

**Journal of the
Quadri- and Poly-Geometry Group
2025**

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Chris van Tienhoven et al.

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Contents

1	Introduction	2
2	Authors	3
2.1	Author Index	4
2.2	Author Information	6
3	Subjects	18
4	Messages	21
4.1	Message Index	21
4.2	Messages	24
5	Keyword Index	368
6	Colophon	371

1 Introduction

This journal is a compilation of messages from the **Quadri- and Poly-Geometry (QPG)** forum, where mathematicians and geometry enthusiasts exchange ideas on the properties of **quadrilaterals, polygons, and curves of n th degree**. The discussions cover a wide range of topics, from classical geometric theorems to new discoveries and insights.

The origins of this journal trace back to the Quadri Figures Group (QFG, available at <https://groups.io/g/Quadri-Figures-Group>), which was active from 2013 until November 2019. In November 2019, the forum transitioned into the Quadri- and Poly-Geometry Group (QPG, available at <https://groups.io/g/Quadri-and-Poly-Geometry>) forum, which continues to facilitate discussions on quadrilaterals, polygons, and related topics. Over the years, these forums have evolved into valuable resources for exploring both well-established results and novel perspectives in geometry. For both forums, an **annual record of all incoming messages** is compiled in this journal.

This journal is available in **PDF format** and includes a **table of contents** that organizes all messages by subject. Navigation is made easy through **hyperlinks** embedded in the message numbers, allowing users to quickly jump between related discussions or return to the table of contents for further reference.

Many of the topics discussed here are closely related to the Encyclopedia of Poly Geometry, available at <https://www.chrisvantienhoven.nl/>, which aims to systematically classify and analyze geometric structures. By collecting these forum messages, this journal serves both as a **historical archive** and as a **source of inspiration** for further research in the fascinating world of geometry.

[↔ Table of Contents](#) [↔ Message Index](#)

2 Authors

This section presents an alphabetical overview of the authors who contributed messages to this volume of the Journal.

- Ángel Montesdeoca - In Memoriam
- Antreas Hatzipolakis
- Bernard Keizer
- Chris van Tienhoven
- Dylan Wyrzykowski
- Eckart Schmidt
- Keita Miyamoto
- M@IMF
- Stanley Rabinowitz
- Tran Quang Hung

2.1 Author Index

This section provides an index of all authors who contributed messages to this volume of the Journal.

Each entry lists the author's name, their identifier, and the message numbers associated with their contributions. The list below shows the authors along with the numbers of related messages. Click on a number to go to the corresponding page.

- **Angel Montesdeoca**
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[#2647](#)
- **Antreas Hatzipolakis**
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[#2593](#) [#2602](#) [#2607](#) [#2614](#) [#2616](#) [#2666](#) [#2671](#)
- **Bernard Keizer**
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[#2562](#) [#2583](#) [#2584](#) [#2596](#) [#2598](#) [#2599](#) [#2609](#) [#2613](#) [#2617](#) [#2619](#)
[#2620](#) [#2622](#) [#2626](#) [#2628](#) [#2634](#) [#2637](#) [#2638](#) [#2639](#) [#2640](#) [#2644](#)
[#2646](#) [#2650](#) [#2652](#) [#2673](#) [#2678](#) [#2703](#) [#2705](#) [#2707](#) [#2714](#) [#2718](#)
[#2719](#) [#2723](#) [#2724](#) [#2726](#) [#2728](#) [#2731](#) [#2735](#) [#2738](#) [#2741](#) [#2743](#)
[#2748](#) [#2750](#) [#2751](#) [#2754](#) [#2756](#) [#2757](#) [#2758](#) [#2760](#) [#2762](#) [#2763](#)
[#2767](#) [#2773](#) [#2775](#) [#2777](#) [#2779](#) [#2780](#) [#2782](#) [#2799](#) [#2808](#) [#2809](#)
[#2810](#) [#2812](#) [#2813](#) [#2814](#) [#2819](#) [#2826](#) [#2827](#) [#2836](#) [#2838](#) [#2840](#)
[#2846](#) [#2851](#) [#2852](#) [#2853](#) [#2859](#) [#2861](#) [#2868](#) [#2869](#) [#2870](#) [#2872](#)
[#2874](#) [#2875](#) [#2877](#) [#2880](#) [#2882](#) [#2883](#) [#2886](#) [#2888](#) [#2889](#) [#2893](#)
- **Chris van Tienhoven**
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[#2577](#) [#2578](#) [#2582](#) [#2601](#) [#2604](#) [#2605](#) [#2610](#) [#2611](#) [#2615](#) [#2618](#)
[#2621](#) [#2627](#) [#2630](#) [#2632](#) [#2636](#) [#2642](#) [#2643](#) [#2647](#) [#2648](#) [#2667](#)
[#2668](#) [#2670](#) [#2672](#) [#2680](#) [#2683](#) [#2710](#) [#2715](#) [#2716](#) [#2720](#) [#2721](#)
[#2722](#) [#2732](#) [#2733](#) [#2744](#) [#2746](#) [#2764](#) [#2784](#) [#2790](#) [#2791](#) [#2795](#)
[#2796](#) [#2803](#) [#2806](#) [#2807](#) [#2816](#) [#2818](#) [#2820](#) [#2822](#) [#2824](#) [#2828](#)
[#2829](#) [#2831](#) [#2834](#) [#2839](#) [#2841](#) [#2856](#) [#2857](#) [#2866](#) [#2878](#) [#2885](#)
[#2890](#) [#2896](#)
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- **Eckart Schmidt**
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[#2568](#) [#2570](#) [#2573](#) [#2595](#) [#2597](#) [#2600](#) [#2603](#) [#2606](#) [#2608](#) [#2625](#)
[#2635](#) [#2641](#) [#2645](#) [#2657](#) [#2660](#) [#2661](#) [#2665](#) [#2669](#) [#2675](#) [#2677](#)
[#2681](#) [#2694](#) [#2696](#) [#2698](#) [#2700](#) [#2702](#) [#2704](#) [#2706](#) [#2708](#) [#2713](#)
[#2717](#) [#2725](#) [#2727](#) [#2730](#) [#2745](#) [#2747](#) [#2749](#) [#2752](#) [#2753](#) [#2755](#)
[#2759](#) [#2761](#) [#2765](#) [#2766](#) [#2768](#) [#2770](#) [#2771](#) [#2772](#) [#2776](#) [#2786](#)
[#2787](#) [#2789](#) [#2793](#) [#2798](#) [#2801](#) [#2802](#) [#2804](#) [#2805](#) [#2811](#) [#2817](#)
[#2821](#) [#2823](#) [#2825](#) [#2830](#) [#2832](#) [#2835](#) [#2837](#) [#2842](#) [#2843](#) [#2847](#)

[#2849](#) [#2850](#) [#2855](#) [#2860](#) [#2862](#) [#2863](#) [#2867](#) [#2871](#) [#2873](#) [#2879](#)
[#2895](#)

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[#2560](#) [#2561](#) [#2565](#) [#2566](#) [#2567](#) [#2575](#) [#2576](#) [#2579](#) [#2580](#) [#2586](#)
[#2592](#) [#2685](#) [#2686](#) [#2689](#) [#2690](#) [#2691](#) [#2693](#)

- **M@IMF**

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[#2569](#) [#2574](#) [#2581](#) [#2585](#) [#2587](#) [#2588](#) [#2590](#) [#2594](#) [#2612](#) [#2623](#)
[#2624](#) [#2631](#) [#2633](#) [#2649](#) [#2651](#) [#2653](#) [#2656](#) [#2658](#) [#2662](#) [#2664](#)
[#2674](#) [#2676](#) [#2679](#) [#2682](#) [#2684](#) [#2687](#) [#2688](#) [#2692](#) [#2695](#) [#2697](#)
[#2699](#) [#2701](#) [#2709](#) [#2711](#) [#2712](#) [#2729](#) [#2734](#) [#2736](#) [#2737](#) [#2739](#)
[#2742](#) [#2769](#) [#2774](#) [#2778](#) [#2781](#) [#2783](#) [#2785](#) [#2788](#) [#2792](#) [#2794](#)
[#2797](#) [#2800](#) [#2815](#) [#2833](#) [#2844](#) [#2845](#) [#2848](#) [#2854](#) [#2858](#) [#2864](#)
[#2865](#) [#2876](#) [#2881](#) [#2884](#) [#2887](#) [#2891](#) [#2892](#) [#2894](#) [#2897](#)

- **Stanley Rabinowitz**

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- **Tran Quang Hung**

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[#2654](#) [#2659](#)

[↔ Table of Contents](#) [↔ Message Index](#)

2.2 Author Information

This section presents background information on the contributing authors. Short biographical notes, areas of interest, and selected publications are included to provide context for their contributions to the Journal. These profiles offer readers an opportunity to become acquainted with the individual behind the names and to appreciate the diverse mathematical backgrounds represented in this volume. Author information is included only insofar as it has been provided or was available.

Ángel Montesdeoca Delgado (1949–2024)

Location

Canary Islands, Spain.

Year of Birth / Generation

1949–2024.

Short Biography

Ángel Montesdeoca Delgado was a highly respected Spanish geometer and former teacher at the Universidad de La Laguna, Canary Islands. He was widely admired for his deep knowledge of projective geometry, his extensive contributions to triangle geometry, and the remarkable clarity and beauty of his mathematical website. Ángel's website, admired for both its content and its elegant layout, remains a testament to his mathematical vision and aesthetic sense. Ángel was known not only for his expertise but also for his kindness, generosity, and willingness to help others. Many members of the geometry community recall his thoughtful explanations, his patient guidance, and his warm, friendly nature. His passing in May 2024 was felt as a profound loss by colleagues, students, and friends around the world. He is remembered with gratitude, respect, and affection by the global geometry community.

Themes and Interests

- Projective geometry
- Triangle geometry
- Geometric constructions and classical configurations
- Mathematical exposition and elegant presentation of results
- Community support, explanation, and mentoring

Selected Publications and Contributions

Ángel produced a large number of geometric results, many of which were shared through his website and through contributions to online geometry communities. His work is frequently cited for its depth, originality, and clarity. A memorial reflection by Francisco Javier García Capitán can be found at: www.garciacapitan.blogspot.com/2025/04/angel-montesdeoca-delgado-1949-2024-un.html

Community Tributes

Members of the geometry community remembered Ángel with great affection:

- His “deep knowledge of Projective Geometry” and “great amount of geometrical results” (F. J. García Capitán).
- His generosity, kindness, and helpful explanations (E. Suppa).
- His profound expertise in triangle geometry (A. P. Hatzipolakis).
- His clarity, thoughtfulness, and warm personality (C. van Tienhoven).

- His influence on many geometers at the beginning of their journey (C. E. Lozada).

Additional Remarks

- Website: <https://amontes.webs.ull.es/>

Antreas P. Hatzipolakis

Location

Lives in Greece.

Year of Birth / Generation

1952.

Short Biography

Antreas P. Hatzipolakis studied mathematics at Athens University. He is the founder of several influential geometry-focused email groups, including *Hyacanthos*, *Anopolis*, and *Euclid*, as well as various Facebook groups dedicated to classical and triangle geometry. For many years, he introduced new problem areas through his email groups, inspiring others to explore, investigate, and solve them. His work has played a significant role in shaping the collaborative culture of modern online geometry communities.

Themes and Interests

- Classical Euclidean geometry
- Triangle geometry
- Problem creation and problem solving

Selected Publications

- Antreas P. Hatzipolakis, Floor van Lamoen, Barry Wolk, and Paul Yiu, *Concurrency of Four Euler Lines*. Forum Geometricorum, Volume 1 (2001), 59–68.
- Antreas P. Hatzipolakis and Paul Yiu, *Reflections in Triangle Geometry*. Forum Geometricorum, Volume 9 (2009), 301–348.

Additional Remarks

Website: <http://www.anthrakitis.blogspot.com/>

Chris van Tienhoven

Global Location

Living in the Netherlands.

Year of Birth

1950.

Short Biography

Chris van Tienhoven graduated in mathematics from Leiden University and has built a career as an entrepreneur working across information technology and graphic design. He also remained active in geometry. Central to his work is a lifelong habit of reducing complexity into simplicity and creating clear, durable structures. He values order, coherence, and long-term vision—principles. All of this eventually led to the creation of the Encyclopedia of Poly Geometry.

Themes, Interests, and Relevant Publications

- Lifelong interest in geometry, beginning in secondary school, with a special fascination for Van Aubel's Theorem.
- Developed the notion of Perspective Fields.
- Initiator of the systematic development and documentation of Quadri Geometry, later expanded into Poly Geometry.
- Founder of the online communities *Quadri Figures Group* and *Quadri and Poly Geometry Group*.
- Editor and compiler of the Annual Journals that collect and preserve the discussions and discoveries of these groups.
- Founder of the Encyclopedia of Poly Geometry (where all entries without external references originate from his own work).

Selected Publications

- Chris van Tienhoven, Dario Pellegrinetti, *Quadrigon Geometry: Circumscribed Squares and Van Aubel Points*. *Journal of Geometry and Graphics*, Vol. 25, No. 1, 2021.

Other Remarks

Website: www.chrisvantienhoven.nl

Biography: www.chrisvantienhoven.nl/header/biography/

Eckart Schmidt

Location

Living in Germany.

Year of Birth / Generation

1939.

Short Biography

Eckart Schmidt is a former teacher of mathematics and physics at a full-time secondary school, with a long-standing interest in geometry. His work spans several decades and includes numerous contributions to geometric constructions, classical geometry, and the study of n -gons and their transformations.

Themes and Interests

- Geometric constructions using CABRI

Selected Publications

- F. Bachmann & E. Schmidt: *n Ecke*. B.I. Hochschultaschenbuch 471/471a, Mannheim/Wien/Zürich, 1970.
- E. Schmidt: *Abbildungen und Klassen von n Ecken*. MNU XXV (1972), pp. 146–150ff.
- E. Schmidt: *Affin reguläre n Ecke und ihre regulären Komponenten*. MNU XXXIX (1986), pp. 193–198ff.
- E. Schmidt: *Mittelsenkrechtenvierecke eines Vierecks*. PM 2/44 (2002), pp. 84–88ff.
- E. Schmidt: *Circumcenters of Residual Triangles*. Forum Geometricorum 3 (2003), 125–134.
- J. Kühl & E. Schmidt: *Husumer Rechenhandschriften und Paul Halckes Mathematisches Sinnen Confect*. Mitteilungen der Mathematischen Gesellschaft in Hamburg XXIII/2 (2004), 111–156.
- E. Schmidt: *Geradenkonstellationen*. MNU 60/1 (2007), 28–29.
- E. Schmidt: *Billardvierecke eines Sehnenvierecks*. MNU 63/5 (2010), 267–269.
- Additional contributions on geometric constructions (see Themen and EQF-notes).

Additional Remarks

- Co-founder of the Encyclopedia of Poly Geometry and one of the principal contributors to QPG.
- Website: www.eckartschmidt.de

Keita Miyamoto

Location

Born and living in Japan.

Year of Birth / Generation

Millennial (approx. 1981–1996).

Short Biography

Keita Miyamoto spent part of his academic training at the Faculty of Science of Kyoto University, after which he continued developing his mathematical interests outside formal education. This independent path has shaped a broad, self-directed approach to geometry. His work reflects a deep curiosity for the foundations and structures of both classical and modern geometric systems.

Themes and Interests

- Euclidean geometry
- Projective geometry
- Spherical geometry
- Geometry in the Lobachevsky plane
- Comparative study of geometric systems

Publications and Contributions

- Explorations and discussions on Euclidean and non-Euclidean geometry.
- Contributions to community exchanges on projective and hyperbolic themes.

M@IMF (Handle)

Location

Japan

Year of Birth / Generation

Generation X (1965 – 1980)

Short Biography

M@IMF is an active contributor to the QPG community and prefers to work anonymously.

Themes and Interests

- Geometry
- Algebraic formulations
- Problem exploration
- Computational experimentation

Stanley Rabinowitz

Location

Living in New Hampshire, USA.

Year of Birth / Generation

1947 (Baby Boomer).

Short Biography

Stanley Rabinowitz is a retired computer programmer with a Ph.D. in Mathematics. Throughout his career he has combined computational thinking with a deep appreciation for classical mathematics, particularly geometry, combinatorics, and number theory. He is the founder and sole proprietor of *MathPro Press*, a small but influential publishing house dedicated to high-quality mathematics problem books, indexes, and reference materials used by educators, problem solvers, and researchers worldwide.

Themes and Interests

- Classical Euclidean geometry
- Problem creation and problem solving
- Combinatorics and number theory
- Mathematical indexing, bibliographic work, and reference compilation
- Computational approaches to mathematical problems

Publications and Contributions

Stanley Rabinowitz enjoys creating elegant and challenging mathematics problems, especially in Euclidean geometry. He is the author of the *Index to Mathematical Problems 1980–1984*, a widely used reference work that reflects his long-standing commitment to organizing and preserving mathematical problem literature. Through MathPro Press, he has contributed to the accessibility of problem-solving resources and supported the broader mathematical community with carefully curated publications.

Selected Publications

- *Algorithmic Manipulation of Fibonacci Identities*, in *Applications of Fibonacci Numbers*, Volume 6, ed. G. E. Bergum et al., Kluwer Academic Publishers, Dordrecht, 1996, pp. 389–408.
- *Arrangement of Central Points on the Faces of a Tetrahedron*, *International Journal of Computer Discovered Mathematics* 5 (2020), 13–41.
- *A Computer Algorithm for Proving Symmetric Homogeneous Triangle Inequalities*, *International Journal of Computer Discovered Mathematics* 7 (2022), 30–62.
- *The Shape of Central Quadrilaterals* (with Ercole Suppa), *International Journal of Computer Discovered Mathematics* 7 (2022), 131–180.

- *Relationships between a Central Quadrilateral and its Reference Quadrilateral* (with Ercole Suppa), *International Journal of Computer Discovered Mathematics* 7 (2022), 214–287.

Additional Remarks

Website: www.stanleyRabinowitz.com

Quang Hung Tran

Location

Born and working in Hanoi, Vietnam.

Year of Birth / Generation

Millennial (approx. 1981–1996).

Short Biography

Quang Hung Tran graduated in Mathematics from the University of Science, Vietnam National University, Hanoi. He is a mathematics teacher at the High School for Gifted Students, VNU University of Science, where he has devoted his career to educating and mentoring mathematically talented students. His primary interest lies in Euclidean geometry, especially in the context of mathematical olympiad training, while his broader research spans higher-dimensional and non-Euclidean geometry, the geometry of the Golden ratio and Fibonacci sequences, and the aesthetic, historical, and logical aspects of mathematics. Outside his academic work, he values family life and enjoys reading and spending time with his two sons.

Themes and Interests

- Euclidean geometry
- Mathematical olympiad problems and gifted student education
- Classical geometric inequalities and triangle geometry
- Notable points, circles, and projective methods (harmonic division, isogonal conjugation)
- Higher-dimensional Euclidean geometry
- Non-Euclidean geometry
- Golden ratio and Fibonacci-related geometric structures
- Aesthetic, historical, logical, and recreational mathematics

Selected Publications (Representative)

- *A Napoleon-like theorem for quadrilaterals*, American Mathematical Monthly, 2022.
- *Another Simple Proof of Pascal's Theorem*, Mathematics Magazine, 2023.
- *A generalization of the Pythagorean theorem via Ptolemy's theorem*, Mathematics Magazine, 2023.
- *A Generalization of de Gua's Theorem with a Vector Proof*, The Mathematical Intelligencer.
- *A family of weighted Erdős–Mordell inequality and applications*, Journal of Geometry, 2021.

- *Some strengthened versions of Klamkin's inequality and applications*, Geometriae Dedicata, 2021.
- *A synthetic proof of the Morley trisector theorem using congruent and similar triangles*, Elemente der Mathematik, 2025.
- *A generalisation of Sylvester's theorem with an application*, The Mathematical Gazette, 2025.
- Tran, Q. H. & Herrera, B., *n-Dimensional Generalizations of a Thébault Conjecture*, Mathematical Notes, 2024.
- *A Generalized Volume Formula for Tetrahedra with Congruent Facet Pairs*, The Mathematical Intelligencer, 2025.

Additional Remarks

He is deeply interested in the geometry of quadrilaterals—whether viewed as configurations of four lines, four points, or four angles—and in polygonal geometry more broadly. He notes that as one moves to higher-order polygons, the complexity of problems increases dramatically. Within this rich field, he is delighted and honored to have contributed to the development of the nL–n–Tf1: nL–Orthopole, documented at:

www.chrisvantienhoven.nl/epg/n-geometry/ngeom/nl-n-tf1/

3 Subjects

The list below shows the subjects along with the numbers of related messages. Click on a number to go to the corresponding page.

- **subject: 2 dual Hesse pencils:**
[#2562](#) [#2577](#) [#2584](#)
- **subject: 4 concurrent circles:**
[#2788](#) [#2791](#) [#2792](#) [#2794](#) [#2795](#)
- **subject: 5L and Lill's method:**
[#2624](#) [#2631](#) [#2633](#)
- **subject: [euclid] Concurrence of four Hatzipolakis Axes:**
[#2666](#) [#2668](#) [#2673](#) [#2678](#) [#2680](#)
- **subject: [euclid] NPCs - Orthologic:**
[#2671](#) [#2672](#)
- **subject: A property of QL-Ci3 Miquel:**
[#2568](#)
- **subject: A property of QL-Ci3 Miquel circle:**
[#2561](#) [#2569](#) [#2574](#) [#2575](#) [#2578](#) [#2579](#) [#2580](#) [#2581](#) [#2582](#) [#2583](#)
[#2585](#) [#2586](#) [#2587](#) [#2588](#)
- **subject: A transformation on Quadrangle points:**
[#2654](#) [#2656](#) [#2657](#) [#2658](#) [#2659](#) [#2660](#) [#2661](#) [#2662](#) [#2664](#) [#2665](#)
[#2667](#) [#2669](#) [#2674](#) [#2675](#) [#2676](#) [#2677](#) [#2679](#) [#2681](#) [#2682](#) [#2683](#)
[#2684](#)
- **subject: About QL-Tr2:**
[#2697](#) [#2698](#) [#2699](#) [#2701](#) [#2712](#) [#2729](#) [#2733](#) [#2734](#)
- **subject: Away for a month:**
[#2648](#)
- **subject: Cayleyan as member of a dual Hesse pencil:**
[#2819](#) [#2820](#) [#2826](#) [#2827](#) [#2829](#)
- **subject: co-tangentials of P:**
[#2722](#) [#2724](#)
- **subject: Congratulations:**
[#2813](#) [#2814](#) [#2815](#) [#2816](#) [#2817](#)
- **subject: Cyclic nP and Hypocycloid:**
[#2590](#)
- **subject: Desmic System / Reye Configuration:**
[#2797](#) [#2799](#) [#2800](#) [#2833](#)
- **subject: Geometry on QA-Cu1:**
[#2700](#)

- **subject: In memoriam Ángel Montesdeoca Delgado:**
#2647
- **subject: Isocubic:**
#2821 #2822
- **subject: Isocubics:**
#2823 #2824 #2825 #2828 #2830 #2831 #2832 #2834 #2835 #2836
#2837 #2838 #2839 #2840 #2841 #2842 #2843 #2844 #2845 #2846
#2847 #2848 #2849 #2850 #2851 #2852 #2853 #2854 #2855 #2856
#2857 #2858 #2859 #2860 #2861 #2862 #2863 #2864 #2865 #2866
#2867 #2868 #2869 #2870 #2871 #2872 #2873 #2874 #2875 #2876
#2877 #2878 #2879 #2880 #2883 #2885 #2886 #2887 #2888 #2895
- **subject: MacCay cubic stelloïd K003 and Van Rees hessian K048:**
#2613 #2617 #2618 #2619 #2620 #2621 #2622 #2625 #2626 #2627
#2628 #2630 #2634 #2636 #2637
- **subject: Miquel Circle:**
#2614 #2615
- **subject: Nine-point Center Quadrangle of Nine-point Center Quadrangle:**
#2591 #2592
- **subject: Partner-cubics:**
#2804 #2806
- **subject: preHessian Form of a cubic, three parameters define a cubic:**
#2632 #2638
- **subject: QA 12-point conic involving orthocenters:**
#2691 #2693 #2696
- **subject: QA 4 concurrent Simson / Steiner lines:**
#2685 #2686 #2687 #2688 #2689 #2692 #2694 #2695
- **subject: QA and Complex Quartic Eq.:**
#2709 #2710 #2711
- **subject: QA-collinear points and QL-concurrent lines:**
#2565 #2570 #2573
- **subject: QG constructing a square from 4 squares:**
#2690
- **subject: Quadrilateral Problem:**
#2593 #2594 #2623
- **subject: quadrilateral, concurrent orthocentric transversals, conjugate mapping:**
#2560

- **subject: Radical axis in quadrilateral:**
[#2602](#) [#2604](#) [#2607](#) [#2608](#) [#2612](#) [#2616](#)
- **subject: Re Real flexline trilateral:**
[#2725](#) [#2726](#) [#2749](#) [#2750](#) [#2751](#)
- **subject: Real flexline trilateral:**
[#2595](#) [#2596](#) [#2597](#) [#2598](#) [#2599](#) [#2600](#) [#2601](#) [#2603](#) [#2605](#) [#2606](#)
[#2609](#) [#2610](#) [#2611](#) [#2635](#) [#2639](#) [#2640](#) [#2641](#) [#2642](#) [#2643](#) [#2644](#)
[#2645](#) [#2646](#) [#2702](#) [#2703](#) [#2704](#) [#2705](#) [#2706](#) [#2707](#) [#2708](#) [#2713](#)
[#2714](#) [#2715](#) [#2716](#) [#2717](#) [#2718](#) [#2719](#) [#2720](#) [#2721](#) [#2723](#) [#2727](#)
[#2728](#) [#2730](#) [#2731](#) [#2732](#) [#2744](#) [#2745](#) [#2746](#) [#2747](#) [#2748](#) [#2752](#)
[#2753](#) [#2754](#) [#2755](#) [#2756](#) [#2757](#) [#2758](#) [#2759](#) [#2760](#) [#2761](#) [#2762](#)
[#2763](#) [#2764](#) [#2765](#) [#2766](#) [#2767](#) [#2768](#) [#2770](#) [#2771](#) [#2772](#) [#2776](#)
[#2786](#) [#2787](#) [#2789](#) [#2790](#) [#2793](#) [#2796](#) [#2798](#) [#2801](#) [#2802](#) [#2803](#)
[#2805](#) [#2807](#) [#2808](#) [#2809](#) [#2810](#) [#2811](#) [#2812](#) [#2881](#) [#2882](#) [#2884](#)
[#2889](#) [#2890](#) [#2891](#) [#2892](#) [#2893](#) [#2894](#)
- **subject: Season's Greetings and a Look Ahead:**
[#2896](#) [#2897](#)
- **subject: Similitude Center of (n-1)Ps formed by Centric Centers:**
[#2649](#) [#2650](#) [#2651](#) [#2652](#) [#2653](#)
- **subject: Simple definition of QL-Tr2:**
[#2735](#) [#2736](#) [#2737](#) [#2738](#) [#2739](#) [#2741](#) [#2742](#) [#2743](#) [#2769](#) [#2773](#)
[#2774](#) [#2775](#) [#2777](#) [#2778](#) [#2779](#) [#2780](#) [#2781](#) [#2782](#) [#2783](#) [#2785](#)
- **subject: Two quadrangles with a common circumconic:**
[#2563](#) [#2564](#) [#2566](#) [#2567](#) [#2571](#) [#2572](#) [#2576](#) [#2663](#) [#2670](#)
- **subject: Update on the new Website:**
[#2784](#) [#2818](#)

[↔ Table of Contents](#) [↔ Message Index](#)

4 Messages

4.1 Message Index

Msg #2560: page 24	Msg #2606: page 66	Msg #2652: page 116
Msg #2561: page 25	Msg #2607: page 67	Msg #2653: page 116
Msg #2562: page 26	Msg #2608: page 68	Msg #2654: page 117
Msg #2563: page 27	Msg #2609: page 69	Msg #2655: deleted msg
Msg #2564: page 28	Msg #2610: page 70	Msg #2656: page 118
Msg #2565: page 29	Msg #2611: page 71	Msg #2657: page 119
Msg #2566: page 30	Msg #2612: page 72	Msg #2658: page 119
Msg #2567: page 31	Msg #2613: page 72	Msg #2659: page 120
Msg #2568: page 32	Msg #2614: page 75	Msg #2660: page 121
Msg #2569: page 33	Msg #2615: page 75	Msg #2661: page 122
Msg #2570: page 33	Msg #2616: page 76	Msg #2662: page 123
Msg #2571: page 34	Msg #2617: page 76	Msg #2663: page 123
Msg #2572: page 34	Msg #2618: page 79	Msg #2664: page 124
Msg #2573: page 35	Msg #2619: page 79	Msg #2665: page 125
Msg #2574: page 35	Msg #2620: page 80	Msg #2666: page 126
Msg #2575: page 36	Msg #2621: page 81	Msg #2667: page 127
Msg #2576: page 37	Msg #2622: page 83	Msg #2668: page 128
Msg #2577: page 37	Msg #2623: page 84	Msg #2669: page 129
Msg #2578: page 38	Msg #2624: page 85	Msg #2670: page 130
Msg #2579: page 39	Msg #2625: page 87	Msg #2671: page 131
Msg #2580: page 40	Msg #2626: page 88	Msg #2672: page 132
Msg #2581: page 41	Msg #2627: page 89	Msg #2673: page 133
Msg #2582: page 43	Msg #2628: page 91	Msg #2674: page 136
Msg #2583: page 45	Msg #2629: deleted msg	Msg #2675: page 138
Msg #2584: page 45	Msg #2630: page 92	Msg #2676: page 138
Msg #2585: page 46	Msg #2631: page 93	Msg #2677: page 138
Msg #2586: page 48	Msg #2632: page 95	Msg #2678: page 139
Msg #2587: page 48	Msg #2633: page 97	Msg #2679: page 139
Msg #2588: page 49	Msg #2634: page 98	Msg #2680: page 141
Msg #2589: deleted msg	Msg #2635: page 101	Msg #2681: page 142
Msg #2590: page 50	Msg #2636: page 102	Msg #2682: page 142
Msg #2591: page 53	Msg #2637: page 102	Msg #2683: page 143
Msg #2592: page 53	Msg #2638: page 103	Msg #2684: page 146
Msg #2593: page 54	Msg #2639: page 104	Msg #2685: page 147
Msg #2594: page 55	Msg #2640: page 105	Msg #2686: page 148
Msg #2595: page 56	Msg #2641: page 106	Msg #2687: page 148
Msg #2596: page 57	Msg #2642: page 107	Msg #2688: page 149
Msg #2597: page 57	Msg #2643: page 109	Msg #2689: page 150
Msg #2598: page 58	Msg #2644: page 109	Msg #2690: page 151
Msg #2599: page 59	Msg #2645: page 110	Msg #2691: page 152
Msg #2600: page 59	Msg #2646: page 111	Msg #2692: page 153
Msg #2601: page 60	Msg #2647: page 112	Msg #2693: page 154
Msg #2602: page 61	Msg #2648: page 113	Msg #2694: page 155
Msg #2603: page 61	Msg #2649: page 113	Msg #2695: page 156
Msg #2604: page 62	Msg #2650: page 114	Msg #2696: page 157
Msg #2605: page 63	Msg #2651: page 115	Msg #2697: page 157

Msg #2698: page 158	Msg #2748: page 216	Msg #2798: page 261
Msg #2699: page 159	Msg #2749: page 217	Msg #2799: page 262
Msg #2700: page 160	Msg #2750: page 217	Msg #2800: page 262
Msg #2701: page 162	Msg #2751: page 218	Msg #2801: page 263
Msg #2702: page 163	Msg #2752: page 218	Msg #2802: page 263
Msg #2703: page 164	Msg #2753: page 219	Msg #2803: page 264
Msg #2704: page 164	Msg #2754: page 219	Msg #2804: page 265
Msg #2705: page 164	Msg #2755: page 220	Msg #2805: page 267
Msg #2706: page 165	Msg #2756: page 221	Msg #2806: page 268
Msg #2707: page 165	Msg #2757: page 222	Msg #2807: page 269
Msg #2708: page 166	Msg #2758: page 222	Msg #2808: page 270
Msg #2709: page 166	Msg #2759: page 223	Msg #2809: page 271
Msg #2710: page 168	Msg #2760: page 224	Msg #2810: page 271
Msg #2711: page 169	Msg #2761: page 225	Msg #2811: page 276
Msg #2712: page 170	Msg #2762: page 226	Msg #2812: page 277
Msg #2713: page 171	Msg #2763: page 227	Msg #2813: page 277
Msg #2714: page 172	Msg #2764: page 228	Msg #2814: page 278
Msg #2715: page 173	Msg #2765: page 230	Msg #2815: page 278
Msg #2716: page 177	Msg #2766: page 231	Msg #2816: page 279
Msg #2717: page 181	Msg #2767: page 231	Msg #2817: page 280
Msg #2718: page 182	Msg #2768: page 232	Msg #2818: page 281
Msg #2719: page 183	Msg #2769: page 232	Msg #2819: page 282
Msg #2720: page 184	Msg #2770: page 234	Msg #2820: page 290
Msg #2721: page 185	Msg #2771: page 235	Msg #2821: page 291
Msg #2722: page 188	Msg #2772: page 236	Msg #2822: page 292
Msg #2723: page 190	Msg #2773: page 237	Msg #2823: page 293
Msg #2724: page 190	Msg #2774: page 238	Msg #2824: page 294
Msg #2725: page 191	Msg #2775: page 239	Msg #2825: page 295
Msg #2726: page 192	Msg #2776: page 240	Msg #2826: page 296
Msg #2727: page 193	Msg #2777: page 240	Msg #2827: page 296
Msg #2728: page 194	Msg #2778: page 241	Msg #2828: page 304
Msg #2729: page 194	Msg #2779: page 242	Msg #2829: page 305
Msg #2730: page 195	Msg #2780: page 242	Msg #2830: page 305
Msg #2731: page 196	Msg #2781: page 243	Msg #2831: page 306
Msg #2732: page 196	Msg #2782: page 244	Msg #2832: page 307
Msg #2733: page 197	Msg #2783: page 245	Msg #2833: page 309
Msg #2734: page 198	Msg #2784: page 246	Msg #2834: page 310
Msg #2735: page 201	Msg #2785: page 247	Msg #2835: page 313
Msg #2736: page 206	Msg #2786: page 248	Msg #2836: page 315
Msg #2737: page 206	Msg #2787: page 249	Msg #2837: page 316
Msg #2738: page 207	Msg #2788: page 250	Msg #2838: page 317
Msg #2739: page 208	Msg #2789: page 251	Msg #2839: page 318
Msg #2740: deleted msg	Msg #2790: page 251	Msg #2840: page 321
Msg #2741: page 209	Msg #2791: page 252	Msg #2841: page 322
Msg #2742: page 210	Msg #2792: page 253	Msg #2842: page 324
Msg #2743: page 211	Msg #2793: page 254	Msg #2843: page 325
Msg #2744: page 211	Msg #2794: page 255	Msg #2844: page 325
Msg #2745: page 214	Msg #2795: page 256	Msg #2845: page 326
Msg #2746: page 214	Msg #2796: page 257	Msg #2846: page 326
Msg #2747: page 215	Msg #2797: page 258	Msg #2847: page 327

Msg #2848: page 328	Msg #2865: page 339	Msg #2882: page 351
Msg #2849: page 328	Msg #2866: page 340	Msg #2883: page 351
Msg #2850: page 329	Msg #2867: page 341	Msg #2884: page 352
Msg #2851: page 330	Msg #2868: page 342	Msg #2885: page 354
Msg #2852: page 330	Msg #2869: page 342	Msg #2886: page 355
Msg #2853: page 331	Msg #2870: page 343	Msg #2887: page 356
Msg #2854: page 332	Msg #2871: page 344	Msg #2888: page 357
Msg #2855: page 333	Msg #2872: page 344	Msg #2889: page 358
Msg #2856: page 333	Msg #2873: page 345	Msg #2890: page 359
Msg #2857: page 334	Msg #2874: page 346	Msg #2891: page 361
Msg #2858: page 334	Msg #2875: page 346	Msg #2892: page 361
Msg #2859: page 335	Msg #2876: page 347	Msg #2893: page 362
Msg #2860: page 336	Msg #2877: page 347	Msg #2894: page 363
Msg #2861: page 336	Msg #2878: page 348	Msg #2895: page 364
Msg #2862: page 337	Msg #2879: page 349	Msg #2896: page 366
Msg #2863: page 337	Msg #2880: page 349	Msg #2897: page 367
Msg #2864: page 338	Msg #2881: page 350	

[↔ Table of Contents](#)

4.2 Messages

Message: #2560

Date: 2025-01-08

From: unidentifiedlethargicorganism@gmail.com

Subject: Re: quadrilateral, concurrent orthocentric transversals,

Dear Eckart Schmidt and all,

Thanks to the remark by Eckart Schmidt, I found that Q is the C_i -antipode of P , where C_i denotes the circle through P and the 2 Plücker points. PH_1 is perpendicular to q_1 , and similarly for q_2, q_3, q_4 . The 2 Plücker points and $\{\text{Newton line}\} \cap \{\text{line at infinity}\}$ are singularities of the mapping $f: P \rightarrow Q$.

Also, any cubic through the 6 vertices of the quadrilateral and 2 plücker points (which also passes through $\{\text{Newton line}\} \cap \{\text{line at infinity}\}$ I think) is a self-conjugate cubic under this conjugation. That is, If P lies on a cubic which belongs to this family, then, Q also lies on it.

Sincerely,
Keita Miyamoto

[Next](#) → ← [Message Index](#) ↑ [Subjects](#)

Message: #2561
Date: 2025-01-12
From: unidentifiedlethargicorganism@gmail.com
Subject: A property of QL-Ci3 Miquel circle

Dear Geometers,

Is the following result well-known?

Let $L_1L_2L_3L_4$ be a general quadrilateral with vertices $P_{12}, P_{13}, P_{14}, P_{23}, P_{24}, P_{34}$. ($P_{ij} = L_i \cap L_j$)

Let $M = QL-P_1 =$ Miquel point and $C_i = QL-Ci_3 =$ Miquel circle.

Let P be a point on C_i .

Let O_1 be the circumcenter of $P_{23}P_{24}P_{34}$. Define O_2, O_3, O_4 cyclically.

Let L_1^* be the QA-orthopolar line (QA-Tf8) of P_{O_1} wrt quadrangle $MP_{23}P_{24}P_{34}$.

Define L_2^*, L_3^*, L_4^* cyclically.

Then, $L_1^*, L_2^*, L_3^*, L_4^*$ are parallel. If P coincides with the C_i -antipode of M , then, these lines are parallel to $QL-L_1 =$ Newton line.

If P moves along C_i , then, each of these lines $L_1^*, L_2^*, L_3^*, L_4^*$ passes through a fixed point on $QL-L_3 = QL$ -pedal line.

Furthermore, Let $K = QL-P_3 =$ Kantor-Hervey point, and M^* be the second intersection point of MK and C_i .

Let H_1, H_2, H_3, H_4 be the orthocenters of the component triangles of the quadrangle $O_1O_2O_3O_4$.

Let L^* be the QA-orthopolar line of MK wrt $H_1H_2H_3H_4$.

If P coincides with M^* , then, $L_1^*, L_2^*, L_3^*, L_4^*, L^*$ are all parallel. L^* passes through $QL-P_5 =$ Clawson center.

In general, if $P_1P_2P_3P_4$ is a cyclic quadrangle with circumcenter O , and L is a line through a fixed point, then, the QA-orthopolar line of L wrt $P_1P_2P_3P_4$ envelopes a deltoid, which degenerates to a point if the fixed point coincides with O .

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2562
Date: 2025-01-12
From: bernard.keizer@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Chris, dear Eckart
Exactly a month ago, I sent you the preceding memo containing the barycentric equation of the cayleyan!
But I got no answer ...
I understand that Eckart has little time for personal reasons and that Chris has many centers of interest, including the maintenance of EQF, which is primordial.
The result for me and for the Hesse pencils is the same.
Meanwhile, I continued to make calculations.
Using Eckart's idea of the barycentric coordinates and Chris' calculations wrt the reference triangle $X_1X_2X_3$, I've applied it not to a variable triangle $H_1H_2H_3$ like Eckart, but to the fixed triangle $X_1X_2X_3$.
We know the coordinates of F_1, F_2, F_3 and P_0 wrt $X_1X_2X_3$, which depends only from a_1, a_2, a_3 and k and we are able to measure the length of all segments or of all surfaces which are needed.
We come to formulas $s_1 = B + tb_1 \dots$, where $b_i = 1/a_i$, $B = \sum b_i$, $t = (k-6)/3$ and $\sum s_i = 1$ and in the end $B(t+3) = 1$, $b_i = (s_i - B)/t \dots$
This system has a solution in a_1, a_2, a_3 for any value of k !
But it is possible to use the same construction, measuring and calculation for the hessian.
The correct value of k is obtained when k and the corresponding k' give the same value of a_1, a_2 and a_3 (up to a multiplying factor).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2563
Date: 2025-01-13
From: Stan.Rabinowitz@comcast.net
Subject: Two quadrangles with a common circumconic

If P is a quadrangle, let $P(n)$ be the quadrangle formed by the $X(n)$ points of the component triangles of P .

It is known that for any quadrangle P , P and $P(4)$ have a common circumconic (QA-Co2).

See <https://chrisvantienhoven.nl/qa-items/qa-conics/qa-co2>.
The center of the conic is the Euler-Poncelet point of both P and $P(4)$.

Are there any other values of n for which P and $P(n)$ have a common circumconic?

Are there any values of n for which P and $P(n)$ have a common inconic?

-

Some interesting things happen if P is a cyclic quadrangle.

Dylan Wyrzykowski has pointed out at <https://www.facebook.com/groups/parmenides52/posts/9041609412619351/> that if P is cyclic, then P and $P(6)$ have a common circumconic.

How is the center of this conic related to P ?

Rabinowitz noted that if P is cyclic, then P and $P(n)$ have a common (non-circular) circumconic when $n=54$ or $n=64$. Ercole Suppa noted that this is true for $n=1173$.

Are these conics known? What can be said about the center of these conics?

Wyrzykowski conjectures that P and $P(n)$ have a common circumconic if P is cyclic, P lies on the Jerabek hyperbola, and the isogonal conjugate of P is a point with constant Shinagawa coefficients.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2564

Date: 2025-01-13

From: Stan.Rabinowitz@comcast.net

Subject: Re: Two quadrangles with a common circumconic

On Mon, Jan 13, 2025 at 04:20 PM, Stanley Rabinowitz wrote:

>

> Correction:

>

> Wyrzykowski conjectures that P and $P(n)$
> have a common circumconic if P is cyclic,
> $X(n)$ lies on the Jerabek hyperbola,
> and the isogonal conjugate of $X(n)$
> is a point with constant Shinagawa coefficients.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2565
Date: 2025-01-13
From: unidentifiedlethargicorganism@gmail.com
Subject: QA-collinear points and QL-concurrent lines

Dear Geometers,

*** QA-version (the dual of the QL-version) ***

Let $P_1P_2P_3P_4$ be a general quadrangle with sides L_{12} , L_{13} , L_{14} , L_{23} , L_{24} , L_{34} . ($L_{ij} = P_i \cap P_j$)

Let P be a random point.

We can construct the circumscribed conic QA-Co for 5 points P_1 , P_2 , P_3 , P_4 , P .

Let L^* be the tangent line to QA-Co at P . L^* is QA-Tf9(P).

Let L_{12}^* be the second tangent line to QA-Co from $L^* \cap L_{12}$.

Define L_{13}^* , L_{14}^* , L_{23}^* , L_{24}^* , L_{34}^* similarly.

Let T_{12} be the touchpoint of L_{12}^* and QA-Co. Define T_{13} , T_{14} , T_{23} , T_{24} , T_{34} cyclically.

Then, the 4 points P , $L_{12}^* \cap L_{34}^*$, $L_{13}^* \cap L_{24}^*$, $L_{14}^* \cap L_{23}^*$ lie on the same line L^{**} .

The 4 lines L^* , $T_{12}T_{34}$, $T_{13}T_{24}$, $T_{14}T_{23}$ concur in a point P' . P and P' are a pair of doublepoints of L^* . (See QA-Tf1)

*** QL-version (the dual of the QA-version) ***

Let $L_1L_2L_3L_4$ be a general quadrilateral with vertices P_{12} , P_{13} , P_{14} , P_{23} , P_{24} , P_{34} . ($P_{ij} = L_i \cap L_j$)

Let L be a random line.

We can construct the inscribed conic QL-Co for 5 lines L_1 , L_2 , L_3 , L_4 , L .

Let P^* be the touchpoint of QL-Co and L . P^* is QL-Tf7(L).

Let P_{12}^* be the second intersection point of QL-Co and P^*P_{12} .

Define P_{13}^* , P_{14}^* , P_{23}^* , P_{24}^* , P_{34}^* similarly.

Let t_{12} be the tangent line at P_{12}^* to QL-Co. Define t_{13} , t_{14} , t_{23} , t_{24} , t_{34} cyclically.

Then, the 4 lines L , $P_{12}^*P_{34}^*$, $P_{13}^*P_{24}^*$, $P_{14}^*P_{23}^*$ concur in a point P^{**} .

The 4 points P^* , $t_{12} \cap t_{34}$, $t_{13} \cap t_{24}$, $t_{14} \cap t_{23}$ lie on the same line L' . L and L' are a pair of doublelines of P^* . (See QL-Tf8).

Are the transformations $P \rightarrow L^{**}$ (in the QA-environment) and $L \rightarrow P^{**}$ (in the QL-environment) already known in EQF / QFG / QPG?

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2566

Date: 2025-01-13

From: unidentifiedlethargicorganism@gmail.com

Subject: Re: Two quadrangles with a common circumconic

Dear Stanley Rabinowitz,

For general quadrangles, I could not find any common circumconics in the range $X(1)$ to $X(100)$ except for $X(4)$. Not algebraically confirmed, but my GeoGebra sketch shows that if a quadrangle is cyclic, then, the center of $X(6)$ -circumhyperbola coincides with $QA-P44$ = least-squared distance point of QA -sidelines. For $X(54)$, $X(64)$ and $X(1173)$, the centers do not correspond to any of the QA -points listed in EQF.

I don't understand the inconic part. By P and $P(n)$ you mean quadrilaterals P and $P(n)$? If P is a quadrangle or quadrilateral, then, it has 3 component quadrilaterals $P(n)$ -3QG. If P and $P(n)$ are both quadrilaterals, then I could not find any common inconics in the range $X(1)$ to $X(100)$. If you mean the inconic of P through the vertices of $P(n)$, I could not find any interesting examples in the range $X(1)$ to $X(100)$.

However, it is known that if a quadrilateral has 2 inconics $Co1$ with foci $F1, F1'$ and $Co2$ with foci $F2, F2'$, then, the quadrilateral formed by $F1F2, F2F1', F1'F2', F2'F1$ and the reference quadrilateral have a pair of confocal inconics with foci $F3, F3'$. In other words, $F1, F1', F2, F2', F3, F3'$ lie on $QL-Cu1$ = QL -quasi isogonal cubic of the reference quadrilateral.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2567

Date: 2025-01-13

From: unidentifiedlethargicorganism@gmail.com

Subject: Re: Two quadrangles with a common circumconic

Dear Stanley Rabinowitz,

Sorry, my answer is confusing. I mean, if P is a quadrangle, then P has 3 component quadrilaterals, and if P is a quadrilateral, then $P(n)$ is a quadrangle, but if you consider inconics of $P(n)$, then, $P(n)$ should be a quadrilateral.

Sincerely,

Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2568
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: A property of QL-Ci3 Miquel

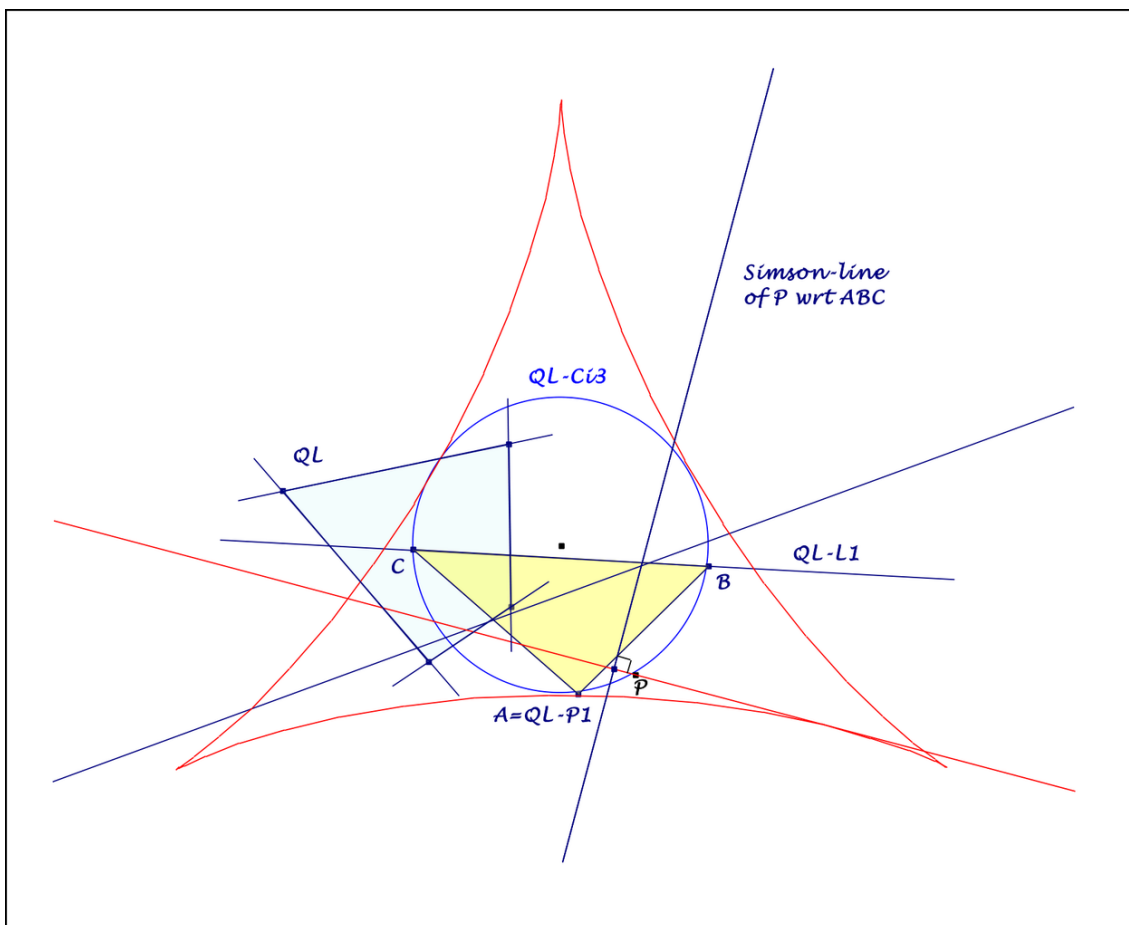
Dear Keita Miyamoto,

with interest I have reproduced your construction
... of the 4 parallel lines Li^* for P on QL-Ci3,
... here a further observation:

These parallels are parallel to the Simson line of P
... wrt a triangle ABC : one vertex A in QL-P1
... with opposite line QL-L1, intersecting QL-Ci3 in B and C .

If you consider perpendiculars through P wrt these Simson lines,
... they envelope a deltoid with inner circle QL-Ci3.

Best regards Eckart



2025-01-14.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2569
Date: 2025-01-14
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Miyamoto-san and Mr. Schmidt,

As for the last part of #2561, is the deltoid expressed in the complex plane like this?

$z = (p + a)/2 + (|p|/2)(bc)^{1/3} (2t + 1/t^2)$, $|t| = 1$,
where $a = p_1 + p_2 + p_3 + p_4$, $b = p_1 p_2 p_3 p_4 / R^4$, $R = |p_j|$
($j=1,2,3,4$), $c = p/|p|$, p is a fixed point.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2570
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: QA-collinear points and QL-concurrent lines

Dear Keita Miyamoto,

the transformation $P \dashrightarrow P'$ (first part of #2565) is QA-Tf2.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2571
Date: 2025-01-14
From: Stan.Rabinowitz@comcast.net
Subject: Re: Two quadrangles with a common circumconic

Ercole Suppa has found that for $n=4, 6, 54, 64, 74, 98, 99, 1173, 3426, 3431, 3527, 3531, 3532$, ABCD and EFGH have a common circumconic.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2572
Date: 2025-01-14
From: Stan.Rabinowitz@comcast.net
Subject: Re: Two quadrangles with a common circumconic

Note that isogonal conjugate of $X(3426)$ is $X(376)$ which has constant Shinagawa coefficients $(2, -3)$.
The isogonal conjugate of $X(3431)$ is $X(381)$ which has constant Shinagawa coefficients $(1, 3)$.
The isogonal conjugate of $X(3527)$ is $X(631)$ which has constant Shinagawa coefficients $(2, -1)$.
The isogonal conjugate of $X(3531)$ is $X(3524)$ which has constant Shinagawa coefficients $(4, -3)$.
The isogonal conjugate of $X(3532)$ is $X(3146)$ which has constant Shinagawa coefficients $(1, -4)$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2573
Date: 2025-01-14
From: eckart_schmidt@t-online.de
Subject: Re: QA-collinear points and QL-concurrent lines

Dear Keita Miyamoto,
the transformation $L \rightarrow L'$ (second part of #2565) is QL-Tf2.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2574
Date: 2025-01-14
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Let me correct #2569:
$$z = (p + a)/2 \cdot (|p|/2)(b/c)^{1/3} (2t + 1/t^2), |t| = 1,$$
where $a = p_1 + p_2 + p_3 + p_4$, $b = p_1 p_2 p_3 p_4 / R^4$, $R = |p_j|$
($j=1,2,3,4$), $c = p/|p|$, p is a fixed point.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2575
Date: 2025-01-14
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear M@IMF and Eckart Schmidt,

Thank you for your interest.
I drew it on GeoGebra.
I can confirm that your formula is correct.
I am not highly educated in mathematics, and I was not able to derive an equation for the deltoid by myself, so thank you very much.

By the way, there is a generalization of the orthopole which is called isopole.

Let ABC be a triangle and L be a line. Let A', B', C' be points on L such that $m\angle(AA', L) = m\angle(BB', L) = m\angle(CC', L) = \theta$.
Let A'', B'', C'' be points on BC, CA, AB , respectively, such that $m\angle(BC, A'A'') = m\angle(CA, B'B'') = m\angle(AB, C'C'') = -\theta$.
Then, the 3 lines $A'A'', B'B'', C'C''$ concur in a point P , which is called the θ -isopole of L wrt ABC .

Analogously, there are generalizations of the orthopolar lines --- both QA-isopolar line (QA-Tf8) and QL-isopolar line (QL-Tf5), and even QL-isopole (QL-Tf4).
That is, define the 4 θ -isopoles of a line L wrt the 4 component triangles of a reference quadrilateral or quadrangle. Then, the 4 isopoles lie on the same line which is called the θ -isopolar line of L .
For QL-isopolar lines, if L is fixed and θ varies, then, the envelope of the isopolar line of L wrt a quadrilateral is a parabola. In particular, if L passes through the Miquel point, then, the envelope is the QL-inscribed parabola.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2576
Date: 2025-01-14
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Two quadrangles with a common circumconic

Dear Stanley Rabinowitz,

I think this property is remarkable.
I am not sure if it is suitable for EQF, for the website is dedicated to general quadri-figures and polygons, but it would be nice to see it documented somewhere...
Hopefully Mr. Chris van Tienhoven will be interested in your result. Apparently he is busy right now.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2577
Date: 2025-01-14
From: van10hoven@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Bernard,

I went through your messages #2539 and #2569.
Congratulations on your discovery of the barycentric equation of the Cayleyan in Hesse's pencil—quite an effort!
Thank you for your beautiful pictures.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2578
Date: 2025-01-14
From: van10hoven@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear M@IMF, Keita Miamoto and Eckart,

M@IMF, regarding your formula for the Deltoid:
For my understanding could you please explain the relationships between the parameters of your expression of the Deltoid in the Complex Plane and the defining parameters of the reference Quadrilateral?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2579

Date: 2025-01-15

From: unidentifiedlethargicorganism@gmail.com

Subject: Re: A property of QL-Ci3 Miquel circle

Dear Chris van Tienhoven, M@IMF, Eckart Schmidt and all,

In the complex plane, let p_1, p_2, p_3, p_4 be 4 points on the same circle centered at the origin 0, and t is a point on the unit circle $|t| = 1$. Let p be a fixed point.

Then, the QA-orthopolar line of a line through p wrt cyclic quadrangle $p_1p_2p_3p_4$ envelopes a deltoid. The equation of the deltoid is:

$$z(t) = (p+p_1+p_2+p_3+p_4)/2 + (|p|/2) * [((p_1*p_2*p_3*p_4)/(|p_1|^4))*(|p|/p)]^{(1/3)} * (2*t+1/(t^2))t = |1|$$

(complex parametrization)

Application:

Let M be the Miquel point QL-P1 of a quadrilateral $L_1L_2L_3L_4$.

Let M' be the QL-Ci3-antipode of M . (QL-Ci3 = Miquel circle)

Let C be the circle centered at M' and passing through M .

Let C_1 be the circumcircle of the triangle bounded by L_2, L_3, L_4 . Define C_2, C_3, C_4 cyclically.

Let P_1 be the second intersection point, other than M , of C and C_1 . Define P_2, P_3, P_4 cyclically.

Then, the QA-orthopolar line of a line through M wrt the cyclic quadrangle $P_1P_2P_3P_4$ envelopes the Kantor-Hervey deltoid QL-Qu2 of $L_1L_2L_3L_4$.

I am sorry if there are any mistakes...

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2580

Date: 2025-01-15

From: unidentifiedlethargicorganism@gmail.com

Subject: Re: A property of QL-Ci3 Miquel circle

Dear all,

Sorry, the correct equation of the deltoid in the complex plane is:

$$z(t) = (p+p_1+p_2+p_3+p_4)/2 - (|p|/2) * [((p_1*p_2*p_3*p_4)/(|p_1|^4))*(|p|/p)]^{1/3} * (2*t+1/(t^2))|t| = 1$$

(complex parametrization.

p_1, p_2, p_3, p_4 lie on the same circle centered at the origin 0, and t moves on the unit circle. p is a fixed point)

Sincerely,

Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2581
Date: 2025-01-16
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Chris and all,

I'll explain the derivation of the equation.
In complex coordinates, the parametric equation of a deltoid is
 $z = z_0 + (\rho/3) (2t + 1/t^2)$, $|t| = 1$, where
 z_0 = center of deltoid
 $|\rho|$ = radius of circumcircle
 $\arg(\rho)$ = direction of vertex (mod $2\pi/3$).

Let points p_j ($j=1,2,3,4$) lie on a circle with radius R and center at the origin O in the complex plane.
Let QA_1 be a cyclic quadrangle formed by them.
Denote the complex number corresponding to point P as $z[P]$, and let $p = z[P]$.
Denote QA_1 -Tf8(L) by $Tf_8(L)$ and let L_0 be a line through O .
When L_0 rotates around O at angular velocity ω , $Tf_8(L_0)$ rotates around QA_1 - P_2 at $-\omega$.
 L_0 and $Tf_8(L_0)$ are parallel if L_0 is parallel or perpendicular to Ax_1 (one of the axes of QA_1 - $2Co_1$).
The direction of Ax_1 is $\phi = \arg(p_1 p_2 p_3 p_4)/4$ (mod $\pi/2$).
Denote QA_1 - P_2 by A and let B be a point such that $z[B] = z[A] + p$.
Since QA_1 - $P_3 = 0$, $z[A] = (p_1 + p_2 + p_3 + p_4)/2$.

If we displace L_0 along its normal by a certain distance, $Tf_8(L_0)$ is displaced by the same distance (same direction).
Let L_p be a line through P parallel to L_0 , then the distance between L_p and L_0 depends on their direction.
As a result, when L_p rotates around P at ω ,
 $Tf_8(L_p)$ rotates around a point D at $-\omega$, and
 D revolves around a point C at 2ω ,
where C is the midpoint of A and B .
 D lies on a circle with diameter AB , and BD is parallel to L_p .
This circle is the incircle of the deltoid, so
 $z_0 = z[C] = (p + p_1 + p_2 + p_3 + p_4)/2$
 $|\rho|/3 = |\rho|/2$.
Please see the attached file. (The figure will tell much more than my English.)

$Tf_8(L_p)$ passes through the vertex of the deltoid when it passes through C ,
which means the angle bisector of $\angle CDB$ is parallel or perpendicular to Ax_1 .

Then we get

$$\angle BCD = (\pi + 4\varphi - 4\arg(p))/3, \text{ and}$$

$$\arg(p) = \pi + (4\varphi - \arg(p))/3 \pmod{2\pi/3}.$$

The equation of the deltoid becomes

$$z = (p + a)/2 - (|p|/2)(b/c)^{1/3} (2t + 1/t^2), \quad |t| = 1,$$

where

$$a = p_1 + p_2 + p_3 + p_4$$

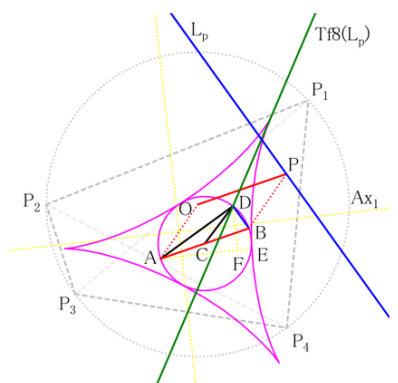
$$b = p_1 p_2 p_3 p_4 / R^4 \text{ (phase factor of } p_1 p_2 p_3 p_4)$$

$$c = p/|p| \text{ (phase factor of } p).$$

Best regards, M@IMF

Deltoid Enveloped by Orthopolar Line wrt Cyclic QA

M@IMF



P_1, P_2, P_3, P_4 are concyclic points.
 QA_1 is a cyclic quadrangle $\{P_1, P_2, P_3, P_4\}$.
 P is a fixed point.
 L_p is a line through P .
 $Tl_8(L_p)$ is $QA_1-Tl_8(L_p)$.
 Ax_1 is one of the axes of QA_1-2Co1 .
 O is $QA_1-P_3 (= QA_1-P_4)$.
 A is QA_1-P_2 .
 B is a point such that vector $AB =$ vector OP .
 C is the midpoint of A and B .
 D is a point such that $CD = CA$ and $BD \parallel L_p$.
 E is a point such that $CE = CA$ and $AE \parallel Ax_1$.
 F is a point such that $DF \perp AE$.

Note that DF is the angle bisector of BD and $Tl_8(L_p)$.
 $Tl_8(L_p)$ envelopes a deltoid (Keita Miyamoto).

Proposition: If $\angle CDF = \angle FDB$, then $\angle BCD = (\pi - 4\angle EAB)/3$.
 (\because)
 $\angle CDF = \pi/2 - (\angle BCD + \angle EAB)$
 $\angle FDB = (1/2)\angle BCD + \angle EAB$.

Let's consider complex plane with the origin O , and denote the complex number corresponding to point Q as $z[Q]$. Let $p = z[P]$ and $p_j = z[P_j]$ ($j=1,2,3,4$). Then the parametric equation of the deltoid is

$$z = (p + a)/2 - (|p|/2)(b/c)^{1/3} (2t + 1/t^2), \quad |t| = 1,$$

where

$$a = p_1 + p_2 + p_3 + p_4$$

$$b = p_1 p_2 p_3 p_4 / R^4 \text{ (phase factor of } p_1 p_2 p_3 p_4)$$

$$R = |p_j|$$

$$c = p/|p| \text{ (phase factor of } p).$$

Message: #2582
Date: 2025-01-17
From: van10hoven@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Keita, M@IMF and Eckart,

I further studied your messages regarding "A property of the QL-Ci3 Miquel circle" and was pleasantly surprised by the deltoids that emerged.

Now, we have:

- * QL-Deltoid-1, which circumscribes the Hervey Circle QL-Ci4, with center QL-P3.
This Deltoid is inscribed in the four lines of the QL.
- * QL-Deltoid-2, which circumscribes the Miquel Circumcircle QL-Ci3, with center QL-P4.

Constructions:

- * QL-Deltoid-1: As Keita mentioned in #2579, "The QA-orthopolar line of a line through M with respect to the cyclic quadrangle P1P2P3P4 envelopes the Kantor-Hervey deltoid QL-Qu2 of L1L2L3L4."
- * QL-Deltoid-2: As Eckart stated in #2568, "If you consider perpendiculars through P with respect to these Simson lines, they envelope a deltoid with inner circle QL-Ci3."

Both circumscribed circles, as well as the QL-Deltoids, are of the same size, and their axes are mutually parallel. See the last property of QL-Qu2 and QFG#514 (<https://groups.io/g/Quadri-Figures-Group/topic/71448762#msg16951>) for reference.

I made a picture of both QL-Deltoids using these constructions. See attachment.

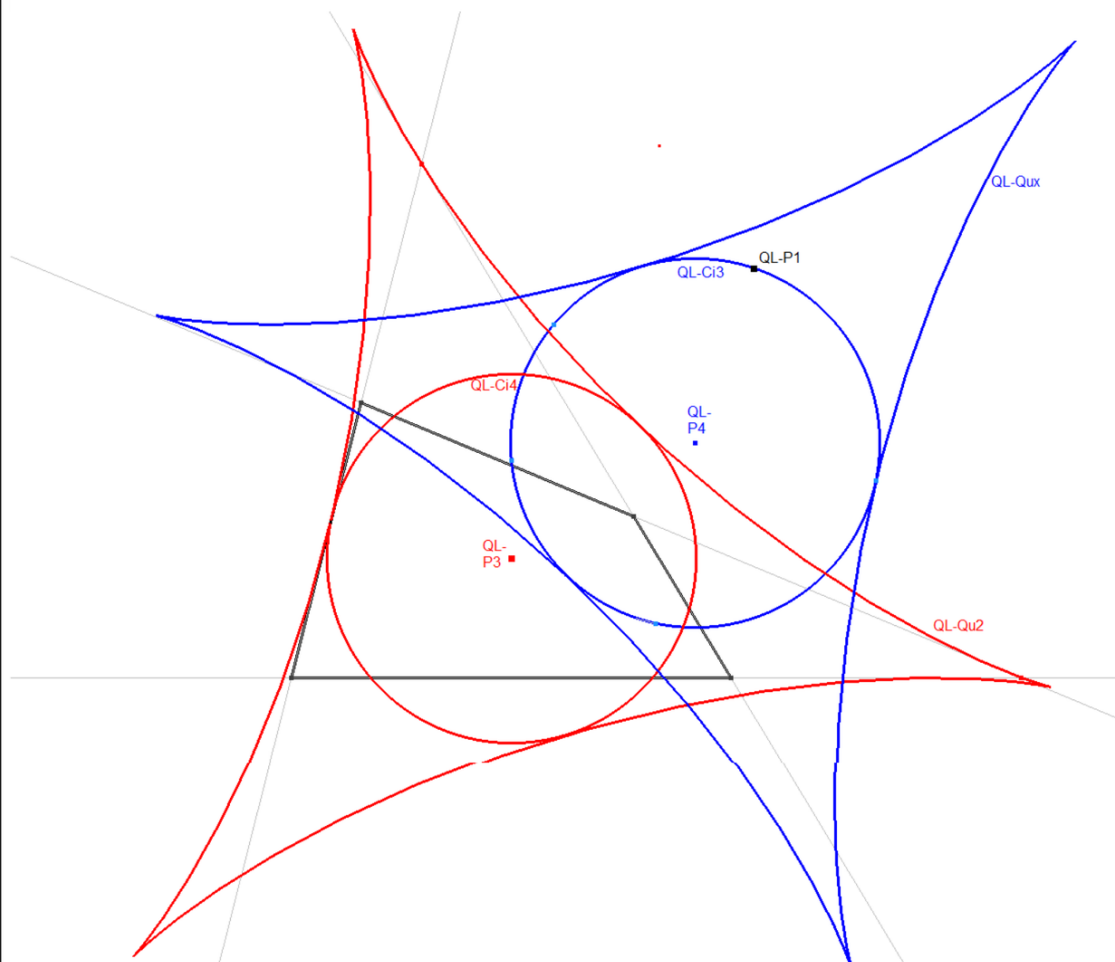
I found M@IMF's equation for QL-Deltoid-2 very intriguing. Providing a proper equation for curves of the 4th degree in barycentric coordinates is challenging, but the polar equation in the complex plane offers a good alternative.

I wonder if there is a similar equation for:

- * QL-Qu2 (Kantor-Hervey Deltoid)?
- * QL-Qu1 (Morley's Mono Cardioid)? This cardioid is inscribed with circles passing through QL-P1, and having its center on QL-Ci3.

Best regards,
Chris

2 QL-Deltoids circumscribing QL-Ci3 and QL-Ci4



Chris van Tienhoven

January 17, 2025

QL-Qu2 and QUX QL-Deltoids-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2583
Date: 2025-01-18
From: bernard.keizer@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear all,
These inscribed circles and QL-deltaoids are symmetric wrt the Clawson center QL-P5.
You may obtain with the same symmetry a cardioid with reference circle the Hervey circle QL-Ci4 ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2584
Date: 2025-01-18
From: bernard.keizer@gmail.com
Subject: Re: 2 dual Hesse pencils

Dear Chris, dear Eckart
I had a dream
Having in EQF the equations of the cubic stelloid QL-Cu2 and it's hessian the Van Rees curve QL-Cu1, Chris was able to calculate the flexes, real and imaginary and to determine the points H1, H2 and H3 and the coefficients a1, a2, a3 and k for QL-Cu2 and k' for QL-Cu1.
As every polar conic wrt QL-Cu1 is a rectangular hyperbola, the poles P0, X1, X2 and X3 of the line of real flexes form an orthocentric QA and in particular, P0 is the orthocenter of X1X2X3, the lines L1, L2 and L3 are the altitudes of this triangle and T1T2T3 is it's orthic triangle (we discussed this configuration with Eckart many many years ago in a time we didn't know about the point P0 and the harmonic polars ...)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2585
Date: 2025-01-18
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear Chris, Miyamoto-san, Mr. Schmidt, Mr. Keizer and all,

Strictly speaking, my equation is for the deltoid defined by a cyclic QA and a point P'.

