EQF-Note 2014-05-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

Orthogonal Hyperbolas through
***QL-P26* (Least Squares Point)**

In *EQF* there is a special orthogonal hyperbola through *QL-P26*; here is a generalization. The studies are only CABRI controlled, analytical calculations are too extensive. We start with

… a quadrilateral $L_1L_2L_3L_4$,
… a line pencil for a point *P* (*), called pivot,
… lines *p* through *P* with variable points *X*,

and construct

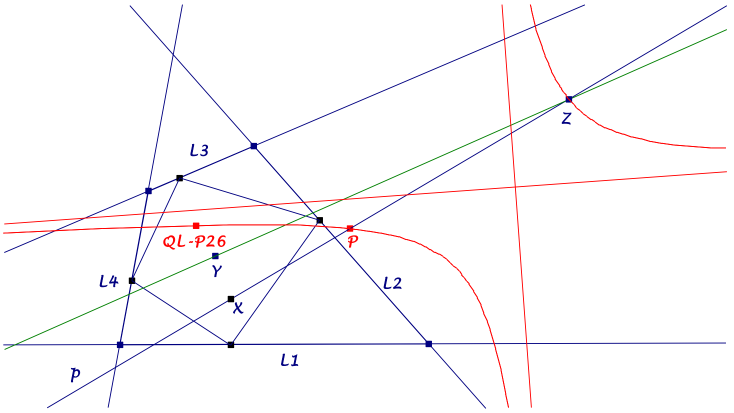
… the quadrangle of the pedal points of *X* wrt L_i,
… the centroid *Y* of the pedal quadrangle

and consider

… the locus of *Y* for variable *X*, which is a line *q*,
… the intersection *Z* of *p* and *q*,
… the locus of Z for the lines *p* through *P*.

(*) For a point at infinity see below.

The result of this construction is an orthogonal hyperbola.

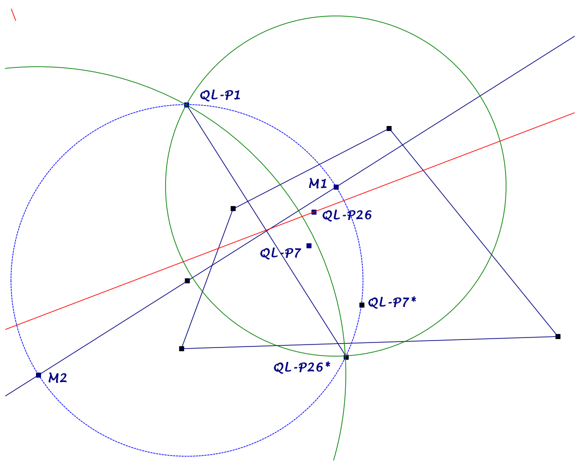


These orthogonal hyperbolas contain always the pivot point *P* and the Least Squares Point *QL-P26*.

For the pivot point *QL-P26* this orthogonal hyperbola degenerates into two orthogonal lines.

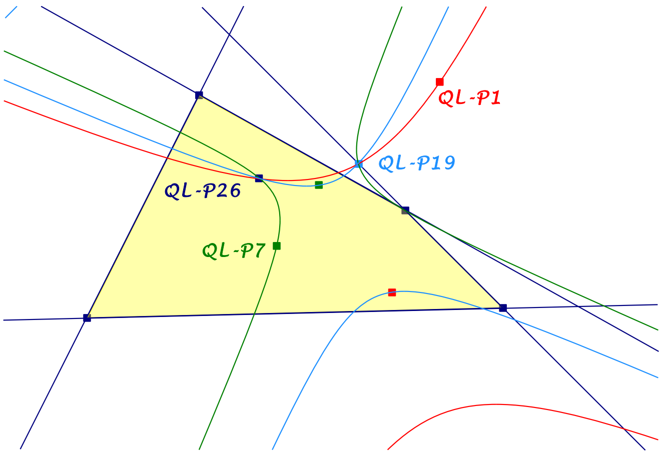
The asymptotes of these orthogonal hyperbolas are parallel in pairs.

