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Chris van Tienhoven et al.

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Messages #2560 - #2897

Contents

1 Introduction

This journal is a compilation of messages from the **Quadri- and Poly-Geometry (QPG)** forum, where mathematicians and geometry enthusiasts exchange ideas on the properties of **quadrilaterals**, **polygons**, and **curves of nth degree**. The discussions cover a wide range of topics, from classical geometric theorems to new discoveries and insights.

The origins of this journal trace back to the Quadri Figures Group (QFG, available at https://groups.io/g/Quadri-Figures-Group), which was active from 2013 until November 2019. In November 2019, the forum transitioned into the Quadri- and Poly-Geometry Group (QPG, available at https://groups.io/g/Quadri-and-Poly-Geometry) forum, which continues to facilitate discussions on quadrilaterals, polygons, and related topics. Over the years, these forums have evolved into valuable resources for exploring both well-established results and novel perspectives in geometry. For both forums, an **annual record of all incoming messages** is compiled in this journal.

This journal is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing users to quickly jump between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at https://www.chrisvantienhoven.nl/, which aims to systematically classify and analyze geometric structures. By collecting these forum messages, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

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2 Authors

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- **subject: Miquel Circle**:
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4.2 Messages

Message: #2560
Date: 2025-01-08
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: quadrilateral, concurrent orthocentric transversals,

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```
Dear Eckart Schmidt and all,

Thanks to the remark by Eckart Schmidt, I found that Q is the
Ci-antipode of P, where Ci denotes the circle through P and the
2 Plücker points. PH1 is perpendicular to q1, and similarly for
q2, q3, q4. The 2 Plücker points and {Newton line} ∩ {line at
infinity} are singularities of the mapping f: P → Q.

Also, any cubic through the 6 vertices of the quadrilateral and
2 plücker points (which also passes through {Newton line} ∩
{line at infinity} I think) is a self-conjugate cubic under this
conjugation. That is, If P lies on a cubic which belongs to this
family, then, Q also lies on it.

Sincerely,
Keita Miyamoto
```

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Message: #2561
Date: 2025-01-12
From: unidentifiedlethargicorganism@gmail.com
Subject: A property of QL-Ci3 Miquel circle

Dear Geometers,

Is the following result well-known?

Let L1L2L3L4 be a general quadrilateral with vertices P12, P13, P14, P23, P24, P34. (Pij = Li ∩ Lj)
Let M = QL-P1 = Miquel point and Ci = QL-Ci3 = Miquel circle.
Let P be a point on Ci.
Let O1 be the circumcenter of P23P24P34. Define O2, O3, O4 cyclically.
Let L1* be the QA-orthopolar line (QA-Tf8) of PO1 wrt quadrangle MP23P24P34.
Define L2*, L3*, L4* cyclically.

Then, L1*, L2*, L3*, L4* are parallel. If P coincides with the Ci-antipode of M, then, these lines are parallel to QL-L1 = Newton line.
If P moves along Ci, then, each of these lines L1*, L2*, L3*, L4* passes through a fixed point on QL-L3 = QL-pedal line.

Furthermore, Let K = QL-P3 = Kantor-Hervey point, and M* be the second intersection point of MK and Ci.
Let H1, H2, H3, H4 be the orthocenters of the component triangles of the quadrangle O1O2O3O4.
Let L* be the QA-orthopolar line of MK wrt H1H2H3H4.
If P coincides with M*, then, L1*, L2*, L3*, L4*, L* are all parallel. L* passes through QL-P5 = Clawson center.

In general, if P1P2P3P4 is a cyclic quadrangle with circumcenter O, and L is a line through a fixed point, then, the QA-orthopolar line of L wrt P1P2P3P4 envelopes a deltoid, which degenerates to a point if the fixed point coincides with O.

Sincerely,
Keita Miyamoto

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Message: #2562
Date: 2025-01-12
From: bernard.keizer@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Chris, dear Eckart
Exactly a month ago, I sent you the preceding memo containing the barycentric equation of the cayleyan !
But I got no answer ...
I understand that Eckart has little time for personal reasons and that Chris has many centers of interest, including the maintenance of EQF, which is primordial.
The result for me and for the Hesse pencils is the same.
Meanwhile, I continued to make calculations.
Using Eckart's idea of the barycentric coordinates and Chris'calculations wrt the reference triangle X1X2X3, I've applied it not to a variable triangle H1H2H3 like Eckart, but to the fixed triangle X1X2X3.
We know the coordinates of F1, F2, F3 and P0 wrt X1X2X3, which depends only from a1, a2, a3 and k and we are able to measure the length of all segments or of all surfaces which are needed.
We come to formulas s1 = B + tb1 ..., where bi = 1/ai, B = \SigmaFix bi, t = (k-6)/3 and \SigmaFix si = 1 and in the end B (t+3) = 1, bi = (si - B)/t ...
This system has a solution in a1, a2, a3 for any value of k !
But it is possible to use the same construction, measuring and calculation for the hessian.
The correct value of k is obtained when k and the corresponding k' give the same value of a1, a2 and a3 (up to a multiplying factor).
Best regards
Bernard

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Message: #2563
Date: 2025-01-13
From: Stan.Rabinowitz@comcast.net
Subject: Two quadrangles with a common circumconic

```
If P is a quadrangle, let P(n) be the quadrangle formed by the
X(n) points of the component triangles of P.

It is known that for any quadrangle P, P and P(4) have a common
circumconic (QA-Co2).
See https://chrisvantienhoven.nl/qa-items/qa-conics/qa-co2.
The center of the conic is the Euler-Poncelet point of both P
and P(4).

Are there any other values of n for which P and P(n) have a
common circumconic?

Are there any values of n for which P and P(n) have a common
inconic?

======

Some interesting things happen if P is a cyclic quadrangle.

Dylan Wyrzykowski has pointed out at https://www.facebook.com/gr
oups/parmenides52/posts/9041609412619351/ that if P is cyclic,
then P and P(6) have a common circumconic.

How is the center of this conic related to P?

Rabinowitz noted that if P is cyclic, then P and P(n) have a
common (non-circular) circumconic when n=54 or n=64. Ercole
Suppa noted that this is true for n=1173.

Are these conics known? What can be said about the center of
these conics?

Wyrzykowski conjectures that P and P(n) have a common
circumconic if P is cyclic, P lies on the Jerabek hyperbola, and
the isogonal conjugate of P is a point with constant Shinagawa
coefficients.
```

Message: #2564
Date: 2025-01-13
From: Stan.Rabinowitz@comcast.net
Subject: Re: Two quadrangles with a common circumconic

```
On Mon, Jan 13, 2025 at 04:20 PM, Stanley Rabinowitz wrote:

>
> Correction:
>

>
> Wyrzykowski conjectures that P and P(n) have a common
circumconic if P is
> cyclic, X(n) lies on the Jerabek hyperbola, and the isogonal
conjugate of
> X(n) is a point with constant Shinagawa coefficients.
>
>
>
>
> ( #window-247567131 )
>
```

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Message: #2565
Date: 2025-01-13
From: unidentifiedlethargicorganism@gmail.com
Subject: QA-collinear points and QL-concurrent lines

Dear Geometers,

*** QA-version (the dual of the QL-version) ***

Let P1P2P3P4 be a general quadrangle with sides L12, L13, L14, L23, L24, L34. (Lij = Pi ∩ Pj)
Let P be a random point.
We can construct the circumscribed conic QA-Co for 5 points P1, P2, P3, P4, P.
Let L* be the tangent line to QA-Co at P. L* is QA-Tf9(P).
Let L12* be the second tangent line to QA-Co from L* ∩ L12.
Define L13*, L14*, L23*, L24*, L34* similarly.
Let T12 be the touchpoint of L12* and QA-Co. Define T13, T14, T23, T24, T34 cyclically.

Then, the 4 points P, L12* ∩ L34*, L13* ∩ L24*, L14* ∩ L23* lie on the same line L**.
The 4 lines L*, T12T34, T13T24, T14T23 concur in a point P'. P and P' are a pair of doublepoints of L*. (See QA-Tf1)

*** QL-version (the dual of the QA-version) ***

Let L1L2L3L4 be a general quadrilateral with vertices P12, P13, P14, P23, P24, P34. (Pij = Li ∩ Lj)
Let L be a random line.
We can construct the inscribed conic QL-Co for 5 lines L1, L2, L3, L4, L.
Let P* be the touchpoint of QL-Co and L. P* is QL-Tf7(L).
Let P12* be the second intersection point of QL-Co and P*P12.
Define P13*, P14*, P23*, P24*, P34* similarly.
Let t12 be the tangent line at P12* to QL-Co. Define t13, t14, t23, t24, t34 cyclically.

Then, the 4 lines L, P12*P34*, P13*P24* P14*P23* concur in a point P**.
The 4 points P*, t12 ∩ t34, t13 ∩ t24, t14 ∩ t23 lie on the same line L'. L and L' are a pair of doublelines of P*. (See QL-Tf8).

Are the transformations P → L** (in the QA-environment) and L → P** (in the QL-environment) already known in EQF / QFG / QPG?

Sincerely,
Keita Miyamoto

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Message: #2566
Date: 2025-01-13
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Two quadrangles with a common circumconic

Dear Stanley Rabinowitz,

For general quadrangles, I could not find any common
circumconics in the range X(1) to X(100) except for X(4).
Not algebraically confirmed, but my GeoGebra sketch shows that
if a quadrangle is cyclic, then, the center of
X(6)-circumhyperbola coincides with QA-P44 = least-squared
distance point of QA-sidelines.
For X(54), X(64) and X(1173), the centers do not correspond to
any of the QA-points listed in EQF.

I don't understand the inconic part. By P and P(n) you mean
quadrigons P and P(n)? If P is a quadrangle or quadrilateral,
then, it has 3 component quadrigons P(n)-3QG. If P and P(n) are
both quadrigons, then I could not find any common inconics in
the range X(1) to X(100).
If you mean the inconic of P through the vertices of P(n), I
could not find any interesting examples in the range X(1) to
X(100).

However, it is known that if a quadrilateral has 2 inconics Co1
with foci F1, F1' and Co2 with foci F2, F2', then, the
quadrilateral formed by F1F2, F2F1', F1'F2', F2'F1 and the
reference quadrilateral have a pair of confocal inconics with
foci F3, F3'.
In other words, F1, F1', F2, F2', F3, F3' lie on QL-Cu1 =
QL-quasi isogonal cubic of the reference quadrilateral.

Sincerely,
Keita Miyamoto

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Message: #2567
Date: 2025-01-13
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Two quadrangles with a common circumconic

```
Dear Stanley Rabinowitz,

Sorry, my answer is confusing. I mean, if P is a quadrangle,
then P has 3 component quadrigons, and if P is a quadrilateral,
then P(n) is a quadrangle, but if you consider inconics of P(n),
then, P(n) should be a quadrigon.

Sincerely,
Keita Miyamoto
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Message: #2568
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: A property of QL-Ci3 Miquel

```
Dear Keita Miyamoto,

with interest I have reproduced your construction

... of the 4 parallel lines Li* for P on QL-Ci3,

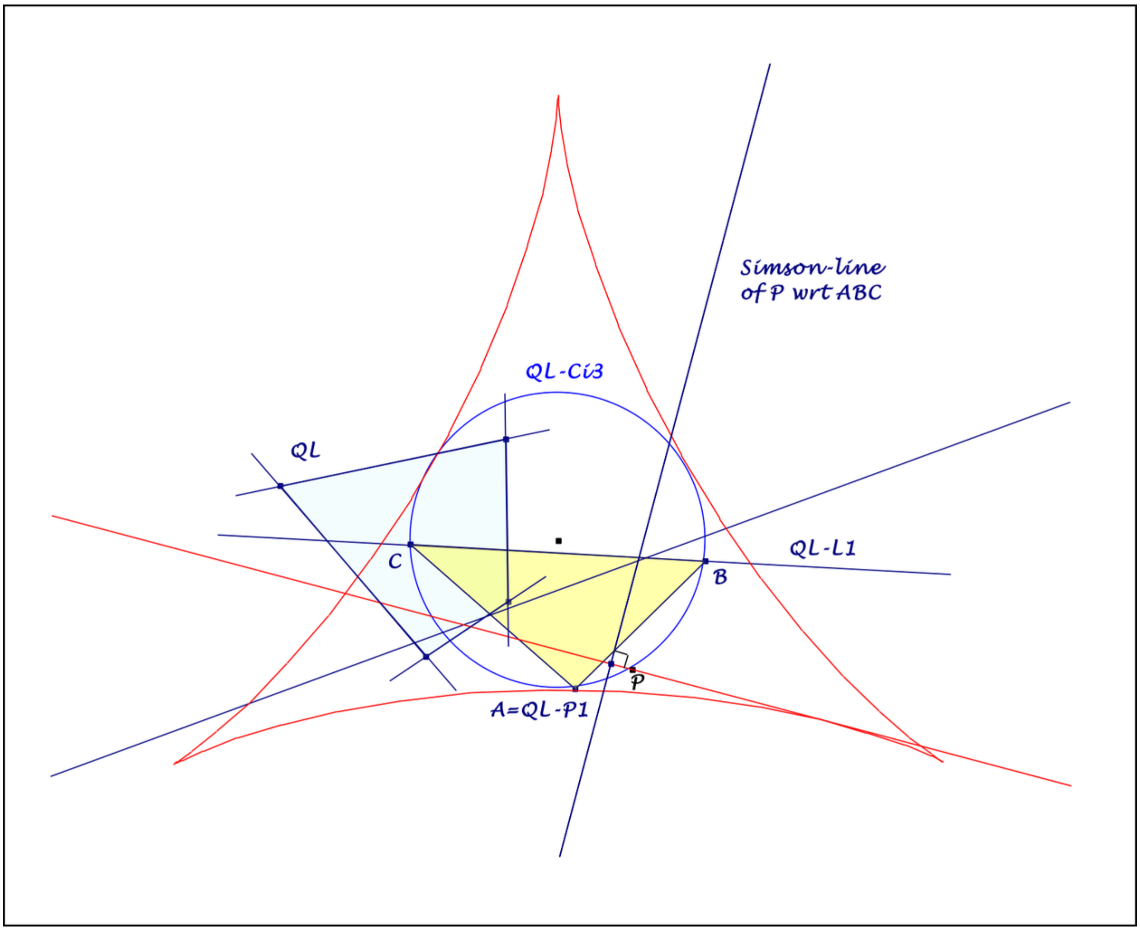
... here a further observation:

These parallels are parallel to the Simson line of P

... wrt a triangle ABC: one vertex A in QL-P1

... with opposite line QL-L1, intersecting QL-Ci3 in B and C.
If you consider perpendiculars through P wrt these Simson lines,
... they envelope a deltoid with inner circle QL-Ci3.

Best regards Eckart
```



2025-01-14.pdf

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Message: #2569
Date: 2025-01-14
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Dear Miyamoto-san and Mr. Schmidt,

As for the last part of #2561, is the deltoid expressed in the
complex plane like this?
z = (p + a)/2 + (|p|/2)(bc)^(1/3) (2t + 1/t^2), |t| = 1,
where a = p1 + p2 + p3 + p4, b = p1p2p3p4/R^4, R = |pj|
(j=1,2,3,4), c = p/|p|, p is a fixed point.

Best regards,
M@IMF
```

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Message: #2570
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: QA-collinear points and QL-concurrent lines

```
Dear Keita Miyamoto,

the transformation P --> P' (first part of #2565) is QA-Tf2.

Best regards Eckart
```

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Message: #2571
Date: 2025-01-14
From: Stan.Rabinowitz@comcast.net
Subject: Re: Two quadrangles with a common circumconic

```
Ercole Suppa has found that for n=4, 6, 54, 64, 74, 98, 99,
1173,  3426, 3431, 3527, 3531, 3532, ABCD and EFGH have a common
circumconic.
```

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Message: #2572
Date: 2025-01-14
From: Stan.Rabinowitz@comcast.net
Subject: Re: Two quadrangles with a common circumconic

```
Note that isogonal conjugate of X(3426) is X(376) which has
constant Shinagawa coefficients (2, -3).
The isogonal conjugate of X(3431) is X(381) which has constant
Shinagawa coefficients (1,3).
The isogonal conjugate of X(3527) is X(631) which has constant
Shinagawa coefficients (2,-1).
The isogonal conjugate of X(3531) is X(3524) which has constant
Shinagawa coefficients ( 4, -3).
The isogonal conjugate of X(3532) is X(3146) which has constant
Shinagawa coefficients (1, -4).
```

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Message: #2573
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: QA-collinear points and QL-concurrent lines

```
Dear Keita Miyamoto,

the transformation L --> L' (second part of #2565) is QL-Tf2.

Best regards Eckart
```

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Message: #2574
Date: 2025-01-14
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Let me correct #2569:
z = (p + a)/2 *-* (|p|/2)(b /c )^(1/3) (2t + 1/t^2), |t| = 1,
where a = p1 + p2 + p3 + p4, b = p1p2p3p4/R^4, R = |pj|
(j=1,2,3,4), c = p/|p|, p is a fixed point.
```

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Message: #2575
Date: 2025-01-14
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear M@IMF and Eckart Schmidt,

Thank you for your interest.
I drew it on GeoGebra. I can confirm that your formula is correct.
I am not highly educated in mathematics, and I was not able to derive an equation for the deltoid by myself, so thank you very much.

By the way, there is a generalization of the orthopole which is called isopole.
Let ABC be a triangle and L be a line. Let A', B', C' be points on L such that m∠(AA', L) = m∠(BB', L) = m∠(CC', L) = θ.
Let A'', B'', C'' be points on BC, CA, AB, respectively, such that m∠(BC, A'A'') = m∠(CA, B'B'') = m∠(AB, C'C'') = -θ.
Then, the 3 lines A'A'', B'B'', C'C'' concur in a point P, which is called the θ-isopole of L wrt ABC.

Analogously, there are generalizations of the orthopolar lines --- both QA-isopolar line (QA-Tf8) and QL-isopolar line (QL-Tf5), and even QL-isopole (QL-Tf4).
That is, define the 4 θ-isopoles of a line L wrt the 4 component triangles of a reference quadrilateral or quadrangle. Then, the 4 isopoles lie on the same line which is called the θ-isopolar line of L.
For QL-isopolar lines, if L is fixed and θ varies, then, the envelope of the isopolar line of L wrt a quadrilateral is a parabola. In particular, if L passes through the Miquel point, then, the envelope is the QL-inscribed parabola.

Sincerely,
Keita Miyamoto

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Message: #2576
Date: 2025-01-14
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Two quadrangles with a common circumconic

Dear Stanley Rabinowitz,

I think this property is remarkable.
I am not sure if it is suitable for EQF, for the website is dedicated to general quadri-figures and polygons, but it would be nice to see it documented somewhere...
Hopefully Mr. Chris van Tienhoven will be interested in your result. Apparently he is busy right now.

Sincerely,
Keita Miyamoto

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Message: #2577
Date: 2025-01-14
From: van10hoven@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Bernard,

I went through your messages #2539 and #2569.

Congratulations on your discovery of the barycentric equation of the Cayleyan in Hesse's pencil–quite an effort!

Thank you for your beautiful pictures.

Best regards,

Chris

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Message: #2578
Date: 2025-01-14
From: van10hoven@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Dear M@IMF, Keita Miamoto and Eckart,

M@IMF, regarding your formula for the Deltoid:
For my understanding could you please explain the relationships
between the parameters of your expression of the Deltoid in the
Complex Plane and the defining parameters of the reference
Quadrilateral?

Best regards,
Chris
```

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Message: #2579
Date: 2025-01-15
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Dear Chris van Tienhoven, M@IMF, Eckart Schmidt and all,

In the complex plane, let p1, p2, p3, p4 be 4 points on the same
circle centered at the origin O, and t is a point on the unit
circle |t| = 1. Let p be a fixed point.
Then, the QA-orthopolar line of a line through p wrt cyclic
quadrangle p1p2p3p4 envelopes a deltoid. The equation of the
deltoid is:

z(t) = (p+p1+p2+p3+p4)/2 + (|p|/2) *
[((p1*p2*p3*p4)/(|p1|^4))*(|p|/p)]^(1/3) * (2*t+1/(t^2))
t=|1|
(complex parametrization)

Application:
Let M be the Miquel point QL-P1 of a quadrilateral L1L2L3L4.
Let M' be the QL-Ci3-antipode of M. (QL-Ci3 = Miquel circle)
Let C be the circle centered at M' and passing through M.
Let C1 be the circumcircle of the triangle bounded by L2, L3,
L4. Define C2, C3, C4 cyclically.
Let P1 be the second intersection point, other than M, of C and
C1. Define P2, P3, P4 cyclically.
Then, the QA-orthopolar line of a line through M wrt the cyclic
quadrangle P1P2P3P4 envelopes the Kantor-Hervey deltoid QL-Qu2
of L1L2L3L4.

I am sorry if there are any mistakes...

Sincerely,
Keita Miyamoto
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Message: #2580
Date: 2025-01-15
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Dear all,

Sorry, the correct equation of the deltoid in the complex plane
is:

z(t) = (p+p1+p2+p3+p4)/2 - (|p|/2) *
[((p1*p2*p3*p4)/(|p1|^4))*(|p|/p)]^(1/3) * (2*t+1/(t^2))
|t| = 1
(complex parametrization. p1, p2, p3, p4 lie on the same circle
centered at the origin O, and t moves on the unit circle. p is a
fixed point)

Sincerely,
Keita Miyamoto
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Message: #2581
Date: 2025-01-16
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Chris and all,

I'll explain the derivation of the equation.

In complex coordinates, the parametric equation of a deltoid is
z = z0 + (ρ/3) (2t + 1/t^2), |t| = 1,
where
z0 = center of deltoid
|ρ| = radius of circumcircle
arg(ρ) = direction of vertex (mod 2π/3).

Let points pj (j=1,2,3,4) lie on a circle with radius R and center at the origin O
in the complex plane. Let QA1 be a cyclic quadrangle formed by them.
Denote the complex number corresponding to point P as z[P], and let p = z[P].

Denote QA1-Tf8(L) by Tf8(L) and let L0 be a line through O.
When L0 rotates around O at angular velocity ω, Tf8(L0) rotates around QA1-P2 at -ω.
L0 and Tf8(L0) are parallel if L0 is parallel or perpendicular to Ax1(one of the axes of QA1-2Co1).
The direction of Ax1 is
φ = arg(p1p2p3p4)/4 (mod π/2).
Denote QA1-P2 by A and let B be a point such that z[B] = z[A] + p.
Since QA1-P3 = O, z[A] = (p1 + p2 + p3 + p4)/2.

If we displace L0 along its normal by a certain distance,
Tf8(L0) is displaced by the same distance (same direction).
Let Lp be a line through P parallel to L0, then the distance between Lp and L0 depends on their direction.
As a result, when Lp rotates around P at ω, Tf8(Lp) rotates around a point D at -ω,
and D revolves around a point C at 2ω, where C is the midpoint of A and B.
D lies on a circle with diameter AB, and BD is parallel to Lp.
This circle is the incircle of the deltoid, so
z0 = z[C] = (p + p1 + p2 + p3 + p4)/2
|ρ|/3 = |p|/2.
Please see the attached file. (The figure will tell much more than my English.)

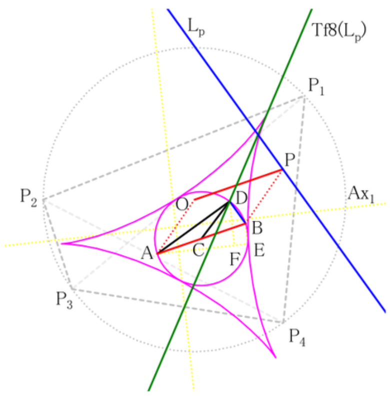
Tf8(Lp) passes through the vertice of the deltoid when it passes through C,
which means the angle bisector of <CDB is parallel or perpendicular to Ax1.
Then we get $\angle BCD = (\pi + 4\varphi - 4\arg(p))/3$, and
$\arg(\rho) = \pi + (4\varphi - \arg(p))/3 \pmod{2\pi/3}$.

The equation of the deltoid becomes
$z = (p + a)/2 - (|p|/2)(b/c)^{(1/3)} (2t + 1/t^2), |t| = 1$,
where
$a = p_1 + p_2 + p_3 + p_4$
$b = p_1p_2p_3p_4/R^4$ (phase factor of $p_1p_2p_3p_4$)
$c = p/|p|$ (phase factor of p).

Best regards,
M@IMF

Deltoid Enveloped by Orthopolar Line wrt Cyclic QA

M@IMF



P_1, P_2, P_3, P_4 are concyclic points.
QA1 is a cyclic quadrangle {P_1, P_2, P_3, P_4}.
P is a fixed point.
L_p is a line through P.
Tf8(L_p) is QA1−Tf8(L_p).
Ax_1 is one of the axes of QA1−2Co1.
O is QA1−P3 (= QA1−P4).
A is QA1−P2.
B is a point such that vector AB = vector OP.
C is the midpoint of A and B.
D is a point such that CD = CA and BD // L_p.
E is a point such that CE = CA and AE // Ax_1.
F is a point such that DF ⊥ AE.

Note that DF is the angle bisector of BD and Tf8(L_p).
Tf8(L_p) envelopes a deltoid (Keita Miyamoto).

Proposition: If ∠CDF = ∠FDB, then ∠BCD = (π − 4∠EAB)/3.
(∵)
∠CDF = π/2 − (∠BCD + ∠EAB)
∠FDB = (1/2)∠BCD + ∠EAB.

Let's consider complex plane with the origin O, and denote the complex number corresponding to point Q as z[Q]. Let p = z[P] and p_j = z[P_j] (j=1,2,3,4). Then the parametric equation of the deltoid is

$z = (p + a)/2 - (|p|/2)(b/c)^{1/3}\ (2t + 1/t^2),\ |t| = 1,$

where

$a = p_1 + p_2 + p_3 + p_4$
$b = p_1p_2p_3p_4/R^4$ (phase factor of $p_1p_2p_3p_4$)
$R = |p_j|$
$c = p/|p|$ (phase factor of p).

Deltoid Enveloped by Orthopolar Line.pdf

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Message: #2582
Date: 2025-01-17
From: van10hoven@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Keita, M@IMF and Eckart,

I further studied your messages regarding "A property of the QL-Ci3 Miquel circle" and was pleasantly surprised by the deltoids that emerged.

Now, we have:

* *QL-Deltoid-1* , which circumscribes the Hervey Circle QL-Ci4, with center QL-P3.

This Deltoid is inscribed in the four lines of the QL.

* *QL-Deltoid-2* , which circumscribes the Miquel Circumcircle QL-Ci3, with center QL-P4.

Constructions:

* *QL-Deltoid-1* : As Keita mentioned in #2579, "The QA-orthopolar line of a line through M with respect to the cyclic quadrangle P1P2P3P4 envelopes the Kantor-Hervey deltoid QL-Qu2 of L1L2L3L4."
* *QL-Deltoid-2* : As Eckart stated in #2568, "If you consider perpendiculars through P with respect to these Simson lines, they envelope a deltoid with inner circle QL-Ci3."

Both circumscribed circles, as well as the QL-Deltoids, are of the same size, and their axes are mutually parallel. See the last property of *QL-Qu2 (http://www.chrisvantienhoven.nl/ql-items/ql-quartics/ql-qu2)* and QFG#514 (https://groups.io/g/Quadri-Figures-Group/topic/71448762#msg16951) for reference.

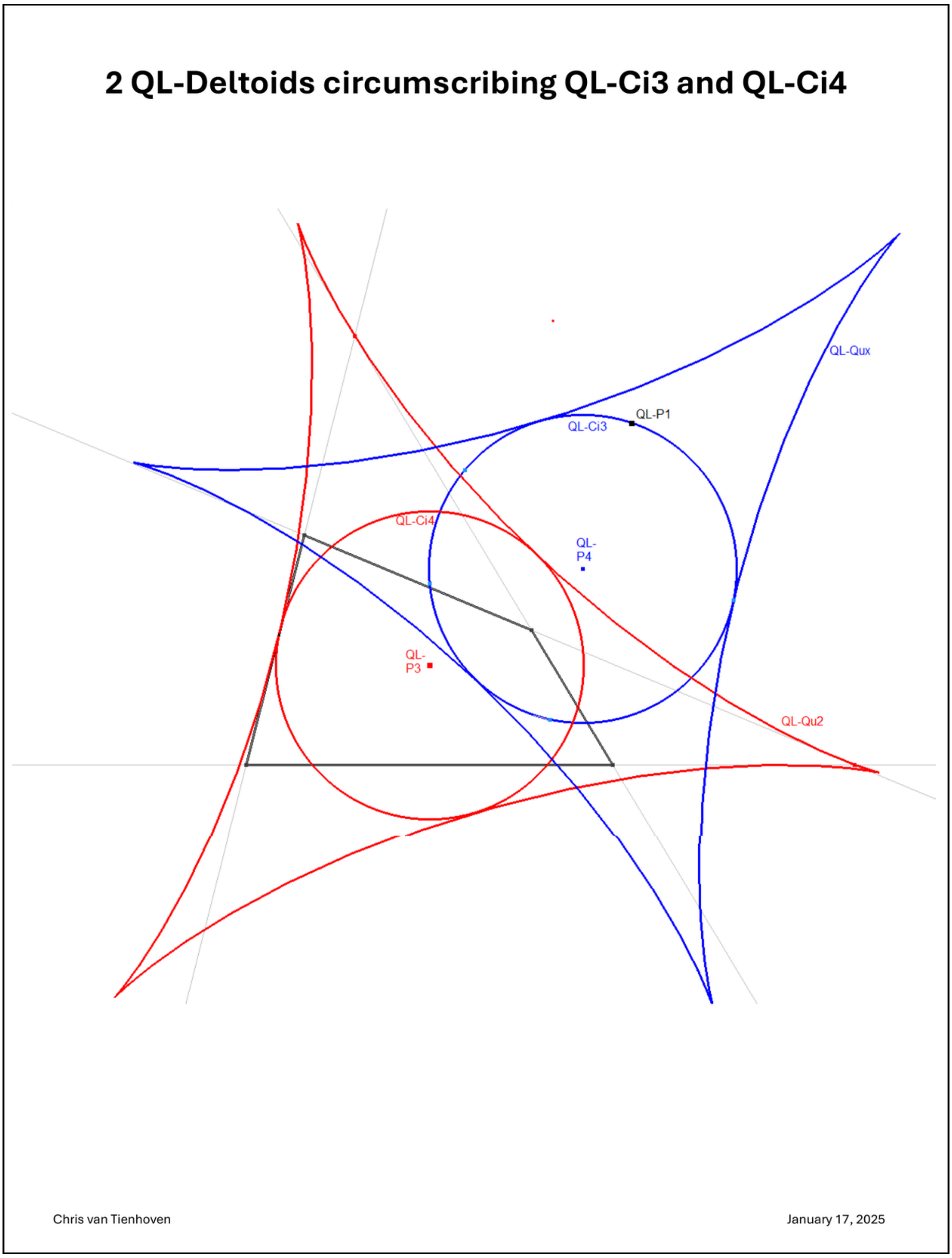
I made a picture of both QL-Deltoids using these constructions. See attachment.

I found M@IMF's equation for QL-Deltoid-2 very intriguing. Providing a proper equation for curves of the 4th degree in barycentric coordinates is challenging, but the polar equation in the complex plane offers a good alternative.

I wonder if there is a similar equation for:

* *QL-Qu2 (
http://www.chrisvantienhoven.nl/ql-items/ql-quartics/ql-qu2)
(Kantor-Hervey Deltoid)* ?
* *QL-Qu1 (
http://www.chrisvantienhoven.nl/ql-items/ql-quartics/ql-qu1)*
(Morley's Mono Cardioid) ? This cardioid is inscribed with
circles passing through QL-P1, and having its center on QL-Ci3.

Best regards,
Chris



QL-Qu2 and QUx QL-Deltoids-01.pdf

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Message: #2583
Date: 2025-01-18
From: bernard.keizer@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear all,
These inscribed circles and QL-deltoïds are symmetric wrt the Clawson center QL-P5.
You may obtain with the same symmetry a cardioïd with reference circle the Hervey circle QL-Ci4 ...
Best regards
Bernard

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Message: #2584
Date: 2025-01-18
From: bernard.keizer@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Chris, dear Eckart
I had a dream
Having in EQF the equations of the cubic stelloïd QL-Cu2 and it's hessian the Van Rees curve QL-Cu1, Chris was able to calculate the flexes, real and imaginary and to determine the points H1, H2 and H3 and the coefficients a1, a2, a3 and k for QL-Cu2 and k' for QL-Cu1.
As every polar conic wrt QL-Cu1 is a rectangular hyperbola, the poles P0, X1, X2 and X3 of the line of real flexes form an orthocentric QA and in particular, P0 is the orthocenter of X1X2X3, the lines L1, L2 and L3 are the altitudes of this triangle and T1T2T3 is it's orthic triangle (we discussed this configuration with Eckart many many years ago in a time we didn't know about the point P0 and the harmonic polars ...)
Best regards
Bernard

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Message: #2585
Date: 2025-01-18
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Chris, Miyamoto-san, Mr. Schmidt, Mr. Keizer and all,

Strictly speaking, my equation is for the deltoid defined by a cyclic *QA* and a point P'.
If P' is on the circumcircle of QA1, and QL1 is a quadrilateral formed by Simson lines of P'
wrt QA1-component triangles, then P' is QL1-P1.
The equation of QL1-Deltoid-1 (QL1-Qu2) is
z = (p' + a)/2 - (R/2)(b/c)^(1/3) (2t + 1/t^2), |t| = 1,
where
a = p1 + p2 + p3 + p4, b = p1p2p3p4/R^4, c = p'/R, R = |p'|,
and the equation of QL1-Deltoid-2 is
z = p'/2 + (R/2)(b/c)^(1/3) (2t + 1/t^2), |t| = 1.

When we use the configuration in #2493,
z[QL1-P1] = (m - q)/2 + i(f + r - p)/2
z[QL1-P2] = -(m + 3q)/4 + i(r - 3f - p)/2
z[QL1-P3] = -(m + 3q)/4 + i(5r - 3f - p)/4 = (p' + a)/2
z[QL1-P4] = (m - q)/4 + i(f + r - p)/4 = p'/2
b/c = -[(m - q) + i(f + r - p)]/√[(m - q)^2 + (f + r - p)^2],
where i is the imaginary unit.
(Note that the origin of xy-plane and that of complex plane differ.)

The equation of a cardioid is like this:
z = z0 + ρ(2t - t^2), |t| = 1.
But I can't determine z0 and ρ because I'm not familier with QL-Qu1.

By the way, is QL-Deltoid-2 Steiner deltoid of the anticomplementary triangle of the triangle ABC below?

On Tue, Jan 14, 2025 at 05:05 PM, Eckart Schmidt wrote:

>
> Dear Keita Miyamoto,
>
> with interest I have reproduced your construction
>
> ... of the 4 parallel lines Li* for P on QL-Ci3,
>
> ... here a further observation:
>

> These parallels are parallel to the Simson line of P
>
> ... wrt a triangle ABC: one vertex A in QL-P1
>
> ... with opposite line QL-L1, intersecting QL-Ci3 in B and C.
>
> If you consider perpendiculars through P wrt these Simson lines,
>
> ... they envelope a deltoid with inner circle QL-Ci3.
>
>
> Best regards Eckart
>

Best regards,
M@IMF

Message: #2586
Date: 2025-01-20
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Dear M@IMF, Chris, Eckart Schmidt, Bernard Keizer and all,

Let m be the Miquel point (= QL-P1), and o be the Miquel
circumcenter (= QL-P4), then, the equation of Morley's mono
cardioid (= QL-Qu1) in the complex plane is:
z = o + (o-m)*(2*t + t^2),  |t|=1

On Sun, Jan 19, 2025 at 04:26 AM, M@IMF wrote:

>
> By the way, is QL-Deltoid-2 Steiner deltoid of the
anticomplementary
> triangle of the triangle ABC below?

Yes.

Sincerely,
Keita Miyamoto
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Message: #2587
Date: 2025-01-20
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

```
Thank you, Keita-san.
I was KUWAZUGIRAI.

M@IMF
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Message: #2588
Date: 2025-01-21
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear all,

More about QL-Qu2.

Given a cyclic quadrangle QA0{Q1,Q2,Q3,Q4}, and let F lie on the circumcircle of QA0.
QL1 is a quadrilateral formed by Simson lines of F wrt QA0-component triangles.
When F revolves around QA0-P3 at angular velocity ω,
QL1-Qu2 rotates around QL1-P3 at -ω/3,
and QL1-P3 revolves around QA0-P2 at ω.
If F is the reflection of Ql wrt QA0-P3, QL1-Qu2 coincides Steiner deltoid of the triangle{Qi,Qj,Qk}, where {i,j,k,l} = {1,2,3,4}.

The locus of the vertex of QL1-Qu2 is an astroid, that is, QL1-Qu2 rolls inside this astroid.
(cf. https://mathworld.wolfram.com/Rotor.html)
To draw the whole curve, F needs to revolve around QA0-P3 three times.
The radius of the astroid is 4 times that of QL1-Ci4.
The direction of the vertex of the astroid is φ + π/4 (mod π/2), where φ is the direction of Ax1(one of the axes of QA0-2Co1).
Note that Steiner axes of QL1 is parallel or perpendicular to Ax1.

I'm going to post a message about cyclic nP and hypocycloid.

Best regards,
M@IMF

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Message: #2590
Date: 2025-01-23
From: contiwa.goma3@gmail.com
Subject: Cyclic nP and Hypocycloid

Dear all,

I defined normalized hypocycloid of a cyclic nP as well as
n-stator and (n-1)-rotor.
Is there any use for them?

< *Rotor and Stator* >
To see what rotor and stator are like, please visit below sites:
https://mathworld.wolfram.com/Rotor.html
https://en.wikipedia.org/wiki/Hypocycloid
https://johncarlosbaez.wordpress.com/2013/12/03/rolling-hypocycloids/
https://demonstrations.wolfram.com/RollingCycloidalCurves/

In complex coordinates, the parametric equation of n-cusped
hypocycloid is
z = z0 + ρ[(n-1)t + 1/t^(n-1)]/n, |t| = 1,
where
z0 = center of hypocycloid
|ρ| = radius of circumcircle
arg(ρ) = direction of vertex (mod 2π/n).
When z0 = 0 and ρ = 1, its vertices are n-th roots of 1.

Define
Hc(ν, z0, ρ, t) = z0 + ρ[(ν-1)t + 1/t^(ν-1)]/ν.
If n-cusped hypocycloid stator (n-stator) is
z = Hc(n, 0, 1, t), |t| = 1,
then (n-1)-cusped hypocycloid rotor ((n-1)-rotor) is
z = Hc(n-1, exp(i (n-1)s)/n, exp(- i s) (n-1)/n, t), |t| = 1,
(0 ≤ s ≤ 2π),
where i is the imaginary unit.
Note that
Hc(n/(n-1), 0, 1, t) = [(n-1)t^(-1/(n-1)) +
1/(t^(-1/(n-1)))^(n-1)]/n,
and the locus of the vertices of (n-1)-rotor is n-stator.

N-stator and (n-1)-rotor of a cyclic nP can be defined
inductively.
(N-1)-rotor of a cyclic nP is defined as (n-1)-rotor which
coincides (n-1)-stator of
each component (n-1)P of nP at a certain moment.

< *Deltoid, Astroid* >

Given 3 points pj (j=1,2,3) such that |pj| = R, and let Tr1 be a triangle formed by them.
Steiner deltoid of Tr1 is
z = (p1 + p2 + p3)/2 + (1/2)(p1p2p3)^(1/3) (2t + 1/t^2), |t| = 1.
Its incircle is the nine-point circle of Tr1, and the directions of its verices are
those of the circumtangential triangle of Tr1.
3-stator of Tr1 is defined as its Steiner deltoid.

Given 4 points pj (j=1,2,3,4) such that |pj| = R, and let QA1 be a quadrangle formed by them.
The center of 3-stator of the componet triangle of QA1 lies on a circle with center a/2
and radius R/2, so 3-rotor of QA1 is defined as
z = a/2 + (R/2) (-b)^(1/4) [exp(3 i s) + exp(- i s) (2t + 1/t^2)], |t| = 1, (0 ≤ s ≤ 2π),
where a = p1 + p2 + p3 + p4, b = p1p2p3p4/R^4.
When exp(i s) = [(-p1p2p3p4)^(1/4) /(pipjpk)^(1/3)], 3-rotor coincides with 3-stator of triangle{pi,pj,pk},
where {i,j,k,l} = {1,2,3,4}.
(See also #2588 for QL-Qu2.)

In the expression of 3-rotor, t = 1 corresponds to one of the vertices of the rolling deltoid, and
z = a/2 + (R/2) (-b)^(1/4) [exp(3 i s) + 3 exp(- i s)], 0 ≤ s ≤ 2π
represents an astroid. Then 4-stator of QA1 is defined as
z = a/2 + (R/2) (-b)^(1/4) (3t + 1/t^3), |t| = 1.

< *Normalized Hypocycloid* >
In general, for n points pj (j=1,...,n) such that |pj| = R, n-stator is defined as
z = Hc(n, a[n]/2, g[n] n/2, t), |t| = 1,
and (n-1)-rotor is defined as
z = a[n]/2 + g[n] Hc(n-1, exp(i (n-1)s)/2, exp(- i s) (n-1)/2, t), |t| = 1, (0 ≤ s ≤ 2π),
where
a[n] = p1 + p2 + ... + pn
g[n] = [-(-1)^n p1p2...pn]^(1/n).
When exp(i s) = g[n]/g[n-1], (n-1)-rotor coincides with (n-1)-stator.

As n becomes larger, the radius of the circumcircle of n-stator becomes larger,
and the relationship between n-stator and nP becomes weaker.
So I defined normalized hypocycloid of a cyclic nP{p1,...,pn} as
z = Hc(n, a[n]/n, g[n], t), |t| = 1,

which is homothetic with n-stator (ratio 2/n, center the
origin).
Its radius of the circumcircle is R and its center is the
centroid of the nP.
(Isn't it nice?)

Best regards,
M@IMF

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Message: #2591
Date: 2025-01-23
From: Stan.Rabinowitz@comcast.net
Subject: Nine-point Center Quadrangle of Nine-point Center Quadrangle

---

Is the following result known?

Let WXYZ be the Nine-point Center Quadrangle of the Nine-point Center Quadrangle of quadrangle ABCD.

Then ABCD and WXYZ are homothetic.

How is the perpector related to quadrangle ABCD?

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Message: #2592
Date: 2025-01-23
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Nine-point Center Quadrangle of Nine-point Center Quadrangle

---

Dear Stanley Rabinowitz,

Yes, it is known.
See QA-P7 (
https://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p7) (=
QA-nine-point homothetic center) and QL-P27 (
https://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p27) (=
nine point homothetic center QL-X(3) quadrangle).
QA-P7 lies on the line joining the QA-centroid and isogonal center.

Sincerely,
Keita Miyamoto

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Message: #2593
Date: 2025-01-26
From: anopolis72@gmail.com
Subject: Quadrilateral Problem

Considérons quatre droites dans un plan: ces quatre droites prises trois ii
trois forment quatre triangles et l'une d'elles, AB par exemple, appartient
à trois de ces triangles. Dans chacun des trois triangles correspondant à
AB, joignons le centre du cercle circonscrit au sommet opposé à AB. 1° Pour
un même côté AB les trois droites ainsi menées concourent en un point I :
2° les quatre points analogues à l et les quatre centres des cercles
circonscrits aux triangles formés par les quatre droites sont sur une même
circonférence.

Journal de mathématiques élémentaires
tome sixième, 1882, p. 37

Google translation
Consider four straight lines in a plane: these four straight lines taken
three by three form four triangles and one of them, AB for example, belongs
to three of these triangles. In each of the three triangles corresponding
to AB, let us join the center of the circumscribed circle to the vertex
opposite AB. 1° For the same side AB, the three straight lines thus drawn
converge at a point I: 2° the four points analogous to l and the four
centers of the circles circumscribed to the triangles formed by the four
straight lines are on the same circumference.

APH

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Message: #2594
Date: 2025-01-26
From: contiwa.goma3@gmail.com
Subject: Re: Quadrilateral Problem

```
Dear APH,

The point I or its analogue (on QL-Ci3) is the reflection of Vl'
in the circumcenter of the triangle{Tjk,Tki,Tij} in #2529,
where (i,j,k,l) = (1,2,3,4) or their permutations.

Best regards,
M@IMF
```

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Message: #2595
Date: 2025-01-28
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

now I am rather convinced, that this is a correct construction

... for the real flexline trilateral H1H2H3 of a cubic CU:

Let F1,F2,F3 be the real flexpoints,

... L1,L2,L3 their harmonic polars with common point Po,

... intersecting the infinite part of CU in P1,P2,P3,

... finally the intersections Ti of Li and the flextangent in Fi.

Start with quadrilaterals QLk = (FiPi,FiTi,FjPj,FjTj),

... which give the regions of Hk, let FiPi^FjPj = Uk,

... and take points P on the flexline F1F2F3

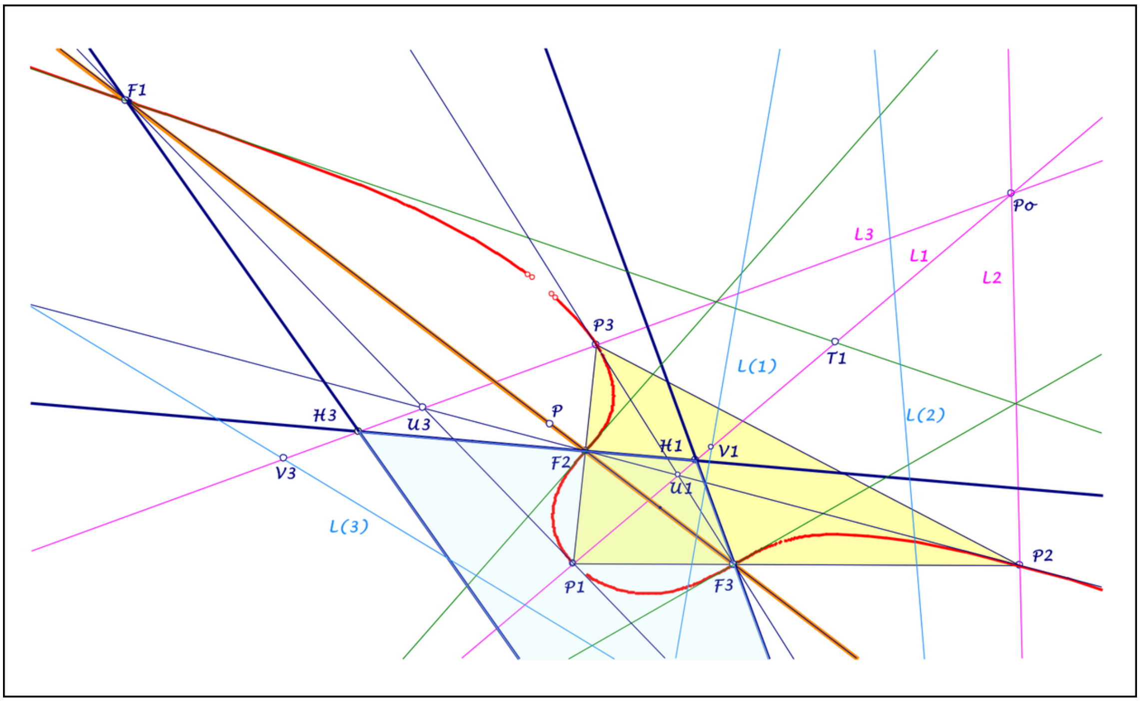
... with their dual lines L(k) wrt QLk (wrt "dual" see QL-8 in EQF),

... the lines L(k) for P on F1F2F3 have a common point Vk on Lk,

... then the 4th harmonic point of Po wrt Uk and Vk is Hk.

I made several drawings with astonishing validation!

Best regards Eckart



2025-01-28.pdf

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Message: #2596
Date: 2025-01-29
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
As I already told you, I'm always bluffed by your tenacity and
your imagination !
I'm very glad to notice that you are stil interested in the item
!
Uk is easy to draw, I suppose Vk is the dual point of the line
of real flexes wrt your QLk
Naturally, UiUj, ViVj and HiHj intersect in Fk and it seems that
UiVj, UjVi and HiHj intersect on Lk in fk, the harmonic of Fk
wrt Hi and Hj
I'll try to check your property as soon as I find enough time to
do it properly
The real proof would be for you to do the same construction with
the hessian (Pi becomes Ti and Ti another point ti on Li)
and to find the same points Hi
It would mean that the points P0 and Hi are harmonic wrt the
copple of points Ui and Vi for any cubic of the pencil, in
particular the cubic and it's hessian
Very beautiful indeed !
Best regards
Bernard
```

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Message: #2597
Date: 2025-01-30
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for interest and remarks,

... I constructed H1,H2,H3 also for the hessian

... and got the same points as for the reference cubic.

Best regards Eckart

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Message: #2598
Date: 2025-01-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I must have misunderstood your construction, as I'm not able to reproduce your property.
Is it correct that your point Vk is the dual of the line of real flexes wrt QLk ?
In this case, considering this QLk, it's DT being selfdual, Vk is the opposite vertice of this line of real flexes.
Considering the intersections of FiPi and FiTi with FjPj and FjTj, we get 4 points Uk, Xk and A and B and Vk should be the intersection of the 2 other diagonals of DT, which are Lk and AB.
Am I wrong somewhere ?
Naturally, if you find the same points H1, H2 and H3 for the cubic and it's hessian, your construction seems correct ...
Can you help me ?
Best regards
Bernard

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Message: #2599
Date: 2025-01-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
On your drawing in message 2595, QL1is formed by the lines F2T2, F2P2, F3T3 and F3P3
F2P2 and F3P3 intersect in U1, F2T2 and F3T3 intersect in X1
F2T2 and F3P3 intersect in A, F2P2 and F3T3 intersect in B
AB (through F1) intersects L1 in V1
I almost reproduced your construction for k = 10 and k = 12 and exactly for k = 100 , but it doesn't work for k = 1
Very strange, indeed !
Best regards
Bernard
PS These are examples of monopartite cubics, you said for bipartite cubics, you take the infinity part ?

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Message: #2600
Date: 2025-02-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

excuse I haven't reacted on your message #2598:

You are right, that Vk is the dual of the real flexline F1F2F3 wrt QLk.

The point Vk can be described without A, B as Vk = Lk^(Fk.FiPi^FjTj) .

Best regards Eckart

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Message: #2601
Date: 2025-02-03
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

You describe in #2595:

Start with quadrilaterals QLk = (FiPi,FiTi,FjPj,FjTj),
... which give the regions of Hk, let FiPi^FjPj = Uk,
... and take points P on the flexline F1F2F3
... with their dual lines L(k) wrt QLk (wrt "dual" see QL-8 in EQF),
... the lines L(k) for P on F1F2F3 have a common point Vk on Lk,

Am I right in my next reasoning:

When you make a QL bounded by the lines (FiPi, FiTi,FjPj,FjTj), then you actually span a QG with 3rd diagonal FiFj (which is F1F2F3).

The dual lines L(k) will have common point Vk.

Am I right when I say that the dual lines are QL-Tf10(P)?

If so, then this common point = the QG-Diagonal Crosspoint QG-P1?

Or stated in another way:

Create a QL from two lines emanating from Fi and two lines emanating from Fj.

This QL has three component Quadrigons,

one of which will have FiFj as its 3rd diagonal QG-L1.

The diagonal crosspoint QG-P1 of this QG will be Vk.

See QL-3QG1.

Best regards,

Chris

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Message: #2602
Date: 2025-02-03
From: anopolis72@gmail.com
Subject: Radical axis in qudrilateral

L'axe radical commun des trois circonférences décrites trois
diagonales
d'un quadrilatère complet comme diamètres passe par le centre du
cercle
circonscrit au triangle formé par ces trois diagonales.
By Chapron in the Solution of a problem by E. Lemoine
Journal de mathematiques elementaires
<https://dn790001.ca.archive.org/0/items/s2journaldemathm05pari/s2journaldemathm05pari.pdf>
1886, p. 144

(The common radical axis of the three circumferences described
by the three
diagonals of a complete quadrilateral as diameters, passes
through the
center of the circle circumscribed to the triangle formed by
these three
diagonals).

APH

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2603
Date: 2025-02-03
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

I cannot understand your reasoning,

... but if you consider the diagonal triangle QLk-Tr1,

... Vk is the opposite point of the sideline F1F2F3.

Vk is not the diagonal crosspoint of your considered QG,

... in #2600 I described Vk as Vk = Lk^(Fk.FiPi^FjTj),

... please have a look at the drawing of #2595.

Best regards Eckart

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Message: #2604
Date: 2025-02-04
From: van10hoven@gmail.com
Subject: Re: Radical axis in qudrilateral

Dear Antreas,

Thanks for the reference. It is an extraordinary report from a remarkable period of continuous research during those days.

The common radical axis of the three circumferences described by the three diagonals of a complete quadrilateral as diameters is QL-L2 (https://www.chrisvantienhoven.nl/ql-items/ql-lines/ql-l2) , the Steiner Line, also called the Aubert Line or Orthocentral Line.

There are several points known to lie on this line:

1. The orthocenters of the four component triangles of the reference quadrilateral.

2. The two common intersection points of the three circumferences described by the three diagonals of a complete quadrilateral as diameters QL-2P1a (https://www.chrisvantienhoven.nl/ql-items/ql-mult-pts-lns/ql-2p1) & QL-2P1b.

3. The circumcenter of the Diagonal Triangle QL-P9 (https://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p2).

4. The Morley Point QL-P2 (https://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p2).

5. The Newton-Steiner Point QL-P7 (https://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p7).

Everything converges in a quadrilateral.

Best regards,

Chris

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Message: #2605
Date: 2025-02-04
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Thanks for the further explanation of the 'dual'.

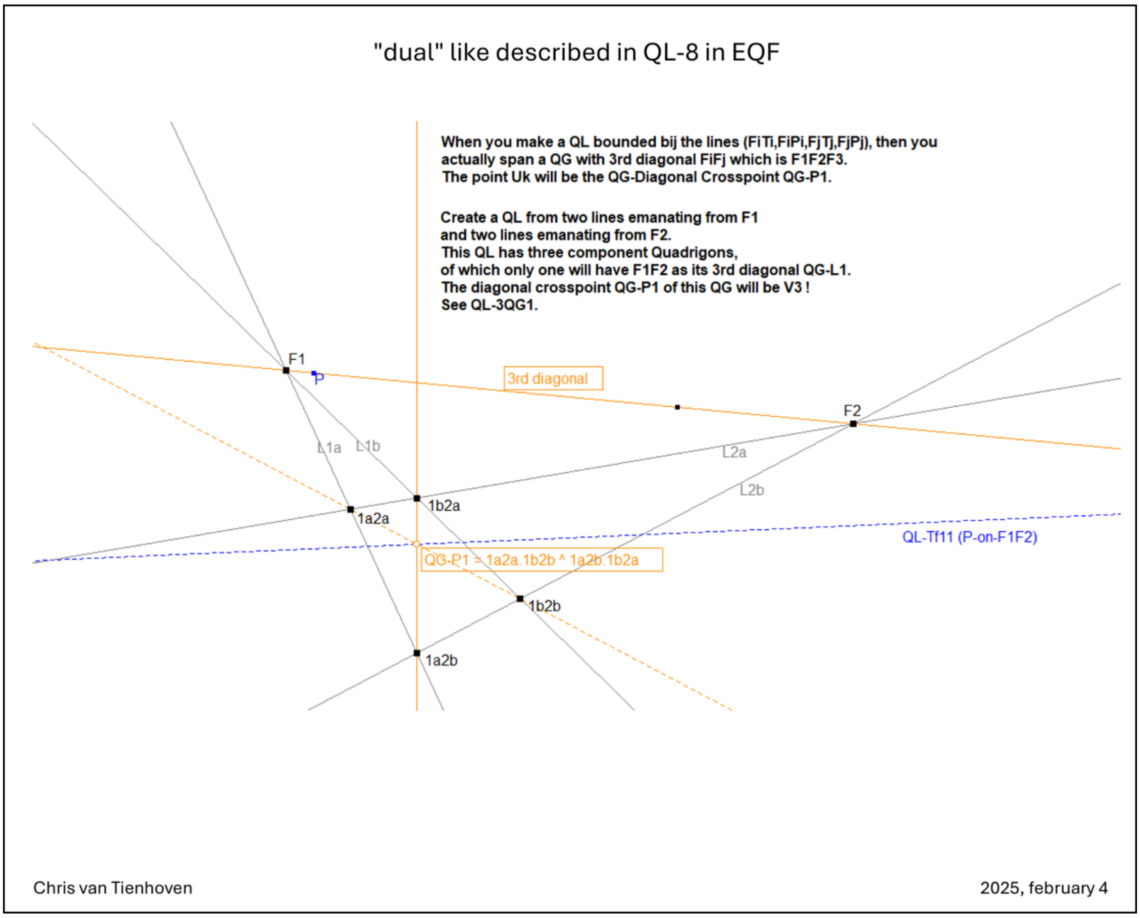
I calculated a bipartite cubic using your method and determined the points (V1, V2, V3) as you described: Vk = Lk^(Fk.FiPi^FjTj). I also calculated these points as the diagonal crosspoints of the component QG with F1F2F3 as the third diagonal. I obtained exactly the same outcome, so I think both methods are valid. I have attached a picture to better explain my method.

I also calculated the points (U1, U2, U3), but I could not confirm the relationship where Hi is the harmonic conjugate of P0 with respect to (Ui, Vi). Since you created different pictures where the relationship holds, I wonder where I am misunderstanding you.

Therefore, I have attached a picture of my calculated results, where you can see that Hi does not match the harmonic conjugate of P0 with respect to (Ui, Vi). Note that I did not use Flexpoints (F1, F2, F3), but instead I used (F1, F4, F7) because it worked out that way in my calculations.

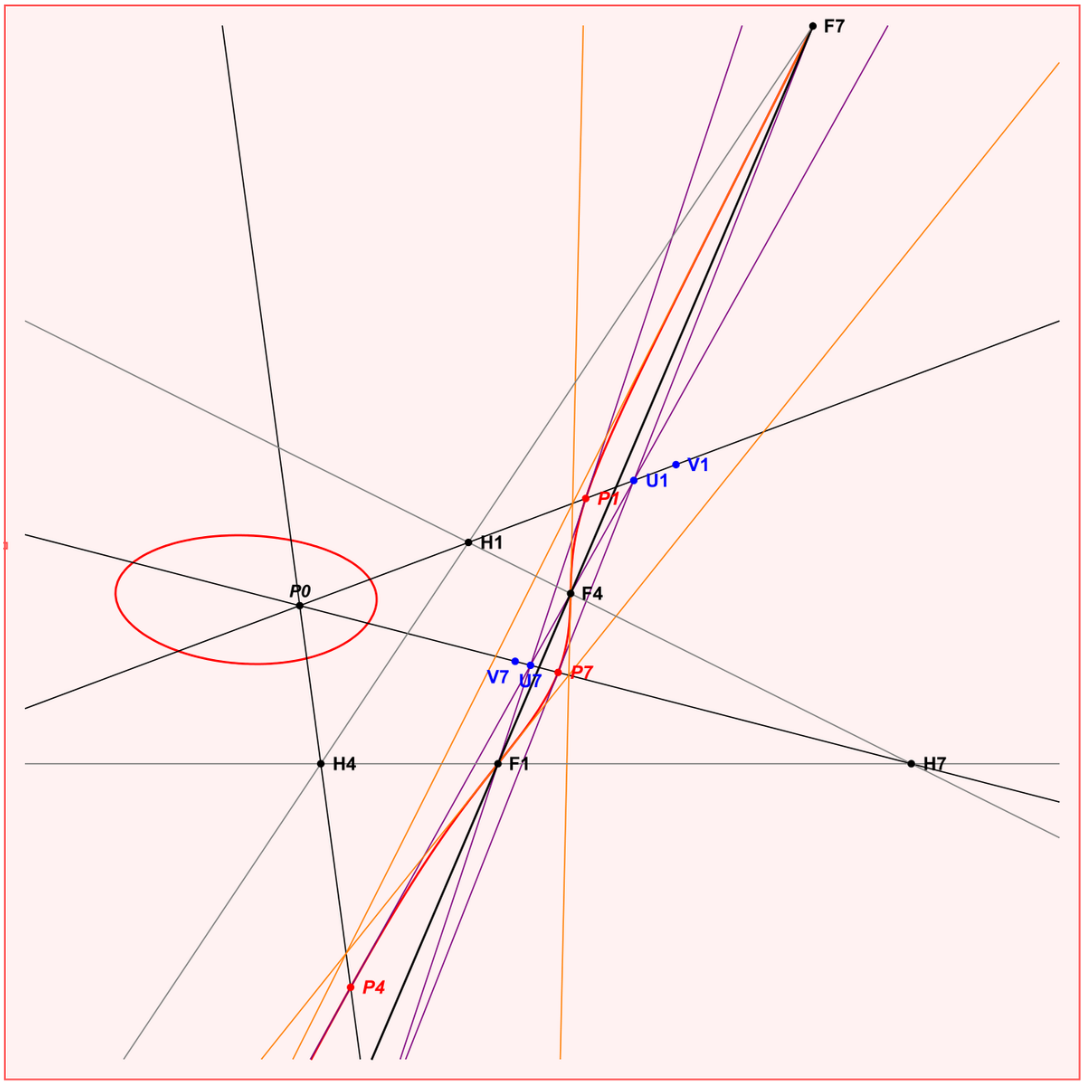
Best regards,

Chris


"dual" like described in QL-8 in EQF
When you make a QL bounded bij the lines (FiTi,FiPi,FjTj,FjPj), then you
actually span a QG with 3rd diagonal FiFj which is F1F2F3.
The point Uk will be the QG-Diagonal Crosspoint QG-P1.
Create a QL from two lines emanating from F1
and two lines emanating from F2.
This QL has three component Quadrigons,
of which only one will have F1F2 as its 3rd diagonal QG-L1.
The diagonal crosspoint QG-P1 of this QG will be V3 !
See QL-3QG1.
F1
P
3rd diagonal
F2
L1a
L1b
L2a
L2b
1b2a
1a2a
QL-Tf11 (P-on-F1F2)
QG-P1 = 1a2a.1b2b ^ 1a2b.1b2a
1b2b
1a2b
Chris van Tienhoven
2025, february 4

QL-Tf11x.pdf

Bipartite Cubic with constructed points (U1,U2,U3) and (V1,V2,V3)



Chris van Tienhoven

2025, Februari 4

CU-HE-12L1abc-21__Eckarts-Case-Numerical.pdf

Message: #2606
Date: 2025-02-05
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

I have to specify my construction in #2595:

The construction of the real flexline trilateral

... seems correct for monopartite cubics,

... but for bipartite cubics there are three triangles P1P2P3

... for intersections of harmonic polars Li and the cubic,

... so that the cubic is invariant wrt isoconjugations with
fixed point Po.

Please take not needful P1P2P3 on the infinite part as I wrote,

... but the triangle P1P2P3, whose lines FiPi and flextangents
FiTi

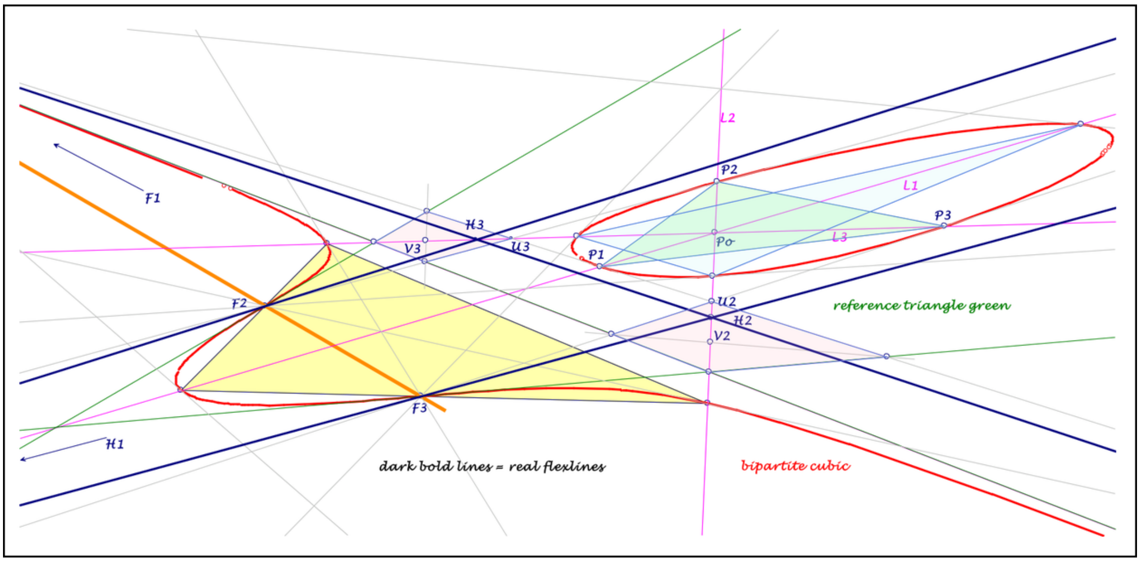
... ... are borders for the allowed real flexlines through Fi,

... attached this is the green triangle, which leads to the
right real flexlines.

Best regards Eckart

PS to Chris: Excuse my wrong remark in # 2603, that Vk is not
QG-P1.

Thanks for further interpretations.



2025-02-05.pdf

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Message: #2607
Date: 2025-02-05
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Radical axis in qudrilateral

---

Dear Chris,,

I don't speak french, but I think this article is interesting

Marchand, J
<http://archive.numdam.org/item/NAM_1927_6_2__320_1.pdf>, Sur
un théorème de Steiner.
Nouvelles annales de mathématiques : journal des candidats aux
écoles
polytechnique et normale, Serie 6, Volume 2 (1927), pp. 320-325

I translate the theorem by google

In each of the four triangles formed by the sides of a complete
quadrilateral, there is an inscribed circle and three
exinscribed circles;
which makes in all sixteen circles, the centers of which are
four by four
on a circumference so as to give birth to eight new circles.
These eight
circles are divided into two groups of four, such that each of
the circles
of one of the groups orthogonally intersects all the circles of
the other,
We conclude that the centers of the circles of the two groups
are on two
straight lines perpendicular to one to the other. " Finally,
these last two
straight lines intersect at the meeting point of the circles
circumscribed
by the four triangles."

APH

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Message: #2608
Date: 2025-02-05
From: eckart_schmidt@t-online.de
Subject: Re: Radical axis in qudrilateral

Dear ATM,

wrt a quadrilateral and its 16 in- and excircles,

... I made a drawing and following observations:

(1)   The 16 centers of the circles give four circles and 12 lines,

... each circle or line bearing 4 centers as triple intersections of the lines.

(2)   The 12 lines intersect pairwise orthogonal in the vertices of the quadrilateral.

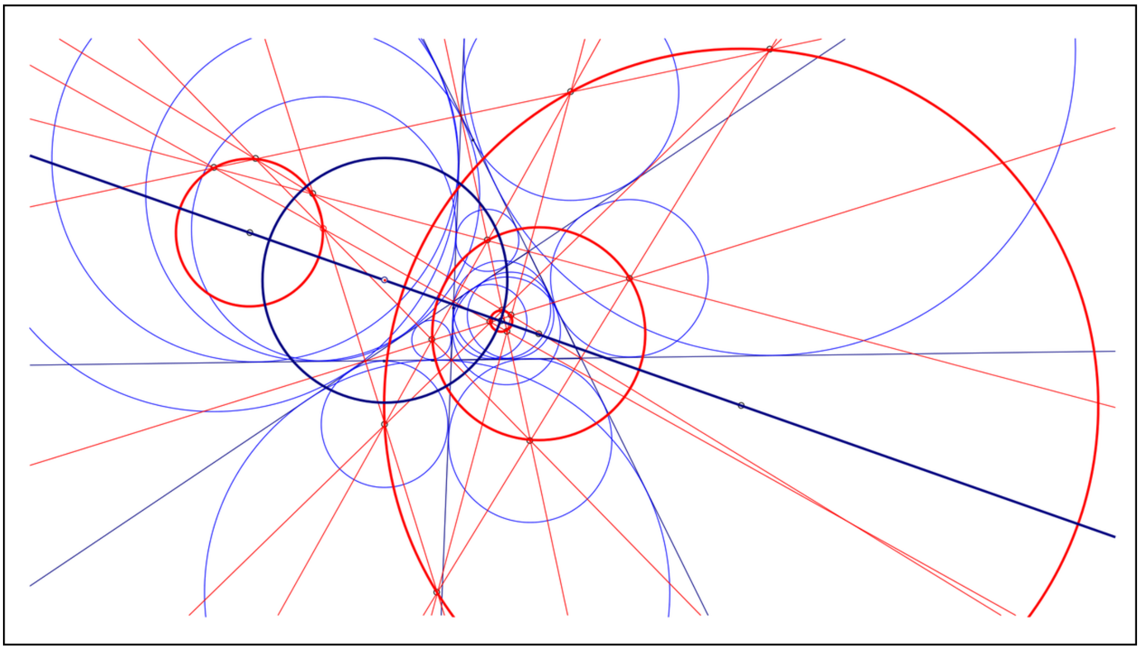
(3)   The 4 circles are CSC-invariant,

... they intersect the CSC inversion circle orthogonal

... and their centers are collinear on the CSC-axis.

I couldn't reproduce all properties in your message.

Best regards Eckart



2025-02-06.pdf

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Message: #2609
Date: 2025-02-06
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart
After a lot of drawings, I'm now convinced that Eckart's last construction is again a very ingenious, but fake construction !
1) for Chris, how do you find your points H1, H2 and H3 ? I suppose it's pure calculation ...
2) for Eckart, how do you know that your points are the real points H1, H2 and H3 ?
3) As I already told you, I use the reverse construction starting with a triangle H1H2H3 and a point P0 with barycentric coordinates (1/a1,1/a2,1/a3).
Then I know the equations of the line of real flexes and of the cubics of the pencil.
Variing k gives the cubics CU and Eckart's construction works only for relatively big values of k (almost, but not quite up from 10 and exact up from 25).
In this case, CU tends to RF and the fact that the construction gives the same result for CU and HE is easy to understand, as RF is it's own hessian ...
Best regards
Bernard
PS Chris, if you finally are able to calculate the coordinates of the points Hi by solving the system CU = 0 and FE = 0, could you please make an application for the MacCay stelloïd cubic K003 and it's Van Rees hessian cubic K048 wrt a triangle ABC ?
Then a1, a2 a3 and k and k' depend only from the 3 values a, b and c (lenghs of the sides of ABC). The equations of the 2 cubics are in Bernard Gibert ... I suppose you also could have the coordinates of A, B and C wrt the triangle H1H2H3 ?

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Message: #2610
Date: 2025-02-06
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

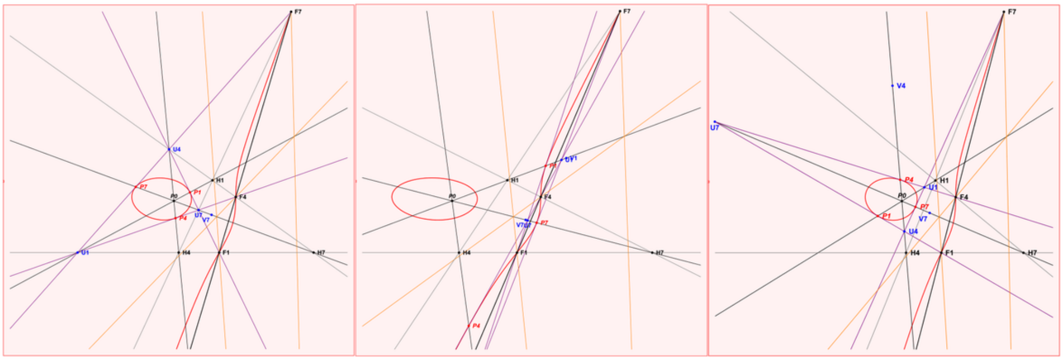
```
Dear Eckart,

I made pictures of the three different sets of (P1,P4,P7) such
that PiPj = Fk.
Unfortunately, according to my calculations, in all cases Hi is
not the Harmonic Conjugate of P0 wrt (Ui,Vi).