If P' is on the circumcircle of QA1, and QL1 is a quadrilateral formed by Simson lines of P'

wrt QA1-component triangles, then P' is QL1-P1.

The equation of QL1-Deltoid-1 (QL1-Qu2) is

$$z = (p' + a)/2 - (R/2)(b/c)^{(1/3)} (2t + 1/t^2), |t| = 1,$$

where

$$a = p_1 + p_2 + p_3 + p_4, b = p_1 p_2 p_3 p_4 / R^4, c = p' / R, R = |p'|,$$

and the equation of QL1-Deltoid-2 is

$$z = p'/2 + (R/2)(b/c)^{(1/3)} (2t + 1/t^2), |t| = 1.$$

When we use the configuration in #2493,

$$z[QL1-P1] = (m - q)/2 + i(f + r - p)/2$$

$$z[QL1-P2] = -(m + 3q)/4 + i(r - 3f - p)/2$$

$$z[QL1-P3] = -(m + 3q)/4 + i(5r - 3f - p)/4 = (p' + a)/2$$

$$z[QL1-P4] = (m - q)/4 + i(f + r - p)/4 = p'/2$$

$$b/c = -[(m - q) + i(f + r - p)] / \sqrt{[(m - q)^2 + (f + r - p)^2]},$$

where i is the imaginary unit.

(Note that the origin of xy-plane and that of complex plane differ.)

The equation of a cardioid is like this:

$$z = z_0 + \rho(2t - t^2), |t| = 1.$$

But I can't determine z_0 and ρ because I'm not familiar with QL-Qu1.

By the way, is QL-Deltoid-2 Steiner deltoid of the anticomplementary triangle of the triangle ABC below?

On Tue, Jan 14, 2025 at 05:05 PM, Eckart Schmidt wrote:

- > Dear Keita Miyamoto,
- > with interest I have reproduced your construction
- > ... of the 4 parallel lines L_i^* for P on QL-Ci3,
- > ... here a further observation:
- > These parallels are parallel to the Simson line of P
- > ... wrt a triangle ABC: one vertex A in QL-P1
- > ... with opposite line QL-L1, intersecting QL-Ci3 in B and C.
- > If you consider perpendiculars through P wrt these Simson lines,
- > ... they envelope a deltoid with inner circle QL-Ci3.
- > Best regards Eckart

Best regards, M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2586
Date: 2025-01-20
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear M@IMF, Chris, Eckart Schmidt, Bernard Keizer and all,

Let m be the Miquel point (= QL-P1),
and o be the Miquel circumcenter (= QL-P4),
then the equation of Morley's mono cardioid (= QL-Qu1)
in the complex plane is:

$$z = o + (o-m) \cdot (2t + t^2), \quad |t|=1$$

On Sun, Jan 19, 2025 at 04:26 AM, M@IMF wrote:

> By the way, is QL-Deltoid-2 Steiner deltoid of the
> anticomplementary triangle of the triangle ABC below?

Yes.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2587
Date: 2025-01-20
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Thank you, Keita-san.
I was KUWAZUGIRAI.

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2588
Date: 2025-01-21
From: contiwa.goma3@gmail.com
Subject: Re: A property of QL-Ci3 Miquel circle

Dear all,

More about QL-Qu2.

Given a cyclic quadrangle $QA_0\{Q_1, Q_2, Q_3, Q_4\}$, and let F lie on the circumcircle of QA_0 .

QL_1 is a quadrilateral formed by Simson lines of F wrt QA_0 -component triangles.

When F revolves around QA_0-P_3 at angular velocity ω , QL_1-Qu_2 rotates around QL_1-P_3 at $-\omega/3$, and QL_1-P_3 revolves around QA_0-P_2 at ω .

If F is the reflection of Q_1 wrt QA_0-P_3 , QL_1-Qu_2 coincides Steiner deltoid of the triangle $\{Q_i, Q_j, Q_k\}$, where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

The locus of the vertex of QL_1-Qu_2 is an astroid, that is, QL_1-Qu_2 rolls inside this astroid.

(cf. <https://mathworld.wolfram.com/Rotor.html>)

To draw the whole curve, F needs to revolve around QA_0-P_3 three times.

The radius of the astroid is 4 times that of QL_1-Ci_4 .

The direction of the vertex of the astroid is $\varphi + \pi/4 \pmod{\pi/2}$, where φ is the direction of Ax_1 (one of the axes of QA_0-2Co_1).

Note that Steiner axes of QL_1 is parallel or perpendicular to Ax_1 .

I'm going to post a message about cyclic nP and hypocycloid.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2590
Date: 2025-01-23
From: contiwa.goma3@gmail.com
Subject: Cyclic nP and Hypocycloid

Dear all,

I defined normalized hypocycloid of a cyclic nP as well as n-stator and (n-1)-rotor.
Is there any use for them?

< Rotor and Stator >

To see what rotor and stator are like, please visit below sites:

<https://mathworld.wolfram.com/Rotor.html>

<https://en.wikipedia.org/wiki/Hypocycloid>

<https://johncarlosbaez.wordpress.com/2013/12/03/rolling-hypocycloids/>

<https://demonstrations.wolfram.com/RollingCycloidalCurves/>

In complex coordinates, the parametric equation of n-cusped hypocycloid is

$$z = z_0 + \rho[(n-1)t + 1/t^{(n-1)}]/n, \quad |t| = 1,$$

where

z_0 = center of hypocycloid

$|\rho|$ = radius of circumcircle

$\arg(\rho)$ = direction of vertex (mod $2\pi/n$).

When $z_0 = 0$ and $\rho = 1$, its vertices are n-th roots of 1.

Define

$$Hc(v, z_0, \rho, t) = z_0 + \rho[(v-1)t + 1/t^{(v-1)}]/v.$$

If n-cusped hypocycloid stator (n-stator) is

$$z = Hc(n, 0, 1, t), \quad |t| = 1,$$

then (n-1)-cusped hypocycloid rotor ((n-1)-rotor) is

$$z = Hc(n-1, \exp(i(n-1)s)/n, \exp(-i s)(n-1)/n, t), \\ |t| = 1, \quad (0 \leq s \leq 2\pi),$$

where i is the imaginary unit.

Note that

$$Hc(n/(n-1), 0, 1, t)$$

$$= [(n-1)t^{(-1/(n-1))} + 1/(t^{(-1/(n-1)))})^{(n-1)}/n,$$

and the locus of the vertices of (n-1)-rotor is n-stator.

N-stator and (n-1)-rotor of a cyclic nP can be defined inductively.

(N-1)-rotor of a cyclic nP is defined as (n-1)-rotor which coincides (n-1)-stator of each component (n-1)P of nP at a certain moment.

< Deltoid, Astroid >

Given 3 points p_j ($j=1,2,3$) such that $|p_j| = R$, and let Tr_1 be a triangle formed by them.

Steiner deltoid of Tr_1 is

$$z = (p_1 + p_2 + p_3)/2 + (1/2)(p_1 p_2 p_3)^{(1/3)} (2t + 1/t^2),$$

$$|t| = 1.$$

Its incircle is the nine-point circle of Tr_1 , and the directions of its vertices are those of the circumtangential triangle of Tr_1 .

3-stator of Tr_1 is defined as its Steiner deltoid.

Given 4 points p_j ($j=1,2,3,4$) such that $|p_j| = R$, and let QA_1 be a quadrangle formed by them.

The center of 3-stator of the component triangle of QA_1 lies on a circle with center $a/2$ and radius $R/2$, so 3-rotor of QA_1 is defined as

$$z = a/2 + (R/2) (-b)^{(1/4)} [\exp(3 i s) + \exp(- i s) (2t + 1/t^2)],$$

$$|t| = 1, \quad (0 \leq s \leq 2\pi),$$

where $a = p_1 + p_2 + p_3 + p_4$, $b = p_1 p_2 p_3 p_4 / R^4$.

When $\exp(i s) = [(-p_1 p_2 p_3 p_4)^{(1/4)} / (p_i p_j p_k)^{(1/3)}]$,

3-rotor coincides with 3-stator of triangle $\{p_i, p_j, p_k\}$,

where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

(See also #2588 for QL-Qu2.)

In the expression of 3-rotor,

$t = 1$ corresponds to one of the vertices of the rolling deltoid, and

$$z = a/2 + (R/2) (-b)^{(1/4)} [\exp(3 i s) + 3 \exp(- i s)],$$

$0 \leq s \leq 2\pi$ represents an astroid.

Then 4-stator of QA_1 is defined as

$$z = a/2 + (R/2) (-b)^{(1/4)} (3t + 1/t^3), \quad |t| = 1.$$

< Normalized Hypocycloid >

In general, for n points p_j ($j=1, \dots, n$) such that $|p_j| = R$, n -stator is defined as

$$z = Hc(n, a[n]/2, g[n] n/2, t), \quad |t| = 1,$$

and $(n-1)$ -rotor is defined as

$$z = a[n]/2 + g[n] Hc(n-1, \exp(i (n-1)s)/2, \exp(- i s) (n-1)/2, t), \quad |t| = 1, \quad (0 \leq s \leq 2\pi),$$

where

$$a[n] = p_1 + p_2 + \dots + p_n$$

$$g[n] = [-(-1)^n p_1 p_2 \dots p_n]^{(1/n)}.$$

When $\exp(i s) = g[n]/g[n-1]$, $(n-1)$ -rotor coincides with $(n-1)$ -stator.

As n becomes larger, the radius of the circumcircle of n -stator becomes larger,

and the relationship between n -stator and nP becomes weaker.

So I defined normalized hypocycloid of a cyclic nP{p1,...,pn} as
 $z = Hc(n, a[n]/n, g[n], t), |t| = 1,$
which is homothetic with n-stator (ratio 2/n, center the
origin).
Its radius of the circumcircle is R and its center is the
centroid of the nP.
(Isn't it nice?)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2591
Date: 2025-01-23
From: Stan.Rabinowitz@comcast.net
Subject: Nine-point Center Quadrangle of Nine-point Center Quadrangle

Is the following result known?

Let WXYZ be the Nine-point Center Quadrangle of the Nine-point Center Quadrangle of quadrangle ABCD.

Then ABCD and WXYZ are homothetic.

How is the perpector related to quadrangle ABCD?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2592
Date: 2025-01-23
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: Nine-point Center Quadrangle of Nine-point Center Quadrangle

Dear Stanley Rabinowitz,

Yes, it is known.

See QA-P7 (= QA-nine-point homothetic center) and QL-P27 (= nine point homothetic center QL-X(3) quadrangle).

QA-P7 lies on the line joining the QA-centroid and isogonal center.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2593
Date: 2025-01-26
From: anopolis72@gmail.com
Subject: Quadrilateral Problem

Considérons quatre droites dans un plan: ces quatre droites prises trois à trois forment quatre triangles et l'une d'elles, AB par exemple, appartient à trois de ces triangles.

Dans chacun des trois triangles correspondant à AB, joignons le centre du cercle circonscrit au sommet opposé à AB.

1° Pour un même côté AB les trois droites ainsi menées concourent en un point I:

2° les quatre points analogues à I et les quatre centres des cercles circonscrits aux triangles formés par les quatre droites sont sur une même circonférence.

Journal de mathématiques élémentaires
tome sixième, 1882, p. 37

Google translation

Consider four straight lines in a plane: these four straight lines taken three by three form four triangles and one of them, AB for example, belongs to three of these triangles. In each of the three triangles corresponding to AB, let us join the center of the circumscribed circle to the vertex opposite AB.

1° For the same side AB, the three straight lines thus drawn converge at a point I:

2° the four points analogous to I and the four centers of the circles circumscribed to the triangles formed by the four straight lines are on the same circumference.

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2594
Date: 2025-01-26
From: contiwa.goma3@gmail.com
Subject: Re: Quadrilateral Problem

Dear APH,

The point I or its analogue (on $QL-Ci3$) is the reflection of Vl' in the circumcenter of the triangle $\{T_{jk}, T_{ki}, T_{ij}\}$ in #2529, where $(i, j, k, l) = (1, 2, 3, 4)$ or their permutations.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

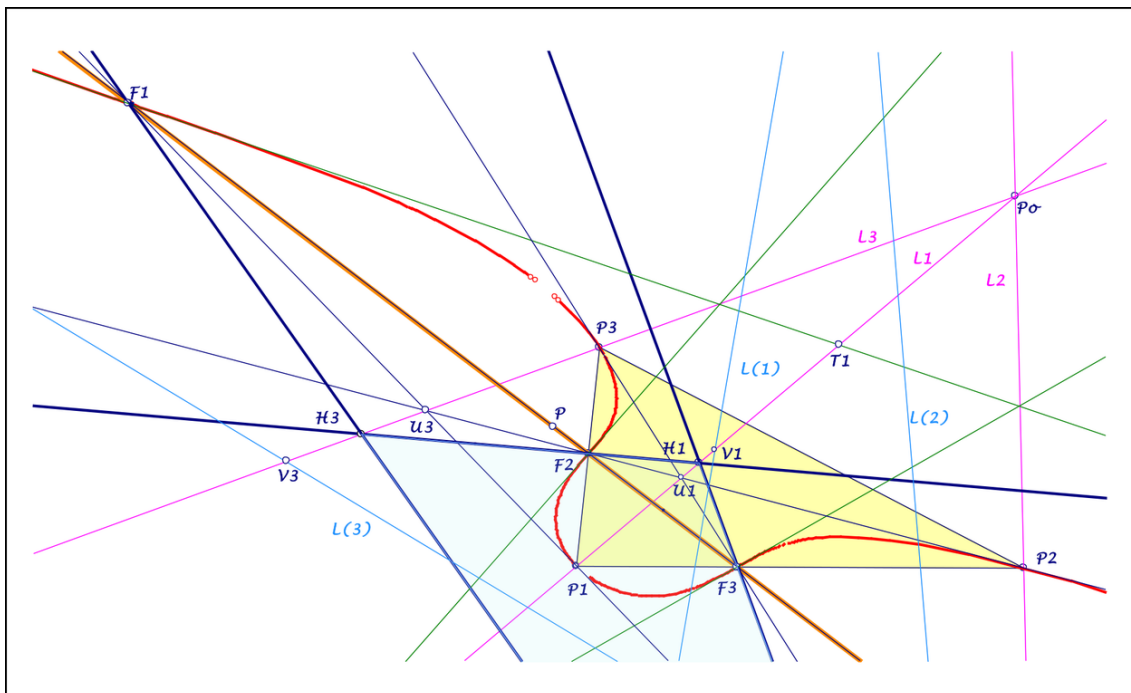
Message: #2595
Date: 2025-01-28
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

now I am rather convinced, that this is a correct construction
... for the real flexline trilateral $H_1H_2H_3$ of a cubic CU :
Let F_1, F_2, F_3 be the real flexpoints,
... L_1, L_2, L_3 their harmonic polars with common point P_0 ,
... intersecting the infinite part of CU in P_1, P_2, P_3 ,
... finally the intersections T_i of L_i
and the flextangent in F_i .

Start with quadrilaterals $QL_k = (F_iP_i, F_iT_i, F_jP_j, F_jT_j)$,
... which give the regions of H_k , let $F_iP_i \wedge F_jP_j = U_k$,
... and take points P on the flexline $F_1F_2F_3$
... with their dual lines $L(k)$ wrt QL_k
(wrt "dual" see QL-8 in EQF),
... the lines $L(k)$ for P on $F_1F_2F_3$ have a common point V_k on L_k ,
... then the 4th harmonic point of P_0 wrt U_k and V_k is H_k .
I made several drawings with astonishing validation!

Best regards Eckart



2025-01-28.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2596
Date: 2025-01-29
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
As I already told you, I'm always bluffed by your tenacity and your imagination!
I'm very glad to notice that you are stil interested in the item!
Uk is easy to draw, I suppose V_k is the dual point of the line of real flexes wrt your QL_k
Naturally, U_iU_j , V_iV_j and H_iH_j intersect in F_k and it seems that U_iV_j , U_jV_i and H_iH_j intersect on L_k in fk , the harmonic of F_k wrt H_i and H_j .
I'll try to check your property as soon as I find enough time to do it properly.
The real proof would be for you to do the same construction with the hessian (P_i becomes T_i and T_i another point t_i on L_i) and to find the same points H_i .
It would mean that the points P_0 and H_i are harmonic wrt the copple of points U_i and V_i for any cubic of the pencil, in particular the cubic and it's hessian.
Very beautiful indeed!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2597
Date: 2025-01-30
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for interest and remarks,
... I constructed H_1, H_2, H_3 also for the hessian
... and got the same points as for the reference cubic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2598
Date: 2025-01-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I must have misunderstood your construction, as I'm not able to reproduce your property.
Is it correct that your point V_k is the dual of the line of real flexes wrt QL_k ?
In this case, considering this QL_k , it's DT being selfdual, V_k is the opposite vertice of this line of real flexes.
Considering the intersections of F_iP_i and F_iT_i with F_jP_j and F_jT_j , we get 4 points U_k , X_k and A and B and V_k should be the intersection of the 2 other diagonals of DT , which are L_k and AB .
Am I wrong somewhere?
Naturally, if you find the same points H_1 , H_2 and H_3 for the cubic and it's hessian, your construction seems correct ...
Can you help me?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2599
Date: 2025-01-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
On your drawing in message 2595, QL1 is formed by the lines F2T2, F2P2, F3T3 and F3P3
F2P2 and F3P3 intersect in U1, F2T2 and F3T3 intersect in X1
F2T2 and F3P3 intersect in A, F2P2 and F3T3 intersect in B
AB (through F1) intersects L1 in V1
I almost reproduced your construction for $k = 10$ and $k = 12$ and exactly for $k = 100$, but it doesn't work for $k = 1$
Very strange, indeed!
Best regards
Bernard

PS These are examples of monopartite cubics, you said for bipartite cubics, you take the infinity part?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2600
Date: 2025-02-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,
excuse I haven't reacted on your message #2598:
You are right,
that V_k is the dual of the real flexline $F_1F_2F_3$ wrt QL_k .
The point V_k can be described without A, B
as $V_k = L_k^{(F_k.F_iP_i^{F_jT_j})}$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2601
Date: 2025-02-03
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

You describe in #2595:

Start with quadrilaterals $QL_k = (F_iP_i, F_iT_i, F_jP_j, F_jT_j)$,
... which give the regions of H_k , let $F_iP_i \wedge F_jP_j = U_k$,
... and take points P on the flexline $F_1F_2F_3$
... with their dual lines $L(k)$ wrt QL_k
 (wrt "dual" see QL-8 in EQF),
... the lines $L(k)$ for P on $F_1F_2F_3$ have a common point V_k on L_k ,

Am I right in my next reasoning:

When you make a QL bounded by the lines $(F_iP_i, F_iT_i, F_jP_j, F_jT_j)$,
then you actually span a QG with 3rd diagonal F_iF_j (which is
 $F_1F_2F_3$).

The dual lines $L(k)$ will have common point V_k .

Am I right when I say that the dual lines are $QL-Tf_{10}(P)$?

If so, then this common point = the QG-Diagonal Crosspoint
 $QG-P_1$?

Or stated in another way:

Create a QL from two lines emanating from F_i and two lines
emanating from F_j .

This QL has three component Quadrigons,

one of which will have F_iF_j as its 3rd diagonal $QG-L_1$.

The diagonal crosspoint $QG-P_1$ of this QG will be V_k .

See QL-3QG1.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2602
Date: 2025-02-03
From: anopolis72@gmail.com
Subject: Radical axis in qudrilateral

L'axe radical commun des trois circonferences décrites trois diagonales d'un quadrilatère complet comme diamètres passe par le centre du cercle circonscrit au triangle formé par ces trois diagonales.

By Chapron in the Solution of a problem by E. Lemoine
Journal de mathematiques elementaires, 1886, p. 144
<<https://dn790001.ca.archive.org/0/items/s2journaldemathm05pari/s2journaldemathm05pari.pdf>>

(The common radical axis of the three circumferences described by the three diagonals of a complete quadrilateral as diameters, passes through the center of the circle circumscribed to the triangle formed by these three diagonals).

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2603
Date: 2025-02-03
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

I cannot understand your reasoning,
... but if you consider the diagonal triangle $QLk-Tr1$,
... Vk is the opposite point of the sideline $F1F2F3$.
 Vk is not the diagonal crosspoint of your considered QG ,
... in #2600 I described Vk as $Vk = Lk^{(Fk.FiPi^{\wedge}FjTj)}$,
... please have a look at the drawing of #2595.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2604
Date: 2025-02-04
From: van10hoven@gmail.com
Subject: Re: Radical axis in quadrilateral

Dear Antreas,

Thanks for the reference. It is an extraordinary report from a remarkable period of continuous research during those days.

The common radical axis of the three circumferences described by the three diagonals of a complete quadrilateral as diameters is QL-L2, the Steiner Line, also called the Aubert Line or Orthocentral Line.

There are several points known to lie on this line:

1. The orthocenters of the four component triangles of the reference quadrilateral.
2. The two common intersection points of the three circumferences described by the three diagonals of a complete quadrilateral as diameters QL-2P1a & QL-2P1b.
3. The circumcenter of the Diagonal Triangle QL-P9.
4. The Morley Point QL-P2.
5. The Newton-Steiner Point QL-P7.

Everything converges in a quadrilateral.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2605
Date: 2025-02-04
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Thanks for the further explanation of the 'dual'.

I calculated a bipartite cubic using your method and determined the points (V_1, V_2, V_3) as you described: $V_k = L_k \wedge (F_k \cdot F_i \Pi \wedge F_j T_j)$. I also calculated these points as the diagonal crosspoints of the component QG with $F_1 F_2 F_3$ as the third diagonal. I obtained exactly the same outcome, so I think both methods are valid. I have attached a picture to better explain my method.

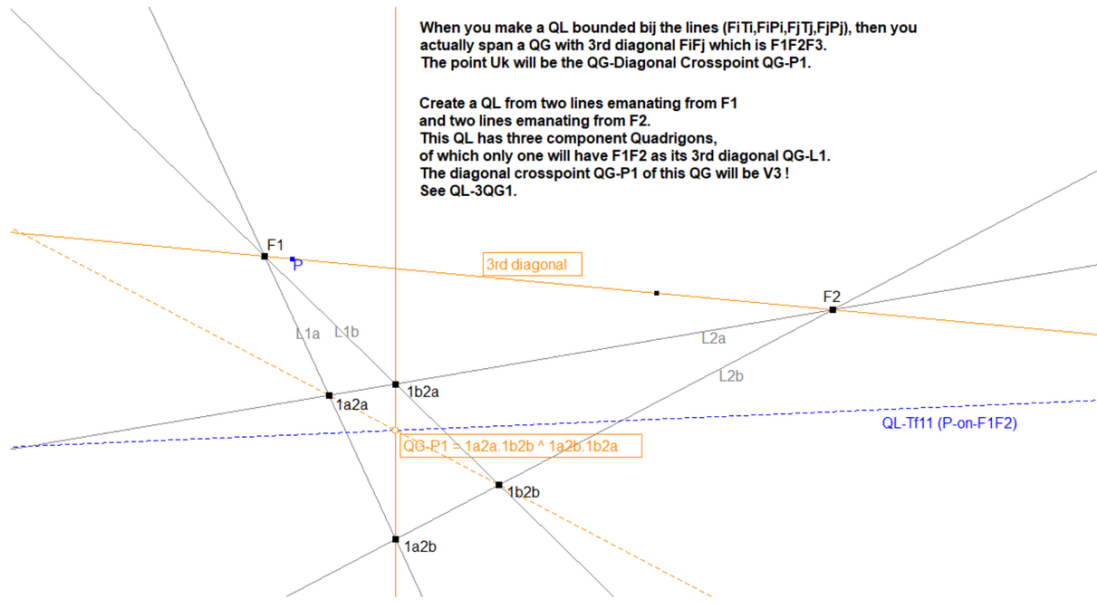
I also calculated the points (U_1, U_2, U_3) , but I could not confirm the relationship where H_i is the harmonic conjugate of P_0 with respect to (U_i, V_i) . Since you created different pictures where the relationship holds, I wonder where I am misunderstanding you.

Therefore, I have attached a picture of my calculated results, where you can see that H_i does not match the harmonic conjugate of P_0 with respect to (U_i, V_i) . Note that I did not use Flexpoints (F_1, F_2, F_3) , but instead I used (F_1, F_4, F_7) because it worked out that way in my calculations.

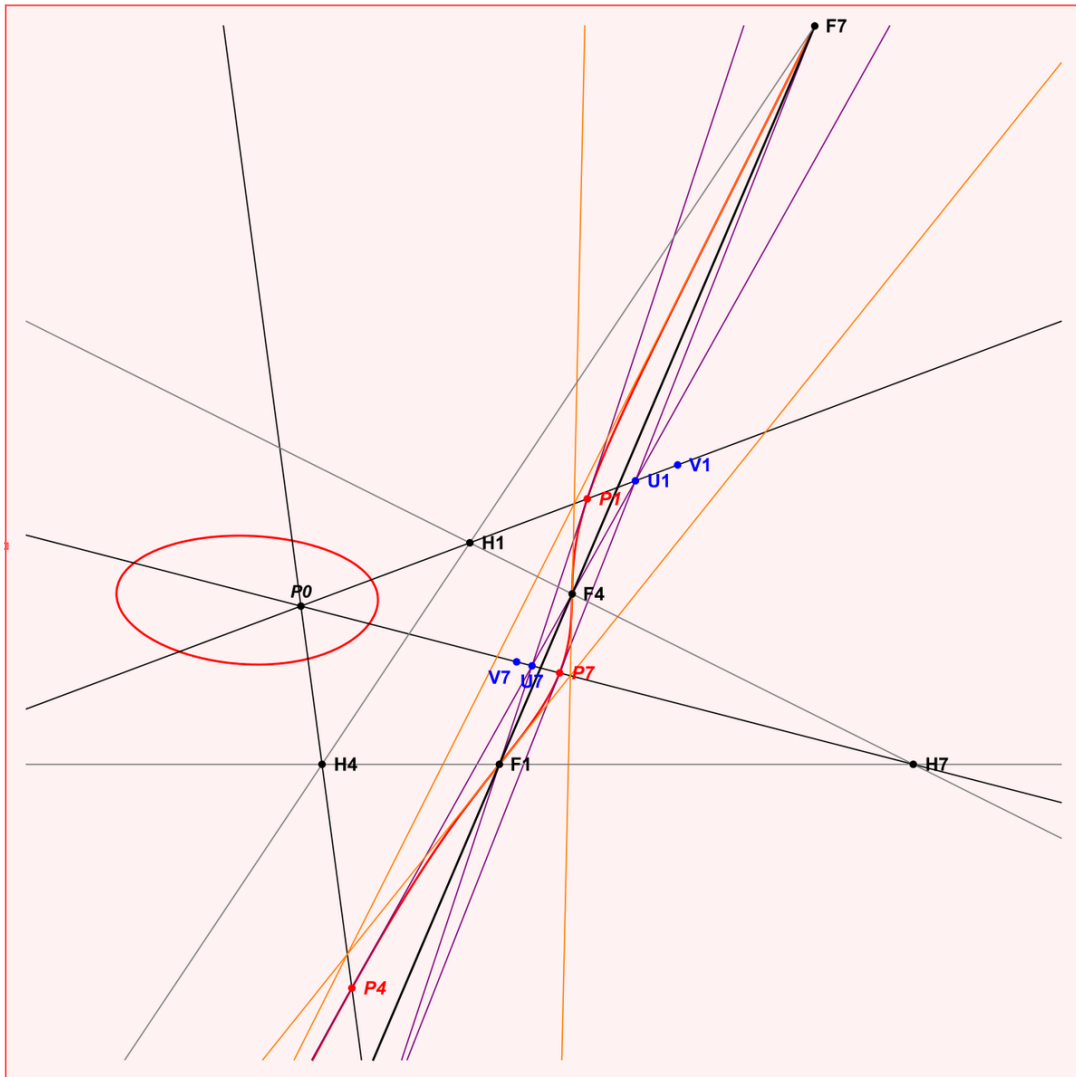
Best regards,

Chris

"dual" like described in QL-8 in EQF



Bipartite Cubic with constructed points (U1,U2,U3) and (V1,V2,V3)



Chris van Tienhoven

2025, Februari 4

CU-HE-12L1abc-21_Eckarts-Case-Numerical.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2606
Date: 2025-02-05
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

I have to specify my construction in #2595:

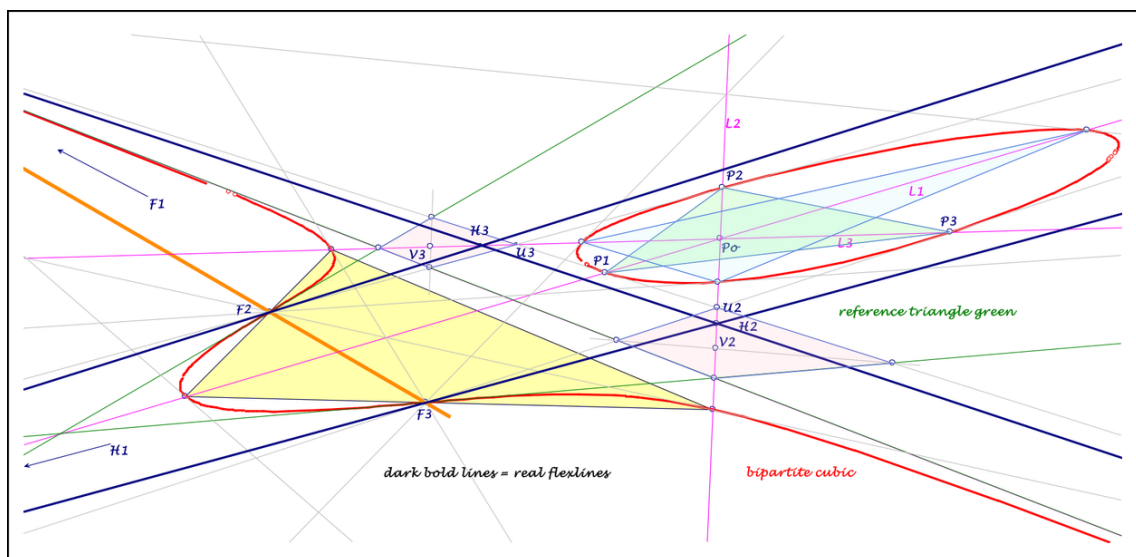
The construction of the real flexline trilateral
... seems correct for monopartite cubics,
... but for bipartite cubics there are three triangles $P_1P_2P_3$
... for intersections of harmonic polars L_i and the cubic,
... so that the cubic is invariant wrt isoconjugations
with fixed point P_0 .

Please take not needful $P_1P_2P_3$ on the infinite part as I wrote,
... but the triangle $P_1P_2P_3$,
whose lines F_iP_i and flextangents F_iT_i
... are borders for the allowed real flexlines through F_i ,
... attached this is the green triangle,
which leads to the right real flexlines.

Best regards Eckart

PS to Chris:

Excuse my wrong remark in # 2603, that V_k is not Q_G-P_1 .
Thanks for further interpretations.



2025-02-05.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2607

Date: 2025-02-05

From: anopolis72@gmail.com

Subject: Re: [Quadri-and-Poly-Geometry] Radical axis in qudrilateral

Dear Chris,,

I don't speak french, but I think this article is interesting

Marchand, J

<http://archive.numdam.org/item/NAM_1927_6_2__320_1.pdf>, Sur un théorème de Steiner.

Nouvelles annales de mathématiques: journal des candidats aux écoles polytechnique et normale, Serie 6, Volume 2 (1927), pp. 320-325

I translate the theorem by google:

In each of the four triangles formed by the sides of a complete quadrilateral, there is an inscribed circle and three exinscribed circles;

which makes in all sixteen circles, the centers of which are four by four on a circumference so as to give birth to eight new circles. These eight circles are divided into two groups of four, such that each of the circles of one of the groups orthogonally intersects all the circles of the other.

We conclude that the centers of the circles of the two groups are on two straight lines perpendicular to one to the other.

"Finally, these last two straight lines intersect at the meeting point of the circles circumscribed by the four triangles."

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2608
Date: 2025-02-05
From: eckart_schmidt@t-online.de
Subject: Re: Radical axis in quadrilateral

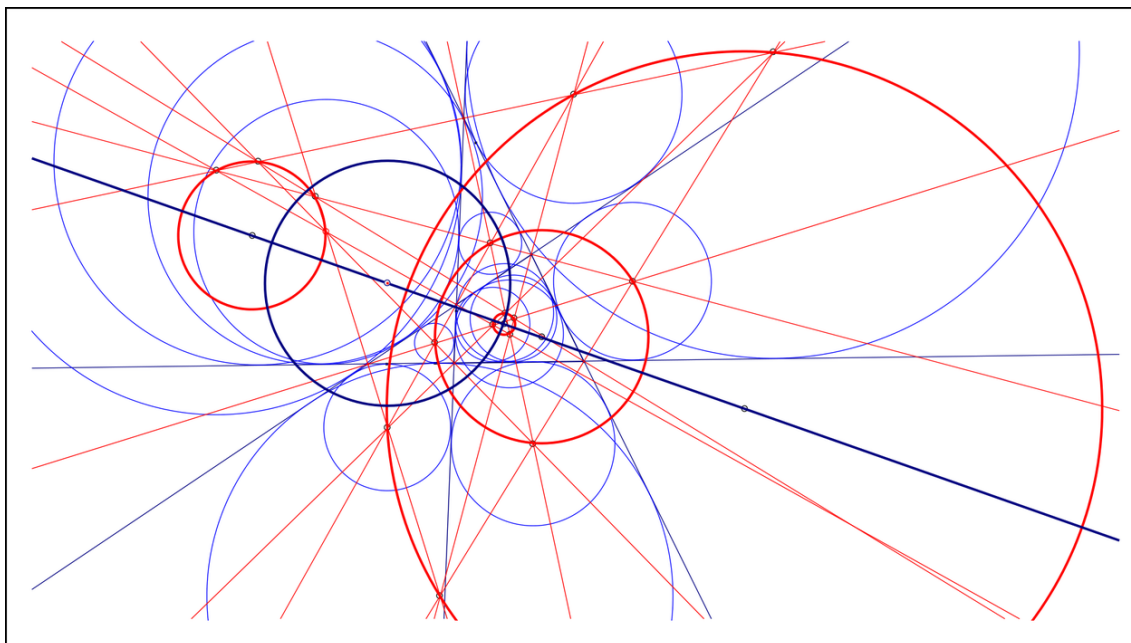
Dear ATM,

wrt a quadrilateral and its 16 in- and excircles,
... I made a drawing and following observations:

- (1) The 16 centers of the circles give four circles and 12 lines,
... each circle or line bearing 4 centers as triple intersections of the lines.
- (2) The 12 lines intersect pairwise orthogonal in the vertices of the quadrilateral.
- (3) The 4 circles are CSC-invariant,
... they intersect the CSC inversion circle orthogonal
... and their centers are collinear on the CSC-axis.

I couldn't reproduce all properties in your message.

Best regards Eckart



2025-02-06.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2609
Date: 2025-02-06
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart

After a lot of drawings, I'm now convinced that Eckart's last construction is again a very ingenious, but fake construction!

1) for Chris, how do you find your points H1, H2 and H3? I suppose it's pure calculation ...

2) for Eckart, how do you know that your points are the real points H1, H2 and H3?

3) As I already told you, I use the reverse construction starting with a triangle H1H2H3 and a point P0 with barycentric coordinates $(1/a_1, 1/a_2, 1/a_3)$.

Then I know the equations of the line of real flexes and of the cubics of the pencil.

Varying k gives the cubics CU and Eckart's construction works only for relatively big values of k (almost, but not quite up from 10 and exact up from 25).

In this case, CU tends to RF and the fact that the construction gives the same result for CU and HE is easy to understand, as RF is its own hessian ...

Best regards

Bernard

PS Chris, if you finally are able to calculate the coordinates of the points H_i by solving the system $CU = 0$ and $FE = 0$, could you please make an application for the MacCay stelloid cubic K003 and its Van Rees hessian cubic K048 wrt a triangle ABC? Then a_1, a_2, a_3 and k and k' depend only from the 3 values a, b and c (lengths of the sides of ABC). The equations of the 2 cubics are in Bernard Gibert ... I suppose you also could have the coordinates of A, B and C wrt the triangle H1H2H3?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

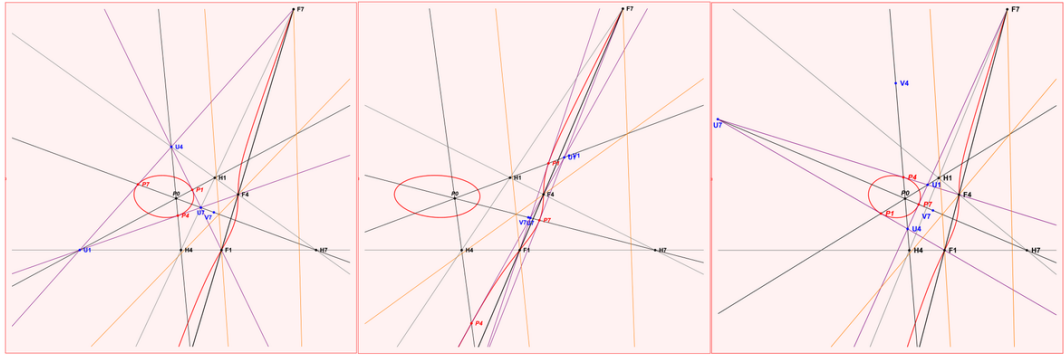
Message: #2610
Date: 2025-02-06
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I made pictures of the three different sets of $(P1, P4, P7)$ such that $P_i P_j = F_k$. Unfortunately, according to my calculations, in all cases H_i is not the Harmonic Conjugate of P_0 wrt (U_i, V_i) . See attached file.

Best regards,
Chris

Three pictures in Eckart's case with different sets of $(P1, P4, P7)$



There are three different sets of $(P1, P4, P7)$ such that $P_i P_j = F_k$.
According to calculations, in all cases H_i is not the Harmonic Conjugate of P_0 wrt (U_i, V_i) .

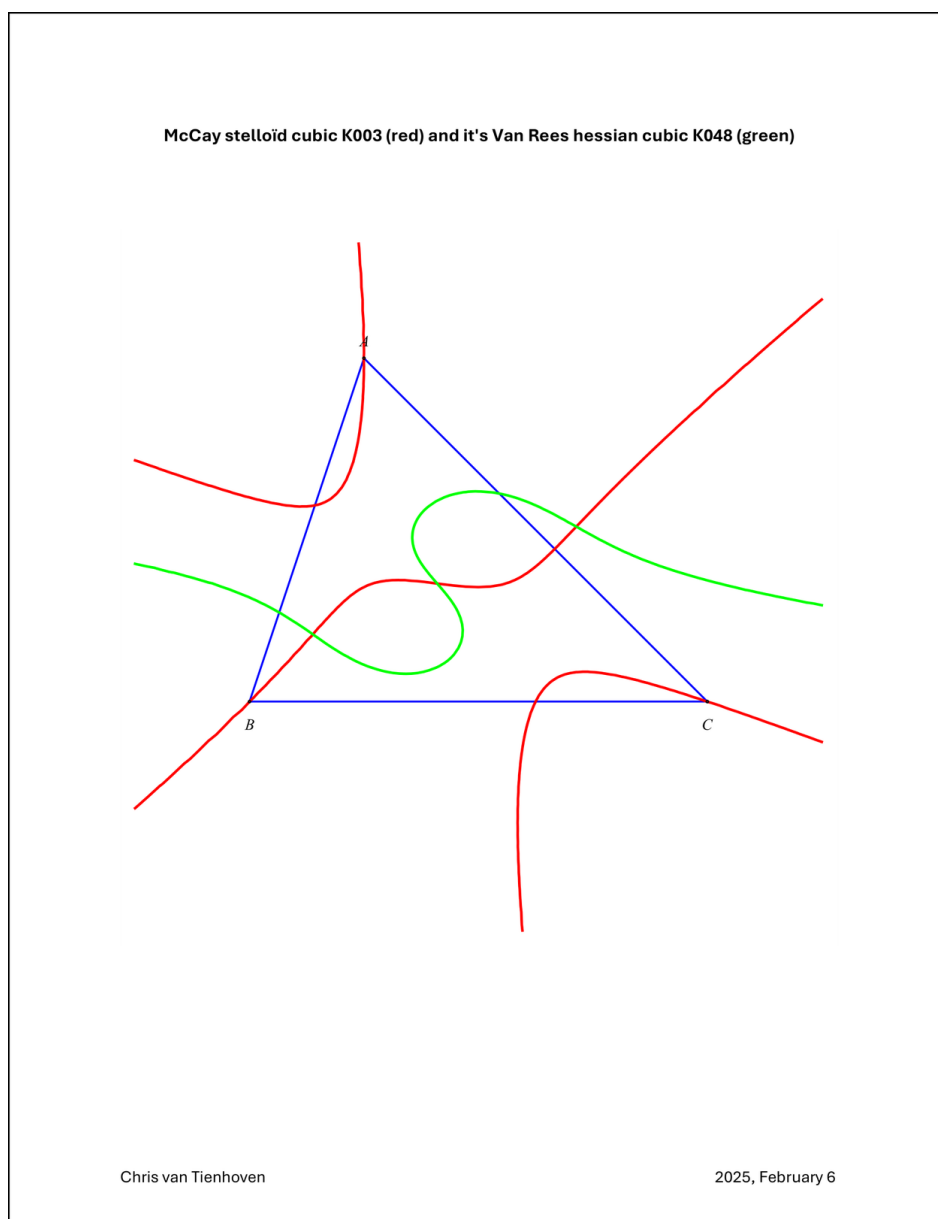
Chris van Tienhoven 2025, February 6

CU-HE-12L1abc-22_Eckarts-Case-Numerical.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2611
Date: 2025-02-06
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Bernard,
Please find enclosed a picture of K003 and K048.
That's not so difficult to plot for me.
However finding the Flexpoints and the real flexlines, that part
will cost a lot of time.
Therefore just a quick and short picture this time.
Best regards,
Chris



K003-K048 cubics-01.pdf

Message: #2612
Date: 2025-02-06
From: contiwa.goma3@gmail.com
Subject: Re: Radical axis in qudrilateral

Dear all,

Generalization of #2602.
If foci of three QL1-inconics are intersection points of QL2 lines,
then pedal circles of the foci wrt QL1 (i.e. auxiliary circles of the QL1-inconics) are coaxial.
The radical axis passes through the midpoint of QL1-P9 and QL2-P9. (cf. QFG#1832)

Best regards,
M@IMF

p.s. Ref-4 in EQF will be helpful for #2607.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2613
Date: 2025-02-07
From: bernard.keizer@gmail.com
Subject: MacCay cubic stelloid K003 and Van Rees hessian K048

Dear Chris, dear Eckart
Here a summary of the most well-known properties of the Mac Cay cubic stelloid K003 and it's Van Rees hessian K048, determining together a syzygetic pencil.
I hope Chris will be able to solve the system of the 2 barycentric equations wrt the triangle ABC, in order to find the triangle H1H2H3 and the point P0.
Best regards
Bernard
PS I'll try to do the same for Kjp and it's hessian

MacCay cubic stelloid K003 and Van Rees hessian cubic K048

1) Triangle ABC and the 2 cubics

Let's consider a triangle ABC with centroid G, circumcenter O, orthocenter H, Lemoine point K, isodynamic points S and S' on the Brocard Line OK and Steiner inellipse with foci F and F'.

The Mac Cay stelloid cubic is the pivotal isocubic wrt ABC with pivot O.

The cubic passes through A, B, C, O, H.

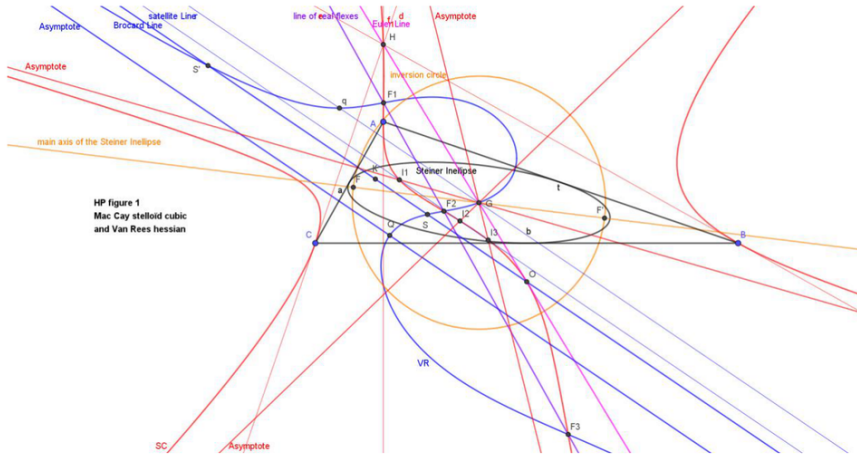
The 3 tangents in A, B and C are the altitudes of the triangle.

The 3 real asymptotes through G make angles of 60° , are parallel to the sides of the Morley triangles of the triangle ABC, intersect the curve in 3 points on the satellite Line of the infinity Line, homothetic to the Brocard Line in the homothety $(G, 2/3)$ and trisect the angle between the main axis of the Steiner inellipse and the parallel in G to the Brocard Line.

The Mac Cay hessian cubic is a Van Rees circular cubic, passing through G, S and S'.

The asymptote of this cubic is homothetic of the Brocard Line in the homothety $(G, 2)$.

The curve is invariant in the Moebius transformation centered in G with fixed points F and F' and swapping S and S'.



2) Syzygetic pencil

The 2 curves determine a syzygetic pencil and intersect in the 9 flexes (3 real and 6 imaginary) on 12 lines (4 real and 8 imaginary).

The 4 real lines are the line of the 3 real flexes and 3 others respectively through each real flex, determining a real triangle with vertices H1, H2 and H3 (this triangle is the cubic RF).

The 9 flexes have 9 harmonic polars, 3 real intersecting in P0 and 6 imaginary.

The 3 flextangents determine a triangle X1X2X3, P0 is the orthocenter of this triangle and the 3 real harmonic polars are the altitudes of this triangle.

3) Triangle H1H2H3

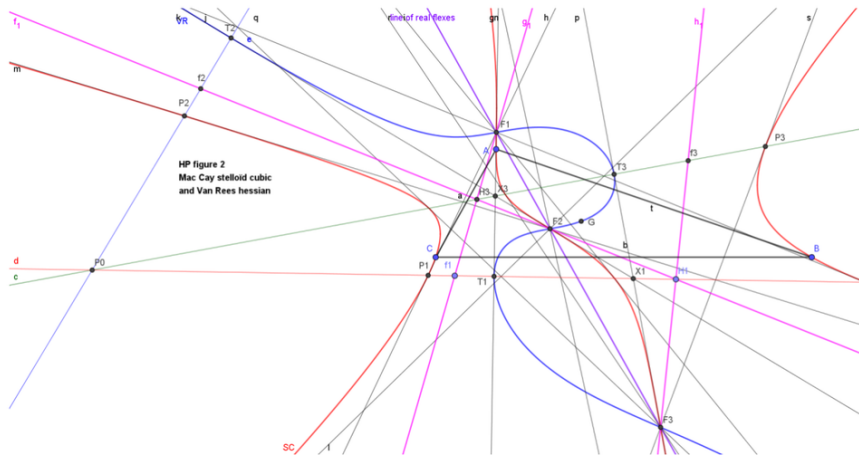
If f_i is the harmonic of F_i wrt H_j and H_k , it must lie on the harmonic polar L_i of F_i between P_i and T_i , intersections of L_i and respectively the 2 curves Mac Cay and it's hessian.

Solving the system of barycentric equations of the 2 curves wrt the triangle ABC should give the equation of the 4 real flexlines and the coordinates of H1, H2 and H3 wrt ABC.

Reversing the calculation should also give the coordinates of A, B and C wrt H1H2H3.

It should give also the coordinates of F1, F2, F3 and P0 wrt ABC and wrt H1H2H3.

If a, b and are the lengths of the sides of ABC and a_1, a_2, a_3, k and k' are the coefficients of the 2 cubics in the syzygetic pencil defined by H1, H2, H3 and P0, there is a relation between the 2 systems.



Mac Cay stellod cubic K003 and Van Rees hessian cubic K048.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2614
Date: 2025-02-07
From: anopolis72@gmail.com
Subject: Miquel Circle

FRANÇOIS GIRAULT
<http://archive.numdam.org/item/NAM_1919_4_19__452_0.pdf>,
Sur le cercle de Miquel. Nouvelles annales de mathématiques 4e
série, tome 19 (1919), p. 452-456
APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2615
Date: 2025-02-07
From: van10hoven@gmail.com
Subject: Re: Miquel Circle

Dear Antreas,

Thanks for your new reference.

M. Francois Girault describes here:

1. the focus F of the parabola
of a quadrilateral $QL-Co1$,
which is $QL-P1$.
2. a circle F passing through the centers of the circumcircles
of the 4 component triangles of a quadrilateral,
which is $QL-Ci3$.
3. a circle passing through the centers of the circles at 2.
of the 5 component quadrilaterals of a pentalateral,
which is $5L-n-Ci1$ for $n=5$).
4. a common point of the circles mentioned at 3.
of the 4 component quadrilaterals,
which is $5L-n-P1$ for $n=5$).

It is a nice introduction for the Encyclopedia of Polygon
Geometry EPG.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2616
Date: 2025-02-09
From: anopolis72@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] Radical axis in qudrilateral

On Wed, Feb 5, 2025 at 11:04 AM Antreas Hatzipolakis
<anopolis72@gmail.com>

wrote:

> Marchand, J
> <http://archive.numdam.org/item/NAM_1927_6_2__320_1.pdf>,
> Sur un théorème de Steiner.
> Nouvelles annales de mathématiques:
> journal des candidats aux écoles polytechnique et normale,
> Serie 6, Volume 2 (1927), pp. 320-325

More references

H. G. Forder, <<https://www.jstor.org/stable/3607453>>
Steiner's Theorems on Incentres and Ecentres
The Mathematical Gazette, Vol. 22, No. 248 (Feb., 1938),
pp. 79-8

Jean-Pierre Ehrmann

<<https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=f741d7b6aaa4a73bc36706b599db740b73ce8538>>,
Steiner's Theorems on the Complete Quadrilateral
Forum Geometricorum Volume 4 (2004) 35-52.

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2617
Date: 2025-02-09
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris, dear Eckart

As promised, here a short addendum to my 1st memo about Kjp and
it's hessian.

I will now wait on Chris's answer ...

Best regards

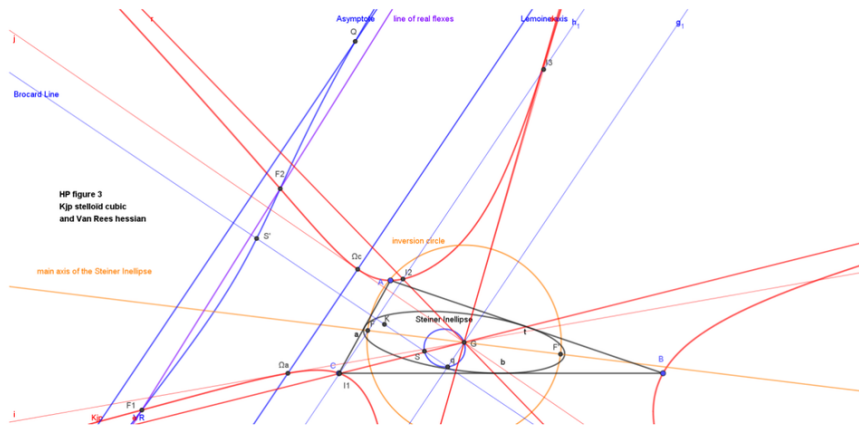
Bernard

Kjp cubic stelloid K024 and Van Rees hessian cubic K193

1) Triangle ABC and the 2 cubics

The properties of the 2 cubic stelloids are similar, both cubics intersect orthogonally in A, B and C and their asymptotes are orthogonal in G.

The properties of the 2 hessians are also similar, both hessians intersect orthogonally in S, S' and G and their asymptotes are orthogonal.

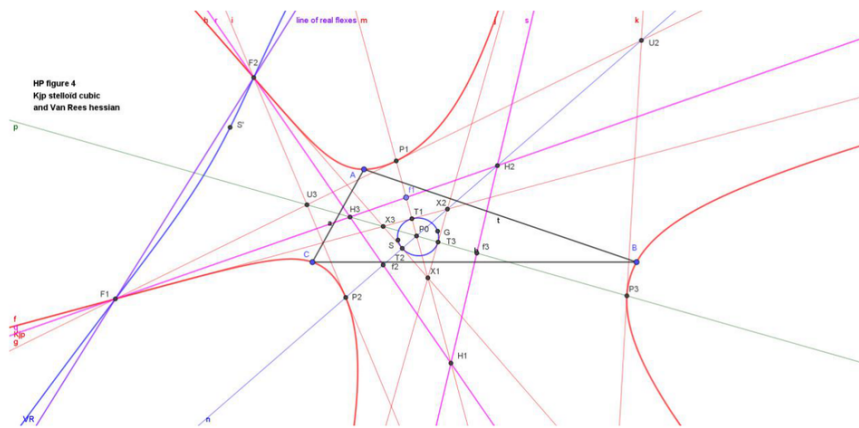


Kjp cubic stelloid K024 and Van Rees hessian cubic K193.pdf

2) Syzygetic pencils and Triangles H1H2H3

Kjp and its hessian determine also a syzygetic pencil and P_0 is also the orthocenter of $X_1X_2X_3$ (property of the stelloids, as the polar conics are equilateral hyperbolas).

The 2 points P_0 , the 2 groups of flexes and the 2 groups of harmonic polars are different as well as the 2 triangles $H_1H_2H_3$ and the 2 syzygetic pencils.



Kjp cubic stellod K024 and Van Rees hessian cubic K193.pdf

Message: #2618
Date: 2025-02-10
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,
What is it exactly you want me to calculate?
Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2619
Date: 2025-02-10
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,
I want you to solve in both cases the system of the 2 equations wrt ABC (all the coefficients are in terms of a, b and c) in order to find the coordinates of F1, F2, F3, H1, H2, H3 and P0 wrt ABC and then by changing the reference triangle, the coordinates of A, B, C, F1, F2, F3 and P0 wrt H1H2H3. Finally, I hope you we will be able to get the equations of the 2 cubics and their hessians in their syzygetic system and if we are lucky, we could find the equation of the 3 real prehessians of MacCay and of the real prehessian of Kjp ...
Many thanks for your attention and best wishes for the realisation of the calculation.

Best regards
Bernard

PS I'm convinced since a certain time that there isn't a simple geometric construction of the points H1, H2 and H3 for a general cubic and that it is pure calculation. This is why I would be so happy if you could make this calculation for these 2 well-known cubic stelloïds and their circular hessians ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2620

Date: 2025-02-14

From: bernard.keizer@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,

I realise that I don't know in fact what you are capable to calculate with Mathematica!

I thought you were able to calculate a_1 , a_2 and $a_3 = f(a,b,c)$.

But you need perhaps real values of a , b , c ...

In this case, I suggest to choose $(a, b, c) = (13, 6, 9)$ like in ETC (see Tables).

Perhaps we would be capable to identify directly the point P_0 !

Please just tell me where you are and what you intend to do.

Many thanks in advance

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2621

Date: 2025-02-14

From: van10hoven@gmail.com

Subject: Re: MacCay cubic stelloid K003 and Van Rees hessian K048

Dear Bernard,

I once again tried to tackle the case of general cubic CU: $c_1 x^3 + c_2 y^3 + c_3 z^3 + c_4 x^2 y + c_5 x^2 z + c_6 x y^2 + c_7 y^2 z + c_8 x z^2 + c_9 y z^2 + c_0 x y z$ wrt a reference triangle ABC and transcribe it to a cubic in Hesse's form $a_1 x^3 + a_2 y^3 + a_3 z^3 + k x y z$ wrt some reference triangle $H_1(X_1, Y_1, Z_1)$, $H_2(X_2, Y_2, Z_2)$, $H_3(X_3, Y_3, Z_3)$.

That is doable.

I found that

$$a_1 = T_1 / (X_1 + Y_1 + Z_1)^3$$

$$a_2 = T_2 / (X_2 + Y_2 + Z_2)^3$$

$$a_3 = T_3 / (X_3 + Y_3 + Z_3)^3$$

$$k = T_0 / ((X_1 + Y_1 + Z_1)(X_2 + Y_2 + Z_2)(X_3 + Y_3 + Z_3)) ,$$

where:

$$T_1 = c_1 X_1^3 + c_4 X_1^2 Y_1 + c_6 X_1 Y_1^2 + c_2 Y_1^3 + c_5 X_1^2 Z_1 + c_0 X_1 Y_1 Z_1 + c_7 Y_1^2 Z_1 + c_8 X_1 Z_1^2 + c_9 Y_1 Z_1^2 + c_3 Z_1^3 ,$$

$$T_2 = c_1 X_2^3 + c_4 X_2^2 Y_2 + c_6 X_2 Y_2^2 + c_2 Y_2^3 + c_5 X_2^2 Z_2 + c_0 X_2 Y_2 Z_2 + c_7 Y_2^2 Z_2 + c_8 X_2 Z_2^2 + c_9 Y_2 Z_2^2 + c_3 Z_2^3 ,$$

$$T_3 = c_1 X_3^3 + c_4 X_3^2 Y_3 + c_6 X_3 Y_3^2 + c_2 Y_3^3 + c_5 X_3^2 Z_3 + c_0 X_3 Y_3 Z_3 + c_7 Y_3^2 Z_3 + c_8 X_3 Z_3^2 + c_9 Y_3 Z_3^2 + c_3 Z_3^3 ,$$

$$T_0 = 6 c_1 X_1 X_2 X_3 + 6 c_2 Y_1 Y_2 Y_3 + 6 c_3 Z_1 Z_2 Z_3 + 2 c_4 (X_2 X_3 Y_1 + X_1 X_3 Y_2 + X_1 X_2 Y_3) + 2 c_5 (X_2 X_3 Z_1 + X_1 X_3 Z_2 + X_1 X_2 Z_3) + 2 c_6 (X_3 Y_1 Y_2 + X_2 Y_1 Y_3 + X_1 Y_2 Y_3) + 2 c_7 (Y_2 Y_3 Z_1 + Y_1 Y_3 Z_2 + Y_1 Y_2 Z_3) + 2 c_8 (X_3 Z_1 Z_2 + X_2 Z_1 Z_3 + X_1 Z_2 Z_3) + 2 c_9 (Y_3 Z_1 Z_2 + Y_2 Z_1 Z_3 + Y_1 Z_2 Z_3) + c_0 (X_3 Y_2 Z_1 + X_2 Y_3 Z_1 + X_3 Y_1 Z_2 + X_1 Y_3 Z_2 + X_2 Y_1 Z_3 + X_1 Y_2 Z_3) .$$

The big question is what are the coordinates of $H_1(X_1, Y_1, Z_1)$, $H_2(X_2, Y_2, Z_2)$, $H_3(X_3, Y_3, Z_3)$ wrt ABC.

In order to find the general coordinates of these points we first need to find the flexpoints. And in order to find the flexpoints we need to know the Hessian. Fortunately that is an easy to calculate cubic.

However to find the intersection points of CU and its Hessian (two equations of 3 rd order) in general form is the limit of what is doable with Mathematica. I tried in vain all kind of tricks, but nothing worked.

So I stopped that path.

That means that I can't calculate flexpoints and flexlines as a function of (a,b,c).

All I can do is getting numerical . . .

What is it you would like me to calculate that way?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2622

Date: 2025-02-16

From: bernard.keizer@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,

Many thanks for the calculations and explanations about what is doable or not!

These 2 calculations (HE in 2436 and a_1, a_2, a_3 and k in 2621) will be my reference up to now.

- 1) Intersection of CU and HE in general terms is not doable
- 2) Calculation of $A_1 = a_1^3$, $A_2 = a_2^3$, $A_3 = a_3^3$ and $K = ka_1a_2a_3$ is doable for any cubic in general terms

(congratulations for this idea!)

(The equations of the 3 sides of $H_1H_2H_3$ are $x=0$, $y=0$ and $z=0$ and the equation of the line of the 3 real flexes is $a_1x+a_2y+a_3z=0$)

I suppose you checked that the calculation for HE gives the same a_1 , a_2 and a_3 and $k' = -(6/k)^2 - k/3 \dots$

- 3) Wrt any triangle ABC, the equation of K003 MacCay cubic stelloïd is $\sum a^2(b^2+c^2-a^2)x(c^2y^2-b^2z^2) \dots$ (cyclic) = 0

The equation of K024 Kjp cubic stelloïd is $\sum a^2x(c^2y^2+b^2z^2) \dots$ (cyclic) = 0

(I don't write the equation of the Hessians, they are in Bernard Gibert)

What are a_1, a_2, a_3 and k ?

Is in this case the calculation of the 9 flexes doable? (there are less coefficients, 6 for CU and 9 for HE instead of 10 and 10)

- 4) If not and if you have to take a numerical example, could you please choose $a=13$, $b=6$ and $c=9$ (ETC reference triangle)

Thanks in advance for these last efforts

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2623
Date: 2025-02-16
From: contiwa.goma3@gmail.com
Subject: Re: Quadrilateral Problem

Dear all,

Is this an already known thing?

Let

$QL1 = QL\{L1, L2, L3, L4\}$

$F = QL1-P1$

$Tij = Li^{\wedge}Lj$

$C1 =$ the circumcircle of triangle $TjkTkiTij$

$O1 =$ the circumcenter of $C1$

$QAo = QA\{O1, O2, O3, O4\}$

$QAo' = QA\{O'1, O'2, O'3, O'4\}$

which is homothetic to QAo with homothetic center F

$Co' =$ the circumcircle of QAo'

$P1 =$ the second intersection point of Co' and $C1$

$QAp = QA\{P1, P2, P3, P4\}$

$QL2 =$ QL formed by Simson lines of F

wrt component triangles of QAp

$L'l =$ perspectrix of triangles $TjkTkiTij$ and $O'io'j0'k$,

where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

Then $QL\{L'1, L'2, L'3, L'4\}$ and $QL2$ are similar.

(Note that when $QAo' = QAo$, $P1$ is I or its analogue in #2593.)

Best regards,

M@IMF

p.s. I didn't recognize that the point I and its analogues are the perspectors of not only component triangles but also cyclic quadrangles themselves.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2624
Date: 2025-02-16
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

Dear all,

Perspectivity like #2623 can be applied to 5L in #2528 which is not a general 5L.

Let

5L2 = 5L formed by L_1, L_2, L_3, L_4, L_5
which are tangents of a parabola

$QL_h = QL\{L_i, L_j, L_k, L_l\}$

$F = QL_h - P_1 (= 5L_2 - n - P_1)$

$T_{ij} = L_i \wedge L_j$

O_{lh} = the circumcenter of triangle $T_{jk}T_{ki}T_{ij}$

$Ch = QL_h - C_i^3$

$Oh = QL_h - P_4$

$5Po = 5P\{O_1, O_2, O_3, O_4, O_5\}$

$5Po' = 5P\{O'_1, O'_2, O'_3, O'_4, O'_5\}$

which is homothetic to $5Po$ with homothetic center F

Co' = the circumcircle of $5Po'$

Ph = the second intersection point of Co' and Ch ,

where $\{i, j, k, l, h\} = \{1, 2, 3, 4, 5\}$.

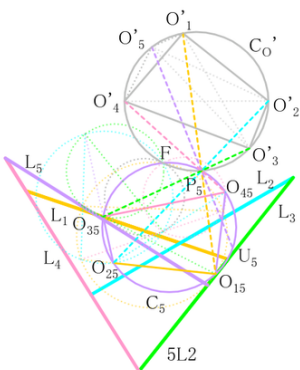
Then $QA\{O_{ih}, O_{jh}, O_{kh}, O_{lh}\}$ and $\{O'_i, O'_j, O'_k, O'_l\}$ are perspective with perspector Ph .

L_h is a Simson line of F wrt $QA\{O'_i, O'_j, O'_k, O'_l\}$ whose homothetic ratio 4.

See #2527 and the attached file.

Best regards,
M@IMF

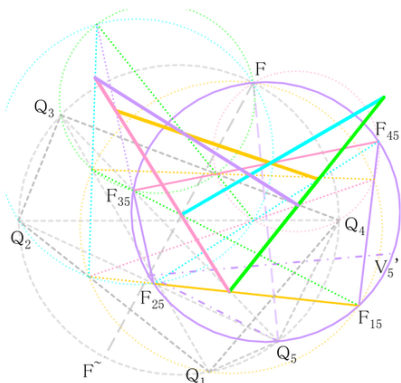
When lines L_1, L_2, L_3, L_4, L_5 are tangents of a parabola, some cyclic 5Ps related to them are perspective and similar.



Let

- $5L2 = 5L\{L_1, L_2, L_3, L_4, L_5\}$
- $QL_h = QL\{L_i, L_j, L_k, L_l\}$
- $F = QL_h - P1 (= 5L2 - n - P1)$
- $T_{ij} = L_i \cap L_j$
- O_{ih} = the circumcenter of triangle $T_{jk}T_{ki}T_{ij}$
- $C_h = QL_h - Ci3$
- $O_h = QL_h - P4$
- $5P_o = 5P\{O_1, O_2, O_3, O_4, O_5\}$
- $5P_o' = 5P\{O'_1, O'_2, O'_3, O'_4, O'_5\}$ which is homothetic to $5P_o$ with homothetic center F
- C_o' = the circumcircle of $5P_o'$
- P_h = the second intersection point of C_o' and C_h ,
- U_h = the second intersection point of line $P_hO'_h$ and C_h ,

where $\{i, j, k, l, h\} = \{1, 2, 3, 4, 5\}$.
 Then $5P\{O_{ih}, O_{jh}, O_{kh}, O_{lh}, U_h\}$ and $\{O'_i, O'_j, O'_k, O'_l, O'_h\}$ are perspective with perspector P_h , and similar with similitude center F .
 (Note that the circumcircle of $5P_o$ is $5L2 - n - Ci1$.)



<Relationship to Simson lines>

Given concyclic points F, Q_1, Q_2, Q_3, \dots ,
 a Simson line of F wrt cyclic nP is inductively defined.

Let

- $Sim[ij] = \text{line } Q_iQ_j$
- F_{ij} = the foot of the perpendicular from F onto $Sim[ij]$
- $Sim[ijk] = \text{a line through } F_{ij}, F_{jk}, F_{ik}$
- F_{ijk} = the foot of the perpendicular from F onto $Sim[ijk]$
- $Sim[ijkl] = \text{a line through } F_{ijk}, F_{jkl}, F_{ikl}, F_{jkl}$
- F_{ijkl} = the foot of the perpendicular from F onto $Sim[ijkl]$
- $Sim[ijklh] = \text{a line through } F_{ijkl}, F_{ijkh}, F_{ijlh}, F_{iklh}, F_{jklh}$
- ...

Then $Sim[j1, \dots, jn]$ is the Simson line of F wrt cyclic $nP\{Q_{j1}, \dots, Q_{jn}\}$.

When $Q_j(j=1, \dots, 5)$ divides the line segment $F.O_j$ in the ratio $4 : -3$,

- O_{ij} = the midpoint of F and F_{ij}
- $T_{ih} = F_{ijk}$
- $L_h = Sim[ijkl]$
- $5P\{F_{ih}, F_{jh}, F_{kh}, F_{lh}, V_h'\}$ and $\{O'_i, O'_j, O'_k, O'_l, O'_h\}$ are perspective and similar,

where

- $V_h' = \text{the reflection of } V_h \text{ in } F.Q_h$
- $V_h = \text{the foot of the perpendicular from } Q_h \text{ onto } FF^\sim$
- $F^\sim = \text{the antipode of } F \text{ wrt the circumcircle of } 5P\{Q_1, \dots, Q_5\}$
- $\{i, j, k, l, h\} = \{1, 2, 3, 4, 5\}$.

In general, $nL - n - P3$ of $nL\{Sim[j1, \dots, j(n-1)], \dots, Sim[j2, \dots, jn]\}$ divides the line segment FF^\sim in the ratio $1 : 2^{n-2} - 1$.

Similarly, $Sim[ijkl]$ s of F^\sim give $5L4$.

Note that $7P\{F, F_{ih}, F_{jh}, F_{kh}, F_{lh}, V_h', Q_h\}, \{F, Q_i, Q_j, Q_k, Q_l, Q_h, F^\sim\}, \{Q_h, F_{ih}, F_{jh}, F_{kh}, F_{lh}, V_h'', F^\sim\}$ are similar, where V_h'' is the reflection of V_h in $F^\sim Q_h$.

Message: #2625

Date: 2025-02-19

From: eckart_schmidt@t-online.de

Subject: Re: MacCay cubic stelloid K003 and Van Rees hessian K048

Dear Bernard, dear Chris,

wrt Chris message #2621:

What about starting with a general cubic
... as nonpivotal isocubic with reference triangle P1P2P3,
... with root Po(u,v,w), also fixpoint for the isoconjugation
... and a point H on the cubic.

