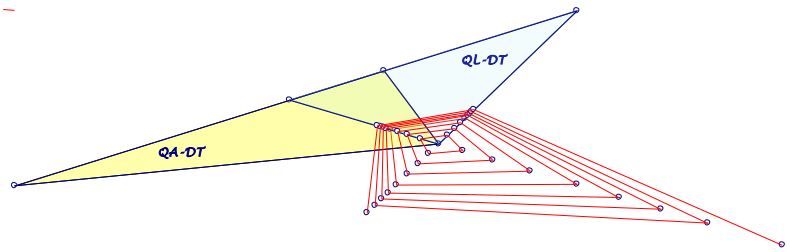
EQF-Note 2014-07-17

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://chrisvantienhoven.nl/

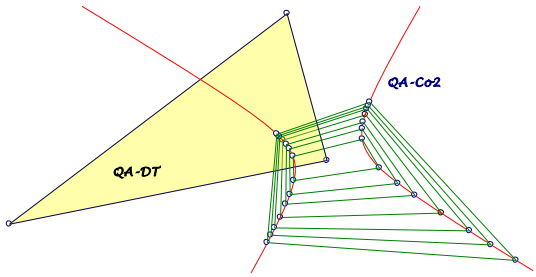
Special Loci for *EQF*-Points

Here are studied the loci of EQF-points for three pencils of quadrigons:

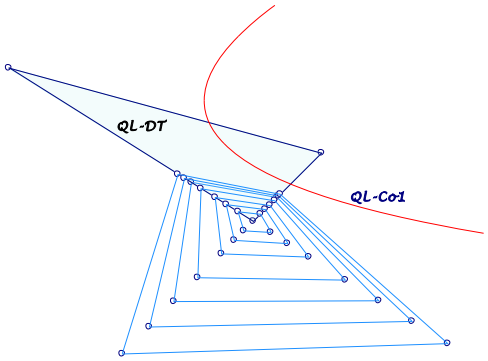
1. ***Quadrigons (with the same Diagonal Crosspoint QG-P1), which have the same QA-Diagonal Triangle QA-DT and the same QL-Diagonal Triangle QL-DT.***



2. ***Quadrigons with the same Diagonal Crosspoint QG-P1, which have the same QA-Diagonal Triangle QA-DT and the same QA-Orthogonal Hyperbola QA-Co2.***



3. ***Quadrigons with the same Diagonal Crosspoint QG-P1, which have the same QL-Diagonal Triangle QL-DT and the same Inscribed Parabola QL-Co1.***



Barycentric coordinates for the equations of the curves are DT-notations (see EQF).

For the second pencil the following shortcuts are used:

$$u=\frac{1}{b^2r^2-c^2q^2},\quad v=\frac{1}{c^2p^2-a^2r^2},\quad w=\frac{1}{a^2q^2-b^2p^2},$$

and the orthogonal hyperbola has the equation

$$\frac{x^2}{u}+\frac{y^2}{v}+\frac{z^2}{w}=0.$$

For the third pencil the following shortcuts are used:

$$u=m^2-n^2,\quad v=n^2-l^2,\quad w=l^2-m^2,$$

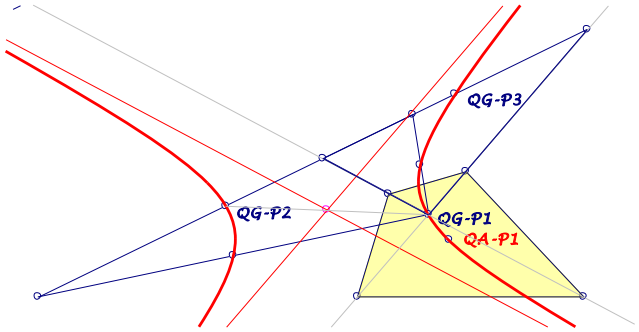
and the inscribed parabola has the equation

$$\frac{x^2}{u}+\frac{y^2}{v}+\frac{z^2}{w}=0.$$

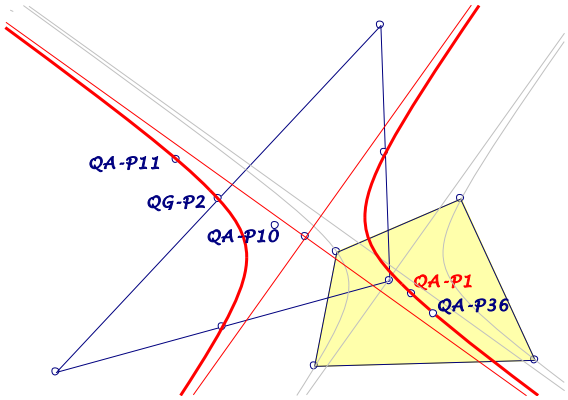
Some remarks: For every EQF-point three loci can be considered, but not all are discussed. Only loci are mentioned, which are a fixed point or a line or a circle or a conic, otherwise you find "------" If a locus is not completely described, you will find "???". Considered are only QA-P1 – QA-P30, QL-P1 – QL-P26 , QG-P1-QG-P19 .

QA-P1 *QA*-Centroid or Quadrangle Centroid

- **Hyperbola:**
  … centered in the midpoint of *QG-P1.QG-P2*,
  … asymptotes parallel to the diagonals,
  … through *QG-P1*, *QG-P2*, *QG-P3*,
  ... equation: $r^2x(x-y)+(p^2-r^2)zx+p^2z(y-z)=0$



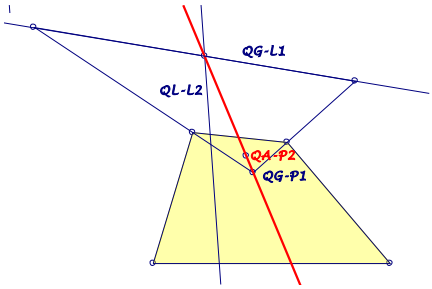
- **Orthogonal hyperbola:**
  … centered in *T(QG-P2, QA-P10, -3:1)*,
  … parallel asymptotes wrt *QA-Co2*,
  … circumscribed *QA-DT* medial triangle,
  … through *QG-P2*, *QA-P11*, *QA-P36*
  … equation:
  $vwx(-x+y+z)+wuy(x-y+z)+uvz(x+y-z)=0$



- **Line *QG-L3***

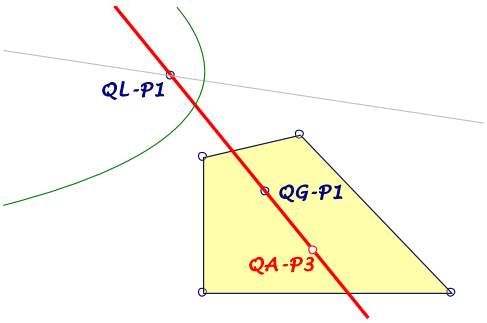
QA-P2 Euler-Poncelet Point

- **Circle *QA-Ci1***
- **Fixed point**
- **Line**
  …through *QG-P1* and the intersection of *QG-L1* and *QL-L2*.
  … equation:
  $(-S_A u + S_B v + S_C w)x + (S_A u + S_B v - S_C w)z = 0$



QA-P3 Gergonne-Steiner Point

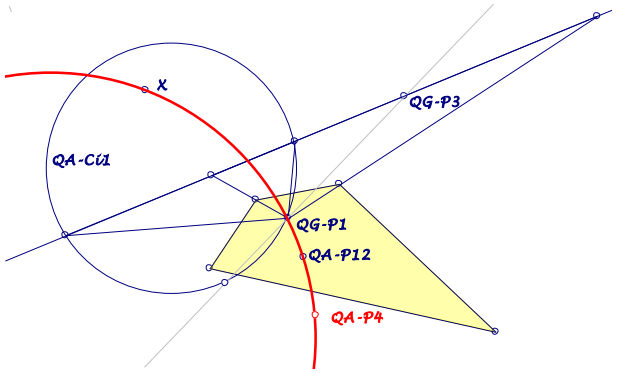
- **------**
- **Orthogonal Hyperbola *QA-Co4***
- **Line**
  ... through *QG-P1* and *QL-P1*
  …equation:
  $(S_A u + S_B v - S_C t)wx - (-S_A u + S_B v + S_C w)rz = 0$



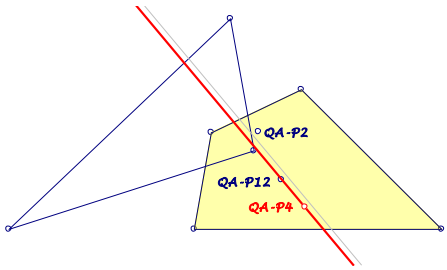
QA-P4 Isogonal Center

- **Circle**
  … through *QG-P1*, *QA-P12* and a point *X*, which is the reflection of the second intersection of *QG-P1.QG-P3* and *QA-Ci1* in *QG-L1*.
  … equation:

$$2r^2a^2S_Ax^2-2p^2c^2S_Cz^2+(S_A-S_C)(p^2c^2+r^2a^2)zx$$
$$+(p^2c^4+r^2a^2(S_A-S_B))xy-(r^2a^4-p^2c^2(S_B-S_C))yz=0$$

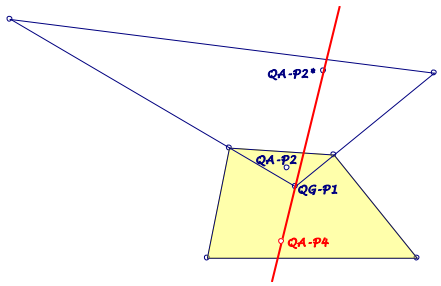


- **Line**
  … through *QA-P12*,
  … parallel to the Simson line of *QA-P2* wrt *QA-DT*,
  … equation: $a^2S_Avwx+b^2S_Bwuy+c^2S_Cuvz=0$