See attached file.

Best regards,
Chris
```

Three pictures in Eckart's case with different sets of (P1,P4,P7)



There are three different sets of (P1,P4,P7) such that PiPj = Fk.

According to calculations, in all cases Hi is not the Harmonic Conjugate of P0 wrt (Ui,Vi).

Chris van Tienhoven

2025, February 6

CU-HE-12L1abc-22__Eckarts-Case-Numerical.pdf

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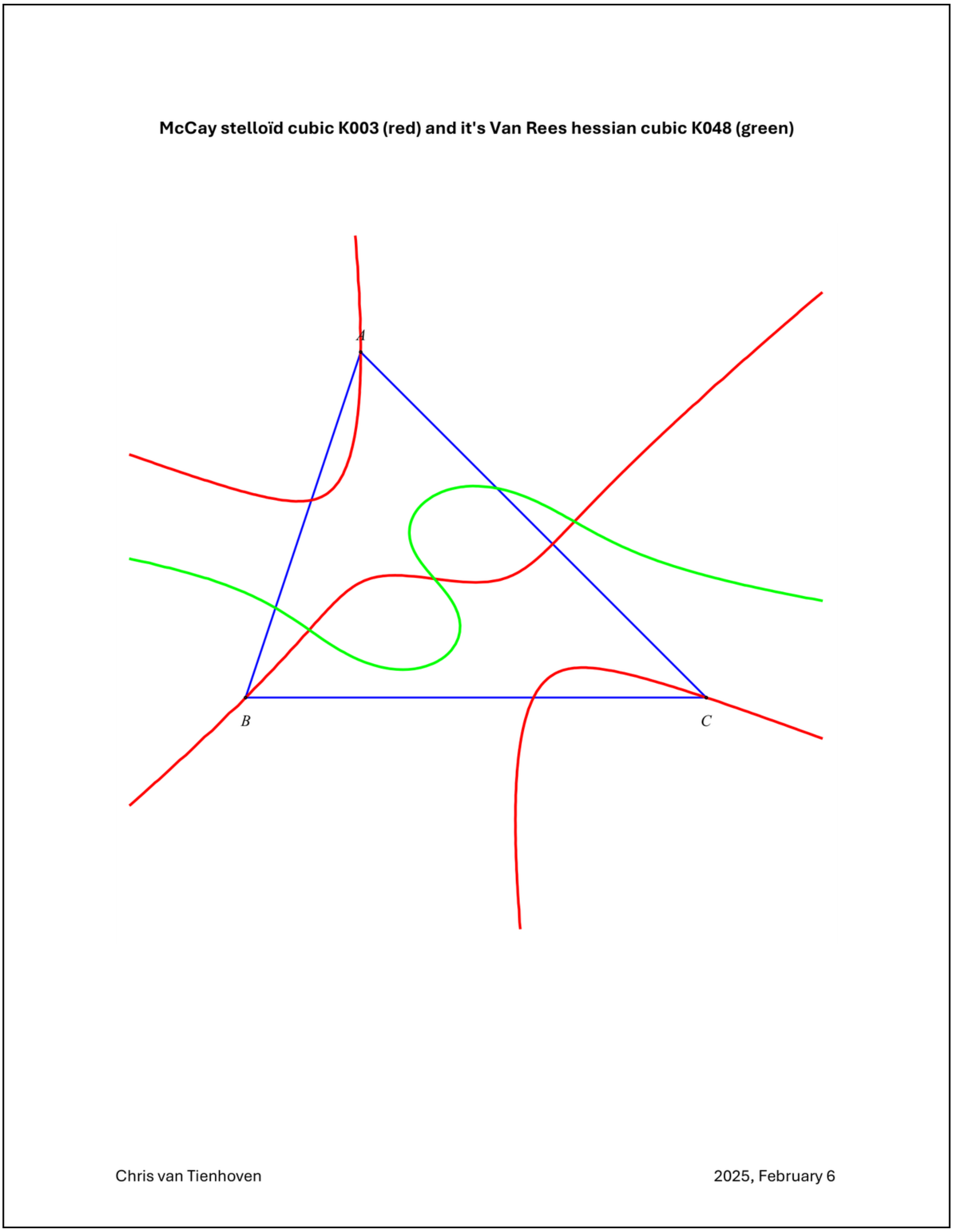
Message: #2611
Date: 2025-02-06
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Bernard,

Please find enclosed a picture of K003 and K048.
That's not so difficult to plot for me.
However finding the Flexpoints and the real flexlines, that part
will cost a lot of time.
Therefore just a quick and short picture this time.

Best regards,
Chris
```



K003-K048 cubics-01.pdf

Message: #2612
Date: 2025-02-06
From: contiwa.goma3@gmail.com
Subject: Re: Radical axis in qudrilateral

Dear all,

Generalization of #2602.

If foci of three QL1-inconics are intersection points of QL2 lines,
then pedal circles of the foci wrt QL1 (i.e. auxiliary circles of the QL1-inconics) are coaxial.
The radical axis passes through the midpoint of QL1-P9 and QL2-P9.
(cf. QFG#1832)

Best regards,
M@IMF

p.s. Ref-4 in EQF will be helpful for #2607.

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Message: #2613
Date: 2025-02-07
From: bernard.keizer@gmail.com
Subject: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris, dear Eckart
Here a summary of the most well-known properties of the Mac Cay cubic stelloïd K003 and it's Van Rees hessian K048, determining together a syzygetic pencil.
I hope Chris will be able to solve the system of the 2 barycentric equations wrt the triangle ABC, in order to find the triangle H1H2H3 and the point P0.
Best regards
Bernard
PS I'll try to do the same for Kjp and it's hessian

MacCay cubic stelloïd K003 and Van Rees hessian cubic K048

1) Triangle ABC and the 2 cubics

Let's consider a triangle ABC with centroid G, circumcenter O, orthocenter H, Lemoine point K, isodynamic points S and S' on the Brocard Line OK and Steiner inellipse with foci F and F'.

The Mac Cay stelloïd cubic is the pivotal isocubic wrt ABC with pivot O.

The cubic passes through A, B, C, O, H.

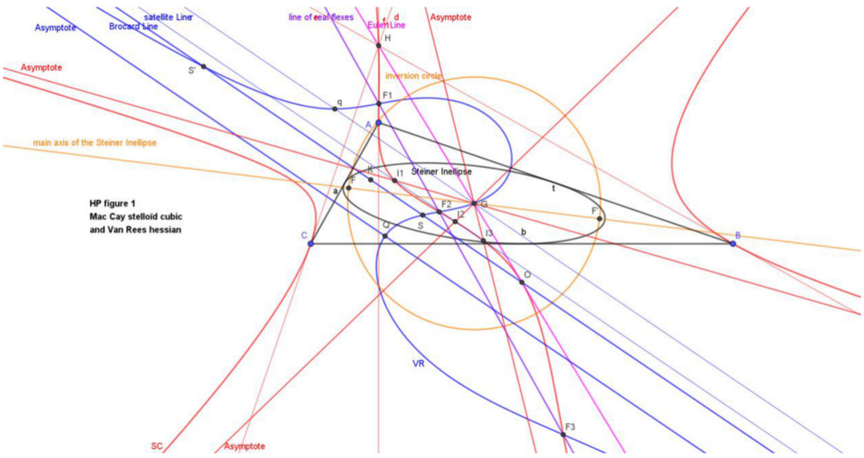
The 3 tangents in A, B and C are the altitudes of the triangle.

The 3 real asymptotes through G make angles of 60°, are parallel to the sides of the Morley triangles of the triangle ABC, intersect the curve in 3 points on the satellite Line of the infinity Line, homothetic to the Brocard Line in the homothety (G,2/3) and trisect the angle between the main axis of the Steiner inellipse and the parallel in G to the Brocard Line.

The Mac Cay hessian cubic is a Van Rees circular cubic, passing through G, S and S'.

The asymptote of this cubic is homothetic of the Brocard Line in the homothety (G,2).

The curve is invariant in the Moebius transformation centered in G with fixed points F and F' and swapping S and S'.



2) Syzygetic pencil

The 2 curves determine a syzygetic pencil and intersect in the 9 flexes (3 real and 6 imaginary) on 12 lines (4 real and 8 imaginary).

The 4 real lines are the line of the 3 real flexes and 3 others respectively through each real flex, determining a real triangle with vertices H1, H2 and H3 (this triangle is the cubic RF).

The 9 flexes have 9 harmonic polars, 3 real intersecting in P0 and 6 imaginary.

The 3 flextangents determine a triangle X1X2X3, P0 is the orthocenter of this triangle and the 3 real harmonic polars are the altitudes of this triangle.

3) Triangle H1H2H3

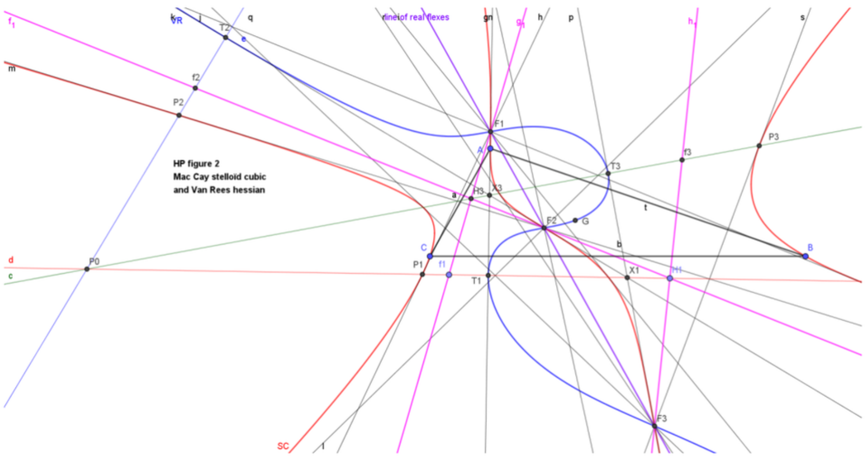
If fi is the harmonic of Fi wrt Hj and Hk, it must lie on the harmonic polar Li of Fi between Pi and Ti, intersections of Li and respectively the 2 curves Mac Cay and it's hessian.

Solving the system of barycentric equations of the 2 curves wrt the triangle ABC should give the equation of the 4 real flexlines and the coordinates of H1, H2 and H3 wrt ABC.

Reversing the calculation should also give the coordinates of A, B and C wrt H1H2H3.

It should give also the coordinates of F1, F2, F3 and P0 wrt ABC and wrt H1H2H3.

If a, b and are the lengths of the sides of ABC and a1, a2, a3, k and k' are the coefficients of the 2 cubics in the syzygetic pencil defined by H1, H2, H3 and P0, there is a relation between the 2 systems.



Mac Cay stellod cubic K003 and Van Rees hessian cubic K048.pdf

Message: #2614
Date: 2025-02-07
From: anopolis72@gmail.com
Subject: Miquel Circle

```
  FRANÇOIS GIRAULT
  <http://archive.numdam.org/item/NAM_1919_4_19__452_0.pdf>,
Sur le cercle de Miquel. Nouvelles annales de mathématiques 4e
série, tome
19 (1919), p. 452-456

APH
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Message: #2615
Date: 2025-02-07
From: van10hoven@gmail.com
Subject: Re: Miquel Circle

Dear Antreas,

Thanks for your new reference.

M. Francois Girault describes here:

1. the focus F of the parabola of a quadrilateral *QL-Co1 (http://www.chrisvantienhoven.nl/ql-items/ql-conics/ql-co1)* , which is *QL-P1 (http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p1)*.

2. a circle F passing through the centers of the circumcircles of the 4 component triangles of a quadrilateral, which is *QL-Ci3 (http://www.chrisvantienhoven.nl/ql-items/ql-circles/ql-ci3)*.

3. a circle passing through the centers of the circles at 2. of the 5 component quadrilaterals of a pentalateral, which is 5L-n-Ci1 (*nL-n-Ci1 (http://www.chrisvantienhoven.nl/nl-items/nl-obj/nl-ci/nl-n-ci1)* for n=5).

4. a common point of the circles mentioned at 3. of the 4 component quadrilaterals, which is 5L-n-P1 (*nL-n-P1 (http://www.chrisvantienhoven.nl/nl-items/nl-obj/nl-pts/nl-n-p1)* for n=5).

It is a nice introduction for the Encyclopedia of Polygon Geometry *EPG (https://www.chrisvantienhoven.nl/mathematics/encyclopedia-of-poly-geometry)*.

Best regards,

Chris

Message: #2616
Date: 2025-02-09
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Radical axis in qudrilateral

---

On Wed, Feb 5, 2025 at 11:04 AM Antreas Hatzipolakis
<anopolis72@gmail.com>
wrote:

>
> Marchand, J
<http://archive.numdam.org/item/NAM_1927_6_2__320_1.pdf>, Sur
> un théorème de Steiner.
> Nouvelles annales de mathématiques : journal des candidats aux
écoles
> polytechnique et normale, Serie 6, Volume 2 (1927), pp.
320-325
>

More references

H. G. Forder, <https://www.jstor.org/stable/3607453> Steiner's
Theorems on
Incentres and Ecentres
The Mathematical Gazette, Vol. 22, No. 248 (Feb., 1938), pp.
79-8

Jean-Pierre Ehrmann
<https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=
f741d7b6aaa4a73bc36706b599db740b73ce8538>,
Steiner's Theorems on the Complete Quadrilateral
Forum Geometricorum Volume 4 (2004) 35–52.

APH

Message: #2617
Date: 2025-02-09
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

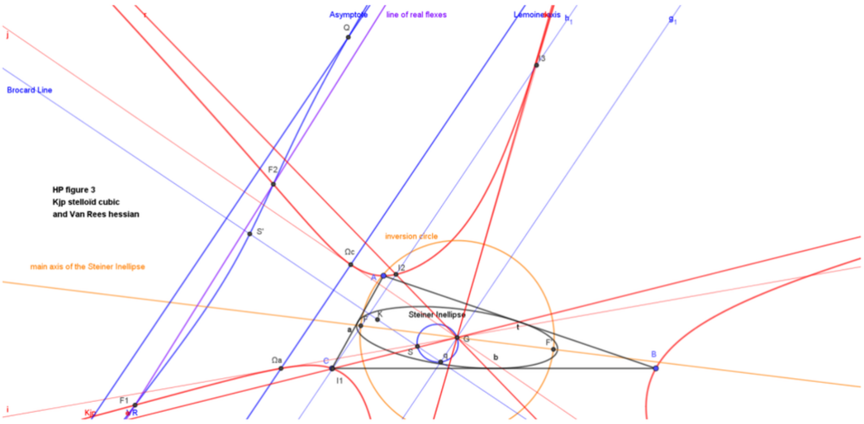
```
Dear Chris, dear Eckart
As promised, here a short addendum to my 1rst memo about Kjp and
it's hessian.
I will now wait on Chris's answer ...
Best regards
Bernard
```

Kjp cubic stelloïd K024 and Van Rees hessian cubic K193

1) Triangle ABC and the 2 cubics

The properties of the 2 cubic stelloïds are similar, both cubics intersect orthogonally in A, B and C and their asymptotes are orthogonal in G.

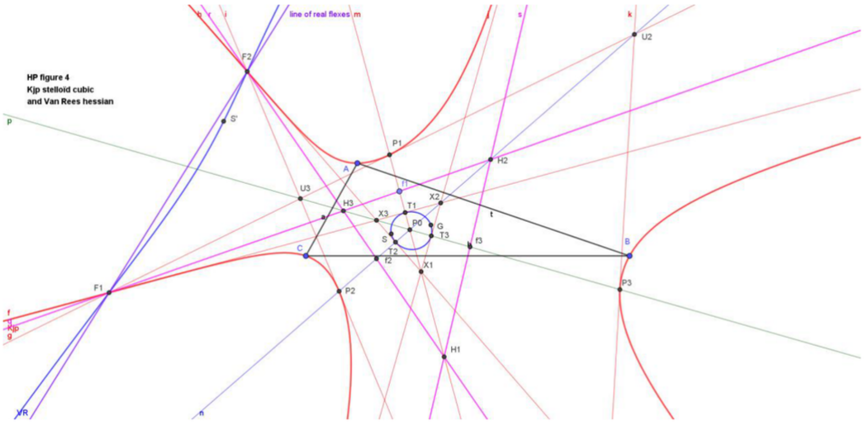
The properties of the 2 hessians are also similar, both hessians intersect orthogonally in S, S' and G and their asymptotes are orthogonal.



2) Syzygetic pencils and Triangles H1H2H3

Kjp and it's hessian determine also a sygyzetic pencil and P0 is also the orthocenter of X1X2X3 (property of the stelloïds, as the polar conics are equilateral hyperbolas).

The 2 points P0, the 2 groups of flexes and the 2 groups of harmonic polars are different as well as the 2 triangles H1H2H3 and the 2 syzygetic pencils.



Message: #2618
Date: 2025-02-10
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

```
Dear Bernard,

What is it exactly you want me to calculate?

Best regards,
Chris
```

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Message: #2619
Date: 2025-02-10
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

```
Dear Chris,
I want you to solve in both cases the system of the 2 equations
wrt ABC (all the coefficients are in terms of a, b and c) in
order to find the coordinates of F1, F2, F3, H1, H2, H3 and P0
wrt ABC and then by changing the reference triangle, the
coordinates of A, B, C, F1, F2, F3 and P0 wrt H1H2H3.
Finally, I hope you we will be able to get the equations of the
2 cubics and their hessians in their syzygetic system and if we
are lucky, we could find the equation of the 3 real prehessians
of MacCay and of the real prehessian of Kjp ...
Many thanks for your attention and best wishes for the
realisation of the calculation.
Best regards
Bernard
PS I'm convinced since a certain time that there isn't a simple
geometric construction of the points H1, H2 and H3 for a general
cubic and that it is pure calculation. This is why I would be so
happy if you could make this calculation for these 2 well-known
cubic stelloïds and their circular hessians ...
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Message: #2620
Date: 2025-02-14
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

```
Dear Chris,
I realise that I don't know in fact what you are capable to
calculate with Mathematica !
I thought you were able to calculate a1, a2 and a3 = f(a,b,c).
But you need perhaps real values of a, b, c ...
In this case, I suggest to choose (a, b, c) = (13, 6, 9) like in
ETC (see Tables).
Perhaps we would be capable to identify directly the point P0 !
Please just tell me where you are and what you intend to do.
Many thanks in advance
Best regards
Bernard
```

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Message: #2621
Date: 2025-02-14
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,

I once again tried to tackle the case of general cubic CU: c1 x^3 + c2 y^3 + c3 z^3 + c4 x^2 y + c5 x^2 z + c6 x y^2 + c7 y^2 z + c8 x z^2 + c9 y z^2 + c0 x y z wrt a reference triangle ABC and transcribe it to a cubic in Hesse’s form a1 x^3 + a2 y^3 + a3 z^3 + k x y z wrt some reference triangle H1(X1,Y1,Z1), H2(X2,Y2,Z2), H3 (X3,Y3,Z3).

That is doable.

I found that

a1 = T1 / (X1+Y1+Z1) ^3

a2 = T2 / (X2+Y2+Z2) ^3

a3 = T3 / (X3+Y3+Z3) ^3

k = T0 / ((X1+Y1+Z1) (X2+Y2+Z2) (X3+Y3+Z3)) ,

where:

T1 = c1 X1 3 +c4 X1 2 Y1+c6 X1 Y1 2 +c2 Y1 3 +c5 X1 2 Z1+c0 X1 Y1 Z1+c7 Y1 2 Z1+c8 X1 Z1 2 +c9 Y1 Z1 2 +c3 Z1 3 ,

T2 = c1 X2 3 +c4 X2 2 Y2+c6 X2 Y2 2 +c2 Y2 3 +c5 X2 2 Z2+c0 X2 Y2 Z2+c7 Y2 2 Z2+c8 X2 Z2 2 +c9 Y2 Z2 2 +c3 Z2 3 ,

T3 = c1 X3 3 +c4 X3 2 Y3+c6 X3 Y3 2 +c2 Y3 3 +c5 X3 2 Z3+c0 X3 Y3 Z3+c7 Y3 2 Z3+c8 X3 Z3 2 +c9 Y3 Z3 2 +c3 Z3 3 ,

T0 = 6 c1 X1 X2 X3 + 6 c2 Y1 Y2 Y3 + 6 c3 Z1 Z2 Z3

+2 c4 (X2 X3 Y1 + X1 X3 Y2 + X1 X2 Y3) + 2 c5 (X2 X3 Z1 + X1 X3 Z2 + X1 X2 Z3) + 2 c6 (X3 Y1 Y2 + X2 Y1 Y3 + X1 Y2 Y3)

+2 c7 (Y2 Y3 Z1 + Y1 Y3 Z2 + Y1 Y2 Z3) + 2 c8 (X3 Z1 Z2 + X2 Z1 Z3 + X1 Z2 Z3) + 2 c9 (Y3 Z1 Z2 + Y2 Z1 Z3 + Y1 Z2 Z3)

+ c0 (X3 Y2 Z1 + X2 Y3 Z1 + X3 Y1 Z2 + X1 Y3 Z2 + X2 Y1 Z3 + X1 Y2 Z3).

The big question is what are the coordinates of H1(X1,Y1,Z1), H2(X2,Y2,Z2), H3 (X3,Y3,Z3) wrt ABC.

In order to find the general coordinates of these points we first need to find the flexpoints. And in order to find the flexpoints we need to know the Hessian. Fortunately that is an easy to calculate cubic.

However to find the intersection points of CU and its Hessian (two equations of 3 rd order) in general form is the limit of what is doable with Mathematica. I tried in vain all kind of tricks, but nothing worked.

So I stopped that path.

That means that I can't calculate flexpoints and flexlines as a function of (a,b,c).

All I can do is getting numerical . . .

What is it you would like me to calculate that way?

Best regards,

Chris

Message: #2622
Date: 2025-02-16
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,
Many thanks for the calculations and explanations about what is doable or not !
These 2 calculations (HE in 2436 and a1,a2,a3 and k in 2621 will be my reference up to now.
1) Intersection of CU and HE in general terms is not doable
2) Calculation of $A1 = a1^3$, $A2 = a2^3$, $A3 = a3^3$ and K = ka1a2a3 is doable for any cubic in general terms (congratulations for this idea !)
(The equations of the 3 sides of H1H2H3 are x=0, y=0 and z=0 and the equation of the line of the 3 real flexes is a1x+a2y+a3z=0)
I suppose you checked that the calculation for HE gives the same a1, a2 and a3 and $k' = -(6/k)^2 - k/3$ ...
3) Wrt any triangle ABC, the equation of K003 MacCay cubic stelloïd is \SigmaFix $a^2(b^2+c^2-a^2)x(c^2y^2-b^2z^2)$... (cyclic) = 0
The equation of K024 Kjp cubic stelloïd is \SigmaFix $a^2x(c^2y^2+b^2z^2)$... (cyclic) = 0
(I don't write the equation of the hessians, they are in Bernard Gibert)
What are a1,a2,a3 and k ?
Is in this case the calculation of the 9 flexes doable ? (there are less coefficients, 6 for CU and 9 for HE instead of 10 and 10)
4) If not and if you have to take a numerical example, could you please choose a=13, b=6 and c=9 (ETC reference triangle)
Thanks in advance for these last efforts
Best regards
Bernard

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Message: #2623
Date: 2025-02-16
From: contiwa.goma3@gmail.com
Subject: Re: Quadrilateral Problem

```
Dear all,

Is this an already known thing?

Let
QL1 = QL{L1,L2,L3,L4}
F = QL1-P1
Tij = Li^Lj
Cl = the circumcircle of triangle TjkTkiTij
Ol = the circumcenter of Cl
QAo = QA{O1,O2,O3,O4}
QAo' = QA{O'1,O'2,O'3,O'4} which is homothetic to QAo with
homothetic center F
Co' = the circumcircle of QAo'
Pl = the second intersection point of Co' and Cl
QAp = QA{P1,P2,P3,P4}
QL2 = QL formed by Simson lines of F wrt component triangles of
QAp
L'l = perspectrix of triangles TjkTkiTij and O'iO'jO'k,
where {i,j,k,l} = {1,2,3,4}.
Then QL{L'1,L'2,L'3,L'4} and QL2 are similar.
(Note that when QAo' = QAo, Pl is I or its analogue in #2593.)

Best regards,
M@IMF

p.s. I didn't recognize that the point I and its analogues are
the perspectors of not only component triangles but also cyclic
quadrangles themselves.
```

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Message: #2624
Date: 2025-02-16
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

Dear all,

Perspectivity like #2623 can be applied to 5L in #2528 which is not a general 5L.

Let
5L2 = 5L formed by L1,L2,L3,L4,L5 which are tangents of a parabola
QLh = QL{Li,Lj,Lk,Ll}
F = QLh-P1 (= 5L2-n-P1)
Tij = Li^Lj
Olh = the circumcenter of triangle TjkTkiTij
Ch = QLh-Ci3
Oh = QLh-P4
5Po = 5P{O1,O2,O3,O4,O5}
5Po' = 5P{O'1,O'2,O'3,O'4,O'5} which is homothetic to 5Po with homothetic center F
Co' = the circumcircle of 5Po'
Ph = the second intersection point of Co' and Ch,
where {i,j,k,l,h} = {1,2,3,4,5}.
Then QA{Oih,Ojh,Okh,Olh} and {O'i,O'j,O'k,O'l} are perspective with perspector Ph.

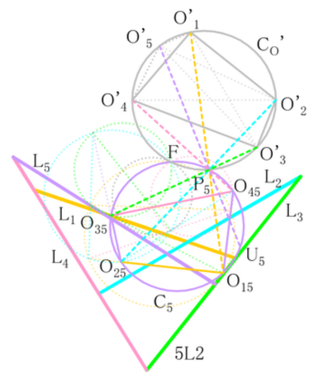
Lh is a Simson line of F wrt QA{O'i,O'j,O'k,O'l} whose homothetic ratio 4.
See #2527 and the attached file.

Best regards,
M@IMF

Perspectivity and Similarity in 5L

M@IMF

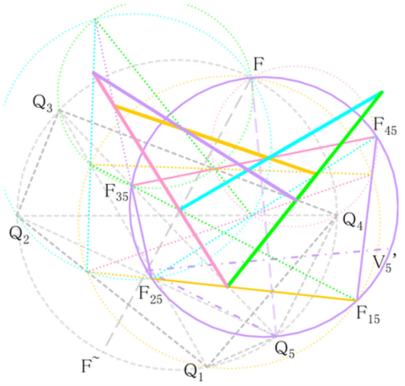
When lines L_1, L_2, L_3, L_4, L_5 are tangents of a parabola, some cyclic 5Ps related to them are perspective and similar.



Let

- $5L2 = 5L\{L_1, L_2, L_3, L_4, L_5\}$
- $QL_h = QL\{L_i, L_j, L_k, L_l\}$
- $F = QL_h\text{-}P1$ (= 5L2-n-P1)
- $T_{ij} = L_i \cap L_j$
- O_{lh} = the circumcenter of triangle $T_{jk}T_{ki}T_{ij}$
- $C_h = QL_h\text{-}Ci3$
- $O_h = QL_h\text{-}P4$
- $5P_o = 5P\{O_1, O_2, O_3, O_4, O_5\}$
- $5P_o' = 5P\{O'_1, O'_2, O'_3, O'_4, O'_5\}$ which is homothetic to $5P_o$ with homothetic center F
- C_o' = the circumcircle of $5P_o'$
- P_h = the second intersection point of C_o' and C_h,
- U_h = the second intersection point of line $P_hO'_h$ and C_h,

where $\{i,j,k,l,h\} = \{1,2,3,4,5\}$.
Then $5P\{O_{ih}, O_{jh}, O_{kh}, O_{lh}, U_h\}$ and $\{O'_i, O'_j, O'_k, O'_l, O'_h\}$ are perspective with perspector P_h, and similar with similitude center F.
(Note that the circumcircle of $5P_o$ is 5L2-n-Ci1.)



<Relationship to Simson lines>
Given concyclic points F, Q_1, Q_2, Q_3, ...,
a Simson line of F wrt cyclic nP is inductively defined.
Let

- Sim[ij] = line Q_iQ_j
- F_{ij} = the foot of the perpendicular from F onto Sim[ij]
- Sim[ijk] = a line through F_{ij}, F_{ik}, F_{jk}
- F_{ijk} = the foot of the perpendicular from F onto Sim[ijk]
- Sim[ijkl] = a line through F_{ijk}, F_{ijl}, F_{ikl}, F_{jkl}
- F_{ijkl} = the foot of the perpendicular from F onto Sim[ijkl]
- Sim[ijklh] = a line through F_{ijkl}, F_{ijkh}, F_{ijlh}, F_{iklh}, F_{jklh}
- ...

Then Sim[j1, ... ,jn] is the Simson line of F wrt cyclic $nP\{Q_{j1, \dots ,} Q_{jn}\}$.

When $Q_j(j=1,\dots,5)$ divides the line segment $F.O_j$ in the ratio 4 : −3,

- O_{ij} = the midpoint of F and F_{ij}
- $T_{lh} = F_{ijk}$
- L_h = Sim[ijkl]
- $5P\{F_{ih}, F_{jh}, F_{kh}, F_{lh}, V_h'\}$ and $\{O'_i, O'_j, O'_k, O'_l, O'_h\}$ are perspective and similar,

where

- V_h' = the reflection of V_h in $F.Q_h$
- V_h = the foot of the perpendicular from Q_h onto $FF^\sim$
- $F^\sim$ = the antipode of F wrt the circumcircle of $5P\{Q_1, \dots ,Q_5\}$
- $\{i,j,k,l,h\} = \{1,2,3,4,5\}$.

In general, nL-n-P3 of nL{Sim[j1,...,j(n−1)], ... , Sim[j2,...,jn]} divides the line segment $FF^\sim$ in the ratio $1 : 2^{n-2} - 1$.

Similarly, Sim[ijkl]s of $F^\sim$ give 5L4.
Note that $7P\{F, F_{ih}, F_{jh}, F_{kh}, F_{lh}, V_h', Q_h\}$, $\{F, Q_i, Q_j, Q_k, Q_l, Q_h, F^\sim\}$, $\{Q_h, F^\sim_{ih}, F^\sim_{jh}, F^\sim_{kh}, F^\sim_{lh}, V_h'', F^\sim\}$ are similar,
where V_h'' is the reflection of V_h in $F^\sim Q_h$.

Perspectivity and Similarity in 5L.pdf

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Message: #2625
Date: 2025-02-19
From: eckart_schmidt@t-online.de
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard, dear Chris,

wrt Chris message #2621:

What about starting with a general cubic

... as nonpivotal isocubic with reference triangle P1P2P3,

... with root Po(u,v,w), also fixpoint for the isoconjugation

... and a point H on the cubic.

Then you can discuss an equation

ux(w^2y^2+v^2z^2)+vy(u^2z^2+w^2x^2)+wz(v^2x^2+u^2y^2)+hxyz =0

... and get the flexpoints F1 = (0,-v,w), F2 = (u,0,-w), F3 = (-u,v,0),

... and the harmonic polars L1 = (0,w,-v), L2 = (w,0,-u), L3 = (-v,u,0),

... and the flexline F1F2F3 vwx+wuy+uvz = 0,

... and the flextangents of F1,F2,F3

(-hvw+2uv^2w^2)x-u^2vw^2y-u^2v^2wz = 0,

-uv^2w^2x+(-huw+2u^2vw^2)y-u^2v^2wz = 0,

-uv^2w^2x-u^2vw^2y+(-huv+2u^2v^2w)z = 0,

... and the intersections T1,T2,T3 of the flextangents and L1,L2,L3,

(2u^2vw,v(2uvw-h),w(2uvw-h)),

(u(2uvw-h),2uv^2w,w(2uvw-h)),

(u(2uvw-h),v(2uvw-h),2uvw^2),

... and the intersections X1,X2,X3 of the flextangents (uvw-h,v^2w,vw^2),

(u^2w,uvw-h,uw^2),

(u^2v,uv^2,uvw-h).

So far, I hope there is no mistake,

... but I am not up to date with your calculations.

Best regards Eckart

PS: Thanks for your messages 2609, 26010,

... I shall give up my constructions of the real flexline trilateral,

... meanwhile I found an example with significant aberrations

... for a low k as Bernard remarks ...

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Message: #2626
Date: 2025-02-20
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris, dear Eckart
It looks like the story of the ουροβοροσ, the snake which bites it's own tail !
In order to know a1, a2, a3 and k, you need the coordinates of H1, H2 and H3.
In order to know the coordinates of H1, H2 and H3, you need the flexes.
In order to know the flexes, you need the intersection of a CU and It's HE ...
In order to know P1, P2 and P3, you need the flexes ...
The equations of K048 and K193 are following :
K048 \SigmaFix $b^2c^2(b^2-c^2)(b^2+c^2-a^2)x^3-a^2(b^2+c^2-2a^2)(c^2(a^2+b^2-c^2)y-b^2(a^2+c^2-b^2)z)yz)$ cyclic = 0
K193 $-6a^2b^2c^2xyz$+\SigmaFix $b^2c^2(b^2+c^2)x^3$ cyclic +\SigmaFix $(b^2+c^2-2a^2)a^2(c^2y+b^2z)yz$ cyclic = 0
Chris, can you solve the system of K003 and K048 or K024 and K193 either in general terms or for a numerical example with a = 13, b = 6 and c = 9 ?
It could help greatly to understand these mysteries !
Best regards
Bernard

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Message: #2627
Date: 2025-02-25
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard and Eckart,

I made a picture with K003 and its Hessian K048 with reference triangle (6,9,13).

Also the Real Flexpoints (F1,F2,F3), the corresponding real Flexlines and the corresponding Harmonic Polars and P0. See attached file.

I saw no possibilities for calculating these items in general format.

Here are the relevant points in numeric barycentric coordinates:

F1 = {-0.563859, -0.801327, 1}

F2 = {3.0014, 4.02205, 1}

F3 = {-0.0918122, -0.162705, 1}

P0 = {1, -1.42454, 0.0616252}

And here are the relevant lines:

F1F2F3 = {1, -0.739163, -0.0284531}

The real Flexlines:

through F1 = {1, -2.0498, -1.0787}

through F2 = {1, -0.855748, 0.440457}

through F3 = {1, -0.22189, 0.0557097}

The Harmonic Polars:

wrt F1 = {1, 0.680691, -0.492151}

wrt F2 = {1, 0.729946, 0.646424}

wrt F3 = {1, 0.812874, 2.5634}

I hope it helps in our elaborations.

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Best regards,

Chris
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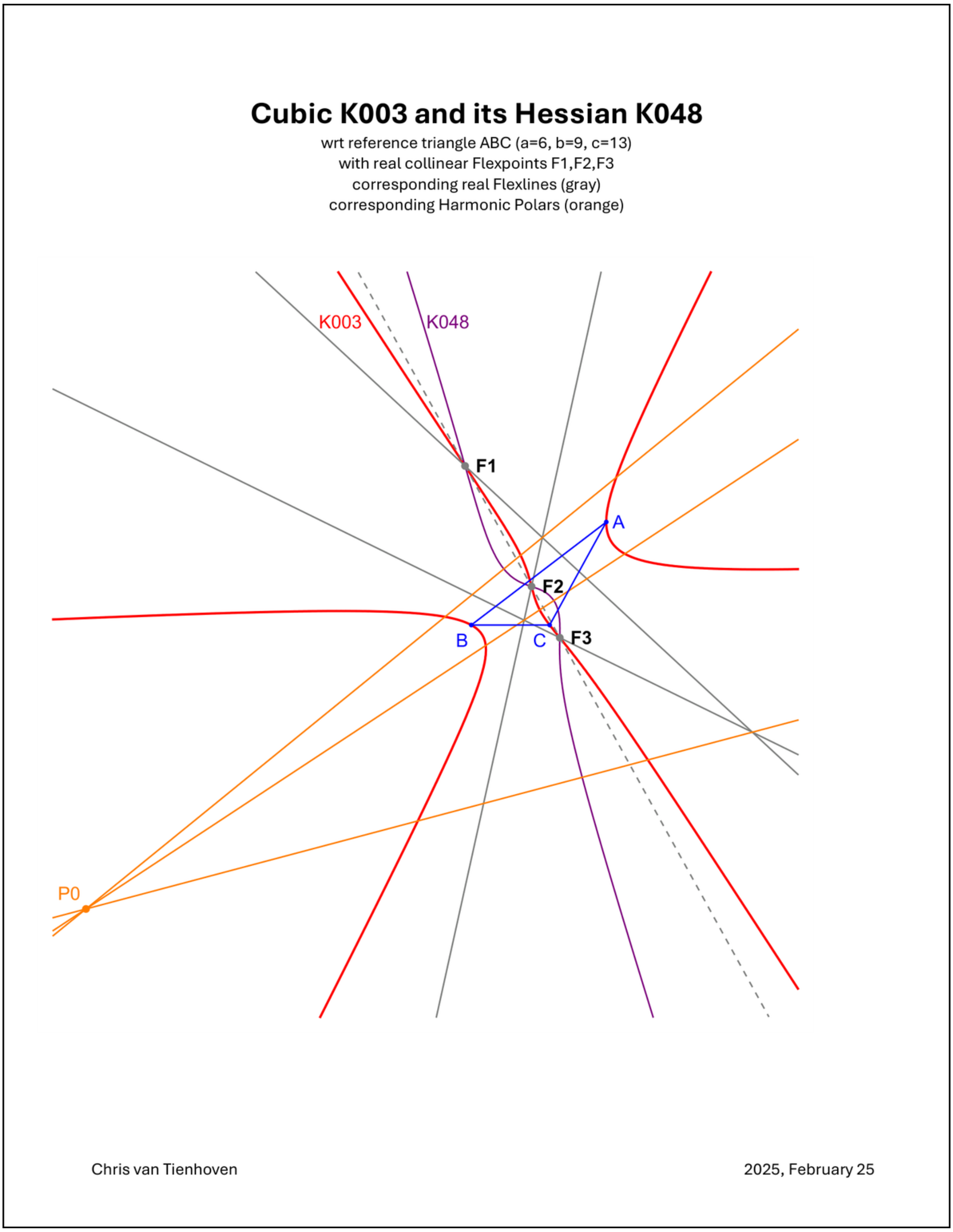

K003-K048 cubics-02.pdf

Message: #2628
Date: 2025-03-01
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

```
Dear Chris,
I wasn't home the last few days and I found your last message
only yesterday evening.
That's very beautiful and it is exactly what I was dreaming of !
Thanks a lot and congratulations.
You didn't finish completely the calculations and we need now
the exact coordinates of H1, H2 and H3 as intersections of the
real flexlines on the corresponding harmonic polars. Then it
would be nice to change the reference triangle in order to have
A, B and C as well as P0 wrt H1H2H3 and finally find a1, a2, a3
and k and k'.
I found approximately k = - 6.3 and k' = 1.2, but naturally, it
lacks on precision.
I wonder if P0 is an ETC point of the triangle ABC ...
Best regards
Bernard
```

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Message: #2630
Date: 2025-03-15
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,

Regarding your messages #2628 and #2629, I performed the following calculations:
H1={0.078976,0.606993,1.}
H2={-0.193416,-0.620606,1.}
H3={-1.5292,-1.27227,1.}
When I change the reference system to H1H2H3, I obtain:
CU = 1. x^3-156.861 y^3-3.05422 z^3 - 48.3472 x y z
HE = 1. x^3-156.861 y^3-3.05422 z^3 + 8.73714 x y z
Accordingly k = -6.17874 and k' = 1.1166, which closely match your results!
However, I could not find P0 as an ETC point of ABC.

Best regards,
Chris

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Message: #2631
Date: 2025-03-18
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

```
Dear all,

To describe *n-Chain* , I defined lines *wL* and points *wP*
inductively.
Let F, F', Qi, Qj, Qk, ... be concyclic.

{Qi,Qj,Qk; F,F'}
*wL* [ijk]
wL[ijk]1 = a line through F perpendicular to wL[ijk]2
wL[ijk]2 = Simson line of F wrt triangle QiQjQk
wL[ijk]3 = Simson line of F' wrt triangle QiQjQk
wL[ijk]4 = a line through F' perpendicular to wL[ijk]3
*wP* [ijk]
wP[ijk]r = wL[ijk]r∩wL[ijk](r+1)  (r=1,2,3)

{Qi,Qj,Qk,Ql; F,F'}
*wL* [ijkl]
wL[ijkl]1 = a line through F perpendicular to wL[ijkl]2
wL[ijkl](r+1) = wP[ijk]r∪wP[ijl]r∪wP[ikl]r∪wP[jkl]r  (r=1,2,3)
wL[ijkl]5 = a line through F' perpendicular to wL[ijkl]4
*wP* [ijkl]
wP[ijkl]r = wL[ijkl]r∩wL[ijkl](r+1)  (r=1,2,3,4)

{Qi,Qj,Qk,Ql,Qh; F,F'}
*wL* [ijklh]
wL[ijklh]1 = a line through F perpendicular to wL[ijklh]2
wL[ijklh](r+1) =
wP[ijkl]r∪wP[ijkh]r∪wP[ijlh]r∪wP[iklh]r∪wP[jklh]r  (r=1,2,3,4)
wL[ijklh]6 = a line through F' perpendicular to wL[ijklh]5
*wP* [ijklh]
wP[ijklh]r = wL[ijklh]r∩wL[ijklh](r+1)  (r=1,2,3,4,5)

...

(I hope "∪" is not garbled. If it is, I will use "_" instead. "L
= P1_P2_ ... _Pn" means points P1,...,Pn are collinear and L is
a line through them.)

Relationship to *6-Chain* (#2550):
F = A0, F' = A6
Qa = a point such that QL-P4 of {5L2b, 5L2c, 5L2d, 5L2e} divides
A0.Qa in the ratio 1:3
wL[abcde]r = line 6L(r+1)  (r=1,2,3,4)
```

```
wP[abcde]r = Ar  (r=1,2,3,4,5)
5Lr = {wL[bcde]r, wL[acde]r, wL[abde]r, wL[abce]r, wL[abcd]r}
 (r=2,3,4)
Br.a = wP[bcde]r  (r=1,2,3,4).

Relationship to *Simson line wrt nP* (#2527,2624):
wL[j1,...,jn]2 = Sim[j1,...,jn; F]
wL[j1,...,jn]n = Sim[j1,...,jn; F']
wP[j1,...,jn]1 = F.j1,...,jn
wP[j1,...,jn]n = F'j1,...,jn.

Best regards,
M@IMF
```

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Message: #2632
Date: 2025-03-18
From: van10hoven@gmail.com
Subject: preHessian Form of a cubic, three parameters define a cubic

---

Dear Bernard and Eckart and friends,

Hesse's Form

I have reviewed for myself the key results concerning the matter of Hesse's form once again.

For the latest summary, see message #2531.

The basics are actually quite simple: a cubic of the form

c1 x^3 + c2 y^3 + c3 z^3 + c4 x^2 y + c5 x^2 z + c6 x y^2 + c7 y^2 z + c8 x z^2 + c9 y z^2 + c0 x y z = 0

can be transformed via a projective transformation into a cubic of the form:

x^3 + y^3 + z^3 + k x y z = 0.

This is Hesse's Form and being described in several scientific papers.

preHessian Form

What we have added to this is an intermediate form. Let's call it the *preHessian Form* , which is given by:

a1^3 x^3 + a2^3 y^3 + a3^3 z^3 + k a1 a2 a3 x y z = 0.

This form can again be transformed via a simple projective transformation into:

x^3 + y^3 + z^3 + k x y z = 0, Hesse's Form.

The key aspect of the *preHessian Form* is that the actual shape of the cubic does not change, but only the reference frame shifts from a triangle *ABC* to a triangle *$H_1H_2H_3$* , which is the triangle formed by the only three real flex lines passing through the only three real flex points of a cubic.

Since the cubic remains the same, but its defining equation changes due to a change in perspective, it follows that any cubic can be described by:

$$a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3 + k\, a_1 a_2 a_3\, x y z = 0.$$

A direct consequence of this is that each cubic is characterized by four numbers: a_1^3, a_2^3, a_3^3, and $k\, a_1 a_2 a_3$.
Since one of these four factors can be normalized to 1, the most natural choice is to set $k\, a_1 a_2 a_3 = 1$, which results in the equation:

$$x^3 / (k\, a_2 a_3) + y^3 / (k\, a_1 a_3) + z^3 / (k\, a_1 a_2) + x y z=0.$$

Thus, every cubic —and consequently its shape— depends on the numbers appearing in this expression: $1 / (k\, a_2 a_3)$, $1 / (k\, a_1 a_3)$, $1 / (k\, a_1a_2)$,

which means that the shape of each cubic depends on three numbers.

Parallel with conics

This is analogous to a conic section, whose shape depends on just *one number* , namely its *eccentricity*.

The eccentricity e of a conic section determines its shape and how "stretched" or "deviated from a circle" it is. It is defined as the ratio of the distance from any point on the conic to a focus, compared to its distance from the corresponding directrix. The value of e classifies the conic section:

$e=0$ → Circle

A circle is a special case where the foci coincide with the center.

$0<e<1$ → Ellipse

An ellipse has two foci, and the closer e is to 1, the more elongated the ellipse appears.

$e=1$ → Parabola

A parabola has exactly one focus and one directrix, and it extends infinitely outward.

$e>1$ → Hyperbola

A hyperbola consists of two separate branches, and the larger e, the more widely spaced the branches are.

In summary:

The closer e is to 0, the more circular the conic section.

The larger e, the more elongated or open the conic becomes.

New questions:

* My question is: Do you agree with my conclusions, or am I overlooking something?
* If all of this is correct, what can be said about the shape of a cubic based on these three cubic parameters?

Best regards,

Chris

Message: #2633
Date: 2025-03-19
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

```
Dear all,

Let me correct "Relationship to 6-Chain" in #2631 as follows:
F = A0, F' = A6
Qa = a point such that QL-P4 of {5L2b, 5L2c, 5L2d, 5L2e} divides
A0.Qa in the ratio 1:3
wL[abcde]r = line 6Lr.f  (r=1,2,3,4,5,6)
wP[abcde]r = Ar  (r=1,2,3,4,5)
5Lr = {wL[bcde]r, wL[acde]r, wL[abde]r, wL[abce]r, wL[abcd]r}
 (r=2,3,4)   (5Lr.a = wL[bcde]r)
Br.a = wP[bcde]r  (r=1,2,3,4).

In general
A[n]0 = F , A[n](n+1) = F~ (antipode of F on circumcircle of
cyclic nP{Qj1,...,Qjn})
A[n]r = wP[j1,...,jn]r  (r=1,...,n)
(n+1)Lr.(n+1) = wL[j1,...,jn]r  (r=1,...,n+1)
nLr.k = wL[j1,...,j(k-1),j(k+1),...,jn]r  (r=1,...,n)
B[n]r.k = wP[j1,...,j(k-1),j(k+1),...,jn]r  (r=1,...,n-1)
B[n]0.k = F , B[n]n.k = F~
(n-1)L-n-P3 of {nL2.j2, ..., nL2.jn} divides F.Qj1 in the ratio
1 : 2^(n-2) -1.

Best regards,
M@IMF
```

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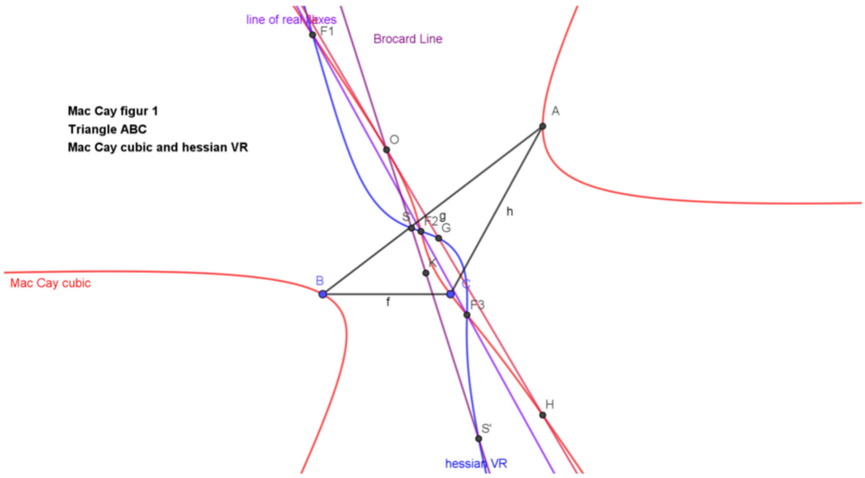
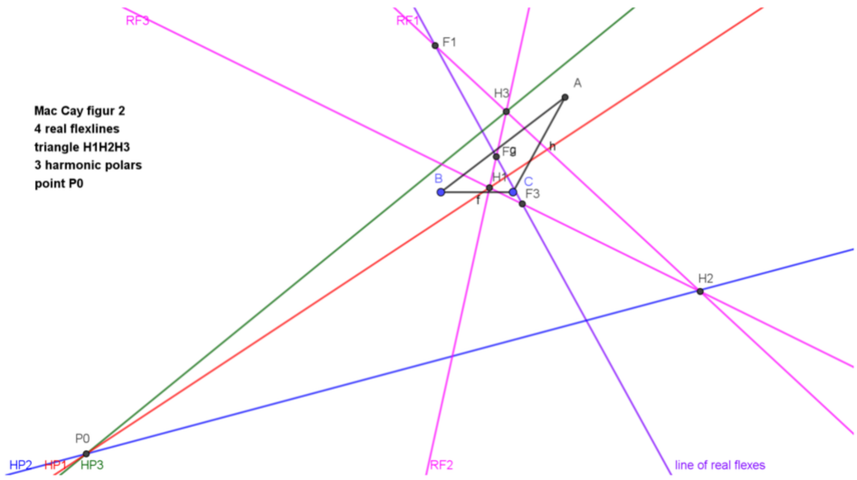
Message: #2634
Date: 2025-03-19
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

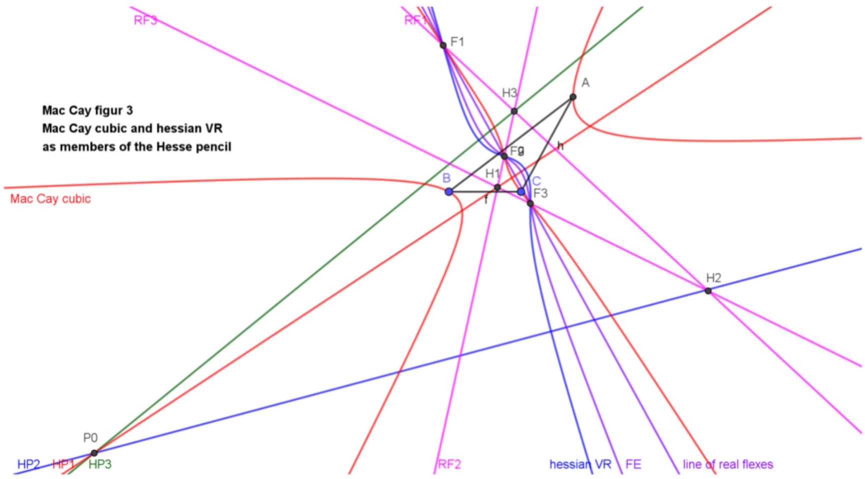
Dear Chris,
Many thanks for these last elements !
I send you a short memo with 4 figures (figur 1 with K003 and K048 wrt triangle ABC, figure 2 drawing of the triangle H1H2H3 and the harmonic polar, figure 3 K003 and K048 wrt H1H2H3, figure 4 cayleyan CA and curve Γ wrt H1H2H3).
As expected, CA is tangent to the hessian in 3 points on the harmonic polars and has 3 cusps on the harmonic polars and Γ has 3 cusps on the harmonic polars in the contact points of the hessian and the cayleyan.
Best regards
Bernard

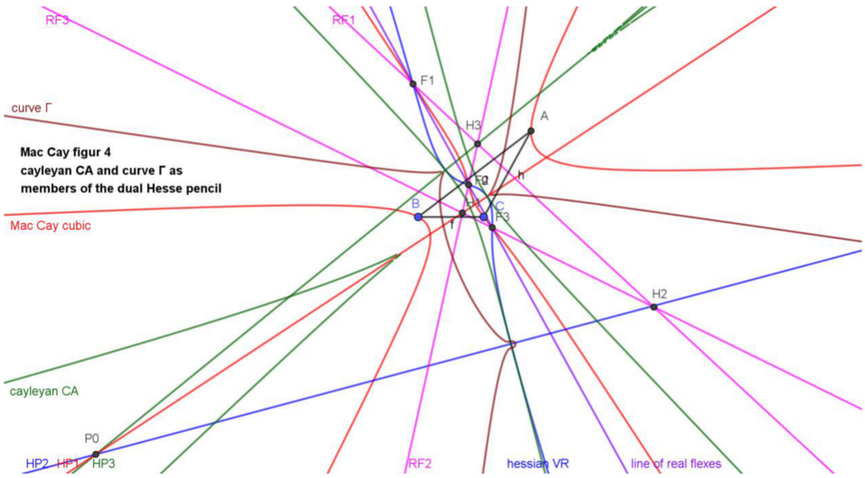
Mac Cay cubic and hessian





Mac Cay.pdf


Mac Cay figur 3
Mac Cay cubic and hessian VR
as members of the Hesse pencil
RF3
RF1
F1
H3
A
Fg
h
B
H1
C
F3
f
Mac Cay cubic
H2
P0
HP2
HP1
HP3
RF2
hessian VR
FE
line of real flexes


Mac Cay figur 4
cayleyan CA and curve Γ as
members of the dual Hesse pencil
RF3
RF1
curve Γ
F1
H3
A
Fg
h
B
C
F3
f
Mac Cay cubic
H2
cayleyan CA
P0
HP2
HP1
HP3
RF2
hessian VR
line of real flexes

Mac Cay.pdf

Message: #2635
Date: 2025-03-20
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

sorry I cannot give up, please control the following method,

... to find the real flexlines for a bipartite cubic:

Consider a bipartite cubic and a cubic point P

... with four contact points of its tangents at the cubic,

... which give a QA with its diagonal triangle TR.