Then you can discuss an equation
 $ux(w^2y^2+v^2z^2)+vy(u^2z^2+w^2x^2)+wz(v^2x^2+u^2y^2)+hxyz = 0$
... and get the flexpoints
F1 = (0,-v,w), F2 = (u,0,-w), F3 = (-u,v,0),
... and the harmonic polars
L1 = (0,w,-v), L2 = (w,0,-u), L3 = (-v,u,0),
... and the flexline F1F2F3 $vwx+wuy+uvz = 0$,
... and the flextangents of F1,F2,F3
 $(-hvw+2uv^2w^2)x - u^2vw^2y - u^2v^2wz = 0$,
 $-uv^2w^2x + (-huw+2u^2vw^2)y - u^2v^2wz = 0$,
 $-uv^2w^2x - u^2vw^2y + (-huv+2u^2v^2w)z = 0$,
... and the intersections T1,T2,T3
of the flextangents and L1,L2,L3,
($2u^2vw, v(2uvw-h), w(2uvw-h)$),
($u(2uvw-h), 2uv^2w, w(2uvw-h)$),
($u(2uvw-h), v(2uvw-h), 2uvw^2$),
... and the intersections X1,X2,X3 of the flextangents
($uvw-h, v^2w, vw^2$),
($u^2w, uvw-h, uw^2$),
($u^2v, uv^2, uvw-h$).

So far, I hope there is no mistake,
... but I am not up to date with your calculations.

Best regards Eckart

PS: Thanks for your messages 2609, 26010,
... I shall give up my constructions
of the real flexline trilateral,
... meanwhile I found an example with significant aberrations
... for a low k as Bernard remarks ...

Message: #2626

Date: 2025-02-20

From: bernard.keizer@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris, dear Eckart

It looks like the story of the ουροβορσ, the snake which bites it's own tail!

In order to know a_1 , a_2 , a_3 and k ,
you need the coordinates of H_1 , H_2 and H_3 .

In order to know the coordinates of H_1 , H_2 and H_3 ,
you need the flexes.

In order to know the flexes,
you need the intersection of a CU and It's HE ...

In order to know P_1 , P_2 and P_3 ,
you need the flexes ...

The equations of K048 and K193 are following:

```
K048 \SigmaFix b^2c^2(b^2-c^2)(b^2+c^2-a^2)x^3
-a^2(b^2+c^2-2a^2)(c^2(a^2+b^2-c^2)y-b^2(a^2+c^2-b^2)z)yz cyclic = 0
K193 -6a^2b^2c^2xyz+\SigmaFix b^2c^2(b^2+c^2)x^3 cyclic
+\SigmaFix (b^2+c^2-2a^2)a^2(c^2y+b^2z)yz cyclic = 0
```

Chris, can you solve the system of K003 and K048 or K024 and K193 either in general terms or for a numerical example with $a = 13$, $b = 6$ and $c = 9$?

It could help greatly to understand these mysteries!

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2627

Date: 2025-02-25

From: van10hoven@gmail.com

Subject: Re: MacCay cubic stelloid K003 and Van Rees hessian K048

Dear Bernard and Eckart,

I made a picture with K003 and its Hessian K048 with reference triangle (6,9,13).

Also the Real Flexpoints (F1,F2,F3), the corresponding real Flexlines and the corresponding Harmonic Polars and P0.
See attached file.

I saw no possibilities for calculating these items in general format.

Here are the relevant points in numeric barycentric coordinates:

F1 = {-0.563859, -0.801327, 1}

F2 = {3.0014, 4.02205, 1}

F3 = {-0.0918122, -0.162705, 1}

P0 = {1, -1.42454, 0.0616252}

And here are the relevant lines:

F1F2F3 = {1, -0.739163, -0.0284531}

The real Flexlines:

through F1 = {1, -2.0498, -1.0787}

through F2 = {1, -0.855748, 0.440457}

through F3 = {1, -0.22189, 0.0557097}

The Harmonic Polars:

wrt F1 = {1, 0.680691, -0.492151}

wrt F2 = {1, 0.729946, 0.646424}

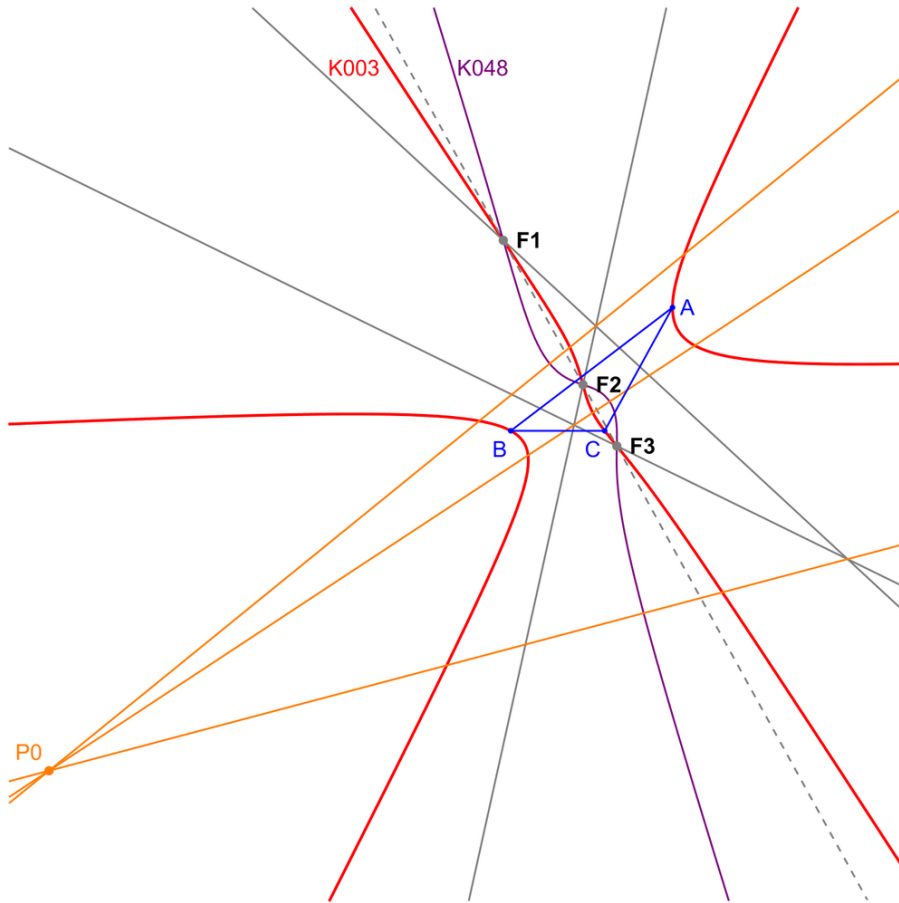
wrt F3 = {1, 0.812874, 2.5634}

I hope it helps in our elaborations.

Best regards,
Chris

Cubic K003 and its Hessian K048

wrt reference triangle ABC ($a=6, b=9, c=13$)
with real collinear Flexpoints $F1, F2, F3$
corresponding real Flexlines (gray)
corresponding Harmonic Polars (orange)



Chris van Tienhoven

2025, February 25

K003-K048 cubics-02.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2628

Date: 2025-03-01

From: bernard.keizer@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,

I wasn't home the last few days and I found your last message only yesterday evening.

That's very beautiful and it is exactly what I was dreaming of!

Thanks a lot and congratulations.

You didn't finish completely the calculations and we need now the exact coordinates of H1, H2 and H3 as intersections of the real flexlines on the corresponding harmonic polars. Then it would be nice to change the reference triangle in order to have A, B and C as well as P_0 wrt H1H2H3 and finally find a_1 , a_2 , a_3 and k and k' .

I found approximately $k = -6.3$ and $k' = 1.2$, but naturally, it lacks on precision.

I wonder if P_0 is an ETC point of the triangle ABC ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2630

Date: 2025-03-15

From: van10hoven@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,

Regarding your messages #2628 and #2629, I performed the following calculations:

H1={0.078976,0.606993,1.}

H2={-0.193416,-0.620606,1.}

H3={-1.5292,-1.27227,1.}

When I change the reference system to H1H2H3, I obtain:

CU = 1. x³-156.861 y³-3.05422 z³ - 48.3472 x y z

HE = 1. x³-156.861 y³-3.05422 z³ + 8.73714 x y z

Accordingly $k = -6.17874$ and $k' = 1.1166$, which closely match your results!

However, I could not find P_0 as an ETC point of ABC.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2631
Date: 2025-03-18
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

Dear all,

To describe *n-Chain* , I defined lines *wL* and points *wP* inductively.

Let F, F', Qi, Qj, Qk, ... be concyclic.

{Qi,Qj,Qk; F,F'}

wL[ijk]

wL[ijk]1 = a line through F perpendicular to wL[ijk]2

wL[ijk]2 = Simson line of F wrt triangle QiQjQk

wL[ijk]3 = Simson line of F' wrt triangle QiQjQk

wL[ijk]4 = a line through F' perpendicular to wL[ijk]3

wP[ijk]

wP[ijk]r = wL[ijk]r ∩ wL[ijk](r+1) (r=1,2,3)

{Qi,Qj,Qk,Ql; F,F'}

wL[ijkl]

wL[ijkl]1 = a line through F perpendicular to wL[ijkl]2

wL[ijkl](r+1) = wP[ijk]r ∩ wP[ijl]r ∩ wP[ikl]r ∩ wP[jkl]r (r=1,2,3)

wL[ijkl]5 = a line through F' perpendicular to wL[ijkl]4

wP[ijkl]

wP[ijkl]r = wL[ijkl]r ∩ wL[ijkl](r+1) (r=1,2,3,4)

{Qi,Qj,Qk,Ql,Qh; F,F'}

wL[ijklh]

wL[ijklh]1 = a line through F perpendicular to wL[ijklh]2

wL[ijklh](r+1) =

wP[ijk]r ∩ wP[ijk]r ∩ wP[ijl]r ∩ wP[ikl]r ∩ wP[jkl]r (r=1,2,3,4)

wL[ijklh]6 = a line through F' perpendicular to wL[ijklh]5

wP[ijklh]

wP[ijklh]r = wL[ijklh]r ∩ wL[ijklh](r+1) (r=1,2,3,4,5)

...

(I hope "U" is not garbled. If it is, I will use "_" instead. "L = P1_P2_ ... _Pn" means points P1,...,Pn are collinear and L is a line through them.)

Relationship to *6-Chain* (#2550):

F = A0, F' = A6

Qa = a point such that QL-P4 of {5L2b, 5L2c, 5L2d, 5L2e} divides A0.Qa in the ratio 1:3

wL[abcde]r = line 6L(r+1) (r=1,2,3,4)

wP[abcde]r = Ar (r=1,2,3,4,5)

5Lr = {wL[bcde]r, wL[acde]r, wL[abde]r, wL[abce]r, wL[abcd]r} (r=2,3,4)

$Br.a = wP[bcde]r \quad (r=1,2,3,4).$

Relationship to Simson line wrt nP (#2527, #2624):

$wL[j_1, \dots, j_n]_2 = \text{Sim}[j_1, \dots, j_n; F]$
 $wL[j_1, \dots, j_n]_n = \text{Sim}[j_1, \dots, j_n; F']$
 $wP[j_1, \dots, j_n]_1 = F.j_1, \dots, j_n$
 $wP[j_1, \dots, j_n]_n = F'.j_1, \dots, j_n.$

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2632

Date: 2025-03-18

From: van10hoven@gmail.com

Subject: preHessian Form of a cubic, three parameters define a cubic

Dear Bernard and Eckart and friends,

Hesse's Form

I have reviewed for myself the key results concerning the matter of Hesse's form once again.

For the latest summary, see message #2531.

The basics are actually quite simple: a cubic of the form

$$c_1 x^3 + c_2 y^3 + c_3 z^3 + c_4 x^2 y + c_5 x^2 z + c_6 x y^2 + c_7 y^2 z + c_8 x z^2 + c_9 y z^2 + c_0 x y z = 0$$

can be transformed via a projective transformation into a cubic of the form:

$$x^3 + y^3 + z^3 + k x y z = 0.$$

This is Hesse's Form and being described in several scientific papers.

preHessian Form

What we have added to this is an intermediate form. Let's call it the preHessian Form, which is given by:

$$a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3 + k a_1 a_2 a_3 x y z = 0.$$

This form can again be transformed via a simple projective transformation into:

$$x^3 + y^3 + z^3 + k x y z = 0, \text{ Hesse's Form.}$$

The key aspect of the *preHessian Form* is that the actual shape of the cubic does not change, but only the reference frame shifts from a triangle *ABC* to a triangle *H₁H₂H₃*, which is the triangle formed by the only three real flex lines passing through the only three real flex points of a cubic.

Since the cubic remains the same, but its defining equation changes due to a change in perspective, it follows that any cubic can be described by:

$$a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3 + k a_1 a_2 a_3 x y z = 0.$$

A direct consequence of this is that each cubic is characterized by four numbers: a_1^3 , a_2^3 , a_3^3 , and $k a_1 a_2 a_3$.

Since one of these four factors can be normalized to 1, the most natural choice is to set $k a_1 a_2 a_3 = 1$, which results in the equation:

$$x^3 / (k a_2 a_3) + y^3 / (k a_1 a_3) + z^3 / (k a_1 a_2) + x y z = 0.$$

Thus, every cubic –and consequently its shape– depends on the numbers appearing in this expression:

$$1 / (k a_2 a_3), 1 / (k a_1 a_3), 1 / (k a_1 a_2),$$

which means that the shape of each cubic depends on three numbers.

Parallel with conics

This is analogous to a conic section, whose shape depends on just one number, namely its eccentricity.

The eccentricity e of a conic section determines its shape and how "stretched" or "deviated from a circle" it is. It is defined as the ratio of the distance from any point on the conic to a focus, compared to its distance from the corresponding directrix. The value of e classifies the conic section:

$e = 0 \rightarrow$ Circle

A circle is a special case where the foci coincide with the center.

$0 < e < 1 \rightarrow$ Ellipse

An ellipse has two foci, and the closer e is to 1, the more elongated the ellipse appears.

$e = 1 \rightarrow$ Parabola

A parabola has exactly one focus and one directrix, and it extends infinitely outward.

$e > 1 \rightarrow$ Hyperbola

A hyperbola consists of two separate branches, and the larger e , the more widely spaced the branches are.

In summary:

The closer e is to 0, the more circular the conic section.

The larger e , the more elongated or open the conic becomes.

New questions:

* My question is: Do you agree with my conclusions, or am I overlooking something?

* If all of this is correct, what can be said about the shape of a cubic based on these three cubic parameters?

Best regards,
Chris

Message: #2633
Date: 2025-03-19
From: contiwa.goma3@gmail.com
Subject: Re: 5L and Lill's method

Dear all,

Let me correct "Relationship to 6-Chain" in #2631 as follows:

$F = A_0$, $F' = A_6$

Q_a = a point such that QL - P_4 of $\{5L_2b, 5L_2c, 5L_2d, 5L_2e\}$ divides $A_0.Q_a$ in the ratio 1:3

$wL[abcde]_r = \text{line } 6L_r.f \quad (r=1,2,3,4,5,6)$

$wP[abcde]_r = A_r \quad (r=1,2,3,4,5)$

$5L_r = \{wL[bcde]_r, wL[acde]_r, wL[abde]_r, wL[abce]_r, wL[abcd]_r\}$
 $(r=2,3,4) \quad (5L_r.a = wL[bcde]_r)$

$Br.a = wP[bcde]_r \quad (r=1,2,3,4).$

In general

$A[n]_0 = F$,

$A[n]_{(n+1)} = F^{\sim}$ (antipode of F on circumcircle of cyclic $nP\{Q_{j1}, \dots, Q_{jn}\}$)

$A[n]_r = wP[j_1, \dots, j_n]_r \quad (r=1, \dots, n)$

$(n+1)L_r.(n+1) = wL[j_1, \dots, j_n]_r \quad (r=1, \dots, n+1)$

$nL_{r.k} = wL[j_1, \dots, j_{(k-1)}, j_{(k+1)}, \dots, j_n]_r \quad (r=1, \dots, n)$

$B[n]_{r.k} = wP[j_1, \dots, j_{(k-1)}, j_{(k+1)}, \dots, j_n]_r \quad (r=1, \dots, n-1)$

$B[n]_{0.k} = F$, $B[n]_{n.k} = F^{\sim}$

$(n-1)L$ - n - P_3 of $\{nL_2.j_2, \dots, nL_2.j_n\}$ divides $F.Q_{j1}$ in the ratio $1 : 2^{(n-2)} - 1$.

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2634

Date: 2025-03-19

From: bernard.keizer@gmail.com

Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,

Many thanks for these last elements!

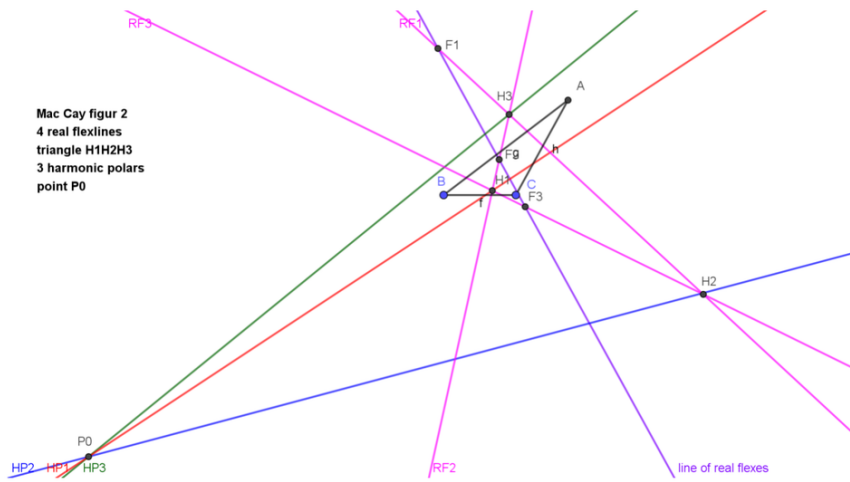
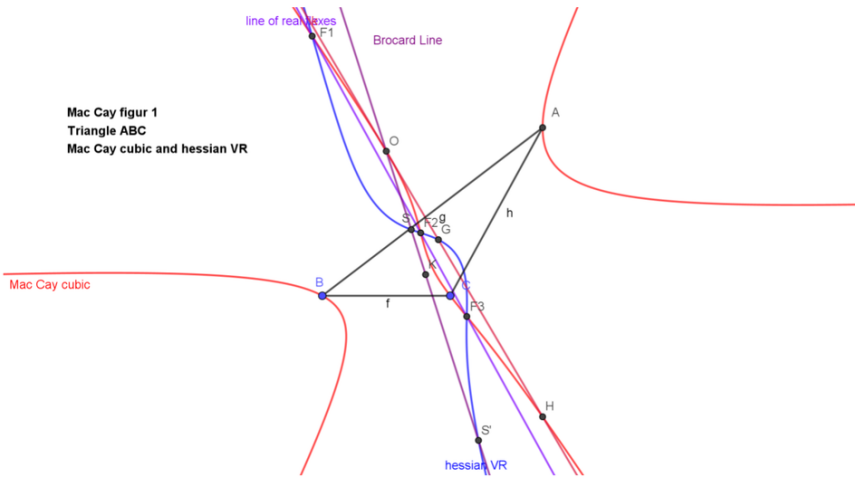
I send you a short memo with 4 figures (figur 1 with K003 and K048 wrt triangle ABC, figure 2 drawing of the triangle H1H2H3 and the harmonic polar, figure 3 K003 and K048 wrt H1H2H3, figure 4 cayleyan CA and curve Γ wrt H1H2H3).

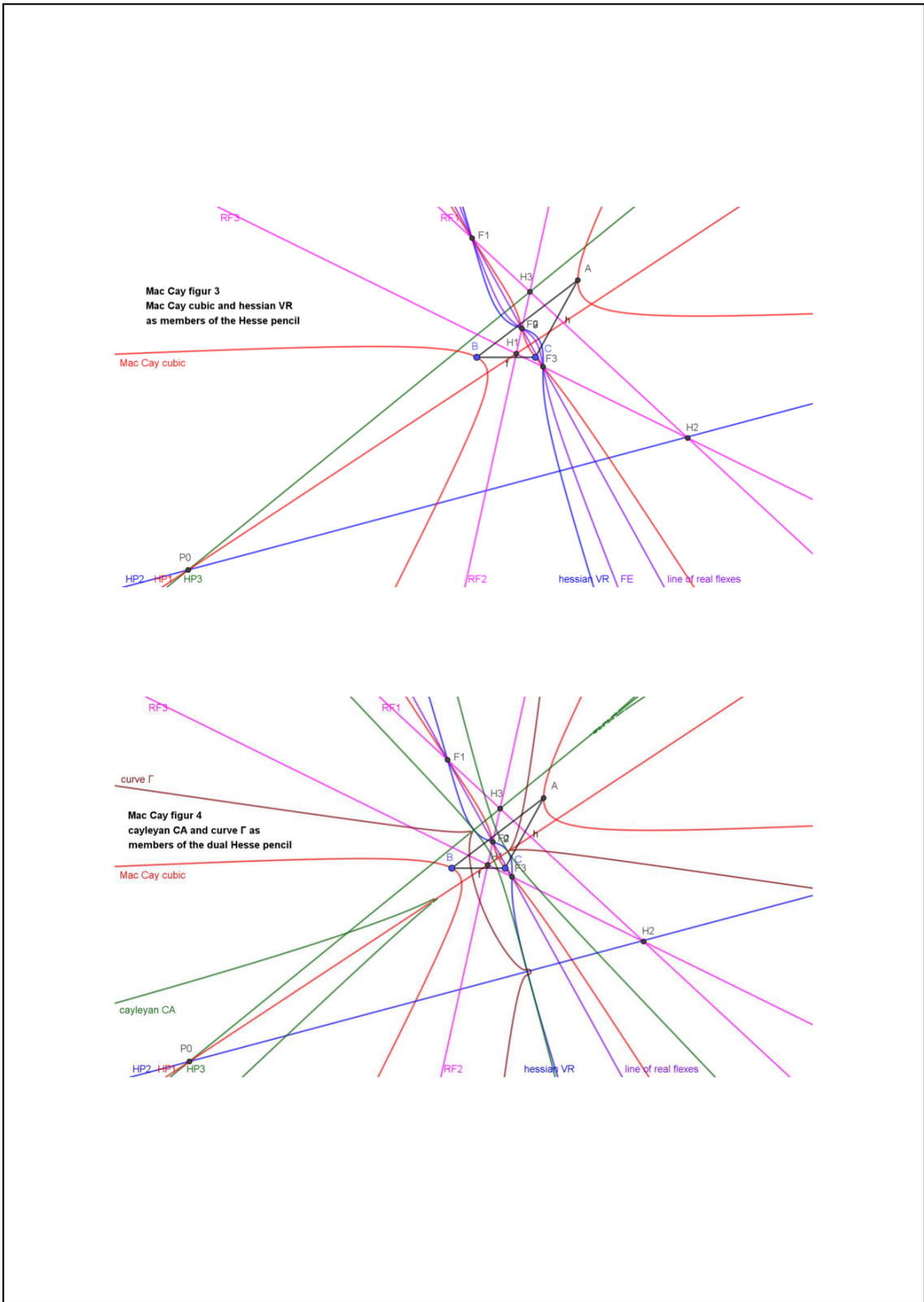
As expected, CA is tangent to the hessian in 3 points on the harmonic polars and has 3 cusps on the harmonic polars and Γ has 3 cusps on the harmonic polars in the contact points of the hessian and the cayleyan.

Best regards

Bernard

Mac Cay cubic and hessian





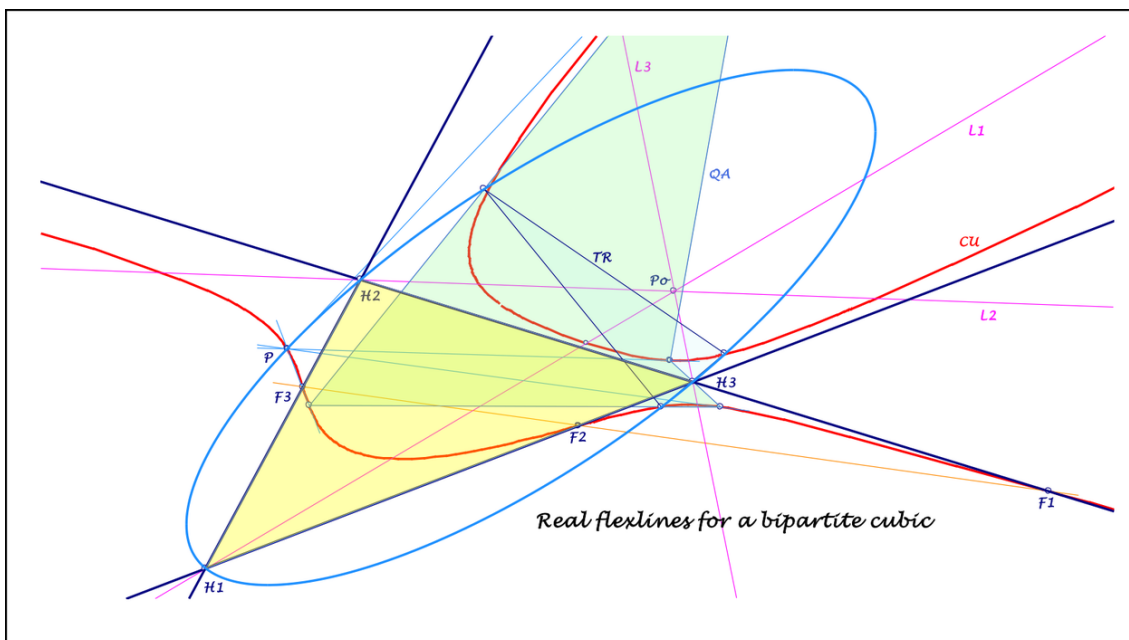
Mac Cay.pdf

Message: #2635
Date: 2025-03-20
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

sorry I cannot give up, please control the following method,
... to find the real flexlines for a bipartite cubic:
Consider a bipartite cubic and a cubic point P
... with four contact points of its tangents at the cubic,
... which give a QA with its diagonal triangle TR.
Consider further a variable triangle H1'H2'H3'
... on the harmonic polars L1,L2,L3 of the real flexpoints,
... with the flexpoints F1,F2,F3 on the sidelines.
You can find a conic, bearing P, the vertices of TR and
H1',H2',H3',
... then Hi' will be the intersections Hi of the real flexlines.
If the cubic is a pivotal isocubic, take the pivot as point P,
... the fixpoints of the isoconjugation give QA ...

Best regards Eckart



2025-03-20.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2636
Date: 2025-03-21
From: van10hoven@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Bernard,

Thanks for the beautiful pictures. Even with two Cayleyans!
Is Γ the Cayleyan of the Hessian?
Why did you draw the Brocard line in the first picture?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2637
Date: 2025-03-21
From: bernard.keizer@gmail.com
Subject: Re: MacCay cubic stelloïd K003 and Van Rees hessian K048

Dear Chris,

As already explained in a previous memo, Γ is the curve of the the 3rd order associated to a cubic CU, which swaps with CU their respective hessian and cayleyan.

In other words, if HE and CA are the hessian and the cayleyan of CU, CA and HE are the hessian and the cayleyan of Γ .

CU and HE belong to the Hesse pencil for k and k' , CA and Γ to the dual Hesse pencil for k'' and k^* .

I drew in the 1rst picture the points O (X3) and H (X4) on the Euler Line, which belong to Mac Cay and G (X2), S (X15) and S' (X16) on the Brocard Line, which belong to it's hessian, in order to check on the 3rd picture that these points are on the curves calculated wrt H1H2H3.

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2638

Date: 2025-03-21

From: bernard.keizer@gmail.com

Subject: Re: preHessian Form of a cubic, three parameters define a cubic

Dear Chris,

I would like to help you, but I'm not an expert in this matter

...

I know only what I read in several references:

the most relevant are

* Salmon

* Schröter (see in particular the pages dedicated to the shape of the cubics)

* Roger Cüppens (see in particular tome 2 chap 17 page 259 ...)

* Artebani

* Bonifant (see in particular figur 10 page 41)

* Mathcurve (see in particular the definition 'cubic')

Your question of the classification of the cubics has been studied by Newton and Chasles and a few others ...

A Hesse pencil is defined by a triangle $H_1H_2H_3$ and a point $P_0(1/a_1, 1/a_2, 1/a_3)$, the line of real flexes being $a_1X + a_2Y + a_3Z = 0$...

Then the particular cubic depends only from one coefficient, which is k (which may be any value between $-\infty$ and $+\infty$, including 0).

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2639
Date: 2025-03-23
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart,
Eckart, it's amazing, it seems you are this time on the target!
It is like the walls of Jericho, they fell only at the 7th
turn!!!

Please accept my deep and sincere congratulations
I can valid your construction

1) For a general bipartite cubic constructed wrt a triangle $H_1H_2H_3$ with according equations in a_1, a_2, a_3 and k , the points P , the vertices of the DT of the tangential QA of P and the points H_1, H_2 and H_3 are coconic.

Such a bipartite cubic is a pivotal isocubic in any of it's points with fixed points of the isoconjugation being the vertices of the tangential QA of the point.

Therefore, you may find an infinity of such conics through any point P and the vertices of the DT of it's tangential QA and H_1, H_2 and H_3 .

For example, P and this DT form the tangential QA of the isopivot P' and the vertices of the DT of this new QA (I named it DDT in old times) are coconic with H_1, H_2, H_3 and the isopivot.

2) In the example of Mac Cay, the cubic is a pivotal isogonal cubic wrt a triangle ABC , the pivot being the circumcenter O and the fixed points the in- and excenters. DT is the triangle ABC and it is easy to check on Chris figure wrt $ABC = (6,9,13)$ that A, B, C, H_1, H_2, H_3 and the circumcenter are coconic.

The cubic is also pivotal with pivot the orthocenter and the fixed points A, B, C and the circumcenter. The DT is made by the 3rd intersections of the sides of ABC with the cubic and it is easy to check that the 3 3rd intersectons are coconic with H_1, H_2, H_3 and the orthocenter.

3) Taking any triangle ABC and considering the Mac Cay cubic and it's hessian VR gives easily the 3 real flexes as intersections as well as their harmonis polars. Using your method allows to find H_1, H_2 and H_3 .

I hope Chris will be able to valid your observation perhaps by calculating the equation of the conic in the Mac Cay example?

I suppose it is now the end of this long quest

Many thanks for this happy conclusion

Best regards

Bernard

PS As I am always curious, how did you come on the idea of this property?

Message: #2640
Date: 2025-03-24
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Last idea this morning!
The conic through the vertices of DT and P is the conjugate of a line through the isopivot P' in the isoconjugation with fixed points the vertices of the tangential QA.
The conjugates of H1,H2 and H3 in this isoconjugation are aligned on a line through the isopivot.
For Mac Cay again, which is an isogonal pivotal cubic wrt ABC with pivot the circumcenter O, the isogonals of H1, H2 and H3 are aligned on a line through the orthocenter.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2641
Date: 2025-03-24
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your certification and further remarks,
... result of one year
with hundreds of drawings looking for properties.

Now I study the possibility to construct the conic through TR,
H1H2H3 and P,
... whose center is the common point (on QA-Co1 of TR plus P)
... of the three loci
for the conic centers of Hi',Hj' plus TR ...

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2642
Date: 2025-03-25
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart, Dear Bernard,

Eckart, congratulations!

I did some checks, and it looks like you are spot on. I am very happy for you that, after a year of studying and experimenting, you have achieved such a great result.

In order to verify it, I first attempted a general approach, but I wasn't able to complete the calculations in Mathematica. Therefore, I looked for a numerical example of a bipartite cubic, and the Neuberg Cubic K001 was a good choice. I also searched for a point P that produces four tangents at tangency points (S_1, S_2, S_3, S_4) on the cubic, and X(14) turned out to be a good example. Then I calculated the vertices (DT_1, DT_2, DT_3) of the Diagonal Triangle of QA (S_1, S_2, S_3, S_4) .

Following your instructions, I calculated the conic through $(X(14), DT_1, DT_2, DT_3, H_i)$ for $i = 1, 2, 3$. The coefficients of the three conics matched exactly up to 13 decimal places. For a visualization, see the attached file.

Of course, this is just one example.

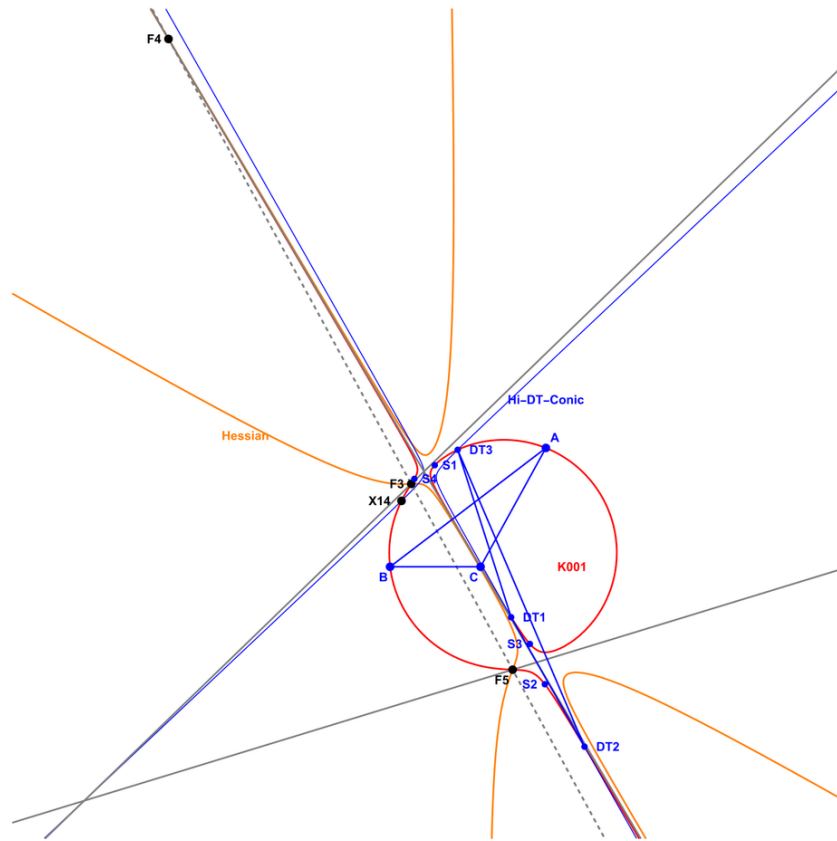
But given that you checked it, Bernard checked it, and I also verified it under different conditions, and we all arrived at the same result, at the very least, we can say that your construction has not been disproven and has a high probability of being correct.

It is also important to note that the theorem has only been tested for a bipartite cubic so far. Personally, I expect it to hold in general for all cubics, but that would require calculations with complex numbers. Perhaps later...

For now, I am very pleased to share my findings with you.

Best regards,
Chris

Construction of the real flexlines according to method Eckart



The reference cubic is the bipartite Neuberg Cubic K001 (in red color).
 The Hessian HE is shown (in orange color).
 (F3,F4,F5) are the real flexpoints as intersection points of K001 and HE.
 As point P is chosen X(14).
 (S1,S2,S3,S4) are the points of tangency of the tangents from X(14).
 (DT1,DT2,DT3) are the vertices of the Diagonal Triangle of QA(S1,S2,S3,S4).
 Hi-DT-Conic is the conic through (X(14), DT1,DT2,DT3, H1).
 Indeed H2 and H3 lie precisely on this very conic.

Chris van Tienhoven

March 25, 2025

Message: #2643
Date: 2025-03-27
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

I was still reflecting on Eckart's beautiful discovery.
Here are some additional remarks:

* The method assumes that we have four points sharing the same tangential (P and vertex points QA-DT(P)). However, this does not yet provide five points to define a fixed conic. Is there a convenient locus to consider for a variable H1 (on Harmonic Polar-1) that determines H2 and H3?

* When CU is not bipartite, we can use HE as a starting point, as it may be bipartite then or structured in such a way that there are situations with four points sharing the same tangential. After all, CU and HE share the same flexpoints and thus also share the three real flexlines.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2644
Date: 2025-03-28
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris,

1) As mentioned by Eckart, let's consider the QA-Co1 of a variable H1 on HP1 and the vertex of QA-DT(P) and the same for the corresponding H2 and H3 on HP2 and HP3 such as H1H2 through F3 and H1H3 through F2 : the 3 conics have a 4th common point on the QA-Co1 of P and the vertex of QA-DT(P) when H1, H2 and H3 are the searched vertices of RF.

2) Naturally, if CU is monopartite, HE will be bipartite ...

Best regards
Bernard

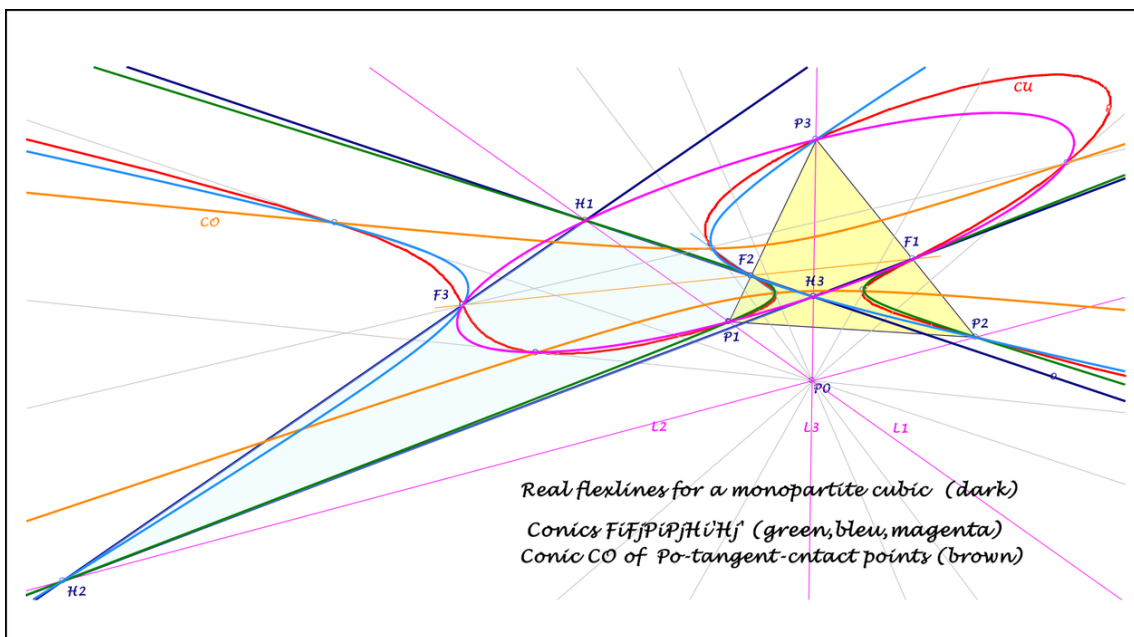
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2645
Date: 2025-03-28
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris, dear Bernard,

thanks to Chris for his acknowledgement and the extensive calculations,
... now I shall look for observations for monopartite cubics.
Here a first test (attached), if six tangents from P_0 to the cubic CU exist,
... the contact points are coconic on a conic CO,
... consider now the variable conics $F_i F_j P_i P_j H_i' H_j'$
... and the constellations,
that they intersect CO further on the cubic,
... there are three of these constellations,
for one holds $H_i' = H_i$???

Best regards Eckart



2025-03-28.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2646
Date: 2025-03-30
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
By definition, your C_0 is the polar conic of P_0 wrt CU and the
polar line of P_0 wrt C_0 is the line of real flexes!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2647
Date: 2025-04-09
From: van10hoven@gmail.com
Subject: In memoriam Ángel Montesdeoca Delgado

Dear friends,

On the Euclid group, I received the following message from Francisco Javier García Capitán:

"I have just been informed that Ángel Montesdeoca Delgado, former teacher at the Universidad de La Laguna, Canary Islands (Spain), passed away on May 17, 2024. We will always remember his deep knowledge of Projective Geometry, the great number of geometrical results he discovered, as well as his impressive website—not only for its content but also for its beautiful layout."

I remember Ángel for his profound knowledge of geometry and his warm, friendly nature.

Whenever things became complicated, I could always count on him for clear and thoughtful answers.

It is sad that we learned of his passing so late.

He also discovered a QA-point, *QA-P38 (<http://www.chrisvantienhoven.nl/qa-items/qa-points/qa-p38>)* , which was named after him.

He made many other contributions to geometry.

Rest in peace, my friend.

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2648
Date: 2025-04-09
From: van10hoven@gmail.com
Subject: Away for a month

Dear friends,

This coming weekend, I'm leaving for a long hiking trip along the GR65 in southern France. I'll be away for at least a month, so don't worry if you don't hear from me for a while. I'll still be reading the messages, though.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2649
Date: 2025-04-22
From: contiwa.goma3@gmail.com
Subject: Similitude Center of $(n-1)P$ s formed by Centric Centers

Dear geometers,

Given $nL\{L_1, \dots, L_n\}$ and let
 $O_i = (n-1)L-n-P_3$ of $\{L_1, \dots, L_{(i-1)}, L_{(i+1)}, \dots, L_n\}$
 $O_{ij} = (n-2)L-n-P_3$ of $\{L_1, \dots, L_{(i-1)}, L_{(i+1)}, \dots, L_{(j-1)}, L_{(j+1)}, \dots, L_n\}$ ($i \neq j$).
Then cyclic $(n-1)P\{O_1, \dots, O_{(i-1)}, O_{(i+1)}, \dots, O_n\}$
and $\{O_{i1}, \dots, O_{i(i-1)}, O_{i(i+1)}, \dots, O_{in}\}$ are similar.
For $n=5$, the similitude center is $5L-o-P_2$.

In general, the similitude center lies on the line $nL-n-P_1.P_3$.
Is this well-known (or obvious)?

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2650

Date: 2025-04-26

From: bernard.keizer@gmail.com

Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

Dear M@IMF,

You may read with interest the 4 references 47-48-49 and 55 of EQF, the 3 1st by Frank Morley and the last one by R.

Goormaghtigh and naturally the messages on the Forum exchanged by Chris, Eckart and myself (mentionned by Chris under nL-n-P3).

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2651
Date: 2025-04-26
From: contiwa.goma3@gmail.com
Subject: Re: Similitude Center of (n-1)Ps formed by Centric Centers

Dear Mr. Keizer,

Thank you for your reply.

I read the ref.48 (Morley) and got the formula of the similitude center (SC).

But I didn't realized that the SC is the inverse of nL-n-P1 (Centric Focus) wrt nL-n-Ci1 (Center Circle) when I posted the message.

After posting, I drew 5L-o-Ci1(Clifford's Circle) and noticed it passes through 5L-n-P1.

(I didn't know even such a thing.)

Then I searched in EPG and found Properties of nL-n-P1.

Now I'm studying QFG#937-953(Center Circles and Clifford's Chain for n Lines and n Penosculant).

I have a question:

5L is determined by $O_i(i=1,\dots,5)$ and 5L-n-P1.

6L is determined by $O_i(i=1,\dots,6)$ and what points?

(I think only 6L-n-P1 is not enough.)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2652
Date: 2025-04-28
From: bernard.keizer@gmail.com
Subject: Re: Similitude Center of $(n-1)$ Ps formed by Centric Centers

Dear M@IMF ,

In fact, 4L and 5L are determined by the O_i and the centric focus.

But I think you are right, it doesn't work for $n \geq 6$!

I don't know the answer to your question ...

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2653
Date: 2025-04-28
From: contiwa.goma3@gmail.com
Subject: Re: Similitude Center of $(n-1)$ Ps formed by Centric Centers

Dear Mr. Keizer,

Thank you for answering.

I was wondering if there is a systematic way to determine nL by using $O_i (i=1, \dots, n)$ and other points.

It will be a great help for me.

On the other hand, I conjecture each nL appeared in Lill's method is described by cyclic nP and one point.

I'm struggling to show that.

Anyway, thanks again.

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2654

Date: 2025-04-29

From: analgeomatica@gmail.com

Subject: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

Dear Geometers,

Let (A_1, A_2, A_3, A_4) be Quadrangle points.

Let P be any point.

Let B_1, B_2, B_3, B_4 be isotomic conjugates of P with respect to the triangles $A_2A_3A_4, A_3A_4A_1, A_4A_1A_2, A_1A_2A_3$, respectively.

Then six intersections of six pairs of line $(A_1A_2, B_3B_4), (A_3A_4, B_1B_2), (A_1A_4, B_2B_3), (A_2A_3, B_1B_4), (A_1A_3, B_2B_4), (A_2A_4, B_1B_3)$ are collinear on line d .

Thus we have a transformation of P to line d .

Is this transformation new?

Sincerely yours,

Tran Quang Hung

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2656
Date: 2025-04-29
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung,

Although I don't know whether your transformation appeared in EQF, QPG and QFG, it seems that isotomic conjugates can be generalized to isoconjugates. (I only checked numerically for some examples.)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2657
Date: 2025-04-29
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung,

your transformation seems not to be discussed in the group,
... it holds also for the isogonal isoconjugation,
... but cannot generalized for all isoconjugations.

A property of your transformation:
Consider a QA and its diagonal triangle TR,
for the points of a TR circumscribed conic
the image lines will envelope an inscribed TR conic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2658
Date: 2025-04-29
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt,

Thank you very much for correcting my mistake.
I checked only isoconjugations like
(p:q:r) -> (aⁿ/p : bⁿ/q : cⁿ/r).
I was careless.

Best regards,
M@IMF
(There is a royal road to geometry. Ask geometry masters.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2659
Date: 2025-04-30
From: analgeomatrica@gmail.com
Subject: Re: [Quadri-and-Poly-Geometry] A transformation on Quadrangle points

Dear Mr Eckart and friend,

Thanks a lot for your attention. I'm glad to know that it seems to be a new transformation, and not only that, it is a generalization of isogonal isoconjugation. I find that very useful. As for the property on triangle TR, I find it very strange and new to me. I will look into it further.

Sincerely yours,
Tran Quang Hung

Vào Th 3, 29 thg 4, 2025 vào lúc 22:48 M@IMF via groups.io
<contiwa.goma3@gmail.com@groups.io> đã viết:

> Dear Mr. Schmidt,
> Thank you very much for correcting my mistake.
> I checked only isoconjugations like
> $(p:q:r) \rightarrow (a^n/p : b^n/q : c^n/r)$.
> I was careless.
> Best regards,
> M@IMF
> (There is a royal road to geometry. Ask geometry masters.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2660
Date: 2025-04-30
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and M@IMF <mailto:M@IMF> ,

thanks for your messages, here another observation:
Consider the transformation for the isogonal isoconjugation
... and a 5P = A1A2A3A4A5,
 use P = Ai and the QA of the remaining 4 points,
... you get 5 lines for a 5L, whose inscribed conic 5L-s-Co1
... is similar to the circumscribed conic 5P-s-Co1
 of the 5P above
... with the same axes ...
I am sure, you can calculate the factor of expansion.

Best regards
Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2661
Date: 2025-05-01
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and M@IMF <mailto:M@IMF>,

what about this modification of your transformation in #2654:
Replace B_i by the isoconjugate of P wrt
... the reference triangle of the points A_j with j unequal i
... and the isoconjugation with fixpoint A_i .
This will give also 6 collinear points
... for a new transformation $P \rightarrow L$.
Points on a QA-side will be mapped to the opposite side ...

Finally a further modification:
Replace B_i by the isoconjugate of A_i wrt
... the reference triangle of the points A_j with j unequal i
... and the isoconjugation with fixpoint P .
Then already the B_i are collinear
... and give the image line $P.QA-TF_2(P)$,
... which is also the final line.
For points P on a line
... these image lines envelope a quartic,
... tangent to the 6 lines of the starting QA.
I hope this are not overhasty observations.

Best regards Eckart

PS. What about the inverses of these transformations?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2662
Date: 2025-05-01
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Herr Schmidt,

Could you extend due date for the homework?
I think the homothetic ratio is $(D+C)/(D-C)$
when the conic is expressed like $Cx^2 + Dy^2 = 1$.
But again, I only checked it numerically for some examples.
I'm trying to show analytically now. (Or it's not correct.)
By the way, please use "MIMF" instead of "M@IMF" if @ is
annoying.

Best regards,
M@IMF
(What should geometry masters do? They'd ask their masters.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2663
Date: 2025-05-28
From: dylanwyrzykowski@gmail.com
Subject: Re: Two quadrangles with a common circumconic

I have managed to prove the above conjecture. This result has
been published in the IJCDM:
https://drive.google.com/file/d/1j-zqKZ-szorvqWjfhF2p2qxZzmRimMU_A/view

My proof relied on a non-constructive computation using Pascal's
Theorem, I was hoping someone has an idea for a more synthetic
proof.

Edit: Unfortunately, I have found a flaw in my proof. The result
seems to still hold true, but it needs to be proved using
techniques from projective geometry.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2664
Date: 2025-05-04
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt, Tran Quang Hung and all,

As for #2660, I didn't realized that if $5P-s-Co1(COp)$ is fixed, the image line of P does not depend on QA :
Reflect the tangent to COp at P in the major axis of COp , expand (or contract) it with scale factor S around the center of COp .

S is $(a^2 + b^2)/(a^2 - b^2)$ when COp is an ellipse and
 S is $(a^2 - b^2)/(a^2 + b^2)$ when COp is a hyperbola,
where a and b are resp. the lengths of the major and minor axes of COp .

If COp is $QA-Co3$ of a quadrangle, $QA-P2.P3/QA-P3.P4 = S$.
I'm wondering this has something to do with the below:
 $AiAjAkAl$ is the circumcenter QA of $BihBjhBkhBlh$,
where Blh is the isogonal conjugate of Al wrt $AiAjAk$,
and $\{i,j,k,l,h\} = \{1,2,3,4,5\}$.
Still under study.

Best regards,
M@IMF

(What if masters of ... masters of geometry masters are not in the present world?
We can learn from their writings although we can't communicate with themselves like Jedi Master.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2665
Date: 2025-05-04
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,

with great interest I have reproduced your remarks in #2664,
they were new for me, thanks.
(I think, the last condition has to be $\{i,j,k,l\}=\{1,2,3,4\}$.)

A further observation:

Consider a 5P = A1A2A3A4A5

... and its quadrangles Q_{A_i} with these vertices unequal A_i,
... further the diagonal triangle DT_i of Q_{A_i},
... then the images B_i of a point P for an isoconjugation
... wrt reference triangle DT and fixpoint A_i
... are collinear on the polar of P
wrt the circumconic of the 5P.

Best regards
Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2666

Date: 2025-05-05

From: anopolis72@gmail.com

Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

On Sun, May 4, 2025 at 6:02 PM Elias M Hagos via groups.io <hagos_em@hotmail.com@groups.io> wrote:

> On Sat, May 3, 2025 at 02:42 AM, Elias M Hagos wrote:
> Dear friends,
> Let ABC be a triangle with circumcenter O and orthocenter H.
> The Hatzipolakis axis is the perpendicular bisector
> of segment OH.
> Let A', B', C' be the points where an arbitrary line L
> intersects BC, CA and AB respectively.
> The Hatzipolakis axes of triangles AB'C', A'BC' and A'B'C
> concur in a point Q which lies on the Hatzipolakis axis ABC.
> This generalizes a result given in Hyacinthos 27613
> <<http://www.hyacinthos.epizy.com/message.php?msg=27613&i=1>>
> when L is the Euler line for which Q = X(18279).
> See also X(18309)
> <<https://faculty.evansville.edu/ck6/encyclopedia/ETCPart10.html#X18039>>
> for a brief on the Hatzipolakis axis.
> Best regards,
> Elias M. Hagos

> Antreas and friends,
> I should have mentioned this in the original post.
> The Hatzipolakis axis is the perpendicular line
> to the Euler line E at the nine-point center N.
> Let P be any point on E.
> Let Z be the perpendicular to E at a point P.
> Let A', B', C' be the points where an arbitrary line L
> intersects BC, CA and AB respectively.
> Let Za, Zb, Zc be the Z of triangles
> AB'C', A'BC' and A'B'C respectively.
> Remarkably, Z, Za, Zb and Zc concur only if P = N.
> Best regards,
> Elias M. Hagos

Hello Elias.

It is a theorem of Quadri-Figures Geometry

Let (a, b, c, d) be a quadrilateral.

The Hatzipolakis axes of the component triangles

(a, b, c), (b, c, d), (c, d, a), (d, a, b) are concurrent.

APH

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2667
Date: 2025-05-04
From: van10hoven@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung, Eckart and MIMF,

When

$A_1 = \{1, 0, 0\}$, $A_2 = \{0, 1, 0\}$, $A_3 = \{0, 0, 1\}$, $A_4 = \{p, q, r\}$
and $P = \{x, y, z\}$,

then the isotomic case gives algebraically this transformation:

QA-Tfx(P) =

$\{x (-q x + p y) (-r x + p z)$
 $(q r y^2 + p^2 y z + 2 p q y z + 2 p r y z + 2 q r y z$
 $+ q r z^2),$

$y (-q x + p y) (r y - q z)$
 $(p r x^2 + 2 p q x z + q^2 x z + 2 p r x z + 2 q r x z$
 $+ p r z^2),$

$- z (-r x + p z) (r y - q z)$
 $(p q x^2 + 2 p q x y + 2 p r x y + 2 q r x y + r^2 x y$
 $+ p q y^2)\}$

the isogonal case gives this transformation:

QA-Tfy(P) =

$\{-q r x (q x - p y) (r x - p z)$
 $(SA y^2 + SB y^2 + 2 SA y z + SA z^2 + SC z^2),$

$-p r y (q x - p y) (-r y + q z)$
 $(SA x^2 + SB x^2 + 2 SB x z + SB z^2 + SC z^2),$

$p q z (r x - p z) (-r y + q z)$
 $(SA x^2 + SC x^2 + 2 SC x y + SB y^2 + SC y^2)\}$

These point-to-line-transformations are both new to me.

Best regards,
Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2668

Date: 2025-05-05

From: van10hoven@gmail.com

Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Elias, dear Antreas,

About:

The Hatzipolakis axes of the component triangles

(a, b, c), (b, c, d), (c, d, a), (d, a, b)

are concurrent.

where:

- * The Hatzipolakis axis is the perpendicular line to the Euler line E at the nine-point center N.
- * This concurring point is QL-P3 in EQF.
- * Also to my surprise this nice result already was mentioned at the 4th bullet at the properties.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2669
Date: 2025-05-05
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear Tran Quang Hung and MIMF,

back to your first transformation in #2654,
... you use the isotomic isoconjugation with fixpoint $X2(1:1:1)$,
... it will also be successful for isoconjugations
with fixpoints
... $X1(a:b:c)$,
 $X6(a^2:b^2:c^2)$,
 $X31(a^3:b^3:c^3)$,
 $X76(1/a^2:1/b^2:1/c^2)$, ...
... as I controlled.

I think you will generalize these examples by calculation.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2670

Date: 2025-05-05

From: van10hoven@gmail.com

Subject: Re: Two quadrangles with a common circumconic

Dear Dylan, Stanley, Keita,

It is a great result Dylan that you describe in your paper.

I had previously looked at the messages from Stanley, Keita, and you, but couldn't find any specific additions to them at that moment.

However, I would now like to provide a more general description of the Shinagawa coefficients.

If multiple points share 2 identical algebraic elements (such as $a^2 SA$ and $SB SC$) in the description of their barycentric coordinates, they are collinear and describe a perspective arrangement along a line. Other algebraic elements describe different perspective arrangements, either along the Euler line or another line.

If multiple points share 3 identical algebraic elements (such as $a^2 SA$, $SB SC$, and $a^2 S$) in the description of their barycentric coordinates, they describe a perspective arrangement of the projective plane, which I have come to call a Perspective Field.

It turns out that many more specific geometric properties are linked to particular perspective arrangements of lines or the entire plane.

See Perspective Fields

(<https://chrisvantienhoven.nl/mathematics/perspective-fields>).

Part-2 deals with the algebraic elements, but for a better understanding also part-1 is very worthwhile.

I hope to contribute something extra to this discussion in this way."

Best regards,

Chris van Tienhoven

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2671

Date: Tue, May 6, 2025 at 1:17 AM

From: αντρέας χατζηπολάκης via groups.io <anopolis72@gmail.com@groups.io>

Subject: Re: [euclid] NPCs - Orthologic

----- Forwarded message -----

To: <euclid@groups.io>

On Tue, May 6, 2025 at 1:12 AM APH wrote:

- > Let ABC be a triangle, L a line, and A', B', C'
- > the intersections of L and BC, CA, AB , resp.
- > Denote
- > N_a, N_b, N_c = the NPC centers of $AB'C', A'BC', A'B'C$, resp.
- > $ABC, N_aN_bN_c$ are orthologic.
- > Orthologic centers for some central lines?
- > APH

In Quadri Figures Geometry

Let (a, b, c, d) be a quadrilateral.

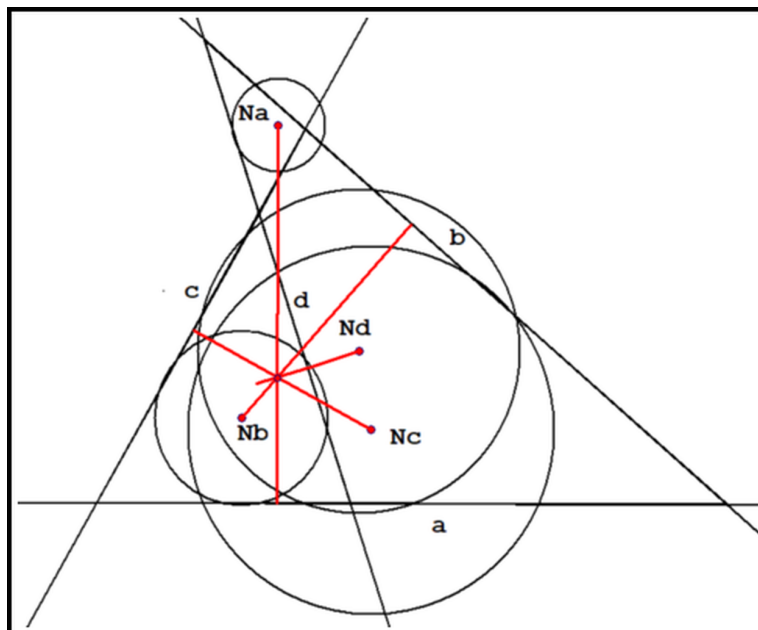
Denote

N_a, N_b, N_c, N_d = the NPC centers of the component triangles $(b, c, d), (c, d, a), (d, a, b), (a, b, c)$, resp.

The perpendiculars from N_a, N_b, N_c, N_d to a, b, c, d , resp. are concurrent.

Point?

APH



NPC.png

Message: #2672
Date: 2025-05-05
From: van10hoven@gmail.com
Subject: Re: [euclid] NPCs - Orthologic

Dear Antreas,

About:

N_a, N_b, N_c, N_d = the NPC centers of the component triangles
(b, c, d), (c, d, a), (d, a, b), (a, b, c), resp.
The perpendiculars from N_a, N_b, N_c, N_d to a, b, c, d , resp.
are concurrent.

Your point in EQF is QL-P2, the Morley Point.

This point was found by Frank Morley naming it the Second Orthocenter in his document Ref-49, paragraph 3.
(<http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html>),
It is described as a recursive point in an n-Line.
In his document he uses the letter "h" for this point.

QL-P2 is also mentioned by J.W. Clawson in Ref-31
(<http://www.chrisvantienhoven.nl/other-quadrangle-objects/9-mathematics/quadrangle-objects/188-references.html>) (pp. 40 and 41)
as the "mean center of gravity of equal masses placed at H_1, H_2, H_3, H_4 ", where H_1, H_2, H_3, H_4 are the orthocenters of the Component Triangles of the Reference Quadrilateral.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2673

Date: 2025-05-07

From: bernard.keizer@gmail.com

Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Chris, dear Elias, dear Antreas

As you seem to rediscover the well-known Kantor-Hervey theorem, I send you on attached file a generalisation of this theorem to the hypocycloïds with $2n+1$ cusps tangent to 4 lines with the help of the Hofstadter points. (I put this attached file 4 years ago on the Forum ...).

Best regards

Bernard

QL and hypocycloids with $2n + 1$ cusps

or

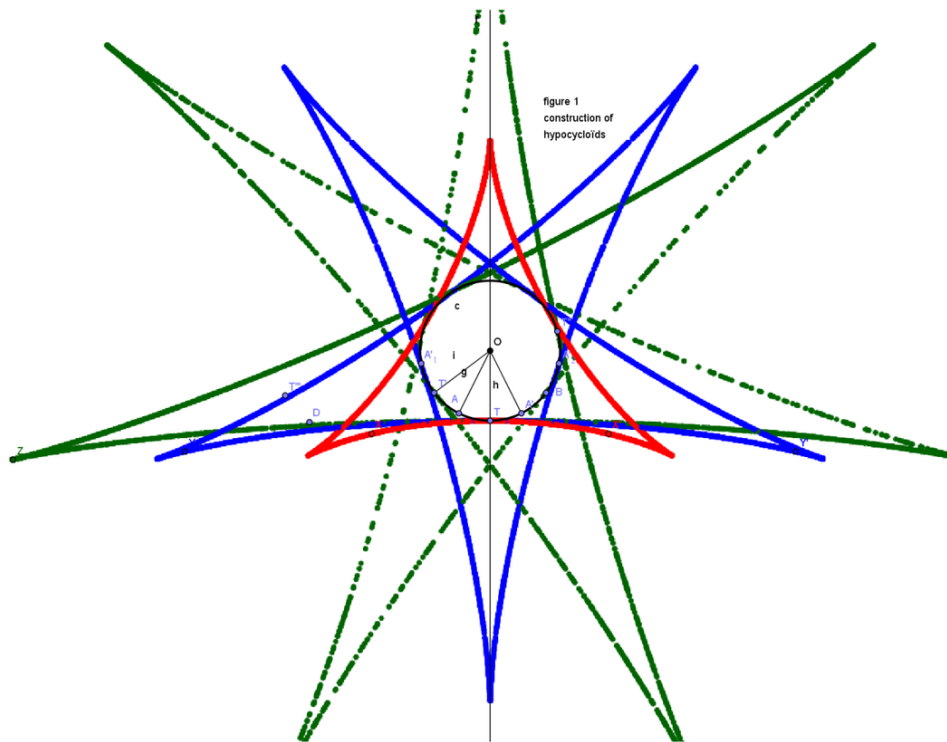
generalization of the Kantor-Hervey theorem

1) Hypocycloids with $2n + 1$ cusps

Let's consider a fixed point T on a fixed circle and 2 points A and B starting from T in opposite directions and describing the circle at speeds in a ratio $n+1/n$ (precisely the first point A being the primary point and the other the secondary point B at a speed $n+1/n$ the speed of the first).

The lines through the 2 points envelop a hypocycloid with $2n + 1$ cusps and the contact point is outside the segment between the 2 points and divides this segment in the same ratio $n+1/n$. (we have $TB/n+1 = TA/n = AB$)

For example, if $n=1$, we obtain the construction of the deltoid, the contact point is the reflection of the secondary point in the primary point. For $n=2$ or 3 , it's a hypocycloid with 5 or 7 cusps ...



2) Hypocycloïds with $2n + 1$ cusps tangent to 4 lines

Let's now consider the Hofstadter points for r integer (ETC X3, X4, X186, X265, X5961, X5962 ...); these points for the reference triangles of the QL derive from the same points for the triangle OjOkOl in the similitude with center QL-P1 which transforms O_i in QL-P4 and the line L_i in the Steiner Line QL-L2.

(This remark allows perhaps the calculation of the barycentric coordinates of the points and the demonstration of the following properties)

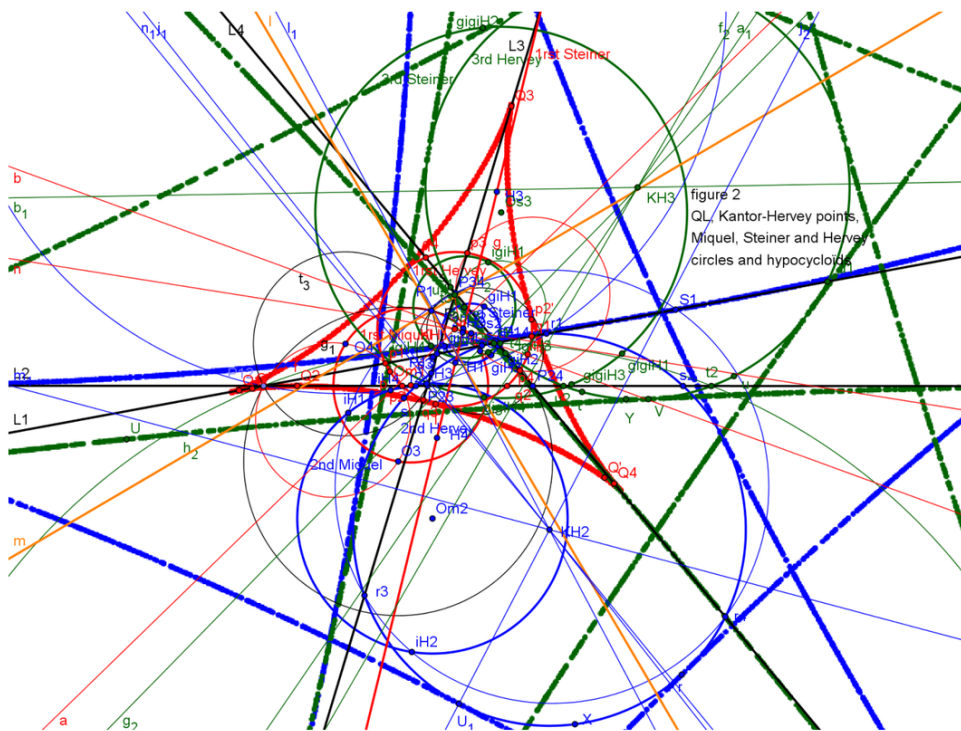
Following observations can be made :

- the same points for the 4 reference triangles are on circles (X3 on the 1st Miquel circle, X4 on the 1st Steiner "circle", which is the Steiner Line, X186 on the 2nd Miquel circle, X265 on the 2nd Steiner circle ...)
- the circles for isogonal points or Miquel and Steiner circles of the same rang are Cl-S partners (property mentioned by Tsihong Lau)
- the perpendicular bisectors of the segments joining 2 isogonal points of rang n in the list of Hofstadter points concur in a point named n th Kantor-Hervey point (QL-P3 for $n=1$)(property mentioned separately by Eckart and me the same day)
- the circles through each vertice P_{ij} of the QL and 2 same Hofstadter points X_k and X_l of the reference triangles having P_{ij} as vertice cut the 2 lines L_i and L_j in 2 points p_i and p_j , q_i and q_j , r_i and r_j , s_i and s_j , t_i and t_j , u_i and u_j ...(p for X3, q for X4, r for X186, s for X265, t for X5961, u for X5962 ...) (construction mentioned by Eckart)
- there are 6 such circles, each circle carrying 2 points X_k and X_l and 2 intersections p_i and p_j (or q_i and q_j ...). Each of the 4 points X and 4 points p (or q ...) belongs to 3 circles. 2 of the 6 circles intersect in one point X and one point p (or q ...).
- the 8 points p_i and q_i are on the 1st Hervey circle with center QL-P3, the 8 points r_i and s_i are on the 2nd Hervey circle with center the 2nd Kantor-Hervey point, the 8 points t_i and u_i are on the 3rd Hervey circle with center the 3rd Kantor-Hervey point
- there are on each socalled Hervey circle $2n + 1$ points T which divide the 4 arcs of circle in the same ratio $n/n+1$:
 - for $n = 1$ 3 points forming an equilateral triangle

- for $n = 2$ 5 points forming a pentagon
- for $n = 3$ 7 points forming a heptagon
- ...
- The construction in part 1 gives from any of these points a hypocycloid with $2n + 1$ cusps tangent to the QL

The QL of 4 lines is tangent to an infinity of hypocycloids with $2n + 1$ cusps and the inner circles of these hypocycloids are the Hervey circles defined above.

This property is a complete generalization of the Kantor-Hervey theorem.



QL and hypocycloids.pdf

Message: #2674
Date: 2025-05-07
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt,

I'm glad I could make a contribution.

(#2665)

In the second paragraph of #2664, I reorganized 20 points B: {Bhi, Bhj, Bhk, Bh1} -> {Bih, Bjh, Bkh, Blh}.

But nothing was obtained at that time.

After all, I'm calculating the scale factor S in the complex plane. (It takes more time than expected.)

(#2669)

I checked also X32, then I thought isotomic conjugates could be generalized to (all) isoconjugates.

I should have checked at least X3 or X4.

It seems that $(f(a) : f(b) : f(c))$ is OK.

Best regards,

M@IMF

p.s. I found C0-Tf3 in ICS.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2675
Date: 2025-05-08
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,
wrt the 2nd remark in #2674:
My conjecture for the transformation in #2654,
... described for the isotomic isoconjugation
... will also hold for isoconjugations with fixpoints
... $(a^n : b^n : c^n)$ with n positive or negative integer.
Best regards Eckart
PS: What about, if n rational?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2676
Date: 2025-05-08
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt,
I checked $f(a) = a^{1/2} + 5a^{2/3}$ and $\sin(a)$ [not $\sin A$], then
it held.
This tempted me to think isoconjugations
 $(p:q:r) \rightarrow (f(a)/p : f(b)/q : f(c)/r)$
would be OK. (I can't prove it, though)
Best regards, M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2677
Date: 2025-05-08
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,
I think, your last example shows
... that you are right with your informed conjecture,
... excuse my careless reading of your last messages.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2678

Date: 2025-05-12

From: bernard.keizer@gmail.com

Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Chris,

As you don't seem to be interested in these hypocycloids with $2n+1$ cusps tangent to 4 lines, I try it another way.

1) For r integer the Hofstadter points in a triangle ABC are defined from X_3 and it's isogonal X_4 as a sequence of inverse in the circumcircle/isogonal wrt ABC .

(see ETC at 359/360). isogonal of H_r is H_{1-r} .

Hence the sequence $X_3 = H_2$ and $X_4 = H_{-1}$, $X_{186} = H_3$ and $X_{265} = H_{-2}$, $X_{5961} = H_4$ and $X_{5962} = H_{-3}$, $X_{5963} = H_5$ and $X_{5964} = H_{-4}$...