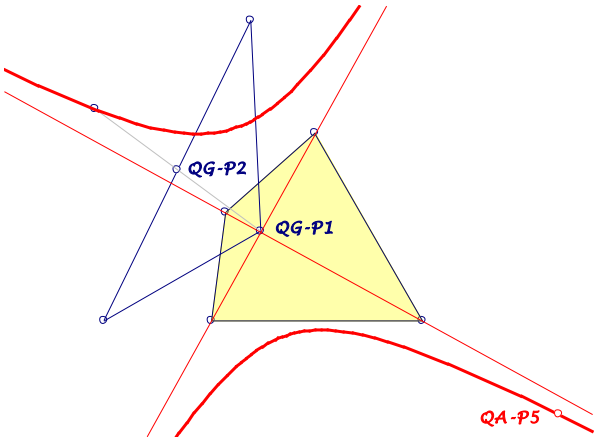
- **Line**
  … through *QG-P1* and the isogonal conjugate of *QA-P2* wrt *QL-DT*,
  … equation:

$$c^2(S_Au+S_Bv-S_Cw)x+a^2(-S_Au+S_Bv+S_Cw)z=0$$

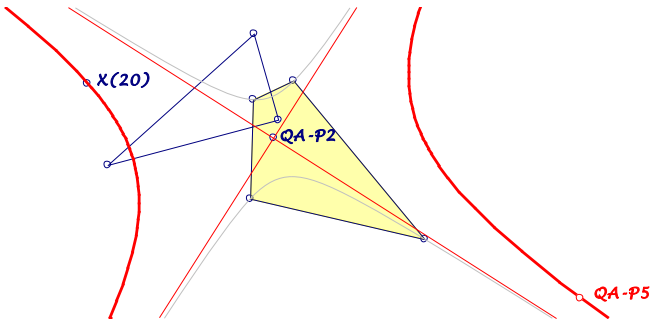


QA-P5 Isotomic Center

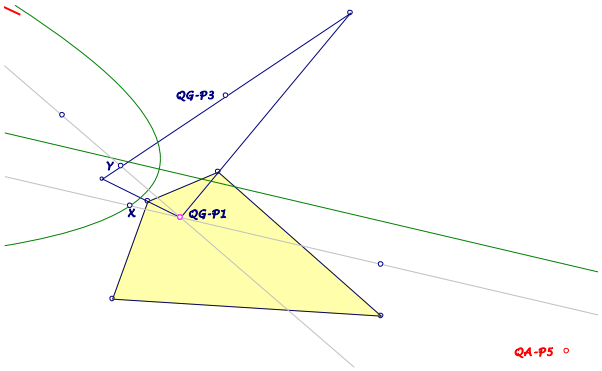
- **Hyperbola**
  … centered in *QG-P1*,
  … asymptotes are the diagonals,
  … through the reflection of *QG-P1* in *QG-P2*,
  … equation:
  $p^2(x^2+y^2)-r^2(y^2+z^2)+2(p^2-r^2)(xy+yz+zx)=0$



- **Orthogonal hyperbola**
  … asymptotes and center as *QA-Co2*,
  … through *X(20)* of *QA-DT*,
  … equation:
  $vw(y+z)(2x+y+z)+wu(z+x)(x+2y+z)$
  $+uv(x+y)(x+y+2z)=0$



- **Line**
  … through *T(QG-P1, X, -4:5)* with *X* intersection of *QL-Co1* and a parallel to *QL-L1* in *QG-P1*,
  … through *T(QG-P1, Y, -2:1)* with *Y* intersection of *QG-L1* and the polar of *QG-P3* wrt *QL-Co1*,
  … equation: $(u-v)wx+2wuy+(w-v)uz=0$



QA-P6 Parabola Axes Crosspoint

- **Conic**

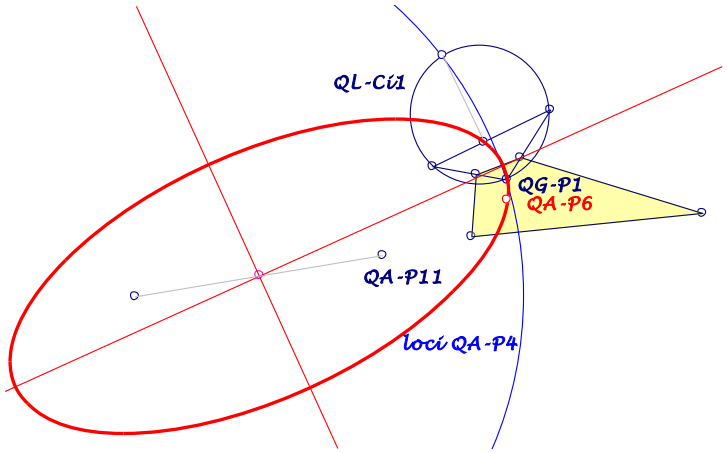
  … centered in the midpoint of *QA-P11* and the center of the circle, which is the loci for *QA-P4*,

  … axes parallel to the angle bisectors of *QL-DT* in *QG-P1*,

  … through *QG-P1* and the pedal point on *QG-L1* of the second intersection of *QL-Ci1* and the circle, which is the loci of *QA-P4*,

  … equation:

$$r^4a^2(r^2a^2-p^2S_B)x^2-p^4c^2(p^2c^2-r^2S_B)z^2$$
$$+p^2r^4a^2S_Axy-p^4r^2c^2S_Cyz$$
$$+p^2r^2(r^2a^2(c^2+S_B)-p^2c^2(a^2+S_B))zx=0$$



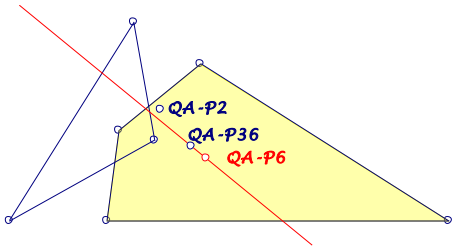
- **Line**

  … Simson line of *QA-P2* wrt *QA-DT*,

  … through *QA-P36*

  … equation:

$$\frac{vwx}{b^2w(u+v)-c^2v(u+w)}+\frac{wuy}{c^2u(v+w)-a^2w(v+u)}$$
$$+\frac{uvz}{a^2v(w+u)-b^2u(w+v)}=0$$



- **Line**
  … through *QG-P1* and a point *Z*, constructed as follows:
  let *X* be the intersection of *QG-L1* and *QL-L2*,
  let *Y* be the intersection of *QG-L3* and a parallel to *QA-L1* through *X*,
  let *Z* be the intersection of a parallel to *QA-L4* through *Y* and a parallel to *QA-L2* through *X*,
  … equation: $c^2(a^2v+b^2u)x+a^2(b^2w+c^2v)z=0$

QA-P7 QA-Nine-point Homothetic Center

- **Hyperbola**
  … through *QG-P1*,
  … ???
- **Hyperbola**
  …through *QA-P11*,
  … ???
- **Line**
  …through *QG-P1*,
  … ???

QA-P8 Midray Homothetic Center

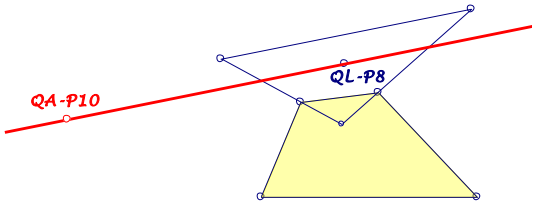
- **Hyperbola**
  ... through *QG-P1*,
  … ???
- **Hyperbola**
  …centered in *QA-P13*,
  ... through *QA-P11*, *QA-P12*,
  …???
- **Line**
  ... through *QG-P1*,
  … ???

QA-P9 *QA*-Miquel Center

- **-----**
- **-----**
- **Line**
  …???

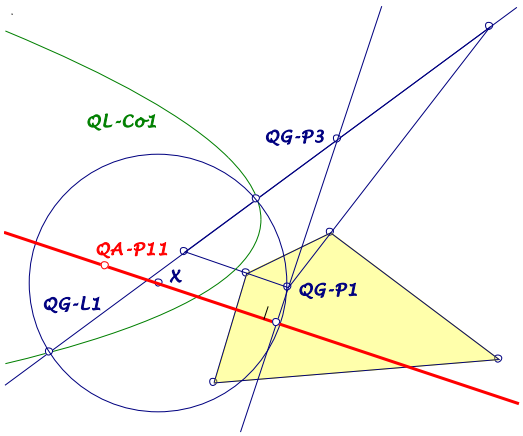
QA-P10 Centroid of the *QA*-Diagonal Triangle

- **Fixed point**
- **Fixed point**
- **Line**
  …parallel to *QG-L1*,
  …through *QL-P8*,
  …equation: $x-2y+z=0$



QA-P11 Circumcenter of the *QA-DT* Triangle

- **Fixed point**
- **Fixed point**
- **Line**
  …orthogonal to *QG-P1*.*QG-P3*,
  …through circumcenter *X* for *QG-P1* and the intersections of *QL-Co1* and *QG-L1*,
  …equation: $c^2x-S_By+a^2z=0$



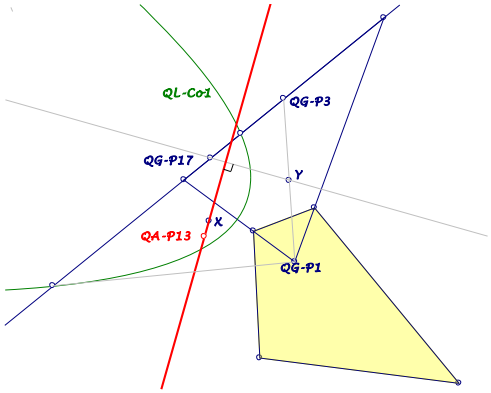
QA-P12 Orthocenter of the *QA-DT* Triangle

- **Fixed point**
- **Fixed point**
- **Line**
  …through *QG-P1* perpendicular *QG-L1*

QA-P13 Nine-Point Center of the *QA-DT* Triangle

- **Fixed point**
- **Fixed point**

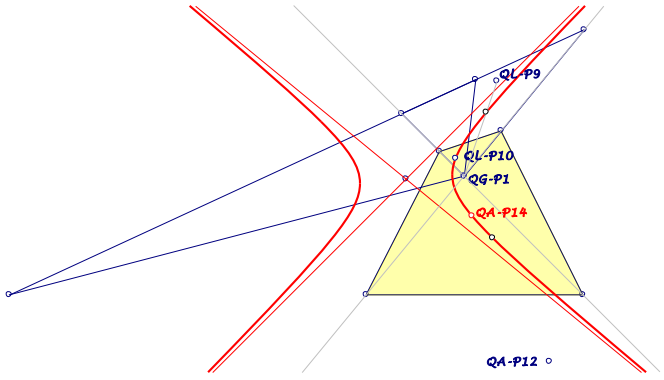
- **Line**
  … through the nine-point center *X* of the triangle *QG-P1* and the intersections of *QL-Co1* and *QG-L1*,
  … orthogonal to *Y.QG-P17* with *Y* midpoint of *QG-P1.QG-P3*,
  … equation:
  $(S^2-2S_A^2+S_AS_C)x-(3S^2+S_AS_C)y+(S^2-2S_C^2+S_AS_C)z=0$



QA-P14 Centroid of the Morley Triangle

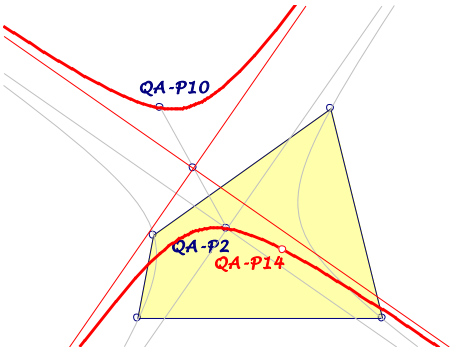
- **Hyperbola**
  … asymptotes orthogonal to the diagonals,
  …through *QL-P10, T(QG-P1, QL-P9, 2:1), T(QG-P1, QA-P12, 1:2)*,
  … equation:

$$(p^2c^2S_A+r^2(S^2+S_BS_C))x^2-(p^2(S^2+S_AS_B)+r^2a^2S_C)z^2$$
$$+S_B(p^2c^2-r^2a^2)y^2-(p^2(S^2+2S_A^2)-r^2(S^2+2S_C^2))xz$$
$$-(2p^2c^4-r^2(S^2+2S_B^2))xy-(p^2(S^2+2S_B^2)-2r^2a^4)yz=0$$

