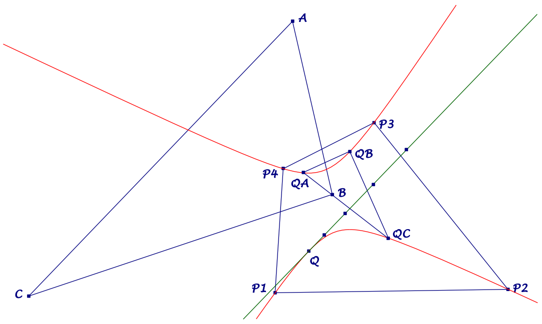
EQF-Note 2013-01-18

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Two Circumscribed Conics of a Quadrangle

A quadrangle $P_1P_2P_3P_4$ and a further point $Q(u:v:w)$ destinate a circumscribed conic. For the reference triangle *QA-Tr1* this conic has the equation

$$(q^2w^2 - r^2v^2)x^2 + (r^2u^2 - p^2w^2)y^2 + (p^2v^2 - q^2u^2)z^2 = 0.$$



The conic has two interesting properties:

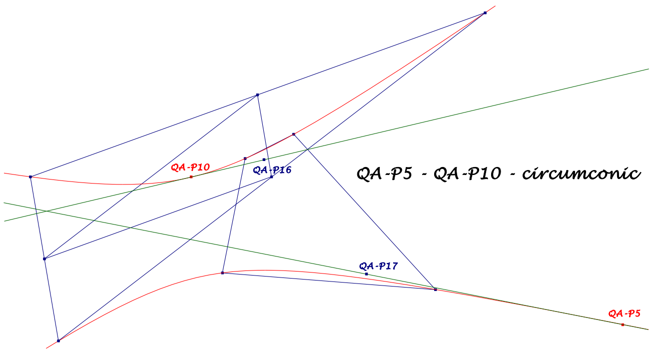
- The vertices of the anticevian triangle of Q wrt *QA-Tr1* lie on this conic.
- Using an isoconjugation wrt this anticevian triangle of Q with fixpoint Q, the images of P_i are collinear on the tangent in Q to the conic.

There are two circumscribed conics of a quadrangle, which contain further *QA*-points.

The *QA-P5-QA-P10*-circumconic of a Quadrangle (*QA-Cox*)

- There exists a circumconic *QA-Cox* of a quadrangle through the points *QA-P5* and *QA-P10* with the equation

$$(q^2 - r^2)x^2 + (r^2 - p^2)y^2 + (p^2 - q^2)z^2 = 0.$$



- The vertices of the circumscribed triangle of *QA-Tr1* lie on *QA-Cox* (anticevian triangle of *QA-P10* wrt *QA-Tr1*).

- *QA-Cox* is the isotomic conjugate of *QA-P10.QA-P16* wrt the circumscribed triangle of *QA-Tr1*.

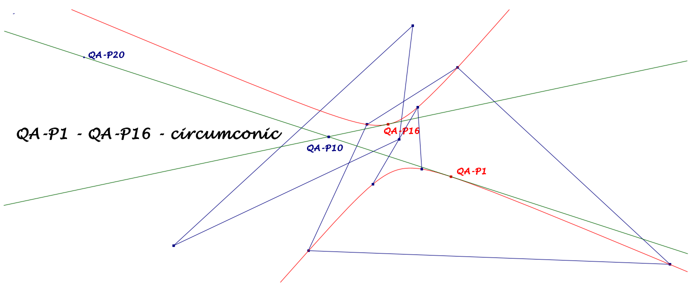
- The tangent in *QA-P5* is *QA-P5.QA-P17* (see below).
  The tangent in *QA-P10* is *QA-P10.QA-P16* (see below).
  (*QA-P17* and *QA-P16* are the *QA-Tf2* images of *QA-P5* and *QA-P10*.)

- The center of *QA-Cox*
  … is the fourth intersection of *QA-Co1* and the circumscribed Steiner ellipse of *QA-Tr1*,
  … is the trilinear pole of *QA-P10.QA-P16*,
  (… is the isotomic conjugate of the infinity point of the trilinear polar of the isotomic conjugate of *QA-P16*).