(It is not surprising that there is no other such circles until $X=4000$, as mentionned in EQF)

2) For a QL , the 4 Hofstadter points of the same rank are concyclic :

X_3 on the Miquel circle centered in $QL-P_4$, X_4 on the Steiner circle (which is an fact the Steiner Line $QL-L_2$, but a line is a particular circle with radius infinite)

X_{186} on the 2nd Miquel circle centered in $QL-P_{28}$ and X_{265} on the 2nd Steiner circle centered in $QL-P_{29}$

X_{5961} and X_{5962} on the 3rd Miquel and Steiner circles

X_{5963} and X_{5964} in the 4th Miquel and Steiner circles

...

The circles of isogonal Hofstadter conjugates are CSC partners (starting with the Miquel circle and the Steiner Line)

3) The perpendicular bisectors of the segments joining 2 isogonal Hofstadter points concur in a point :

Kantor-Hervey point $QL-P_3$ for X_3 and X_4

2nd, 3rd and 4th Kantor-Hervey points for the copples X_{186} and X_{265} , X_{5961} and X_{5962} and X_{5963} and X_{5964}

4) These Kantor-Hervey points are the centers of the 1st, 2nd, 3rd and 4th Hervey circles, which are the inner circles of the hypocycloids with 3, 5, 7 and 9 cusps tangent to the 4 lines and so on ...

Best regards

Bernard

PS It seems the property of the perpendiculars to the 4 lines in the points X_5 doesn't hold for the next isogonal Hofstadter points

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2679
Date: 2025-05-14
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear all,

I think isogonal version of Tran Quang Hung's transformation becomes $QA-Tfy(P) = C0-Tf1(C0-Tf3^{(-1)}(P))$, where $C0 = 5P-s-Co1$ of $\{QA, P\}$.

Is this correct?

If so, this can be an answer for the "homework" of #2660.

Here's another answer (only formulas for ellipse):

$5P-s-Co1: z = [(a+b)t + (a-b)/t]/2, |t|=1$

$Aj: zj = [(a+b)tj + (a-b)/tj]/2, |tj|=1$

$Bh1$ (isogonal conjugate of Ah wrt $AiAjAk$):

$zh + \{ (a-b)(1/th + U.th) - (a+b)s1[ijk] + (a-b)s2[ijk]th - (a+b)s3[ijk](th^2 - U) \} / 2(s3[ijk]th - 1)$

$Xijkl$ (intersection point of $AiAj$ and $BhkBh1$):

$\{(a+b)[(2S)titjth - (ti+tj)] - (a-b)[2S - (ti+tj)th]th\} / 2(titjth^2 - 1)$

Lh (line through $Xijkl, Xikjl, \dots, Xklij$):

$[(a+b)th - (a-b)/th]z - [(a-b)th - (a+b)/th]z^* = 4abS$

$5L-s-Co1: z = S[(a+b)/t + (a-b)t]/2, |t|=1$

where

$U = 4ab/(a^2 - b^2), S = (a^2 + b^2)/(a^2 - b^2)$

$s1[ijk] = ti+tj+tk, s2[ijk] = tjtk+tkti+titj, s3[ijk] = titjtk$

"^*" means complex conjugate and $\{i,j,k,l,h\} = \{1,2,3,4,5\}$.

Best regards,

M@IMF

(May the force be with you. Oh, it's already 14th.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2680

Date: 2025-05-14

From: van10hoven@gmail.com

Subject: Re: [euclid] Concurrence of four Hatzipolakis Axes

Dear Bernard,

Thank you for your fantastic contribution on Epi- and Hypocycloids.

I have already expressed my deep appreciation for it, including in message #1, and I still stand by that.

Unfortunately, I cannot give it a place in the EQF due to my priorities in the many projects, as mentioned in message #2548. That said, I am working on a yearly Journal compiling messages from our forum, so that all contributions will in any case be preserved for the future.

And this applies to everyone: if you have personal comments about the QPG forum or the EQF/EPG encyclopedias, please email me separately to keep content-related and personal messages separate.

Each email includes below the option "contact group owner". This way, you can always reach me outside the group for procedural or non-content-related questions or remarks.

Kind regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2681
Date: 2025-05-15
From: eckart_schmidt@t-online.de
Subject: Re: A transformation on Quadrangle points

Dear MIMF,

the isogonal version of Tran Quang Hung's transformation
... can also be described, using
... the $5P = QA$ plus P , its $5P-s-Co1$ and $5P-s-Tf3$
... as tangent in $5P-s-Tf3(P)$ to the conic $5P-s-Tf3(5P-s-Co1)$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2682
Date: 2025-05-15
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Dear Mr. Schmidt,

Thank you for the reply.
I found $CO-Tf3$ was first discussed in QFG#(932=)935,36,44,47,48.
Accidentally, I was studying QFG#937-953(Center Circles and
Clifford's Chain for nL ...) last month.
Although there is no attachment in QFG#2792(Loci wrt
 $5P-s-Tf1,2,3,4$), I can see it on your site.
Figures are very helpful.

Anyone can access these pieces of information beyond time and
space.
That's great.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2683

Date: 2025-05-17

From: van10hoven@gmail.com

Subject: Re: A transformation on Quadrangle points

Dear MIMF,

Back in the days that the Quadri Figures Group was supported by Yahoo Groups, attached files were often not properly included in the Yahoo system (so frustrating!).

Fortunately, they were attached to the accompanying email.

So, I searched for it in my mail archive.

Just to be sure, I'm enclosing the attachment for QFG#2792.

Perhaps you've already found it on Eckart's site.

Best regards,

Chris

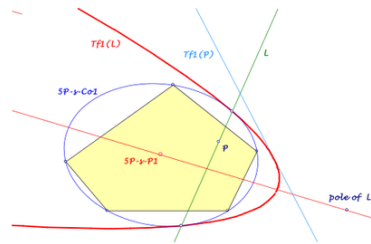
EQF-Note 2017-12-30

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
<http://www.chrisvantienhoven.nl/>

Loci wrt 5P-s-Tf1,2,3,4

Tf1,2 map points to lines, Tf3 maps points to points and Tf4 maps lines to lines. So there are several possibilities, to study loci wrt points on a line or lines through a point.

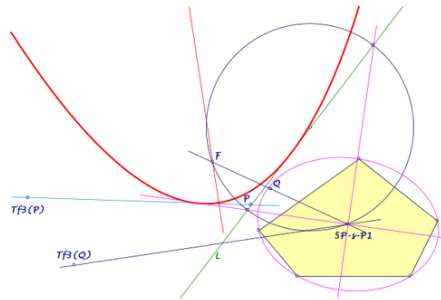
***Tf1 of points P on a line L give a parabola,
... tangent to 5P-s-Co1 in the intersections with L ,
... with axis through 5P-s-P1 and pole of L wrt 5P-s-Co1.***



***Tf2 of points P on a line L give a line pencil
... for the pole of L wrt 5P-s-Co1.***

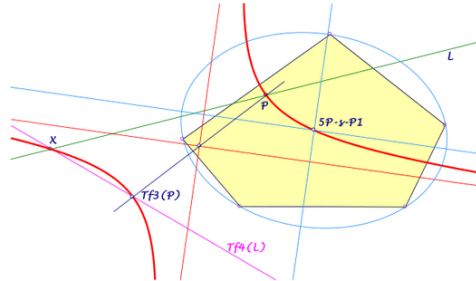
Tf3 of points P on a line L give the line $Tf4(L)$.

***Tf4 of lines L through a point P
... give the line pencil of $Tf3(P)$.***

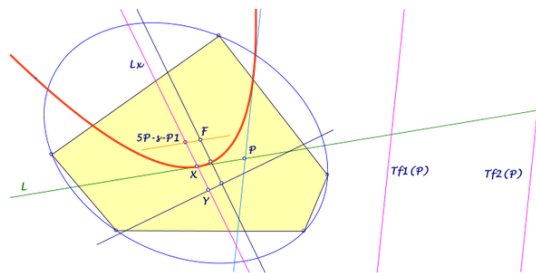


***Lines $P.Tf3(P)$ for points P on a line L give a parabola
... tangent to L and the axes of 5P-s-Co1
... directrix 5P-s-P1. $Tf3(Q)$ with $Q =$ pole of L wrt 5P-s-Co1
... and focus in the 2nd intersection of $Q.5P-s-P1$
... .. and the circumcircle of the triangle
... .. with sidelines L and the axes of 5P-s-Co1.***

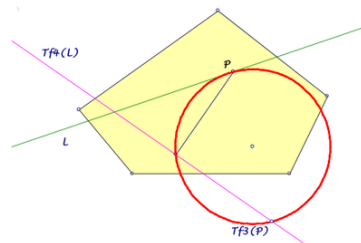
Points $L \cap Tf4(L)$ for a line pencil of point P
 ... give an orthogonal hyperbola,
 ... through $5P-s-P1$, P and $Tf3(P)$,
 ... centered in the midpoint of P and $Tf3(P)$
 ... with asymptotes parallel to the axes of $5P-s-Co1$.



Parallels to $Tf1,2(P)$ through P for points P on a line L
 ... give a parabola,
 ... tangent to L in X , the intersection with the line L_x
 ... through $5P-s-P1$ and pole of L wrt $5P-s-Co1$,
 ... with directrix orthogonal L_x
 ... through Y , the reflection of $5P-s-P1$ in X ,
 ... with focus F in the reflection of Y in L .



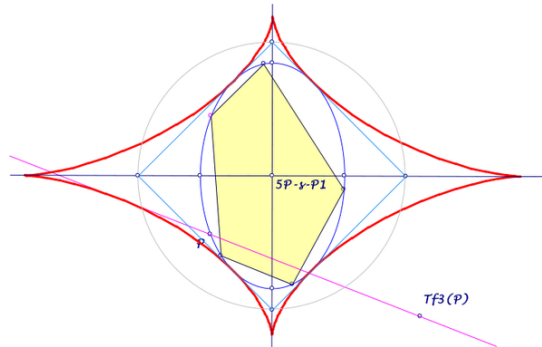
Pedal points of P on $Tf4(L)$ for lines L through P
 ... give a circle with diameter $P.Tf3(P)$.



Final remarks

- $Tf3$ maps a conic to a conic, also the centers of the conics.

Lines $P.Tf_3(P)$ for points P on $5P-s-Co1$ give an astroid
... with the same axes as $5P-s-Co1$
... and common tangents with $5P-s-Co1$,
... .. which form a square,
... and cusps inverse to the apexes of $5P-s-Co1$ wrt the
circumcircle of the square.



Eckart Schmidt
<http://eckartschmidt.de>
eckart_schmidt@t-online.de

2017-12-30.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2684
Date: 2025-05-17
From: contiwa.goma3@gmail.com
Subject: Re: A transformation on Quadrangle points

Thank you, Chris.
That's what I found.
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2685
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: QA 4 concurrent Simson / Steiner lines

Dear Geometers,

I'm sorry if the following results are already known...
Let $P_1P_2P_3P_4$ be a general quadrangle.
Denote by $M_{i,j}$ the midpoint of P_iP_j . ($i, j = \{1, 2, 3, 4\}, i < j$)
Let P be the Gergonne-Steiner point ($QA-P_3$) of $P_1P_2P_3P_4$.
Let L_1 be the Simson line of P wrt $M_{1,2}M_{1,3}M_{1,4}$. Define L_2, L_3, L_4 cyclically.
Then, L_1, L_2, L_3, L_4 are concurrent.
The point of concurrency is the reflection of $QA-P_6$ in $QA-P_1$. It lies on $QA-L_4$.
If we take Steiner lines instead of the Simson lines, we get another 4 concurrent lines.
In this case, the point of concurrency lies on the line joining $QA-P_1$ and $QA-P_7$.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2686
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

In the case of the Steiner lines, the point of concurrency is the reflection of QA-P4 in QA-P1.
Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2687
Date: 2025-05-21
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita-san,

Maybe this is the pair of your result:
Let P' be the Euler-Poncelet point (QA-P2) of $P_1P_2P_3P_4$.
Let L'_1 be the Simson line of P' wrt $M_{\{3,4\}}M_{\{2,4\}}M_{\{2,3\}}$. Define L'_2, L'_3, L'_4 cyclically.
Then, L'_1, L'_2, L'_3, L'_4 are concurrent at QA-P6.

Let's consider another way.
Let 4 circles C_1, C_2, C_3, C_4 be concurrent at P .
Let T_{ij} be the second intersection point of C_i and C_j .
Let L_l be the Simson line of triangle $T_{jk}T_{ki}T_{ij}$,
where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.
I'm wondering when L_1, L_2, L_3, L_4 are concurrent.
I want to look up in KIKAGAKU DAIJITEN, but the library is closed until tomorrow.
If 4 circles are circumcircles of component triangles of QL , 4 Simson lines are identical.

Best regards,
M@IMF

p.s. I didn't realize QA-Co1 is QA-Co3 of $\{DT, QA-P3\}$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2688
Date: 2025-05-21
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita-san and all,

After posting #2687, I found "the pair of your result" in Properties of QA-P6. I didn't know that.
Correction: Let l_l be the Simson line of P wrt triangle $T_{jk}T_{ki}T_{ij}$, where $\{i,j,k,l\} = \{1,2,3,4\}$.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2689
Date: 2025-05-21
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear M@IMF,

Obviously the 4 nine-point circles and 4 midray circles are congruent and homothetic (homothetic center = QA-P1), hence it's true for the nine-point circles as well.

I can't figure out what is the necessary and sufficient condition for the more general case you have mentioned. In a quadrangle, the following is known.

Let $P_1P_2P_3P_4$ be a quadrangle, and P a point. Let $Q_{i,j}$ be the orthogonal projection of P onto P_iP_j . Let L_i be the Simson line of P wrt $Q_{i,j}Q_{i,k}Q_{i,l}$. The locus of P such that L_1, L_2, L_3, L_4 are concurrent is QA-Cu1.

Shiko Iwata's Kikagaku Daijiten is voluminous and contains many interesting theorems about Simson line(s), its generalizations and relating concepts for sure (Carnot's theorem, Longchamps-Langley's theorem, Turner's theorem, Aubel's theorem, Seimiya's theorem, generalizations by Hisao Mori, θ -Simson circle, Simson line for conics, Simson line-related Newton line, pseudo-Simson line etc.).

Maybe I will pick up some of the theorems relating to quadri-figure and polygon geometry from Kikagaku Daijiten in the future.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2690

Date: 2025-05-22

From: unidentifiedlethargicorganism@gmail.com

Subject: QG constructing a square from 4 squares

Dear Geometers,

The following problem appears in a Chinese plane geometry exercise called 正方形问题. The proposer is unknown.

Problem 63:

Let $P_1P_2P_3P_4$ be a general convex quadrigon.

Erect 4 squares $P_1P_2P_{12}P_{21}$, $P_2P_3P_{23}P_{32}$, $P_3P_4P_{34}P_{43}$, $P_4P_1P_{41}P_{14}$ outwardly.

Let

$$A_{12} = P_1P_3 \cap P_2P_4$$

$$A_{23} = P_2P_4 \cap P_3P_1$$

$$A_{34} = P_3P_1 \cap P_4P_2$$

$$A_{41} = P_4P_2 \cap P_1P_3$$

$$B_{12} = P_1P_3 \cap P_2P_4$$

$$B_{23} = P_2P_4 \cap P_3P_1$$

$$B_{34} = P_3P_1 \cap P_4P_2$$

$$B_{41} = P_4P_2 \cap P_1P_3$$

$$C_{12} = A_{12}B_{23} \cap A_{41}B_{12}$$

$$C_{23} = A_{23}B_{34} \cap A_{12}B_{23}$$

$$C_{34} = A_{34}B_{41} \cap A_{23}B_{34}$$

$$C_{41} = A_{41}B_{12} \cap A_{34}B_{41}$$

To prove: the quadrigon $C_{12}C_{23}C_{34}C_{41}$ is a square.

Does anyone know a reference to this theorem?

Sincerely,

Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2691
Date: 2025-05-22
From: unidentifiedlethargicorganism@gmail.com
Subject: QA 12-point conic involving orthocenters

Dear Geometers,

Is the following result well-known?

Let $P_1P_2P_3P_4$ be a quadrangle.
Let H_1 be the orthocenter of $P_2P_3P_4$.
Define H_2, H_3, H_4 cyclically.

Let

$$A_{1,2} = P_1P_2 \cap H_1H_2$$

$$A_{1,3} = P_1P_3 \cap H_1H_3$$

$$A_{1,4} = P_1P_4 \cap H_1H_4$$

$$A_{2,3} = P_2P_3 \cap H_2H_3$$

$$A_{2,4} = P_2P_4 \cap H_2H_4$$

$$A_{3,4} = P_3P_4 \cap H_3H_4$$

$$B_{1,2} = P_1P_2 \cap H_3H_4$$

$$B_{1,3} = P_1P_3 \cap H_2H_4$$

$$B_{1,4} = P_1P_4 \cap H_2H_3$$

$$B_{2,3} = P_2P_3 \cap H_1H_4$$

$$B_{2,4} = P_2P_4 \cap H_1H_3$$

$$B_{3,4} = P_3P_4 \cap H_1H_2$$

Then, these 12 points lie on the same conic.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2692
Date: 2025-05-22
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita-san and all,

In the case of pedal circles of P_1 wrt $P_iP_jP_k$ (concurrent at QA-P2),
4 Simson lines are concurrent at $(QA-P2 + QA-P23 - QA-P6)$.
As is often the case, it seems that θ -generalized Simson lines
can be applied.
QA-Cu1 will be a hint. Thank you.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2693
Date: 2025-05-23
From: unidentifiedlethargicorganism@gmail.com
Subject: Re: QA 12-point conic involving orthocenters

Dear Geometers,

I think:

- 1) The center of this conic is QA-P23.
- 2) QA-P2 lies on this conic.
- 3) The axes of this conic are parallel to the axes of the nine-point conic.
- 4) If the conic is an ellipse, then it is similar to the nine-point conic.
- 5) Suppose that this conic and the nine-point conic have 4 real intersection points. One of them is QA-P2. Let A, B, C be the other intersection points. Then, QA-P2, A, B, C are concyclic. The Steiner line of QA-P2 wrt ABC is the line QA-P4 QA-P8. The orthocenter of ABC is QA-P23.

I am sorry if it turns out to be false.

Sincerely,
Keita Miyamoto

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2694
Date: 2025-05-27
From: eckart_schmidt@t-online.de
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear Keita,

I haven't studied all your results,
... but perhaps of interest an addition to #2689:
Let $P_1P_2P_3P_4$ be a quadrangle, and P a point.
Let $Q_{\{i,j\}}$ be the orthogonal projection of P onto P_iP_j .
Let L_i be the Simson line of P wrt $Q_{\{i,j\}}Q_{\{i,k\}}Q_{\{i,l\}}$.
The locus of P such that L_1, L_2, L_3, L_4
are concurrent is $QA-Cu_1$.

If we ask for for points P on $QA-Cu_1$ such that the lines L_i are
parallel,
... we get the contact points
of tangents from $QA-P_3$ at the cubic.

What about their directions?

Perhaps helpful:

Consider $QA-Co_1$ as pivotal isocubic wrt
... pivot $QA-P_3$,
... reference triangle with vertices $QG-P_{16}$
of the QA -quadrilaterals,
... isoconjugation, which swaps vertices of $QA-Tr_1$ and $QA-Tr_2$
... with fixpoints in the contact points
of tangents from $QA-P_3$ at the cubic.
(This is mentioned in my QFG-message #1770 (30.06.2016),
not in EQF.)

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2695
Date: 2025-05-28
From: contiwa.goma3@gmail.com
Subject: Re: QA 4 concurrent Simson / Steiner lines

Dear all,

Thanks to Keita-san, my question in #2687 was solved.
Given 4 circles C_1, C_2, C_3, C_4 concurrent at P .

Let

C_{ij} = the second intersection point of C_i and C_j

L_{pl} = the Simson line of P wrt triangle $C_{il}C_{jl}C_{kl}$

P_i = the antipode of P wrt C_i

$QAp = \{P_1, P_2, P_3, P_4\}$,

where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

Since C_{ij} is the orthogonal projection of P on P_iP_j ,

$L_{p1}, L_{p2}, L_{p3}, L_{p4}$ are concurrent iff P lies on $QAp-Cu1$

(QFG#1534).

According to the 12th property in QA-Cu1 (Bernard Gibert, March 15, 2015),

this means 3 lines $C_{14}C_{23}, C_{24}C_{31}, C_{34}C_{12}$ are concurrent.

Best regards,

M@IMF

p.s. Correction(again) in #2687,8: Let L_l be the Simson line of P wrt triangle $T_{il}T_{jl}T_{kl}$, where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2696
Date: 2025-05-29
From: eckart_schmidt@t-online.de
Subject: Re: QA 12-point conic involving orthocenters

Dear Keita,

I have reproduced your #2691,
... here a further property for #2693:
6. The Simson line of QA-P2 wrt triangle ABC
... is a parallel to the line QA-P4,8,23,32,42.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2697
Date: 2025-06-05
From: contiwa.goma3@gmail.com
Subject: About QL-Tr2

Dear all,

Is this well-known?
The contact points of QL-Co1 with QL-Tr2 are the intersection
points of QL-Co1 and QA-Co1,
where QA is formed by the contact points of QL-Co1 with QL.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2698
Date: 2025-06-06
From: eckart_schmidt@t-online.de
Subject: Re: About QL-Tr2

Dear MIMF,

I have reproduced your constructions,
... a good addition to QFG #1460 of 25.01.2016,
... here attached:

- > ... I had a look at the contact points of the 3 sides
of QL-Tr2 with the parabola,
which shall give the triangle QL-Trx:
- > There is a perspector of QL-Tr2 and QL-Trx, point Y in #678.
- > This point Y has a lot of interesting properties:
- > ... Point Y is the QL-Tr2-isotomic conjugate
of the point at infinity of QL-L1.
- > ... Point Y lies on the QL-Tr2-Steiner ellipse
(circumscribed).
- > ... The tangent in Y at the QL-Tr2-Steiner ellipse
is Y.QL-P8. (orthogonal QL-L5).
- > ... The point Y is the contact point of two Steiner ellipses:
QL-Tr2 circumscribed and QL-Trx-inscribed.
- > ... Point Y is the reflection of point X in #678 in QL-P12.
- > ... Point Y lies on a QL-Tr2-circumconic
through QL-P8, QL-P13, QL-P24
- > ... Point Y is the Brianchon point of QL-Co1 wrt QL-Tr2.
- > ... The point Y is collinear with QL-P7 and the 4th
intersection of QL-Co1 and the QL-Trx-circumcircle.
- > ... The trilinear polar of Y wrt QL-Tr2 is ...
- > ... also the trilinear polar of Y wrt QL-Trx,
- > ... the polar of Y wrt QL-Co1,
- > ... the line for the collinear trilinear poles
of the QL-lines wrt QL-Tr2,
- > ... which is the image of QL-Co1 wrt an isoconjugation
of QL-Trx with fixed point Y.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2699
Date: 2025-06-06
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear Mr. Schmidt,

Thank you for your reply.

The polar curve of DQL-Co1(anticomplement of QL-Co3 wrt QL-Tr1) wrt QL-Co1 is QA-Co1(QA = contact points quadrangle).

So the poles of the common tangents of QL-Co1 and DQL-Co1 wrt QL-Co1 are the intersection points of QL-Co1 and QA-Co1.

I checked it analytically (not numerically).

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2700
Date: 2025-06-06
From: eckart_schmidt@t-online.de
Subject: Geometry on QA-Cu1

Dear Keita, dear all,

may I invite you to an geometric excursion on QA-Cu1,
... starting with the property,
mentioned in Keita's message #2689:

Let $P_1P_2P_3P_4$ be a quadrangle, and P a point.
Let $Q_{\{i,j\}}$ be the orthogonal projection of P onto P_iP_j .
Let L_i be the Simson line of P wrt $Q_{\{i,j\}}Q_{\{i,k\}}Q_{\{i,l\}}$.
The locus of P such that L_1, L_2, L_3, L_4 are concurrent is
QA-Cu1.

(1) Points on QA-Cu1 with parallel lines L_i (see #2694)
... are the 4 contact points U of tangents
from QA-P3 at the cubic.

Let L_u be a line through $P=U$ as further parallel to the lines L_i .

(2) There are 4 cubic points $P=V$ whose 4 lines L_i are
... each parallel to one of the 4 lines L_u .

(3) These points V are the in- and ex-centers
of the Miquel Triangle QA-Tr2,
... their QA-Cu1- tangents are parallel to the asymptote.

(4) A connection-line of an U - and an V -point
bears a QA-vertex,
... a connection-line of two V -points bears a QA-Tr2-vertex.

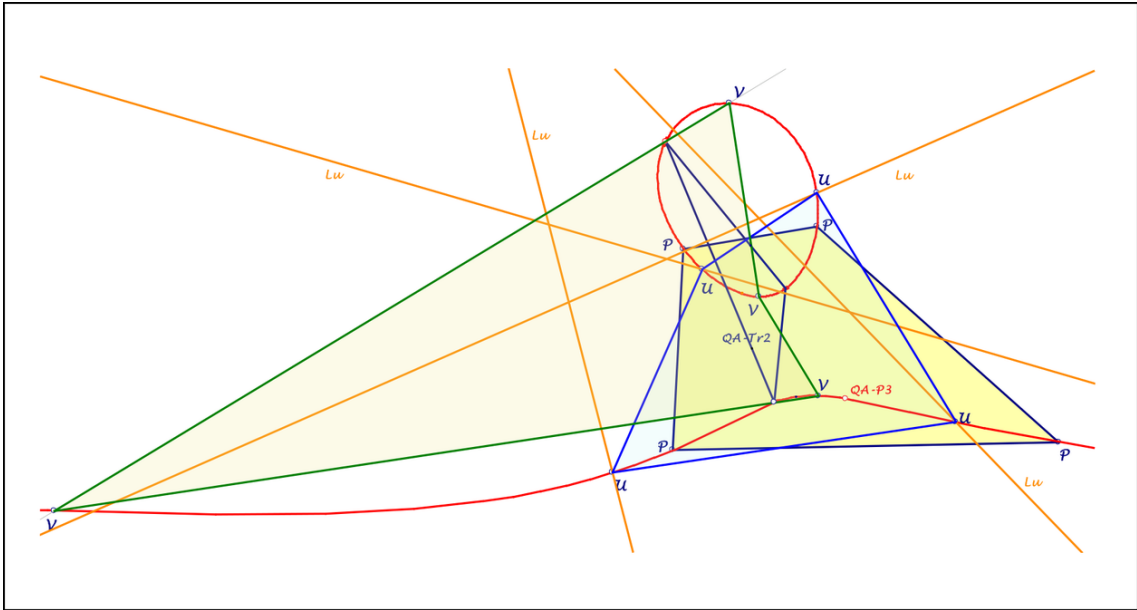
(5) Two connection-lines of different V -pairs
are perpendicular

... and have an intersection on the cubic
... in a vertex of the Miquel triangle QA-Tr2.

But what about the four directions of the lines L_u ?

Best regards Eckart

PS: Attached only the main points and lines.



2025-06-06.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2701
Date: 2025-06-08
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear all,

I found Mr. Schmidt's QFG message(#1335, 18/10/2015):

... The 6 contact points of the parabolas to the triangles $S_1S_2S_3$ lie on the conic QA-Co1.

The 6 contact points are 3 contact points of QL1-Co1 with QL1-Tr2 and those of QL2-Co1 with QL2-Tr2, where QL1 and QL2 are QA-2Co1.

I had searched "QL-Tr2" & "QA-Co1" in QFG (also QPG), but couldn't find above message.

That's because QL-Tr2 was registered in 2016!

By the way, the QA formed by the centroids of the component triangles of QL1 and that of QL2 are symmetric wrt QA-P10. I wonder this was already mentioned.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2702
Date: 2025-06-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

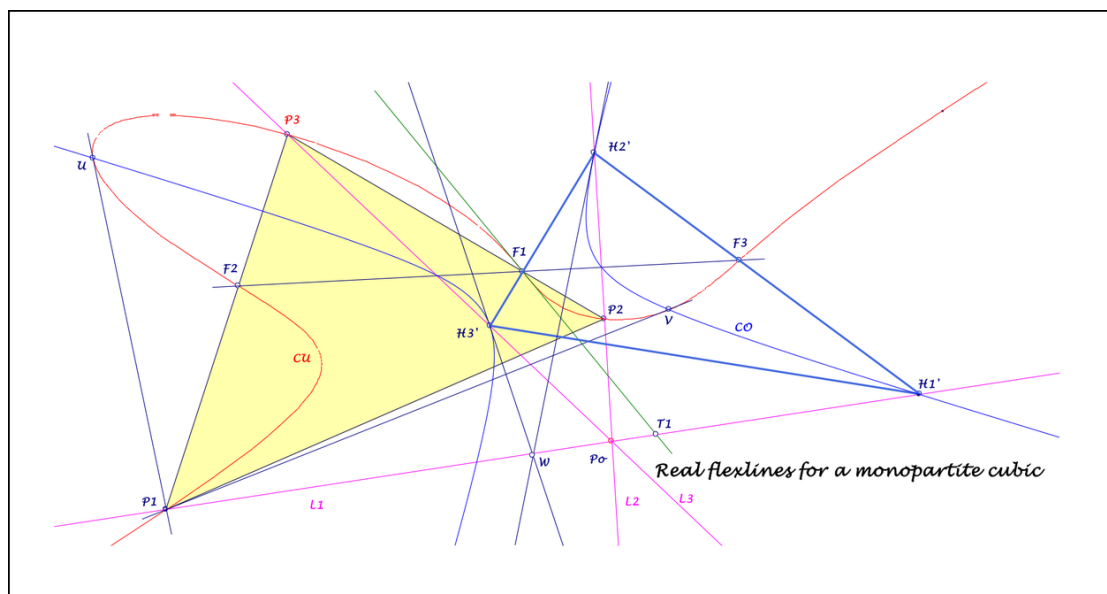
Dear Bernard, dear Chris,

in #2635 I offered a "construction"
 ... for the real flexline trilateral of a bipartite cubic,
 ... perhaps you can confirm the following
 for a monopartite cubic:

Consider a monopartite cubic CU
 ... their real flexpoints $F1,2,3$, harmonic polars $L1,2,3$,
 ... bearing the cubic-intersections $P1,2,3$ and
 ... the intersections $T1,2,3$ with the real flextangents.
 Start with the point P_i and its two tangents at the cubic
 ... with contact points U,V ,
 ... further with variable triangles $H1'H2'H3'$ on $L1,L2,L3$
 ... with $F1,F2,F3$ on their sidelines.
 Consider the conic CO through $U,V,H1',H2',H3'$
 ... and the conic tangents in H_j',H_k' ,
 ... which intersect in a point W on L_i .
 If H_i' is the 4th harmonic of W wrt P_i,T_i ,
 ... then $H_i' = H_i$, vertex of the real flexline trilateral.

Best regards Eckart

PS: Attached $P_i = P1$.
 I made several drawings without a significant anomaly.



2025-06-12.pdf

← Previous Next → ↔ Message Index ↑ Subjects

Message: #2703
Date: 2025-06-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
This time, I cannot confirm your construction.
The property that the point W, intersection on L1 of the tangents in H2 and H3 to the conic through H1, H2, H3 and U and V is the harmonic of H1 wrt P1 and T1 simply doesn't hold! (U and V being the tangentials of P1 are aligned with F1, which is the tangential of P1).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2704
Date: 2025-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,
sorry, your last remark is wrong,
... have a look on the drawing.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2705
Date: 2025-06-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
In fact, P1 being the tangential of U and V and F1 the tangential of P1, U and V are aligned with F1 seems a more correct formulation, but the conclusion holds!
Best regards Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2706
Date: 2025-06-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,
we can dispute about better formulations,
... but can you give me a drawing with a significant aberration
... or another reason for your strict opinion?
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2707
Date: 2025-06-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I use always the same kind of figures already explained in
several old messages.
Giving a triangle $H_1H_2H_3$ and a point $P_0 (1/a_1, 1/a_2, 1/a_3)$,
the barycentric equation of the line of real flexes is
$$a_1X + a_2Y + a_3Z = 0$$
and the equation of a cubic of the syzygetic pencil is
$$a_1^3X^3 + a_2^3Y^3 + a_3^3Z^3 + ka_1a_2a_3 XYZ = 0.$$
For $k \leq -3$, the cubics are bipartite, for $k \geq -3$ they are
monopartite.
I was very glad to confirm your construction for a bipartite
cubic.
The used property is that 4 points having the same tangential,
in other terms the vertices of a tangential quadrangle, are
coconic with H_1, H_2 and H_3 .
The problem with a monopartite cubic is that we have only pairs
of points having the same tangential and we have to find another
property of the conics.
Checking your construction for varying k shows easily that your
property doesn't hold for $k = 0, 1, 2, 3, 6 \dots$
It begins to be an acceptable approximation for bigger values of
 k (10, 12, 100 ...)
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2708
Date: 2025-06-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your explanation,
... so I have to study it furthermore,
... repeating the constellation in #2702 for $k = 4.2$
... with CABRI and greatest care there is a small aberration,
... my other drawings have all a greater k ... you are right.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2709
Date: 2025-06-16
From: contiwa.goma3@gmail.com
Subject: QA and Complex Quartic Eq.

Dear all,

First, in the name of MIMF, I have to write about complex cubic eq. and triangle although this is Quadri-and-Poly-Geometry.

< Cubic Eq.>

Let z_1, z_2, z_3 be the solutions of a cubic equation
 $z^3 + pz + q = 0$.

Define

$$u_k = (\omega^k)u, \quad v_k = v/(\omega^k) \quad (k=1,2,3),$$

where

$$u = -(z_3 + z_1/\omega + \omega z_2)/3$$

$$v = -(z_3 + \omega z_1 + z_2/\omega)/3$$

$$\omega = (-1 + i\sqrt{3})/2 \quad (\text{cube root of } 1),$$

then

$$z_k = -(u_k + v_k).$$

When z_1, z_2, z_3 form a triangle in the complex plane,

$u_k(v_k)$ is the vertice of outer(inner) Napoleon triangle and

$$X(15) = (v^2)/u$$

$$X(16) = (u^2)/v$$

$$X(187) = (u^3 + v^3)/2uv = -3q/2p$$

$$PU(118) = \pm\sqrt{uv} = \pm\sqrt{-p/3}.$$

< Quartic Eq.>

Let z_1, z_2, z_3, z_4 be the solutions of a quartic equation
 $z^4 + bz^2 + cz + d = 0$.

Define

$$m_{ij} = (z_i + z_j)/2$$

$$\kappa_{ij} = -(z_i + z_j)(z_k + z_l) = 4m_{ij}^2,$$

where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

Then

$$z_l = m_{il} + m_{jl} + m_{kl},$$

and κ_{ij} satisfies the cubic equation

$$\kappa^3 + 2b\kappa^2 + (b^2 - 4d)\kappa - c^2 = 0.$$

κ_{ij} is dimension 2, so let's define a triangle Tr_k in the complex plane as $\{\kappa_{14}/\sigma, \kappa_{24}/\sigma, \kappa_{34}/\sigma\}$, where σ is a certain point except the origin.

The triangle $\{\kappa_{il}/\sigma, \kappa_{jl}/\sigma, \kappa_{kl}/\sigma\}$ is inversely similar to the pedal triangle of point z_l wrt triangle $\{z_i, z_j, z_k\}$.

It relates to cross ratio $(z_j - z_k)(z_i - z_l)/[(z_i - z_k)(z_j - z_l)]$.

(I didn't realize that pedal triangles of circumcenter QA are similar to those of the reference QA.)

Since $c/(2\kappa_{ij})$ is the vertice of QA-Tr2 of $\{z_1, z_2, z_3, z_4\}$, Tr_k is obtained from QA-Tr2 by QL-Tf1 with the origin as QL-P1, $\pm\sqrt{c/(2\sigma)}$ as QL-2P3.

My question is what σ is appropriate.

When $\sigma = \pm 2\sqrt{-b/6}$, σ is the centroid of Tr_k .

But I don't know the geometrical meaning of $\pm\sqrt{c/(2\sigma)}$.

I'm searching σ which is relevant to QA/QL geometry, not necessarily expressed by only b, c, d .

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2710
Date: 2025-06-16
From: van10hoven@gmail.com
Subject: Re: QA and Complex Quartic Eq.

Dear MIMF,

Nice approach to the three roots of a cubic equation in the complex plane.

This group also explores curves of the n th degree, so your contribution is very welcome.

I try to describe your approach.

You created a mapping of the roots of a cubic equation in the complex plane, which together form a triangle. Using the elements of these roots, you then describe the inner/outer vertices of the Napoleon Triangle, the ETC points $X(15)$ and $X(16)$, as well as the bicentric pair $PU(118)$. Very nice! After that, you applied the same method to the roots of a quartic equation.

You created an image of the roots of a quartic equation in the complex plane, which together form a quadrangle QA. Using the elements of these roots, you then describe the vertices of QA-Tr2, QL-P1, and QL-2P3 (I hope I have described this correctly).

I haven't verified all the calculations, so I can't adequately answer your question. But I really like your approach. It could serve as an interesting step toward new insights in Triangle Geometry and Quadrilateral Geometry.

Best regards, Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2711
Date: 2025-06-17
From: contiwa.goma3@gmail.com
Subject: Re: QA and Complex Quartic Eq.

Dear Chris,

Thank you for your reply.
I used the terms QL-Tf1,P1,2P3 to describe a (involutive)
Moebius transformation.
So far, I can't find such a QL, but finding it might be a hint.

Below is more about triangle (although ...).

- 1) Psi transform of X(187) is
 $X(353) = 2(p^2)/9q$.
We can obtain z_1, z_2, z_3 from X(2), X(187) and X(353)
by addition, subtraction, multiplication, division,
and extraction of n-th roots (n=2,3).
- 2) If we use u^* and v^* (the complex conjugates of u and v),
we get
 $X(13) = u(v/u)^*$, $X(14) = v(u/v)^*$
 $X(3) = (v^2 u^* - u^2 v^*) / (|u|^2 - |v|^2)$
 $X(6) = (v^2 u^* + u^2 v^*) / (|u|^2 + |v|^2)$.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2712
Date: 2025-06-30
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear all,

Is this already known?

Let

C1 = the Brianchon point of QL1-Co1 wrt QL1-Tr2

C2 = the Brianchon point of QL2-Co1 wrt QL2-Tr2

D1 = the Brianchon point of DQL1-Co1 wrt QL1-Tr2

D2 = the Brianchon point of DQL2-Co1 wrt QL2-Tr2,

then

C1D1 is parallel to QL1-L1

C2D2 is parallel to QL2-L1

C1C2 is parallel to QL1-L8(=QL2-L8).

QL1, QL2, etc. are as before:

QL1-Co1 = QA-2Co1a

QL2-Co1 = QA-2Co1b

QA = a convex quadrangle (not a trapezoid)

QL1 = a quadrilateral formed by the tangents to QL1-Co1 at the vertices of QA

QL2 = a quadrilateral formed by the tangents to QL2-Co1 at the vertices of QA

DQL1-Co1 = anticomplement of QL2-Co1(=QL1-Co3) wrt DT

DQL2-Co1 = anticomplement of QL1-Co1(=QL2-Co3) wrt DT

DT = QA-Tr1 = QL1-Tr1 = QL2-Tr1.

I wonder why this holds.

Best regards,
M@IMF

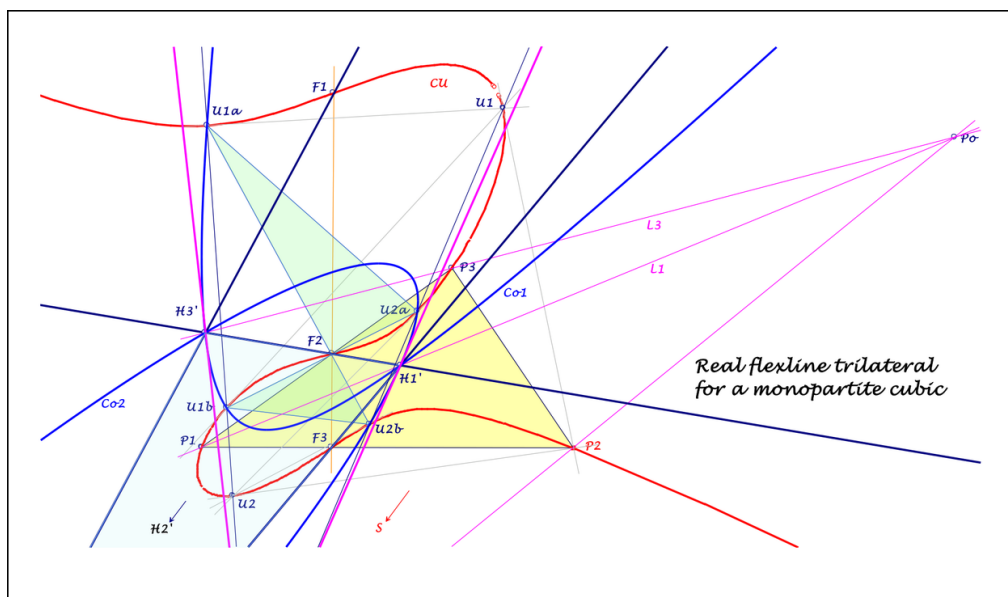
[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2713
Date: 2025-07-04
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

in #2635 I offered a "construction"
 ... for the real flexline trilateral of a bipartite cubic,
 ... I try it once more for a monopartite cubic:
 Consider a monopartite cubic CU
 ... their real flexpoints F1,2,3, harmonic polars L1,2,3,
 ... bearing the cubic-intersections P1,2,3 .
 Start with a point Pi (attached Pi = P2)
 ... and its two tangents to the cubic
 ... with contact points U1,U2,
 ... further the tangents from U1, U2 to the cubic
 ... with contact points U1a,U1b,U2a,U2b.
 The lines U1aU1bU2 and U2aU2bU1
 ... intersect in a new point S on Li.
 Use further variable triangles H1'H2'H3' on L1,L2,L3
 ... with F1,F2,F3 on their sidelines.
 Consider the conics
 ... Co1(H1',H2',H3',U1a,U1b) and Co2(H1',H2',H3',U2a,U2b).
 Now vary the triangles H1'H2'H3':
 If SHj' and SHk' are tangents to the conics in Hj' and Hk',
 ... then Hi' = Hi, vertices of the real flexline trilateral.

Best regards Eckart



2025-07-03.pdf

Message: #2714
Date: 2025-07-08
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Wunderbar, lieber Eckart

This time, it seems you have reached your goal!

I checked carefully with one of my cubics of the pencil and I can confirm your property of the point S , intersection of U_1aU_1b and U_2aU_2b , that S belongs to one tangent in H_2 to a conic and one tangent in H_3 to the other conic. The 2 other tangents in H_2 and H_3 to the 2 conics intersect in a point T , also on the line L_i (L_2 on your figur).

What about this point T ?

I suppose it musn't be too difficult (only a little bit boring), starting with the equation of the cubic given in message 2707, to calculate in barycentric coordinates wrt $H_1H_2H_3$ the coordinates of P_i , U_i and V_i and U_{ia} and b and V_{ia} and b and S_i on L_i , then the equation of the 2 conics (circumconics of H_1 , H_2 and H_3) and to check your property.

Sincere congratulations

Best regards

Bernard

PS For me, as I had found the barycentric equation of the cayleyan wrt $H_1H_2H_3$ (which is the same as the tangential equation of the cubic) and since you have found the construction of the 3 points H_1 , H_2 and H_3 in both cases mono- and bipartite, the game is now definitely over. Many thanks again for our mutual collaboration.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2715
Date: 2025-07-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

I studied Eckart's construction of the real Flex-trilateral in QPG#2713 and Bernard's response in QPG#2714.

Bernard said "I suppose it musn't be too difficult (only a little bit boring), starting with the equation of the cubic given in message 2707, to calculate in barycentric coordinates wrt $H_1H_2H_3$ the coordinates of P_i , U_i and V_i and U_{ia} and b and V_{ia} and b and S_i on L_i , then the equation of the 2 conics (circumconics of H_1 , H_2 and H_3) and to check your property."

Well, I tried to do that, but I already failed at the first step: calculating P_i in general. You have to solve the intersection of a reference cubic (third degree) and a polar conic (second degree). Even with the cubic simplified in Hesse's form, Mathematica was not able to solve it.

Therefore, the only way to proceed was by trying a numerical approach.

I used Mathematica to calculate two numerical cases – based on Hesse's form – with a monopartite reference cubic starting from a point P_2 , which is the only real intersection of the second Harmonic Polar L_2 and the monopartite reference cubic.

Since calculating the real Flex-trilateral $H_1H_2H_3$ is relatively straightforward, my approach was to construct the two conics passing through (H_1, H_2, H_3) and observe whether two of the tangents to these conics at H_1, H_2, H_3 coincide with H_1S and H_3S .

See attached pictures.

The outcomes were:

Numerical case 1:

- * The line H_1S appears to be the tangent at H_1 to Co_2 ($H_1, H_2, H_3, U_{2a}, U_{2b}$) – almost (value difference: 99.999156%)
- * The line H_3S appears to be the tangent at H_3 to Co_1 ($H_1, H_2, H_3, U_{1a}, U_{1b}$) – almost (value difference: 100.000844%)

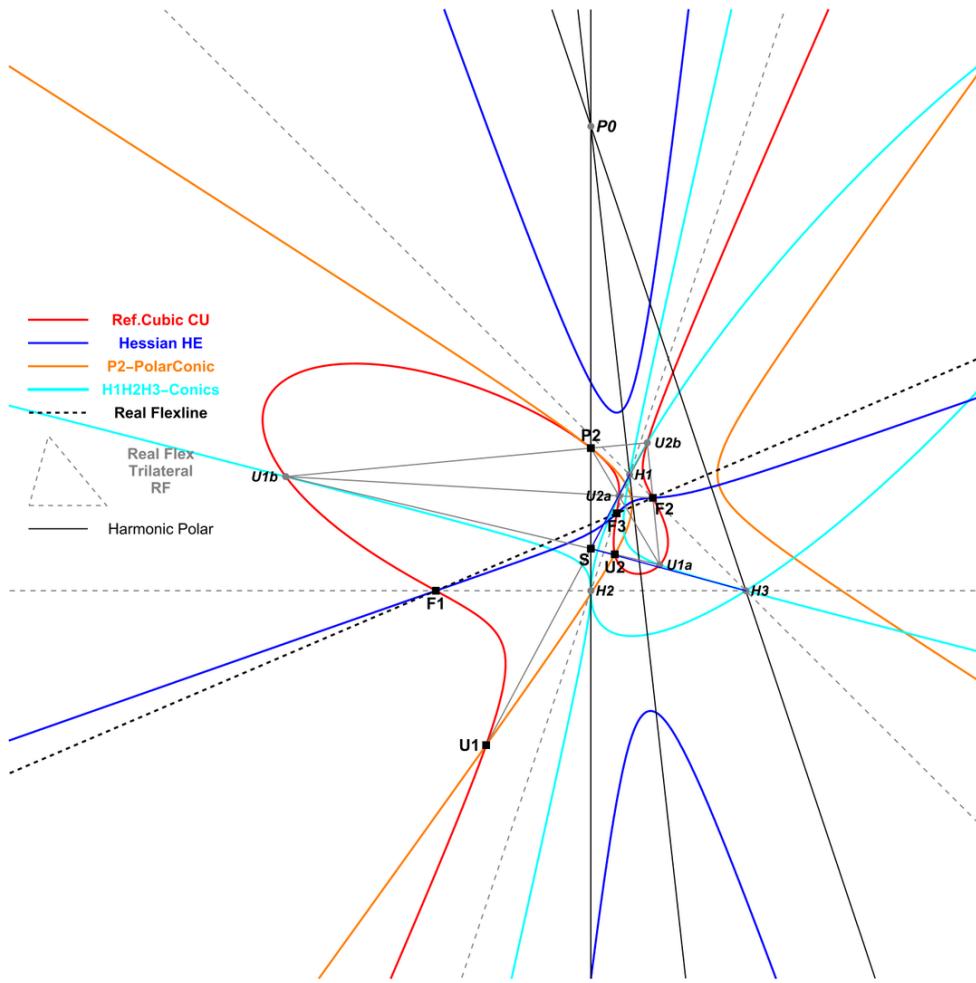
Numerical case 2:

- * The line H_1S appears to be the tangent at H_1 to Co_2 ($H_1, H_2, H_3, U_{2a}, U_{2b}$) – almost (value difference: 99.3387%)
- * The line H_3S appears to be the tangent at H_3 to Co_1 ($H_1, H_2, H_3, U_{1a}, U_{1b}$) – almost (value difference: 100.666%)

As a matter of fact, I find these results not very promising. They show that the tangents are indeed almost in the right place, but not exactly. I often encounter values around 99.9999999% in other calculations. Nevertheless, the performed computations go many levels deep, so I think it's reasonable to say the validity of the construction remains uncertain. The best way to proceed, I suppose, is by trying more test cases, especially exceptional ones. Maybe not the prettiest message.

Best regards,
Chris

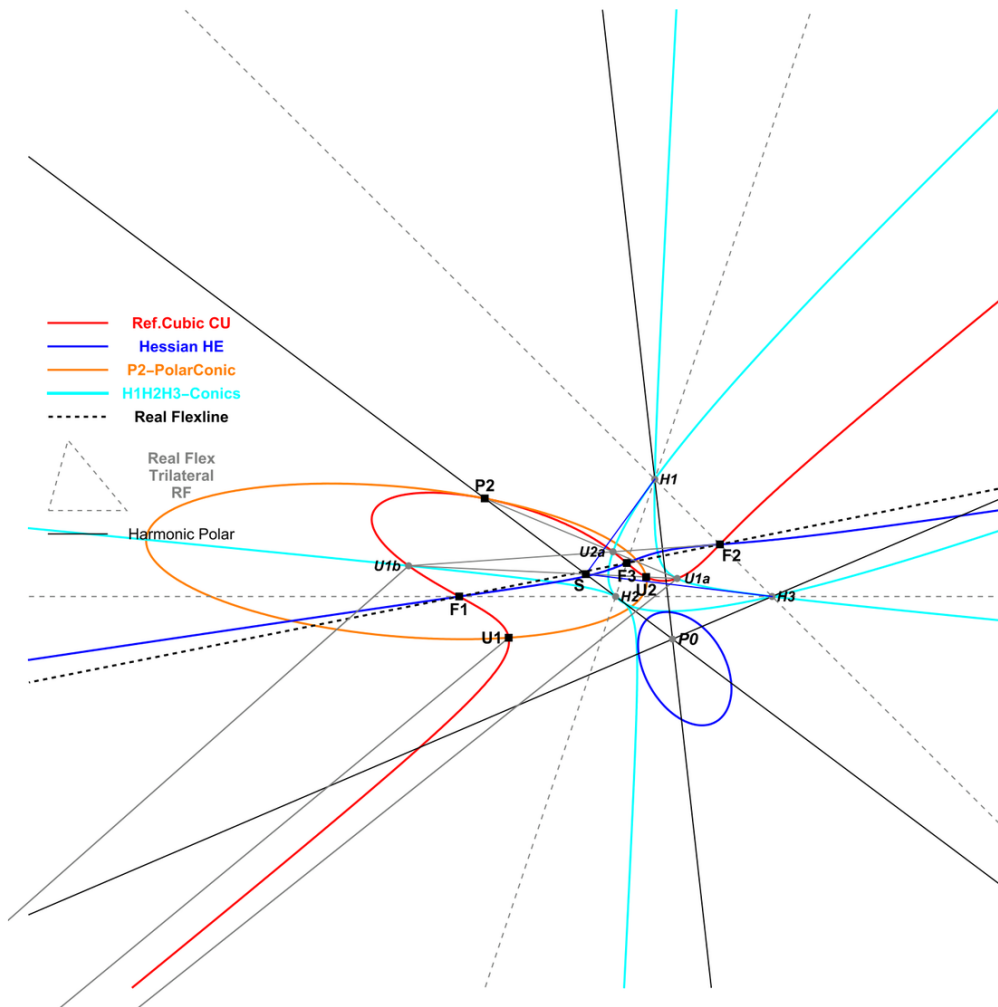
Numerical case-1



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.999156%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.000844%)

Numerical case-2



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.3387%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.666%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-2.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2716
Date: 2025-07-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart and Bernard,

Being in the flow, I created another third numerical case – a more extreme one.

Because the resulting cubic was fairly 'flat', I had to stretch it to see the details.

See attachment.

This time it came out:

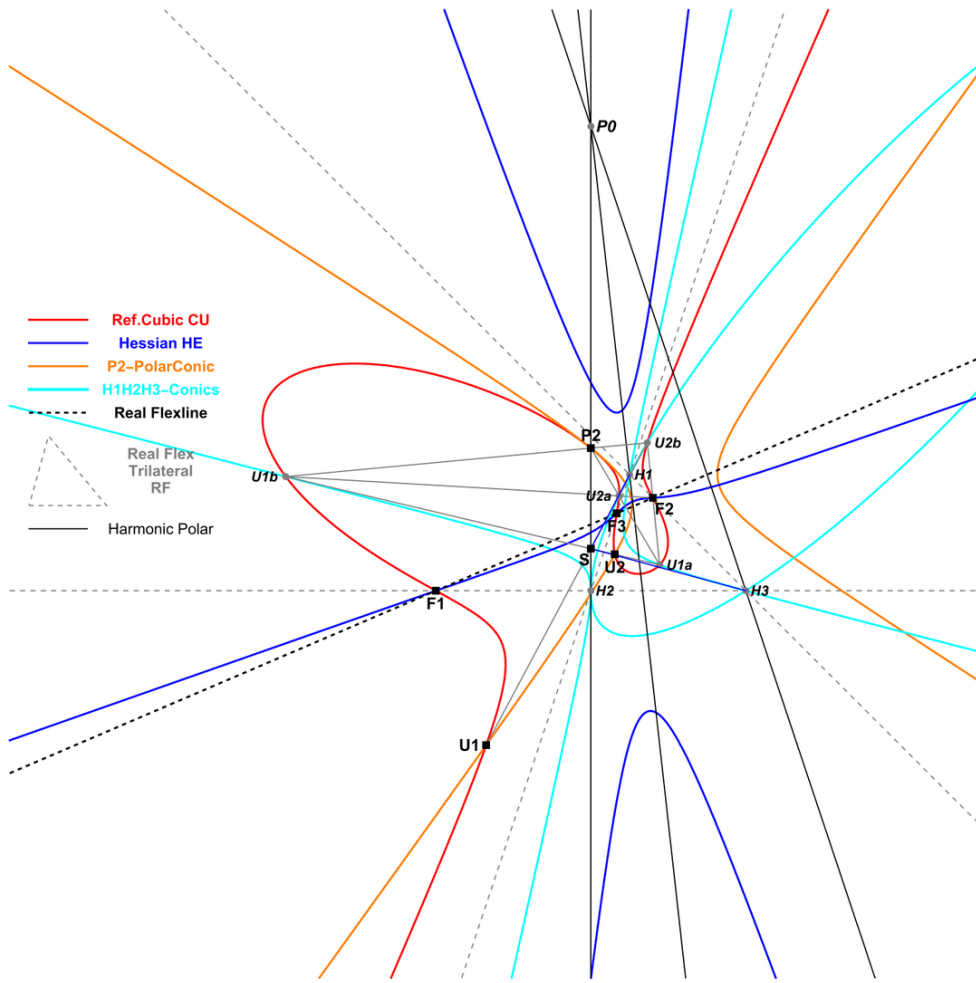
- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) – almost (value difference: 95.0132%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) – almost (value difference: 105.249%)

These value differences are too large. If I haven't made any mistakes, this disproves the conjecture.

To my regret . . .

Best regards,
Chris

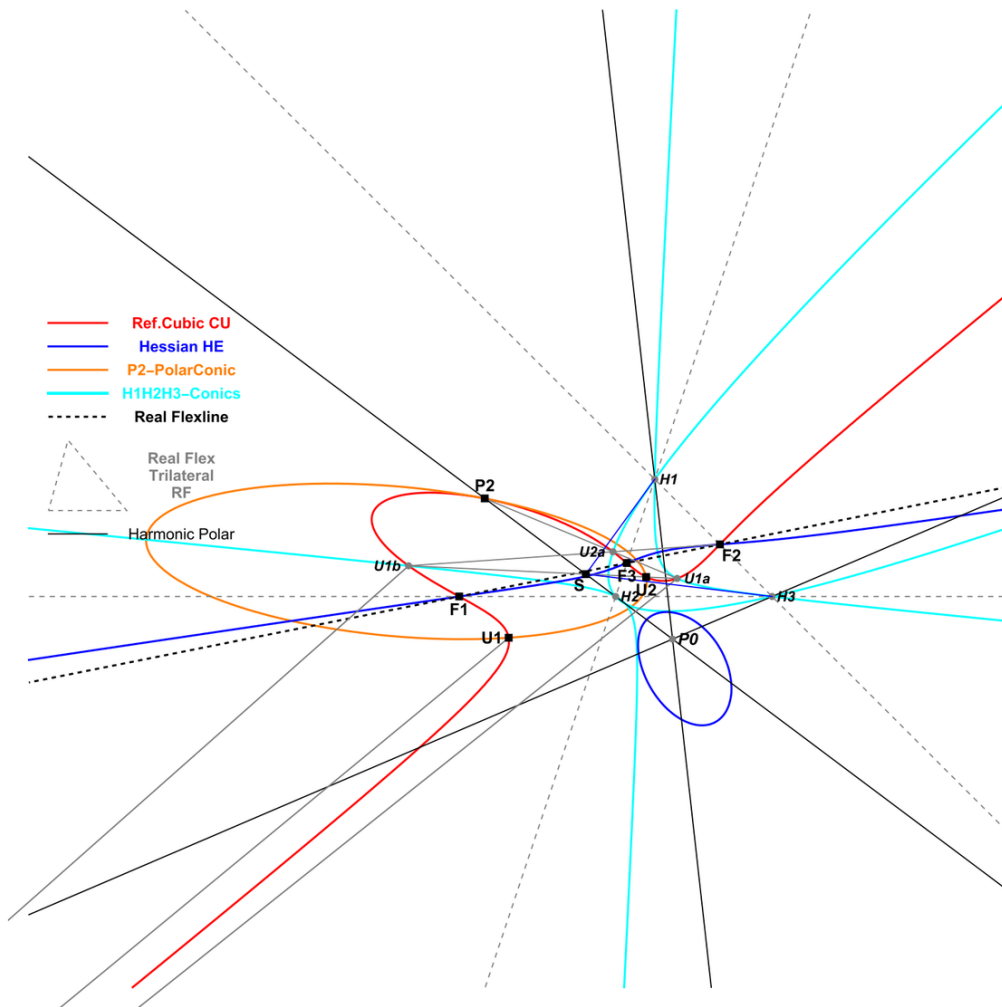
Numerical case-1



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.999156%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.000844%)

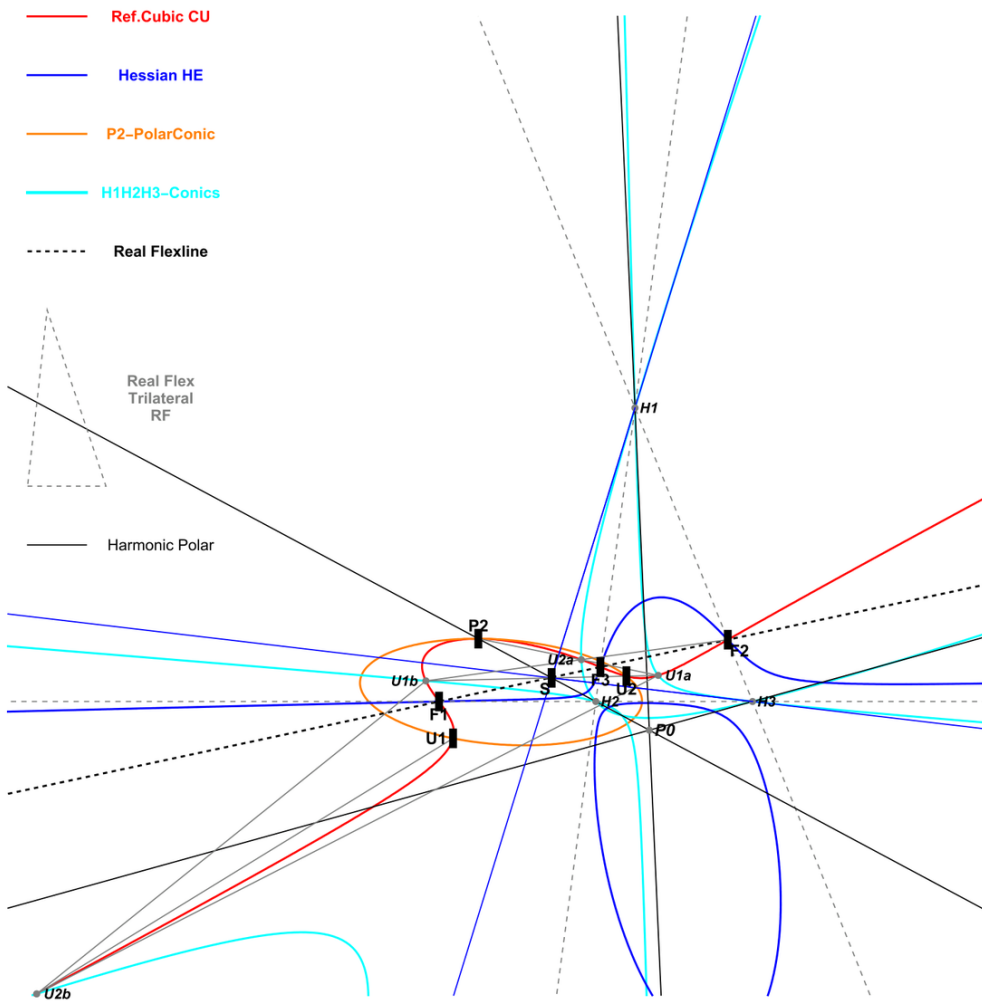
Numerical case-2



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 99.3387%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 100.666%)

Numerical case-3 (stretched picture)
 $CU = 3375x^3 - 8y^3 + 125xyz - 64z^3$



Result:

- The line H1S appears to be the tangent at H1 to Co2 (H1, H2, H3, U2a, U2b) — almost (value difference: 95.0132%)
- The line H3S appears to be the tangent at H3 to Co1 (H1, H2, H3, U1a, U1b) — almost (value difference: 105.249%)

9P-s-Cu1 Hessian-plus Flexlines-71-Test ES-Flextrilateral-02.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2717
Date: 2025-07-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

thanks to your judgements of my construction,
... especially to Bernard for his friendly message,
... but I think Chris will be right,
... special thanks for his complex calculations
... and his refusal ...

Chris 3rd example is near to a bipartite cubic,
... I researched such example and observed
... noticeable aberrations,
... so I have to retire my assumption.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2718
Date: 2025-07-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart,
My humblest apologies for my mistake!
Thanks to Chris for correcting my error ...
On my figures, the lines SiH_j and SiH_k are not tangent to the conics through H_1 , H_2 and H_3 , but the aberrations are sometimes not very significant.
Anyhow, I should have checked more precisely on several different figures ...
Now a last question, already asked in an old message :
For a bipartite cubic, the vertices of any tangential QA (4 points having the same tangential) are coconic with the 3 vertices H_1 , H_2 and H_3 .
For a monopartite cubic, what can be said about the conics through the vertices of a tangential segment (2 points having the same tangential) and the 3 vertices H_1 , H_2 and H_3 ? (For example, locus of their centers ...)
The fight goes on and I cannot retire, as I hoped ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2719
Date: 2025-07-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart,
I think finally that the 2 cases mono- or bipartite have the same solution!
Chris says often that the calculation with imaginary values are the same as with real values.
Then, we may find for any point of the cubic the 4 intersections of it's polar conic with the cubic other than the point itself. These 4 points are real for a bipartite cubic, 2 are real and 2 imaginary for a monopartite cubic.
The 3 vertices H1, H2 and H3 are on the 3 harmonic polars with the flexes F1, F2 and F3 on the sides H2H3, H1H3 and H1H2 and are coconic with the 4 points.
This gives a geometrical solution for a bipartite cubic, but there is no need to search a geometrical solution for a monopartite cubic, there isn't any!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2720
Date: 2025-07-13
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Bernard, dear Eckart,

I agree with you, Bernard, that algebraically there are no differences in how many real branches a cubic has. There are always just as many solutions to a question. The only difference lies in how many of those solutions are real or imaginary. After all, a cubic is a third-degree curve.

So algebraically, there's really no issue. Geometric constructions, however, suffer from the number of available real components, because imaginary components can't be used in drawings. If a particular construction only works when all components are real, that doesn't mean there are no alternative constructions that work around the presence of imaginary components.

Let's hope that one day, an alternative workaround construction for the 'real Flextrilateral' will be found.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2721
Date: 2025-07-14
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Bernard,

In #2718 you said:

"For a bipartite cubic, the vertices of any tangential QA (4 points having the same tangential) are coconic with the 3 vertices H1, H2 and H3.

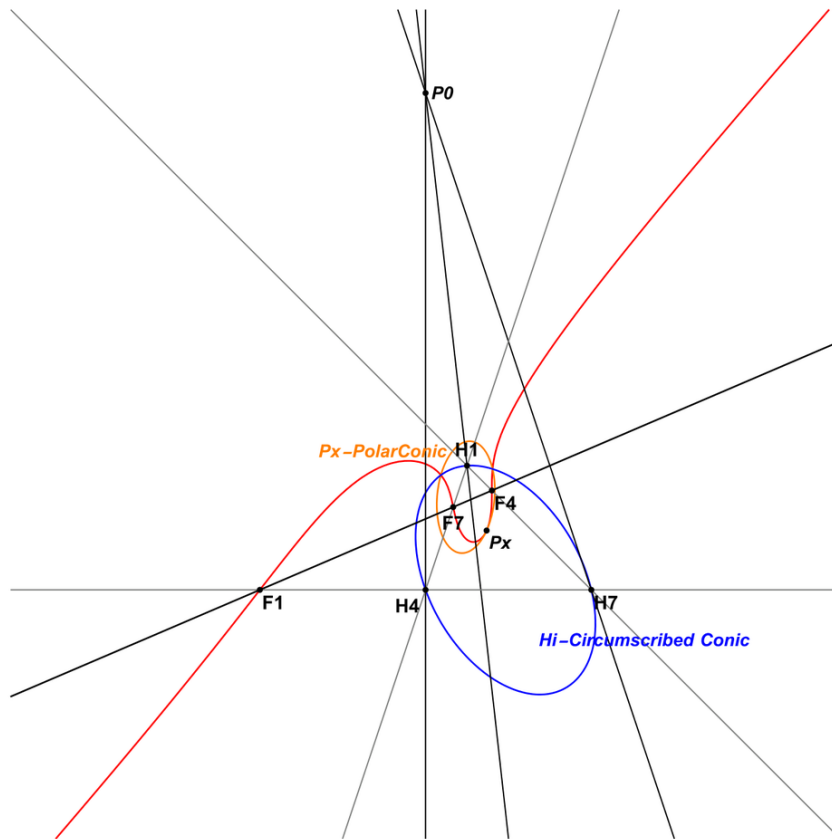
For a monopartite cubic, what can be said about the conics through the vertices of a tangential segment (2 points having the same tangential) and the 3 vertices H1, H2 and H3? (For example, locus of their centers ...)"

I created a picture to spark our imagination for both a monopartite and a bipartite cubic. See attachment.

Best regards,

Chris

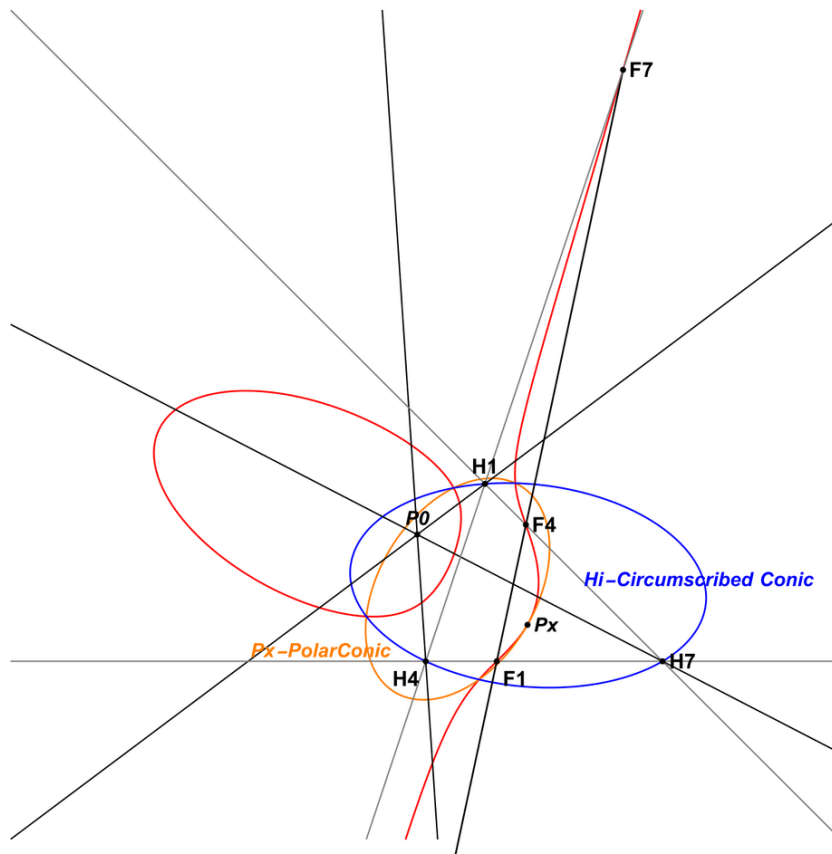
point P_x on a MONOPARTE cubic (**red**)
yielding a P_x -tangential quadrangle
circumscribed by the P_x -Polar Conic (**orange**)
and circumscribed by the H_i -Circumscribed Conic (**blue**)



Chris van Tienhoven

2025, July 14

point P_x on a BIPARTE cubic (**red**)
 yielding a P_x -tangential quadrangle
 circumscribed by the P_x -Polar Conic (**orange**)
 and circumscribed by the H_i -Circumscribed Conic (**blue**)



Chris van Tienhoven

2025, July 14

CU-HE-12L1abc-30-mono-bipartite cubic.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2722
Date: 2025-07-14
From: van10hoven@gmail.com
Subject: co-tangentials of P

Dear Eckart, dear Bernard,

Inspired by the construction of Eckart, I investigated the tangential quadrangle of a tangential point of P.