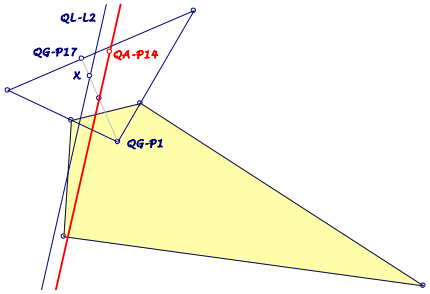


- **Orthogonal hyperbola**
  … parallel asymptotes wrt *QA-Co2*,
  … centered in the midpoint of *QA-P2.QA-P10*,
  … through *QA-P2*, *QA-P10*,
  … equation:

$$2vwx^2+2wuy^2+2uvz^2-w(u+v)xy-v(w+u)zx-u(v+w)yz=0$$



- **Line**
  … parallel *QL-L2*,
  … through *T(QG-P1, X, 2:1)* with *X* intersection of *QG-P1.QG-P17* and *QL-L2*,
  … equation:
  $(-2S_A u + 2S_B v + S_C w)x + (S_A u - S_B v + S_C w)y$
  $+ (S_A u + 2S_B v - 2S_C w)z = 0$



QA-P15 Orthocenter of the Morley Triangle

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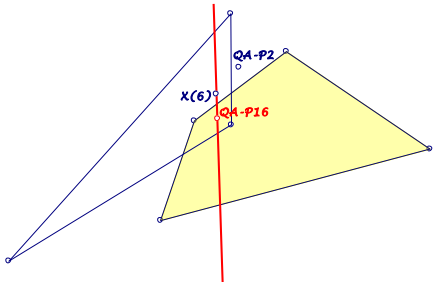
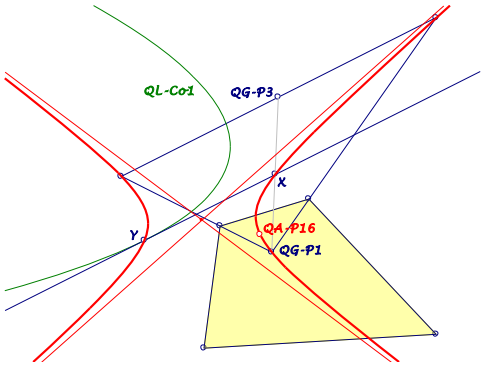
- ------
- **Line**
  …???

QA-P16 *QA*-Harmonic Center

- **Line** *QG-L2*
- **Line**
  … trilinear polar of *QA-P2* wrt *QA-DT*,
  … through *X(6)* of *QA-DT*,
  … equation: $vwx + wuy + uvz = 0$



- **Hyperbola**
  … circumscribed *QL-DT*,
  … through midpoint *X* of *QG-P1.QG-P3*,
  … *Y* contact point of the tangent at *QL-Co1* through *X* (parallel to *QG-L1*),
  … equation: $uyz + 2vzx + wxy = 0$



QA-P17 Involutary Conjugate of *QA-P5*

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QA-P18 Involutary Conjugate of *QA-P19*

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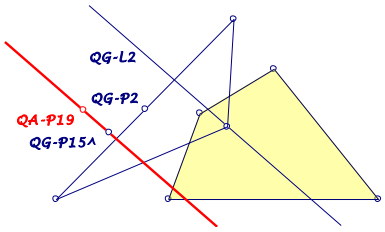
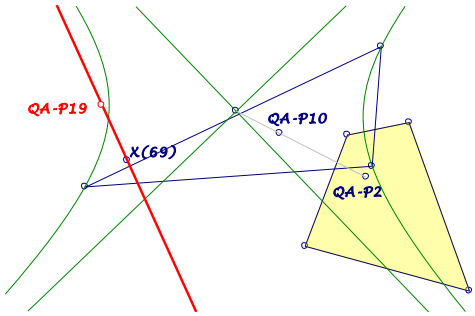
QA-P19 AntiComplement of *QA-P16* wrt *QA-DT*

- **Line**
  … reflection of *QG-L2* in *QG-P2*,
  … through isotomic conjugate of *QG-P15* wrt *QA-DT*,
  … equation: $p^2x + (p^2 - r^2)y - r^2z = 0$



- **Line**
  … isotomic conjugate of an orthogonal hyperbola, circumscribed *QA-DT*, centered in *T(QA-P2, QA-P10,-3:1)* through *X(69)* of *QA-DT*,
  … equation: $u(v+w)x + v(w+u)y + w(u+v)z = 0$



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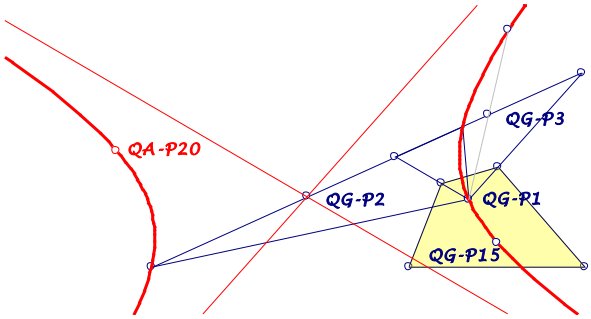
QA-P20 Reflection of *QA-P5* in *QA-P1*

- **Hyperbola**
  … *QA-DT*-isotomic conjugate of the locus of *QA-P19*,
  … asymptotes parallel to the diagonals,
  … centered in *QG-P2*,
  …through *QG-P1*, *QG-P15*, reflection of *QG-P1* in *QG-P3*,
  … equation: $p^2(x+y)z-r^2(y+z)x=0$



- **Orthogonal Hyperbola *QA-Co4***
- **Line**
  … parallel *QL-L1* through *QG-P1*

QA-P21 Reflection of *QA-P16* in *QA-P1*

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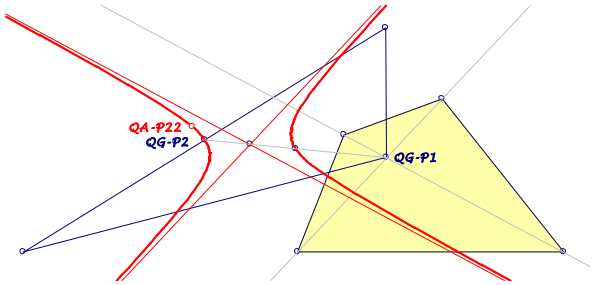
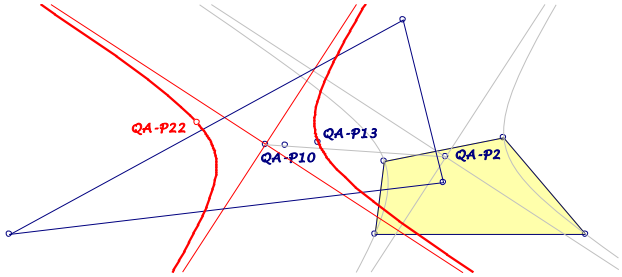
QA-P22 Midpoint *QA-P1* and *QA-P20*

- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in *T(QG-P1, QG-P2, 3:1)*,
  … through *QG-P2* and midpoint *QG-P1.QG-P2*
  … equation:
  $p^2(x+y-z)(x+y-3z)-r^2(-x+y+z)(-3x+y+z)=0$

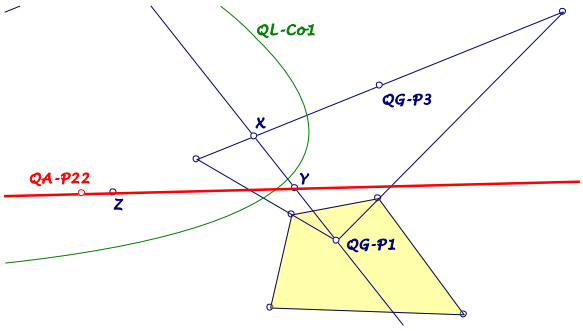


- **Orthogonal hyperbola**
  … parallel asymptotes wrt *QA-Co2*,
  … centered *T(QA-P2, QA-P10, -9:1)*,
  … through *QA-P13*,
  … equation:

$$(uv+uw+3vw)x^2+(vw+vu+3wu)y^2+(wu+wv+3uv)z^2$$
$$+2(vw-2u(v+w))yz+2(wu-2v(w+u))zx+2(uv-2w(u+v))xy=0$$

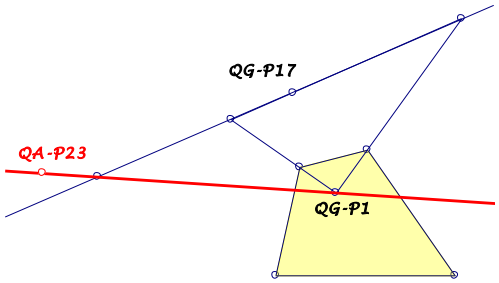


- **Line**
  … through the midpoint *Y* of *QG-P1* and *X* with *X* intersection of *QG-L1* and the polar of *QG-P3* wrt *QL-Co1*,
  … through the 4th harmonic point *Z* of *X* wrt *QL-2P3*,
  … equation: $(w-u)(wx-uz)+2wuy=0$

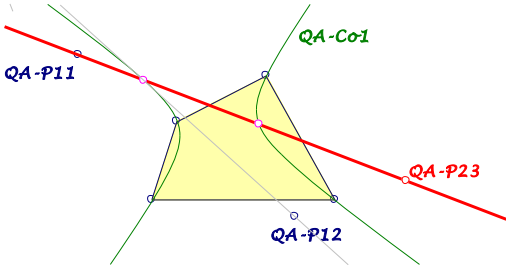


QA-P23 Inscribed Square Axes Crosspoint

- **Line**
  … through *QG-P1* and the 4th harmonic point of *QG-P17* wrt *QG-2P3*,
  … equation: $p^2S_Az-r^2S_Cx=0$



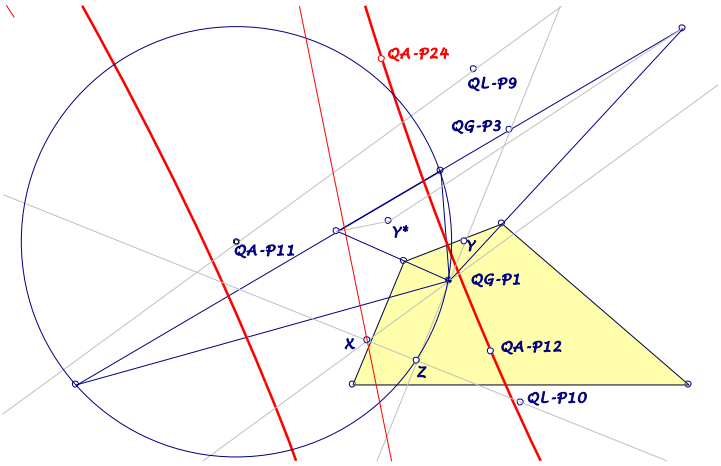
- **Line**
  … polar of *QA-P12* wrt *QA-Co2*,
  … through *QA-P11*,
  … equation: $vwS_BS_Cx+wuS_CS_Ay+uvS_AS_Bz=0$