Consider further a variable triangle H1'H2'H3'

... on the harmonic polars L1,L2,L3 of the real flexpoints,

... with the flexpoints F1,F2,F3 on the sidelines.

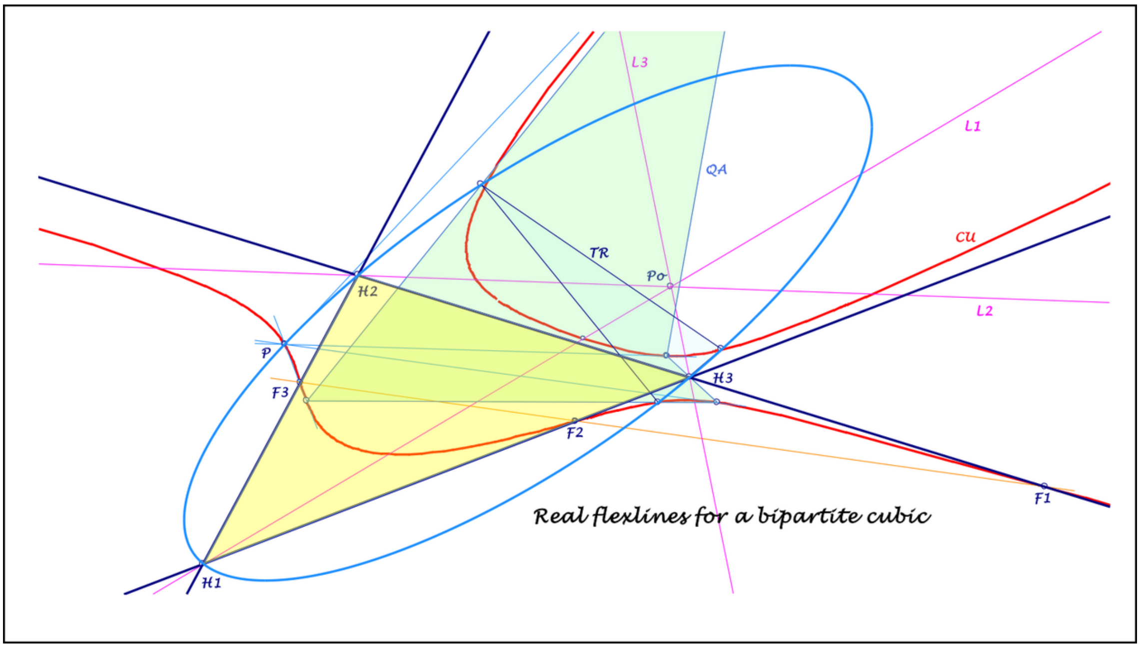
You can find a conic, bearing P, the vertices of TR and
H1',H2',H3',

... then Hi' will be the intersections Hi of the real flexlines.

If the cubic is a pivotal isocubic, take the pivot as point P,

... the fixpoints of the isoconjugation give QA ...

Best regards Eckart



2025-03-20.pdf

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Message: #2636
Date: 2025-03-21
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,

Thanks for the beautiful pictures. Even with two Cayleyans!

Is Γ the Cayleyan of the Hessian?

Why did you draw the Brocard line in the first picture?

Best regards,

Chris

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Message: #2637
Date: 2025-03-21
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,
As already explained in a previous memo, Γ is the curve of the
the 3rd order associated to a cubic CU, which swaps with CU
their respective hessian and cayleyan.
In other words, if HE and CA are the hessian and the cayleyan of
CU, CA and HE are the hessian and the cayleyan of Γ.
CU and HE belong to the Hesse pencil for k and k', CA and Γ to
the dual Hesse pencil for k'' and k*.
I drew in the 1rst picture the points O (X3) and H 'X4) on the
Euler Line, which belong to Mac Cay and G (X2), S 'X15) and S'
'X16) on the Brocard Line, which belong to it's hessian, in
order to check on the 3rd picture that these points are on the
curves calculated wrt H1H2H3.
Best regards
Bernard

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Message: #2638
Date: 2025-03-21
From: bernard.keizer@gmail.com
Subject: Re: preHessian Form of a cubic, three parameters define a cubic

---

Dear Chris,
I would like to help you, but I'm not an expert in this matter ...
I know only what I read in several references : the most relevant are

* Salmon
* Schröter (see in particular the pages dedicated to the shape of the cubics)
* Roger Cüppens (see in particular tome 2 chap 17 page 259 ...)
* Artebani
* Bonifant (see in particular figur 10 page 41)
* Mathcurve (see in particular the definition ''cubic'')

Your question of the classification of the cubics has been studied by Newton and Chasles and a few others ...
A Hesse pencil is defined by a triangle H1H2H3 and a point P0(1/a1,1/a2,1/a3), the line of real flexes being a1X + a2Y + a3Z = 0 ...
Then the particular cubic depends only from one coefficient, which is k (which may be any value between -∞ and +∞, including 0).
Best regards
Bernard

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Message: #2639
Date: 2025-03-23
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart,
Eckart, it's amazing, it seems you are this time on the target !
It is like the walls of Jericho, they fell only at the 7th turn
!!!
Please accept my deep and sincere congratulations
I can valid your construction
1) For a general bipartite cubic constructed wrt a triangle
H1H2H3 with according equations in a1, a2, a3 and k, the points
P, the vertices of the DT of the tangentialQA of P and the
points H1, H2 and H3 are coconic.
Such a bipartite cubic is a pivotal isocubic in any of it's
points with fixed points of the isoconjugation being the
vertices of the tangential QA of the point.
Therefore, you may find an infinity of such conics through any
point P and the vertices of the DT of it's tangential QA and H1,
H2 and H3.
For example, P and this DT form the tangential QA of the
isopivot P' and the vertices of the DT of this new QA (I named
it DDT in old times) are coconic with H1,H2,H3 and the isopivot.
2) In the example of Mac Cay, the cubic is a pivotal isogonal
cubic wrt a triangle ABC, the pivot being the circumcenter O and
the fixed points the in- and excenters. DT is the triangle ABC
and it is easy to check on Chris figure wrt ABC = (6,9,13) that
A,B,C,H1,H2,H3 and the circumcenter are coconic.
The cubic is also pivotal with pivot the orthocenter and the
fixed points A,B,C and the circumcenter. The DT is made by the
3rd intersections of the sides of ABC with the cubic and it is
easy to check that the 3 3rd intersectons are coconic with
H1,H2,H3 and the orthocenter.
3) Taking any triangle ABC and considering the Mac Cay cubic and
it's hessian VR gives easily the 3 real flexes as intersections
as well as their harmonis polars. Using your method allows to
find H1, H2 and H3.
I hope Chris will be able to valid your observation perhaps by
calculating the equation of the conic in the Mac Cay example ?
I suppose it is now the end of this long quest
Many thanks for this happy conclusion
Best regards
Bernard
PS As I am always curious, how did you come on the idea of this
property ?

Message: #2640
Date: 2025-03-24
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Last idea this morning !
The conic through the vertices of DT and P is the conjugate of a line through the isopivot P' in the isoconjugation with fixed points the vertices of the tangential QA.
The conjugates of H1,H2 and H3 in this isoconjugation are aligned on a line through the isopivot.
For Mac Cay again, which is an isogonal pivotal cubic wrt ABC with pivot the circumcenter O, the isogonals of H1, H2 and H3 are aligned on a line through the orthocenter.
Best regards
Bernard

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Message: #2641
Date: 2025-03-24
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your certification and further remarks,

... result of one year with hundreds of drawings looking for properties.

Now I study the possibility to construct the conic through TR, H1H2H3 and P,

... whose center is the common point (on QA-Co1 of TR plus P)

... of the three loci for the conic centers of Hi',Hj' plus TR ...

Best regards Eckart

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Message: #2642
Date: 2025-03-25
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart, Dear Bernard,

Eckart, congratulations!
I did some checks, and it looks like you are spot on.
I am very happy for you that, after a year of studying and experimenting, you have achieved such a great result.

In order to verify it, I first attempted a general approach, but I wasn't able to complete the calculations in Mathematica.
Therefore, I looked for a numerical example of a bipartite cubic, and the Neuberg Cubic K001 was a good choice.
I also searched for a point P that produces four tangents at tangency points (S_1, S_2, S_3, S_4) on the cubic, and X(14) turned out to be a good example. Then I calculated the vertices (DT_1, DT_2, DT_3) of the Diagonal Triangle of QA (S_1, S_2, S_3, S_4).

Following your instructions, I calculated the conic through (X(14), DT_1, DT_2, DT_3 , H_i) for i = 1, 2, 3.
The coefficients of the three conics matched exactly up to 13 decimal places.
For a visualization, see the attached file.

Of course, this is just one example.

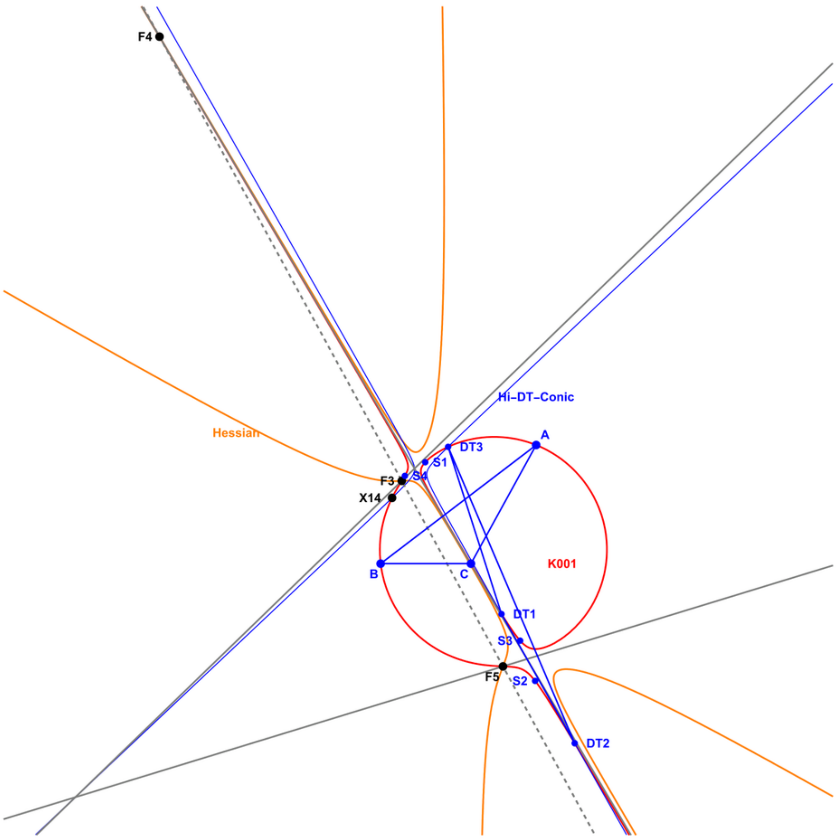
But given that you checked it, Bernard checked it, and I also verified it under different conditions, and we all arrived at the same result,
at the very least, we can say that your construction has not been disproven and has a high probability of being correct.

It is also important to note that the theorem has only been tested for a bipartite cubic so far.
Personally, I expect it to hold in general for all cubics, but that would require calculations with complex numbers. Perhaps later…

For now, I am very pleased to share my findings with you.

Best regards,
Chris

Construction of the real flexlines according to method Eckart



The reference cubic is the bipartite Neuberg Cubic K001 (in red color).
The Hessian HE is shown (in orange color).
(F3,F4,F5) are the real flexpoints as intersection points of K001 and HE.
As point P is chosen X(14).
(S1,S2,S3,S4) are the points of tangency of the tangents from X(14).
(DT1,DT2,DT3) are the vertices of the Diagonal Triangle of QA(S1,S2,S3,S4).
Hi-DT-Conic is the conic through (X(14), DT1,DT2,DT3, H1).
Indeed H2 and H3 lie precisely on this very conic.

Chris van Tienhoven March 25, 2025

CU-12L1-Real-Flextangents-10.pdf

Message: #2643
Date: 2025-03-27
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

I was still reflecting on Eckart's beautiful discovery.
Here are some additional remarks:

* The method assumes that we have four points sharing the same tangential (P and vertex points QA-DT(P)). However, this does not yet provide five points to define a fixed conic. Is there a convenient locus to consider for a variable H1 (on Harmonic Polar-1) that determines H2 and H3?
* When CU is not bipartite, we can use HE as a starting point, as it may be bipartite then or structured in such a way that there are situations with four points sharing the same tangential. After all, CU and HE share the same flexpoints and thus also share the three real flexlines.

Best regards,
Chris

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Message: #2644
Date: 2025-03-28
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris,
1) As mentionned by Eckart, let's consider the QA-Co1 of a variable H1 on HP1 and the vertex of QA-DT(P) and the same for the corresponding H2 and H3 on HP2 and HP3 such as H1H2 through F3 and H1H3 through F2 : the 3 conics have a 4th common point on the QA-Co1 of P and the vertex of QA-DT(P) when H1, H2 and H3 are the searched vertices of RF.
2) Naturally, if CU is monopartite, HE will be bipartite ...
Best regards
Bernard

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Message: #2645
Date: 2025-03-28
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris, dear Bernard,

thanks to Chris for his acknowledgement and the extensive
calculations,

... now I shall look for observations for monopartite cubics.

Here a first test (attached), if six tangents from Po to the
cubic CU
exist,

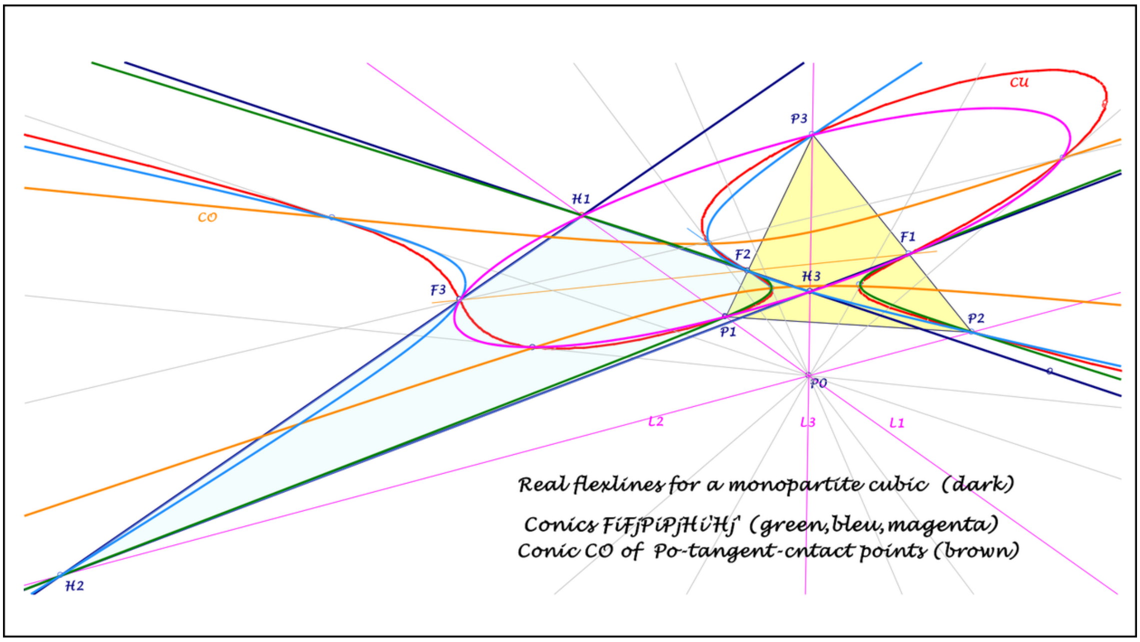
... the contact points are coconic on a conic CO,

... consider now the variable conics FiFjPiPjHi'Hj'

... and the constellations, that they intersect CO further on
the cubic,

... there are three of these constellations, for one holds Hi' =
Hi ???

Best regards Eckart
```



2025-03-28.pdf

Message: #2646
Date: 2025-03-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
By definition, your CO is the polar conic of P0 wrt CU and the
polar line of P0 wrt CO is the line of real flexes !
Best regards
Bernard
```

Message: #2647
Date: 2025-04-09
From: van10hoven@gmail.com
Subject: In memoriam Ángel Montesdeoca Delgado

Dear friends,

On the Euclid group, I received the following message from Francisco Javier García Capitán:

“I have just been informed that Ángel Montesdeoca Delgado, former teacher at the Universidad de La Laguna, Canary Islands (Spain), passed away on May 17, 2024.
We will always remember his deep knowledge of Projective Geometry, the great number of geometrical results he discovered, as well as his impressive website—not only for its content but also for its beautiful layout.”

I remember Ángel for his profound knowledge of geometry and his warm, friendly nature.
Whenever things became complicated, I could always count on him for clear and thoughtful answers.
It is sad that we learned of his passing so late.
He also discovered a QA-point, *QA-P38 (http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p38)* , which was named after him.
He made many other contributions to geometry.

Rest in peace, my friend.

Chris

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Message: #2648
Date: 2025-04-09
From: van10hoven@gmail.com
Subject: Away for a month

```
Dear friends,

This coming weekend, I'm leaving for a long hiking trip along
the GR65 in southern France.

I'll be away for at least a month, so don't worry if you don't
hear from me for a while.

I'll still be reading the messages, though.

Best regards,

Chris
```

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Message: #2649
Date: 2025-04-22
From: contiwa.goma3@gmail.com
Subject: Similitude Center of (n-1)Ps formed by Centric Centers

---

```
Dear geometers,

Given nL{L1,...,Ln} and let
Oi = (n-1)L-n-P3 of {L1, ..., L(i-1), L(i+1), ..., Ln}
Oij = (n-2)L-n-P3 of {L1, ..., L(i-1), L(i+1), ..., L(j-1),
L(j+1), ..., Ln}  (i≠j).
Then cyclic (n-1)P{O1, ..., O(i-1), O(i+1), ..., On} and {Oi1,
..., Oi(i-1), Oi(i+1), ..., Oin} are similar.
For n=5, the similitude center is 5L-o-P2.

In general, the similitude center lies on the line nL-n-P1.P3.
Is this well-known (or obvious)?

Best regards,
M@IMF
```

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Message: #2650
Date: 2025-04-26
From: bernard.keizer@gmail.com
Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

---

```
Dear M@IMF ,
You may read with interest the 4 references 47-48-49 and 55 of
EQF, the 3 1rst by Frank Morley and the last one by R.
Goormaghtigh and naturally the messages on the Forum exchanged
by Chris, Eckart and myself (mentionned by Chris under nL-n-P3).
Best regards
Bernard
```

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Message: #2651
Date: 2025-04-26
From: contiwa.goma3@gmail.com
Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

---

```
Dear Mr. Keizer,

Thank you for your reply.

I read the ref.48(Morley) and got the formula of the similitude
center (SC).
But I didn't realized that the SC is the inverse of
nL-n-P1(Centric Focus)
wrt nL-n-Ci1(Center Circle) when I posted the message.
After posting, I drew 5L-o-Ci1(Clifford's Circle) and noticed it
passes through 5L-n-P1.
(I didn't know even such a thing.)
Then I searched in EPG and found Properties of nL-n-P1.
Now I'm studying QFG#937-953(Center Circles and Clifford's Chain
for n Lines and n Penosculant).

I have a question.
5L is determined by Oi(i=1,...,5) and 5L-n-P1.
6L is determined by Oi(i=1,...,6) and what points?
(I think only 6L-n-P1 is not enough.)

Best regards,
M@IMF
```

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Message: #2652
Date: 2025-04-28
From: bernard.keizer@gmail.com
Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

---

```
Dear M@IMF ,
In fact, 4L and 5L are determined by the Oi and the centric
focus.
But I think you are right, it doesn't work for n>=6 !
I don't know the answer to your question ...
Best regards
Bernard
```

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Message: #2653
Date: 2025-04-28
From: contiwa.goma3@gmail.com
Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

---

```
Dear Mr. Keizer,

Thank you for answering.
I was wondering if there is a systematic way to determine nL by
using Oi(i=1,...,n) and other points.
It will be a great help for me.
On the other hand, I conjecture each nL appeared in Lill's
method is described by cyclic nP and one point.
I'm struggling to show that.
Anyway, thanks again.

Best regards,
M@IMF
```

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Message: #2654
Date: 2025-04-29
From: analgeomatica@gmail.com
Subject: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Geometers,

Let (A1, A2, A3, A4) be Quadrangle points.

Let P be any point.

Let B1, B2, B3, B4 be isotomic conjugates of P with respect to the
triangles A2A3A4, A3A4A1,A4A1A2, A1A2A3, respectively.

Then six intersections of six pairs of line (A1A2,B3B4), (A3A4,B1B2),
(A1A4,B2B3), (A2A3,B1B4), (A1A3,B2B4), (A2A4,B1B3) are collinear on line d.

Thus we have a transformation of P to line d.

Is this transformation new?

Sincerely yours,
Tran Quang Hung

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Message: #2656
Date: 2025-04-29
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Tran Quang Hung,

Although I don't know whether your transformation appeared in
EQF, QPG and QFG,
it seems that isotomic conjugates can be generalized to
isoconjugates.
(I only checked numerically for some examples.)

Best regards,
M@IMF
```

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Message: #2657
Date: 2025-04-29
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung,

your transformation seems not to be discussed in the group,

... it holds also for the isogonal isoconjugation,

... but cannot generalized for all isoconjugations.

A property of your transformation:

Consider a QA and its diagonal triangle TR,

for the points of a TR circumscribed conic

the image lines will envelope an inscribed TR conic.

Best regards Eckart

Message: #2658
Date: 2025-04-29
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Mr. Schmidt,

Thank you very much for correcting my mistake.
I checked only isoconjugations like
(p:q:r) -> (a^n/p : b^n/q : c^n/r).
I was careless.

Best regards,
M@IMF
(There is a royal road to geometry. Ask geometry masters.)
```

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Message: #2659
Date: 2025-04-30
From: analgeomatica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

---

Dear Mr Eckart and friend,

Thanks a lot for your attention. I'm glad to know that it seems
to be a new
transformation, and not only that, it is a generalization of
isogonal
isoconjugation. I find that very useful. As for the property on
triangle
TR, I find it very strange and new to me. I will look into it
further.

Sincerely yours,
Tran Quang Hung

Vào Th 3, 29 thg 4, 2025 vào lúc 22:48 M@IMF via groups.io
<contiwa.goma3=
gmail.com@groups.io> đã viết:

> Dear Mr. Schmidt,
>
> Thank you very much for correcting my mistake.
> I checked only isoconjugations like
>   (p:q:r) -> (a^n/p : b^n/q : c^n/r).
> I was careless.
>
> Best regards,
> M@IMF
> (There is a royal road to geometry. Ask geometry masters.)
>
>
>
>

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Message: #2660
Date: 2025-04-30
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and M@IMF <mailto:M@IMF> ,

thanks for your messages, here another observation:

Consider the transformation for the isogonal isoconjugation

... and a 5P = A1A2A3A4A5, use P = Ai and the QA of the remaining 4 points,

... you get 5 lines for a 5L, whose inscribed conic 5L-s-Co1

... is similar to the circumscribed conic 5P-s-Co1 of the 5P above

... with the same axes ...

I am sure, you can calculate the factor of expansion.

Best regards

Eckart

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Message: #2661
Date: 2025-05-01
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and M@IMF <mailto:M@IMF> ,

what about this modification of your transformation in #2654:

Replace Bi by the isoconjugate of P wrt

... the reference triangle of the points Aj with j unequal i

... and the isoconjugation with fixpoint Ai.

This will give also 6 collinear points

... for a new transformation P --> L.

Points on a QA-side will be mapped to the opposite side ...

Finally a further modification:

Replace Bi by the isoconjugate of Ai wrt
... the reference triangle of the points Aj with j unequal i
... and the isoconjugation with fixpoint P.

Then already the Bi are collinear

... and give the image line P.QA-TF2(P),

... which is also the final line.

For points P on a line

... these image lines envelope a quartic,

... tangent to the 6 lines of the starting QA.

I hope this are not overhasty observations.

Best regards Eckart

PS. What about the inverses of these transformations?

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Message: #2662
Date: 2025-05-01
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Herr Schmidt,

Could you extend due date for the homework?
I think the homothetic ratio is (D+C)/(D-C)
when the conic is expressed like Cx^2 + Dy^2 = 1.
But again, I only checked it numerically for some examples.
I'm trying to show analytically now. (Or it's not correct.)
By the way, please use "MIMF" instead of "M@IMF" if @ is
annoying.

Best regards,
M@IMF
(What should geometry masters do? They'd ask their masters.)
```

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Message: #2663
Date: 2025-05-28
From: dylanwyrzykowski@gmail.com
Subject: Re: Two quadrangles with a common circumconic

```
I have managed to prove the above conjecture. This result has
been published in the IJCDM:

https://drive.google.com/file/d/1j-zqKZ-szorvqWjfhF2p2qxZzmRimMU
A/view

My proof relied on a non-constructive computation using Pascal's
Theorem, I was hoping someone has an idea for a more synthetic
proof.

Edit: Unfortunately, I have found a flaw in my proof. The result
seems to still hold true, but it needs to be proved using
techniques from projective geometry.
```

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Message: #2664
Date: 2025-05-04
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Mr. Schmidt, Tran Quang Hung and all,

As for #2660, I didn't realized that if 5P-s-Co1(COp) is fixed,
the image line of P does not depend on QA:
Reflect the tangent to COp at P in the major axis of COp,
expand (or contract) it with scale factor S around the center of
COp.
S is (a^2 + b^2)/(a^2 - b^2) when COp is an ellipse
and S is (a^2 - b^2)/(a^2 + b^2) when COp is a hyperbola,
where a and b are resp. the lengths of the major and minor axes
of COp.

If COp is QA-Co3 of a quadrangle, QA-P2.P3/QA-P3.P4 = S.
I'm wondering this has somthing to do with the below:
AiAjAkAl is the circumcenter QA of BihBjhBkhBlh,
where Blh is the isogonal conjugate of Al wrt AiAjAk,
and {i,j,k,l,h} = {1,2,3,4,5}.
Still under study.

Best regards,
M@IMF
(What if masters of ... masters of geometry masters are not in
the present world?
We can learn from their writings although we can't communicate
with themselves like Jedi Master.)
```

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Message: #2665
Date: 2025-05-04
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,

with great interest I have reproduced your remarks in #2664,

they were new for me, thanks.

(I think, the last condition has to be {i,j,k,l}={1,2,3,4}.)

A further observation:

Consider a 5P = A1A2A3A4A5

... and its quadrangles QAi with these vertices unequal Ai,

... further the diagonal triangle DTi of QAi,

... then the images Bi of a point P for an isoconjugation

... wrt reference triangle DT and fixpoint Ai

... are collinear on the polar of P wrt the circumconic of the 5P.

Best regards

Eckart

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Message: #2666
Date: 2025-05-05
From: anopolis72@gmail.com
Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

```
On Sun, May 4, 2025 at 6:02 PM Elias M Hagos via groups.io
<hagos_em=
hotmail.com@groups.io> wrote:

> On Sat, May 3, 2025 at 02:42 AM, Elias M Hagos wrote:
>
> Dear friends,
>
> Let ABC be a triangle with circumcenter O and orthocenter H.
The
> Hatzipolakis axis is the perpendicular bisector of segment OH.
>
> Let A', B', C' be the points where an arbitrary line L
intersects BC, CA
> and AB respectively. The Hatzipolakis axes of triangles AB'C',
A'BC' and
> A'B'C concu in a point Q which lies on the Hatzipolakis axis
ABC. This
> generalizes a result given in Hyacinthos 27613
> <http://www.hyacinthos.epizy.com/message.php?msg=27613&i=1>
when L is the
> Euler line for which Q = X(18279). See also X(18309)
> <https://faculty.evansville.edu/ck6/encyclopedia/ETCPart10.htm
l#X18039>
> for a brief on the Hatzipolakis axis.
>
> Best regards,
> Elias M. Hagos
>
>
> Antreas and friends,
>
> I should hae mentioned this in the original post.
>
> The Hatzipolakis axis is the perpendicular line to the Euler
line E at the
> nine-point center N. Let P be any point on E. Let Z be the
perpendicular to
> E at a point P. Let A', B', C' be the points where an
arbitrary line L
> intersects BC, CA and AB respectively. Let Za, Zb, Zc be the Z
of triangles
> AB'C', A'BC' and A'B'C respectively. Remarkably, Z, Za, Zb and
Zc concur
```

```
> only if P = N.
>
> Best regards,
> Elias M. Hagos
>
```

Hello Elias.

It is a theorem of Quadri-Figures Geometry

Let *(a, b, c, d) *be a quadrilateral.

The Hatzipolakis axes of the component triangles
(a, b, c), (b, c, d), (c, d, a), (d, a, b) are concurrent.
APH

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Message: #2667
Date: 2025-05-04
From: van10hoven@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung, Eckart and MIMF,

When A1 = {1, 0, 0}, A2 = {0, 1, 0}, A3 = {0, 0, 1}, A4 = {p, q, r} and P = {x, y, z}, then

the isotomic case gives algebraically this transformation:

QA-Tfx(P) =

{x (-q x + p y) (-r x + p z) (q r y^2 + p^2 y z + 2 p q y z + 2 p r y z + 2 q r y z + q r z^2),

y (-q x + p y) (r y - q z) (p r x^2 + 2 p q x z + q^2 x z + 2 p r x z + 2 q r x z + p r z^2),

- z (-r x + p z) (r y - q z) (p q x^2 + 2 p q x y + 2 p r x y + 2 q r x y + r^2 x y + p q y^2)}

the isogonal case gives this transformation:

QA-Tfy(P) =

{-q r x (q x - p y) (r x - p z) (SA y^2 + SB y^2 + 2 SA y z + SA z^2 + SC z^2),

-p r y (q x - p y) (-r y + q z) (SA x^2 + SB x^2 + 2 SB x z + SB z^2 + SC z^2),

p q z (r x - p z) (-r y + q z) (SA x^2 + SC x^2 + 2 SC x y + SB y^2 + SC y^2)}

These point-to-line-transformations are both new to me.

Best regards,

Chris van Tienhoven

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Message: #2668
Date: 2025-05-05
From: van10hoven@gmail.com
Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Elias, dear Antreas,

About:

The Hatzipolakis axes of the component triangles (a, b, c), (b, c, d), (c, d, a), (d, a, b) are concurrent.

where:

The Hatzipolakis axis is the perpendicular line to the Euler line E at the nine-point center N.

This concurring point is *QL-P3 (http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p3)* in EQF.

Also to my surprise this nice result already was mentioned at the 4 th bullet at the properties.

Best regards,

Chris

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Message: #2669
Date: 2025-05-05
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

```
Dear Tran Quang Hung and MIMF,

back to your first transformation in #2654,

... you use the isotomic isoconjugation with fixpoint X2(1:1:1),

... it will also be successful for isoconjugations with
fixpoints

... X1(a:b:c), X6(a^2:b^2:c^2), X31(a^3:b^3:c^3),
X76(1/a^2:1/b^2:1/c^2),
...

... as I controlled.

I think you will generalize these examples by calculation.

Best regards Eckart
```

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Message: #2670
Date: 2025-05-05
From: van10hoven@gmail.com
Subject: Re: Two quadrangles with a common circumconic

Dear Dylan, Stanley, Keita,

It is a great result Dylan that you describe in your paper.

I had previously looked at the messages from Stanley, Keita, and you, but couldn't find any specific additions to them at that moment

However, I would now like to provide a more general description of the Shinagawa coefficients.

If multiple points share *2* identical algebraic elements (such as a^2 SA and SB SC) in the description of their barycentric coordinates, they are collinear and describe a perspective arrangement along a line. Other algebraic elements describe different perspective arrangements, either along the Euler line or another line.

If multiple points share *3* identical algebraic elements (such as a^2 SA, SB SC, and a^2 S) in the description of their barycentric coordinates, they describe a perspective arrangement of the projective plane, which I have come to call a Perspective Field.

It turns out that many more specific geometric properties are linked to particular perspective arrangements of lines or the entire plane.

See Perspective Fields (https://chrisvantienhoven.nl/mathematics/perspective-fields). Part-2 deals with the algebraic elements, but for a better understanding also part-1 is very worthwhile.

I hope to contribute something extra to this discussion in this way."

Best regards,

Chris van Tienhoven

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Message: #2671
Date: Tue, May 6, 2025 at 1:17 AM
From: αντρέας χατζηπολάκης via groups.io <anopolis72=gmail.com@groups.io>
Subject: Re: [euclid] NPCs - Orthologic

---------- Forwarded message ---------
To: <euclid@groups.io>

On Tue, May 6, 2025 at 1:12 AM APH wrote:

> Let ABC be a triangle, L a line, and A', B', C' the
intersections of L
> and BC, CA, AB,resp.
>
> Denote
>
> Na, Nb, Nc = the NPC centers of AB'C',A'BC', A'B'C, resp.
>
> ABC, NaNbNc are orthologic.
>
> Orthologic centers for some central lines ?
>
> APH
>

In Quadri Figures Geometry

Let *(a, b, c, d)* be a quadrilateral.

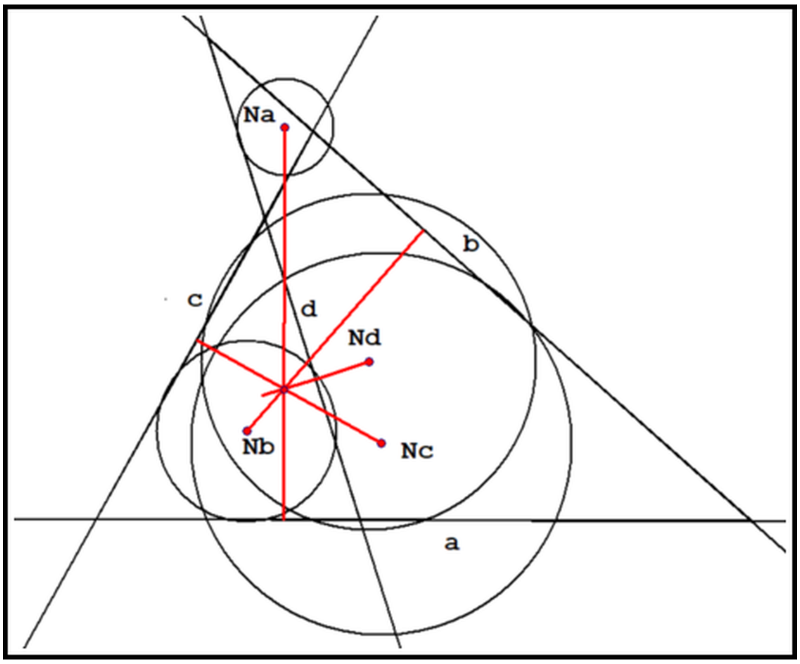
Denote

*Na, Nb, Nc , Nd *= the *NPC* centers of the component triangles
*(b, c,
d), (c, d, a), (d, a, b), (a, b, c), *resp.

The perpendiculars from *Na, Nb, Nc, Nd* to* a, b, c, d,* resp.
are
concurrent.

Point?

APH



NPC.png

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Message: #2672
Date: 2025-05-05
From: van10hoven@gmail.com
Subject: Re: [euclid] NPCs - Orthologic

Dear Antreas,

About:

Na, Nb, Nc , Nd = the NPC centers of the component triangles (b, c, d), (c, d, a), (d, a, b), (a, b, c), resp. The perpendiculars from Na, Nb, Nc, Nd to a, b, c, d, resp. are concurrent.

Your point in EQF is *QL-P2 (http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p2)* , the Morley Point.

This point was found by Frank Morley naming it the Second Orthocenter in his document Ref- (http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html) 49, paragraph 3. It is described as a recursive point in an n-Line. In his document he uses the letter “h” for this point.

QL-P2 (http://www.chrisvantienhoven.nl/ql-items/ql-points/ql-p2) is also mentioned by J.W. Clawson in Ref- (http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html) 31 (pp. 40 and 41) as the “mean center of gravity of equal masses placed at H1, H2, H3, H4”, where H1, H2, H3, H4 are the orthocenters of the Component Triangles of the Reference Quadrilateral.

Best regards,

Chris

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Message: #2673
Date: 2025-05-07
From: bernard.keizer@gmail.com
Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Chris, dear Elias, dear Antreas
As you seem to rediscover the well-known Kantor-Hervey theorem,
I send you on attached file a generalisation of this theorem to
the hypocycloïds with 2n+1 cusps tangent to 4 lines with the
help of the Hofstadter points. (I put this attached file 4 years
ago on the Forum ...).
Best regards
Bernard

QL and hypocycloïds with 2n + 1 cusps

or

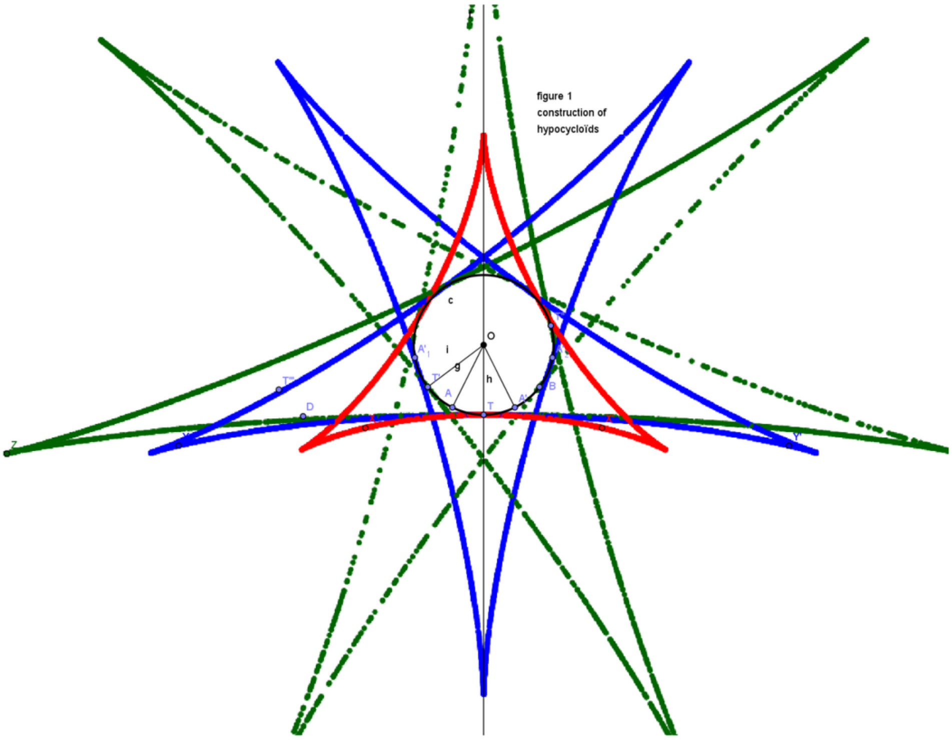
generalization of the Kantor-Hervey theorem

1) Hypocycloïds with 2n + 1 cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio n+1/n (precisely the first point A being the primary point and the other the secondary point B at a speed n+1/n the speed of the first).

The lines through the 2 points envelop a hypocycloid with 2n + 1 cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio n+1/n.(we have TB/n+1 = TA/n = AB)

For example, if n=1, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For n=2 or 3, it's a hypocycloid with 5 or 7 cusps …



QL and hypocyclods.pdf

2) Hypocycloïds with 2n + 1 cusps tangent to 4 lines

Let's now consider the Hofstadter points for r integer (ETC X3, X4, X186, X265, X5961, X5962 …) ; these points for the reference triangles of the QL derive from the same points for the triangle OjOkOl in the similitude with center QL-P1 which transforms Oi in QL-P4 and the line Li in the Steiner Line QL-L2.
(This remark allows perhaps the calculation of the barycentric coordinates of the points and the demonstration of the following properties)

Following observations can be made :

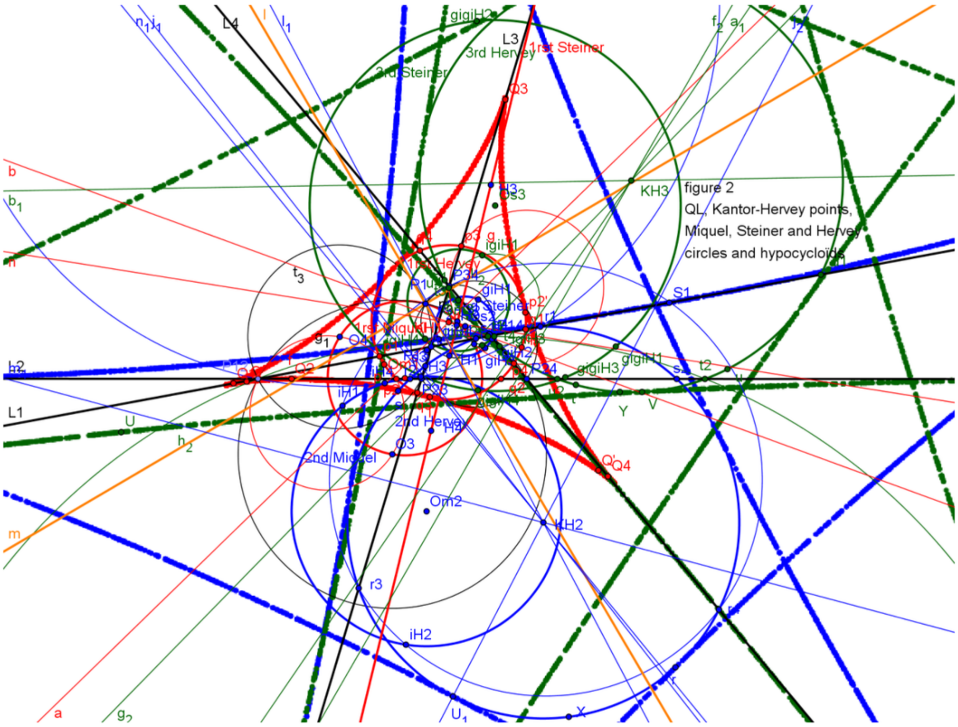
- the same points for the 4 reference triangles are on circles (X3 on the 1rst Miquel circle, X4 on the 1rst Steiner ''circle'', which is the Steiner Line, X186 on the 2nd Miquel circle, X265 on the 2nd Steiner circle …)
- the circles for isogonal points or Miquel and Steiner circles of the same rang are Cl-S partners (property mentioned by Tsihong Lau)
- the perpendicular bisectors of the segments joining 2 isogonal points of rang n in the list of Hofstadter points concur in a point named nth Kantor-Hervey point (QL-P3 for n=1)(property mentioned separately by Eckart and me the same day)
- the circles through each vertice Pij of the QL and 2 same Hofstadter points Xk and Xl of the reference triangles having Pij as vertice cut the 2 lines Li and Lj in 2 points pi and pj, qi and qj, ri and rj, si and sj, ti and tj, ui and uj …(p for X3, q for X4, r for X186, s for X265, t for X5961, u for X5962 …) (construction mentioned by Eckart)
- there are 6 such circles, each circle carrying 2 points Xk and Xl and 2 intersections pi and pj (or qi and qj …). Each of the 4 points X and 4 points p (or q …) belongs to 3 circles. 2 of the 6 circles intersect in one point X and one point p (or q…).
- the 8 points pi and qi are on the 1rst Hervey circle with center QL-P3, the 8 points ri and si are on the 2nd Hervey circle with center the 2nd Kantor-Hervey point, the 8 points ti and ui are on the 3rd Hervey circle with center the 3rd Kantor-Hervey point
- there are on each socalled Hervey circle 2n + 1 points T which divide the 4 arcs of circle in the same ratio n/n+1 :
    - for n = 1 3 points forming an equilateral triangle

- for n = 2 5 points forming a pentagon
- for n = 3 7 points forming a heptagon
- …

- The construction in part 1 gives from any of these points a hypocycloïd with 2n + 1 cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloïds with 2n + 1 cusps and the inner circles of these hypocycloïds are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocyclods.pdf

Message: #2674
Date: 2025-05-07
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt,

I'm glad I could make a contribution.

(#2665)
In the second paragraph of #2664, I reorganized 20 points B:
{Bhi,Bhj,Bhk,Bhl} -> {Bih,Bjh,Bkh,Blh}.
But nothing was obtained at that time.
After all, I'm calculating the scale factor S in the complex plane.
(It takes more time than expected.)

(#2669)
I checked also X32, then I thought isotomic conjugates could be generalized to (all) isoconjugates.
I should have checked at least X3 or X4.
It seems that (f(a) : f(b) : f(c)) is OK.

Best regards,
M@IMF

p.s. I found CO-Tf3 in ICS.

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Message: #2675
Date: 2025-05-08
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

```
Dear MIMF,

wrt the 2nd remark in #2674:

My conjecture for the transformation in #2654,

... described for the isotomic isoconjugation

... will also hold for isoconjugations with fixpoints

... (a^n:b^n:c^n) with n positive or negative integer.

Best regards Eckart

PS: What about, if n rational?
```

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Message: #2676
Date: 2025-05-08
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Mr. Schmidt,

I checked f(a) = a^(1/2) + 5a^(2/3) and sin(a) [not sinA], then
it held.
This temped me to think isoconjugations
(p:q:r) → (f(a)/p : f(b)/q : f(c)/r)
would be OK.
(I can't prove it, though)

Best regards,
M@IMF
```

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Message: #2677
Date: 2025-05-08
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

```
Dear MIMF,

I think, your last example shows

... that you are right with your informed conjecture,

... excuse my careless reading of your last messages.

Best regards Eckart
```

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Message: #2678
Date: 2025-05-12
From: bernard.keizer@gmail.com
Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

```
Dear Chris,
As you don't seem to be interested in these hypocycloïds with
2n+1 cusps tangent to 4 lines, I try it another way.
1) For r integer the Hofstadter points in a triangle ABC are
defined from X3 and it's isogonal X4 as a sequence of inverse in
the circumcircircle/isogonal wrt ABC.
(see ETC at 359/360). isogonal of Hr is H1-r.
Hence the sequence X3 = H2 and X4=H-1, X186 = H3 and X265 = H-2,
X5961 = H4 and X5962 = H-3, X5963 = H5 and X5964 = H-4 ...
(It is not surprising that there is no other such circles until
X=4000, as mentionned in EQF)
2) For a QL, the 4 Hofstadter points of the same rank are
concyclic :
X3 on the Miquel circle centered in QL-P4, X4 on the Steiner
circle (which is an fact the Steiner Line QL-L2, but a line is a
particular circle with radius infinite)
X186 on the 2nd Miquel circle centered in QL-P28 and X265 on the
2nd Steiner circle centered in QL-P29
X5961 and X5962 on the 3rd Miquel and Steiner circles
X5963 and X5964 in the 4th Miquel and Steiner circles
...
The circles of isogonal Hofstadter conjugates are CSC partners
(starting with the Miquel circle and the Steiner Line)
3) The perpendicular bisectors of the segments joining 2
isogonal Hofstadter points concur in a point :
Kantor-Hervey point QL-P3 for X3 and X4
2nd, 3rd and 4th Kantor-Hervey points for the copples X186 and
X265, X5961 and X5962 and X5963 and X5964
4) These Kantor-Hervey points are the centers of the 1rst, 2nd,
3rd and 4th Hervey circles, which are the inner circles of the
hypocycloïds with 3,  5, 7 and 9 cusps tangent to the 4 lines
and so on ...
Best regards
Bernard
PS It seems the property of the perpendiculars to the 4 lines in
the points X5 doesn't hold for the next isogonal Hofstadter
points
```

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Message: #2679
Date: 2025-05-14
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear all,

I think isogonal version of Tran Quang Hung's transformation
becomes
QA-Tfy(P) = CO-Tf1( CO-Tf3^(-1)(P) ),
where CO = 5P-s-Co1 of {QA, P}.
Is this correct?
If so, this can be an answer for the "homework" of #2660.

Here's another answer (only formulas for ellipse):
5P-s-Co1:  z = [(a+b)t + (a-b)/t]/2, |t|=1
Aj:  zj = [(a+b)tj + (a-b)/tj]/2, |tj|=1
Bhl(isogonal conjugate of Ah wrt AiAjAk):
zh + { (a-b)(1/th + U.th) - (a+b)s1[ijk] + (a-b)s2[ijk]th -
(a+b)s3[ijk](th^2 - U) }/2(s3[ijk]th - 1)
Xijkl(intersection point of AiAj and BhkBhl):
{ (a+b)[(2S)titjth - (ti+tj)] - (a-b)[2S - (ti+tj)th]th
}/2(titjth^2 - 1)
Lh(line through Xijkl, Xikjl, ..., Xklij):
[(a+b)th - (a-b)/th]z - [(a-b)th - (a+b)/th]z^* = 4abS
5L-s-Co1:  z = S[(a+b)/t + (a-b)t]/2, |t|=1
where
U = 4ab/(a^2 - b^2), S = (a^2 + b^2)/(a^2 - b^2)
s1[ijk] = ti+tj+tk, s2[ijk] = tjtk+tkti+titj, s3[ijk] = titjtk
"^*" means complex conjugate and {i,j,k,l,h} = {1,2,3,4,5}.

Best regards,
M@IMF
(May the force be with you. Oh, it's already 14th.)
```

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Message: #2680
Date: 2025-05-14
From: van10hoven@gmail.com
Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

```
Dear Bernard,

Thank you for your fantastic contribution on Epi- and
Hypocycloids.
I have already expressed my deep appreciation for it, including
in message #1, and I still stand by that.

Unfortunately, I cannot give it a place in the EQF due to my
priorities in the many projects, as mentioned in message #2548.

That said, I am working on a yearly Journal compiling messages
from our forum, so that all contributions will in any case be
preserved for the future.

And this applies to everyone: if you have personal comments
about the QPG forum or the EQF/EPG encyclopedias, please email
me separately to keep content-related and personal messages
separate.

Each email includes below the option “contact group owner”. This
way, you can always reach me outside the group for procedural or
non-content-related questions or remarks.

Kind regards,

Chris
```

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Message: #2681
Date: 2025-05-15
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,

the isogonal version of Tran Quang Hung's transformation

... can also be described, using

... the 5P = QA plus P, its 5P-s-Co1 and 5P-s-Tf3

... as tangent in 5P-s-Tf3(P) to the conic 5P-s-Tf3(5P-s-Co1).

Best regards Eckart

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Message: #2682
Date: 2025-05-15
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear Mr. Schmidt,

Thank you for the reply.
I found CO-Tf3 was first discussed in QFG#(932=)935,36,44,47,48.
Accidentally, I was studying QFG#937-953(Center Circles and
Clifford's Chain for n L ...) last month.
Although there is no attachment in QFG#2792(Loci wrt
5P-s-Tf1,2,3,4),
I can see it on your site. Figures are very helpful.

Anyone can access these pieces of information beyond time and
space.
That's great.

Best regards,
M@IMF
```

Message: #2683
Date: 2025-05-17
From: van10hoven@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Dear MIMF,

Back in the days that the Quadri Figures Group was supported by
Yahoo Groups,
attached files were often not properly included in the Yahoo
system (so frustrating!).
Fortunately, they were attached to the accompanying email.
So, I searched for it in my mail archive.
Just to be sure, I’m enclosing the attachment for QFG#2792.
Perhaps you’ve already found it on Eckart’s site.

Best regards,
Chris
```

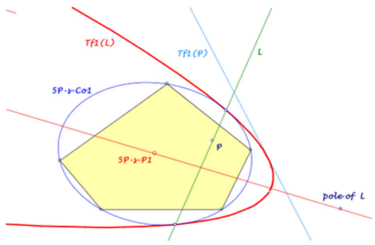
EQF-Note 2017-12-30

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

Loci wrt 5P-s-Tf1,2,3,4

Tf1,2 map points to lines, Tf3 maps points to points and Tf4 maps lines to lines. So there are several possibilities, to study loci wrt points on a line or lines through a point.

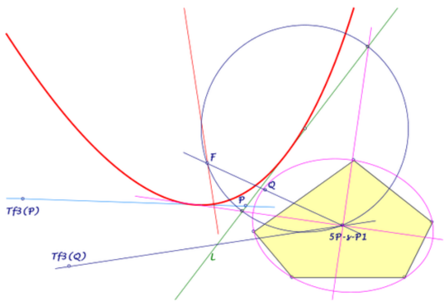
Tf1 **of points** ***P*** **on a line** ***L*** **give a parabola,**
… tangent to ***5P-s-Co1*** **in the intersections with** ***L*****,**
… with axis through ***5P-s-P1*** **and pole of** ***L*** **wrt** ***5P-s-Co1*****.**



Tf2 **of points** ***P*** **on a line** ***L*** **give a line pencil**
… for the pole of ***L*** **wrt** ***5P-s-Co1*****.**

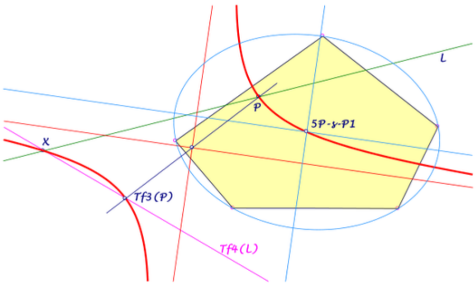
Tf3 **of points** ***P*** **on a line** ***L*** **give the line** ***Tf4(L)*****.**

Tf4 **of lines** ***L*** **through a point** ***P***
… give the line pencil of ***Tf3(P)*****.**

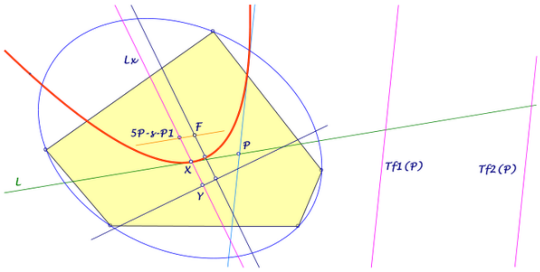


Lines ***P.Tf3(P)*** **for points** ***P*** **on a line** ***L*** **give a parabola**
… tangent to ***L*** **and the axes of** ***5P-s-Co1***
… directrix ***5P-s-P1.Tf3(Q)*** **with** ***Q*** **= pole of** ***L*** **wrt** ***5P-s-Co1***
… and focus in the 2nd intersection of ***Q.5P-s-P1***
… … and the circumcircle of the triangle
… … with sidelines ***L*** **and the axes of** ***5P-s-Co1*****.**

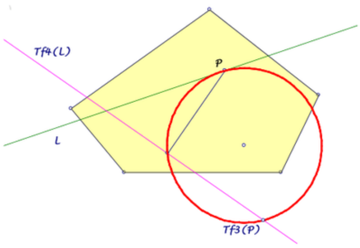
Points $L \cap$ *Tf4(L)* for a line pencil of point *P*
… give an orthogonal hyperbola,
… through *5P-s-P1*, *P* and *Tf3(P)*,
… centered in the midpoint of *P* and *Tf3(P)*
… with asymptotes parallel to the axes of *5P-s-Co1*.



Parallels to *Tf1,2(P)* through *P* for points *P* on a line *L*
… give a parabola,
… tangent to *L* in *X*, the intersection with the line L_x
… … through *5P-s-P1* and pole of *L* wrt *5P-s-Co1*,
… with directrix orthogonal L_x
… … through *Y*, the reflection of *5P-s-P1* in *X*,
… with focus *F* in the reflection of *Y* in *L*.



Pedal points of *P* on *Tf4(L)* for lines *L* through *P*
… give a circle with diameter *P.Tf3(P)*.

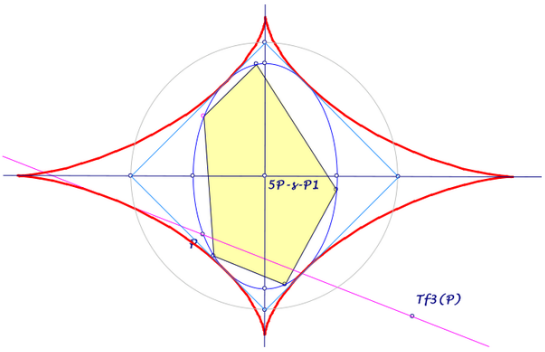


Final remarks

- ***Tf3* maps a conic to a conic, also the centers of the conics.**

2017-12-30.pdf

Lines *P.Tf3(P)* for points *P* on *5P-s-Co1* give an astroid
… with the same axes as *5P-s-Co1*
… and common tangents with *5P-s-Co1*,
… … which form a square,
… and cusps inverse to the apexes of *5P-s-Co1* wrt the circumcircle of the square.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de

2017-12-30.pdf

Message: #2684
Date: 2025-05-17
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

```
Thank you, Chris.
That's what I found.
M@IMF
```

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Message: #2685
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: QA 4 concurrent Simson / Steiner lines

```
Dear Geometers,

I'm sorry if the following results are already known...

Let P1P2P3P4 be a general quadrangle.
Denote by M{i,j} the midpoint of PiPj. (i, j={1, 2, 3, 4}, i<j)
Let P be the Gergonne-Steiner point (QA-P3) of P1P2P3P4.
Let L1 be the Simson line of P wrt M{1,2}M{1,3}M{1,4}. Define
L2, L3, L4 cyclically.
Then, L1, L2, L3, L4 are concurrent.

The point of concurrency is the reflection of QA-P6 in QA-P1. It
lies on QA-L4.

If we take Steiner lines instead of the Simson lines, we get
another 4 concurrent lines.
In this case, the point of concurrency lies on the line joining
QA-P1 and QA-P7.

Sincerely,
Keita Miyamoto
```

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Message: #2686
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

```
In the case of the Steiner lines, the point of concurrency is
the reflection of QA-P4 in QA-P1.

Sincerely,
Keita Miyamoto
```

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Message: #2687
Date: 2025-05-21
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

```
Dear Keita-san,

Maybe this is the pair of your result:
Let P' be the Euler-Poncelet point (QA-P2) of P1P2P3P4.
Let L'1 be the Simson line of P' wrt M{3,4}M{2,4}M{2,3}. Define
L'2, L'3, L'4 cyclically.
Then, L'1, L'2, L'3, L'4 are concurrent at QA-P6.

Let's consider another way.
Let 4 circles C1,C2,C3,C4 be concurrent at P.
Let Tij be the second intersection point of Ci and Cj.
Let Ll be the Simson line of triangle TjkTkiTij, where {i,j,k,l}
= {1,2,3,4}.
I'm wondering when L1,L2,L3,L4 are concurrent.
I want to look up in KIKAGAKU DAIJITEN, but the library is
closed until tomorrow.
If 4 circles are circumcircles of component triangles of QL, 4
Simson lines are identical.

Best regards,
M@IMF

p.s. I didn't realize QA-Co1 is QA-Co3 of {DT, QA-P3}.
```

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Message: #2688
Date: 2025-05-21
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

```
Dear Keita-san and all,

After posting #2687, I found "the pair of your result" in
Properties of QA-P6.
I didn't know that.
Correction: Let Ll be the Simson line of P wrt triangle
TjkTkiTij, where {i,j,k,l} = {1,2,3,4}.

Best regards,
M@IMF
```

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Message: #2689
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear M@IMF,

Obviously the 4 nine-point circles and 4 midray circles are congruent and homothetic (homothetic center = QA-P1), hence it's true for the nine-point circles as well.

I can't figure out what is the necessary and sufficient condition for the more general case you have mentioned. In a quadrangle, the following is known.
Let P1P2P3P4 be a quadrangle, and P a point. Let Q{i,j} be the orthogonal projection of P onto PiPj. Let Li be the Simson line of P wrt Q{i,j}Q{i,k}Q{i,l}. The locus of P such that L1, L2, L3, L4 are concurrent is QA-Cu1.

Shiko Iwata's Kikagaku Daijiten is voluminous and contains many interesting theorems about Simson line(s), its generalizations and relating concepts for sure (Carnot's theorem, Longchamps-Langley's theorem, Turner's theorem, Aubel's theorem, Seimiya's theorem, generalizations by Hisao Mori, θ-Simson circle, Simson line for conics, Simson line-related Newton line, pseudo-Simson line etc.).
Maybe I will pick up some of the theorems relating to quadri-figure and polygon geometry from Kikagaku Daijiten in the future.

Sincerely,
Keita Miyamoto

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Message: #2690
Date: 2025-05-22
From: unidentifiedlethargicorganism@gmail.com
Subject: QG constructing a square from 4 squares

```
Dear Geometers,

The following problem appears in a Chinese plane geometry
exercise called □□□□□□□. The proposer is unknown.

Problem 63:
Let P1P2P3P4 be a general convex quadrigon.
Erect 4 squares P1P2P12P21, P2P3P23P32, P3P4P34P43, P4P1P41P14
outwardly.
Let
A12 = P1P32 ∩ P2P43
A23 = P2P43 ∩ P3P14
A34 = P3P14 ∩ P4P21
A41 = P4P21 ∩ P1P32
B12 = P1P34 ∩ P2P41
B23 = P2P41 ∩ P3P12
B34 = P3P12 ∩ P4P23
B41 = P4P23 ∩ P1P34
C12 = A12B23 ∩ A41B12
C23 = A23B34 ∩ A12B23
C34 = A34B41 ∩ A23B34
C41 = A41B12 ∩ A34B41
To prove: the quadrigon C12C23C34C41 is a square.

Does anyone know a reference to this theorem?

Sincerely,
Keita Miyamoto
```

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Message: #2691
Date: 2025-05-22
From: unidentifiedlethargicorganism@gmail.com
Subject: QA 12-point conic involving orthocenters

```
Dear Geometers,

Is the following result well-known?

Let P1P2P3P4 be a quadrangle.
Let H1 be the orthocenter of P2P3P4. Define H2, H3, H4
cyclically.
Let
A{1,2} = P1P2 ∩ H1H2
A{1,3} = P1P3 ∩ H1H3
A{1,4} = P1P4 ∩ H1H4
A{2,3} = P2P3 ∩ H2H3
A{2,4} = P2P4 ∩ H2H4
A{3,4} = P3P4 ∩ H3H4
B{1,2} = P1P2 ∩ H3H4
B(1,3) = P1P3 ∩ H2H4
B{1,4} = P1P4 ∩ H2H3
B{2,3} = P2P3 ∩ H1H4
B{2,4} = P2P4 ∩ H1H3
B{3,4} = P3P4 ∩ H1H2
Then, these 12 points lie on the same conic.

Sincerely,
Keita Miyamoto
```

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Message: #2692
Date: 2025-05-22
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita-san and all,

In the case of pedal circles of Pl wrt PiPjPk (concurrent at QA-P2),
4 Simson lines are concurrent at (QA-P2 + QA-P23 - QA-P6).
As is often the case, it seems that θ-generalized Simson lines can be applied.
QA-Cu1 will be a hint. Thank you.

Best regards,
M@IMF

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Message: #2693
Date: 2025-05-23
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 12-point conic involving orthocenters

Dear Geometers,

I think:
1) The center of this conic is QA-P23.
2) QA-P2 lies on this conic.
3) The axes of this conic are parallel to the axes of the nine-point conic.
4) If the conic is an ellipse, then it is similar to the nine-point conic.
5) Suppose that this conic and the nine-point conic have 4 real intersection points. One of them is QA-P2. Let A, B, C be the other intersection points. Then, QA-P2, A, B, C are concyclic. The Steiner line of QA-P2 wrt ABC is the line QA-P4 QA-P8. The orthocenter of ABC is QA-P23.

I am sorry if it turns out to be false.

