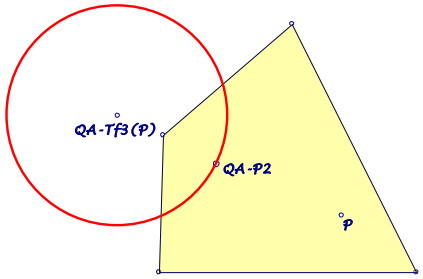
EQF-Note 2018-02-09

Background for these notes is:
Chris van Tienhoven:
Encyclopedia of Quadri-Figures and Poly Geometry
http://www.chrisvantienhoven.nl/

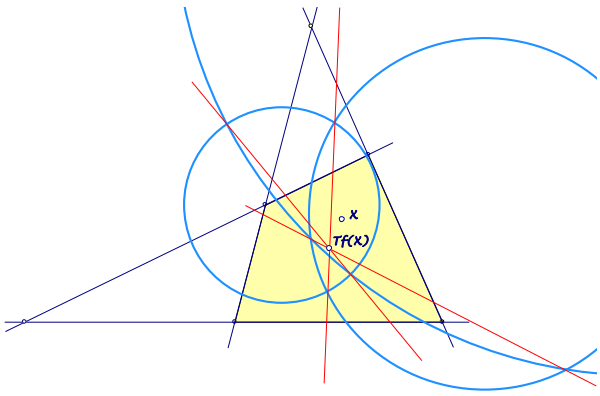
QA-Orthopole-Circles for a QL

The QA-orthopole-circle of a point P is not explicit in EQF, but mentioned under QA-Tf3, found by Antreas P. Hatzipolakis. These circles lead to a QL-transformation, considering the radical center of the three QA-orthopole-circles of a point P wrt the quadrigon-components of the quadrilateral.



The *QA*-orthopole-circle of a point *P* is a circle round *QA-Tf3(P)* through *QA-P2*. For a quadrilateral we define the following transformation $X \rightarrow Tf(X)$ with

- ***Tf(X)* is the radical center of the *QA*-orthopole-circles of *X* wrt the three quadrigon components of the quadrilateral.**

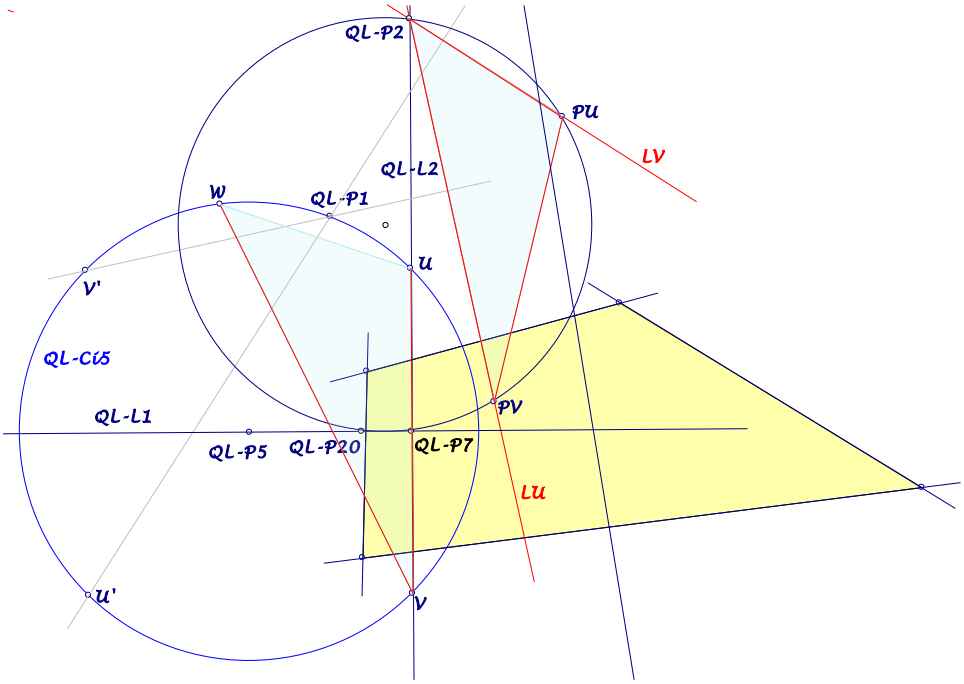


This transformation is not reciprocal, iterated using leads to the point *QL-P2*, which is a fixed point.

- **For points *X* on *QL-L2*: *Tf(X)* = *QL-P2*.**

- **For points *X* on the Plücker-circle *QL-Ci5* *Tf(X)* are points at infinity.**
  Diametral points on *QL-Ci5* give orthogonal directions.

For points on the Plücker-circle the radical axes are parallel:
… for *QL-P1* orthogonal *QL-L1*,
… for *QL-P1*, reflected in *QL-P5*, parallel *QL-L1*,
… and for the Plücker points *U, V (QL-2P1)* the radical axes coincide and give two lines L_U, L_V through *QL-P2*, orthogonal *V´.QL-P1*, *U´.QL-P1* with *U´*, *V´* diametral points of *U, V* on *QL-Ci5*.



Let us consider additional a circle *Ci* through *QL-P2*, *QL-P7*, *QL-P20*, centered in the midpoint of *QL-P1.QL-P20*, with 2nd intersections P_V, P_U on L_U, L_V, further the point

$$W = CSC(CSC(QL\text{-}Ci5) \cap QL\text{-}L2),$$

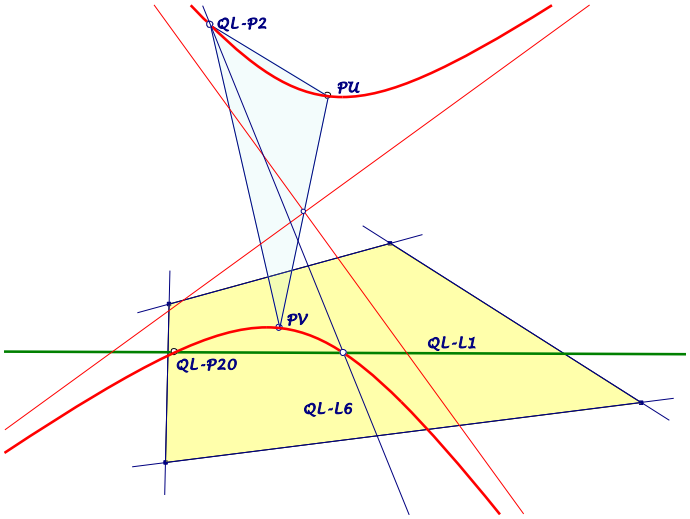
which is the 2nd intersection of *QL-Ci5* and *QL-Ci6*.

- **For points *X* on *WU*: *Tf(X)* = P_V,**
  … for points *X* on *WV*: *Tf(X)* = P_U.

- **The triangles *UVW* and P_UP_V*QL-P2* are spiral similar**
  … with ratio of the radii of *QL-Ci5* and *Ci*.

- **The transformation *Tf* maps lines (not *QL-L2*, L_U, L_V) to conics through the points P_U, P_V, *QL-P2*.**

- **The transformation *Tf* maps the Newton line *QL-L1* to an orthogonal hyperbola,**
  … centered in the midpoint of P_UP_V,
  … through P_U, P_V, *QL-P2*
  … and *QL-P20*, *QL-L6*∩*QL-L1*.



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