymptotes.
The CB-point 8P-s-P1 of QA plus A, B, C plus any point X of the cubic
... is again a point Y of the cubic with a fixed 3rd collinear
    point Z on the cubic.

Wrt (3):
The reference triangle ABC for the 5P-circumcubics
... has 5P-s-Tf(P) as centroid,
    which is the common point of the asymptotes.
The CB-point 8P-s-P1 of 5P plus A, B, C is again a point of the cubic.

Wrt (4)
The reference triangle ABC for the 6P-circumcubics
... has 6P-s-Px as centroid,
    which is the common point of the asymptotes.
The CB-point 8P-s-P1 of 6P plus two points on the cubic is again a point of the cubic.

I hope, there are no fake results,
... but CABRI-observations can have their limits.
Who can lighten the background?

Best regards Eckart



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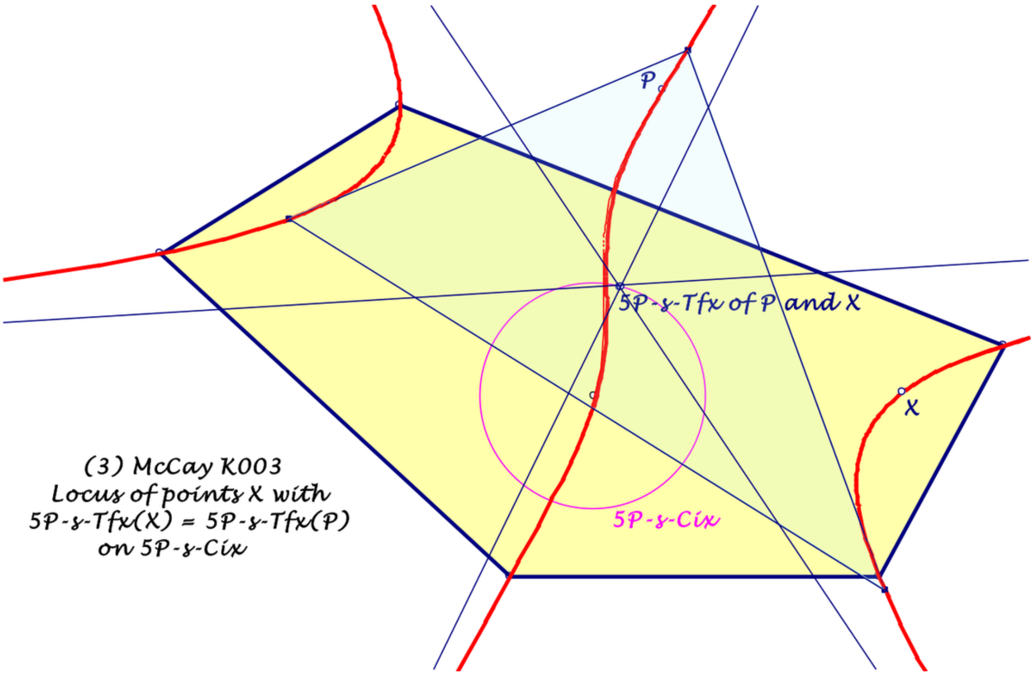

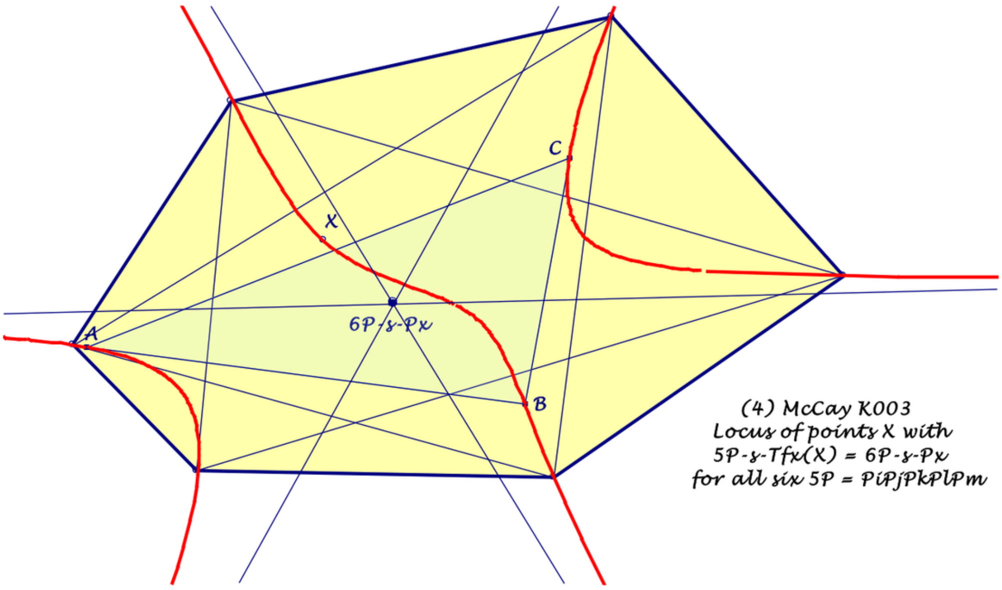

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Message: #1393
Date: 2022-06-11
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A properties of n point on a conic

---

Dear geometers,

I see this property:
On the conic (C), we choose n points A1,A2,...An such that n-gon A1,A2,...An has area S.
Let P and Q be two points on (C).
Circumcircles of triangles A1PQ, A2PQ,... AnPQ meet (C) again at B1,B2,..., Bn, respectively.
Then, n-gon B1,B2...Bn also has area S.
I have a hard time describing n-gon "has area". Maybe that means it's a non-self-cutting polygon?
Please give me more comments.

Sincerely yours,
Tran Quang Hung

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Message: #1394
Date: 2022-06-11
From: ivan.pavlov@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A properties of n point on a conic

---

Dear Tran Quang Hung,

You can define the area of a self-intersecting polygon algebraically like this:
https://mathworld.wolfram.com/PolygonArea.html

Your theorem holds for self-intersecting polygons as well.

Kind Regards,
Ivan Pavlov

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Message: #1395
Date: 2022-06-11
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A properties of n point on a conic

---

```
Dear  Ivan Pavlov,

Thank you for the link. It seems to be the Shoelace formula, it
is very useful for this my theorem.

Sincerely yours,
Tran Quang Hung

Vào Th 7, 11 thg 6, 2022 vào lúc 21:02 Ivan Pavlov
<ivan.pavlov@gmail.com>
đã viết:

> Dear Tran Quang Hung,
>
> You can define the area of a self-intersecting polygon
algebraically like
> this:
> https://mathworld.wolfram.com/PolygonArea.html
>
> Your theorem holds for self-intersecting polygons as well.
>
> Kind Regards,
> Ivan Pavlov
```

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Message: #1396
Date: 2022-06-11
From: eckart_schmidt@t-online.de
Subject: 5G-s-P2

```
Dear Chris,

5G-s-P2 is 5G-s-P1 of Pi+1,Pi-2,Pi,Pi+2,Pi-1,
... for example P1,P3.P5,P2.P4.

Best regards Eckart
```

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Message: #1397
Date: 2022-06-11
From: eckart_schmidt@t-online.de
Subject: 7P-s-P1

```
Dear Chris,

is the following property of 7P-s-P1 in EPG correct?
"6P-s-Ci1
<https://www.chrisvantienhoven.nl/np-items/np-geninf/np-0/23-mathematics/encyclopedia-of-poly-figures/np-objects/artikelen-np/438-6p-s-ci1>
common-circles-center of component 6P's"
I think, it must be:
"6P-s-Ci1
<https://www.chrisvantienhoven.nl/np-items/np-geninf/np-0/23-mathematics/encyclopedia-of-poly-figures/np-objects/artikelen-np/438-6p-s-ci1>
common-circles-intersection of component 6P's"

Best regards Eckart
```

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Message: #1398
Date: 2022-06-11
From: eckart_schmidt@t-online.de
Subject: Re: 5G-s-P2

```
Dear Chris,

excuse, there is a mistake:
5G-s-P2 is 5G-s-P5 of Pi+1,Pi-2,Pi,Pi+2,Pi-1,
5G-s-P3 is 5G-s-P4 of Pi+1,Pi-2,Pi,Pi+2,Pi-1.

Best regards Eckart
```

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Message: #1399
Date: 2022-06-13
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
I remember I found this old item with 5P-s-Cix very interesting!
I reproduced without difficulty the construction of the circle
and, in the case of the QA, an approximate cubic stelloïd Mac
Cay 003 or Kjp 024.
(The circumcenter of Cix is the perpendcular bisector of PQ).
And I read again the developments of Bernard Gibert, but I can't
see any reference to an inscribed QA. For a 5P, there are 5 such
curves.
The property I prefer is following :
The foci of the Steiner inellipse of ABC are the fixed points of
a Cl-S swapping the circumcircle and the Brocard circle.
For each point X of the curve, you may draw the ellipse centered
in P having the same foci and through the middle of PX (and it's
reflexion in P).
This ellipse is the Steiner inellipse of another pivot triangle
of the curve with one vertice in X and the 2 others on the curve
easy to determinate.
This construction is given by Bernard Gibert in his note
Eckart's cubic.
The interest of this Cl-S is that the Cl-S transform of any
circle Cix through P is a line through Cl-S(Q) ...
Best regards
Bernard
PS How do you find the reference triangle ABC ? Is it through
approximations ?

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Message: #1400
Date: 2022-06-14
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
I progress slowly, but surely !
When the curve is Mac Kay, one tangent from P to the curve
(inner part) gives the circumcenter O = X3 as contact point and
a 2nd intersection which is the orthocenter H = X4 (PH = 2 PO).
H is the tangential of O and of the 3 vertices of the triangle
ABC. OH is the Euler Line.
The circumcircle of ABC cuts the curve in 3 other points,
vertices of an equilateral triangle ; the perpendiculars from P
to the sides of this 2nd triangle are the asymptotes of the
curve.
The Brocard Line OK (K = X6) cuts the curve in the 2 isodynamic
points X15 and X16, which are isogonal wrt ABC and Cl-S partners
in the Cl-S centered in P with fixed points the foci of the
Steiner inellipse of ABC.
I'm looking now for your point Z, center of the CB
transformation wrt the vertices of the QA and the triangle ABC.
Best regards
Bernard

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Message: #1401
Date: 2022-06-14
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
Sorry for my mistake, naturally the isodynamic points X15 and
X16 are not on the stelloïd Mac Kay 003, but on it's hessian the
Van Rees monopartite curve 048.
Best regards
Bernard
PS I guess confusely there is a link between special QA's
inscribed in a Mac Cay curve and QL's inscribed in it's hessian
...

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Message: #1402
Date: 2022-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

```
Dear Bernard,

thanks for interest and observations.
You start with a reference triangle ABC for a McCay cubic
... and study further properties of this curve.
But my problem is another:
I start with a K003/K024-cubic,
... constructed in any way,
... looking for a reference triangle.
My reference triangles in #1392 are approximately put in,
... controlled with their properties.

Here an example for a more systematic way, to get back
... an approximate reference triangle of a cubic K024
(attached):
Consider an arbitrary 6P on K024 with the point 6P-s-Px,
... which has to be the centroid P of the reference triangle ABC
... and intersection of the asymptotes,
... which can easy very precisely approximately be drawn.
The asymptotes intersect the cubic collinear in U, V, W,
... whose tangents at the cubic contact in U', V', W',
... vertices of an equilateral triangle,
... whose circumcircle intersects the cubic further in A, B, C,
... which are the vertices of the reference triangle of K024.

Finally a consequence of (2) and (3) in #1392:
For every 5P on K003 or K024 the circles 5P-s-Cix
... bear the centroid of the reference triangle of the cubic.

Best regards Eckart
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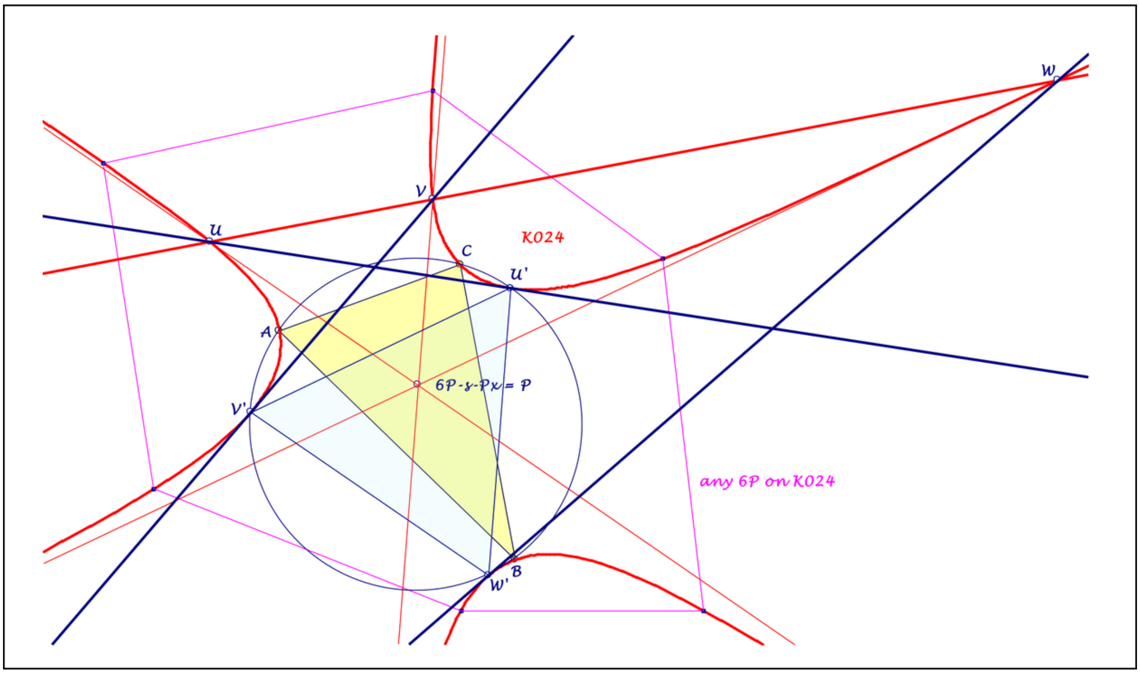

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Message: #1403
Date: 2022-06-15
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
If I understand correctly, the key property is that for Mac Kay K003 or Kjp K024 the circle 5P-s-Cix of any 5P on the curve pass through the point P.
For any 6P on the curve, the 6 5P-s-Cix intersect in P.
Very amazing property indeed !
P is the centroïd of the reference triangle ABC.
Knowing only the curve gives a possibility of having this point P.
For Mac Kay, my construction gives then successively the circumcenter, the orthocenter and the vertices of ABC. The circumcircle of ABC gives a 2nd (equilateral) triangle with sides perpendicular to the asymptotes (so you don't need to know the asymptotes).
For Kjp, I didn't find such a simple construction and it helps in fact to have the asymptotes (this time, they are parallel to the sides of the 2nd (equilateral) triangle U'V'W' on your figure.
Beautiful
Best regards
Bernard
PS I started with a QA and the point P, as mentionned in your point (1) in 1392. I found the curve by searching points X having the mentionned property that the circle 5P-s-Cix of the 5P formed by the QA vertices and X pass trhough P.

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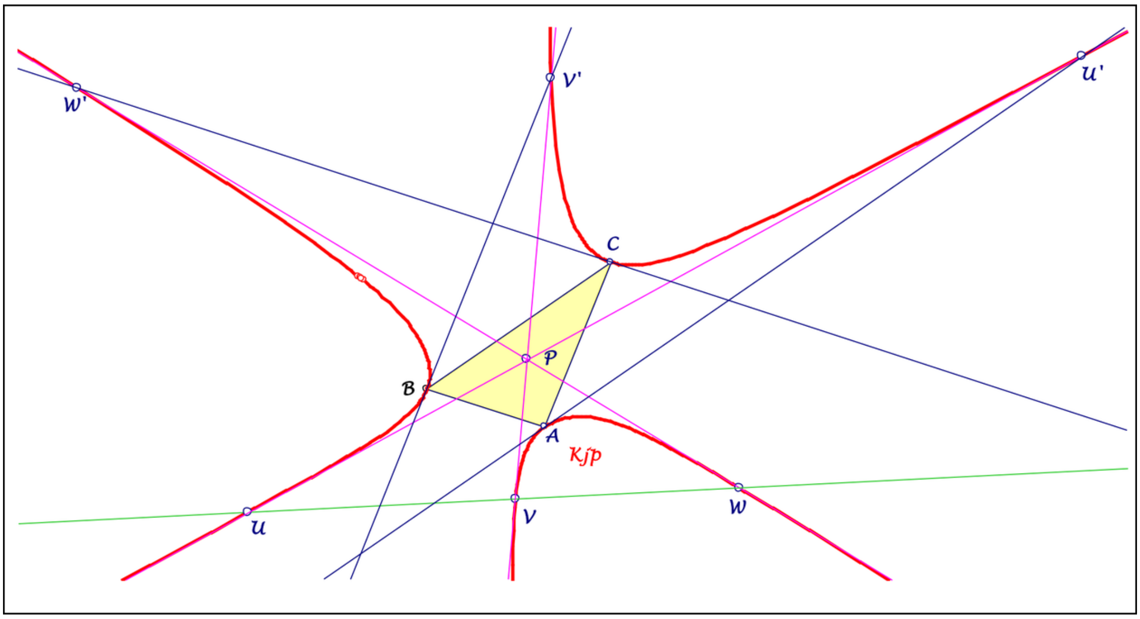
Message: #1404
Date: 2022-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Bernard,

thanks for simplifying the way back to the reference triangle
... for the McCay-cubic without using the asymptotes.
For the Kjp-cubic this is also possible:
Consider an arbitrary 6P on Kjp with the point 6P-s-Px,
... which has to be the centroid P of the reference triangle
ABC.
There are 3 tangents from P to the cubic with contact points U,
V, W.
Let U', V', W' be the 3rd intersections of UP, VP, WP and the
cubic,
... the 2nd tangents from U', V', W' to the cubic have the
contact points A, B, C,
... which are the vertices of the reference triangle
for the Kjp-cubic.

Best regards Eckart



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Message: #1405
Date: 2022-06-16
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
Thanks for this construction of the triangle ABC for Kjp !
Your property of 5P-s-Ci1 passing through the center P for any 5P on a Mac Cay or Kjp stelloïd cubic opens a wide field of possibilities for EQF.
This cubic can always be followed by it's hessian which is a Van Rees mono- (for Mac Cay) or bipartite (for Kj with focus in P and fixed points of the Cl-S the foci of the Steiner inellipse of ABC.
So I see at last 3 ideas for EQF :
1) definition of 5P-s-Cix in 5P conics
2) mention of the property in QL-Cu2 (Eckart's cubic), which is always Mac Cay or Kjp
3) examples of QA-Mac Cay or Kjp cubics for P in different QA-points
a) QA-P1 (possible link with the 3 foci of the QA ?)
b) QA-P4 (possible link with the Van Rees curve with focus QA-P4 and QL-2P2 in QA-P3 and QA-P9 ?)
c) QA-P41 (possible link with the Van Rees curve QA-P7 with focus in QA-P41 and QL-2P2 in QA-P2 and QA-P4 ?)
There are surely plenty of other possibilities ...
Best regards
Bernard

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Message: #1406
Date: 2022-06-16
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

```
Dear Bernard,

thanks for further suggestions,
... here first observations wrt your point 2) in #1405:
If QL-Cu2 is a McCay-cubic,
... the circumcircle of the reference triangle is centered
... in the intersection of the cubic and QL-L1.
If QL-Cu2 is a Kjp-cubic,
... the circumcircle of the reference triangle is centered
... on a perpendicular of QL-L1 through QL-L1^QL-L6.
The CSC-images of both cubics bear the center
... of the circumcircle of the reference triangle.
The 1st Steiner axis of the QL is the main axis
... of the inscribed Steiner ellipse of the reference triangle.
Question:
Are the CSC-fixed points the foci of the inscribed Steiner
ellipse?

Now we can "construct" the reference triangle,
... if QL-Cu2 is a McCay-cubic:
Consider the intersection M of QL-L1 and QL-Cu2
... and the circle CI round M through CSC(M),
... intersecting QL-Cu2 in 6 points,
... three give an equilateral triangle,
... the other three the reference triangle.

Best regards Eckart
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Message: #1407
Date: 2022-06-16
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
Here I am in a terra cognita !
Yes, of course, the fixed points of the CSC are the foci of the Steiner inellipse.
The isodynamic points are X15 and X16, which are the QL-2P2.
For Mac Cay, the Newton Line is the Brocard axis through X3 = O, X6 = K, and X15 and X16.
For Kjp, the Newton Line is the Lemoine axis, perpendicular bisector of X15X16.
X110, which is CSC(O) is on the circumcircle and on Mac Cay, but, if I'm not wrong, not on Kjp.
Hence your construction holds for Mac Cay, but not for Kjp.
For Kjp, you have to use the intersections of the Lemoine axis with the curve, which are the centers of the 3 Apollonian circles.
Each Apollonian circle through X15 and X16 cuts the curve in 4 other points, 3 vertices of an equilateral triangle and one vertice of ABC.
Best regards
Bernard

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Message: #1408
Date: 2022-06-16
From: amontes@ull.es
Subject: Symmedians and orthologic centers

Let Gb, Kb be the orthogonal projections of B to AG and AK respectively, and Gc, Kc be the orthogonal projections of C to AG and AK respectively. Let A1 (on BC) be the intersection of GbKc and GcKb, and define B1 and C1 cyclically. Let A2 (on AH) be the intersection of GbKb and GcKc, and define B2 and C2 cyclically. The triangles A1B1C1 and A2B2C2 are orthologic; the orthologic center of A1B1C1 wrt A2B2C2 is:

W = midpoint of the orthocenters of this triangles: 4th Brocard and circumsymmedial
=(a^2-2 (b^2+c^2)) (20 a^4+a^2 (b^2+c^2)-(b^2+c^2)^2) : :
= X(31961) + X(31962)
= lies on these lines: {2,11166}, {51,17430}, {184,15534}, {353,47075}, {524,47074}, {597,10183}, {3906,11123}, {31961,31962}
= 6-9-13 search numbers [-1.69553404949354, 3.07457137760065, 2.29466924333480]

The orthologic center od A2B2C2 wrt A1B1C1 is:
Z = reflection of X(2) in X(47589)
= 20 a^10-19 a^8 (b^2+c^2)+a^6 (-145 b^4+122 b^2 c^2-145 c^4) +a^4 (149 b^6-153 b^4 c^2-153 b^2 c^4+149 c^6)+a^2 (17 b^8-100 b^6 c^2+198 b^4 c^4-100 b^2 c^6+17 c^8) -2 (b^2-c^2)^2 (11 b^6-21 b^4 c^2-21 b^2 c^4+11 c^6) ::
= X(2) - 2 X(47589)
= lies on these lines: {2,47589}, {3,47591}, {4,47590}, {30,31748}, {381,13378}, {3091,47592}, {3543,5032}, {14269,46673}
= reflection of X(2) in X(47589)
= 6-9-13 search numbers [-7.69395212637534, -7.74792499463209, 12.5556673826721]

Details in: HG120622 (https://amontes.webs.ull.es/otrashtm/HGT2022.htm#ortologico120622)

Angel Montesdeoca

Message: #1409
Date: 2022-06-16
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

```
Dear Eckart,
In fact, X110 is neither on Mac Cay, nor on Kjp (but on the
circumcircle ).
Noone of the CSC of the 2 cubics can bear the circumcenter !
Best regards
Bernard
```

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Message: #1410
Date: 2022-06-16
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Symmedians and orthologic centers

---

```
Dear Angel

Your posting should be posted to Euclid

APH

On Thu, Jun 16, 2022 at 6:46 PM Angel Montesdeoca
<amontes@ull.es> wrote:
>
> Let Gb, Kb be the  orthogonal projections of B to AG and AK
respectively,
> and Gc, Kc be the  orthogonal projections of C to AG and AK
respectively.
> Let A1 (on BC) be the intersection of   GbKc and GcKb, and
define B1 and C1
> cyclically.   Let A2 (on AH) be the intersection of   GbKb and
GcKc, and
> define B2 and C2 cyclically. The triangles A1B1C1 and A2B2C2
are
> orthologic; the orthologic center of A1B1C1 wrt A2B2C2 is:
>
>   W = midpoint of the orthocenters of this triangles:  4th
Brocard  and
> circumsymmedial
>  =(a^2-2 (b^2+c^2)) (20 a^4+a^2 (b^2+c^2)-(b^2+c^2)^2) : :
>  = X(31961) + X(31962)
>  = lies on these lines: {2,11166}, {51,17430}, {184,15534},
{353,47075},
> {524,47074}, {597,10183}, {3906,11123}, {31961,31962}
>  = 6-9-13 search numbers [-1.69553404949354, 3.07457137760065,
> 2.29466924333480]
>
> The orthologic center od A2B2C2 wrt A1B1C1 is:
> Z =  reflection of X(2) in X(47589)
>  = 20 a^10-19 a^8 (b^2+c^2)+a^6 (-145 b^4+122 b^2 c^2-145 c^4)
+a^4 (149
> b^6-153 b^4 c^2-153 b^2 c^4+149 c^6)+a^2 (17 b^8-100 b^6
c^2+198 b^4
> c^4-100 b^2 c^6+17 c^8) -2 (b^2-c^2)^2 (11 b^6-21 b^4 c^2-21
b^2 c^4+11
> c^6) ::
>  = X(2) - 2 X(47589)
```

```
>  = lies on these lines: {2,47589}, {3,47591}, {4,47590},
{30,31748},
> {381,13378}, {3091,47592}, {3543,5032}, {14269,46673}
> = reflection of X(2) in X(47589)
>  = 6-9-13 search numbers [-7.69395212637534,
-7.74792499463209,
> 12.5556673826721]
>
>  Details in: HG120622
> <https://amontes.webs.ull.es/otrashtm/HGT2022.htm#ortologico12
0622>
>
> Angel Montesdeoca
```

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Message: #1411
Date: 2022-06-18
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

```
Dear Bernard,

thanks for your "construction" of the reference triangle
... for QL-Cu2 in the case of a Kjp-cubic.
Here an abbreviated version without your explanations:

The Newton line intersects the cubic QL-Cu2 in 3 points,
... which are the centers of the Apollonius circles,
... whose common two points are CSC-partner
... on a perpendicular to QL-L1 through the intersection with
QL-L6.
Each Apollonius circle intersects QL-Cu2 in four points,
... three points of an equilateral triangle
... and the fourth is vertex of the reference triangle.

Best regards Eckart
```

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Message: #1412
Date: 2022-06-21
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
In message 815, I made a short survey of the links between a QL, it's associated Mac Kay or Kjp QL-Cu2 and it's main pivot triangle, the hessian QL-Cu1 and the psi (after Bernard Gibert) or mu (after Benedetto Scimemi) or Cl-S (in EQF) transformation, with the foci of the Steiner inellipse of the triangle as fixed points of the Cl-S QL-2P3.
I gave the references of the 2 articles by B. G. and B. S.
More interesting, I mentionned the links between the directions of the sides of the triangle and of the QL and the asymptotes of Mac Cay and Kjp.
I attach the file again, so you won't need to search it ...
New for me was your beautiful and amazing property that for 5 points on these curves, the circle 5P-s-Cix pass through the point P.
In message 1205, I asked for examples of QA-Mac Cay or Kjp with P in some well-known QA-points (QA-P1, P4 or P41). Did you try these constructions ?
I suppose you use macros and you will get the results more quickly than me ...
Best regards
Bernard

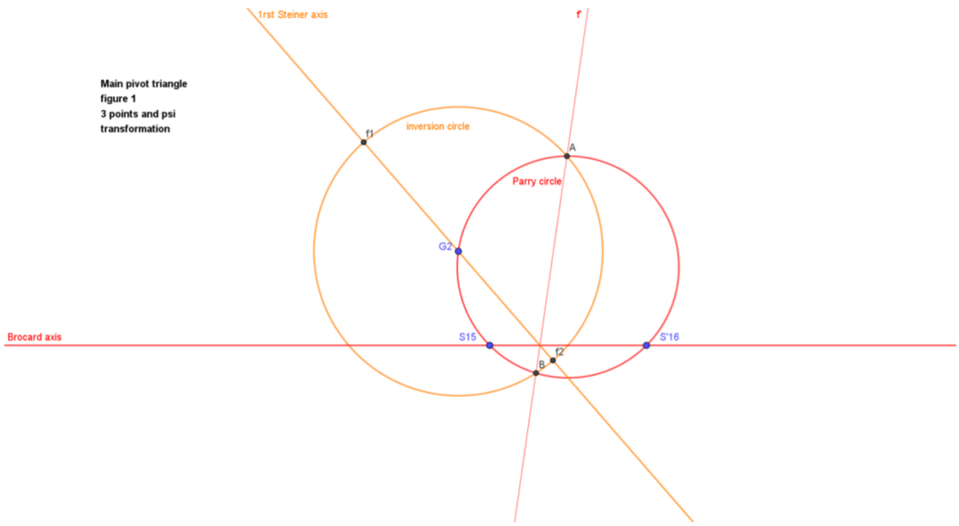
Main pivot triangle of QL-Cu2

A Moebius transformation, named psi by Bernard Gibert (*) (followed by ETC) or mu by Benedetto Scimemi (**) and CSC in EQF associates an axial symmetry and an inversion wrt a circle centered on the axis of symmetry. This transformation can be defined by 3 points, one being the center of the transformation and the 2 others the transformed points. In a triangle, the choosen points will be the centroïd G2 and the isodynamic points S15 and S'16 and the fixed points are the foci of the Steiner inellipse. For a QL, the points will be QL-P1 and the points QL-2P2a and b and the fixed points are QL-2P3a and b. This item shows the links between the triangle and the QL having the same transformation ; it involves the cubics QL-Cu1 and QL-Cu2.

A. 3 points and psi transformation

Having choosen 3 points, one as the center of the transformation G2 and the 2 others as 2 transformed points S15 and S'16, the internal bisector of the angle SGS' is the 1rst Steiner axis, the reflexion of the line SS' in this axis intersects the circle through G, S and S' in 2 points A and B equidistant from G and the inversion circle is the circle with center G through A and B. The 1rst Steiner axis and the inversion circle intersect in 2 points f1 and f2, fixed points of the transformation.

The psi transformation centered in G with fixed points f1 and f2 swaps S and S' and the line through S and S' and the circle through G, S and S'.



(*) Bernard Gibert Orthocorrespondance and Orthopivotal Cubics Forum Geometricorum 2003

(**) Benedetto Scimemi Simple relations regarding the Steiner inellipse of a triangle Forum Geometricorum 2010

Main pivot triangle of QL-Cu2.pdf

B. Triangle and psi transformation

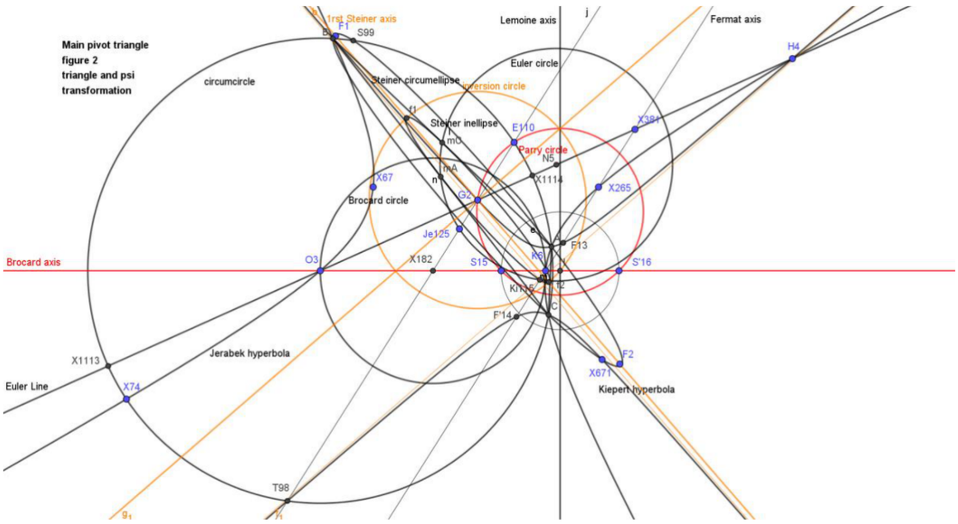
Having the 3 points G2, S15 and S’16, we have immediately the Brocard axis through S and S’ and the Parry circle through G, S and S’ and we find the foci of the Steiner inellipse f1 and f2 by using the preceding construction ; it follows the foci of the Steiner circumellipse F1 and F2, homothetic of f1 and f2 in the homothety (G2, 2). We have also the Lemoine axis as bisector of SS’.

If you have the QL-Cu2 when QL-Cu1 is monocursal, the Brocard Line cuts the cubic QL-Cu2 in the circumcircle O3. If we have only the 3 points G, S and S’, we can use an approximate solution.

For any point O on the Brocard axis, K is the harmonic of O wrt S and S’, ψO is the point E110 and ψK the point X111. The points E110 and X111 are the intersections of the circumcircle and the Parry circle. O is in the position of the circumcenter O3 when the psi transform of the circle with diameter OK or Brocard circle is the circumcircle.

It’s possible to use other alternative constructions. For example, the Stammler hyperbola of the searched triangle ABC is centered in E110 and passes through the in- and excenters of both triangles ABC and GSS’ and through the points O3 and K6. For any point O on the Brocard axis, having the same way K and ψO, O is in the position of O3 when the rectangular hyperbola centered in ψO through the in- and excenters of GSS’ passes through O and K.

It follows the Euler Line with the orthocenter H4 and the center of the Euler circle N5. ψH is JE125, the center of the rectangular Jerabek hyperbola through O, K and H and their reflexions in Je125, respectively X265, X67 and X74. The Jerabek hyperbola cuts the circumcircle in X74 and the 3 vertices on the searched triangle ABC.



