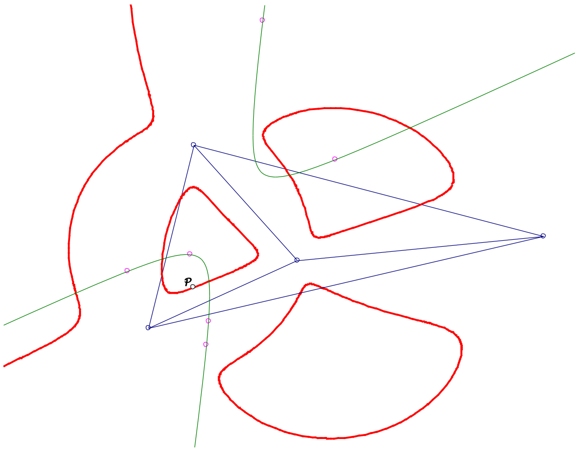
EQF-Note 2014-04-14

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Coconic Reflections of Points in P_iP_j

This problem is mentioned by Randy Hutson in QFG-message 387: What is the locus of points whose reflections in the six lines P_iP_j of a quadrangle lie on a conic? In QFG-message 391 Angel Montesdeca has given the very extensive equation for the curve of degree seven. For this curve a construction will be described.

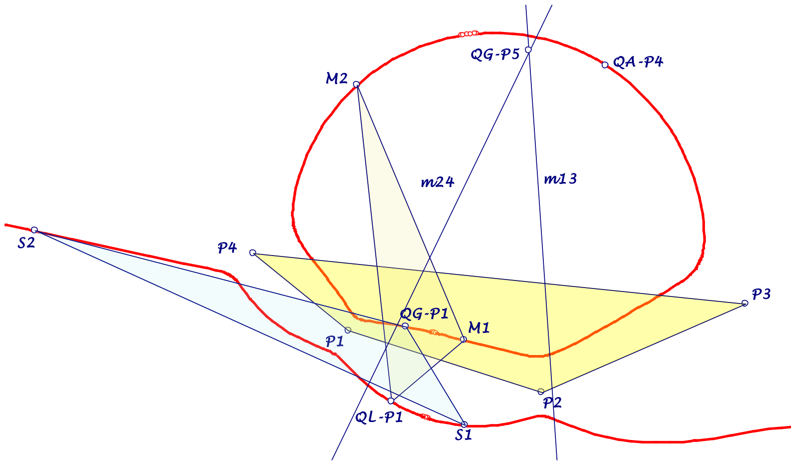


Let P_1, P_2, P_3, P_4 be the points of a quadrangle, but we shall describe the constellation wrt a quadrigon version $P_1P_2P_3P_4$ with

… the Diagonal Triangle *QG-P1*, S_1, S_2,
… the Miquel Triangle *QL-P1*, M_1, M_2.
… the perpendicular bisectors m_{13}, m_{24} of the diagonals,
… the *QL*-mapping *CSC* (Clawson-Schmidt Conjugate *QL-Tf1*),
… the *QG*-mapping *CIC*, which gives for a point X the second intersection of the circles $Ci(X,P_1,P_3)$ and $Ci(X,P_2,P_4)$. This transformation is already mentioned in *EQF*-Note *2013-12-05*.

Properties:

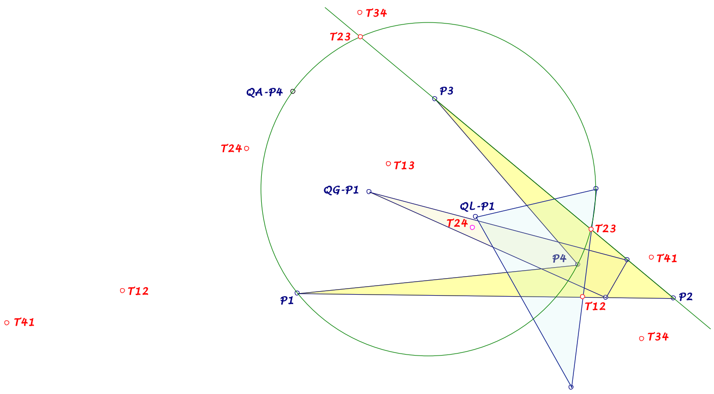
- The basic 7 points of the curve are (see *QFG* 388, 389)
  … the vertices of the Diagonal triangle,
  … the vertices of the Miquel Triangle,
  … the Isogonal Center *QA-P4*.
- The asymptote is perpendicular to the Steiner Line *QL-L2* (see *QGF* 389).



- *QA-P4* lies either inside of all circumcircles of the triangle components or outside. In the first case the curve is bipartite, in the second case the curve is quadripartite.