Sincerely,
Keita Miyamoto

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Message: #2694
Date: 2025-05-27
From: eckart_schmidt@t-online.de
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita,

I haven't studied all your results,

... but perhaps of interest an addition to #2689:

> Let P1P2P3P4 be a quadrangle, and P a point.
>
> Let Q{i,j} be the orthogonal projection of P onto PiPj.
>
> Let Li be the Simson line of P wrt Q{i,j}Q{i,k}Q{i,l}.
>
> The locus of P such that L1, L2, L3, L4 are concurrent is QA-Cu1.

If we ask for for points P on QA-Cu1 such that the lines Li are parallel,

... we get the contact points of tangents from QA-P3 at the cubic.

What about their directions?

Perhaps helpful:

Consider QA-Co1 as pivotal isocubic wrt
... pivot QA-P3,
... reference triangle with vertices QG-P16 of the QA-quadrigons,
... isoconjugation, which swaps vertices of QA-Tr1 and QA-Tr2
... with fixpoints in the contact points of tangents from QA-P3 at the cubic.
(This is mentioned in my QFG-message #1770 (30.06.2016), not in EQF.)

Best regards Eckart

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Message: #2695
Date: 2025-05-28
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

```
Dear all,

Thanks to Keita-san, my question in #2687 was solved.
Given 4 circles C1,C2,C3,C4 concurrent at P.
Let
Cij = the second intersection point of Ci and Cj
Lpl = the Simson line of P wrt triangle CilCjlCkl
Pi = the antipode of P wrt Ci
QAp = {P1,P2,P3,P4},
where {i,j,k,l} = {1,2,3,4}.
Since Cij is the orthogonal projection of P on PiPj,
Lp1, Lp2, Lp3, Lp4 are concurrent iff P lies on QAp-Cu1
(QFG#1534).
According to the 12th property in QA-Cu1 (Bernard Gibert, March
15, 2015),
this means 3 lines C14C23, C24C31, C34C12 are concurrent.

Best regards,
M@IMF

p.s. Correction(again) in #2687,8: Let Ll be the Simson line of
P wrt triangle TilTjlTkl , where {i,j,k,l} = {1,2,3,4}.
```

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Message: #2696
Date: 2025-05-29
From: eckart_schmidt@t-online.de
Subject: Re: QA 12-point conic involving orthocenters

Dear Keita,

I have reproduced your #2691,

... here a further property for #2693:

6. The Simson line of QA-P2 wrt triangle ABC

... is a parallel to the line QA-P4,8,23,32,42.

Best regards Eckart

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Message: #2697
Date: 2025-06-05
From: contiwa.goma3@gmail.com
Subject: About QL-Tr2

Dear all,

Is this well-known?
The contact points of QL-Co1 with QL-Tr2 are the intersection points of QL-Co1 and QA-Co1,
where QA is formed by the contact points of QL-Co1 with QL.

Best regards,
M@IMF

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Message: #2698
Date: 2025-06-06
From: eckart_schmidt@t-online.de
Subject: Re: About QL-Tr2

Dear MIMF,

I have reproduced your constructions,
... a good addition to QFG #1460 of 25.01.2016,
... here attached:

> ... I had a look at the contact points of the 3 sides of
QL-Tr2 with the parabola, which shall give the triangle QL-Trx:
> There is a perspector of QL-Tr2 and QL-Trx, point Y in #678.
> This point Y has a lot of interesting properties:
> ... Point Y is the QL-Tr2-isotomic conjugate of the point at
infinity of QL-L1.
> ... Point Y lies on the QL-Tr2-Steiner ellipse
(circumscribed).
> ... The tangent in Y at the QL-Tr2-Steiner ellipse is
Y.QL-P8. (orthogonal QL-L5).
> ... The point Y is the contact point of two Steiner ellipses:
QL-Tr2 circumscribed and QL-Trx-inscribed.
> ... Point Y is the reflection of point X in #678 in QL-P12.
> ... Point Y lies on a QL-Tr2-circumconic through QL-P8,
QL-P13, QL-P24
> ... Point Y is the Brianchon point of QL-Co1 wrt QL-Tr2.
> ... The point Y is collinear with QL-P7 and the 4th
intersection of QL-Co1 and the QL-Trx-circumcircle.
> ... The trilinear polar of Y wrt QL-Tr2 is ...
> ... ... also the trilinear polar of Y wrt QL-Trx,
> ... ... the polar of Y wrt QL-Co1,
> ... ... the line for the collinear trilinear poles of the
QL-lines wrt QL-Tr2,
> ... ... which is the image of QL-Co1 wrt an isoconjugation of
QL-Trx with fixed point Y.

Best regards Eckart

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Message: #2699
Date: 2025-06-06
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

```
Dear Mr. Schmidt,

Thank you for your reply.
The polar curve of DQL-Co1(anticomplement of QL-Co3 wrt QL-Tr1)
wrt QL-Co1 is QA-Co1(QA = contact points quadrangle).
So the poles of the common tangents of QL-Co1 and DQL-Co1 wrt
QL-Co1 are the intersection points of QL-Co1 and QA-Co1.
I checked it analytically (not numerically).

Best regards,
M@IMF
```

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Message: #2700
Date: 2025-06-06
From: eckart_schmidt@t-online.de
Subject: Geometry on QA-Cu1

Dear Keita, dear all,

may I invite you to an geometric excursion on QA-Cu1,

... starting with the property, mentioned in Keita's message #2689:

Let P1P2P3P4 be a quadrangle, and P a point.

Let Q{i,j} be the orthogonal projection of P onto PiPj.

Let Li be the Simson line of P wrt Q{i,j}Q{i,k}Q{i,l}.

The locus of P such that L1, L2, L3, L4 are concurrent is QA-Cu1.

(1)   Points on QA-Cu1 with parallel lines Li (see #2694)

... are the 4 contact points U of tangents from QA-P3 at the cubic.

Let Lube a line through P=U as further parallel to the lines Li.

(2)   There are 4 cubic points P=V whose 4 lines Li are

... each parallel to one of the 4 lines Lu.

(3)   These points V are the in- and ex-centers of the Miquel Triangle QA-Tr2,

... their QA-Cu1- tangents are parallel to the asymptote.

(4)   A connection-line of an U- and an V-point bears a QA-vertex,

... a connection-line of two V-points bears a QA-Tr2-vertex.

(5)   Two connection-lines of different V-pairs are perpendicular

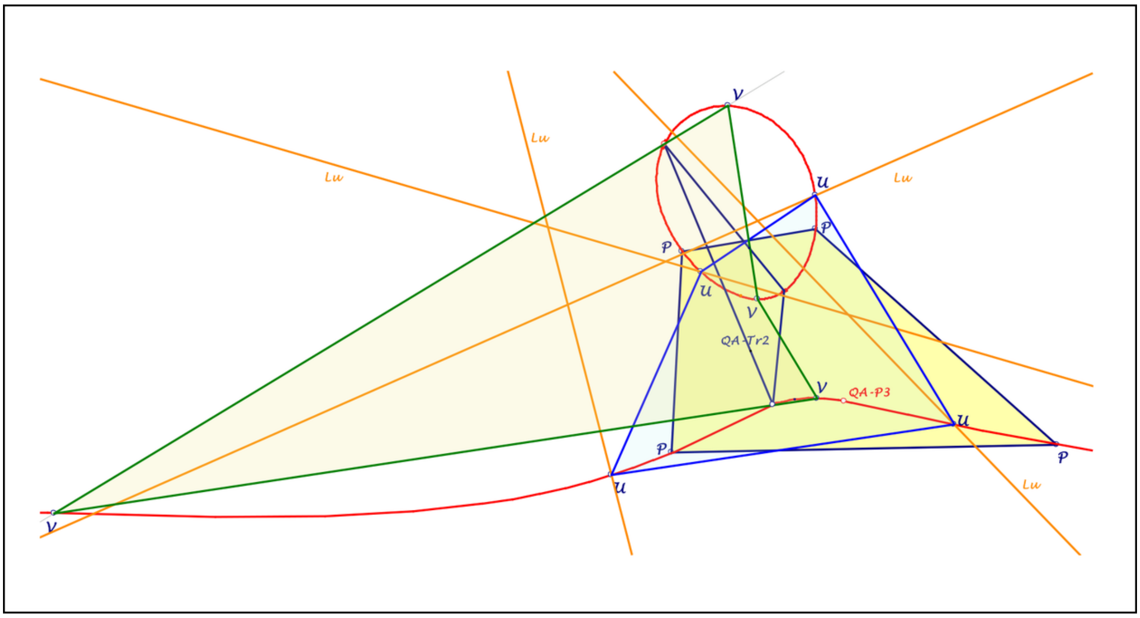
... and have an intersection on the cubic

... in a vertex of the Miquel triangle QA-Tr2.

But what about the four directions of the lines Lu?

Best regards Eckart

PS: Attached only the main points and lines.



2025-06-06.pdf

Message: #2701
Date: 2025-06-08
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear all,

I found Mr. Schmidt's QFG message(#1335, 18/10/2015):

... The 6 contact points of the parabolas to the triangles
S1S2S3 lie on the conic QA-Co1.

The 6 contact points are 3 contact points of QL1-Co1 with
QL1-Tr2
and those of QL2-Co1 with QL2-Tr2, where QL1 and QL2 are
QA-2Co1.
I had searched "QL-Tr2" & "QA-Co1" in QFG (also QPG), but
couldn't find above message.
That's because QL-Tr2 was registered in 2016!

By the way, the QA formed by the centroids of the component
triangles of QL1 and that of QL2 are symmetric wrt QA-P10.
I wonder this was already mentioned.

Best regards,
M@IMF

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Message: #2702
Date: 2025-06-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

in #2635 I offered a "construction"

... for the real flexline trilateral of a bipartite cubic,

... perhaps you can confirm the following for a monopartite cubic:

Consider a monopartite cubic CU

... their real flexpoints F1,2,3, harmonic polars L1,2,3,

... bearing the cubic-intersections P1,2,3 and

... the intersections T1,2,3 with the real flextangents.

Start with the point Pi and its two tangents at the cubic

... with contact points U,V,

... further with variable triangles H1'H2'H3' on L1,L2,L3

... with F1,F2,F3 on their sidelines.

Consider the conic CO through U,V,H1',H2',H3'

... and the conic tangents in Hj',Hk',

... which intersect in a point W on Li.

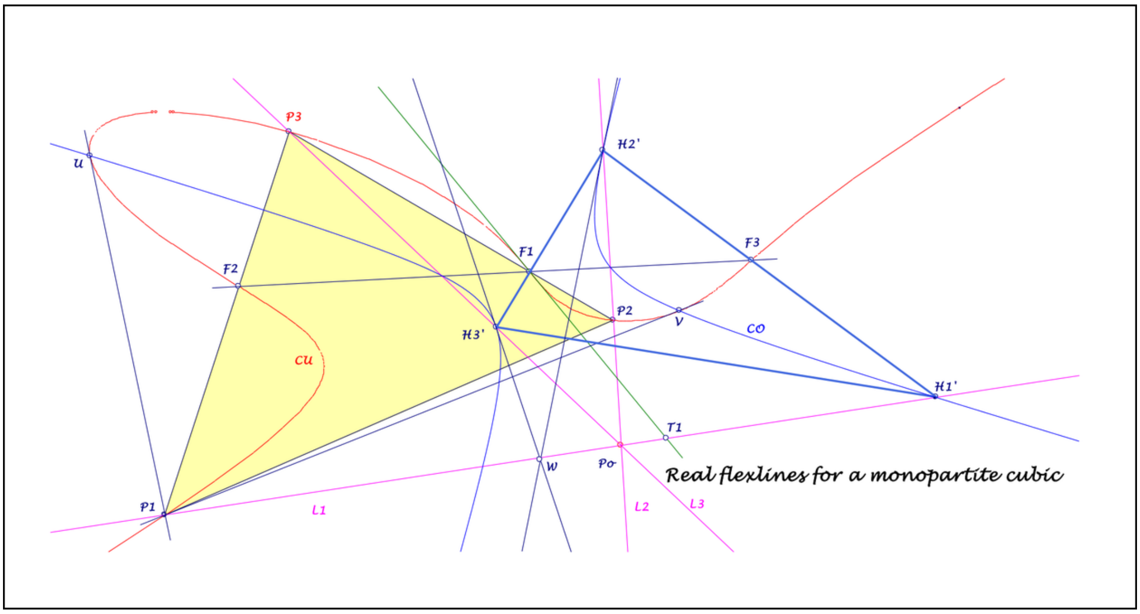
If Hi' is the 4th harmonic of W wrt Pi,Ti,

... then Hi' = Hi, vertex of the real flexline trilateral.

Best regards Eckart

PS: Attached Pi = P1.

I made several drawings without a significant anomaly.



2025-06-12.pdf

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Message: #2703
Date: 2025-06-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
This time, I cannot confirm your construction.
The property that the point W, intersection on L1 of the
tangents in H2 and H3 to the conic through H1, H2, H3 and U and
V is the harmonic of H1 wrt P1 and T1 simply doesn't hold ! (U
and V being the tangentials of P1 are aligned with F1, which is
the tangential of P1).
Best regards
Bernard
```

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Message: #2704
Date: 2025-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard,

sorry, your last remark is wrong,

... have a look on the drawing.

Best regards Eckart
```

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Message: #2705
Date: 2025-06-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
In fact, P1 being the tangential of U and V and F1 the
tangential of P1, U ad V are aligned with F1 seems a more
correct formulation, but the conclusion holds !
Best regards
Bernard
```

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Message: #2706
Date: 2025-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

we can dispute about better formulations,
... but can you give me a drawing with a significant aberration
... or another reason for your strict opinion?

Best regards Eckart

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Message: #2707
Date: 2025-06-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I use always the same kind of figures already explained in several old messages.
Giving a triangle H1H2H3 and a point P0 (1/a1,1/a2,1/a3), the barycentric equation of the line of real flexes is a1X + a2Y +a3Z = 0 and the equation of a cubic of the syzygetic pencil is $a1^3X^3 + a2^3Y^3 + a3^3Z^3 + k a1a2a3\, XYZ = 0$.
For k <= -3, the cubics are bipartite, for k>=-3 they are monopartite
I was very glad to confirm your construction for a bipartite cubic.
The used property is that 4 points having the same tangential, in other terms the vertices of a tangential quadrangle, are coconic with H1,H2 and H3.
The problem with a monopartite cubic is that we have only pairs of points having the same tangential and we have to find another property of the conics.
Checking your construction for variing k shows easily that your property doesn't hold for k = 0, 1, 2, 3, 6 ...
It begins to be an acceptable approximation for bigger values of k (10, 12, 100 ...)
Best regards
Bernard

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Message: #2708
Date: 2025-06-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your explanation,

... so I have to study it furthermore,

... repeating the constellation in #2702 for k = 4.2

... with CABRI and greatest care there is a small aberration,

... my other drawings have all a greater k .... you are right.

Best regards Eckart

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Message: #2709
Date: 2025-06-16
From: contiwa.goma3@gmail.com
Subject: QA and Complex Quartic Eq.

Dear all,

First, in the name of MIMF, I have to write about complex cubic eq. and triangle
although this is Quadri-and-Poly-Geometry.

< *Cubic Eq*.>
Let z1,z2,z3 be the solutions of a cubic equation
z^3 + pz + q = 0.
Define
uk = (ω^k)u, vk = v/(ω^k) (k=1,2,3),
where
u = -(z3 + z1/ω + ω z2)/3
v = -(z3 + ω z1 + z2/ω)/3
ω = (-1 + i √3)/2 (cube root of 1),
then
zk = -(uk + vk).

When z1,z2,z3 form a triangle in the complex plane,
uk(vk) is the vertice of outer(inner) Napoleon triangle
and
X(15) = (v^2)/u , *X(16)* = (u^2)/v
X(187) = (u^3 + v^3)/2uv = -3q/2p
PU(118) = ±√(uv) = ±√(-p/3).

< *Quartic Eq*.>
Let z1,z2,z3,z4 be the solutions of a quartic equation
z^4 + bz^2 + cz + d = 0.
Define
m_ij = (zi + zj)/2
κij = -(zi + zj)(zk + zl) = 4m_ij^2,
where {i,j,k,l} = {1,2,3,4}.
Then
zl = m_il + m_jl + m_kl,
and κij satisfies the cubic equation
κ^3 + 2bκ^2 + (b^2 - 4d)κ - c^2 = 0.

κij is dimension 2, so let's define a triangle Trκ in the complex plane as {κ14/σ, κ24/σ, κ34/σ}, where σ is a certain point except the origin.
The triangle {κil/σ, κjl/σ, κkl/σ} is inversely similar to the pedal triangle of point zl wrt triangle {zi, zj, zk}.
It relates to cross ratio (zj - zk)(zi - zl)/[(zi - zk)(zj - zl)].
(I didn't realize that pedal triangles of circumcenter QA are similar to those of the reference QA.)

Since c/(2κij) is the vertice of *QA-Tr2* of {z1,z2,z3,z4}, Trκ is obtained from QA-Tr2 by QL-Tf1 with the origin as QL-P1, ±√[c/(2σ)] as QL-2P3.

My question is what σ is appropriate.
When σ = ±2√(-b/6), σ is the centroid of Trκ.
But I don't know the geometrical meaning of ±√[c/(2σ)].
I'm searching σ which is relevant to QA/QL geometry,
not necessarily expressed by only b, c, d.

Best regards,
M@IMF

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Message: #2710
Date: 2025-06-16
From: van10hoven@gmail.com
Subject: Re: QA and Complex Quartic Eq.

Dear MIMF,

Nice approach to the three roots of a cubic equation in the complex plane.

This group also explores curves of the nth degree, so your contribution is very welcome.

I try to describe your approach.

You created a mapping of the roots of a cubic equation in the complex plane, which together form a triangle. Using the elements of these roots, you then describe the inner/outer vertices of the Napoleon Triangle, the ETC points X(15) and X(16), as well as the bicentric pair PU(118). Very nice!

After that, you applied the same method to the roots of a quartic equation.

You created an image of the roots of a quartic equation in the complex plane, which together form a quadrangle QA. Using the elements of these roots, you then describe the vertices of QA-Tr2, QL-P1, and QL-2P3 (I hope I have described this correctly).

I haven't verified all the calculations, so I can't adequately answer your question. But I really like your approach. It could serve as an interesting step toward new insights in Triangle Geometry and Quadrilateral Geometry.

Best regards, Chris

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Message: #2711
Date: 2025-06-17
From: contiwa.goma3@gmail.com
Subject: Re: QA and Complex Quartic Eq.

```
Dear Chris,

Thank you for your reply.
I used the terms QL-Tf1,P1,2P3 to describe a (involutive)
Moebius transformation.
So far, I can't find such a QL, but finding it might be a hint.

Below is more about triangle (although ...).

1) Psi transform of X(187) is
X(353) = 2(p^2)/9q.
We can obtain z1,z2,z3 from X(2), X(187) and X(353) by addition,
subtraction,
multiplication, division, and extraction of n-th roots (n=2,3).

2) If we use u^* and v^* (the complex conjugates of u and v), we
get
X(13) = u(v/u)^* , X(14) = v(u/v)^*
X(3) = (v^2 u^* - u^2 v^*)/(|u|^2 - |v|^2)
X(6) = (v^2 u^* + u^2 v^*)/(|u|^2 + |v|^2).

Best regards,
M@IMF
```

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Message: #2712
Date: 2025-06-30
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

```
Dear all,

Is this already known?
Let
C1 = the Brianchon point of QL1-Co1 wrt QL1-Tr2
C2 = the Brianchon point of QL2-Co1 wrt QL2-Tr2
D1 = the Brianchon point of DQL1-Co1 wrt QL1-Tr2
D2 = the Brianchon point of DQL2-Co1 wrt QL2-Tr2,
then
C1D1 is parallel to QL1-L1
C2D2 is parallel to QL2-L1
C1C2 is parallel to QL1-L8(=QL2-L8).

QL1, QL2, etc. are as before:
QL1-Co1 = QA-2Co1a
QL2-Co1 = QA-2Co1b
QA = a convex quadrangle (not a trapezoid)
QL1 = a quadrilateral formed by the tangents to QL1-Co1 at the
vertices of QA
QL2 = a quadrilateral formed by the tangents to QL2-Co1 at the
vertices of QA
DQL1-Co1 = anticomplement of QL2-Co1(=QL1-Co3) wrt DT
DQL2-Co1 = anticomplement of QL1-Co1(=QL2-Co3) wrt DT
DT = QA-Tr1 = QL1-Tr1 = QL2-Tr1.

I wonder why this holds.

Best regards,
M@IMF
```

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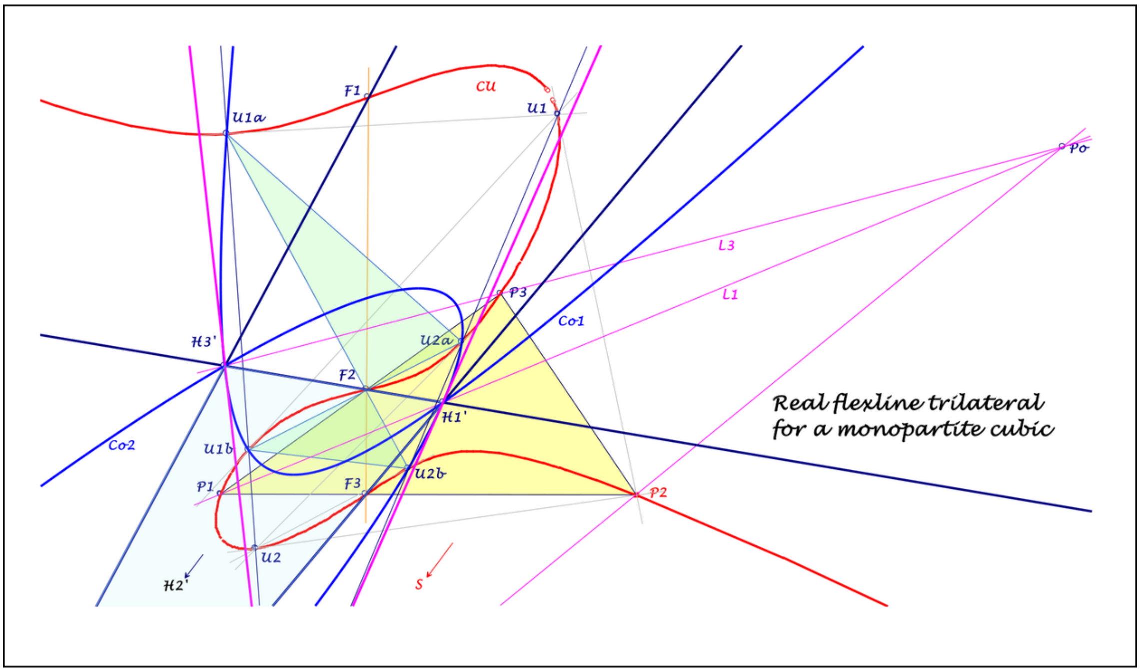
Message: #2713
Date: 2025-07-04
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard, dear Chris,

in #2635 I offered a "construction"
... for the real flexline trilateral of a bipartite cubic,
... I try it once more for a monopartite cubic:
Consider a monopartite cubic CU
... their real flexpoints F1,2,3, harmonic polars L1,2,3,
... bearing the cubic-intersections P1,2,3 .

Start with a point Pi (attached Pi = P2)
... and its two tangents to the cubic
... with contact points U1,U2,
... further the tangents from U1, U2 to the cubic
... with contact points U1a,U1b,U2a,U2b.
The lines U1aU1bU2 and U2aU2bU1
... intersect in a new point S on Li.
Use  further variable triangles H1'H2'H3' on L1,L2,L3
... with F1,F2,F3 on their sidelines.
Consider the conics
... Co1(H1',H2',H3',U1a,U1b) and Co2(H1',H2',H3',U2a,U2b).
Now vary the triangles H1'H2'H3':
If SHj' and SHk' are tangents to the conics in Hj' and Hk',
... then Hi' = Hi, vertices of the real flexline trilateral.

Best regards Eckart
```



2025-07-03.pdf

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Message: #2714
Date: 2025-07-08
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Wunderbar, lieber Eckart
This time, it seems you have reached your goal !
I checked carefully with one of my cubics of the pencil and I can confirm your property of the point S, intersection of U1aU1b and U2aU2b, that S belongs to one tangent in H2 to a conic and one tangent in H3 to the other conic. The 2 other tangents in H2 and H3 to the 2 conics intersect in a poit T, also on the line Li (L2 on your figur).
What about this point T ?
I suppose it musn't be too difficult (only a little bit boring), starting with the equation of the cubic given in message 2707, to calculate in barycentric coordinates wrt H1H2H3 the coordinates of Pi, Ui and Vi and Uia and b and Via and b and Si on Li, then the equation of the 2 conics (circumconics of H1, H2 and H3) and to check your property.
Sincere congratulations
Best regards
Bernard
PS For me, as I had found the barycentric equation of the cayleyan wrt H1H2H3 (which is the same as the tangential equation of the cubic) and since you have found the construction of the 3 points H1, H2 and H3 in both cases mono- and bipartite, the game is now definitely over. Many thanks again for our mutual collaboration.

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Message: #2715
Date: 2025-07-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

I studied Eckart’s construction of the real Flex-trilateral in QPG#2713 and Bernard’s response in QPG#2714.

Bernard said “I suppose it musn't be too difficult (only a little bit boring), starting with the equation of the cubic given in message 2707, to calculate in barycentric coordinates wrt H1H2H3 the coordinates of Pi, Ui and Vi and Uia and b and Via and b and Si on Li, then the equation of the 2 conics (circumconics of H1, H2 and H3) and to check your property.”

Well, I tried to do that, but I already failed at the first step: calculating Pi in general. You have to solve the intersection of a reference cubic (third degree) and a polar conic (second degree). Even with the cubic simplified in Hesse’s form, Mathematica was not able to solve it.

Therefore, the only way to proceed was by trying a numerical approach.
I used Mathematica to calculate two numerical cases – based on Hesse’s form – with a monopartite reference cubic starting from a point P2, which is the only real intersection of the second Harmonic Polar L2 and the monopartite reference cubic.

Since calculating the real Flex-trilateral H1H2H3 is relatively straightforward, my approach was to construct the two conics passing through (H1, H2, H3) and observe whether two of the tangents to these conics at H1, H2, H3 coincide with H1S and H3S.

See attached pictures.

The outcomes were:

Numerical case 1:
* The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) – almost (value difference: 99.999156%)
* The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) – almost (value difference: 100.000844%)

Numerical case 2:

* The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) – almost (value difference: 99.3387%)
* The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) – almost (value difference: 100.666%)

As a matter of fact, I find these results not very promising. They show that the tangents are indeed almost in the right place – but not exactly. I often encounter values around 99.9999999% in other calculations.

Nevertheless, the performed computations go many levels deep, so I think it's reasonable to say the validity of the construction remains uncertain.

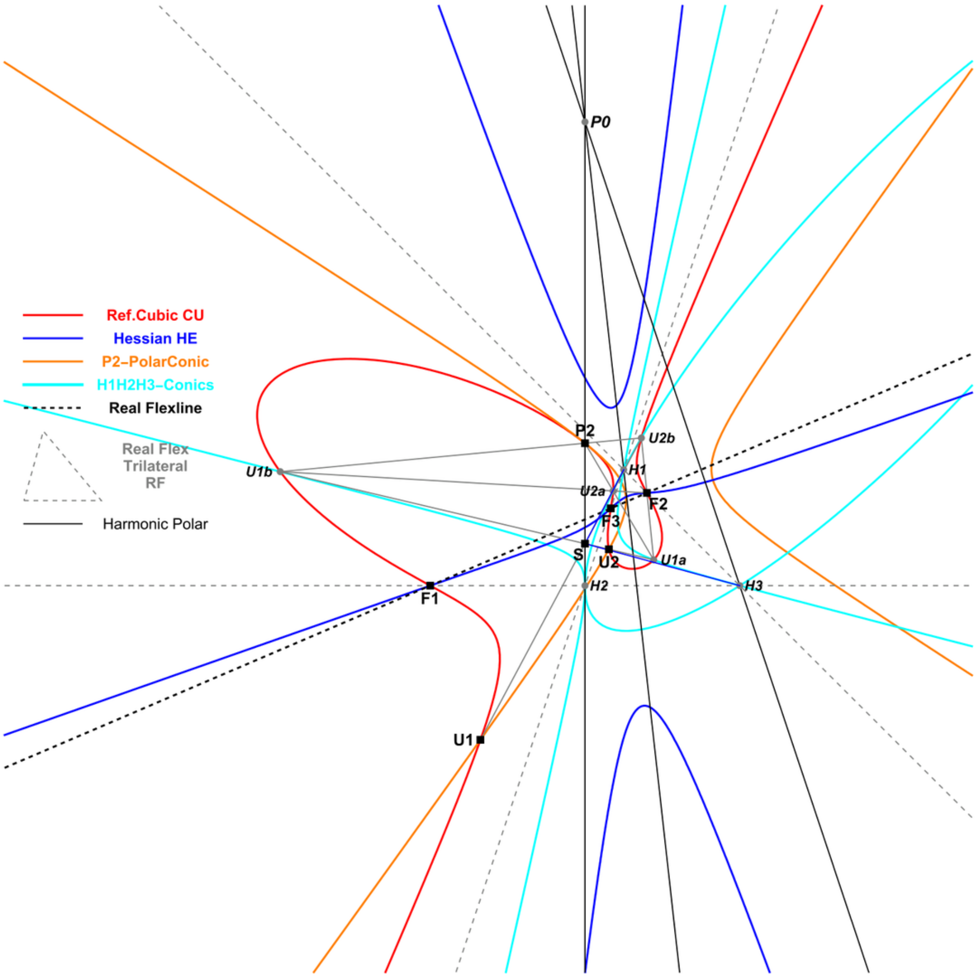
The best way to proceed, I suppose, is by trying more test cases – especially exceptional ones.

Maybe not the prettiest message.

Best regards,

Chris

Numerical case-1

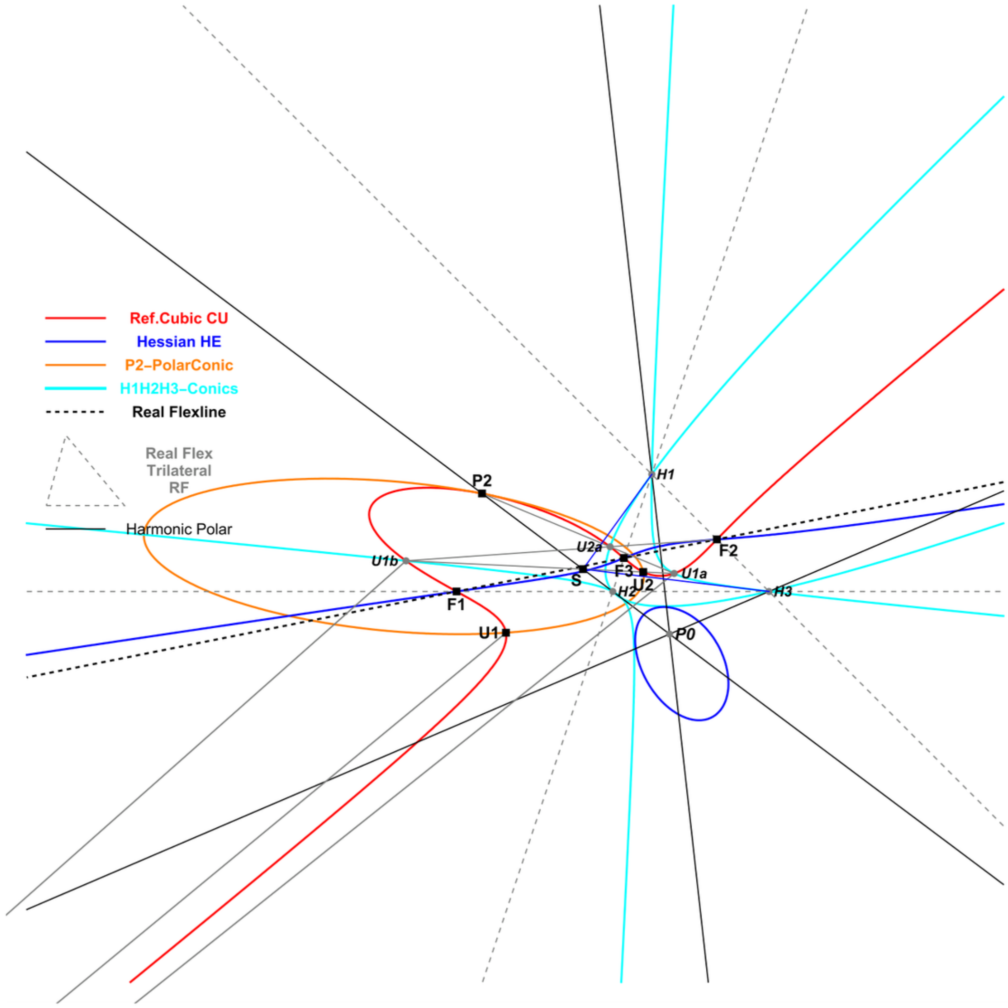


Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.999156%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.000844%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-2.pdf

Numerical case-2



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.3387%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.666%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-2.pdf

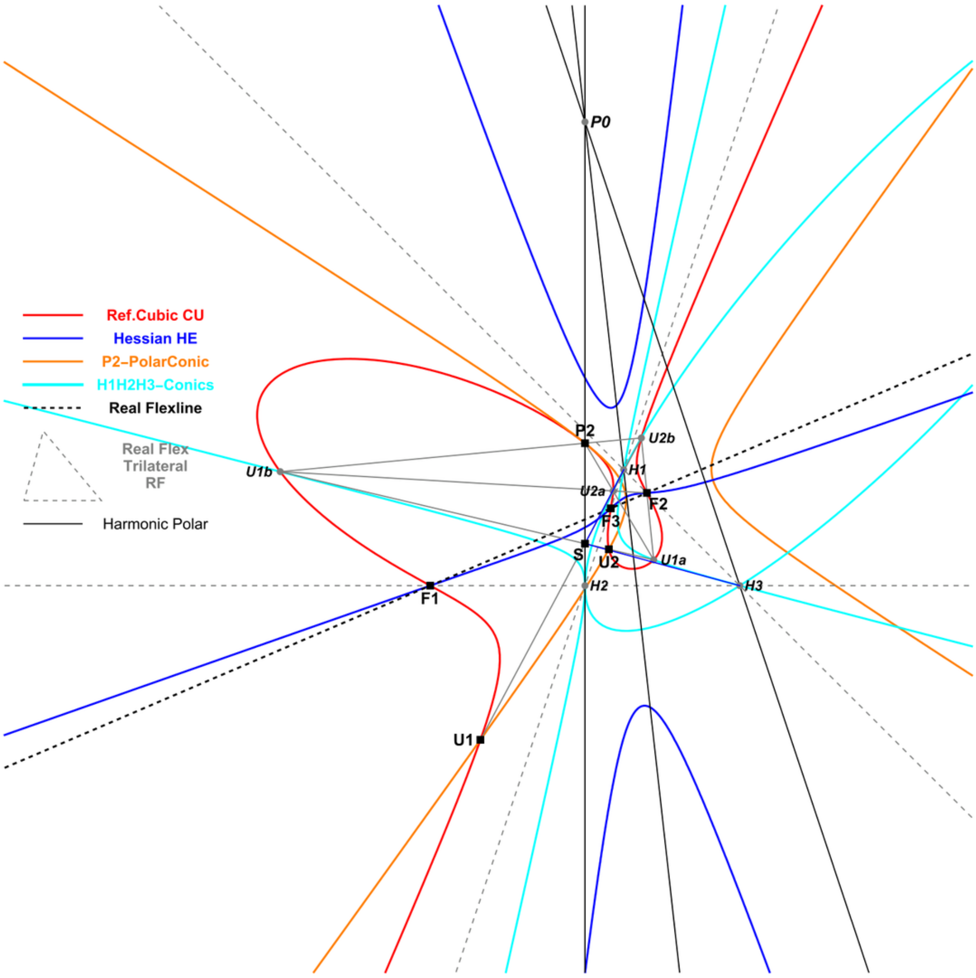
Message: #2716
Date: 2025-07-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

Being in the flow, I created another third numerical case – a more extreme one.
Because the resulting cubic was fairly 'flat', I had to stretch it to see the details.
See attachment.
This time it came out:
• The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) – almost (value difference: 95.0132%)
• The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) – almost (value difference: 105.249%)
These value differences are to large. If I haven’t made any mistakes, this disproves the conjecture.
To my regret . . .

Best regards,
Chris

Numerical case-1

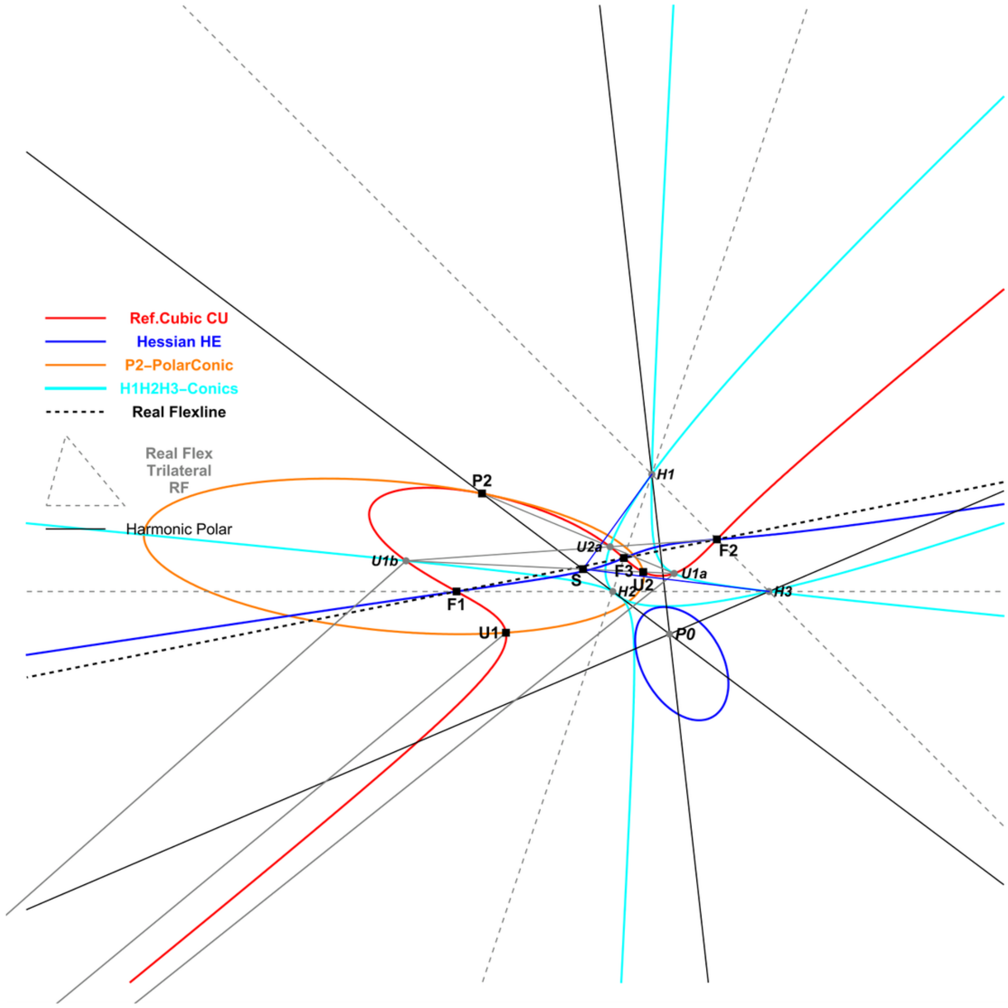


Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.999156%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.000844%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-02.pdf

Numerical case-2



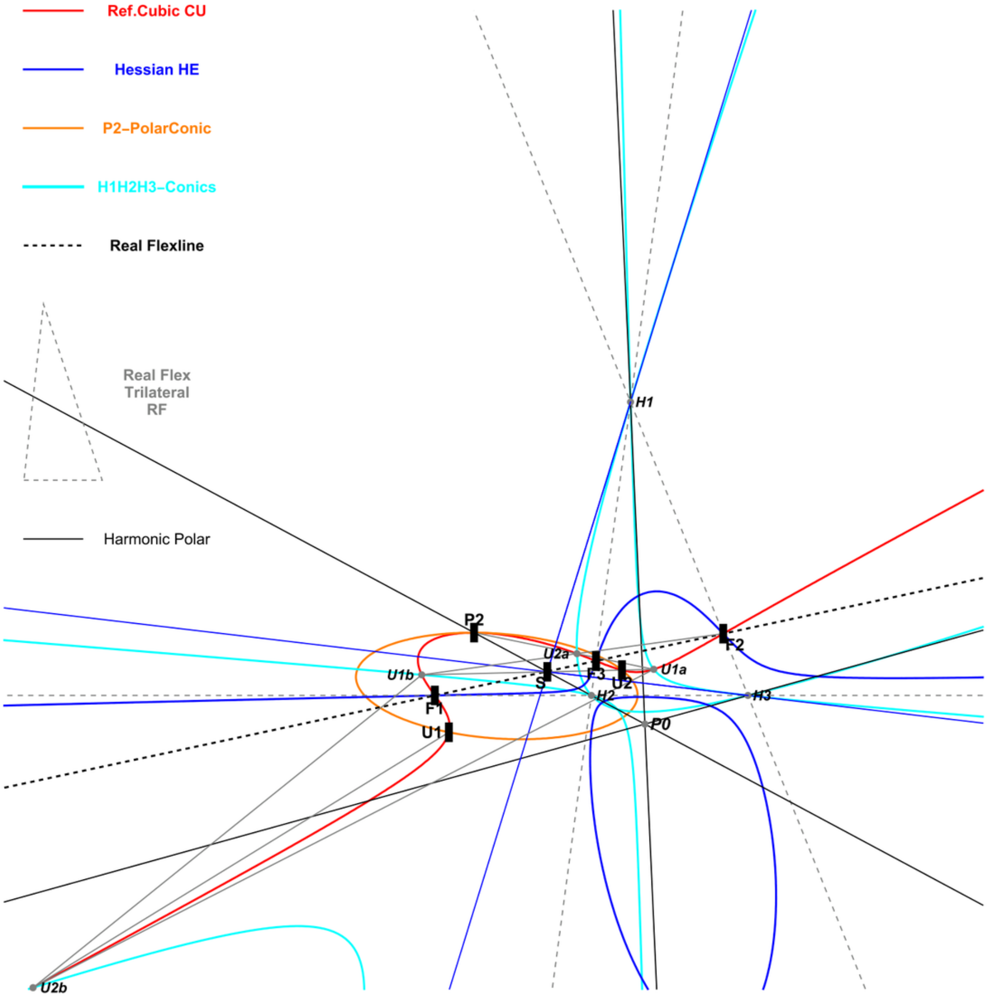
Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.3387%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.666%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-02.pdf

Numerical case-3 (stretched picture)

CU = $3375x^3 - 8y^3 + 125xyz - 64z^3$



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 95.0132%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 105.249%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-02.pdf

Message: #2717
Date: 2025-07-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

thanks to your judgements of my construction,

... especially to Bernard for his friendly message,

... but I think Chris will be right,

... special thanks for his complex calculations

... and his refusal ...

Chris 3rd example is near to a bipartite cubic,

... I researched such example and observed

... noticeable aberrations,

... so I have to retire my assumption.

Best regards Eckart

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Message: #2718
Date: 2025-07-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart,
My humblest apologies for my mistake !
Thanks to Chris for correcting my error ...
On my figures, the lines SiHj and SiHk are not tangent to the conics through H1, H2 and H3, but the aberrrations are sometimes not very significant.
Anyhow, I should have checked more precisely on several different figures ...
Now a last question, already asked in an old message :
For a bipartite cubic, the vertices of any tangential QA (4 points having the same tangential) are coconic with the 3 vertices H1, H2 and H3.
For a monopartite cubic, what can be said about the conics through the vertices of a tangential segment (2 points having the same tangential) and the 3 vertices H1, H2 and H3 ? (For example, locus of their centers ...)
The fight goes on and I cannot retire, as I hoped ...
Best regards
Bernard

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Message: #2719
Date: 2025-07-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris and Eckart,
I think finally that the 2 cases mono- or bipartite have the
same solution !
Chris says often that the calculation with imaginary values are
the same as with real values.
Then, we may find for any point of the cubic the 4 intersections
of it's polar conic with the cubic other than the point itself.
These 4 points are real for a bipartite cubic, 2 are real and 2
imaginary for a monopartite cubic.
The 3 vertices H1, H2 and H3 are on the 3 harmonic polars with
the flexes F1, F2 and F3 on the sides H2H3, H1H3 and H1H2 and
are coconic with the 4 points.
This gives a geometrical solution for a bipartite cubic, but
there is no need to search a geometrical solution for a
monopartite cubic, there isn't any !
Best regards
Bernard
```

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Message: #2720
Date: 2025-07-13
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Bernard, dear Eckart,

I agree with you, Bernard, that algebraically there are no differences in how many real branches a cubic has. There are always just as many solutions to a question. The only difference lies in how many of those solutions are real or imaginary. After all, a cubic is a third-degree curve.

So algebraically, there's really no issue. Geometric constructions, however, suffer from the number of available real components, because imaginary components can't be used in drawings. If a particular construction only works when all components are real, that doesn't mean there are no alternative constructions that work around the presence of imaginary components.

Let's hope that one day, an alternative workaround construction for the 'real Flextrilateral' will be found.

Best regards,

Chris

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Message: #2721
Date: 2025-07-14
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Bernard,

In #2718 you said:

“For a bipartite cubic, the vertices of any tangential QA (4 points having the same tangential) are coconic with the 3 vertices H1, H2 and H3.

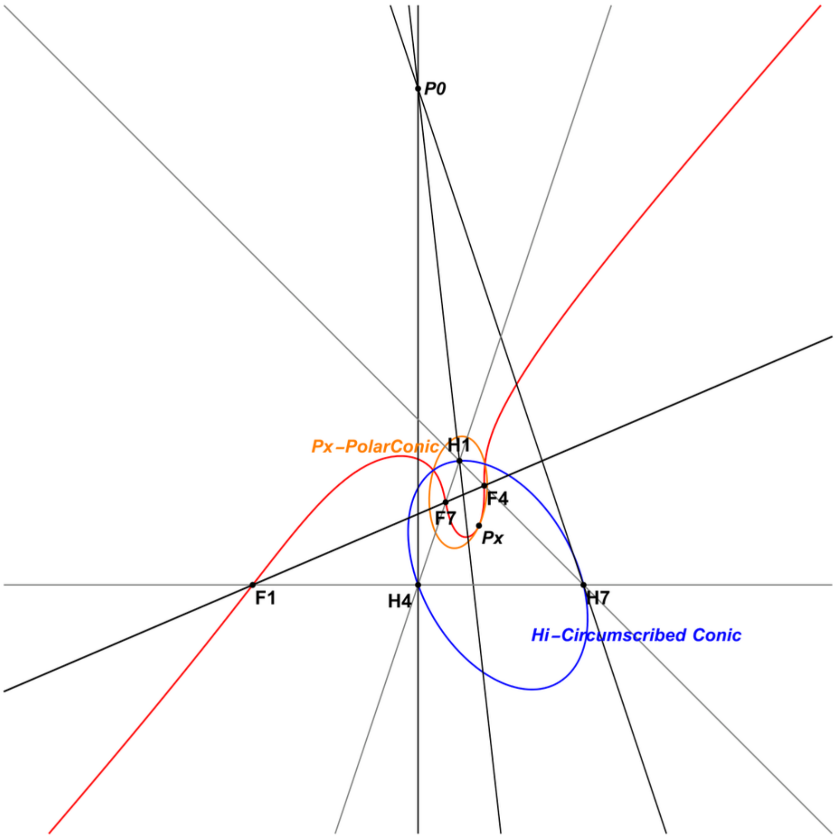
For a monopartite cubic, what can be said about the conics through the vertices of a tangential segment (2 points having the same tangential) and the 3 vertices H1, H2 and H3 ? (For example, locus of their centers ...) ”

I created a picture to spark our imagination for both a monopartite and a bipartite cubic. See attachment.

Best regards,

Chris

point Px on a MONOPARTE cubic (**red**)
yielding a Px-tangential quadrangle
circumscribed by the Px-Polar Conic (**orange**)
and circumscribed by the Hi-Circumscribed Conic (**blue**)

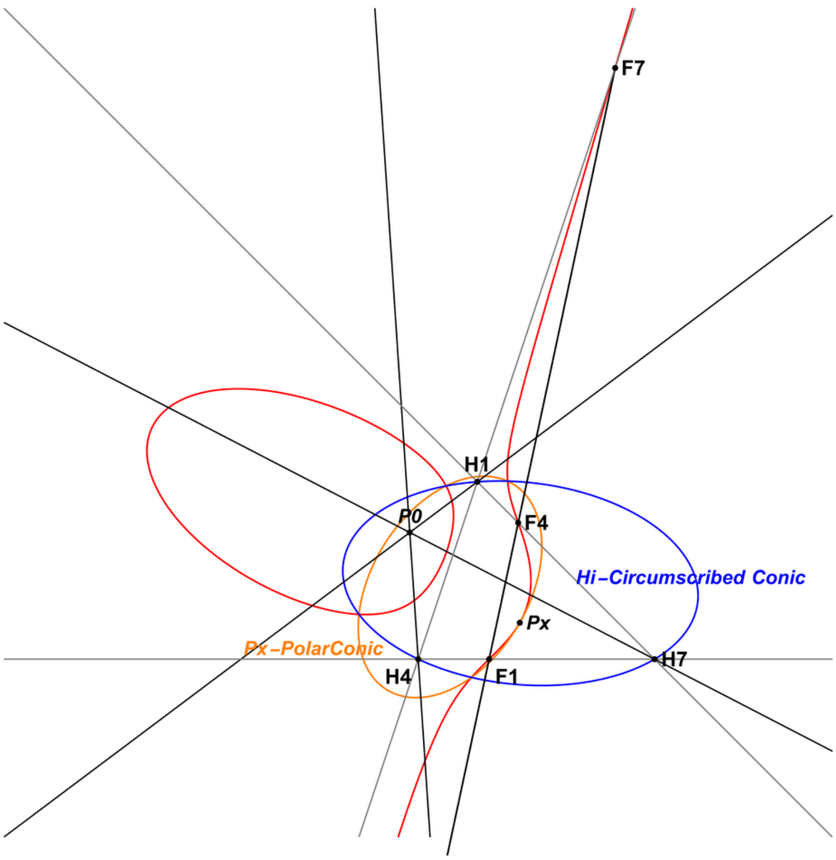


Chris van Tienhoven

2025, July 14

CU-HE-12L1abc-30-mono-bipartite cubic.pdf

point Px on a BIPARTE cubic (**red**)
yielding a Px-tangential quadrangle
circumscribed by the Px-Polar Conic (**orange**)
and circumscribed by the Hi-Circumscribed Conic (**blue**)



Chris van Tienhoven 2025, July 14

CU-HE-12L1abc-30-mono-bipartite cubic.pdf

Message: #2722
Date: 2025-07-14
From: van10hoven@gmail.com
Subject: co-tangentials of P

Dear Eckart, dear Bernard,

Inspired by the construction of Eckart, I investigated the tangential quadrangle of a tangential point of P.

I don’t know if it has been noted before, but I believe this is a special property.

· Let P be an arbitrary point on CU, and let (P1,P2,P3,P4) be the four points of tangency of the tangents from P to CU (also called tangentials).

· Let (P1a,P1b,P1c,P1d) be the four tangentials of P1.

· Now it turns out that the set (P2,P3,P4) coincides with the vertices (Q1a,Q1b,Q1c) of the Diagonal Triangle of the Quadrangle QA(P1a,P1b,P1c,P1d).

It provides us with a method to construct the co-tangentials of some point P.

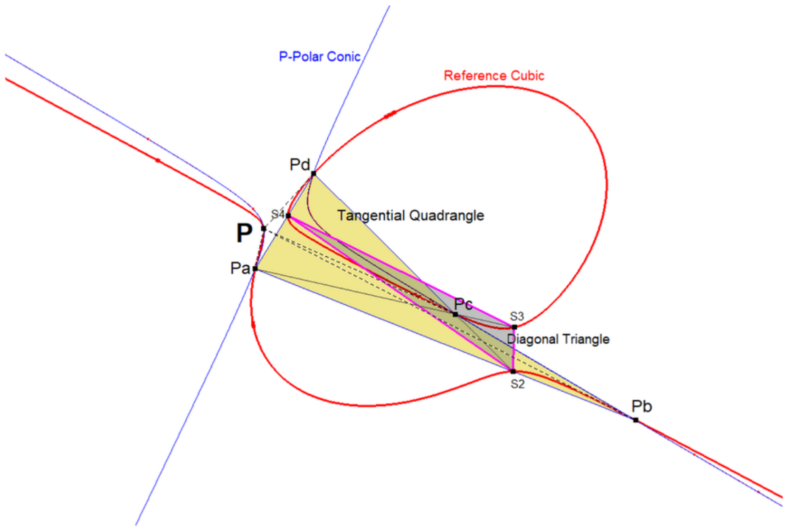
This set of points -sharing the same tangential as P- consists of the vertices of the Diagonal Triangle of the Quadrangle formed by the four tangentials of P.

See attached picture.

Best regards,

Chris

The co-tangentials of P



(S1,S2,S3) are the co-tangentials of P

Chris van Tienhoven

2025, July 14

CU_P-3P1 P-points with same tangential-01.pdf

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Message: #2723
Date: 2025-07-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris,
Thanks for your interest !
I'm glad that we perfectly understand eachother.
Your figures are the perfect illustation of my observation for a
bipartite cubis and my conjecture for a monopartite cubic as
explained in my message 2719.
Best regards
Bernard
```

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Message: #2724
Date: 2025-07-14
From: bernard.keizer@gmail.com
Subject: Re: co-tangentials of P

```
Dear Chris,
All this is already well-known and explained in EQF !
A bipartite cubic is a pivotal isocubic in any of it's points,
the point being the pivot and the tangential of the 4 fixed
points of the isoconjugation.
The DT of this tangential QA is the reference triangle of the
isoconjugation
The isopivot is in turn the tangential of the pivot and of the
DT vertices of the tangential QA of the fixed points ...
Take as example the Mac Cay cubic of a triangle ABC : it is a
pivotal isogonal cubic with pivot the circumcenter and fixed
points the in- and excenters.
The isopivot is the orthocenter, which is the tangential of the
circumcenter and of the vertices of the triangle ABC, which is
the DT of the QA of the in- and excenters.
Best regards
Bernard
```

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Message: #2725
Date: 2025-07-18
From: eckart_schmidt@t-online.de
Subject: Re Real flexline trilateral

Dear Bernard, dear Chris,

nevertheless a further proposal

... for the real flexline trilateral in the monopartite case,

... with the nomination of #2713,

... perhaps the same background:

Start with a fixed Pi (attached P1),

... which give U1,U1a,U1b,U2,U2a,U2b,

... vary H1'H2'H3' so,

,,, that the line PiFi intersect the line U1U2aU2b on Hi'Hj'

... and the line U2U1aU1b on Hi'Hk',

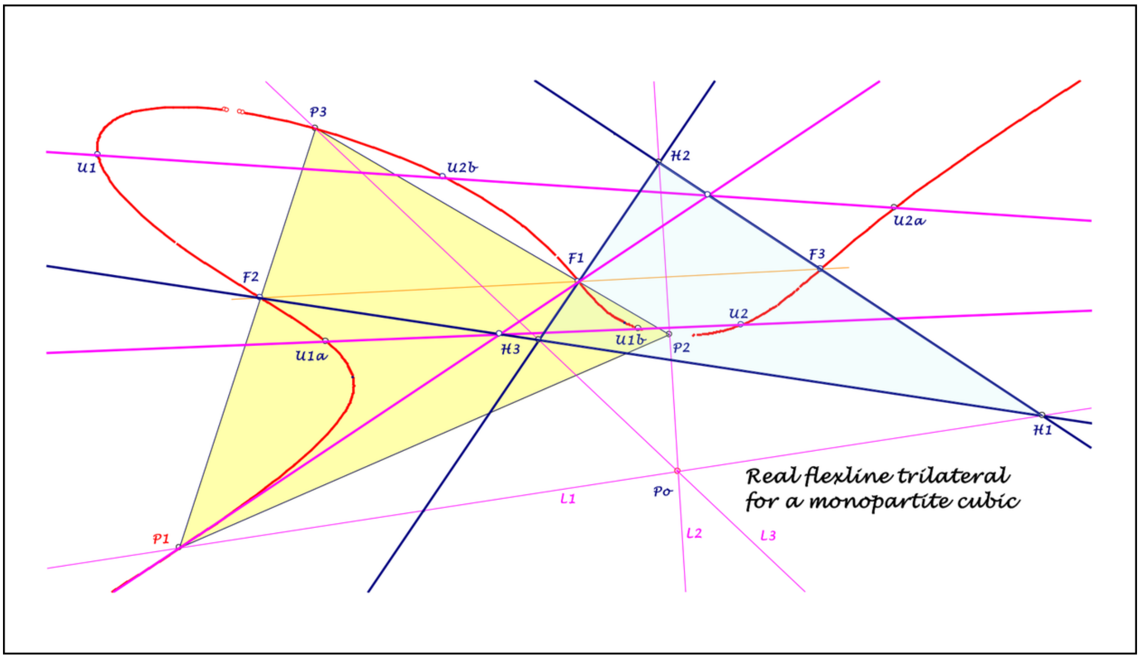
... then H1'H2'H3' will be H1H2H3.

Thanks in advance for a test of my observation.

Best regards Eckart

PS: Please use H1'H2'H3',

... which intersect the cubic only in F1,F2,F3!



2025-07-18.pdf

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Message: #2726
Date: 2025-07-18
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

```
Dear Eckart,
This property doesn't hold !
Best regards
Bernard
```

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Message: #2727
Date: 2025-07-19
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

my "construction" proposal in #2725

... for the real flexline trilateral monopartite

... has the following structure:

Start with a real flexpoint Fi (attached F3),

... then for Fi its tangent-contact Pi,

... then for Pi its tangent-contacts U1,U2,

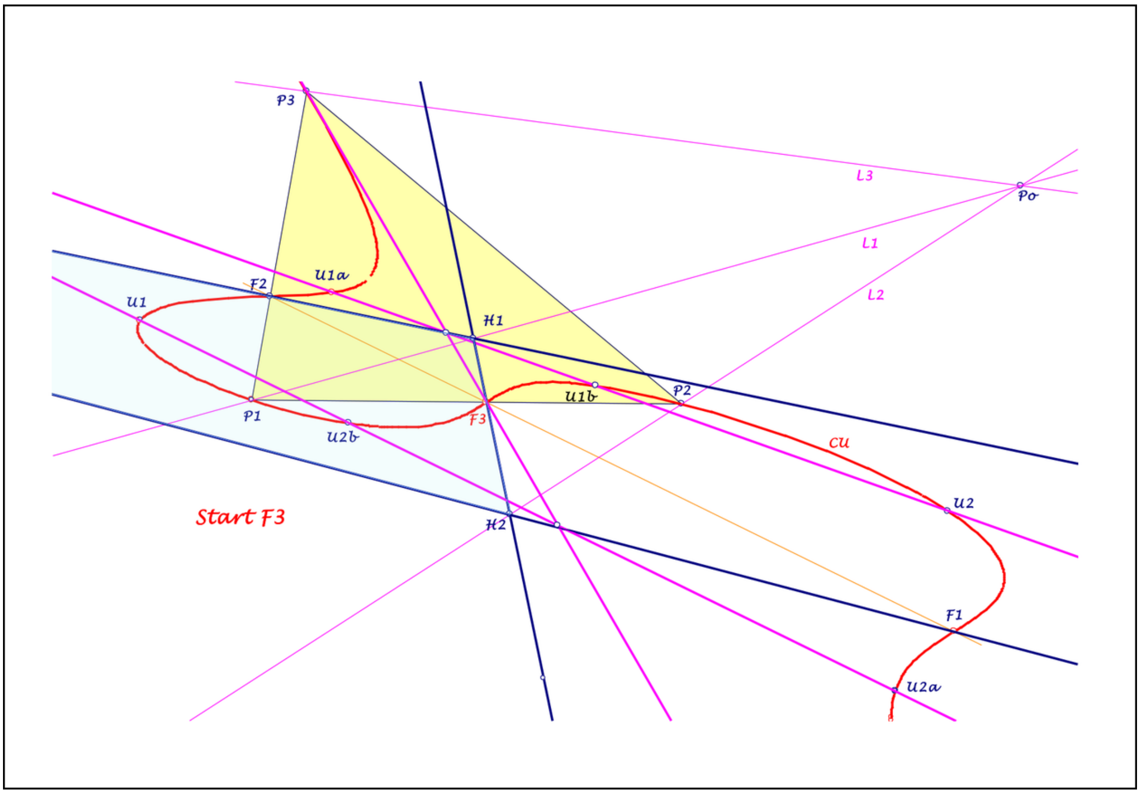
... then for U1,U2 their tangent-contacts U1a,b and U2a,b

... and finally the intersections of U1aU1b and U2aU2b

... with the connection FiPi (of the initial points)

... on lines of the real flexline trilateral H1H2H3.

Best regards Eckart



2025-07-19.pdf

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Message: #2728
Date: 2025-07-19
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
As already mentionned in 2726, this property doesn't hold !
Best regards
Bernard
```

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Message: #2729
Date: 2025-07-21
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear all,

For me, one of mysteries of QL-Tr2 is this:
Let QL1 be formed by 4 lines
y = tjx - f(tj^2) (j=1,2,3,4),
which are tangents of a parabola x^2 = 4fy (QL1-Co1).
When tj satisfies the quartic equation
ft^4 + mt^3 + pt^2 + qt + r = 0,
the slopes of the sides of QL1-Tr2 are the solutions of the
cubic equation
4ft^3 + 3mt^2 + 2pt + q = 0.
(cf. #2517,2509)

I'll state differently.
Let QA0 be formed by 4 points
(xj, (xj^2)/4f) (j=1,2,3,4),
where xj satisfies the quartic equation
x^4 + (2m)x^3 + (4pf)x^2 + (8qf^2)x + (16rf^3) = 0.
The equation of QA0-Co1 is
(x + m/2)(y' + p'/2) = -fq'/2,
where
y' = y + (m/4f)x, p' = p - (m^2)/4f, q' = q - (m/2f)p'.
(Please see #2532.)
Then the x-coordinates of the intersection points of QL1-Co1 and
QA0-Co1 are
the solutions of the cubic equation
4x^3 + 3(2m)x^2 + 2(4pf)x + (8qf^2) = 0.

Here's my interpretation (*the method of Lagrange multiplier*
).
Let
h(x) = x^4 + ax^3 + bx^2 + cx + d,
where
a = 2m, b = 4pf, c = 8qf^2, d = 16rf^3.
The critical points of h(x) are the solutions of the cubic
equation dh(x)/dx = 0.
Finding them is equivalent to finding the critical points of
L(x,y) subject to g(x,y) = 0,
where
L(x,y) = h(x,y) + λg(x,y)   (λ: Lagrange multiplier)
h(x,y) = (4fy)^2 + ax(4fy) + b(4fy) + cx + d
g(x,y) = x^2 - 4fy.
L(x,y) can be written in the form

```
C(x - x0)^2 + D(y' - y'0)^2 + E,
where C, D, E, x0, y'0 are real constants.
Then ∂L/∂x = ∂L/∂y = 0 give x = x0 and y' = y'0, which is the
center of the conic
C(x - x0)^2 + D(y' - y'0)^2 + const. = 0.
Since L(x,y) = 0 is a circumconic of QA0, the critical points
are on QA0-Co1.