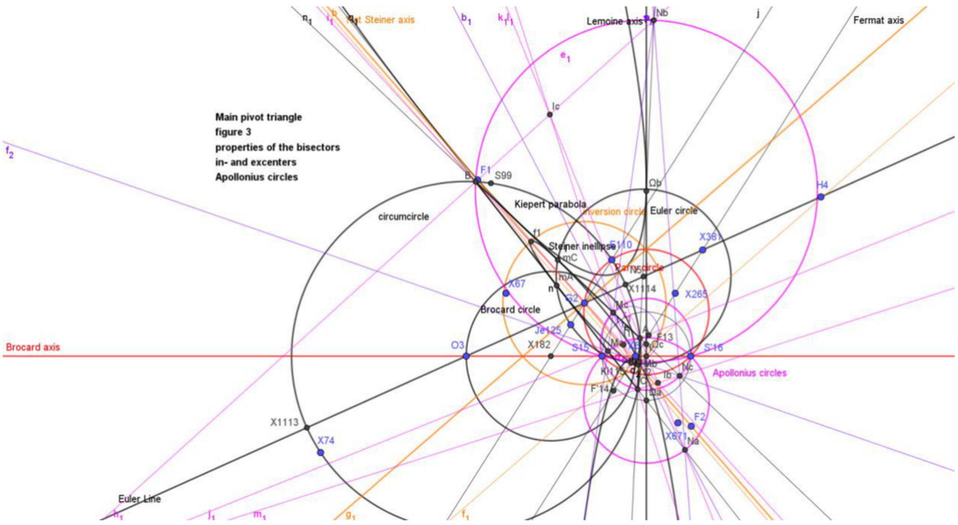
We may use an alternative construction by drawing the reflexion of the Euler Line in the 1rst Steiner axis ; this line is ψEuler Line through E110, G2, Je125 and X182 (middle of OK and center of the Brocard circle) and cuts the circumcircle in a 2[nd] point T98, the Tarry point. The diametral point of T98 on the circumcircle is S99, the Steiner point and the ellipse with foci F1 and F2 through the Steiner point is the Steiner circumellipse, which cuts the circumcircle in 3 other points being the vertices of the searched triangle ABC.

Perhaps a last well known rectangular hyperbola of the triangle ABC : Ki115 is the Kiepert point, middle of H4T98. The Kiepert hyperbola, centered in this point, through the vertices of the triangle ABC, the centroïd G2 and the orthocenter H4, as well as through their reflexions in Ki115, respectively X671 and T98, has it's asymptotes parallel to the Steiner axes and passes through the Fermat points F13 and F'14 and through the Napoleon points Np17 and Np'18, copples of ψpartners.

C. Triangle and properties of the bisectors

Let's draw the 6 bisectors of the triangle ABC :

- they intersect in the in- and excenters of ABC, I, Ia , Ib and Ic
- each of the lines cuts the opposite side of the triangle Ma, Mb and Mc for the internal bisectors, Na, Nb and Nc for the external bisectors ; the middles of MaNa, MbNb and McNc are Ωa, Ωb and Ωc and the circles with diameters MaNa, Mb,Nb and McNc are the Apollonius circles through the opposite vertice of the triangle ABC, which intersect in S15 and S'16. The 6 points Ma, Mb, Mc, Na , Nb and Nc are the vertices of a QL with QL-P1 in Ki115 and QL-P17 in E110, DT is the triangle ABC, the Newton Line is the Lemoine axis, the Steiner Line is the Brocard axis and S15 and S'15 are the Plücker points.
  The CSCdiag centered in E110 swaps the Plücker points S15 and S'16, the Brocard axis and the Parry circle and the circumcircle of the triangle ABC and the Lemoine axis. The parabola with focus Ki 115 and directrix the Brocard axis is QL-Co1, tangent to the 4 lines of this QL and the parabola with focus E110 and directrix the Euler Line is DQL-Co1, tangent to the 3 sides of the triangle ABC, to the Lemoine axis and to the Steiner axes.



The interest of the points I, Ia, Ib and Ic is that they are on the Mac Cay cubic stelloïd.

The interest of the points Ωa, Ωb and ΩC is that they are on the Kjp cubic stelloïd.

D. Properties of the cubic stelloïd, it's hessian and it's cayleyan

1. The stelloïd QL-Cu2

The cubic stelloïd associated to a QL is the curve having as apolar conics the inscribed conics of the QL and as polar conics the rectangular hyperbolas cutting harmonically the sides of the diagonal triangle of the QL and the main axes of all inscribed conics (diagonals of QL's inscribed in the hessian)

It is the locus of the 27 centers of the inscribed cardioïds.

The 3 asymptotes through QL-P1 trisect the angle between the axes of the parabola QL-Co1 and the cardioïd QL-Qu1 ; they cut the curve in 3 points aligned on a parallel to the Newton Line, homothetic in a homothety (QL-P1, 2/3).

This curve has an infinity of pivots triangles, all having QL-2P3a and b as foci of the Steiner inellipse. The main pivot triangle is ABC, which has QL-P1 as centroïd G2, QL-2P3a and b as foci of the Steiner inellipse and QL-2P2a and b as isodynamic points S15 and S'16.

Wrt this main pivot triangle, QL-Cu2 is either a Mac Cay cubic stelloïd, if the Newton Line is the Brocard axis, or a Kjp cubic stelloïd, if the Newton Line is the Lemoine axis.

The Mac Cay cubic stelloïd is K003 in Bernard Gibert's catalogue ; it is pK(K6,O3), an isogonal pK with pivot O3 and isopivot H4, which passes through the in- and excenters of the triangle ABC.

The Kjp cubic stelloïd is K024 in Bernard Gibert's catalogue ; it is nK0+(K6,K6), a non-pivotal isogonal cubic with root in K6, which passes through the centers of the Apollonius circles of the triangle ABC.

2. The hessian QL-Cu1

The hessian of the cubic is the locus of the points for which the rectangular hyperbola is degenerated in 2 orthogonal lines. 2 conjugate points of the hessian are the foci of an inscribed conic and a pair of vertices of an infinity of QL's inscribed in the hessian.

In each point of the hessian pass 2 orthogonal lines, which form the degenerated polar conic of it's conjugate wrt the cubic stelloïd. These 2 lines cut the stelloïd in the contact points of the tangent to the stelloïd from the conjugate point on the hessian.

For example, the Miquel point QL-P1 is the conjugate of the infinity point, the 2 Steiner axes are the degenerated rectangular hyperbola of the infinity point wrt the stelloïd and cut the stelloïd in points where the tangent is parallel to the Newton Line.

The 2 perpendicular lines in each point cut the axis of the parabola (main axis of an inscribed conic) in 2 points , which are harmonic conjugates wrt the Miquel point and the infinity point and therefore symmetric wrt the Miquel point QL-P1, which makes easy to find the second one if we know the first.

Main pivot triangle of QL-Cu2.pdf

The 2 points QL-2P3a and b, intersections of the inversion circle and the 1rst Steiner axis are invariant in the Clawson-Schmidt transformation QL-Tf1, which is the transformation ψ of the main pivot triangle of QL-Cu2.

The hessian of the Mac Cay cubic stelloïd is K048 in Bernard Gibert's catalogue ; it is monocursal and passes through the points X1113 and X1114, on the Euler Line of the triangle ABC.

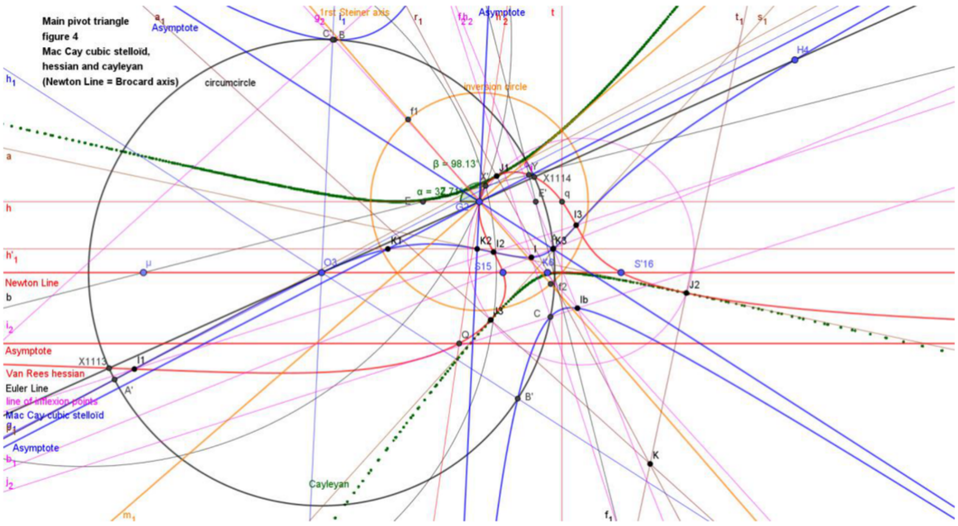
The hessian of the Kjp cubic stelloïd is K193 in Bernard Gibert's catalogue ; it is bicursal.

3. The cayleyan

The cayleyan is the enveloppe of all the lines forming these degenerated rectangular hyperbolas or the enveloppe of the lines through 2 conjugate points (main axis of an inscribed conic). The contact point is the harmonic conjugate of the 3rd intersection point of a line trough 2 conjugates.

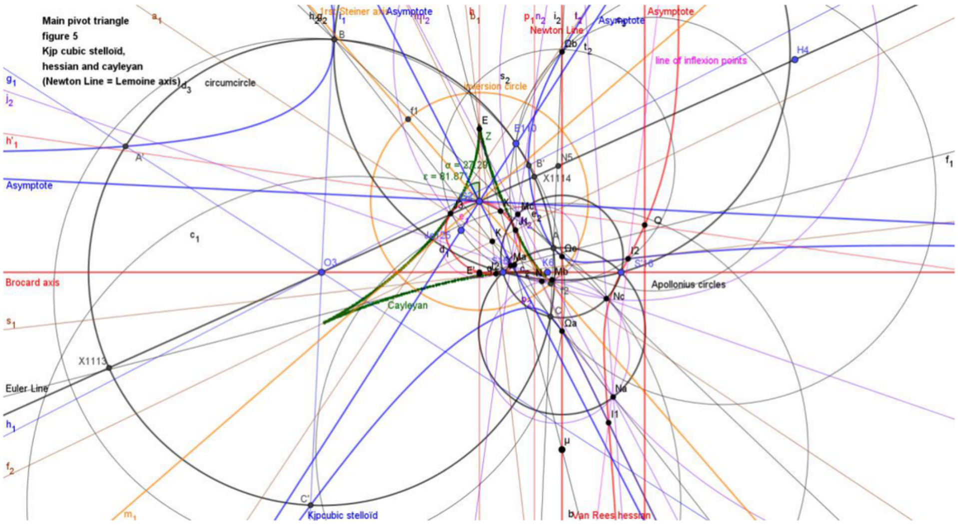
This makes easy to draw the curve : the line joining 2 conjugate points cuts the axis of the parabola in a point ; the perpendicular to this line in the reflection of this point in QL-P1 gives the 3rd point of intersection between the line and the hessian. Both lines form the degenerated hyperbola of the tangential point of the 2 conjugate points, which is the conjugate of the 3rd intersection.

For example, the axis of the parabola is tangent to the cayleyan in a point , which is the harmonic conjugate of the infinity point, the reflection of the 2nd intersection point in QL-P1 and, in the unicursal curve, the Newton Line QL-L1 is tangent to the cayleyan in a point, which is the harmonic conjugate of the infinity point, the middle of QL-2P2a and b.



Last, the cayleyan and the hessian are tangents in 3 points on a circle trough QL-P1 and the conjugates of these 3 points are also their tangentials, the 3 inflexion points and intersection points of the stelloïd and the hessian, which lie on a line conjugate of this circle. The 6 points are therefore the vertices of a QL inscribed in the hessian.

The 3 common tangents to the hessian and the cayleyan pass through the corresponding inflexion points and form the DT of this QL. The normals to the hessian and the cayleyan in the 3 contact points intersect in a point K.



E. Properties of the QL's inscribed in QL-Cu1

The fundamental property of the QL's inscribed in a given curve QL-Cu1 is that the sum of the directions of the 4 lines wrt a fixed direction is constant given by the curves QL-Cu1 and Cu2.

If St is the 1rst Steiner axis of the psi transformation, Si are the directions of the sides of the triangle ABC and Di are the directions of the 4 lines, Br and Le the directions of the Brocard and Lemoine axes, we have Σ (St,Si) = (St,Le) = π/2 + (St,Br) and Σ(St,Di) = 2*Σ (St,Di) = 2*(St,Le) = 2*(St,Br).

The sides of the Morley triangles of ABC have as direction wrt St 1/3*Σ(St,Di) m° π/3 and the asymptotes of the Mac Cay cubic are orthogonal to these lines whereas the asymptotes of the Kjp cubic are parallel to these lines ; they are rotated from one to the other in an angle π/6.

The axes of the deltoïds QL-Qu2 of the QL's inscribed in the corresponding hessians are parallel to these asymptotes.

Message: #1413
Date: 2022-06-26
From: eckart_schmidt@t-online.de
Subject: nP-n-2Px

```
Dear all,

in #689 Chris describes Möbius transformations
... nP-n-Tf1, their centers nP-n-P5 and swapped points.
Starting with 4P-n-Tf1 = QA-Tf4,
... center 4P-n-P5 = QA-P4,
... swapping Pi with 3P-n-P5 of PjPkPl,
... with 3P-n-P5 = circumcenter of PjPkPl.
This can be generalized for n > 4 (see #689).
For a nP and any point P we can consider
... the (n-1)P-n-Tf1(P) for the (n-1)P
    omitting one vertex of nP.
There exist two points P so that these n points coincide,
... these two points nP-n-2Px are nP-n-Tf1-partner.
nP-n-2Px are defined for n > 4,
... for n = 5 they are 5P-Tf8- and csc3-partner (see old#3679),
... elements on the 5P-quartic (see #923)
    and the cubic of its 5P-s-Tf8-image,
... center symmetric elements of the orthogonal hyperbola in
    old#3579.

What about 6P-n-2Px?

Best regards Eckart
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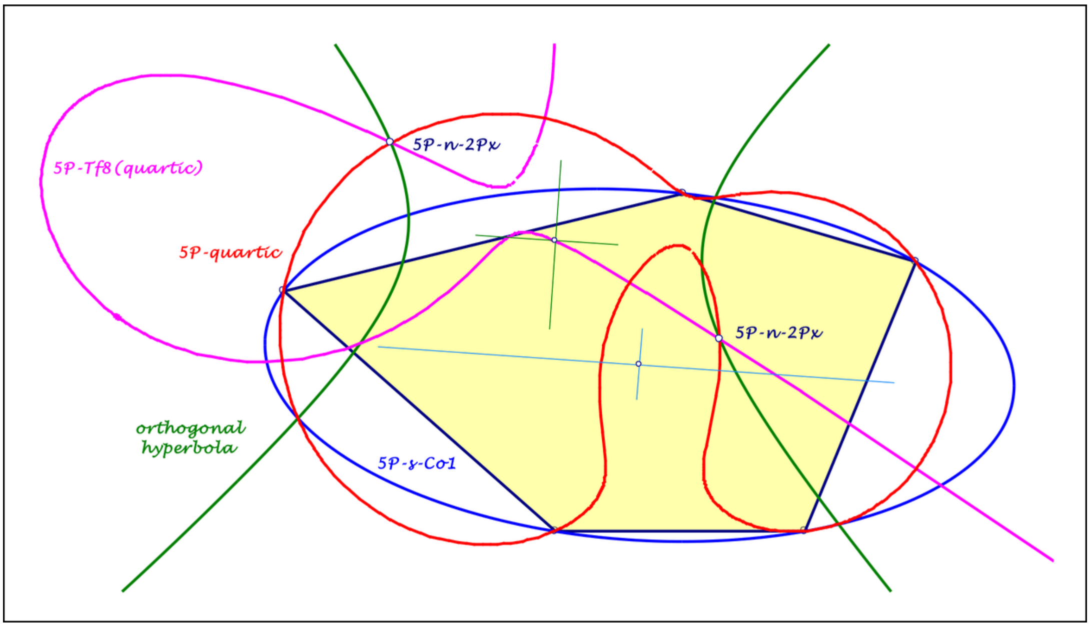

2022-06-27.pdf

Message: #1414
Date: 2022-06-27
From: eckart_schmidt@t-online.de
Subject: 6P-s-Px

```
Dear all,

6P-s-Tf1 maps the plane to the circle 6P-s-Ci1,
...here another aspect of this circle:
There is a special point 6P-s-Px,
... whose 5P-s-Tf7-circles wrt the 6 component 5P
... coincide in 6P-s-Ci1.
What about this point 6P-s-Px?

Best regards Eckart
```

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Message: #1415
Date: 2022-06-27
From: van10hoven@gmail.com
Subject: Re: 6P-s-Px

```
Dear Eckart,

It's a long time ago I looked to this.
In the properties of 6P-s-Tf1 it says:
• When we consider a circular cubic Cux = 7P-s-Cu1
(P1,P2,P3,P4,P5,P6,Px), where Px is some point on 6P-s-Ci1, then
6P-s-Tf1(X) = 6P-s-Tf1(Px) for all points X on Cux. Hence we
have a pencil of circular cubics, all passing through 6P-s-P2
and each corresponding with its own unique point on circle
6P-s-Ci1. See Ref-66, QPG-message #653 and #657.

Therefore this point 6P-s-Px should be 6P-s-P2, being the
Cayley-Bacharach Point of the 6P. It is the only common point of
the pencil of cubics whose whole set of points yield a
transformational point on 6P-s-Ci1.

Best regards,
Chris
```

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Message: #1416
Date: 2022-06-28
From: eckart_schmidt@t-online.de
Subject: Re: 6P-s-Px

```
Dear Chris,
thanks for interest and identification of 6P-s-Px = 6P-s-P2.
Best regards Eckart
```

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Message: #1417
Date: 2022-06-28
From: eckart_schmidt@t-online.de
Subject: Circular 6P-circumcubics with knot

```
Dear all,

if we look for circular 6P-circumcubics with knot
... we get 6 knots with the property,
... that 5P-s-Tf6(K) = K holds
... for the six 5P components of 6P.
As well as:
if we look for 8P-circumcubics with knot
... we get 8 knots with the property,
... that 7P-s-Tf1(K) = K holds
... for the eight 7P components of 8P.

Best regards Eckart
```

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Message: #1418
Date: 2022-06-28
From: van10hoven@gmail.com
Subject: Re: 7P-s-P1

```
Dear Eckart,
I changed EPG according to your remark about 7P-s-P1.
Best regards,
Chris
```

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Message: #1419
Date: 2022-06-28
From: van10hoven@gmail.com
Subject: Re: 5G-s-P2

```
Dear Eckart,
I changed EPG according to your remarks about 5G-s-P2, 5G-s-P3,
5G-s-P4, 5G-s-P5.
Also I added 5G-s-P8 and 5G-s-P9.
Thanks for all your comments.
Best regards,
Chris
```

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Message: #1420
Date: 2022-06-28
From: van10hoven@gmail.com
Subject: Re: 6P-s-Px

```
Dear Eckart, dear Bernard,
Finally I added nP-n-Tf1 (nP-Moebius Conjugate) and nP-n-P5
(nP-Moebius Center) to EPG.
Also I added nP-n-Tf2 and nL-n-Tf5 (nP-Linear Pole/nL-Linear
Polar) to EPG.
These last items I found a long time ago, but never published.
Best regards,
Chris
```

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Message: #1421
Date: 2022-06-28
From: eckart_schmidt@t-online.de
Subject: Fixed points of 6P-s-Tf1

```
Dear all,

there are three points A, B, C on 6P-s-Ci1,
... each with six concyclic 5P-s-Tf3-images
... wrt the 5P-components of the 6P,
... A, B, C are the fixed points of 6P-s-Tf1.

What about the circumcubic of 6P plus A, B, C?

Best regards Eckart
```

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Message: #1422
Date: 2022-06-29
From: eckart_schmidt@t-online.de
Subject: Re: 6P-s-Px

```
Dear Chris,

5P-n-Tf1 is already used as csc3
... since old#3660, old#3679 (Aug. 2019).
5P-n-Tf1 maps the 5P-quartic (not in EPG) to itself,
...  locus of the foci of focal circular circumscribed cubics of
5P (not in EPG).

Best regards Eckart
```

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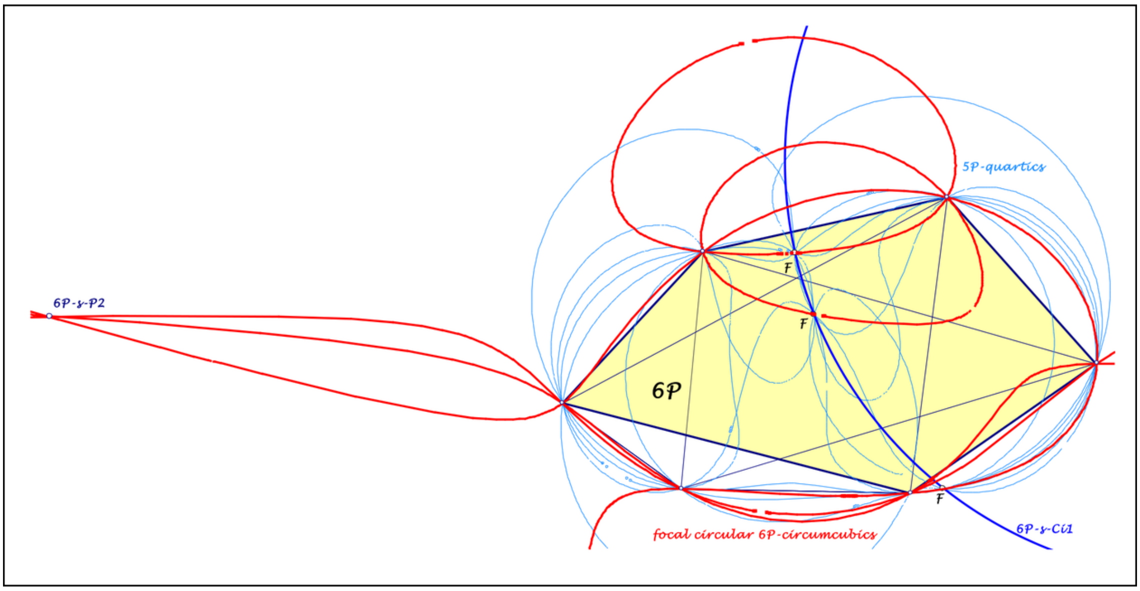
Message: #1423
Date: 2022-06-29
From: eckart_schmidt@t-online.de
Subject: Re: Fixed points of 6P-s-Tf1

```
Dear all,

I have to correct my last message:
There are up to three fixed points of 6P-s-Tf1 on 6P-s-Ci1.

Perhaps new:
These fixed points are the intersections
... of the 5P-quartics of the 5P-components,
... they are the only foci
... of focal circular circumcubics of 6P
... with common point 6P-s-P2.

Best regards Eckart
```



2022-06-29.pdf

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Message: #1424
Date: 2022-06-30
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
I regret that you didn't answer my last message (lack of interest for this Moebius transformation applied to the QL and to the main pivot triangle of the stelloïd QL-Cu2 associated to the QL ?).
You didn't answer either my questionning about QA Mac Cay or Kjp with P in particular QA-points (generation of a whole family of such curves).
Anyhow, I realised 2 QA curves with P in QA-P1 and QA-P4 ; I checked that in the last case, the Moebius transformation is not QA-Tf4.
Best regards
Bernard

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Message: #1425
Date: 2022-07-01
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Bernard,

excuse, not having answered your last message,
... there was no time, to study your previous paper once more.

I have drawn QA-circumcubics (2) for P = QA-P1 and QA-P4,
... but found no new aspects.
What Moebius transformation do you mean in #1424 in this context?

I think the main result of my #1392 is the special 6P-circumcubic (4).

Finally:
Forget #1295, there are mistakes wrt 5P-circumcubics (3),
.. please replace #1295 by #1392, sorry!

Best regards Eckart

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Message: #1426
Date: 2022-07-02
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
The Moebius transformation I mean in 1424 is precisely the one centered in P (centroïd of ABC) with fixed points in the foci of the Steiner inellipse of ABC !
If P = QA-P4, the foci are not the same as the the foci of QA-Tf4 ...
Best regards
Bernard
PS Another interesting property of these foci of the Steiner inellipse of ABC is that there are an infinity of pivot triangles on the stelloïd cubic all having the same points as foci of their Steiner inellipse. For any point X on the curve with center P, you may find the 2 other vertices of the correspondind pivot triangle by drawing the ellipse with the 2 foci as foci through the middle mx of PX (and it's reflexion m'x in P) and the 2 tangents in X to the ellipse, which intersect the tangent in m'x to the ellipse in the 2 other vertices of the searched triangle (also naturally on the same curve) ...

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Message: #1427
Date: 2022-07-03
From: eckart_schmidt@t-online.de
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear all,

is the following property evident or well known?
Any 6-Point on a cubic K003 or K024
... has the same 6P-s-Px of #1392,
... which is the centroid of the reference triangle.

Best regards Eckart

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Message: #1428
Date: 2022-07-03
From: bernard.keizer@gmail.com
Subject: Re: Circumcubics K003/K024 for 4P/5P/6P

Dear Eckart,
I'm lost with your definitions !
The main property is that any 5 points on a SC (cubic stelloïd Mac Kay K003 or Kjp K024) define a circle 5P-s-Cix passing through the center P of the SC (centroïd of the reference triangle ABC).
So the rest follows :

* 4 points + a point P define a SC centered in P.
* 5P + a point P on it's 5P-s-Cix define a SC with center P.
* 6P define a SC centered in P, intersection of the 6 5P-s-Cix.
(any 6P defines a unique SC)

Best regards
Bernard

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Message: #1429
Date: 2022-07-06
From: bernard.keizer@gmail.com
Subject: New QL-point and cubic

```
Dear Eckart,
As you didn't answer my last message, I consider the mentionned
properties as correct (in fact, I'm rather surprised by you
apparent lack of interest, as this was your item).
I applied it to the 6 points of a QL.
It worked and I found 7 points, the 3 qa-p4 of the 3 QA's as
Bar((QA-P4,QA-P1),(-1,4)) and the 4 triple qa-p4 of the 12
remaining degenerated QA's, with the same definition.
For a QA formed by a point and 3 other aligned points, the point
is the QA-P4 and the definition gives qa-p4 ; it happens that
for the QL, the 3 QA's formed by 3 aligned points and another of
the 3 remaining points have the same qa-point : there are
therefore 4 such triple points, one on each QL line.
The 6 circles intersect in a common point P, which is the center
of a unique SC through the 6 QL vertices.
This new circumcubic intersect QL-Cu1 in 3 points forming with
the 6 vertices of the QL a CB system.
I don't know if the point P and the 3 other points are already
known QL points.
Best regards
Bernard
```

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Message: #1430
Date: 2022-07-06
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

```
Dear Eckart,
The 4 triple qa-p4 on the 4 QL lines are obviously the
barycenters of the 3 vertices on this line.
Their barycenter is therefore QL-P12.
Best regards
Bernard
```

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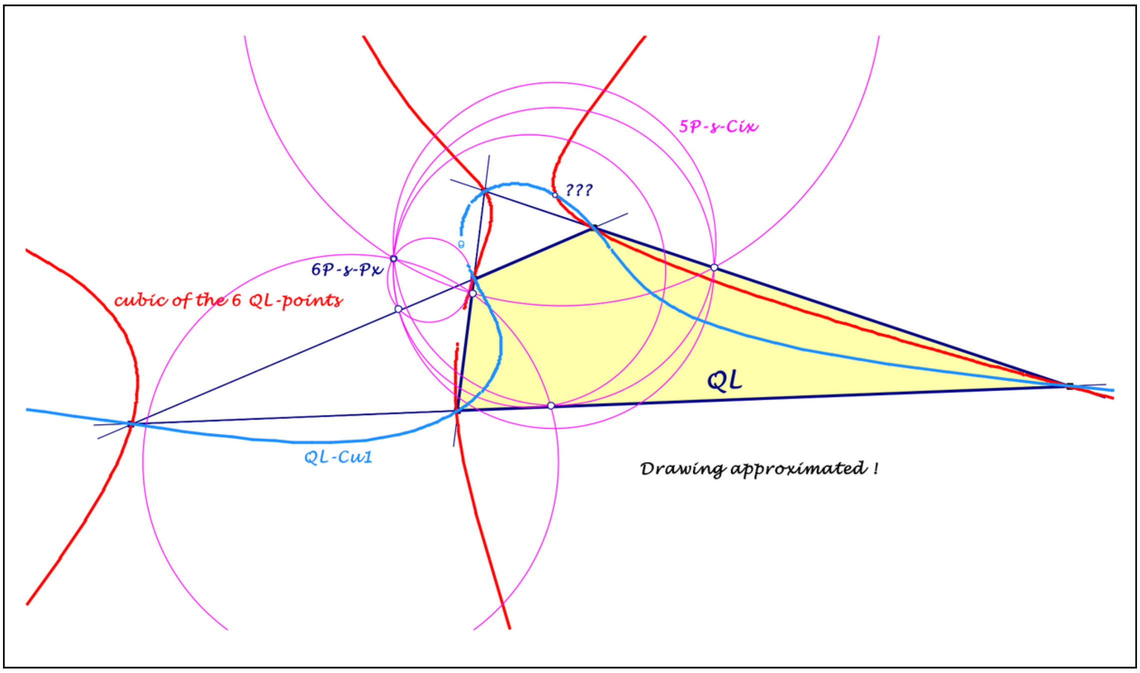
Message: #1431
Date: 2022-07-07
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard,

that is an interesting application wrt the 6 QL-points!
Attached an approximate drawing of your observations.
But your discussion of the QA-P4 I couldn't understand,
... and my drawing shows only one and not three
... intersections of QL-Cu1 and the new QL-circumcubic.
Sorry, up to now I cannot give properties of the new QL-objects.

Best regards Eckart
```



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Message: #1432
Date: 2022-07-07
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard.

just observed:
For the 6 points of a QL
... 6P-s-Ci1 = QL-Ci3,
... 6P-s-Tf1(QL-Cu1) = QL-P1,
... 6P-n-P5 = QL-P17.

Best regards Eckart
```

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Message: #1433
Date: 2022-07-08
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard,

further remarks for the 6 points of a QL:
Any circular cubic through the 6 QL-points
... is CSC-invariant,
... bears QL-P1 = 6P-s-P2,
... mapped by 6P-s-Tf1 to a fixed point on QL-Ci3 = 6P-s-Ci1
... with QL-P4 = 6P-s-P1.

What about 6P-s-Px?

Best regards Eckart
```

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Message: #1434
Date: 2022-07-08
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

Dear Eckart,
Thanks for answer, interest, observations and drawing !
Particularly interesting, the Moebius transformation centered in QL-P17 swaps QL vertices and Miquel points (other than QL-P1) of the 3 QA's of the QL (these 6 points being the 5P-n-P5).
It is the transformation I named CSCdiag in old times (CSC of the QLdiag formed by the DT of the QL and the Newton Line), which swaps the DT vertices and the middles of the 3 segments joining 2 CSC partners ; it also swaps in particular the 2 Plücker points.
I didn't identify 6P-s-Px, center of the unique circumSC of the 6 QL vertices.
Best regards
Bernard
PS These properties are QL properties and could or should appear in EQF (good example of Chris recent addition)

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Message: #1435
Date: 2022-07-08
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

Dear Bernard,

the Möbius transformation 6P-n-Tf1 for the 6P of the QL-points
... is centered in QL-P17
... and swaps QL-P16 and the intersection of QL-L1 and QL-L6,
... example: 6P-n-Tf1(QL-L1) = QL-Ci1.

Best regards Eckart

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Message: #1436
Date: 2022-07-08
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

Dear Bernard,

6P-n-Tf1 for the 6P of the QL-vertices
... is CSC for the QL = QL-Tr1 plus QL-L1,
... see your cited observation under QL-P17.

Best regards Eckart

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Message: #1437
Date: 2022-07-08
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

Dear Eckart,
This transformation 6P-n-Tf1 swaps the Plücker points (this
property is not in EQF).
But nevermind.
What about the 7P-n-Tf1 of the 6 vertices of a QL and the point
QL-P1 ?
I think it is centered in the point CSC(QL-P17) = CSCdiag(QL-P1)
and swaps QL-P1 and QL-P17 and again the QL vertices and the
Miquel points (other than QL-P1) of the 3 QAs of the QL.
This time, QL-P17 and these 6 Miquel points are the 6P-n-P5 of
the 7P.
Do you agree with this ?
Best regards
Bernard
PS If this is true, it is again a good example of Chris recent
addition ...

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Message: #1438
Date: 2022-07-09
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

```
Dear Eckart,
Here we have worked with the 3 QAs of a QL and found as
interesting points QL-P1, QL-P17, CSC(QL-P17) and the 6 Miquel
points (other than QL-P1) of these 3 QAs.
I suppose it is also possible to work with the 3 QLs of a QA and
define the same way the 3 Miquel points and the 3 QL-P17.
We would find a 7P-n-Tf1 for the 4 vertices of a QA and the 3
vertices of it's DT, swapping in particular the 3 DT vertices
and the 3 QL-P17 ...
Best regards
Bernard
```

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Message: #1439
Date: 2022-07-09
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard,

very interesting your properties of 6P-n-Tf1 for the 6P of
QL-vertices:
6P-n-Tf1 as Möbius transformation, centered in QL-P17,
... swapping the Plücker points QL-2P1
... or: swapping QL-vertices and Miquel points.

Wrt the 7P of the 6 QL-points plus QL-P1:
I can confirm that 7P-n-Tf1 swaps QL-P1 and QL-P17,
... but not the other properties.

