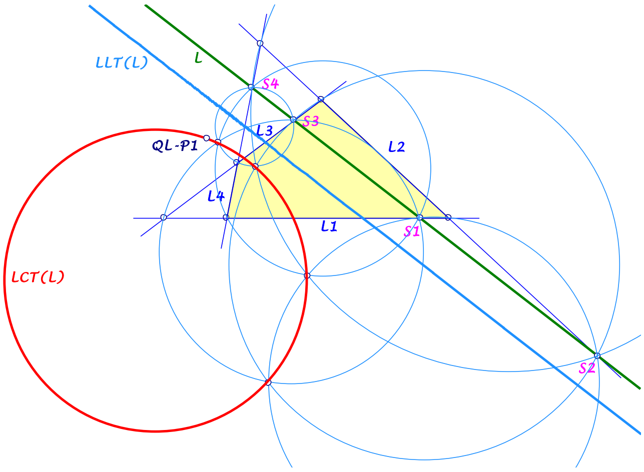
EQF-Note 2015-06-30

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Line-Circle Transformation for a QL

Starting with a Line-Circle Transformation (LCT), which gives circles, containing the Miquel Point QL-P1, and using the CSC-transformation QL-Tf1, there is a Line-Line Transformation (LLT) with relevant properties for EQF-objects. – The results are only CABRI-controlled.



Let L_1, L_2, L_3, L_4 be the lines of a quadrilateral and S_{ij} their intersections. Let L be a line and S_i its intersections with the lines L_i of the quadrilateral.

- **The circumcircles of S_i, S_j, S_{ij} have four concyclic triple intersections.**

Beside the already known transformation *QL-Tf1* = *CSC* this property leads to another line-circle transformation *LCT* with the following properties:

- **The circle *LCT(L)* contains the Miquel Point *QL-P1*.**
- **The circle *LCT(L)* contacts the circle *CSC(L)* in *QL-P1*.**
- **For tangents *L* at the inscribed parabola *QL-Co1* the circle *LCT(L)* degenerates in *QL-P1*.**
- **The center of the circle *LCT(L)* is collinear with *QL-P1* and the *CSC*-image of the reflection of *QL-P1* in *L*.**

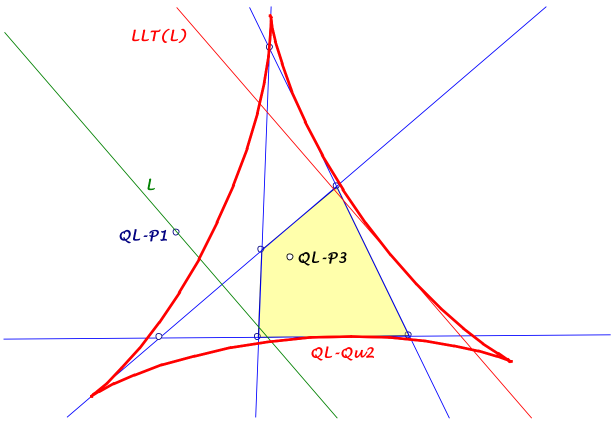
The circle *LCT(L)* has the point *QL-P1*, so its *CSC*-image is a line and we get a line-line transformation *LLT(L)* = *CSC(LCT(L)).*

- **The line *LLT(L)* is a parallel to *L*.**
- **The line-line transformation *LLT* is involuntary.**
- **The Newton Line *QL-L1* is a fixed line of *LLT*.**
- **For parallels *L* to *QL-L1* the *LLT*-image is the reflection of *L* in *QL-L1*.**
- ***LLT(QL-L9)* is a parallel through *QL-P13*.**
- **For *L* = *LLT(T)* − *T* line through *QL-P1* – the circle *LCT(L)* degenerates to a line tangent in *QL-P1* to the circle *LCT(T)*.**
- **For lines *L* parallel to a line L_i of the quadrilateral holds *LLT(L)* = L_i and *LCT(L)* is the circumcircle of the remaining triangle component.**
- **The asymptotes of inscribed hyperbolas of the quadrilateral are fixed lines of *LLT*.**
- **For a line *L* with *LCT(L)* = *Ci* holds *LLT(CSC(Ci)))* = *L*.**

Now we consider lines *L* of a pencil with common point *QL-P1*:

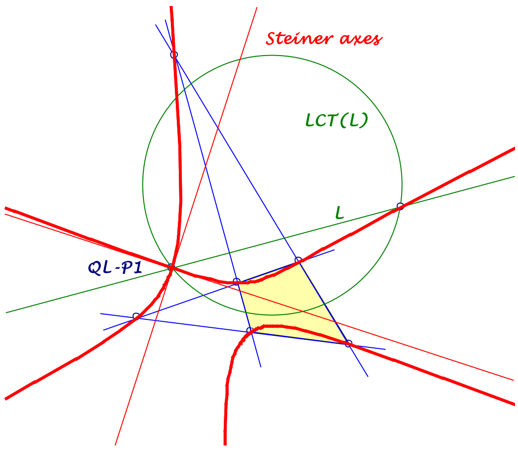
- **For lines *L* of the pencil of the Miquel Point *QL-P1* the lines *LLT(L)* envelope the Kantor-Hervey Deltoid *QL-Qu2*.**

The axes of *QL-Qu2* are lines through *QL-P3*, whose *LLT*-image contains *QL-P1*.

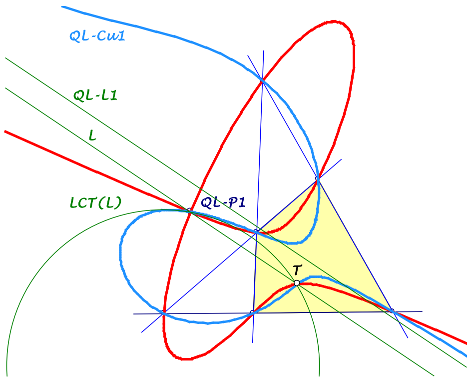


- **For lines *L* of the pencil of the Miquel Point *QL-P1* the 2nd intersection with the circle *LCT(L)* gives a cubic**

  … through the six intersections of the lines of the quadrilateral,
  … through the Miquel-Point *QL-P1* as knot
  … with the Steiner axes as tangents,
  … through the intersection *T* with *QL-Cu1*.
  … Lines *L* through *QL-P1* with *L = LLT(L)* are parallel to the asymptotes of the cubic.



Wrt the point *T*: The here discussed cubic and the *QL*-Quasi-Isogonal Cubic *QL-Cu1* have 8 common points: the 6 vertices of the quadrilateral, the Miquel Point *QL-P1* and a further intersection *T*, which is the *CSC*-image of the intersection of *QL-Cu1* and its asymptote (see *EQF*). On the other hand *T* is the 2nd intersection of a parallel to *QL-L1* through *QL-P1* and its *LCT*-circle.



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