Let
Co2 = a circumconic of QA0
Co3 = the polar curve of Co2 wrt QL1-Co1.
When the center of Co2 is on QL1-Co1, the side of QL1-Tr2
through this point
and QL1-L1 are the conjugate diameters of Co3.

Best regards,
M@IMF
```

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Message: #2730
Date: 2025-07-24
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris,

can you confirm Bernard's unsubstantiated answers

... in his last two messages?

Best regards Eckart
```

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Message: #2731
Date: 2025-07-25
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
I can understand your deception and your disappointment, but I
think I don't deserve your nasty commentar about my
unsubstantiated answers.
I've checked already patiently a dozent of your fake
constructions.
You asked me to check your last construction in your message
2725.
I've explained my method in the message 2707 : I deal with exact
cubics for which I know the exact points H1, H2 and H3.
Therefore, it's not too difficult to check your properties ; as
soon as I find any significant aberration, I simply mention that
the property doesn't hold.
What do you want more ?
Best regards
Bernard
PS Again, as mentionned in my message 2719, the algebraic
solution is the same in both cases mono- or bipartite !
```

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Message: #2732
Date: 2025-07-25
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,

I will be testing your new method as soon as possible. I just
need a bit of time.

It will be a pleasure to explore your approach.

Best regards,

Chris
```

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Message: #2733
Date: 2025-07-25
From: van10hoven@gmail.com
Subject: Re: About QL-Tr2

Dear MIMF,

Many thanks for yet another impressive contribution on QA/QL and a 4th degree function.

It connects various aspects of Quadri Geometry that I wasn't aware of.

I can't immediately follow every detail, but I truly admire your perseverance.

Best regards,

Chris

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Message: #2734
Date: 2025-07-26
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear Chris,

Thank you for your kind message.
I don't intend to demand a quick response.
(Of course, any comments are always welcome.)
If I posted a message, it might happen to be someone's help someday.

By the way, let's complexify #2729.
Given 4 complex numbers z1,z2,z3,z4 such that
z1 + z2 + z3 + z4 = 0,
and let
b = (z1 + z2)(z3 + z4) + z1z2 + z3z4
c = -(z1 + z2)z3z4 - (z3 + z4)z1z2
d = z1z2z3z4.
Then the z-coordinates of the intersection points of 2 complex conics
w = z^2 and z(w + b/2) = -c/4 are the solutions of the cubic equation
4z^3 + 2bz + c = 0.
This will have something to do with foci of Siebeck sextic,
but I'm not familier with cubics nor higher degree(class) curves.

P.S. Stereograms in the attached file are for parallel viewing.

Best regards,
M@IMF

Let QA0 be formed by 4 points (zj, wj) (j=1,2,3,4) which are the intersection points of 2 conics

$w = z^2$ (1)

$w^2 + bw + cz + d = \lambda(z^2 - w)$. (2)

(1) is QL1-Co1 where QL1 is formed by 4 tangents to (1) at the vertices of QA0.
(2) is a circumconic of QA0, which is written as

$[w + (\lambda + b)/2]^2 - \lambda(z - c/2\lambda)^2 = [\lambda^3 + 2b\lambda^2 + (b^2 - 4d)\lambda - c^2]/4\lambda$,

so its center is $(c/2\lambda, -(\lambda+b)/2)$, and QA0-Co1 is

$z(w + b/2) = -c/4$. (3)

We can obtain the intersection points of QL1-Co1 and QA0-Co1 from

$4z^3 + 2bz + c = 0$.

Let

$z = x + iy$, $w = u + iv$

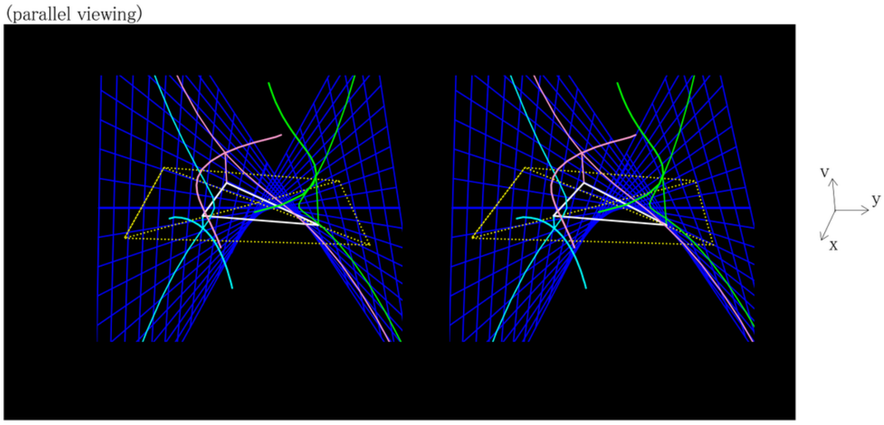
$b = b' + ib''$, $c = c' + ic''$,

where x,y,u,v,b',b'',c',c'' are real and i is the imaginary unit.
Then we get

QL1-Co1		QA0-Co1	
$u = x^2 - y^2$	(1')	$x(u + b'/2) - y(v + b''/2) = -c'/4$	(3')
$v = 2xy$	(1'')	$x(v + b''/2) + y(u + b'/2) = -c''/4$.	(3'')

When u is a constant, (3') and (3'') are hyperbolic paraboloids as well as (1'').

(parallel viewing)



The blue lines indicate v = 2xy.
The 3 curves on this surface are QL1-Co1(other branches are not shown).
The other 3 curves are (part of) QA0-Co1.
Note that the curves in different colors correspond to the different values of u which is the real part of z^2, where z is the solution of $4z^3 + 2bz + c = 0$ (the vertice of the white triangle).
The yellow dotted lines indicate the quadrangle {z1,z2,z3,z4}.

Appx. Foci of Ellipse

Let's complexify an ellipse

$Cx^2 + Dy^2 = 1$ $(0<C<D)$,

that is,

$x \to x + i\,\xi$, $y \to y + i\,\eta$

$C(x^2 - \xi^2) + D(y^2 - \eta^2) = 1$

$Cx\xi + Dy\eta = 0$.

Then we get

Tangents through circular points at infinity

$x - \eta = +f$, $\xi + y = 0$

$x - \eta = -f$, $\xi + y = 0$

$x + \eta = +f$, $\xi - y = 0$

$x + \eta = -f$, $\xi - y = 0$

Points of tangency

$x = +1/fC$, $\eta = +1/fD$, $\xi = y = 0$

$x = -1/fC$, $\eta = -1/fD$, $\xi = y = 0$

$x = +1/fC$, $\eta = -1/fD$, $\xi = y = 0$

$x = -1/fC$, $\eta = +1/fD$, $\xi = y = 0$

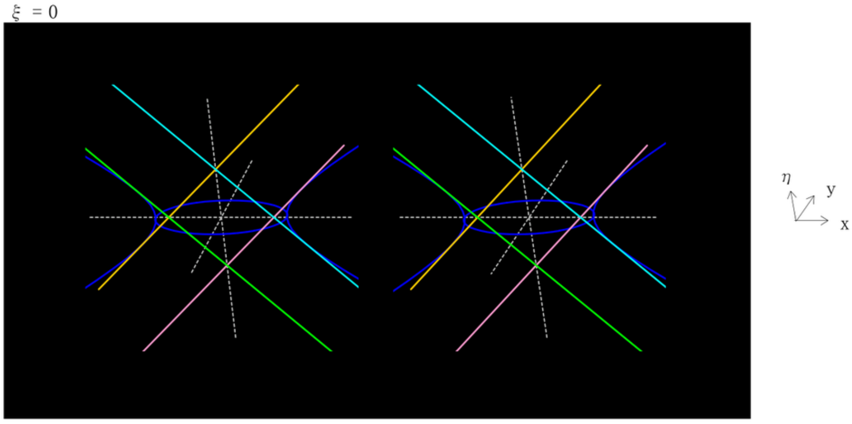
Foci

real: $x = \pm f$, $\eta = \xi = y = 0$

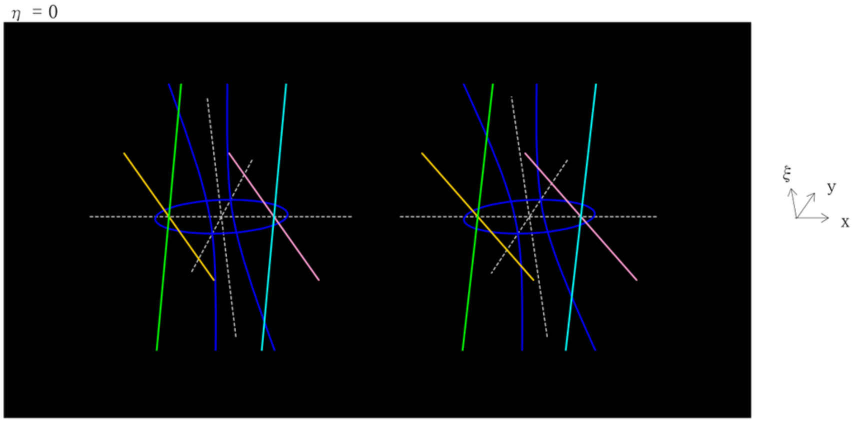
imaginary: $\eta = \pm f$, $x = \xi = y = 0$,

where $f = \sqrt{(1/C - 1/D)}$.

$\xi = 0$



$\eta = 0$



Complex Conics.pdf

Message: #2735
Date: 2025-07-27
From: bernard.keizer@gmail.com
Subject: Simple definition of QL-Tr2

Dear Chris, dear Eckart, dear M@IMF
First, many thanks to M@IMF for revisiting the triangle of the S-points QL-Tr2.
Some time ago, I've put on the forum a memo about Hessian and generalised S-points for a pivotal isocubic.
Here only a few remarks :
1) There are as many S-points as points on the cubic : they are always the 3 intersections (other than the point taken as pivot) of the polar conics of the point and it's conjugate (which is the isopivot). These 2 conics are the duals of 2 conics inscribed in the QL and in the DT, which leads to the usual well-known properties ...
2) These S-points may be used in order to draw the hessian of the cubic (see Schröter).
3) Let's consider the cubic QA-Cux with pivot QA-P16 and isopivot QA-P10 (pK(QA-P16,QA-P16) according to Bernard Gibert).
Then the vertices of QL-Tr2 are the 3 intersections (other than QA-P16) of the polar conics of QA-P16 and QA-P10 wrt the cubic QA-Cux.
The duals of these 2 conics are the parabolas QL-Co1 and DQL-Co1 ...
Best regards
Bernard

Hessian and generalised S-points for a pivotal isocubic

A. S-points and points of the hessian
   (see figur1)

Let's start with a pivotal isocubic wrt a triangle T1T2T3 with pivot P and isopivot P'.

The fixed points of the isoconjugation are P1, P2, P3 and P4.

The cevian triangle of P wrt T1T2T3 is u1u2u3.

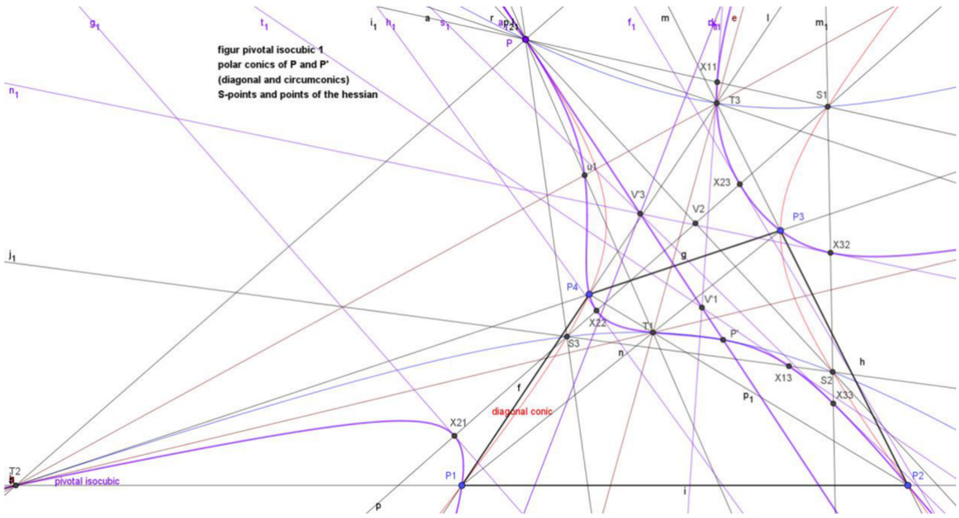
The polar conic of P wrt the cubic is the diagonal conic through P, P1, P2, P3 and P4 (in red).

The polar conic of P' wrt the cubic is the circumconic through P, P', T1, T2 and T3 (in blue).

Both conics intersect in P and 3 S-points S1, S2 and S3; the 4 points are the poles of PP' wrt the cubic and the polar conics of all points of the line PP' pass through the 4 points.

The DT vertices of the QA of the 4 points, V1, V2 and V3 belong to the hessian of the cubic, as well as their corresponding points (on the hessian), V'1, V'2 and V'3 on PP' having as polar conics the lines through V1, V2 and V3 and the 4 points P, S1, S2 and S3.

(In order to find V'1, V'2 and V'3, it's easy to draw the lines of the QA and the tangents in their intersections with the cubic …)



B. Hessian and flexes
(see figur 2)

P is the tangential of P’ (P/P’ in Bernard Gibert) and of u1, u2 and u3.

The cubic is also a pivotal isocubic wrt u1u2u3 with pivot P’ and isopivot p.

It’s possible to do the same construction and to find the 4 poles of the line P’p as intersection of the polar conics of P’ (unchanged) and of p through p, P’, u1, u2 and u3 (in green). (note that the polar conic of P’ is now a diagonal conic).

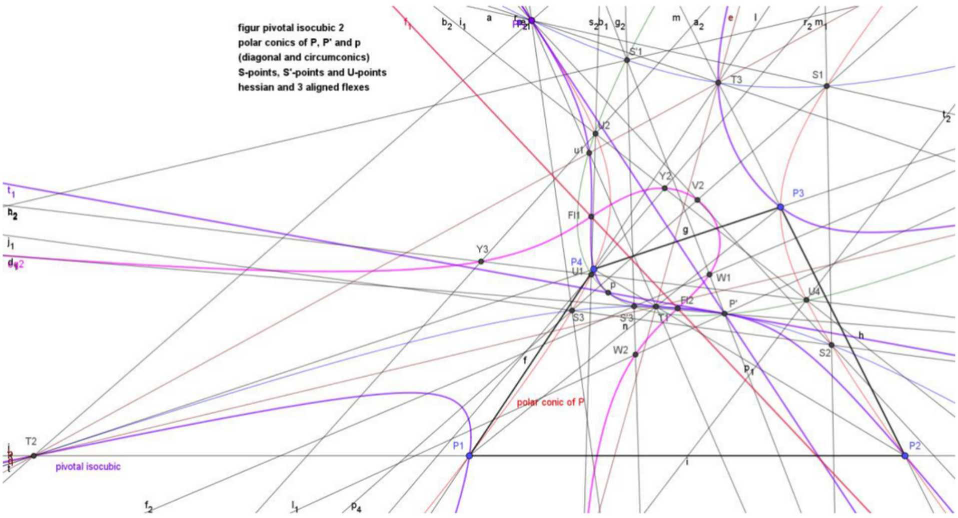
We find the same way the 3 points S’1, S’2 and S’3, the DT vertices W1, W2 and W3 of the QA P’S’1S’2S’3 and their corresponding points W’1, W’2 and W’3 on P’p.

We have now 12 points of the hessian.

We may naturally also consider the QA of the 4 intersections U1, U2 U3 and U4 of the polar conics of P and p and the vertices Y1, Y2 and Y3 of it’s DT, which also lie on the hessian …

Note that the points U1, U2, U3 and U4 are not S-points, as P and p are not isoconjugates.

The 3 real intersections of the cubic and it’s hessian are the (aligned) flexes of the 2 cubics.



C. S-points

Now, we consider the S-points S1, S2 and S3, intersection of the polar conics of the pivot P and the isopivot P'.

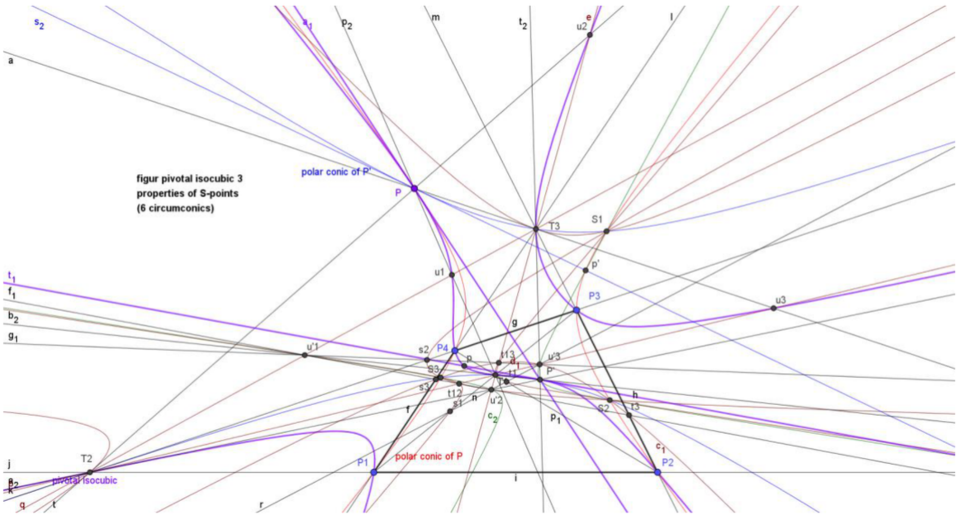
Let u'1u'2u'3 be the cevian triangle of P' wrt T1T2T3 and p' be the point 2nd intersection of the tangent in P to the polar conic of P' with the polar conic of P (P/P' of Bernard Gibert).

u'2u'3 intersects T2T3 in t23 and P1P3 and P2P4 (diagonals of the QA through P1) respectively in s1 and t1 and so on.

There are 4 more conics through these 3 points :

- first one through P', p', u'1, u'2 and u'3 (in green)
- 3 others through T1, u'1, s1, t1, t12 and t13 and the like (in brown)

making altogether 6 circumconics.



Naturally, the traditional S-points (vertices of QL-Tr2) are obtained for P = QA-P16 and by definition P' = QA-P10.

(pole = pivot according to Bernard Gibert)

I suppose Eckart will have recognized his 3 conics 1), 2), 3) and the 3 conics in 4) in his now old but always fascinating note on the S-points.

D. Last properties with twin pivotal isocubics

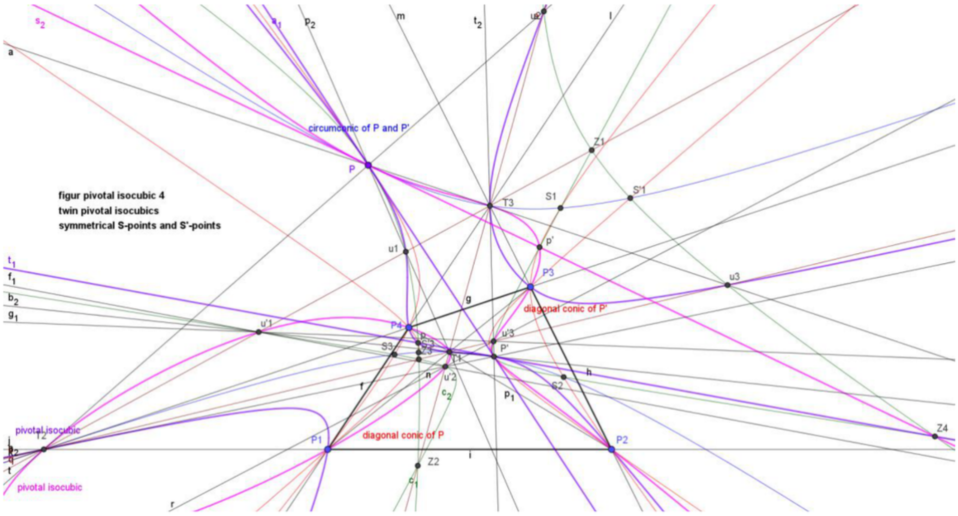
We consider now the twin pivotal isocubics having the same copple of isoconjugate points as pivot and isopivot ; P1, P2, P3, P4, T1, T2,T3, P and P’ are CB partners.

The 1rst cubic passes through p, u1, u2 and u3 and the 2nd through p’, u’1, u’2 and u’3.

There are 2 diagonal conics (in red), but only one circumconic (in blue) common to the 2 cubics ; the intersections between the circumconic and the 2 diagonal conics give the symmetric S-points for the 1rst cubic and S’-points for the 2nd.

There are 2 conics in green, the 1rst passes through P, p, u1, u2 and u3 and the S’-points and the 2nd through P’, p’, u’1, u’2 and u’3 and the S-points.

It is remarkable that these 2 last conics intersect in 4 points Z1, Z2 and Z3 on each side of the DT T1T2T3 and the 4th Z4 on the intersection of Pp’ and P’p.



Hessian and generalised S-points for a pivotal isocubic.pdf

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Message: #2736
Date: 2025-07-27
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer,

Thank you very much for your message.
It seems very difficult, so I have to study from cubics.
(It might take some time...)

For me, one of mysteries of QL-Tr2 (deja vu?) is QL1-QL2
symmetry.
(cf. #2712, and of course, QFG#1315-38 "ETC and EQF")
What will become of this?
Please forgive me if I ask a misguided question.

Best regards,
M@IMF
(Please use "MIMF" if @ is annoying.)
```

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Message: #2737
Date: 2025-07-28
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer and all,

I was overwhelmed by #2735, but I realized some of them are
understandable even for me.
I could follow 3), and the answer to my question is simply
perspectors of 2 QLs are isotomic wrt DT as usual.

Best regards,
M@IMF
```

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Message: #2738
Date: 2025-07-29
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Many thanks for your interest
On pages 30-31 of his well-known memo about Special Isocubics in a Triangle Plane, Bernard Gibert defines the 2 considered conics for a pivotal isocubic as diagonal and circumscribed polar conics (the 1rst through pivot and fixed points, the 2nd through pivot, isopivot and reference triangle).
The beauty of the given definition of the S-points is naturally it's simplicity : 3 intersections other than the pivot of these 2 conics.
The vertices of QL-Tr2 are a special case, when P is QA-P16 and the cubic pK(QA-P16,QA-P16) with pivot=pole (therefore, the duals of the 2 conics are parabolas).
In fact, we don't need the cubic, only the vertices of the QA as fixed points of the isoconjugation and the DT as reference triangle.
I wonder what the locus of the S-points could be :
1) for a given cubic, variing P, it's tangential QA and it's DT
2) for a given QA and it's DT, variing P and the cubics
Any idea ?
Best regards
Bernard

Message: #2739
Date: 2025-07-30
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer and all,

Wrt 1) in #2738, let me comfirm my understanding.
Given a QA and a point P, and let
Cu = pK(QA-P16, P) wrt QA-Tr1
P' = a point on Cu
P'~ = the tangential of P' wrt Cu
COd' = the polar conic of P' wrt Cu
COc' = the polar conic of P'~ wrt Cu
S'j (j=1,2,3) = the intersection points of COd' and COc' except
P'.
Then S-points are S'1,S'2,S'3.

When P is parametrized by t, the S-point is expressed by t:
(u'j(t):v'j(t):w'j(t)).
Eliminating t from
x = u'j(t), y = v'j(t), z = w'j(t)
gives the locus of the S-point in principle, but this will be
unrealistic.
One method I came up with is to derive differential equations:
dx/dt = du'j(t)/dt, dy/dt = dv'j(t)/dt, dz/dt = dw'j(t)/dt.
Solving cubic eq. is cumbersome, so how about finding the locus
of the centroid
of the 3 S-points for starters?
If we are lucky, the differential eq. can be solved analytically
(otherwise numerically).

Wrt 2), how does P vary?
I'm also interested in QLx-Tr2 where QLx is formed by tangents
to a circumconic of (convex) QA0 at its vertices.

Best regards,
M@IMF
```

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Message: #2741
Date: 2025-07-31
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
In 1) Cu is defined by the QA and P and Cu is pK(QA-P16,P),
QA-P16 being the pole and P being the pivot.
A variable P' on Cu defines a new QA (it's tangential QA) and
it's DT as well as a new isoconjugation wrt this DT
P'* is the isoconjugate of P' in thi isoconjugation and P' and
the vertices of this DT have P' as tangential
The 2 conics are the polar conics of P' and P'*
In 2) The QA, it's DT are fixed, but P is a random variable
point and Cu is defined as pK(QA-P16,P) (as for example QA-Cu1
to QA-Cu6 in EQF)
P* is the isoconjugate of P, P is the tangential of the QA
vertices and P* is the tangential of P and of the DT vertices
The 2 conics are the polar conics of P and P*
in 3) P is QA-P16 (pivot = pole), P* is QA-P10
The 2 conics are the polar conics of QA-P16 and QA-P10
Best regards
Bernard

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Message: #2742
Date: 2025-07-31
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer,

Thank you for the explanation.

Wrt 1), I assumed new isoconjugation is QA'-Tf2:
Given a QA and a point P, and let
Cu = pK(QA-P16, P) wrt QA-Tr1
P' = a point on Cu
Q'j (j=1,2,3,4) = the points of tangency of P' to Cu
(pretangentials of P'?)
QA' = {Q'1,Q'2,Q'3,Q'4}.
I thought
Cu = pK(QA'-P16, P') wrt QA'-Tr1,
then
P'* = QA'-Tf2(P') = P'~(the tangential of P' wrt Cu)
S'j (j=1,2,3) = the intersection points of COd' and COc' except
P',
where
COd' = the polar conic of P' wrt Cu
COc' = the polar conic of P'~ wrt Cu.

Wrt 2), I thought when P is a random variable point (not lies on
a certain curve),
the locus of S-point becomes 2-dimensional.

Best regards,
M@IMF
```

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Message: #2743
Date: 2025-08-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
We agree
The Q'1,2,3,4 are in fact the 4 pretangentials of P', which is their common tangential
P' and the vertices of QA'-Tr1 are the 4 pretangentials of P'*, which is their common tangential
Best regards
Bernard
PS Having the equations of the 2 polar conics given in Bernard Gibert, I hoped it would be possible to solve the system and, knowing that P is a solution, to find the 3 other

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Message: #2744
Date: 2025-08-01
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I have reviewed your latest configuration, as outlined in QPG#2725 and QPG#2727, utilizing the same setups from my previous analysis.

In both setups, the line F2P2 appears as a dashed orange line.

The lines U1aU1b and U2aU2b are also orange, but not dashed.

In the first drawing, you can see that the intersection points W1 = F2P2 ∩ U1aU1b and W2 = F2P2 ∩ U2aU2b are nearly aligned with H2H3 and H2H1.

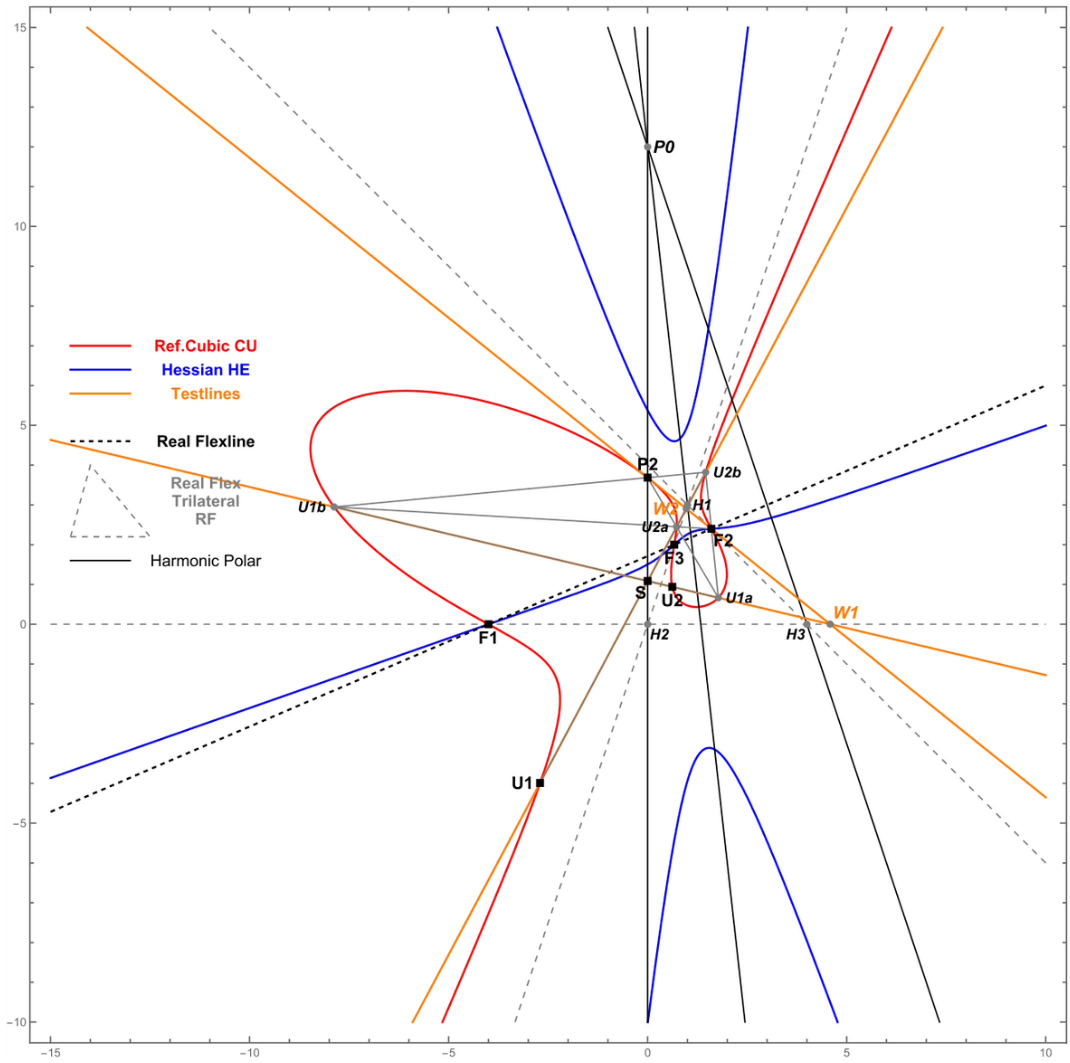
In the second drawing, only W2 is visible, but it’s clearly not positioned on H2H1.

I hope there are no errors on my part, but please double-check my drawings to be sure.

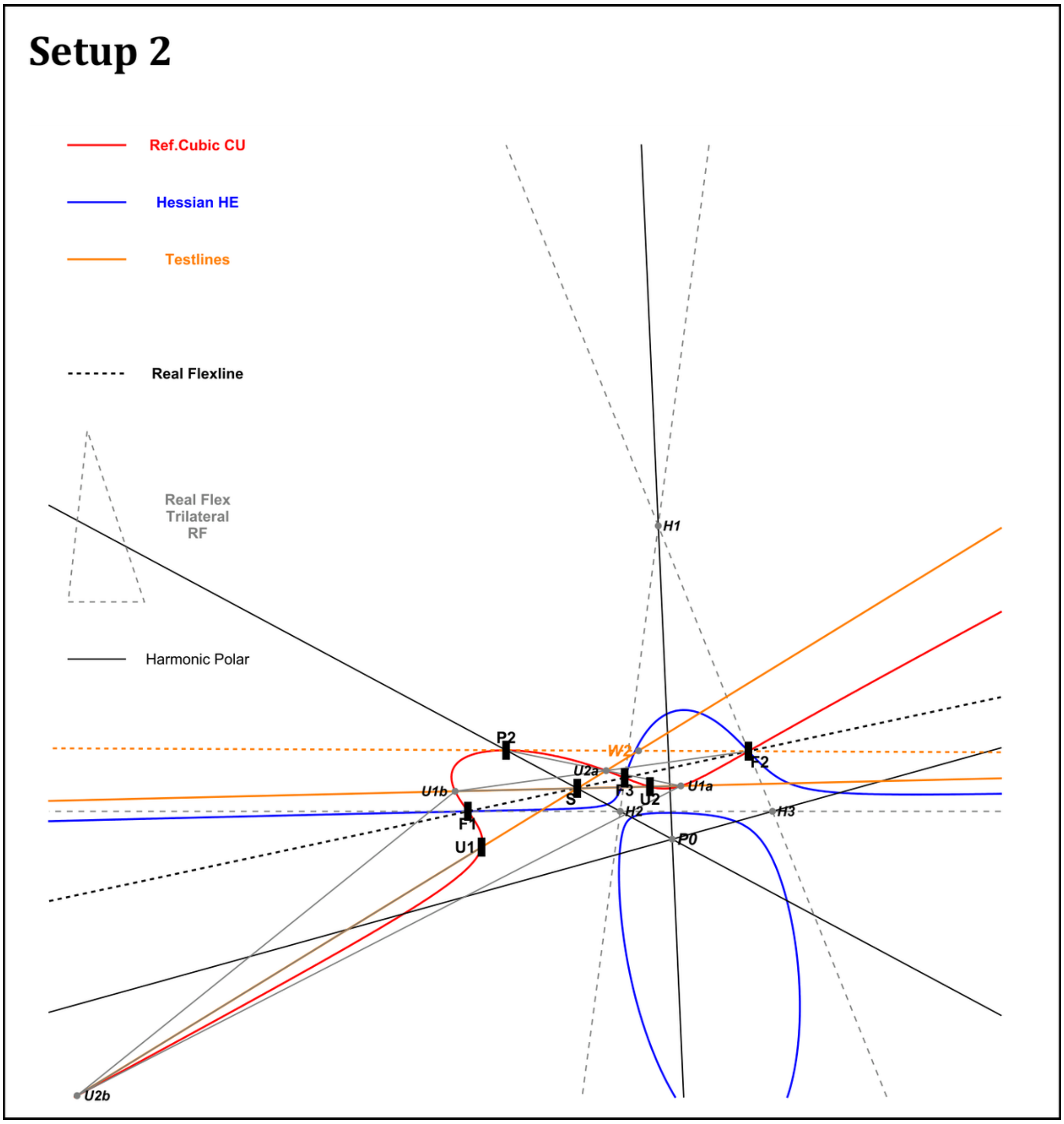
Best regards,

Chris

Setup 1



9P-s-Cu1 Hessian-plus Flexlines-80-Test ES-Flextrilateral.pdf



9P-s-Cu1 Hessian-plus Flexlines-80-Test ES-Flextrilateral.pdf

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Message: #2745
Date: 2025-08-02
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris,

thanks a lot for your second drawing,

... which does not confirm my "construction".

I made a further similar to your second

... and similar to my in #2725 with small aberrations,

... but my proving of H1H2H3 is not the best.

These cubics, where you show aberrations,

... are often nearly bipartite with low k.

Thanks once more for your time consuming help.

Best regards Eckart
```

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Message: #2746
Date: 2025-08-02
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,

Always happy to help.
Your perseverance is more stable than the Flexline Trilateral!
Really hope you'll find the solution to this remarkable problem
someday.

Best regards,

Chris
```

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Message: #2747
Date: 2025-08-06
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

now I can offer a method to find the real flexline triangle

... in the same way for mono- and bipartite cubics,

... using the variable triangle H1H2H3 with vertices

... on the harmonic polars L1,L2,L3 of the real flexpoints

... with the flexpoints F1,F2,F3 on their sidelines.

Start with a point X on the cubic (see attached figure)

... and consider two X-tangents, contacting CU in U1,U2,

... construct a first conic as polar conic of X, bearing U1,U2,

... and a second conic through U1,U2,H1,H2,H3.

Consider for a fixed Hi the following three lines:

(1) cubic tangent in Hi,

(2) polar of Hi wrt the first conic,

(3) polar of X wrt the second conic.

Finally you find a constellation of the variable triangle H1H2H3,

... so that the three lines intersect in a common point on HjHk,

... and H1H2H3 will be the real flexline triangle.

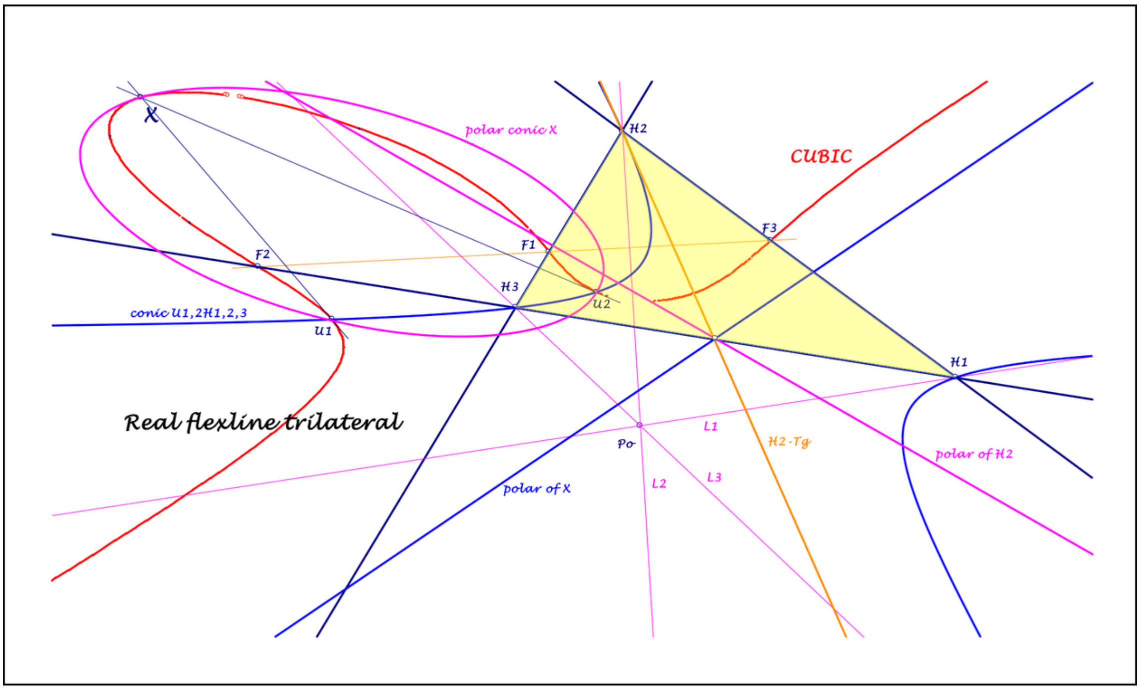
I hope someone can confirm this method.

Best regards Eckart

PS: For a bipartite cubic there are four X-tangents,

... it doesn't matter which two you take,

... the two conics are the same.



2025-08-06.pdf

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Message: #2748
Date: 2025-08-07
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
I've checked your ultime construction on my cubics for variing
points X in both cases mono- and bipartite.
I'm glad to confirm your property.
I suppose Chris will agree and possibly prove it analytically.
This time, the game is over !
Sincere congratulations (you deserve them now).
Best regards
Bernard
PS I think there is a typo in your message : the 1rst line is
the tangent in Hi to the 2nd conic (not the cubic tangent)
```

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Message: #2749
Date: 2025-08-08
From: eckart_schmidt@t-online.de
Subject: Re Real flexline trilateral

Dear Bernard,

thanks for your confirmation

... and correcting the typo, we have to replace

... in (1) "cubic tangent..." by "tangent to the 2nd conic...",

... it was good that I made a drawing.

You had already earlier indicated ,

... that there would be the same construction

... for mono- and bipartite cubics.

Best regards Eckart

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Message: #2750
Date: 2025-08-08
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

Dear Eckart,
Perhaps interesting (or obvious ?)
The polar of X wrt the 2nd conic is the trilinear polar of X wrt
the triangle H1H2H3.
It's intersection with the line of real flexes is therefore the
trinear pole of the line XP0.
Best regards
Bernard

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Message: #2751
Date: 2025-08-09
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

Dear Eckart,
I checked again by variing the triangle H'1H'2H'3 !
If I'm not wrong, the property that the polar of X wrt the conic through H'1, H'2, H'3, U1 and U2 is the trilinear polar of X wrt the triangle H'1H'2H'3 holds only for H'1, H'2 and H'3 in the right place of H1, H2 and H3.
I hope you will confirm that.
Best regards
Bernard

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Message: #2752
Date: 2025-08-09
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for interest in the drawing of #2747,

... I can confirm your first observation,

... but not the second:

The trilinear poles of lines through X wrt H1H2H3

... give a conic, which doesn't bear

... the intersection of F1F2F3 and the polar of X wrt the 2nd conic.

Best regards Eckart

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Message: #2753
Date: 2025-08-09
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard,

I think your observation in #2750 is not evident,

... and my figure confirms your further observation in #2751,

... so there is a fourth line for a necessary common point

... wrt Hi' on Hj'H3k', to get H1H2H3.

Best regards Eckart
```

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Message: #2754
Date: 2025-08-10
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,
Sorry, I tried in vain to understand this last message !
You confirm that the polar line of X wrt the 2nd conic is the
trilinear polar of X wrt H1H2H3 only for the right triangle.
But what is this 4th line ? What are the 3 other lines ?
Best regards
Bernard
PS I'm always convinced that these properties should be proved
analytically
```

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Message: #2755
Date: 2025-08-10
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard,

yes, I confirmed in my figure, that
... " the polar line of X wrt the 2nd conic

... is the trilinear polar of X wrt H1H2H3

... only for the right triangle".

Then there are now 5 lines with a common point:

(1) tangent in Hi to the second conic,
(2) polar of Hi wrt the first conic,
(3) polar of X wrt the second conic,
(4) trilinear polar of X wrt H1H2H3,
(5) HjHk.

Best regards Eckart
```

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Message: #2756
Date: 2025-08-11
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Wrt H1H2H3 :
The equation of the cubic is a1³X³ + ... + ka1a2a3XYZ = 0
(monopartite if k >=-3, bipartite if k<=-3)
The equation of the polar conic of X is X(3a1³x² + ka1a2a3yz) + ... (1rst conic)
The equation of the circumconic to H1H2H3 through U1 and U2 is
Xyz + Yxz + Zxy = 0 (2nd conic)
The tangent in H1 to the 2nd conic is Zy + Yz = 0 (1rst line),
which intersects H2H3 (5th line) in the point (0, Y, -Z)
The polar of H1 wrt the 1rst conic is 6a1³Xx + ka1a2a3(Yz + Zy) = 0 (2nd line), which passes through the point (0, Y, -Z)
The polar of X wrt the 2nd conic is X(Yz +Zy) + ... = 0, id est
xYZ + yXZ + zXY = 0 (3rd line), which passes through the point (0, Y, -Z)
The trilinear polar of X is x/X + y/Y + z/Z = 0 (4th line),
which is the same line as the 3rd
Best regards
Bernard
PS Can you in particular confirm the equation of the 2nd conic ?

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Message: #2757
Date: 2025-08-11
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris, dear Eckart
Any point (X,Y,Z) in the plane defines a unique value of k and a
unique cubic of the pencil
The polar conic of X wrt any cubic of the pencil is given by
X(3a1³x² + ka1a2a3yz) + ... = 3(a1³Xx² + a2³Yy² +a3³Zz²) +
ka1a2a3(Xyz + Yxz + Zxy) = 0
The 1rst part in brackets is the equation of the polar conic of
X wrt the cubic FE (for k = 0).
The 2nd part in brackets is the equation of the polar conic of X
wrt the cubic RF (for k = ∞)
For the k defining the cubic of the pencil, we have your 1rst
conic, for k = ∞, we have your 2nd conic
It is remarkable that the points U1 and U2 (in the monopartite
case) or U1,U2,U3 and U4 (in the bipartite case) belong to the
polar conics of all the cubics of the pencil
Best regards
Bernard
```

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Message: #2758
Date: 2025-08-12
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris, dear Eckart
Is the property that the polar conics of a point wrt a pencil of
cubics form a pencil of conics well-known or obvious ?
Any reference ?
Best regards
Bernard
```

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Message: #2759
Date: 2025-08-13
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

excuse, if I don't react on your messages #2756 and #2757:

... with the background of construction

... I describe cubics as nonpivotal isocubics:

... reference triangle P1P2P3

... with Po as root and fixpoint of the isoconjugation

... and a given cubic point Q,

... varying Q, we get a set of cubics (not syzygetic).

I think, Chris will be your partner for the calculations,

... but I don't understand his silence.

Sorry I can't understand the last sentence of #2757,

... if Ui are contact points of tangents from a cubic point X to the cubic.

Best regards Eckart

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Message: #2760
Date: 2025-08-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Can you please answer my last message 2758 ?
A pencil of cubics is defined by 9 points (CB partners), not necessary all real.
(I don't know if your construction of not pivotal isocubics wrt P1P2P3 with root and fixed point P0 and variing Q forms a pencil ?)
A pencil of conics is defined by 4 points, not necessary all real).
For the syzygetic pencil, the 9 points are the flexes (3 real and 6 imaginary).
For a pencil of circles, 2 of the 4 points are the circular points.
For a point X in the plane, the 2 points U1 and U2 are defined as contact points of X to the unique cubic of the pencil passing through X.
These 2 points belong to the polar conics of X to all cubics of the pencil, in particular the cubic passing through X and the cubic formed by the sides of the triangle H1H2H3. These 2 conics are your 1rst and your 2nd conic.
Best regards
Bernard
PS I can't understand either Chris silence

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Message: #2761
Date: 2025-08-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

independent of the calculations

... I try in vain to understand your last two messages:

(1)   Is it correct, that all cubics through 8 given points

... and their CB-point give a "pencil" of cubics?

(2)   Can it be, that the last sentence in #2757

... describes only the wellknown property,

... that U1,U2 or U1,U2.U3,U4 as contact points of tangents

... from a cubic point X to the cubic

... lie on the polar conic of X?

Excuse my lack of comprehension.

Best regards Eckart

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Message: #2762
Date: 2025-08-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Thanks for trying to understand and excuse if I'm not clear (I wasn't a teacher) !
1) That is correct, these 9 CB partners (8 +1) define a pencil of cubics
2) Taking a point X in the plane, not among these 9 points, consider the polar conics of X wrt the cubics of the pencil.
X belongs to only one cubic of the pencil , which determines the points U1 and U2 or U1,U2,U3 and U4 as contact points of the tangents from X to this cubic.
These points belong to the polar conics of X to all cubics of the pencil, which form a pencil of conics.
I wouldn't be surprised if this property was in Schröter ...
Best regards
Bernard
PS If F(x,y,z) is a homogenous equation of a curve, then equation of the 1rst polar curve of a point (X,Y,Z) wrt this curve is given by
$X\partial F/\partial x + Y\partial F/\partial y + Z\partial F/\partial z = 0$ (see Wikipedia)

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Message: #2763
Date: 2025-08-16
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Perhaps a useful precision
As I said, the point X (not one of the 9 CBpartners) is only on one cubic of the pencil
For this cubic, the contact points of the tangents from X are U1 and U2 and the polar conic of X passes through X and U1 and U2.
For any other cubic of the pencil, the contact points of the tangents from X are 2 different point V1 and V2 and the polar conic of X don't pass through X, but passes through V1 and V2 and through U1 and U2.
So the V1 and V2 are all different, but the points U1 and U2 belong to all polar conics.
Is it clearer this way ?
Apparently, you didn't know this property, neither did I, I just discover it by calculation !
Best regards
Bernard

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Message: #2764
Date: 2025-08-16
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Back from holiday, I reviewed your latest method for identifying the real flexline triangle in QPG#2747.
This approach involves two conics:

* Co1 – the X-Polar Conic, which touches the reference cubic CU in point X through (U1, U2)
* Co2 – the conic passing through (U1, U2, H1, H2, H3)

For each point Hi (representing H1, H2, H3), we consider three lines:

* (1) The tangent to Co2 at Hi
* (2) The polar of Hi with respect to the first conic Co1
* (3) The polar of X with respect to the second conic Co2

The points Hi are ‘gliding’ points along the harmonic polars Li (representing L1, L2, L3), such that the sides of triangle H1H2H3 pass through the real flexpoints F1, F2, F3.

When H1H2H3 form the vertices of the real flexline trilateral, the three lines (1), (2), and (3) concur at a point lying on one of the sidelines of the trilateral.
I verified this statement using a numerical example in Mathematica, and in all cases, the concurrence held true to at least 14 decimal places. This gives me strong confidence in the validity of the result.

As far as I know, this is the first constructive property of the real flexline trilateral.
Sincere congratulations!

That said, I believe there remains the challenge of finding a precise construction in which H1, H2, H3 arise from intersections of well-defined lines or curves. The notion of ‘gliding’ along a line lacks geometric precision.

Compare this with the ‘gliding’ construction for flexpoints you mentioned in QPG#2046:
Start with an arbitrary point F on the cubic and draw three secants,
... yielding 3×2 intersections with the cubic,

... if these six points lie on a conic CO , then F is an inflection point.
[Schröter's book, page 242]

I've been thinking about this further:

* Gliding H1, H2, H3 along L1, L2, L3 produces the three lines (1), (2), and (3), forming a triangle. Each vertex of this triangle defines a locus. Perhaps one of these loci is a line or conic, which could help track the points of concurrence.
* Instead of choosing an arbitrary point X on the reference cubic CU , we might select a specific point—such as one of the points at infinity on CU. In that case, Co1 becomes the IP-Polar Conic or Diametral Conic. This might open up new directions for exploration.

Best regards,
Chris

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Message: #2765
Date: 2025-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

wrt your last sentences in #2757 and #2758:

I started with 8 points and its CB-point,

... which define a pencil of cubics,

... then I took a point X in the plain,

... which defines a special cubic of the pencil

... and considered U1,U2 as contact points

... of tangents from X to the cubic,

... then I took another cubic of the pencil,

... not bearing X (but the initial 8 points and CB)

... and constructed its polar conic,

... but unexpected not bearing U1,U2 ...

Is there something wrong?

Best regards Eckart

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Message: #2766
Date: 2025-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

excuse, my last message was sent

... before reading your last message...

...I will answer later.

Best regards Eckart

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Message: #2767
Date: 2025-08-17
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I feal ashamed !
Unfortunately, it seems you're right and my property doesn't
hold, at least for pencils of cubics through 9 CB partners not
being the flexpoints.
I keep a last hope for the syzygetic pencils, the 9 CB partners
being the flexpoints.
Maybe Chris will be able to help me to understand my barycentric
calculations and their interpretation.
With my apologies
Best regards
Bernard

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Message: #2768
Date: 2025-08-17
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

Thanks for confirmation of the property,

... I observed for the real flexline triangle.

Of course I am aware of the fact,

... that this is no real construction,

... so I already experimented in the meaning of your last sentences.

Best regards Eckart

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Message: #2769
Date: 2025-08-24
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer and all,

Wrt 2) in #2738, I calculated the *locus of S-points* when P
lies on a cubic.
Given a quadrangle QA and a point P0, and let
Cu0 = pK(QA-P16, P0) wrt QA-Tr1
QA-P16 = (p^2 : q^2 : r^2)
P0 = (u0:v0:w0)
P(u:v:w) = a point on Cu0
Cu(P) = pK(QA-P16, P) wrt QA-Tr1.

What we should solve are system of equations:
*Locus of P0* (= Cu0)
u0u[(rv)^2 - (qw)^2] + v0v[(pw)^2 - (ru)^2] + w0w[(qu)^2 -
(pv)^2] = 0
*Polar conic of P wrt Cu(P)*
[(rv)^2 - (qw)^2]x^2 + [(pw)^2 - (ru)^2]y^2 + [(qu)^2 -
(pv)^2]z^2 = 0
*Polar conic of QA-Tf2(P) wrt Cu(P)*
(p^2)u[(rv)^2 - (qw)^2]yz + (q^2)v[(pw)^2 - (ru)^2]zx +
(r^2)w[(qu)^2 - (pv)^2]xy = 0.
By introducing
X = rx/pz, Y = ry/qz
U = ru/pw, V = rv/qw, U0 = ru0/pw0, V0 = rv0/qw0,
we get
(1)   (V^2 - 1)(U0U - 1) - (U^2 - 1)(V0V - 1) = 0
(2)   (V^2 - 1)(X^2 - 1) - (U^2 - 1)(Y^2 - 1) = 0
(3)   (V^2 - 1)(U/X - 1) - (U^2 - 1)(V/Y - 1) = 0.
For having solutions other than U^2 = 1 and V^2 = 1,
(4)   (X^2 - 1)(V0V - 1) - (Y^2 - 1)(U0U - 1) = 0
(5)   (X^2 - 1)(V/Y - 1) - (Y^2 - 1)(U/X - 1) = 0,
which give
(6)   U = (V0/Y - 1)X(X^2 - Y^2)/(Y^2 - 1)(U0X - V0Y)
(7)   V = (U0/X - 1)Y(X^2 - Y^2)/(X^2 - 1)(U0X - V0Y).
Substituting (6),(7) into (2) yields
(8)   [(U0X - 1)^2 (Y^2 - 1)^2 - (V0Y - 1)^2 (X^2 - 1)^2](X^2 -
Y^2)
- [(U0X - 1)(Y^2 - 1) - (V0Y - 1)(X^2 - 1)]^2 = 0.
Then we obtain
(9)   (U0X - 1)(Y^2 - 1) - (V0Y - 1)(X^2 - 1) = 0,
which is the Locus of P0, and
(10)   (U0X - 1)(Y^2 - 1)(X^2 - Y^2 - 1) + (V0Y - 1)(X^2 -
1)(X^2 - Y^2 + 1) = 0,
```

which is the Locus of S-points.

In DT-coordinates, (10) becomes
*(u0x/p^2)[(y/q)^2 - (z/r)^2][(x/p)^2 - (y/q)^2 - (z/r)^2] +
cyclic permutations = 0*.
It bears the vertices of QA, QA-Tr1 and QA-Tf2(P0).
I checked numerically some S-points lie on this curve when Cu(P)
= QA-Cu2,3,4,5,
but I'm not sure my result is correct because I can't draw
quintics (or even cubics).

Best regards,
M@IMF

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Message: #2770
Date: 2025-08-26
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

perhaps no more of interest, but:

... I would be glad, if you can confirm the following method

... to find the real flexline trilateral for a monopartite cubic,

... using the variable triangle H1'H2'H3' as in #2747:

Consider the conic through P1,P2,F1,F2,H1',H2',

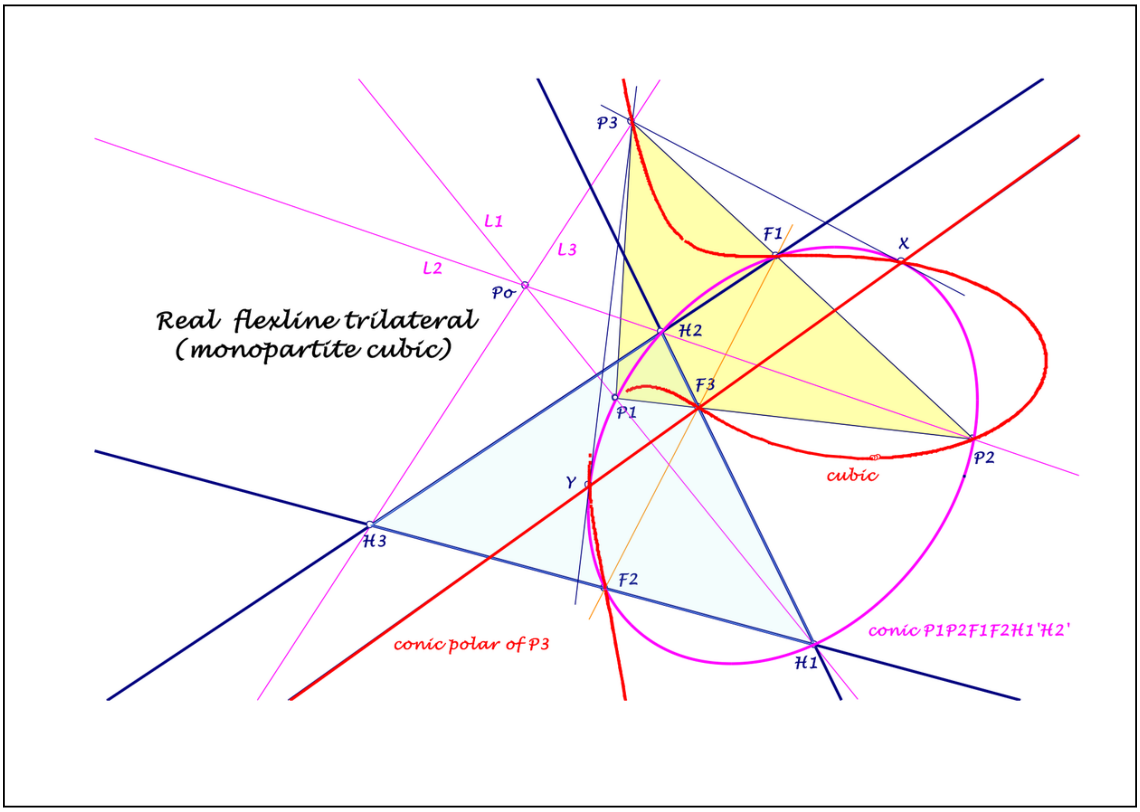
... which intersects the cubic in two further points X, Y.

If the line XY is the conic-polar of P3,

... you have the real flexline trilateral H1H2H3 = H1'H2'H3'.

Best regards Eckart

PS. It seems, that the points X, Y doesn't exist in the bipartite case???



2025-08-26.pdf

Message: #2771
Date: 2025-08-27
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard, dear Chris,

The PS in my last message is not correct!

Excuse.

Best regards Eckart
```

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Message: #2772
Date: 2025-08-29
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

perhaps a new method to find

... the real flexline triangle for a bipartite cubic,

... using the variable triangle H1'H2'H3' as in #2747:

Let P1,P2,P3 be the intersections of L1,L2,L3

... and the infinity part of the bipartite cubic CU.

Draw the three conics COk through PiPjFiFjHi'Hj',

... each with two further CU-intersections,

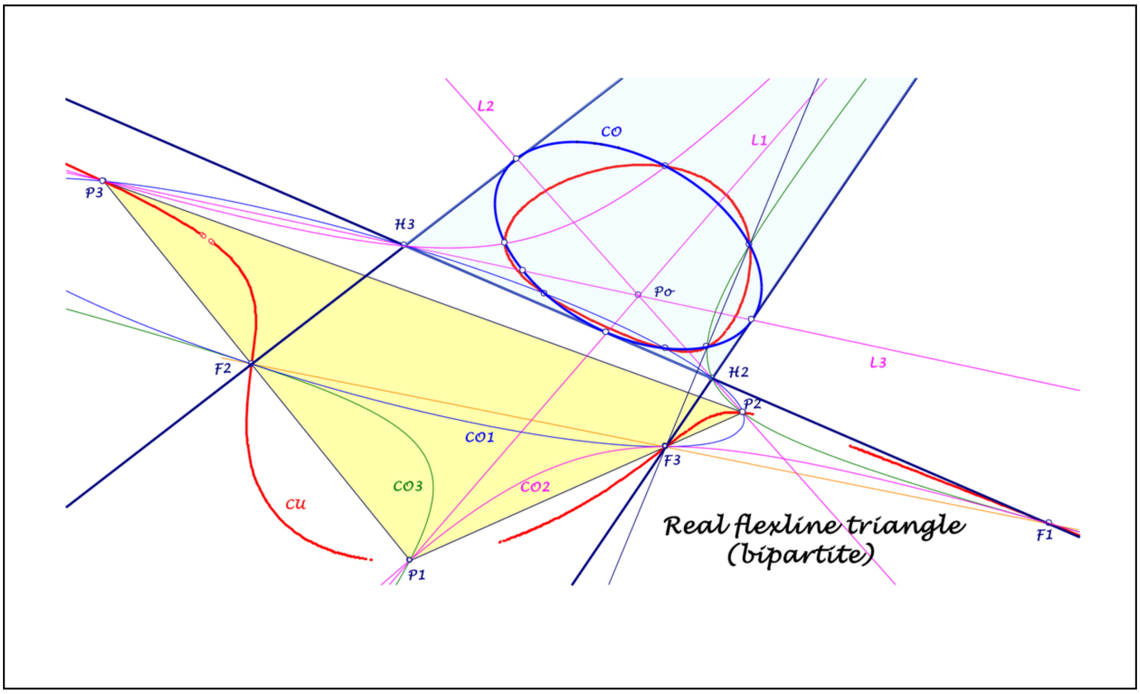
... which are coconic on a conic CO.

If CO is inscribed H1'H2'H3'

... with contact points in Lk ^ Hi'Hj',

... then H1'H2'H3' = H1H2H3.

Best regards Eckart



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Message: #2773
Date: 2025-08-31
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear M@IMF
Many thanks for your interest and congratulations for your
calculations.
This quintic derived from a cubic is very interesting !
In particular, if P0 = QA-P16 (pole = pivot) and Cu the
corresponding Cux (unfortunately not in EQF),
for P = QA-P16 and QA-Tf2(P) = QA-P10, you should find the
S-points as vertices of QL-Tr2 (where QL is the dual of QA)
belonging to your corresponding quintic.
I suppose it is also possible to find the quintic for a
well-known cubic such as Mac Cay cubic for example
(it is a pivotal isogonal cubic wrt a triangle ABC with pivot
the circumcenter, isopivot the orthocenter and fixed points the
in- and excenters).
Best regards
Bernard
```

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Message: #2774
Date: 2025-08-31
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer,

Thank you for the message.
Let me correct #2769:
"Locus of P0" must be replaced by "Locus of P".

As I wrote in #2739, I'm interested in QLx-Tr2 where QLx is
formed by tangents
to a circumconic of convex QA0 at its vertices.
I realized that the locus of QLx-P13 is QA0-Tf2 of Steiner
inellipse of QA0-Tr1.
But I can't draw the quartic, either.

By the way, wrt PS in #2743, do you mean like this?
The equations of the 2 polar conics (cf. (2),(3) in #2769) are
(V^2 - 1)X^2 + (1 - U^2)Y^2 + (U^2 - V^2) = 0
(V^2 - 1)U/X + (1 - U^2)V/Y + (U^2 - V^2) = 0,
from which we can get
(U^2 - V^2)X^4 + 2(V^2 - 1)UX^3 + (1 - U^2)(U^2 + V^2 + 1)X^2 +
2(U^2 - V^2)UX + (V^2 - 1)U^2 = 0.
Dividing by (X-U) gives
(U^2 - V^2)X^3 + (U^2 + V^2 - 2)UX^2 + (1 + V^2 - 2U^2)X + (1 -
V^2)U = 0,
and its solutions are
Xk = -A/3 - Dω^k - E/ω^k (k=1,2,3),
where
A = (U^2 + V^2 - 2)U/(U^2 - V^2)
D = {C'/2 + √[(C'/2)^2 + (B'/3)^2]}^(1/3)
E = (-B'/3)/D
B' = B - (A^2)/3
C' = C - BA/3 + 2(A/3)^3
B = (1 + V^2 - 2U^2)/(U^2 - V^2)
C = (1 - V^2)U/(U^2 - V^2)
ω = cube root of 1.
Then *S-points* are (pXk : qYk : r), where
Yk = V(U^2 - 1)/[(V^2 - 1)U/Xk + (U^2 - V^2)].

Best regards,
M@IMF
```

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Message: #2775
Date: 2025-09-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
I'm rather confused now !
Reading your message 2769 again, I understand :

* a certain pivotal isocubic wrt QA-Tr1 with pivot P0 and fixed points the vertices of QA Cu0 = pK(QA-P16, P0)
* P is a point on Cu0 and Cu(P) = pK(QA-P16,P)
* the S-points are the intersections other than P* = QA-Tf2(P) of the polar conics of P and P* wrt Cu(P)

Is this correct ?
Then you take as example QA-Cu2,3,4,5
But the pivots of these cubics (your points P) are not on the same cubic !
It seems you have to choose :

* you start with a QA, it's QA-Tr1 and QA-Tf2 and take P random in the plane (then Cu(P) is defined and gives 3 S-points)
* you start with a bipartite cubic and take P on the cubic, but P* is the tangential of P, giving 3 S-points for each point P on the cubic

(a bipartite cubic is a pivotal isocubic in each of it's points, but the QA-Tf2 are all different ...)
In the 1rst case, the locus of the S-points refers to a particular QA, in the 2nd case, it refers to a cubic.
Best regards
Bernard

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Message: #2776
Date: 2025-09-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

... my conclusion in message 2772 doesn't hold,

... excuse, but it is difficult for me,

... to find a drawing or calculation with significant aberrations.

