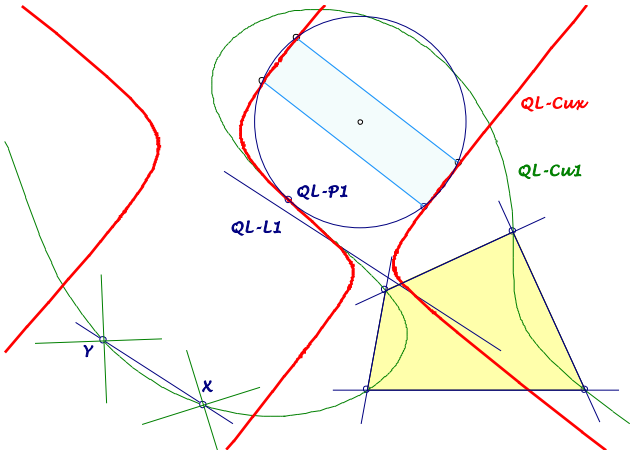
**EQF-Note 2016-05-16**

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

## Cubic derived from QL-Cu1 wrt QL-Tf3

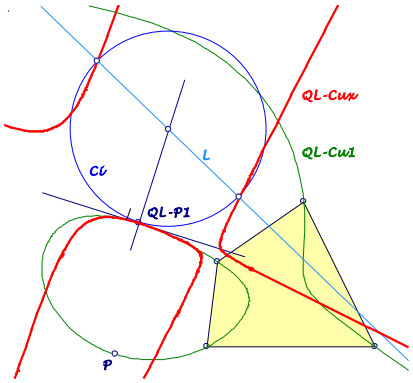
*The angle bisectors for points on the cubic QL-Cu1 wrt opposite QL-points have QL-Tf3-images on a cubic, which shall be tested in this note. The properties are only CABRI-controlled.*




Let *X*, *Y* be points on *QL-Cu1* with *XY* parallel *QL-L1*.
… take the angle bisectors at *X* and *Y* wrt opposite points of the quadrilateral:
… Their *QL-Tf3*-images give a rectangle:
… with sides parallel to the Steiner axes,
… with circumcircle through *QL-P1*,
… centered on a perpendicular to *QL-P1*.*QL-P4* through *QL-P1*.
… **The locus of the rectangle vertices is a cubic *QL-Cux***.

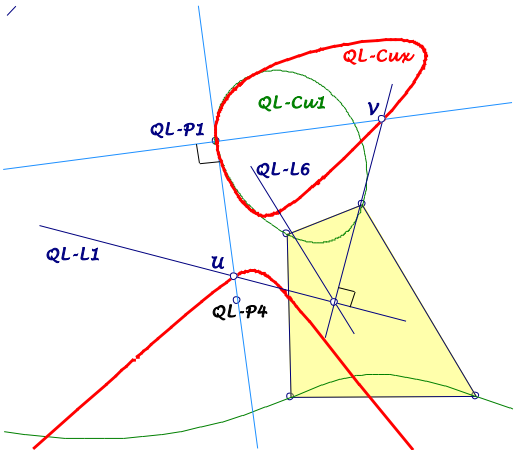
The construction can be simplified, taking the 2nd intersections of the angle bisectors at *X* and *Y* with the circumcircle of *X*, *Y*, *QL-P1*.

There is a further alternative construction of the cubic *QL-Cux*:

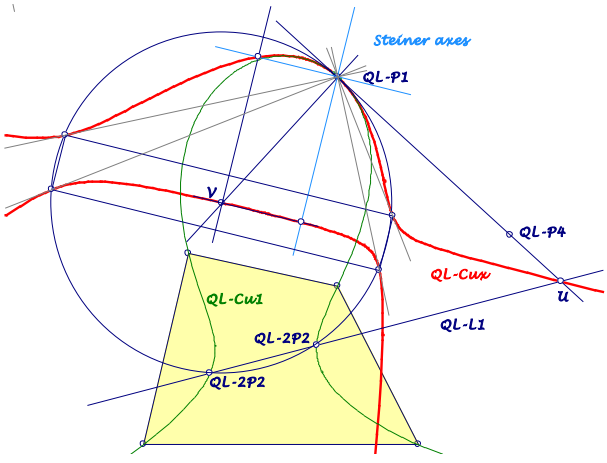


… Let *P* be point on *QL-Cu1* and *L* the line with *QL-Tf3(L)=P*.
… Let *Ci* be a circle through *QL-P1*, centered in the intersection of *L* and a perpendicular to *QL-P1.QL-P4* in *QL-P1*.
… The intersections of *L* and *Ci* give the cubic *QL-Cux*.

