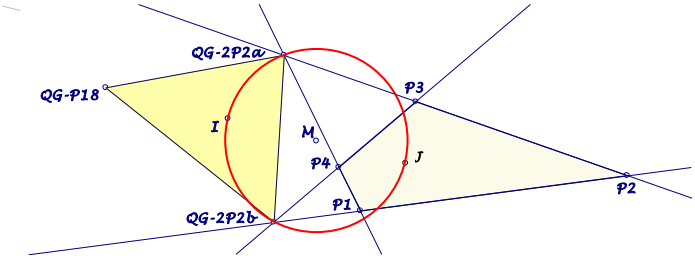
EQF-Note 2013-11-20

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A new Circle for Quadrigons

Here a new QG-circle will be described connected with the angle bisectors of a quadrigon. – Reference triangle is the Quasi Isogonal Triangle QG-Tr3. But most of the properties are only tested with CABRI.



Preliminary Remarks

Reference triangle is the Quasi Isogonal Triangle *QG-Tr3* with vertices in the intersections of opposite sides *QG-2P2a,b* and the Quasi Isogonal Crosspoint *QG-P18*. We use the Incenter *I*, the excenter *J* wrt *QG-P18* and the corresponding isogonal conjugate (*QG-Tf2*).

The new *QG*-circle is the Thales circle about *IJ*.

If we chose the reference triangle *ABC* with

$$A = QG - 2P2a, \quad B = QG - 2P2b, \quad C = QG - P18$$

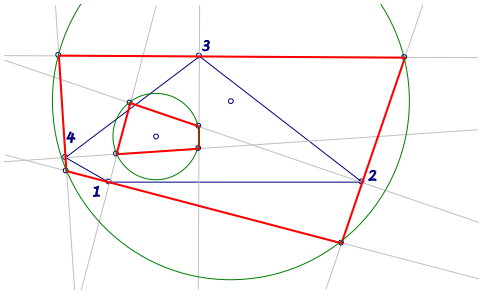
and give P_4 the coordinates *u*, *v*, *w*, then we get

$$P_1(a^2w : c^2uv : c^2uw), \quad P_2(a^2vw : b^2uw : c^2uv),$$

$$P_3(c^2uv : b^2w^2 : c^2vw), \quad P_4(u : v : w)$$

and the equation of the new *QG*-circle is

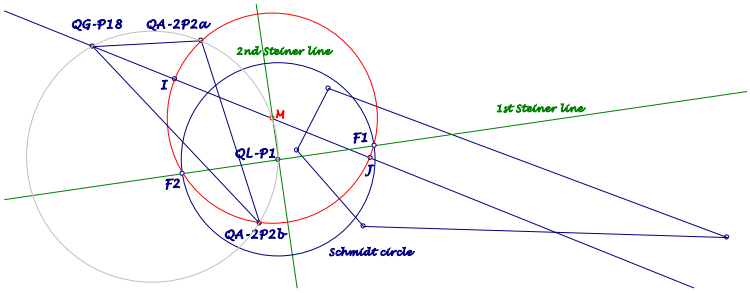
$$a^2yz + b^2zx + c^2xy - abz(x + y + z) = 0.$$



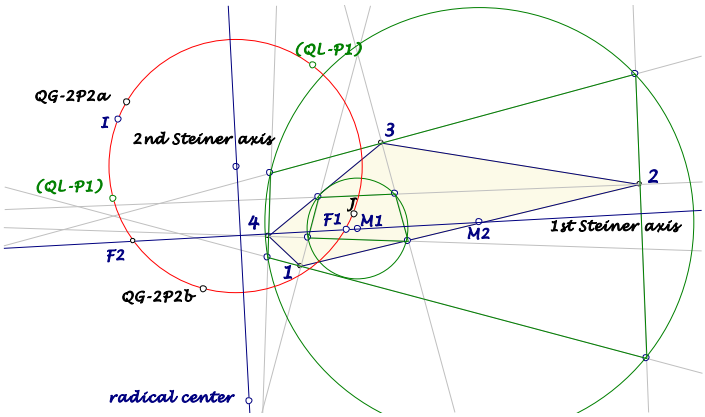
Furthermore we consider the two cyclic quadrigons, constructed with the inner and outer angle bisectors of the quadrigon.

Properties of the new *QG*-Circle *QG-Cix*

- *QG-Cix* contains the intersections *I* and *J* of the inner and outer angle bisectors of opposite sidelines of the quadrigon.
- *QG-Cix* contains *QG-2P2a,b* the intersections of opposite sidelines of the quadrigon.
- *QG-Cix* contains the fixed points F_1, F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.
- The midpoint *M* of *QG-Cix* is the intersection of the 2^{nd} Steiner axis (see *QL-Tf1*) and the circumcircle of *QG-Tr3* (beside *QL-P1*).



- *QG-Cix* is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.
- *QG-Cix* is invariant wrt the Isogonal Conjugate for *QG-Tr3* (*QG-Tf2*).
- *QG-Cix* cuts the circumcircles of the cyclic angle-bisector-quadrigons orthogonal.
- *QG-Cix* contains the Miquel points (*QL-P1*) of the cyclic angle-bisector-quadrigons.
- The radical center of *QG-Cix* and the circumcircles of the angle-bisector-quadrigons is the intersection of *QG-L1* and the 2^{nd} Steiner axis.
- The polar of the radical center wrt *QG-Cix* is the 1^{st} Steiner axis.



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