Best regards Eckart

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Message: #2777
Date: 2025-09-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Sorry, please read The S-points are the intersections other than P of the polar conics of P and P* = QA-Tf2(P) wrt Cu(P) !
With your definition, each cubic Cu(P) has a twin cubic Cu(P*), the polar conic of P* wrt Cu(P) is the same as the polar conic of P wrt Cu(P*).
This conic carries 3 S-points for Cu(P) and 3 other for Cu(P*).
Best regards
Bernard

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Message: #2778
Date: 2025-09-01
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

I'm very sorry to confuse you. I have to correct the last part of #2769:
"when Cu(P) = QA-Cu2,3,4,5" must be replaced by "when Cu0 = QA-Cu2,3,4,5".

What I calculated is 1st case in #2775 (2nd case in #2738), but P is taken on a cubic (Cu0) not whole plane.
I think S-points of P wrt QA are the intersections (except P) of the polar conic of P wrt Cu(P) and
the polar conic of QA-Tf2(P) wrt Cu(P), where Cu(P) is pK(QA-P16, P) wrt QA-Tr1.

I checked the expression of (10) in #2769 as follows (in the case of Cu0 = QA-Cu2):
0. Given QA.
1. Let P0 = QA-P5, which gives U0 and V0.
2. Let P = QA-P1, which gives U and V.
3. Solve (2),(3) as shown #2774, which gives Xk and Yk.
4. Substitute U0,V0,Xk,Yk into the left side of (10).
If it is 0 within the numerical error, (10) is valid.
Similarily let P = QA-P5,P17,P20.

Best regards,
M@IMF

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Message: #2779
Date: 2025-09-02
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Thanks for this clarification !
In fact, your calculation refers to 1rst case 2775 or 2nd case 2738 with a given QA and variables Cu(P) with P on a certain cubic Cu0.
You take as Cu0 QA-Tf2 invariant cubics like QA-Cu2 and the like.
I suppose you could also choose as locus of P a line (then P* describes a conic circum QA-Tr1) or a conic or any other cubic ...
I agree that P cannot be choosen random in the plane.
The 2nd case 2775 or 1rst case 2738 remains open (this is the case used by Schröter in order to draw the hessian of a cubic).
Beautiful work even without possibility of drawing your curve !
Best regards
Bernard

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Message: #2780
Date: 2025-09-03
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
I made several drawings with P on P10P16 or P1P20 and the locus of the S-points looks in fact like a cubic ...
Perhaps you will be able to confirm that by calculation ?
Best regards
Bernard
PS With your definition, a point P is always associated with 3 S-points, for example P16 with the vertices of QL-Tr2
The locus of the S-points for P on any line or conic or curve through this point contains these 3 particular S-points

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Message: #2781
Date: 2025-09-04
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear Mr. Keizer,

When P lies on a line, the system of equations are
(1') AU + BV + C = 0
(2) (V^2 - 1)(X^2 - 1) - (U^2 - 1)(Y^2 - 1) = 0
(3) (V^2 - 1)(U/X - 1) - (U^2 - 1)(V/Y - 1) = 0.
where A,B,C are constants.
(5) in #2769 and (1') gives
(6') U=[BY(Y^2 - X^2) - C(X^2 - 1)]X/[AX(X^2 - 1) + BY(Y^2 - 1)]
(7') V=[AX(X^2 - Y^2) - C(Y^2 - 1)]Y/[AX(X^2 - 1) + BY(Y^2 - 1)]
Substituting (6'),(7') into (2) yields
(8') [BY(Y^2 - 1) - (BY + C)X^2 - AX]^2 - (X^2)(AX + BY + C)^2
= [AX(X^2 - 1) - (AX + C)Y^2 - BY]^2 - (Y^2)(AX + BY + C)^2.
Then we obtain
(9') AX + BY + C = 0,
which is the locus of P, and
(10') AX(X^2 - Y^2 - 1) + BY(Y^2 - X^2 - 1) - C(X^2 + Y^2 -
1)=0,
which is the locus of S-points.

In DT-coordinates, (10') becomes
A(x/p)[(x/p)^2 - (y/q)^2 - (z/r)^2] + cyclic permutaions = 0.
Although I can't check the validity of this equation yet, you
will do graphically.
For P10P16,
A/p = q^2 - r^2
B/q = r^2 - p^2
C/r = p^2 - q^2.
For P1P20(QA-L3),
A/p = (q^2 - r^2)(-p^2 + q^2 + r^2)
B/q = (r^2 - p^2)(-q^2 + r^2 + p^2)
C/r = (p^2 - q^2)(-r^2 + p^2 + q^2).

Best regards,
M@IMF
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Message: #2782
Date: 2025-09-04
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

```
Dear M@IMF ,
Many thanks for these calculations ...
Interesting perhaps : the QA-Tf2 of the S-points/vertices of
QL-Tr2 are on the line P10P16 !
Best regards
Bernard
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Message: #2783
Date: 2025-09-04
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

If I don't misunderstand, S-points of P wrt QA lie on QA-Tf2 of line PP*,
which is the circumconic of QA-Tr1 that bears P and P*(QA-Tf2 of P).
Yes, polar conic of P* wrt pK(QA-P16, P). (So do S'-points.)

Wrt 2nd case in #2775 (1st case in #2738):
Given a quadrangle QA0 and a point P0, and let
Cu0 = pK(QA0-P16, P0) wrt QA0-Tr1
QA0-P16 = (p0^2 : q0^2 : r0^2)
P0 = (u0:v0:w0)
P'(u:v:w) = a point on Cu0.
Note that
Cu0 = pK(QA'-P16, P') wrt QA'-Tr1
P'˜(the tangential of P' wrt Cu0) = QA'-Tf2(P'),
where QA' is formed by the pretangentials of P' wrt Cu0 (cf. #2742).

Then the system of equations are
Locus of P ' (= Cu0)
u0u[(r0v)^2 - (q0w)^2] + v0v[(p0w)^2 - (r0u)^2] + w0w[(q0u)^2 - (p0v)^2] = 0
Polar conic of P' wrt Cu0
[(r0^2)v0v - (q0^2)w0w]x^2 - 2(p0^2)(v0w - w0v)yz + cyclic permutations = 0
Polar conic of P'˜ wrt Cu0
[(r0^2)v0η - (q0^2)w0ζ]x^2 - 2(p0^2)(v0ζ - w0η)yz + cyclic permutations = 0,
where (ξ:η:ζ) is the DT0-coordinate of P'˜, which I can't obtain yet.
(Note that these polar conics are not circumscribed to QA0 nor QA0-Tr1.)
To solve these equations will be much harder than the previous case.

Best regards,
M@IMF

Message: #2784
Date: 2025-09-05
From: van10hoven@gmail.com
Subject: Update on the new Website

Dear friends,

As announced last year, I’ve been moving the encyclopedias to a new platform, since the old one could no longer keep up with current technology. At the same time, I’m merging the Encyclopedia of Quadri-Figures (EQF) , the Encyclopedia of Polygon Geometry , and recent work on cubics into a brand-new Encyclopedia of Poly Geometry (EPG). The term “poly” reflects both the versatility of n points and n lines, and the study of n -degree curves. Old links to the former encyclopedias should continue to work. The good news: all pages are now transferred into the new format, and the new cubic material has been added. That’s a milestone worth sharing!

This has been quite a big job, so thanks for your patience if I haven’t always been as active in QPG discussions.

I expect the new website to be ready and online in some more months.

Best,
Chris

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Message: #2785
Date: 2025-09-23
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear all,

Although someone might have already calculated,
here is *DT* -coordinates of the vertices of *QL-Tr2* :
((α + 2Re[(ω^k)β1])/l^2 : -(α + 2Re[(ω^k)β2])/m^2 : 3(l^2 - m^2)) (k=1,2,3),
where
α = (l^2 + m^2)n^2 - 2(lm)^2
β1 = [(δ^2 - 9εl^4) + i (3√3)(l^2)(l^2 - m^2)(l^2 - n^2)δ]^(1/3)
β2 = [(δ^2 - 9εm^4) + i (3√3)(m^2)(m^2 - l^2)(m^2 - n^2)δ]^(1/3)
δ^2 = [(mn)^2 + (nl)^2 + (lm)^2]^3 - 27(lmn)^4
ε = (mn)^4 + (nl)^4 + (lm)^4 - (l^2 + m^2 + n^2)(lmn)^2
ω = cube root of 1
i = imaginary unit
Re[z] = real part of z.

Correction in #2774 (
https://groups.io/g/Quadri-and-Poly-Geometry/message/2774) :
D = {C'/2 + √[(C'/2)^2 + (B'/3)^ 3]}^(1/3)

Best regards,
M@IMF

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Message: #2786
Date: 2025-10-13
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

our group seems dead, nevertheless, I think

... the following is a further method to find the real flexline trilateral:

The cubic attached is drawn as isocubic

... reference triangle P1P2P3,

... isoconjugation with fixpoint Po and root Po,

... bearing a given point

... and using a variable triangle H1H2H3 as in #2747.

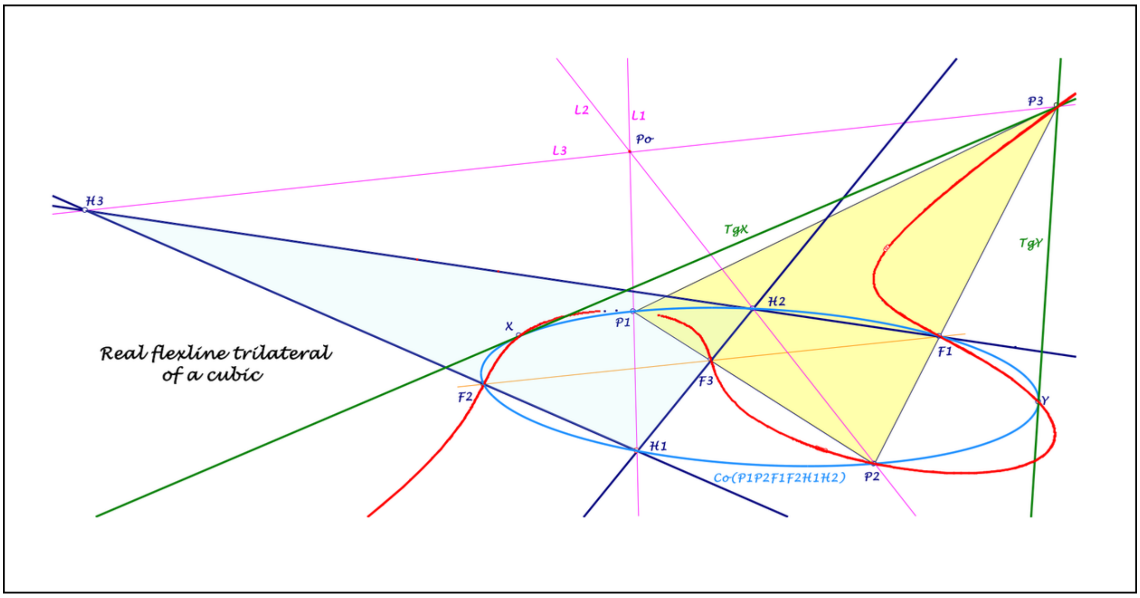
Start with a conic through PiPjFiFjHiHj,

... intersecting the cubic in two further points X and Y,

... if their conic-tangents TgX and TgY intersect in Pk

... you have the searched real flexline trilateral H1H2H3.

Best regards Eckart



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Message: #2787
Date: 2025-10-14
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard, dear Chris,

excuse, it seems that my last message #2786

... doesn't hold for bipartite cubics.

Best regards Eckart
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Message: #2788
Date: 2025-10-14
From: contiwa.goma3@gmail.com
Subject: 4 concurrent circles

```
Dear geometers,

Do I understand correctly?
Given 4 circles C1,C2,C3,C4 concurrent at P.
Let
Cij = the second intersection point of Ci and Cj
Dkl = Cij
Dl = circle through Dil,Djl,Dkl,
where {i,j,k,l} = {1,2,3,4}.
Then 4 circles D1,D2,D3,D4 intersect at a common point Q.
(Clifford's Circle Theorem)
Let
CSC1 = Moebius conjugate swapping Ci and Di (i=1,2,3,4)
F = the center of CSC1
CSCp = Moebius conjugate centered in P, swapping F and Q
CSCq = Moebius conjugate centered in Q, swapping F and P
QL1 = CSCp({C1,C2,C3,C4}) = CSCq({D1,D2,D3,D4}),
then CSC1 = QL1-Tf1 (F = QL1-P1).

When 3 lines C14D14, C24D24, C34D34 are concurrent at E,
P and Q lie on QL1-Qu3.
(P and CSCp(E), Q and CSCq(E) are CSCdiag ( https://groups.io/g/
Quadri-and-Poly-Geometry/topic/78474426#msg527 ) partners,
respectively.)

Best regards,
M@IMF
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Message: #2789
Date: 2025-10-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard, dear Chris,

I found also significant aberrations
... wrt the result in #2786 for monopartite cubics,
... please excuse and forget my overhasty message.

Best regards Eckart
```

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Message: #2790
Date: 2025-10-18
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Eckart,

So good to see you're still chasing improvements to the
Flextrilateral construction.

Hats off for sticking with it. Your determination and geometric
tenacity are impressive.

Keep going — the breakthrough feels close.

Best regards,

Chris
```

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Message: #2791
Date: 2025-10-19
From: van10hoven@gmail.com
Subject: Re: 4 concurrent circles

Dear M@IMF,

Thanks for your construction involving 4 circles converging at a single common point.

You say

CSC1 = Moebius conjugate swapping Ci and Di (i=1,2,3,4)

F = the center of CSC1

How do you determine this conjugate?

Best regards,

Chris

p.s. Your reference to Clifford's Circle Theorem has inspired me to explore some special applications in Poly Geometry. I'll address that in a separate thread.

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Message: #2792
Date: 2025-10-19
From: contiwa.goma3@gmail.com
Subject: Re: 4 concurrent circles

Dear Chris,

Thank you for the reply.
First, note that triangles CilCjlCkl and DilDjlDkl are circumcyclologic (https://groups.io/g/euclid/message/790) , i.e. their cyclologic centers are on the circumcircles of them:
P = cyclologic center of CilCjlCkl wrt DilDjlDkl
Q = cyclologic center of DilDjlDkl wrt CilCjlCkl,
where {i,j,k,l} = {1,2,3,4}. (There are 4 pairs.)

3 (circum)cyclologic triagnles and Moebius conjugates are already mentioned:
QFG#997 (https://groups.io/g/Quadri-Figures-Group/message/17437) (3 cyclologies of Diagonal Triangle, Miquel Triangle and Triple T) and previous/next
QFG#1989 (https://groups.io/g/Quadri-Figures-Group/topic/71470802#msg18432) (Generalized Cyclologic Center Chain in ...)
QFG#2000 (https://groups.io/g/Quadri-Figures-Group/message/18443) (QA-Tr-4).

By the way, I'm not sure the meaning of "this Cyclologic relation isn't always reciprocal"(QA-Tr-4 (https://www.chrisvantienhoven.nl/qa-items/qa-triangles/qa-tr-4) in EQF).

Best regards,
M@IMF

p.s. Other than nP-e-Tf1,2 and nP-o-Tf1,2,3,4?

Message: #2793
Date: 2025-10-19
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Bernard, dear Chris,

perhaps another method to find the real flexline trilateral:
The cubic attached is drawn as nonpivotal isocubic
... reference triangle P1P2P3,
... isoconjugation * with fixpoint Po and root Po,
... bearing a given point Q
... and using a variable triangle H1H2H3 as in #2747.

Consider the polar conic CO1 of a cubic point X,
... intersecting the  cubic in U, V (on the infinite part),
... and the conic CO2 through H1,H2,H3,U,V,
... if the polars of X* wrt CO1 and CO2 coincide,
... you get the real flexline trilateral H1H2H3,
... controlled for mono- and bipartite cubics
... in 4 examples without significant aberrations.

Best regards Eckart

PS: Thanks to Chris for his encouraging mail.
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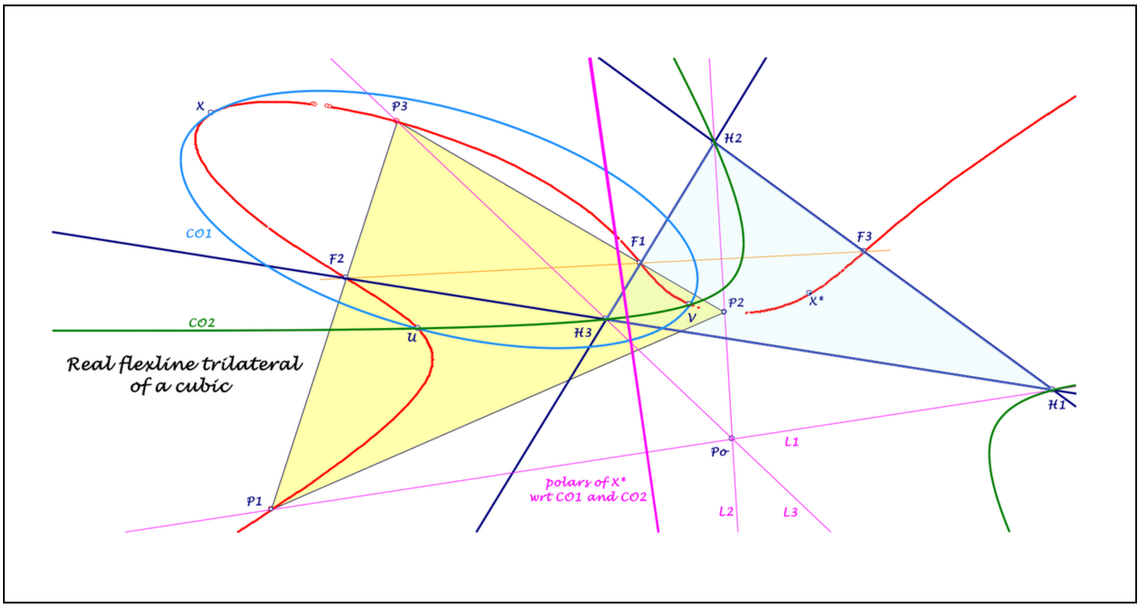

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Message: #2794
Date: 2025-10-20
From: contiwa.goma3@gmail.com
Subject: Re: 4 concurrent circles

Dear Chris and all,

It seems that I was overhasty. Let me explain once again.
(Circumcyclologic is OK. I should have dealt with 3 triangles carefully.)

I first considered inversion not Moebius conjugate.
Let
InvP = inversion wrt a circle with center P through Q
InvQ = inversion wrt a circle with center Q through P,
then
InvP({C1,C2,C3,C4}) = QL2 (quadrilateral with Miquel point Q)
InvP({D1,D2,D3,D4}) = circumcircles of component triangles of QL2
InvQ({D1,D2,D3,D4}) = QL3 (quadrilateral with Miquel point P)
InvQ({C1,C2,C3,C4}) = circumcircles of component triangles of QL3.
There are two QLs. But if we use CSCp and CSCq, we have QL1 only.
Moreover CSCp*CSCq (centered in F) swaps not only P and Q,
but also Cj and Dj (j=1,2,3,4). This is CSC1(= QL1-Tf1).

Let's define Ek4 = CSCp(Dk4) = CSCq(Ck4) (k=1,2,3).
When triangles C14C24C34 and D14D24D34 are QA-Triple Triangles of QL-P1 and QG-P5, respectively,
P = QA-P9, Q = QA-P3, F = QA-P4, CSC1 = QA-Tf4
CSC1 swaps QL-Ci3 and QG-Ci4
Ek4 is the vertice of QA-Triple Triangle of QG-Px (https://groups.io/g/Quadri-Figures-Group/attachment/18437/0/2016-10-16.pdf).
If we express this as <QG-Px> = {{QL-P1, QG-P5}}, then
<QG-Px> = {{QL-P1, QG-P5}} = {{QG-P5, QL-P1}}
<QL-P1> = {{QG-P5, QG-Px}} = {{QG-Px, QG-P5}}
<QG-P5> = {{QG-Px, QL-P1}} = {{QL-P1, QG-Px}}
hold. (Note that F is cyclologic center of E14E24E34 wrt C14C24C34 and D14D24D34.)
Maybe this is true for 3 triangles which are pairwise circumcyclologic and perspective (i.e. generalized cyclologic?).
By the way, QFG#1989 is not about circumcyclologic. Sorry.

An counterexample is QA-Triple Triangles of QL-P1, QG-P9, QG-Py.
(P = QA-P9, Q = QA-P1, QG-Py = CSCp(QG-P9) = CSCq(QL-P1).)
In this case, <QG-Py> = {{QL-P1, QG-P9}} = {{QG-P9, QL-P1}},
but {{QG-P9, QG-Py}} is not <QL-P1>. {{QL-P1, QG-Py}} is not <QG-P9>, either.

```
Best regards,
M@IMF

p.s. QA-Triple Triangle of QG-Py is similar to Trκ in #2709 ( QA
and Complex Quartic Eq. ( https://groups.io/g/Quadri-and-Poly-Ge⌋
ometry/topic/113684476#msg2711 ) )
I'm wondering what property F has.
```

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Message: #2795
Date: 2025-10-30
From: van10hoven@gmail.com
Subject: Re: 4 concurrent circles

```
Dear M@IMF,

Thank you for your explanation.

As promised, I took a closer look at Clifford’s Circle Theorem.

However, it didn’t yield the insights I was hoping for, so I’ve
decided to set it aside.

Best regards,

Chris
```

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Message: #2796
Date: 2025-10-30
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I've looked over your latest construction. I haven't checked it algebraically, as that would take quite a bit of time.

Your approach actually resembles earlier ones, working with a variable triangle H1H2H3 whose sides pass through F1, F2, F3.

It remains somewhat tricky to arrive at a precise construction.

Best regards,

Chris

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Message: #2797
Date: 2025-10-30
From: contiwa.goma3@gmail.com
Subject: Desmic System / Reye Configuration

Dear all,

I drew some stereograms of desmic systems. A desmic system (https://en.wikipedia.org/wiki/Desmic_system) is formed by 3 tetrahedra.
(Orthogonal projection of 3 tetrahedra onto a plane are 3 quadrangles.
Please see QA-Tr-1 (https://www.chrisvantienhoven.nl/qa-items/qa-triangles/qa-tr-1) of EQF.)
Let the vertices of the 3 tetrahedra be {X1,X2,X3,X4}, {Y1,Y2,Y3,Y4}, {Z1,Z2,Z3,Z4}, and
TrX4 = {X1,X2,X3}
TrY4 = {Y1,Y2,Y3}
TrZ4 = {Z1,Z2,Z3}.
When TrX4, TrY4, TrZ4 are pairwise perspective, the perspectors of TrY4 and TrZ4,
TrZ4 and TrX4, TrX4 and TrY4 are X4, Y4, Z4, respectively. (X4, Y4, Z4 are collinear.)
These 3 triangles have a common perspectrix, which is the intersection of 3 planes
determined by the 3 triangles.
There are such 16 perspectrices and they form another Reye configuration,
that is "transpose Reye" or "hidden Desmic" (QFG#2039 (https://groups.io/g/Quadri-Figures-Group/topic/71471025#msg18482)).
12 points of this configuration and the 3 tetrahedra form (24_3, 18_4) configuration.

Best regards,
M@IMF

p.s. 3 tetrahedra in my first figure look similar to 3 QAs of QFG#2025 (https://groups.io/g/Quadri-Figures-Group/message/18468) rather than QFG#2041.

Desmic System of 3 Tetrahedra

M@IMF

Let TetX = {X1,X2,X3,X4}, TetY = {Y1,Y2,Y3,Y4}, TetZ = {Z1,Z2,Z3,Z4}.
3 tetrahedra TetX, TetY, TetZ which satisfy below table form a desmic system.

Collinear points (*)			Pairwise perspective triangles			Perspectrix
X1,Y1,Z4	A1,A2,D3	P2,Q2,R1	X4X3X2,	Y4Y3Y2,	Z1Z2Z3	U4_V3_W3
X1,Y2,Z3	A1,B2,C3	P2,Q3,R4	X4X3X2,	Y3Y4Y1,	Z2Z1Z4	U1_V2_W3
X1,Y3,Z2	A1,C2,B3	P2,Q4,R3	X4X3X2,	Y2Y1Y4,	Z3Z4Z1	U1_V3_W2
X1,Y4,Z1	A1,D2,A3	P2,Q1,R2	X4X3X2,	Y1Y2Y3,	Z4Z3Z2	U4_V2_W2
X2,Y1,Z3	B1,A2,C3	P3,Q2,R4	X3X4X1,	Y4Y3Y2,	Z2Z1Z4	U2_V1_W3
X2,Y2,Z4	B1,B2,D3	P3,Q3,R1	X3X4X1,	Y3Y4Y1,	Z1Z2Z3	U3_V4_W3
X2,Y3,Z1	B1,C2,A3	P3,Q4,R2	X3X4X1,	Y2Y1Y4,	Z4Z3Z2	U3_V1_W2
X2,Y4,Z2	B1,D2,B3	P3,Q1,R3	X3X4X1,	Y1Y2Y3,	Z3Z4Z1	U2_V4_W2
X3,Y1,Z2	C1,A2,B3	P4,Q2,R3	X2X1X4,	Y4Y3Y2,	Z3Z4Z1	U2_V3_W1
X3,Y2,Z1	C1,B2,A3	P4,Q3,R2	X2X1X4,	Y3Y4Y1,	Z4Z3Z2	U3_V2_W1
X3,Y3,Z4	C1,C2,D3	P4,Q4,R1	X2X1X4,	Y2Y1Y4,	Z1Z2Z3	U3_V3_W4
X3,Y4,Z3	C1,D2,C3	P4,Q1,R4	X2X1X4,	Y1Y2Y3,	Z2Z1Z4	U2_V2_W4
X4,Y1,Z1	D1,A2,A3	P1,Q2,R2	X1X2X3,	Y4Y3Y2,	Z4Z3Z2	U4_V1_W1
X4,Y2,Z2	D1,B2,B3	P1,Q3,R3	X1X2X3,	Y3Y4Y1,	Z3Z4Z1	U1_V4_W1
X4,Y3,Z3	D1,C2,C3	P1,Q4,R4	X1X2X3,	Y2Y1Y4,	Z2Z1Z4	U1_V1_W4
X4,Y4,Z4	D1,D2,D3	P1,Q1,R1	X1X2X3,	Y1Y2Y3,	Z1Z2Z3	U4_V4_W4

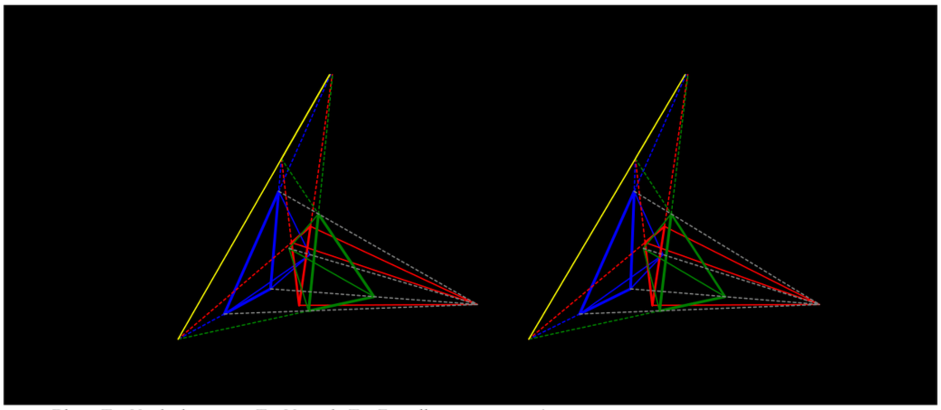
(*) Corresponding points in QA-Tr-1 of EQF are also shown.

Uj, Vj, Wj (j=1,2,3,4) are defined as follows:

U1 = X2X3∩Y1Y4∩Z1Z4, U2 = X1X4∩Y2Y3∩Z1Z4, U3 = X1X4∩Y1Y4∩Z2Z3, U4 = X2X3∩Y2Y3∩Z2Z3
V1 = X3X1∩Y2Y4∩Z2Z4, V2 = X2X4∩Y3Y1∩Z2Z4, V3 = X2X4∩Y2Y4∩Z3Z1, V4 = X3X1∩Y3Y1∩Z3Z1
W1 = X1X2∩Y3Y4∩Z3Z4, W2 = X3X4∩Y1Y2∩Z3Z4, W3 = X3X4∩Y3Y4∩Z1Z2, W4 = X1X2∩Y1Y2∩Z1Z2
(Conversely, X1 = U2U3∩V1V4∩W1W4, X2 = U1U4∩V2V3∩W1W4, ...)

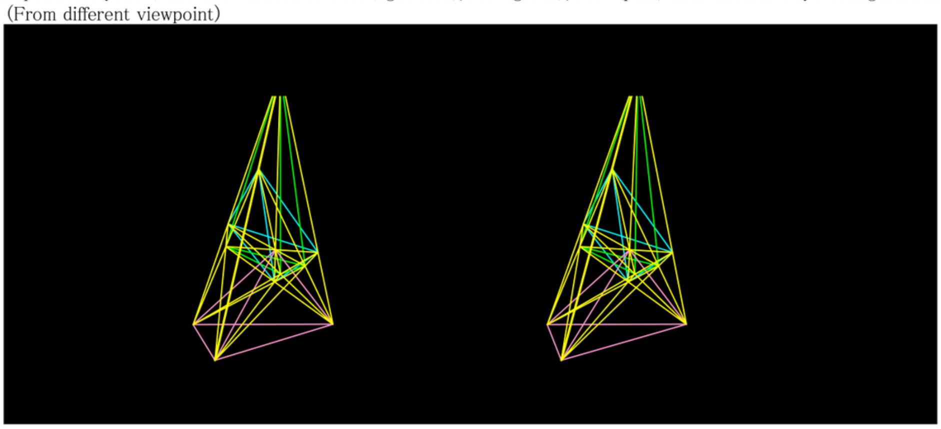
These points form 3 tetrahedra: TetU = {U1,U2,U3,U4}, TetV = {V1,V2,V3,V4}, TetW = {W1,W2,W3,W4}.

Let TrXl = {Xi,Xj,Xk}, TrYl = {Yi,Yj,Yk}, TrZl = {Zi,Zj,Zk}, where {i,j,k,l} = {1,2,3,4}.
When TrXa, TrYb, TrZc are pairwise perspective, the perspector of TrYb and TrZc,
TrZc and TrXa, TrXa and TrYb are Xa, Yb, Zc, respectively.

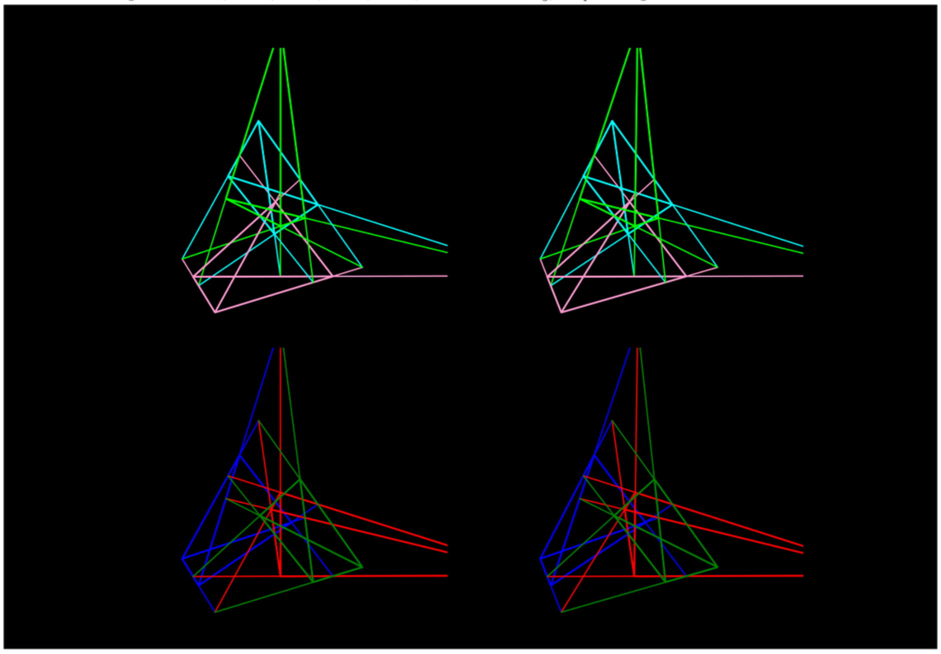


Blue: TetX, dark green: TetY, red: TetZ, yellow: perspectrix

16 perspectrices(yellow) and the vertices of TetU(light blue),TetV(green),TetW(pink) form another Reye configuration.
(From different viewpoint)



The vertices and edges of TetX,TetY,TetZ,TetU,TetV,TetW form $(24_3, 18_4)$ configuration.



The opposite edges of a tetrahedron don't intersect in 3D space,
while a quadrangle in 2D plane has its diagonal triangle.

X2X3∩X1X4 = U1U4∩U2U3	Y2Y3∩Y1Y4 = U2U4∩U3U1	Z2Z3∩Z1Z4 = U3U4∩U1U2	DTu
X3X1∩X2X4 = V1V4∩V2V3	Y3Y1∩Y2Y4 = V2V4∩V3V1	Z3Z1∩Z2Z4 = V3V4∩V1V2	DTv
X1X2∩X3X4 = W1W4∩W2W3	Y1Y2∩Y3Y4 = W2W4∩W3W1	Z1Z2∩Z3Z4 = W3W4∩W1W2	DTw
DTx	DTy	DTz	

Desmic System of 3 Tetrahedra.pdf

Message: #2798
Date: 2025-11-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

if you accept CABRI-intersections

... of a line and any curve as construction,

... I can finish my conjecture in #2793

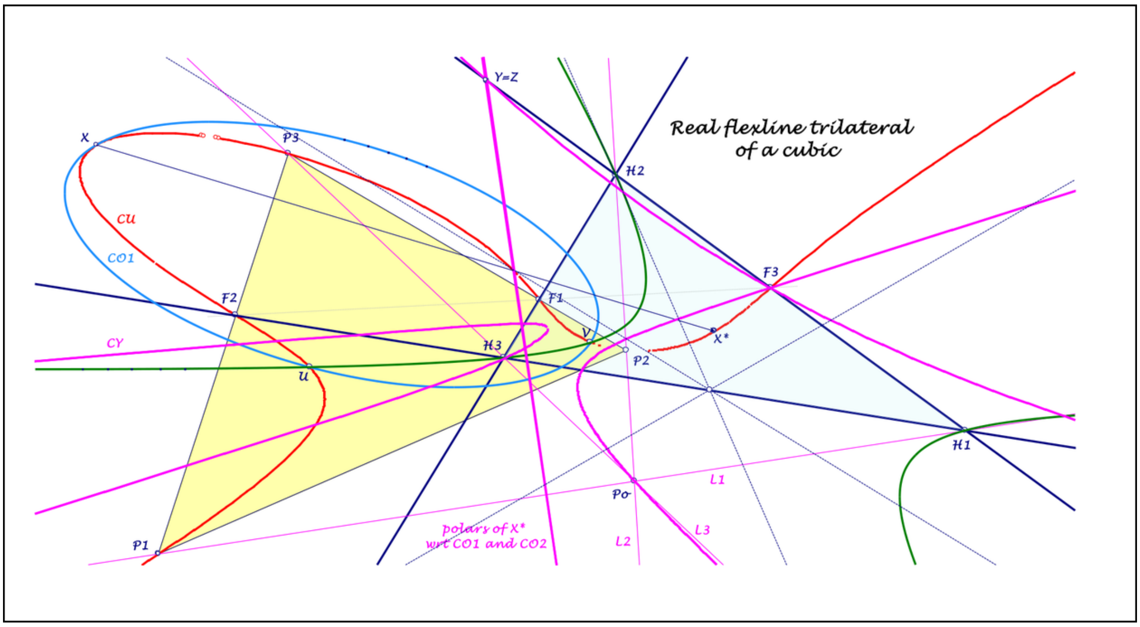
... to a construction of the real flexline trilateral:

Consider the polar conic CO1 of a cubic point X,
... intersecting the cubic in U, V (on the infinite part),
... and the conic CO2 through H1,H2,H3,U,V,
... if the polars of X* wrt CO1 and CO2 coincide,
... you get the real flexline trilateral H1H2H3.

Let the polar of X* wrt CO2 intersect HiHj in Y,
... consider the locus of Y varying triangle H1H2H3,
... that is a curve CY of higher unknown degree (5?),
... take the intersection Z of CY and the polar of X* wrt CO1,
... then ZFk = HiHj is a sideline of the real flexline
trilateral ...

Best regards Eckart

PS: Attached HiHj = H1H2,
... further attached the "construction" of #2747 for H2.



2025-11-01.pdf

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Message: #2799
Date: 2025-11-03
From: bernard.keizer@gmail.com
Subject: Re: Desmic System / Reye Configuration

```
Dear M@IMF ,
This Desmic/Reye configuration is well known, but I'm surprised
that you don't mention the main property, that the 12 points Xi,
Yi and Zi for i=1 to 4 belong to the same cubic ...
Best regards
Bernard
```

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Message: #2800
Date: 2025-11-03
From: contiwa.goma3@gmail.com
Subject: Re: Desmic System / Reye Configuration

```
Dear Mr. Keizer,

Thank you for the message.
Of course, there are many interesting properties about cubics.
But I think "hidden Desmic/transpose Reye" is worthwhile to show
without them.
Someone who become interested in the topic will visit QFG and
know its fascination.
By the way, I didn't realize that triangles in a desmic system
are DTs of QLs formed by their perspectrices.

Best regards,
M@IMF
```

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Message: #2801
Date: 2025-11-06
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris,

now I am quite surprised,

... is the following property correct, evident or anywhere
mentioned?

The vertices H1,H2,H3 of the real flexline trilateral

... are contact points of tangents

... from the flexpoints F1,F2,F3 to the hessian of the cubic,

... (for a bipartite hessian take the infinite part).

Best regards Eckart
```

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Message: #2802
Date: 2025-11-07
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris,

excuse, excuse excuse, the property in #2801 is nonsense,

... I confused cubics in my drawing,

... the hessian doesn't bear its points Hi,

... I hope to escribe the right cubic.

Best regards Eckart
```

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Message: #2803
Date: 2025-11-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I studied your construction in #2798.

At first, you repeat your approximation construction from #2747, valid for both monopartite cubics and bipartite cubics.

Then you introduce a new point X* (what are its specifications? I suppose it lies on CU– is it chosen randomly or is it a conjugate?). This yields a curve of higher degree, which is then intersected with the polar of X* with respect to C01. The intersection point lies on a sideline of the real flexline trilateral.

It is a bit difficult for me to understand thoroughly. Nevertheless, the intersection of a higher-degree locus curve with a line is not easy to construct. But –when correct– it represents a step forward toward finding for example a construction as the intersection of a line with a conic.

Best regards,

Chris

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Message: #2804
Date: 2025-11-12
From: eckart_schmidt@t-online.de
Subject: Partner-cubics

Dear Bernard, dear Chris,

perhaps of interest:

Let us start with a non degenerated reference cubic CU

... as isocubic of a triangle P1P2P3 with root Po

... and isoconjugation with fixpoint also Po,

and consider further cubics for its points

... F1,F2,F3 real flexpoints

... X1,X2,X3 intersections of the real flextangents,

... T1,T2,T3 their intersections

... ... with the harmnonic polars of the flexpoints F1,F2,F3,

... H1,H2,H3 vertices of the real flexline trilateral.

A cubic CU1 through F1,F2,F3,T1,T2,T3,H1,H2,H3

... is invariant for the isoconjugation * with ref-triangle H1H2H3

... and fixpoints Po plus FiHi^FjHj,

and

... is invariant for the isoconjugation ^ with ref-triangle T1T2T3

... and fixpoints Po plus X1,X2,X3.

A cubic CU2 through P1,P2,P3,F1,F2,F3,H1,H2,H3

... is invariant for the first isoconjugation *.

A cubic CU3 through P1,P2,P3,F1,F2,F3,T1,T2,T3

... is invariant for the second isoconjugation ^.

Finally common tangents:

In Pi contact CU,CU2,CU3 with common tangent FiPi,

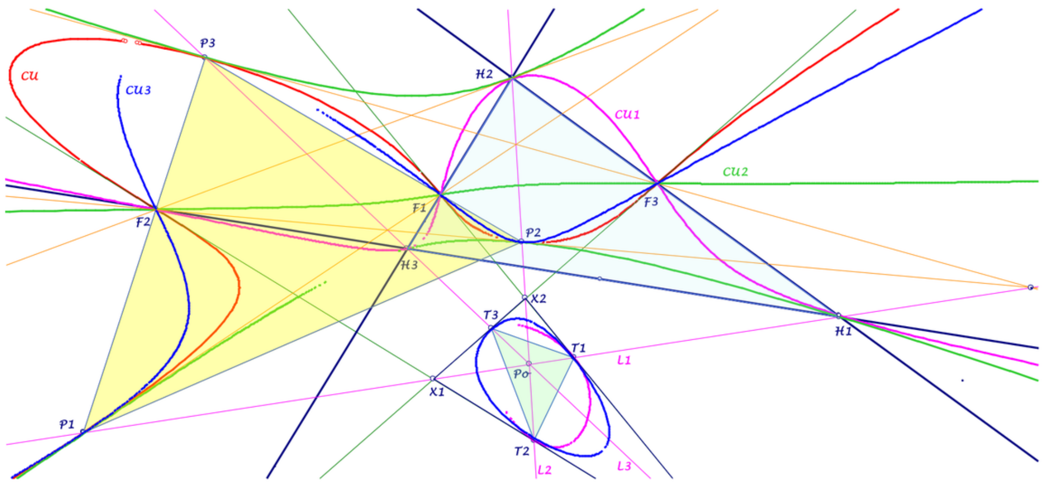
in Ti contact CU1 and CU3 with common tangent FiTi,

in Hi contact CU1 and CU2 with common tangent FiHi.

Best regards Eckart

PS: Excuse the breaks in the attached figure,

... they exist not in the CABRI drawing.



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Message: #2805
Date: 2025-11-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

```
Dear Chris,

thanks for your message #2803,

you are right, excuse, that I indirect repeat my approximation
from #2747,

but in #2798 there is a reference to #2793,

where the isoconjugation * is defined

wrt triangle P1P2P3 and fixpoint Po.

In face of all my fake constructions,

I found a lot of properties and relations

not really relevant for the real flexline trilateral ...

Best regards Eckart
```

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Message: #2806
Date: 2025-11-13
From: van10hoven@gmail.com
Subject: Re: Partner-cubics

Dear Eckart,

Interesting configuration you sketched in #2804.

It appears that the cubics involved are all members of Hesse’s pencil.

Particularly noteworthy are the pairwise isoconjugations, which constitute the most distinctive aspect of the construction.

Best regards,

Chris

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Message: #2807
Date: 2025-11-13
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Regarding your remarks in message #2805:

1. “You are right, excuse me, that I indirectly repeated my approximation from #2747.”

I actually appreciated that you repeated it. For you it is engraved in memory, but for me it is not.

2. ‘The isoconjugation * is defined with respect to triangle P1P2P3 and fixpoint Po.”

The same applies here: you mentioned it earlier, but whenever I dive back into the subject I still need this definition.

3. “In face of all my fake constructions, I found a lot of properties and relations not really relevant for the real flexline trilateral ….”

Absolutely true, your provisional constructions have yielded several beautiful new features. My compliments. I very much enjoy them!

Best regards,

Chris

Message: #2808
Date: 2025-11-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart
I'm also full of admiration for Eckart's tenacity !
I tried myself several ways of finding a geometrical construction, but in vain.
My remarks, which are a repetition of earlier messages :
1) a bipartite cubic is a pivotal isocubic QA-Cux with pivot in any of it's points, fixed points of the isoconjugation the 4 points having the pivot as common tangential and vertices of the reference triangle the 3 points having the same tangential as the pivot (this point is the isopivot
2) analytically, the equation of a cubic QA-Cux is in EQF and it is possible to calculate the equation of it's hessian, to solve the system and to find the flexes (3 real and 6 imaginary) and the flexlines (4 real and 8 imaginary), as well as P0 and the harmonic polars. (These calculations would naturally improve EQF greatly !)
3) geometrically, it is possible to construct the pivotal isocubic with pivot P through the QA vertices, the DT vertices, the pivot P and the isopivot P' and the feet of the cevians of P wrt DT, which are u1, u2 and u3.
We have then 3 polar conics of P, P' and P" wrt this cubic.
Let's consider the polar conic of P' (through P and the DT vertices) ; the intersections with the 2 other conics (one through P and the QA vertices and the other through P', P'' and u1, u2 and u3) form 2 qa's (P,S1,S2 and S3 and P',S'1,S'2 and S'3).
Let's consider now the dt's of the 2 qa's ; their vertices belong to the hessian, as well as the intersections of their sides with the lines PP' and P'P''.
This gives 12 points, which is more than enough in order to determinate the hessian.
4) the intersection of the cubic and the hessian gives without difficulty the 3 real flexes, the point P0 and the 3 harmonic polars.
5) Using Eckart's property, we may start with a point H1 on L1, giving H2 on L2 and H3 on L3.
H1 is in the right place when the 3 points H1, H2 and H3 are coconic with the vertices of the QA (having P as tangential), the points P and the vertices of DT (having P' as tangential) and the points P', u1, u2 and u3 (having P'' as tangential).
Considering H'1, H'2 and H'3 the QA-Tf1 of H1, H2 and H3, they must lye on a line through P', which is the QA-Tf1 of the conic through P and the vertices of DT.

```
It is remarkable that H'1 lies on H2H3, H'2 on H1H3 and H'3 on
H1H2. (By the way, QA-Tf1(P0) lies on the line of real flexes).
I hope Eckart will agree with these observations ...
Best regards
Bernard
PS Dear Eckart, naturally our forum is dead since a long time,
but apparently, the 3 of us are still alive and that's the only
thing which matters ...
```

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Message: #2809
Date: 2025-11-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris and Eckart,
Please read QA-Tf2 instead of QA-Tf1
Best regards
Bernard
```

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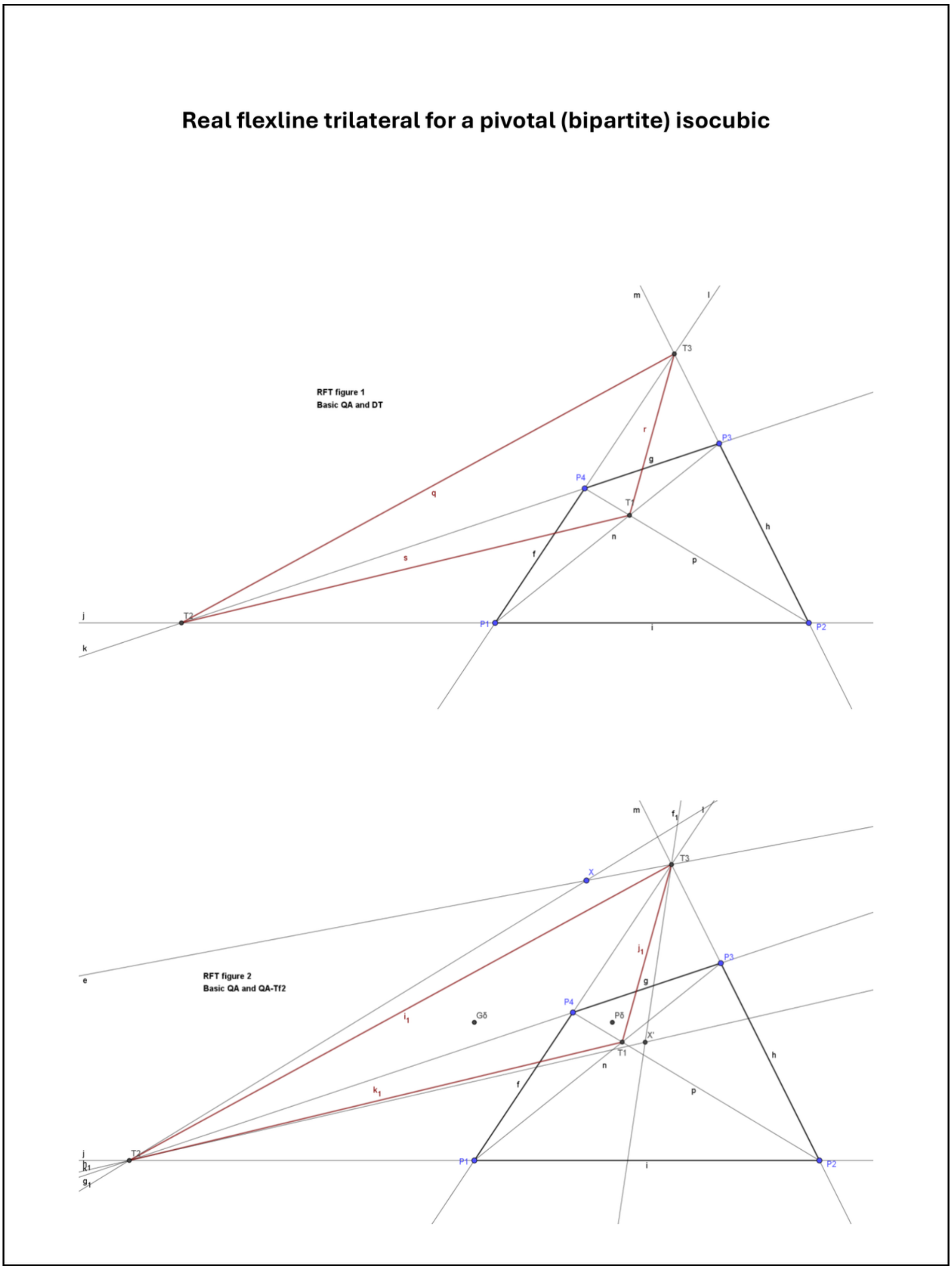
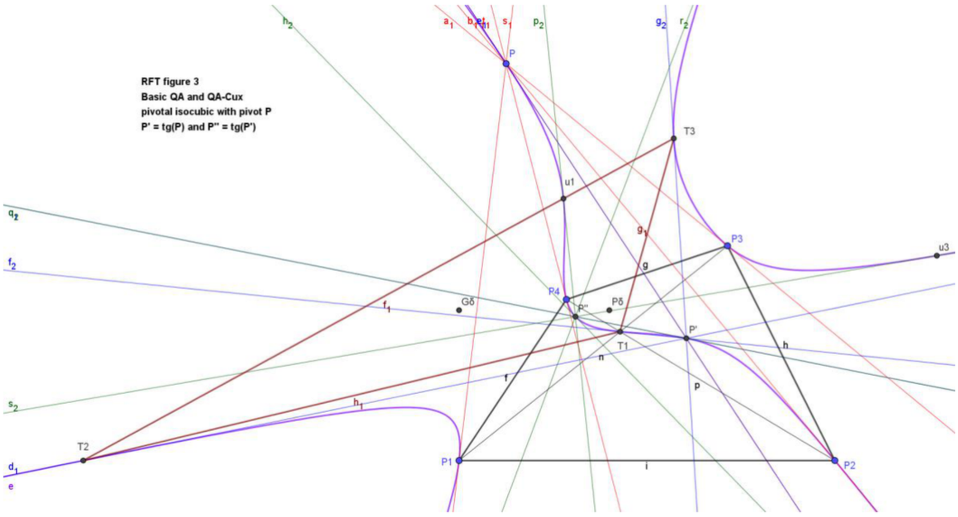
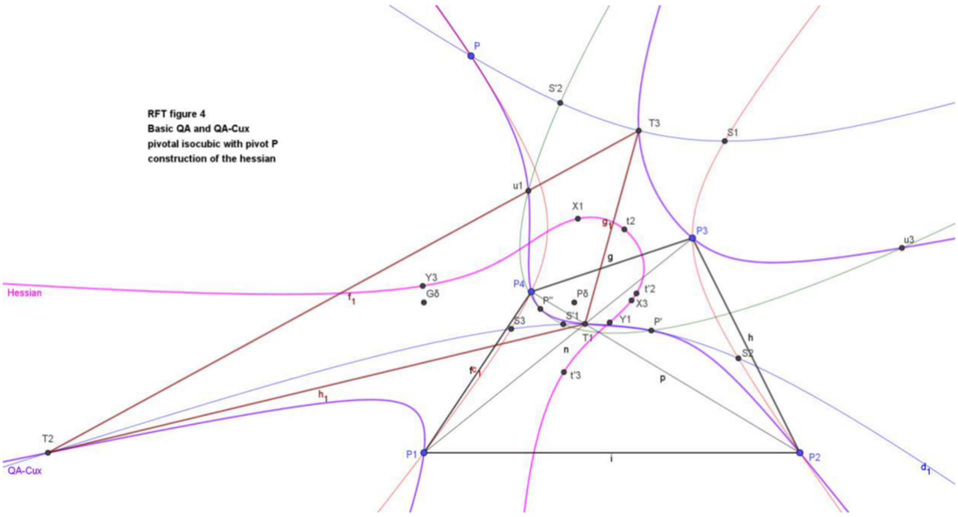
Message: #2810
Date: 2025-11-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris and Eckart
Here an illustration of my last message
I hope everything is clear (Gδ is QA-P10 and Pδ is QA-P16)
Best regards
Bernard
```

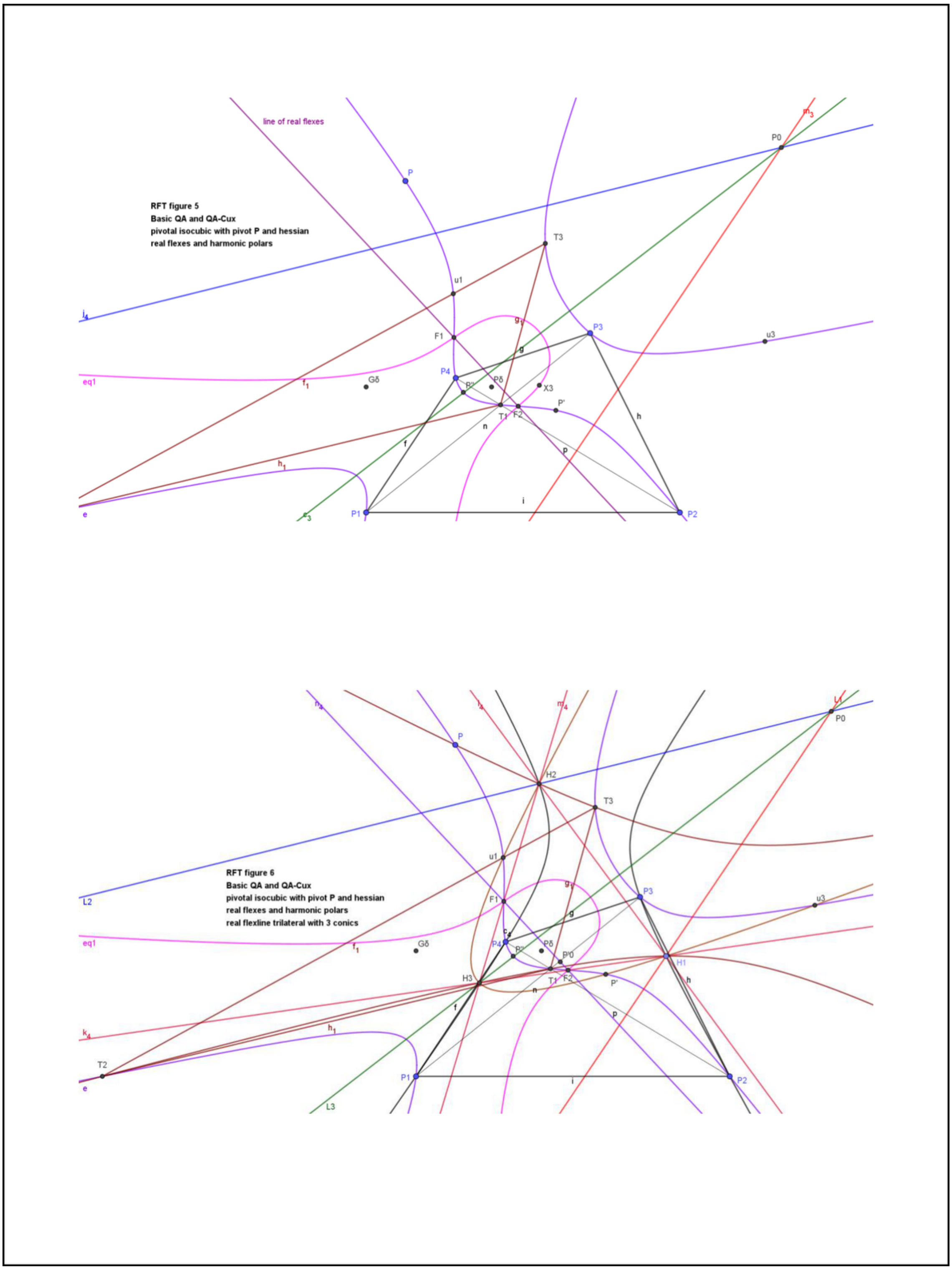


RFT for a pivotal (bipartite) isocubic.pdf

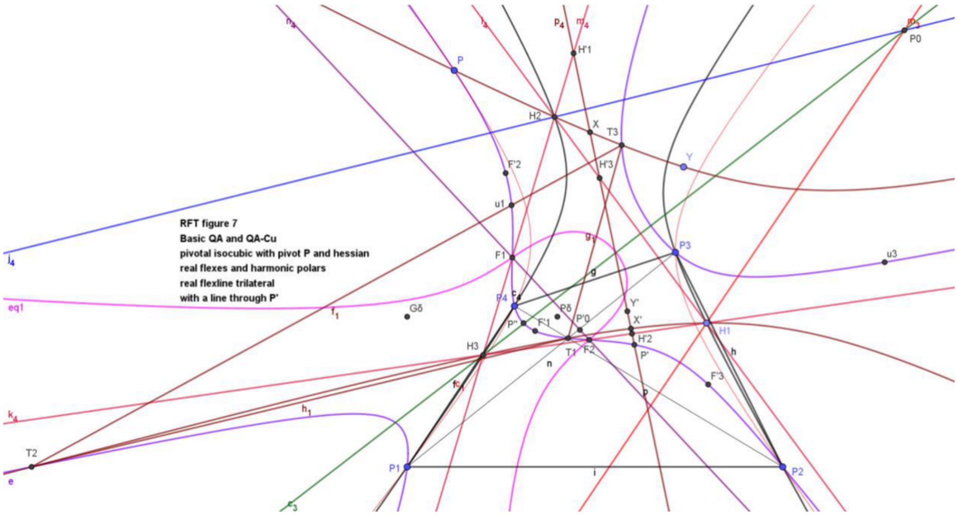




RFT for a pivotal (bipartite) isocubic.pdf



RFT for a pivotal (bipartite) isocubic.pdf



RFT for a pivotal (bipartite) isocubic.pdf

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Message: #2811
Date: 2025-11-17
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your extensive messages.

You restrict your observations

to bipartite isocubics, which are pivotal, as you describe.

My observations based on nonpivotal isocubics,

including mono- and bipartite isocubics.

I hope my following sight is correct:

For an isocubic a reference triangle and an isoconjugation must be given.

Isocubics can be mono- or bipartite.

Bipartite cubics are pivotal,

but can also be described as nonpivotal isocubics

with a root Po, which is the tripol for the line F1F2F3 of real flexpoints,

which are the intersections of reference cubic

and sidelines of the reference triangle.

So any isocubic can be drawn with a reference triangle P1P2P3,

a point Po as root and also fixpoint for an isoconjugation

with a starting cubic-point Q.

If I am not wrong,

you construct the hessian in 3) (extensive),

to get F1,F2,F3,Po and L1,L2,L3 in 4),

which are the starting elements in my sight.

Nevertheless your observation in 5) is interesting and new for me:

For a pivotal isocubic the 4 tangentials of a cubic-point P

are coconic with the vertices of the real flexline trilateral.

Best regards Eckart

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Message: #2812
Date: 2025-11-18
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Thanks for interest and recation !
In the last point, it's the 4 pretangentials (having the same tangential) of a point P of a bipartite cubic.
Best regards
Bernard

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Message: #2813
Date: 2025-11-19
From: bernard.keizer@gmail.com
Subject: Congratulations

```
Dear Chris,
I've just visited the site of EQF and have been redirected on
your new site !
I'm amazed, it's a fantastic work !!!
Congratulations
Naturally, I will visit it in details, but give me time to
discover all the jewels ...
I've started with cubics and Hesse pencil ...
Very beautiful indeed
Best regards
Bernard
```

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Message: #2814
Date: 2025-11-20
From: bernard.keizer@gmail.com
Subject: Re: Congratulations

```
Dear Chris,
I can't really find the appropriate words.
It's really astounding or amazing !
My 1rst contribution : on the page Quadrangle items, replace
Quadrilateral by Quadrangle (General Information as well as
Centers).
Best regards
Bernard
```

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Message: #2815
Date: 2025-11-20
From: contiwa.goma3@gmail.com
Subject: Re: Congratulations

```
Dear Chris,

Congratulations on renewal of your website.
I admire your outstanding ability to execute great works.
I've just exploired QFG to learn about cubics yesterday.
Now I can visit the well-organized new website.
It will be very helpful.

Best regards,
M@IMF
```

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Message: #2816
Date: 2025-11-20
From: van10hoven@gmail.com
Subject: Re: Congratulations

Dear Bernard and M@IMF,

Thank you very much for your kind compliments.

