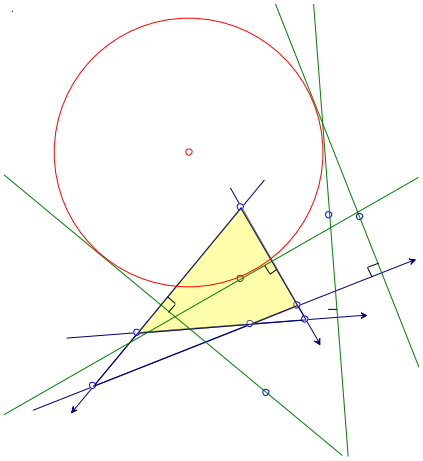
EQF-Note 2014-12-19

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Hodgson's Directed n-Lines II

*These are further remarks wrt an article of J. E. Hodgson: "Orthocentric Properties of the Plane Directed n-Line"(1912) (*see also *EQF-Note 2014-12-15 in #908*).

2nd Circles of directed 4-lines



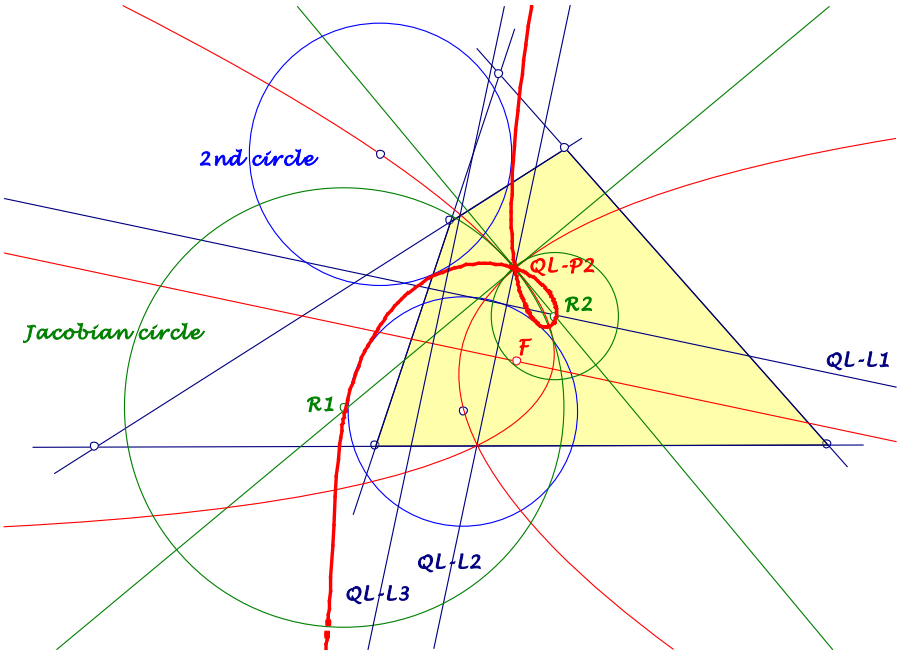
(1) For directed 4-lines Hodgson mentioned a second circle (p.204) inscribed the four perpendiculars through the equidistant points (of three lines) wrt the line left out.

(2) A quadrilateral has eight directed 4-lines, so there are eight 2nd circles. They are divided into two groups of four. Take one directed 4-line for reference and three other changing two directions, then you get one group; the other group contains the directed 4-lines with one direction changed (p.220).

(3) Each group has a common point $R_{1,2}$ for radical axes.

(4) The centers of group-circles are with the Morley Point *QL-P2* (point *c* by Hodgson on p. 222) on a parabola.

(5) The two parabolas have the same axis parallel to the Newton Line *QL-L1* with the same focus (p.222). They intersect orthogonally on the Steiner Line *QL-L2* (e.g. in *QL-P2*).



(6) The points $R_{1,2}$ lie on the orthogonal tangents in *QL-P2* to the two parabolas.

(7) Circles round $R_{1,2}$ through *QL-P2* (Jacobian circles p.225) intersect the circles of a group orthogonally.

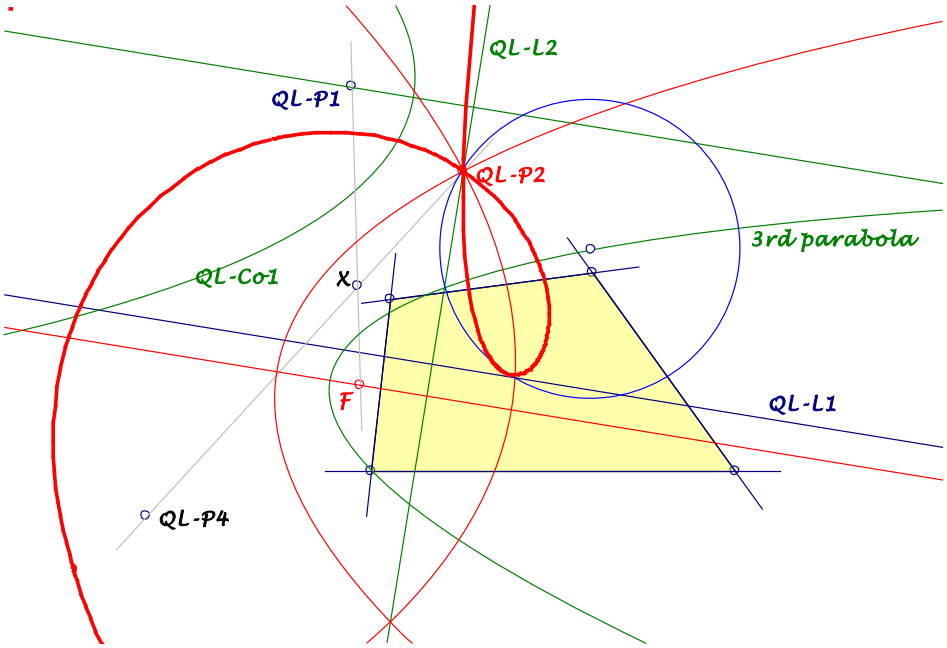
(8) The envelope of all circles, centered on one parabola and orthogonal intersecting the correspondent Jacobian circle give a cubic, which is the same for both parabolas (p.226).

(9) This cubic has a double point in *QL-P2*, contains the points $R_{1,2}$ and its asymptote is the Pedal Line *QL-L3* (p.226). The cubic is anallagmatic, invariant to inversions wrt the Jacobian circles.

(10) Not mentioned by Hodgson: The common focus of the two parabolas can be simply constructed as follows:
… let *X* divide *QL-P2.QL-P4* with ratio 1:2,
… the common focus *F* divides *QL-P1.X* with ratio -3:1.

(11) Hodgson mentioned a third parabola (p.227): Circles through *QL-P2* centered on this parabola envelop the cubic above.

(12) Not mentioned by Hodgson: This third parabola has the same focus as the two other parabolas and is homothetic to the inscribed parabola *QL-Co1* of the quadrilateral: center point *X* (see above), factor -1/2.



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