- The curve is *CSC*-invariant (see *QFG* 388, 392):
  … *CSC(QG-P1) = QA-P4*, $CSC(S_{1,2}) = S_{2,1}$,
  … *CSC(QL-P1) not defined*, $CSC(M_{1,2}) = M_{2,1}$,
  … *CSC(QA-P4) = QG-P1*.

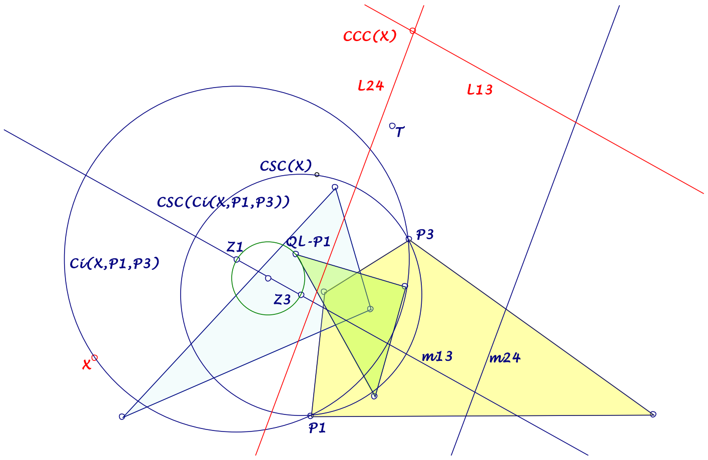
- The curve is *CIC*-invariant:
  … *CIC(QG-P1) not defined*, $CIC(S_{1,2}) = M_{2,1}$,
  … *CIC(QL-P1) = QA-P4*, $CIC(M_{1,2}) = S_{2,1}$,
  … *CIC(QA-P4) = QL-P1*.



- There are further 12 special points on the curve (not all real): $T_{ij} = P_iP_j \cap Ci(QA\text{-}P4,P_k,P_l)$ intersections of lines P_iP_j and the circumcircle through the remaining vertices P_k, P_l and *QA-P4*.

Construction:

For construction we need a transformation *CCC* as follows:

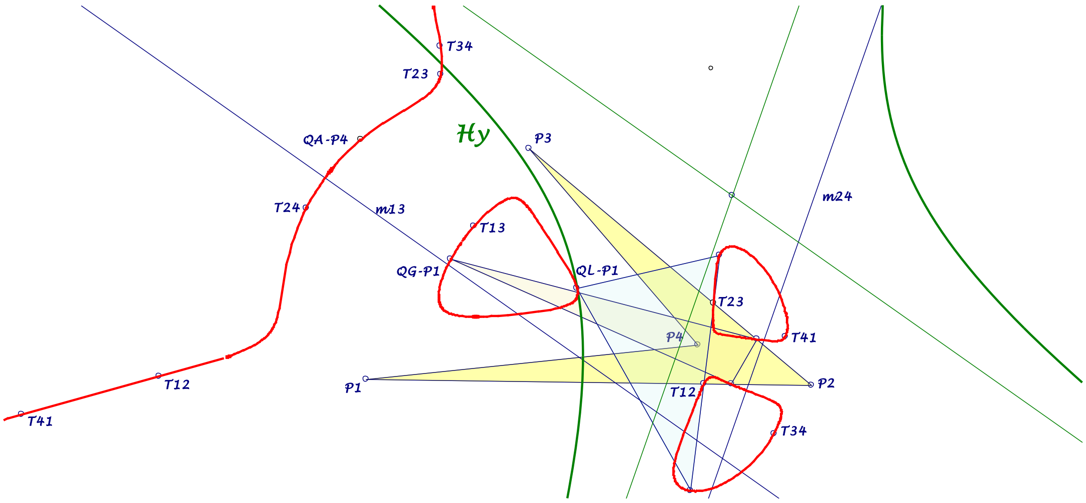


Let X be a variable point,
… Z_1 midpoint of the circle $Ci(X,P_1,P_3)$ on m_{13},
… Z_3 midpoint of $CSC(Ci(X,P_1,P_3)$ on m_{13},
… circle $Ci(Z_1,Z_3,QL\text{-}P1)$,
… CSC of this circle gives a line L_{13} parallel m_{13}.
… Analog we get a line L_{24}.
… The intersection of L_{13} and L_{24} is $CCC(X)$.

$CCC(QG\text{-}P1) = CCC(QA\text{-}P4) = CCC(QL\text{-}P1) = QL\text{-}P1$,
$CCC(S_1) = CCC(S_2) = CCC(M_1) = CCC(M_2) = T$ (new point),
$CCC(T_{13}) = CCC(T_{24}) = QL\text{-}P1$.

- The *CCC*-images of the 7 basic points and the 12 special points (see above) lie on a hyperbola *Hy* with asymptotes parallel to the perpendicular bisectors m_{13}, m_{24} of the diagonals.

- The inverse transformation of *CCC* for points on *Hy* give the locus for points with coconic reflections in P_iP_j.



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