It feels like a reward for all the effort I put in.

I have been working very hard on my new site (
https://www.chrisvantienhoven.nl/) for almost a year,
implementing numerous improvements.

That's why I've been so quiet on the forum lately.

Bernard, I also added your related papers at the EPG treasury (
https://www.chrisvantienhoven.nl/epg/explore-epg/treasury/).

I promptly adapted the Quadrangle feature as well.

I will make an official announcement of the renewed site in
another post.

Best regards,

Chris

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Message: #2817
Date: 2025-11-20
From: eckart_schmidt@t-online.de
Subject: Re: Congratulations

Dear Chris,

WOW, congratulations!

I have to study your admirable work,

especially "cubics", my current interest.

What a development ,

remembering your e-mail of 02.09.2011,

where you announced your first quadrilateral observations,

later asking for a review of your results,

what now would be absolute impossible for me,

my geometrical horizon goes down age-related.

Best regards and Good Luck

Eckart

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Message: #2818
Date: 2025-11-20
From: van10hoven@gmail.com
Subject: Re: Update on the new Website

□ *A New Chapter: My Website Has Launched*

I'm pleased to announce the launch of my brand-new website — a major step forward in both design and content.

What's New?

* *A Modern Foundation*
The outdated Joomla-based engine has been replaced with a fresh, streamlined WordPress platform, offering improved performance and a more intuitive user experience.
* *A Unified Encyclopedia*
The two encyclopedias previously developed with the Quadri-Figures Group (QFG) and the Quadri- and Poly Geometry Group (QPG) — the Encyclopedia of Quadri-Figures and the Encyclopedia of Polygon Geometry — have now been merged into a single, comprehensive resource: the *Encyclopedia of Poly Geometry (EPG)*.
Poly Geometry is envisioned as a new branch of geometry, shaped by:

* Configurations of n points and/or n lines
* Curves of degree n
The name "Poly" reflects this broader, more inclusive structure.

* *New Focus on Cubics*
A substantial new section has been added, dedicated to *cubic curves* — not necessarily tied to triangles or quadrilaterals, but studied as third-degree curves in their own right. These insights, developed over the past two years within the QPG Forum by Eckart Schmidt, Bernard Keizer, and myself, have led to several surprising discoveries.
* *Expanded Downloads Section*
The site now includes a significantly enriched *downloads area*, featuring:

* Macros for Cabri II Plus
* Notebooks for Mathematica
* The complete EPG encyclopedia, available in downloadable PDF segments

* *Forum Access & Journals*
Direct access is now available to the QFG and QPG forums.

Even more exciting: *Journals* will soon be released — annual PDF compilations of forum discussions and illustrations, making it easier to preserve and explore the collective knowledge.
* *EPG Treasury*

A dedicated section has been established for papers related to Poly Geometry. Scholars and researchers who have authored work in this field are invited to submit their papers, thereby contributing to a collective treasury of knowledge that will remain accessible for as long as the site is maintained. Please note, however, that I cannot guarantee preservation in perpetuity.

Most components of the new website are complete now, though a few elements are still pending. I aim to finalize everything as soon as possible.

If you have any questions, comments, or suggestions, I'd love to hear from you.

Best regards,
Chris

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Message: #2819
Date: 2025-11-21
From: bernard.keizer@gmail.com
Subject: Cayleyan as member of a dual Hesse pencil

Dear Chris,
I've modified the message 2539 sent a year ago about the 2 dual Hesse pencils.
You will find the barycentric equation of the cayleyan
It's easy to identify your coefficients u, v and w, which depend only of k''
I hope this could help you to complete the item CU-5
Best regards
Bernard

2 dual Hesse pencils, 4 curves and calculations

1. 2 dual Hesse pencils wrt a triangle H1H2H3

The 1rst pencil contains the cubics having the same 9 flexes (3 real and 6 imaginary). The real flexes are F1, F2 and F3

The 2^{nd} pencil contains the curves of the 3rd order tangent to the 9 harmonic polars (3 real) The real harmonic polars are L1, L2 and L3 ; they intersect in P0

2. 4 curves

2 curves belong to the 1rst pencil, the cubic CU and it's hessian HE

2 curves belong to the 2^{nd} pencil, the cayleyan CA and a curve Γ associated to the cubic CU

CA is the hessian of Γ and HE is it's cayleyan (in other words, CU and Γ swap their hessian and cayleyan)

Both pencils have the same structure, with a Core curve and a residual curve

3. Calculations

The barycentric coordinates of P0 are 1/a1, 1/a2 and 1/a3 and the barycentric equation of the line of the 3 real flexes is a1x + a2y + a3z = 0.

All the curves of both pencils have an equation of the form $x^3 + y^3 + z^3 + kxyz = 0$, considering it as a barycentric equation for the 1rst pencil and replacing x, y and z by a1x, a2y and a3z and as a tangential equation for the 2^{nd} pencil and replacing x, y and z by u/a1, v/a2 and w/a3.

In particular, if the cubic CU has the constant k, then HE has the constant $k' = -(6/k)^2 - k/3$, CA has the constant $k'' = 6/k - (k/3)^2$ and Γ has the constant $k^* = -18/k$.

It's obvious that $k'(k^*) = k''(k)$ and $k''(k^*) = k'(k)$, which explains the swapping of HE and CA for the curves CU and Γ.

It's possible to find the tangential equation of a curve defined by a barycentric equation and vice-versa, the calculation being exactly the same. We find equations of the 6th degree[1].

In particular, the barycentric equation of CA is

$$27\,(a1^3x^3 + a2^3y^3 + a3^3z^3)^2 - 4\,(27 + k''^3)\,(a1^3a2^3\,x^3y^3 + a1^3a3^3\,x^3z^3 + a2^3\,a3^3\,y^3z^3)$$

$$- 18k''^2\,xyz\,(a1^3x^3 + a2^3y^3 + a3^3z^3) - k''(108 + k''^3)\,a1^2a2^2a3^2x^2y^2z^2 = 0$$ and the same for Γ

replacing k'' by k*.

2 dual Hesse pencils.pdf

The cayleyan of the Core cubic is the Residual curve of the 2nd pencil (k = 0 and k* = ∞) and the cayleyan of the residual cubic is the Core curve of the 2nd pencil (k = ∞ and k* = 0).

The barycentric equation of this Core curve is

$(a1^3x^3 + a2^3y^3 + a3^3z^3)^2 - 4\,(a1^3a2^3\,x^3y^3 + a1^3a3^3\,x^3z^3 + a2^3\,a3^3\,y^3z^3) = 0$

The barycentric equation of this Residual curve is $x^2y^2z^2 = 0$ and it's tangential curve uvw = 0

It's remarkable that this Residual curve is 2 times the triangle forming the residual cubic !

4. Properties

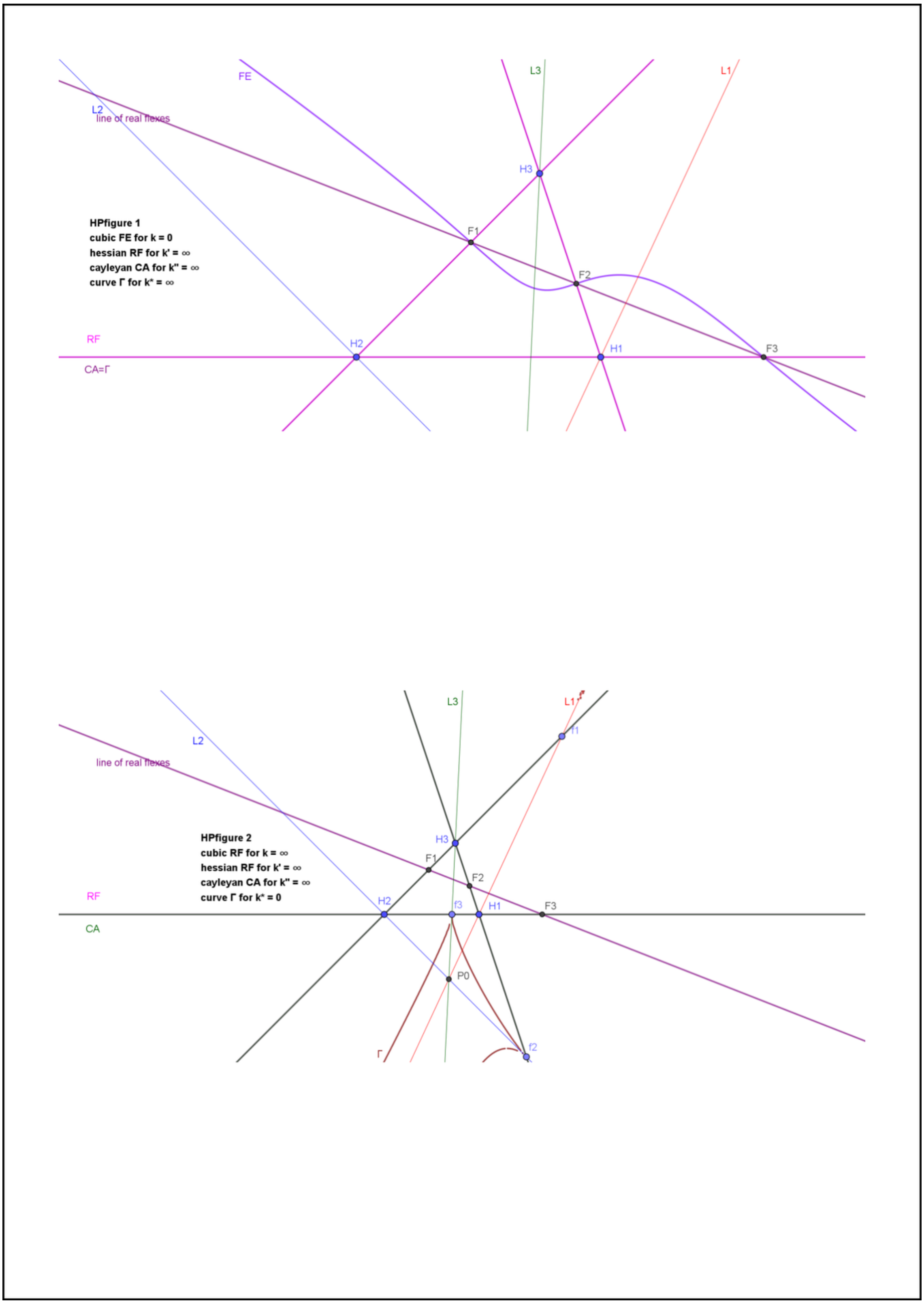
Drawing this way a certain number of curves, it appears that the cusps of the curves Γ are the contact points of HE and CA, which seems to be the dual property of the common tangents to CU and HE being the tangents in the intersections between CU and CA …

1 The calculations are long and boring !

Considering the tangential equation G(u,v,w) of the curve and the tangent ux + vy + wz = 0, we know that G'u/x = G'v/y = G'w/z and must eliminate u, v and w in order to find the barycentric equation F(x,y,z) of the curve.

For example, for $G(u,v,w) = u^3 + v^3 + w^3 = 0$ (k'' = 0), it's immediate that $u^2/x = v^2/y = w^2/z$ and $x^{3/2} + y^{3/2} + z^{3/2} = 0$. Then $z^{3/2} = -\,(x^{3/2} + y^{3/2})$, $z^3 = (x^{3/2} + y^{3/2})^2 = x^3 + y^3 + 2\,u^{3/2}v^{3/2}$ and last we find $(2\,x^{3/2}y^{3/2})^2 = (z^3 - x^3 - y^3)^2$, $x^6 + y^6 + z^6 = 2(x^3y^3 + x^3z^3 + y^3z^3)$ or $(x^3 + y^3 + z^3)^2 = 4(x^3y^3 + x^3z^3 + y^3z^3)$.

2 dual Hesse pencils.pdf



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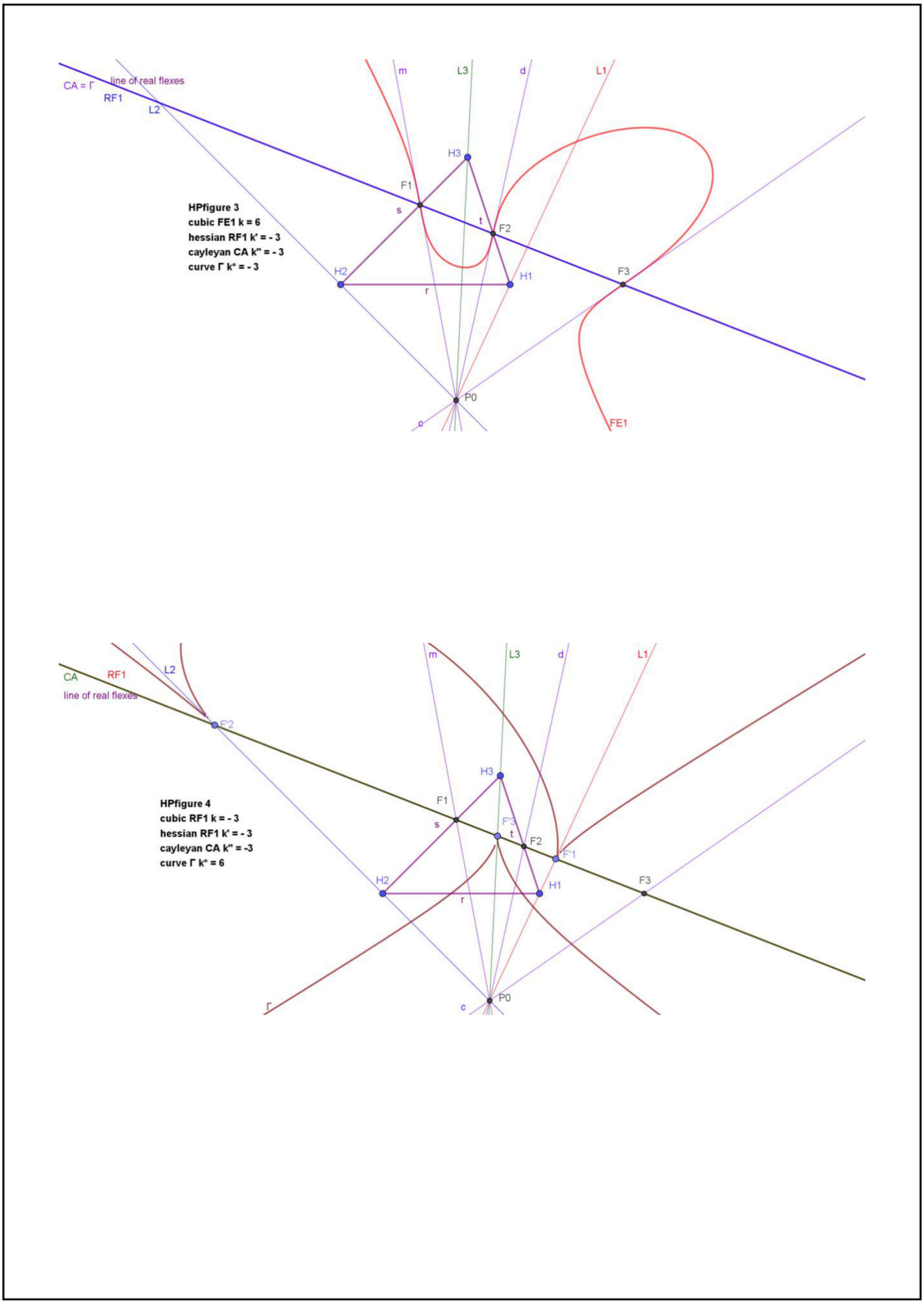

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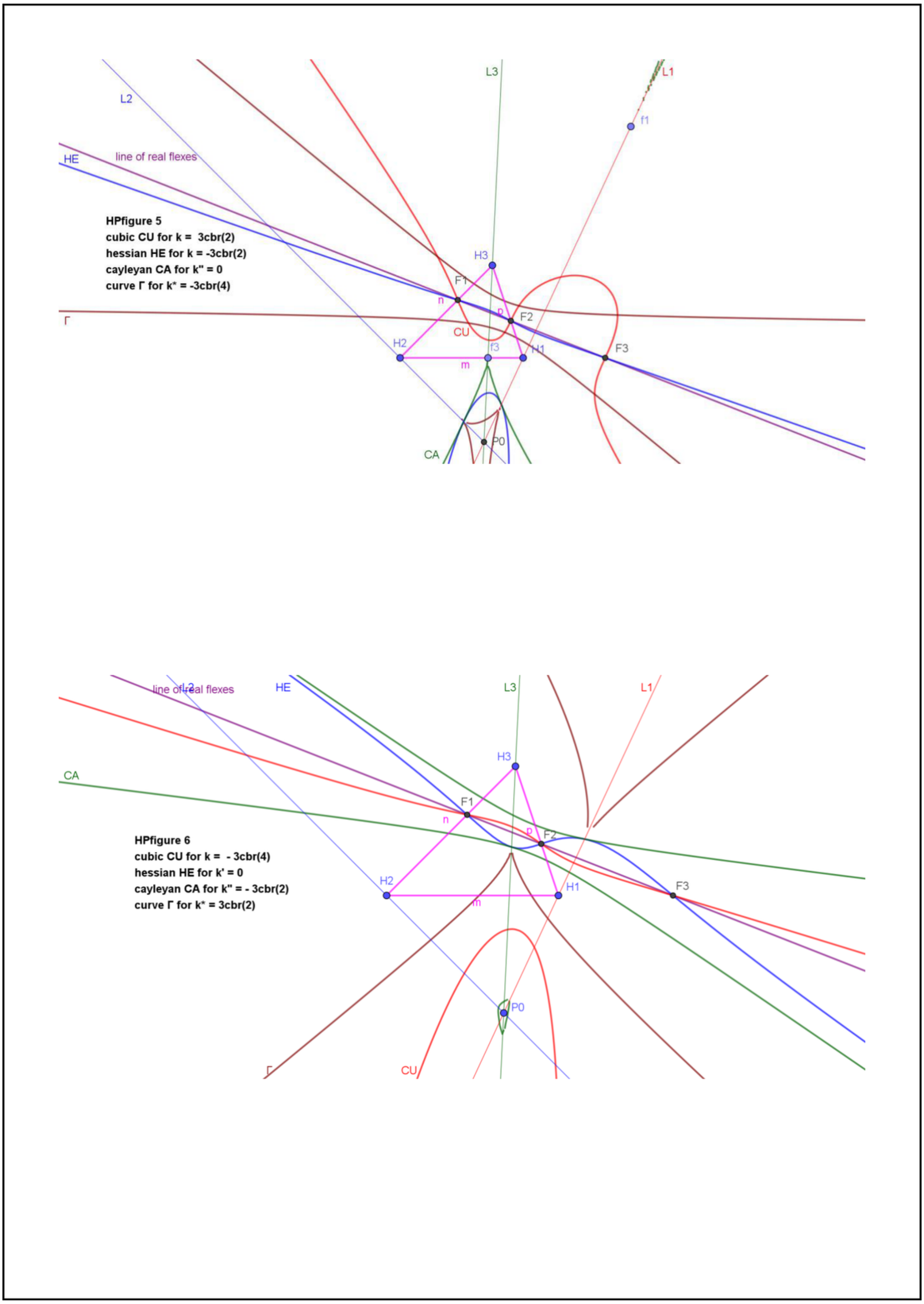

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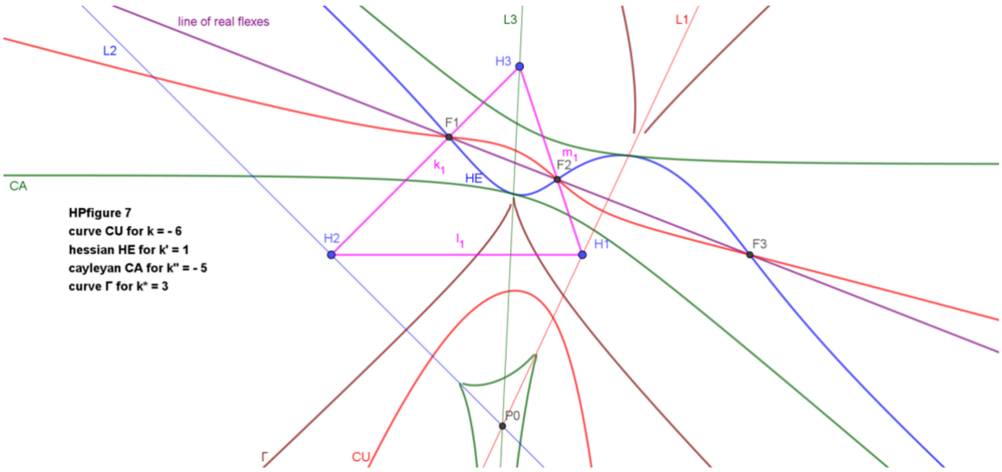

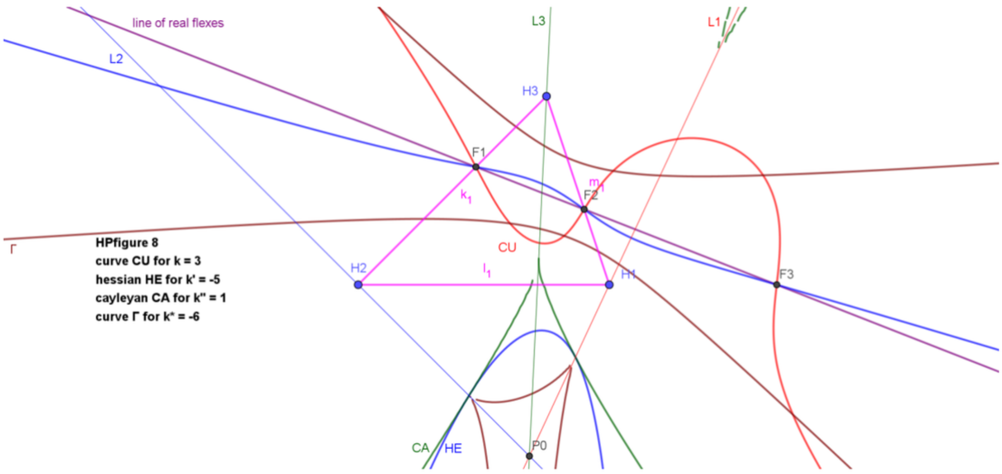

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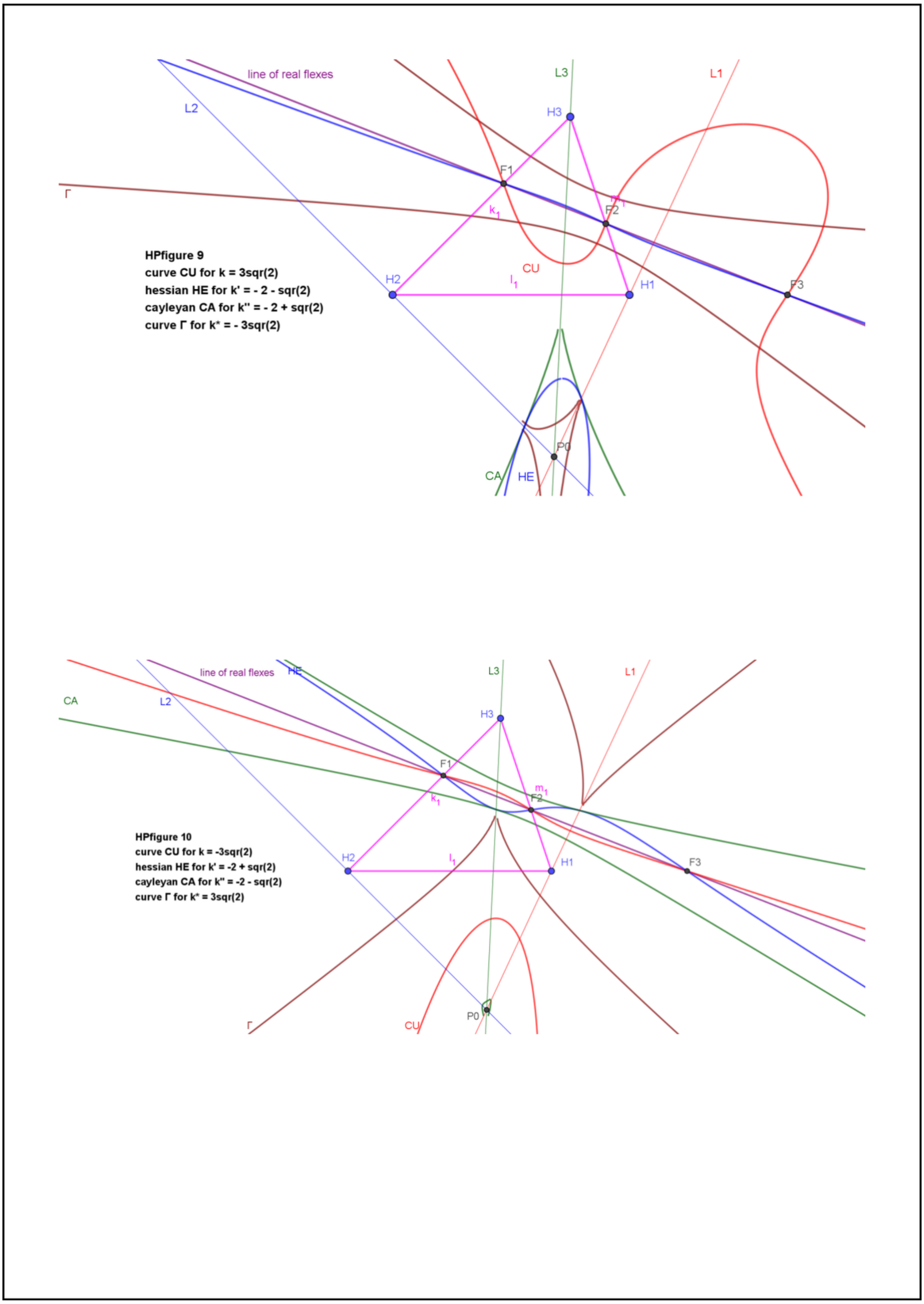

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Message: #2820
Date: 2025-11-24
From: van10hoven@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Bernard,

Thanks for your document 2 dual Hesse pencils, 4 curves and calculations.

I checked in a numerical example in Mathematica your equation for the Cayleyan:

CA = 27 (a1^3 x^3+ a2^3 y^3 + a^3 z^3)^2 - 4 (27 + k''^3) (a1^3 a2^3 x^3 y^3 + a1^3 a3^3 x^3 z^3 + a2^3 a3^3 y^3 z^3) - 18 k''^2 x y z (a1^3 x^3 + a2^3 y^3 + a^3 z^3) - k'' (108 + k''^3) a1^2 a2^2 a3^2 x^2 y^2 z^2 = 0

And partly it fits, only for the part - 18 k''2 x y z (a1^3 x^3 + a2^3 y^3 + a3^3 z^3) there is no match.

It can be some typo of mine, but could you please check it again for me?

Then some questions:

1. Is there a reference for working with this second pencil (curves of 3 rd order tangent to the 9 harmonic polars). Or what is the reason for working with this second pencil.

2. Of what degree is this curve gamma?

3. How did you derive the formula of CA ?

Best regards,

Chris

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Message: #2821
Date: 2025-11-24
From: eckart_schmidt@t-online.de
Subject: Isocubic

Dear Chris,

where can I find in your new webside the possibility

... to describe a 9P-cubic as isocubic

with ref-triangle P1P2P3

... (Pi intersections of the infinite part of the cubic

... and the harmonic polars Li of the real flexpoints Fi),

with an isoconjugation with fixpoint Po (common point of Li),

also root Po of the cubic

and with a starting cubic-point P ?

This is a good constellation for constructions

... starting with a triangle and two points Po and P.

Best regards Eckart

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Message: #2822
Date: 2025-11-24
From: van10hoven@gmail.com
Subject: Re: Isocubic

Dear Eckart,

In the Encyclopedia of Poly Geometry (EPG), we generally describe only constructions that start from n arbitrary points or lines.

The five points you mention, however, are not entirely arbitrary—two of them serve specific roles.

In such a case, we could present it as a 3P-Transformation with two input points that produce a cubic.

However this falls within triangle geometry, as described, among others, by Bernard Gibert.

Since this has already been treated extensively, I have limited the section of EPG-Triangle Geometry (https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/) only to references pointing to sources where it is covered.

Unless you believe your construction is genuinely new.

Could you provide a solid step-by-step construction?

Best regards,

Chris

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Message: #2823
Date: 2025-11-25
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

you wrote in #2822: "However this falls within triangle geometry",

... that is really an absurd interpretation,

... it is a possibility, to study cubics constructive.

Let us start with a triangle P1P2P3 and points

... Po as root and P as cubic-point.

Let R1,R2,R3 be the anticevians of Po wrt P1P2P3,

... then PoPi = Li and RiRj^PiPj = Fk

... and consider finally Po as fixpoint of an isoconjugation,

... then Bernard Gibert's construction of a nonpivotal isocubic (1.5.4)

... gives the cubic invariant under the isoconjugation with fixpoint Po.

A macro wrt a further hessian point P for the ref-triangle

... of intersections of Fi-flextangents and Li gives the hessian ...

Best regards Eckart

PS: Especially the isoconjugations

... are interesting tools to study cubics.

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Message: #2824
Date: 2025-11-25
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

1. Where would you like it to be mentioned in EPG?

2. Bernard Gibert's construction of a nonpivotal isocubic (1.5.4) uses other terminology. Could you translate your terminology to his terminology for the innocent reader.

3. I do not understand your remark ‘A macro wrt a further hessian point P for the ref-triangle

... of intersections of Fi-flextangents and Li gives the hessian ...’. Again make a clear stepwise description please.

Best regards,

Chris

Message: #2825
Date: 2025-11-25
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Chris,

wrt 2) Here is a description of my construction
... in  Bernard Gibert's terminology,
... preparing his construction (1.5.4) for a nonpivotal
isocubic.

Let us start with a triangle ABC and points
...  P as root and Q as cubic-point.
The trilinear polar of P meets ABC sidelines at U,V,W,
... which are the real flexpoints
... and PA,PB,PC their harmonic polars.
Consider finally P as fixpoint of an isoconjugation,
... then Bernard Gibert's construction 1.5.4 of a nonpivotal
isocubic
... gives the cubic invariant under the isoconjugation with
fixpoint P.

wrt 3) Excuse my bad formulation,
... it should describe the same construction for the hessian,
... if you have a hessian point for Q,
... using the same root P, but for ABC the triangle
... for intersections of tangents and harmonic polars of the
flexpoints.

Best regards Eckart
```

Message: #2826
Date: 2025-11-25
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Chris,
Many thanks for interest and quick answer !
My apologise, I forgot the term a1a2a3 befor xyz (the equation is of the 6th degree, homogenous in a1x, a2y and a3z).
1) All this is explained in Salmon or Artebani, which you already put in your bibliography
2) Like all the curves of the 2nd pencil, CA, Γ as well as the core and the residual curves are sextics of the 3rd order
3) I tried to explain the general method in the footnote 1 page 2 of my document
Best regards
Bernard

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Message: #2827
Date: 2025-11-26
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Chris,
Sorry, I forgot to send you the corrected version of my document!
Best regards
Bernard

.

2 dual Hesse pencils, 4 curves and calculations

1. 2 dual Hesse pencils wrt a triangle H1H2H3

The 1rst pencil contains the cubics having the same 9 flexes (3 real and 6 imaginary). The real flexes are F1, F2 and F3.

Cubics are curves of 3rd degree and 6th order.

The 2nd pencil contains the curves of the 3rd order tangent to the 9 harmonic polars (3 real) The real harmonic polars are L1, L2 and L3; they intersect in P0.

These curves of 3rd order are sextics of 6th degree.

2. 4 curves

2 curves belong to the 1rst pencil, the cubic CU and it's hessian HE

2 curves belong to the 2nd pencil, the cayleyan CA and a curve Γ associated to the cubic CU

CA is the hessian of Γ and HE is it's cayleyan (in other words, CU and Γ swap their hessian and cayleyan)

Both pencils have the same structure, with a Core curve and a residual curve.

3. Calculations

The barycentric coordinates of P0 are 1/a1, 1/a2 and 1/a3 and the barycentric equation of the line of the 3 real flexes is a1x + a2y + a3z = 0.

All the curves of both pencils have an equation of the form $x^3 + y^3 + z^3 + kxyz = 0$, considering it as a barycentric equation for the 1rst pencil and replacing x, y and z by a1x, a2y and a3z and as a tangential equation for the 2nd pencil and replacing x, y and z by u/a1, v/a2 and w/a3.

In particular, if the cubic CU has the constant k, then HE has the constant $k' = -(6/k)^2 - k/3$, CA has the constant $k'' = 6/k - (k/3)^2$ and Γ has the constant $k^* = -18/k$.

It's obvious that $k'(k^*) = k''(k)$ and $k''(k^*) = k'(k)$, which explains the swapping of HE and CA for the curves CU and Γ.

It's possible to find the tangential equation of a curve defined by a barycentric equation and vice-versa, the calculation being exactly the same. We find equations of the 6th degree[1].

2 dual Hesse pencils.pdf

In particular, the barycentric equation of CA is

$27\ (a1^3x^3 + a2^3y^3 + a3^3z^3)^2 - 4\ (27 + k''^3)\ (a1^3a2^3\ x^3y^3 + a1^3a3^3\ x^3z^3 + a2^3\ a3^3\ y^3z^3)$

$- 18k''^2\ a1a2a3xyz\ (a1^3x^3 + a2^3y^3 + a3^3z^3) - k''(108 + k''^3)\ a1^2a2^2a3^2x^2y^2z^2 = 0$

and the same for Γ replacing k'' by k*.

The cayleyan of the Core cubic is the Residual curve of the 2nd pencil (k = 0 and k* = ∞) and the cayleyan of the residual cubic is the Core curve of the 2nd pencil (k = ∞ and k* = 0).

The barycentric equation of this Core curve is

$(a1^3x^3 + a2^3y^3 + a3^3z^3)^2 - 4\ (a1^3a2^3\ x^3y^3 + a1^3a3^3\ x^3z^3 + a2^3\ a3^3\ y^3z^3) = 0$

The barycentric equation of this Residual curve is $x^2y^2z^2 = 0$ and it's tangential curve uvw = 0

It's remarkable that this Residual curve is 2 times the triangle forming the residual cubic !

4. Properties

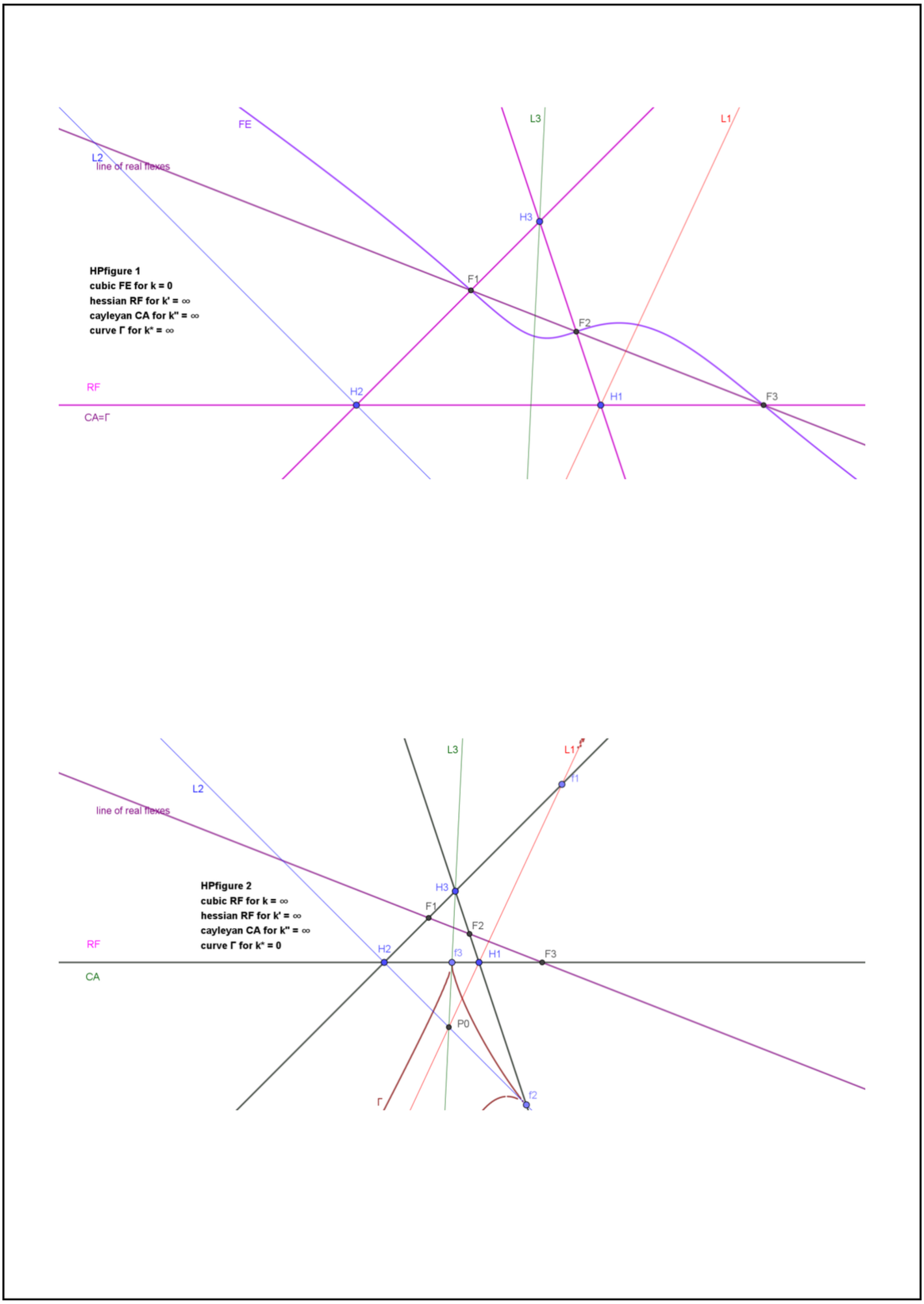
Drawing this way a certain number of curves, it appears that the cusps of the curves Γ are the contact points of HE and CA, which seems to be the dual property of the common tangents to CU and HE being the tangents in the intersections between CU and CA …

1 The calculations are long and boring !

Considering the tangential equation G(u,v,w) of the curve and the tangent ux + vy + wz = 0, we know that G'u/x = G'v/y = G'w/z and must eliminate u, v and w in order to find the barycentric equation F(x,y,z) of the curve.

For example, for $G(u,v,w) = u^3 + v^3 + w^3 = 0$ (k'' = 0), it's immediate that $u^2/x = v^2/y = w^2/z$ and $x^{3/2} + y^{3/2} + z^{3/2} = 0$. Then $z^{3/2} = -(x^{3/2} + y^{3/2})$, $z^3 = (x^{3/2} + y^{3/2})^2 = x^3 + y^3 + 2\ u^{3/2}v^{3/2}$ and last we find $(2\ x^{3/2}y^{3/2})^2 = (z^3 - x^3 - y^3)^2$, $x^6 + y^6 + z^6 = 2(x^3y^3 + x^3z^3 + y^3z^3)$ or $(x^3 + y^3 + z^3)^2 = 4(x^3y^3 + x^3z^3 + y^3z^3)$.

For G(u,v,w) = uvw = 0 (k'' = ∞), we get vw/x = uw/y = uv/z and the solution of the system is given by u = yz, v = xz and w = xy. It follows $uvw = x^2y^2z^2 = 0$.



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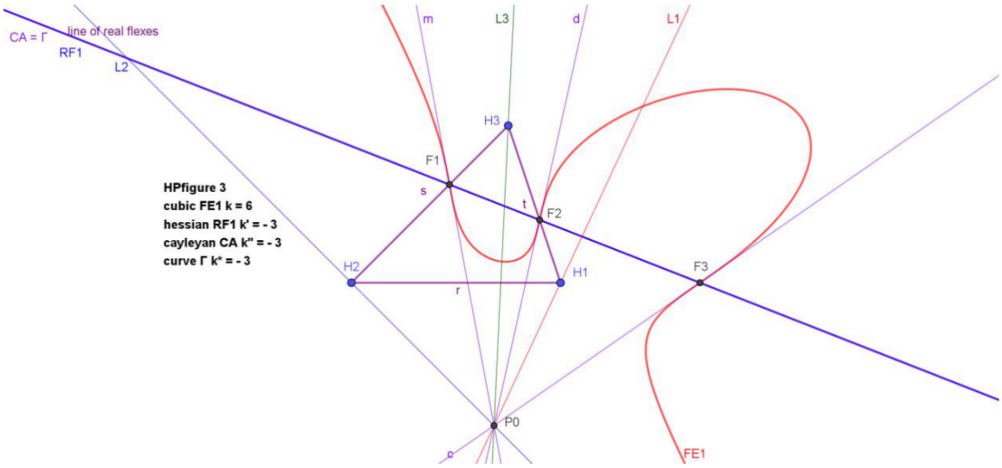

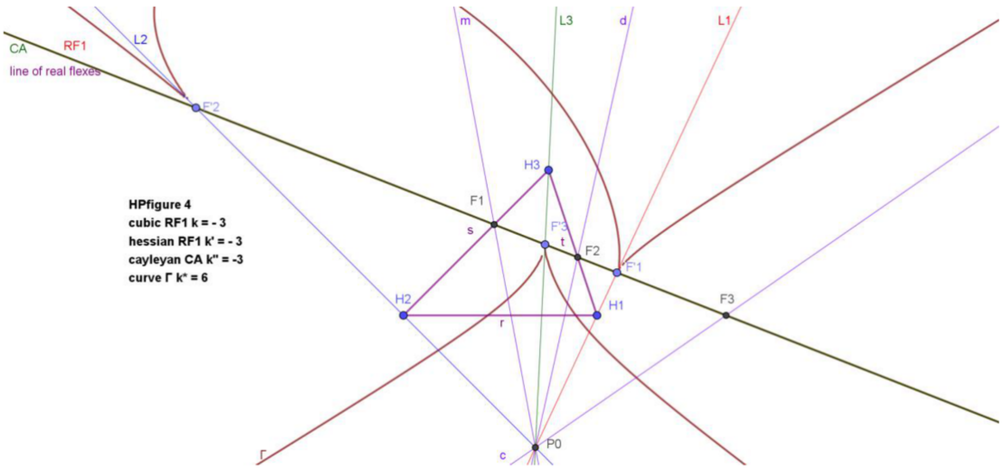

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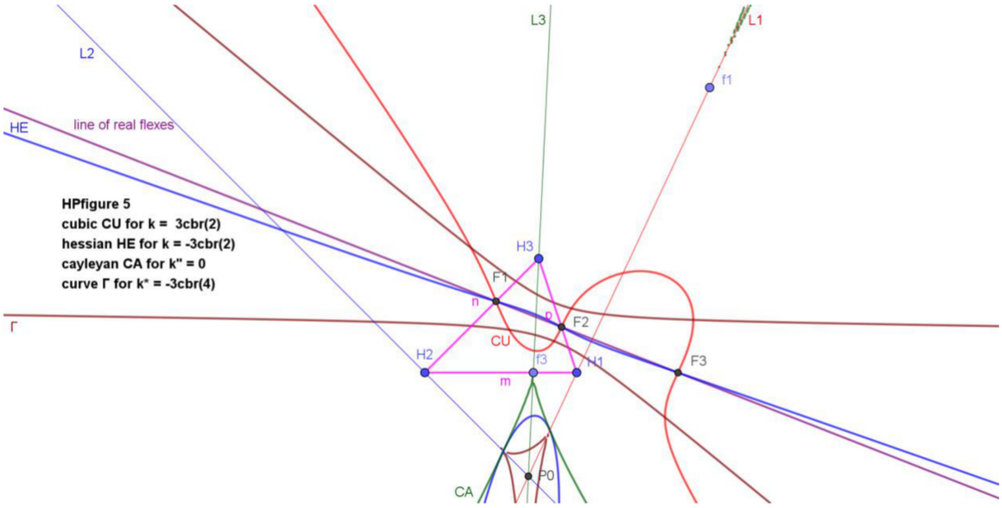

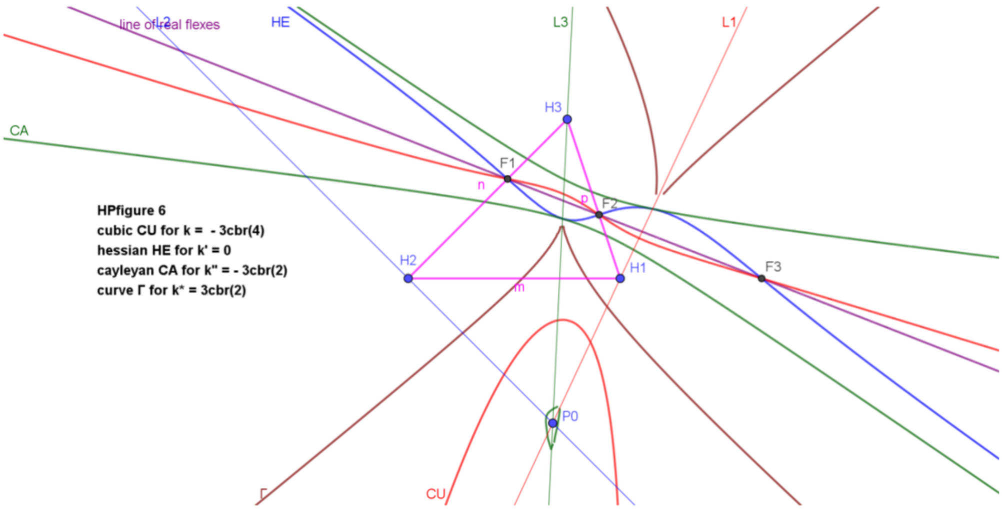

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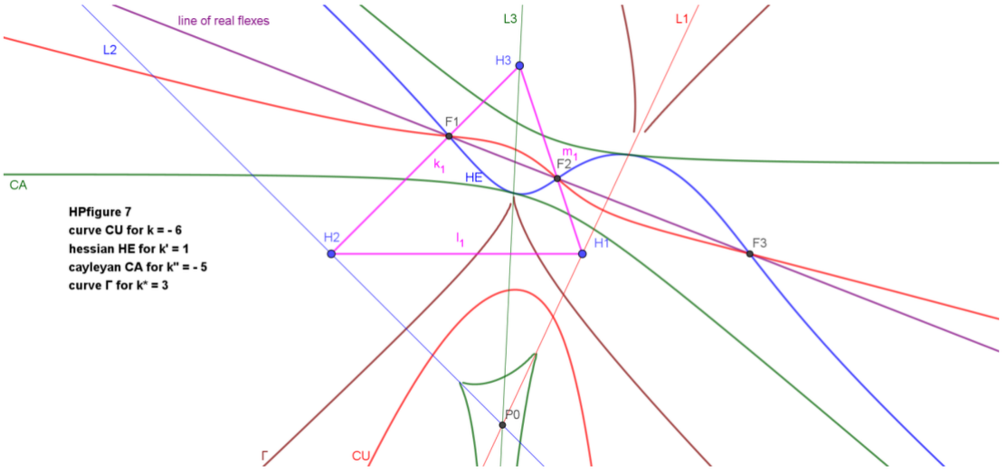

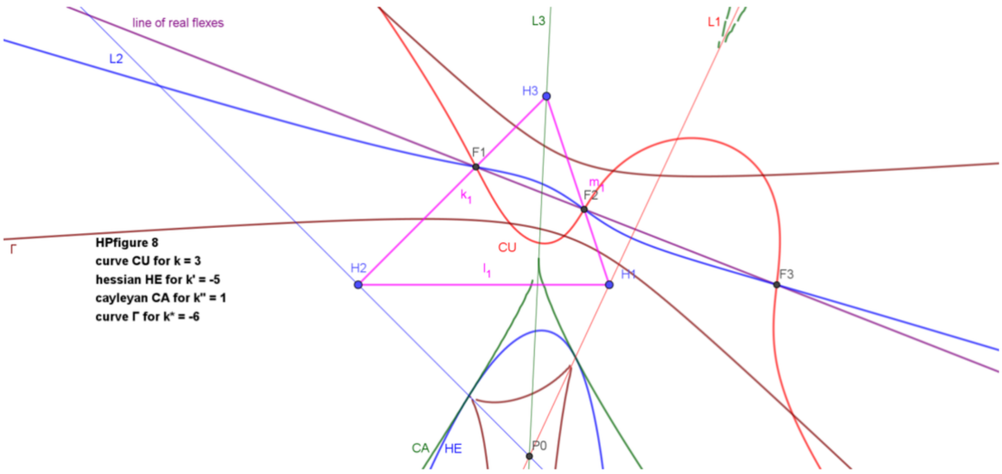

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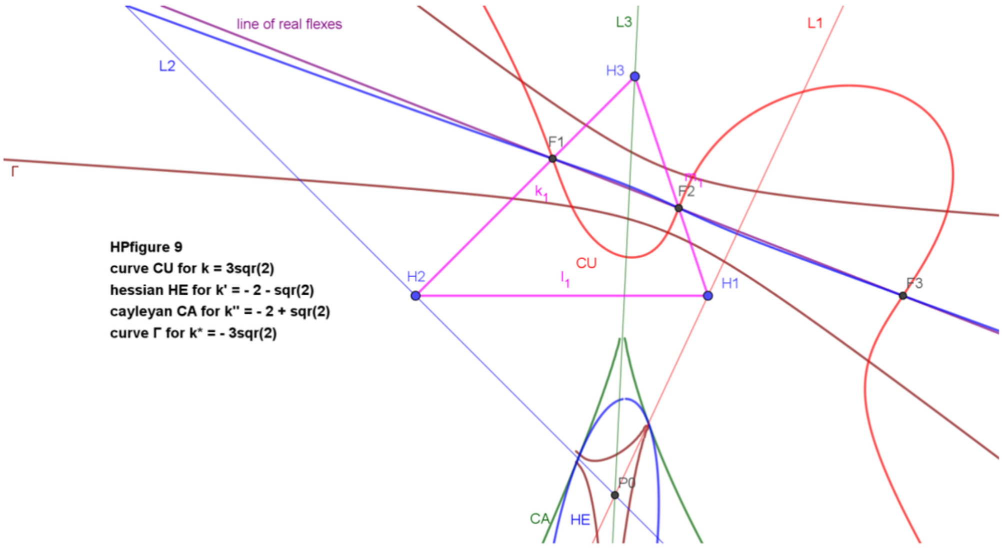

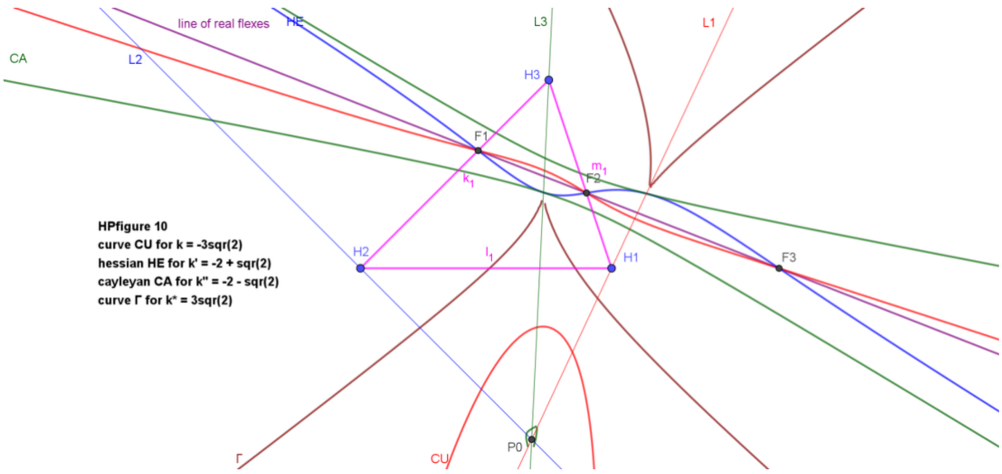

2 dual Hesse pencils.pdf

Message: #2828
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Thank you for the clarification.

You asked me to describe a 9P-cubic as an isocubic with reference triangle P1P2P3, defined by an isoconjugation with fixed point/root P0 and a starting cubic point P.

For the EPG-reader, it's important to have a simple, step-by-step construction with minimal references. However, we can note that the isoconjugation is given by QA–Tf2(X), where QA = ABCP. These are the ingredients for constructing the cubic.

I have reviewed Bernard Gibert's construction of the cubic in Section 1.5.4, and it remains a significant hurdle. Could you provide a step-by-step description, following BG's principles (with a proper reference to his paper), in the simplest possible form?

I think this is worthwhile, as you pointed out, because it would also give us an easy construction of the flex points and harmonic polars. I would then add an extra section to EPG: "CU-6: Different Constructions of a Cubic."

What I need is a straightforward, step-by-step construction given triangle ABC, the isoconjugation fixed point P (root), and another point Q on the cubic, where the isoconjugation is defined as QA–Tf2(X) with QA = ABCP.

It's particularly important to explain how the locus is generated.

Next you could describe the construction of the Hessian.

As an expert in this matter, could you kindly provide that?

Best regards,

Chris

Message: #2829
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Bernard,

I checked the normalized equation of the Cayleyan again and this time I obtained a good result in a numerical example.

I have updated the equation of the Cayleyan in CU-5 (
https://www.chrisvantienhoven.nl/epg/polyn-curves/cubics/cu-5/
).

Thank you very much for the beautiful result.

Best regards,

Chris

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Message: #2830
Date: 2025-11-26
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

sorry, my construction description

... is not the place to explain "isoconjugation with fixpoint"

... nor "construction of nK-isocubic"

... in a better way as Bernard Gibert.

Your interpretation of isoconjugation with fixpoint P is wrong:

... " the isoconjugation is given by QA-Tf2(X), where QA=ABCP."

You have to take the QA with vertices P and anticevians of P wrt ABC,

... if you define isoconjugations with QA-geometry.

My further remark wrt the hessian is of no importance,

... it is an analog application of the described construction,

... but we don't have a usable point Q for the hessian

... without further constructions.

Best regards Eckart

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Message: #2831
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

In EPG I try to present everything in the most elementary way possible.

I personally find BG's description of the construction rather difficult.

I noticed that he also described the isoconjugation, which we can refer to as QA-Tf2.

That is why I had hoped you could provide, with your expertise, a simpler step-by-step description of the remaining steps without going into every detail.

Anyone who wishes to study the subject more deeply can of course consult BG's paper.

Unfortunately, that does not seem possible at the moment.

Best regards,

Chris

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Message: #2832
Date: 2025-11-28
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

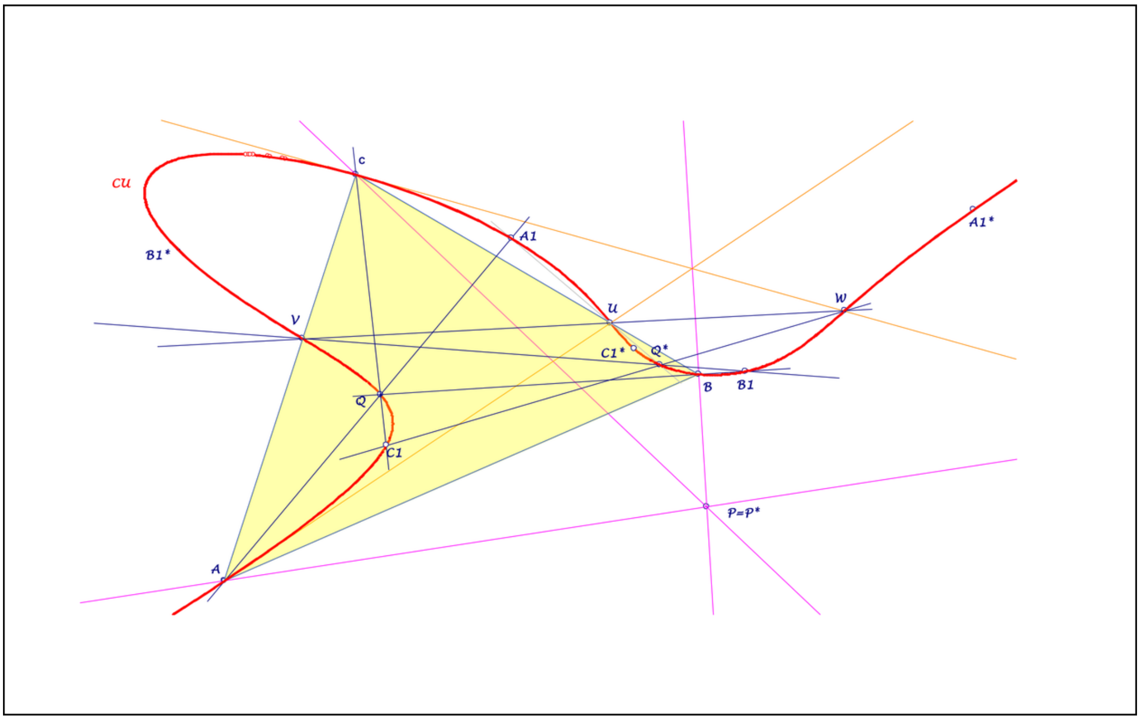
let us furthermore define an isoconjugation wrt a triangle ABC
... (see Wolfram MathWorld or ETC),
... then the fixpoints give a QA with diagonal triangle ABC.

You are right, GB's nK-construction is rather difficult,
... here shortened, but finally using the 9P-construction:

Let us start with a triangle ABC and points
... P as root and fixpoint of an isoconjugation *,
... further a cubic point Q.
Let the tripole of P wrt ABC intersect the sidelines in U,V,W,
... which will be the real flexpoints.
Get the cubic points (see BG 1.5.4)
... A1 = QA^Q*U, B1 =QB^Q*V, C1 = QC^Q*W
... and their isoconjugates A1*,B1*, C1*.
Are this enough points for a 9P-construction?
A,B,C,A1,A2,A3,A1*,B1*,C1*,Q,Q*,U,V,W.

Using my old 9P-macro and approaching for a 9th point
... I can draw the cubic with every precision,
... but exact for the 9th point my macro fails,
... perhaps it is not perfect or there are bad relations for the points.

Best regards Eckart



2025-11-27.pdf

Message: #2833
Date: 2025-11-29
From: contiwa.goma3@gmail.com
Subject: Re: Desmic System / Reye Configuration

```
Dear all,

I didn't realized the circumcubic of hidden desmic is expressed
as a non-pivotal isocubic
(p - P)x(rRy^2 + qQz^2) + (q - Q)y(pPz^2 + rRx^2) + (r -
R)z(qQx^2 + pPy^2) + 2(PQR - pqr)xyz = 0
when the circumcubic of reference desmic is a pivotal isocubic
(p + P)x(rRy^2 - qQz^2) + (q + Q)y(pPz^2 - rRx^2) + (r +
R)z(qQx^2 - pPy^2) = 0.
The vertices of DTs are parametrized as follows:
DTux = (1 : 0 : 0) , DTuy = (P : q : r) , DTuz = (p : Q : R)
DTvx = (0 : 1 : 0) , DTvy = (p : Q : r) , DTvz = (P : q : R)
DTwx = (0 : 0 : 1) , DTwy = (p : q : R) , DTwz = (P : Q : r).

Please regard tetrahedra in #2797 (
https://groups.io/g/Quadri-and-Poly-Geometry/attachment/2797/0/D
esmic%20System%20of%203%20Tetrahedra.pdf ) as quadrangles:
reference desmic = {QAx, QAy, QAz}
hidden desmic = {QAu, QAv, QAw}
QAx = {X1,X2,X3,X4}, QAy = {Y1,Y2,Y3,Y4}, QAz = {Z1,Z2,Z3,Z4}
QAu = {U1,U2,U3,U4}, QAv = {V1,V2,V3,V4}, QAw = {W1,W2,W3,W4}
U1 = X2X3∩Y1Y4∩Z1Z4, ...,
then
QAx-Tr1 = {DTux, DTvx, DTwx} , QAu-Tr1 = {DTux, DTuy, DTuz}
QAy-Tr1 = {DTuy, DTvy, DTwy} , QAv-Tr1 = {DTvx, DTvy, DTvz}
QAz-Tr1 = {DTuz, DTvz, DTwz} , QAw-Tr1 = {DTwx, DTwy, DTwz}
DTux = X2X3∩X1X4 = U1U4∩U2U3, ....

The circumcubic of the reference desmic is
pK(QAx-P16, Px) wrt QAx-Tr1 = pK(QAy-P16, Py) wrt QAy-Tr1 =
pK(QAz-P16, Pz) wrt QAz-Tr1,
and that of the hidden desmic is
pK(QAu-P16, Pu) wrt QAu-Tr1 = pK(QAv-P16, Pv) wrt QAv-Tr1 =
pK(QAw-P16, Pw) wrt QAw-Tr1,
where
QAx-P16 = (pP : qQ : rR)
Px = (p + P : q + Q : r + R), Py = (P : Q : R), Pz = (p : q : r)
Pu = (0 : Q - q : r - R), Pv = (p - P : 0 : R - r), Pw = (P - p
: q - Q : 0).