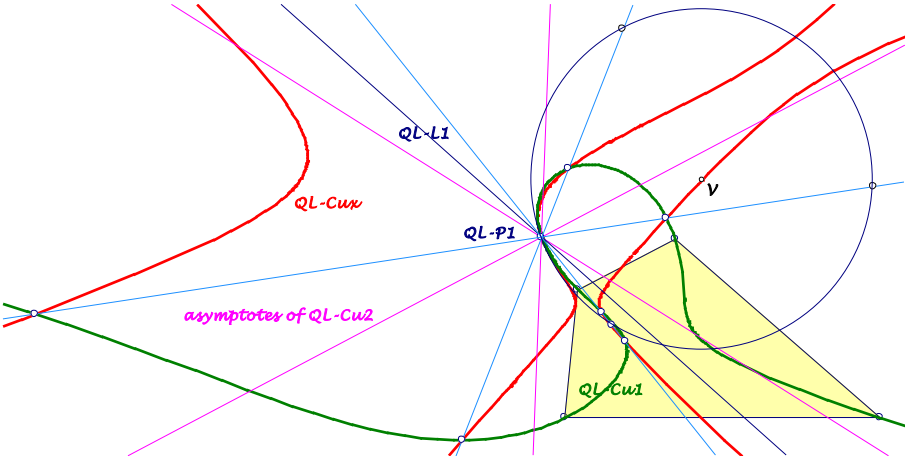
- ***QL-Cu1* and *QL-Cux* have a common tangent in *QL-P1* (*QL-P1.QL-P4*).**
- **The intersection *U* of the common tangent in *QL-P1* and *QL-L1* is a point of the cubic *QL-Cux*.**
- **The intersection *V* of the common normal in *QL-P1* and a *QL-L1*-perpendicular line in the intersection with *QL-L6* is a point of the cubic *QL-Cux*.**
- ***V* = *QL-Tf3(QL-L1)*.**
- ***UV* is tangent to *QL-Cux* in *V*.**
- ***QL-Tf3(UV)* = *QL-L1* ∩ *QL-L6*.**



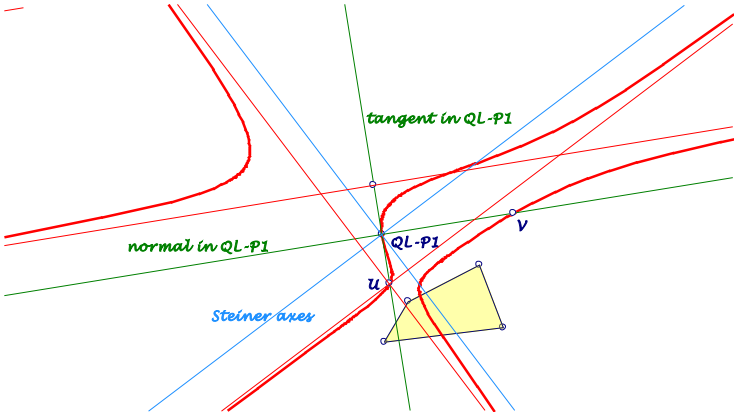

- **If *QL-Cu1* is unipartite, *V* is the center of the circumcircle *Ci* of *QL-P1* and *QL-2P2*, which intersects *QL-Cux* in rectangle-points, whose tangents bear *QL-P1*.**
- **Parallels to the Steiner axes through *V* intersect the Steiner axes on the cubic *QL-Cux*.**



- **Six further intersections of *QL-Cu1* and *QL-Cux* lie in pairs collinear with *QL-P1*.**
- **The lines of these collinearities are the reflections of the asymptotes of *QL-Cu2* in the first Steiner axis (intersecting with 60°).**
- **The 4th harmonic points of *QL-P1* wrt the pairs of intersection of *QL-Cu1* and *QL-Cux* lie on a circle round *V* through *QL-P1*.**




- **The asymptotes of *QL-Cux* are**
  **… parallels to the Steiner axes through *U***
  **… and a perpendicular line to *U.QL-P1* through the reflection of *U* in *QL-P1*.**
- **The triangle of the asymptotes is rectangular:**
  **… altitude *QL-P1.QL-P4*,**
  **… Euler line parallel *QL-L2*,**
  **… Apollonius circle through *U* centered on *QL-L1*.**




- **On lines *L* through *QL-P1* the intersections with *QL-Cu1* and *QL-Cux* define a line involution with center in the pedal point of *V* on *L* (one double point is *QL-P1*).**

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