I don't know if it has been noted before, but I believe this is a special property.

- Let P be an arbitrary point on CU, and let (P_1, P_2, P_3, P_4) be the four points of tangency of the tangents from P to CU (also called tangentials).
- Let $(P_{1a}, P_{1b}, P_{1c}, P_{1d})$ be the four tangentials of P_1 .
- Now it turns out that the set (P_2, P_3, P_4) coincides with the vertices (Q_{1a}, Q_{1b}, Q_{1c}) of the Diagonal Triangle of the Quadrangle $QA(P_{1a}, P_{1b}, P_{1c}, P_{1d})$.

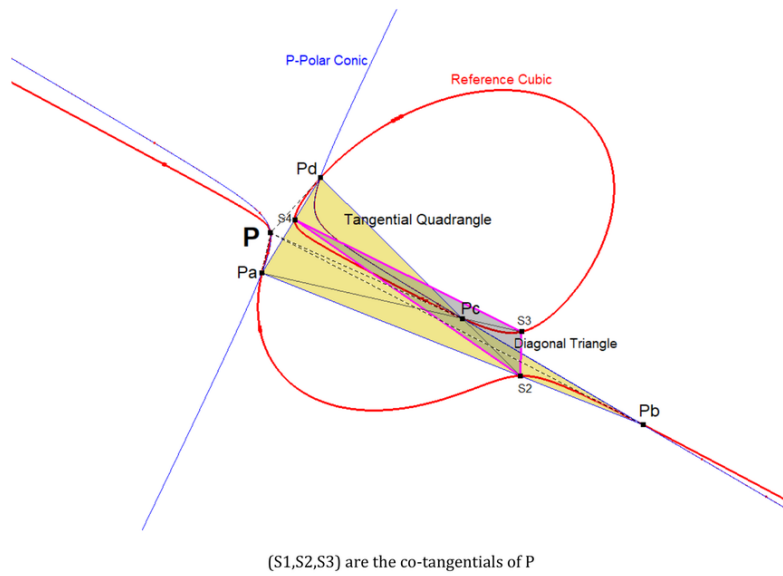
It provides us with a method to construct the co-tangentials of some point P.

This set of points -sharing the same tangential as P- consists of the vertices of the Diagonal Triangle of the Quadrangle formed by the four tangentials of P.

See attached picture.

Best regards,
Chris

The co-tangentials of P



Chris van Tienhoven

2025, July 14

CU_P-3P1 P-points with same tangential-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2723
Date: 2025-07-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris,
Thanks for your interest!
I'm glad that we perfectly understand eachother.
Your figures are the perfect illustration of my observation for a bipartite cubic and my conjecture for a monopartite cubic as explained in my message 2719.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2724
Date: 2025-07-14
From: bernard.keizer@gmail.com
Subject: Re: co-tangentials of P

Dear Chris,
All this is already well-known and explained in EQF!
A bipartite cubic is a pivotal isocubic in any of its points, the point being the pivot and the tangential of the 4 fixed points of the isoconjugation.
The DT of this tangential QA is the reference triangle of the isoconjugation
The isopivot is in turn the tangential of the pivot and of the DT vertices of the tangential QA of the fixed points ...
Take as example the Mac Cay cubic of a triangle ABC : it is a pivotal isogonal cubic with pivot the circumcenter and fixed points the in- and excenters.
The isopivot is the orthocenter, which is the tangential of the circumcenter and of the vertices of the triangle ABC, which is the DT of the QA of the in- and excenters.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2725
Date: 2025-07-18
From: eckart_schmidt@t-online.de
Subject: Re Real flexline trilateral

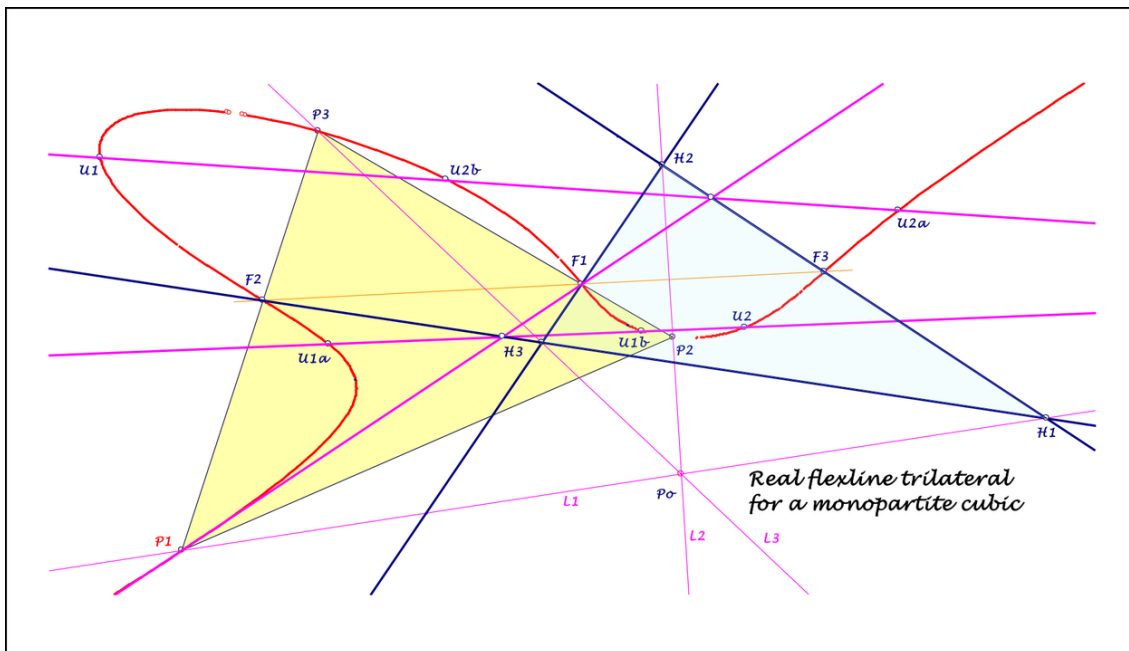
Dear Bernard, dear Chris,

nevertheless a further proposal
... for the real flexline trilateral in the monopartite case,
... with the nomination of #2713,
... perhaps the same background:

Start with a fixed P_i (attached P_1),
... which give $U_1, U_{1a}, U_{1b}, U_2, U_{2a}, U_{2b}$,
... vary H_1, H_2, H_3 so,
... that the line $P_i F_i$ intersect the line $U_1 U_{2a} U_{2b}$ on H_i, H_j '
... and the line $U_2 U_{1a} U_{1b}$ on H_i, H_k ',
... then H_1, H_2, H_3 will be $H_1 H_2 H_3$.

Thanks in advance for a test of my observation.
Best regards Eckart

PS: Please use H_1, H_2, H_3 ,
... which intersect the cubic only in F_1, F_2, F_3 !



2025-07-18.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2726
Date: 2025-07-18
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

Dear Eckart,
This property doesn't hold!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

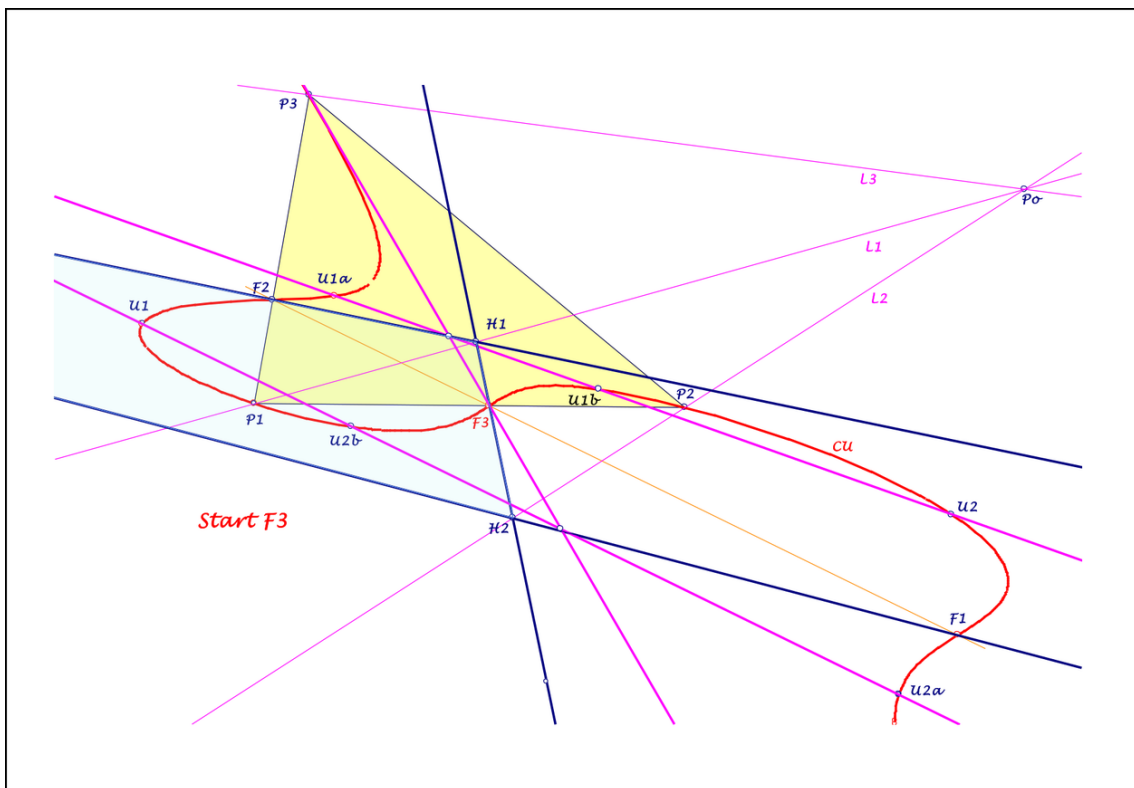
Message: #2727
Date: 2025-07-19
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

my "construction" proposal in #2725
... for the real flexline trilateral monopartite
... has the following structure:

Start with a real flexpoint F_i (attached F_3),
... then for F_i its tangent-contact P_i ,
... then for P_i its tangent-contacts U_1, U_2 ,
... then for U_1, U_2 their tangent-contacts $U_{1a,b}$ and $U_{2a,b}$
... and finally the intersections of $U_{1a}U_{1b}$ and $U_{2a}U_{2b}$
... with the connection F_iP_i (of the initial points)
... on lines of the real flexline trilateral $H_1H_2H_3$.

Best regards Eckart



2025-07-19.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2728
Date: 2025-07-19
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
As already mentioned in 2726, this property doesn't hold!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2729
Date: 2025-07-21
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear all,

For me, one of mysteries of QL-Tr2 is this:
Let QL1 be formed by 4 lines
 $y = tjx - f(tj^2)$ ($j=1,2,3,4$),
which are tangents of a parabola $x^2 = 4fy$ (QL1-Co1).
When tj satisfies the quartic equation
 $ft^4 + mt^3 + pt^2 + qt + r = 0$,
the slopes of the sides of QL1-Tr2 are the solutions of the
cubic equation
 $4ft^3 + 3mt^2 + 2pt + q = 0$.
(cf. #2517,2509)

I'll state differently.
Let QA0 be formed by 4 points
 $(xj, (xj^2)/4f)$ ($j=1,2,3,4$),
where xj satisfies the quartic equation
 $x^4 + (2m)x^3 + (4pf)x^2 + (8qf^2)x + (16rf^3) = 0$.
The equation of QA0-Co1 is
 $(x + m/2)(y' + p'/2) = -fq'/2$,
where
 $y' = y + (m/4f)x$, $p' = p - (m^2)/4f$, $q' = q - (m/2f)p'$.
(Please see #2532.)
Then the x-coordinates of the intersection points of QL1-Co1 and
QA0-Co1 are
the solutions of the cubic equation
 $4x^3 + 3(2m)x^2 + 2(4pf)x + (8qf^2) = 0$.

Here's my interpretation (the method of Lagrange multiplier).

Let

$$h(x) = x^4 + ax^3 + bx^2 + cx + d,$$

where

$$a = 2m, b = 4pf, c = 8qf^2, d = 16rf^3.$$

The critical points of $h(x)$ are the solutions of the cubic equation $dh(x)/dx = 0$.

Finding them is equivalent to finding the critical points of $L(x,y)$ subject to $g(x,y) = 0$,

where

$$L(x,y) = h(x,y) + \lambda g(x,y) \quad (\lambda: \text{Lagrange multiplier})$$

$$h(x,y) = (4fy)^2 + ax(4fy) + b(4fy) + cx + d$$

$$g(x,y) = x^2 - 4fy.$$

$L(x,y)$ can be written in the form

$$C(x - x_0)^2 + D(y' - y'_0)^2 + E,$$

where C, D, E, x_0, y'_0 are real constants.

Then $\partial L/\partial x = \partial L/\partial y = 0$ give $x = x_0$ and $y' = y'_0$, which is the center of the conic

$$C(x - x_0)^2 + D(y' - y'_0)^2 + \text{const.} = 0.$$

Since $L(x,y) = 0$ is a circumconic of QA_0 , the critical points are on QA_0 -Co1.

Let

Co2 = a circumconic of QA_0

Co3 = the polar curve of Co2 wrt QL_1 -Co1.

When the center of Co2 is on QL_1 -Co1, the side of QL_1 -Tr2 through this point

and QL_1 -L1 are the conjugate diameters of Co3.

Best regards, M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2730

Date: 2025-07-24

From: eckart_schmidt@t-online.de

Subject: Re: Real flexline trilateral

Dear Chris,

can you confirm Bernard's unsubstantiated answers

... in his last two messages?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2731
Date: 2025-07-25
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I can understand your deception and your disappointment, but I think I don't deserve your nasty commentar about my unsubstantiated answers.
I've checked already patiently a dozent of your fake constructions.
You asked me to check your last construction in your message #2725.
I've explained my method in the message #2707: I deal with exact cubics for which I know the exact points H1, H2 and H3.
Therefore, it's not too difficult to check your properties ; as soon as I find any significant aberration, I simply mention that the property doesn't hold.
What do you want more?
Best regards
Bernard
PS Again, as mentionned in my message #2719, the algebraic solution is the same in both cases mono- or bipartite!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2732
Date: 2025-07-25
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I will be testing your new method as soon as possible. I just need a bit of time.
It will be a pleasure to explore your approach.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2733
Date: 2025-07-25
From: van10hoven@gmail.com
Subject: Re: About QL-Tr2

Dear MIMF,

Many thanks for yet another impressive contribution on QA/QL and a 4th degree function.

It connects various aspects of Quadri Geometry that I wasn't aware of.

I can't immediately follow every detail, but I truly admire your perseverance.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2734
Date: 2025-07-26
From: contiwa.goma3@gmail.com
Subject: Re: About QL-Tr2

Dear Chris,

Thank you for your kind message.
I don't intend to demand a quick response.
(Of course, any comments are always welcome.)
If I posted a message, it might happen to be someone's help
someday.

By the way, let's complexify #2729.
Given 4 complex numbers z_1, z_2, z_3, z_4 such that
 $z_1 + z_2 + z_3 + z_4 = 0$,

and let

$$\begin{aligned} b &= (z_1 + z_2)(z_3 + z_4) + z_1z_2 + z_3z_4 \\ c &= -(z_1 + z_2)z_3z_4 - (z_3 + z_4)z_1z_2 \\ d &= z_1z_2z_3z_4. \end{aligned}$$

Then the z -coordinates of the intersection points of 2 complex
conics

$$\begin{aligned} w &= z^2 \text{ and } z(w + b/2) = -c/4 \\ &\text{are the solutions of the cubic equation} \\ 4z^3 + 2bz + c &= 0. \end{aligned}$$

This will have something to do with foci of Siebeck sextic,
but I'm not familiar with cubics nor higher degree(class)
curves.

P.S. Stereograms in the attached file are for parallel viewing.

Best regards,
M@IMF

Let QA0 be formed by 4 points (z_j, w_j) ($j=1,2,3,4$) which are the intersection points of 2 conics

$$w = z^2 \quad (1)$$

$$w^2 + bw + cz + d = \lambda(z^2 - w). \quad (2)$$

(1) is QL1-Co1 where QL1 is formed by 4 tangents to (1) at the vertices of QA0.

(2) is a circumconic of QA0, which is written as

$$[w + (\lambda + b)/2]^2 - \lambda(z - c/2\lambda)^2 = [\lambda^3 + 2b\lambda^2 + (b^2 - 4d)\lambda - c^2]/4\lambda,$$

so its center is $(c/2\lambda, -(\lambda + b)/2)$, and QA0-Co1 is

$$z(w + b/2) = -c/4. \quad (3)$$

We can obtain the intersection points of QL1-Co1 and QA0-Co1 from

$$4z^3 + 2bz + c = 0.$$

Let

$$z = x + iy, w = u + iv$$

$$b = b' + ib'', c = c' + ic''$$

where $x, y, u, v, b', b'', c', c''$ are real and i is the imaginary unit.

Then we get

QL1-Co1

$$u = x^2 - y^2 \quad (1')$$

$$v = 2xy \quad (1'')$$

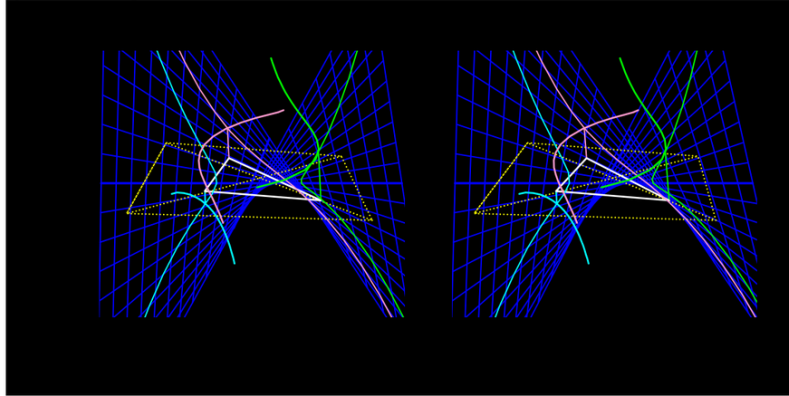
QA0-Co1

$$x(u + b'/2) - y(v + b''/2) = -c'/4 \quad (3')$$

$$x(v + b''/2) + y(u + b'/2) = -c''/4. \quad (3'')$$

When u is a constant, (3') and (3'') are hyperbolic paraboloids as well as (1').

(parallel viewing)



The blue lines indicate $v = 2xy$.

The 3 curves on this surface are QL1-Co1 (other branches are not shown).

The other 3 curves are (part of) QA0-Co1.

Note that the curves in different colors correspond to the different values of u which is the real part of z^2 , where z is the solution of $4z^3 + 2bz + c = 0$ (the vertex of the white triangle).

The yellow dotted lines indicate the quadrangle $\{z_1, z_2, z_3, z_4\}$.

Appx. Foci of Ellipse

Let's complexify an ellipse

$$Cx^2 + Dy^2 = 1 \quad (0 < C < D),$$

that is,

$$x \rightarrow x + i\xi, \quad y \rightarrow y + i\eta$$

$$C(x^2 - \xi^2) + D(y^2 - \eta^2) = 1$$

$$Cx\xi + Dy\eta = 0.$$

Then we get

Tangents through circular points at infinity

$$x - \eta = +f, \quad \xi + y = 0$$

$$x - \eta = -f, \quad \xi + y = 0$$

$$x + \eta = +f, \quad \xi - y = 0$$

$$x + \eta = -f, \quad \xi - y = 0$$

Points of tangency

$$x = +1/fC, \quad \eta = +1/fD, \quad \xi = y = 0$$

$$x = -1/fC, \quad \eta = -1/fD, \quad \xi = y = 0$$

$$x = +1/fC, \quad \eta = -1/fD, \quad \xi = y = 0$$

$$x = -1/fC, \quad \eta = +1/fD, \quad \xi = y = 0$$

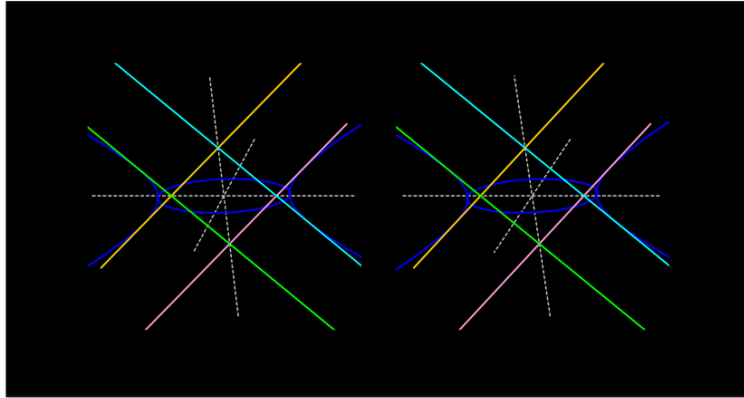
Foci

$$\text{real: } x = \pm f, \quad \eta = \xi = y = 0$$

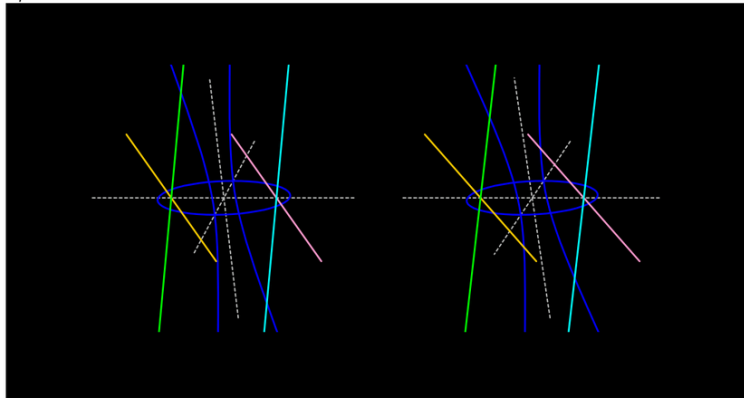
$$\text{imaginary: } \eta = \pm f, \quad x = \xi = y = 0,$$

where $f = \sqrt{1/C - 1/D}$.

$\xi = 0$



$\eta = 0$



Message: #2735
Date: 2025-07-27
From: bernard.keizer@gmail.com
Subject: Simple definition of QL-Tr2

Dear Chris, dear Eckart, dear M@IMF,

First, many thanks to M@IMF for revisiting the triangle of the S-points QL-Tr2.

Some time ago, I've put on the forum a memo about Hessian and generalised S-points for a pivotal isocubic.

Here only a few remarks :

1) There are as many S-points as points on the cubic : they are always the 3 intersections (other than the point taken as pivot) of the polar conics of the point and it's conjugate (which is the isopivot). These 2 conics are the duals of 2 conics inscribed in the QL and in the DT, which leads to the usual well-known properties ...

2) These S-points may be used in order to draw the hessian of the cubic (see Schröter).

3) Let's consider the cubic QA-Cux with pivot QA-P16 and isopivot QA-P10 ($pK(QA-P16,QA-P16)$ according to Bernard Gibert). Then the vertices of QL-Tr2 are the 3 intersections (other than QA-P16) of the polar conics of QA-P16 and QA-P10 wrt the cubic QA-Cux.

The duals of these 2 conics are the parabolas QL-Co1 and DQL-Co1 ...

Best regards
Bernard

Hessian and generalised S-points for a pivotal isocubic

A. S-points and points of the hessian
(see figur1)

Let's start with a pivotal isocubic wrt a triangle $T_1T_2T_3$ with pivot P and isopivot P' .

The fixed points of the isoconjugation are P_1, P_2, P_3 and P_4 .

The cevian triangle of P wrt $T_1T_2T_3$ is $u_1u_2u_3$.

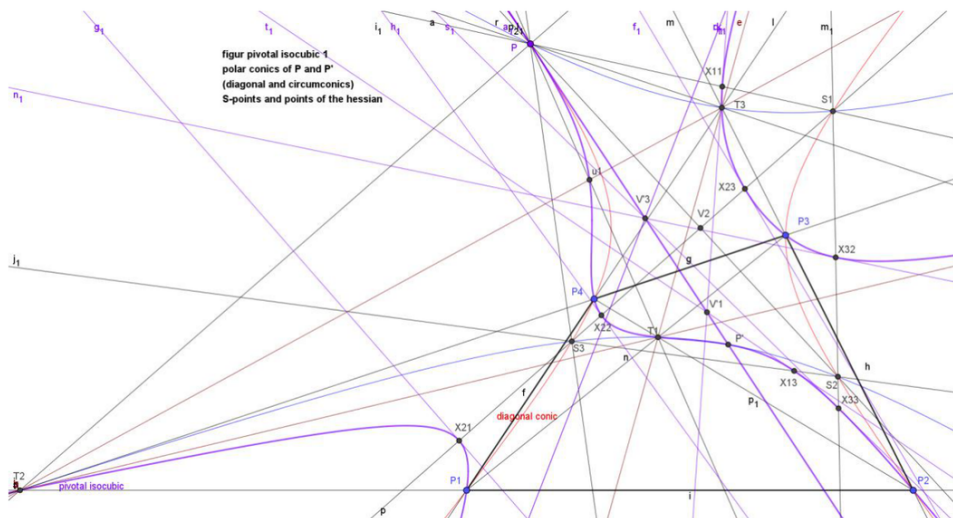
The polar conic of P wrt the cubic is the diagonal conic through P, P_1, P_2, P_3 and P_4 (in red).

The polar conic of P' wrt the cubic is the circumconic through P, P', T_1, T_2 and T_3 (in blue).

Both conics intersect in P and 3 S-points S_1, S_2 and S_3 ; the 4 points are the poles of PP' wrt the cubic and the polar conics of all points of the line PP' pass through the 4 points.

The DT vertices of the QA of the 4 points, V_1, V_2 and V_3 belong to the hessian of the cubic, as well as their corresponding points (on the hessian), V'_1, V'_2 and V'_3 on PP' having as polar conics the lines through V_1, V_2 and V_3 and the 4 points P, S_1, S_2 and S_3 .

(In order to find V'_1, V'_2 and V'_3 , it's easy to draw the lines of the QA and the tangents in their intersections with the cubic ...)



Hessian and generalised S-points for a pivotal isocubic.pdf

B. Hessian and flexes
(see figure 2)

P is the tangential of P' (P/P' in Bernard Gibert) and of u_1, u_2 and u_3 .

The cubic is also a pivotal isocubic wrt $u_1u_2u_3$ with pivot P' and isopivot p .

It's possible to do the same construction and to find the 4 poles of the line $P'p$ as intersection of the polar conics of P' (unchanged) and of p through p, P', u_1, u_2 and u_3 (in green). (note that the polar conic of P' is now a diagonal conic).

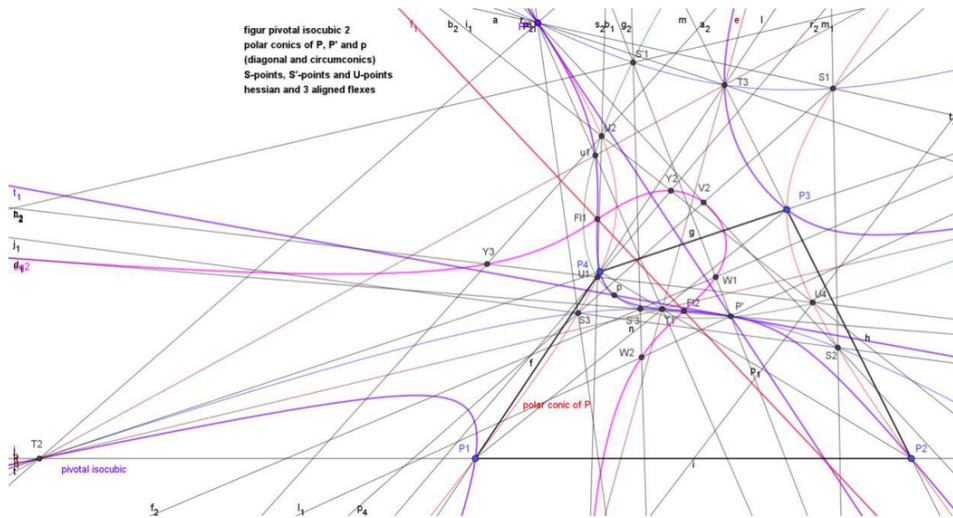
We find the same way the 3 points $S'1, S'2$ and $S'3$, the DT vertices $W1, W2$ and $W3$ of the QA $P'S'1S'2S'3$ and their corresponding points $W'1, W'2$ and $W'3$ on $P'p$.

We have now 12 points of the hessian.

We may naturally also consider the QA of the 4 intersections $U1, U2, U3$ and $U4$ of the polar conics of P and p and the vertices $Y1, Y2$ and $Y3$ of it's DT, which also lie on the hessian ...

Note that the points $U1, U2, U3$ and $U4$ are not S-points, as P and p are not isoconjugates.

The 3 real intersections of the cubic and it's hessian are the (aligned) flexes of the 2 cubics.



Hessian and generalised S-points for a pivotal isocubic.pdf

C. S-points

Now, we consider the S-points S_1, S_2 and S_3 , intersection of the polar conics of the pivot P and the isopivot P' .

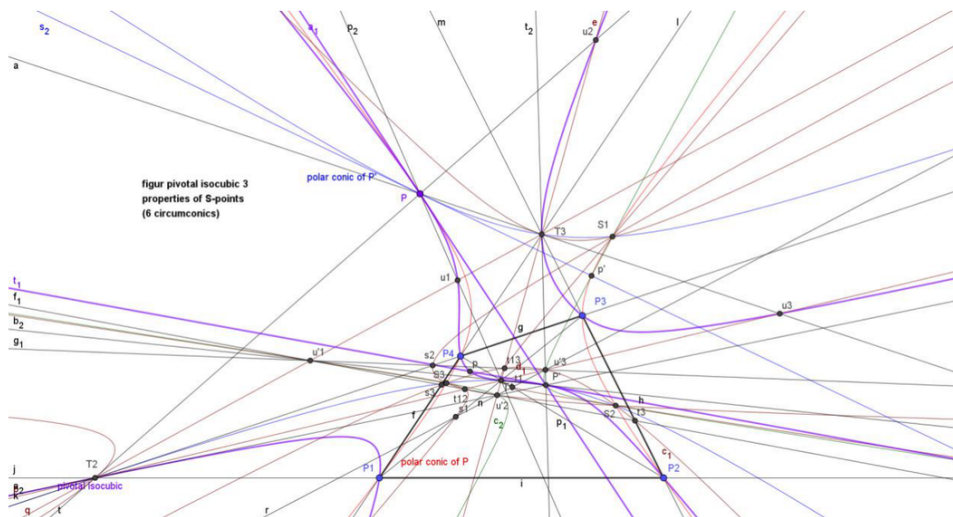
Let $u'1u'2u'3$ be the cevian triangle of P' wrt $T_1T_2T_3$ and p' be the point 2nd intersection of the tangent in P to the polar conic of P' with the polar conic of P (P/P' of Bernard Gibert).

$u'2u'3$ intersects T_2T_3 in t_{23} and P_1P_3 and P_2P_4 (diagonals of the QA through P_1) respectively in s_1 and t_1 and so on.

There are 4 more conics through these 3 points :

- first one through $P', p', u'1, u'2$ and $u'3$ (in green)
- 3 others through $T_1, u'1, s_1, t_1, t_{12}$ and t_{13} and the like (in brown)

making altogether 6 circumconics.



Naturally, the traditional S-points (vertices of $QL-Tr_2$) are obtained for $P = QA-P_{16}$ and by definition $P' = QA-P_{10}$.

(pole = pivot according to Bernard Gibert)

I suppose Eckart will have recognized his 3 conics 1), 2), 3) and the 3 conics in 4) in his now old but always fascinating note on the S-points.

D. Last properties with twin pivotal isocubics

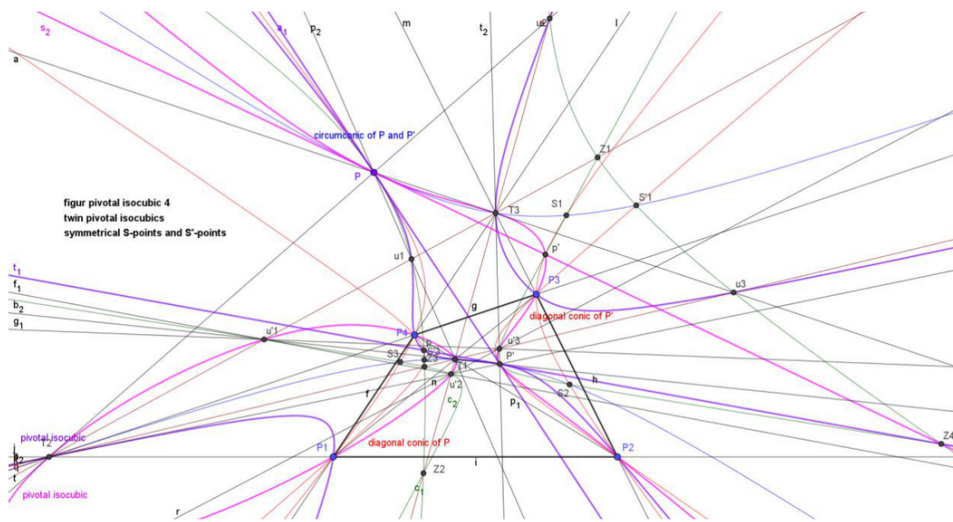
We consider now the twin pivotal isocubics having the same couple of isoconjugate points as pivot and isopivot ; $P_1, P_2, P_3, P_4, T_1, T_2, T_3, P$ and P' are CB partners.

The 1st cubic passes through p, u_1, u_2 and u_3 and the 2nd through p', u'_1, u'_2 and u'_3 .

There are 2 diagonal conics (in red), but only one circumconic (in blue) common to the 2 cubics ; the intersections between the circumconic and the 2 diagonal conics give the symmetric S -points for the 1st cubic and S' -points for the 2nd.

There are 2 conics in green, the 1st passes through P, p, u_1, u_2 and u_3 and the S' -points and the 2nd through P', p', u'_1, u'_2 and u'_3 and the S -points.

It is remarkable that these 2 last conics intersect in 4 points Z_1, Z_2 and Z_3 on each side of the $DT_1T_2T_3$ and the 4th Z_4 on the intersection of Pp' and $P'p$.



Hessian and generalised S -points for a pivotal isocubic.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2736
Date: 2025-07-27
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

Thank you very much for your message.
It seems very difficult, so I have to study from cubics.
(It might take some time...)
For me, one of mysteries of QL-Tr2 (deja vu?) is QL1-QL2
symmetry.
(cf. #2712, and of course, QFG#1315-38 "ETC and EQF")
What will become of this?
Please forgive me if I ask a misguided question.

Best regards,
M@IMF
(Please use "MIMF" if @ is annoying.)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2737
Date: 2025-07-28
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer and all,

I was overwhelmed by #2735, but I realized some of them are
understandable even for me.
I could follow 3), and the answer to my question is simply
perspectors of 2 QLs are isotomic wrt DT as usual.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2738
Date: 2025-07-29
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,

Many thanks for your interest

On pages 30-31 of his well-known memo about Special Isocubics in a Triangle Plane, Bernard Gibert defines the 2 considered conics for a pivotal isocubic as diagonal and circumscribed polar conics (the 1st through pivot and fixed points, the 2nd through pivot, isopivot and reference triangle).

The beauty of the given definition of the S-points is naturally it's simplicity: 3 intersections other than the pivot of these 2 conics.

The vertices of QL-Tr2 are a special case, when P is QA-P16 and the cubic $pK(QA-P16, QA-P16)$ with pivot=pole (therefore, the duals of the 2 conics are parabolas).

In fact, we don't need the cubic, only the vertices of the QA as fixed points of the isoconjugation and the DT as reference triangle.

I wonder what the locus of the S-points could be :

- 1) for a given cubic, varying P, it's tangential QA and it's DT
- 2) for a given QA and it's DT, varying P and the cubics

Any idea?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2739
Date: 2025-07-30
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer and all,

Wrt 1) in #2738, let me confirm my understanding.

Given a QA and a point P, and let

$C_u = pK(QA-P16, P)$ wrt QA-Tr1

P' = a point on C_u

$P' \sim$ = the tangential of P' wrt C_u

COd' = the polar conic of P' wrt C_u

COc' = the polar conic of $P' \sim$ wrt C_u

$S'j$ ($j=1,2,3$) =

the intersection points of COd' and COc' except P' .

Then S-points are $S'1, S'2, S'3$.

When P is parametrized by t, the S-point is expressed by t:

$(u'j(t):v'j(t):w'j(t))$.

Eliminating t from

$x = u'j(t), y = v'j(t), z = w'j(t)$

gives the locus of the S-point in principle, but this will be unrealistic.

One method I came up with is to derive differential equations:

$dx/dt = du'j(t)/dt, dy/dt = dv'j(t)/dt, dz/dt = dw'j(t)/dt$.

Solving cubic eq. is cumbersome, so how about finding the locus of the centroid

of the 3 S-points for starters?

If we are lucky, the differential eq. can be solved analytically (otherwise numerically).

Wrt 2), how does P vary?

I'm also interested in QLx-Tr2 where QLx is formed by tangents to a circumconic of (convex) QA0 at its vertices.

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2741
Date: 2025-07-31
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,

In 1) Cu is defined by the QA and P and Cu is $pK(QA-P16,P)$, QA-P16 being the pole and P being the pivot.

A variable P' on Cu defines a new QA (it's tangential QA) and it's DT as well as a new isoconjugation wrt this DT

P'* is the isoconjugate of P' in thi isoconjugation and P' and the vertices of this DT have P' as tangential

The 2 conics are the polar conics of P' and P'*

In 2) The QA, it's DT are fixed, but P is a random variable point and Cu is defined as $pK(QA-P16,P)$ (as for example QA-Cu1 to QA-Cu6 in EQF)

P* is the isoconjugate of P, P is the tangential of the QA vertices and P* is the tangential of P and of the DT vertices

The 2 conics are the polar conics of P and P*

in 3) P is QA-P16 (pivot = pole), P* is QA-P10

The 2 conics are the polar conics of QA-P16 and QA-P10

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2742
Date: 2025-07-31
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

Thank you for the explanation.

Wrt 1), I assumed new isoconjugation is QA'-Tf2:

Given a QA and a point P, and let

$C_u = pK(QA-P16, P)$ wrt QA-Tr1

P' = a point on C_u

Q'_j ($j=1,2,3,4$) = the points of tangency of P' to C_u
(pretangentials of P' ?)

$QA' = \{Q'_1, Q'_2, Q'_3, Q'_4\}$.

I thought

$C_u = pK(QA'-P16, P')$ wrt QA'-Tr1,

then

$P'^* = QA'-Tf2(P') = P' \sim$ (the tangential of P' wrt C_u)

S'_j ($j=1,2,3$) = the intersection points of COd' and COc'
except P' ,

where

COd' = the polar conic of P' wrt C_u

COc' = the polar conic of $P' \sim$ wrt C_u .

Wrt 2), I thought when P is a random variable point (not lies on a certain curve),
the locus of S-point becomes 2-dimensional.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2743
Date: 2025-08-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF,
We agree.
The $Q'_{1,2,3,4}$ are in fact the 4 pretangentials of P' , which is their common tangential
 P' and the vertices of $QA'-Tr1$ are the 4 pretangentials of P'^* , which is their common tangential.
Best regards
Bernard

PS Having the equations of the 2 polar conics given in Bernard Gibert, I hoped it would be possible to solve the system and, knowing that P is a solution, to find the 3 other

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

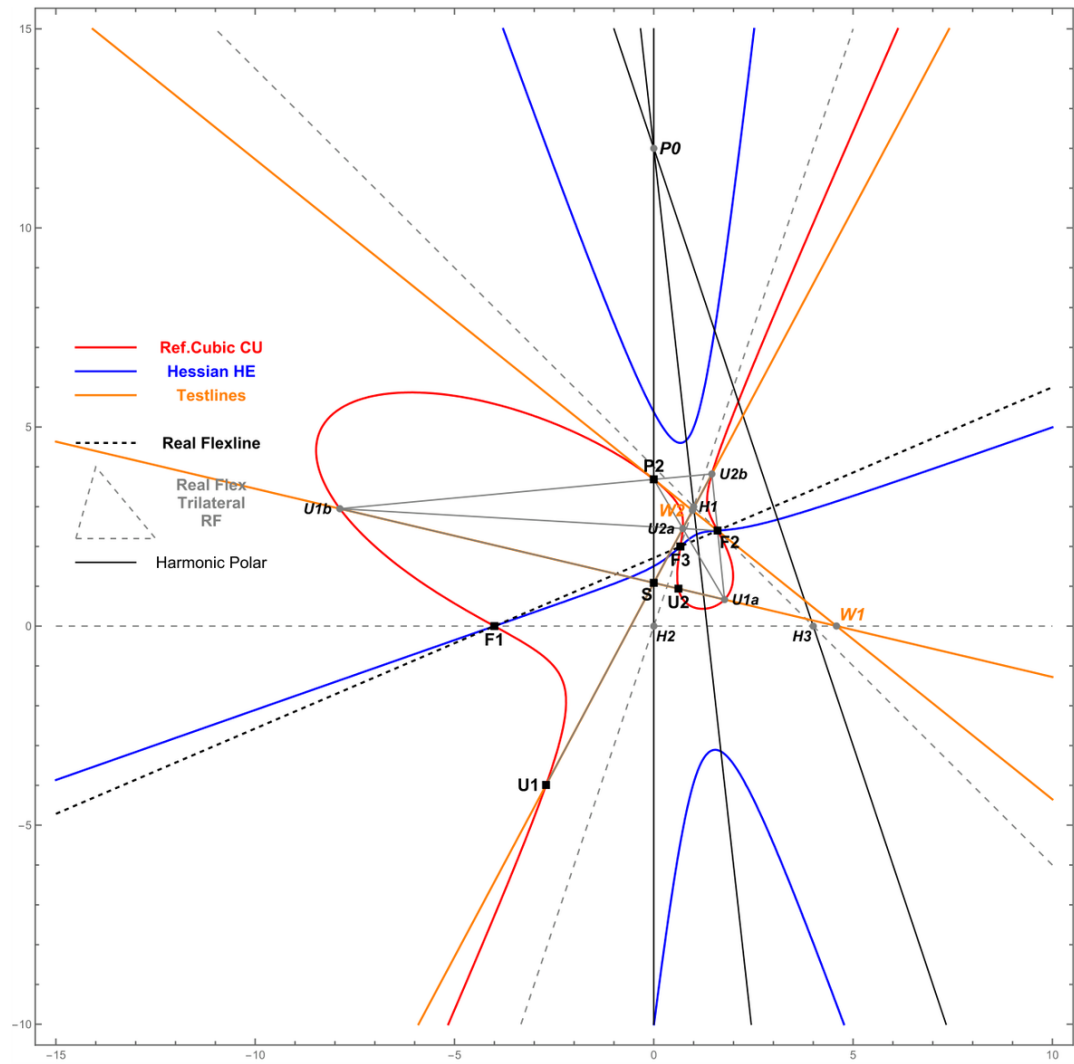
Message: #2744
Date: 2025-08-01
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I have reviewed your latest configuration, as outlined in QPG#2725 and QPG#2727, utilizing the same setups from my previous analysis.
In both setups, the line $F2P2$ appears as a dashed orange line. The lines $U1aU1b$ and $U2aU2b$ are also orange, but not dashed. In the first drawing, you can see that the intersection points $W1 = F2P2 \cap U1aU1b$ and $W2 = F2P2 \cap U2aU2b$ are nearly aligned with $H2H3$ and $H2H1$.
In the second drawing, only $W2$ is visible, but it's clearly not positioned on $H2H1$.
I hope there are no errors on my part, but please double-check my drawings to be sure.

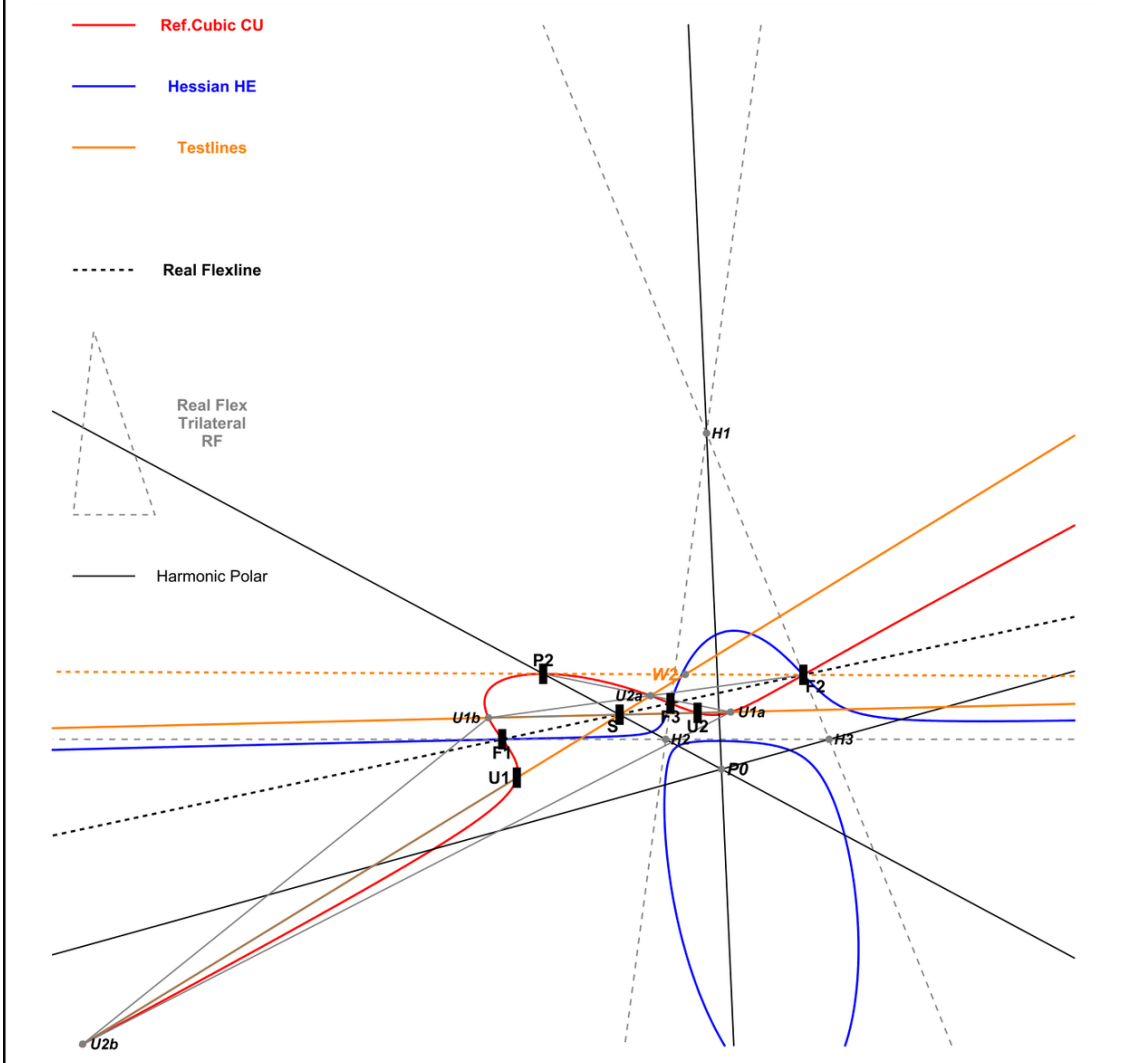
Best regards,
Chris

Setup 1



9P-s-Cu1 Hessian-plus Flexlines-80-Test ES-Flextrilateral.pdf

Setup 2



9P-s-Cu1 Hessian-plus Flexlines-80-Test ES-Flextrilateral.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2745
Date: 2025-08-02
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

thanks a lot for your second drawing,
... which does not confirm my "construction".

I made a further similar to your second
... and similar to my in #2725 with small aberrations,
... but my proving of H1H2H3 is not the best.

These cubics, where you show aberrations,
... are often nearly bipartite with low k.

Thanks once more for your time consuming help.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2746
Date: 2025-08-02
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Always happy to help.
Your perseverance is more stable than the Flexline Trilateral!
Really hope you'll find the solution to this remarkable problem
someday.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2747
Date: 2025-08-06
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

now I can offer a method to find the real flexline triangle
... in the same way for mono- and bipartite cubics,
... using the variable triangle $H_1H_2H_3$ with vertices
... on the harmonic polars L_1, L_2, L_3 of the real flexpoints
... with the flexpoints F_1, F_2, F_3 on their sidelines.

Start with a point X on the cubic (see attached figure)
... and consider two X -tangents, contacting CU in U_1, U_2 ,
... construct a first conic as polar conic of X , bearing U_1, U_2 ,
... and a second conic through U_1, U_2, H_1, H_2, H_3 .

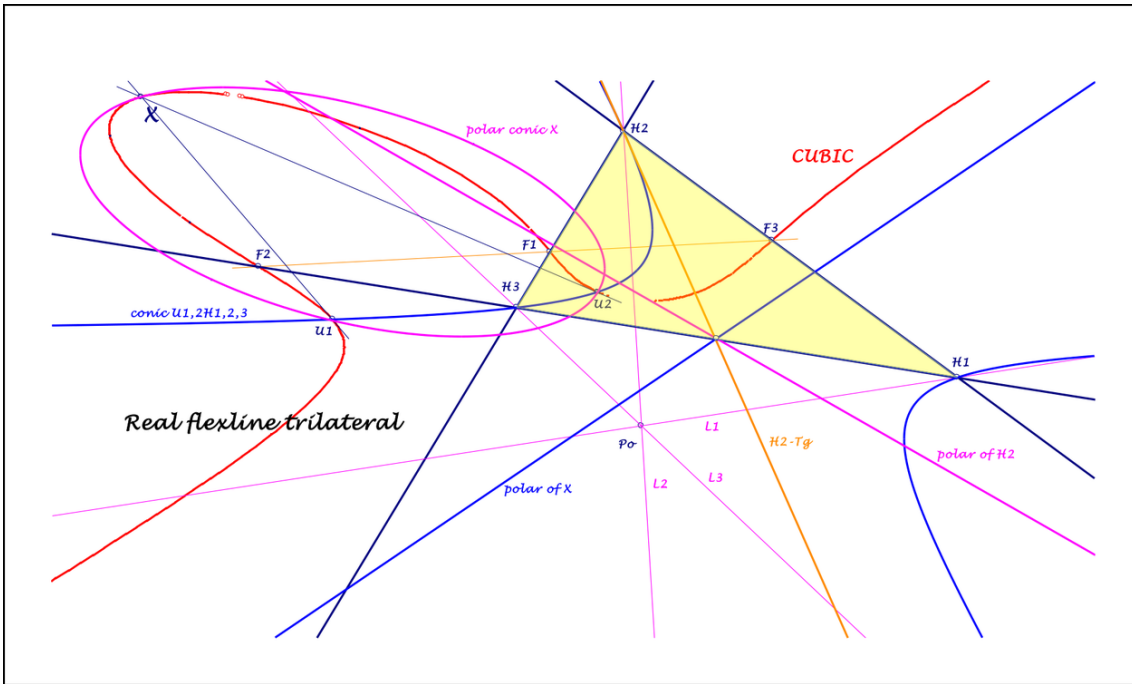
Consider for a fixed H_i the following three lines:

- (1) cubic tangent in H_i ,
- (2) polar of H_i wrt the first conic,
- (3) polar of X wrt the second conic.

Finally you find a constellation of the variable triangle
 $H_1H_2H_3$,
... so that the three lines intersect in a common point on H_jH_k ,
... and $H_1H_2H_3$ will be the real flexline triangle.
I hope someone can confirm this method.

Best regards Eckart

PS: For a bipartite cubic there are four X -tangents,
... it doesn't matter which two you take,
... the two conics are the same.



2025-08-06.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2748
Date: 2025-08-07
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I've checked your ultime construction on my cubics for variing points X in both cases mono- and bipartite. I'm glad to confirm your property. I suppose Chris will agree and possibly prove it analytically. This time, the game is over! Sincere congratulations (you deserve them now).

Best regards

Bernard

PS I think there is a typo in your message : the 1rst line is the tangent in Hi to the 2nd conic (not the cubic tangent)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2749
Date: 2025-08-08
From: eckart_schmidt@t-online.de
Subject: Re Real flexline trilateral

Dear Bernard,

thanks for your confirmation
... and correcting the typo, we have to replace
... in (1) "cubic tangent..." by "tangent to the 2nd conic...",
... it was good that I made a drawing.

You had already earlier indicated ,
... that there would be the same construction
... for mono- and bipartite cubics.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2750
Date: 2025-08-08
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

Dear Eckart,
Perhaps interesting (or obvious?)
The polar of X wrt the 2nd conic is the trilinear polar of X wrt
the triangle $H_1H_2H_3$.
It's intersection with the line of real flexes is therefore the
trilinear pole of the line XP_0 .
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2751
Date: 2025-08-09
From: bernard.keizer@gmail.com
Subject: Re: Re Real flexline trilateral

Dear Eckart,
I checked again by varying the triangle $H'1H'2H'3$!
If I'm not wrong, the property that the polar of X wrt the conic through $H'1, H'2, H'3, U1$ and $U2$ is the trilinear polar of X wrt the triangle $H'1H'2H'3$ holds only for $H'1, H'2$ and $H'3$ in the right place of $H1, H2$ and $H3$.
I hope you will confirm that.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2752
Date: 2025-08-09
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for interest in the drawing of #2747,
... I can confirm your first observation,
... but not the second:
The trilinear poles of lines through X wrt $H1H2H3$
... give a conic, which doesn't bear
... the intersection of $F1F2F3$ and the polar of X
wrt the 2nd conic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2753
Date: 2025-08-09
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

I think your observation in #2750 is not evident,
... and my figure confirms your further observation in #2751,
... so there is a fourth line for a necessary common point
... wrt H_i on $H_j'H_3k'$, to get $H_1H_2H_3$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2754
Date: 2025-08-10
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Sorry, I tried in vain to understand this last message!
You confirm that the polar line of X wrt the 2nd conic is the
trilinear polar of X wrt $H_1H_2H_3$ only for the right triangle.
But what is this 4th line? What are the 3 other lines?

Best regards

Bernard

PS I'm always convinced that these properties should be proved
analytically

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2755
Date: 2025-08-10
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

yes, I confirmed in my figure, that
... "the polar line of X wrt the 2nd conic
... is the trilinear polar of X wrt $H_1H_2H_3$
... only for the right triangle".

Then there are now 5 lines with a common point:

- (1) tangent in H_i to the second conic,
- (2) polar of H_i wrt the first conic,
- (3) polar of X wrt the second conic,
- (4) trilinear polar of X wrt $H_1H_2H_3$,
- (5) H_jH_k .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2756
Date: 2025-08-11
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Wrt H1H2H3 :

The equation of the cubic is

$$a_1^3 X^3 + \dots + k a_1 a_2 a_3 X Y Z = 0$$

(monopartite if $k \geq -3$, bipartite if $k \leq -3$).

The equation of the polar conic of X is

$$X(3a_1^3 x^2 + k a_1 a_2 a_3 y z) + \dots \text{ (1st conic)}$$

The equation of the circumconic to H1H2H3 through U1 and U2 is

$$X y z + Y z x + Z x y = 0 \text{ (2nd conic)}$$

The tangent in H1 to the 2nd conic is

$$Z y + Y z = 0 \text{ (1st line),}$$

which intersects H2H3 (5th line) in the point $(0, Y, -Z)$

The polar of H1 wrt the 1st conic is

$$6a_1^3 X x + k a_1 a_2 a_3 (Y z + Z y) = 0 \text{ (2nd line),}$$

which passes through the point $(0, Y, -Z)$.

The polar of X wrt the 2nd conic is

$$X(Y z + Z y) + \dots = 0, \text{ id est } x Y Z + y X Z + z X Y = 0 \text{ (3rd line),}$$

which passes through the point $(0, Y, -Z)$.

The trilinear polar of X is

$$x/X + y/Y + z/Z = 0 \text{ (4th line), which is the same line as the 3rd.}$$

Best regards

Bernard

PS Can you in particular confirm the equation of the 2nd conic?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2757
Date: 2025-08-11
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart
Any point (X,Y,Z) in the plane defines a unique value of k and a unique cubic of the pencil
The polar conic of X wrt any cubic of the pencil is given by
 $X(3a_1^3x^2 + ka_1a_2a_3yz) + \dots = 3(a_1^3Xx^2 + a_2^3Yy^2 + a_3^3Zz^2) + ka_1a_2a_3(Xyz + Yxz + Zxy) = 0$
The 1st part in brackets is the equation of the polar conic of X wrt the cubic FE (for $k = 0$).
The 2nd part in brackets is the equation of the polar conic of X wrt the cubic RF (for $k = \infty$)
For the k defining the cubic of the pencil, we have your 1st conic, for $k = \infty$, we have your 2nd conic
It is remarkable that the points U_1 and U_2 (in the monopartite case) or U_1, U_2, U_3 and U_4 (in the bipartite case) belong to the polar conics of all the cubics of the pencil
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2758
Date: 2025-08-12
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart
Is the property that the polar conics of a point wrt a pencil of cubics form a pencil of conics well-known or obvious?
Any reference?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2759
Date: 2025-08-13
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

excuse, if I don't react on your messages #2756 and #2757:
... with the background of construction
... I describe cubics as nonpivotal isocubics:
... reference triangle $P_1P_2P_3$
... with P_0 as root and fixpoint of the isoconjugation
... and a given cubic point Q ,
... varying Q , we get a set of cubics (not syzygetic).

I think, Chris will be your partner for the calculations,
... but I don't understand his silence.

Sorry I can't understand the last sentence of #2757,
... if U_i are contact points of tangents
 from a cubic point X to the cubic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2760
Date: 2025-08-13
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Can you please answer my last message 2758?

A pencil of cubics is defined by 9 points (CB partners), not necessary all real.

(I don't know if your construction of not pivotal isocubics wrt $P_1P_2P_3$ with root and fixed point P_0 and varying Q forms a pencil?)

A pencil of conics is defined by 4 points, not necessary all real).

For the syzygetic pencil, the 9 points are the flexes (3 real and 6 imaginary).

For a pencil of circles, 2 of the 4 points are the circular points.

For a point X in the plane, the 2 points U_1 and U_2 are defined as contact points of X to the unique cubic of the pencil passing through X .

These 2 points belong to the polar conics of X to all cubics of the pencil, in particular the cubic passing through X and the cubic formed by the sides of the triangle $H_1H_2H_3$. These 2 conics are your 1st and your 2nd conic.

Best regards

Bernard

PS I can't understand either Chris silence

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2761
Date: 2025-08-15
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

independent of the calculations

... I try in vain to understand your last two messages:

(1) Is it correct, that all cubics through 8 given points

... and their CB-point give a "pencil" of cubics?

(2) Can it be, that the last sentence in #2757

... describes only the wellknown property,

... that U_1, U_2 or U_1, U_2, U_3, U_4 as contact points of tangents

... from a cubic point X to the cubic

... lie on the polar conic of X ?

Excuse my lack of comprehension.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2762
Date: 2025-08-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Thanks for trying to understand and excuse if I'm not clear (I wasn't a teacher)!

1) That is correct, these 9 CB partners (8 +1) define a pencil of cubics

2) Taking a point X in the plane, not among these 9 points, consider the polar conics of X wrt the cubics of the pencil. X belongs to only one cubic of the pencil, which determines the points U1 and U2 or U1,U2,U3 and U4 as contact points of the tangents from X to this cubic.

These points belong to the polar conics of X to all cubics of the pencil, which form a pencil of conics.

I wouldn't be surprised if this property was in Schröter ...

Best regards

Bernard

PS If $F(x,y,z)$ is a homogenous equation of a curve, then equation of the 1rst polar curve of a point (X,Y,Z) wrt this curve is given by

$X\partial F/\partial x + Y\partial F/\partial y + Z\partial F/\partial z = 0$ (see Wikipedia)

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2763
Date: 2025-08-16
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Perhaps a useful precision
As I said, the point X (not one of the 9 CBpartners) is only on one cubic of the pencil
For this cubic, the contact points of the tangents from X are U_1 and U_2 and the polar conic of X passes through X and U_1 and U_2 .
For any other cubic of the pencil, the contact points of the tangents from X are 2 different point V_1 and V_2 and the polar conic of X don't pass through X , but passes through V_1 and V_2 and through U_1 and U_2 .
So the V_1 and V_2 are all different, but the points U_1 and U_2 belong to all polar conics.
Is it clearer this way?
Apparently, you didn't know this property, neither did I, I just discover it by calculation!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2764
Date: 2025-08-16
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Back from holiday, I reviewed your latest method for identifying the real flexline triangle in QPG#2747.

This approach involves two conics:

- * Co1 - the X-Polar Conic, which touches the reference cubic CU in point X through (U1, U2)
- * Co2 - the conic passing through (U1, U2, H1, H2, H3)

For each point H_i (representing H1, H2, H3), we consider three lines:

- * (1) The tangent to Co2 at H_i
- * (2) The polar of H_i with respect to the first conic Co1
- * (3) The polar of X with respect to the second conic Co2

The points H_i are 'gliding' points along the harmonic polars L_i (representing L1, L2, L3), such that the sides of triangle $H_1H_2H_3$ pass through the real flexpoints F1, F2, F3.

When $H_1H_2H_3$ form the vertices of the real flexline trilateral, the three lines (1), (2), and (3) concur at a point lying on one of the sidelines of the trilateral.

I verified this statement using a numerical example in Mathematica, and in all cases, the concurrence held true to at least 14 decimal places. This gives me strong confidence in the validity of the result.

As far as I know, this is the first constructive property of the real flexline trilateral.

Sincere congratulations!

That said, I believe there remains the challenge of finding a precise construction in which H1, H2, H3 arise from intersections of well-defined lines or curves. The notion of 'gliding' along a line lacks geometric precision.

Compare this with the 'gliding' construction for flexpoints you mentioned in QPG#2046:

Start with an arbitrary point F on the cubic and draw three secants,

... yielding 3×2 intersections with the cubic,

... if these six points lie on a conic C0, then F is an inflection point.

[Schröter's book, page 242]

I've been thinking about this further:

* Gliding H_1, H_2, H_3 along L_1, L_2, L_3 produces the three lines (1), (2), and (3), forming a triangle. Each vertex of this triangle defines a locus. Perhaps one of these loci is a line or conic, which could help track the points of concurrence.

* Instead of choosing an arbitrary point X on the reference cubic CU , we might select a specific point—such as one of the points at infinity on CU . In that case, Co_1 becomes the IP-Polar Conic or Diametral Conic. This might open up new directions for exploration.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2765
Date: 2025-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

wrt your last sentences in #2757 and #2758:
I started with 8 points and its CB-point,
... which define a pencil of cubics,
... then I took a point X in the plain,
... which defines a special cubic of the pencil
... and considered U1,U2 as contact points
... of tangents from X to the cubic,
... then I took another cubic of the pencil,
... not bearing X (but the initial 8 points and CB)
... and constructed its polar conic,
... but unexpected not bearing U1,U2 ...
Is there something wrong?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2766
Date: 2025-08-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

excuse, my last message was sent
... before reading your last message...
... I will answer later.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2767
Date: 2025-08-17
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
I feel ashamed!
Unfortunately, it seems you're right and my property doesn't hold, at least for pencils of cubics through 9 CB partners not being the flexpoints.
I keep a last hope for the syzygetic pencils, the 9 CB partners being the flexpoints.
Maybe Chris will be able to help me to understand my barycentric calculations and their interpretation.
With my apologies
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2768
Date: 2025-08-17
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

Thanks for confirmation of the property,
... I observed for the real flexline triangle.
Of course I am aware of the fact,
... that this is no real construction,
... so I already experimented
 in the meaning of your last sentences.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2769
Date: 2025-08-24
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer and all,

Wrt 2) in #2738, I calculated the *locus of S-points* when P lies on a cubic.

Given a quadrangle QA and a point P0, and let

$Cu0 = pK(QA-P16, P0)$ wrt QA-Tr1

$QA-P16 = (p^2 : q^2 : r^2)$

$P0 = (u0:v0:w0)$

$P(u:v:w)$ = a point on $Cu0$

$Cu(P) = pK(QA-P16, P)$ wrt QA-Tr1.

What we should solve are system of equations:

Locus of P0 (= Cu0)

$$u0u[(rv)^2 - (qw)^2] + v0v[(pw)^2 - (ru)^2] + w0w[(qu)^2 - (pv)^2] = 0$$

Polar conic of P wrt Cu(P)

$$[(rv)^2 - (qw)^2]x^2 + [(pw)^2 - (ru)^2]y^2 + [(qu)^2 - (pv)^2]z^2 = 0$$

Polar conic of QA-Tf2(P) wrt Cu(P)

$$(p^2)u[(rv)^2 - (qw)^2]yz + (q^2)v[(pw)^2 - (ru)^2]zx + (r^2)w[(qu)^2 - (pv)^2]xy = 0.$$

By introducing

$$X = rx/pz, Y = ry/qz$$

$$U = ru/pw, V = rv/qw, U_0 = ru_0/pw_0, V_0 = rv_0/qw_0,$$

we get

$$(1) (V^2 - 1)(U_0U - 1) - (U^2 - 1)(V_0V - 1) = 0$$

$$(2) (V^2 - 1)(X^2 - 1) - (U^2 - 1)(Y^2 - 1) = 0$$

$$(3) (V^2 - 1)(U/X - 1) - (U^2 - 1)(V/Y - 1) = 0.$$

For having solutions other than $U^2 = 1$ and $V^2 = 1$,

$$(4) (X^2 - 1)(V_0V - 1) - (Y^2 - 1)(U_0U - 1) = 0$$

$$(5) (X^2 - 1)(V/Y - 1) - (Y^2 - 1)(U/X - 1) = 0,$$

which give

$$(6) U = (V_0/Y - 1)X(X^2 - Y^2)/(Y^2 - 1)(U_0X - V_0Y)$$

$$(7) V = (U_0/X - 1)Y(X^2 - Y^2)/(X^2 - 1)(U_0X - V_0Y).$$

Substituting (6),(7) into (2) yields

$$(8) [(U_0X - 1)^2 (Y^2 - 1)^2 - (V_0Y - 1)^2 (X^2 - 1)^2](X^2 - Y^2) - [(U_0X - 1)(Y^2 - 1) - (V_0Y - 1)(X^2 - 1)]^2 = 0.$$

Then we obtain

$$(9) (U_0X - 1)(Y^2 - 1) - (V_0Y - 1)(X^2 - 1) = 0,$$

which is the Locus of P_0 , and

$$(10) (U_0X - 1)(Y^2 - 1)(X^2 - Y^2 - 1) + (V_0Y - 1)(X^2 - 1)(X^2 - Y^2 + 1) = 0,$$

which is the Locus of S-points.

In DT-coordinates, (10) becomes

$$(u_0x/p^2)[(y/q)^2 - (z/r)^2][(x/p)^2 - (y/q)^2 - (z/r)^2] + \text{cyclic permutations} = 0.$$

It bears the vertices of QA, QA-Tr1 and QA-Tf2(P_0).

I checked numerically some S-points lie on this curve when $Cu(P) = QA-Cu_{2,3,4,5}$,

but I'm not sure my result is correct because I can't draw quintics (or even cubics).

Best regards,

M@IMF

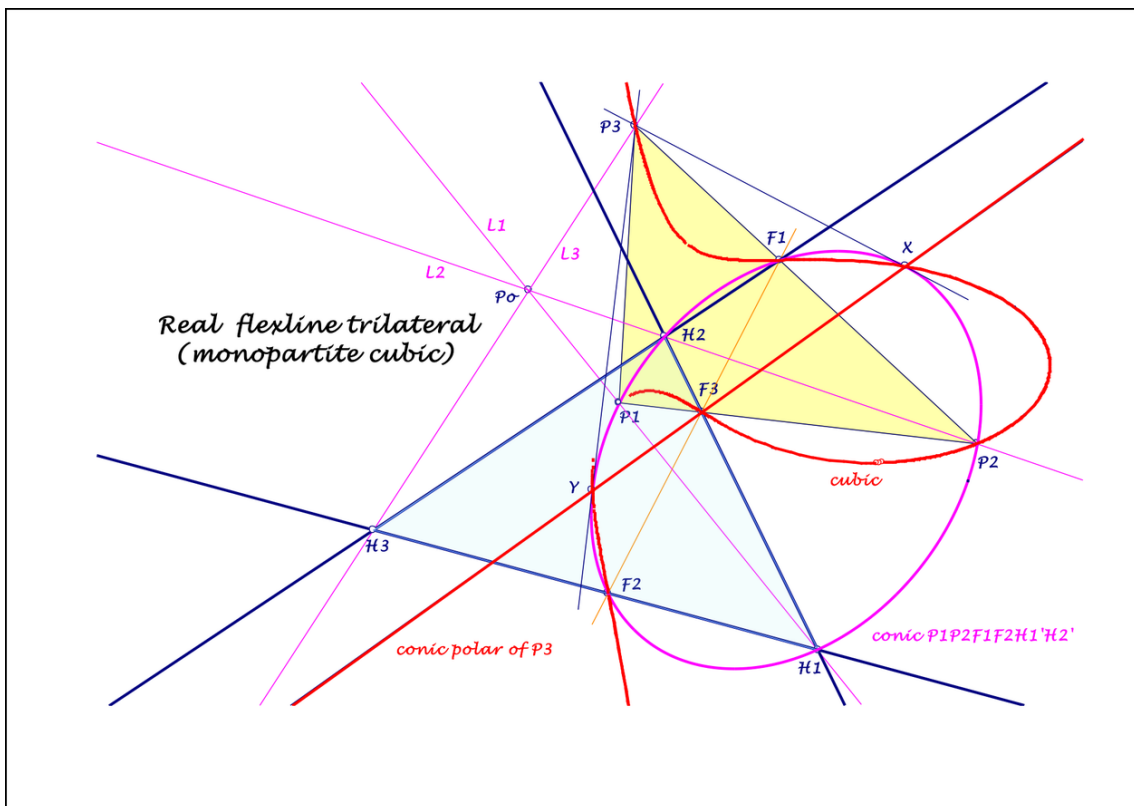
Message: #2770
Date: 2025-08-26
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

perhaps no more of interest, but:
 ... I would be glad, if you can confirm the following method
 ... to find the real flexline trilateral
 for a monopartite cubic,
 ... using the variable triangle $H_1'H_2'H_3'$ as in #2747:
 Consider the conic through $P_1, P_2, F_1, F_2, H_1', H_2'$,
 ... which intersects the cubic in two further points X, Y .
 If the line XY is the conic-polar of P_3 ,
 ... you have the real flexline trilateral $H_1H_2H_3 = H_1'H_2'H_3'$.