- DT-coordinates: $\left(\frac{1}{q^2-r^2}:\frac{1}{r^2-p^2}:\frac{1}{p^2-q^2}\right)$.

The *QA-P1-QA-P16*-circumconic of a Quadrangle (*QA-Coy*)

- There exists a circumconic *QA-Coy* of a quadrangle through the points *QA-P1* and *QA-P16* with the equation
  $q^2r^2(q^2-r^2)x^2+r^2p^2(r^2-p^2)y^2+p^2q^2(p^2-q^2)z^2=0$.



- The vertices of the anticevian triangle of *QA-P16*, which are the *QA*-versions of *QG-P12*, lie on *QA-Coy*.

- *QA-Coy* is the image of *QA-P10.QA-P16* for the isoconjugation with fixpoint *QA-P16* wrt the *QG-P12* triangle (the anticevian triangle of *QA-P16*).

- The tangent in *QA-P1* is *QA-P1.QA-P20* (see below).
  The tangent in *QA-P16* is *QA-P16.QA-P10* (see below).
  (*QA-P20* and *QA-P10* are the *QA-TF2* images of *QA-P1* and *QA-P16*.)

- The center of *QA-Coy*
  … lies on *QA-Co1*,

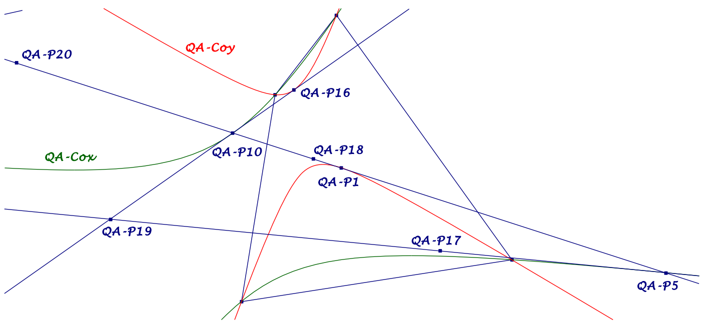
... is the trilinear pole of *QA-P1.QA-P16*,
(…is the *QA-Tf2* image of the infinity point of the trilinear polar of the isotomic conjugate of *QA-P16*).

- DT-coordinates: $(\frac{p^2}{q^2-r^2} : \frac{q^2}{r^2-p^2} : \frac{r^2}{p^2-q^2})$.

Further remarks:

- The two conics have a common tangent *QA-P10.QA-P16*.
- The center of *QA-Coy* is the *QA-Tf2* image of the isotomic conjugate of the center of *QA-Cox*.

The geometry of these two conics seems to be the geometry of the *QA-Tf2* pairs *(QA-P1, QA-P20), (QA-P5, QA-P17), (QA-P10, QA-P16), (QA-P18, QA-P19)*. All these points lie on the sidelines of the triangle *QA-P5.QA-P10.QA-P19*. For their polars wrt *QA-Cox* and *QA-Coy* (which can be tangents) see the table below:



point	on QA-Cox	on QA-Coy	polar wrt *QA-Cox*	polar wrt *QA-Coy*	*QA-Tf2*
QA-P1		x	QA-P19.P20	QA-P1-P5-P10-P18-P20	QA-P20
QA-P5	x		QA-P5.P17.P19	QA-P1.P17	QA-P17
QA-P10	x		QA-P10.P16.P19	QA-P1.P16	QA-P16
QA-P16		x	QA-P10. ?	QA-P10.P16.P19	QA-P10
QA-P17			QA-P5. ?	QA-P5. ?	QA-P5
QA-P18			QA-P19. ?	QA-P1.P19	QA-P19
QA-P19			QA-P1-P5-P10-P18-P20	QA-P16.P18	QA-P18
QA-P20			QA-P1.P19	QA-P1. ?	QA-P1

Not only for these points holds:

- The polars of an arbitrary point wrt *QA-Cox* and *QA-Coy* concur in the *QA-Tf2* image of this point.

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