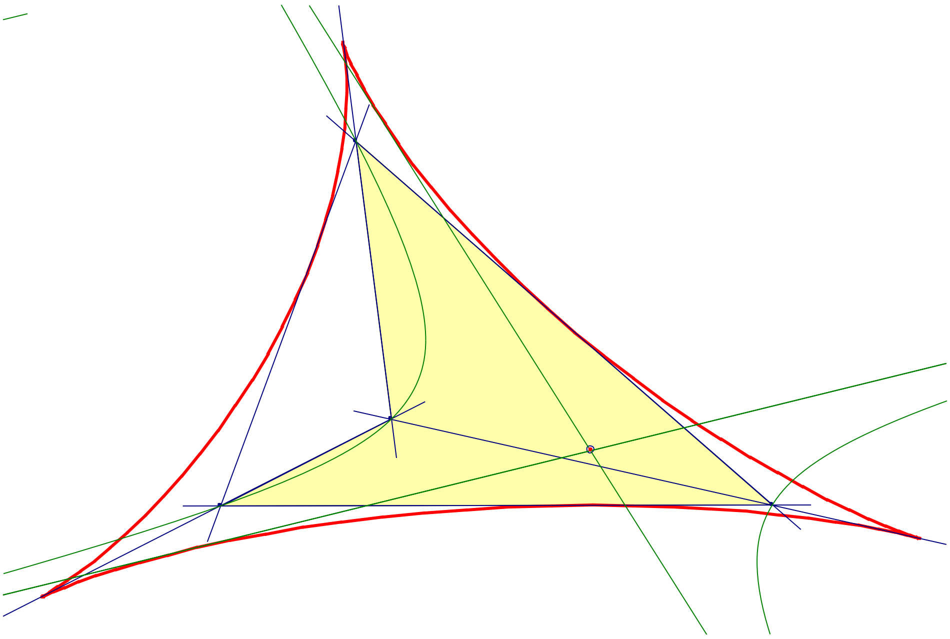
EQF-Note 2017-01-10

Background for these notes is:
Chris van Tienhoven: Encyclopedia of Quadri-Figures
http://www.chrisvantienhoven.nl/

QA-Inscribed Quartic *QA-Tf9(L*$_\infty$*)*

The asymptotes of QA-circumscribed hyperbolas envelop a quartic, which is the QA-Tf9-image of the line at infinity. This quartic is already described in QFG-message 141 but wrt other aspects.



Review (see *QFG*-message *141*)

Consider a quadrilateral $L_1L_2L_3L_4$ (with intersections S_{ij}) and a point P. Let L_{ij} be the reflection of the line PS_{ij} in the angle bisector of L_i, L_j. The intersections $L_{12}\cap L_{34}$, $L_{23}\cap L_{41}$, $L_{13}\cap L_{24}$ coincide for points P on the cubic *QL-Cu1* in the *QL-Tf1*-image of P.
There is an analogon for a quadrangle $P_1P_2P_3P_4$ (with lines L_{ij}) and a line L. Let S_{ij} be the reflections of $L\cap L_{ij}$ in the midpoint of P_iP_j. The lines $S_{12}S_{34}$, $S_{23}S_{41}$, $S_{13}S_{24}$ coincide for lines, which envelop the considered quartic.

Calculations

Here are used barycentric coordinates wrt the *QA*-diagonal triangle (in *EQF DT*-notation). For lines $L(e,f,g)$ with $S_{12}S_{34}$ = $S_{23}S_{41}$ = $S_{13}S_{24}$ holds

$$\frac{ep^2}{f-g}+\frac{fq^2}{g-e}+\frac{gr^2}{e-f}=0\,.$$

This is valid for the lines P_iP_j of the quadrangle and for example *QA-P3.QA-P4*. For every infinity point $P_{inf}(u:v:w)$ with $u+v+w=0$ there is a line through this point satisfying this condition:

$$L_{uvw} = (u(q^2w^2 - r^2v^2), v(r^2u^2 - p^2w^2), w(p^2v^2 - q^2u^2)).$$

These lines envelop the discussed quartic. Contact points are:

$$(q^2r^2u^4 - p^4v^2w^2 + p^2u(v-w)(r^2v^2 - q^2w^2):$$
$$r^2p^2v^4 - q^4w^2u^2 + q^2v(w-u)(p^2w^2 - r^2u^2):$$
$$p^2q^2w^4 - r^4u^2v^2 + r^2w(u-v)(q^2u^2 - p^2v^2))$$