Best regards Eckart

PS. It seems, that the points X, Y doesn't exist in the bipartite case???



2025-08-26.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2771
Date: 2025-08-27
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

The PS in my last message is not correct!
Excuse.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2772
Date: 2025-08-29
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

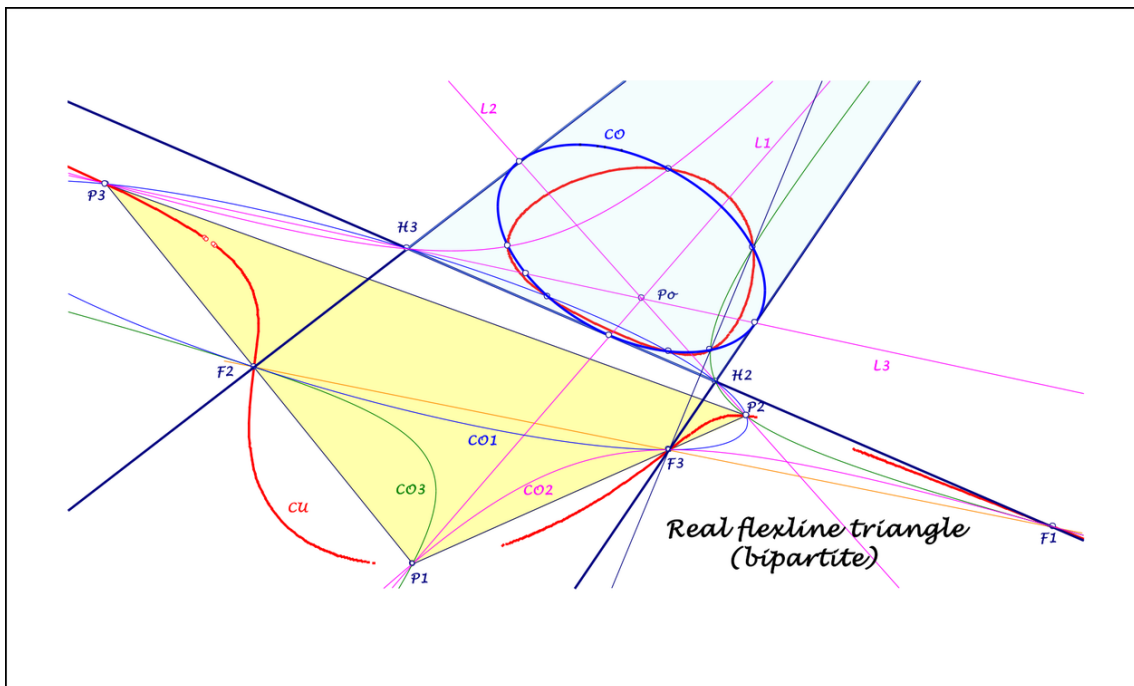
perhaps a new method to find
 ... the real flexline triangle for a bipartite cubic,
 ... using the variable triangle $H_1'H_2'H_3'$ as in #2747:

Let P_1, P_2, P_3 be the intersections of L_1, L_2, L_3
 ... and the infinity part of the bipartite cubic CU .

Draw the three conics CO_k through $P_i P_j F_i F_j H_i' H_j'$,
 ... each with two further CU -intersections,
 ... which are coconic on a conic CO .

If CO is inscribed $H_1'H_2'H_3'$
 ... with contact points in $L_k \wedge H_i' H_j'$,
 ... then $H_1'H_2'H_3' = H_1 H_2 H_3$.

Best regards Eckart



2025-08-29.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2773
Date: 2025-08-31
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF

Many thanks for your interest and congratulations for your calculations.

This quintic derived from a cubic is very interesting!
In particular, if $P_0 = QA-P16$ (pole = pivot) and C_u the corresponding C_{ux} (unfortunately not in EQF),
for $P = QA-P16$ and $QA-Tf2(P) = QA-P10$, you should find the S-points as vertices of QL-Tr2 (where QL is the dual of QA) belonging to your corresponding quintic.

I suppose it is also possible to find the quintic for a well-known cubic such as Mac Cay cubic for example (it is a pivotal isogonal cubic wrt a triangle ABC with pivot the circumcenter, isopivot the orthocenter and fixed points the in- and excenters).

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2774
Date: 2025-08-31
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

Thank you for the message.

Let me correct #2769:

"Locus of P0" must be replaced by "Locus of P".

As I wrote in #2739,

I'm interested in QLx-Tr2 where QLx is formed by tangents to a circumconic of convex QA0 at its vertices.

I realized that the locus of QLx-P13 is QA0-Tf2 of Steiner inellipse of QA0-Tr1.

But I can't draw the quartic, either.

By the way, wrt PS in #2743, do you mean like this?

The equations of the 2 polar conics (cf. (2),(3) in #2769) are

$$(V^2 - 1)X^2 + (1 - U^2)Y^2 + (U^2 - V^2) = 0$$

$$(V^2 - 1)U/X + (1 - U^2)V/Y + (U^2 - V^2) = 0,$$

from which we can get

$$(U^2 - V^2)X^4 + 2(V^2 - 1)UX^3 + (1 - U^2)(U^2 + V^2 + 1)X^2 + 2(U^2 - V^2)UX + (V^2 - 1)U^2 = 0.$$

Dividing by (X-U) gives

$$(U^2 - V^2)X^3 + (U^2 + V^2 - 2)UX^2 + (1 + V^2 - 2U^2)X + (1 - V^2)U = 0,$$

and its solutions are

$$X_k = -A/3 - D\omega^k - E/\omega^k \quad (k=1,2,3),$$

where

$$A = (U^2 + V^2 - 2)U/(U^2 - V^2)$$

$$D = \{C'/2 + \sqrt{[(C'/2)^2 + (B'/3)^2]\}^{(1/3)}$$

$$E = (-B'/3)/D$$

$$B' = B - (A^2)/3$$

$$C' = C - BA/3 + 2(A/3)^3$$

$$B = (1 + V^2 - 2U^2)/(U^2 - V^2)$$

$$C = (1 - V^2)U/(U^2 - V^2)$$

ω = cube root of 1.

Then *S-points* are (pXk : qYk : r), where

$$Y_k = V(U^2 - 1)/[(V^2 - 1)U/X_k + (U^2 - V^2)].$$

Best regards,

M@IMF

Message: #2775
Date: 2025-09-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
I'm rather confused now!
Reading your message 2769 again, I understand:
* a certain pivotal isocubic wrt QA-Tr1 with pivot P_0 and fixed points the vertices of QA $Cu_0 = pK(QA-P16, P_0)$
* P is a point on Cu_0 and $Cu(P) = pK(QA-P16, P)$
* the S-points are the intersections other than $P^* = QA-Tf2(P)$ of the polar conics of P and P^* wrt $Cu(P)$

Is this correct?
Then you take as example QA-Cu2,3,4,5
But the pivots of these cubics (your points P) are not on the same cubic!
It seems you have to choose:
* you start with a QA, it's QA-Tr1 and QA-Tf2 and take P random in the plane (then $Cu(P)$ is defined and gives 3 S-points)
* you start with a bipartite cubic and take P on the cubic, but P^* is the tangential of P , giving 3 S-points for each point P on the cubic
(a bipartite cubic is a pivotal isocubic in each of it's points, but the QA-Tf2 are all different ...)
In the 1rst case, the locus of the S-points refers to a particular QA, in the 2nd case, it refers to a cubic.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2776
Date: 2025-09-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,
... my conclusion in message 2772 doesn't hold,
... excuse, but it is difficult for me,
... to find a drawing or calculation
 with significant aberrations.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2777
Date: 2025-09-01
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Sorry, please read The S-points are the intersections other than P of the polar conics of P and $P^* = QA-Tf2(P)$ wrt $Cu(P)$!
With your definition, each cubic $Cu(P)$ has a twin cubic $Cu(P^*)$,
the polar conic of P^* wrt $Cu(P)$ is the same as the polar conic
of P wrt $Cu(P^*)$.
This conic carries 3 S-points for $Cu(P)$ and 3 other for $Cu(P^*)$.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2778
Date: 2025-09-01
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

I'm very sorry to confuse you.
I have to correct the last part of #2769:
"when $Cu(P) = QA-Cu_{2,3,4,5}$ "
must be replaced by "when $Cu_0 = QA-Cu_{2,3,4,5}$ ".

What I calculated is 1st case in #2775 (2nd case in #2738), but P is taken on a cubic (Cu_0) not whole plane.
I think S -points of P wrt QA are the intersections (except P) of the polar conic of P wrt $Cu(P)$ and the polar conic of $QA-Tf_2(P)$ wrt $Cu(P)$, where $Cu(P)$ is $pK(QA-P_{16}, P)$ wrt $QA-Tr_1$.

I checked the expression of (10) in #2769 as follows (in the case of $Cu_0 = QA-Cu_2$):

0. Given QA .
1. Let $P_0 = QA-P_5$, which gives U_0 and V_0 .
2. Let $P = QA-P_1$, which gives U and V .
3. Solve (2),(3) as shown #2774, which gives X_k and Y_k .
4. Substitute U_0, V_0, X_k, Y_k into the left side of (10).
If it is 0 within the numerical error, (10) is valid.
Similarly let $P = QA-P_5, P_{17}, P_{20}$.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2779
Date: 2025-09-02
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Thanks for this clarification!
In fact, your calculation refers to 1st case 2775 or 2nd case 2738 with a given QA and variables Cu(P) with P on a certain cubic Cu0.
You take as Cu0 QA-Tf2 invariant cubics like QA-Cu2 and the like.
I suppose you could also choose as locus of P a line (then P* describes a conic circum QA-Tr1) or a conic or any other cubic ...
I agree that P cannot be chosen random in the plane.
The 2nd case 2775 or 1st case 2738 remains open (this is the case used by Schröter in order to draw the hessian of a cubic).
Beautiful work even without possibility of drawing your curve!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2780
Date: 2025-09-03
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
I made several drawings with P on P10P16 or P1P20 and the locus of the S-points looks in fact like a cubic ...
Perhaps you will be able to confirm that by calculation?
Best regards
Bernard
PS With your definition, a point P is always associated with 3 S-points, for example P16 with the vertices of QL-Tr2
The locus of the S-points for P on any line or conic or curve through this point contains these 3 particular S-points

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2781
Date: 2025-09-04
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

When P lies on a line, the system of equations are

$$(1') AU + BV + C = 0$$

$$(2) (V^2 - 1)(X^2 - 1) - (U^2 - 1)(Y^2 - 1) = 0$$

$$(3) (V^2 - 1)(U/X - 1) - (U^2 - 1)(V/Y - 1) = 0.$$

where A,B,C are constants.

(5) in #2769 and (1') gives

$$(6') U = [BY(Y^2 - X^2) - C(X^2 - 1)]X / [AX(X^2 - 1) + BY(Y^2 - 1)]$$

$$(7') V = [AX(X^2 - Y^2) - C(Y^2 - 1)]Y / [AX(X^2 - 1) + BY(Y^2 - 1)]$$

Substituting (6'),(7') into (2) yields

$$(8') [BY(Y^2 - 1) - (BY + C)X^2 - AX]^2 - (X^2)(AX + BY + C)^2 = [AX(X^2 - 1) - (AX + C)Y^2 - BY]^2 - (Y^2)(AX + BY + C)^2.$$

Then we obtain

$$(9') AX + BY + C = 0,$$

which is the locus of P, and

$$(10') AX(X^2 - Y^2 - 1) + BY(Y^2 - X^2 - 1) - C(X^2 + Y^2 - 1) = 0,$$

which is the locus of S-points.

In DT-coordinates, (10') becomes

$$A(x/p)[(x/p)^2 - (y/q)^2 - (z/r)^2] + \text{cyclic permutaions} = 0.$$

Although I can't check the validity of this equation yet, you will do graphically.

For P10P16,

$$A/p = q^2 - r^2$$

$$B/q = r^2 - p^2$$

$$C/r = p^2 - q^2.$$

For P1P20(QA-L3),

$$A/p = (q^2 - r^2)(-p^2 + q^2 + r^2)$$

$$B/q = (r^2 - p^2)(-q^2 + r^2 + p^2)$$

$$C/r = (p^2 - q^2)(-r^2 + p^2 + q^2).$$

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2782
Date: 2025-09-04
From: bernard.keizer@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear M@IMF ,
Many thanks for these calculations ...
Interesting perhaps : the QA-Tf2 of the S-points/vertices of
QL-Tr2 are on the line P10P16!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2783
Date: 2025-09-04
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear Mr. Keizer,

If I don't misunderstand, S-points of P wrt QA lie on QA-Tf2 of line PP*, which is the circumconic of QA-Tr1 that bears P and P*(QA-Tf2 of P).

Yes, polar conic of P* wrt pK(QA-P16, P). (So do S'-points.)

Wrt 2nd case in #2775 (1st case in #2738):

Given a quadrangle QA0 and a point P0, and let

$$Cu0 = pK(QA0-P16, P0) \text{ wrt } QA0-Tr1$$

$$QA0-P16 = (p0^2 : q0^2 : r0^2)$$

$$P0 = (u0:v0:w0)$$

$$P'(u:v:w) = \text{a point on } Cu0.$$

Note that

$$Cu0 = pK(QA'-P16, P') \text{ wrt } QA'-Tr1$$

$$P' \sim (\text{the tangential of } P' \text{ wrt } Cu0) = QA'-Tf2(P'),$$

where QA' is formed by the pretangentials of P' wrt Cu0 (cf. #2742).

Then the system of equations are

* Locus of P* ' (= Cu0)

$$u0u[(r0v)^2 - (q0w)^2] + v0v[(p0w)^2 - (r0u)^2] + w0w[(q0u)^2 - (p0v)^2] = 0$$

* Polar conic of P' wrt Cu0*

$$[(r0^2)v0v - (q0^2)w0w]x^2 - 2(p0^2)(v0w - w0v)yz + \text{cyclic permutations} = 0$$

* Polar conic of P'~ wrt Cu0*

$$[(r0^2)v0\eta - (q0^2)w0\zeta]x^2 - 2(p0^2)(v0\zeta - w0\eta)yz + \text{cyclic permutations} = 0,$$

where ($\xi:\eta:\zeta$) is the DT0-coordinate of P'~, which I can't obtain yet.

(Note that these polar conics are not circumscribed to QA0 nor QA0-Tr1.)

To solve these equations will be much harder than the previous case.

Best regards,
M@IMF

Message: #2784
Date: 2025-09-05
From: van10hoven@gmail.com
Subject: Update on the new Website

Dear friends,

As announced last year, I've been moving the encyclopedias to a new platform, since the old one could no longer keep up with current technology. At the same time, I'm merging the Encyclopedia of Quadri-Figures (EQF) , the Encyclopedia of Polygon Geometry , and recent work on cubics into a brand-new Encyclopedia of Poly Geometry (EPG). The term "poly" reflects both the versatility of n points and n lines, and the study of n -degree curves. Old links to the former encyclopedias should continue to work. The good news: all pages are now transferred into the new format, and the new cubic material has been added. That's a milestone worth sharing!

This has been quite a big job, so thanks for your patience if I haven't always been as active in QPG discussions. I expect the new website to be ready and online in some more months.

Best,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2785
Date: 2025-09-23
From: contiwa.goma3@gmail.com
Subject: Re: Simple definition of QL-Tr2

Dear all,

Although someone might have already calculated,
here is *DT* -coordinates of the vertices of *QL-Tr2* :
 $((\alpha + 2\text{Re}[(\omega^k)\beta_1])/l^2 : -(\alpha + 2\text{Re}[(\omega^k)\beta_2])/m^2 : 3(l^2 - m^2))$ ($k=1,2,3$),

where

$$\alpha = (l^2 + m^2)n^2 - 2(lm)^2$$

$$\beta_1 = [(\delta^2 - 9\varepsilon l^4) + i (3\sqrt{3})(l^2)(l^2 - m^2)(l^2 - n^2)\delta]^{(1/3)}$$

$$\beta_2 = [(\delta^2 - 9\varepsilon m^4) + i (3\sqrt{3})(m^2)(m^2 - l^2)(m^2 - n^2)\delta]^{(1/3)}$$

$$\delta^2 = [(mn)^2 + (nl)^2 + (lm)^2]^3 - 27(lmn)^4$$

$$\varepsilon = (mn)^4 + (nl)^4 + (lm)^4 - (l^2 + m^2 + n^2)(lmn)^2$$

ω = cube root of 1

i = imaginary unit

$\text{Re}[z]$ = real part of z .

Correction in #2774 (

<https://groups.io/g/Quadri-and-Poly-Geometry/message/2774>) :

$$D = \{C'/2 + \sqrt{[(C'/2)^2 + (B'/3)^3]}\}^{(1/3)}$$

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2786
Date: 2025-10-13
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

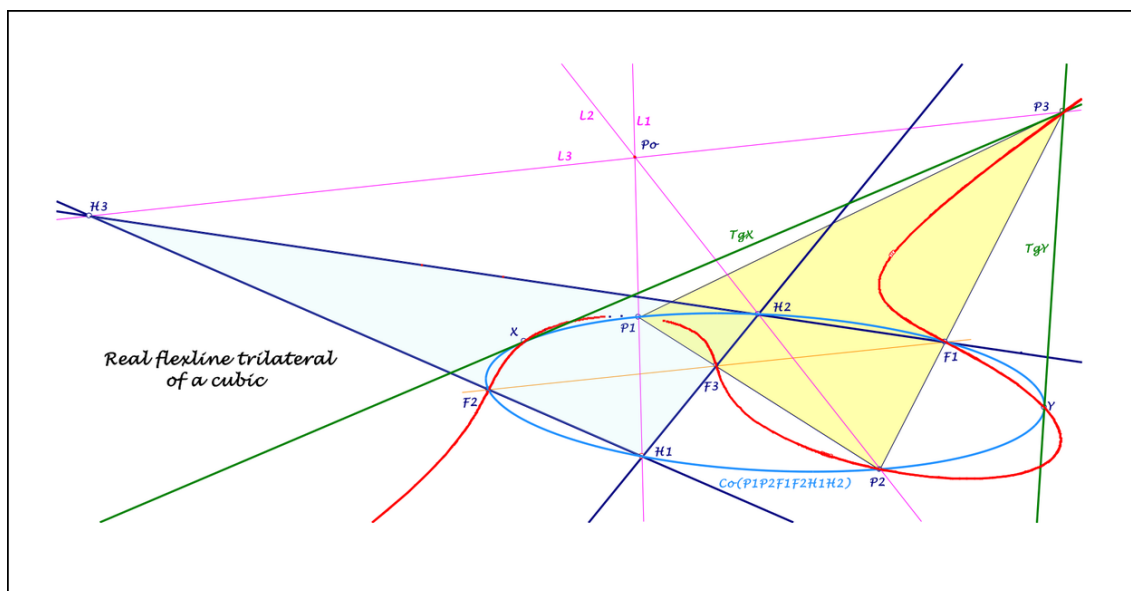
Dear Bernard, dear Chris,

our group seems dead, nevertheless, I think
... the following is a further method to find the real flexline
trilateral:

The cubic attached is drawn as isocubic
... reference triangle $P_1P_2P_3$,
... isoconjugation with fixpoint P_o and root P_o ,
... bearing a given point
... and using a variable triangle $H_1H_2H_3$ as in #2747.

Start with a conic through $P_iP_jF_iF_jH_iH_j$,
... intersecting the cubic in two further points X and Y ,
... if their conic-tangents TgX and TgY intersect in P_k
... you have the searched real flexline trilateral $H_1H_2H_3$.

Best regards Eckart



2025-10-13.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2787
Date: 2025-10-14
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

excuse, it seems that my last message #2786
... doesn't hold for bipartite cubics.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2788
Date: 2025-10-14
From: contiwa.goma3@gmail.com
Subject: 4 concurrent circles

Dear geometers,

Do I understand correctly?

Given 4 circles C_1, C_2, C_3, C_4 concurrent at P .

Let

C_{ij} = the second intersection point of C_i and C_j

$D_{kl} = C_{ij}$

D_l = circle through D_{il}, D_{jl}, D_{kl} ,

where $\{i, j, k, l\} = \{1, 2, 3, 4\}$.

Then 4 circles D_1, D_2, D_3, D_4 intersect at a common point Q .

(Clifford's Circle Theorem)

Let

CSC_1 = Moebius conjugate swapping C_i and D_i ($i=1, 2, 3, 4$)

F = the center of CSC_1

CSC_p = Moebius conjugate centered in P , swapping F and Q

CSC_q = Moebius conjugate centered in Q , swapping F and P

$QL_1 = CSC_p(\{C_1, C_2, C_3, C_4\}) = CSC_q(\{D_1, D_2, D_3, D_4\})$,

then $CSC_1 = QL_1 - Tf_1$ ($F = QL_1 - P_1$).

When 3 lines $C_{14}D_{14}$, $C_{24}D_{24}$, $C_{34}D_{34}$ are concurrent at E ,

P and Q lie on $QL_1 - Qu_3$.

(P and $CSC_p(E)$, Q and $CSC_q(E)$ are CSC_{diag} (<https://groups.io/g/Quadri-and-Poly-Geometry/topic/78474426#msg527>) partners, respectively.)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2789
Date: 2025-10-16
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard, dear Chris,

I found also significant aberrations
... wrt the result in #2786 for monopartite cubics,
... please excuse and forget my overhasty message.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2790
Date: 2025-10-18
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

So good to see you're still chasing improvements to the
Flextrilateral construction.
Hats off for sticking with it. Your determination and geometric
tenacity are impressive.
Keep going – the breakthrough feels close.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2791
Date: 2025-10-19
From: van10hoven@gmail.com
Subject: Re: 4 concurrent circles

Dear M@IMF,

Thanks for your construction involving 4 circles converging at a single common point.

You say

CSC1 = Moebius conjugate swapping C_i and D_i ($i=1,2,3,4$)

F = the center of CSC1

How do you determine this conjugate?

Best regards,
Chris

p.s. Your reference to Clifford's Circle Theorem has inspired me to explore some special applications in Poly Geometry. I'll address that in a separate thread.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2792
Date: 2025-10-19
From: contiwa.goma3@gmail.com
Subject: Re: 4 concurrent circles

Dear Chris,

Thank you for the reply.

First, note that triangles $CilCjlCkl$ and $DilDjldkl$ are circumcyclogologic (<https://groups.io/g/euclid/message/790>), †)

i.e. their cyclogologic centers are on the circumcircles of them:

P = cyclogologic center of $CilCjlCkl$ wrt $DilDjldkl$

Q = cyclogologic center of $DilDjldkl$ wrt $CilCjlCkl$,

where $\{i,j,k,l\} = \{1,2,3,4\}$. (There are 4 pairs.)

3 (circum)cyclogologic triagnles and Moebius conjugates are already mentioned:

QFG#997 (<https://groups.io/g/Quadri-Figures-Group/message/17437>)
(3 cyclogologies of Diagonal Triangle, Miquel Triangle and Triple T) and previous/next

QFG#1989 (https://groups.io/g/Quadri-Figures-Group/topic/7147080_2#msg18432) (Generalized Cyclogologic Center Chain in ...)

QFG#2000

(<https://groups.io/g/Quadri-Figures-Group/message/18443>)
(QA-Tr-4).

By the way, I'm not sure the meaning of "this Cyclogologic relation isn't always reciprocal" (QA-Tr-4 in EQF).

Best regards,
M@IMF

p.s. Other than $nP-e-Tf_{1,2}$ and $nP-o-Tf_{1,2,3,4}$?

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

†) Editorial note: The referenced message(s) can be found as listed in [EPG-References \[72\]](#).

Message: #2793
Date: 2025-10-19
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

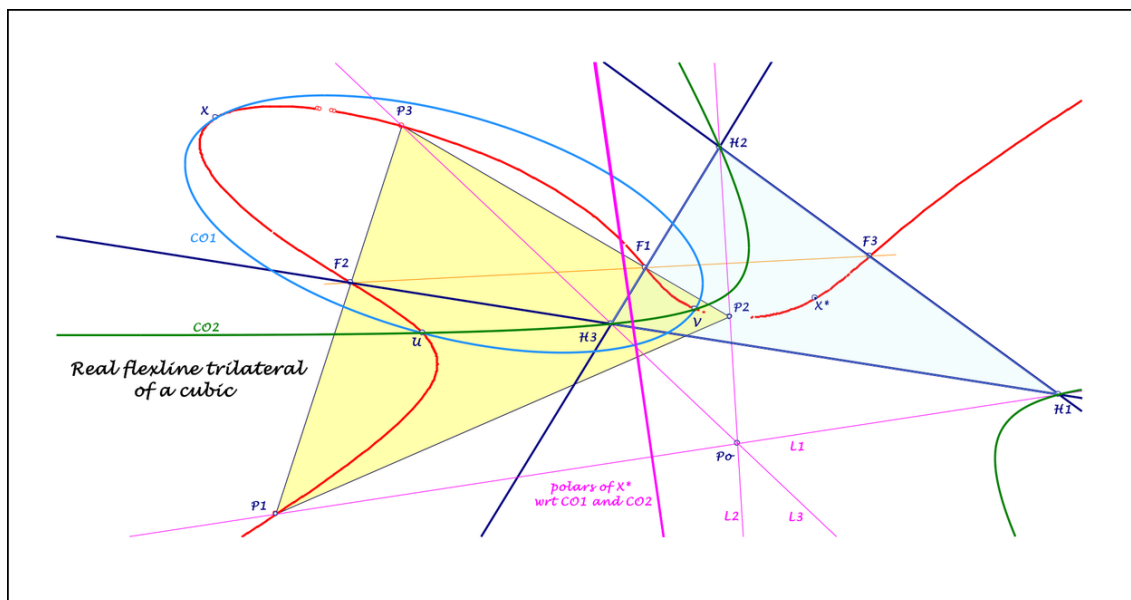
Dear Bernard, dear Chris,

perhaps another method to find the real flexline trilateral:
 The cubic attached is drawn as nonpivotal isocubic
 ... reference triangle $P_1P_2P_3$,
 ... isoconjugation $*$ with fixpoint P_o and root P_o ,
 ... bearing a given point Q
 ... and using a variable triangle $H_1H_2H_3$ as in #2747.

Consider the polar conic C_{01} of a cubic point X ,
 ... intersecting the cubic in U, V (on the infinite part),
 ... and the conic C_{02} through H_1, H_2, H_3, U, V ,
 ... if the polars of X^* wrt C_{01} and C_{02} coincide,
 ... you get the real flexline trilateral $H_1H_2H_3$,
 ... controlled for mono- and bipartite cubics
 ... in 4 examples without significant aberrations.

Best regards Eckart

PS: Thanks to Chris for his encouraging mail.



2025-10-19.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2794
Date: 2025-10-20
From: contiwa.goma3@gmail.com
Subject: Re: 4 concurrent circles

Dear Chris and all,

It seems that I was overhasty. Let me explain once again.
(Circumcyclogic is OK. I should have dealt with 3 triangles carefully.)

I first considered inversion not Moebius conjugate.

Let

InvP = inversion wrt a circle with center P through Q

InvQ = inversion wrt a circle with center Q through P,

then

InvP({C1,C2,C3,C4}) = QL2 (quadrilateral with Miquel point Q)

InvP({D1,D2,D3,D4}) = circumcircles of component triangles
of QL2

InvQ({D1,D2,D3,D4}) = QL3 (quadrilateral with Miquel point P)

InvQ({C1,C2,C3,C4}) = circumcircles of component triangles
of QL3.

There are two QLS. But if we use CSCp and CSCq, we have QL1
only.

Moreover CSCp*CSCq (centered in F) swaps not only P and Q,
but also Cj and Dj (j=1,2,3,4). This is CSC1(= QL1-Tf1).

Let's define Ek4 = CSCp(Dk4) = CSCq(Ck4) (k=1,2,3).

When triangles C14C24C34 and D14D24D34 are QA-Triple Triangles
of QL-P1 and QG-P5, respectively,

P = QA-P9, Q = QA-P3, F = QA-P4, CSC1 = QA-Tf4

CSC1 swaps QL-Ci3 and QG-Ci4

Ek4 is the vertice of QA-Triple Triangle of QG-Px

(<https://groups.io/g/Quadri-Figures-Group/attachment/18437/0/2016-10-16.pdf>).

If we express this as $\langle QG-Px \rangle = \{\{QL-P1, QG-P5\}\}$, then

$\langle QG-Px \rangle = \{\{QL-P1, QG-P5\}\} = \{\{QG-P5, QL-P1\}\}$

$\langle QL-P1 \rangle = \{\{QG-P5, QG-Px\}\} = \{\{QG-Px, QG-P5\}\}$

$\langle QG-P5 \rangle = \{\{QG-Px, QL-P1\}\} = \{\{QL-P1, QG-Px\}\}$ hold.

(Note that F is cyclologic center of E14E24E34 wrt C14C24C34 and D14D24D34.)

Maybe this is true for 3 triangles which are pairwise
circumcyclogic and perspective (i.e. generalized cyclologic?).
By the way, QFG#1989 is not about circumcyclogic. Sorry.

An counterexample is QA-Triple Triangles of QL-P1, QG-P9, QG-Py.

(P = QA-P9, Q = QA-P1, QG-Py = CSCp(QG-P9) = CSCq(QL-P1))

In this case, $\langle QG-Py \rangle = \{\{QL-P1, QG-P9\}\} = \{\{QG-P9, QL-P1\}\}$,

but $\{\{QG-P9, QG-Py\}\}$ is not $\langle QL-P1 \rangle$. $\{\{QL-P1, QG-Py\}\}$ is not $\langle QG-P9 \rangle$, either.

Best regards,
M@IMF

p.s. QA-Triple Triangle of QG-Py is similar to Trk in #2709 (QA and Complex Quartic Eq. (<https://groups.io/g/Quadri-and-Poly-Geometry/topic/113684476#msg2711>))
I'm wondering what property F has.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2795
Date: 2025-10-30
From: van10hoven@gmail.com
Subject: Re: 4 concurrent circles

Dear M@IMF,

Thank you for your explanation.
As promised, I took a closer look at Clifford's Circle Theorem. However, it didn't yield the insights I was hoping for, so I've decided to set it aside.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2796
Date: 2025-10-30
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I've looked over your latest construction. I haven't checked it algebraically, as that would take quite a bit of time. Your approach actually resembles earlier ones, working with a variable triangle $H_1H_2H_3$ whose sides pass through F_1, F_2, F_3 . It remains somewhat tricky to arrive at a precise construction.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2797
Date: 2025-10-30
From: contiwa.goma3@gmail.com
Subject: Desmic System / Reye Configuration

Dear all,

I drew some stereograms of desmic systems. A desmic system (https://en.wikipedia.org/wiki/Desmic_system) is formed by 3 tetrahedra.

(Orthogonal projection of 3 tetrahedra onto a plane are 3 quadrangles.

Please see QA-Tr-1 (<https://www.chrisvantienhoven.nl/qa-items/qa-triangles/qa-tr-1>) of EQF.)

Let the vertices of the 3 tetrahedra be $\{X1, X2, X3, X4\}$, $\{Y1, Y2, Y3, Y4\}$, $\{Z1, Z2, Z3, Z4\}$, and

$$\text{Tr}X4 = \{X1, X2, X3\}$$

$$\text{Tr}Y4 = \{Y1, Y2, Y3\}$$

$$\text{Tr}Z4 = \{Z1, Z2, Z3\}.$$

When $\text{Tr}X4$, $\text{Tr}Y4$, $\text{Tr}Z4$ are pairwise perspective, the perspector of $\text{Tr}Y4$ and $\text{Tr}Z4$,

$\text{Tr}Z4$ and $\text{Tr}X4$, $\text{Tr}X4$ and $\text{Tr}Y4$ are $X4$, $Y4$, $Z4$, respectively. ($X4$, $Y4$, $Z4$ are collinear.)

These 3 triangles have a common perspectrix, which is the intersection of 3 planes determined by the 3 triangles.

There are such 16 perspectrices and they form another Reye configuration, that is "transpose Reye" or "hidden Desmic" (QFG#2039 (https://groups.io/g/Quadri-Figures-Group/topic/71471_025#msg18482)).

12 points of this configuration and the 3 tetrahedra form (24_3, 18_4) configuration.

Best regards,

M@IMF

p.s. 3 tetrahedra in my first figure look similar to 3 QAs of QFG#2025

(<https://groups.io/g/Quadri-Figures-Group/message/18468>) rather than QFG#2041.

Desmic System of 3 Tetrahedra

M@IMF

Let $\text{Tet}X = \{X1, X2, X3, X4\}$, $\text{Tet}Y = \{Y1, Y2, Y3, Y4\}$, $\text{Tet}Z = \{Z1, Z2, Z3, Z4\}$.
 3 tetrahedra $\text{Tet}X$, $\text{Tet}Y$, $\text{Tet}Z$ which satisfy below table form a desmic system.

Collinear points (*)	Pairwise perspective triangles			Perspectrix
X1,Y1,Z4	A1,A2,D3	P2,Q2,R1	X4X3X2, Y4Y3Y2, Z1Z2Z3	U4,V3,W3
X1,Y2,Z3	A1,B2,C3	P2,Q3,R4	X4X3X2, Y3Y4Y1, Z2Z1Z4	U1,V2,W3
X1,Y3,Z2	A1,C2,B3	P2,Q4,R3	X4X3X2, Y2Y1Y4, Z3Z4Z1	U1,V3,W2
X1,Y4,Z1	A1,D2,A3	P2,Q1,R2	X4X3X2, Y1Y2Y3, Z4Z3Z2	U4,V2,W2
X2,Y1,Z3	B1,A2,C3	P3,Q2,R4	X3X4X1, Y4Y3Y2, Z2Z1Z4	U2,V1,W3
X2,Y2,Z4	B1,B2,D3	P3,Q3,R1	X3X4X1, Y3Y4Y1, Z1Z2Z3	U3,V4,W3
X2,Y3,Z1	B1,C2,A3	P3,Q4,R2	X3X4X1, Y2Y1Y4, Z4Z3Z2	U3,V1,W2
X2,Y4,Z2	B1,D2,B3	P3,Q1,R3	X3X4X1, Y1Y2Y3, Z3Z4Z1	U2,V4,W2
X3,Y1,Z2	C1,A2,B3	P4,Q2,R3	X2X1X4, Y4Y3Y2, Z3Z4Z1	U2,V3,W1
X3,Y2,Z1	C1,B2,A3	P4,Q3,R2	X2X1X4, Y3Y4Y1, Z4Z3Z2	U3,V2,W1
X3,Y3,Z4	C1,C2,D3	P4,Q4,R1	X2X1X4, Y2Y1Y4, Z1Z2Z3	U3,V3,W4
X3,Y4,Z3	C1,D2,C3	P4,Q1,R4	X2X1X4, Y1Y2Y3, Z2Z1Z4	U2,V2,W4
X4,Y1,Z1	D1,A2,A3	P1,Q2,R2	X1X2X3, Y4Y3Y2, Z4Z3Z2	U4,V1,W1
X4,Y2,Z2	D1,B2,B3	P1,Q3,R3	X1X2X3, Y3Y4Y1, Z3Z4Z1	U1,V4,W1
X4,Y3,Z3	D1,C2,C3	P1,Q4,R4	X1X2X3, Y2Y1Y4, Z2Z1Z4	U1,V1,W4
X4,Y4,Z4	D1,D2,D3	P1,Q1,R1	X1X2X3, Y1Y2Y3, Z1Z2Z3	U4,V4,W4

(*) Corresponding points in QA-Tr-1 of EQF are also shown.

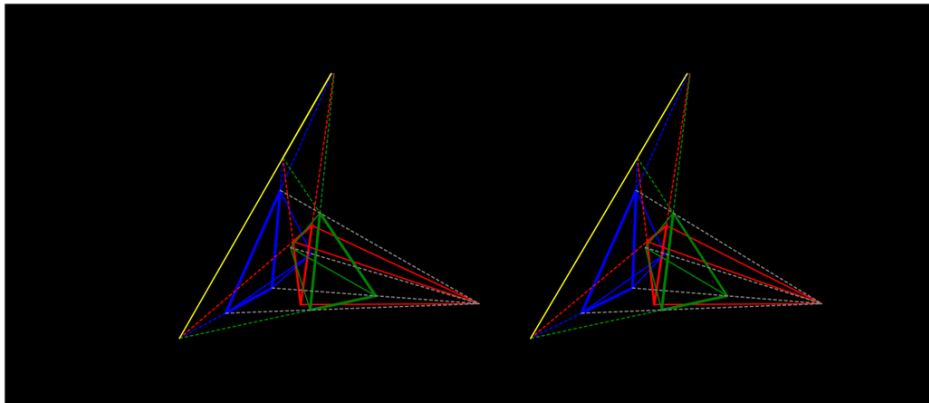
U_j, V_j, W_j ($j=1,2,3,4$) are defined as follows:

$$\begin{aligned}
 U1 &= X2X3 \cap Y1Y4 \cap Z1Z4, & U2 &= X1X4 \cap Y2Y3 \cap Z1Z4, & U3 &= X1X4 \cap Y1Y4 \cap Z2Z3, & U4 &= X2X3 \cap Y2Y3 \cap Z2Z3 \\
 V1 &= X3X1 \cap Y2Y4 \cap Z2Z4, & V2 &= X2X4 \cap Y3Y1 \cap Z2Z4, & V3 &= X2X4 \cap Y2Y4 \cap Z3Z1, & V4 &= X3X1 \cap Y3Y1 \cap Z3Z1 \\
 W1 &= X1X2 \cap Y3Y4 \cap Z3Z4, & W2 &= X3X4 \cap Y1Y2 \cap Z3Z4, & W3 &= X3X4 \cap Y3Y4 \cap Z1Z2, & W4 &= X1X2 \cap Y1Y2 \cap Z1Z2
 \end{aligned}$$

(Conversely, $X1 = U2U3 \cap V1V4 \cap W1W4$, $X2 = U1U4 \cap V2V3 \cap W1W4$,)

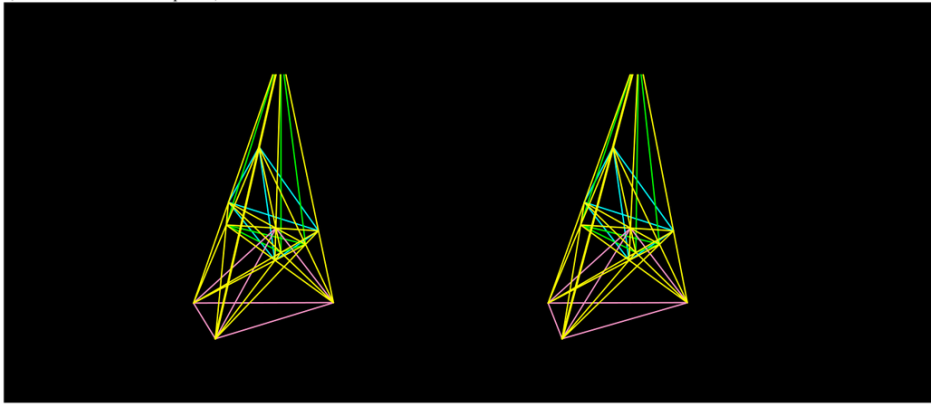
These points form 3 tetrahedra: $\text{Tet}U = \{U1, U2, U3, U4\}$, $\text{Tet}V = \{V1, V2, V3, V4\}$, $\text{Tet}W = \{W1, W2, W3, W4\}$.

Let $\text{Tr}Xi = \{Xi, Xj, Xk\}$, $\text{Tr}Yi = \{Yi, Yj, Yk\}$, $\text{Tr}Zi = \{Zi, Zj, Zk\}$, where $\{i,j,k,l\} = \{1,2,3,4\}$.
 When $\text{Tr}Xa, \text{Tr}Yb, \text{Tr}Zc$ are pairwise perspective, the perspector of $\text{Tr}Yb$ and $\text{Tr}Zc$, $\text{Tr}Zc$ and $\text{Tr}Xa$, $\text{Tr}Xa$ and $\text{Tr}Yb$ are Xa, Yb, Zc , respectively.

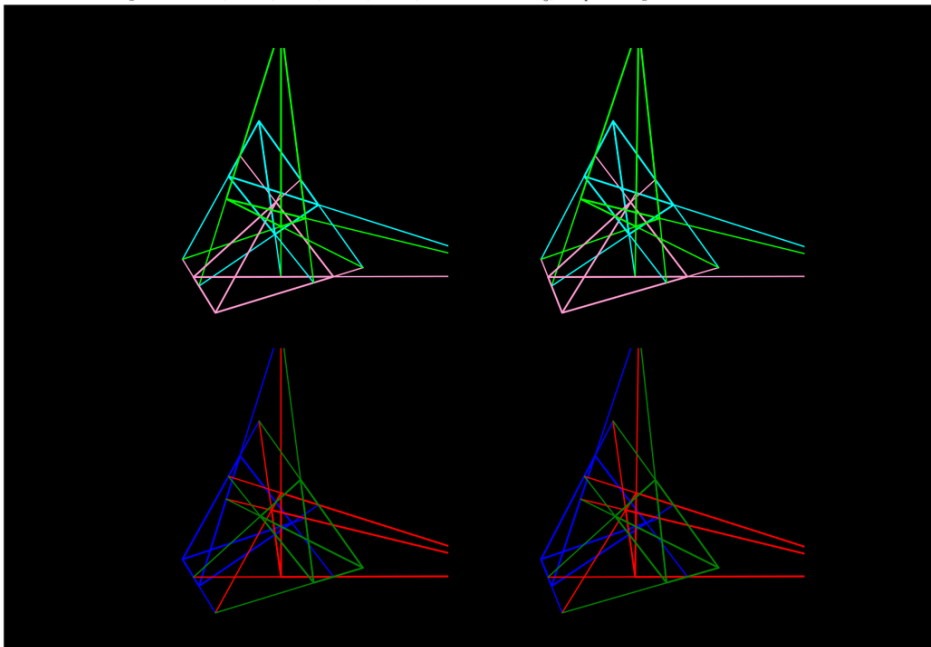


Blue: TetX, dark green: TetY, red: TetZ, yellow: perspectrix

16 perspectrices(yellow) and the vertices of TetU(light blue),TetV(green),TetW(pink) form another Reye configuration.
(From different viewpoint)



The vertices and edges of TetX,TetY,TetZ,TetU,TetV,TetW form $(24_3, 18_4)$ configuration.



The opposite edges of a tetrahedron don't intersect in 3D space,
while a quadrangle in 2D plane has its diagonal triangle.

$X_2X_3 \cap X_1X_4 = U_1U_4 \cap U_2U_3$	$Y_2Y_3 \cap Y_1Y_4 = U_2U_4 \cap U_3U_1$	$Z_2Z_3 \cap Z_1Z_4 = U_3U_4 \cap U_1U_2$	DT _u
$X_3X_1 \cap X_2X_4 = V_1V_4 \cap V_2V_3$	$Y_3Y_1 \cap Y_2Y_4 = V_2V_4 \cap V_3V_1$	$Z_3Z_1 \cap Z_2Z_4 = V_3V_4 \cap V_1V_2$	DT _v
$X_1X_2 \cap X_3X_4 = W_1W_4 \cap W_2W_3$	$Y_1Y_2 \cap Y_3Y_4 = W_2W_4 \cap W_3W_1$	$Z_1Z_2 \cap Z_3Z_4 = W_3W_4 \cap W_1W_2$	DT _w
DT _x	DT _y	DT _z	

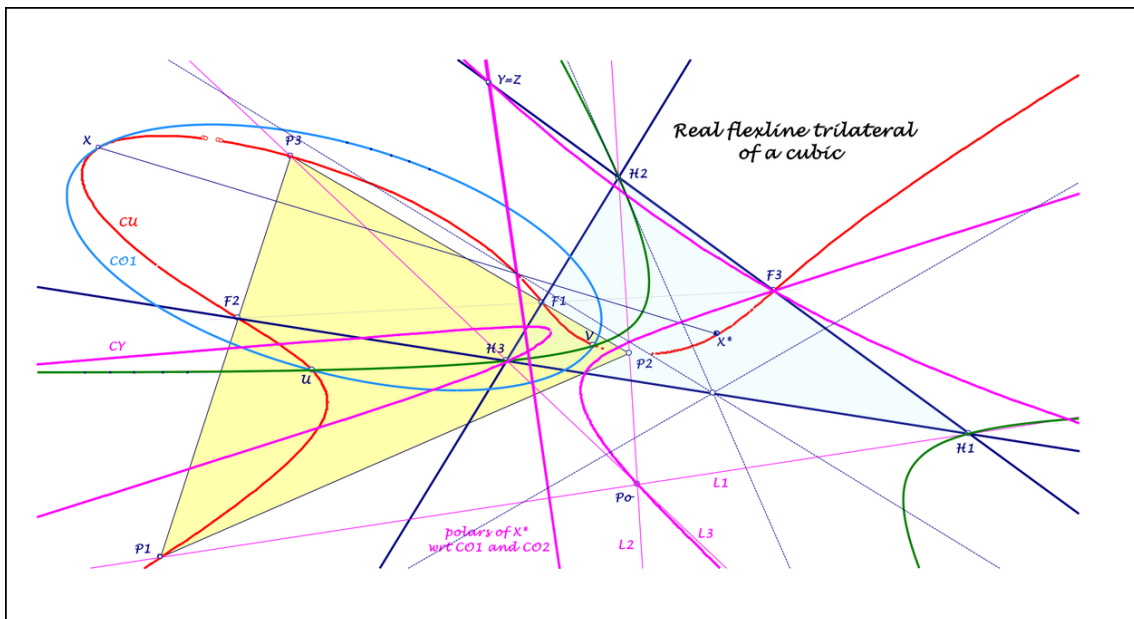
Message: #2798
Date: 2025-11-01
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

if you accept CABRI-intersections
 ... of a line and any curve as construction,
 ... I can finish my conjecture in #2793
 ... to a construction of the real flexline trilateral:
 Consider the polar conic $C01$ of a cubic point X ,
 ... intersecting the cubic in U, V (on the infinite part),
 ... and the conic $C02$ through $H1, H2, H3, U, V$,
 ... if the polars of X^* wrt $C01$ and $C02$ coincide,
 ... you get the real flexline trilateral $H1H2H3$.
 Let the polar of X^* wrt $C02$ intersect H_iH_j in Y ,
 ... consider the locus of Y varying triangle $H1H2H3$,
 ... that is a curve CY of higher unknown degree (5?),
 ... take the intersection Z of CY and the polar of X^* wrt $C01$,
 ... then $ZF_k = H_iH_j$ is a sideline
 of the real flexline trilateral ...

Best regards Eckart

PS: Attached $H_iH_j = H1H2$,
 ... further attached the "construction" of #2747 for $H2$.



2025-11-01.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2799
Date: 2025-11-03
From: bernard.keizer@gmail.com
Subject: Re: Desmic System / Reye Configuration

Dear M@IMF ,
This Desmic/Reye configuration is well known, but I'm surprised that you don't mention the main property, that the 12 points X_i , Y_i and Z_i for $i=1$ to 4 belong to the same cubic ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2800
Date: 2025-11-03
From: contiwa.goma3@gmail.com
Subject: Re: Desmic System / Reye Configuration

Dear Mr. Keizer,

Thank you for the message.
Of course, there are many interesting properties about cubics.
But I think "hidden Desmic/transpose Reye" is worthwhile to show without them.
Someone who become interested in the topic will visit QFG and know its fascination.
By the way, I didn't realize that triangles in a desmic system are DTs of QLs formed by their perspectrices.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2801
Date: 2025-11-06
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

now I am quite surprised,
... is the following property correct,
evident or anywhere mentioned?

The vertices H_1, H_2, H_3 of the real flexline trilateral
... are contact points of tangents
... from the flexpoints F_1, F_2, F_3 to the hessian of the cubic,
... (for a bipartite hessian take the infinite part).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2802
Date: 2025-11-07
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

excuse, excuse excuse, the property in #2801 is nonsense,
... I confused cubics in my drawing,
... the hessian doesn't bear its points H_i ,
... I hope to escribe the right cubic.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2803
Date: 2025-11-11
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

I studied your construction in #2798.
At first, you repeat your approximation construction from #2747, valid for both monopartite cubics and bipartite cubics. Then you introduce a new point X^* (what are its specifications? I suppose it lies on CU — is it chosen randomly or is it a conjugate?). This yields a curve of higher degree, which is then intersected with the polar of X^* with respect to $C01$. The intersection point lies on a sideline of the real flexline trilateral.

It is a bit difficult for me to understand thoroughly. Nevertheless, the intersection of a higher-degree locus curve with a line is not easy to construct. But —when correct— it represents a step forward toward finding for example a construction as the intersection of a line with a conic.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2804
Date: 2025-11-12
From: eckart_schmidt@t-online.de
Subject: Partner-cubics

Dear Bernard, dear Chris,

perhaps of interest:

Let us start with a non degenerated reference cubic CU
... as isocubic of a triangle P1P2P3 with root Po
... and isoconjugation with fixpoint also Po,

and consider further cubics for its points
... F1,F2,F3 real flexpoints
... X1,X2,X3 intersections of the real flextangents,
... T1,T2,T3 their intersections
... .. with the harmonic polars of the flexpoints F1,F2,F3,
... H1,H2,H3 vertices of the real flexline trilateral.

A cubic CU1 through F1,F2,F3,T1,T2,T3,H1,H2,H3
... is invariant for the isoconjugation *
 with ref-triangle H1H2H3
... and fixpoints Po plus $F_i H_i \wedge F_j H_j$,
and
... is invariant for the isoconjugation ^
 with ref-triangle T1T2T3
... and fixpoints Po plus X1,X2,X3.

A cubic CU2 through P1,P2,P3,F1,F2,F3,H1,H2,H3
... is invariant for the first isoconjugation *.

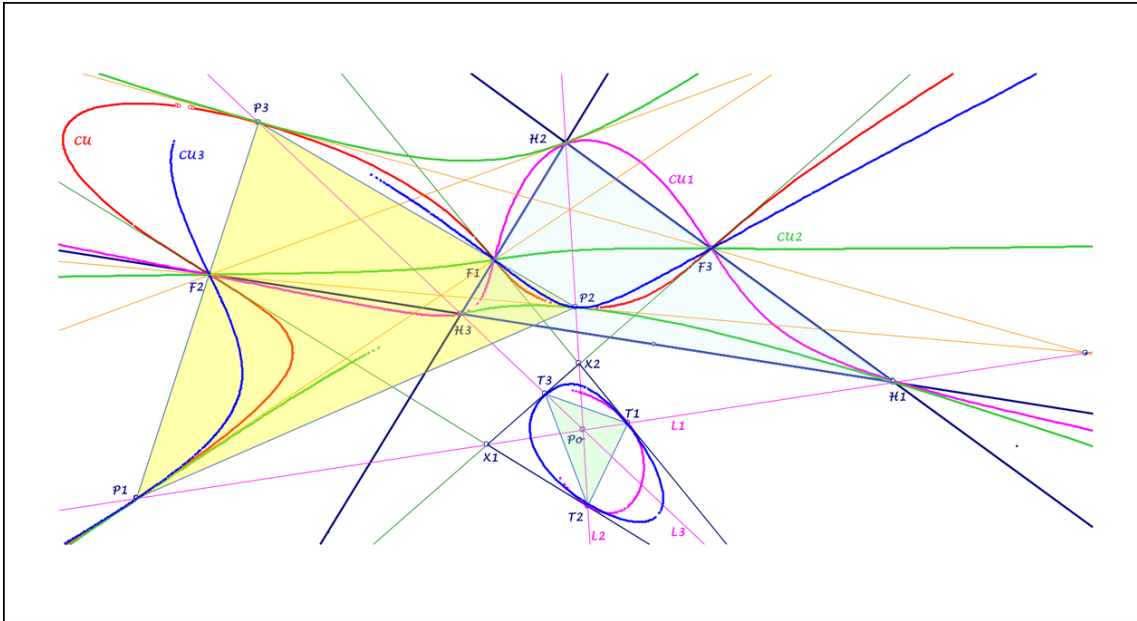
A cubic CU3 through P1,P2,P3,F1,F2,F3,T1,T2,T3
... is invariant for the second isoconjugation ^.

Finally common tangents:

In Pi contact CU, CU2, CU3 with common tangent FiPi,
in Ti contact CU1 and CU3 with common tangent FiTi,
in Hi contact CU1 and CU2 with common tangent FiHi.

Best regards Eckart

PS: Excuse the breaks in the attached figure,
... they exist not in the CABRI drawing.



2025-11-12.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2805
Date: 2025-11-12
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Chris,

thanks for your message #2803,
you are right, excuse, that I indirect repeat my approximation
from #2747,
but in #2798 there is a reference to #2793,
where the isoconjugation $*$ is defined
wrt triangle $P_1P_2P_3$ and fixpoint P_o .

In face of all my fake constructions,
I found a lot of properties and relations
not really relevant for the real flexline trilateral ...

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2806
Date: 2025-11-13
From: van10hoven@gmail.com
Subject: Re: Partner-cubics

Dear Eckart,

Interesting configuration you sketched in #2804.
It appears that the cubics involved are all members of Hesse's pencil.
Particularly noteworthy are the pairwise isoconjugations, which constitute the most distinctive aspect of the construction.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2807
Date: 2025-11-13
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,

Regarding your remarks in message #2805:

1. "You are right, excuse me, that I indirectly repeated my approximation from #2747."

I actually appreciated that you repeated it. For you it is engraved in memory, but for me it is not.

2. 'The isoconjugation * is defined with respect to triangle $P_1P_2P_3$ and fixpoint P_o .'

The same applies here: you mentioned it earlier, but whenever I dive back into the subject I still need this definition.

3. "In face of all my fake constructions, I found a lot of properties and relations not really relevant for the real flexline trilateral ..."

Absolutely true, your provisional constructions have yielded several beautiful new features. My compliments. I very much enjoy them!

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2808
Date: 2025-11-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris, dear Eckart

I'm also full of admiration for Eckart's tenacity!

I tried myself several ways of finding a geometrical construction, but in vain.

My remarks, which are a repetition of earlier messages:

1) a bipartite cubic is a pivotal isocubic QA-Cux with pivot in any of it's points, fixed points of the isoconjugation the 4 points having the pivot as common tangential and vertices of the reference triangle the 3 points having the same tangential as the pivot (this point is the isopivot

2) analytically, the equation of a cubic QA-Cux is in EQF and it is possible to calculate the equation of it's hessian, to solve the system and to find the flexes (3 real and 6 imaginary) and the flexlines (4 real and 8 imaginary), as well as P_0 and the harmonic polars. (These calculations would naturally improve EQF greatly!)

3) geometrically, it is possible to construct the pivotal isocubic with pivot P through the QA vertices, the DT vertices, the pivot P and the isopivot P' and the feet of the cevians of P wrt DT, which are u_1 , u_2 and u_3 .

We have then 3 polar conics of P, P' and P'' wrt this cubic. Let's consider the polar conic of P' (through P and the DT vertices); the intersections with the 2 other conics (one through P and the QA vertices and the other through P', P'' and u_1 , u_2 and u_3) form 2 qa's (P, S₁, S₂ and S₃ and P', S'₁, S'₂ and S'₃).

Let's consider now the dt's of the 2 qa's ; their vertices belong to the hessian, as well as the intersections of their sides with the lines PP' and P'P''.

This gives 12 points, which is more than enough in order to determinate the hessian.

4) the intersection of the cubic and the hessian gives without difficulty the 3 real flexes, the point P_0 and the 3 harmonic polars.

5) Using Eckart's property, we may start with a point H₁ on L₁, giving H₂ on L₂ and H₃ on L₃.

H1 is in the right place when the 3 points H1, H2 and H3 are coconic with the vertices of the QA (having P as tangential), the points P and the vertices of DT (having P' as tangential) and the points P', u1, u2 and u3 (having P'' as tangential). Considering H'1, H'2 and H'3 the QA-Tf1 of H1, H2 and H3, they must lie on a line through P', which is the QA-Tf1 of the conic through P and the vertices of DT.

It is remarkable that H'1 lies on H2H3, H'2 on H1H3 and H'3 on H1H2. (By the way, QA-Tf1(P0) lies on the line of real flexes). I hope Eckart will agree with these observations ...

Best regards
Bernard

PS Dear Eckart, naturally our forum is dead since a long time, but apparently, the 3 of us are still alive and that's the only thing which matters ...

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2809
Date: 2025-11-14
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

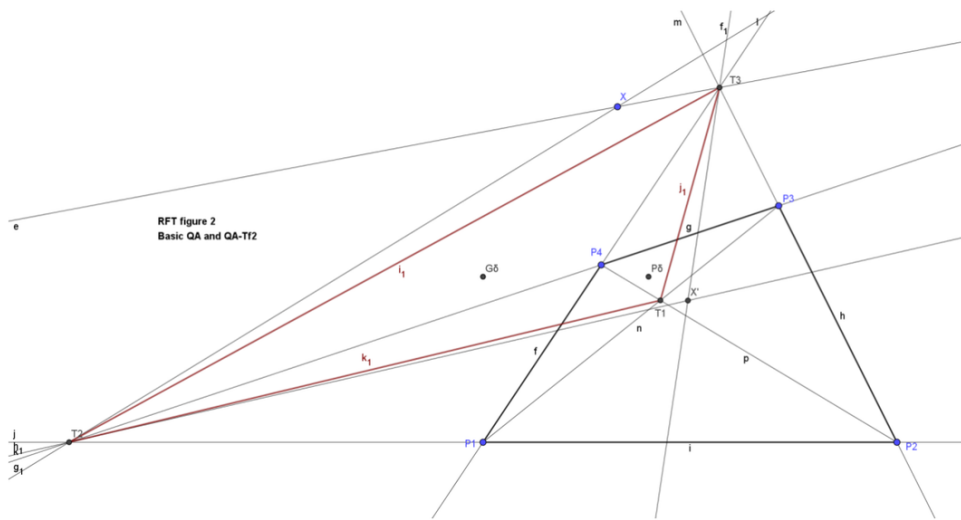
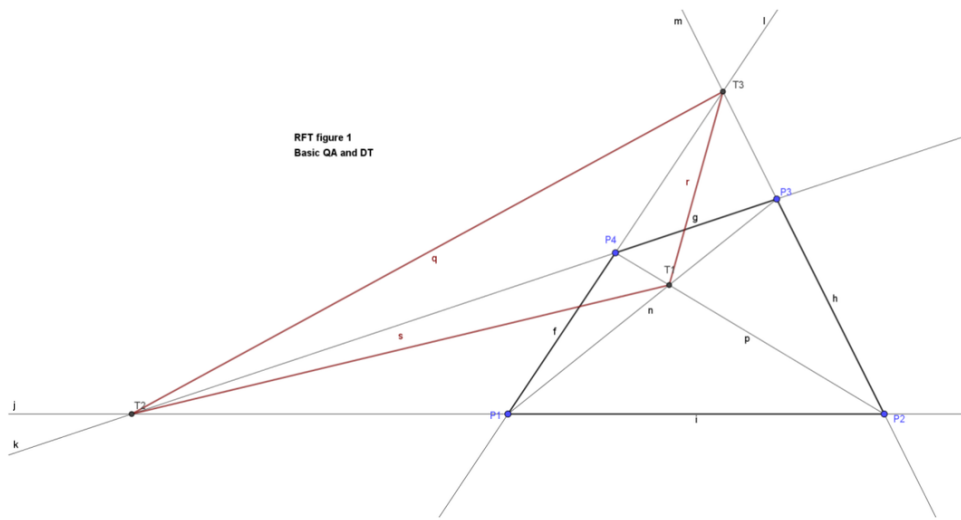
Dear Chris and Eckart,
Please read QA-Tf2 instead of QA-Tf1
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

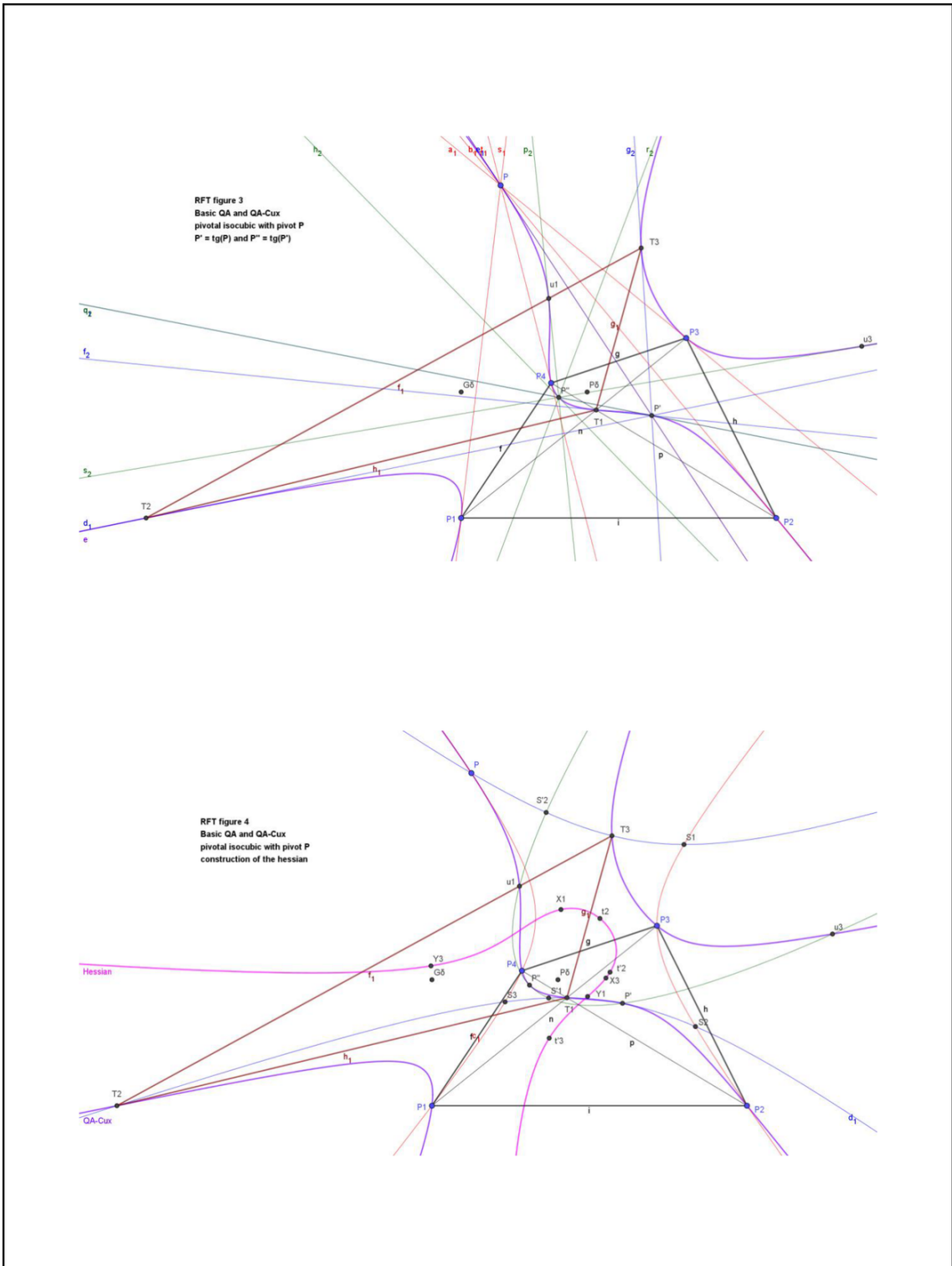
Message: #2810
Date: 2025-11-15
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and Eckart
Here an illustration of my last message
I hope everything is clear (Gδ is QA-P10 and Pδ is QA-P16)
Best regards
Bernard

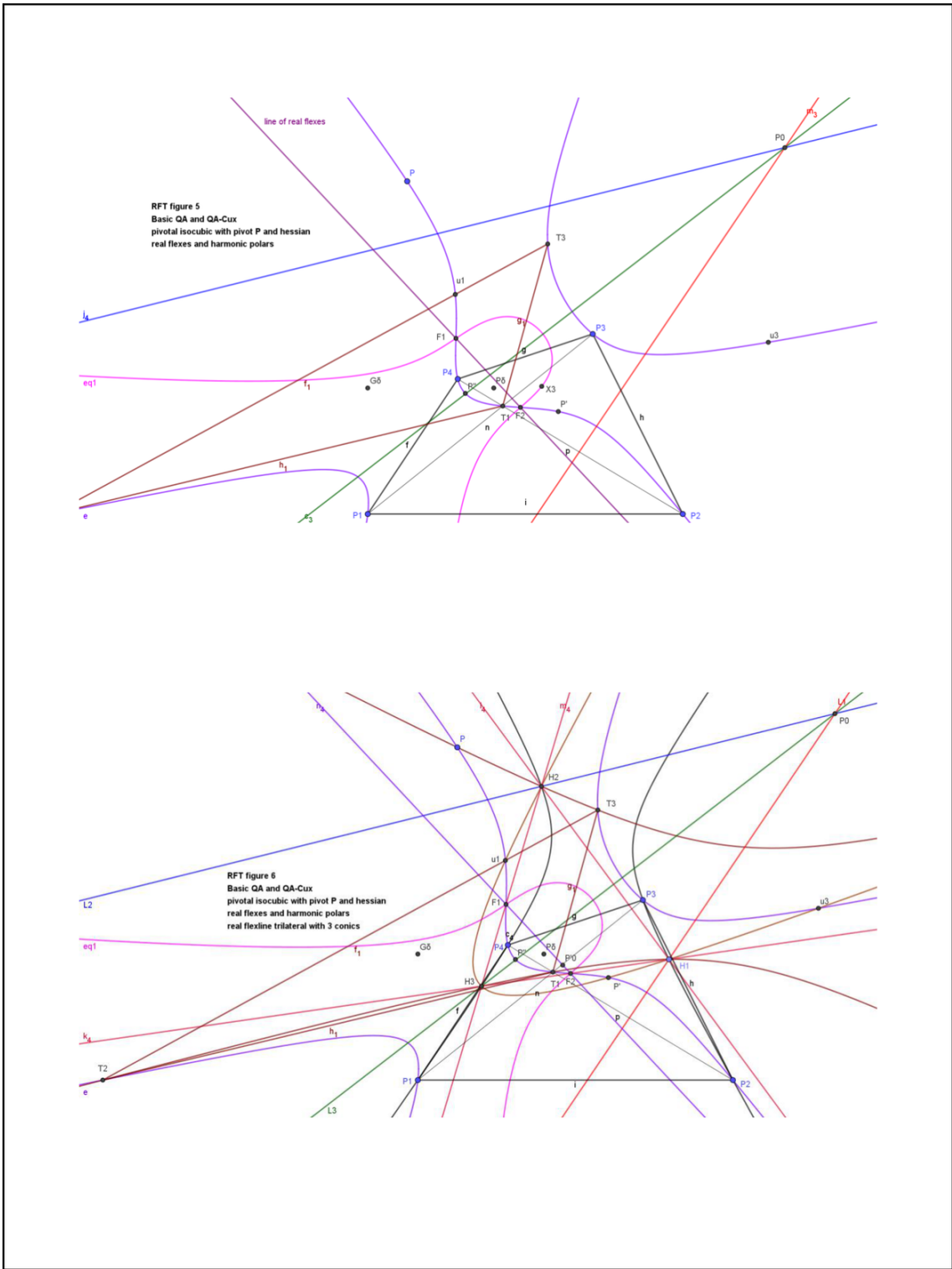
Real flexline trilateral for a pivotal (bipartite) isocubic



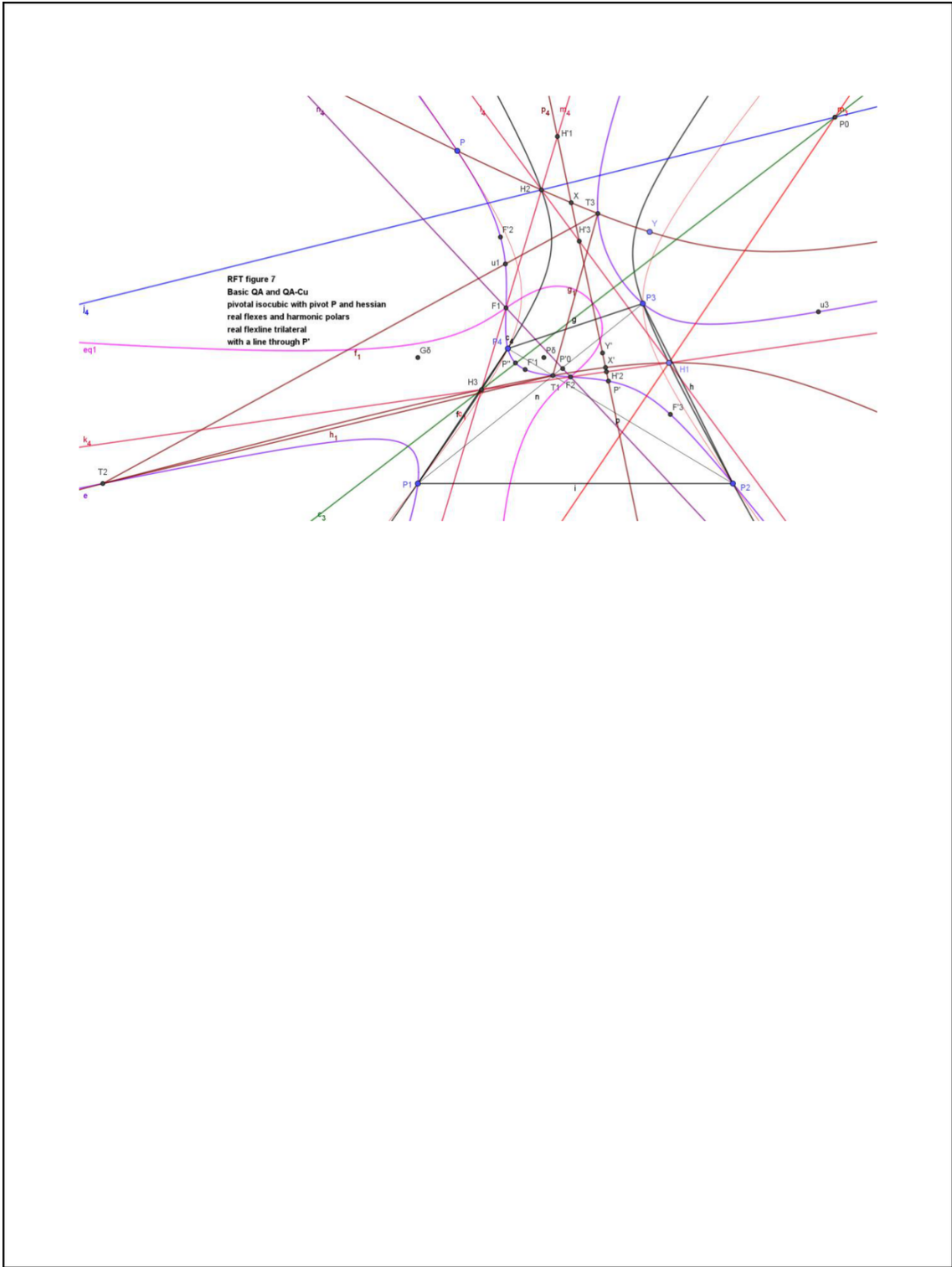
RFT for a pivotal (bipartite) isocubic.pdf



RFT for a pivotal (bipartite) isocubic.pdf



RFT for a pivotal (bipartite) isocubic.pdf



RFT for a pivotal (bipartite) isocubic.pdf

[← Previous](#)
 [Next →](#)
 [↔ Message Index](#)
 [↑ Subjects](#)

Message: #2811
Date: 2025-11-17
From: eckart_schmidt@t-online.de
Subject: Re: Real flexline trilateral

Dear Bernard,

thanks for your extensive messages.
You restrict your observations to bipartite isocubics, which are pivotal, as you describe.
My observations based on nonpivotal isocubics, including mono- and bipartite isocubics.