Best regards Eckart
```

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Message: #1440
Date: 2022-07-10
From: eckart_schmidt@t-online.de
Subject: Re. New QL-point and cubic

Dear Bernard,

I can confirm your observation:
"We would find a 7P-n-Tf1 for the 4 vertices of a QA and the 3
vertices of it's DT,
swapping in particular the 3 DT vertices and the 3 QL-P17 ..."

Best regards Eckart

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Message: #1441
Date: 2022-07-10
From: bernard.keizer@gmail.com
Subject: Re: Re. New QL-point and cubic

Dear Eckart,
Thanks for this confirmation !
Which is in this case this new QA-point or 7P-n-P5 ?
Best regards
Bernard

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Message: #1442
Date: 2022-07-10
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

Dear Eckart,
I'm sad that you don't confirm these observations !
May be I was too elliptic.
The center of this transformation is CSC(QL-P17) or
CSCdiag(QL-P1), which swaps in fact QL-P1 and QL-P17 ; you seem
to agree with that.
But this transformation swaps also the QL vertices and the
Miquel points other than QL-P1 of the 3 QAs of the QC.
For a copple of vertices CSCpartners, there are 2 such Miquel
points other than QL-P1 ; one is the 5P-n-P5 of the 5P formed by
5/6 vertices.
But the other one is the 6P-n-P5 of the 6P formed by the same
5/6 points and QL-P1 ; it swaps in particular QL-P1 and the 1rst
one !
The 2 points are again respectively 6P-n-P5 and 5P-n-P5 for the
5/6 points obtained by replacing the omitted point by it's CSC
partner (the QA is the same).
Best regards
Bernard
PS In older times, I named this 7P-n-Tf1 CSCter, which is in
fact CSC*CSCdiag ; your wonderful quartic QL-Qu3 is invariant in
the 3 transformations !

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Message: #1443
Date: 2022-07-11
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard,

wrt the 7P of the QL-vertices plus QL-P1 in #1437 my
observation:
7P-n-Tf1 is not centered in CSC(QL-P1) = CSCdiag(QL-P1),
but swaps QL-P1 and QL-P17, further the Plücker points,
but swaps not QL-vertices and Miquel points,
the discussed Miquel points are not 6P-n-P5.

Best regards Eckart
```

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Message: #1444
Date: 2022-07-11
From: eckart_schmidt@t-online.de
Subject: Re: New QL-point and cubic

```
Dear Bernard,

excuse, I forgot in my last message:
For the 7P of the 6 QL-points and QL-P1 the transformation
7P-n-Tf1
... is centered in 7P-n-P5 = Miquel point for the quadrigon
... of QL-P1, QL-P17 and the Plücker points QL-2P1 as opposite
points.

Best regards Eckart
```

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Message: #1445
Date: 2022-07-11
From: bernard.keizer@gmail.com
Subject: Re: Re. New QL-point and cubic

```
Sorry, I withdraw or change my question :
the point is perfectly known from the 3 copples of partners
(more than enough) !
But is it a QA well-known point or a new point ?
Best regards
Bernard
```

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Message: #1446
Date: 2022-07-11
From: bernard.keizer@gmail.com
Subject: Re: New QL-point and cubic

```
Dear Eckart,
Apologise for my mistake and many thanks for your patience and
this ultime precision, which makes and end to this interesting
item !
Best regards
Bernard
```

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Message: #1447
Date: 2022-07-13
From: danjohnhardisky@yahoo.com
Subject: 7 pts on the line joining the centroid and diag. Intersection

---

```
I have discovered 7 collinear points in rational ratios
for quadrilaterals similar to Euler's line on triangles.

E,V,I,R,S,T,F

These are on the line joining the centroid and the
intersection of the diagonals as shown.

Drawn with GeoGebra. https://drive.google.com/file/d/1YSUAlLeBOC⌋
ZGv-Ex5_CyZ4qBP4eYCFd1/view?usp=drivesdk
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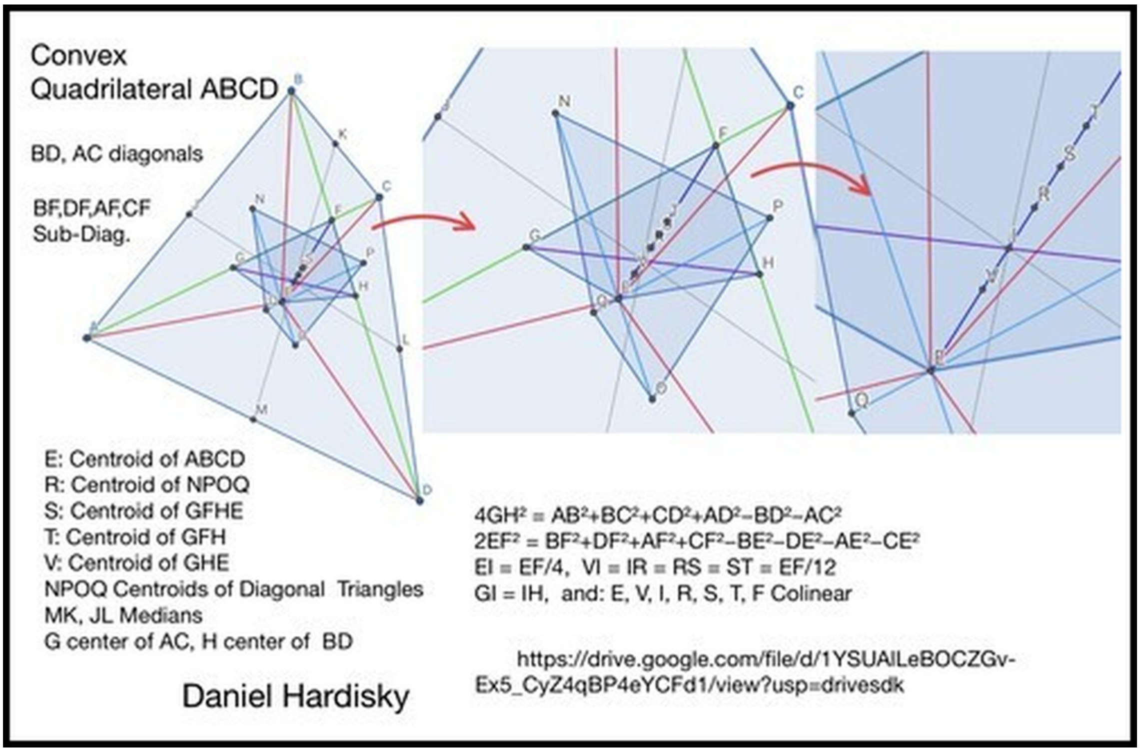

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Message: #1448
Date: 2022-07-15
From: danjohnhardisky@yahoo.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

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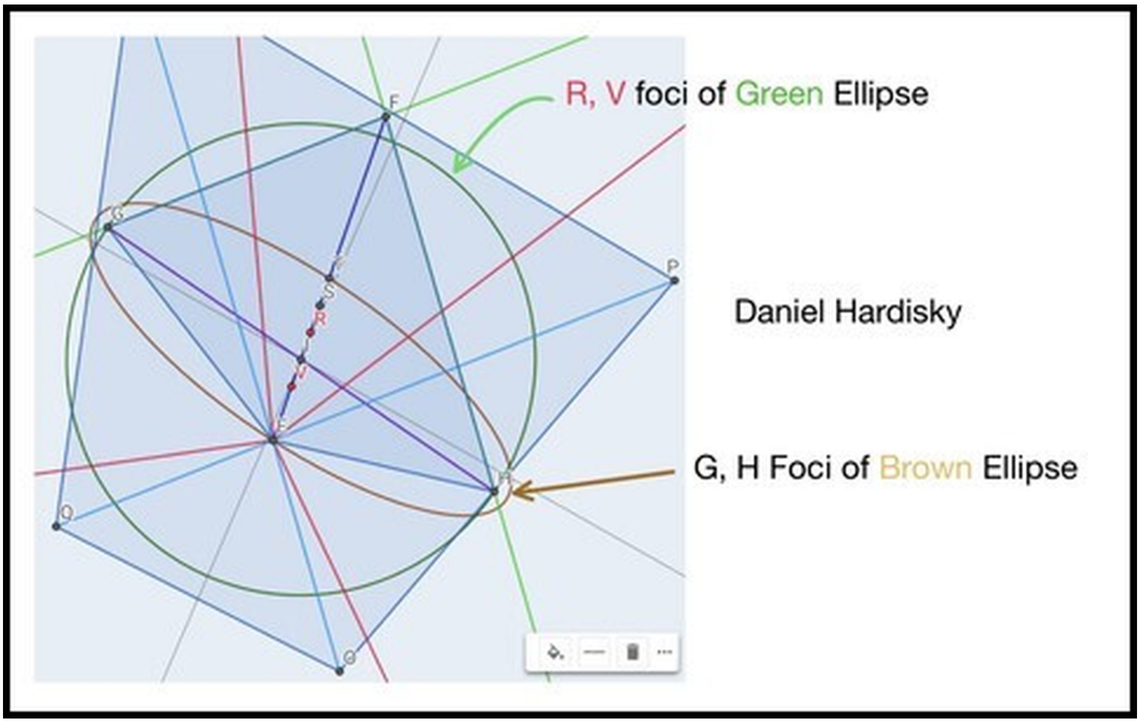
```
There are two ellipses connecting these points
```



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Message: #1449
Date: 2022-07-15
From: van10hoven@gmail.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

Dear Daniel,

Thank you for your contribution!
The line joining the centroid and the intersection of the diagonals is QG-L3: The QG-Centroids Line.
QG-L3 is the QG-version of the 3 Centroid Lines in Quadri Figures:
1. for a Quadrangle (reference figure of 4 random points) we have the Centroids Line QA-L3,
2. for a Quadrilateral (reference figure of 4 random lines) we have a Centroids Line QL-L8 and
3. for a Quadrigon (reference figure of 4 consecutive points and 4 consecutive connecting lines) we have the Centroids Line QG-L3.
In EQF we have this notation for your points
E = QA-P1: Centroid of the reference quadrangle
R = QG-P4: 1st QG-Quasi Centroid
T = QG-P5.QG-P11 ^ QG-P6.QG-P9

Other EQF-points on QG-L3 are:
QG-P15: Kirikami Center, center of the parallelogram defined by the lines parallel to the diagonals through the vertices of the quadrigon not on the diagonal. It is also the reflection of QG-P1 about QA-P1.
QG-P8: 2nd QG-Quasi Centroid

Your points V,I,S,T do not occur in EQF.

Best regards,

Chris

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Message: #1450
Date: 2022-07-15
From: danjohnhardisky@yahoo.com
Subject: Re: [Quadri-and-Poly-Geometry] 7 pts on the line joining the centroid

---

```
Chris
  So, have I added anything new???

> Dear Daniel,
>
> Thank you for your contribution!
> The line joining the centroid and the intersection of the diagonals is QG-L3: The QG-Centroids Line.
> QG-L3 is the QG-version of the 3 Centroid Lines in Quadri Figures:
> 1. for a Quadrangle (reference figure of 4 random points) we have the Centroids Line QA-L3,
> 2. for a Quadrilateral (reference figure of 4 random lines) we have a Centroids Line QL-L8 and
> 3. for a Quadrigon (reference figure of 4 consecutive points and 4 consecutive connecting lines) we have the Centroids Line QG-L3.
>
> In EQF we have this notation for your points
> E = QA-P1: Centroid of the reference quadrangle
> R = QG-P4: 1st QG-Quasi Centroid
> T = QG-P5. QG-P11 ^ QG-P6. QG-P9
>
> Other EQF-points on QG-L3 are:
> QG-P15: Kirikami Center, center of the parallelogram defined by the lines parallel to the diagonals through the vertices of the quadrigon not on the diagonal. It is also the reflection of QG-P1 about QA-P1.
> QG-P8: 2nd QG-Quasi Centroid
>
> Your points V,I,S,T do not occur in EQF.
>
> Best regards,
>
> Chris
>
```

Message: #1451
Date: 2022-07-16
From: van10hoven@gmail.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

Dear Daniel,

I withdraw my interpretation of your points E, R, T.
I think there might be some misinterpretations in your description.
For me the centroid of a 4-point-figure is as described in QA-P1. Do you agree?
For me the diagonal crosspoint of a cyclic 4-point-figure is as described in QG-P1. Do you agree?

You state:
E=Centroid ABCD; in your drawing E = the diagonal crosspoint of ABCD
R=Centroid NPOQ; in your drawing R is not the centroid of NPOQ
S=Centroid GFHE; in your drawing S is not the centroid of GFHE
T=Centroid GFH; ok
V=Centroid GHE; ok
NPOQ=Centroids Diagonal Triangles; in EQF we call them the Component Triangles of the Quadrangle ABCD.
Could you reconsider/restate the points in your drawing?

Best regards,
Chris

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Message: #1452
Date: 2022-07-16
From: danjohnhardisky@yahoo.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

On Sat, Jul 16, 2022 at 04:27 AM, Chris wrote:

> *Dear Daniel,
>
> *I withdraw my interpretation of your points E, R, T.*
> *I think there might be some misinterpretations in your description.*
> *For me the centroid of a 4-point-figure is as described in QA-P1. Do you agree?*
> *For me the diagonal crosspoint of a cyclic 4-point-figure is as described
> in QG-P1. Do you agree?*
> *You state:*
> *E=Centroid ABCD; in your drawing E = the diagonal crosspoint of ABCD*
> *R=Centroid NPOQ; in your drawing R is not the centroid of NPOQ*
> *S=Centroid GFHE; in your drawing S is not the centroid of GFHE*
> *T=Centroid GFH; ok*
> *V=Centroid GHE; ok*
> *NPOQ=Centroids Diagonal Triangles; in EQF we call them the Component
> Triangles of the Quadrangle ABCD.*
> *Could you reconsider/restate the points in your drawing?*
>
> Best regards,
> Chris

Chris
GeoGebra says E is the centroid of the Quadtilateral ABCD and intersection of centroids of diagonal triangles .
GeoGebra says NPOQ has centroid R
GeoGebra says GHFE has Centroid S
why do you not agree?
if this is wrong then GoGebra is wrong. It works for all convex quadrilaterals.

Message: #1453
Date: 2022-07-16
From: danjohnhardisky@yahoo.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

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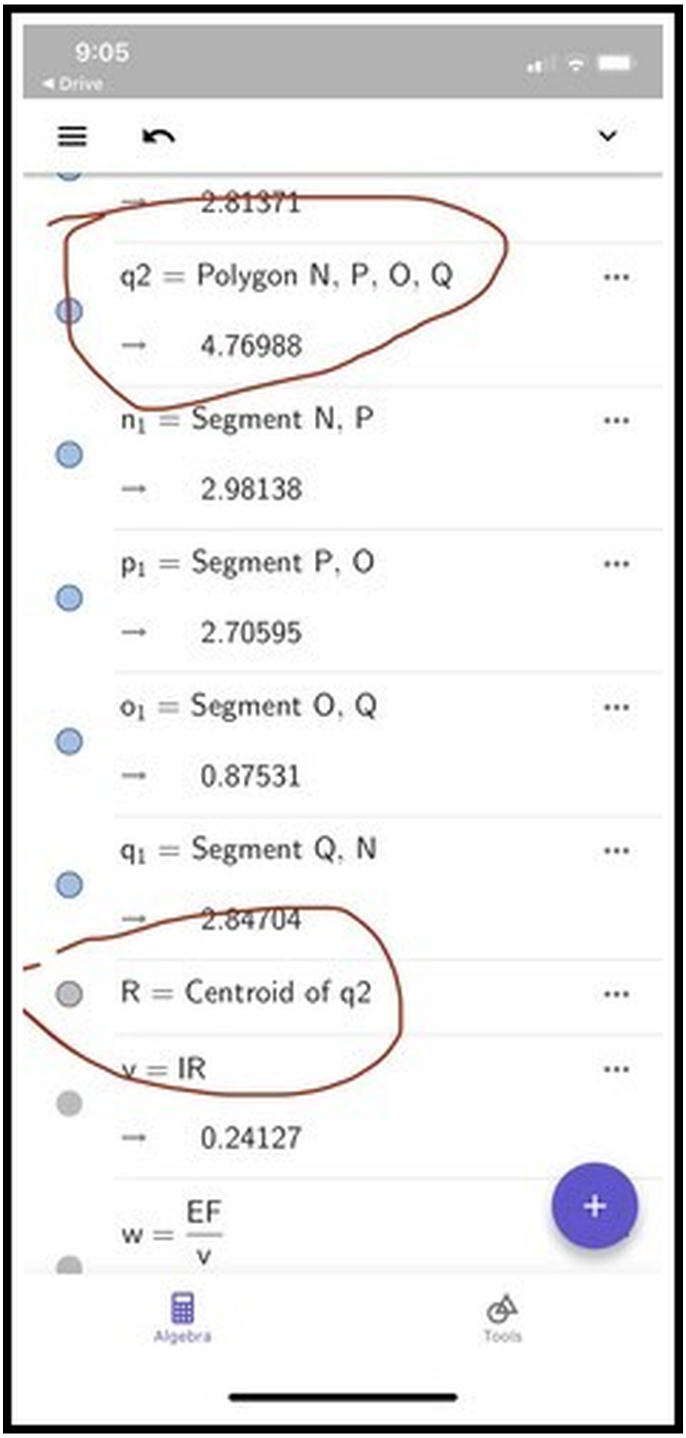

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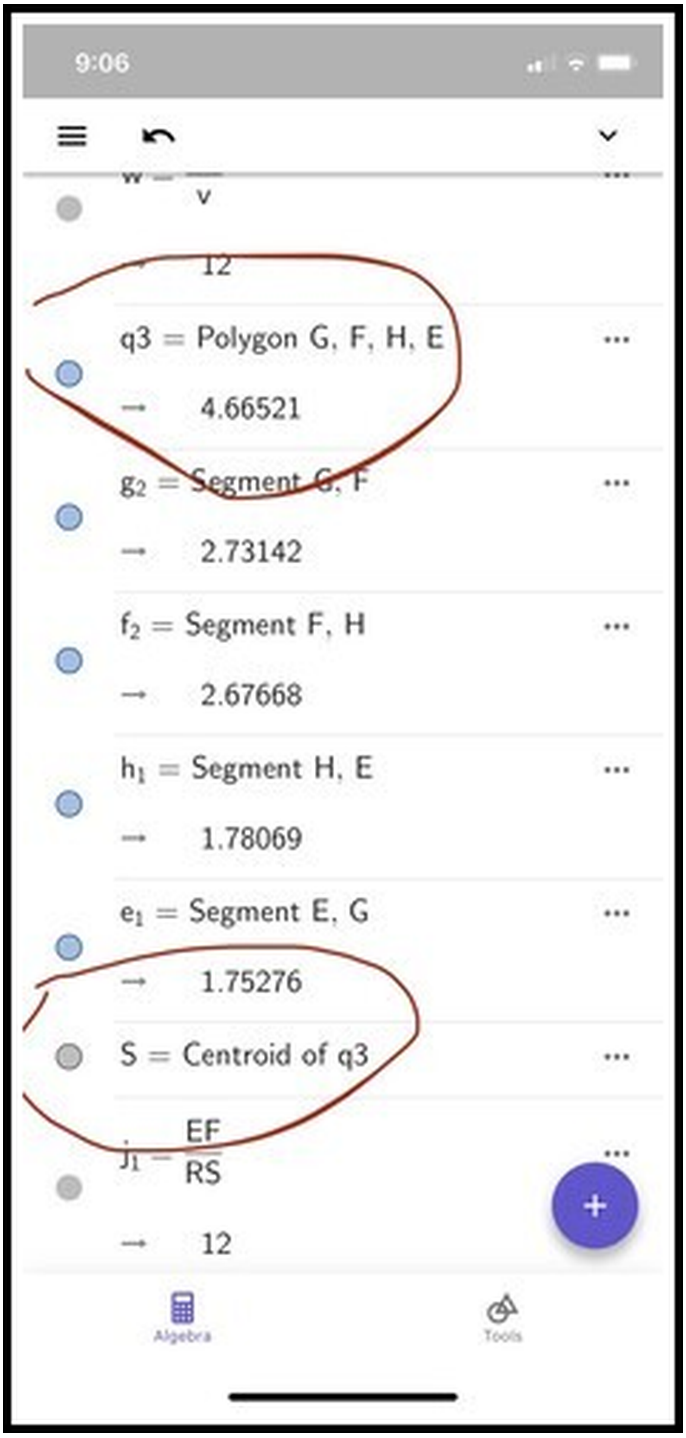

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Message: #1454
Date: 2022-07-16
From: garciacapitan@gmail.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

```
Dear Daniel and Chris,

Maybe the cause of the problem is that Geogebra's centroid of
the quadrilateral is returning the center of mass of the region,
like Mathematica's RegionCentroid.

Centroid[p_]:=Total[p]/Length[p];
ptA={0,0};
ptB={4,0};
ptC={6,3};
ptD={2,5};
RegionCentroid[Polygon[{ptA,ptB,ptC,ptD}]]
{26/9,19/9}
ptE=Centroid[{ptA,ptB,ptC,ptD}]
{3,2}

Francisco Javier.
```

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Message: #1455
Date: 2022-07-17
From: garciacapitan@gmail.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

```
Dear friends,

Here it is a figure with some minor changes of notation for
symmetry.

Francisco Javier.
```

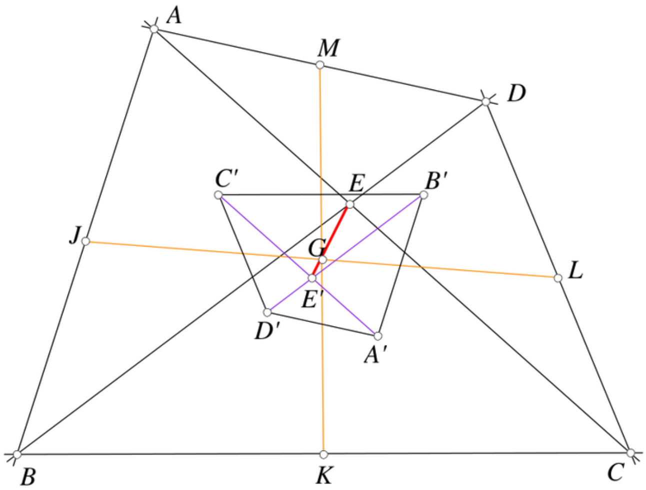
Centroids

If ABC is a triangle, we can undestand the centroid of ABC in several ways:

- We have three ball with the same height at the vertices of the triangle and the centroid is the center of mass of the system formed by them.
- We have a triangular lamina of metal and the centroid is the center of mass of the lamina.

The two points of wiew coincide in the case of a triangle. However, for a convex quadrilateral $ABCD$ this is not true.

$ABCD$	Given quadrilateral
E	Intersection of AC and BD
J, K, L, M, F, H	Midpoints of AB, BC, CD, DA, AC, BD.
A', B', C', D'	Centroids of BCD, CDA, DBA, ABC.
G	Intersection of JL and KM
E'	Centroid of region $ABCD$.



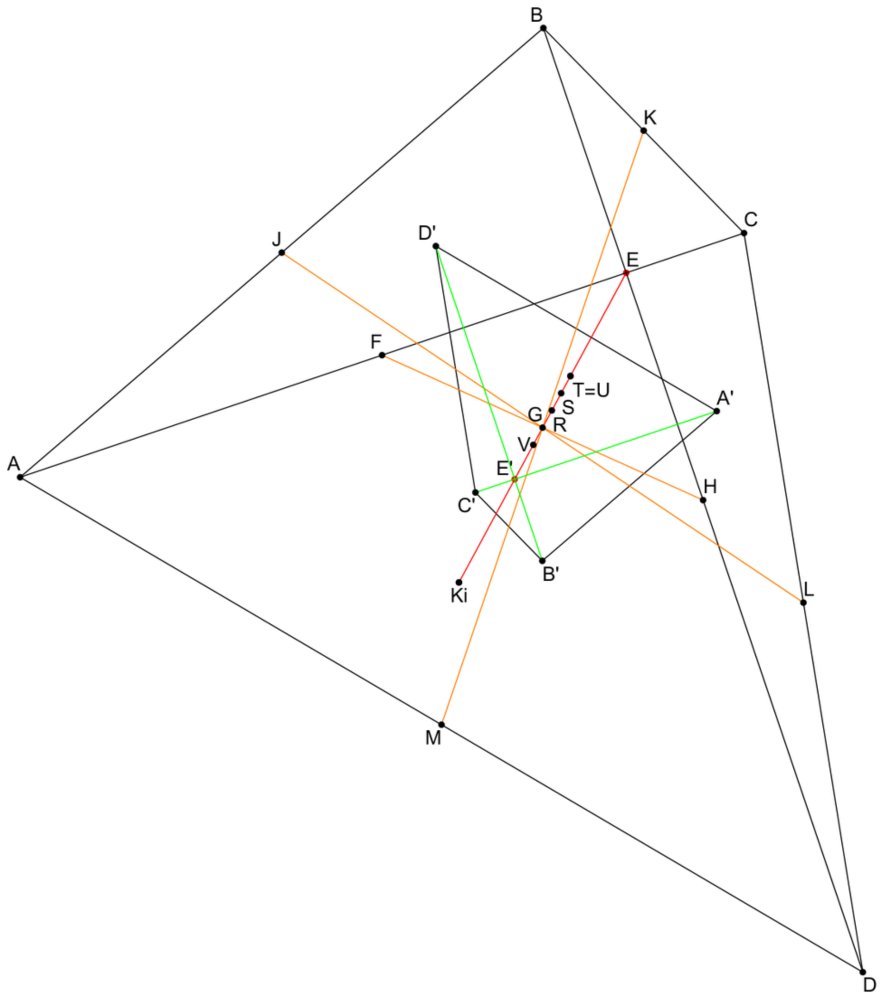
From $2J+2L = (A+B)+(C+D) = (A+D)+(B+C) = 2K+2L$, the lines JL and KM meet at the point G such that $4G = A+B+C+D$, the centrid of the quadrangle $ABCD$.

From $A + 3A' = A + (B + C + D) = 4P$, P divides the segment AA' in the ratio $3 : 1$, and the same with segments BB', CC', DD', thus $A'B'C'D'$ is the image of $ABCD$ by a homothety with center G and ratio $-\frac{1}{3}$. Therefore, E' defined as the centroid of region $ABCD$, that is the intersection of $A'C'$ and $B'D'$, is the image of E by the homothety and we have $EG : GE' = 3 : 1$.

Remark. Since A', C' are the centroids of triangles BDC, ABD, the center of mass of quadrilateral region $ABCD$ must lie on line $A'C'$ and, in the same way, on line $B'D'$.

The following figure is the result of a change of notation and some adding to the figure proposed by Daniel Hardisky.

$ABCD$	Given quadrilateral
E	Intersection of AC and BD
J, K, L, M	Midpoints of AB, BC, CD, DA.
A', B', C', D'	Centroids of BCD, CDA, DBA, ABC.
G	Intersection of JL and KM
E'	Centroid of region $ABCD$.
R	Centroid of region $A'B'C'D'$.
S	Centroid of region $EFE'H$.
$T = U$	Centroid of EFH = midpoint of EE'.
V	Centroid of $E'FH$.
Ki	Kirikami center



$KiE' : E'V : VG : GR : RS : ST : TE = 6 : 2 : 1 : 1 : 1 : 1 : 6.$

danielcentroids.pdf

Points already in EQF:

G	$QA-P1$	Quadrangle centroid
E	$QG-P1$	Diagonal crosspoint
E'	$QG-P4$	1st QG-Quasi Centroid
$T=U$	$QG-P8$	2nd QG-Quasi Centroid
Ki	$QG-P15$	Kirikami center

Francisco Javier García Capitán, July 17, 2022.

danielcentroids.pdf

Message: #1456
Date: 2022-07-17
From: van10hoven@gmail.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

Dear Daniel,

It is good that we found out the difference between the 2 types of centroids.

See also: https://sites.math.washington.edu/~king/java/gsp/center-mass-quad.html.

You asked: So, have I added anything new???

Yes sure. Your points R,S,V are new in EQF. Of course I cannot say there have not been earlier mentioning in literature of one or more of these points.

But your setup emphasises the function of QG-L3, being a Line of many Centroids in a Quadrigon.

The calculation of the distances between the centroids related to the sidelengths are new to me. Did you work them out yourself, or did you find them some place in literature?

Best regards,
Chris

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Message: #1457
Date: 2022-07-17
From: ”Francisco Javier García Capitán” <garciacapitan@gmail.com>
Subject: Re: [Quadri-and-Poly-Geometry] 7 pts on the line joining the centroid and diag. Intersection

---

Dear Daniel & Francisco

This looks very nice! As also mentioned by Chris, the collinearity of all the points on the line in Fig 2 is neat. If I'm not mistaken, it seems to follow more or less directly from the homothetic relation between the two quadrilaterals?

There is of course a 3rd centroid for a quadrilateral just consisting of rods of equal density, which I like to call the 'perimeter centroid'. In the case of the triangle, the perimeter centroid coincides with the Spieker centre of the triangle. It might also be interesting to investigate the relation of this centroid with that of the mass & the lamina centroids of a quadrilateral, if any?

Regards
Michael

To: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>
Sent: Sunday, July 17, 2022 1:19:57 PM

Dear friends,

Here it is a figure with some minor changes of notation for symmetry.

Francisco Javier.

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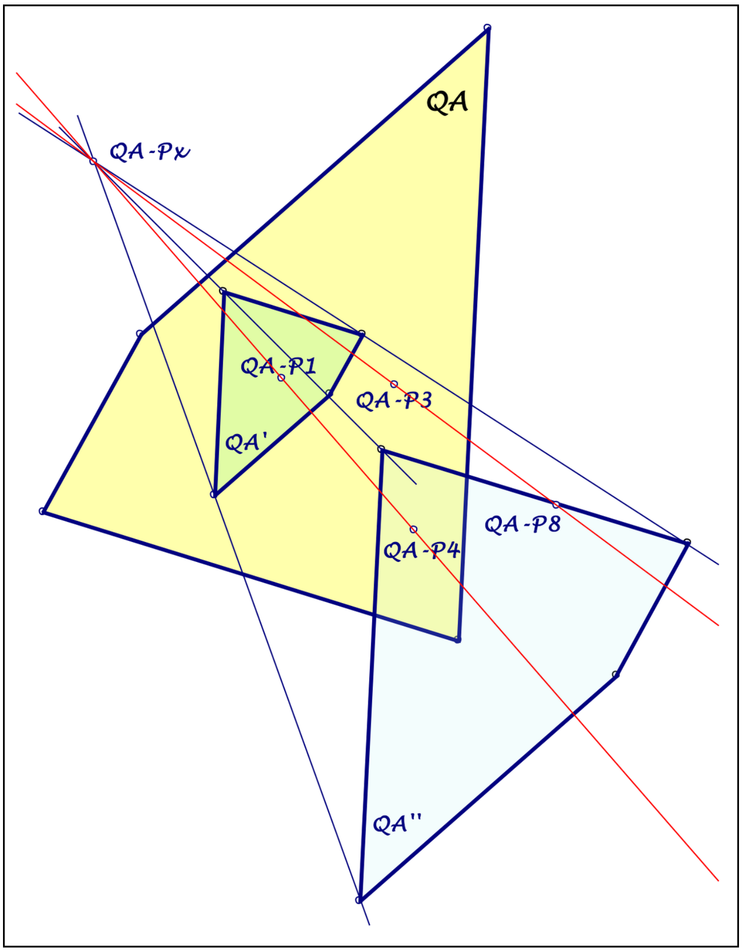
Message: #1458
Date: 2022-07-18
From: eckart_schmidt@t-online.de
Subject: New QA-point

```
Dear all,

perhaps in relation to the discussion in the last messages:
Consider for a quadrangle QA
... the quadrangle QA'
    of the centroids of its triangle components
... and a quadrangle QA'',
... which is the second quadrangle
    of the circumcenters for the triangle components.
QA' is homothetic QA wrt QA-P1, ratio -1/4,
QA'' is homothetic QA wrt QA-P4, ratio Sigma (see old#1471),
QA'' is homothetic QA' wrt a new QA-point QA-Px, ratio -3*Sigma,
... QA-Px is the intersection QA-P1.QA-P4 and QA-P3.QA-P8.