The two orthogonal lines through *QL-P26*, which are parallel to the asymptotes of the hyperbolas, can be constructed as follows. The Clawson-Schmidt Conjugate (*QL-Tf1*) will be abbreviated with *CSC*.
The circumcircle of *QL-P1*, *CSC(QL-P26)* and *CSC(QL-P7)* intersects the perpendicular bisector of *QL-P1.CSC(QL-P26)* in M_1 and M_2. The *CSC*-images of the circles round M_1 and M_2 through *QL-P1* are the searched orthogonal lines through *QL-P26*.



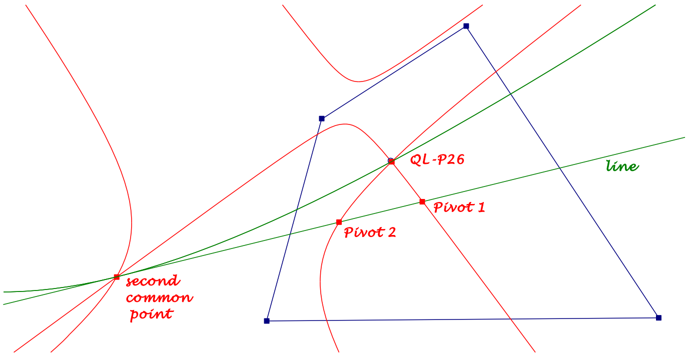
Taking for pivot *QL*-points, there are three special examples:

In addition to *QL-P26* the orthogonal hyperbolas with pivot *QL-P1* and *QL-P7* contain *QL-P19*.



There are some generalizations:

Pivots on a line have hyperbolas, which contain with *QL-P26* a second common point on the line. The line is tangent to the hyperbola for the second common point as pivot.



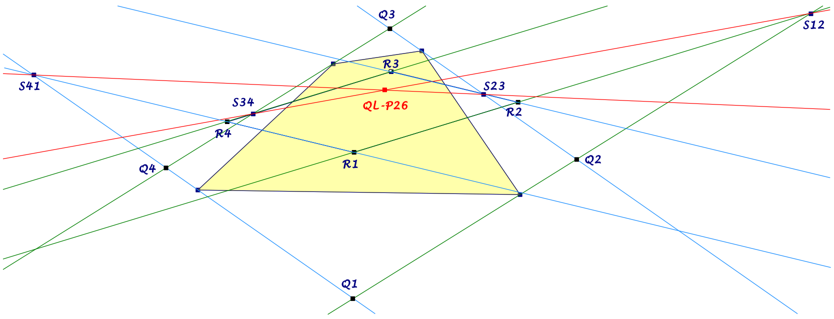
Example: For pivots on the line *QL-L6* the second common point will be *QL-L6* ^ *QL-P26.QL-P19*.

For pivots on a line through *QL-P26* the hyperbolas contact. For pivots on lines through a fixed point the second common points lie on the hyperbola of the fixed point.

Pivots on a line have hyperbolas with collinear centers.

(*) If the pivot is a point at infinity, the result of the construction at the beginning will not be a hyperbola but a line connecting *QL-P26* with the second common points for pivots on parallel lines through the point at infinity. For the point at infinity of *QL-L5,6* this line will be *QL-P26.QL-P19.*

This leads to the following clearly arranged construction of *QL-P26*:



Take a quadrigon component of the quadrilateral
… with the circumscribed parallelogram $Q_1Q_2Q_3Q_4$,
… the centroids R_i of the pedal quadrangles of Q_i,
… the intersections S_{ij} of R_iR_j and Q_iQ_j
… and $S_{12}.S_{34}$ ^ $S_{23}.S_{41}$ = *QL-P26*.

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