EQF-Note 2013-01-21

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Two Circumscribed *QA-Tf2* Isocubics of a Quadrigon

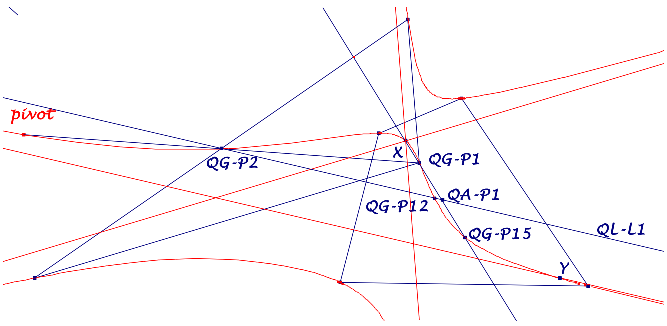
The two isocubics have the same reference triangle and the same isoconjugation, but different pivots:

reference triangle *QA-Tr1*, isoconjugation *QA-Tf2*.

1. Pivot: Reflection of *QG-P1* in *QG-P2.*

Properties:

- This isocubic contains the vertices of the quadrigon and the vertices of the *QA*-Diagonal Triangle *QA-Tr1*.
- Further points on this isocubic are: *QG-P1*, *QG-P2*, *QG-P12*, *QG-P15* and the pivot point.
- The equation of this isocubic:
  $$p^2(y+z)yz+q^2(z-x)zx-r^2(x+y)xy=0.$$



- Two asymptotes are parallel to the leg lines of *QA-Tr1*. The third asymptote is parallel to *QL-L1*.
- The intersection point *X* of the first two asymptotes is the third intersection point of *QG-L3* and the cubic: the fourth harmonic point of *QA-P1* wrt *QG-P1* and the intersection of *QG-L1* and *QG-L3*:
  $$X\,(\,p^2(-p^2+q^2+r^2):-q^2(p^2-q^2+r^2):r^2(p^2+q^2-r^2))\,.$$
- The intersection *X* of the first two asymptotes is Railway Watcher for *QL-L1* and the third asymptote. The

intersection point Y of the third asymptote and the isocubic is

$$Y\left(\frac{p^2(-p^2+q^2+r^2)}{(-p^2+q^2+r^2)^2-2(-p^4+q^4+r^4)}\right.$$
$$\left.:\frac{-q^2(p^2-q^2+r^2)}{(p^2-q^2+r^2)^2-2(p^4-q^4+r^4)}:\frac{r^2(p^2+q^2-r^2)}{(p^2+q^2-r^2)^2-2(p^4+q^4-r^4)}\right)$$

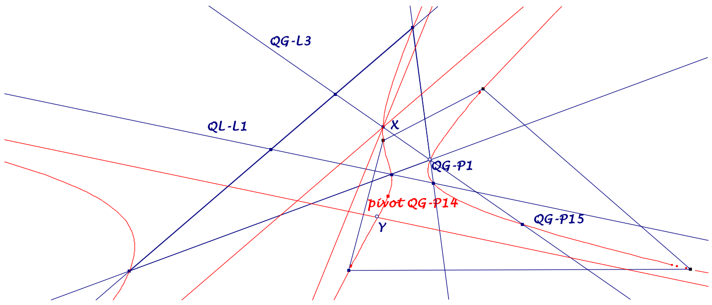
- The tangents in the vertices of the quadrigon concur in the pivot point.
- The tangents in the vertices of the reference triangle and the pivot point concur in *QG-P12*.
- The tangent in *QG-P1* is *QG-L2* (inflection tangent).
- The tangent in *QG-P2* is *X.QG-P2*.
- The tangent in *QG-P12* is *X.QG-P12*.
- The tangent in X is XY.
- The isocubic and *QA-Co1* intersect in *QG-P1*, *QG-P15* and contact in *QG-2P2a,b*.

2. Pivot: *QG-P14*

Properties:

- This isocubic contains the vertices of the quadrigon and the vertices of the *QA*-Diagonal Triangle *QA-Tr1*.
- Further points on this isocubic are: *QG-P1*, *QG-P14*, *QG-P15* and the intersection points of the leg lines of *QA-Tr1* with *QL-L1*.
- The equation of this isocubic:

$$(r^2x^2-p^2z^2)q^2y-(r^2-p^2)(p^2y^2z-q^2(z+x)zx+r^2xy^2)=0.$$



- There are two asymptotes parallel to *QG-L1* and *QG-P1.QG-P3*. The third asymptote is parallel to *QL-L1*.

- The intersection point X of the first two asymptotes is the third intersection point of *QG-L3* and the cubic: the midpoint of *QG-P1* and the intersection of *QG-L1* and *QG-L3:*
$$X(p^2(-p^2+q^2+r^2):q^2(r^2+p^2)-(r^2-p^2)^2:r^2(p^2+q^2-r^2)).$$

- The intersection X of the first two asymptotes is Railway Watcher for *QL-L1* and the third asymptote. The intersection point Y of the third asymptote and the isocubic is
$$Y(\frac{p^2(-p^2+q^2+r^2)}{(p^2-r^2)(p^2+q^2-r^2)^2-4p^2q^4}$$
$$:\frac{q^2(p^2+r^2)-(p^2-r^2)^2}{(p^2-r^2)(3(-p^2+q^2+r^2)(p^2+q^2-r^2)+2q^2(p^2-q^2+r^2))}$$
$$:\frac{r^2(p^2+q^2-r^2)}{(p^2-r^2)(-p^2+q^2+r^2)^2+4r^2q^4}).$$

- The tangents in the vertices of the quadrigon concur in the pivot point *QG-P14*.

- The tangents in the vertices of the reference triangle and the pivot point *QG-P14* are parallel to *QG-P1.QG-P3*.

- The tangents in the intersection points of *QL-L1* and the leg lines of *QA-Tr1* concur in point X.

- The tangent in X is XY.

- The isocubic and *QA-Co1* intersect in *QG-P14*, *QG-P15*, *QA-2P2a,b* and contact in *QG-P1*.

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