Best regards Eckart
```

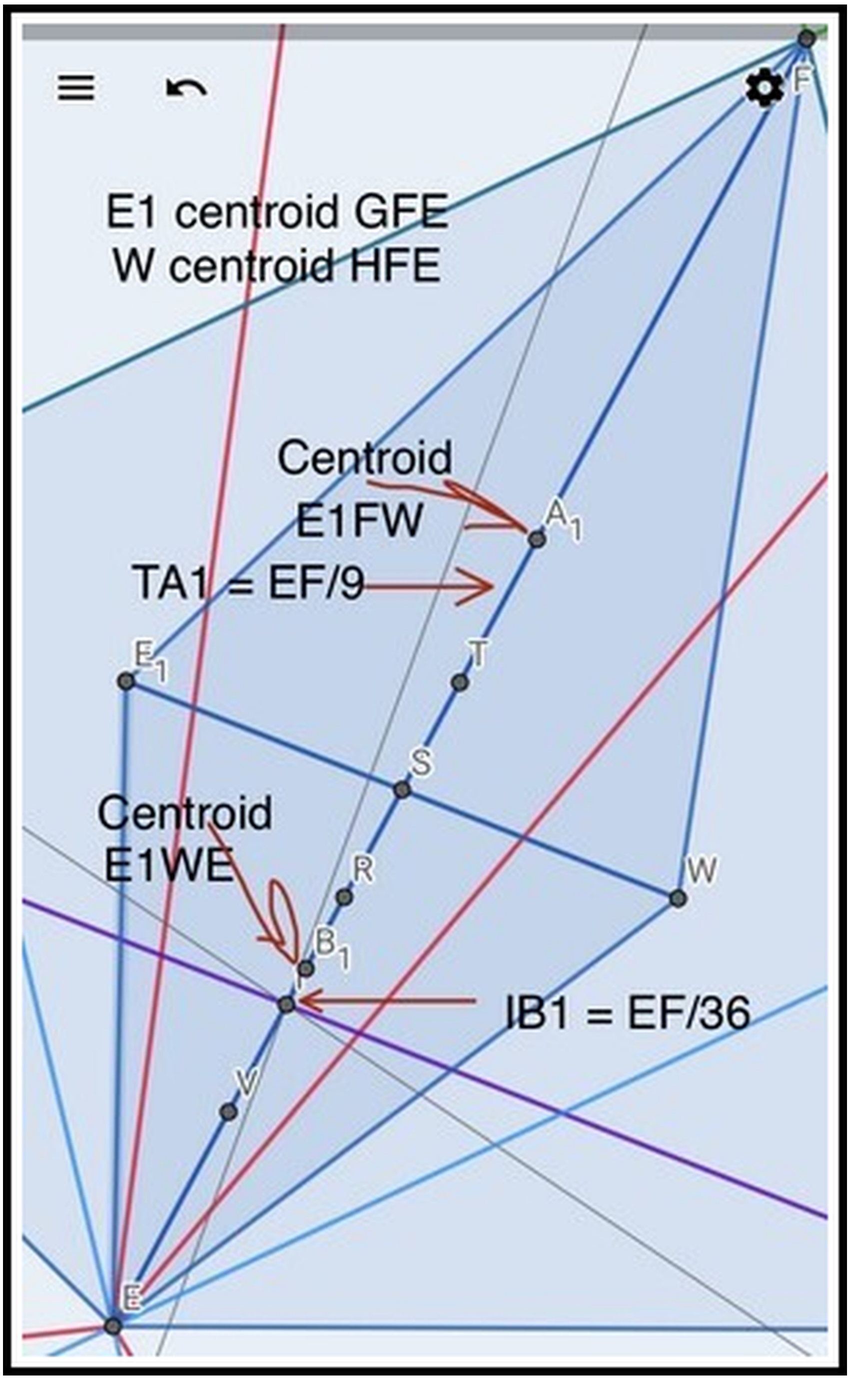


2022-07-18.pdf

Message: #1459
Date: 2022-07-18
From: danjohnhardisky@yahoo.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

```
Everything here is computed with GeoGebra and verified by
Mathematica. Not proof, but that usually follows much later.
This region is filled with even more strange points and Area
ratios, many integral.
```

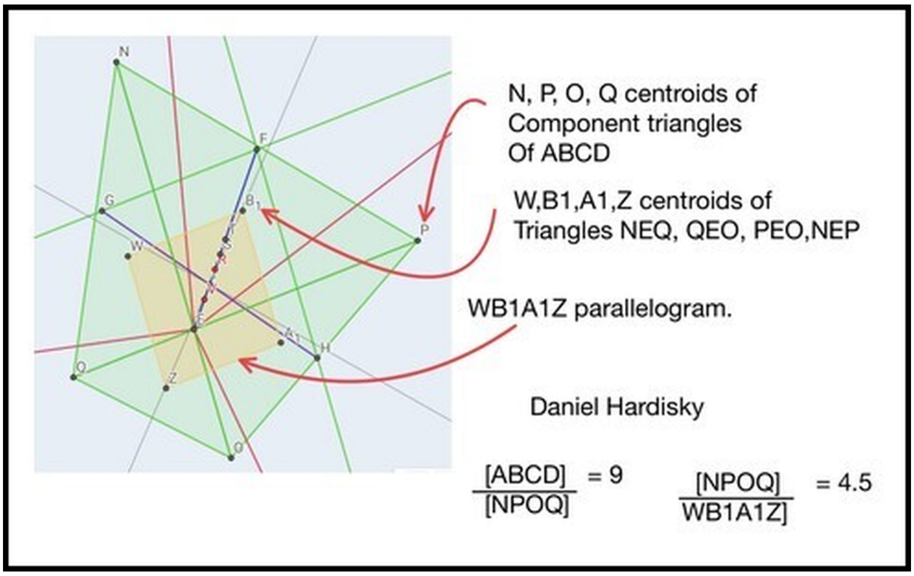


FC6B131B-B0CA-428E-897B-651E84C180B5.jpeg

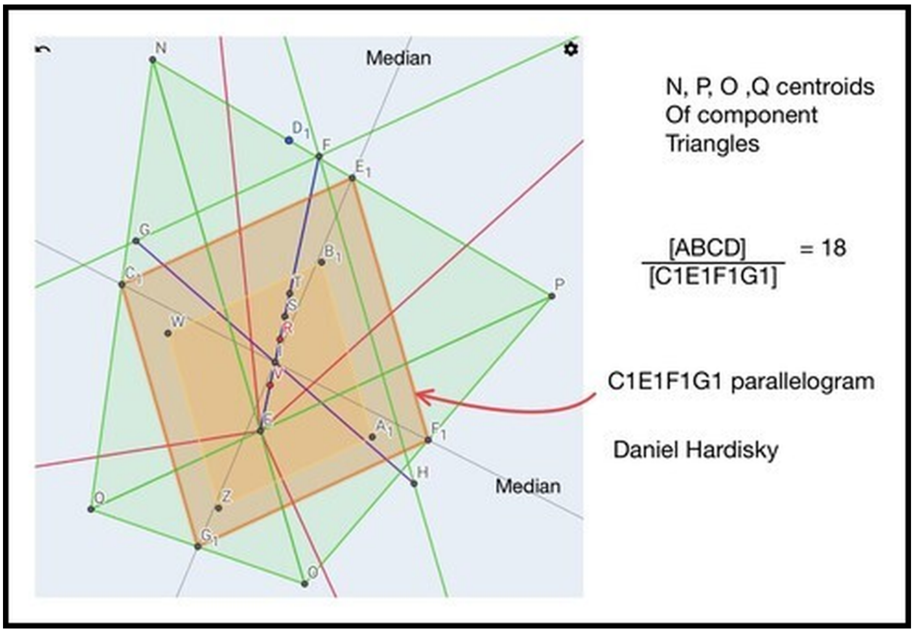
Message: #1460
Date: 2022-07-18
From: danjohnhardisky@yahoo.com
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

A few more. This “region” which includes centroids that create other triangles and their centroids needs to be studied. The points seem endless.



C1E36A4D-01A2-45CA-AA27-3902BF3251AA.jpeg



E6672FF3-87F0-4D46-8A18-CF13DC6FAE94.jpeg

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Message: #1461
Date: 2022-07-18
From: eckart_schmidt@t-online.de
Subject: Re: 7 pts on the line joining the centroid and diag. Intersection

---

```
Dear Daniel, dear Chris,

perhaps of interest, two aspects of QG-L3:

(1) For a quadrigon the line QG-L3 = QG-P1.QA-P1
... intersects the QG-sides in ratios with product 1
    (well known),
... intersects opposite sides in contact points for two
    inscribed conics
... ... with further contact points in the 4th harmonic points
        on the sides.
The 4 contact points of such a conic
... and the 2 intersections of opposite sides QG-2P2 lie on
    another conic.

(2) For the three QG of a QL
... the lines QG-L3 intersect the opposite lines of QL-Tr1
    collinear,
... the corresponding line bears QL-P12
... and is QL-Tf2 of QL-P8.QL-P12.

Best regards Eckart
```

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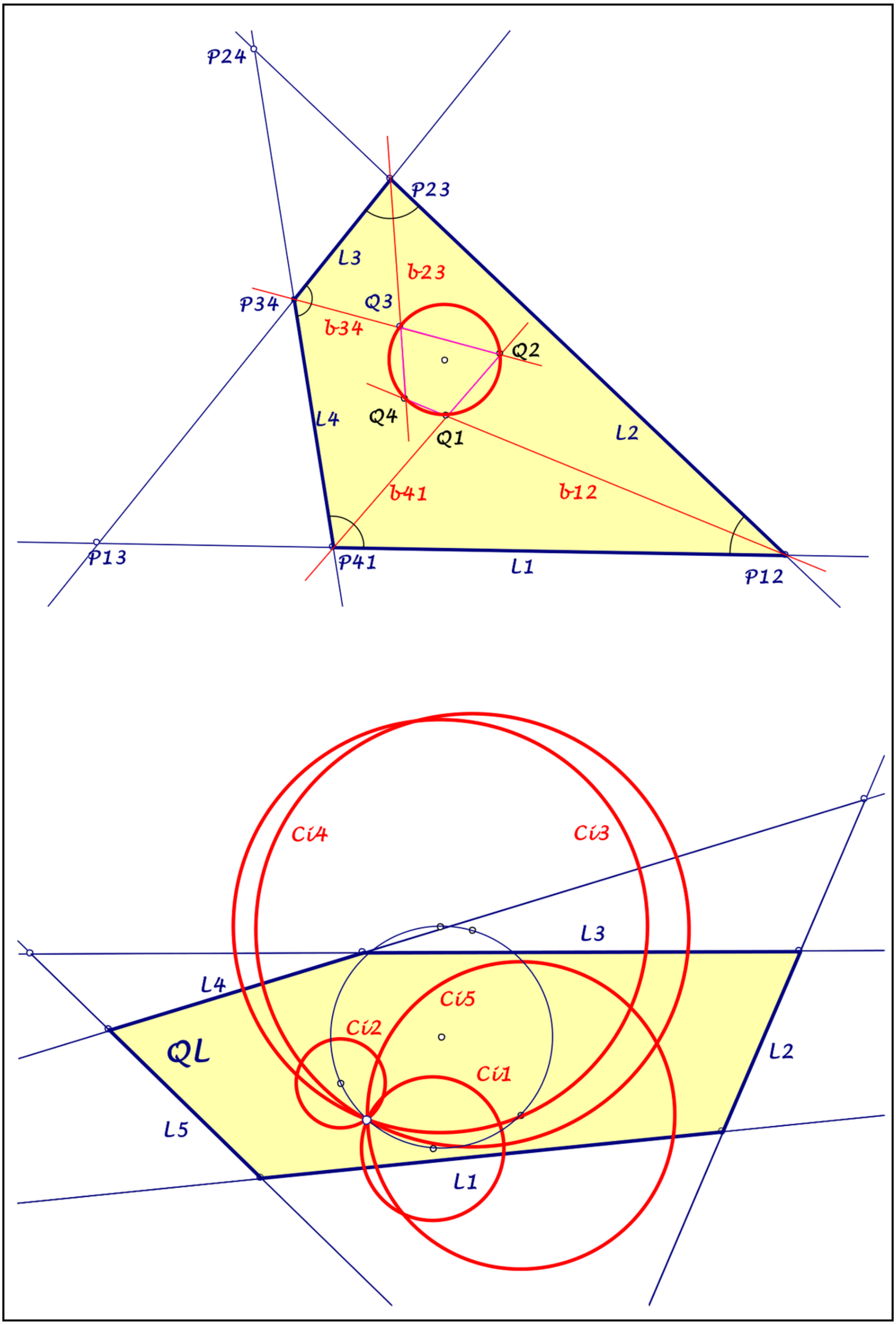
Message: #1462
Date: 2022-07-23
From: eckart_schmidt@t-online.de
Subject: Angle bisectors for a QG-circle

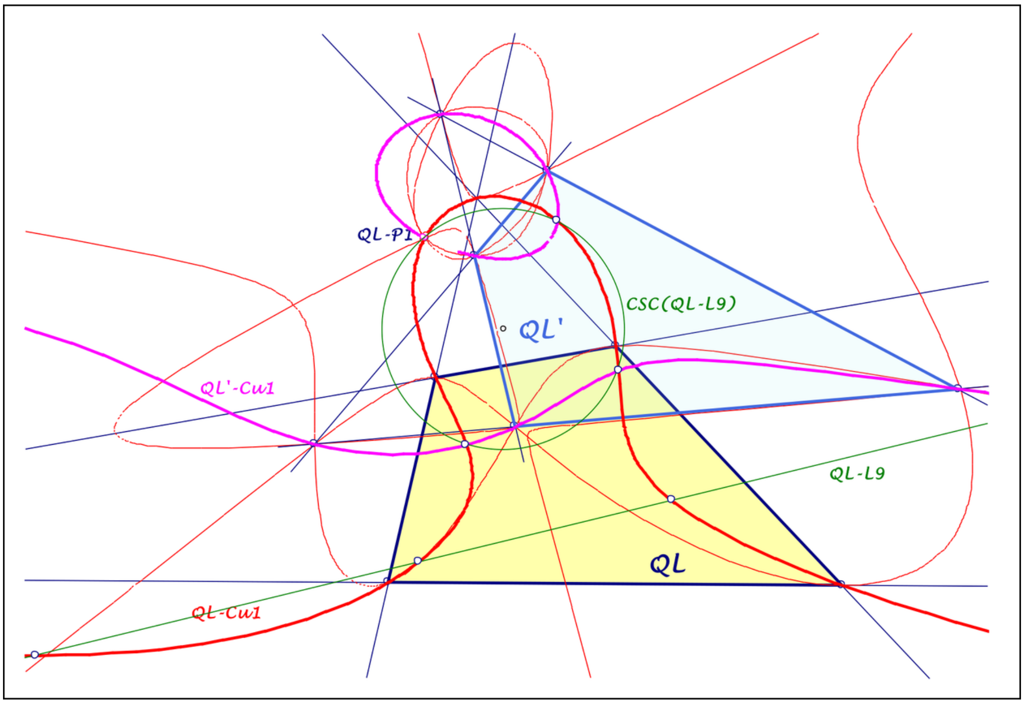
```
Dear all,

the following quadrigon circle CI seems not to be in EQF,
... but I think the circle must be well known.

Consider a quadrigon, defined by four lines Li and vertices
P(i,i+1) = Li ^ Li+1
... and the bisectors b(i,i+1)
    of the angles <(P(i-1,i),P(i,i+1),P(i+1,i+2)),
... which intersect concyclic in the points
    Qi = b(i-1,i)^b(i,i+1).
The CB-point 8P-s-P1 from  the QG-vertices and Qi is QG-P1.
Analog to this "inner" bisectors with a circle CI
... the "outer" bisectors give another circle CI'.

For a QL there are 3 circles CI for the QG-components:
The circles CI for the convex and the nonconvex but not
overturned QG-components coincide.
The three circles CI are centered on the 1st Steiner axis,
... they are CSC-invariant each with a fixed point for the lines
    of CSC-partners,
... the fixed point is the inverse of QL-P1 wrt the CI-circle,
... center and fixed point are CSC-partner.
If we consider for a QL the 3 CI and 3 CI',
... we get three pairs of coinciding circles.

An interesting property for 5G, defined analog with 5 lines Li,
... but only observed for the case,
... that the 5 trilaterals Li-1, Li, Li+1 have the same
    orientation:
The circles CIi for the 5 quadrangles Li+1,Li+2,Li-2,Li-1
... have a common point and concyclic centers (see attached).

Best regards Eckart
```



2022-07-23.pdf

Message: #1463
Date: 2022-07-25
From: eckart_schmidt@t-online.de
Subject: Modification of QA-Cu7

```
Dear all,

for QA-Cu7 we started with QA-P4,
... now let us start with any fixed point P
... and consider the three QG for a QL:
For each QG take orthogonal lines L, L' through P,
... the intersections of the conic QA-Tf2(L) and the line L'
... give a cubic for each QG,
... with seven triple intersections of the three cubics,
    one is P.
If P is a point on QL-Cu1,
... the 6 triple intersections unequal P
    define a quadrilateral QL'.
Let us finally consider this related quadrilateral QL' for P =
QL-P1:
... QL and QL' have the same Miquel point QL-P1,
... parallel Newton lines QL-L1 with the same QL-L2
... and so the same inscribed parabola QL-Co1.
... The cubics QL-Cu1 for QL and QL'  intersect in QL-P1
... and the CSC-images of the intersections of QL-L9 and QL-Cu1.

Best regards Eckart
```



2022-07-27.pdf

Message: #1464
Date: 2022-07-26
From: bernard.keizer@gmail.com
Subject: Re: Modification of QA-Cu7

```
Dear Eckart,
Very interesting properties !
If I understand correctly, for any QL formed by 4 tangents to a
given parabola, the circle CSC(QL-L9) and the  CSC of the 3
intersections of QL-L9 with QL-Cu1 are the same.
What are these 3 points exactly for the parabola ?
Best regards
Bernard
```

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Message: #1465
Date: 2022-07-27
From: eckart_schmidt@t-online.de
Subject: Re: Modification of QA-Cu7

```
Dear Bernard,

I have difficulties with your question.
If you take another QL of tangents at a given parabola,
... you get not the same circle CSC(QL-L9)
... and not the same intersections of QL-Cu1 and QL-L9.

Perhaps of interest:
The 3 intersections X1,X2,X3 of QL-Cu1 and QL-L9 beside QL-P1
... must not all be real,
... Xi, CSC(Xj), CSC(Xk) are collinear,
... the Simson line of QL-P1 for the triangle
CSC(X1)CSC(X2)CSC(X3)
... ... is parallel QL-L2, intersecting QL-L9 on QL-P1.QL-P7.

Best regards Eckart
```

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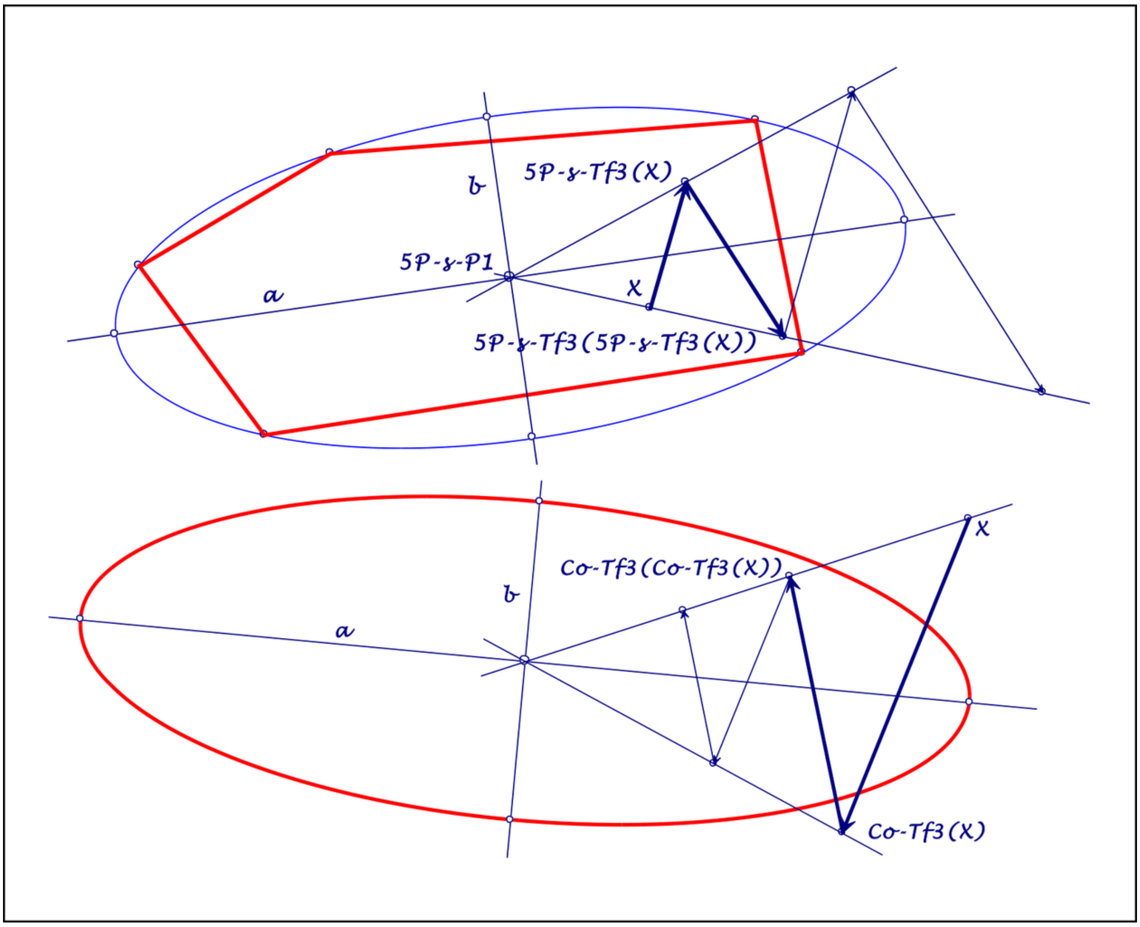
Message: #1466
Date: 2022-07-27
From: eckart_schmidt@t-online.de
Subject: 5P-s-Tf3 and its inverse Co-Tf3

```
Dear Chris, dear Scimemi,

I think, the following property isn't in EPG:
If we apply 5P-s-Tf3 twice, we get an expansion,
... centered in 5P-s-P1 with ratio ((a^2+b^2)/(a^2-b^2))^2,
... with a, b axes of 5P-s-Co1.

Analog for CO-Tf3 = 5P-s-Tf3inv.

Best regards Eckart
```



2022-07-27a.pdf

Message: #1467
Date: 2022-07-27
From: van10hoven@gmail.com
Subject: Re: 5P-s-Tf3 and its inverse Co-Tf3

```
Dear Eckart,
It is more or less described in the definition of CO-Tf3.
Best regards,
Chris
```

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Message: #1468
Date: 2022-07-27
From: bernard.keizer@gmail.com
Subject: Re: Modification of QA-Cu7

```
Dear Eckart,
Thanks for the answer !
In fact, your property wrt CSC(QL-L9) works not for any QL'(P),
but only for QL'(QL-P1) !
Best regards
Bernard
```

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Message: #1469
Date: 2022-08-02
From: Stan.Rabinowitz@comcast.net
Subject: Re: central quadrilaterals

```
The final paper has been published.

http://www.journal-1.eu/2022/9.%20Stanley%20Rabinowitz,%20Ercole
%20Suppa.%20Relationships%20between%20a%20Central%20Quadrilatera
l%20and%20its%20Reference%20Quadrilateral,%20pp.%20214-287..pdf?
fbclid=IwAR3-QCxg5NBz5ofc66pOTKwtTkmH-Ak90S0ed5HU4lnZVN1nHYBXc3R
vIZI
```

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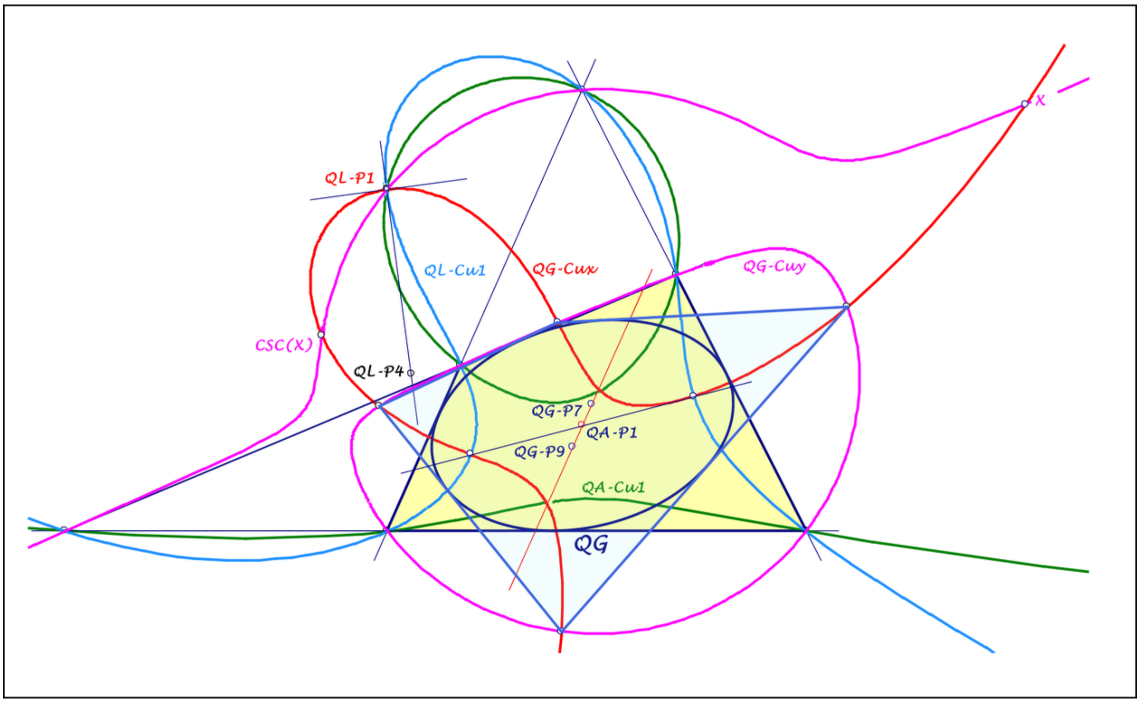
Message: #1470
Date: 2022-08-08
From: eckart_schmidt@t-online.de
Subject: QG-cubics

```
Dear all,

perhaps of no interest, but nevertheless:

A quadrigon QG can be interpreted as quadrangle QA or
quadrilateral QL,
... this are the reference figures for the used nomination,
... well known the cubics QA-Cu1 and QL-Cu1,
... intersecting in QL-P1 and the 6 QL-points
    (attached a drawing).

Now let us consider the circular QG-cubic QG-Cux,
... defined by the Möbius transformation QL-Tf1
... and a Newton line QG-P7.QA-P1.QG-P9,
... (orthogonal to the reference Newton line QL-L1).

This cubic is already described in old#414 (but not in EQF), as
locus of points,
... whose products of distances to opposite QG-vertices
    are equal.

QG-Cux intersects QL-Cu1 three times orthogonal
... in QL-P1 with tangent orthogonal QL-P1.QL-P4,
... and in two CSC-partners, foci of an inscribed QG-conic CO
centered in QA-P1.

Let us finally consider circular cubics QG-Cuy,
... defined by two CSC-partners on QG-Cux,
    the vertices of QG and QL-P1:
QG-Cuy has 4 further intersections with QG-Cux,
... which give with their tangents to CO a quadrigon,
    whose QL-Cu1 = QG-Cux.

The three QG-Cux for a QA
... have three intersections in the three QG-P5.
The three QG-Cux for a QL
... contact in QL-P1 with a common tangent orthogonal
    QL-P1.QL-P4.

Best regards Eckart
```



2022-08-08.pdf

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Message: #1471
Date: 2022-08-11
From: eckart_schmidt@t-online.de
Subject: New QA-Px

```
Dear all,

for a 6P the transformation 6P-s-Tf1
... maps the plane to the circle 6P-s-Ci1,
... centered in 6P-s-P1, bearing the 6 5P-s-P5.

There is an analogon for 5P:
Consider the new QA-Point QA-Px (not in EQF),
... which divides QA-P1.QA-P4 with ratio -1:4,
... the 5 QA-Px for a 5P are concyclic on a circle 5P-s-Cix,
... centered in a point 5P-s-Px.

5P-s-Px and 5P-s-Cix are already described in #1392,
... the transformation 5P-s-Tfx mentioned there
... maps the plane to 5P-s-Cix.

Best regards Eckart
```

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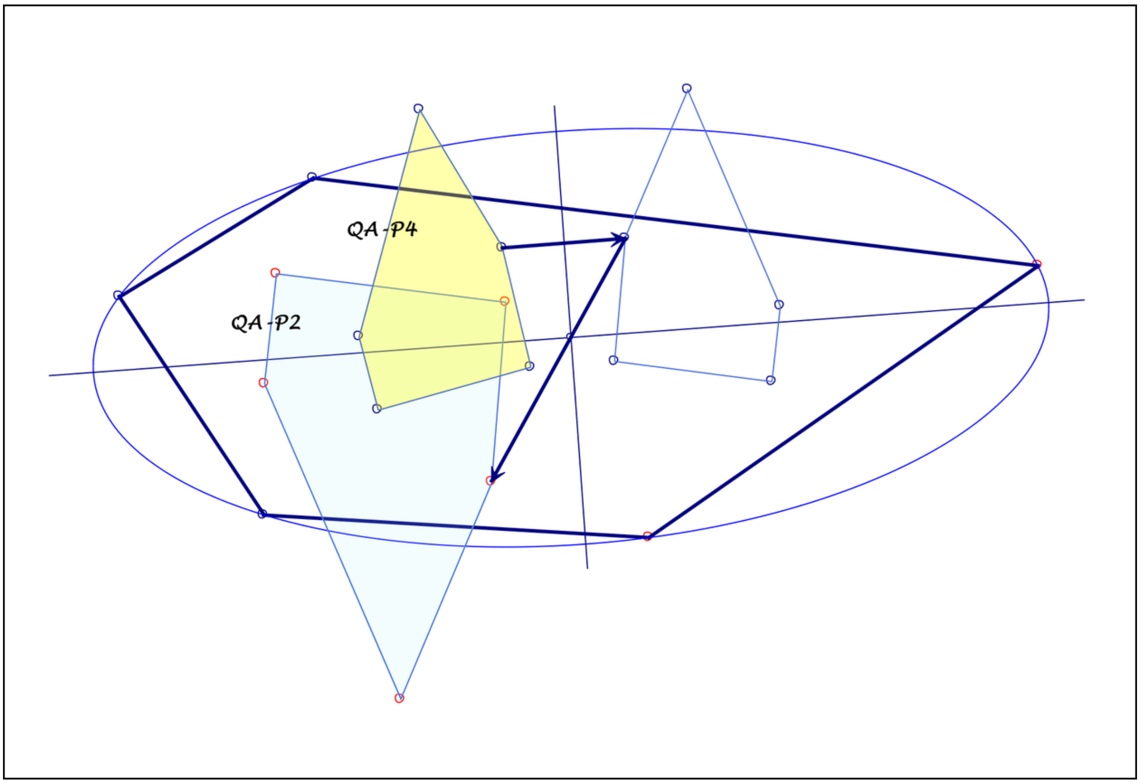
Message: #1472
Date: 2022-08-14
From: eckart_schmidt@t-online.de
Subject: QA-P2 and QA-P4 for pentangle

Dear all,

perhaps of interest (see attachment):
The QA-P2- and the QA-P4-pentangle for a 5P are inversely similar.
Starting with a 5P, its circumconic 5P-s-Co1, center 5P-s-P1 and axes a, b,
... consider the QA-P4-pentangle of its QA components,
... reflect it in the minor axis of 5P-s-Co1
... and stretch it from 5P-s-P1
    with factor -(a^2+b^2)/(a^2-b^2),
... then you get the QA-P2-pentangle.

Best regards Eckart



2022-08-14.pdf

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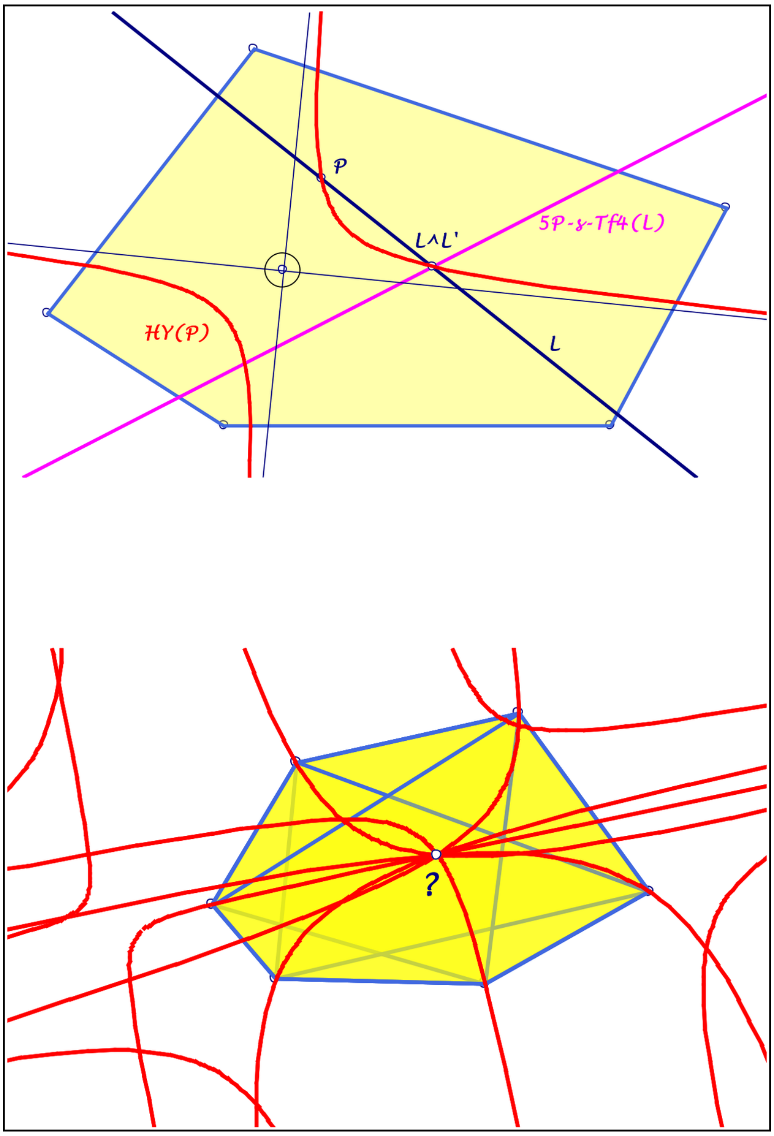
Message: #1473
Date: 2022-08-19
From: eckart_schmidt@t-online.de
Subject: New 6P-s-point ???

```
Dear Chris,

is the following point new for EPG?
Starting with  a 5P plus any point P,
... consider lines L through P
... and their 5P-s-Tf4-image line L'
... locus of L^L' is an orthogonal hyperbola HY(P).
For a 6P with vertices Pi wrt the remaining 5P
... we get 6 hyperbolas HY(Pi) with a common point.

Best regards Eckart
```



2022-08-19.pdf

Message: #1474
Date: 2022-08-20
From: eckart_schmidt@t-online.de
Subject: Re: [Quadri-and-Poly-Geometry] New 6P-s-point ???

```
Dear Chris,

please apologize,
... the last observation in my final message doesn't hold.

Best regards Eckart




-----Original-Nachricht-----
Betreff: [Quadri-and-Poly-Geometry] New 6P-s-point ???
Datum: 2022-08-19T21:34:05+0200
Von: "Eckart Schmidt" <eckart_schmidt@t-online.de>
An: "Quadri-and-Poly-Geometry"
<Quadri-and-Poly-Geometry@groups.io>

Dear Chris,

is the following point new for EPG?
Starting with  a 5P plus any point P,
... consider lines L through P
... and their 5P-s-Tf4-image line L'
... locus of L^L' is an orthogonal hyperbola HY(P).
For a 6P with vertices Pi wrt the remaining 5P
... we get 6 hyperbolas HY(Pi) with a common point.

Best regards Eckart
```

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Message: #1475
Date: 2022-08-30
From: van10hoven@gmail.com
Subject: Re: New 6P-s-point ???

```
Dear Eckart,

About your statement:
Starting with  a 5P plus any point P,
... consider lines L through P
... and their 5P-s-Tf4-image line L'
... locus of L^L' is an orthogonal hyperbola HY(P).

Nice feature!
I noticed also that:
1. HY(P) passes through the center of the circumscribed 5P-conic
5P-s-P1.
2. All lines 5P-s-Tf4(L) have a common point Q on OH for alle
lines through a fixed point P, being the reflection of P about
5P-s-P1.

Best regards,
Chris

P.s. Next 6 weeks I will be on holiday.
```

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Message: #1476
Date: 2022-08-31
From: eckart_schmidt@t-online.de
Subject: Re: New 6P-s-point ???

```
Dear Chris,

Thanks for interest and further observations,
... but there will be a typo at the end of your message:
... replace "5P-s-P1" by "center of HY(P) = OH".

Best regards Eckart
```

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Message: #1477
Date: 2022-09-01
From: eckart_schmidt@t-online.de
Subject: QA-Splitter

```
Dear Chris,

are the following two properties already mentioned?
(1) The three circles QL-Cu1 for a QA have radical center
QL-P16.
(2) The three circles QL-Ci5 for a QA
... have collinear centers on a line parallel QA-L4
... through the center of the circumcircle of the Miquel
triangle QA-Tr2.

