EQF-Note 2016-04-07

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Alternating Sum of Distance Squares

For quadrigons here are researched the loci for points X or lines L with the following properties:

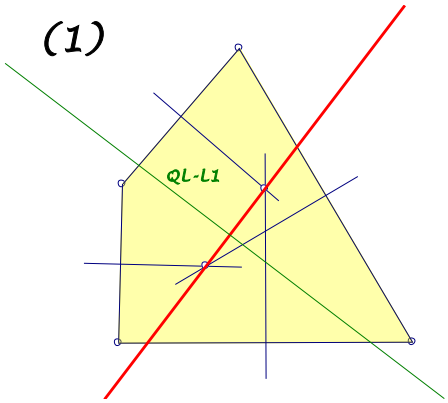
(1) $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = zero\ or\ const.$

(2) $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = zero\ or\ const.$

(3) $LP_1^2 - LP_2^2 + LP_3^2 - LP_4^2 = zero\ or\ const.$

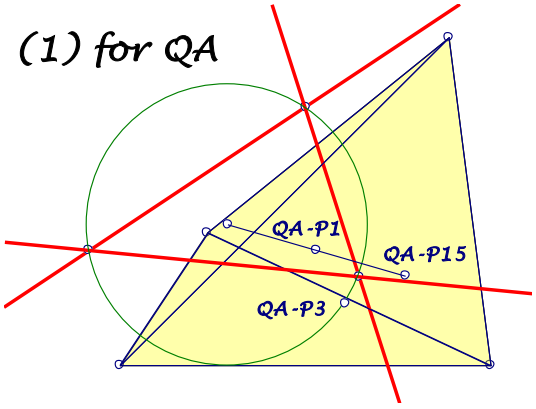
This note is an extension of EQF-Note 2014-01-12 in QFG-message 402.

(1) $XP_1^2 - XP_2^2 + XP_3^2 - XP_4^2 = zero\ or\ const.$



- **The locus for points with alternating sum zero is a perpendicular line to the Newton line *QL-L1*, containing the intersections of the perpendicular bisectors of opposite sides.**

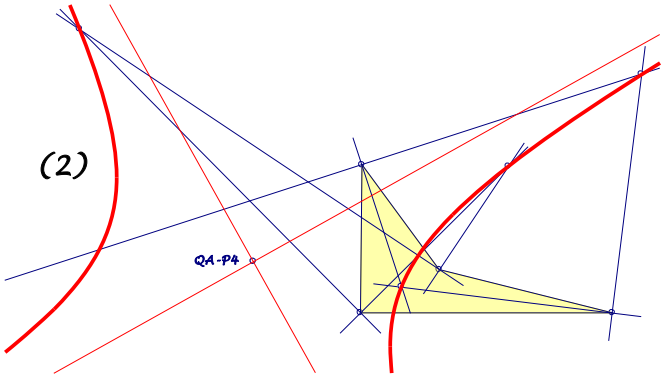
- **The loci for points with constant alternating sum are perpendiculars to the Newton line *QL-L1*.**



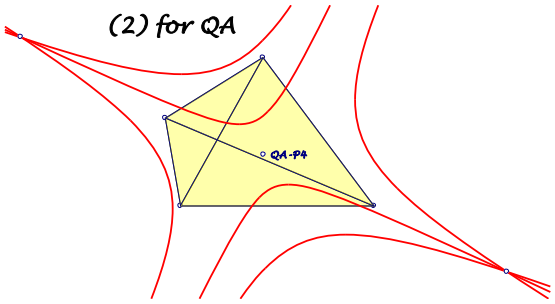
- **For a quadrangle the three lines for alternating sum zero give a triangle with circumcircle through *QA-P3*, centered in the reflection of *QA-P15* in *QA-P1*.**

(2) $XL_1^2 - XL_2^2 + XL_3^2 - XL_4^2 = zero\ or\ const.$

- **The locus of points with alternating sum zero is an orthogonal hyperbola, centered in *QA-P4*, containing the intersections of the inner or outer angle bisectors at opposite vertices.**

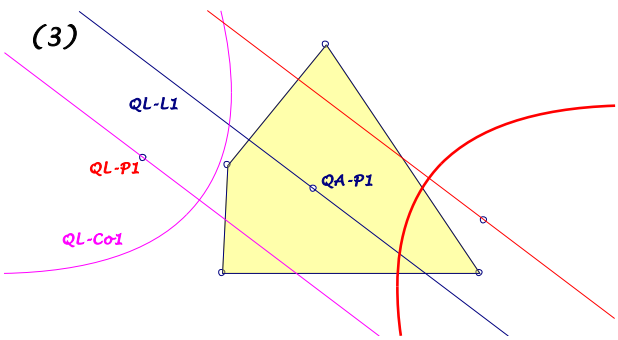


- **The loci of points with constant alternating sum are orthogonal hyperbolas, centered in *QA-P4* with the same asymptotes.**



- **For a quadrangle the three hyperbolas for alternating sum zero have two common points with midpoint *QA-P4*.**

(3) $LP_1^2 - LP_2^2 + LP_3^2 - LP_4^2 = zero\ or\ const.$



- **The lines with alternating sum zero envelop a parabola, which is the reflection of *QL-Co1* in *QA-P1*.**

- **The lines with constant alternating sum envelop parabolas with focus and axis of *QL-Co1* reflected in *QA-P1*.**

- **For quadrilaterals the three parabolas for alternating sum zero have as axis the asymptote of *QL-Cu1*.**

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de