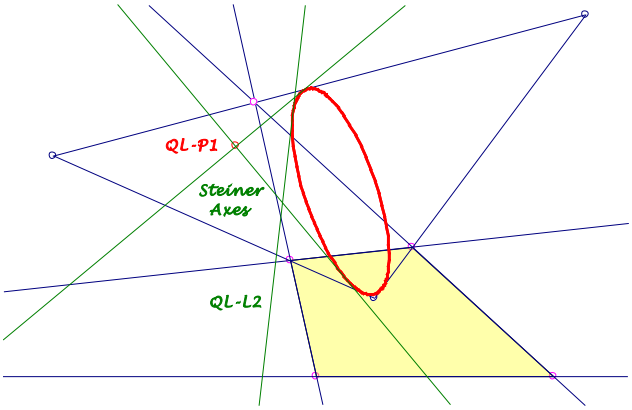
EQF-Note 2014-03-17

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Inscribed QL-DT-Conic tangent to the Steiner Axes

There is a special conic Co for a quadrilateral, inscribed in the QL-Diagonal Triangle and tangent to the Steiner Axes and the Steiner Line QL-L2. The Steiner Axes are the angle bisectors at the Miquel Point QL-P1 wrt two opposite points of the quadrilateral (see EQF QL-Tf1). This conic Co is already mentioned in QFG-message 465 and in EQF-Note 2014-02-27. – Reference triangle for barycentric coordinates is QL-DT.



Properties: It is worth to have a special look on tangents and polars wrt this conic *Co*; there seem to be no relevant *EQF*-points on *Co*. Some properties are already mentioned in *QFG*-message 465 and in *EQF-Note 2014-02-27*.

- Defining tangents of *Co*: The diagonals of the quadrilateral and the Steiner Axes (see *QL-Tf1*).

- Equation of *Co* wrt reference triangle *QL-DT* and one line (l,m,n):

$$U^2 x^2 + V^2 y^2 + W^2 z^2 - 2 V W y z - 2 W U z x - 2 U V x y = 0;$$
$$U := l^2 (m^2 - n^2) (b^2 (l^2 - m^2) + c^2 (n^2 - l^2));$$
$$V := m^2 (n^2 - l^2) (c^2 (m^2 - n^2) + a^2 (l^2 - m^2));$$
$$W := n^2 (l^2 - m^2) (a^2 (n^2 - l^2) + b^2 (m^2 - n^2)).$$

By the way, the coordinates of the Miquel Point are:

$$\left\{\frac{U}{l^2}, \frac{V}{m^2}, \frac{W}{n^2}\right\}$$

- The Steiner Line *QL-L2* is tangent to *Co*. Point of tangency:

$$\{l^4 (m^2 - n^2)^2 V W, m^4 (n^2 - l^2)^2 W U, n^4 (l^2 - m^2)^2 U V\}$$

- The center of *Co*

$$\{V + W, W + U, U + V\}$$

lies diametral to *QL-P6* on the circumcircle of *QL-P1, QL-P6, QL-P17*.
The Brianchon point of *Co* wrt *QL-DT* is

$$\{V W, W U, U V\}$$

- The polars of *QL-P1* wrt inscribed conics of the quadrilateral envelope the conic *Co*.

- *Co* is also the envelope of the *QL-Tf2* images of lines through *QL-P1*. Example: The line parallel *QL-P3.4.5.6* through *QL-P1* gives *QL-L2* as tangent for *Co*.

- Consider a quadrigon component $S_{12}S_{23}S_{34}S_{41}$ of the quadrilateral with Miquel Point M_1=*QL-P1* and the further Miquel Points M_2 of $S_{12}S_{23}S_{41}S_{34}$ and M_3 of $S_{12}S_{34}S_{23}S_{41}$, then the lines M_2S_{24} and M_3S_{13} are tangent to *Co*.

- The polar of *QL-P1* wrt *Co* is the Clawson-Schmidt Conjugate *QL-Tf1* (shortened *CSC*) of the Dimidium Circle *QL-Ci6*.

- The polar of *QL-P1* wrt *Co* contains the Least Squares Point *QL-P26* and *CSC(QL-P17), CSC(QL-P24).* Coefficients of the polar:

$$\{U (-m^2 n^2 U^2 + n^2 l^2 V^2 + l^2 m^2 W^2),$$
$$V (m^2 n^2 U^2 - n^2 l^2 V^2 + l^2 m^2 W^2),$$
$$W (m^2 n^2 U^2 + n^2 l^2 V^2 - l^2 m^2 W^2)\}$$

- The *CSC*-images for reflections of *QL-P1* in tangents at *Co* give an orthogonal hyperbola, centered in *QL-P6* with asymptotes parallel to the Steiner Axes containing *QL-P4* and *QL-P5*.

- The perpendicular bisector of *QL-P1.CSC(QL-P5)* is tangent to *Co*. This line is the *CSC*-image of the Plücker Circle *QL-Ci5*.

- Consider inscribed conics of a quadrilateral centered in points on *QL-L1*: The points of tangency lie with each pair of opposite points of the quadrilateral on a conic. The polars of *QL-P1* wrt these three conics have a common point and the locus for this point is the discussed conic *Co*.

- The intersections of tangents *T* at *Co* and *QL-Tf2(T)* give a conico-pivotal isocubic (see *EQF-Note 2014-02-27*).

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