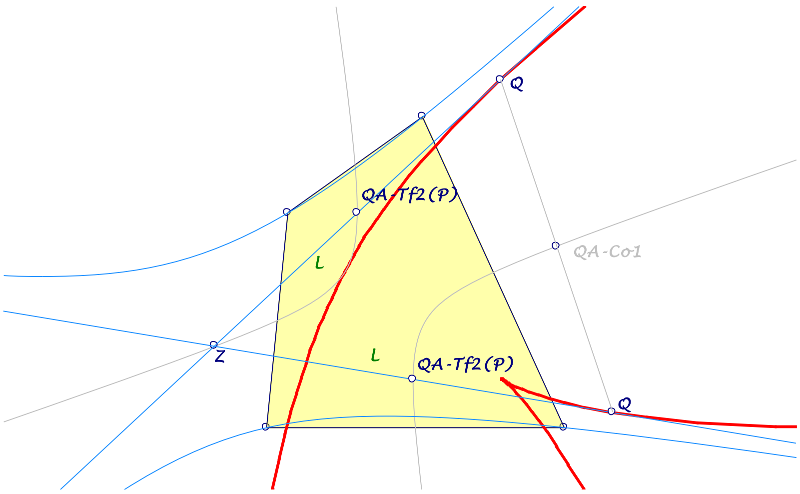
The other two points of intersection with the quartic have midpoint $QA\text{-}Tf2(P_{inf})$ = $(p^2vw : q^2wu : r^2uv)$ on *QA-Co1*.

The very extensive equation of the quartic can be found in *QFG-*message *141*.

Constructions

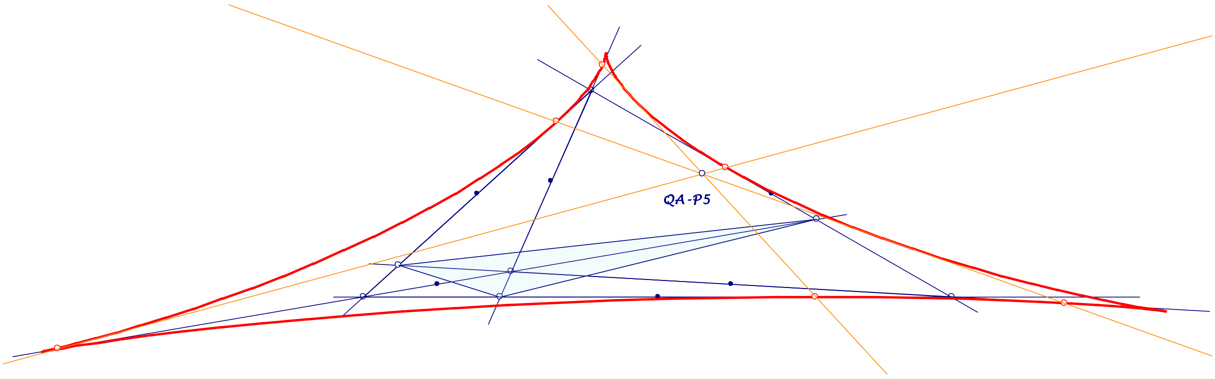
- **The asymptotes of *QA*-circumscribed hyperbolas envelop the quartic.**

Let *L* be an asymptote of a *QA*-circumscribed conic *Co* with center *Z* and infinity point *P*: The contact point *Q* of *L* to the quartic is the reflection of *Z* in *QA-Tf2(P)*.



The construction can be simplified:
Let *X, Y* be two center-symmetric points on *QA-Co1*
… and *Z* the 4^{th} intersection of *QA-Co1* and circle *(X,Y,QA-P3)*,
… *ZX, ZY* are tangents of the quartic
… with contact points in the reflection of *Z* in *X, Y*.

The contact points T_{ij} of the lines P_iP_j and the quartic are the reflections of the *QA-Tr1*-vertices in the midpoints of P_iP_j. The lines $T_{12}T_{34}$, $T_{23}T_{41}$, $T_{13}T_{24}$ are also tangents to the quartic and have a common point in *QA-P5*.



Eckart Schmidt
http://eckartschmidt.de
eckart_schmidt@t-online.de