- **Line *QA-L4***

QA-P24 Anticomplement of *QA-P1* wrt Morley Triangle

- **Hyperbola**
  … centered in the intersection *X* of a parallel to *QA-P11.QL-P9* through *QG-P1* and a perpendicular line to *QG-P1.QG-P3* through *QL-P10*,
  … asymptotes orthogonal wrt the legs of a triangle about *QL-2P2* with the vertex *Y**, which is the *QL-DT*-isogonal conjugate of the fourth harmonic point *Y* of the second intersection *Z* of *QG-P1.QG-P3* and *QA-Ci1* wrt *QG-P1.QG-P3*
  … through *QA-P12*

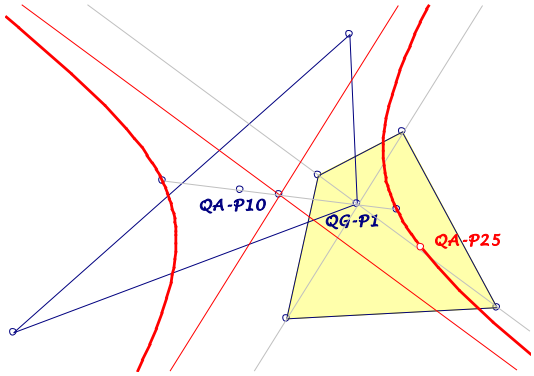


- **Hyperbola**
  … centered in *QA-P2*,
  … through *QA-P12*,
  … ???
- **Line**
  … ???

QA-P25 1st QA-Quasi Centroid

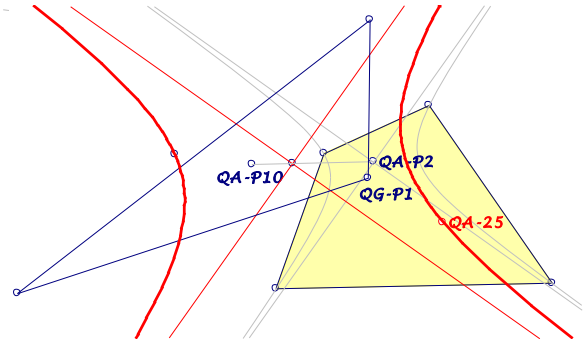
- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in *T(QG-P1, QA-P10, 2:1)*,
  … through *T(QG-P1,QA-P10,-1:4)* and *T(QG-P1,QA-P10,-5:2)*,
  … equation:

$$p^2(5x+5y-4z)(x+y+10z)-r^2(-4x+5y+5z)(10x+y+z)=0$$



- **Orthogonal hyperbola**
  … asymptotes parallel wrt *QA-Co2*,
  … centered *T(QA-P2, QA-P10, 2:1)*,
  …through *T(QG-P1, QA-P10, -5:2)*,
  … equation:

$$vw(-4x+5y+5z)(10x+y+z)$$
$$+wu(5x-4y+5z)(x+10y+z)$$
$$+uv(5x+5y-4z)(x+y+10z)=0$$



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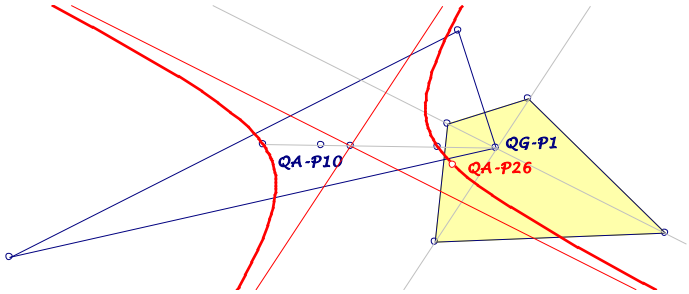
QA-P26 2nd QA-Quasi Centroid

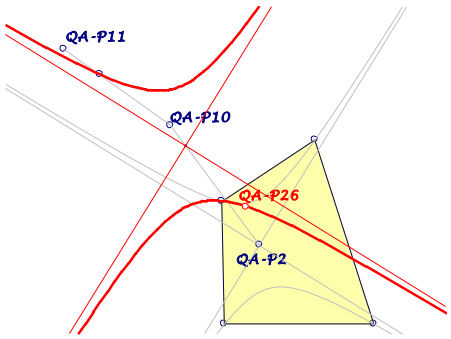
- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in *T(QG-P1, QA-P10, 5:1)*,
  … through *T(QG-P1, QA-P10, 1:2)* and *T(QG-P1, QA-P10, -4:1)*,
  … equation:

$$p^2(x+y-8z)(4x+4y-5z)-r^2(-5x+4y+4z)(-8x+y+z)=0$$



- **Orthogonal hyperbola**
  … parallel asymptotes wrt *QA-Co2*,
  … centered in *T(QA-P2, QA-P10, 5:1)*,
  … through *T(QA-P10, QA-P11, 2:1)*,
  … equation:

$$vw(-5x+4y+4z)(-8x+y+z)+wu(4x-5y+4z)(x-8y+z)$$
$$+uv(4x+4y-5z)(x+y-8z)=0$$



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QA-P27 M3D Center

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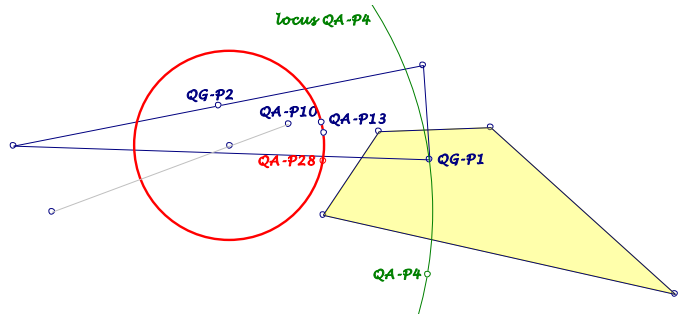
QA-P28 Midpoint of the Foci of the QA-Parabolas

- **Circle**
  … $h_{QA\text{-}P10}(Ci,1/4)$ with the circle *Ci* locus for *QA-P4*,
  … through *QA-P13* and midpoint *QG-P1.QG-P2*,

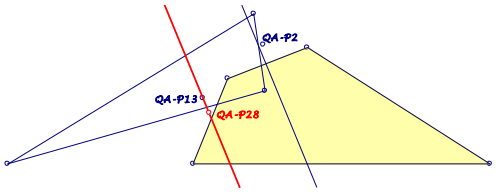
… equation:

$$r^2a^2(2S_A x(-3x+y+z)-S_B(x-3y+z)(x-y+z) \\ -S_C(x+y-3z)(x+y-z)) \\ +p^2c^2(S_A(-3x+y+z)(-x+y+z)+S_B(x-3y+z)(x-y+z) \\ -2S_C z(x+y-3z))=0$$



- **Line**
  … parallel to the Simson line of *QA-P2* wrt *QA-DT*,
  … through *QA-P13*,
  … equation:

$$(b^2c^2u(v+w)-2a^2S_A vw)x+(c^2a^2v(w+u)-2b^2S_B wu)y \\ +(a^2b^2w(u+v)-2c^2S_C uv)z=0$$



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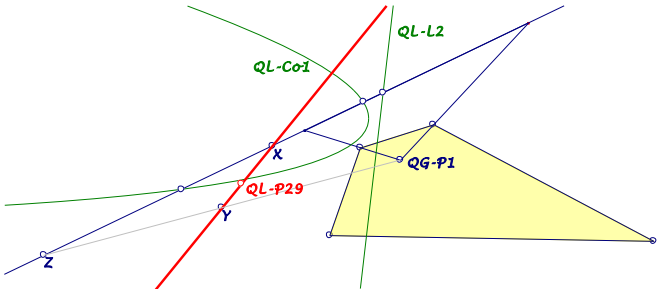
QA-P29 Complement of *QA-P2* wrt *QA-DT*

- **Circle *QA-Ci2***
- **Fixed point**
- **Line**
  … through the midpoint *X* of the intersections of *QG-L1* and *QL-Co1*,
  … through the midpoint *Y* of *QG-P1* and the 4th harmonic point *Z* of *QG-L1*∩*QL-L2* wrt *QG-2P3*,
  … equation:

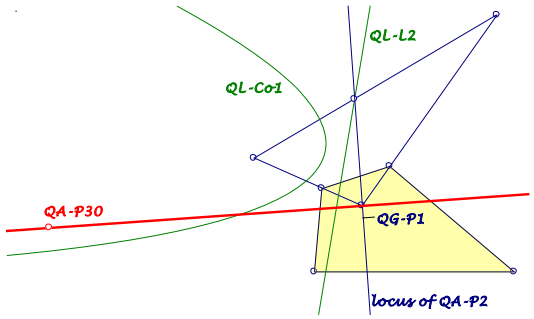
$$2S_B(wx-uz)-(a^2w-c^2u)y=0$$



QA-P30 Reflection of *QA-P2* in *QA-P11*

- **Circle *QA-Ci1***
- **Fixed point**
- **Line**

  … through *QG-P1*,
  … orthogonal to the line, which is the locus of *QA-P2*,
  … equation: $(S_A^2 u - S^2 w + S_A S_B u - S_B S_C w)x$
  $-(S_C^2 w - S^2 u - S_A S_B u + S_B S_C w)z = 0$

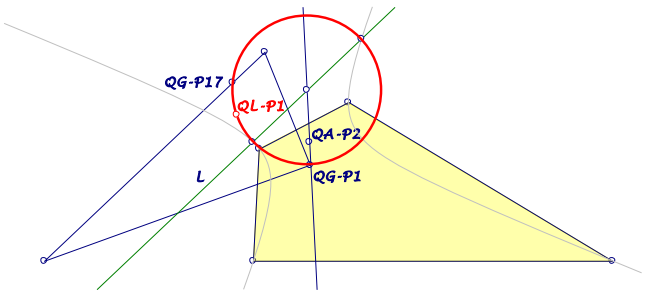


QL-P1 Miquel point

- **Circle *QL-Ci2***
- **Circle**

  … centered in the intersection of *QG-P1.QA-P2* and a parallel *L* to *QG-L1* half the distance to *QG-P1*,
  …through *QG-P1*, *QG-P17*, intersections of *QA-Co2* and *L*,
  …equation:

$$wS_A x^2 + uS_C z^2 - (uc^2 + wS_B)xy - (wa^2 + uS_B)yz - (uS_A + wS_C)zx = 0$$