Best regards Eckart
```

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Message: #1478
Date: 2022-09-04
From: eckart_schmidt@t-online.de
Subject: Re: QA-Splitter

```
Dear Chris,

I just notice, that in my last message #1477 is a typo:
Please replace QL-Cu1 by QL-Ci1, excuse.

Best regards Eckart
```

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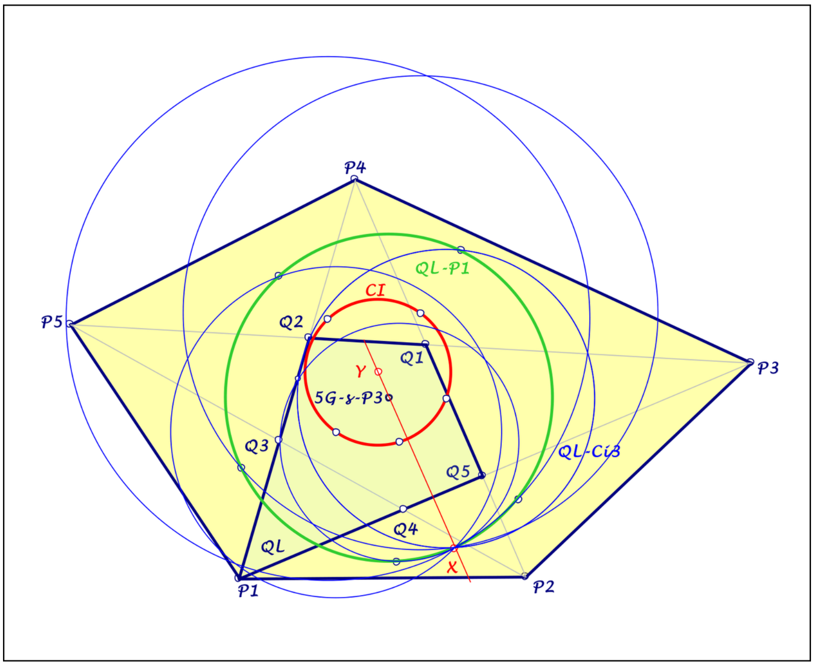
Message: #1479
Date: 2022-09-05
From: eckart_schmidt@t-online.de
Subject: New 5G-elements

```
Dear all,
let us consider an "inner" 5G with vertices Qi = Pi+1Pi-2 ^
Pi-1Pi+2
... and the circumscribed quadrigons PiQi-1QiQi+1,
    considered as QL,
... this is another sight as in EPG for 5G-s-P1,3.
(1) The Newton lines QL-L1 of these quadrigons
... have also the common point 5G-s-P1.
(2) The Miquel points lie on a circle,
... same as mentioned in 5G-s-P3.
(3) The circles QL-Ci1 of these quadrigons
... have radical axes with the common point 5G-s-P1.
(4) The circles QL-Ci3 of these quadrigons
... have a common point X (new)
... and centers on a circle CI (new) centered in Y (new),
... X is the inverse of 5P-s-P3 wrt the circle CI,
... X is a point on the circle of the Miquel points.
Finally:
(5) If we take the CSC-images of a point P wrt the quadrigons,
... we get a circle Ci(P), which degenerates to a line,
... if P is a point on the circle of the Miquel points.
These lines envelope the inscribed conic of the "inner" 5G.
Best regards Eckart
```



2022-09-05.pdf

Message: #1480
Date: 2022-09-05
From: eckart_schmidt@t-online.de
Subject: Re: New 5G-elements

```
Dear all,

wrt my last message (1) and (2):
It is not "another sight as in EPG for 5G-s-P1,3",
for different quadrigons of the same QL are studied.

Best regards Eckart
```

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Message: #1481
Date: 2022-09-10
From: eckart_schmidt@t-online.de
Subject: EQF and EPG

```
Dear Chris,

is there a reason not to get connected with EQF and EPG since
several days?

Best regards Eckart
```

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Message: #1482
Date: 2022-09-12
From: van10hoven@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] EQF and EPG

```
Dear Eckart,
Thanks for mentioning!
Things should be ok now.
Best regards,
Chris

Op za 10 sep. 2022 14:58 schreef Eckart Schmidt
<eckart_schmidt@t-online.de
> Dear Chris,
>
> is there a reason not to get connected with EQF and EPG since
several days?
>
> Best regards Eckart
>
```

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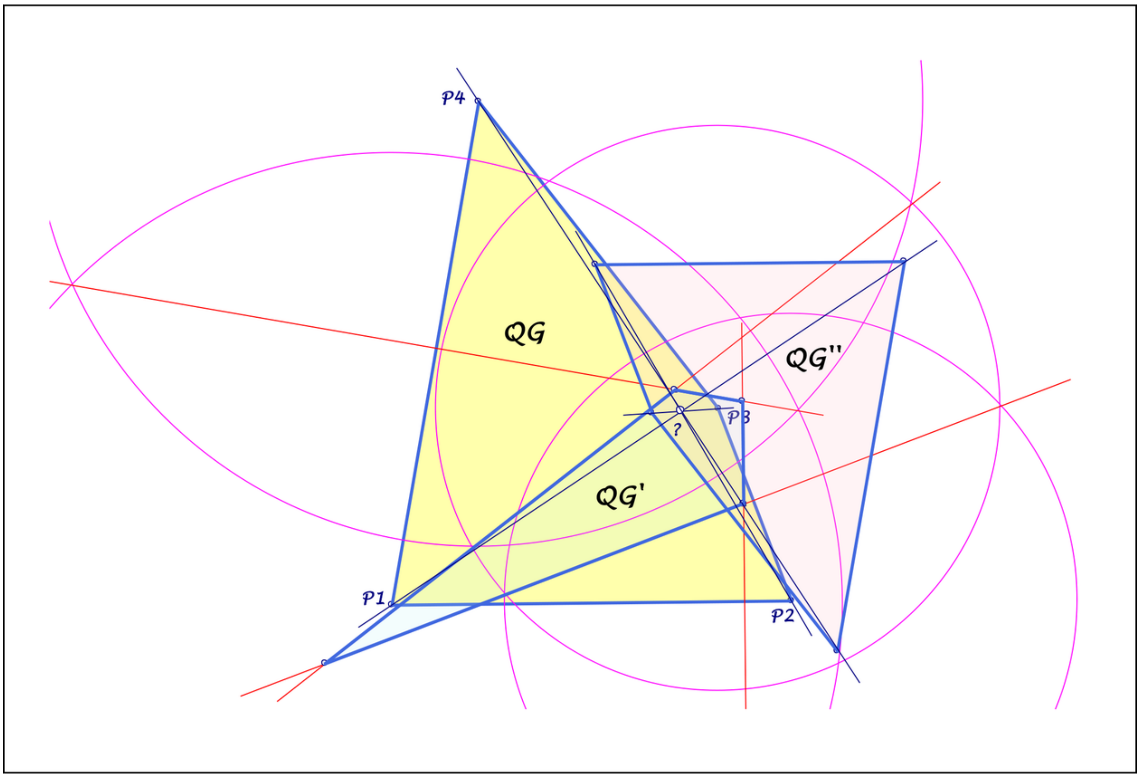
Message: #1483
Date: 2022-09-13
From: eckart_schmidt@t-online.de
Subject: New QG-point

```
Dear all,

if we consider the 2nd X(3)-QG of a QG,
... we get a homothetic QG to the reference QG,
... here another example:
Start with a QG = P1P2P3P4 and consider circles
... round the vertices Pi with radius sqr(PiPi-1*PiPi+1)
... and you get radical axes of neighbored circles as sidelines
of a new QG'.
QG'' as 2nd QG' is homothetic to the reference QG
... and the homothetic center will be a new QG-point,
... but I found no properties wrt other EQF-elements.

Best regards Eckart
```



2022-09-13.pdf

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Message: #1484
Date: 2022-09-15
From: eckart_schmidt@t-online.de
Subject: Circle geometry for quadrangle

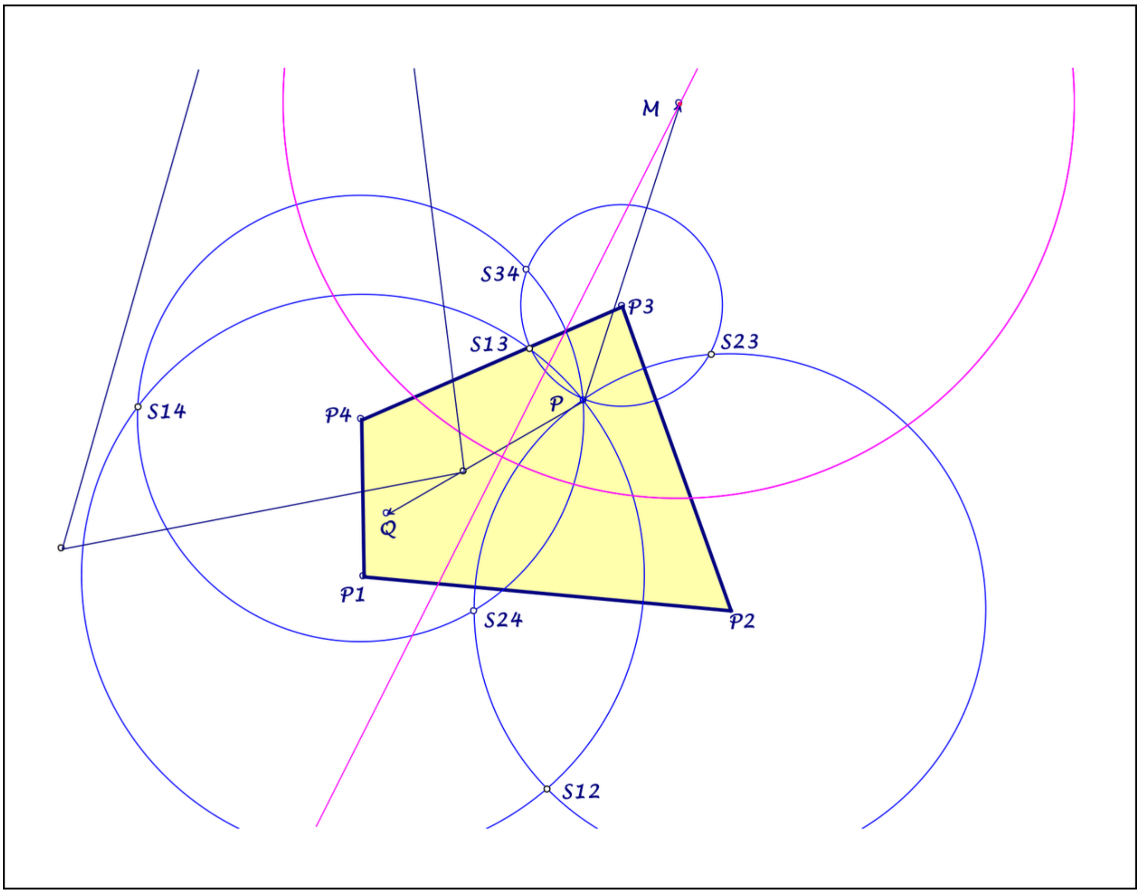
Dear all,

circle geometry is rare in QA-geometry,
... perhaps new the following circle constellation:
Consider QA = P1P2P3P4 and a fixed point P,
... draw the circles Ci(Pi,P) round Pi through P,
... with the six 2nd intersections Sij = Ci(Pi,P)^Ci(Pj,P).
There is a Möbius transformation,
... swapping the three pairs S12<-->S34, S13<-->S24, S41<-->S23.
Any point P defines such a Möbius transformation,
... for P = QA-P4 this transformation degenerates in a reflection in QA-P2,
... but I found no properties wrt other EQF-elements.

Let M be the center of the Möbius transformation and Q the image of P,
... M is the Miquel point e.g. of the QG = S12S23S34S41,
... Q is common point of the six circles (Sij,Sjk,Sik).
Let us consider a QA-transformation TF, which maps P to Q,
... which is no Möbius transformation,
... maps QA-P4 to its reflection in QA-P2,
... has as fixed points the vertices of QA-Tr1.
Let us consider a QA-transformation TF, which maps P to M,
... which is no Möbius transformation,
... maps QA-Tr1-vertices to their QA-Tr1-pedal points,
... has as fixed points the vertices of QA.

Finally an application for the transformation TF = P-->Q:
The TF-image of a QA-circumconic CO is a new conic
... with the same axes and foci as CO,
... the orthogonal circumconic QA-Co2 is TF-invariant,
... point and image center-symmetric.

Best regards Eckart



2022-09-15.pdf

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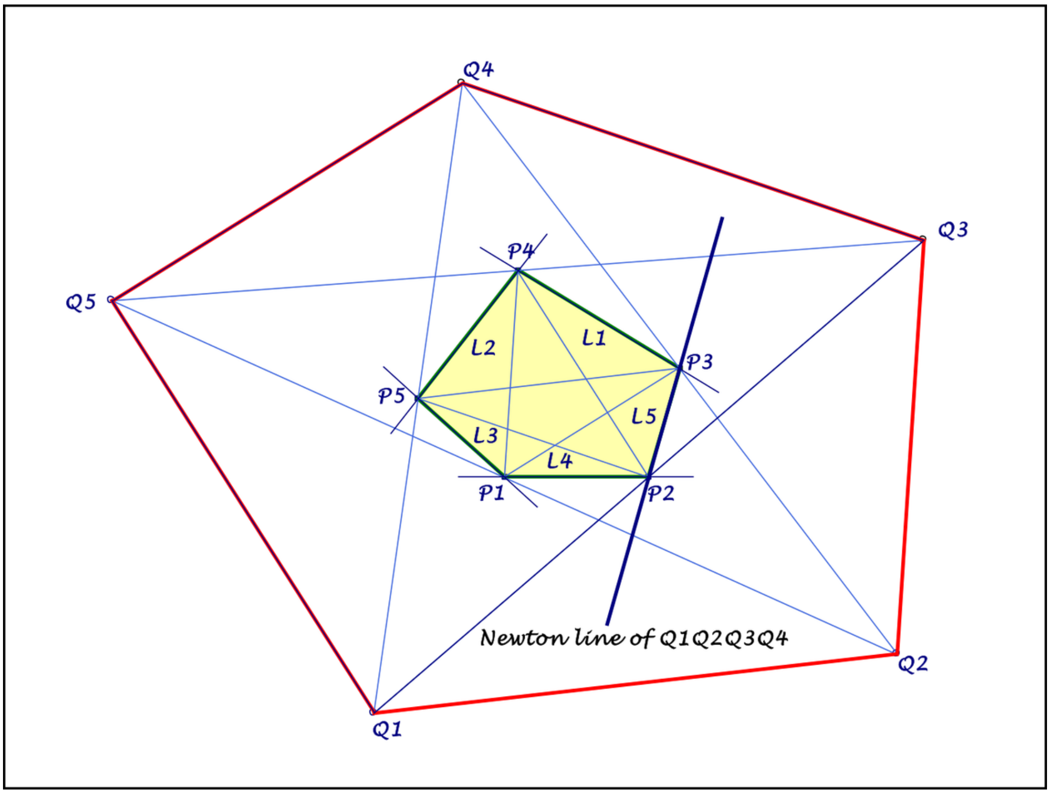
Message: #1485
Date: 2022-09-24
From: eckart_schmidt@t-online.de
Subject: Just for fun

Dear all,

5 lines Li define a 5-gon with vertices Pi = Li-2^Li+2:
... how to find a 5-gon Q1Q2Q3Q4Q5
... whose QG-components Qi+1Qi+2Qi+3Qi+4 have the lines Li as Newton lines?
Solution: Start with the 5-gon 5G' = P1P3P5P2P4,
... and reflect a point round 5G',
... there is one circumscribed 5-gon,
... use the nomination: Qi-1 and Qi+1 with midpoint Pi,
... and you get the solution Q1Q2Q3Q4Q5.

Best regards Eckart

PS. If not well known, the construction of the circumscribed 5-gon of ABCDE:
... Take X as 4th parallelogram point of ABC,
... take Y as 4th parallelogram point of XDE
... and reflect Y round ABCDE.



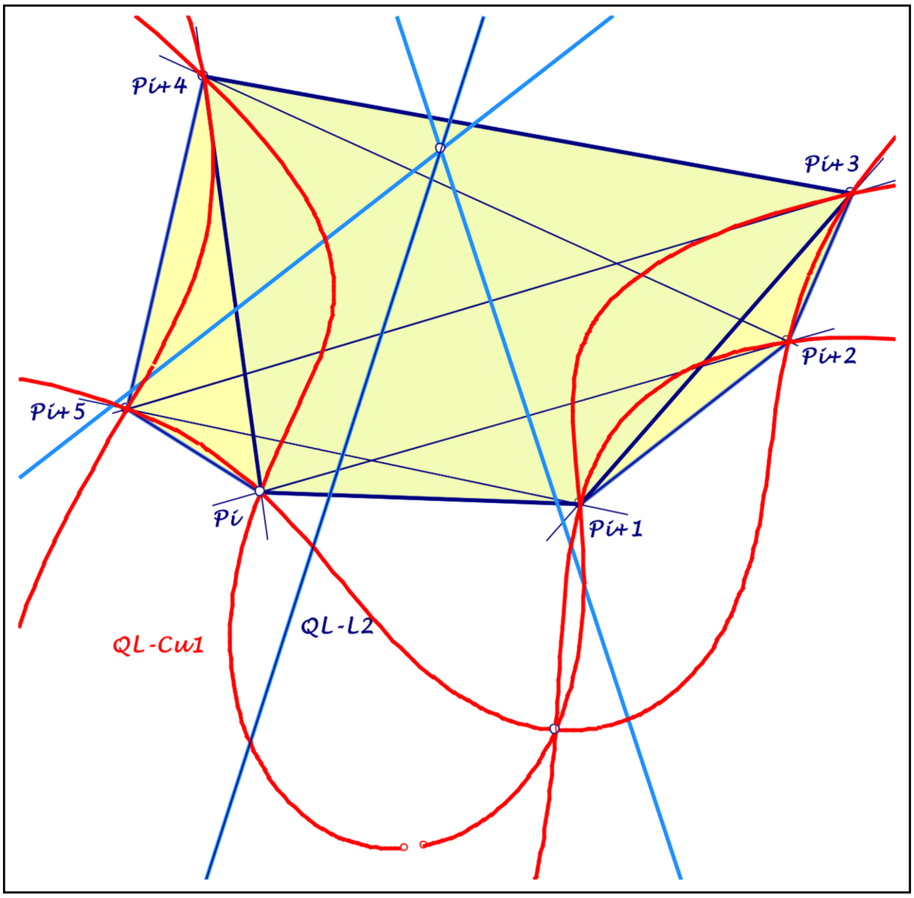
2022-09-23.pdf

Message: #1486
Date: 2022-09-25
From: eckart_schmidt@t-online.de
Subject: New 6G-points

```
Dear all,

consider a 6G = P1...P6 and the 3 quadrigons
... PiPi+1Pi+3Pi+4 (6G-vertices without two opposite vertices).
(1) The lines QL-L2 of the 3 QGs have a common point.
(2) The cubics QL-Cu1 of the 3 QGs have a common point,
... whose pedal QA wrt the 3 QGs are cyclic
... with two common points of the corresponding circles ...

Best regards Eckart
```



2022-09-25.pdf

Message: #1487
Date: 2022-09-26
From: ivan.pavlov@gmail.com
Subject: Re: New 6G-points

```
Dear Eckart,

Your first point has also appeared in the context of triangle
geometry. See the preamble to X(45010) in ETC, where Cezar
Lozada and myself called it the 1st Aubert point of two
triangles.

Kind Regards,
Ivan
```

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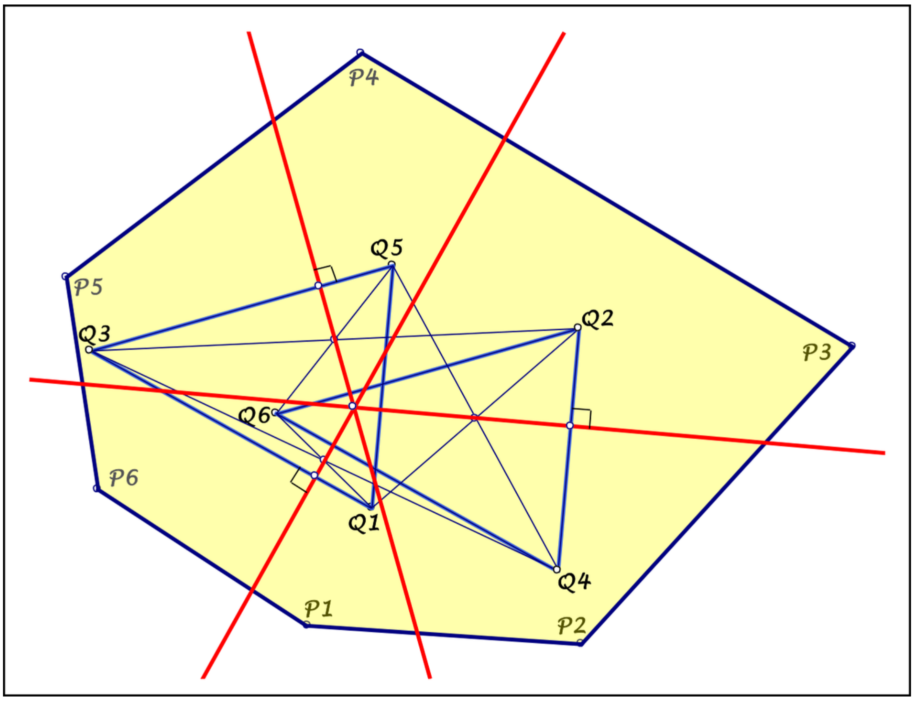
Message: #1488
Date: 2022-09-26
From: eckart_schmidt@t-online.de
Subject: Re: New 6G-points

```
Dear Ivan Pavlov,

thanks for information, the ETC-reference was new for me,
... so  my point is the 1st Aubert point of the triangles P1P3P5
    and P4P6P2.
There is a further 1st Aubert point for a 6G = P1...P6:
Let Qi be the 4th parallelogram point of Pi-1,Pi,Pi+1,
... the triangles Q1Q3Q5 and Q4Q6Q2 are congruent parallel
    connected,
... their (degenerated) Steiner lines have a common point,
... which will be a new 6G-point, attached a drawing.

Best regards Eckart
```



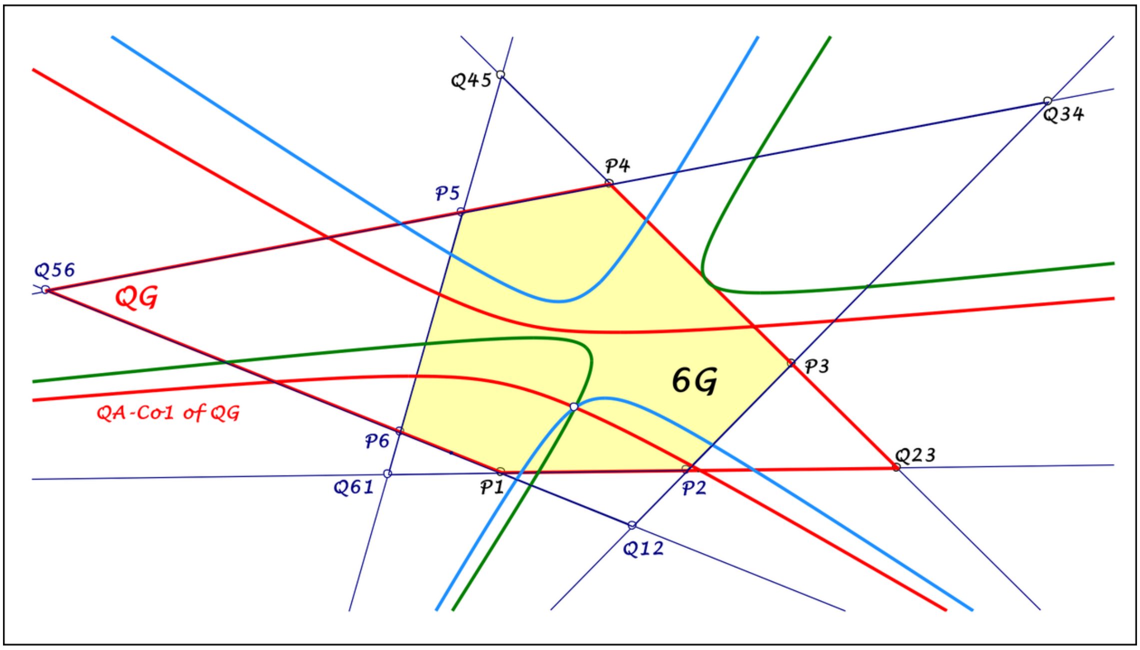
2022-09-26.pdf

Message: #1489
Date: 2022-09-27
From: eckart_schmidt@t-online.de
Subject: Re: New 6G-points

```
Dear all,

after two new 6G-points in # 1486 and a third in #1488 here a
fourth:
(4) Starting with a 6G = P1...P6
... and the intersections Qi,i+1 = Pi-1Pi^Pi+1Pi+2,
... consider the 3 QGs P1Q23P4Q56, P2Q34P5Q61 and P3Q45P6Q12,
... their conics QA-Co1 have a common point (see attached).

Best regards Eckart
```



2022-09-27.pdf

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Message: #1490
Date: 2022-09-28
From: bernard.keizer@gmail.com
Subject: Some reflexions with QL-Qu3

Dear Eckart,
I tried to make some order in my files, which is not easy !
I worked again on the transformation 7P-n-Tf1 for the 6 vertices of a QL and QL-P1.
You said in your message 1444 that 7P-n-P5 is the point swapping in the same CSC QL-P1 and QL-P17 and the 2 Plücker points.
I understand that it necessary swaps QL-P1 and QL-P17, as QL-P17 is 6P-n-P5 for the 6 vertices of the QL, but why the Plücker points ?
This point is also 6P-s-Px on your figure in the message 1431. I suppose it is then pure observation.
Then I had a look again at your wonderful quartic QL-Qu3.
I have a question about the defining conic : it is tangent to the 3 sides of QL DT, to the Steiner axes of the CSC and to the Steiner Line.
(By the way, the diagonal circle is a Poncelet curve for the inscribed conic).
The quartic is CSC invariant (as mentionned in EQF), but also CSCdiaginvariant (with CSCdiag the CSC of the QLdiag formed by the 3 DTsides and the Newton Line).
This 2nd property in not mentionned in EQF.
The quartic passes through the fixed points of the CSC (which are CSCdiag partners) and through the fixed points of CSCdiag (which are CSC partners).
My question is simple : are the Steiner axes of the CSCdiag also tangent to the conic ? (I think so, but I can't be sure, as I don't know how to draw the conic).
If this is the case, it is possible only with the 4 points QL-P1, QL-P17 and the Plücker points to determine CSCdiag (from the Plücker points) and CSC.
Then, the Newton Line and the Steiner Line being given by the 2 Plücker points, it s possible to determine QL-Cu1, the conic and finally QL-Qu3.
If this is true, the orthoptic circle of the conic passes through QL-P1 and QL-P17 (like the Dimidium circle).
Many thanks in advance for your attention, your answer will be very important for me !
Best regards Bernard
The next step is to find a construction for the SC circonscribed to the 6 vertices of DT and centered in the point 7P-n-P5 or 6P-s-Px ...

Message: #1491
Date: 2022-09-28
From: bernard.keizer@gmail.com
Subject: Re: Some reflexions with QL-Qu3

```
Dear Eckart,
In fact, I checked that the CSC circles of the Steiner axes of
CSCdiag are centered on the rectangular hyperbola centered in
QL-P6 through QL-P4 and QL-P5.
You mentionned this RH somewhere in the forum, but it is not in
EQF.
So I suppose my assumption holds !
Best regards
Bernard
```

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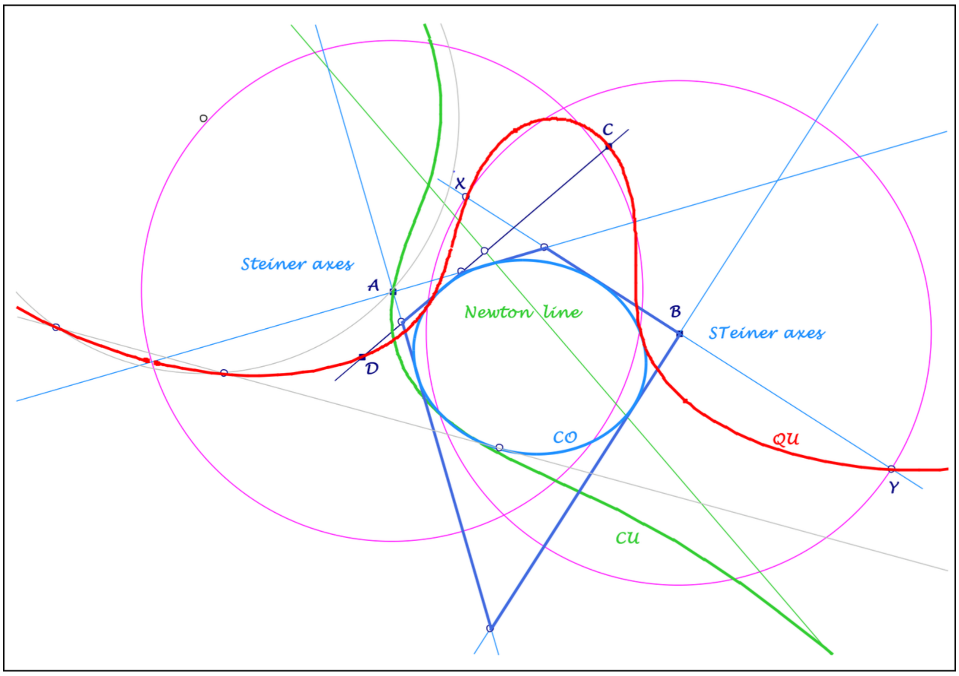
Message: #1492
Date: 2022-09-29
From: eckart_schmidt@t-online.de
Subject: Re: Some reflexions with QL-Qu3

```
Dear Bernard,

with interest I have reproduced your observations,
... they are correct, confirmed with CABRI.
You give a nice geometry for four arbitrary points A, B, C, D
(attached)
... (background: A = QL-P1, B = QL-P17, C and D the Plücker
    points QL-2P1),
... and two Möbius transformations
... Tf1 centered in A, swapping the fixed points X, Y of Tf2
    (background CSC).
... Tf2 centered in B, swapping C and D
    (background your CSCdiag).

Tf1 and the bisector of C and D as Newton line define a cubic CU
(QL-Cu1).
The Steiner lines of Tf1 and Tf2 and CD define a 5L with
inscribed conic CO,
... whose tangents intersect their Tf1-circle in points of a
    quartic QU (QL-Qu3).

Best regards Eckart
```



2022-09-29.pdf

Message: #1493
Date: 2022-09-29
From: eckart_schmidt@t-online.de
Subject: Re: Some reflexions with QL-Qu3

```
Dear Bernard,

sorry, I don't understand the PS of your message #1490,
... thanks for explaining
    "... construction for the SC
         circonscribed to the 6 vertices of DT ...".

