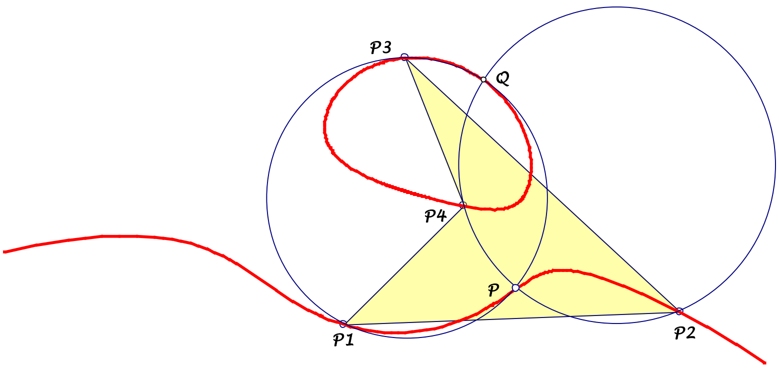
EQF-Note 2013-12-06

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Second Quartic for Quadrigons

For a quadrigon circles through opposite vertices are tangent in points on a quartic (see EQF-Note 2013-12-05). Here another quartic is described as locus of points, where circles through opposite vertices intersect perpendicular.



Let $P_1P_2P_3P_4$ be a quadrigon and $P_1P_2P_3$ the reference triangle for barycentric coordinates with $P_4(p:q:r)$. Then the locus for points, where circles through opposite vertices intersect perpendicular, is a quartic with the equation

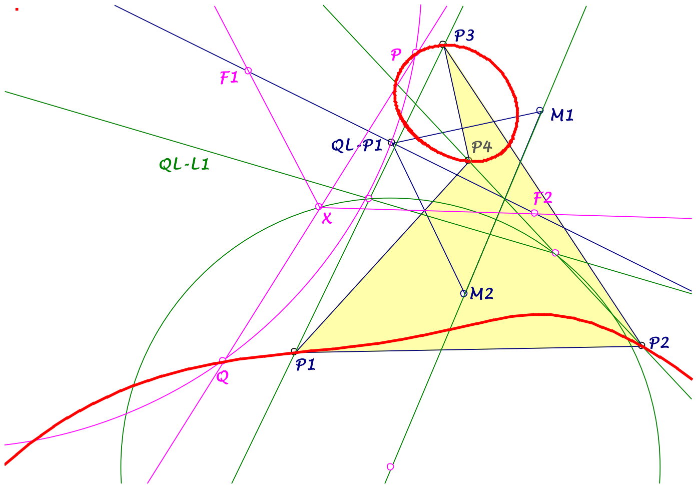
$$(p+r)\,(a^4\,z\,Z\,y^2+a^2\,c^2\,y^2\,(X\,z+x\,Z)+c^4\,x\,X\,y^2)$$
$$-a^2\,b^2\,z\,(p\,Z\,y\,(-x+y+z)\;-r\,(X\,y\,z-x\,Y\,(x+y+z)\,+x\,y\,Z))$$
$$-b^2\,c^2\,x\,(r\,X\,y\,(x+y-z)-p\,(X\,y\,z-Y\,z\,(x+y+z)\,+x\,y\,Z))$$
$$-b^4\,x\,z\,(r\,X\,(x+y)\,+p\,Z\,(y+z))\;==0$$

with

$$X:=x\,(q+r)-p\,(y+z)\,;\;Y:=y\,(r+p)-q\,(z+x)\,;\;Z:=z\,(p+q)-r\,(x+y)$$

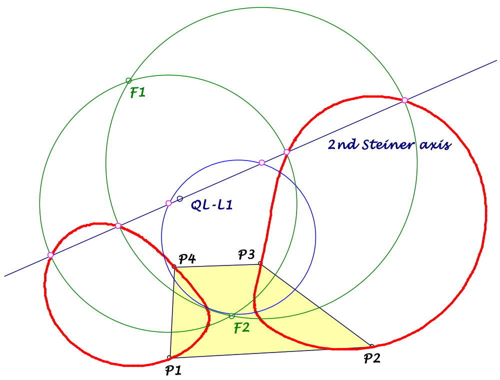
Construction of the quartic: For the construction we need the vertices M_1 and M_2 (unequal *QL-P1*) of the Miquel triangle and the fixed points F_1 and F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.

1. Circle through the intersections of the Newton Line *QL-L1* and the diagonals with center on M_1M_2.
2. For points X on this circle consider the angle bisector L_x of $\angle F_1XF_2$.
3. P, Q as intersections of L_x and its image-circle wrt *QL-Tf1* are points of the quartic.

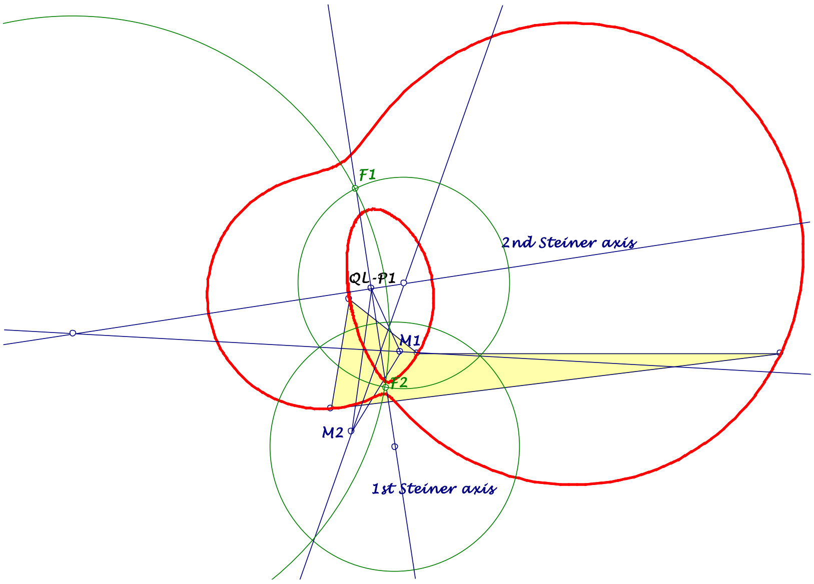


Properties:

- The quartic is the locus of points, where circles through opposite vertices of the quadrigon intersect perpendicular.
- The quartic is a circumquartic of the quadrigon.
- The quartic is invariant wrt *QL-Tf1*.
- The quartic is invariant wrt the transformation described in *EQF-Note 2013-12-05*.
- For *QL-Tf1* partners on the quartic the midpoints lie on a circle through the intersections of *QL-L1* and the diagonals, centered on M_1M_2.
- The 2^{nd} Steiner axis cuts this circle in two points not always real). Circles round these points through F_1, F_2 cut the 2^{nd} Steiner axis on the quartic.



- The quartic is anallagmatic, that means: The quartic is invariant wrt a reflection in a circle. There are three those circles (not always real): Two circles with midpoints in the intersections of the 2^{nd} Steiner axis with the angle bisectors of the Miquel triangle at M_1 and M_2, containing F_1 and F_2, and one circle perpendicular to the Schmidt circle with midpoint in the excenter of the Miquel triangle corresponding *QL-P1*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de