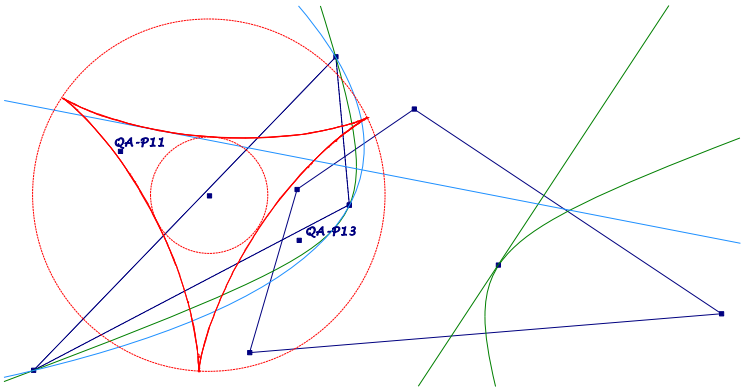
EQF-Note 2013-03-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Steiner-Hypocycloid for Quadrangles with the same *QA*-Diagonal Triangle

The Simson-lines of a triangle envelop a Steiner-hypocycloid [1]. In this EQF-note quadrangle geometry leads to a Steiner hypocycloid of the medial triangle of the QA-Diagonal Triangle. − The results are only Cabri controlled, the calculations with barycentric coordinates are too extensive.



We start with a triangle and a quadrangle with this triangle as diagonal triangle *QA-DT*.
Consider the Nine-Point Conic *QA-Co1* of this quadrangle and its tangents.
The Involutary Conjugates (*QA-Tf2*) of *QA-Co1*-points are infinity points.
The Involutary Conjugates (*QA-Tf2*) of lines are circumscribed conics of *QA-DT*.
The Involutary Conjugates (*QA-Tf2*) of tangents at *QA-Co1* are circumscribed parabolas of *QA-DT*.

The axes of circumparabolas of a triangle are Simson lines of the medial triangle and envelop a Steiner-hypocycloid.

Properties of this Steiner-hypocycloid:

- **The center is the midpoint of *QA-P11* and *QA-P13* (*X*(140) of *QA-DT*).**

- **The circumcircle has a radius of $\frac{3}{4}r$ (*r* Radius of the *QA*-*DT*- circumcircle).**
- **The incircle has a radius of $\frac{r}{4}$ (*r* Radius of the *QA*-*DT*- circumcircle).**
- **This Steiner-hypocyloid of a quadrangle is the envelope of the Simson-lines of the medial triangle of *QA*-*DT*.**
- **All quadrangles with the same *QA*-Diagonal Triangle have the same Steiner-hypocycloid.**

So the Steiner-hypocycloid of a quadrangle isn´t a real quadrangle object, for it depends only on the *QA*-Diagonal Triangle.

References:

[1] E. H. Lockwood: A Book of Curves. – Cambridge, At the University Press, 1961.

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