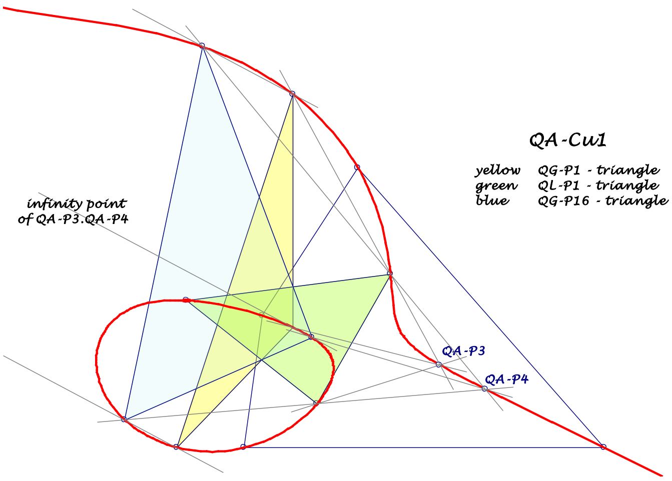
EQF-Note 2016-10-25

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

Cubics for Perspective QA-Triple Triangles

Starting with two perspective QA-triple triangles (see QA-Tr-2) there is a third QA-triple triangle, perspective with both and a cubic, bearing the 3 x 3 vertices of the perspective triangles and their perspectors. The cubic QA-Cu1 is of this type.



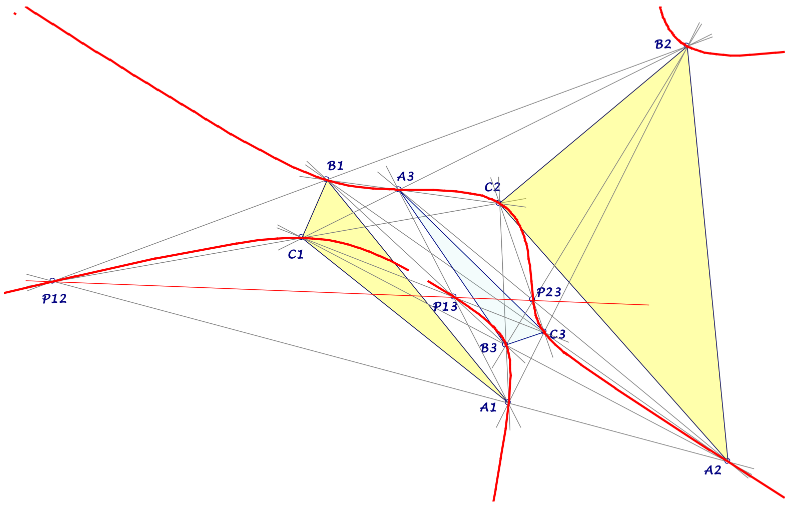
Wellknown Example QA-Cu1

The diagonal triangle *QA-Tr1* and the Miquel triangle *QA-Tr2* are perspective wrt *QA-P3*. The *QG-P16*-triangle is perspective to *QA-Tr1* wrt the infinity point of *QA-P3.QA-P4* and perspective to *QA-Tr2* wrt *QA-P4*.
The isoconjugation *QA-Tf2* wrt *QA-Tr1* swaps the vertices of *QA-Tr2* and the *QG-P16*-triangle. The isogonal conjugate wrt *QA-Tr2* swaps the vertices of *QA-Tr1* and the *QG-P16*-triangle. There is a third isoconjugation wrt the *QG-P16*-triangle, which swaps the vertices of *QA-Tr1* and *QA-Tr2*. Fixed points of this isoconjugation are the points of the cubic *QA-Cu1* with tangential *QA-P3*.
Summary: The three triangles are pairwise perspective and pairwise isoconjugated wrt the third triangle. Their vertices and the perspectors lie on a cubic. This can be generalized.