Cf. Mathews, " Cubic Curves and Desmic Surfaces (
https://www.ams.org/journals/tran/1926-028-03/S0002-9947-1926-15
01362-3/S0002-9947-1926-1501362-3.pdf )."
```

(I don't intend to step into desmic surfaces.)

Best regards,
M@IMF

p.s. The links of EPG-Ref [11](Hyacinthos) are wrong: "epizy"s are replaced by "99on"s.

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Message: #2834
Date: 2025-11-30
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Thanks for your shortened version of the construction of an nK cubic.
I have drafted a new section in EPG. Could you please review it?
See attachment.

Best regards,
Chris

CU-6 Construction of Isocubics

1. Construction of an nK cubic

A non-pivotal isocubic is a cubic curve in the plane of a reference triangle defined by an isoconjugation with respect to a fixed pole, but which does not possess a pivot point. Such cubics are commonly abbreviated as nK.

The notion of *isoconjugation* arises from projective geometry and generalizes classical conjugations such as isogonal and isotomic conjugation.
In trilinear coordinates, let P=(p:q:r) be a fixed point (called the *pole*) and U=(u:v:w) any point in the plane of triangle ABC. The P-isoconjugate of U is the point

$$U^* = (qrvw: rpwu: pquv).$$

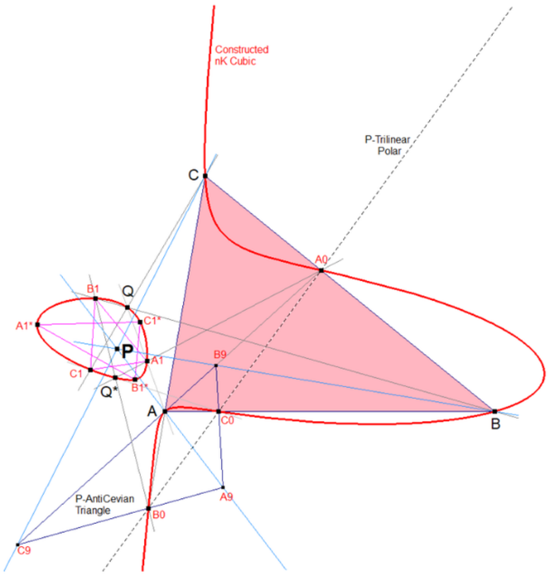
See [13].
This mapping is involutive: applying the isoconjugation twice returns the original point U.
For constructions of the P-isoconjugate see CU-P-Tf1.
The P-Isoconjugate of X is also QA-Tf2(X) wrt QA=PA0B0C0, where A0B0C0 = P-AntiCevian Triangle wrt ABC.

For the construction of an nK cubic we adopt, in part, the method of Bernard Gibert as described in [17b], paragraph1.5.4. Eckart Schmidt proposed in [66], QPG#2832, applying the first steps of this construction and then continuing with the construction with the 9P-method (see 9P-s-Cu1).

The construction then becomes:

1. Let ABC be the reference triangle and point P a fixed point of the isoconjugation. Furthermore, let Q be an additional point intended to lie on the nK cubic.
2. Let Q* be the isoconjugate of Q. It can be constructed as QA-Tf2(Q), with QA=(P + vertices P-Anticevian Triangle wrt ABC).
3. Let L0 be the trilinear polar of P wrt ABC intersecting ABC in points A0, B0, C0.
4. Construct the following nK-points:
   A1 = QA^Q*A0, B1 =QB^Q*B0, C1 = QC^Q*C0.
5. Perform the 9-Point-constructions for the cubic using nine points from the set
   (A, B, C, A0, B0, C0, A1, B1, C1, Q, Q*),
   as described at 9P-s-Cu1.

CU-6 nK-construction-01.pdf



6. Eventually, the isoconjugates of A1, B1, C1, denoted A1*, B1*, C1*, can also be constructed using QA-Tf2. These points likewise lie on the nK-cubic.

Advantage of the Construction
This construction has the great advantage that:

a. A0, B0, C0 are the three real flex points of this cubic (see CU-9P1).
b. The lines PA, PB, PC are the three real harmonic polars of the cubic (see 9P-9L1).

CU-6 nK-construction-01.pdf

Message: #2835
Date: 2025-12-02
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

thanks for your elaboration,

... let me use my nomination in #2832,
... proposing 14 point for a 9P-construction of nK,
... finally I succeeded also with my macro in confirming.

Anymore the points give an interesting constellation,

... for example these two further cubic-points:

Z1 = AB1* ^ BC1* ^ CA1* ^ A1W ^ B1U ^ C1W,

Z2 = AC1* ^ BA1* ^ CB1* ^ A1V ^ B1W ^ C1U.

Lines through Q or Q*, Z1 or Z2:

QAA1, QBB1, QCC1, QA1*U, QB1*V, QC1*W,

Q*AA1*, Q*BB1*, Q*CC1*, Q*A1U, Q*B1V, Q*C1W,

Z1AB1*, Z1BC1*, Z1CA1*, Z1A1W, Z1B1U, Z1C1V,

Z2AC1*, Z2BA1*, Z2CB1*, Z2A1V, Z2B1W, Z2C1U.

Conics

Z1,Z2,A,A1,A1*,U,

Z1,Z2,B,B1,B1*,V,

Z1,Z2,C,C1,C1*,W,

Z1,Z2,A1,B1,C1,Q*,

Z1,Z2,A1,B1,C1*,Q,

Z1,Z2,B1,C1,A1*,Q,

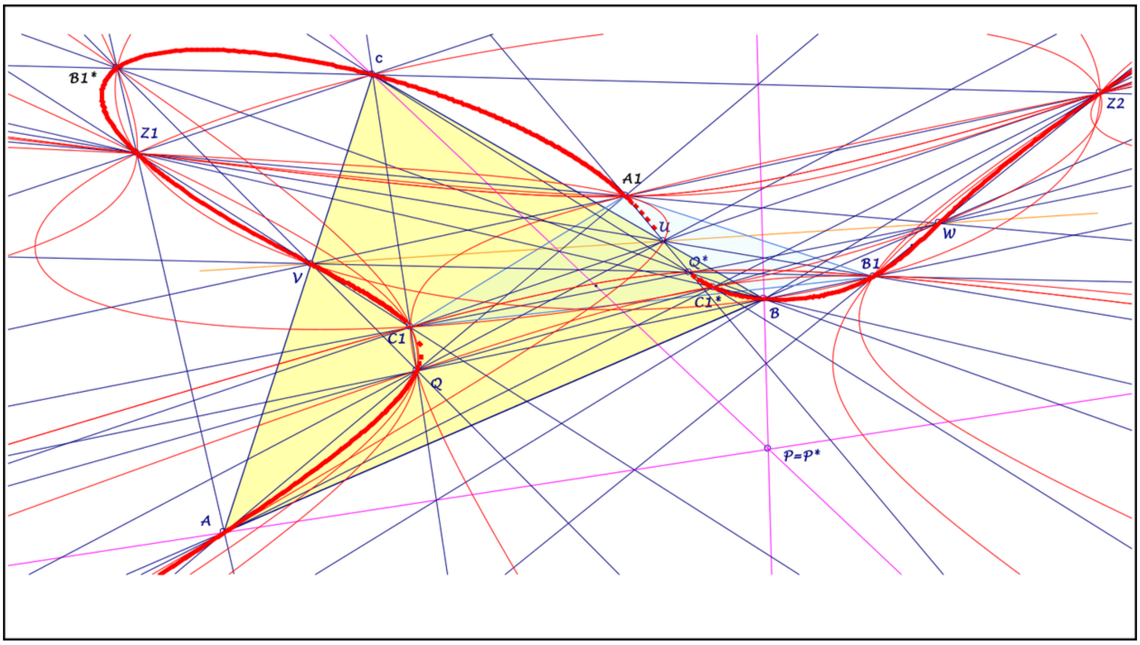
Z1,Z2,B1,C1,A1,Q*,

Z1,Z2,A1,C1,B1*,Q,

Z1,Z2,A1,C1,B1,Q*.

Amazing the diversity of relations!

Best regards Eckart



2025-12-02.pdf

Message: #2836
Date: 2025-12-02
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

```
Dear Chris, dear Eckart
I read carefully the draft of CU-6.
Here some remarks :
The nK construction refers to a triangle ABC and an
isoconjugation with a pole and 4 fixed points.
If A0, B0 and C0 are the 3rd intersections of the sides of ABC
with the cubic, they are aligned and the root of the nK is the
trilinear pole of this line.
The QA-Tf2 of this line is a conic tritangent to the cubic in A,
B and C.
Note that neither the root nor the fixed points are necessary on
the cubic)
1) A nK can be mono- or bipartite.
A bipartite cubic is not pivotal *wrt this triangle and this
isoconjugation* , but can be pivotal with any of it's points
with fixed points the 4 pretangential of the pivot.
(For example, QA-Cu7 is a nK wrt the reference QA ...)
2) The isoconjugate is pvw : qwu : ruv
3) A0B0C0 is not the anticevian triangle of P (you have to
choose another naming)
4) Last and most important, *A0, B0 and C0 are the real flexes
of the cubic only if the root is one of the 4 fixed points*.
I suggest that you procede in 2 steps, describing 1rst the nK
construction of Bernard Gibert (in which the root has an
isoconjugate) and 2nd Eckart's special case with the root in one
of the 4 fixed points (for a given triangle and a pole inside
the triangle, there are 4 real fixed points and 4 different
lines of real flexes, which are the trilinear polars of the
fixed points).
Best regards
Bernard
```

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Message: #2837
Date: 2025-12-05
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

trying to understand Bernard's message #2836

... I studied once more your description of CU-6.

An isoconjugation can be given

... by a reference triangle ABC and a fixed point P

... P is not the pole, defined as isoconjugate of the centroid.

For a pole (p,q,r) holds U(u,v,w) --> U*(pvw,quw,ruv) (see GB 1.2.1).

For a pivotal isocubic we need the pivot for construction.

... for a non pivotal cubic we need the root for construction,

... and in both cases we need a further cubic point U.

You don't mention the root in 1) of the construction description,

... but you use the fixed point P as root in 2) and 3),

... for the root has to be a fixed point of the isoconjugation in my construction.

Perhaps this is the background for Bernard's message,

... which I try in vain to understand.

Best regards Eckart

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Message: #2838
Date: 2025-12-05
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
I'm very surprised that you don't understand my message, as you say approximately quite the same as I do !
Provided you have a reference triangle and an isoconjugation with pole isoconjugate of the centroïd
1) For a pivotal isocubic you need the pivot, *but you don't need a further point U on the cubic*.
You have the 4 fixed points, the DT vertices, the pivot and the isopivot and the 3 traces of the pivot on DT.
2) For a non pivotal isocubic, you need the vertices of DT, the root and it's isoconjugate and a further point U on the cubic and it's isoconjugate.
For example QA-Cu7 is a nK wrt the reference QA (the vertices of the QA or fixed points of the isoconjugation are not on the cubic).
3) Your construction is a special case of the non pivotal isocubic with the root in one of the fixed points of the isoconjugation.
In this case and only in this case, the trilinear polar of the root wrt the reference triangle intersects the sides of the triangle in the flexpoints of the cubic.
What is not clear in these 3 points ?
Best regards
Bernard

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Message: #2839
Date: 2025-12-06
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart and Bernard,

Thanks for both responses to my short article on the construction of the non-pivotal isocubic nK.
The topic of isocubics is somehow difficult to comprehend, which is why I also now seek to describe them accurately.

Isoconjugation
It all begins with isoconjugation. Eckart once gave some fine descriptions, which I mentioned at CU-P-Tf1 (https://www.chrisvantienhoven.nl/epg/polyn-curves/cubics/cu-p-tf1/). An isoconjugate always requires an additional defining point W in order to map any point X to its conjugate point X*. For this reason, the isoconjugate is sometimes referred to as the W-isoconjugate when W plays a relevant role.

Isocubic
We then define an *isocubic* as a cubic curve that is invariant under a given isoconjugation. This means that if a point X lies on the cubic, then its isoconjugate X* also lies on the cubic.

Pivotal and Non-pivotal Isocubics
We distinguish between pivotal isocubics (pK) and non-pivotal isocubics (nK).
* Pivotal isocubics are those in which X and X* always align with another fixed point on the cubic, called the pivot.
* Non-pivotal isocubics are those in which X and X* do not align with a fixed point.

It is important to note that this distinction depends on the chosen reference triangle and the isoconjugation associated with it. It may happen that with another reference triangle and another isoconjugation, the same cubic appears pivotal in that constellation.

Why every cubic can be described as Pivotal
Every cubic can be described as pivotal because:
* Given a point X on the cubic, there are four points of tangency arising from tangents drawn from X to the cubic. These points may be real or imaginary.

* These four points form a quadrangle QA, for which QA-Tf2 serves as the isoconjugate. The diagonal triangle of QA (whose vertices also lie on the cubic) can then be regarded as the reference triangle, with QA-Tf2 as its isoconjugate.
* When imaginary components occur, there may be no real four points of tangency, and the processes of isoconjugation also do exist, but take place only in the imaginary domain.
So why can a cubic be classified as non-pivotal? Because not every triangle inscribed in a cubic is the diagonal triangle of a quadrangle of tangency points from a given point on the cubic. Therefore, in the construction of an nK (as described in BG's paper SITP on isocubics, §1.5.4), one seeks another reference triangle and corresponding isoconjugate that allow the cubic to qualify as a pivotal isocubic (pK). From there, the cubic may or may not be constructible as a pK.

Conclusion
My conclusion is that the classification of a cubic as pivotal or non-pivotal is misleading. Whether a cubic is pivotal or non-pivotal depends not on the cubic itself, but on the combination of the cubic and the reference triangle. Meanwhile, the notion of the isoconjugate depends only on the reference triangle and a pole. Every triangle has its own unique -isoconjugate, just as every triangle has its own isogonal conjugate.

BG's Notation
In BG's notation we find the following dependencies:
* Pivotal isocubic pK(W,P) : a pK with pole W and pivot P.
* Non-pivotal isocubic nK(W,P,M) : an nK with pole W, root P and passing through M.
These are the defining factors for pivotal and non-pivotal isocubics.

Construction of pK(W,P)
Here a reference triangle is given, together with a pole W that defines the W-isoconjugate. Any point P may be chosen as pivot, from which a pK can be constructed as described in §1.4.3.

Construction of nK(W,P,M)
Here again a reference triangle is given, together with a pole W defining the W-isoconjugate.
Next, there is a root P , intended as the trilinear pole of the line through the "third" intersections of the curve with the sidelines of ABC (see SITP §1.5.1).
Finally, a point M on the cubic must be specified to complete the construction.

```
That concludes my contribution for today regarding the
phenomenon of isocubics.

Best regards,

Chris
```

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Message: #2840
Date: 2025-12-06
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
I think you should mention that the pole of the isoconjugation is the isoconjugate of the centroïd of the reference triangle. This explains why the reference triangle and the pole define together the isoconjugation.
Perhaps also worth mentionning : for a pK, the pivot and the the fixed points of the isoconjugation are on the cubic, but for a nK, the root and the fixed points are not necessarely on the cubic ...
Last and most important, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic.
Best regards
Bernard

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Message: #2841
Date: 2025-12-06
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

Thanks for the additional relevant information you gave.
I have revised my description:

ISOCUBICS

Isocubic
We define an isocubic as a cubic curve that is invariant under a given isoconjugation. This means that if a point X lies on the cubic, then its isoconjugate X* also lies on the cubic.

Isoconjugation
The isoconjugation constitutes a projective transformation.
The isoconjugate always requires an additional defining point W in order to map any point X to its conjugate point X*. For this reason, the isoconjugate is sometimes referred to as the W-isoconjugate when W plays a relevant role.
The isoconjugate can be defined in the context of a reference triangle by specifying a source point and its isoconjugate. From this information each point in the field can be ‘isoconjugated’ to a corresponding point. The isoconjugate of the centroid, obtained in this manner, is referred to as the pole W. See BG-description Isoconjugation (https://bernard-gibert.fr/gloss/isoconjugation.html).
The same isoconjugation may alternatively be described using a point K, around which a source point is harmonically reflected. This approach was described by Günther Pickert (see CU-P-Tf1 (https://www.chrisvantienhoven.nl/epg/polyn-curves/cubics/cu-p-tf1/)).
There are 4 fixed points for which source point and its isoconjugate coincide. These fixed points are K (as designated by Günther Pickert) and the three vertices of the K-anticevian triangle.

Pivotal and Non-pivotal Isocubics
We distinguish between pivotal isocubics (pK) and non-pivotal isocubics (nK).
* Pivotal isocubics are those in which X and X* always align with another fixed point on the cubic, called the pivot.
* Non-pivotal isocubics are those in which X and X* do not align with a fixed point.

It is important to note that this distinction depends on the chosen reference triangle and the isoconjugation associated with it. It may happen that with another reference triangle and another isoconjugation, the same cubic appears pivotal in that constellation.

Why every cubic can be described as Pivotal
Every cubic can be described as pivotal because:
* Given a point X on the cubic, there are four points of tangency arising from tangents drawn from X to the cubic. These points may be real or imaginary.
* These four points form a quadrangle QA, for which QA-Tf2 serves as the isoconjugate. The diagonal triangle of QA (whose vertices also lie on the cubic) can then be regarded as the reference triangle, with QA-Tf2 as its isoconjugate.
* When imaginary components occur, there may be no real four points of tangency, but the processes of isoconjugation still do exist, but take place only in the imaginary domain.
So why can a cubic be classified as non-pivotal? Because not every triangle inscribed in a cubic is the diagonal triangle of a quadrangle of tangency points from a given point on the cubic. Therefore, in the construction of an nK (as described in BG’s paper SITP on isocubics, §1.5.4), one seeks another reference triangle and corresponding isoconjugate that allow the cubic to qualify as a pivotal isocubic (pK). From there, the cubic may or may not be constructible as a pK.

An Isocubic is not a special type of cubic
Considered in that perspective the classification of a cubic as pivotal or non-pivotal is misleading. Whether a cubic is pivotal or non-pivotal depends not on the cubic itself, but on the combination of the cubic and the reference triangle. Meanwhile, the notion of the isoconjugate depends only on the reference triangle and a pole. Every triangle has its own unique -isoconjugate, just as every triangle has its own isogonal conjugate.

BG’s Notation
In Bernard Gibert’s notation we find the following dependencies:
* Pivotal isocubic pK(W,P) : a pK with pole W and pivot P.
* Non-pivotal isocubic nK(W,P,M) : an nK with pole W, root P and passing through M.

These are the defining factors for pivotal and non-pivotal isocubics.

Construction of pK(W,P)

Here a reference triangle is given, together with a pole W that defines the W-isoconjugate. Any point P may be chosen as pivot, from which a pK can be constructed as described in §1.4.3.

Construction of nK(W,P,M)
Here again a reference triangle is given, together with a pole W defining the W-isoconjugate.
Next, there is a root P , intended as the trilinear pole of the line through the “third” intersections of the curve with the sidelines of ABC (see SITP §1.5.1).
Finally, a point M on the cubic must be specified to complete the construction.
For a pK, the pivot and the fixed points of the isoconjugation are on the cubic, but for a nK, the root and the fixed points are not necessarily on the cubic.
Last but not least, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic.

Best regards,

Chris

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Message: #2842
Date: 2025-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard,

thanks for clearance.

 Wrt 1) in #2838: Evidently you are right, excuse my
 carelessness.

 Wrt 3) in # 2838: Finally I understood your remark,

... but for me remains the question:

Can I consider every cubic

... as isocubic with root in a fixpoint of the isoconjugation?

Best regards Eckart

PS: Now I shall study your messages 2839-41.
```

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Message: #2843
Date: 2025-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

with great interest I have read your description of isocubics,

... my acceptance, this was necessary already years ago, thanks!

Isogonality will be X(6) -isoconjugation, alright?

I think, you can forget my message #2837.

Best regards Eckart

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Message: #2844
Date: 2025-12-06
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear geometers,

Excuse me for intruding, but are the fixed points of nK
the vertices of dual QA of QL formed by the reference triangle
and the trilinear polar of the root?

By the way, I can't reach QL-8(The Dual QA/QL-configuration) in
EPG.
(QA-8 is OK.)

Best regards,
M@IMF

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Message: #2845
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear geometers,

I'm sorry. Let me correct my question:
Are the fixed points of nK the root and the vertices of DT of QL
formed by the reference triangle and the trilinear polar of the
root?
When the root is Cross Point of 3 real Harmonic Polars and the
reference triangle is Real Flex Trilateral,
3 real flexes lie on the trilinear polar of the root, I think.

Best regards,
M@IMF
```

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Message: #2846
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

```
Dear Eckart,
I'm not sure I understand your question in 2842.
Every cubic is an isocubic wrt any of it's inscribed triangles
with a particular isoconjugation wrt this triangle.
The 3rd intersections of the sides of the triangle with the
cubic are aligned and the root is the trilinear pole of this
line.
If the triangle is the triangle of the pretangentials of the
flexpoints (one triangle for a monopartite cubic, 3 for a
bipartite cubic), naturally the root is the trilinear pole of
the line of the real flexes and, according to your statement,
one of the fixed points of the isoconjugation.
But it is a selfdefining property !
Is that what you mean ?
Best regards
Bernard
```

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Message: #2847
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear M@IMF <mailto:M@IMF> ,

if we consider a triangle ABC and a point P

... with its trilinear polar p

... and its anticevians Pa,Pb,Pc,

... which are with P the fixed points of an isoconjugation,

... then the dual of Pa wrt the QL(AB,BC,CA,p) is PbPc

... and PaPbPc is the QL-diagonal.

I can confirm your final observation.

Best regards Eckart

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Message: #2848
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear Mr. Schmidt,

Thank you for the reply.
The colinearity of "3rd intersections" of nK and reference
triangle was
a bit surprise for me although a cubic can bear any 9 points.

Best regards,
M@IMF
```

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Message: #2849
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard

you wrote wrt my question in #2842:

"Every cubic is an isocubic wrt any of it's inscribed triangles
with a
particular isoconjugation wrt this triangle.
The 3rd intersections of the sides of the triangle with the
cubic are
aligned and the root is the trilinear pole of this line."

I think, this is not correct.
How do you find a reference triangle for a 9 point-cubic
... its root and isoconjugation for any inscribed triangle?

Best regards Eckart
```

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Message: #2850
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris, dear Bernard,

let me describe a way from a 9P-cubic

... to a reference triangle, root and isoconjugation:

Starting with a 9P-cubic CU,

... for the flexpoints Fi three secants

... ... must have 6 coconic further CU-intersections.

Varying a CU-point A with B = AF1 ^ CU, C = BF2 ^ CU, D = CF3 ^ CU,

... we get for D = A the only ref-triangle ABC.

The trilinear pole of F1F2F3 wrt ABC is the root Po,

... fixpoint of an ABC-isoconjugation,

... which lets the cubic invariant.

Excuse, that it is not an exact construction.

Best regards Eckart

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Message: #2851
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
I no longer understand you !
If you know the flexpoints on your cubic, you already have the reference triangle (vertices are the pretangentials of the Fi) as well as the root as trilinear pole of the line of real flexes, hence the isoconjugation ...
If you don't know the flexpoints, any inscribed triangle ABC can serve as reference triangle of an isoconjugation. Take a variable point P as fixed points, the 3 other are the vertices of it's anticevian triangle and there is a position of P giving an isoconjugation wrt ABC in which the cubic is invariant. The cubic is then a nK with this isoconjugation wrt this triangle and the root is the trilinear polar of the line A0B0C0 of the 3rd intersections of the sides of ABC with the cubic.
Note that the isoconjugation swaps the line A0B0C0 with a conic tritangent in A, B and C to the cubic.
Best regards
Bernard

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Message: #2852
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
Sorry, I have to correct my message.
An inscribed triangle ABC is the reference triangle of an isoconjugation only if the tangentials A1, B1 and C1 of A, B and C are aligned.
In this case, the 3rd intersections A0, B0 and C0 are also aligned (as a result of the calculations on a cubic).
Best regards
Bernard

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Message: #2853
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart and M@IMF ,
Back to the question in 2844 and 2845 and the answer in 2847 !
The dual QA of the QL formed by ABC and the trilinear polar of P wrt ABC is formed by the perspectors of the DT and the 4 triangles formed by 3/4 lines.
One vertice of the QA is P, obviously perspector of ABC and PaPbPc.
The 3 other vertices are the vertices of the anticevian triangle of P wrt PaPbPc.
It's remarkable that we have to consider 2times the anticevian triangle of P, 1rst wrt ABC, in order to get PaPbPc and 2nd wrt PaPbPc !
Best regards
Bernard

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Message: #2854
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear Mr. Keizer,

I usually consider in DT-coordinates, so it came to my mind that
a line of QL is the trilinear polar of the perspector of DT and
the component triangle formed by the other 3 lines wrt DT.
That's why I mixed up and had to correct my quesiton.
For me, a nK's bearing vertices of a QL is an eye-opener.

Last month I understood what Flexlines were at last.
The very next day new EPG launched.
This month I'm learning Cubics without tears.

I was overwhelmed by your "Hessian and generalised S-points
for a pivotal isocubic" again when I studied Hessian and
PreHessian.
I skipped that part before(#2736,37).

Best regards,
M@IMF
```

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Message: #2855
Date: 2025-12-08
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard,

thanks for your last three messages,

... your numerous observations show a good overview.

Best regards Eckart
```

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Message: #2856
Date: 2025-12-09
From: van10hoven@gmail.com
Subject: Re: Isocubics

```
Dear M@IMF,

It is impressive how well you have familiarized yourself with
the subject of cubics without prior involvement, and I
particularly appreciate the fresh perspectives you bring.

Regarding QL-8, the information is the same as for QA-8.
Previously, an incorrect link was attached to QL-8, but it has
now been fixed and links correctly.

Best regards,

Chris
```

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Message: #2857
Date: 2025-12-09
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard, Eckart and M@IMF,

Some questions.

QPG#2840 Bernard

Last and most important, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic.

Can you explain or proof this?

QPG#2850 Eckart

Varying a CU-point A with B = AF1 ^ CU, C = BF2 ^ CU, D = CF3 ^ CU,

... we get for D = A the only ref-triangle ABC.

That's quite a statement. Can you proof, that it is the *only* ref-triangle ABC on CU that defines an nK?

Best regards,

Chris

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Message: #2858
Date: 2025-12-09
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear Chris,

Wow! Thank you.
To tell the truth, I misunderstood Mr. Schmidt's message #2847.
(Please don't ask what it is. How stupid I was.)

Best regards,
M@IMF
```

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Message: #2859
Date: 2025-12-09
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
All these properties are linked !
Any triangle ABC inscribed in CU can be the reference triangle of a nK, provided the tangentials A1, B1 and C1 of A, B and C are aligned.
But I don't know how to construct the fixed points.
In this case, the 3rd intersections A0, B0 and C0 are also aligned, A1, B1 and C1 being their tangentials.
There is a conic tritangent in A, B and C to the cubic, which is the isoconjugate of the line A0B0C0 (BG page 11).
If A0 = A1, it is a flex, the same for Bo = B1 or C0 = C1.
If the lines A0B0C0 and A1B1C1 coïncide in the line of real flexes, ABC is Eckart's triangle, but it is not the only triangle for which the cubic is a nK
Let's use the calculations on the cubic.
It follows from the definition that A + B + C0 = M and so on and that 2A + A1 = M and so on ...
If A1 + B1 + C1 = M, 2A + 2B +2C = 2M (conic tritangent to the cubic).
Then in turn, A0 + B0 + C0 = M.
2A = 2 A0 and A and A0 have the same tangential, which is A1 and so on ...
If A0 = A1, we have 3A0 = M, which means that A0 is a flex ...
I don't know why, if the line A0B0C0 (or A1B1C1) is the line of real flexes, the trilinear pole of A0B0C0 is one of the fixed points of the isoconjugation.
You have to ask Eckart, as it is his statement since the beginning !
Best regards
Bernard

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Message: #2860
Date: 2025-12-10
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

all my results are only observations of drawings,

... I think, there is only one solution for the triangle.

Best regards Eckart

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Message: #2861
Date: 2025-12-10
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
Is it correct that the isoconjugate of the line of real flexes
is a conic tritangent to the cubic in A, B and C where A, B and
C are the pretangentials of the flexpoints and one fixed point
of the isoconjugation is the root, trilinear pole of the line of
real flexes ?
Best regards
Bernard

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Message: #2862
Date: 2025-12-10
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard,

your description of our discussed configuration is correct.

Best regards Eckart
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Message: #2863
Date: 2025-12-11
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard,

an observation:

For a cubic we consider
... a ref-triangle ABC,
... an isoconjugation with fixed points,
... a root Po
... a further cubic-point Q.

The isoconjugation can be described
... with the root Po as fixpoint.

Or the isoconjugation can be described
... with a fixpoint, not necessary the root,
... for the root has to be that fixpoint,
... which don't lie on a cubic-tangent for A,B,C.

Best regards Eckart
```

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Message: #2864
Date: 2025-12-11
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear all,

I hope this will be some help. (cf. SITP 1.5.1)
Let the pole of nK be (uu' : vv' : ww'), then
nK: vw(w'y + v'z)x^2 + wu(u'z + w'x)y^2 + uv(v'x + u'y)z^2 + kxyz = 0
vertices of reference triangle: A(1 : 0 : 0), B(0 : 1 : 0), C(0 : 0 : 1)
root and its isoconjugate: P = (u : v : w), P* = (u' : v' : w')
their trilinear polars: IP(P) = (1/u : 1/v : 1/w), IP(P*) = (1/u' : 1/v' : 1/w')
feet of IP(P): U = (0 : v : -w), V = (-u : 0 : w), W = (u : -v : 0)
feet of IP(P*): U' = (0 : v' : -w'), V' = (-u' : 0 : w'), W' = (u' : -v' : 0)
circumconic IP(P)*: u'/x + v'/y + w'/z = 0
tangents of nK at A,B,C: (0 : w' : v'), (w' : 0 : u'), (v' : u' : 0)
tangent of nK at U: (u(wv' + vw') - k : -uu'w : -uu'v)
tangent of nK at V: (-vv'w : v(uw' + wu') - k : -vv'u)
tangent of nK at W: (-ww'v : -ww'u : w(vu' + uv') - k)
tangential of A: A˜ = (uu'(wv' - vw') : v'[u(wv' + vw') - k] : -w'[u(wv' + vw') - k])
tangential of B: B˜ = (-u'[v(uw' + wu') - k] : vv'(uw' - wu') : w'[v(uw' + wu') - k])
tangential of C: C˜ = (u'[w(vu' + uv') - k] : -v'[w(vu' + uv') - k] : ww'(vu' - uv'))
line through A˜,B˜,C˜: ([u(wv' + vw') - k]/u' : [v(uw' + wu') - k]/v' : [w(vu' + uv') - k]/w')
IP(P)∩IP(P*): (uu'(wv' - vw') : vv'(uw' - wu') : ww'(vu' - uv'))
(U = A0, V = B0, W = C0, A˜ = A1, B˜ = B1, C˜ = C1 in #2859.)

When P = P* (i.e. the root is a fixed point of isoconjugation), the nK becomes
(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z' = 0,
and its Hessian is
2(k'-1)(x'^3 + y'^3 + z'^3 - 3x'y'z') + (k'^2)[(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z'] = 0 ,
where
x' = x/u, y' = y/v, z' = z/w, k' = k/2uvw.
The real flexes are (0 : v : -w), (-u : 0 : w), (u : -v : 0), that is, U,V,W.

```
If k'=1, the Hessian coincides nK
(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2x'y'z' = 0,
i.e.
(wy + vz)(uz + wx)(vx + uy) = 0.
I think this will be Real Flex Trilateral.
Its vertices are (-u : v : w), (u : -v : w), (u : v : -w).
I'm trying to calculate other flexes now.

Best regards,
M@IMF
```

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Message: #2865
Date: 2025-12-12
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear all,

I'm afraid the last paragraph of #2864 is misleading.
It only says (singlar/degenerated) nK
(wy + vz)(uz + wx)(vx + uy) = 0
is its Real Flex Trilateral. (Maybe trivial.)
I thought it has something to do with nK with k' <> 1, but it
doesn't.
Their flexes are different, only real ones are same.

I'm wondering a triangle formed by 3 real flextangents will play
an important role.
For the nK whose root is a fixed point, its vertices are
(uvw - k : wv^2 : vw^2), (wu^2 : uvw - k : uw^2), (vu^2 : uv^2 :
uvw - k).
Note that if it is the reference triangle, the cubic is no
longer isocubic.

Best regards,
M@IMF
```

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Message: #2866
Date: 2025-12-12
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

In QPG #2859 you write: ‘Any triangle ABC inscribed in CU can be the reference triangle of a nK, provided the tangentials A1, B1 and C1 of A, B and C are aligned.’

Some comments:

1. I inscribed an arbitrary triangle ABC an arbitrary 9-point cubic, but in such a way that the third intersections of the sidelines of the inscribed triangle are collinear on a line L. From this line L I constructed the trilinear pole W. According to the theory, W is then the isoconjugate of the centroid of ABC. This determines the isoconjugation, and therefore allows us to determine other isoconjugates as well.

However, when I take a point X on the cubic and compute its isoconjugate, that point does not lie on the cubic. This shows that an arbitrary inscribed triangle in a general cubic does not automatically make the cubic an nK. This raises the question: what conditions must an inscribed triangle satisfy, within a general cubic, in order for the cubic to become an nK?

2. You also said you don't know how to construct the fixed points in the configuration of an inscribed triangle ABC provided the tangentials A1, B1 and C1 of A, B and C are aligned.

I think this can be done by combining the different constructions of the isoconjugate.

The isoconjugate as described by BG has a pole. The isoconjugate as described by GP has a point K, which I will refer to as the isoconjugate center. This point is one of the four fixed points. The other three fixed points can be obtained as the vertices of the anticevian triangle of K with respect to ABC.

Thus the question becomes: how can we determine the isoconjugate center K from the pole W?

There is another construction of the isoconjugate, which I have called the Perspective Conjugate (TR-Tf3 (https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf3/)) within the theory of Perspective Fields. The idea is that the traces of the two isoconjugated points P and Q on the sidelines of the triangle are perspectively reflected around the trace, on the same sideline, of the isoconjugate center M.

Now, when we know X2 and W (isoconjugated points) and want to determine M (the isoconjugate center), the construction tells us that the traces of X2 and W are reflections of each other around the trace of M. Thus the trace of M is the perspective center of the traces of X2 and W. To determine this midpoint, we need the vanishing point V on the corresponding sideline, which is obtained as the intersection of that sideline with the trilinear polar of M with respect to ABC.

This is constructible, and therefore M is determined, and with it the four fixed points.

See TR-Tf4 (https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf4/) for the construction.

Best regards,

Chris

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Message: #2867
Date: 2025-12-13
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

I think, I have answered your first #2866-question to Bernard in #2850.

Best regards Eckart

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Message: #2868
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
In point 1, thetrilinear pole of the line is not the pole' but
thr root of the nK !
In point 2, thanks for this construction I didn't know ...
I'm not home, I'll make you a complete answer as soon as
possible.
Best regards
Bernard

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Message: #2869
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear M@IMF ,
I'm impressed by your barycentric calculations.
The triangle mentionned in the end is not the triangle of real
flexlines !
It is the triangle with sides the tangents in A, B and C to the
cubic.
Furthermore, it is the anticevian triangle of the root P wrt the
triangle ABC and it is perspective to this triangle with
perspector the isoconjugate P* of P.
This gives the 2 isoconjugate points wrt ABC needed in order to
déterminante the 4 fixed points of the isoconjugation (not
necessary real and not necessary on the cubic).
Best regards
Bernard

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Message: #2870
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,
I think it is now possible to make a short summary of the mystery of the cubic.
II hope it will answer all the last remaining questions.
1) 2 points X and X* are said conjugate (Salmon or Schroter) if they share the same tangential X1, which is the tangential of the 3rd intersection of the line XX* with the cubic.
2X = 2X* = M - X1 and 2(M- X -X*) = 2(M - 2X) = 2X1
The transformation swapping X and X* is a QMT (quasi-Moebius transformation) studied earlier in the Forum. There are 3 such transformations on each cubic (one real for monopartite and 3 for bipartite).
2) All these copples of conjugates are isoconjugate wrt any triangle ABC inscribed in the cubic, provided the tangentials A1, B1 and C1 of A, B and C are aligned.
In this case, the 3rd intersections A0, B0 and C0 are the isoconjugates of A, B and C and are also aligned and the root P is the trilinear pole of this line. The triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of P and is perspective to ABC with perspector P*' the isoconjugate of P.
The 2 points P and P* determine the isoconjugation wrt ABC and it's possible to find the 4 fixed points.
3) If for one particular triangle ABC the root P is one of the fixed points of the isoconjugation, P = P* and A0, B0 and C0 and A1, B1 and C1 coïncide, which means that these 3 points are the flexes of the cubic.
This observation of Eckart is here fully explained and proved.
Best regards
Bernard

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Message: #2871
Date: 2025-12-14
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

```
Dear Bernard,

wrt 2) in #2870: I repeat my conjecture,

... that there is only one triangle with your condition,

... see #2850, which can also be formulated:

... AF2 ^ CU and AF3 ^ CU have to be collinear with F1.

Best regards Eckart
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Message: #2872
Date: 2025-12-14
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

```
Dear Eckart,
Um Gottes Willen, can you read the message completely !
Your triangle is my point 3, but there are an infinity of
triangles in point 2 wrt which the cubic is a nK and *A0, B0 and
C0 are not the flexes* !!!
In this case, the root has an isoconjugate (see BG).
I'm really desperate, as I don't see how to make you understand
what I mean.
Best regards
Bernard
```

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Message: #2873
Date: 2025-12-15
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

already the first passage of your #2870 seems wrong,

... I think the sentence has to end:

... "whose tangential is the 3rd intersection of the line XX* with the cubic."

I made a drawing of this constellation, starting with a 9P-cubic

... and constructed then as described in #2850 the isoconjugation,

... which gave the same result as your corrected 1).

Best regards Eckart

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Message: #2874
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
Thanks for your attention, I prefer you this way ...
1) The correct sentence is which is the *conjugate* (and not tangential) of the 3rd intersection of XX* with the cubic
2) There is another mistake in point 2 the triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of P* (and not P) and is perspective to ABC with perspector P*, P* being the isoconjugate of P.
3) Naturally in point 3, as P = P* being a fixed point of the isoconjugation, the anticevian triangles of P and P* coïncide and their vertices are the 3 other fixed points.
Best regards
Bernard

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Message: #2875
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear M@IMF ,
I made the same mistake in my messages 2869 and 2870
The triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of *P** (and not P).
If P = P* in your calculation, the pole is (u^2,v^2,w^2) and the root and fixed point is (u,v,w), the 3 other fixed points being (-u,v,w), (u,-v,w) and (u,v,-w).
Best regards
Bernard

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Message: #2876
Date: 2025-12-15
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear Mr. Keizer,

Thank you for the message.
Since I'm studying mainly P=P* case, I didn't notice the
mistake.
Po(=P=P*) is the intersection of 3 real Harmonic Polars
as well as the trilinear pole of the main real Flexline
(, which through 3 real flexes) wrt the reference triangle.
The anticevian triangle of Po is flextangent trilateral of
preHessian, isn't it?

Best regards,
M@IMF

p.s.  What I feared happened. That's why I posted #2865.
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Message: #2877
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

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Dear M@IMF ,
Your last proposition is correct
Best regards
Bernard
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Message: #2878
Date: 2025-12-15
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

I am trying to understand your messages #2870 and #2874.

I began by drawing a random 9-point cubic together with an inscribed triangle ABC, and I constructed the corresponding tangential points A1, B1, C1, arranging them so that they are collinear.

Next, from the sidelines AB, BC, CA, I constructed the third intersection points C0, A0, B0, which indeed turn out to be collinear.

From the line A0B0C0 I then constructed the trilinear pole P with respect to triangle ABC.

According to #2870/#2874, the following should hold: the triangle formed by the tangents at A, B, C to the cubic is the anticevian triangle of P*.

My question is: why is this point P* (the isoconjugate of P)? What is the underlying theory?

Once this is understood and verified, the two points P and P* determine the isoconjugation with respect to ABC, and it becomes possible indeed to locate the four fixed points.

This can be done by using BG's construction in https://bernard-gibert.fr/gloss/isoconjugation.html, as well as the construction in TR-Tf4 (https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf4/) like I noted before.

I have not yet carried out this construction to verify the validity of the statements above.

These are my findings so far.

Best regards,
Chris

Message: #2879
Date: 2025-12-16
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

finally I have now reproduced your constructions in #2870.

In my message #2850 I described the case,

... that the tangentials of A, B, C are the collinear real flexpoints.

Best regards Eckart

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Message: #2880
Date: 2025-12-16
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,
The simplest way to find a suitable triangle ABC inscribed in a general cubic (here monopartite) is to draw a line, which intersect the cubic in 3 points A1, B1 and C1.
The tangents from A1, B1 and C1 contact the cubic in A and A0, B and B0 and C and C0 (3 copples of conjugate points).
It appears that these 6 points form a QL and there are in fact 4 possible triangles, let say ABC, AB0C0, A0BC0 and A0B0C, the root being the trilinear pole of the 4th line A0B0C0, A0BC, AB0C and ABC0.
For Chris, the properties mentionned for P and P* are explained page 11 of Bernard Gibert.
They can be proved easily with a barycentric calculation like the one made by I@MIF...
Best regards
Bernard

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Message: #2881
Date: 2025-12-16
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear all,

I calculated real flexline trilateral of nK whose root is a fixed point of isoconjugation.
When the nK is
(y/v + z/w)(x/u)^2 + (z/w + x/u)(y/v)^2 + (x/u + y/v)(z/w)^2 + 2k'xyz/uvw = 0
(cf. #2864 (
https://groups.io/g/Quadri-and-Poly-Geometry/message/2864)),
the vertices of real flexline trilateral are
(-u(1 + ω)(1 + ηω)/(1 + η/ω) : v : w)
(u : -v(1 + ω)(1 + ηω)/(1 + η/ω) : w)
(u : v : -w(1 + ω)(1 + ηω)/(1 + η/ω)),
where ω is a primitive cubic root of unity and
η is a unit complex number which satisfies
η^3 + (3 - 2k'ω)/(1/ω - ω) η^2 - (3 - 2k'/ω)/(ω - 1/ω) η - 1 = 0.
I hope there is no mistake.

Best regards,
M@IMF

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Message: #2882
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF ,
This time, I'm no longer only impressed, but I'm totally bluffed by your barycentric calculations !
Sincere congratulations
I have now 2 questions :
1) Could you explain how you managed this (at last a starting point, in order to understand where this strange η comes from)
(I was only able to verify that H1 is on the P0A and that H2H3 passes through F1)
2) I would be completely satisfied if you could express the equation of the nK with the referential H1(1,0,0), H2(0,1,0), H3(0,0,1),
id est to reverse from the coordinates of H1,H2 and H3 wrt the referential A,B,C to the coordinates of A,B,C wrt H1,H2,H3
(in the referential H1, H2, H3, as you know P0 has coordinates 1/a1,1/a2,1/a3, the line of real flexes is a1x + a2y +a3z = 0 and the cubic is $a1^3x^3 + a2^3y^3 + a3^3z^3$ + ka1a2a3xyz = 0)
Many thanks in advance
Best regards
Bernard

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Message: #2883
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,
Using the preceding property, it is perhaps interesting to notice that the cubic QL-Cu1 is a nK wrt each of the 4 reference triangles with root the trilinear pole of the 4th line wrt this triangle ...
Best regards
Bernard

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Message: #2884
Date: 2025-12-17
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear Mr. Keizer and all,

Let me explain 1) in #2882.

< *nK and Hessian* >
We start from 2 equaions in #2864 (https://groups.io/g/Quadri-and-Poly-Geometry/topic/116466734?page=2#msg2865) :
(1)   (y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z' = 0
(2)   2(k'-1)(x'^3 + y'^3 + z'^3 - 3x'y'z') + (k'^2)[(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z'] = 0.
(The primes are annoying, but I retain them.)

< *Flexes* >
The flexes are the intersections of (1) and (2). We solve (1) and the next equation
(3)   x'^3 + y'^3 + z'^3 - 3x'y'z' = 0,
which leads to
(3a)   x' + y' + z' = 0
(3b)   x' + y'ω + z'/ω = 0
(3c)   x' + y'/ω + z'ω = 0,
where ω is a primitive cubic root of unity.

>From (3a) and (1), we get
(x' : y' : z') = (0 : 1 : -1), (-1 : 0 : 1), (1 : -1 : 0),
which are real flexes.

>From (3b),
x' = -y'ω - z'/ω
is obtained and substituting it into (1) yields
(4)   y'^3 + (3 - 2k'ω)/(1/ω - ω) z'y'^2 - (3 - 2k'/ω)/(ω - 1/ω) y'z'^2 - z'^3 = 0.
Let Yj (j=1,2,3) be the solutions of 3rd degree equation
(5) *Y^3 + (3 - 2k'ω)/(1/ω - ω) Y^2 - (3 - 2k'/ω)/(ω - 1/ω) Y - 1 = 0* ,
then
(x' : y' : z') = (-Yjω - 1/ω : Yj : 1).

>From (3c) and (1), we get
(6)   y'^3 + (3 - 2k'/ω)/(ω - 1/ω) z'y'^2 - (3 - 2k'ω)/(1/ω - ω) y'z'^2 - z'^3 = 0,
and
(x' : y' : z') = (-1/Yjω - ω : 1/Yj : 1).

< *Flexlines* >
A line through (-Yjω - 1/ω : Yj : 1) and (-1/Yjω - ω : 1/Yj : 1) is
(l' : m' : n') = (ω(1 + Yj)/(1 + Yj/ω) : 1 : 1)
which is also through (0 : 1 : -1). Other flexlines are
(1 : ω(1 + Yj)/(1 + Yj/ω) : 1), (1 : 1 : ω(1 + Yj)/(1 + Yj/ω)).

If 1/Yj = Yj^* (i.e. |Yj| = 1), ω(1 + Yj)/(1 + Yj/ω) is real,
where ^* means complex conjugate.
>From (5), we can show 1/Yj and Yj^* satisfy the same equation,
therefore {1/Y1, 1/Y2, 1/Y3} = {Y1^*, Y2^*, Y3^*}. Since
(1/Y1, 1/Y2, 1/Y3) = (Y2^*, Y3^*, Y1^*) and (1/Y1, 1/Y2, 1/Y3) = (Y3^*, Y1^*, Y2^*)
are impossible, *at least 1 solution is a unit complex number*.
Let *η* denote it.
(I don't know whether (1/Y1, 1/Y2, 1/Y3) = (Y1^*, Y2^*, Y3^*) is possible.)

< *Real Flexline Trilateral* >
The real flexlines are (1 : 1 : 1) and
(ω(1 + η)/(1 + η/ω) : 1 : 1)
(1 : ω(1 + η)/(1 + η/ω) : 1)
(1 : 1 : ω(1 + η)/(1 + η/ω)).
Finally, the vertices of real flexline trilateral are
(-(1 + ω)(1 + ηω)/(1 + η/ω) : 1 : 1)
(1 : -(1 + ω)(1 + ηω)/(1 + η/ω) : 1)
(1 : 1 : -(1 + ω)(1 + ηω)/(1 + η/ω)).

When I calculated flexes before(#2865), I didn't realize the property of (5).
Then I tried to let the flextangent trilateral mediate ABC and H1H2H3.
But I came up with above calculation, so I didn't do that.
Please give me some time to prepare for answering 2).

Best regards,
M@IMF

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Message: #2885
Date: 2025-12-17
From: van10hoven@gmail.com
Subject: Re: Isocubics

Beste Bernard, M@IMF and Eckart,

There is something I cannot understand. In SITP 1.2.1 it says:

“We now define the pole of the isoconjugation as the isoconjugate Ω = G* of the centroid G. In other words, Ω is the intersection of the two polar lines of G in the two conics γ (P) and γ (P*). This shows that there is no need of coordinates to define an isoconjugation. Nevertheless, since a lot of computation is needed for this paper, we will make use of barycentric coordinates and, if Ω = (p : q : r), the isoconjugation with pole Ω is the mapping : φ Ω :M(x:y :z) → M* p x : q y : r z ~ (p y z : q z x : r x y)”

Günther Pickert’s Isoconjugate (QPG #2068) uses a fixed point K, and after carrying out his construction the result is (p^2 y z : q^2 z x : r^2 x y).

I therefore conclude that Pickert’s point K is different from the pole described by BG. Can you confirm this?

And one more question for M@IMF: In your message #2864 you begin with “Let the pole of nK be (uu' : vv' : ww')”. Why do you start with this combined expression?

Best regards,

Chris

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Message: #2886
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
If the pole of the isoconjugation is (p^2,q^2,r^2), then the fixed points are (p,q,r), (-p,q,r), p,-q,r) and (p,q,-r) !
In EQF, the pole is QA-P16, isoconjugate of the centoïd QA-P10
...
BG uses the notion of pole of the isoconjugation, I suppose GP uses a fixed point of the same isoconjugation.
Best regards
Bernard

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Message: #2887
Date: 2025-12-17
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

```
Dear Chris and all,

Let the pole be (P:Q:R), then the isoconjugate of P(u:v:w) is
(P/u : Q/v : R/w).
When P* = (u':v':w'), we get P = uu', Q = vv', R = ww'.
Note that the pole depends on the coordinate system.
(Maybe this is "Shaka ni Seppou" (Preaching to the choir).)
Barycentric:
isogonal conjugate (a^2/x : b^2/y : c^2/z)
pole = X6(a^2 : b^2 : c^2)
fixed points: X1(a:b:c), the excenters
isotomic conjugate (1/x : 1/y : 1/z)
pole = X2(1:1:1)
fixed points: X2(1:1:1), the vertices of anti-medial triangle
Trilinear:
isogonal conjugate (1/x' : 1/y' : 1/z')
pole = X1(1:1:1)
fixed points: X1(1:1:1), the excenters
isotomic conjugate (1/x'a^2 : 1/y'b^2 : 1/z'c^2)
pole = X75(1/a^2 : 1/b^2 : 1/c^2)
fixed points: X2(1/a : 1/b : 1/c), the vertices of anti-medial
triangle

Best regards,
M@IMF

p.s. Another example
   x-coordinate   x'-coordinate
A   (1 : 0 : 0)   (-(uK + v + w)(K+1) : (u + vK + w) : (u + v +
wK))
B   (0 : 1 : 0)   ((uK + v + w) : -(u + vK + w)(K+1) : (u + v +
wK))
C   (0 : 0 : 1)   ((uK + v + w) : (u + vK + w) : -(u + v +
wK)(K+1))
A'  (uK : v : w)  (1 : 0 : 0)
B'  (u : vK : w)  (0 : 1 : 0)
C'  (u : v : wK)  (0 : 0 : 1)
Po  (u : v : w)   ((uK + v + w) : (u + vK + w) : (u + v + wK))
When the pole of isoconjugation wrt ABC is (u^2 : v^2 : w^2),
that wrt A'B'C' is ((uK + v + w)^2 : (u + vK + w)^2 : (u + v +
wK)^2). Po is a fixed point.
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Message: #2888
Date: 2025-12-18
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
I answered your message immediately, without checking anything !
But it appears GP's construction is exactly the same as BG's construction page 7 Remark 1
With a triangle and

* a fixed point K (GP), you have the 3 other fixed points as vertices of the anticevian triangle of K wrt the triangle.

You may then have the isoconjugate of any point by using GP's construction from any of the fixed points (Ka and K'a are the same ...)
In particular, the isoconjugate of the centroïd G is the pole Ω

* 2 conjugate points (in particular G and Ω), you have the fixed points by using BG's construction

Note that the 4 fixed points are real only if Ω lies inside the triangle, hence p, q, r >0
Best regards
Bernard

Message: #2889
Date: 2025-12-19
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF ,
Many thanks for these explanations !
I've tried to reproduce the calculations.
The 9 flexes (3 real and 6 imaginary) are on 12 lines (4 real and 8 imaginary), forming 4 triangles, the 1rst one H1HH3 has 3 real sides and is the real flextrilateral.
The 2nd has only one real side, which is the line of real flexes and only one real vertex, which is the point P0, fixed point of the isoconjugation and root of the cubic.
The 3rd and 4th have 3 imaginary sides and 3 imaginary vertices.
All this is well-known !
It is remarkable that the equations of the 1rst and 2nd triangles wrt the referential H1H2H3 are precisely

* $x'y'z' = 0$ and
* $x'^3 + y'^3 + z'^3 - 3x'y'z' = 0$ (your equation (3) wrt ABC)

Each of these 4 triangles forms a cubic belonging to the Hesse pencil and each of these 4 cubics is it's own hessian ...
I'm waiting impatiently for your equation of the pencil of cubics wrt H1H2H3 as a Christmas gift.
Best regards
Bernard

Message: #2890
Date: 2025-12-19
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF, Bernard and Eckart,

Thank you all for the wonderful messages and answers.

So many different insights, and all of them very welcome.

I will now give my own interpretation.

We have a general cubic CU with a general equation.

$c_1 x^3 + c_2 y^3 + c_3 z^3 + c_4 x^2y + c_5 x^2z + c_6 xy^2 + c_7 y^2z + c_8 xz^2 + c_9 yz^2 + c_{10} xyz$

We always inscribe a triangle in CU in whatever way is convenient, in order to transform the general equation into a nicer one, for the sake of simplification or applicability.

Flextrilateral as inscribed reference triangle

When we use the real flextrilateral as the reference triangle, the general equation changes into the first normal form:

$a_1 x^3 + a_2 y^3 + a_3 z^3 + k a_1 a_2 a_3 x y z$

which, through a projective transformation ($a_1x \to x$; $a_2y \to y$; $a_3z \to z$), can be converted into *Hesse's Form* :

$x^3 + y^3 + z^3 + k x y z$

nK-construction as inscribed reference triangle

When we deal with an nK-construction, in which we inscribe a reference triangle ABC in such a way that the third intersection points of the ABC-sidelines are collinear.
We then obtain a normal form (as described by BG in SITP):

$u x (r y^2 + q z^2) + v y (p z^2 + r x^2) + w z (q x^2 + p y^2) + k x y z$

where the *root* of the isoconjugation is P(u:v:w) and the *pole* of the isoconjugation is (p:q:r).

Eventually it turns out that there is a relation between root and pole: (p~u 2 , q~v 2 , r~w 2).

By substituting this, the following normal form arises, as named by M@IMF in QPG#2881:

$(x^2/u^2)(y/v + z/w) + (y^2/v^2)(z/w + x/u) + (z^2/w^2)(x/u + y/v) + 2 k x y z / (u v w)$

which can be transformed projectively accordingly (also noted by M@IMF in QPG#2884) into:

$(x + y) z^2 + y^2 (x + z) + x^2 (y + z) + 2 x y z k$

With this, just as with the first normal form for Hesse's Form, all kinds of calculations can be applied.

It is good to remember that all these equations describe the same cubic. The only difference is the inscribed reference triangle from which the calculations are carried out.

I am considering renaming these normal forms,

for example the *1st and 2nd Flex-Normal Form* ,

and the *1st and 2nd nK-Normal Form*.
It all revolves around a general cubic that is presented differently depending on the inscribed reference triangle that is chosen.

There can be more interesting normal forms.

Best regards,

Chris

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Message: #2891
Date: 2025-12-19
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Chris and all,

I'm afraid the real flextrilateral isn't an inscribed triangle
in the cubic,
although it is a member of the family of triangles such that
vertices lie on
real Harmonic Polars and edges bear real flexes.

When the root is (p^2 : q^2 : r^2), pK is written as
(u/p)(x/p)[(y/q)^2 - (z/r)^2] + (v/q)(y/q)[(z/r)^2 - (x/p)^2] +
(w/r)(z/r)[(x/p)^2 - (y/q)^2] = 0.
(2nd pK-Normal Form?)
pK is a circumcubic of QA, nK is a circumcubic of QL.
Interesting!

I'm interested in flextangent trilateral as a reference
triangle.
The cubic has a general form but cofficients are not
independent.

Best regards,
M@IMF

p.s. My message #2887 might be a bit beside the point.
Also sorry for using confusing symbols.
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Message: #2892
Date: 2025-12-19
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

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Sorry, the root should be the pole. M@IMF
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Message: #2893
Date: 2025-12-20
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris,
Nice summary !
4 remarks :
1) The real flextrilateral H1H2H3 is not inscribed in the cubic
(as already pointed by I@IMF)
(Neither are the 3 other triangles through the 9 flexes)
2) In the general nK, there is no relation between the pole and
the root, which are totally independant !
For example, QL-Cu1 is a nK wrt the 4 reference triangles with
root in the trilinear pole of the 4th line ; isoconjugation will
be the Moebius transformation ...
3) If and only if the root is one of the fixed points of the
isoconjugation, then it holds that $p = u^2$, $q = v^2$ and $r = w^2$
This happens only for a particular triangle, formed by the
contact points of the tangents from the flexpoints to the cubic
(other than the flextangent)
In this case, the lines A0B0C0 and A1B1C1 coïncide with the line
of real flexes.
4) A pK can always be described as a nK, but I think a nK can be
described as a pK only if it is bipartite
For example, QA-Cu7 is a perfect nK wrt the DT, with fixed
points of the isoconjugation the vertices of the reference QA ;
but it is also a QL-Cu1 as well as a QA-Cu1 ...
As you say, all depends of the reference triangle.
Best regards
Bernard
PS Merry Christmas and Happy New Year to all of you !

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Message: #2894
Date: 2025-12-21
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

```
Dear Mr. Keizer and all,

As for 2) in #2882, let
H1 = (-ξu : v : w)
H2 = (u : -ξv : w)
H3 = (u : v : -ξw),
where
ξ = (1 + ω)(1 + ηω)/(1 + η/ω),
which satisfies
(ξ^3 - 3ξ^2 + 9ξ - 5) - 2k'(ξ^2 - ξ + 1) = 0
When the reference triangle is changed from ABC to H1H2H3,
the coordinates of A,B,C become
A = ( (ξ - 1)(-ξu + v + w) : (u - ξv + w) : (u + v - ξw) )
B = ( (-ξu + v + w) : (ξ - 1)(u - ξv + w) : (u + v - ξw) )
C = ( (-ξu + v + w) : (u - ξv + w) : (ξ - 1)(u + v - ξw) ),
and the equation of nK
(y/v + z/w)(x/u)^2 + (z/w + x/u)(y/v)^2 + (x/u + y/v)(z/w)^2 +
2k'(x/u)(y/v)(z/w) = 0
becomes
CU:   (a1x")^3 + (a2y")^3 + (a3z")^3 + 3g(a1x")(a2y")(a3z") = 0,
where
(1/a1 : 1/a2 : 1/a3) = ((-ξu + v + w) : (u - ξv + w) : (u + v -
ξw))
3g = 6 - k'(ξ^3 - 3ξ - 2)/[ξ^2 - (k'+1)ξ + 1].

On the other hand,  the intersection of CU and
its Harmonic Polar (0 : a2 : -a3) is (-ξ"/a1 : 1/a2 : 1/a3),
where ξ" satisfies
ξ"^3 + 3gξ" - 2 = 0.
When 3g is given as above, (1 - ξ) satisfies this equation.

Best regards,
M@IMF
```

Message: #2895
Date: 2025-12-22
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard, dear Chris,

an observation for QL-Cu1

... and the line L of its real flexpoints:

CSC(L) is a circle CI, intersecting CU-Cu1

... in QL-P1 and three points A,B,C,

... which give a ref-triangle for QL-Cu1 as nK,

... using the trilinear pole Po of L as fixpoint

... of an isoconjugation *.

L* is a conic CO through QL-P1

... contacting the cubic in A,B,C

... with tangents through the real flexpoints.

For QL-Cu1-points the isoconjugate * and CSC coincide.

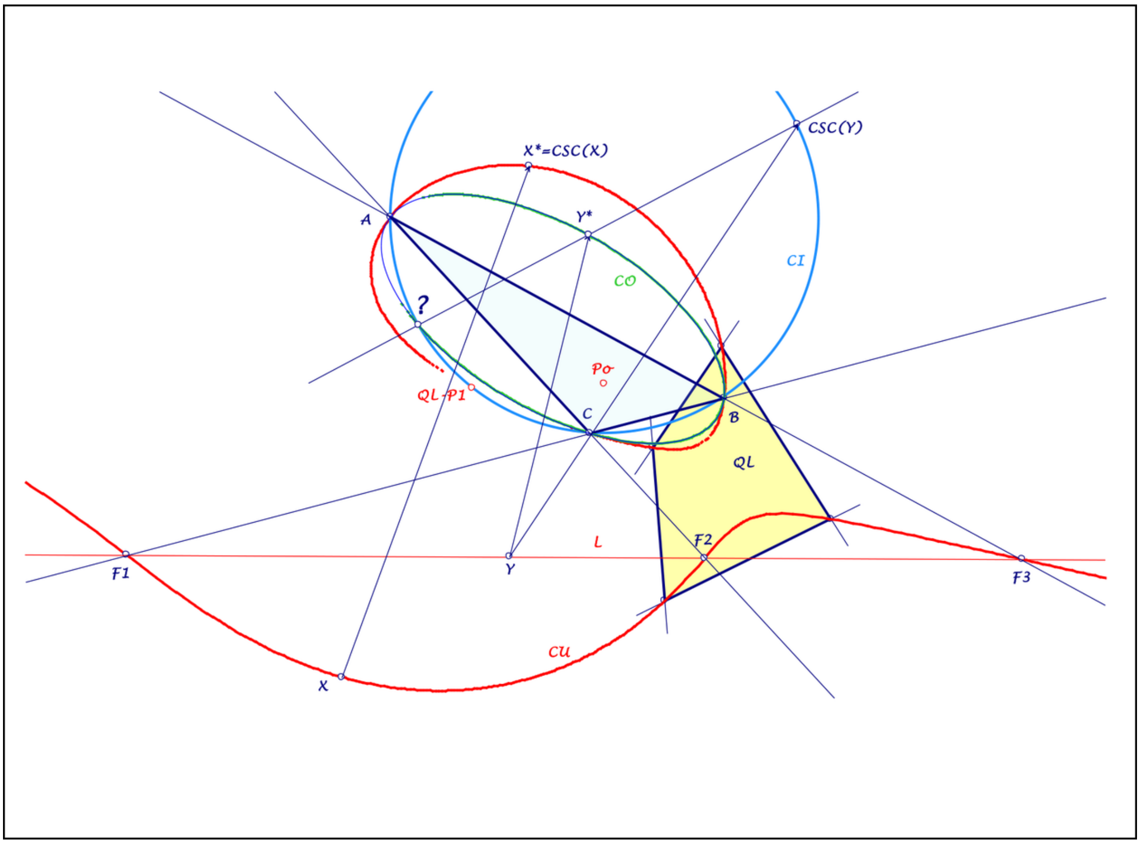
For L-points X the lines X*.CSC(X) have a common point

... in the 4th intersection of conic and circle:

... what about this point?

... Its Simson-line wrt ABC is orthogonal L

Best regards Eckart



2025-12-22.pdf

Message: #2896
Date: 2025-12-23
From: van10hoven@gmail.com
Subject: Season's Greetings and a Look Ahead

Dear friends,

As the year draws to a close, it feels like the right moment to pause and look back.

The forum saw a modest level of activity, yet it brought several remarkable and engaging contributions.

Besides myself, the following people shared their insights and work: Eckart Schmidt, Bernard Keizer, M@IMF, Keita Miyamoto, Antreas Hatzipolakis, Dylan Wyrzykowski, Tran Quang Hung, and Stanley Rabinowitz. My thanks to all of them for their thoughtful and special contributions. Our community now consists of around 30 members.

We also received the sad news of the passing of Angel Montesdeoca, who was once part of our group.

One of the highlights this year was the launch of the new website, which also brought together the former encyclopedias for Quadri-Figures and Poly-Figures and introduced a new section dedicated to n-curves (conics, cubics, and higher-degree curves).

I also intend to create Annual Journals of our posts. The work is already well underway, and I will share more as soon as it is ready.

It would be nice if, in the coming year, we could begin further developing the higher-degree curves, much as we once developed the Poly-Figures from the Quadri-Figures. I already have a few ideas in mind, which I will share when the time is right. For those waiting for a new opportunity to join in, this may also be a good moment.

I wish everyone a peaceful holiday season a good year ahead in whatever form it takes.

Best regards,

Chris

Message: #2897
Date: 2025-12-23
From: contiwa.goma3@gmail.com
Subject: Re: Season’s Greetings and a Look Ahead

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Dear all,

I wish you a Merry Christmas and a Happy New Year.
(No one point out about #2854. I should have said
"Wham! Thank you." in #2858.)

Best regards,
M@IMF
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5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

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6 Colophon

Sources & Contact

Web address (QPG Forum): https://groups.io/g/Quadri-and-Poly-Geometry
EPG Encyclopedia (content reference): https://www.chrisvantienhoven.nl
Editorial correspondence: van10hoven@gmail.com

Journal of the Quadri- and Poly-Geometry Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom

Published Volumes:

- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
- Volume 1 (Nov. 2019–Dec. 2019), messages #1–#60

Predecessor Journal:
Journal of the Quadri-Figures Group

ISSN: (to be assigned)
Published by: Uitgeverij Varenboom

Volumes of the predecessor journal:

- Volume 7 (Jan. 2019–Oct. 2019), messages #3280–#3906
- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393