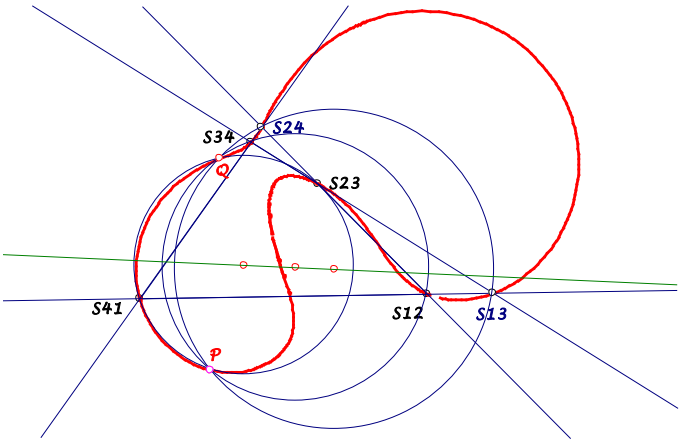
EQF-Note 2013-12-08

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

A Quartic for Quadrilaterals

For a quadrilateral there are three pairs of intersections and for each pair a circle through a given point. The points, for which these circles are coaxal give a quartic.



Let L_1, L_2, L_3, L_4 define a quadrilateral, then there are three pairs of intersections:

$$S_{12} = L_1 \cap L_2, S_{34} = L_3 \cap L_4 \quad and \quad S_{23} = L_2 \cap L_3, S_{41} = L_4 \cap L_1$$
$$and \quad S_{13} = L_1 \cap L_3, S_{24} = L_2 \cap L_4 \ .$$

We take for each pair the circumcircle through a point P and consider the locus of P for collinear centers of these three circles. This curve give a quartic.
If we chose the diagonal triangle *QL-DT* as reference triangle for barycentric coordinates and give L_4 the coefficients l, m, n, then the quartic has the equation

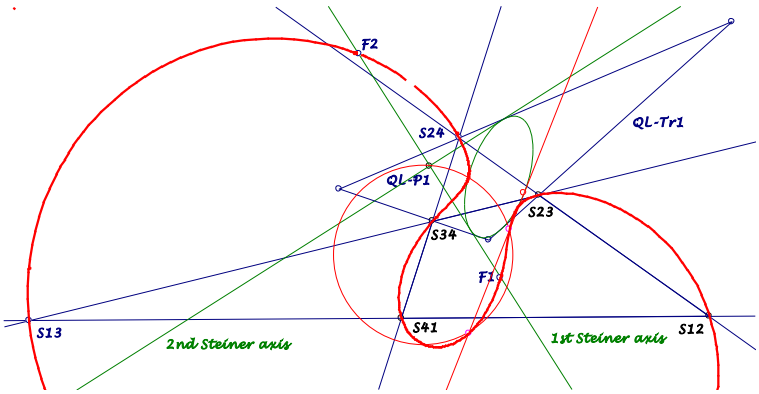
$$\begin{aligned}
&a^2 b^2 (l^2 - m^2) z (l^2 n^2 x X + m^2 n^2 y Y - n^4 z Z + 2 l^2 m^2 x y (x + y)) \\
&+ b^2 c^2 (m^2 - n^2) x (-l^4 x X + l^2 m^2 y Y + l^2 n^2 z Z + 2 m^2 n^2 y z (y + z)) \\
&+ a^2 c^2 (n^2 - l^2) y (l^2 m^2 x X - m^4 y Y + m^2 n^2 z Z + 2 l^2 n^2 x z (x + z)) \\
&+ a^4 (l^2 - m^2) (n^2 - l^2) y z (-m^2 y (x + y) + n^2 z (x + z)) \\
&+ b^4 (l^2 - m^2) (m^2 - n^2) x z (-n^2 z (y + z) + l^2 x (x + y)) \\
&+ c^4 (m^2 - n^2) (n^2 - l^2) x y (-l^2 x (x + z) + m^2 y (y + z)) == 0
\end{aligned}$$

with

$$X := 2 y z + x (x + y + z); \quad Y := 2 x z + y (x + y + z);$$
$$Z := 2 x y + z (x + y + z)$$

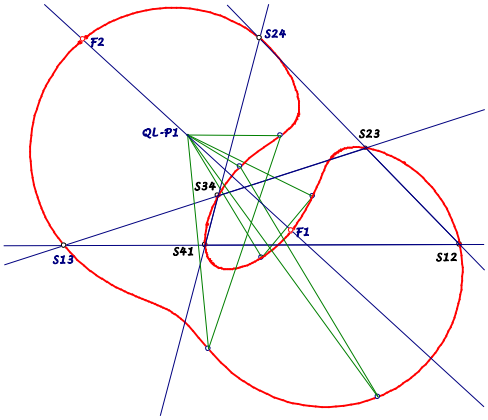
Construction of the quartic: For the construction we need the *1st* and *2nd* Steiner axis (see *QL-Tf1*):

1. Construct an inscribed conic of the Diagonal Triangle *QL-Tr1* with the *1st* and *2nd* Steiner axis (and Steiner Line *QL-L2*) as tangents.
2. The intersections of tangents at this conic and their image-circle wrt the Clawson-Schmidt Conjugate *QL-Tf1* will give points of the quartic.



Properties:

- This quartic contains the six intersections of the quadrilateral.
- The quartic contains the vertices M_1, M_2 (unequal *QL-P1*) of the Miquel Triangles *QA-Tr2* of the three quadrigon components.
- The quartic contains the fixed points F_1, F_2 of the Clawson-Schmidt Conjugate *QL-Tf1*.



- The quartic is invariant wrt the Clawson-Schmidt Conjugate *QL-Tf1*.
- The quartic is invariant wrt the transformation described in *EQF-Note 2013-12-05*.

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