General Observations

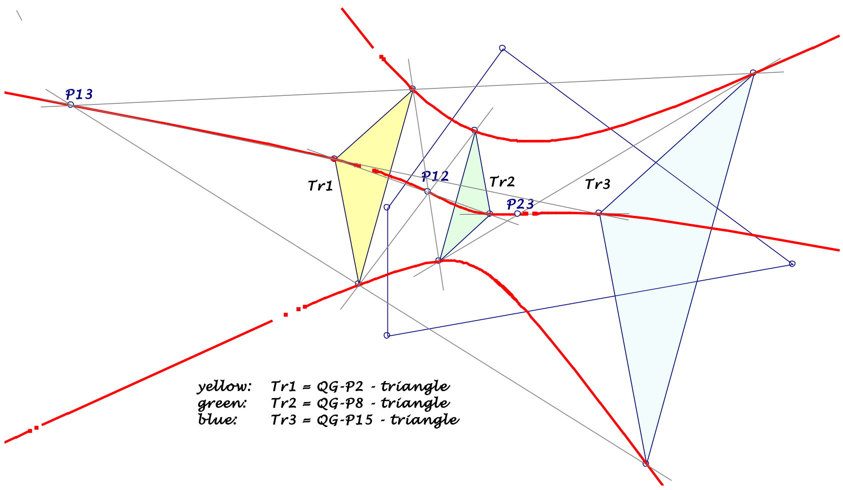
Two perspective triangles $A_1B_1C_1$, $A_2B_2C_2$ have a third triangle $A_3B_3C_3$
… with $A_3 = B_1C_2 \cap B_2C_1$, $B_3 = A_1C_2 \cap A_2C_1$, $C_3 = B_1A_2 \cap B_2A_1$,
… perspective to $A_1B_1C_1$ in $P_{13} = A_1A_3 \cap B_1B_3 \cap C_1C_3$,

… perspective to $A_2B_2C_2$ in $P_{23} = A_2A_3 \cap B_2B_3 \cap C_2C_3$,
… with collinear perspectors,
… with a pivotal isocubic:
… … one triangle as reference triangle,
… … isoconjugation swapping the vertices of the other two triangles,
… … or: isoconjugation swapping the perspectors with the other two triangles,
… … pivot in the perspector of the other two triangles,
… … bearing the vertices of the three triangles
… … and their perspectors.

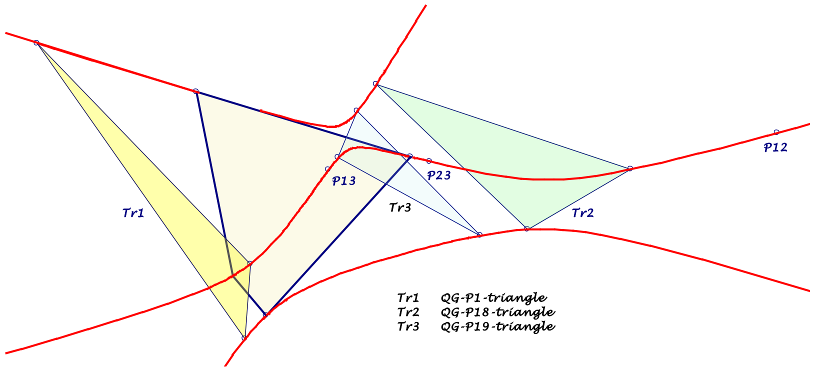


Generalization for perspective QA-triple triangles

In *QA-Tr-2* perspective *QA*-triple triangles are listened, which can be considered in the described manner.



Example 1:
The *QA*-triple triangles *Tr1* of *QG-P2* and *Tr2* of *QG-P8*
… have as perspector a point P_{12}, which divides *QA-P1.QA-P10* with ratio *3:2*.
... The third triangle *Tr3* is the *QA*-triple triangle of *QG-P15*,
… perspector P_{13} = *QA- P20*, perspector P_{23} = *QA-P1*.



Example 2:
This is a very special example, for the cubic contains also the vertices of the quadrangle.
The *QA*-triple triangles *Tr1* of *QG-P1, Tr2* of *QG-P18* and *Tr3* of *QG-P19* have the perspectors (found by Chris van Tienhoven)
… P_{12} = *QA-Tf2*(P_{13}),
… P_{13} = *QA-P12.QA-P24*∩*QA-P6.QA-Tf2(QA-P11)*,
… P_{23} = $P_{12}.P_{13}$∩*QA-P4.QA-Tf4(QA-P12)*.

The corresponding cubic
… contains a lot of points:
… … 4 vertices of the quadrangle,
… … 3x3 vertices of the three perspective triangles,
… … 3 perspectors (pivots),
… … 3x3 cevian points of the perspectors wrt the third triangle,
… is invariant wrt isoconjugations for the *QA*-triple triangles
… … of *QG-P1*: *QA-Tf2*,
… … of *QG-P18*: isogonal conjugation,
… … of *QG-P19*: swapping *QA-P2* and *QA-P41*.

Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de