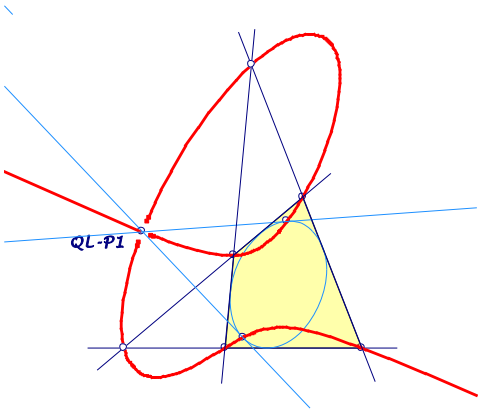
EQF-Note 2015-07-04

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

A new Cubic for a Quadrilateral

This cubic is already mentioned in 12.3 on my homepage. There are further aspects in QFG-messages 1175, 1205, 1212 and 1213. Here is a summary of its properties. – Reference triangle for barycentric calculations is the QL-Diagonal Triangle QL-Tr1.



There are three possibilities, to describe the cubic:

- **The cubic is the locus for the contact points of tangents from the Miquel Point *QL-P1* to inscribed conics of the quadrilateral (# 1213).**

- **The cubic is the locus for the intersections of lines through *QL-P1* and their *QL-Tf2*-image (#1175).**

- **The cubic is the locus for the 2nd intersections of lines through *QL-P1* and their *LCT*-image (#1205).**

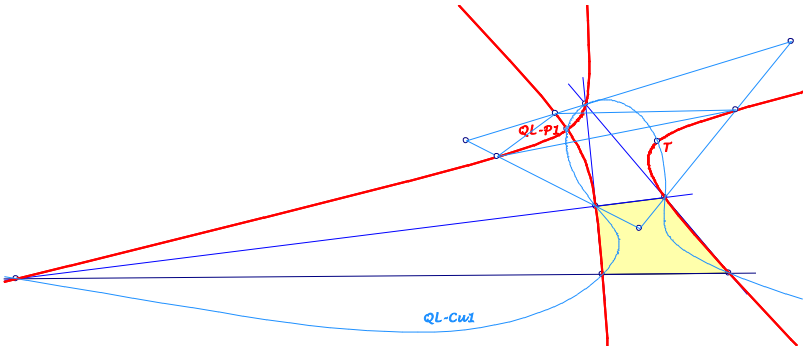
For the *QL*-Diagonal Triangle *QL-Tr1=ABC* as reference triangle for barycentric coordinates and a line (l,m,n) of the quadrilateral:

- **The cubic has the equation**

$$\frac{l^2 x}{wy-vz}+\frac{m^2 y}{uz-wx}+\frac{n^2 z}{vx-uy}=0$$

with $QL-P1=(u:v:w)$.

- **The cubic contains the following points:**
  … the Miquel Point *QL-P1* as knot,
  … the six intersections of the lines of the quadrilateral,
  … the vertices of the Ceva triangle of *QL-P1* wrt *QL-Tr1* (this is the diagonal triangle of *A, B, C, QL-P1),*
  … the intersection *T* with *QL-Cu1* (*CSC*-image of the intersection of *QL-Cu1* and its asymptote).



- **Tangents to the cubic in *QL-P1* are the Steiner axes.**

- **The cubic is invariant wrt an isoconjugation * with fixed point *QL-P1* and the Ceva triangle of *QL-P1* wrt *QL-Tr1*:**

$$X(x:y:z) \quad \rightarrow$$
$$X^*((-vwx+wuy+uvz)x$$
$$:(vwx-wuy+uvz)y$$
$$:(vwx+wuy-uvz)z)$$

- **This isoconjugation …**
  … has the fixed points *QL-P1* and the vertices of *QL-Tr1*,
  … swaps opposite intersections of the lines of the quadrilateral,
  … swaps the contact points of tangents from *QL-P1* to inscribed conics
  … and gives for the line at infinity the Nine Point Conic of *A, B, C, QL-P1*.

- **The angle bisectors at *QL-P1* wrt two conjugate points *X* and *X** on the cubic are the Steiner axes.**

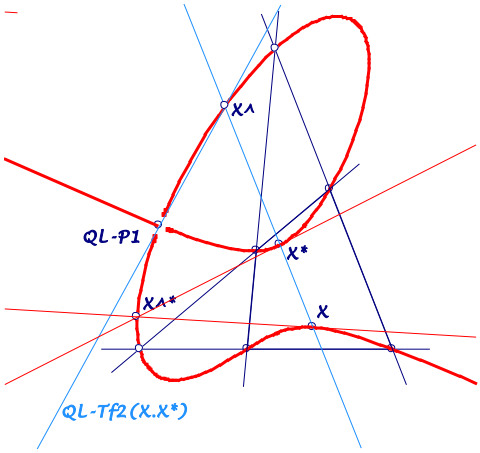
- **For points of the cubic the lines *XX** envelope an inscribed conic tangent to the Steiner axes and *QL-L2* (see #481, #488):**

$$u^2l^4x^2+v^2m^4y^2+w^2n^4z^2$$
$$-2(uvl^2m^2xy+vwm^2n^2yz+wun^2l^2zx)=0\ .$$

- **This conic is also the locus of the 4th harmonic points of *X.X** wrt the 3rd intersections with the cubic.**

- **The 3rd intersection *X^* of *X.X** with the cubic is the intersection with *QL-Tf2(X.X*).***

- **The tangents in *X* and *X** at the cubic intersect in the isoconjugate * of *X^* on the cubic.**



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