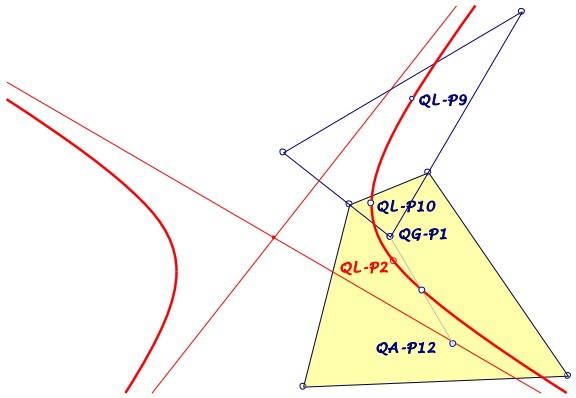
- **Fixed point**

QL-P2 Morley Point

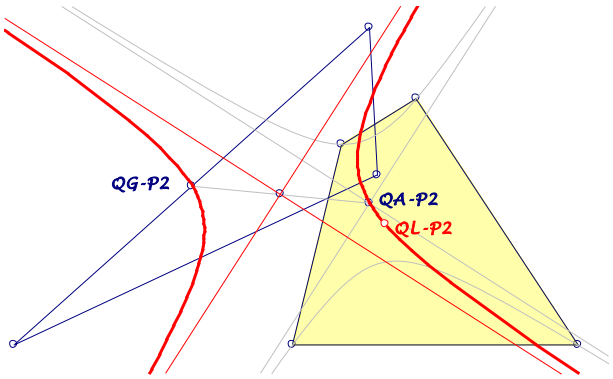
- **Hyperbola**

  … asymptotes orthogonal to the diagonals,
  … through *QL-P9*, *QL-P10* and midpoint of *QG-P1.QA-P12*,
  … equation:

$$l^2S_A(S_B-S_C)x^2+S_B(l^2a^2-n^2c^2)y^2+n^2S_C(S_A-S_B)z^2$$
$$-(l^2(S_Aa^2+S_B(S_B-S_C))-n^2S_Bc^2)xy+(l^2S_Aa^2-n^2S_Cc^2)zx$$
$$-(l^2S_Ba^2-n^2(S_Cc^2+S_B(S_B-S_A))yz=0$$



- **Orthogonal hyperbola**
  … parallel asymptotes wrt *QA-Co1*,
  … centered in midpoint *QA-P2.QG-P2*,
  … through *QA-P2*, *QG-P2*,
  … equation: $\frac{x^2}{u}+\frac{2y^2}{v}+\frac{z^2}{w}-\frac{x(y+z)}{u}-\frac{z(x+y)}{w}=0$



- **Line *QL-L2***

QL-P3 Kantor-Hervey Point

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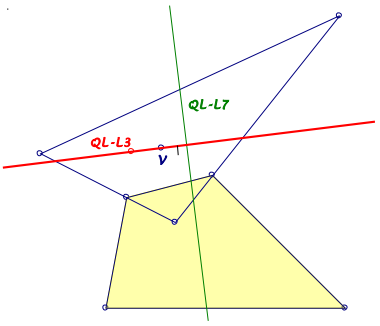
- ---
- **Line**
  …orthogonal to *QL-L7*,
  … through point *V* (see remark *QL*-P22),
  … equation:

$$(8S^2S_Auvw+T)x+(8S^2S_Buvw+T)x+(8S^2S_Cuvw+T)x=0$$
$$with \quad T=(a^2v+b^2u)(b^2w+c^2v)(c^2u+a^2w)$$



QL-P4 Miquel Circumcenter

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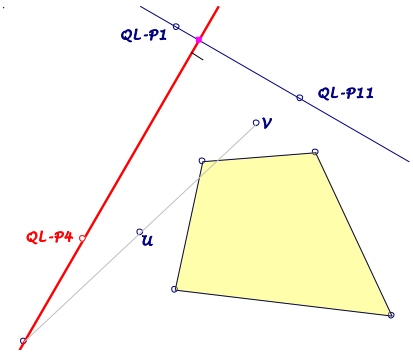
- **Line**
  … orthogonal to *QL-P1.QL-P11*,
  … through *T(U, V, -1:2)* with *U*, *V* see remark *QL*-P22,
  … equation very extensive



QL-P5 Clawson Center

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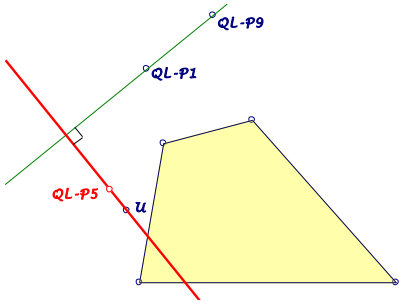
- **Line**
  … orthogonal to *QL-P1.QL-P9*,
  … through point *U* (see remark *QL*-P22),
  … equation:

$$(a^4vw+b^4wu+c^4uv+4a^2S_Avw)x+(a^4vw+b^4wu+c^4uv+4b^2S_Bwu)y$$
$$+(a^4vw+b^4wu+c^4uv+4c^2S_Cuv)z=0$$



QL-P6 Dimidium Point

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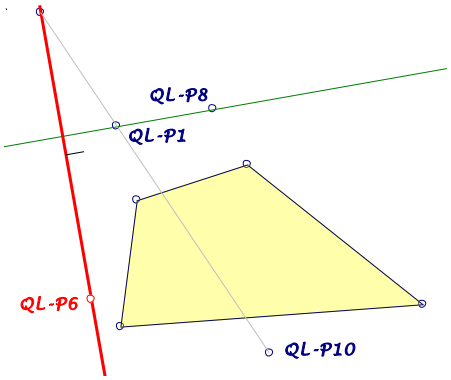
- **Line**
  … orthogonal *QL-P1.QL-P8*,
  … through *T(QL-P1, QL-P10, -1:3)*,
  … equation:

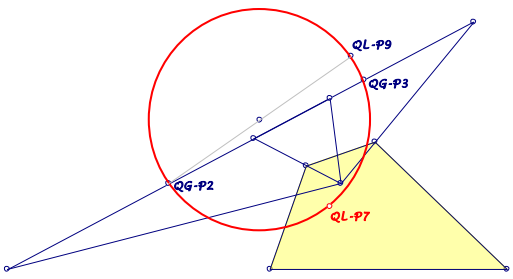
$$((S_A-3S_B)c^2v^2+2(S_A^2+3S^2)vw+(S_A-3S_C)b^2w^2)x$$
$$+((S_B-3S_A)c^2u^2+2(S_B^2+3S^2)wu+(S_B-3S_C)a^2w^2)y$$
$$+((S_C-3S_A)b^2u^2+2(S_C^2+3S^2)uv+(S_C-3S_B)a^2v^2)z=0$$



QL-P7 Newton-Steiner Point

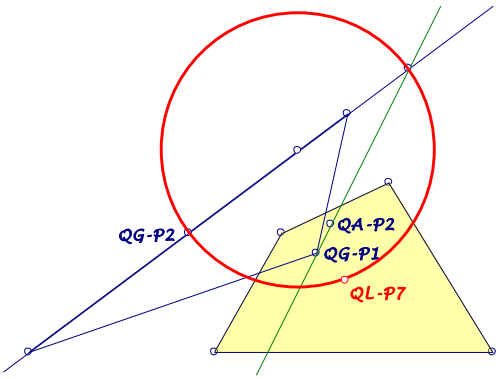
- **Circle**
  … Thales circle about *QG-P2.QL-P9*,
  … through *QG-P3*,
  … equation:

$$b^2(z-x)(l^2x+n^2z)+(n^2c^2-l^2a^2)y^2$$
$$+(l^2(c^2-2S_C)-n^2c^2)xy+(l^2a^2-n^2(a^2-2S_A))yz=0$$



- **Circle**
  … Thales circle about *QG-P2* and the intersection of *QG-L1* and *QG-P1.QA-P2*,
  … equation:

$$b^2v(x-z)(wx-uz)+(S_Bv(u+w)-b^2wu)y^2$$
$$+(3S_Cw-S_Au-b^2w)vxy+(3S_Au-S_Cw-b^2u)vyz=0$$



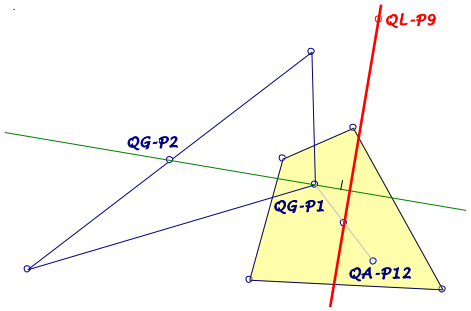
- **Line *QL-L2***

QL-P8 **Centroid of QL-Diagonal Triangle**

- **Fixed point**
- **Line**
  … parallel to *QG-L1* through *QA-P10*,
  … equation: $x-2y+z=0$
- **Fixed point**

QL-P9 **Circumcenter QL-Diagonal Triangle**

- **Fixed point**
- **Line**
  … orthogonal to *QG-P1.QG-P2*,
  … through midpoint of *QG-P1.QA-P12*,
  … equation: $c^2x-2S_By+a^2z=0$



- **Fixed point**

QL-P10 **Orthocenter QL-Diagonal Triangle**

- **Fixed point**
- **Line *QG-P1.QA-P12***
- **Fixed point**

QL-P11 **Nine-Point Center QL-Diagonal Triangle**

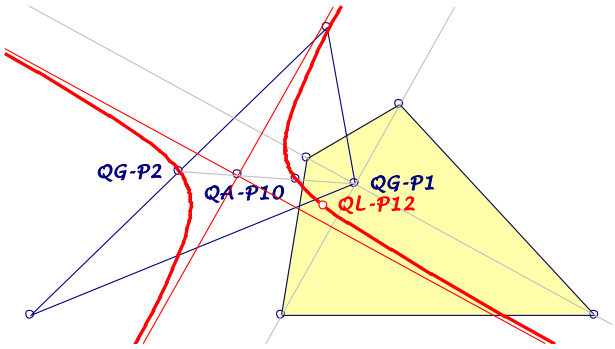
- **Fixed point**

- **Line**
  … ???
- **Fixed point**

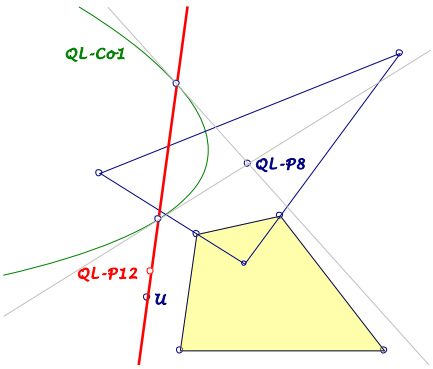
QL-P12 QL-Centroid or Lateral Centroid

- **Hyperbola**
  … centered in *QA-P10*,
  … asymptotes parallel to the diagonals,
  … through *QG-P2* and midpoint *QG-P1.QA-P10*,
  … equation:

$$2(l^2x+n^2z)^2-l^2n^2(x^2-y^2+z^2)+l^4(2y-z)x+n^4(2y-x)z=0$$

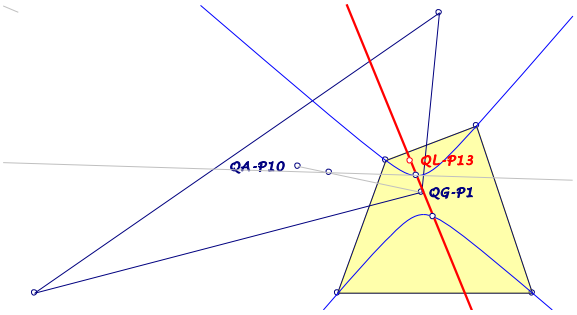


- **Orthogonal hyperbola**
  … asymptotes parallel to those of *QA-Co2*,
  … through *QG-P2*,
  … ???
- **Line**
  … polar of *QL-P8* wrt *QL-Co1*,
  … through point *U* (see remark *QL*-P22),
  … equation: $vwx+wuy+uvz=0$