Best regards Eckart
```

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Message: #1494
Date: 2022-09-30
From: bernard.keizer@gmail.com
Subject: Re: Some reflexions with QL-Qu3

Dear Eckart,
Thanks for your quick answer !
I had 2 points in my message 1490
1) the 1rst was about the Steiner axes of CSCdiag tangent to the same conic ; you confirm this property.
Then I was curious and tried to find a QL as intersection of QL-Cu1 and QL-Qu3.
On my ugly figure, it worked ! You just need QL-Ci1 as CSCdiag(QL-L1) and QL-Co1 given by it's focus QL-P1 and it's directrix QL-L2.
Unfortunately, on your figure there is only a copple of CSC partners (one visible, the other not) ; the 2 other copples are not real.
>From your visible point (let say P12), you may have it's CSC partner P34, the middle of P12P34 (on the Newton Line) and it's CSCdiag partner, which is a vertice of DT.
P12P34 is a diagonal, tangent to your conic, but it doesn't cut QL-Co1 ...
Nevermind, it was only a curiosity, just for fun, as you often say
2) My 2nd point was the construction of a SC (cubic stelloïd Mac Cay K003 or Kjp024) circumscribed to the 6 real vertices of a given existing QL.
I refer to our discussion in the messages 1392 to 1446, in particular the messages 1392, 1431 and 1444.
The center of the SC (point where the 3 asymptotes concur) was 6P-s-Px of the 6 vertices (obtained as intersection of the 6 5P-s-Cix) (message 1392 and figure 1431).
I understand that this point was also 7P-n-P5 for the 7P formed by the 6 QL vertices and QL-P1 and that this point was the Miquel point of the QG of the 4 mentionned points (QL-P1, QL P17 and the 2 Plücker points).
Is this correct ? How do you construct the QL-circumscribed SC centered in this point ? I suggest to name this SC QL-Cu3
Thanks again for your attention
Best regards
Bernard

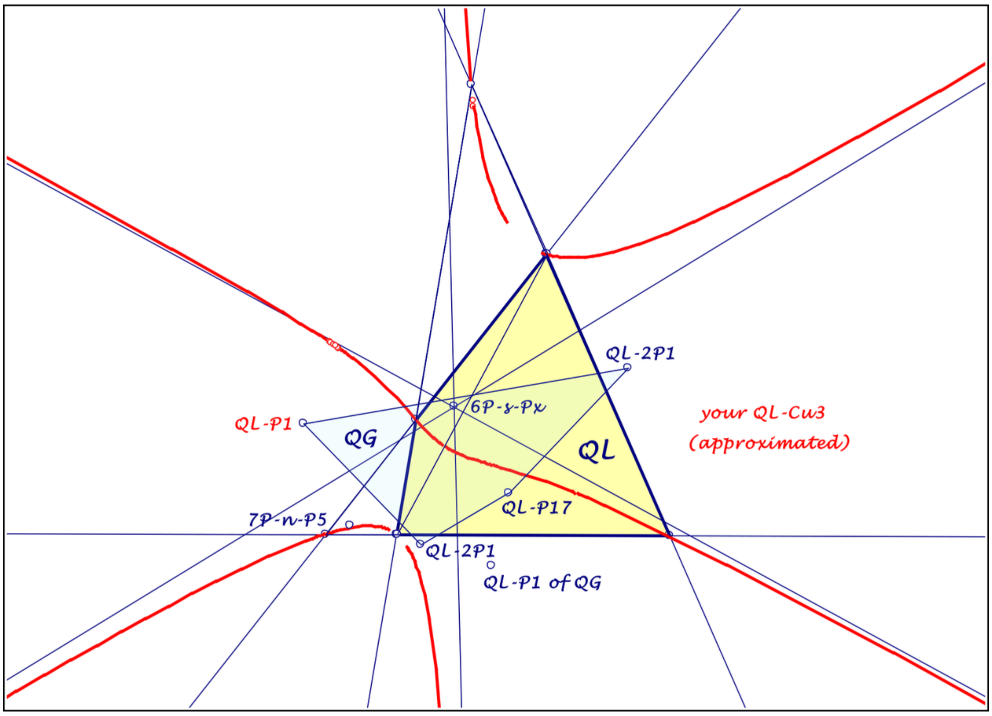
Message: #1495
Date: 2022-10-01
From: eckart_schmidt@t-online.de
Subject: Re: Some reflexions with QL-Qu3

Dear Bernard,

I try to remember the results of our observations
... wrt the cubic, circumscribed QL in terms of #1392.
This cubic bears not necessary QL-P1,
... is centered in 6P-s-Px (intersection of the 3 asymptotes).
The point 6P-s-Px is not 7P-n-P5 of the 6 QL-vertices and QL-P1.
7P-n-P5 is not a point on the cubic
... and not Miquel point of the QG of QL-P1, QL-P17 and the Plücker points,
... for there are mistakes in messages #1443 and #1444, excuse,
... which we didn't realize:
In #1443: 7P-n-Tf1 swaps not the Plücker points.
In #1444: 7P-n-P5 is not QL-P1 of the QG of QL-P1,QL-P17 and QL-2P1.
Wrt a construction of the cubic,
... I can only give an approximate drawing (attached).

Best regards Eckart

PS. I hope, this gives not new irritations.



2022-10-1.pdf

Message: #1496
Date: 2022-10-02
From: bernard.keizer@gmail.com
Subject: Re: Some reflexions with QL-Qu3

```
Dear Eckart,
On a contrary !
I'm rather released, as I couldn't reproduce the property that
the 3 points were the same !!!
In fact, you confirm that the 3 points are different and I thank
you for that.
It remains 1) my game with the 4 points QL-P1,P17 and 2P1 and
the construction of QL-Cu1 and QL-Qu3
2) the 6 QL vertices and their 6P-s-Px, center of the unique
circumscribed SC QL-Cu3
I reproduced already your approximate construction, but I would
be more satisfied with an exact construction
The fight goes on !
Best regards
Bernard
```

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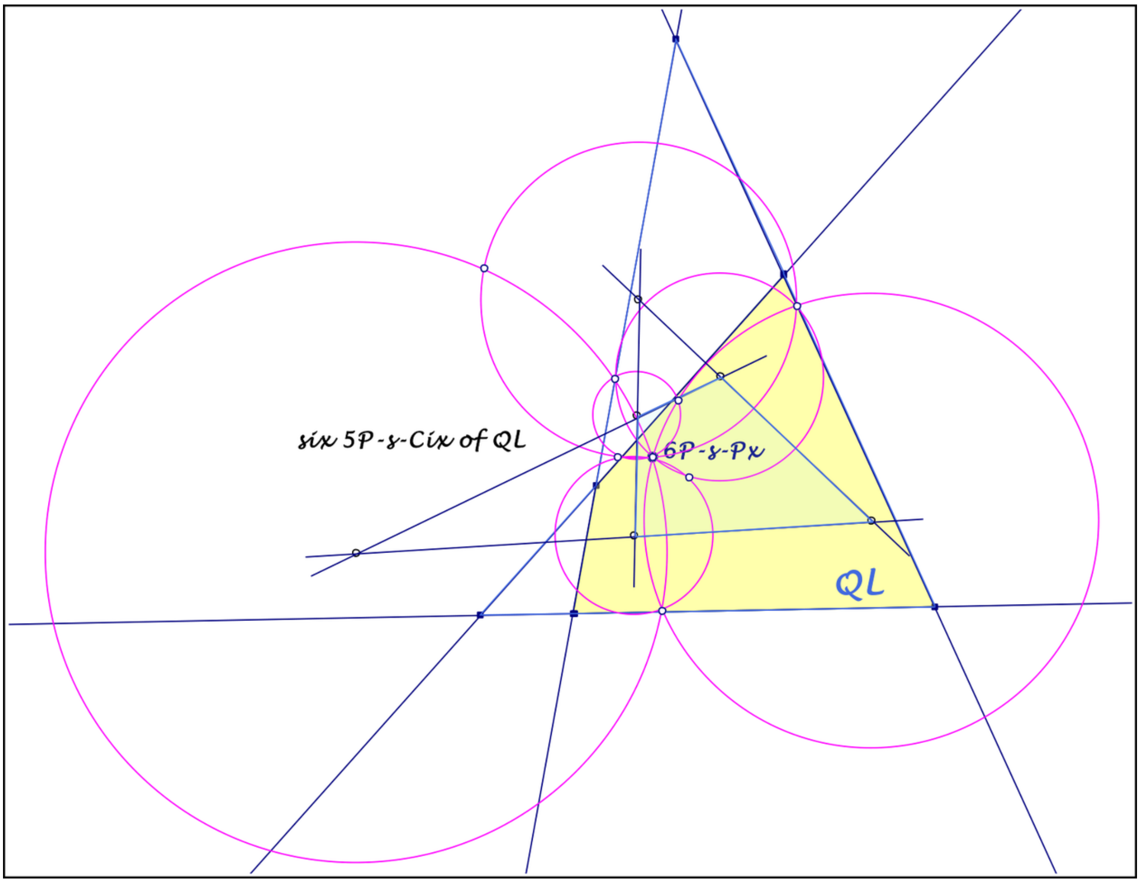
Message: #1497
Date: 2022-10-04
From: eckart_schmidt@t-online.de
Subject: Re: Some reflexions with QL-Qu3

```
Dear Bernard,

perhaps helpful to study your QL-Cu3:
The 6 circles 5P-s-Cix for the 5P-components of the 6 QL-points
(1)  ... have the common point 6P-s-Px (already mentioned),
(2)  ... have on each QL-line a triple intersection,
(3)  ... have three double intersections wrt the QG-components
         of QL as QG-points Q
         (see #1392(2)) dividing QA-P1.QA-P4 with ratio -1:4,
(4)  ... have centers, which give the vertices of a new QL ...

Best regards Eckart

PS. Wrt 5P-s-Px, 5P-s-Cix, 6P-s-Px see #1392,
... this elements seem relevant!
```



2022-10-03.pdf

Message: #1498
Date: 2022-10-04
From: bernard.keizer@gmail.com
Subject: Re: Some reflexions with QL-Qu3

```
Dear Eckart,
Of course I read your message 1392 several times !
I mentionned the properties 2) and 3) in my message 1429,
with the precision in 1430 that the triple intersections on each
QL side is the barycenter of the 3 vertices on this side.
4) is a consequence of 2) as the sides of the 2nd QL are the
perpendicular bisectors of the segments joining QL-s-Px to these
4 triple intersections.
What else can be said about this 2nd QL ?
Best regards
Bernard
```

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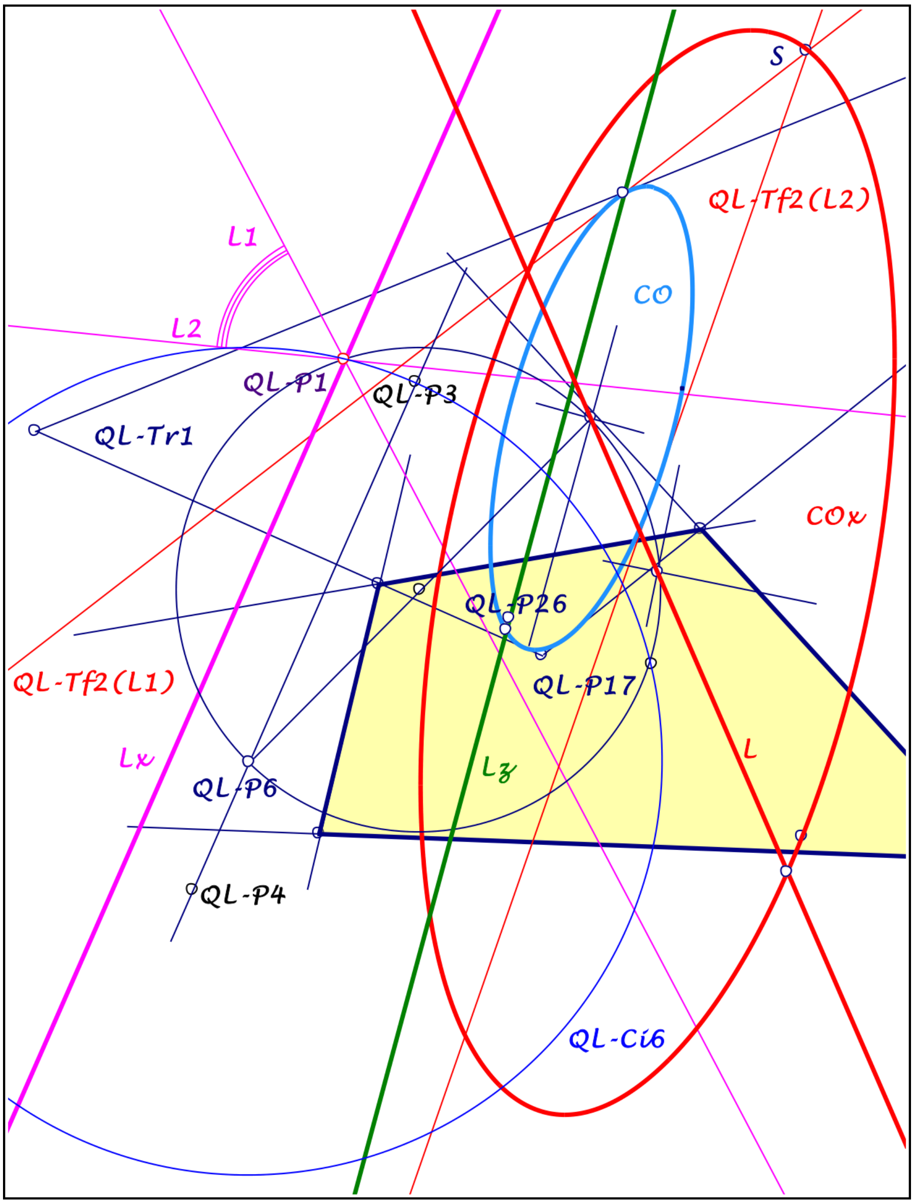
Message: #1499
Date: 2022-10-17
From: eckart_schmidt@t-online.de
Subject: New aspect in QL-geometry

```
Dear all,

let us start with a quadrilateral QL and lines L through the
Miquel point
... and study an application of QL-Tf2, which maps lines to
    lines.
The images QL-Tf2(L) envelope a wellknown conic CO (see QFG
old#481),
... inscribed QL-Tr1, tangent to the Steiner axes and QL-L2.

Now let us consider any pairs of lines L1, L2 through QL-P1
... with a constant angle of intersection,
... and look for the locus of the intersections S =
QL-Tf2(L1)^QL-Tf2(L2).

If the angle is 90°, the locus is a line Lx
... through QL-P1 parallel to QL-P3.QL-P4.

Else the locus of S is a conic COx,
... centered on a line L (new), bearing the center of CO,
... which lies diametral to QL-P6 on the circumcircle of
    QL-P1,6,17,
... and the pole of Lx wrt CO,
... which is CSC of the diametral point of QL-P1 on QL-Ci6.
... L intersects COx in two points with tangents parallel Lx.

The line L is independent of the angle between the lines L1 and
L2,
... also the polar Lz (new) of QL-P1 wrt COx is a fixed line,
... CSC of QL-Ci6, bearing QL-P26.

Perhaps some interesting new elements, attached a drawing..

Best regards Eckart
```



2022-10-17.pdf

Message: #1500
Date: 2022-10-18
From: eckart_schmidt@t-online.de
Subject: Re: New aspect in QL-geometry

Dear all,

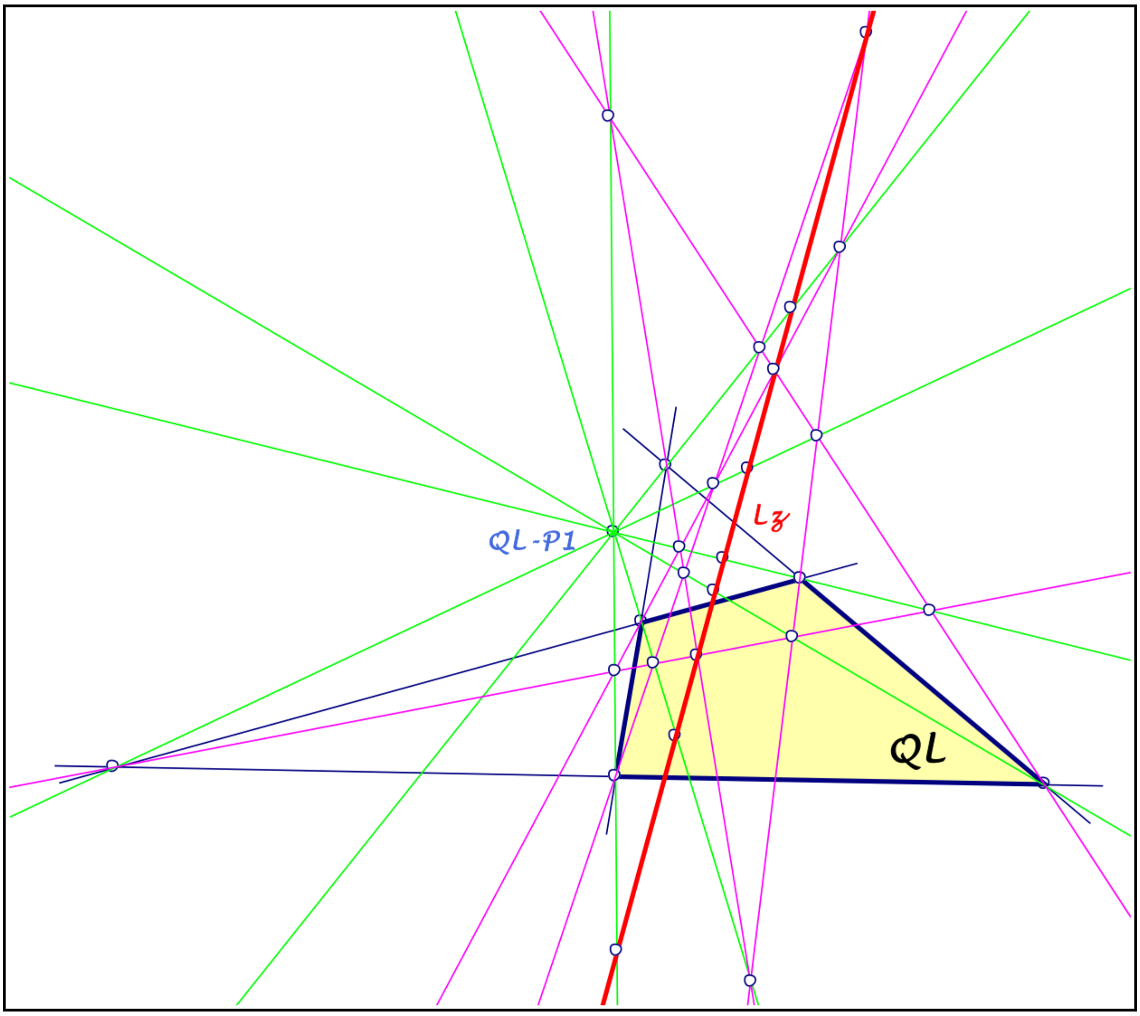
two further observations to my last message:

(1) The line Lz in #1499 with another construction:
Connect QL-P1 with lines Li to the 6 vertices Pi of QL
... and consider the image lines Mi = QL-Tf2(Li),
... Mi and Mj of opposite QL-points Pi and Pj intersect on Lz,
... others intersect on the first lines Li,
... two times on each of the 6 lines.
The fourth harmonic points of QL-P1
... wrt these two points are also collinear on Lz,
... finally Lz bearing 9 intersections (attached).

(2) Let us consider 60°-line pencils of three lines through QL-P1,
... their QL-Tf2-image lines give triangles,
... whose vertices and the vertices of QL-Tr1 lie on a conic.
The loci of triangle points often give conics,
.. examples: loci for X(2), X(3), X(4) are conics,
... but the isodynamic points X(15) and X(16) give two circles
... and the Fermat points X(13) and X(14) give also two circles,
... which contact in QL-P1 with tangent through QL-P4.

Certainly there are more interesting loci of triangle points.

Best regards Eckart



2022-10-18.pdf

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Message: #1501
Date: 2022-10-19
From: eckart_schmidt@t-online.de
Subject: Re: New aspect in QL-geometry

```
Dear all,

in #1499 and #1500 I described a line Lz wrt QL-P1 for a QL,
... here is a simpler view and a generalization:
Consider for a QL and a point P
... the 6 lines Li,j =  P.Li^Lj = P.Sij from P to the vertices
Sij of the QL,
... the QL-Tf2-images of these lines to opposite QL-points
... intersect collinear on Lz.

If we take a quadrangle with its 3 QG-components,
interpreted as QL,
... the three lines Lz for a point P have a common point in
    QA-Tf2(P).
Or:
For a point P and a QL the line Lz
... is the line of the collinear QA-Tf2(P) for the 3
    QG-components.

Best regards Eckart
```

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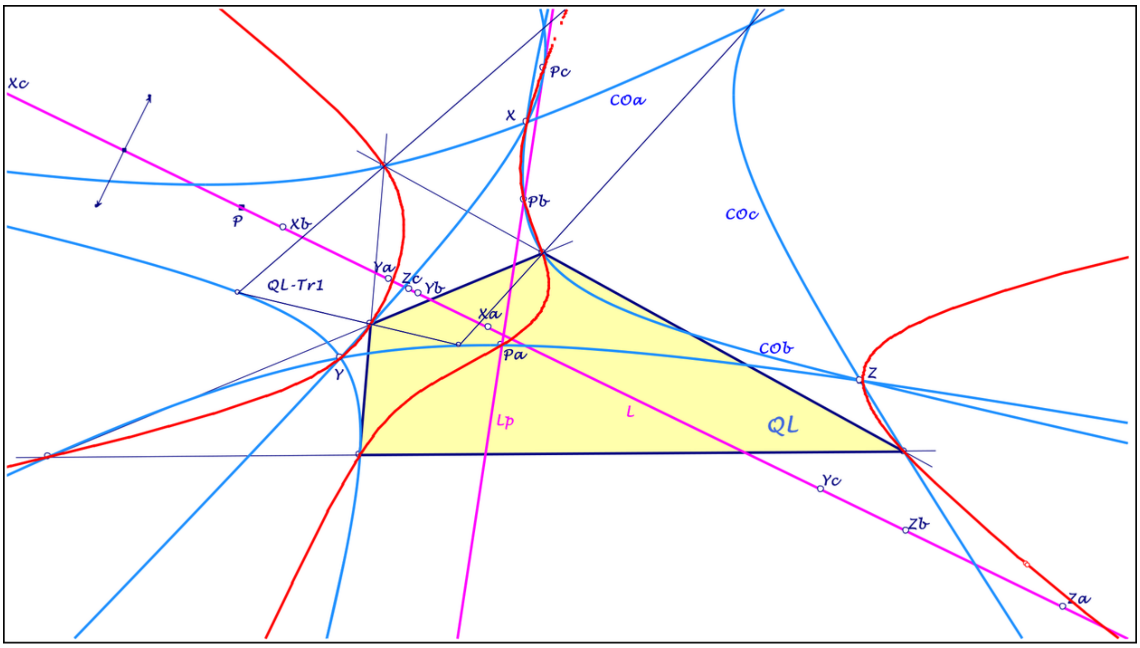
Message: #1502
Date: 2022-10-22
From: eckart_schmidt@t-online.de
Subject: Re: New aspect in QL-geometry

Dear all,

may I invite you to a short round tour in QG-/QA-/QL-geometry?
Let us start with a QL and its QG-versions QGa,b,c
... and consider a point P and a line L through P,
... the QA-Tf2-images Pa,b,c of P wrt QGa,b,c are collinear
on a line Lp,
... QGa-vertices and Pb,c and P lie on a conic ...
The QA-Tf2-images of L wrt QGa,b,c are conics COa,b,c with three
triple intersections X, Y, Z.
The QA-Tf2-images of X, Y, Z wrt QGa,b,c give 9 collinear points
Xa,b,c, Ya,b,c, Za,b,c on L.
Rotating L round P the locus of X, Y, Z is a cubic CU,
... bearing the 6 QL-vertices and the points Pa,b,c.
This QL-cubic is the locus for points,
... whose QA-Tf2-images wrt QGa,b,c are collinear with the
defining point P.
Example: For P = QL-P1 the points Pa,b,c are QG-P16a,b,c.
Best regards Eckart

PS: The double intersections of COa,b,c
... lie collinear on the sidelines of QL-Tr1.



2022-10-22.pdf

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Message: #1503
Date: 2022-11-03
From: eckart_schmidt@t-online.de
Subject: QL-L2

```
Dear Chris,

is the following observation well known?
QL-L2 is the locus of points,
... which are the radical center
... of their 3 nine-point circles
... wrt opposite QL-vertices.

Best regards Eckart
```

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Message: #1504
Date: 2022-11-04
From: van10hoven@gmail.com
Subject: Re: QL-L2

```
Dear Eckart,
I do not understand your remark.
Could you explain what you mean with "nine-point circles wrt
opposite QL-vertices".
Since you mention nine-point circles I miss wrt what triangles.
Since you talk about a locus it should be variable triangles.
I think I miss the meaning of your remark.
Best regards,
Chris
```

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Message: #1505
Date: 2022-11-04
From: eckart_schmidt@t-online.de
Subject: QL-L2

```
Dear Chris,

excuse my short formulation, I try it once more:

Let X be a point and Y, Z a pair of opposite vertices of a QL,
... consider the three triangles XYZ
... and their nine-point circles with radical center S.
The locus of X with X = S is QL-L2.

Best regards Eckart
```

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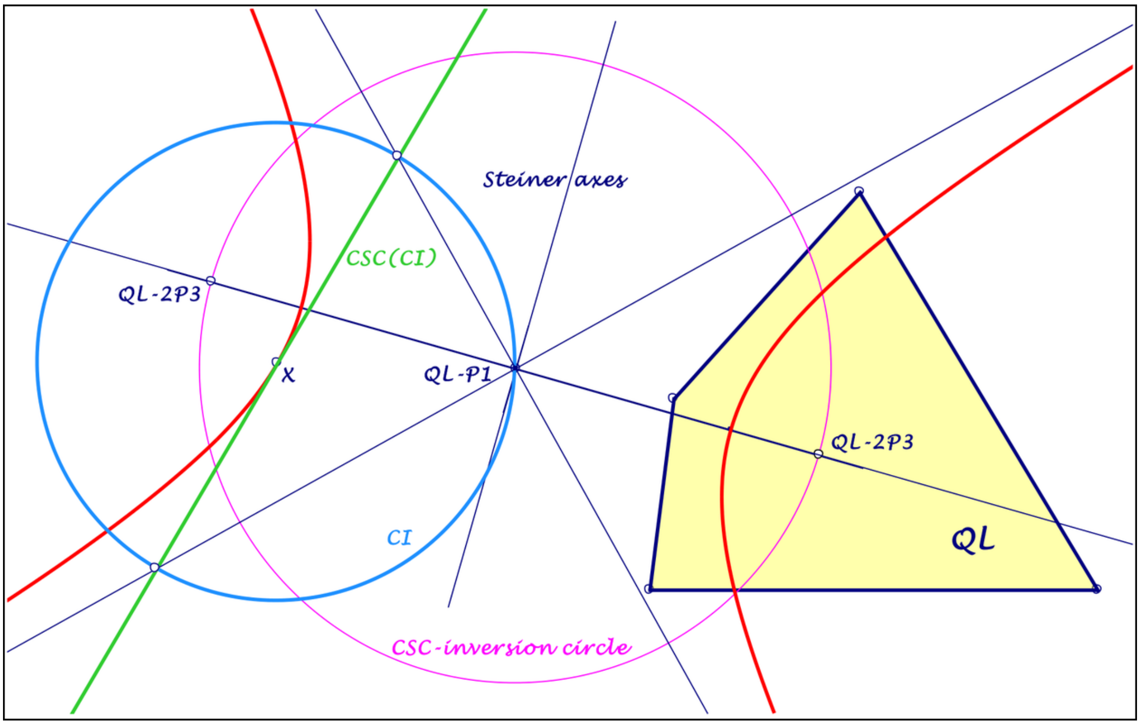
Message: #1506
Date: 2022-11-05
From: eckart_schmidt@t-online.de
Subject: Curious orthogonal QL-hyperbola

```
Dear all,

the locus of centers X for circles through QL-P1,
... whose CSC-line bears X, is an orthogonal hyperbola,
... centered in QL-P1 with Steiner axes,
... and the fixed CSC-points QL-2P3 as foci.
The tangents at this hyperbola
... intersect their CSC-circles on the asymptotes.

Best regards Eckart
```



2022-11-05.pdf

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Message: #1507
Date: 2022-11-05
From: eckart_schmidt@t-online.de
Subject: Re: Curious orthogonal QL-hyperbola

```
Dear all,
the orthogonal QL-hyperbola is the CSC-image
... of the lemniscate for QL-2P3.
Best regards Eckart
```

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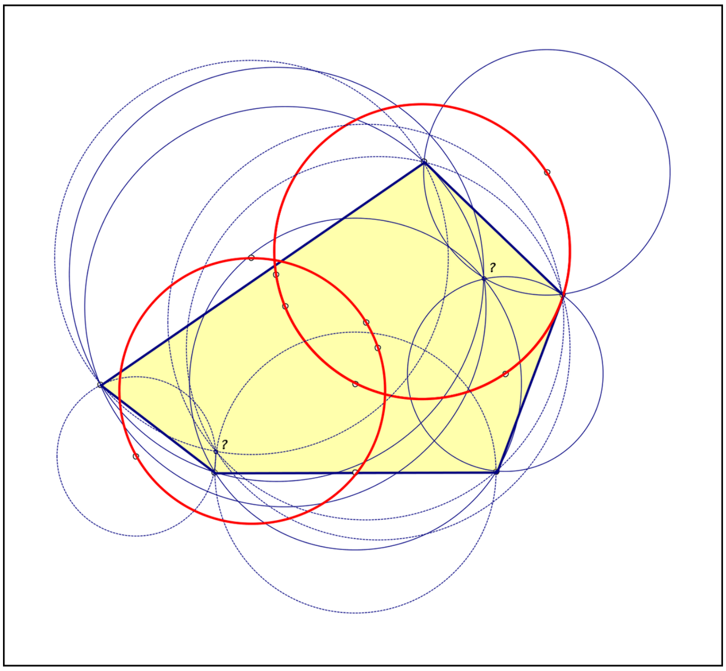
Message: #1508
Date: 2022-11-07
From: eckart_schmidt@t-online.de
Subject: 5G-s-2Px?

```
Dear all,
if we consider for a QG = P1,2,3,4 and a point P
... circles through P and two succeeding vertices Pi, Pi+1
... and ask for concyclic centers for the circles,
... we get P on QL-Cu1 (see EQF).
If we consider a 5G, there are only two points P
... with concyclic centers of the circles (P,Pi,Pi+1).
What about these two points (see attached)?
Best regards Eckart
```



2022-11-07.pdf

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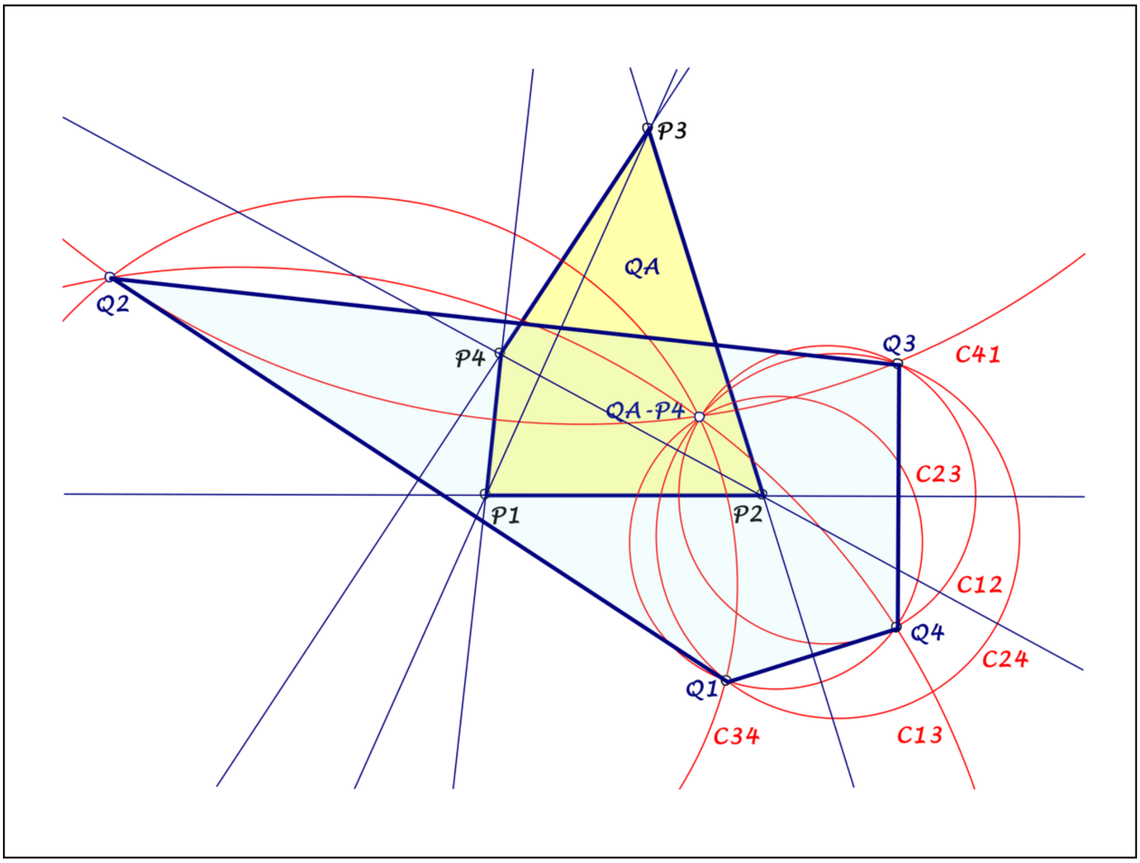
Message: #1509
Date: 2022-11-09
From: eckart_schmidt@t-online.de
Subject: QA-P4

```
Dear all,

this is a further unexpected property of the main QA-point
QA-P4:
Consider for a QA = P1,2,3,4 (see attached)
... the 6 Apollonius circles Ci,j through QA-P4 wrt Pi, Pj,
... which have 4 triple intersections Qi of Cj,k, Ck,l, Cj,l,
... the bisectors of QiQj
    are the lines PkPl of the reference QA.

Best regards Eckart
```



2022-11-09.pdf

Message: #1510
Date: 2022-11-13
From: james.cooper@jku.at
Subject: Brahmagupta

```
The classical Brahmagupta formula (which we write in the form
$16 F^2=(-a+b+c+d)(a-b+c+d)(a+b+c-d)(a+b+c-d)$) for the area of
a cyclic quadrilateral is only valid in the convex case.
However, if we add an additional term $-16abcd$ to the right
hand side, we obtain one which is valid in the non convex case.
This is, presumably, known. Can anybody provide a reference?
```

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Message: #1511
Date: 2022-11-14
From: van10hoven@gmail.com
Subject: Re: QL-L2

```
Dear Eckart,
Thanks for clarification.
I think this feature is new.
Best regards,
Chris
```

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Message: #1512
Date: 2022-11-14
From: bernard.keizer@gmail.com
Subject: Droz-Farny theorem for QL, 5L and QA

Dear Chris, dear Eckart
I've just discovered the Droz-Farny theorem.
For a triangle, consider a point P, it's isogonal P' and the projections p1, p2 and p3 of P on the 3 sides.
Draw the 3 circles through P' centered in p1, p2 and p3 ; each circle intersects the corresponding side in 2 points.
The 6 intersections are on a circle centered in P.
Doing the same with P' gives a 2nd circle centered in P' ; both circles have the same radius.
For a QL, any point P on QL-Cu1 has concyclic projections on the 4 sides, the center of the circle being the CSC partner on QL-Cu1 and isogonal conjugate of P wrt the 4 triangles of the QL. We get 2 circles centered in P and P' with the same radius, each through 8 points.
For a 5L, only the 2 foci of the inscribed conic have this property, being the centers of 2 circles with the same radius, each through 10 points.
(the 2 points are the intersection of 5 QL-Cu1).
For a QA, we deal with 6 lines and 3 QL-Cu1 of the 3 QL's of the QA. The 3 QL-Cu1 intersect in the 4 vertices of the QA and in the circular points and have dobble intersection in the vertices of the DT, but the 2 last intersections are not triple.
Best regards
Bernard

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Message: #1513
Date: 2022-11-15
From: bernard.keizer@gmail.com
Subject: A gem found in the Olympiade

Dear Chris, dear Eckart
Problem 6 in the Olympiade in 2011 in Amsterdam
Given a triangle XYZ, a point P on it's circumcircle and the tangent l in P to this circumcircle.
The reflexions of l in the sides of XYZ determine a triangle ABC ; the Steiner Line of P wrt XYZ cuts the sides of ABC in A',B' and C'.
The QL formed by the sides of ABC and the line A'B'C' has a point M as Miquel point.
The 3 circles of the QL (apart of ABC) pass respectively through the points X, Y and Z (AB'C' through X ...).
The figur has several beautiful properties :

* CSC(P) is on the circumcircle of XYZ
* PCSC(P) is parallel to A'B'C'
* XYZ and ABC are in perspective with perspector CSC(P)
* the circumcircles of ABC and XYZ are tangent in M

Best regards
Bernard

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Message: #1514
Date: 2022-11-21
From: eckart_schmidt@t-online.de
Subject: Round QG-P5

```
Dear all,

may I invite you to an excursion in QG-geometry
... round QG-P5, which is the intersection of QG-diagonal
    bisectors.
Let us consider "inversion circles" (X,Y,Z) of two points X, Y
wrt a third point Z,
... that means circles centered in Z with radius sqr(XZ*YZ).
Not in EQF: The 4 inversion circles (Pi,Pi+1,QG-P5) coincide for
a QG,
... which is a new QG-Cix ...