I hope my following sight is correct:
For an isocubic a reference triangle and an isoconjugation must be given.
Isocubics can be mono- or bipartite.
Bipartite cubics are pivotal, but can also be described as nonpivotal isocubics with a root P_0 , which is the tripol for the line $F_1F_2F_3$ of real flexpoints, which are the intersections of reference cubic and sidelines of the reference triangle.

So any isocubic can be drawn with a reference triangle $P_1P_2P_3$, a point P_0 as root and also fixpoint for an isoconjugation with a starting cubic-point Q .

If I am not wrong,
you construct the hessian in 3) (extensive),
to get F_1, F_2, F_3, P_0 and L_1, L_2, L_3 in 4),
which are the starting elements in my sight.
Nevertheless your observation in 5) is interesting and new for me:
For a pivotal isocubic the 4 tangentials of a cubic-point P are coconic with the vertices of the real flexline trilateral.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2812
Date: 2025-11-18
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Eckart,
Thanks for interest and recation!
In the last point, it's the 4 pretangentials (having the same tangential) of a point P of a bipartite cubic.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2813
Date: 2025-11-19
From: bernard.keizer@gmail.com
Subject: Congratulations

Dear Chris,
I've just visited the site of EQF and have been redirected on your new site!
I'm amazed, it's a fantastic work!!!
Congratulations
Naturally, I will visit it in details, but give me time to discover all the jewels ...
I've started with cubics and Hesse pencil ...
Very beautiful indeed
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2814
Date: 2025-11-20
From: bernard.keizer@gmail.com
Subject: Re: Congratulations

Dear Chris,
I can't really find the appropriate words.
It's really astounding or amazing!
My 1rst contribution: on the page Quadrangle items, replace
Quadrilateral by Quadrangle (General Information as well as
Centers).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2815
Date: 2025-11-20
From: contiwa.goma3@gmail.com
Subject: Re: Congratulations

Dear Chris,

Congratulations on renewal of your website.
I admire your outstanding ability to execute great works.
I've just exploired QFG to learn about cubics yesterday.
Now I can visit the well-organized new website.
It will be very helpful.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2816
Date: 2025-11-20
From: van10hoven@gmail.com
Subject: Re: Congratulations

Dear Bernard and M@IMF,

Thank you very much for your kind compliments.
It feels like a reward for all the effort I put in.
I have been working very hard on my new site
(<https://www.chrisvantienhoven.nl/>) for almost a year,
implementing numerous improvements.
That's why I've been so quiet on the forum lately.

Bernard, I also added your related papers at the EPG treasury
(<https://www.chrisvantienhoven.nl/epg/explore-epg/treasury/>).
I promptly adapted the Quadrangle feature as well.
I will make an official announcement of the renewed site in
another post.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2817
Date: 2025-11-20
From: eckart_schmidt@t-online.de
Subject: Re: Congratulations

Dear Chris,

WOW, congratulations!
I have to study your admirable work,
especially "cubics", my current interest.

What a development,
remembering your e-mail of 02.09.2011,
where you announced your first quadrilateral observations,
later asking for a review of your results,
what now would be absolute impossible for me,
my geometrical horizon goes down age-related.

Best regards and Good Luck
Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2818
Date: 2025-11-20
From: van10hoven@gmail.com
Subject: Re: Update on the new Website

A New Chapter: My Website Has Launched

I'm pleased to announce the launch of my brand-new website – a major step forward in both design and content.

WHAT'S NEW ?

* A Modern Foundation

The outdated Joomla-based engine has been replaced with a fresh, streamlined WordPress platform, offering improved performance and a more intuitive user experience.

* A Unified Encyclopedia

The two encyclopedias previously developed with the Quadri-Figures Group (QFG) and the Quadri- and Poly Geometry Group (QPG) – the Encyclopedia of Quadri-Figures and the Encyclopedia of Polygon Geometry – have now been merged into a single, comprehensive resource: the Encyclopedia of Poly Geometry (EPG).

Poly Geometry is envisioned as a new branch of geometry, shaped by:

- * Configurations of n points and/or n lines
- * Curves of degree n

The name "Poly" reflects this broader, more inclusive structure.

* New Focus on Cubics

A substantial new section has been added, dedicated to cubic curves – not necessarily tied to triangles or quadrilaterals, but studied as third-degree curves in their own right. These insights, developed over the past two years within the QPG Forum by Eckart Schmidt, Bernard Keizer, and myself, have led to several surprising discoveries.

* Expanded Downloads Section

The site now includes a significantly enriched downloads area, featuring:

- * Macros for Cabri II Plus
- * Notebooks for Mathematica
- * The complete EPG encyclopedia, available in downloadable PDF segments

* Forum Access & Journals

Direct access is now available to the QFG and QPG forums.

Even more exciting: *Journals* will soon be released – annual PDF compilations of forum discussions and illustrations, making it easier to preserve and explore the collective knowledge.

* EPG Treasury

A dedicated section has been established for papers related to Poly Geometry. Scholars and researchers who have authored work in this field are invited to submit their papers, thereby contributing to a collective treasury of knowledge that will remain accessible for as long as the site is maintained. Please note, however, that I cannot guarantee preservation in perpetuity.

Most components of the new website are complete now, though a few elements are still pending. I aim to finalize everything as soon as possible.

If you have any questions, comments, or suggestions, I'd love to hear from you.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2819

Date: 2025-11-21

From: bernard.keizer@gmail.com

Subject: Cayleyan as member of a dual Hesse pencil

Dear Chris,

I've modified the message 2539 sent a year ago about the 2 dual Hesse pencils.

You will find the barycentric equation of the cayleyan

It's easy to identify your coefficients u , v and w , which depend only of k

I hope this could help you to complete the item CU-5

Best regards

Bernard

2 dual Hesse pencils, 4 curves and calculations

1. 2 dual Hesse pencils wrt a triangle H1H2H3

The 1st pencil contains the cubics having the same 9 flexes (3 real and 6 imaginary).
The real flexes are F1, F2 and F3

The 2nd pencil contains the curves of the 3rd order tangent to the 9 harmonic polars (3 real)
The real harmonic polars are L1, L2 and L3 ; they intersect in P0

2. 4 curves

2 curves belong to the 1st pencil, the cubic CU and it's hessian HE

2 curves belong to the 2nd pencil, the cayleyan CA and a curve Γ associated to the cubic CU

CA is the hessian of Γ and HE is it's cayleyan (in other words, CU and Γ swap their hessian and cayleyan)

Both pencils have the same structure, with a Core curve and a residual curve

3. Calculations

The barycentric coordinates of P0 are $1/a_1, 1/a_2$ and $1/a_3$ and the barycentric equation of the line of the 3 real flexes is $a_1x + a_2y + a_3z = 0$.

All the curves of both pencils have an equation of the form $x^3 + y^3 + z^3 + kxyz = 0$, considering it as a barycentric equation for the 1st pencil and replacing x, y and z by a_1x, a_2y and a_3z and as a tangential equation for the 2nd pencil and replacing x, y and z by $u/a_1, v/a_2$ and w/a_3 .

In particular, if the cubic CU has the constant k , then HE has the constant $k' = -(6/k)^2 - k/3$, CA has the constant $k'' = 6/k - (k/3)^2$ and Γ has the constant $k^* = -18/k$.

It's obvious that $k'(k^*) = k''(k)$ and $k''(k^*) = k'(k)$, which explains the swapping of HE and CA for the curves CU and Γ .

It's possible to find the tangential equation of a curve defined by a barycentric equation and vice-versa, the calculation being exactly the same. We find equations of the 6th degree¹.

In particular, the barycentric equation of CA is

$$27 (a_1^3x^3 + a_2^3y^3 + a_3^3z^3)^2 - 4 (27 + k''^3) (a_1^3a_2^3x^3y^3 + a_1^3a_3^3x^3z^3 + a_2^3a_3^3y^3z^3)$$

$$- 18k''^2 xyz (a_1^3x^3 + a_2^3y^3 + a_3^3z^3) - k''(108 + k''^3) a_1^2a_2^2a_3^2x^2y^2z^2 = 0$$

and the same for Γ replacing k'' by k^* .

The cayleyan of the Core cubic is the Residual curve of the 2nd pencil ($k = 0$ and $k^* = \infty$) and the cayleyan of the residual cubic is the Core curve of the 2nd pencil ($k = \infty$ and $k^* = 0$).

The barycentric equation of this Core curve is

$$(a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3)^2 - 4(a_1^3 a_2^3 x^3 y^3 + a_1^3 a_3^3 x^3 z^3 + a_2^3 a_3^3 y^3 z^3) = 0$$

The barycentric equation of this Residual curve is $x^2 y^2 z^2 = 0$ and it's tangential curve $uvw = 0$

It's remarkable that this Residual curve is 2 times the triangle forming the residual cubic !

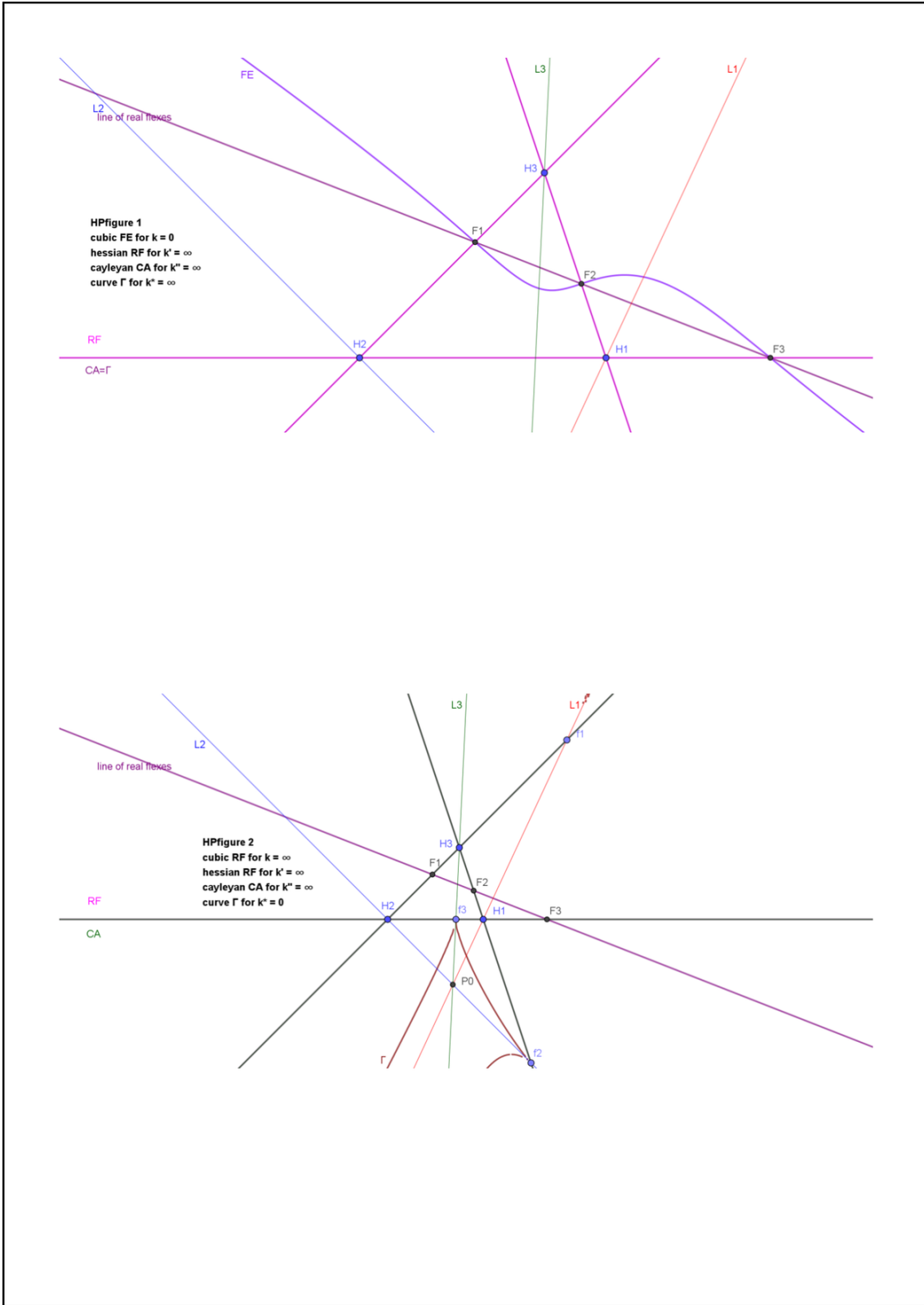
4. Properties

Drawing this way a certain number of curves, it appears that the cusps of the curves Γ are the contact points of HE and CA, which seems to be the dual property of the common tangents to CU and HE being the tangents in the intersections between CU and CA ...

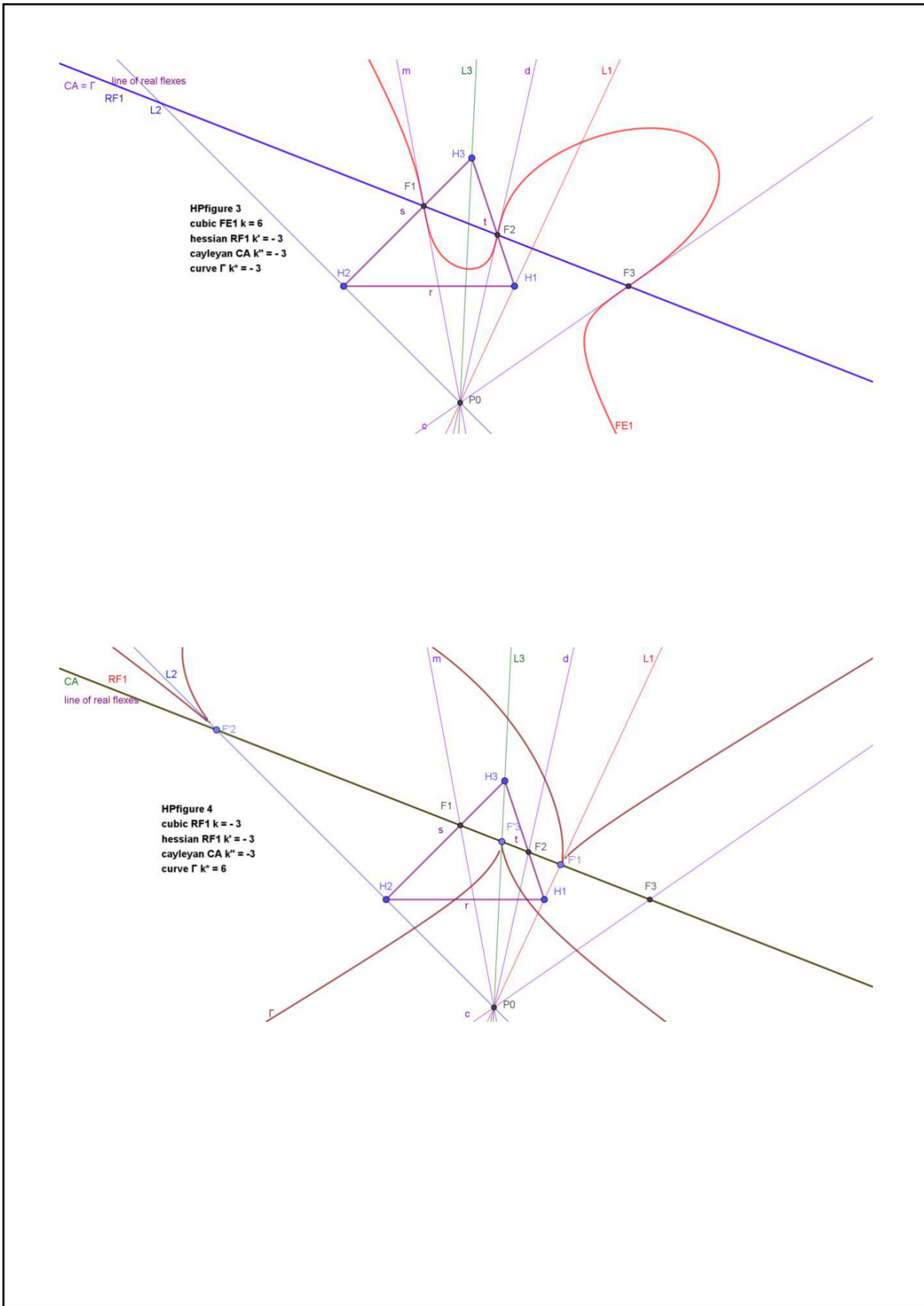
1 The calculations are long and boring !

Considering the tangential equation $G(u,v,w)$ of the curve and the tangent $ux + vy + wz = 0$, we know that $G'u/x = G'v/y = G'w/z$ and must eliminate u, v and w in order to find the barycentric equation $F(x,y,z)$ of the curve.

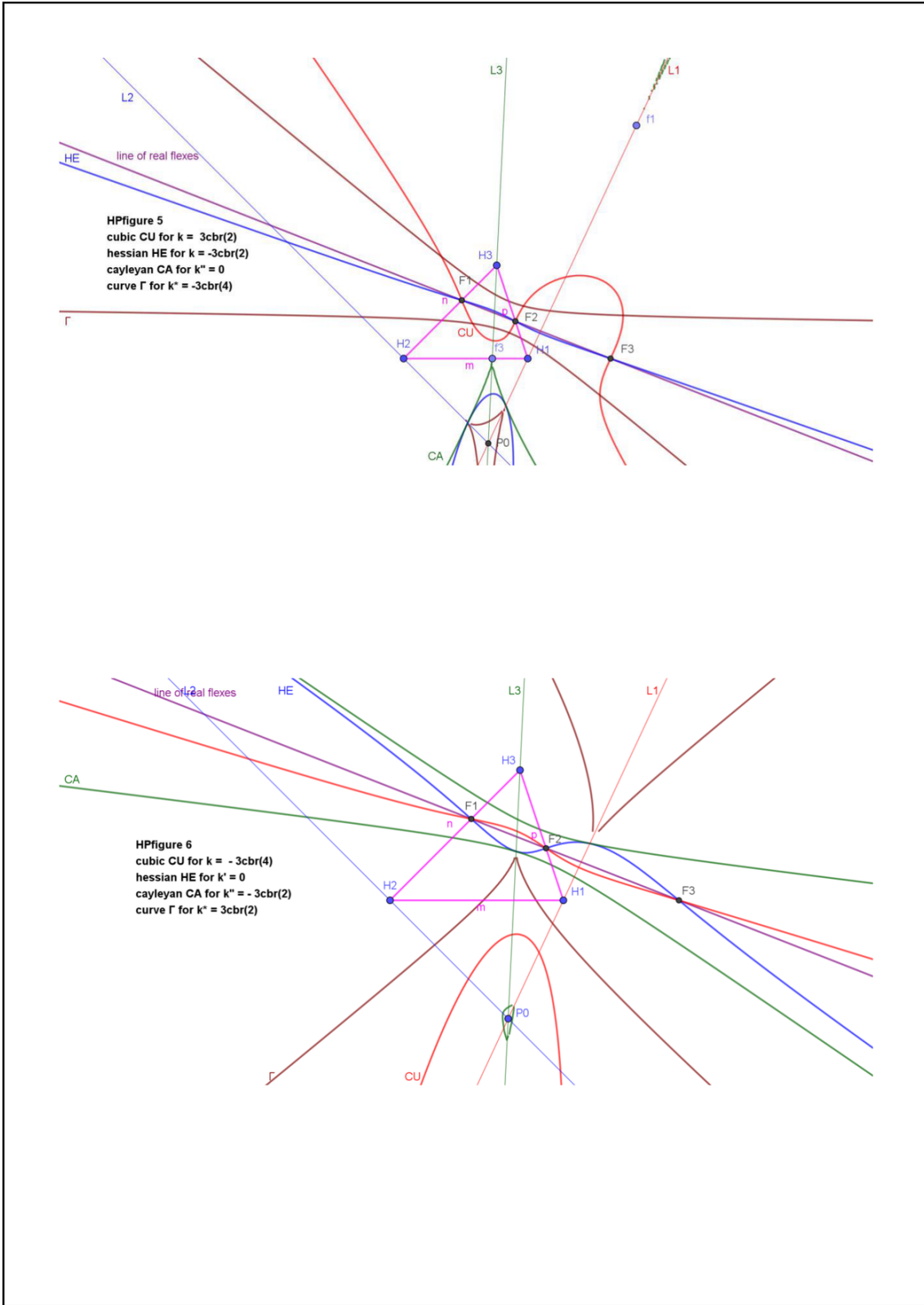
For example, for $G(u,v,w) = u^3 + v^3 + w^3 = 0$ ($k'' = 0$), it's immediate that $u^2/x = v^2/y = w^2/z$ and $x^{3/2} + y^{3/2} + z^{3/2} = 0$. Then $z^{3/2} = -(x^{3/2} + y^{3/2})$, $z^3 = (x^{3/2} + y^{3/2})^2 = x^3 + y^3 + 2 u^{3/2} v^{3/2}$ and last we find $(2 x^{3/2} y^{3/2})^2 = (z^3 - x^3 - y^3)^2$, $x^6 + y^6 + z^6 = 2(x^3 y^3 + x^3 z^3 + y^3 z^3)$ or $(x^3 + y^3 + z^3)^2 = 4(x^3 y^3 + x^3 z^3 + y^3 z^3)$.



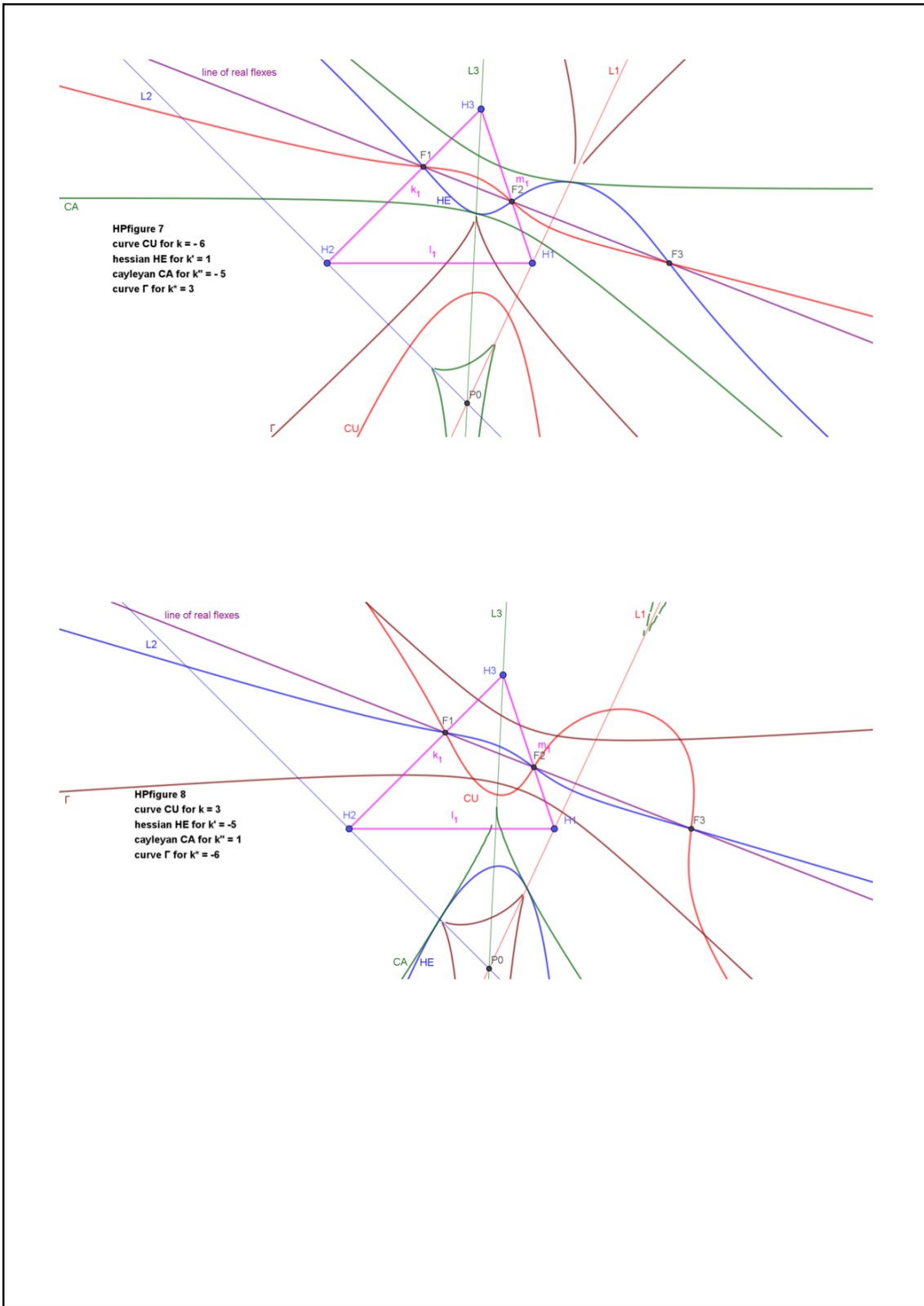
2 dual Hesse pencils.pdf

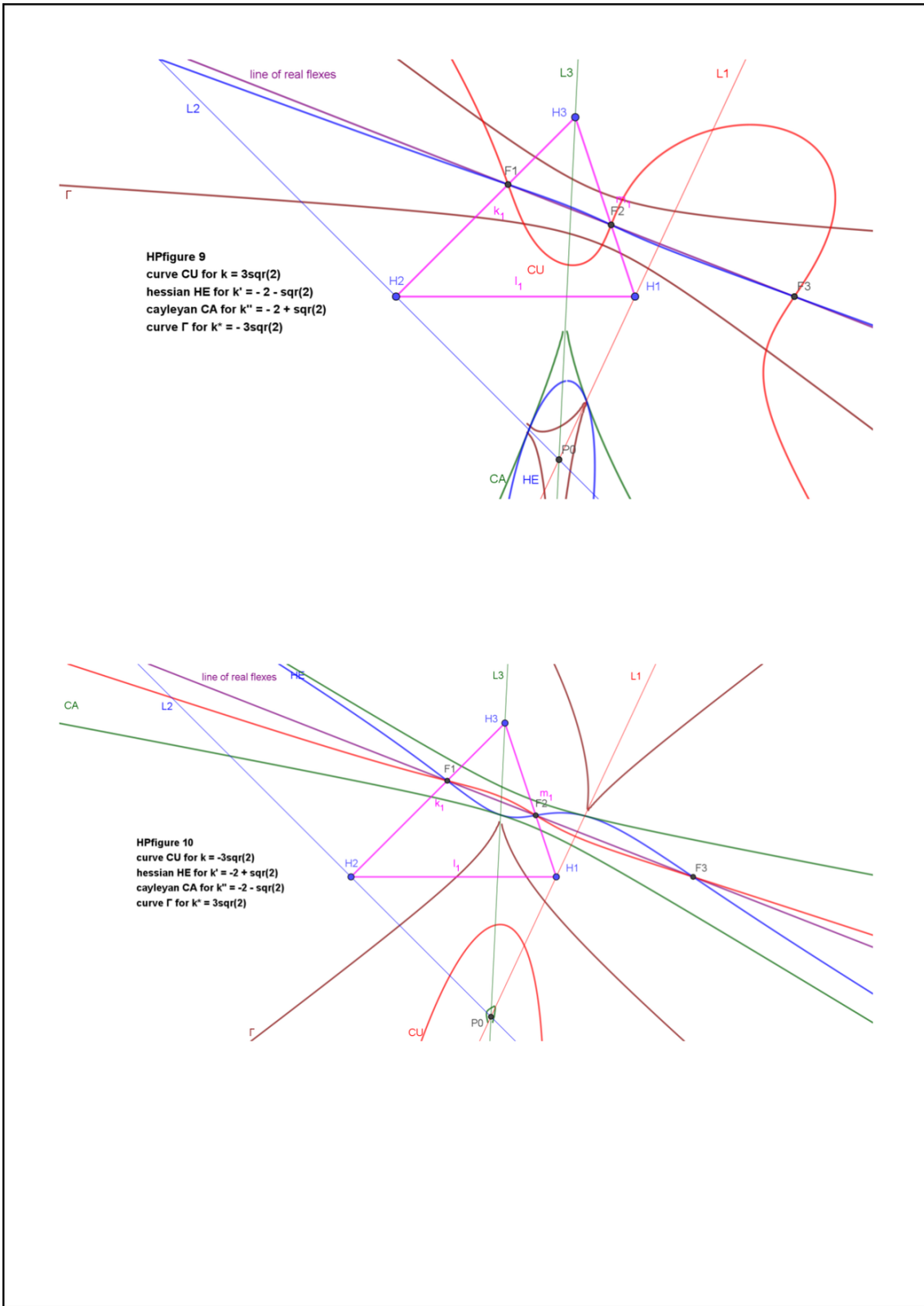


2 dual Hesse pencils.pdf



2 dual Hesse pencils.pdf





2 dual Hesse pencils.pdf

Message: #2820

Date: 2025-11-24

From: van10hoven@gmail.com

Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Bernard,

Thanks for your document 2 dual Hesse pencils, 4 curves and calculations.

I checked in a numerical example in Mathematica your equation for the Cayleyan:

$$CA = 27 (a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3)^2 - 4 (27 + k''^3) (a_1^3 a_2^3 x^3 y^3 + a_1^3 a_3^3 x^3 z^3 + a_2^3 a_3^3 y^3 z^3) - 18 k''^2 x y z (a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3) - k'' (108 + k''^3) a_1^2 a_2^2 a_3^2 x^2 y^2 z^2 = 0$$

And partly it fits, only for the part $- 18 k''^2 x y z (a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3)$ there is no match.

It can be some typo of mine, but could you please check it again for me?

Then some questions:

1. Is there a reference for working with this second pencil (curves of 3rd order tangent to the 9 harmonic polars). Or what is the reason for working with this second pencil.
2. Of what degree is this curve gamma?
3. How did you derive the formula of CA?

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2821
Date: 2025-11-24
From: eckart_schmidt@t-online.de
Subject: Isocubic

Dear Chris,

where can I find in your new webside the possibility
... to describe a 9P-cubic as isocubic
with ref-triangle $P_1P_2P_3$
... (P_i intersections of the infinite part of the cubic
... and the harmonic polars L_i of the real flexpoints F_i),
with an isoconjugation with fixpoint P_o (common point of L_i),
also root P_o of the cubic
and with a starting cubic-point P ?
This is a good constellation for constructions
... starting with a triangle and two points P_o and P .

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2822
Date: 2025-11-24
From: van10hoven@gmail.com
Subject: Re: Isocubic

Dear Eckart,

In the Encyclopedia of Poly Geometry (EPG), we generally describe only constructions that start from n arbitrary points or lines.

The five points you mention, however, are not entirely arbitrary—two of them serve specific roles. In such a case, we could present it as a 3P-Transformation with two input points that produce a cubic. However this falls within triangle geometry, as described, among others, by Bernard Gibert. Since this has already been treated extensively, I have limited the section of EPG-Triangle Geometry (<https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/>) only to references pointing to sources where it is covered. Unless you believe your construction is genuinely new. Could you provide a solid step-by-step construction?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2823
Date: 2025-11-25
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

you wrote in #2822: "However this falls within triangle geometry",
... that is really an absurd interpretation,
... it is a possibility, to study cubics constructive.

Let us start with a triangle $P_1P_2P_3$ and points
... P_o as root and P as cubic-point.

Let R_1, R_2, R_3 be the anticevians of P_o wrt $P_1P_2P_3$,
... then $P_oP_i = L_i$ and $R_iR_j \wedge P_iP_j = F_k$
... and consider finally P_o as fixpoint of an isoconjugation,
... then Bernard Gibert's construction
of a nonpivotal isocubic (1.5.4)
... gives the cubic invariant
under the isoconjugation with fixpoint P_o .

A macro wrt a further hessian point P for the ref-triangle
... of intersections of F_i -flectangents and L_i
gives the hessian ...

Best regards Eckart

PS: Especially the isoconjugations
... are interesting tools to study cubics.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2824
Date: 2025-11-25
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

1. Where would you like it to be mentioned in EPG?
 2. Bernard Gibert's construction of a nonpivotal isocubic (1.5.4) uses other terminology. Could you translate your terminology to his terminology for the innocent reader.
 3. I do not understand your remark
'A macro wrt a further hessian point P for the ref-triangle ... of intersections of Fi-flex tangents and Li gives the hessian ...'.
- Again make a clear stepwise description please.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2825
Date: 2025-11-25
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

wrt 2) Here is a description of my construction
... in Bernard Gibert's terminology,
... preparing his construction (1.5.4)
for a nonpivotal isocubic.

Let us start with a triangle ABC and points
... P as root and Q as cubic-point.
The trilinear polar of P meets ABC sidelines at U,V,W,
... which are the real flexpoints
... and PA,PB,PC their harmonic polars.
Consider finally P as fixpoint of an isoconjugation,
... then Bernard Gibert's construction 1.5.4
of a nonpivotal isocubic
... gives the cubic invariant under the isoconjugation
with fixpoint P.

wrt 3) Excuse my bad formulation,
... it should describe the same construction for the hessian,
... if you have a hessian point for Q,
... using the same root P, but for ABC the triangle
... for intersections of tangents and harmonic polars
of the flexpoints.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2826
Date: 2025-11-25
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Chris,
Many thanks for interest and quick answer!
My apologise, I forgot the term $a_1a_2a_3$ before xyz (the equation is of the 6th degree, homogenous in a_1x , a_2y and a_3z).
1) All this is explained in Salmon or Artebani, which you already put in your bibliography
2) Like all the curves of the 2nd pencil, CA , Γ as well as the core and the residual curves are sextics of the 3rd order
3) I tried to explain the general method in the footnote 1 page 2 of my document
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2827
Date: 2025-11-26
From: bernard.keizer@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Chris,
Sorry, I forgot to send you the corrected version of my document!
Best regards
Bernard

2 dual Hesse pencils, 4 curves and calculations

1. 2 dual Hesse pencils wrt a triangle H1H2H3

The 1st pencil contains the cubics having the same 9 flexes (3 real and 6 imaginary). The real flexes are F1, F2 and F3.

Cubics are curves of 3rd degree and 6th order.

The 2nd pencil contains the curves of the 3rd order tangent to the 9 harmonic polars (3 real) The real harmonic polars are L1, L2 and L3; they intersect in P0.

These curves of 3rd order are sextics of 6th degree.

2. 4 curves

2 curves belong to the 1st pencil, the cubic CU and it's hessian HE

2 curves belong to the 2nd pencil, the cayleyan CA and a curve Γ associated to the cubic CU

CA is the hessian of Γ and HE is it's cayleyan (in other words, CU and Γ swap their hessian and cayleyan)

Both pencils have the same structure, with a Core curve and a residual curve.

3. Calculations

The barycentric coordinates of P0 are $1/a_1$, $1/a_2$ and $1/a_3$ and the barycentric equation of the line of the 3 real flexes is $a_1x + a_2y + a_3z = 0$.

All the curves of both pencils have an equation of the form $x^3 + y^3 + z^3 + kxyz = 0$, considering it as a barycentric equation for the 1st pencil and replacing x , y and z by a_1x , a_2y and a_3z and as a tangential equation for the 2nd pencil and replacing x , y and z by u/a_1 , v/a_2 and w/a_3 .

In particular, if the cubic CU has the constant k , then HE has the constant $k' = -(6/k)^2 - k/3$, CA has the constant $k'' = 6/k - (k/3)^2$ and Γ has the constant $k^* = -18/k$.

It's obvious that $k'(k^*) = k''(k)$ and $k''(k^*) = k'(k)$, which explains the swapping of HE and CA for the curves CU and Γ .

It's possible to find the tangential equation of a curve defined by a barycentric equation and vice-versa, the calculation being exactly the same. We find equations of the 6th degree¹.

In particular, the barycentric equation of CA is

$$27 (a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3)^2 - 4 (27 + k''^3) (a_1^3 a_2^3 x^3 y^3 + a_1^3 a_3^3 x^3 z^3 + a_2^3 a_3^3 y^3 z^3) - 18 k''^2 a_1 a_2 a_3 x y z (a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3) - k'' (108 + k''^3) a_1^2 a_2^2 a_3^2 x^2 y^2 z^2 = 0$$

and the same for Γ replacing k'' by k^* .

The cayleyan of the Core cubic is the Residual curve of the 2nd pencil ($k = 0$ and $k^* = \infty$) and the cayleyan of the residual cubic is the Core curve of the 2nd pencil ($k = \infty$ and $k^* = 0$).

The barycentric equation of this Core curve is

$$(a_1^3 x^3 + a_2^3 y^3 + a_3^3 z^3)^2 - 4 (a_1^3 a_2^3 x^3 y^3 + a_1^3 a_3^3 x^3 z^3 + a_2^3 a_3^3 y^3 z^3) = 0$$

The barycentric equation of this Residual curve is $x^2 y^2 z^2 = 0$ and it's tangential curve $uvw = 0$

It's remarkable that this Residual curve is 2 times the triangle forming the residual cubic !

4. Properties

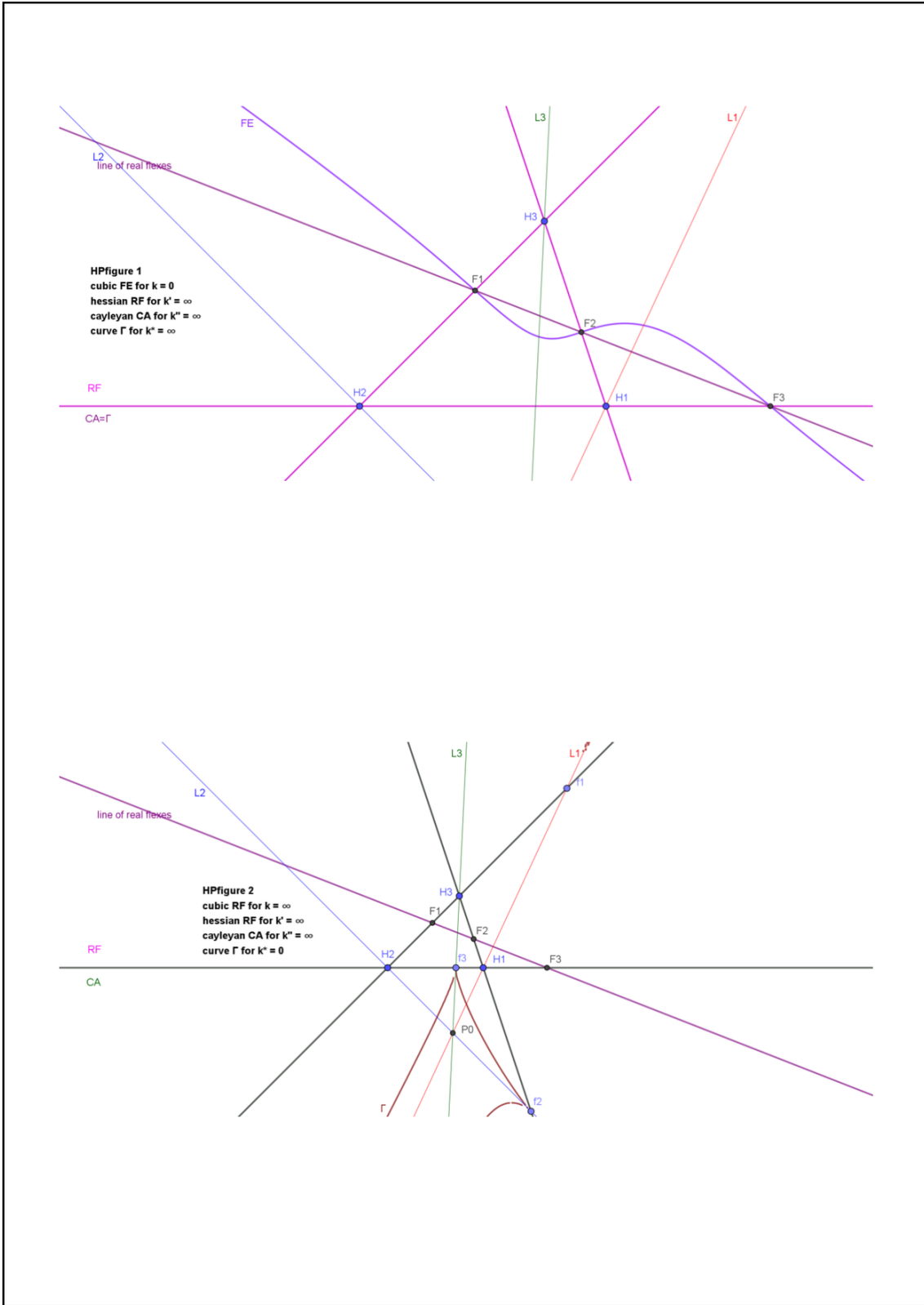
Drawing this way a certain number of curves, it appears that the cusps of the curves Γ are the contact points of HE and CA, which seems to be the dual property of the common tangents to CU and HE being the tangents in the intersections between CU and CA ...

1 The calculations are long and boring !

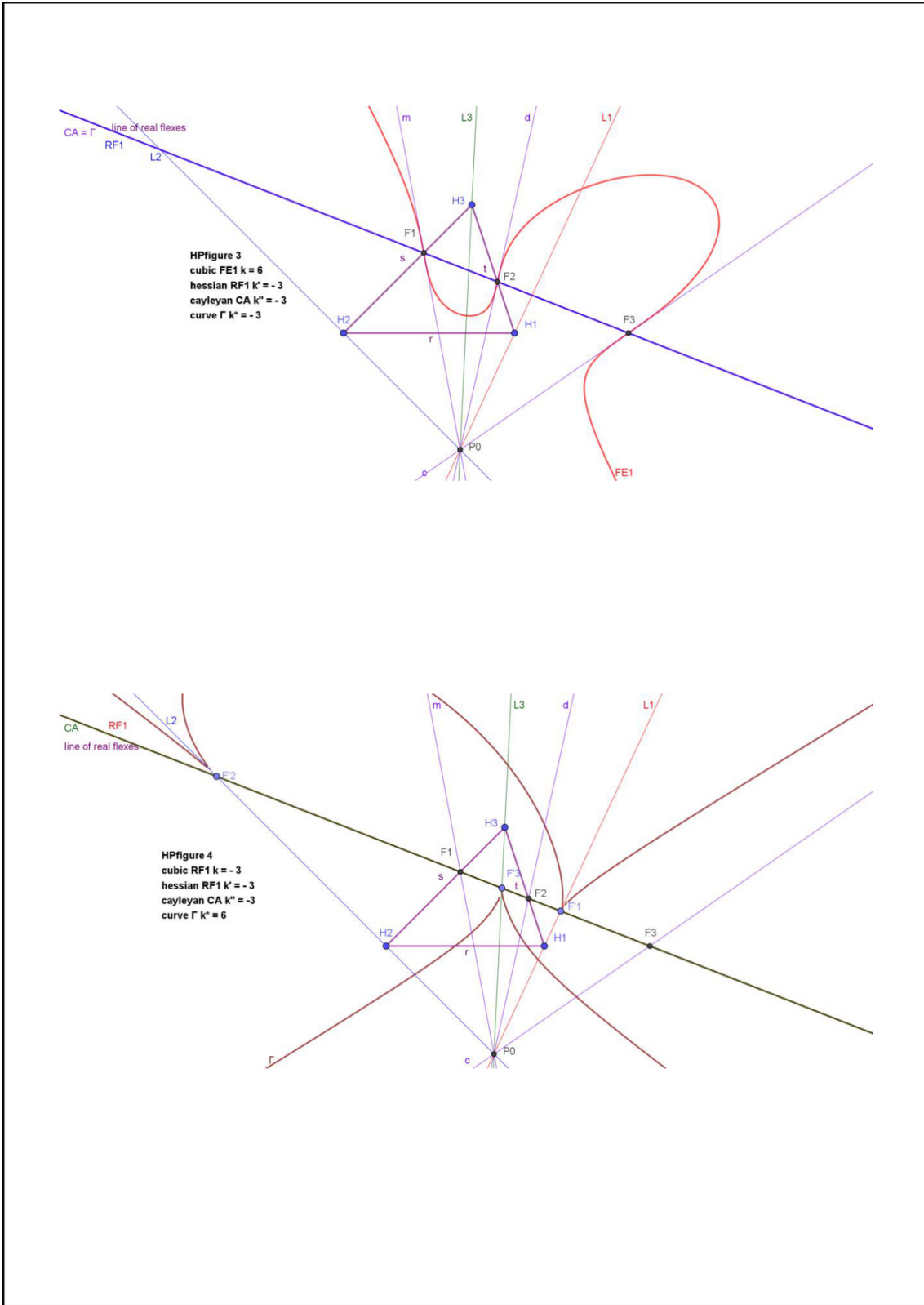
Considering the tangential equation $G(u,v,w)$ of the curve and the tangent $ux + vy + wz = 0$, we know that $G'u/x = G'v/y = G'w/z$ and must eliminate u, v and w in order to find the barycentric equation $F(x,y,z)$ of the curve.

For example, for $G(u,v,w) = u^3 + v^3 + w^3 = 0$ ($k'' = 0$), it's immediate that $u^2/x = v^2/y = w^2/z$ and $x^{3/2} + y^{3/2} + z^{3/2} = 0$. Then $z^{3/2} = - (x^{3/2} + y^{3/2})$, $z^3 = (x^{3/2} + y^{3/2})^2 = x^3 + y^3 + 2 u^{3/2} v^{3/2}$ and last we find $(2 x^{3/2} y^{3/2})^2 = (z^3 - x^3 - y^3)^2$, $x^6 + y^6 + z^6 = 2(x^3 y^3 + x^3 z^3 + y^3 z^3)$ or $(x^3 + y^3 + z^3)^2 = 4(x^3 y^3 + x^3 z^3 + y^3 z^3)$.

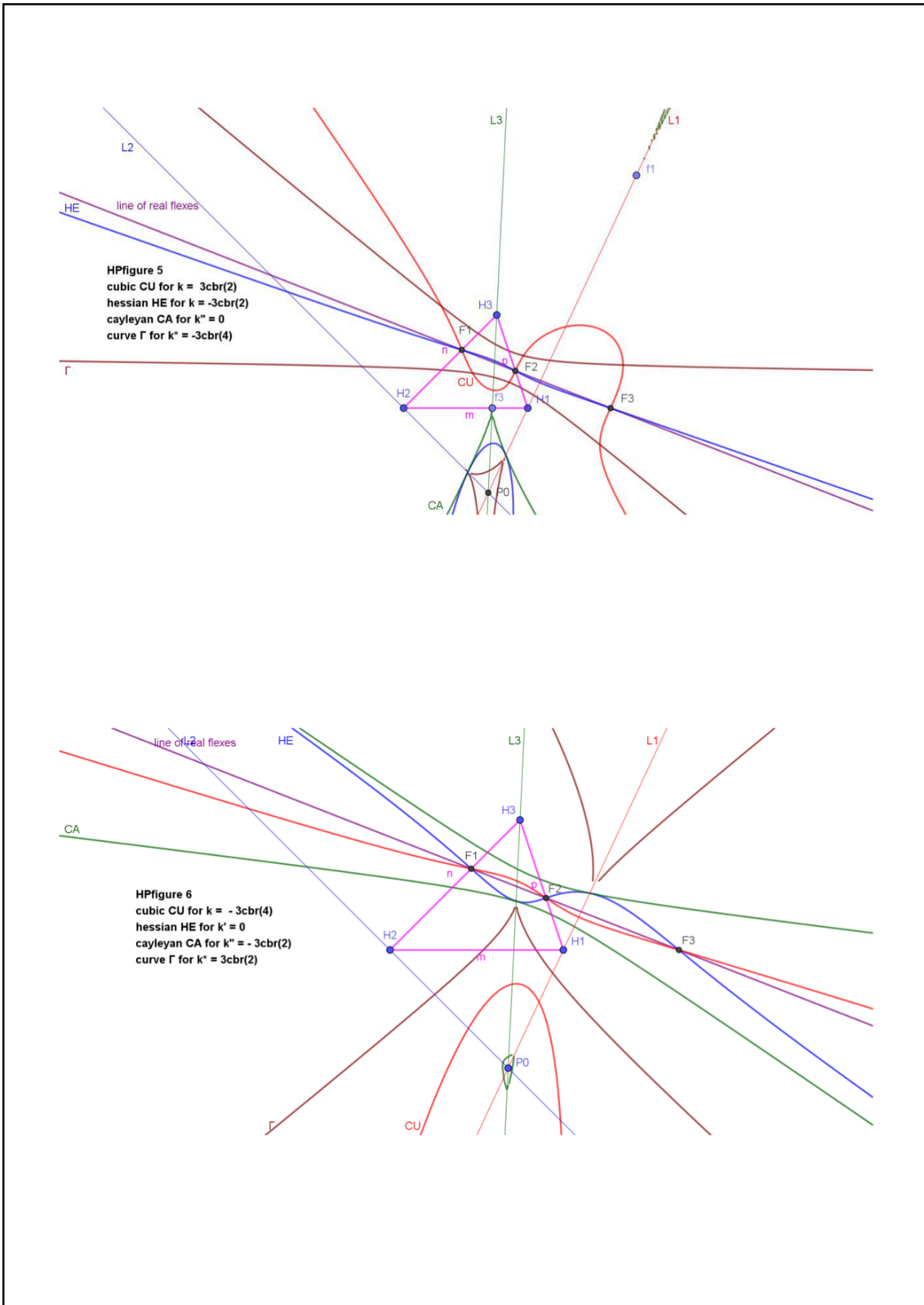
For $G(u,v,w) = uvw = 0$ ($k'' = \infty$), we get $vw/x = uw/y = uv/z$ and the solution of the system is given by $u = yz, v = xz$ and $w = xy$. It follows $uvw = x^2 y^2 z^2 = 0$.



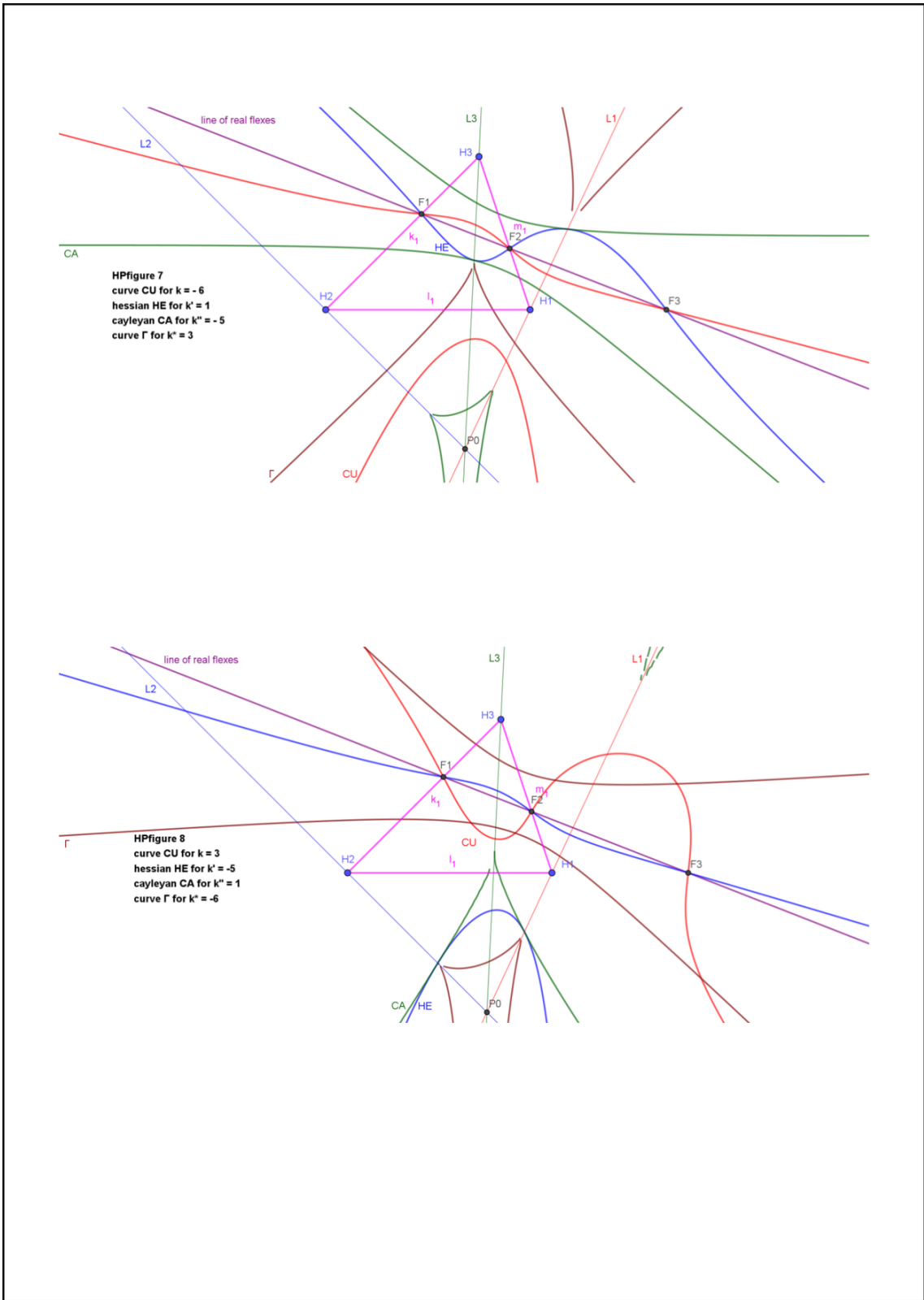
2 dual Hesse pencils.pdf



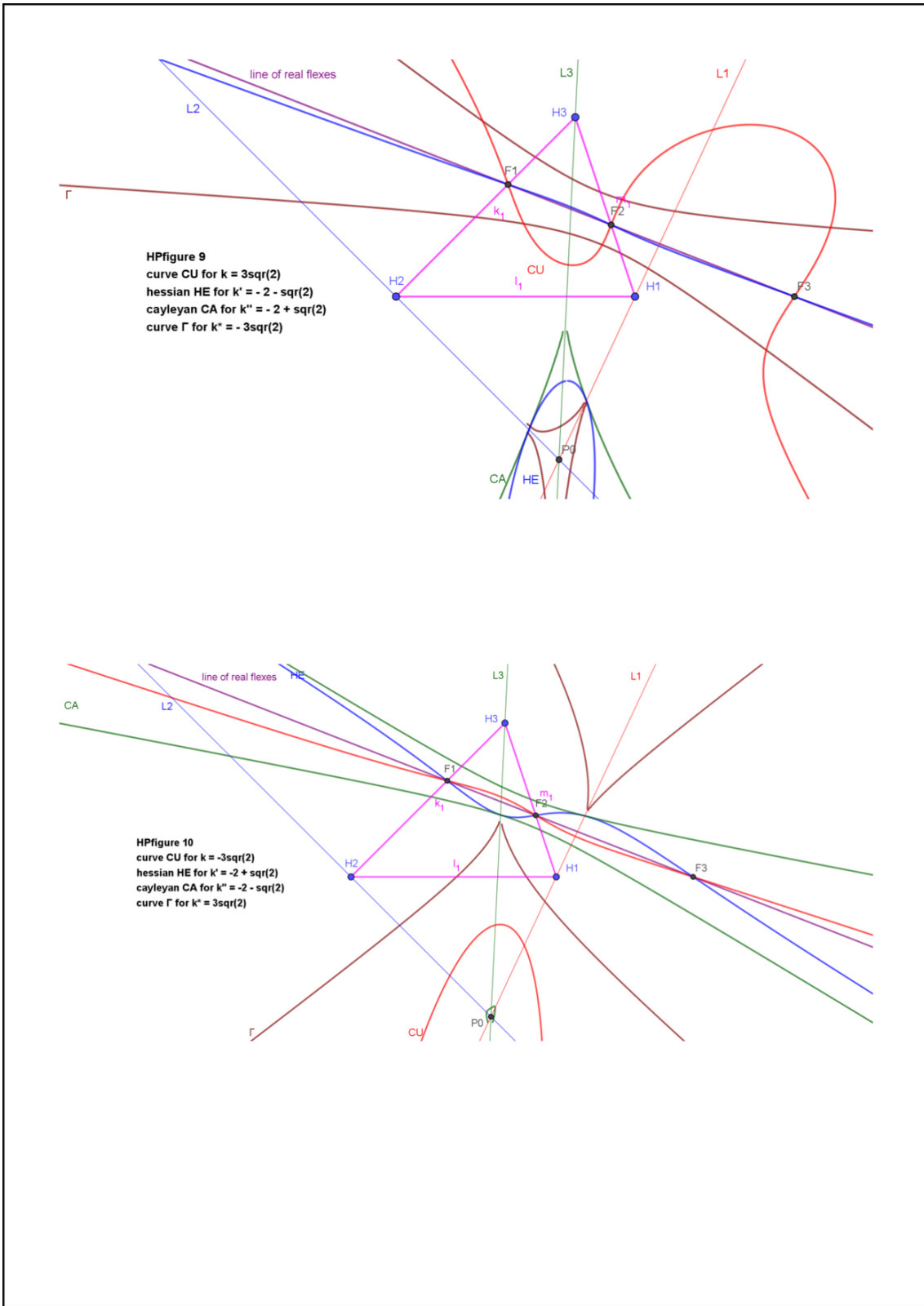
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2 dual Hesse pencils.pdf



2 dual Hesse pencils.pdf



2 dual Hesse pencils.pdf

Message: #2828
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Thank you for the clarification.

You asked me to describe a 9P-cubic as an isocubic with reference triangle $P_1P_2P_3$, defined by an isoconjugation with fixed point/root P_0 and a starting cubic point P .

For the EPG-reader, it's important to have a simple, step-by-step construction with minimal references. However, we can note that the isoconjugation is given by $QA-Tf_2(X)$, where $QA = ABCP$. These are the ingredients for constructing the cubic.

I have reviewed Bernard Gibert's construction of the cubic in Section 1.5.4, and it remains a significant hurdle. Could you provide a step-by-step description, following BG's principles (with a proper reference to his paper), in the simplest possible form?

I think this is worthwhile, as you pointed out, because it would also give us an easy construction of the flex points and harmonic polars. I would then add an extra section to EPG: "CU-6: Different Constructions of a Cubic."

What I need is a straightforward, step-by-step construction given triangle ABC , the isoconjugation fixed point P (root), and another point Q on the cubic, where the isoconjugation is defined as $QA-Tf_2(X)$ with $QA = ABCP$.

It's particularly important to explain how the locus is generated.

Next you could describe the construction of the Hessian.

As an expert in this matter, could you kindly provide that?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2829
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Cayleyan as member of a dual Hesse pencil

Dear Bernard,

I checked the normalized equation of the Cayleyan again and this time I obtained a good result in a numerical example. I have updated the equation of the Cayleyan in CU-5. Thank you very much for the beautiful result.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2830
Date: 2025-11-26
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

sorry, my construction description
... is not the place to explain "isoconjugation with fixpoint"
... nor "construction of nK-isocubic"
... in a better way as Bernard Gibert.

Your interpretation of isoconjugation with fixpoint P is wrong:
... "the isoconjugation is given by $QA-Tf_2(X)$, where $QA=ABCP$."
You have to take the QA with vertices P and anticevians of P wrt ABC,
... if you define isoconjugations with QA-geometry.

My further remark wrt the hessian is of no importance,
... it is an analog application of the described construction,
... but we don't have a usable point Q for the hessian
... without further constructions.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2831
Date: 2025-11-26
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

In EPG I try to present everything in the most elementary way possible.

I personally find BG's description of the construction rather difficult.

I noticed that he also described the isoconjugation, which we can refer to as QA-Tf2.

That is why I had hoped you could provide, with your expertise, a simpler step-by-step description of the remaining steps without going into every detail.

Anyone who wishes to study the subject more deeply can of course consult BG's paper.

Unfortunately, that does not seem possible at the moment.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2832
Date: 2025-11-28
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

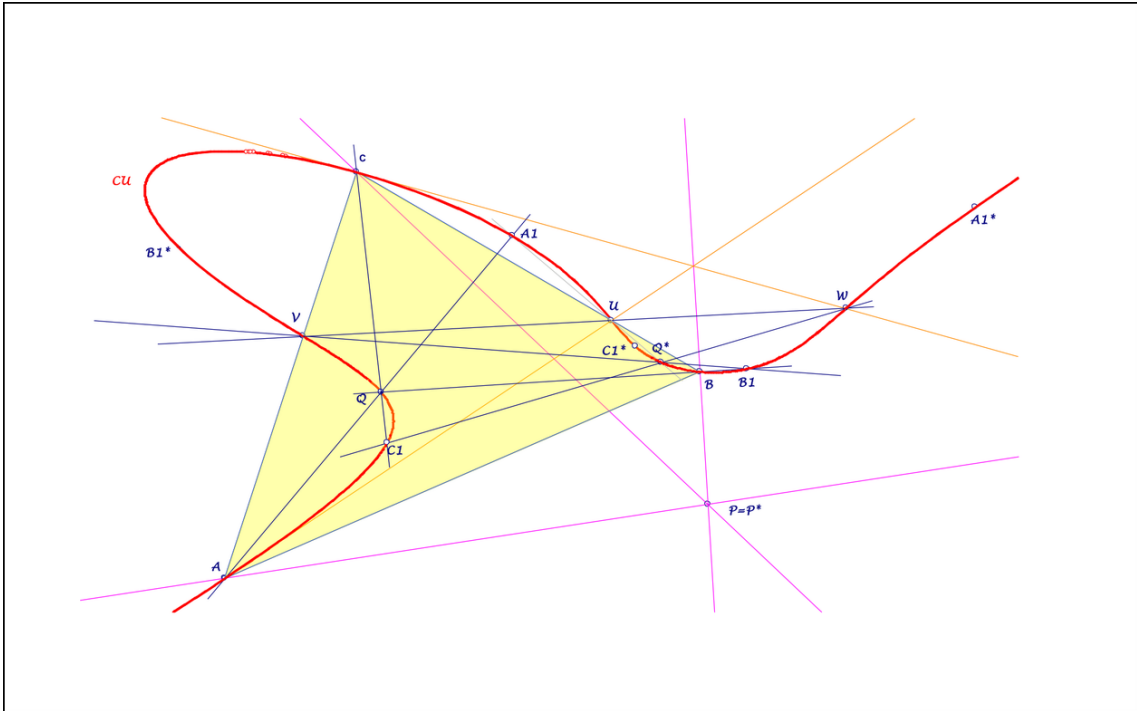
let us furthermore define an isoconjugation wrt a triangle ABC
... (see Wolfram MathWorld or ETC),
... then the fixpoints give a QA with diagonal triangle ABC.

You are right, GB's nK-construction is rather difficult,
... here shortened, but finally using the 9P-construction:

Let us start with a triangle ABC and points
... P as root and fixpoint of an isoconjugation *,
... further a cubic point Q.
Let the tripole of P wrt ABC intersect the sidelines in U,V,W,
... which will be the real flexpoints.
Get the cubic points (see BG 1.5.4)
... $A1 = QA^Q*U$, $B1 = QB^Q*V$, $C1 = QC^Q*W$
... and their isoconjugates $A1^*$, $B1^*$, $C1^*$.
Are this enough points for a 9P-construction?
 $A,B,C,A1,A2,A3,A1^*,B1^*,C1^*,Q,Q^*,U,V,W$.

Using my old 9P-macro and approaching for a 9th point
... I can draw the cubic with every precision,
... but exact for the 9th point my macro fails,
... perhaps it is not perfect or there are bad relations
for the points.

Best regards Eckart



2025-11-27.pdf

[← Previous](#)
[Next →](#)
[↔ Message Index](#)
[↑ Subjects](#)

Message: #2833
Date: 2025-11-29
From: contiwa.goma3@gmail.com
Subject: Re: Desmic System / Reye Configuration

Dear all,

I didn't realized the circumcubic of hidden desmic is expressed as a non-pivotal isocubic

$$(p - P)x(rRy^2 + qQz^2) + (q - Q)y(pPz^2 + rRx^2) + (r - R)z(qQx^2 + pPy^2) + 2(PQR - pqr)xyz = 0$$

when the circumcubic of reference desmic is a pivotal isocubic

$$(p + P)x(rRy^2 - qQz^2) + (q + Q)y(pPz^2 - rRx^2) + (r + R)z(qQx^2 - pPy^2) = 0.$$

The vertices of DTs are parametrized as follows:

$$\begin{aligned} DTux &= (1 : 0 : 0) , DTuy = (P : q : r) , DTuz = (p : Q : R) \\ DTvx &= (0 : 1 : 0) , DTvy = (p : Q : r) , DTvz = (P : q : R) \\ DTwx &= (0 : 0 : 1) , DTwy = (p : q : R) , DTwz = (P : Q : r). \end{aligned}$$

Please regard tetrahedra in #2797 (

https://groups.io/g/Quadri-and-Poly-Geometry/attachment/2797/0/D_ismic%20System%20of%203%20Tetrahedra.pdf) as quadrangles:

$$\begin{aligned} \text{reference desmic} &= \{QAx, QAy, QAz\} \\ \text{hidden desmic} &= \{QAU, QAV, QAW\} \\ QAx &= \{X1, X2, X3, X4\}, QAy = \{Y1, Y2, Y3, Y4\}, QAz = \{Z1, Z2, Z3, Z4\} \\ QAU &= \{U1, U2, U3, U4\}, QAV = \{V1, V2, V3, V4\}, QAW = \{W1, W2, W3, W4\} \\ U1 &= X2X3 \cap Y1Y4 \cap Z1Z4, \dots, \end{aligned}$$

then

$$\begin{aligned} QAx-Tr1 &= \{DTux, DTvx, DTwx\} , QAU-Tr1 = \{DTux, DTuy, DTuz\} \\ QAy-Tr1 &= \{DTuy, DTvy, DTwy\} , QAV-Tr1 = \{DTvx, DTvy, DTvz\} \\ QAz-Tr1 &= \{DTuz, DTvz, DTwz\} , QAW-Tr1 = \{DTwx, DTwy, DTwz\} \\ DTux &= X2X3 \cap X1X4 = U1U4 \cap U2U3, \dots \end{aligned}$$

The circumcubic of the reference desmic is

$$pK(QAx-P16, Px) \text{ wrt } QAx-Tr1 = pK(QAy-P16, Py) \text{ wrt } QAy-Tr1 = pK(QAz-P16, Pz) \text{ wrt } QAz-Tr1,$$

and that of the hidden desmic is

$$pK(QAU-P16, Pu) \text{ wrt } QAU-Tr1 = pK(QAV-P16, Pv) \text{ wrt } QAV-Tr1 = pK(QAW-P16, Pw) \text{ wrt } QAW-Tr1,$$

where

$$\begin{aligned} QAx-P16 &= (pP : qQ : rR) \\ Px &= (p + P : q + Q : r + R), \\ Py &= (P : Q : R), \\ Pz &= (p : q : r) \\ Pu &= (0 : Q - q : r - R), \\ Pv &= (p - P : 0 : R - r), \\ Pw &= (P - p : q - Q : 0). \end{aligned}$$

Cf. Mathews, " Cubic Curves and Desmic Surfaces (<https://www.ams.org/journals/tran/1926-028-03/S0002-9947-1926-1501362-3/S0002-9947-1926-1501362-3.pdf>)."
(I don't intend to step into desmic surfaces.)