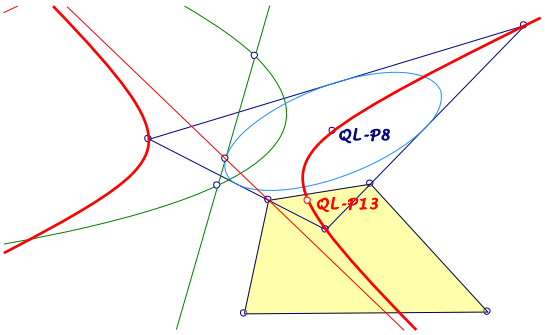


QL-P13 Lateral Harmonic Center

- **Line *QG-L2***
- **Line**
  … polar of *T(QG-P1, QA-P10, 3:1)* wrt *QA-Co2*
  … equation: $vwx+2wuy+uvz=0$



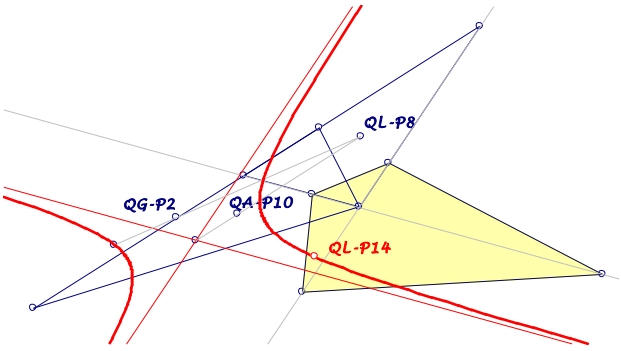
- **Hyperbola**
  … circumscribed *QL-DT*,
  … centered in the contact point of the polar of *QL-P8* wrt *QL-Co1* and the inscribed Steiner ellipse of *QL-DT*,
  … through *QL-P8*
  … equation: $wxy+uyz+vzx=0$



QL-P14 1st QL-Quasi Centroid

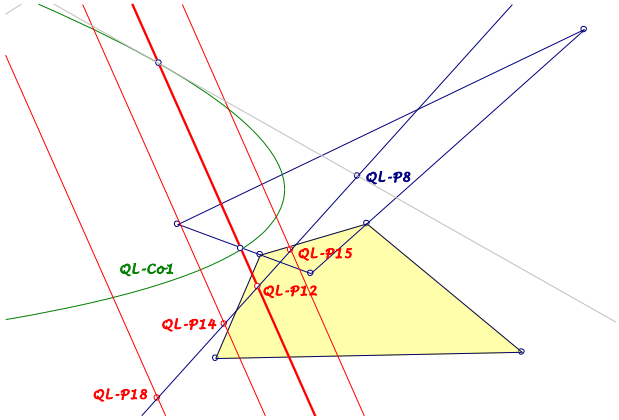
- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in *T(QA-P10, QL-P8, -1:4)*,
  … through *T(QG-P2, QL-P8, -4:1)*,
  … equation:

$$-2(-35l^4+10l^2n^2+n^4)x^2+(7l^4+34l^2n^2+7n^4)y^2$$
$$-2(l^4+10l^2n^2-35n^4)z^2+(5l^4+14l^2n^2+77n^4)yz$$
$$-(13l^4-122l^2n^2+13n^4)zx+(77l^4+14l^2n^2+5n^4)xy=0$$



- **-----**
- **Line**

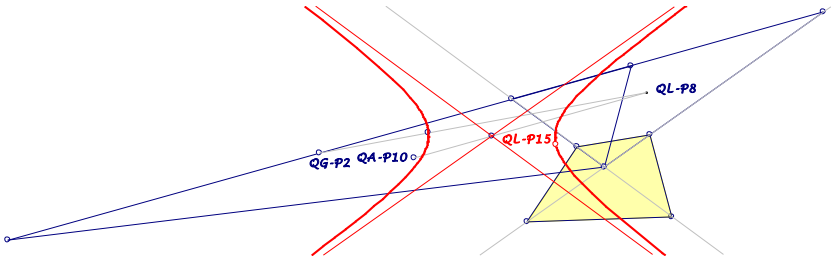
… parallel to the polar of *QL-P8* wrt *QL-Co1* (see *QL-P12*),
… 4/3 distance wrt *QA-P8*,
… equation:
$(u^2-10vw)x+(v^2-10wu)y+(w^2-10uv)z=0$



QL-P15 2nd QL-Quasi Centroid

- **Hyperbola**
  … asymptotes parallel to the diagonals,.
  … centered in *T(QA-P10, QL-P8, 1:2)*,
  …through *T(QG-P2, QL-P8, 1:2)*,
  … equation:

$$4(10l^4-8l^2n^2+n^4)x^2-5(l^4-22l^2n^2+5n^4)y^2+4(l^4-8l^2n^2+10n^4)z^2$$
$$-(l^4+10l^2n^2-35n^4)yz-(37l^4-98l^2n^2+37n^4)zx-(-35l^4+10l^2n^2+n^4)xy=0$$



- ---
- **Line**
  … parallel to the polar of *QL-P8* wrt *QL-Co1* (see *QL-P12, 14*),
  … 2/3 distance wrt *QA-P8*,
  … equation: $(u^2+8vw)x+(v^2+8wu)y+(w^2+8uv)z=0$

QL-P16 QL-Quasi Circumcenter

- **Circle *QL-Ci1***
- ---
- **Fixed point**

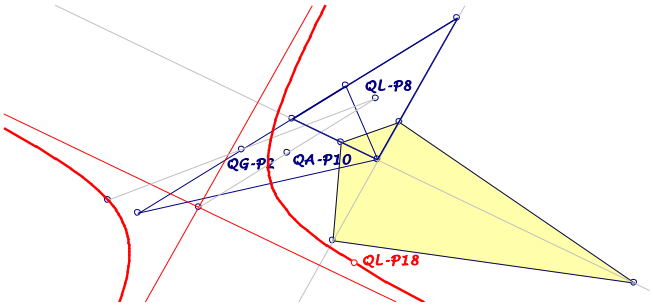
QL-P17 QL-Adjunct Quasi Circumcenter

- **Circle *QL-Ci1***
- **Kreis**
  … centered on *QG-L1*,
- **Circle *QL-Ci1***

QL-P18 Reflection of *QL-P8* in *QL-P12*

- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in the reflection of *QL-P8* in *QA-P10*,
  … through the reflection of *QL-P8* in *QG-P2*,
  … equation:

$$4l^4x^2+(l^2+n^2)y^2+4n^4z^2+(l^4+2l^2n^2+5n^4)yz$$
$$+(l^4+6l^2n^2+n^4)zx+(5l^4+2l^2n^2+n^4)xy=0$$

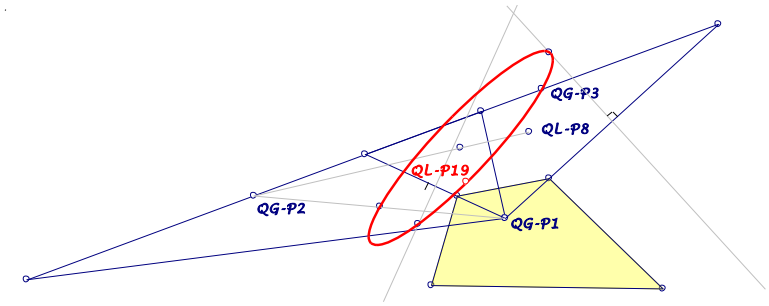


- **-----**
- **Line**
  … parallel to the polar of *QL-P8* wrt *QL-Co1* (see *QL-P12, 14, 15*),
  … double distance wrt *QA-P8*,
  … equation: $(v-w)^2x+(w-u)^2y+(u-v)^2z=0$

QL-P19 Midpoint of *QL-P1* and *QL-P7*

- **Ellipse**
  … centered in *T(QG-P2, QL-P8, 3:1)*,
  … through *QG-P3*, midpoint *QG-P1.QG-P2* and its pedal points on the bisectors of the *QL-DT*-legs,
  … equation:

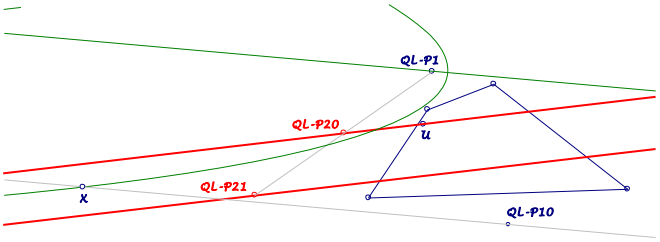
$$n^6c^2(-x+y+z)^2-l^6a^2(x+y-z)^2$$
$$+l^2n^4(-x+y+z)(2S_Ax+S_B(3x+y-3z)$$
$$-l^4n^2(x+y-z)(2S_Cz+S_B(-3x+y+3z)=0$$



- **Ellipse**
  … axes parallel to the asymptotes of *QA-Co2*,
  … through intersection of *QG-L1* and *QG-P1.QA-P2*,
  … ???
- **Line *QL-L3***

QL-P20 Orthocenter Homothetic Center

- ---
- ---
- **Line**
  … through *U* (see remark *QL*-P22),
  … parallel to the tangent in *X* at *QL-Co1* with *X* intersection of *QL-Co1* and a parallel to *QL-L1* through *QL-P10*,
  … equation:
  $(b^2w+c^2v)^2x+(c^2u+a^2w)^2y+(a^2v+b^2u)^2z=0$



QL-P21 Adjunct Orthocenter Homothetic Center

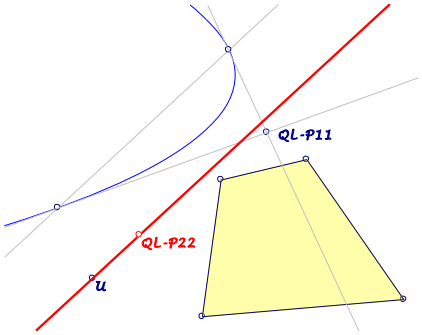
- ---
- ---
- **Line**
  … parallel to the locus-line of *QL-P20*,
  … with double distance to *QL-P1*