(1) This leads to four Möbius transformations (attached),
... centered in QG-P5, swapping Pi and Pi+1,
... which give for a point P four concyclic images
    on a circle CI(P),
... centered in QG-P5 through the inverse of P wrt QG-Cix.
The image QG of a point wrt the 4 Möbius transformations
... has parallel diagonals and the same bisector
    for opposite vertices,
... the image QGs for all points P are similar.
The circles CI(Pi) for QG-vertices Pi
... bear the neighbored vertices Pi-1, Pi+1
    and the inverse of Pi wrt QG-Cix.

(2) For points P on a line L the Möbius transformations give 4
circles
... with the same radius through QG-P5, centered on a circle
... which is CI(Q) for Q reflection of QG-P5 in L.
The two pairs of opposite circles
... define with their 2nd intersections a line L1
... which is the same for parallel lines L.
The four pairs of neighbored circles
... have collinear 2nd intersections on a line L2,
    perpendicular L1.

(3) For lines L through a fixed point P
... the locus of the intersections L^L1
    is an orthogonal hyperbola HY(P)
... through P, centered in the midpoint of P.QG-P5.
The locus for intersections of L1 and L2 is a line L(P).
The lines L2 envelope a parabola PB(P) with focus QG-P5
... and apex tangent L(P).
```

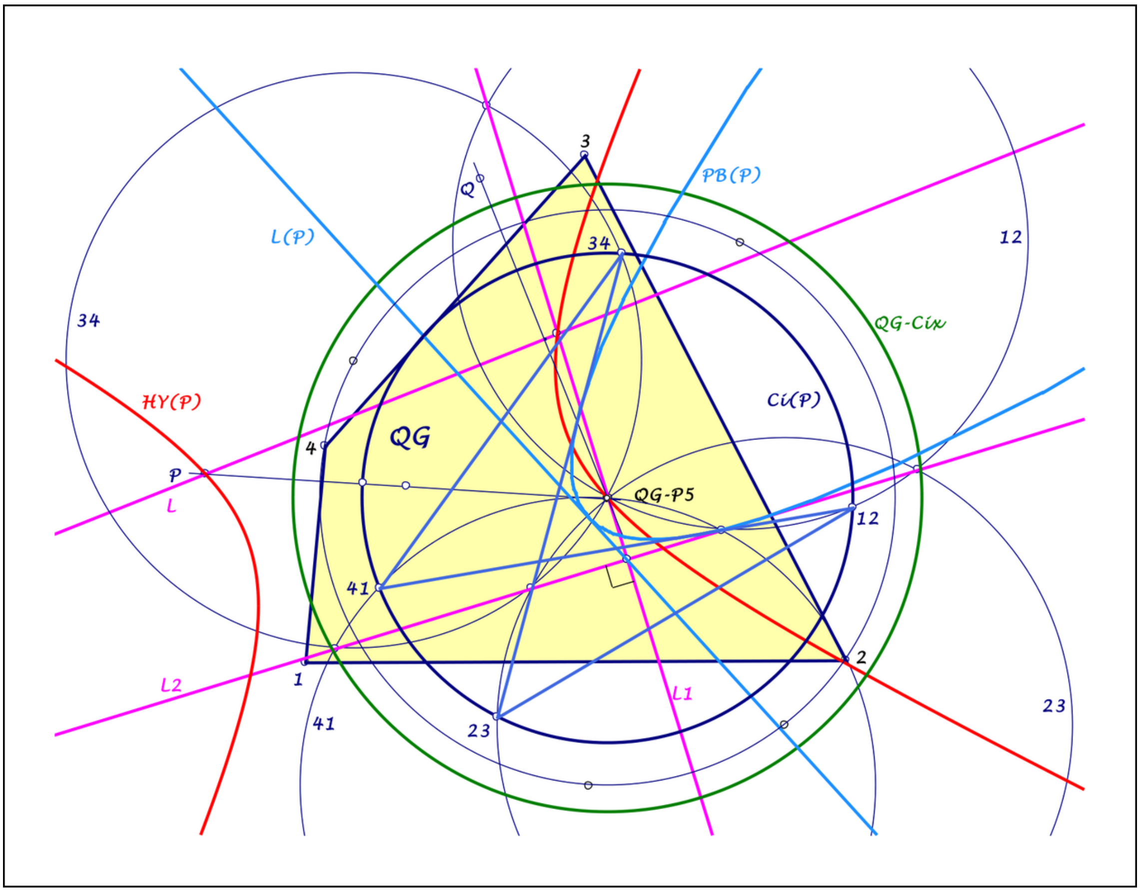
(4) Finally: For a QL there are 3 orthogonal hyperbolas
    HY(QL-P1)
... with 3 further triple intersections (attached),
... whose circumcircle bears the 3 QG-P5,
... orthogonal intersecting QL-Ci1 in QL-P16 and QL-P17.
For a QL there are 3 lines L(QL-P1) with a common point.

(5) There will be more properties with QL-geometry, example:
The line QL-L1 generates three lines L2,
... intersecting on QL-Ci1 diametral to QL-P16 on
    QL-P1.QL-P8.QL-P24,
... each bearing a vertex of QL-Tr1.

What about these new QL-points?

Best regards Eckart

PS: I hope, there will be no fake observations!



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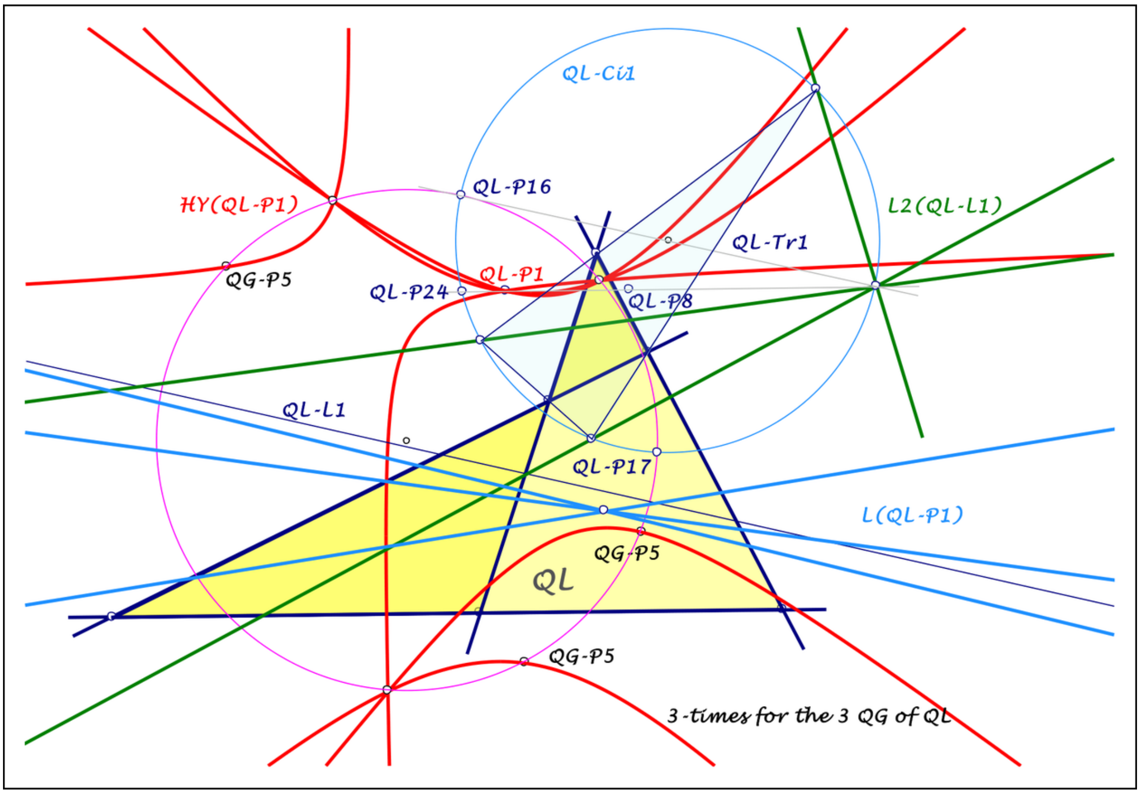


2022-11-21b.pdf

Message: #1515
Date: 2022-11-27
From: bernard.keizer@gmail.com
Subject: Re: Round QG-P5

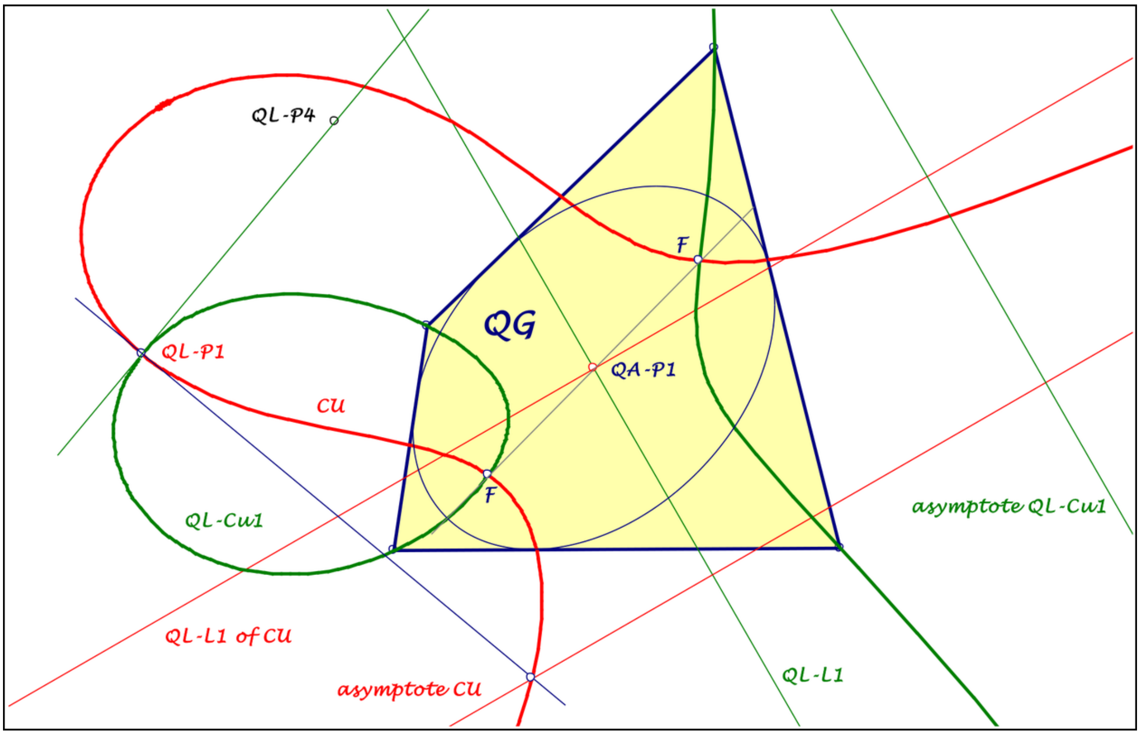
```
Dear Eckart,
I checked almost all your beautiful properties.
Nice work, indeed !
Congratulations
Best regards
Bernard
```

Message: #1516
Date: 2022-11-28
From: eckart_schmidt@t-online.de
Subject: New QG-cubic

```
Dear Bernard,

thanks for your interest in "inversion circles",
... here is another application, an addition to message 1483:
Let us once more consider "inversion circles" (P,X,Y),
... that means circles round P with radius sqr(PX*PY),
... especially for a QG round QL-P1 wrt opposite vertices X, Y,
... and ask for the locus of points,
    whose two inversion circles coincide.
The locus is a circular cubic CU through QL-P1, CSC-invariant,
... constructible as "QL-Cu1", CSC-invariant with Newton line
... ... through QA-P1 orthogonal to the reference QL-L1.
The orthogonal intersections of CU and QL-Cu1 unequal QL-P1
... are the foci F of the inscribed conic of QG,
    centered in QA-P1.
A perpendicular to the QL-Cu1-tangent QL-P1.QL-P4 in QL-P1
... intersects CU on its asymptote.

Best regards Eckart
```



2022-11-27.pdf

Message: #1517
Date: 2022-11-28
From: bernard.keizer@gmail.com
Subject: Re: New QG-cubic

```
Dear Eckart,
Very interesting this 2nd circular focal cubic with the same
QL-Cu1, the same CSC and Newton Line and asymptote perpendicular
to those of QL-Cu1 !
Best regards
Bernard
PS You continue to accumulate material for EQF ...
```

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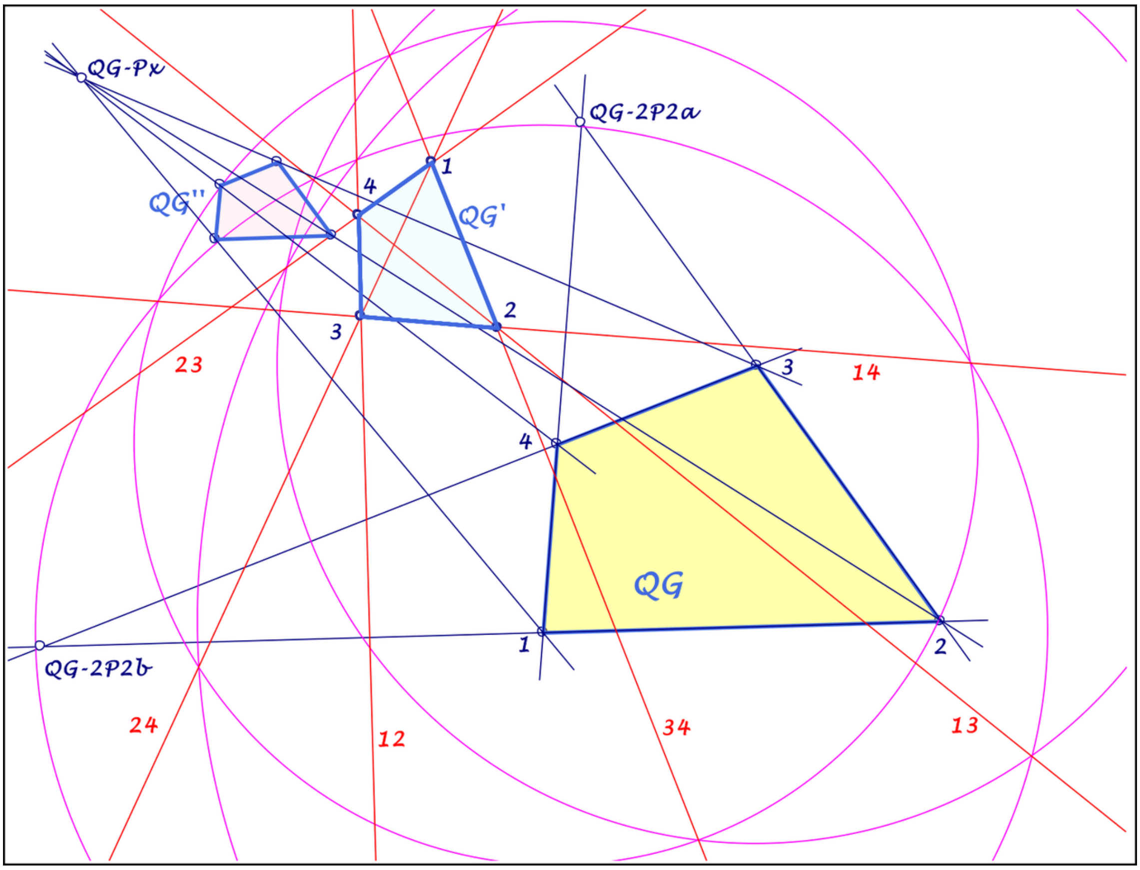
Message: #1518
Date: 2022-11-29
From: eckart_schmidt@t-online.de
Subject: New QG-point

```
Dear Bernard,

a further offer for your interest:

Let us once more consider "inversion circles" (P,X,Y),
... that means circles round P with radius sqr(PX*PY),
... especially for a QG round QG-vertices Pi wrt QG-2P2,
... and look for the radical axes Lij
    of corresponding inversion circles.
The triple intersections Qi = Ljk^Lkl^Llj define a new quadrigon
QG',
... the 2nd new quadrigon QG''
    is perspective to the reference QG.
What about the perspector of QG and QG''?

Best regards Eckart
```



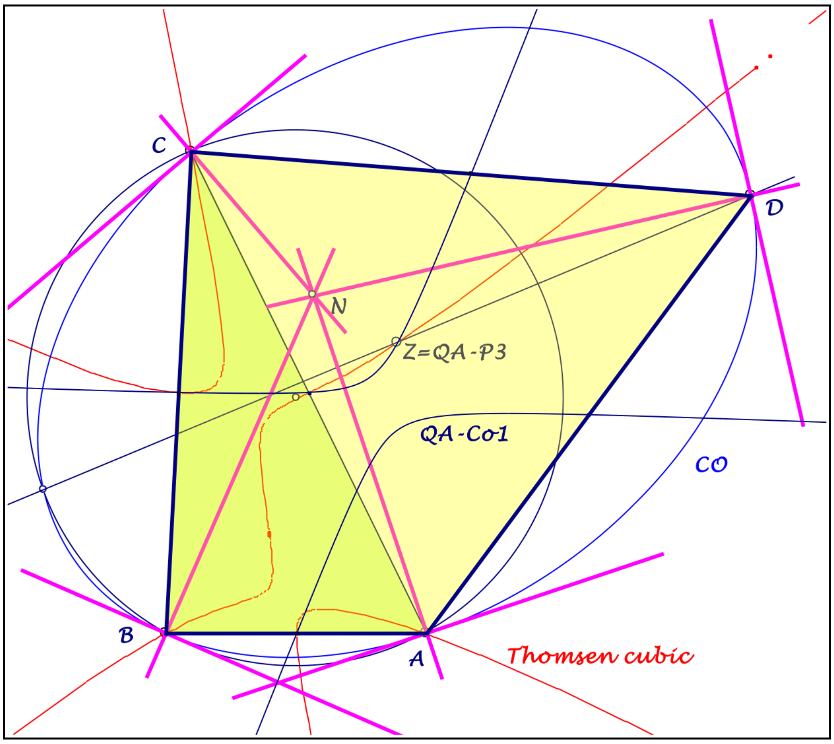
2022-11-28.pdf

Message: #1519
Date: 2022-11-29
From: eckart_schmidt@t-online.de
Subject: N-Vierecke

```
Dear all,

perhaps someone is interested in special quadrangles:
Let N-QA be quadrangles, which have a circumconic CO,
... whose normals  for the vertices have a common point N.
Possible construction (attached):
Three points A, B, C can be complemented to a N-QA,
... choosing a conic center Z for CO on the Thomson cubic,
... reflecting in Z the 4th intersection S of the cubic
    and the triangle circumcircle,
... to get the 4th vertex D of a N-QA  ABCD.
Property:
A quadrangle is exact then a N-quadrangle,
... if QA-Co2 bears QA-P3, the center of the circumconic CO.
The points Z = QA-P3 and N of a N-QA
... lie on the cubic QA-Cu1 as intersections with QA-Co2
... collinear with QA-P11 and QA-P41.
Best regards Eckart

PS.
This is a result of correspondence with Roland Stärk in 2003.
```



2022-11-29.pdf

Message: #1520
Date: 2022-11-30
From: bernard.keizer@gmail.com
Subject: Re: New QG-cubic

Dear Eckart,
I suppose with this construction there are for an ordinary QL 3 such focal circular cubics orthogonal to QL-Cu1 in 3 points QL-P1 and the foci of the inscribed conics in the 3 QAs centered in the 3 QA-P1. The 3 cubics are tangent in QL-P1 and have parallel Newton Lines and asymptotes, orthogonal to QL-L1.
Best regards
Bernard

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Message: #1521
Date: 2022-12-01
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A circle passes through QA-P1 and

---

```
Dear geometers,

Let P1.P2.P3.P4 be a Quadrigon.
S1 = P3P4 meets P1P2
S2 = P1P3 meets P2P4
S3 = P1P4 meets P2P3.
Mij is the midpoint of PiPj.

We obtain
- Circles (S1,M12,M34), (S2,M13,M24), (S3,M14,M23)
  go through QA-P3.
- Let X1, X2, X3 be the centers of circles (S1,M12,M34),
  (S2,M13,M24), (S3,M14,M23), reps,
  then circle (X1,X2,X3) goes through QA-P1.
  Is it a new QA circle?
- QA-P3 is the orthocenter of triangle X1X2X3.
- Let Y1, Y2, Y3 be the intersections (other than QA-P3)
  of pairs circles
  (S2,M13,M24), (S3,M14,M23); (S3,M14,M23), (S1,M12,M34);
  (S1,M12,M34), (S2,M13,M24).
Line Y1Y2 meets the circles (S2,M13,M24), (S1,M12,M34) again at
Z32, Z31,
resp. Let d3 be the perpendicular bisector of segment Z32Z31.
Define similarly lines d2 and d1. Then lines d1, d2, d3 are
concurrent at a QA-point. Which is this point?

Sincerely yours,
Tran Quang Hung
```

Message: #1522
Date: 2022-12-02
From: eckart_schmidt@t-online.de
Subject: Re: A circle passes through QA-P1 and concurrent lines

---

```
Dear Tran Quang Hung,

with interest I have reproduced your constructions.
Your points Xi are QG-P9
... of the quadrigon versions for a QA with vertices
    P1,P2,P3,P4.
Your circle through QA-P1 and 3 QG-P9
... is a new QA-circle, not in EQF.

Your result "QA-P3 is the orthocenter of triangle X1X2X3."
doesn't hold.
The orthocenter of X1X2X3 is the reflection of QA-P15 in QA-P5.

Best regards Eckart
```

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Message: #1523
Date: 2022-12-02
From: eckart_schmidt@t-online.de
Subject: Re: A circle passes through QA-P1 and concurrent lines

---

```
Dear Tran Quang Hung,

your first 4 circles are centered in X1,X2,X3, which are the
QG-P9 of the
QA-quadrigons,
... bearing QA-P1 and QG-P5 (see last message).
Your points Y1,Y2,Y3 are the Miquel points of the QA-quadrigons,
... the lines d1,d2,d3 are orthogonal to the sides
    of the Miquel triangle.
Your last point is new, not in EQF.

Best regards Eckart
```

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Message: #1524
Date: 2022-12-03
From: eckart_schmidt@t-online.de
Subject: QA-Tf16 as isoconjugation on QA-Cu1

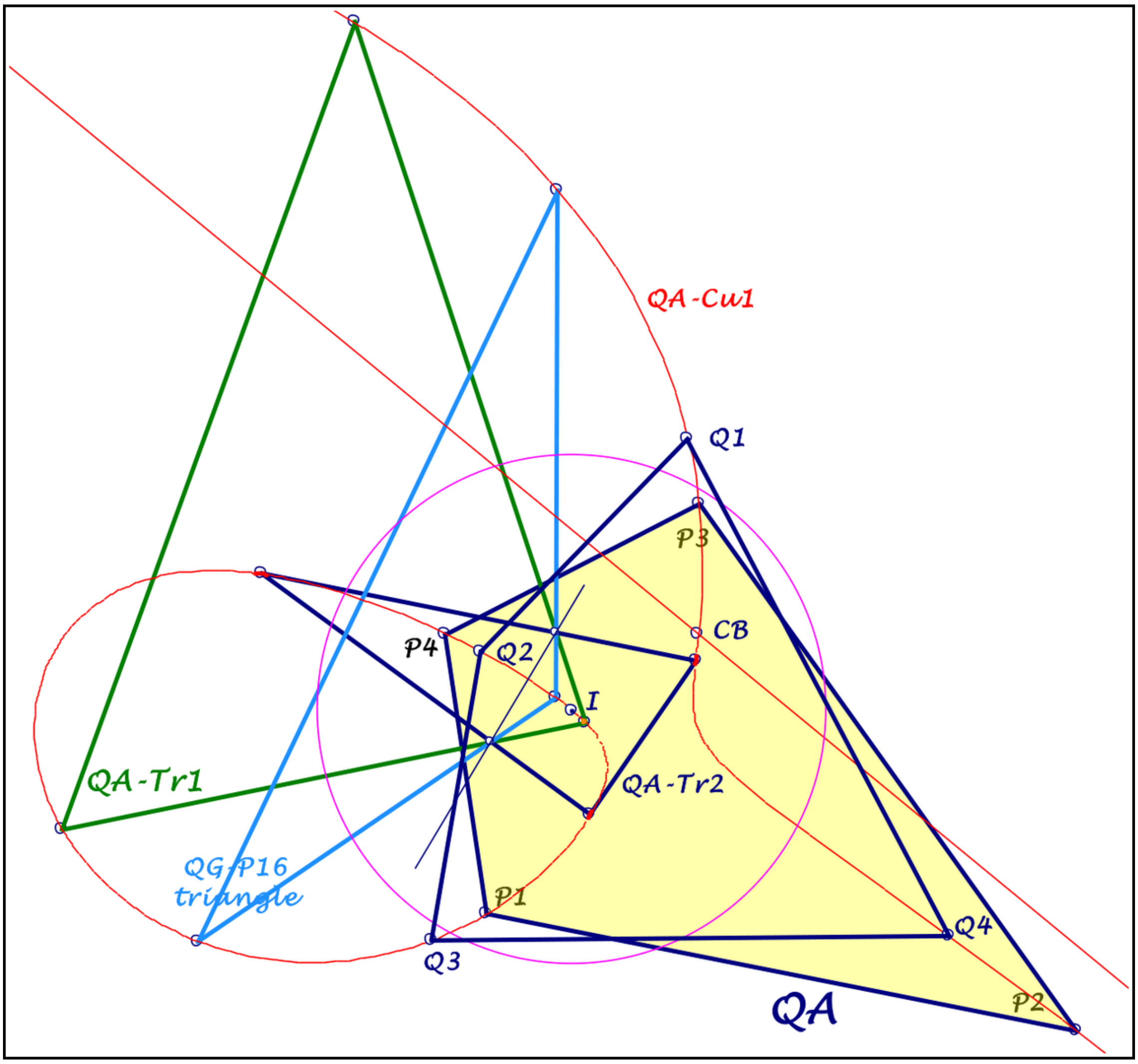
```
Dear all,

the main property of QA-Tf16 is,
... that it maps the diagonal triangle to the Miquel triangle.
There is also an isoconjugation with the same property:
Reference triangle is the QG-P16-triangle,
... fixed points are the 3rd intersections Qi of Pi.I and QA-Cu1
... with Pi vertex of QA and I incenter of the Miquel triangle,
... so it is QA-Tf2 for the quadrangle Q1Q2Q3Q4
... and coincides with QA-Tf16 for points on QA-Cu1.
This Q-quadrangle is 4 times perspective to the reference QA
... wrt the in- and ex-centers of the Miquel triangle.
For the incenter with Pi and Qi there is a common "inversion"
circle,
... the point Qi is the center-reflected inverse of Pi.
The Q-quadrangle has the same Miquel triangle and cubic QA-Cu1.
The Cayley-Bacharach point CB of P- plus Q-quadrangle
... is the intersection of the cubic QA-Cu1 and its asymptote.

The triangles QA-Tr1, QA-Tr2 and the QG-P16-triangle
... have three collinear triple intersections of their sides
... on the QA-Tr1-tripolar of the reflection of QA-P4 in QA-P3,
... also QA-Tr2-tripolar of the midpoint of QA-P3 and QA-P4.
The dual of this new QA-line - in terms of QA-8/QL8 -
... is a point Z on the QA-Cu1-asymptote,
... QA-Tf2 of the reflection of QA-P4 in QA-P3,
or: intersection of the QA-Cu1-asymptote and tangent in QA-P3 at
QA-Cu1.

Fascinating EQF-geometry!

Best regards Eckart
```



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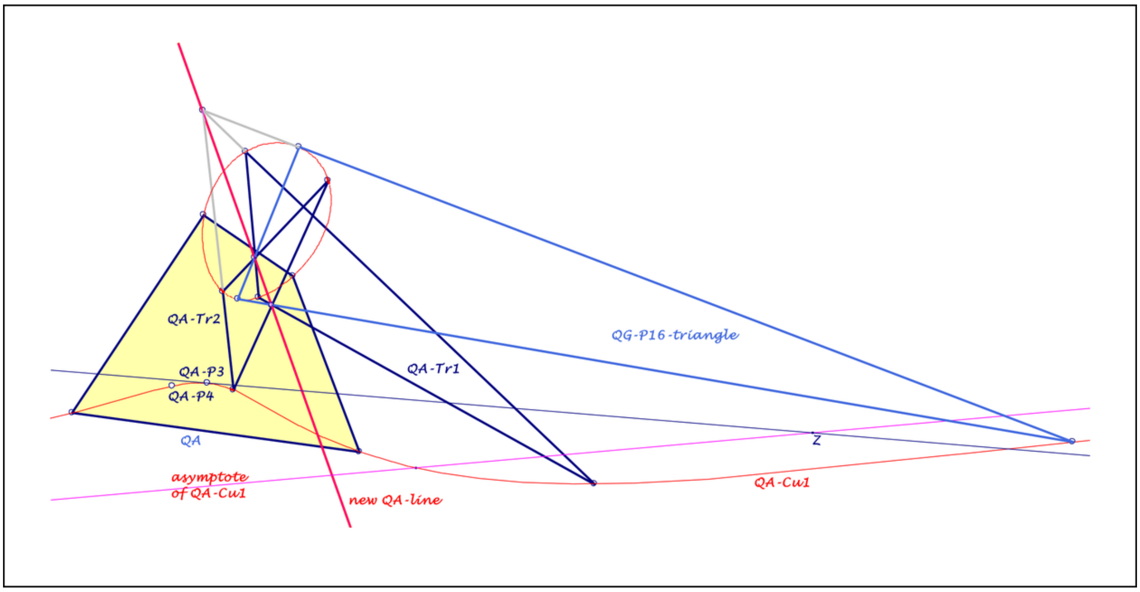

2022-12-02.pdf

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Message: #1525
Date: 2022-12-03
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf16 as isoconjugation on QA-Cu1

Dear Eckart,
The 12 points P vertices, Q vertices and in- and excenters of QA-Tr2 are in a Reye configuration.
The cubic QA-Cu1 contains the 12 points.
The Reye configuration extends for the tangentials of these points on the cubic, which gives P-QA-Tr1 with QA-P4, Q-QA-Tr1 with QA-P3 and QA-Tr2 with the infinity point of the asymptote...
One step more and you get the tangentials of P4, P3 and the infinity point of the asymptote, id est P41=tgP4, your point Z = tgP3 and the intersection of the cubic with it's asymptote, which is tg(infinity point). The 3 tangentials are aligned.
Best regards
Bernard

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Message: #1526
Date: 2022-12-04
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A circle passes through QA-P1 and

---

```
Dear Eckart,

Thank you very much for your contribution. Your knowledge is
amazing.
I'm happy to find a new circle and a new point in QA.

Sincerely yours,
Tran Quang Hung

Vào Th 6, 2 thg 12, 2022 vào lúc 22:50 Eckart Schmidt <
eckart_schmidt@t-online.de> đã viết:

> Dear Tran Quang Hung,
>
> your first 4 circles are centered in X1,X2,X3, which are the
QG-P9 of the
> QA-quadrigons,
>
> ... bearing QA-P1 and QG-P5 (see last message).
>
> Your points Y1,Y2,Y3 are the Miquel points of the
QA-quadrigons,
>
> ... the lines d1,d2,d3 are orthogonal to the sides of the
Miquel triangle.
>
> Your last point is new, not in EQF.
>
> Best regards Eckart
>
```

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Message: #1527
Date: 2022-12-04
From: eckart_schmidt@t-online.de
Subject: Re: A circle passes through QA-P1 and concurrent lines

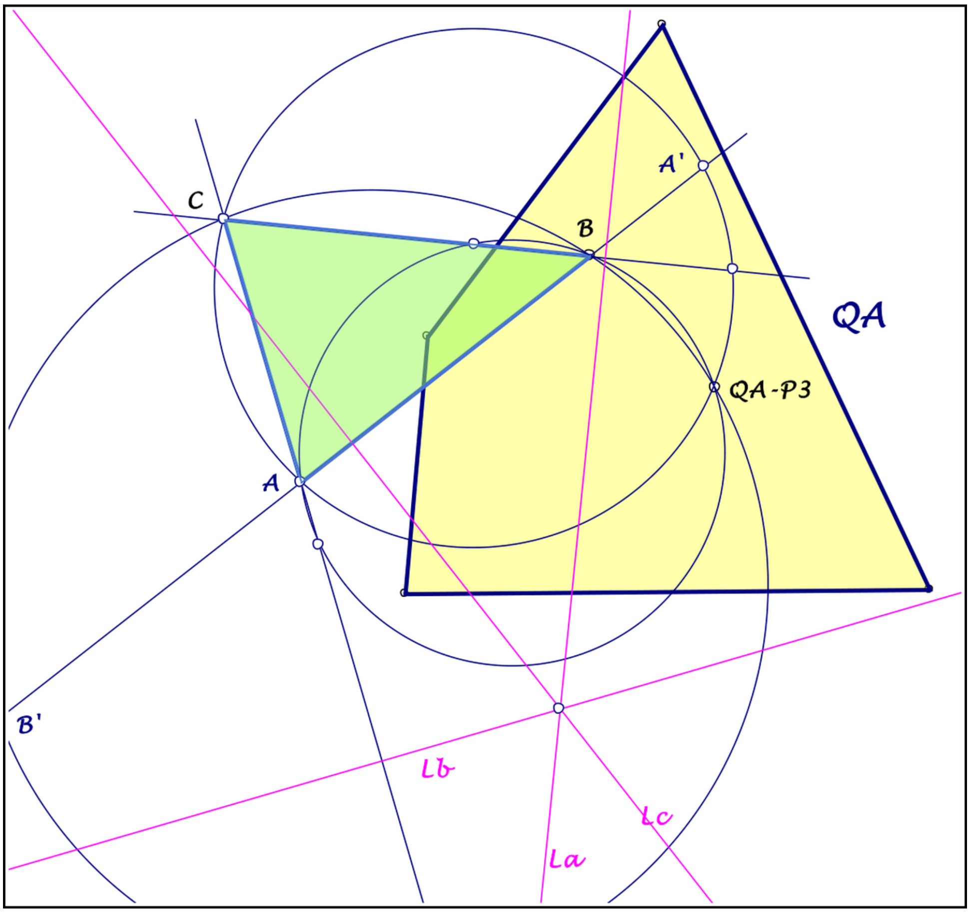
---

```
Dear Tran Quang Hung,

there is a generalization of your construction attached,
... replacing the Miquel triangle by any triangle,
... to get a point for a triangle wrt a reference quadrangle.

