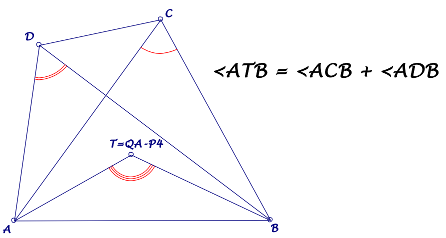
EQF-Note 2013-03-03

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Some Remarks for Angle Properties of *QA-P4*

There is a basic angle property for a quadrangle wrt the Isogonal Center *QA-P4.*



This property is mentioned in:

1. *H. V. Mallison: Pedal Circles and the quadrangles, The Mathematical Gazette, Vol. 42, No. 339 (Feb. 1958), 17-20*

Returning to P. W. Wood's article, quoted above, several results were obtained and quoted about the *isoptic point* of $ABCD$, which we call here E to avoid confusion. E is the common point of the circles of similitude of pairs of the circles ABC, BCD, CDA and DAB.

An additional property of the isoptic point has been pointed out by Mr. T. McHugh, who also supplied a simple geometrical proof:

Any two of four coplanar points A, B, C, D subtend at their isoptic point E an angle which is equal to the sum or the difference between the angles which the two points subtend at the remaining two points, e.g.

$$\angle AEB = \angle ACB + \angle ADB.$$

for the points A, B, with similar results for any other pair from A, B, C, D.

The use of directed angles avoids any ambiguity in the sign.

It may be observed that if A, B, C, D are concyclic the above definition of the isoptic point is not clear, whereas the property given above shows that E is at the centre of the circle $ABCD$.

2. *Praxis der Mathematik in der Schule 1/44, Jg 2002, 19-27*

Ein merkwürdiger Punkt des Vierecks

Roland Stärk und Daniel Baumgartner

Gegeben sind in der Ebene vier Punkte A, B, C, D. Man zeige, dass es einen Punkt T gibt, der die folgenden Winkelbedingungen (orientierte Winkel modulo 180°) erfüllt:

$$\measuredangle ATB = \measuredangle ADB + \measuredangle ACB, \quad \measuredangle BTC = \measuredangle BAC + \measuredangle BDC,$$

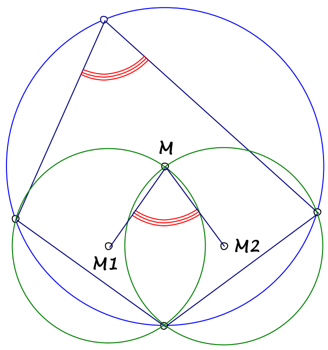
$$\measuredangle CTD = \measuredangle CBD + \measuredangle CAD, \quad \measuredangle DTA = \measuredangle DCA + \measuredangle DBA.$$

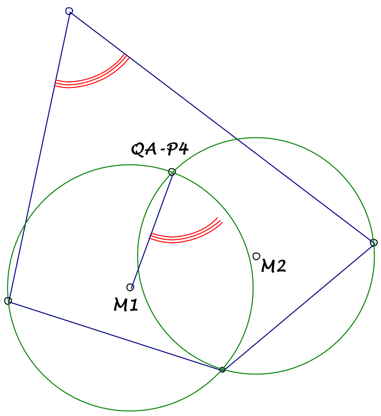
For a cyclic quadrangle *QA-P4* is the circumcenter and the property above is well known. In *EQF* we find an interesting geometrical comparison:

QA-P4 has equivalent functions as the Circumcenter in a Cyclic Quadrangle.

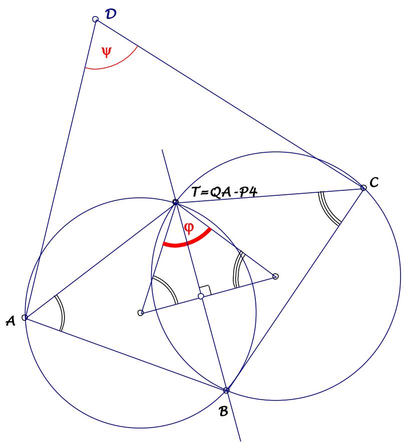
1. In addition to the given examples in *EQF* here is a further angle property:

See also my homepage:
12-1 Winkel am Tangentialpunkt eines Vierecks.





Synthetic proof (private note Roland Stärk):



Triangle: $\varphi + \angle BAT + \angle TCB \equiv 0$

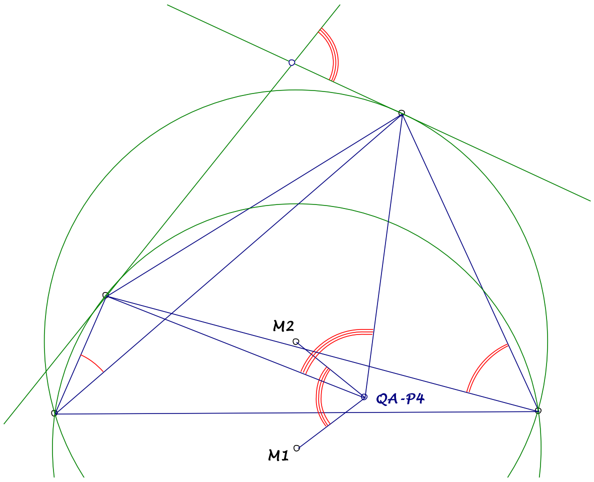
Quadrigon *ABCT*: $\angle BAT + \angle TCB + \angle CBA + \angle ATC \equiv 0$

$\varphi \equiv \angle CBA + \angle ATC$

Property of *T=QA-P4*: $\angle ATC \equiv \angle ADC + \angle ABC$

$\varphi \equiv \angle ADC \equiv \psi$

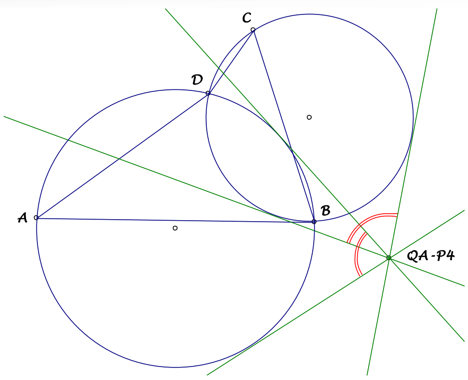
2. An example, which shows the variety of angle relations wrt *QA-P4.*



3. *QA-P4* has different names. The earliest one will be "isoptic point", related to the following property, to be found in

H. M. Cundy and C. F. Parry:
Geometrical Properties
of some Euler and Circular Cubics. Part 2.
Journal of Geometry, 2000, 63-64

owes its name to the property:
(a) it is the unique point at which the four circumcircles of the triangles ABC, BCD, CAD ABD of a quadrangle ABCD subtend equal angles.



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