QL-P22 QL-Nine-Point Center Homothetic Center

- ---
- ---
- **Line**
  … through *U* (see remark),
  … parallel to the polar of *QL-P11* wrt *QL-Co1*

… equation:

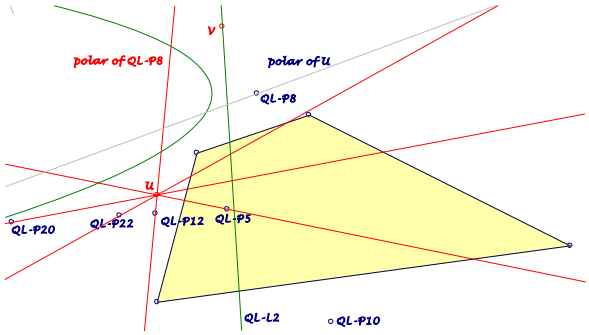
$$(4S^2vw+(-S_Au+S_Bv+S_Cw)^2)x+(4S^2wu+(S_Au-S_Bv+S_Cw)^2)y$$
$$+(4S^2uv+(S_Au+S_Bv-S_Cw)^2)z=0$$



Remark wrt *U* and *V*: The points *QL-P5*, *QL-P12*, *QL-P20*, *QL-P22* are collinear on *QL-L1*. Their loci are lines with a common point *U* on the polar of *QL-P8* wrt *QL-Co1*:

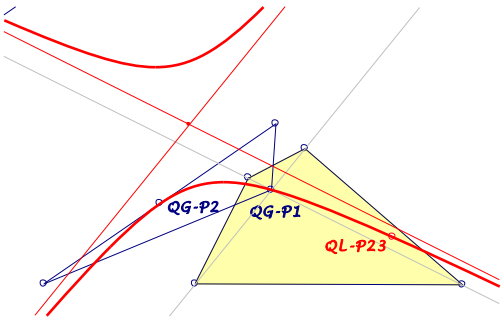
$$U(u(v(a^2w+c^2u)^2-w(a^2v+b^2u)^2)$$
$$:v(w(a^2v+b^2u)^2-u(b^2w+c^2v)^2):w(u(b^2w+c^2v)^2-v(a^2w+c^2u)^2))$$

A perpendicular line to the polar of *U* wrt *QL-Co1* cuts *QL-L2* in the point *V*.

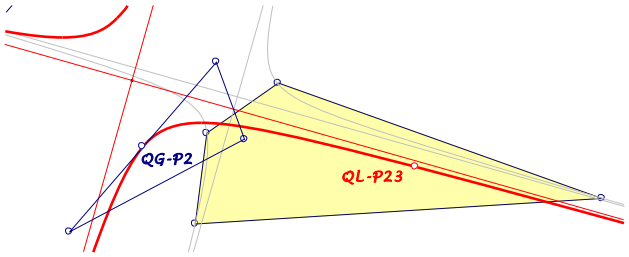


QL-P23 Center of the Inscribed Midline Hyperbola

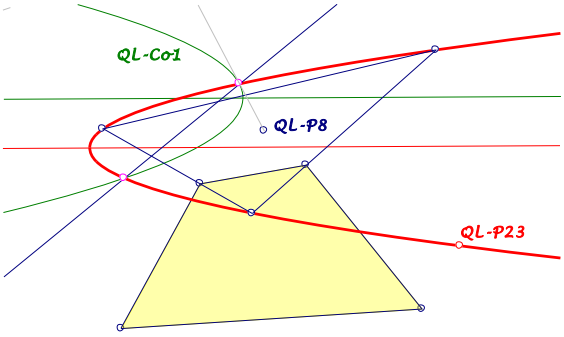
- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … through *QG-P1*, *QG-P2*,
  … *QG-L1* is tangent in *QG-P2*,
  … equation: $l^4(x+y)x+2l^2n^2xz+n^4(y+z)z=0$



- **Orthogonal hyperbola**
  … parallel asymptotes wrt *QA-Co2*,
  … through *QG-P2*,
  … *QG-L1* tangent in *QG-P2*



- **Parabola**
  … circumscribed *QL-DT*,
  … axis parallel to the axis of *QL-Co1*, half the distance to *QL-P8* on the other side,
  … through the intersections of *QL-Co1* and the polar of *QL-P8*,
  … equation: $u^2yz + v^2zx + w^2xy = 0$



QL-P24 **Intersection *QL-P1.QL-P8* ^ *QL-P13.QL-P17***

- **Circle *QL-Ci1***
- **-----**
- **Fixed point**

QL-P25 **2nd *QL*-Parabola Focus**

- **Circle *QL-Ci2***
- **-----**
- **Circle *QL-Ci2***

QL-P26 **Least Squares Point**

- **-----**
- **-----**
- **-----**

QG-P1 Diagonal Crosspoint

- **Fixed point**
- **Fixed point**
- **Fixed point**

QG-P2 Midpoint 3rd *QA*-Diagonal

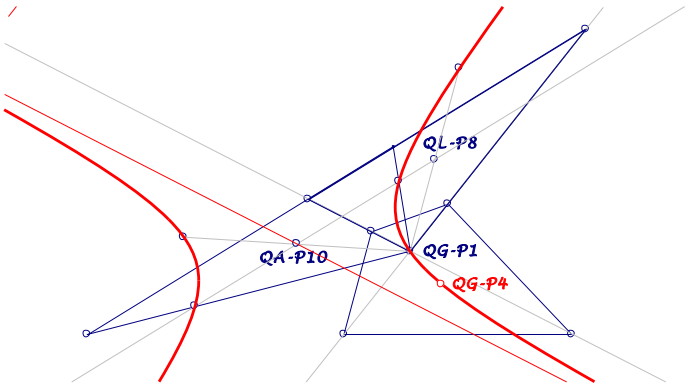
- **Fixed point**
- **Fixed point**
- **Line *QG-L1***

QG-P3 Midpoint 3rd *QL*-Diagonal

- **Fixed point**
- **Line *QG-L1***
- **Fixed point**

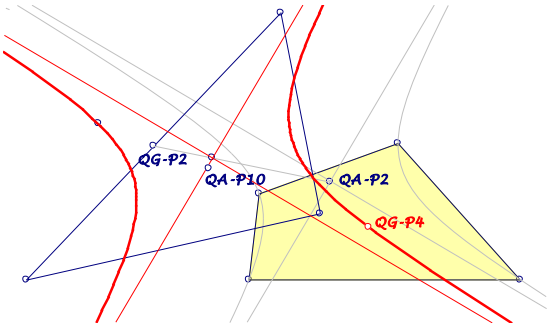
QG-P4 1st QG-Quasi Centroid

- **Hyperbola**
  … asymptotes parallel to the diagonals,
  … centered in *QA-P10*,
  … through *QG-P1* and reflections of *QG-P1* in *QA-P10* or *QL-P8*,
  … intersections of *QA-P10.QL-P8* and the legs of *QA-DT*,
  … equation:
  $r^2x^2 - p^2z^2 - 2r^2xy + 2(p^2 - r^2)zx + 2p^2yz = 0$



- **Orthogonal hyperbola**
  ... parallel asymptotes wrt *QA-Co2*,
  … centered in *T(QA-P2, QG-P2, 2:1)*,
  … through the reflection of *QA-P10* in *QG-P2*,,
  … equation:

$$3vwx(-x+2v+2w)+3uvz(2x+2y-z)$$
$$+uw(x-2y+z)(x+4y+z)=0$$

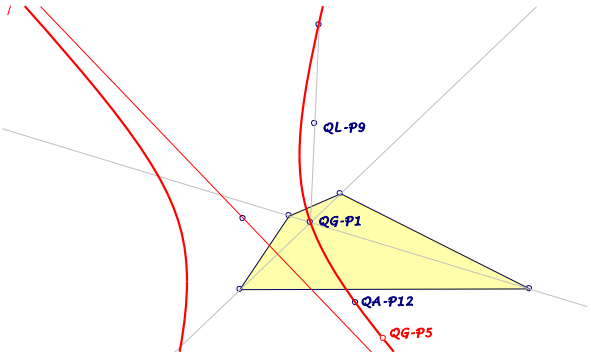


- **Line *QG-L3***

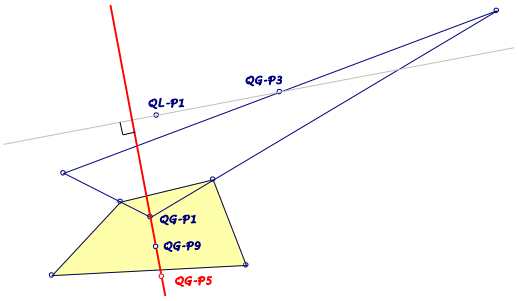
QG-P5 1st QG-Quasi Circumcenter

- **Hyperbola**
  … asymptotes orthogonal to the diagonals,
  … through *QG-P1*, *QA-P12* and the reflection of *QG-P1* in *QL-P9*,
  … equation:

$$-(p^2c^2-r^2a^2)(S_Ax^2+S_Cz^2)$$
$$+(p^2c^2(S_B+c^2)+r^2(a^2S_A-2S_B^2))xy$$
$$-(p^2(S^2-S_A^2)-r^2(S^2-S_C^2))zx$$
$$-(r^2a^2(S_B+a^2)+p^2(c^2S_C-2S_B^2))yz=0$$



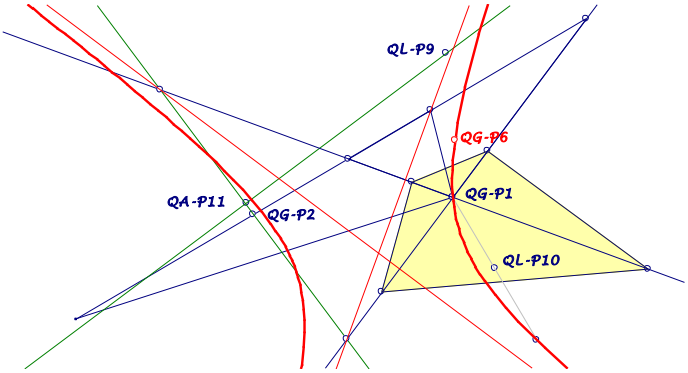
- **Hyperbola**
  … centered in *QG-P2*,
  … through *QA-P12*,
  … ???
- **Line**
  … orthogonal *QG-P3.QL-P1*,
  … through *QG-P1*, *QG-P9*
  … equation: $c^2(S_Cw-S_Bv)x+a^2(S_Au-S_Bv)z=0$