Best regards,
M@IMF

p.s. The links of EPG-Ref[11] (Hyacinthos) are wrong: "epizy"s are replaced by "99on"s.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2834
Date: 2025-11-30
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Thanks for your shortened version of the construction of an nK cubic.
I have drafted a new section in EPG. Could you please review it?
See attachment.

Best regards,
Chris

CU-6 Construction of Isocubics

1. Construction of an nK cubic

A non-pivotal isocubic is a cubic curve in the plane of a reference triangle defined by an isoconjugation with respect to a fixed pole, but which does not possess a pivot point. Such cubics are commonly abbreviated as nK.

The notion of *isoconjugation* arises from projective geometry and generalizes classical conjugations such as isogonal and isotomic conjugation.

In trilinear coordinates, let $P=(p:q:r)$ be a fixed point (called the *pole*) and $U=(u:v:w)$ any point in the plane of triangle ABC . The P-isoconjugate of U is the point

$$U^* = (qrvw:rpwu:pquv).$$

See [13].

This mapping is involutive: applying the isoconjugation twice returns the original point U .

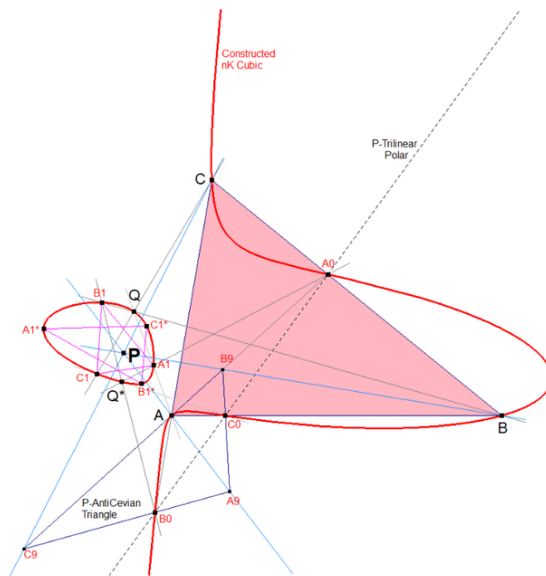
For constructions of the P-isoconjugate see CU-P-Tf1.

The P-Isoconjugate of X is also QA-Tf2(X) wrt QA=PA0B0C0, where A0B0C0 = P-AntiCevian Triangle wrt ABC.

For the construction of an nK cubic we adopt, in part, the method of Bernard Gibert as described in [17b], paragraph 1.5.4. Eckart Schmidt proposed in [66], QPG#2832, applying the first steps of this construction and then continuing with the construction with the 9P-method (see 9P-s-Cu1).

The construction then becomes:

1. Let ABC be the reference triangle and point P a fixed point of the isoconjugation. Furthermore, let Q be an additional point intended to lie on the nK cubic.
2. Let Q^* be the isoconjugate of Q. It can be constructed as QA-Tf2(Q), with QA=(P + vertices P-Anticevian Triangle wrt ABC).
3. Let L_0 be the trilinear polar of P wrt ABC intersecting ABC in points A_0, B_0, C_0 .
4. Construct the following nK-points:
 $A_1 = QA^*Q^*A_0, B_1 = QB^*Q^*B_0, C_1 = QC^*Q^*C_0$.
5. Perform the 9-Point-constructions for the cubic using nine points from the set $(A, B, C, A_0, B_0, C_0, A_1, B_1, C_1, Q, Q^*)$, as described at 9P-s-Cu1.



6. Eventually, the isoconjugates of A_1, B_1, C_1 , denoted A_1^*, B_1^*, C_1^* , can also be constructed using QA-Tf2. These points likewise lie on the nK-cubic.

Advantage of the Construction

This construction has the great advantage that:

- a. A_0, B_0, C_0 are the three real flex points of this cubic (see CU-9P1).
- b. The lines PA, PB, PC are the three real harmonic polars of the cubic (see 9P-9L1).

Message: #2835
Date: 2025-12-02
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

thanks for your elaboration,
... let me use my nomination in #2832,
... proposing 14 point for a 9P-construction of nK,
... finally I succeeded also with my macro in confirming.

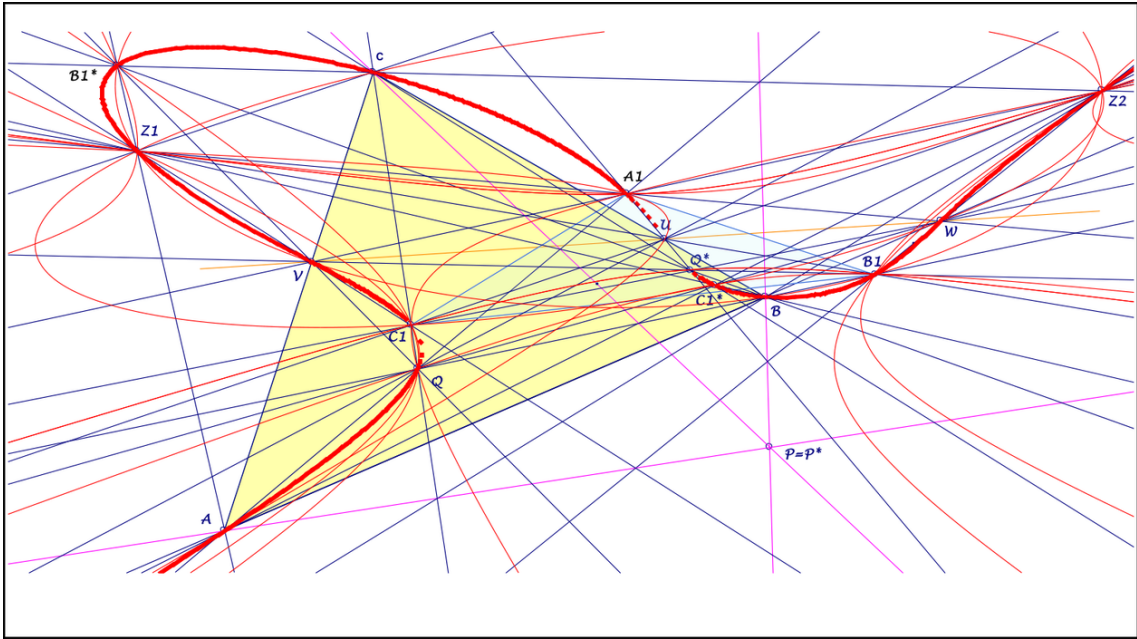
Anymore the points give an interesting constellation,
... for example these two further cubic-points:
Z1 = AB1* ^ BC1* ^ CA1* ^ A1W ^ B1U ^ C1W,
Z2 = AC1* ^ BA1* ^ CB1* ^ A1V ^ B1W ^ C1U.

Lines through Q or Q*, Z1 or Z2:
QAA1, QBB1, QCC1, QA1*U, QB1*V, QC1*W,
Q*AA1*, Q*BB1*, Q*CC1*, Q*A1U, Q*B1V, Q*C1W,
Z1AB1*, Z1BC1*, Z1CA1*, Z1A1W, Z1B1U, Z1C1V,
Z2AC1*, Z2BA1*, Z2CB1*, Z2A1V, Z2B1W, Z2C1U.

Conics
Z1,Z2,A,A1,A1*,U,
Z1,Z2,B,B1,B1*,V,
Z1,Z2,C,C1,C1*,W,
Z1,Z2,A1,B1,C1,Q*,
Z1,Z2,A1,B1,C1*,Q,
Z1,Z2,B1,C1,A1*,Q,
Z1,Z2,B1,C1,A1,Q*,
Z1,Z2,A1,C1,B1*,Q,
Z1,Z2,A1,C1,B1,Q*.

Amazing the diversity of relations!

Best regards Eckart



2025-12-02.pdf

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2836
Date: 2025-12-02
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, dear Eckart

I read carefully the draft of CU-6.

Here some remarks :

The nK construction refers to a triangle ABC and an isoconjugation with a pole and 4 fixed points.

If A_0 , B_0 and C_0 are the 3rd intersections of the sides of ABC with the cubic, they are aligned and the root of the nK is the trilinear pole of this line.

The QA-Tf2 of this line is a conic tritangent to the cubic in A, B and C.

Note that neither the root nor the fixed points are necessary on the cubic)

1) A nK can be mono- or bipartite.

A bipartite cubic is not pivotal *wrt this triangle and this isoconjugation* , but can be pivotal with any of it's points with fixed points the 4 pretangential of the pivot.

(For example, QA-Cu7 is a nK wrt the reference QA ...)

2) The isoconjugate is $pvw : qwu : ruv$

3) $A_0B_0C_0$ is not the anticevian triangle of P (you have to choose another naming)

4) Last and most important, A_0 , B_0 and C_0 are the real flexes of the cubic only if the root is one of the 4 fixed points*.

I suggest that you procede in 2 steps, describing 1st the nK construction of Bernard Gibert (in which the root has an isoconjugate) and 2nd Eckart's special case with the root in one of the 4 fixed points (for a given triangle and a pole inside the triangle, there are 4 real fixed points and 4 different lines of real flexes, which are the trilinear polars of the fixed points).

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2837
Date: 2025-12-05
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

trying to understand Bernard's message #2836
... I studied once more your description of CU-6.

An isoconjugation can be given
... by a reference triangle ABC and a fixed point P
... P is not the pole, defined as isoconjugate of the centroid.

For a pole (p,q,r) holds
 $U(u,v,w) \rightarrow U^*(pvw,quw,ruv)$ (see GB 1.2.1).

For a pivotal isocubic we need the pivot for construction.
... for a non pivotal cubic we need the root for construction,
... and in both cases we need a further cubic point U.

You don't mention the root in 1) of the construction
description,
... but you use the fixed point P as root in 2) and 3),
... for the root has to be a fixed point of the isoconjugation
in my construction.

Perhaps this is the background for Bernard's message,
... which I try in vain to understand.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2838
Date: 2025-12-05
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,

I'm very surprised that you don't understand my message, as you say approximately quite the same as I do!

Provided you have a reference triangle and an isoconjugation with pole isoconjugate of the centroid

1) For a pivotal isocubic you need the pivot, but you don't need a further point U on the cubic.

You have the 4 fixed points, the DT vertices, the pivot and the isopivot and the 3 traces of the pivot on DT.

2) For a non pivotal isocubic, you need the vertices of DT, the root and it's isoconjugate and a further point U on the cubic and it's isoconjugate.

For example QA-Cu7 is a nK wrt the reference QA (the vertices of the QA or fixed points of the isoconjugation are not on the cubic).

3) Your construction is a special case of the non pivotal isocubic with the root in one of the fixed points of the isoconjugation.

In this case and only in this case, the trilinear polar of the root wrt the reference triangle intersects the sides of the triangle in the flexpoints of the cubic.

What is not clear in these 3 points?

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2839
Date: 2025-12-06
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Eckart and Bernard,

Thanks for both responses to my short article on the construction of the non-pivotal isocubic nK. The topic of isocubics is somehow difficult to comprehend, which is why I also now seek to describe them accurately.

Isoconjugation

It all begins with isoconjugation. Eckart once gave some fine descriptions, which I mentioned at CU-P-Tf1 (<https://www.chrisvanantienhoven.nl/epg/polyn-curves/cubics/cu-p-tf1/>). An isoconjugate always requires an additional defining point W in order to map any point X to its conjugate point X^* . For this reason, the isoconjugate is sometimes referred to as the W -isoconjugate when W plays a relevant role.

Isocubic

We then define an *isocubic* as a cubic curve that is invariant under a given isoconjugation. This means that if a point X lies on the cubic, then its isoconjugate X^* also lies on the cubic.

Pivotal and Non-pivotal Isocubics

We distinguish between pivotal isocubics (pK) and non-pivotal isocubics (nK).

* Pivotal isocubics are those in which X and X^* always align with another fixed point on the cubic, called the pivot.

* Non-pivotal isocubics are those in which X and X^* do not align with a fixed point.

It is important to note that this distinction depends on the chosen reference triangle and the isoconjugation associated with it. It may happen that with another reference triangle and another isoconjugation, the same cubic appears pivotal in that constellation.

Why every cubic can be described as Pivotal

Every cubic can be described as pivotal because:

* Given a point X on the cubic, there are four points of tangency arising from tangents drawn from X to the cubic. These points may be real or imaginary.

* These four points form a quadrangle QA, for which QA-Tf2 serves as the isoconjugate. The diagonal triangle of QA (whose vertices also lie on the cubic) can then be regarded as the reference triangle, with QA-Tf2 as its isoconjugate.

* When imaginary components occur, there may be no real four points of tangency, and the processes of isoconjugation also do exist, but take place only in the imaginary domain.

So why can a cubic be classified as non-pivotal? Because not every triangle inscribed in a cubic is the diagonal triangle of a quadrangle of tangency points from a given point on the cubic. Therefore, in the construction of an nK (as described in BG's paper SITP on isocubics, §1.5.4), one seeks another reference triangle and corresponding isoconjugate that allow the cubic to qualify as a pivotal isocubic (pK). From there, the cubic may or may not be constructible as a pK.

Conclusion

My conclusion is that the classification of a cubic as pivotal or non-pivotal is misleading. Whether a cubic is pivotal or non-pivotal depends not on the cubic itself, but on the combination of the cubic and the reference triangle. Meanwhile, the notion of the isoconjugate depends only on the reference triangle and a pole. Every triangle has its own unique -isoconjugate, just as every triangle has its own isogonal conjugate.

BG's Notation

In BG's notation we find the following dependencies:

- * Pivotal isocubic $pK(W,P)$: a pK with pole W and pivot P.
- * Non-pivotal isocubic $nK(W,P,M)$: an nK with pole W, root P and passing through M.

These are the defining factors for pivotal and non-pivotal isocubics.

Construction of $pK(W,P)$

Here a reference triangle is given, together with a pole W that defines the W-isoconjugate. Any point P may be chosen as pivot, from which a pK can be constructed as described in §1.4.3.

Construction of $nK(W,P,M)$

Here again a reference triangle is given, together with a pole W defining the W-isoconjugate.

Next, there is a root P, intended as the trilinear pole of the line through the "third" intersections of the curve with the sidelines of ABC (see SITP §1.5.1).

Finally, a point M on the cubic must be specified to complete the construction.

That concludes my contribution for today regarding the phenomenon of isocubics.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2840
Date: 2025-12-06
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,

I think you should mention that the pole of the isoconjugation is the isoconjugate of the centroid of the reference triangle. This explains why the reference triangle and the pole define together the isoconjugation.

Perhaps also worth mentioning : for a pK , the pivot and the the fixed points of the isoconjugation are on the cubic, but for a nK , the root and the fixed points are not necessarily on the cubic ...

Last and most important, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2841
Date: 2025-12-06
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

Thanks for the additional relevant information you gave.
I have revised my description:

ISOCUBICS

Isocubic

We define an isocubic as a cubic curve that is invariant under a given isoconjugation. This means that if a point X lies on the cubic, then its isoconjugate X^* also lies on the cubic.

Isoconjugation

The isoconjugation constitutes a projective transformation. The isoconjugate always requires an additional defining point W in order to map any point X to its conjugate point X^* . For this reason, the isoconjugate is sometimes referred to as the W -isoconjugate when W plays a relevant role. The isoconjugate can be defined in the context of a reference triangle by specifying a source point and its isoconjugate. From this information each point in the field can be 'isoconjugated' to a corresponding point. The isoconjugate of the centroid, obtained in this manner, is referred to as the pole W . See BG-description Isoconjugation (<https://bernard-gibert.fr/gloss/isoconjugation.html>). The same isoconjugation may alternatively be described using a point K , around which a source point is harmonically reflected. This approach was described by Günther Pickert (see CU-P-Tf1 (https://www.chrisvantienhoven.nl/epg/polyn-curves/cubics/cu-p-t_f1/)). There are 4 fixed points for which source point and its isoconjugate coincide. These fixed points are K (as designated by Günther Pickert) and the three vertices of the K -anticevian triangle.

Pivotal and Non-pivotal Isocubics

We distinguish between pivotal isocubics (pK) and non-pivotal isocubics (nK).

* Pivotal isocubics are those in which X and X^* always align with another fixed point on the cubic, called the pivot.

* Non-pivotal isocubics are those in which X and X^* do not align with a fixed point.

It is important to note that this distinction depends on the chosen reference triangle and the isoconjugation associated with it. It may happen that with another reference triangle and another isoconjugation, the same cubic appears pivotal in that constellation.

Why every cubic can be described as Pivotal

Every cubic can be described as pivotal because:

* Given a point X on the cubic, there are four points of tangency arising from tangents drawn from X to the cubic. These points may be real or imaginary.

* These four points form a quadrangle QA , for which $QA-Tf2$ serves as the isoconjugate. The diagonal triangle of QA (whose vertices also lie on the cubic) can then be regarded as the reference triangle, with $QA-Tf2$ as its isoconjugate.

* When imaginary components occur, there may be no real four points of tangency, but the processes of isoconjugation still do exist, but take place only in the imaginary domain.

So why can a cubic be classified as non-pivotal? Because not every triangle inscribed in a cubic is the diagonal triangle of a quadrangle of tangency points from a given point on the cubic. Therefore, in the construction of an nK (as described in BG's paper SITP on isocubics, §1.5.4), one seeks another reference triangle and corresponding isoconjugate that allow the cubic to qualify as a pivotal isocubic (pK). From there, the cubic may or may not be constructible as a pK .

An Isocubic is not a special type of cubic

Considered in that perspective the classification of a cubic as pivotal or non-pivotal is misleading. Whether a cubic is pivotal or non-pivotal depends not on the cubic itself, but on the combination of the cubic and the reference triangle. Meanwhile, the notion of the isoconjugate depends only on the reference triangle and a pole. Every triangle has its own unique -isoconjugate, just as every triangle has its own isogonal conjugate.

BG's Notation

In Bernard Gibert's notation we find the following dependencies:

* Pivotal isocubic $pK(W,P)$: a pK with pole W and pivot P .

* Non-pivotal isocubic $nK(W,P,M)$: an nK with pole W , root P and passing through M .

These are the defining factors for pivotal and non-pivotal isocubics.

Construction of $pK(W,P)$

Here a reference triangle is given, together with a pole W that defines the W -isoconjugate. Any point P may be chosen as pivot, from which a pK can be constructed as described in §1.4.3.

Construction of $nK(W,P,M)$

Here again a reference triangle is given, together with a pole W defining the W -isoconjugate.

Next, there is a root P , intended as the trilinear pole of the line through the "third" intersections of the curve with the sidelines of ABC (see SITP §1.5.1).

Finally, a point M on the cubic must be specified to complete the construction.

For a pK , the pivot and the fixed points of the isoconjugation are on the cubic, but for a nK , the root and the fixed points are not necessarily on the cubic.

Last but not least, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2842

Date: 2025-12-06

From: eckart_schmidt@t-online.de

Subject: Re: Isocubics

Dear Bernard,

thanks for clearance.

Wrt 1) in #2838: Evidently you are right, excuse my carelessness.

Wrt 3) in #2838: Finally I understood your remark,

... but for me remains the question:

Can I consider every cubic

... as isocubic with root in a fixpoint of the isoconjugation?

Best regards Eckart

PS: Now I shall study your messages #2839-#2841.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2843
Date: 2025-12-06
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

with great interest I have read your description of isocubics,
... my acceptance, this was necessary already years ago, thanks!
Isogonality will be $X(6)$ -isoconjugation, alright?
I think, you can forget my message #2837.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2844
Date: 2025-12-06
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear geometers,

Excuse me for intruding, but are the fixed points of nK
the vertices of dual QA of QL formed by the reference triangle
and the trilinear polar of the root?
By the way, I can't reach QL-8(The Dual QA/QL-configuration) in
EPG.
(QA-8 is OK.)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2845
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear geometers,

I'm sorry. Let me correct my question:
Are the fixed points of nK the root and the vertices of DT of QL formed by the reference triangle and the trilinear polar of the root?

When the root is Cross Point of 3 real Harmonic Polars and the reference triangle is Real Flex Trilateral, 3 real flexes lie on the trilinear polar of the root, I think.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2846
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
I'm not sure I understand your question in 2842.
Every cubic is an isocubic wrt any of it's inscribed triangles with a particular isoconjugation wrt this triangle.
The 3rd intersections of the sides of the triangle with the cubic are aligned and the root is the trilinear pole of this line.
If the triangle is the triangle of the pretangentials of the flexpoints (one triangle for a monopartite cubic, 3 for a bipartite cubic), naturally the root is the trilinear pole of the line of the real flexes and, according to your statement, one of the fixed points of the isoconjugation.
But it is a selfdefining property!
Is that what you mean?
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2847
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear M@IMF <mailto:M@IMF> ,

if we consider a triangle ABC and a point P
... with its trilinear polar p
... and its anticevians Pa,Pb,Pc,
... which are with P the fixed points of an isoconjugation,
... then the dual of Pa wrt the QL(AB,BC,CA,p) is PbPc
... and PaPbPc is the QL-diagonal.
I can confirm your final observation.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2848
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear Mr. Schmidt,

Thank you for the reply.
The colinearity of "3rd intersections" of nK and reference triangle was a bit surprise for me although a cubic can bear any 9 points.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2849
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard

you wrote wrt my question in #2842:
"Every cubic is an isocubic wrt any of it's inscribed triangles with a particular isoconjugation wrt this triangle.
The 3rd intersections of the sides of the triangle with the cubic are aligned and the root is the trilinear pole of this line."

I think, this is not correct.
How do you find a reference triangle for a 9 point-cubic ... its root and isoconjugation for any inscribed triangle?

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2850
Date: 2025-12-07
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris, dear Bernard,

let me describe a way from a 9P-cubic
... to a reference triangle, root and isoconjugation:
Starting with a 9P-cubic CU,
... for the flexpoints F_i three secants
... .. must have 6 coconic further CU-intersections.
Varying a CU-point A with $B = AF_1 \wedge CU$, $C = BF_2 \wedge CU$, $D = CF_3 \wedge CU$,
... we get for $D = A$ the only ref-triangle ABC.
The trilinear pole of $F_1F_2F_3$ wrt ABC is the root P_o ,
... fixpoint of an ABC-isoconjugation,
... which lets the cubic invariant.
Excuse, that it is not an exact construction.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2851
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
I no longer understand you!
If you know the flexpoints on your cubic, you already have the reference triangle (vertices are the pretangentials of the F_i) as well as the root as trilinear pole of the line of real flexes, hence the isoconjugation ...
If you don't know the flexpoints, any inscribed triangle ABC can serve as reference triangle of an isoconjugation. Take a variable point P as fixed points, the 3 other are the vertices of it's anticevian triangle and there is a position of P giving an isoconjugation wrt ABC in which the cubic is invariant. The cubic is then a nK with this isoconjugation wrt this triangle and the root is the trilinear polar of the line $A_0B_0C_0$ of the 3rd intersections of the sides of ABC with the cubic.
Note that the isoconjugation swaps the line $A_0B_0C_0$ with a conic tritangent in A, B and C to the cubic.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2852
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
Sorry, I have to correct my message.
An inscribed triangle ABC is the reference triangle of an isoconjugation only if the tangentials A_1 , B_1 and C_1 of A, B and C are aligned.
In this case, the 3rd intersections A_0 , B_0 and C_0 are also aligned (as a result of the calculations on a cubic).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2853
Date: 2025-12-07
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart and M@IMF ,
Back to the question in 2844 and 2845 and the answer in 2847!
The dual QA of the QL formed by ABC and the trilinear polar of P wrt ABC is formed by the perspector of the DT and the 4 triangles formed by 3/4 lines.
One vertice of the QA is P, obviously perspector of ABC and PaPbPc.
The 3 other vertices are the vertices of the anticevian triangle of P wrt PaPbPc.
It's remarkable that we have to consider 2times the anticevian triangle of P, 1rst wrt ABC, in order to get PaPbPc and 2nd wrt PaPbPc!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2854
Date: 2025-12-07
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear Mr. Keizer,

I usually consider in DT-coordinates, so it came to my mind that a line of QL is the trilinear polar of the perspector of DT and the component triangle formed by the other 3 lines wrt DT. That's why I mixed up and had to correct my question. For me, a nK's bearing vertices of a QL is an eye-opener.

Last month I understood what Flexlines were at last. The very next day new EPG launched. This month I'm learning Cubics without tears.

I was overwhelmed by your "Hessian and generalised S-points for a pivotal isocubic" again when I studied Hessian and PreHessian. I skipped that part before(#2736,37).

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2855
Date: 2025-12-08
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

thanks for your last three messages,
... your numerous observations show a good overview.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2856
Date: 2025-12-09
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear M@IMF,

It is impressive how well you have familiarized yourself with the subject of cubics without prior involvement, and I particularly appreciate the fresh perspectives you bring.

Regarding QL-8, the information is the same as for QA-8. Previously, an incorrect link was attached to QL-8, but it has now been fixed and links correctly.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2857
Date: 2025-12-09
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard, Eckart and M@IMF,

Some questions.

QPG#2840 Bernard

Last and most important, if the root is one of the fixed points, the "3rd" intersections of the curve with the sidelines of ABC are the real flexes of the cubic. Can you explain or proof this?

QPG#2850 Eckart

Varying a CU-point A with $B = AF1 \wedge CU$, $C = BF2 \wedge CU$, $D = CF3 \wedge CU$,
... we get for $D = A$ the only ref-triangle ABC.
That's quite a statement. Can you proof, that it is the **only** ref-triangle ABC on CU that defines an nk?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2858
Date: 2025-12-09
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear Chris,

Wow! Thank you.
To tell the truth, I misunderstood Mr. Schmidt's message #2847.
(Please don't ask what it is. How stupid I was.)

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2859
Date: 2025-12-09
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
All these properties are linked!
Any triangle ABC inscribed in CU can be the reference triangle of a nK, provided the tangentials A1, B1 and C1 of A, B and C are aligned.
But I don't know how to construct the fixed points.
In this case, the 3rd intersections A0, B0 and C0 are also aligned, A1, B1 and C1 being their tangentials.
There is a conic tritangent in A, B and C to the cubic, which is the isoconjugate of the line A0B0C0 (BG page 11).
If A0 = A1, it is a flex, the same for B0 = B1 or C0 = C1.
If the lines A0B0C0 and A1B1C1 coincide in the line of real flexes, ABC is Eckart's triangle, but it is not the only triangle for which the cubic is a nK
Let's use the calculations on the cubic.
It follows from the definition that $A + B + C0 = M$ and so on and that $2A + A1 = M$ and so on ...
If $A1 + B1 + C1 = M$, $2A + 2B + 2C = 2M$ (conic tritangent to the cubic).
Then in turn, $A0 + B0 + C0 = M$.
 $2A = 2A0$ and A and A0 have the same tangential, which is A1 and so on ...
If $A0 = A1$, we have $3A0 = M$, which means that A0 is a flex ...
I don't know why, if the line A0B0C0 (or A1B1C1) is the line of real flexes, the trilinear pole of A0B0C0 is one of the fixed points of the isoconjugation.
You have to ask Eckart, as it is his statement since the beginning!
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2860
Date: 2025-12-10
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

all my results are only observations of drawings,
... I think, there is only one solution for the triangle.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2861
Date: 2025-12-10
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Is it correct that the isoconjugate of the line of real flexes is a conic tritangent to the cubic in A, B and C where A, B and C are the pretangentials of the flexpoints and one fixed point of the isoconjugation is the root, trilinear pole of the line of real flexes?

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2862
Date: 2025-12-10
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,
your description of our discussed configuration is correct.
Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2863
Date: 2025-12-11
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

an observation:

For a cubic we consider
... a ref-triangle ABC,
... an isoconjugation with fixed points,
... a root P_o
... a further cubic-point Q.
The isoconjugation can be described
... with the root P_o as fixpoint.
Or the isoconjugation can be described
... with a fixpoint, not necessary the root,
... for the root has to be that fixpoint,
... which don't lie on a cubic-tangent for A,B,C.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2864
Date: 2025-12-11
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear all,

I hope this will be some help. (cf. SITP 1.5.1)

Let the pole of nK be $(uu' : vv' : ww')$, then

$nK: vw(w'y+v'z)x^2 + wu(u'z+w'x)y^2 + uv(v'x+u'y)z^2 + kxyz = 0$
 vertices of reference triangle:

$$A(1 : 0 : 0), B(0 : 1 : 0), C(0 : 0 : 1)$$

root and its isoconjugate:

$$P = (u : v : w), P^* = (u' : v' : w')$$

their trilinear polars:

$$IP(P) = (1/u : 1/v : 1/w), IP(P^*) = (1/u' : 1/v' : 1/w')$$

feet of $IP(P)$:

$$U = (0 : v : -w), V = (-u : 0 : w), W = (u : -v : 0)$$

feet of $IP(P^*)$:

$$U' = (0 : v' : -w'), V' = (-u' : 0 : w'), W' = (u' : -v' : 0)$$

circumconic $IP(P)^*$:

$$u'/x + v'/y + w'/z = 0$$

tangents of nK at A, B, C :

$$(0 : w' : v'), (w' : 0 : u'), (v' : u' : 0)$$

tangent of nK at U :

$$(u(wv' + vw') - k : -uu'w : -uu'v)$$

tangent of nK at V :

$$(-vv'w : v(uw' + wu') - k : -vv'u)$$

tangent of nK at W :

$$(-ww'v : -ww'u : w(vu' + uv') - k)$$

tangential of A :

$$A\tilde{=} = (uu'(wv' - vw') : v'[u(wv' + vw') - k] : -w'[u(wv' + vw') - k])$$

tangential of B :

$$B\tilde{=} = (-u'[v(uw' + wu') - k] : vv'(uw' - wu') : w'[v(uw' + wu') - k])$$

tangential of C :

$$C\tilde{=} = (u'[w(vu' + uv') - k] : -v'[w(vu' + uv') - k] : ww'(vu' - uv'))$$

line through $A\tilde{=}, B\tilde{=}, C\tilde{=}$:

$$([u(wv' + vw') - k]/u' : [v(uw' + wu') - k]/v' : [w(vu' + uv') - k]/w')$$

$IP(P) \cap IP(P^*)$:

$$(uu'(wv' - vw') : vv'(uw' - wu') : ww'(vu' - uv'))$$

($U = A0, V = B0, W = C0, A\tilde{=} = A1, B\tilde{=} = B1, C\tilde{=} = C1$ in #2859.)

When $P = P^*$ (i.e. the root is a fixed point of isoconjugation), the nK becomes

$$(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z' = 0,$$

and its Hessian is

$$2(k'-1)(x'^3 + y'^3 + z'^3 - 3x'y'z') + (k'^2)[(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z'] = 0 ,$$

where

$$x' = x/u, y' = y/v, z' = z/w, k' = k/2uvw.$$

The real flexes are $(0 : v : -w)$, $(-u : 0 : w)$, $(u : -v : 0)$, that is, U, V, W .

If $k'=1$, the Hessian coincides nK

$$(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2x'y'z' = 0,$$

i.e.

$$(wy + vz)(uz + wx)(vx + uy) = 0.$$

I think this will be Real Flex Trilateral.

Its vertices are $(-u : v : w)$, $(u : -v : w)$, $(u : v : -w)$.

I'm trying to calculate other flexes now.

Best regards, M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2865

Date: 2025-12-12

From: contiwa.goma3@gmail.com

Subject: Re: Isocubics

Dear all,

I'm afraid the last paragraph of #2864 is misleading.

It only says (singular/degenerated) nK

$$(wy + vz)(uz + wx)(vx + uy) = 0$$

is its Real Flex Trilateral. (Maybe trivial.)

I thought it has something to do with nK with $k' <> 1$, but it doesn't.

Their flexes are different, only real ones are same.

I'm wondering a triangle formed by 3 real flextangents will play an important role.

For the nK whose root is a fixed point, its vertices are

$$(uvw - k : vw^2 : vw^2), (wu^2 : uvw - k : uw^2), (vu^2 : uv^2 : uvw - k).$$

Note that if it is the reference triangle, the cubic is no longer isocubic.

Best regards, M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2866
Date: 2025-12-12
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

In QPG #2859 you write: 'Any triangle ABC inscribed in CU can be the reference triangle of a nK, provided the tangentials A1, B1 and C1 of A, B and C are aligned.'

Some comments:

1. I inscribed an arbitrary triangle ABC an arbitrary 9-point cubic, but in such a way that the third intersections of the sidelines of the inscribed triangle are collinear on a line L. From this line L I constructed the trilinear pole W. According to the theory, W is then the isoconjugate of the centroid of ABC. This determines the isoconjugation, and therefore allows us to determine other isoconjugates as well.

However, when I take a point X on the cubic and compute its isoconjugate, that point does not lie on the cubic. This shows that an arbitrary inscribed triangle in a general cubic does not automatically make the cubic an nK. This raises the question: what conditions must an inscribed triangle satisfy, within a general cubic, in order for the cubic to become an nK?

2. You also said you don't know how to construct the fixed points in the configuration of an inscribed triangle ABC provided the tangentials A1, B1 and C1 of A, B and C are aligned.

I think this can be done by combining the different constructions of the isoconjugate.

The isoconjugate as described by BG has a pole. The isoconjugate as described by GP has a point K, which I will refer to as the isoconjugate center. This point is one of the four fixed points. The other three fixed points can be obtained as the vertices of the anticevian triangle of K with respect to ABC.

Thus the question becomes: how can we determine the isoconjugate center K from the pole W?

There is another construction of the isoconjugate, which I have called the Perspective Conjugate (TR-Tf3 (<https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf3/>)) within the theory of Perspective Fields. The idea is that the traces of the two isoconjugated points P and Q on the sidelines of the triangle are perspectively reflected around the trace, on the same sideline, of the isoconjugate center M.

Now, when we know X2 and W (isoconjugated points) and want to determine M (the isoconjugate center), the construction tells us that the traces of X2 and W are reflections of each other around the trace of M. Thus the trace of M is the perspective center of the traces of X2 and W. To determine this midpoint, we need the vanishing point V on the corresponding sideline, which is obtained as the intersection of that sideline with the trilinear polar of M with respect to ABC.

This is constructible, and therefore M is determined, and with it the four fixed points.

See TR-Tf4 (<https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf4/>) for the construction.

Best regards,

Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2867
Date: 2025-12-13
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Chris,

I think, I have answered your first #2866-question to Bernard in #2850.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2868
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,
In point 1, the trilinear pole of the line is not the pole' but
the root of the nK !
In point 2, thanks for this construction I didn't know ...
I'm not home, I'll make you a complete answer as soon as
possible.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2869
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear M@IMF ,
I'm impressed by your barycentric calculations.
The triangle mentioned in the end is not the triangle of real
flexlines!
It is the triangle with sides the tangents in A, B and C to the
cubic.
Furthermore, it is the anticevian triangle of the root P wrt the
triangle ABC and it is perspective to this triangle with
perspector the isoconjugate P^* of P.
This gives the 2 isoconjugate points wrt ABC needed in order to
determine the 4 fixed points of the isoconjugation (not
necessarily real and not necessarily on the cubic).
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2870
Date: 2025-12-13
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,

I think it is now possible to make a short summary of the mystery of the cubic.

II hope it will answer all the last remaining questions.

1) 2 points X and X^* are said conjugate (Salmon or Schroter) if they share the same tangential X_1 , which is the tangential of the 3rd intersection of the line XX^* with the cubic.

$$2X = 2X^* = M - X_1 \text{ and } 2(M - X - X^*) = 2(M - 2X) = 2X_1$$

The transformation swapping X and X^* is a QMT (quasi-Moebius transformation) studied earlier in the Forum. There are 3 such transformations on each cubic (one real for monopartite and 3 for bipartite).

2) All these couples of conjugates are isoconjugate wrt any triangle ABC inscribed in the cubic, provided the tangentials A_1, B_1 and C_1 of A, B and C are aligned.

In this case, the 3rd intersections A_0, B_0 and C_0 are the isoconjugates of A, B and C and are also aligned and the root P is the trilinear pole of this line. The triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of P and is perspective to ABC with perspector P^* the isoconjugate of P .

The 2 points P and P^* determine the isoconjugation wrt ABC and it's possible to find the 4 fixed points.

3) If for one particular triangle ABC the root P is one of the fixed points of the isoconjugation, $P = P^*$ and A_0, B_0 and C_0 and A_1, B_1 and C_1 coincide, which means that these 3 points are the flexes of the cubic.

This observation of Eckart is here fully explained and proved.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2871
Date: 2025-12-14
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

wrt 2) in #2870: I repeat my conjecture,
... that there is only one triangle with your condition,
... see #2850, which can also be formulated:
... $AF2 \wedge CU$ and $AF3 \wedge CU$ have to be collinear with $F1$.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2872
Date: 2025-12-14
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,
Um Gottes Willen, can you read the message completely!
Your triangle is my point 3, but there are an infinity of
triangles in point 2 wrt which the cubic is a nK and $*A0, B0$ and
 $C0$ are not the flexes* !!!
In this case, the root has an isoconjugate (see BG).
I'm really desperate, as I don't see how to make you understand
what I mean.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2873
Date: 2025-12-15
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

already the first passage of your #2870 seems wrong,
... I think the sentence has to end:
... "whose tangential is the 3rd intersection
of the line XX^* with the cubic."

I made a drawing of this constellation, starting with a 9P-cubic
... and constructed then as described in #2850
the isoconjugation,
... which gave the same result as your corrected 1).

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2874
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Eckart,

Thanks for your attention, I prefer you this way ...

- 1) The correct sentence is which is the **conjugate** (and not tangential) of the 3rd intersection of XX^* with the cubic
- 2) There is another mistake in point 2 the triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of P^* (and not P) and is perspective to ABC with perspector P^* , P^* being the isoconjugate of P.
- 3) Naturally in point 3, as $P = P^*$ being a fixed point of the isoconjugation, the anticevian triangles of P and P^* coincide and their vertices are the 3 other fixed points.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2875
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear M@IMF ,

I made the same mistake in my messages 2869 and 2870

The triangle formed by the tangents in A, B and C to the cubic is the anticevian triangle of **P*** (and not P).

If $P = P^*$ in your calculation, the pole is (u^2, v^2, w^2) and the root and fixed point is (u, v, w) , the 3 other fixed points being $(-u, v, w)$, $(u, -v, w)$ and $(u, v, -w)$.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2876
Date: 2025-12-15
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear Mr. Keizer,

Thank you for the message.

Since I'm studying mainly $P=P^*$ case, I didn't notice the mistake.

$Po(=P=P^*)$ is the intersection of 3 real Harmonic Polars as well as the trilinear pole of the main real Flexline (, which through 3 real flexes) wrt the reference triangle. The anticevian triangle of Po is flextangent trilateral of preHessian, isn't it?

Best regards,
M@IMF

p.s. What I feared happened. That's why I posted #2865.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2877
Date: 2025-12-15
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear M@IMF ,
Your last proposition is correct
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2878
Date: 2025-12-15
From: van10hoven@gmail.com
Subject: Re: Isocubics

Dear Bernard,

I am trying to understand your messages #2870 and #2874. I began by drawing a random 9-point cubic together with an inscribed triangle ABC, and I constructed the corresponding tangential points A1, B1, C1, arranging them so that they are collinear.

Next, from the sidelines AB, BC, CA, I constructed the third intersection points C0, A0, B0, which indeed turn out to be collinear.

From the line A0B0C0 I then constructed the trilinear pole P with respect to triangle ABC.

According to #2870/#2874, the following should hold: the triangle formed by the tangents at A, B, C to the cubic is the anticevian triangle of P*.

My question is: why is this point P* (the isoconjugate of P)? What is the underlying theory?

Once this is understood and verified, the two points P and P* determine the isoconjugation with respect to ABC, and it becomes possible indeed to locate the four fixed points.

This can be done by using BG's construction in <https://bernard-gibert.fr/gloss/isoconjugation.html>, as well as the construction in TR-Tf4 (<https://www.chrisvantienhoven.nl/epg/n-geometry/triangle-geometry/persp-fields/tr-tf4/>) like I noted before.

I have not yet carried out this construction to verify the validity of the statements above.

These are my findings so far.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2879
Date: 2025-12-16
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard,

finally I have now reproduced your constructions in #2870.
In my message #2850 I described the case,
... that the tangentials of A, B, C are the collinear real
flexpoints.

Best regards Eckart

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2880
Date: 2025-12-16
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,
The simplest way to find a suitable triangle ABC inscribed in a
general cubic (here monopartite) is to draw a line, which
intersect the cubic in 3 points A1, B1 and C1.
The tangents from A1, B1 and C1 contact the cubic in A and A0, B
and B0 and C and C0 (3 couples of conjugate points).
It appears that these 6 points form a QL and there are in fact 4
possible triangles, let say ABC, AB0C0, A0BC0 and A0B0C, the
root being the trilinear pole of the 4th line A0B0C0, A0BC, AB0C
and ABC0.
For Chris, the properties mentionned for P and P* are explained
page 11 of Bernard Gibert.
They can be proved easily with a barycentric calculation like
the one made by I@MIF...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2881
Date: 2025-12-16
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear all,

I calculated real flexline trilateral of nK whose root is a fixed point of isoconjugation.

When the nK is

$$(y/v + z/w)(x/u)^2 + (z/w + x/u)(y/v)^2 + (x/u + y/v)(z/w)^2 + 2k'xyz/uvw = 0$$

(cf. #2864

(<https://groups.io/g/Quadri-and-Poly-Geometry/message/2864>),

the vertices of real flexline trilateral are

$$(-u(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega) : v : w)$$

$$(u : -v(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega) : w)$$

$$(u : v : -w(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega)),$$

where ω is a primitive cubic root of unity and

η is a unit complex number which satisfies

$$\eta^3 + (3 - 2k'/\omega)/(1/\omega - \omega) \eta^2 - (3 - 2k'/\omega)/(\omega - 1/\omega) \eta - 1 = 0.$$

I hope there is no mistake.

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2882
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF ,
This time, I'm no longer only impressed, but I'm totally bluffed
by your barycentric calculations!
Sincere congratulations
I have now 2 questions :
1) Could you explain how you managed this (at last a starting
point, in order to understand where this strange η comes from)
(I was only able to verify that H1 is on the P0A and that H2H3
passes through F1)
2) I would be completely satisfied if you could express the
equation of the nK with the referential H1(1,0,0), H2(0,1,0),
H3(0,0,1),
id est to reverse from the coordinates of H1,H2 and H3 wrt the
referential A,B,C to the coordinates of A,B,C wrt H1,H2,H3
(in the referential H1, H2, H3, as you know P0 has coordinates
1/a1,1/a2,1/a3, the line of real flexes is $a_1x + a_2y + a_3z = 0$
and the cubic is $a_1^3x^3 + a_2^3y^3 + a_3^3z^3 + ka_1a_2a_3xyz = 0$)
Many thanks in advance
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2883
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris, Eckart and I@IMF ,
Using the preceding property, it is perhaps interesting to
notice that the cubic QL-Cu1 is a nK wrt each of the 4 reference
triangles with root the trilinear pole of the 4th line wrt this
triangle ...
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2884
Date: 2025-12-17
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear Mr. Keizer and all,

Let me explain 1) in #2882.

< nK and Hessian >

We start from 2 equations in #2864 (https://groups.io/g/Quadri-a_nd-Poly-Geometry/topic/116466734?page=2#msg2865) :

$$(1) (y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z' = 0$$

$$(2) 2(k'-1)(x'^3 + y'^3 + z'^3 - 3x'y'z') + (k'^2)[(y'+z')x'^2 + (z'+x')y'^2 + (x'+y')z'^2 + 2k'x'y'z'] = 0.$$

(The primes are annoying, but I retain them.)

< Flexes >

The flexes are the intersections of (1) and (2). We solve (1) and the next equation

$$(3) x'^3 + y'^3 + z'^3 - 3x'y'z' = 0, \text{ which leads to}$$

$$(3a) x' + y' + z' = 0$$

$$(3b) x' + y'\omega + z'/\omega = 0$$

$$(3c) x' + y'/\omega + z'\omega = 0,$$

where ω is a primitive cubic root of unity.

>From (3a) and (1), we get

$$(x' : y' : z') = (0 : 1 : -1), (-1 : 0 : 1), (1 : -1 : 0),$$

which are real flexes.

>From (3b),

$$x' = -y'\omega - z'/\omega$$

is obtained and substituting it into (1) yields

$$(4) y'^3 + (3 - 2k'\omega)/(1/\omega - \omega) z'y'^2 - (3 - 2k'/\omega)/(\omega - 1/\omega) y'z'^2 - z'^3 = 0.$$

Let Y_j ($j=1,2,3$) be the solutions of 3rd degree equation

$$(5) Y^3 + (3 - 2k'\omega)/(1/\omega - \omega) Y^2 - (3 - 2k'/\omega)/(\omega - 1/\omega) Y - 1 = 0,$$

$$\text{then } (x' : y' : z') = (-Y_j\omega - 1/\omega : Y_j : 1).$$

>From (3c) and (1), we get

$$(6) y'^3 + (3 - 2k'/\omega)/(\omega - 1/\omega) z'y'^2 - (3 - 2k'\omega)/(1/\omega - \omega) y'z'^2 - z'^3 = 0,$$

$$\text{and } (x' : y' : z') = (-1/Y_j\omega - \omega : 1/Y_j : 1).$$

< Flexlines >

A line through $(-Y_j\omega - 1/\omega : Y_j : 1)$ and $(-1/Y_j\omega - \omega : 1/Y_j : 1)$ is

$(l' : m' : n') = (\omega(1 + Y_j)/(1 + Y_j/\omega) : 1 : 1)$

which is also through $(0 : 1 : -1)$.

Other flexlines are

$(1 : \omega(1 + Y_j)/(1 + Y_j/\omega) : 1)$, $(1 : 1 : \omega(1 + Y_j)/(1 + Y_j/\omega))$.

If $1/Y_j = Y_j^*$ (i.e. $|Y_j| = 1$), $\omega(1 + Y_j)/(1 + Y_j/\omega)$ is real, where * means complex conjugate.

>From (5), we can show $1/Y_j$ and Y_j^* satisfy the same equation,

therefore $\{1/Y_1, 1/Y_2, 1/Y_3\} = \{Y_1^*, Y_2^*, Y_3^*\}$. Since

$(1/Y_1, 1/Y_2, 1/Y_3) = (Y_2^*, Y_3^*, Y_1^*)$ and $(1/Y_1, 1/Y_2, 1/Y_3) = (Y_3^*, Y_1^*, Y_2^*)$

are impossible, *at least 1 solution is a unit complex number*.

Let η denote it.

(I don't know whether $(1/Y_1, 1/Y_2, 1/Y_3) = (Y_1^*, Y_2^*, Y_3^*)$ is possible.)

< Real Flexline Trilateral >

The real flexlines are

$(1 : 1 : 1)$ and

$(\omega(1 + \eta)/(1 + \eta/\omega) : 1 : 1)$

$(1 : \omega(1 + \eta)/(1 + \eta/\omega) : 1)$

$(1 : 1 : \omega(1 + \eta)/(1 + \eta/\omega))$.

Finally, the vertices of real flexline trilateral are

$(-(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega) : 1 : 1)$

$(1 : -(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega) : 1)$

$(1 : 1 : -(1 + \omega)(1 + \eta\omega)/(1 + \eta/\omega))$.

When I calculated flexes before(#2865), I didn't realize the property of (5).

Then I tried to let the flex-tangent trilateral mediate ABC and H1H2H3.

But I came up with above calculation, so I didn't do that.

Please give me some time to prepare for answering 2).

Best regards,

M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2885
Date: 2025-12-17
From: van10hoven@gmail.com
Subject: Re: Isocubics

Beste Bernard, M@IMF and Eckart,

There is something I cannot understand. In SITP 1.2.1 it says:

“We now define the pole of the isoconjugation as the isoconjugate $\Omega = G^*$ of the centroid G . In other words, Ω is the intersection of the two polar lines of G in the two conics $\gamma (P)$ and $\gamma (P^*)$. This shows that there is no need of coordinates to define an isoconjugation. Nevertheless, since a lot of computation is needed for this paper, we will make use of barycentric coordinates and, if $\Omega = (p : q : r)$, the isoconjugation with pole Ω is the mapping :

$\varphi_{\Omega} : M(x:y :z) \rightarrow M^* p x : q y : r z \sim (p y z : q z x : r x y)$ ”

Günther Pickert’s Isoconjugate (QPG #2068) uses a fixed point K , and after carrying out his construction the result is $(p^2 y z : q^2 z x : r^2 x y)$.

I therefore conclude that Pickert’s point K is different from the pole described by BG . Can you confirm this?

And one more question for M@IMF: In your message #2864 you begin with “Let the pole of nK be $(uu' : vv' : ww')$ ”. Why do you start with this combined expression?

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2886
Date: 2025-12-17
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,

If the pole of the isoconjugation is (p^2, q^2, r^2) , then the fixed points are (p, q, r) , $(-p, q, r)$, $(p, -q, r)$ and $(p, q, -r)$!

In EQF, the pole is QA-P16, isoconjugate of the centroid QA-P10

...

BG uses the notion of pole of the isoconjugation, I suppose GP uses a fixed point of the same isoconjugation.

Best regards

Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2887
Date: 2025-12-17
From: contiwa.goma3@gmail.com
Subject: Re: Isocubics

Dear Chris and all,

Let the pole be $(P:Q:R)$, then the isoconjugate of $P(u:v:w)$ is $(P/u : Q/v : R/w)$.

When $P^* = (u':v':w')$, we get $P = uu'$, $Q = vv'$, $R = ww'$.

Note that the pole depends on the coordinate system.

(Maybe this is "Shaka ni Seppou" (Preaching to the choir).)

Barycentric:

isogonal conjugate $(a^2/x : b^2/y : c^2/z)$

pole = X6($a^2 : b^2 : c^2$)

fixed points: X1($a:b:c$), the excenters

isotomic conjugate $(1/x : 1/y : 1/z)$

pole = X2($1:1:1$)

fixed points: X2($1:1:1$), the vertices of anti-medial triangle

Trilinear:

isogonal conjugate $(1/x' : 1/y' : 1/z')$

pole = X1($1:1:1$)

fixed points: X1($1:1:1$), the excenters

isotomic conjugate $(1/x'a^2 : 1/y'b^2 : 1/z'c^2)$

pole = X75($1/a^2 : 1/b^2 : 1/c^2$)

fixed points: X2($1/a : 1/b : 1/c$), the vertices of anti-medial triangle

Best regards,

M@IMF

p.s. Another example

	x-coordinate	x'-coordinate
A	$(1 : 0 : 0)$	$(-(uK+v+w)(K+1) : (u+vK+w) : (u+v+wK))$
B	$(0 : 1 : 0)$	$((uK+v+w) : -(u+vK+w)(K+1) : (u+v+wK))$
C	$(0 : 0 : 1)$	$((uK+v+w) : (u+vK+w) : -(u+v+wK)(K+1))$
A'	$(uK : v : w)$	$(1 : 0 : 0)$
B'	$(u : vK : w)$	$(0 : 1 : 0)$
C'	$(u : v : wK)$	$(0 : 0 : 1)$
Po	$(u : v : w)$	$((uK+v+w) : (u+vK+w) : (u+v+wK))$

When the pole of isoconjugation wrt ABC is $(u^2 : v^2 : w^2)$, that wrt A'B'C' is $((uK+v+w)^2 : (u+vK+w)^2 : (u+v+wK)^2)$.

Po is a fixed point.

Message: #2888
Date: 2025-12-18
From: bernard.keizer@gmail.com
Subject: Re: Isocubics

Dear Chris,

I answered your message immediately, without checking anything!
But it appears GP's construction is exactly the same as BG's
construction page 7 Remark 1
With a triangle and

* a fixed point K (GP), you have the 3 other fixed points as
vertices of the anticevian triangle of K wrt the triangle.
You may then have the isoconjugate of any point by using GP's
construction from any of the fixed points
(K_a and K'_a are the same ...)
In particular, the isoconjugate of the centroid G is the pole
 Ω

* 2 conjugate points (in particular G and Ω),
you have the fixed points by using BG's construction

Note that the 4 fixed points are real only if Ω lies inside the
triangle, hence $p, q, r > 0$

Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2889
Date: 2025-12-19
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF ,
Many thanks for these explanations!
I've tried to reproduce the calculations.
The 9 flexes (3 real and 6 imaginary) are on 12 lines (4 real and 8 imaginary), forming 4 triangles, the 1st one H1HH3 has 3 real sides and is the real flextrilateral.
The 2nd has only one real side, which is the line of real flexes and only one real vertex, which is the point P0, fixed point of the isoconjugation and root of the cubic.
The 3rd and 4th have 3 imaginary sides and 3 imaginary vertices.
All this is well-known!
It is remarkable that the equations of the 1st and 2nd triangles wrt the referential H1H2H3 are precisely

* $x'y'z' = 0$ and
* $x'^3 + y'^3 + z'^3 - 3x'y'z' = 0$ (your equation (3) wrt ABC)

Each of these 4 triangles forms a cubic belonging to the Hesse pencil and each of these 4 cubics is it's own hessian ...
I'm waiting impatiently for your equation of the pencil of cubics wrt H1H2H3 as a Christmas gift.
Best regards
Bernard

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2890
Date: 2025-12-19
From: van10hoven@gmail.com
Subject: Re: Real flexline trilateral

Dear M@IMF, Bernard and Eckart,

Thank you all for the wonderful messages and answers.
 So many different insights, and all of them very welcome.
 I will now give my own interpretation.

We have a general cubic CU with a general equation.
 $c_1 x^3 + c_2 y^3 + c_3 z^3 + c_4 x^2y + c_5 x^2z + c_6 xy^2 + c_7 y^2z + c_8 xz^2 + c_9 yz^2 + c_{10} xyz$

We always inscribe a triangle in CU in whatever way is convenient, in order to transform the general equation into a nicer one, for the sake of simplification or applicability.

Flextrilateral as inscribed reference triangle
 When we use the real flextrilateral as the reference triangle, the general equation changes into the first normal form:
 $a_1 x^3 + a_2 y^3 + a_3 z^3 + k a_1 a_2 a_3 x y z$
 which, through a projective transformation ($a_1x \rightarrow x$; $a_2y \rightarrow y$; $a_3z \rightarrow z$), can be converted into *Hesse's Form* :
 $x^3 + y^3 + z^3 + k x y z$

nK-construction as inscribed reference triangle
 When we deal with an nK-construction, in which we inscribe a reference triangle ABC in such a way that the third intersection points of the ABC-sidelines are collinear. We then obtain a normal form (as described by BG in SITP):
 $u x (r y^2 + q z^2) + v y (p z^2 + r x^2) + w z (q x^2 + p y^2) + k x y z$
 where the root of the isoconjugation is $P(u:v:w)$ and the pole of the isoconjugation is $(p:q:r)$.

Eventually it turns out that there is a relation between root and pole: $(p \sim u^2, q \sim v^2, r \sim w^2)$.

By substituting this, the following normal form arises, as named by M@IMF in QPG#2881:
 $(x^2/u^2)(y/v + z/w) + (y^2/v^2)(z/w + x/u) + (z^2/w^2)(x/u + y/v) + 2 k x y z / (u v w)$
 which can be transformed projectively accordingly (also noted by M@IMF in QPG#2884) into:
 $(x + y) z^2 + y^2 (x + z) + x^2 (y + z) + 2 x y z k$
 With this, just as with the first normal form for Hesse's Form, all kinds of calculations can be applied.

It is good to remember that all these equations describe the same cubic. The only difference is the inscribed reference triangle from which the calculations are carried out. I am considering renaming these normal forms, for example the 1st and 2nd Flex-Normal Form, and the 1st and 2nd nK-Normal Form. It all revolves around a general cubic that is presented differently depending on the inscribed reference triangle that is chosen. There can be more interesting normal forms.

Best regards,
Chris

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2891
Date: 2025-12-19
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris and all,

I'm afraid the real flextrilateral isn't an inscribed triangle in the cubic, although it is a member of the family of triangles such that vertices lie on real Harmonic Polars and edges bear real flexes.

When the root is $(p^2 : q^2 : r^2)$, pK is written as $(u/p)(x/p)[(y/q)^2 - (z/r)^2] + (v/q)(y/q)[(z/r)^2 - (x/p)^2] + (w/r)(z/r)[(x/p)^2 - (y/q)^2] = 0$.
(2nd pK -Normal Form?)
 pK is a circumcubic of QA , nK is a circumcubic of QL .
Interesting!

I'm interested in flextangent trilateral as a reference triangle.
The cubic has a general form but coefficients are not independent.

Best regards,
M@IMF

p.s. My message #2887 might be a bit beside the point.
Also sorry for using confusing symbols.

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2892
Date: 2025-12-19
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Sorry, the root should be the pole. M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2893
Date: 2025-12-20
From: bernard.keizer@gmail.com
Subject: Re: Real flexline trilateral

Dear Chris,
Nice summary!
4 remarks:

1) The real flextrilateral $H_1H_2H_3$ is not inscribed in the cubic (as already pointed by I@IMF)
(Neither are the 3 other triangles through the 9 flexes)

2) In the general nK , there is no relation between the pole and the root, which are totally independent!
For example, $QL-Cu_1$ is a nK wrt the 4 reference triangles with root in the trilinear pole of the 4th line ; isoconjugation will be the Moebius transformation ...

3) If and only if the root is one of the fixed points of the isoconjugation,
then it holds that $p = u^2$, $q = v^2$ and $r = w^2$
This happens only for a particular triangle, formed by the contact points of the tangents from the flexpoints to the cubic (other than the flextangent)
In this case, the lines $A_0B_0C_0$ and $A_1B_1C_1$ coincide with the line of real flexes.

4) A pK can always be described as a nK , but I think a nK can be described as a pK only if it is bipartite
For example, $QA-Cu_7$ is a perfect nK wrt the DT , with fixed points of the isoconjugation the vertices of the reference QA ;
but it is also a $QL-Cu_1$ as well as a $QA-Cu_1$...
As you say, all depends of the reference triangle.

Best regards
Bernard

PS Merry Christmas and Happy New Year to all of you!

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2894
Date: 2025-12-21
From: contiwa.goma3@gmail.com
Subject: Re: Real flexline trilateral

Dear Mr. Keizer and all,

As for 2) in #2882, let

$$H1 = (-\xi u : v : w)$$

$$H2 = (u : -\xi v : w)$$

$$H3 = (u : v : -\xi w),$$

where

$$\xi = (1 + w)(1 + \eta w)/(1 + \eta/w),$$

which satisfies

$$(\xi^3 - 3\xi^2 + 9\xi - 5) - 2k'(\xi^2 - \xi + 1) = 0$$

When the reference triangle is changed from ABC to H1H2H3, the coordinates of A,B,C become

$$A = ((\xi - 1)(-\xi u + v + w) : (u - \xi v + w) : (u + v - \xi w))$$

$$B = ((-\xi u + v + w) : (\xi - 1)(u - \xi v + w) : (u + v - \xi w))$$

$$C = ((-\xi u + v + w) : (u - \xi v + w) : (\xi - 1)(u + v - \xi w)),$$

and the equation of nK

$$\begin{aligned} (y/v + z/w)(x/u)^2 + (z/w + x/u)(y/v)^2 \\ + (x/u + y/v)(z/w)^2 + 2k'(x/u)(y/v)(z/w) \\ = 0 \end{aligned}$$

becomes

$$CU: (a_1x'')^3 + (a_2y'')^3 + (a_3z'')^3 + 3g(a_1x'')(a_2y'')(a_3z'') = 0,$$

where

$$(1/a_1 : 1/a_2 : 1/a_3) =$$

$$((-\xi u + v + w) : (u - \xi v + w) : (u + v - \xi w))$$

$$3g = 6 - k'(\xi^3 - 3\xi - 2)/[\xi^2 - (k'+1)\xi + 1].$$

On the other hand, the intersection of CU and its Harmonic Polar $(0 : a_2 : -a_3)$ is $(-\xi''/a_1 : 1/a_2 : 1/a_3)$, where ξ'' satisfies

$$\xi''^3 + 3g\xi'' - 2 = 0.$$

When $3g$ is given as above, $(1 - \xi)$ satisfies this equation.

Best regards,
M@IMF

[← Previous](#) [Next →](#) [↔ Message Index](#) [↑ Subjects](#)

Message: #2895
Date: 2025-12-22
From: eckart_schmidt@t-online.de
Subject: Re: Isocubics

Dear Bernard, dear Chris,

an observation for QL-Cu1
... and the line L of its real flexpoints:

CSC(L) is a circle CI, intersecting CU-Cu1
... in QL-P1 and three points A,B,C,
... which give a ref-triangle for QL-Cu1 as nK,
... using the trilinear pole Po of L as fixpoint
... of an isoconjugation *.

L* is a conic C0 through QL-P1
... contacting the cubic in A,B,C
... with tangents through the real flexpoints.

For QL-Cu1-points the isoconjugate * and CSC coincide.

For L-points X the lines X*.CSC(X) have a common point
... in the 4th intersection of conic and circle:
... what about this point?
... Its Simson-line wrt ABC is orthogonal L

Best regards Eckart

Message: #2896
Date: 2025-12-23
From: van10hoven@gmail.com
Subject: Season's Greetings and a Look Ahead

Dear friends,

As the year draws to a close, it feels like the right moment to pause and look back.

The forum saw a modest level of activity, yet it brought several remarkable and engaging contributions.

Besides myself, the following people shared their insights and work: Eckart Schmidt, Bernard Keizer, M@IMF, Keita Miyamoto, Antreas Hatzipolakis, Dylan Wyrzykowski, Tran Quang Hung, and Stanley Rabinowitz. My thanks to all of them for their thoughtful and special contributions. Our community now consists of around 30 members.

We also received the sad news of the passing of Angel Montesdeoca, who was once part of our group.

One of the highlights this year was the launch of the new website, which also brought together the former encyclopedias for Quadri-Figures and Poly-Figures and introduced a new section dedicated to n-curves (conics, cubics, and higher-degree curves).

I also intend to create Annual Journals of our posts. The work is already well underway, and I will share more as soon as it is ready.

It would be nice if, in the coming year, we could begin further developing the higher-degree curves, much as we once developed the Poly-Figures from the Quadri-Figures. I already have a few ideas in mind, which I will share when the time is right. For those waiting for a new opportunity to join in, this may also be a good moment.

I wish everyone a peaceful holiday season a good year ahead in whatever form it takes.

Best regards,

Chris

Message: #2897

Date: 2025-12-23

From: contiwa.goma3@gmail.com

Subject: Re: Season's Greetings and a Look Ahead

Dear all,

I wish you a Merry Christmas and a Happy New Year.
(No one point out about #2854. I should have said
"Wham! Thank you." in #2858.)

Best regards,
M@IMF

[← Previous](#) [↔ Message Index](#) [↑ Subjects](#)

5 Keyword Index

The list below shows several keywords along with numbers of related messages. Click on a number to go to the corresponding page.

- **CSC:**
[#2608](#) [#2678](#) [#2895](#)
- **Cardioid, cardioïd:**
[#2582](#) [#2583](#) [#2585](#) [#2586](#)
- **Cayley-Bacharach, CB:**
[#2760](#) [#2761](#) [#2762](#) [#2765](#) [#2767](#)
- **Cayleyan:**
[#2562](#) [#2577](#) [#2634](#) [#2636](#) [#2637](#) [#2714](#) [#2819](#) [#2820](#) [#2829](#)
- **Cycloid, cycloïd:**
[#2673](#) [#2678](#)
- **Cyclologic:**
[#2792](#) [#2794](#)
- **Deltoid, deltoïd:**
[#2561](#) [#2568](#) [#2569](#) [#2575](#) [#2578](#) [#2579](#) [#2580](#) [#2581](#) [#2582](#) [#2583](#) [#2585](#)
[#2586](#) [#2588](#) [#2590](#)
- **Desmic:**
[#2797](#) [#2799](#) [#2800](#) [#2833](#)
- **Flexline:**
[#2595](#) [#2600](#) [#2601](#) [#2606](#) [#2625](#) [#2702](#) [#2713](#) [#2725](#) [#2727](#) [#2746](#) [#2747](#)
[#2764](#) [#2768](#) [#2770](#) [#2772](#) [#2786](#) [#2793](#) [#2798](#) [#2801](#) [#2803](#) [#2804](#) [#2805](#)
[#2807](#) [#2811](#) [#2876](#) [#2881](#) [#2884](#)
- **Flexpoint:**
[#2727](#)
- **Harmonic Polar:**
[#2584](#) [#2595](#) [#2606](#) [#2625](#) [#2627](#) [#2628](#) [#2634](#) [#2635](#) [#2643](#) [#2702](#) [#2713](#)
[#2715](#) [#2719](#) [#2747](#) [#2764](#) [#2808](#) [#2820](#) [#2821](#) [#2825](#) [#2828](#) [#2845](#) [#2876](#)
[#2891](#) [#2894](#)
- **Hessian:**
[#2562](#) [#2584](#) [#2596](#) [#2597](#) [#2598](#) [#2609](#) [#2613](#) [#2617](#) [#2621](#) [#2627](#) [#2632](#)
[#2634](#) [#2636](#) [#2637](#) [#2639](#) [#2735](#) [#2779](#) [#2801](#) [#2802](#) [#2808](#) [#2811](#) [#2823](#)
[#2824](#) [#2825](#) [#2828](#) [#2830](#) [#2854](#) [#2864](#) [#2884](#) [#2889](#)
- **Hypocycloid, hypocycloïd:**
[#2588](#) [#2590](#) [#2673](#) [#2678](#)
- **Isoconjugation, isoconjugate:**
[#2625](#) [#2635](#) [#2639](#) [#2640](#) [#2657](#) [#2659](#) [#2660](#) [#2661](#) [#2665](#) [#2669](#) [#2675](#)
[#2694](#) [#2698](#) [#2724](#) [#2738](#) [#2741](#) [#2742](#) [#2759](#) [#2786](#) [#2793](#) [#2804](#) [#2805](#)
[#2807](#) [#2808](#) [#2811](#) [#2821](#) [#2823](#) [#2825](#) [#2828](#) [#2830](#) [#2831](#) [#2832](#) [#2836](#)

#2837 #2838 #2839 #2840 #2841 #2842 #2843 #2846 #2847 #2849 #2850
#2851 #2852 #2859 #2861 #2863 #2864 #2866 #2869 #2870 #2872 #2873
#2874 #2878 #2881 #2885 #2886 #2887 #2888 #2889 #2890 #2893 #2895

- **Isocubics:**
#2738 #2759 #2760 #2811 #2839 #2841 #2843
- **Miquel Point:**
#2561 #2575 #2579 #2586 #2794
- **Miquel-point, Miquel Point:**
#2561 #2575 #2579 #2586 #2794
- **Miquel-triangle, Miquel Triangle:**
#2700 #2792
- **Newton Line:**
#2560 #2561 #2689
- **Orthologic:**
#2671
- **Perspective Fields:**
#2670 #2866
- **Pivotal Isocubic:**
#2625 #2635 #2639 #2694 #2724 #2735 #2738 #2759 #2760 #2775 #2793
#2808 #2811 #2823 #2824 #2825 #2833 #2837 #2838 #2839 #2841 #2854
- **Plücker points:**
#2560
- **Prehessian:**
#2619 #2632 #2854 #2876
- **QA-Cu1:**
#2689 #2692 #2694 #2695 #2700 #2741 #2893
- **QA-Cu7:**
#2836 #2838 #2893
- **QA-P3:**
#2664 #2685 #2687 #2694 #2700 #2794
- **QA-P4:**
#2686 #2693 #2696 #2794
- **QL-Cu1:**
#2566 #2584 #2883 #2893 #2895
- **Quartic:**
#2661 #2709 #2710 #2729 #2774 #2794
- **Quintic:**
#2773

- **Reye configuration:**
[#2797](#) [#2799](#)
- **Sextic:**
[#2734](#)
- **Simson line:**
[#2568](#) [#2582](#) [#2585](#) [#2588](#) [#2623](#) [#2624](#) [#2631](#) [#2685](#) [#2687](#) [#2688](#) [#2689](#)
[#2692](#) [#2694](#) [#2695](#) [#2696](#) [#2700](#)
- **Steiner Axes:**
[#2588](#)
- **Steiner Line:**
[#2604](#) [#2678](#) [#2685](#) [#2686](#) [#2693](#)
- **Stelloid, stelloïd:**
[#2584](#) [#2609](#) [#2613](#) [#2619](#) [#2622](#)
- **Tangential:**
[#2639](#) [#2640](#) [#2643](#) [#2703](#) [#2705](#) [#2707](#) [#2714](#) [#2718](#) [#2721](#) [#2722](#) [#2724](#)
[#2738](#) [#2739](#) [#2741](#) [#2742](#) [#2743](#) [#2775](#) [#2783](#) [#2808](#) [#2812](#) [#2859](#) [#2864](#)
[#2870](#) [#2873](#) [#2874](#) [#2878](#)
- **Triple Triangle:**
[#2794](#)
- **Van Rees:**
[#2584](#) [#2609](#) [#2613](#)
- **circular points:**
[#2760](#)

[↔ Table of Contents](#) [↔ Message Index](#)

6 Colophon

Sources & Contact

Web address (QPG Forum): <https://groups.io/g/Quadri-and-Poly-Geometry>

EPG Encyclopedia (content reference): <https://www.chrisvantienhoven.nl>

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Journal of the Quadri- and Poly-Geometry Group

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- Volume 7 (2025), messages #2560–#2897
- Volume 6 (2024), messages #2052–#2559
- Volume 5 (2023), messages #1545–#2051
- Volume 4 (2022), messages #1295–#1544
- Volume 3 (2021), messages #631–#1294
- Volume 2 (2020), messages #61–#630
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- Volume 6 (2018), messages #2780–#3299
- Volume 5 (2017), messages #2170–#2799
- Volume 4 (2016), messages #1403–#2169
- Volume 3 (2015), messages #917–#1402
- Volume 2 (2014), messages #394–#916
- Volume 1 (2013), messages #1–#393