Let ABC be any triangle and QA a reference quadrangle,
... consider the circles (A,B,QA-P3), (B,C,QA-P3), (A,C,QA-P3)
... and the bisector Lc of the intersections A', B'
    unequal QA-P3
... of the line AB with the circles (A,C,QA-P3), (B,C,QA-P3),
... analog for the bisectors La, Lb ...
... then the bisectors have a common point,
... defined by a triangle wrt a reference quadrigon.

Best regards Eckart
```



2022-12-04.pdf

Message: #1528
Date: 2022-12-05
From: eckart_schmidt@t-online.de
Subject: Re: A circle passes through QA-P1 and concurrent lines

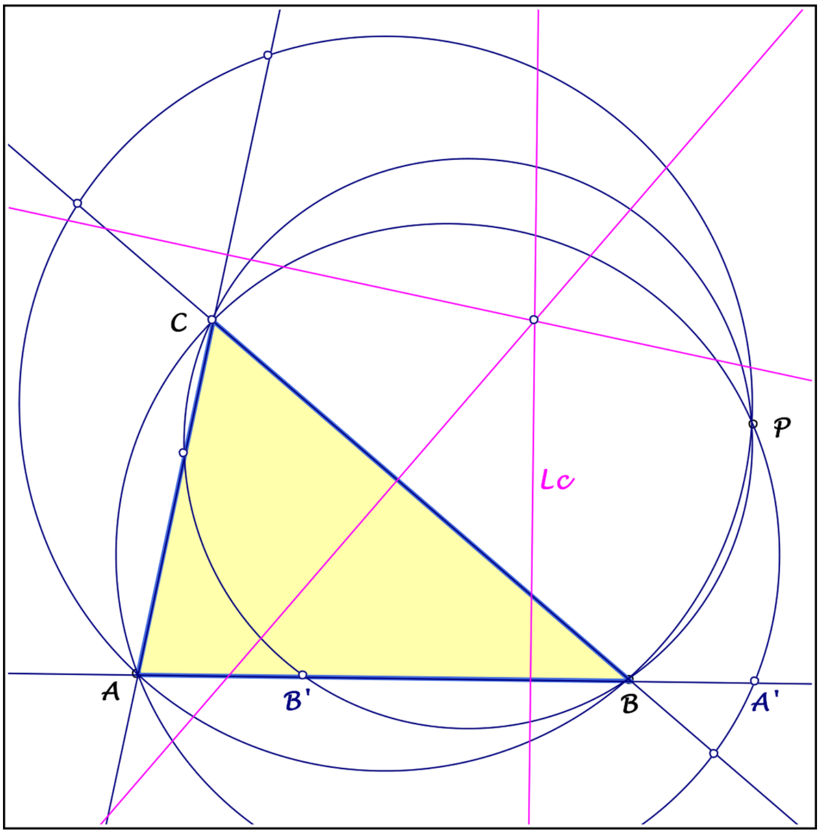
---

Dear Tran Quang Hung,

excuse, there is an evident further generalization of your construction,
... replace in my last message QA and its QA-P3 by any point P,
... then the following construction gives an image point Q for P wrt the triangle,
... independent of any quadrigon, only triangle geometry.

Let ABC be a triangle and P an arbitrary point,,
... consider the circles (A,B,P), (B,C,P), (A,C,P)
... and the bisector Lc of the intersections A', B' unequal P
... of the line AB with the circles (A,C,P), (B,C,P),
... analog for the bisectors La, Lb ...
... then the bisectors have a common point Q,
... defined by P wrt a reference triangle.

Best regards Eckart



2022-12-05.pdf

Message: #1529
Date: 2022-12-06
From: van10hoven@gmail.com
Subject: Re: A circle passes through QA-P1 and concurrent lines

---

Dear Eckart and Tran Quang Hung,

With interest I read the generalized Triangle Transformation Tf [P] of Eckart.

Having general point P(x:y:z) I found:
Tf [P]=
{a^2 c^2 p^2 q^2 - b^2 c^2 p^2 q^2 + c^4 p^2 q^2 - a^4 p^2 q r + 2 a^2 b^2 p^2 q r - b^4 p^2 q r + 2 a^2 c^2 p^2 q r + 2 b^2 c^2 p^2 q r - c^4 p^2 q r + a^2 b^2 p^2 r^2 + b^4 p^2 r^2 - b^2 c^2 p^2 r^2 - 2 a^4 q^2 r^2,

-a^2 c^2 p^2 q^2 + b^2 c^2 p^2 q^2 + c^4 p^2 q^2 - a^4 p q^2 r + 2 a^2 b^2 p q^2 r - b^4 p q^2 r + 2 a^2 c^2 p q^2 r + 2 b^2 c^2 p q^2 r - c^4 p q^2 r - 2 b^4 p^2 r^2 + a^4 q^2 r^2 + a^2 b^2 q^2 r^2 - a^2 c^2 q^2 r^2,

-2 c^4 p^2 q^2 - a^2 b^2 p^2 r^2 + b^4 p^2 r^2 + b^2 c^2 p^2 r^2 - a^4 p q r^2 + 2 a^2 b^2 p q r^2 - b^4 p q r^2 + 2 a^2 c^2 p q r^2 + 2 b^2 c^2 p q r^2 - c^4 p q r^2 + a^4 q^2 r^2 - a^2 b^2 q^2 r^2 + a^2 c^2 q^2 r^2}

I also found:
· Tf [X(1)] = X(1)
· Tf [X(2)] = X(3)
· Tf [X(4)] = X(382)
· Tf [X(6)] = X(576)

Best regards,

Chris

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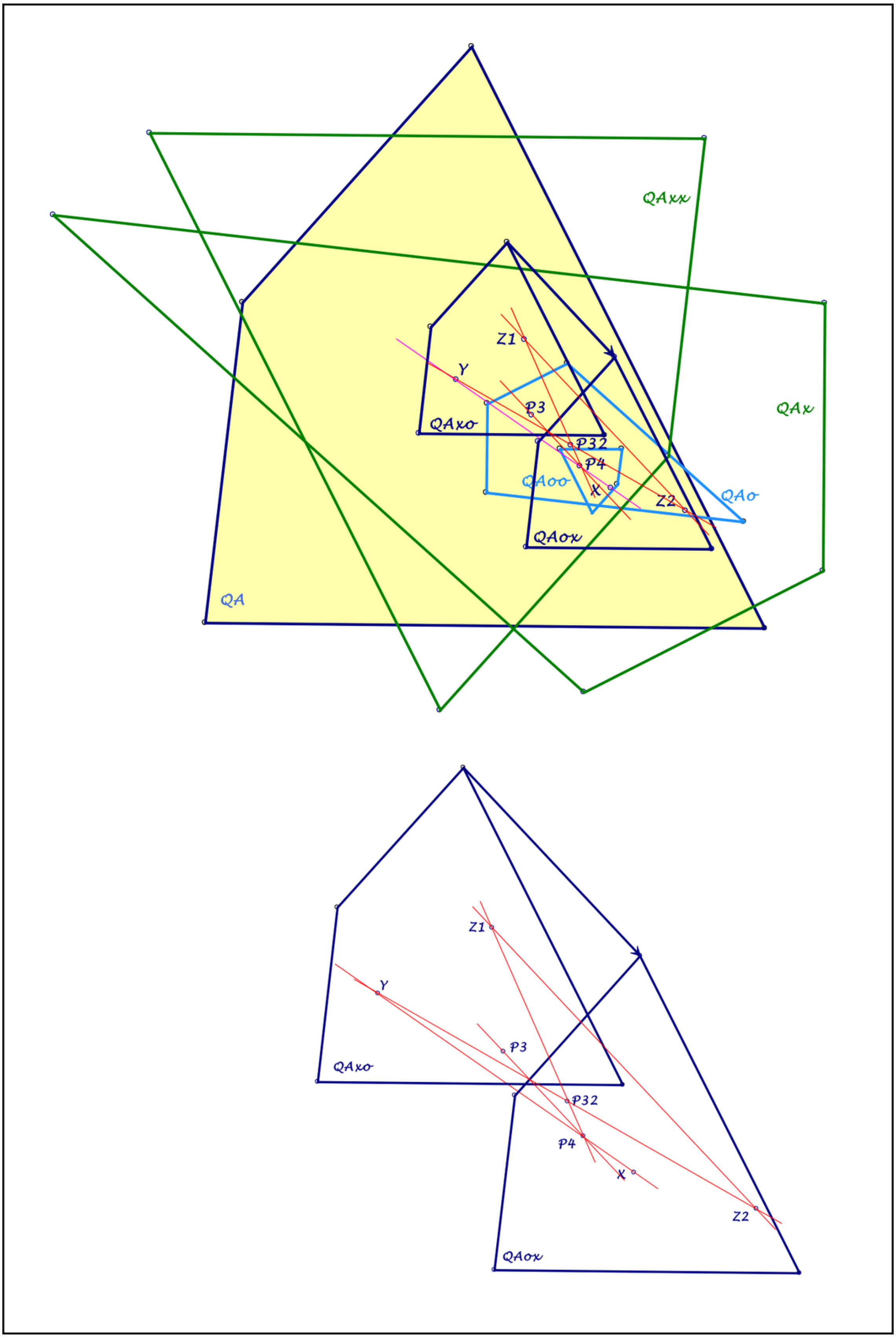
Message: #1530
Date: 2022-12-07
From: eckart_schmidt@t-online.de
Subject: Homothetic quadrangles

```
Dear all,

let us start with a well-known configuration:
For a QA consider QAo, the quadrangle of the X(3) for the
QA-triangles,
... the 2nd version QAoo is homothetic to the reference QA,
... center QA-P4, ratio s (Stärk's Sigma, Benedetto's Lambda).

We shall compare this with another construction:
In message 1528  a modified construction of Tran Quang Hung
... is used for a triangle transformation Tf: P-->Q:
For a QA consider QAx the quadrangle of the Tf-images of the
QA-vertices
... wrt the triangle of the remaining QA-vertices,
... the 2nd version QAxx is also homothetic to the reference QA.

(a)  QAx is homothetic QAo: center QA-P32, ratio 3:1.
(b)  QAxx is homothetic QAoo: center new QA-point X,
     ratio 9:1.
(c)  QAoo is homothetic refQA: center QA-P4, ratio s
     (see above).
(d)  QAxx is homothetic refQA: center new QA-point Y,
     ratio 9s:1.
(e)  The centers X, Y, QA-P4 are collinear, new QA-line.

Finally:
(f)  QAxo and QAox are further homothetic to the reference QA,
...  QAxo: center Z1, ratio -3s:1; QAox: center Z2, ratio -3s:1,
...  Z1.Z2 is parallel to QA-P3.QA-P4
     (parallel to the asymptote of QA-Cu1),
...  QAxo and QAox are parallel positioned,
     vector 2s*QA-P3.QA-P4,
...  QA-P4, QA-P32, Z1 are collinear, QA-P32, Y, Z2 are
     collinear ...

Perhaps someone can lighten the background,
... I hope, there are no fake observations,
... thanks to Tran Quang Hung for the idea of the construction,
... thanks to Chris for calculation.

Best regards Eckart
```



2022-12-07.pdf

Message: #1531
Date: 2022-12-07
From: tungvtt@gmail.com
Subject: Re: A circle passes through QA-P1 and concurrent lines

---

```
Dear all,

I found a further generalization:

Let ABC be a triangle

Consider 3 circles:  (Oa) passing through B,C;   (Ob) passing
through C,A,   (Oc) passing through A,B.
(Ob), (Oc) intersects BC at two points different to B,C; and let
La be the perpendicular bisector of these two points.
Define Lb, Lc similarly. Then La, Lb, Lc concur.

Best regards,
Vu Thanh Tung
```

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Message: #1532
Date: 2022-12-07
From: eckart_schmidt@t-online.de
Subject: Re: A circle passes through QA-P1 and concurrent lines

---

```
Dear Vu Thanh Tung,

I tried your construction generalization
... for the outer X(8) Nagel points Na,b,c
... with the circles (Na,B,C), (Nb,A,C), (Nc,A,B)
... and got the point X(18525).
The constructions for the outer X(7) and X(9)
... seem to give analog new ETC-points.

Best regards Eckart
```

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Message: #1533
Date: 2022-12-07
From: van10hoven@gmail.com
Subject: Is there a proof

```
Dear friends,

Given a conic Co(P1,P2,P3,P4,P5).
Is there a proof that Co(P1,P2,P3,P4,P5) = Co(P2,P1,P3,P4,P5),
apart from the algebraical proof?

Best regards,
Chris
```

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Message: #1534
Date: 2022-12-08
From: eckart_schmidt@t-online.de
Subject: Re: QA-Tf16 as isoconjugation on QA-Cu1

```
Dear Bernard,

excuse my late answer, but thanks for studying my message 1524.
Sorry, I cannot confirm Z = tgP3, so the last collinearity is
... Q (intersection of QA-Cu1 and its asymptote),
    tgP3 and P41 = tgP4.

Best regards Eckart
```

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Message: #1535
Date: 2022-12-08
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A circle passes through QA-P1 and

---

```
Dear all

Thank you very much for your contributions on this problem. The
generalization of Vu Thanh Tung was appeared here
https://artofproblemsolving.com/community/c6h2887866p25688041

Best regards
Tran Quang Hung

Vào 22:31, Th 4, 7 thg 12, 2022 Eckart Schmidt
<eckart_schmidt@t-online.de>
đã viết:

> Dear Vu Thanh Tung,
>
> I tried your construction generalization
>
> ... for the outer X(8) Nagel points Na,b,c
>
> ... with the circles (Na,B,C), (Nb,A,C), (Nc,A,B)
>
> ... and got the point X(18525).
>
> The constructions for the outer X(7) and X(9)
>
> ... seem to give analog new ETC-points.
>
> Best regards Eckart
```

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Message: #1536
Date: 2022-12-11
From: bernard.keizer@gmail.com
Subject: Re: QA-Tf16 as isoconjugation on QA-Cu1

```
Dear Eckart,
Sorry for my mistake and many thanks for your attention and your
correction !
Best regards
Bernard
```

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Message: #1537
Date: 2022-12-16
From: eckart_schmidt@t-online.de
Subject: Neuberg-Mineur circle

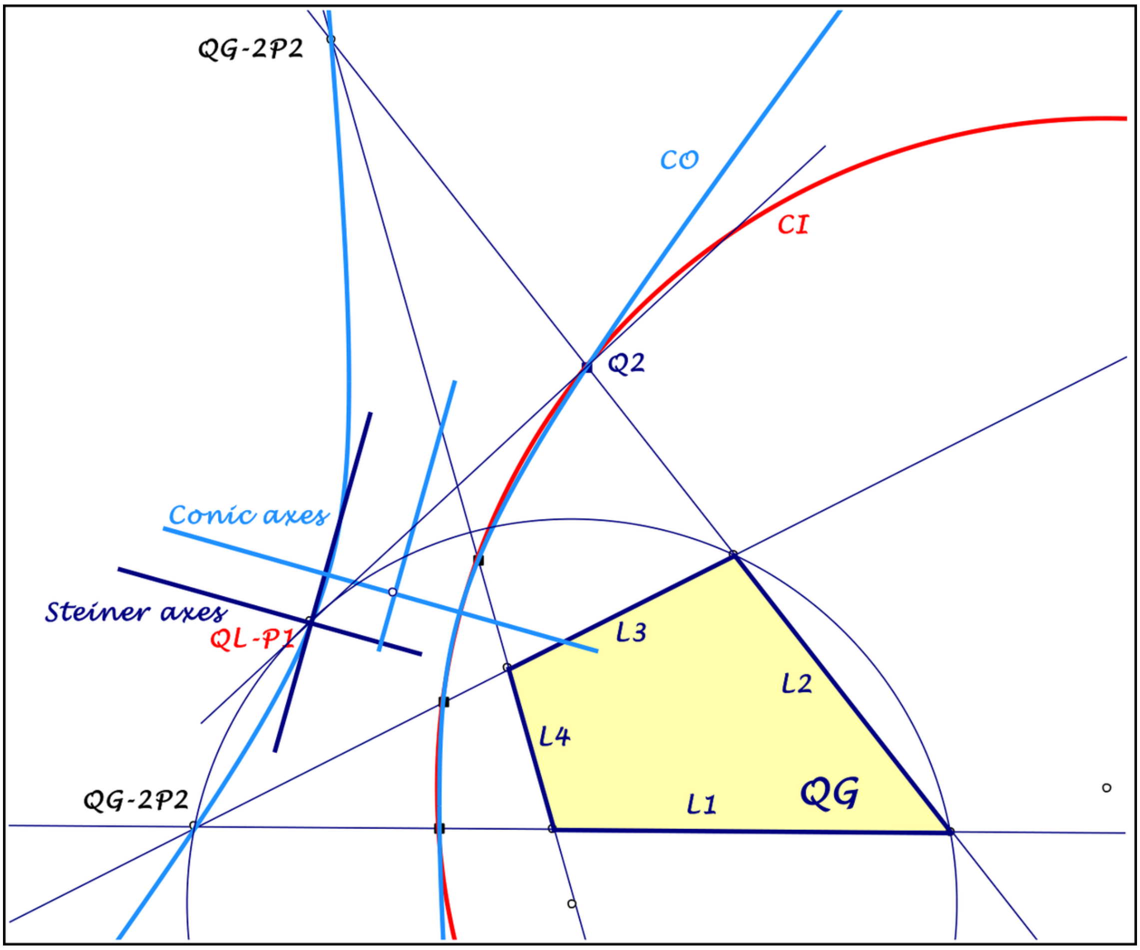
```
Dear Chris,

the following QG-circle  should be mentioned in EQF:
Ref:<https://www.cip.ifi.lmu.de/~grinberg/NeubergMineur.pdf>
Theorem 3, the Neuberg-Mineur theorem.
Let ABCD be a quadrilateral,
and let X; Y; Z; W be the points on the lines AB; BC; CD; DA;
which divide the sides AB; BC; CD; DA externally
in the ratios of the squares of the adjacent sides,
i.e. which satisfy
    AX/XB=-DA^2/BC^2,
    BY/YC=-AB^2/CD^2,
    CZ/ZD=-BC^2/DA^2,
    DW/WA=-CD^2/AB^2.:
Then, the points X; Y; Z; W lie on one circle.

Grinberg gives a detailed synthetic prove
... and a simple construction of the four concyclic points:
Let us use EQF-nomination
... and define the QG with the lines L1,L2,L3,L4,
... consider the circumcircles of Li-1LiLi+1,
... bearing QL-P1 with tangent Tgi, intersecting Li in Qi,
... which give the four concyclic points of the Neuberg-Mineur
    circle CI.

Studying this constellation (see attached drawing)
```

... I found this not mentioned property:
The four concyclic points Qi
... and QL-P1 and QG-2P2 lie on a conic CO ,
... whose axes are parallel to the Steiner axes,
... 1st Steiner axis parallel to the main conic axis,
    if the conic is a hyperbola,
... 2nd Steiner axis parallel to the main conic axis,
    if the conic is an ellipse.

Finally:
The 3 conics CO for the QG-versions of a QA
... have 3 common points A, B, C
... on the circumcircle of the Miquel triangle.
The 6 Simson lines of A, B, C wrt the Miquel triangle
... and of the Miquel points wrt ABC
... have a common point S.
7P-s-Cu1 of QA plus A, B, C
... bears the intersection Q of QA-Cu1 and its asymptote.

There remain a lot of questions wrt these new EQF elements.
The conic CO seems to be more relevant as the circle CI.

Best regards Eckart



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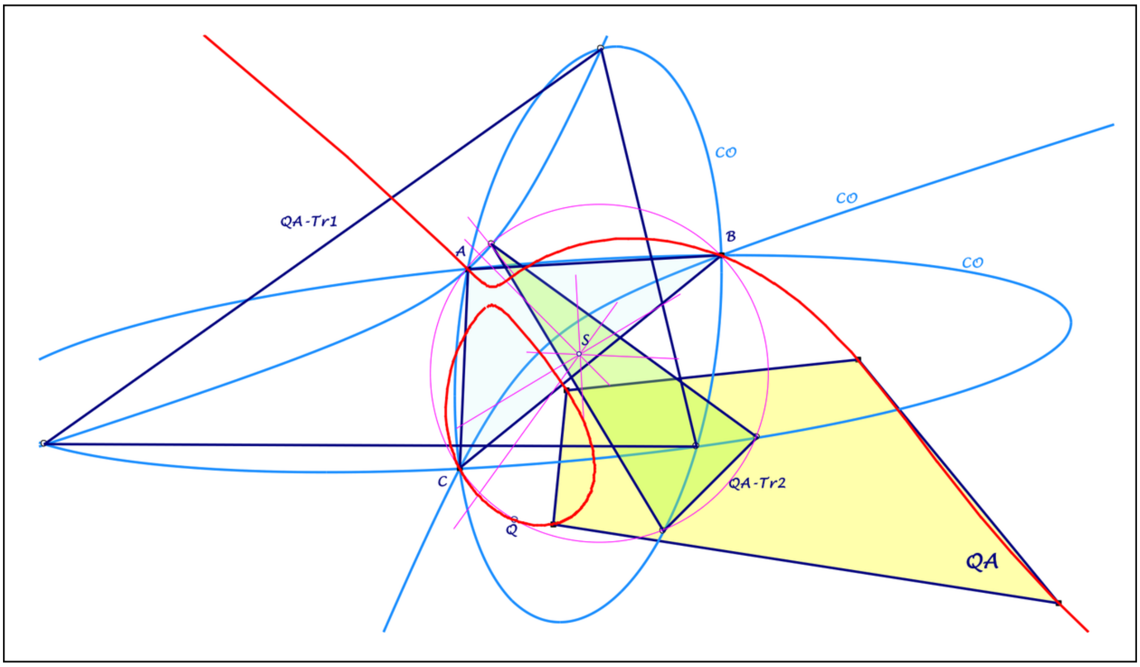

2022-12-16b.pdf

Message: #1538
Date: 2022-12-17
From: james.cooper@jku.at
Subject: Re Neuberg- Mineur

```
As a coda: if we split up the sides with parameters l,m,n,p (
i.e., X=(1-l)A+l B and so on, then the condition that XYZW be
cyclic
is equivalent to these points lying on a quartic surface in
4-space (which, of course, includes those of the Neuberg-Mineur
case).
This quartic can be explicitly computed, given the shape of
ABCD. Is this a known result?
```

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Message: #1539
Date: 2022-12-17
From: bernard.keizer@gmail.com
Subject: Re: Neuberg-Mineur circle

```
Dear Eckart,
Amazing properties !
I had never heard about this Neuberg-Mineur circle before.
A QG is the intersection of a QA and a QL.
You chose the 3 QL's of a QA.
That's the same for a QL and it's 3 QA's : we find 3 such
circles and 3 such conics, each time through 2 opposite
vertices.
But I couldn't find interesting properties.
Best regards
Bernard
```

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Message: #1540
Date: 2022-12-17
From: bernard.keizer@gmail.com
Subject: Re: Neuberg-Mineur circle

```
Dear Eckart,
You may also visit the site of Jean-Louis Aymé (in 2008 le
cercle Neuberg-Mineur).
Best regards
Bernard
```

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Message: #1541
Date: 2022-12-18
From: eckart_schmidt@t-online.de
Subject: Re: Neuberg-Mineur circle

```
 Dear Bernard,

wrt QA/QL/QG in your message 1539:
The Neuberg-Mineur circle CI is a QG-circle,
... the related conic CO a QG-conic.
For a QA we get three CO for the QG-versions,
... each bearing two QA-Tr1-vertices,
... ... which are the QG-2P2 of a QG-version.
For a QL we get three CO for the QG-versions,
... all bearing the Miquel point QL-P1,
... ... each bearing two opposite QL-vertices.
I found no properties for the three circles CI
... for the QG-versions of a QA/QL.

Best regards Eckart
```

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Message: #1542
Date: 2022-12-20
From: van10hoven@gmail.com
Subject: Re: Neuberg-Mineur circle

Dear Eckart, James Bell Cooper and Bernard,

Interesting configuration.

I studied it in relationship with other features of EQF and did some tryouts, but could not find any more than you already did.

Best regards,

Chris

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Message: #1543
Date: 2022-12-31
From: eckart_schmidt@t-online.de
Subject: 16 in-/excircles of a quadrilateral

Dear all,

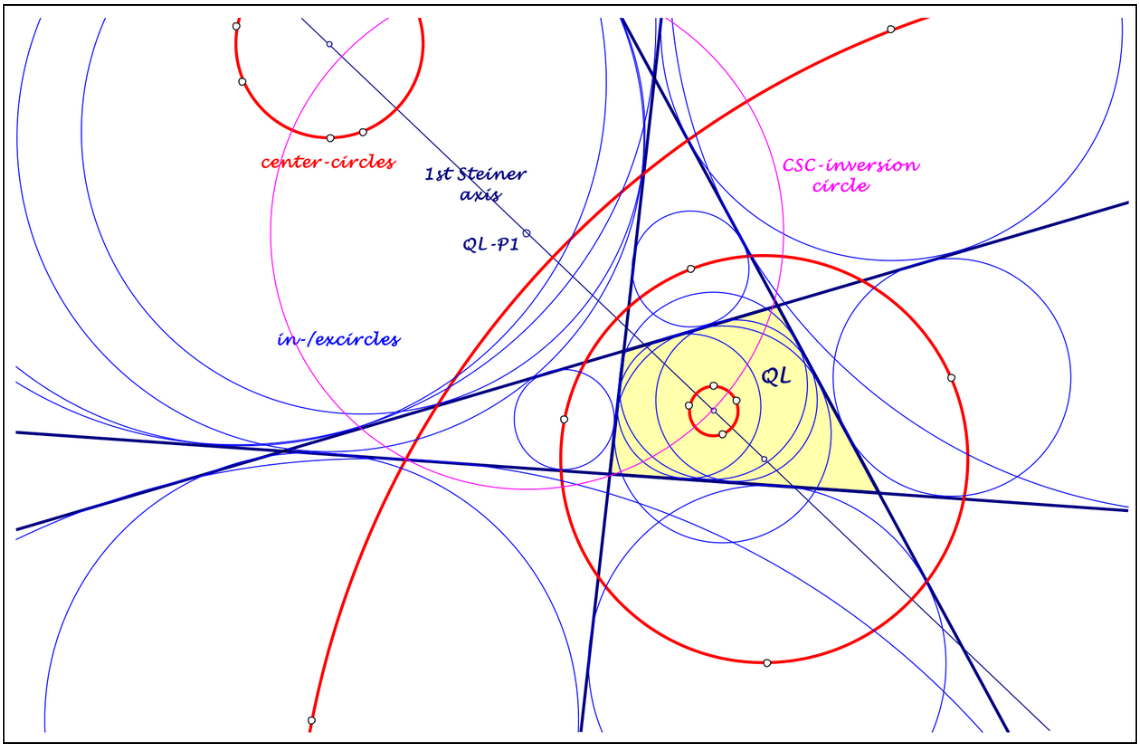
a quadrilateral, defined by four lines, gives four trilaterals,
... each with an incircle and three excircles,
... so we have 16 in-/excircles (1st attached).
The centers of the 16 in-/excircles lie on 4 center-circles.

Construction of the center-circle for one of the 16
in-/excircles:
Let Z be the center of a chosen in-/excircle (2nd attached)
... and L the bisector of Z.CSC(Z),
... intersecting the 1st Steiner axis in M,
... then the circle round M through Z is the center-circle,
... bearing 3 other centers for the in-/excircles.
Starting with the in- and excircles of one trilateral,
... we get the four center-circles,
    centered on the 1st Steiner axis,
... orthogonal to the CSC-inversion circle.

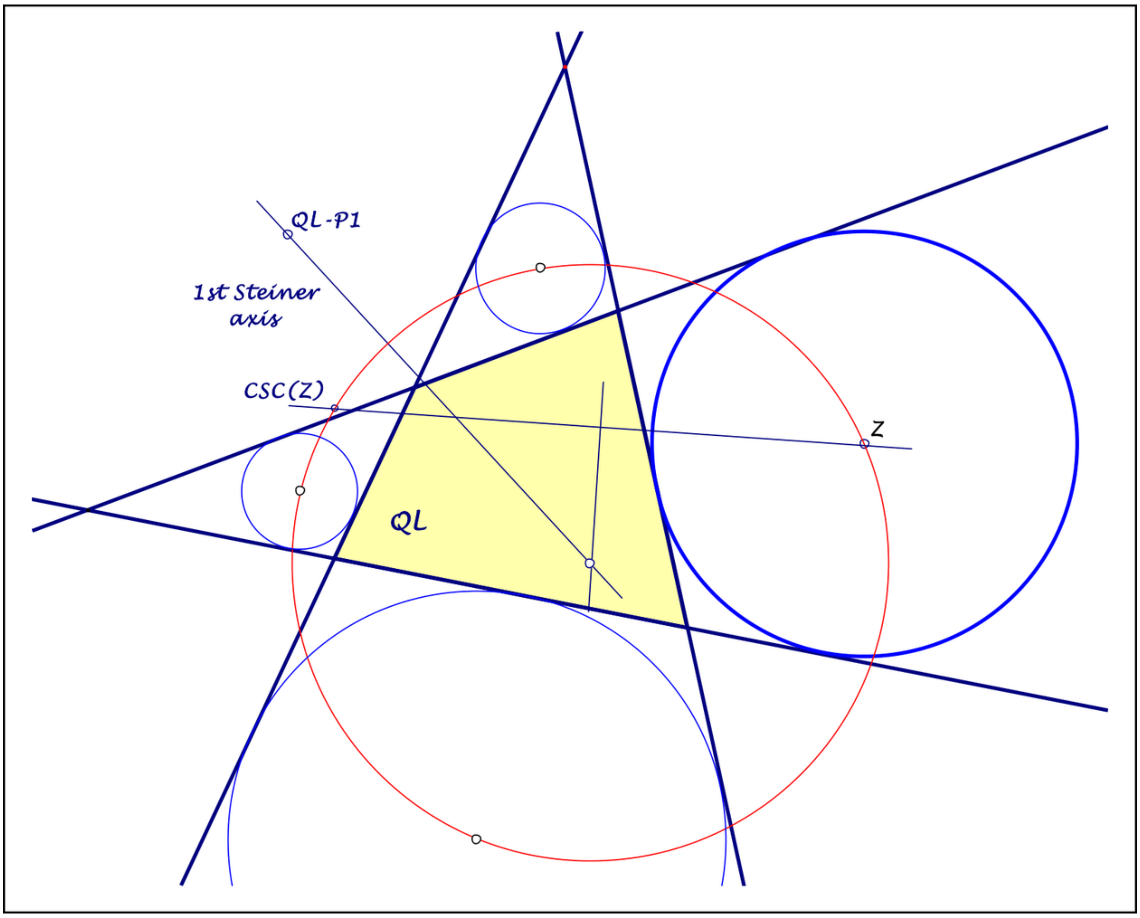
Finally an interesting application for 5L (3rd attached):
The 20 center-circles for the 5 QL of a 5L
... have one 5-times intersection
... concyclic with the centers of 5 center-circles.
What about this special 5L-point?

Happy New Year!
Best regards Eckart

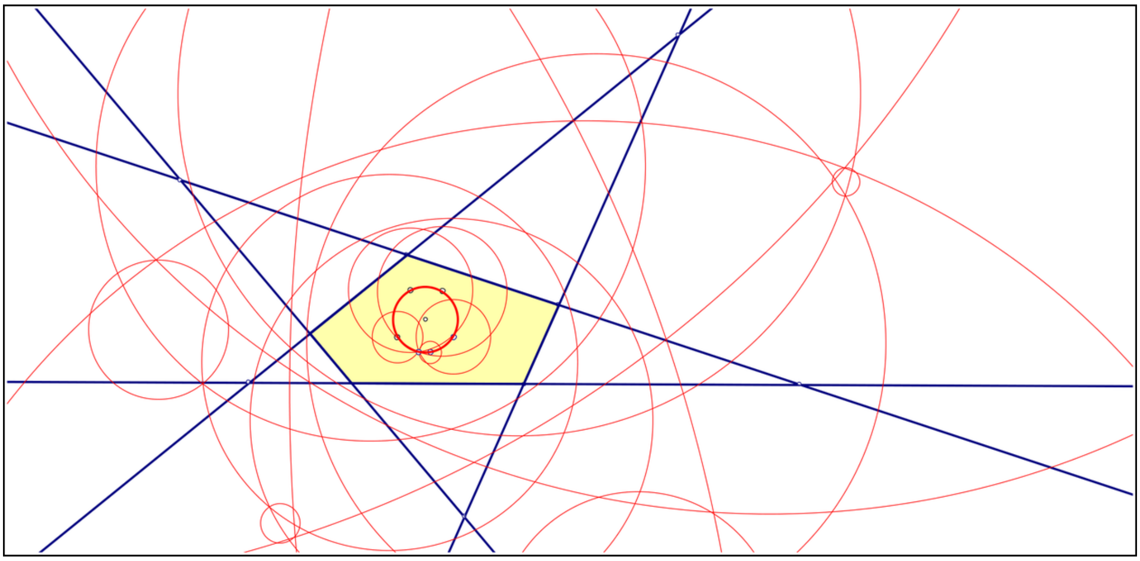
PS. A special center-circle is examined by
Darij Grinberg: A tour around Quadrilateral Geometry
<http://www.cip.ifi.lmu.de/~grinberg/TourQuadriPDF.zip>



2022-12-31a.pdf



2022-12-31b.pdf



2022-12-31c.pdf

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Message: #1544
Date: 2022-12-31
From: bernard.keizer@gmail.com
Subject: Re: 16 in-/excircles of a quadrilateral

Dear Eckart,
The 16 in- and excenters lie on 2 orthogonal sets of 4 circles centered on both Steiner axes (see QL-8P1 in EQF) ; the 8 circles are CSC invariant.
Less known property (not in EQF) : the 2nd intersections (other than in- or excenters) of each circle of one set with the 4 circles of the 2nd set form a 2nd group of 16 points on the same 8 circles by definition, but also on 6 pairs of orthogonal circles intersecting in QL-P1 and a QL vertice.
Happy New Year to you !
Best regards
Bernard

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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom
Editorial Board: Chris van Tienhoven

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393