QG-P6 1st *QG*-Quasi Orthocenter

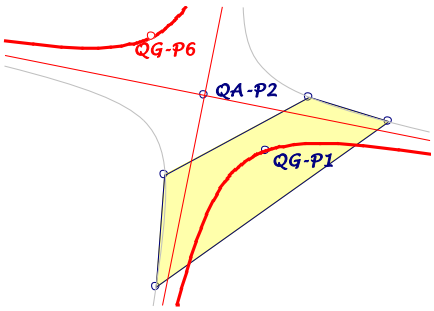
- **Hyperbola**
  … asymptotes orthogonal to the diagonals,
  … an asymptote cuts the non-orthogonal diagonal on an orthogonal line through *QG-P2* wrt *QA-P11.QL-P9*,
  … through *QG-P1*, reflection of *QG-P1* in *QL-P10*,
  … equation:

$$(p^2c^2S_B - r^2(S^2 + a^2S_B))x^2 - (r^2a^2S_B - p^2(S^2 + c^2S_B))z^2$$
$$+ (p^2c^2 - r^2a^2)((a^2 + S_B)yz + (a^2 + c^2)zx + (c^2 + S_B)xy) = 0$$

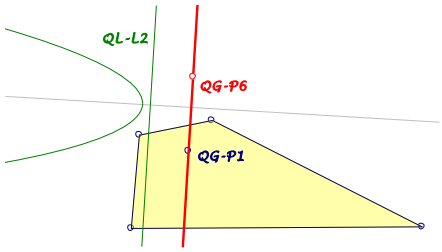


- **Orthogonal hyperbola**
  … asymptotes and center as *QA-Co2*,
  … through *QG-P1*,
  … equation:

$$w(u - v)x^2 + u(w - v)z^2 + 2wu(xy + yz + zx) = 0$$

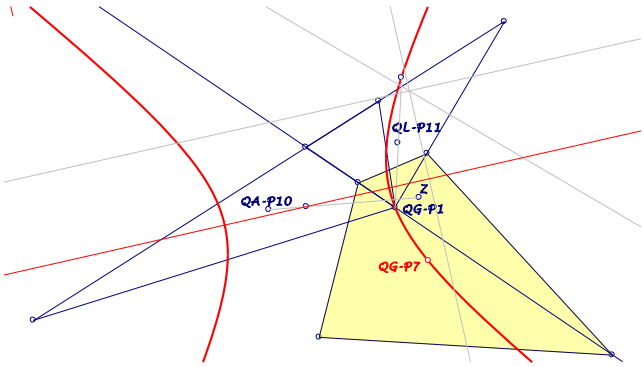


- **Line**
  … parallel to *QL-L2*,
  … through *QG-P1*,
  … equation: $(S_A u - S_B v)x + (S_C w - S_B v)z = 0$



QG-P7 1st QG-Quasi Nine-point Center

- **Conic**
  … axes: parallel to the angle bisectors of the bisectors of the *QL-DT*-legs,
  … center *T(QA-P10, Z, 1:3)* with *Z* center of the locus-hyperbola of *QG-P6*,
  … through *QG-P1* and its reflection in *QL-P11*.



- **Hyperbola**
  …through *QA-P11*,
  …???
- **Line *QG-P1.QG-P11***

QG-P8 2nd QG-Quasi Centroid

- **Hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P4*,1/2)
- **Orthogonal hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P4*,1/2)
- **Line *QG-L3***

QG-P9 2nd QG-Quasi Circumcenter

- **Hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P5*,1/2)
- **Hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P5*,1/2)
- **Line**
  … $h_{QG\text{-}P1}$(locus *QG-P5*,1/2)

QG-P10 2nd QG-Quasi Orthocenter

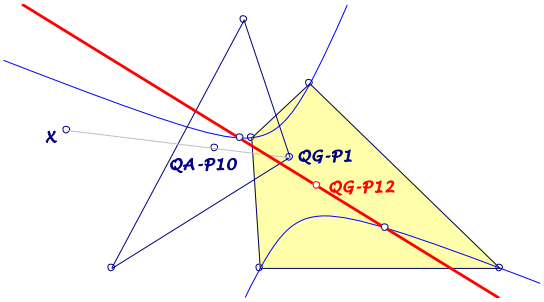
- **Hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P6*,1/2)
- **Orthogonal hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P6*,1/2)
- **Line**
  … $h_{QG\text{-}P1}$(locus *QG-P6*,1/2)

QG-P11 2nd QG-Quasi Nine-point Center

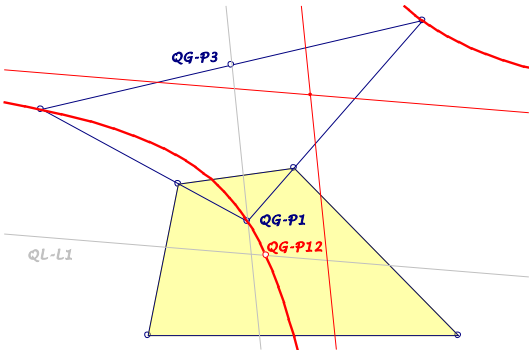
- **Conic**
  … $h_{QG\text{-}P1}$(locus *QG-P7*,1/2)
- **Hyperbola**
  … $h_{QG\text{-}P1}$(locus *QG-P7*,1/2)
- **Line**
  … $h_{QG\text{-}P1}$(locus *QG-P7*,1/2)

QG-P12 Inscribed Harmonic Conic Center

- **Line *QG-L2***
- **Line**
  … polar of *X=T(QG-P1, QA-P10, -3:2)* wrt *QA-Co2*,
  … equation: $vwx - wuy + uvz = 0$

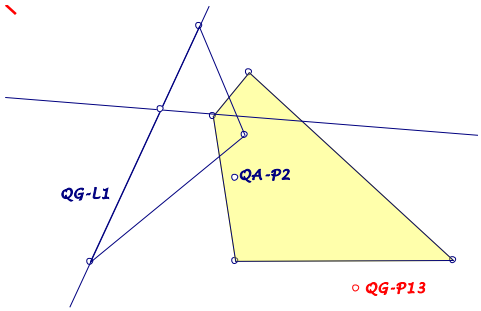


- **Hyperbola**
  … circumscribed *QL-DT*,
  … asymptotes: parallels to *QG-P1*.*QG-P3* and *QL-L1*,
  … equation: $uyz - 2vzx + wxy = 0$

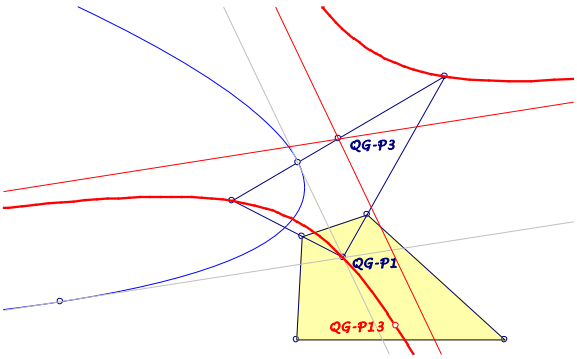


QG-P13 Circumscribed Harmonic Conic Center

- **Line *QG-L2***
- **Line**
  ... through *QA-P2* and the intersection of *QG-L1* with the trilinear polar of *QA-P2* wrt *QA-DT*,
  ... equation: $vwx-2wuy+uvz=0$



- **Hyperbola**
  ... circumscribed *QL-DT*,
  ... centered in *QG-P3*,
  ... asymptotes parallel to the tangents from *QG-P1* to *QL-Co1*,
  ... equation: $uyz-vzx+wxy=0$

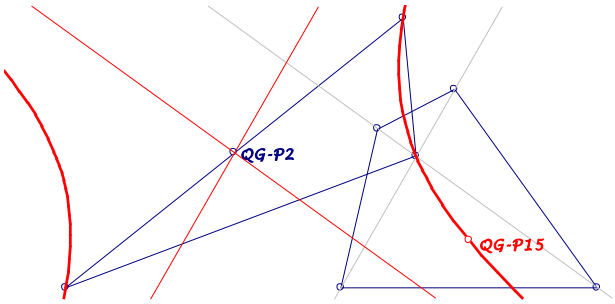


QG-P14 Center of the M3D Hyperbola

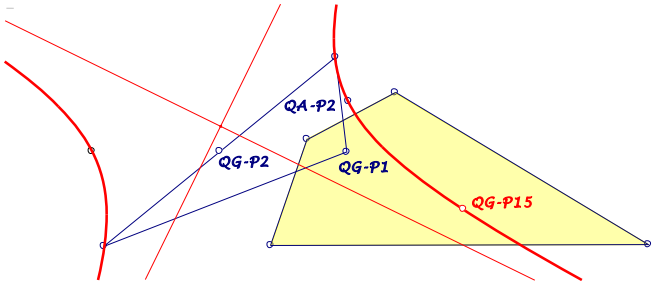
- **Line**
  ... parallel *QG-L1* through *QG-P1*
- **Line**
  ... parallel *QG-L1* through *QG-P1*
- **Line**
  ... parallel *QG-L1* through *QG-P1*

QG-P15 Kirikami Center

- **Hyperbola**
  ... circumscribed *QA-DT*,
  ... centered in *QG-P2*,
  ... asymptotes parallel to the diagonals



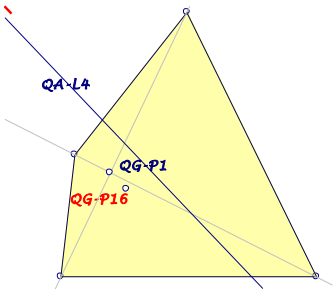
- **Orthogonal hyperbola**
  ... points: *QG-2P2*, *QA-P2*, reflection of *QG-P1* in *QG-P2*,
  ... equation:
  $$(u-v)wxy-(w+u)vzx+(w-v)uyz+2uwy^2=0$$



- **Line *QG-L3***

QG-P16 Schmidt Point

- **-----**
- **-----**
- **Line**
  ... parallel to *QA-L4*,
  ... through *QG-P1*,
  ... equation: $w(a^2v+b^2u)x+u(b^2w+c^2v)z=0$

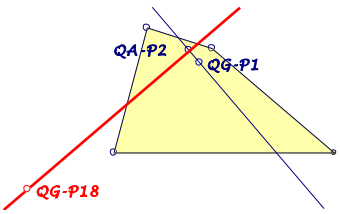


QG-P17 Projection Point of *QG-P1* on *QG-L1*

- **Fixed point**
- **Fixed point**
- **Fixed point**

QG-P18 Quasi Isogonal Crosspoint

- **Line**
  ... ???
- **Line**
  ... orthogonal to *QG-P1.QA-P2*
  ... through *QA-P2*,
  ... equation:
  $S_C(c^2q^2-b^2r^2)x+b^2(c^2p^2-a^2r^2)y+S_A(b^2p^2-a^2q^2)z=0$



- **Line**
  ... ???

QG-P19 Quasi Isogonal Conjugate of QG-P1

- **Conic**
  ... circumscribed *QA-DT*, *???*
- **Conic**
  ... through *QG-2P2a,b*,
  ... ???
- **Conic**
  ... through *QG-P1*

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